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**A Frequency Analysis of the rapidly oscillating Ap star HD 101065**

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**Thesis submitted for the Degree of Master of Science at the University of Cape Town**

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## Introduction

The study of pulsating stars is a mature and important field of stellar astrophysics. The recent discovery that main sequence stars such as the Sun and the cool Ap stars oscillate with a large number of normal modes has given rise to *asteroseismology*, a new approach which promises to yield accurate knowledge of the interior structure and dynamics of these stars. Although the techniques of asteroseismology have yet to be perfected, they will provide us with extremely powerful tools to test theories of stellar structure and evolution and to provide detailed knowledge of stellar mass, age, internal rotation, magnetism and convection. They may also provide information on the elemental abundances and mixing and indicate the presence of low-mass companions.

In asteroseismological studies, the primary data are the frequencies of the normal modes present in the object of interest. This thesis describes an attempt to perform a definitive frequency analysis of the rapidly oscillating Ap star HD 101065. The results of the intense observing program and the subsequent frequency analysis have been published and we reproduce them here in their entirety. The disadvantage in this approach is that the terseness expected by the editor of a scientific journal is sometimes a stumbling block for the reader not fully acquainted with the field. It is thus the purpose of Part I of this thesis to supplement the papers presented in Part II and the Appendix and to provide a more general background against which they can be read and understood.

The discovery by Don Kurtz in 1978 of the new class of variable star called the *Rapidly Oscillating Ap Stars* is one of those flukes with which the history of astronomy abounds. In order to understand these variables properly we devote Section 1 to a discussion of some of the properties and problems of the A-type stars. The discussion will focus on the chemically peculiar Am and Ap stars, the possible origins of these spectral peculiarities, the pulsations in the A stars, and the question of pulsation in the presence of metallicity. In the HR diagram, our region of interest is where the Cepheid instability strip crosses the main sequence among the late A and early F stars. We will see why it is surprising that rapidly oscillating Ap stars were only discovered in 1978 and why, when they were discovered, it was almost for the wrong reason. In Section 2 we will review the status of HD 101065 as an Ap star. In Section 3 we give a brief introduction to the rapidly oscillating Ap stars before discussing in Section 4 the previous work on HD 101065 as a rapidly oscillating Ap star.

Part II of this thesis contains the paper entitled *New observations and a frequency analysis of the extremely peculiar rapidly oscillating Ap star HD 101065*, which has been accepted for publication in the *Monthly Notices of the Royal Astronomical Society*. In it we

present 138 hr of new high-speed photometric observations of HD 101065 obtained in 1988. These observations reveal that HD 101065 pulsates with at least three frequencies near 1.37 mHz which cannot all be identified with consecutive overtones. These frequencies completely describe the oscillations down to the 0.40 millimagnitude level. There is strong evidence of more frequencies below the 0.35 mmag level, but the complexity of the frequency spectrum is such that we are unable to determine any of those frequencies securely. Although an application of the oblique pulsator model of the rapidly oscillating Ap stars is not yet possible, a tentative application of the techniques of asteroseismology indicates that HD 101065 is only slightly evolved off the zero age main sequence and suggests that its radius, mass and luminosity are similar to those of the other Ap stars. We find a significant secular variation in the principal frequency using two independent techniques and we argue that it cannot be interpreted as arising from evolutionary effects in HD 101065. We then examine the possibility that an unresolved frequency is involved and we show that a binary star model in which a low-mass companion orbits about HD 101065 is consistent with the observations. Finally, we propose that a multi-site observing campaign may offer the best hope of further deciphering the frequency spectrum of HD 101065.

The Appendix contains a paper entitled *HD 116763 - a false alarm?* which was published in the *Monthly Notices of the Royal Astronomical Society*. This paper illustrates some of the difficulties encountered when searching for new rapidly oscillating Ap stars.

**Part I**

**Chemically Peculiar A stars, Stellar Oscillations and HD 101065**

## 1. The Chemically Peculiar Stars of the Upper Main Sequence

For over half a century it has been known that the upper main sequence between B2 and F2 is partly populated by stars with peculiar spectra. For any sufficiently large sample of stars with appropriate  $T_{\text{eff}}$  and luminosity, the peculiar stars will represent at least 30% of the sample. These peculiar stars are classified into several well-defined groups such as the Bp, Ap, Am, Fp and Fm stars (Fig. 1.1). The most promising attempts to construct a unified description of all of these groups of peculiar stars have focussed on the particularly high stability which seems to prevail in their atmospheres. The reasons for the importance of this stability will emerge later. Because of their particular relevance to our understanding of HD 101065, this discussion will focus on the chemically peculiar A stars with references to the other chemically peculiar stars where appropriate. There are two distinct groups of chemically peculiar A stars, the Ap stars and the Am stars. These stars can be distinguished at classification dispersions since the characteristically strong (or weak) absorption lines differ for the two groups. There are additional differences. The Ap stars have strong ( $\approx$  kG) magnetic fields; they are spectrum, magnetic, photometric and radial velocity variables. They also have an unusually low binary frequency. In contrast, the Am stars are non-magnetic, usually constant in luminosity and spectral appearance, but with an unusually high binary frequency. We will discuss these features in more detail below.<sup>1</sup>

### 1.1a The Ap stars

Of the peculiar stars, the Ap stars exhibit the highest degree of spectral diversity. These stars all have anomalously strong lines in their spectra of several of the following elements: Si, Mn, Sr, Cr, Eu and/or the other rare earths. The peculiarities correlate with  $T_{\text{eff}}$  in that the coolest of these stars have very enhanced Sr-Cr-Eu lines while the hotter ones appear to be Si over-abundant (Fig 1.2). Five groups of Ap stars are recognized ( $\lambda$ 4700 Si, Si, SiCrEu, SrCrEu and Sr from hottest to coolest) but membership in any of the groups is insufficient to specify the full range of spectral peculiarities in any given star.<sup>2</sup> Indeed, the salient feature of Ap star spectra is their diversity.

The Ap stars form part of a sequence of magnetic stars extending along the main sequence from F0 up to around B2. The earlier spectral types exhibit a wide range of magnetic field strengths ( $\approx$  1-6 kG) while the later spectral types are confined to a narrower range of lower field strengths

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1 Lest the reader gain the impression that the occurrence of chemically peculiar stars is confined to the upper main sequence, we hasten to add that many chemically peculiar stars appear elsewhere in the HR diagram. The review by Vauclair & Vauclair (1982) provides an introduction to these other groups of chemically peculiar stars and their literature.

2 The Bp stars are likewise divided into several groups (Fig. 1.1), but we defer the description of these groups to Section 1.1b.

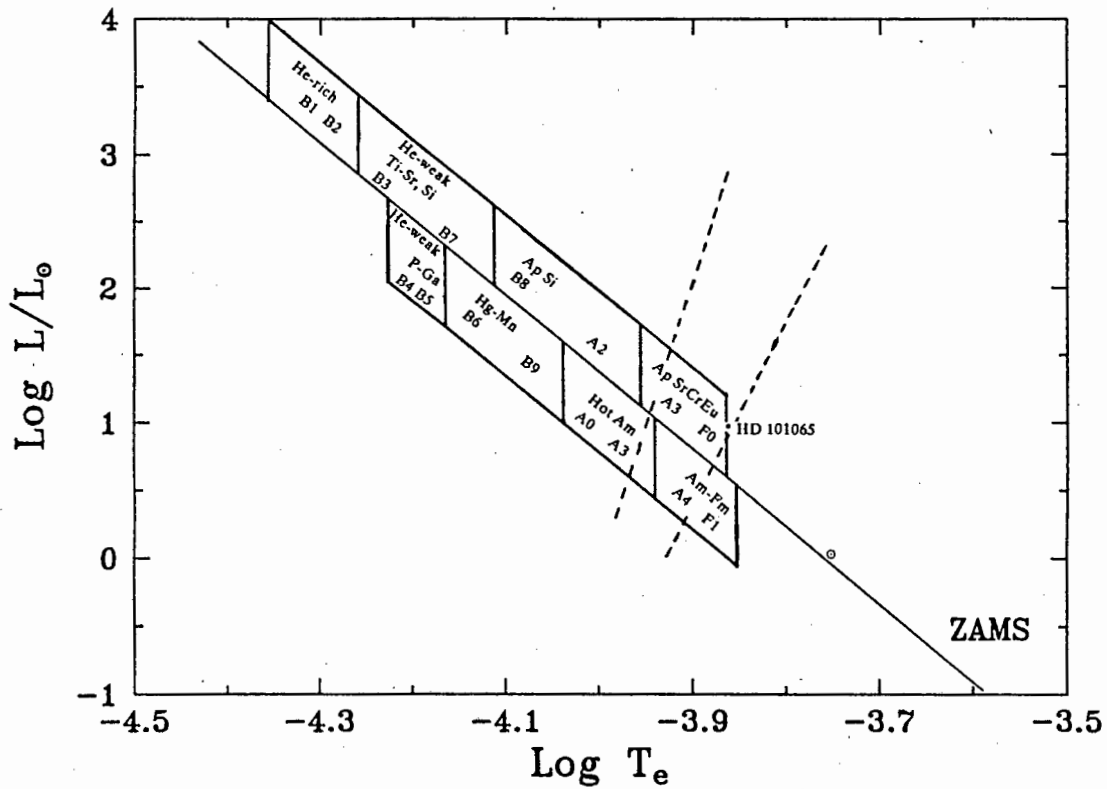


Fig. 1.1 A schematic HR diagram of the chemically peculiar stars of the upper main sequence. I have placed the magnetic stars above the ZAMS and the non-magnetic stars below it simply for clarity; the luminosity borders are purely schematic. The stippled lines indicate the red and blue borders of the  $\delta$  Scuti instability strip. The probable position of HD 101065 is also indicated.

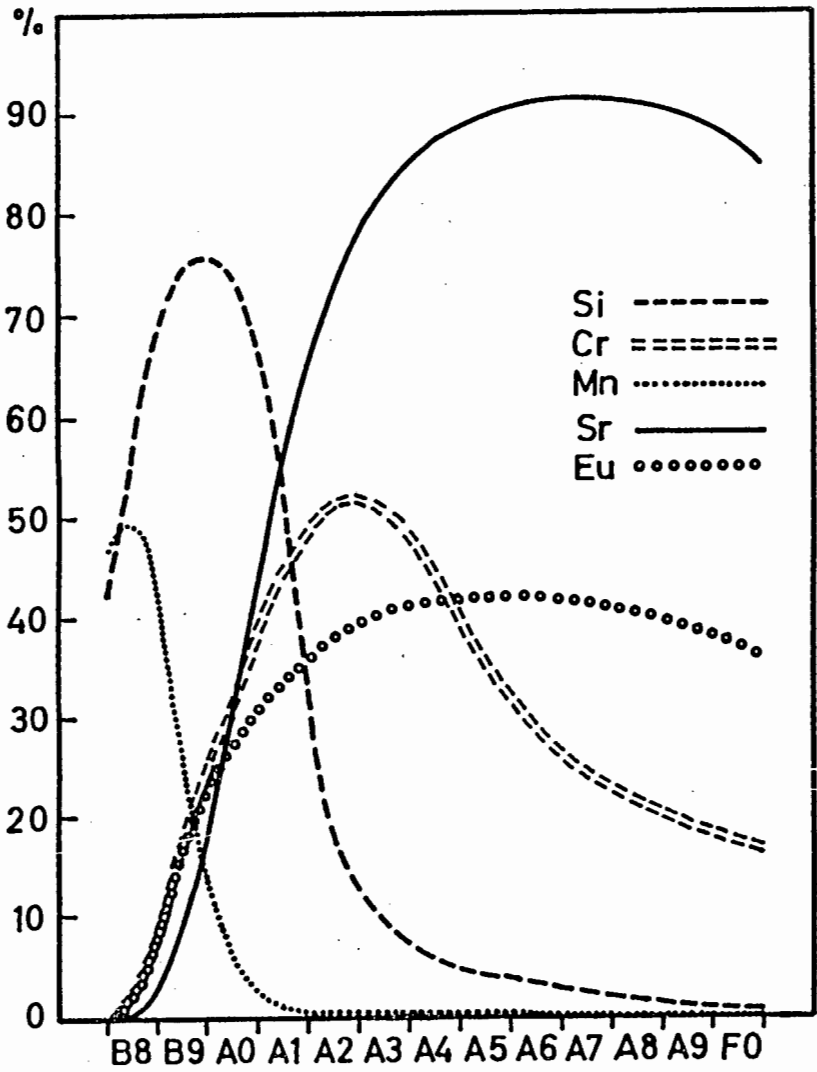
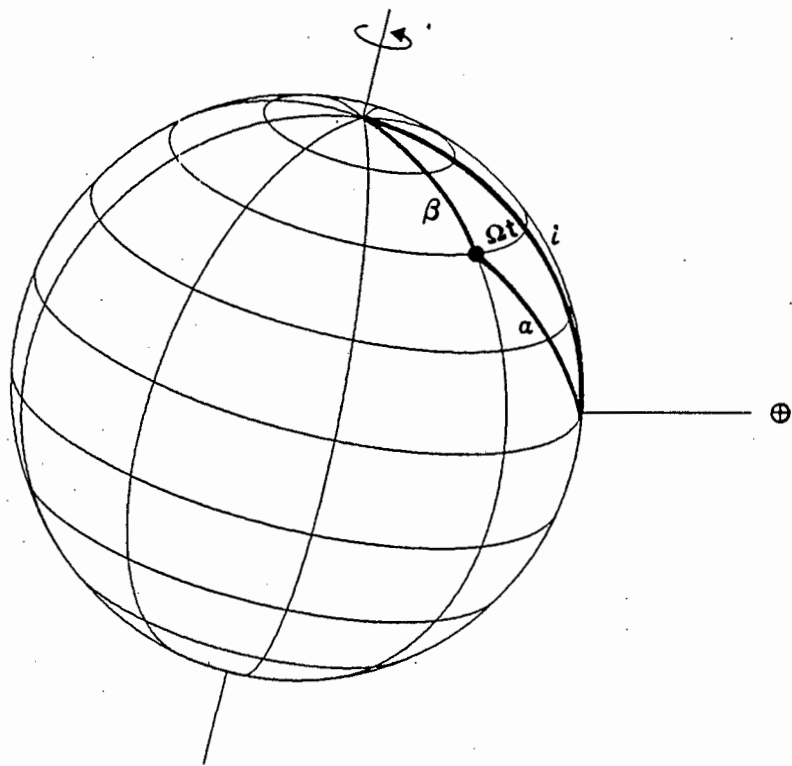


Fig. 1.2 Relative percentage per spectral class of magnetic stars showing especially strong overabundances of Si, Sr, Cr, Eu & Mn (from Ledoux & Renson 1966).



**Fig. 1.3** The geometry of the Oblique Rotator Model of the magnetic Ap stars. This is also the geometry of the oblique pulsator model of the rapidly oscillating Ap stars which we will discuss in later Sections.

( $\approx 1\text{-}2$  kG). There are notable exceptions to these ranges of field strengths and spectral types, however. For example, the B5 star HD 215441 has a mean surface magnetic field of about 34 kG (Borra & Landstreet 1978). Observational evidence suggests that the magnetic geometries are predominantly dipolar (Preston 1971). This is easily understood in terms of the physical requirement that the net magnetic flux through the surface must vanish; fields as strong as 1 kG which are measured in the integrated light of the stellar disk are unlikely to arise from significantly higher-order magnetic geometries. The magnetic fields are subject to variations on a time-scale of days. The amplitudes of the magnetic variations range from a few hundred to a few thousand gauss and in many cases polarity reversals are observed.

The Ap stars are also photometric, spectrum and radial velocity variables. One of the key observational results is that these attributes all vary with the *same* period as the magnetic variations. Typical periods are several days, although some instances of much longer periods (e.g. 21 yr in the case of HR 465 (Rice, 1988) ) are also known. The variable lines include the anomalously strong lines mentioned above and although not all elements vary in phase, lines of a given ion always do. For all elements, line strength maximum occurs as the radial velocity passes through its mean value.

The Ap stars have an unusually low binary frequency. Abt & Snowden (1973) found that only 20% of the Si & SrCrEu Ap stars in their survey are in spectroscopic binary systems, whereas for normal A stars, this frequency is closer to 40%. They did, however, find a normal frequency of Ap stars in visual binaries. In a few cases, (Abt *et al.* 1968, van Dessel 1972) it is possible to derive the mass of the Ap star from the orbital parameters. The masses derived in this way are essentially normal for the star's spectral type.

The standard model which accounts for the variability observed in the Ap stars is the oblique rotator model (Stibbs 1950) in which a dipole magnetic field is inclined at an angle  $\beta$  to the axis of rotation (Fig 1.3). This magnetic field is assumed to be frozen into the star which means that these stars have negligible differential rotation and thus rotate like rigid bodies. The appellation 'rigid rotator' has firm observational support. Some Ap stars have been observed spectroscopically for over half a century and no period or phase changes have been detected in their spectral variations (Wolff 1983). The usual interpretation of the spectral observations is to conjecture an inhomogeneous distribution of elements over the stellar surface so that the period of magnetic, spectral and luminosity variations is simply the period of rotation.

Obviously, the most challenging problem posed by the Ap stars is to explain the origin of the line-strength anomalies. The first step is to determine whether these anomalous line intensities arise

because of some combination of peculiar atmospheric structures and departures from LTE or whether they arise because of peculiar abundances. When considering the former possibility, one starts with an atmosphere having a solar composition and modifies the atmospheric structure in an attempt to produce the observed H-line profiles, line-strengths and colours. Thus far, no one has produced self-consistent spectra which match the observations and there is almost universal agreement that the spectral peculiarities are produced by real abundance anomalies (Tomley *et al.* 1970). However, there are several compelling reasons to believe that these anomalies are confined to the surface. Firstly, the abundance anomalies do not persist among the giants, some of which must be evolved Ap stars. Secondly, as Wolff (1983) pointed out, if the interiors of these stars have an anomalous composition then virtually the entire galactic abundance of rare earths is confined to the Ap stars. Thirdly, the presence of Ap stars in clusters and binary systems with other normal A stars argues against an anomalous interior composition. Strittmatter & Norris (1971) present evidence in favour of the hypothesis that the abundance anomalies in Ap stars are confined to the surface layers. They also examine the effects of meridional circulation, convection, accretion & mass loss on the surface abundance anomalies.

If the anomalously abundant elements are distributed inhomogeneously, it becomes important to determine what effect the magnetic geometry has on the surface distribution of these elements. Several attempts to map the surface distribution of elements have been made, but the solutions produced are not unique (Megessier *et al.* 1979); the effects of the spot sizes, abundances and latitudes cannot be separated. Such maps can only reveal the number of spots and their longitudes, as well as the asymmetric distribution of elements with respect to the magnetic equator. A common feature of these investigations is that they attempted to study the abundance distribution with no reference to the magnetic field geometry. Landstreet (1988) has suggested that the abundance distribution cannot be determined reliably in total ignorance of the magnetic field geometry. By studying both the magnetic and spectral variations in 53 Cam, he showed that it is possible to derive essentially unique low-resolution abundance distributions for this star.

Nuclear processing, magnetic accretion (from the interstellar medium) and separation of elements have all been proposed as the dominant mechanism responsible for the anomalous surface abundances. The magnetic accretion models (Havnes & Conti 1971) have the potential to explain why the Ap stars rotate slowly as a group and why the surface distribution of elements is inhomogeneous, but they cannot explain the co-existence of normal and peculiar A stars in clusters and binary systems. The existence of Ap stars in even the youngest associations is in conflict with the time-scales required

by these models to produce the peculiarities observed. The nucleosynthetic models all need an appropriate site at which the nuclear processing occurs. This site can be the stellar interior (Fowler *et al.* 1965), the surface of the star (in which case strong magnetic fields accelerate particles to energies high enough for nuclear reactions to occur), or a former companion which evolved into a supernova (Guthrie 1971). Fowler *et al.* (1965) proposed that Ap stars are highly evolved objects which have returned to the main sequence and that the abundance anomalies are dredged up from the interior. However, this model cannot explain the presence of Ap stars in even the youngest associations. It also predicts that He should be over-abundant, whereas it is actually under-abundant in many Ap stars (Norris 1971). Multiple-star models which invoke the supernova explosion of a companion face the difficulty of explaining why there are binary systems with one normal and one chemically peculiar star or why there is a dependence of peculiarity on  $T_{\text{eff}}$ . Also, if chemically peculiar stars were really formed in this way, the required rate of supernova explosions in the galaxy would be significantly higher than is observed. Thus far, no one has been able to employ any of the nucleosynthetic models to account in detail for the abundance peculiarities in any of the Ap stars.

A very promising non-nucleosynthetic model of the origin of the abundance anomalies invokes the separation of elements, or diffusion, in the atmosphere. The astrophysical significance of diffusion processes has been recognized for a long time. In his seminal work, *The Internal Constitution of the Stars*, which was published in 1926, Sir Arthur Eddington discussed early ideas of the astrophysical significance of diffusion with references to the earlier literature. However, Michaud (1970) was the first to invoke diffusion as a possible explanation for the abundance anomalies in the Ap stars.

The basic tenets of diffusion are easily grasped. In a slowly rotating star with a stable atmosphere devoid of turbulence and convection, the dominant force on a given ion is determined by the imbalance between the gravitational and radiative forces. Elements with many lines near flux maximum will experience a net upwards force. These elements will rise through the atmosphere and concentrate in the line-forming regions, thus appearing to be overabundant. Low-abundance elements with many lines near flux maximum will only depress the flux slightly and will tend to be preferentially supported. Michaud (1970) shows how this hypothesis predicts overabundances of Mn, Y, Zr, Sr and other rare earths in the temperature ranges in which they are observed. Elements with few lines near flux maximum or with saturated lines will tend to sink in a sea of hydrogen and therefore appear under-abundant and, indeed, deficiencies of He, N, O and Ne are observed (Sargent *et al.* 1969).

Although the diffusion velocities are very low ( $\approx 1 \text{ cm s}^{-1}$ ), element separation is highly effective in the absence of disruptive processes such as turbulence and convection. Unhindered diffusion processes can lead to marked surface composition anomalies on a time-scale of  $\approx 10^4$  years (Michaud 1970), much shorter than the nuclear evolution time-scale. This is consistent with the existence of Ap stars in even very young associations (Abt 1979). The key note here is that according to the diffusion hypothesis, chemical peculiarity is the hallmark of a stable atmosphere. The atmospheric stability criterion can be used to predict the red and blue edges of the Ap phenomenon. Sufficiently stable atmospheres are expected for stars of spectral types B2-F2. Stars earlier than B2 suffer from extensive mass loss while stars later than F2 are expected to have extensive surface convection zones.

Alecian (1986) has devised a diagram (Fig. 1.4) which neatly summarizes the development of Ap stars within the context of the diffusion theory. Prior to the onset of peculiarity, these stars must somehow shed angular momentum via some sort of braking mechanism as they contract onto the main sequence. The type of braking experienced by the star determines which of the subgroups of Ap stars (e.g. Ap Si, Ap SrCrEu, or HgMn) it will ultimately fall into. Magnetic stars will experience magnetic braking and this will allow the gravitational settling of He under the He II convection zone to proceed, until eventually this convection zone disappears. With the atmosphere stabilized, diffusion processes are free to produce the abundance anomalies at the surface. The exact run of abundance anomalies will depend on the hydrodynamics, residual turbulence,  $T_{\text{eff}}$  and the magnetic field structure<sup>3</sup> and possibly other unknown factors.

In the non-magnetic stars, braking can be achieved via tidal interactions in binary systems. Once braking has been achieved, the development of spectral peculiarities follows essentially the same course as in the magnetic stars except that the emergence of spectral peculiarities should take longer to occur. This is because the magnetic field has a stabilizing effect and one thus expects peculiarities to develop more rapidly in the magnetic stars than in non-magnetic ones. This probably explains Abt's (1979) observation of a higher incidence of chemically peculiar magnetic stars as opposed to chemically peculiar non-magnetic stars in young clusters.

The stability requirement also agrees well with the observation that, as a group, the Ap stars are slow rotators. However, stability, not rotation, is the primary parameter. Because a rotating star is spheroidal in shape, the temperature at the poles exceeds the temperature at the equator. This

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<sup>3</sup> We will discuss the effect of the magnetic field on diffusion processes shortly.

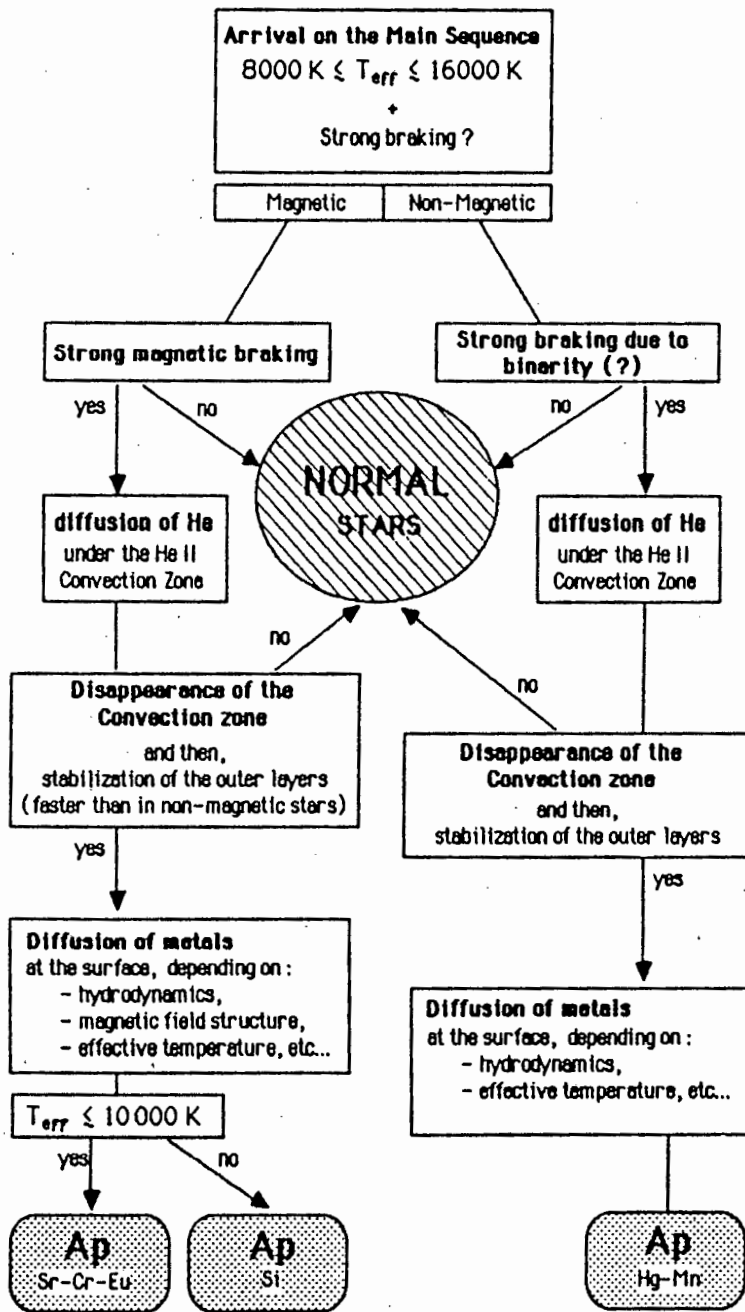


Fig. 1.4 Possible scenarios invoking diffusion for the development of the Ap phenomenon in stars (from Alccian, 1986).

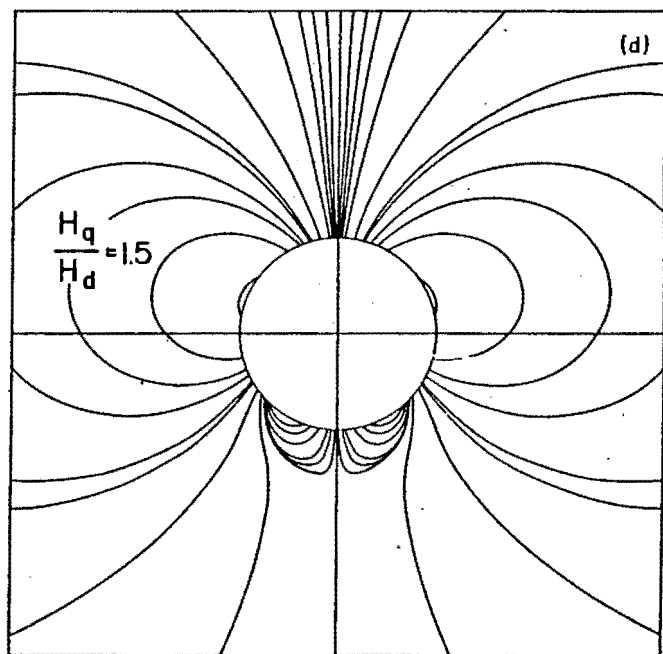
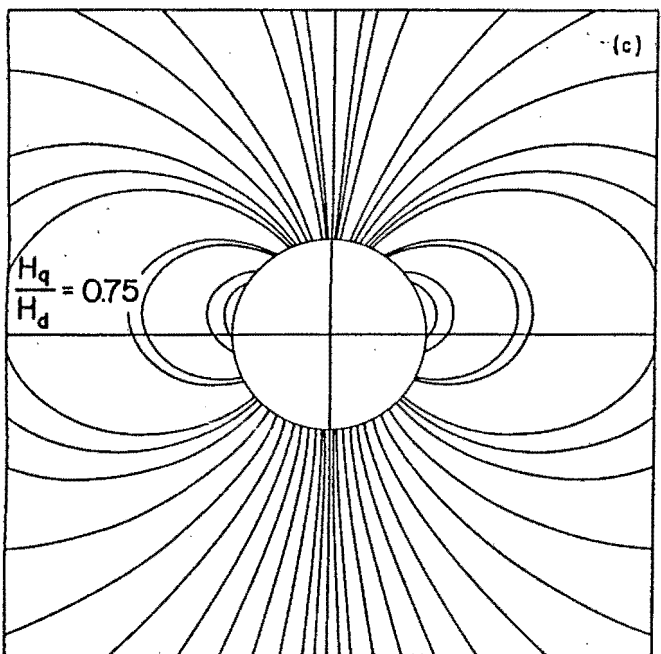
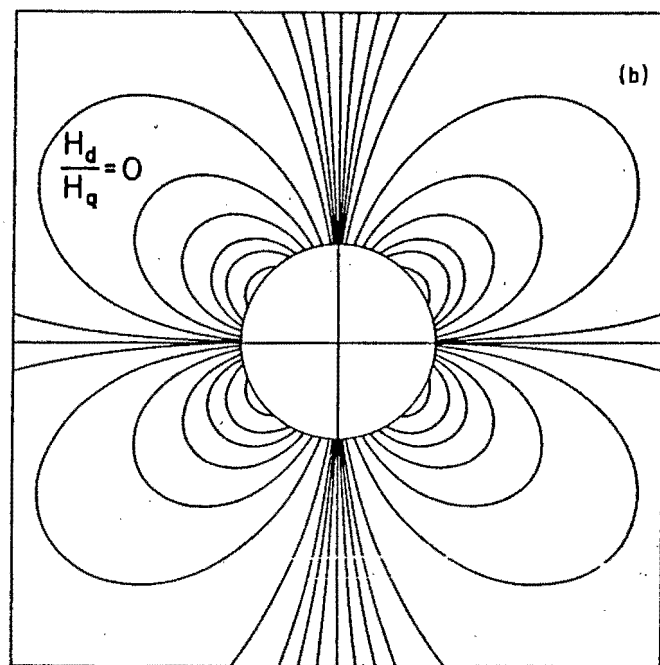
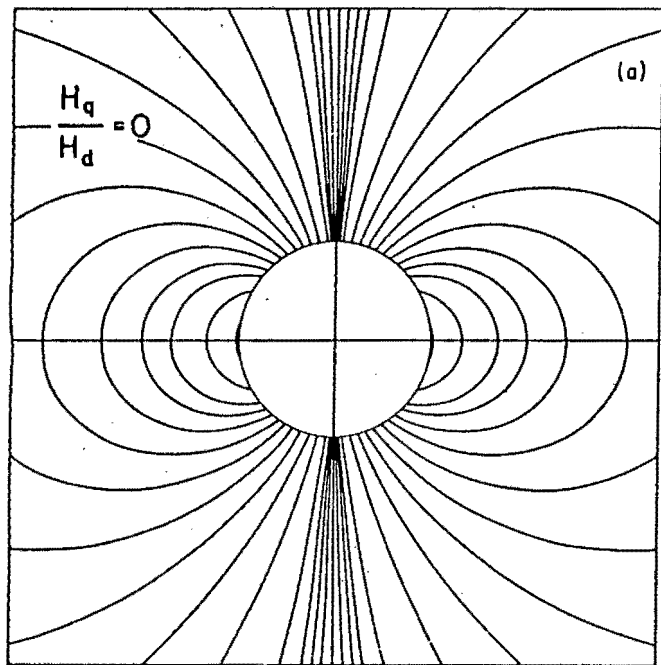
induces *meridional circulation*, a flux of material (and energy) from the poles to the equator and back to the poles via internal flows. Early simplified treatments of meridional circulation (which did not take proper account of the boundary conditions) indicated that the circulation velocity in the outer envelope depends inversely on the density  $\rho$  and may reach anywhere from tens to hundreds of metres per second. Obviously such velocities would destroy the effects of chemical separation and for many years this was one of the arguments levelled against diffusion theories.

The hydrodynamical problem of meridional circulation in the radiative zone of a non-magnetic star has been more rigorously formulated by Tassoul & Tassoul (1982) who consider especially the outer boundary conditions. They show that the  $\rho^{-1}$  dependence of the circulation velocity does not arise in the outer envelope and instead they find a uniformly slow motion in the outer layers with turbulence decreasing towards the surface. Assuming the meridional circulation to be laminar,<sup>4</sup> Michaud (1982) has used their treatment to compute the vertical circulation velocity which he compares with the He diffusion velocity below the He convection zone. In this way he is able to show that the He II convection zone will disappear only in stars with equatorial velocities below  $90 \text{ km s}^{-1}$ , although this result is sensitive to the atmospheric gravity. As soon as the equatorial velocity is below  $90 \text{ km s}^{-1}$ , the He settles out of the He II convection zone, the convection zone vanishes and radiative diffusion proceeds uninhibited. This cut-off velocity is in excellent agreement with the cut-off velocity actually observed for the HgMn stars (Wolff & Preston 1978). The effects of slight turbulence on chemical separation are not known. The proper treatment of diffusion in the presence of mild turbulence poses an interesting theoretical challenge but it also promises the commensurate reward of being able to specify how such turbulence affects the abundance anomalies.

A treatment of diffusion processes in Ap stars cannot neglect the strong magnetic fields mentioned earlier. Such a treatment is given by Michaud *et al.* (1981). The effective magnetic fields can be described adequately with simple combinations of dipoles and quadrupoles (Fig 1.5). Magnetic fields influence diffusion by influencing the motion of ionized elements and via Zeeman splitting which desaturates lines and alters the radiation pressure on these elements. From the diffusion point of view, the regions of special interest are those in which the magnetic field is either horizontal or vertical. In the absence of a magnetic field, or for neutral states of ionization, diffusion

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4 Michaud's viewpoint is that until turbulence is properly understood, it is reasonable to assume *a priori* that turbulence is small enough to permit diffusion processes to occur. The success of numerous diffusion calculations supports this approach.



**Fig. 1.5** Magnetic field lines for a dipole (a), a quadrupole (b) and aligned dipoles plus quadrupoles (c and d). For diffusion calculations, the regions on the star where the magnetic field is horizontal or vertical are the most important. This varies considerably as the ratio of the quadrupole to dipole field strength varies from 0.75 to 1.50, as allowed by the magnetic field observations (from: Michaud, G. *et al.* 1981).

is predominantly vertical and is governed by the imbalance between the gravitational force and the radiation pressure.

For non-neutral states of ionization, the magnetic field acts differently on the components of the diffusion velocity parallel and perpendicular to the field lines. The parallel component of the diffusion velocity is not modified by the magnetic field, but the perpendicular component is modified in a way which depends on the strength of the field, the collision frequency, temperature and the charge-to-mass ratio of the given ion. Michaud *et al.* (1981) show that the magnetic field can guide elements effectively only once they are pushed above the region where the continuum forms. Where the continuum forms, the high number density (and hence collision frequency) means that the perpendicular velocity is comparable to the parallel velocity and so the magnetic field does not significantly influence diffusion. Consequently all suitable elements are pushed upwards into the region where they can be seen. As the ion diffuses into the upper regions of the atmosphere, the lower density reduces the importance of the perpendicular diffusion velocity and the ion is able to follow the field lines more and more closely. Diffusion ceases when the ion reaches the level in which the magnetic field is horizontal. Such elements are bound to the star and accumulate in regions where the magnetic field is horizontal. Because of the cylindrical symmetry of the magnetic field, the elements tend to concentrate in spots or rings centred on the magnetic axis. The latitudes of these features will depend on the ratio of the quadrupole to dipole field-strengths (Fig. 1.5).

Changing the strength of the magnetic field changes the depth at which the elements accumulate, but Michaud *et al.* show that realistically strong fields ( $\leq 4 \times 10^5$  G) cannot prevent ionized elements from rising to the surface. In the absence of a magnetic field, ionized elements will leave the star on a time-scale of  $10^4$  yr, whereas in the presence of a strong magnetic field, this time-scale is greater than the stellar life-time. In summary, the magnetic field cannot prevent ionized elements from appearing on the stellar surface, but it does prevent the diffusion-induced loss of such elements.

Michaud *et al.* have also examined the effect of turbulence on diffusion processes in a magnetic field. They argue that large-scale mass motions associated with turbulence, meridional circulation or convection will be suppressed by a vertical magnetic field while a horizontal field will permit mass motions which roll around the field lines. They thus expect the abundance anomalies to concentrate in regions with a vertical magnetic field, essentially forming spots or rings near the magnetic poles. It is also expected that those elements on which the radiation force is weakest will only be supported very near the magnetic poles where the turbulence is most effectively suppressed.

Other elements on which the radiation pressure is higher, will be supported further away from the magnetic poles.

The most significant feature of Michaud *et al.*'s models is that they lead to specific relations between line profiles, equivalent widths, effective and surface magnetic field strength variations. A thorough test of their models requires a complete set of observations of all these attributes. Unfortunately, such a set of observations is not yet available for any star.

### 1.1b The Bp stars.

Before proceeding to describe the other groups of chemically peculiar A stars, we present, for completeness, a brief taxonomical overview of the Bp stars. In the visible region, the line-strength anomalies are more conspicuous for the Ap stars than for the Bp stars and consequently the Ap stars have been scrutinized more carefully than the Bp stars. However, the difference in line-strengths may only be a consequence of the higher temperatures of the Bp stars. In the visible region, a Bp star will have less conspicuous lines of the ionized rare earths than an Ap star of identical composition because of the higher stages of ionization in the B star's atmosphere. Also, from the stellar atmospheric point of view, the division at A0 is an artificial one and it is now accepted that the Ap phenomenon extends to spectral types as early as B2.

The diversity of Bp star spectra has led to the definition of several groups of Bp stars (Fig 1.1). The hottest group comprises the *He-rich* (also called *He-strong*) stars which have an abnormally high atmospheric abundance of He and are mostly confined to spectral type B2. They also have strong magnetic fields and they exhibit luminosity, spectrum and magnetic variations in common with many Ap stars.

The He-rich stars are followed by a cooler group, the *He-weak* stars, which have spectral types in the range B3 to B7. This group is further subdivided into the *P-Ga*, *Ti-Sr* and *Si* He-weak subgroups. The names of these subgroups allude to the anomalously strong spectral lines which characterize them. Like the He-rich stars and the Ap stars, the latter two subgroups have magnetic fields and exhibit magnetic, photometric and spectral variability (Borra *et al.* 1983). Very little is known about the binary frequency among the He-rich and He-weak stars.

Wolff & Wolff (1976) argue that there are two distinct sequences of peculiar stars in the range  $10000 \text{ K} < T_{\text{eff}} < 16000 \text{ K}$  (Fig. 1.1) and that the differences between these two groups mirrors the differences between the Ap and Am stars. Thus the coolest of the Bp stars, the HgMn stars which have spectral types in the range B6 to B9, appear to be non-magnetic counterparts of the Ap Si stars (Borra & Landstreet 1980, Borra *et al.* 1983). They are slow rotators, non-magnetic, non-variable

and appear to have a much higher frequency of spectroscopic binaries than the Ap Si stars. One might say that they are hotter analogues of the Am stars. The P-Ga subgroup of the He-weak stars seems to constitute a hotter analogue of the HgMn stars. Like the HgMn stars, the P-Ga stars are non-variable and non-magnetic (Borra *et al.* 1983). A more detailed description of the Bp stars with references to the literature has been given by Wolff (1983) in her book on the A stars.

Having introduced the Ap-Bp phenomenon, we now proceed to a brief discussion of two other important groups of A stars, the Am and the  $\delta$  Scuti stars.

### 1.2 The Am stars.

The *classical Am stars* are those main sequence stars of spectral type A4 - F1 in which the metallic-line and Ca II K-line spectral types differ by 5 or more spectral subclasses. The hydrogen line types range from A4-F1 and are intermediate between the metal and K-line types: The spectra of the Am stars are characterized by anomalously strong metallic lines and marked deficiencies of Ca, C, O, Mg and Sc (Boyarchuk & Savanov 1986). In addition, Am spectra are less diverse than Ap spectra although several groups of Am stars are also defined. *Marginal Am stars* (designated Am:) are stars of spectral appearance similar to the classical Am stars, but in which the metallic and K-line spectral types differ by less than the required 5 subclasses. The *Hot Am stars* have H-line spectral types from A0-A3 and show pronounced abundance anomalies but are not classified as classical Am stars owing to the general weakness of their line strength anomalies. This weakness is a consequence of the weaker metal lines which prevail at these higher temperatures. The  *$\delta$  Delphini stars* are A-type stars lying above the main sequence which have metallic line spectra. They are defined spectroscopically (Kurtz 1976) to be those late-A and early-F subgiants and giants with spectra similar to  $\delta$  Del. Kurtz (1976) considers the  $\delta$  Del stars to be evolved Am stars although this view is not universally accepted (Wolff 1983). Gray and Garrison (1989) argue that the  $\delta$  Del class is spectroscopically inhomogeneous and that most stars classified as  $\delta$  Del in the literature are either normal A-type, Am: or Am stars. They propose that the  $\delta$  Del designation be dropped. Gray and Garrison have also identified a class of late, apparently evolved, Am stars which they designate the  $\rho$  Puppis stars after the bright prototype.

As a group, the Am stars are slow rotators (Abt & Moyd 1973, Abt 1975). Roughly, the Am stars have  $v \sin i < 100 \text{ km s}^{-1}$  while spectroscopically normal stars have  $v \sin i > 100 \text{ km s}^{-1}$ . Moreover, most Am stars are to be found in binary systems. Abt (1961, 1965) showed that 22 out of 25 Am stars were members of spectroscopic binaries with  $P < 100$  days. Abt & Bidelman (1969) later suggested that all main sequence stars in the spectral range A4-F1 that are primaries in binaries with periods in

the range  $2.5 < P < 100$  days have Am spectra while stars outside this period range are chemically normal with a few exceptions. Carcuillat (Vauclair 1983) has subsequently demonstrated the existence of 8 normal A stars with periods in the  $2.5 < P < 100$  day range. Since slow rotation and a high binary frequency are both associated with the Am phenomenon and since slow rotation may be a consequence of binarity in many cases, it is important to test whether rotation is the key parameter that determines whether a given star will have a normal or an Am spectrum. In an attempt to establish whether *all* Am stars and *no* normal A stars are members of binary systems, Conti & Barker (1973) searched for and found radial velocity variations in only 2 out of 5 stars with Am characteristics in the Coma cluster. Unfortunately their sample is too small to rule out inclination effects.

The existence of Am stars in even the youngest associations (Abt 1979) indicates that the time required for the emergence of the Am phenomenon is substantially less than the nuclear evolution time-scale. Also, metallicity appears to be a main sequence phenomenon since Am stars are only found fairly close to the main sequence.

Several observers have sought variability in the Am stars. Initial variability studies indicated that the lack of spectrum and brightness variability and the Am star phenomenon go together. The lack of spectrum variability on time-scales of days seemed to exclude surface inhomogeneities as in the Ap stars, whilst the absence of brightness variability on time-scales of minutes to hours seemed to exclude pulsation. In fact, some  $\delta$  Del and Am: stars *do* pulsate (Kurtz 1976, 1978a, 1984) and so does at least one classical Am star (Kurtz 1989). We will return to this issue of pulsation and metallicity after introducing the  $\delta$ -Sct stars.

It is not surprising that similar models for the production of the abundance anomalies have been considered for the Am stars as for the Ap stars. Watson (1970) and Smith (1971) were the first to apply the diffusion hypothesis to the Am phenomenon and it has subsequently become the favoured working hypothesis of the origin of the abundance anomalies in the Am stars. The diffusion hypothesis provides a natural explanation for the absence of metallicity in rapid rotators; strong meridional circulation currents in rapid rotators destroy the atmospheric stability required for diffusion to operate effectively. The existence of Am stars in very young associations is consistent with the theoretical expectation that unimpeded diffusion can produce marked abundance anomalies in only  $10^4$  yr. For further details on the successes and problems of diffusion models of Am stars consult the excellent reviews by Wolff (1983) and Vauclair (1983) as well as the proceedings of IAU Colloquium 32 (Weiss *et al.* 1975), IAU Colloquium 90 (Cowley *et al.* 1985) and Liège Colloquium 23 (Renson 1981).

### 1.3 The $\delta$ Scuti stars

The lower portion of the Cepheid instability strip crosses the main sequence around the late-A and early-F spectral types. The pulsating variables located here are called the  $\delta$  Scuti stars. They are population I low-amplitude ( $A < 0.8$  mag), short-period ( $1/2 \text{ hr} < P < 7 \text{ hr}$ ) pulsating variables of spectral types A and F which lie within 3 magnitudes of the main sequence. The distribution of  $v \sin i$  in the  $\delta$  Scuti stars is also of interest. While the  $\delta$  Scuti giants have a distribution of  $v \sin i$  which is essentially the same as that for normal A stars, there is a dearth of slow rotators near the main sequence (Wolff 1983). This complements the observation that the chemically peculiar dwarfs are predominantly slow rotators. Because of the complexity of their light curves, some  $\delta$ -Scuti stars have attracted extensive observational investigations and it is not yet clear whether such complexity arises from the beating of several stable modes or from a single unstable mode (Kurtz 1980, 1988, Wolff 1983).

There is plenty of observational material on these variables. Breger (1979) lists 129  $\delta$  Scuti stars in his review and new candidates are being added to the list all the time. It is interesting that only about a third of the stars in this section of the instability strip pulsate with *detectable* amplitudes. The reason why I emphasize detectable pulsation is that the majority of  $\delta$  Scuti stars pulsate with very low amplitudes. The histogram of pulsation amplitudes in Breger's (1979) review reveals that the number of pulsators increases exponentially with decreasing amplitude. This point is worth pursuing with a survey which should have a limiting amplitude considerably less than the 0.01 mag limit of existing surveys.

Note that the definition of a  $\delta$  Scuti star does not imply anything spectroscopic about the star. This means that the definition of a  $\delta$  Scuti star does not formally exclude the Ap, Am and Am: stars from admission to this class of variables. Indeed, observations suggest that some Ap, Am and Am: stars exhibit  $\delta$  Scuti variability and we will return to this issue when we discuss pulsation in the presence of metallicity in Section 1.5.

The  $\delta$  Scuti stars are thought to share a common pulsation-driving mechanism, called the  $\kappa$ -mechanism, with the Cepheids. This driving mechanism applies to any sufficiently abundant element for which the next stage of ionization is characterized by a sharp drop in opacity (and hence the appellation  $\kappa$ -mechanism). Another important condition to be satisfied is that the ionization region of the element in question must coincide with the transition region between the quasi-adiabatic interior and the non-adiabatic outer portions of the envelope. In the  $\delta$  Sct stars, the He II ionization zone satisfies these conditions. The temperature of the He II ionization zone is  $\approx 50000$  K which means that a significant fraction of the radiation lies in the ultraviolet part of the spectrum. He II is opaque to

this UV radiation, so the He II ionization zone dams up radiation to the extent that sufficient pressure is created to lift the overlying layers and the atmosphere expands. At maximum expansion, this zone is mainly composed of He III which is transparent to the UV flux. This UV flux flows freely through the ionization region, allowing the temperature and pressure to drop. Consequently the weight of the overlying layers starts to compress the star. Recombinations of He III plus an electron are facilitated and such recombinations also cool the gas further. The star returns to the state of maximum compression and the next cycle begins. It is the relative positions of the ionization and transition regions that determine the red and blue borders of the instability strip. If the ionization region is too deep, the driving is insufficient to lift the weight of the overlying layers. This marks the red border of the instability strip. In hotter stars, the ionization zone moves to the outermost part of the envelope where there is little mass. Thus the blue border is determined by those stars where the weight of the overlying layers is insufficient to force a recompression.

#### 1.4 The Description of Pulsation in Stars

Oscillatory motions may arise in a star in response to slight perturbations from its equilibrium state. The major restoring forces in stellar oscillation are pressure and buoyancy. Pressure fluctuations tend to dominate at high frequencies to produce acoustic waves while buoyancy dominates at low frequencies to produce gravity waves. Standing acoustic waves are known as  $p$ -modes and standing gravity waves are known as  $g$ -modes. These global acoustic and gravity modes are visible at the surface through the surface displacements and temperature variations which they cause. The surface displacements are detectable through Doppler shifts and line profile variations while the temperature, opacity and projected area variations are detectable through luminosity variations. In our study of the oscillations in HD 101065, we elected to study the luminosity variations using high-speed photometry.

The key to using pulsation as a probe of the stellar interior is to identify the modes of oscillation. In the usual mathematical formulation of the spherical harmonic oscillator problem, the angular variations of the physical variables are considered to be proportional to  $Y_{\ell m}(\theta, \phi)$ , the spherical harmonic of degree  $\ell$  and azimuthal order  $m$ . Some of the low-order spherical harmonics are illustrated in Fig. 1.6. Physically, the degree  $\ell$  is the number of surface nodal lines and the azimuthal order  $m$  is the number of nodal lines that cross the equator. Purely radial pulsation constitutes alternating expansion and contraction and this corresponds to the mode  $\ell=0$ . A mode with  $\ell=1$  is called the dipole mode,  $\ell=2$ , a quadrupole mode, and so on. In a non-rotating star, the  $m$ -modes are degenerate and each segment of the stellar surface will oscillate with a frequency  $\nu(n, \ell)$

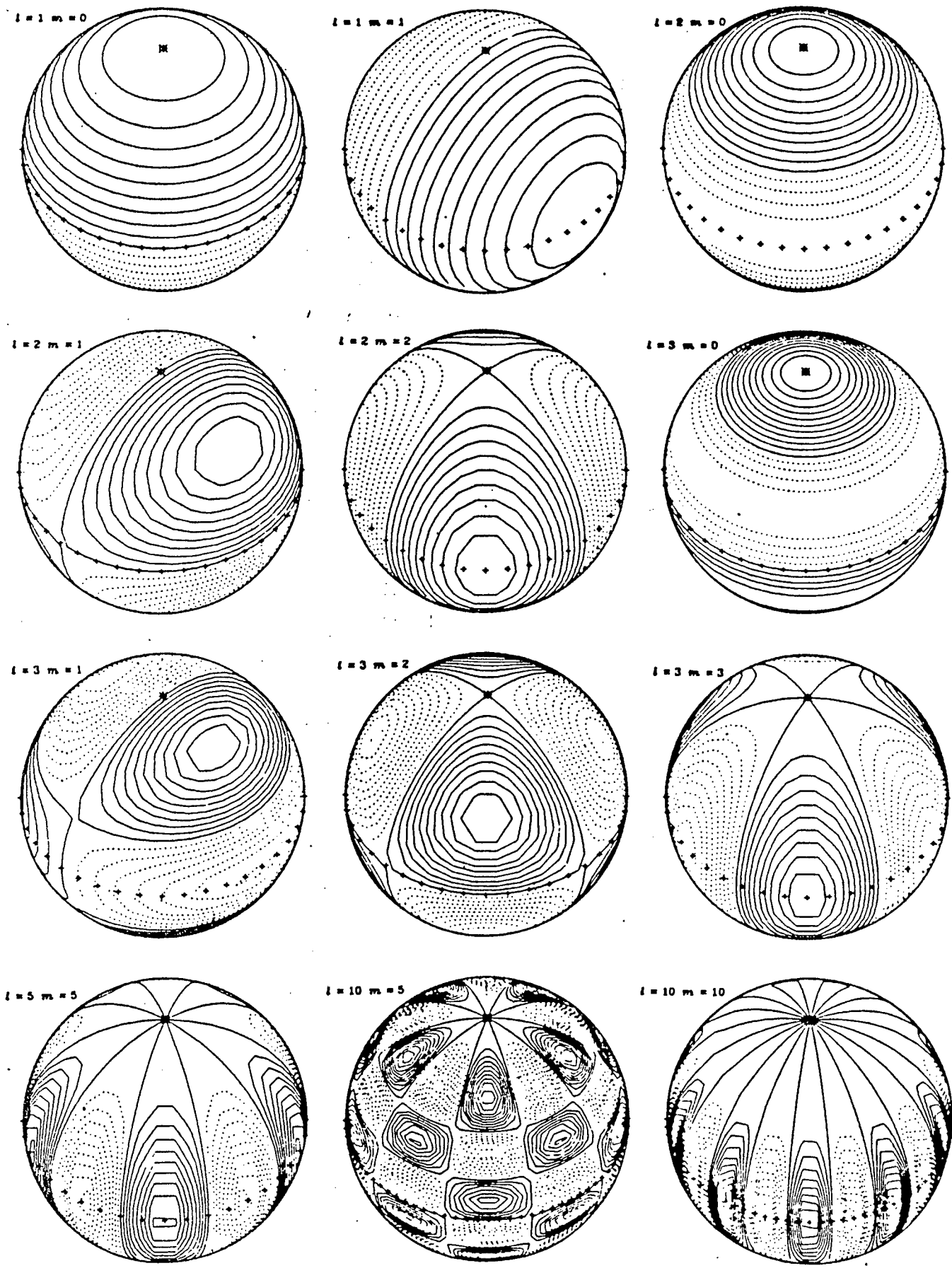


Fig. 1.6 A sample of surface expressions of the spherical harmonics (from Christensen-Dalsgaard, 1982). Using photometric techniques, it is possible only to detect modes with  $l \leq 3$ . For higher-degree modes, the star presents too many sectors pulsating out of phase in order for there to be a detectable net signal in the integrated light of the stellar disk.

determined by  $n$  and  $\ell$  only. In real stars this degeneracy is lifted by rotation through the Coriolis force and an inertial observer will see each  $\nu(n, \ell)$  split into  $(2\ell + 1)$   $m$ -modes whose frequencies are given by the Ledoux (1951) relation

$$\nu(n, \ell, m) = \nu(n, \ell, 0) - m\Omega(1 - C_{n, \ell}) \quad (1.1)$$

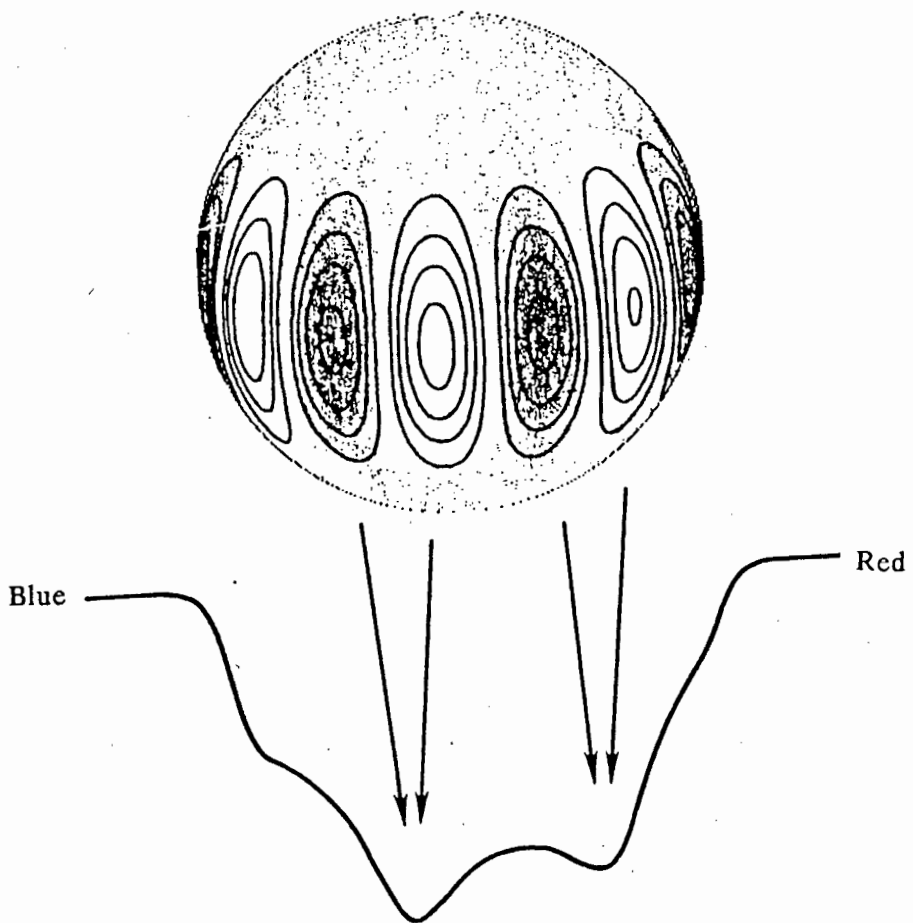
where  $C_{n, \ell}$  is a constant dependent on the stellar structure and pulsation mode and  $\Omega$  is the rotation frequency.

The radial structure of the oscillations is characterized by the radial overtone  $n$ , which is usually the number of nodes along a radius of the star. The first of these values,  $n=0$ , is called the fundamental radial mode,  $n=1$ , the first overtone,  $n=2$ , the second overtone, and so forth. Unlike the angular component of the non-radial oscillations, the radial component is not directly accessible to observation. Nonetheless, once the degree  $\ell$  is known, it may be possible to determine  $n$  by appealing to theoretical treatments of pulsation. We will do this later for our HD 101065 observations.

In the terminology we have just developed, the  $\delta$  Scuti stars may be described as low-degree, low-overtone  $p$ -mode pulsators. In contrast, the short period ( $P \approx 5-15$  min) rapidly oscillating Ap stars (among which HD 101065 is numbered) are described as low-degree, high-overtone ( $n \approx 10-40$ )  $p$ -mode pulsators. Since in most cases the stellar disk is unresolved, asteroseismologists doing high-speed photometry can only detect low-degree modes with  $0 \leq \ell \leq 4$ . For higher  $\ell$  values, the surface is divided into too many sectors pulsating out of phase with each other for there to be a detectable signal in the integrated light of the stellar disk.

However, observations of line profile variations allow the detection of modes with  $\ell \geq 4$ . Line profile mode typing is based on the one-to-one relationship between positions across the stellar disk and the wavelength positions in a rotationally broadened line profile (assuming solid body rotation). This is indicated schematically in Fig. 1.7 (Vogt & Penrod 1983) where the arrows depict red (right) and blue (left) shifts for a given absorption line in the different pulsation zones. The degree of inclination of the arrows from the vertical is an indication of the strength of the Doppler shift. The dips in the line profile arise where the red and blue shifts add to produce greater absorption than that expected for a line affected only by rotational broadening. When the lines are Doppler shifted away from a certain wavelength, a hump will arise at that wavelength because of the lower absorption there. The motion of these features across the line profile is governed by the motion of the pulsation pattern across the stellar surface as well as the rotation of the stellar surface itself.

Vogt & Penrod (1983) have studied the line profile variations in the main sequence star  $\zeta$  Oph and they find that an  $\ell=8$ ,  $m=-8$  mode fits their data well. In the B star  $\epsilon$  Per, the modal degree,



**Fig. 1.7** Illustration of the formation of distortions in the line profiles of a rapidly rotating non-radially pulsating star. The line profile is essentially a velocity map of the star. The width of the line profile has been scaled to match the diameter of the star thus reflecting the mapping which occurs between a position across the disk and a position across the profile. The dark regions indicate material moving away from the observer, while the lightest regions indicate material moving toward the observer (from Vogt & Penrod, 1983).

oscillation amplitude and  $v \sin i$  all combine to produce fortuitously large line profile variations. Indeed, this star exhibits such extreme line profile variations that it has been mistaken for a double-lined spectroscopic binary with an unusually short period. Smith *et al.* (1987) tentatively identify the dominant oscillation frequency with an  $\ell=4$ ,  $m=-4$  mode and a secondary frequency with an  $\ell=6$ ,  $m=-6$  mode. Even so, reliable mode identifications are usually difficult to obtain and very few stars have secure mode identifications.

Several techniques of mode identification can be applied to photometric stellar oscillation data. For brevity, we will discuss only the technique of Balona and Stobie (1979) since this is the technique that was applied to HD 101065 by Kurtz (1980a). Balona and Stobie showed how light and colour variations can be used as a mode discriminant. One starts with an estimate of the ratio,  $f$ , of the flux-to-radius variation and the phase shift,  $\psi$ , between these variations. Balona & Stobie (1979) give relations which enable one to compute the phase shift  $\Delta\phi(V, B-V)$  between the light and colour variations for different assumed spherical harmonics. The values of  $\Delta\phi(V, B-V)$  differ sufficiently to permit their use as mode discriminants for even, low-order  $\ell$  modes. For odd values of  $\ell$ ,  $\Delta\phi(V, B-V)=0$  independent of  $f$ , which means that this technique cannot discriminate between modes with odd  $\ell$ . If  $\Delta\phi(V, B-V) < 0$ , an  $\ell=0$  mode is indicated. Typical values of  $f$  and  $\psi$  are given by Balona and Stobie (1979) and references therein. For the  $\delta$  Scuti stars,  $f \approx 10$  and  $\psi \approx 124^\circ$ . This technique has been applied to the phase shifts in the  $\delta$  Scuti star 1 Mon by Balona & Stobie (1979) and to the very high metallicity  $\delta$  Scuti star HD 188136 by (Kurtz 1980b). We defer an illustration of the use of this technique until Section 4 where we discuss Kurtz's (1980a) identification of  $\ell=2$  pulsation in HD 101065.

### 1.5 Metallicism and Pulsation

The definitions of the  $\delta$  Del and Am stars imply nothing about pulsation in such stars and, conversely, the definition of a  $\delta$  Scuti star implies nothing spectroscopic about such a star. As all three classes of object populate the same region of the HR diagram, one might suppose that pulsation and metallicism could coexist in some stars. In fact, for many years an apparent exclusion between the  $\delta$  Scuti stars and the Am stars was observed (Breger 1970). That is, an exclusion between pulsation and metallicism was observed. The diffusion hypothesis is usually invoked to explain this observation. It is argued that in slowly rotating stars, He II diffuses out of the He II ionization zone and the star is stabilized against pulsation. If the star is magnetic, it becomes an Ap star, otherwise it becomes an Am star. On the other hand, rapid rotation gives rise to turbulent meridional circulation currents which destroy the effects of diffusion and leave the He II ionization zone with sufficient He to drive

pulsation. According to this simple picture, all slowly rotating A stars should be chemically peculiar and should not pulsate. The rapidly rotating A stars should all be chemically normal and pulsate.

However, the situation is not this simple, since some  $\delta$  Del and Am: stars *do* pulsate (Kurtz, 1976, 1978a, 1984) and so does at least one classical Am star (Kurtz 1989). One might argue that the Am, Am: or  $\delta$  Del classifications of these stars are incorrect, but such an argument becomes much less tenable as the number of candidates increases. Thus Cox, King and Hodson (1979) responded to the need for theoretical studies addressing the problem of pulsation in the presence of metallicity. They found that the Am: and  $\delta$  Del stars may possess sufficient residual He to drive pulsation in a narrow instability strip only 200-500 K wide situated in the coolest one-third of the  $\delta$  Scuti instability strip. They conjectured that the residual He exists because of incomplete settling, or because of a small recent upward mixing of He.<sup>5</sup> Cox *et al.* argue that the classical Am and Ap stars should *not* pulsate because their He II ionization zones will be fully depleted of He. In the decade that has followed the publication of Cox *et al.*'s paper, the whole issue of pulsation and metallicity has become considerably clouded with exceptions to this rule.

The first pulsating metal-rich A star to be discovered was the extremely peculiar star HD 101065 (Kurtz 1978b). This star proved to be instrumental in the discovery of a new (and entirely unexpected) class of rapidly oscillating Ap stars. These stars oscillate with periods in the range 4-15 min and amplitudes of a few millimagnitudes (mmag). Kurtz (1979, 1980b) also discovered low-overtone multi-periodic  $\delta$  Scuti pulsation in the metal-rich star HD 188136. It could be argued that since this star is above the main sequence and displays enhanced metallicity it is a  $\delta$  Del star, but Wegner's (1981) abundance analysis indicates that HD 188136 is spectroscopically akin to HD 101065. This means that HD 188136 is probably an Ap star which exhibits  $\delta$  Scuti pulsation. Kreidl (1986) has reviewed the observational evidence for  $\delta$  Scuti variability in about 20 candidate Ap stars. Although there do not seem to be any clear-cut cases of  $\delta$  Scuti variability in a star with an unambiguous Ap designation, further observational work in this direction is justified. A most exciting recent development is Kurtz's (1989) discovery of  $\delta$  Scuti pulsation in the classical Am star HD 1097. Unless the Am classification of this star can be shown to be incorrect, Kurtz's observations constitute evidence that classical Am stars can pulsate.

It seems that just about the only general statement that can still be made about pulsation and metallicity is that where they co-exist, low pulsation amplitudes ( $A \leq 0.05$  mag) prevail. This suggests

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<sup>5</sup> The co-existence of pulsation and mild metallicity in the Am: stars could also be a consequence of the metals diffusing upwards faster than the He diffuses downwards (Vauclair *et al.* 1978).

that diffusion is still a viable underlying mechanism for producing the spectral peculiarities, although one then requires that the turbulent velocities remain much lower than the diffusion velocity of  $\approx 1 \text{ cm s}^{-1}$  while pulsation generates radial velocities as high as several  $\text{km s}^{-1}$  (Kurtz 1989, Matthews *et al.* 1987b). The whole question of pulsation in the presence of metallicity is far from being resolved.

## 2 HD 101065 as an Ap star

HD 101065 may easily be described as one of the most peculiar non-degenerate stars in the sky. For a decade, this star's remarkable spectrum fueled a controversy regarding its spectral type with the debate raging from the late-A to the early-G types. Its B5 classification in the Henry Draper catalogue was shown to be incorrect by Przybylski (1961) who first drew attention to the extremely complex nature of its spectrum. Although many of the spectral features appeared to be attributable to the rare earths, he also noted a great weakness in the lines of the iron-peak elements. On the basis of its UBV colours (Przybylski 1961), HD 101065 might be classified as a K0 star with an ultraviolet excess. Kron & Gordon (1961) obtained 6-colour photometry of HD 101065 and four other late-F and early-G stars. From this, they concluded that HD 101065 is an extremely heavily blanketed F8 or G0 dwarf. Such stars have extensive surface convection zones and it was thought that HD 101065 might represent an advanced stage of stellar evolution with the spectral peculiarities arising as a consequence of processed material being dredged up from the interior.

Przybylski (1963a,b, 1966) performed the first abundance analyses of HD 101065 in the blue photographic region of the spectrum, where the mutual blending is so severe that the continuum cannot be reliably identified. In the range 3650-4820 Å alone, he measured around 3000 lines. He drew attention to the great enhancement ( $\approx 10^5$ ) in the abundances of the ionized rare earths such as Ho II, Dy II, Nd II and Eu II. Strangely, there was also an apparent absence of elements such as Fe, Ti, Cr, Mn, Na and Ca (which he found to be under-abundant by a factor of 1000). Working in the region 5000-6880 Å, Warner (1966) demonstrated the presence of lithium and strontium and confirmed that "nearly every rare earth ion line appears in HD 101065." He also noted that since any surface Li dragged into the interior by convection would be destroyed, it had to be replenished by equilibrium spallation processes on the surface of the star. This motivated him to suggest that HD 101065 might have a significant magnetic field.

We have already seen that the coolest Ap and Am stars are to be found at around F2, a point coincident with the emergence of an extensive surface convection zone. If the spectral identification of HD 101065 as F8, or even G0, is correct, it has to be a unique pathological object totally unrelated to the chemically peculiar stars of the upper main sequence. To several astronomers, such an extreme

view seemed less attractive than a reconsideration of the spectral type of HD 101065. After all, it seemed reasonable to expect that a star with such a complex spectrum might not easily be accommodated in classification schemes based on selected absorption features. The first challenge to Przybylski's placement of HD 101065 near spectral class F8 came from Wegner & Petford (1974). From the Balmer line strengths and the excitation temperature derived from the curve of growth, they deduced that HD 101065 is probably an F0p dwarf. Moreover, they suggested the presence of the Fe-peak elements in essentially cosmic abundances.

Przybylski (1975,1976) voiced his skepticism of the F0p classification and contested the line identifications on which Wegner & Petford's abundance analysis was based. In support of the later spectral type, he cited an independent determination of  $T_{eff} \approx 6300$  K by Hyland *et al.* (1975) from measurements of the  $V-J$ ,  $V-H$  and  $V-K$  colours. This, they reasoned, was a reliable temperature indicator because the complicating effect of blanketing is minimal beyond  $1\mu\text{m}$ . However, such temperature determinations are strongly sensitive to errors in colour and interstellar reddening is often a major source of such errors. In addressing this issue, Hyland *et al.* utilized an unpublished polarization measurement which indicated that  $E(B-V) < 0.01$  and concluded that interstellar reddening does not significantly affect the colours of HD 101065. Wegner (1976) contested this assertion on the grounds that polarization measurements of individual stars do not yield secure determinations of reddening. He thus determined the polarization in the direction of HD 101065 from  $UBV$  observations of 60 nearby field stars, found an increase in  $E(B-V)$  with distance modulus, and concluded that Hyland *et al.*'s estimate of  $T_{eff}$  could be too low by  $\approx 1000$  K. On the basis of this and the  $H\alpha$  and  $H\beta$  profiles, he argued that  $T_{eff}$  is probably nearer 7000 K. Przybylski (1977b), who was reluctant to accept the view that HD 101065 is strongly reddened, argued that an unacceptably high reddening correction of  $E(B-V) = 0.23$  would be required to make HD 101065 an F0 star.

The identification of HD 101065 as an Ap star initially faced several difficulties, but these have largely fallen away. One of these difficulties was the extreme over-abundance of Ho and Dy, elements thought to be rare in Ap stars (Adelman 1973a,b). However, these elements were demonstrated to be present with substantial abundances in the known Ap stars HD 51418 by Jones *et al.* (1974) and HR 465 by Cowley & Cowley (Jones *et al.* 1974, Rice 1988). Another significant difficulty facing the Ap classification of HD 101065 was the apparent absence of the Fe-peak elements which were shown by Adelman (1973a,b) and others to be over-abundant in Ap stars. Cowley *et al.* (1977) attempted to resolve the element identification controversy with the aid of wavelength-coincidence statistical techniques. They were able to demonstrate indisputably the

presence of the Fe-peak elements and although they concurred with Przybylski that Fe is underabundant by "orders of magnitude", they did not exclude the possibility that HD 101065 might be an extreme Ap star. The discovery by Wolff & Hagen (1976) of a -2200 G magnetic field in HD 101065 provided strong support for its proposed Ap nature since if it were as cool as suggested by Przybylski, it would have an extensive surface convection zone and thus be unlikely to support a net longitudinal magnetic field as strong as -2200 G.

Several observers also searched for spectral and photometric observations of the sort normally seen in Ap stars. Cowley *et al.* (1977), Wegner & Petford (1974) and Wolff & Hagen (1976) all conclude that HD 101065 is not a spectrum variable on time-scales between a year and a decade. Because of its B5 classification in the Henry Draper catalogue, the possibility that HD 101065 might be a very long-term spectrum variable (as HR 465, say) was examined by Cowley *et al.* and Wolff & Hagen. Both groups conclude that at classification dispersions, the myriad lines merge into a pseudo-continuum with only the H and Ca II lines clearly visible. It seems probable that the weakness of the Ca K line motivated the early HD classifier to choose the B5 classification. Heck *et al.* (1976) found no evidence for systematic photometric variations on the time-scale of a week. Przybylski (1977c) discovered the presence of small changes in visual brightness ( $<0.03\text{mag}$ ) from 1969 to 1977. It was not possible for him to determine whether these variations were irregular or periodic and unfortunately there are no complementary spectral or magnetic observations.

Kurtz & Wegner (1979) reasoned that if the higher  $T_{eff}$  were correct, HD 101065 would lie near the cool border of the  $\delta$  Scuti instability strip and might therefore show  $\delta$  Scuti-like variability. Indeed, they discovered variability in HD 101065, but with a period of 12.14 min, a period of unprecedented shortness in  $\delta$  Scuti stars whose shortest periods are  $\approx \frac{1}{2}$  hr. The very short period in HD 101065 implies high-overtone  $p$ -mode pulsation. Because the red edge of the  $\delta$  Scuti instability strip extends only from  $T_{eff} \approx 7500$  K on the ZAMS to  $T_{eff} \approx 6950$  K at about a magnitude above the ZAMS (Breger 1979), Kurtz & Wegner reasoned that the pulsations of HD 101065 are a strong argument in favour of the higher value of  $T_{eff}$ . They also obtained the first infrared spectra of HD 101065 in the region 7600 Å-9000 Å in an attempt to get away from the intense blanketing in the visible region. Figure 2.1 shows their spectra of HD 101065 and a few comparison stars. Notice that from G2 to F5, the Paschen lines are weak and the Ca II triplet is strong. For F0, and hotter types, the H lines strengthen and blend with the Ca lines which die out in the early A stars. Using the Paschen 12 (P12) line as a qualitative temperature indicator, Kurtz and Wegner concluded that HD 101065 cannot possibly be later than F5 and that its spectrum bears closer resemblance to a late-A or F0

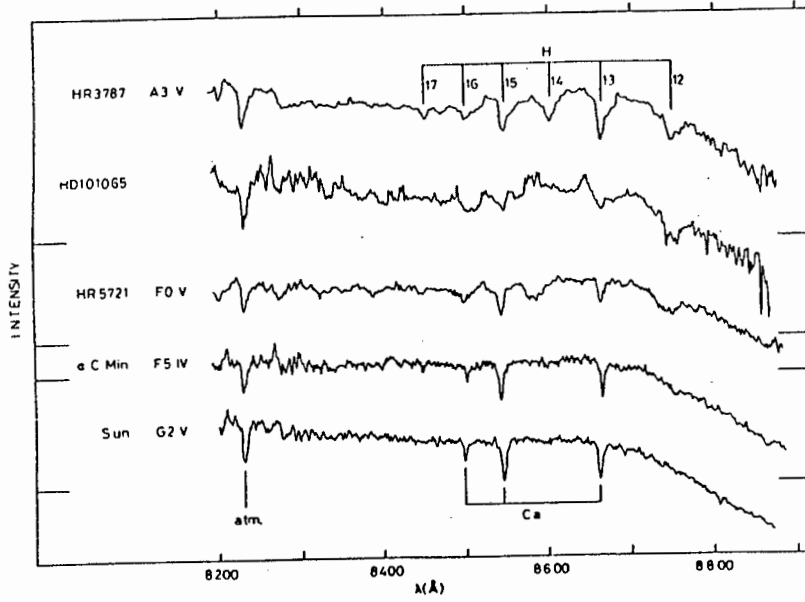


Fig. 2.1 Infrared spectra of HD 101065 and stars of known MK spectral type. Note that the P12 line in HD 101065 has a strength similar to that of the F0 V and A3 V stars (from Kurtz & Wegner, 1979).

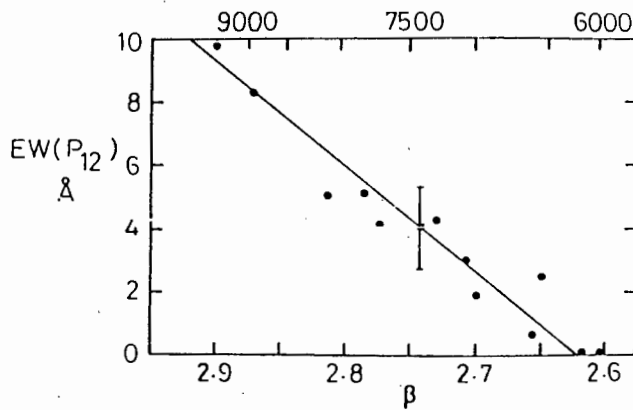


Fig. 2.2 The equivalent widths,  $EW(P_{12})$ , of the P12 line plotted against  $\beta$  for luminosity class V to III stars. The line is a least-squares fit to the data. The position of HD 101065 is indicated with the error bar (from Kurtz & Wegner, 1979).

spectrum. They then correlated the equivalent width of the P12 line with a well calibrated temperature index,  $H\beta$ , for 12 comparison stars (of different luminosity class). The derived relation was linear and independent of the luminosity class (Fig. 2.2), whence they concluded that  $T_{eff} \approx 7400 \pm 300$  K in agreement with the lower bound imposed on the temperature by the pulsation of HD 101065. Glass (1982) performed *JHK* photometry of HD 101065 and several comparison stars nominated by Przybylski. He then looked at the infrared colours *J-H*, *H-K* and *J-K* and found that they resemble those of the F0 dwarfs closely. The purely infrared colours may be a more reliable temperature indicator than the *V-J*, *V-H* and *V-K* colours used by Hyland *et al.* (1975) because interstellar reddening and absorption are unlikely to produce noticeable changes in these colours.

Wegner *et al.* (1983) went on to extend the spectral coverage of HD 101065 to the ultraviolet with the *IUE* satellite. Their observations extend from 1200 Å-3200 Å. Again, simple comparisons with the spectra of other stars suggest that HD 101065 has the energy distribution of an F0V star; there is simply too much ultraviolet flux to be consistent with a  $T_{eff}$  as low as 6300 K. They also conclude that the "clear dominance of the Fe peak elements in the ultraviolet spectrum of HD 101065 supports the higher temperature estimates for this star."

Przybylski (1977b, 1979, 1982) remained unmoved by all arguments in support of the higher temperature estimates and normal abundances of the Fe-peak elements although he never responded to the challenges presented by the strong magnetic field or the rapid oscillations. He (Przybylski 1982) disputed the higher  $T_{eff}$  estimates based on the P12 line on the grounds that he could not confirm Kurtz & Wegner's estimate of an equivalent width of 4 Å for this line. Przybylski (1977b) also used the  $H\alpha$  and  $H\beta$  line profiles to estimate  $T_{eff}$  and his results differ widely from those of Wegner (1976). In his last paper on this star (Przybylski 1982), written *before* the *IUE* observations, he presented 6 arguments for the lower  $T_{eff}$  and the under-abundance of the Fe-peak elements. The reader skeptical of the Ap identification of HD 101065 is referred to that paper and the references therein.

### 3 Rapidly oscillating Ap stars

It would not be an exaggeration to say that were it not for HD 101065, the rapidly oscillating Ap stars would probably still lie undiscovered. The belief that HD 101065 is an extreme member of the cool Ap stars motivated Kurtz (1978b) to search for  $\delta$  Scuti-like oscillations in HD 101065. The initial searches were performed with standard differential photometry. The cycle time for HD 101065 and the two comparison stars, HD 101066 and HD 101388, was 7 min and while the comparison stars were constant within a scatter  $\sigma = 0.002$  mag, HD 101065 had a "suspiciously regular" scatter of  $\sigma = 0.003$

mag about the mean value (Kurtz & Wegner 1979). Subsequent observations with continuous 10-s integrations revealed that HD 101065 is indisputably a variable star with a dominant 12.14-min oscillation having an amplitude of 0.01 mag peak-to-peak.

It is important to appreciate that at the time there was no theoretical or empirical motivation to believe that such rapid oscillations might be common in cool Ap stars. Indeed, the theoretical expectation was that such stars should *not* pulsate since this would be incompatible with the atmospheric stability required for the diffusion mechanism to produce the anomalous abundances of these stars. Nonetheless, on the belief that HD 101065 is an extreme Ap star, Kurtz decided to search for rapid oscillations in other cool Ap stars and whether his reasoning was correct or not, the fact remains that his search was highly successful and as of this writing 14 such stars have been discovered (Table 3.1). The most recent comprehensive review of the rapidly oscillating Ap stars is that of Kurtz (1990). Although the hardware and techniques used to study the rapidly oscillating Ap stars date back to the late 1960's, these variables were not discovered until the late 1970's simply because there was no motivation to search for oscillations in them. Were it not for Kurtz and Wegner's idea to search for non-spectroscopic evidence in favour of the higher  $T_{eff}$  estimates for HD 101065, the rapidly oscillating Ap stars would probably still lie undiscovered.

All the known rapidly oscillating Ap stars have periods in the range 4-15 min and  $B$  semi-amplitudes  $< 8$  mmag. They are cool, magnetic Ap stars with SrCrEu line-strength peculiarities and the phenomenon does seem to be confined to the  $\delta$  Scuti instability strip (Kurtz 1988b).<sup>6</sup> Some of them are known oblique rotators and the favoured explanation for these stars requires that they all be such. The oscillations are modulated in amplitude on time-scales of days. The modulation periods are equal to the periods of the magnetic, spectral and photometric variations and oscillation amplitude maxima always coincide with magnetic extrema. Magnetic cross-over is also accompanied by an oscillation-phase-flip of  $\pi$  radians in two stars, HR 3831 (Kurtz & Shibahashi 1986) and HD 6532 (Kurtz & Cropper 1987).

Because the oscillation amplitude is modulated with rotation and because rotation periods of years, or even many decades (as in HR 465), are not uncommon in Ap stars, one cannot conclude from a few null results that a given star really is constant. As a specific example, we note  $\gamma$  Equ which was found to be constant by Kurtz on several occasions before he successfully identified rapid oscillations in this star (Kurtz 1983).

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6 Not all cool Ap SrCrEu stars in the  $\delta$  Scuti instability strip are rapid oscillators, however.

Table 3.1 Some properties of the rapidly oscillating Ap stars (adapted from Kurtz, 1990).

HD	Spectral Type	$B_{\text{eff}}$ (G)	$\nu$ (mHz)
6532	Ap SrCrEu		2.396215
			2.402165
			2.408120
			4.804299
24712	A5p, Ap SrCr(Eu)	+400 to +1300	2.61965
			2.65294
			2.68760
			2.72085
			2.75569
			2.80568
60435	Ap SrEu, Fp SrEu	$ B_{\text{eff}}  < 1000$	0.9 - 4.2
80316	Ap SrEu <sup>a</sup>		2.2516
			2.2460
83368	Ap SrCrEu <sup>b</sup>	-700 to +700	1.42395445
			1.42801257
			1.43207097
			2.84790845
			2.85602534
			2.86414166
			4.28809652
101065	Controversial	-2200	1.3728660
			1.3699260
			1.3150340
			2.7457355
128898	Ap SrCrEu, Ap SrEu	-300 (variable)	2.442041
			2.4395
134214	Ap SrEu(Cr), F0 SrEu		2.94960
137949	Ap SrCrEu, Fp SrCrEu	+1400 to +1800	2.01482
			4.02964
166473	Ap SrCrEu		1.892
			1.824
			1.928
176232	F0p SrEu		1.435996
			1.385373
			1.239261
201601	F0p	+500 to -800	1.339
			1.366
			1.427
203932	Ap SrEu		2.8051
217522	Ap SiCr <sup>c</sup> , Fp SrEu		1.2151

Notes: <sup>a</sup> very strong Sr, <sup>b</sup> strong Sr, <sup>c</sup> may be Eu rather than Si

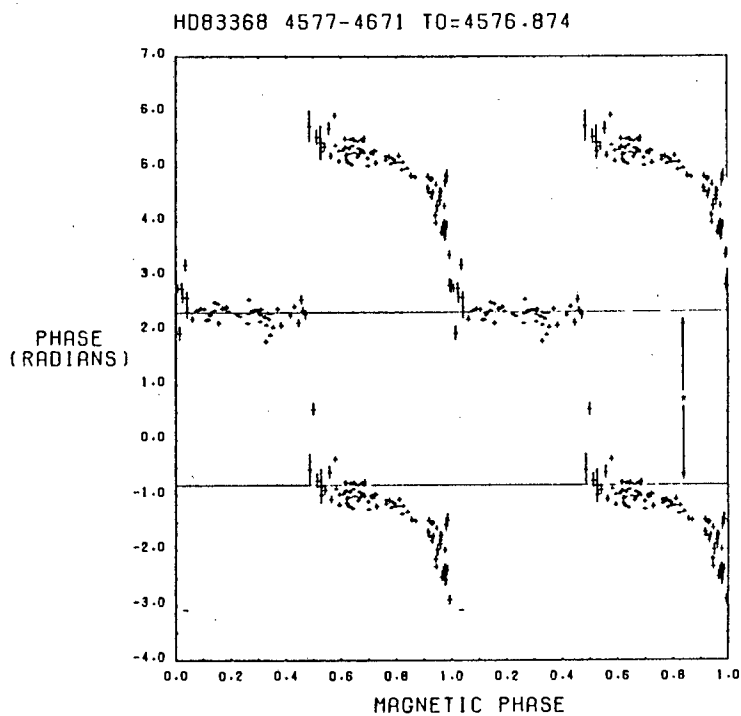
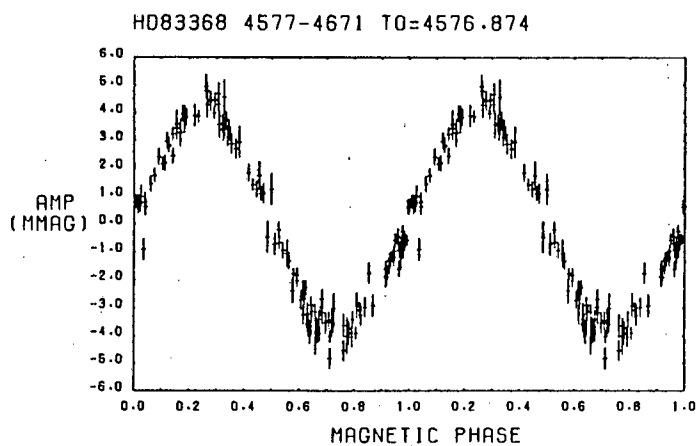


Fig. 3.1 The variation in the amplitude and phase of the dominant oscillation in the rapidly oscillating Ap star HR 3831 which oscillates primarily in an  $\ell = 1$  mode. The amplitudes and phases were derived by fitting the dominant frequency to 1-hr segments of the data. The error bars are  $\pm 1\sigma$ . Refer to the text for an explanation of how these patterns arise (from Kurtz & Shibahashi, 1986).

Unfortunately, the small sample of known rapid oscillators has made it difficult to determine all of the distinguishing characteristics of these stars. For instance, we still do not know what distinguishes the cool rapidly oscillating Ap stars from other cool magnetic Ap stars. Hence the search for new candidates remains a fundamental priority in the study of this class. Since these stars are all bright, they can be studied with telescopes as small as 0.5 m. Often 1-m, or larger, telescopes are required, not because they collect more photons, but because they average over many seeing cells and thus lower the scintillation noise. Our 1988 observations of HD 101065 afford an example of this. The detection of rapid oscillations in these stars requires a superior photometric site, meticulous care in acquiring the observations, a stable, reliable photometer and *many* hours of observing time.

HD 116763 affords a good example of how time consuming it can be to identify rapid oscillations in these stars. This star was tentatively identified as a rapidly oscillating Ap star by Matthews *et al.* (1988) with 9 hr of high-speed photometry collected over 4 nights at CTIO. In 1988, I observed HD 116763 for 11 hr spread over 4 nights at SAAO and found it to be constant within 0.3 mmag. Although my observations of HD 116763 do not, by themselves, constitute a conclusive proof of the constancy of HD 116763, I argued (Martinez 1989) that Matthews *et al.* over-interpreted their data and that both the SAAO and CTIO observations support the notion that HD 116763 does not yet show evidence of rapid oscillations. The Appendix contains a paper which I published in this regard.

The short oscillation periods in the rapidly oscillating Ap stars exclude both low overtone *p*-mode as well as *g*-mode descriptions of the oscillations. The Fourier spectra of these rapid oscillations often show equally spaced frequency multiplets suggestive of the *m*-mode splitting expected in rotating non-radial pulsators. The frequencies of such rotationally split *m*-modes are given by the Ledoux (1951) relation

$$\nu(n, \ell, m) = \nu(n, \ell, 0) - m\Omega(1 - C_{n, \ell})$$

where  $C_{n, \ell}$  is a constant dependent on the stellar structure and pulsation mode and  $\Omega$  is the rotation frequency.

Shibahashi & Saio's (1985) A star models indicate that  $10^{-3} \leq C_{n\ell} \leq 10^{-2}$  in the rapidly oscillating Ap stars. By studying the coincidence of magnetic and oscillation amplitude maxima in HR 1217, Kurtz *et al.* (1989) placed upper limits on  $C_{n\ell}$  of  $C_{n\ell} \leq 0.0006$  at the  $3\sigma$  level of confidence. This coincidence of magnetic and oscillation amplitude maxima is also seen in HR 3831 by Kurtz & Shibahashi (1986) whose observations indicate that  $C_{n\ell} \leq 10^{-5}$  in that star. If  $C_{n\ell}$  is not exactly zero, such agreement in the times of pulsation and magnetic maxima is purely fortuitous and it is expected that these two maxima should drift apart on a time-scale  $\approx C_{n\ell}^{-1}$ . That such a drift has not been seen

in either HR 1217 or HR 3831 after more than 5 years (Kurtz 1989, Kurtz & Shibahashi 1986) suggests that it is not possible for rotational splitting of  $m$ -modes to produce the frequency multiplets seen in the rapidly oscillating Ap stars. Kurtz (1982) also considered magnetic splitting of  $m$ -modes, but rejected that possibility because the required magnetic field strengths are far stronger than the observed field strengths in the Ap stars. He then developed a model which explains the observed frequency splittings. His oblique pulsator model (Kurtz 1982) proposes that a rapidly oscillating Ap star is a pulsating oblique rotator whose pulsation axis is coincident with the magnetic axis. This very simple model can explain the rotational amplitude modulation, the phase shifts and the equally-spaced frequency multiplets whilst preserving our understanding of the magnetic, spectral and mean light variations in these Ap stars.

We will illustrate the oblique pulsator model for the case of dipole ( $\ell=1$ ) pulsation. The luminosity variations over the surface of the star are given by (Kurtz 1982)

$$\Delta L/L \propto P_\ell(\cos \alpha) \cos(\omega t + \phi)$$

where  $P_\ell(\cos \alpha)$  is an appropriate Legendre polynomial,  $\omega=2\pi\nu$  is the oscillation frequency and  $\alpha$  is the angle between the pulsation pole and the line of sight. As the star rotates, this angle changes in the fashion

$$\cos \alpha = \cos i \cos \beta + \sin i \sin \beta \cos \Omega t$$

where  $i$  is the rotational inclination,  $\beta$  the magnetic obliquity,  $\Omega$  the rotation frequency and  $t=0$  is defined at magnetic maximum (see Fig. 1.3). For an  $\ell=1$  mode,  $P_\ell(\cos \alpha) = \cos \alpha$  so that

$$\Delta L/L \propto A_0 \cos(\omega t + \phi) + A_1 \cos[(\omega - \Omega)t + \phi] + A_1 \cos[(\omega + \Omega)t + \phi]$$

where

$$A_0 = \cos i \cos \beta$$

and

$$A_1 = \frac{1}{2} \sin i \sin \beta.$$

Thus we see that an oblique pulsator pulsating in an  $\ell=1$  mode will give rise to a frequency triplet with a spacing which is equal to the rotation frequency. This type of frequency splitting has nothing to do with  $m$ -mode splitting. In the general case, each  $\ell$  mode in an oblique pulsator will appear in the frequency spectrum as a  $2\ell + 1$  frequency multiplet.

It is also possible to investigate the magnetic geometry of a system with this model. For a dipole oblique rotator,

$$\tan i \tan \beta = (1-r)/(1+r)$$

where  $r = H_{e \text{ min}}/H_{e \text{ max}}$  is an observable quantity. Using the oblique pulsator model, for an  $\ell=1$  mode we may write

$$A_1/A_0 = \frac{1}{2} \tan \iota \tan \beta,$$

whence

$$r = (1-2A_1/A_0)/(1+2A_1/A_0)$$

where  $A_1/A_0$  is an observable quantity. In the few cases where  $r$  values have been obtained from magnetic measurements and from the rapid oscillations there is good agreement in the results. The oblique pulsator model also predicts the phase reversal of the oscillations at magnetic cross-over. For a dipole mode we have  $\Delta L/L \propto \cos \alpha \cos(\omega t + \phi)$  which may be regarded as a sinusoid of constant phase  $\phi$  until  $\cos \alpha$  changes sign, which is equivalent to a phase-flip of  $\phi$  by  $\pi$  radians. Physically, what is happening is that the visible hemisphere is initially dominated by one of the pulsation poles and the observer sees an oscillation of constant phase  $\phi$ , but modulated in amplitude. At quadrature, the amplitude passes through zero and increases again with constant phase  $\phi + \pi$  as the other pulsation pole comes into view. These effects are well illustrated (Fig. 3.1) in HR 3831 (HD 83368) (Kurtz & Shibahashi 1986) which presents the best case for the basic oblique pulsator model. The basic model has been refined by Dziembowski & Goode (1985, 1986) and Kurtz & Shibahashi (1986) to take into account the effects of rotation and an oblique magnetic field.

The driving mechanism in the rapidly oscillating Ap stars is not yet known. Because they are situated in the  $\delta$  Scuti instability strip, the  $\kappa$ -mechanism has been proposed as the driving mechanism despite the expectation that the He II ionization zones should be depleted in these stars. This begs the question of why the rapidly oscillating Ap stars pulsate in such high overtones in comparison with the  $\delta$  Scuti stars. Matthews (1988) has avoided the He depletion problem by proposing a  $\kappa$ -mechanism based on the Si IV ionization zone. The idea sounds promising, but depends critically on how much Si is required in the Si IV zone to drive pulsation with the observed amplitudes. Shibahashi (1983) has invoked magnetic overstability in which the magnetic field lines provide the restoring force against convective motions to generate periods in the observed range. If this is the case, then the occurrence of rapid oscillators may not be confined to the instability strip.

The rapidly oscillating Ap stars are the first main sequence stars other than the Sun in which rapid oscillations have been indisputably identified. The motivation for obtaining frequency solutions for these stars is that the frequency spectra contain information on the physical conditions in the interiors of these stars. For instance, if a star pulsates in a single radial mode, the pulsation frequency provides a measure of its mean density. As the number of modes increases, the amount of

information also increases so that the non-radial pulsations in the Sun and the rapidly oscillating Ap stars are a mine of astrophysical data. The low-order high-overtone  $p$ -modes penetrate deep into the stellar interior and thus provide a powerful probe of stellar interior structure and dynamics.

For high-overtone ( $n \gg \ell \approx 1$ )  $p$ -mode pulsation, the frequencies are given by (Tassoul 1980)

$$\nu_{n,\ell} = \Delta\nu(n + \frac{1}{2}\ell + \epsilon) + \delta\nu, \quad (3.1)$$

where

$$\Delta\nu = [2\int_0^R c^{-1} dr]^{-1}$$

is the return-travel time for sound with speed  $c(r)$  from the surface to the centre and  $\epsilon$  is a small constant dependent on the structure of the star. The first term in eqn (3.1) indicates that frequencies of  $p$ -modes which are consecutive overtones of a given  $\ell$  will have a spacing  $\Delta\nu$ . Frequencies of modes with alternating even and odd  $\ell$  will have a spacing  $\frac{1}{2}\Delta\nu$ . To lowest order, eqn (3.1) is degenerate for modes  $(n,\ell)$  and  $(n-1,\ell+2)$  or  $(n+1,\ell-2)$ . The  $\delta\nu$  term lifts this degeneracy by introducing a small separation between the frequencies associated with these modes. The A-star models of Shibahashi & Saio (1985), Gabriel *et al.* (1985) and Heller & Kawaler (1988) indicate (Fig. 3.2) that  $\Delta\nu \approx 50$ - $60 \mu\text{Hz}$  for slightly evolved A stars of mass  $\approx 2M_\odot$ . The frequency spacing between the principal frequencies is not expected to be exactly uniform because of the  $\delta\nu$  term. Shibahashi & Saio (1985) have shown that there should be systematic inequalities:

$$(\nu_{n,1} - \nu_{n,0}) < (\nu_{n+1,0} - \nu_{n,1})$$

and

$$(\nu_{n,1} - \nu_{n-1,2}) > (\nu_{n,2} - \nu_{n,1}).$$

These inequalities can be used to constrain the mode identifications and they have been applied to the frequency spacings in HR 1217 (Kurtz *et al.* 1989) and to HD 60435 by Matthews, Kurtz & Wehlau (1987). Refer to these papers for further details.

It is worth reviewing the progress that has been made in the observational determination of  $\delta\nu$  and  $\Delta\nu$  in the rapidly oscillating Ap stars. Thus far, there are only two tentative determinations of  $\delta\nu$ ;  $\delta\nu = 2.6 \mu\text{Hz}$  in  $\alpha$  Cir (Kurtz & Balona 1984) and  $\delta\nu = 4 \mu\text{Hz}$  or  $\delta\nu = 8 \mu\text{Hz}$  in HD 203932<sup>7</sup> (Kurtz 1988a). These values are 3-7 times lower than Christensen-Dalsgaard's theoretical expectations. In HR 1217, there are six principal frequencies separated by about  $34 \mu\text{Hz} \approx 3 \text{ day}^{-1}$  which means that observations from a single observatory suffer from alias ambiguities. Moreover, since the principal

<sup>7</sup> We note that these values of  $\delta\nu$  are not confirmed with the latest photometry of HD 203932 (Martinez, Kurtz & Heller *in preparation*). Except for the principal frequency, we do not detect the other frequencies which were present in Kurtz's 1987 data and upon which the above determinations of  $\delta\nu$  are based. This suggests that some of the oscillation frequencies in HD 203932 are as short-lived as those in HD 60435.

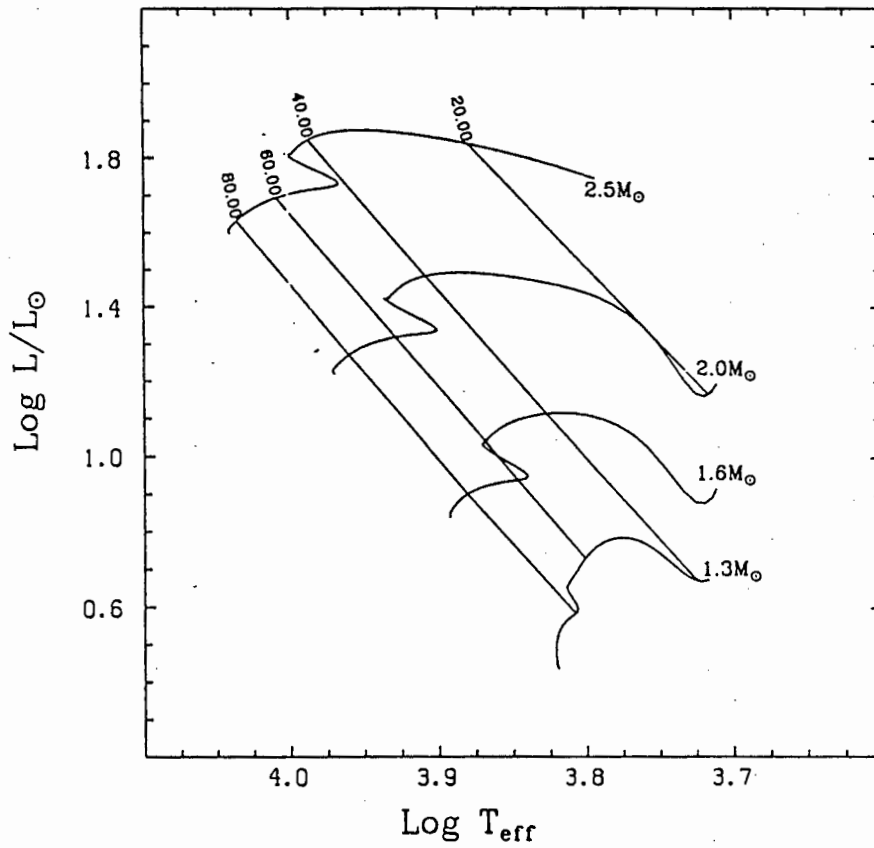


Fig. 3.2 Evolutionary tracks for the A-star models of Heller & Kawaler with  $X=0.73$  and  $Z=0.02$ . Lines of constant  $\Delta\nu$ , in  $\mu\text{Hz}$ , are shown (from Heller and Kawaler, 1988).

frequencies are rotationally split into  $(2\ell + 1)$  frequency multiplets, the true oscillation frequencies and their aliases can become hopelessly confused. The only way to overcome these problems is to acquire contemporaneous multi-site observations of HR 1217. In 1986, I participated in such an observing campaign which was coordinated by Prof. D.W. Kurtz in the last three months of that year. Fifteen observers participated at 9 different sites and acquired a total of 365 hr of high-speed photometry. Kurtz *et al.* (1989) confirmed that HR 1217 is an oblique pulsator which pulsates with 6 principal frequencies and they find evidence that these frequencies are not completely stable. They also find that the frequency spacing takes on alternating values of  $\Delta\nu \approx 33.3 \mu\text{Hz}$  and  $\Delta\nu \approx 34.7 \mu\text{Hz}$  for the first five frequencies. The correct interpretation of this frequency pattern is not clear: Kurtz *et al.* are unable to discriminate between the possibility that they are seeing alternating even and odd  $\ell$  modes with  $\Delta\nu = 68 \mu\text{Hz}$  and the possibility that they are seeing dipole modes spaced by  $\approx 34 \mu\text{Hz}$ .

HD 166473 pulsates with 3 frequencies, two of which are separated by  $\Delta\nu = 68 \mu\text{Hz}$  (Kurtz & Martinez 1987). This is consistent with theoretical expectations (Shibahashi & Saio 1985) for a cool A star near the main sequence. HD 60435 is a rapidly oscillating Ap star with an extremely complex amplitude spectrum. Multi-site observations are required because the frequency spacing of  $\Delta\nu = 26 \mu\text{Hz}$  is near that of the  $2 \text{ day}^{-1}$  aliases. Matthews, Kurtz & Wehlau (1987) discovered a rich  $p$ -mode spectrum in HD 60435 after 3 multi-site observing campaigns covering a total of 95 nights. They also found very strong indications of the transience of some of the oscillation frequencies in HD 60435. The spacing of  $26 \mu\text{Hz}$  is too small to correspond to the spacing of consecutive overtones of like degree. Thus Matthews *et al.* conclude that in HD 60435 we are actually seeing  $\frac{1}{2}\Delta\nu$ , the spacing between alternating even and odd  $\ell$  modes. This is similar to the situation that obtains in HR 1217. HD 203932 is another rapidly oscillating Ap star for which possible values of  $\frac{1}{2}\Delta\nu = 36 \mu\text{Hz}$  or  $\frac{1}{2}\Delta\nu = 25 \mu\text{Hz}$  have been discussed by Kurtz (1988a). Again, such values of  $\Delta\nu$  are reasonable for a cool A star just slightly off the ZAMS. Unfortunately, Kurtz's observations were beset by an alias ambiguity, so he organized a multi-site campaign in an attempt to secure definitive measurements of  $\Delta\nu$  and  $\delta\nu$  in this star. In the third quarter of 1988, D.W. Kurtz, C. Heller (Yale University) and I obtained 14 nights of high-speed photometric observations of HD 203932, half of which were acquired contemporaneously at SAAO and CTIO. An exciting result of this campaign is an indication in the multi-site data of at least 4 frequencies with a spacing  $\approx 33 \mu\text{Hz}$  (Martinez, Kurtz & Heller, in preparation). The first harmonic of the principal frequency has also been detected. There is an indication of further frequencies in the amplitude spectrum of this star, but the aliases are too severe for us to pursue the analysis any further. There is also strong evidence for the transience of some of

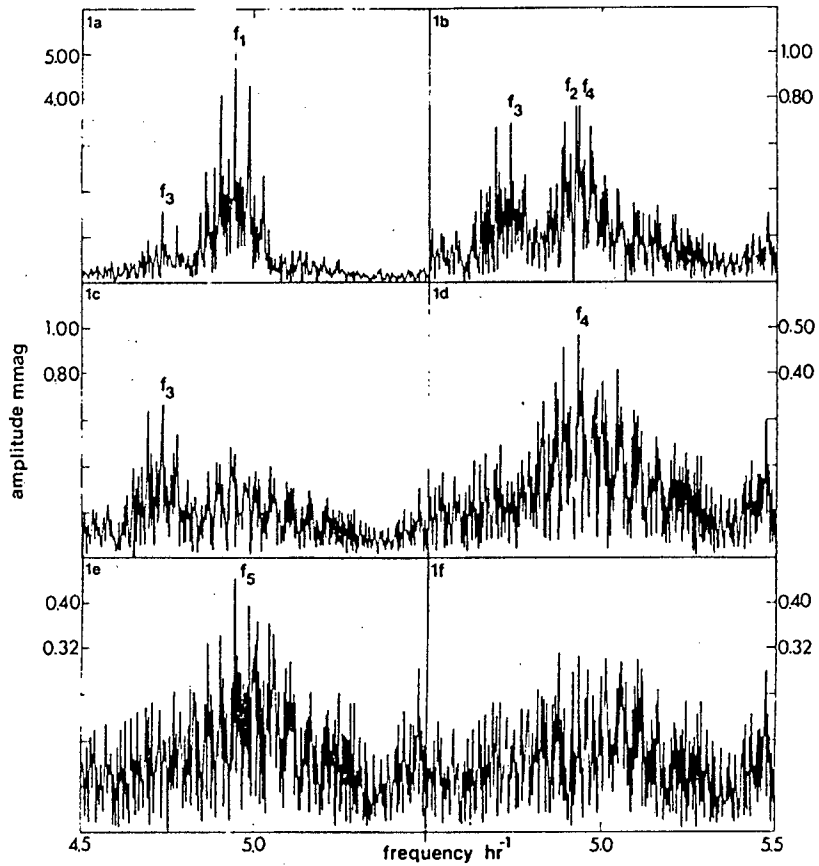
the oscillation frequencies since three of the frequencies discussed by Kurtz (1988a) do not appear in the latest data. Qualitatively, the oscillation spectrum of HD 203932 appears to be similar to that of HD 60435. In part II of this thesis we describe the application of similar ideas to HD 101065.

Heller & Kawaler (1988) have investigated evolutionary period changes in the rapidly oscillating Ap stars. They find that the asymptotic frequency spacings  $\Delta\nu$  depend on stellar mass, metallicity and age. For a given evolutionary stage, the spacing increases with decreasing mass and metallicity. They find that  $d(\ln \nu)/dt \approx 10^{-17}\text{s}^{-1}$  for a variety of A-star models with masses in the range  $1.6M_{\odot} < M < 2.0M_{\odot}$  prior to exhaustion of core hydrogen. After core hydrogen exhaustion, these stars evolve rapidly off the main sequence towards the red giant branch and  $d(\ln \nu)/dt$  increases to  $\approx 10^{-16}\text{s}^{-1}$ . A detection threshold of  $\approx 10^{-17}\text{s}^{-1}$  is attainable with about 10 years worth of observations.

One of the problems posed by the rapidly oscillating Ap stars is why so few modes are selectively excited when the Sun exhibits so many  $p$ -modes in its global low-degree ( $\ell < 3$ ) light variations. However, recent work on some of the rapidly oscillating Ap stars (e.g. HR 1217 (Kurtz *et al.* 1989), HD 60435 (Matthews *et al.* 1987)) suggests that these stars have far richer  $p$ -mode spectra than was previously suspected to be the case. This is reminiscent of the rich solar  $p$ -mode spectrum and augurs well for the future of asteroseismology.

#### 4 HD 101065 as a Rapidly Oscillating Ap Star

Rapid variability in HD 101065 was discovered by Kurtz (1978a) who also performed the first detailed frequency analyses of its oscillations (Kurtz 1980a) based on data from 4 nights in 1978 and 8 nights in 1979. The four frequencies (Fig. 4.1) which he tentatively identified are listed in Table 4.1. Fig. 4.1 suggests that while  $\nu_1$  and  $\nu_3$  are quite convincing,  $\nu_2$ ,  $\nu_4$  and  $\nu_5$  require confirmation. Kurtz extended his observations of HD 101065 in 1980 with a further 8 nights of high-speed photometry. He then used all of the 1978 to 1980 observations to determine the principal frequency  $\nu_1 = 1.372866 \pm 0.000006$  mHz. The uncertainty quoted for  $\nu_1$  is derived from the Gaussian half-width of the tallest peak in the frequency spectrum of the 1978-1980 observations. This is slightly larger than the half-width of the central peak in the spectral window of these data. Noise, unresolved secondary frequencies or a gradual frequency shift can account for such a difference. He also discovered evidence of variations in the amplitude and phase of  $\nu_1$  in the yearly data sets. Although the 1980 data confirmed that  $\nu_1$  and  $\nu_3$  do not form a complete frequency solution, Kurtz was unable to confirm his earlier tentative identifications of  $\nu_2$ ,  $\nu_4$  and  $\nu_5$ . Because of the clear need to extend the high-speed observations of this star, Kurtz observed HD 101065 again in 1981, 1984 and 1986, but he did not publish these data because the analyses did not yield consistent frequency solutions beyond  $\nu_1$



**Fig 4.1** The amplitude spectra of data subset C (Table 4.1) in Kurtz's 1979 data. The panels in this figure should be viewed left-to-right and top-to-bottom. The sequence of panels shows the residuals at successive stages of dewindowing. Note that the scale of the ordinates varies from panel to panel (from Kurtz 1980).

Dates	$n$	$f$ (hr <sup>-1</sup> )	$A$ (mmag)	$\phi$	Subset
1979		$\pm 0.19$ (0.06)			
March 8	1	4.94	5.66	2.652	A
	2	4.73	0.61	-1.477	
March 6, 7, 8		$\pm 0.027$ (0.009)			B
	1	4.943	5.81	-2.382	
	2	4.927	0.90	2.699	
March 6-19		$\pm 0.0047$ (0.0016)			C
	1	4.9424	5.42	2.276	
	2	4.9229	0.81	0.824	
	3	4.7343	0.65	0.113	
1978 + 1979 data		$\pm 0.00013$ (0.00004)			
	1	4.94232	5.52	-0.413	
	1	4.94218	5.48	1.201	

The components given in this table fit the equation

$$\Delta B = \sum_i A_i \cos(2\pi f_i t + \phi_i)$$

where  $t$  is the Heliocentric Julian Date - 2440000. Note that for each data subset the principal frequency is slightly different which results in very different phases for that frequency due to the long time base to the zero point of JD 2440000.

Table 4.1 Frequencies identified by Kurtz in the 1978-1980 high-speed observations of HD 101065.

From: Kurtz, D.W. *Mon. Not R. astr. Soc.*, **191**, 115, 1980.

The Phase Shift  $\Delta\phi(V, B-V)$  in degrees given by the Balona & Stobie (1979) theory for  $\psi = 90^\circ$ .

$f$	2	5	10	20	50
0	-45	-22	-11	-6	-2
2	39	28	17	9	4
4	167	149	122	85	40
6	174	166	152	127	77
8	177	172	164	149	110

Table 4.2

From: Kurtz, D.W., *Mon. Not R. astr. Soc.*, **191**, 115, 1980.

and  $\nu_3$ . We thus developed the suspicion that the frequency spectrum of HD 101065 was too complex for the observations at hand and that a secure frequency solution would emerge from the noise near  $\nu_1$  only after prolonged scrutiny. This strongly motivated us to embark on an extensive observational program in 1988.

Mode identifications were another unresolved issue. Kurtz (1980) measured a phase shift  $\Delta\phi(V, B-V) = 39^\circ \pm 3^\circ$  between the light and colour curves for  $\nu_1$ . He compared this with the values of  $\Delta\phi(V, B-V)$  as predicted by Balona & Stobie (1979) for various values of  $\ell$  and  $f$  (the ratio of flux-to-radius variation) assuming a phase lag of  $\psi = 90^\circ$  which is appropriate for adiabatic pulsation (Table 4.2). This simple comparison strongly suggests that  $\nu_1$  can be identified with  $\ell = 2$  (quadrupole) mode. Using this identification, Shibahashi & Saio (1985) argued on the basis of their A-star models that  $\nu_3$  could be identified as an  $\ell = 0$  mode. The problem with Kurtz's  $\ell = 2$  mode identification is that Kurtz & Balona (1984) showed that it is not possible to use  $\Delta\phi(V, B-V)$  in HR 3831 and  $\alpha$  Cir to get consistent results. Similarly, Kreidl & Kurtz (1986) discussed the phase shifts in HD 134214 and HD 6532 and concluded that the correct interpretation of  $\Delta\phi(V, B-V)$  in the rapidly oscillating Ap stars will have to await further theoretical developments. Thus it appears that the earlier  $\ell = 2$  mode identification was premature and the question of mode identifications in HD 101065 is still open.

The frequency spacing of  $\Delta\nu = 57 \mu\text{Hz}$  between  $\nu_1$  and  $\nu_3$  has also received attention. Assuming  $T_{\text{eff}} = 7400 \text{ K}$ , a spacing of  $\Delta\nu = 57 \mu\text{Hz}$  indicates that HD 101065 has slightly evolved off the main sequence with a mass  $1.6M_\odot \leq M \leq 2.0M_\odot$ . Gabriel *et al.* (1985) consider the possibility that what we are actually seeing a  $\frac{1}{2}\Delta\nu$  as in HR 1217. This makes  $\Delta\nu = 114 \mu\text{Hz}$  and places HD 101065 on the ZAMS with a mass  $\leq 1.5M_\odot$ .

The advantages of a complete frequency solution for HD 101065 are manifold. In the ideal case, the application of the oblique pulsator model to such a solution would provide information on the rotational inclination  $\iota$ , magnetic obliquity  $\beta$ , rotation period, constraints on the magnetic field strength variations and possible mode identifications. In addition to this, the techniques of asteroseismology can be applied to the spacings between the eigenfrequencies (i.e. the central components of the rotationally split frequency multiplets) to yield information on the mass, luminosity and age of HD 101065. With these objectives in mind, D.W. Kurtz and I embarked on a project to collect an extensive set of high-speed photometric observations well concentrated in time using the largest possible telescopes in order to reduce the scintillation noise. This project culminated in the paper *New observations and a frequency analysis of the extremely peculiar rapidly oscillating Ap star HD 101065* which is reproduced verbatim in Part II of this thesis.

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**Part II**

**New Observations and a Frequency Analysis of HD 101065**

**New observations and a frequency analysis of the extremely peculiar rapidly oscillating Ap star HD 101065.**

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**Summary.** We present 138 hr of new high-speed photometric observations of HD 101065 obtained with the 1-m telescope of the South African Astronomical Observatory. These observations reveal that HD 101065 pulsates with at least three frequencies near 1.37 mHz which cannot all be identified with consecutive overtones. These frequencies completely describe the oscillations down to the 0.40 mmag level. There is very strong evidence of many more frequencies below the 0.35 mmag level, but the complexity of the spectrum is such that we are unable to determine any of those frequencies securely. Definitive mode identifications and a straightforward application of the oblique pulsator model to HD 101065 are not yet possible. A tentative application of the techniques of asteroseismology indicates that HD 101065 is only slightly off the zero age main sequence and suggests that its radius, mass and luminosity are similar to those of the other Ap stars. There is also evidence which suggests that the second order quantity in the asymptotic relation for pulsation in high-overtone p-modes,  $\delta\nu$ , is 3-7 times lower in the rapidly oscillating Ap stars than current theoretical expectations. We find a significant secular variation in the principal frequency using two independent techniques and we argue that it cannot be interpreted as arising from evolutionary effects in HD 101065. We then examine the possibility that an unresolved frequency is involved and we show that a binary star model in which a low-mass companion orbits about HD 101065 is consistent with the available observations. We have also detected the presence of two secondary frequencies which describe the long-term amplitude modulation of the principal frequency. These frequencies indicate that the amplitude of the dominant frequency may not be completely stable. Finally, we propose that a multi-site observing campaign may offer the best hope of further deciphering the frequency spectrum of HD 101065.

## 1 Introduction

### 1.1 THE Ap STAR MODEL OF HD101065

Przybylski's star, HD 101065, is surely one of the most peculiar non-degenerate stars in the sky. Its exceedingly complex spectrum which is dominated by the lines of the rare earths (Przybylski 1961, 1963, 1966, Cowley *et al.* 1977) has fueled a long-standing controversy regarding its nature (Wegner & Petford 1974, Wegner 1976, Cowley *et al.* 1977, Przybylski 1977a,b; 1982, Wegner *et al.* 1983). Particularly contentious issues have been the effective temperature and the question of whether the iron peak elements are deficient. The absolute magnitude of HD 101065 is also uncertain. A determination of its trigonometric parallax by Churms (Kurtz & Wegner, 1979) of  $\pi = 0''.004 \pm 0''.006$  yields a  $3\sigma$  upper limit of  $\pi \leq 0''.022$ , which only constrains the absolute magnitude to  $M_V \leq 4.7$ . The low proper motion and radial velocity are suggestive of a position on the Population I main sequence. However, from the fact that it is an isolated field star, we infer that it is probably not on the zero age main sequence (ZAMS).

Since the discovery of: (1) qualitatively similar spectra in the known magnetic Ap stars HD 51418 (Jones *et al.* 1974) and HR 465 (Hartoog *et al.* 1973), (2) a -2200 gauss magnetic field by Wolff and Hagen (1976) and (3) rapid light variations by Kurtz (1978) in common with many other cool Ap stars (Kurtz 1982, 1986), HD 101065 has generally been supposed to be an extreme Ap star. Thus several observers have searched for long-period mean light<sup>\*</sup>, spectrum and magnetic variations. Such variations are observed in many Ap stars and occur on time-scales ranging from days to years. We will briefly review the evidence for each of these types of variation in HD 101065 in turn.

#### a) Spectral variations

A detailed line-by-line comparison of 1974 and 1975 coude plates by Cowley *et al.* (1977) suggests that if the spectrum varied substantially, it must have returned to the same phase in one year. Both Wolff & Hagen (1976) and Wegner & Petford (1974) remark that a comparison of their spectra with Warner's (1966) line list yields no outstanding differences. This, of course, does not immediately rule out spectral variations on longer, or shorter, time-scales. Indeed, HD 101065 was assigned a B5 classification in the Henry Draper Catalogue. Wolff and Hagen (1976) addressed this problem by examining a reproduction of the original low-resolution spectrogram. They conclude that HD 101065 was not a normal B5 star at that time and that the weakness of the K-line probably led to a misclassification.

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\* Throughout this paper we will use the phrase 'mean light variation' to refer to light variability occurring on a time-scale of the rotation in order to distinguish it from the rapid pulsational variability which occurs on a time-scale of minutes.

#### b) Magnetic field strength variations

Wolff & Hagen (1976) report no significant variations in the magnetic field strength from their measurements of three Zeeman spectrograms; one of the spectrograms was taken in April, 1974 and the other two in March, 1975.

#### c) Radial velocity variations

There is convincing evidence that HD 101065 has a constant radial velocity to within several  $\text{km s}^{-1}$ . The heliocentric radial velocities determined independently by Cowley *et al.* (1977), Wolff & Hagen (1976) and Wegner & Petford (1974) from the blue-region spectrograms are in very close agreement when one considers that they were obtained with different spectrographs and in different wavelength regions.

#### d) Mean light variations

Photometric mean light *uvby* observations by Heck *et al.* (1976), Renson *et al.* (1976) and Heck *et al.* (1987) appear to exclude systematic variations on a time-scale of a week. Przybylski (1977c) reported "possible small long-period brightness variations on the order of 0.02 to 0.03 magnitudes" in Johnson V. His observations span the period 1969 to 1977 and unfortunately there is no complementary set of spectral or magnetic observations.

### 1.2 RAPID LIGHT VARIATIONS IN HD 101065

We turn now to a more thorough discussion of the rapid light variations in HD 101065. Typical light curves in Johnson U and B are presented by Kurtz and Wegner (1979). The dominant oscillation,  $\nu_1$ , has a period of 12.14 min and an amplitude of  $\approx 5$  millimagnitudes (mmag). Kurtz (1980) was able to derive another frequency,  $\nu_3 = 1.315$  mHz (the reason for this appellation will become clear in Section 3), well removed from the frequencies near  $\nu_1 = 1.3729$  mHz in independent data sets, and its existence is regarded as secure. However, he noted that its alias pattern had a more pronounced  $-1$  day $^{-1}$  alias than  $+1$  day $^{-1}$  alias in all of the data sets that he chose to analyse. He discovered that the frequency perturbing the  $\nu_3$  alias pattern had an amplitude too low to be seen in the noise of the amplitude spectrum of the residuals when  $\nu_1$  and  $\nu_3$  were removed from the data. In addition, he later discovered (in the same data set) a fourth frequency,  $2\nu_1 = 2.746$  mHz, well removed from the frequencies near  $\nu_1$ . Within the errors, this is the first harmonic of  $\nu_1$ , which is why we denote it by  $2\nu_1$ .

Because of its good signal-to-noise, the principal frequency  $\nu_1$  has been studied in greater detail. Kurtz (1981) found  $\nu_1$  to be stable in frequency over the three-year time-span of his 1978-1980

data. This is a significant finding because the only known cause of frequency modulation in stellar pulsation is evolution and thus a measure of  $d\nu/dt$  is essentially a measure of the evolutionary rate of HD 101065\*. The principal frequency is also amplitude modulated on an unknown (long) time-scale. Kurtz (1981) could not determine whether the amplitude modulation arises from beating with further unresolved frequencies or from a real variation in the amplitude of the mode giving rise to  $\nu_1$ . There is another possible cause of such amplitude modulation which was not known to Kurtz at that time. This is the amplitude modulation which arises as a natural consequence of rotation within the context of the oblique pulsator model which he later developed.

That model was developed by Kurtz (1982) in order to account for the empirical properties of the rapidly oscillating Ap stars. It is a simple extension of the well-established oblique magnetic rotator model of magnetic Ap stars. An oblique pulsator is essentially a pulsating oblique magnetic rotator in which the pulsation axis is coincident with the magnetic axis which is itself inclined to the rotational axis. This model thus preserves our understanding of the long-term magnetic, spectral and mean light variations which arise from the rotation of the star, while at the same time also enabling rotation to account for the phase relationships and amplitude modulation observed in the rapidly oscillating Ap stars. The rapidly oscillating Ap stars have been recently reviewed by Kurtz (1986), Matthews (1986), Weiss (1986) and Shibahashi (1986). Kurtz, Matthews and Weiss emphasize the observations while Shibahashi reviews the theoretical developments in greater detail. Further enhancements to the oblique pulsator model are given by Dziembowski & Goode (1985,1986) and Kurtz & Shibahashi (1986)

By applying the Baade-Wesselink technique of Balona and Stobie (1979) to the phase shift,  $\Delta\phi(V,B-V) = \phi(V) - \phi(B-V)$ , between the light and colour curves of HD 101065 and HR 3831, Kurtz (1980) identified  $\nu_1$  with an  $\ell=2$  pulsation mode. The oblique pulsator model predicts that a frequency belonging to a pulsation mode of degree  $\ell$  is split into a multiplet of  $2\ell+1$  equally spaced components in the frequency spectrum. This multiplet is centred on the actual pulsation frequency. Thus an  $\ell=1$  mode will give rise to a triplet, an  $\ell=2$  mode will give rise to a quintuplet, and so on. Since there were no equally spaced multiplets in the amplitude spectrum of HD 101065, the oblique pulsator model could not be used for an independent verification of the  $\ell=2$  mode identification for  $\nu_1$  and to confirm the applicability of the technique of Balona and Stobie to the rapidly oscillating Ap

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\* Note that we distinguish between *stellar pulsation* frequencies and *light oscillation* frequencies. The latter can, of course, be modulated by processes which are not necessarily of an evolutionary nature. We will address this further in Section 3.6.

stars. In fact, Kurtz and Balona (1984) subsequently showed that it is not possible to use  $\Delta\phi(V,B-V)$  in HR 3831 and  $\alpha$  Cir and get consistent results. Similarly, Kreidl & Kurtz (1986) discussed the phase shift in HD 134214 and HD 6532 and concluded that the correct interpretation of  $\Delta\phi(V,B-V)$  in the rapidly oscillating Ap stars will have to await further theoretical developments. We now think that the earlier  $\ell=2$  mode identification by Kurtz (1980) for  $\nu_1$  was premature. This leaves the question of mode identifications in HD 101065 open.

Even when all securely determined frequencies were removed from the data, there was still evidence of an unresolved complex of further frequencies of very low amplitude ( $<0.5$  mmag) centered on  $\nu_1$ . Kurtz was unable to secure a complete frequency solution because any further frequencies which he determined depended on which particular subset of the data was analysed.

One of the problems posed by the rapidly oscillating Ap stars is why so few modes are selectively excited when the Sun exhibits so many p-modes in its global low degree ( $\ell < 3$ ) light variations. This dichotomy may be breaking down from the observational side, however. The results of recent continuous-coverage observing campaigns suggest that the amplitude spectra of HD 60435 (Matthews *et al.* 1987) and HR 1217 (Kurtz *et al.* 1989, in press) are qualitatively similar to the rich p-mode spectrum of the Sun.

In view of the above, it is not unreasonable to suppose that the same may hold for the frequency spectrum of HD 101065. If HD 101065 is indeed a rich multi-mode oblique pulsator, each frequency will be split into  $2\ell + 1$  components, each of which has its own alias pattern. For frequencies having low amplitudes, the alias patterns will be hopelessly confused in data sets covering short time-spans (because of the low resolution) or in data sets having large gaps which result in complex alias patterns. Indeed, one of us (DWK) collected further observations of HD 101065 in 1981, 1982 and 1984 (see Table 1 below), but did not publish them because the analyses did not yield consistent frequency solutions beyond the already known frequencies.

We thus had the suspicion that the frequency spectrum of HD 101065 appeared to be unsolvable only because it was too complex for the observations then at hand and that a complete frequency solution would emerge from the noise near  $\nu_1$  only after prolonged scrutiny. Consequently, we decided to collect a larger data set more concentrated in time using the largest possible telescopes in order to reduce the scintillation noise.

The advantages of a complete frequency solution for HD 101065 are manifold. In the ideal case, the application of the oblique pulsator model to such a solution would provide information on the rotational inclination  $i$ , magnetic obliquity (co-latitude)  $\beta$ , rotation period, constraints on the

magnetic field strength variations and possible mode identifications. In addition to this, the techniques of asteroseismology can be applied to the frequency spacings between the eigenfrequencies (*i.e.* the central components of the rotationally split frequency multiplets) to yield information on the mass, luminosity and age of HD 101065.

## 2 Observations

New high-speed photometric observations of HD 101065 were obtained on 33 nights between January and May 1988 at the Sutherland site of the South African Astronomical Observatory (SAAO). Where possible, we used the St. Andrews Photometer attached to the SAAO 1.0-m telescope in order to reduce the scintillation noise although several runs were collected with different SAAO telescope/photometer combinations. See Table 1 for details. All observations were made by using continuous 10-s integrations through a Johnson B filter with occasional (aperiodic) interruptions for sky measurements. Apertures of 30 arcsec, or larger, were used in order to minimize the effects of small tracking drifts.

Instrumental B magnitudes were produced by correcting the observations for mean extinction, sky background, coincidence-counting losses and (where necessary) long term drifts probably due to sky transparency variations. No transformation to the standard system was attempted since experience has shown that the results obtained from these different SAAO telescope/photometer combinations are interchangeable for our purposes without any further corrections. The time at the middle of each integration was converted to the Heliocentric Julian Date to an accuracy of  $10^{-5}$  day and the observations were averaged to produce 80-s integrations after we ascertained that there was nothing but noise in the frequency range  $6 < \nu < 12.5$  mHz; the highest known frequency in HD 101065,  $2\nu_1 = 2.74$  mHz, is well below the Nyquist frequency  $\nu_N = 6.25$  mHz for 80-s integrations.

Table 1 is a complete journal of all the high-speed photometric observations of HD 101065 obtained by us from 1978 to 1988. It lists the Heliocentric Julian date of the first observation on each night, the number of 80-s integrations obtained, the duration of the observations in hours, the standard deviation  $\sigma$  (in mmag) of one observation with respect to the mean for the night, the telescope/photometer combination and the observer. The standard deviation  $\sigma$  is a rough indicator of the quality of the night. It includes contributions from the photon noise, scintillation noise, residual sky transparency variations and, of course, the actual light variations in HD 101065 itself. Only the 1978-1980 observations have been published before by Kurtz (1980, 1981). We present here observations collected in 1981, 1982, 1984 and 1986 as well as the 1988 observations.

**Table 1.** Complete journal of high-speed B observations of HD 101065 obtained during the period 1978-1988.

JD 2440000+	$N_{80}$	t hr	$\sigma$ mmag	Tel m	Phot	Obs
1978						
3643.21902	162	4.13	4.6	0.5	PP	DWK
3644.21693	174	4.44	4.9	0.5	PP	DWK
3686.20772	96	2.68	3.7	1.0	StAP	DWK
3690.20751	100	2.74	4.8	0.5	PP	DWK
1979						
3939.29861	207	5.96	5.2	1.0	StAP	DWK
3940.29498	288	8.22	4.0	1.0	StAP	DWK
3941.29361	295	8.28	4.4	1.0	StAP	DWK
3944.31775	205	6.09	4.4	1.0	StAP	DWK
3946.27194	257	7.55	4.3	1.0	StAP	DWK
3948.49796	75	2.11	3.5	1.0	StAP	DWK
3949.26313	204	5.86	3.8	1.0	StAP	DWK
3952.49274	85	2.39	4.8	1.0	StAP	DWK
1980						
4331.37695	41	1.02	3.7	0.5	PP	DWK
4335.26987	37	0.93	3.6	0.5	PP	DWK
4336.28675	29	0.71	4.4	0.5	PP	DWK
4353.25635	53	1.34	3.8	1.0	PP	DWK
4354.26540	71	1.75	3.3	1.0	PP	DWK
4356.26812	36	0.89	4.5	1.0	PP	DWK
4357.22985	63	1.55	4.6	1.0	PP	DWK
4358.22190	68	1.66	3.8	1.0	PP	DWK
1981						
4692.45409	140	3.14	5.1	0.75	UCTP	AP
4693.42753	105	2.34	3.7	0.75	UCTP	AP
4724.35049	86	2.34	4.8	0.75	UCTP	MSC
4725.34983	103	2.27	3.9	0.75	UCTP	MSC
4730.32464	38	0.93	4.2	0.5	PP	DWK
4731.32150	23	0.56	5.1	0.5	PP	DWK
4734.30635	36	0.88	3.9	0.5	PP	DWK
4735.28399	36	0.88	5.3	0.5	PP	DWK
1984						
5725.58279	21	0.51	3.5	0.5	PP	DWK
5726.53483	65	1.50	5.1	0.5	PP	DWK
5728.57003	40	0.92	3.8	0.5	PP	DWK
5730.56397	44	0.97	4.4	0.5	PP	DWK
5731.54379	69	1.54	5.0	0.5	PP	DWK
5732.54622	67	1.49	3.6	0.5	PP	DWK
5733.53699	55	1.33	4.1	0.5	PP	DWK
5734.53822	76	1.69	4.5	0.5	PP	DWK
5735.52799	76	1.70	4.8	0.5	PP	DWK
5736.54862	70	1.56	3.2	0.5	PP	DWK
5759.33067	52	1.17	5.2	0.5	PP	DWK
5760.41329	232	5.24	4.4	0.5	PP	DWK
5762.29002	367	8.28	5.0	0.5	PP	DWK
5763.30212	354	7.97	4.1	0.5	PP	DWK
5764.30119	55	1.22	3.6	0.5	PP	DWK
5773.37914	251	6.30	4.0	1.9	RPP	DWK
5774.26374	371	9.07	4.8	1.9	RPP	DWK
5775.25894	329	7.96	4.4	1.9	RPP	DWK
5776.35156	274	6.82	4.9	1.9	RPP	DWK

5779.45030	114	2.78	3.6	1.9	RPP	DWK
5780.50414	39	0.89	3.4	0.5	PP	DWK
5783.51698	76	1.73	4.4	0.5	PP	DWK
5786.29234	233	5.35	5.2	0.5	PP	DWK
1986						
6576.20293	50	1.17	3.7	1.0	StAP	DWK
1988						
7166.48771	115	2.61	4.2	1.0	StAP	PM
7169.47300	123	2.89	3.7	1.0	StAP	PM
7170.47191	125	2.81	4.7	1.0	StAP	PM
7171.46423	140	3.24	5.1	1.0	StAP	PM
7172.45979	133	3.11	3.8	1.0	StAP	PM
7194.37950	259	5.84	4.8	1.0	StAP	DWK
7195.37528	259	5.86	3.9	1.0	StAP	DWK
7198.41495	74	1.65	4.0	1.0	StAP	DWK
7199.56664	61	1.36	4.6	1.0	StAP	DWK
7223.55346	92	2.08	4.6	1.0	StAP	PM
7224.31414	305	8.03	3.1	1.0	StAP	PM
7228.49002	43	0.95	3.8	1.0	StAP	PM
7243.27539	66	1.49	2.8	1.0	UCTP	PM
7244.26453	39	0.87	4.5	1.0	UCTP	PM
7246.25693	187	4.39	4.6	1.0	UCTP	PM
7250.27030	191	4.38	4.7	1.0	StAP	PM
7251.24508	319	8.30	3.4	1.0	StAP	PM
7252.26616	302	7.13	3.2	1.0	StAP	PM
7260.23992	85	2.59	3.5	1.0	StAP	PM
7261.35497	209	5.04	5.1	1.0	StAP	PM
7262.24200	267	5.98	4.7	1.0	StAP	PM
7263.22997	355	7.97	3.4	1.0	StAP	PM
7278.24112	114	3.25	4.0	1.0	StAP	DWK
7279.25394	278	6.23	2.9	1.0	StAP	DWK
7280.23216	297	6.66	3.8	1.0	StAP	DWK
7281.21651	303	7.03	4.2	1.0	StAP	DWK
7282.21799	298	6.93	4.1	1.0	StAP	DWK
7283.21646	306	6.90	2.7	1.0	StAP	DWK
7284.21504	306	6.92	3.9	1.0	StAP	DWK
7288.23529	183	4.38	4.5	1.0	UCTP	PM
7289.21431	228	5.77	4.4	1.0	UCTP	PM
7290.21532	56	1.77	3.1	1.0	UCTP	PM
7291.21552	79	3.90	3.2	1.0	UCTP	PM

#### Notes

- 1) StAP = St Andrews Photometer
- 2) UCTP = University of Cape Town Photometer
- 3) PP = Peoples Photometer
- 4) RPP = Radcliffe Peoples Photometer
- 5) AP = Anthony Putman
- 6) MSC = Mark Cropper
- 7) PM = Peter Martinez
- 8) DWK = Don Kurtz

### 3 Frequency Analysis

Frequency analyses of several combinations of the data were performed using Kurtz's (1985) faster implementation of Deeming's (1975) Discrete Fourier Transform (DFT) for unevenly spaced data. The computing time required to perform a DFT is determined by the number of observations, the resolution required and the frequency range of interest. For instance, the computing time required for some of the frequency spectra presented in this paper was  $\approx 17.4$  hr on a VAX 8550 and this is why we chose to work with 80-s integrations rather than the 10-s integrations used by Kurtz in his earlier studies.

Also, in order to minimise the number of DFTs to be computed, we used the technique of Gray and Desikachary (1973) in which the subtraction of frequencies is carried out in the frequency domain. We thus had only to compute one DFT of the data in the range of interest and the spectral window of the data in the same range. The spectral window is easily computed with the FORTRAN code given by either Deeming or Kurtz simply by setting all of the magnitudes of the input time-series to unity.

In order to facilitate the detection of multiple frequency components present in the data we employed a method which we will refer to as dewindowing. This involves selecting the tallest alias pattern in the spectrum and fitting the frequency of the central peak by least squares in order to optimize its amplitude and phase. The spectral window is then centred on the central peak of this alias pattern, scaled by the optimized amplitude and subtracted from the spectrum. We then examine the amplitude spectrum of the residuals and repeat the process until no further alias patterns can be convincingly identified above the level of the noise in the spectrum\*.

#### 3.1 THE NIGHTLY AMPLITUDE SPECTRA

The first step in our analysis was to generate amplitude spectra for all of the individual light curves out to the Nyquist frequency of  $\nu_N = 6.25$  mHz for 80-s integrations. In Figure 1 we present a few examples of such spectra. Spectra of such short data strings are necessarily of modest frequency resolution, but they do not suffer from the aliasing problems which arise when several nights of data are analysed together, as in later sections of this paper. They are also important for monitoring any night-to-night amplitude modulation, the consistency of the presence of the first and higher harmonics of the oscillation frequencies and also the presence of resolved transient frequencies (for which Matthews *et al.* (1987) found fairly strong evidence in HD 60435). The possible presence of

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\* The term 'prewhitening' is commonly used in the literature when referring to this process. However, we prefer the use of the more descriptive (and more accurate) term 'dewindowing'.

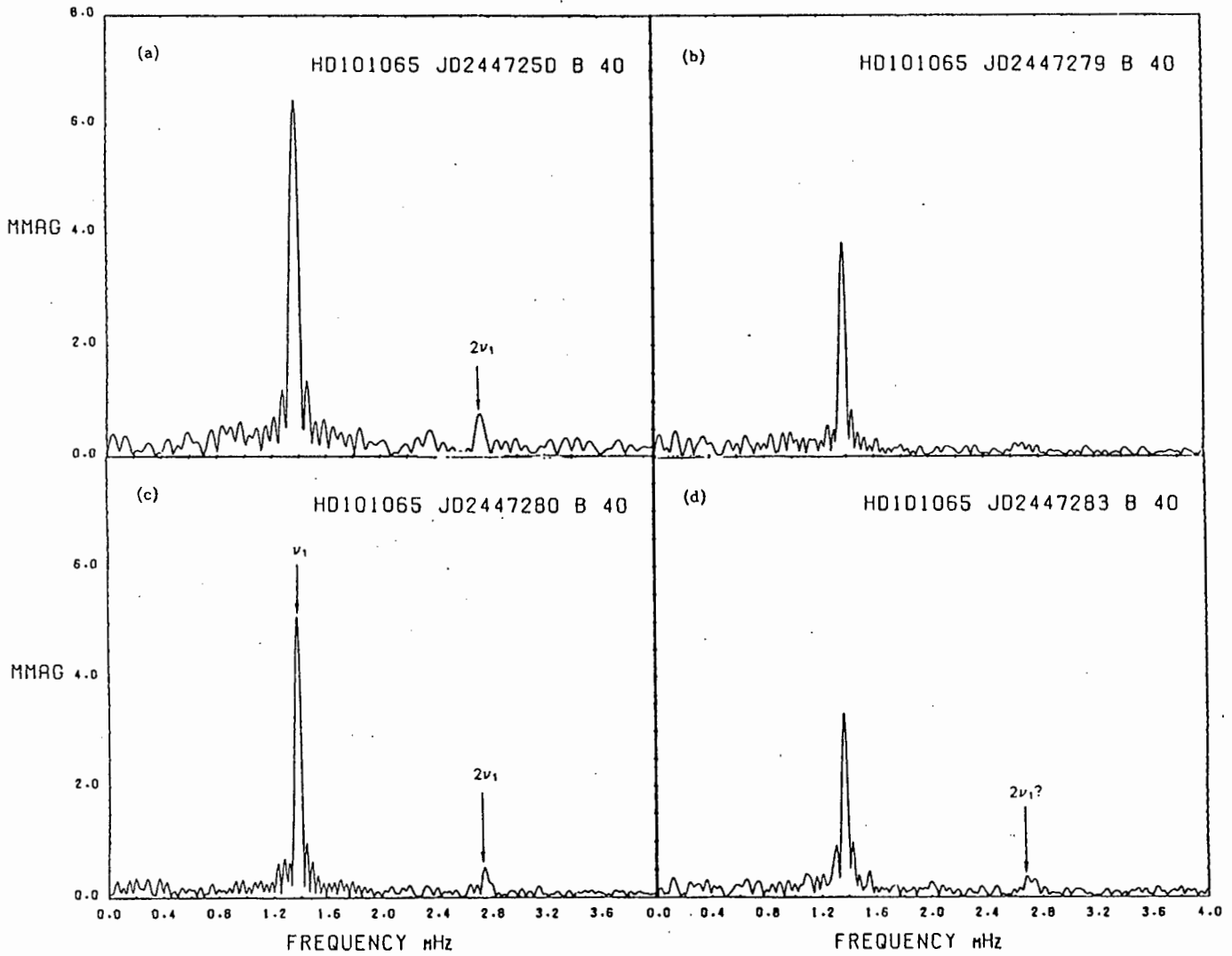


Figure 1. A selection of nightly amplitude spectra of high-speed Johnson *B* observations of HD 101065 acquired during 1988. Note the amplitude modulation of  $\nu_1$  from night to night as well as the presence of the first harmonic of  $\nu_1$  on several nights [e.g. panels (a), (c) and possibly (d)].

such transient frequencies in the oscillations of HD 101065 might also explain the lack of a complete solution to the frequency spectrum because such a solution would not exist.

The variations in frequency resolution and noise levels in Fig. 1 arise from the differing nightly coverage and differing observing conditions. Kurtz (1984) discusses the sources of noise in this type of photometry in greater detail and uses the older HD 101065 data to illustrate his discussion. The spectra in Fig. 1 were selected because they illustrate a few noteworthy features regarding the oscillations of HD 101065:

(i) The principal oscillation frequencies are confined to a small range around 1.37 mHz. The  $\nu_1 = 1.3729$  mHz oscillation discovered by Kurtz (1978) is still present in the most recent photometry.

(ii) The dominant oscillation frequency,  $\nu_1$ , is amplitude modulated on a time-scale of days. In the Introduction we mentioned several possible sources of such modulation and we will examine this modulation of  $\nu_1$  more closely in Section 3.2.

(iii) On some, but not all, nights there is an indication of significant amplitude at the first harmonic of  $\nu_1$  (e.g. panels a, c and (possibly) d in Fig. 1). We have not detected the second harmonic in any of the nightly amplitude spectra.

(iv) We have not convincingly detected the presence of transient frequencies sufficiently resolved from  $\nu_1$ . We will not speculate on the significance of this result here, but we will return to the issue of transient frequencies in later Sections of this paper.

(v) These spectra illustrate the general quality of photometric data obtainable at Sutherland on good, but not outstanding nights.

### 3.2 AMPLITUDE AND PHASE BEHAVIOUR

Our next step was to study the amplitude modulation and phase behaviour of the dominant oscillation frequency  $\nu_1$  in greater detail. Such a study, while formally equivalent to performing a straightforward Fourier analysis, is desirable because it can illustrate graphically the amplitude and phase behaviour of the oscillations with rotation in the oblique pulsator model. For instance, Kurtz and Shibahashi (1986) made use of the same sort of diagrams that we will employ in this section to illustrate graphically that HR 3831 must be an oblique pulsator pulsating primarily with an  $\ell=1$  (dipole) mode.

We began by fitting an accurately determined value of  $\nu_1 = 1.37286$  mHz (Kurtz 1986) to two-cycle segments of the data in order to determine optimum values of its amplitude and phase in each segment. For 80-s integrations, a continuous two-cycle segment of data should contain 18 integrations. There will, of course, always be a small number of segments with considerably fewer

than 18 points. These are the cases where the data are gapped and/or do not span two contiguous cycles. We conservatively rejected all such cases in which the least-squares fit was done for fewer than 12 observations. We thus produced 329 oscillation amplitude and oscillation phase points for the 1988 data and 388 such points for the 1978-1984 data which we shall hereafter refer to as the new and old modulation data sets respectively.

Figure 2 shows the amplitude  $A(t)$  of the oscillations in HD 101065 during the week JD 2447278 - JD 2447284 of the 1988 data. A four-day period is suggested in this diagram and similar results obtain for many other well-sampled weeks in both the old and new data sets. Note that panels b, c and d in Fig. 1 are nightly subsets of these data. This is also the same week that we will analyse in detail in Section 3.3.

We present the amplitude spectrum of all the 1988 modulation data in Fig. 3. For reasons of scale we present only the abscissae in the range  $0 < \nu < 1 \text{ day}^{-1}$ ; although we searched the frequency space of both the old and new modulation data sets out to their Nyquist frequencies of  $30 \text{ day}^{-1}$  we found no peaks taller than those shown in Fig. 3. We select the highest peak,  $\nu_m = 0.2540 \text{ day}^{-1}$ , as the modulation frequency. The other peak at  $\nu \approx 0.75 \text{ day}^{-1}$  is the  $+1 \text{ day}^{-1}$  alias of the negative counterpart of  $\nu_m$ .

One might argue that the detection of  $\nu_m$  is at best marginal because of the poor signal-to-noise ratio. Undoubtedly, a naïve application of Scargle's (1982) False Alarm criterion will indicate that  $\nu_m$  is almost certainly spurious. However, we have also Fourier analysed the old modulation data set and we are able to secure exactly the same value of  $\nu_m$  within the errors. We regard this to be independent confirmation of the reality of  $\nu_m$ . In this instance, such confirmation is preferable to an application of Scargle's criterion which is highly sensitive to the power signal-to-noise ratio. The problem in applying this test is to assess correctly the level of the noise (Kurtz & Marang 1987, Martinez 1989). This problem is exacerbated at very low frequencies by the complex spectral window and the spill-over from the mirror-image of the window pattern at the negative frequencies. When we dewindow  $\nu_m$  from the modulation data, we are left with a flat spectrum of the residuals in the range  $0 \leq \nu \leq 1 \text{ day}^{-1}$  as shown in the lower panel of Fig. 3. There is at least one other alias pattern at frequencies higher than  $1 \text{ day}^{-1}$ , but we do not pursue that pattern any further here as it is unlikely to be due to the rotation of HD 101065; we will discuss it in detail in the following sections.

The rotation period of HD 101065 is not known. If the four-day period is indeed the rotation period, HD 101065 should show magnetic field strength variations with the same periodicity. Unfortunately, because of the paucity of regular high-precision spectrum, magnetic and mean light

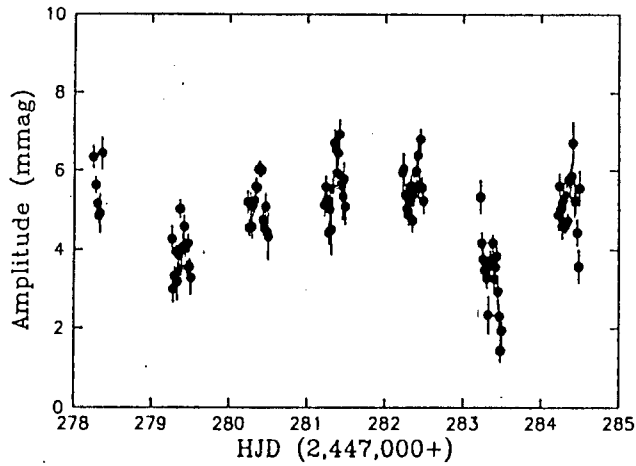


Figure 2. A plot of the oscillation amplitude  $A(t)$  of  $\nu_1$  for the data collected during the week JD 2447278-7284. Each point corresponds to a fit of  $\nu_1 = 1.37286$  mHz to a two-cycle segment of the data. A four-day modulation period is suggested.

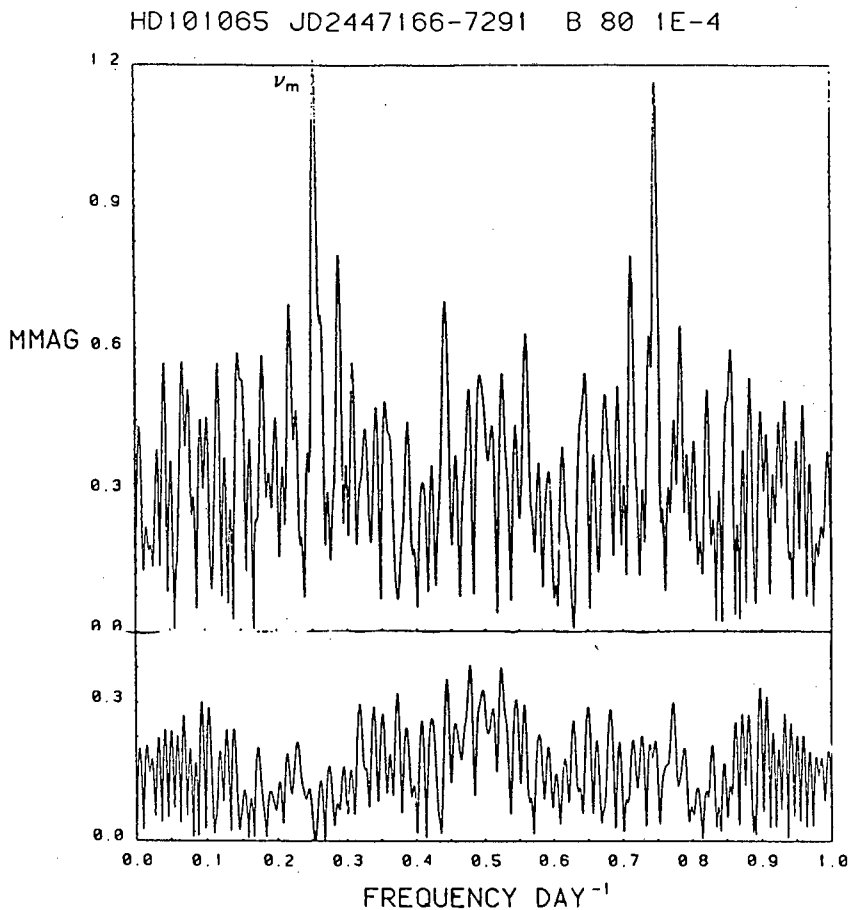


Figure 3. The upper panel shows the amplitude spectrum of the 1988 modulation data. The tallest peak in the upper panel,  $\nu_m = 0.2540 \text{ d}^{-1}$ , corresponds to the four-day modulation period seen in Fig. 2. The other peak at  $\nu \approx 0.75 \text{ d}^{-1}$  is the  $+1 \text{ d}$  alias of the negative counterpart of  $\nu_m$ . The spectrum in the lower panel results when the spectrum in the upper panel is dewindowed by  $\nu_m$ .

observations we cannot immediately exclude or confirm the possibility of a 4-day rotation period. For instance, Wolff and Hagen's (1976) three magnetic measurements are separated very nearly by multiples of four days. Fortunately, for systems with a favourable geometry, the  $2\ell + 1$  frequency splitting predicted by the oblique pulsator model can provide very good rotation periods for non-radial ( $\ell \geq 1$ ) modes. Unfortunately, as we will show in later sections of this analysis, this frequency splitting does not emerge from any of the data sets that we analyse.

Our next step was to examine the oscillation phase diagrams of the modulation data in greater detail. For a dominant dipole ( $\ell = 1$ ) oscillation we expect to see constant pulsation phase from magnetic (rotational) quadrature to quadrature. This is because the visible hemisphere is dominated by one of the pulsation poles. At quadrature, the observed oscillation phase should undergo a phase shift of  $\pi$  radians as the other pulsation pole comes into view. It should then remain constant between magnetic quadratures where it would then phase-reverse back by  $\pi$  radians as the first pulsation pole returns into view.

In Fig. 4 we present the observed oscillation phase  $\phi(t)$  for the same data as in Fig. 2. From this it can be seen that the above-mentioned pattern of phase shifts is clearly not occurring in HD 101065 on a 4-day time-scale. We have also searched the other modulation data and we find no evidence of phase reversals by  $\pi$  radians in any of the ten years of our observations. This does not mean that we exclude  $\ell = 1$  pulsation, however. All we are entitled to conclude is that if HD 101065 is pulsating in an  $\ell = 1$  mode, then we see only one pulsation pole. Such a situation arises if  $i + \beta < 90^\circ$  for HD 101065 and/or if its rotation period is several decades, or longer.

We turn now to a brief discussion of the long-term amplitude modulation noted by Kurtz (1981). We fitted  $\nu_1 = 1.372865$  mHz by least squares to the yearly data sets which have time-spans long enough to resolve all known oscillation frequencies in HD 101065. The results of the least-squares fits are shown in Table 2. The amplitudes of the 1978-1980 data are somewhat lower than those tabulated by Kurtz (1981) because he used 10-s integrations whereas we are working with 80-s integrations\*. Figure 5 comprises the beginnings of a long-term amplitude-modulation curve for HD 101065 plotted from the amplitudes listed in Table 2. We confirm with the 1981 to 1988 data sets that there are differences at the  $3\sigma$  level of confidence in the internal errors of the yearly data sets. However, the data are still too scanty to assign even a tentative period. Clearly several more years'

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\* It is easy to show that for an integration time  $\tau$ , the observed amplitude of a pure sinusoid of frequency  $\nu$  is reduced by a factor  $\text{sinc}(\nu\tau)$ .

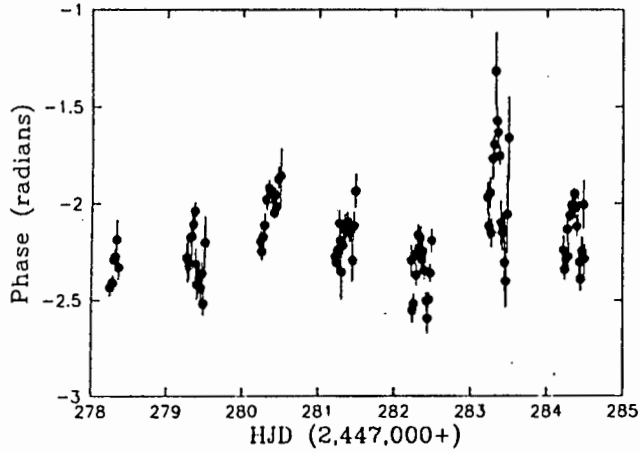


Figure 4. The pulsation phase behaviour,  $\phi(t)$ , of the  $\nu_m$  oscillation for the same data as in Fig. 2. The phases are computed relative to  $t_0 = \text{JD } 2447000.00$ . There is no evidence of phase reversals by  $\pi$  radians on a time-scale of four days. This suggests that the amplitude modulation seen in Fig. 2 is probably not rotational in origin. Similar results are obtained for the other modulation data.

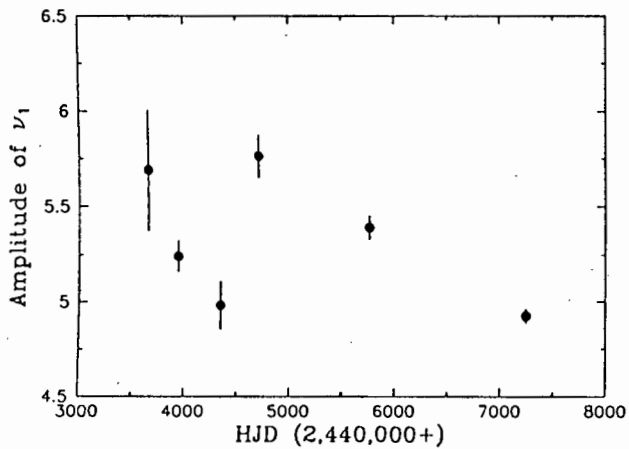


Figure 5. The long-term amplitude modulation behaviour of  $\nu_1$  plotted from the values given in Table 2. Each point is a fit of  $\nu_1 = 1.372865$  mHz to all of the data in a yearly dataset (see Table 1). The times plotted are the centroids of the observations in the yearly datasets.

Table 2. Least squares fits of  $\nu_1=1.372865$  mHz to each of the yearly data sets. The parameters in this and subsequent Tables fit the relation  $A(t)=A\cos(2\pi\nu(t-t_0)+\phi)$  where  $t_0=JD2446081.46$ .

Year	$N_{80}$	$A(\text{mmag})$	$\phi$
1978	532	$5.69\pm 0.13$	$2.108\pm 0.024$
1979	1616	$5.24\pm 0.08$	$2.232\pm 0.015$
1980	398	$4.98\pm 0.13$	$1.775\pm 0.025$
1981	567	$5.76\pm 0.11$	$0.988\pm 0.020$
1984	3330	$5.39\pm 0.06$	$0.922\pm 0.011$
1988	6197	$4.92\pm 0.03$	$1.865\pm 0.007$

Table 3. Least squares fit of frequencies secured in the weekly data set JD 2447278 - JD 2447284. The phases are relative to  $t_0=JD 2446081.46$

	$\nu$ (mHz) $\pm 0.00094$	$A(\text{mmag})$ $\pm 0.4$	$\phi$
$\nu_1$	1.37289	4.83	$-1.899\pm 0.009$
$\nu_2$	1.36963	0.90	$0.154\pm 0.047$
$\nu_3$	1.31507	0.62	$-2.395\pm 0.070$
$2\nu_1$	2.74567	0.22	$2.317\pm 0.194$

Table 4. Least squares fit of frequencies secured in the yearly data set JD 2447166 - JD 2447291. The phases are relative to  $t_0=JD 2446081.46$

	$\nu(\text{mHz})$ $\pm 0.000048$	$A(\text{mmag})$ $\pm 0.3$	$\phi$
$\nu_1$	1.372865	4.98	$2.640\pm 0.006$
$\nu_2$	1.369921	1.07	$-2.302\pm 0.028$
$\nu_3$	1.315034	0.66	$0.103\pm 0.046$
$2\nu_1$	2.745730	0.21	$-0.303\pm 0.140$

Table 5. Least squares fit of frequencies secured in the 10-year data set JD 2443643 - JD 2447291. The phases are relative to  $t_0=JD 2446081.46$

	$\nu(\text{mHz})$ $\pm 0.0000013$	$A(\text{mmag})$ $\pm 0.3$	$\phi$
$\nu_1$	1.3728660	4.50	$1.435\pm 0.006$
$\nu_{1-}$	1.3728630	1.98	$-0.992\pm 0.015$
$\nu_2$	1.3699260	0.90	$0.774\pm 0.029$
$\nu_{1+}$	1.3728685	1.06	$0.872\pm 0.027$
$\nu_3$	1.3150340	0.62	$0.062\pm 0.041$
$2\nu_1$	2.7457355	0.18	$2.525\pm 0.141$

worth of observations are required before we will be able to characterize this long-term variability reliably.

### 3.3 THE JD 2447278-7284 DATA

As the next step in our frequency analysis we combined the nightly light curves into weekly data sets. For weeks with above-average photometric conditions such data sets have superior resolution, lower noise levels, a high duty cycle and suffer 'only' from  $1 \text{ day}^{-1}$  aliasing problems. Another reason for this combination is that it allows us to verify the constancy, or otherwise, of the relative amplitudes of the different frequencies present in the data. In addition, if  $P_{\text{rot}}$  really is four days, such data sets have sufficient frequency resolution to reveal the rotational sidelobes predicted by the oblique pulsator model. There is a total of 8 such data sets in the 1988 (JD 2447166-7291) observations and a total of 11 such data sets in the earlier observations (JD 2443643 - 5786).

Frequency analyses of these data sets confirm that all oscillation frequencies occur in the region of  $\nu_1$  and  $2\nu_1$ . Since the results of these analyses are all similar, we present here in detail only one of these, JD 2447278-7284, with brief references to the other data sets where appropriate. This data set was acquired during the week of 1988 April 26 to May 2. It spans 168 hr during which a total of 42 hr of observations were obtained, giving a duty cycle of 25%. In terms of coverage, this is about as complete a data set as can be obtained from a one-week run at a single observatory.

Figure 6 shows an amplitude spectrum of these data in the range  $1.23 \leq \nu \leq 1.53 \text{ mHz}$  which is broad enough to show all of the oscillation frequencies near  $\nu_1$  as well as a short section of noise on both sides. The  $1 \text{ day}^{-1}$  and  $2 \text{ day}^{-1}$  aliases are particularly severe and respectively have amplitudes of 87.5% and 56.6% of the amplitude of the central peak in the spectral window. As in the other weekly data sets which we analysed, the first harmonic is present, but not the second.

The choice of  $\nu_1 = 1.37289 \pm 0.00094 \text{ mHz}$  is unambiguous. In this paper, the uncertainty that we quote for any given frequency is the Gaussian half-width of the central peak in the spectral window. In the absence of noise, frequency changes, or further unresolved frequencies, it is possible to determine the centroid of an alias pattern to a significantly higher precision than this. However, as we shall see, several factors complicate the analysis of the frequency spectrum of HD 101065 and we thus favour this more conservative estimate of the uncertainty. Dewindowing by  $\nu_1$  leaves us with the spectrum shown in Fig. 7 which shows the presence of at least two further frequencies,  $\nu_2 = 1.36963 \text{ mHz}$  and  $\nu_3 = 1.31507 \text{ mHz}$ . Although our choice of  $\nu_2 = 1.36963 \text{ mHz}$  instead of its  $+1 \text{ day}^{-1}$  alias is not motivated by the Figure, we are guided in this by our analyses of the other data

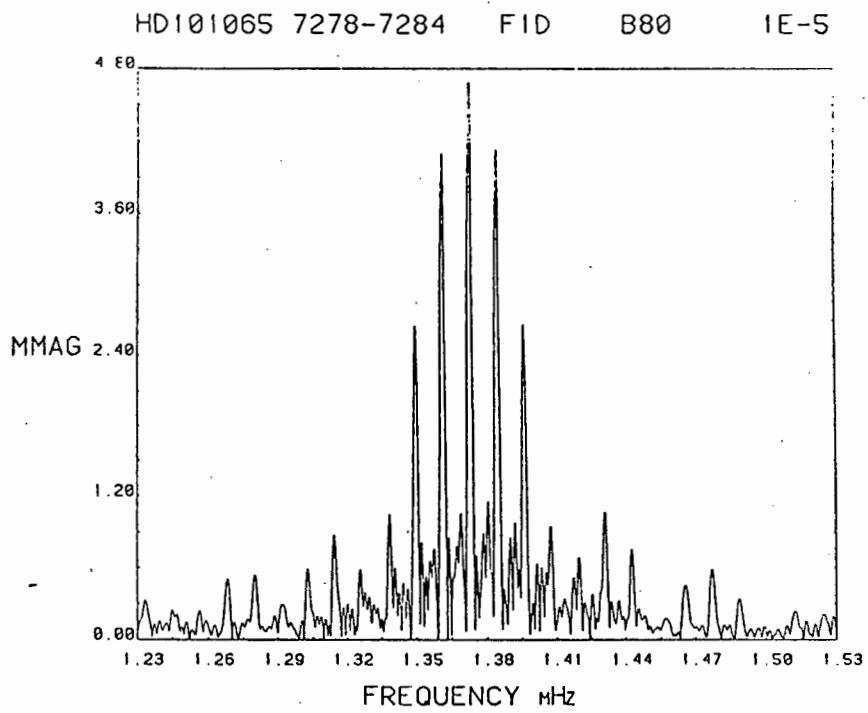


Figure 6. An amplitude spectrum of the high-speed *B* observations of HD 101065 during the week JD 2447278-7284. These are the same data as those depicted in Figs 2 and 4 and the choice of  $\nu_1 = 1.37289$  mHz is unambiguous.

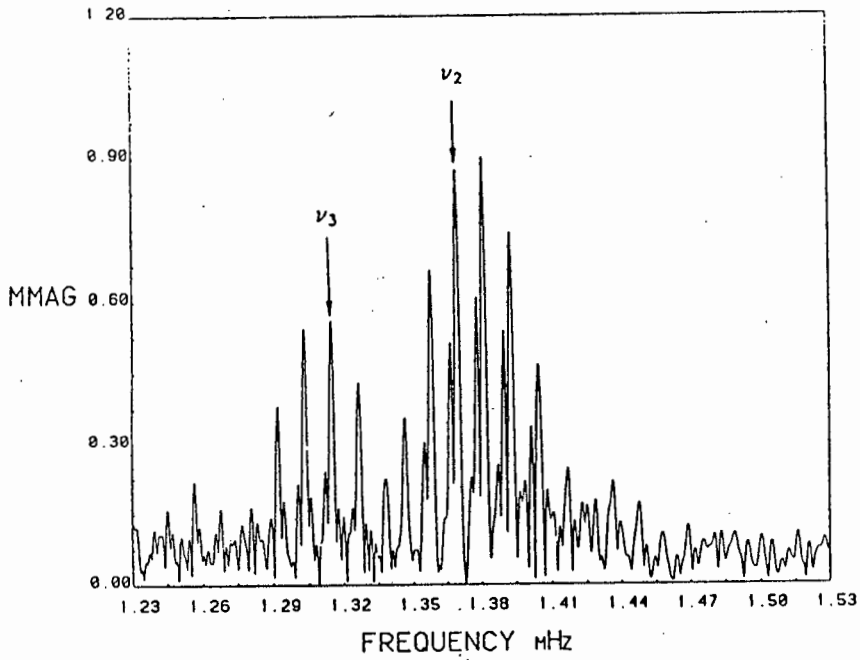


Figure 7. The amplitude spectrum which results when  $\nu_1 = 1.37289$  mHz is dewindowed from the spectrum in Fig. 6. At least two further frequencies ( $\nu_2$  and  $\nu_3$ ) are present in the oscillations. Note the difference from Fig. 6 in the scale of the ordinates.

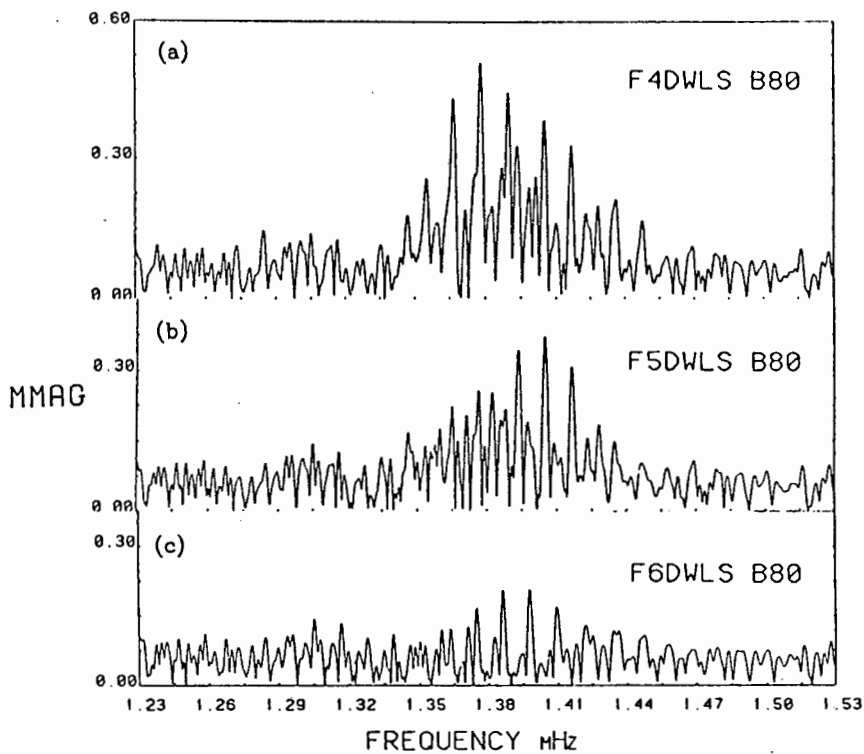


Figure 8. The amplitude spectrum of the residuals on dewindowing by  $\nu_1$ ,  $\nu_2$  and  $\nu_3$  (a). There is residual amplitude well above the level of the noise and we pursue the analysis to find  $\nu_4 = 1.37796$  mHz (a),  $\nu_5 = 1.40489$  mHz (b) and  $\nu_6 = 1.37648$  mHz (c). However, these values are not obtained in the other datasets and hence we do not regard them as secure.

sets. In the remainder of this paper when referring to these three frequencies ( $\nu_1$ ,  $\nu_2$  and  $\nu_3$ ) we shall use the expedient abbreviation  $\nu_{123}$ .

We immediately note that the frequency separation  $\nu_1 - \nu_2 = 3.26 \mu\text{Hz}$  can be identified with the modulation frequency  $\nu_m$  which we discussed in Section 3.2, in which case  $\nu_2$  is possibly a rotational sidelobe of  $\nu_1$ . This means that there are at least two ( $\nu_1$  and  $\nu_3$ ) but probably three ( $\nu_{123}$ ) eigenfrequencies in the light variations of HD 101065. The reason why we favour three or more independent frequencies will become clear with the following remarks concerning the possible 4-day rotation period.

(i) All modes with  $\ell > 1$  should exhibit  $(2\ell + 1)$ -fold rotational splitting. No rotational sidelobes about  $\nu_3$  are evident. This is possibly an indication that  $\nu_3$  is a radial ( $\ell = 0$ ) mode. Radial modes are spherically symmetric and hence show no amplitude modulation with aspect. If, however,  $\nu_3$  is a non-radial mode then we may either exclude a rotation period short enough to be resolved in seven days or conclude that both rotational sidelobes have amplitudes too low to be seen above the level of the noise.

(ii) We find no evidence of a rotational sidelobe at  $\nu = \nu_1 + (\frac{1}{4} \text{ day}^{-1})$ . Again, we may argue that this sidelobe is lost in the noise and complexity of the alias patterns. To test this we dewindowed  $\nu_2$  and  $\nu_3$  to arrive at the amplitude spectrum presented in the top panel of Fig. 8. There is no evidence for the presence of rotational sidelobes about  $\nu_1$  or  $\nu_3$  in this data set.

(iii) The same results obtain for all combinations of data that we analysed, including the yearly, and longer, data sets.

(iv) Attempts to search for the 'missing' sidelobes by force-fitting them to the data by least-squares methods were also unsuccessful. In so doing, we considered the model of Dziembowski and Goode (1985, 1986; Kurtz & Shibahashi 1986; Shibahashi 1986) which examines the relative importance of rotational splitting to magnetic splitting in a rotating oblique pulsator. They find that for an  $\ell = 1$  mode, as the ratio of magnetic to rotational splitting increases, the rotation sidelobe structure becomes increasingly asymmetric until one of the sidelobes may even exceed the height of the central peak. We thus tried force-fitting the frequency triplets  $(\nu_2, \nu_1, \nu_1 + \frac{1}{4} \text{ day}^{-1})$  (Fig. 9a),  $(\nu_2, \nu_2 + 1/2(\frac{1}{4} \text{ day}^{-1}), \nu_1)$  (Fig. 9b) and  $(\nu_2 - \frac{1}{4} \text{ day}^{-1}, \nu_2, \nu_1)$  (Fig. 9c). In all three cases the amplitude of the conjectured sidelobe was no higher than the local level of the noise. The simplest interpretation of these results consistent with the other observations of HD 101065 is that the data argue against a 4-day rotation period.

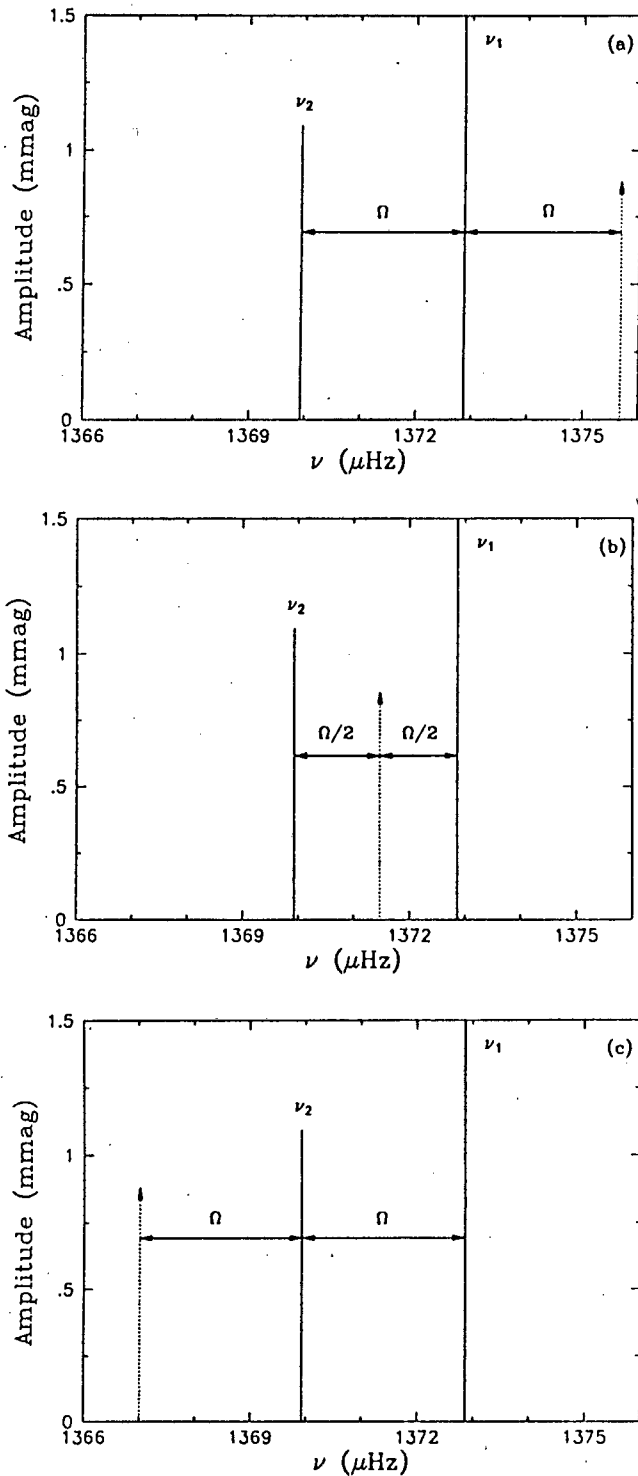


Figure 9. A schematic view of the frequency spectrum in the region of  $\nu_1$  and  $\nu_2$ . The panels show the three possible rotation/sideloobe structures that we are testing for. Each spectral line in these diagrams represents an observed frequency component of frequency  $\nu$  and amplitude  $A$ ; the  $\nu_1$  line runs off the top of the plots up to a height of 5 mmag. In each of the panels the dotted line represents the frequency that we are force-fitting to the data. The frequency spacing  $\Omega = 0.2540 \text{ d}^{-1}$ .

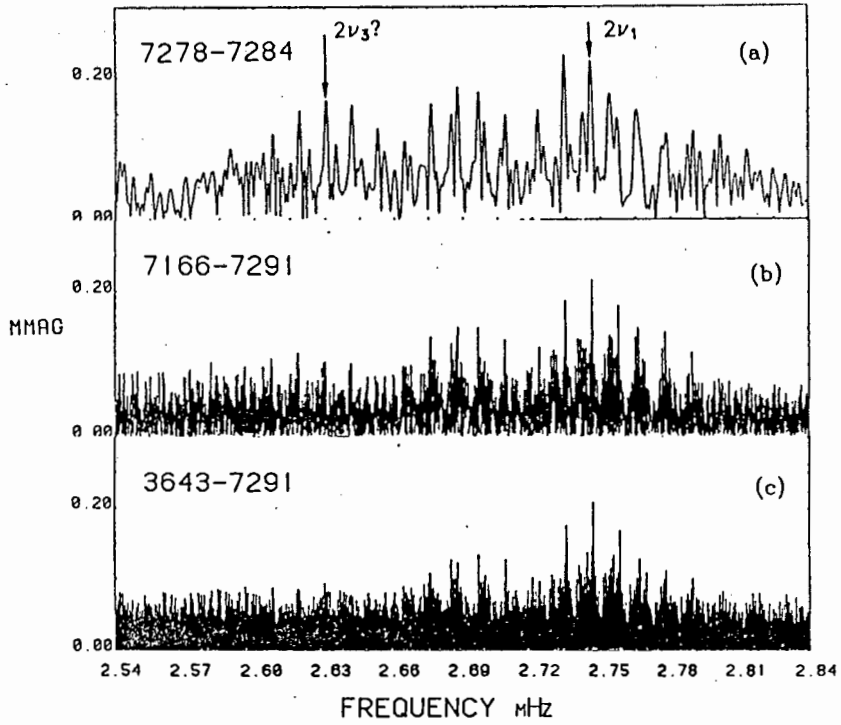


Figure 10. Frequency spectra in the region of the first harmonic for three of the datasets analysed in this paper. Note that  $2\nu_1$  shows up clearly in all three panels. The top panel contains an indication of  $2\nu_3$ , but this harmonic does not appear in the other two panels. The feature centred roughly on  $\nu \approx 2.687$  MHz shows up in all three panels. It does not correspond on a 2:1 basis to any secure frequencies near  $\nu_1$ .

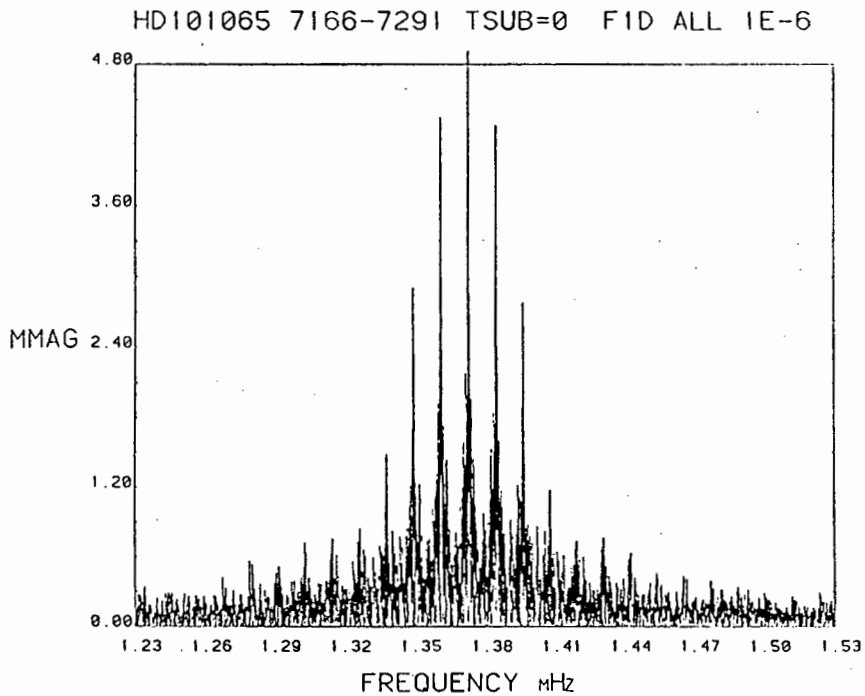


Figure 11. The amplitude spectrum of all the 1988 high-speed  $B$  observations of HD 101065. We find  $\nu_1 = 1.372865$  MHz in excellent agreement with earlier observations.

rapidly oscillating Ap stars (Kurtz 1986; Kurtz 1988b). However, in Fig. 10 (top panel) there is an indication of 2 other alias patterns. The alias pattern marked ' $2\nu_3$ ?' lies, within the errors, where we would expect to find the first harmonic of  $\nu_3$ . The other alias pattern centred roughly on  $\nu \approx 2.687$  mHz is not in a 2:1 correspondence with any of the peaks near  $\nu_1$  determined for this data set. These features have such a low signal-to-noise that we require their appearance in independent data sets before regarding them as secure. However, they certainly deserve some attention because, if they are real, these frequencies pose a difficulty with regard to the critical frequency,  $\nu_{\text{crit}}$ , which we will touch on in Section 4.

### 3.4 THE JD 2447166-7291 DATA

The lack of a complete frequency solution motivated the intense observational program reported in this paper. During the 1988 observing season we acquired as many new data as we already had obtained for the period 1978-1986. These new data span 125 days, during which 137.71 hr of observations were obtained. Given the realities of the weather and telescope scheduling, this is about as complete a data set as is likely to be obtained from a single site during a single observing season. However, as we shall see, it also is inadequate to solve the frequency spectrum of HD 101065.

Figure 11 shows the amplitude spectrum of the entire 1988 data set. This spectrum was sampled at 300,000 frequencies with a resolution of 1 nHz. The spectral window of these data has substantial  $1 \text{ day}^{-1}$  and  $2 \text{ day}^{-1}$  aliases with amplitudes of 83% and 55% of the central peak respectively. The higher order aliases are all significantly lower in amplitude than these. In this spectrum,  $\nu_1 = 1.372865$  mHz is easily determined although, as in the weekly data set JD 2447278-7284, there is an alias problem in determining  $\nu_2$  (Fig. 12). Again we select  $\nu_2 = 1.369921$  mHz, which in this case is only marginally higher than its  $+1 \text{ day}^{-1}$  alias, and dewindow this  $\nu_2$ . The choice of  $\nu_3 = 1.315034$  mHz is then unambiguous (Fig. 13, top panel). The amplitudes and phases of the secured frequencies for this data set are given in Table 4. One perversity of this data set is that the sense of asymmetry of the  $\nu_3$  alias pattern is reversed; here it is the  $+1 \text{ day}^{-1}$  alias that is higher than the  $-1 \text{ day}^{-1}$  alias. Upon dewindowing  $\nu_3$ , we are left with the spectrum shown in the lowest panel of Fig. 13.

Again, there is significant amplitude above the level of the noise. We conservatively estimate the level of the noise to be 0.13 mmag while the tallest peaks in the lower panel of Fig. 13 have amplitudes  $\approx 0.35$  mmag. We can, of course, lower the height of this residual mound of amplitude by selecting further frequencies, but these are unique to this particular data set and thus we do not quote them. While the 1988 data set is clearly as complete as is likely to be obtained in one season from a

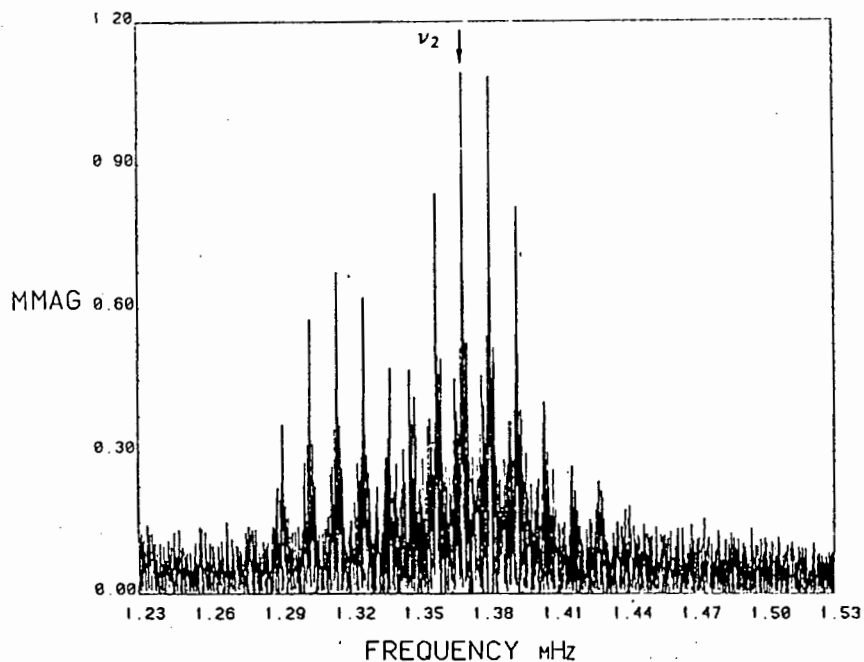


Figure 12. The spectrum which results when  $\nu_1 = 1.372865$  mHz is dewindowed from Fig. 11. Our particular choice of  $\nu_2 = 1.369921$  mHz is motivated by the analyses of several other datasets. Again, note the change in the scale of the ordinates.

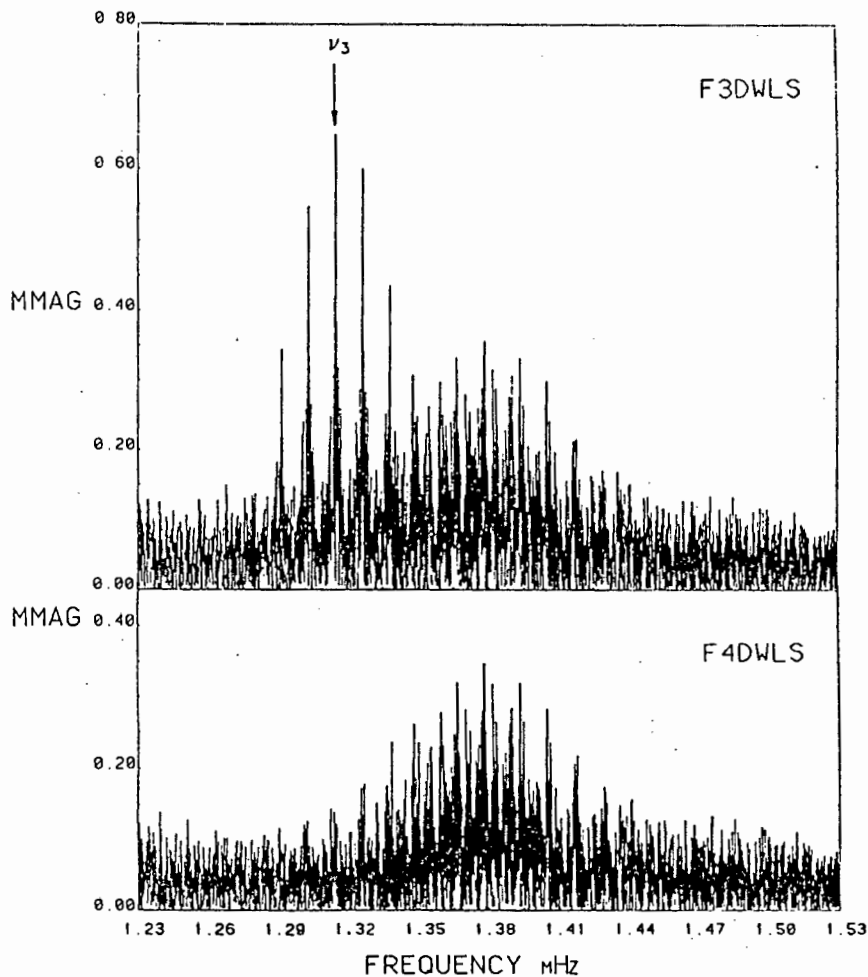


Figure 13. The choice of  $\nu_3 = 1.315034$  mHz is unambiguous once we have dewindowed by  $\nu_1$  and  $\nu_2$ . The lower panel shows the residuals of dewindowing by  $\nu_3$ . There is residual amplitude at the 0.35 mmag level, but further pursuit of the frequency analysis is essentially fruitless as far as establishing secure frequencies.

single observatory, it is inadequate to unravel the oscillation spectrum of HD 101065 below the level of 0.4 mmag. Perhaps only contemporaneous multi-site observations will permit further decoding of this frequency spectrum.

As far as the harmonics of  $\nu_1$  are concerned, we find the first harmonic, but not the second. Dwindowing the first harmonic leaves nothing but noise in the residuals in the frequency range of  $2\nu_1$ . There is no evidence for the presence of the first harmonics of  $\nu_2$  and  $\nu_3$ , but this is not surprising given the significantly lower amplitudes of  $\nu_2$  and  $\nu_3$ . There is enhanced amplitude at  $\nu \approx 2.687$  mHz in common with the JD 2447278-7284 data set (Fig. 10, middle panel). We do not regard this as evidence of the reality of this pattern, however, because the JD 2447278-7284 data set is the best-sampled week in the 1988 observations (JD 2447166-7291) and its frequency spectrum at the first harmonic possibly dominates that of the 1988 observations.

### 3.5 THE JD 2443643-7291 DATA

We then performed a frequency analysis of all of the data listed in Table 1. These data span 3648 days and comprise 12690 80-s integrations. We searched 600,000 frequencies in the range  $1.23 < \nu < 1.53$  mHz at a resolution of 0.5 nHz. The purpose of this analysis was to refine the accuracy of the frequencies already determined and to search for the secondary frequencies which must be present in order to describe the long-term amplitude modulation discussed in Section 3.2. We were only partially successful in both instances since in this data set we had to contend with fairly severe daily, weekly, monthly and yearly aliases which give rise to an extremely complex spectral window.

By the process of dwindowing we secured  $\nu_{123}$  and  $2\nu_1$  as given in Table 5\*. Again, we found enhanced amplitude in the region of the first harmonic at  $\nu \approx 2.687$  mHz (Fig. 10, bottom panel). We also discovered two new frequencies giving a closely spaced ( $\approx 3$  nHz) frequency triplet centred on  $\nu_1$ . We denote the low-frequency component of this triplet as  $\nu_{1-}$  and the high-frequency component as  $\nu_{1+}$ . We shall also refer to these frequencies as the secondary frequencies. These were not discovered in previous analyses since they were not resolved in those shorter data sets. They would only have affected the relative amplitudes of  $\nu_{123}$ . When  $\nu_3$  is dwindowed, it appears as though there may be another such triplet centred on  $\nu_3$ , but the alias patterns are too severe for us to be certain of this. However, there is certainly residual amplitude in the region of  $\nu_3$ , something not noted in the other data sets. This may explain why the relative amplitudes of  $\nu_2$  and  $\nu_3$  also vary on long time-scales.

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\* The choice of  $\nu_2 = 1.3699260$  mHz is unambiguous for this data set.

The spacing of the triplet centred on  $\nu_1$  corresponds to a beat period of 10.6 yr, close to the 10 yr time-span of the data and we caution that these frequencies are *not* resolved in terms of the  $1.5/T$  criterion of Loumos & Deeming (1978) where  $T$  is the time-span of the data set. Thus we suspect that the secondary frequencies may be spaced by  $\approx 3$  nHz from  $\nu_1$  only because that is the characteristic resolution of the entire data set. That is,  $\nu_{1-}$  and  $\nu_{1+}$  may only be the DFT's way of modelling what little it sees of the long-term amplitude modulation rather than real oscillation modes in the star, rotational sidelobes or even some form of hyperfine structure in the spectrum of  $\nu_1$ . If this supposition is correct, the positions of the secondary frequencies will depend upon the particular data set under consideration. We have examined the shorter data set JD 2443643-5786 and find no evidence of any secondary frequencies analogous to  $\nu_{1+}$  and  $\nu_{1-}$ , but the proper way to test this supposition is to analyse a longer data set at higher resolution.

### 3.6 THE SECULAR EVOLUTION OF THE PRINCIPAL FREQUENCY

In the foregoing analyses we implicitly assumed that the effects of any secular frequency variations are negligible. This is a valid assumption for all but the longest data sets extending over several years. The residual mound of amplitude can be interpreted in two ways. It either arises because of further unresolved frequencies or because of secular changes of frequency. If the latter case holds, we ought to be able to measure  $dP_1/dt$ , the secular rate of change of the principal (728 s) oscillation. It is instructive to estimate the smallest value of  $dP_1/dt$  measurable with the JD2443643-7291 data set. The uncertainty in the phase of  $\nu_1$ ,  $\Delta\phi = 0.006$  (Table 5), allows us to estimate the uncertainty in (O-C) from  $\Delta(O-C) \approx \Delta\phi/2\pi\nu \approx 0.7$  s, whence (Heller & Kawaler, 1988), for a data set spanning  $T = 10$  yr,

$$dP_1/dt \approx 2\Delta(O-C)/\nu_1 T^2 \approx 1.0 \times 10^{-14} \text{ s s}^{-1}.$$

Thus, the data are sensitive enough to permit a comparison of the measured  $dP_1/dt$  with the evolutionary models of Heller & Kawaler (1988), which show that  $dP/dt$  should be measurable over a few years if HD 101065 is a post-main sequence object.

We thus computed  $d\nu_1/dt$  for the data set JD 2443643-7291 by introducing a  $d\nu/dt$  term into our FORTRAN non-linear least-squares fitting routine. This enabled us to fit the form

$$\Delta B = \sum_i A_i \cos(2\pi(t-t_0)(\nu_i + \frac{1}{2}(t-t_0)d\nu_i/dt) + \phi_i)$$

for  $\nu_i = \nu_{123}$  to these data. We found

$$[dP_1/dt]_{(nls)} = (-8.48 \pm 0.09) \times 10^{-12} \text{ s s}^{-1},$$

where we note that, because of the incompleteness of the solution, the formal error quoted is probably an underestimate of the true error. We also fitted a parabola to the (O-C) diagram of the

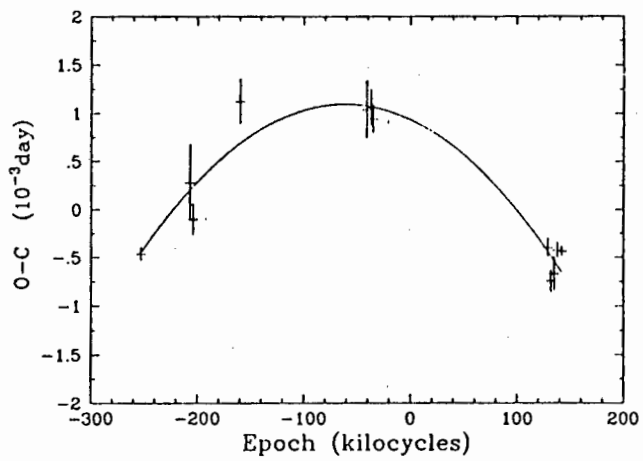


Figure 14. The (O-C) diagram of the 728-s period for the JD 2443643-7291 dataset.

same data set (Fig. 14). In order to obtain a meaningful phase for the 728 s period in the complicating presence of the other nearby periods, we worked with (weekly) subsets which are long enough to resolve  $\nu_{123}$ . As in the earlier sections, the phases were computed relative to  $t_0 = \text{JD}2446081.46000$ .

The best-fit parabola of the form  $(O-C) = B_0 + B_1E + B_2E^2$  is given by

$$B_0 = (0.937 \pm 0.118) \times 10^{-3} \text{ day},$$

$$B_1 = (-5.187 \pm 0.566) \times 10^{-9} \text{ day},$$

$$B_2 = (-4.20 \pm 0.46) \times 10^{-14} \text{ day}.$$

Since  $dP/dt = 2B_2/P$ , we find

$$[dP_1/dt]_{(O-C)} = (-9.97 \pm 1.10) \times 10^{-12} \text{ s s}^{-1}.$$

We suspect that the somewhat higher value of  $dP/dt$  from the  $(O-C)$  residuals arises because of the complicating presence of  $\nu_{1-}$  and  $\nu_{1+}$  which are obviously not resolved in the weekly data sets used to determine the phase of  $\nu_1$ . These values of  $dP_1/dt$  cannot be easily interpreted in terms of the models of Heller & Kawaler (1988). Compare their Fig. 2 with our values of  $[d(\ln\nu_1)/dt]_{(nls)} = 1.165 \pm 0.013 \times 10^{-14} \text{ s}^{-1}$  and  $[d(\ln\nu_1)/dt]_{(O-C)} = 1.37 \pm 0.15 \times 10^{-14} \text{ s}^{-1}$ . Not only are these values roughly a factor of 10 greater than the highest values produced by Heller & Kawaler's models, but they also indicate that  $\nu_1$  is *increasing*, whereas we would expect it to be *decreasing* as a result of evolutionary frequency changes prior to core hydrogen exhaustion unless, for some unknown reason, HD 101065 happens to be in a stage of contraction. The rate of frequency change for post-main sequence evolution is much greater and  $[d(\ln\nu_1)/dt]$  may be positive for some evolutionary stages. Heller & Kawaler remark that for higher values of  $T_{\text{eff}}$ , their models produce greater absolute values for  $[d(\ln\nu)/dt]$ . Models with suitably adjusted parameters might bring the expected and observed values of  $[d(\ln\nu)/dt]$  into better agreement; further model calculations are required. However, given the rough factor of 10 difference between these values at present, it seems unlikely that an upward revision of  $T_{\text{eff}}$  alone will do the job; the required upward revision of  $T_{\text{eff}}$  is probably too great.

Fortunately, we are not restricted to evolutionary processes in order to explain our values of  $dP_1/dt$ . Indeed, *any* suitable unresolved frequency may be invoked, but it is difficult to attribute such a frequency to a pulsation mode or its associated fine structure since the required separation from  $\nu_1$  is too small. An exciting possibility is that HD 101065 has an unseen low-mass companion, in which case the  $(O-C)$  diagram would follow a sinusoid\* rather than a parabola since the orbital

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\* Here we assume a circular orbit for simplicity.

motion of HD 101065 would produce a Doppler shift of the pulsation frequencies which varies sinusoidally with the orbital period. We thus attempted to model the observed variation in ( $O-C$ ) by fitting a sinusoid of the form  $(O-C) = A_0 + A_1 \cos(2\pi t/p + \phi)$  to the data by least squares. We obtain a best fit for  $p = 9.78$  yr and  $A_1 = 82.8$  s = 0.166 AU. Here  $A_1 = a_1 \sin i$  is the projection of the radius of the orbit along the line of sight, where  $i$  is the angle between the plane of the sky and the orbital plane. It was earlier stated (Section 1.1(c)) that HD 101065 is known to have a constant radial velocity to within several  $\text{km s}^{-1}$ . The range of radial velocity variations in a binary is given by the standard relation

$$K_1 = 2\pi a_1 \sin i / p(1-e^2)^{1/2}$$

where all of the symbols have their usual meanings. Taking  $e = 0$  for simplicity and substituting our values of  $a_1 \sin i$  and  $p$ , it may be readily verified that the expected radial velocity variations do not exceed several  $\text{km s}^{-1}$ .

We now attempt to find a crude estimate of the mass of the conjectured companion. We begin by expressing the orbital relations  $(M_1 + M_2)p^2 = a^3$ ,  $a = a_1 + a_2$  and  $a_1 M_1 = a_2 M_2$  in the form

$$p^2/a_1^3 = (1 + M_1/M_2)^3/(M_1 + M_2)$$

whence, for  $M_2 \ll M_1$ ,

$$(M_1^{1/3} p^{2/3}/a_1) - 1 \approx M_1/M_2.$$

In practice, we find our approximation  $M_2 \ll M_1$  to be valid for all but the smallest values of  $\sin i$ . Fortunately, this relation is not strongly dependent on the mass of the primary (HD 101065), so we select  $M_1 \approx 1.8M_\odot$ , a mass typical of the late A stars. We also substitute  $p = 9.78$  yr and  $A_1 = a_1 \sin i = 0.166$  AU to find

$$M_1/M_2 \approx 34 \sin i - 1.$$

This implies that for a wide range of orbital inclinations a secondary of mass  $M \leq 0.1M_\odot$  will suffice to explain the ( $O-C$ ) results and will be consistent with the observed lack of radial velocity variations larger than several  $\text{km s}^{-1}$ . The observational confirmation of this model is straightforward; the ( $O-C$ ) values must rise within the next few years if they are following a sinusoid.

#### 4 Discussion

In Section 3 we showed that there are at least three stable frequencies in the region of  $\nu_1 = 1.37$  mHz in the light variations of HD 101065. In addition, the existence of the first harmonic of  $\nu_1$  in this star is now well established. However, when the data are dewindowed by  $\nu_{123}$  there is always considerable residual amplitude in the region of  $\nu_1$ . Further frequencies can always be selected, but these will vary from data set to data set and so we do not regard them as secure. We are thus unable to provide a

complete frequency solution for the light variations of HD 101065. This means that the interpretation of  $\nu_{123}$  must be attempted with considerable caution.

The three frequencies  $\nu_{123}$  are not equally spaced and hence cannot be straightforwardly interpreted in terms of the oblique pulsator model wherein a frequency triplet describes rotational amplitude modulation of an  $\ell=1$  mode. In Section 3.3 we argued that there is no convincing evidence that  $\nu_2$  is a rotational sidelobe of  $\nu_1$ . Moreover, the 57.8  $\mu\text{Hz}$  spacing between  $\nu_1$  and  $\nu_3$  cannot be interpreted as rotational splitting of an  $\ell=1$  mode in an oblique pulsator as this would imply an impossibly short (for an Ap star) rotation period of 4.8 hr. In short, the biggest obstacle to a meaningful application of the oblique pulsator model is the lack of a rotation period in the (admittedly few) magnetic, spectral, mean light and rapid oscillation data. But this is at least consistent with Wegner's (1979) remark that the sharpness of the spectral lines indicates that  $v \sin i \leq 7 \text{ km s}^{-1}$ .

The asymptotic relation for high-overtone p-modes is (Tassoul 1980)

$$\nu_{n,\ell} = \Delta\nu (n + \ell/2 + \epsilon) + \delta\nu \quad (1)$$

where  $n$  is the radial overtone,  $\ell$  is the spherical degree, and  $\epsilon$  is a constant of order unity which characterizes the outer layers of the envelope. To lowest order, eq. (1) is degenerate for modes  $(n,\ell)$  and  $(n-1,\ell+2)$  or  $(n+1,\ell-2)$ . The  $\delta\nu$  term lifts this degeneracy by introducing a small separation between the frequencies associated with these modes. The quantity  $\Delta\nu$  is directly related to the return sound travel time from the surface to the centre

$$\Delta\nu = [2 \int_0^R c^{-1} dr]^{-1} \quad (2)$$

( $c$  being the sound speed) and is largely a measure of the mean density (Ulrich 1986).

According to equation (1), at the crudest level, the data will contain only an indication of  $\Delta\nu = \nu_{n+1,\ell} - \nu_{n,\ell}$ , the frequency spacing between consecutive overtones of like degree, or  $\frac{1}{2}\Delta\nu = \nu_{n,\ell+1} - \nu_{n,\ell}$ , the spacing between eigenfrequencies of modes with  $\ell$  differing by 1 with identical radial overtones. The problem is to decide whether the observed frequency spacing corresponds to  $\Delta\nu$  or  $\frac{1}{2}\Delta\nu$ . The spacing of 57.8  $\mu\text{Hz}$  between  $\nu_1$  and  $\nu_3$  is in good agreement with the computed values of  $\Delta\nu$  for a variety of A star models. Applying the models of Shibahashi & Saio (1985), Gabriel *et al.* (1985) and Heller & Kawaler (1988) we find that  $\Delta\nu = 57.8 \mu\text{Hz}$  is consistent with a cool A star slightly evolved off the ZAMS. Assuming  $T_{\text{eff}} = 7400\text{K}$ , these models suggest that HD 101065 is about 0.9 magnitudes above the ZAMS and has  $R \approx 2.1R_{\odot}$  with a mass  $1.6M_{\odot} < M < 2.0M_{\odot}$ . These results are consistent with what is generally known about the masses and radii of the other Ap stars. A more definitive estimate of the luminosity of HD 101065 will have to

await an accurate parallax, perhaps from the HIPPARCOS satellite. On the assumption that HD 101065 has  $T_{\text{eff}}=7400\text{K}$  and is about a magnitude above the main sequence, its apparent magnitude of 8.0 indicates an expected parallax of  $\pi \approx 0''.0066$ , well within the reach of HIPPARCOS.

Gabriel *et al.* (1985) have suggested that  $\nu_1$  and  $\nu_3$  might be associated with values of  $\ell$  differing by 1 (as in HD 60435 and HR 1217), in which case  $\Delta\nu = 116 \mu\text{Hz}$ . Using the same  $T_{\text{eff}}$ , these models place HD 101065 on the ZAMS with a mass  $1.3M_{\odot} < M < 1.5M_{\odot}$ . However, we feel that this case is unlikely since HD 101065 is an isolated field star; it is unusual to find an isolated ZAMS star like this.

The assumption that  $\nu_1$  and  $\nu_3$  are consecutive overtones of the same degree,  $\ell$ , allows us to use eq. (1) with  $\epsilon \approx 1.0$  (Christensen-Dalsgaard 1984) to compute rough values for the radial overtones. Such calculations yield  $n \approx 20 - 25$ . These are obviously not definitive values of the radial overtones; they are merely an indication that these fairly high overtones are consistent with the models we are using. Also, especially in the case of HD 101065, it is reassuring to see that similar values of  $n$  obtain for the other rapidly oscillating Ap stars.

The spacing of  $\nu_1 - \nu_2 = 2.94 \mu\text{Hz}$  is too small to be associated with modes of alternating even and odd  $\ell$ ; such modes are expected to have a spacing of  $\frac{1}{2}\Delta\nu \approx 29 \mu\text{Hz}$  (Shibahashi & Saio 1985). We have also argued that  $\nu_2$  is unlikely to be a rotational sidelobe of  $\nu_1$ . We thus consider the possibility that  $\nu_1$  and  $\nu_2$  are associated with eigenmodes of  $(n, \ell)$  and  $(n-1, \ell+2)$ . This implies that  $\delta\nu = 2.94 \mu\text{Hz}$  in HD 101065, whereas Christensen-Dalsgaard's (1987) models predict values 3 - 7 times higher than this. A similar situation obtains for  $\alpha$  Cir which may have  $\delta\nu = 2.6 \mu\text{Hz}$  (Kurtz & Balona 1984) and HD 203932 (Kurtz 1988a). It appears that  $\delta\nu$  may be significantly smaller in the rapidly oscillating Ap stars than current models predict but the strong magnetic field is an additional factor not yet included in these models.

It was argued in the Introduction that the  $\ell=2$  mode identification by Kurtz (1980) was probably premature. The only general constraint that we can place on the observed modes is that they must probably be of low degree ( $\ell \leq 3$ ) in order to be visible in the integrated light of the stellar disk. The low value of  $v \sin i$ , the lack of pulsation phase reversals, the lack of magnetic, spectral and mean light variations as well as the lack of rotational sidelobes all point toward a pole-on orientation for HD 101065 with  $i + \beta < 90^\circ$ , unless HD 101065 happens to have a rotation period of many decades. Definitive mode identifications will have to await further observational developments.

The critical frequency  $\nu_{\text{crit}}$  is the frequency above which no standing oscillation can occur in a star. It arises because, at sufficiently high frequencies, the finite scale heights begin to be

comparable to the wavelength of the oscillation and the maintenance of phase coherence on reflection (and hence of a standing wave) becomes impossible. Shibahashi and Saio (1985) have computed  $\nu_{crit}$  for various A star models. Applying their models to our value of  $\Delta\nu = 57.8 \mu\text{Hz}$  we find  $1.7 \text{ mHz} \leq \nu_{crit} \leq 2.0 \text{ mHz}$  for HD 101065. The only secure frequency above  $\nu_{crit}$  is  $2\nu_1 = 2.7457 \text{ mHz}$ , the first harmonic of  $\nu_1$ . It is still not known whether this harmonic is an independent pulsation mode or whether it indicates nonlinearity in  $\nu_1$ .

Thus far, only the harmonics of the dominant oscillation in several rapidly oscillating Ap stars have been convincingly detected. This is not surprising when one considers that the amplitudes of the other frequencies in their spectra are substantially lower than that of the dominant frequency. Our marginal detection of  $2\nu_3$  in the JD 2447278-7284 data needs to be confirmed in an independent data set. Also, the occurrence at the first harmonic of the same spectral features in the JD 2447278-7284, JD 2447166-7291 and JD 2447166-7291 data sets indicates that these structures deserve further attention. A more detailed study of the frequency spectrum in the region of  $2\nu_1$  would have to be done with larger telescopes in order to minimize the scintillation noise. The problem of then having too many photons can be solved by reducing the operating voltage of the photomultiplier tube or introducing suitable neutral-density or narrow-band filters. It is our experience with the other rapidly oscillating Ap stars that, of the standard filters, the Strömgren  $\nu$  filter yields the highest signal-to-noise ratio. A study conducted through a Strömgren  $\nu$  filter might reveal the presence of the first harmonics of  $\nu_2$  and  $\nu_3$ , if they exist. In some cases, there is a significant phase-shift with filter in the rapidly oscillating Ap stars (Kurtz & Balona 1984, Weiss 1986, Watson 1988). A wideband filter such as Johnson B may be contributing phase-smearing to the amplitude spectrum of the residuals in Fig. 13. If so, then Strömgren  $\nu$  observations may reduce the phase-smearing component sufficiently to bring out further frequency components.

The residual mound of amplitude in the lower panel of Fig. 13 illustrates graphically that even the intense observational effort in 1988 was insufficient to decipher the frequency spectrum of HD 101065. This is an unexpected result and its interpretation is not yet clear. Perhaps there is a rich complex of low-amplitude oscillations beating against each other to produce the amplitude spectra observed. Perhaps variable, or even stochastic (as in the Sun), driving play a role. Also, recall that in Section 3.2, we demonstrated that no transient modes with observational amplitudes arise at frequencies sufficiently resolved from  $\nu_1$  in a single night's observations. This, of course, does not exclude the possibility of such modes existing very close to  $\nu_1$ . Of course, the resolution with which such possible transient frequencies may be measured will be governed by their life-spans, rather than

by the time-span of the observations. The width of the central peak of the alias pattern of such a transient frequency would yield a rough measure of its life-span. Moreover, the analysis of separate frames of data will permit variations in phase and amplitude to be followed. If there really are transient frequencies in the rapidly oscillating Ap stars, their growth and decay times as well as their lifetimes will be of great relevance to the search for the driving and excitation mechanisms, which are not yet known. The  $1 \text{ day}^{-1}$  alias asymmetries of  $\nu_2$  and  $\nu_3$  are another unsolved problem. The only way to investigate these issues further is to conduct a continuous-coverage observing campaign on HD 101065, probably of some two weeks in duration.

The significant secular frequency variation  $d\nu_1/dt$  deserves further investigation. We argued that it is unlikely to be a measure of evolutionary effects in HD 101065. It thus seemed probable that an unresolved frequency is involved and we showed that a low-mass companion orbiting about HD 101065 could account for the (O-C) observations whilst not conflicting with the observed lack of radial velocity variations. Companions in the  $0.05M_{\odot}$ - $0.08M_{\odot}$  range are especially interesting because this is in the mass range of the brown dwarfs. A rapidly oscillating Ap star in a binary system with a brown dwarf would be a very rich prize indeed. Unfortunately, the prospects of detecting the conjectured companion photometrically by measuring any excess infrared flux above that expected for an F0V star are very poor. We stress that this is an insecure model and we will not speculate further on the nature of the secondary. An observing campaign such as the one proposed above, if conducted within the next several years, could also reveal whether the (O-C)'s lie on a parabola or on a sinusoid as required by the binary star hypothesis. It appears that HD 101065 is poised to remain an enigmatic object for quite some time yet.

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## **Part III**

### **Summary and Future Work**

## Summary and Future Work

### RAPIDLY OSCILLATING $A_p$ STARS

The  $A_p$  stars form part of a sequence of magnetic stars from B2 to F2. These stars have anomalously strong lines of Mn, Si, Sr, Cr, Eu and/or the other rare earths. It is thought that the anomalous line intensities arise from patches or rings of anomalous composition above the level in which the continuum forms in these stars. The existence of the surface patches of anomalous composition is widely ascribed to the action of diffusion processes in the atmospheres of these stars. The distribution of the patches is governed by the magnetic field structure. Since diffusion processes can lead to chemically differentiated atmospheres only in the absence of turbulence or convection, the existence of pulsating chemically peculiar stars is not yet fully understood.

The rapidly oscillating  $A_p$  stars are cool magnetic  $A_p$  stars with SrCrEu line-strength anomalies. They are multi-mode non-radial, high-overtone ( $n \gg \ell \approx 1$ )  $p$ -mode pulsators with periods in the range 4 to 15 minutes and amplitudes of a few mmag. Some of the rapidly oscillating  $A_p$  stars are known oblique rotators and in explaining the empirical properties of these stars, Kurtz has extended the well-established oblique rotator model by proposing that in a rapidly oscillating  $A_p$  star the magnetic and pulsation axes coincide. This is known as the *oblique pulsator model*. In principle, this model allows one to constrain the rotational inclination  $i$  and the magnetic obliquity  $\beta$  as well as providing information on the period of rotation and the internal magnetic field strength.

The rapidly oscillating  $A_p$  stars are the first main sequence stars other than the Sun in which rapid oscillations have been indisputably identified. Initially it was thought that they differ from the Sun in that only a few  $p$ -modes are excited. After much work, it has become apparent that at least some of them have rich  $p$ -mode spectra somewhat akin to the solar  $p$ -mode spectrum. Although in most cases the dominant oscillation frequencies appear to be stable on a time-scale of years, there is a suggestion in two of these stars that some of the oscillation frequencies last only for a few days. An accurate knowledge of the stability, or otherwise, of the oscillation frequencies is essential to the proper understanding of the driving mechanism, which is still not known. The modal lifetimes, their growth and decay rates and the driving mechanism are all related.

The low-order  $p$ -modes penetrate deep into the interiors of the rapidly oscillating  $A_p$  stars and thus provide a powerful probe of their interior structure and dynamics. Since some of these stars have been observed for the better part of a decade and since the frequency resolution goes as  $T^{-1}$ , their pulsation frequencies are very well determined. By applying the techniques of asteroseismology to the frequency spacings, it is possible to obtain information on the mass, luminosity and age of these

stars. In addition, measurements of secular changes in their pulsation frequencies can be used to determine their evolutionary status. An exciting alternative source of frequency modulation is the Doppler shift in the pulsation frequencies which is caused by the presence of a low-mass companion. Thus, studies of the rapidly oscillating Ap stars may lead to serendipitous detections of low-mass companions.

Much work has been done to determine the pulsation modes in the rapidly oscillating Ap stars. Photometric studies can indicate  $\ell$  and constrain  $n$ , but very few secure mode identifications exist;  $\ell = 1$  pulsation has been identified unambiguously for two stars, HR 3831 and HD 6532. Some of these stars pulsate in alternating even and odd  $\ell$  modes simultaneously. Other techniques of mode identification can also be applied to the rapidly oscillating Ap stars. The phase shift between the light and colour curves  $\Delta\phi(V, B-V)$  has been used successfully as a mode discriminant for some  $\delta$  Scuti stars and Cepheids. Unfortunately no consistent applications of this technique to the rapidly oscillating Ap stars are possible and the correct interpretation of the phase shift  $\Delta\phi(V, B-V)$  in these stars will have to await further theoretical developments. Line profile variations can potentially be used for mode identification the Ap stars, but no mode identifications have yet been secured with this technique; one is limited to working on bright stars with large telescopes in order to achieve a respectable signal-to-noise ratio.

#### *HD 101065 AS A RAPIDLY OSCILLATING Ap STAR*

HD 101065 is one of the most peculiar non-degenerate stars in the sky. It has an exceedingly complex spectrum which is dominated by the lines of the rare earth elements and which has fueled a long-standing controversy regarding its nature. Particularly contentious issues have been  $T_{\text{eff}}$  and the question of whether the Fe peak elements are deficient. There is a considerable body of opinion which holds that HD 101065 is an extreme Ap star. Indeed, it is this belief that led Kurtz to the discovery of the rapidly oscillating Ap stars.

This project has focussed on HD 101065 as a rapidly oscillating Ap star. In order to apply the oblique pulsator model or the techniques of asteroseismology, a complete frequency solution of this star's oscillations is required. Previous work on the oscillation spectrum of this star indicated that it is a multi-mode pulsator, although a complete frequency solution could not be obtained with the available data. We had the suspicion that the frequency spectrum of HD 101065 appeared to be unsolvable only because it was too complex for the observations then at hand and that a complete frequency solution would emerge only after prolonged scrutiny of this star's oscillations.

We thus obtained 138 hr of high-speed photometric observations of HD 101065 on 33 nights from January to June 1988. Frequency analyses of these data indicates that HD 101065 pulsates with at least three frequencies near 1.37 mHz which cannot be identified with consecutive overtones. These three frequencies completely describe the oscillations down to the 0.40 mmag level. There is strong evidence of several more frequencies below the 0.35 mmag level, but the complexity of the spectrum is such that we are unable to determine any of those frequencies securely. We also confirmed the presence of the first harmonic of the principal frequency and found an indication of the first harmonic of  $\nu_3$  in one of the weekly data sets, but this needs to be confirmed. In addition, a frequency at  $\nu \approx 2.687$  mHz shows up in several data sets. This frequency does not correspond on a 2:1 basis to any secure frequencies near  $\nu_1$ .

One of HD 101065's basic parameters which remains unknown is its rotation rate. The low value of  $v \sin i$ , the lack of pulsation phase reversals, the lack of magnetic, spectral and mean light variations as well as the lack of rotational sidelobes all indicate that either  $i$  or  $\beta \approx 0^\circ$ , unless HD 101065 happens to have a rotation period of many decades.

Unfortunately, a straightforward application of the oblique pulsator model to HD 101065 is not yet possible and thus definitive mode identifications cannot be secured. A tentative application of the techniques of asteroseismology indicates that HD 101065 is only slightly off the zero age main sequence and suggests that its radius, mass and luminosity are similar to those of the other Ap stars. There is also evidence which suggests that the second order quantity in the asymptotic relation for pulsation in high-overtone  $p$ -modes,  $\delta\nu$ , is 3-7 times lower in the rapidly oscillating Ap stars than current theoretical expectations.

The above-mentioned frequency analyses were all performed on the explicit assumption that all of the component frequencies are stable. In order to test this assumption, we searched for a significant secular variation in the principal frequency using two independent techniques, the ( $O-C$ ) diagram and non-linear least-squares fitting of frequencies with a  $d\nu/dt$  term. Over the 10-year time-span of our longest data set we found a significant secular variation in the dominant pulsation frequency. This variation is roughly a factor of ten higher than is expected from evolutionary considerations and we argued that it cannot be interpreted as arising from evolutionary effects in HD 101065. We then examined the possibility that an unresolved frequency is involved and we showed that a binary star model in which a low-mass companion orbits about HD 101065 is consistent with the observations.

We also detected the presence of two secondary frequencies which describe some sort of long-term amplitude modulation of the principal frequency. We do not believe that these frequencies are associated with fine structure of the primary frequency because they are separated from  $\nu_1$  by a spacing equivalent to the resolution of the data. These frequencies thus indicate that the amplitude of the dominant frequency may not be completely stable.

#### *FUTURE WORK*

It is obvious that even the intense observational effort in 1988 was insufficient to decipher the frequency spectrum of HD 101065. This is an unexpected result and its interpretation is not yet clear. Perhaps there is a rich complex of low-amplitude oscillations beating against each other to produce the amplitude spectra observed. Perhaps variable, or even stochastic (as in the Sun), driving play a role. If there are transient frequencies in the rapidly oscillating Ap stars, their growth and decay times as well as their lifetimes will be of great relevance to the search for the driving and excitation mechanisms, which are not yet known. The  $1 \text{ day}^{-1}$  alias asymmetries of  $\nu_2$  and  $\nu_3$  are another unsolved problem.

Our tentative determination of  $\delta\nu = 2.94 \text{ } \mu\text{Hz}$  in HD 101065, is substantially lower (3 - 7 times) than Christensen-Dalsgaard's theoretical expectations. A similar situation obtains for  $\alpha$  Cir (and possibly for HD 203932). It appears that  $\delta\nu$  may be significantly smaller in the rapidly oscillating Ap stars than current models predict but the strong magnetic field is an additional factor not yet included in these models. We stress that this is really no more than a suggestion; much work needs to be done to obtain secure determinations of  $\delta\nu$ .

Thus far, only the harmonics of the dominant oscillation in several rapidly oscillating Ap stars have been convincingly detected. This is not surprising when one considers that the amplitudes of the other frequencies in their spectra are substantially lower than that of the dominant frequency. Our marginal detection of  $2\nu_3$  in the JD 2447278-7284 data needs to be confirmed in an independent data set. Also, the occurrence at the first harmonic of the same spectral features in the JD 2447278-7284, JD 2447166-7291 and JD 2443643-7291 data sets indicates that these structures deserve further attention. A more detailed study of the frequency spectrum in the region of  $2\nu_1$  would have to be done with larger telescopes in order to minimize the scintillation noise. The problem of then having too many photons can be solved by reducing the operating voltage of the photomultiplier tube or introducing suitable neutral-density or narrow-band filters. It is our experience with the other rapidly oscillating Ap stars that, of the standard filters, the Strömgren  $\nu$  filter yields the highest signal-to-noise ratio. A study conducted through a Strömgren  $\nu$  filter might reveal the presence of the first

harmonics of  $\nu_2$  and  $\nu_3$ , if they exist. In some cases, there is a significant phase-shift with filter in the rapidly oscillating Ap stars. A wideband filter such as Johnson B may be contributing phase-smearing to the amplitude spectra. If so, then Strömberg  $\nu$  observations may reduce the phase-smearing component sufficiently to bring out further frequency components.

The significant secular frequency variation  $d\nu_1/dt$  deserves further investigation. In Part II, we argued that it is unlikely to be a measure of evolutionary effects in HD 101065. It thus seemed probable that an unresolved frequency is involved and we showed that a low-mass companion orbiting about HD 101065 could account for the (*O-C*) observations whilst not conflicting with the observed lack of radial velocity variations. Companions in the  $0.05M_{\odot}$ - $0.08M_{\odot}$  range, the mass range of brown dwarfs, are of special interest. Unfortunately, the prospects of detecting the conjectured companion photometrically by measuring any excess infrared flux above that expected for an F0V star are very poor. We stress that this is an insecure model and we will not speculate further on the nature of the secondary.

At this point the reader's impression may be that the oscillation spectrum of HD 101065 is as complex as its optical spectrum! Yet, the only way to answer the issues raised above is to conduct an international continuous-coverage observing campaign, probably of some two weeks in duration. Such a campaign, if conducted within the next several years, could also reveal whether the (*O-C*)'s lie on a parabola or on a sinusoid as required by the binary star hypothesis.

We close with a brief comment concerning future work on the rapidly oscillating Ap stars in general. As of this writing, only 14 rapidly oscillating Ap stars are known. Such a small number of rapid oscillators has made it difficult to determine all of the distinguishing characteristics of these stars. For instance, we still do not know what distinguishes the rapidly oscillating Ap stars from other cool magnetic Ap stars. Strictly speaking, it remains to be shown that there exist cool magnetic Ap stars which are *stable* against high-overtone pulsation. This would be a difficult result to secure since it is harder to demonstrate constancy as opposed to variability in any given star. This problem is exacerbated by the amplitude modulation in the rapidly oscillating Ap stars. For Ap stars of unknown magnetic geometry and rotation period, an observation of constancy only entitles the observer to conclude that the star was constant at the time of the observation. It is thus important to observe Ap stars with known magnetic (rotation) periods so that they can be observed at magnetic (pulsation) maxima.

Because of the paucity of known rapid oscillators, the temperature and luminosity ranges of these variables are not well determined. The accurate determination of these ranges is crucial to the

understanding of the physical processes driving pulsation in these stars. Moreover, asteroseismological investigations of a greater number of rapid oscillators are highly desirable. Thus, a fundamental priority in the study of the rapidly oscillating Ap stars is the discovery of more members of this class. Given the sound growth potential of this field of research, the next logical step towards maturity would be to conduct an unbiased, large-scale systematic survey of the Ap-Bp stars. In order to discount the possibility of a null detection owing to rotational amplitude modulation, each candidate would have to be observed for several hours a night on many widely separated nights with telescopes of the 1-m class in order to reduce the scintillation noise.

Recent work has shown that some of these stars (such as HD 203932 and HD 60435) do not have stable frequency spectra. It can thus be as important to extend observations of well-studied rapidly oscillating Ap stars as it is to discover new ones.

The luminosities of the rapidly oscillating Ap stars are not well determined. Thus another priority is the discovery of some of these variables in well-studied clusters of known distance, reddening and age. The author has been awarded telescope time to pursue this topic in the near future.

**Appendix**

**HD 116763 - A False Alarm?**

## HD 116763 - A false alarm?

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**Summary.** The southern cool Ap star HD 116763 has recently been proposed by Matthews *et al.* to be a possible rapidly oscillating Ap star. We present 11.2 hr of high-speed photometric observations obtained on four nights in July 1988 which fail to confirm the presence of oscillations. We present an independent frequency analysis of the data of Matthews *et al.* which leads to the conclusion that the proposed pulsation frequencies are probably spurious. We discuss the differences between these analyses in terms of the difficulty of correctly assessing the appropriate noise level to use when calculating the False Alarm Probability defined by Scargle.

### 1 Introduction

Rapid photometric oscillations have recently been discovered in twelve cool magnetic Ap stars with SrCrEu line strength peculiarities. These *rapidly oscillating Ap stars* are high overtone ( $n \approx 10-30$ ) *p*-mode pulsators which lie mostly in the  $\delta$  Scuti instability strip. Some are well known magnetic oblique rotators and in explaining the empirical properties of these stars Kurtz has extended the well established oblique rotator model by proposing that a rapidly oscillating Ap star is also an oblique pulsator. That is, the star undergoes pulsations aligned with its magnetic axis. For a detailed guide to the literature of these stars consult the recent reviews by Kurtz (1986), Weiss (1986) and Shibahashi (1986).

Unfortunately the small sample of known rapid oscillators has made it difficult to determine all of the distinguishing characteristics of these stars. For instance, we still do not know what distinguishes the rapidly oscillating Ap stars from other cool magnetic Ap stars. Hence the search for new candidates remains a fundamental priority in the study of this class. During a recent search for rapidly oscillating Ap stars Matthews *et al.* (1988) observed the  $V=8.8$  cool Ap star HD 116763. Since no Strömgren photometry of this star is available, it is not known whether its Strömgren indices fall within the ranges of the indices of the known rapidly oscillating Ap stars (Kurtz 1986). Matthews *et al.* selected HD 116763 purely on the basis of its Ap CrSrEu classification by Houk (1978) and tentatively identified the presence of oscillations with periods of

19.8 and 7.1 minutes. They have called for further observations to verify whether HD 116763 is indeed a rapidly oscillating Ap star. We present the results of 11.2 hr of high-speed photometry which fail to confirm the presence of these periods.

## 2 Observations

High-speed photometric observations were obtained on four nights in July 1988 with the St. Andrews Photometer attached to the 1.0-m telescope of the South African Astronomical Observatory (SAAO) in Sutherland. We used continuous 10-s integrations through a Johnson B filter with occasional (aperiodic) interruptions for sky measurements through the same filter as were necessary depending on the phase and position of the moon. All observations were obtained through 30-arcsec, or larger, apertures.

Since these were single channel measurements and the program star was being monitored for periods as short as 8 minutes, no comparison star was employed and we were thus unable to compensate for any slow changes in sky transparency other than mean extinction. However, previous observations (e.g., Kurtz 1982) have amply demonstrated that coherent oscillations with periods under about 20 minutes such as those suggested by Matthews *et al.* for HD 116763 ought to be readily detectable through the analysis of such non-differential photometry.

The observed counts were corrected for coincidence losses, sky background and mean extinction and were then converted to instrumental magnitudes normalized to the nightly means. Finally, the data were converted to 40-s integrations by taking non-overlapping four-point averages. Frequency analyses of each night's data were then performed using the faster algorithm (Kurtz 1985) for a discrete Fourier transform (DFT) based on Deeming's (1975) DFT algorithm. The amplitude spectra thus produced contain high amplitude ( $\approx 3$  mmag) peaks at low frequencies ( $< .35$  mHz) which are almost certainly due to residual sky transparency variations which were not removed from these non-differential data by the reduction procedure. Kurtz (1984) discusses the sources of noise in this type of photometry in greater detail. It is important to remove these low-frequency peaks for two reasons: (1) A relatively small bump on the wing of a high amplitude sky transparency peak may compete with the signal that we are interested in. (2) Applications of linear least-squares methods (for fitting frequencies to the data) or of Scargle's (1982) False Alarm criterion assume a white noise spectrum while the transparency-generated noise has a definite frequency dependence. In all cases the peaks removed from our data were for frequencies less than 0.6 mHz and amplitudes lower than 3 mmag, except for the first night where we prewhitened

**Table 1.** Journal of 1988 SAAO B observations of HD 116763.

Date(1988)	JD2440000+	$t$ (hr)	$N_o(40-s)$	$\sigma$ (mmag)
June 30/01	7343.22483	4.22	359	2.29
July 01/02	7344.28727	1.50	126	1.63
July 04/07	7347.20838	2.55	225	0.86
July 30/31	7373.22831	2.89	243	1.21
$\Sigma$		11.16	953	

**Table 2.** Journal of 1987 CTIO B observations of HD 116763 by Matthews *et al.*

Date (1987)	JD2440000+	$t$ (hr)	$N_o(60-s)$	$\sigma$ (mmag)
Feb 25/26	6852.79443	2.26	131	2.23
Mar 05/06	6860.76849	2.48	146	2.23
Mar 08/09	6863.75055	2.69	155	5.00
Mar 10/11	6865.75916	1.47	85	2.95
$\Sigma$		8.90	517	
$\Sigma$ excluding 6863		6.21	362	

out a very low frequency peak (0.046 mHz) of amplitude 13.5 mmag\*. Table 1 lists the nights on which observations were obtained, the heliocentric Julian dates of the first observation on each of these nights, the number of 40-s integrations obtained and the standard deviation,  $\sigma$ , of one observation with respect to the mean for that night after removal of the sky transparency peaks. The standard deviation is a rough indicator of the quality of the night. It contains contributions from the photon noise, scintillation noise, residual sky transparency variations and any actual variations in the star itself. Generally if  $\sigma \geq 5$  mmag we regard this as evidence of a non-photometric night and reject the data from further analysis. Typically,  $\sigma \leq 2.5$  mmag for SAAO observations on good nights.

### 3 Frequency analysis

#### 3.1 THE 1988 SAAO DATA

Individual nights and the entire data set were examined out to the Nyquist frequency of 12.5 mHz for evenly spaced 40-s integrations. The nightly amplitude spectra of these data (nearly out to the Nyquist frequency) are presented in Fig. 1 while Fig. 3 (lower panel) shows the amplitude spectrum of the entire data set. In these prewhitened amplitude spectra, we have no information for  $\nu \leq 0.6$  mHz. The unprewhitened spectra do not suggest the presence of any signals above the noise level in this range which is best studied using conventional standard differential photometry. For  $0.6 \leq \nu \leq 2.0$  mHz we have some information, but it is difficult to assess the degree of contamination by residual sky transparency variations. Clearly, though, no peaks stand proud of the generally higher noise level expected in this frequency range. Neither do any of these peaks persistently occur in our amplitude spectra at the frequencies obtained by Matthews *et al.* from their 1987 data. We thus conservatively attribute all of these low-frequency peaks to residual sky transparency variations. For  $\nu > 2$  mHz the spectrum is flat out to the Nyquist frequency. Thus, if we conservatively adopt a noise level of 0.3 mmag out to the Nyquist frequency we may confidently claim that HD 116763 exhibited no significant periodic variations of more than 0.6 mmag peak-to-peak in the frequency range  $2 < \nu < 12.5$  mHz during the time that we observed it.

By inspection of our nightly amplitude spectra we conclude that the more pronounced sky transparency variations on the first two nights dominate the low-frequency region  $\nu < 2.0$  mHz of the amplitude spectrum in Fig. 3 (lower panel); these peaks are not present on the other nights. This raises the possibility that the low-frequency peaks described by Matthews *et al.* are also

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\* We shall also refer to data filtered in this way as being "prewhitened".

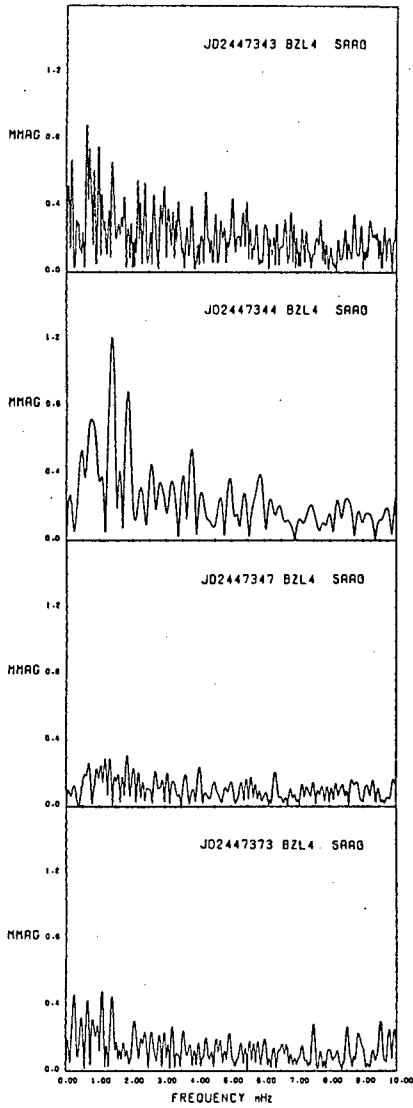


Figure 1. Prewhitened  $B$  amplitude spectra of the 1988 SAAO observations of HD 116763 out to nearly the Nyquist frequency of 12.5 mHz. Each panel corresponds to an individual night listed in Table 1.

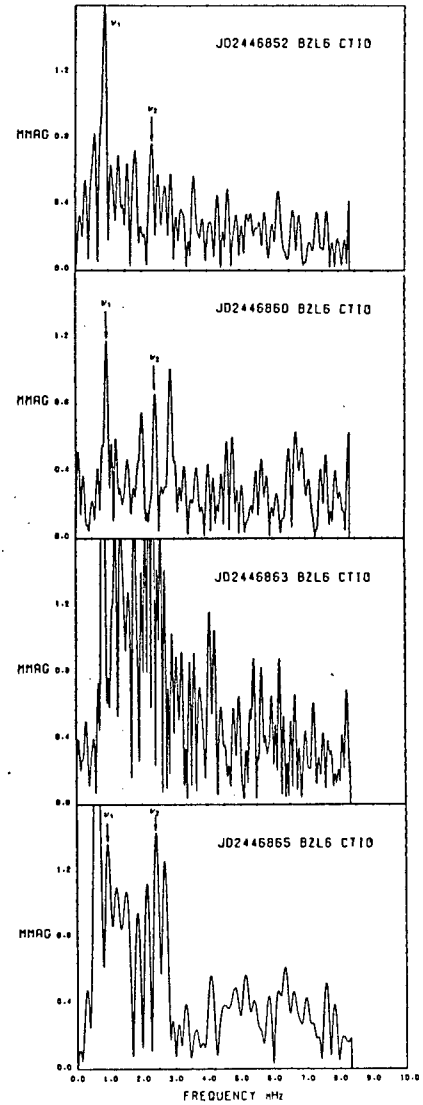


Figure 2. Prewhitened  $B$  amplitude spectra of the 1987 CTIO observations of Matthews *et al.* up to the Nyquist frequency of 8.33 mHz. Each panel corresponds to an individual night listed in Table 2, with the annotated arrows indicating the peaks nearest the frequencies of interest. Notice that while  $\nu_1$  appears to be marginally convincing, the peaks marked  $\nu_2$  are not particularly prominent on any of these nights. The noise level in the third night is so high that we have rejected it from further analysis.

simply residual sky transparency variations. The greater height of those peaks would then be accounted for by the higher noise level in their data. Matthews *et al.* also remark that their amplitude spectra show considerable amplitude modulation from night to night. Such rapid amplitude modulation may arise from rotation or from the beating of several unresolved closely-spaced frequencies. However, we judge this to be unlikely since on none of the nights that we observed HD 116763 was a statistically significant signal detected.

One must be cautioned against the immediate interpretation of this null result as proof of the nonexistence of oscillations in HD 116763. In the oblique pulsator model (Kurtz 1982, Dziembowski & Goode 1985a,b; Kurtz & Shibahashi 1986) the pulsation amplitude of nonradial modes is modulated by the rotation of the star and may drop below the level of the noise or go to zero at some rotation phases. Hence it is possible that we have coincidentally observed HD 116763 near its amplitude minima on two occasions. If we interpret our observations in this context, then it is immediately clear that  $\nu_1 = 0.8403 \text{ mHz}$  and  $\nu_2 = 2.3519 \text{ mHz}$  (the two frequencies proposed by Matthews *et al.*) cannot be due to pure  $\ell = 0$  (radial) pulsation. Radial pulsation is spherically symmetric and hence should undergo no amplitude modulation with aspect. The rotation period could be about four weeks or some period commensurate with four weeks. However, since these speculations hinge on the reality of the oscillations detected by Matthews *et al.* we decided to analyse their 1987 data independently.

### 3.2 THE 1987 CTIO DATA

Matthews *et al.* kindly supplied us with a copy of their 1987 data which we have reanalysed. We began by partitioning their unfiltered data into individual nights from which we then filtered out the transparency-generated low-frequency peaks in exactly the same way as with our own data (see Table 2). The nightly amplitude spectra thus obtained are presented in Fig. 2 and we note that the suggested pulsation frequencies (marked as  $\nu_1$  and  $\nu_2$  in Fig. 2) are not cleanly separated from the other low-frequency peaks on all of the nights. From our experience in searching for new rapidly oscillating Ap stars we usually regard this as evidence of a marginally photometric night and hence reject the data from further analysis. In particular, we would reject the observations in the third night of their data (see Fig. 2 and Table 2). While  $\nu_1$  appears to be marginally convincing, we note that in all four of these amplitude spectra the  $\nu_2$  peak is indistinguishable from the other peaks in the low-frequency noise "continuum". The amplitude spectrum for the entire prewhitened 1987 data set (barring the third night) is presented in Fig. 3 (top panel). Although there is no

compelling evidence for the existence of  $\nu_2$  in the individual nights' data some degree of phase coherence is suggested by the presence of peaks slightly above the noise level at  $\nu_1$  and  $\nu_2$  in this figure.

Matthews *et al.* also addressed the concern that the third night of their data was non-photometric. They removed that night from their analysis and the observed peaks persisted at somewhat reduced amplitudes. Indeed, it is on the basis of this and the apparent phase coherence suggested by their Table III that Matthews *et al.* tentatively identified  $\nu_1$  and  $\nu_2$ . We turn now to a closer examination of the reality of  $\nu_1$  and  $\nu_2$  in the CTIO data.

In general, the reality of features in an amplitude spectrum may be tested by altering the sampling time. If we increase the effective sampling time of their data to 180 s by taking non-overlapping three-point averages we effectively lower the variance and Nyquist frequency, but any real peaks should remain, albeit at a slightly reduced amplitude.\* Moreover, for a given Nyquist frequency  $\nu_N$ , the number of independent frequencies  $N$  is roughly given by  $N \approx \nu_N \Delta T$  where  $\Delta T$  is the total time-span of the data set.  $N$  is also an indication of the statistical penalty (Scargle 1982) associated with searching many frequencies. Hence, for a lower Nyquist frequency (i.e. lower  $N$ ), a given spectral peak becomes more significant. A search of the nightly amplitude spectra of the averaged data (and of the entire averaged CTIO data set) reveals no persistent peaks at the locations of  $\nu_1$  and  $\nu_2$ . We thus conclude that  $\nu_1$  and  $\nu_2$  are probably artefacts of their analysis.

#### 4 Discussion

We must examine the reasons for the difference between Matthews *et al.*'s and our analyses of the 1987 data. Matthews *et al.* secured their frequency identifications by appealing to the Scargle (1982) "False Alarm" criterion which gives the probability  $F$  of finding a peak at a signal-to-noise ratio of  $z$  in a power spectrum of white noise as

$$F = 1 - (1 - e^{-z})^N \quad (1)$$

where  $N$  is the number of independent frequencies searched. However, extreme care must be exercised in the application of this test; the difficulty often lies in correctly assessing the level of the noise. Figures 4 and 5 clearly show the importance of this assessment. The trajectories in these Figures trace the growth of the false alarm probability for different values of  $z$  as the number of independent frequencies  $N$  rises. For illustrative purposes let us consider a 2.5-hr run

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\* The amplitude reduction factor of a real frequency  $\nu$  due to the sampling time  $\tau$  is easily shown to be  $\text{sinc}(\nu\tau)$ . In the case of  $\tau \approx 180$ -s integrations and  $\nu \approx 0.8403$  mHz this correction is small;  $\text{sinc}(\nu\tau) \approx 0.96$ .

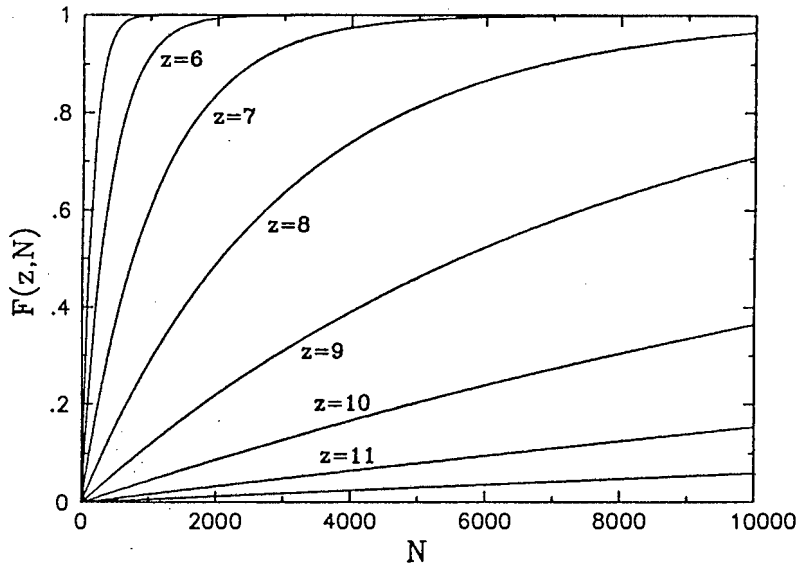


Figure 4. This diagram shows the sensitivity of the false alarm function  $F(z, N)$  to the assessment of the noise level ( $z$ -trajectory). The trajectories are drawn up to the values of  $N$  that were obtained for roughly 1 week of observations of a rapidly oscillating Ap star using 30-s integrations.

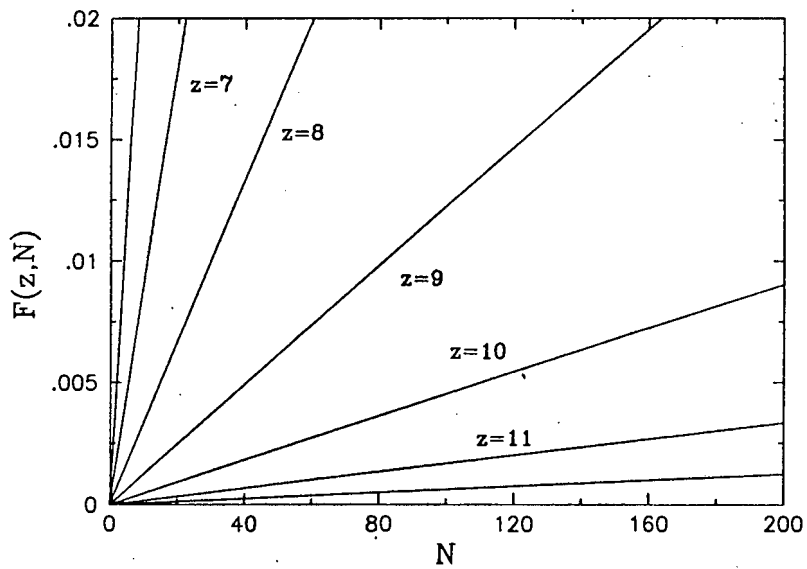


Figure 5. The behaviour of  $F(z, N)$  is shown in greater detail for the nightly values of  $N$  appropriate to the CTIO data discussed in this paper.

of 60-s integrations which has  $N = 75$ . Notice that in order to obtain a false alarm probability of 1% from these data Fig. 5 indicates that the peak of interest must have  $z \approx 9$ ; in order to retain this 1% uncertainty for the same peak with one week's worth of data ( $N \approx 5000$ ) Figure 4 shows that  $z$  will have to be even higher. These  $z$  values may at first glance appear to be very high, but it must be borne in mind that we are dealing with amplitude spectra which scale as  $z^{1/2}$ . Thus a slight underestimate of the level of the noise may result in a severe underestimate of the false alarm probability. The opposite situation may also arise. For instance, in determining the rotation period of HD 6532 Kurtz and Marang (1987) showed how the naive application of Scargle's criterion indicated that their signal peak was almost certainly spurious whereas a realistic assessment of the noise showed it to be highly significant. In the case of HD 116763, the difficulty in correctly applying the Scargle criterion has probably led Matthews *et al.* to over-interpret their data. In particular, more conservative estimates of the noise levels in their data are called for since even the prewhitened amplitude spectra (Fig. 2) are not flat out to the Nyquist frequency. There are still markedly higher peaks at the lower frequencies. Unfortunately, there is no rigorous prescription for correctly estimating the level of the noise. However, it is clear that because of the non-Gaussian nature of the noise we cannot always find a single estimate of the noise which is appropriate to the entire amplitude spectrum. For frequencies above about 2.5 mHz, the dominant source of noise is scintillation and this appears to be the appropriate noise level to use. For the lower frequencies we obviously cannot use the level of the scintillation noise. Neither can we use error estimates derived from a least-squares fit which assumes a white noise spectrum. Here the appropriate noise level to use may be determined from the level of the "continuum" near the frequency of interest. Thus two peaks of identical height, but in different parts of the amplitude spectrum, will generally lie on different  $z$ -trajectories in Fig. 4 which may lead to very different false alarm probabilities.

The other important parameter in the false alarm calculation is the number of independent frequencies  $N$ . Some use has been made of the empirical relation of Horne and Baliunas (1986) for  $N_o$  evenly sampled data

$$N = -6.362 + 1.193N_o + 0.00098N_o^2. \quad (2)$$

However, this relation should be applied with caution since Horne and Baliunas derived it by simulating a large number of data sets of pseudo-Gaussian noise whereas the amplitude spectra presented in this paper do not strictly satisfy this condition. Moreover, it is not only for strongly gapped or unevenly sampled data that equation (2) will overestimate the number of independent

frequencies. A six-hour run consisting of 10-s integrations has  $N_0 \approx 2160$  and eq. (2) yields  $N = 7143$ . These frequencies will obviously be much more closely spaced than the frequency resolution  $\Delta T^{-1} \approx 46 \mu\text{Hz}$ . We feel that a more reasonable indicator of the number of independent frequencies is  $N \approx \nu_N \Delta T \approx N_0/2$  which, in this example yields  $N = 1080$ . This more conservative estimate of  $N$  will yield a lower false alarm probability for a given power signal-to-noise ratio  $z$ . However, the false alarm probability calculation is still largely dominated by the choice of  $z$  as Fig. 4 clearly shows. A recalculation of the false alarm probability with more stringent noise levels and  $N$  determined in this way leads to the conclusion that  $\nu_1$  and  $\nu_2$  are probably spurious. Once again these results emphasize the extreme care with which the false alarm criterion ought to be applied. In general, we prefer not to invoke statistical arguments to justify marginal detection of pulsation in new candidates.

## 5 Conclusions

The 1988 data presented in this paper show that HD 116763 did not vary in brightness by more than 0.6 mmag peak-to-peak in the frequency range  $2 < \nu < 12.5$  mHz during 11 hr of observations on four nights. At frequencies less than 2 mHz, there are larger amplitude peaks which we attribute to residual sky transparency variations. A more conservative reanalysis of the 1987 data of Matthews *et al.* suggests that their  $\nu_1$  and  $\nu_2$  are probably spurious. This emphasizes again the caution required in the application of Scargle's false alarm criterion.

Because of the small sample of known rapidly oscillating Ap stars, we are still uncertain of all of the distinguishing characteristics of these stars and hence the detection of new members of this class is yet of fundamental importance. However, to be useful, new members should exhibit clear and reproducible evidence of pulsation. Unfortunately, HD 116763 does not yet satisfy this criterion.

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