

**AN APPLICATION OF AN OPTION PRICING
MODEL TO EVALUATE THE COST OF A
GOVERNMENT LOAN GUARANTEE:
AN HYPOTHETICAL CASE BASED ON
ESKOM**

by Abrahams Mutedi M'pasi

**Thesis submitted
in partial fulfilment of the requirements for the degree
of Master of Commerce in Economics**

**School of Economics
University of Cape Town
1995**

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ABSTRACT

In the late 1930s, the Great Depression and its consequences led to the U.S federal government's intervention in credit assistance and insurance programmes. The main reason for this intervention was that there was a general desire to rescue individuals and businesses which were unable to repay their debts when due.

Considerable debate has focused on the determination of **the magnitude** of the government liabilities resulting from guaranteed loan re-payments. Today, most nations, including South Africa, employ such government guarantees, but they are often improperly valued; that is, one has no idea whether such guarantees are 'good' or 'bad' policy tools.

This paper illustrates how Put option pricing models may be used to estimate the 'real' cost to the South African government of a loan guarantee to Eskom, which is investing a large hydroelectric project in Mozambique, hypothetically assuming that Eskom has been privatized.

While the paper recognises the importance of the insurance premium which could be charged by the government for its loan guarantee, the results under the hypothetical case show that the Eskom is able to readily repay the promised payment and, thus, the loan guarantee provides value to Eskom's owners. In this regard, one can argue that parties involved in such a project, such as the South African government, Eskom and the European agencies may benefit from the loan guarantee programme. Thus, a loan guarantee programme may be seen as a 'good' policy tool to resolve conflicts between lenders and borrowers, to encourage investment and to meet a broader public interest.

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Abrahams Mutedi M'pasi,

Cape Town, 1995.

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INTRODUCTION

In the late 1930s, the Great Depression and its consequences led to the U.S federal government's intervention in credit assistance and insurance programmes. The main reason for this intervention was that there was a general desire to rescue individuals and businesses or companies which were unable to repay their promised payment when due. Since then, loan guarantee programmes have been considerably expanded to meet a range of social and economic requirements. These may be classified into two major categories: (1) direct loans of the government funds to borrowers, and (2) loan guarantees.

Direct loans have been a key method by which governments have provided credit assistance. Under this form of credit, the government lends its own funds to borrowers, i.e., individuals, or companies, which in turn agree to repay the loan at a specified date with interest charged by the government.

Loan guarantees are agreements by which a government, or an agency thereof, guarantees to pay all or part of the loan principal and interest to lenders in the event of a default¹. Like direct loans, government uses loan guarantees to redirect, economic resources by allowing borrowers to obtain credit at more favourable terms than those which would have been available in the financial market.

Under loan guarantee programmes many corporations such as International Computer Limited in the United Kingdom, Lockheed and the Chrysler Corporation in the United State have been assisted during times of financial crises. For example, in 1971, the Lockheed Corporation was in financial crises. It was nearly out of cash after absorbing heavy cost overruns on military contracts and, at the same time, committing more than \$800 million to the development of the L1011 TriStar airliner. Lockheed was on the

¹*Default* means failure to meet any obligation or term of a credit agreement that causes the lender to accelerate demand on a borrower.

verge of bankruptcy. With a loan guarantee up to \$250 million² by the U.S government, the Lockheed Corporation has been able to repay the promised principal payment with interest.

Nevertheless, this type of assistance necessarily results in increase governmental costs which may be offset by fees or premiums charged under these programmes. Considerable debate focuses on the determination of the magnitude of this cost. In other words, most nations, including South Africa, employ such government guarantees but they are often improperly valued or not at all; that is, one has no idea whether such guarantees are 'good' or 'bad' policy tools.

Given the importance of properly valuing the contingent liabilities of government, the goal of the paper is to apply the option pricing model to evaluate the cost of a particular government loan guarantee programme: the cost of governmental loan guarantees to an hypothetical case based on Eskom which, in turn, is investing in Mozambique. To achieve this goal, **Put Option Pricing** models were employed because they are especially useful in valuing corporate liabilities.

This problem is particularly illustrated by Merton(1977). Merton's model of corporate debt is subject to Put option on a common stock. In other words, the owner has the right to sell a specified number of shares of a given stock at a specified price per share on a specified date. Since the value of the Put at the maturity date depends on the stock price, its value prior to expiration will depend on the probability distribution of the stock prices on the expiration date.

In order to apply the methods described it has been assumed that Eskom has been privatized. This is because Eskom has no share capital. This assumption allows us to retain

²Reinhardt (1973), Break-Even analysis for Lockheed's TriStar:An application of financial theory, The Journal of Finance, Vol.28,(p.821-838)

throughout the paper the original Black-Scholes assumptions to estimate the cost of government loan guarantee.

In the hypothetical case study, Eskom has reached agreement for the renewed supply of power from Cahora Bassa, beginning in 1997³. Work is proceeding to rehabilitate the transmission line which was extensively sabotaged during the civil war in Mozambique. To deal with the risk of expropriation which could lead to Eskom defaulting the payment of the promised principal and interest, a government loan guarantee has been employed to reduce the potential losses to Eskom and thus to encourage it to undertake the project.

Following existing literature, for example, Merton (1977), Jones and Masson (1980), and Chen *et al* (1986), this paper makes a contribution to the academic literature and to a public policy. From an academic point of view, this paper provides an explicit methodology for the valuation of the cost of loan guarantees and shows the importance of a particular government intervention in the economy. This can be seen from the economic benefits resulting from such intervention, i.e. the loan guarantees are simply a form of subsidy to a specific economic activity. From a public policy standpoint, this paper emphasises that social welfare may result from such a loan guarantee⁴.

This paper reviews the theoretical background to the valuation of loan guarantees and extends this to the theoretical valuation of the currency options. In other words, the contribution of this paper lies in its combination of Put Option Pricing techniques and the theory of Currency Options. In particular, this paper illustrates this combination with the hypothetical case of Eskom guaranteed investment in Mozambique.

³Financial Mail, (26 May 1995, pp.99)

⁴It is argued that private economic activities which cost more than the sum of the benefit accruing to private participants but less than their aggregate social benefit, could suggest some form of government financial assistance, i.e. government loan guarantees or direct credit programmes and subsidies. See Jones and Masson (1980)

The paper is structured as follows: Chapter One reviews the general theory of option pricing and explains the Black-Scholes pricing model in detail. In the second Chapter, the theory of the valuation of loan guarantees and models thereof are presented, and certain hypotheses are examined as to the possible benefit to the national output, to shareholders, and to senior bondholders that arises when the company has succeeded in obtaining such a government loan guarantee. Chapter Three presents a profile of Eskom and considers briefly the Cahora Bassa hydroelectric project. The emphasis is on Eskom's balance sheet and the nature of the loan guarantee to company. Finally, in Chapter Four, an estimate of the cost of the loan guarantee is presented on the basis of an assumed share price series and some implications of these result are considered. The findings of the paper are then summarised in the conclusion.

CHAPTER ONE

THE THEORY OF OPTION PRICING

1. INTRODUCTION

Prior to considering the Eskom-Government Loan Guarantee programme, (GLG), it is necessary to consider the literature regarding option pricing, generally, and the application of this literature to the case of a GLG. As can be seen in the brief summary which follows, the approaches to option pricing are broad, indeed. Only by invoking various simplifying assumptions is it possible to adequately treat various dimensions of option pricing. It will be seen, *inter alia*, that it has often been necessary to adapt concepts which originated in the Black-Scholes model to the problem of valuation of government liabilities.

In a GLG model, there are typically three parties: a private company or a firm; a bank, and a third-party such as the government or its agency. A company issues junior debt with a total promised payment of **B** dollars. The terms of the guarantee are such that, if the firm defaults on the promised payment to the junior bondholders, the guarantor (the government) will make payment. By guaranteeing the firm's debt, the government has acted as **writer of a Put option**. Viewed differently, it has, in essence, issued an insurance policy at no charge. Just as outstanding policies represent contingent liabilities to insurance companies, so outstanding loan guarantees represent contingent liabilities to a government.

It will be seen below that Put-Call options trading may cause very large gains or losses if the options are not correctly priced. Thus, this paper begins with a discussion of the determination of option pricing and the attempts to approximate a 'fair' value to the cost of a GLG. This is crucial since it is argued⁵ that all corporate liabilities may be

⁵In a seminal paper, Black-Scholes (1973: hereafter B-S) developed an explicit formula for the valuation of a call option and discussed the pricing of a firm's common stock and bonds when the stock is viewed as an option on the value of the firm. More recently, Myers (1977) has also suggested that corporate investment opportunities may be represented as options.

viewed as options; thus, analysis of option pricing is applicable to corporate liabilities, in general and to corporate debt, specifically.

In this chapter we consider three major issues:

- (i) the development of traded Call-Put options, and preliminary considerations about such options;
- (ii) the development of the two major option pricing models: one a continuous-time, and other a discrete-time model; and , finally,
- (iii) the application of option pricing models to a wide variety of financial instruments such as corporate bonds, futures, and insurance policies.

1.2. TRADED CALL-PUT OPTIONS

The long history of the theory of option pricing began in 1900 when the French mathematician Louis Bachelier deduced an option pricing formula based on the assumption that stock prices followed a Brownian motion with zero drift. Since that time others have contributed to the theory of option pricing, but it is only in relatively recent years that the development of options has undergone rapid expansion.

Traded options enable investors to control risk. They can be considered as a type of insurance against price movements. In such markets, an investor who opens a position by selling a traded option is termed the writer of that option. The maximum profit to be made by an option writer is simply the **premium** received.

Nevertheless, in such markets, traded options are frequently mispriced. When the market is not efficient⁶ investors can enjoy abnormal returns. In an attempt to improve

⁶In an efficient market the current prices of securities represent unbiased estimates of the 'fair' or 'intrinsic' value of the securities. If all securities are fairly valued then there are no under-or overpriced securities. Thus, the degree to which markets are efficient has important implications for investors. In other words, time and money spent on security analysis will be wasted. See Fama (1991) for an excellent analysis of efficient capital markets.

market efficiency, the first major options trading exchange: the Chicago Board Options Exchange was created so as to form a centralised market for traded Call options (options which grant the holder the right to buy listed stocks⁷). The American Stock Exchange, the Philadelphia Stock Exchange, and the Pacific Stock Exchange were established soon afterwards. In March 1977, the trading of Put options, which grant the holder the right to sell a share on listed stocks, began.

Trading of Put options has received relatively little attention in the past because they are less popular than Call options and it has been argued by many financial economist (among them Black and Scholes) that, given the price of a Call option and the underlying common stock, the value of a Put is uniquely determined.

By the early 1980s, Put and Call options on a large number of listed stocks were traded. This increase in the number of options permitted them to be employed with most financial instruments; and, additionally, enabled researchers such as Merton, Cox and Rubinstein⁸, to test, and contest, the Black-Scholes assumptions and formulation⁹ which they viewed as too general.

1.3. PRELIMINARY CONSIDERATION OF CALL-PUT OPTIONS

Some of the basic uses of Call and Put options will be illustrated in this section. Before discussing options further, some concepts¹⁰ discussed in this section need to be defined.

⁷ Robert Nathan, R., Public Policy Aspects of a Futures- type Market in Option on Securities, (The Chicago Board of Trade, Vol. 1 &2, November 1969)

⁸ Copeland and Weston, Financial Theory and Corporate Policy, (Second edition, Addison-Wesley, 1983, pp. 273)

⁹Most financial economists recognize that the seminal paper of B-S is a general framework for contingent claim analysis in finance. However, some researchers such as Rubinstein (1976) have demonstrated the shortcoming of the B-S framework by including the effect of transaction costs in their models.

¹⁰All the contracts may differ with respect to other provisions such as exercise price changes, etc. Other option contracts such as straddles, straps and strips, are combinations of put and call options.

The **option contract** is a right to buy or to sell an asset at a given price within a specified period of time. Warrants to purchase common stock, and Put-Call options, are common examples of option contracts.

A **Call option** is a contract to buy, which gives its owner the right to purchase an asset at a fixed price, called the **exercise price** (or the **strike price**) for a fixed period of time, called the **maturity period**.

A **Put option** is exactly the opposite of call option. The owner of a put has the right to sell an asset at the exercise price up to the maturity date of the contract. These contracts are truly options in the sense that they do not oblige the owner to buy or sell.

The options just described are **American options**¹¹. Such options give the owner the right to exercise the option at anytime up to the date the option expires. Options that may be exercised only at the maturity date are referred to as **European options**.

In order to illustrate how Call-Put options are priced, it is essential to explore the environments in which option contracts can be priced. Thus, the following section considers three major issues:

- (i) the put-call parity relationship;
- (ii) the boundary price conditions that largely dictate an option price; and, finally,
- (iii) the relationship between the option value and the determinants of the option premium.

All of these are particularly useful, because they lead to the B-S option pricing model.

¹¹Note that in South Africa, most options are of the American type

1.3.1. Put-Call Parity (PCP)

The PCP relationship was developed by Stoll (1969). It is called *PCP* because it gives the value of a corresponding Put if the value of Call is known. In other words, there is a fixed relationship between the price of Put and Call options with the same maturity date and the same exercise price on a single underlying asset. This explicitly shows that the PCP concerns the valuation of European options. Thus, the price of European options may be expressed as follows¹²:

$$C^E - P^E = S - Xe^{-r_f t} \quad (1.1)$$

where r_f is the annual risk-free rate, t is the time to maturity of the Put and Call options, X is the exercise price of both the Put and the Call and S is the current stock price.

Equation 1.1 represents the difference between the price of a European Call (C^E) and Put (P^E) and is equal to the difference between the market price, with time to maturity ($= t$) and the discounted strike price. Thus, if C^E is known, the value of P^E can be determined, and conversely.

This relationship holds when there are strong economic forces keeping the price of the Call and the Put in parity with each other. There are special times, however, when these forces work inefficiently, thus allowing prices to move out of parity¹³.

¹²Equation 1.1 is applicable when no dividends are to be paid before the option expires. See Haugen (1990, p. 451) for details in the case a dividend is known to be paid.

¹³As a result a Call option may be *in-the-money* or *out-of-the-money*. If it is *in-the-money*, then the owner can sell the stock short, sell the Put and buy the Call. If the Put option has a positive value, then the Call option is *out-of-the-money*. As a result of this, the holder buys the stock and the Put, and he sells the Call.

1.3.2. Boundary Price for Call-Put Options

In this section we consider the boundaries for Call-Put options. If we know the boundary price of Call-Put options, then we can readily follow the derivation of the B-S option pricing model and thus solve a problem of the GLG. We here consider, then, European Call-Put and American Call-Put options, and show a range within which these options should be priced.

1.3.2.1. American Call Options (ACO):

An ACO should sell for at least zero or the difference between the underlying asset price and the exercise price, whichever is greater. It may be represented as follows:

$$S \geq C^A \geq \text{Max} [0, S-X] \quad (1.2)$$

where C^A is the price of an American Call option, S is the underlying asset price, and X is the option's exercise price.

For the American Call options, the process of arbitrage ensures the boundary price, above, may necessitate a premature exercise. However, European Call options do not; therefore, we cannot conclude that $C^E \geq \text{Max} [0, S-X]$.

1.3.2.2. European Call Options (ECO):

The boundary price of a ECO is often applied to an asset that pays no dividend or interest. This explains why most of the literature on option pricing has considered the ECO since it is a simple option with which to deal. The price of a ECO ranges over the following interval:

$$C^E \geq \text{MAX} [0, Se^{-r_f t}] \quad (1.3)$$

where C^E is the price of an European Call option, t is the time to maturity, and r_f is the risk free rate. This equation indicates that an ECO should sell for at least zero, or the difference between the underlying asset price and the discounted present value of the exercise price, whichever is greater.

For European Call options, there are no losses as a result of the process of arbitrage. If the option expires *in-the-money*, the payoff is S , so the arbitrageur can reverse a short position in the asset. If it expires out-of-the-money, the pay-off is X . Since $X \geq S$, the arbitrageur can again reverse the short position in the asset. Thus, the process of arbitrage can alter such option's movement so as to accurately value the ECO.

1.3.2.3. American Put Options (APO):

An APO should sell for at least zero, or the difference between the exercise price and the underlying asset, whichever is greater. The boundary price of an APO is expressed as follows:

$$X \geq P^A \geq \text{Max} [0, X-S] \quad (1.4)$$

where P^A denotes the price of an APO.

1.3.2.4. European Put Options (EPO):

The inequality of the boundary price of an ECO is as follows:

$$Xe^{-r_f t} \geq P^E \geq \text{Max}[0, Xe^{-r_f t} - S] \quad (1.5)$$

This implies that an EPO price cannot exceed the discounted exercise price. An EPO should sell for at least zero, or the difference between the discounted exercise price and the stock price, which ever is greater.

From equations 1.6 and 1.7, the following inequalities can be deduced:

$$P^E \leq P^A \quad (1.6)$$

$$C^A \geq C^E$$

(1.7)

These explain that an APO or ACO will be at least as valuable as its EPO or ECO counterpart.

Analysis of the boundary price indicates the range within which an option will be priced. However, violations of boundary price conditions may allow an investor to earn abnormal profits or to earn a return in excess of the riskless interest rate. To avoid this, arbitrage ensures that the boundary price obtains in the presence of disturbances in the capital markets, e.g. transaction costs.

We turn now to the relationship between an option's value and the factors in the market for options. This section flows from the boundary price of Call-Put options in the sense that it specifies factors which should accurately reflect the value of option pricing. In this framework, B-S (1973) derived a practical model for the valuation of European Call option, which Merton (1977) extended in a GLG model.

1.3.3. The Relationship between an Option's Value and Factors in the Market for Options

A number of complex mathematical models have been formulated to value options. The most widely used is the B-S option valuation formula. It is derived by calculating the price at which an option would have to stand in the market to allow a risk-free "hedge" between the option and the underlying asset. In this framework, the following factors establish the sort of relationship one can expect between an option's value and the security's price.

1.3.3.1. The Underlying Asset Price

The relationship between an option's value and the price of the underlying stock is a positive one. *Ceteris paribus*, the higher is the underlying asset price relative to the strike price of a Call option, the higher is the value of a Call option and the lower is the value of a Put option.

Given the relationship between the Call option's value and the underlying asset price, two important measures are noted¹⁴: *delta* (Δ) and *gamma* (γ).

Delta measures the change in the value of the call option as a result of a one-unit change in the price of the asset. It is expressed as follows:

$$\Delta = \partial P / \partial S \quad (1.8)$$

where P is the value of the Call option and S represents the underlying asset's price. This equation (Eq. 1.9) indicates that the value of a Call option will range between 0 and 1, and, conversely, a Put option ranges between 0 and -1. While a delta value ≤ 1 indicates that the Call option is *in-the-money*, a delta of value ≤ -1 reflects that the Put option is *in-the-money*.

Gamma represents the change in the delta as a result of a small change in the asset price. It is, therefore, the second derivative of the call option value with respect to the asset price¹⁵:

$$\gamma = \partial^2 P / \partial S^2 \quad (1.9)$$

¹⁴Falkena *et al.* The Options Market (Second edition, 1989, pp. 11-13)

¹⁵See Falkena *et al.* (1989) for some examples, pp 10-11

1.3.3.2. The Strike Price

This is part of the contract specification and its relationship with an option's value is positive. The higher is the strike price, the lower is the intrinsic value¹⁶ of a call option and the higher is the intrinsic value of a put option.

1.3.3.3. Risk-Free Interest Rates

The relationship between the risk-free interest rate and the value of the Call option indicates that the value of the Call option increases as a function of the risk-free rate of return¹⁷. This implies a positive relationship between the riskless short-term rate of interest, and the value of the Call option, and may be expressed as follows:

$$\rho = \partial P / \partial r_f \quad (1.10)$$

where r_f is the risk-free rate and ρ is positive for Call options and negative for Put options. Equation 1.11 represents the first derivative of the value of the Call option with respect to the risk-free rate. An increase in the risk-free rate may generate a benefit to holding a Call option relative to buying the underlying instrument and of buying a Put option. As a result, when the yield on the underlying asset is greater than the cost of funds, then Put options will be more expensive than Call options, and *vice versa*.

¹⁶The *intrinsic value* is the amount that the buyer **would** recover if he exercised the option immediately.

¹⁷Black-Scholes (1973) have shown that it is possible to create a risk-free hedged position consisting of long position in the stock and a short position in the option. This insight allowed them to argue that the rate of return on the equity in the hedged position is nonstochastic. Therefore, the appropriate rate is the risk-free rate" See in Copeland and Weston , op.cit. (pp. 373)

1.3.3.4. The Time to Maturity

The expiry date is also specified in the option contract. For contracts within the same class and having the same exercise price, the longer is the time to maturity, the greater is an option's value. This holds for both Call and Put options. In other words, the longer is the time to maturity, the greater is the chance that the stock price will exceed the exercise price. Thus, options with a longer maturity date result in a higher value of Call and Put options, *ceteris paribus*.

1.3.3.5. The Variance

Other things equal, a volatile market will generate a higher value of Call-Put options than will a stable market. This is because a higher volatility increases the probability of the holder making large gains.

This is the only factor affecting option pricing that is not directly observable; yet it has a significant effect on an option's value. As one can observe, volatility cannot be predicted accurately; it is often approximated by the standard deviation (an issue discussed in Chapter Four). It is also measured by the K^* of an option. K^* is defined as the first derivative of the Call price with respect to the asset's price volatility. It is expressed as follows:

$$K^* = \partial P / \partial \sigma \quad (1.11)$$

where P is the Call price and σ is the asset's price volatility.

The preceding description of the relationship between an option's value and factors which affect prices of Call-Put options demonstrates a positive relationship for Call options, on the one hand, and a negative one for Put options, on the other. These factors are also fundamental for modelling European options. Thus, B-S make use of

them and, coupled with additional assumptions, they thus represent the European Call price as follows:

$$C^E = f (S, X, \sigma^2, T, R_f) \quad (1.12)$$

Examination of this equation shows that the value of a European Call option is a function of the stock price (=S), the exercise price (=X), the variance (=σ²), the maturity date (=T), and the risk-free rate (=R_f). It also shows that the option's value increases continuously as either time, or the variance, or the risk-free interest rate increase. In each case, it approaches, as a maximum, the stock's price.

We turn now to the theoretical valuation of option contracts. This discussion deals with models which were propounded by B-S. Merton (1973) indicates that the B-S model is particularly attractive because it is a complete general equilibrium formulation of an option's value. Since the final formulation is a function of observable variables and the standard deviation, then the model may be subjected to direct empirical tests.

1.4. THE THEORETICAL VALUATION OF OPTIONS PRICING MODEL

In addition to the five factors described above, other factors-- bankruptcy, variability of interest rates, and taxes--can also affect an option's value. For simplicity, B-S (1972) invoked six major assumptions. We discuss below these assumptions, hedged positions, and some general applications of the B-S model. These outline the necessary conditions for the theoretical valuation of options.

1.4.1. Assumptions

Black and Scholes assume that:

- (i) the short-term interest rate is known and is constant over time¹⁸. It is considered as the rate of borrowing and lending for investors;
- (ii) stock prices follows a random walk in continuous-time with a variance proportional to the square of the stock price. This implies that the distribution of possible stock prices at the end of any finite interval will be log-normal. The variance rate of the return on the stock is assumed constant¹⁹;
- (iii) there are no dividends nor does the exercise price change over the life of the contract;
- (iv) there are no transaction costs in buying or selling the underlying stock. The model may be extended to account for transaction costs, such as tax rates; however, the difficulty is that there is not one unique tax rate;
- (v) there are no restrictions on short selling. A seller will agree to settle with the buyer at some future date by paying him an amount equal to the price of the security on that date; and, finally,
- (vi) trading takes place continuously in time.

1.4.2. Hedged Positions

The value of an option, given these assumptions, will depend only on the stock price, Time and on those variables that are known constants. In this regard, there is a unique value that will always equilibrate an option's value and the price of the underlying stock. This sets up a risk-free hedged position.

Price movements in an option will be offset by opposite price movements in the stock. In other words, when one holds a combination of stocks and options, movements in the

¹⁸See Merton (1973) for details on the effects of variable interest rates.

¹⁹If the variance is not constant, the hedge will not be riskless. This is true only in a static, but not a dynamic, setting. See Chapter Two for details on the dynamic structure.

price of the stock are offset by opposite movements in the value of the option. Thus, one could expect to earn the risk-free rate (=RFR) on a perfectly hedged²⁰ position.

1.4.3. Option Valuation Models

1.4.3.1. The Black-Scholes(B-S) Model

The above argument, together with that discussed in section 1, inspired B-S to construct a continuous-time model. This model is based on the principle that there are no arbitrage opportunities in the market and it is applied exclusively to European options. B-S assumed that the probability distribution for expected rates of return on the stock over time was normal.

Given this, B-S developed the following model to calculate the value of an ECO on a stock that pays no dividends:

$$CP = SN(d_1) - Xe^{-rt} N(d_2) \quad (1.13)$$

where,

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r_f + 0.5\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

CP = Call option price

S = Current asset price

X = Exercise price

t = Time to expiration

σ = Standard deviation of the stock price

e = Napierian constant (e = 2.71828)

²⁰A hedger has the objective of offsetting any unfavourable price movement in a long position in the stock and a short position in the option.

$N(d_1)$ and $N(d_2)$ indicate cumulative probability density. This indicates the probability of getting a deviation that is below d (a movement up and down in the stock price).

1.4.3.2. Use of the B-S model

The B-S model is relatively easy to use in practice. The standard deviation is not directly observable but can be estimated. To compute the value of a Call option using the B-S model, the values for d_1 and d_2 are first computed. Then the values for $N(d_1)$ and $N(d_2)$ are found in statistical tables of the normal deviate. Finally, $N(d_1)$ is multiplied by the current stock price, and $N(d_2)$ is multiplied by the present value of the exercise price and the two products are netted.

It should be mentioned that there is another model for the determination of option values. This is the Binomial model. The method is, however, much more difficult than the B-S model; that is, the computing time is considerably longer.

The following section presents the Binomial model and its limitations.

1.4.3.3. Binomial Option Pricing Model (BOPM) against B-S Model

The description of the B-S option pricing model indicates explicitly that the model considers only terminal boundary conditions; thus, this model is ultimately applicable only to European options which do not pay dividends. However, the model loses its efficiency and applicability when employed with American options. This is because an American option may be exercised at anytime on or before the expiration date. This limitation for solving the option pricing problem using the B-S model led Merton (1973) to derive theorems about the properties of option prices based on assumptions sufficiently weak to gain universal support. To do so, Merton added structure to the

problem by invoking additional assumptions, such as that the option to be priced must be neither a dominant nor a dominated security in perfect or imperfect markets.

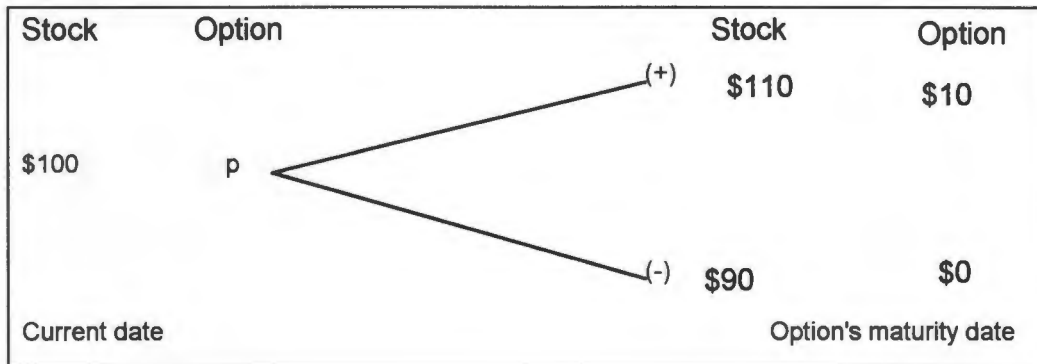
Considering the shortcomings of the B-S model, Rendleman *et al* (1979) indicate a preference for the BOPM, as it makes allowance for the above-mentioned factors. The model provides solutions not only for a closed-form European option price but also for American options on dividend-paying shares. In order to do so, it assumes that the underlying asset price obeys a binomial generating process. This means that in any possible states the asset price can either go up or down at each different period of time. These price movements over all time periods will then generate a binomial distribution of possible prices of the underlying asset at expiration. Since the value of an option on such an underlying asset is either zero or its *intrinsic value*, the premium can be calculated by adding up all the possible option values at expiration, weighted by the probability of each value.

In practice, the BOPM is really most useful as a pedagogical tool since its process (without involving the complexity of stochastic differential equations) appears easy to follow. The BOPM makes possible the accurate valuation of American options on dividend-paying shares because the option's value can be calculated at the beginning of each intermediate time period. If no early exercise is permitted, the binomial method converges to the B-S model.

As a mere example of the one-period BOPM, suppose a Call option on a stock with an exercise price of \$100. Assume that the current price of the stock is \$100 and the possible movements of the price on the option's maturity date are \$110 (when it rises) and \$90 (when it falls such that the option would expire worthless). Thus, the present prices and the end-of-period pay-offs of the stock and option may be represented as follows:

Figure 1.1

The One-Period BOPM



This model can be extended to n-periods in which the price of the underlying stock can take one of two values at any given time t , given the price of the stock at $t-1$. Although this model assumes a binomial distribution for the returns of the stock, it nevertheless gives an approximation to option pricing. Thus, the model may be used to solve continuous time option pricing problems.

1.4.3.4. Extensions of the B-S Model

As demonstrated previously, the derivation of the B-S option pricing model involves six highly restrictive assumptions: (i) there are no transactions costs, taxes or restrictions on short sales; (ii) the risk-free rate of interest is constant; (iii) the log-normality of future asset prices; (iv) the stock pays no dividends; (v) the option is an European option; (vi) and that trading takes place continuously.

Subsequently, a number of researchers have relaxed these assumptions. They have done so since they found the assumptions both too weak and too general. Some such simplifications have been:

- (i) Cox and Ross's(1975) relaxation of the short sale constraint assumption;

- (ii) Merton's(1973) relaxation of non-dividend paying stock and the stochastic interest rate assumptions; and,
- (iii) Geske's(1977) analysis of the effects of differing tax rates (transaction costs) on capital gains and dividends.

All these variants have been undertaken in an attempt to extend the B-S model.

In spite of some criticisms, the B-S model is particularly attractive because it is a complete general equilibrium formulation of the problem of option pricing and because, in its ultimate formulation, it includes observable variables, thus making the model subject to direct empirical tests.

1.4.4. Applications of Option Pricing Model

The option pricing model is an important tool in the field of finance. Since the option is a particularly simple type of contingent-claim liability, the theory of option pricing may lead to a general theory of contingent-claims pricing. Brennan (1979) emphasises that all securities can be expressed as combinations of basic option contracts, and as such, a theory of option pricing constitutes a theory of contingent-claims pricing.

In recent years, the option pricing model has been used to evaluate a wide range of contracts including the insurance implicit in government loan guarantees, the valuation of market timing advice, and the efficiency of dynamic portfolio strategies, as well as the effects of the tax code on company performance.

As a mere example of this application, B-S (1973) suggested that the equity in a levered firm can be thought of as a Call option. When shareholders issue bonds, it is equivalent to selling the assets of the firm to the bondholders in return for cash and a Call option on the firm.

Given the B-S assumptions, the value of the equity is equal to the discounted value of the bonds and a Call option. If, on the maturity date, the value of the firm ($=V$) exceeds the value of the bonds ($=B$) the shareholders will exercise their Call option, by paying off the bonds and keeping the excess. On the other hand, if the value of the firm is less than the face value of the bonds, the shareholders will default on the debt by failing to exercise their option. Therefore, at the maturity date, the shareholders' wealth ($=S$) is:

$$S = \text{MAX}[0, V-B] \quad (1.14)$$

This is, of course, a ECO, and we can use the B-S model to value it.

The bondholder's wealth, ($=D$) is:

$$D = \text{MIN}[V, B] \quad (1.15)$$

1.5. CONCLUSION

The theory of option pricing has been shown to be applicable to contingent liabilities of a firm. By extension, then, the theory can be employed to evaluate a GLG. It is to that which we now turn.

CHAPTER TWO

**THE THEORY OF THE VALUATION
OF
LOAN GUARANTEES**

2.1. THE EFFECTS OF LOAN GUARANTEES

In this section, the applicability of the option pricing model to government loan guarantees is considered. The section begins with a review of an early and 'classical' case: the valuation of loan guarantees to the Chrysler Corporation. In the Seventies, the Chrysler Corporation experienced a financial crisis and it appeared as if it would go bankrupt. However, the U.S. government intervened and provided a 'rescue package' to avoid bankruptcy on the ground that would be desirable from a social point of view.

Despite this, some of Chrysler's creditors stated:

" Some people feel bankruptcy is the preferable solution. A reorganization could bring 70 cents to 80 cents on the dollar. With the government in, we do not know what our chances of recovery would be" Chen *et al* (1986, p.102)

This Chapter reviews the literature which flows from this classic case and which laid the theoretical framework for the analysis of the cost of government loan guarantees. The analysis does not strictly follow this classic case, but rather attempts to include the technique of hedging exchange rates and the use of currency options, and considers the case of fully--or partially--guaranteed loans in order to value the cost of loan guarantees. These techniques will be applied in Chapter Four, below.

After a review of the theoretical literature regarding the value of loan guarantees in section 2.2, 2.3 considers the economic and financial impact of a government loan guarantee. Section 2.4 then presents a general overview of the government's loan guarantee on the outstanding financial claims of Chrysler and of International Computers Limited in the United Kingdom.

2.2. THE THEORETICAL VALUE OF LOAN GUARANTEES

As noted previously, there are two competing general models for the valuation of contingent claims. These are the continuous-time and the discrete-time models. The pioneering work of Merton (1977), followed by that of Jones and Masson (1980), and of Sosin (1980), analysed loan guarantees and deposit insurance within a continuous-time model with a single homogeneous debt issue. The literature reveals that:

- (i) Merton concentrated on laying the foundation for the cost of government loan guarantees. He outlined the development of a deposit insurance pricing model by noting the relationship between deposit insurance and common stock Put options. He derived the payoff structure of a GLG based on reasonably universal assumptions regarding Put option pricing. Then, he established the following relations: (a) the promised payment (in the loan guarantee model) corresponded to the exercise price (or the Put option's price); (b) the value of the firm's assets corresponded to the common stock's price in the Put option model; and, he concluded that the guarantor, by guaranteeing the debt issue, has issued a Put option on the assets of the firm which gives management the right to sell those assets for B dollars at the maturity date of the debt.

Noting the relationship between the values of the risk premium [$= R(T)-r$], the variance rates, the firm's values and the variety of maturity dates, he demonstrated that the cost of loan guarantees can be substantial.

- (ii) Using option pricing theory within a continuous-time framework, Jones and Masson developed various contingent claim models of a GLG for (a) a fully and partially guaranteed issue of non-callable coupon debt, and (b) junior and senior non-callable debt with guarantees. Jones and Masson based their analysis on the assumptions invoked by B-S (1973) and Merton (1973, 1974). Some of their

findings were: that the guaranteed debt will always trade like a riskless bond, independent of the risk level of the firm, i.e., that the guaranteed debtholders will have no incentive to monitor the actions of the firm; that the value of the guarantee would become zero or worthless as the market value of the firm tends to infinity; and, that the relationship between the value of a GLG and the risk of the firm is positive; i.e., *ceteris paribus*, the greater the value of the GLG, the greater the risk of the firm. However, Jones and Masson also included stochastic interest rates, tax effects and callable convertible debt into their analysis of a GLG. Thus, their results are regarded as low estimates of the true value of loan guarantees.

(iii) Sosin also illustrated how option pricing theory could be used to estimate the purely pecuniary costs of Federal loan guarantees and how these would affect the firm's senior and junior debt. He suggested that guarantees should be employed if the present value profitability index were than less unity, $\xi < 1$. He introduced this into his analysis in order to consider a discrepancy between the cost and the market value of a project. He concluded that the value of loan guarantees increased with an increase in the percentage of the loan that is guaranteed. From this result, Sosin established the functional relationship between the percentage guarantee required to make a firm indifferent between undertaking, and not undertaking, a project. The relationship reveals that as the standard deviation, σ , increases the costs of guarantees and the interest savings increase dramatically. Similar results have been shown between the profitability index, ξ , and the increase in the standard deviation.

(iv) On the other hand, Brennan (1979), Rubinstein (1976), and Stapleton and Subrahmanyam (1984) valued contingent claims using discrete-time models. Their approach eliminated the need for the construction of hedged positions as propounded by B-S. In other words, the debate between B-S and Brennan occurred

since B-S placed no restrictions on the investor's preferences and assumed that asset trading took place continuously, whereas Brennan imposed stronger restrictions on investor's preferences and assumed that assets were traded in a discrete-time framework.

Following the work of Brennan, Chen *et al* (1986) focussed on the discrete Risk-Neutral Valuation Relationship, (RNVR), to analyse the effects of a GLG on the outstanding claims of the Chrysler Corporation. Their analysis tested three hypotheses:

- (i) senior bondholders should gain when the firm has succeeded in obtaining a GLG;
- (ii) chrysler's shareholders should also benefit from the loan guarantee from the government; and,
- (iii) the risk associated with Chrysler's common stock after the GLG should be lower than before the government loan guarantee.

They found that loan guarantees could have a positive effect upon a firm's valuation. They also suggested that future loan guarantee programmes should be structured so as to provide fees to the government in exchange for loan guarantees since they provide value to the firm's owners and creditors. This is very important because some fees and premiums charged by these programmes are not sufficient to offset the costs of insuring losses. In this regard, it was argued that no one, at the time, really knew how much those losses would be. For example, the evidence revealed²¹ that:

"the past fiscal years in the USA have been characterized by an increase defaults on guaranteed loans. Federal exposure to potential risk from these programs--which range from defaults on guaranteed student loans and mortgages to making good on deposit insurance in banks and savings and loan association--has risen dramatically in recent years. Only two

²¹Federal Credit Assistance and Insurance Programs, (Serial No. 101-103, November 16 1989, US Government Printing Office, Washington 1990).

decades ago, federal credit and insurance programs totaled just over \$400 billion. Today, that total is more than \$5 trillion.... Losses from these programs have already cost the taxpayers a billions of dollars and the government does not today know the full magnitude of the losses incurred...."(Federal Credit Assistance and Insurance Programs, November 6 1989, p. 10-11)

This suggests that federal financial assistance beyond that already provided by the government would be needed to pay for growing losses. It is the intent of the section to show how one can obtain a 'real' cost of such losses.

Prior to the estimation of the cost of loan guarantees, the following section briefly introduces *Contingent Claims Analysis, (CCA)* and the formulation of the problem to be treated.

2.2.1. Contingent Claims Analysis (CCA)

This section presents CCA as a basis for the pricing of corporate liabilities. This was developed by B-S (1973) and by Merton (1974, 1977), and is a general equilibrium methodology for the pricing of corporate liabilities. Following these authors, the usual assumptions made in the CCA literature are:

- (i) frictionless markets: capital markets are perfect with no transactions costs, no taxes and there is equal access to information for all investors;
- (ii) trading in assets takes place continuously in time;
- (iii) the riskless short-term rates ($= r$) are known and constant over time;
- (iv) the instantaneous variance of return ($= \sigma^2$) on asset value ($=V$) is constant over time;

(v) the dynamics for the value of the firm ($=V$) through time can be described by a diffusion-type stochastic process with a stochastic differential equation:

$$dV = (\alpha V - C)dt + \sigma V dz$$

where,

V = the value of the firm. It follows a lognormal diffusion processes.

α and σ^2 are the instantaneous expected rates of return and variance of returns on all assets. Both α and σ^2 are assumed to be constant;

C = the total cash outflow per unit time, e.g., dividends or interest payments;

and,

dz = is a standard Gauss-Wiener process with zero mean and variance = dt ;

(vi) All bonds issued by the firm are discount bonds maturing in year t ;

(vii) The firm liquidates, at time t , at which time bond holders are to receive the full face value of their bonds, and equity holders are to receive any residual value; and, finally,

(viii) There exists perfect bankruptcy protection: Firms cannot file for protection from creditors except when they are unable to make required cash payments.

Looking carefully at these assumptions, one can deduce-- on the one hand-- that the value of a particular issue of corporate debt depends essentially on three factors: (a) the required rate on riskless (in terms of default) debt, e.g., government bonds; (b) the various provisions and restrictions contained in the indenture, e.g, maturity date, coupon rate, seniority in the event of default etc; and, (c) the probability of default. On the other hand, the value of corporate debt must satisfy the partial differential equation expressed above²².

²²A description of Ito's Lemma-- a stochastic differential equation-- can be seen in Mc Kean *et al* (1965)

In contrast, Rubinstein (1976) and Brennan (1979) employed the assumptions below in a discrete-time framework:

- (i) Non-satiety: this implies that the larger the payoff in any state, the greater is the current price of a security;
- (ii) The capital market is perfectly competitive with no transaction costs, no taxes and equal access to information by all investors;
- (iii) The representative investor exhibits constant proportional risk aversion;
- (iv) There are no dividends or coupon payments. There are no payouts from either the firm or its guarantor to the shareholders and/or bondholders before the maturity date of the discount debt;
- (v) The law of one price obtains: all securities or portfolios of securities with identical payoffs sell at the same price;
- (vi) The debt issuer's asset values and the guarantor's asset value and aggregate wealth, are multivariate lognormally distributed; and,
- (viii) The conditions for aggregation are met so that securities are priced as though all investors had the same characteristics as a representative investor.

Thus we may make the general observation that an important distinction between the discrete-time and continuous-time variants of pricing models of corporate liabilities is that regarding the investor's expectations about the variance. In a continuous-time model, the investors agree about the variance but there is no such agreement in the discrete-time model. In this respect, it can be argued that the investors will take infinite positions in the formation and maintenance of a riskless hedge when disagreement occurs.

2.2.2. Quantitative Analysis

The literature illustrates that a number of studies have derived the value of corporate liabilities within the general equilibrium assumptions put forward by B-S. To mention but one, Merton(1977) recognized that the same basic assumptions could be applied in evaluating deposit insurance and, thus, loan guarantees. The essential terms of the model constructed in Merton's paper make it equivalent to a European Put option (EPO)²³ on a common stock. That is, the owner has the right to sell an underlying asset at a predetermined price, called the exercise price, on a specified date--the maturity date. Translating into a GLG model, the government will exercise the underlying option at the promised payment in **B** dollars, if the firm is unable to satisfy all the indenture requirements at the maturity date. However, as a matter of practical interest, the more interesting question is what will happen if there is 'premature' default? This issue extends the analysis to that of American Put options. Unfortunately, the contingent claims formulation of this problem has not been widely developed in the literature but one the solution can be approximated using various combinations of assumptions. One recent result is that of Saunders and Allen (1993)²⁴.

These authors criticized Merton's model on the basis of two assumptions: (a) that of self-closure; and, (b) that of forbearance or regulatory closure. While *the self-closure* assumption explains the situation whereby the shareholders of the company choose to exit voluntarily via the sale of the company, *the forbearance* assumption, in contrast, allows the company to take advantage of the extra time so as to return to solvency, i.e., the case of 'premature' bankruptcy.

²³Insurance contracts can also be viewed as EPOs. These protect insured parties from losses such as natural disasters, foreign political risk, and pension benefit losses, e.g., Pension Benefit Guaranty Corporation in the U.S.A. (see Su-Jane Hsieh *et al*, (1994))

²⁴Allen, L., and A. Saunders, "Forbearance and Valuation of Deposit Insurance as a Callable Put", Journal of Banking and Finance , (Vol. 17, 1993, pp. 629-643)

Given this divergence between voluntary closure and forbearance, Allen and Saunders generated a model of loan guarantees for a compound option. Such an option is one that is an American Put held by a company and a Call option retained by the insurer-government.

In general, then, the basic assumption of the American Put option discussed previously, shows that the option can be exercised before and up to the maturity date. This suggests that the company's shareholders will benefit by renewing the Put option and remaining in operation until the next regularly-scheduled maturity date.

From the above, it may be concluded that the American Put option is more complex than is the European Put option. Different maturities observed with American Put options make them more valuable than an EPO. In contrast, if we assume that: (a) the Put writer-government loan guarantor is unwilling to write a new option to replace the old-expired option; and (b) the GLG may not forcibly close the company in case of default, such as the case of a noncallable American Put option, then the model propounded by Saunders and Allen is weakened. This results from the fact that whenever the government--Put writer enforces premature exercise of the option by calling the Put option and closing the company, the guarantee will have a zero value. As a result, the value of the American Put option may be minimised relative to a European Put option.

2.2.2.1. Loan Guarantees Model

Risky-debt guarantees are used in a variety of situations. In project financing, it is often the case that a third-party guarantees a loan. While the third-party is sometimes a government or its agency, it can also be a private company.

In order to analyze the loan guarantee model, we specify the assumptions underlying the model. The firm is contractually obliged to meet the indenture requirement of the debt contract at the maturity date. In the event of default, this means that if the firm does not make the promised payment to the bondholders, the government or its agency will meet these payments. Thus the guaranteed debt corresponds to a 'risky' discount bond, and the value of the debt guarantee is equal to the value of an EPO. This reflects a conclusion derived by Merton (1974) in attempting to establish the relationship between deposit insurance and the value of loan guarantees.

2.2.2.2. Formulation of Loan Guarantees Model

Below we examine a simple model of loan guarantees in the continuous-time and we illustrate the model of loan guarantees for the common stock of Chrysler in a discrete-time framework. In both cases, the following assumptions²⁵ underlie the model of loan guarantees; however, one would add the assumption of a Risk-Neutral Valuation Relationship (RNVR) for a discrete-time model. This is examined below.

In general, the assumptions of the problem of loan guarantees may be summarised as follows:

- (i) the firm borrows money by issuing a single homogeneous debt issue;
- (ii) at the maturity date ($= t$) the firm promises to pay a total of B dollars²⁶ to the bondholders;
- (iii) there is a positive probability that the value of the assets on the maturity date will be less than or equal to the promised payment, B dollars. Thus, the debt is risky;

²⁵In an abbreviated form, if $t=0$ and $V > X$ then the guarantee has no value. This implies that $G(V,0) = \text{Max}(0, V-X)$. If $V < X$, then the guarantee is worth the difference between X and V ; that is $G(V,0) = \text{Max}(0, X-V)$.

²⁶Note that this paper will use dollars to avoid exchange rate problems; see Chapter Four.

- (iv) there exists a third-party guarantee of the payment to the bondholders in the event of default. This implies that the government has an incentive to monitor the firm's behavior and therefore, the guaranteed debtholders have little or no incentive to monitor the actions of the firm; and,
- (v) the government has agreed to fully or partially guarantee a new issue of debt.

To determine the value and the cost of the government loan guarantees, we begin by evaluating the payoffs to equity, E, and to debt, D, for the 'risky' discount bonds. We make use of the formula developed by Merton (1974, 1977). He suggests that the value of the equity and debt after undertaking the project may be expressed as follows:

$$E = f(V, t) = VN(X_1) - Be^{-rt}N(X_2) \quad (2.2)$$

and,
$$D = F(V, t) = Be^{-rt} [N(H_2) + \frac{1}{d}N(H_1)] \quad (2.3)$$

where,

$$X_1 = \frac{\log\left(\frac{V}{B}\right) + (R - 0.5\sigma^2)T}{\sigma\sqrt{t}}; \text{ and, } X_2 = X_1 - \sigma\sqrt{t}$$

$$H_{1,2} = \frac{-0.5\sigma^2 t \log(d)}{\sigma\sqrt{t}}$$

where $X_1(H_1)$ and $X_2(H_2)$ represents the instantaneous standard deviation of the returns on all assets. Further,

V = the current market value of the assets of the firm

B = the promised payment (which corresponds to the exercise price); and,

d = the debt to firm value ratio = $\frac{Be^{-rt}}{V}$ = the firm's quasi-leverage ratio²⁷

Now, the value of the government loan guarantee is given by:

²⁷It is the ratio of the present value of the promised payment, discounted at the riskless rate, to the current value of the firm.

$$G(V, T) = Be^{-rt} N(d_2) - VN(d_1) \quad (2.4)$$

where,

$$d_1 = \frac{\log\left(\frac{B}{V}\right) - (R + 0.5\sigma^2)T}{\sigma\sqrt{t}}; \text{ and, } d_2 = d_1 + \sigma\sqrt{t}$$

After solving these, one can apply the following equation so as to determine the correct insurance premium for the government loan guarantee. This is a fraction of the amount of money guaranteed and is expressed as follows²⁸:

$$\frac{G(V, T)}{Be^{-rt}} = 1 - e^{-(R(t) - r)t} \quad (2.5)$$

According to the discrete-time model, the risk-neutral valuation relationship (RNVR) may be used. This eliminates the construction of a riskless hedge and modifies the formulation of the current market value of equity and senior bondholders as follows:

$$V_E^P = [V_0^P - BR^{-1}]N(-K'_B) + R^{-1}\sigma_{Fb}(-K'_b) \quad (2.6)$$

and,

$$V_S^P = D_1R^{-1}N(-K'_1) + V_0^P[N(-k'_0) - N(-K'_1)] + R^{-1}\sigma_F[n(-k'_0) - n(-k'_1)]$$

where,

$$K'_B = \frac{B - V_0^P R}{\sigma_F}, \quad K'_1 = \frac{D_1 - V_0^P R}{\sigma_F}, \quad K'_0 = \frac{-V_0^P R}{\sigma_F}$$

and,

$$V_0^P = \text{the post-investment current value of the firm;}$$

²⁸where, $R(t) - r$ is the risk premium. It is defined as the risk structure of interest rates under the assumption that the term structure is 'flat' and 'nonstochastic'.

σ_F^2 = the variance of the post-investment;

D_1 = the face value of the discount bond at maturity;

$R = 1$ plus the risk-free interest rate; and,

$n(-k'_j)$ = the standard normal density function.

Note that the current market value of the firm is calculated as follows:

$$V_0 = V_E + V_S \quad (2.7)$$

The preceding may be applied to value of the claims of a firm that has stockholders and a single class of bondholders. In the case of Chrysler and the GLG, the ending payoffs to the junior bondholders ($= \tilde{Y}_J^P$) can be expressed as:

$$\tilde{Y}_J^P = \begin{cases} D_2 & \text{if } V_3 \geq B \\ D_2 & \text{if } D_1 < V_3 < B \\ D_2 & \text{if } V_3 \leq D_1 \end{cases}$$

where V_3 represents the postinvestment cash flows of the firm. It is equal to the sum of the cash flow generated from the new risky project and the end-of-period cash flow of the firm in the absence of a loan guarantee.

Thus, according to the RNVR, the value of a loan guarantee, ($= V_G$)²⁹, may be calculated as follows:

$$V_G = D_2 R^{-1} [1 - N(-K'_2)] + [V_0^P - D_1 R^{-1} \sigma_F] [N(-k'_1) - N(-k'_B)] - R^{-1} \sigma_F [n(-k'_1) - n(-k'_B)] \quad (2.8)$$

²⁹Within B-S (1973) option pricing, the value of a loan guarantee is the value of the Put option issued by the guarantor or issuer.

2.3. RELATIONSHIP BETWEEN THE GLG, WEALTH TRANSFERS AND INCENTIVES TO INVEST

This relationship was discussed by Selby *et al* (1988). Throughout his inquiry into the CCA, Selby examined the relationship between GLG, wealth transfers and incentives to invest within the International Computer Limited (ICL). Like the Chrysler case, the British government announced in the early 1980s that it would guarantee 200 million Pounds Sterling of new borrowings advanced by banks to ICL. As discussed previously, any default by ICL would compel the government to pay bondholders at the maturity date. Fortunately, ICL performed well and delayed this adverse economic consequence.

As far as the relationship between GLG, wealth transfers and incentives to invest was concerned, Selby *et al* found that the guaranteed loan to ICL was significant as this benefit raised the market value of its shares. They noted that market value was a function of the maturity structure of the existing loans and their priority relative to the newly guaranteed loan; as a result they argued that compound options should be used rather than a single period option model. The following equation shows how to value loan guarantees using compound options techniques³⁰:

The value of a short senior bond ($=B_1$) is given by:

$$B_1 = \left\{ \frac{S_s}{S_s + S_L} \right\} (V_0 - BS(V_0, S_s + S_L)) \quad (2.9)$$

where,

S_s = the face value of the short senior bonds; and,

S_L = the face value of long senior bonds

³⁰The model assumes the existence of short and long bonds. While the short bond has one year of maturity, a long senior bond has five years of maturity. In other words, the model can be applied when we have multiple loans outstanding associated with different maturity dates.

The valuation formula for the long senior B_3 is given by:

$$\begin{aligned}
 B_3 = & e^{-rt_1} \left\{ \frac{S_L}{S_S + S_L} \right\} \int_0^{S_S + S_L} V_1 f_1(v_1) dv_1 + e^{-rt_1} S_L \int_{S_S + S_L}^{\bar{V}_1} f_1(v_1) dv_1 \\
 & + e^{-rt_2} \int_{\bar{V}_1}^{\infty} \int_0^{S_L} v_2 f_2(v_1, v_2) dv_1 dv_2 + e^{-rt_2} S_L \int_{\bar{V}_1}^{\infty} \int_{S_L}^{\bar{V}_2} f_2(v_1, v_2) dv_1 dv_2 \\
 & + \dots
 \end{aligned} \tag{2.10}$$

where $f_i(V_1 \dots V_i)$ is the i^{th} dimensional normal probability density function; \bar{v}_i , is the i^{th} critical value of the firm, is determined as the solution to the equation; $E(\bar{V}_i) = x_i$, where $E(V)$ is the value of the equity (including any possible accrued interest) due at maturity $t = i$.

The first term on the RHS of the equation is the long senior bond's proportion of the discounted expected value of the firm if default occurs at maturity $t=1$ when $v_1 \leq (s_s + s_l)$. The second term is the expected discounted value of S if default occurs and when $v_1 \leq s_s + s_l \leq \bar{v}_1$. The third term is the discounted expected value of the firm if default occurs at time $t = 2$ (conditional on a default not occurring at $t = 1$ and $v_2 \leq s_s + s_l \leq \bar{v}_2$ etc.)

As far as wealth transfers and incentives to invest are concerned, Selby *et al* showed that the wealth transfers could be large and, therefore, could affect the shareholders' incentives to invest. In this context, Myers(1977) argued that the fact of avoiding bankruptcy in order to raise new investment finance could affect the firm's capital structure decision. It might lead to a lower amount of debt, or to a requirement for less restrictive covenants in loan guarantees.

2.4. ECONOMIC AND FINANCIAL IMPACT OF A GOVERNMENT LOAN GUARANTEE (GLG)

This section considers the consequences of GLG at a macroeconomic level. It examines:

- (i) the impact of government intervention on financial markets, following the lines of Merton (1974) and Sosin (1980); and,
- (ii) the impact on labour markets following Parsons(1980) and Lapan (1976), and finally, the impact on credit market using Fried's (1983) analysis.

2.4.1. The Impact of a GLG on Financial Markets

Merton and Sosin observe that government intervention in financial markets (by providing loan guarantees) affects the probability and timing of default of the firm and the distribution of wealth among bondholders.

2.4.1.1. The Impact of GLG on the Probability and Timing of Default of the Firm

To see this, we assume that the new project is fully or partly financed by new debt. This implies that a loan guaranteed by the government substitutes a new debt (junior debt) issue for the firm's existing debt (senior debt). In this respect, a loan guarantee affects the probability of default in two ways: (a) via the leverage effect; and (b) a maturity effect.

2.4.1.1.1. The leverage effect

This means that, in the event of default by the firm, the government will meet the promised payment. In other words, the holders of the guaranteed loan shift the default risk of the firm to the government. Thus, the firm's securities will be priced to return the riskless rate of interest.

2.4.1.1.2. The maturity effect

In general, the effect of the loan guarantee can be seen on the components of the firm, e.g. its value, V , its expected growth rate and the risk of its existing debt, e.g., its discount rate, and the time to maturity, and, the expiration date of the guaranteed loan.

For a given quasi-leverage ratio, a change in maturity can affect the probability of default (Merton, 1974). To see this, suppose the maturity date of the guaranteed loan increases, then the probability of default will decrease to zero. This is because the promised payment to the debt will grow, as maturity increases, at a rate that is less than the rate that would exist without the guarantee, since the government absorbs the risk of default. However, for some finite values of the maturity of the guaranteed loan, it is possible for the probability of default to increase. This will depend on the firm's initial leverage and the relative maturity of the government loan guarantee.

2.4.1.2. The Impact of GLG on the Distribution of Wealth

It is argued that GLG may be used as an inducement for stockholders and bondholders to reorganize the firm's operations and capital structure. This means that the loan guarantee introduces a new claimant into the analysis--the junior bondholders-- and this affects the value of current stockholders' and senior bondholders' claims. To understand this, one can compare the value of the debt prior to the government guarantee (Merton, 1974) and the present value of the guarantee valued as a European Put, (Merton, 1977). In this regard, senior bondholders gain as long as the change in the current market value of the firm is greater than zero. In other words, the difference between the present value of the guarantee and the value of unguaranteed debt must be greater than zero. Thus, senior bondholders' wealth will increase since the injection of new funds increases the likelihood that senior bondholders will be paid off in full at the maturity, and therefore, it minimizes their risk of not paying off.

Now it is known that shareholders' wealth maximization is consistent with all projects with positive net present values, (NPVs). Thus, if the injection of new funds minimizes the risk of senior bondholders not being paidoff (and thereby contributes equity to the firm), then there will be a profit so that stockholders will not lose from the issue of subordinated debt. Thus, one can conclude that the common stockholders will gain if the project undertaken has a positive NPV³¹.

2.4.2. The Impact of GLG on the Labour Markets and the Present Value of National Output

Given the usual assumptions about financial markets, i.e., that capital markets are perfect, the GLG would have no impact on output. The present value of the cost to taxpayers would be offset by an increase in wealth to security holders. However, labour markets are typically imperfect ones. It is the intent of this section to consider the implications of such imperfections on the decision to provide loans which are guaranteed by the government.

In economic theory, it is usually assumed that there are two factors of production: labour and capital. Labour is typically identified as mobile while capital is assumed fixed in the short-term. If we assume two sectors in the economy--a manufacturing sector and the rest of the economy-- then a labour force once unemployed in the manufacturing sector will be absorbed slowly into the rest of the economy. By definition, the rate at which labour is employed is assumed to be proportional to the level of unemployment in the economy. This is characterized by the following labour transfer function:

$$\frac{dL}{dt} = \alpha U \quad (2.11)$$

³¹If the NPV is negative, it is obvious that the stockholders would have been willing to call the debt without the aid of the government.

where,

L = the total level of employment, outside of the manufacturing sector;

α = the instantaneous rate at which unemployed labor is absorbed into the rest of the economy; and,

U = the level of unemployment

Given this, it can be argued that such imperfect markets would lead to a significant increase in unemployment as a result of a default of the firm. Therefore, a loan with a government guarantee can be used to reduce or delay the probability of default. This will affect the magnitude and timing of unemployment and therefore, increase the present value of national output.

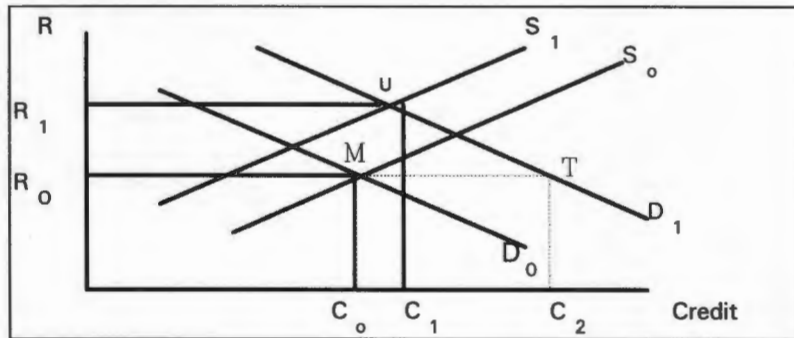
2.4.3. The Impact of GLG on the Credit Market

In practice, the market for credit is characterized by the supply and demand for credit. These are functions of the loan rate of interest and the quantity of credit. As depicted in Figure 2-1, D_0 represents the demand for the loans by the private sector and S_0 reflects the increasing marginal costs of lending³² (i.e., it is the credit supply curve). The point M indicates the equilibrium level and corresponds to the quantity of credit, C_0 and the rate of interest, R_0 .

³²These costs consist of the operating costs of lending and the costs of obtaining funds to lend.

Figure 2-1:

The Supply and Demand of the Market for Credit



Following Fried's (1983) view, suppose that the government institutes a GLG and a fixed subsidy rate. With a loan guarantee, the government makes private loans less risky and, therefore, we expect that the demand for credit to rise to D_1 . Specifically, D_1 describes the increase in demand for credit by all potential borrowers who are eligible for government loans. This generates excess demand (i.e, MT in Fig.2) which will lead to a higher loan rate of interest, R_1 , raise prices in the capital goods market, and induce banks to issue more loans by selling government securities and thus shift the credit supply curve to the left, S_1 . The new equilibrium level of loans occurs at the point, U , and corresponds to the quantity of credit, C_2 .

This analysis thus far has explained that loan guarantees have a positive impact on national output and on financial markets. Loan guarantees may provide wealth transfers to existing debtholders and may affect the shareholders' incentives to invest. This conclusion may be important in other situations where, for example, firms in financial crisis are unable to restructure their existing loans. The question is whether loan guarantees provide efficient incentives to maintain the firm and its operations?

To answer this question, we examine the impact of loan guarantees to companies in the event of financial deterioration. In particular, we look at the Chrysler Corporation.

2.4.4. The Impact of GLG to the Chrysler Corporation (CC)³³

In Dec. 1979, the US Congress passed the CC loan guarantee bill in response to Chrysler's rapidly deteriorating financial position which was a result of a general decline in automobile sales due to a shift in consumer preferences towards small cars. This decline of sales, estimated at 7 percent at the end of the third quarter of 1979, had an immediate impact on the economic activity of the USA. Banks refused to lend CC money, fearing that it would not be able to repay. In spite of rejection by the banks, several negotiations were attempted and some important terms of an agreement were reached as a result of third-party intervention. This intervention resolved the conflict under the following terms:

- (i) The government: provided guarantees for up to \$1.5 billion in new loans over a 10 year period;
- (ii) Management: sold \$300 million in assets, raised \$50 million in new equity, and distributed \$762.5 million in equity to labour;
- (iii) Creditors: conceded \$100 million in existing obligations and provided at least \$ 5.5 million in new credit; and,
- (iv) Labour: conceded \$ 587.5 million in wages and salaries.

These concessions enabled CC to pay back the loans without going bankrupt, and got the company got back on its feet.

Two main reasons explain the US government's intervention in the Chrysler case:

³³Chen, Chen and Sears "The value of loan guarantees: "The case of Chrysler Corporation" Research in Finance, Vol. 6 (1986, pp.108-109); and Stiglitz, J.E. Economics, (New York: w.w. Norton and Company, 1993, pp. 6-7)

- (a) The guarantee programme was designed to help the continuity of the operation of at least some of the Chrysler plants and reduce the rate at which Chrysler's labour force would be redeployed;
- (b) The government stood to lose quite a large sum through an insurance programme it had set up several years before to secure and protect worker's pensions³⁴.

From the above brief history of CC we see that a loan guarantee by the government may be looked upon as a means of reorganizing a firm which is financially deteriorating and thus encourage new investment.

2.5. CONCLUSION

With respect to South Africa's economic and fiscal prospects, loan guarantees may be more appropriate than direct loans. One reason why loan guarantees may be a more attractive form of aid than direct loan is that contingent liabilities do not require government outlays until the firm defaults and the guarantee is called. This is an important result and it will be examined in the following chapter. While Chapter Three presents a brief profile of the Eskom and the nature of the Cahora Bassa peoject, Chapter Four estimates the cost of GLG and presents our results.

Let turn now to the profile of the Eskom.

³⁴The insurance programme guaranteed that even if a company went bankrupt , workers would still receive their pensions.

CHAPTER THREE

A BRIEF PROFILE OF ESKOM

3.1. BRIEF OVERVIEW OF ESKOM

In Chapter 2, the theory of a GLG was discussed. Three sections were reviewed to assess the impact of a GLG in the financial market. In this chapter, we present a profile of the Eskom and some background to the Cahora Bassa Hydroelectric project.

3.1.1. Capacity Management of Eskom

Eskom supplies more than 95 % of the electricity consumed in South Africa. The combined performance of power stations and associated collieries has thus far enabled Eskom to meet the growth in demand and in energy sales. Nevertheless, changes in demand patterns in the domestic sector, and expected growth in the industry sector, have caused Eskom to re-assess load management and surplus capacity. For this reason, in July 1994, Eskom adopted an International Electricity Plan (IEP). The plan lays out an optimal policy for combining methods of supply so as to meet the expected growth in demand³⁵.

Within this plan, construction programmes for various power stations are under review; and the Ingagane, Highveld and Taaibos power stations have been decommissioned. In addition, the plan takes cognisance of the creation of the Southern African Power Pool, e.g. the Cahora Bassa project, in order to import hydroelectric power from the north to meet the expected increase in demand.

3.1.1.1. Payments and Cost of Electricity

There is a growing realisation that customers are not paying for services supplied to them. Total outstanding electricity arrears at 31 December 1994 were R 923 million³⁶.

³⁵"Power for the people", Eskom Annual Report, (1994, p. 22)

³⁶Ibid., (p. 22)

This is one of the most significant financial threats facing Eskom. However, considerable attention is being applied to the problem, including cutting off service, implementing normal metered tariffs, and involving community leaders in the process of fixing prices.

Despite the high costs of electrification, the average 1994 price increase was 2% below the rate of inflation. This was a significant achievement for Eskom and occurred as a result of improvements in the utilisation of existing capacity, improvements in business and significant productivity gains which occurred during the year. In line with the Reconstruction Development Programme (RDP), Eskom has undertaken to reduce the real price of electricity by 15 % between 1995 and 2000³⁷. This will result in an additional 3% real reduction of price over the period 1997 to 2000.

3.1.2. Funding and Balance sheet of Eskom

As Table 3-1 shows, the major portion of Eskom's funding requirements are met in the local capital and the money markets. However, it can be observed that fully R800 million is to be raised in the foreign capital market during 1995; this will be by way of newly issued electrification bonds denominated in US dollars.

³⁷ Eskom Annual Report, (1993, p. 10)

Table 3-1

Eskom's Funding (in millions of Rand)

	Actual	Planned (1994)	Planned (1995)
Domestic Market			
Money market	3000	2500	2400
Capital market	1575	1800	1200
Foreign market			
Bond issues	-	500	800
Exports credits	75	100	350
Bank loans	-	-	100
Property finance	-	-	100
Total	4650	4900	4950

Source: Eskom Annual Report, (1994, p. 16)

At the end of 1994 Eskom had total assets of R47.4 billion, while revenue was R15.417 billion and net income for the same year was R2.3 billion. Eskom is entirely owned by the South African government although it is a separate legal entity³⁸, and it is funded entirely by debt (see Table 3-2 below) and accumulated reserves. Accumulated reserves increased to R16 005 million³⁹ after a transfer of R100 million to an insurance reserve to cover potential, abnormal self-insured losses not covered externally.

Table 3-2 provides an abbreviated balance sheet for Eskom. A quick reading of the 1993 and 1994 accounts shows a net increase in total assets of R2967m (R47364-R44397= R2967) and a net decrease in current liabilities of R 868 m, falling from R2582 in 1993 to R1714 in 1994.

³⁸It should be remembered that in order to illustrate the continuous-time model it has been assumed that Eskom has been privatized.

³⁹Ibid., (p. 15)

Table 3-2

Balance Sheet of Eskom

31 December	1994	1993
	Rm	Rm
Capital employed		
Accumulated Reserves	16 105	13 837
Net interest-bearing debt	2788	28027
Employment of capital		
Fixed assets	40711	38605
Non-current assets	4074	3762
Current assets	2579	2030
Total assets	47364	44397
Interest- free liabilities	2637	2137
Net assets	44727	42260

Source: ABSA BANK, The Investors' Guide, (1995, pp.154-155)

3.1.3. Investment Instruments of Eskom

Eskom offers the investor a wide variety of debt instruments. At present, it has outstanding a total of 77 bonds from which the investor can choose. These bonds (see Table 3-3 below) differ in their maturity dates and coupons offered. The Eskom Participation Note (EPN) and the Commercial Paper Bill, (CPB)⁴⁰ have been created for investors who wish to invest in short-term instruments, i.e. those maturing in 1-24 months. A well developed options market rounds out of the range of financial products offered to the investor, with Eskom making a market in options on the E172, E168 and E170 bonds.

Eskom bonds are among the most liquid of those traded on the stock exchange⁴¹ with the E168 generally considered as being the 'bell-weather' stock for long-term

⁴⁰The Commercial Paper Bill (CPB) is Eskom's prime money market instrument offered to prospective investors. Due to the extremely liquid nature of this instrument, it represents an ideal way for investors to invest short-term cash surpluses (from 1 month up to 24 months) at competitive rates with the assurance of holding a marketable security. Eskom makes a market in these instruments and it quotes two-way prices at all times.

⁴¹We refer to bonds which are traded in the stock exchange

borrowings in the South African market. Table 3-3 shows some of the more liquid bonds as well as their maturity dates, coupons, and the volume issued in the market.

Table 3-3

Investment Instruments

Loan	Redemption date	Coupon rate	Volume Issued in 1993
E167	1996	12%	1577
E169	1998	15%	2471
E172	2001	8%	791
E171	2002	-	254
E168	2008	1 1%	13815
E170	2020	13.5%	1323

Source: ABSA BANK, The Investment's Guide, (1995, pp.155)

3.1.4. Government Export-Import Credit Guarantees

At times it has been difficult to find comprehensive data on domestic sources of financing and loan guarantees. The South African government publishes only limited data on overseas liabilities. Lind *et al* (1988) use data on South Africa's imports to estimate the level of overseas trade credits supplied to South Africa in 1985 through 1987. They conclude that between \$3.75 billion and \$ 6.25 billion of overseas trade credits were needed to finance the whole of South Africa's imports⁴².

It may noted that, on the one hand, Eskom has raised \$550 m of overseas finance for a new power station, of which \$350 m is guaranteed by the UK ECGD for 20 years for the purchase of 6 GEC Turbines⁴³. On the other hand, KFW (the German Development Bank) has provided DM 32 m of supplier's credit to South Africa for the purchase of Siemens machinery⁴⁴.

⁴²This result may force importers to substitute overseas credit to domestic sources.

⁴³Euromoney Trade Finance, (April 1983)

⁴⁴Euromoney Supplement, (March 1989)

Regarding the Cahora Bassa project, existing data⁴⁵ reveals that a lump-sum loan guarantee of \$48 m was given by various European agencies. In the following, we briefly discuss this issue.

3.1.5. Eskom's Cooperation with Southern Africa

Eskom's 19 power stations have a nominal capacity of 37 840 megawatts (MW). The total network comprises 239 457 km of power lines. Nevertheless, the interconnection between the Botswana Power Corporation, (BPC), the Zimbabwe Electricity Supply Authority (ZESCO), and the Zambia Electricity Supply Corporation Limited, (SADC), will provide a transmission capacity of 500 megawatt (MW) which will improve the reliability of supply in the short term and will further provide an opportunity for sharing power in the region in the long term.

Eskom has taken the initiative to consult with other electricity utilities in Southern Africa so as to promote closer co-operation. Moreover, a number of other African countries, including Egypt, Kenya, and Zaire, have approached Eskom to explore yet other areas of possible co-operation.

3.1.5.1. Mozambique: Cahora Bassa Hydroelectric dam

The Cahora Bassa is a 2000 MW dam on the Zambezi river and was completed in 1977. Its transmission lines were a favourite and a frequent target of Renamo rebels in the early 1980s, and more than 200 km of lines were destroyed. Cahora Bassa is now operating at only 1.5 % capacity⁴⁶.

⁴⁵Africa Research Bulletin, (1994, p. 11583)

⁴⁶Ibid, (p. 11583-84)

The end of the country's 17- year civil war and the willingness to rehabilitate the dam by countries such as Italy, Norway, France and of the European Development Bank (EDB) may increase export returns to the Mozambican economy. According to Mr. Manuel Lopes da Costa's expectation⁴⁷, Cahora Bassa will be Mozambique's biggest export earner with estimated revenues of \$56 m a year. Portugal, the former colonial ruler of the country, has agreed to finance \$27 m for the procurement of cables and isolators. Italy had agreed to contribute \$35 m but withdrew its offer in 1993 when its foreign programme came under investigation for corruption.

Recently, in November 1994, Eskom announced⁴⁸ that funding had been found to start the rehabilitation of the Cahora Bassa Hydro-electric power project. Funds were made available by some agencies such as NORAD (The Norwegian Aid Organisation), the European Investment Bank (EIB) and the European Community (EC). These funds make up the shortfall of \$40-\$50 m caused by the withdrawal of promised funding by the Italian government. Fernando Juliao said⁴⁹ that reconstruction will begin in March 1995 and should be completed in 1997.

3.2. CONCLUSION

The two major issues highlighted in this chapter were the balance sheet of the Eskom and the nature of the rehabilitation of the Cahora Bassa Hydroelectric dam. While the chapter briefly discussed these issues, it has highlighted necessary elements for a loan guarantees model which is discussed in the Chapter Four.

⁴⁷Ibid, (p. 11584)

⁴⁸Eskom in perspective, (1994 , p. 22)

⁴⁹Africa Research Bulletin, op.cit. (p.11882)

Of these, only the total value of Eskom's asset (= **\$44 billion**) and the total value of loan (= **\$48 m**) to Mozambique for the rehabilitation of the Cahora Bassa hydroelectric can be used for the estimation of cost of the GLG. It is to that which we now turn.

CHAPTER FOUR

**THE ESTIMATION OF THE COST
OF
LOAN GUARANTEES**

4.1. LOAN GUARANTEES WITH THE DYNAMIC PROCESS

As observed in Chapter two, most financial economists have adopted the B-S model as the dominant model for pricing options and for hedging. Despite this, the basic model applied in this paper follows that of Merton. This is because the solution provided by Merton adds an additional assumption to that of the B-S model. Specifically, it assumes that stock prices are governed by a diffusion process. This assumption is sufficiently general to cover the usual hypotheses involving a geometric Brownian movement of stock prices, rather than assuming that the variance of the rate of the return on shares stock is constant.

As regards the two basic models, it can be said that both are simultaneously complex and intuitive to many users. The main difficulty in employing the models results from one unobservable parameter-- *the volatility of stock prices*. In this regard, this chapter provides an obvious deviation from the two basic models regarding the estimation of the volatility of stock prices. It blends the assumption of 'no arbitrage opportunities' used in the B-S model with the assumption that the stock's returns follows a diffusion stochastic process with a constant variance per unit of time.

Given this, we shall assume here the diffusion process will provide the 'best' estimator for a model of stock price dynamics. Nevertheless, in section 4.2, below, we present the procedure underlying option hedging within the B-S model. This is important in that it helps to distinguish the dynamic from the static process. In making this deviation, we invoke the use of a log-normal assumption for this dynamic process and we estimate the volatility of **hypothetical** stock prices in section 4.3. Section 4.4 estimates the value of the loan guarantees to the Eskom and explains the nature of the data used, while section Four presents the methodology of Merton's model and provides the results therefrom.

4.2. OPTION HEDGING IN THE B-S MODEL

This section presents a simple way of estimating the risk, i.e. *the standard deviation*, of option values. Since different variants of the B-S model have been presented, this section follows the recent work of Brenner and Subrahmanyam (1994)⁵⁰ which focuses largely on one component of the value of European options-- *the insurance premium*⁵¹-- and assumes that the traded options are *at-the-money* on a forward basis, i.e., the spot price equals the present value of the strike price, rather than the strike price itself.

Given this assumption the following relationship is implied:

$$S = Xe^{-rt} \quad (4.1)$$

where the term e^{-rt} indicates the present value factor used to discount in continuous time. This equation tells us that the present value of the strike price equals the current stock price.

The second assumption is that:

$$F = Se^r = X \quad (4.2)$$

where F is the forward price of S.

Invoking these two assumptions, it may be said that the option value has the property of being always *at-the-money* on a forward basis. This has an important implication for the value of the Call and Put option. Specifically, the value of the Call and Put at the strike price will be identical because the time value and the intrinsic-value components

⁵⁰Brenner, M. and M.G. Subrahmanyam "A simple approach to option valuation and Hedging in the B-S model", *Financial Analysts Journal*, (March-April 1994, p. 25-28)

⁵¹In spite of other components-- such as the intrinsic value and the time value of money or the strike price of the option-- the authors argue that the insurance premium is the '*raison d'etre*' of options. This is because of the distinction it makes between spot-cash assets and forward contracts.

are neutralized. In particular, if volatility is zero, the Call and the Put have no insurance value because there is no risk to be insured.

Given this, the B-S model, discussed previously, and in particular the volatility of stock prices in case of a private corporation, may be rewritten as follows:

$$d_1 = \sigma\sqrt{0.5t} \quad \text{and} \quad d_2 = -\sigma\sqrt{0.5t} \quad (4.3)$$

Applying a Taylor expansion of the normal density function on d yields the following cumulative normal density function:

$$N(d) = 0.5 + \frac{1}{\sqrt{2\Pi}} \left(d - \frac{d^3}{6} + \frac{d^5}{40} - \dots + \dots \right) \quad (4.5)$$

Hence, as a first approximation, i.e., dropping all terms of third or higher order, we may write the value of a Call and a Put as follows:

$$C = P \approx 0.4S\sigma_t \quad (4.6)$$

where $\sigma_t \equiv \sigma\sqrt{t}$ indicates the volatility which is assumed to prevail over the period until expiration.

As stated above, this equation (Eq.4.6) demonstrates that the value of Call and Put options will be identical under the above assumptions. In this context, the hedge ratio, or *delta* of the Call and Put options may be approximated as follows:

$$\Delta_c = N(d_1) \approx 0.5 + 0.4d_1 = 0.5 + 0.2\sigma_t \quad (4.7)$$

$$\Delta_p = N(d_1) - 1 \approx 0.2\sigma_t - 0.5 \quad (4.8)$$

where Δ_c , Δ_p are the hedge ratios of the call and put options, respectively.

Following this argument, the implied standard deviation (ISD) can be estimated as:

$$ISD = \sigma \approx \frac{2.5}{\sqrt{t}} \left(\frac{C}{S} \right) = \frac{2.5}{\sqrt{t}} \left(\frac{P}{S} \right) \quad (4.9)$$

We observe, then, that the techniques for the derivation of the hedge ratios in the B-S model are simple to follow in practice. This results from the following: (a) the intrinsic value and time, are both neutralized; (b) the insurance premium component, and the assumption that traded options are *at -the- money* on a forward basis, are taken into account in the case of European options; and, (c) the variance rate of the return on the stock is assumed to be constant.

These assumptions have economic meaning; and thus they may be used to obtain accurate estimates of option values.

4.3. EFFICIENT VOLATILITY ESTIMATES OF STOCK PRICES

A complete model of loan guarantees necessitates further specification of the above argument. Specifically, it requires an efficient volatility estimate since the dynamics for the value of the firm through time can be described by a diffusion stochastic process. In other words, a model of loan guarantees must be treated from a **dynamic** point of view, where the stock price (or the bond price) must satisfy a distributional assumption.

Given the distributional assumptions regarding stock prices and riskless bond prices, a number of methods based on complex approaches have been suggested in the financial literature; however, it is not the purpose of this section to describe such findings. Rather the section below describes a methodology suggested by Rogers *et al* (1994)⁵² to explain how to estimate volatility using a method based on *high-low prices* from daily data. Specifically, this method assumes that the distribution of returns on stock

⁵²L.C.G. Rogers, S.E. Satchell and Y. Yoon "Estimating the volatility of stock prices: A comparison of methods that use high and low prices", Applied Financial Economics (Vol. 4, 1994, p.241-247)

prices is log-normal; it requires only the opening, closing, high and low stock prices from time series to estimate volatility.

Following their argument, Rogers *et al* assumed the following:

- (i) $h(t)$ and $l(t)$ are, respectively, the high and low price on day t ;
- (ii) $c(t)$ and $o(t)$ are, respectively, the closing and the opening price

To make the high, $h(t)$, low, $l(t)$ and closing, $c(t)$ price independent across a day, they divide $h(t)$, $l(t)$ and $c(t)$ by the opening price as follows:

$$H(t) = \text{Log}\left(\frac{h(t)}{o(t)}\right) \quad (4.10)$$

$$C(t) = \text{Log}\left(\frac{c(t)}{o(t)}\right) \quad (4.11)$$

$$L(t) = \text{Log}\left(\frac{l(t)}{o(t)}\right) \quad (4.12)$$

From these above three equations, Rogers *et al* suggest an estimator that has the attractive property that is unbiased regardless of the value of α (the instantaneous expected rate of return). This is given by the following :

$$\sigma^2 = H(H - S) + L(L - S) \quad (4.13)$$

According to Rogers, these simplified high / low estimators seem to perform well for a changing random walk with drift and variability in the level of market activity. Thus, it will be tested in the following section.

4.4. VALUING ESKOM'S LOAN GUARANTEE

This section follows our discussion of the theoretical value of loan guarantees of Chapter Two, above, and develops a simple model which involves three parties: (a) the Eskom; (b) a third-party guarantor, such as the South African government; and (c) the

ultimate lenders, such as the Norwegian Aid Organisation, (NORAD), the European Investment Bank, (EIB), and, the European Community, (EU).

Among the major considerations which bear importantly on the valuation of the loan guarantee to Eskom are:

- (i) A political constraint and the Reconstruction Development Programme (RDP). The South African Eskom has published ten 'commitments'⁵³ demonstrating its support of the RDP. For example, in rural areas, Eskom is committed to improving the quality of life of rural households via expanded electricity connections. Eskom's rural electrification will raise the number of households served by electricity from the present 12 % to fully 30% in 1999 and it aspires to reduce the real price of electricity by 15% between 1995 and the year 2000.

In order to achieve these objectives and to assist economic development, Eskom has undertaken the project of rehabilitation of the Cahora Bassa Hydroelectric dam.

- (ii) The South African government has concluded that the project is economically and socially desirable in light of its own RDP, and so has agreed to guarantee the loan to Eskom as an integral part of its public policy. In this respect, the loan guarantees are contingent liabilities of the South African government.
- (iii) To assist in this project, Eskom has obtained loans from various European agencies. A financial package of \$105 m⁵⁴ was made available by such agencies in order to rebuild 1.800 pylons destroyed by South African- backed rebels during the Mozambiquan civil war.

⁵³ Eskom Annual Report, (1994, p.14)

⁵⁴ Africa Research Bulletin, op.cit

Merton (1977) and Jones and Masson (1978) evaluated certain loan guarantees, as well as the associated benefits and incentives accruing to the participants from such loans. In order to obtain an explicit solution to the simple model above, this paper invokes the standard assumptions described by Merton, i.e., frictionless market assumptions and additional ones employed by Jones and Masson, i.e. that the government has agreed to fully or partially guarantee a new issue of debt.

As a classical application of the formulation of a problem involving governmentally guaranteed loans, we note the following regarding corporate debt pricing: suppose that the company holds business assets; it borrows money by issuing a single homogeneous debt issue, i.e. junior debt, to finance a new risky project. The terms of the debt are that the firm promises to pay a total of B dollars at maturity. Suppose there is a third-party, e.g., the government or its agencies, which guarantee payment to junior bondholders. In the event that the promised payment is not made⁵⁵, the guarantor (government) will make payment. This means that the guarantor has issued a put option on the assets of the firm which gives the junior bond-holders the right to sell the assets for B dollars at maturity of the debt.

There are other relationships in this model which bear consideration in using futures to hedge risk. A *hedge* protects against random fluctuations which the futures market does not, and cannot, predict. Those fluctuations can cause compensating changes in option price. Nevertheless, it should be noted that more often than not, when performance is

⁵⁵If $V(T) \subseteq B$, then the firm will not make the payment. Thus the value of a Put on the guarantee, $G(V,0)$ and the initial condition for the debt, $D(V,0)$ can be written as:
 $G(V,0) = \text{Max} [0, B - V]$, and $D(V,0) = \text{Min} [V, B]$.

In the case of partial guarantees, the terminal condition may be modified as follows:

$G(V,0) = \text{Max} [0, \phi B - V]$, which indicates that if the asset's value, $V > \phi B$, then the guarantee is worth zero. If the asset's value is less than ϕB , then the guarantee is worth the difference between ϕB and V .

measured in local currency, the gains from hedging are similar to the gains from international diversification because hedging reduces risk for both sides⁵⁶.

Since the model constructed below leads to a hedge against changes in the value of the real exchange rate (i.e., in the relative values of domestic and foreign assets), there is a need to introduce a currency hedge ratio into the 'classical' model of government loan guarantees. This is the contribution we emphasised in the introduction to this paper and the manner in which we have slightly modified the 'classical' model of Merton. Thus, for a futures hedge, one can use currency options.

A *currency option* is a contract which grants the holder of the option the right, but not the obligation, to buy or sell a fixed amount of a specified currency, at an agreed exchange rate, on or before a specified date⁵⁷. Currency options are used as an alternative to forward exchange rates, since we have assumed away the possibility of speculation in currencies. In other words, currency options are ideal for contingent liabilities for overseas contracts⁵⁸.

Given this background, we now we turn to the methodology and results for valuing the cost of the South African GLG to Eskom, as well as to some issues related to partial versus full guarantees.

⁵⁶See F.Black (1995) for an excellent presentation of the details of hedging.

⁵⁷A call option grants the holder of the option the right, but not the obligation, to buy a specified currency; whereas a put option grants the holder of the option the right but not the obligation, to sell a specified currency.

⁵⁸As an example of the application of currency options to hedging position, consider a firm investing in a foreign land. Suppose the firm is obliged to pay for investment in US Dollars, at a specified date, i.e., in six months' time. The firm expects the Dollar to strengthen against the Rand and wishes to hedge its position. Thus, it buys a \$ 10 million six month Dollar European call option from a South African Bank. Assume, for mere example, that the strike price is R 2.845 and the premium is priced at 5.5% of the contract size, i.e., US\$ 550.000 (R 1569700).

If we further assume the spot rate to rise R 3.05 after six months, the importer would exercise the option at the strike of R 2.8540. The \$ 10 million would cost the firm R 28.540.000 plus the premium amount of R 156 9700, i.e., R 30.109 700, which implies a saving of R 390 300 against the current spot rate of R3.05.

4.5. METHODOLOGY AND RESULTS

As one might expect, the weak assumptions of the B-S model are not accepted by all financial economists. This is because the assumptions are not sufficient to uniquely determine a 'rational theory of option pricing'. However, the work of Merton (1973) laid the foundations for such a theory. Making additional assumptions to the B-S model, Merton (1977) derived an explicit formula for the valuation of the cost of loan guarantees.

To be consistent with Merton's model, we assume the following: (a) there are no interim or coupon payments attached to the debt, and so the debt is a discount bond; (b) the bond price changes follow a stationary random walk in continuous-time. This means that the bond's price distribution at the end of a finite interval is log-normal. Merton describes the bond price dynamics as follows:

$$\frac{dP}{P} = \mu(\tau)dt + \delta(\tau)dq(t; \tau) \quad (4.14)$$

where μ is the instantaneous expected return, δ^2 is the instantaneous variance, and $dq(t; \tau)$ is the standard Gauss-Wiener process. However, δ is otherwise assumed to be non-stochastic and independent of the level of P . In the special case when the interest rate is nonstochastic and constant over time, $\delta \equiv 0$, $\mu = r$, and $P(\cdot) = e^{-r t}$.

Following the work of Merton and focussing on the special case where the interest rate is described as non-stochastic, we can estimate the cost of government loan guarantees to Eskom.

4.5.1. Data Used for the Calculation

In order to estimate a 'fair' risk premium for the Eskom guarantee, one must determine the parameters of Eskom's financial structure. Specifically, we require knowledge of the standard deviation of the rate of return of the assets of the firm, the exercise price

(which corresponds to the promised payment), the time to maturity, the current value of the firm's assets, the values for the equity and debt as well as the riskless interest rate (or the Risk- Free Rates).

A principal feature of Merton's model is that it does not depend on the expected return to common stock⁵⁹, or the risk preferences of investors, or the aggregate supply of assets. It does depend on the rate of interest (an observable variable) and the total variance of the return on the common stock, which is often estimated using time series data.

Given this, the steps used to obtain these estimates may now be explained.

4.5.1.1. Calculation of the standard deviation

To estimate the standard deviation of the rate of return, we apply the efficient volatility estimation technique propounded by Rogers *et al.* In fact, the efficient volatility estimation model becomes relatively easy to apply in practice⁶⁰ since daily, weekly, and in some cases, monthly highs and lows are published for every stock.

Since we do not have price data for our hypothetical case we will, for illustrative purposes only use the yields on Eskom loan stock as a **proxy**⁶¹ for share price. The yields used were based on the closing prices of each day over the year-1994⁶².

In order to apply Roger's model, we assume that the opening prices are an average of the high and low prices. This means that if $H(t)$ indicates the high and $L(t)$ represents

⁵⁹This is an important result because the expected return is not directly observable and estimates from the past data are poor because of nonstationarity.

⁶⁰One can also use the extreme value method for estimating the volatility. See Parkinson (1980) for an excellent description of the model.

⁶¹Although it may be a bit ambitious to assume that the standard deviation of the yield on loan stocks would be similar to that on shares, the assumption has been made in order to illustrate the method.

⁶²I thank Dr. H. High for providing me with the data for this estimation.

the low price in the period of one year--1994, then the opening price, $O(t)$, may be calculated as follows:

$$O(t) = [H(t) + L(t)] / 2 \quad (4.15)$$

This gives one of the time series values we require for applying Roger's model. Then, the mean of the high, low and closing prices was calculated, using the following formula:

$$\bar{x} = \frac{1}{192} \sum_{i=1}^{192} x_i \quad (4.16)$$

where the number of observation over the year 1994 is 192⁶³. This formula was used to capture the true variance of the rate of return of the common stock over a unit time interval.

Thus, using our data, we find:

$$\begin{aligned} \bar{H} &= 0.0070944 \\ \bar{L} &= -0.0126785 \\ \bar{C} &= 0.0040667; \end{aligned}$$

and the standard deviation of the Eskom loan stock E168, $\sigma_i = 0.0152\%$ is a function of the Yield-to-Maturity on a daily basis. This result approximates the estimation value of 0.01% of the minimum price movements suggested by Falkena (1989).

4.5.1.2. Determining the Risk -Free Rate (RFR) and the time to maturity

Using the assumption of the Capital Asset Pricing Model, CAPM, any portfolio with a zero ('beta') market risk must have an expected return equal to the RFR, i.e., government bonds. Consequently, an equilibrium condition can be established between

⁶³The number of observations depends on the volume of data collected. In our time series, we encountered missing data which reduces the number of observation to 192.

the expected return on the option, the expected return on the stock, and the riskless rate. This is a reasonable argument in view of Merton's model. In this regard, we need information on the short-term interest rate, i.e. Treasury bills. However, one can assume that the fluctuations in the short-term interest rate will positively correlate with the yields for the Eskom 168 11 percent 2008.

In effect, those fluctuations which are not correlated are ignored. This is not as extreme as the usual assumption for stock options, which ignores all fluctuations in the short-term interest rate. However, while it is possible to consider models in which such fluctuations are accounted for, this paper will not attempt to do so since assuming the RFR of 11 % as a known variable will satisfy the general assumptions within the continuous-time model and CCA.

Moreover, the 'guarantee' period considered in this paper is two years. This is derived from the period over 1995 to 1997 with respect to the starting and ending periods for the Cahora Bassa hydroelectric dam project.

4.5.1.3. Calculation the postinvestment current market value of the firm

Before calculating the value of the loan guarantee it is convenient to explain how one obtains the value of the pre-existing financial claims (the debt and the equity) of the firm.

Since the gilts are traded on the basis of their yields, we can observe their market value and thus can measure the standard deviation of the yields. The former may be collected from the balance sheet of Eskom. We observed this data in Chapter Three, above. The data available reveals an amount of \$44.397 m (≈R122m) in 1993 in total assets prior to the announcement of the loan guarantee--November 1993. This is the first variable required for the model constructed below.

Using the information associated with the value of the standard deviation discussed above, we can estimate the value of equity using the following equation:

$$E(v, \tau) = V\Phi(X_1) - Be^{-r\tau}\Phi(X_2) \quad (4.17)$$

where,

$$X_1 = \left[\log\left(\frac{V}{B}\right) + (r + 0.5\sigma^2)\tau \right] / \sigma\sqrt{\tau}$$

$$X_2 = X_1 - \sigma\sqrt{\tau}$$

4.5.1.4. Determination of the Promised Payment (= B)

The second variable required for the model refers to the amount of debt. Existing information⁶⁴ reveals that a lump sum loan of **\$48 m** was given by overseas agencies such NORAD, EU etc., to Eskom in order to supply new electricity pylon in Mozambique. Portugal will finance the procurement of cables and isolators in the amount of **\$27 m**.

Using this information, one can readily calculate the future compounded value given that the RFR is 11 percent per a year for years 1 and 2. We obtain the following results:

$$\begin{aligned} PV &= \Sigma (PV_{11\%, 2}) & (4.18) \\ PV &= \$48 \text{ m} (1.11) + \$48\text{m} (1.11)^2 \\ PV &= 53\,280\,000 + 59\,140\,800 = \$112\,420\,800 \end{aligned}$$

Hence, the promised payment at maturity will be approximately **\$ 112 m (R408 m)**.

In the section below we consider the use of currency option.

⁶⁴See Africa Research Bulletin, op.cit.

4.5.1.5. Hedging Exchange Rates

As explained above, Eskom can hedge its position by buying a Dollar European Call option from a South African Bank, e.g. First National Bank. To do so, it can buy a fixed amount of \$48m at an agreed exchange rate. Assuming the Dollar-Rand rate is R 3.63 and the premium is priced at 5.5%, then it will cost Eskom a total of \$2,64m ($\$48m * 5.5/100$) on such a contract. However, if the spot rate were to rise to R3.70 after two years, then the importer (Eskom) would exercise the option at the strike rate of R3.63. If we assume a loan of \$48 m then it would cost Eskom R174 m ($\$48m * 3.63$) plus the premium amount of R9,58 m ($\$48m * 0.055 * 3.63$), then subtracting the promised payment on the original loan which reflects RFR of 11 %, yields **\$199m**. This is a positive cash flow and can be saved by Eskom.

Substituting all the estimated parameters above into the following equation, we obtain⁶⁵ the value of equity and we derive the post-investment current market value of Eskom:

$$\begin{aligned} E &= 44.397m\Phi(X_1) - 112me^{-0.11*2}\Phi(X_2) \\ E &= 44.397m * 0.636399 - 112m * 0.8025 * 0.636319 \\ E &\approx \mathbf{\$28.196\ m} \end{aligned}$$

Now, the post-investment current market value of the firm is calculated as follows:

$$V(t) = \text{the value of equity} + \text{the value of debt} \quad (4.19)$$

Thus,

$$\begin{aligned} V(t) &= \$ 28\ 196\ 799\ 171 + \$112\ 420\ 800 \\ V(t) &\approx \mathbf{\$28.309\ m} \end{aligned}$$

This result shows that $V(t)$ is greater than the promised payment, B. This implies that the firm (Eskom) will make the payment and will not default to the bondholders. In other words, Eskom's stockholders will gain since the value of equity is not zero.

⁶⁵All the calculations were performed using Microsoft Excel. One can use the command NORMSDIST to determine the cumulative normal probability

4.5.1.6. Calculation of the value and cost of the South African loan guarantees

The determinants of the value of loan guarantee include: (a) the post-investment current market value ($V(T) = \$28.309 \text{ m}$); (b) the RFR of 11% on the yield for E168; and (c) the standard deviation, $\sigma = 0.0152 \%$.

According to Merton(1977), the value and cost of the loan guarantees may be expressed as follows:

$$G(V, T) = Be^{-RFR \cdot T} \Phi(X_2) - V\Phi(X_1) \quad (4.20)$$

where,

$$X_1 = \frac{\text{LOG}(B/V) - (RFR + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$X_2 = X_1 + \sigma\sqrt{\tau}$$

Substituting all the parameters estimated above we obtain the following value of loan guarantee:

$$G(V, T) = 112 \text{ m} \cdot 0.8025 \cdot 0.36817 - 28\,309 \text{ m} \cdot 0.36809$$

$$G(V, T) \approx -\$28.276 \text{ m}$$

This yields a negative value for the South African GLG. This result implies that the value of debt will approach the value of a riskless bond. That is, $V(t)$ tends to infinity and, thus, the value of the guarantee would become worthless,

$$G(\infty, t) = 0 \quad (4.21)$$

Hence, the cost of the loan guarantee is calculated as follows:

$$\frac{G(V, T)}{Be^{-RFR \cdot T}} = 1 - e^{-(R(T)-r)T} \quad (4.22)$$

where,

$\frac{G(V, T)}{Be^{-RFR \cdot T}} = Y$ is the value of the South African loan guarantee as a fraction of the value of the loan covered by the guarantee; and,

$1 - e^{-(R(T)-r)T} = X$ is the value above the risk-free interest rate which European agencies will charge to cover the additional risk involved in lending funds to Eskom.

Considering the value of the South African loan guarantee as a fraction of the value of the loan covered by the guarantee (=\$561.342 m), one can argue that Eskom's benefits (= \$28.276m) outweigh the guarantee fees that the South African government could charged. Like Chrysler case, Eskom's stock price(under the assumption above) should rise. In other words, Eskom's stockholders benefit from the South African GLG.

Since the government is liable for the present value of the future promised payment, the risk premium calculated at the riskless interest rate, RFR which is associated with the volatility of the firm is given by:

$$R(T)-r = \$ 178.935m$$

In the table below, we summarise our results:

Table 4-1

**The Value of the South African GLG
(in millions of U.S Dollars)**

(for T=2 years, RFR=2, and the promised payment (=\$48m))

σ	Present value	Hedging \$/R at R 3.63	Value of equity	Post-investment current market value	Real value of the South African GLG	$\frac{G(V,T)}{Be^{-RFR \cdot T}} - y$	R(T)-r
0.0152	\$112	\$119	\$28.196	\$28.309	-\$28.276	\$561.342	\$178.93

Source: Summarized from estimates, above.

These are interesting results and there are a number of possible extensions which can be readily observed. For instance, in our calculations it was assumed that the government is the guarantor and therefore there is no risk associated with the payment of the guaranteed loan. A possible extension would be to allow for a risky guarantor such as a

private agency or another firm. Tax effects may also be incorporated into the analysis. As noted by Merton (1973), the value of the loan guarantees calculated under nonstochastic interest rates are low estimates of the true value. In our model, the guarantor is assumed to guarantee only one loan such as the junior bond and have capital to guarantee this loan. A more general model would allow the guarantor to guarantee many loans with default, e.g. loans associated with risk probabilities positively correlated. In this regard, the results of this paper are in the spirit of the B-S(1973), Jones and Masson(1980) and specifically of Merton(1974; 1977). Thus, the results demonstrate that loan guarantees can create additional value to the firm and its security holders. Given this, government charges for the provision of such a guarantee would appear justified.

CHAPTER FIVE

CONCLUSION

This paper has addressed a long debated issue regarding the 'fairness' or the 'real' cost of the GLG. Using an option pricing model with the promised payment of \$48m (which corresponds to exercise price in the standard option pricing model) and empirical data from yields on the E168 as a proxy for share price volatility, Put option values for loan guarantees were estimated.

Prior to obtaining these results, we outlined, in chapter One, the option pricing model which could be applied in the estimation of the cost of loan guarantees. The literature on option pricing, such as that of Jones and Masson(1980), Sossin(1980), Chen, Chen, and Sears(1986) and, specifically, Merton(1977) generally assumed a nonstochastic interest rate, as we have here.

We then examined the economic effects of the GLG on markets--financial and credit markets, and considered the classical case of Chrysler bailouts in the United States. In this regard, we noted that the presence of monitoring and negotiation costs resulted in a public interest concern regarding potential bankruptcy. The US federal government's objective of maximising the present value of national output was seemingly furthered through its federal loan guarantees to Chrysler.

It should be noted that chapter Two focused on the possible benefits of a GLG while ignoring possible costs. A government loan guarantee programme no doubt has high costly monitoring and administrative costs. Furthermore, another important public policy consideration is the danger of a moral hazard problem developing as result of expectations of loan guarantees to other firms approaching bankruptcy.

Although the model of a GLG has referred to the important elements of the Chrysler case, we justified in, chapter Three, the South African government's intervention into the risky project of the rehabilitation of the Cahora Bassa Hydroelectric dam. Specifically, a complex capital structure and the risk of expropriation made monitoring

and negotiation between lenders (European agencies) and the borrower (Eskom) costly. Thus, one could rely on a third-party (South African government) to resolve conflicts of interest and lower monitoring costs to the ultimate lenders.

As far as our results are concerned, it was shown that the value of the South African guarantee is worth nothing to Eskom. This because the total market value at the expiration date (= \$28 billion) is far greater than the promised payment (= \$48 m). Thus, Eskom should be able to easily make good its promised payment.

Because the South African government's exposure was so large under this loan guarantee, it is important that it be aware of the 'real' costs thereof. One of the factors which influences these costs is the amount of fees and premiums it receives from providing such loan guarantees to Eskom. Whether such fees need to be adjusted in magnitude to approximate the Put value which corresponds to the value such of loan guarantees, is a policy decision that detailed consideration of the programme characteristics and objectives.

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