# Risk Parity as an Asset Allocation Technique: Evidence from the South African Capital Markets

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**Abstract:**

This paper investigates the asset allocation technique known as Risk Parity, whereby assets are allocated such that they contribute equal amounts of risk to the overall risk of the portfolio. It is a relatively new technique and one which has grown in popularity and stature amongst Hedge Fund and asset managers alike. Academics have recently also come to fore, documenting the flaws with the mean-variance framework and have begun looking towards portfolio construction techniques based solely on predicted risk, not expected return. The prior literature on the topic is exclusively done abroad and finds that an unlevered Risk Parity portfolio, despite being inferior to other portfolios from a return perspective, is superior in terms of its risk-adjusted return, or Sharpe Ratio. Many academics propose the idea of levering up the Risk Parity portfolio so that its standard deviation matches that of another, riskier portfolio. This paper analyses five portfolio strategies, namely an unlevered and levered Risk Parity, a traditional 60/40, a minimum variance and a maximum Sharpe Ratio (tangency) portfolio. The first part of the paper will categorise the equity asset class into the FINDI and RESI indices, as well as using the ALBI and South African Property Index (PROP). The second part of this paper, Test 2, will subcategorize the equity portion of the strategies into Value and Momentum, using style indices, thus testing for evidence of style anomalies on the JSE. It will still use the ALBI and PROP as the other asset classes. The key findings are that for both tests, the unlevered and levered Risk Parity strategies underperform the FTSE/ALSI benchmark over the sample period, concluding that at the given level of risk free rates, Risk Parity as an asset allocation technique is not superior. Furthermore, Test 2 provides superior annualized returns for all but one of the strategies, indicating the possibility of style anomalies on the JSE.

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## Chapter 1: Introduction

One of the cornerstones of market inefficiency is that financial markets move according to surprises that take place: changes in conditions relative to those that have been priced in. The larger and more relevant the surprise, the larger the change in the market. It is based on this tenet of finance, that the concept of Risk Parity was founded. The idea of holding a portfolio that will perform well across all economic environments is inextricably linked to holding a portfolio whereby all the components contribute an equal amount to overall risk. Before the advent of Risk Parity portfolios, many of the world's largest pension funds and asset managers used equity-dominant asset allocations, resulting in most of the risk being stock-based. By virtue of it being a much riskier asset class, the resulting portfolio risk was completely dominated by equities. This translates into a bullish bet on stocks and a belief that there will be positive surprises attached to the equity market.

This paper will endeavour to explore the asset allocation technique of Risk Parity in the context of the South African Capital Markets, and evaluate its effectiveness therein. Using the methodologies of prior literature, it will compare both unlevered and levered Risk Parity portfolios, across four asset classes, namely the All Bond Index, the FINDI, the RESI and the South African Property Index. This will be called Test 1.

The second part of the paper, known as Test 2, will subcategorise the equity component into Value and Momentum Indices, in a separate test of style anomalies. The debt and property asset classes will remain the same as Test 1. Comparisons will be made with other traditional asset allocation strategies, namely the minimum variance, maximum Sharpe Ratio and traditional 60/40 portfolios. The paper has been structured as follows: firstly, there will be a review of the prominent literature on the topic, focusing specifically on the papers that calculate Risk Parity on multi-asset portfolios, not just the standard two-asset-class portfolio of Equity and Bonds. The theoretical groundwork of existing asset allocation and portfolio construction techniques will also be laid here. Secondly, the methodology and Data selection will be discussed, followed by an analysis of findings of both tests, the limitations of the study & areas of further research and a conclusion.

## Chapter 2: Theoretical overview and review of literature:

### *2.1 Problems with the mean-variance framework*

Asset allocation has long been at the forefront of the agendas of portfolio managers and investors alike. Bound by the investor's goals, objectives and constraints, how to efficiently allocate one's capital is of paramount importance in any investing scenario. Markowitz (1952, 1956) first postulated in his mean-variance framework, that a rational investor would aim to maximize their expected return, given a level of risk. This optimal portfolio, known as the market or tangency portfolio, is the tangency point on the efficient frontier and represents the greatest utility, given the investor's constraints and the risk-free rate in the market. Maillard, S., Roncalli, T., Teiletche, J. (2009) identify numerous problems with this solution. Firstly, this optimal portfolio is often overly concentrated in a small subset of the greater universe of assets or securities. Secondly, they argue that the mean-variance solution is extremely sensitive to input parameters. This can be corroborated by Merton (1980), who found that changing expected returns can lead to particularly large variations in a portfolio's asset allocation.

Asness, C.S., Frazzini, A. and Pedersen, L.H. (2012) also document some relevant downfalls of the market portfolio, particularly from the perspective of risk. They argue that because equities are historically more volatile than bonds, stock market movement dominates bond market movement. Therefore, from a risk perspective, the market portfolio is chiefly an equity-weighted portfolio because most of the variation in its performance is explained by the stock market variation. It therefore offers little risk diversification, despite its deceivingly diversified nature from a market capitalization point of view. The same is argued for the traditional 60/40 pension fund strategy, with 60% allocated to equities and 40% allocated to bonds. When viewed as Rands invested in each asset class, the portfolio might be perceived to be well-balanced. But from the perspective of risk, these portfolios offer little diversification.

Chaves, D.B., Hsu, J., Li, F., Shakernia, O. (2010) argue that due to the "time-varying nature of asset class risk premiums and their joint covariance", expected returns become increasingly difficult to estimate under the mean-variance framework. Clarke, R., De Silva, H., & Thorley, S. (2013) highlight the popularity of portfolio construction techniques based solely on predicted risk, not expected return. These findings will be examined and explored further in due course.

*2.2 Minimum variance and equally-weighted portfolios*

Despite the drawbacks of the mean-variance framework, many investors adopt two well-known strategies for their computational simplicity and perceived robustness; namely the minimum variance and equally-weighted portfolios (Maillard *et al*, 2009). The former strategy is a portfolio which lies on the Efficient Frontier and contains a unique solution: it is simply the portfolio with the smallest variance. The perceived robustness of this strategy therefore comes from the fact that it does not depend on expected returns. Maillard *et al* (2009) argue, however, that a chief drawback of this strategy is its portfolio concentration, specifically in low-risk of debt instruments. They describe the simple and widely-used method of dealing with this issue, namely the equally-weighted or *1/n* portfolio, which has exhibited out-of-sample efficiency by DeMigeul, V., Garlappi, L. & Uppal, R. (2009). The primary drawback of this strategy is its failure to address risk diversification, especially if individual asset class risks differ significantly (Maillard *et al*, 2009). This is the fundamental basis of Risk Parity.

Clarke *et al* (2011 & 2013) present various equations used to construct the minimum variance portfolio. Equation 1 provides the individual asset weights for such a portfolio:

$$w_{MV,i} = \frac{\sigma_{MV}^2}{\sigma_{\varepsilon,i}^2}\left(1 - \frac{\beta_i}{\beta_L}\right) \; for \; \beta_i < \beta_L; else = 0 \tag{1}$$

where $\beta_L$is a long-only threshold, $\sigma_{MV}$is the risk of the minimum variance portfolio and $\sigma_{\varepsilon,i}^2$is the idiosyncratic risk of the asset class. As can be seen by this equation, subject to a long-only constraint, an asset class can only be included if its Beta is less than the threshold Beta. Furthermore, a high level of idiosyncratic risk will result in a lower weighting. As can be seen from the second term of the equation, the highest weight will be given to the asset class with the lowest Beta.

The objective function of this portfolio, as stated by Clarke *et al* (2013), is the minimization of ex ante portfolio risk, according to equation 2 below:

$$\sigma_P^2 = w'\Omega w \tag{2}$$

where $w$ is an N-by-1 vector of asset class weights and $\Omega$ is an N-by-N asset covariance matrix. Clarke *et al* (2013) provide a well0known solution to this optimization problem, presented as equation 3 below:

$$w_{mv} = \sigma_{MV}^2 \Omega^{-1} \iota \tag{3}$$

where $\iota$ is an N-by-1 vector of ones and $w_{MV}$ is the weight of the entire minimum variance portfolio, which according to the long-only constraint imposed by Clarke *et al* (2013), must sum to one.

*2.3 Introducing the concept of Risk Parity*

Given the aforementioned problems highlighted about the market portfolio and the mean-variance framework, investors and academics alike have searched for alternative approaches to asset allocation. One such method that has grown in stature and popularity, particularly amongst passive fund managers, is Risk Parity: the concept of allocating asset classes so that they contribute the same amount of risk to the total portfolio risk. This is also known as a risk-diversified portfolio. Qian (2011) believes that capital diversification needs to be improved upon through risk diversification. He goes on to prove that despite this risk-diversified portfolio being inferior to the market portfolio (or indeed a 60/40 mix, made up of 60% equities and 40% bonds) in terms of total return, its risk-adjusted return, as numerated by the Sharpe Ratio, is superior. This can be illustrated on a simple Efficient Frontier diagram, whereby the Risk Parity portfolio lies below and to the left of the less-diversified portfolios, despite offering superior risk-adjusted returns. Qian therefore poses the question of whether or not it is possible to achieve both a greater total return and superior diversification. Much of the literature reviewed in this paper presents the answer to this question as applying leverage to the Risk Parity portfolio.

Asness *et al* (2012) further looks at the expected return-versus-risk of different asset allocations. They state that a Risk Parity strategy cannot be deemed superior simply because it is better diversified from a risk perspective. Investors adopting this strategy also need to believe that they are not being adequately compensated for being concentrated in equities, as in a market or 60/40 portfolio. In other words, Asness *et al* (2012, p. 48) state that a “Risk Parity investor should say, ‘We do not believe expected returns are high enough to give them a disproportionate part of our risk budget.’”

Chaves *et al* (2010) found that while a Risk Parity strategy is able to achieve a higher Sharpe Ratio than other well-established strategies such as a minimum-variance or 60/40 portfolio, it is unable to consistently outperform such portfolios.

*2.4 Evolution of the Risk Parity definition*

According to Clarke *et al* (2013), an asset allocation was initially deemed to be in Risk Parity when the weights were "proportional to asset-class inverse volatility." A portfolio made up of equities and bonds with volatilities of 15 percent and 5 percent respectively, will have Risk Parity weights of 75 percent and 25 percent respectively (three times as much). Clarke *et al* (2013) caution that this early definition ignored the correlations between assets classes, even as the concept applied to more than two classes.

Qian (2006) and later Maillard *et al* (2009) introduced the idea of a risk budget, whereby assets weights are adjusted so that each class contributes an equal amount to total portfolio risk. Lee (2011) postulated that asset class weights must be proportional to the inverse of the asset beta in order to achieve Risk Parity.

Chaves *et al* (2012) further argue that theoretically, only if the assets classes have the same Sharpe Ratios and correlations, can the naïve weighting method of *1/vol* be deemed optimal under the mean-variance framework. Maillard *et al* (2010), in an extension of their working paper, tried to address this problem by introducing a more accommodating correlation assumption, but Chaves *et al* (2012) still believe that the numeracy required to calculate these optimal weights is time-consuming and often requires special mathematical software. They present two simple algorithms which they believe don't require such optimization techniques and can be solved relatively simply using Matrix algebra.

Clarke *et al* (2013) present a somewhat simple equation for calculating Risk Parity weights, which they believe to be instantaneous in its application, as well as applicable to large investment sets. It is presented as Equation 4 below:

$$\omega_{RP,i} = \frac{\sigma_{RP}^2}{\sigma_{\varepsilon,i}^2}\left[\left(\frac{\beta_i^2}{\gamma^2} + \frac{1}{N}\frac{\sigma_{\varepsilon,i}^2}{\sigma_{RP}^2}\right)^{1/2} - \frac{\beta_i}{\gamma}\right] \tag{4}$$

Here, $\gamma$ is a constant across all assets, $\sigma_{RP}$ is the Risk Parity portfolio risk, $\beta_i$ is the asset's Beta and *N* is the number of assets. The idea of underweighting risker assets is evident in this equation, whereby higher idiosyncratic risk will lead to a smaller weight of the asset. The authors do, however, caution that in Equation 5, the portfolio constants of $\gamma$ and $\beta_i$, which are embedded in the right hand side of the equation, depend of the final asset weights (Clarke *et al*, 2013). This makes the weights endogenously determined and thus no closed-form solution is possible. Other literature has also arrived at similar conclusions. Maillard *et al* (2009) present an equation for determining Risk Parity weights whereby the weight of each asset is inversely proportional to its Beta. The endogeneity can clearly be seen here whereby the asset weight, $x_i$, is a function of its Beta, which in turn is dependent on the portfolio $x$. Chaves *et al* (2012) describe this problem in Layman's terms: "One cannot find the Betas without the weights, but the weights depend on the Betas."

A generally accepted method of overcoming the problem of endogeneity by all of the aforementioned authors is through a process of iteration. Here, one assumes an initial equal weight in assets, and then iteratively calculates the various parameters in equation 5 and the resulting weights until they converge (Clarke *et al,* 2013).

*2.5 Levered Risk Parity*

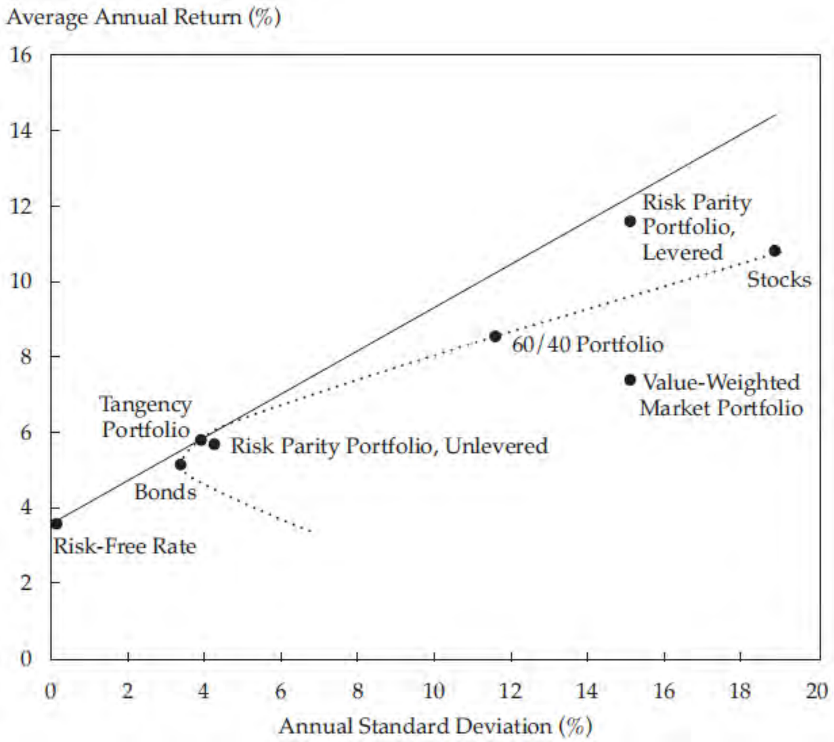
Asness *et al* (2012) postulate that applying leverage to the Risk Parity portfolio increases both its expected return and its risk to desired levels. This can be illustrated on an Efficient Frontier, whereby levering a Risk Parity portfolio increases the expected return, for a given level of risk, thus moving the portfolio directly above its unlevered counterpart. This concept can be illustrated in Figure 2, which is taken from Qian (2011). Here, various portfolios are presented along the Efficient Frontier and the Risk Parity line, along which all portfolios are in Risk Parity and with the same Sharpe Ratio as the tangency 25/75 Risk Parity portfolio. If one desires a Risk Parity portfolio with the same risk as the 60/40 portfolio (9.6%), one would need to lever the 25/75 portfolio upwards, arriving at the Risk Parity 9.6% portfolio. Simple arithmetic can be used to determine the level of leverage, whereby the standard deviation of the 60/40 portfolio is divided by that of the unlevered Risk Parity portfolio. This method will be used as a guideline for this paper's tests; however, as will be discussed in the methodology, due to the importance of providing the computing package with sufficient leeway to run its optimizations, this level of leverage is not always attained.

Naturally, the use of leverage presents unique problems of its own and is constrained by the investors' risk aversion. These issues will be dealt with and reviewed separately in due course.

Figure 1 below graphs the efficient frontier of various portfolios, taken from Asness *et al* (2012). The levered Risk Parity portfolio can be seen in the far right, directly above the value-weighted market portfolio. This is indicative that the authors levered this portfolio up, using the same standard deviation as the value-weighted portfolio, not the 60/40 portfolio as was done by Qian (2011). However, this levered portfolio still lies on the same Capital Market Line as its unlevered counterpart, as can be seen if one extends a line from the same risk-free rate and through the unlevered Risk Parity portfolio. This indicates that the same arithmetic method used by Qian (2011) is used by Asness *et al* (2012); namely if one takes the ratio of standard deviations of the unlevered Risk Parity and 60/40 portfolios, and levers up the Risk Parity portfolio by that scale factor, the resulting portfolio will lie on the same

Capital Market Line as its unlevered counterpart, meaning their Sharpe Ratios will be the same. The fact that the two authors used different reference portfolios for the target standard deviation seems trivial.

**Figure 1: Efficient Frontier of various portfolios**




*Source: Asness et al (2012)*

Anderson *et al* (2012) use a series of equations for the construction of their Risk Parity portfolios. The volatility of each asset class is estimated each month, using a rolling window of the previous 36 months. The time *t* volatility for asset class *i* is therefore given by

$$\hat{\sigma}_{i,\,t} = \text{std}(r_{i,\,t\text{-}36}, \ldots, r_{i,\,t\text{-}1}) \tag{5}$$

The time *t* portfolio weight for asset class *i* is given by

$$w_{i,t}^{u} = \delta_t \hat{\sigma}_{i,t}^{-1} \tag{6}$$

where

$$\delta_t = \frac{1}{\Sigma_i \hat{\sigma}_{i,t}^{-1}} \qquad (7)$$

As can be seen from the above equations and in line with the literature, this simple strategy overweights less volatile assets (bonds) and underweights more volatile assets (equities). This is one of the tenants of the Risk Parity strategy.

The levered Risk Parity strategy is obtained by levering up this strategy so that its volatility matches the *ex post* volatility of the value-weighted strategy. This will be done on a conditional basis, whereby the leverage constant *k,* will change at each rebalancing date, as discussed in the Literature Review above. It will be calculated as the quotient of the volatility estimate for the value-weighted portfolio and the unlevered Risk Parity portfolio:

$$l_t = \frac{\widehat{\sigma_{v,t}}}{\hat{\sigma}_{u,t}} \qquad (8)$$

The time *t* portfolio weight for asset class *i* is simply the unlevered weight scaled by the leverage constant $l_t$

$$w_{i,t}^{l} = l_t w_{i,t}^{u} \qquad (9)$$

The rebalancing is done on a monthly basis, more specifically on the last trading day of each month. The target standard deviation is based on a three-year rolling window period.

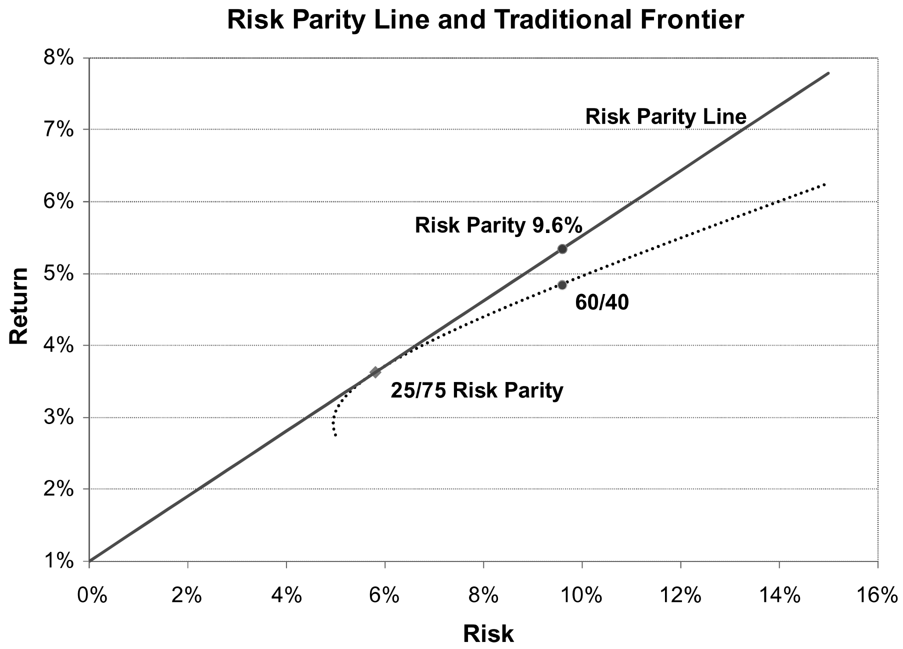
The resulting portfolio has the notional exposure of 41% in stocks and 124% in bonds. Its expected return would be higher than that of 60/40, thus offering both a higher risk-adjusted and a higher total return!

$124\% \times 2.75\%$ - $65\% \times 1\%$, which makes the leverage cost explicit—65 basis points in this hypothetical case. We have effectively borrowed an additional 65% at the short-term risk-free rate and invested it in the 25/75

**Figure 2**

EXHIBIT 3

**Risk Parity Portfolio Line and the Traditional Frontier**




*Source:* Qian (2011)





*2.6 Risk aversion*

In the working paper by Frazzini and Pedersen (2010), it was found that “if some investors are averse to leverage, low-beta assets provide higher risk-adjusted returns and high-beta assets provide lower risk-adjusted returns.” These leverage-averse investors will tend to overweight riskier assets to avoid leverage and as a result, the prices of these assets will rise, causing their expected returns to drop. The opposite will be true for safer assets. As a result, investors who are willing and able to use leverage can earn higher risk-adjusted returns by overweighting the relatively cheaper safe assets and underweighting the relatively more expensive risky assets. Extending this finding, it can be seen that, contrary to the theory of CAPM, the highest risk-adjusted return is achieved not by the market portfolio, but by a portfolio that overweights safer assets, namely the Risk Parity portfolio. The results of this paper concluded that an investor who is less averse to or constrained by leverage could benefit by overweighting low-beta assets, underweighting high-beta assets and levering up the resulting portfolio (Frazzini and Pedersen, 2010). These finding were subsequently found to be empirically consistent. Asness *et al* (2012) conclude that leverage risk is rewarded in equilibrium through the relative pricing of securities, which is why the tangency (theoretical market) portfolio includes a disproportionate amount of safer assets.” Its composition will therefore vary according to how many leverage-averse investors there are: a figure which cannot be quantified *ex ante.* The concept of Risk Parity is put forward as a solution to this, in line with the Leverage Aversion Theory. Anderson *et al* (2012) make an important disclaimer in that despite empirical evidence pointing to levered Risk Parity outperforming the market consistently, this is in an 'idealised setting' which does not take into account backtesting biases or market frictions. This will be elaborated on in the following paragraphs.

*2.7 The backtesting period and assumptions about market frictions*

Contrary to Frazzini and Pedersen's (2010) theory of risk aversion, Anderson *et al* (2012) found that the performance of a Risk Parity portfolio depends materially on two primary factors, namely the backtesting period and assumptions about market frictions. If some of the securities that make up a successful trading strategy today were not available in the past, then the strategy has no true antecedent. Proxies must then be used for these securities, which can distort the measured returns. Furthermore, adding more securities today could change the profitability of a strategy. Anderson *et al* (2012) were able to confirm this notion in their study titled *'Will My Risk Parity Strategy Outperform?'* Here, they evaluated various trading strategies over an 85-year long sample, which was broken up into four subperiods. All four subperiods were not only exposed to vastly different economic conditions, but also different available securities and borrowing rates. They consequently found that different subperiods had different superior trading strategies.

Chaves *et al* (2010) also allude to backtesting biases when they discuss sensitivity of the asset inclusion decision with the Risk Parity strategy. They postulate that it is not the process of including more assets which will improve results: including lower-risk bond instruments, which generally have higher Sharpe Ratios, can lead to better backtested results. However, there is no guarantee that these Sharpe Ratios can persist *ex ante* (Chaves *et al,* 2010).

Secondly, Anderson *et al* (2012) found that when accounting for market frictions, such as transaction costs, the levered Risk Parity strategy underperforms both value-weighted and 60/40 strategies. They argued that this could be a result of the "high degree of leverage in the levered risk parity strategy and the fact that their strategy contains look-ahead bias and is thus uninvestable", (Anderson *et al*, 2012). To improve the robustness of their results, they evaluated their various trading strategies under three sets of assumptions about transaction costs. The base case used the risk-free rate as the borrowing rate. The middle case used the three-month Eurodollar deposit rate from 1971 and the risk-free rate plus 60 basis points before 1971. This is due to the availability of Eurodollar data. The final case used the middle case's borrowing assumptions and added 'turnover-induced trading costs', which varied over the different subperiods.

*2.8 Constructing the levered Risk Parity portfolio*

There are two contrasting methods of constructing the levered Risk Parity strategy presented in the papers by Asness *et al* (2012) and Anderson *et al* (2012). The former's strategy is described as unconditional: it uses a constant scale factor, *k*, such that the annualized volatility of the Risk Parity strategy matches the *ex post* volatility of the benchmark portfolio, which in these studies comprised a value-weighted market portfolio. Asness *et al* (2012) argue that because *k* is constant over time, it does not affect the statistical conclusions drawn and is therefore unconditional. The latter study mentioned above describes their methodology for constructing the levered Risk Parity portfolio as conditional: the portfolio is rebalanced at each rebalancing date such that the *ex post* volatility of the benchmark and Risk Parity portfolios match. Thus the variable *k,* which is essentially a leverage constant, will change at each rebalancing date. Anderson *et al* (2012) argue that the unconditional version of levered Risk Parity is uninvestable, as *k* can only be computed once the entire study period has elapsed. Furthermore, the target standard deviation of each asset class is also not known until the end of the study period and is therefore not a realistic portfolio construction technique.

These two methods of constructing a levered Risk Parity portfolio yielded two vastly different results in the two studies spoken about. When comparing the graphs of cumulative log returns over the sample periods, the results of Anderson *et al* (2012) show the levered Risk Parity portfolio's cumulative return to be roughly half that of Asness *et al's* (2012). Naturally, there could be many other variables at play here, but it certainly draws attention to the issue of portfolio construction.

*2.9 Criticisms of Risk Parity*

The portfolio construction technique of Risk Parity would seem too good to be true if it weren't for criticisms. Clare, A., Seaton, J., Smith, P. and Thomas, S. (2015) compare a buy-and-hold Risk Parity strategy to a trend following technique, which uses a simple rule of trading out of risky assets and into cash if the former is in a downtrend. They believe that trend following circumvents many of the behavioural biases that investors are subject to, specifically regret and herding. The same feature heralded by academics as superior for the Risk Parity technique, namely not relying on expected returns, is deemed inadequate by Clare *et al* (2015). They argue that financial market momentum can offer explanatory power to future market returns.

Clare *et al* (2015) found that the buy-and-hold risk parity approach outperformed an equally-weighted methodology (each of the five asset classes tested are weighted equally by market capitalisation) on a risk-adjusted or Sharpe Ratio basis. They argue that this could be largely a result of the bull run of bonds over the sample period. However, when compared with trend following, risk parity underperformed. They found that the greatest benefit of trend following was being disinvested in the market during period of downturn. Furthermore, when combined with momentum, trend following returned a better risk-adjusted performance, smaller drawdowns and a less negative skew than just the former.

Chapter 3: Data and methodology

The results presented in this paper are based on stock, bond and property monthly returns data, denominated in South African Rand, for the period 1 January 2005- 30 September 2015 from the *A-Dex* Fund Manager's online database. For the levered Risk Parity strategy, the RMB cash index data is used over the same time period. All of the strategies were rebalanced on a monthly basis, with a training period of 12 months, meaning the first 12 months of the given time period isn't used in the calculations. All of the strategies hold an initial constraint of setting the maximum weight of any one asset class to 60% of the entire portfolio, as well as a fully invested constraint[1].

*3.1 Strategies:*

This paper will be composed of two key tests. Each of the tests will involve five trading strategies representing different asset allocation techniques. The difference between the two tests lies in the composition of the equity asset class. The first part of this paper, Test 1, will subcategorise the equity component into the FINDI 30 and RESI 20. Test 2 will subcategorise the equity component into Value and Momentum indices.

The first trading strategy will be the traditional 60/40; a fully-invested strategy which allocates 60% of the portfolio to equity and 40% to debt. This strategy will use the FTSE/ALSI and the ALBI indices for the equity and debt components respectively. It will be an identical portfolio in Test 1 and 2, and thus its results will not be included in the write-up for Test 2. It is a common pension fund strategy and will thus mimic one of the largest pension funds in South Africa. The remaining four strategies will be a minimum variance, maximum Sharpe Ratio, unlevered and levered Risk Parity. These strategies are based solely on predicted risk and thus do not require the calculation of expected returns. For Test 1, these portfolios will be built using four asset classes, namely the South African All Bond Index (ALBI), the FINDI 30, the RESI 20 and the South African Property Index. These indices contain the largest (by market capitalization) and most liquid stocks and bonds, thus reflecting a realistic investing scenario. All of these strategies will be subject to a fully invested constraint, a "box" constraint which stipulates the maximum weighting of any given asset class and various objectives, depending on the strategy.

The second part of this paper, known as Test 2, will subcategorize the equity portion of the above strategies into Value and Momentum Indices, taken from the Salient Index Funds. The Salient Value Index Fund tracks the proprietary Salient Value Index. It is constructed through a pre-defined rules-based strategy to select and weight stocks on their degree of cheapness as measured by price

[1] The program *R Studio* will be used, with its *Portfolio Analytics* package.

relative to a composite of headline earnings, book value and dividends. It consists of a set of 25-30 stocks chosen from the 60 largest and most liquid stocks listed on the JSE. In this manner, the Value effect in share price returns is offered in a low cost indexed form.

The Salient Momentum Fund tracks the proprietary Salient Momentum Index. It is constructed through a pre-defined rules-based strategy to select and weight stocks on their recent performance as measured by a composite of price and earnings acceleration metrics. It consists of a set of 25-30 stocks chosen from the 60 largest and most liquid stocks listed on the JSE. In this manner, the Momentum effect in share price returns is offered to investors in a low cost indexed form[2].

The debt and property asset classes will be identical to that of Test 1. The objective of Test 2 is to determine if, when composing the equity asset class according to style indices, the superior returns from these anomalies filter into the various strategies. Furthermore, by looking at the asset class weights obtained through the optimization process, one can identify potential style anomalies that exist on the Johannesburg Stock Exchange. The data obtained is only available in real-time from 1 January 2010, drastically reducing the number of observations from 118 to 58. This could cause biases in the results and investors should note this with caution. Both Test 1 and Test 2 calculate the annualized return on cash and use this figure as a proxy for the Risk Free rate. Due to the different sample periods for Test 1 and 2, two different rates will be used in the tests.

### *3.2 Minimum-variance*

This portfolio lies on the leftmost side of the Efficient Frontier, and whilst being suboptimal from a Sharpe ratio point of view, could represent a portfolio of a highly risk-averse investor. It will be created by setting an objective to minimize portfolio risk, and rebalanced monthly, with a training period of 12 months.

### *3.3 Maximum Sharpe Ratio*

The tangency portfolio on the Capital Market Line will be constructed by adding the *MaxSR* objective on *Portfolio Analytics*. The original box constraint will remain, thereby restricting each asset class' weight to a maximum of 60%. As the results in the following section will show, this constraint is pushed to the limit in order to maximize the Sharpe Ratio, resulting in the RESI and PROP indices constituting almost 50% of the portfolio each. This would empirically pose many issues for fund managers' mandates and will thus only be considered as a theoretical portfolio, in line with the reviewed literature. A solution to this problem, known as *shrinking the covariance matrix*, is discussed below.

[2] Index descriptions taken from Salient Quants website.
Available at: http://salientquants.com/images/FundSheets/SalientSAValueIndexFundFacts-0216.pdf

*3.4 Shrinking the sample covariance matrix*

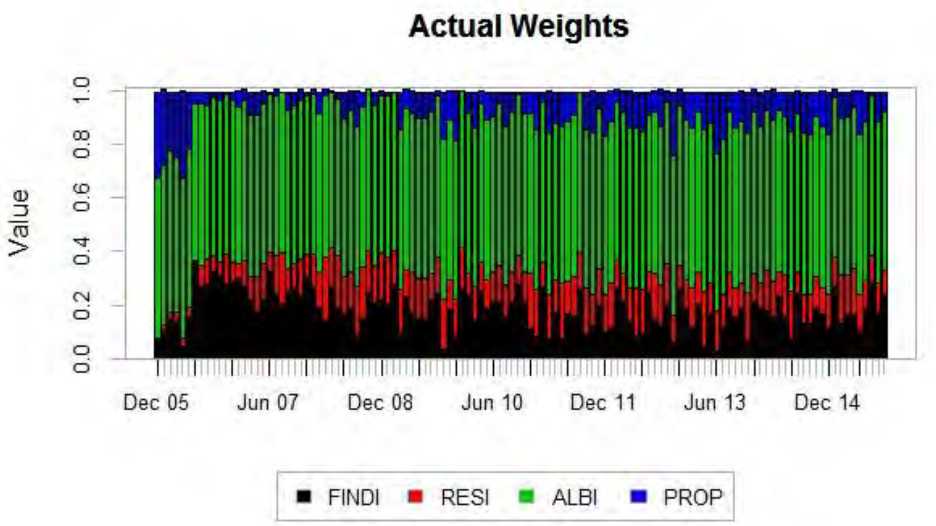
According to Ledoit, O. and Wolf, M. (2003), using a sample covariance matrix for portfolio optimisation techniques will create significant estimation error. They advocate for the use of a matrix obtained through a process called *shrinkage*. This process "tends to pull the most extreme coefficients towards more central values, thereby systematically reducing estimation error where it matters most," (Ledoit *et al*, 2003, p. 1). The result is a reduction in tracking error relative to a benchmark index and a greater realized information ratio for the active portfolio manager.

When the portfolio optimisation was initially done for the minimum variance and maximum Sharpe Ratio portfolios in this paper, the resulting weights were hugely volatile, as is evident in Figure 3 and 4. Of particular concern is the significant fluctuation of portfolio weights for the Maximum Sharpe Ratio portfolio, with its composition ranging from almost entirely RESI-PROP to a FINDI-PROP dominant composure from the latter parts of 2012. Therefore, a custom moment function is created in *Portfolio Analytics*, which shrinks the covariance matrix every rebalancing period. The shrinkage intensity is a number ranging from 0 to 1 and is estimated using an analytic formula from Opgen-Rhein, R. and Strimmer, K. (2007). This formula is derived below:

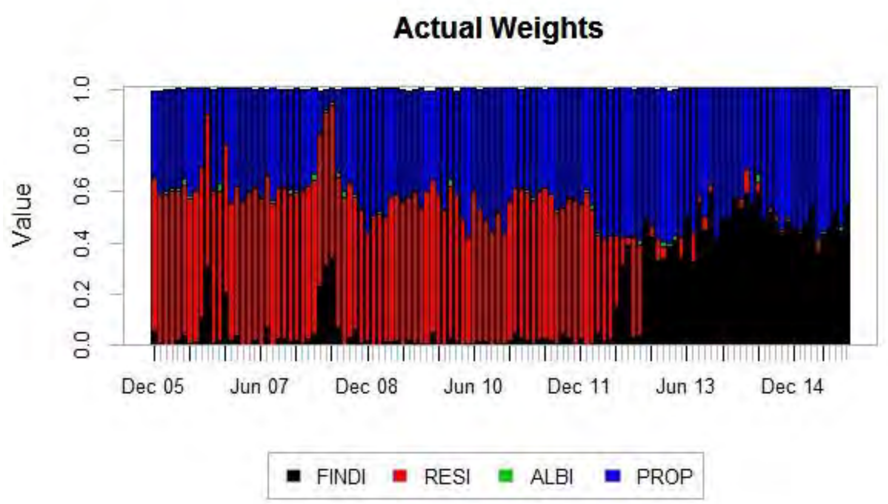
$$\begin{aligned} \delta^{\lambda} &= \delta^{0} - \lambda\Delta \\ &= \hat{\theta} - \lambda\,(\hat{\theta} - \hat{\theta}^{Target}) \end{aligned} \tag{10}$$

The shrinkage estimate $\delta^{\lambda}$ is the linear combination $\lambda\hat{\theta}^{Target} + (1-\lambda)\hat{\theta}$ of the original estimator $\hat{\theta}$ and a target estimate $\hat{\theta}^{Target}$. The parameter $\lambda$ determines the extent to which these estimates are pooled together and therefore the intensity of the shrinkage. If $\lambda = 1$ then the target estimate dominates completely, whereas if $\lambda = 0$, the original estimator dominates and no shrinkage occurs (Opgen-Rhein *et al,* 2007.

**Figure 3 and 4: Actual weights of the Minimum Variance and Maximum Sharpe Ratio Portfolios before shrinking the covariance matrix (Test 1)**




*Notes:* Figure 3 represents the actual weights of the four asset classes tested, for the **Minimum Variance** strategy, before the shrinking of the covariance matrix. This is a fully invested strategy, with 'box' constraints set on the different asset classes of a maximum of 60%. An objective is added to minimize the portfolio standard deviation, with rebalancing done on a monthly basis. The volatility of the weights is a result of using a sample covariance matrix, which contains estimation error that violates the mean-variance optimizer (Lediot *et al,* 2003).




*Notes:* Figure 4 represents the actual weights of the four asset classes tested, for the **maximum Sharpe Ratio** strategy, before the shrinking of the covariance matrix. This is a fully invested strategy, with 'box' constraints set on the different asset classes of a maximum of 60%. An objective is added to maximise the Sharpe Ratio, with rebalancing done on a monthly basis. The extreme volatility of the weights can clearly be seen here.

### *3.5 Levered and unlevered Risk Parity*

The unlevered Risk Parity strategy is a fully invested one, whereby the *ex post* risk contributions of the asset classes are equal. Equation 4, which is taken from Clarke *et al* (2013), has been used in much of the literature, in one form or another. As was discussed in the review of literature, this equation suffers from endogeneity, whereby the portfolio constants contained within the equation, depend on the final asset class' weights. The *Portfolio Analytics* package solves this problem with its *DEoptim* solver. The objective *Risk Budget* is added and the optimization is run, using a training period of 12 months, with the weights rebalanced monthly so that they sum to 1 at each rebalancing. This fully invested constraint is constructed using a window of 0.99-1.01 to give the solver some leeway. The same leeway is given when constructing the leverage constraints for the levered portfolio, and is discussed in due course.

The levered Risk Parity strategy adds the RMB cash index as an asset class and a shorting constraint, to lever up the other asset classes. As Qian (2011) postulates, when examining the relative placement of the levered Risk Parity and 60/40 portfolios in Figure 2, an investor is faced with a dilemma: The Risk Parity portfolio offers a better Sharpe ratio but a lower return, whereas the traditional 60/40 portfolio offers a slightly better return, at the expense of a lesser Sharpe ratio. This is confirmed in the results section of this paper. This is where the concept of leveraging the entire Risk Parity portfolio along its own Capital Market Line comes into play. Along this line, all portfolios are Risk Parity, with the same Sharpe ratios (Qian, 2011). The resulting leveraged portfolio will merely have a higher return, for the same level of risk as the 60/40 portfolio.

Qian (2011) calculates this level of leverage by dividing the annualized standard deviation of the 60/40 portfolio by that of the unlevered Risk Parity. For Test 1, we obtain a level of 116% (=9.98/8.58). This means that the minimum weight of cash is set to -16%. The portfolio leverage constraint, which is the sum of the absolute value of each of the asset weights, is given a leeway of lying between 0.99 (99%) and 1.16 (116%). An objective is then added in *Portfolio Analytics* to match the *ex post* monthly standard deviation to that of the 60/40 portfolio, and subject to the aforementioned leverage constraints. For Test 2, the same 60/40 portfolio is used, so we obtain a level of 139% (=9.98/8.58), which will be round up to 140%. This means cash will be allowed a short position of 40%, which represents a significantly higher level of leverage compared with Test 1; the results will corroborate this. Once again, the leverage constraint is given leeway and lies between 0.99 (99%) and 1.39 (139%).

The above strategies are all compared against the FTSE/ALSI benchmark.

## Chapter 4 Results: Test 1

The following section presents the results of the five different strategies tested, namely the unlevered and levered Risk Parity, the minimum variance, maximum Sharpe Ratio and 60/40 pension fund for Test 1. A comparison with the FTSE/ALSI benchmark is also made. Direct comparisons are also made for each strategy's counterpart in Test 2, which simply categorizes the equity asset class differently, namely into Value and Momentum indices. Below is a table of the annualized returns and standard deviations of the four asset classes used in Test 1, with their corresponding Sharpe Ratios. A risk free rate of 7.58% is used, which is the annualized return of the cash index used in the levered strategy.

Table 1: Descriptive Statistics of the FINDI, RESI, ALBI and PROP indices for Test 1

| | **Annualized Return** | **Annualized Standard Deviation** | **Annualized Sharpe Ratio** |
|---|---|---|---|
| FINDI | 0.2012 | 0.1498 | 0.8368 |
| RESI | 0.0810 | 0.2475 | 0.0210 |
| ALBI | 0.0818 | 0.0692 | 0.0874 |
| PROP | 0.2091 | 0.1668 | 0.7992 |

The FINDI and the PROP indices have the highest Sharpe Ratios, which would explain their relative overweighting in the maximum Sharpe Ratio strategy (see below). The ALBI surprisingly doesn't have the lowest Sharpe Ratio, which is in part due to its low risk. However, it is hardly robust at 0.0874, and could partially explain the underperformance of the Risk Parity strategies. The RESI index, comprising the largest and most liquid resource stocks on the JSE, has an extremely poor annualized return with a disproportionate amount of risk, led chiefly by a volatile exchange rate, fluctuating commodity prices and labour unrest in these companies. The result is a Sharpe Ratio of 0.0210, the lowest of the asset classes tested.

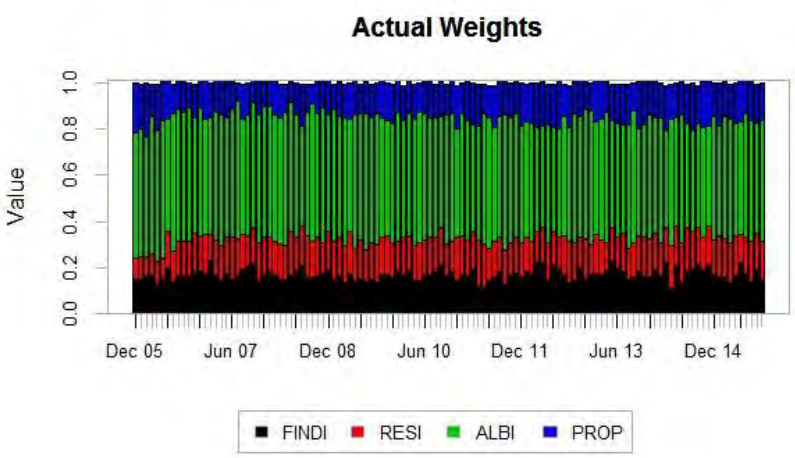
### *4.1 Unlevered Risk Parity*

A summary of results is given in Table 2. As can be seen, the arithmetic mean return is a meagre 0.98% with a standard deviation of 2.4%. The annualised return and standard deviation are 11.46% and 8.58% respectively, producing a Sharpe Ratio of 0.45. This is calculated using the annualized risk free rate of 7.58%. While the annualized return figure is greater than its counterpart in Test 2, the greater standard deviation and risk free rate in Test 1 leads to a lesser Sharpe Ratio.

The annualized risk and return figures underperform every strategy except the minimum variance and confirm the idea that weighting asset classes according to Risk Parity will result in an overweighting of low-risk assets, and an underweighting of higher-risk assets. This can be confirmed when looking at Figure 5; a graph of the actual weights of the different asset classes. The relatively lower-risk ALBI is weighted the most over the entire sample period, and the higher-risk equity and property indices are given lesser weightings. The resulting portfolio therefore has a relatively smaller return.

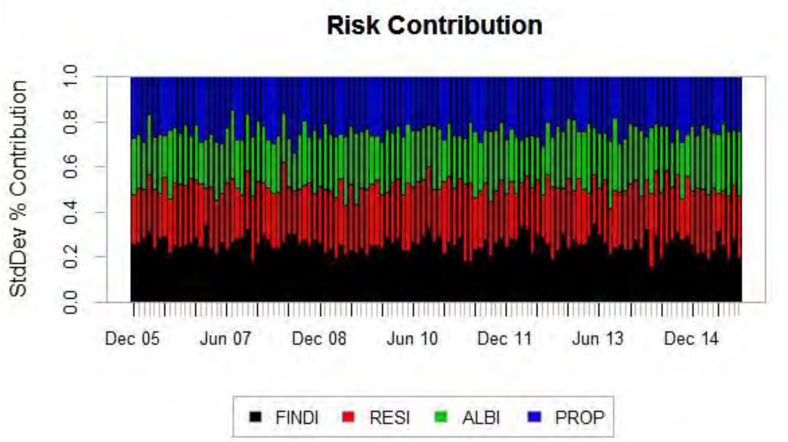
Figure 6 shows the risk contribution, in percentage, of the different asset classes, as measured by standard deviation. It is clear to see the concept of Risk Parity in this figure. Table 3 presents a summary of downside risk measures for the strategies in question. The semi-deviation, calculated by square rooting the deviations of values less than the mean, is 1.82%. The gain deviation measures the fund's average return for periods with a gain only, and then measures the variation of the winning periods around this gain mean. The loss deviation is calculated in the same manner, except for periods of loss only. This strategy has a gain and loss deviation of 1.51% and 1.54% respectively.

**Figure 5: Actual weights of the unlevered Risk Parity portfolio for Test 1**




*Notes:* Figure 5 represents the actual weights of the four asset classes tested, for the **unlevered Risk Parity** portfolio. It is a fully invested portfolio, with 'box' constraints imposed on the asset classes at a maximum of 60%. A 'risk-budget' objective is added on *PortfolioAnalytics* to create Risk Parity.

**Figure 6: Risk contribution of the unlevered Risk Parity portfolio, measured by Standard Deviation**



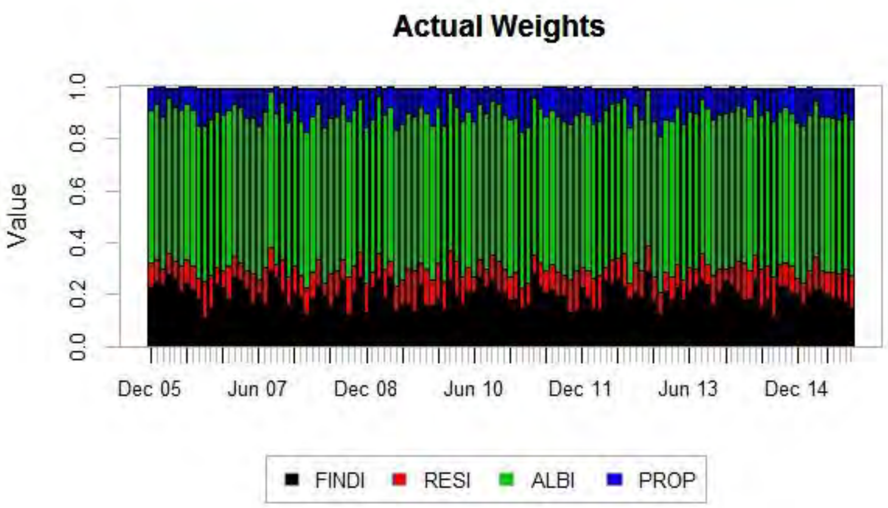

*Notes:* Figure 6 represents the risk contribution (in percentage) of the four asset classes tested, for the **unlevered Risk Parity** portfolio. It is a fully invested portfolio, with 'box' constraints imposed on the asset classes at a maximum of 60%. A 'risk-budget' objective is added on *PortfolioAnalytics* to create Risk Parity.

### *4.2 Minimum variance*

It is not surprising that the arithmetic mean return of the minimum variance portfolio sits below that of the unlevered Risk Parity, at 0.31%. Its corresponding standard deviation is 2.33%; the smallest of all the portfolios tested. The annualized return and standard deviation are also the smallest of the strategies tested, at 2.72% and 8.08% respectively, resulting in a Sharpe Ratio of -0.602. This is significantly more negative than its counterpart in Test 2, driven chiefly by a smaller annualized return. This once again confirms its relative placing on the efficient frontier in Figure 10, where it lies below the risk free rate.

When looking at Figure 7, a graph of the actual weights of the different asset classes *after* the shrinking of the covariance matrix, the overweighting of bonds can explicitly be seen. In almost every month, the weight of the ALBI is over 40% of the entire portfolio, with the relatively more volatile Property and Resource Indices being significantly underweighted. The relative stability of the weights over the time period, compared with Figure 3, is also apparent, providing evidence of a shrunken and more accurate sample covariance matrix.

**Figure 7: Actual weights of the minimum variance portfolio, after shrinking the covariance matrix, for Test 1**




*Notes:* Figure 7 represents the actual weights of the four asset classes tested, for the **minimum variance** strategy, after the shrinking of the covariance matrix. This is a fully invested strategy, with 'box' constraints set on the different asset classes of a maximum of 60%. An objective is added to minimize the portfolio standard deviation, with rebalancing done on a monthly basis.

*4.3 Maximum Sharpe Ratio*

Table 2 shows the Maximum Sharpe ratio portfolio yielding an arithmetic return of 2.06% and a standard deviation of 4.27%. The annualized return and standard deviation are 26.87% and 14.71% respectively, yielding a healthy Sharpe Ratio of 1.31. This is the highest Sharpe Ratio of the unlevered strategies in Test 1, but still underperforms its counterpart in Test 2. This is in part due to the higher risk free rate used in Test 1. The level of risk is significantly higher than the other strategies, and can be explained by the heavy weightings of the FINDI and PROP indices, as shown in Figure 8**.** Figure 9 compares the weightings of the FINDI and PROP indices for the maximum Sharpe Ratio and unlevered Risk Parity strategies. It is clear to see the vast difference in weightings of the indices for the two strategies. It is unsurprising that for the maximum Sharpe Ratio portfolio, the weightings of the two indices are far higher, with the PROP index remaining at around 40% and the FINDI index increasing to around that level. Contrast that with the unlevered Risk Parity portfolio, whereby the two indices oscillate around the 15% mark for the entire sample period. This relative overweighting of the two indices, which individually have the highest Sharpe Ratios of the asset classes tested, could explain the higher return of the this strategy.

This strategy also contains the greatest negative skewness of -0.454. This means that it is more volatile when losing money than it is when gaining money; a fundamental tenet of behavioural finance and a common characteristic amongst managers. This robust negative skew is corroborated by 3.04% loss deviation: the greatest out of the tested strategies. Loss deviation essentially measures the volatility of downside performance, which can be illustrated by a fat left tail in the distribution.

**Figure 8: Actual weights of the maximum Sharpe Ratio portfolio, after shrinking the covariance matrix, for Test 1**

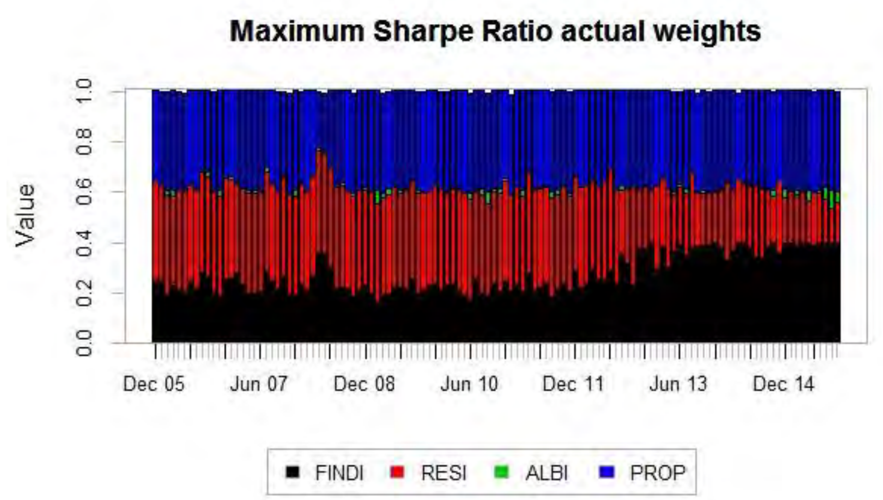
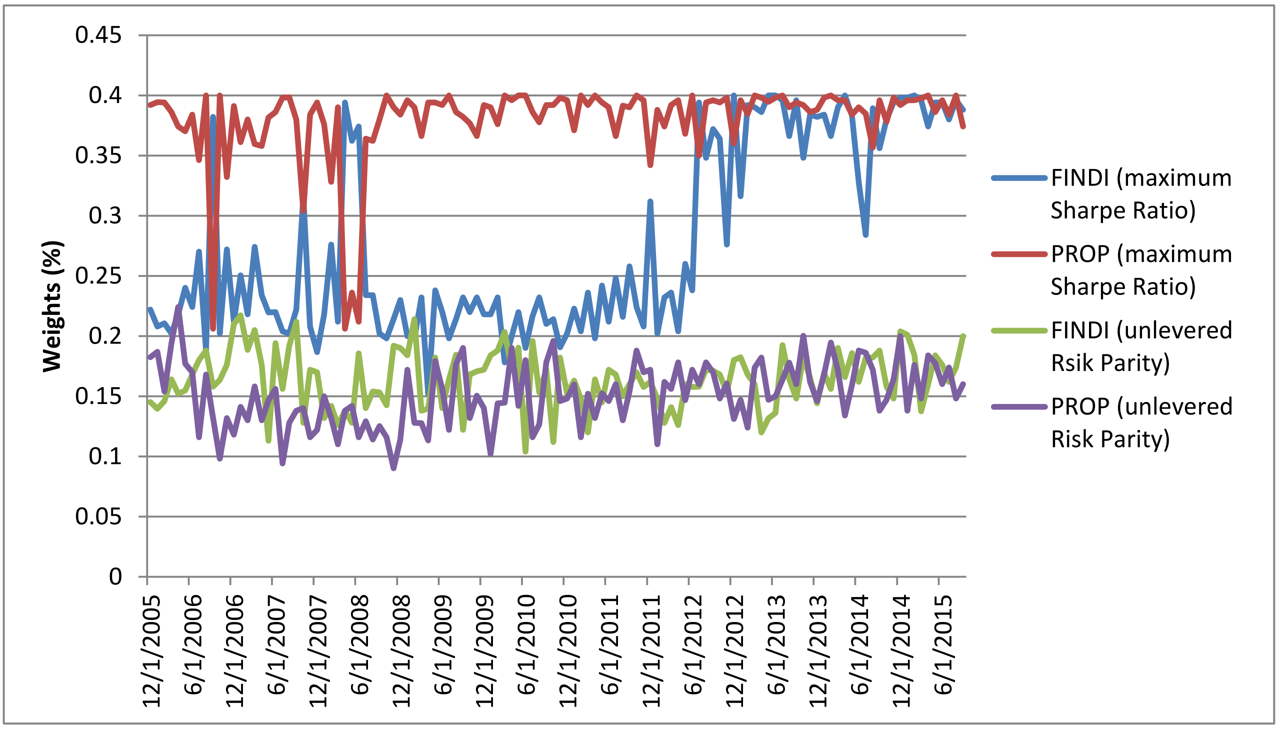



**Figure 9: Weights of the FINDI and PROP indices for the maximum Sharpe Ratio and unlevered Risk Parity strategies (Test 1)**

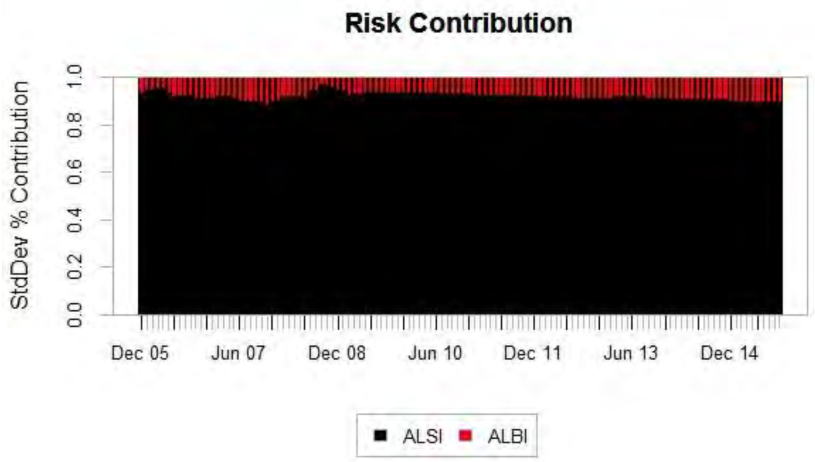


*4.4 60/40 Pension Fund*

This popular Pension Fund strategy returned an arithmetic mean of 1.08% with a standard deviation of 2.92%. Its annualized return and standard deviation are 11.86% and 9.98% respectively, resulting in a Sharpe Ratio 0.43 and hence sits below the Capital Market line in Figure 10**.** If a line had to be extended from the risk free rate through the unlevered Risk Parity portfolio, whereby all portfolios on that line would be in Risk Parity, the 60/40 portfolio would lie marginally below this line. This is similar to the findings of Asness *et al* (2012), illustrated on the efficient frontier in Figure 1. This relative underperformance of the 60/40 portfolio is in part due to the greater risk it possesses, by virtue of it holding 60% of its capitalisation in equities. This added risk has not been commensurately compensated for in the Johannesburg Stock Exchange.

This strategy has a negative skewness of -0.271, although the unlevered Risk Parity strategy is the only one with a more positive figure of -0.1173. This indicates that, despite the riskiness of this 60/40 asset allocation and the risk-seeking nature of these types of investors, there is relative safety in pension funds compared with the other strategies. Figure 10 shows the risk contribution in percentage of the ALSI and the ALBI. The lack of risk diversification, as spoken about in the review of literature, is clearly evident in this figure. For the entire sample period, the ALSI's risk contributes more than 80% to the overall portfolio risk, thus indicating the inherently risky nature of equities.

**Figure 10: Risk contribution of the unlevered Risk Parity portfolio, measured by Standard Deviation**




*Notes:* Figure 10 represents the risk contribution (in percentage) of the ALSI and ALBI, for the **60/40** portfolio. It is a fully invested portfolio, with the 60% and 40% weightings rebalanced monthly.

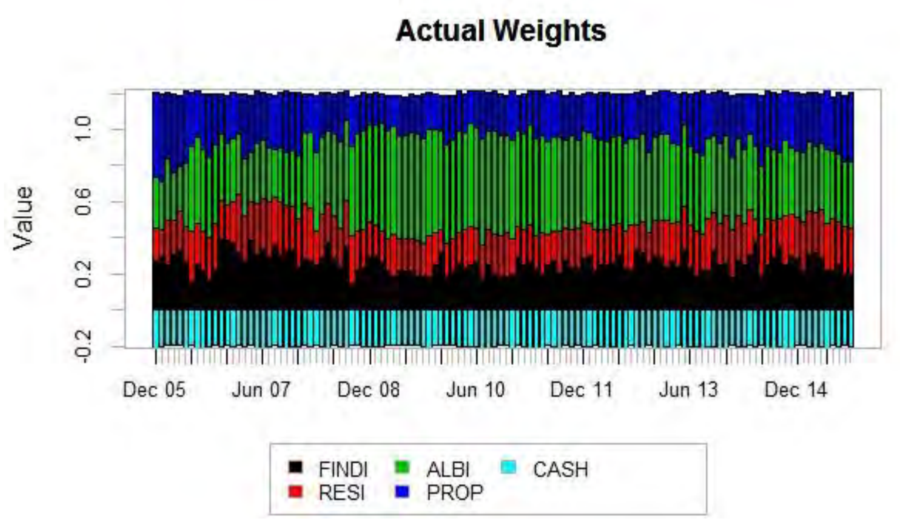
*4.5 Levered Risk Parity*

The final portfolio examined in this paper yields an arithmetic return of 3.69% with a corresponding standard deviation of 4.29%. The annualized return and standard deviation are 12.08% and 9.98% respectively. The resulting Sharpe Ratio is 0.4509, which is almost identical to its unlevered counterpart. This is unsurprising since it has been levered up along the Risk Parity Capital Market Line. It should be recalled that the level of leverage undertaken was done so using the simple arithmetic calculation (as conducted by Qian (2011)) of dividing the annualized standard deviation of the 60/40 portfolio by that of the unlevered Risk Parity portfolio. The resulting figure will be a levered portfolio weight, which should in theory still lie along the unlevered Risk Parity portfolio's Capital Market Line. The annualized return figure can therefore be calculated using a simple straight line formula, whereby the intercept is the risk free rate of 7.58%, the gradient is the unlevered Risk Parity Sharpe Ratio of 0.4513 and the so-called 'x' value is the 60/40 portfolio's standard deviation of 9.98%. Plugging in these values will yield an annualized return of 12.08%.

Due to the nature of the *Portfolio Analytics* package, when setting the leverage constraints, one cannot be too restrictive; else the solver will be unable to return results with the exact weights. Therefore, for the total portfolio weight, the constraint was set with a band of 99% and 116%, and the individual cash weights (the asset being shorted) were given minimum and maximum values of -17% and -15% respectively. Figure 11 graphs the actual weights of the asset classes for this strategy. Providing this leeway could result in a leverage ratio different from that of the 116% obtained by simple arithmetic. This does undermine the robustness of the results; however cognizance must be given to the objective of creating a levered portfolio. It was to ascertain whether applying leverage to a Risk Parity portfolio, within an acceptable range, can provide superior returns to the other portfolios and to a benchmark. Changing the leverage constraints will indeed change the fundamental statistics of the portfolio. This will be discussed in due course.

Another interesting result of this strategy is its positive skewness of 0.29. This means that the portfolio is more volatile when making money than it is when losing money and should be noted with caution by managers. It is the only strategy explored in this paper which yields a positive skew. This observation is corroborated by the gain deviation of 3.55%; the largest of the strategies tested. The gain deviation is a measure of volatility of upside performance, illustrated by a fat right tail, or positive skewness.

**Figure 11: Actual weights of the levered Risk Parity portfolio for Test 1**




*Notes:* Figure 11 represents the actual weights of the four asset classes tested, for the **levered Risk Parity** strategy. This is a fully invested strategy, with 'box' constraints set on the different asset classes of a maximum of 60% and a maximum shorting constraint on cash of -20%. A leeway is provided for the leverage constraint, setting the total portfolio weight between 0.99 and 1.19. The risk parity objective is added, with rebalancing done on a monthly basis.

**Table 2: Descriptive Statistics of the unlevered and levered Risk Parity, Minimum Variance, Maximum Sharpe, 60/40 and ALSI Portfolios**

| | Unlevered Risk Parity | Minimum Variance | Maximum Sharpe | 60/40 | Levered Risk Parity | ALSI |
|---|---|---|---|---|---|---|
| Minimum | -0.0488 | -0.0678 | -0.1392 | -0.0755 | -0.0716 | -0.1324 |
| Quartile 1 | -0.0052 | -0.0126 | -0.0071 | -0.0097 | 0.0053 | -0.0189 |
| Median | 0.0099 | 0.0067 | 0.0235 | 0.0123 | 0.0319 | 0.0160 |
| Arithmetic Mean | 0.0098 | 0.0031 | 0.0206 | 0.0108 | 0.0369 | 0.0122 |
| Geometric Mean | 0.0095 | 0.0028 | 0.0197 | 0.0104 | 0.0361 | 0.0112 |
| Quartile 3 | 0.0293 | 0.0174 | 0.0452 | 0.0293 | 0.0636 | 0.0361 |
| Maximum | 0.0615 | 0.0545 | 0.1265 | 0.0760 | 0.1332 | 0.1245 |
| SE Mean | 0.0022 | 0.0021 | 0.0039 | 0.0027 | 0.0040 | 0.0041 |
| LCL Mean (0.95) | 0.0055 | -0.0011 | 0.0128 | 0.0055 | 0.0291 | 0.0040 |
| UCL Mean (0.95) | 0.0142 | 0.0074 | 0.0284 | 0.0162 | 0.0448 | 0.0203 |
| Variance | 0.0006 | 0.0005 | 0.0018 | 0.0009 | 0.0018 | 0.0020 |
| Stdev | 0.0240 | 0.0233 | 0.0427 | 0.0292 | 0.0292 | 0.0445 |
| Skewness | -0.1173 | -0.3262 | -0.4538 | -0.2707 | 0.2916 | -0.2833 |
| Kurtosis | -0.5283 | -0.1508 | 1.1754 | 0.3434 | -0.3017 | 0.8698 |
| Sharpe Ratio | 0.4513 | -0.6015 | 1.3113 | 0.4289 | 0.4509 | 0.4344 |
| Annualized portfolio rebalancing return | 0.1146 | 0.0272 | 0.2687 | 0.1186 | 0.1208 | 0.1428 |
| Annualized portfolio Standard Deviation | 0.0858 | 0.0808 | 0.1471 | 0.0998 | 0.0998 | 0.1541 |

*Notes:* This table is a summary of key statistics, taken from *PortfolioAnalytics* for **Test 1**. *Unlevered Risk Parity* portfolio is risk-budgeted portfolio whereby the four asset classes (RESI, FINDI, ALBI and SA PROP) are weighted such that they contribute an equal amount to total portfolio risk, as measured by standard deviation. This portfolio is rebalanced monthly to maintain these weights. *Minimum variance* portfolio is constructed using the objective of the same name in *PortfolioAnaltyics*, and represents the leftmost portfolio on the Efficient Frontier. Rebalancing takes places monthly. *Maximum Sharpe* is constructed using a similar constraint, and represents the tangency portfolio on the efficient frontier. Rebalancing takes place monthly. *60/40* allocates 60% to equities (taken from the FTSE/ALSI) and 40% to bonds, taken from the ALBI. Rebalancing is done on a monthly basis to maintain these weights. *Levered Risk Parity* uses the quotient of its unlevered counterpart and 60/40 standard deviations to determine an appropriate level of risk. This portfolio is then levered up from the same standard deviation as the 60/40 portfolio.

| Table 3: Downside risk measures of the unlevered and levered Risk Parity, Minimum Variance, Maximum Sharpe, 60/40 and ALSI Portfolios | | | | | |
|---|---|---|---|---|---|
| | Unlevered Risk Parity | Minimum Variance | Maximum Sharpe | 60/40 | Levered Risk Parity |
| Semi Deviation | 0.0182 | 0.0167 | 0.0309 | 0.021 | 0.0289 |
| Gain Deviation | 0.0151 | 0.0141 | 0.0293 | 0.0178 | 0.0355 |
| Loss Deviation | 0.0154 | 0.0145 | 0.0304 | 0.017 | 0.0173 |
| Downside Deviation (MAR=10%) | 0.0176 | 0.0201 | 0.0249 | 0.0202 | 0.0148 |
| Downside Deviation (Rf=0%) | 0.0136 | 0.0154 | 0.0216 | 0.0161 | 0.0117 |
| Downside Deviation (0%) | 0.0136 | 0.0154 | 0.0216 | 0.0161 | 0.0117 |
| Maximum Drawdown | 0.1387 | 0.1748 | 0.2839 | 0.2285 | 0.1116 |
| Historical VaR (95%) | -0.0288 | -0.0367 | -0.0432 | -0.0319 | -0.0273 |
| Historical ES (95%) | -0.0436 | -0.0447 | -0.0746 | -0.0541 | -0.0451 |
| Modified VaR (95%) | -0.0335 | -0.0363 | -0.0527 | -0.0394 | -0.0313 |
| Modified ES (95%) | -0.0467 | -0.0463 | -0.087 | -0.0541 | -0.044 |

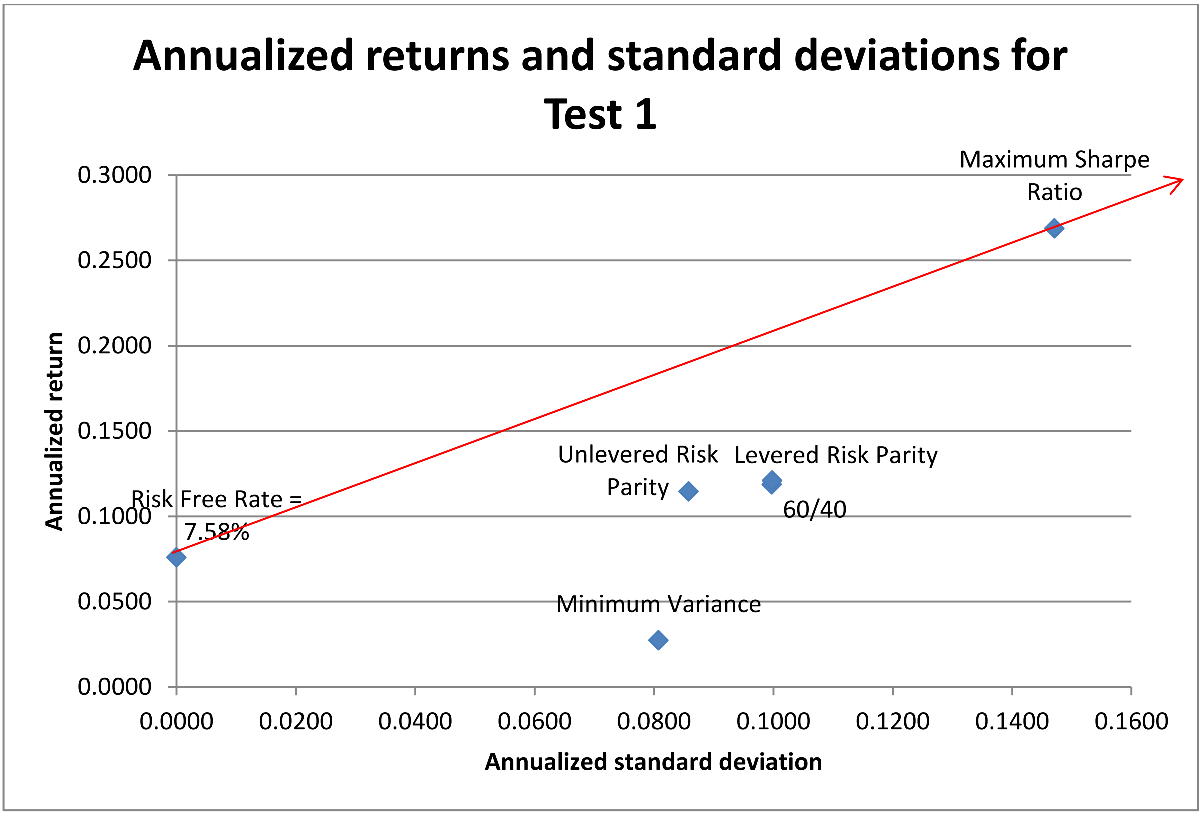
*Notes:* This table is a summary of key downside risk statistics, taken from *PortfolioAnalytics* for **Test 1**. The five trading strategies are included here.

The results of Test 1 are very similar to those generated by Asness *et al* (2013) and their theory of leverage aversion. The Risk Free rate was calculated as the annualized return on the cash index used for the levered Risk Parity strategy. A figure of 7.58% was obtained. Looking at Figure 12 below, which plots the annualized returns and standard deviations of the five strategies tested, we can see the unlevered Risk Parity and 60/40 portfolios having very similar Sharpe Ratios, with both portfolios lying within the Efficient Frontier. The former has a Sharpe Ratio of 0.45 and the latter, 0.43. Due to their similarity, the levered portfolio doesn't earn a significantly higher return than the reference 60/40 portfolio. On the efficient frontier in Figure 1, which is taken from Asness *et al* (2012), the levered portfolio lies far higher above the reference (Value-Weighted Market) portfolio, due to the unlevered Risk Parity portfolio having a much great Sharpe Ratio.

As was discussed in the literature review, and postulated by Asness *et al* (2013, p. 51), "leverage risk is rewarded in equilibrium through the relative pricing of securities, which is why the tangency portfolio includes a disproportionate amount of safer assets." In other words, investors who are able to use leverage, such as those utilising the tangency portfolio, can earn higher risk-adjusted returns by overweighting safer assets (Asness *et al,* 2013). The intuition behind this is that by overweighting safer assets, their price will increase, thus reducing their expected return. Furthermore, the levered

Risk Parity portfolio in this paper lies in a similar position to that in Figure 1: by leveraging it up from the 60/40 portfolio, it earns a higher return for the given level of risk.

**Figure 12: The Efficient Frontier (Test 1)**



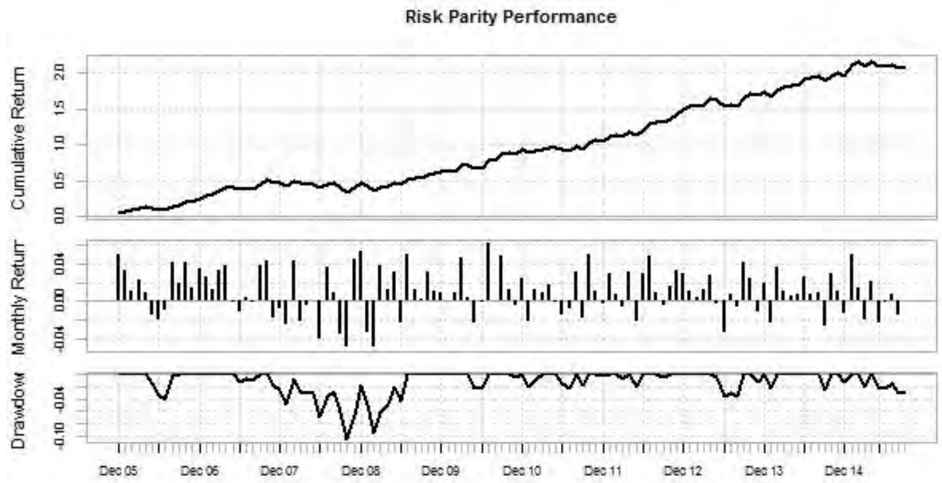

*Notes:* Figure 12 plots the annualized returns and standard deviations of the five tested strategies. The tangency portfolio (maximum Share Ratio) is constructed using a risk free rate of 6.06%, calculated as the annualized return of the cash index used

*4.6 Comparing with a benchmark*

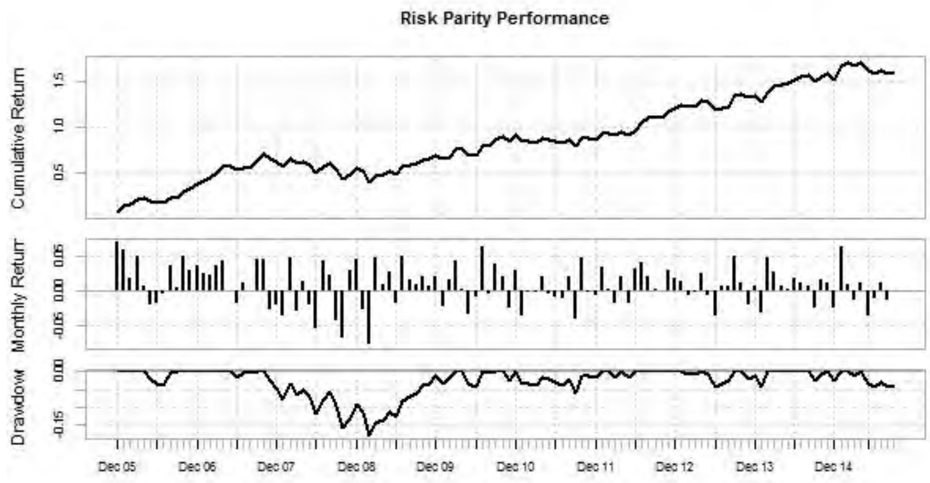
The FTSE/ALSI was used as a common benchmark in this paper, despite the presence of a bond class in all of the portfolios. It returned a meagre 1.2% arithmetic mean against a robust standard deviation of 4.5%. Its annualized return and standard deviation were 14.28% and 15.41% respectively, underperforming only the maximum Sharpe Ratio portfolio. This highlights the sustained Bull Run experienced on the JSE over the sample period.

Figures 13-17 display the cumulative returns, monthly returns and drawdowns of the levered and unlevered Risk Parity, minimum variance, maximum Sharpe Ratio and FTSE/ALSI portfolios. As can be seen in Figure 17**,** the cumulative return reaches just below 700% for the 10 year sample period, which is below that of the maximum Sharpe Ratio. The large dip and corresponding drawdown sustained during the 2008 Financial Crisis is clearly evident in the figure. The relatively flat returns of 2014 and 2015 can also be seen. It can therefore be stated that, whilst the unlevered Risk Parity portfolio underperforms this benchmark, despite levering up the Risk Parity portfolio along its Capital Market Line, to a level of risk akin to the 60/40 portfolio, one still cannot achieve superior risk-adjusted returns.

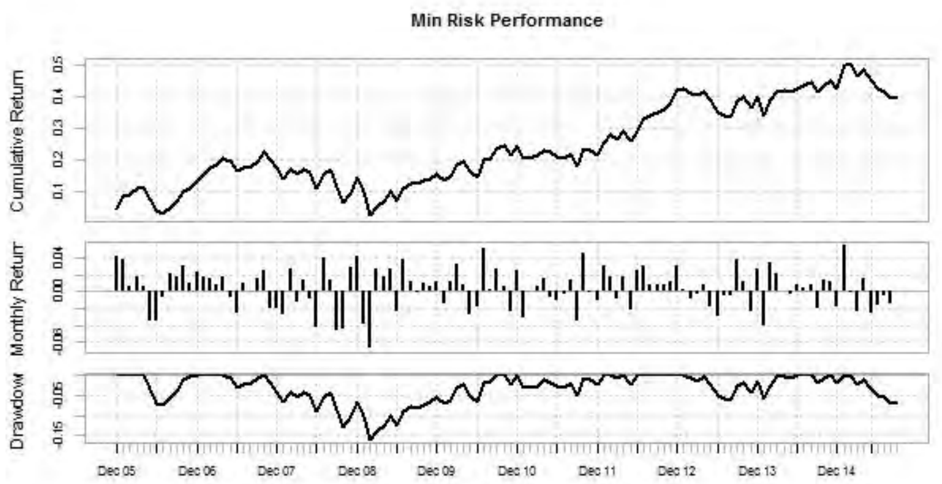
### Figures 13-17: Performance of the unlevered and levered Risk Parity, minimum Variance, maximum Sharpe Ratio and FTSE/ALSI portfolios for Test 1



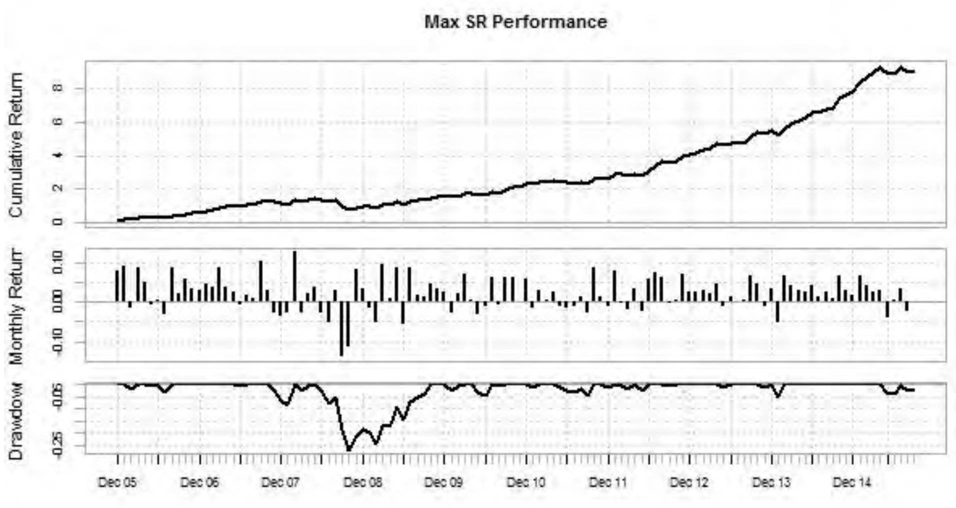

*Notes:* Figure 13 represents the cumulative returns, monthly returns and drawdown for the **unlevered Risk Parity** strategy.



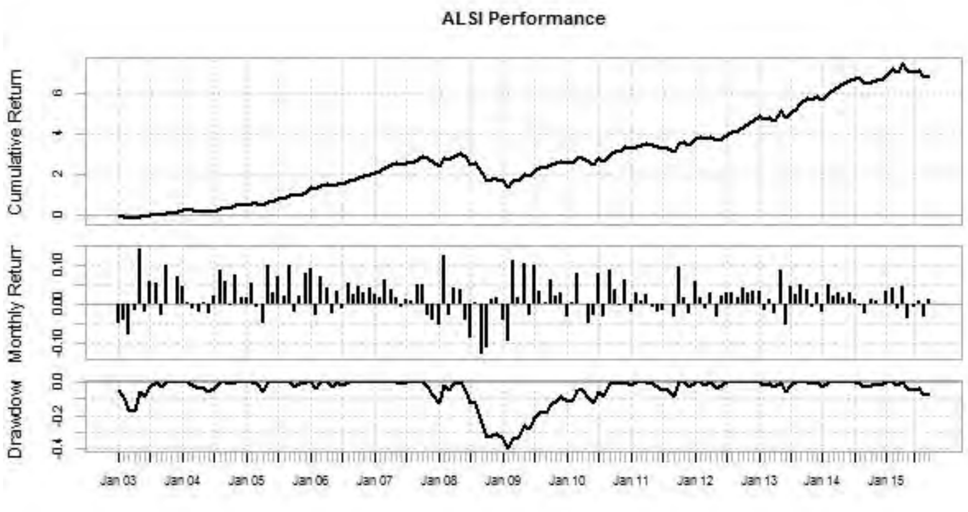

*Notes:* Figure 14 represents the cumulative returns, monthly returns and drawdown for the **levered Risk Parity** strategy.




*Notes:* Figure 15 represents the cumulative returns, monthly returns and drawdown for the **minimum variance** strategy.




*Notes:* Figure 16 represents the cumulative returns, monthly returns and drawdown for the **maximum Sharpe Ratio** strategy.




*Notes:* Figure 17 represents the cumulative returns, monthly returns and drawdown for the **FTSE/ALSI** benchmark.

Chapter 5 Results: Test 2

Below is a table of the descriptive statistics of the four asset classes used in Test 2, namely the Momentum, Value, ALBI and PROP indices. The risk free rate used is 6.06%, calculated as the annualized return of the cash index used in the levered strategy. The difference in rates between Test 1 and 2 comes from the differing sample periods used.

Table 4: Descriptive Statistics of the FINDI, RESI, ALBI and PROP indices for Test 2

| | **Annualized Return** | **Annualized Standard Deviation** | **Annualized Sharpe Ratio** |
|---|---|---|---|
| MOMENTUM | 0.1895 | 0.1261 | 1.0219 |
| VALUE | 0.1312 | 0.1145 | 0.6169 |
| ALBI | 0.0818 | 0.0692 | 0.3072 |
| PROP | 0.2091 | 0.1668 | 0.8903 |

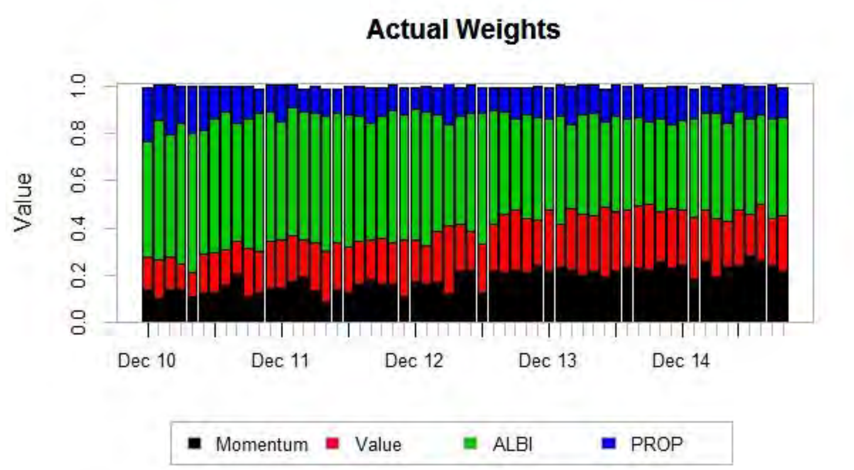
The Momentum index earns the highest Sharpe Ratio, in part due to its robust return. This is unsurprising, given that the index is comprised of the best performing stocks using price and earnings acceleration metrics. The next highest Sharpe Ratio is the PROP index at 0.89. The maximum Sharpe Ratio portfolio is virtually only comprised of these two indices and earned the highest return out of the strategies tested. Interesting to note is the large jump in Sharpe Ratio of the ALBI from 0.087 in Test 1 to 0.307 in Test 2, due to the change in risk free rate. As the Risk Parity strategies have a relative overweighting of bonds, one would expect the Test 2 Risk Parity strategies to therefore outperform their Test 1 counterparts. However, only the levered strategy earns a higher return, and this is partially due to the lower borrowing cost used in the shorting of the cash index.

*5.1 Unlevered Risk Parity*

A summary of results is displayed in Table 3. The unlevered Risk Parity strategy earns an arithmetic return of 1.1%, underperforming the FTSE/ALSI benchmark by 0.1%. It has a standard deviation of 2.1%. Its annualised return and standard deviation are 14.5% and 7.57%. Using these figures, and an annualized risk free rate of 6.06% (calculated from the Cash Index used in the levered Risk Parity portfolio), an annualized Sharpe Ratio of 0.677 is obtained. This is significantly better than the Sharpe Ratio obtained in Test 1, which is largely driven by a smaller standard deviation and risk free rate.

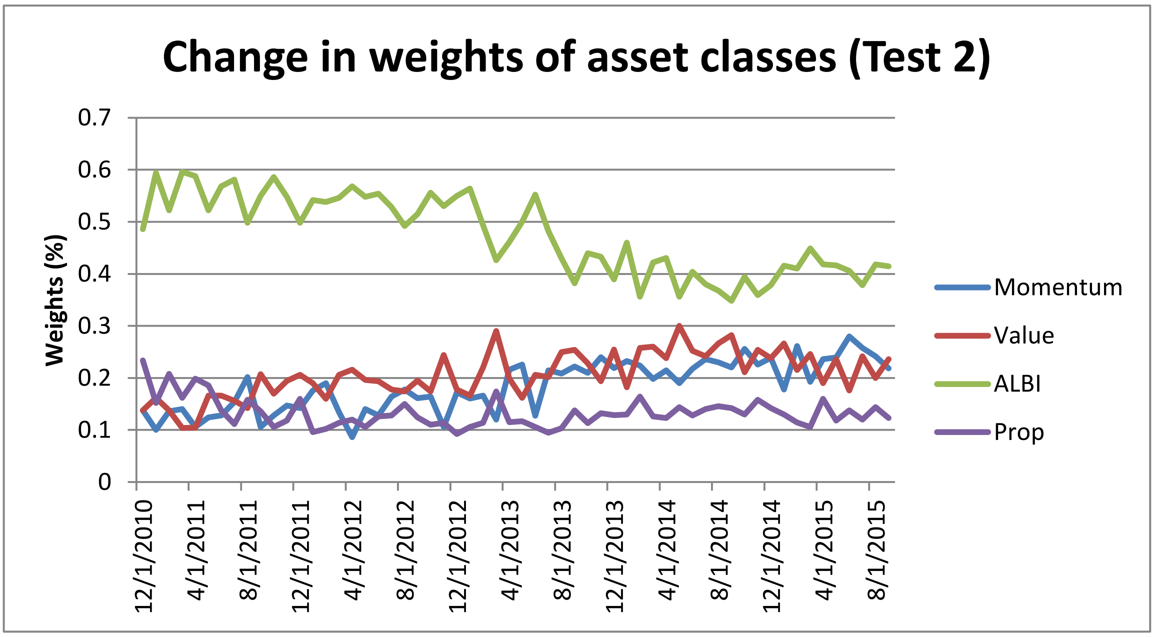
The asset class weights breakdown, as shown in Figure 18, display a similar story to that of Test 1, namely an overweighting of the lower-risk ALBI. However, from the latter stages of 2013 the ALBI's weight begins to wane, giving way to an increase in the Momentum index. This could be an indication of the relative stability of this index over this period and its consequent underperformance, relative to the other asset classes. This seems plausible given the volatile environment of South African equities over this time. Despite valuations being relatively high, the FTSE JSE All Share Index returned 7.17% in 2013 before dropping to a -2.60% return in 2015 (Financial Times Markets, 2016). Alternatively, it could be indicative of a bullish environment for South African bonds. Figure 19 plots the change in weights of the four asset classes over the sample period. It is clear to see the steady decline in weight of the ALBI, giving way to a gradual increase in weights of the two style indices. The Property Index remains relatively flat throughout the sample period.

**Figure 18: Actual weights of the unlevered Risk Parity portfolio for Test 2**




*Notes:* Figure 18 represents the actual weights of the four asset classes tested, for the **unlevered Risk Parity** strategy, for Test 2. This is a fully invested strategy, with ‘box’ constraints set on the different asset classes of a maximum of 60%. The risk parity objective is added, with rebalancing done on a monthly basis.

**Figure 19: Change in weights of Momentum, Value, ALBI and Property Indices for the unlevered Risk Parity strategy (Test 2)**



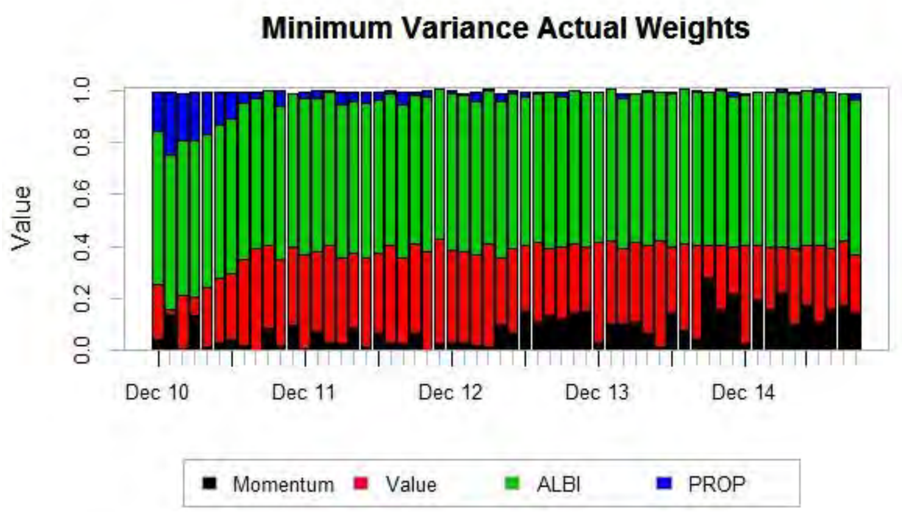

The semi-deviation of this portfolio is 1.52%, less than the 1.82% obtained in Test 1, indicating less downside risk. The gain and loss deviation are 1.54% and 1.09% respectively, compared with 1.51% and 1.54% for Test 1. This means that the unlevered Risk Parity portfolio for Test 2 has a greater volatility of upside performance and thus a more positive skew. This is confirmed by its skewness of -0.006, compared with -0.1173 for Test 1.

*5.2 Minimum variance*

Unsurprisingly, this strategy once again returns the lowest, with an arithmetic mean of 0.9%, significantly below the ALSI. It has a standard deviation of 1.8%. Its annualized return and standard deviation is 5.50% and 6.64% respectively, with a resulting Sharpe Ratio of -0.085. This portfolio therefore performs better on a risk-adjusted basis to its Test 1 counterpart, which returned a Sharpe Ratio of -0.602. Akin to Test 1, the annualized return and standard deviation figures are the lowest out of the strategies in question.

Unlike Test 1 however, this portfolio is the only one with a positive skew, which is 0.17. This indicates that the portfolio is more volatile when making money than it is when losing money. This follows intuition, as risk-averse investors do not enjoy negative skewness. Figure 20 plots the actual weights of the four asset classes for the minimum variance portfolio. The overweight of the ALBI, which has been a recurring theme throughout this paper, is even more robust for this strategy. Furthermore, the almost-nonexistence of the Property and Momentum Index indicate these indices' relative volatility and inherent riskiness.

**Figure 20: Actual weights of the minimum variance portfolio for Test 2**




*Notes:* Figure 20 represents the actual weights of the four asset classes tested, for the **minimum variance** strategy, for Test 2. This is a fully invested strategy, with 'box' constraints set on the different asset classes of a maximum of 60%. The minimum variance objective is added, with rebalancing done on a monthly basis. The covariance shrinkage objective is also set.

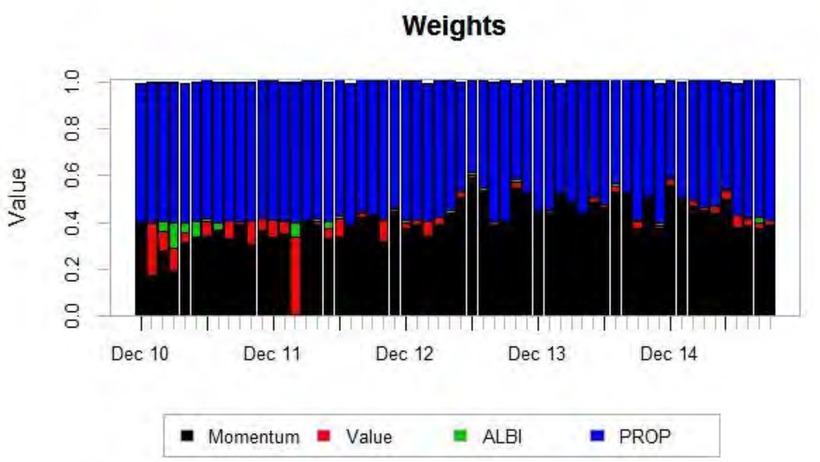
*5.3 Maximum Sharpe Ratio*

The arithmetic mean of this strategy is 1.1%, the same as the unlevered Risk Parity. The standard deviation is slightly more, at 2.2%; the highest out of the unlevered strategies. The annualised return and standard deviation are 28.49% and 13.22% respectively. The resulting Sharpe Ratio is 1.696, which is superior to its counterpart in Test 1. This is in part due to the lower risk free rate and annualized standard deviation obtained in Test 2. Unsurprisingly, this figure is the greatest of the unlevered strategies in Test 2.

This strategy has the highest semi-deviation and loss deviation, at 3.09% at 3.04% respectively, indicating the highest level of downside risk. These figures are complemented by the strategy's negative skew of -0.0458, although it is not the smallest (most negative) of the strategies tested.

Figure 21 below charts the actual weights of the four asset classes over the sample period, after shrinking the covariance matrix. As can be seen, the weights are extremely polarised, with the Momentum and Property indices making up most of the portfolio weight for the entire sample period. The 'box' constraints set on all the strategies in *PortfolioAnalytics* can be seen here, with the Property Index making up the full 60% limit for the early stages of the sample period. This could be the reason for the polarisation in weights and should be further explored. Furthermore, the robust Sharpe Ratio of 1.696, the largest of the unlevered strategies in Test 2, is apparently achieved through the extensive overweighting of the high returning Momentum and Property indices, which individually have the highest Sharpe Ratios of the asset classes tested. These weights seem unrealistic as far as a manager's mandate is concerned; however the portfolio is included for theoretical purposes and is useful when analysing the efficient frontier.

**Figure 21: Actual weights of the maximum Sharpe Ratio portfolio for Test 2**




*Notes:* Figure 21 represents the actual weights of the four asset classes tested, for the **maximum Sharpe Ratio** strategy, for Test 2. This is a fully invested strategy, with 'box' constraints set on the different asset classes of a maximum of 60%. The maximum Sharpe Ratio objective is added, with rebalancing done on a monthly basis. The covariance shrinkage objective is also set.

### *5.4 Levered Risk Parity*

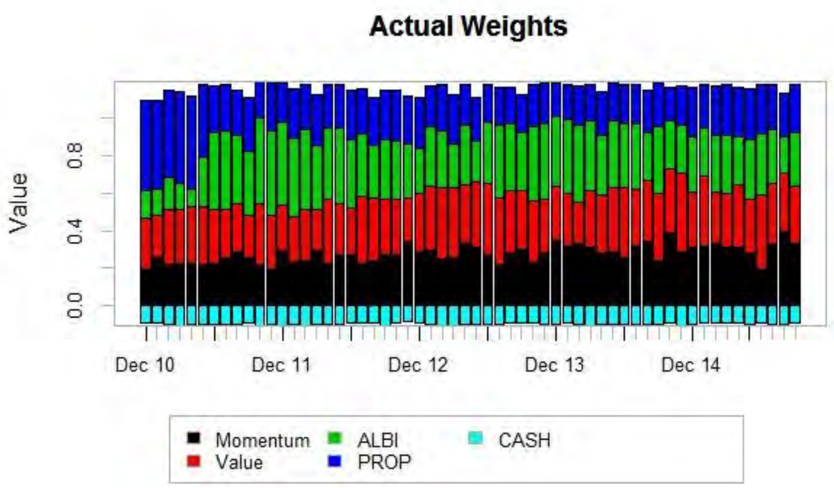
This strategy returns a modest arithmetic mean of 1.46% and a corresponding monthly standard deviation of 2.92%, identical to that of the 60/40 portfolio. The annualised return is a 12.82%, slightly higher than that of its Test 1 counterpart, but still underperforming the 14.28% of the ALSI. The outperformance of Test 1 could be due to the fact that the momentum index includes 25-30 of the best performing stock on the JSE, based on recent price and earnings acceleration metrics. It could also be in part due to the Value Effect on the JSE, whereby the relatively cheap stocks included in the Value Index have outperformed over the sample period. The same annualized return figure can be obtained using the straight line formula method, as explained for Test 1. The intercept is, however, slightly lower for Test 2, at 6.08. The resulting Sharpe Ratio is 0.6774, indicating the portfolio has been levered along the unlevered Risk Parity's Capital Market Line. This is a significant outperformance on the Test 1 counterpart, which was only able to achieve a Sharpe Ratio of 0.45, and can explain the slightly higher return achieved in Test 2.

Figure 22 plots the weights of the five asset classes (including the shorted cash index), rebalanced monthly over the sample period. The short position in cash can be clearly seen, however, interesting to note is the more evenly-weighted long asset classes. The ALBI, which was over weighted for the unlevered Risk Parity strategy, loses its dominant weighting, surrendering it to the Value and Momentum Indices. The increased weighting of these higher risk, higher return asset classes partly explains the significant outperformance of this strategy.

As was evident in Test 1, when constructing the portfolio constraints in *PortfolioAnalytics*, it is imperative to allow the computer some leeway in the asset weights. Therefore the level of leverage obtained arithmetically (140%) will not necessarily be the same as that of results. This could explain the slight difference in Sharpe Ratios between the unlevered and levered Risk Parity portfolios. Figure 23, which graphs the annualized risks and returns of the given strategies, shows the levered portfolio lying directly above the 60/40 and on the same Capital Line as its unlevered counterpart. It still underperforms the tangency portfolio and the ALSI benchmark.

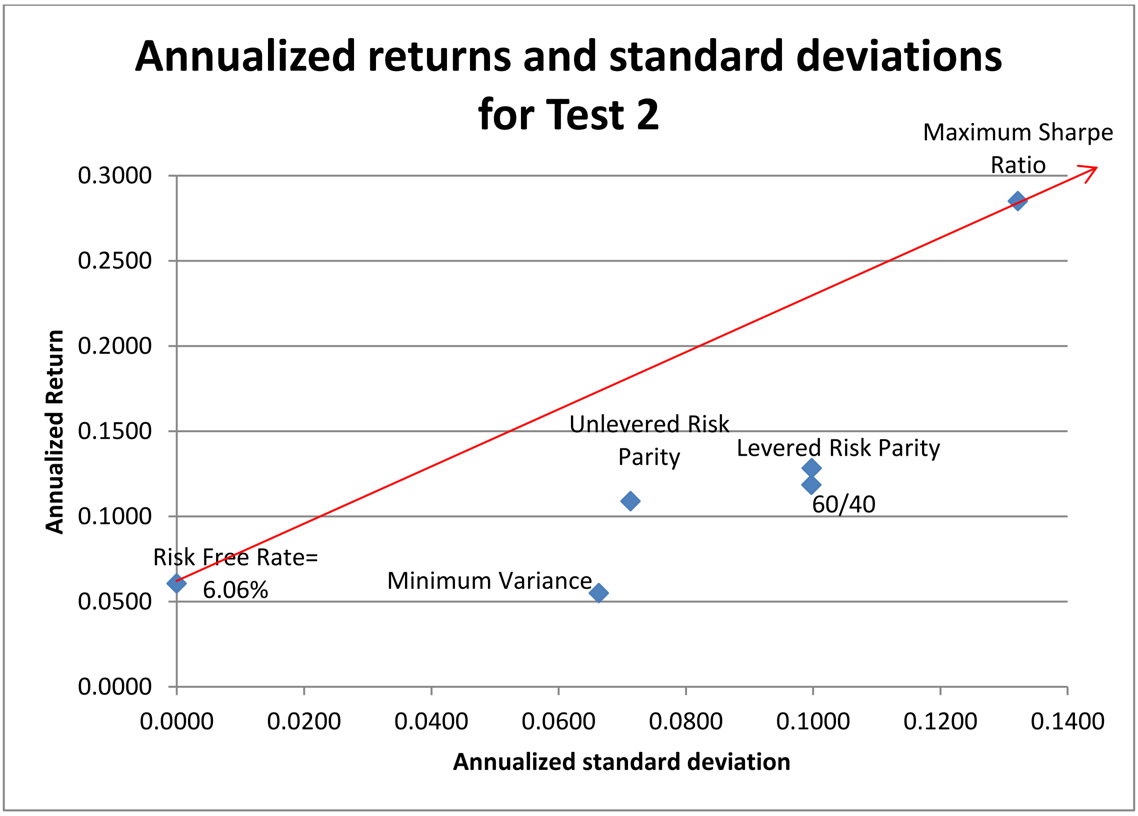
The standard deviation is more closely aligned to the ALSI's, but given its exuberant use of leverage, it is unlikely to be accepted in many mangers' mandates. As the riskiest portfolio out of the strategies tested, it is unsurprising that is has the most negative skew, excluding the ALSI, of -0.0989. This indicates the investor's risk-seeking behaviour. The other portfolios which have a large negative skews are the benchmark and the maximum Sharpe Ratio, which are the first and third riskiest portfolios by standard deviation, respectively. Surprisingly, the levered Risk Parity portfolio doesn't have the greatest loss aversion; this is a title borne by the maximum Sharpe Ratio portfolio; but it is still relatively significant at 1.73%.

**Figure 22: Actual weights of the levered Risk Parity portfolio for Test 2**




*Notes:* Figure 22 represents the actual weights of the four asset classes tested, for the **levered Risk Parity** strategy, for Test 2. This is a fully invested strategy, with 'box' constraints set on the different asset classes of a maximum of 60% and a maximum shorting constraint on cash of -40%. A leeway is provided for the leverage constraint, setting the total portfolio weight between 0.99 and 1.39. The risk parity objective is added, with rebalancing done on a monthly basis.

**Figure 23: The Efficient Frontier (Test 2)**




*Notes:* Figure 23 plots the annualized returns and standard deviations of the five tested strategies. The tangency portfolio (maximum Share Ratio) is constructed using a risk free rate of 6.06%, calculated as the annualized return of the cash index used.

### *5.5 Comparisons with a benchmark*

The FTSE/ALSI returned a meagre 1.2% arithmetic mean against a robust standard deviation of 4.5%. Its annualized return and standard deviation were robust at 14.28% and 15.41% respectively, underperforming only the maximum Sharpe Ratio portfolios. This highlights the sustained Bull Run experienced on the JSE over the sample period. It also draws the conclusion that even when categorizing the equity asset class according to Value and Momentum Style indices, levering a Risk Parity portfolio cannot provide superior returns. From a risk-adjusted perspective, the ALSI has a Sharpe Ratio of 0.533, which is inferior to all the strategies except the minimum variance. This draws attention to the large standard deviation (which is in fact the largest of all the strategies) and the inherent riskiness of the ALSI and South African economy as a whole.

Attention must be drawn here to the different risk free rates used in Test 1 and 2. The two rates differed only because of their sample periods: Test 2's data was only available for the past five years, thus five year data for the cash index was used to calculate the risk free rate. Interestingly, if one uses the risk free rate of Test 1 (7.58%), the resulting return for the unlevered Risk Parity would be

14.34%, which is marginally above the benchmark. However, after adjusting for trading and brokerage fees, this outperformance is likely to be eroded away.

**Table 5: Descriptive Statistics of the unlevered and levered Risk Parity, Minimum Variance, Maximum Sharpe and ALSI Portfolios**

| | Unlevered Risk Parity | Minimum Variance | Maximum Sharpe | Levered Risk Parity | ALSI |
|---|---|---|---|---|---|
| Minimum | -0.0435 | -0.0275 | -0.0374 | -0.0462 | -0.1324 |
| Quartile 1 | -0.0057 | -0.0021 | -0.0038 | -0.0045 | -0.0189 |
| Median | 0.0135 | 0.0104 | 0.0138 | 0.0169 | 0.0160 |
| Arithmetic Mean | 0.0106 | 0.0090 | 0.0111 | 0.0146 | 0.0122 |
| Geometric Mean | 0.0104 | 0.0089 | 0.0108 | 0.0142 | 0.0112 |
| Quartile 3 | 0.0229 | 0.0204 | 0.0253 | 0.0335 | 0.0361 |
| Maximum | 0.0628 | 0.0586 | 0.0687 | 0.0774 | 0.1245 |
| SE Mean | 0.0028 | 0.0023 | 0.0029 | 0.0038 | 0.0041 |
| LCL Mean (0.95) | 0.0051 | 0.0043 | 0.0054 | 0.007 | 0.0040 |
| UCL Mean (0.95) | 0.0162 | 0.0137 | 0.0168 | 0.0223 | 0.0203 |
| Variance | 0.0004 | 0.0003 | 0.0005 | 0.0009 | 0.0020 |
| Stdev | 0.0211 | 0.0179 | 0.0217 | 0.0292 | 0.0445 |
| Skewness | -0.0060 | 0.1666 | -0.0458 | -0.0989 | -0.2833 |
| Kurtosis | -0.0137 | 0.1605 | 0.1014 | -0.4674 | 0.8698 |
| Sharpe Ratio | 0.6772 | -0.0848 | 1.6964 | 0.6774 | 0.5332 |
| Annualized portfolio rebalancing return | 0.1089 | 0.0549 | 0.2849 | 0.1282 | 0.1428 |
| Annualized portfolio Standard Deviation | 0.0713 | 0.0663 | 0.1322 | 0.0998 | 0.1541 |

*Notes:* This table is a summary of key statistics, taken from *PortfolioAnalytics* for **Test 2**. The same strategies are used as in Table 1; however the equity asset class is subcategorized into momentum and value styles, according to the Citadel Multi Factor Fund. The ALBI is used as the low volatility component to ensure consistency across the paper. The 60/40 strategy is not included in this table as its composition and construction is identical to that of the first part of the paper and Table 1. The ALSI is, however, included here as a benchmark.

**Table 6: Downside risk measures of the unlevered and levered Risk Parity, Minimum Variance and Maximum Sharpe Portfolios**

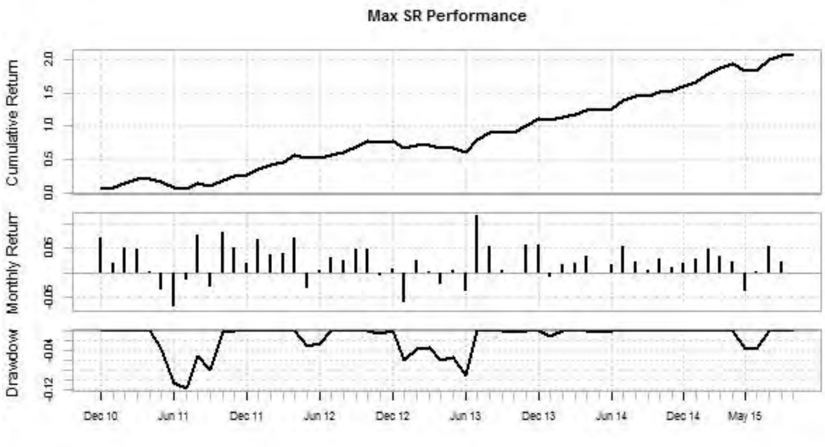
| | Unlevered Risk Parity | Minimum Variance | Maximum Sharpe | Levered Risk Parity |
|---|---|---|---|---|
| Semi Deviation | 0.0152 | 0.0167 | 0.0309 | 0.0289 |
| Gain Deviation | 0.0154 | 0.0141 | 0.0293 | 0.0355 |
| Loss Deviation | 0.0109 | 0.0145 | 0.0304 | 0.0173 |
| Downside Deviation (MAR=10%) | 0.0134 | 0.0201 | 0.0249 | 0.0148 |
| Downside Deviation (Rf=0%) | 0.0094 | 0.0154 | 0.0216 | 0.0117 |
| Downside Deviation (0%) | 0.0094 | 0.0154 | 0.0216 | 0.0117 |
| Maximum Drawdown | 0.0694 | 0.1748 | 0.2839 | 0.1116 |
| Historical VaR (95%) | -0.0262 | -0.0367 | -0.0432 | -0.0273 |
| Historical ES (95%) | -0.0301 | -0.0447 | -0.0746 | -0.0451 |
| Modified VaR (95%) | -0.0231 | -0.0363 | -0.0527 | -0.0313 |
| Modified ES (95%) | -0.0312 | -0.0463 | -0.087 | -0.044 |

*Notes:* This table is a summary of key downside risk statistics, taken from *PortfolioAnalytics* for **Test 2**. The four trading strategies, excluding the 60/40 which is presented in Table 2, are included here.

Figures 24-27 display the cumulative returns, monthly returns and drawdowns of the levered and unlevered Risk Parity, minimum variance, maximum Sharpe Ratio and FTSE/ALSI portfolios for Test 1. As can be seen in Figure 24**,** the levered Risk Parity cumulative return reaches just above 80% for the five year sample period, which is below that of the maximum Sharpe Ratio and levered Risk Parity portfolios. These are the same findings as that of Test 1. Much of this can be attributed to the significant losses sustained during the 2008 Financial Crisis and the subpar returns of 2014 and 2015. It can therefore be stated that, whilst the unlevered Risk Parity portfolio underperforms this benchmark on a cumulative return basis, by levering up this portfolio along its Capital Market Line, one still cannot achieve superior risk-adjusted returns. If using the slightly higher risk free rate of Test 1, however, the levered portfolio can provide marginal outperformance. This is likely to disappear, however, after factoring in trading and brokerage fees.

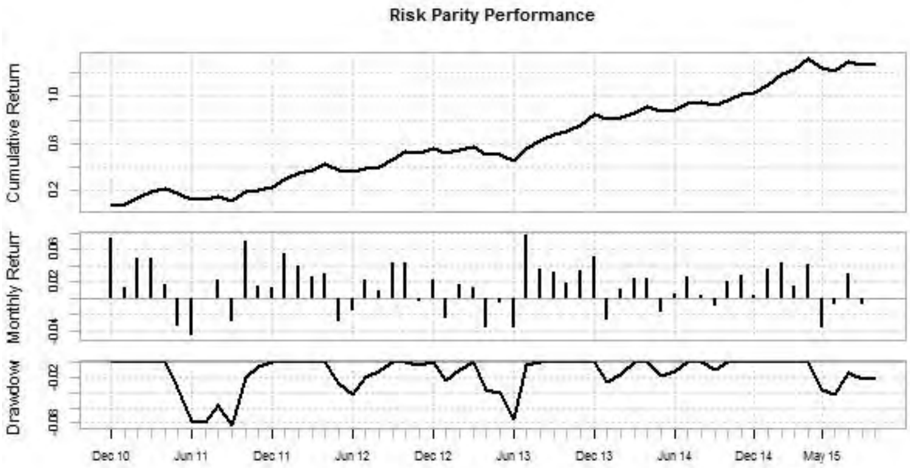
### Figures 24-27: Performance of the unlevered and levered Risk Parity, minimum Variance, maximum Sharpe Ratio and FTSE/ALSI portfolios for Test 2


*Notes:* Figure 24 represents the cumulative returns, monthly returns and drawdown for the **unlevered Risk Parity** portfolio.




*Notes:* Figure 25 represents the cumulative returns, monthly returns and drawdown for the **maximum Sharpe Ratio** portfolio.


*Notes:* Figure 26 represents the cumulative returns, monthly returns and drawdown for the **minimum variance** portfolio.




*Notes:* Figure 27 represents the cumulative returns, monthly returns and drawdown for the **levered Risk Parity** portfolio.

Chapter 6: Conclusion, limitations of the study and areas of further research

Asset allocation has long been at the fore of the agendas of fund managers and academics alike. In recent times, the concept of diversifying a portfolio based on risk, rather than number of asset classes has gained traction in overseas markets. This paper explored the concept of Risk Parity in a South African context, comparing it with a number of other real-world and theoretical portfolios, namely the minimum variance, maximum Sharpe Ratio and 60/40 pension fund portfolios. It used equities, categorised by the FINDI & RESI indices in Test 1 and Value & Momentum indices in Test 2, the ALBI and the South African Property Index to determine which of the aforementioned strategies produced the greatest risk adjusted returns over a historic sample period. A levered Risk Parity portfolio, with a short position in cash, was also constructed to evaluate the effect of leverage on risk diversification. For Test 1, the unlevered Risk Parity portfolio earned a respectable risk-adjusted return, with a Sharpe Ratio of 0.45. The only other strategies to produce superior risk-adjusted returns were the maximum Sharpe Ratio, which produced a Sharpe Ratio of 1.31. These were driven largely by its robust annualized returns of 26.87 and its overweighting of the FINDI and PROP indices. Against the benchmark of the FTSE/ALSI, only the maximum Sharpe Ratio outperformed on an annualized return basis, concluding that even after levering up the Risk Parity strategy along its Capital Market Line, one still cannot earn superior returns. For Test 2 similar results were obtained. Once again, the unlevered Risk Parity portfolio had a Sharpe Ratio of 0.68, surpassed only by the maximum Sharpe Ratio, which yielded 1.70. This portfolio was almost exclusively comprised of the Momentum and PROP indices, which individually earned the highest Sharpe Ratios. Once again, this portfolio was the only one to outperform the benchmark in Test 2. Test 2 produced greater annualized returns across the four of the five strategies, leading to evidence of style anomalies on the JSE. Furthermore, while an unlevered Risk Parity does not outperform the benchmark on a return basis, applying a level of leverage along its Risk Parity Capital Line will still fail to earn superior returns for both Test 1 and Test 2. If, however, the higher risk free rate of Test 1 is used in Test 2, the levered portfolio earns a marginally higher return than the benchmark. However, after factoring in transaction and brokerage fees, this excess return is likely to disappear.

*Limitations of the study and areas of further research:*

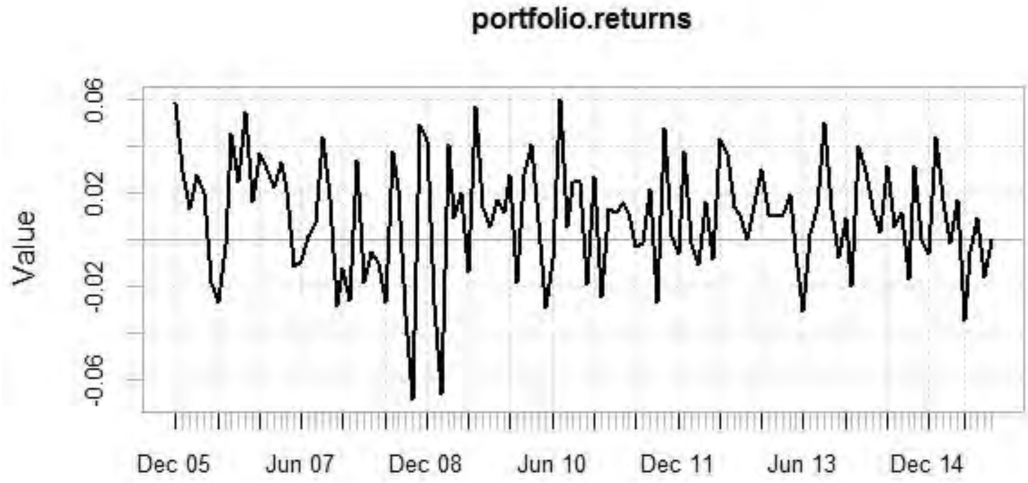
As this study is a first of its kind in a South African context, there are various limitations which should be taken into account. Firstly, the sample period of the study could be considered too short. The availability of the data only allowed for a ten year period, which despite containing all elements of the business cycle, doesn't compare with the considerably longer sample periods of foreign publications. The economic nature of sample period must also be considered; the extended Bull Run for equities over the sample period would mean an overly pessimistic picture for risk parity.

Secondly, as mentioned previously, the level of leverage is chosen using simple arithmetic division. When using leverage constraints in *Portfolio Analytics*, it doesn't yield the exact Sharpe Ratio or Standard Deviation. This is likely to be the result of placing bands on the leverage constraints to create leeway for the solver. An alternative way of constructing the levered Risk Parity portfolio is discussed in the literature review. Postulated by Clarke *et al* (2013), the strategy is obtained by levering up its unlevered counterpart so that its volatility matches the *ex post* volatility of the value weighted strategy. Furthermore, the method above is described by Asness *et al* (2012) as unconditional: by using a constant scale factor, such that the annualised volatility of the unlevered Risk Parity strategy matches the *ex post* volatility of the benchmark portfolio, it does not affect the conclusions drawn. This method has been argued to be unavertable as the leverage factor can only be determined once the entre study period has elapsed. An avenue for further study would therefore be to adopt the *conditional* method, as postulated by Anderson *et al* (2012), whereby the leverage factor will change at each rebalancing date.

Thirdly, market frictions such as transaction costs were not accounted for when constructing the levered Risk Parity portfolio. Taking a short position in a cash index would incur borrowing costs that would erode the high returns of the levered strategy. Anderson *et al* (2012) actually found that the levered Risk Parity portfolio underperforms the value-weighted and 60/40 strategies, arguing that the look-ahead bias created makes this portfolio un-investable. They incorporate three different assumptions about transaction costs into their strategies, which could be replicated in a South African context. The first case could use the South African 10- year bond rate, commonly used as the proxy for the risk free rate. The middle case could use the Johannesburg Interbank Agreed Rate (JIBAR), which is the money market rate used in South Africa, calculated as the average rate at which banks buy and sell money. And the final case could use the middle case's borrowing assumptions and add turnover-induced trading costs, as proposed by Anderson *et al* (2012).

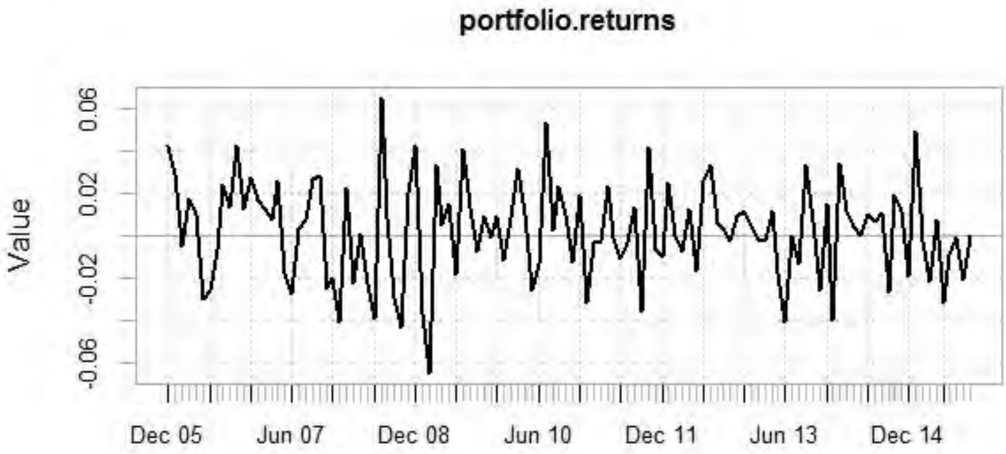
*Appendix*

## Figures 1-5: Time Series returns of the unlevered and levered Risk Parity, minimum variance, maximum Sharpe Ratio and 60/40 portfolios, for Test 1



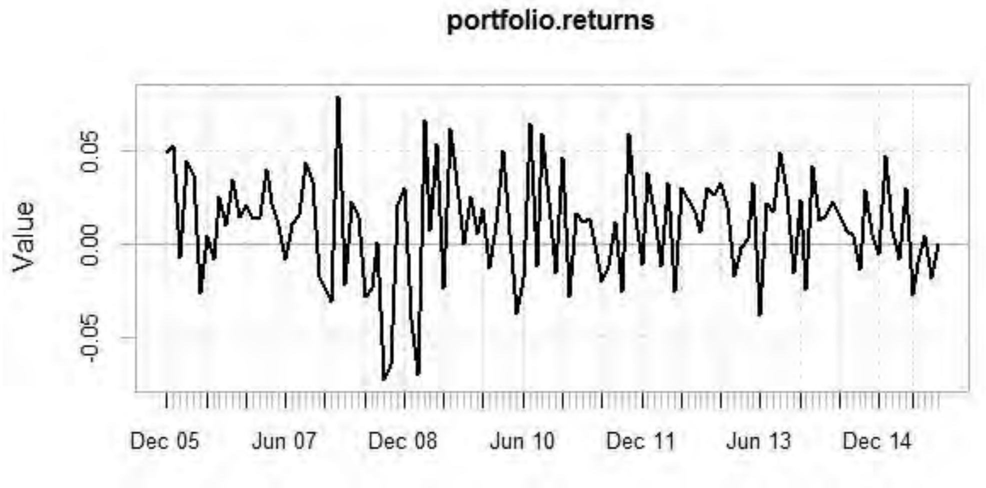

*Notes:* Figure 1 represents the monthly returns of the **unlevered Risk Parity** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2005 and the last rebalancing date is 30/09/2015.


*Notes:* Figure 2 represents the monthly returns of the **levered Risk Parity** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2005 and the last rebalancing date is 30/09/2015.




*Notes:* Figure 3 represents the monthly returns of the **minimum variance** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2005 and the last rebalancing date is 30/09/2015.

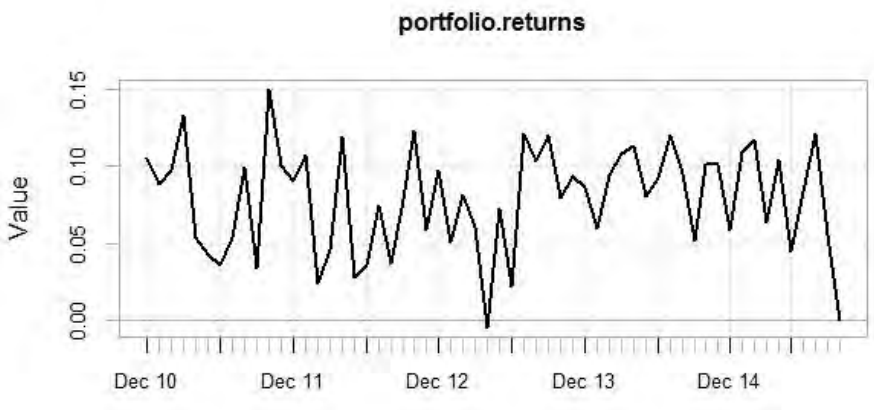

*Notes:* Figure 4 represents the monthly returns of the **maximum Sharpe Ratio** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2005 and the last rebalancing date is 30/09/2015.




*Notes:* Figure 5 represents the monthly returns of the **60/40** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2005 and the last rebalancing date is 30/09/2015.

**Figures 6-9: Time Series returns of the unlevered and levered Risk Parity, minimum variance and maximum Sharpe Ratio portfolios, for Test 2**


*Notes:* Figure 6 represents the monthly returns of the **unlevered Risk Parity** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2010 and the last rebalancing date is 30/09/2015.

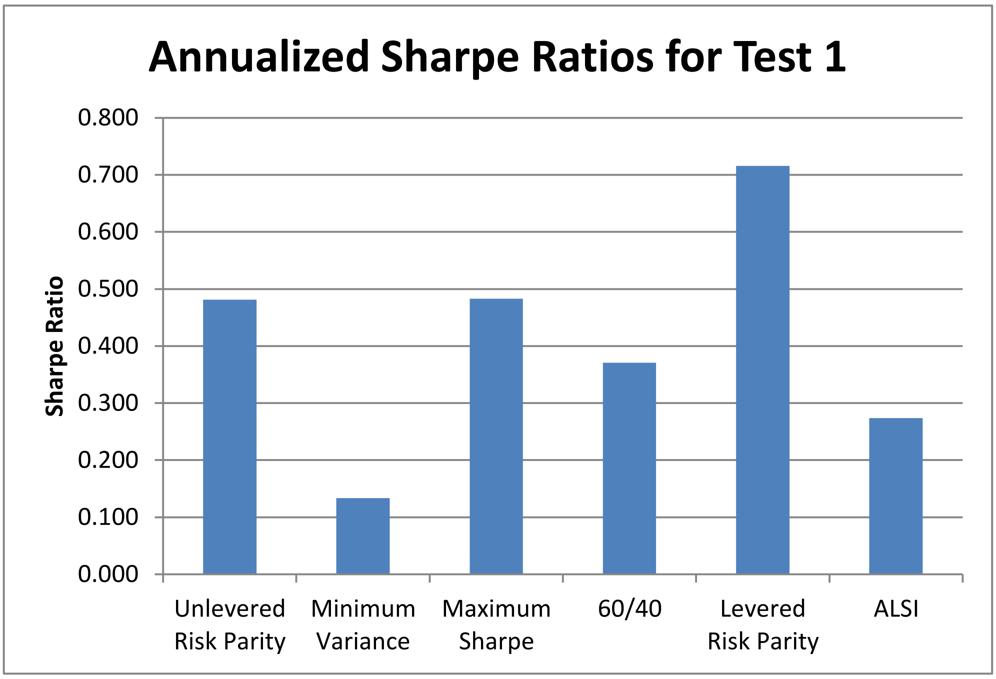


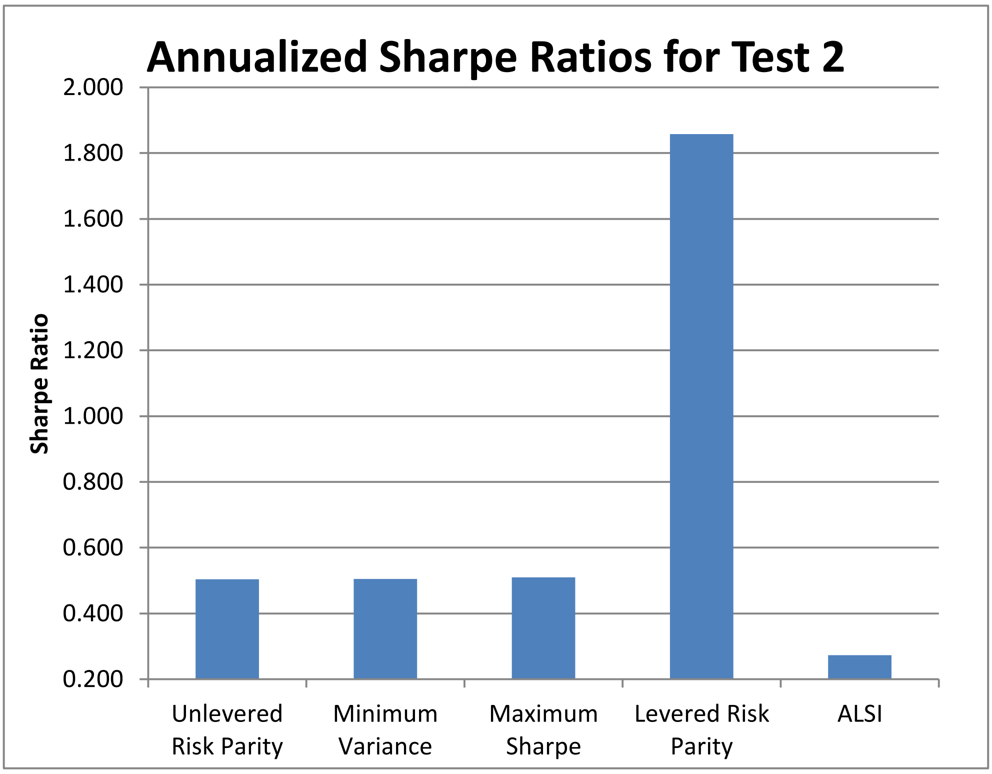

*Notes:* Figure 7 represents the monthly returns of the **levered Risk Parity** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2010 and the last rebalancing date is 30/09/2015.


*Notes:* Figure 8 represents the monthly returns of the **minimum variance** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2010 and the last rebalancing date is 30/09/2015.


*Notes:* Figure 9 represents the monthly returns of the **maximum Sharpe Ratio** portfolio. Portfolio rebalancing was done on a monthly basis, with a training period of 12 months. The first rebalancing date is 31/12/2010 and the last rebalancing date is 30/09/2015.

**Figures 11-12: Annualised Sharpe Ratios for Test 1 and 2**



Code used in R (*PortfolioAnalytics*):

```
#Load Packages############################################
library(PerformanceAnalytics)
library(PortfolioAnalytics)
library(DEoptim)
library(ROI)
library(ROI.plugin.glpk)
require(ROI.plugin.quadprog)
library(tawny)
library(xts)
library(quadprog)
#######################################################
###Change the directory to where you have saved your data###
data=read.csv("C:/Users/Nick/Documents/Masters/4 asset classes.csv",row.names= 1)

#Convert to xts/zoo format#
data = as.xts(data)
data = data/100
#Check Data#
head(data)
data[2,2]
########################################################
#Initial Portfolio#
funds <- colnames(data)
head(funds)
initial <- portfolio.spec(assets=funds)
initial
```

*#Constraints#*

```
initial = add.constraint(portfolio=initial, type="leverage",min_sum = 0.99,max_sum=1.01)

initial = add.constraint(portfolio = initial,type = "box",min=0.00,max=0.60)
```

***Unlevered Risk Parity Portfolio:***

```
#Objective#

port1 = add.objective(portfolio = initial,type = "risk_budget",name =
"StdDev",arguments=list(p=0.95),min_concentration = TRUE)

port1

#####Optimisation#####

risk_parity = optimize.portfolio.rebalancing(R=data,portfolio = port1,

                                rebalance_on = "months",

                                optimize_method = "DEoptim",

                                training_period = 12,trace = TRUE,

                                itermax = 999,

                                search_size = 4000)

###data etc.###

risk_parity

print.default(risk_parity)


weights_risk_parity = extractWeights(risk_parity)

weights_risk_parity


risk_parity_ret = Return.rebalancing(data,weights = weights_risk_parity,

                          rebalance_on = "months")

risk_parity_ret

charts.PerformanceSummary(R = risk_parity_ret,main = "Risk Parity Performance")


chart.TimeSeries(R = risk_parity_ret)

chart.RiskBudget(risk_parity, risk.type = "percentage", match.col = "StdDev")
```

```
chart.Weights(risk_parity,main = "Actual Weights")

print.default(risk_parity)

################################################################
```

***Minimum Variance Portfolio:***

```
#Objective for Min Variance#

port2 = add.objective(portfolio = initial,type = "risk",name =
"StdDev",arguments=list(p=0.95),min_concentration = TRUE)

port2

##############Min Variance#######################

min_risk = optimize.portfolio.rebalancing(R=data,portfolio = port2,

                         rebalance_on = "months",

                         optimize_method = "DEoptim",

                         training_period = 12,trace = TRUE,

                         itermax = 999,

                         search_size = 4000)

min_risk

print.default(min_risk)


weights_min_risk = extractWeights(min_risk)

weights_min_risk


min_risk_ret = Return.rebalancing(data,weights = weights_min_risk,

                     rebalance_on = "months")

min_risk_ret

charts.PerformanceSummary(R = min_risk_ret,main = "Min Risk Performance")


chart.TimeSeries(R = min_risk_ret)

chart.RiskBudget(min_risk, risk.type = "percentage", match.col = "StdDev")

chart.Weights(min_risk,main = "Actual Weights")
```

```
print.default(min_risk)

########################
```

***Maximum Sharpe Ratio Portfolio:***

```
########Max SR##########

port3 = add.objective(portfolio = initial,type = "return",name = "mean",maxSR = TRUE)


max_SR = optimize.portfolio.rebalancing(R=data,portfolio = port3,
                                        rebalance_on = "months",
                                        optimize_method = "DEoptim",
                                        training_period = 12,trace = TRUE,
                                        itermax = 999,
                                        search_size = 4000)

weights_max_SR = extractWeights(max_SR)


maxSR_ret = Return.rebalancing(data,weights = weights_max_SR,
                               rebalance_on = "months")
```

***Levered Risk Parity Portfolio***

```
#####################Levered RP####

###Change the directory to where you have saved your data###

data=read.csv("C:/Users/Nick/Documents/Masters/Assets and cash.csv",row.names= 1)

#Convert to xts/zoo format#

data = as.xts(data)

data = data/100

#Check Data#

head(data)

data[2,2]
```

```
#Initial Portfolio#

funds <- colnames(data)

head(funds)

initial <- portfolio.spec(assets=funds)

initial


#Constraints#

#Notes: leverage is not exactly 120% - must give optimizer leeway

#119% and 121% allows is space to solve#

initial= add.constraint(portfolio=initial, type="leverage",min_sum = 0.99,max_sum=1.09)

#Box: Cash MUST be shorted(borrowed), min amount is -21% and max 19% - links to the leverage
constraint

initial = add.constraint(portfolio = initial,type = "box",min=c(0, 0, 0, 0, -0.11), max=c(0.6, 0.6, 0.6, 0.6,
-0.09))


##Objectives#

port1 = add.objective(portfolio = initial,type = "risk_budget",name =
"StdDev",arguments=list(p=0.95),min_concentration = TRUE)

port1


##Match Risk to MaxSR

table.Stats(MAXSRET)

#StdDev is approx.  0.029per month#

port1 = add.objective(portfolio = port1 ,type = "portfolio_risk",name = "StdDev",target = 0.029)

port1


#Optimize#

#Must use DEoptim because risk parity is not a quadratic problem#

Levered_RP = optimize.portfolio.rebalancing(R=data,portfolio = port1,

                              rebalance_on = "months",
```

```
                              optimize_method = "DEoptim",
                              training_period = 12,trace = TRUE,
                              itermax = 999)

Levered_RP
max_SR

#Get Weights and Returns#
weights = extractWeights(Levered_RP)
RP_RET = Return.rebalancing(data,weights = weights, rebalance_on = "months" )
#Stats Table#
table.Stats(RP_RET)
table.Stats(MAXSRET)
#Sharpe Ratio#
SharpeRatio(RP_RET)
SharpeRatio(MAXSRET)
RP_RET
MAXSRET

#Compare StdDev#
StdDev(RP_RET)
StdDev(MAXSRET)
#almost the same#

charts.PerformanceSummary(R = RP_RET,main = "Risk Parity Performance")
chart.TimeSeries(R = Max_Levered_RP)
chart.RiskBudget(Levered_RP, risk.type = "percentage", match.col = "StdDev")
chart.Weights(Levered_RP,main = "Actual Weights")
print.default(Levered_RP)
```

***Value weighted (60/40)***

```
###Change the directory to where you have saved your data###
data=read.csv("C:/Users/Nick/Documents/Masters/Value weighted.csv",row.names= 1)


#Convert to xts/zoo format#
data = as.xts(data)
data = data/100
#Check Data#
head(data)
data[2,2]
#######################################################
#Initial Portfolio for Value weighted#
funds <- colnames(data)
head(funds)
initial <- portfolio.spec(assets=funds)
initial
w= c(0.6, 0.4)
> ValW= Return.portfolio(R= data, weights = w )
> table.Stats(R= ValW)
##############Done##############
###########Contraints##########
initial= add.constraint(portfolio=initial, type="leverage",min_sum = 0.99,max_sum=1.01)
initial = add.constraint(portfolio = initial,type = "box",min=c(0.59,0.39), max=c(0.61,0.41))
ValW = optimize.portfolio.rebalancing(R=data,portfolio = initial,
                    rebalance_on = "months",
                    optimize_method = "DEoptim",
                    training_period = 12,trace = TRUE,
                    itermax = 999,
                    search_size = 4000)
```

```
weights_ValW = extractWeights(ValW)

ValW_ret = Return.rebalancing(data,weights = weights_ValW,

                              rebalance_on = "months")

table.Stats(R= ValW_ret)
```

***Test 2 (Equity Asset Class categorized into Value and Momentum Indices)***

```
#Load Packages##########################################

library(PerformanceAnalytics)

library(PortfolioAnalytics)

library(DEoptim)

library(ROI)

library(ROI.plugin.glpk)

require(ROI.plugin.quadprog)

library(tawny)

library(xts)

#####################################################

###Change the directory to where you have saved your data###

data=read.csv("C:/Users/Nick/Documents/Masters/Value and momentum .csv",row.names= 1)


#Convert to xts/zoo format#

data = as.xts(data)

data = data/100

#Check Data#

head(data)

data[2,2]


######################################################

#Initial Portfolio#
```

```
funds <- colnames(data)

head(funds)

initial <- portfolio.spec(assets=funds)

initial


#Constraints#

initial = add.constraint(portfolio=initial, type="leverage",min_sum = 0.99,max_sum=1.01)

initial = add.constraint(portfolio = initial,type = "box",min=0.00,max=0.60)
```

***Unlevered Risk Parity portfolio:***

```
#Objective#

port1 = add.objective(portfolio = initial,type = "risk_budget",name =
"StdDev",arguments=list(p=0.95),min_concentration = TRUE)

port1


#####Optimisation#####

risk_parity = optimize.portfolio.rebalancing(R=data,portfolio = port1,

                              rebalance_on = "months",

                              optimize_method = "DEoptim",

                              training_period = 12,trace = TRUE,

                              itermax = 999,

                              search_size = 4000)

###data etc.###

risk_parity

print.default(risk_parity)


weights_risk_parity = extractWeights(risk_parity)

weights_risk_parity


risk_parity_ret = Return.rebalancing(data,weights = weights_risk_parity,
```

```
                    rebalance_on = "months")
risk_parity_ret
charts.PerformanceSummary(R = risk_parity_ret,main = "Risk Parity Performance")


chart.TimeSeries(R = risk_parity_ret)
chart.RiskBudget(risk_parity, risk.type = "percentage", match.col = "StdDev")
chart.Weights(risk_parity,main = "Actual Weights")
print.default(risk_parity)
################################################################
```

***<u>Maximum Sharpe Ratio portfolio:</u>***

```
########Max SR##########
#Constraints#
##Notes##
#Fully Invested...i.e we do not allow borrowing here?#
#Are the box constraints correct? Change if they are wrong#
initial = add.constraint(portfolio=initial, type="leverage",min_sum = 0.99,max_sum=1.01)
initial = add.constraint(portfolio = initial,type = "box",min=0.00,max=0.60)


#Objectives#
port3 = add.objective(portfolio = initial,type = "return",name = "mean",maxSR = TRUE)
port3 = add.objective(portfolio = port3,type = "risk",name = "StdDev")
port3
#Optimize#
#Notes: You can use ROI solver here because it is a quadratic problem#
max_SR = optimize.portfolio.rebalancing(R=data,portfolio = port3,
                    rebalance_on = "months",
                    training_period = 12,
                    optimize_method = "ROI")
#ROI is much faster - optimization will take a second or 2#
max_SR
```

```
#Max SR now saved & completed#

###Get weights and returns##

weightsMS = extractWeights(max_SR)

MAXSRET = Return.rebalancing(data, weightsMS)

##############################################################################
```

***Levered Risk Parity portfolio:***

```
###Change the directory to where you have saved your data###

data=read.csv("C:/Users/Nick/Documents/Masters/Value, Momentum, Cash.csv",row.names= 1)


#Convert to xts/zoo format#

data = as.xts(data)

data = data/100

#Check Data#

head(data)

data[2,2]


#Levered RP#

#Initial Portfolio#

funds <- colnames(data)

head(funds)

initial <- portfolio.spec(assets=funds)

initial


#Constraints#

#Notes: leverage is not exactly 120% - must give optimizer leeway

#119% and 121% allows is space to solve#

initial= add.constraint(portfolio=initial, type="leverage",min_sum = 0.99,max_sum=1.09)

#Box: Cash MUST be shorted(borrowed), min amount is -21% and max 19% - links to the leverage
constraint
```

```
initial = add.constraint(portfolio = initial,type = "box",min=c(0, 0, 0, 0, -0.11), max=c(0.6, 0.6, 0.6, 0.6, -0.09))


##Objectives#

port1 = add.objective(portfolio = initial,type = "risk_budget",name = "StdDev",arguments=list(p=0.95),min_concentration = TRUE)

port1


#StdDev is approx. 0.0373 per month#

port1 = add.objective(portfolio = port1 ,type = "portfolio_risk",name = "StdDev",target = 0.029)

port1


#Optimize#

#Must use DEoptim because risk parity is not a quadratic problem#

Levered_RP = optimize.portfolio.rebalancing(R=data,portfolio = port1,

                              rebalance_on = "months",

                              optimize_method = "DEoptim",

                              training_period = 12,trace = TRUE,

                              itermax = 999)


Levered_RP

max_SR

#Get Weights and Returns#

weights = extractWeights(Levered_RP)

RP_RET = Return.rebalancing(data,weights = weights, rebalance_on = "months" )

#Stats Table#

table.Stats(RP_RET)


#Sharpe Ratio#

SharpeRatio(RP_RET)
```

```
RP_RET

charts.PerformanceSummary(R = RP_RET,main = "Risk Parity Performance")

chart.TimeSeries(R = Max_Levered_RP)

chart.RiskBudget(Levered_RP, risk.type = "percentage", match.col = "StdDev")

chart.Weights(Levered_RP,main = "Actual Weights")

print.default(Levered_RP)
```

***ALSI portfolio:***

```
data=read.csv("C:/Users/Nick/Documents/Masters/ALSI returns.csv",row.names= 1)


ALSI_returns = as.xts(data)

ALSI_returns = data/100


charts.PerformanceSummary(cbind(risk_parity_ret,ALSI_returns))

head(data)

data[2,2]

ALSI_returns= as.Date(as.character(ALSI_returns), format= "%Y%m%d")
```

***Asset Class Descriptive Statistics:***

```
data=read.csv("C:/Users/Nick/Documents/Masters/FINDI.csv",row.names= 1)

FINDI_returns = as.xts(data)

FINDI_returns = data/100


data=read.csv("C:/Users/Nick/Documents/Masters/RESI.csv",row.names= 1)

RESI_returns = as.xts(data)

RESI_returns = data/100


data=read.csv("C:/Users/Nick/Documents/Masters/ALBI.csv",row.names= 1)
```

```
ALBI_returns = as.xts(data)

ALBI_returns = data/100

data=read.csv("C:/Users/Nick/Documents/Masters/PROP.csv",row.names= 1)

PROP_returns = as.xts(data)

PROP_returns = data/100


data=read.csv("C:/Users/Nick/Documents/Masters/MOM.csv",row.names= 1)

MOM_returns = as.xts(data)

MOM_returns = data/100


data=read.csv("C:/Users/Nick/Documents/Masters/VAL.csv",row.names= 1)

VAL_returns = as.xts(data)

VAL_returns = data/100
```

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