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PORTFOLIO CONSTRUCTION USING ROBUST WEIGHT FUNCTIONS



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1 ABSTRACT

The Markowitz portfolio selection model has formed the foundation from which all the other portfolio selection models are formulated. The Sharpe Single Index and the Improved Sharpe Single Index models have been formulated in a bid to form better performing models. In the optimization algorithms, these models tend to not select highly volatile shares and thus eliminate the possibility of making better returns in the event these shares perform very well. The Huber and Tukey Bisquare weights are considered in this project to enhance these models in capturing these outlying observations. The Huber weights in the Improved Sharpe (Troskie-Hossain) Single Index model are found to be giving a better and more realistic optimal portfolio compared to the Sharpe Single Index model.

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2 Introduction

An investor wanting to form an investment portfolio has the challenge of deciding on the amount to invest in each stock included in the portfolio. This challenge is guided by the objective of maximizing the returns on investment while minimizing the risk of the portfolio. The first mathematical model to solve this problem was formulated by Markowitz(1952). He showed that given the amount of risk an investor is willing to absorb and the minimal returns he can accept, the optimal portfolio can be formed by solving a quadratic optimization problem.

Shortfalls of this model have led to its further development by William Sharpe (1970) (the Sharpe Single Index model). Sharpe's model was further improved by Troskie et al (2009) and is known as the Improved Sharpe Single Index model. Michaud (1998) quotes what seems to be a summary of the portfolio construction problem, "*Although Markowitz efficiency is a convenient and useful theoretical framework for portfolio optimality, in practice it is an error prone procedure that often results in error-maximized and investment-irrelevant portfolios.*" Goldfarb et al (2002) suggest that this behaviour is a reflection of the fact that solutions of optimization problems are very sensitive to perturbations in the parameters of the problem.

In the mean-variance model by Markowitz (1952), stocks with high level of risk for less returns are assigned zero weights in the portfolio selection process. In as much as high volatility is unfavourable here, such stocks need not necessarily be neglected. These stocks exhibit high positive and very low negative returns. We need the positive side of the stock in as much as we dislike its negative side. To accommodate this stocks several weighting functions have been proposed, whereby outlying observations are weighted prior the stock selection process. For this paper, the Huber and Tukey bisquare robust weight functions are considered using the two single index models.

We shall first look at the development of the Markowitz model, Sharpe Single Index model and the Improved Sharpe Single Index (Troskie-Hossain) model. The mean and the variance-covariance matrix estimates of the asset returns shall be estimated for each of the above models. Secondly, the Huber weighted least squares estimates of the Sharpe and the Improved Sharpe single index models will be used to estimate the mean and the covariance matrix. The efficient frontiers of these are compared with those estimated using the ordinary least squares method. In the third chapter, we also look at the effect of using a different weighting function under the weighted least squares method. The Tukey Bisquare weighting function is considered in this case. Lastly, we look at the performance of each of the optimal portfolios resulting from all the above models. Through comparing them we try to pick an optimal portfolio that gives more realistic and better results from a portfolio manager's point of view.

3 Index Models

3.1 Markowitz Portfolio Theory

Harry Markowitz (1952), developed a nobel prize winning formulation which underlies the basic principles behind choosing optimal proportions of asset composition in portfolio construction. His formulation forms the baseline for most asset selection methods available in portfolio construction. The basic concept used is that a rational investor who makes investment decisions based on asset risk (standard deviation of the asset) and returns will want a portfolio that provides the highest possible returns for a given certain level of risk. The amount of risk in this portfolio is measured by the standard deviation of the returns of the portfolio.

3.1.1 Markowitz formulation

Given a portfolio consisting of n stocks with the price of each stock denoted as P_{it} , the log returns of the stocks be defined as;

$$R_{i,t} = \ln \left(\frac{P_{i,t}}{P_{i,t-1}} \right) \quad (1)$$

Let the weights for each of the n stocks be denoted by w_i , the expected returns of each stock by \bar{R}_i , the covariance by σ_{ij} for $i, j = 1, 2, \dots, n$, and the portfolio's expected returns \bar{R}_p . Markowitz (1952) aims at minimizing the variance and maximizing the returns, hence he formulated his problem as

$$\text{minimize } \sum_i \sum_j w_i w_j \sigma_{ij}$$

subject to

$$\bar{R}_p = \sum_{i=1}^n w_i \bar{R}_i$$

where

$$\sum_{i=1}^n w_i = 1$$

and $0 \leq w_i \leq 1$

By changing the weights of the assets, the expected return and variance of the portfolio is changed. Hence this problem is solved once the optimal portfolio weights, giving higher returns for a given amount of variance, are found. This can be solved numerically to obtain a numerical solution. Several computer packages and softwares can be used to compute numerical solutions.

3.2 The Sharpe Single Index Model

The Sharpe Single Index model further develops what the Markowitz model does by explaining how the asset returns are related in a way that a better understanding of the market

dynamics is achieved.

This model is formulated as

$$R_{it} = \alpha_i + \beta_i I_t + e_{it}, \forall i = 1, \dots, q; t = 1, \dots, n \quad (2)$$

where

R_{it} = rate of return on security at time t,

α_i = component of the security that is independent of the market,

β_i = coefficient measuring the response factor of R_{it} to change in I_t ,

I_t = rate of return on market index at time t.

e_{it} = i-th residual error at time t.

The following assumptions are made on the residual errors:

- The variance of security i is given by $E(e_{it}^2) = \sigma_{ei}^2 = \sigma_i^2 \quad \forall t$.
- There is virtually no correlation between the residual errors of the same security at different time horizons.
- The security's residuals and the benchmark index are not related.
- The residual errors of different shares are uncorrelated

Given the above assumptions, the portfolio selection problem is solved using the estimated parameters for α_i, β_i , and σ_i^2 . The expected returns $E(R_{it})$ and variance $var(R_{it})$ for asset i are given by

$$E(R_{it}) = \alpha_i + \beta_i \mu_I \quad (3)$$

$$var(R_{it}) = \beta_i^2 \sigma_I^2 + \sigma_{ei}^2 \quad (4)$$

where μ_I and σ_I^2 are the expected return and variance of the market (JSE Overall Index) returns.

The residual covariance matrix is

$$cov(e) = \begin{bmatrix} \sigma_{e1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{eq}^2 \end{bmatrix}$$

This implies that

$$cov(R) = \sigma_I^2 \beta \beta' + \begin{bmatrix} \sigma_{e1}^2 & \cdots & 0 \\ \vdots & \ddots & \cdots \\ 0 & \cdots & \sigma_{eq}^2 \end{bmatrix} \quad (5)$$

These equations give us all the inputs for the portfolio selection problem

Dutt (2003) quotes, "The single index model has been criticised because of its assumption that stock prices move together only because of common co-movement with the market. Many

researchers have found that there are influences beyond the market, like industry-related factors, that cause securities to move together. Empirical evidence, however reveal that even the more complex models have not been able to consistently outperform the single index in terms of their ability to predict the covariance's between stock." It is for this reason that we still consider this model as our base for further investigations in portfolio construction.

3.3 Improved Sharpe Single Index (Troskie-Hossain) model

The Sharpe Single Index model, was further improved by Troskie, Hossain, and Guo. They, argue in their paper that the residuals of the returns are correlated and hence the Sharpe Single Index model gives inaccurate estimates of the covariance matrix. This model is formulated similarly to the Sharpe Single Index model but differs in the assumptions on the residuals. Note that similar notation as in the paper Troskie et al (2009) is used in the formulation below. Given a portfolio consisting of q stocks

$$R_{it} = \alpha_i + \beta_i I_t + e_{it}, \forall i = 1, \dots, q; t = 1, \dots, n$$

where α_i , β_i , I_t , R_{it} and e_{it} are as defined for equation 2. The assumptions on the residuals are:

$$E(e_{it}^2) = \sigma_{ei}^2 = \sigma_i^2 = \sigma_{ii} \quad (6)$$

$$E(e_{it}e_{is}) = 0; t \neq s = 1, \dots, n \quad (7)$$

$$E(e_{it}I_t) = 0; t = 1, \dots, n \quad (8)$$

$$E(e_{it}e_{jt}) = \sigma_{ij}; t = 1, \dots, n; i, j = 1, \dots, q \quad (9)$$

This model assumes that the errors e_{it} of the different stocks are correlated. As proved by Troskie et al (2009), strong evidence suggests that this is indeed the case on the JSE (Johannesburg Stock Exchange). The correlations are not small, and could also be negative or positive.

Thus

$$E(ee') = \Omega = \begin{bmatrix} \sigma_{e1}^2 & \sigma_{12} & \cdots & \sigma_{1q} \\ \sigma_{21} & \sigma_{e2}^2 & \cdots & \sigma_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{q1} & \cdots & \cdots & \sigma_{eq}^2 \end{bmatrix} \quad (10)$$

and

$$\text{cov}(R_t) = \sigma_I^2 \beta \beta' + \Omega = \Phi \quad (11)$$

For a given portfolio $P = W' R$ we have

$$E(P) = W' (\alpha + \beta \mu_I) = \mu_q \quad (12)$$

and

$$\text{var}(p) = \sigma_I^2 W' \beta \beta' + W' \Omega W = \sigma_q^2$$

The second term $W'\Omega W$, as shown by Haines (2008), plays an important role in the formulation of the risk, $var(P)$, of a portfolio, e.g. If we let $\Omega = [\sigma_{ij}]$, $i, j = 1, \dots, q$ then

$$W'\Omega W = \sum_i^q w_i^2 \sigma_i^2 + \sum_{i \neq j}^q w_i w_j \sigma_{ij}$$

Thus as outlined in Troskie et al (2009) there is a difference in the term $W'\Omega W$ for the Troskie-Hossain and Sharpe Index Models depending on the presence or absence of correlations between the different stock's residuals. Hence the variance of the portfolio $\sigma_q^2 = var(P)$ and the estimated variances $\hat{\sigma}_q^2 = \hat{var}(p)$ will also differ.

3.4 The Efficient Frontier and the Optimal Portfolio

The efficient frontier is a 2-dimensional plot of all the possible portfolio weight combinations that give the best return for different given amounts of risk. It is a risk- return plot where each point on the curve represents a portfolio of weight combination of all assets that give the highest expected return for a given level of risk.

Efficient frontiers help in the evaluation of portfolios of different asset combination. After solving the optimization problem and the appropriate weight combination have been established, an efficient frontier can be plotted by calculating the portfolio return and variance for the different weight combinations.

In reality, investors do not only invest in risky assets, they also hold a risk-free asset. Therefore when choosing the best portfolio combination, a rational investor will choose a portfolio combination that gives the highest returns for a reasonable amount of risk. This point along the efficient frontier is often referred to as the optimal portfolio. A combination of the optimal portfolio and the risk-free asset result in what is called a capital allocation line. This is a straight line, tangent to the efficient frontier at the optimal portfolio point, to the y-axis at the risk-free rate.

3.5 Empirical Investigation: Markowitz, Sharpe Single Index and Troskie-Hossain Models

3.5.1 Objectives of the Investigation

The main objectives of this investigation are as follows:

- First, we would like to use the empirical data to construct the efficient frontier using the Markowitz model, which shall serve as our base for comparison with the Sharpe Single Index and Troskie Hossain models.
- Secondly, we shall estimate the alpha, beta and variance of both the Sharpe Single Index and Troskie-Hossain models. These will be used to estimate the mean and covariance matrix of the asset returns for both models, which shall be used in the construction of the efficient frontiers for both models.

- The efficient frontiers of all the three models will be compared mainly to look at the shifts in the positions of the optimal portfolio within these models.
- Lastly, if it's possible to explain these differences we would also like to consider their impact on making investment decisions.

3.5.2 Data Used In the Investigation

The data used in the analysis is the weekly share prices of 15 selected stocks from the Johannesburg Stock Exchange (JSE). This data contains 263 observations covering the last 5 years from November 2004 to November 2009. It was obtained from the Statistics department of the University of Cape Town.

This data was first converted into log returns by differencing the log prices. A risk-free rate of 6 percent and a borrowing rate of 8 percent were assumed. Since we are dealing with weekly log returns, the weekly risk-free rate for log returns is 0.00112. The 15 shares used in this investigation consists of:

1. Three of the large banking institutions in South Africa. These provide a range of banking services, banking insurance and wealth management products and services:
 - Absa
 - Capitec
 - Nedbank
2. Two non-banking financial institutions:
 - Sanlam: is a leading financial service provider, specialising in a range of wealth management products and services.
 - Firststrand: is an Integrated Financial services group, structured with critical mass to take advantage of the blurring boundaries in the financial services industry.
3. Three investment firms:
 - Remgro: is an investment holding company, specialising in investments on tobacco products, building and motor components, medical services, mining, petroleum, printing and packaging, food, and other various products.
 - Acucap: Acucap Properties Limited is a property holding company through the ownership of investment properties by its wholly owned subsidiaries.
 - Altech: is an investment holding company involved in telecommunications, multimedia and information technology industries.
4. Two infrastructure development and construction companies:
 - Aveng: is an infrastructure development company, providing a range of construction, infrastructure and engineering products, services and solutions.

- Group 5: Group 5 Limited provides integrated building, infrastructure and engineering solutions globally.
5. Illovo: is a leading, global sugar producing company
 6. Advtech: specialises in the sustainable development of human capital; education, training, skills and carrier development.
 7. Anglo: Anglo American is one of the largest diversified mining and natural resource groups in the world and owns a range of quality assets.
 8. Foschini: is a clothing company specialising in the retailing of a wide range of woman clothing footwear, and accessories.
 9. KgMedia: is a media and advertising firm specialising in advertising and direct marketing, brochure and collateral design, illustration and logo design, website and user interface design, tradeshow graphics creative project management.

3.5.3 Methodology

Parameter estimation was undertaken using Eviews 5. The constrained optimization procedure was performed in Matlab 7. From Matlab we get the efficient frontier and the weight combination of every portfolio point in the frontier. Also the optimal portfolio is calculated and plotted in Matlab 7. For the Markowitz model, we used the maximum likelihood estimate of the covariance matrix.

The expected returns vector and the covariance matrix of the Sharpe Index model was estimated using equations 9 and 10 while equation 9 and 11 was used for the calculation of the Troskie-Hossain parameter.

3.5.4 Results

Table 3 gives the estimated coefficient values of the regression of the two index models (Sharpe and Improved Sharpe). The beta coefficients are essential input arguments in the estimation of portfolio risk. The R-squared values of the model indicates the proportion of variation that can be explained by the market. The *T-stat* values of the α and β coefficients indicate that 60% of these coefficients are significant. The R^2 values are all very low, which suggests that the model $R_{it} = \beta_i + e_{it}$ might be an equivalent model.

Figure 1 displays the efficient frontiers obtained from the analysis for all three models under consideration. The points marked along the efficient frontiers are the optimal portfolios. This point has the highest Return to Risk ratio. By holding this portfolio and the risk free asset at proportions depending on the investor's risk appetite, the investor will be able maximise his returns.

Table 1: *Parameter Estimates for the Index models*

Share	α	β	T – stat	σ	R²
Absa	0.002	0.020	2.184	0.042	0.017
Acucap	0.003	-0.021	-2.863	0.035	0.030
Advtech	0.006	-0.046	-4.549	0.048	0.073
Altech	0.002	-0.007	-0.750	0.043	0.002
Anglo	0.003	-0.002	-0.140	0.062	0.00008
Aveng	0.005	-0.044	-3.259	0.062	0.039
Capitec	0.007	-0.006	-0.567	0.047	0.001
Foschini	0.001	-0.002	-0.211	0.045	0.0002
Firstrand	0.002	0.011	1.059	0.049	0.004
Group 5	0.004	-0.041	-3.396	0.056	0.042
Illovo	0.005	0.044	4.446	0.047	0.070
Kgmedia	0.002	-0.032	-3.505	0.043	0.045
Remgro	0.004	-0.023	-3.001	0.036	0.033
Sanlam	0.003	-0.019	-2.168	0.039	0.018
Nedbank	0.002	0.010	1.010	0.043	0.004

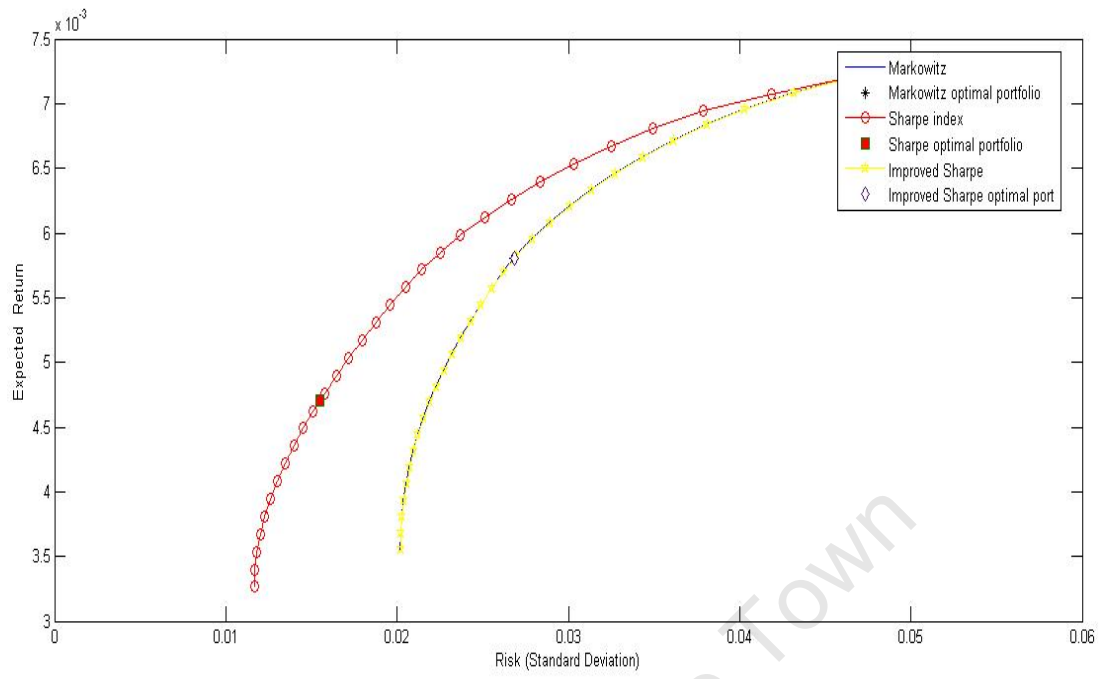
The weight composition of each of the optimal portfolios is shown in Table 2. The expected return and standard deviation of the optimal portfolios is given in Table 3. In figure 1 the circular-marked line represents the Sharpe Single index while the unmarked and the diamond-marked line represents the Markowitz and Improved Sharpe Single Index models respectively.

Evidently from figure 1, table 2 and table 3, the Markowitz model together with the Improved Sharpe Single Index model seem to agree in most cases. Their efficient frontiers are almost the same and hence the asset weight allocation in their optimal portfolios is the same. There exists only a slight difference in their optimal portfolio risk.

The Sharpe efficient frontier is shifted towards the left compared to both Markowitz and Improved Sharpe. For the same expected return Sharpe's model gives lower estimates of risk. The weight allocation of its optimal portfolio is very different from both the Markowitz and Improved Sharpe Single Index model. These three models only agree on the weight of two assets while the Markowitz and Improved Sharpe tend to agree on all of the assets.

The difference between the Sharpe and the Troskie-Hossain models is due to the assumption of the correlations between the assets. Table 4 shows the presence of both positive and negative correlations of the residuals. However, the Sharpe Single Index model assumes that these are zero hence the difference in risk estimation. The presence of more positive correlations explains why the Troskie-Hossain model tends to give higher estimates of risk and the Sharpe model tends to underestimate the risk.

Figure 1: Efficient Frontiers for the Models



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Table 2: *Optimal Portfolio Percentage Weights of the Models*

Share	Markowitz	Sharpe	Improved Sharpe
Absa	0	4.43	0
Acucap	0.26	7.20	0.26
Advtech	20.54	14.09	20.54
Altech	0	3.69	0
Anglo	0	3.08	0
Aveng	4.76	6.25	4.76
Capitec	34.42	19.33	34.42
Foschini	0	0	0
Firststrand	0	2.45	0
Group5	0	3.97	0
Illovo	22.56	14.01	22.56
Kgmedia	0	0	0
Remgro	17.46	14.68	17.46
Sanlam	0	3.93	0
Nedbank	0	2.89	0

Table 3: *Risk and Returns of the Models' Optimal portfolios*

	Markowitz	Sharpe	Improved Sharpe
Expected return	0.0058	0.0047	0.0058
Standard Deviation	0.0267	0.0154	0.0268

Table 4: *Residual Correlation Matrix of the Returns*

1														
0.26	1													
0.36	0.22	1												
0.31	0.24	0.29	1											
0.32	0.12	0.18	0.14	1										
0.34	0.29	0.28	0.24	0.42	1									
0.24	0.2	0.21	0.1	0.12	0.11	1								
0.74	0.27	0.29	0.28	0.39	0.44	0.22	1							
0.61	0.42	0.32	0.32	0.21	0.29	0.23	0.52	1						
0.46	0.28	0.28	0.25	0.37	0.63	0.12	0.5	0.36	1					
0.02	-0.02	0.03	0.04	0.1	-0.03	0.13	0.01	0.12	0.05	1				
0.05	0.11	0.13	0.09	0.15	0.07	0.04	0.12	0.2	0.19	0.01	1			
0.47	0.2	0.29	0.32	0.4	0.36	0.11	0.52	0.38	0.32	0.07	0.01	1		
0.55	0.32	0.33	0.28	0.31	0.3	0.24	0.48	0.44	0.31	0.01	0.09	0.38	1	
0.69	0.28	0.32	0.28	0.27	0.35	0.22	0.67	0.55	0.43	0.06	0.03	0.49	0.53	1

A formal statistical test was performed to test for the significant correlations. The null hypothesis is

$$H_0 : \rho = 0,$$

and the alternative is

$$H_1 : \rho \neq 0.$$

The test statistic T is given by

$$T = r \sqrt{\frac{n-2}{1-r^2}} \approx t_{n-2}$$

where $n=263$. The two sided critical level for r at 5% is 0.1307 while at 1% its 0.1707. Out of the 105 pairwise correlations in the table, 77 were significant at 5% level while 73 were significant at 1%. Therefore almost three quarters of the pairwise correlations (73%) were significant at the 5% level.

3.5.5 Conclusion

In studying the three models, Markowitz, Sharpe Single Index, and Improved Sharpe Single Index model, using the identified shares we can conclude as follows:

- The Sharpe Single Index model gives an efficient frontier that is different from the other two models, which produce similar efficient frontiers and that seem to agree in terms of the optimal portfolio weight allocation.
- Correlation is a major factor in the estimation of risk, and as such accounts for the difference between the Sharpe Single Index and the Improved Sharpe Single model. By relaxing the assumption of zero correlation in the Sharpe Single Index model and using the Improved Sharpe Single Index model we can get a different efficient frontier.
- Positive correlations in the residuals results in the Sharpe Index model underestimating risk while negative correlations in the residuals results in the Sharpe model overestimating risk.

4 Weighted Least Squares Regression

4.1 Introduction

In this chapter the weighted least squares regression technique is employed as one of the robust regression methods that is being used in managing the effects of outliers in modelling. In reducing these effects, the Huber and Tukey Bisquare weighting functions are considered in this chapter. These functions are introduced in the next sections and an empirical investigation is carried out to determine the different effects these have on outliers. In a way a comparison is established between the ordinary least squares method used in the previous chapter and the weighted least squares method.

4.2 M-estimation in Robust Regression

Huber (1964) proposed the so called M-estimates, a family of robust regression estimates. Given the regression model

$$Y_i = X_i' \beta + e_i \quad (13)$$

The M-estimates are derived from the minimization of the objective function:

$$\sum_{i=1}^n \rho(e_i) = \sum_{i=1}^n \rho(Y_i - X_i' \hat{\beta}) \quad (14)$$

The function $\rho(e_i)$ is appropriately chosen to achieve robustness. Ordinary least squares are actually a special case of the M-estimates where $\rho(e_i) = e_i^2$. According to Fox (2002) a reasonable function ρ should have the following properties: $\forall i$,

- $\rho(e_i) \geq 0$
- $\rho(0) = 0$
- $\rho(e_i) = \rho(-e_i)$
- $\rho(e_i) \geq \rho(-e_{i'})$ for $|e_i| \geq |e_{i'}|$

If we let $\psi = \rho'$ (the derivative of ρ), then by differentiating the objective function and setting the partial derivatives to zero we get the solution

$$\sum_{i=1}^n \psi(Y_i - X_i' \hat{\beta}) X_i' = 0 \quad (15)$$

By choosing a weighting function $w(e_i)$ that depends on the residuals, the ordinary least squares estimator can be improved by down-weighting the outlying observation to achieve robustness. The estimating equation may thus be written as

$$\sum_{i=1}^n w_i (Y_i - X_i' \hat{\beta}) X_i' = 0 \quad (16)$$

4.3 Robust Weighting Functions

Weighted least squares regression (wls) requires a weighting function to cater for the presence of potential outlying observations. The inclusion of a weighting function in a regression is sometimes called bounded influence regression, as it aims at putting an upperbound on the ability of outlying observations to influence the overall fit of a regression model. In the case of the ordinary least squares each observation is given the same weight, hence an equal potential to affect overall model fit. (Mupambirei (2008))

Weighting outlying observations instead of completely eliminating them is very useful. This is mainly because outlying observations may be due to stock market crashes which is a true representation of the stock market dynamics. Deleting such an observation, does not only violate the market dynamics but also results in the underestimation of the portfolio's variance and hence the risk of the portfolio.

A number of weighting functions have been proposed, but our focus will mainly be on the Huber weights and Tukey bisquare weights.

Huber (1981) proposed the following weights

$$w_H(\hat{e}_i, h_{ii}) = \frac{1}{\max(1, |\hat{k}_i|)}$$

with the value of k_i equal to

$$\hat{k}_i = \frac{\hat{e}_i}{1.345s\sqrt{(1 - h_{ii})}}$$

with \hat{e}_i being the residual, h_{ii} is the leverage which is derived from the hat matrix and s is a robust estimate of the standard deviation of the residuals. For efficiency, a constant value of $1.345s$ called a tuning constant is chosen. (Mupambirei (2008))

For a higher value of k_i , less weight will be assigned to the i^{th} observation and thus there will be less influence on the overall fit of the model. The second equation portrays that observations with larger residuals will have higher k_i values and hence less weight will be assigned in the estimation. This is very useful and can be used to down weight potential observations that may violate the regression assumptions.

To measure the potential influence of an observation, the diagonal elements of the hat matrix are used. The second equation clearly indicates that an observation with a high h_{ii} will receive a lower weighting in the regression equation. It must be noted that the first equation is a function of \hat{e}_i and h_{ii} , mainly to stress the fact that weights depend not only on the residuals, but also on the leverage.

The Tukey Bisquare weights as proposed by Tukey (1960) are defined as

$$w_{TB}(\hat{e}_i, h_{ii}) = (1 - \hat{k}_i^2)^2, |k_i| < 1 \quad (17)$$

$$w_{TB}(\hat{e}_i, h_{ii}) = 0, |k_i| \geq 1 \quad (18)$$

with k now defined as,

$$\hat{k}_i = \frac{\hat{e}_i}{4.685s\sqrt{(1 - h_{ii})}}$$

The tuning constant in this case is 4.685s.

Iteratively Reweighting Algorithm

The weights in the weighted least squares regression depend on the residuals and the residuals depend on the estimated parameters which themselves, depend on the weights used to estimate them. This means that fitting a set of weights can be an iterative process because once the first set of weights is used the model can be improved by using the new sets of weights that depend on the residuals from the first iteration. Fox (2002), proposed the following algorithm steps:

1. select initial estimates $\beta^{(0)}$, such as the least squares estimates
2. at each iteration m , use the residuals from the previous iteration $e_i^{(m-1)}$ to calculate its associated weights $w_i^{(m-1)} = w(e_i^{(m-1)})$.
3. use these weight to solve for the new weighted least squares estimates $\beta^{(m)} = [X'W^{(m-1)}X]^{-1}X'W^{(m-1)}Y$

Repeat steps 2 and 3 until all the parameters converge.

4.4 Empirical Investigation using the Huber and Tukey Bisquare Weights

4.4.1 Objectives of the Investigation

This investigation is aimed at looking at the effect of Huber and Tukey Bisquare weights using the weighted least squares in estimating the regression coefficients of both the Sharpe and Improved Sharpe single models. We want to compare the beta estimates and the efficient frontiers obtained first when using the Huber weighted least squares with those obtained using the Tukey Bisquare weighted least squares regression method. Secondly, the estimates obtained using the weighted least square (wls) method will be compared with those obtained using the ordinary least square (ols). Bearing in mind that Huber weights down-weights outlying observations while Tukey Bisquare weights assign zero weights to outlying observations, we would like to see the effect this would have to these estimates and efficient frontiers. It would also be very interesting to see the effect these weighting methods have on the residual correlations and hence the risk estimation between the Sharpe and Improved Sharpe Index models.

4.4.2 Method of Investigation

The same data used for the Sharpe and Improved Sharpe Single Index models will be used in estimating the regression coefficients using the Huber and Tukey Bisquare weighted least squares. The estimation will be done using the iteratively re-weighting algorithm. These computations will be done in Matlab 7 and the regression will be done for the same 15 shares used in chapter 2. These estimates will be used as inputs in the portfolio selection problem and the construction of the efficient frontier.

4.4.3 Results

Table 5 shows a comparison of the estimated parameters from the Huber weighted least squares and the ordinary least squares. With the exception of one share, Altech, the table shows that the beta estimates from the Huber weighted least squares method of all the other shares are very different from those computed using the ordinary least squares(ols) method. Also evident from Table 5, is the fact that the residual risk from the Huber-weighted least squares method is much lower than that from the ordinary least squares method. From these differences in the parameter estimates hence the portfolio inputs, it is expected that the resulting efficient frontiers will differ.

The figure 2 shows the efficient frontiers for the two models, Sharpe and Improved Sharpe, under the two methods, ordinary least squares (ols) and Huber weighted least squares(wls). From the left; the unmarked line shows the efficient frontier produced from the Sharpe Single Index model using the Huber weighted least squares, the dot-marked line shows efficient frontier from the Sharpe Index using the ordinary least squares. The cross-marked and the triangular-marked line depicts the efficient frontier from the Improved Sharpe model using the Huber weighted least squares and ordinary least squares respectively.

The weighted least squares has resulted in left shifts in the efficient frontiers of both the Sharpe and Improved Sharpe Single Index models compared to the ordinary least squares. With both models the weighted least squares produces higher returns for a smaller amount of risk. Also worth noting is the fact that for lower returns the Sharpe Single Index model ordinary least squares estimates of risk are lower than that of the Improved Sharpe weighted least squares. However, for higher returns the Improved Sharpe weighted least squares estimates of risk tend to be lower than that of the Sharpe Single Index model least squares estimates.

Table 6 displays a comparison of the estimated parameters from the Tukey Bisquare weighted least squares and the Huber weighted least squares. The beta estimates from the Tukey Bisquare weighted least squares method are very different from those computed using the Huber weighted least squares method. Also the residual risks from the Tukey Bisquare weighted least squares method are much lower than those from the Huber weighted least squares method.

Figure 3 illustrates the differences between the Huber and Tukey bisquare weights when

plotted against the residuals. The Bisquare weights gradually decrease and reach a point at which residuals greater than a certain particular value are given zero weights. The Huber weights are less drastic in that no observation is given a weight of zero, instead the weights gradually decrease as the residuals become large.

In figure 4, the efficient frontiers for the Huber and Tukey Bisquare weighted least square together with the ordinary least squares are shown. These are contrasted for both the Sharpe and the Improved Sharpe Single Index models. The Tukey Bisquare weighted frontiers are shifted more to the left compared to the Huber weighted ones. The optimal portfolios for these give better returns for less risk. However, there is a point above which the portfolios formed give no better return for more risk taken. Beyond this point the Huber weighted efficient frontiers outperform the Tukey Bisquare weighted efficient frontiers.

Figure 2: Efficient Frontiers for Huber Weighted and OLS estimates

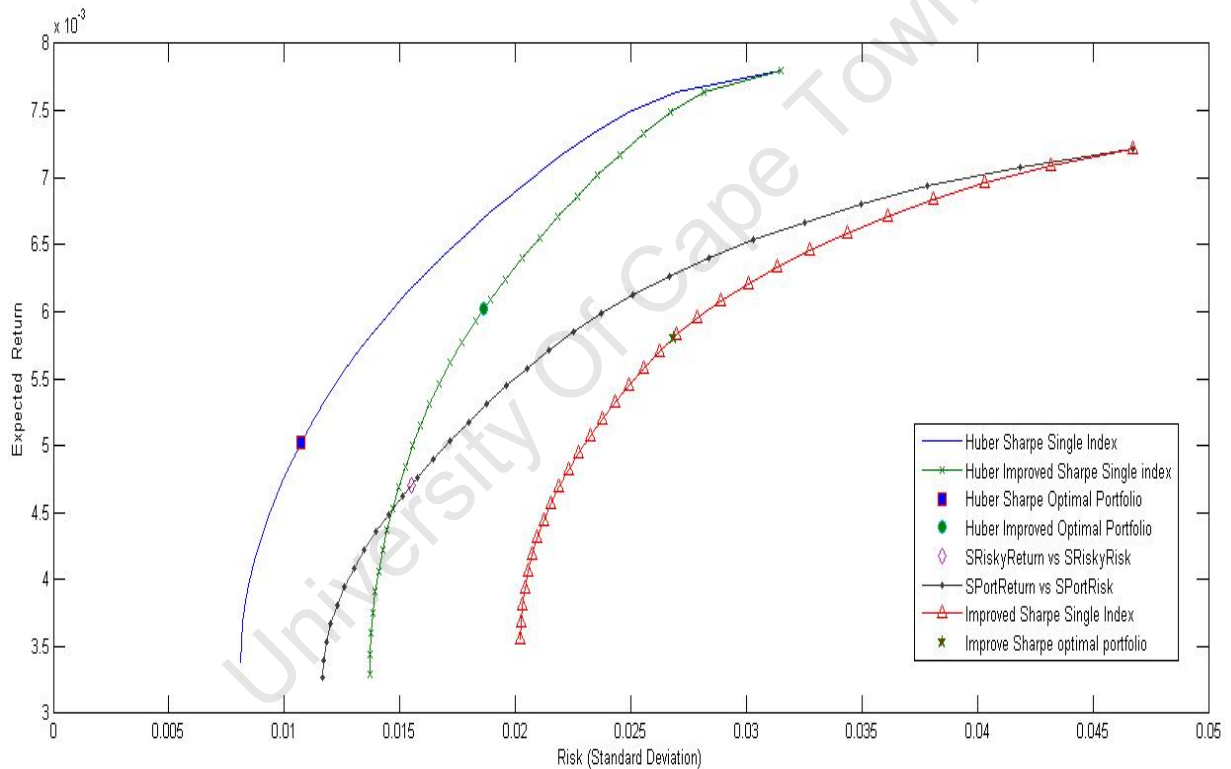


Figure 3: Huber and Tukey Bisquare weighting functions

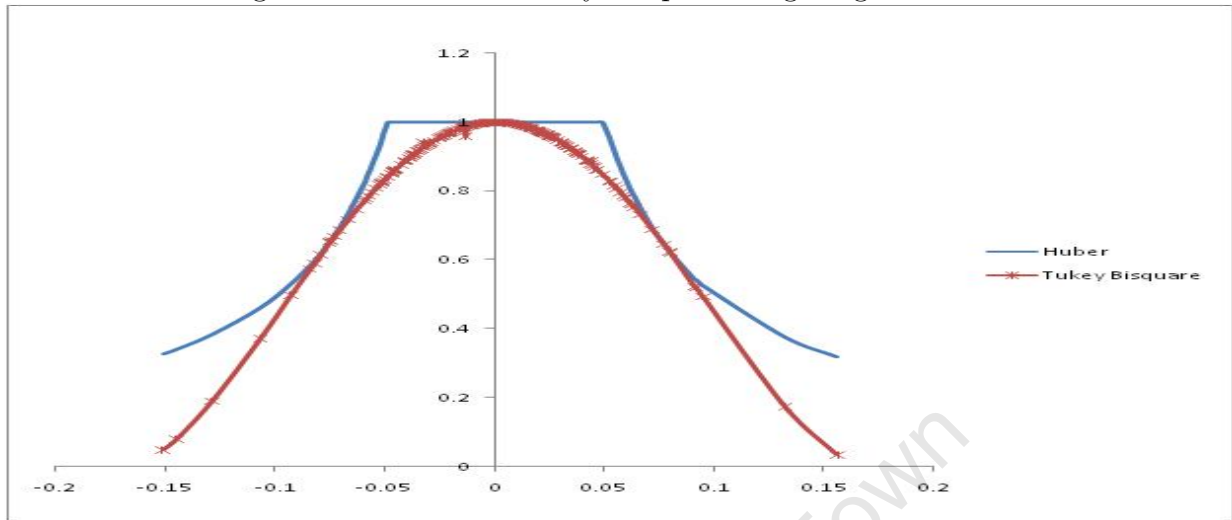


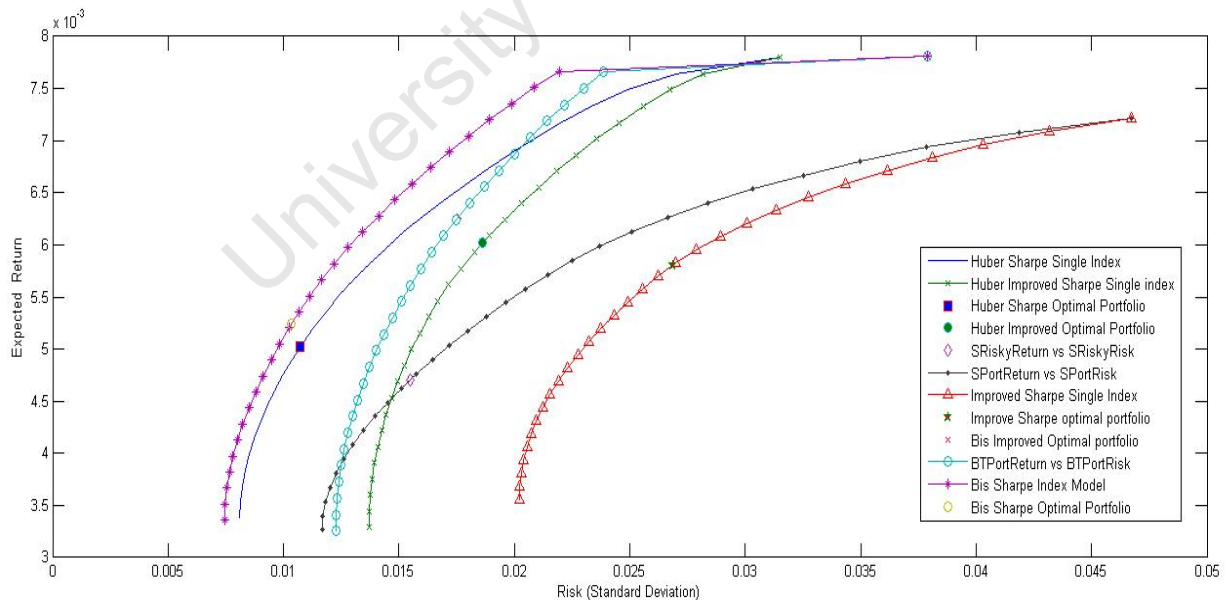
Table 5: *Beta estimates from the ordinary least squares and Huber weighted least squares*

Share	β_{ols}	β_{wls}	σ_{ols}	σ_{wls}
Absa	0.020	0.017	0.042	0.039
Acucap	-0.021	-0.008	0.035	0.031
Advtech	-0.046	-0.023	0.048	0.044
Altech	-0.007	-0.007	0.043	0.040
Anglo	-0.002	-0.004	0.062	0.053
Aveng	-0.044	-0.019	0.062	0.053
Capitec	-0.006	-0.004	0.047	0.043
Foschini	-0.002	0.009	0.045	0.044
Firststrand	0.011	-0.004	0.049	0.040
Group5	-0.041	-0.017	0.056	0.050
Illovo	0.044	0.077	0.047	0.042
Kgmedia	-0.032	-0.025	0.043	0.031
Remgro	-0.023	-0.004	0.036	0.032
Sanlam	-0.019	0.006	0.039	0.036
Nedbank	0.010	0.007	0.043	0.043

Table 6: Beta estimates from the Huber and Tukey Bisquare weighted least squares

Share	β_{hwls}	β_{twls}	σ_{hwls}	σ_{twls}
Absa	0.017	0.0166	0.039	0.039
Acucap	-0.008	0.0064	0.031	0.029
Advtech	-0.023	-0.0051	0.044	0.042
Altech	-0.007	-0.0081	0.040	0.040
Anglo	-0.004	-0.0085	0.053	0.055
Aveng	-0.019	0.0056	0.053	0.053
Capitec	-0.004	-0.0040	0.043	0.043
Foschini	0.009	0.0085	0.044	0.045
Firststrand	-0.004	-0.0057	0.040	0.040
Group5	-0.017	0.0056	0.050	0.048
Illovo	0.077	0.0981	0.042	0.041
Kgmedia	-0.025	-0.0002	0.031	0.030
Remgro	-0.004	0.0108	0.032	0.032
Sanlam	0.006	0.0252	0.036	0.036
Nedbank	0.007	0.0065	0.043	0.042

Figure 4: Efficient frontiers from ols, Huber and Tukey Bisquare wls estimates



4.4.4 Conclusion

After undertaking the investigation on the effect of the weighted least squares on the Sharpe and Improved Sharpe index models, the following can be concluded:

- The Huber weighted least squares method results in less risky portfolios (efficient frontiers are shifted more towards the left) compared to the ordinary least squares method. Weighting outlying observation can help in the construction of better portfolios.
- The estimates of the mean and the variance-covariance matrix used in the Sharpe and Sharpe improved index models can be greatly influenced by outlying observations.
- The efficient frontiers and the corresponding optimal portfolios obtained from using the Huber-weighted estimates are different from those obtained using the Tukey Bisquare weighted estimates.
- The Tukey Bisquare weights assigns zero weights to outlying observations, hence the risk estimates obtained are lower resulting in efficient frontiers shifted more to the left than those obtained using the Huber weights.
- By deleting outliers, the Tukey Bisquare weights results in estimates that are a poor reflection of the actual stock price dynamics. As a result estimates from the Huber weighted least squares are still preferable.

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5 Portfolio Performance

5.1 Introduction

Several factors must be considered when looking at a method that can give a better performing portfolio. Risk and return are the driving forces behind these factors. We want to maximise the returns for lower amounts of risk. Hence most of our efforts have been in the reduction of portfolio risk. We will compare the performance of the portfolios constructed in the previous chapters based on the Sharpe's risk adjusted measure and Treynor's risk adjusted measure. Also, we will consider PDI (Portfolio Diversification Index) measure of these portfolios.

5.2 Sharpe's Risk Adjusted Measure

Sharpe's risk adjusted performance measure calculates the reward to risk ratio, that is the reward for each unit of risk taken. It is given by

$$\text{Sharpe's Measure} = \frac{R_{pt} - R_f}{\sigma_p}$$

where

R_{pt} = return on portfolio at time t

R_f = risk-free rate at time t

σ_p = standard deviation of R_{pt} . (Sharpe(1966))

Risk according to Sharpe is defined as the standard deviation of the return. This total risk is composed of market risk and unique risk, where the market risk is defined as the component of the volatility of the portfolio that is attributable to the portfolio's exposure to the market and Unique risk is defined as the volatility attributable to the fund manager's ability to select stock.

Graphically, Sharpe's ratio is understood as the gradient of the capital allocation line (CAL), the line joining the risk-free rate and the optimal portfolio on the efficient frontier of Markowitz's risk-return plot. Sharpe measure penalises a manager who has not fully diversified.

5.3 Treynor's Risk Adjusted Measure

This measure adjusts returns according to the market risk. It is the reward to market risk ratio defined

$$\text{Treynor's Measure} = \frac{R_{pt} - R_f}{\beta_p}$$

where R_{pt} and R_f are as defined above and β_p is the portfolio's beta (sensitivity of portfolio to market return). (Treynor (1965))

Treynor's measure does not penalise a manager for not fully diversifying. Therefore, for a fully diversified portfolios the Sharpe and Treynor's measure will have equivalent ratings. This measure is ideal for specialist funds where full diversification is not a requirement.

5.4 Portfolio Diversification Index

The PDI is a measure of how well diversified a portfolio is. Diversification is mainly concerned with the number of unrelated sources of return, hence a well diversified portfolio is the one which has a number of uncorrelated sources of return contributing to its volatility. On the other hand a portfolio which has fewer and perhaps more concentrated sources of volatility dominating its make-up, is likely to be far less diversified.

Risk and diversification are very related, hence the reason we are more concerned about diversification as a rating on how well our portfolios will perform. Research has shown that highly diversified portfolios have less risk compared to less diversified portfolios. In their paper, Rudin and Morgan (2006) outline the following procedure for calculating the PDI of a portfolio:

- The data required to establish the current PDI of a portfolio is the current composition, on a stock level, as well as a return history of the stocks held.
- A column consisting of a time series of returns multiplied by the respective weight is constructed for each of the n stocks in the portfolio.
- A principal components analysis (PCA) is then conducted on the covariance matrix of these series to quantify all the uncorrelated sources of risk and their relative magnitudes.
- The PCA produces a series of uncorrelated factors describing the portfolio's return volatility
- The factors are ordered from the most significant to least significant.
- These are then substituted into the formula:

$$PDI = 2 \sum_{k=1}^n k \lambda_k - 1$$

where n is the number of assets and λ_k is the percentage contribution of factor k to volatility.

Bradfield et al (2006) defines the PDI as a centre of gravity or balancing point between independent factors. For independent factors lined up from left to right starting in a decreasing order, the balancing point will lean towards the heavier side. If these factors are equal the balancing point tends to be in the middle. Hence Bradfield et al (2006) summarizes;

- For a completely undiversified portfolio which is dominated by a single factor the PDI=1.
- For a completely diversified portfolio, the PDI is equal to the number of assets in the portfolio.
- The smaller the PDI measure the less diversified the portfolio.
- Conversely the larger the PDI measure the more diversified the portfolio.

5.5 Investigating Portfolio Performance

5.5.1 Objectives of Investigation

Using the optimal portfolios obtained from the efficient frontiers constructed in the previous chapters, we compare their performance using the two performance measures, Sharpe's measure and Treynor's measure. We also investigate which of these portfolios is better diversified using the Portfolio Diversification Index with the JSE overall index as a proxy for the market index. Lastly we would like to view these portfolios from an investor's point of view and most likely be able to select a better portfolio among these.

5.5.2 Methodology

For the seven efficient frontiers constructed using the models; Markowitz model, Sharpe Single Index models, Improved Sharpe Single Index model, Huber Sharpe Single Index model, Huber Improved Sharpe Single Index model, Tukey Bisquare Sharpe Single Index model, Tukey Bisquare Improved Sharpe Single Index model, Sharpe's measure and Treynor's measure will be computed for the optimal portfolios using the equation stated in the previous sections. The PDI will be calculated following the procedure outlined previously. The principal components and eigenvalues will be computed in Matlab7. Table 9 displays the weight composition of the optimal portfolios to be used for the 15 shares used in the previous investigations.

5.5.3 Results

According to Sharpe's measure the Sharpe Single Index model under ordinary least squares, Huber weighted least squares, and Tukey Bisquare weighted least squares outperforms the Improved Sharpe Single Index and Markowitz model. This is observed from Table 8 as the optimal portfolio from the Sharpe Single Index has the highest Sharpe ratio. Treynor's measure agrees with Sharpe's measure for the ordinary least squares and Huber weighted least squares estimates but differs in the case of Tukey Bisquare weighted least squares. The Tukey Bisquare Improved Sharpe Single Index model outperforms the Tukey Bisquare Sharpe single index model and the rest of the models.

The PDI measure from Table 7 depicts the Huber Sharpe Single index model as the model with the most diversified optimal portfolio. The Sharpe Single Index model also outperforms the Improved Sharpe and Markowitz under the ordinary least square and weighted

Table 7: *PDI measure of the optimal portfolios*

Model	Huber Sharpe	Huber Improved	Bisquare Sharpe	Bisquare Improved	Markowitz	Sharpe	Improved
PDI	4.26	2.48	3.92	2.20	2.66	4.21	2.66

Table 8: *Sharpe and Treynor's performance measures*

Model	Port returns	Risk-free	β_p	σ_p	Sharpe's Measure	Treynor's Measure
Markowitz	0.006	0.001	-0.008	0.027	0.175	-0.607
Sharpe	0.005	0.001	-0.010	0.005	0.762	-0.347
Improved Sharpe	0.006	0.001	-0.008	0.027	0.174	-0.607
Huber-Sharpe	0.005	0.001	-0.001	0.011	0.364	-5.243
Huber-Improved Sharpe	0.006	0.001	-0.001	0.019	0.263	-6.018
Bisquare-Sharpe	0.005	0.001	0.004	0.010	0.398	1.039
Bisquare-Improved Sharpe	0.006	0.001	0.003	0.018	0.293	1.587

least square estimates in as far as diversification is concerned. The optimal portfolios from Huber weights are more diversified than those from the Tukey Bisquare weights.

Based on Sharpe's measure the Sharpe Single Index model gives the best optimal portfolio. On the other hand Treynor's measure suggests the Tukey Bisquare Improved Sharpe Single Index model as the best. Therefore for managers concerned about being fully diversified, the optimal portfolio from the Sharpe Single Index model is the best. However for index trackers, Treynor's measure gives us the optimal portfolio from the Turkey Bisquare Improved Sharpe Single Index model as the best portfolio to invest in. This measure calculates performance based on the exposure of the portfolio to market risk, hence is best for managers who want to outperform the market.

However, from Chapter 2 it was shown that the Sharpe Single Index model underestimates the risk as there exists correlations between the errors of the asset returns that this model assumes are zero and hence the Improved Sharpe Single Index model was found to be giving better risk estimates. From chapter 3 a couple of facts were established. Firstly, the Huber weighted least squares gives lower risk estimate and hence better efficient portfolios compared to the ordinary least squares estimates. Secondly the Tukey Bisquare weighted least squares renders lower risk estimates and hence results in better efficient portfolios than the Huber weighted least squares. These are however unrealistic as they portray a poor reflection on the real stock price dynamics hence the Huber weighted least square estimates were preferred.

This therefore leaves us with the Huber weighted Improved Sharpe Index model, which the PDI measure has shown to be only better diversified than the Tukey bisquare Improved Sharpe Single Index model, as our preferred optimal portfolio.

Table 9: *Optimal portfolio weights*

Share	Markowitz	Sharpe	Improved Sharpe	H-Sharpe	H-Improved Sharpe	B-Sharpe	B-Improved Sharpe
Absa	0.000	0.044	0.000	0.036	0.000	0.036	0.000
Acucap	0.003	0.072	0.003	0.138	0.123	0.147	0.121
Advtech	0.205	0.141	0.205	0.122	0.143	0.113	0.111
Altech	0.000	0.037	0.000	0.011	0.000	0.000	0.000
Anglo	0.000	0.031	0.000	0.055	0.000	0.074	0.029
Aveng	0.048	0.063	0.048	0.100	0.136	0.125	0.190
Capitec	0.344	0.193	0.344	0.206	0.374	0.224	0.397
Foschini	0.000	0.000	0.000	0.003	0.000	0.003	0.000
Firststrand	0.000	0.025	0.000	0.013	0.000	0.013	0.000
Group5	0.000	0.040	0.000	0.040	0.000	0.044	0.000
Illovo	0.226	0.140	0.226	0.083	0.117	0.066	0.083
Kgmedia	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Remgro	0.175	0.147	0.175	0.125	0.107	0.114	0.070
Sanlam	0.000	0.039	0.000	0.026	0.000	0.009	0.000
Nedbank	0.000	0.029	0.000	0.042	0.000	0.033	0.000

5.5.4 Conclusion

Having considered Sharpe's measure, Treynor's measure, and the PDI the following can be concluded:

- The Sharpe Single Index models give better and highly performing optimal portfolios in as far as Sharpe's measure is concerned. Treynor's measure suggests the Tukey Bisquare Improved Sharpe model as the model that gives the better performing optimal portfolio.
- The Sharpe Single Index model underestimates the amount of risk.
- By taking into account of the correlation between residuals, the Improved Sharpe Index model gives more realistic portfolios.
- The Huber weighted index models results in better diversified portfolios than the Tukey Bisquare weighted index models.
- Huber weighting function has been found to be more appropriate for financial data compared to Tukey Bisquare weighting function as it down-weights outlying observations instead of deleting them. Therefore a more realistic better portfolio can be obtained using the Huber Improved Sharpe Single Index model.

6 Conclusion

The three portfolio construction models, Markowitz, Sharpe Single Index, and Improved Sharpe Single Index models, were investigated. We found that the Markowitz and the Improved Sharpe Single Index model produced similar efficient frontiers that were different from the one resulting from the Sharpe Single model. Correlations in the residuals, are the major factor that accounts for the difference in the efficient frontiers of the Sharpe and Improved Sharpe Single Index models. The presence of positive correlations in the residuals results in the Sharpe Single model underestimating risk, while negative correlation results in the overestimation of risk.

In comparing the ordinary least squares (ols) estimates with the weighted least squares (wls) estimates of these models, we found that the latter method (wls) produces less risky portfolios compared to the ols method. Comparisons of the Huber and Tukey Bisquare weighting functions gave different efficient frontiers. The Tukey Bisquare weights assigns zero weights to outlying observations, hence the risk estimates obtained are lower resulting in efficient frontiers shifted more to the left than those obtained using the Huber weights. By deleting outliers, the Tukey Bisquare weights results in estimates that are a poor reflection of the actual stock price dynamics. This makes the estimates from huber weights to remain preferable.

Based on Sharpe's measure, Treynor's measure and the Portfolio Diversification Index, the performance of each of the optimal portfolios obtained from the above models were compared. The Sharpe Single index models give better performing optimal portfolios in as far as Sharpe's measure is concerned. On the other hand Treynor's measure suggests the Tukey Bisquare Improved Sharpe model as the best. By taking into account the fact that there exists correlations in the residuals and the fact that Tukey Bisquare weights delete outliers, we can conclude that more realistic better portfolios can be obtained using the Huber Improved Sharpe Single Index model.

There exists other factors and performance measures that need to be considered for a more realistic and better performing portfolio to be constructed. Factors such as liquidity of stocks, concentration on stock markets, need to be considered also.

7 References

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8 APPENDIX

8.1 A-Eviews code

```
'Eviews code for Markowitz, Sharpe single Index and Troskie-Hossain models
scalar n
n=263

vector(15)marko_mean
for %k %y 1 r1 2 r2 3 r3 4 r4 5 r5 6 r6 7 r7 8 r8 9 r9 10 r10 11 r11
  12 r12 13 r13 14 r14 15 r15
marko_mean(%k)=@mean({%y})
next
matrix marko_cov=@cov(marko)*n/(n-1)      'Markowitz Covariance Matrix

,
,
''''

scalar n
n=263

scalar rm
      scalar vm
              rm=@mean(jse)
              vm=@var(jse)*n/(n-1)
,
,
,
''''

for %y r1 r2 r3 r4 r5 r6 r7 r8 r9 r10 r11 r12 r13 r14 r15
      equation {%y}_ls.ls {%y} c jse
      {%y}_ls.makesresid {%y}_residuals
      stom({%y}_residuals,{%y}_residvector)
next
group trosres r1_residuals r2_residuals r3_residuals r4_residuals
  r5_residuals r6_residuals r7_residuals r8_residuals r9_residuals
  r10_residuals r11_residuals r12_residuals r13_residuals r14_residuals
  r15_residuals
matrix residcor=@cor(trosres)

vector(15)beta
vector(15)beta_tstat
vector(15)beta_serr
vector(15)sereg
```

```

vector(15)alpha
vector(15)adjusted_R
vector(15)Rsquare

for %k %y 1 r1 2 r2 3 r3 4 r4 5 r5 6 r6 7 r7 8 r8 9 r9 10 r10
    11 r11 12 r12 13 r13 14 r14 15 r15
        alpha(%k)={%y}_ls.c(1)
        beta(%k)={%y}_ls.c(2)
beta_tstat(%k)={%y}_ls.@tstats(2)
beta_serr(%k)={%y}_ls.@stderrs(2)
sereg(%k)={%y}_ls.@se
adjusted_R(%k)={%y}_ls.@rbar2
Rsquare(%k)={%y}_ls.@r2

next
vector(15) sharp_mean
for !i=1 to 15
    sharp_mean(!i)=alpha(!i)+beta(!i)*rm
next !i

matrix(15,15) phi

phi=beta*@transpose(beta)*vm

matrix(15,15) phi2
matrix(15,263) e 'e is the residual matrix E
for %k %y 1 r1 2 r2 3 r3 4 r4 5 r5 6 r6 7 r7 8 r8 9 r9 10 r10
    11 r11 12 r12 13 r13 14 r14 15 r15
        phi2(%k,%k)={%y}_ls.@se^2
        rowplace(e,@transpose({%y}_residvector),{%k})
next
matrix(15,15) phi3
phi3=e*@transpose(e)/(n-2)          'tros hos single index

matrix(15,15) sharp_covariance
    sharp_covariance=phi+phi2

matrix(15,15) troshos
troshos=phi+phi3

,
,
,

```

8.2 B-Matlab code PCA

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
Principal Component Analysis for PDI calculation %%%  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
a = xlsread('pca_A.xls');  
b = xlsread('pca_B.xls');  
c = xlsread('pca_C.xls');  
d = xlsread('pca_D.xls');  
e = xlsread('pca_E.xls');  
f = xlsread('pca_F.xls');  
g = xlsread('pca_G.xls');  
  
h = cov(a);  
i = cov(b);  
j = cov(c);  
k = cov(d);  
l = cov(e);  
m = cov(f);  
n = cov(g);  
  
[coeff1,latent1,explained1]=pcacov(h);  
[coeff2,latent2,explained2]=pcacov(i);  
[coeff3,latent3,explained3]=pcacov(j);  
[coeff4,latent4,explained4]=pcacov(k);  
[coeff5,latent5,explained5]=pcacov(l);  
[coeff6,latent6,explained6]=pcacov(m);  
[coeff7,latent7,explained7]=pcacov(n);  
  
eg1 = eig(h);  
eg2 = eig(i);  
eg3 = eig(j);  
eg4 = eig(k);  
eg5 = eig(l);  
eg6 = eig(m);  
eg7 = eig(n);
```

8.3 C-Matlab code for Tukey Bisquare Weights

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
% Robust Iteratively Re-Weighted Regression(Tukey Bisquare weights)%%  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
jsew = importdata('jsew.txt');  
r1=jsew(:,1);  
r2=jsew(:,2);  
r3=jsew(:,3);  
r4=jsew(:,4);  
r5=jsew(:,5);  
r6=jsew(:,6);  
r7=jsew(:,7);  
r8=jsew(:,8);  
r9=jsew(:,9);  
r10=jsew(:,10);  
r11=jsew(:,11);  
r12=jsew(:,12);  
r13=jsew(:,13);  
r14=jsew(:,14);  
r15=jsew(:,15);  
jse=jsew(:,16);  
  
[b1,stat1]=robustfit(jse,r1,'bisquare');  
[b2,stat2]=robustfit(jse,r2,'bisquare');  
[b3,stat3]=robustfit(jse,r3,'bisquare');  
[b4,stat4]=robustfit(jse,r4,'bisquare');  
[b5,stat5]=robustfit(jse,r5,'bisquare');  
[b6,stat6]=robustfit(jse,r6,'bisquare');  
[b7,stat7]=robustfit(jse,r7,'bisquare');  
[b8,stat8]=robustfit(jse,r8,'bisquare');  
[b9,stat9]=robustfit(jse,r9,'bisquare');  
[b10,stat10]=robustfit(jse,r10,'bisquare');  
[b11,stat11]=robustfit(jse,r11,'bisquare');  
[b12,stat12]=robustfit(jse,r12,'bisquare');  
[b13,stat13]=robustfit(jse,r13,'bisquare');  
[b14,stat14]=robustfit(jse,r14,'bisquare');  
[b15,stat15]=robustfit(jse,r15,'bisquare');
```

8.4 D-Matlab code for Huber Weights

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
% Robust Iteratively Re-Weighted Regression(Huber weights)%%  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
jsew = importdata('jsew.txt');  
r1=jsew(:,1);  
r2=jsew(:,2);  
r3=jsew(:,3);  
r4=jsew(:,4);  
r5=jsew(:,5);  
r6=jsew(:,6);  
r7=jsew(:,7);  
r8=jsew(:,8);  
r9=jsew(:,9);  
r10=jsew(:,10);  
r11=jsew(:,11);  
r12=jsew(:,12);  
r13=jsew(:,13);  
r14=jsew(:,14);  
r15=jsew(:,15);  
jse=jsew(:,16);  
  
[b1,stat1]=robustfit(jse,r1,'huber');  
[b2,stat2]=robustfit(jse,r2,'huber');  
[b3,stat3]=robustfit(jse,r3,'huber');  
[b4,stat4]=robustfit(jse,r4,'huber');  
[b5,stat5]=robustfit(jse,r5,'huber');  
[b6,stat6]=robustfit(jse,r6,'huber');  
[b7,stat7]=robustfit(jse,r7,'huber');  
[b8,stat8]=robustfit(jse,r8,'huber');  
[b9,stat9]=robustfit(jse,r9,'huber');  
[b10,stat10]=robustfit(jse,r10,'huber');  
[b11,stat11]=robustfit(jse,r11,'huber');  
[b12,stat12]=robustfit(jse,r12,'huber');  
[b13,stat13]=robustfit(jse,r13,'huber');  
[b14,stat14]=robustfit(jse,r14,'huber');  
[b15,stat15]=robustfit(jse,r15,'huber');
```

8.5 E-Matlab code Efficient Frontier

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%           Matlab code for plotting Effient Frontier           %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% for a 3 asset portfolio
ExpReturn = [0.002680214 0.004023772 0.005676392 ];

ExpCovariance = [2.343167969e-05 -1.09621893e-05 -3.0968184e-05
                 -2.57474421e-05 -4.850768778e-06 1.2373571e-05
                 -5.40984043e-06 -2.39660864e-05 0.000105472 ];

NumPorts = 30;

[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...
ExpCovariance, NumPorts);

RisklessRate = 0.00112;
BorrowRate   = 0.00148;
RiskAversion  = 3;

portalloc (PortRisk, PortReturn, PortWts, RisklessRate,...
BorrowRate, RiskAversion);

[RiskyRisk, RiskyReturn, RiskyWts,RiskyFraction, OverallRisk,...
OverallReturn] = portalloc (PortRisk, PortReturn, PortWts,...
RisklessRate, BorrowRate, RiskAversion);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```