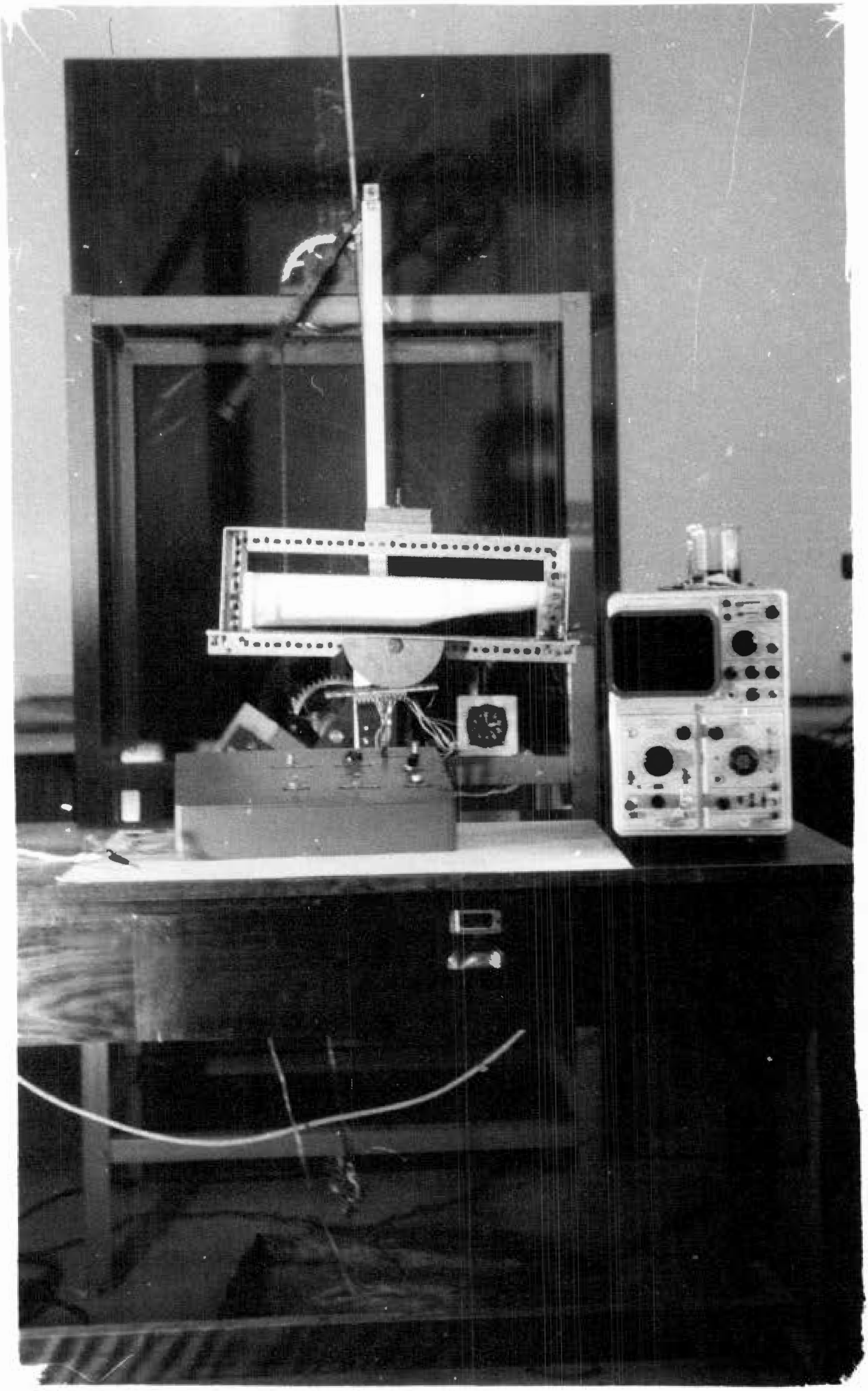
" AN ANALYSIS OF SHIP ROLL DAMPING BY RECTANGULAR FREE-SURFACE TANKS "



The simulator in action.

" AN ANALYSIS OF SHIP ROLL DAMPING BY RECTANGULAR FREE-SURFACE TANKS "

by

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Submitted to the department of Mechanical Engineering, University of Cape Town, in September 1966, for consideration for the award of the M.Sc.(Eng) degree.

P R E F A C E

The present investigation attempts to analyse the action of rectangular free-surface tanks as passive ship roll dampers.

In order to develop the necessary equations, the characteristics of ocean waves and the motion of ships among them are discussed in detail. A simulator was designed to reproduce ship rolling conditions at sea. With the aid of this apparatus, the mode of liquid movement in a rectangular tank fitted on a ship was observed and actual angles of roll for various tanks and liquid depths were recorded.

Using the above-mentioned observations, equations describing the complete motion of ship and liquid in the free-surface tank were derived. The results of these equations were compared with the figures obtained experimentally and the agreement was found to be good enough to permit the use of these equations for comparing the damping performance of rectangular tanks of different dimensions and varying liquid depths.

Consequently the equations were used to compile an "Optimum Stabilization Chart". By means of this Chart, the tank dimensions and liquid depth for best possible stabilization can be obtained for any ship whose particulars are known.

As an example, the Chart was used to determine the tank dimensions and water depth of a stabilizer giving optimum damping to the new U.C.T. research vessel, the "T.B. Davie". Unfortunately the figures available for this vessel were only approximate but the simple calculations can easily be repeated if the true dimensions differ appreciably from those used.

Acknowledgements. Finally, I should like to express my thanks to a number of kind persons: to my wife, Wendy, for her admirable performance in doing the typing to a close deadline; to Prof. P. Metcalf, Mr. R.M. Stegen and the whole academic staff of the Mechanical Engineering department for their advice and encouragement; to Mrs. C.P. Hart for filling in most of the symbols at the expense of other pressing work; and to Mr. W. Bettesworth and his staff for their cooperation in building the apparatus.

Edmond Széchényi

CONTENTS

TABLE OF SYMBOLS

Following is a list of the symbols most widely used in this thesis. Other symbols are used only in one or two sections and are introduced in the text; their inclusion here would only cause confusion as they are sometimes repeated with different meanings in different chapters.

$\doteq$ denotes 'approximately equal to'.

$\sim$ denotes 'proportional to'.

B centre of buoyancy

$C', "$ $C_{1,2,3,}$ } damping constants (ft lb sec)

D ratio of water depth to tank length ($^{h_o}/_b$)

G centre of gravity of the ship (or model)

$\overline{GM}$ metacentric height (ft)

I moment of inertia of ship (or model) about its centre of gravity (lb ft sec^2)

$K_{1,2,3,4}$ constants

L ocean wave length

M moment caused by moving bore (lb ft)

M' restoring moment on rolling ship (lb ft)

P ratio of weight of liquid in tank to dead weight of ship ($^{m_1 g}/_{\Delta}$)

R radius of rolling circle generating trochoidal profile of ocean waves (ft)

R' radius of rollers on the model (ft)

S height of tank from the centre of gravity of ship (or model) (ft)

T harmonic period (sec)

V Volts

b	tank length (ft)
g	constant of conversion from lb mass to lb force
h	ocean wave height (ft)
h_o	still water depth in tank (ft)
k	radius of gyration of ship (or model) (ft)
m_1	mass of water in tank (from Ch.5 onwards)
r	radius of orbit of ocean wave surface particles (= $\frac{1}{2}$ wave height) (ft)
t	time elapsed (sec)
w	tank width (ft)
Δ	dead weight of ship (or model) (lb)
α	slope of ocean wave
α_o	amplitude of ocean wave slope
ϵ	phase angle between bore and rolling angle
ϵ_s	phase angle between disturbing wave and rolling
θ	angle of roll
θ_o	amplitude of angle of roll
ϕ	some angle (= ωt)
ω	angular velocity (rad/sec)
ω_n	angular velocity at the natural frequency of the ship (or model) (rad/sec)
ω_n'	angular velocity at the natural frequency of the still-water surface-disturbance in the tank (rad/sec)
ρ	density of liquid in tank (lb/ft^3)
σ	scale factor for ship models

ERRATA

Page

14 The last equation before Section 2 - 5 should read:-

$$k^2\ddot{\theta} + \overline{GM}\,\theta = \overline{GM}\,\frac{\alpha_o \sin\omega t}{1 + \alpha_o \cos\omega t}$$

41 The last equation on this page should read:-

$$\omega_1 = \sqrt{10}\,\omega = 3\cdot16\,\omega$$

52 Equation (3 - 14a) should read:-

$$d_1 = \frac{56 - 0\cdot31m_t \pm\sqrt{(56 - 0\cdot31m_t)^2 + 4(130m_t - 55\cdot6)(m_t - 5\cdot12)}}{2\,(m_t - 5\cdot12)}$$

Equation (3 - 14b) should read:-

$$m_1 = \frac{11\cdot25m_t - 57\cdot6}{d_1 + 11\cdot25}$$

85 The 5^{th}, 6^{th} and 7^{th} lines respectively should read:-

$$C_1 = \dot{x}/_g\,\rho\, a w S^2$$

$$C_2 = \dot{x}/_g\,\rho\, a w x^2$$

Now C_1 is constant and only C_2 needs altering.

86 The equation for a_o near the bottom of the page should read:-

$$a_o = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2} \frac{b^2}{\pi^2}\beta^2\, d\beta$$

88 The 1^{st} line of the paragraph starting in the middle of the page should read:-

"The constant $^1/_2\,\rho w a$ will not be ......"

90 Equation (5 - 12) should read:-

$$I\ddot{\theta} + \left[C + \frac{\rho w a x}{g}\,(S^2 + \frac{b^2}{12})\right]\dot{\theta} + \Delta\,\overline{GM}\,\theta = \Delta\,\overline{GM}\,\alpha_o \sin(\omega t + \epsilon_s) - M\cos(\omega t - \epsilon)$$

I - INTRODUCTION

Historical background - The action of rectangular free-surface tank ship stabilizers - The scope of this investigation.

1 - 1. Historical background:

During the second half of the nineteenth century the transition from sail to steam as a means of ship propulsion brought about its own particular problems. One of these was ship rolling, which hitherto had beem damped by the sails.

The first recorded investigation into a mechanical means of damping the roll is a paper read by Philip Watts to a meeting of the Royal Institute of Naval Architects in 1883. In this and a subsequent paper read two years later, Watts described how the movement of a fluid in an open rectangular tank placed across a ship creates a moment which damps the ship's rolling. However, there was little practical interest in this idea and it was soon forgotten.

Ship stabilization was again spoken about in 1912 when Frahm proposed a tank which consisted essentially of two wing tanks connected by a pipe to form a U-tube arrangement. The moment of the oscillating liquid column counteracts the rolling motion. This type of stabilizer has been fitted to a number of ships with a fair measure of success but has never really become popular.

A number of modifications of Frahm's tank have been fitted to ships but their effectiveness has generally been very similar to that of the original U-tube device.

The period between the World Wars saw the development of a mechanical stabilizer which used the torque resulting from the forced precession of a gyroscope to oppose the disturbing moment. This invention has been used successfully in only one ship, a luxury liner. Its lack of popularity is due to the fact that a very large gyroscopic flywheel is required for effective stabilization.

With the end of World War II came the fin stabilizers. These have proved extremely successful and are very widely used today. However they have the disadvantage that the quality of stabilization is very much dependent on the speed of the ship, the fins having no effect at all when the vessel is stationary.

In more recent years the trend has been back to stabilization by means of liquid transfer. In 1964 a patent was granted to J.McMullen Associates for their 'flume-type stabilizer'. This stabilizer consists of two wingtanks connected by a flume (or channel) with a free fluid surface. The system acts partly like a Frahm tank in that pressure differences are created by viscous effects and partly like Watts' original tank because the wave action, although hampered, is still present. This stabilizer has been fitted to a number of ships with (according to the manufacturers) a good deal of success.

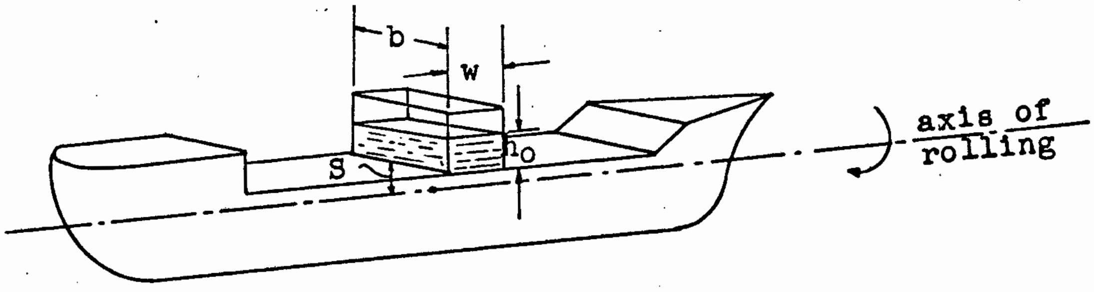
All the above-mentioned tank stabilizers are of the passive type. In 1965 Muirhead-Brown devised an activated tank stabilizer in which the transfer of liquid from side to side is effected by means of an axial flow pump. A gyroscopic sensing device similar to that used for fin and gyroscopic stabilizers regulates the flow direction and volume.

Until recently researchers have rather neglected Watts' original idea of using the moment caused by the transfer of liquid in a passive rectangular open tank for ship stabilization. However, since 1964 the shipbuilding laboratories of the Technische Hogeschool in Delft have been working on this subject. Their published work to date consists mainly of experimental data which, it is hoped, can be used in the design of such tanks.

1 - 2. The action of passive rectangular free-surface tank stabilizers:

From now on only passive rectangular tanks are discussed and for convenience will be referred to merely as 'free-surface tanks' or just 'tanks'.

Fig. 1 - 1



Following is a brief outline of the principles underlying the use of 'sloshing' liquid in a tank as a ship stabilizer:

The tank is mounted across the beam of the ship extending more or less from side to side (Fig.1 - 1). The main stabilizing action is created by a bore travelling to and fro along the length of the tank (across the beam of the ship). This bore is set up by the rolling oscillation of the ship. The liquid in the tank must be shallow so that a bore is generated - if the liquid is too deep a wave motion replaces the bore.

The moving bore transfers varying masses of liquid from side to side, applying an oscillatory couple on the ship. This couple acts against the disturbing moment due to the waves, hence reducing the moment causing rolling.

1 - 3. The scope of the present investigation:

At present no theoretical analysis of the moment caused by the moving bore or its ship stabilizing effect is available. This investigation will attempt to present such an analysis.

The first requirement in the execution of this work must be an apparatus to simulate a rolling ship carrying a rectangular tank, otherwise the value of the theoretical analysis cannot be objectively gauged.

With the help of experimental observations and results, at least an approximate theoretical analysis should be obtainable. The equations derived will be used to obtain a criterion for designing tanks that will give optimum stabilization.

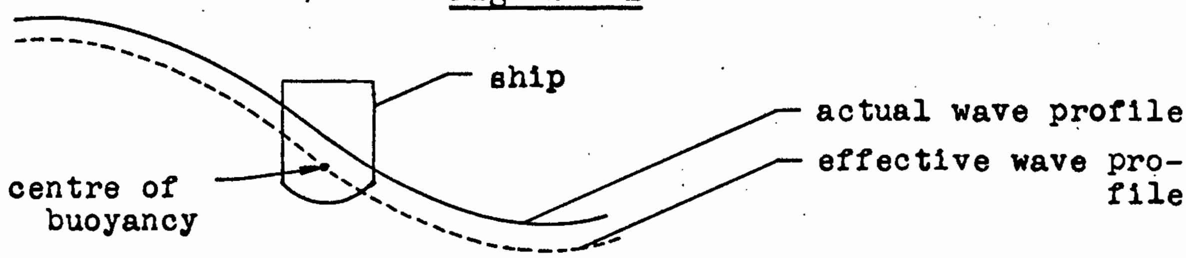
2 - THE ROLLING MOTION OF SHIPS

Introduction - Ocean waves - Characteristics of surge type waves - Froude's equation of rolling - Solution to Froude's equation of rolling motion - Krilov's equation of rolling - - The effect of Krilov's term - Natural roll damping - The equation of damped Rolling - Summary.

2 - 1. Introduction:

Ship rolling is a low frequency oscillatory motion caused by progressive waves moving past a ship in a beamwise direction. The variation of the slope of the wave profile at the depth of the centre of buoyancy of the ship shifts this point, causing the disturbing moment.

Fig. 2 - 1



The response of ships to this 'forcing function' has been studied by a number of people to a greater or lesser extent. The most widely accepted and simplest equation of rolling motion is that due to Froude or sometimes attributed to Rankine - both of whom obtain the same expression working on different basic assumptions. Krilov, the foremost Russian authority on the subject, has exposed inaccuracies incurred as a result of the assumptions made by the aforementioned researchers and offers a more accurate and complex expression based on hydrodynamic principles. However he is also forced to make broad assumptions in order to keep his equation as easy to handle as possible.

Since waves are the cause of ship rolling, the first few sections of this chapter are devoted to a brief survey of ocean waves, their geometry and characteristics.

2 - 2. Ocean Waves:

Ocean waves can generally be classified under three headings, viz.: wind-waves, surge and breakers.

Wind-waves, as the name suggests, are those waves that are actually whipped up by the force of winds. These waves are usually much steeper on the leeward than on the windward side and tend to form frothy peaks at their crests.

Waves thus generated are damped out very slowly and frequently move into areas of no wind. In such cases the waves settle down to a regular two-dimensional pattern called surge or swell. This is the only type of wave that can be expressed mathematically with accuracy and is also the pattern of ocean movements most generally encountered.

Breakers, the third category of waves, occur when a surge type wave moves into water where depth does not exceed half the wave length. The crest of the wave then rises, the sides becoming steeper until the peak breaks. These are the waves encountered at the coast.

In the study of ship motions, only the disturbance due to surge type waves is analysed because of the simple geometry of these waves.

2 - 3. Characteristics of surge type waves:

(a) A swell moves two-dimensionally as plane progressive waves. The profile of these waves is thoroughly analysed in a wide variety of literature and hence it is known to be very nearly trochoidal. The small difference between a true trochoid and a wave profile is that the wave tends to 'lean over' slightly away from the direction of motion, while the trochoid is perfectly symmetrical cf. Fig. 2 - 2 and Ref. 1 (pg.14).

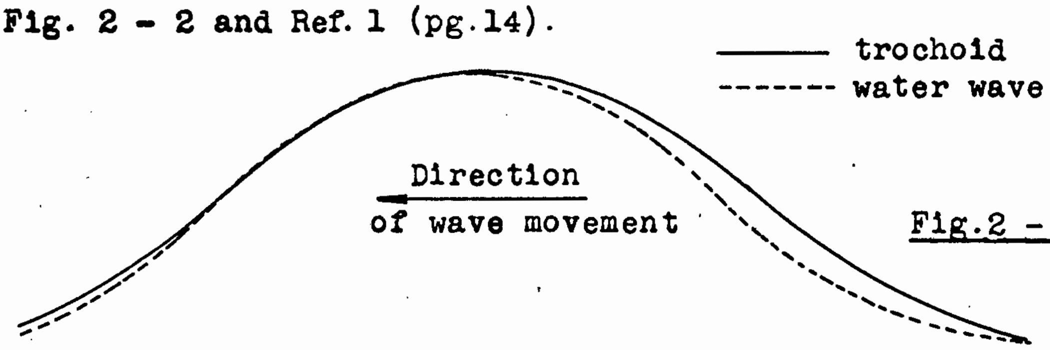


Fig.2 - 2

In the motion of plane progressive waves, the water particles do not move with the waves but perform circular orbits of decreasing radius with increasing depth below the surface (Fig. 2 - 3).

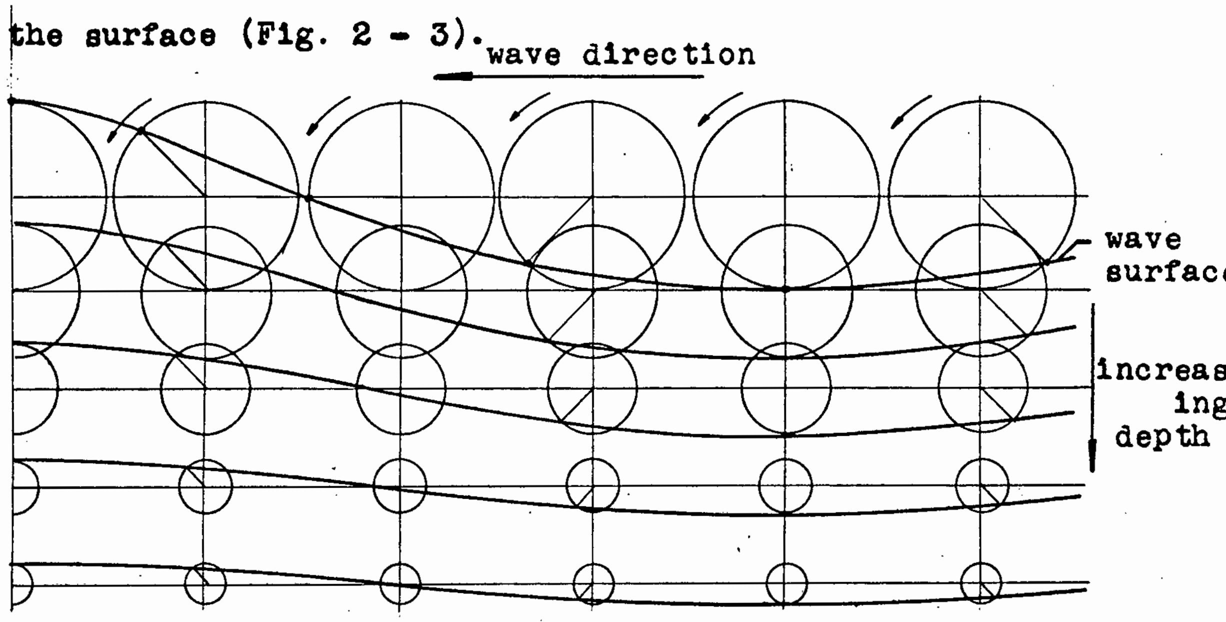


Fig. 2 - 3

Each particle, as a point on a fictitious rolling circle, describes a trochoid whose amplitude depends on the radius of the particle's orbit. Each successive trochoid is a line of constant pressure. Hence the orbiting surface particles also form a trochoid and are all at the same hydrostatic pressure.

(b) The trochoid: Let us now analyse this surface profile.

A trochoid is the curve traced by any point on a disc whose circumference is rolling on a straight line (Fig. 2 - 4).

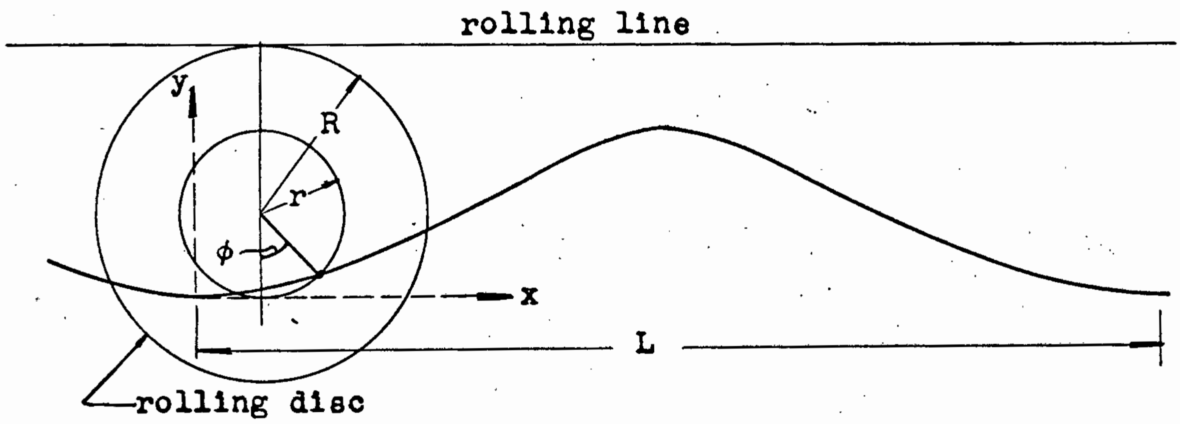


Fig. 2 - 4

h is the wave height

R is the radius of the rolling circle

r is the particle orbit radius, i.e.

for a surface particle $r = \frac{1}{2}$ wave height

L is the wave length (peak to peak)

ω is the angular velocity of rolling

t is time elapsed

α is the slope of the curve

For convenience, let angle $\omega t = \phi$

From Fig. 2 - 4,

At any instant

$$x = R\phi + r\sin\phi \qquad \ldots. (2 - 1)$$

$$y = R - r\cos\phi \qquad \ldots. (2 - 2)$$

Also $\quad 2\pi R = L$

From (2 - 1) and (2 - 2) we have:-

$$\frac{dx}{d\phi} = R + r\cos\phi$$

and $$\frac{dy}{d\phi} = r\sin\phi$$

The slope of the trochoid, α

$$= \frac{dy}{dx}$$

Now we assume α small,

$$\therefore \quad \tan\alpha \doteq \alpha = \frac{r \sin\phi}{R + r\cos\phi} \qquad \ldots\ldots\ldots\ldots (2 - 3)$$

$$= \frac{{}^{r}/_{R} \sin\phi}{1 + {}^{r}/_{R} \cos\phi}$$

In rolling, the maximum disturbing moment will be caused by the maximum wave slope.

Let the amplitude of wave slope be α_o,

This occurs when

$$\frac{d\alpha}{d\phi} = 0$$

$$\text{i.e. when} \quad R\cos\phi + r = 0$$

$$\text{or} \quad \cos\phi = -{}^{r}/_{R}$$

Substituting in (2 - 3),

$$\alpha_o = \frac{r}{\sqrt{R^2 + r^2}}$$

$$= \frac{1}{\sqrt{({}^{R}/_{r})^2 + 1}}$$

As will be seen later, for the average wave the ratio ${}^{L}/_{h} \doteq 30$

$$\therefore \quad {}^{R}/_{r} = \tfrac{1}{2\pi} L/\tfrac{1}{2}h = {}^{30}/_{\pi} \doteq 10$$

$$\therefore \quad ({}^{R}/_{r})^2 >> 1$$

$$\therefore \quad \alpha_o = {}^{r}/_{R} \qquad \ldots\ldots\ldots\ldots (2 - 4)$$

Substituting (2 - 4) in (2 - 3):-

$$\alpha = \frac{\alpha_o \sin\phi}{1 + \alpha_o \cos\phi} \qquad \ldots\ldots\ldots (2 - 5)$$

$$\text{or} \quad \alpha = \frac{\alpha_o \sin\omega t}{1 + \alpha_o \cos\omega t}$$

(c) <u>The wave period to length relationship:</u>

Consider any particle on the surface of a wave (Fig. 2 - 5 (a)). The pressure at all points on the surface is the same. Let m be the mass of the particle:-

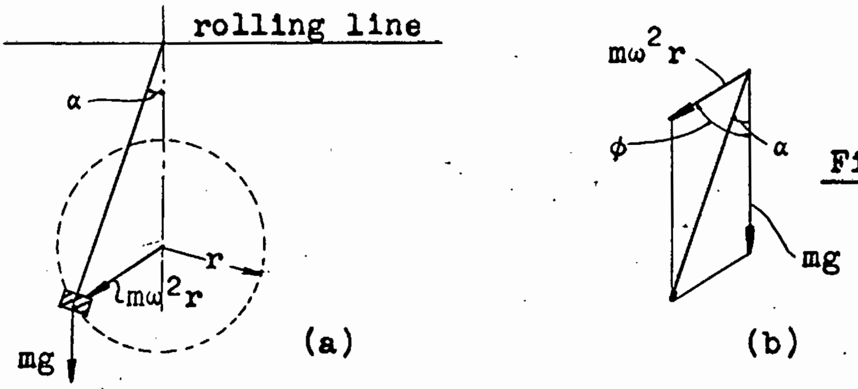


Fig 2 - 5

The forces acting on the particle are

(i) the centrifugal force, $m\omega^2 r$

(ii) the gravity force, mg

Since the particle does not move relative to the surface, the resultant force must act normal to the slope of the wave.

Hence the triangle of forces, Fig. 2 - 5 (b), yields

$$mg \sin \alpha = mr\omega^2 \sin (\phi - \alpha)$$

$$\therefore \quad \omega^2 = g/_r \; \frac{\sin \alpha}{\sin (\phi - \alpha)}$$

But $\alpha \doteq \sin \alpha$ and $\cos \alpha \doteq 1$

Then by substituting equation (2 - 5):-

$$\omega^2 = \frac{g \sin \phi}{R \sin \phi} = g/_R$$

But $R = L/_{2\pi}$

$$\therefore \quad \omega^2 = 2\pi g/_L \quad \ldots\ldots\ldots\ldots\ldots \quad (2 - 6)$$

Rewriting this equation in terms of the period, T:-

$$T = \sqrt{\frac{2\pi L}{g}}$$ *approx. Simple harmonic for free pendulum of length R (for small angles)*

(d) <u>The wave length to height relationship:</u>

In theory the limit to the ratio $L/_h$ must occur when

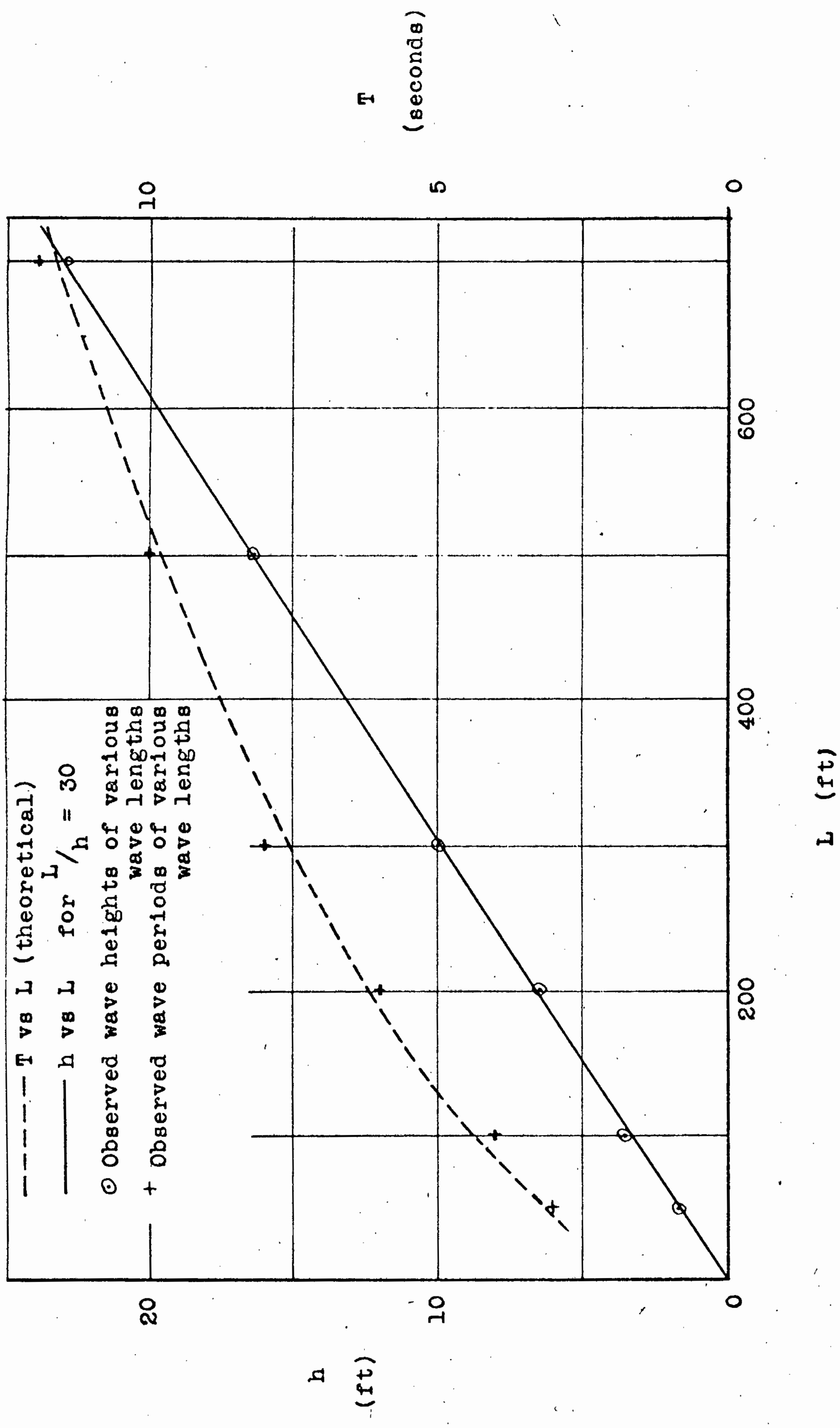

T (seconds)
h (ft)
L (ft)
0
5
10
20
200
400
600
T vs L (theoretical)
h vs L for L/h = 30
⊙ Observed wave heights of various wave lengths
+ Observed wave periods of various wave lengths

Fig. 2 - 6

The figures of Table I may give the impression that progressive waves are all identical to each other and that the period of the waves is constant. This is in fact not true. The figures in the Table are average; the actual figures vary at random but never too greatly e.g. the observed waves of surge No. 5 were found to vary in period from about 7 sec to 9 sec (Ref. 2).

2 - 4. Froude's equation of rolling:

The assumptions made in this analysis are:-

(i) The waves are long compared to the beam of the ship.

(ii) The form of the wave profile is a sine curve.

(iii) The ship is broadside to the waves.

(iv) The ship behaves as a particle of the water surface and therefore performs an orbital motion in space.

(v) The 'effective' wave surface which passes through the centre of buoyancy is the actual wave surface.

(vi) The motion of the ship is completely free of damping.

(vii) The buoyancy force on the ship always passes through the meta-centre. This is only strictly true at small angles of roll.

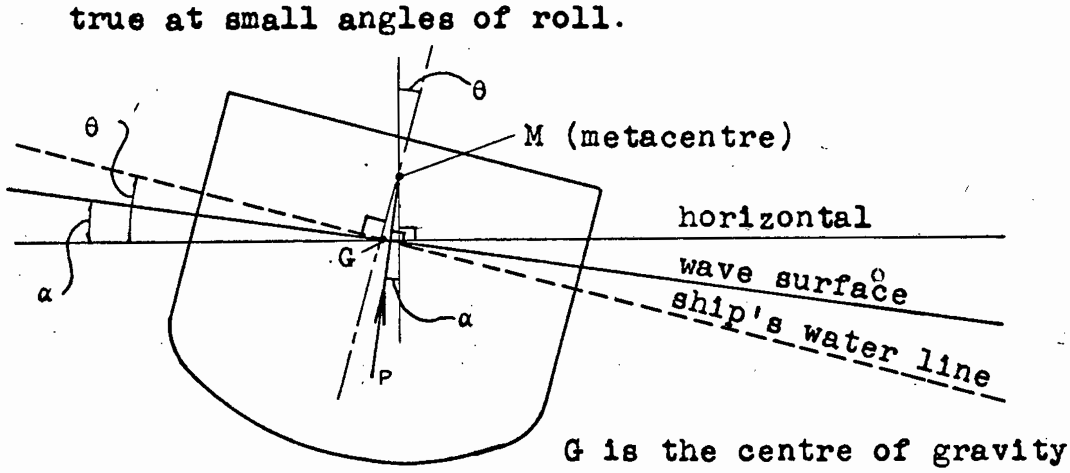


Fig. 2 - 7

From the previous section we have that the forces acting on an orbiting particle of a wave surface are:-

(i) the gravity force, Δg

(ii) the centrifugal force, $\Delta\omega^2 r$

(iii) the reaction normal to the wave surface due to the above two forces.

The reaction force for a ship must be the buoyancy P, which is normal to the wave slope.

$$\therefore \quad P = \Delta \cos\alpha + \Delta/g\, \omega^2 r \cos(\phi - \alpha)$$

As before, α is small

$$\therefore \quad P = \Delta + \frac{\Delta\omega^2 r}{g}(\cos\phi + \alpha\sin\phi) \quad \ldots\ldots\ldots\ldots (2-7)$$

The equation of motion of the ship is of the form:-

$$I\ddot{\theta} + M' = 0 \quad \ldots\ldots\ldots\ldots (2-8)$$

where I is the moment of inertia of the ship

M' is the restoring moment

From Fig. 2 - 7, we see that

$$M' = P\,\overline{GM}(\theta - \alpha)$$

where $\overline{GM}$ is the metacentric height.

Substituting from equation (2 - 7):-

$$M' = \overline{GM}(\theta - \alpha)\left[1 + \frac{\omega^2 r}{g}(\cos\phi + \alpha\sin\phi)\right]$$

From assumption (ii) we have

$$\alpha = \alpha_o \sin\phi \quad \ldots\ldots\ldots\ldots (2-9)$$

Also from equations (2 - 4) and (2 - 6) we have

$$\alpha_o = r/_R \quad \text{and} \quad \omega^2/g = 1/_R$$

Then by substitution,

$$M' = \Delta\,\overline{GM}(\theta - \alpha_o\sin\phi)\left[1 + \alpha_o\cos\phi + \alpha_o^2\sin^2\phi\right]$$

θ and α are small. Neglecting all terms of the second order of smallness (i.e. α_o^2 and $\theta\alpha_o$) as very small compared to unity:-

$$M' = \Delta\,\overline{GM}\theta - \Delta\,\overline{GM}\,\alpha_o\sin\phi$$

Substituting in equation (2 - 8):-

$$I\ddot{\theta} + \Delta\,\overline{GM}\theta = \Delta\,\overline{GM}\alpha_o \sin \omega t \qquad \text{..........}(2 - 10)$$

$$(\omega t = \phi)$$

This is Froude's equation of rolling and may be rewritten by dividing through by Δ :-

$$k^2\ddot{\theta} + \overline{GM}\theta = \overline{GM}\alpha_o \sin \omega t \qquad \text{..........}(2 - 10a)$$

where k is radius of gyration of ship.

Or else, since $\alpha = \alpha_o \sin \omega t$,

$$k^2\ddot{\theta} + \overline{GM}\,\theta = \overline{GM}\,\alpha \qquad \text{..........}(2 - 10b)$$

In these equations,

$$k^2\ddot{\theta} + \overline{GM}\theta = 0 \qquad \text{..........}(2 - 11)$$

is the equation of free rolling and

$\overline{GM}\,\alpha$ is the forcing function.

Using (2-5) the approximate trochoidal solution for α , the equation becomes:-

$$k^2\ddot{\theta} + \overline{GM}\theta = \overline{GM}\;\frac{\alpha_o \sin \omega t}{1 - \alpha_o \cos \omega t}$$

2 - 5. Solution to Froude's equation of rolling motion:

The general solution of equation (2 - 10) is that of forced vibration for an undamped single degree of freedom system and may be written down as:-

$$\theta = C_1 \sin \omega_n t + C_2 \cos \omega_n t + \theta_o \sin \omega t$$

(ω_n is the angular velocity at the natural frequency) where the first two terms on the right are the free oscillation and the third is the forced oscillation.

The free oscillation is of little significance here because in a damped system it dies out very quickly with time and in actual fact a rolling ship is a damped oscillation, as will be seen in section 2 - 8.

Hence we consider the particular solution

$$\theta = \theta_o \sin \omega t \qquad ....(2 - 12)$$

where θ_o is the amplitude of oscillation.

Substituting the above in (2 - 10a):-

$$-k^2\omega^2\theta_o + \overline{GM}\theta_o = \overline{GM}\alpha_o$$

$$\therefore \quad \theta_o = \frac{(\overline{GM}/k^2)\alpha_o}{(\overline{GM}/k^2) - \omega^2}$$

From equations (2 - 11) and (2 - 12) we have that:-

$$\overline{GM}/k^2 = \omega_n^2$$

$$\therefore \quad \theta_o = \frac{\alpha_o}{1 - (\omega/\omega_n)^2} \qquad ......(2 - 13)$$

This solution shows that at $\omega = \omega_n$, $\theta \rightarrow \infty$ i.e. resonance occurs.

The general solution mentioned above includes the terms for free oscillation which will set up a beat in the vicinity of the natural frequency until the damping causes this transient to die out.

Although equation (2 - 10) represents rolling in a very simplified and linearized form, the true motion of a ship, qualitatively at least, follows the same pattern as that given by Froude's equation. Quantitatively the true motion may vary considerably from the results of Froude.

2 - 6. Krilov's equation of rolling:

Krilov's derivation is also for small ships. He makes essentially the same assumptions as Froude except that he does not find it necessary to assume the ship as merely a particle of a wave surface.

As opposed to Froude's purely dynamic analysis of the motion, Krilov's approach is the consideration of the hydrodynamic forces in detail.

Taking the basic equation of motion (2 - 8):-

$$I\ddot{\theta} + M' = 0$$

M' is determined as the sum of the moments caused by the hydrodynamic force on elements of the area of the hull.

The detailed derivation of this is very long and can be found in ref. 2 section 32. The final solution will be sufficient and is given as

$$M' = \Delta\overline{GM}\,(\theta - \alpha) - I_V\ddot{\alpha} \quad \text{.........(2 - 14)}$$

where I_V is the moment of inertia of the displaced mass of water.

Substituting the above in equation (2 - 8):-

$$I\ddot{\theta} + \Delta\overline{GM}\theta = \Delta\overline{GM}\alpha + I_V\ddot{\alpha}$$

In Krilov's assumption, as in Froude's,

$$\alpha = \alpha_o \sin \omega t$$

$$\therefore \quad I\ddot{\theta} + \Delta\overline{GM}\theta = \Delta\overline{GM}\alpha_o \sin \omega t - I_V\omega^2\alpha_o \sin \omega t \quad \text{..(2 - 15)}$$

It is evident that this equation differs from that of Froude (2 - 10) by the term $I_V\omega^2\alpha_o \sin \omega t$ in the forcing function. Krilov explains that this term is due to the nonuniformity of the ship's vertical acceleration. Froude obtained a corresponding term in his analysis but neglected it since it was of the second order of smallness.

The magnitude of I_V is generally in the region of 40% to 50% of I and therefore is not negligible.

Although equation (2 - 15) is still subject to a number of assumptions, it should be used in preference to equation (2 - 10) in studying the rolling of ships.

According to Krilov, the addition of an appropriate damping term to his equation brings the solution very close indeed to actually observed rolling.

2 - 7. The effect of Krilov's term:

For equation (2 - 14) assume the particular solution

$$\theta = \theta_o \sin \omega t$$

and neglect the superimposed free oscillation.

Then:

$$(\Delta \overline{GM} - I\omega^2)\theta_o = \alpha_o (\Delta \overline{GM} - I_v\omega^2)$$

$$\therefore \quad \theta_o = \frac{(\Delta \overline{GM} - I_v \omega^2)\alpha_o}{\Delta \overline{GM} - I\omega^2}$$

Dividing top and bottom by $\Delta \overline{GM}$ and putting $\overline{GM}/I = \omega_n{}^2$ (see section 2 - 5), we have:-

$$\theta_o = \frac{\alpha_o}{1 - (\omega/\omega_n)^2} \left[1 - {}^{I_v}/_I \, (\omega/\omega_n)^2 \right]. \quad (2 - 16)$$

Equation (2 - 13) gave Froude's solution for θ_o as

$$\theta_o = \frac{\alpha_o}{1 - (\omega/\omega_n)^2}$$

Comparing the above two solutions it is obvious that the angle of roll according to Froude is larger than that found through Krilov's more accurate solution by the amount

$$I_v/_I \left(\omega/\omega_n\right)^2 \quad \frac{\alpha_o}{1 - (\omega/\omega_n)^2}$$

Now $I_v/_I$ is generally in the region of 0·4 to 0·5 (see section 2 - 6).

Therefore Froude's angle exceeds Krilov's by an

error $\doteq \dfrac{0{\cdot}5\ \alpha_o}{(\omega_n/\omega)^2 - 1}$

Hence we see that at a low frequency ($\omega_n/_\omega >> 1$) the error is small and negligible. However the error increases with ω tending towards a maximum at $\omega = \omega_n$. Resonance occurs at this frequency so that the angle of roll tends to infinity whether considering Froude or Krilov.

Fig. 2 - 8 compares the angles of roll as obtained by the two solutions.

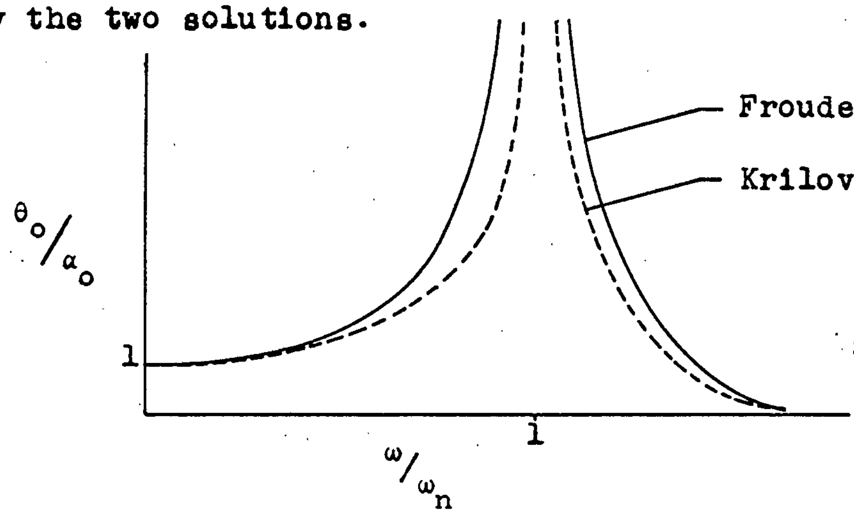


Fig. 2 - 8

To summarize, Froude's solution (equation 2 - 13) leads us to expect a much larger angle of roll than actually occurs.

2 - 8. Natural roll damping:

The natural damping of ship roll is due to:-

(i) the skin friction between water and hull.

(ii) the loss of energy of the ship in the generation of waves.

(iii) the moments set up by vortices and other flow characteristics determined by the shape of the hull.

(iv) the forward velocity of the ship.

(i) Skin friction:

The damping moment due to the skin friction, M_1, is necessarily a function of the angular velocity of roll, $\dot{\theta}$, because it is caused solely by the shearing force between the water particles at the hull and the particles immediately adjacent.

i.e. $M_1 = C_1 \dot{\theta}$

where C_1 is some constant.

(ii) Wave generation:

If the ship were purely a rolling cylinder this effect would be nonexistent because the waves are generated by the "paddle" effect of the vertical sides of the hull. It is well known that the force exerted on paddles in fluids is a function of the square of the speed. Hence the damping moment due to wave generation (M_2) may be expressed as:-

$$M_2 = C_2 \dot{\theta}^2$$

where C_2 is some constant.

(iii) Hull shape:

Any fullness of form outside the circular shape, besides generating waves, has the effect of causing "streamline" water pressures opposing the the rolling motion. These pressures become particularly marked about hull protrusions such as bilge keels (Fig. 2 - 9).

The magnitude of this damping moment has been found experimentally to depend mainly on the

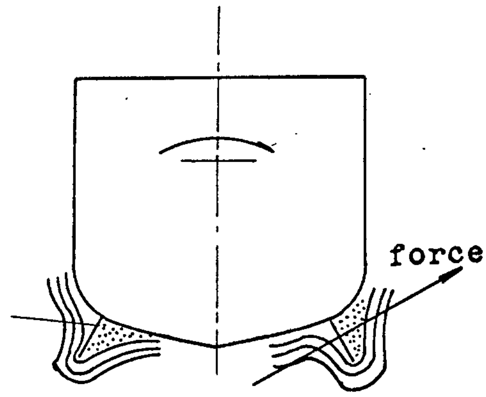


Fig. 2 - 9

angular velocity of roll, $\dot{\theta}$, although it is also a function of $\dot{\theta}^2$ particularly at large angles of roll. The exact relationship between the moment and the angular velocity is characteristic of individual ships and cannot be acurately predicted. The complexity of this effect will be realized when considering that experimenters cannot agree on the powers of $\dot{\theta}$ affecting the moment; some maintain that the moment is also a function of $\dot{\theta}^3$ or even $\dot{\theta}^4$.

(iv) Forward velocity of ship:

Although the speed of a ship contributes considerably to the roll damping, we will not consider this effect here and concentrate purely on the rolling of stationary ships.

The independent damping action of the first three components mentioned above has been investigated by Dr. G.S. Baker, among others (Ref. 1). He gives the following average comparisons between their magnitudes:-

Skin friction	13·5%
Wavemaking	16·0%
Hull shape	70·5%

Obviously it is very difficult - wellnigh impossible - to derive an exact general expression for the natural damping moment on rolling ships. Each ship has its own characteristics depending on the nature and shape of the hull among other parameters.

Experimenters such as Froude, Bertin and Watt used data obtained by observing the still water rolling (free rolling) of numerous ships to lay down the following general expression:-

$$\delta\theta = a\theta + b\theta^2 \qquad \text{..........(2 - 17)}$$

where $\delta\theta$ is the decrement of roll (over one cycle).

and θ is the mean angle of roll.

a and b are constants determined experimentally for any particular ship.

Although a and b are purely and solely dependent on the characteristics of the ship concerned, their relative magnitudes are usually of a similar order in cases where no anti-rolling devices such as bilge keels are used:-

$b \doteq 7\%$ of a (<u>no</u> bilge keels)

The fitting of bilge keels increases b considerably while the change in a is comparatively small. The magnitude of the increase in b is impossible to define generally but it has been observed to increase by as much as tenfold.

Our aim is to obtain an expression for the damping moment for inclusion in the equation of rolling. This expression must somehow incorporate the effects discussed at the beginning of this section and at the same time must satisfy Eqn(2 - 17)

We have seen that the overall damping moment is primarily dependent on $\dot{\theta}$ and $\dot{\theta}^2$. Therefore let us assume

$$M_d = C_3\dot{\theta} + C_4\dot{\theta}^2$$

where C_3 and C_4 are some constants.

If we now assume rolling motion with small angles of roll only, θ is small. Hence $\dot{\theta}$ will be small over the range of wave frequencies (Period = 2 to 12 sec.). Then $\dot{\theta}^2$ will be of the second order of smallness and negligible.

∴ for small angles

$$M_d = C\dot{\theta} \qquad \text{..........(2 - 18)}$$

For the sake of linearity we shall henceforth assume this relationship even for angles which are not exactly small. Incidentally, the above relationship underestimates the damping.

The constant C in equation (2 - 18) must be chosen so as to satisfy equation (2 - 17) as closely as possible. To do this, the decrement of roll due to the damping moment $C\dot{\theta}$ must be found via the solution of the equation of damped rolling and compared to (2 - 17).

2 - 9. <u>Equation of damped rolling:</u>

Consider equation (2 - 11):-

$$k^2 \ddot{\theta} + \overline{GM}\theta = 0$$

If we now introduce damping according to equation (2 - 18), then

$$I\ddot{\theta} + C\dot{\theta} + \Delta\overline{GM}\theta = 0 \qquad \text{..........(2 - 19)}$$

is the equation of damped rolling <u>in still water</u>. The solution of this equation is well known (Ref. 4 pg.39), and yields

$$\frac{\theta_{n+1}}{\theta_n} = e^{-\pi C/Iq} = e^{-\delta}$$

where q is the damped natural frequency

and θ_n and θ_{n+1} are consecutive peaks of θ_o. It is well known that in the case of ships with no anti-rolling devices, the damping can be considered small.

$$\therefore \frac{\pi C}{Iq} \text{ is small}$$

$$\text{i.e. } \delta \text{ is small}$$

$$\therefore e^{-\delta} \doteq 1 - \delta$$

$$\text{also } q \doteq \omega_n$$

$$\therefore \frac{\theta_{n+1}}{\theta_n} = 1 - \frac{\pi C}{I\omega_n}$$

$\therefore$ the decrement of roll over one cycle, $\delta\theta$, is

$$\theta_n - \theta_{n+1} = \theta_n - \theta_n \left(1 - \frac{\pi C}{I\omega_n}\right)$$

$$= \frac{\theta_n \pi C}{I\,\omega_n}$$

$$\text{i.e. } \delta\theta = K\theta \qquad \text{..........(2 - 20)}$$

where K is a constant $= \frac{\pi C}{I\omega_n}$

To compare this solution with equation (2 - 17), consider the curves of $\delta\theta$ vs θ, Fig. 2 - 10(a). In order to determine K and hence C as accurately as possible, equate (2 - 17) and (2 - 20):-

$$K\theta = a\theta + b\theta^2$$

$$\therefore K = a + b\theta$$

Hence the best value of K will be that obtained for a mean overall value of θ.

Therefore for any known values of a and b, and assuming a mean angle of roll θ', the best value of the constant K is obtained. Fig. 2 - 10(b).

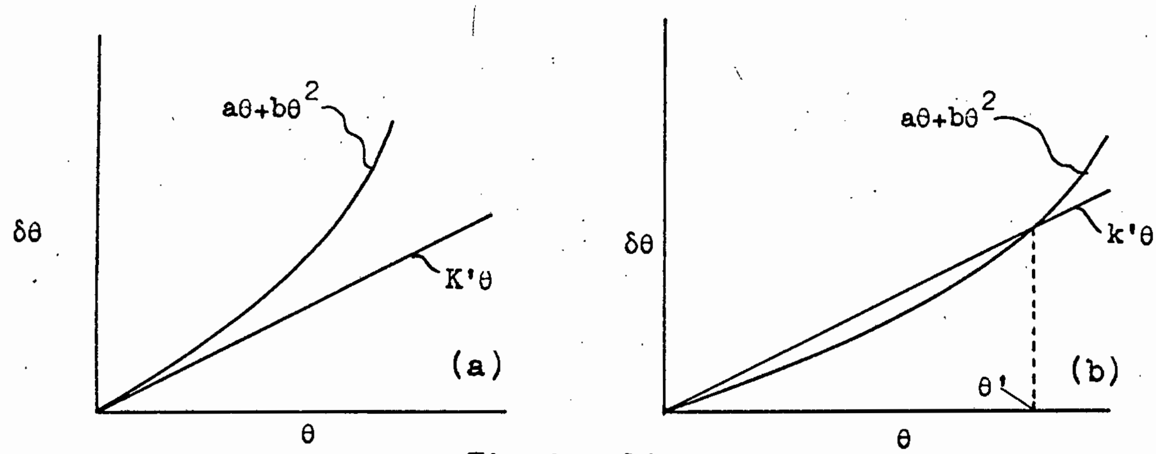


Fig.2 - 10

Hence the damping constant is determined:-

$$C = \frac{KI\omega_n}{\pi} \qquad \text{..........(2 - 21)}$$

When refering to ship damping, this constant is frequently quoted in dimensionless form and called μ :-

$$\mu = \frac{C}{2I\,\omega_n}$$

i.e. $$\mu = \frac{K}{2}$$

μ is known as the coefficient of decay of the rolling motion.

Now having determined the damping moment, we can introduce it into the equation of rolling:-

$$I\ddot{\theta} + C\dot{\theta} + \Delta\overline{GM}\theta = \Delta\overline{GM}\alpha_o \sin\omega t \qquad \text{..........(2 - 22)}$$

for Froude's simplified equation.

If Krilov's hydrodynamic term is considered, the equation becomes:-

$$I\ddot{\theta} + C\dot{\theta} + \Delta\overline{GM}\theta = \Delta\overline{GM}\alpha_o \sin\omega t - I_v\omega^2\alpha_o \sin\omega t \qquad \text{..........(2 - 23)}$$

Finally, if the true trochoidal shape of the wave is considered, equation(2 - 23)becomes:-

$$I\ddot{\theta} + C\dot{\theta} + \Delta\overline{GM}\theta = \frac{(\Delta\overline{GM} - I_v\omega^2)\,\alpha_o \sin \omega t}{1 + \alpha_o \cos \omega t}$$

Equation (2 - 22) is the complete linearized equation of rolling motion used in this work. The other equations will be found too cumbersome (section 3 - 3).

2 - 10. Summary:

(i) The profile of ideal progressive ocean waves is trochoidal.

(ii) The change in the position of the centre of buoyancy of ships caused by the changing slope of passing waves induces the rolling in ships.

(iii) The basic equation of undamped rolling of ships on the assumption of a sinusoidal wave profile is of the form derived by Froude, i.e. Equation (2 - 10).

(iv) This equation was found to be incomplete by Krilov, who adds a further hydrodynamic term to the equation.

(v) Both the abovementioned equations are formulated without the inclusion of a damping term, which is very real in practice.

The damping moment is mathematically complex and is conveniently simplified and linearized to give a term of pure and simple first order viscous damping. The constant which then governs the damping depends entirely on the shape and size of the ship concerned.

(vi) The consideration of the damping moment leads to the equation which completely describes the rolling motion of ships (in a very simplified form, of course).

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3 - DESIGN OF SHIP ROLL SIMULATOR & SUITABLE FREE-SURFACE TANK

Introduction - The wave motion simulator - The dynamic ship model - Mechanical design of the wave simulator - - Mechanical design of the ship model - Relation between ship weight and free surface tank width - Weight distribution in the model - Instrumentation - Damping - - Summary.

3 - 1. Introduction:

A ship roll simulator must consist essentially of two main components:

(i) a sea wave simulator

(ii) a dynamic ship model

The combined motion of these two must obey the equation of rolling of a ship as closely as possible so that the amplitude of roll of the model resembles that of a ship under similar conditions.

To obtain a versatile simulator, it is necessary so to design the components that the wave and ship characteristics may be altered and set to simulate a range of desired conditions.

The result to be obtained from experiments on the simulator is principally the angle of roll at various wave configurations. Therefore the apparatus must be instrumented so as to enable continuous, quick and accurate angular measurements while the model is actually rolling. Furthermore, it is important that the liquid motion in the tank should be easy to observe.

3 - 2. The wave motion simulator:

The problem is to design a mechanism which will simulate the changing slope of a wave while performing the orbital motion of surface particles. These requirements are satisfied by a small flat plate performing vertical circular orbits, while its slope is made to conform to a true wave slope at the corresponding instant of the orbit.

This type of mechanism (Fig. 3 - 1) was first devised by the Italian Capt. Russo round about the beginning of this century.

Considering Fig. 3 - 1: the angle of the wave surface plate must be at α to the horizontal, where α is given by equation (2 - 3).

The length of the crank arm, r, is half the height of the simulated wave - to some predetermined scale.

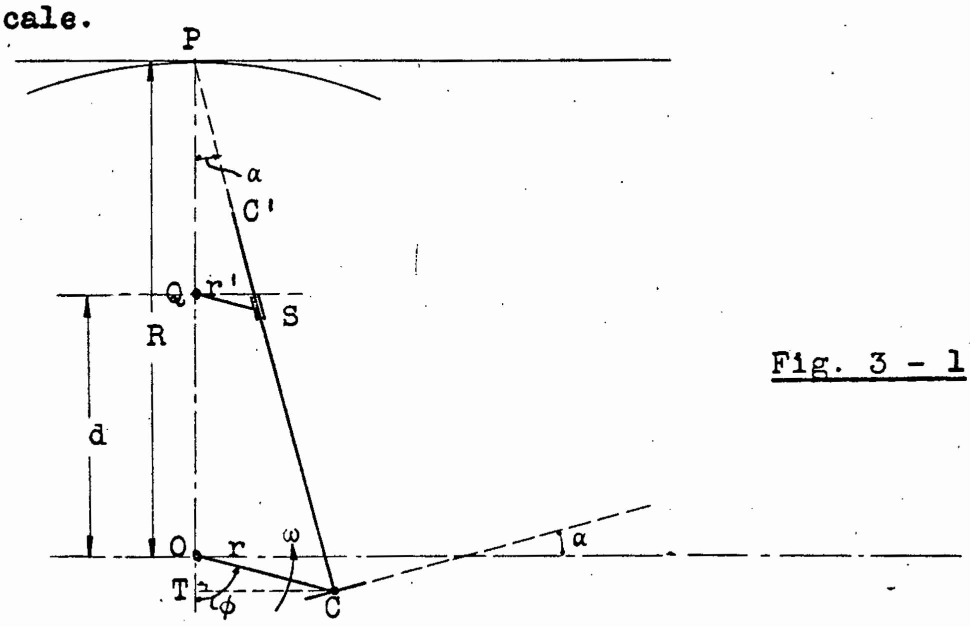


Fig. 3 - 1

Briefly, the mechanism consists of two arms OC and QS turning about centres O and Q in a vertical plane and always parallel to each other. Q is vertically above O.

The link CC' is fixed perpendicular to the wave surface plate and passes through a swivelling slide at S.

This arrangement ensures that CS always passes through the same point in space, P, whatever the angle ϕ. To prove this, consider the linkage at two different positions ϕ_1 and ϕ_2 in Fig.3 - 2:

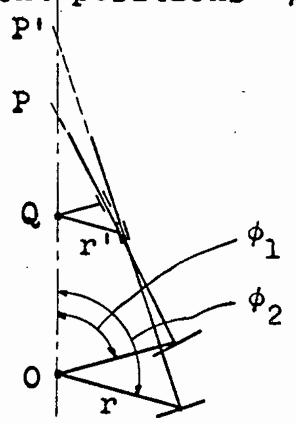


Fig. 3 - 2

$$\frac{r}{r'} = \frac{OP}{OP - OQ} \quad \text{for } \phi_1$$

and $$\frac{r}{r'} = \frac{OP'}{OP' - OQ} \quad \text{for } \phi_2$$

but r and r' are fixed,

$$\therefore \quad \frac{OP}{OP - OQ} = \frac{OP'}{OP' - OQ}$$

i.e. $$\frac{OQ}{OP} = \frac{OQ}{OP'}$$

$$\therefore \quad OP = OP'$$

Then OP must represent the radius of the rolling circle of a trochoid generator. OC represents the radius of the tracing circle. The rolling line is through P perpendicular to OP.

The slope of the surface plate is α, where $\tan\alpha = CT/TP \approx \alpha$, since α is small.

But $TP = R + r\cos\phi$

$CT = r\sin\phi$

The length d must be fixed to some value convenient for construction.

The required speed of the mechanism is obtained from equation (2 - 6) where

$$\omega = \sqrt{\frac{2\pi g}{L}}$$

$\therefore$ for wave length L_1,

the speed $\omega_1 = \sqrt{\frac{2\pi g}{L_1}} = \sqrt{\frac{2\pi g}{L\sigma}}$ ....(3 - 2)

where ω_1 is the angular velocity of the simulator arms.

$$\therefore \quad \omega_1 = \frac{\omega}{\sqrt{\sigma}}$$

i.e. The wave speed is scaled according to the square root of the linear scale.

To summarize, the mechanism of Fig. 3 - 1 will simulate exactly the movement of a segment of a wave surface.

3 - 3. The dynamic ship model:

The "model ship" required must be such that it will roll on the surface plate of the simulator, subject to an equation of motion identical to the equation of rolling of ships.

To begin with, let us assume a circular rolling profile, rolling on the surface plate, Fig. 3 - 3. The surface plate is moving in a circular orbit as discussed in the previous section.

Using the assumption of Froude that the buoyancy force passes through the meta-centre, the centre of the rolling profile must be at M, (which is

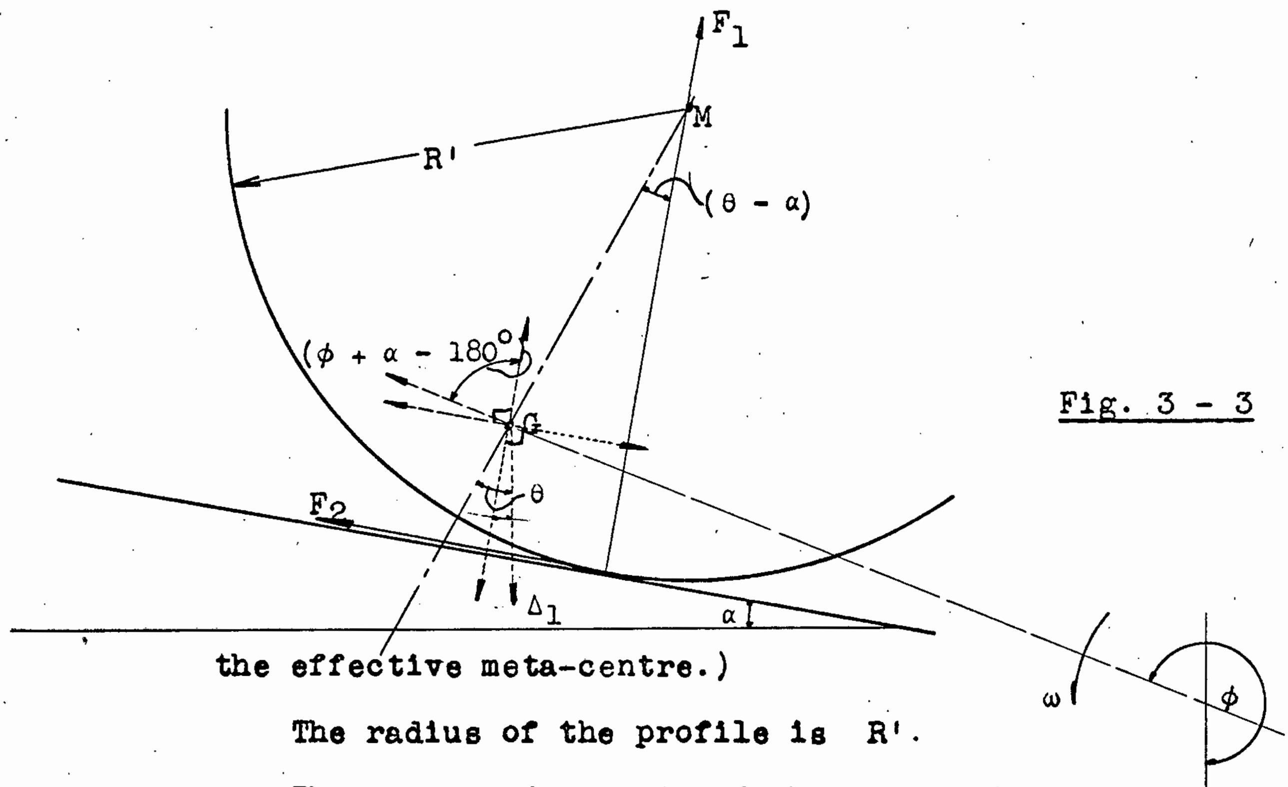


Fig. 3 - 3

the effective meta-centre.)

The radius of the profile is R'.

The centre of gravity of the model, G, lies vertically below M on the axis of symmetry so that the height, GM, is the metacentric height known as $\overline{GM}$, to scale σ.

θ is the angle of roll from the vertical.

Δ_1 is the weight of the model.

I_1 is the moment of inertia of the model about G.

The motion is assumed as pure rolling (no slipping takes place). Then the equation of rolling is:

$$I_1\ddot{\theta} + M_1 = 0 \qquad ....(3 - 3)$$

where M_1 is the total moments acting on the model.

From Fig. 3 - 3 we see that the forces acting on the model are:-

(i) The reaction force normal to the surface and passing through M, F_1.

(ii) The gravity force Δ_1 perpendically down.

(iii) The friction force along the surface plate, F_2.

(iv) The centrifugal force at G due to the orbital motion $\frac{\Delta_1 \omega_1^2 r}{g}$ at ϕ to the vertical.

Assuming pure rolling, the vector sum of forces must balance.

Hence:-

$$F_1 - \Delta_1 \cos \alpha + \frac{\Delta_1 \omega_1^2 r}{g} \cos (\phi + \alpha - 180) = 0$$

But α is small,

$$\therefore \quad F_1 = \Delta_1 \left[1 + \frac{\omega_1^2 r}{g} (\cos \phi - \alpha \sin \phi) \right]$$

and

$$F_2 - \Delta_1 \sin \alpha + \frac{\Delta_1 \omega_1^2 r}{g} \sin (\phi + \alpha - 180) = 0$$

$$\therefore \quad F_2 = \Delta_1 \left[\alpha + \frac{\omega_1^2 r}{g} (\sin \phi + \alpha \cos \phi) \right]$$

From equation (3 - 2) we have

$$\frac{\omega_1^2}{g} = \frac{2\pi}{L_1}.$$

$$\text{But} \quad L_1 = 2\pi R$$

$$\therefore \quad \frac{\omega_1^2 r}{g} = \frac{r}{R}$$

We have already seen that $r/_R = \alpha_o$

$$\therefore \quad \frac{\omega_1^2 r}{g} = \alpha_o \qquad \text{..........(3 - 4)}$$

Therefore we can write

$$\left.\begin{aligned} F_1 &= \Delta_1 \left[1 + \alpha_o (\cos \phi - \alpha \sin \phi) \right] \\ F_2 &= \Delta_1 \left[\alpha + \alpha_o (\sin \phi + \alpha \cos \phi) \right] \end{aligned}\right\} \qquad \text{..........(3 - 5)}$$

These two forces F_1 and F_2 cause couples:-

(i) $F_1 \overline{GM} \sin(\theta - \alpha)$..... anticlockwise in Fig. 3 - 3

(ii) $F_2\left[R' - \overline{GM}\cos(\theta - \alpha)\right]$....clockwise in Fig. 3 - 3

The above two couples are the only disturbances causing the motion of the model.

Then, considering anticlockwise as the positive direction,

$$M_1 = -F_2\left[R' - \overline{GM}(\cos\theta + \alpha\sin\theta)\right] + F_1\overline{GM}(\sin\theta - \alpha\cos\theta) \quad \text{..........(3 - 6)}$$

Substituting (3 - 5) in (3 - 6) and simplifying:

$$M_1 = -\Delta_1 R'\alpha - \Delta_1\overline{GM}_1\left[R'\alpha_o\sin\phi + R'\alpha\,\alpha_o\cos\phi - \alpha_o\sin\phi\cos\theta - \alpha_o\alpha^2\cos\phi\sin\theta - \sin\theta - \alpha_o\sin\theta\cos\phi - \alpha^2\alpha_o\sin\phi\cos\theta\right]$$

We have assumed θ as small in the analysis of ship motion,

$$\therefore \sin\theta \doteq \theta \quad \text{and} \quad \cos\theta \doteq 1$$

Also, θ is of the same order of magnitude as α so that any product of θ and α is of the second order of small quantities and negligible.

$$\therefore M_1 = \Delta_1\overline{GM}_1\left[(\alpha_o\sin\phi + \theta) - R'(\alpha + \alpha_o\sin\phi)\right]$$

Rewriting this equation -

$$M_1 = -\Delta_1\left[R'\alpha - \overline{GM}_1\theta + (R' - \overline{GM}_1)\alpha_o\sin\phi\right]$$

Substituting in (3 - 3):-

$$I_1\ddot{\theta} - \Delta_1(R' - \overline{GM}_1)\alpha_o\sin\phi + \Delta_1\overline{GM}\theta - \Delta_1 R'\alpha = 0 \quad \text{..........(3 - 7)}$$

Comparing this equation to Froude's equation of rolling (2 - 10) we see that the form of the equation is similar when $R' = \overline{GM}_1$.

i.e. rewriting equation (3 - 7) with $R' = \overline{GM}_1$:-

$$I_1\ddot{\theta} + \Delta_1\overline{GM}_1\theta = \Delta_1\overline{GM}_1\alpha$$

and dividing through by Δ_1:-

$$k_1^2\ddot{\theta} + \overline{GM}_1\theta = \overline{GM}_1\alpha \qquad \text{..........(3 - 8)}$$

where k_1 is the radius of gyration of the model.

This equation is identical to (2 - 10b).

We have seen in section 3 - 2 that

$$\alpha = \frac{\alpha_o \sin\phi}{1 + \alpha_o \cos\phi}$$

and ϕ is the angle turned through by the simulator arm in a certain length of time.

Therefore we can write:

$$\phi = \omega_1 t_1$$

As before, ω_1 is the speed of the simulator and t_1 is the time elapsed.

$$\therefore \quad \alpha = \frac{\alpha_o \sin\omega_1 t_1}{1 + \alpha_o \cos\omega_1 t_1}$$

Substituting in (3 - 8), the equation of motion of the model is:

$$k_1^2\ddot{\theta} + \overline{GM}_1\theta = \overline{GM}_1 \frac{\alpha_o \sin\omega_1 t_1}{1 + \alpha_o \cos\omega_1 t_1} \qquad \text{......(3 - 9)}$$

and gives a motion slightly more accurate than Froude's equation in that the forcing function is true and not simplified to sinusoidal. The difference will always be very small since $\alpha_o \mathrel{\hat{=}} 0{\cdot}1$ and rarely exceeds 0·2. Therefore the neglect of

$\alpha_o \cos \omega_1 t_1$ yields a maximum error of 8% for $\alpha_o = 0{\cdot}1$.

Hence the use of Froude's equation for the model is not unreasonable and in fact, for the sake of simplicity in calculation, we shall assume $\alpha = \alpha_o \sin \omega_1 t_1$ for the simulator.
i.e. The equation of motion of the model subsequently used is:

$$k_1^2 \ddot{\theta} + \overline{GM}_1 \theta = \overline{GM}_1 \alpha_o \sin \omega_1 t_1 \qquad \text{..........(3 - 10)}$$

although equation (3 - 9) describes the motion exactly.

It has already been discussed in section 2 - 6 that Froude's equation of rolling does not describe the rolling of a ship entirely satisfactorily. A further term $I_v \ddot{\alpha}$ should appear in the equation.
No conceivable alteration to the simple model assumed above will give a term which is a function of $\ddot{\alpha}$. The closest to obtaining such a term would be to assume $\alpha \mathrel{\hat{=}} \theta$ (at frequencies below resonance only, because a phase of 180° appears at resonance), since they have been taken of the same order.
Then $\ddot{\alpha} \mathrel{\hat{=}} \ddot{\theta}$ and the equation could be written:-

$$(k_1^2 - I_v/\Delta_1)\ddot{\theta} + \overline{GM}_1 \theta = \overline{GM}_1 \alpha_o \sin \omega_1 t_1$$

However this becomes completely invalid when the existing damping is introduced in the equation because then a phase angle exists between α and θ and the terms $\ddot{\alpha}$ and $\ddot{\theta}$ can no longer be added scalarly. The assumption $\alpha \mathrel{\hat{=}} \theta$ is also completely invalid when frequencies approach resonance and θ increases considerably.

Since the only assumption conceivably possible has no validity at all, there is no obvious or simple way of introducing the extra hydrodynamic term by means of the model on the wave simulator. The only possibility might be to have a ship model in actual water waves.

Consequently we must be satisfied to simulate Froude's equation of rolling which gives an exaggerated angle of rolling as shown in section 2 - 7.

In a study of ship stabilization such as this, the use of exaggerated angles of roll in practical work is not necessarily detrimental, since the successful damping of this rolling will result in even better stabilization on an actual ship.

Therefore the model of Fig. 3 - 3 with $R' = \overline{GM}$ is suitable enough to simulate the rolling of ships. However for equation (3 - 10) to represent the rolling of an actual ship, the coefficients must all be scaled to the scale of the model:-

for ship $\quad k^2\ddot{\theta} + \overline{GM}\,\theta = \overline{GM}\,\alpha_o \sin \omega t$

for model $\quad k_1^2\ddot{\theta} + \overline{GM}_1\theta = \overline{GM}_1\,\alpha_o \sin \omega_1 t_1$

θ is the same for both equations and

$$\omega t = \omega_1 t_1$$

for any instant.

∴ The equation of the model can be obtained by multiplying the equation of the ship by the scale σ:-

$$\frac{\overline{GM}_1}{\overline{GM}} = \sigma \quad \text{as mentioned before}$$

and $$\frac{k_1^2 \ddot{\theta}}{k^2 \ddot{\theta}} = \sigma$$

$\ddot{\theta}$ in each case being a function of ω^2 and ω_1^2, respectively.

$$\therefore \quad \left[\frac{\omega_1 k_1}{\omega k}\right]^2 = \sigma$$

But from (3 - 2), $\left(\frac{\omega_1}{\omega}\right)^2 = \frac{1}{\sigma}$

$$\therefore \quad \frac{k_1^2}{k^2} = \sigma^2$$

i.e. $$\frac{k_1}{k} = \sigma$$

Hence for exact simulation of rolling, the dimensions to be scaled are as follows:-

(i) the length and height of waves (X scale)

(ii) the radius of gyration (X scale)

(iii) the metacentric height (X scale)

(iv) the angular velocity (or speed) of the waves (X $\frac{1}{\sqrt{\text{scale}}}$).

At this stage it might be mentioned that although the simplification of the trochoidal characteristic in equation (3 - 9) to give (3 - 10) makes the somewhat elaborate mechanism of the simulator seem unnecessary, this is not so, as only the trochoidal wave profile will give (3 - 4) and hence (3 - 9). Also, the simulator contributes the orbital motion to give the realistic movement of a ship in space which will be necessary to study the forces acting on the liquid in a free surface stabilizer.

Note: For simplicity we will now omit the subscript "1" denoting the model's dimensions as opposed to those of a ship.

3 - 4. Mechanical design of the wave simulator:

Firstly it is necessary to determine the scale to which the apparatus is to be built.

Consider an average wave of height 10 ft. (section 2 - 3). This height is represented by the diameter of the orbit described by the surface plate of the simulator. A practical dimension for this could be 1 ft. i.e. a scale of $^1/_{10}$.

Let us consider this as the scale:-

$$\sigma = {}^1/_{10}$$

Let the range of the mechanism be such as to simulate waves with heights from 8 ft. to 30 ft. and $^h/_L$ from $^1/_{30}$ to $^1/_{20}$.

$^h/_L = {}^1/_{30}$ is the general average value (see section 2 - 3) at which experiments will be carried out.

Then, applying equation (3 - 1 (a)) for $^h/_L = {}^1/_{30}$:

$$r' = \frac{h\sigma}{2} - \frac{\pi d}{30}$$

$$\text{But } \frac{h\sigma}{2} = r$$

$$\therefore \quad r' = r - 0{\cdot}1047d$$

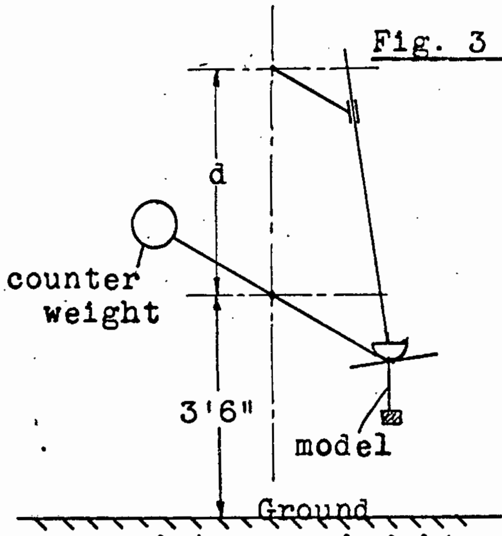
Now $2r$ varies from $0{\cdot}8$ ft. to 3 ft. -

$$\therefore \quad r'_{min} = 0{\cdot}4 - 0{\cdot}1047d$$

$$\text{and} \quad r'_{max} = 1{\cdot}5 - 0{\cdot}1047d$$

The plate carrying arm must have an arm to carry a counterbalance weight on the other side of the axis of rotation (Fig. 3 - 4 overleaf).

Fig. 3 - 4



Therefore d must be large enough to accomodate r'_{max} and the counterbalance arm simultaneously. If this arm is of the same length as the maximum wave height, then:

$$d > 1 \cdot 5 - 0 \cdot 1047d + 1 \cdot 5$$

$$\therefore d > \frac{3 \cdot 0}{1 \cdot 1047} = 2 \cdot 72 \text{ ft.}$$

∴ To give adequate clearance, say:

$$d = 32''$$

Then r' will vary from 1·3" to 14·5".

The complete design of the simulator is shown on the drawings, Fig. 3 - 5.

The drive is such that a variable speed is obtained ranging from very slow to well above the natural frequency of the ship to be tested. A maximum speed of 6 rad/sec for the $^1/_{10}{}^{th}$ scale simulator will be adequate for any realistic tests. It is important that the drive be capable of operating at very low speeds.

The drive used in this apparatus is a $^1/_6$ HP DC motor with a variac as a speed control. To make this possible, the speed is reduced through a 10:1 ratio worm gear box and then further stepped down through a 8:1 ratio belt drive. This allows the motor to run at a reasonable speed at all times preventing it from overloading.

The height of the main shaft of the simulator above the ground is determined as follows:-

Consider the model on the surface plate which is set for a maximum wave height. When the model is in the wave trough, the pendular weight of the model at a maximum of 20" below the wave surface (see section 3 - 5) must clear the ground. (Fig. 3 - 4).

Therefore the minimum height of the shaft above ground is:

$$18 + 20 = 38''$$

For good clearance this height is taken as 3' 6".

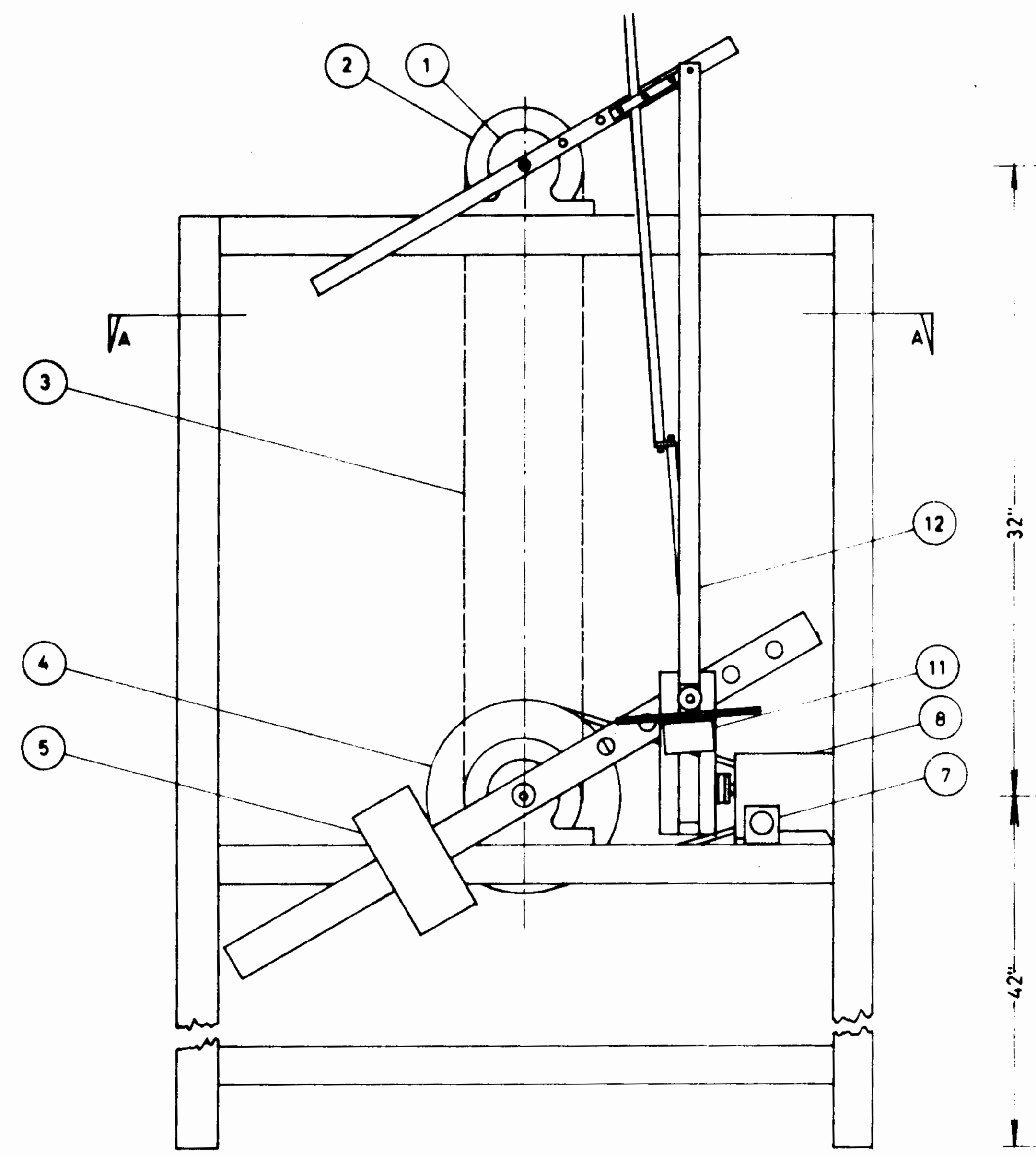
The speed of the mechanism is measured by means of a small dynamo driven directly off the motor whose output is measured on a voltmeter. This equipment is of standard Air Force type so that the calibrated units on the meter have no significance in themselves. However by calibrating this meter against a rev. counter measuring the main shaft speed, it was found that -

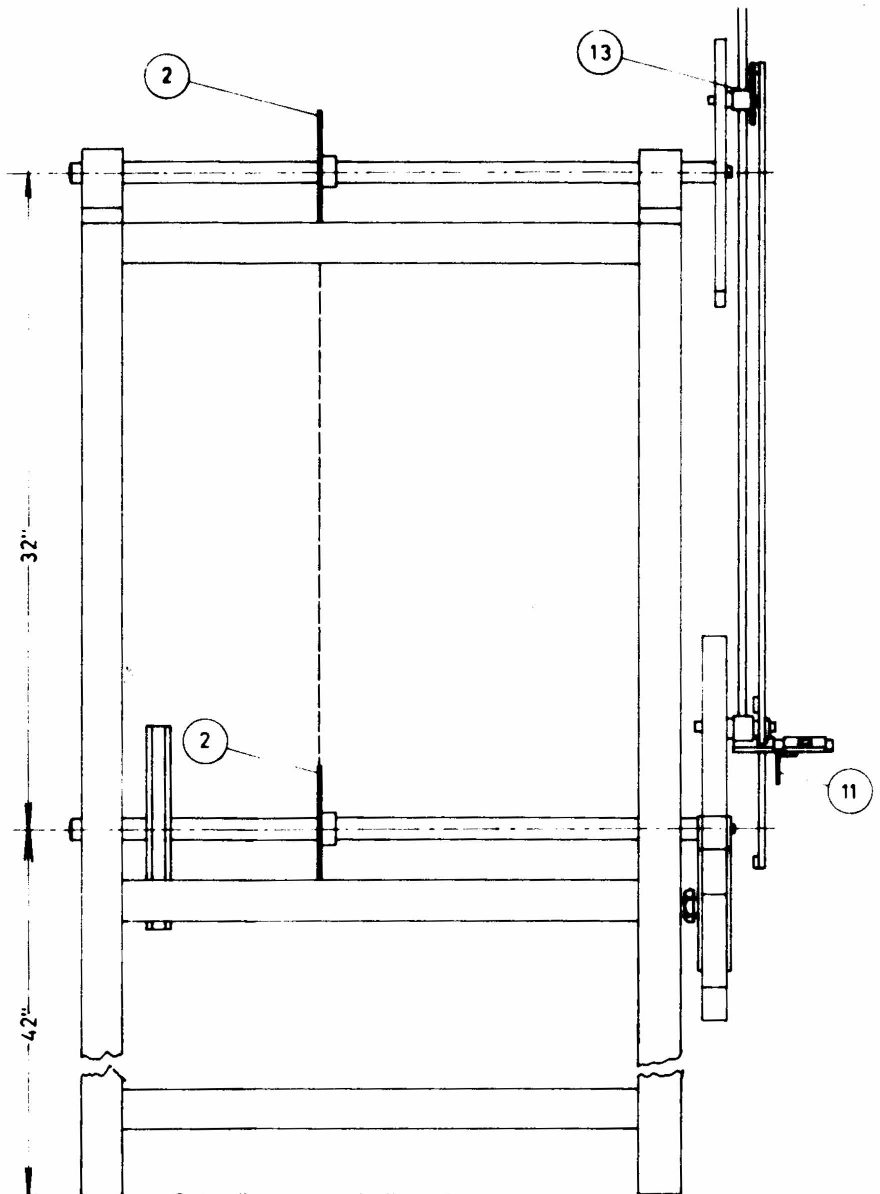
$$\text{Simulator speed} = \frac{\text{meter reading}}{8 \cdot 46} \text{ R.P.M.}$$

For $^1/_{10}{}^{th}$ scale, the speed at which the simulator must be driven is:-

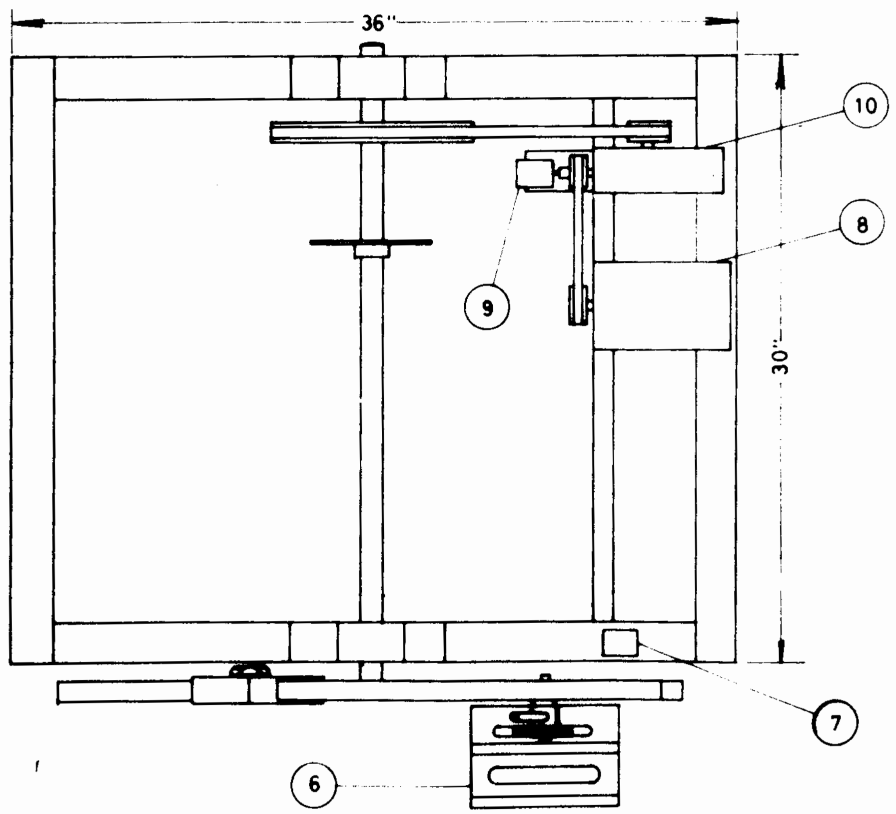
$$\omega_1 = \sqrt{10}\,\omega = 3 \cdot 16\ \omega$$

Finally, it might be mentioned that the overall height of the simulator could be greatly reduced mainly by decreasing the length of the counterbalance arm and hence the height d. Further, the extreme case of a wave of height 30 ft. is unnecessary and the surface plate carrying arm could be shorter.

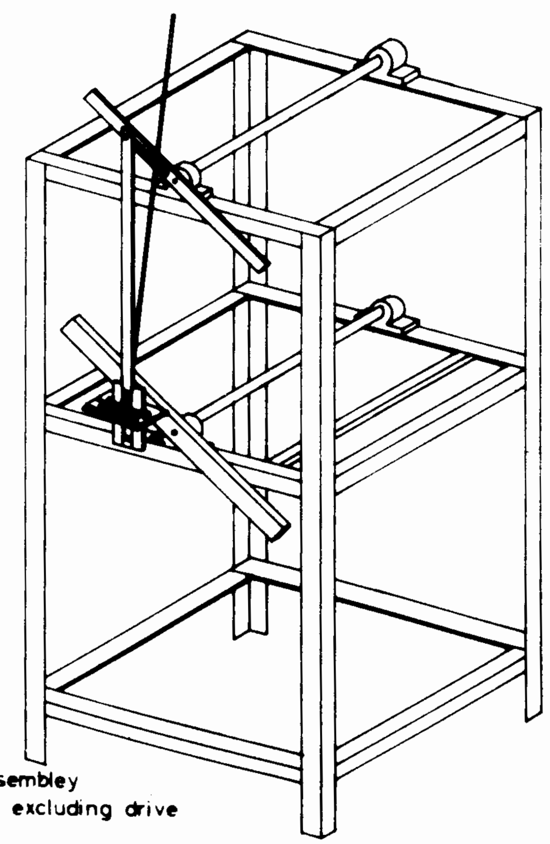




Side View — excluding drive



Plan of Section A-A — showing drive



Sketch of Assembley
excluding drive

1 Plummer Blocks
2 Bicycle Gears
3 Bicycle Chain
4 Driving Pulley
5 Counter weight
6 Surface Plate
7 Turning-speed Meter
8 Motor
9 Speed measuring Dynamo
10 Gear Box
11 Bracket to carry Damper
12 Vertical reference Link
13 Slider

SEA WAVE SIMULATOR

Fig 3-5

3 - 5. The mechanical design of the ship model:

To design a model we must first obviously determine of what the model is to be a replica. In this case the approximate dimensions of the new U.C.T. research vessel, the T.B. Davie, will be taken.

It was shown in section 3 - 3 that the necessary dimensions are the radius of gyration and the metacentric height.

These figures for the "T.B.Davie" are:

Radius of gyration $(k) \doteq 9\frac{1}{2}$ ft.

Metacentric height $(\overline{GM}) \doteq 3$ ft.

Hence according to section 3 - 3, the respective dimensions for the $^1/_{10}{}^{th}$ size model will be:-

$$\overline{GM} \doteq 3{\cdot}6 \text{ in.}$$

$$k \doteq 11{\cdot}5 \text{ in.}$$

In work on stabilizing tanks, the beam of the ship is also of importance since a tank placed across a ship will have a length approximately equal to the beam.

The beam of the "T.B. Davie" is 20 - 21 ft. Allowing for a 2 ft. clearance on either side of the tank, a tank length of approximately $16\frac{1}{2}$ ft. can be assumed.

Then, to $^1/_{10}{}^{th}$ scale, the tank on the model must be:

$$\frac{16{\cdot}5}{10} \text{ ft.} \doteq 20 \text{ inches long.}$$

Henceforth this tank length is denoted as b.

So far we have that the radius of the rollers must be 3·6" (= $\overline{GM}$) and the width [length] of the tank 20". The weight of the model is not fixed but it must be distributed so as to give a radius of gyration for the model of 11·4" about the centre of gravity. Further, it would be of obvious experimental advantage if the weight of the model could be altered without too much difficulty while still retaining the same radius of gyration and same centre of gravity.

This can be done by the use of pendular weights at the top and bottom of the model (Fig. 3 - 6), one of the weights being movable.

Considering Fig. 3 - 6:-

m is the weight of the tank, rollers etc. whose centre of gravity is at distance d from the centre of gravity of the model.

I is the inertia of this section about its c. of g.

m_1 is the pendular weight at the bottom of the model, d_1 from the c. of g. of the model.

m_2 is the top pendular weight at d_2 from the c. of g. of the model.

Now $$m_1 d_1 = md + m_2 d_2 \quad \text{..........(i)}$$
to maintain the same centre of gravity.

and $$m_1 d_1^2 + m_2 d_2^2 + I = 11{\cdot}4^2 \Delta \quad \text{.....(ii)}$$
where Δ is the weight of the model to maintain a constant $k = 11{\cdot}4"$

also $$m_1 + m_2 + m = \Delta \quad \text{..........(iii)}$$

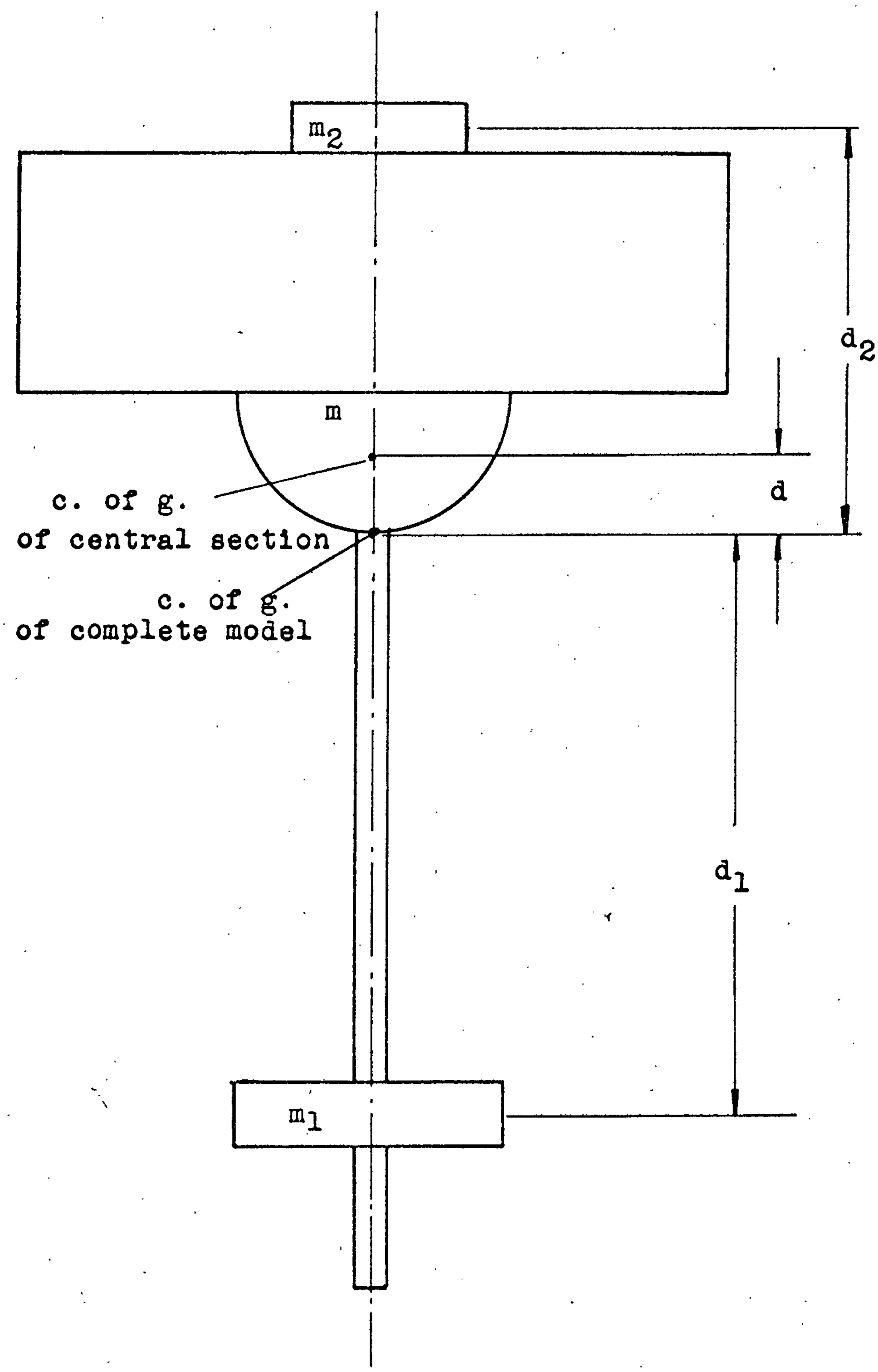


Fig. 3 - 6

By varying m_1, m_2 and d_1, Δ can be varied, while equations (i), (ii) and (iii) remain true. Also m, d, d_2 and I remain fixed.

The weight Δ will be determined by the tank width since the greater the weight, the greater the tank width required for the same stabilization.

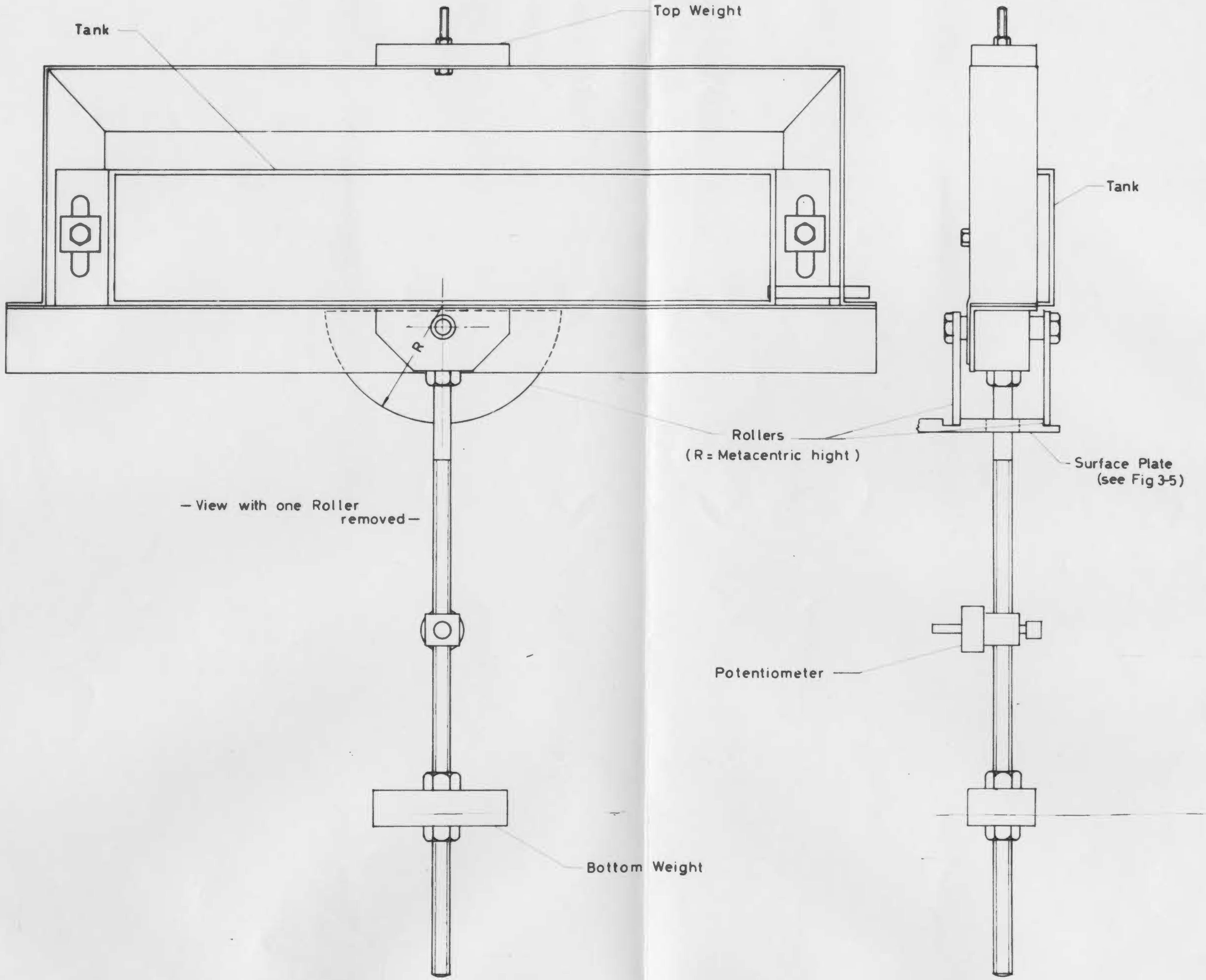
For design convenience we shall work with tank widths ranging from 1" to 2½". The model weights for these tank widths will be determined in section 3 - 6 and can be adjusted on the model merely by adding or removing weights at the top and bottom and resetting the position of the bottom weight.

The length d_1 is a function of Δ which is as yet unknown. However rough calculations show that $d_{1\ max} < 20"$ for the purpose of this experiment.

Hence we have the overall design of the model as shown in Fig. 3 - 7. The potentiometer indicated in this drawing will be discussed later.

3 - 6. Relationship between ship weight and free-surface tank width:

Consider the model on the horizontal surface plate and liquid in the tank to a level h_o. If the model is disturbed from its vertically upright position, it should return to this position when released. However the free surface effect of the liquid collecting on one side may well have a greater moment than the restoring couple, so that the model remains permanently at some angle to the



DYNAMIC SHIP MODEL

Fig. 3-7

vertical. Obviously this condition is not tolerable in a ship and the limiting weight of the ship giving an upright stability position must be determined.

The weight of water in the tank $= \rho b h_o w$ lb.

where:

- b is length of tank
- w is width of tank
- h_o is depth of water when ship is vertical
- ρ is density of liquid

Let us first consider the particular case illustrated in Fig. 3 - 8 where the angle of inclination, θ', is such that the cross-section of the liquid just forms a triangle, i.e. $\tan\theta' = \frac{2h_o}{b}$

Then the moment arm of the liquid is:

$$\frac{b}{2} - \frac{b}{3} + \frac{2}{3} h_o \sin\theta' + S \sin\theta' - \overline{GM}\sin\theta'$$

(for $\cos\theta' \doteq 1$)

where S is the height of tank above c.of g. of the model,

and the moment arm of the c.of g. of the model is:

$$\overline{GM}\sin\theta'$$

$\therefore$ Equating moments:-

$$\Delta\overline{GM}\sin\theta' = \left(\frac{b}{6} + \frac{2}{3} h_o \sin\theta' + S\sin\theta' - \overline{GM}\sin\theta'\right) b h_o w$$

However θ' is small,

$$\therefore \sin\theta' = \theta' = \tan\theta' = \frac{2h_o}{b}$$

$$\therefore \frac{2\Delta\overline{GM}}{b} = \left[\frac{b}{6} + \frac{4h_o^2}{3b} + (S - \overline{GM})\frac{2h_o}{b}\right]\rho b w$$

$$\therefore \quad \Delta = \frac{b^2 w \rho}{12\,\overline{GM}} \left[b + \frac{8h_o^2}{b} + (s - \overline{GM}) \frac{2h_o}{b} \right]$$

but h_o/b is of the order of $1/20^{th}$ as discussed in Chapter 8

$$\therefore \quad b^2 \gg 8h_o^2$$

$$\text{and} \quad b^2 \gg 2(s - \overline{GM})h_o, \text{ particularly if}$$

$s \doteq \overline{GM}$.

$$\therefore \quad \Delta = \frac{\rho b^3 w}{12\,\overline{GM}} \qquad \text{..........(3 - 11)}$$

where the constant '12' is nondimensional.

Applying equation (3 - 11) to the model with water in the tank:-

$$\Delta = 3 \times 10^{-3} \; \frac{b^3 w}{\overline{GM}} \text{ lb.}$$

where b, w and $\overline{GM}$ are in inches.

(ρ for water = 62·5 lb/ft^3)

$\therefore$ For the angle of inclination of the model at rest to be less than θ',

$$\Delta > 3 \times 10^{-3} \; \frac{b^3 w}{\overline{GM}} \qquad \text{..........(3 - 12)}$$

By analysing any condition of stable angle of inclination $\theta < \theta'$ it can be shown that the equation for Δ is identical to equation (3 - 11). i.e. Equation (3 - 11) gives the limiting value of Δ above which the stable position of a ship will always be vertically erect.

At this point it is of interest to note that the limiting weight of the ship as given by equation (3 - 11) is dependent on the width of the tank only and NOT on the depth of liquid in

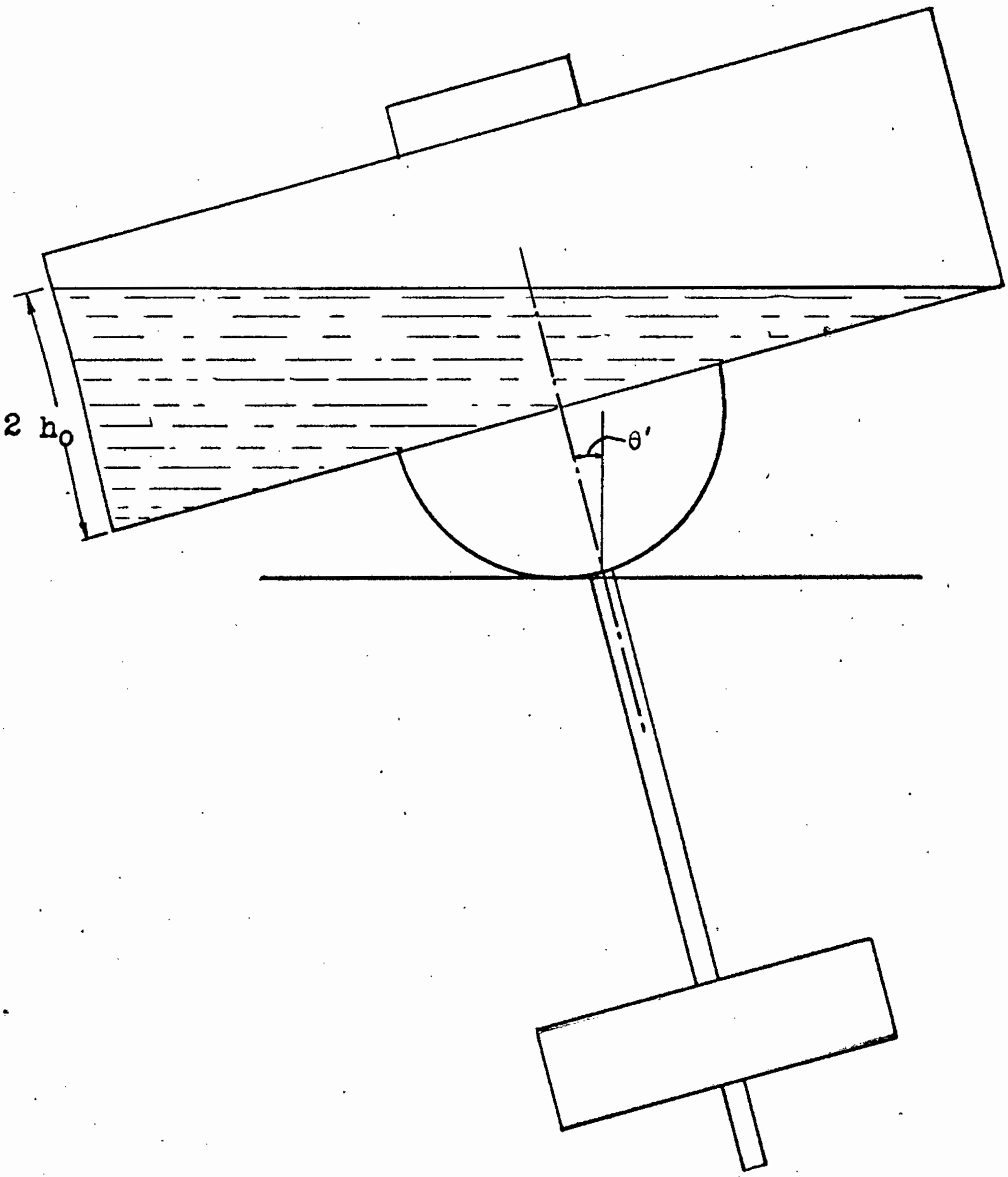


Fig. 3 - 8

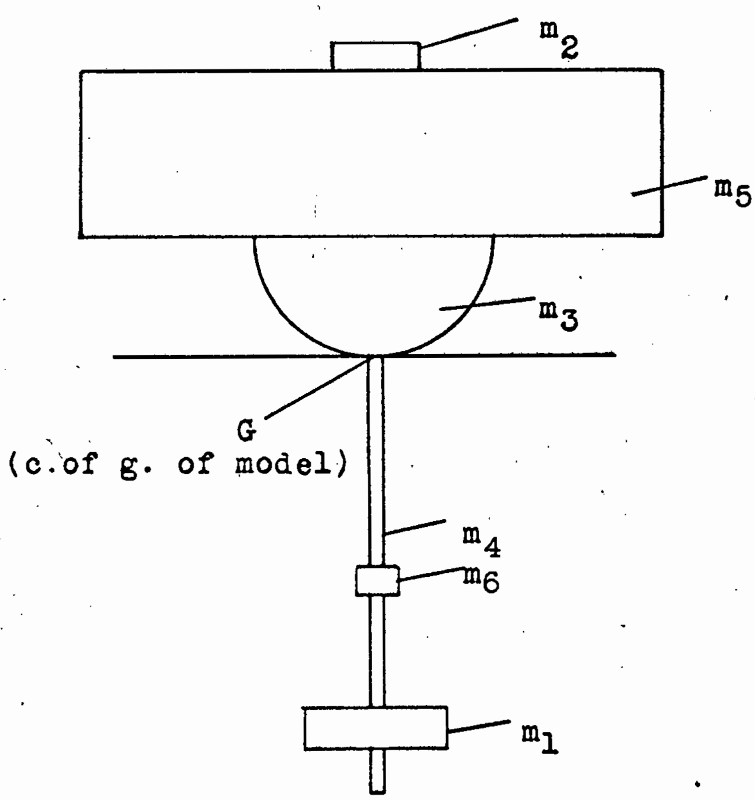


Fig. 3 - 9

the tank.

Returning to the example using water in the model, in equation (3 - 12)

$$\frac{b^3}{\overline{GM}} = \frac{20^3}{3 \cdot 6} = 2220$$

$$\therefore \quad \Delta > 3 \times 10^{-3} \times 2220w \text{ lb.}$$

$$> 6 \cdot 7\, w \text{ lb.}$$

where w is in inches.

Then for the slenderest tank of 1" width -

the weight $\Delta > 6 \cdot 7$ lb.

and for the widest tank of $2\frac{1}{2}$" width -

the weight $\Delta > 17$ lb.

Due to the fact that a small term has been neglected, limiting Δ will be slightly larger than indicated above. To take a safe figure for Δ, the calculated values are increased by a 'safety factor' of say 25%.

Then Δ_{max} for the model is $\hat{=}$ 21 lb.

Δ_{min} for the model is $\hat{=}$ $8\frac{1}{2}$ lb.

Introducing this 25% safety factor in the general equation (3 - 11), we obtain a useful and workable equation for limiting Δ:-

$$\Delta = \frac{b^3 w}{9 \cdot 5\, \overline{GM}} \qquad \text{..........(3 - 13)}$$

It may be mentioned here that if S is negative, i.e. the tank is below the centre of gravity, the previously neglected term involving $(S - \overline{GM})$ will become more significant and the limiting Δ will become smaller. However this change is small, as may be seen by comparing b^2 to $2(S - \overline{GM})h_o$.

3 - 7. Weight distribution on the model:

When weighing the parts of the constructed model, the following results were obtained:-

(the symbols refer to Fig.3 - 9, m being the weight of a part and d the distance of its c.of g. from the c.of g. of the model).

m_1 and m_2 are the bottom and top pendular weights and are to be determined.

$m_3 = 1{\cdot}08$ lb.

$m_4 = 0{\cdot}94$ lb.

$m_5 = 4{\cdot}76$ lb.

$m_6 = 0{\cdot}60$ lb.

d_1 is to be determined

$d_2 = 11{\cdot}25''$

$d_3 = 3{\cdot}00''$

$d_4 = 7{\cdot}00''$

$d_5 = 6{\cdot}75''$

$d_6 = 6{\cdot}00''$

Also, the moment of inertia of each part about G (Fig.3 - 9) was found:

$I_1 = m_1 d_1^2$ lb.in.^{2}

$I_2 = 126{\cdot}5\ m_2$ lb.in.^{2}

$I_3 \simeq 1$ lb.in.^{2}

$I_4 = 46$ lb.in.^{2}

$I_5 = 235$ lb.in.^{2}

$I_6 = 21{\cdot}5$ lb.in.^{2}

The conditions to be fulfilled were discussed in section 3 - 5 and are repeated here for convenience:

(i) Total weight, $m_t = \Delta$ as required.

(ii) The centre of gravity must be at G (Fig. 3 - 9).

(iii) The moment of inertia about G must be $11 \cdot 4^2 \, m_t$.

To satisfy (i):-

$$m_1 + m_2 = m_t - 7 \cdot 38 \quad \text{..........(i)}$$

To satisfy (ii):-

$$m_1 d_1 + 3 \cdot 6 + 6 \cdot 55 = 11 \cdot 25 m_2 + 3 \cdot 24 + 32 \cdot 2$$

$$\therefore m_1 d_1 - 11 \cdot 25 m_2 = 25 \cdot 3 \quad \text{..........(ii)}$$

To satisfy (iii):-

$$m_1 d_1^2 + 126 \cdot 5 m_2 + 303 \cdot 6 = (11 \cdot 4)^2 m_t \quad \text{......(iii)}$$

Substituting (i) in (ii):-

$$m_1 d_1 - 11 \cdot 25 (m_t - m_2 - 7 \cdot 38) = 25 \cdot 3$$

$$\therefore m_1 (d_1 + 11 \cdot 25) = 11 \cdot 25 m_t - 57 \cdot 6 \quad \text{..........(iv)}$$

Substituting (i) in (iii):-

$$126 \cdot 5 (m_t - m_1 - 7 \cdot 38) + d_1^2 m_1 = 130 m_t - 303 \cdot 6$$

$$\therefore m_1 (126 \cdot 5 - d_1^2) = -(3 \cdot 5 m_t + 630) \quad \text{..........(v)}$$

Equating (iv) and (v):-

$$\frac{11 \cdot 25 m_t - 57 \cdot 6}{d_1 + 11 \cdot 25} = \frac{630 + 3 \cdot 5 m_t}{d_1^2 - 126 \cdot 5}$$

Hence:

$$d_1 = \frac{56 - 0 \cdot 31 m_t \pm \sqrt{(56 - 0 \cdot 31 m_t)^2 + 4(130 m_t - 55 \cdot 6)(m_t - 5 \cdot 12)}}{2(m_t - 5 \cdot 12)} \quad \text{.......... (3 - 14a)}$$

Using this result, from (iv):-

$$m_1 = \frac{11 \cdot 25 m_t - 57 \cdot 6}{d_1 + 11 \cdot 25} \quad \text{.......... (3 - 14b)}$$

Then from (i):-

$$m_2 = m_t - m_1 - 7 \cdot 38 \quad \text{.......... (3 - 14c)}$$

By solving equations (3 - 14) for values of

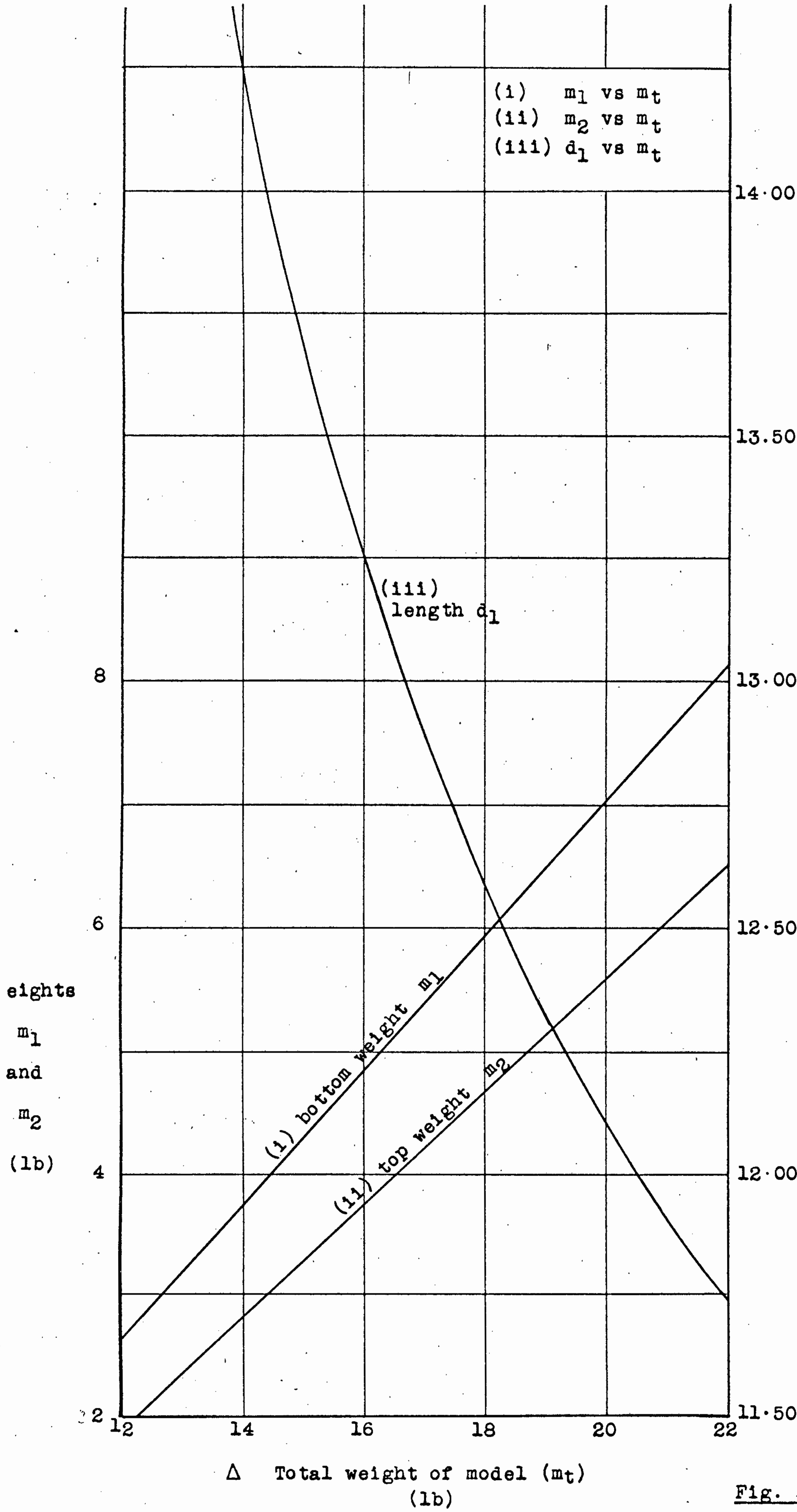

(i) m_1 vs m_t
(ii) m_2 vs m_t
(iii) d_1 vs m_t
(iii) length d_1
(i) bottom weight m_1
(ii) top weight m_2
eights
m_1
and
m_2
(lb)
2
4
6
8
12
14
16
18
20
22
11·50
12·00
12·50
13·00
13.50
14·00
d_1"
Δ Total weight of model (m_t)
(lb)

Fig. 3 - 10

m_t (= Δ) between 8·5 and 21 lb., the necessary weight distribution for the model is determined. This is shown in the graph of Fig.3 - 10. Compliance with these values will always ensure that the conditions laid down above are always obeyed.

3 - 8. Instrumentation:

The essential instrumentation for this apparatus is a method of determining the angle of roll of the model.

Firstly, there must be some reference marker. The link No. 12 in Fig.3 - 5 is arranged to be always vertical and hence is a vertical reference marker.

A slider is fitted in the portion of link No.12 below the surface plate. This slider carries a small radial potentiometer (Fig.3 - 11).

A second identical potentiometer is fixed to the lower part of the model (m_6 in Fig.3 - 9 and shown in Fig.3 - 7).

The two potentiometer wipers are rigidly connected by a small link (Fig 3 - 11).

As the model rolls, the potentiometer settings change, altering the potential difference between the two wipers. This change in potential is recorded and represents the amplitude of roll.

That the potential changes truly represent the angle of roll is shown as follows:-

The potentiometers used are linear; therefore the potential difference between wiper and terminals changes linearly with the angle turned through by

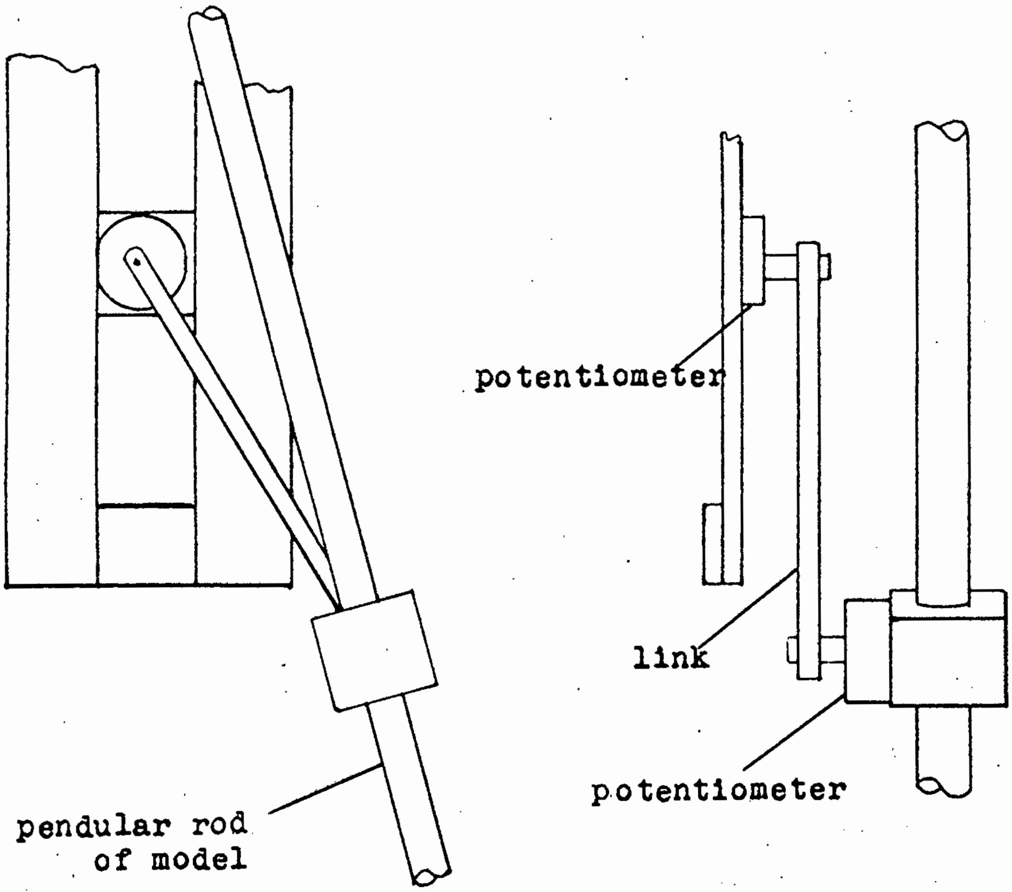


Fig. 3 - 11

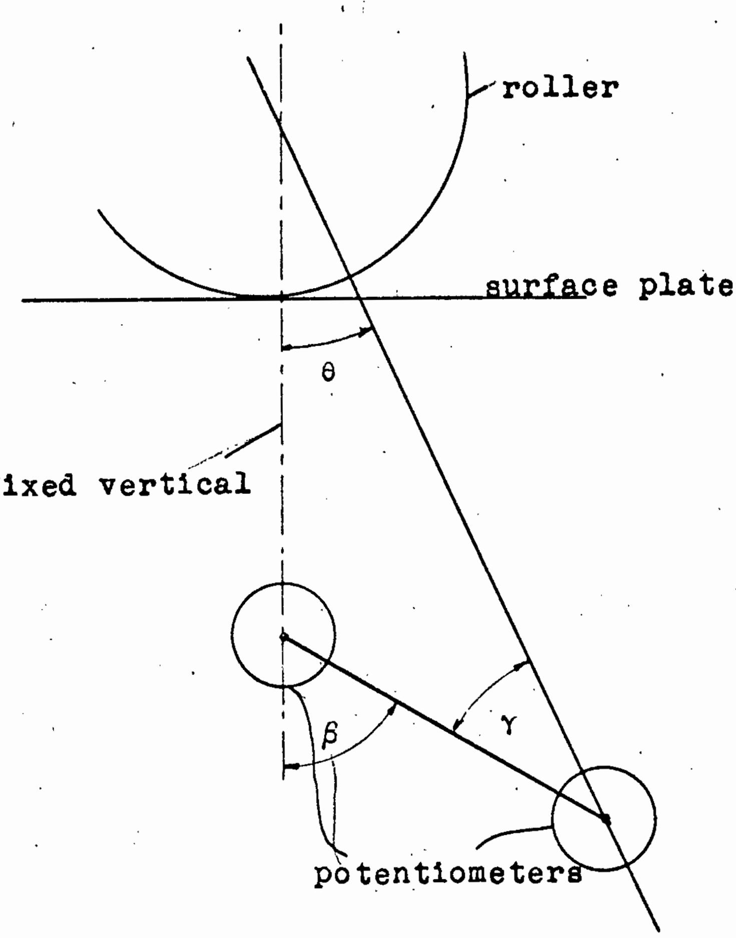


Fig. 3 - 12

the wiper shafts:-

$$\beta \sim \delta V \quad \text{and} \quad \gamma \sim \delta V$$

where β and γ are the angles turned through by the two potentiometers respectively from the vertical position and δV_1 and δV_2 are the changes in potential due to these angles of rotation.

Now consider Fig.3 - 12. By geometry -

$$\beta = \gamma + \theta$$

$$\therefore \quad \theta = \beta - \gamma$$

Hence by connecting the potentiometers so that the difference between potential changes can be read, $\delta V_1 - \delta V_2 = \delta V \sim \theta$

This arrangement allows for any lateral movement of the model without affecting the true reading of the angle of roll.

The complete wiring diagram of the potentiometers and control panel is shown in Fig.3 - 13.

This diagram immediately shows a source of error in the 200 ohm zero-setting resistance. However, this error is small since the maximum addition of 200 ohms to the terminal of the 15 K ohm potentiometer introduces an error of less than 2%. Moreover the usefulness of this resistance as a fine adjustment of the zero (no angle of roll) position greatly exceeds its nuisance value in inaccuracy.

The zero is initially set roughly by fixing the potentiometers in such an angular position that at the erect position there is little or no potential difference between their wiper terminals.

It is important to note that the potentiometers must be wired so that the <u>difference</u> and not the

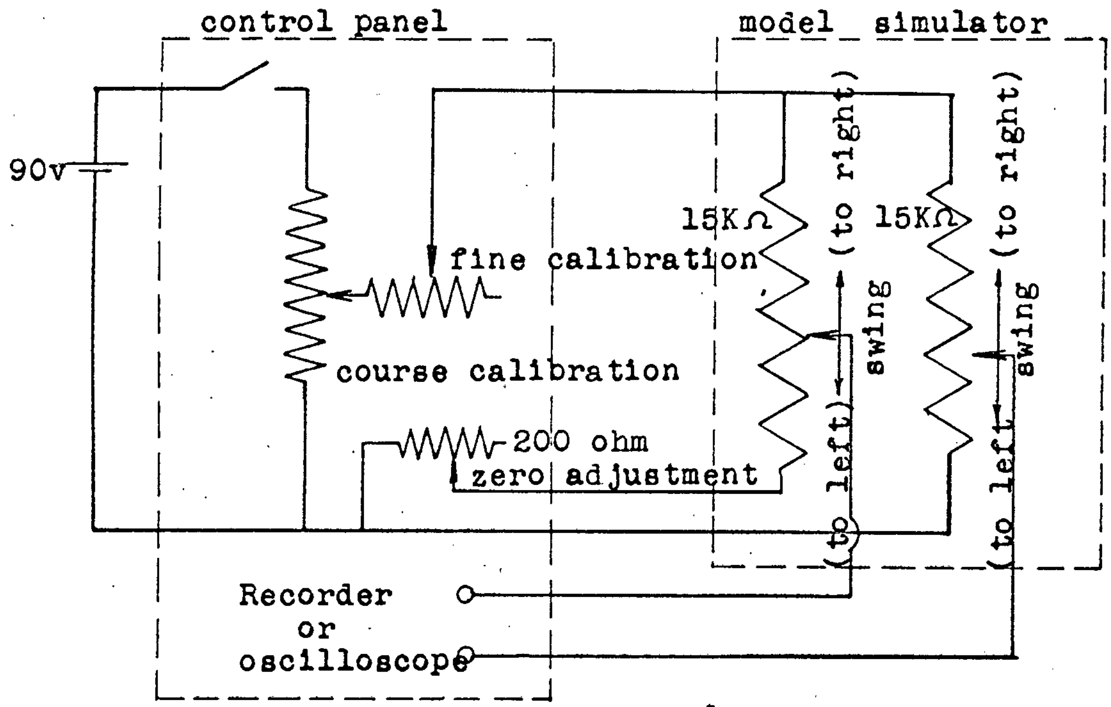


Fig. 3 - 13

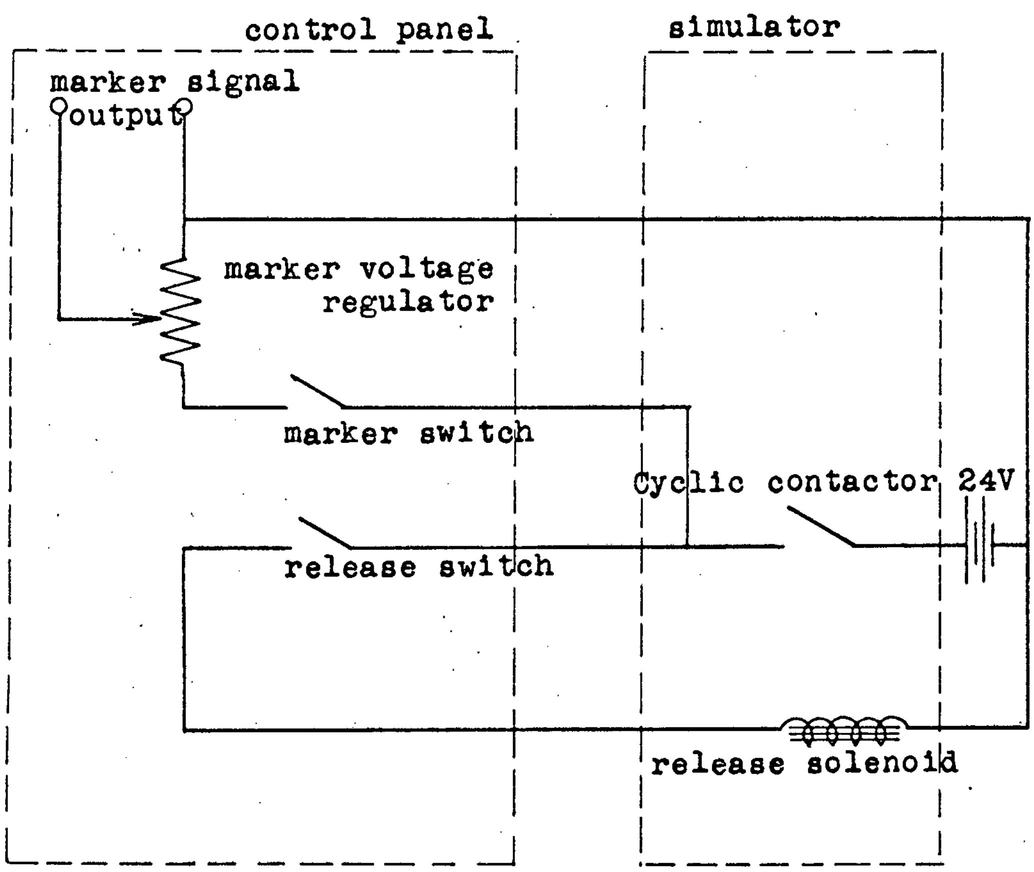


Fig. 3 - 14

sum of the potential changes occurs between the wiper terminals.

The output signal which is the potential difference between the wiper terminals is recorded while the model is in motion and the amplitude of this signal represents the angle of roll.

Less important instrumentation includes an event marker and a release mechanism. These two are supplied by the same 24 volt batteries. The wiring diagram is shown in Fig.3 - 14.

The release mechanism consists of an electro-magnet fixed to the vertical marker link which withdraws a locking pin from the model on receiving a signal. This is particularly useful at higher frequency tests when the system must be taken through resonance first.

The signal for release is only transmitted at one specific instant in every cycle, the same instant at which the event marker is excited. The bottom of the wave trough was selected as this instant of release.

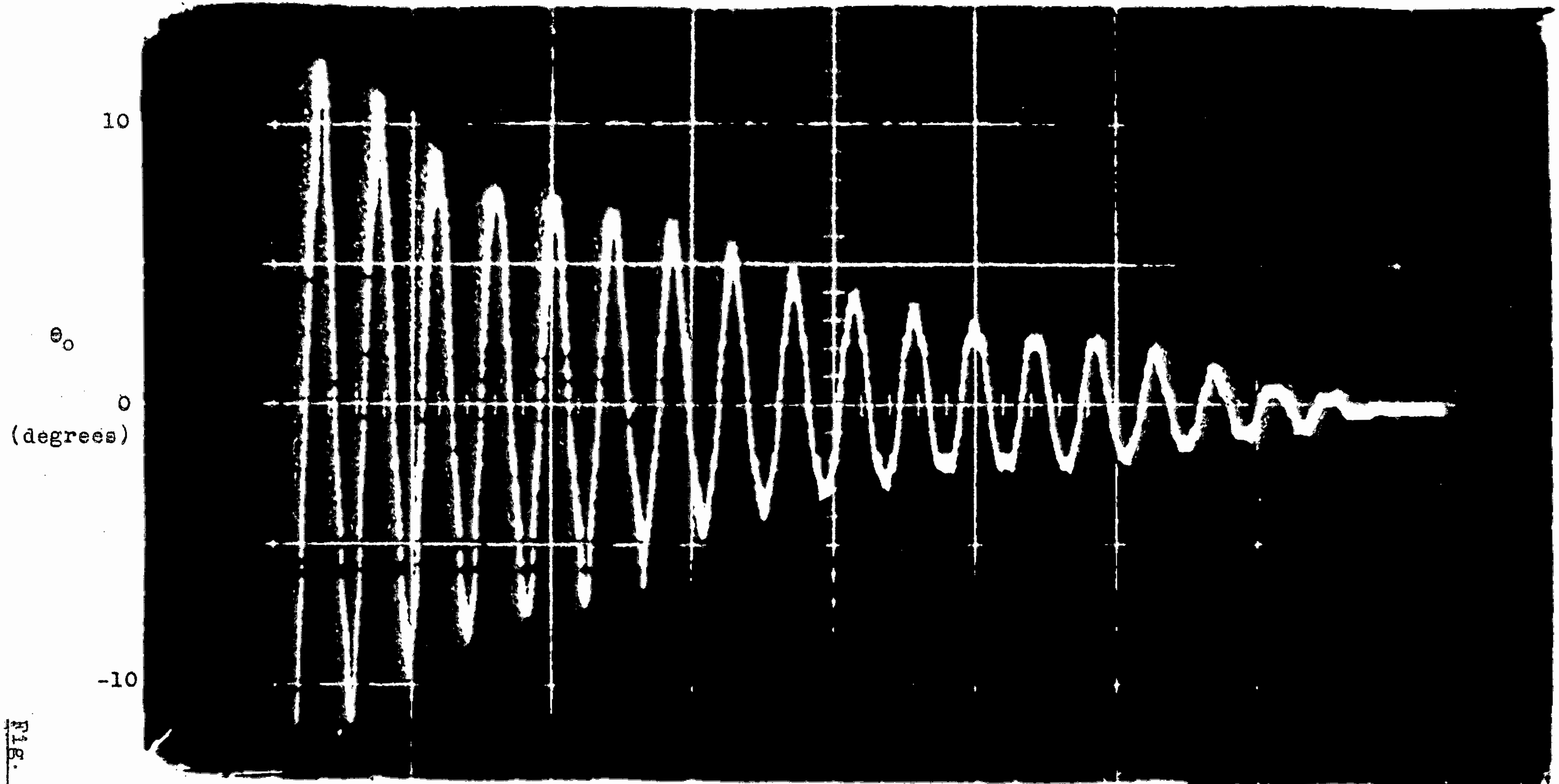
These two refinements are not essential.

3 - 9. Damping:

The simulation of rolling would not be complete without the introduction of some damping to the rolling of the model.

In section 2 - 9 it was shown that the damping required must be viscous or nearly so. This is best supplied by a simple oil damper.

However, experiments with the model as constructed



Curve of roll decay of the model in 'still water'.

Fig. 3 - 15.

show that the potentiometers and, to a lesser extent, the rollers provide a substantial amount of damping as shown in the curve of decay of the model Fig.3 - 15. The curve shows that the decay tends to be exponential rather than linear, i.e. it is more or less viscous.

Fig.3 - 15 is obtained for the model weighing 22 lb. From the curve it can be determined that the effective damping constant for this case is approximately 0·58 lb.in.sec. Is this damping adequate?

The amount of damping actually required is given by the equation

The damping constant $C_1 = \dfrac{(a + b\theta)\, I\, \omega_n}{\pi}$

where a and b are constants as indicated in chapter 2, and ω_n, the natural frequency $= \sqrt{\dfrac{\overline{GM}}{gk^2}}$

That the constants a and b are the same for both the actual ship and the model can be shown as follows -

Froude's equation of damped motion may be rewritten as:

$$I\ddot{\theta} + \frac{(a + b\theta)I}{\pi}\sqrt{\frac{\overline{GM}}{gk^2}}\,\dot{\theta} + \Delta\,\overline{GM}\,\theta = \Delta\,\overline{GM}\,\alpha_o \sin\phi$$

$$\therefore\; k^2\ddot{\theta} + \frac{(a + b\theta)k}{\pi}\sqrt{\frac{\overline{GM}}{g}}\,\dot{\theta} + \overline{GM}\,\theta = \overline{GM}\,\alpha_o \sin\phi$$

For a scale σ,

$$\sigma = \frac{(\overline{GM})_m}{(\overline{GM})_s} = \frac{\left[(a + b\theta)k\sqrt{\overline{GM}}\,\dot{\theta}\right]_m}{\left[(a + b\theta)k\sqrt{\overline{GM}}\,\dot{\theta}\right]_s}$$

where subscripts m and s stand for model and ship respectively.

Now $\dot{\theta}$ is a function of ω

$$\therefore \quad \sigma = \frac{\left[(a + b\theta)k\sqrt{GM}\,\omega\right]_m}{\left[(a + b\theta)k\sqrt{GM}\,\omega\right]_s}$$

From section 3 - 3 we know that:-

$$\frac{(\sqrt{GM})_m}{(\sqrt{GM})_s} = \sqrt{\sigma}, \quad \frac{(k)_m}{(k)_s} = \sigma \quad \text{and} \quad \frac{(\omega)_m}{(\omega)_s} = \frac{1}{\sqrt{\sigma}}$$

$$\therefore \quad \sigma = \frac{(a + b\theta)_m}{(a + b\theta)_s} \cdot \frac{\sqrt{\sigma}}{\sqrt{\sigma}}\,\sigma$$

$$\therefore \quad \frac{(a + b\theta)_m}{(a + b\theta)_s} = 1$$

Since the angle of roll θ is the same for ship and model, the number $(a + b\theta)$ remains the same whatever the size of the model.

Then, introducing values, the damping constant for the model becomes:-

$$C_1 = \frac{\Delta(a + b\theta)\ 11\cdot4\sqrt{3\cdot6}}{\pi\,.\sqrt{386}}$$

$$= 0\cdot35\,\Delta\,(a + b\theta)$$

The values of a and b for the "T.B.Davie" can only be guessed. A somewhat similar type of ship with bilge keels gave:

$$a = 0\cdot035, \quad b = 0\cdot050$$

As this is the most similar ship to be found in the available literature, these values will be assumed.

As will be seen later, the exact value for damping is unimportant in this work, since its effect is almost negligible compared to the damping due to free-surface tanks.

Assume the mean angle of roll, $\theta = 10^{\circ}$.

Then $C_1 = 0{\cdot}35\,\Delta\,(0{\cdot}035 + (0{\cdot}174 \times 0{\cdot}05))$

$= 1{\cdot}53 \times 10^{-2}\,\Delta$

In section 3 - 6, Δ was determined as varying from 9 lb. to 21 lb.

$\therefore$ C_1 varies from 0·138 to 0·322 lb.in.sec.

Considering that the damping moment is very small compared to free-surface tank effect which is being studied and also that it is very small compared to the inertia and restoring moments, a fairly large error in C will virtually not affect the results of the experiments to be conducted.

Therefore, if we accept the damping in the mechanism as being adequate, we obtain the still water decay of Fig.3 - 15 and we are as close to the true rolling of a ship as we are likely to get.

This assumption greatly simplifies matters and makes a damper unnecessary; however, in order to give the apparatus as broad an application as possible, a simple oil damper was constructed - Fig.3 - 16. (see next page).

Due to the broad tolerances of the design so far, the 'gaps' on either side of the paddle in the oil tank should be as large as possible.

To give damping constants of the order of those encountered in ships, the gaps were calculated to be 25 thou for an immersed paddle area ranging from 2 in.^{2} to 6 in.^{2}, the viscous fluid being standard gearbox oil (2,500 centistokes at 60°F). In subsequent experiments this damper was found to act quite satisfactorily besides causing a

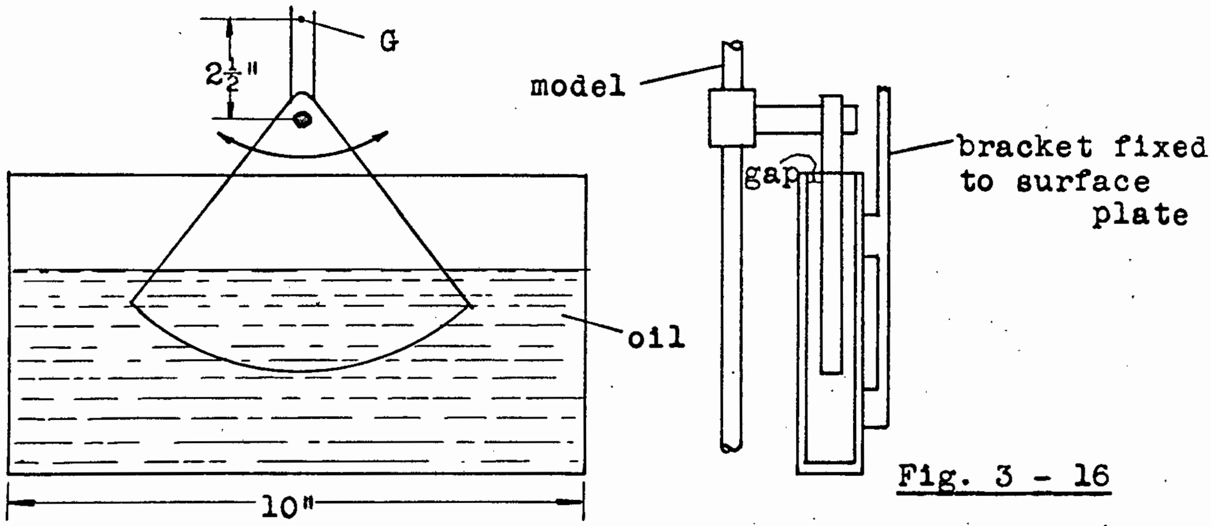


Fig. 3 - 16

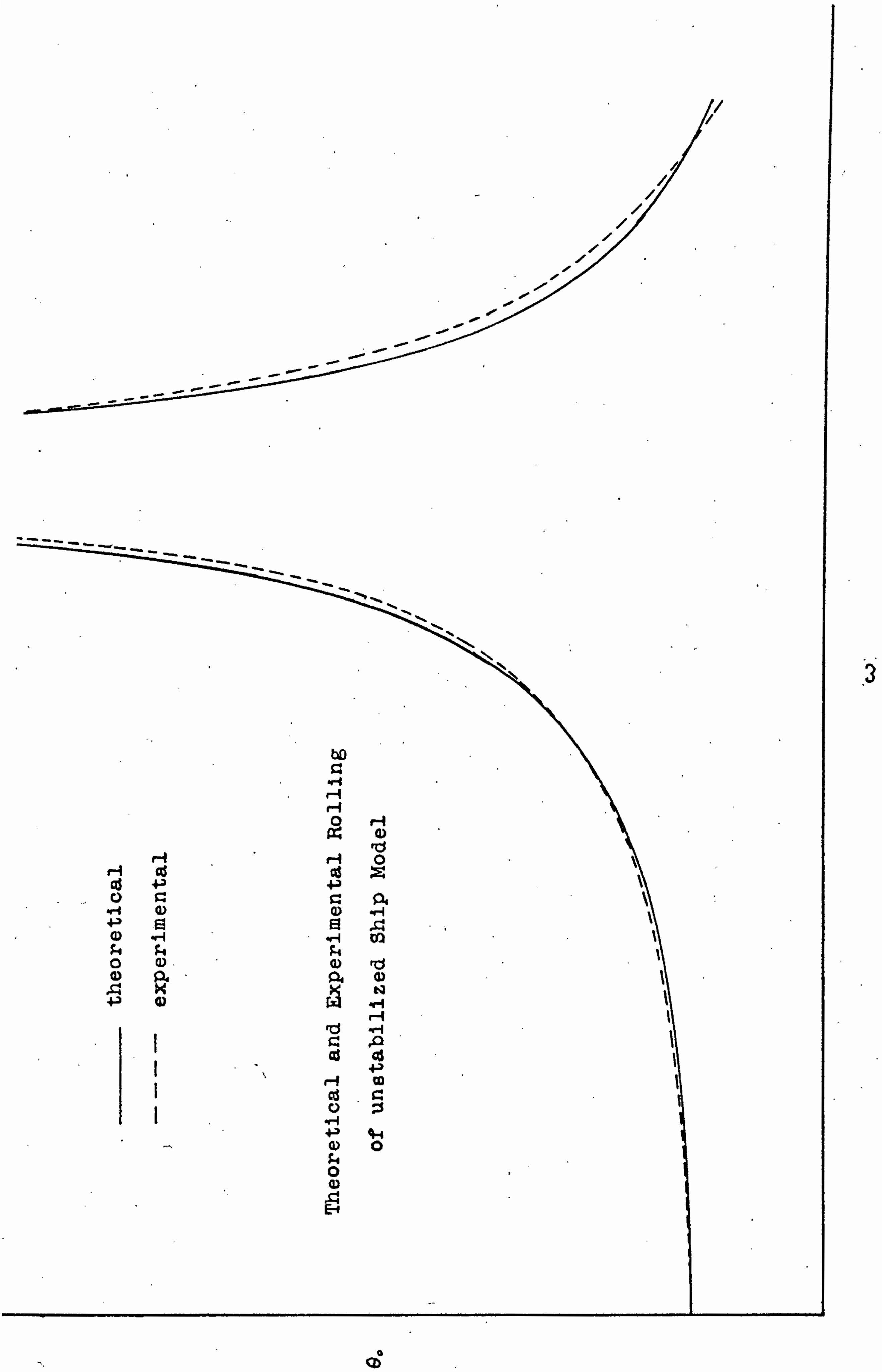
good deal of oil spillage. However for the free surface tank tests discussed here, the damper had a negligible effect and was dispensed with.

3 - 10. Summary:

(i) A simple sea-wave simulator and appropriate rolling ship model are designed to obey Froude's equation of damped rolling motion exactly. Fig.3 - 17 shows curves of angle of roll against period of wave (a) obtained by calculation from Froude's equation and (b) obtained practically by means of the simulator and model.

(ii) The simulation of Krilov's more complete equation of rolling motion cannot readily be performed mechanically.

(iii) The measurement of the angle of roll is done electrically by means of rotation sensing potentiometers giving an output signal which is either recorded by a pen recorder or else a storage oscilloscope.



Theoretical and Experimental Rolling of unstabilized Ship Model

Fig. 3 - 17

4 - THE THEORETICAL MOVEMENT OF SHALLOW LIQUID IN OSCILLATING RECTANGULAR TANKS

Introduction - Shallow liquid in a horizontally oscillating tank - Shallow liquid in a tank on a rolling ship.

4 - 1. Introduction:

The principal purpose of this chapter is to emphasize the complexity of the movement of shallow liquids in an oscillating tank such as a free surface tank on a ship.

The motion of liquid in a tank is to some extent predictable (section 4 - 2); however a tank with angular oscillations gives rise to very intricate two dimensional flow problems. The combination of these two effects as in the case of a liquid in a tank mounted on a rolling ship requires a hydrodynamic analysis which is well beyond the scope of this work and should be tackled preferably by a mathematician.

To bypass this difficulty an analysis based partly on assumptions obtained from physical observations is attempted in Chapter 5.

4 - 2. Shallow liquid in a horizontally oscillating tank:

The following discussion attempts to show the mode of oscillation of a liquid in a horizontally moving tank and how this motion is generated.

Assume purely horizontal tank oscillations.

Consider the case where the tank begins to move cosinusoidally from rest, the liquid being initially completely still. The effect of the tank motion of the still liquid is identical to that obtained by a paddle advancing into the liquid on one end of the tank and another paddle moving away at the other end (Fig.4 - 1). At any instant the liquid particles in contact with a moving tank wall will be given the velocity of the wall at that instant. This causes a disturbance in the liquid surface which becomes a bore (moving hydraulic jump) of small height. The height of the bore depends on the change of velocity of the tank wall during some lapse of time.

In Fig.4 - 1 consider planes a and b. The plane b will have been given a greater velocity than the plane a, since the tank movement is cosinusoidal and the speed is increasing.

Therefore the surface velocity of the segment between a and b must be greater than that ahead of a. This can only be if the liquid depth between a and b is greater than that ahead of a, since the surface velocity $C = \sqrt{2gh}$, where h is the depth and $C_b > C_a$

$$\therefore \; h_b > h_a$$

Considering the 'compression' end only (left hand end in Fig.4 - 1), the wall is accelerating or increasing its speed over the first $\frac{1}{4}$ cycle; therefore during this time the consecutive planes will have surface disturbances of increasing velocity, i.e. $\dot{x}_1 < \dot{x}_2 < \dot{x}_3$ etc.

The sections 1 to 5 were all generated with the same width δx; however, since the following planes

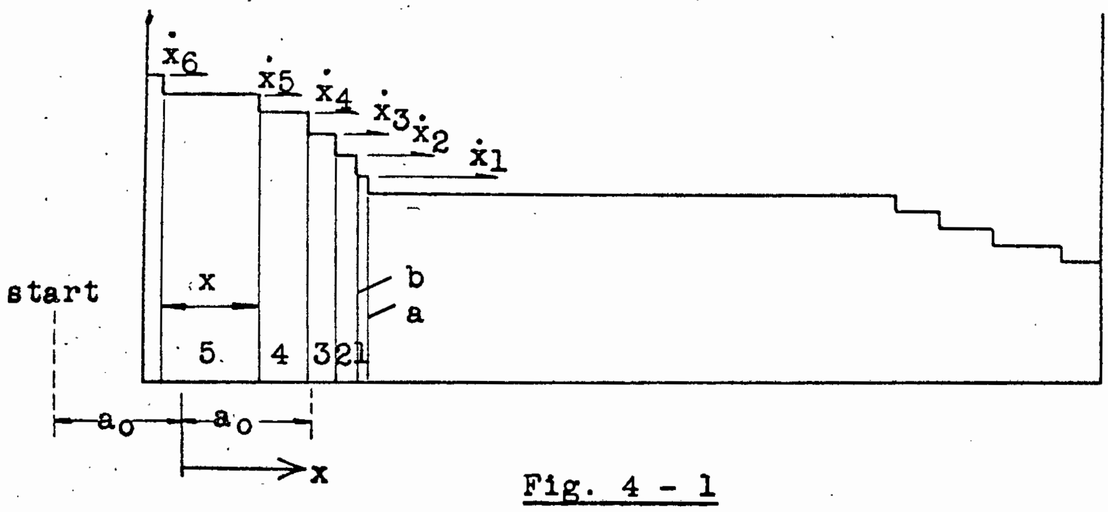


Fig. 4 - 1

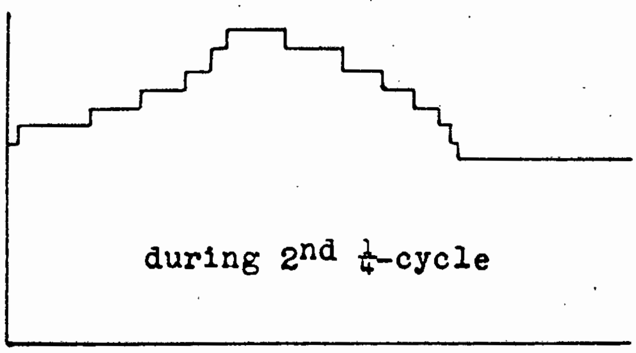
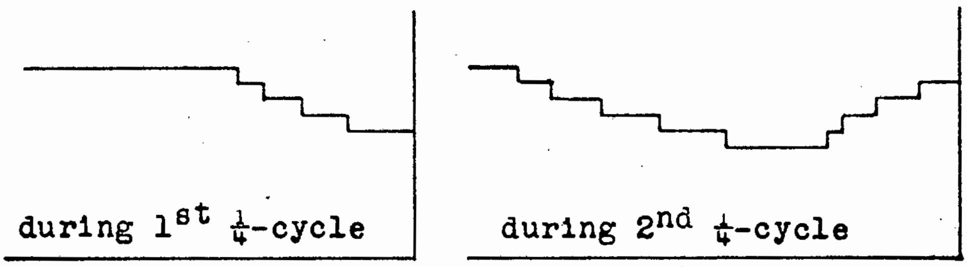
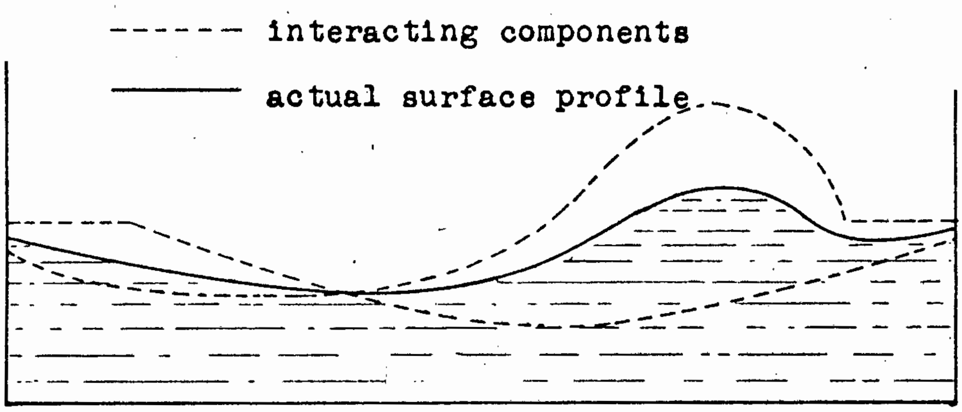


Fig. 4 - 2



(a) (b)

Fig. 4 - 3



Effect of interaction

Fig. 4 - 4

are of increasing speed, the sections become narrower with time.

When $\dot{x}_{max}$ is reached at $x = 0$, the pattern changes slightly since the increments following are now s l o w e r than $\dot{x}_{max}$ and hence a depression occurs (Fig.4 - 2).

The 'rarefaction' end of the tank (right hand end, Fig.4 - 1) acts in the same way as the compression side with the exception that now the depression occurs in the first $\frac{1}{4}$ cycle and a build up or 'compression' in the second $\frac{1}{4}$ cycle (Fig.4 - 3) — exactly the reverse of the compression side.

After approximately a $\frac{1}{4}$ cycle, the disturbances from either end of the tank meet somewhere near the centre of the tank and interaction begins taking place.

The result of the interaction, as shown in Fig.4 - 4, can be determined by superposition.

Up to this point the equation of the surface profile can be found. However after sufficient time some of the sections 1 to 5 in Fig.4 - 1 will catch up the one ahead so that the wave front will become steeper and steeper, finally changing from a weak shock front to a discontinuous shock wave or, in hydraulic terms, a fully developed bore.

If the shock front reaches the end of the tank before the bore is fully developed, a reflection takes place which will follow the pattern of superposition and thence continue till the 'steep shock' appears. The reflection of a fully developed bore will follow the same law as a shock wave reflection in a gas and can also be solved by using the principle of superposition.

In the 3rd and 4th ¼ cycles the sinusoidal motion will superimpose another wave onto the reflecting waves so that the resulting interaction will depend largely on the difference between the frequency of the tank and the natural speed of the wave. The latter is directly a function of the liquid depth.

To show the mathematical complexity of this b a s i c movement, the equation describing the fluid surface profile of the 'compression' wave during the first ¼ cycle is derived below:-

Fig.4 - 5 shows a tank with liquid to a still surface depth h_o. The tank is oscillating in a linear horizontal direction.

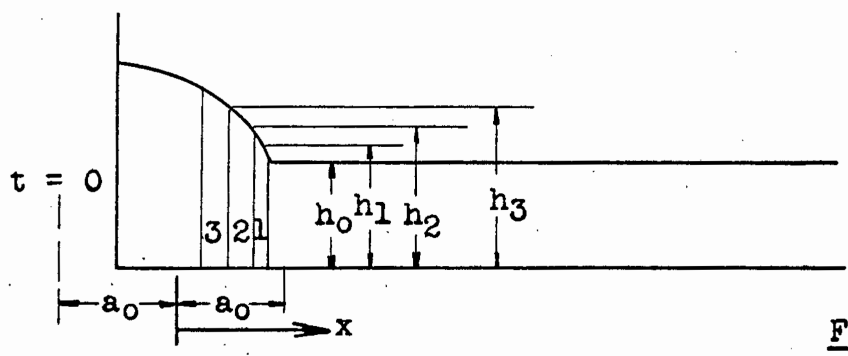


Fig. 4 - 5

$$a = a_o \cos \omega t$$

where ω is the angular velocity

t is the time from rest

a_o is the amplitude of oscillation

The speed of a surface disturbance on a fluid of depth h is given by

$$\dot{x} = \sqrt{gh} \qquad \text{..........(4 - 1)}$$

For the first section, element 1, the surface speed is

$$\dot{x}_1 = \sqrt{gh_o}$$

where h_o is the still water depth.

The velocity w_1 of the particles of fluid vertically below this surface is given as

$$w_1 = (1 - {}^{h_o}/_{h_1})\sqrt{gh_1}$$

w_o being equal to zero.

Writing this more generally, the particle velocity of the n^{th} element relative to that of the element immediately ahead is:-

$$= \left[1 - \frac{h_{n-1}}{h_n}\right]\sqrt{gh_n}$$

where h_n is the depth of the element and

h_{n-1} is the depth of the element immediately ahead.

Therefore the absolute velocity of the n^{th} element is the sum of the particle velocities of all the elements ahead of it -

$$w_n = w_{n-1} + w_{n-2} \cdots\cdots + w_2 + w_1 + w_o$$

$$\therefore \quad w_n = \sum_{n=o}^{n}\left[1 - \frac{h_{n-1}}{h_n}\right]\sqrt{gh_n}$$

$$= \sum_{n=o}^{n}\sqrt{g}\left[\frac{h_n - h_{n-1}}{\sqrt{h_n}}\right]$$

Now $h_n - h_{n-1}$ is a small change in depth = h

$$\therefore \quad w_n = \sqrt{g}\sum_{n=o}^{n}\frac{\delta h}{\sqrt{h_n}}$$

$$\doteqdot \sqrt{g}\int_{h_o}^{h_n}\frac{dh}{\sqrt{h}}$$

$$= 2(\sqrt{gh_n} - \sqrt{gh_o}) \qquad \cdots\cdots\cdots\cdots(4 - 2)$$

Now the surface velocity of the n^{th} element is equal to $\sqrt{gh_n}$ r e l a t i v e to the liquid into which it is advancing.

Therefore the absolute velocity of the surface is

$$\dot{x}_n = \sqrt{gh_{n-1}} + w_{n-1}$$

where w_{n-1} is the particle velocity of the $(n-1)^{th}$ element and h_{n-1} is the depth into which the surface element is moving.

From equation (4 - 2),

$$w_{n-1} = 2(\sqrt{gh_{n-1}} - \sqrt{gh_o})$$

$$\therefore \quad \dot{x}_n = \sqrt{gh_{n-1}} + 2\sqrt{gh_{n-1}} - 2\sqrt{gh_o}$$

$$\text{i.e.} \quad \dot{x}_{n+1} = 3\sqrt{gh_n} - 2\sqrt{gh_o} \qquad \text{..........}(4 - 3)$$

Therefore the absolute distance travelled by the bore between the n^{th} and the $(n+1)^{th}$ element in time t' is:

$$A = (3\sqrt{gh_n} - 2\sqrt{gh_o})t'$$

The position at which this bore was generated by the tank wall at time t was $a = a_o \cos \omega t$. At this position the velocity of the tank wall was $a_o \omega \sin \omega t$ and this is the speed imparted to the liquid particles of the element in contact with the wall i.e. the n^{th} element.

$$\therefore \quad a_o \omega \sin \omega t = w_n$$

$$\therefore \quad a_o \omega \sin \omega t = 2(\sqrt{gh_n} - \sqrt{gh_o})$$

$$\therefore \quad t = \frac{1}{\omega} \text{arc sin}\left[\frac{2\sqrt{g}}{a_o\omega}(\sqrt{h} - \sqrt{h_o})\right]$$

i.e. t is the time until generation of the n^{th} element and t' is the time from that instant.

Therefore the overall time elapse is:

$$t_o = t + t'$$

and the distance of the n^{th} element from the centre line of oscillation (a = o) after time t_o is:

$$x = A - a_o \cos \omega t$$

$$= (3\sqrt{gh_n} - 2\sqrt{gh_o})(t_o - t) - a_o \cos \omega t \qquad \text{..........(4 - 4)}$$

However,

$$\sin \omega t = \frac{2\sqrt{g}}{a_o \omega}(\sqrt{h_n} - \sqrt{h_o})$$

$$\therefore \quad \cos \omega t = \sqrt{1 - \frac{2\sqrt{g}}{a_o \omega}(\sqrt{h_n} - \sqrt{h_o})^2}$$

Then substituting for t and cos ωt in equation (4 - 4) we obtain the equation of the surface profile of the 'compression' disturbance before any interaction has taken place.

The equation relates the coordinates (horizontal and vertical) x and h. Since the coordinate h referred to has the subscript n throughout, this subscript can be left out (h_o is a constant):

$$x = (3\sqrt{gh} - 2\sqrt{gh_o})\left[t_o - \frac{\text{arc sin}}{\omega}\left\{\frac{2\sqrt{g}}{a_o \omega}(\sqrt{h} - \sqrt{h_o})\right]\right.$$

$$- a_o\sqrt{1 - \frac{2\sqrt{g}}{a_o \omega}(\sqrt{h} - \sqrt{h_o})^2} \qquad \text{..........(4 - 5)}$$

A similar equation may be found for the other end of the tank and the sum of the h's in the two equations give the depth after interaction of the two disturbances has taken place. However h cannot easily be taken out of the equations and hence the solution during interaction cannot be simply written down.

After a certain time, reflection of the disturbances takes place and further complicates the issue.

4 - 3. Shallow liquid in a tank on a rolling ship:

For a tank on a rolling ship, the oscillation is far from purely horizontal; in fact the angular motion contributes more to the movement of the liquid than the horizontal displacement (assuming the tank not at the centre of rolling). This was found experimentally for a tank mounted well above the centre of rotation (centre of gravity).

It seems at first sight that this problem should be solved by Bernoulli's equation based on the assumption that the pressure on all surface particles is the same. However Bernoulli's equation requires that there be no energy loss in the system and this is incompatible with the present problem, since the concept of free surface stabilization is by means of a moving bore which constitutes an energy loss. Other highly mathematical methods for a solution exist but are neither essential nor within the scope of this work. The problem is to find some solution for the damping effect of free surface tanks and this can be done with good accuracy by using experimentally determined results.

5 - EQUATION OF ROLLING WITH FREE-SURFACE TANK STABILIZATION

Introduction - Observations and Assumptions - - Moments acting on ship due to moving liquid - - The equation of motion of ship and tank - The solution of the equation of motion of ship and tank - Some ommissions and corrections - A possible value for M - Possible values of ϵ - Summary.

5 - 1. Introduction:

We have seen in the previous chapter that a theoretical analysis of liquid behaviours in an oscillating tank will not be attempted because of the mathematical complexity. A practical and logical solution can presumably be obtained by observing the behaviour of the liquid and working on the results as basic assumptions. On introducing these into the equation of rolling, an equation of motion which is not too difficult to solve and whose terms have some physical significance should be obtainable.

To ascertain the accuracy of these assumptions the solution of the equation will be compared in a later chapter to experimentally determined results of ship rolling.

5 - 2. Observations and assumptions:

The apparatus described in Chapter 3 was run to observe the bahaviour of shallow water in the free-surface tank. The following results were obtained:-

(i) The movement of shallow water quickly develops into a bore with a steep wave front and it keeps moving as such (Fig.5 - 1). The exception to this occurs at very low frequencies.

(ii) On reaching the tank wall, the bore reflects to repeat an identical half cycle in the opposite direction.

(iii) The speed of the bore is approximately constant and such that it has the same period as the tank.

(iv) The height of the bore varies with -

a). the angle of roll

b) the frequency

c) the depth of water in the tank.

(v) The phase angle between bore and tank varies with -

a) the frequency

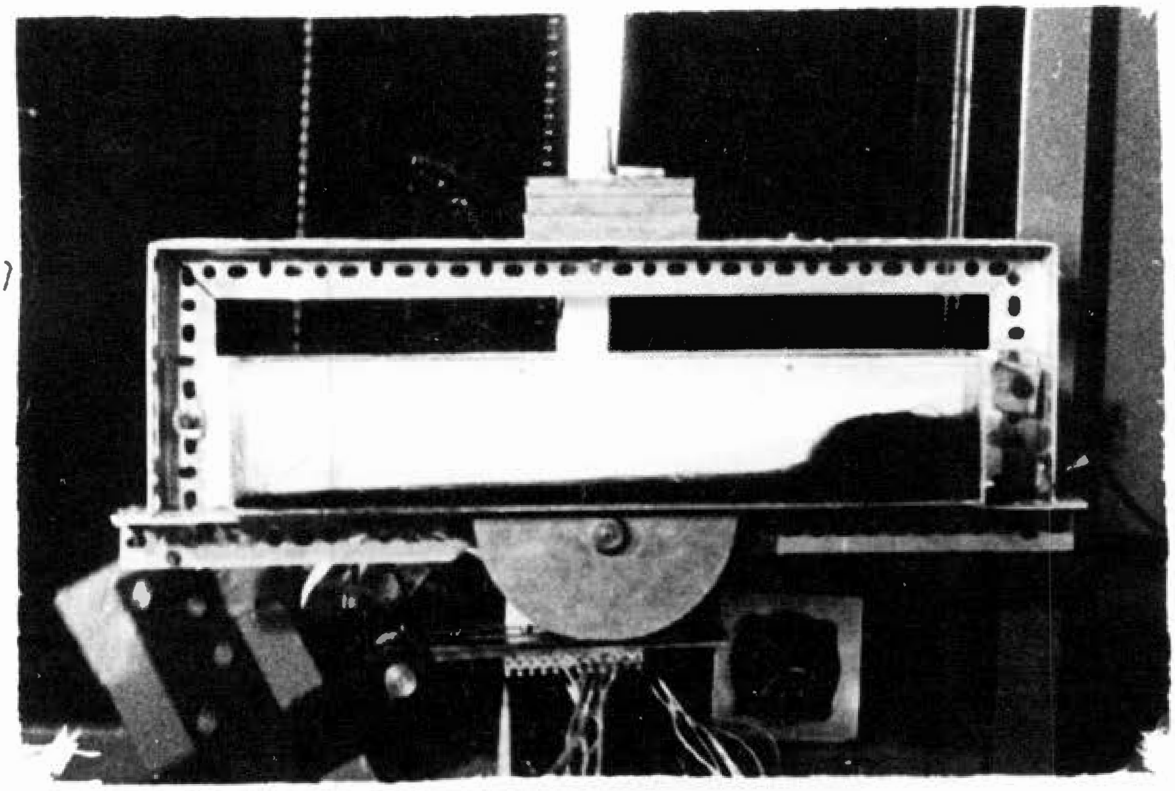
b) the angle of roll

c) the depth of water in the tank.

From these observations some judicious assumptions must be made. The relationships between the various factors in observations (iv) and (v) will be dealt with later.

At first, simply assume the phase angle to be $\underline{\varepsilon}$ and the bore height to be $\underline{a}$.

Now assume the bore to be a perfectly vertical wave front moving at constant linear velocity $\dot{x}$ on a water level which is always parallel to the tank base (Fig.5 - 2). These simple assumptions will be used to find the characteristics of the water as a



View of moving bore in rolling tank.

Fig. 5 - 1

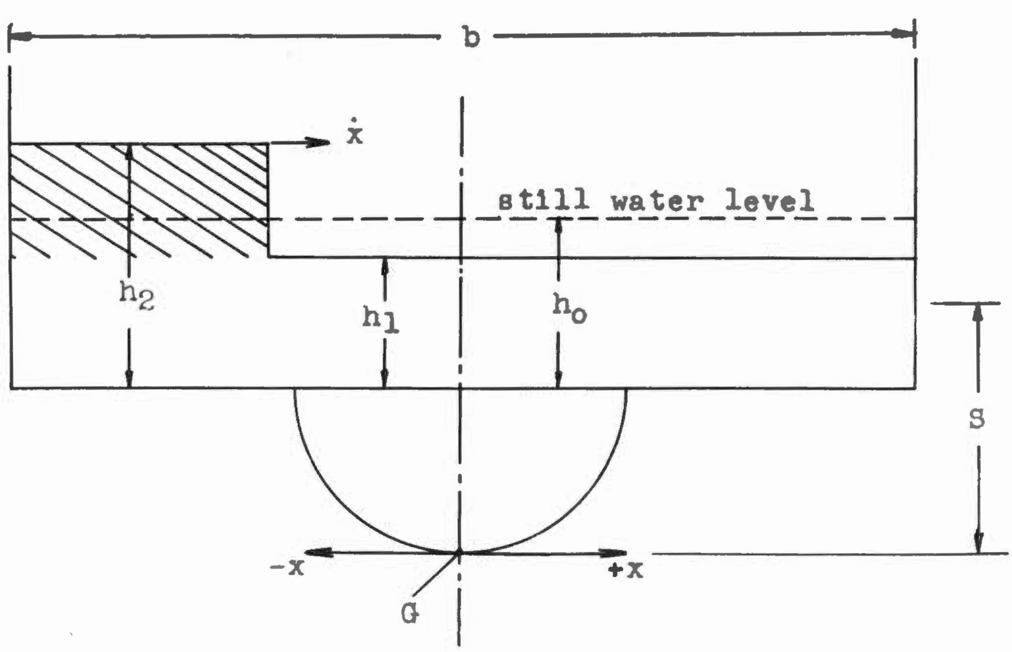


Fig. 5 - 2

damper of a rolling ship. The value of the actual damping effect will depend on values of ϵ and a which are as yet undefined and will be determined by further observations.

5 - 3. Moments acting on ship due to moving liquid:

Consider the tank in Fig.5 - 2 as fitted on the ship model described in Chapter 3. The tank has:-

length b

width w

S is the height of the mean liquid depth above the centre of gravity, G, of the e m p t y (water-less) ship and tank.

h_o is the still liquid depth

h_1 and h_2 are the liquid depths on either side of the moving bore.

To obtain a general solution, the density of the liquid is ρ.

The position of the bore, x, is measured horizontally from the centre of gravity of the ship. For simplicity the lever arm of the moving water will be measured from this point and not from the instantaneous pivot point. This simplification is valid because the maximum distance of the instantaneous pivot point from the centre of gravity is $\theta_o \overline{GM}$ where θ_o is small by definition and therefore $\theta_o \overline{GM}$ is small compared to the lever arm which largely depends on the tank length b.

The liquid causes moments about the centre of gravity due to:-

(i) the rate of change of angular momentum caused by the transfer of liquid,

(ii) the unbalanced weight of liquid acting at some lever arm.

Consider the case of the bore moving from left to right as shown in Fig.5 - 2:-

The mass of the liquid causing a moment about G is: (i.e. the shaded portion in Fig. 5 - 2).

$$m = \frac{\rho wa}{g}\left(\frac{b}{2} + \frac{x}{\cos\theta}\right)$$

Now throughout this analysis we assume θ small, $\therefore \cos\theta \doteq 1, \sin\theta = \theta$

$$\therefore \quad m = \frac{\rho wa}{g}\left(\tfrac{1}{2}b + x\right) \qquad \text{..........(5 - 1)}$$

Inertia I_m of this mass about the centre of gravity G is:

$$I_m = m\left[\frac{1}{3}\left(\frac{a}{2}\right)^2 + \frac{1}{3}\left(\frac{\frac{1}{2}b + x}{2}\right)^2 + s^2 + \left(\frac{\frac{1}{2}b - x}{2}\right)^2\right] \qquad \text{.....(5 - 2)}$$

As mentioned above, part of the moment caused by the moving liquid is due to the rate of change of angular momentum.

The angular momentum of the liquid is:

$$I_m\dot{\theta} = \frac{\rho wa}{g}\left(\tfrac{1}{2}b + x\right)\left[{}^1/_3\left(\tfrac{1}{2}a\right)^2 + {}^1/_3\left(\frac{\frac{1}{2}b + x}{2}\right)^2 + s^2 + \left(\frac{\frac{1}{2}b - x}{2}\right)^2\right]\dot{\theta}$$

$$\therefore \quad I_m = \frac{\rho wa}{g}\left[{}^{a^2}/_{12}\left(\tfrac{1}{2}b + x\right) + {}^1/_{12}\left(\tfrac{1}{2}b + x\right)^3 + s^2\left(\tfrac{1}{2}b + x\right) + \tfrac{1}{4}\left(\tfrac{1}{4}b^2 - x^2\right)\left(\tfrac{1}{2}b - x\right)\right]\dot{\theta}$$

The rate of change of angular momentum is:

$$\frac{d(I_m\dot{\theta})}{dt} = I_m \frac{\partial\dot{\theta}}{\partial t} + \dot{\theta}\frac{\partial I_m}{\partial t}$$

$$\therefore \frac{d(I_m\dot{\theta})}{dt} = \frac{\rho wa\ddot{\theta}}{g}\Big[\frac{a^2}{12}(\tfrac{1}{2}b + x) + \frac{1}{12}(\tfrac{1}{2}b + x)^3 + S^2(\tfrac{1}{2}b + x) + \tfrac{1}{4}(\tfrac{1}{4}b^2 - x^2)(\tfrac{1}{2}b - x)\Big]$$

$$+ \frac{\rho wa\dot{\theta}}{g}\Big[\frac{a^2}{12}\dot{x} + \frac{\dot{x}}{4}(\tfrac{1}{2}b + x)^2 + \dot{x}S + \tfrac{1}{4}(3\dot{x}x^2 - bx\dot{x} - \tfrac{1}{4}b^2\dot{x})\Big] \quad \text{..........(5 - 3)}$$

This is the moment due to the rate of change of angular momentum.

A further couple is caused by the "unbalance" weight of a portion of the liquid at same lever-arm:-

moment = mg x lever arm

The lever arm, x_1, is the horizontal distance from the centre of gravity G to the centre of gravity of the unbalance portion of liquid,

$$x_1 = \tfrac{1}{2}(\tfrac{1}{2}b - x)$$

$$\therefore \text{Moment} = \tfrac{1}{2}\rho wa(\tfrac{1}{2}b + x)(\tfrac{1}{2}b - x)$$

$$= \tfrac{1}{2}\rho wa\ (\tfrac{1}{4}b^2 - x^2)$$

The direction, or sense, of this moment is not evident from the expression. For the case shown in Fig.5 - 2 the moment acts anticlockwise; however if the bore is in the same position but moving in the opposite direction (from right to left), the moment is clockwise.

Therefore, the direction of the moment depends on the direction of motion.

Now assuming clockwise moments +ve and $\dot{x}$ from left to right +ve, then the above expression may be written as:

$$\text{moment} = -\frac{\dot{x}}{|\dot{x}|}\ \tfrac{1}{2}\rho w a(\tfrac{1}{4}b^2 - x^2) \qquad \text{..........}(5 - 4)$$

and now the expression determines the direction of the moment.

The total moment, M'', caused by the moving liquid is the sum of the moments of expressions (5 - 3) and (5 - 4).

Then, using the sign convention assumed above,

$$\text{Total moment } M'' = w\rho a\left\{\frac{\ddot{\theta}}{g}(\tfrac{1}{2}b + x)\left[\frac{a^2}{12} + \frac{1}{12}(\tfrac{1}{2}b + x)^2 + \tfrac{1}{4}(\tfrac{1}{2}b - x)^2 + s^2\right] + \frac{\dot{x}\dot{\theta}}{g}\left[\frac{a^2}{12} + \tfrac{1}{4}(\tfrac{1}{2}b + x)^2 + s^2 + \tfrac{1}{4}(3x + \tfrac{1}{2}b)(x - \tfrac{1}{2}b)\right] - \frac{\dot{x}}{|\dot{x}|}\ \tfrac{1}{2}(\tfrac{1}{4}b^2 - x^2)\right\} \qquad \text{.......}(5 - 5)$$

5 - 4. <u>The equation of motion of ship and tank</u>:

If we now consider a tank carrying a shallow liquid mounted on a ship or, in this case, on the model, then the equation of motion of the system becomes:

$$I\ddot{\theta} + C\dot{\theta} + \Delta\overline{GM}\theta = \Delta\overline{GM}\ \alpha_o \sin\phi + M'' \qquad \text{..........}(5 - 6)$$

from Froude's equation of rolling (2 - 10).

By substituting from equation (5 - 5) for M'' the complete equation may be written down.

However this equation can be simplified:-

(i) The coefficient of $\ddot{\theta}$ is now

$$I + \frac{w\rho a}{g}(\tfrac{1}{2}b + x)\left[\frac{a^2}{12} + \frac{1}{12}(\tfrac{1}{2}b + x)^2 + \tfrac{1}{4}(\tfrac{1}{2}b - x)^2 + s^2\right]$$

which we will rewrite as:

$$I + \frac{\rho w\ a}{g} A$$

By inspection $A_{max} < b(^{b^2}/_4 + s^2)$

assuming $^{a^2}/_{12} \ll b^2$

$\therefore$ The maximum coefficient of $\ddot{\theta}$ is

$$< \frac{\Delta}{g}\left[k^2 + \frac{w\rho a}{\Delta} b(\tfrac{1}{4}b^2 + s^2)\right]$$

Now $\frac{w\rho h_o b}{\Delta}$, the ratio of liquid weight to ship weight should not exceed about 8% in practice, and a is of the same order of magnitude as h_o,

$\therefore$ the coefficient $< \frac{\Delta}{g}\left[k^2 + 0{\cdot}05(\tfrac{1}{4}b^2 + s^2)\right]$

Now s is usually small compared to b, also $\tfrac{1}{2}b$ is of the same order of magnitude as k,

$\therefore \quad k^2 \gg 0{\cdot}05\ (\tfrac{1}{4}b^2 + s^2)$

This permits the omission of the later part of the coefficient which then takes back its original value of I.

i.e. The term in M'' which is a coefficient of $\ddot{\theta}$ can be neglected.

(ii) The coefficient of $\dot{\theta}$ is:

$$C + \frac{w\rho a\dot{x}}{g}\left(\frac{a^2}{12} + \frac{b^2}{16} + \frac{bx}{4} + \frac{x^2}{4} + S^2 + \frac{3x}{4} - \frac{bx}{4} - \frac{b^2}{16}\right)$$

$$= C + \frac{w\rho a\dot{x}}{g}\left(\frac{a^2}{12} + S^2 + x^2\right)$$

Now $\underline{a}$ is of the same order of magnitude as h_o and $h_o \ll x$ except when $x \doteq 0$ which is only an instant in each cycle.

$\therefore$ We can neglect $\frac{a^2}{12}$ and the coefficient of $\dot{\theta}$ becomes:

$$C + \frac{w\ a\dot{x}}{g}(S^2 + x^2)$$

Using the above two simplifications, the equation of motion describing the system may be written as:

$$I\ddot{\theta} + \left[C + \frac{w\rho a\dot{x}}{g}(S^2 + x^2)\right]\dot{\theta} + \Delta\,\overline{GM}\,\theta = \Delta\,\overline{GM}\,\alpha_o\,\sin\phi$$

$$- \frac{\rho a w\dot{x}}{2\,|\dot{x}|}\left(\tfrac{1}{4}b^2 - x^2\right) \qquad \text{..........(5 - 7)}$$

As it stands, this equation does not appear to be linear and some substitutes for x^2 will have to be found to solve it.

5 - 5. Solution of the equation of motion of ship & tank:

Equation (5 - 7) describes the motion of a ship with a rectangular free surface tank.

The variable x determines the position of the bore and is proportional to time elapsed since the speed of the bore has been assumed constant.

In chapter 2, ϕ was made equal to ωt. However for convenience we shall take t as the oscillating time of the ship;

Then $$\phi = \omega t + \epsilon_s$$

where ϵ_s is the leading phase angle between the forcing term (the waves) and the ship.

Also the equation for x may be written as:

$$x = (\omega t - \epsilon)^{b}/\pi$$

over each half cycle (see Fig.5 - 3a), where ϵ is the lagging phase angle of the bore motion behind the ship motion.

The form of equation (5 - 7) suggests a solution for θ of:

$$\theta = \theta_o \sin \omega t$$

This will certainly be true if x can be successfully linearized. Therefore we assume this solution.

Using the above, the equation can be linearized term by term:

(i) The damping term -

Consider the damping term in equation (5 - 7):

$$\left[C + \frac{w\rho a}{g}(S^2 + x^2)\right]\dot{\theta}$$

To linearize this term, the coefficient of $\dot{\theta}$ must be made constant.

The variables in the coefficient are a and x^2. The variation of a is unknown and will, for simplicity, be assumed as constant. The reasonableness of this will be seen later when the insignificance

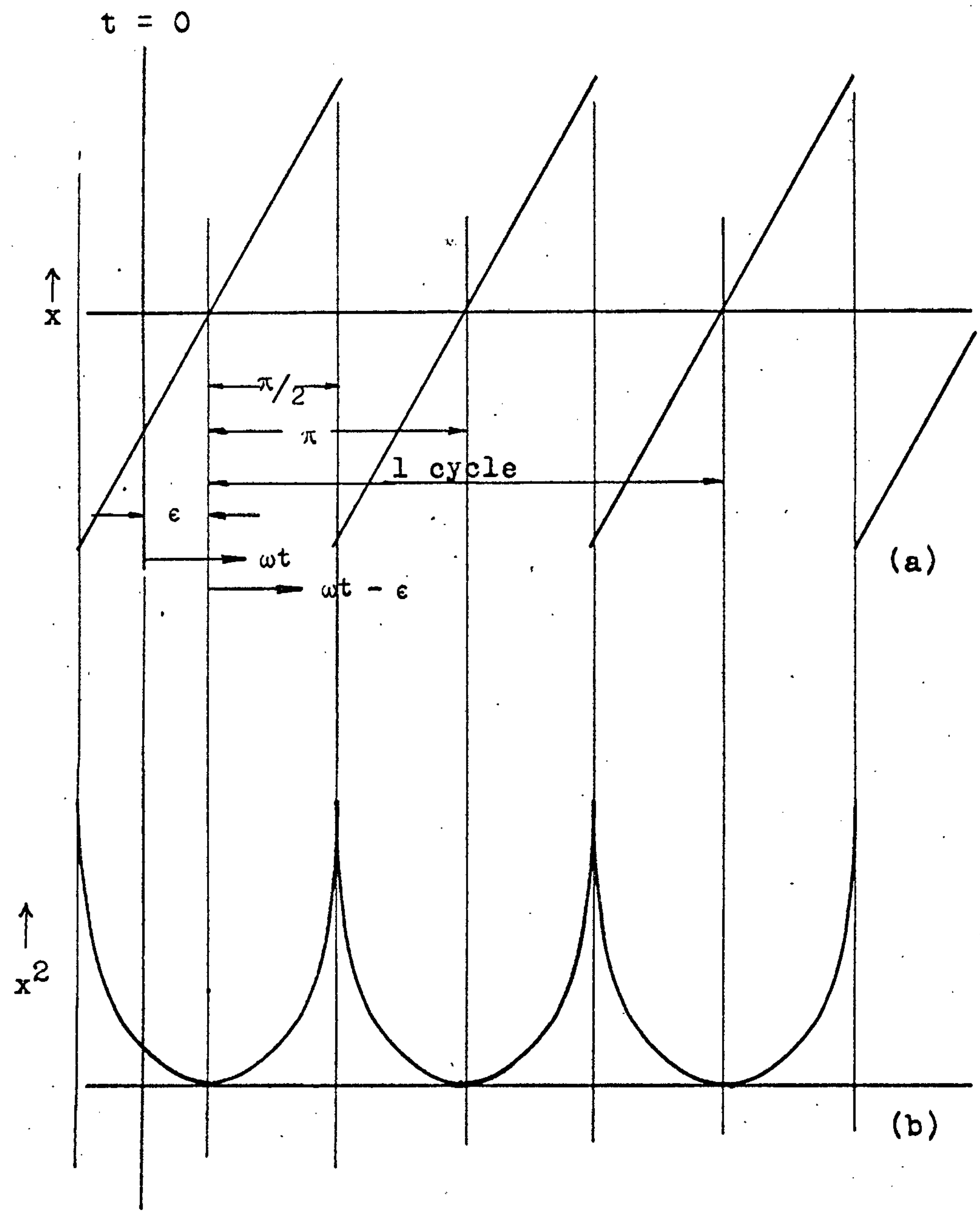


Fig. 5 - 3

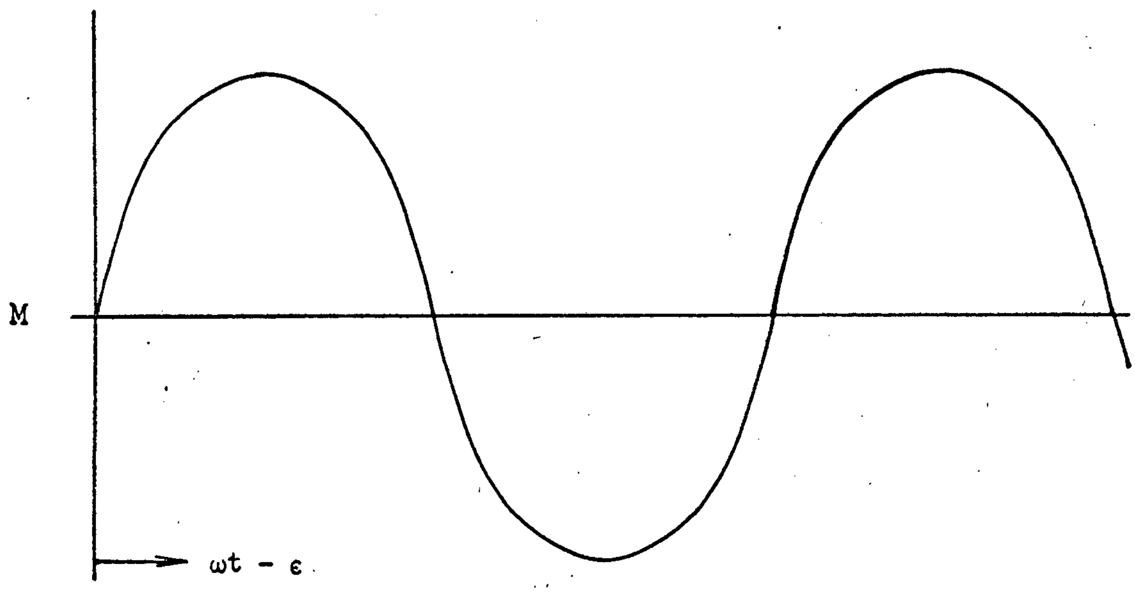


Fig. 5 - 4

of this damping term is shown with respect to other relevant terms.

Assume the coefficient of $\dot{\theta}$ to be:

$$C + C_1 + C_2$$

where $C_1 = \dot{x}/g \, \rho a w \, s^2$

and $C_2 = \dot{x}/g \, \rho a w \, x^2$

Now C_1 is constant and only C_2 needs altering.

An approximate value of C_2 may be found by equating the work done by C_2 and the work done by some constant C_3. Then for the purposes of our equation $C_2 \simeq C_3$.

Work done by C_2 in one cycle is:

$$W.D. = \int C_2 \dot{\theta} d\theta$$

Now $\theta = \theta_o \sin \omega t$

$$\therefore \quad \frac{d\theta}{d(\omega t)} = \theta_o \cos \omega t$$

and $\dot{\theta} = \theta_o \omega \cos \omega t$

$$\therefore \quad W.D. = \omega \int \theta_o^2 C_2 \cos^2 \omega t \, d(\omega t)$$

Then introducing limits:-

$$W.D. = \frac{\rho a w \omega \theta_o^2 \dot{x}}{g} \int_0^{2\pi} x^2 \cos^2 \omega t \, d(\omega t) \qquad \text{........(5 - 8)}$$

The work done by some damping constant C_3

$$= \theta_o^2 \omega \int_0^{2\pi} C_3 \cos^2 \omega t \, d(\omega t)$$

$$= \theta_o^2 \omega \pi C_3$$

We want the same work done by C_2, therefore equate to equation (5 - 8):-

$$\therefore \quad C_3 = \frac{\rho a w \dot{x}}{g \pi} \int_0^{2\pi} x^2 \cos^2 \omega t \, d(\omega t) \quad \ldots\ldots\ldots\ldots (5 - 9)$$

This equation can be solved by expanding x^2 in a Fourier series:-

The graphical representations of x and x^2 are shown in Figs. 5 - 3(a) and 5 - 3(b).

We have $x = {}^b/_\pi \, (\omega t - \epsilon)$ for $\frac{1}{2}$ cycle

for simplicity put $\omega t - \epsilon = \beta$

Then $x^2 = \dfrac{b^2 \beta^2}{\pi^2}$

Note. The Fourier series of x^2 will ultimately be multiplied by $\cos^2 \omega t$ before integration in equation (5 - 8) between limits 0 to 2π.

Therefore all terms involving cos and sin functions of the Fourier series will become zero in the final integration and can therefore be immediately ignored.

That means that only the first term of the series (a_o) will have a value after the integration of (5 - 9).

Using the standard notation for Fourier Series:

$$a_o = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \frac{b^2}{\pi^2} \beta^2 \, d\beta$$

$$= \frac{b^2}{6}$$

Substituting in (5 - 8):-

$$C_3 = \frac{\rho a w \dot{x}}{\pi g} \int_0^{2\pi} \frac{b^2}{12} \left(\frac{1}{2} + \frac{\cos 2 \omega t}{2}\right) d(\omega t)$$

$$C_3 = \frac{\rho\, aw\dot{x}b^2}{12g} \doteq C_2$$

$\therefore$ the coefficient $C + C_1 + C_2 = C + \frac{w\rho a\dot{x}}{g}(S^2 + \frac{b^2}{12})$

And the damping term may be written as:-

$$\underline{C'\,\dot{\theta}}$$

where C' is the above constant coefficient.

(ii) <u>The 'moment' term</u>:-

The term $\frac{w\rho a\dot{x}}{2|\dot{x}|}(\frac{1}{4}b^2 - x^2)$, which we shall call the 'moment' term N, also contains x^2. However it cannot be treated in the same way as the damping since only a term 90^o out of phase with the motion does 'work'.

To linearize, the term must be expressed either in terms of θ or in a sinusoidal form in terms of $\omega t - \epsilon$. For the present assume $\underline{a}$ as constant.

We see in Fig. 5 - 3(b) that

$$x^2 = \frac{b^2\beta^2}{\pi^2} \quad \text{for} \quad -\tfrac{1}{2}\pi < \beta < \tfrac{1}{2}\pi$$

$$\text{where} \quad \beta = \omega t - \epsilon$$

$\therefore$ The moment term $N = -\frac{\rho\, wab^2}{2}(\frac{1}{4} - \frac{\beta^2}{\pi^2})$

for $-\frac{1}{2}\pi < \beta < \frac{1}{2}\pi$ in the right hand side of equation (5 - 7).

For the next half cycle,

$$x^2 = \frac{b^2}{\pi^2}(\beta - \pi)^2 \quad \text{(see Fig.5 - 3(b))}$$

and the sign of the moment term is reversed ($\dot{x}$ is reversed),

$$\therefore \quad N = \frac{\rho\, wab^2}{2}\left[\frac{1}{4} - \frac{(\beta - \pi)^2}{\pi^2}\right]$$

for $\frac{1}{2}\pi < \beta < 3\pi/2$

Therefore the graphical representation of the moment term appears almost sinusoidal, Fig. 5 - 4.

A Fourier series of this curve will express it in a sinusoidal or cosinusoidal form. The equation is -

$$N = \begin{cases} \dfrac{\rho wab^2}{2}\left(\dfrac{\beta^2}{\pi^2} - \dfrac{1}{4}\right) & \text{----} \quad -\frac{1}{2}\pi < \beta < \frac{1}{2}\pi \\ \dfrac{\rho wab^2}{2}\left[\dfrac{1}{4} - \dfrac{(\beta - \pi)^2}{\pi^2}\right] & \text{----} \quad \frac{1}{2}\pi < \beta < 3\pi/2 \end{cases}$$

Let a_o, a_n and b_n be the usual Fourier series notations.

The constant $\frac{1}{2}\rho wab^2$ will not be repeated throughout the Fourier analysis but will be reintroduced into the final solution of the moment.

$$a_o = \frac{b^2}{\pi}\int_{-\pi/2}^{\pi/2}\left(\frac{\beta^2}{\pi^2} - \frac{1}{4}\right) d\beta + \frac{b^2}{\pi}\int_{\pi/2}^{3\pi/2}\left[\frac{1}{4} - \left(\frac{\beta - \pi}{\pi}\right)^2\right] d\beta$$

$$= \frac{b^2}{\pi}\left[\frac{\beta}{4} - \frac{\beta^3}{3\pi^2}\right]_{\pi/2}^{3\pi/2} + \frac{b^2}{\pi}\left[\frac{\beta^3}{3\pi^2} - \frac{\beta}{4}\right]_{\pi/2}^{3\pi/2}$$

$$= 0$$

$\longrightarrow$

$$a_n = \frac{b^2}{2\pi}\int_{-\pi/2}^{\pi/2}\left(\frac{\beta^2}{\pi^2} - \frac{1}{4}\right)\cos n\beta d\beta - \frac{b^2}{2\pi}\int_{\pi/2}^{3\pi/2}\left(\frac{\beta^2}{\pi^2} - \frac{2\beta}{\pi} + \frac{3}{4}\right)\cos n\beta d\beta$$

$$= \frac{b^2}{2\pi}\left[\frac{\beta^2 \sin n\beta}{\pi^2 n} - \frac{\sin n\beta}{4n} \quad \frac{2\beta \cos n\beta}{\pi^2 n^2} - \frac{2 \sin n\beta}{\pi^2 n^3}\right]_{-\pi/2}^{\pi/2}$$

$$- \frac{b^2}{2\pi}\left[\frac{3 \sin n\beta}{4n} + \frac{\beta^2 \sin n\beta}{\pi^2 n} + \frac{2\beta \cos n\beta}{\pi^2 n^2} + \frac{2 \sin n\beta}{\pi^2 n^3}\right.$$

$$\left. - \frac{2\beta \sin n\beta}{\pi n} - \frac{2 \cos n\beta}{\pi n^2}\right]_{\pi/2}^{3\pi/2}$$

$$a_n = -\frac{4b^2}{\pi^3 n^3} \sin n\,{}^{\pi}/_2$$

$\therefore$ for n odd -

$$a_n = -\frac{4b^2}{\pi^3 n^3} \quad (n = 1,5,9\ldots)$$

$$a_n = \frac{4b^2}{\pi^3 n^3} \quad (n = 3,7,11..)$$

for n even - $a_n = 0$

$$b_n = \frac{b^2}{2\pi}\int_{-\pi/2}^{\pi/2}\left(\frac{\beta^2}{\pi^2} - \frac{1}{4}\right)\sin n\beta\, d\beta + \int_{\pi/2}^{3\pi/2}\left(\frac{2\beta}{\pi} - \frac{\beta^2}{\pi^2} - \frac{3}{4}\right)\sin n\beta\, d\beta$$

All cosine terms will become = 0

$$\therefore b_n = \frac{b^2}{2\pi}\left\{\left[\frac{2\beta \sin n\beta}{\pi^2 n^2}\right]_{-\pi/2}^{\pi/2} + \left[\frac{2 \sin n\beta}{\pi n^2} - \frac{2\beta \sin n\beta}{\pi^2 n^2}\right]_{\pi/2}^{3\pi/2}\right\}$$

$$= 0$$

$\therefore$ The series for N is -

$$N = -\rho\, aw \frac{4b^2}{\pi^3}\left(\cos\beta - \frac{\cos 3\beta}{27} + \frac{\cos 5\beta}{125} ----\right) \qquad \ldots\ldots\ldots\ldots(5 - 10)$$

We are looking for a sinusoidal function and can find it in equation (5 - 10) where the neglect of all higher terms is an inaccuracy of negligible proportion.

$\therefore$ The moment term may be written as -

$$N = -\frac{4\beta awb^2}{\pi^3}\cos(\omega t - \epsilon)$$

However, $\frac{4\rho w b^2}{\pi^3}$ is a constant and a is a variable (depending mainly on h_o and θ_o, as will be seen later) whose magnitude is unknown.

Therefore call $\frac{4\rho w b^2 a}{\pi^3}$ some variable M

Then $$N = - M\cos(\omega t - \epsilon) \quad \text{..........(5 - 11)}$$

Later on the magnitude of variable M will be discussed as a whole and a will no longer be mentioned.

Equation (5 - 7) can briefly be written as:

$$I\ddot{\theta} + C'\dot{\theta} + \Delta\overline{GM}\theta = \Delta\overline{GM}\alpha_o \sin(\omega t + \epsilon_s) + N$$

Now substituting equations (5 - 8) and (5 - 11) into this:

$$I\ddot{\theta} + \left[C + \frac{w\rho a\dot{x}}{g}(s^2 + \frac{b^2}{12})\right]\dot{\theta} + \Delta\overline{GM}\theta =$$

$$\Delta\overline{GM}\alpha_o \sin(\omega t + \epsilon_s) - M\cos(\omega t - \epsilon) \quad \text{........(5 - 12)}$$

If a in the damping term is still assumed constant (the value of this will be discussed later) then equation (5-12) is linear with the solution

$$\theta = \theta_o \sin \omega t$$

as previously assumed.

$$\therefore \quad \dot{\theta} = \omega\cos\omega t \quad \text{and} \quad \ddot{\theta} = -\omega^2 \sin\omega t$$

Substituting the above in equation (5 - 12):-

$$-\theta_o I\omega^2 \sin\omega t + C'\theta_o\omega\cos\omega t + \theta_o\Delta\overline{GM}\sin\omega t =$$
$$\Delta\overline{GM}\alpha_o(\cos\epsilon_s \sin\omega t + \sin\epsilon_s\cos\omega t) - M(\cos\omega t\cos\epsilon + \sin\omega t\sin\epsilon)$$

Separating the cos and sin terms:-

$$- \theta_o I\omega^2 + \theta_o \Delta \overline{GM} = \Delta\overline{GM}\alpha_o \cos \epsilon_s - M \sin \epsilon \quad ....(i)$$

$$C'\theta_o \omega = \Delta\overline{GM}\alpha_o \sin \epsilon_s - M \cos \epsilon \quad ....(ii)$$

Hence:

$$\theta_o = \frac{\Delta\overline{GM}\alpha_o}{\sqrt{(\Delta\overline{GM} - I\omega^2 + \frac{M}{\theta_o}\sin\epsilon)^2 + (C'\omega + \frac{M}{\theta_o}\cos\epsilon)^2}} \quad ....(5 - 13)$$

and by dividing equation (i) into equation (ii):-

$$\tan \epsilon_s = \frac{C'\omega + \frac{M}{\theta_o}\cos\epsilon}{\Delta\overline{GM} - I\omega^2 + \frac{M}{\theta_o}\sin\epsilon} \quad ....(5 - 14)$$

Equations (5 - 13) and (5 - 14) are the solutions of the motion based on the assumptions and simplifications already mentioned. However no numerical solution is obtainable until values of M and ϵ can be found.

Furthermore, some factors of seeming importance have not been considered and will be discussed in the following section.

5 - 6. <u>Some omissions and corrections:</u>

(i) An apparently significant omission is a term in the original angular momentum equation which describes the horizontal component of the tank movement when $S \neq 0$ (i.e. when the tank does not lie on the centre of gravity of the ship).

If this term were considered, the inertia equation (5 - 2) would become -

$$I_m = m\left[\frac{a^2}{12} + \frac{1}{12}(\tfrac{1}{2}b + x)^2 + S^2 + (S\tan\theta + \tfrac{1}{4}b - \tfrac{1}{2}x)^2\right]$$

the new term being $S \tan \theta$ in the last bracket.

The significant effect of this term is an addition to the damping term of $\frac{S\theta_o \hat{w} \rho a b^2}{g\,\pi^2}$ found in the same manner as the linearization of the damping term in section 5 - 5.

The magnitude of this factor is generally small compared to the remainder of the damping term, but it is not necessarily small enough to be negligible. However for simplification the addition will be neglected while it must be noted that its significance is that increasing θ_o and ω increase the damping.

(ii) The force on the liquid due to the orbital motion of the ship has not yet been considered:-

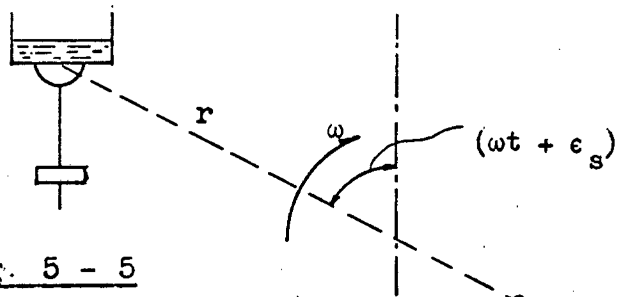


Fig. 5 - 5

Centrifugal force on the liquid = $m_1 r\,\omega^2$

where $m_1 = \frac{\rho b w h_o}{g}$, the mass of water in the tank.

Only the horizontal component of this force is of interest here. This component is

$$= m_1 r \omega^2 \sin(\omega t + \epsilon_s)$$

and acts on the ship at a distance S from the centre of gravity.

∴ the couple on the ship due to the centrifugal force on the liquid is

$$s m_1 r \omega^2 \sin(\omega t + \epsilon_s)$$

which opposes the forcing function.

Hence the nett forcing function becomes -

$$(\Delta \overline{GM}\,\alpha_o - S m_1 r\,\omega^2) \sin(\omega t + \epsilon_s)$$

This correction is very small, but since it is easy enough to handle it is introduced in equation (5 - 13) which becomes:

$$\theta_o = \frac{\Delta \overline{GM} \alpha_o - S m_1 r \omega^2}{\sqrt{(C'\omega + \frac{M}{\theta_o}\cos\epsilon)^2 + (\overline{GM} - I\omega^2 + \frac{M}{\theta_o}\sin\epsilon)^2}} \qquad \ldots\ldots\ldots\ldots(5 - 15)$$

Equation (5 - 14) is not altered by this correction.

5 - 7. A possible value for M:

The variable M represents a couple whose magnitude varies with one or more of the parameters θ, ω, h_o, b, w and ρ.

This couple can be determined mathematically only at $\omega \longrightarrow 0$ when the liquid surface is horizontal. Although in such a case no bore actually exists, the phase angle ϵ can still be assumed to have a value.

It is known that at very low frequencies, $\epsilon = -90^o$, that is, the couple due to the liquid leads the ship motion by 90^o(this is discussed in section 5 - 8).

Then for $\omega \longrightarrow 0$ and $\epsilon = -90^o$ equation (5 -15) gives:

$$\theta_o = \frac{\Delta \overline{GM} \alpha_o}{\Delta \overline{GM} - \frac{M}{\theta_o}}$$

Hence

$$M = \Delta \overline{GM}(\theta_o - \alpha_o) \qquad \ldots\ldots\ldots\ldots(5 - 16)$$

Now consider Fig. 5 - 6 showing a ship or model on the maximum slope of a wave (α_o) and consequently having the maximum inclination (θ_o) from the vertical.

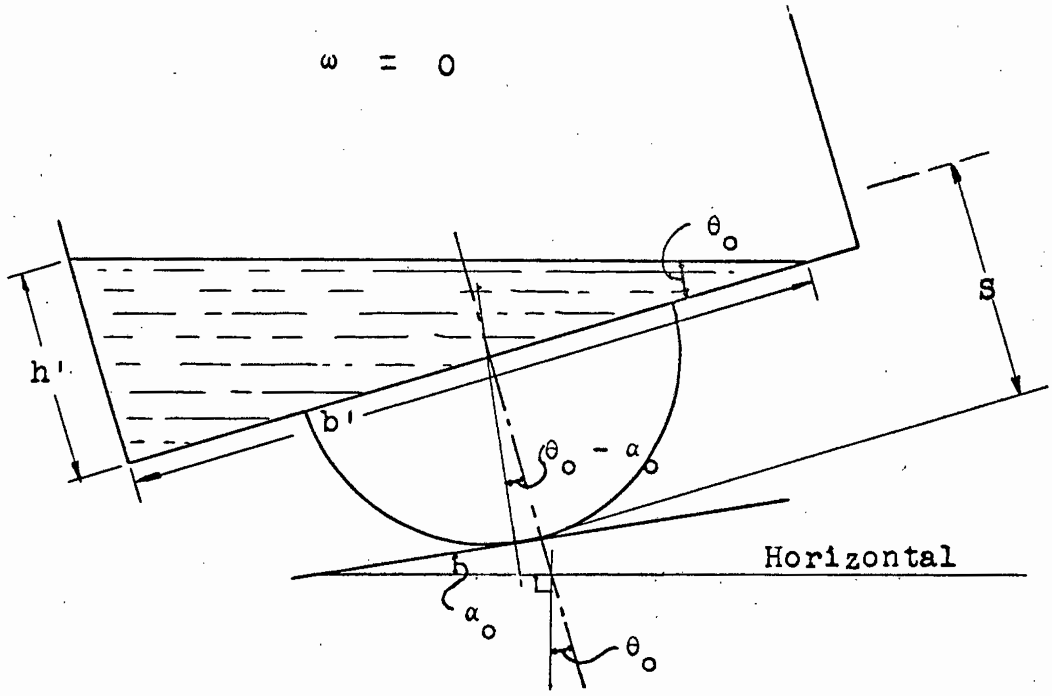


Fig. 5 - 6

$$\frac{h'}{b'} = \tan \theta_o$$

and $\qquad \frac{1}{2}b'h' = bh_o$

$\therefore \qquad b'^2 \tan \theta_o = 2bh_o$

$\therefore \qquad b' = \sqrt{\dfrac{2bh_o}{\tan \theta_o}}$

Neglecting the effect of S (the height of the tank above the centre of gravity), the moment arm of the liquid is:

$$\text{arm} = \left(\frac{b}{2} - \frac{b'}{3}\right) \cos \theta_o$$

the weight of the liquid = ρwbh_o

$\therefore$ Taking moments about the instantaneous centre of rolling (or centre of buoyancy) B:-

$$\rho wbh_o \left(\frac{b}{2} - \frac{b'}{3}\right) \cos \theta_o = \Delta \overline{GM} \sin (\theta_o - \alpha_o)$$

Substituting for b':

$$\left(\frac{b}{2} - \sqrt{\frac{2bh_o}{9 \tan \theta_o}}\right) \rho wbh_o \cos \theta_o = \Delta \overline{GM} \sin (\theta_o - \alpha_o)$$

The assumption of θ_o and α_o both being small is again applied here.

Then $\tan\theta_o \doteq \sin\theta_o \doteq \theta_o$

$\sin(\theta_o - \alpha_o) \doteq \theta_o - \alpha_o$

$\cos\theta_o \doteq 1$

Substituting in the above equation:

$$\left(\frac{b}{2} - \sqrt{\frac{2bh_o}{9\,\theta_o}}\right)\rho w b h_o = \Delta\overline{GM}\,(\theta_o - \alpha_o)$$

But we have from equation (5 - 15) that:

$$M = \Delta\overline{GM}\,(\theta_o - \alpha_o)$$

$$\therefore \quad M = \left(\frac{b}{2} - \sqrt{\frac{2bh_o}{9\,\theta_o}}\right)\rho w b h_o \qquad \text{..........(5 - 17)}$$

This gives a value of M in terms of the liquid weight, the liquid depth, the tank length and the angle of roll. The value is certainly correct for $\omega \to 0$ but whether it is equally applicable at higher frequency or whether the characteristic changes, will be determined by comparing the effect of M practically with calculated values.

From equation (5 - 17) a possible value for a can be obtained - this is required for the damping term.

In equation (5 - 11) we wrote

$$M = \frac{4\rho w b^2 a}{\pi^3}$$

$$\therefore \quad a = \frac{M\pi^3}{4\rho w b^2} \qquad \text{..........(5 - 18)}$$

This may not be the exactly correct value for a; however the damping effect of the damping term is so small compared to that of the 'moment' term that any error becomes negligible.

5 - 8. Possible values for ϵ:

No simple mathematical expression for ϵ can readily be derived from known conditions. So far we know that $\epsilon = -90^o$ at low frequency. From observations actually carried out it is clear that ϵ increases, and $\epsilon = 0$ at a frequency where $\dot{x}$ is in the vicinity of $\dot{x} = \sqrt{gh_o}$, the surface disturbance speed. The phase angle increases still further and most probably reaches $\epsilon = 90^o$.

Observations also show that the angle of roll plays a definite role in the magnitude of the phase angle. Furthermore it is noticeable that ϵ does not increase immediately from -90^o as ω begins to increase above 0.

A possible variation of ϵ with the frequency is shown in Fig. 5 - 7 for a constant angle of roll. The assumption of a straight line is purely for the sake of using the simplest relationship known.

If the above assumption is correct then a family of lines can be drawn to represent various values of θ_o (angle of roll), Fig. 5 - 8, for any particular tank and liquid depth.

J.J. Van den Bosch and J H. Vugts (Ref.3) have done some experimental work on phase angles in oscillating tanks. A set of their results is shown in graphical form in Fig. 5 - 9 for a tank with constant liquid depth.

The resemblance between Figs. 5 - 8 and 5 - 9

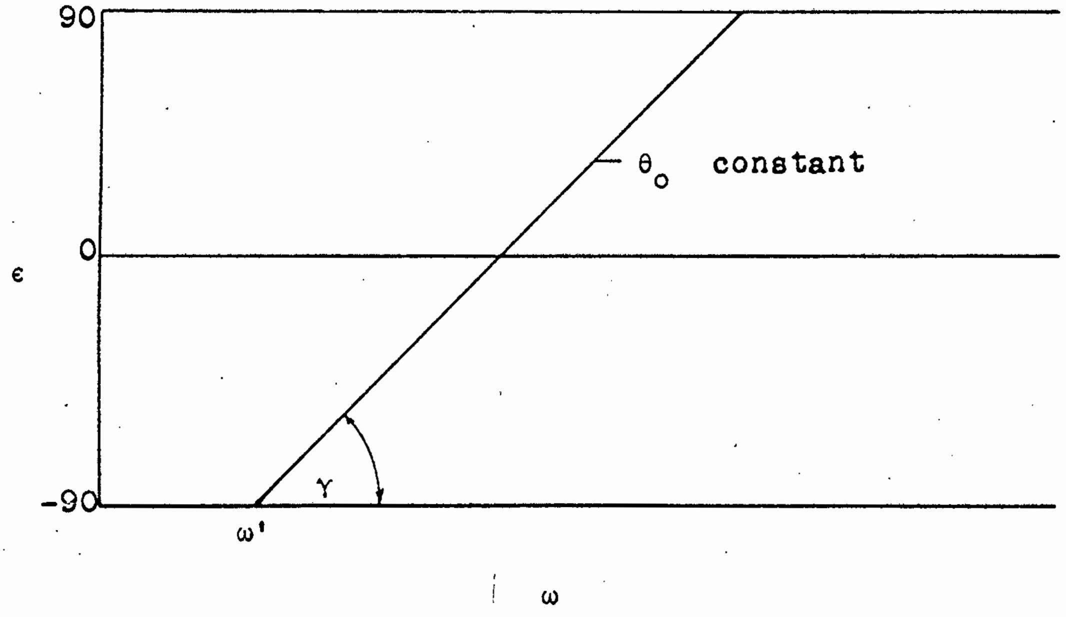


Fig. 5 - 7

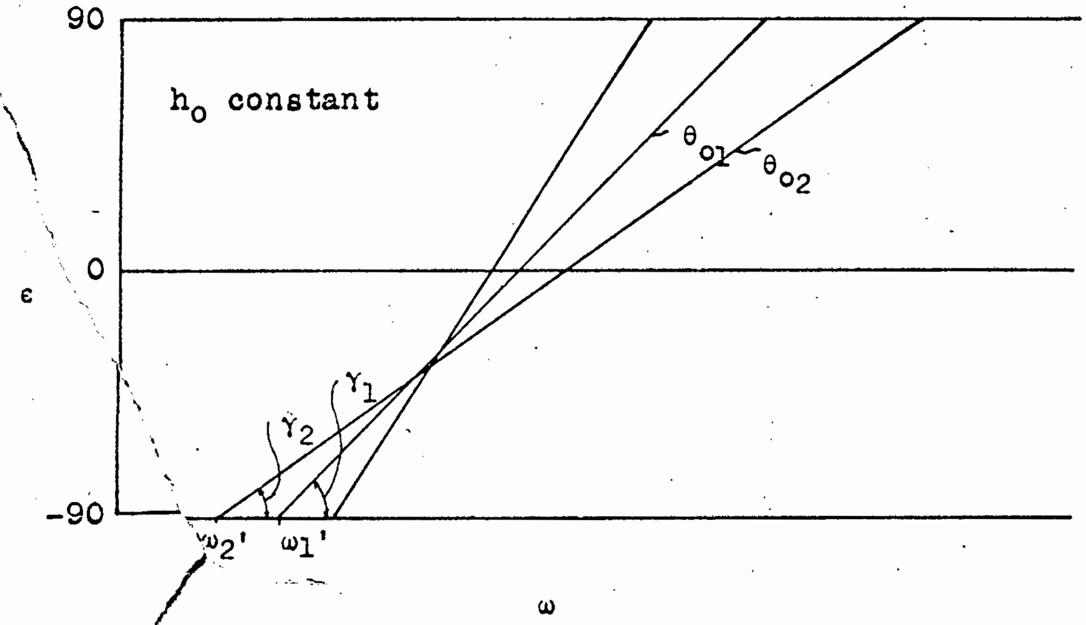


Fig. 5 - 8

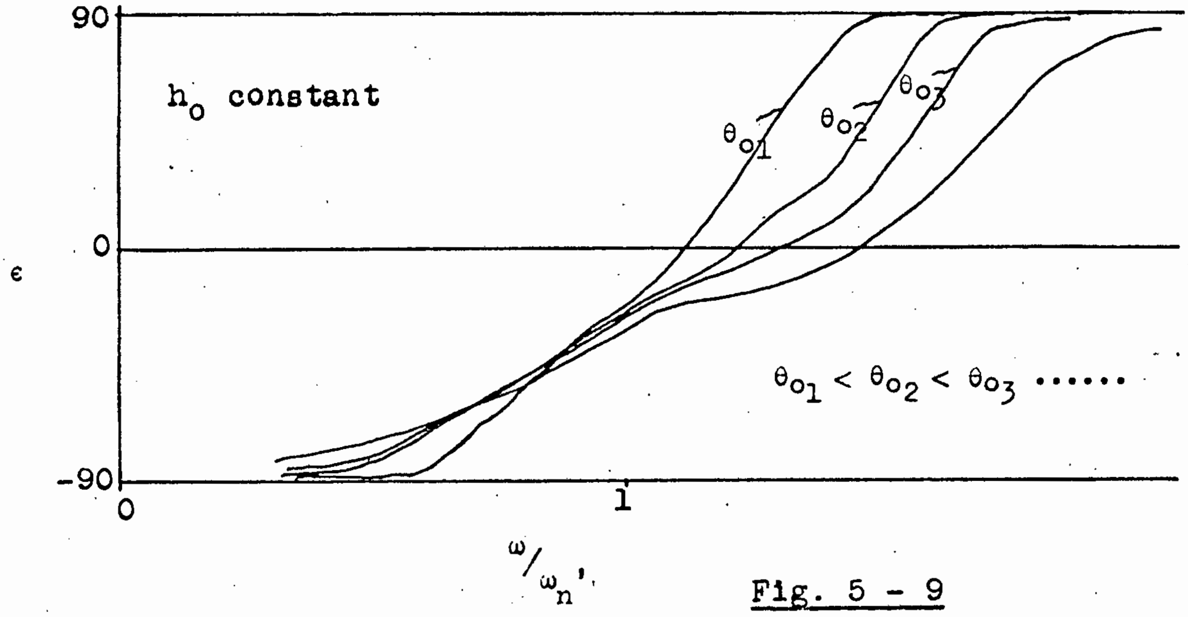


Fig. 5 - 9

can easily be seen. The curves obtained by the experimenters do not have any definite mathematical shape but are not unduly distorted by the assumption of straight lines, particularly in the lower frequency ranges. However it must be emphasixed that a fairly small error in ϵ can alter the term $M \cos \epsilon$ or $M \sin \epsilon$ very greatly and hence change the solution θ_o considerably.

Fig. 5 - 9 can give a very good indication of the possible relationships between θ_o, ϵ and ω and also the frequency at which ϵ begins to increase above -90^o.

From the idealised form of Fig. 5 - 9 (Fig.5 - 8) an empirical formula for ϵ was obtained as follows:

The basic pattern of frequencies must be a function of some particular frequency appertaining to the prevalent conditions such as liquid depth and tank length. Obviously the frequency which is governed by the changes in depth and length is the 'natural frequency' of the liquid surface, that is, when $\dot{x} = \sqrt{gh_o}$.

But $\dot{x}$ is constant

$$\therefore \quad T = \frac{2b}{\sqrt{gh_o}}$$

where T is the period.

$$\therefore \quad \omega_n' = \frac{\pi\sqrt{gh_o}}{b} \qquad \text{..........(5 - 19)}$$

where ω_n' is the natural frequency of the liquid surface.

Although Fig. 5 - 9 was originally published as ϵ vs ω for a certain tank length and depth, it is reproduced here in the nondimensional form of ϵ vs ω/ω_n' which must have more significance if different conditions are considered.

It is assumed that the curves of ϵ vs ω/ω_n' at certain angles of roll are absolute and will not change with b or h_o.

From Fig. 5 - 9 we can draw straight lines to approximate the curves (Fig. 5 - 8) and hence plot a curve of graph slope, γ, against θ_o, Fig. 5 - 10. This curve is approximately hyperbolic so that we can say:

$$\gamma = \frac{K_1}{\theta_o} + K_2 \qquad \text{..........(5 - 20)}$$

where K_1 and K_2 are constants and θ_o is in radians.

The assumed straight lines cut the $\epsilon = -90^0$ line at ω'. The relationships between the values of ω' and θ_o are plotted in Fig.5 - 11. ω' is plotted nondimensionally as a fraction of ω' i.e. $\omega'/\omega_n' = K'$.

This curve can easily be approximated by a straight line whose equation will be:

$$\omega'/\omega_n' \, K_3 + K_4\theta_o = K_3K_4$$

where K_3 and K_4 are constants and θ_o is the angle of roll in radius.

$$\therefore \quad \omega'/\omega_n' = \frac{K_4(K_3 - \theta_o)}{K_3} \qquad \text{..........(5 - 21)}$$

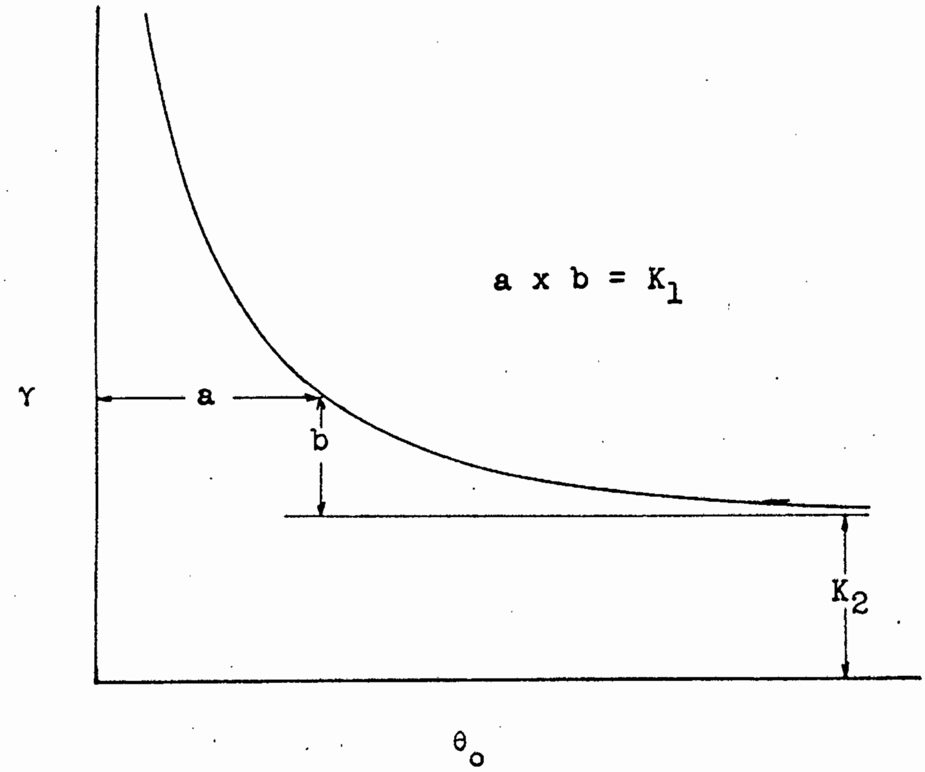


Fig 5 - 10

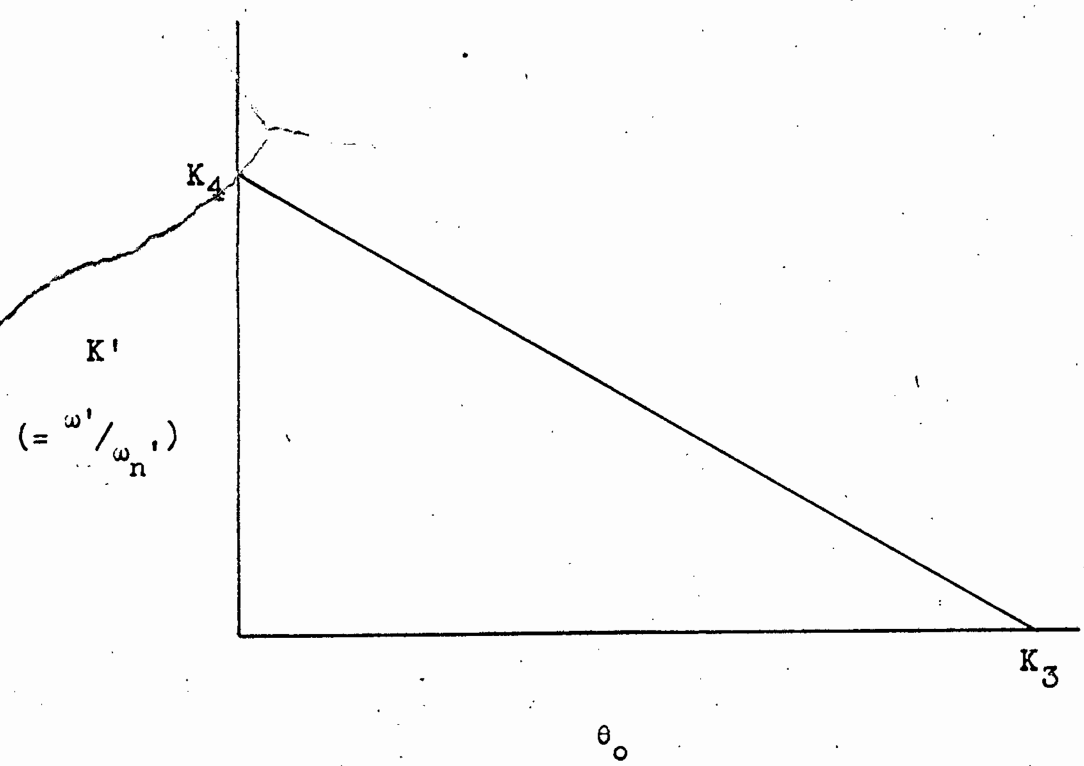


Fig. 5 - 11

We now know the slope (γ) of the lines relating ε and ω/ω_n as well as the coordinants of one point through which each of these lines pass, i.e. $(\omega'/\omega_n', -90^\circ)$ in equations (5 - 21) and (5 - 20):- Hence we have the equation:

$$\frac{\varepsilon + 90}{\omega/\omega_n' - \omega'/\omega_n'} = \gamma$$

$$\therefore \omega_n'(\varepsilon + 90) = \left[\frac{K_1}{\theta_o} + K_2\right]\left[\omega - \frac{K_4}{K_3}(K_3 - \theta_o)\,\omega_n'\right]$$

i.e. $$\varepsilon = \left[\frac{K_1}{\theta_o} + K_2\right]\left[\omega/\omega_n' - \frac{K_4}{K_3}(K_3 - \theta_o)\right] - 90 \qquad \ldots\ldots\ldots\ldots(5 - 22)$$

The values of constants K_1, K_2, K_3 and K_4 can be estimated from Van den Bosch and Vugtes' results. However, due to the artificial linearization of the experimental curves, the constants governing the position and slope of the straight lines cannot be fixed accurately merely from the experimental curves.

The deduction of reasonably good values for these constants will be discussed in Chapter 6.

5 - 9. Summary:

(i) Three equations (5 - 15), (5 - 17) and (5 - 22) have been derived to give the amplitude of rolling of a ship fitted with a free surface tank stabilizer.

The three equations involve three unknowns - θ_o, ε and M of which only the solution of θ_o is of real interest. However θ_o is involved in each equation to such an extent that it is impossible to rearrange them algebraically so as to give a simple equation in θ_o only.

Therefore the only ~~possible~~ practicable method of solution is by 'trial and error' or reiteration, which is quick and effective using the computer.

For the sake of clarity, the three equations mentioned above are repeated here:-

$$\theta_o = \frac{\Delta \overline{GM} \alpha_o - S m_1 r \omega^2}{\sqrt{\left(C'\omega + \frac{M}{\theta_o} \cos \epsilon\right)^2 + \left(\Delta \overline{GM} - I\omega^2 + \frac{M}{\theta_o} \sin \epsilon\right)^2}}$$

$$M = \left[\frac{b}{2} - \sqrt{\frac{2bh_o}{9\theta_o}}\right] \rho w b h_o$$

$$\epsilon = \left[\frac{K_1}{\theta_o} + K_2\right]\left[\omega/\omega_n' - \frac{K_4}{K_3}(K_3 - \theta_o)\right] - 90$$

where all the symbols have already been defined either in the table of symbols or else in the section in which each equation was derived.

(ii) An equation has been derived which will give the phase angle between the disturbing wave and the rolling ship (equation (5 - 14)).

6 - EXPERIMENT AND THEORY: A COMPARISON

Introduction - Experimental results of stabilized ship rolling - Theoretical results of stabilized ship rolling - Accuracy and discussion of the equations - - Further comparison between theory and practice - Summary.

6 - 1. Introduction:

Equations for the rolling of ships with free-surface tanks have now been developed. Their accuracy can be ascertained only by comparing theoretical results obtained by means of the equations and experimental results.

Through this comparison, the unknown constants (K_1, K_2, K_3, K_4) can be given fairly realistic values.

6 - 2. Experimental results of stabilized ship rolling:

Experiments of the angle of roll at various frequencies and liquid depths were carried out by means of the apparatus described in Chapter 3. The liquid used was water ($\rho = 62{\cdot}5$ lb/in^2).

A specific case was first chosen by means of which the unknown constants were to be determined.

width of tank, $w = 2\frac{1}{2}''$

weight of model, $\Delta = 21$ lb.

half wave height, $r = 5\frac{1}{2}''$

maximum wave slope, $\alpha_o = 5^o$

tank position above c.of g., $S = 4''$

As described in Chapter 3, the parameters b (tank length) and $\overline{GM}$ (metacentric height) are fixed for this apparatus.

$$b = 20''$$
$$\overline{GM} = 3{\cdot}6''$$

The procedure of the experimental work was to determine the angle of roll at frequencies ranging from $\omega = 0$ to a speed well above the natural rolling frequency of the model for different water depths in the tank.

The water depths selected had to be such as to give a wide range of values while not exceeding the amount of water which would be the limit, practically speaking, in actual ships. Naval architects would generally like to limit the weight of fluid used in stabilization to 5% of the dead weight of the ship and would never allow it to exceed 10%. Consequently in selecting the depth of water, the amount was limited to give a maximum ratio $^{gm_1}/_{\Delta}$ of 10% approximately. (m_1 is the mass of liquid in the tank).

The above procedure was repeated three times in order to obtain a mean value which would overcome many of the inherent inaccuracies.

These inaccuracies arise as follows:

(i) Due to the low power of the driving motor, the speed of the mechanism fluctuates if the arms are not perfectly balanced.

(ii) The speed reading fluctuates continuously and cannot be maintained absolutely steady.

(iii) The potentiometers yield inaccuracies due to the thickness of the wire in the windings.

(iv) The water depth can vary fractionally owing to splashing, leakage and inaccuracy in reading.

The results obtained from the above experiment are shown in Table II and are presented graphically in Figs.6 - 1 (a),(b),(c) and (d).

Tests with different model weights and tank widths were also conducted and will be discussed in section 6 - 5.

6 - 3. Theoretical results of stabilized ship rolling:

Before solving equations (5 - 15), (5 - 17) and (5 - 22) it is necessary to determine some approximate values for the contacts K_1, K_2, K_3 and K_4.

From Van den Bosch and Vugts' results and from Figs.5 - 10 and 5 - 11 the following starting values for the constants were assumed:-

$$K_1 = 0{\cdot}17$$
$$K_2 = 0{\cdot}8$$
$$K_3 = 0{\cdot}6$$
$$K_4 = 0{\cdot}5$$

The three equations (5 - 15), (5 - 17) and (5 - 22) were programmed for the computer to give a solution for θ_o by a reiterative process. The above constants were introduced in such a way as to make them easily changeable.

The parameters in the equations were fixed to the same values as for the model in the experimental work (section 6 - 2) and ω was made to vary from 0 to 4·6 rad/sec (well above the natural frequency of the model).

TABLE II (Experimental angle of roll)

Depth h_o inches	Frequency of oscillation (rad/sec)												
	0·6	1·2	1·5	1·8	2·1	2·4	2·7	3·0	3·3	3·6	3·9	4·2	
1·00	15·5	15·8	15·5	14·5	10·5	6·0	4·5	5·0	7·0	8·5	9·5	9·0	Angles
0·75	12·7	13·0	12·7	11·2	8·0	5·5	6·1	8·0	9·5	10·0	9·5	7·5	of roll
0·50	10·5	10·5	10·0	8·0	6·0	5·7	6·5	8·7	14·0	12·5	9·0	6·7	in
0·25	7·7	6·8	6·0	5·8	6·5	7·5	10·5	19·0	-	20·0	9·5	7·0	degrees

The water depth, h_o, was given the same values as the experimental depths i.e. 0·25", 0·5", 0·75" and 1·00".

With the constants as indicated above, the theoretical results thus obtained varied rather unpleasantly from the experimental values although the shapes of the curves had the same characteristics.

The similarity in shape suggested that the basic principles of the equations were correct and that the divergence was due principally to errors in the constants.

Therefore a number of systematic changes in the constants were made to ascertain whether or not values could be obtained which would make the equations acceptable as they stand. Results of this attempt seemed encouraging and consequently the program was rearranged so as to determine the best values for the constants as easily as possible.

The following values for the constants were obtained:

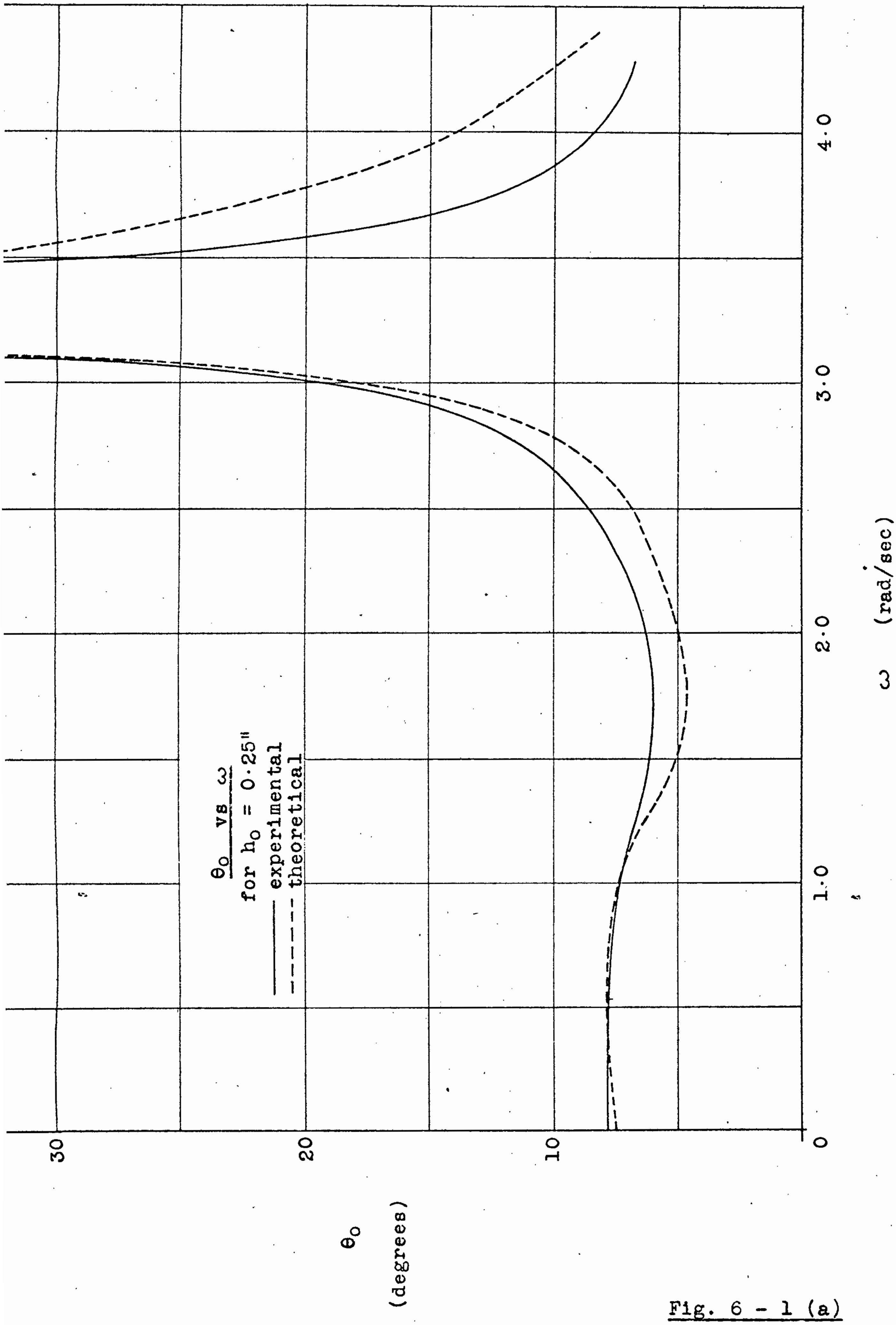
$$\left.\begin{array}{l} K_1 = 0{\cdot}20 \\ K_2 = 0{\cdot}86 \\ K_3 = 0{\cdot}76 \\ K_4 = 0{\cdot}42 \end{array}\right) \quad \ldots\ldots\ldots\ldots\ldots(6-1)$$

Giving the solutions for θ_o which are listed in Table III and plotted alongside the experimental results in Figs.6 - 1 (a) to (d) in order to facilitate comparison.

It is interesting to note that only very slight changes in some of the constants radically affect the values of the angle of roll.

TABLE III (Theoretical angle of roll)

Depth h_o inches	Frequency of oscillation (rad/sec)												
	0·6	1·2	1·5	1·8	2·1	2·4	2·7	3·0	3·3	3·6	3·9	4·2	
1·00	13·2	14·9	15·2	14·9	12·7	7·4	6·0	5·7	6·0	6·5	7·2	7·8	Angles
0·75	11·7	12·6	12·6	11·7	8·7	5·7	5·2	5·8	7·0	8·6	9·5	9·7	of roll
0·50	9·7	10·1	9·3	7·4	5·0	4·6	5·5	7·8	13·4	15·7	14·5	12·3	in
0·25	7·7	6·6	5·0	4·6	5·2	6·2	8·5	18·2	-	27·2	16·0	10·5	degrees



θo vs ω
for ho = 0·25"
—— experimental
- - - theoretical
30
20
10
0
1·0
2·0
3·0
4·0
ω (rad/sec)
θo
(degrees)

Fig. 6 - 1 (a)

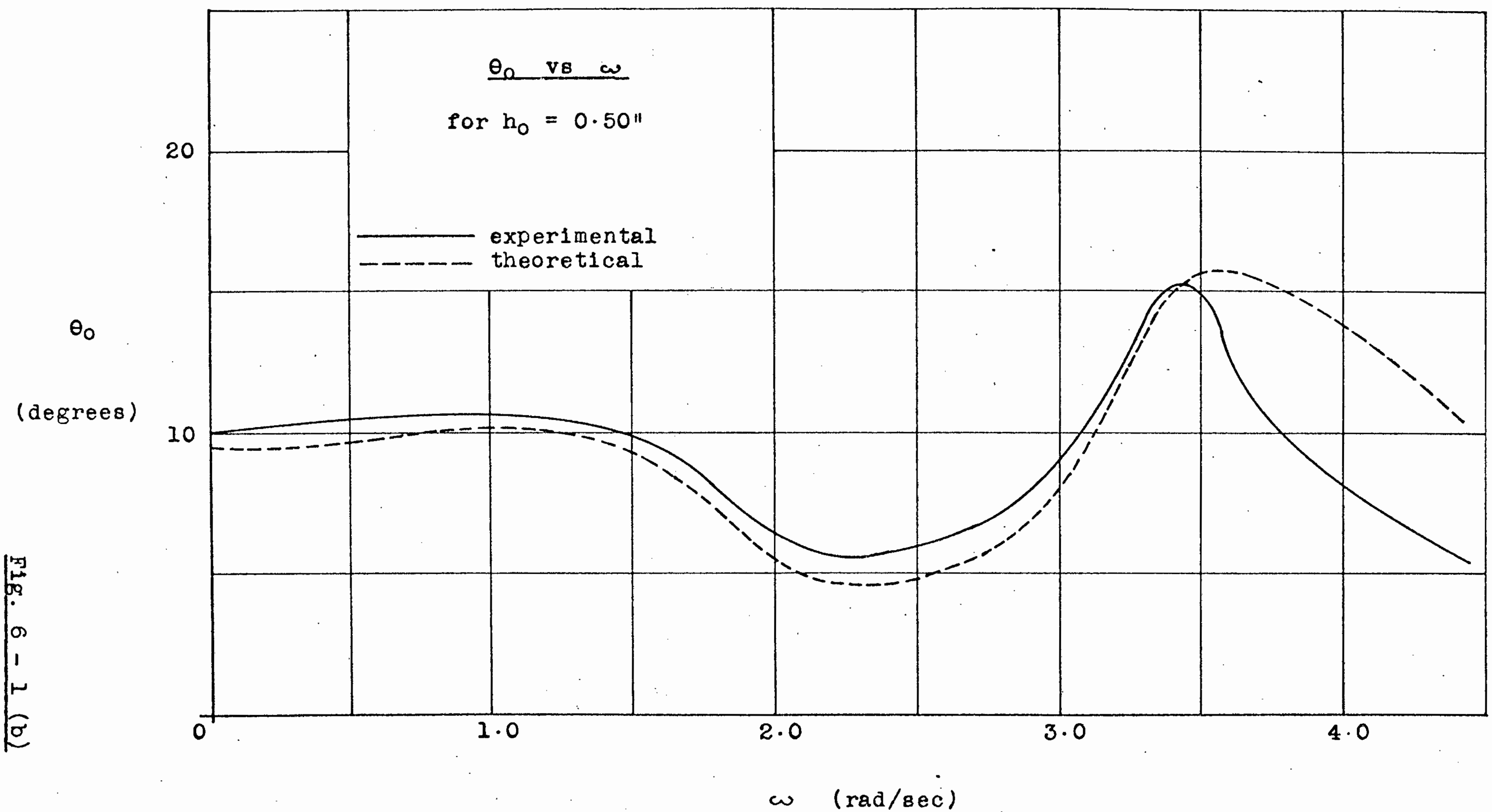

θo vs ω
for ho = 0·50"
experimental
theoretical
20
10
0
θo
(degrees)
1·0
2·0
3·0
4·0
ω (rad/sec)

Fig. 6 - 1 (b)

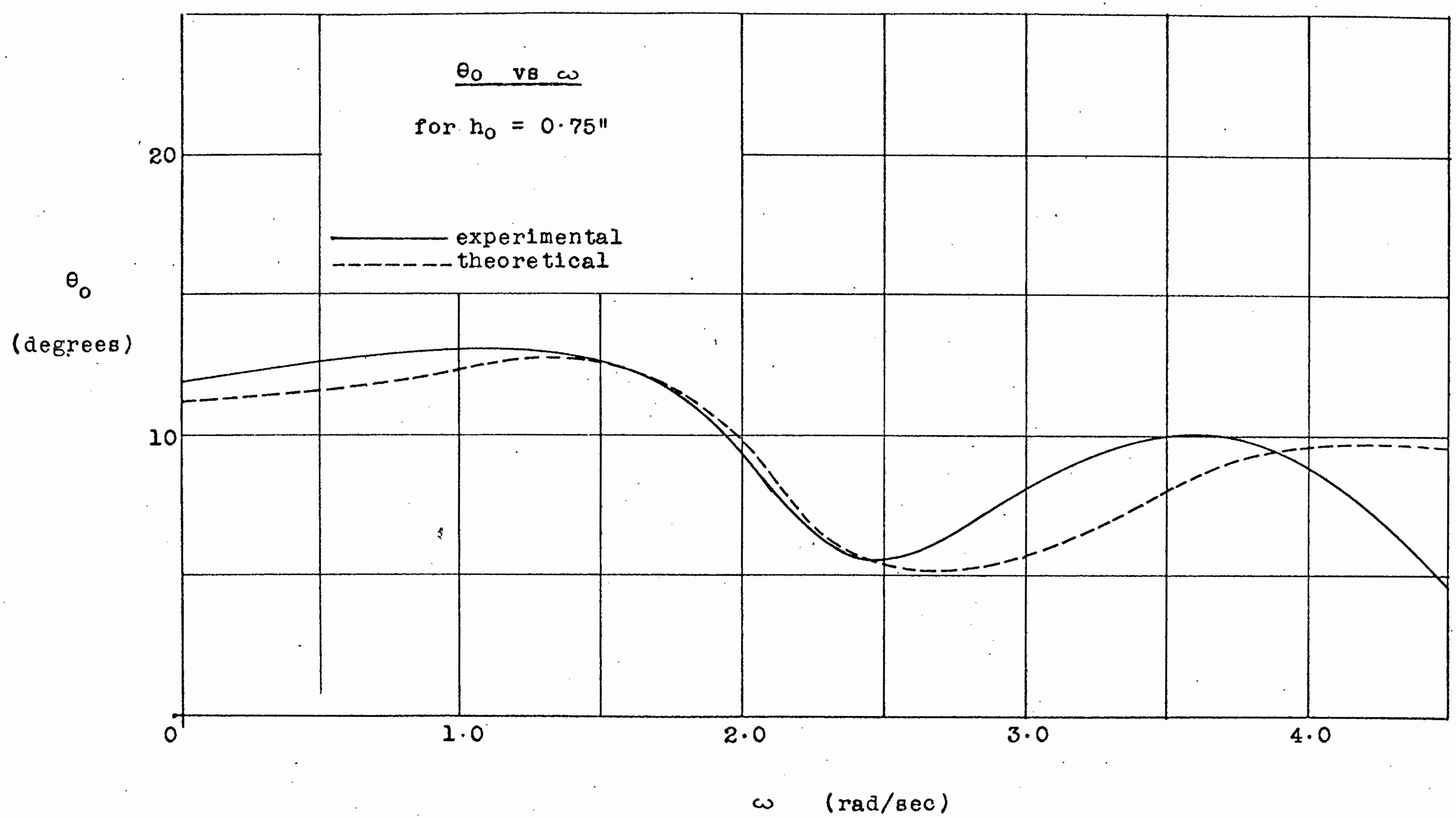

θ_o vs ω
for h_o = 0·75"
experimental
theoretical
θ_o
(degrees)
20
10
0
1·0
2·0
3·0
4·0
ω (rad/sec)

Fig. 6 - 1 (c)

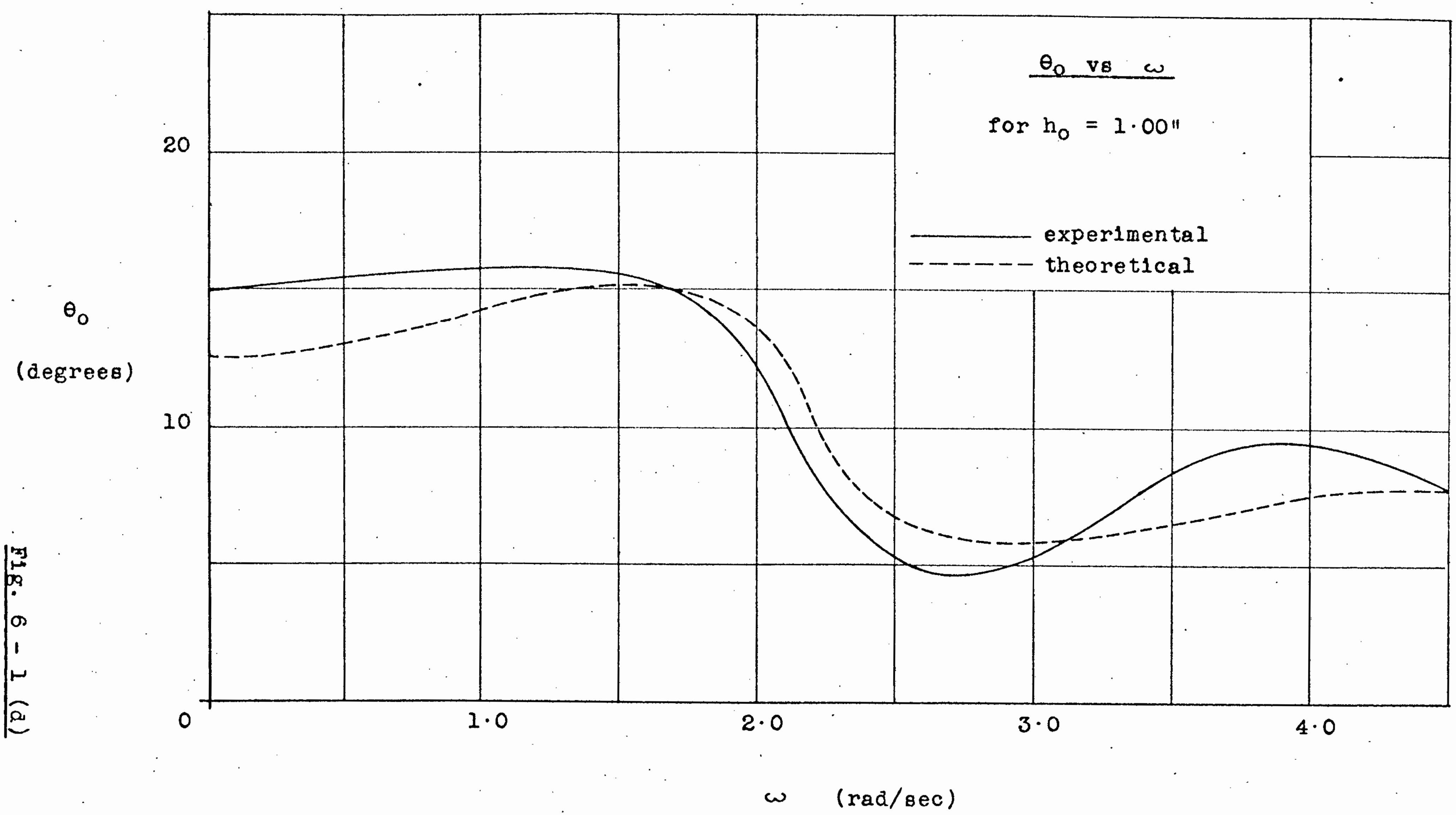

θo vs ω
for ho = 1·00"
experimental
theoretical
θo
(degrees)
20
10
0
1·0
2·0
3·0
4·0
ω (rad/sec)

Fig. 6 - 1 (d)

This shows how sensitive the final result is to changes in the phase angle ϵ. Consequently, the linearization of the ϵ vs ω/ω_n' curves must introduce some fairly serious errors.

Henceforth equations (6 - 1) will be assumed to give the true constants, and therefore the full description of the motion is known.

6 - 4. <u>Accuracy and discussion of the results</u>:

The curves in Figs.6 - 1 compare the experimental with the theoretical solution of the angle of roll. It cannot be pretended that the curves are identical, particularly at higher frequencies. However it is obvious that the characteristics of the curves are the same and that the maximum and minimum angles of roll are very similar (particularly at the lower frequencies).

The discrepancy between experimental and theoretical becomes increasingly marked as the frequency rises. This is easily explained by the increasing inaccuracy of ϵ at these frequencies (as mentioned in section 6 - 3).

However we can consider that a sea wave with a period less than 5 to 6 secs will in practice have little effect on rolling because it is too small in comparison to the dimensions of an average ship. This speed is equal to about 3·3 rad/sec at a $1/10^{th}$ scale. Now, from the graphs we can see that up to this speed the theoretical curve is fairly true.

Therefore we can safely conclude that the theoretical results are accurate enough to be useful for wave conditions that normally cause heavy rolling.

At this stage it must be repeated that both the experimental and theoretical work is based on Froude's equation of rolling, the merits of which were discussed in Chapter 2.

As mentioned before, the inaccuracies incurred by using this equation tend to make θ_o appear larger than it should be. Furthermore, it must be noted that when the equations give a value for θ_o which is large, the solution is unacceptable since the derivations are for small angles of roll only. Actual ships are unable to roll to very large angles because of the shape of the hull which causes a large damping force at large angles of roll. Most ships have a rolling limit of $\theta_o \leqslant 30^o$ approximately.

Therefore these equations can be valid only when their solution gives a fairly small θ_o, certainly not greater than 30°.

6 - 5. Further comparisons between theory and practice:

In order to give the validity of the theoretically derived solutions more weight, further experimental results were obtained for different model weights and tank widths. It is of interest to present one such set of results for -

$$w = 1''$$

$$\Delta = 14 \text{ lb.}$$

$$\alpha_o = 5^o$$

(as shown, § 7 - 1, the half wave height, r, is immaterial).

The same conditions were applied to the theoretical equations.

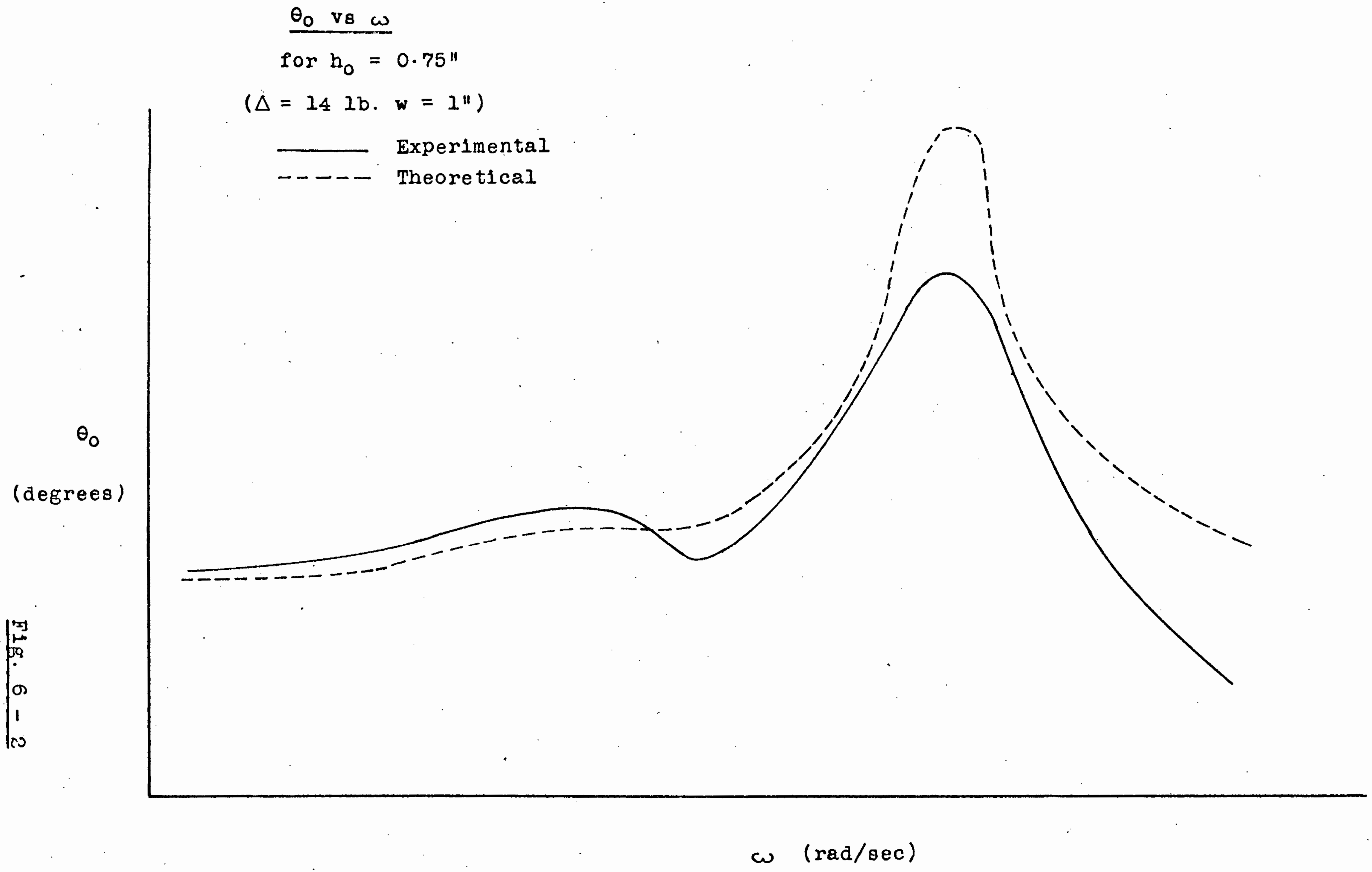
Both the experimental and theoretical results are plotted in Fig. 6 - 2 for a water depth chosen at random and the agreement can be easily ascertained.

6 - 6. Summary:

Experimental results of ship rolling with a free-surface tank were obtained with the apparatus of Chapter 3. These results were compared to theoretically derived values using equations (5 - 15) (5 - 17) and (5 - 22) and arbitrary values for the unknown constants.

The constants were slightly altered until the theoretical and experimental results agreed fairly well. With these constants, the rolling with free-surface tanks is completely known and the equations and constants are used to show the validity of the theory at some other tank width and ship weight.

Although the theoretical and experimental solutions are not always identical, the characteristics are so similar that the basis of the derivations is most probably correct. The errors appearing in the theoretical results can be attributed to the inaccuracy of one or more of the many assumptions and simplifications made in obtaining the equations.



θo vs ω
for ho = 0·75"
(Δ = 14 lb. w = 1")
Experimental
Theoretical
θo
(degrees)
ω (rad/sec)

Fig. 6 - 2

7 - GENERAL DISCUSSION

Discussion of the apparatus - Discussion of the derived equations - The quality of roll stabilization - - Possibilities for further investigation.

7 - 1. Discussion of the apparatus:

The apparatus used (design in Chapter 3) proved very satisfactory in conducting the necessary experimental work. However, some criticism of a few aspects of the apparatus will not be out of place.

(i) The wave simulator was built to simulate waves of different heights. This was found to be unnecessary since, in this work, the effect of the wave height on the angle of roll is negligible.

(ii) The simulator was designed so that different maximum wave slopes could be simulated. Again this was found to be superfluous as an almost linear relationship exists between this angle and the amplitude of the angle of roll.

(iii) Although both a release mechanism and an event marker were wired into the apparatus, it was subsequently found that neither was essential for the particular work being carried out.

(iv) An oil damper was constructed but not used because the inherent friction supplied all the natural damping required.

The above four points show that the apparatus is too elaborate for the purpose of this investigation. However, as it allows for the alteration of many variables, it can be considered very useful equipment for other experimental work in the rolling of ships.

7 - 2. Discussion of the derived equations:

The derived equations (5 - 15), (5 - 17), (5 - 22) and (6 - 1) are open to a good deal of criticism as well as improvement. Following is a brief appraisal of their qualities and failings:-

(i) Equation (5 - 15) is correct if Froude's equation of rolling is recognized, since no consequential assumptions are made besides the neglect of a few terms due to their smallness.

(ii) Equation (5 - 17) can not be passed quite as satisfactorily. The value of M is derived for a particular known condition and its behaviour is assumed to be consistent for other conditions. The possible correctness of this assumption is shown by the closeness between the theoretically derived and experimentally observed angles of roll.

A major shortcoming of this equation as it stands may be that M is entirely independent of S (the height of the tank above the centre of gravity of the ship). Throughout the derivation, the increase or decrease of the moment arm due to S has been neglected. The expected effect of this is that M appears smaller than it should be for S positive (tank above the centre of gravity) and vice versa for S negative (tank below the centre of gravity). No error should exist for S = 0.

The magnitude of this neglected effect could be determined either by introducing the neglected terms in the derivation or else by altering the construction of the apparatus so that the relationship between S and θ_o can be tested.

(iii) Equation (5 - 17) is almost entirely based on experimental observations and somewhat wide assumptions and approximations. Again, its accuracy can be judged only by comparing theoretical and experimental results. Relatively small changes in the phase angle can cause large changes in the solution to the angle of roll. Therefore little store can be set by this equation and most deviations between theory and experiment are probably due to its inaccuracy.

(iv) Equation (6 - 1) is entirely dependent on all the foregoing relationships and may not even exist in fact but for the assumptions made particularly in equation (5 - 22). The values given by equation (6 - 1) could probably by revised even subject to the already assumed equations because these is no guarantee that the best figures were obtained.

7 - 3. The quality of roll stabilization:

It is quite evident that roll stabilization by means of rectangular free-surface tanks is considerable. To obtain a clear picture of the extent of stabilization, Fig. 7 - 1 shows the response of the undamped model plotted on the same axes as that of the model at more or less its optimum damping (by means of a free-surface tank). The low frequency angle of roll

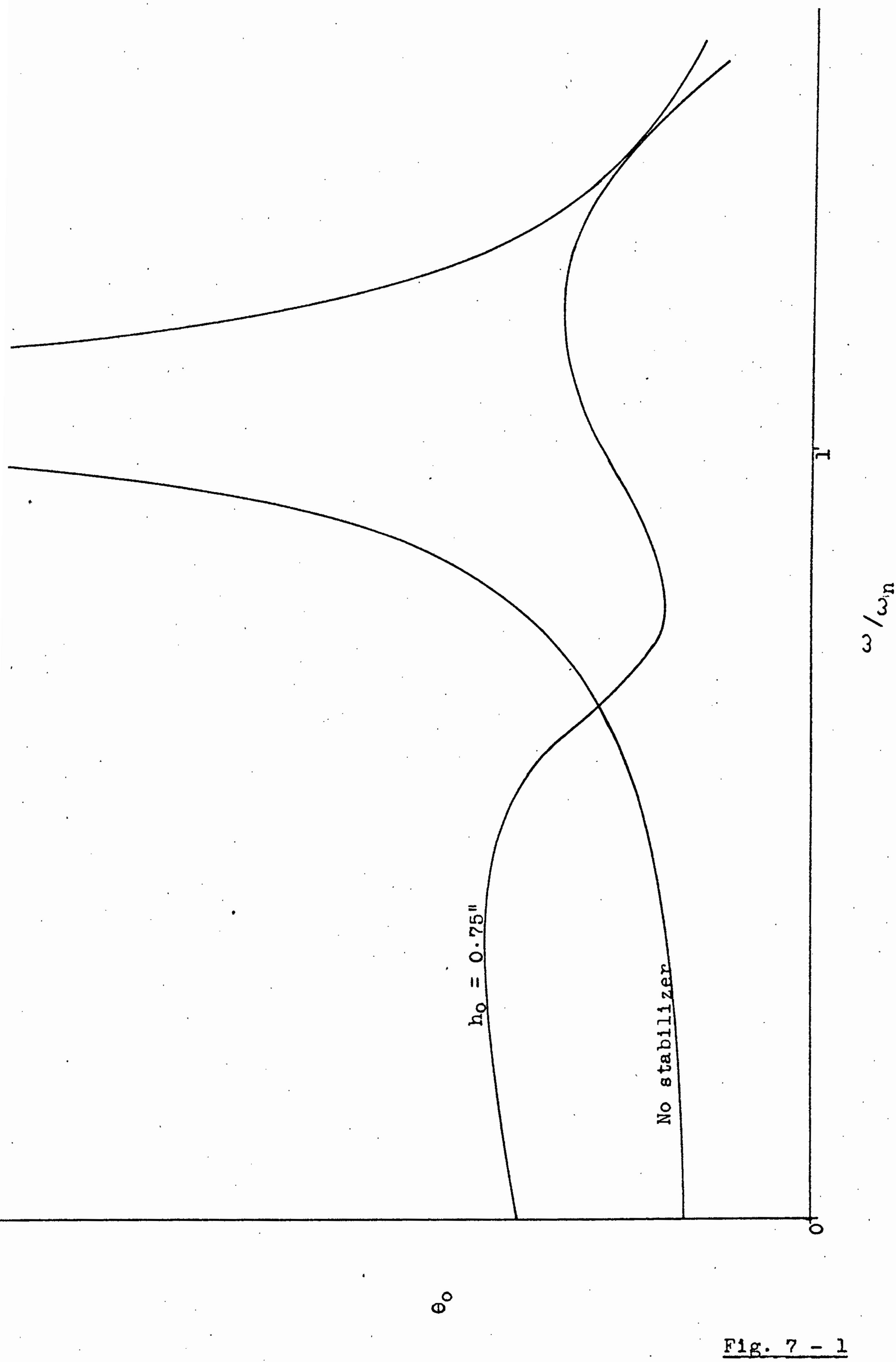


Fig. 7 - 1

is increased but there is no danger at all that the roll will build up when the wave frequency approaches the natural frequency of the vessel. Hence there is no possibility of dangerous unpredicted rolling angles when waves of an unusual frequency occur.

The actual angles of roll and the percentage reduction in roll angle can not be determined accurately by the analysis or experiment in this work. The failings in the analysis have been discussed in the previous section. The inaccuracies entering the experimental work are primarily the simulation of Froude's idealised equation of rolling and the assumption of the damping as a first order term at all times.

A further large inaccuracy in determining the actual angle of roll of a ship is the assumption that the periods of a train of disturbing waves are uniform. In practice this is not true at all. An analysis, to determine the response to some realistic spectrum of wave periods, must be carried out in order to make good this inaccuracy.

Consequently, the results of this investigation are useful only for comparing various tank dimensions and liquid depths and hence determining values at which stabilization is optimum. What the actual quantitative response is at this condition can be estimated only approximately.

Tets on actual ships or scale models would give a clearer indication of the difference between the response found by the analysis and that of an actual vessel.

However it is fairly safe to assume that the actual angles of roll will always be smaller than the results obtained analytically (cf. section 2 - 7).

7 - 4. Possibilities for further investigation:

To date little research has been done in the field of free-surface tank stabilizers and the possibilities for further research are almost limitless. Following is a short list of possible investigations. Some of these are a logical sequence to this work.

(i) The use of two or more rectangular tanks in parallel, each having a different liquid depth.

(ii) Modifying the tank shape and introducing flow constrictions. Of course, this has an infinite number of possibilities and some intuition is needed in order to obtain worthwhile results.

(iii) The accurate determination of the phase angle between the moment due to the moving liquid and the moment due to the oscillation of the ship.

(iv) A novel, yet interesting, research would be to investigate the possibilities of making profitable use of some of the energy of the moving water in a free-surface tank. This could be a source of auxiliary power at sea.

8 - A GUIDE TO THE DESIGN OF STABILIZING TANKS.

Introduction - Conditions for best stabilization - - Design parameters - Determination of optimum stabilization - Discussion of the 'Optimum Stabilization Chart' - Use of the Chart and tank design - The position of the tank - A tank for the "T.B. Davie".

8 - 1. Introduction:

Having obtained a theoretical equation to describe the action of a free-surface stabilizing tank on the rolling of ships, some use must be made of it. In this chapter an attempt is made at co-ordinating all the varying parameters involved in this problem and determining an optimum condition that could be used as a basis for design.

8 - 2. Conditions for best stabilization:

Figs. 6 - 1 show quite clearly that best stabilization does not necessarily occur with the greatest liquid level. The stabilization at h_o = 0·5" (Fig. 6 - 1(b)) is obviously better than at h_o = 1·00" (Fig. 6 - 1(d)). Although it is not possible to lay down a hard and fast rule of what the best possible stabilization means, it is reasonable to take this condition as occurring when the mean angle of roll over a significant range of frequencies is at a minimum i.e. best stabilization occurs when

$$\int_{\omega_1}^{\omega_2} \theta_o \, d\omega \quad \text{is a minimum.}$$

The range of wave frequencies, ω_1 to ω_2 must be representative of the waves generally encountered at sea. It has been said before that waves with a period less than 5 sec. are small and not particularly harmful, while waves with a period larger than 12 sec. are unusual and can be considered as freak.

Therefore the range over which the mean angle of roll is determined should be -

period: 5 sec. - 12 sec.

or 1·25 rad/sec - 0·52 rad/sec

Then stabilization is optimum when

$\int_{0\cdot 52}^{1\cdot 25} \theta_o \, d\omega$ is a minimum for any particular conditions.

8 - 3. Design parameters:

The angle of roll, θ_o, is given by equations (5 - 15), (5 - 17), (5 - 22) and (6 - 1).

Substituting (5 - 17) in (5 - 15) we have -

$$\theta_o = \frac{\Delta \overline{GM} \alpha_o - Sm_1 \omega^2 r}{\sqrt{\left[C'\omega + \left(\frac{b}{2} - \sqrt{\frac{2bh_o}{9\theta_o}}\right)\frac{\rho w b h_o}{\theta_o} \cos \epsilon\right]^2 + \left[\Delta \overline{GM} - I\omega^2 + \left(\frac{b}{2} - \sqrt{\frac{2bh_o}{9\theta_o}}\right)\frac{\rho w b h_o}{\theta} \sin \epsilon\right]^2}}$$

Now $C'\omega$ is small compared to the other term in the same bracket (except when $\cos \epsilon = 0$, but then the magnitude of the term is insignificant) and can be neglected.

Also, $\Delta \overline{GM} \alpha_o \gg Sm_1\omega^2 r$

Therefore the latter term can be neglected.

Hence the equation can be rewritten as:

(see overleaf)

$$\theta_o = \frac{\alpha_o}{\sqrt{\left[\left(\frac{1}{2} - \sqrt{\frac{2h_o}{9b\theta_o}}\right) \frac{\rho w b^2 h_o}{\Delta GM \theta_o} \cos \epsilon\right]^2 + \left[1 - \left(\omega/_{\omega_n}\right)^2 \left(\frac{1}{2} - \sqrt{\frac{2h_o}{9b\,\theta_o}}\right) \frac{\rho w b^2 h_o}{\Delta GM\ \theta_o} \sin \epsilon\right]^2}} \quad \text{..........} \ (8 - 1)$$

In Chapter 3 it is explained that there is a limiting ratio of $^w/_\Delta$ for which a ship will retain its erect position when undisturbed. This ratio is given by equation (3 - 11):-

$$\Delta = \frac{\rho b^3 w}{12\ \overline{GM}}$$

Since this equation gives the limit it is suggested in Chapter 3 that a safety factor of 25% should be included so that

$$^w/_\Delta < \frac{10\ \overline{GM}}{\rho\ b^3}$$

Rewriting this more generally -

$$^w/_\Delta = \frac{F\ \overline{GM}}{\rho b^3} \quad \text{..........} \ (8 - 2)$$

where F is some constant between 0 and 10.

Substituting (8 - 2) in (8 - 1) and putting $D = \frac{h_o}{b}$, where D is a dimensionless number:-

$$\theta_o = \frac{\alpha_o}{\sqrt{\left[\left(\frac{1}{2} - \sqrt{\frac{2D}{9\theta_o}}\right) \frac{FD}{\theta_o} \cos \epsilon\right]^2 + \left[1 - \left(\omega/_{\omega_n}\right)^2 + \left(\frac{1}{2} - \sqrt{\frac{2D}{9\ \theta_o}}\right) \frac{FD}{\theta_o} \sin\ \epsilon\right]^2}} \quad \text{..........} \ (8 - 3)$$

Equation (8 - 3) gives θ_o in terms of the parameters:- α_o, D, F, $^\omega/_{\omega_n}$ and ϵ.

When obtaining $\int \theta_o d\omega$ over a certain range of frequencies, ω is known and α_o is assumed (its value is not important since it is directly proportional to θ_o). Then θ_o is only dependent on ω_n, F, D and ϵ.

However, substituting (6 - 1) in (5 - 22) gives

$$\epsilon = (\frac{0\cdot 2}{\theta_o} + 0\cdot 86)\left[{}^{\omega}/_{\omega_n'} - \frac{0\cdot 42}{0\cdot 76}(0\cdot 76 - \theta_o)\right]$$

and from equation (5 - 19),

$$\omega_n' = \frac{\pi\sqrt{gh_o}}{b} = \pi\sqrt{\frac{g}{b}D}$$

$$\therefore \quad \epsilon = (\frac{0\cdot 2}{\theta_o} + 0\cdot 86)\left[\frac{\omega}{\pi\sqrt{D\ {}^g/_b}} - 0\cdot 55(0\cdot 76 - \theta_o)\right] \quad \ldots\ldots\ldots\ldots (8 - 4)$$

Therefore $\epsilon = f(\theta_o, D, b)$

but $\theta_o = f(F, D, \epsilon, \omega_n)$

$\therefore$ $\theta_o = f(F, D, b, \omega_n)$ and is given by equations (8 - 3) and (8 - 4).

Hence we can conclude that the quality of stabilization depends on the parameters ω_n, F, D and b.

8 - 4. Determination of optimum stabilization:

If a stabilizing tank is to be designed for a particular ship, the variables that become fixed are ω_n (fixed for any ship) and b (approximately equal to the beam of the ship). Then optimum stabilization depends only on F and D.

By integrating θ_o graphically optimum F and D can be found for any particular values of ω_n and b.

However these results will be completely independent of the liquid weight to ship weight ratio consideration. This may cause the optimum stabilization to occur for a weight ratio well above that permissible for any particular vessel.

Therefore the optimum stabilization at limited weight ratio should also be found:-

$$\text{weight ratio, } P = \frac{\rho\, b w h_o}{\Delta}$$

Using equation (8 - 2),

$$P = \frac{F\,\overline{GM}\,D}{b}$$

or $$FD = \frac{Pb}{\overline{GM}} = Q \qquad \text{.......... (8 - 5)}$$

Now Q is fixed for any ship with a limited P. By taking a number of actual ships as examples it can easily be shown that Q generally varies from 0·2 to 1.

$$\text{i.e.} \quad 0{\cdot}2 \leqslant Q \leqslant 1$$

It has been shown in section 8 - 3 that

$$0 \leqslant F \leqslant 10$$

The range of values for D can not be proved as convincingly, but experiments have shown that D = 0·15 is more or less the maximum value to obtain the liquid behaviours discussed in this work. Furthermore, it is interesting to note that experimenters Vugts and van den Bosch do not consider any values of D above 0·11.

$$\therefore \quad 0 \leqslant D \leqslant 0{\cdot}15$$

Graphical integration (by means of computer) of θ_o from equation (8 - 3) and (8 - 4) at various values of ω_n and b and for varying F, D and Q, yielded mean angles of roll whose

"OPTIMUM STABILIZATION CHART"

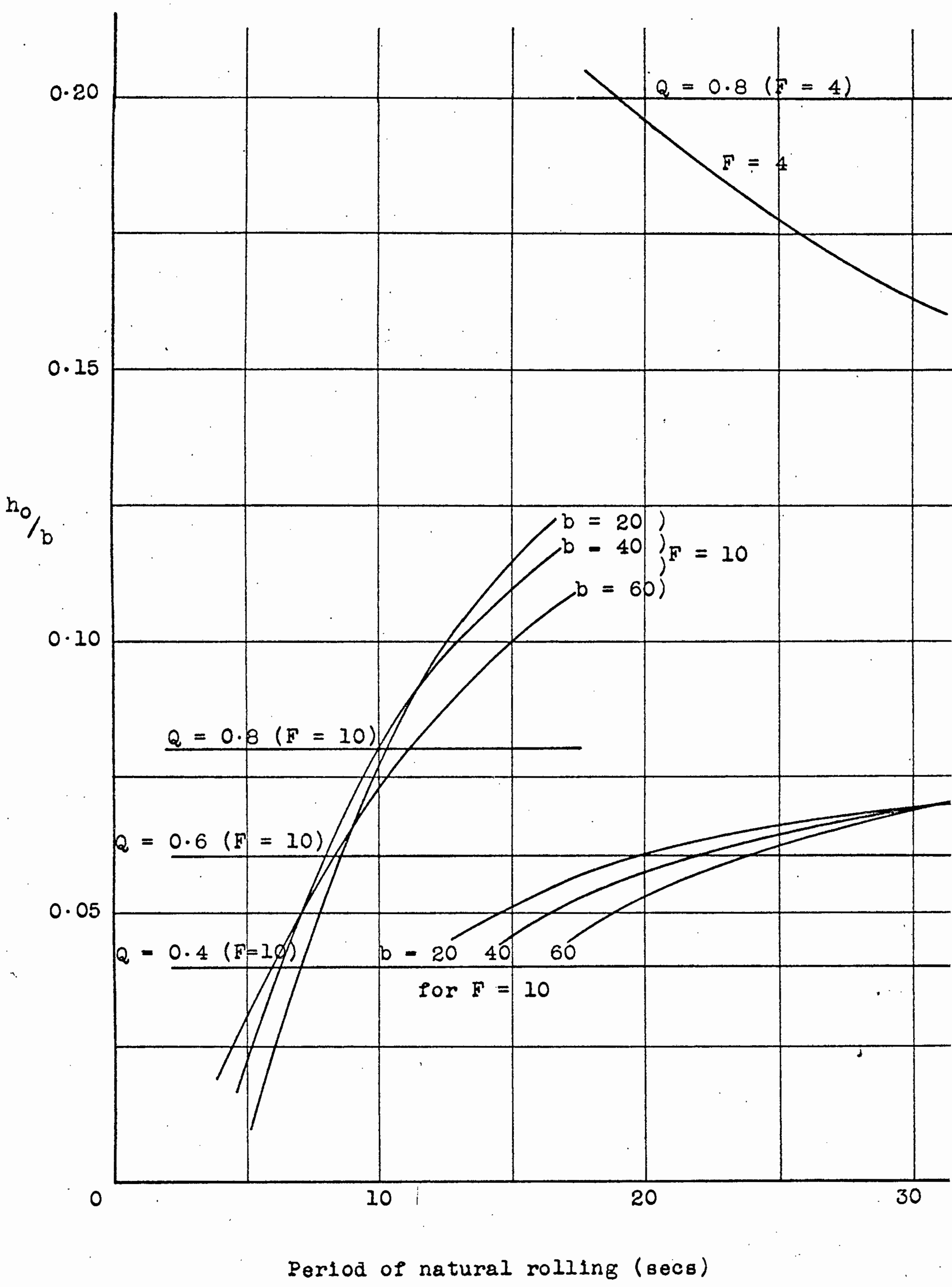


Fig. 8 - 1

minimums were plotted to give the 'Optimum Stabilization Chart', Fig. 8 - 1.

8 - 5. Discussion of the 'Optimum Stabilization Chart':

Over the whole range of natural rolling periods considered, the optimum stabilization actually occurs at a value of F greater than 10. Therefore the values of D at $F = 10$ must be taken (this is the maximum tank width). (Note that the optimum stabilization lies on two separate curves).

Since $Q = FD$ and $F = 10$, optimum stabilization for limited liquid weight will be at a constant D $(= \frac{Q}{10})$ for any particular value of Q.

The surprizing exception to the above is an additional optimum curve for $F \hat{=} 4$ at very low natural frequencies. However this curve gives values of D above 0·15 and therefore cannot be entirely trusted. This curve is tentatively given in Fig.8 - 1 up to $D = 0{\cdot}2$.

Fig. 8 - 1 shows that optimum D does not vary a great deal with changes in b so that interpolation for values inbetween those given should be easy. Since the curve for $F = 4$ is purely tentative, no attempt has been made to plot it for different values of b.

8 - 6. Use of the Chart and tank design:

In order to design a tank for a ship:-

(i) b must be known (approximately equal to the beam)

(ii) The natural period of roll must be known. This can be found by: $T = 2\pi\sqrt{\frac{gGM}{k^2}}$ where k is the radius of gyration and $\overline{GM}$ is the metacentric height.

(iii) The dead weight of the ship must be known.

(a) If there is no limit to the liquid weight to dead weight ratio:-

The optimum value of D is found from the optimum stabilization curve (F = 10) for the known values of b and T.

Then the required tank dimensions will be -

the tank width, $w = \frac{10\,\overline{GM}\,\Delta}{\rho\, b^3}$ (from eqn.8 - 2).

the liquid depth, $h_o = Db$

and the tank length = b.

(b) If the maximum permissible liquid weight to ship weight ratio is P:-

Q is found from equation (8 - 5) -

$$Q = \frac{Pb}{\overline{GM}}$$

The optimum value of D is found from the optimum stabilization curve (F = 10) for the known values of b and T.

Now the maximum permissible value of D is $\frac{Q}{10}$

$\therefore$ if $D_{opt} > \frac{Q}{10}$, then take $D = \frac{Q}{10}$.

But if $\frac{Q}{10} > D_{opt}$, take $D = D_{opt}$.

Hence the required dimemsions are -

$$\text{tank length} = b$$

$$\text{tank width, } w = \frac{10\,\overline{GM}\,\Delta}{\rho\, b^3}$$

$$\text{liquid depth, } h_o = Db$$

(Note. The curve of F = 4 may also be usable at very low natural frequencies).

8 - 7. The position of the tank:

From the discussion of Chapter 7, we see that the tank performance improves as the tank position is raised.

Therefore the tank should be fitted as high as possible in the ship.

Along the fore and aft axis the tank should be placed so that its moment is in the same plane as the disturbing moment, otherwise the couple formed will tend to swing the ship. This plane is perpendicular to the longitudinal axis and lies approximately at the centre of gravity of the ship.

Therefore the tank should be mounted as near the centre of gravity as possible and as high above it as possible.

8 - 8. A Tank for the "T.B. Davie":

To conclude, an example of interest might be to determine the optimum tank dimensions for the "T.B. Davie". It is assumed that there is no limit to the weight ratio.

Although the final figures for the dimensions of the vessel are not available, the following values will be used -

$$\overline{GM} = 3 \text{ ft.}$$

$$k = 9\tfrac{1}{2} \text{ ft.}$$

$$\text{beam} \doteq b = 20 \text{ ft.}$$

$$\Delta = 200 \text{ tons.}$$

The stabilizing liquid is water, $\rho = 62 \cdot 4 \text{ lb/ft}^3$.

$$\therefore \quad T = 2\pi\sqrt{\frac{32{\cdot}2 \times 3}{9{\cdot}5^2}}$$

$$= 6{\cdot}5 \text{ sec.}$$

From the optimum stabilization curve, for $T = 6{\cdot}5$ sec and $b = 20$ft -

$$D = 0{\cdot}0325$$

$$\text{The tank width, } w = \frac{10 \times 3 \times 200 \times 2000}{62{\cdot}4 \times 20^3} \text{ ft.}$$

$$= 24 \text{ ft.}$$

$$\text{The tank length} = 20 \text{ ft.}$$

$$\text{The water depth} = 20 \times 0{\cdot}0325 \text{ ft.}$$

$$= 0{\cdot}65 \text{ ft} \quad (7\tfrac{3}{4}'')$$

To check whether this gives a reasonable weight ratio -

$$P = \frac{24 \times 20 \times 0{\cdot}65 \times 62{\cdot}4}{200 \times 2000}$$

$$= 0{\cdot}049$$

$$\simeq 5\%$$

This seems a very reasonable percentage and is probably acceptable for this research vessel.

$\therefore$ The suggested dimensions of a free-surface stabilizing tank for the "T.B. Davie", based on the available data (given above), are:

length = 20 ft.

width = 24 ft

water depth = 0.65 ft.

The best position for the tank is as high as possible and longitudinally as near the centre of gravity as possible.

This design will give a minimum mean angle of roll over the range of frequencies of waves commonly encountered.

However, the angle of roll at very low frequencies will be larger than without stabilizing tanks; therefore when rolling is caused by very long waves (low frequency), the water in the tank should be let out or reduced.

The average wave has a period of 8 sec. and for this and higher frequencies the stabilizing tank should reduce the rolling.

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