THE CONSTRUCTION OF OPTIMAL SOCIALLY RESPONSIBLE INVESTMENT PORTFOLIOS IN SOUTH AFRICA USING TRADITIONAL AND ARTIFICIAL INTELLIGENCE TECHNIQUES

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DECLARATION

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ABSTRACT

For decades, scholars and practitioners have sought optimal portfolio construction methods. Traditional approaches, like mean-variance, face challenges with complex non-linear and non-convex models. Recently, meta-heuristic artificial intelligence (AI) algorithms have enhanced portfolio construction by addressing such constraints. Socially responsible investment (SRI) has gained popularity for its focus on sustainability, but using Environmental, Social and Governance (ESG) criteria in constructing SRI portfolios can introduce estimation risks, increasing the uncertainty of the input parameters and reducing diversification compared to non-SRI portfolios. This study evaluates six portfolio construction methods for SRI portfolios in South Africa, including traditional (mean-variance, naïve and risk parity) and AI (particle swarm optimization, simulated annealing and genetic algorithm) methods. Portfolios are compared based on risk-adjusted returns, diversification and stability.

On average, AI algorithms produced optimal SRI portfolios with higher risk-adjusted returns. During a period of positive market returns, the genetic algorithm approach performed best, while the mean-variance approach dominated during a period marked by downturns in the market. Across all periods, the genetic algorithm consistently outperformed other methods for SRI portfolios. In contrast, for non-SRI portfolios, the mean-variance method led, followed by genetic algorithm and simulated annealing. Overall, meta-heuristic approaches yielded superior performance for both constrained (SRI) and non-constrained (non-SRI) portfolios, although with higher concentration and less stable weights. Based on the Sharpe ratio, SRI portfolios initially outperformed non-SRI portfolios but lagged in the second period. Non-SRI portfolios ultimately outperformed, suggesting that while AI approaches enhance portfolio construction, SRI strategies may not always match conventional investments.

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LISTS OF ABBREVIATIONS

AI – Artificial Intelligence

CSR – Corporate Social Responsibility

ESG – Environmental, Social and Governance

GA – Genetic Algorithm

MV – Mean-Variance

non-SRI – non-Socially Responsible Investing

PSO – Particle Swarm Optimization

RP – Risk Parity

SA – Simulated Annealing

SIF – Sustainable and Responsible Investment

SRI – Socially Responsible Investing

U.K – United Kingdom

U.S – United States

1/N – Naïve

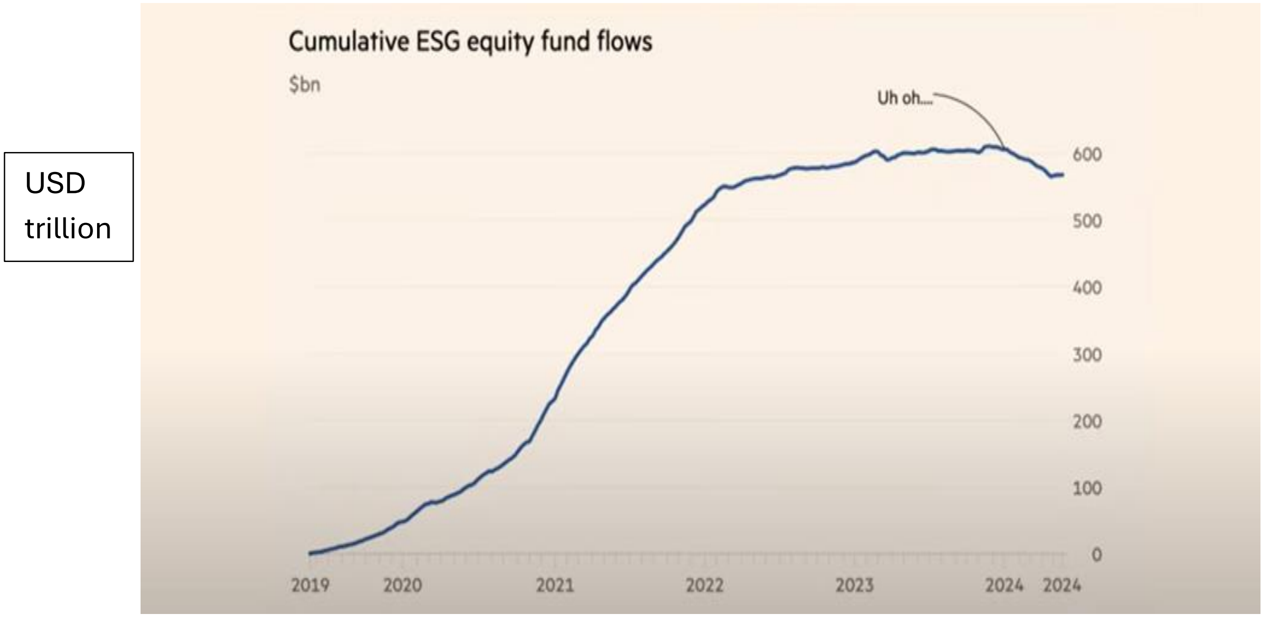
CHAPTER 1. INTRODUCTION

1.1 Background

The United Nations Principles for Responsible Investment (UN PRI) (2018) reports that investors play an important role in achieving sustainable development goals by ensuring that capital is raised and allocated to appropriate investments. The use of Environmental, Social and Governance (ESG) ratings in integrating sustainability into investment analysis is called Socially Responsible Investment (SRI). SRI and non-SRI investors are said to consider ESG factors when choosing an investment (Lean et al., 2015) but the difference is that SRI investors manage their funds in a sustainable manner and hence are said to be doing well while doing good (Peerbhai and Naidoo, 2022). In 2006, the UN PRI was introduced which seeks to advance SRI by encouraging investors to consider and incorporate ESG factors into their investment decisions (UN PRI, 2018). According to Amel-Zadel and Serafeim (2018) and UN PRI (2018), the intended outcomes are to ensure that investors are better able to manage risk and obtain sustainable long-term returns.

SRI investing has surged in recent years as more investors prioritize non-financial factors when making investing decisions. In Figure 1, the graph illustrates the cumulative equity funds flowing into ESG funds from 2019 to 2024. It shows a steady and sharp increase in investments in ESG funds from 2019 through to late 2022, as investors increasingly prioritized sustainable and ethical investment strategies. However, beginning in 2023, the trend reversed, with cumulative flows showing a decline for the first time, marking a significant shift in investor behavior. Although there was a re-evaluation of energy, resource, and defense which are independence as being social ills, the ESG investments dropped in July 2024. Researchers' growing interest in SRI has matched trends in industry over the past two decades. Widyawati (2019) shows the meteoric increase in published articles indexed in the Web of Science focused on SRI from 2002 to 2016. The *Journal of Business Ethics* and the *Journal of Banking and Finance* published 31 articles and one article in this area from 1997 to 2006, respectively compared to 112 and 16 in the period 2007 to 2016, respectively. Searches for the keyword "SRI" in Google Scholar, yields approximately 283 000 articles. It is thus evident that SRI investing is not merely a trend, its wide adoption and strong growth show that it is here to stay.

Figure 1: Total global ESG assets



(Source: Financial Times, 2024)

Although the UN PRI provides a global framework for SRI, it is voluntary meaning that the implementation and support of SRI at the country-level is driven by local parties or authorities. In South Africa there have been several key initiatives which have contributed to an increase in the incorporation of non-financial information by institutional investors into their investment decision making strategies and processes (Atkins and Maroun, 2014; Serafeim, 2015; Amel-Zadeh and Serafeim, 2018). These include amendments to Regulation 28 of the Pension Funds Act and the introduction of the Code for Responsible Investing in South Africa (CRISA), which supports the adoption of a responsible investment approach to deploying capital into markets that will earn adequate risk-adjusted returns suitable for a fund's specific member profile, liquidity and liabilities. Non-financial information, which enables SRI, has been widely available for several years through various forms of corporate reporting, from environmental and sustainability reporting to what is now known as integrated reporting (Weber 2014; Cohen et al., 2015). King IV requires institutional investors to integrate ESG into company integrated reports to assist investors in their investment decisions (Atkins and Maroun, 2014; IODSA, 2016). South Africa was the first country in Africa to mandatorily require listed companies to produce an integrated report in 2011, which resulted in ESG reporting gaining greater prominence (Serafeim 2015; Baboukardos and

Rimmel, 2016). South Africa has positioned itself as an international proponent of SRI and is the single largest market for SRI in the southern African region (Peerbhai and Naidoo, 2022).

Although there is commonality in definitions of SRI, Sandberg et al. (2008) explain that what SRI is in one market might not be recognized in another market. This causes serious hurdles that need to be overcome, starting with transparency and standardization of ESG measures. The mechanisms of SRI are heterogenous (Wdyawati, 2019), however, studies on SRI generally agree that there are three main SRI mechanisms namely, screening (including/excluding stocks based on specific ESG criteria), shareholder activism (shareholders in SRI funds actively engaging with firms through various channels to encourage positive ESG practices) and community investment (direct investments to bring about change) (Haigh and Hazelton, 2004; Sandberg et al., 2008; Schueth, 2003; Sparkes and Cowton, 2004).

The Forum for Sustainable and Responsible Investment argues that some investors embrace SRI strategies to manage risk and fulfil fiduciary duties. Chen et al. (2021) state that investors adopt ESG criteria to assess the management quality and the likelihood of resilience of their portfolio companies when dealing with challenges in the future. Orsato et al. (2015) confirm that ESG factors enable asset managers and investors to actively seek investments that may bring important social or environmental benefits (e.g. community development loan funds or clean technology portfolios). Dam and Scholtens (2015), Derwall et al. (2011), Doskeland and Pedersen (2015), Revelli and Viviani (2015) and Riedl and Smeets (2015) argue that ESG-orientated portfolios are most likely to achieve long-term financial returns due to the reduction in potential risk, such as litigation risk, tax risk, compliance risk and reputation risk. Scholars argue that the inclusion of ESG considerations in investment processes can enhance market efficiency, as prices will more accurately capture the risks and opportunities related to investment decisions and corporate practices (Busch and Friede 2018; Friede et al., 2015).

The empirical analysis of SRI portfolios dates to 1972 (Moskowitz, 1972), with numerous studies evaluating the performance of SRI and conventional or non-SRI investments in both developed and emerging markets (Humphrey and Lee 2011; Jones et al. 2008; Kempf and Osthoff, 2007; Scholtens, 2005). There have been contradictory findings from different markets and dependent upon financial market cycles. Many studies report that SRI stocks and SRI funds tend to either outperform or achieve returns at least equivalent to non-SRI stocks and non-SRI funds (Bauer et al., 2005; Brzeszczyński and McIntosh, 2013; Derwall et al., 2005; Kempf and Osthoff, 2007; Scholtens, 2005). However, a few studies also report evidence of SRI funds underperforming conventional funds; although this is attributed to poor fund management rather than the poor performance of high ESG stocks (Geczy et al., 2005; Trinks and Scholtens, 2017).

1.2 Problem Statement

Markowitz (1952) defines portfolio management as an analytical process involving the selection of a group of investment assets, with the proportions of the allocated investments being continuously adjusted to optimize the expected return. Markowitz's mean-variance (MV) model forms the foundation of portfolio theory and is widely applied in practice (Sharpe and Markowitz, 1989). The MV model aims to determine the optimal portfolio from a set of possible assets by considering only the expected return and risk (Markowitz, 1952). The expected return of a portfolio is calculated as the weighted average of the expected returns of its components, while risk is measured by the extent to which the returns deviate from the mean (Markowitz, 1952; 1959; Grinold and Kahn, 2000). When selecting an efficient portfolio, it is assumed that the expected return and risk measures are reliable, and that a rational investor will choose a portfolio that offers the lowest possible risk for a given level of expected return, or the highest expected return for an acceptable level of risk (Kaplan and Siegel, 1994; Pfleiderer, 2012; Sharpe, 1994).

However, there are two primary concerns with applying the classical MV model to SRI portfolios. First, as noted by Lean et al. (2015), SRI contradicts the MV model's central principle that an efficient portfolio should consist of diversified, uncorrelated stocks to maximize expected returns by spreading risk. SRI portfolios tend to be less diversified due to the screening process during portfolio construction, making them potentially riskier investments, which raises questions about the MV model's suitability for these portfolios (Chegut et al., 2011; Lean et al., 2015). Second, the MV model relies on expected return and risk as inputs to generate optimal portfolios (Beheshti, 2018). SRI portfolios, however, are associated with greater uncertainty in these inputs due to the complexity of measuring ESG scores and the discretionary nature of ESG reporting (Bernadi et al., 2016). This could lead to higher estimation risk in SRI portfolios constructed using the MV model, thereby questioning the model's appropriateness for such portfolios. Given these potential limitations, it is necessary to explore alternative optimization techniques that may result in portfolios with better risk-return profiles (Oikonomou et al., 2018).

Hirschberger et al. (2013) extended the Markowitz model to a three-criterion portfolio selection model by including ESG rating scores and proposed a general rule for calculating the non-dominated set algorithm. Their study demonstrates that this algorithm can outperform standard portfolio strategies for decision-makers who consider multiple criteria, aiding socially responsible investors in making SRI investments. Ballestero et al. (2012), Drut (2012), and Utz et al. (2014) also expanded the MV technique by incorporating SRI preferences as a constraint, either by including them in the objective function or by altering the MV efficient frontier through sustainability screening. In contrast, some studies have explored alternative optimization techniques for constructing SRI portfolios instead of extending the MV model.

With the advancement of artificial intelligence (AI), the range of portfolio construction methods has expanded significantly. Chen et al. (2020) points out that introducing constraints in portfolio optimization, such as limiting the number of assets or restricting short selling, turns the problem into a mixed-integer nonlinear quadratic programming issue for which effective computational algorithms are not yet available. Moreover, as the number of assets increases, so does the complexity of determining the optimal trade-off between return and risk. Erwin and Engelbrecht (2023) suggest that meta-heuristic approaches, including evolutionary and swarm intelligence algorithms, can be effectively employed to optimize complex portfolio problems by enabling a broader search within their methodologies. Chang et al. (2000) compared the performance of genetic algorithms (GA), simulated annealing (SA), and tabu search techniques in optimizing portfolios using data from the Hang Seng, DAX100, FTSE 100, S&P 100, and Nikkei 225 indices. They found that each algorithm had different advantages depending on the dataset, but all were applicable for efficient portfolio selection. Similarly, Cura (2009) found that while no single technique outperformed the others, particle swarm optimization (PSO) generally produced better portfolios with lower-risk investments.

Given the potential limitations of the MV approach to portfolio construction with SRI screened stocks, it is imperative for investors to know what alternatives can be used and how these alternatives perform. A few studies have examined alternative approaches that can be used to construct an optimal SRI portfolio using United States of (U.S), Asian and European data (Chen et al., 2021; Jin, 2022; Qi et al., 2013), but very little attention has been given to creating optimal SRI portfolios in emerging markets. Conclusions drawn from developed markets may not be appropriate for developing markets due to the slower growth in SRI, different regulations surrounding ESG, varying attitudes of investors to SRI and varying market conditions among other differences. Moreover, of those studies that have examined SRI portfolio construction techniques, only a few have compared methods (e.g. Benedetti et al., 2021; Oikonomou et al., 2018; Zhang and Chen, 2021) and where comparisons have occurred, these have largely been limited to a comparison of traditional methods or AI approaches rather than a comparison of traditional versus AI methods.

As mentioned previously, the evidence on the performance of SRI portfolios relative to non-SRI portfolios is mixed (positive, negative and no difference). Some of these mixed results may be due to the use of non-optimal portfolio construction methods. This study seeks to contribute to the literature by evaluating the optimal method to construct an SRI portfolio using an array of traditional and AI techniques for one of the largest and most liquid emerging markets, namely South Africa. South Africa is chosen given the wide adoption of SRI in this country. The study then goes on to examine whether the same optimal technique identified for SRI stocks is applicable for a broader sample of stocks (that are not screened by ESG scores) and, finally, to ascertain how the SRI portfolios perform relative to the non-SRI portfolios using the best construction methods.

This study provides valuable insights for South African investors seeking to follow an SRI mandate by identifying the best approach to constructing an optimal portfolio using traditional or AI approaches and evaluating their performance. The findings are crucial for investors aiming to grow SRI and make it more accessible which in turn can contribute to achieving sustainable development.

1.3 Research Questions

The research questions underpinning this study are:

- Which portfolio optimization method yields an optimal SRI portfolio for the South African market using both traditional and AI approaches?
- Which portfolio optimization method yields an optimal portfolio of non-SRI (conventional) stocks for the South African market using both traditional and AI approaches?

- Using optimal construction methods, does the SRI portfolio outperform or underperform the non-SRI portfolio on a risk-adjusted basis?

An optimal portfolio is defined as one which yields the highest risk-adjusted return while also considering diversification of the portfolio and the stability of the weightings of each asset. For the classical methods, the MV, naïve, and risk parity approaches are employed while the AI methods used include GA, SA, and PSO. The sample comprises securities in the FTSE/JSE Responsible Investment Index (J113) with a minimum rating of 2.9 from the period February 2012 to January 2024.

1.4 Significance of the Research

Most previous research on portfolio optimization in the SRI framework has been conducted in developed markets and in the context of comparing conventional methods or AI methods. Few have compared both traditional and AI approaches to establish which approach forms an optimal SRI portfolio. In fact, most academic research in the area of SRI focuses on whether SRI portfolios outperform or underperform conventional portfolios and do not consider the choice of construction method.

There has been growing awareness, development in reporting and involvement of emerging markets including South Africa in SRI. Peerbhai and Naidoo (2022) and Viviers and Els (2017) assess SRI in South Africa while Chen et al. (2021), Johnson (2020) and Hawg et al. (2021) investigate ESG performance in emerging markets, but none of these studies compare portfolio construction approaches. This study thus makes a notable contribution to both understanding the performance of traditional versus AI approaches to portfolio construction and, whether the choice of construction method, influences the findings regarding the performance of SRI versus conventional portfolios.

The findings of this study are of use to investors. If different optimization techniques lead to different SRI portfolio performance, this suggests that, apart from ESG screening criteria, investors need to carefully consider the choice of portfolio construction method. The results can help investors to recognize which optimization methods yield better results within SRI, which may enhance the growth of the SRI sector, leading to a larger share within developing financial markets.

1.5 Structure of the Study

The remainder of the study is structured as follows. Chapter 2 reviews the relevant literature on both traditional and AI portfolio optimization approaches, the theory of SRI and SRI portfolio construction. The chapter also investigates the empirical findings related to portfolio optimization with a specific focus on the construction and performance of SRI portfolios. Chapter 3 describes the portfolio construction methods used in the study and Chapter 4 discusses the performance metrics utilized to compare the portfolios created, the data collected and the sample period. Chapter 5 presents and analyses the results. Finally, Chapter 6 summarizes the findings and provides recommendations and for future research.

CHAPTER 2. LITERATURE REVIEW AND HYPOTHESIS DEVELOPMENT

2.1 Introduction

This study examines the efficacy of traditional and AI techniques for the construction of SRI portfolios. This chapter provides the context for this study drawing from theory and the findings of prior empirical studies. First, an overview of traditional and AI portfolio construction methods is provided, including a review of their performance from empirical tests. Thereafter, the background to SRI is reviewed and the empirical evidence on the performance of SRI and conventional funds is discussed. The application of portfolio construction methods to SRI portfolios is considered along with findings from empirical studies in this area. Finally, drawing from the literature, the hypotheses tested in this study are specified.

2.2 Portfolio Construction

2.2.1 *Traditional approaches*

As discussed in Chapter 1, Markowitz (1952) introduced the MV approach for portfolio construction. This model is based on the premise that a rational investor aims to maximize returns while minimizing the risk associated with their portfolio. It involves creating an efficient frontier, which represents all possible combinations of assets that achieve the highest return for a given level of risk. Investors can then choose from this frontier based on their individual level of risk tolerance (Elton and Gruber, 1997; Markowitz, 1952). Risk tolerance refers to an individual's willingness to endure potential losses. This contrasts with the broader measure of an individual's risk profile, which is a comprehensive assessment of their overall approach to risk, reflecting various dimensions that influence how they perceive and respond to risk in financial and other life decisions including risk tolerance, risk capacity, risks requirements and behavioral factors.

Since the MV approach simultaneously aims to maximize returns and minimize risk, it tackles a multi-objective conflicting stochastic problem, where more volatile assets generally tend to offer higher returns on average (Abdelaziz et al., 2007; Silva et al., 2019). Achieving an optimal solution thus requires a compromise, where the best possible outcome for one objective is traded off against the other (Masmoudi and Abdelaziz, 2018; Usta and Kantar, 2011). This balance is known as the Pareto optimal solution. According to Porter and Gaumnitz (1972), the problem is considered stochastic because both return and risk are uncertain and must be estimated.

Since the introduction of Markowitz's (1952) portfolio selection theory, mean-variance (MV) analysis has been widely utilized in both academic research and practical portfolio management. It is well known that the exact distributions of security returns are unknown. Consequently, applying Markowitz’s portfolio model in real-world scenarios necessitates estimating mean returns and variances using historical data, where risk is quantified by return variance. However, due to estimation errors in these input parameters, the calculated MV efficient portfolio can differ significantly from the actual efficient portfolio. This issue is especially evident in dynamic trading strategies that are frequently rebalanced (Zhang et al., 2017). As Chopra (1993) points out, even minor inaccuracies in mean or variance estimates can lead to substantially different outcomes for MV optimal portfolios. Such discrepancies are primarily due to estimation errors, which are often treated as if the estimated parameters are the true values, without accounting for potential inaccuracies.

Furthermore, the MV approach does not incorporate subjective investor insights or specific market behaviors into the model, potentially resulting in portfolios that are heavily weighted in overvalued assets and underweighted in undervalued ones (Britten-Jones, 1999). Additionally, assets with high expected returns are often overemphasized, diminishing the diversification benefits typically achieved through optimization in portfolio construction (Schmidt, 2019). Numerous studies have documented that estimated MV efficient portfolios often perform poorly out-of-sample (Broadie, 1993; Chopra and Ziemba, 1993; Jorion, 1985; Kan and Smith, 2008; Klein and Bawa, 1976; Michaud, 1989). Kan and Smith (2008) further demonstrate that the sample minimum-variance frontier is a biased estimator of the true population frontier.

To address the impact of estimation errors in returns and variances, several methods have been suggested in the literature. For instance, Chopra (1993) and Jagannathan and Ma (2002) recommend implementing a no short-selling constraint to minimize these errors when optimizing mean-variance (MV) portfolio selection strategies. Michaud (1998) proposed a statistical resampling method that indirectly deals with estimation errors by averaging the outcomes of multiple optimal portfolios. Kan and Zhou (2007) introduced a three-fund optimal portfolio approach, which includes a risk-free asset, the sample tangency portfolio, and the sample global minimum-variance portfolio. They advocate that incorporating the economic objective function into the parameter estimation process is a valuable method for examining the MV optimal portfolio selection problem.

Another strategy for handling estimation errors in return estimates is the Black-Litterman model, which combines two types of information: implied (neutral) returns and subjective return estimates, known as "views" (Bessler et al., 2017). One of the key benefits of the Black-Litterman model is that it allows investors to provide return estimates for assets they are confident about, while remaining neutral on others (Black and Litterman, 1992). Additionally, the model allows for incorporating the reliability of each return estimate, enabling investors to differentiate between robust estimates and speculative ones. The Black-Litterman approach encourages investors to deviate from a benchmark portfolio, like the market portfolio, only if they possess credible forecasts of future returns. In this model, implied returns serve as priors, calculated based on the weights of assets in the reference portfolio. These implied excess returns are obtained through reverse optimization, as initially proposed by Black and Litterman (1992), where the observed weights of the market or benchmark portfolio are the outcome of mean-variance optimization.

Another technique used is the Bayesian approach, which considers estimation uncertainty in the portfolio selection process, potentially improving out-of-sample performance relative to traditional methods (Jiang et al., 2019). This approach assumes that unknown parameters follow a prior distribution, from which the predictive distribution of asset returns is derived. Optimal portfolio weights within the Bayesian framework are determined by maximizing utility based on the predictive distribution. Early versions of this approach used uninformative diffuse priors (Barry, 1974) or Bayes-Stein shrinkage priors (Jobson and Korkie, 1980; Jorion, 1986, 1991), while more recent research incorporates asset pricing models to develop prior beliefs (Pastor, 2000; Pastor and Stambaugh, 2000). Although Bayesian portfolio analysis effectively reduces estimation risk, it still necessitates estimates for both the means and the covariance matrix of asset returns. It is well-known that estimating accurately is more challenging than estimating the covariance matrix, and errors in mean estimates tend to have a more substantial impact on portfolio weights than errors in the covariance matrix (Merton, 1980; Best and Grauer, 1992; Black and Litterman, 1992). As the minimum-variance portfolio depends solely on covariance matrix estimates, many studies favour the MV approach over the Bayesian method.

A straightforward way to diversify portfolio risk is the equal-weighting strategy, where each asset in the portfolio is given the same weight, often referred to as the naïve or 1/N approach (Swade et al., 2023). This method is considered an active strategy because it requires periodic rebalancing to maintain equal weights (DeMiguel et al., 2009b; Perold and Sharpe, 1988). Equal-weighted portfolios are widely recognized in the literature (Benartzi and Thaler, 2001; Li et al., 2020; Windcliff and Boyle, 2004) and are noted for being a competitive alternative to MV portfolios regarding out-of-sample performance (DeMiguel et al., 2009b; Duchin and Levy, 2009; Kritzman et al., 2010; Tu & Zhou, 2011). The primary advantage of the naïve approach is that it does not require estimating asset return moments or optimizing a specific objective function with constraints. DeMiguel et al. (2009b) examined the performance of fourteen different optimization models, including the baseline MV model, the naïve approach, the Bayesian approach, and portfolios with moment restrictions and short-sale constraints, across seven datasets. They found that none of the models significantly outperformed the 1/N rule, highlighting the need for incorporating diversification into the portfolio solution. They also observed that the out-of-sample Sharpe ratio of the MV model was substantially lower than that of the 1/N rule and hypothesized that the errors in estimating means and covariances contribute to the MV model's lower performance. Consequently, DeMiguel et al. (2009a) conclude that the equal-weighted strategy is a strong contender for portfolio construction. Erb and Harvey (2006) associate naïve investing with the rebalancing literature, which suggests benefits from the act of rebalancing, such as diversification of returns, volatility, or risk (Willenbrock, 2011) or the rebalancing premium (Bouchey et al., 2012). Pflug and Pichler (2012) demonstrate that the naïve portfolio approach is asymptotically the best choice when uncertainty is high. However, Maillard et al. (2010) note that if a market includes securities with significantly different intrinsic risks, this approach may lead to a portfolio that is not well-balanced in terms of risk allocation across securities. In other words, the downside of the naïve approach is that it may result in limited risk diversification if individual asset risks vary greatly.

In contrast to equal-weighting, a value-weighted portfolio is considered a passive buy-and-hold strategy that reflects market drifts. Fama and French (1992) state that the value-weighted approach can be less volatile compared to an equal-weighted portfolio because large companies tend to be more stable and less volatile. However, Chan et al. (1999) state that value-weighted portfolios become heavily concentrated in a few large companies which tends to increase the risk for the portfolio if those companies underperform or face financial difficulties. The allocation of more capital to large companies may result in the portfolio being diversified across smaller companies, sectors and industries (DeMiguel et al., 2009a). Historically, investing in equal-weighted portfolios yields higher returns than investing in the corresponding value-weighted portfolios (Swade et al. 2023). Plyakha et al. (2021) document a monotonic relationship between the equal- and value-weighted return spread and factors such as size, price, liquidity, and idiosyncratic volatility, suggesting that equal-weighted portfolios earn more due to higher risk exposure but badly in terms of risk-adjusted return as they are associated with high volatility. Similarly, Liu et al. (2023) find that the higher returns of the equal-weighted portfolios arise due to systematically higher exposures to market, size and value factors. However, even after adjusting for risk, the equal-weighted portfolios yield higher returns compared to value-weighted portfolios. The authors, however, rationalize this profit as a reward for the need for frequent rebalancing to maintain equal portfolio weights.

An alternative to the naïve equal-weighting strategy is the risk parity approach, which uses portfolio volatility as the key measure of risk. In this method, portfolio weights are determined so that each asset contributes equally to the overall portfolio's standard deviation. The concept of risk parity was first introduced by Qian (2005), who proposed allocating an equal amount of risk to both stocks and bonds within a portfolio. This idea was further formalized by Maillard et al. (2010), who sought to ensure that the total risk contribution of each security in the portfolio is the same. This approach typically involves solving a set of nonlinear equations and inequalities (Maillard et al., 2010; Spinu, 2013), although it can also be expressed as a nonlinear convex optimization problem. Asness et al. (2012) adds that with risk parity focusing on risk rather than returns, it helps to mitigate against severe drawdowns during market downturns. Comparing risk parity to the MV approach, risk parity portfolios are found to outperform MV optimized portfolios, especially when the estimates of expected returns and covariance are noisy (Roncalli, 2013). When compared to an equal-weighted portfolio, risk parity also outperforms by achieving better risk-adjusted return as the 1/N portfolio can expose the portfolio to higher volatility due to equal allocation of capital to all assets regardless of their risk (Plykha et al., 2012).

Although the risk measure commonly used in the risk parity approach is volatility, alternative risk measures have also been considered (such as by Boudt et al., 2013; Roncalli, 2014). For example, Braga et al. (2023) explores the use of kurtosis, with the goal of the homogeneous distribution of portfolio kurtosis, rather than its minimization.

2.2.2 Artificial intelligence approaches

Given the complexity of the portfolio optimization problem, it is evident that any portfolio management technique requires a dynamic decision-making process. Hence, reinforcement-based learning techniques are highly suitable for effectively managing portfolios (Wu et al., 2021). Investors, especially institutional investors, focus on portfolio construction in a highly dimensional space, using fewer observations than the number of assets to construct a diversified portfolio. Researchers have utilized heuristic methods since they can deal with this complexity. A heuristic algorithm refers to the method of searching according to empirical rules when solving problems, rather than solving them according to determined steps. This approach allows for not only tackling more realistic problems but also examining the extent to which simplifications are acceptable and how stable certain approaches are (Chang et al., 2000; Woodside-Oriakhi et al., 2011). Typically, the core of these methods is an iterative principle that includes stochastic elements in generating new candidate solutions and/or in deciding whether these replace their predecessors, while still incorporating some mechanism that prefers and encourages improvements. Their interest arose from key factors, including: (i) dynamic market factors; (ii) changing constraints like liquidity, taxes, transaction costs, management fees etc., and (iii) the requirement for intensive computational capacity to ascertain the appropriate distribution.

Given the complexity involved in portfolio optimization, it is clear that any portfolio management strategy must incorporate a dynamic decision-making process. This makes reinforcement learning techniques particularly well-suited for effective portfolio management (Wu et al., 2021). Investors, particularly institutional investors, often construct portfolios in a high-dimensional space, where the number of assets exceeds the number of observations, necessitating diversification with limited data. To address this complexity, researchers have turned to heuristic methods. A heuristic algorithm involves searching based on empirical rules rather than following predetermined steps, which allows for addressing more realistic problems and evaluating the acceptability and stability of various simplifications (Chang et al., 2000; Woodside-Oriakhi et al., 2011). These methods typically rely on an iterative process that incorporates stochastic elements to generate new candidate solutions and determine whether these should replace existing ones, while also promoting and favoring improvements. Interest in these techniques has grown due to several factors: (i) dynamic market conditions; (ii) evolving constraints such as liquidity, taxes, transaction costs, and management fees; and (iii) the necessity for significant computational power to determine the optimal asset allocation efficiently (Gunjan and Bhattacharyya, 2022).[1]

To solve the portfolio optimization problem, Schaerf (2002) developed local search heuristics. However, it was found that these methods often became trapped in local optima. Therefore, meta-heuristic methods, which undertake broader searches than heuristic methods, have been preferred to efficiently optimize complex portfolio optimization problems (Erwin and Engelbrecht, 2023). The best-known meta-heuristic methods are GA, PSO, SA and tabu search techniques. When dealing with a large number of stocks and a vast searchable space, special techniques like GA simplify the optimization process (Quang, 2022). GA, the preferred meta-heuristic method, was first used by Shoaf and Foster (1998) for portfolio optimization. These algorithms simulate evolution and natural selection on a computer. Engelbrecht (2007) describes evolution as an optimization of an organism's ability to survive in dynamic and competitive environments.

Evolutionary algorithms are inspired by this process and mimic its characteristics to seek global optima for computational optimization problems. The survival strength of individuals within a population is determined by the objective function to be optimized. Individuals are then selected based on their survival strength for reproduction, which produces offspring for the next generation of individuals. GA starts with a random population. It then uses an adaptive function to select good individuals and reject bad ones, working with variable codes rather than directly on the variables. Unlike most optimization techniques that search from a single vertex, GA operate on a set of vertices (optimal points), which increases the chances of reaching a global optimum and avoids premature convergence at local points (Hamdia et al., 2020; Yang, 2006). Quang (2022) states that GA evaluates the objective function to guide the search process, making them applicable to any

[1] There are other approaches used in portfolio construction based on machine learning methods which offer new ways to enhance traditional approaches by leveraging vast amounts of data and sophisticated algorithms. They include reinforcement learning, deep learning, support vector machine, clustering algorithm, Bayesian optimisation, principal component analysis and random forest, among others (Fischer and Krauss, 2018; Huang et al., 2005; Jiang and Liang, (Murtagh and Contreras, 2012; Shahriari et al., 2016). This study, however, focuses exclusively on meta-heuristic approaches.

optimization problem (continuous or discrete). The algorithm is considered a subclass of probabilistic algorithms, incorporating randomness into the processing flow.

Yang (2006) compares the GA, MV and Bayesian methods to construct portfolios using MSCI Country Total Return indices from 1975 to 2004. The study found that portfolios optimized with GA had lower standard deviations and higher average out-of-sample returns compared to the MV and Bayesian approaches. The algorithm integrates historical data with future uncertainties, enhancing the accuracy of average estimates and overcoming computational challenges associated with the Bayesian approach. Moreover, Cheong et al. (2017) developed portfolios of Korean stocks based on investor preferences using GA. This method was employed to construct portfolios by selecting stocks that institutional or foreign investors prefer more heavily. The portfolios were composed of the top 90 companies in Kospi 200 by market capitalization, covering the period from September 2007 to May 2014. Cheong et al. (2017) found that long-term information is more effective than medium- or short-term information in the Korean stock market, and the portfolio weights vary when using GA, impacting portfolio performance. The study specifically showed that GA performs well and contributes to understanding investor behaviors and patterns that can inform investment strategies. Investors typically build portfolios according to their risk-return profiles based on the average variance framework, and leveraging the strategies of other confident investors can improve portfolio performance. However, the authors also highlight that institutional investors expect higher returns with lower risks in the Korean stock market, while foreign investors are primarily concerned with portfolio risk.

The results of Liu et al. (2017), based on a comprehensive sample of 2,317 Chinese stocks over the period February 2016 to February 2017, confirm the findings of prior researchers that GA results in optimal portfolio strategies. They show that a portfolio constructed using GA produces results that are more reliable without the need to introduce or mention each country's data (developing or developed markets). The authors discuss the impact of each stock's time series length on portfolio construction and demonstrate how to enhance algorithmic efficiency in fundamental stock analysis. Adam (2021) verifies that GA results in high Sharpe ratios resulting from the improved increase in the number of iterations, meaning that the ability to achieve excess returns increases gradually.

SA is another meta-heuristic approach, described as a local search algorithm (Eglese, 1990; Henderson and Jacobson, 2003). It is easy to implement, converges well and uses hill-climbing moves to escape local optima, making it a popular technique for the past two decades (Alexander and Sheldon, 2010). The algorithm mimics the physical annealing process of solids, gradually cooling a crystalline solid to achieve a defect-free crystal lattice (Romeo and Sangiovanni-Vincentelli, 1991; Alexander and Sheldon, 2010). Lang et al. (2022) states that if the cooling schedule is sufficiently slow, the final configuration results in a structurally superior solid, establishing a connection between thermodynamic behavior and the search for global minimum in discrete optimization problems. In portfolio optimization, SA is powerful due to its flexibility and ability to approach global optimality better than other local search methods (Delahaye et al., 2018).

Dogan et al. (2024) use SA with data from Hong Kong, comparing it to particle swarm optimization and hybrid methods. They observe that the developed SA algorithm produces comparable reasonable results compared to the MV approach, even though it is not the best solution presented. The study then made a special application of the algorithm with the BIST30 data set which demonstrated a portfolio created with different numbers of assets from the Hang-Seng benchmark. The results show an optimal number of assets that investors could keep in their portfolios at different risk levels to obtain an optimal portfolio using SA. Using the BIST-30 data, Dogan et al (2024) conclude that the optimal number of assets in the portfolio should be 10. Akyer et al. (2018) use the SA algorithm to form portfolios using Turkey's Borsa Istanbul top 30 index constituents. They determined that the optimal number of assets in a portfolio should be ten which balances risk and return.

Fernandez and Gomez (2007), using data from several countries, examined artificial neural networks, GA, tabu search and SA methods for portfolio optimization. They concluded that the artificial neural network gives better results in low-risk investments. In contrast, Coutino-Gomez et al. (2003) compared the GA, greedy algorithm and SA methods in portfolio optimization and found that the SA method yielded better results in low-risk environments. Chang et al. (2000) compared the Hang Seng, DAX100, FTSE 100, S&P 100 and Nikkei 225 data with GA, SA and tabu search techniques for portfolio optimization. The study finds that each algorithm showed different advantages over different data sets, and they showed that all these algorithms can be applied for efficient portfolio selection. The SA method is also used by Fastrich and Winker (2012), Sen et al. (2014), Qodsi et al. (2015), Lukovac et al. (2017), Kumar et al. (2018), Moradi et al. (2021).

PSO, developed by Kennedy and Eberhart (1995a, 1995b), is a stochastic optimization technique based on population dynamics, inspired by the flocking behavior of birds. It is rooted in swarm intelligence, which involves individuals in a group working together to solve a problem by sharing locally available information (Engelbrecht, 2007). Unlike evolutionary computation techniques like GA, PSO does not use typical evolutionary operators such as selection, crossover, or mutation. Instead, particles in PSO navigate through the search space, aiming to find an optimal solution by adjusting their positions according to their velocities and the distance to the best local and global positions (Blum and Li, 2008). PSO is widely used for optimization problems because of its strong exploration capabilities, simplicity, and reliability (Chen et al., 2006). However, the standard PSO can sometimes become stuck in sub-optimal regions, prompting the development of various enhanced PSO variants. To address this, Shi and Eberhart (1998) introduced inertia weights to balance global and local search capabilities. Clerc and Kennedy (2002) further improved convergence by adding a constriction factor to the velocity update formula. Koshino et al. (2007) combined inertia weights with constriction factors for improved performance, while Pram et al. (2003) developed a fitness-distance-ratio-based PSO that incorporates near-neighbor interactions, considering an additional particle, n-best, that has a higher fitness value and is in closer proximity.

Cura (2009) compared the PSO, GA, SA and tabu search techniques with data from Hong Kong, the U.S., United Kingdom (U.K.)., Germany and Japan. The study found that no technique consistently outperformed the others, although PSO performed better in low-risk portfolios. Subsequent studies, such as that by Zhu and Wang (2011), Kamali (2014), Wang et al. (2015), Ni et al. (2017), Abuelfadl (2017) and Akyer et al. (2018), confirm PSO's effectiveness in various optimization scenarios, demonstrating its adaptability and reliability across different datasets and risk levels.

2.2.3 Conclusion

The review reveals that traditional approaches to portfolio construction, such as MV and the naïve approach, are subject to limitations and may not perform well out-of-sample. Meta-heuristic AI approaches have been developed as alternatives to handle complex portfolio optimization problems. Most studies comparing the performance of these meta-heuristic methods to construct an optimal portfolio suggest that there is no one meta-heuristic method that is preferred over all the others. However, the issue of portfolio construction is complicated by considering SRI criteria in the formation of a portfolio. The following section outlines SRI, the performance of SRI funds compared to conventional funds and then considers how traditional and meta-heuristic portfolio construction methods have been expanded to account for this third investment objective.

2.3 Socially Responsible Investing (SRI)

In this section, the theoretical arguments and empirical findings reported in the literature on SRI are discussed. Given the focus of this study on the construction of optimal SRI portfolios, it is crucial to understand the foundation of this approach to investing and the findings of the performance of this investment approach relative to conventional investing.

2.3.1 Theoretical background

The concept of SRI has conflicting theoretical arguments, with the earliest dating back to Milton Friedman. Friedman (1962) posited that the social responsibility of businesses is to increase their profits and avoid any social initiatives that could decrease shareholder value. According to this theory, known as shareholder theory, management is expected to act in the best interests of shareholders, prioritizing their financial interests over those of other stakeholders. Under this theory, ESG is only undertaken if it continues to support the maximization of shareholder value. Advocates of this theory point out the increased costs and opportunity costs associated with ESG which will lower financial performance. As such, they support the exclusive focus on shareholder maximization (Benabou and Tirole, 2010; Kruger, 2015; Renneboog et al., 2008).

However, Freeman (1984) suggested that firms should consider the interests of all parties affected by their actions, not only shareholders. This includes employees, customers, suppliers and the community. Freeman's theory, known as stakeholder theory, recommends that a company's performance should be measured not only by its financial outcomes but also by its impact on various stakeholders. The theory suggests that effective stakeholder-management relationships act as mechanisms for monitoring and enforcement, which can result in benefits such as increased efficiency in responding to external demands and enhanced financial performance (Clarkson, 1995; Donaldson and Preston, 1995; Jensen, 2001). Similarly, slack resources theory posits that strong financial performance enables companies to engage more in socially responsible activities by providing the additional resources required for ESG initiatives (Barney, 1991; Bramer et al., 2007; Margolis et al., 2009). Positive reputation effects, such as enhanced employee goodwill, can subsequently boost a company's financial performance.

The roles of risk rating agencies and ESG effectiveness rating agencies are distinct yet complementary when constructing portfolios. Understanding their differences and the importance of each can significantly influence portfolio strategies, particularly in modern investment environments that emphasize both financial returns and sustainability. Risk rating agencies, such as Moody's, S&P Global, and Fitch, evaluate the creditworthiness and financial stability of entities. These credit ratings are primarily used to assess the likelihood of default and the associated risk of investing in an asset. While ESG rating agencies, such as MSCI, Sustainalytics, and ISS ESG, evaluate an entity's performance and effectiveness in managing ESG issues. These ratings assess sustainability practices and their alignment with stakeholder expectations. The inconsistency among ESG ratings across different agencies arises from variations in their measurement criteria, methodologies, and weighting schemes (Capizzi et al., 2021).

This divergence has significant implications for investors, researchers, and policymakers. Investor confusion and decision-making challenges which cause discrepancies in ESG ratings can lead to confusion among investors attempting to integrate ESG factors into their investment decisions (Berg et al., 2022). Inconsistent ratings may result in misinformed choices, potentially affecting portfolio performance and alignment with sustainability goals. Gibson et al. (2021) states that regulators and policymakers aim to promote sustainable finance, inconsistent ESG ratings pose challenges in setting standards and benchmarks. The lack of uniformity undermines efforts to create transparent and comparable ESG disclosures.

As mentioned in chapter 1, ESG scores capture the extent to which a firm minimizes its negative impact on the environment, supports internal and external stakeholders and upholds good levels of governance in its operations. SRI funds include firms with high ESG scores (minimum of 2.9 score and exclude firms with low ESG scores). If the stakeholder and slack resource theories hold, then SRI funds should outperform conventional funds which include firms with both high and low ESG scores. In contrast, if the shareholder theory is held, SRI funds will underperform conventional funds because firms with high ESG scores will earn lower returns due to the higher costs of supporting their ESG initiatives. However, there are also other considerations for an SRI investor which may impede the performance of an SRI fund based on economic principles (Minor, 2007). First, according to the theory of supply and demand, the low demand for a stock with a low ESG score with equivalent financial performance to a stock with a high ESG score will lead to an increase in its return, causing SRI funds that reject this stock to underperform relative to conventional funds without ESG constraints (Ali and Szyszka, 2006). Second, asset selection constraints limit investment opportunities, reducing diversification benefits and returns. Third, externalities such as fines may reduce a company's profits for socially responsible investors, but not for conventional investors. Thus, by divesting from certain stocks, socially responsible investors may achieve lower returns (Brzeszczyński and McIntosh, 2013). Accordingly, while some theories predict a positive relationship between corporate ESG performance and financial performance leading SRI funds to outperform conventional funds, there are also limitations on the performance of SRI stocks in the stock market.

2.3.2 Empirical findings on SRI and conventional funds

Dating back to Moskowitz (1972), a plethora of studies have examined the performance of SRI compared to conventional investing strategies. The results, however, are very mixed – with many studies showing no significant difference, others showing the outperformance of SRI funds and others the underperformance of SRI funds. For instance, Goldreyer and Diltz (1999) compared the performance of Socially Responsible Investing (SRI) funds to conventional funds in the U.S. from 1981 to 1997 and found no definitive performance advantage for either. Similarly, in Australia, Cummings (2000) reported no significant performance difference between SRI funds and conventional market indexes during 1986 to 1996. Bauer et al. (2007) found comparable outcomes for Canadian funds from 1994 to 2003. Likewise, Scholtens (2005) reported no differences in risk-adjusted returns between SRI and non-SRI funds in the Netherlands. In the U.S., Bello (2005) noted that, after accounting for portfolio diversification, the performance of SRI mutual funds was similar to that of conventional funds. Using a multi-factor model, Bauer et al. (2005) evaluated the performance of mutual funds in Germany, the U.K., and the U.S. from 1990 to 2001 and found no significant differences in risk-adjusted returns between ethical and conventional funds when investment style was considered. Their findings also indicated that ethical funds experienced a period of convergence before achieving financial returns comparable to those of conventional funds.

In the U.K., Mill (2006) observed that the average risk-adjusted performance remained consistent when a conventional fund adopted SRI principles, showing no difference compared to funds that maintained their original investment criteria. Brzeszczyński and McIntosh (2013) evaluated the performance of portfolios consisting of British SRI stocks from 2000 to 2010, selecting stocks from the Global-100 Most Sustainable Corporations (MSC) list. Their findings indicated that SRI portfolios generally produced higher total returns than their benchmarks, but these differences were not statistically significant after accounting for risk. Cortez et al. (2009) analyzed SRI fund performance in seven European countries between August 1996 and February 2007 using both unconditional and conditional models, finding that these funds performed on par with both conventional and socially responsible benchmarks. A comprehensive analysis by Renneboog et al. (2008), covering 17 countries across Europe, North America, and Asia Pacific from 1991 to 2003, revealed that SRI funds in the U.S., the U.K., and several other continental European and Asia Pacific nations generally lagged behind their domestic benchmarks.

The risk-adjusted returns of SRI funds were generally not significantly different from those of conventional funds, with some exceptions in countries like France, Japan, and Sweden. In a similar vein, Schroder (2007) analyzed 29 international SRI indexes and found no notable over- or under-performance compared to conventional indexes, excluding factors like transaction costs, manager skills, and selection process expenses, thus offering a straightforward performance comparison. Sauer (1997) examined the Domini Social Index (DSI) against the S&P 500 and the CRSP Value-Weighted Market Indexes and found that the DSI underperformed both on a risk-adjusted basis from January 1986 to April 1990. However, when the back-test results were combined with the DSI's actual performance from May 1990 to December 1994, its aggregate risk-adjusted returns were higher than those of the conventional benchmarks. Statman (2000) noted that from 1990 to 1998, the DSI's performance was comparable to that of the S&P 500. While the DSI's raw returns were slightly above those of the S&P 500, its risk-adjusted returns were marginally lower, making the difference statistically insignificant. Managi et al. (2012) applied the Markov switching model to analyses the performance of SRI and conventional stock indexes in the U.S., the U.K., and Japan, finding no significant differences in average returns and volatilities, but clear evidence of similar movements between these indexes.

Contrary to earlier findings that showed similar performance between SRI and conventional funds, some studies have reported the outperformance of SRI investments. For example, Orlitzky et al. (2003), in their review of 52 studies with a combined sample of 33,878 observations, concluded that corporate social performance is generally positively correlated with financial performance, a conclusion supported by more recent studies. Kempf and Osthoff (2007) found that a strategy of investing in companies with high ESG scores and shorting those with low ESG scores led to a net outperformance of 4% or abnormal returns of up to 8.7% annually. They attributed this outperformance of SRI firms to their ability to attract cheaper costs, recruit well-suited employees, and market products more effectively due to a stronger reputation. Kempf and Osthoff (2007)'s findings of significant outperformance of ethical funds were also confirmed by Statman and Glushkov (2009) and Derwall et al. (2005). These studies suggested that tilting a portfolio towards firms with high social responsibility scores can be advantageous and compensate for excluding "sin" stocks. Diltz (1995) and Derwall et al. (2005) corroborated these findings with hypothetical portfolios.

Consolandi et al. (2008) examined the Dow Jones Sustainability Stoxx Index (DJSSI) from 2002 to 2006 and found that it slightly outperformed conventional benchmarks. Rathner (2012) demonstrated that European SRI funds significantly outperformed their conventional benchmarks, though this trend was not observed for U.S. funds. Gil-Bazo et al. (2010) found that U.S. SRI funds delivered higher after-fee risk-adjusted returns than comparable conventional funds. They noted no significant performance difference between SRI and conventional funds managed by the same company, indicating that company-level factors might explain the differences. SRI funds managed by specialized companies outperformed similar conventional funds by more than 2.6% annually.

On the other hand, some studies report underperformance of SRI funds and indexes. Geczy et al. (2005) observed that SRI fund investors receive significantly lower risk-adjusted returns. Trinks and Scholtens (2017) suggested that negative screening might negatively affect portfolio performance. Riedl and Smeets (2017) argued that socially responsible investors are willing to accept lower returns and higher fees to align with their social responsibility values.

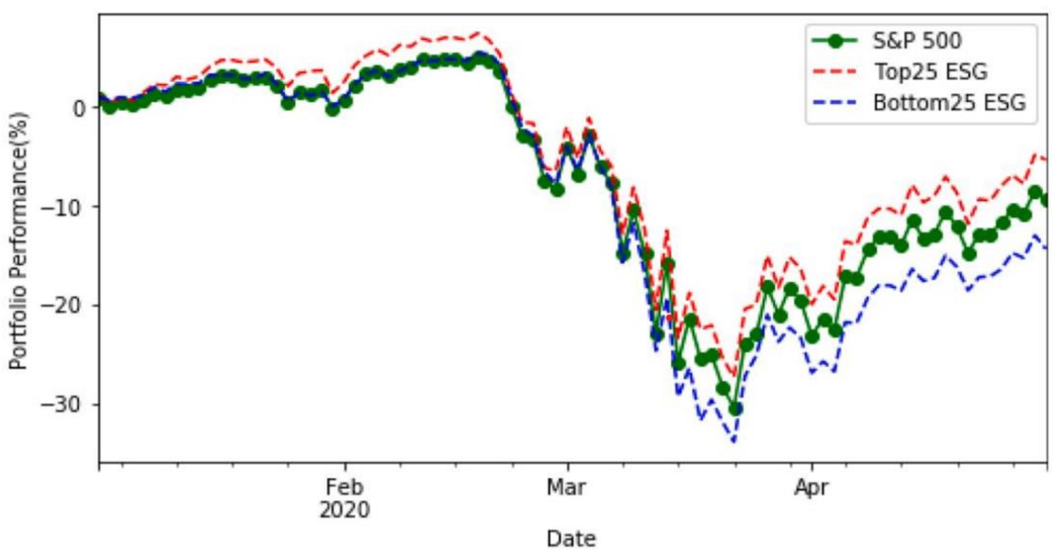
Lee (2011) argued that the underperformance of SRI funds might stem from their management style rather than their socially responsible nature. This view aligns with research by Davis et al.

(2020), Huang and Wang (2015), and Cohen et al. (2018), who found that actively managed funds generally fail to outperform broader market indices. On the other hand, Smith (2013) suggested that higher returns in SRI portfolios might be due to the superior skills of their managers. However, Johnson and Patel (2017) found no evidence to support the notion that SRI managers possess exceptional skills. According to Martinez and Rodriguez (2012), the limited selection of available stocks could explain the higher returns associated with SRI investments. This is further supported by research from Gupta and Li (2016) and Thompson and White (2019), who found that fund families with more focused investment strategies and mutual funds with concentrated portfolios often achieve better performance. Capelle-Blancard and Monjon (2014) also highlighted that the effectiveness and depth of the screening process have a significant influence on fund performance, potentially accounting for differing outcomes. In addition, Rahman (2022) showed that ESG factors are crucial risk components, suggesting that investment decisions in socially responsible funds should consider ESG criteria, as evidenced by higher Sharpe ratios when compared to traditional portfolios.

Similar studies have been conducted in emerging markets. Sherwood and Pollard (2018) found that ESG indices in emerging markets outperform non-ESG indices on a risk-adjusted basis. Ortas et al. (2012) analyzed the Brazilian Corporate Sustainability Index (BSCI) versus the Bovespa Index and concluded that investing in the BSCI did not result in a risk or return disadvantage during bullish markets. Studies by Viviers et al. (2008) and Peerbhai and Naidoo (2021) provided evidence of a learning effect, showing improved performance of SRI funds over time as fund managers became more familiar with socially responsible investments in the South African market. These studies also noted that SRI funds tended to perform better than conventional funds during periods of high market volatility and vice versa during periods of low volatility. Gladysek and Chipeta (2012) conducted an event study and found no significant abnormal returns for investors in a South African SRI index, with the SRI index outperforming the JSE ALSI only in 2004 due to initial enthusiasm for the index's establishment.

Several studies emphasize that SRI portfolio performance varies across market cycles (Nofsinger and Varma, 2014). Kim et al. (2014) investigated the relationship between corporate social responsibility and the risk of future stock price crashes, using data from two stock market boom periods (1995-1999 and 2003-2007) followed by crashes. They found that socially responsible companies with high transparency in financial reporting were less prone to stock price crashes during market crises. Nofsinger and Varma (2014) compared the performance of 240 U.S. SRI funds to similar conventional funds during the Global Financial Crisis (GFC) from 2007 to 2009 and during non-crisis periods. Their findings indicated that conventional funds outperformed SRI funds by 0.67-0.95% during non-crisis periods, but SRI funds outperformed by 1.61-1.70% annually during the crisis, attributing this to positive screening resulting in positive alphas. Additionally, SRI funds typically invested in younger, smaller, more profitable, and less volatile companies than conventional funds. Díaz et al. (2022) examined the relationship between ESG and firm financial performance during the Covid-19 crisis (January to April 2020) using daily data from the S&P500. They found that portfolios consisting of the top 25 ESG-rated stocks outperformed S&P500, while those with the bottom 25 ESG-rated stocks underperformed, similar to Nofsinger and Varma's (2014) observations for the GFC. Their results highlighted those social and environmental factors contributed to the outperformance, as highlighted in Figure 2.

Figure 2: ESG top 25 from the S&P 500 portfolio and the S&P 500



(Source: Díaz et al., 2022)

Hwang et al. (2021) also examined the performance of portfolios during the Covid-19 period in Korean firms. Their results showed that ESG activities helped protect firms from sharp declines in financial performance due to the pandemic. In the first quarter of 2020, firms' earnings dropped significantly, but those with higher ESG performance experienced smaller declines. The results imply that in uncertain environments, a firm's ESG activities positively influence its financial outcomes which benefit SRI investors. The empirical evidence reviewed in this section provides no consensus on whether SRI funds outperform conventional funds. However, there is some evidence to suggest that the best ESG practices may not be appreciated by investors during normal times but are rewarded during periods of increased uncertainty or market turmoil.

2.4 SRI Portfolio Construction

As outlined in Section 2.2, when constructing a portfolio, an investor typically focuses on risk and return. However, Zopounidis and Doumpos (2013) and Steuer et al. (2007) acknowledge the growing inclusion of non-financial criteria in portfolio selection models to better reflect the real preferences of investors. SRI requires investors to balance the competing priorities of enhancing a portfolio's ESG score and maximizing its risk-adjusted return. The key issue in SRI portfolio optimization is how to choose a set of reasonable asset weights while considering social responsibility. A commonly applied and simple method is negative screening, which involves omitting unwanted securities from the universe of interest and then weighting the remaining securities in proportion to their market capitalization (value weighting), equally or optimizing the Markowitz objective function (Jin, 2022). As mentioned in Section 1.2, this application under the MV approach has two key drawbacks. First, it contradicts the principles underpinning the MV model that a portfolio should be fully diversified to yield the optimal risky portfolio – excluding stocks results in a less diversified and less efficient portfolio (Lean, Ang and Smyth, 2015). Second, SRI portfolios are associated with a greater level of uncertainty in their inputs due to the difficulty in measuring ESG scores (Bernadi et al., 2016). Consequently, SRI portfolios formed according to the MV model may be subject to greater estimation risk. The MV approach is already sensitive to estimation risk (see Section 2.2.1), and thus this amplifies the existing problem. Therefore, it is important to study portfolio construction using approaches that account for additional estimation risk arising from including ESG considerations into the portfolio construction decision and to compare their characteristics and performance with existing methods.

Some researchers have adapted optimization problems by incorporating ESG performance metrics of individual securities and preference parameters as constraints or utility factors within the objective function. These adjustments often require complex non-linear programming algorithms to over- or underweight securities that align with a specific ESG orientation. For instance, Hirschberger et al. (2013) extended the traditional two-dimensional Markowitz portfolio optimization model to a three-criterion model by including linear ESG rating scores. They introduced a general method called the Custom Investment Objective Solver (CIOS) for calculating the non-dominated set algorithm. Qi et al. (2013) expanded the traditional portfolio selection model to multiple objectives and solved the problem analytically using a GA. Utz et al. (2014) applied the CIOS algorithm to a utility model for optimizing SRI mutual fund portfolios and determined the risk tolerance of the funds through inverse optimization. Gasser et al. (2017) integrated firms' ESG scores into the investment process using a posteriori method to create a capital allocation plane in a three-dimensional space of return, risk, and social responsibility.

Jin (2022) proposed an alternative framework for integrating ESG into portfolio optimization, acknowledging that systematic ESG risk can drive the co-movement of security prices. This framework includes a double-index model, a two-layer grouping, and an extended criteria decision rule for optimal portfolio selection. It considers that SRI investors have dual objectives and suggests incorporating systematic ESG risk, rather than the risk of individual ESG assets, into portfolio optimization, akin to modern portfolio theory. Jin's (2022) framework offers a straightforward decision rule as a practical alternative to complex non-linear programming algorithms, clearly highlighting the desirable characteristics of securities. Applied to U.S. equity mutual funds, this approach demonstrated how investors could better understand the relevance of systematic ESG risk to future risks and returns, strategically manage this risk, and enhance the portfolio’s risk-adjusted returns. The framework provides an empirical method that aligns with theoretical analyses on ESG factor investing (Jin, 2022).

Some of the ESG evaluation methods referenced, when applied in portfolio construction, feature characteristics that account for multiple indicators. Charnes et al. (1978) and Banker et al. (1984) introduced Data Envelopment Analysis (DEA), a data-driven approach designed to manage multiple inputs and outputs. This approach has been adopted in portfolio optimization by researchers such as Murthi et al. (1997), Briec et al. (2007), Branda (2015), Liu et al. (2015), and Zhou et al. (2018). DEA is used to assess stocks from different perspectives, often involving a multi-input and multi-output framework. However, traditional DEA methods have some drawbacks in stock selection. First, when evaluating publicly listed companies, decision-makers frequently depend on available financial indicators, which can create challenges if there are no explicit inputs to consider. Lovell and Pastor (1999) were among the first to explore a radial DEA model that does not require explicit inputs or outputs, while Despotis (2005) developed a DEA model that focuses solely on output indicators. Liu et al. (2011) laid out a theoretical foundation for DEA models without explicit inputs. Second, ESG evaluations require the concurrent assessment of all three ESG dimensions, which traditional DEA models lack. To address this, Kuosmanen and Post (2002) and Yang et al. (2014) suggested DEA models that incorporate quadratic terms from an extended utility framework. Expanding on this concept, Chen et al. (2021) created a DEA model without explicit inputs and with quadratic terms, allowing for a more accurate recalibration of ESG scores. This research integrated the DEA cross-efficiency model with the mean-variance approach for asset selection, incorporating ESG scores as a measure of social responsibility to reduce risks associated with fluctuations in ESG scores. Additionally, the study proposed a portfolio optimization model that integrates ESG scores with selected assets.

2.5 Empirical Evidence on SRI Portfolio Construction Methods

Most studies on the optimization techniques for SRI portfolios focus not on the effectiveness of the techniques in creating well-performing portfolios but rather on providing portfolio complexity frameworks that integrate financial, social and environmental considerations. These studies investigate whether there is a financial cost or benefit to including additional ESG considerations and whether SRI portfolios tend to outperform or underperform otherwise similar conventional portfolios, as mentioned in Section 2.4. Below is a short overview of relevant SRI-related studies, focusing on multi-criteria portfolio optimization models from a portfolio management perspective.

Amon et al. (2021) developed a range of SRI asset-allocation strategies and evaluated them against naive and value-weighted portfolios, using these as benchmarks for the U.S. and European markets. The strategies were designed to be passive, minimizing the impact of management skills and associated fees. The study found a negative correlation between ESG scores and returns, which was more pronounced in the European market than in the U.S. This difference was attributed to Europe's stricter corporate social responsibility (CSR) standards, leading to generally lower ESG scores across the distribution in Europe compared to the U.S. The findings suggested that a straightforward ESG-weighted asset allocation approach outperformed both naive and value-weighted strategies in terms of ESG performance, although there was no significant difference in financial performance among the portfolios. The research also concluded that socially responsible investors might be willing to accept slightly higher transaction costs to rebalance their portfolios in alignment with their social responsibility preferences, as opposed to following a naive strategy.

Oikonomou et al. (2018) applied six conventional optimization methods—Markowitz, Black-Litterman, robust estimation, 1/N, RP, and reward-to-risk—to develop SRI portfolios for the U.S. market. Their findings indicated that formal optimization techniques like Black-Litterman, Markowitz, and robust estimation produced SRI portfolios with lower risk and better risk-adjusted returns compared to simpler methods such as naive, risk parity, and reward-to-risk. Of all the methods, the Black-Litterman approach generally provided the best results, while the naive approach tended to underperform. Despite generating portfolios with favorable risk-adjusted outcomes, the advanced optimization techniques resulted in less stable asset allocations and reduced diversification. A comparative analysis showed that these findings were more pronounced for SRI portfolios than for portfolios created from a broader range of stocks, likely due to the higher estimation risk associated with SRI portfolios, which sophisticated models are better equipped to handle. Although these advanced techniques struggled with diversification and the consistency of asset allocations over time, the study noted that this did not pose significant practical problems, as their returns remained superior, even after considering transaction costs. In a related study, Branch et al. (2019) evaluated the effectiveness of quantitative approaches in constructing ESG portfolios, including cap-weighted exclusion and optimized exclusion strategies, against a diversified benchmark. Their analysis found that while these methods were effective in reducing risk, they also had the potential to create undesirable exposure.

Very few studies have applied AI techniques to the construction of SRI portfolios. Zhang and Chen (2021) proposed two SRI construction models for double screening and portfolio optimization. These approaches combine ESG screening, where stocks with inferior ESG scores are excluded to align with investors' beliefs, with return potential screening, where stocks with high-return potential are included in the portfolio. The model introduces stock return predictions as inputs into the portfolio. A genetic algorithm extreme learning model (GA-ELM) is employed to predict the prices of the screened stocks, which are then included in the portfolio as high-quality stocks with high ESG and return potential (Zhang and Chen, 2021). The extended global minimum variance model (GMV) or extended maximum Sharpe ratio model (MSPR) is utilized to weigh the screened high-quality stocks. When comparing conventional social screening responsible investment-GMV (or MSPR) to double screening socially responsible portfolio models I or II (DSSRI I or II), superior performance is achieved on both ESG and financial measures. DSSRI I and II are shown to obtain higher annualized returns and ESG scores than any alternative models based on advanced AI models (Zhang and Chen, 2021).

2.6 Hypothesis Specification

Over the last two decades, SRI portfolios have gained widespread popularity in developed countries due to the increased focus on sociocultural rights, economic and environmental sustainability, and other stakeholder concerns. However, very few studies have compared the performance of AI and traditional approaches to portfolio construction for SRI portfolios. ESG practices remain in their development stages in emerging markets where corporations tend to invest less heavily in ESG strategies (Manrique and Martí-Ballester, 2017). Due to differences in the business environment and legislation, the implications and impact of SRI portfolio performance may vary between developed and emerging countries when using either traditional or AI techniques. This study therefore seeks to fill these gaps in the literature by analyzing the performance of AI and traditional portfolio construction techniques in the formation of SRI portfolios in the context of an emerging market, namely South Africa. Building from literature and in line with the research objectives established in chapter 1, the hypotheses are as follows:

H1. AI meta-heuristic optimization methods result in a better performing South African SRI portfolio.

H2. AI meta-heuristic optimization methods result in a better performing South African non-SRI (conventional) portfolio.

H3. The SRI portfolios outperform non-SRI (conventional) portfolios using the optimal methods.

CHAPTER 3. METHODOLOGY

3.1 Introduction

Determining the weightings of securities in a portfolio is a critical consideration in the investment process. Numerous approaches exist to determine these weightings. The goal of portfolio construction is to select the optimal weightings which will yield the highest returns and lowest risk during the investment period. Markowitz's mean-variance approach, being the foundation of portfolio optimization, is one of the models used in this study. Two other traditional approaches are employed, namely the naïve approach and the risk-parity approach. While traditional approaches are used in portfolio optimization, they are not computationally effective for solving complex portfolio models. Therefore, researchers often use meta-heuristic AI approaches to approximate optimal solutions in an efficient manner. These meta-heuristic approaches have been used to efficiently optimize complex portfolio optimization problems and the most popular algorithms of choice are the genetic algorithm, simulated annealing and particle swarm optimization which are used in this study. This chapter outlines each of these approaches.

3.2 Traditional Approaches

3.2.1 Markowitz Mean-Variance (MV)

As mentioned in Chapter 2, the MV portfolio construction approach of Markowitz is widely used by researchers and practitioners because it is easy to understand and introduces the concepts of return and risk in a straight-forward manner (Martinez-Nieto et al., 2021). The mean-variance approach can be described as a two-objective non-linear quadratic programming problem. It recognizes the importance of diversification, with the objectives of the approach being to (i) minimize risk (variance) for a given expected return and (ii) maximize expected return for a given level of risk (Markowitz, 1952; Stuart and Markowitz,1959). Markowitz's (1959) analysis is very useful and easy to implement when the distribution function is not quadratic in nature. The MV rule allows investors to find an optimal balance between return and risk, given their risk tolerance (Markowitz, 1952; Stuart and Markowitz, 1959). A risk tolerance value, λ, must lie between $(0,1)$ and a variety of efficient portfolios with different return and risk attributes can be identified with diverse λ values within the identified range. Risk tolerance values close to zero mean returns are prioritised whereas values closer to one increase the importance of risk avoidance (Stuart and Markowitz, 1959). Hence when λ equals zero, risk is ignored and return is maximized, whereas when λ equals one return is ignored, and risk is minimized. For this study the traditional mean-variance approach will be used as it minimizes variance, focusing entirely on risk reduction. Investors wish to maximize their investment performance and minimize their risk as follows:

$$max\ \mu_p = \sum_{i=1}^{n} w_i \mu_i \qquad \ldots(1)$$

$$min\ \sigma_p^2 = \sum_{i=1}^{n} w_i w_j \sigma_{ij}$$

subject to $\sum_{i=1}^{n} x_i = 1$ where n is the total number of assets in the portfolio, w_i and w_j are the weightings of assets i and j in the portfolio respectively σ_p^2 is the total portfolio variance and μ_p is the total portfolio return and $\mu_p = E(R_P)$ is the expected return of the portfolio (Stuart and Markowitz, 1959). From the data, individual mean and variance/covariances are computed as follows (Markowitz, 1952):

$$\mu_i = E(R_i) \qquad \ldots(2)$$

$$= \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$$

$$\sigma_{ij} = cov(R_i, R_j)$$

$$= \frac{1}{T-1}\sum_{t=1}^{T} (R_{i,t} - \mu_i)(R_{j,t} - \mu_j)$$

where T is the total number of observations in the dataset, $\mu_i = E[\,R_i]$ is the expected return for each asset i (and R_i is the random return), $\sigma_{ij} = cov(R_i, R_j)$ is the covariance between the returns of assets i and j and σ_p^2 is the variance of the portfolio's expected return. An alternative way of calculating the mean and variance of a portfolio is (Markowitz, 1952):

$$\max\ \ \mu_p = \frac{1}{T}\sum_{t=1}^{T} (R_{p,t}) \qquad \ldots(3)$$

$$\min\ \ \sigma_p^2 = \frac{1}{T-1}\sum_{t=1}^{T} (R_p - \mu_p)^2$$

where $$R_{p,t} = \sum_{i=1}^{n} w_i R_i$$

According to Markowitz (1952), the approach requires all assets in the portfolio to have zero or a positive weight and the total weight of all assets included in the portfolio must be equal to one, which is referred to as the budget constraint. These are shown as follows:

$$w_i \geq 0 \qquad \ldots(4)$$
$$\sum_{i=1}^{n} w_i = 1.$$

However, there are substantial shortcomings associated with the MV approach to portfolio construction. It has two multi-objectives which is a non-trivial optimization problem (Chen et al., 2020). As previously stated in Section 2.2.1, no single solution exists that can simultaneously satisfy two objectives optimally and as a result there are several optimal solutions called Pareto optimal solutions (Chen et al., 2020, Liu and Yin, 2018). The naïve or equally weighted approach solves the MV drawback of portfolio concentration as it allocates equal weights to all assets. No objective functions can be improved while not degrading other objective functions.

3.2.2 Naïve (1/N Rule)

Naïve diversified portfolios are based on a simple asset allocation strategy such as the 1/N rule, which suggests an equal split between the available investment opportunities. It does not attempt to assign asset weights to optimize the risk-return trade-off. As highlighted in chapter 2, the strength of the naive approach lies in its simplicity as it does not require the estimation of expected returns, covariances or higher moments of asset returns. Widely used in literature (such as Benartzi and Thaler, 2001; Lie et al., 2020; Windcliff and Boyle, 2004), the naïve approach has shown a competitive advantage to MV portfolios with respect to out-of-sample results (DeMiguel et al., 2009b; Duchin and Levy, 2009; Kritzman et al., 2010; Tu and Zhou, 2011).

The naïve or equal-weighted strategy that this study uses involves a portfolio weight $w_t^{ew} = \frac{1}{N}$ in each of *N* risky assets (DeMiguel et al., 2009b; Li et al., 2021). The naïve approach does not necessitate any optimization or estimation and completely ignores the characteristics of the data.

3.2.3 Risk Parity

The risk parity (RP) approach does not attempt to identify an optimal portfolio with the lowest possible risk for a given level of expected return as per the MV method but instead constructs a portfolio such that the risk contribution to the portfolio of each asset is approximately equal (Fabozzi et al., 2021). This approach was proposed by Qian (2005) and formalized by Maillard et al. (2010) for volatility risk measures. Sometimes viewed as a heuristics approach to portfolio selection without a sound theoretical justification, the RP approach is a result of an industry-driven effort to attain diversification and increase resistance to market downturns (Bellini et al., 2020). Kolm et al. (2014) and Maillard et al. (2010) advocate the best-known version of RP which is the equally weighted risk contributions (EWRC) portfolio approach, which quantifies the total risk contribution of each asset in the portfolio by considering the partial derivative of the portfolio's risk function with respect to the weight of each asset. As mentioned in Section 2.2.1, Miallard et al. (2010) states that the RP approach attempts to overcome the drawbacks of Markowitz's approach by excluding the use of expected returns, thus reducing the reliance on noisy estimated parameters (arising from their estimation errors) that mislead the optimization.

The risk contribution of an asset is defined by the product of its weight in a portfolio and its marginal risk contribution (Gambeta and Kwon, 2020). Euler's homogenous functions theorem is used to decompose the total portfolio risk measure, standard deviation (σ_p), into individual asset risk contributions, σ_i (Roncolli and Weisang, 2016; Bai et al., 2016). The asset's marginal risk contribution defines how individual assets are approached in the optimization process beyond just using the total portfolio risk which is asset agnostic in MV (Maillard et al., 2010). The sum of the assets' absolute risk contribution equates to total portfolio risk, expressed as:

$$\sigma_i(x) = \frac{x_i(\Sigma_x)_i}{\sqrt{x^T \sum x}} - \frac{\sqrt{x^T \Sigma x}}{n} \qquad \ldots(5)$$

$$\sigma_p = \sqrt{x^T \Sigma x} = \sum_{i=1}^{n} \sigma_i\,(x)$$

The risk parity approach is traditionally a least square fourth-order objective problem defined as:

$$min \sum_{i=1}^{N} \sum_{j=1}^{N} \left(x_i(\Sigma x)i - x_j(\Sigma x)j\right)^2 \qquad \ldots(6)$$

$$s \cdot t \cdot 1^T x = 1$$

$$x_i \geq 0$$

where $x_i(\Sigma x)i$ is the risk contribution for each asset and is compared to another asset's risk contribution $x_j(\Sigma x)j)$ (Fency and Palomar, 2015; Bail et al., 2016). This model compares the risk contribution of each asset with every other asset and reaches an objective value of zero indicating all risk contributions are the same. Equation 6 is a polynomial objective which is non-linear and non-convex, raising concerns over its numerical complexity when considering adding other constraints into the model (Maillard et al., 2010).

In addressing this concern, Lobo et al. (1998) discusses the application of a second-order cone programming model (SOCP). Mausser and Romanko (2014) show a clear implementation to risk parity as an equivalent SOCP formulation, which is shown as:

$$\begin{aligned} &\min_{x} \ \psi - \gamma && \ldots(7) \\ \text{s}\cdot t \cdot \ &\zeta_i = (\Sigma x)i \qquad i = 1 \cdot \cdots n && \\ &x^T \Sigma_x \leq n\psi^2 && \\ &x_i \zeta_i \geq \gamma^2 && \\ &1^T x = 1 && \\ &x_i, \zeta_i \geq 0 \ i = 1 \cdot \cdots n && \\ &\psi, \gamma \geq 0 && \end{aligned}$$

Equation 7 shows that, as the linear objective of the SOCP formulation approaches zero, the risk contribution between each asset becomes equal. γ_i is each asset's absolute risk contribution, which is compared to the average risk of the portfolios $\psi. \psi$ is also denoted as an upper bound on the risk contribution of an individual asset and dictates that any one asset's risk contribution should be equal to or less than the average risk (Gambeta and Kwon, 2014). Mausser and Romanko (2014) state that the average risk of the portfolio then equates to an equal risk magnitude to ensure the objective is effectively enforced to the RP allocations. To ensure this objective is met, the lower bound of an individual asset's risk contribution is γ, enforcing that $\psi \geq \gamma$. Using the marginal risk contribution, the total risk contribution of each asset is computed, and the asset weights are adjusted through the constraints in lines 2 and 4 of equation 7 respectively. Gambeta and Kwon (2020) continue to show that if $\psi = \gamma$, a RP portfolio is formed, the model becomes a minimization of the difference between these two risk values.

The RP portfolio achieves neither minimum risk nor maximum expected returns due to the ability to enforce diversity and it shows that the performance lies between the minimum-variance and equal-weight portfolios (Maillard et al., 2010). Risk parity has been seen as both a meaningful and effective approach to portfolio construction as it alleviates the concern of instability and balances return with diversification of risk, thereby avoiding concentration into risky assets (Bail et al., 2016; Maillard et al. 2015; Mausser and Romanko, 2014). However, RP has its drawbacks including the overweighting of low volatility assets (Maillard et al., 2010), sensitivity to interest rates (Asness et al., 2012), reliance on leverage (Anderson and Carverhill, 2012) and neglect of expected returns (Roncalli, 2013).

3.3 Artificial Intelligence Approaches

Traditional approaches, such as those described above, are limited when solving for portfolio models with constraints that introduce non-linear and non-convexity (such as boundary constraints and cardinality constraints). Furthermore, as the number of assets considered increases, determining which combination of assets yield an optimal trade-off between return and risk is more challenging (Erwin and Engelbrecht, 2023). As discussed in Section 2.2.2, the complexity of the portfolio optimization problem requires any portfolio management technique to follow a dynamic decision-making process. Hence, reinforcement-based learning techniques are highly suitable for effectively managing portfolios and meta-heuristic AI techniques can be used to fulfil this mandate. Meta-heuristic AI techniques in portfolio optimization refer to high-level problem-solving frameworks that provide a set of guidelines or strategies to develop heuristic algorithms for solving complex optimization problems, especially when traditional methods are inefficient or infeasible (Sadeghi et al., 2010). Meta-heuristic algorithms are designed to explore the solution space effectively and efficiently by combining exploration (global search) and exploitation (local search) techniques. They are often used in portfolio optimization to find near-optimal solutions in reasonable timeframes, especially in cases involving large, non-linear, or non-convex optimization problems (Gilli et al., 2009; Erwin and Engelbrecht, 2023). All three meta heuristic methods used for portfolio construction in this study focus on maximizing returns and minimizing risk and include the same constraints as Markowitz's mean-variance approach.

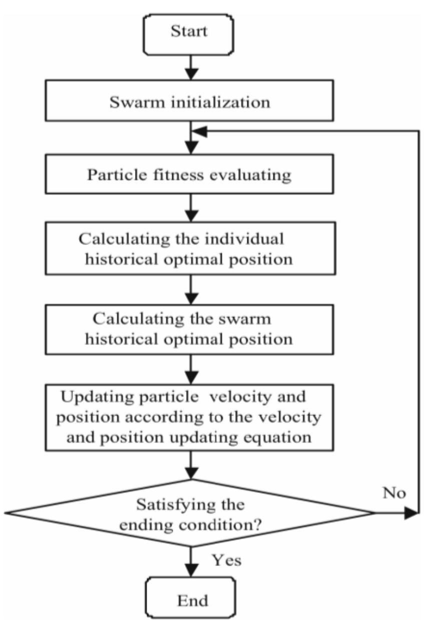
3.3.1 Particle Swarm Optimization (PSO)

3.3.1.1 Background

Eberhart and Kennedy (1995) describe Particle Swarm Optimization (PSO) as a distributed behavioral algorithm that conducts multidimensional searches by simulating the movements of a flock of birds or a school of fish seeking food. Zhu et al. (2011) confirm that PSO is inspired by swarm intelligence, which is derived from studying how natural creatures operate in groups. Through their interactions with one another or their environment, these creatures can accomplish tasks of much greater complexity than they could alone. The PSO algorithm starts by initializing a population of random particles, each associated with position and velocity (Zhu et al., 2011). The algorithm then adjusts particle velocities based on the historical behavior of each particle and its neighbors as they navigate the search space. At each step, positions and velocities are updated, progressively refining the search towards better solutions.

Initially, PSO identifies certain particles as the best in a neighborhood, primarily based on their fitness. All particles are then accelerated in the direction of these optimal particles, as well as towards their own best solutions previously discovered (Bratton and Kennedy, 2007). As particles explore the search space, they sometimes overshoot their target beyond the current best particles. This dynamic allows all particles in the algorithm the opportunity to discover better solutions along the way, prompting others to change direction towards these newly found optimal particles. Because most functions exhibit some degree of continuity, there is a high likelihood that a good solution will be surrounded by equally good or superior solutions. Consequently, particles are likely to approach the current best solution from various directions in the search space, allowing neighboring optimal solutions to be discovered by some particles (Zhu et al., 2011). Figure 3 illustrates the flowchart of the PSO algorithm.

Figure 3: Flowchart for PSO



(Source: Wang et al., 2018).

3.3.1.2 Portfolio Optimization

For portfolio optimization, each particle is a possible weight of security in the portfolio (Chen et al., 2021). It is denoted as the *k*th particle (where $k = 1, \dots., K$ particles) at the *l*th iteration (where $l = 1, \cdot \cdots, L$ iterations) as $x_k^l = \langle x_1, x_2 \dots x_n \rangle_k^l$ is a vector with *n* elements (Zhu et al., 2011). In a feasible area, the particles are randomly dropped and at each iteration, the particle x_k^l is updated consistently to velocity v_k^l, which is also a vector with *n* dimensions (Chen et al., 2021; Wang et al., 2017; Zhu et al., 2011). The mathematical expression can be written as:

$$x_k^{l+1} = x_k^l + v_k^l \qquad \dots(8)$$
$$V_k^{l+1} = wV_k^l + c_1 r_1 \left(p_k^l - X_k^l\right) + c_2 r_2 \left(g_1 - X_k^l\right)$$

where the velocity is decided by equation V_k^{l+1} and is the essential part of the PSO algorithm (Chen et al., 2021). r_1 and r_2 are uniform random numbers and c_1 and c_2 are positive acceleration coefficients (Chen et al., 2021; Wang et al. 2017). A vector of values $\langle x_1, x_2 \dots x_n \rangle$ represents each particle which is valued by the target function $F\left(X_k^l\right)$. g_1 is the best global position (at iteration *l*) that the whole swarm has experienced since the first iteration. Finally, *w* is the decay factor for previous velocity. Velocity is comprised of three components: the previous velocity, a velocity toward the personal best position and a velocity toward the global best position. The parameters, *w*, c_1 and c_2, control the PSO algorithm. As *w* measures the weight for the previous velocity, if w ≤ 1, the velocity decays over time until it reaches zero, while if w > 1, then velocity continues accelerating and does not revert (Chen et al., 2021). The constant, c_1, measures the contribution of personal cognitive experience, where a larger c_1 means the particle is confident about itself when compared to c_2 and will focus on a local search. Homogeneously, c_2 measures the contribution of swarm social experience where a larger c_2 shows that the particle is confident about the swarm and pays more attention to a global search which results in a trade-off between a global and local search when balancing c_1 and c_2 (van den Bergh and Engelbrecht, 2006).

The coefficients of PSO control the trade-off between a global and local search as mentioned above. At the initial stage, the particles are randomly dropped in the feasible space and must focus on a global search. When converging, the particles pay more attention to a local search (Chen et al., 2021) and the coefficients can decrease or increase linearly as iterations increase to reach the optimal goal. These linear expressions can be formulated as:

$$w = (w\,min - w_{max})\frac{l-k}{l} + w_{max} \quad \ldots(9)$$

$$c_1 = (c_{1\,min} - c_{1max})\frac{k}{l} + c_{1,max}$$

$$c_2 = (c_{2max} - c_{2min})\frac{k}{l} + c_{1,min}$$

3.3.1.3 Constraints satisfaction

Cura (2009) and Chen et al. (2021) suggests that for portfolio optimization, there is a necessary constraint that the sum of weights is equal to 1 and an optional short-selling constraint which are given in the linear expression in equation 10 (however in this study there will be no short selling – see Section 4.2.1):

$$X_{cm} = \sum_{k=1}^{n_s} F(X_k) \sum_{k=1}^{n_s} \frac{1}{F(X)_k} \otimes X_k \quad \ldots(10)$$

where X_k and X_{cm} are vectors and n_s is the number of particles in the swarm. The symbol, $\otimes$, represents element-wise multiplication (e.g., the value of $\frac{1}{F(X)_k}$ is multiplied by each element in the vector, X_k) (Chen et al., 2021). The PSO particle moves randomly in most cases, which may result in the particle not providing a feasible solution after a position update, but Cura (2009) proposed an algorithm that assists to ensure that the particle is feasible. To this end, the vector $x_k^l = \langle x_1, x_2 \ldots x_n \rangle_k^l$ contains *n* assets weights for portfolio optimization. Cura (2009) states that x_i represents the *i*th element in x_k^l (or the weight for the *i*th asset and the subscript *k* in x_k^l means the *k*th particle in the swarm which is different from the *i* in x_i). To obtain each particle that is feasible in the swarm and assure equation 10 holds, four different measures ($\delta^*, \varepsilon^*, \eta$ and $\emptyset$) that are the sum of the difference between the original weight, x_i, and bounds u_i or ℓ_i are clarified as (Cura, 2009):

$$\delta^* = \sum_{i=1}^{n} max\{0, u_i - x_i\} \quad \ldots(11)$$

$$\varepsilon^* = \sum_{i=1}^{n} \max\{0, u_i - \ell_i\}$$

$$\eta = \sum_{i=1}^{n} max\{0,\ x_i - u_i\}$$

$$\emptyset = \sum_{i=1}^{n} max\{0,\ \ell_i - x_i\}$$

If the upper or lower bound on any dimension is exceeded by the particle, then it will be arranged as follows (Chen et al., 2021):

$$x_i\{ \ell_i \qquad \ldots(12)$$

$$x_i\{u_i$$

$$x_i\{x_i + \frac{u_i - x_i}{\delta^*}\eta$$

$$x_i\{x_i + \frac{x_i - \ell_i}{\varepsilon^*}\phi$$

3.3.1.4 Portfolio selection problem

Each particle i at iteration j contains n such that weights $x_k^l = \langle x_1, x_2 \ldots x_n \rangle_k^l$ are satisfied (Chen et al., 2021). Chen et al. (2021) states that each particle's best history will be recorded (as its personal best p_k^l) as the loop through each iteration for each particle and the best of the personal bests will be the global best g_l. The global best is updated at each iteration and convergence can be reached after a large enough number of iterations. When using GA for optimization problems, the is no single solution that can simultaneously satisfy the two objectives for the MV. This is then referred to as the pareto optimal solution and none of the objective functions can be improved while not degrading other object functions (Chen et al., 2021). Therefore, the efficient frontier is interpolated by all the pareto optimal solutions. Chen et al. (2021), using a Sharpe ratio as an objective function (as discussed in section 4.2.3), subject to the constraint $\sum_{i=1}^{n} X_i = 1$, shows the Sharpe ratio as a single objective portfolio optimization problem. Constraints on the weights of assets are placed for a restricted portfolio optimization in PSO and it can be formulated as follows:

$$\ell_i < x_i < u_i \qquad \ldots(13)$$

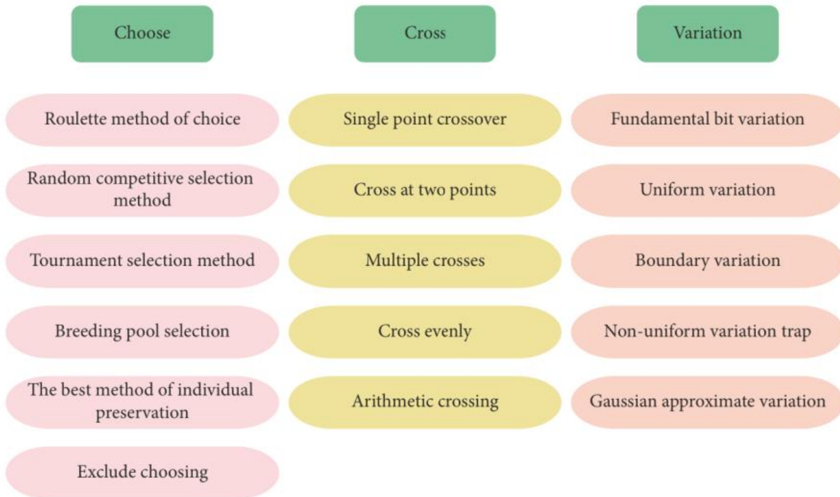
3.3.2 Genetic Algorithm (GA)

3.3.2.1 Background

Holland (1975) describes GA as a global optimization technique that mimics the natural processes of evolution, including natural selection and the survival of the fittest. In GA, each potential solution to a problem is represented as a chromosome, which forms a population of chromosomes suited to the specific constraints of the problem (Li and Shi, 2022). Each solution, or individual, is assessed using a predefined objective function to determine its fitness level. Those with higher fitness, indicating better adaptation to their environment, have a greater chance of survival. Initially, a set of candidate solutions, or individuals, is randomly generated. These candidates are then recombined based on the "survival of the fittest" principle to create offspring, which inherit advantageous traits from their parents and are therefore better suited than the previous generation. This iterative process leads the population of chromosomes to evolve towards an optimal solution. Through genetic operations, such as gene mutation during evolution, new offspring that are better adapted to the environment may be produced, enhancing the search for a global optimum (Tang and Wu, 2021).

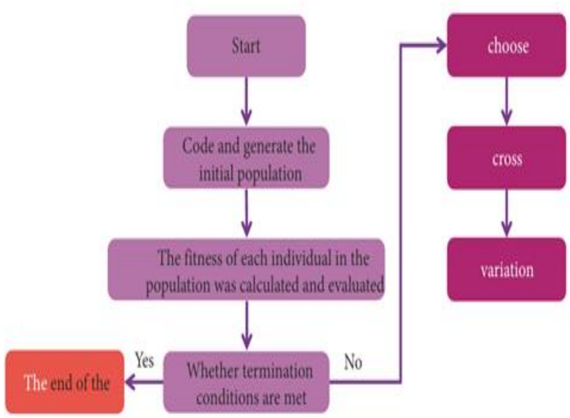
The GA methodology primarily consists of four key components: chromosome encoding, initialization of the population, definition of a fitness function, and the design of genetic operations like crossover and mutation. Encoding involves converting the solution framework of a practical problem into a chromosome format. To optimize solutions using GA, three fundamental operations are typically employed: selection, crossover, and mutation. The choice of selection strategy impacts the speed of the GA optimization process. While high selection pressure can enhance the convergence rate, it may also result in premature convergence to suboptimal solutions (Chang et al., 2009). Therefore, it is crucial to select an appropriate selection method. Selection focuses on choosing the fittest individuals and giving them a higher likelihood of being carried over to the next generation, while less fit individuals have a lower probability of survival. This process aligns the fitness values of individuals closer to the optimal solution. Crossover involves exchanging and combining parts of the genetic material from two parents to create new offspring, thereby enhancing the efficiency of the GA by generating novel solutions in subsequent generations. However, depending exclusively on crossover may lead to convergence on local optima rather than a global optimum. Mutation introduces random changes to individuals at a certain probability, altering vector values among individuals with a low occurrence rate. This random variation can generate entirely new structures within a population, significantly increasing the chances of finding a global optimum (Cuellar et al., 2021; Hung et al., 2021). The process of mutation thereby enhances the algorithm's exploration capabilities, as illustrated in Figures 4 and 5, showing the convergence towards a global optimal solution over successive generations.

Figure 4: The operation method for GA



(Source: Lin and Shi, 2022)

Figure 5: Flowchart for GA



(Source: Lin and Shi, 2022)

An evaluation function is formed to assess the fitness for each chromosome, which defines how good a solution the chromosome represents. By using crossover, mutation values and natural selection, the population will converge to one containing only chromosomes with good fitness. The larger the fitness value, the better the objective function value the solution has. The basic steps in GA are shown in Figures 4 and 5, however the flowcharts can also be illustrated in steps as follows:

1. Initialize a randomly generated population.

2. Evaluate the fitness of an individual in the population.
3. Apply elitist selection which entails carrying on the best individuals to the next generation from reproduction, crossover and mutation.
4. Replace the current population with the new population.
5. If the termination condition is satisfied then stop, else go to Step 2.

3.3.2.2 Portfolio constraints and portfolio selection under the GA algorithm

The GA approach for portfolio optimization is based on the steps outlined above. Following Chang et al. (2009), this study implements a population size of 100. Parents are chosen by binary tournament selection which works by forming two pools of individuals, each consisting of two individuals drawn from the population randomly. The individuals with the best fitness, one taken from each of the two tournament pools, are chosen to be parents. In evaluating the fitness objective function, the mean-variance objective function is used to identify a feasible solution. The algorithm follows the MV function in equation 2. However, equation 16 shows that in the case of λ = 0, it represents minimum risk and λ = 1 represents maximum expected return. The values of λ must satisfy 0 < λ < 1, representing an explicit trade-off between risk and return, generating solutions between the two extremes (Chang et al., 2009). GA will use the MV objective function as a feasible solution in the portfolio optimization problem as follows:

$$\min \lambda \, [\Sigma_{i=Q} \Sigma_{j=Q} w_1 w_j \sigma_{ij}] - (1-\lambda)[\Sigma_{i \epsilon Q} w_i u_i] \qquad \ldots(14)$$

The chromosome representation of a solution has two distinct parts, a set Q of k distinct assets and k real numbers $S_i (0 \leq S_i \leq 1)$, $i \in Q$. When given a set Q of k assets, a fraction $\sum_{j \in Q} \varepsilon_j$ of the total portfolio is already accounted for and so S_i can be interpreted as relating to the share of free portfolio proportion 1 - $\sum_{j \in Q} \varepsilon_j$ associated with asset $i \in Q$. The parameters in the cardinality constrained model *K* represent the number of assets in the portfolio, ε_i the minimum weight of the assets i in portfolio, and δ_i maximum weight of assets i in portfolio. The GA evaluation automatically ensures that the constraints relating to the lower limits, ε_i, are satisfied in a single algorithm step (Chang et al., 2009; Goldberg, 1997; Lin et al., 2008; Lin and Si, 2022). However, the study requires an iterative procedure to ensure that the constraints relating to the upper limits, δ_i, are satisfied. Hence the proportion associated with i in the portfolio is given by:

$$w_i = \varepsilon_i + \left(s_i \Big/ \sum_{j \in Q} s_j \right) \left(1 - \sum_{j \in Q} \varepsilon_j \right) \qquad \ldots(15)$$

which is the minimum proportion plus the appropriate share of the free portfolio proportion. There is a constraint of $\varepsilon_i z_i \leq w_i \leq \delta_i z_i$, $i = 1 \ldots \ldots N$ relating to the limits on the proportion of an asset that can be held. ε_i is the minimum proportion that must be held of asset i if any asset i is held and δ_i is the maximum proportion that can be held of an asset i; $z_i = 1$ if any of asset i is held otherwise 0 (Khader et al., 2020). In terms of portfolio optimization problems, each chromosome represents the weight of an individual stock in the portfolio and is optimised to reach a possible solution (Chang et al., 2009).

For the third step, children in GA are generated by uniform crossovers. In the crossover, two parents have a single child. If an asset i is present in both parents, it is present in the child (with an associated value s_i randomly chosen from one or other parent). But if asset i is present in just one parent, then the probability of it being in the child is 0.5. The algorithm's children are subjected to mutation, which is randomly chosen by the model with equal probability of the value $(\varepsilon_i + s_i)$ of randomly selected asset i and this mutation corresponds to decreasing or increasing by 0.01 (Chang et al., 2009). When the model is in the replacement strategy, the algorithm uses a steady-state population replacement strategy. This strategy allows each new child to be placed in the population as soon as it is generated, and the algorithm chooses to replace the member of the population with the worst objective function value. Lastly, the termination criterion with regards to all the computational results will examine different λ values and the number of iterations for the GA approach used. These values of λ means that the meta heuristic evaluates 100N solutions for each value of λ (Chang et al., 2009).

3.3.3 Simulated Annealing (SA)

3.3.3.1 Background

Developed in the 1980s, the SA algorithm is a trajectory-based heuristic search method that performs stochastic neighborhood searches independently of each other (Kirkpatric et al., 1983;

Cerny, 1985). The SA algorithm provides good solutions for combinatorial optimization problems, and it does not get stuck at the local optimum while searching for a global optimum compared to traditional local search methods. The development of the SA algorithm was inspired by the physical annealing process of solids in physics (Kirkpatrick, 1983). The annealing consists of two steps. First, the temperature of the heat bath is increased to the highest value a metal can melt, such that the atoms gain the necessary energy and have the freedom to move. Second, the appropriate crystal structure is obtained by cooling and freezing (annealing) the metal in a controlled manner until it enters a low energy phase.

The neighbor of the randomly selected initial solution is generated with a suitable mechanism of action in the SA algorithm (Chang et al., 2000). If the new solution creates an improvement in the objective function, then it is considered the current solution. The algorithm's process continues until a better solution cannot be obtained amount the neighbors of the current solution (Kirkpatrick, 1983). This procedure is a descent algorithm and ends with the best local solution. An important step for the algorithm, its characteristics removes the disadvantage of the descent algorithm, and the algorithm also allows/accepts bad solution but in a controlled manner Dogan et al. (2024). This decision ensures that the algorithm avoids local optimum, and this action is determined at random within a plan. The algorithm uses a probability of acceptance function with an objective to calculate the move that causes deuteration. Metropolis et al. (1953) suggest using the Boltzman-Gibbs distribution as a probability function to make the acceptance function and the acceptance function is determined by:

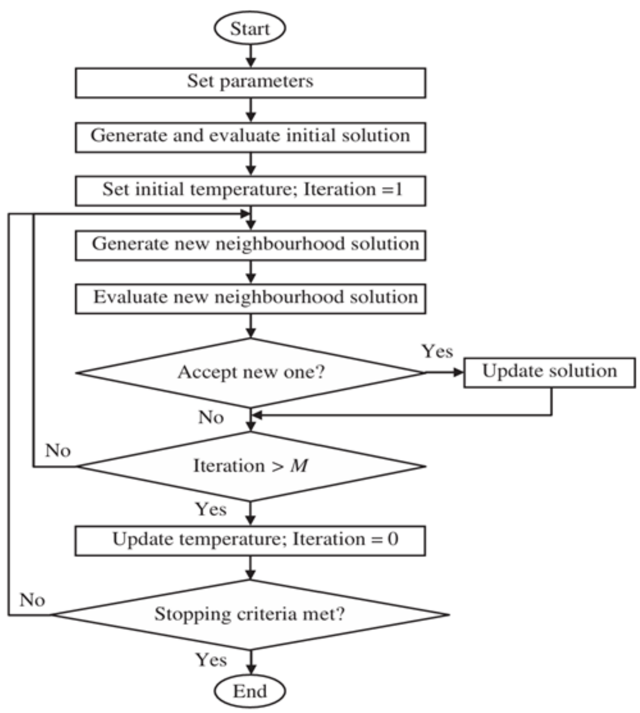
$$P(acceptance) = e^{(-\Delta/T)} \qquad \ldots(16)$$

where T is a control parameter corresponding to the temperature at the physical annealing and is the difference between the current solution and the neighboring solution. Chang et al. (2000) states that a fixed number of neighboring solutions are sought at each temperature value and after each stage, the temperature is reduced by a constant factor $\alpha \in (0.9)$. The SA algorithms differ from each other according to various factors such as neighborhood search, cooling (annealing) schedule and termination criteria.

3.3.3.2 Portfolio constraints and portfolio selection under the SA algorithm

In this study, a constrained portfolio model is adopted for this algorithm. The SA starts the problem with a random solution point coded as 0-1 and the neighboring solutions of the current solution that can be reached with a single action are produced by operating the neighboring structures unique to the SA. The initial temperature value is generated from the initial solution value as in the study of Chang et al. (2000) and at each temperature, neighboring solutions consider four times the total number of assets (4*N) in the portfolio to find the perfect solution. Experiments are carried out with values of 0.80, 0.85, 0.90 and 0.95 for the α parameter used for the cooling plan and 2*N, 3*N and 4*N values for the number of neighbors. The best result is obtained when the number of neighbors is 4*N and α is 0.90. The flow chart and the pseudo code that the algorithm follows is shown in Figures 6 and 7.

Figure 6: Flowchart for SA



(Source: Sadeghi et al., 2010)

Figure 7: Pseudo-code for SA

```
Begin;
    Choose a configuration α_0;
    Select the initial and final temperatures T_0, T_f > 0;
    Select the temperature schedule;
    Set α_1 := α_0 and T := T_0;
    Repeat:
        Repeat:
            Choose a new configuration α_2 from the neighbourhood of α_1;
            If f(α_2) ≥ f(α_1) then α_1 := α_2;
            Else
                Choose a random r uniformly in the range (0,1);
                If r < exp[ (f(α_2) – f(α_1) ) / T] then α_1 := α_2, else α_1 := α_1;
        Until iteration count = M;
        Decrease T according to temperature schedule;
    Until stopping criterion = true;
    α_1 is the approximation to optimal configuration;
End.
```

(Source: Sadeghi et al., 2010)

For the SA feasible solutions to be obtained in the constrained model, the algorithm must meet the Markowitz efficient frontier constraint. The feasible solution must also meet the constraints in equation 14, where SA follows the GA function in the previous section. Then it should be $0 \leq \varepsilon_i \leq \delta_i \leq 1$ and the decision variable $z_i[1]$, asset i is in the portfolio or $z_i[0]$, asset i is not in the portfolio. Hence, the objective function in equation 17 is added to the cardinality constrained model as follows:

$$\sum_{i=1}^{N} z_i = k \qquad \ldots(17)$$

$$\varepsilon_i z_i \leq w_i \leq \delta_i z_i,\ i=1,\ 2,\ 3,\ldots,N$$

$$z_i \epsilon\ [0,1],\ i = 1,\ 2,\ 3,\ldots.N$$

Starting from λ = 0 with a chart corresponding to risk aversion and risk-bearing behavior for a certain level of return, an efficient frontier is created with different weight values until λ = 1 with an increase of 0.01 each time.

3.4 Conclusion

This chapter outlines the six portfolio construction methods used in this study to form SRI portfolios. The methods include three traditional approaches – MV, RP and the 1/N approach and three meta-heuristic methods – SA, GA and PSO. The study uses PyCharm to execute all six portfolios optimization approaches. In PyCharm, portfolio optimization models function as black boxes when users rely on prebuilt libraries and default settings without delving into their methodologies. To mitigate this, this study explores the algorithm logic, log intermediate steps, and customize key elements like the objective function or constraints of each approach. By doing so, the process becomes more transparent, fostering better understanding and reproducibility.

Packages used when solving the optimization problem in PyCharm were prebuilt functions that include SciPy, NumPy, optimize-minimize, cvxpy solvers and Pandas, which are free and open-sources in the Python library used for scientific and technical computing for sequential least-squares programming (SLSQP). SLSQP is an optimizer that uses traditional quasi-Newton method to obtain optimal solutions (Chen et al., 2021). For portfolio optimization, fitness is measured on a backward-looking basis within the in-sample (testing sample) period which involves using historical data to estimate the parameters that drive the optimization. For robust portfolio performance, out-of-sample testing or forward-looking evaluation should follow to validate the results. In the following chapter, the performance metrics that are used to assess portfolio performance are described as well as the data and sample period.

CHAPTER 4. PERFORMANCE MEASUREMENT AND DATA DESCRIPTION

4.1 Introduction

This chapter describes the performance evaluation metrics which are used to compare the SRI portfolios constructed utilizing the portfolio optimization approaches described in the preceding chapter, as well as evaluating the performance of the SRI optimal portfolios against the non-SRI optimal portfolios. The metrics used for this purpose include the portfolio's annual return, risk, risk-adjusted returns (based on the Sharpe, Modified Sharpe and Sortino ratios), the level of diversification of the portfolio and the stability of the portfolio asset weights. Thereafter, the sample period and data that is used in the construction of the portfolios is described. Finally, an explanation is provided of how the sample period is separated into distinct estimation and investment windows.

4.2 Performance Metrics

4.2.1 Return

The annualized return over the investment period is an essential measure for investors as it helps them in comparing the performance of different portfolios. It is calculated as the weighted sum of the return of each asset in the portfolio on day *t*, as shown in eq. (18) line 1, and then annualized as per equation (18) line 2:

$$\begin{aligned} r_{pt} &= \textstyle\sum_{i=1}^{n} w_i \cdot r_{it} \\ R_p &= \left(1 + \bar{r}_p\right)^{252} - 1 \end{aligned} \qquad \ldots(18)$$

where w_i is weight of asset *i* in the portfolio, r_{it} is the return on asset *i* on day *t*, *n* is the number of assets in the portfolio, r_{pt} is the portfolio return on day *t*, R_p is the annualized portfolio return, $\bar{r}_p$ is the average daily portfolio return over the investment period and 252 is the approximate number of trading days in a year. The weights represent the proportion of the total investment allocated to each asset and must sum to one. The weights are determined under each of the optimization approaches outlined in the preceding chapter. In implementing all six approaches, the possibility of short selling (as represented by a negative weighting) was ruled out on the assumption that investors who undertake SRI do not engage in short selling. Motivated by Chopra (1993) and Jagannathan and Ma (2002) where these studies impose a no short-selling constraint to reduce estimation errors in optimizing MV portfolio selection strategies. It also follows the assumptions by Oikonomou et al. (2018).

4.2.2 Risk

The study uses the annualized standard deviation of the portfolio returns as a measure of total risk. Annualized portfolio volatility (standard deviation) provides investors with an estimate of how much the returns of the portfolio have fluctuated over the investment period. It is calculated as follows:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \, \sigma_i \sigma_j \rho_{ij}} * \sqrt{252} \qquad \ldots(19)$$

where $\sigma_i \sigma_j \rho_{ij}$ is the covariance for assets i and j, ρ_{ij} is the correlation for assets i and j and σ_p is the annualized portfolio standard deviation. Higher volatility indicates greater risk, as the returns can vary significantly from the average, whereas lower volatility suggests more stable returns.

Campell et al. (2001) highlight that employing standard deviation as the measure of risk implies that investors weigh the probability of negative returns equally against positive returns. However, it is a stylized fact that the distribution of many financial return series is non-normal, with skewness and kurtosis pervasive. Furthermore, Oikonomou et al. (2018) argue that there is ample evidence that investors often treat losses and gains asymmetrically and there is a wealth of experimental evidence for loss aversion (Kahneman et al., 1991). Hence, researchers and practitioners favor utilizing a measure of risk that incorporates non-normality in the return distribution. For this purpose, the mean downside standard deviation is frequently used, where the negative tail of the distribution is assessed separately. Effectively it focuses only on the returns that fall below a certain threshold (Markowitz, 1991). When a threshold of zero is used, the mean downside standard deviation measures the negative returns in a portfolio and thus, downside volatility provides a more focused view on the risk arising from losses. Downside deviation thus analyses the lower partial moments of the return distribution. A higher downside standard deviation indicates greater negative returns, while lower downside volatility suggests more stable, and fewer negative, portfolio returns. This study also utilizes this downside measure, with the threshold specified as zero.

4.2.3 *Risk-adjusted performance*

Risk-adjusted performance metrics quantify the return a portfolio generates relative to the risk of the portfolio. These metrics help investors understand if portfolio returns justify the risk level. In assessing the risk-adjusted performance of the SRI portfolios, this study employs the Sharpe, Modified Sharpe and Sortino ratios. Sharpe's (1966) ratio is defined as:

$$SR_p = \frac{R_p - r_f}{\sigma_p} \qquad \ldots(20)$$

where SR_p is the Sharpe ratio of portfolio p and r_f is the risk-free rate. This ratio provides a measure of the excess return an investor earns from the extra volatility endured for holding a riskier asset. A higher Sharpe ratio indicates better risk-adjusted performance. The ratio rests on the assumption that returns are normally distributed and/or that the investor has a utility function whose only arguments are return and variance (Bessler et al., 2014). The advantages of this ratio are its simplicity, ease of interpretation and that it cannot be manipulated by leverage (Sharpe, 1994).

A notable limitation of the Sharpe ratio is that when excess returns are negative, the ratio can be non-intuitive and misleading. A negative Sharpe ratio indicates that the investment has underperformed, yielding a return lower than the risk-free rate, but it does not indicate the degree of the underperformance effectively (Bacon, 2008; Bodie et al., 2014). An extension of the Sharpe ratio was developed by Israelsen (2005) to overcome the difficulty in interpreting negative Sharpe ratios, known as the Modified Sharpe ratio, which is used in this study. This ratio accounts for the absolute value of the returns, which helps mitigate the effects of extreme values and outliers in the return distribution. This modification provides a more robust measure of risk-adjusted performance by considering the magnitude of returns rather than their direction. It is computed as follows:

$$MSR_p = \frac{R_p - r_f}{\sigma_p^{ER/|ER|}} \qquad \ldots(21)$$

where MSR_p is the Modified Sharpe ratio of portfolio p and ER is the excess return $(R_p - r_f)$. A higher ratio indicates better risk-adjusted performance, considering both the magnitude and consistency of returns.

The Sortino ratio, developed by Sortino and Satchell (2001), is also used to quantify risk-adjusted performance. It is based on the downside standard deviation meaning that it measures the performance of an investment portfolio relative to the risk of negative returns and hence is also known as the reward-to-lower partial moments ratio. This ratio thus differs from the Sharpe ratio, which considers both upside and downside volatility. It is calculated as:

$$SNR_p = \frac{R_p - r_f}{\sigma_d} \quad \ldots(22)$$

where SNR_p is the Sortino ratio of portfolio p and σ_d is the annualized downside standard deviation. A higher Sortino ratio indicates better risk-adjusted performance. It is a particularly useful metric for conservative or risk-averse investors who are more concerned with minimising losses than achieving high returns (Sortino and Satchell, 2001; Watanabe, 2006; Ziemba, 2005; Ziemba, 2012).

The Sharpe and Sortino ratios are also implemented using the total portfolio return rather than the excess return as the numerator. These versions of the ratios measure how much return is earned for each unit of risk without considering a benchmark or minimum acceptable return. Higher values indicate better risk-adjusted performance. Sharpe (1994) regards this measure as the simplest ratio to calculate risk-adjusted performance on the assumption of a zero risk-free rate. The Sortino ratio in this case evaluates how much return is earned for each unit of downside risk (Biglova et al., 2004; Lo, 2022; Van Der Meer and Yudistira, 2003).

4.3 Stability and diversification

Oikonomou et al. (2018) highlight that the ambiguity in companies' corporate social performance scores makes it more likely that the screening process (non-financial criteria) may lead to greater estimation risk in the inputs to the portfolio. Therefore, it is necessary to examine the way in which each portfolio optimization process also influences such characteristics in this analysis. For this purpose, two measures are employed namely, mean diversification and the mean stability metrics. The mean diversification metric captures the spread of the asset weights in the portfolio by summing up the squared portfolio weights for each asset and estimation period (Blume and Friend, 1975, Oikonomou et al. 2018), as shown:

$$D_t = \sum_{i=1}^{n} w_{i,t}^2 \qquad \ldots(23)$$

where D_t is the diversification measure at time t. A lower value for the measure shows higher diversification whereas a higher value reflects concentration (higher weightings in fewer assets).

The stability metric is calculated for the whole investment (across the investment sub-periods) as it captures the magnitude of changes in portfolio weights from one period to the next period. A smaller value suggests a more stable portfolio, meaning the asset weight changes are low over the sub-periods of the investment holding period. A sense of how stable the portfolio is over time is shown by averaging S_t over multiple periods. This is crucial for ensuring consistent investment performance in real-world scenarios where perfect data accuracy is rarely available. The stability measure is calculated as:

$$S_t = \frac{1}{T-1} \sum_{t=2}^{T} \sum_{i=1}^{n} \left(w_{i,t} - w_{i,t-1}\right)^2 \qquad \ldots(24)$$

where S_t is the stability of a portfolio in each period and the average over *T-1* period.

4.4 Sample and Data

The JSE was the first emerging market to provide an SRI index in 2004 (Bloomberg, 2023). Moreover, the market has been recognized for its contribution to the fostering of sustainable and transparent business practices. The SRI index has evolved over time in response to global sustainability initiatives and the King IV governance code (JSE, 2018). In 2015, the JSE partnered with FTSE Russell, the global index provider, to progress the JSE's work around promoting leading corporate sustainability practices. This saw the launch of two FTSE/JSE Responsible indices – the J113 and J110. The J113 index comprises the top 55 companies while J110 consists of the top 30 companies that comply with the standards of the FTSE Russell ESG ratings. For a company to be considered for inclusion in these indices, it must have a minimum ESG score of 2.9 based on the FTSE Russell ESG metrics system, which is derived from an analysis of the firm's environmental impact, social responsibility and corporate governance practices. For the purposes of this study, the constituents of the J113 FTSE/JSE Responsible Index (top 55 companies) as at

January 2024 are utilized as the assets for the construction of the SRI portfolio as these have already been screened based on their ESG scores.

FTSE Russell is one of many data providers who construct firm ESG scores; each data provider uses their own methodology and weightings of constituents. For example, Bloomberg, Refinitiv, MSCI and S&P Global all provide ESG scores for firms. To ensure that the stocks included in the SRI portfolios constructed in this study are top ESG performers (as reflected by the FTSE Russell ESG scores), the constituents of the MSCI South Africa ESG Leaders Index for South Africa were also examined. All stocks in the MSCI index are also included in the J113 index (although the MSCI index comprises less constituents, that is 28 constituents). Given this confirmation, the constituents of the J113 index formed the SRI assets for the portfolio construction.

Data for all 55 companies was extracted from Bloomberg in 2004 when the JSE first introduced their SRI index. This starting date was chosen as it represents a notable milestone in the evolution of SRI investing in South Africa. However, data was missing from many companies. Due to missing observations, the sample was shortened, commencing in February 2012 and ending in January 2024. However, even with this chosen period, data was only available for 40 companies from February 2012. As such, the SRI portfolios were constructed with 40 assets. The company names are listed in Table A1 in the appendix. All company daily stock prices for this period were obtained. Using the price data, the daily return on each stock was determined as the natural logarithm of the change in the stock's price as follows:

$$r_{it} = ln\left(\frac{p_{t+1}}{p_t}\right) \qquad \ldots(25)$$

where r_{it} is the return on stock *i* on day *t*, ln is the natural logarithm and p_t is the price of the asset on day *t*. For the risk-free rate, the yield on the South African three-month treasury bills was used. This is an annualized rate, with the quarterly rate computed as follows:

$$r_{f_{annual}} = \left(1 + r_{f_{quartely}}\right)^4 - 1 \qquad \ldots(26)$$

where $r_{f_{annual}}$ is the annualized risk-free rate is, calculated from the quarterly risk-free rate.

The final stage of the analysis involves comparing the performance of the SRI optimal portfolios to optimal portfolios constructed using companies that do not necessarily comply with the ESG criterion (i.e., conventional stocks). These are termed non-SRI portfolios. For this purpose, the FTSE/JSE Top 40 Index constituents are used. This index consists of the 40 largest companies ranked by investable market value. The FTSE/JSE Top 40 Index is the flagship index for the JSE capturing more than 80% of the total market capitalization of all the shares listed on the main board (Kotze, 2017). It is noted that many of the companies that are specified as SRI assets, extracted from the J113 index, are also in the JSE Top 40 Index and hence included in the non-SRI portfolios constructed in this study (there is an 80% overlap of companies as seen in Table A1 in the appendix). However, the results illustrate that there is sufficient diversity in constituents between the SRI and non-SRI portfolios to yield meaningfully different results and enable a comparison of their respective performance.

4.5 Estimation and Investment Periods

Following Oikonomou et al. (2018), out-of-sample overlapping estimation and non-overlapping investment periods are employed in constructing and evaluating the SRI portfolios. The use of the out-of-sample investment approach simulates a real investment scenario. It is assumed that a real-time investor applies portfolio weights derived from the training set (the estimation window). This simulation assumes the presence of groups of investors who are identical except for the different times they begin their investment periods (Lan, 2015). By implementing this technique, the study tries to naturally incorporate the adverse effect of estimation risk in the evaluation of portfolio performance.

With the study period of twelve years, two estimation periods are specified, each of six years in duration, and two investment periods are used, of three years each. These are outlined in Tables 1 and 2 which illustrate that each three-year out-of-sample investment period is preceded by its six-year estimation period. The estimation period (six years) is used to train the models (MV, RP, PSO, SA and GA) and then the performance of the portfolios is tracked during the investment period using the performance metrics explained in the previous section. This process is repeated for two periods.

Table 1: Five-year overlapping estimation periods

Period(t)	Start	End	Length in years
Estimation period 1	02/2012	01/2018	6
Estimation period 2	02/2015	01/2021	6

Table 2: Three-year non-overlapping investment periods

Period(t)	Start	End	Length in years
Investment period 1	02/2018	01/2021	3
Investment period 2	02/2021	01/2024	3

Drawing from the literature, a five- to ten-year estimation period is appropriate. This study follows Platinakis and Sutcliffe (2017) and Oikonomou et al. (2018) in the choice of six-year estimation periods. Given the relatively short data period, using a longer period restricts the analysis. The use of a relatively long estimation period provides more data points, which leads to more stable and statistically reliable estimates of parameters including returns, volatilities and correlations (Merton, 1980). It also reduces the risk of overfitting and makes the estimates more robust. Fabozzi et al. (2002) advocate for longer estimation periods, stating that they capture multiple market cycles, and it provides a more comprehensive view of asset behavior across different economic conditions. In another note, Jorion (2007) states that a shorter estimation period allows approaches to be more sensitive to recent market trends and conditions. It also helps with quickly adapting to regime shifts, where the market dynamics change drastically. Ang and Bekaert (2002) state that this can be useful for strategies that aim to capitalize on short-term trends or market anomalies. However, this study opts to use the six-year estimation period in line with a longer investment approach associated with SRI portfolio construction.

There are two reasons for the choice of a three-year out-of-sample investment period. First, Oikonomou et al. (2018) states that the demand for SRI is by long-term institutional investors such as pension and insurance funds. The earlier work of Ryan and Schneider (2002) supports the idea that such investors have long investment horizons with the intent of implementing a buy-and-hold strategy. Second, literature shows that prioritizing social performance by firms leads to the creation of comparative advantages which become economically valuable in the long run, but which may not yield any tangible financial benefits to the firm and investors in the short run (Cox et al., 2004; Hillman and Keim, 2001). Oikonomou et al. (2014) confirm that these benefits of social initiatives take longer than a year and, at times, even longer than five years to be visible. Luo and Bhattacharya (2006) also show that CSR activities lead to higher corporate performance in the long run. Similarly, Chopra and Wu (2016) find that environmental performance is positively correlated with improved operating performance for U.S. firms in the computer and electronics sector over the long term. This is consistent with Hart's (1995) natural resource-based theory that the take-up of efficient new technologies is competitively beneficial to firms. However, while revenue does eventually grow, it may take up to two years for the investment to pay off. Kassinis and Soteriou (2003) observe that good environmental performance yields improved revenues, market share and profitability because of increased product demand and customer loyalty after a year. Given this evidence, using a short investment period would not necessarily be appropriate. However, given the sample period constraints, a three-year period was selected to try and capture the longer-term orientation of SRI investors and the time taken for the benefits of high corporate social performance to materialize.

This study also examines the performance of the portfolios over the whole investment period by combining the sub-period performance (Oikonomou et al., 2018). This will provide a more comprehensive understanding of the portfolio's behavior and allows for a more nuanced assessment of its performance and risk characteristics. Campbell and Thompson (2008) express that analyzing the entire period provides a complete picture of the portfolio's long-term performance and it also allows investors to assess whether the portfolio's strategy has delivered its objectives over the full investment horizon. This helps ensure that short-term fluctuations or anomalies do not obscure the portfolio's overall performance, providing confidence in the strategy's sustainability (Fabozzi et al., 2002). Analyzing the whole period also allows analysts to evaluate the consistency of returns and risk management. Consistency across periods indicates a robust strategy, while significant variation may suggest vulnerability to specific conditions and the need to improve investment processes (Ang and Berkuert, 2002; Grinold and Kahn, 2000).

4.6 Conclusion

This chapter outlines the performance metrics that are used in the study to compare the portfolios across the different optimization methods. These metrics include return, risk (standard deviation and downside deviation), risk-adjusted performance (the Sharpe, Modified Sharpe and Sortino ratios), diversification and stability. These metrics are computed for two non-overlapping three-year investment periods, with the inputs computed over the relevant six-year estimation window. The stocks which comprise the SRI portfolios are those highly ranked on the FTSE/ JSE Russell ESG methodology. The next chapter presents and discusses the portfolio performance across all the investment periods.

CHAPTER 5. RESULTS AND DISCUSSION

5.1 Introduction

This chapter presents the results of the analysis described in chapters 3 and 4. The performance metrics are presented for the SRI portfolios formed using each of the portfolio optimization methods over the two investment periods and the whole investment period. Thereafter, the results for the non-SRI portfolios are examined and compared to those for the SRI portfolios. The results are reviewed in the context of theory and prior empirical findings.

5.2 SRI Portfolio Results and Discussion

The results for the first investment period (February 2018 to January 2021 inclusive) are presented in Table 3. This period includes the Covid-19 crisis which saw stock markets globally, and in South Africa, stumble in March 2020 upon Covid-19 being declared a pandemic and the implementation of lockdowns in most countries around the world. Despite this crisis, the South African stock market recovered by the end of 2020, as shown in Figure A1 (in the appendix). However, prior to the crisis, the market had been relatively flat, in line with South Africa's weak growth levels. Accordingly, the strong performance of SRI stocks over this period (as seen in Table 3), where all portfolios constructed with the SRI constituents yielded positive returns, shows stellar performance for these companies with high corporate social performance despite the general stagnation in the market.

Returns were highest for the naïve approach (12.58%), followed by portfolios formed using genetic algorithm, simulated annealing and the particle swarm optimization (11.05%, 10.60% and 9.48%, respectively). In contrast, the mean-variance portfolio had the lowest return over the period of 5.97%. The portfolio constructed using the naïve approach had the highest risk (standard deviation and downside standard deviation) while the mean-variance approach produced a portfolio with the lowest risk (standard deviation and downside standard deviation). The genetic algorithm, simulated annealing and the particle swarm optimization approaches had the highest risk-adjusted returns with Sharpe ratios of 0.5081, 0.3895 and 0.3107, respectively and Sortino ratios of 0.7036 and 0.5668 and 0.4388, respectively. In contrast, the worst performing portfolios on a risk-adjusted basis were the portfolios formed using the naïve approach (despite the high returns) and the mean-variance approach (despite having the lowest risk measures). This suggests that the AI methods yield more optimal portfolios from a risk-adjusted return perspective than traditional approaches during this investment period, with genetic algorithm the best.

The naïve approach results in the most diversified portfolio as shown by the lowest diversification score of 0.025, followed by the risk parity and simulated annealing approaches with values of 0.029 and 0.0615, respectively. The high diversification of the naïve approach is to be expected given that all assets are equally weighted. The most concentrated portfolio is constructed using the genetic algorithm approach. Thus, although the genetic algorithm approach yields the best outcome from a risk-adjusted perspective, it does so at the expense of diversification which an investor must be aware of. Simulated annealing, however, performs better from a diversification perspective and is second in terms of risk-adjusted performance.

Table 3: SRI portfolios for investment period 1

Performance Metrics	**MV**	**1/N**	**RP**	**PSO**	**SA**	**GA**
Annualized Return	0.0597	**0.1258**	0.0801	0.0948	0.1060	0.1105
Std dev.	**0.1950**	0.5490	0.2781	0.2827	0.2544	0.2038
Downside dev.	**0.1193**	0.3949	0.1679	0.2001	0.1748	0.1472
Sharpe ratio	0.2705	0.2166	0.2629	0.3107	0.3895	**0.5081**
MSR	0.2705	0.2166	0.2629	0.3107	0.3895	**0.5081**
Sortino ratio	0.4422	0.3010	0.4355	0.4388	0.5668	**0.7036**
Sharpe ratio (without r_f)	0.3061	0.2291	0.3589	0.3353	0.4167	**0.5422**
Sortino ratio (without r_f)	0.5004	0.3186	0.4771	0.4738	0.6064	**0.7507**
Diversification	0.0819	**0.0250**	0.0290	0.0757	0.0615	0.1182

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

The performance metrics for the portfolios constructed over the second investment period are shown in Table 4. As indicated in chapter 4, this period covers February 2021 to January 2024. As shown in Figure A1, this period did see overall growth in the South African stock market but also several notable declines including September 2021, April 2022 and August 2022. For example, the April 2022 decline coincides with the consequence of the Russian-Ukraine war (such as rises in energy prices, supply chain disruptions, shortages of critical foods etc.). The poor overall performance of the SRI portfolios (with all exhibiting negative returns) suggests that these companies with high ESG scores were not immune to the market downturns experienced. The portfolios constructed using mean-variance, simulated annealing and particle swarm optimization performed best (the least negative returns) with returns of -0.49%, -0.71% and -1.41%, respectively. The naïve approach generated the most negative returns of -9.69%. The standard deviations for the portfolios were less than in the first investment period across all six construction methods suggesting lower volatility, with the portfolios constructed using mean-variance, risk parity and particle swarm optimization having the lowest standard deviations of 13.64%, 15.69% and 15.75% respectively. Notably, the mean-variance approach again yields the lowest risk portfolio.

Table 4: SRI portfolio for investment period 2

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	**-0.0049**	-0.0969	-0.0532	-0.0141	-0.0071	-0.0233
Std dev.	**0.1364**	0.4084	0.1569	0.1575	0.1630	0.1769
Downside dev.	**0.0802**	0.2894	0.0937	0.1216	0.1194	0.1337
Sharpe ratio	-0.0880	-0.2547	-0.3844	-0.1350	**-0.0863**	-0.1714
MSR	**-0.0016**	-0.0425	-0.0094	-0.0033	-0.0023	-0.0053
Sortino ratio	-0.1344	-0.3594	-0.6438	-0.1748	**-0.1178**	-0.2268
Sharpe ratio (without r_f)	**-0.0359**	-0.2373	-0.3391	-0.0895	-0.0435	-0.1317
Sortino ratio (without r_f)	-0.0610	-0.3348	-0.5678	-0.1159	**-0.0595**	-0.1742
Diversification	0.1157	**0.0250**	0.0300	0.1278	0.0636	0.1637

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve, (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

The simulated annealing, mean-variance and particle swarm optimization approaches yield portfolios with the best Sharpe ratios of -0.0863, -0.0880 and -0.1350, respectively. These approaches also performed better based on the Sortino ratio with ratios of –0.1178, -0.1344 and -0.1748 for the simulated annealing, mean-variance and particle swarm optimization approaches respectively. However, the negative historical returns yield negative Sharpe ratios which become difficult to interpret (see Section 4.2.3) and thus the Modified Sharpe ratio is used to rank the portfolios in this investment period. Using this metric, the ranking of the approaches from first to last, are mean-variance, simulated annealing, particle swarm optimization, genetic algorithm, risk parity and the naïve approach. As such, the portfolio constructed using mean-variance is the best performing (i.e., the lowest underperforming) followed by the portfolios constructed using simulated annealing, particle swarm optimization and genetic algorithm based on risk-adjusted performance. In terms of diversification, the portfolio formed based on the naïve approach is best followed by that formed using the risk parity and simulated annealing methods while the portfolio constructed using genetic algorithm is again the worst performing portfolio. Accordingly, these results still attest to the value of the AI approaches (as they represent three of the top four) but it does illustrate that one approach (i.e. genetic algorithm) does not always produce the best portfolio (as it did in investment period 1).

Table 5 shows the performance metrics for the whole period for all six approaches, with the stability of the portfolio weights also included. This period's metrics are calculated as the average of the first and the second investment periods, as per Oikonomou et al. (2018). Both investment periods include notable but relatively short-lived crisis periods followed by a market recovery. The first period included the Covid-19 crisis, and the second period covered Russia's invasion of Ukraine. Over the entire investment period, however, all the SRI portfolios produce positive returns driven by the substantial positive returns in the first period attesting to the value of this investment strategy over a long investment period. The simulated annealing approach yields the portfolio with the best return across the whole period of 4.94%. The portfolios were constructed using the genetic algorithm, particle swarm optimization and mean-variance approaches follow with returns of 4.36%, 4.03% and 2.74%, respectively. The naïve and risk parity approaches result in portfolios with low returns of 1.44% and 1.34%. The mean-variance approach has the lowest risk with a standard deviation of 16.57% and a downside deviation of 9.97%. In terms of risk-adjusted returns, the genetic algorithm produces the best performing portfolio, with a Sharpe ratio of 0.1683 and a Sortino ratio of 0.4768. Based on the Sharpe ratio, this is followed by portfolios constructed using genetic algorithm, mean-variance and particle swarm optimization. The Sortino ratio yields a slightly different ranking after genetic algorithm, as mean-variance is the next best, then genetic algorithm and particle swarm optimization. The naïve approach produces the most diversified and stable portfolio. The risk parity, simulated annealing and mean-variance constructed portfolios follow with better diversification than the remaining methods of particle swarm optimization and genetic algorithm, while the risk parity, mean-variance and genetic algorithm have relatively stable weights across the two investment periods.

Table 5: SRI portfolios for the whole period

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	0.0274	0.0144	0.0134	0.0403	**0.0494**	0.0436
Std dev.	**0.1657**	0.4787	0.2175	0.2201	0.2087	0.1903
Downside dev.	**0.0997**	0.3421	0.1308	0.1608	0.1471	0.1404
Sharpe ratio	0.0912	-0.0190	-0.0607	0.0878	0.1516	**0.1683**
MSR	0.0912	0.0870	0.1267	0.0878	0.1516	**0.1683**
Sortino ratio	0.3078	-0.0292	-0.1041	0.1320	0.2245	**0.4768**
Diversification	0.0988	**0.0250**	0.0295	0.1017	0.0625	0.1409
Stability	0.0327	**0.0000**	0.0024	0.1732	0.0802	0.0707

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization 0.070(PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

The key issue of SRI portfolio optimization is how to choose a set of reasonable asset weights while considering social responsibility. To answer the first research question of this study, which portfolio optimization method yields an optimal SRI portfolio using both traditional and AI approaches for the South African market, it is of value to summarize the results in terms of risk-adjusted performance (using the Sharpe/ modified Sharpe ratio) as done in the table below:

Table 6: Top 3 SRI ranking for optimization approach both sub and whole period

	Best portfolio construction method
Investment Period 1	1. Genetic algorithm 2. Simulated annealing 3. Particle swarm optimization
Investment Period 2	1. Mean-variance 2. Simulated annealing 3. Particle swarm optimization
Overall Investment Period	1. Genetic algorithm 2. Simulated Annealing 3. Mean-variance

In the period of positive market returns (investment period 1), the genetic algorithm yielded the best portfolio in terms of risk-adjusted returns followed by simulated annealing and particle swarm optimization. During the period of negative market returns (investment period 2), the mean-variance-constructed portfolio dominated followed by the simulated annealing and particle swarm optimization constructed portfolios. For the whole period, the highest risk-adjusted performance was obtained by the portfolios formed using the genetic algorithm approach, followed by the simulated annealing and mean-variance methods. Drawing from this summary, several conclusions follow: First, there is no one single best approach. Second, the naïve and risk-parity approaches do not produce portfolios with the highest risk-adjusted returns for SRI portfolios in any periods. Third, the AI methods consistently outperform traditional approaches. The only exception to this is the mean-variance approach, which performs best in the second investment period and third when considering the whole investment period. Thus overall, the results do suggest that AI approaches perform well in the construction of SRI portfolios. However, investors must be aware that some of these portfolios are associated with relatively more concentration (as reflected by higher diversification scores).

In terms of the choice between the three AI approaches, the evidence is less clear. Genetic algorithms do best overall but perform poorly when the market is in a downturn (investment period 2). Simulated annealing, however, is more consistent, maintaining second position in constructing an optimal portfolio in both sub-periods and overall. In contrast, the particle swarm optimization does relatively well in the two sub-periods (third position) but is outperformed for the entire period by the mean-variance approach. The pattern across the AI approaches may be a unique feature of the time period studied but it is worth further investigation in future research. As such, the choice of AI approach to use when constructing an SRI portfolio is left to the discretion of the investor and, possibly, their market expectations (high or low returns). For investors focused purely on risk, the mean-variance may be the best suited construction method as this method consistently produced the SRI portfolio with the lowest risk.

This study concludes that meta-heuristic algorithms perform well, on average, when used in the construction of portfolios for stocks with high ESG scores compared to traditional methods. These findings are similar to other studies of portfolio construction optimization, both for non-constrained portfolios (i.e., there is no initial screening of stocks based on ESG) and constrained portfolios (i.e., SRI stocks). Sadeghi et al. (2009) show that simulated annealing creates optimal portfolios when comparing it to the genetic algorithm approach, which is like the findings of this study for the second investment period. Sadeghi et al. (2009) conclude by stating that the results clearly demonstrate the efficiency and effectiveness of simulated annealing when creating a portfolio. On the other hand, Chang et al (2000) shows that the genetic algorithm can find higher-quality portfolios than simulated annealing and tabu search approach. This is consistent with the observations for this study over the first investment period and the whole period. Busetti (2006) confirms the superiority of the genetic algorithm over tabu search and simulated annealing. In a follow-up study, Sadeghi et al. (2011) go on to demonstrate that meta-heuristics algorithm override the need for a large sample size, and they can analyze data with minimal knowledge of its structure or the relation between inputs and outputs, including nonlinear relations.

Yang et al. (2022) also further confirms that AI portfolio optimization methods help to mitigate biases. Dogan et al. (2024) found that simulated annealing and genetic algorithm performed better than the mean-variance approach when constructing portfolios using stocks in the Hang Seng index of Hong Kong. The results were robust to the use of stocks from the Borsa Istanbul 30 index. Erwin and Engelbrecht (2023), reviewing 140 papers, confirm that genetic algorithm and particle swarm optimization are the most popular meta-heuristic approaches to portfolio optimization and yield effective results. With specific consideration of SRI stocks, Chen et al. (2021) finds that meta-heuristic strategies result in portfolios of high performing socially responsibility stocks which outperform those based on traditional portfolio construction strategies using the S&P 500 index components from October 2017 to October 2018. The conclusion of this study is that the AI methods namely particle swarm optimization, simulated annealing and genetic algorithm yield the optimal portfolios for high-ranking SRI companies is thus consistent with prior studies.

5.3 Non-SRI Portfolio Results and Discussion

As indicated in chapter 4, the approaches used to construct portfolios comprising SRI stocks are also used to construct portfolios comprising conventional South African stocks (which may have high or low ESG scores). This enables a comparison of the performance of SRI and non-SRI portfolios. The results for the first and second investment periods are shown in Tables 7 and 8 respectively while those for the whole investment period are shown in Table 9.

Table 7: Non-SRI portfolios for investment period 1

Performance Metrics	**MV**	**1/N**	**RP**	**PSO**	**SA**	**GA**
Annualized Return	0.0409	0.0176	-0.0041	-0.0185	**0.0436**	0.0325
Std dev.	**0.0819**	0.2268	0.2048	0.1426	0.2106	0.0891
Downside dev.	**0.0538**	0.1549	0.1410	0.1155	0.1734	0.0714
Sharpe ratio	**0.4145**	0.0469	-0.0539	-0.1788	0.1740	0.2864
MSR	**0.4145**	0.0469	-0.0022	-0.0036	0.1740	0.2864
Sortino ratio	**0.6310**	0.0688	-0.0778	-0.2207	0.2114	0.3824
Sharpe ratio (without r_f)	**0.4994**	0.0776	-0.0200	-0.1297	0.2070	0.3647
Sortino ratio (without r_f)	**0.7602**	0.1136	-0.0291	-0.1602	0.2514	0.4552
Diversification	0.3333	**0.0250**	0.2882	0.1589	0.0901	0.2849

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

In the first investment period, the portfolio constructed using simulated annealing exhibits the highest return followed by the mean-variance and genetic algorithm (4.36%, 4.09% and 3.25%, respectively). The risk parity and the particle swarm optimization underperformed with negative returns (-0.41% and -1.85% respectively). The portfolio constructed using the mean-variance rule has the lowest risk with a standard deviation of 8.19% and a downside standard deviation of 5.38%. This is similar to the rankings based on risk for the SRI portfolios. Notably, this low risk gives rise to this portfolio earning the highest risk-adjusted returns compared to all approaches, with a Sharpe ratio of 0.4145 and a Sortino ratio of 0.6310. The genetic algorithm and simulated annealing-based portfolios follow Sharpe ratios of 0.2864 and 0.1740, and Sortino ratios of 0.3824 and 0.2114, respectively. As some portfolios have negative returns, portfolio risk-adjusted performance is also evaluated according to the Modified Sharpe ratio which ranks the portfolio from first to last as mean-variance, genetic algorithm, simulated annealing, naïve, risk parity and particle swarm optimization. With regards to diversification, the mean-variance, genetic algorithm and risk-parity approaches yield the most concentrated portfolios (highest diversification measures). Thus, although the mean-variance approach produced the portfolio with the highest risk-adjusted performance, it was the least diversified portfolio.

Table 8: Non-SRI portfolios for investment period 2

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	0.0359	0.0434	0.0430	0.0394	0.0450	**0.0564**
Std dev.	0.1628	0.1630	**0.1478**	0.1690	0.1545	0.1836
Downside dev.	0.1052	0.0987	**0.0867**	0.1193	0.1148	0.1290
Sharpe ratio	0.1778	0.2227	0.2432	0.1913	0.2453	**0.2686**
MSR	0.1778	0.2227	0.2432	0.1913	0.2453	**0.2686**
Sortino ratio	0.2752	0.3791	**0.4147**	0.2711	0.3303	0.3824
Sharpe ratio (without r_f)	0.2205	0.2662	0.2909	0.2331	0.2912	**0.3072**
Sortino ratio (without r_f)	0.3412	0.4535	**0.4959**	0.3302	0.3920	0.4372
Diversification	0.5000	**0.0250**	0.2888	0.3341	0.0698	0.3624

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

The second investment period shows an improvement in portfolio returns compared to the first investment period for all except the mean-variance constructed portfolio. This suggests that these top performing stocks on the JSE generated positive returns despite major downturns in the market due to, among other things, the Russian-Ukraine war. The portfolios constructed using the genetic algorithm, simulated annealing and naïve approaches had the highest returns of 5.64%, 4.50% and 4.34% while the portfolio constructed using the risk parity approach had the lowest standard deviation of 14.78% and the lowest downside standard deviation of 8.67%. In terms of risk-adjusted performance, the genetic algorithm, simulated annealing and risk parity approaches produced portfolios with the highest Sharpe ratios of 0.2686, 0.2453 and 0.2432, respectively. The Sortino ratio did produce a different ordering where the risk parity approached ranked first followed by genetic algorithm and the naïve approach with values of 0.4147, 0.3824, 0.3791 respectively. Portfolios exhibited greater concentration over this period, with only simulated annealing and the naïve approach producing well-diversified portfolios.

Looking at the non-SRI portfolios across the whole period (Table 9), the averages across investment periods 1 and demonstrate that all portfolios attained positive returns; the genetic algorithm exhibiting the highest returns followed by the simulated annealing and mean-variance approaches (4.44%, 4.43% and 3.84%, respectively). The mean-variance approach continues to exhibit the lowest risk with a 12.23% standard deviation and 7.95% downside standard deviation. According to the Sharpe and Sortino ratios, the mean-variance produced the best performing portfolio, followed by the genetic algorithm, simulated annealing and naïve constructed portfolios (Sharpe ratios of 0.5923, 0.2775, 0.2096 and 0.1301, respectively and Sortino ratios of 0.4531, 0.3824, 0.2708 and 0.2239, respectively). The naïve approach continued to produce the most well-diversified portfolio followed by simulated annealing and the particle swarm optimization method. The stability of the portfolio's weights was best for the naïve approach (0.000 as weights do not change), followed by the risk parity (0.0218), simulated annealing (0.0797) and mean-variance (0.1667) constructed portfolios. The mean-variance and genetic algorithm approaches yield the least diversified portfolios.

Table 9: Non-SRI portfolios for the whole period

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	0.0384	0.0305	0.0176	0.0104	0.0443	**0.0444**
Std dev.	**0.1223**	0.1818	0.1869	0.1558	0.1827	0.1363
Downside dev.	**0.0795**	0.1268	0.1301	0.1174	0.1441	0.1002
Sharpe ratio	**0.5923**	0.1348	0.0687	0.0062	0.2096	0.2775
MSR	**0.5923**	0.1348	0.0687	0.0062	0.2096	0.2775
Sortino ratio	**0.4531**	0.2239	0.0966	0.0252	0.2708	0.3824
Diversification	0.4166	**0.0250**	0,2888	0.2465	0.0799	0.3236
Stability	0.1667	**0.0000**	0.0218	0.1820	0.0797	0.2172

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

To answer the second research question of this study – the effectiveness of the best portfolio optimization method for the SRI portfolio in producing optimal portfolios for the non-SRI portfolio is evaluated based on risk-adjusted performance. Where differences are noted between the Sharpe and Sortino ratios, the Sharpe ratio-which accounts for total risk-is used for ranking. A summary of these results is provided in Table 10.

Table 10: Top 3 SRI and non-SRI ranking for optimization approaches both sub and whole period

	SRI Portfolios	**Non-SRI Portfolios**
Investment Period 1	1. Genetic algorithm 2. Simulated annealing 3. Particle swarm optimization	1. Mean-variance 2. Genetic algorithm 3. Simulated annealing
Investment Period 2	1. Mean-variance 2. Simulated annealing 3. Particle swarm optimization	1. Genetic algorithm 2. Simulated annealing 3. Risk parity
Overall Investment Period	1. Genetic algorithm 2. Simulated annealing 3. Mean-variance	1. Mean-variance 2. Genetic algorithm 3. Simulated annealing

In the first period, for the non-SRI portfolio, where some portfolios generated positive returns and others negative returns, the mean-variance approach yielded the best portfolio in terms of risk-adjusted returns. During the second period, where all portfolios generated positive returns, the genetic algorithm performed best and for the whole period, the highest risk-adjusted performance was obtained by the mean-variance approach, followed by the genetic algorithm and simulated annealing methods. The AI meta-heuristic approaches tend not to be as dominant in the non-SRI portfolios compared to the SRI portfolios as the particle swarm optimization is not part of the top three approaches for the non-SRI portfolio in any investment period. However, the simulated annealing and genetic algorithm continue to perform well for the non-SRI portfolios as with the SRI portfolios, but they are outperformed by mean-variance in the second investment period and for the overall period. The success of the mean-variance approach for the non-SRI portfolios in period 1 and the whole period shows some similarity to the success of the mean-variance approach in investment period 2 for the SRI portfolios. This suggests that the metaheuristic approaches do not consistently outperform the traditional optimization approach of Markowitz. There is similarity in that the top performers across both SRI and non-SRI portfolios are genetic algorithm, simulated annealing and mean-variance although these portfolios do have higher concentration and less stable weights. These findings are similar to Oikonomou et al. (2018), where their study found that the Black-Litterman approach tended to perform best, while the naïve approach usually performed worse. Although they found that sophisticated approaches produced favorable portfolios in terms of risk-adjusted performance, they had more unstable asset allocations and lower diversification.

The relative success of the AI meta-heuristic approaches in portfolio construction relative to traditional techniques is in line with prior literature, as cited in the preceding section (Adam, 2021; Cheong et al., 2017; Delahaye et al., 2018; Dogan et al., 2024; Erwin and Engelbrecht, 2023; Liu et al., 2017; Quang, 2022; Wang et al., 2015). Cheong et al. (2017) states that meta-heuristic approaches are not limited by the convexity assumptions of portfolio optimization and these approaches explore a broader solution space, avoid local optima and find better global solutions which replicate real-world financial markets. Quang (2022) shows that genetic algorithm is less sensitive to estimation errors because it searches for good solutions through iterative improvements and does not rely on precise parameter estimates. This contributes to the robustness of the algorithm in a practical real-world application, where data is often noisy and uncertain (Delahaye et al., 2018). The results are similar to the finding from the efficient use of meta-heuristic approaches when constructing both SRI and non-SRI portfolios from this study.

5.4 Comparison of SRI and non-SRI Portfolios

In accordance with the third objective of this study, which is to compare the performance of SRI and non-SRI portfolios, this section assesses their relative risk-adjusted performance. The focus is thus no longer on the best construction methods. Using the Sharpe ratio to assess risk-adjusted returns and diversification, the performance of SRI and non-SRI portfolios across both sub-periods and the entire period are compared. Tables 11 and 12 summarize these measures for each portfolio construction approach.

Table 11: Sharpe ratios for SRI and non-SRI portfolios (sub and whole period)

	Period 1		Period 2		Whole Investment Period	
	SRI	**Non-SRI**	**SRI**	**Non-SRI**	**SRI**	**Non-SRI**
MV	0.2705	0.4145	-0.088	0.1778	0.0912	0.5923
1/N	0.2166	0.0469	-0.2547	0.2227	-0.019	0.1348
RP	0.2629	-0.0539	-0.3844	0.2432	-0.0607	0.0687
PSO	0.3107	-0.1788	-0.135	0.1913	0.0878	0.0062
SA	0.3895	0.1740	-0.0863	0.2453	0.1516	0.2096
GA	0.5081	0.2864	-0.1714	0.2686	0.1683	0.2775

Table 12: Diversification for SRI and non-SRI portfolios

	Period 1		Period 2		Whole Investment Period	
	SRI	**Non-SRI**	**SRI**	**Non-SRI**	**SRI**	**Non-SRI**
MV	0.0819	0.3333	0.1157	0.5	0.0988	0.4166
1/N	0.025	0.025	0.025	0.025	0.025	0.025
RP	0.029	0.2882	0.03	0.2888	0.0294	0.288
PSO	0.0757	0.1589	0.1278	0.3341	0.1017	0.2465
SA	0.0615	0.0901	0.0636	0.0698	0.0625	0.0799
GA	0.1182	0.2849	0.1637	0.36	0.1409	0.3236

Using the Sharpe ratio, the SRI portfolio outperforms the non-SRI portfolio in the first period across all methods except mean-variance. Notably, the SRI portfolios seem to be better diversified compared to the non-SRI portfolios. However, this conclusion is not the same for the second investment period. The non-SRI portfolios yielded higher risk-adjusted returns across all six construction methods. However, the SRI portfolios are more diversified. Looking at the whole period, the non-SRI portfolios outperform the SRI portfolios. The risk for non-SRI portfolios is less than the risk experienced by SRI portfolios regardless of SRI portfolios exhibiting lower returns. The highest Sharpe ratio of 0.5923 is achieved for the SRI portfolio (based on mean-variance), while the highest Sharpe ratio for the non-SRI portfolio is of 0.1683 based on the genetic algorithm approach. However, the SRI portfolios are found to have more stable portfolio weights and exhibit greater diversification. Overall, the results point to SRI portfolios underperforming non-SRI portfolios. However, the former do exhibit periods of outperformance (investment period 1). As such, these results suggest that SRI portfolios experience periods of outperformance and underperformance and thus neither the shareholder theory nor stakeholder theory discussed in Chapter 2 dominates.

Notably, the results also reveal that the choice of portfolio construction method has little effect on the relative performance of SRI versus non-SRI portfolios i.e., non-SRI portfolios outperform (underperform) SRI portfolios over an investment period irrespective of the portfolio construction method used. The only exception is the use of the mean-variance approach in the first period which did result in a higher risk-adjusted performance for the non-SRI portfolio whereas all other portfolio construction methods yielded the opposite conclusion. Accordingly, the results in this study suggest that the mixed evidence of SRI performance compared to conventional investments in literature cannot necessarily be a function of the use of a non-optimal weighting method (like equal weighting). However, it must be acknowledged that this is only one study for this conclusion, and more research is needed to corroborate these findings.

The conclusions of SRI investment underperforming in South Africa are similar to the findings of Peerbhai and Naidoo (2022) who evaluated the risk-adjusted performance of South African SRI and conventional funds over the period January 2008 to December 2018. Their study found that SRI funds underperformed relative to conventional funds in earlier periods but outperformed or exhibited no significant performance difference in latter periods. In related work, Gladysek and Chipeta (2012) found that investors do not earn significant abnormal results by investing in an SRI index, with the JSE SRI index only outperforming the JSE ALSI in 2004, which was attributed to the enthusiasm associated with the establishment of the index. However, the findings of this study do differ from those of Viviers et al. (2008) who found that South African SRI portfolio returns are equivalent to those of South African non-SRI portfolios and provide similar levels of diversification. It must also be noted that the findings of non-SRI outperformance, and some periods of SRI dominance, in this South African study do differ from broader emerging market findings. For example, Sherwood and Pollard (2018) found that emerging market indices that integrate ESG considerations outperform non-ESG integrated indices on a risk-adjusted basis. Similarly, Rehman et al. (2021), using sustainable and conventional indices for Brazil, Russia, Indian, China and South Africa, documented that sustainable and conventional indices earn similar returns.

Comparing the results to international evidence, the finding of SRI underperformance is not unique (as indicated in Chapter 2). These results are supported by Jones et al. (2008) who found that SRI funds significantly underperform conventional funds in the market in Australia, particularly in the sample period 2000–2005. A similar study by Renneboog et al. (2008) that considers the performance of SRI funds in 17 countries from Europe, Noth America and Asia-Pacific notes that funds in the U.K, U.S, some of Europe and Asia Pacific underperform the conventional domestic benchmarks. However, the risk-adjusted returns of SRI funds were not statistically different from the performance of conventional funds, except for some countries such as France, Japan and Sweden. Cortez et al. (2009) also observed that SRI funds in seven European countries exhibited neutral performance in relation to both conventional and socially responsible benchmarks.

It must be also noted, however, that the finding that SRI portfolios performed well during the first investment period which included the Covid-19 market crash is broadly consistent with some international evidence, where market conditions are observed to influence the outperformance/ underperformance of SRI and non-SRI portfolios. For instance, Cortez and Leite (2015) show that while under normal conditions ESG indices underperform, during tumultuous periods such as the 2007 GFC, SRI funds outperform conventional indices because they play an insurance role (see also Varma and Nofsinger 2014; Becchetti et al. 2015). But SRI portfolios seemed to play less of an insurance role during the Russia-Ukraine crisis which occurred during the second investment period of this study.

5.5 Robustness Tests

To confirm the robustness of the results, the SRI and non-SRI portfolios were reconstructed and evaluated over different estimation and investment horizons. Three (as opposed to two) overlapping estimation windows of five-years in length were selected and three investment periods (rather than two) were evaluated, still of three years in length but therefore overlapping (rather than independent and consecutive as per the main analysis). The details of these periods are outlined in Tables A2 and A3 in the appendix. The results for the SRI portfolios are shown in Tables A4 to A6, and those for the non-SRI portfolios are shown in Tables A7 to A9. For this analysis, the performance metrics used were historical returns, risk and risk-adjusted returns only. For the SRI portfolios, the results obtained were similar to the ones shown in Tables 3 and 4, with these portfolios yielding high returns and risk-adjusted performance in the first period but the performance gradually declined in the two succeeding periods. The opposite was true for the non-SRI portfolios (see Tables 7 and 8), confirmed in these robust tests where in the first period the non-SRI portfolio returns, and risk-adjusted returns were lower but increased gradually in the following two periods.

For the non-SRI portfolios, simulated annealing resulted in the best performing portfolio across all three periods. Particle swarm optimization was the only other meta-heuristic approach in the top three across the three periods, with the second position held inter-changeably with the risk parity and genetic algorithm approaches. For the SRI portfolios, in two of the three periods, AI approaches were the top performers (simulated annealing in investment period 1 and particle swarm optimization in investment period 2) but in the third period the naïve approach produced the optimal portfolio. In contrast to the main findings, the traditional methods perform quite well in these robust tests particularly the naïve and risk parity approaches (ranking second or third). Overall, these findings do confirm the conclusion that meta-heuristic approaches do perform better, on average, than traditional approaches but the choice of best method among the three examined in this study is not clear. However, there is some evidence to suggest that in difficult periods for SRI portfolios, traditional approaches (especially the naïve method) can still be a useful approach to consider.

5.6 Conclusion

This chapter presented the findings from the construction of the SRI portfolios based on the six approaches outlined in chapter 3. Risk, return and risk-adjusted metrics were used to evaluate the performance of the portfolios. While there are some mixed results overall, they suggest that genetic algorithm, simulated annealing and particle swarm optimization (meta-heuristic approaches) yield the best portfolios comprising only SRI stocks and using a broader sample of non-SRI stocks.

Interestingly, the findings also suggest that, on average, portfolios constructed using a broader sample of stocks (non-SRI) outperform those that comprising SRI stocks only. The following chapter presents the conclusion of the study, the limitations of this research and recommendations for future research in this area.

CHAPTER 6. CONCLUSION

Portfolio optimization is an important consideration for practitioners and scholars alike and limiting the scope of constituent assets to only those with high ESG performance adds additional complexities to portfolio construction. This study investigates which portfolio optimization technique yields an optimal SRI portfolio using both traditional approaches (mean-variance, naïve and risk parity) and AI approaches (particle swarm optimization, simulated annealing and genetic algorithm) for the South African market. South Africa was investigated given the prominence of this market in SRI among emerging markets. Moreover, the study also assesses whether the same portfolio construction approaches used for the SRI stocks yield the optimal portfolio of a broader sample of stocks (non-SRI portfolios). Finally, using the two sets of portfolios, this study adds to the evidence on whether SRI stocks outperform non-SRI stocks but through the lens of portfolio construction i.e., is the finding of whether SRI stocks outperform or underperform influenced by the choice of portfolio construction technique.

With regards to the first research question, the study found that AI approaches on average outperform traditional approaches in constructing an SRI portfolio. However, there is no clearly best approach among the three meta-heuristic methods examined. Ranking the top three approaches in finding optimal portfolios, the genetic algorithm performs best followed by simulated annealing and mean-variance. This suggests that the best portfolio optimization approaches are the meta-heuristic approaches (on average) for SRI portfolios although these portfolios do have higher concentration and less stable weights than those constructed based on some of the traditional approaches. The mean-variance approach does also perform well especially when the market is in a downturn. For the second objective, the results showed that the approaches that form optimal portfolios for SRI stocks, also largely form optimal non-SRI portfolios – with mean-variance, genetic algorithm and simulated annealing the best approaches. However, the AI metaheuristic methods do not consistently outperform the traditional approaches, with particle swarm optimization, not ever among the best performers. With respect to the last objective, the study found non-SRI portfolios outperformed SRI portfolios, although the SRI portfolios did experience periods of outperformance (in the first investment period). The robust results also confirmed that the SRI portfolios cannot outperform the non-SRI portfolio in consecutive periods.

The main analysis was limited to two estimation periods and two investment periods to allow for non-overlapping investment periods (given the sample length and chosen investment window). Being able to use a longer period and include more non-overlapping investment periods would ensure the robustness of the conclusions drawn in this study. Relatedly, it would be of value to consider investment periods that directly coincide with crises whereas the investment periods in this study are quite long so while they do include crises (the first includes Covid-19 and the second, the Russia-Ukraine war) they also typically include the recovery period thereafter as well, mitigating the direct crisis effects. Many previous studies rely on data collected from the GFC (Aho., 2023; Bondera, 2014; Chen at al., 2021; Diaz et al., 2022; Nakai et al., 2016; Peerbhai and Naidoo, 2021) so adding specific (shorter) investment periods for the pandemic and Russia-Ukraine war could be of value. Although such analyses would be shorter in duration and hence may be less applicable for institutional investors, they would be of interest for retail investors. Additionally, including other optimization techniques would allow for further choice in selecting the optimal portfolio construction method. Such approaches could include the extensions of the mean-variance, Black-Litterman, artificial neutral network, support vector machine, Bayesian and random forest approaches.

Another limitation was when interpreting the stability metric, it evaluates portfolios by rebalanced in the evaluation period. For future research, this limitation could be mitigated by incorporate rebalancing in periodic or threshold-based rebalancing where it could be simulated to evaluate its impact on portfolio stability and performance. Also, the use of dynamic adaptation to test how the portfolio reacts to market shocks or significant events with and without rebalancing and the use of benchmark comparisons where portfolios are comparing static portfolios with rebalanced ones and with benchmarks that assume rebalancing. Lastly, research could be expanded by examining the impact of social responsibility on the performance of multiple emerging markets.

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APPENDIX

Table A1: Companies included in the study

Firms included in the SRI portfolios	Firms included in the non-SRI portfolios
ABSA Group Ltd	ABSA Group Ltd
Anglo American	Anglo American
Anglo American Platinum	Anglo American Platinum
Anglogold Ashanti	Anglo American Ashanti ADR
Aspen Pharmacare Holdings	Anheuser Busch Inbev
African Rainbow Minerals Ltd	Aspen Pharmacare Holdings
Anheuser Busch Inbev	BHP Group Ltd
AVI	Bid Corp
Bidvest Group	Bidvest Group
Capitec Bank Holding Ltd	British American Tobacco
Compagnie Financiere Richemont AG	Capitec Bank Holding Ltd
Clicks Group Ltd	Clicks Group Ltd
FirstRand Ltd	Discovery
Gold Fields	Exxaro
Growthpoint Properties Ltd	FirstRand Ltd
Harmony	GlenCore
Impala Platinum Holdings	Gold Fields
Investec Ltd	Growthpoint Properties Ltd
Kumba Iron Ore	Impala Platinum
Life Healthcare Group Holdings	Investec
Momentum Metropolitan Holdings	Investec Ltd
Mondi Ltd	MTN
Mr Price Group	Mondi Ltd
Naspers	Mr Price Group
Nedbank Group	MultiChoice
Netcare	Nepi Rockcastle
Northam Platinum Holding	Naspers
Pick N Pay Stores	Nedbank Group
Redefine Properties	Northam Platinum Holdings
Remgro	Old Mutual
Royal Bafokeng Platinum	Prosus
Sappi	RMB
Standard Bank Group	Reinet Invest
Shoprite	Remgro
Sanlam	Sanlam
Sasol	Sasol
Tiger Brands	Shoprite
The Foschini Group Ltd	Standard Bank Group
Vodacom Group	Vodacom Group
Woolworths Holdings	Woolworths Holdings

Table A2: Five-year overlapping estimation periods

Period(t)	Start	End	Length in years
Estimation period 1	02/2012	01/2017	5
Estimation period 2	02/2014	01/2019	5
Estimation period 3	02/2016	01/2021	5

Table A3: Three-year overlapping investment periods

Period(t)	Start	End	Length in years
Estimation period 1	02/2017	01/2020	3
Estimation period 2	02/2019	01/2022	3
Estimation period 3	02/2021	01/2024	3

Table A4: SRI portfolios for investment period 1

Performance Metrics	**MV**	**1/N**	**RP**	**PSO**	**SA**	**GA**
Annualized Return	**0.1615**	0.1022	0.1143	0.1388	0.1181	0.1137
Std dev.	0.6857	0.4741	0.3989	0.5213	**0.2014**	0.5641
Downside dev.	0.4571	0.3170	0.2634	0.4641	**0.1164**	0.4094
Sharpe ratio	0.2236	0.1985	0.2667	0.2663	**0.5461**	0.1872
MSR	0.2236	0.1985	0.2667	0.2663	**0.5461**	0.1872
Sortino ratio	0.3355	0.2969	0.4031	0.2991	**0.9445**	0.2580

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation and MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

Table A5: SRI portfolios for investment period 2

Performance Metrics	**MV**	**1/N**	**RP**	**PSO**	**SA**	**GA**
Annualized Return	-0.0059	**0.0548**	-0.0002	0.0625	0.0123	0.0139
Std dev.	0.1942	0.4741	0.2631	0.2789	0.2801	0.2093
Downside dev.	**0.1170**	0.3443	0.1596	0.1929	0.2033	0.1490
Sharpe ratio	-0.0707	0.0991	-0.0304	**0.2239**	0.0159	0.0292
MSR	-0.0027	0.0991	-0.0021	**0.2239**	0.0159	0.0292
Sortino ratio	-0.1173	0.1365	-0.0502	**0.3237**	0.0219	0.0410

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

Table A6: SRI portfolios for investment period 3

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	-0.0107	**0.0548**	-0.0002	0.0354	-0.0665	-0.0139
Std dev.	**0.1362**	0.4741	0.2631	0.1432	0.1553	0.1379
Downside dev.	**0.0798**	0.3443	0.1596	0.0929	0.1165	0.1098
Sharpe ratio	-0.1374	**0.0991**	-0.0304	-0.2475	-0.4795	-0.1593
MSR	-0.0025	**0.0991**	-0.0021	-0.0051	-0.0116	-0.0030
Sortino ratio	-0.2345	**0.1365**	-0.0502	-0.3813	-0.6394	-0.2002

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

Table A7: Non-SRI portfolios for investment period 1

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	-0.0115	0.0320	0.0168	0.0190	**0.0322**	0.0139
Std dev.	**0.0284**	0.1211	0.1048	0.1000	0.1013	0.0357
Downside dev.	**0.0226**	0.0733	0.0646	0.0699	0.0872	0.0352
Sharpe ratio	-0.6910	0.1975	0.0832	0.1899	**0.2310**	0.1621
MSR	-0.0005	0.1975	0.0832	0.1899	**0.2310**	0.1621
Sortino ratio	-0.8683	**0.3265**	0.1350	0.2716	0.2762	0.1646

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

Table A8: Non-SRI portfolios for investment period 2

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	0.0055	0.0900	0.0735	0.0095	**0.1045**	0.0340
Std dev.	0.1091	0.2274	0.2071	0.1164	0.2218	**0.0920**
Downside dev.	**0.0699**	0.1560	0.1433	0.0968	0.1838	0.0774
Sharpe ratio	-0.0213	0.3614	0.3170	0.0817	**0.4358**	0.2842
MSR	-0.0002	0.3614	0.3170	0.0817	**0.4358**	0.2842
Sortino ratio	-0.0333	0.5268	0.4583	0.0983	**0.5260**	0.3379

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV) naïve, (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

Table A9: Non-SRI portfolios for investment period 3

Performance Metrics	MV	1/N	RP	PSO	SA	GA
Annualized Return	0.0355	0.0611	0.0567	0.0618	**0.0722**	0.0642
Std dev.	0.1619	0.1629	0.1482	0.1685	**0.1447**	0.1785
Downside dev.	0.1048	0.0954	**0.0867**	0.1222	0.1128	0.1272
Sharpe ratio	0.1698	0.3259	0.3289	0.3669	**0.4434**	0.3146
MSR	0.1698	0.3259	0.3289	0.3669	**0.4434**	0.3146
Sortino ratio	0.2622	0.5566	0.5624	0.5059	**0.5687**	0.4415

Notes: Std dev. refers to standard deviation, Downside dev. refers to downside standard deviation, MSR is the Modified Sharpe ratio. The portfolio construction methods are: mean-variance (MV), naïve (1/N), risk parity (RP), particle swarm optimization (PSO), simulated annealing (SA) and genetic algorithm (GA). All values are expressed in decimals. Values denoted in bold signal the best performing portfolio on that metric.

Figure A1. The JSE All Share Index total return from February 2014 to January 2024.

(Source: Bloomberg Intelligence, 2024)