UNIVERSITY OF CAPE TOWN

FACULTY OF EDUCATION

AN EXPERIMENT IN THE PREDICTION OF ACHIEVEMENT IN SENIOR CERTIFICATE HIGHER GRADE MATHEMATICS

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presented in fulfilment
of the requirements for the Degree of

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DECLARATION

I, JUNE ELIZABETH JOHNSTON ....

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ABSTRACT

This study seeks to determine the nature of the intellectual demands of the Higher Grade course in Mathematics with a view to early, more accurate prediction of individual pupil success in this course. The need for such early prediction is clearly indicated by the interest shown on the part of parents and pupils alike during the Standard Seven year where the realisation exists that Matriculation Mathematics is a subject sometimes found to be "overwhelmingly difficult". The "drop out" figure from the Higher Grade course to the Standard Grade course in most schools further demonstrates the need for more careful selection at the Standard Seven level.

Both old (1973) and new (1984) syllabuses are analysed to determine the nature of the content and the intellectual level at which this should be taught. In addition, a series of past Cape Senior Certificate examination papers are investigated to reveal information about the nature and level of examining. Mental processes involved in the examination items are classified and the general composition of the examination papers is discussed.

A test device suitable for Standard Seven pupils is developed on the basis of the composition of the Higher Grade Matriculation examination papers analysed. The object of this test is to provide that early indication to pupils of their ability to cope with the level of mental process required by the Higher Grade course in Mathematics.

The investigation describes the construction, administration and further development of the test device and, furthermore, seeks to show its predictive validity for the Matriculation examination in Mathematics by comparing test results with successive school examination results over a three year period.

The possibility of sex differences in Mathematics achievement and prediction are also investigated on the basis of the results obtained during the course of this experiment.

General conclusions are drawn, the difficulties encountered are discussed and some suggestions for further research are offered.

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June E. Johnston
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CHAPTER 1

INTRODUCTION TO THE EXPERIMENT

1.1 PREAMBLE

1.2 THE PROBLEM

1.3 THE AIM OF THE STUDY

1.1 PREAMBLE

In 1973 the Differentiated Syllabus for the Senior Certificate Examination was introduced for the first time. One of the many consequences of the new system is that pupils have to make difficult subject choice decisions one year earlier, i.e., at the end of the Standard 7 year. Barely two years after their entry into the secondary school they have to commit themselves to three years of study in both official languages as well as in four electives, which in turn, will probably have some bearing on their tertiary education or career options.

Many fourteen-year-olds have absolutely no idea of what they might do after they matriculate, and those who have given the matter some thought are usually bewildered by the myriad of choices that confront them or are totally unaware of the extent of present-day opportunity. Others may have given life after school no consideration at all and make their choices on the basis of, for example, whether or not the course is a "swot subject", whether or not he likes the teacher, or whether or not his friend intends to take the course.

Schools tend to advise a curriculum that will lay a foundation for a well-rounded and balanced general education with early specialisation being discouraged. The rationale here is, first, to produce a young adult and citizen who knows a little about as many facets of his culture as is possible in a short school career; secondly, such an approach, presumably, leaves all his options open until he is a little older and is in a better position to make wise decisions for the future. Given our present educational system, this would seem to be fairly sound advice. However, there is one school subject that merits very special attention.

In South Africa today, there is an ever increasing demand for technically and scientifically skilled and qualified people. We stand on the threshold of an era in which computer technology is evident in more and more diverse spheres of activity, and it is no longer sufficient just to be literate; the job seekers of today must be numerate too. Entrance to so many careers and training institutions depends upon the successful completion of Senior Certificate Mathematics that to fail this subject, or to omit to study it altogether, automatically restricts severely the pupil's range of opportunities. It is imperative that as many children as possible continue with Mathematics. It could be argued that this should be the third compulsory subject, but such a ruling would place an unfair burden on those individuals whose talents lie elsewhere, e.g., in Music or the Arts. To force the carrying of a subject like Mathematics, in which extremely low scores are quite possible, would be to prejudice the pupil's aggregate score in the Senior Certificate Examination and, perhaps, jeopardise the Senior Certificate pass altogether.

1.2 THE PROBLEM

The decision to take Mathematics, or not to take it, is a critical one for the pupil. To decide against this subject is to close the door firmly on a large range of careers; to decide in favour of it is to embark upon a course of study which many pupils find too exacting after the first year of the three-year syllabus. Taking or not taking Mathematics is not the only decision to be made, however - the grade, Higher or Standard, must also be decided. Frequently, grade selections are made without proper consideration being given to what the different courses actually involve or to what the capabilities of the individual pupil are. Although these decisions are not irreversible, the mathematically weak pupil, who attempts the Higher Grade course, may become so confused and disillusioned that he or she simply "gives up", and finds it is impossible to pick up the threads again even on the Standard Grade course.

The problem for the school teacher is to provide pupils with reliable information about the demands of the senior courses in Mathematics, as well as the extent and nature of the individual's mathematical talent. It is with the latter that teachers have the greatest difficulty - particularly with reference to the large group of pupils sandwiched between the obviously talented and the desperately weak mathematicians.

1.3 THE AIM OF THE STUDY

In order to improve the quality of the advice that we give our pupils, it is necessary to have more detailed and specific information about the objectives and demands of the Higher and Standard Grade syllabuses and examinations, as well as the capabilities of each individual pupil. This study envisages the detailed analyses of these syllabuses and examinations with a view to

(a) revealing the specific abilities called forth by these, and

(b) the development of a Standard 7 test in Mathematics that would have predictive validity for the Senior Certificate course in Mathematics.

It is hoped that this test might provide a prognostic device that would enable teachers to give more objective and detailed information to pupils concerning their chances of achieving success in Senior Certificate Mathematics, and the choice of grade best suited to their particular capabilities.

CHAPTER 2

ANALYSIS OF REQUIREMENTS FOR THE SENIOR COURSE IN MATHEMATICS:

2.1 STATED DEMANDS OF SYLLABUS

- 2.1.1 Nature of the Topics Covered
- 2.1.2 General Aims and Method of Teaching the Senior Course
- 2.1.3 Comparison of Higher and Standard Grade Syllabuses

2.2 ACTUAL DEMANDS OF FINAL EXAMINATION

- 2.2.1 Classification of Mental Processes
- 2.2.2 Analysis of Ten Past Papers
- 2.2.3 Limitations of the Analysis

2.3 OTHER DEMANDS

- 2.3.1 Understanding of Previous Work
- 2.3.2 Motivation and Incentive
- 2.3.3 Pupil's Work and Study Habits
- 2.3.4 Persistence, Tenacity and Determination

2.4 SUMMARY

2.1 STATED DEMANDS OF THE SYLLABUS

In any consideration of the requirements of a course of study, the point of departure must be the official syllabus. It is on this document that teachers base their instruction and examiners base their selection of test items in the final examination. The syllabus enumerates not only the subject-matter to be examined but also the aims to be pursued and the guidelines to be followed with respect to teaching method. This investigation must take into account four syllabuses, viz., the 1973 syllabus (both Higher and Standard Grade) on which all subsequent analyses are based, and the 1984 syllabus (both Higher and Standard Grade) which was implemented in Std 8 for the first time in 1985. Copies of the relevant syllabuses, in which the detailed list of content topics and general aims may be studied, can be found in Appendix A.

2.1.1 Nature of the topics covered

The content of the 1973 syllabus may be summarised by the following list of broad headings:

1. Indices and Logarithms
2. Quadratics
3. Functions and Graphs
4. Series and Induction
5. Trigonometry
6. Euclidean Geometry
7. Vectors

During 1976 and 1977 a further section on Groups and Fields was examinable but this was later designated for enrichment purposes only and does not appear in later examinations.

This subdivision of topics holds for both the Higher and Standard Grades. The Standard Grade course does not have the depth of the Higher Grade course and certain subsections are omitted altogether, e.g., mathematical induction, inverse functions, compound angles in Trigonometry and the dot product and vector equations of a straight line. Quantitatively, the content difference between the two grades is marked and the effect of this in the classroom is to give the Standard Grade candidate much more time to spend on each topic.

The subject content of the 1984 syllabus is largely the same with the following exceptions:

Excluded - Vectors
Mathematical induction (HG)
Sum and product of roots (HG)
Theory of logarithms (SG)
Calculation by common logarithms

Introduced - Analytic geometry
Differential calculus
Remainder theorem
Calculation by electronic calculator

The question immediately arises, does the change in syllabus affect the various analyses and conclusions of this discussion? It is the writer's opinion that the actual content covered makes little difference to the difficulty of the subject for the pupil since all mathematical topics tend to be abstract and demanding of logical, sequential thinking. The fact that the syllabus has changed, therefore, should not alter the intellectual demands made on the candidates.

It must be said, however, that the content of the syllabus is not entirely neutral with respect to the difficulty of the subject. On the whole, the senior course flows out of, and is an extension of the junior secondary course in Mathematics. Pupils with a history of difficulties with this subject, or pupils with too many gaps in their knowledge of previous work, cannot expect to do well in this more advanced course.

The pupils' perception of the content causes certain difficulties too. Perhaps this is an indictment of our teaching, but it seems that pupils view the subject matter of Mathematics as "very dry stuff". One can enthuse for hours on the intrinsic beauty of Mathematics, its usefulness as a tool and concise language in all scientific endeavour and its aesthetic value in Art, but still relatively few pupils really seem to enjoy getting to grips with the subject. Essentially Mathematics is a pursuit of the mind - girls especially, cannot see themselves ever using what they are being taught even though it is quite possible that some of them may eventually pursue activities in technical and scientific fields or may take up sailing and be required to navigate the seas. To many pupils Mathematics is simply uninterrupted hard work that everyone claims has value, but that value may be largely incomprehensible to the pupil. Perhaps we can lay a little blame at the feet of television and video games - so much of our pupils' entertainment these days is intellectually passive, that there is a tendency, among some youngsters, to resent coping with anything that requires too much thought.

All this does not really make Mathematics any more demanding, but it does affect the individual's capacity to tackle the subject with the persistence and determination which are almost indispensable for success in this sphere.

2.1.2 General Aims and Method of Teaching the Senior Course

The complete list of general aims and teaching methods can be found in the syllabuses mentioned before and an accompanying extract of a Cape Education Department Guide also tabled in Appendix A. A comparison of the Higher and Standard Grade courses is undertaken in Section 1.3 of this chapter, but as far as intellectual requirements are concerned there seems to be a common thread running through all four syllabuses.

The main thrust of the stated aims and methods is to avoid "magic" or "recipe" Mathematics, i.e., pupils must <u>understand</u> what they are doing and why. There is a distinct emphasis on the <u>development</u> of insight and logical deduction.

The importance of the verb "to develop" should be noted. In the official syllabus it occurs several times. Let us consider the meaning of the word:

Chamber's Twentieth Century Dictionary (1972 edition) yields, inter alia:

> **to develop**:- to unroll; to lay open by degrees; to bring out what is latent or potential in; to bring to a more advanced or more highly organised state; to make more available; to exploit the natural resources of; etc.

Thus <u>to develop</u> clarity of thought, powers of logical deduction and mathematical insight implies that such abilities already exist (albeit in an embryonic stage) in the individual pupil and the duty of the teacher is to facilitate their unfolding.

It seems clear then that pupils must be taught in a manner which will promote insight and understanding and so the "monkey say, monkey do" method in which less able pupils feel most secure is obviously inappropriate. Candidates will have to be led to do their own thinking, be challenged constantly to produce possible solutions to problems, which solutions, they must then learn to evaluate in terms of accuracy, mathematical efficiency and elegance. The aims of the syllabus thus demand that the candidate is not only <u>willing</u> to study and learn in this manner, but also that he has <u>the potential to manage in such a situation</u>, i.e., that he has the necessary pre-knowledge and possesses within him the seeds of the ability to think clearly, logically and insightfully about Mathematics. Herein lies the essence of this investigation - to establish whether or not the pupil has the necessary qualities to succeed at a higher level of Mathematics, i.e., whether or not he can meet the demands made by the aims of the syllabus.

2.1.3 Comparison of Higher and Standard Grades

The reasoning behind the differentiated syllabus in Mathematics springs essentially from three needs: First, to provide a basis for further study in scientific fields; second, to aid in the selection of prospective university students; and third, to equip pupils with the mathematical knowledge and problem-solving techniques that they may find useful in their daily lives. The following statement by the syllabus designers is noteworthy:

> The syllabuses lend themselves particularly to preparing and selecting prospective university students. Candidates who take Mathematics on the higher grade should not only be better equipped for any intended course of study, but are specially prepared for further study in Mathematics, Physical Science and other courses of study for which mathematical methods and techniques are essential. The standard grade is meant to make mathematics accessible to as many pupils as possible.

It is not always appreciated by pupils and parents that the two grades of Mathematics are fundamentally different, in depth, scope, intent and approach.

The content of the Higher Grade syllabus is physically greater and within each topic the candidate is required to explore more detail and to work with greater mathematical rigour. The intention, in this course is to develop the skills of analytical thinking necessary for university degree courses, and this, in turn, means that the approach to the subject involves a more inductive teaching style with less emphasis on classroom drill and greater emphasis on self-activity and discovery. The teacher of a higher grade group should not merely expect to "get through" the syllabus, but should be expecting and demanding greater innovation, flexibility of thought and mathematical finesse of the pupils. Higher grade candidates should be encouraged to work ahead on occasions so that the insights they obtain on topics are uniquely their own; these might then be critically appraised in discussion with the rest of the class. Only in this sort of way can it be hoped to prepare pupils for further education in scientific fields.

The Standard Grade candidate, on the other hand, has less ground to cover and so has more time to assimilate the syllabus material and to spend practising the various exercises presented. In addition, Standard Grade problems tend to be less abstract in their setting and thus more obviously relevant to everyday life.

A topic which points up the difference between the two courses very well is the section on logarithms: Standard Grade pupils come to this after a discussion of the Laws of Indices. The tedium of arithmetic with large numbers is deplored, the elegance of working with powers is noted, eventually John Napier's work is mentioned and Common Logarithms are launched. Copious exercises follow because it is important that the pupil can handle awkward arithmetic in the section on Trigonometry.

The Higher Grade introduction to logarithms grows out of an examination of the inverse of the exponential function, i.e., what happens to the graph of $y = a^x$ if one reflects it in the $y = x$ axis. Pupils then explore the features of this new function, discover and prove the Laws of Logarithms, investigate changes of base and make the necessary connections to the Common Logarithms they were taught to manipulate in Std 8. Clearly a much more theoretical approach for the Higher Grade candidate. It is interesting to note that, in the new syllabus (1984), Standard Grade pupils no longer need to study logarithms because of the introduction of electronic calculators, whereas the topic remains for the Higher Grade students who must view it not as a handy tool in arithmetic, but rather as an inverse function.

It is important, then, for pupil, parent and teacher to think carefully about choice of grade - Standard Grade Mathematics is not just a convenient place to fall if the pupil cannot achieve the necessary 160 marks on the Higher Grade paper - it is a course with its own objectives, designed to meet the needs of those people who are not going to be the scientists of tomorrow, but a course which allows the mainstream of pupils access to those techniques and concepts they will require to live, and prosper, in a technologically advanced society.

2.2 ACTUAL DEMANDS OF THE FINAL EXAMINATION

The device that ultimately determines whether or not a candidate is successful is the final examination. The syllabus provides a route map for the course, but no investigation of the demands of Senior Mathematics would be complete without a study of the type of test items that have been used to measure the achievement of pupils in this subject. Critical analysis of past examination papers is a fascinating task on its own, and one in which matriculation pupils indulge enthusiastically each year in search of an answer to the question, <u>what does the examination demand</u>?

Obviously, the examination covers the content of the syllabus and adheres to the mark allocation of the various topics as prescribed. Of much greater interest, both for matriculation students and for this investigation, is the manner in which topics are tested. Do certain questions appear every year; are there visible trends in the testing of topics and to what extent is the syllabus preoccupation with understanding and insight realised in the test papers?

In order to reach any conclusions about the demands of the examination it was necessary to consider the following:

(a) Can test items be classified?

(b) Which papers should be analysed?

(c) What are the limitations of the analysis?

(d) How do the Higher and Standard Grade papers differ?

2.2.1 Classification of Mental Processes

Anyone who has ever written an examination can testify to the fact that some questions are more difficult than others to answer, and that the critical factor determining the level of difficulty is not what is asked but rather how it is asked. In order to decide exactly what an examination demands of an examinee intellectually, it was necessary to find a suitable system for categorising different types of question with respect to the mental processes required.

Bloom's Taxonomy of Educational Objectives for the cognitive domain (BLOOM 1956) has offered some researchers (VENTER 1985) a useful starting point for this task. This six-level hierarchy of learning outcomes, namely,

1. Knowledge
2. Comprehension
3. Application
4. Analysis
5. Synthesis
6. Evaluation

appears, at first, to offer a fairly straightforward solution to the problem. In practice, however, like Norton (1983), the writer found that this system proved to be a little too general to be applied simply and consistently. Bloom's Taxonomy was designed to be comprehensive, in its scope, and mentions very few mathematical objectives specifically; for this reason, its consistent application, in the context of this study, proved problematic. A taxonomy was needed, which had been specifically designed for Mathematics. The classification that offered the fewest difficulties of application was the one listed in Gronland (1971) from the Educational Testing Services:

6 Mental Processes Enumerated for Mathematics

1. RECALL FACTUAL KNOWLEDGE - restricted to questions which require only the recall of a definition, fact or theorem without doing anything with it.

2. PERFORM MATHEMATICAL MANIPULATION - used for items regardless of complexity, that call for the application of a technique that has been learned, where no decision is required on how to approach the solution.

3. SOLVE ROUTINE PROBLEMS - includes questions in which a choice of the technique to be used is necessary, or a definition or theorem recalled and applied, but where there is a straightforward technique available which is commonly taught.

4. DEMONSTRATE COMPREHENSION OF IDEAS AND CONCEPTS - includes questions which require some understanding of the underlying concepts necessary to complete the item. The student must not only decide what to do, but how to do it.

5. SOLVE NON-ROUTINE PROBLEMS REQUIRING INSIGHT OR INGENUITY - concerned with questions which require the student to develop his own technique for solving a problem which he has probably not met in a text-book. The solution may be straightforward and simple, but some insight should be needed to find it.

6. APPLY "HIGHER" MENTAL PROCESSES - used to classify questions testing generalisation, evaluation, the nature of proof, induction, logical inference and decisions about the sufficiency of data.

This system of classification clearly echoes Bloom's Taxonomy but has the advantage of dealing specifically with mathematical processes and therefore leaves less room for doubt when assigning a particular item to a particular level in the hierarchy. Furthermore, the extent to which questions fall into categories 4, 5 or 6 should indicate whether the aims of the syllabus, in respect of the development of understanding, insight and logical thought, are indeed being followed through in the testing device. Ultimately, for the student, it is what the examination demands that counts rather than more or less vague statements in the preamble to the syllabus. The wary candidate will examine past papers carefully and adjust his methods of study according to what he perceives to be important in those papers.

2.2.2 Analysis of Ten Past Papers

In order to discern any possible trends in the allocation of marks to different levels of mental process in examinations, it was decided that as many papers as possible should be analysed. At the time this was undertaken there were ten sets of final examinations available to the writer, viz., those Cape Senior Certificate papers written in November and March from 1976 to 1981 (both Higher and Standard Grade).

Each test item was scrutinised and then classified according to the system mentioned above. The mark allocation of the item was noted and set down against the appropriate mental process. When all the items in each paper had been suitably classified, the marks in each ability class were added and represented as a percentage (correct to the nearest whole number) of the total mark allocation for that paper (400 marks in the case of Higher Grade papers and 300 in the case of Standard Grade papers).

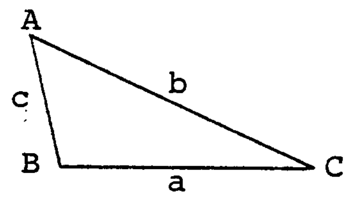
The classification of examination items according to the mental processes they involve is a highly subjective exercise. Very often, it is difficult to distinguish between levels of the hierarchy of mental processes, making any attempt at classification rather tentative and open to dispute. In order to give some indication of the writer's application of the aforementioned classification, two sample items from each level of the hierarchy will be discussed:

Level 1 - Recall Factual Knowledge

Example 1: Prove that $\log mn = \log m + \log n$

This examination item is classified at the lowest level of the hierarchy because the proofs of the laws of logarithms are required bookwork in the higher grade syllabus and, therefore, this item would have been practised many times before, probably committed to memory on the express advice of the teacher.

Example 2: In the given figure, ABC is a triangle with angle ABC obtuse.
Prove that
$c^2 = a^2 + b^2 - 2ab.\cos C$



Once again, this item is required theory for the examination, and, as such, should have been drilled and memorised. In neither of these examples was any attempt made, on the part of the examiner, to disguise or complicate the question, and so, it was considered that, these items presented the lowest level of mental challenge to candidates.

Level 2 - Perform Mathematical Manipulation

Example 3: Solve for x where, $\sqrt{4x + 17} = x + 3$

Here, the instruction given is quite explicit, i.e., solve. Solution of equations involving surds is commonly taught and drilled. The command to solve, together with the appearance of the square root sign, should signal the commencement of a well-rehearsed routine, in which the only variables are the numbers used in this particular example.

Example 4: Determine the values of x and y for which

$$x - 2y = 3 \quad \text{and}$$

$$x^2 - xy + y^2 = 3$$

Put in a lengthier format, but nevertheless, just as explicit as before, the words "determine the values of ..." should provide a readily recognisable signal for another well-practised drill, i.e., the solution of a linear-quadratic system.

Both of the above items exhibit clear and explicit instructions so that there is no decision required by the candidate; both items call for well-practised, recipe-type solutions. Candidates need only be able to recognise the required technique and perform with accuracy.

Level 3 - Solve Routine Problems

Example 5: Prove $\frac{1 - \sin\theta}{1 + \sin\theta} = (\sec\theta - \tan\theta)^2$

Here, there is no direct instruction indicating the method of solution. The candidate is expected to use known relationships between the trigonometrical functions to transform one side of the equation into the same expression as the one given on the other side; techniques for accomplishing this are commonly taught and well-drilled.

Example 6: One root of the equation $x^2 - ax + b = 0$ is three times the other; a and b are real constants. Prove that $b = \frac{3a^2}{16}$

In this problem, the presence of the quadratic equation and the mention of "roots" could signal one of a few techniques. The candidate is expected to examine the question carefully and then decide to apply his knowledge of the sum and product of roots. Fairly extensive practice on problems of this sort is normally provided in class.

Problems at this level of the hierarchy are characterised, firstly, by inexplicit instructions, i.e., the candidate must decide what to do from a number of possible techniques, and, secondly, by the familiarity of the problem, i.e., the type of problem is well-known to the candidate.

Level 4 - Demonstrate Comprehension of Ideas and Concepts

Example 7: For which real values of x will $\frac{1}{\sqrt{x^2 - 16}}$ be real?

In order to complete the above item, the candidate must understand the concept of the real number; he must then realise that the value of the denominator poses a threat to the "realness" of the number if

1. $x^2 - 16 = 0$, thus producing an undefined quantity, or
2. $x^2 - 16 < 0$, thus producing an imaginary denominator.

Once the candidate has made this realisation, the solution of the equation $x^2 - 16 > 0$ to give the correct answer is a relatively routine matter.

Example 8: The sum of the squares of two successive positive odd integers is 290. Use an algebraic method to determine the numbers.

The understanding required here is that there must be a "translation" from English into mathematical symbols. Moreover, the concept of "successive" and "odd" must be grasped, i.e., that consecutive odd numbers are two units apart, and that odd numbers are numbers one bigger than, or one smaller than even numbers, which, in turn, are multiples of 2. Again, once this realisation has been made, the setting up and solution of the equation

$$(2n + 1)^2 + (2n - 1)^2 = 290$$

is not difficult.

Level 5 - Solve Non-routine Problems Involving Insight and Ingenuity

Example 9: **In the equation (a - cos x) (a - tan x) = 0, a is constant. Determine one value for sin x if the equation has equal roots. The answer may be left in surd form.**

One of the difficulties for the candidate, in the above examination item, is the simultaneous application of trigonometrical and algebraic techniques. The words "equal roots" possibly induce thoughts of Quadratic Theory and discriminant problems in the candidate, while a solution for sin x is required - and sin x appears nowhere in the equation! Quick solution hinges on the fact that a is constant, and so, for equal roots

$$\cos x = \tan x = a$$

Recalling the Quotient and Pythagorean identities, the equation easily reduces to

$$1 - \sin^2 x = \sin x$$

and this equation may be solved using the quadratic formula.

Example 10:

Prove that $\sin(x+y) + \sin(x-y) = 2\sin x.\cos y$ and, hence, deduce a formula for $\sin A + \sin B$.

The first part of this problem is a direct application of the compound angle formulae for the sine function. Finding a solution to the second part depends upon the ability of the candidate to "see" the connection between the form of the just proved identity and the expression $\sin A + \sin B$, i.e., that A has replaced $(x+y)$ and B has replaced $(x-y)$. Finding x and y in terms of A and B is not difficult, and the required formula is then easily deduced.

Problems at this level of the hierarchy are characterised by their unfamiliarity, i.e., the candidates have not seen them previously, and the fact that there are no "recipe" solutions; candidates must pursue their own mathematical intuition and apply what they know in new situations.

Level 6 - Apply "Higher Mental Processes"

Example 11: Prove by means of mathematical induction that

$$1.2 + 2.2^2 + 3.2^3 + \ldots + n.2^n = 2 + (n-1).2^{n+1}$$

The solution of this type of problem demands the complete understanding of the principle of mathematical induction together with the application of the candidate's knowledge of algebraic expression, his insight in grasping what is required and his ingenuity in reaching the desired endpoint - it requires a process of generalisation, i.e., the candidate must be able to proceed from the particular, in this case, the statement is true if n = 1, to the general, in this case, suppose the statement is true for n = k, then is it true for n = k+1? If this can be shown to hold then the desired result is obtained.

Example 12: AD is a tangent to circle BCD and CB is produced to meet AD in A. BX ∥ CD and BD is joined.

(a) Prove $AD^2 = AC.AB$

(b) If BD = BC, show that

(i) BX is a tangent to the circle, and

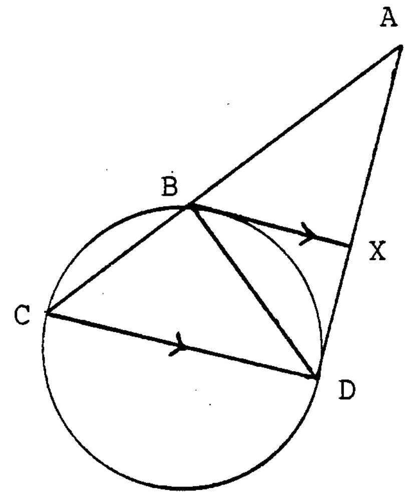
(ii) BX = DX

(c) If further AB = BC, deduce that

(i) $BD = \sqrt{2}\,XD$,

(ii) BX ⊥ AD, and

(iii) CD is a diameter of the circle.



This geometry rider would be classified at the highest level of the hierarchy, because the solution demands the recall and application of theorems as well as constant decisions on the part of the candidate as to the sufficiency of the data available for the particular statement he wishes to make, or conclusion he wishes to draw. In order to prove the statement in (a), the candidate must first prove the similarity of the triangles ABX and ADB; to do this he needs to recall that the equiangularity of the triangles is a sufficient condition and then prove that at least two angles are equal by recalling a further theorem and checking again that minimum conditions are met. The whole process of completing such an item is one of logical deduction, in which the candidate must progress from step to step, repeatedly checking the sufficiency of his information and then applying a suitable theorem. Not all geometry riders would be classified at this level of mental process, as some of them are so well known that they have become "classics"; familiarity with such problems would reduce them to the level of the routine.

Items classified in this highest level of the hierarchy are characterised by the need to apply knowledge and insight in a complex way which demands not only that the candidate pioneers his own route to the final solution, but also that he must use his mathematical judgement in selecting a suitable and, if possible, mathematically elegant route.

Using the kind of thinking indicated above, examination questions from ten past Cape Senior Certificate papers were categorised; no claim is made that the resulting analysis is unassailable, particularly in view of the impossibility of knowing the precise mathematical background of the candidates involved - although all pupils study a common syllabus, their mathematical experiences are very varied. This problem is by no means unique to this study, but is an echo of the problem, voiced by Bloom (1956), in his discussion on the usefulness of his taxonomy:

> ... test material can be satisfactorily classified by means of the taxonomy only when the context in which the test problems were used is known or assumed.

The following tables show a summary of the analyses for the Higher Grade papers and, secondly, the

Standard Grade papers:

TABLE 1: PERCENTAGE MARK ALLOCATION TO THE DIFFERENT LEVELS OF MENTAL PROCESS INVOLVED IN THE EXAMINATION ITEMS OF TEN PAST CAPE SENIOR CERTIFICATE HIGHER GRADE MATHEMATICS PAPERS

Date of Paper	MENTAL PROCESSES					
	Recall %	Manipulation %	Routine Problems %	Comprehension %	Insight %	Apply "HMP" %
Nov'76	8	26	34	19	13	0
Mar'77	7	24	35	29	6	0
Nov'77	8	21	23	25	20	4
Mar'78	6	23	37	15	3	17
Nov'78	9	11	39	14	13	14
Mar'79	6	24	37	12	13	8
Nov'79	4	16	42	29	7	2
Mar'80	5	11	48	14	17	5
Nov'80	7	16	34	18	13	12
Mar'81	7	9	43	20	12	9
Average	7	17	37	20	12	7

The analysis of Higher Grade papers shows that, from year to year, a fair degree of variability occurs in the balance of mental processes represented in the examination; however, the main thrust of the examination is clearly in the ability to solve routine problems with Comprehension of Concepts enjoying the edge over Manipulation. It will be noted that all ability levels are generally represented in the Higher Grade examination.

The fact that, on average, 39% of higher grade marks are allocated to the top three stages of the ability hierarchy surely indicates that the examiners are, indeed, following through the aims of the syllabus with respect to the development of understanding, insight and logical thought. It also means that no candidate will excel on the basis of hard work alone. Nevertheless, it must be remembered that items which can be mastered by diligence amount to 61% (on average) and so the hard worker should certainly be able to pass the Higher Grade examination.

Similar analysis of Standard Grade papers, from the same period, also shows that the composition of examinations, with respect to mental processes, varies considerably from year to year; however, there is a tendency discernible, for the emphasis to be placed more firmly on the lower levels of the ability hierarchy, and the highest levels, namely Insight and Application of "Higher Mental Processes", are hardly represented at all.

The standard grade papers show a marked difference in the balance of mental processes with the ability to manipulate mathematically running a close second to the ability to cope with routine problems. In fact, for standard grade candidates it seems that consistent, hard work would be sufficient even to do very well since only 16% of the marks, on average, are allocated to the top three ability levels. Again it seems that examiners are following through on the aims of the syllabus - in this case, that the Standard Grade course should be accessible to as many pupils as possible.

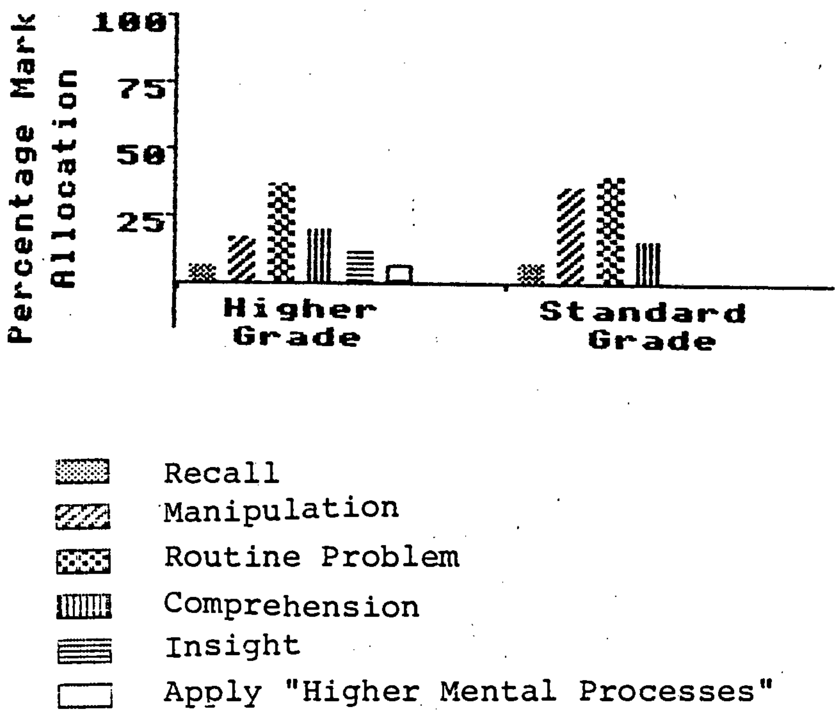
TABLE 2: PERCENTAGE MARK ALLOCATION TO THE DIFFERENT LEVELS OF MENTAL PROCESS INVOLVED IN THE EXAMINATION ITEMS OF TEN PAST CAPE SENIOR CERTIFICATE STANDARD GRADE MATHEMATICS PAPERS

	MENTAL PROCESSES					
Date of Paper	Recall %	Manipulation %	Routine Problems %	Comprehension %	Insight %	Apply "HMP" %
Nov'76	10	39	39	10	2	0
Mar'77	7	43	38	12	0	0
Nov'77	9	26	47	18	0	0
Mar'78	8	26	45	21	0	0
Nov'78	8	38	39	13	2	0
Mar'79	9	36	36	19	0	0
Nov'79	7	44	33	16	0	0
Mar'80	6	35	48	11	0	0
Nov'80	8	35	33	24	0	0
Mar'81	7	39	41	13	0	0
Average	8	36	40	16	0	0

The different emphasis on the weighting of the mental processes is illustrated by the bar chart that follows:

Figure 1

COMPARISON OF THE DISTRIBUTION OF MENTAL PROCESSES IN HIGHER AND STANDARD GRADE CAPE SENIOR CERTIFICATE MATHEMATICS EXAMINATIONS 1976 TO 1981



From the chart, it can be seen clearly that Higher Grade candidates are expected to be able to cope with a much wider range of mental processes than Standard Grade candidates, and that whilst diligence will be required of the Higher Grade pupil in order to cope with the bulk of routine and manipulative techniques covered, diligence alone will not be sufficient to excel in examinations which contain such noticeable proportions of the higher mental processes. In contrast, Standard Grade candidates are expected to cope with mostly "reproductive" mathematics, i.e., the examinations seem to concentrate on that which is familiar to the pupil and demand less in the way of innovative thinking.

There are two problems which come to mind at this point. First of all, it has been assumed that in order for a pupil to operate successfully at the lower levels of the hierarchy all that he needs to do is to work consistently, listen in class and learn carefully those manipulative techniques and routine procedures prescribed by the teacher; but, how much understanding does a pupil need in order to follow through the various methods of solution taught in the senior mathematics course? To what extent can students learn by rote at this level of mathematics? There is probably no answer to these questions because the capacity to remember "meaningless material" varies from one individual to another, but it is certain that many pupils will need a basic understanding of principles before they are able to exercise some mathematical procedures.

Secondly, how reliable is the classification system that was used to analyse the examination papers? Although the descriptions of the various categories of mental process are very clear and specific, the analysis of items was, nevertheless, a task fraught with difficulties. These will be discussed fully in the section that follows.

2.2.3 Limitations of the Analysis

Apart from the previously mentioned problem of not knowing exactly how much understanding might be required to perform manipulations and routine techniques, there exists the difficulty of distinguishing between the routine problem (which is familiar to the pupil) and the non-routine problem requiring insight and ingenuity. The main difference between these two hinges on whether or not the problem has been encountered before. In this regard, there are at least three, probably different, perspectives; the candidate's, the teacher's and the examiner's. The outcome of such an analysis depends entirely upon the subjective bias of the person undertaking the task.

Items that from the examiner's point of view are higher order non-routine problems may, in fact, have been practised by some teachers thereby reducing those items to the third level of the hierarchy, i.e., to the level of the routine problem, for some candidates. Attitudes towards the final examination differ from school to school and even from teacher to teacher - some, pursuing "good" examination results assiduously, may drive and drill their pupils to such an extent that all problems are reduced to the realms of the routine; others, scornful of such "spoon-feeding" practices, may be content to teach the syllabus, leave final revision in the hands of their pupils and "let the chips fall where they may"; and others still may operate between these two extremes.

Even between a teacher and his pupils there would be argument on the question of what has and has not been <u>seen before</u>. Some pupils have poorer memories than others and some are confused easily by slightly different wording, so that what <u>should</u> be familiar to them is not. In this way items intended as routine problems are upgraded to the level of the insight and ingenuity in the hierarchy. On the other hand, the proofs of theorems and derivations of formulae, which would be intended to demand only low-level mental processes such as recall or manipulation in an examination, might present as problems of a much higher level for pupils who have failed to revise and learn these "bookwork pieces" carefully.

The foregoing analysis, then, is not absolutely reliable, and it is likely that if repeated by another researcher that a different outcome would result. The particular bias of the writer is that of a practising teacher whose opinion it is that a balance has to be struck between the interest of the pupil in obtaining the greatest number of marks possible and the true spirit of education, which demands that eventually the student must take on more responsibility and rely less on the "whip-cracking" teacher.

2.3 OTHER DEMANDS

In common with every other school subject, Mathematics makes hidden demands on the pupil. "Hidden" only in the sense that they are not specially mentioned in writing in the syllabus; they are, however, obvious and very few pupils will achieve success without meeting these demands.

2.3.1 Understanding of Previous Work

The nature of Mathematics is such that subject matter is cumulative - one must start at the beginning and progress logically step by step; it is not possible to "pick up" anywhere in the middle. Mathematics can be compared to a brick wall, its strength depends upon a solid foundation and the construction of successive layers of precisely laid-down, overlapping small blocks. The content of the senior syllabus is an extension of everything that has gone before; not just from the junior secondary level, but right back through the primary phases. The pupil's success in the senior course depends first on the extent of his pre-knowledge.

Teachers at the senior secondary level have to assume an understanding of the previous work - their syllabus is too full for looking back. All too often, however, we know that there are large gaps in the pupil's mathematical foundations and this is clearly demonstrated in the research of Kathleen Hart (1981) and the CSMS Mathematics Team.

> Very often secondary school teachers are reteaching what was first presented in the primary school. In many cases it was not understood when first presented, or even when 'taught' the second time ... Is there really any point in teaching something we know most children will not understand? One reason given for doing this is that the child will become familiar with the idea and understand it later. We have no proof of this, in fact our results show that the understanding does not 'come'.

Hart (1981) suggests that the only way around this problem is to reteach, from scratch, topics which are inadequately understood. Unfortunately our system, in South Africa, is rather more rigid than the British system and there is no time, certainly in Standards nine and ten, for casting backward glances.

It seems clear, then, that success in the senior course of Mathematics requires that candidates should have reached an adequate level of understanding with respect to previous work by the end of their second year in the high school. To proceed without this surely will lead to frustration, a decrease in the pupil's self-confidence and, probably, eventual failure in Matriculation Mathematics.

2.3.2 Motivation and Incentive

After standard seven, Mathematics is no longer a compulsory school subject. Pupils studying Mathematics have chosen to do so. In any sphere of life, it is true that we do best that which we enjoy doing, or, that which we have powerful incentives to do. It follows that a pupil's reasons for choosing Mathematics will have a marked effect on his approach to the subject and his eventual success.

One of the most often quoted reasons for studying Mathematics is that it is an entrance requirement for many avenues of tertiary education or career fields. The extent to which this reasoning provides a powerful motivating force depends largely on how convinced of its truth the pupil is. A pupil who is really determined to enter some field of endeavour that he knows requires achievement in Mathematics will probably move mountains to succeed. On the other hand, a pupil with no specific plans for the future, who is simply conforming to parental wishes, may not have the same drive to do well.

Another common reason, among pupils, for taking the senior course in Mathematics is that it is not considered to be a "learning subject". These pupils hold the opinion that Mathematics is not something to be worked at consistently and studied with determination, but rather that one learns to perform a few tricks with numbers and then leaves the "understanding bit" to the ingenuity of the teacher. With such an attitude it must be an ordeal to sit through three years of Mathematics which, except to those who love it, can be a very dry subject; furthermore, the realisation that the examination is more than just a few tricks must be a painful one.

Students' incentives to study Mathematics are many and varied, but the important thing is that the motivation must be strong enough to keep the pupil battling on over a period of three years, and through all the difficult and abstract material that constitutes the course. Extrinsic motivating factors, such as promises of material rewards or threats of beatings or other punishments will probably not stand the test of time. Even a desire to please the teacher, very often a powerful driving force, may fail if the pupil has too many changes of teacher between standard eight and standard ten.

In order to do well in Mathematics it is necessary that the pupil finds enjoyment and fascination in Mathematics; or, that he feels that this subject will lead him into other fields, in which he is deeply interested, that are unattainable except through the portals of Mathematics.

2.3.3 Work and Study Habits

Given an understanding of basics and the right kind of motivation the pupil still has to cope with the content of the syllabus. White children in the Cape

Peninsula are provided with a textbook covering the work to be completed, classroom situations which are seldom overcrowded and a qualified teacher (usually). Different teaching methods may be applied but, in general, he is taught and has access to expert help. All these are external aids to learning, the pupil also must provide an input. It is the pupil's duty to be attentive in class, ask questions when necessary, complete homework assignments, undertake revision for tests and, if he is aiming at excellence in the subject, to research topics rather more fully by consulting other books and doing more difficult exercises.

Model study habits are, unfortunately, not handed out with the textbooks at the beginning of standard eight; they have to be acquired by practise in earlier years. Children learn good work habits by observing the adults around them and having their own work supervised, checked and praised until they themselves begin to take pride in a job well done. Adolescents, however, are prey to many distractions and not all of them come from homes where the value of disciplined hard work is acknowledged. It is difficult for the easily led and those who receive scant encouragement at home to acquire the necessary study routines.

Others curiously at risk in this respect are the very bright. They manage their school work with such ease and achieve such outstanding results right from the beginning that no one suspects that they have been "coasting". A few can even "coast" through the Senior Certificate examination, but many discover their inability to work in standard nine and cannot understand what has suddenly gone wrong - they expect to read and conquer as they always have done - they have never learned to struggle for understanding through repeated exercises and repeated revision of a technique until they make it their own.

For the great majority of candidates consistent hard work, i.e. keeping up with the rest of the class every day, making sure that all assignments are completed and planning periodic revision sessions is a prerequisite of success in the final examination. Without a fairly substantial personal input no pupil will do well in Matriculation Mathematics.

2.3.4 Persistence, Tenacity and Determination

The ability to apply one's efforts steadily over a protracted period is obviously an advantage in any field of study. Determination, that stubborn refusal to give up trying, can make the difference between passing and failing for the less intelligent child, or between passing and distinction for the more intelligent.

Nothing spurs us on to greater effort like success - the better we do, the harder we try. However, many students of Mathematics find that they have to work for long periods without the encouragement of visible progress, and it is this group that requires persistence the most.

Persistence is enhanced by the possession of the attributes mentioned above, e.g., if one has a certain understanding of basic principles, one at least has some sort of "life-raft" to cling to when wallowing in a sea of mathematical confusion; if one has a powerful incentive to succeed, then one at least has a goal pointing the way ahead; if one has established a pattern of working and studying, then persistence may be reduced to a matter of routine.

There are those, of course, who just have the kind of nature that is patient and persevering and this is just as valuable for Matriculation Mathematics as that indefinable quality "mathematical flair" that is evident in a few pupils.

Summing up the results of the CSMS research, Kathleen Hart (1981) said, "The overwhelming impression obtained is that mathematics is a very difficult subject for most children." It is clear then that the majority of pupils can succeed only through perseverence and determination. This cannot be applied externally, it must come from within.

2.4 SUMMARY

In brief, the prerequisites for success in the Senior Certificate course in Mathematics may be given as follows:

(a) a firm understanding of previous work on which to base more advanced material;

(b) a genuine desire to succeed, i.e., an incentive to study Mathematics which is powerful enough to see the candidate through to the end in spite of difficulties along the way;

(c) a willingness to work consistently and with determination;

(d) mathematical ability - the ability to manipulate figures with precision; the ability to grasp abstract concepts; the ability to solve mathematical problems by the application of learned techniques and, sometimes, by a synthesis of various techniques.

The extent to which a candidate possesses these qualities will determine his degree of success in the final examination.

No one would dream of encouraging the hopelessly unco-ordinated girl to continue with the study of Ballet to the senior level; no one would suggest that a tone deaf boy persists with the violin to Matriculation; however, such is the importance of Mathematics today that pupils, and their parents, are extremely reluctant to leave this subject out of the curriculum, irrespective of whether or not the pupil is able to meet the demands of the course.

There are two courses open to schools - to allow any pupil to take Mathematics without any consideration of his chances of success, or to select only those who are considered likely to succeed. The former course of action is very popular among standard seven pupils and their parents, but it places great strain on Mathematics Departments, most of which are understaffed, or are using under-qualified teachers to fill the gaps; youngsters in large classes receive less individual attention, and so pupils, who might have managed, "go under" because the teacher had to spend so much time with students who were set to fail from the beginning.

To select only those who seem likely to succeed is, in the writer's opinion, a much wiser course of action, but one which is far more difficult to carry out.

The basis for such a selection process would have to be a consideration of the extent to which the individual pupil meets the demands of the senior course in Mathematics as listed above, but the question remains, how can we tell beforehand who has what it takes? How can we predict who is likely to be successful in the final examination?

CHAPTER 3

PREDICTION OF SUCCESS IN SENIOR CERTIFICATE MATHEMATICS

Very rough predictions can be made about the likely mathematical fortunes of any group of standard seven pupils attending any ordinary Cape Education Department school, viz., that there will be a number of pupils who will do very well, a large number who will pass comfortably and a number who have no chance of success.

These different groups will correspond roughly to the top, the middle and the bottom of the academic merit list. Selection processes, however, have to be based on more refined predictions. Teachers with the responsibility of advising students really need to be able to make predictions about individuals. Parents ask, "Should my child continue with Mathematics? On which grade should my child study Mathematics?" We need a reliable predictive device which informs us about the individual pupil's status vis-a-vis the prerequisites for the course, as mentioned in the previous chapter, and which has credibility for pupils and their parents.

Prediction of academic achievement is a serious issue and should be undertaken by those skilled in the use of the various predictive devices devised by the psychologists. A survey of some of the literature concerning prediction is given in Appendix F. However, classroom teachers are called upon to make rough predictions about their pupils' success in Matriculation mathematics and the following devices would be available to them:

1. Intelligence Quotient

Most teachers of Mathematics would agree that a good mathematical ability is usually supported by a good general ability, but measures of intelligence tell us nothing about the candidate's understanding of previous work, motivation or willingness to work hard. A score on a test of intelligence might indicate how well the pupil can reason, and that might be fairly useful, but it is unlikely that the intelligence quotient of the pupil will provide an adequate measure of mathematical potential.

2. Parent Intuition

Few parents are professional mathematicians, and when they do venture to make predictions about their child's mathematical success these are often based on hopes rather than realities. Some rely on the pupil's previous "track record", which is understandable, but they seldom have considered that the senior course might be qualitatively different from the junior course. Some, whose offspring have not performed well previously, put this down to naughtiness, and assert that with greater maturity and, perhaps a different teacher, all will yet be fine.

The area in which information from the parent is most reliable is in respect of the pupil's motivation. They are aware of any burning ambitions within their children that could provide the powerful incentives needed for the less able to succeed. Teachers very often have no way of finding out about these driving forces, particularly from quiet and shy youngsters.

3. Teacher Intuition

This is probably the most reliable predictor of all because the teacher's opinion is formed over the period of the school year, perhaps two years, and is based on a knowledge of the student's previous performance and present status, as well as observation of the pupil in class. The student's behaviour in class betrays his attitude towards the subject and, also, his ability to think and make mathematical connections; the questions he asks indicate the level of his understanding and his ability to deal creatively with problems; the way he works shows whether he cares about accuracy and whether he has the commitment to persevere with a task until it has been completed.

In short, the teacher who knows the pupil, the quality and potential of his work, and who also has the advantage of knowing exactly what lies ahead in the senior syllabus, can give the most accurate and well-balanced prediction of the pupil's chances in the matriculation examination. Most parents value the teacher's opinion and accept it, provided that what they are told is what they want to hear; as soon as the teacher's advice is in conflict with their own feelings on the matter they may no longer be interested in opinions, but want <u>proof</u>. The trouble with an intuition is that it cannot be quantified; a statement about a pupil's ability without a number attached to it seems too vague for many parents - and what is vague, is possibly untrue or, at least, open to doubt.

The teacher's intuition, although the most useful of the predictors, lacks credibility in the eyes of parents and pupils because it cannot be expressed numerically. We need to find some quantitative way of supporting this opinion.

4. Standard Seven Mathematics Examination

Reporting on an extensive investigation into predicting success in algebra, E.G. Begle (1979) remarked:

> The best predictor of mathematics achievement is previous mathematics achievement. Only rarely does any other cognitive variable contribute significantly to prediction. Almost never do affective, non-intellective, or teacher variables add anything to the predictive power of previous mathematics achievement measures.

It is certain that the promotion examination provides the most valuable information about the student's knowledge and understanding of junior course Mathematics. This examination also contains a measure of the pupil's regular classwork throughout the year as the final examination result includes a "year mark" which is a summary of all test marks. Furthermore, the outcome of this examination is expressed as a number which means that parents and pupils tend to accept this evidence more readily and more wholeheartedly than mere verbal statements made by the teacher.

It would seem, then, that the obvious predictor may be the most acceptable and the best one; however, a note of warning must be sounded here. Mathematics is a compulsory subject in Standard Seven. Most schools have a certain proportion of academically very weak students who struggle with even the easiest elements of the junior syllabus and who find it almost impossible to pass Mathematics. If these pupils are to enjoy their mathematics lessons at all, if they are not to have their self-confidence utterly crushed, then they must be taught at what Skemp (1979) would call the level of "instrumental understanding", i.e., the level of rules without reasons. To a certain extent, these pupils can learn skill - the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. For the benefit of these pupils some standard seven examinations tend to be biased in favour of mechanical manipulative techniques and this will prejudice the reliability of this examination as a predictor of success in Higher Grade Mathematics, which we have seen is biased in favour of what Skemp (1979) would call "relational understanding", i.e., comprehension, knowing not only how but also why, or the ability to deduce specific rules or procedures from more general mathematical relationships.

We cannot base predictions about the ability to cope with mathematics in an insightful way on the pupil's ability to cope with mathematical recipes. Begle (1979) also comments:

> ..., the best predictor of computational achievement is previous computational achievement, and the best predictors of achievement at higher cognitive levels are previous mathematics achievement at higher cognitive levels.

Before we lend too much weight to the standard seven school examination as a predictor, we must be sure that that examination contains the correct balance of mental processes to reflect the demands of the senior course. We must also question the fairness of deliberately using this examination as a predictor; after all, some pupils are aware of their limitations and have no intention of continuing with Mathematics - they simply want to be tested on what they have been taught and do as well as they possibly can so that they can escape out of compulsory mathematics into standard eight where they may be able to choose less academically demanding subjects. Perhaps it would be better to use some test of Mathematics other than the promotion examination for the purposes of selection.

5. A Separate Teacher-made Test of Mathematics

Such a test need not be the sole predictor of success but rather play a supporting role. We have the regular school examination to inform us about the pupil's present standing, we have the parent's information about the pupil's motivation and we have the power of the teacher's intuition to inform us about the pupil's willingness to work and his determination. This new test need provide only a quantitative indication of the pupil's ability to deal insightfully with problems in Mathematics and we would surely have a battery of predictors acceptable to all.

This new test would serve only the process of selection and the results would not be reflected as part of the student's final examination mark, and so would not adversely affect the results of poor students. Moreover, such a test would not be restricted by the bounds of the standard seven syllabus, but could range more widely in its attempts to call forth mathematical insight. The only possible restriction on this test would be that it should reflect the nature of the matriculation examination in its balance of mental processes in order to be consistent with the

> finding of Begle's investigation that achievement is best predicted by previous achievement of the same intellectual level.

To sum up, then, we have considered various possible predictors, all of which would be readily available to the classroom teacher. Some of them will be more reliable than others, but all should be helpful although Begle (1979) insists:

> ...the best predictors...are measures of the student's previous success in mathematics, as measured by his grades in mathematics courses or by the opinions of his mathematics teachers. General intellectual ability, as measured by IQ, and reading ability seem to have little value...

Nevertheless, the writer feels that there is a need for an additional test which could yield specific information about the pupil's potential ability at a higher level of mathematics understanding.

The construction of such a test is described in the chapter that follows.

CHAPTER 4

DEVELOPMENT OF THE TESTS

4.1 CHAPTER OUTLINE

In this chapter, the construction and development of a multiple-choice test of Mathematics are described. Owing to the repetitive nature of the process of developing an efficient test, it was considered necessary to set down, in brief, the procedures followed by the writer. It is hoped that with such an overview, confusion may be avoided.

This test, which it was hoped would have predictive validity for the Higher Grade Senior Certificate examination, was developed in three stages and administered three times. Each successive test was a refinement or a revision of the previous test, i.e., the items of the successive tests were either wholly or partially the same as the test that went before. To avoid confusion in the discussion of these tests, they shall be referred to as the pilot test, the revised test and the new test respectively. The Table 3 lists the administration details of the three tests.

<u>TABLE 3</u>: THE NAMES, ADMINISTRATION DATES AND DETAILS OF THREE MULTIPLE-CHOICE TESTS DESIGNED TO PREDICT SUCCESS IN SENIOR CERTIFICATE MATHEMATICS

Test	Date	No. of Items	Sample Size	Sample Level	No. of Schools
Pilot	Sept'83	70	129	Std 7	1
Revised	Nov '83	35	676	Std 7	5
New	Nov '84	30	458	Std 7	3
			404	Std 8	4
			369	Std 9	3

The results of each test are recorded and compared to results of school examinations. In addition, the items of each test are subjected to scrutiny in order to determine their efficiency in separating those pupils with superior mathematical ability from the others.

Before proceeding with the discussion of the tests, it is necessary to mention briefly the nature of the test sample and certain problems encountered with the sample.

The schools in the sample for testing were all under the administration of the Cape Education Department and the pupils attending these schools are white, urban, middle-class, English-speaking boys and girls.

Originally, there were six participating high schools and roughly 800 standard seven Peninsula school children wrote the pilot test or the revised test. One year later, the new test was written. The size of the sample, at this stage, was greatly decreased owing to the withdrawal of two of the schools from the experiment, and the fact that some pupils did not continue with Mathematics in standard eight. Approximately 400 of the original candidates were retested at the end of their standard eight year. In addition to these, standard seven and standard nine pupils at three of the remaining four schools wrote the new multiple-choice test.

To concentrate on the 400 retested pupils for a moment, the size of this sample is not quite as large as it seems. Since comparisons are made with results of school examinations, it must be remembered that not only do different schools write their own internal examinations, but within each school there is a separation of Higher Grade and Standard Grade examining. This means that the sample size is whittled down again and each grade in each school must be treated separately in the various analyses.

There is a further problem concerning diminishing sample size. Gradually, candidates, who started out on the Higher Grade syllabus, found it too difficult and changed to Standard Grade. The result of this is that the Higher Grade group becomes progressively smaller, and, it is not possible to compare new analyses with old because the nature of the group has changed.

These problems are highlighted again as the analyses of the various test results proceed.

At the outset there were three principal decisions to be made:

1. What form should the test take?
2. How long should the test be?
3. How could the test items be balanced to cater for both Higher and Standard Grade predictions?

The multiple-choice format was chosen for this test because of the ease of scoring and analysing items from such a test, particularly as it was envisaged that the number of candidates involved in writing the test would be quite large. Furthermore, it was not intended that this test should provide information about the pupil's skill in writing solutions down, but rather it was intended simply to reveal the presence of insight.

The length of the test is an important issue. Theoretically, the longer the test the greater is the reliability of the test; however, there are practical considerations to be taken into account. Pupils have a limited concentration span and, since multiple-choice items can be very demanding, care has to be taken not to make the test so long that the candidates become exhausted and unable to think clearly. In addition, the school day is very full and it might be difficult to persuade School Principals to set aside more than an hour or two for the writing of this test. In order to settle the question of length, it was decided that a pilot study should be undertaken at just one school and that this could serve as a basis for the test in its final form.

The construction of this test demanded a strict adherence to the proportions and balance of the various mental processes that were seen to occur in matriculation examinations. The question was, to the composition of which paper should the test conform - Higher Grade or Standard Grade? The differences, with respect to the more complex mental processes, between these two was quite marked. Since Standard Grade papers concentrate heavily on manipulation and routine problems, it was considered to have more in common with the standard seven examination than did the Higher Grade Senior Certificate examinations. Consequently, it was decided that this test should conform to the requirements of the Higher Grade papers because it is in prospective Higher Grade students that we need to detect mathematical insight. Consequently, the ambit of the study is restricted to an investigation of possible prediction for the Higher Grade examination.

4.2 THE PILOT TEST

The pilot test was devised, as mentioned before, to settle the question of the length of the test. It would also serve to estimate, more accurately, the actual level of difficulty of the test items. In a test of this nature it is important that the items discriminate clearly between the able and less able student. To accomplish this task of discrimination the chosen items should fall in the middle difficulty range; the level of difficulty can be determined only by a trial run of the items. The pilot study allows the truly valuable items to be identified and retained. Less efficient items, for the purposes of the investigation, are discarded.

4.2.1 Construction of the Pilot Test

During the analysis of the Higher Grade Senior Certificate examinations the average proportions of the various mental processes was found to be: Recall 7%; Manipulation 17%; Routine problems 37%; Comprehension 20%; Insight 12%; Application of higher mental processes 7%. The pilot test was designed to conform to these same proportions. Many of the items are regular standard seven syllabus questions simply converted to a multiple-choice format with five foils to minimise the probability of guessing. These items cover algebraic and geometric material roughly in the same relative quantities as the normal school examination. Some items are not ordinary syllabus material but test thinking ability and insight at a level not beyond the reach of standard seven mathematics.

Test items, suitable for the level of knowledge of standard seven pupils, were sought and classified according to the mental process they involved. An example from each level of the hierarchy of mental processes is given below to illustrate the writer's train of thought in the classification of items.

Level 1 - Recall

Two angles, measuring 47° and 43° respectively, may be described as .... angles.

A adjacent
B co-interior
C complementary
D right
E supplementary

This item demands nothing more of the candidates than the simple recall of a definition, and so it was classified at the lowest level of the hierarchy.

Level 2 - Manipulation

Multiply: $(2p - 7q)(5p + 3q)$

A $10p^2 - 41pq - 21q^2$
B $10p^2 - 35pq - 21q^2$
C $10p^2 + 29pq - 21q^2$
D $10p^2 - 29pq - 21q^2$
E $10p^2 - 21q^2$

Here, the instruction is quite explicit, i.e., multiply, the pupil is not expected to decide what has to be done. The technique of finding the product of two binomials is taught and drilled in standard seven, therefore this procedure should be well known to the candidates - they simply have to perform the procedure accurately.

Level 3 - Routine Problems

The interior angles of a quadrilateral are y, 2y, y + 20° and 2y + 40°. Find the value of y.

A 45°

B 40°

C 55°

D 50°

E Insufficient information

No explicit instruction is provided for the candidate - he must decide what has to be done. The fact that the angles of a quadrilateral have a sum of 360° must be recalled, an equation set up and solved. However, similar examples are commonly found in textbooks and pupils should be quite familiar with this problem.

Level 4 - Comprehension

Which of the following numbers is the largest?

A $2 \times 2 \times 2$

B 22^2

C 222

D 2^{2^2}

E 2^{22}

The candidate must demonstrate his comprehension of algebraic notation by deciding how to convert these numbers from their given form to a form in which they may be compared, i.e., to numbers whose exponent is one. Further understanding will be required to deal with the number in alternative E, which is too large for quick calculation - the pupil must comprehend the magnitude of this number by his ability to imagine the rate of increase of successive powers of two.

Level 5 - Insight

The cube root of 62 570 773 is .....

A 383
B 397
C 429
D 517
E 20 851

Although the finding of cube roots is covered in the standard six syllabus, the size of the given number should discourage a process of factorisation and this leaves the candidate to decide what else might be attempted. Insight will be required of the standard seven pupil to use the units digit of the proferred solutions to narrow the possibilities down to alternatives B and D. Once this is achieved the answer may be obtained by estimation. This item is classified as a non-routine problem because, not only must the candidate decide what must be done, he must find a new way to solve a problem he has not met before.

Level 6 - Apply "Higher" Mental Processes

How much information do you need to answer the question, "Is x a two digit number?", if you are told:

I x^2 is a three digit number
II 10x is a three digit number
(Assume x is a whole number)

A I alone is sufficient but II alone is not.
B II alone is sufficient but I alone is not.
C I and II together are sufficient, but neither statement alone is sufficient.
D Each statement alone is sufficient.
E More information is needed to answer the question.

This item is classified at the highest level of the hierarchy because it demands that the candidate apply his understanding of the size of the numbers given, in an insightful way, to decide on the sufficiency of data supplied.

Eventually a battery of seventy items was assembled, classified according to the level of mental process required, as illustrated above and graded according to the estimated level of difficulty of the individual item. In accordance with the proportions of mental processes represented in Higher Grade Senior Certificate examinations, the test was constructed as follows:

Items 1 - 4 involve Recall

Items 5 - 17 involve Manipulation

Items 18 - 43 involve solving Routine problems

Items 44 - 57 involve the demonstration of Comprehension

Items 58 - 65 involve solving Non-routine problems by insight

Items 66 - 70 involve the application of the "Higher" mental processes.

Working on an average of just under two minutes per item, it was decided that the duration of the test should be two hours. It was thought that this would probably be the limit of the pupils' endurance on such a test, and the limit of time that the school authorities might feel pleased to yield.

The positions of the correct responses to the items were randomly selected and the test was typed. A copy of this pilot test may be found in Appendix B.

4.2.2 Administration of the Pilot Test

In 1983, during the third school quarter, 129 standard seven boys attending a local high school wrote the pilot test described in the foregoing section. At this stage of the year most of the syllabus content would have been covered by teachers, and it was considered that these boys were not at any major disadvantage in comparison with those who wrote the later test at the end of the school year.

Clear instructions about how to answer items were printed on the cover of the test paper and these were reiterated by the invigilator before the test began. Additional instructions, concerning erasures and changes to answer choices, were given verbally by the invigilator. Approximately ten minutes before the end of the test, candidates were informed about the time remaining to them and were advised to complete items by intelligent guesswork if necessary so that no item should be left unanswered. The invigilator reported that, towards the end of the test, the boys were showing signs of fatigue and that some of them had to rush through the last few items.

The test papers were returned to the writer for scoring and item analysis.

4.2.3 Analysis of the Pilot Test

The scoring of the test was carried out by the writer on the basis of one mark per correct response and no mark for an incorrect response. There was no correction made for guessing as the candidates had been encouraged to make intelligent guesses where necessary. At the time the test was written, the invigilator discovered that, owing to an error in typing, there was no correct response offered for item number 28. This item was then excluded from the analysis which, effectively, reduced the number of items in the pilot test to sixty-nine.

Each candidate was given a code number and the results of the test were recorded. The bulk of test results yielded by this study is extremely large. Consequently all test results are summarised by tables of test statistics.

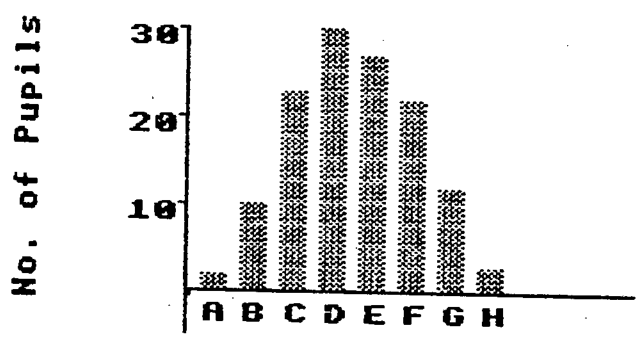
TABLE 4: TEST STATISTICS OF THE PILOT TEST ADMINISTERED TO A CAPE PENINSULA BOYS' HIGH SCHOOL DURING THE THIRD SCHOOL TERM OF 1983

No. of pupils tested	129
Maximum score possible	69
Mean	34,2 (49,5%)
Median	35 (50,7%)
Standard Deviation	10,6 (15,4%)

Figure 2 further illustrates the distribution of Pilot Test scores.

Figure 2

SYMBOL DISTRIBUTION OF PILOT TEST SCORES



The symbols used are the usual school symbols allocated to school examinations, i.e. A reflects scores greater than 80%, B reflects scores greater than 70%, C reflects marks greater than 60%, etc.

An item analysis was undertaken in order to ascertain the level of difficulty of each question, its discriminating value or efficiency and the appropriateness of the foils or distracters - this with a view to improving the test for administration to a much wider sample of pupils. The separation of the pupils into an upper and a lower group was accomplished by removing the large group of candidates whose scores were clustered around the median, and then using the top 35 scorers as the Upper group and the 37 least scoring candidates as the Lower group.

In the item analysis that follows, the level of difficulty of the item is given by p, which is the number of correct responses divided by the total number of responses; the efficiency of the item as a discriminator is given by the ratio U/L, i.e. the ratio of upper group correct responses to lower group correct responses. The grounds on which an item will be rejected or modified are:

1. p < ,26 - the item was too difficult
2. p > ,69 - the item was too easy
3. U/L < 1 - does not discriminate between the able and less able properly.
4. Foils are confusing or unconvincing.

The item analysis follows:

ITEM 1

	A	B	C*	D	E
U35	3	1	23	1	7
L35	3	2	21	5	4

p = ,63
U/L = 23/21

ITEM 2

	A	B	C*	D	E
U35	1	0	32	1	1
L35	2	5	14	9	5

p = ,66

U/L = 32/14

ITEM 3

	A*	B	C	D	E
U35	21	0	0	0	14
L35	5	0	1	6	23

p = ,37

U/L = 21/5

Distracters B and E confusing

ITEM 4

	A*	B	C	D	E
U35	28	3	2	0	2
L35	17	9	4	1	4

p = ,64

U/L = 28/17

ITEM 5

	A	B	C	D*	E
U35	0	0	0	35	0
L35	3	1	3	26	2

p = ,87

U/L = 35/26

ITEM 6

	A	B*	C	D	E
U35	3	27	2	0	3
L35	9	9	6	1	10

p = ,51

U/L = 27/9

ITEM 7

	A	B	C*	D	E
U35	O	O	27	7	1
L34	3	3	15	9	4

p = ,61
U/L = 27/15

ITEM 8

	A	B*	C	D	E
U35	3	26	1	5	O
L34	10	9	4	7	4

p = ,51
U/L = 26/9

ITEM 9

	A	B	C*	D	E
U35	4	1	23	4	2
L35	11	5	9	5	5

p = ,46
U/L = 23/9

ITEM 10

	A	B	C	D*	E
U34	1	O	5	27	1
L35	11	4	2	13	5

p = ,58
U/L = 27/13

ITEM 11

	A	B*	C	D	E
U/34	O	17	12	2	3
L/35	2	3	17	8	5

p = ,29
U/L = 17/3

ITEM 12

	A*	B	C	D	E
U34	29	0	5	0	0
L35	5	14	15	0	1

p = ,49
U/L = 29/5

ITEM 13

	A*	B	C	D	E
U35	18	1	12	3	1
L35	2	2	13	9	9

p = ,29
U/L = 18/2

ITEM 14

	A*	B	C	D	E
U35	32	1	1	0	1
L35	20	4	8	2	1

p = ,74
U/L = 32/20

ITEM 15

	A	B	C	D*	E
U35	1	2	2	30	0
L35	9	4	2	11	9

p = ,59
U/L = 30/11

ITEM 16

	A*	B	C	D	E
U35	16	11	2	5	1
L35	7	13	2	6	7

p = ,33
U/L = 16/7

ITEM 17

	A	B	C*	D	E
U35	2	9	22	O	2
L35	12	5	5	1	12

p = ,39
U/L = 22/5

ITEM 18

	A	B	C	D*	E
U35	O	O	1	34	O
L35	3	8	6	16	2

p = ,71
U/L = 34/16

ITEM 19

	A	B	C	D*	E
U35	1	O	O	33	1
L35	1	4	4	12	14

p = ,64
U/L = 33/12

ITEM 20

	A*	B	C	D	E
U35	32	1	1	O	1
L35	10	5	8	8	4

p = ,60
U/L = 32/10

ITEM 21

	A	B	C	D	E*
U35	O	10	O	2	23
L35	2	13	1	13	6

p = ,41
U/L = 23/6

ITEM 22

	A	B	C	D*	E
U35	1	3	1	23	7
L35	13	3	0	4	15

p = ,39

U/L = 23/4

ITEM 23

	A	B*	C	D	E
U35	0	27	4	1	3
L35	3	15	8	3	6

p = ,60

U/L = 27/15

ITEM 24

	A	B	C	D*	E
U35	0	1	1	32	1
L34	2	1	3	27	1

p = ,86

U/L = 32/27

ITEM 25

	A	B*	C	D	E
U35	4	6	19	4	2
L35	13	4	13	3	2

p = ,14

U/L = 6/4

ITEM 26

	A	B	C	D*	E
U35	4	0	3	28	0
L35	13	4	4	8	6

p = ,51

U/L = 28/8

ITEM 27

	A	B	C*	D	E
U35	1	4	24	3	3
L34	4	6	10	11	3

p = ,49
U/L = 24/10

ITEM 28

	A	B	C	D	E
U					
L					

p =
U/L =
No correct answer

ITEM 29

	A*	B	C	D	E
U35	23	8	2	2	0
L35	10	8	5	9	3

p = ,47
U/L = 23/10

ITEM 30

	A*	B	C	D	E
U35	28	1	0	2	4
L35	18	3	2	1	11

p = ,66
U/L = 28/18

ITEM 31

	A	B	C	D	E*
U34	4	3	2	4	21
L35	5	6	8	8	8

p = ,42
U/L = 21/8

ITEM 32

	A	B*	C	D	E
U35	O	33	1	O	1
L35	6	16	3	4	6

p = ,70
U/L = 33/16

ITEM 33

	A	B	C	D	E*
U35	7	O	O	2	26
L35	10	4	4	2	15

p = ,59
U/L = 26/15

ITEM 34

	A	B*	C	D	E
U35	7	25	O	1	2
L35	16	8	9	1	1

p = ,47
U/L = 25/8

ITEM 35

	A*	B	C	D	E
U35	23	6	1	4	1
L35	9	6	1	15	4

p = ,46
U/L = 23/9

ITEM 36

	A*	B	C	D	E
U35	20	7	7	O	1
L35	6	5	24	O	O

p = ,37
U/L = 20/6

ITEM 37

	A	B*	C	D	E
U35	2	31	2	0	0
L35	3	11	10	2	9

p = ,60
U/L = 31/11

ITEM 38

	A*	B	C	D	E
U35	28	1	0	5	1
L35	10	5	5	14	1

p = ,54
U/L = 28/10

ITEM 39

	A	B	C	D	E*
U35	0	0	0	0	35
L35	9	5	3	1	17

p = ,74
U/L = 37/17

ITEM 40

	A	B	C	D*	E
U35	0	0	4	31	0
L35	1	7	10	15	2

p = ,66
U/L = 31/15

ITEM 41

	A	B*	C	D	E
U35	0	32	0	0	3
L35	1	6	0	1	27

p = ,54
U/L = 32/6

ITEM 42

	A	B	C	D	E*
U35	1	6	O	O	28
L35	4	14	7	5	5

p = ,47
U/L = 28/5

ITEM 43

	A	B	C	D	E*
U35	O	1	O	O	34
L35	O	8	4	O	23

p = ,81
U/L = 34/23

ITEM 44

	A	B	C*	D	E
U35	3	6	21	5	O
L35	7	9	4	12	3

p = ,36
U/L = 21/4

ITEM 45

	A	B*	C	D	E
U35	1	22	2	3	7
L35	2	5	10	3	15

p = ,39
U/L = 22/5

ITEM 46

	A	B	C	D*	E
U35	O	O	4	29	2
L35	3	5	10	15	2

p = ,63
U/L = 29/15

ITEM 47

	A	B	C	D*	E
U35	O	5	4	22	4
L35	4	10	8	8	5

p = ,43
U/L = 22/8

ITEM 48

	A	B	C*	D	E
U35	O	O	35	O	O
L35	O	O	19	5	11

p = ,77
U/L = 35/19

ITEM 49

	A	B*	C	D	E
U35	2	31	O	2	O
L35	3	13	5	13	1

p = ,63
U/L = 31/13

ITEM 50

	A	B	C*	D	E
U35	O	O	35	O	O
L35	3	6	24	O	2

p = ,84
U/L = 35/24

ITEM 51

	A	B*	C	D	E
U35	3	26	2	3	1
L35	2	15	8	8	2

p = ,59
U/L = 26/15

ITEM 52

	A	B	C	D	E*
U34	0	5	6	2	21
L34	11	6	7	7	3

p = ,35
U/L = 21/3

ITEM 53

	A	B	C*	D	E
U35	3	4	23	2	3
L35	2	8	8	7	10

p = ,44
U/L = 23/8

ITEM 54

	A	B	C	D	E*
U35	1	1	2	4	27
L35	12	3	5	7	8

p = ,50
U/L = 27/8

ITEM 55

	A	B*	C	D	E
U34	1	30	1	2	0
L34	9	11	6	5	3

p = ,60
U/L = 30/11

ITEM 56

	A	B	C	D	E*
U34	22	0	8	0	4
L35	18	2	5	4	6

p = ,14
U/L = 4/6

ITEM 57

	A	B*	C	D	E
U34	1	21	5	4	3
L35	7	14	5	3	6

p = ,51
U/L = 21/14

ITEM 58

	A	B*	C	D	E
U33	4	20	1	4	4
L35	0	12	4	6	13

p = ,47
U/L = 20/12

ITEM 59

	A*	B	C	D	E
U35	22	0	6	1	6
L35	6	3	14	7	5

p = ,40
U/L = 22/6

ITEM 60

	A	B	C*	D	E
U35	9	12	13	0	1
L35	17	10	6	2	0

p = ,27
U/L = 13/6

ITEM 61

	A	B*	C	D	E
U35	3	9	12	6	5
L35	1	2	18	9	5

p = ,16
U/L = 9/2

ITEM 62

	A	B	C	D	E*
U35	3	4	5	2	21
L35	8	7	7	4	9

p = ,43
U/L = 21/9

ITEM 63

	A*	B	C	D	E
U35	8	9	5	4	9
L35	18	3	1	5	8

p = ,37
U/L = 8/18

ITEM 64

	A*	B	C	D	E
U35	21	4	4	2	4
L35	5	6	16	3	5

p = ,37
U/L = 21/5

ITEM 65

	A	B*	C	D	E
U35	15	4	7	6	3
L35	18	3	4	9	1

p = ,10
U/L = 4/3

ITEM 66

	A	B	C	D	E*
U35	7	5	3	15	5
L35	3	3	6	19	4

p = ,13
U/L = 5/4

ITEM 67

	A	B	C	D*	E
U35	3	2	3	22	5
L35	0	4	18	9	4

p = ,44
U/L = 22/9

ITEM 68

	A	B	C*	D	E
U35	5	7	15	4	2
L35	8	6	8	7	6

p = ,34
U/L = 15/8

ITEM 69

	A*	B	C	D	E
U35	20	4	8	1	2
L35	14	6	4	6	5

p = ,49
U/L = 20/14

ITEM 70

	A	B	C	D*	E
U35	1	4	12	16	2
L35	8	3	9	6	9

p = ,31
U/L = 16/6

The item analysis yields the following distribution of difficulty levels:

Figure 3

THE DISTRIBUTION OF PILOT TEST ITEM DIFFICULTY LEVELS, p, CALCULATED ON THE BASIS OF THE TOP 35 SCORERS AS THE UPPER GROUP AND THE 35 LEAST SCORING CANDIDATES AS THE LOWER GROUP

< ,16	****	4
,16 - ,25	*	1
,26 - ,35	*******	7
,36 - ,45	***************	15
,46 - ,55	****************	16
,56 - ,65	**************	14
,66 - ,75	*******	7
,76 - ,85	***	3
> ,85	**	2

MEAN p = ,50 STANDARD DEVIATION = ,17

Some of the items proved to be rather more difficult than estimated and a few proved to be too easy. In order to achieve maximum discrimination, these items would have to be excluded from the test. Items rejected on this basis were numbers 5, 14, 18, 24, 25, 32, 39, 40, 43, 48, 50, 56, 61, 65 and 66.

Items rejected because of the U/L ratio, i.e. because they did not discriminate properly between the able and the less able, were 1, 56, 63 and 66. The only item excluded on the grounds of confusing distracters was number 3. A further five items were rejected simply to shorten the test, namely, items 2, 10, 16, 60 and 61.

Apart from the information gained about the items, the pilot test also provided information about the length of the test and the mechanical business of scoring. Two hours had obviously been too long a spell for many of the boys to concentrate but still not enough time to finish all the questions. It was decided that the next test should be shorter in duration, but allow more time per item. Reducing the number of items, of course, would mean a decrease in reliability; however, too much guessing in order to finish would likewise reduce reliability - a balance would have to be found.

The boys who wrote the pilot test had not been provided with an answer sheet, but had made their responses on the question paper. The reasoning behind this had been that there would be less chance of errors being made in matching the item with its answer since the candidate's eye would not have to travel to a separate sheet. However, scoring this had proved to be tedious and inefficient because of the necessity to turn pages constantly. It was decided that, for the next test, an answer sheet would be provided.

The pilot test was a very valuable exercise which greatly facilitated the construction and administration of the revised test, a description of which follows in the next section.

4.3 THE REVISED TEST

The pilot test had provided information about the length of the test, the level of difficulty of the items and the desirability of having an answer sheet. The revised test would have to solve the problem of maximising reliability by striking a balance between providing an adequate sample of suitable items whilst not exhausting the candidates. The problem of maximising the test's discriminating power would be addressed by having item difficulty levels more closely clustered around ,50 through the exclusion of items which had proved to be too easy or too difficult, e.g.,

Item 5: (p = ,87)

Add: $5x - x^2 + 6$; $3x^2 + x + 8$; $-4x + 2x^2$

A typical manipulation taught to standard six pupils in introductory algebra - this item proved to be much too easy for these standard seven boys and, consequently, nearly everyone chose the correct answer, thereby providing no distinction between the able and the less able groups.

Item 25: (p = ,14)

Which of the following are rational numbers?

I $\sqrt{3}$ II $\sqrt{4}$ III $\sqrt{2\frac{1}{4}}$

This item proved to be very difficult (perhaps the boys had not yet been taught the definition of "rational"), and practically no one answered correctly - again providing no distinction between able and less able pupils.

By excluding such items from the revised test, only those middle-difficulty items, which provide an opportunity for maximum discrimination between the upper and lower groups of candidates, would remain.

4.3.1 Construction of the Revised Test

It was decided that the revised test would use only items that had already been tried out in the pilot test. In this way it would be possible to extract scores on the revised test for the pupils who had written the pilot test.

Certain items had already been rejected on the basis of the results of the foregoing item analysis. This left 47 items which were suitable for re-use. However, the rejection of questions had disturbed the balance of the various ability types among the items and it was important that the revised test conform to the same proportions of types of mental process as the senior certificate examinations. To serve this purpose, items of the pilot test were scrutinised again and a further 12 items were rejected. Most of the questions excluded at this stage had worked quite well in the original test, and were left out simply because they were similar to another item or, because they performed slightly less well than other items. The test items rejected on this basis were numbers 4, 8, 20, 27, 30, 33, 35, 45, 46, 49, 52 and 70.

The remaining items were rearranged to place some of the later questions of the pilot test nearer the beginning of the revised test to determine what effect the guessing of the slower candidates had had on the difficulty levels of these items - it will be remembered that a number of boys had experienced difficulty in finishing, and that ten minutes before the end of the test, they had been advised to use intelligent guesswork in order to complete all items. It was thought that this might have affected adversely the difficulty levels of those items near the end of the test paper. Other items were presented in ascending order of difficulty according to the levels of p calculated in the item analysis of the previous section. Again the positioning of correct responses among the foils was selected randomly.

The time allocation per item was increased to two minutes which meant that the duration of the test would be seventy minutes. This seemed a reasonable time span from the point of view of the candidates' concentration, the school timetable and the purposes of the investigation.

The revised test was typed and printed along with an answer sheet, and a copy of each of these is tabled separately in Appendix C.

4.3.2 Administration of the Revised Test

The co-operation of five local high schools was obtained, three co-educational and two girls' schools. It was agreed that, in order to ensure that pupils viewed this test with the utmost seriousness, it should be written at the same time as the ordinary end-of-year promotion examinations. It was not possible to arrange that the test would be written on the same day in each school, and so the test was administered throughout the month of November 1983 to the standard seven pupils of four of the schools. Standard eight pupils in the fifth school wrote the test immediately after school recommenced in January 1984. This group of candidates was different from the others in that they all had chosen to study Mathematics at the senior level; however, nearly all of the pupils attending this particular school do opt for this subject, and so the composition of the group still represents a wide range of talent, but excludes the very weakest pupils. This had the effect of inflating the mean score of these pupils a little. All test material was kept secure before the writing of the test and was then completely removed from the schools after writing. There is, therefore, no reason to believe that there was any information, concerning the content of the test, passed between pupils of different schools.

The test was administered by four or five different teachers in each separate school. In order to ensure some sort of standard procedure in administration, invigilator's instructions were sent with the test in the hope that this would leave less room for doubt on the part of those teachers, and so produce similar test conditions for all candidates. A copy of these instructions is tabled separately in Appendix C.

Administered together with the revised multiple-choice test was a short test of mathematical reasoning which was designed to indicate which pupils could find their own solutions to problems which involved complex thought processes or complicated verbal statement. This test is discussed more fully in Section 4.4.

Reliable feedback from invigilators was difficult to obtain as it was not possible to make contact with all of the teachers involved; however, comments from a few indicated that lack of time had been a problem for some of the candidates, in spite of the extra time allocated to each item. An invigilator from school C reported that some of the pupils had not seemed to take the test seriously and that some results might be open to question. This had been a potential problem at school E, the students of which wrote the test in January; however, the pupils were told that the outcome of this test would determine subject setting, and, according to invigilators' reports, this seemed to have the desired effect.

4.3.3 Analysis of Results, Reliability and Validity

All scoring of answer sheets was undertaken by the writer and one assistant, again on the basis of one mark per correct response with no adjustment made for guessing - it was mentioned previously that intelligent guessing had been encouraged as an alternative to leaving an item unanswered. The results, on the 35 items of the revised test, of the boys who wrote the pilot test were also extracted so that they could be compared with the others. Note that school H is the school at which the pilot test was run. The code letter H was used to denote this school simply to set it a little apart from the others because the test written at this school was not the same as that written at the other schools. The table that follows gives a summary of the test statistics school by school:

TABLE 5: COMPARISON OF REVISED TEST STATISTICS FOR THE STANDARD SEVEN PUPILS OF SIX CAPE PENINSULA HIGH SCHOOLS

	SCHOOLS					
	H	A	B	C	D	E
N	129	131	100	127	176	142
TYPE	Boys	Girls		Co-educational		
MEAN	15,95	13,34	9,96	10,44	11,54	13,33
SD	6,63	5,44	4,92	5,18	5,04	5,04
MEDIAN	16	13	9	9	11	13
HIGH	32	29	28	28	27	28
LOW	3	2	1	2	3	3

It is interesting that, with the exception of school H, the top scorers in each school had very similar results; there is a corresponding similarity at the bottom of the range too. Schools A, B, E and H are all, geographically very close, while schools C and D are some distance away from the others. Most interesting, perhaps, is the difference in results between school A and school H. These schools serve exactly the same residential area and, to a large extent, the same families; it is noticeable that the boys have fared better, on average, than the girls. The possibility that boys are better mathematicians than girls is investigated in Chapter 6.

Apart from the above observations, these test results tell us little. We need to know whether this test is reliable, and whether it is valid for its purpose, i.e., does this test have predictive validity for the Senior Certificate examination in Mathematics?

Reliability:

Since this test is intended to select candidates capable of succeeding in matriculation mathematics, we need to have an estimate of reliability for this test which includes a measure of candidate stability, i.e., we need to know to what extent the results of candidates would be consistent should they rewrite the test. For this reason, it was decided that pupils would indeed rewrite the test at some later stage. This is discussed in Section 4.5 of this chapter. However, it was possible, at this stage, to obtain a measure of internal consistency by using the method of Split-Halves. A rough split of items of the pilot test had been obtained when the boys' scores on the 35-item test had been extracted; their scores on the 35 items were correlated with their scores on the remaining 34 items using the Product Moment correlation coefficient. The reliability estimate was calculated to be

$$r_r = +0{,}74$$

in the case of the revised test and, using the Spearman-Brown formula for a test twice the length, we obtain a reliability estimate of

$$r_p = +0{,}85$$

in the case of the pilot test. These measures reflect a fair degree of internal consistency especially when it is remembered that the split of items used was not, perhaps, the best one, but rather a convenient one. A more dependable measure of the reliability of the test was obtained when candidates were retested.

Validity:

We are interested in the ability of this multiple-choice test of mathematics to predict achievement in the Senior Certificate examination in mathematics, i.e., we are concerned here with criterion-related validity. The criterion, in this case, is the matriculation examination. Is it, then, necessary to wait three years to find out whether the test has predictive validity? Logically, if the demands of the senior and junior syllabuses are different, then somewhere between standard seven and standard ten there must be a transition. It has been the writer's experience, in the past, that this transition occurs during the first quarter of standard nine. The nature of much of the standard eight syllabus material lends itself to the same sort of mechanical handling as the junior study topics, and there are no time pressures. At the beginning of the standard nine year, the quality and quantity of the syllabus content changes noticeably and, therefore, so do the level of thinking required by the pupils, the pace of lessons and the nature of tests and examinations. Standard nine tests begin to look more and more like the final Senior Certificate examination.

The introduction of the new syllabus (1984) may bring this transition forward into the standard eight year, as yet it is too early to tell; certainly a few more difficult topics are now to be studied one year earlier. Thus, if successive school examinations are increasingly similar to the matriculation examination, then the correlation between the multiple-choice test and each successive examination must also increase, assuming that the multiple-choice test does have predictive validity for the Senior Certificate paper. Conversely, if the standard seven examination tests essentially some different ability than the matriculation examination, then the correlation between it and the later school examinations should decline. To pursue this line of reasoning, it would be necessary to follow the progress of the students tested for at least two years, in order to determine whether or not such trends would materialise.

In the construction of the multiple-choice test there had been no attempt to cover the same ground as the standard seven examination, but both tests are tests of mathematical ability and so one might expect a positive correlation between them. It was not considered worthwhile, at this stage, to calculate coefficients for all schools, since many of the candidates would not continue with mathematics, and therefore, would be lost to the investigation. However, the Product-Moment correlation coefficients were calculated for two schools, viz., schools A and H. The results obtained are given in Table 6.

TABLE 6: PRODUCT MOMENT CORRELATION BETWEEN THE REVISED MULTIPLE-CHOICE TEST AND THE STANDARD SEVEN EXAMINATIONS IN TWO HIGH SCHOOLS

SCHOOL	SAMPLE	r	STD ERROR
A	130 girls	+0,78	0,056
H	125 boys	+0,73	0,062

One girl and four boys were excluded because there was no standard seven examination result available. The strength of the positive correlation is interesting, but it must be remembered that the range of talent within these two groups of pupils was considerable and that this would tend to inflate the correlation coefficient. Nevertheless, it is obvious that these two tests, although designed for different purposes, are testing some common abilities.

Since, at this stage, it was not possible to address the question of reliability and validity, it was decided that a further more rigorous investigation of the test items be undertaken with a view to improving the test for re-administration at the end of the standard eight year. In order to maximise reliability and validity it would be necessary to alleviate the pressure of time in the test and to seek out those questions which were the best middle-difficulty discriminators. This investigation is described in the section that follows.

4.3.4 Further Analyses

As was mentioned before, many pupils were lost to the investigation either because they did not continue to study Mathematics or, as in a few cases, they left the school. It was unfortunate, too, that at this point school C withdrew from the experiment. The table that follows indicates how many pupils continued with Mathematics and the grade on which they continued:

TABLE 7: COMPARISON OF THE NUMBERS OF STANDARD SEVEN PUPILS OF SIX CAPE PENINSULA HIGH SCHOOLS CONTINUING WITH SENIOR COURSE MATHEMATICS

SCHOOL	SEX	STD 7	SENIOR MATHS	HIGHER GRADE	STANDARD GRADE
H	Boys	125	125	107	18
A	Girls	130	105	74	31
B	Girls	100	54	42	12
C	Co-ed	127	99	68	31
D	Co-ed	176	142	56	86
E	Co-ed	170	137	101	26

It is clear that in all the schools a high percentage (about 80%) of pupils are continuing with Mathematics; in fact in school H (a boys' school) it appears that all students proceed to the senior course. Only in school B (a girls' school) does there seem to be no great pressure felt to opt for Matriculation Mathematics. The division of grades is also very similar from school to school; except in school D, at least two-thirds of the pupils, who take Mathematics, take it on the Higher Grade. In school D, a process of selection for the Higher Grade group was fairly rigidly adhered to, and this probably accounts for the different proportions of Higher and Standard Grade candidates. An interesting innovation, at this school, was the introduction in 1983 of the so-called Terminal Mathematics. This was a one-year course designed to meet the computational needs of pupils who wished to study Standard Grade Physical Science but who could not manage the regular course in Mathematics for the full three years of its duration. It also covered the syllabus of the N.T.C. I examination, so that boys entering apprenticeship might be exempt from this first hurdle. Ten pupils opted for this course at the beginning of 1984 and a few more transferred to this class from the Standard Grade class a little later in the year. This group of students does not constitute part of this investigation, but the existence of such a group within an ordinary school situation highlights the desperate need that pupils feel to expose themselves to as much Mathematics as they possibly can.

On the whole, as might be expected, the pupils who do not continue with Mathematics are those whose previous record in the subject has been poor. Among the girls, there are a few who managed the junior course well enough but simply have no interest in Mathematics and so do not pursue it in the senior secondary phase. Conversely there are pupils who continue with the subject whose previous record does not give any encouragement.

The following tables indicate the shifts in test statistics when the pupils who did not continue with Mathematics are excluded:

TABLE 8: COMPARISON OF REVISED TEST STATISTICS FOR THE STANDARD SEVEN PUPILS OF SIX PENINSULA HIGH SCHOOLS WHO CONTINUED WITH MATHEMATICS IN STANDARD EIGHT

	SCHOOLS					
	H	A	B	C	D	E
N	125	105	54	99	142	137
TYPE	Boys	Girls		Co-educational		
MEAN	16,06	14,05	12,17	11,37	12,37	13,39
SD	6,62	5,42	4,78	5,40	5,03	4,88
MEDIAN	16	13	13	10	11	13
HIGH	32	29	28	28	27	28
LOW	3	3	2	2	4	3

TABLE 9: COMPARISON OF REVISED TEST STATISTICS FOR THE STANDARD SEVEN PUPILS OF SIX PENINSULA HIGH SCHOOLS WHO CONTINUED WITH <u>HIGHER GRADE</u> MATHEMATICS IN STANDARD EIGHT

	SCHOOLS					
	H	A	B	C	D	E
N	107	74	42	68	56	101
TYPE	Boys	Girls		Co-educational		
MEAN	17,05	15,88	13,79	13,04	16,52	14,84
SD	6,19	4,89	4,52	5,43	4,70	4,62
MEDIAN	17	15	13	12	17	14
HIGH	32	29	28	28	27	28
LOW	5	8	6	2	7	6

It is interesting that, after the stringent selection of higher grade candidates at school D, the median has increased to the same level as the median score at school H, where the great majority of pupils are higher grade candidates.

In order to find out more about the difficulty and discriminating power of the test items for the group continuing with Mathematics, another item analysis was carried out. Only the pupils from schools A and H were used for this purpose since the number of candidates is comparable, the home background of the pupils is the same and a separation of the sexes is obtained.

Instead of splitting pupils into an upper and a lower group, candidates were divided into quintile groups, on the basis of their scores on the multiple-choice test. It was thought that such a split might provide more detailed information about the way that items discriminate between pupils whose scores place them in the middle of the distribution rather than at the extremes. After all, these are the pupils about whom this investigation revolves - for the top group and the lowest group prediction is not such a problem.

The girls from school A were divided into five roughly equal groups of 22 pupils, 19 pupils, 28 pupils, 16 pupils and 20 pupils, in descending order of achievement on the revised multiple-choice test. The boys from school H were split into five groups of 28 pupils, 25 pupils, 25 pupils, 24 pupils and 27 pupils. Absolute parity in groups could not be achieved because of the necessity of placing candidates with the same score in the same group.

The item analysis follows:

SCHOOL A

ITEM 1

A	B	C	D*	E
0	1	0	17	4
0	1	1	14	3
0	2	0	13	12
1	4	1	4	5
2	2	2	3	10

p = ,50
D = +0,54

ITEM 2

A	B*	C	D	E
0	20	1	0	1
0	13	2	0	4
0	18	5	0	4
1	7	6	0	2
2	10	4	3	1

p = ,65
D = +0,33

ITEM 3

A	B	C	D*	E
2	2	1	17	0
1	1	0	17	0
3	2	2	20	0
2	1	2	11	0
3	4	2	10	1

p = ,72
D = +0,24

SCHOOL H

ITEM 1

A	B	C	D*	E
1	0	0	26	1
0	1	0	22	2
0	1	2	19	3
0	1	1	14	8
1	4	3	8	11

p = ,69
D = +0,47

ITEM 2

A	B*	C	D	E
2	25	1	0	0
1	21	2	0	1
0	21	1	0	3
3	9	5	1	6
3	9	7	1	7

p = ,66
D = +0,51

ITEM 3

A	B	C	D*	E
1	1	1	25	0
1	2	2	19	1
1	3	0	19	2
6	7	1	9	1
5	3	1	9	9

p = ,63
D = +0,47

SCHOOL A / SCHOOL H

ITEM 4

School A:

A	B	C*	D	E
0	2	17	3	0
0	2	17	0	0
1	2	19	6	0
0	3	7	2	4
0	5	4	6	5

p = ,61
D = +0,51

School H:

A	B	C*	D	E
0	0	24	4	0
0	0	14	8	3
0	2	15	7	0
0	2	11	6	3
4	3	10	6	4

p = ,59
D = +0,28

ITEM 5

School A:

A*	B	C	D	E
17	0	0	4	1
12	0	0	6	1
13	1	4	9	1
8	0	0	5	1
5	1	1	8	3

p = ,54
D = +0,28

School H:

A*	B	C	D	E
24	1	0	2	1
16	0	1	7	0
14	2	1	7	1
13	2	2	7	0
5	4	5	12	1

p = ,56
D = +0,40

ITEM 6

School A:

A	B*	C	D	E
3	13	1	3	2
3	8	3	2	1
6	9	8	2	1
2	2	3	6	1
4	5	4	4	1

p = ,38
D = +0,32

School H:

A	B*	C	D	E
2	20	1	4	1
2	16	3	2	2
5	12	2	5	1
2	16	1	2	3
3	9	6	8	1

p = ,56
D = +0,18

SCHOOL A

ITEM 7

A	B	C*	D	E
0	1	20	1	0
0	2	15	2	0
0	2	20	3	3
0	1	9	4	1
0	1	14	4	0

p = ,76
D = +0,19

SCHOOL H

A	B*	C	D	E
0	21	4	1	2
1	19	4	0	1
0	12	5	3	5
0	10	7	2	5
3	10	6	3	5

p = ,55
D = +0,36

ITEM 8

A	B	C	D*	E
5	0	0	16	1
5	0	1	12	1
14	0	0	13	1
8	0	1	6	1
10	0	0	8	2

p = ,52
D = +0,30

A	B	C	D*	E
3	0	2	23	0
6	1	2	16	0
4	1	3	16	1
4	1	5	9	4
13	3	2	7	2

p = ,55
D = +0,41

ITEM 9

A	B*	C	D	E
3	11	2	2	3
3	10	2	1	3
4	11	2	4	2
3	5	1	1	4
0	4	4	5	4

p = ,44
D = +0,23

A	B*	C	D	E
0	25	1	1	0
1	14	1	6	3
3	11	5	3	3
5	7	6	3	2
7	11	3	3	3

p = ,53
D = +0,39

SCHOOL A / SCHOOL H

ITEM 10

School A

A	B	C	D	E*
0	0	1	3	17
0	0	1	3	15
0	0	3	6	18
1	0	2	3	10
3	2	1	6	7

p = ,66
D = +0,31

School H

A	B*	C	D	E
2	22	1	0	3
2	19	1	0	3
6	13	0	0	6
7	7	2	0	8
4	6	5	1	11

p = ,52
D = +0,55

ITEM 11

School A

A	B*	C	D	E
1	13	4	1	3
4	10	2	0	3
0	15	5	4	2
1	6	3	1	5
4	5	4	1	3

p = ,49
D = +0,23

School H

A	B*	C	D	E
1	17	4	1	4
0	17	4	3	1
6	14	3	0	2
7	11	1	2	3
6	8	4	3	6

p = ,52
D = +0,28

ITEM 12

School A

A	B*	C	D	E
4	7	5	1	4
1	6	3	1	6
4	6	4	1	9
5	2	1	0	6
2	3	3	1	10

p = ,25
D = +0,19

School H

A	B*	C	D	E
3	16	1	4	2
2	12	2	3	5
1	18	1	2	3
2	10	1	4	7
0	9	3	3	12

p = ,51
D = +0,19

SCHOOL A

ITEM 13

A*	B	C	D	E
19	2	0	0	1
12	2	2	1	2
10	3	4	2	8
6	4	1	2	3
5	5	3	2	4

p = ,50

D = +0,43

ITEM 14

A	B	C*	D	E
2	1	18	1	0
3	1	13	2	0
4	2	15	1	4
7	0	5	2	1
10	3	3	2	2

p = ,53

D = +0,51

ITEM 15

A	B	C	D	E*
0	2	2	1	17
5	0	1	4	8
10	4	0	2	9
5	2	1	1	7
1	2	5	3	7

p = ,48

D = +0,19

SCHOOL H

ITEM 13

A*	B	C	D	E
15	3	7	2	1
15	4	3	0	2
14	1	3	4	3
11	1	3	4	5
10	7	2	4	4

p = ,51

D = +0,17

ITEM 14

A	B	C*	D	E
1	1	20	3	2
4	2	16	2	1
2	2	13	5	3
10	1	11	1	1
6	7	4	4	6

p = ,50

D = +0,39

ITEM 15

A	B	C	D	E*
0	0	1	3	24
6	1	1	2	15
5	1	3	5	11
7	1	5	1	10
10	2	5	5	5

p = ,50

D = +0,43

SCHOOL A | SCHOOL H

ITEM 16

School A:

A	B*	C	D	E
0	18	1	1	2
0	7	0	2	9
2	13	1	3	9
0	10	0	2	4
1	9	1	4	5

p = ,55
D = +0,07

School H:

A	B*	C	D	E
0	27	0	0	1
0	12	0	0	13
0	11	0	1	13
1	7	0	1	15
1	3	0	0	23

p = ,47
D = +0,52

ITEM 17

School A:

A	B	C*	D	E
1	1	15	4	1
1	1	11	3	3
0	11	6	5	6
0	2	5	4	5
1	3	1	6	9

p = ,36
D = +0,45

School H:

A	B	C*	D	E
1	2	21	2	2
2	7	12	2	2
0	10	11	1	3
0	7	7	5	5
2	4	6	8	7

p = ,45
D = +0,36

ITEM 18

School A:

A	B	C	D	E*
0	3	2	1	16
1	6	2	2	7
1	10	5	7	3
1	7	3	3	1
0	6	6	6	1

p = ,28
D = +0,50

School H:

A	B	C	D	E*
0	6	0	0	22
1	7	0	0	17
0	15	2	0	8
1	9	5	2	7
4	9	7	3	4

p = ,45
D = +0,52

SCHOOL A — SCHOOL H

ITEM 19

School A:

A	B	C	D	E*
1	7	3	0	11
2	3	2	3	9
6	9	2	3	7
5	3	2	2	4
4	7	3	1	4

p = ,34
D = +0,25

School H:

A	B	C	D	E*
1	4	5	1	17
3	3	3	2	14
4	4	1	0	16
4	8	2	6	4
7	5	7	2	6

p = ,45
D = +0,39

ITEM 20

School A:

A	B	C	D*	E
3	2	3	11	2
1	2	8	3	4
4	3	3	8	10
2	2	2	5	5
3	2	6	2	7

p = ,28
D = +0,14

School H:

A	B	C	D*	E
1	4	2	19	2
0	2	4	13	6
4	5	1	11	4
4	5	4	7	4
5	6	7	6	3

p = ,44
D = +0,35

ITEM 21

School A:

A*	B	C	D	E
14	3	0	4	1
8	4	2	3	1
11	1	3	10	2
3	1	5	5	1
4	2	5	6	1

p = ,40
D = +0,33

School H:

A*	B	C	D	E
19	5	1	3	0
12	5	3	5	0
10	5	1	7	2
10	3	2	6	3
4	8	6	6	3

p = ,43
D = +0,30

SCHOOL A

ITEM 22

A	B*	C	D	E
4	13	2	2	O
6	6	2	3	1
11	8	4	3	O
7	5	3	O	1
12	2	3	2	1

p = ,34
D = +O,27

ITEM 23

A*	B	C	D	E
19	1	2	O	O
10	2	6	1	O
12	3	12	1	O
5	3	8	O	O
4	10	6	O	O

p = ,48
D = +O,44

ITEM 24

A	B	C*	D	E
O	O	22	O	O
O	1	18	O	O
1	O	23	4	O
O	O	12	2	1
O	2	12	4	2

p = ,84
D = +O,28

SCHOOL H

ITEM 22

A	B*	C	D	E
5	20	1	O	2
7	14	O	2	2
9	7	5	4	O
9	6	6	2	1
13	7	7	O	O

p = ,41
D = +O,38

ITEM 23

A*	B	C	D	E
24	O	3	O	O
14	2	7	1	1
6	3	13	2	1
3	5	15	O	1
4	14	8	1	O

p = ,40
D = +O,59

ITEM 24

A	B	C*	D	E
3	6	15	4	O
2	4	15	4	O
O	5	13	4	2
2	6	5	9	2
6	8	4	7	2

p = ,40
D = +O,39

SCHOOL A / SCHOOL H

ITEM 25

SCHOOL A

A	B	C	D*	E
1	0	3	13	4
2	0	5	6	5
2	2	6	7	9
1	1	8	1	5
5	0	7	1	7

p = ,28
D = +0,42

SCHOOL H

A	B	C	D*	E
2	1	3	18	4
2	3	5	12	3
2	3	8	10	2
1	3	10	6	4
0	5	13	5	4

p = ,40
D = +0,34

ITEM 26

SCHOOL A

A	B	C	D	E*
3	2	1	3	13
2	5	1	1	8
0	3	10	7	7
1	1	6	3	4
2	6	5	4	3

p = ,35
D = +0,32

SCHOOL H

A	B	C	D	E*
4	3	1	3	16
4	3	3	2	13
5	3	6	2	9
5	1	3	7	8
3	5	6	8	5

p = ,39
D = +0,30

ITEM 27

SCHOOL A

A	B	C	D	E*
1	2	0	3	16
1	4	0	4	8
2	4	3	6	11
1	11	0	2	2
3	8	2	5	2

p = ,39
D = +0,49

SCHOOL H

A	B	C	D	E*
0	6	0	2	20
0	12	0	0	13
1	14	0	3	7
0	10	1	7	6
3	9	1	11	3

p = ,38
D = +0,44

ITEM 28

SCHOOL A

A*	B	C	D	E
11	1	7	0	3
5	1	5	2	3
6	4	13	1	4
2	3	3	3	4
1	0	6	5	7

p = ,25
D = +0,32

SCHOOL H

A*	B	C	D	E
18	0	4	1	5
14	1	5	1	4
7	5	5	4	4
4	1	11	3	5
4	3	10	5	5

p = ,37
D = +0,44

ITEM 29

SCHOOL A

A*	B	C	D	E
7	2	9	1	3
3	2	6	1	4
4	7	8	4	1
1	2	7	5	0
0	5	9	2	4

p = ,15
D = +0,22

SCHOOL H

A*	B	C	D	E
18	2	4	2	2
10	4	5	2	4
6	4	10	4	1
8	3	9	2	2
2	6	12	2	5

p = ,34
D = +0,32

ITEM 30

SCHOOL A

A	B	C*	D	E
6	3	6	5	2
2	4	3	2	6
14	3	3	1	7
7	2	0	3	3
6	3	0	4	6

p = ,12
D = +0,23

SCHOOL H

A	B	C*	D	E
0	6	20	1	1
3	5	10	2	5
9	2	7	4	3
8	5	3	2	6
8	5	3	0	11

p = ,33
D = +0,44

SCHOOL A

ITEM 31

A	B	C*	D	E
1	5	4	8	3
4	3	2	2	1
6	2	3	7	6
2	4	3	O	5
2	3	2	8	4

p = ,16
D = +0,03

SCHOOL H

A	B	C*	D	E
3	6	13	3	2
3	3	14	4	O
1	8	7	5	4
4	3	7	5	5
7	6	1	6	7

p = ,33
D = +0,37

ITEM 32

SCHOOL A

A*	B	C	D	E
5	1	8	2	2
6	O	5	1	3
7	4	12	2	O
1	O	8	3	2
1	2	9	4	3

p = ,22
D = +0,28

SCHOOL H

A*	B	C	D	E
15	O	10	3	O
11	1	9	1	3
8	1	8	4	4
4	1	6	6	7
2	3	9	8	5

p = ,31
D = +0,37

ITEM 33

SCHOOL A

A	B	C	D*	E
2	1	1	12	5
1	2	1	4	7
7	3	2	3	11
3	O	1	2	8
8	2	O	1	8

p = ,23
D = +0,33

SCHOOL H

A	B	C	D*	E
2	1	1	20	4
O	4	1	6	14
6	3	1	8	7
6	3	1	4	10
11	3	O	1	12

p = ,30
D = +0,37

SCHOOL A / SCHOOL H

ITEM 34

School A:

A*	B	C	D	E
11	5	6	0	0
2	3	9	1	0
5	2	18	2	0
2	4	7	1	0
0	7	12	0	0

p = ,21
D = +0,25

School H:

A*	B	C	D	E
20	4	3	0	1
7	6	12	0	0
3	8	13	0	1
2	6	16	0	0
6	4	17	0	0

p = ,30
D = +0,35

ITEM 35

School A:

A	B*	C	D	E
0	16	5	0	1
1	5	10	0	0
3	8	11	2	2
0	2	6	3	3
3	1	12	2	1

p = ,33
D = +0,43

School H:

A	B*	C	D	E
0	17	8	1	1
0	6	15	1	3
0	5	14	3	3
3	4	10	1	6
3	1	12	7	4

p = ,26
D = +0,33

Item difficulty levels were calculated as before. Naturally, the p-levels obtained from the boys' scores were virtually unchanged from the original analysis; however, the girls reacted quite differently to some items and some of the levels of difficulty obtained from their scores differ greatly from the p-levels in the original analysis.

TABLE 10: ITEM DIFFICULTY LEVELS, p, AND ITEMS RANKED ACCORDING TO DIFFICULTY VALUES, FOR TWO SCHOOLS, A AND H

ITEM	SCHOOL A p	SCHOOL H p	SCHOOL A Rank	SCHOOL H Rank
1	0,50	0,69	24,5	35
2	0,65	0,66	31	34
3	0,72	0,63	33	33
4	0,61	0,59	30	32
5	0,54	0,56	28	30,5
6	0,38	0,56	17	30,5
7	0,76	0,55	34	28,5
8	0,52	0,55	26	28,5
9	0,44	0,53	20	27
10	0,66	0,52	32	25,5
11	0,49	0,52	23	25,5
12	0,25	0,51	7,5	23,5
13	0,50	0,51	24,5	23,5
14	0,53	0,50	27	21,5
15	0,48	0,50	21,5	21,5
16	0,55	0,47	29	20
17	0,36	0,45	16	18
18	0,28	0,45	10	18
19	0,34	0,45	13,5	18
20	0,28	0,44	10	16
21	0,40	0,43	19	15
22	0,34	0,41	13,5	14
23	0,48	0,40	21,5	12
24	0,84	0,40	35	12
25	0,28	0,40	10	12
26	0,35	0,39	15	10
27	0,39	0,38	18	9
28	0,25	0,37	7,5	8
29	0,15	0,34	2	7
30	0,12	0,33	1	5,5
31	0,16	0,33	3	5,5
32	0,22	0,31	5	4
33	0,23	0,30	6	2,5
34	0,21	0,30	4	2,5
35	0,33	0,26	12	1

One would hardly expect difficulty values to be exactly the same for two different groups, but there does seem to be a fair degree of correspondence between them. A measure of this correspondence is given by the Rank-Difference Correlation Coefficient, which was calculated to be:

$$\text{rho} = +,74$$

There are eleven items for which the p-levels of the two groups are very different, and the nature of these items will be discussed in due course. The girls of school A found some of the items to be much easier than did the boys, but on the whole the test presented greater difficulty to the girls. The frequency distributions below reveal the greater spread of item difficulty levels for school A as compared with school H.

<u>Figure 4</u>

DISTRIBUTION OF REVISED TEST ITEM DIFFICULTY LEVELS, p, FOR THE GIRLS OF SCHOOL A

< ,16	**	2
,16 - ,25	******	6
,26 - ,35	*******	7
,36 - ,45	*****	5
,46 - ,55	*********	9
,56 - ,65	**	2
,66 - ,75	**	2
,76 - ,85	**	2
> ,85		

Mean = 0,42   Standard Deviation = 0,18

<u>Figure 5</u>

DISTRIBUTION OF REVISED TEST ITEM DIFFICULTY LEVELS, p, FOR THE BOYS OF SCHOOL H

< ,16		
,16 - ,25		
,26 - ,35	*******	7
,36 - ,45	************	12
,46 - ,55	**********	10
,56 - ,65	****	4
,66 - ,75	**	2
,76 - ,85		
> ,85		

Mean = 0,46   Standard Deviation = 0,11

Three of the items that the girls found considerably easier than did the boys were numbers 7, 10 and 24. Both item 7 and item 10 are straightforward manipulative techniques taught and drilled in standard seven, viz., algebraic long division and subtraction; once the pupil knows the technique or has established the rule and can work through the process with precision, these items should present no problem. It has been the writer's experience that girls, especially, feel very secure with this level of Mathematics, most of them take pleasure in completing such exercises and performing them perfectly. It is possible that boys have less patience with this type of procedure and, in haste, make mistakes. It may be that the girls' love of precision helped them through item 24 too, while the boys seemed to be quite confused about this question:

$$\text{If } 5^2 - 4^2 = \sqrt[x]{81}, \text{ then } x = \ldots$$

Some of the boys seemed reluctant to choose the answer x = 2 on the grounds that when symbolising a square root it is common practice to omit the two, whereas with cube roots, fourth roots, etc., the order of the surd must be given. Many of the boys, therefore, felt that x = 2 could not be the correct response. Girls, on the other hand, frequently show a little chagrin with the practice of omitting certain <u>understood</u> numbers in algebraic notation. It is quite common to see the younger girls cling to notations such as 1x instead of x, or, even $1x^1$ for x. Similarly, with surds, standard six and seven girls often prefer square roots and cube roots to be reinforced verbally as "two'th" and "three'th" roots.

The items that the girls found considerably more difficult than did the boys were items 1, 6, 12, 18, 20, 29, 30 and 31. Of these eight items, four involve geometry, viz., items 1, 6, 30 and 31; three involve estimation or working with numbers in an abstract way, viz., items 12, 20 and 29; and, item 18 asks the candidates to visualise the construction of a wire framework. As far as the geometrical items and item 18 are concerned, the superiority of the boys is due, probably, to a more highly developed spatial ability - boys tend to make things to a far greater extent than girls. The estimation items may have given the girls more trouble because, once again, they tend to prefer to know precisely with which numbers they are dealing.

These results tend to be in line with Krutetskii's (1976) generalisation when summing up various studies of sex differences in mathematical ability:

> ..., boys excel in ability for logical reasoning, and girls excel in precision, rigour, accuracy, a kind of "punctiliousness" of thought.

In addition to providing information about the difficulty of the items, the analysis also yields data concerning the discriminating power of each item for the various quintile groups. By scanning the pattern of correct responses, it was hoped that certain trends might become evident that would assist in the choosing of the most effective discriminators among the items. These would be items which showed a decreasing correct response tally from the top group to the lower groups. It was hoped that certain items would show themselves to be good discriminators for both groups of pupils, and that by studying the nature of these items it might be possible to concentrate on these in the new edition of the test, thereby facilitating the classification of pupils' scores more accurately.

A visual inspection of the correct response columns was made, but this did not prove to be very fruitful;

all that was revealed was that most of the items seemed to be working quite well, and this was not really surprising as the split of pupils into five groups had been based on their scores on this same test. It was decided that, in order to dispel vagueness, for each item an index of discrimination should be calculated, i.e., that there should be some quantitative way of expressing the extent to which a test item separates the able students from the less able. The index of discrimination selected for this purpose was D, which was calculated using the formula

$$D = (U - L) \div \tfrac{1}{2}N$$

where U represents the number of correct responses of the top scoring 40% of candidates, L the number of correct responses of the least scoring 40% of candidates and N the number of candidates involved.

In effect, D is simply the difference between the ratios of correct responses from an agreed upon upper and lower group. It follows that the larger the index D, the greater is the efficiency of the item as a discriminator.

The table below lists the index of discrimination for each item:

TABLE 11: COMPARISON OF THE INDICES OF DISCRIMINATION OF THE REVISED TEST ITEMS AS CALCULATED FOR SCHOOLS A AND H

ITEM	SCHOOL A D	SCHOOL H D
1	+0,54	+0,47
2	+0,33	+0,51
3	+0,24	+0,47
4	+0,51	+0,28
5	+0,28	+0,40
6	+0,32	+0,18
7	+0,19	+0,36
8	+0,30	+0,41
9	+0,23	+0,39
10	+0,31	+0,55
11	+0,23	+0,28
12	+0,19	+0,19
13	+0,43	+0,17
14	+0,51	+0,39
15	+0,19	+0,43
16	+0,07	+0,52
17	+0,45	+0,36
18	+0,50	+0,52
19	+0,25	+0,39
20	+0,14	+0,35
21	+0,33	+0,30
22	+0,27	+0,38
23	+0,44	+0,59
24	+0,28	+0,39
25	+0,42	+0,34
26	+0,32	+0,30
27	+0,49	+0,44
28	+0,32	+0,44
29	+0,22	+0,32
30	+0,23	+0,44
31	+0,03	+0,37
32	+0,28	+0,37
33	+0,33	+0,37
34	+0,25	+0,35
35	+0,43	+0,33

It is clear that these items are much more powerful discriminators for the boys than they are for the girls, especially if one considers the following statistics:

TABLE 12: COMPARISON OF EXTREME AND MEDIAN INDICES OF DISCRIMINATION IN TWO SCHOOLS

	HIGHEST D	LOWEST D	MEDIAN D
SCHOOL A	+0,54	+0,03	+0,28
SCHOOL H	+0,59	+0,17	+0,37

The extent to which these D-indices would rank the items, similarly, as discriminators does not appear to be very great. Item number 16 is one of the most powerful discriminators for the boys, but one of the weakest for the girls; similarly, item number 13 works in reverse order for boys and girls. The D-indices of items 1 and 18 are noticeably in agreement. A full list of ranked indices of discrimination is given in Table 16.

Still in pursuit of information that might indicate which items would differentiate in the same way for everyone, it was decided that a second set of indices should be calculated, using the candidates' mid-year school examination scores to divide them into groups; these two sets could then be compared and trends sought out. Furthermore, since separating the sexes had produced no obvious patterns, it was decided to repeat the calculations for schools D and E, where the results would include both boys and girls.

At this stage, it was no longer possible to deal, simultaneously, with all pupils taking Mathematics, because, from this point on, Higher Grade candidates and Standard Grade candidates would write different examination papers. All further analyses, therefore, will be based on the results of the Higher Grade students only.

As soon as the school examination results were available, the calculations of the new indices of discrimination were completed in the same way, except that the upper and lower groups represent the top 33% and lowest 33% of pupil scores. The results are summarised in the tables that follow:

TABLE 13: COMPARISON OF INDICES OF DISCRIMINATION OF REVISED TEST ITEMS FOR SCHOOLS A AND H, BASED ON TWO DIFFERENT UPPER AND LOWER GROUPINGS OBTAINED BY USING THE REVISED TEST SCORES AND THE SCHOOL EXAMINATION SCORES

	REVISED TEST GROUPED		MID-YEAR TEST GROUPED	
ITEM	SCHOOL A	SCHOOL H	SCHOOL A	SCHOOL H
1	+0,46	+0,46	+0,42	+0,31
2	+0,29	+0,51	+0,21	+0,43
3	+0,13	+0,49	+0,13	+0,29
4	+0,33	+0,29	+0,21	+0,09
5	+0,33	+0,34	+0,17	+0,29
6	+0,25	+0,29	+0,17	+0,23
7	+0,17	+0,34	+0,04	+0,09
8	+0,29	+0,43	+0,25	+0,29
9	+0,29	+0,37	+0,13	+0,26
10	+0,13	+0,49	+0,21	+0,29
11	+0,00	+0,34	+0,00	+0,11
12	+0,17	+0,11	+0,08	+0,14
13	+0,50	+0,20	+0,04	+0,17
14	+0,33	+0,29	+0,33	+0,26
15	+0,38	+0,51	+0,25	+0,31
16	+0,21	+0,71	-0,04	+0,43
17	+0,42	+0,40	+0,29	+0,31
18	+0,67	+0,57	+0,54	+0,43
19	+0,29	+0,51	+0,13	+0,40
20	+0,25	+0,34	+0,04	+0,09
21	+0,54	+0,26	+0,21	+0,20
22	+0,33	+0,51	+0,08	+0,46
23	+0,33	+0,60	+0,38	+0,66
24	+0,29	+0,31	+0,08	+0,46
25	+0,38	+0,26	+0,25	+0,23
26	+0,33	+0,29	+0,21	+0,11
27	+0,54	+0,40	+0,21	+0,34
28	+0,38	+0,46	+0,33	+0,37
29	+0,25	+0,31	+0,04	+0,34
30	+0,21	+0,51	+0,29	+0,34
31	-0,04	+0,23	-0,13	+0,26
32	+0,21	+0,43	+0,00	+0,20
33	+0,46	+0,57	+0,46	+0,49
34	+0,29	+0,46	+0,29	+0,17
35	+0,46	+0,37	+0,33	+0,40
Highest D	+0,67	+0,71	+0,54	+0,66
Lowest D	-0,04	+0,11	-0,13	+0,09
Median D	+0,29	+0,37	+0,21	+0,29

TABLE 14: COMPARISON OF INDICES OF DISCRIMINATION OF REVISED TEST ITEMS FOR SCHOOLS D AND E, BASED ON TWO DIFFERENT UPPER AND LOWER GROUPINGS OBTAINED BY USING THE REVISED TEST SCORES AND THE MID-YEAR EXAMINATION SCORES

ITEM	REVISED TEST GROUPED		MID-YEAR TEST GROUPED	
	SCHOOL D	SCHOOL E	SCHOOL D	SCHOOL E
1	+0,19	+0,29	+0,37	+0,31
2	+0,24	+0,20	+0,21	+0,11
3	-0,05	+0,26	+0,00	+0,31
4	+0,33	+0,29	+0,32	+0,11
5	+0,43	+0,26	-0,11	-0,08
6	+0,05	+0,39	-0,42	+0,26
7	+0,10	+0,23	-0,11	+0,36
8	+0,19	+0,31	0,00	+0,19
9	+0,33	+0,12	0,00	+0,09
10	+0,14	+0,43	+0,11	+0,37
11	+0,24	+0,49	-0,21	+0,22
12	+0,19	+0,57	0,00	+0,15
13	+0,43	+0,26	+0,05	+0,03
14	+0,24	+0,46	+0,32	+0,25
15	+0,38	+0,35	+0,26	+0,25
16	+0,33	-0,03	+0,21	+0,03
17	+0,19	+0,38	-0,05	+0,23
18	+0,67	+0,29	+0,26	+0,31
19	+0,10	+0,35	+0,05	+0,22
20	+0,33	+0,40	+0,11	+0,03
21	+0,43	+0,37	+0,26	+0,11
22	+0,38	+0,12	+0,42	+0,14
23	+0,33	+0,14	+0,11	+0,25
24	+0,14	+0,32	+0,05	+0,25
25	+0,38	+0,36	+0,16	+0,06
26	+0,33	+0,26	+0,11	+0,03
27	+0,43	+0,46	+0,58	+0,08
28	+0,29	+0,31	+0,05	+0,17
29	+0,48	+0,30	+0,16	+0,03
30	+0,29	+0,06	+0,11	+0,14
31	+0,05	+0,21	-0,11	+0,12
32	+0,43	+0,25	+0,32	+0,36
33	+0,00	+0,22	+0,05	+0,03
34	+0,19	+0,18	+0,16	+0,06
35	+0,49	+0,22	+0,21	+0,03
Highest D	+0,67	+0,57	+0,58	+0,37
Lowest D	-0,05	-0,03	-0,21	-0,08
Median D	+0,29	+0,29	+0,11	+0,14

The degree of correspondence between the two different groupings for each school was calculated using the Rank-Difference Correlation Coefficient (ranks are shown in Tables 16 and 17), and the results are as follows:

TABLE 15: RANK-DIFFERENCE CORRELATION OF THE INDICES OF DISCRIMINATION OF REVISED TEST ITEMS BASED ON REVISED TEST SCORES AND THOSE BASED ON THE MID-YEAR EXAMINATION SCORES

SCHOOL	N	rho
A	74	+0,64
H	107	+0,69
D	56	+0,55
E	101	+0,25

Although the discriminating power of items was generally reduced when grouping was carried out according to the mid-year school examination results, it seems that, with the exception of school E, there is, nevertheless, a fairly strong positive correlation between the two sets of D-indices. This is particularly noticeable in schools A and H.

In order to obtain a measure of the extent to which these four schools agree on the merit order of the test items as discriminators, the items were ranked and Kendall's Coefficient of Concordance, W was calculated.

The Tables 16 and 17 show the ranking of items for the four schools, first, using upper and lower groups based on the multiple-choice test; and secondly, upper and lower groups according to the May school examination.

TABLE 16: REVISED TEST ITEMS RANKED AS DISCRIMINATORS ACCORDING TO THE REVISED MULTIPLE-CHOICE TEST GROUPING OF SCORES

ITEM	H	A	D	E
1	23	30	11	18
2	29	15,5	15	7
3	25,5	3,5	1	14,5
4	7,5	21,5	21,5	18
5	13,5	21,5	30	14,5
6	7,5	11	3,5	29
7	13,5	5,5	5,5	11
8	20,5	15,5	11	21,5
9	16,5	15,5	21,5	3,5
10	25,5	3,5	7,5	31
11	13,5	2	15	34
12	1	5,5	11	35
13	2	32	30	14,5
14	7,5	21,5	15	32,5
15	29	26	26	24,5
16	35	8	21,5	1
17	18,5	28	11	28
18	32,5	35	35	18
19	29	15,5	5,5	24,5
20	13,5	11	21,5	30
21	4,5	33,5	30	27
22	29	21,5	26	3,5
23	34	21,5	21,5	5
24	10,5	15,5	7,5	23
25	4,5	26	26	26
26	7,5	21,5	21,5	14,5
27	18,5	33,5	30	32,5
28	23	26	17,5	21,5
29	10,5	11	33	20
30	29	8	17,5	2
31	3	1	3,5	8
32	20,5	8	30	12
33	32,5	30	2	9,5
34	23	15,5	11	6
35	16,5	30	34	9,5

Kendall's Coefficient of Concordance, W = 0,25
Critical values of W depend both on m, the number of sets of ranks, and on N, the number of ranks in each set. For $N > 7$, a Chi-square test may be applied, where

$$\text{Chi-square} = m(N - 1)W$$

This has a Chi-square distribution with N-1 degrees of freedom.

Thus, Chi-square = 4(35 - 1) 0,25

i.e., Chi-square = 34 with 34 degrees of freedom.

Tables of Chi-square yield

$$P > 0,05$$

i.e., if this experiment were repeated 100 times, these results could occur more than 5 times by chance alone.

There is no statistically significant association.

TABLE 17: REVISED TEST ITEMS RANKED AS DISCRIMINATORS ACCORDING TO THE MID-YEAR SCHOOL EXAMINATION GROUPING OF SCORES

ITEM	H	A	D	E
1	21	33	33	31
2	30	19,5	25	14
3	17,5	13	8,5	31
4	2	19,5	31	14
5	17,5	15,5	4	1
6	11,5	15,5	1	29
7	2	6,5	4	33,5
8	17,5	24	8,5	21
9	14	13	8,5	12
10	17,5	19,5	18	35
11	4,5	3,5	2	22,5
12	6	10	8,5	19
13	7,5	6,5	13	5
14	14	30	31	26,5
15	21	24	28	26,5
16	30	2	25	5
17	21	27	6	24
18	30	35	28	31
19	27,5	13	13	22,5
20	2	6,5	18	5
21	9,5	19,5	28	14
22	32,5	10	34	17,5
23	35	32	18	26,5
24	32,5	10	13	26,5
25	11,5	24	22	9,5
26	4,5	19,5	18	5
27	24	19,5	35	11
28	26	30	13	20
29	24	6,5	22	5
30	24	27	18	17,5
31	14	1	4	16
32	9,5	3,5	31	33,5
33	34	34	13	5
34	7,5	27	22	9,5
35	27,5	30	25	5

Kendall's Coefficient of Concordance, W = 0,37

Thus, Chi-square = 50,43 with 34 degrees of freedom.

There is statistically significant association at the 0,05 level.

Note: For both Tables 16 and 17, a high rank indicates a powerful discriminator, while a low rank indicates a weak discriminator.

It seems the association that exists is very marginal. Even a brief examination of the rankings shows that some items fluctuate wildly, in their discriminating power, from school to school, and even between different groupings in the same school. However, no matter how marginal, a noticeable degree of correspondence was recorded. Consequently, an attempt was made, roughly, to classify items as "consistently good", "consistently fair", "consistently poor" and "fluctuating" discriminators. Items were further classified according to subject material, viz., Algebra, Geometry, Algebra/Geometry or Other; the mental process involved, viz., Manipulation, Routine problem, Comprehension and Non-routine problem; and, the level of difficulty recorded (on average), viz., Fairly easy, Middle difficulty, More difficult or Very difficult.

In the lists that follow, an attempt has been made to place items in some sort of merit order. The figures given are the levels of difficulty recorded for schools A, H, D and E respectively.

1. "Consistently Good" Discriminators:

Item 18: Algebra/Geometry Non-routine problem
Middle difficulty (,35; ,54; ,52; ,39)

Item 1: Algebra/Geometry Routine problem
Fairly easy (,52; ,69; ,57; ,68)

Item 23:	Algebra	Manipulation
	Middle difficulty	(,58; ,48; ,69; ,39)
Item 15:	Geometry	Comprehension
	Middle difficulty	(,54; ,54; ,48; ,61)
Item 22:	Algebra	Routine problem
	Middle difficulty	(,43; ,46; ,43; ,35)
Item 27:	Algebra	Comprehension
	Middle difficulty	(,56; ,40; ,64; ,54)
Item 28:	Geometry	Non-routine problem
	More difficult	(,28; ,43; ,29; ,31)
Item 35:	Algebra	Manipulation
	More difficult	(,46; ,33; ,34; ,14)

2. "Consistently Fair" Discriminators:

Item 33:	Geometry	Non-routine problem
	Very difficult	(,34; ,37; ,10; ,17)
Item 32:	Algebra	Manipulation
	More difficult	(,22; ,31; ,40; ,35)
Item 19:	Geometry	Non-routine problem
	More difficult	(,36; ,43; ,29; ,38)
Item 8:	Algebra	Routine problem
	Fairly easy	(,50; ,59; ,71; ,59)
Item 4:	Algebra	Manipulation
	Fairly easy	(,61; ,59; ,79; ,74)
Item 16:	Geometry	Non-routine problem
	Middle difficulty	(,55; ,47; ,45; ,44)
Item 29:	Algebra	Comprehension
	More difficult	(,17; ,44; ,33; ,24)

3. "Consistently Poor" Discriminators:

Item 6:	Geometry	Comprehension
	Middle difficulty	(,44; ,54; ,45; ,61)
Item 7:	Algebra	Manipulation
	Fairly easy	(,79; ,60; ,86; ,71)
Item 12:	Arithmetic	Non-routine problem
	More difficult	(,28; ,50; ,24; ,34)
Item 31:	Geometry	Non-routine problem
	Very difficult	(,24; ,32; ,22; ,20)

4. "Fluctuating" Discriminators:

Item 14:	Algebra	Manipulation
	Middle difficulty	(,53; ,50; ,74; ,49)
Item 10:	Algebra	Manipulation
	Fairly easy	(,70; ,56; ,93; ,68)
Item 2:	Geometry	Comprehension
	Middle difficulty	(,38; ,38; ,55; ,70)
Item 21:	Geometry	Comprehension
	Middle difficulty	(,40; ,43; ,40; ,44)
Item 17:	Algebra	Comprehension
	Middle difficulty	(,46; ,46; ,38; ,39)
Item 25:	Algebra	Non-routine problem
	More difficult	(,36; ,41; ,33; ,27)
Item 24:	Algebra	Comprehension
	Middle difficulty	(,84; ,40; ,12; ,74)
Item 3:	Algebra	Manipulation
	Fairly easy	(,73; ,64; ,74; ,57)

Item 34:	Geometry	Non-routine problem
	Very difficult	(,21; ,30; ,29; ,09)
Item 9:	Numbers	Comprehension
	Fairly easy	(,45; ,60; ,74; ,71)
Item 26:	Algebra	Routine problem
	More difficult	(,35; ,39; ,45; ,28)
Item 11:	Numbers	Non-routine problem
	Middle difficulty	(,57; ,46; ,50; ,48)
Item 20:	Algebra	Comprehension
	More difficult	(,43; ,49; ,31; ,37)
Item 30:	Geometry	Routine problem
	Very difficult	(,12; ,33; ,14; ,12)
Item 5:	Algebra/Geometry	Routine problem
	Middle difficulty	(,45; ,60; ,60; ,53)
Item 13:	Graph	Comprehension
	Middle difficulty	(,60; ,47; ,45; ,54)

Summary

1. By subject material:

	Algebra	Geometry	Alg/Geo	Other
"Good"	4	2	2	-
"Fair"	4	3	-	-
"Poor"	1	2	-	1
"Fluctuating"	8	4	1	3

2. By mental process:

	Manipulation	Routine	Comprehension	Non-Routine
"Good"	2	2	2	2
"Fair"	2	1	1	3
"Poor"	1	-	1	2
"Fluctuating"	3	3	7	3

3. By level of difficulty:

	Easy	Middle difficulty	More difficult	Very difficult
"Good"	1	5	2	-
"Fair"	2	1	3	1
"Poor"	1	1	1	1
"Fluctuating"	3	8	3	2

In general, there seem to be no visible trends as far as subject material and mental process are concerned. Items which required a combination of algebraic and geometric thought appear to do better than the others, but there are not sufficient of these items to be sure that this did not occur by chance. The mental processes seem to be well represented at all levels of discrimination but it is possible that this is a reflection of the difficulties that were experienced in classifying questions according to mental processes, i.e., that for groups of pupils taught in different schools, or, even, in the same school but by different teachers, the levels of mental process may vary depending upon whether or not the item is familiar, or, the extent to which this type of item has been emphasized and drilled by the teacher. This area of difficulty was discussed earlier, when the past examination papers were analysed, and it is obvious that it may be a restricting factor in the present analysis as well. In order to discover the existence of trends among the different types of mental process, it would be necessary for each individual teacher to reclassify questions in the light of her own teaching. Even if this were feasible, it would not provide the perfect classification because of the individual pupil perceptions of each item, which, as was mentioned before, can also vary to a marked extent, even among pupils taught in the same class.

The middle difficulty items, as expected, are obviously the best represented among the better discriminating items. In the construction of the new test, this was kept in mind and a further determined effort made to centralise difficulty levels as some of these still seemed to be more extreme than desired. The last few items of the revised test tended to be more difficult than was expected, and this may have been due to the time factor. These items would have to be carefully considered.

Before proceeding to the construction of the third and last multiple-choice test, the "reasoning" test, which was administered along with the revised test must be discussed. The naming of this test as the Reasoning Test was a simple convenience in order to distinguish it from the other multiple-choice tests.

4.4 THE REASONING TEST

After writing a Higher Grade Mathematics examination, and subsequently discussing the solutions with their teachers, pupils frequently complain that their problems more often stem from not knowing what a particular question is "getting at", rather than being able to complete the solution. Among a certain number of these pupils, this may indicate an immature reading ability; others may have an inadequate comprehension of the work and try to depend too heavily on recognising a particular "recipe" solution; however, too many pupils make this kind of comment to explain it away on the grounds of reading ability or lack of basic understanding. It is possible that finding the solution to a non-routine mathematical problem requires some special quality of thought from the candidate. Mathematical statements are always extremely concise, but some problems are posed using many words and the quantity of information seems overpowering at the first reading; other problems are so concisely stated, in so few words, that there seems to be no information to work on at all; yet other problems present a combination of diagrammatic and verbal information. The pupil who is able to sift through questions and discover their essence quickly and successfully, must be at a considerable advantage over those who, for one or other reason, cannot do this. For this reason, it was decided that a short test, consisting of non-routine items, involving all of the above-mentioned methods of presenting information, would be constructed and administered together with the multiple-choice test described in the previous section.

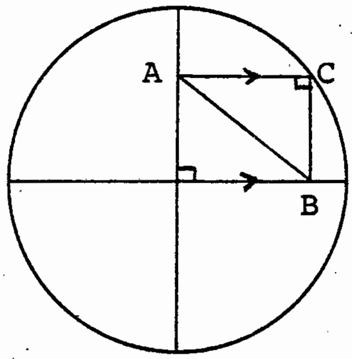
Example 1: 32 men are employed to do a piece of work, and, after 15 days have completed four-ninths of the work. 12 of the men are then drafted to another job, and the work proceeds for 8 days with only 20 men. How many extra men must then be employed to finish the whole job in 10 days more?

This problem involves a great deal of verbal information and requires careful reading before the solution may be attempted. The solution is fairly complex and requires that the candidate determine what part of the work has been completed after the initial 23 days, but also that he can devise some sort of unit of measurement for the job - probably the "man-day". Additional hazards to be negotiated in the text are words such as "<u>extra</u> men" and "in 10 days <u>more</u>".

Example 2: Find the word in brackets:
53(DICE)94
54(....)16

In this item the verbal information is extremely limited. The solution of this problem depends only on the candidate's ability to make the correct connections between letters and numbers in contrast to the previous problem where the ability to read and comprehend was very important.

Example 3: If the circle has diameter 10cm., how long is AB?



This item presents a combination of verbal and diagramatic information. Neither the statement nor the sketch is sufficient on its own for the solution - the candidate is forced to use both to arrive at an answer.

This test would not be in the multiple-choice format, but would offer the candidates an opportunity to show their thoughts and working along with their answers. A copy of this test is tabled separately in Appendix C.

The duration of this ten-item test was thirty minutes, and the questions were unlikely to have been drilled, or even encountered in the classroom. This test of reasoning, then, presented the candidates with quite a considerable challenge. Participating schools, i.e., schools A, B, C, D and E, were informed that this test need not be written on the same day as the multiple-choice test, if it was difficult to set aside the necessary time all on one day. In fact, however, all schools adhered to the suggestion that the two tests be written consecutively with a break of ten to fifteen minutes between them. Candidates were instructed to show answers and relevant working on the question papers, and, then, these were returned to the writer for scoring.

The allocation of marks to the items of this test posed certain difficulties. To offer one mark for a correct response would not create sufficient separation of able and less able candidates; on the other hand, detailed marking of working given, would seriously reduce the facility and speed of scoring so many scripts. The following, fairly simple procedure was adopted: one mark given for making a start that looked hopeful; two marks given for completely correct reasoning shown with an arithmetically incorrect answer or for almost complete and correct reasoning not followed through to a conclusion; and, three marks given for a correct response, with or without working. Guessing of correct answers was not considered to be an issue in this test, and the writer had to assume that tests had been carefully invigilated and so cheating could, likewise, not have taken place.

Scoring the test, in practice, was easier than had been expected. Many candidates were so overwhelmed by the problems that they did not attempt any of them; others offered incorrect answers supported by no working at all; others, still, gave no final answer but only some illegible and incomprehensible scribbles next to the question. The question arises, what was the cause of this large-scale failure to arrive at answers? It could be that time was so short that candidates panicked, but then one would expect that the last few questions would be noticeably less well done than the first few - this was not the case on many scripts; many candidates simply seemed to freeze, and then either would not, or could not, persevere with any one of the problems. It is interesting that this petrification of examinees occurs in school mathematics examinations as well, fortunately not on such a large scale. Perhaps, if we could determine the reason for this phenomenon, we would discover exactly why mathematics is such an "overwhelmingly difficult" subject for many high school pupils.

The results of this test were recorded; however, such was the consternation among students, about the reasoning test, that these results could in no way be considered as a reliable estimate of ability, and no conclusions were drawn from them. For the sake of completeness, and interest, a summary of the test statistics is given in the Tables 18 to 20.

TABLE 18: COMPARISON OF REASONING TEST STATISTICS FOR THE STANDARD SEVEN PUPILS OF FOUR PENINSULA HIGH SCHOOLS

	SCHOOLS			
	A	B	C	D
N	131	100	127	176
TYPE	girls		co-educational	
MEAN	8,02	5,70	5,79	6,44
SD	4,11	3,31	4,04	4,31
MEDIAN	8	6	5	6
HIGH	19	14	19	18
LOW	0	0	0	0

TABLE 19: COMPARISON OF REASONING TEST STATISTICS FOR THOSE STANDARD SEVEN PUPILS WHO CONTINUED WITH MATHEMATICS IN STANDARD EIGHT IN FIVE PENINSULA HIGH SCHOOLS

	SCHOOLS				
	A	B	C	D	E
N	105	54	99	142	137
TYPE	Girls		Co-educational		
MEAN	8,06	6,74	6,34	6,82	7,37
SD	4,05	3,28	4,26	4,33	4,28
MEDIAN	8	6,5	6	6	7
HIGH	19	14	19	18	23
LOW	0	0	0	0	0

TABLE 20: COMPARISON OF REASONING TEST STATISTICS FOR THOSE STANDARD SEVEN PUPILS OF FIVE PENINSULA HIGH SCHOOLS WHO CONTINUED WITH HIGHER GRADE MATHEMATICS IN STANDARD EIGHT

	SCHOOLS				
	A	B	C	D	E
N	74	42	68	56	101
TYPE	Girls		Co-educational		
MEAN	8,78	7,24	7,24	9,41	8,02
SD	4,09	3,08	4,41	4,12	4,36
MEDIAN	9	7	7	9	8
HIGH	19	14	19	18	23
LOW	1	1	0	0	0

Note that the boys of school H did not write the Reasoning Test; pupils at school E wrote the test at the beginning of their standard eight year, and those tested were all continuing with Mathematics.

As a matter of curiosity, the results of the reasoning test were correlated with the results of the multiple-choice test. The Product-Moment correlation coefficient, r, was used for this purpose, and the outcome is shown below:

TABLE 21: PRODUCT-MOMENT CORRELATION BETWEEN THE MULTIPLE-CHOICE TEST SCORES AND THE REASONING TEST SCORES FOR THE STANDARD SEVEN PUPILS IN FIVE PENINSULA HIGH SCHOOLS

SCHOOL	N	r
A	131	+0,38
B	100	+0,40
C	127	+0,47
D	176	+0,42
E	142	+0,47

The association that exists is positive, not strong and varies little from school to school.

It was thought that these reasoning test results might correlate more closely with the pupils' Intelligence Quotients. Since these were made available to the writer in two schools, viz., A and D, it was considered worthwhile to examine the results of such a correlation. Not all pupils in the two schools A and D had completed intelligence tests, consequently the sample numbers are slightly smaller than given in Table 21. Again the Product-Moment correlation coefficient was used for the calculation:

TABLE 22: PRODUCT-MOMENT CORRELATION BETWEEN THE REASONING TEST SCORES AND THE INTELLIGENCE QUOTIENTS OF STANDARD SEVEN PUPILS IN SCHOOLS A AND D

SCHOOL	N	r
A	119	+0,39
D	171	+0,46

Since the r-values obtained are very similar to those obtained from the previous calculation, the question is raised, to what extent do the multiple-choice test scores correlate with the intelligence quotients?

TABLE 23: PRODUCT-MOMENT CORRELATION BETWEEN THE MULTIPLE-CHOICE TEST SCORES AND INTELLIGENCE QUOTIENTS

SCHOOL	N	r
A	119	+0,59
D	171	+0,62

General ability seems to play a greater part in the multiple-choice test than it does in the reasoning test, and this is true of the standard seven examination as well.

TABLE 24: PRODUCT-MOMENT CORRELATION BETWEEN THE STANDARD SEVEN SCHOOL EXAMINATION SCORES AND THE INTELLIGENCE QUOTIENTS

SCHOOL	N	r
A	119	+0,57
D	171	+0,60

The results of the reasoning test, then, as stated before, do not reveal much about the pupil's ability. The horror struck in the hearts and minds of many pupils by this test was curious; not all of the items were so very difficult, although the time allotted was short. It would be interesting to determine exactly how much this apparent fear of verbose problems, or questions that appear to be complicated, affects the student's ability to perform well; and the extent to which the Senior Certificate examination papers have this effect on candidates.

This train of thought was not pursued by the writer in this investigation; however, it might provide an interesting avenue for future study.

4.5 THE REVISED TEST FURTHER REVISED

The purpose of this revision of the Revised Test, or the New Test, was twofold. Firstly, it would provide an estimate of reliability for the multiple-choice test, as most of its items would come from the Revised Test. This estimate of reliability would be a measure of internal consistency as well as a measure of candidate stability because of the intervening time lapse of one year. During the course of the year, pupils would have added to their knowledge and so it could be expected that they would do slightly better on average. Secondly, the items retained in this test would be those which had shown themselves to be efficient discriminators in the previous test, and so, it was hoped that this test would indicate more clearly which pupils were able to perform with insight.

4.5.1 Construction of the New Test

The items of the Revised Test retained in the New Test were those which had been classified as "good" or "fair" discriminators - only number 16 was omitted as its efficiency had seemed rather erratic. In addition, seven of the better "fluctuators" were included. One item from the Pilot Test, which had not been included in the Revised Test was resurrected for the New Test. These twenty-two items did not provide an adequate sample for the New Test, and so a further eight items, which had not been tried before, but which seemed to offer the right kind of challenge, were also included in the New Test. Item numbers were re-assigned and the Table 25 shows the item numbers of those questions which had been retained from previous tests:

TABLE 25: NEW TEST ITEM NUMBERS AS THEY APPEARED IN THE REVISED AND PILOT TESTS

NEW TEST	REVISED TEST	PILOT TEST
1	27	21
2	32	13
3	34	36
4	8	26
5	35	11
6	14	9
7	33	22
8	29	64
9	26	31
11	28	59
12	-	45
13	22	34
14	18	42
15	1	19
19	23	12
20	25	67
21	2	37
22	4	7
23	21	29
24	15	54
25	17	53
26	19	62

Items 10, 16, 17, 18, 27, 28, 29 and 30 in the New Test were not tried out in either of the previous tests.

The duration of the test was extended to 75 minutes, i.e., 2,5 minutes per item, in order to give each candidate a fair chance of completing all of the items.

The New Test was typed, and a copy thereof can be found in Appendix D.

4.5.2 Administration of the New Test

The New Test was administered to standard eight pupils at schools A, B, E and H; these pupils had all written either the Revised Test or the Pilot Test during the previous year. Unfortunately school C had withdrawn from the experiment earlier in the year, and school D did not respond to the request for their pupils to write this test. In addition, the standard seven and standard nine pupils of schools A, E and H wrote the New Test, although, none of these students had been tested before. It was foreseen that the investigator might follow the progress of the standard nine pupils until their matriculation examination, while the new standard seven class would provide a basis for comparison with the original standard seven group.

As before, with the exception of school H, the test was written as part of the end-of-year school examination, to ensure that pupils made every effort to offer their best attempt. The pupils of school H wrote the New Test very early in the new academic year. Once again, the test was accompanied by a set of invigilator's instructions and separate answer sheets. Copies of these can be found in Appendix D.

4.5.3 Analysis of Results

All test materials were returned to the writer, who scored the tests, with one assistant, on the basis of one mark for a correct response and nil for an incorrect response, omission or multiple response. The following tables give a summary of the test statistics and some indication of the increase in score from standard seven to standard nine. In the case of standards eight and nine, the figures quoted include both higher and standard grade candidates.

TABLE 26: COMPARISON OF NEW TEST STATISTICS FOR THE 1984 STANDARD SEVEN PUPILS OF THREE PENINSULA HIGH SCHOOLS

	SCHOOLS		
	H	A	E
N	144	143	171
TYPE	Boys	Girls	Co-educational
MEAN	12,7	12,9	10,2
SD	5,99	5,91	5,46
MEDIAN	12	12	9
HIGH	29	27	25
LOW	2	1	2

TABLE 27: COMPARISON OF NEW TEST STATISTICS FOR THE 1984 STANDARD EIGHT MATHEMATICS PUPILS OF FOUR PENINSULA HIGH SCHOOLS

	SCHOOLS			
	H	A	B	E
N	116	96	54	130
TYPE	Boys	Girls		Co-educational
MEAN	18,9	14,7	13,7	13,9
SD	6,32	5,66	5,37	5,96
MEDIAN	19	14	13	13
HIGH	30	28	27	29
LOW	4	2	5	2

TABLE 28: COMPARISON OF NEW TEST STATISTICS FOR THE 1984 STANDARD NINE MATHEMATICS PUPILS OF THREE PENINSULA HIGH SCHOOLS

	SCHOOLS		
	H	A	E
N	131	110	128
TYPE	Boys	Girls	Co-educational
MEAN	18,3	15,9	14,7
SD	6,33	6,68	6,03
MEDIAN	19	15	14
HIGH	30	30	30
LOW	5	5	3

Note that small discrepancies in sample numbers in some schools are due to the absence of a few pupils on the day of the test and further to the fact that some pupils left the school.

Note that the New Test had a maximum possible score of 30, while the Revised Test had a maximum possible of 35. Comparing the test statistics of both tests reveals that, at school A, the standard seven pupils fared slightly better than did their predecessors on the Revised Test; at schools E and H, the standard seven pupils did less well than the previous standard seven class. It must be remembered, however, that the students, at school E, who first wrote the multiple-choice test were not representative of all of the standard sevens but were those who had already chosen to continue with Mathematics.

The results of the standard eight and nine pupils show the expected increase in scores owing to their greater knowledge and experience of Mathematics.

The Tables 29 and 30 give a summary of the New Test statistics for Higher Grade candidates only in standards eight and nine.

TABLE 29: COMPARISON OF NEW TEST STATISTICS FOR THE 1984 HIGHER GRADE STANDARD EIGHT PUPILS OF FOUR PENINSULA HIGH SCHOOLS

	SCHOOLS			
	H	A	B	E
N	100	57	39	86
TYPE	Boys	Girls		Co-educational
MEAN	20,0	17,5	15,5	16,1
SD	5,68	4,61	4,99	5,32
MEDIAN	20,5	17	14	15,5
HIGH	30	30	27	29
LOW	4	10	7	5

TABLE 30: COMPARISON OF NEW TEST STATISTICS FOR THE 1984 HIGHER GRADE STANDARD NINE PUPILS OF THREE PENINSULA HIGH SCHOOLS

	SCHOOLS		
	H	A	E
N	97	62	57
TYPE	Boys	Girls	Co-educational
MEAN	20,3	20,2	19,4
SD	5,42	5,14	4,41
MEDIAN	20	21	19
HIGH	30	30	30
LOW	7	6	10

One important reason for running this New Test, was to provide an estimate of reliability for the multiple-choice test which would include a measure of candidate stability. With the results of the

New Test processed this could now be calculated. The Product-Moment correlation coefficient was used for this estimate of reliability, and Table 31 shows the result for each school when all candidates' scores are included, i.e., Higher and Standard Grade:

TABLE 31: PRODUCT-MOMENT CORRELATION BETWEEN THE REVISED TEST SCORES AND THE NEW TEST SCORES OF STANDARD EIGHT MATHEMATICS PUPILS

SCHOOL	N	r	STD ERROR
A	96	+0,81	0,061
B	54	+0,72	0,096
E	130	+0,77	0,056
H	116	+0,81	0,056

This procedure was repeated using only the Higher Grade candidates' scores as a basis for the correlation, and the results are given in Table 32.

TABLE 32: PRODUCT-MOMENT CORRELATION BETWEEN THE REVISED TEST SCORES AND THE NEW TEST SCORES FOR HIGHER GRADE STANDARD EIGHT PUPILS

SCHOOL	N	r	STD ERROR
A	57	+0,72	0,091
B	39	+0,67	0,122
E	86	+0,76	0,071
H	100	+0,76	0,066

These r-values show that the multiple-choice test produces fairly consistent results when one considers the considerable lapse of time between tests. Unfortunately numbers in the individual school samples are not large, and so the estimates are perhaps less dependable than desired; on the other hand, however, the various r-values for the different schools are quite similar, indicating that if one r-value for all pupils was calculated (thus, giving a sample size of 396 or 242 higher grade students) the resulting estimate of reliability would not be very different from the values we have for the individual schools.

The decreases in r when only the Higher Grade candidates are considered are, probably, due to the reduction in the range of talent for this group. It is obvious from the test statistics in Table 27 that the presence of the Standard Grade pupils reduces the mean and median scores of all of the school groups noticeably.

The question of validity must still hang in the balance at this stage, because we have only two examinations, viz., the May and November papers of the standard eight year, with which to compare the multiple-choice tests. It is the writer's opinion, that the standard eight syllabus does not demand the same high degree of insightful thinking as does the standard nine syllabus, and so, it might be expected that the correlation between the standard seven examination scores and the standard eight examination scores would be stronger than the correlation between the multiple-choice test scores and the standard eight examination scores. This hypothesis was tested using the results of schools A, B, E and H for Higher Grade candidates only. In the Tables 33 to 36, the Product-Moment correlation coefficient, r, has been calculated for each pair of tests, and is shown in the appropriate cell of the matrix.

TABLE 33: PRODUCT-MOMENT CORRELATION OF MULTIPLE-CHOICE TEST AND HIGHER GRADE MATHEMATICS EXAMINATION RESULTS FOR 1984 STANDARD EIGHT PUPILS

SCHOOL H: BOYS N = 100

	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984
Revised Test 1983	1	+,76	+,69	+,73	+,71
New Test 1984		1	+,62	+,67	+,72
Std 7 Exam 1983			1	+,76	+,81
Mid-year HG Exam 1984				1	+,90
Std 8 HG Exam 1984					1

TABLE 34: PRODUCT-MOMENT CORRELATION OF MULTIPLE-CHOICE TEST AND HIGHER GRADE MATHEMATICS EXAMINATION RESULTS FOR 1984 STANDARD EIGHT PUPILS

SCHOOL A: GIRLS N = 57

	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984
Revised Test 1983	1	+,72	+,68	+,58	+,71
New Test 1984		1	+,51	+,49	+,69
Std 7 Exam 1983			1	+,80	+,80
Mid-year HG Exam 1984				1	+,74
Std 8 HG Exam 1984					1

TABLE 35: PRODUCT-MOMENT CORRELATION OF MULTIPLE-CHOICE TEST AND HIGHER GRADE MATHEMATICS EXAMINATION RESULTS FOR 1984 STANDARD EIGHT PUPILS

SCHOOL B: GIRLS N = 39

	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984
Revised Test 1983	1	+,67	+,59	+,45	+,56
New Test 1984		1	+,57	+,60	+,63
Std 7 Exam 1983			1	+,71	+,78
Mid-year HG Exam 1984				1	+,86
Std 8 HG Exam 1984					1

TABLE 36: PRODUCT-MOMENT CORRELATION OF MULTIPLE-CHOICE TEST AND HIGHER GRADE MATHEMATICS EXAMINATION RESULTS FOR 1984 STANDARD EIGHT PUPILS

SCHOOL E: CO-EDUCATIONAL N = 86

	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984
Revised Test 1983	1	+,76	+,63	+,58	+,74
New Test 1984		1	+,59	+,53	+,69
Std 7 Exam 1983			1	+,76	+,80
Mid-year HG Exam 1984				1	+,84
Std 8 HG Exam 1984					1

From the Tables 33 to 36 it is clear that, in each school, the standard seven examination results correspond more closely with the end-of-year standard eight examination. Although the r-values of both multiple-choice tests generally increase, they still cannot compete with the strength of the correlation between the two school examinations; also noteworthy is the fact that this correlation appears to be strengthening as well, except in school A where it remains constant.

In order to determine whether these initial trends would be continued, it was decided to repeat the procedure using the scores obtained from the Higher Grade standard nine pupils in Schools A and H. The necessary results of past examinations were obtained from the official records of the two schools and, as before the Product-Moment correlation coefficient, r, was calculated for each pair of tests indicated by the cells in the Tables 37 and 38.

TABLE 37: PRODUCT-MOMENT CORRELATION OF MULTIPLE-CHOICE TEST AND HIGHER GRADE EXAMINATION SCORES FOR 1984 STANDARD NINE PUPILS

SCHOOL H: BOYS N = 97

	New Test 1984	Std 7 Exam 1982	May Exam 1983	Std 8 Exam 1983	May Exam 1984	Std 9 Exam 1984
New Test 1984	1	+,51	+,50	+,75	+,79	+,77
Std 7 Exam 1982		1	+,68	+,72	+,56	+,54
May Exam 1983			1	+,68	+,58	+,58
Std 8 Exam 1983				1	+,85	+,80
May Exam 1984					1	+,90
Std 9 Exam 1984						1

TABLE 38: PRODUCT-MOMENT CORRELATION OF MULTIPLE-CHOICE TEST AND HIGHER GRADE EXAMINATION SCORES FOR 1984 STANDARD NINE PUPILS

SCHOOL A:			GIRLS			N = 53
	New Test 1984	Std 7 Exam 1982	May Exam 1983	Std 8 Exam 1983	May Exam 1984	Std 9 Exam 1984
New Test 1984	1	+,55	+,67	+,63	+,62	+,55
Std 7 Exam 1982		1	+,79	+,80	+,68	+,66
May Exam 1983			1	+,87	+,80	+,77
Std 8 Exam 1983				1	+,84	+,84
May Exam 1984					1	+,86
Std 9 Exam 1984						1

Note that of the 62 girls tested at school A, 9 were either new-comers to the school and their results could, therefore, not be traced back, or they had missed an examination through illness.

From Tables 37 and 38, two different pictures emerge. In school A, the correspondence between the multiple-choice test and the successive school examinations peaks in May of the standard eight year and then decreases throughout the standard nine year to its original strength; the pattern of correspondence between the standard seven school examination and later examinations is the same, except that the final r-value is considerably greater than that of the multiple-choice test with the same school examination.

of the multiple-choice tests and the school examinations has indicated that, in general, the correlation between the standard seven school examination and subsequent school examinations is higher than that between the multiple-choice tests and the school examinations. There is also some evidence of an increasing trend for the correlation coefficients of the multiple-choice tests and a decreasing trend for the standard seven school examination. The 1985 school examination results would provide further information.

4.5.4 Analysis of New Test Items

Most of the items of the New Test had been used previously and, therefore, their efficiency had been investigated before; however, their relative positions in the test had been altered and it was necessary to determine what effect this had had on the difficulty level and discriminating power of these items. It was thought that certain items, previously placed near the end of the test may have been subject to overmuch guessing owing to a lack of time in the earlier test. The new items in the test had not yet been analysed, and had been chosen simply on the grounds that they appeared to require the same kind of thought processes as the tried and tested items. Another item analysis was undertaken and the details are summarised in Tables 39 and 44.

In order to maximise discriminating power, items of middle difficulty had been sought out in the previous item analysis and used again in the New Test. The pupils, however, had had one more year's experience of Mathematics and had learned a few more advanced

In school H, the r-values for the multiple-choice test increase noticeably over the two-year period, while the r-values for the standard seven school examination remain, more or less, constant throughout standard eight and then show a marked decrease in standard nine, falling well below the strength of the correlation coefficients for the multiple-choice test.

It must be remembered that, apart from the multiple-choice test, the examinations written by the two groups were internal school examinations, i.e., they were completely different tests of Mathematics, with different subject material and varying quantities of that subject material - some teachers preferring to move quickly through the syllabus to finish early for thorough revision, others moving more slowly and carefully to finish just before the final examination; however, all teachers of senior Mathematics have to ensure that their pupils are prepared for the same final matriculation examination and, therefore, there must be a certain degree of similarity between the papers of different schools.

Another possible reason for differences shown in the Tables 37 and 38 might be that prediction for girls is different from prediction for boys. This possibility is discussed in Chapter 6 when taking into account the data subsequently collected.

To sum up on the questions of reliability and validity:

It seems that the multiple-choice tests show a reasonable and acceptable degree of reliability, i.e., these tests yield consistent results. The validity of the test cannot properly be estimated at this stage of the investigation because of the lack of a suitable criterion which is common for all pupils. However, a comparison of the results techniques, which had the effect of making some of the items rather less problematic for them than they had been in standard seven. A study of Table 39 and the frequency distributions shown in Figures 6 to 9, indicates a definite shift in the mean difficulty levels for the different schools.

TABLE 39: NEW TEST ITEM DIFFICULTY LEVELS, p, FOR HIGHER GRADE STANDARD EIGHT PUPILS OF 1984

ITEM	SCHOOL H p	SCHOOL A p	SCHOOL B p	SCHOOL E p
1	0,73	0,67	0,87	0,72
2	0,51	0,36	0,31	0,48
3	0,62	0,50	0,36	0,26
4	0,76	0,64	0,82	0,66
5	0,42	0,66	0,46	0,34
6	0,74	0,84	0,54	0,67
7	0,45	0,31	0,32	0,49
8	0,67	0,47	0,28	0,45
9	0,49	0,40	0,18	0,42
10	0,78	0,90	0,79	0,58
11	0,67	0,55	0,51	0,59
12	0,47	0,36	0,58	0,31
13	0,53	0,48	0,54	0,44
14	0,77	0,70	0,67	0,48
15	0,91	0,84	0,76	0,83
16	0,80	0,69	0,56	0,80
17	0,65	0,50	0,38	0,44
18	0,53	0,29	0,44	0,38
19	0,73	0,88	0,87	0,73
20	0,76	0,57	0,51	0,59
21	0,85	0,81	0,56	0,69
22	0,76	0,69	0,67	0,76
23	0,73	0,59	0,51	0,60
24	0,89	0,78	0,67	0,65
25	0,69	0,67	0,49	0,57
26	0,64	0,48	0,36	0,57
27	0,81	0,64	0,67	0,55
28	0,67	0,53	0,33	0,50
29	0,29	0,31	0,13	0,29
30	0,65	0,50	0,36	0,57

Figure 6

DISTRIBUTION OF NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE BOYS OF SCHOOL H

< ,16		0
,16 - ,25		0
,26 - ,35	*	1
,36 - ,45	**	2
,46 - ,55	*****	5
,56 - ,65	****	4
,66 - ,75	**********	10
,76 - ,85	******	6
> ,85	**	2

Mean = 0,67 Standard Deviation = 0,15

Figure 7

DISTRIBUTION OF NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE GIRLS OF SCHOOL A

< ,16		0
,16 - ,25		0
,26 - ,35	***	3
,36 - ,45	***	3
,46 - ,55	********	8
,56 - ,65	****	4
,66 - ,75	******	6
,76 - ,85	****	4
> ,85	**	2

Mean = 0,59 Standard Deviation = 0,18

Figure 8

DISTRIBUTION OF NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE GIRLS OF SCHOOL B

< ,16	*	1
,16 - ,25	*	1
,26 - ,35	****	4
,36 - ,45	*****	5
,46 - ,55	*******	7
,56 - ,65	***	3
,66 - ,75	****	4
,76 - ,85	***	3
> ,85	**	2

Mean = 0,52 Standard Deviation = 0,20

Figure 9

DISTRIBUTION OF NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE PUPILS OF SCHOOL E

< ,16		0
,16 - ,25		0
,26 - ,35	****	4
,36 - ,45	*****	5
,46 - ,55	*****	5
,56 - ,65	********	8
,66 - ,75	******	6
,76 - ,85	**	2
> ,85		0

Mean = 0,55 Standard Deviation = 0,15

Clearly, the New Test was much less difficult for these standard eight pupils than it had been the previous year. It must not be forgotten that these pupils were all Higher Grade candidates, if Standard Grade pupils had been included, the p-levels would not have shown such a marked increase.

The standard seven pupils of 1984, on the other hand, experienced just as much difficulty with the New Test as their predecessors had had with the Revised Test the year before. Table 40 lists difficulty levels for these students in schools H, A and E:

TABLE 40: NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE STANDARD SEVEN PUPILS OF 1984 AT THREE PENINSULA HIGH SCHOOLS

ITEM	SCHOOL H p	SCHOOL A p	SCHOOL E p
1	0,41	0,39	0,32
2	0,26	0,31	0,16
3	0,28	0,37	0,18
4	0,72	0,58	0,38
5	0,32	0,43	0,27
6	0,54	0,59	0,29
7	0,24	0,25	0,15
8	0,40	0,33	0,22
9	0,48	0,49	0,34
10	0,70	0,61	0,55
11	0,33	0,35	0,27
12	0,34	0,22	0,32
13	0,34	0,35	0,37
14	0,35	0,32	0,22
15	0,67	0,69	0,57
16	0,70	0,69	0,67
17	0,35	0,31	0,18
18	0,33	0,40	0,21
19	0,41	0,54	0,28
20	0,37	0,34	0,29
21	0,56	0,58	0,49
22	0,57	0,72	0,54
23	0,39	0,49	0,41
24	0,44	0,52	0,46
25	0,38	0,38	0,39
26	0,34	0,36	0,32
27	0,58	0,53	0,40
28	0,41	0,34	0,21
29	0,24	0,14	0,22
30	0,32	0,37	0,30

Figure 10

DISTRIBUTION OF NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE 1984 STANDARD SEVEN PUPILS OF SCHOOL H

< ,16		0
,16 - ,25	**	2
,26 - ,35	***********	11
,36 - ,45	********	8
,46 - ,55	**	2
,56 - ,65	***	3
,66 - ,75	****	4
,76 - ,85		0
> ,85		0

Mean = 0,43 Standard Deviation = 0,14

Figure 11

DISTRIBUTION OF NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE 1984 STANDARD SEVEN PUPILS OF SCHOOL A

< ,16	*	1
,16 - ,25	**	2
,26 - ,35	********	8
,36 - ,45	*******	7
,46 - ,55	*****	5
,56 - ,65	****	4
,66 - ,75	***	3
,76 - ,85		0
> ,85		0

Mean = 0,43 Standard Deviation = 0,15

Figure 12

DISTRIBUTION OF NEW TEST ITEM DIFFICULTY LEVELS, p, FOR THE 1984 STANDARD SEVEN PUPILS OF SCHOOL E

< ,16	*	1
,16 - ,25	********	8
,26 - ,35	**********	10
,36 - ,45	*****	5
,46 - ,55	****	4
,56 - ,65	*	1
,66 - ,75	*	1
,76 - ,85		0
> ,85		0

Mean = 0,33 Standard Deviation = 0,13

The levels of difficulty given in Figures 10 to 12 were based on the scores of all standard seven pupils in schools H, A and E during 1984. As mentioned earlier, one cannot expect the level of difficulty of an item to remain constant from school to school, but the distributions do appear to be fairly similar and there is considerable agreement among the three groups as to the relative difficulty of the items. A measure of this correspondence, between any pair of school groups, is given by the Rank-Difference correlation coefficient, which may be obtained if the test items are ranked according to their level of difficulty as in Table 41.

TABLE 41: NEW TEST ITEMS RANKED ACCORDING TO THEIR LEVEL OF DIFFICULTY

ITEM	SCHOOL H	SCHOOL A	SCHOOL E
1	19	16	17
2	3	4,5	2
3	4	13,5	3,5
4	30	24,5	21
5	5,5	18	10,5
6	23	26	13,5
7	1,5	3	1
8	17	7	8
9	22	19,5	19
10	28,5	27	28
11	7,5	10,5	10,5
12	10	2	17
13	10	10,5	20
14	12,5	6	8
15	27	28,5	29
16	28,5	28,5	30
17	12,5	4,5	3,5
18	7,5	17	5,5
19	19	23	12
20	14	8,5	13,5
21	24	24,5	26
22	25	30	27
23	16	19,5	24
24	21	21	25
25	15	15	22
26	10	12	17
27	26	22	23
28	19	8,5	5,5
29	1,5	1	8
30	5,5	13,5	15

The correlation coefficients obtained are given in Table 42:

TABLE 42: RANK-DIFFERENCE CORRELATION BETWEEN RANKED LEVELS OF DIFFICULTY FOR SCHOOLS A, E AND H

School A vs School H	rho = +,79
School E vs School H	rho = +,75
School A vs School E	rho = +,75

These correlation coefficients show almost exactly the same degree of correspondence among schools as was found previously, when the difficulty levels of the Revised Test were compared in schools A and H (rho = +,74).

It is interesting that, in a few items, the standard seven groups experienced no more difficulty than the standard eight Higher Grade groups; in fact, in items 9, 18 and 22 the standard seven girls of school A managed with greater ease than did the standard eight girls, which, considering the nature of these particular items, is difficult to explain.

In addition to item difficulty levels, the item analysis provided a measure of the discriminating power of each item. These D-indices were calculated as described before, except that the upper group, U, represents the "top" 50% of candidates, and the lower group, L, represents the "bottom" 50% of candidates. Grouping was carried out both on the basis of the New Multiple-Choice Test results and on the basis of the

November Higher Grade examination for standard eight. The results are summarised in the Tables 43 and 44.

TABLE 43: COMPARISON OF THE INDICES OF DISCRIMINATION, D, FOR TWO DIFFERENT UPPER AND LOWER GROUPS IN SCHOOLS A AND H

ITEM	New M-C Test Grouped		November Exam Grouped	
	SCHOOL A	SCHOOL H	SCHOOL A	SCHOOL H
1	+0,03	+0,42	+0,11	+0,34
2	+0,10	+0,10	-0,07	-0,06
3	+0,24	+0,40	+0,32	+0,12
4	+0,24	+0,04	+0,07	-0,04
5	+0,34	+0,44	+0,25	+0,48
6	-0,03	+0,28	+0,21	+0,36
7	+0,41	+0,42	+0,29	+0,20
8	+0,45	+0,26	+0,25	+0,38
9	+0,31	+0,34	+0,18	+0,38
10	0,00	+0,04	0,00	0,00
11	+0,21	+0,42	+0,21	+0,30
12	+0,03	+0,38	-0,04	+0,18
13	+0,21	+0,30	+0,04	+0,30
14	+0,39	+0,26	+0,44	+0,22
15	+0,10	+0,18	+0,04	+0,10
16	+0,34	+0,24	-0,14	+0,08
17	+0,31	+0,50	+0,04	+0,34
18	+0,24	+0,34	+0,18	+0,30
19	-0,03	+0,42	+0,04	+0,26
20	+0,45	+0,32	+0,36	+0,20
21	+0,10	+0,22	+0,04	+0,22
22	+0,21	+0,20	+0,14	+0,16
23	+0,28	+0,42	+0,29	+0,34
24	+0,17	+0,14	0,00	+0,10
25	+0,17	+0,34	+0,11	+0,26
26	+0,21	+0,48	+0,18	+0,24
27	+0,31	+0,14	+0,32	+0,06
28	+0,31	+0,50	+0,21	+0,34
29	-0,07	+0,22	+0,07	+0,10
30	+0,24	+0,30	+0,29	+0,18
Highest D	+0,45	+0,50	+0,44	+0,48
Lowest D	-0,07	+0,04	-0,07	-0,06
Median D	+0,23	+0,31	+0,16	+0,22

TABLE 44: COMPARISON OF THE INDICES OF DISCRIMINATION, D, FOR TWO DIFFERENT UPPER AND LOWER GROUPS IN SCHOOLS B AND E

ITEM	New M-C Test Grouped		November Exam Grouped	
	SCHOOL B	SCHOOL E	SCHOOL B	SCHOOL E
1	+0,10	+0,23	0,00	+0,12
2	+0,31	+0,35	+0,14	+0,22
3	+0,31	+0,19	+0,11	+0,19
4	+0,10	+0,26	-0,07	+0,21
5	+0,41	+0,16	+0,32	+0,29
6	+0,36	+0,28	+0,11	+0,21
7	+0,32	+0,21	+0,14	+0,17
8	+0,36	+0,47	+0,11	+0,22
9	+0,26	+0,14	+0,25	+0,05
10	+0,26	+0,23	-0,04	-0,10
11	+0,41	+0,21	+0,07	+0,17
12	+0,42	+0,30	+0,36	+0,19
13	+0,46	+0,45	+0,29	+0,22
14	+0,21	+0,40	-0,07	+0,14
15	+0,26	+0,26	+0,15	+0,12
16	+0,31	+0,12	+0,18	-0,07
17	+0,36	+0,14	+0,32	-0,02
18	+0,15	+0,30	+0,11	+0,31
19	+0,21	+0,16	+0,11	+0,17
20	+0,31	+0,44	+0,14	+0,10
21	+0,21	+0,35	-0,04	+0,05
22	-0,10	+0,26	0,00	+0,24
23	+0,10	+0,33	+0,07	+0,31
24	+0,31	+0,21	+0,11	+0,12
25	+0,46	+0,26	+0,14	+0,29
26	+0,41	+0,35	+0,14	+0,31
27	+0,41	+0,40	+0,14	0,00
28	+0,36	+0,49	+0,07	+0,29
29	+0,05	+0,26	+0,04	+0,12
30	+0,31	+0,30	+0,18	+0,17
Highest D	+0,46	+0,49	+0,36	+0,31
Lowest D	-0,10	+0,12	-0,07	-0,10
Median D	+0,31	+0,26	+0,11	+0,17

Once again, the degree of correspondence between the two different groupings for each school was calculated using the Rank-Difference correlation coefficient, and the results are given in Table 45:

TABLE 45: RANK-DIFFERENCE CORRELATIONS BETWEEN NEW TEST BASED INDICES OF DISCRIMINATION AND SCHOOL EXAMINATION BASED INDICES OF DISCRIMINATION

SCHOOL	N	rho
A	57	+0,69
H	100	+0,71
B	39	+0,63
E	86	+0,36

As before (see Tables 13, 14 and 15), with the exception of school E, there is a strong positive correlation between the indices of discrimination obtained from the two different groupings in each school. Indeed, even in school E, a slight strengthening of the correlation is evident. Could this indicate that the internal school examinations and the multiple-choice tests are beginning to call forth more similar thought processes?

The discriminating power of items, in general, seems to have been reduced by comparison to the D-indices determined in the item analysis of the revised test, but this is probably due to the fact that, at least for these standard eight pupils, individual items seem to be less difficult.

The items of the test had been ranked as discriminators, and in order to determine the extent to which the four schools agreed on these rankings, Kendall's coefficient of concordance was calculated.

The Tables 46 and 47 show the ranking of the indices of discrimination, D, of the thirty items for the various schools, first, grouped according to the new multiple-choice test, and secondly, grouped according to the November Higher Grade school examinations.

TABLE 46: NEW TEST ITEMS RANKED AS DISCRIMINATORS ACCORDING TO THE GROUPS YIELDED BY THE NEW MULTIPLE-CHOICE TEST

ITEM	SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
1	5,5	24	4	10,5
2	8	3	15,5	23
3	17,5	21	15,5	6
4	17,5	1,5	4	14
5	25,5	27	25,5	4,5
6	2,5	13	21,5	17
7	28	24	19	8
8	29,5	11,5	21,5	29
9	22,5	18	11	2,5
10	4	1,5	11	10,5
11	13,5	24	25,5	8
12	5,5	20	28	19
13	13,5	14,5	29,5	28
14	27	11,5	8	25,5
15	8	6	11	14
16	25,5	10	15,5	1
17	22,5	29,5	21,5	2,5
18	17,5	18	6	19
19	2,5	24	8	4,5
20	29,5	16	15,5	27
21	8	8,5	8	23
22	13,5	7	1	14
23	20	24	4	21
24	10,5	4,5	15,5	8
25	10,5	18	29,5	14
26	13,5	28	25,5	23
27	22,5	4,5	25,5	25,5
28	22,5	29,5	21,5	30
29	1	8,5	2	14
30	17,5	14,5	15,5	19

Kendall's Coefficient of Concordance, W = 0,35
Thus, Chi-square = 40,6 with 29 degrees of freedom.

There is no statistically significant association.

TABLE 47: NEW TEST ITEMS RANKED AS DISCRIMINATORS ACCORDING TO THE GROUPS YIELDED BY THE NOVEMBER SCHOOL EXAMINATION (STANDARD EIGHT HIGHER GRADE)

ITEM	SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
1	12,5	24,5	5,5	9,5
2	1	1	19,5	22
3	27,5	9	13,5	17,5
4	10,5	2	1,5	19,5
5	22,5	30	28,5	26
6	20	27	13,5	19,5
7	25	13,5	19,5	14,5
8	22,5	28,5	13,5	22
9	17	28,5	26	5,5
10	3,5	3	3,5	1
11	20	21	9	14,5
12	2	11,5	30	17,5
13	7	21	27	22
14	30	15,5	1,5	12
15	7	7	23	9,5
16	14,5	5	24,5	2
17	7	24,5	28,5	3
18	17	21	13,5	29
19	7	18,5	13,5	14,5
20	29	13,5	19,5	7
21	7	15,5	3,5	5,5
22	14,5	10	5,5	24
23	25	24,5	9	29
24	3,5	7	13,5	9,5
25	12,5	18,5	19,5	26
26	17	17	19,5	29
27	27,5	4	19,5	4
28	20	24,5	9	26
29	10,5	7	7	9,5
30	25	11,5	24,5	14,5

Kendall's Coefficient of Concordance, W = 0,36

Thus, Chi-square = 41,76 with 29 degrees of freedom.

There is no statistically significant association.

Note: For both tables, a high rank indicates a powerful discriminator, while a low rank indicates a weak discriminator.

Once again, it appears that items operate with varying degrees of efficiency from school to school and that no statistically significant conclusions may be drawn, although there does seem to be some degree of association. There are some items, however, that do seem to show a certain amount of stability, and in a last attempt to discover any hidden trends present, another method was devised to classify items as "good", "fair" or "poor". Note that these terms are relative to the particular set of D-indices under scrutiny.

Thus, for each method of grouping in each school, test items were classified as "good" discriminators if their indices of discrimination were among the highest within that group, "fair" if their D-indices were clustered round the middle of the distribution and "poor" if their D-indices were among the lowest within that group.

TABLE 48: "GOOD" DISCRIMINATORS ACCORDING TO THE NEW TEST GROUPING

SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
20;8	17;28	13;25	28
7	26	12	8
14	5	5;11;26;27	13
5;16	1;7;11;19;23		20
9;17;27;28			14;27
			2;21;26

TABLE 49: "GOOD" DISCRIMINATORS ACCORDING TO THE NOVEMBER EXAMINATION GROUPING

SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
14	5	12	18;23;26
20	8;9	5;17	5;25;28
3;27	6	13	22
7;23;30	1;17;23;28	9	
		16;30	
		15	

TABLE 50: "FAIR" DISCRIMINATORS ACCORDING TO THE NEW TEST GROUPING

SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
23	3	6;8;17;28	23
3;4;18;30	12	7	12;18;30
11;13;22;26	9;18;25	2;3;16;20;24;30	6
24;25	20	9;10;15	4;15;22;25
2;15;21	13;30	14;19;21	29
	6		1;10
	8;14		7;11;24
	16		
	21;29		

TABLE 51: "FAIR" DISCRIMINATORS ACCORDING TO THE NOVEMBER EXAMINATION GROUPING

SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
5;8	11;13;18	2;7;20;25;26;27	2;8;13
6;11;28	19;25	3;6;8;18;19;24	4;6
9;18;26	26	11;23;28	3;12
16;22	14;21	29	7;11;19;30
1;25	7;20		14
4;29	12;30		1;15;24;29
13;15;17;19;21	22		
	3		
	15;24;29		

TABLE 52: "POOR" DISCRIMINATORS ACCORDING TO THE NEW TEST GROUPING

SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
1;12	22	18	3
10	15	1;4;23	5;19
6;19	24;27	29	9;17
29	2	22	
	4;10	22	

TABLE 53: "POOR" DISCRIMINATORS ACCORDING TO THE NOVEMBER EXAMINATION GROUPING

SCHOOL A	SCHOOL H	SCHOOL B	SCHOOL E
10;24	16	1;22	20
12	27	10;21	9;21
2	10	4;14	27
	4		17
	2		16
			10

In order to accentuate those items which, most consistently, discriminate efficiently between upper and lower groups, a simple point system was devised. Two points were allocated to items classified as "good" discriminators, one point to items classified as "fair" discriminators and no points (zero) to those classified as "poor" discriminators. This procedure was followed with both sets of classifications, i.e., items classified according to the new multiple-choice test upper and lower groups, and the school examination upper and lower groups. Each item, then, obtained a score as a discriminator; the larger the item's score, the more consistent the item in its power to discriminate between top scoring pupils and least scoring pupils.

TABLE 54: POINTS OBTAINED BY ITEMS OF THE NEW TEST ACCORDING TO THEIR EFFICIENCY TO DISCRIMINATE BETWEEN THE ABLE AND LESS ABLE IN 4 HIGH SCHOOLS

Item	New Test Grouped	School Examination Grouped	Combined Score
1	3	4	7
2	4	2	6
3	3	5	8
4	2	2	4
5	6	7	13
6	3	5	8
7	6	5	11
8	6	5	11
9	4	5	9
10	2	0	2
11	6	4	10
12	4	4	8
13	6	5	11
14	6	4	10
15	3	5	8
16	4	3	7
17	5	5	10
18	4	5	9
19	3	4	7
20	6	4	10
21	5	2	7
22	2	4	6
23	4	7	11
24	3	3	6
25	5	5	10
26	7	5	12
27	6	3	9
28	7	6	13
29	2	4	6
30	4	6	10

By this somewhat artificial device, it becomes clear that items 5, 28, 26, 7, 8, 13 and 23 are, over-all, the most promising of the test questions. Other items also perform well for one or two of the schools, but lack the stability of the above-mentioned items; among these are questions 9, 11, 12, 14, 15, 17, 20 and 27. Items 3 and 6 are not among the best discriminators, but are consistently "fair". Some items vary in their power to differentiate between the able and the less able - in one school being very effective, but not at all effective in another. Only two items, 4 and 10, are consistently "poor" discriminators and the probable reason for this is that they proved to be too easy for pupils at the standard eight level.

On the basis of the scores in Table 54, items may be grouped together under the headings "Good" for those items which have been consistently efficient as discriminators, "Fair" for those which have shown more fluctuation in their effectiveness or have been consistently fair discriminators, and "Poor" for those items which have not been efficient discriminators.

TABLE 55: NEW TEST ITEMS CLASSIFIED ACCORDING TO THEIR EFFICIENCY AS DISCRIMINATORS AMONG HIGHER GRADE STANDARD EIGHT PUPILS IN FOUR SCHOOLS

GOOD	FAIR	POOR
5;28	11;14;17;20;25;30	2;22;24;29
26	9;18;27	4
7;8;13;23	3;6;12;15	10
	1;16;19;21	

These groupings were then examined with a view to determining the distribution of mental processes involved. The results are given in Table 56.

TABLE 56: OCCURRENCE OF MENTAL PROCESSES AMONG THE GOOD, FAIR AND POOR DISCRIMINATORS

Mental Process	"GOOD"		"FAIR"		"POOR"	
	No.	%	No.	%	No.	%
Manipulation	1	14,3	3	17,6	2	33,3
Routine Problem	1	14,3	4	23,5	1	16,7
Comprehension	3	42,9	4	23,5	2	33,3
Non-routine problem	2	28,6	6	35,3	1	16,7

The distribution of mental processes in the new multiple-choice test is given in Table 57.

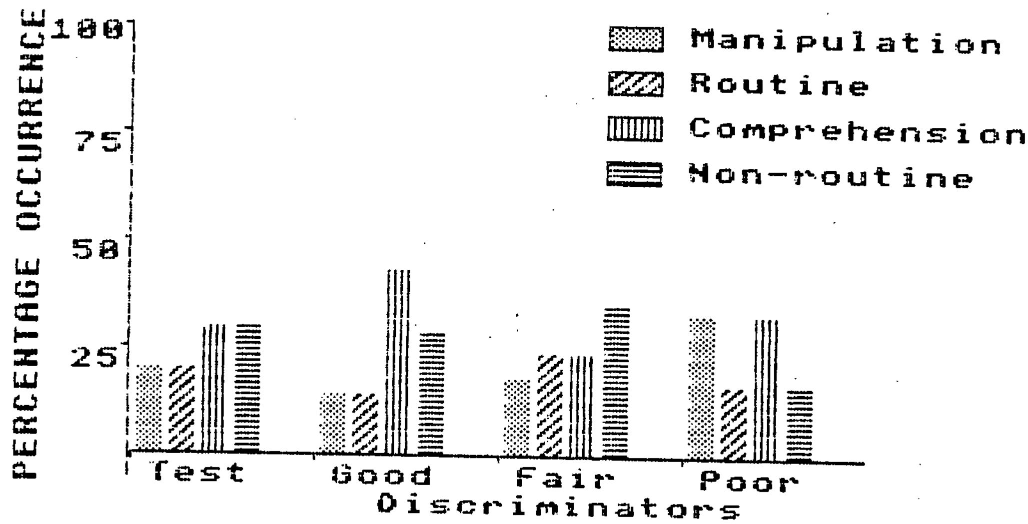
TABLE 57: THE DISTRIBUTION OF MENTAL PROCESSES IN THE NEW MULTIPLE-CHOICE TEST

Mental Process	NEW TEST CONTENT	
	No. of Items	%
Manipulation	6	20
Routine problem	6	20
Comprehension	9	30
Non-routine problem	9	30

The information in Tables 55, 56 and 57, together with the diagram in Figure 13, indicates that, on the whole, the mental processes occur in the same proportions among the different levels of discrimination as they do in the new multiple-choice test. Three notable exceptions are the high percentage of Comprehension among the "good" discriminators, the high percentage of Manipulation among the "poor" discriminators and the slightly higher than expected occurrence of the Non-routine types of problem among the "fair" discriminators.

Figure 13

DISTRIBUTION OF MENTAL PROCESSES AMONG THE DIFFERENT LEVELS OF DISCRIMINATOR



MENTAL PROCESSES INVOLVED

In the foregoing classification, the combined scores of items were used to categorise items as "good", "fair" or "poor" discriminators; if the process is repeated using the scores obtained from the new test groupings and the school examination groupings, the picture remains much the same. Tables 58 and 59 and Figures 14 and 15 illustrate this.

Note that the lowest and the highest levels of the ability hierarchy, i.e., Recall and Apply "higher mental processes" do not appear in these latest analyses. The reason for this is that the new test did not include any Recall type questions and the number of items of the highest level was so small that it was decided to combine these with the Insight type to form the Non-routine problem category.

TABLE 58: DISTRIBUTION OF MENTAL PROCESSES AMONG DIFFERENT LEVELS OF DISCRIMINATOR ACCORDING TO NEW TEST GROUPING OF PUPILS

Mental Process	"GOOD"		"FAIR"		"POOR"	
	No.	%	No.	%	No.	%
Manipulation	1	10	4	25	1	25
Routine problem	1	10	4	25	1	25
Comprehension	3	30	5	31,3	1	25
Non-routine problem	5	50	3	18,7	1	25

TABLE 59: DISTRIBUTION OF MENTAL PROCESSES AMONG DIFFERENT LEVELS OF DISCRIMINATOR ACCORDING TO THE SCHOOL EXAMINATION GROUPING OF PUPILS (STANDARD EIGHT HIGHER GRADE)

Mental Process	"GOOD"		"FAIR"		"POOR"	
	No.	%	No.	%	No.	%
Manipulation	1	25	4	18,2	1	25
Routine problem	0	0	5	22,7	1	25
Comprehension	3	75	5	22,7	1	25
Non-routine problem	0	0	8	36,4	1	25

Figure 14

DISTRIBUTION OF MENTAL PROCESSES AMONG THE DIFFERENT LEVELS OF DISCRIMINATOR

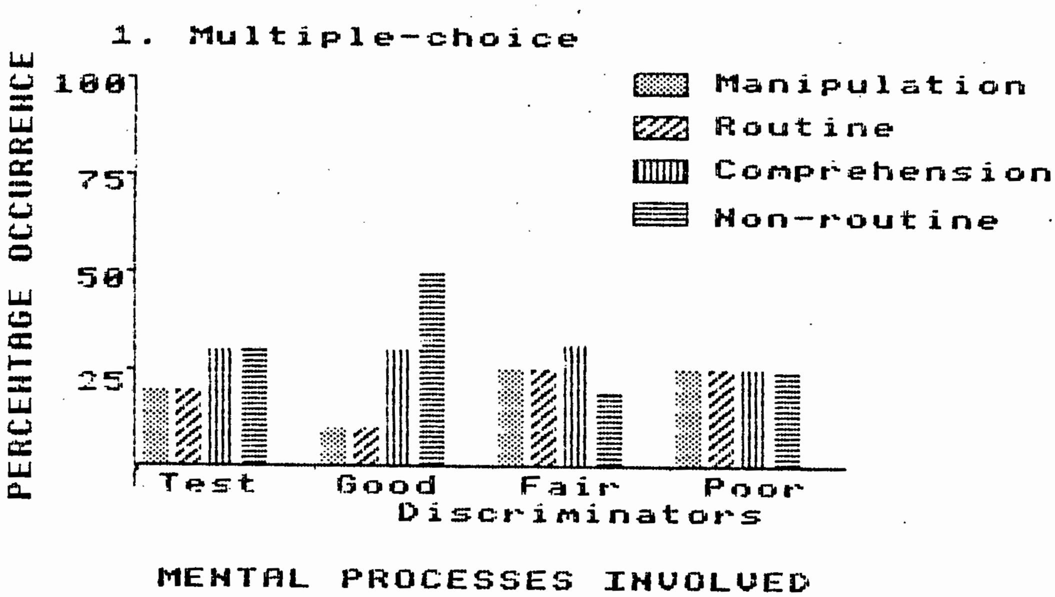
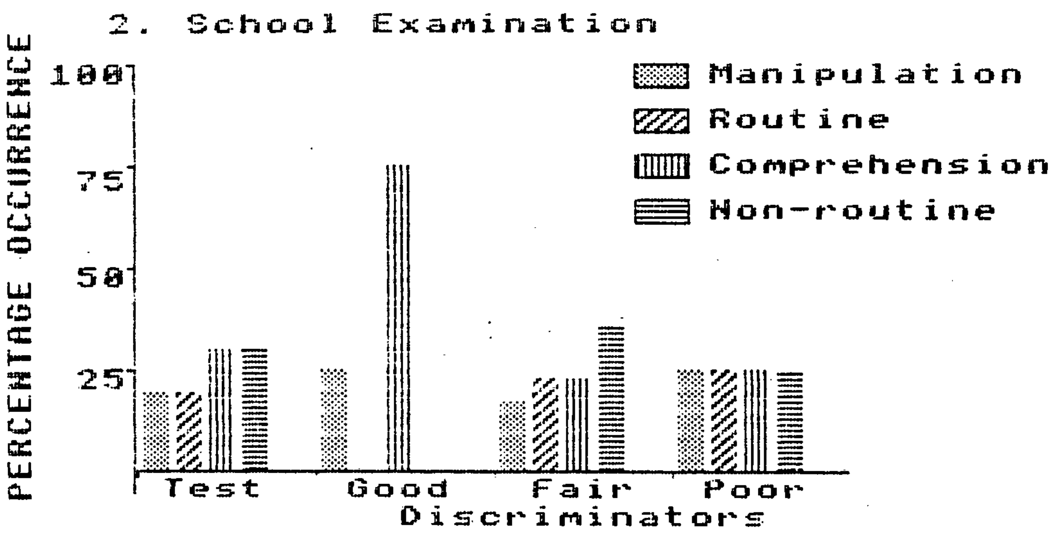


Figure 15

DISTRIBUTION OF MENTAL PROCESSES AMONG
THE DIFFERENT LEVELS OF DISCRIMINATOR



MENTAL PROCESSES INVOLVED

Once again it appears that the occurrence of the various mental processes among the different levels of discriminator simply echoes the proportions of their occurrence in the new multiple-choice test. The apparent dominance of the Comprehension type of item among the "good" discriminators of the school examination grouping is merely a function of the very small number of items that could be classified as "good", i.e., since there are only four items in this particular category, small variations from the expected frequencies produce an exaggerated distribution. The dominance of the Non-routine problems among the "good" discriminators of the new multiple-choice test grouping is slightly more impressive owing to the larger number of items in this category, viz., ten items.

Bearing in mind the difficulties involved in classifying a test item according to the mental process involved to complete it successfully, this analysis has indicated that although Comprehension and Non-routine problems may make marginally better discriminators than the lower levels of mental ability, i.e., Manipulation and Routine problems, there is not sufficient evidence to prove that the occurrence of the mental processes among the different levels of discriminator is any different from their pattern of occurrence in the test itself.

The school examinations used in the foregoing analysis were the internal standard eight Higher Grade promotion examinations, and it is pertinent to ask to what extent these examinations resemble the final matriculation examination? Could it be possible that the upper and lower groups of student scores would change to such an extent over the standard nine and ten years so as to produce a different distribution of mental processes among the various categories of discriminator? This line of thought is pursued in Chapter 5 when the pupils' progress at a more advanced level of the Higher Grade course is discussed.

Another method of investigating the efficiency of individual test items was undertaken, viz., each item was submitted to a Chi-square test. For promising items, two-by-two contingency tables were constructed comparing those above the median score with those below on the correctness of their response to the item. Again, two upper and lower groups were obtained by making use of the results from both the new multiple-choice test and the standard eight higher grade school examinations. Note that Chi-square was corrected for continuity and that the null hypothesis is as follows:

> Ho: There is no difference in the ability to complete this item successfully of those above the median and those below the median.

The reader should note that the median in question is the median score either of the New multiple-choice test or of the standard eight Higher Grade school examination depending upon which grouping of pupils is under discussion.

TABLE 60: SUMMARY OF RESULTS OF CHI-SQUARE TEST OF SIGNIFICANCE APPLIED TO SOME ITEMS OF THE NEW MULTIPLE-CHOICE TEST WHERE THE NEW TEST GROUPING OF PUPILS WAS USED

ITEM	SCHOOL	TYPE	CHI-SQUARE	P	Ho
5	H	Boys	18,10	0,00	Reject
	A	Girls	8,12	0,00	Reject
	B	Girls	4,41	0,04	Reject
	E	Co-ed	1,87	0,17	
28	H	Boys	26,05	0,00	Reject
	A	Girls	5,71	0,02	Reject
	B	Girls	3,71	0,05	Reject
	E	Co-ed	18,61	0,00	Reject
26	H	Boys	22,96	0,00	Reject
	A	Girls	2,46	0,12	
	B	Girls	4,92	0,03	Reject
	E	Co-ed	9,30	0,00	Reject
7	H	Boys	15,57	0,00	Reject
	A	Girls	10,89	0,00	Reject
	B	Girls	3,04	0,08	
	E	Co-ed	4,03	0,04	Reject
8	H	Boys	6,51	0,01	Reject
	A	Girls	11,60	0,00	Reject
	B	Girls	4,14	0,04	Reject
	E	Co-ed	18,00	0,00	Reject
13	H	Boys	7,87	0,01	Reject
	A	Girls	2,46	0,17	
	B	Girls	5,75	0,02	Reject
	E	Co-ed	16,27	0,00	Reject
23	H	Boys	20,29	0,00	Reject
	A	Girls	4,21	0,04	Reject
	B	Girls	0,02	0,88	
	E	Co-ed	8,22	0,00	Reject
18	H	Boys	10,28	0,00	Reject
	A	Girls	3,33	0,07	
	B	Girls	0,26	0,61	
	E	Co-ed	7,08	0,01	Reject
6	H	Boys	8,78	0,00	Reject
	A	Girls	ft < 5		
	B	Girls	3,08	0,08	
	E	Co-ed	6,41	0,01	Reject

TABLE 61: SUMMARY OF RESULTS OF CHI-SQUARE TEST OF SIGNIFICANCE APPLIED TO SOME ITEMS OF THE NEW MULTIPLE-CHOICE TEST WHERE THE STANDARD EIGHT HIGHER GRADE EXAMINATION GROUPING OF PUPILS WAS USED

ITEM	SCHOOL	TYPE	CHI-SQUARE	P	Ho
5	H	Boys	21,72	0,00	Reject
	A	Girls	2,87	0,09	
	B	Girls	7,13	0,01	Reject
	E	Co-ed	6,48	0,01	Reject
28	H	Boys	11,58	0,00	Reject
	A	Girls	1,79	0,18	
	B	Girls	0,13	0,72	
	E	Co-ed	5,76	0,02	Reject
26	H	Boys	5,25	0,02	Reject
	A	Girls	1,14	0,28	
	B	Girls	1,05	0,31	
	E	Co-ed	6,96	0,01	Reject
7	H	Boys	3,26	0,07	
	A	Girls	4,01	0,05	Reject
	B	Girls	0,90	0,34	
	E	Co-ed	2,29	0,13	
8	H	Boys	14,65	0,00	Reject
	A	Girls	2,57	0,11	
	B	Girls	ft < 5		
	E	Co-ed	2,78	0,10	
13	H	Boys	7,87	0,01	Reject
	A	Girls	0,00	1,00	
	B	Girls	5,51	0,02	Reject
	E	Co-ed	2,78	0,10	
23	H	Boys	12,99	0,00	Reject
	A	Girls	3,08	0,08	
	B	Girls	0,11	0,74	
	E	Co-ed	7,19	0,01	Reject
18	H	Boys	7,87	0,01	Reject
	A	Girls	1,35	0,25	
	B	Girls	0,45	0,50	
	E	Co-ed	4,99	0,03	Reject
6	H	Boys	15,02	0,00	Reject
	A	Girls	ft < 5		
	B	Girls	0,45	0,50	
	E	Co-ed	3,49	0,06	

The results of the test of significance for a few of the items are summarised in Tables 60 and 61.

Note that when the null hypothesis is rejected, the conclusion is that it is likely that those pupils with above median scores will be more successful in that particular item than the others with the probability as given by P.

The application of the Chi-square test to individual items reveals nothing new, but rather, once again, points up the variability of the efficiency of the items as discriminators from school to school and between the different upper and lower groupings produced by the multiple-choice test and the school examination. Whether this would change during the standard nine year remained to be seen as the investigator followed the progress of these students, but at this stage, it did not seem possible to isolate particular types of item which would discriminate efficiently for all groups of pupils.

To sum up: The purpose of this new test was, primarily, to provide a check on the reliability of the multiple-choice test. The measure of reliability obtained was found to be satisfactory.

In addition, it was hoped that further item analyses, using the results of the higher grade candidates only, might lead to the discovery of trends of discriminating power among certain types of item, thereby allowing deductions to be made about the sort of predictive instrument that could be devised to test the suitability of standard seven pupils for participation in higher grade Mathematics. Unfortunately, the various analyses have provided no evidence of such trends, and it seems that, as far as the standard eight pupils are concerned, any kind of item is capable of being a "good" discriminator, provided that it is neither too easy nor too difficult. However, it is possible that the standard eight examination is too similar, in its intellectual demands, to the standard seven examination to yield the information about the items for which it had been hoped. Groupings of candidate scores on future, more advanced Mathematics examinations may provide this information.

In the next section, the progress of the candidates is followed into standards nine and ten. The final matriculation examination is based directly on the syllabus taught during these last two years of the secondary course, and so, it might be expected that, school examination papers, internally set, should begin to follow the pattern of the externally set papers more closely. If this is true then any predictive value, possessed by the multiple-choice tests, should become apparent by a tendency for the correlation, between these tests and the school examinations, to increase as pupils approach the final examination.

CHAPTER 5

THE PROGRESS OF STUDENTS IN STANDARDS NINE AND TEN

5.1 CHAPTER OUTLINE

5.2 THE SAMPLE

5.3 THE 1985 MID-YEAR SCHOOL EXAMINATIONS

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5.4 THE 1985 END-OF-YEAR STANDARD NINE SCHOOL EXAMINATIONS

5.5 THE 1985 CAPE SENIOR CERTIFICATE EXAMINATION

5.1 CHAPTER OUTLINE

In the chapter that follows the progress of candidates, originally tested as standard seven pupils in 1983, as well as candidates tested, once only, as standard nine pupils in 1984, is pursued in 1985, their standard nine and final years of school respectively. The results of the multiple-choice tests written previously are compared with the Higher Grade results of the internally set May examinations for standard nine and ten and the November promotion examination for standard nine; in addition, the results of the 1985 standard ten pupils in the, externally set, Cape Senior Certificate examination were obtained, and these are also compared with the pupils' scores on the multiple-choice test that they wrote at the end of their standard nine year.

The main thrust of the analyses will be:

1. To establish the extent to which the multiple-choice tests, developed earlier, may have predictive validity for the Senior Certificate course in Mathematics by examining the correlation between these tests and successive school examinations.

2. To discover whether any particular type of item (vis-a-vis the mental process involved) provides a maximally efficient discriminator between those who, ultimately, will be successful in senior Mathematics and those who will not.

5.2 THE SAMPLE

It has been mentioned in previous sections that the size of the sample of pupils under investigation steadily decreased from the outset of the study. This was due firstly, to the non-compulsory status of Mathematics in the senior high school years; secondly, to the normal movement of a few pupils out of the school for various reasons or their absence during an examination; and, thirdly, to the tendency of candidates to abandon the Higher Grade course in favour of the Standard Grade course, when they feel that they can no longer cope with its greater content and complexity. Table 62 indicates the extent to which the investigation sample was depleted over a two year period. This table refers to the standard nine pupils of 1985.

TABLE 62: SAMPLE DEPLETION AMONG HIGHER GRADE PUPILS AT FIVE SCHOOLS FROM EARLY STANDARD EIGHT TO THE END OF STANDARD NINE

SCHOOL	MID-YEAR 1984	NOVEMBER 1984	MID-YEAR 1985	NOVEMBER 1985
H	107	100	97	86
A	74	57	54	49
B	42	39	34	31
D	56	51	44	42
E	101	86	58	36

During the standard ten year, the decline in numbers of pupils taking the Higher Grade course continues as candidates take stock of their mathematical status and measure it against the demands of the syllabus with which, at this stage, they are almost completely acquainted. Table 63 indicates the decrease in numbers of Higher Grade candidates in three schools during the final year of study.

TABLE 63: SAMPLE DEPLETION AMONG HIGHER GRADE PUPILS AT THREE SCHOOLS DURING THE FINAL YEAR OF STUDY

SCHOOL	NOVEMBER 1984	NOVEMBER 1985
H	97	80
A	53	35
E	57	26

The writer wishes to stress that in the analyses that follow, two different groups of students are discussed. The first group consists of those pupils who first wrote the multiple-choice test at the end of 1983 when they were in standard seven, and then were retested, with the exception of pupils at school D, at the end of 1984. These candidates are referred to as "the standard nine" pupils, because during the year under consideration in this chapter, 1985, these pupils were in standard nine. The second group of students wrote only one multiple-choice test in November 1984, when they were at the end of their standard nine year; these pupils will be referred to as "the standard ten" pupils, because in 1985 these pupils were in standard ten.

5.3 THE 1985 MID-YEAR SCHOOL EXAMINATIONS

The work directly examinable by the Cape Senior Certificate mathematics papers is taught during the course of the standard nine and ten years. It follows, then, that teachers, faced with the task of setting internal examination papers for these senior pupils, would pay close attention to the construction and particular bias of previous external papers and reflect these in their own question papers, so that their pupils might become accustomed to and practised in the writing of such examinations. If this is the case, then, there is reason to believe that any predictive validity possessed by the multiple-choice test should begin to become apparent by a tendency to correlate more closely with ever more advanced school examination papers. Furthermore, the different examinations set in the various schools should submit more readily to comparison with each other as they are written progressively closer to the final external paper. However, this is possibly less true for the mid-year standard nine examinations, than for other examinations. Especially in standard nine, the quality and quantity of the work covered by different school groups, could vary considerably, depending upon the teacher's starting point in the syllabus, of which there are several logical ones, and final revision policy, viz., to finish as quickly as possible and then undertake extensive revision programmes, or to teach more slowly and carefully, leaving final revision to the pupils themselves.

In the sections that follow, pupils' results, on their current examinations, are compared to their results on previously written multiple-choice tests, not only to produce a measure of validity for the multiple-choice tests, but also to try to detect any increasing trends that might exist in these correlation coefficients as the school examinations are set more and more in the image of the final matriculation papers. Individual items are also examined with respect to their ability to discriminate between the able and less able groups of pupils as these emerge from the more advanced school examination papers.

5.3.1 Correlation of the Multiple-choice Tests with School Examinations

5.3.1.1 Standard nine candidates' scores:

During the month of May, Cape Peninsula schools hold their mid-year examinations. Papers are set internally and cover the work completed in class from the beginning of the school year. The examination results of the Higher Grade candidates under observation were obtained from the official school records of schools H, A, B, D and E.

These latest scores were correlated with five sets of previously obtained scores for these pupils, viz., two multiple-choice tests, written at the end of 1983 and 1984, the standard seven and standard eight promotion examinations also written at the end of 1983 and 1984, and the mid-year standard eight examination of 1984. In the case of school D, it will be remembered that there was no score on the 1984 multiple-choice test.

The Product-Moment correlation coefficient, r, was calculated for each pair of test or examination results sets to produce the correlation matrices given in Tables 64 to 68.

TABLE 64: TABLE OF CORRELATIONS BETWEEN SIX SETS OF TEST AND HIGHER GRADE EXAMINATION RESULTS

SCHOOL H:	BOYS			N = 97		
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984	Mid-year HG Exam 1985
Revised Test 1983	1	+,74	+,67	+,71	+,69	+,73
New Test 1984		1	+,61	+,64	+,68	+,75
Std 7 Exam 1983			1	+,75	+,81	+,75
Mid-year HG Exam 1984				1	+,89	+,83
Std 8 HG Exam 1984					1	+,90
Mid-year HG Exam 1985						1

In the School H (boys), the multiple-choice tests, particularly, the later of the two, show an encouraging increase in their correlation with successive school examinations. The standard seven examination shows a strong positive correlation with later school examinations but it is impossible to discern any upward or downward trend from the figures. The school examinations at School H show a quite remarkable degree of consistency.

TABLE 65: TABLE OF CORRELATIONS BETWEEN SIX SETS OF TEST AND HIGHER GRADE EXAMINATION RESULTS

SCHOOL A:	GIRLS				N = 54	
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984	Mid-year HG Exam 1985
Revised Test 1983	1	+,74	+,72	+,60	+,73	+,66
New Test 1984		1	+,55	+,49	+,70	+,61
Std 7 Exam 1983			1	+,81	+,81	+,85
Mid-year HG Exam 1984				1	+,74	+,74
Std 8 HG Exam 1984					1	+,82
Mid-year HG Exam 1985						1

In school A (girls), the correlation coefficients of the multiple-choice tests fluctuate. The New Test coefficients show a slight tendency to increase, but the pattern is disturbed by the mid-year examinations. The standard seven school examination, on the other hand, shows a very strong positive correlation with later examinations and, moreover, the coefficient shows an increase in the mid-year standard nine examination. School examinations, in school A, are also quite consistent one with the other, but not quite so remarkably so as in school H; this could be due to the smaller number of candidates involved with a more restricted range of talent.

TABLE 66: TABLE OF CORRELATIONS BETWEEN SIX SETS OF TEST AND HIGHER GRADE EXAMINATION RESULTS

SCHOOL B:	GIRLS				N = 33	
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984	Mid-year HG Exam 1985
Revised Test 1983	1	+,70	+,59	+,45	+,59	+,38
New Test 1984		1	+,57	+,60	+,71	+,66
Std 7 Exam 1983			1	+,71	+,78	+,36
Mid-year HG Exam 1984				1	+,86	+,38
Std 8 HG Exam 1984					1	+,52
Mid-year HG Exam 1985						1

In school B (girls), the revised test correlation coefficients fluctuate, but the coefficients of the New Test show an encouraging tendency to increase with successive school examinations. The coefficients of the standard seven school examination first increase, and then decrease sharply; other school examinations show less consistency one with the other than in schools H and A. This could be due to the relatively small number of candidates involved at school B, again with a more restricted range of talent.

TABLE 67: TABLE OF CORRELATIONS BETWEEN SIX SETS OF TEST AND HIGHER GRADE EXAMINATION RESULTS

SCHOOL E:	CO-EDUCATIONAL			N = 58		
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984	Mid-year HG Exam 1985
Revised Test 1983	1	+,76	+,63	+,59	+,76	+,46
New Test 1984		1	+,59	+,56	+,69	+,41
Std 7 Exam 1983			1	+,76	+,80	+,65
Mid-year HG Exam 1984				1	+,84	+,64
Std 8 HG Exam 1984					1	+,79
Mid-year HG Exam 1985						1

In school E (boys and girls), the correlation coefficients of the multiple-choice tests fluctuate and no trends can be discerned. The same is true for the standard seven school examination, although individual coefficients are more strongly positive for these than for either of the multiple-choice tests.

TABLE 68: TABLE OF CORRELATIONS BETWEEN FIVE SETS OF TEST AND HIGHER GRADE EXAMINATION RESULTS

SCHOOL D: CO-EDUCATIONAL N = 44

	Revised Test 1983	Std 7 Exam 1983	Mid-year HG Exam 1984	Std 8 HG Exam 1984	Mid-year HG Exam 1985
Revised Test 1983	1	+,44	+,33	+,36	+,12
Std 7 Exam 1983		1	+,61	+,70	+,45
Mid-year HG Exam 1984			1	+,74	+,41
Std 8 HG Exam 1984				1	+,65
Mid-year HG Exam 1985					1

In school D (boys and girls) the multiple-choice test shows a definite decreasing trend in its correlation with the successive school examinations. The internal school examinations themselves seem to lack the same degree of consistency that has been noted in schools H, A and E; however, once again the number of pupils involved here is rather small, and since the pupils at school D were subjected to a fairly stringent selection process before being admitted to the Higher Grade course, the range of talent represented here is probably quite restricted.

From the Tables 64 to 68 it is extremely difficult to draw any conclusions at all about the ability of the multiple-choice tests to predict performance in Higher Grade Mathematics - in one school the correlation coefficients increase, in one they decrease and in the other schools they fluctuate. The mid-year examinations, based as they are on small amounts of work, seem to disturb the relationships between tests and the examinations and, even, among the examinations themselves.

In order to determine exactly what effect the mid-year examinations have on the pattern of correlation coefficients it would be necessary to set up the same kind of matrices for the standard ten pupils. It must be remembered that, although more examination history is available for these candidates, they wrote only one multiple-choice test and they took this test at the end of their standard nine year. In other words, the multiple-choice test presented much less of a challenge to these candidates than it did for the younger pupils, and there was hardly any time lapse between the writing of the multiple-choice test and the senior course examinations.

5.3.1.2 Standard ten candidates' scores:

Previous examination results for these pupils were traced back, as far as the end-of-year standard seven promotion examination, through the official school records of two schools only, at this stage, viz., schools H and A. As before, the Product-Moment correlation coefficients were calculated for each pair of results sets to produce tables of correlations. These are the Tables 69 and 70.

TABLE 69: TABLE OF CORRELATIONS OF SIX SETS HIGHER GRADE EXAMINATION RESULTS AND THE 1984 MULTIPLE-CHOICE TEST

SCHOOL H:	BOYS				N = 82		
	New Test 1984	Std 7 Exam 1982	May Exam 1983	Std 8 Exam 1983	May Exam 1984	Std 9 Exam 1984	May Exam 1985
New Test 1984	1	+,46	+,41	+,69	+,73	+,70	+,70
Std 7 Exam 1982		1	+,65	+,71	+,52	+,50	+,61
May Exam 1983			1	+,63	+,53	+,55	+,58
Std 8 Exam 1983				1	+,82	+,77	+,79
May Exam 1984					1	+,87	+,83
Std 9 Exam 1984						1	+,90
May Exam 1985							1

Once again, the tendency for the correlation between the multiple-choice test and the successive school examinations is seen to be an increasing one among the boys of school H. The correlation between the standard seven examinations and later examinations showed a decrease up to the end of the standard nine year, but shows an increase in May of the standard ten year. However, at this stage, the multiple-choice test would be a slightly better predictor than the standard seven examination.

TABLE 70: TABLE OF CORRELATIONS OF SIX SETS HIGHER GRADE EXAMINATION RESULTS AND THE 1984 MULTIPLE-CHOICE TEST

SCHOOL A:			GIRLS		N = 45		
	New Test 1984	Std 7 Exam 1982	May Exam 1983	Std 8 Exam 1983	May Exam 1984	Std 9 Exam 1984	May Exam 1985
New Test 1984	1	+,57	+,65	+,60	+,56	+,47	+,58
Std 7 Exam 1982		1	+,81	+,82	+,68	+,67	+,58
May Exam 1983			1	+,87	+,80	+,76	+,73
Std 8 Exam 1983				1	+,83	+,83	+,71
May Exam 1984					1	+,84	+,84
Std 9 Exam 1984						1	+,86
May Exam 1985							1

Among the girls of school A, the correlation coefficients of the multiple-choice test and the school examinations remain fairly constant over-all except for a peak in the standard 8 May examination and a low point in the end-of-year standard nine examination. The coefficients of the standard seven examination, in contrast, show a marked tendency to decline; however, at this stage, the standard seven examination would make just as efficient a predictor as the multiple-choice test.

As yet there has been no conclusive evidence that the multiple-choice test would make a better predictor of success in the Senior Certificate examination than would the standard seven school examination. Certainly for the boys of school H, the multiple-choice test seems to be more efficient, but this is not the case among the girls of school A. Again the question arises, is this indicative of some underlying difference between the mathematical ability of boys and girls, the predictability of boys and girls or, even, the teaching of the different sexes? Or, could it be, that we are trying to predict the largely unpredictable? Could it be that there are so many uncontrollable variables that precision prediction is impossible? Before drawing any conclusions on these issues, it was decided to gather one last set of examination results for both the standard nine and standard ten pupils, viz., the end-of-year standard nine Higher Grade examination and the final Cape Senior Certificate Higher Grade examination for 1985. It was hoped that obtaining scores on an external, and therefore, common examination, for pupils of three different schools might shed more light on the predictive value of the multiple-choice test. The outcome of this further investigation is discussed in Sections 5.4 and 5.5 of this chapter.

5.3.2 Analysis of Multiple-Choice Test Items Using the 1985 Mid-year Examinations as a Basis for Grouping Pupil Scores

The issue of whether a certain type of item makes an optimum efficiency discriminator between the able and less able candidates still has to be resolved. The previous item analysis was conducted at the end of the standard eight year, and it was thought that a possible reason for the lack of evidence to support the notion, that the items involving the more complex mental processes would make better discriminators, was that the standard eight syllabus material and examination did not place sufficient emphasis on higher level work and tended, still, to concentrate on manipulative skills. It was considered worthwhile to carry out one more item analysis, this time using the mid-year standard nine Higher Grade examination results as a basis for separating pupil scores into quartile groups.

The items analysed were those included in the most recent multiple-choice test, which has been referred to as the New Test. The pupil scores involved will be those from schools H, A, B and E.

In order to reduce the physical bulk of this study the item analysis grids are not included, but levels of difficulty calculated are summarised in Table 71 and indices of discrimination are listed in Table 72.

5.3.2.1 Item difficulty levels, p:

Although, at this point, it is the discriminating index of each item that is of interest, it is, nevertheless, necessary to review the item difficulty values, as it is known that the difficulty level of an item has a bearing on its power to discriminate. In schools H, A and B, relatively few candidates have been lost since the standard eight examination and the recorded levels of item difficulty, p, should not have changed much; however, in school E, the number of Higher Grade candidates has been reduced from 86, at the end of the standard eight year, to 58 at the time of their mid-year examination. It seems reasonable that those pupils, who dropped out of the Higher Grade course, were the weaker students, and so their absence may have caused a considerable shift in the p-levels for this school; this in turn might affect the indices of discrimination of the items.

Table 71 lists the item difficulty levels, p, for the four schools, H, A, B and E.

The upward shift in the mean p-levels for schools B and E, indicates that for the pupils remaining in the Higher Grade course during the mid-year examination, the test items had presented rather less difficulty. This could result in a decrease in the discriminating power of individual items.

5.3.2.2 Index of Discriminating Power, D:

Indices of discrimination, D, were calculated for each item in exactly the same way as before, i.e., U represents the upper 50% of scores and L the lower 50% of scores. For ease of comparison, the D-indices obtained previously, in which grouping of upper and lower scores was based on the multiple-choice test, is repeated in the Table 72.

TABLE 71: ITEM DIFFICULTY LEVELS FOR THIRTY ITEMS OF THE NEW TEST USING THE MID-YEAR STANDARD NINE EXAMINATION AS A BASIS FOR GROUPING PUPIL SCORES

ITEM	SCHOOL H p	SCHOOL A p	SCHOOL B p	SCHOOL E p
1	0,74	0,70	0,88	0,78
2	0,51	0,36	0,31	0,53
3	0,63	0,49	0,41	0,29
4	0,79	0,64	0,78	0,69
5	0,43	0,66	0,47	0,36
6	0,77	0,85	0,56	0,78
7	0,48	0,34	0,32	0,31
8	0,67	0,27	0,31	0,56
9	0,50	0,42	0,22	0,41
10	0,79	0,92	0,81	0,60
11	0,68	0,53	0,59	0,62
12	0,49	0,36	0,65	0,31
13	0,54	0,47	0,56	0,51
14	0,79	0,64	0,69	0,55
15	0,93	0,91	0,77	0,86
16	0,81	0,70	0,63	0,79
17	0,66	0,53	0,47	0,43
18	0,54	0,30	0,41	0,45
19	0,74	0,89	0,91	0,74
20	0,77	0,58	0,56	0,63
21	0,85	0,79	0,56	0,71
22	0,78	0,68	0,66	0,88
23	0,73	0,62	0,50	0,69
24	0,91	0,77	0,72	0,72
25	0,71	0,66	0,53	0,64
26	0,65	0,49	0,44	0,64
27	0,81	0,64	0,72	0,52
28	0,69	0,55	0,34	0,60
29	0,30	0,34	0,09	0,32
30	0,65	0,51	0,44	0,64

Figure 16

DISTRIBUTION OF DIFFICULTY LEVELS, p, OF NEW TEST ITEMS IN SCHOOL H

< ,16		0
,16 - ,25		0
,26 - ,35	*	1
,36 - ,45	*	1
,46 - ,55	******	6
,56 - ,65	***	3
,66 - ,75	********	8
,76 - ,85	*********	9
> ,85	**	2

Mean = 0,68 Standard Deviation = 0,15

Figure 17

DISTRIBUTION OF DIFFICULTY LEVELS, p, OF NEW TEST ITEMS IN SCHOOL A

< ,16		0
,16 - ,25		0
,26 - ,35	****	4
,36 - ,45	***	3
,46 - ,55	*******	7
,56 - ,65	*****	5
,66 - ,75	*****	5
,76 - ,85	***	3
> ,85	***	3

Mean = 0,59   Standard Deviation = 0,18

Figure 18

DISTRIBUTION OF DIFFICULTY LEVELS, p, OF NEW TEST ITEMS IN SCHOOL B

< ,16	*	1
,16 - ,25	*	1
,26 - ,35	****	4
,36 - ,45	****	4
,46 - ,55	****	4
,56 - ,65	*******	7
,66 - ,75	****	4
,76 - ,85	***	3
> ,85	**	2

Mean = 0,54   Standard Deviation = 0,20

Figure 19

DISTRIBUTION OF DIFFICULTY LEVELS, p, OF NEW TEST ITEMS IN SCHOOL E

< ,16		0
,16 - ,25		0
,26 - ,35	****	4
,36 - ,45	****	4
,46 - ,55	****	4
,56 - ,65	********	8
,66 - ,75	*****	5
,76 - ,85	***	3
> ,85	**	2

Mean = 0,59   Standard Deviation = 0,17

TABLE 72: COMPARISON OF D-INDICES FOR SCHOOLS A AND H BASED ON THE GROUPINGS ACCORDING TO THE NEW TEST AND THE MID-YEAR STANDARD NINE EXAMINATION

ITEM	NEW M-C TEST GROUPED		MID-YEAR EXAM GROUPED	
	School A	School H	School A	School H
1	+0,03	+0,42	+0,11	+0,40
2	+0,10	+0,10	-0,04	+0,10
3	+0,24	+0,40	+0,23	+0,04
4	+0,24	+0,04	+0,15	-0,06
5	+0,34	+0,44	+0,34	+0,48
6	-0,03	+0,28	+0,11	+0,33
7	+0,41	+0,42	+o,23	+0,13
8	+0,45	+0,26	+0,30	+0,38
9	+0,31	+0,34	0,00	+0,46
10	0,00	+0,04	-0,04	+0,04
11	+0,21	+0,42	0,00	+0,27
12	+0,03	+0,38	-0,04	+0,23
13	+0,21	+0,30	+0,19	+0,13
14	+0,39	+0,26	+0,23	+0,25
15	+0,10	+0,18	+0,15	+0,06
16	+0,34	+0,24	+0,26	+0,08
17	+0,31	+0,50	-0,08	+0,36
18	+0,24	+0,34	+0,15	+0,33
19	-0,03	+0,42	+0,04	+0,31
20	+0,45	+0,32	+0,19	+0,13
21	+0,10	+0,22	+0,08	+0,13
22	+0,21	+0,20	+0,08	+0,17
23	+0,28	+0,42	+0,46	+0,25
24	+0,17	+0,14	+0,15	+0,10
25	+0,17	+0,34	+0,26	+0,17
26	+0,21	+0,48	+0,15	+0,21
27	+0,31	+0,14	+0,23	+0,04
28	+0,31	+0,50	+0,04	+0,33
29	-0,07	+0,22	+0,15	+0,09
30	+0,24	+0,30	+0,26	+0,13
Highest D	+0,45	+0,50	+0,46	+0,48
Lowest D	-0,07	+0,04	-0,08	-0,06
Median D	+0,23	+0,31	+0,15	+0,17

Indices of discrimination, D, remain much the same for schools A and H with only a small decrease in the median index of D evident. In both of these schools, the sample of pupils was practically unchanged from November 1984 to May 1985, and the latest examination has obviously caused little disturbance to the merit order of candidates, suggesting that the November examination and the May examination call forth much the same abilities, or that, even if the abilities called forth are different, the same ranking of pupils occurs. It was mentioned previously that it could be expected that top scoring pupils in the junior Mathematics course would probably be the most successful pupils in the senior course too, and that the least scoring pupils would, similarly, be least successful in the more advanced course; however, it was hoped that more precise information about that broad band of "average" candidates and their ability to succeed would emerge as higher levels of mathematical thinking and complexity were presented to them. Thus far, there has been no evidence that such information can be obtained.

In schools B and E, greater changes in pupil sample occurred between November 1984 and May 1985, namely, a 12,8% loss of pupils in school B and a 32,6% loss in school E. In the girls' school B, the effectiveness of test items as discriminators showed an increased range of values, as well as, an increase in the median index of D. The items contributing most to this increased efficiency were numbers 3, 4, 7, 8, 14, 16, 20, 23, 26, 28 and 30; of these none are classified as manipulation types, two are routine problems (18,2%), four are comprehension types (36,4%) and five are classified as non-routine problems (45,4%). In the co-educational school, E, the efficiency of items as discriminators also shows an increase in the range of D-indices, but a decrease in the median index of D. Contributing most to this decline in effectiveness were items 1, 2, 4, 5, 7, 14, 16, 17, 19, 23, 28 and 30; of these four were classified as manipulation types (32,3%), three were routine problems (25%), three were comprehension types (25%) and two were non-routine problems (16,7%). There is a temptation to conclude that the items involving the higher mental processes are improving in efficiency compared to those involving mere manipulation or routine problem types; however, this is not supported by the shifts in D-indices recorded in schools A and H, e.g., in school H, 50% of items showing an increase in efficiency were manipulation types, and 50% of items showing a decrease in their D-indices were non-routine problems. It seems, then, that even midway through the standard nine year, the test items involving higher levels of mathematical thought cannot be said to make consistently better discriminators between the able and less able than those involving more mechanical or routine skills. Of course, this part of the analysis is haunted by the spectre of the previously encountered classification difficulties, namely, that individual items might be classified differently from school to school because of the different teaching styles, practices and philosophies of all the teachers involved. This factor alone could obscure any trends that do exist; it is impossible to tell from the data available here.

For the sake of completeness, and to make quite certain that no correspondence was overlooked, the indices of discrimination of the four schools were ranked and Kendall's coefficient of concordance was calculated. The ranks and the value of W are listed in Table 73. As expected, no significant correspondence was forthcoming.

TABLE 73: ITEMS RANKED ACCORDING TO MAY 1985 STANDARD NINE HIGHER GRADE SCHOOL EXAMINATION

ITEM	SCHOOLS			
	A	H	B	E
1	11,5	28	14	6,5
2	3	8,5	14	13,5
3	22,5	3	29	24,5
4	15,5	1	18,5	6,5
5	29	30	18,5	18,5
6	11,5	24	6,5	28,5
7	22,5	12	24	3
8	28	27	27,5	16
9	5,5	29	18,5	30
10	3	3	2	11
11	5,5	21	9,5	22
12	3	18	14	16
13	19,5	12	22	20
14	22,5	19,5	14	3
15	15,5	5	14	9
16	26	6	27,5	1
17	1	26	9,5	18,5
18	15,5	24	25,5	26,5
19	7,5	22	18,5	11
20	19,5	12	22	16
21	9,5	12	6,5	11
22	9,5	15,5	1	22
23	30	19,5	22	13,5
24	15,5	8,5	4,5	26,5
25	26	15,5	9,5	24,5
26	15,5	17	30	28,5
27	22,5	3	4,5	3
28	7,5	24	25,5	6,5
29	15,5	7	9,5	22
30	26	12	3	6,5

Kendall's Coefficient of Concordance, W = 0,31

Thus, Chi-square = 35,96 with 29 degrees of freedom.

Although there is no statistically significant association, as before in Section 4.3.4, it may be noted that W = 0,31 does show some noticeable degree of association.

In the previous chapter, a simple points system was devised to enable items to be categorised as "good", "fair" or "poor" discriminators. This method was employed once more, to determine to what extent item categories would change or remain stable, after pupils had been exposed to a portion of the syllabus material directly examinable by the final Senior Certificate examination. Two points were assigned for items ranking in the top ten discriminators, one point for those ranking between 10 and 20 and no points for the rest. Scores from each school were added to give the total scores, and then items were classified as "good" if they obtained six or more points, "fair" if they obtained three, four or five points and "poor" if they obtained less than three points. The classification is given in Table 74.

TABLE 74: NEW TEST ITEMS CLASSIFIED ACCORDING TO THEIR EFFICIENCY AS DISCRIMINATORS AMONG HIGHER GRADE STANDARD NINE PUPILS IN FOUR SCHOOLS

Good	Fair	Poor
8;18	6;7;9;20;25	2;4;15;21;27
3;5;13;23;26	1;11;14;16;19;28	10
	12;17;22;24;29;30	

These groupings were analysed according to the occurrence of the various mental processes involved in the items. The result is given in Table 75.

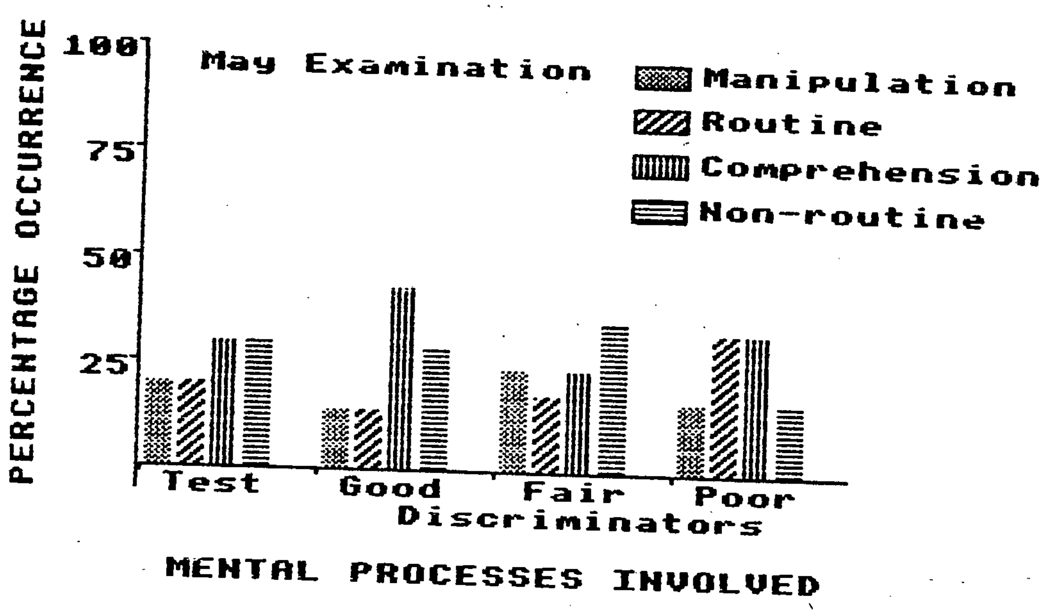
TABLE 75: OCCURRENCE OF THE MENTAL PROCESSES AMONG THE GOOD, FAIR AND POOR DISCRIMINATORS AS DETERMINED BY THE GROUPING OF PUPILS OBTAINED FROM THE MID-YEAR STANDARD NINE EXAMINATION

	"Good"		"Fair"		"Poor"	
	No.	%	No.	%	No.	%
Manipulation	1	14,3	4	23,5	1	16,7
Routine problem	1	14,3	3	17,6	2	33,3
Comprehension	3	42,8	4	23,5	2	33,3
Non-Routine problem	2	28,6	6	35,3	1	16,7

The occurrence of the various mental processes among the different categories of discriminator may be compared with their percentage occurrence in the multiple-choice test by means of the bar graph shown in Figure 20.

Figure 20

DISTRIBUTION OF MENTAL PROCESSES AMONG THE DIFFERENT LEVELS OF DISCRIMINATOR



Although a few individual items have changed categories, either improving slightly or deteriorating slightly as discriminators, the over-all distribution of the mental processes among the different categories has not changed much. As before, there appears to be a preponderance of comprehension and non-routine problems among the "good" discriminators and "fair" discriminators, respectively, but not to any statistically significant extent. There is no evidence to suggest that this distribution of mental processes is any different from the distribution of the various item types in the construction of the test.

Earlier, the question was posed, "Would the distribution of mental processes change as pupils progressed further into the senior course?". It was thought that the standard eight syllabus might not provide the same degree of challenge as the standard nine and ten syllabus, and that this might have been obscuring the true efficacy of the items involving the higher mental processes. The foregoing item analysis based on the groupings of pupil scores yielded by the mid-year school examination has not produced results much different from those produced by the previous item analysis, and therefore, it must be concluded that comprehension and non-routine problem types do not make better discriminators, between the able and less able to cope with Senior Certificate Mathematics, than do the type of items involving manipulation and routine problems.

No further item analyses will be undertaken, but, in the sections that follow, the progress of the standard nine pupils will be recorded up to the end-of-year examination. The progress of the standard ten pupils will be pursued to the final Cape Senior Certificate examination also written at the end of the 1985 academic year.

5.4 THE 1985 END-OF-YEAR STANDARD NINE SCHOOL EXAMINATIONS

During November, Cape Peninsula schools conduct their promotion examinations. The results of candidates in different schools have not previously been comparable because of the great differences possible in the setting of internal examinations. However, at the end of the standard nine year, it is probably safe to say that most teachers would have covered at least half of the two-year syllabus, examination questions included in the papers would be similar to the kinds of problem presented in past Senior Certificate papers, and that the tests should provide a fair challenge to pupils to ensure that they are ready for promotion to the final year of study. For these reasons the end-of-year standard nine promotion examinations of the different schools, although internally set, are probably more comparable, one with the other, than examinations in standards seven and eight have been.

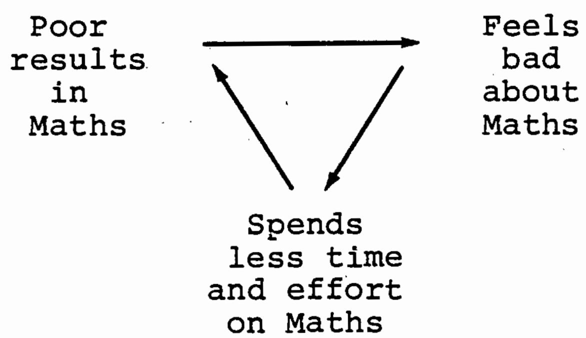
The examination results of all candidates were obtained from the official school records. As mentioned in a previous section, the sample size of higher grade candidates has been reduced still further since the May examination.

TABLE 76: PERCENTAGE REDUCTION IN SAMPLE SIZE FROM MAY TO NOVEMBER OF THE STANDARD NINE HIGHER GRADE GROUPS IN FIVE SCHOOLS

SCHOOL	DECREASE
H	11,3%
A	9,3%
B	8,8%
D	4,5%
E	37,9%

Judging by the above figures and the reduction in numbers of pupils noted earlier in the year, it seems that quite a substantial proportion of candidates do feel that the pressure of the Higher Grade syllabus is too great during standard nine and, consequently, decide to change to the less demanding Standard Grade course. It has been the writer's experience that the fortunes of this "Higher Grade drop out" group are rather mixed; some pupils seem to derive great benefit from a year or two in the

Higher Grade course, and, even though they may have had a desperate struggle to keep up, eventually, when they make the change in grade, do exceedingly well in the Standard Grade course. Other pupils, in contrast, show no sign of benefit from the more demanding course, and continue to achieve at the same low level when they switch grades. Many possible explanations for these phenomena come to mind, e.g., intellectually very sound students, with no particular interest in the theoretical bias of the Higher Grade course, may find the practical approach of the Standard Grade course more to their taste and, since their problem was not one of inadequate understanding, they bring their deeper knowledge and increased interest into the less demanding course of study, experience greater satisfaction and, thus, achieve better results. Weaker students, on the other hand, may be so confused and demoralised from their experience with the Higher Grade course, that their feelings of hopeless inadequacy and lack of self-confidence preclude success in Mathematics, even Standard Grade Mathematics - they have spent too long in the vicious "mathematical triangle":



Still other pupils may consider the Standard Grade course to be so much less demanding intellectually and content-wise, that they feel at liberty to reduce their own efforts substantially, and so achieve no greater success than they had managed before on the more difficult Higher Grade course.

It is with these latter two groups of "Higher Grade drop outs" that this study is principally concerned - these are the pupils who need to be identified early and persuaded that the Standard Grade course holds more hope of maximum achievement for them. Doggedly persevering with the Higher Grade course, when it is obvious that they cannot succeed is detrimental not only to their own self-image but also places restraints on those pupils who can achieve success. Very often, bright pupils become bored and impatient with the low-level explanations that must be offered to the stragglers, and pupils of marginal ability, i.e., those pupils who are capable of success with a little extra help, are deprived of the attention of the teacher in favour of the hopeless cases.

Returning to the November examinations, it was suggested earlier that any predictive validity, possessed by the multiple-choice tests, should be revealed by a tendency for the correlation between these tests and the successive school examinations to increase. Accordingly, the Product-Moment correlation coefficient, r, was calculated for each pair of test or examination results from standard seven to the end of standard nine, using only the scores of those pupils still involved in the Higher Grade course. These values of r are recorded in the correlation matrices given in Tables 77 to 81.

TABLE 77: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS

SCHOOL H:			BOYS		N = 86		
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1	+,72	+,63	+,67	+,66	+,67	+,63
New Test 1984		1	+,53	+,58	+,59	+,66	+,58
Std 7 Exam 1983			1	+,73	+,78	+,70	+,67
May Exam 1984				1	+,89	+,79	+,78
Std 8 Exam 1984					1	+,86	+,86
May Exam 1985						1	+,90
Std 9 Exam 1985							1

In school H, the correlation coefficients between the multiple-choice tests and successive school examinations remain fairly constant or, in the case of the New Test, show a slight tendency to increase. The standard seven examination coefficients, although ultimately still more strongly positive than those of the multiple-choice tests, definitely tend to decrease.

TABLE 78: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS

SCHOOL A:	GIRLS			N = 49			
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1	+,75	+,74	+,61	+,74	+,69	+,73
New Test 1984		1	+,55	+,50	+,72	+,61	+,65
Std 7 Exam 1983			1	+,79	+,78	+,80	+,83
May Exam 1984				1	+,72	+,73	+,77
Std 8 Exam 1984					1	+,81	+,81
May Exam 1985						1	+,82
Std 9 Exam 1985							1

The correlation coefficients of the multiple-choice tests again remain fairly constant in the case of the Revised Test, and show a slight tendency to increase in the case of the New Test. However, the coefficients of the standard seven examination show a marked inclination to increase.

TABLE 79: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS

SCHOOL B:	GIRLS			N = 31			
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1	+,71	+,59	+,46	+,61	+,38	+,63
New Test 1984		1	+,56	+,60	+,72	+,65	+,73
Std 7 Exam 1983			1	+,71	+,78	+,34	+,46
May Exam 1984				1	+,86	+,36	+,55
Std 8 Exam 1984					1	+,51	+,64
May Exam 1985						1	+,88
Std 9 Exam 1985							1

In school B, the correlation coefficients of the multiple-choice tests with successive school examinations show sharply rising tendencies, particularly in the case of the New Test. In contrast, the coefficients of the standard seven examination decline sharply. These were the expected trends which, unfortunately, were not as clearly discernible in any of the other schools.

TABLE 80: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS

SCHOOL E:	CO-EDUCATIONAL			N = 36			
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1	+,81	+,55	+,53	+,74	+,38	+,50
New Test 1984		1	+,59	+,55	+,73	+,37	+,54
Std 7 Exam 1983			1	+,76	+,77	+,64	+,64
May Exam 1984				1	+,84	+,66	+,69
Std 8 Exam 1984					1	+,74	+,80
May Exam 1985						1	+,87
Std 9 Exam 1985							1

In school E, the correlation coefficients between the multiple-choice tests and successive school examinations fluctuate considerably, but over-all show a decreasing pattern. The coefficients of the standard seven examination also tend to decrease but, ultimately, are more strongly positive than those of either of the multiple-choice tests.

TABLE 81: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS

SCHOOL D:		CO-EDUCATIONAL			N =	42	
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1		+,44	+,34	+,39	+,20	+,30
New Test 1984							
Std 7 Exam 1983			1	+,59	+,70	+,49	+,50
May Exam 1984				1	+,72	+,43	+,57
Std 8 Exam 1984					1	+,64	+,71
May Exam 1985						1	+,68
Std 9 Exam 1985							1

In school D, the correlation coefficients between the Revised Test and successive school examinations show a fairly steady decline. The coefficients of the standard seven examination fluctuate, but over-all tend to decrease. In this school, coefficients are noticeably weaker than in the other schools, perhaps owing to the rather restricted range of talent.

The expected trends in the pattern of correlation coefficients between the multiple-choice tests and successive school examinations, and between the standard seven school examination and later school examinations, namely, that the multiple-choice scores should correlate increasingly closely with the examinations, while the standard seven examination scores should correlate less and less closely with later examinations, was only fully realised in school B, where the multiple-choice tests proved to be better predictors of success in later Higher Grade examinations than did the standard seven school examination. In school H, the expected pattern, though less marked, was noted, but the standard seven examination, in this school, remains a more accurate predictor of achievement in the senior years than the multiple-choice tests. In school A, although the multiple-choice test correlation coefficients remained fairly constant and fairly strongly positive, they could in no way compete with the increasing strength of the correlation between the standard seven school examinations and later school examinations. The position in the co-educational schools, E and D, is less clear as correlation coefficients tend to fluctuate to a greater extent. However, in the end, it seems that the standard seven examination, again, is the better predictor of achievement in the senior years, although, in both schools, these examinations also tend to correspond less and less well with the later examinations.

To sum up: Using the scores of these standard nine pupils on both versions of the multiple-choice test and successive school examinations from standard seven to standard nine, measures of validity for the tests have been established. The strength of the correlation coefficients is quite encouraging (school D excepted), although less dependable than is desired owing to the relatively small number of pupils left in the study. In other words, it seems that the multiple-choice tests do have predictive validity for the Higher Grade course in Mathematics. However, the standard seven examination in all of the schools, except B, proved to be a better predictor still. This phenomenon was not expected, as it was thought that this examination would, in most cases, concentrate on mechanical, drilled skills which could not be expected to draw out the mathematical insight required to succeed in the Higher Grade course. Four possible explanations spring to mind:

1. The standard seven examinations set at schools H, A, D and E contain considerable amounts of material demanding insight and understanding.

2. Teachers are unconsciously teaching toward their examination papers, so that items normally involving the higher mental processes are reduced to drilled, routine items. This is difficult to avoid as teachers are very aware of the importance of good results in Mathematics to the pupils. Very poor class averages are often a bone of contention at parents' meetings and pupils have become expert at subtly "pumping" teachers just before examinations.

3. Success in Higher Grade Mathematics does not require ability alone, but also motivation, sustained interest in the subject, determination and dedication to hard work. Pupils who do well in standard seven probably have these

qualities, and these are sufficient to carry them through successfully to the final matriculation examination. Pupils who have performed well in junior Mathematics possibly have managed to build sufficient self-confidence to allow them to weather the mathematical storms they encounter when they enter the senior Higher Grade course.

4. The writer first became concerned about the problem of the lack of success among certain pupils in a school not unlike school B, i.e., a girls' school, not particularly noted for high academic achievement - a school whose pupils were fairly "average" mathematicians. Perhaps the problem only exists for the middle stratum of high school girls. In school A, also a girls' school, but particularly noted for the achievement of its pupils, the multiple-choice tests did not perform as well as predictors (compared to the standard seven examination) as in the case of school B.

There is no way of determining whether any of the above possibilities have any validity, with the exception, perhaps, of number 2. When pupils write the, externally set, final Senior Certificate paper there is no possibility of unconscious coaching on the part of the teacher. In the next section, when the progress of the standard ten pupils is investigated, correlation coefficients will be calculated between the multiple-choice test and the 1985 Cape Senior Certificate examination, as well as the standard seven school examinations and this external examination; it may be possible, then, to detect some common decrease in the high degree of correspondence between the school examinations and this latest examination.

Before the investigation passes on to the standard ten pupils, it may be helpful to summarise the foregoing tables of correlation so that the situation may be more easily and clearly appreciated. In the summary Tables 82 to 84, correlation coefficients for mid-year examinations have been omitted to accentuate the upward or downward trends in the values of r. All tables appear on the same page so as to facilitate comparison.

TABLE 82: COMPARISON OF CORRELATION COEFFICIENTS BETWEEN THE REVISED TEST SCORES AND HIGHER GRADE SCHOOL EXAMINATION SCORES IN FIVE HIGH SCHOOLS

School	Type of School	N	MC Test vs Std 7	MC Test vs Std 8	MC Test vs Std 9
H	Boys	86	+0,63	+0,66	+0,63
A	Girls	49	+0,74	+0,74	+0,73
B	Girls	31	+0,59	+0,61	+0,63
D	Co-ed	42	+0,44	+0,39	+0,30
E	Co-ed	36	+0,55	+0,74	+0,50

TABLE 83: COMPARISON OF CORRELATION COEFFICIENTS BETWEEN THE NEW TEST SCORES AND HIGHER GRADE SCHOOL EXAMINATION SCORES IN FOUR HIGH SCHOOLS:

School	Type of School	N	MC Test vs Std 7	MC Test vs Std 8	MC Test vs Std 9
H	Boys	86	+0,53	+0,59	+0,58
A	Girls	49	+0,55	+0,72	+0,65
B	Girls	31	+0,56	+0,72	+0,73
E	Co-ed	36	+0,59	+0,73	+0,54

TABLE 84: COMPARISON OF CORRELATION COEFFICIENTS BETWEEN THE STANDARD SEVEN EXAMINATION SCORES AND HIGHER GRADE SCHOOL EXAMINATION SCORES IN FIVE HIGH SCHOOLS:

School	Type of School	N	Std 7 vs Std 8	Std 7 vs Std 9
H	Boys	86	+0,78	+0,67
A	Girls	49	+0,78	+0,83
B	Girls	31	+0,78	+0,46
D	Co-ed	42	+0,70	+0,50
E	Co-ed	36	+0,77	+0,64

5.5 THE 1985 CAPE SENIOR CERTIFICATE EXAMINATION

The 1985 matriculation candidates of three schools, namely, H, A and E wrote the new multiple-choice test at the end of their standard nine year. The examination results of the pupils at schools H and A were traced back through the school records as far as standard seven and their progress was followed through to the final matriculation examination at the end of 1985. A similar procedure was adopted for pupils attending school E, but, unfortunately, no mid-year examination results were recorded, and so the information for this school will appear in the summary only.

During the period between the May examination and the final examination, only two boys, in school H, changed from Higher Grade to Standard Grade, whilst, in the case of the school A, ten girls made a grade change, i.e. a further loss to the sample (in school A) of 22,2%. School E had a sample loss of 31 pupils during the course of the standard ten year, i.e., a loss of 54,4% of their Higher Grade pupils between the end of standard nine and the end of standard ten.

Examination results for pupils at these three schools were obtained from the official school records. A comparison of the actual examination statistics is undertaken in Chapter 6, but it must be mentioned here that the candidates are assigned symbols, not marks, in the final examination. In order to carry out statistical calculations, the symbols obtained by pupils have been translated as the mid-marks of the particular class-interval in question. The symbols allocated, the class-intervals represented by these and the mid-marks assigned to those symbols are listed in the Table 85.

TABLE 85: SYMBOLS ALLOCATED TO CANDIDATES IN THE CAPE SENIOR CERTIFICATE EXAMINATION, THEIR CLASS-INTERVALS AND MID-MARKS TO BE USED IN ALL CALCULATIONS

Symbol	Class-Interval	Mid-Mark
A	320 - 400	360
B	280 - 319	300
C	240 - 279	260
D	200 - 239	220
E	160 - 199	180
E*	140 - 159	150
F*	120 - 139	130
G	80 - 119	100

The * notation indicates failure to obtain a Higher Grade pass, but a Standard Grade pass was awarded. Candidates obtaining G symbols fail Mathematics completely.

In the Tables 86 to 88 the Product-Moment correlation coefficients between all possible pairs of examinations and the multiple-choice test are given for those pupils who wrote the Higher Grade Senior Certificate paper at the end of 1985 and whose scores could be traced back through the records to standard seven.

TABLE 86: TABLE OF CORRELATION COEFFICIENTS BETWEEN SEVEN SETS HIGHER GRADE EXAMINATION RESULTS AND THE 1984 MULTIPLE-CHOICE TEST

SCHOOL H:			BOYS			N = 80		
	New Test 1984	Std 7 Exam 1982	May Exam 1983	Std 8 Exam 1983	May Exam 1984	Std 9 Exam 1984	May Exam 1985	Final Exam 1985
New Test 1984	1	+,44	+,41	+,70	+,73	+,70	+,71	+,64
Std 7 Exam 1982		1	+,67	+,71	+,51	+,46	+,58	+,46
May Exam 1983			1	+,62	+,53	+,54	+,58	+,42
Std 8 Exam 1983				1	+,81	+,76	+,78	+,70
May Exam 1984					1	+,88	+,83	+,79
Std 9 Exam 1984						1	+,89	+,85
May Exam 1985							1	+,86
Final Exam 1985								1

In school H, the correlation coefficients between the multiple-choice test and successive examinations shows a very definite tendency to increase, while the standard seven examination coefficients show an over-all tendency to decrease. In the case of these boys, it seems that the multiple-choice test would make a more efficient predictor of success in the final examination than would the standard seven school examination.

TABLE 87: TABLE OF CORRELATION COEFFICIENTS BETWEEN SEVEN SETS HIGHER GRADE EXAMINATION RESULTS AND THE 1984 MULTIPLE-CHOICE TEST

SCHOOL A:				GIRLS			N = 35	
	New Test 1984	Std 7 Exam 1982	May Exam 1983	Std 8 Exam 1983	May Exam 1984	Std 9 Exam 1984	May Exam 1985	Final Exam 1985
New Test 1984	1	+,52	+,70	+,57	+,49	+,39	+,44	+,51
Std 7 Exam 1982		1	+,79	+,80	+,66	+,60	+,46	+,64
May Exam 1983			1	+,87	+,81	+,73	+,70	+,75
Std 8 Exam 1983				1	+,81	+,78	+,62	+,70
May Exam 1984					1	+,83	+,83	+,80
Std 9 Exam 1984						1	+,84	+,80
May Exam 1985							1	+,81
Final Exam 1985								1

In school A, the correlation coefficients between the multiple-choice test and later examinations fluctuate quite considerably, and remain at all times much less strongly positive than those of the school standard seven examination which, for these girls, appears to make the more efficient predictor of success in the final Senior Certificate examination.

It is extremely difficult to draw, from these results, any valid conclusions, since the tendencies in the two schools appear to pull in opposite directions. It has been mentioned before that, in many ways, the pupils in schools H and A are readily comparable, coming from the same residential area (in many cases, from the same families) and representative of much the same ability spectrum - both of these schools regularly produce several distinction candidates in the external examinations. Had it not been for the fact that the multiple-choice test seems to make a fairly good predictor for the standard nine girls of school B, there might have been justification for concluding that the multiple-choice test works well for boys, but less so for girls. Sex differences in the study of Mathematics are discussed further in the next Section. However, in an attempt to clarify the situation, a similar set of correlations was carried out for the boys and girls of the co-educational school, E. Unfortunately, the sample size at this school was reduced to only twenty-six candidates and no mid-year results were obtained from the school.

TABLE 88: TABLE OF CORRELATION COEFFICIENTS BETWEEN FOUR SETS HIGHER GRADE EXAMINATION RESULTS AND THE 1984 MULTIPLE-CHOICE TEST

SCHOOL E:		CO-EDUCATIONAL		N = 26	
	New Test 1984	Std 7 Exam 1982	Std 8 Exam 1983	Std 9 Exam 1984	Final Exam 1985
New Test 1984	1	+,43	+,59	+,71	+,57
Std 7 Exam 1982		1	+,85	+,75	+,53
Std 8 Exam 1983			1	+,87	+,62
Std 9 Exam 1984				1	+,64
Final Exam 1985					1

In school E, once again, the correlation coefficients of the multiple-choice test and successive school examinations show a definite tendency to increase - the decline at the end is also noted in the other two school matrices. In contrast, the standard seven examination coefficients show a decreasing trend, but ultimately, there is not much difference in the respective predictive abilities of the multiple-choice test and the standard seven examination for the pupils of school E.

In order to facilitate comparison among the three schools the information in Tables 86 - 88 is summarised in the Tables 89 and 90.

TABLE 89: COMPARISON OF CORRELATION COEFFICIENTS BETWEEN THE MULTIPLE-CHOICE TEST SCORES AND HIGHER GRADE SCHOOL EXAMINATION SCORES IN THREE HIGH SCHOOLS:

School	Type of School	N	MC Test vs Std 7	MC Test vs Std 8	MC Test vs Std 9	MC Test vs Final
H	Boys	80	+0,44	+0,70	+0,70	+0,64
A	Girls	35	+0,52	+0,57	+0,39	+0,51
E	Co-ed	26	+0,43	+0,59	+0,71	+0,57

TABLE 90: COMPARISON OF CORRELATION COEFFICIENTS BETWEEN THE STANDARD SEVEN SCHOOL EXAMINATION SCORES AND HIGHER GRADE SCHOOL EXAMINATION SCORES IN THREE HIGH SCHOOLS:

School	Type of School	N	Std 7 vs Std 8	Std 7 vs Std 9	Std 7 vs Final
H	Boys	80	+0,71	+0,46	+0,46
A	Girls	35	+0,80	+0,60	+0,64
E	Co-ed	26	+0,85	+0,75	+0,53

The general impression created by the results in Tables 89 and 90 tends to confirm the value of the multiple-choice test as a predictor of success in Senior Certificate Mathematics. However, it should be remembered that the standard ten pupils wrote the multiple-choice test only one year before their final matriculation examination, whereas there was a time lapse, between the standard seven paper and the final paper, of three years. Thus, there is really no conclusive evidence that the multiple-choice test has greater predictive validity for the final matriculation examination than does the standard seven examination.

In the previous section, it was suggested that one of the reasons for the particularly, and consistently, high degree of correspondence among all of the school examinations, might be that there was an element of unconscious coaching for internal examinations. Certainly, in the correlation matrices for the standard ten pupils, there is evidence of a decrease in the correlation coefficients of schools H and E. However, it is impossible to say how much this was due to the unaccustomed novelty of the paper - certainly, candidates frequently remark, on emerging from their final examination, that they found the paper "strange" or "different" in some vague, undefined way.

It is interesting that the correlation coefficient between the standard nine promotion examination and the final matriculation examination, in both schools H and A, is remarkably high (+0,85 and +0,80). This indicates that by the end of the standard nine year, the pupils of these two schools have been examined in such a way that they should have a fairly accurate idea of their standing with respect to the final examination.

Before proceeding to the concluding section of this dissertation, the items of the 1984 version of the multiple-choice test were subjected to scrutiny once more, this time on the basis of the combined Senior Certificate results of the pupils of schools H, A and E. Results were collected into one merit list - this was permissible since all pupils wrote a common examination - 29 pupils with median scores, i.e., 180 marks, were excluded, and then two-by-two contingency tables were constructed for each test item. The cells of the table represent those above and below the median, correct and incorrect responses to the item. Chi-square was calculated for each item, and a full record of these is given in Table 91. Of the thirty items in the test, only fourteen produced values of Chi-square large enough to allow the rejection of the null hypothesis:

> Ho: There is no difference in the ability to complete this item successfully of those who pass and those who fail the Senior Certificate examination.

Note that since the median was 180 marks, i.e., an E symbol, those below the median were, in fact, those who failed the Higher Grade course.

TABLE 91: SUMMARY OF RESULTS OF CHI-SQUARE TEST OF SIGNIFICANCE APPLIED TO ITEMS OF THE NEW MULTIPLE-CHOICE TEST WHERE THE 1985 CAPE SENIOR CERTIFICATE HIGHER GRADE MATHEMATICS GROUPING OF PUPILS WAS USED

ITEM	CHI-SQUARE	P	Ho
1	ft < 5		
2	0,248	0,62	
3	1,146	0,28	
4	5,007	0,03	Reject
5	4,278	0,04	Reject
6	14,897	0,00	Reject
7	6,422	0,01	Reject
8	2,256	0,13	
9	3,560	0,06	
10	0,181	0,67	
11	1,822	0,18	
12	3,510	0,06	
13	20,937	0,00	Reject
14	4,944	0,03	Reject
15	ft < 5		
16	0,019	0,89	
17	0,009	0,96	
18	13,521	0,00	Reject
19	12,336	0,00	Reject
20	6,415	0,01	Reject
21	0,529	0,47	
22	4,768	0,03	Reject
23	2,306	0,13	
24	2,450	0,12	
25	9,910	0,00	Reject
26	0,796	0,37	
27	0,179	0,67	
28	17,916	0,00	Reject
29	10,122	0,00	Reject
30	8,762	0,00	Reject

The items for which the null hypothesis could be rejected were, in order of merit: Items 13, 28, 6, 18, 19, 29, 25, 30, 7, 20, 4, 14, 22 and 5. The distribution of the mental processes involved in these items is shown by Table 92.

TABLE 92: THE DISTRIBUTION OF THE MENTAL PROCESSES AMONG THE GOOD DISCRIMINATORS ACCORDING TO THE COMBINED MATRICULATION RESULTS OF SCHOOLS H, A AND E

Mental Process	OBSERVED		EXPECTED	
	No.	%	No.	%
Manipulation	4	28,6	2,8	20
Routine problem	2	14,3	2,8	20
Comprehension	4	28,6	4,2	30
Non-Routine problem	4	28,6	4,2	30

As before, when the scores of the standard nine pupils were used, the mental processes are distributed among the good discriminators in almost exactly the same proportions as they are in the construction of the test. Nothing new was learned from this analysis.

CHAPTER 6

SEX DIFFERENCES IN THE STUDY OF HIGHER GRADE MATHEMATICS

6.1 ARE THERE DIFFERENCES IN THE MATHEMATICAL ABILITIES OF BOYS AND GIRLS?

6.2 ARE BOYS MORE PREDICTABLE, MATHEMATICALLY-SPEAKING, THAN GIRLS?

6.3 INTEREST, INCLINATION AND TRADITION

On several occasions in earlier sections, test results and results of correlations have given rise to questions concerning the nature of mathematical abilities in boys and girls, and the possibility that prediction of future achievement may be easier among boys than among girls. Certainly some teachers of Mathematics comment that they perceive differences in teaching only boys or only girls, but are these differences real, or imagined; and, if they are real, to what are they due? Are girls less able mathematically than boys, are they less inclined towards Mathematics, or are they just less interested?

6.1 ARE THERE DIFFERENCES IN THE MATHEMATICAL ABILITIES OF BOYS AND GIRLS?

The Tables 93 and 94 give the examination statistics for the 1985 Cape Senior Certificate examination in Higher Grade Mathematics.

TABLE 93: SENIOR CERTIFICATE EXAMINATION STATISTICS FOR HIGHER GRADE MATHEMATICS IN SCHOOLS H, A AND E

	School H Boys	School E Boys	School E Girls	School A Girls
N	80	12	14	35
Mean	208	270	242,86	241,71
Median	180	260	220	220
High Score	360	360	360	360
Low Score	100	180	180	130
Std Deviation	75,01	53,60	57,57	59,28

Since schools H and A are schools of roughly the same size, and school E is only slightly larger, with respect to the number of scholars, it is clear, from the large number of boys involved in the Higher Grade course, that the attitude towards Higher Grade Mathematics in school H is very different to that in schools A and E. In the boys' school H, it seems that pupils are encouraged, not only to study Mathematics on the Higher Grade, but also to persist with it even when the mid-year standard ten examination results indicate that the pupil is in danger of failing. In schools A and E, weak candidates are persuaded to change grades almost as soon as their examination results show signs of dropping below the official pass mark of 160 marks. The effect of this policy is to ensure that, on the whole, those candidates who eventually write the final examination on the Higher Grade do, in fact, pass on the Higher Grade, whereas, in school H, there are a number of boys who fail or obtain converted Standard Grade passes. Table 94 illustrates the different spread of pupil results in the three schools.

TABLE 94: 1985 CAPE SENIOR CERTIFICATE HIGHER GRADE MATHEMATICS SYMBOL DISTRIBUTION AMONG BOYS AND GIRLS OF SCHOOLS H, A AND E

Symbol	School H Boys %	School E Boys %	School E Girls %	School A Girls %	
A	10,0	16,7	14,3	8,6	
B	6,3	16,7		14,3	
C	15,0	41,7	28,6	28,6	Higher Grade
D	16,3	16,7	35,7	25,7	Pass
E	21,3	8,3	21,4	11,4	
E*	15,0			8,6	Standard Grade
F*	5,0			2,9	Pass
G	11,3				Fail

Another interesting feature of Table 94 is that the top mark in all schools, for both sexes, is the same, i.e., the maximum possible, 360 marks. This would seem to indicate that boys and girls have equal capabilities in

Mathematics. Although it may be just a chance result, if the scores of the top pupil in each school are traced back, it is evident that their marks are consistently of the same order.

TABLE 95: RECORDED MULTIPLE-CHOICE TEST AND EXAMINATION SCORES FOR THE TOP PUPILS OF SCHOOLS H, A AND E

Pupil Code	XH72	XEO1	XE13	XA32
School	H	E	E	A
Pupil's Sex	Boy	Boy	Girl	Girl
New M-C Test	28	30	29	29
Std 7 Examination	260	288	276	286
Std 8 Examination	382	364	352	331
Std 9 Examination	379	345	359	369
Senior Certificate	360	360	360	360

Although pupil scores in different schools may not be compared, directly, because the examinations they wrote were different, nevertheless, it is clear that the top boys and girls are achieving the same degree of distinction on different tests covering a common syllabus. This is reinforced by the recorded results of the top boy and girl at school E, who were taught in the same class and were graded on the same examinations.

Even if the top 20% of pupils is considered, there is not much difference in the mean scores of boys and girls in the Senior Certificate examination:

TABLE 96: MEAN SCORES OF THE TOP 20% OF BOYS AND GIRLS (A) IN SCHOOLS H, A AND E IN THE 1985 CAPE SENIOR CERTIFICATE HIGHER GRADE MATHEMATICS EXAMINATION

	School H Boys	School E Boys	School E Girls	School A Girls
Mean Score	322,5	340,0	326,7	325,7
Std Deviation	41,2	34,6	57,7	32,1

There is no evidence that the capabilities of top scoring boys are different from the top scoring girls. However, the mean scores of boys and girls, who wrote the Higher Grade examination, at school E, do show an interesting phenomenon, over the years. In the final matriculation examination the boys at school E had a mean score higher than the mean score produced by the girls of school E in this examination. In every other, previous, examination, the mean score of these girls had been higher than the mean score of the boys. Table 96 illustrates these differences.

TABLE 96: (B) MEAN EXAMINATION SCORES AND MULTIPLE-CHOICE TEST SCORES OF THE 12 HIGHER GRADE BOYS AND 14 HIGHER GRADE GIRLS AT SCHOOL E

	Boys	Girls
Std 7 Examination	231,42	253,43
Std 8 Examination	244,33	273,79
Std 9 Examination	215,58	236,79
M-C Test 1984	21,58	21,50
Final Examination	270,00	242,86

If, as has already been stated, there is no evidence to suggest that the mathematical capabilities of boys and girls are different, why do the school E girls suddenly do less well, on average, than the boys? The pattern was also discernible among the standard nine pupils of the co-educational schools, D and E, i.e., the mean scores of girls, almost without exception, greater than the mean scores of boys on internally set papers, with the opposite holding for the externally set multiple-choice papers. Much research has been carried out in this field and a review of some of that research is given in Appendix F.

TABLE 97: MEAN EXAMINATION SCORES AND MULTIPLE-CHOICE TEST SCORES OF THE 17 HIGHER GRADE BOYS AND 19 HIGHER GRADE GIRLS IN STANDARD NINE AT SCHOOL E

	Boys	Girls
Std 7 Examination	209,94	220,00
M-C Test 1983	18,94	16,82
Std 8 Mid-year	229,41	249,47
Std 8 Examination	251,41	261,26
M-C Test 1984	19,65	18,32
Std 9 Mid-year	194,53	210,05
Std 9 Examination	222,82	232,63

TABLE 98: MEAN EXAMINATION SCORES AND MULTIPLE-CHOICE TEST SCORES OF THE 26 HIGHER GRADE BOYS AND 16 HIGHER GRADE GIRLS IN STANDARD NINE AT SCHOOL D

	Boys	Girls
Std 7 Examination	227,65	229,13
M-C Test 1983	17,85	16,25
Std 8 Mid-year	226,31	250,50
Std 8 Examination	217,54	217,38
Std 9 Mid-year	245,38	247,25
Std 9 Examination	215,12	206,19

One explanation might be that the tendency among girls, noted previously, to prefer the security of work that requires precision and accuracy of technique, rather than logical reason or innovation, may reduce their flexibility somewhat, so that when confronted by an examination, set in an unfamiliar style, their confidence is a little shaken to the detriment of their performance. This effect has been noticed, by the writer, even in internally set examinations - in girls' schools at least, the pupils, whose teacher sets the examination, seem to have a slight advantage over the others, simply because the style of thinking in the question paper is familiar to them. The greater flexibility of the boys, perhaps renders them less sensitive to such changes in style.

The girls' love of precision and accuracy, often, has another undesirable effect; namely, that many girls are unwilling to commit their mathematical thoughts to paper, if they are unsure that they are quite correct. They feel that they have to present solutions that are perfect in every way, and tend to discard their rough work on unfinished problems, preferring to leave a blank page rather than present a "mess", which they define to be any tentative attempts or half-completed solutions. Even in the everyday activity of the classroom, without the pressure of an examination, it is often difficult to convince some girls that they can think mathematically. They seem to lack the self-assurance necessary to penetrate the world of intuitive mathematics, and certainly show no signs of feeling at all cheated if the teacher explains everything carefully and, then, provides that panacea for all their mathematical ills, "the method".

To sum up: Although individually girls and boys have equal mathematical potential, it may be that, because of their preference for the known above the uncharted unknown, <u>collectively</u>, girls may have more difficulty achieving really well than do boys.

6.2 ARE BOYS MORE PREDICTABLE, MATHEMATICALLY-SPEAKING, THAN GIRLS?

Intuitively, it seems quite reasonable that prediction of achievement in Mathematics might be more difficult for girls than for boys. Emotionally, there seem to be more disturbing influences for the girl in Mathematics.

During the first two years of high school, most girls are quite enchanted with the subject, and derive much satisfaction from its tidiness, strict adherence to rules and the possibility of having hard work rewarded by very high marks - in Mathematics, unlike the languages, it is quite possible to score 100% and there is security in the thought that answers are either right or wrong, with no bewildering shades of grey. However, as they begin to discover their femininity, they may believe that success in a subject, that they perceive to be more masculine in nature, will detract from this new-found attribute and may even attempt to hide their talent. This new disinclination may coincide with the onset of the more difficult, less mechanical part of the syllabus, and so even the satisfaction derived from conscientious hard work may be "under fire" as increased efforts produce less visible rewards. In trying to predict the level of a girl's success in Higher Grade Senior Certificate Mathematics, it becomes more a task of predicting how she will deal with growing up than with determining the strength of her mathematical ability. For boys, on the other hand, there would appear to be no such conflicts with the study of Mathematics. If a boy has great mathematical ability, there is no reason to feel embarrassed about it, and if he is less able, he probably still tries to do his best work because he appreciates the importance of Mathematics in nearly all career areas. In general, boys seem to be more relaxed in their attitude towards Mathematics and less afraid of making mistakes. Consequently, they are more inclined to use their mathematical "flair". Thus, it might be expected that a boy's performance in Mathematics in successive tests would be a more pure measure of his mathematical ability, less affected by emotional factors.

All this is, of course, conjecture. If this view had any validity, it might be expected that the correspondence between early school examinations and later examinations would be weaker for girls than for boys. To put this to the test, scores for boys and girls were separated in the results of the co-educational schools, and correlation matrices were constructed as before. This was carried out for both standard nine and ten pupils at school E, but only standard nine pupils at school D. Results are shown in Tables 99 to 104.

TABLE 99: TABLE OF CORRELATION COEFFICIENTS BETWEEN FOUR SETS HIGHER GRADE EXAMINATION RESULTS AND THE 1984 MULTIPLE-CHOICE TEST FOR STANDARD TEN GIRLS

SCHOOL E:		CO-EDUCATIONAL		N = 14	
	New Test 1984	Std 7 Exam 1982	Std 8 Exam 1983	Std 9 Exam 1984	Final Exam 1985
New Test 1984	1	+,72	+,85	+,81	+,71
Std 7 Exam 1982		1	+,78	+,73	+,59
Std 8 Exam 1983			1	+,84	+,79
Std 9 Exam 1984				1	+,72
Final Exam 1985					1

The correlation coefficients between the multiple-choice test and successive school examinations, for the standard ten girls of school E, are marginally higher than the correlation coefficients between the standard seven examination and later examinations.

TABLE 100: TABLE OF CORRELATION COEFFICIENTS BETWEEN FOUR SETS HIGHER GRADE EXAMINATION RESULTS AND THE 1984 MULTIPLE-CHOICE TEST FOR STANDARD TEN BOYS

SCHOOL E:		CO-EDUCATIONAL		N = 12	
	New Test 1984	Std 7 Exam 1982	Std 8 Exam 1983	Std 9 Exam 1984	Final Exam 1985
New Test 1984	1	+,21	+,40	+,61	+,43
Std 7 Exam 1982		1	+,89	+,75	+,85
Std 8 Exam 1983			1	+,91	+,72
Std 9 Exam 1984				1	+,72
Final Exam 1985					1

The correlation coefficients between the multiple-choice test and successive examinations, for the standard ten boys of school E, fluctuate considerably, but given the small sample size and a restricted range of talent, this might have been expected. The strength of the correlation between the standard seven examination and later examinations is noteworthy.

TABLE 101: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS FOR STANDARD NINE GIRLS

SCHOOL E:	CO-EDUCATIONAL				N = 19		
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1	+,75	+,67	+,68	+,74	+,22	+,37
New Test 1984		1	+,65	+,67	+,70	+,25	+,54
Std 7 Exam 1983			1	+,77	+,82	+,67	+,71
May Exam 1984				1	+,89	+,51	+,68
Std 8 Exam 1984					1	+,65	+,79
May Exam 1985						1	+,83
Std 9 Exam 1985							1

For the standard nine girls of school E, the coefficients between both multiple-choice tests and successive school examinations show decreasing trends, although they fluctuate, noticeably, from one examination to the next. For this group of girls, the standard seven examination seems to be a more accurate predictor of success in the senior course, the correlation coefficients being more stable and more strongly positive.

TABLE 102: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS FOR STANDARD NINE BOYS

SCHOOL E:			CO-EDUCATIONAL			N = 17
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1	+,88	+,57	+,52	+,85	+,61	+,70
New Test 1984		1	+,62	+,48	+,81	+,58	+,58
Std 7 Exam 1983			1	+,75	+,76	+,63	+,59
May Exam 1984				1	+,78	+,81	+,70
Std 8 Exam 1984					1	+,83	+,80
May Exam 1985						1	+,91
Std 9 Exam 1985							1

For the standard nine boys of school E, the correlation coefficients between the multiple-choice tests and successive school examinations are fairly erratic, but, nevertheless, finish just as strongly positive as the coefficients of the standard seven examination.

TABLE 103: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS FOR STANDARD NINE GIRLS

SCHOOL D:			CO-EDUCATIONAL			N = 16	
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1		+,13	+,42	+,25	+,23	+,40
New Test 1984							
Std 7 Exam 1983			1	+,62	+,83	+,48	+,58
May Exam 1984				1	+,80	+,53	+,61
Std 8 Exam 1984					1	+,80	+,81
May Exam 1985						1	+,85
Std 9 Exam 1985							1

The correlation coefficients between the multiple-choice test and later school examinations fluctuate and are only weakly positive. The coefficients for the standard seven examination are more strongly positive, but also fluctuate noticeably. Once again, the sample size is small and the range of talent among the standard nine girls of school D is very restricted.

TABLE 104: TABLE OF CORRELATIONS BETWEEN THE MULTIPLE-CHOICE TESTS AND THE EXAMINATION RESULTS OF HIGHER GRADE PUPILS FOR STANDARD NINE BOYS

SCHOOL D:	CO-EDUCATIONAL				N = 26		
	Revised Test 1983	New Test 1984	Std 7 Exam 1983	May Exam 1984	Std 8 Exam 1984	May Exam 1985	Std 9 Exam 1985
Revised Test 1983	1		+,62	+,40	+,48	+,20	+,24
New Test 1984							
Std 7 Exam 1983			1	+,58	+,61	+,50	+,46
May Exam 1984				1	+,69	+,37	+,61
Std 8 Exam 1984					1	+,52	+,65
May Exam 1985						1	+,58
Std 9 Exam 1985							1

The correlation coefficients for the standard nine boys of school D, fluctuate less noticeably than those of the girls in school D. The coefficients between the multiple-choice test and later examinations show a marked decline from standard seven to standard nine. The standard seven examination appears to be a marginally better predictor of success in the senior course than does the multiple-choice test.

The Tables 99 to 104 compared boys and girls, who were taught in the same classes and wrote the same internal examinations. However, these tables provide no shred of evidence that girls' achievement is any more difficult to predict than boys' achievement. In fact, judging by the variety of patterns of correlation, it may only be concluded that the achievement of students in general, is impossible to predict. Of course the overriding factor in this part of the investigation is the fact that so few pupils have continued in the Higher Grade Mathematics course that the correlation results obtained can hardly be considered dependable to any extent. Another factor could be the very restricted range of talent among the boys and girls under consideration - it was noted earlier that, in school D, pupils were very carefully selected at the beginning of standard eight, and in schools A and E, large numbers of mathematically weaker students had been persuaded to change to Standard Grade, leaving small samples of candidates possessing above average aptitude for Mathematics. The only schools, in the investigation, which offer a broader spectrum of talent, are boys' school H and girls' school B. If the various correlations for these two schools are examined in the summary Tables 105 to 114, it will be seen that, although the standard seven examination did not prove to be such an efficient predictor of later achievement among the girls, the 1983 multiple-choice test was equally efficient as a predictor in both schools, and the 1984 test was more efficient for the girls. Again, there is no evidence that success is easier to predict for boys than for girls.

In the summary Tables 105 to 114, Product-moment correlation coefficients, between the multiple-choice tests and end-of-year examinations and between the standard seven school examination and later end-of-year examinations, are shown for boys and girls separately, in all schools.

TABLE 105: CORRELATION BETWEEN THE REVISED MULTIPLE-CHOICE TEST AND LATER SCHOOL EXAMINATIONS FOR STANDARD 9 GIRLS

SCHOOL	N	MC 7 / MC 8	MC 7 / SE 7	MC 7 / SE 8	MC 7 / SE 9
A	49	+,75	+,74	+,74	+,73
B	31	+,71	+,59	+,61	+,63
D	16		+,67	+,74	+,37
E	19	+,75	+,13	+,25	+,40

TABLE 106: CORRELATION BETWEEN THE REVISED MULTIPLE-CHOICE TEST AND LATER SCHOOL EXAMINATIONS FOR STANDARD 9 BOYS

SCHOOL	N	MC 7 / MC 8	MC 7 / SE 7	MC 7 / SE 8	MC 7 / SE 9
H	86	+,72	+,63	+,66	+,63
D	26		+,62	+,48	+,24
E	17	+,88	+,57	+,85	+,70

TABLE 107: CORRELATION BETWEEN THE NEW MULTIPLE-CHOICE TEST AND LATER SCHOOL EXAMINATIONS FOR STANDARD 9 GIRLS

SCHOOL	N	MC 8 / SE 7	MC 8 / SE 8	MC 8 / SE 9
A	49	+,55	+,72	+,65
B	31	+,56	+,72	+,73
E	19	+,65	+,70	+,54

TABLE 108: CORRELATION BETWEEN THE NEW MULTIPLE-CHOICE TEST AND LATER SCHOOL EXAMINATIONS FOR STANDARD 9 BOYS

SCHOOL	N	MC 8 / SE 7	MC 8 / SE 8	MC 8 / SE 9
H	86	+,53	+,59	+,58
E	17	+,62	+,81	+,58

TABLE 109: CORRELATION BETWEEN THE STANDARD 7 SCHOOL EXAMINATION AND LATER SCHOOL EXAMINATIONS FOR STANDARD 9 GIRLS

SCHOOL	N	SE 7 / SE 8	SE 7 / SE 9
A	49	+,78	+,83
B	31	+,78	+,46
D	16	+,83	+,58
E	19	+,82	+,71

TABLE 110: CORRELATION BETWEEN THE STANDARD 7 SCHOOL EXAMINATION AND LATER SCHOOL EXAMINATIONS FOR STANDARD 9 BOYS

SCHOOL	N	SE 7 / SE 8	SE 7 / SE 9
H	86	+,78	+,67
D	26	+,61	+,46
E	17	+,76	+,59

TABLE 111: CORRELATION BETWEEN THE NEW MULTIPLE-CHOICE TEST AND LATER SCHOOL EXAMINATIONS FOR STANDARD 10 GIRLS

SCHOOL	N	MC / SE 7	MC / SE 8	MC / SE 8	MC / SC
A	35	+,52	+,57	+,39	+,51
E	14	+,72	+,85	+,81	+,71

TABLE 112: CORRELATION BETWEEN THE NEW MULTIPLE-CHOICE TEST AND LATER SCHOOL EXAMINATIONS FOR STANDARD 10 BOYS

SCHOOL	N	MC / SE 7	MC / SE 8	MC / SE 8	MC / SC
H	80	+,44	+,70	+,70	+,64
E	12	+,21	+,40	+,61	+,43

TABLE 113: CORRELATION BETWEEN THE STANDARD 7 SCHOOL EXAMINATION AND LATER SCHOOL EXAMINATIONS FOR STANDARD 10 GIRLS

SCHOOL	N	SE 7 / SE 8	SE 7 / SE 9	SE 7 / SC
A	35	+,80	+,60	+,64
E	14	+,78	+,73	+,59

TABLE 114: CORRELATION BETWEEN THE STANDARD 7 SCHOOL EXAMINATION AND LATER SCHOOL EXAMINATIONS FOR STANDARD 10 BOYS

SCHOOL	N	SE 7 / SE 8	SE 7 / SE 9	SE 7 / SC
H	80	+,71	+,46	+,46
E	12	+,89	+,75	+,85

6.3 INTEREST, INCLINATION AND TRADITION

During the course of this experiment, certain variations in results led the writer to consider the possibility that some of the variations might be due to differences, between the sexes, in mathematical ability or predictability. In the preceding two sections, this possibility was explored by using the collected test and examination data for standard nine and ten candidates with the sexes separated. The data available provided no evidence of such differences between boys and girls in Mathematics.

If perceived differences in the teaching of, and the collective achievement of the different sexes are not due to fundamental differences in mathematical ability, then the fact that advanced Mathematics is a predominantly male pursuit, must be the result of other factors. Chief among these must be interest in Mathematics and willingness or inclination to study an abstract subject, the usefulness of which is often obscure to teenage girls. Unfortunately, Mathematics is a difficult subject to "pick up later", and so most mathematicians begin their career in high school, cultivating a love for the subject and learning very basic techniques necessary to scale the theoretical heights ahead - without this grounding it is impossible to proceed to more advanced work, which includes not only pure Mathematics, but also, applied Mathematics and all spheres of activity which require the manipulation of numbers, e.g., engineering, astronomy, architecture, computer science, quantity surveying, navigation, land surveying, statistics, marketing and all research fields. However, these pursuits must be uppermost in the minds of only very few high school girls, who, probably due to peer pressure and the pressure of society, are more concerned with the more traditional feminine pursuits of fashion, romance and the Arts. At this stage in a girl's life, an open interest in things mathematical marks her as being different or tomboyish, and puts her on the outside of her friendship group. To be too interested in Mathematics creates a conflict situation for the girl that it does not for her male counterpart.

It has been mentioned before that the last two years of the Higher Grade syllabus demand a less mechanical, more insightful approach on the part of both teacher and student. This seems to be less to the taste of girls than to boys - girls prefer to have nice, neat books of set methods and procedures which can be perfected and honed to a fine art, rather than exercise books which record mathematical explorations, some showing more success than others. Girls often resist this new approach to Mathematics, and when they discover that success depends on the ability to think with more insight and flexibility, may become disenchanted with the subject.

At the moment when the serious study of Mathematics demands maximum interest, persistence and inclination to think mathematically of the student, girls are experiencing a period of diminished interest and inclination owing partly to their own nature and partly to the dictates of tradition and peer pressure. When the difficulties of adolescence are over, and the individual in the young woman asserts itself, it may be impossible to return to mathematical studies, because the foundations she should have laid in high school are long forgotten. Thus, women tend not to enter the world of Mathematics, and so preserve the notion that only men make mathematicians.

In a country so desperately in need of professionally qualified scientists and technicians, it would be short-sighted to allow the mathematical potential of women to go to waste. Teachers of Mathematics have a duty to dispel the idea many girls have that an interest in Mathematics detracts somehow from their femininity. By teaching in a way that makes the usefulness of Mathematics undeniable to teenage girls, and by encouraging girls to show more boldness in their Mathematics, so that they become less content slavishly to follow set routines and methods, it may yet be possible to convince society that women are the mathematical equals of men.

To sum up: The results of this study confirm that there is no difference in mathematical ability between boys and girls. Nor is there any evidence that the mathematical achievement of boys is more easily predicted than that of girls. The fact that fewer girls stay with Higher Grade Mathematics is probably due to a disinclination to master the subject. This disinclination possibly has its origin in peer pressure on girls to avoid a subject which society views as the preserve of males.

CHAPTER 7

SUMMARY AND CONCLUSIONS OF THE EXPERIMENT

7.1 SUMMARY OF THE EXPERIMENT

7.2 CONCLUSIONS

7.3 SUGGESTIONS FOR FURTHER STUDY

7.1 SUMMARY OF THE EXPERIMENT

The aim of this experiment was to improve the quality of information given to pupils and their parents concerning the intellectual demands of the senior course in Mathematics and the probability of success in this subject for each individual student.

To accomplish this, both syllabuses and examination papers were analysed in an attempt to isolate and reveal the exact nature of the mental processes required by the senior Higher Grade Mathematics candidate. The classification of the mental processes involved in examination items proved to be a highly subjective task and the end result, which was based on the writer's own experience and intuition, is open to debate. However, the emphasis given to the importance of teaching and testing for insight in the syllabuses did seem to be reflected in the weighting of higher order questions presented in the examination papers analysed by the writer. To reveal the individual's ability (or inability) to deal with such examinations, a test was designed which, it was hoped, would call forth the same mental processes.

To be of any use, the test had to be administered before pupils made their final subject choice. Items chosen had, then, to be suitable for, and not beyond the mathematical expertise of standard seven pupils. Multiple-choice items were classified according to the mental processes thought to be involved and a test was constructed to echo the weighting of the various mental processes noted in the Senior Certificate (Higher Grade) examination papers. Once again the classification of the test items had to be accomplished intuitively and the compilation of the multiple-choice test was open to subjective bias. This multiple-choice test was administered to a fairly large group of standard seven pupils in several Cape Peninsula schools. The mathematical progress of this group of students was followed to the end of their standard nine year as a means of comparing their actual ability to cope with senior course Mathematics, as measured by their achievement in successive school examinations, and their predicted ability to cope, as recorded by their result in the multiple-choice test. The extent to which both sets of results dovetailed determined the validity and usefulness of the multiple-choice test as a predictive device. The reliability of the multiple-choice test was estimated by repeated administration of the test. The efficiency of individual items was checked by means of item analysis, comparing the responses of top-scoring candidates with least-scoring candidates. Furthermore, the multiple-choice test was administered once to pupils entering their final year of study so that the results could be compared with the results of an actual Senior Certificate examination externally set and common to pupils from different schools. This was an attempt to cancel the effects of varying standards and examination policies existing in the different schools.

From the results of this experiment it is not possible to pinpoint exactly which students "have what it takes" and which students do not. Nor is it possible to determine the level of mental process at which a successful candidate must be able to operate. However, it is possible to draw some positive conclusions from the study and these are discussed in the section that follows. Moreover, since the initial problem still remains, the experiment at least has the merit of pointing up the difficulties involved in such a task.

7.2 CONCLUSIONS

This investigation was prompted by the need felt for an additional predictive device to assist pupils, their parents and teachers, in their decision concerning the suitability of Higher Grade Mathematics as a matriculation subject. Young people on the threshold of their working lives are cognisant of the fact that there will be an increasing demand for the technically skilled and a diminishing area of opportunity for the unskilled. Senior Certificate Mathematics is an essential stepping stone to any technical qualification. Bearing in mind the importance of Mathematics in countless career areas, as well as the difficult, often abstract, nature of the subject, it is imperative that the decisions that pupils make, concerning Mathematics, are based on realities such as the extent of their mathematical ability and an awareness of the exact demands of the senior syllabus, rather than fantasies such as the widely held pupil belief that "Maths is not a swot subject", or irrelevancies such as "all my friends are taking Maths", "Maths is better than History" and "I don't like the Needlework teacher".

It is important that as many students as possible continue with Mathematics, preferably, on the Higher Grade, so as to keep all career fields open. However, those pupils who have no chance of passing on the Higher Grade, should not be permitted to enlarge classes unnecessarily and cause able youngsters to be delayed or denied their share of the teacher's time.

The Standard Grade course provides adequate mathematical training for those students who will not proceed to careers of a scientific or technical nature. Very weak mathematicians should be excluded from the subject altogether, unless there is some shorter course of Mathematics available, which does not commit the pupil to three years of study in a theoretical subject for which he has little liking or aptitude.

In short, it is desirable that through careful selection, pupils be placed in a Mathematics course from which they can derive maximum benefit, neither holding back those who are more able, nor having their own self-confidence and morale shattered by attempting a course of study beyond their intellectual capacity. In order to achieve this goal, there must be a process of selection. This process must be accurate, it must be credible and it must be easily applicable during the course of the standard seven year, when pupils need to be advised on their Senior Certificate subject choice.

The multiple-choice test designed for this experiment could form part of such a selection process. The use of this test has clear advantages. It is quick and easy to administer, requiring only a double period to complete; scoring may be done in a short time and may be undertaken by non-expert clerical staff. The test items are not syllabus-bound which allows that the test may be written long before the end of the school year, thereby providing earlier feedback to pupils and parents. The multiple-choice test is clearly a test of Mathematics and therefore is likely to be an acceptable measuring instrument in the eyes of pupils and parents. Finally, although the multiple-choice test is perhaps not always as good a predictor as the standard seven examination, nevertheless, this test also appears to possess a highly significant degree of predictive validity for the final matriculation examination, and it is possible that the validity of this test could be increased if some of the limitations, which beset this study, could be eliminated or, at least, reduced. The problems encountered were:

1. An ever diminishing number of pupils in the sample owing to pupils changing grade. Although a large number of pupils were tested, their results had to be treated separately in school groups. Thus, instead of being based on one large sample, the investigation was reduced to a basis of several small samples each, ultimately, reflecting a rather restricted range of talent.

2. The subjective nature of the classification of test and examination items in terms of the mental process involved.

How these limitations might be overcome is discussed in the last section of this chapter.

The question remains, how should prospective Higher Grade students be selected to achieve maximum success for the greatest number of pupils? A popular means of selection in some schools is the "cut-off point", i.e., the selection of only those students who obtain above a certain predetermined score in the final standard seven examination. Proponents of this method claim that pupils are forewarned and are aware of exactly what must be achieved; lazy students, are, therefore, more highly motivated to work steadily in order to exceed the target score.

The results of this experiment have shown the standard seven promotion examination to be a valuable predictor of success in matriculation Mathematics, but there are dangers inherent in the "cut-off point" system. To be visibly fair in the eyes of the pupils, the target score should remain constant from year to year. Consequently, the standard of examinations should be carefully monitored so that these are equivalent from year to year. The target score itself should not be some arbitrary figure, but should be calculated so that it serves the purpose of selection adequately. Another disadvantage of this system is that it creates confusion among pupils and parents when they hear of other schools using different "cut-off points". This, of course, is unavoidable, because schools set examinations of varying levels and standards, depending on the philosophy of the teacher in charge and, to a large extent, the ability spectrum of pupils attending the school.

A better means of selection, in the writer's opinion, is one which utilises the teacher's recommendation reinforced quantitatively by the pupil's score in the multiple-choice test and the standard seven promotion examination. Provided that she has taught the pupil for an adequate length of time, the teacher's recommendation is, probably, the most accurate predictor available. This verbal assessment, together with the multiple-choice test score, would provide a strong indication to pupils, at an early stage of the decision year, of their likelihood of success in the senior Mathematics course. The end-of-year examination could provide a final check on the pupils selected to continue. This approach has several advantages.

Parents start asking advice about subject choice issues shortly after the mid-year test series, believing that this examination offers some sort of predictor for future success. However, these examinations may have been based on a very meagre amount of work and for many pupils do not provide a particularly accurate measure of potential. The pupil's score on a wider ranging test such as the multiple-choice test provides a firmer basis for parent-teacher discussion concerning the student's chances of successfully completing the Higher Grade course in Mathematics.

Early, well-informed decision making by pupils facilitates timetable and staff allocation planning undertaken by School Principals during the third school term, leaving only minimal adjustments necessary at the end of the year. In addition, pupils, whose true ability is difficult to assess by mere observation, are subjected to an extra test. These students might include those with ability who through laziness underachieve, and those conscientious souls who produce excellent results in standard seven through unremitting hard work rather than by true mathematical ability. It is for these pupils that the multiple-choice test may have some particular value.

The results of the experiment showed that the only school, in which the multiple-choice test proved to be a better predictor than the school examination, was the girls' school B - a school of largely "average" mathematicians. It is for this middle section of the ability spectrum that there is the greatest concern in the selection process. It is important to include those who are capable of succeeding, but exclude those who are incapable of passing the Higher Grade course. The multiple-choice test is a test that depends on thinking ability rather than a period of past diligence and so may be able to reveal inadequacies normally hidden by extreme zeal and industry, or real mathematical ability carefully concealed by teen-age apathy or rebelliousness.

To sum up: This study has highlighted the need for careful selection of Higher Grade pupils. The large drop-out rate in all schools, except school H, shows that teacher time as well as pupil time is being wasted when too many students struggle for a year or two with the Higher Grade syllabus, only to change to Standard Grade eventually. It will never be possible to make absolutely accurate predictions concerning individual pupil achievement, and there will always be those students, who show early promise which does not come to fruition. However, selection could minimise the number of students incorrectly placed. If the greatest number of students were placed in the Mathematics course that best suits their ability, then, surely, the success rate would be maximised. Thus fewer pupils would experience the demoralisation and frustration associated with the performance of seemingly meaningless tasks in the Mathematics classroom because they are unable to cope intellectually with the levels of thinking and understanding required by Higher Grade Mathematics.

The process of selection should pay due regard to the teacher's recommendation, the pupil's achievement in the school examination and some measure of the pupil's ability to think at higher levels, measured by a test written earlier in the standard seven year. A test could be designed along the lines of the multiple-choice test, which concentrates on the mathematical insight of the pupil.

The test developed for this study certainly shows a marked degree of predictive validity for the Matriculation examination and presents clear advantages in ease of administration to standard seven pupils. It is possible that the predictive validity of this test might be increased and this possibility is explored in the section that follows.

7.3 SUGGESTIONS FOR FURTHER STUDY

The most limiting factor in this study was the subjectivity involved in two crucial phases of the experiment. Central to the entire investigation was the determination of the mental processes that are involved in Senior Certificate Mathematics. Syllabus specification makes quite clear the desire that Higher Grade students should be encouraged to think insightfully, as well as being skilled in various taught mathematical procedures. However, to determine a general level of thought for a Senior Certificate examination item is impossible. Unless a candidate's past mathematical experience with respect to particular questions and the way he was taught in the classroom is known, any attempt to classify those questions according to mental process can at best be speculative. Classification was no less vexing a problem when items were being sought or constructed in the compilation of the multiple-choice test device. It would be interesting to determine whether the predictive validity of the test, or the discriminating efficiency of the different types of item would change to any great extent, if similar tests were to be devised by individual teachers for administration to their own pupils, thereby partially eliminating the problem of the test designer not knowing what kind of problems the candidates have already encountered in class, and therefore having absolutely no idea at all at which level of mental process individual items are being received by candidates.

The problem of obtaining and then keeping a large enough sample to produce dependable results could be overcome, to a certain extent perhaps, by the collaboration of several teachers in different schools. Each one testing and examining his own pupils along the same lines as the others; each using different, but equivalent testing devices, and then comparing results over the three year period covered by the senior course.

Apart from the problem of selecting pupils for the Higher Grade course in Mathematics, there remains the problem of selecting pupils for the Standard Grade course. What can be done, mathematically, for those pupils who are not capable of passing a course that contains three years' work? Courses, such as the one-year "Terminal Mathematics" initiated at school D, which have been designed especially to meet the specific, and practical, further training and career needs of the less able pupil, would provide the education system with some much needed flexibility, and pupils with a platform from which they could, more confidently, enter the working world of commerce and industry. If free enterprise is to be broadened in South Africa, then the schools must play their part in providing useful skills and knowledge for those pupils who are not intellectually equipped to benefit from a lengthy stay in school studying academic subjects. Entrance to numerous career fields depends on the successful completion of matriculation Mathematics. It might be instructive to establish exactly how much mathematical knowledge is required in these fields; perhaps, Mathematics is being used merely as a sieve to maintain some exclusivity. It is possible that the talents of many are lost to society simply because of their inability to negotiate the barrier of Mathematics.

To live, and prosper, in today's world everyone needs some Mathematics; however, not all people have the same capacity for the subject. Individuals and society will be served best when teachers of Mathematics can channel their pupils into Mathematics courses, in which maximum achievement, satisfaction and self-fulfilment can be realised. Thus, frustration, confusion and demoralisation among students might be reduced to a minimum. In order to realise this ambition, it will be necessary for educators to pursue all the available means of determining the exact nature and potential of their pupils' mathematical ability.

APPENDIX A

SYLLABUS MATERIAL

PROVINCIAL ADMINISTRATION OF THE CAPE OF GOOD HOPE

DEPARTMENT OF EDUCATION

SENIOR SECONDARY COURSE

SYLLABUS FOR MATHEMATICS

(Higher Grade)

1973

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS (HIGHER GRADE)

The following syllabus for Mathematics (Higher Grade) for the Senior Secondary Course will be introduced as from 1st January, 1974.

The syllabus will be introduced in Standard 8 in 1974, and the first Senior Certificate Examination on this syllabus will be held in November/December, 1976.

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS (HIGHER GRADE)

1. *General remarks*

1.1 The subject-matter dealt with in the Standard 8 syllabus is regarded as *pre-knowledge* to the syllabus for Standards 9 and 10. This knowledge is again used either directly or indirectly in the syllabus for the last two school years and, therefore, also in the Senior Certificate examination, but during these years the emphasis should fall on the actual syllabus for Standards 9 and 10.

1.2 In tests and examinations more stress should be laid on insight and understanding than on mechanical reproduction of formal knowledge.

1.3 Where applicable, the slide rule may be used.

2. *General aims*

2.1 To acquaint the pupils with the part played by Mathematics in the modern world in which man is constantly required to handle the quantitative and spatial aspects of situations;

2.2 to contribute to the general education of the pupils, with special emphasis on the development of logical thought and of systematic, accurate and neat methods of working;

2.3 to cultivate in pupils appreciation of the structure and the central theme of each section of the syllabus, as well as of the underlying relation between certain sections;

2.4 to acquaint pupils with, and train them in, mathematical methods of thought and work;

2.5 to give pupils a clear insight into, and a thorough knowledge and understanding of those basic mathematical principles which will prepare and equip them for daily life and further study in Mathematics, the pure sciences and certain applied sciences.

3. *Remarks*

3.1 In all sections of the subject pupils must be guided to tackle and solve each problem or theorem systematically by

3.1.1 paying close attention to the data and what is required;

3.1.2 accounting for all facts and theorems that can possibly serve as a key to the solution, irrespective of the section of Mathematics in which they occur;

3.1.3 starting with the most obvious methods and then considering other possibilities;

3.1.4 comparing the different methods of solution and making a choice;

3.1.5 paying close attention to necessary and sufficient requirements with respect to wording and reasoning when writing down the solution.

3.2 Pupils must be trained in the correct use of notation and terminology, the exact formulation of statements and the making of accurate deductions.

THE SYLLABUS

4. STANDARD 8

4.1 Algebra.

4.1.1 Sets (consolidation).

4.1.1.1 The set concept.

4.1.1.2 Elements of a set, the empty set, subset, universal set, complement of a set, intersection and union of sets, Cartesian product.

4.1.1.3 Venn diagrams and their applications as aids to illustrate the solution of problems.

4.1.2 Number concept.

4.1.2.1 An outline of the structure of the system of real numbers, as developed from the natural numbers, with emphasis on the irrational numbers.

4.1.2.2 Relations of order between numbers and the relevant laws; operations with numbers, rules for operations, closure in respect of operations.

4.1.2.3 The number line. One-to-one correspondence between points on the number line and real numbers.

4.1.3 Products of the following types by inspection:

4.1.3.1 $(a\pm b)(c\pm d)$.

4.1.3.2 $(ax\pm b)(cx\pm d)$

4.1.3.3 $(ax\pm b)^2$

4.1.3.4 $(ax+b)(ax-b)$

4.1.4 Factors of polynomials of the following types:

4.1.4.1 $ax\pm bx\pm ay\pm by$

4.1.4.2 $ax^2\pm bx\pm c$ and $ax^2\pm bxy\pm cy^2$

4.1.4.3 a^2-b^2

4.1.4.4 The foregoing types, with the inclusion of a common factor.

4.1.5 L.C.M. of polynomials by factorisation only.

4.1.6 Algebraic fractions (Simple examples only).

4.1.6.1 Simplification.

4.1.6.2 Operations.
4.1.7 Solution sets.
4.1.7.1 Determining the solution sets of linear equations in one unknown with numerical and literal coefficients.
4.1.7.2 Determining the solution sets of linear inequalities in one unknown with numerical coefficients only.
4.1.7.3 The solution of simple problems with the aid of linear equations and inequalities in one unknown.
4.1.7.4 Determining the solution sets of simple quadratic equations and inequalities in one unknown by means of factors.
4.1.8 Formulae.
4.1.8.1 Construction of formulae from data.
4.1.8.2 Changing the subject of a formula.
4.1.8.3 Substitution in formulae.
4.1.9 Relations and functions.
4.1.9.1 Relation: the mapping according to a given rule of the elements of one set on the elements of another set.
4.1.9.2 Sets of ordered pairs.
4.1.9.3 The function concept.
4.1.9.4 Representation of ordered pairs by points in the Cartesian plane.
4.1.9.5 The function defined by $y=mx+c$ and its graphical representation; intercept and gradient.
4.1.10 Systems of linear equations and inequalities.
4.1.10.1 Algebraic solution of systems of linear equations in two unknowns. (Numerical coefficients only).
4.1.10.2 Graphical illustration of the solution sets of systems of linear equations in two unknowns.
4.1.10.3 Graphical illustration of the solution sets of systems of inequalities in two unknowns.
4.1.10.4 Application of systems of linear equations in the solution of simple problems.
4.1.11 Logarithms.
4.1.11.1 Definition of a^n for n an integer ($a>o$).
4.1.11.2 Use of logarithmic tables for:
4.1.11.2.1 multiplication;
4.1.11.2.2 division;
4.1.11.2.3 raising to a power and taking roots (negative indices excluded). (Simple examples only.)

4.2 *Synthetic Geometry*

N.B. Although all theorems must be proved, formal proofs of theorems in examinations must be limited. Only proofs of theorems denoted by an asterisk in the following list, will be required. However, applications (including constructions) of any theorem, definition or axiom in this list may be set.

4.2.1 If two lines intersect, the sum of any pair of adjacent angles is equal to 180°, and conversely, if the sum of two adjacent angles is 180°, the outer arms form a straight line. (Axiom)
*4.2.2 When two lines intersect, the vertically opposite angles are equal. (Theorem)
4.2.3 Two lines are parallel if, and only if, an intersecting transversal forms equal corresponding angles. (Definition)
4.2.4 If a transversal intersects two lines, these two lines are parallel if, and only if, alternate angles are equal. (Theorem)
4.2.5 If a transversal intersects two lines, these two lines are parallel if, and only if, the sum of the interior angles on the same side of the transversal is 180°. (Theorem)
4.2.6 Lines which are parallel to the same line, are parallel to each other. (Theorem)
*4.2.7 The exterior angle of a triangle is equal to the sum of the interior opposite angles. (Theorem)
*4.2.8 The sum of the angles of a triangle is 180°. (Theorem)
4.2.9 The concept of congruence.
4.2.10 If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another, the two triangles are congruent. (Axiom)
4.2.11 If three sides of one triangle are respectively equal to three sides of another, the triangles are congruent. (Axiom)
4.2.12 If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another, the two triangles are congruent. (Theorem without proof)
*4.2.13.1 If two sides of a triangle are equal, the angles opposite them are equal. (Theorem)
*4.2.13.2 If two angles of a triangle are equal, the sides opposite them are equal. (Theorem)
4.2.14 If in two right-angled triangles the hypotenuse and one side of the one are respectively equal to the hypotenuse and one side of the other, the triangles are congruent. (Theorem)
4.2.15 Definitions of: quadrilateral, parallelogram, rhombus, rectangle, square and trapezium.
*4.2.16.1 The opposite sides and angles of a parallelogram are equal. (Theorem)
*4.2.16.2 A quadrilateral is a parallelogram if the opposite sides are equal. (Theorem)
*4.2.16.3 A quadrilateral is a parallelogram if the opposite angles are equal. (Theorem)
*4.2.17 A diagonal of a parallelogram bisects the area of the parallelogram. (Theorem)
*4.2.18.1 The diagonals of a parallelogram bisect each other. (Theorem)
*4.2.18.2 If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram. (Theorem)
*4.2.19 If two opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram. (Theorem)
4.2.20 The diagonals of a rectangle are equal to each other. (Theorem)
4.2.21 The diagonals of a rhombus bisect each other at right angles, and bisect the angles of the rhombus. (Theorem)

*4.2.22 A parallelogram and a rectangle on the same base and between the same parallels, have equal areas (theorem), with the following corollaries;

4.2.22.1 The area of a parallelogram = base × height.

4.2.22.2 The area of a triangle = $\frac{1}{2}$ base × height.

4.2.22.3 The area of a trapezium = $\frac{1}{2}$ (sum of the parallel sides) × (the perpendicular distance between the parallel sides).

*4.2.23 The line segment joining the mid-points of two sides of a triangle, is parallel to the third side, and is equal to half the third side. (Theorem)

*4.2.24 The line drawn through the mid-point of one side of a triangle, parallel to another side, bisects the third side. (Theorem)

*4.2.25 If three or more parallel lines make equal intercepts on a given transversal, they make equal intercepts on any other transversal. (Theorem)

4.2.26 The theorem of Pythagoras and its converse.

4.2.27 Calculations in connection with lengths, areas and volumes of

4.2.27.1 right prisms on triangular and rectangular bases;

4.2.27.2 right pyramids on rectangular bases;

4.2.27.3 right cylinders and cones on circular bases; and

4.2.27.4 spheres.

(Direct calculations and calculations involving changing the subject of a formula. Only simple numerical applications, where the data and results are rational numbers, are required. Deductions of formulae are not required.)

THE EXAMINATION

Two papers will be set

First Paper: 2 hours

Algebra	125 marks
Second Paper: $1\frac{1}{2}$ hours	
Geometry	75 marks
Cumulative year mark	100 marks
Total	300 marks

5. STANDARDS 9 AND 10

N.B. (*a*) Where applicable, the language and notation of sets should be used throughout.
(*b*) When the universal set is not specified, the set of real numbers is implied.
(*c*) No point-by-point plotting of graphs will be required in examinations.
(*d*) Any mathematically valid solution of a problem is acceptable.

5.1 *Algebra*

5.1.1 Number concept: summary of the operations and rules for operations in the various number sets, only as introduction to the following:

5.1.1.1 The principle of mathematical induction with simple applications to summation of series.

5.1.1.2 The structures, groups and fields, to illustrate properties of familiar number systems.

5.1.1.2.1 Definition of a group; examples of groups in both the additive and multiplicative structures of the familiar number systems; applications limited to equivalence classes (residue classes) and symmetries of simple geometric figures.

5.1.1.2.2 Definition and recognition of a field in a number system; fields for equivalence classes modulo m (m a natural number).

5.1.2 Relations and functions.

5.1.2.1 Mapping, rule, formula, one-to-one, one-to-many, many-to-one and many-to-many mappings, relations and functions, domain and range, variable, dependent variable, independent variable.

5.1.2.2 The inverses of the functions defined by $y=ax+b$, $y=ax^2$ and $y=a(x+p)^2$.

5.1.2.3 The quadratic expression ax^2+bx+c and its conversion to the form $a(x+p)^2+q$.

5.1.2.4 Function of a function.

5.1.2.5 Graphical representation of relations, including examples of relations defined by equations (with rational coefficients) of at most the second degree; symmetry, intercepts with the axes and shape.

5.1.2.6 Special attention to the following relations:
$\{(x;y)|ax+by+c=o\}$;
$\{(x;y)|x^2+y^2=r^2\}$; $\{(x;y)|y=ax^2+bx+c\}$;
$\{(x;y)|xy=k\}$, and absolute values, such as
$\{(x;y)|y=|ax+b|\}$, where x and y are real.
The deduction of the characteristics of each relation from its equation and graphical representation in respect of domain, range, intercepts with the axes, shape and symmetry. Slope and gradient of $\{(x;y)|ax+by+c=o\}$

5.1.2.7 Equalities and inequalities in respect of relations from 5.1.2.6, including
$\{(x;y)|y \gtreqless \sqrt{r^2-x^2}\}$; $\{(x;y)|y \gtreqless -\sqrt{r^2-x^2}\}$; $\{(x;y)|y \gtreqless \frac{k}{x}\}$; $\{(x;y)|y \gtreqless |ax+b|\}$.

5.1.3 Quadratic equations and inequalities

5.1.3.1 The roots of $ax^2+bx+c=o$; relationship between b^2-4ac and the nature of the roots. The sum and product of the roots.

5.1.3.2 The solution of $ax^2+bx+c \gtreqless o$.

5.1.3.3 The solution of two simultaneous equations in two unknowns of which one equation is of the first degree and the other of the second.

5.1.3.4 Problems which lead to quadratic equations and inequalities of the above types.

5.1.4 Exponents.

N.B. For the proofs of laws the base must be positive.

5.1.4.1 Definition of a^n for n a natural number, with deduction of the laws:

$a^m.a^n=a^{m+n}$

$a^m \div a^n=a^{m-n}$ for $m>n$ or $\frac{1}{a^{n-m}}$ for $m<n$

$(ab)^n=a^nb^n$

$(a^m)^n=a^{mn}$

5.1.4.2 Extension of the definition of a^n to include all integers and rational numbers as exponents by defining

$a^o=1$; $a^{-n}=\frac{1}{a^n}$; $\left(a^{\frac{p}{q}}\right)^q=a^p$

5.1.4.3 Relation between surds and exponents and notation for the corresponding basic properties:

$\sqrt[n]{a}.\sqrt[n]{b}=\sqrt[n]{ab}$; $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}$; $(\sqrt[m]{a})^n=\sqrt[m]{a^n}$. $(a,b>o)$.

5.1.5 Logarithms.

5.1.5.1 The exponential function defined by $y=a^x$, $a>o$; its graph, shape and position; deductions from graph.

5.1.5.2 The logarithmic function defined by $y=\log_a x$, $a>o$, and $a\neq 1$, as inverse of the exponential function defined by $y=a^x$; its characteristics and graph; deductions from the graph.

5.1.5.3 The basic properties of logarithms. (Proofs required.)

5.1.5.4 Change of base. (Proof required.)

5.1.5.5 The use of logarithmic tables as needed for relevant calculations.

5.1.6 Sequences and series.

5.1.6.1 Characteristics and the general terms of arithmetic and geometric sequences; the Σ-notation; the sum to n terms of arithmetic and geometric series; calculation of number of terms.

5.1.6.2 The convergence of a geometric series and its sum to infinity.

5.1.6.3 Simple applications.

5.2. *Synthetic Geometry*

N.B. Although all theorems must be proved, only proofs of theorems denoted with an asterisk (and their converses where mentioned or implied) in the following list will be required in examinations. Applications (including constructions) of any definition, axiom or theorem in this list or in the list for Standard 8 may be set. Not more than *three-tenths* of the marks will be allocated to bookwork in the examination.

*5.2.1 The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord, and, conversely, the perpendicular bisector of a chord passes through the centre of the circle. (Theorem)

*5.2.2 The angle which an arc of a circle subtends at the centre is double the angle it subtends at any point on the circumference. (Theorem)

5.2.3 The angle in a semi-circle is a right angle. (Theorem)

5.2.4 Angles in the same segment of a circle are equal, and, conversely, if a line segment joining two points subtends equal angles at two other points on the same side of the line segment, these four points lie on the circumference of a circle. (Theorem)

*5.2.5 A quadrilateral is a cyclic quadrilateral if, and only if, opposite angles are supplementary. (Theorem)

5.2.5.1 The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. (Corollary)

5.2.6 Triangles (parallelograms) on the same base (or equal bases) and on the same side of it (them), are equal in area if, and only if, they lie between the same parallel lines. (Theorem)

5.2.7 Definition of similarity.

*5.2.8 Any line divides two sides of a triangle proportionally if, and only if, it is parallel to the third side. (Theorem)

*5.2.9 Two triangles are similar if, and only if, corresponding sides are proportional. (Theorem)

*5.2.10 The theorem of Pythagoras and its converse.

5.2.11 A line through a point on the circumference of a circle is a tangent to the circle if, and only if, it is perpendicular to the radius. (Theorem)

*5.2.12 A line through a point on the circumference of a circle is a tangent to the circle if, and only if, the angle between the line and a chord through the point is equal to the angle in the alternate segment. (Theorem)

*5.2.13 If a secant and a tangent to a circle are drawn from a point outside the circle, the square of the length of the tangent is equal to the product of the lengths of the segments of the secant. (Theorem)

*5.2.14 If two chords of a circle intersect inside or outside the circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other. (Theorem)

5.2.15 The internal bisectors of the angles of a triangle are concurrent. (Theorem)

5.2.16 The perpendicular bisectors of the sides of a triangle are concurrent. (Theorem)

5.2.17 The medians of a triangle are concurrent. (Theorem)

5.2.18 The altitudes of a triangle are concurrent. (Theorem)

5.3 *Vector algebra*

5.3.1 Definition of a vector as an ordered number pair; its magnitude and direction.
5.3.1.1 Corollary: directed line segments are representations of vectors.
5.3.2 Equality of vectors.
5.3.3 Special vectors.
5.3.3.1 The null vector.
5.3.3.2 Unit vector in the direction of a given vector.
5.3.3.3 The negative of a vector.
5.3.4 Addition of vectors.
5.3.4.1 Geometric representation.
5.3.4.2 Deductions.
5.3.4.2.1 Commutative law.
5.3.4.2.2 Associative law.
5.3.4.2.3 Additive identity.
5.3.4.2.4 Additive inverse; subtraction of vectors; geometric representation.
5.3.5 Multiplication of a vector by a scalar.
5.3.5.1 Deductions: the two distributive laws and the associative law.
5.3.6 The inner product of two vectors and the angle between two vectors.
5.3.6.1 Deductions and corollaries from the definitions.
5.3.7 Vector equation of a straight line and the division of a line segment in the ratio $k : l$.
5.3.8 Elementary applications to geometric problems.

5.4 *Trigonometry*

5.4.1 Definitions of the six trigonometric ratios for any angle (positive or negative), with the aid of co-ordinates in respect of a set of rectangular axes.
5.4.2 Deduction of the relations between the trigonometric ratios for any angle.
5.4.3 Sin, cos and tan as functions, with a complete description of the distinct domains, ranges and periodicity, the latter as it becomes evident from the function values for $\theta \pm n \times 360°$.
5.4.4 Function values for 0°, 30°, 45°, 60°, 90° and the corresponding angles in the other quadrants.
5.4.5 Sketches of the curves of the sin, cos and tan functions, in respect of a set of rectangular axes. (Also for compound angles.)
5.4.6 Function values for $180° \pm \theta$, $360° \pm \theta$ and $\theta \pm n \times 360°$ for $n = \pm 1; \pm 2; \pm 3; \ldots$ expressed in terms of θ.
5.4.7 Application of the trigonometric functions to angles in a triangle. Deduction of the sine rule, cosine rule and the formula: area of a triangle $ABC = \frac{1}{2}ab \sin C$. Solution of triangles. Problems in two and three dimensions in which these concepts are applied.
5.4.8 Identities.
5.4.8.1 Elementary examples obtained from the relations between the trigonometric ratios for any angle.
5.4.8.2 The identities:
(i) $\sin^2\theta + \cos^2\theta = 1$
(ii) $\sec^2\theta = \tan^2\theta + 1$
(iii) $\operatorname{cosec}^2\theta = \cot^2\theta + 1$
5.4.8.3 Proof of the identity $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$ and deductions of identities for $\cos(\alpha + \beta)$, $\sin(\alpha \pm \beta)$, $\cos(90° \pm \theta)$, $\sin(90° \pm \theta)$, $\cos 2\theta$, $\sin 2\theta$, $\tan(\alpha \pm \beta)$ and $\tan 2\theta$.
5.4.9 General solutions of elementary trigonometric equations.

THE EXAMINATION

STANDARD 10

The examination at the end of Standard 10 will consist of two papers, each of three hours duration. Both papers will be set on the syllabus for Standards 9 and 10.

The total marks for each paper will be 150. The distribution of marks among the various sections of the syllabus, will be as follows:

Paper 1

Algebra, and one question of a miscellaneous nature which may cover all the sections of the syllabus and for which at most 15 marks may be allocated: .. .. 150 marks.

Paper 2

Synthetic Geometry	50 marks
Vector algebra	40 marks
Trigonometry	60 marks
Total	150 marks

N.B. The marks for each section of the second paper may be increased or decreased by a maximum of 10.

STANDARD 9

The examination at the end of Standard 9 will consist of two papers. The total marks for each paper will be 150. Schools themselves decide on the marks allocated to each sub-section, as well as on the duration of the examination.

SYMBOLS

$=$	is equal to.
$\neq$	is not equal to.
$\approx$	is approximately equal to.
$>$	is greater than.
$<$	is less than.
$\ngtr$	is not greater than.
$\nless$	is not less than.
$\geq$	is greater than or equal to.
$\leq$	is less than or equal to.
$\equiv$	is congruent to; is identical to.
$\vert\vert\vert$	is similar to.
$\vert\vert$	is parallel to.
$\perp$	is perpendicular to.
$\therefore$	therefore.
$\because$	because.
$\Rightarrow$	implies.
$\Leftrightarrow$	implies and is implied by.
$\vert x\vert$	absolute value of x.
$\in$	is an element of.
$\notin$	is not an element of.
$\subset$	is a subset of.
$\supset$	includes.
$\{\ \}$ or ϕ	the empty set.
$\simeq$	is equivalent to.
$\not\subset$	is not a subset of.
$A \cap B$	the intersection of A and B.
$A \cup B$	the union of A and B.
A'	the complement of A.
$n(A)$	the cardinal number of A.
$\{x\vert\ldots\}$ or $\{x:\ldots\}$	the set of all x, such that ....
$A \times B$	A cross B; the cross product (Cartesian product) of A and B.
f^{-1}	inverse of f.
U	universal set.
$f: x \rightarrow 2x+1$	the function mapping x onto $2x+1$.
$f \circ g$	f of g.
$f(g(1))$	the value of the function f at $g(1)$.
$2,\dot{3}$	2 comma 3 recurring.
$[-360°; 360°]$	closed interval
$\sum_{1}^{n}$	sum to n terms.
$\sum_{1}^{\infty}$	sum to infinity.
$\underline{a}$	vector a.
$\vert\underline{a}\vert$	length of vector a.

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PROVINCIAL ADMINISTRATION OF THE CAPE OF GOOD HOPE

DEPARTMENT OF EDUCATION

SENIOR SECONDARY COURSE

SYLLABUS FOR MATHEMATICS
(Standard Grade)

1973

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS (STANDARD GRADE)

The following syllabus for Mathematics (Standard Grade) for the Senior Secondary Course will be introduced as from 1st January, 1974.

The syllabus will be introduced in Standard 8 in 1974, and the first Senior Certificate Examination on this syllabus will be held in November/December, 1976.

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS (STANDARD GRADE)

1. *General remarks*

1.1 The subject-matter dealt with in the Standard 8 syllabus is regarded as *pre-knowledge* to the syllabus for Standards 9 and 10. This knowledge is again used either directly or indirectly in the syllabus for the last two school years, and, therefore, also in the Senior Certificate Examination, but during these years the emphasis should fall on the actual syllabus for Standards 9 and 10.

1.2 In tests and examinations more stress should be laid on insight and understanding than on mechanical reproduction of formal knowledge; nevertheless the subject must be made accessible to as many pupils as possible, by avoiding unrealistic demands in the examination.

1.3 Where applicable, the slide rule may be used.

2. *General aims*

2.1 To acquaint the pupils with the part played by Mathematics in the modern world in which man is constantly required to handle quantitative and spatial aspects of situations;

2.2 to contribute to the general education of the pupils, with special emphasis on the development of logical thought and of systematic, accurate and neat methods of working;

2.3 to cultivate in pupils appreciation of the structure and the central theme of each section of the syllabus, as well as of the underlying relation between certain sections;

2.4 to acquaint pupils with, and train them in, mathematical methods of thought and work;

2.5 to give pupils a clear insight into, and a thorough knowledge and understanding of those basic mathematical principles which will prepare and equip them for daily life and further study.

3. *Remarks*

3.1 In all sections of the subject pupils must be guided to tackle and solve each problem or theorem systematically by

3.1.1 paying close attention to the data and what is required;

3.1.2 accounting for all facts and theorems that can possibly serve as a key to the solution, irrespective of the section of Mathematics in which they occur;

3.1.3 paying close attention to necessary and sufficient requirements with respect to wording and reasoning when writing down the solution.

3.2 Pupils must be trained in the correct use of notation and terminology, the exact formulation of statements and the making of accurate deductions.

THE SYLLABUS

4. STANDARD 8

4.1 *Algebra*

4.1.1 Sets (consolidation).

4.1.1.1 The set concept.

4.1.1.2 Elements of a set, the empty set, subset, universal set, complement of a set, intersection and union of sets, Cartesian product.

4.1.1.3 Venn diagrams and their application as aids to illustrate the solution of problems.

4.1.2 Number concept.

4.1.2.1 An outline of the structure of the system of real numbers, as developed from the natural numbers, with emphasis on the irrational numbers.

4.1.2.2 Relations of order between numbers and the relevant laws; operations with numbers, rules for operations, closure in respect of operations.

4.1.2.3 The number line. One-to-one correspondence between points on the number line and the real numbers.

4.1.3 Products of the following types by inspection:

4.1.3.1 $(a \pm b)(c \pm d)$.

4.1.3.2 $(ax \pm b)(cx \pm d)$.

4.1.3.3 $(ax \pm b)^2$.

4.1.3.4 $(ax + b)(ax - b)$.

4.1.4 Factors of polynomials of the following types:

4.1.4.1 $ax \pm bx \pm ay \pm by$.

4.1.4.2 $ax^2 + bx + c$ and $ax^2 + bxy + cy^2$

4.1.4.3 $a^2 - b^2$

4.1.4.4 The foregoing types, with the inclusion of a common factor

4.1.5 L.C.M. of polynomials by factorisation only.

4.1.6 Algebraic fractions (simple examples only).

4.1.6.1 Simplification

4.1.6.2 Operations.

4.1.7 Solution sets.

4.1.7.1 Determining the solution sets of linear equations in one unknown with numerical and literal coefficients.

4.1.7.2 Determining the solution sets of linear inequalities in one unknown with numerical coefficients only.
4.1.8 Formulae.
4.1.8.1 Changing the subject of a formula. (Simple examples only.)
4.1.8.2 Substitution in formulae.
4.1.9 Relations and functions.
4.1.9.1 Relation; the mapping according to a given rule of the elements of one set on the elements of another set.
4.1.9.2 Sets of ordered pairs.
4.1.9.3 The function concept.
4.1.9.4 Representation of ordered pairs by points in the Cartesian plane.
4.1.9.5 The function defined by $y=mx+c$ and its graphical representation; intercept and gradient.
4.1.10 Systems of linear equations and inequalities.
4.1.10.1 Algebraic solution of systems of linear equations in two unknowns. (Numerical coefficients only.)
4.1.10.2 Graphical illustration of the solution sets of systems of linear equations in two unknowns.
4.1.10.3 Graphical illustration of the solution sets of systems of inequalities in two unknowns.
4.1.11 Logarithms.
4.1.11.1 Definition of a^n for n an integer.
4.1.11.2 Use of logarithmic tables for:
4.1.11.2.1 multiplication;
4.1.11.2.2 division;
4.1.11.2.3 raising to a power and taking roots (negative indices excluded). (Simple examples only.)

4.2 *Synthetic Geometry*

N.B.—Although all theorems must be proved, formal proofs of theorems in examinations must be limited. Only proofs of theorems (but not their converses) denoted by an asterisk in the following list, will be required. However, applications (including constructions), of any definition, axiom or theorem in this list may be set.

4.2.1 If two lines intersect, the sum of any pair of adjacent angles is equal to 180°, and conversely, if the sum of two adjacent angles is 180°, the outer arms form a straight line. (Axiom)
*4.2.2 When two lines intersect, the vertically opposite angles are equal. (Theorem)
4.2.3 Two lines are parallel if, and only if, an intersecting transversal forms equal corresponding angles. (Definition)
4.2.4 If a transversal intersects two lines, these two lines are parallel if, and only if, alternate angles are equal. (Theorem)
4.2.5 If a transversal intersects two lines, these two lines are parallel if, and only if, the sum of the interior angles on the same side of the transversal is 180°. (Theorem)
4.2.6 Lines which are parallel to the same line, are parallel to each other. (Theorem)
*4.2.7 The exterior angle of a triangle is equal to the sum of the interior opposite angles. (Theorem)
*4.2.8 The sum of the angles of a triangle is 180°. (Theorem)
4.2.9 The concept of congruence.
4.2.10 If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another, the two triangles are congruent. (Axiom)
4.2.11 If three sides of one triangle are respectively equal to the three sides of another, the triangles are congruent. (Axiom)
4.2.12 If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another, the two triangles are congruent. (Theorem without proof)
*4.2.13.1 If two sides of a triangle are equal, the angles opposite them are equal. (Theorem)
4.2.13.2 If two angles of a triangle are equal, the sides opposite them are equal. (Theorem)
4.2.14 If in two right-angled triangles the hypotenuse and one side of the one are respectively equal to the hypotenuse and one side of the other, the triangles are congruent. (Theorem)
4.2.15 Definitions of: quadrilateral, parallelogram, rhombus, rectangle, square and trapezium.
*4.2.16.1 The opposite sides and angles of a parallelogram are equal. (Theorem)
4.2.16.2 If the opposite angles or sides of a quadrilateral are equal, the quadrilateral is a parallelogram. (Theorem)
*4.2.17 A diagonal of a parallelogram bisects the area of the parallelogram. (Theorem)
*4.2.18.1 The diagonals of a parallelogram bisect each other. (Theorem)
4.2.18.2 If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram. (Theorem)
*4.2.19 If two opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram (Theorem)
4.2.20 The diagonals of a rectangle are equal to each other. (Theorem)
4.2.21 The diagonals of a rhombus bisect each other at right angles, and bisect the angles of the rhombus. (Theorem)
*4.2.22 A parallelogram and a rectangle on the same base and between the same parallel lines have equal areas (theorem), with the following corollaries:
4.2.22.1 The area of a parallelogram = base × height.
4.2.22.2 The area of a triangle = ½ base × height.
4.2.22.3 The area of a trapezium = ½ (sum of the parallel sides) × (the perpendicular distance between the parallel sides).
*4.2.23 The line segment joining the mid-points of two sides of a triangle, is parallel to the third side, and is equal to half the third side. (Theorem)
4.2.24 The line drawn through the mid-point of one side of a triangle, parallel to another side, bisects the third side. (Theorem)

4.2.25 If three or more parallel lines make equal intercepts on a given transversal, they make equal intercepts on any other transversal. (Theorem)

4.2.26 The theorem of Pythagoras and its converse.

4.2.27 Calculations in connection with lengths, areas and volumes of

4.2.27.1 right prisms on triangular and rectangular bases;

4.2.27.2 right pyramids on rectangular bases;

4.2.27.3 right cylinders and cones on circular bases, and

4.2.27.4 spheres.

(Direct calculations not involving changing the subject of a formula. Only simple numerical applications, where the data and results are rational numbers, are required. Deductions of formulae are not required.)

THE EXAMINATION

Two papers will be set.

First Paper: 2 hours.

Algebra	125 marks
Second Paper: $1\frac{1}{2}$ hours.	
Geometry	75 marks
Cumulative year mark	100 marks
Total	300 marks

5. STANDARDS 9 AND 10

N.B.—(a) Where applicable, the language and notation of sets should be used throughout.
(b) When the universal set is not specified, the set of real numbers is implied.
(c) No point-by-point plotting of graphs will be required in examinations.
(d) Any mathematically valid solution of a problem is acceptable.

5.1 *Algebra*

5.1.1 Relations and functions.

5.1.1.1 Mapping, rule, formula, one-to-one, one-to-many, many-to-one and many-to-many mappings, relations and functions, domain and range, variable, dependent variable, independent variable.

5.1.1.2 The quadratic expression ax^2+bx+c and its conversion to the form $a(x+p)^2+q$.

5.1.1.3 Graphical representation of relations, including examples of relations defined by equations (with rational coefficients) of at most the second degree; symmetry, intercepts with the axes and shape.

5.1.1.4 Special attention to the following relations:
$\{(x;y)|ax+by+c=o\}$; $\{(x;y)|x^2+y^2=r^2\}$; $\{(x;y)|y=ax^2+bx+c\}$; and $\{(x;y)|xy=k\}$;
where x and y are real. The deduction of the characteristics of each relation from its equation and graphical representation with respect to domain, range, intercepts with the axes, shape and symmetry. Slope and gradient of $\{(x;y)|ax+by+c=o\}$.

5.1.1.5 Equality relations between the linear relation $\{(x;y)|ax^2+by+c=o\}$ and one of the following relations: $\{(x;y)|y=ax^2+bx+c\}$; $\{(x;y)|x^2+y^2=r^2\}$ and $\{(x;y)|xy=k\}$ where x and y are real numbers. Equality relations between 2 of the latter 3 relations are excluded.

5.1.2 Quadratic equations.

5.1.2.1 The roots of $ax^2+bx+c=o$; the relationship between b^2-4ac and the nature of the roots.

5.1.2.2 Solution of two simultaneous equations in two unknowns of which one equation is of the first degree and the other of the second.

5.1.3 Exponents.

5.1.3.1 Definition of a^n for n a natural number, with deduction of the laws:
$a^m.a^n=a^{m+n}$
$a^m \div a^n=a^{m-n}$ for $m>n$ or $\frac{1}{a^{n-m}}$ for $m<n$
$(ab)^n=a^nb^n$
$(a^m)^n=a^{mn}$.
(No formal proofs will be required in the examination.)

5.1.3.2 Extension of the definition of a^n to include all integers and rational numbers as exponents by defining

$$a^o=1;\ a^{-n}=\frac{1}{a^n};\ \left(a^{\frac{p}{q}}\right)^q=a^p(a>o).$$

5.1.3.3 Relation between surds and exponents and the notation for the corresponding basic properties:

$$\sqrt[n]{a}.\sqrt[n]{b}=\sqrt[n]{ab};\ \sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a};\ (\sqrt[m]{a})^n=\sqrt[m]{a^n}\quad (a,b>o).$$

5.1.4 Logarithms.

5.1.4.1 Definition of a logarithm.

5.1.4.2 The basic properties of logarithms, excluding change of base (no formal proofs required).

5.1.4.3 Application of logarithms to calculations with the aid of logarithmic tables.

5.1.5 Sequences and series.

5.1.5.1 Characteristics and the general terms of arithmetic and geometric sequences; the Σ-notation; the sum to n terms of arithmetic and geometric series; calculation of number of terms in arithmetic series.

5.1.5.2 Simple applications.

5.2 *Synthetic Geometry*

N.B.--Although all theorems must be proved, only proofs of theorems denoted with an asterisk (but not of their converses) in the following list will be required in examinations. Applications (including constructions) of any definition, axiom or theorem in this list or in the list for Standard 8 may be set. Not more than *two-fifths* of the marks will be allocated to bookwork in the examination. Simple problems only.

*5.2.1.1 The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. (Theorem)

5.2.1.2 The line from the centre of a circle, perpendicular to a chord, bisects the chord. (Theorem)

5.2.1.3 The perpendicular bisector of a chord passes through the centre of the circle. (Theorem)

*5.2.2 The angle which an arc of a circle subtends at the centre, is double the angle it subtends at any point on the circumference. (Theorem)

5.2.3 The angle in a semi-circle is a right angle. (Theorem)

5.2.4 Angles in the same segment of a circle are equal: and, conversely, if a line segment joining two points subtends equal angles at two other points on the same side of the line segment, these four points lie on the circumference of a circle. (Theorem)

*5.2.5.1 The opposite angles of a cyclic quadrilateral are supplementary. (Theorem)

5.2.5.2 If the opposite angles of a quadrilateral are supplementary, it is a cyclic quadrilateral. (Theorem)

5.2.5.3 The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and the converse. (Theorem)

5.2.6 Triangles (parallelograms) on the same base (or equal bases) and on the same side of it (them), are equal in area if, and only if, they lie between the same parallel lines. (Theorem)

5.2.7 Definition of similarity.

*5.2.8.1 Any line parallel to one side of a triangle divides the other two sides proportionally. (Theorem)

5.2.8.2 A line dividing two sides of a triangle proportionally is parallel to the third side. (Theorem)

*5.2.9.1 Corresponding sides of equiangular triangles are proportional. (Theorem)

5.2.9.2 Two triangles are equiangular if their corresponding sides are proportional. (Theorem)

*5.2.10.1 The theorem of Pythagoras. (Theorem)

5.2.10.2 The converse of the theorem of Pythagoras. (Theorem)

*5.2.11 A line through a point on the circumference of a circle is a tangent to the circle if, and only if, it is perpendicular to the radius. (Theorem)

*5.2.12.1 The angle which a tangent to a circle makes with a chord drawn to the point of contact is equal to the angle in the alternate segment. (Theorem)

5.2.12.2 If from one end of a chord a straight line is drawn to make with the chord an angle equal to the angle in the alternate segment, this straight line is a tangent to the circle. (Theorem)

5.2.13 The internal bisectors of the angles of a triangle are concurrent (Theorem)

5.2.14 The perpendicular bisectors of the sides of a triangle are concurrent. (Theorem)

5.2.15 The medians of a triangle are concurrent. (Theorem)

5.2.16 The altitudes of a triangle are concurrent. (Theorem)

5.3 *Vector algebra*

5.3.1 Definition of a vector as an ordered number pair; its magnitude and direction.

5.3.1.1 Corollary: directed line segments are representations of vectors.

5.3.2 Equality of vectors.

5.3.3 Special vectors.

5.3.3.1 The null vector.

5.3.3.2 Unit vector in the direction of a given vector.

5.3.3.3 The negative of a vector.

5.3.4 Addition of vectors.

5.3.4.1 Geometric representation.

5.3.4.2 Deductions:

5.3.4.2.1 Commutative law.

5.3.4.2.2 Associative law.

5.3.4.2.3 Additive identity.

5.3.4.2.4 Additive inverse: subtraction of vectors; geometric representation.

5.3.5 Multiplication of a vector by a scalar

5.3.5.1 Deductions: the two distributive laws and the associative law.

5.3.6 Elementary applications to geometric problems.

5.4 *Trigonometry*

5.4.1 Definitions of the six trigonometric ratios for any angle over the interval [0°; 360°] with the aid of co-ordinates in respect of a set of rectangular axes.

5.4.2 Deduction of the relations between the trigonometric ratios.

5.4.3 Sin, cos and tan as functions, with a complete description of the distinct domains, ranges and periodicity over the interval [0°; 360°]

5.4.4 Function values for 0°, 30°, 45°, 60°, 90° and the corresponding angles in the other quadrants.

5.4.5 Sketches of the curves of the sin, cos and tan functions in respect of a set of rectangular axes. (No compound angles and only over the interval [0°; 360°])

5.4.6 Expression of the sine, the cosine and the tangent of $90° \pm \theta$, $180° \pm \theta$ and $360° \pm \theta$ in terms of θ, where θ is acute.

5.4.7 Application of the trigonometric functions to angles in a triangle. Deduction of the sine rule, cosine rule and the formula: area of a triangle $ABC = \frac{1}{2}ab \sin C$. Solution of triangles. Problems in two dimensions in which these concepts are applied.

THE EXAMINATION

STANDARD 10

The examination at the end of Standard 10 will consist of two papers, each of three hours duration. Both papers will be set on the syllabus for Standards 9 and 10.

The total marks for each paper will be 150.

The distribution of marks among the various sections of the syllabus, will be as follows:

Paper 1

Algebra, and one question of a miscellaneous nature which may cover all the sections of the syllabus and for which at most 15 marks may be allocated; .. .. 150 marks.

Paper 2

Synthetic Geometry	50 marks
Vector algebra	40 marks
Trigonometry	60 marks
Total	150 marks

N.B.—The marks for each section of the second paper may be increased or decreased by a maximum of 10.

STANDARD 9

The examination at the end of Standard 9 will consist of two papers. The total marks for each paper will be 150. Schools themselves decide on the marks allocated to each subsection, as well as on the duration of the examination.

SYMBOLS

$=$	:	is equal to.
$\neq$	:	is not equal to.
$\approx$	:	is approximately equal to.
$>$	:	is greater than.
$<$	:	is less than.
$\ngtr$	:	is not greater than.
$\nless$	:	is not less than.
$\geq$	:	is greater than or equal to.
$\leq$	:	is less than or equal to.
$\equiv$	:	is congruent to; is identical to.
$\vert\vert\vert$	:	is similar to.
$\parallel$	:	is parallel to.
$\perp$	:	is perpendicular to.
$\therefore$	:	therefore.
$\because$	:	because.
$\Rightarrow$	:	implies.
$\Leftrightarrow$	:	implies and is implied by.
$\vert x \vert$	:	absolute value of x.
$\in$	:	is an element of.
$\notin$	:	is not an element of.
$\subset$	:	is a subset of.
$\supset$	:	includes.
$\{\ \}$ or ϕ	:	the empty set.
$\simeq$	:	is equivalent to.
$\not\subset$	:	is not a subset of.
$A \cap B$	:	the intersection of A and B.
$A \cup B$	:	the union of A and B.
A'	:	the complement of A.
$n(A)$	:	the cardinal number of A.
$\{x : \ldots\}$ or $\{x \vert \ldots\}$	:	the set of all x, such that ....
$A \times B$	:	A cross B; the cross product (Cartesian product) of A and B.
f^{-1}	:	inverse of f.

U	:	universal set.
$f: x \to 2x+1$	:	the function mapping x onto $2x+1$.
$f \circ g$	:	f of g.
$f(g(1))$	:	the value of the function f at $g(1)$.
$2,\dot{3}$	:	2 comma 3 recurring.
$[-360°; 360°]$	:	closed interval.
$\sum_1^n$	:	sum to n terms.
$\sum_1^\infty$	:	sum to infinity.
$\underline{a}$	:	vector a.
$\lvert\underline{a}\rvert$	:	length of vector a.

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SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS (STANDARD GRADE): AMENDMENTS

The attention of principals of high schools is drawn to the following amendments to the syllabus for Mathematics (Standard Grade) for Standards 8, 9 and 10. The amendments take effect immediately and the first Senior Certificate examination on the amended syllabus will be written during November/December 1977:

Standard 8

Paragraph 4.1.1.1: Add the following in brackets: "Revision, but not to be examined".
Paragraph 4.1.1.2: Add the following in brackets: "Revision, but not to be examined".
Paragraph 4.1.1.3: Delete the whole sentence.

Standards 9 *and* 10

Paragraph 5.1.3.3: Add the following in brackets: "Rationalisation of multinomials as denominators is excluded".

PROVINCIAL ADMINISTRATION OF THE CAPE OF GOOD HOPE

DEPARTMENT OF EDUCATION

SENIOR SECONDARY COURSE

SYLLABUS

FOR

MATHEMATICS

HIGHER GRADE

1985

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS HIGHER GRADE

The following syllabus for Mathematics Higher Grade for the Senior Secondary Course will be introduced as from 1 January 1985.

The syllabus will be introduced in Standard 8 in 1985 and the first Senior Certificate Examination in this subject will be held in November/December 1987.

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS HIGHER GRADE

1. REMARKS

1.1 The arrangement of the content of the syllabus and its subdivision is not necessarily an indication of the sequence in which the work must be handled.

1.2 Non-programmable pocket calculators may be used where necessary and applicable to develop mathematical concepts and for calculation. Basic instruction in the practical use of pocket calculators must be given.

1.3 The breakdown for standards 9 an 10 is suggested but is not prescriptive.

2. AIMS

2.1 To develop a love for, an interest in and a positive attitude towards Mathematics, by presenting the subject meaningfully

2.2 To enable pupils to gain mathematical knowledge and proficiency.

2.3 To develop clarity of thought and the ability to make logical deductions.

2.4 To develop accuracy and mathematical insight.

2.5 To instil in pupils the habit of estimating answers where applicable and, where possible, of verifying answers

2.6 To develop the ability of the pupils to use mathematical knowledge and methods in other subjects and in their daily life.

2.7 To provide basic training for future study and careers.

3. EXAMINATION

3.1 Standards 8 and 9 are examined internally.

3.2 Standard 10

3.2.1 The external Senior Certificate papers will be set on the syllabus for Standard 9 and Standard 10.

3.2.2 There will be two three-hour papers of equivalent value with the allocation of marks as follows:

3.2.2.1 First paper

• Algebra	: 75% (± 5%)
• Differential Calculus	: 25% (± 5%)

3.2.2.2 Second paper

• Questions of a miscellaneous nature covering two or more sections of the whole syllabus may be asked	: ± 5%
• Euclidean Geometry	: 30% (± 5%)
• Analytical Geometry	: 25% (± 5%)
• Trigonometry	: 40% (± 5%)

4. STANDARD 8

4.1 **Products**

The following types by inspection:

4.1.1 $(a \pm b)(c \pm d)$

4.1.2 $(ax \pm b)(cx \pm d)$ and $(ax \pm by)(cx \pm dy)$

4.1.3 $(ax \pm b)^2$ and $(ax \pm by)^2$

4.1.4 $(ax + b)(ax - b)$ and $(ax + by)(ax - by)$

4.1.5 $(ax + b)(a^2x^2 - abx + b^2)$ and $(ax + by)(a^2x^2 - abxy + b^2y^2)$

4.1.6 $(ax - b)(a^2x^2 + abx + b^2)$ and $(ax - by)(a^2x^2 + abxy + b^2y^2)$

4.2 **Factors**

Factorization of the following types:

4.2.1 Quadrinomials by grouping

4.2.2 Quadratic trinomials

4.2.3 Difference of squares

4.2.4 Sum and difference of cubes

4.2.5 Preceding types including a common factor

4.3 **Algebraic fractions**

4.3.1 Simplification

4.3.2 Main operations

4.4 **Equations and inequalities in one unknown**

4.4.1 Solution of linear equations with numerical and literal coefficients

4.4.2 Solution of linear inequalities with numerical coefficients

4.4.3 Solutions of problems with the aid of linear equations

4.4.4 Solutions of quadratic equations by means of factors in which only integers may occur as coefficients

4.5 **Formulae**

4.5.1 Construction of formulae for area and volume of right prisms and right cylinders

4.5.2 Substitution in formulae

4.5.3 Changing the subject of formulae

4.6 **Functions**

4.6.1 Concept of a function, functional notation and values of a function

4.6.2 Domain and range of a function

4.6.3 Graphical representation of the functions (and deduction of the characteristics of each from its equation and graphical representation) defined by:

4.6.3.1 $ax + by + c = 0$

4.6.3.2 $y = \sqrt{r^2 - x^2}$ (r rational and $r \neq 0$)

4.6.3.3 $y = -\sqrt{r^2 - x^2}$ (r rational and $r \neq 0$)

4.6.3.4 $xy = k$ (k an integer and $k \neq 0$)

4.6.3.5 $y = ax^2 + c$ (a en c rational numbers)

4.7 **Systems of linear equations in two unknowns**

4.7.1 Solution of systems of linear equations — graphically and algebraically

4.7.2 Application of systems of linear equations in the solution of problems

4.8 **Exponents (Pocket calculators may not be used)**

4.8.1 The meaning of a^{-m}, a^0 en $a^{\frac{m}{n}}$, $a \neq 0$ (m, n natural numbers), (for $a^{\frac{m}{n}}$ examples $a>0$ only)

4.8.2 Intuitive extension of the laws of exponents to include integers and rational exponents

4.9 **Euclidean Geometry**

(i) The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the list for Standard 7 may be used as reasons for statements in solving riders.

(ii) Although all theorems must be proved only proofs of theorems denoted with an asterisk (*) (and their converses where mentioned) will be required for examination purposes.

(iii) Applications of any axiom or theorem in this list or in the list for Standard 7, may be set (No constructions for examination purposes).

(iv) Not more than *three tenths* of the marks for Geometry will be given for "bookwork" in the examination.

(v) A logical order of parr. 4.9.1 to 4.9.12 must be adhered to.

*4.9.1 The opposite sides and angles of a parallelogram are equal, and conversely, if the opposite sides or angles of a quadrilateral are equal, the quadrilateral is a parallelogram (Theorem).

*4.9.2 The diagonals of a parallelogram bisect each other, and conversely, if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram (Theorem).

*4.9.3 If two opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram (Theorem).

4.9.4 The diagonals of a rectangle are equal (Theorem).

4.9.5 The diagonals of a rhombus bisect each other at right angles, and bisect the angles of the rhombus (Theorem).

*4.9.6 A diagonal of a parallelogram bisects the area of the parallelogram (Theorem).

*4.9.7 A parallelogram and rectangle on the same base and between the same parallels have equal areas (Theorem).

4.9.8 Triangles (parallelograms) on the same base (or equal bases) and on the same side thereof are equal in area if they lie between the same parallel lines, and conversely, if triangles (parallelograms) lying on the same base (or equal bases) and on the same side thereof, have equal areas, they lie between the same parallel lines (Theorem).

*4.9.9 The line segment joining the mid-points of two sides of a triangle is parallel to the third side, and equal to half the third side (Theorem).

*4.9.10 The line passing through the mid-point of one side of a triangle, parallel to another side, bisects the third side (Theorem)..

*4.9.11 If three or more parallel lines make equal intercepts on a given transversal, they make equal intercepts on any other transversal (Theorem).

4.9.12 The Theorem of Pythagoras and its converse.

4.10 **Trigonometry**

4.10.1 Definitions of the six trigonometric functions for an angle θ and $\theta \in [0^\circ; 90^\circ]$

4.10.2 Applications for the six trigonometric functions in a right-angled triangle

4.10.3 Solution of right-angled triangles

4.10.4 The definitions of the six trigonometric functions for any angle in terms of co-ordinates with respect to perpendicular axes

5. **STANDARD 9**

5.1 **Algebra**

5.1.1 *A brief intuitive review of the real numbers*

5.1.2 *Absolute value*

5.1.2.1 Definition

5.1.2.2 Algebraic solution of $|x - a| \lesseqgtr b$

5.1.3 *Functions*

(No point-by-point plotting of graphs will be required for examination purposes.)

5.1.3.1 Graphical reprsentation of the functions defined by:

(a) $y = ax^2 + bx + c$ $(a \neq 0)$

(b) $y = |x|$, $y = |x - a|$ and $y = |x| + a$, a is a rational number.

5.1.3.2 The deduction of the characteristics of the functions in 5.1.3.1 from their equations and graphical representation

5.1.3.3 Graphical representation of simultaneous equalities with respect to functions from 5.1.3.1, including their intersection with $ax + by + c = 0$

5.1.3.4 The inverses of the functions defined by $y = mx + c$, $y = ax^2$, $xy = k$, $y = |x|$

5.1.4 *Linear programming*

5.1.4.1 Graphical representation of $ax + by + c \lesseqgtr 0$

5.1.4.2 Problem solving by means of programming

5.1.5 *Quadratic equations and inequalities*

5.1.5.1 The roots of $ax^2 + bx + c = 0$ where a, b and c are rational

(a) The solution of $ax^2 + bx + c = 0$

(b) Conditions for which the equation is solvable on the set of real numbers

(c) Equal and unequal roots; rational and irrational roots; real and non-real roots.

5.1.5.2 The solution of $ax^2 + bx + c \lesseqgtr 0$

5.1.5.3 Problems which lead to quadratic equations

5.1.6 *The remainder and factor theorem*

Applications including solution of equations of the third degree

5.1.7 *Systems of equations*

5.1.7.1 Solving simultaneous equations in two unknowns of which one equation is of the first and the other of the second degree

5.1.7.2 Solving of problems which lead to equations as shown in 5.1.7.1

5.1.8 *Exponents*

5.1.8.1 Solving of equations of the form

$ax^{\frac{m}{n}} - b = 0$ where m and n are integers, $n \neq 0$

5.1.8.2 Relationship between surds and exponents and the corresponding basic properties where a and b are positive and m and n are positive integers:

$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$(\sqrt[m]{a})^n = \sqrt[m]{a^n}$

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

5.1.8.3 Rationalization of surds

5.1.8.4 Solving of simple exponential equations in one variable

5.2 **Trigonometry**

5.2.1 The definitions of the six trigonometric functions for any angle in terms of co-ordinates with respect to perpendicular axes

5.2.2 Graphs of $y = \sin\theta$, $y = \cos\theta$ en $y = \tan\theta$

5.2.3 Function values for $(90° - \theta)$; $(180° \pm \theta)$ and $(360° \pm \theta)$, expressed in function values for θ, where $\theta \in [0°; 90°]$

5.2.4 Function values for 0°, 30°, 45° and multiples thereof without the use of calculators

5.2.5 *Identities*

5.2.5.1 The mutual relationships between the trigonometric function values

5.2.5.2 (a) $\sin^2\theta + \cos^2\theta = 1$

(b) $\tan^2\theta + 1 = \sec^2\theta$

(c) $\cot^2\theta + 1 = \text{cosec}^2\theta$

5.2.6 *Formulae*

5.2.6.1 The sine formula

5.2.6.2 The cosine formula

5.2.6.3 Area of a triangle $ABC = \frac{1}{2}\, ab\, \sin C$

5.2.6.4 Application of the above formulae in the solution of

(a) triangles;

(b) problems in two and three dimensions.

5.3 **Euclidean Geometry**

(i) The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the list of standards 7 and 8 may be used as reasons for statements in solving riders.

(ii) Although all theorems must be proved only proofs of theorems denoted with an asterisk (*) (and their converses where mentioned) in the following list will be required for examination purposes.

(iii) Applications of any axiom or theorem in this list or in the lists for Standard 7 and Standard 8 may be set (No constructions for examination purposes).

(iv) Not more than *three tenths* of the marks for Geometry will be given for "bookwork" in the examination.

(v) A logical order of parr. 5.3.1 to 5.3.11 must be adhered to.

5.3.1 The Theorem of Pythagoras and its converse.

*5.3.2 The line segment joining the centre of a circle to the mid-point of a chord is perpendicular to the cord, and conversely, the line segment drawn from the centre of a circle, perpendicular to a chord bisects the chord (Theorem).

5.3.2.1 Corollary: The perpendicular bisector of a chord passes through the centre of a circle.

5.3.2.2 A unique circle can be drawn through any three points not in a straight line.

*5.3.3 The angle which an arc of a circle subtends at the centre is double the angle it subtends at any point on the circumference (Theorem).

5.3.4 The angle at the circumference of a circle subtended by a diameter is a right angle, and conversely if a chord of a circle subtends a right angle on the circumference, the chord is a diameter (Theorem).

5.3.5 Angles in the same segment of a circle are equal and conversely, if a line segment joining two points subtends equal angles at two other points on the same side of the line segment, these four points are concyclic (Theorem)

- Angles in equal segments of a circle, or of equal circles, are equal (Theorem).

*5.3.6 The opposite angles of a cyclic quadrilateral are supplementary, and conversely, if a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic (Theorem).

5.3.7 The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle, and conversely, if an exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral is cyclic (Theorem).

5.3.8 A tangent to a circle is perpendicular to the radius at the point of contact, and conversely, a line drawn perpendicular to a radius at the point where it meets the circumference is a tangent to the circle (Theorem).

5.3.9 If two tangents are drawn to a circle through a common point, then the distances between this point and the points of contact are equal (Theorem).

*5.3.10 The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment, and conversely, if a line is drawn through the end point of a chord making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle (Theorem).

5.3.11 The following theorems:

5.3.11.1 The bisectors of the angles of a triangle are concurrent.

5.3.11.2 The perpendicular bisectors of the sides of a triangle are concurrent.

5.3.11.3 The medians of a triangle are concurrent.

5.3.11.4 The altitudes of a triangle are concurrent.

6. STANDARD 10

6.1 Algebra

6.1.1 *Logarithms*

6.1.1.1 The exponential function $y = a^x$, $a > 0$; its graph and deductions from the graph

6.1.1.2 The logarithmic function $y = \log_a x$, $a > 0$ and $a \neq 1$; its graph and deductions from the graph

6.1.1.3 The basic properties of logarithms

6.1.1.4 Change of base of a logarithm

6.1.1.5 Simple logarithmic equations and inequalities

6.1.2 *Sequences and series*

6.1.2.1 Characteristics and the general terms of arithmetic and geometric sequences

6.1.2.2 The Σ-notation

6.1.2.3 Calculations involving the sum to n terms of arithmetic and geometric series

6.1.2.4 Convergence of a geometric series and its sum to infinity

6.1.2.5 Solving simple problems using the above

6.2 Differential calculus

6.2.1 *The average gradient of a curve between two points; average speed*

6.2.2 *Limits*

6.2.2.1 Intuitive approach to the concept of a limit

6.2.2.2 Determining $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, for $f(x)$ one of the following functions: k; $\frac{k}{x}$; ax; $ax + b$; ax^2; $ax^2 + bx + c$ and ax^3

6.2.2.3 The derivative of a function; the notations: D_x; $\frac{d}{dx}$; and $f'(x)$

6.2.2.4 The gradient of a curve at any point on the curve

6.2.3 $D_x[x^n] = nx^{n-1}$; n real (without proof)

6.2.4 *Rules for differentiating*

6.2.4.1 $D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]$

6.2.4.2 $D_x[k.f(x)] = k.D_x[f(x)]$

6.2.5 *Applications*

6.2.5.1 The equations of tangents to graphs

6.2.5.2 Turning-points and sketches of polynomials of at most the third degree.

6.2.5.3 Simple practical problems in connection with maxima and minima and rates of change

6.3 **Trigonometry**

6.3.1 Function values for $-\theta$ and $(\theta + 360° n)$ where n is an integer, expressed in functionvalues for θ, and $\theta \in [0°; 90°]$

6.3.2 The sine, cosine and tangent functions

6.3.2.1 Description of domain and range

6.3.2.2 Maximum and minimum function values and period

6.3.2.3 Sketches of curves of the following types:
(where a is an integer or a fraction of the form $\frac{1}{n}$; n an integer)
(a) $y = a \sin \theta$, $y = a \cos \theta$, $y = a \tan \theta$
(b) $y = \sin a\theta$, $y = \cos a\theta$, $y = \tan a\theta$
(c) $y = a + \sin \theta$, $y = a + \cos \theta$, $y = a + \tan \theta$
(d) $y = \sin(\alpha + \theta)$, $y = \cos(\alpha + \theta)$

6.3.3 $\text{Cos}(A - B) = \cos A \cos B + \sin A \sin B$, and identities for
(a) $\cos(A + B)$
(b) $\sin(A \pm B)$
(c) $\tan(A \pm B)$
(d) $\sin 2\theta$
(e) $\cos 2\theta$
(f) $\tan 2\theta$

6.3.4 General and specific solutions of elementary trigonometric equations (Equations of the type $a \sin x + b \cos x = c$ are included only if $c = 0$)

6.4 **Euclidean Geometry**

(i) The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the lists for Standards 7, 8 and 9 may be used as reasons for statements in solving riders.

(ii) Although all theorems must be proved only proofs of theorems denoted with an asterisk (*) (and their converses where mentioned) will be required for examinations purposes.

(iii) Applications of any axiom or theorem in this list or in the list for Standards 7, 8 and 9 may be set (No constructions for examination purposes).

(iv) Not more than *three tenths* of the marks for Geometry will be given for "bookwork" in the examination.

(v) A logical order of parr. 6.4.1 to 6.4.6 must be adhered to.

*6.4.1 A line parallel to one side of a triangle divides the two other sides proportionally and conversely, if a line divides two sides of a triangle proportionally, it is parallel to the third side (Theorem).

6.4.2 Definition of similarity

*6.4.3 If two triangles are equiangular, the corresponding sides are proportional and conversely, if the corresponding sides of two triangles are proportional, the triangles are equiangular (Theorem).

*6.4.4 Equiangular traingles are similar, and if the corresponding sides of two triangles are proportional, the triangles are similar (Corollaries).

*6.4.5 The perpendicular drawn from the vertex of the right angle of a right-angled triangle to the hypotenuse, divides the triangle into two triangles which are similar to each other and to the original triangle (Theorem).

6.4.6 The Theorem of Pythagoras and its converse (Theorem).

6.5 **Analytical Geometry in a Plane**

6.5.1 The distance between two points

6.5.2 The mid-point of a line segment

6.5.3 Gradient of a line
6.5.4 Equation of a line and its sketch
6.5.5 Perpendicular and parallel lines (no proofs)
6.5.6 Collinear points and intersecting lines
6.5.7 Intercepts made by a line on the axes
6.5.8 Equations of circles with any given centre and given radius
6.5.9 Points of intersection of lines and circles
6.5.10 Equation of the tangent to a circle at a given point on the circle
6.5.11 Other loci with respect to straight lines and circles

PROVINCIAL ADMINISTRATION OF THE CAPE OF GOOD HOPE

DEPARTMENT OF EDUCATION

SENIOR SECONDARY COURSE

SYLLABUS

FOR

MATHEMATICS

STANDARD GRADE

1985

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS STANDARD GRADE

The following syllabus for Mathematics Standard Grade for the Senior Secondary Course will be introduced as from 1 January 1985.

The syllabus will be introduced in Standard 8 in 1985 and the first Senior Certificate Examination in this subject will be held in November/December 1987.

SENIOR SECONDARY COURSE: SYLLABUS FOR MATHEMATICS STANDARD GRADE

1 REMARKS

1.1 The arrangement of the content of the syllabus and its subdivision is not necessarily an indication of the sequence in which the work must be handled.

1.2 Non-programmable pocket calculators may be used where necessary and applicable to develop mathematical concepts and for calculation. Basic instruction in the practical use of pocket calculators must be given.

1.3 The breakdown for Standards 9 an 10 is suggested but is not prescriptive.

2. AIMS

2.1 To develop a love for, an interest in and a positive attitude towards Mathematics, by presenting the subject meaningfully

2.2 To enable pupils to gain mathematical knowledge and proficiency

2.3 To develop clarity of thought, so that mathematical techniques may be well understood

2.4 To develop accuracy

2.5 To instil in pupils the habit of estimating answers where applicable and, where possible, of verifying answers

2.6 To develop the ability of the pupils to use mathematical knowledge and methods in other subjects and in their daily life

2.7 To provide basic training for future study and careers

3. EXAMINATION

3.1 Standards 8 and 9 are examined internally.

3.2 Standard 10

3.2.1 The external Senior Certificate papers will be set on the syllabus for Standard 9 and Standard 10.

3.2.2 There will be two three-hour papers of equivalent value with the allocation of marks as follows:

3.2.2.1 First paper

• Algebra	: 75% (± 5%)
• Differential Calculus	: 25% (± 5%)

3.2.2.2 Second paper

• Questions of a miscellaneous nature covering two or more sections of the whole syllabus may be asked	: ± 5%
• Euclidean Geometry	: 30% (± 5%)
• Analytical Geometry	: 25% (± 5%)
• Trigonometry	: 40% (± 5%)

4. STANDARD 8

4.1 **Products**

The following types by inspection:

4.1.1 $(a \pm b)(c \pm d)$

4.1.2 $(ax \pm b)(cx \pm d)$ and $(ax \pm by)(cx \pm dy)$

4.1.3 $(ax \pm b)^2$ en $(ax \pm by)^2$

4.1.4 $(ax + b)(ax - b)$ and $(ax + by)(ax - by)$

4.2 Factors

Factorization of the following types:

4.2.1 Quadrinomials by grouping in pairs

4.2.2 Quadratic trinomials

4.2.3 Difference of squares

4.2.4 Preceding types including a common factor

4.3 **Algebraic fractions**

4.3.1 Simplification

4.3.2 Main operations

4.4 **Equations and inequalities in one unknown**

4.4.1 Solution of linear equations with numerical and literal coefficients

4.4.2 Solution of linear inequalities with numerical coefficients

4.4.3 Solutions of problems with the aid of linear equations

4.4.4 Solutions of quadratic equations by means of factors in which only integers may occur as coefficients

4.5 **Formulae**

4.5.1 Construction of formulae for area and volume of right prisms and right cylinders

4.5.2 Substitution in formulae

4.5.3 Changing the subject of formulae

4.6 **Functions**

4.6.1 Concept of a function, functional notation and values of a function

4.6.2 Domain and range of a function

4.6.3 Graphical representation of the functions (and deduction of the characteristics of each from its equation and graphical representation) defined by:

4.6.3.1 $ax + by + c = 0$

4.6.3.2 $y = \sqrt{r^2 - x^2}$ (r rational and $r \neq 0$)

4.6.3.3 $y = -\sqrt{r^2 - x^2}$ (r rational and $r \neq 0$)

4.6.3.4 $xy = k$ (k an integer and $k \neq 0$)

4.6.3.5 $y = ax^2 + c$ (a and c rational numbers)

4.7 **Solution of systems of linear equations — graphically and algebraically**

4.8 **Exponents (Pocket calculators may not be used)**

4.8.1 The meaning of a^{-m}, a^0 en $a^{\frac{m}{n}}$, $a \neq 0$ (m, n natural numbers), (for $a^{\frac{m}{n}}$ examples $a > 0$ only)

4.8.2 Intuitive extension of the laws of exponents to include integers and rational exponents

4.9 **Euclidean Geometry**

(i) The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the list for Standard 7 may be used as reasons for statements in solving riders.

(ii) Although all theorems must be proved only proofs of theorems (but not of their converses) denoted with an asterisk (*) in the following list will be required for examination purposes.

(iii) Applications of any axiom or theorem in this list or in the list for Standard 7, may be set (No constructions for examination purposes).

(iv) Not more than *three tenths* of the marks for Geometry will be given for "bookwork" in the examination.

(v) A logical order of parr. 4.9.1 to 4.9.12 must be adhered to.

*4.9.1 The opposite sides and angles of a parallelogram are equal, and conversely, if the opposite sides or angles of a quadrilateral are equal, the quadrilateral is a parallelogram (Theorem).

*4.9.2 The diagonals of a parallelogram bisect each other, and conversely, if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram (Theorem).

*4.9.3 If two opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram (Theorem).

4.9.4 The diagonals of a rectangle are equal (Theorem).

4.9.5 The diagonals of a rhombus bisect each other at right angles, and bisect the angles of the rhombus (Theorem).

*4.9.6 A diagonal of a parallelogram bisects the area of the parallelogram (Theorem).

*4.9.7 A parallelogram and rectangle on the same base and between the same parallels have equal areas (Theorem).

4.9.8 Triangles (parallelograms) on the same base (or equal bases) and on the same side thereof are equal in area if they lie between the same parallel lines, and conversely, if triangles (parallelograms) lying on the same base (or equal bases) and on the same side thereof, have equal areas, they lie between the same parallel lines (Theorem).

*4.9.9 The line segment joining the mid-points of two sides of a triangle is parallel to the third side, and equal to half the third side (Theorem).

*4.9.10 The line passing through the mid-point of one side of a triangle, parallel to another side, bisects the third side (Theorem)..

*4.9.11 If three or more parallel lines make equal intercepts on a given transversal, they make equal intercepts on any other transversal (Theorem).

4.9.12 The Theorem of Pythagoras and its converse.

4.10 **Trigonometry**

4.10.1 Definitions of the six trigonometric functions for an angle θ and $\theta \in [0^\circ; 90^\circ]$

4.10.2 Application of the six trigonometric ratios in a rightangled triangle

4.10.3 Solution of right-angled triangles

5. STANDARD 9

5.1 Algebra

5.1.1 *A brief intuitive review of the real numbers*

5.1.2 *Functions*

(No point-by-point plotting of graphs will be required for examination purposes.)

5.1.2.1 Graphical representation of the functions defined by:

$y = ax^2 + bx + c \ (a \neq 0)$|

5.1.2.2 The deduction of the characteristics of the functions in 5.1.2.1 from its equations and graphical representation.

X 5.1.2.3 Graphical representation of simultaneous equalities with respect to functions from 5.1.2.1, and the function defined by $ax + by + c = 0$.

5.1.3 *Quadratic equations*

The roots of $ax^2 + bx + c = 0$ where a, b and c are rational

(a) The solution of $ax^2 + bx + c = 0$

(b) Conditions for which the equation is solvable on the set of real numbers

(c) Equal and unequal roots; rational and irrational roots; real and non-real roots.

5.1.4 *The remainder and factor theorem*

Applications including solution of equations of the third degree

5.1.5 *Systems of equations*

X Solving simultaneous equations in two unknowns of which one equation is of the first and the other of the second degree.

5.1.6 *Exponents*

5.1.6.1 Solving of equations of the form

$ax^{\frac{m}{n}} - b = 0$ where m and n are integers, $n \neq 0$

5.1.6.2 Relationship between surds and exponents and the corresponding basic properties where a and b are positive and m and n are positive integers:

$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$(\sqrt[m]{a})^n = \sqrt[m]{a^n}$

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

5.1.6.3 Rationalization of surds (denominations of fractions limited to monomials)

5.1.6.4 Solving of simple exponential equations in one variable

5.2 **Trigonometry**

5.2.1 Expansion of the definitions of the six trigonometric functions for any angle over [0°; 360°] in terms of co-ordinates with respect to perpendicular axis

5.2.2 Graphs of $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$

5.2.3 Function values for $(90° - \theta)$, $(180° \pm \theta)$ and $(360° - \theta)$ expressed in function values for θ, where $\theta \in [0°; 90°]$

5.2.4 Function values for 0°, 30°, 45° and multiples thereof over [0°; 360°]

5.2.5 The mutual relationships between the trigonometric function values

5.2.5.1 Reciprocals e.g. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

5.2.5.2 Quotients e.g. $\frac{\sin\theta}{\cos\theta} = \tan\theta$

5.2.5.3 Quadratic formulae:

(a) $\sin^2\theta + \cos^2\theta = 1$

(b) $\tan^2\theta + 1 = \sec^2\theta$

(c) $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

5.2.6 Formulae

5.2.6.1 The sine formula;

5.2.6.2 The cosine formula;

5.2.6.3 Area of a triangle $ABC = \frac{1}{2} ab \sin C$;

5.2.6.4 Application of the above formulae in the solution of

(a) triangles;

(b) problems in two dimensions.

5.3 **Euclidean Geometry**

(i) The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the list for Standards 7 and 8 may be used as reasons for statements in solving riders.

(ii) Although all theorems must be proved only proofs of theorems (but not of their converses) denoted with an asterisk (*) in the following list will be required for examination purposes.

(iii) Applications of any axiom or theorem in this list or in the lists for Standards 7 and 8 may be set (No constructions for examination purposes).

(iv) Not more than *three tenths* of the marks for Geometry will be given for "bookwork" in the examination.

(v) A logical order of parr. 5.3.1 to 5.3.10 must be adhered to.

5.3.1 The Theorem of Pythagoras and its converse

*5.3.2 The line segment joining the centre of a circle to the mid-point of a chord is perpendicular to the cord, and conversely, the line segment drawn from the centre of a circle, perpendicular to a chord bisects the chord (Theorem).

5.3.2.1 Corollary: The perpendicular bisector of a chord passes through the centre of a circle.

5.3.2.2 A unique circle can be drawn through any three points not in a straight line.

*5.3.3 The angle which an arc of a circle subtends at the centre is double the angle it subtends at any point on the circumference (Theorem).

5.3.4 The angle at the circumference of a circle subtended by a diameter is a right angle, and conversely, if a chord of a circle subtends a right angle on the circumference, the chord is a diameter (Theorem).

5.3.5 Angles in the same segment of a circle are equal and conversely, if a line segment joining two points subtends equal angles at two other points on the same side of the line segment, these four points are concyclic (Theorem)

- Angles in equal segments of a circle, or of equal circles, are equal (Theorem).

*5.3.6 The opposite angles of a cyclic quadrilateral are supplementary, and conversely, if a pair of opposite angles of a quadrilateral is supplementary, it is cyclic (Theorem).

5.3.7 The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle, and conversely, if an exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral is cyclic (Theorem).

5.3.8 A tangent to a circle is perpendicular to the radius at the point of contact, and conversely, a line drawn perpendicular to a radius at the point where it meets the circumference is a tangent to the circle (Theorem).

*5.3.9 If two tangents are drawn to a circle through a common point, then the distances between this point and the points of contact are equal (Theorem).

*5.3.10 The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment, and conversely, if a line is drawn through the end point of a chord making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle (Theorem).

6 **STANDARD 10**

6.1 **Algebra**

6.1.1 *Compound Increase and Decrease*

6.1.1.1 Calculation of initial and final sum

6.1.1.2 Calculation of rate

6.1.1.3 Calculation of intervals

6.1.2 *Sequences and series*

6.1.2.1 Characteristics and the general terms of arithmetic and geometric sequences

6.1.2.2 The writing of a series in expanded form when given in Σ-notation, but not the converse

6.1.2.3 Calculations involving the sum to n terms of arithmetic and geometric series

6.1.2.4 Solving simple problems using the above

6.2 **Differential calculus**

6.2.1 *The average gradient of a curve between two points; average speed*

6.2.2 *Limits*

6.2.2.1 Intuitive approach to the concept of a limit

6.2.2.2 Determining $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, for $f(x)$ one of the following functions: $k, ax, ax + b$ and ax^2

6.2.2.3 The derivative of a function; the notations: D_x, $\frac{d}{dx}$, $f'(x)$

6.2.2.4 The gradient of a curve at any point on the curve

6.2.3 $D_x[x^n] = nx^{n-1}$; n real (without proof)

6.2.4 *Rules for differentiating (no proofs)*:

6.2.4.1 $D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x g(x)]$

6.2.4.2 $D_x[k.f(x)] = k.D_x[f(x)]$

6.2.5 *Applications*

6.2.5.1 Turning-points and sketches of polynomials of at most the third degree

6.2.5.3 Simple practical problems in connection with maxima and minima

6.3 **Trigonometry**

6.3.1 The sine, cosine and tangent functions

6.3.1.1 Description of the range with [0°; 360°] as domain:

6.3.1.2 Sketches of curves of the following types:

$y = a \sin x$, $y = a \cos x$, $y = a \tan x$; $x \in [0°; 360°]$

$y = \sin ax$, $y = \cos ax$; (a an integer and $ax \in [0°; 360°]$)

6.3.3 Solving elementary trigonometric equations as stated in 6.3.1.2

6.4 **Euclidean Geometry**

(i) The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the lists for Standards 7, 8 and 9 may be used as reasons for statements in solving riders.

(ii) Although all theorems must be proved only proofs of theorems (but not of their converses) denoted with an asterisk (*) will be required for examinations purposes.

(iii) Applications of any axiom or theorem in this list or in the list for Standards 7, 8 and 9 may be set (No constructions for examination purposes).

(iv) Not more than *three tenths* of the marks for Geometry will be given for "book work" in the examination

(v) A logical order of parr. 6.4.1 to 6.4.4 must be adhered to.

*6.4.1 A line parallel to one side of a triangle divides the two other sides proportionally and conversely, if a line divides two sides of a triangle proportionally, it is parallel to the third side (Theorem).

6.4.2 Definition of similarity.

*6.4.3 If two triangles are equiangular, the corresponding sides are proportional, and conversely, if the corresponding sides of two triangles are proportional, the triangles are equiangular (Theorem).

*6.4.4 Equiangular traingles are similar, and if the corresponding sides of two triangles are proportional, the triangles are similar (Corollaries).

6.5 Analytical Geometry in a Plane

6.5.1 The distance between two points

6.5.2 The mid-point of a line segment

6.5.3 Gradient of a line

6.5.4 Equation of a line and its sketch

6.5.5 Perpendicular and parallel lines (no proofs)

6.5.6 Collinear points and intersecting lines

6.5.7 Intercepts made by a line on the axes

6.5.8 Equations of circles with centre (0; 0) and given radius

6.5.9 Points of intersection of lines and circles

6.5.10 Other simple loci with respect to straight lines and circles

THE AIMS, TUITION AND EXAMINATION OF THE ACCOMPANYING SYLLABUSES

1. Introduction

A syllabus naturally specifies the prescribed subject-matter in broad outline, but inevitably also comprises the standard to be maintained as well as the structure of each section round its central theme. In compiling a syllabus both the general and the special needs of the pupils for whom it is intended must be taken into account very thoroughly, as well as the acquired knowledge and intellectual maturity of the pupils concerned.

Besides the content and the arrangement of the subject-matter the teaching methods are also of the greatest importance. This determines the success or failure of the teaching. Through the teaching the understanding and insight of the pupils are promoted, and in this way their interest in and love of the subject are developed.

The examination technique is just as important as are the subject-matter and the teaching methods. Stereotyped questions and examination papers encourage stereotyped techniques in the teaching. Such examinations promote memory work on the part of the pupils and excessive drilling by the teacher. Then it pays the teacher and the pupil to concentrate on formulae and mechanical techniques, at the expense of understanding and insight.

2. General Aims

(a) Choice of subject-matter

(1) The syllabuses link up with the existing syllabuses, with essential changes.

(2) With the content and choice of items due regard was paid to the general mathematical needs of an educated person who has to deal with aspects of quantity and space in all fields.

(3) The syllabuses lend themselves particularly to preparing and selecting prospective university students. Candidates who take Mathematics on the higher grade should not only be better equipped for any intended course of study, but are specially prepared for further study in Mathematics, Physical Science and other courses of study for which mathematical

methods ....2/.....

methods and techniques are essential. The standard grade is meant to make mathematics accessible to as many pupils as possible.

(4) With effective tuition and examination both syllabuses should also lend themselves to preparing students and to the selection of prospective university students by the Joint Matriculation Board.

(b) Examination practice

(1) Mechanical reproduction in examinations of memorised definitions, formulae and techniques should be avoided as far as possible.

(2) Examination questions should call for solutions based on understanding and insight. A candidate should be able to apply his knowledge and to make deductions.

(3) Seeing that final examination papers have such a great influence on the tuition, papers should be set in such a manner that they promote correct teaching methods. It should pay the teacher to concentrate on understanding and insight rather than on excessive drill. Stereotyped papers should be be avoided.

CAPE EDUCATION DEPARTMENT

THE TEACHING AND EXAMINATION OF MATHEMATICS IN STANDARDS 8, 9 AND 10 ON THE HIGHER GRADE (HG) AS WELL AS ON THE STANDARD GRADE (SG)

1. AIMS

1.1 The general aims are explained clearly in the sullabuses of both the SG- and the HG-mathematics. (Teachers are advised to make a thorough study of these syllabuses)

1.2 STANDARD GRADE

In the presentation of the subject on the SG the following additional aspect of the aims should be considered:

1.2.1 In the presentation the value of the subject in the modern world must be emphasized and the fear of mathematics experienced by some pupils must be erased.

1.2.2 As many pupils as possible must be able to take the subject. This is one of the reasons why mathematics is offered on the SG.

1.2.3 Teachers must guard against the learning of certain "recipes" which have no mathematical value. In the teaching of the subject the following such be drilled: basic principles, insight, understanding, correct methods and the logical structure.

1.2.4 In examinations, questions should be set which are of more direct nature and which will avoid unrealistic demands.

1.3 HIGHER GRADE

In the presentation of the subject on the HG the following aspects of the aims should be considered:

1.3.1 Pupils must be given a sound concept and understanding as well as the necessary knowledge to equip them for further mathematical studies.

1.3.2 Emphasis on the logical structure of the subject

1.3.3 Opportunities should be created for self-activity, through which pupils can attempt more advanced examples and thus discover new principles.

1.3.4 Development of the pupils' mathematical and original thinking ability must be encouraged.

2./....

APPENDIX B

COPY OF PILOT TEST

NAME: .... CLASS: ....

MATHEMATICS STANDARD 7
MULTIPLE-CHOICE TEST
TIME: 2 HOURS

READ ALL THE INSTRUCTIONS BEFORE YOU BEGIN:

1. Print your name and class clearly in the space provided.
2. Select the ONE alternative which CORRECTLY completes the statement or answers the question.
3. Indicate the answer you think is correct by drawing a RING around the LETTER next to it, as shown in the examples below. No other mark should be made on the question paper - all necessary calculation should be done on scrap paper.
4. Answer ALL the questions. If you are uncertain, make an intelligent guess - you will NOT be penalised for incorrect answers.

Example 1.: $2a + 3a + 4a = ...$

A 7a
(B) 9a
C 10a
D 15a
E 24a

Example 2.: $2x + 1 = 11$ if $x = ...$

A 2
B 3
C 4
(D) 5
E 6

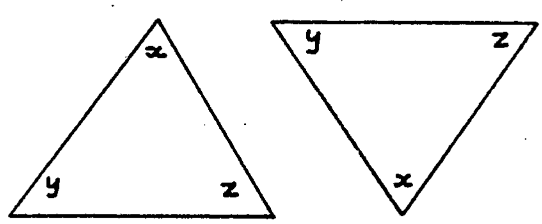
1. 2^3 is ....

A a base
B an index
C a power
D a variable
E a polynomial

2. Two angles, measuring 47° and 43° respectively, may be described as .... angles.

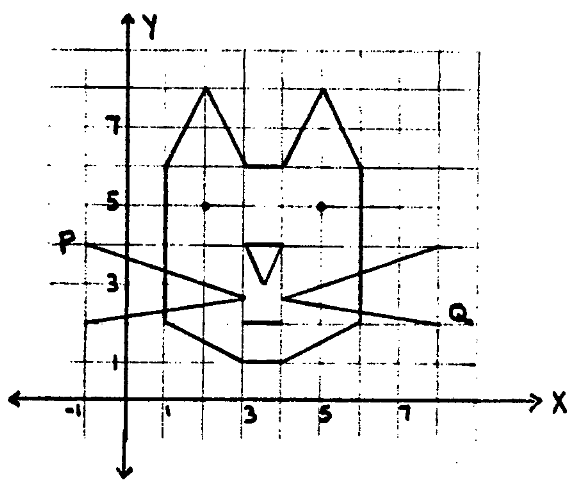
A adjacent
B co-interior
C complementary
D right
E supplementary

3. Which of the following words BEST describes the triangles in the sketch?



A similar
B right-angled
C isosceles
D equilateral
E congruent

4. This is Felix, the Mathematical Cat. What are the coordinates of the ends of his whiskers, marked P and Q?



A P(-1;4) and Q(8;2)
B P(4;-1) and Q(2;8)
C P(-1;-4) and Q(8;2)
D P(-4; -1) and Q(2;8)
E P(-1;3½) and Q(2;7½)

5. Add: $5x - x^2 + 6$; $3x^2 + x + 8$; $-4x + 2x^2$

A $20x^3$
B $6x^2 + 14$
C $4x^6 + 2x^3 + 14$
D $4x^2 + 2x + 14$
E $2x^2 + 4x + 14$

6. Subtract $y^2 - 4$ from $-3y + 1$

A $y^2 + 3y - 5$
B $-y^2 - 3y + 5$
C $4y^2 - 5$
D $-4y^2 + 5$
E None of the above

7. Multiply: $(-2a^3)^4 \times -4a^3$

A $-8a^9$
B $-32a^{10}$
C $-64a^{15}$
D $+64a^{15}$
E $+32a^{10}$

8. Divide: $2(k^6 + k^4 - k^2) \div 4k^2$

A $\frac{1}{2}(k^3 + k^2 - k)$
B $\frac{1}{2}(k^4 + k^2 - 1)$
C $\frac{1}{2}(k^3 + k^2)$
D $2(k^4 + k^2 - 1)$
E $2(k^3 + k^2)$

9/........

9. Simplify $(p^2q^4 \div pq^2)^2 + p^2q^4$

A p^4q^8
B $2p^4q^8$
C $2p^2q^4$
D $p^4q^4 + p^2q^4$
E $2pq^2 + p^2q^4$

10. Multiply: $(2p - 7q)(5p + 3q)$

A $10p^2 - 41pq - 21q^2$
B $10p^2 - 35pq - 21q^2$
C $10p^2 + 29pq - 21q^2$
D $10p^2 - 29pq - 21q^2$
E $10p^2 - 21q^2$

11. Express as a single fraction: $\frac{2}{y} - \frac{3}{x} - \frac{x - 4}{xy}$

A $\frac{1}{xy}$
B $\frac{x - 3y + 4}{xy}$
C $\frac{x - 3y - 4}{xy}$
D $\frac{x - 3y + 4}{x^2y^2}$
E $\frac{x - 3y - 4}{x^2y^2}$

12. Multiply $\frac{a + m}{x - y}$ by $\frac{y - x}{m + a}$

A -1
B 0
C 1
D $-xy$
E xy

13. Find the value of the following expression if $p = -2$ and $q = -1$:

$$\sqrt{4p^2 - 24pq + 36q^2}$$

A 2
B 3
C 4
D 10
E 100

14. Factorise $4x^3 + 6x^2 - 2x$

A $2x(2x^2 + 3x - 1)$
B $2x(2x^2 + 4x - 1)$
C $2x(2x^2 + 3x)$
D $2x(2x^2 + 4x)$
E $2x(2^3 + 4^2)$

15. Solve for k: $14k - 5(2k + 1) = 3$

A ½
B −½
C 1
D 2
E −2

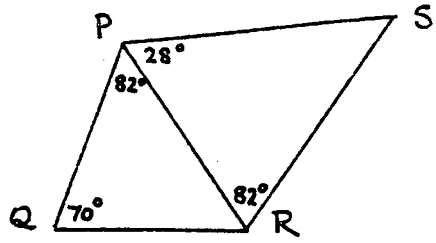
16. The solution of $2x + 7 = \frac{4x + 5}{3} - \frac{x - 5}{5}$

is between ....

A −7 and −4
B 4 and 7
C −1 and 1
D 2 and 5
E 0 and 3

17/............

17. What kind of quadrilateral is PQRS ?



A kite
B trapezium
C parallelogram
D rhombus
E no special kind of quadrilateral

18. Which of the following fractions is the nearest in value to 1/3 ?

A ¼
B ⅜
C 3/16
D 5/16
E 7/16

19. The interior angles of a quadrilateral are y, $2y$, $y + 20^\circ$ and $2y + 40^\circ$. Find the value of y.

A 45°
B 40°
C 55°
D 50°
E Insufficient information

20. What must be added to $2a + a^2 - 3$ to obtain $a^2 - 2a - 5$?

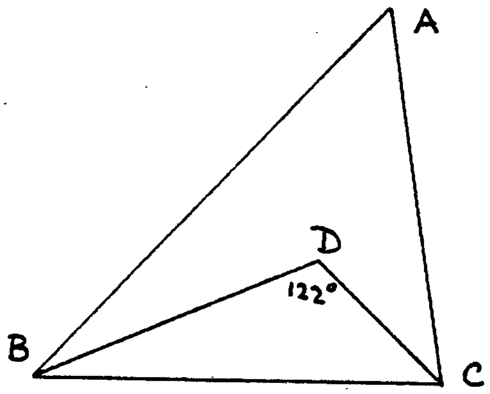
A $-4a - 2$
B $4a - 2$
C $4a + 2$
D $-a^2 - 3a - 2$
E $a^2 + 3a + 2$

21. Simplify $2 \times 2^4 \times 2^7$

A 6^{12}
B 8^{12}
C 8^{26}
D 2^{11}
E 2^{12}

22. DB bisects $\hat{B}$ and DC bisects $\hat{C}$.
If $\hat{D} = 122^\circ$, then $\hat{A} = \ldots.$

A 58°
B 61°
C 62°
D 64°
E Insufficient information

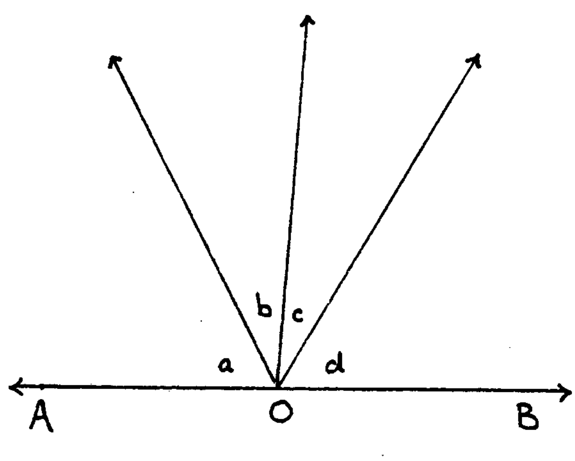


23. Divide $(6x^3 - x^2 - 4x - 1)$ by $(2x + 1)$

A $3x^2 - 1$
B $3x^2 - 2x - 1$
C $3x^2 - x - 1$
D $3x^2 + 2x - 1$
E $3x^2 + x - 1$

24. AOB is a straight line, a = d and b = c.
Which one of the following statements is FALSE ?

A $a + b = 90^\circ$
B $a + c = 90^\circ$
C $b + d = 90^\circ$
D $b + c = 90^\circ$
E $d + c = 90^\circ$



25. Which of the following are rational numbers ?

I $\sqrt{3}$ II $\sqrt{4}$ III $\sqrt{2\frac{1}{4}}$

A I and II
B II and III
C II only
D III only
E I, II and III

26. Find the H.C.F. (Highest Common Factor) of $4x^2yz$, $6xy^2z$ and $8xyz^2$.

A $24x^2y^2z^2$
B $4x^2y^2z^2$
C $4xyz$
D $2xyz$
E $2x^2y^2z^2$

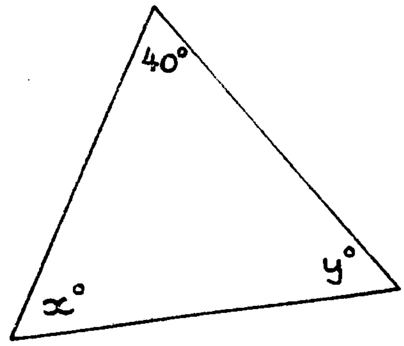
27. What is the L.C.M. (Least Common Multiple) of $2x^2y$, $3xz$, $4y^2z^2$ and $5xyz$?

A $x^2y^2z^2$
B xyz
C $60x^2y^2z^2$
D $60xyz$
E $120x^2y^2z^2$

28. When an expression is divided by $3x + 15$, the answer obtained is $2x - 3$ and there is a remainder of 2. What is the expression ?

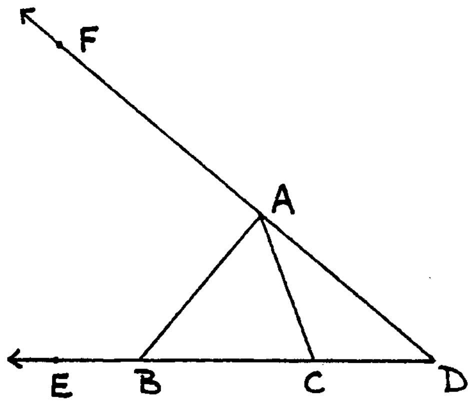
A $6x^2 - 13$
B $6x^2 - 17$
C $6x^2 - x - 13$
D $6x^2 + x - 13$
E $6x^2 + x - 15$

29. If $60 < y < 100$, then ....



A $40 < x < 80$
B $40 \leqslant x \leqslant 80$
C $60 \leqslant x \leqslant 100$
D $60 < x < 100$
E $80 < x < 120$

30. If $E\hat{B}A = 120^\circ$ and $D\hat{C}A = 140^\circ$, then $F\hat{A}B + D\hat{A}C$ = ... ?



A 100°
B 120°
C 140°
D 160°
E Insufficient information

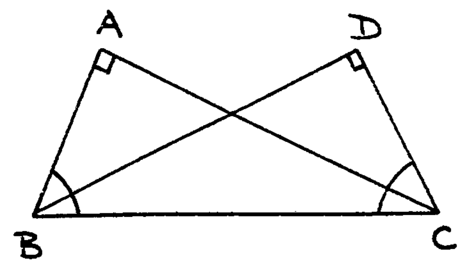
31. The equation $\frac{x + 4}{3} - \frac{x + 7}{3} = 0$ has ....

A an infinity of solutions
B a real, irrational solution
C one solution, $x = 0$
D two solutions, $x = -4$, $x = -7$
E no solution

32. Each of the equal angles of an isosceles triangle is four times as big as the third angle. Find the third angle.

A 15°
B 20°
C 25°
D 30°
E 36°

33. If $\hat{A} = 90^\circ = \hat{D}$ and $A\hat{B}C = D\hat{C}B$, then $\triangle ABC \equiv \triangle \ldots (\ldots\ldots)$



A DCB (90°, H, S)
B DBC (90°, H, S)
C DCB (S, S, S)
D DBC ($\angle$, $\angle$, S)
E DCB ($\angle$, $\angle$, S)

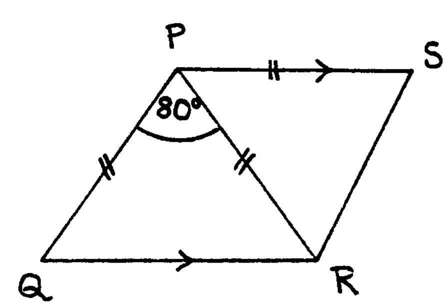
34. A certain number is 5 more than half the number subtracted from 40. What is the number ?

A 15
B 30
C 35
D 70
E 90

35. All the pupils in Std 7E play hockey or tennis; 18 play both games; 21 play hockey and 23 play tennis. How many pupils are there in Std 7E ?

A 26
B 44
C 53
D 62
E Information insufficient

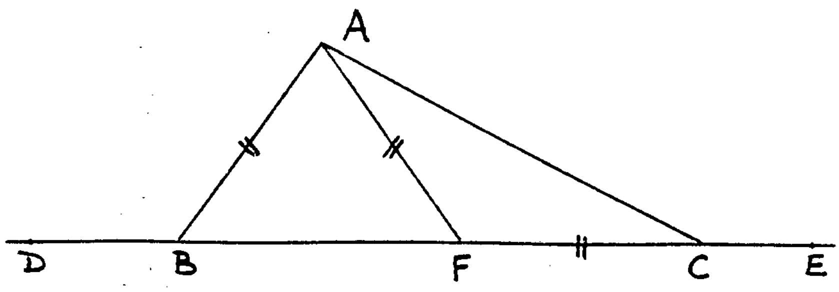
36. In quadrilateral PQRS, PQ // QR and PQ = PR = PS. If $Q\hat{P}R = 80^\circ$, find $P\hat{R}S$.



A 65°
B 50°
C 80°
D 45°
E 55°

37/.............

37. If $\hat{ACE} = 140^\circ$ and $AB = AF = FC$, find $\hat{ABD}$.



A 80°
B 100°
C 140°
D 160°
E Impossible without more information.

38. x and y are the magnitudes of a pair of adjacent angles. If $x = 72^\circ$ and $y = \frac{3}{2}x$, then $x + y$ is ....

A a straight angle
B a right angle
C an acute angle
D an obtuse angle
E a reflex angle

39. Which of the following is the sum of three consecutive natural numbers ?

I 9 II 13 III 15

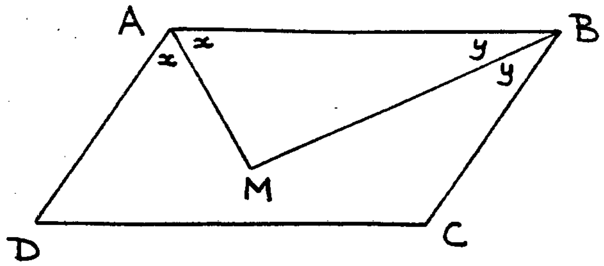
A I only
B II only
C I and II
D II and III
E I and III

40. Find the three terms that correctly complete:

$(2x + 3y)(3x - \ldots) = \ldots + 5xy - \ldots$

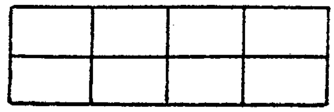
A 4; $6x^2$; $12y$
B $4y$; $5x^2$; $7y^2$
C $4y$; $6x^2$; $12y^2$
D $2y$; $6x^2$; $6y^2$
E $2y$; $5x^2$; $5y^2$

41. If ABCD is a parallelogram, what is the size of $A\hat{M}B$?



A 45^o
B 90^o
C $x - y$
D $2x + 2y$
E $180^o - x + y$

42. This is a sketch of part of Kit's circuitry. The length of the network is $2x + 5$ and the width is $x - 4$. The total length of wire, T, required to make this network is ......



A $T = 4x^2 - 6x - 40$
B $T = 2x^2 - 3x - 20$
C $T = 3x - 1$
D $T = 6x - 2$
E $T = 11x - 5$

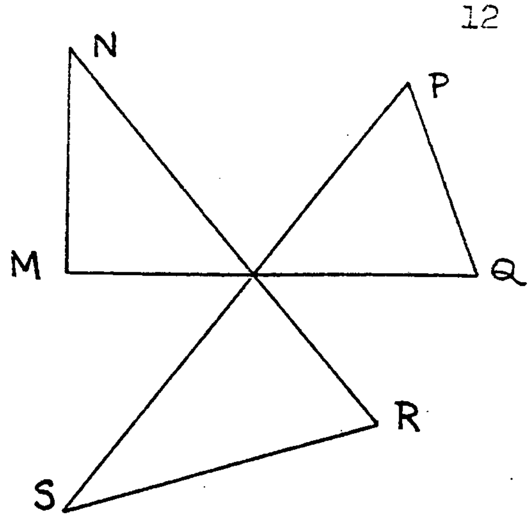
43. Which of the following numbers is the largest ?

A $2 \times 2 \times 2$
B 22^2
C 222
D 2^{2^2}
E 2^{22}

44. If $5^2 - 4^2 = \sqrt[x]{81}$, then $x = ...$

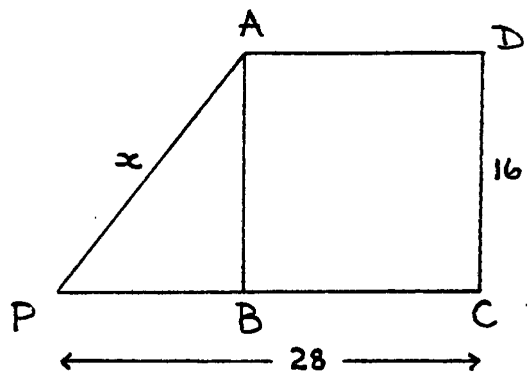
A 0
B 1
C 2
D 3
E 4

45. What is the sum of the sizes of angles M, N, P, Q, R and S shown in the sketch ?



A 180^0
B 360^0
C 540^0
D 720^0
E Insufficient information

46. ABCD is a square. CB is produced to P and PA is joined. Calculate the value of x.



A 12
B 14
C 18
D 20
E 25

47. If h, k, m and n are positive integers and $k > m$ and $n > h$, which of the following are correct ?

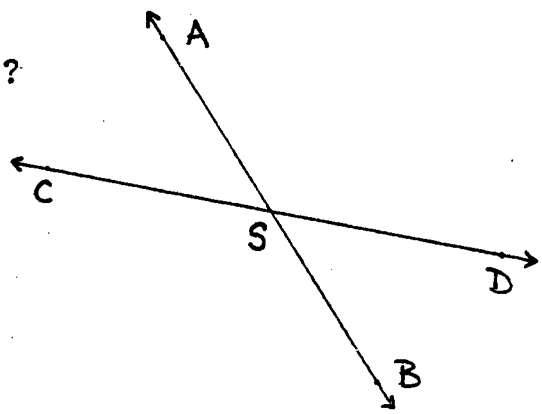
I $n + h$ can be equal to $k + m$
II $k + h$ can be equal to $n + m$
III $k + n$ can be equal to $m + h$

A I only
B II only
C I and III
D I and II
E I, II and III

48. Three boxes, each $3 \times 4 \times 5$, are piled on top of one another. The lowest box lies on its 3×4 side; the middle box lies on its 3×5 side, and the topmost box lies on its 4×5 side. What is the height of the pile of boxes ?

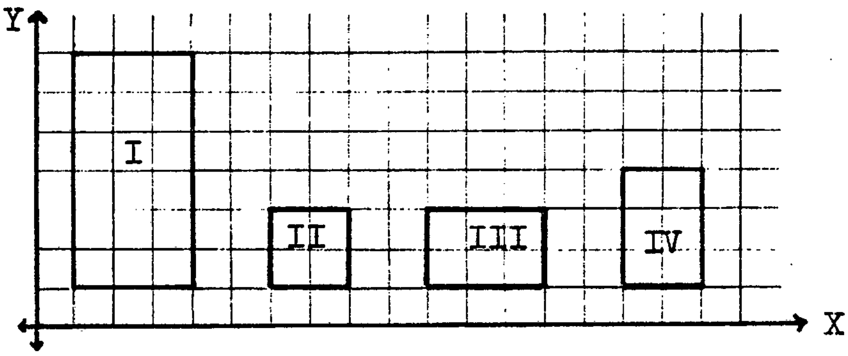
A 7
B 9
C 12
D 15
E 60

49. Straight lines AB and CD intersect at S. $A\hat{S}C$ varies from 20° to 150°. Between which limits will $C\hat{S}B$ vary ?



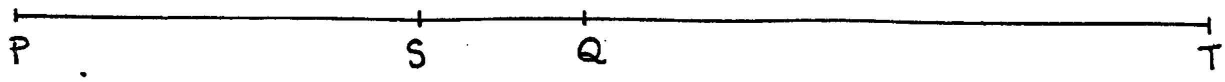
A 70° to 140°
B 30° to 160°
C 10° to 110°
D 20° to 150°
E 60° to 130°

50. Every unit along the X-axis represents 3 metres and each unit along the Y-axis represents 2 metres. Which figures are squares ?



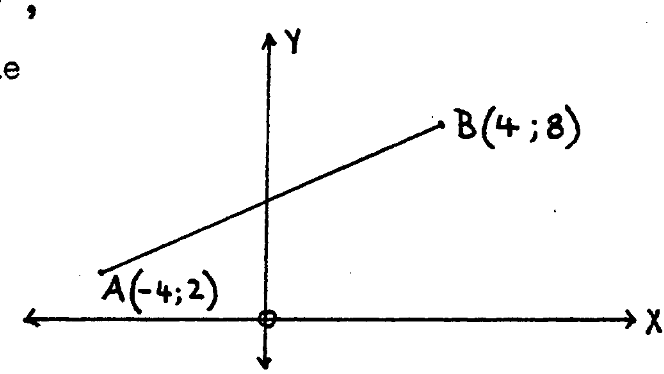
A I only
B II only
C IV only
D I and II
E III and IV

51. If PS = 2SQ, PT = 2QT and PQ = $3\frac{1}{3}$, find $\frac{PS}{QT}$



A ½
B 2/3
C 1
D 3/2
E 2

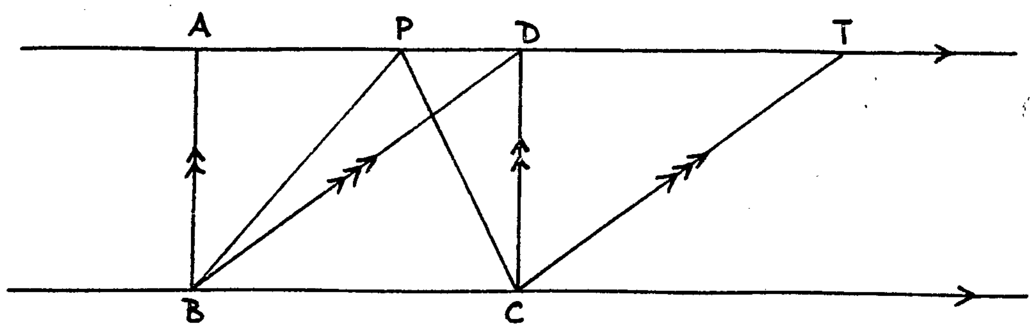
52. If Gradient = $\frac{\text{Vertical change}}{\text{Horizontal change}}$,
what is the gradient of AB in the sketch ?



A -2/1
B -4/3
C 4/3
D -3/4
E 3/4

53. x and y are both integers. Both are divisible by 7. Which ONE of the following is NOT necessarily true ?

A x + y is divisible by 7
B x - y is divisible by 7
C x + y is divisible by 14
D xy is divisible by 49
E $x^2 + y^2$ is divisible by 49

54. 

Which ONE of the following is NOT true ?

A $\triangle$DBC = $\triangle$DAB = $\triangle$PBC in area
B ABCD = $\triangle$DBC + DCT in area
C ABCD = DBCT in area
D $\triangle$PBC = $\triangle$CTD in area
E $\triangle$PBC = $\triangle$PTC in area

55. Find the missing number:

+11	-45	-5
-29		+3
-21	+19	-37

A +13
B -13
C +39
D -39
E -17

56. The difference between two numbers is larger than their sum. From this can be deduced that:

A The numbers must have different signs
B Both numbers must be positive
C Both numbers must be negative
D At least one number is positive
E At least one number is negative

57/..................

57. The missing number is ....

2	6
54	18

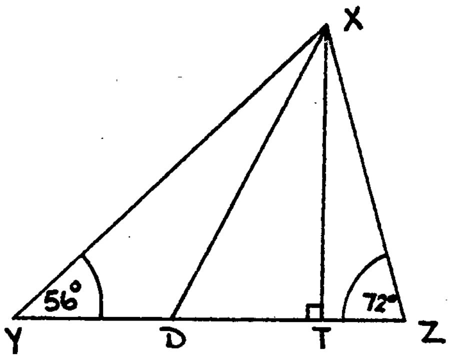
	4
64	16

A 0
B 1
C 2
D 3
E 4

58. The cube root of 62 570 773 is ....

A 383
B 397
C 429
D 517
E 20 851

59. If $XT \perp YZ$, $X\hat{Y}Z = 56^\circ$, $X\hat{Z}Y = 72^\circ$ and XD bisects $Y\hat{X}Z$, find $D\hat{X}T$.



A 8°
B 14°
C 18°
D 24°
E 26°

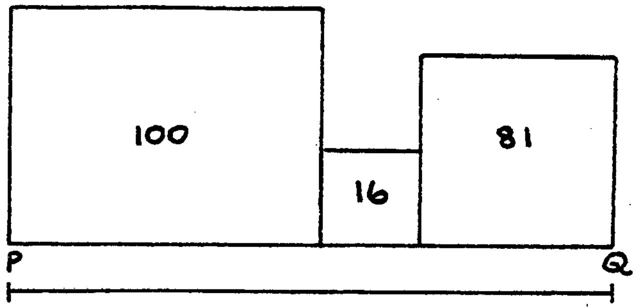
60. What is the smallest positive number h so that the sum of $(x + 1) + (x + 2) + \dots\dots\dots + (x + h)$ is even for any integer x ?

A 2
B 3
C 4
D 5
E 6

61. If 17^{15} is calculated, what is the value of the last digit, ie the "units" digit ?

A 1
B 3
C 5
D 7
E 9

62. Three squares with areas of 100, 16 and 81 lie side by side as shown. By how much must the area of the middle square be reduced in order that the total length, PQ, of the resulting three squares be 21 ?



A $\sqrt{2}$
B 2
C 4
D 8
E 12

63. A sack, W, contains 5kg white maize and a sack, Y, contains 5kg yellow maize. 1kg of yellow maize is taken from Y and mixed with the white maize in W. Then 1kg of the mixture is put back into sack Y.
What is the ratio $\frac{\text{white maize}}{\text{yellow maize}}$ in Y ?

A 1/5
B 24/125
C 4/21
D ¼
E 6/25

64/..............

64. If x has a value between 4 and 8, and y has a value between 20 and 40, then $y \div x$ has a value between .....

A $2\frac{1}{2}$ and 10
B 4 and 40
C 5 and 10
D 8 and 20
E $2\frac{1}{2}$ and 5

65. How many factors of 2 are there in $(10^{17} + 2^{17})$?

A 34
B 18
C 17
D 2
E 1

66. If the base and the index of p ($p \neq 0$) are doubled, and the result is equal to the product of p and y, then y is equal to ....

A 2
B 4
C p
D 2p
E 4p

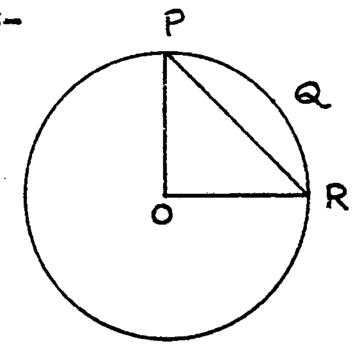
67. If $x > 1$, which of the following will increase in size if x increases in size ?

I $x - \frac{1}{x}$ II $\frac{1}{x^2 - x}$ III $4x^3 - 2x^2$

A I only
B II only
C III only
D I and III
E I, II and III

68/.................

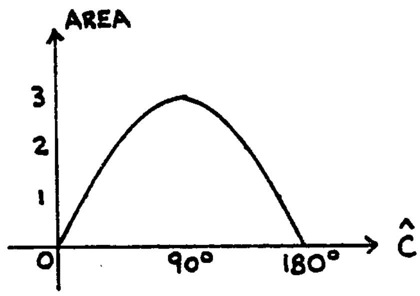
68. What is the length of arc PQR of a circle with centre O, if the area of the right-angled triangle POR is 32 ?



A π
B 2π
C 4π
D 8π
E 16π

69. $\triangle ABC$ has $a = 3$cm, $b = 2$cm and $\hat{C}$ variable. The graph shows how the area of $\triangle ABC$ varies as $\hat{C}$ varies. From the graph, decide which of the following statements are TRUE.

I The area increases as $\hat{C}$ increases
II The area is a maximum when $\hat{C} = 90^\circ$
III The area decreases as $\hat{C}$ increases from 90° to 180°



A II and III
B I and II
C III only
D II only
E I only

70. How much information do you need to answer the question, " Is x a two digit number? ", if you are told:

I x^2 is a three digit number
II 10x is a three digit number
(Assume x is a whole number)

A I alone is sufficient but II alone is not.
B II alone is sufficient but I alone is not.
C I and II together are sufficient, but neither statement alone is sufficient.
D Each statement alone is sufficient .
E More information is needed to answer the question .

APPENDIX C

COPY OF "REVISED" MULTIPLE-CHOICE TEST

ANSWER SHEET

INVIGILATOR'S INSTRUCTIONS

COPY OF "REASONING" TEST

STANDARD 7
MATHEMATICS

MULTIPLE-CHOICE TEST
TIME: 70 MINUTES

READ ALL THE INSTRUCTIONS BEFORE YOU BEGIN:

1. Print your name and class clearly in the space provided on the ANSWER SHEET.

2. Select the ONE alternative which CORRECTLY completes the statement or answers the question.

3. Indicate the answer you think is correct by drawing a PENCIL RING around the appropriate LETTER on the ANSWER SHEET.

4. Make no marks on this question paper - all necessary calculations should be done on scrap paper.

5. You may NOT use a calculator.

6. Answer ALL the questions - if you are uncertain, make an intelligent guess. You will NOT be penalised for incorrect answers.

EXAMPLE:

1. $2a + 3a + 4a = ...$

A. 7a
B. 9a
C. 10a
D. 15a
E. 24a

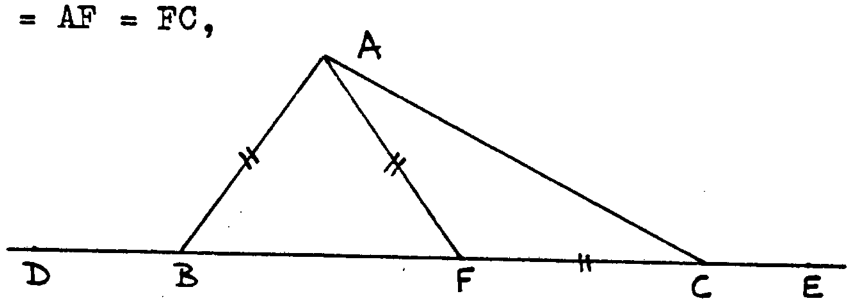
ANSWER SHEET

1. A (B) C D E

1. The interior angles of a quadrilateral are y, $2y$, $y + 20^\circ$ and $2y + 40^\circ$. Find the value of y.

A. 45°
B. 40°
C. 55°
D. 50°
E. Insufficient information

2. If $\widehat{ACE} = 140^\circ$ and $AB = AF = FC$, find $\widehat{ABD}$.



A. 80°
B. 100°
C. 140°
D. 160°
E. Impossible without more information

3. Solve for k: $14k - 5(2k + 1) = 3$

A. ½
B. −½
C. 1
D. 2
E. −2

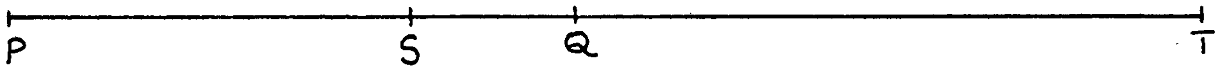
4. Multiply: $(-2a^3)^4 \times -4a^3$

A. $-8a^9$
B. $-32a^{10}$
C. $-64a^{15}$
D. $+64a^{15}$
E. $+32a^{10}$

5. x and y are the magnitudes of a pair of adjacent angles. If $x = 72^{\circ}$ and $y = \frac{3}{2}x$, then $x + y$ is ....

A. a straight angle
B. a right angle
C. an acute angle
D. an obtuse angle
E. a reflex angle

6. If $PS = 2SQ$, $PT = 2QT$ and $PQ = 3\frac{1}{3}$, find $\frac{PS}{QT}$



A. ½
B. 2/3
C. 1
D. 3/2
E. 2

7. Divide $(6x^3 - x^2 - 4x - 1)$ by $(2x + 1)$

A. $3x^2 - 1$
B. $3x^2 - x - 1$
C. $3x^2 - 2x - 1$
D. $3x^2 + x - 1$
E. $3x^2 + 2x - 1$

8. Find the H.C.F. (Highest Common Factor) of $4x^2yz$, $6xy^2z$ and $8xyz^2$.

A. $24x^2y^2z^2$
B. $4x^2y^2z^2$
C. $4xyz$
D. $2xyz$
E. $2x^2y^2z^2$

9. Find the missing number:

+11	-45	-5
-29		+3
-21	+19	-37

A. +13
B. -13
C. +39
D. -39
E. -17

10. Subtract $y^2 - 4$ from $-3y + 1$

A. $2y + 5$
B. $4y^2 - 5$
C. $-4y^2 + 5$
D. $y^2 + 3y - 5$
E. $-y^2 - 3y + 5$

11. The missing number is ....

2	6
54	18

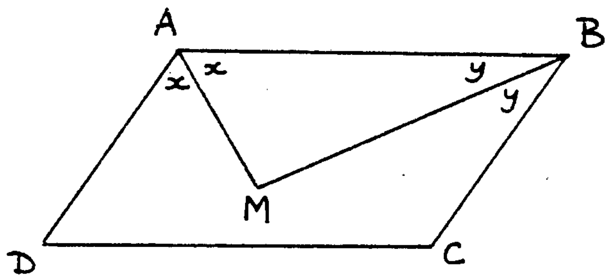
	4
64	16

A. 0
B. 1
C. 2
D. 3
E. 4

12. The cube root of 62 570 773 is ....

A. 383
B. 397
C. 429
D. 517
E. 20 851

16. If ABCD is a parallelogram, what is the size of $\hat{AMB}$?

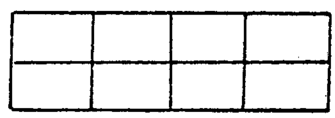


A. 45°
B. 90°
C. $2x + 2y$
D. $180^{\circ} - y + x$
E. $180^{\circ} - x + y$

17. x and y are both integers. Both are divisible by 7. Which ONE of the following is NOT necessarily true ?

A. $x + y$ is divisible by 7
B. $x - y$ is divisible by 7
C. $x + y$ is divisible by 14
D. xy is divisible by 49
E. $x^2 + y^2$ is divisible by 49

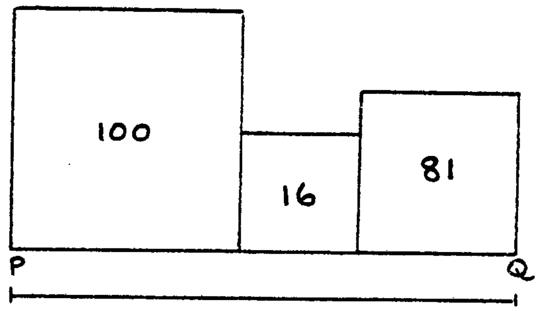
18. This is a sketch of part of Kit's circuitry. The length of the network is $2x + 5$ and the width is $x - 4$. The total length of wire, T, required to make this network is ....



A. $T = 4x^2 - 6x - 40$
B. $T = 2x^2 - 3x - 20$
C. $T = 3x - 1$
D. $T = 6x - 2$
E. $T = 11x - 5$

19. Three squares, with areas of 100, 16 and 81, lie side by side as shown. By how much must the area of the middle square be reduced in order that the total length, PQ, of the resulting three squares be 21 ?

A. $\sqrt{2}$
B. 2
C. 4
D. 8
E. 12



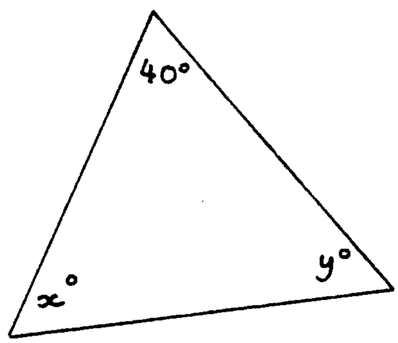
20. If h, k and n are positive integers, $k > m$ and $n > h$, which of the following are correct ?

I $n + h$ can be equal to $k + m$
II $k + h$ can be equal to $n + m$
III $k + n$ can be equal to $m + h$

A. I only
B. II only
C. I and III
D. I and II
E. I, II and III

21. If $60 < y < 100$, then ....

A. $40 < x < 80$
B. $40 \leqslant x \leqslant 80$
C. $60 \leqslant x \leqslant 100$
D. $60 < x < 100$
E. $80 < x < 120$



22. A certain number is 5 more than half the number subtracted from 40. What is the number ?

A. 15
B. 30
C. 35
D. 70
E. 90

23. Multiply $\frac{a + m}{x - y}$ by $\frac{y - x}{m + a}$

A. -1
B. 0
C. 1
D. -xy
E. xy

24. If $5^2 - 4^2 = \sqrt[x]{81}$, then x = ...

A. 0
B. 1
C. 2
D. 3
E. 4

25. If $x > 1$, which of the following will increase in size if x increases in size ?

I $x - \frac{1}{x}$ II $\frac{1}{x^2 - x}$ III $4x^3 - 2x^2$

A. I only
B. II only
C. III only
D. I and III
E. I, II and III

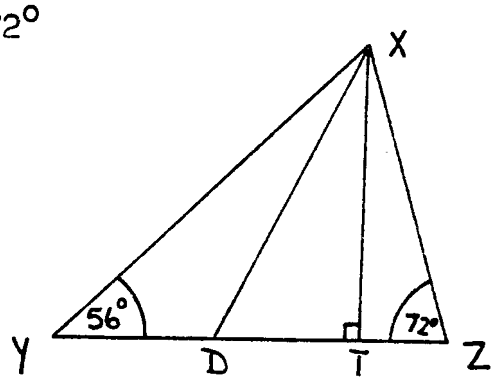
26. The equation $\frac{x + 4}{4} - \frac{x + 7}{4} = 0$ has ....

A. an infinity of solutions
B. a real irrational solution
C. one solution, $x = 0$
D. two solutions, $x = -4$, $x = -7$
E. no solution

27. Simplify $2 \times 2^4 \times 2^7$

A. 6^{12}
B. 8^{12}
C. 8^{26}
D. 2^{11}
E. 2^{12}

28. If $XT \perp YZ$, $X\hat{Y}Z = 56^\circ$, $X\hat{Z}Y = 72^\circ$ and XD bisects $Y\hat{X}Z$, find $D\hat{X}T$.

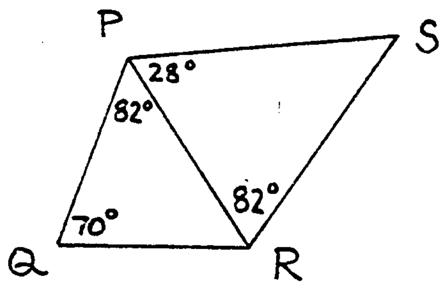


A. 8°
B. 14°
C. 18°
D. 24°
E. 26°

29. If x has a value between 4 and 8, and y has a value between 20 and 40, then $y \div x$ has a value between ....

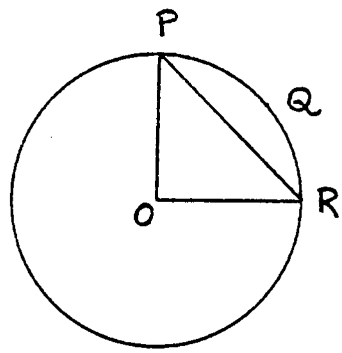
A. 2½ and 10
B. 4 and 40
C. 5 and 10
D. 8 and 20
E. 2½ and 5

30. What kind of quadrilateral is PQRS ?



A. kite
B. trapezium
C. parallelogram
D. rhombus
E. no special kind of quadrilateral

31. What is the length of arc PQR of a circle with centre O, if the area of the right-angled triangle POR is 32 ?

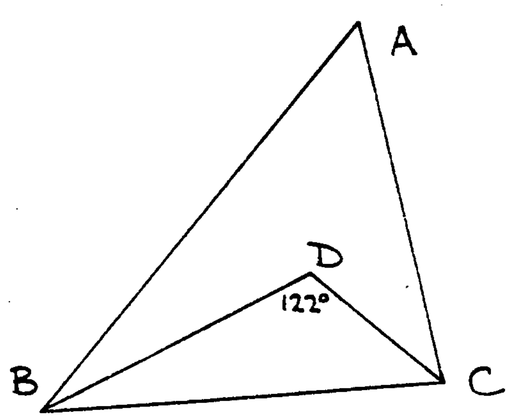


A. π
B. 2π
C. 4π
D. 8π
E. 16π

32. Find the value of the following expression if $p = -2$ and $q = -1$: $\sqrt{4p^2 - 24pq + 36q^2}$

A. 2
B. 3
C. 4
D. 10
E. 100

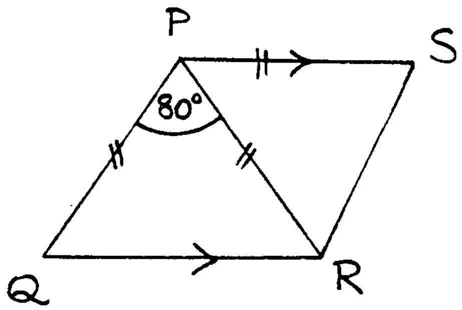
33. DB bisects $\hat{B}$ and DC bisects $\hat{C}$. If $\hat{D} = 122^\circ$, then $\hat{A} = \ldots.$



A. 58°
B. 61°
C. 62°
D. 64°
E. Insufficient information

34/.........

34. In quadrilateral PQRS, PS // QR and PQ = PR = PS. If $Q\hat{P}R = 80^\circ$, find $P\hat{R}S$.



A. 65°
B. 50°
C. 80°
D. 75°
E. 55°

35. Express as a single fraction:

$$\frac{2}{y} - \frac{3}{x} - \frac{x-4}{xy}$$

A. $\frac{1}{xy}$
B. $\frac{x - 3y + 4}{xy}$
C. $\frac{x - 3y - 4}{xy}$
D. $\frac{x - 3y + 4}{x^2y^2}$
E. $\frac{x - 3y - 4}{x^2y^2}$

MULTIPLE-CHOICE TEST ANSWER SHEET

NAME: .... CLASS: .... CODE: ....

Indicate the answer you think is correct by drawing a PENCIL RING around the LETTER representing that answer on this answer sheet.

1.	A	B	C	D	E	19.	A	B	C	D	E
2.	A	B	C	D	E	20.	A	B	C	D	E
3.	A	B	C	D	E	21.	A	B	C	D	E
4.	A	B	C	D	E	22.	A	B	C	D	E
5.	A	B	C	D	E	23.	A	B	C	D	E
6.	A	B	C	D	E	24.	A	B	C	D	E
7.	A	B	C	D	E	25.	A	B	C	D	E
8.	A	B	C	D	E	26.	A	B	C	D	E
9.	A	B	C	D	E	27.	A	B	C	D	E
10.	A	B	C	D	E	28.	A	B	C	D	E
11.	A	B	C	D	E	29.	A	B	C	D	E
12.	A	B	C	D	E	30.	A	B	C	D	E
13.	A	B	C	D	E	31.	A	B	C	D	E
14.	A	B	C	D	E	32.	A	B	C	D	E
15.	A	B	C	D	E	33.	A	B	C	D	E
16.	A	B	C	D	E	34.	A	B	C	D	E
17.	A	B	C	D	E	35.	A	B	C	D	E
18.	A	B	C	D	E						

Have you answered ALL of the questions? If you have missed some, return to these and make an intelligent guess.

INSTRUCTIONS FOR INVIGILATORS

1. PLEASE READ THROUGH THE INSTRUCTIONS ON THE PAPER WITH PUPILS

2. PENCILS SHOULD BE USED TO MARK ANSWERS. CHANGES MAY BE ERASED OR CROSSED OUT AS INDICATED:

   A B Ø D Ⓔ

   IN THIS CASE "E" WOULD BE REGARDED AS THE SELECTED ANSWER.

3. AFTER 70 MINUTES. TELL PUPILS TO CHECK THAT THEY HAVE ANSWERED ALL THE QUESTIONS. IF THEY HAVE OMITTED TO ANSWER ANY QUESTION/S, THEY MUST RETURN TO THESE AND GUESS.

   <u>IT IS VERY IMPORTANT THAT EVERY QUESTION IS ATTEMPED.</u>

4. PUPILS WILL REQUIRE ONE OR TWO SHEETS OF ROUGH WORKING PAPER. PLEASE EMPHASISE THAT NO MARKS SHOULD BE MADE ON QUESTION PAPERS.

THANK YOU FOR YOUR HELP AND COOPERATION.

STANDARD 7 | REASONING TEST | TIME: 30 MINUTES

NAME: .... CLASS: .... CODE: ....

1. Print your name and class clearly in the space provided.

2. Unless you are instructed otherwise, insert your FINAL answer in the BOX provided.

3. You may do any necessary calculation in the space below each question.

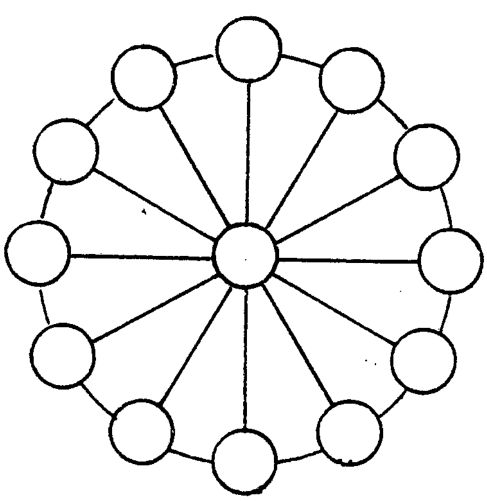
4. You may NOT use a calculator.

---

1. Of three men, Jones, Smith and Davis, two eat beef, two eat pork and two eat mutton. One of them eats neither pork nor mutton, and the one who doesn't eat mutton, doesn't eat beef either. If Jones eats either of the two and Smith does likewise, describe the meat eating habits of the three men.

Jones eats ....
Smith eats ....
Davis eats ....

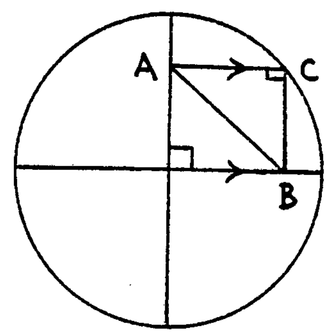
2. Arrange the first 13 natural numbers in the 13 circles so that each set of 3 numbers in a straight line along any diameter has a sum of 21.



3/.............

3. The cold water tap can fill a bath in 6 minutes 40 seconds. The hot water tap can fill the bath in 8 minutes. If the bath is full, it takes 13 minutes 20 seconds to drain when the stopper is removed. How long will it take to fill the bath if both taps are fully open and the stopper is out ?

4. If the circle has diameter 10cm., how long is AB ?



5. Find the missing number: 8; 10; 15; 26; ...?.

6. Find the word in brackets: 53(DICE)94
54(....)16

7. 20 men stand in a line at regular intervals of 4 paces apart. They are told to spread out from the left, (that is the left-hand man is to remain where he is) till they are 6 paces apart. How many paces should the right-hand man move ?

8/..........

8. 32 men are employed to do a piece of work, and, after 15 days have completed four-ninths of the work. 12 of the men are then drafted to another job, and work proceeds for 8 days with only 20 men. How many extra men must then be employed to finish the whole job in 10 days more ?

9. 3 ducks are worth 5 chickens, and 4 geese are worth 9 ducks. If a chicken is worth R2,04, what is the value of a goose ?

10. Three boys, A, B and C run a 100m race; A beats B by 5m and C by 16m. If B takes 11,2 seconds for the full distance, how long does C take ?

&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&

APPENDIX D

COPY OF "NEW" MULTIPLE-CHOICE TEST

INVIGILATOR'S INSTRUCTIONS

ANSWER SHEET

MATHEMATICS

Multiple-Choice Test

Time: 75 minutes

READ ALL THE INSTRUCTIONS BEFORE YOU BEGIN:

1. Print your name and standard clearly in the space provided on the ANSWER SHEET.

2. Select the ONE alternative which CORRECTLY completes the statement or answers the question.

3. Indicate the answer you think is correct by drawing a PENCIL RING around the appropriate LETTER on the ANSWER SHEET.

4. Make no marks on this question paper - all necessary calculations should be done on scrap paper.

5. You may NOT use a calculator.

6. N.B.: You must answer ALL of the questions - if you are uncertain, make an intelligent guess. You will NOT be penalised for incorrect answers.

EXAMPLE:

1. $2a + 3a + 4a = \ldots\ldots\ldots$

   A. $7a$
   B. $9a$
   C. $10a$
   D. $15a$
   E. $24a$

ANSWER SHEET

1. A (B) C D E

1. Simplify $2 \times 2^4 \times 2^7$

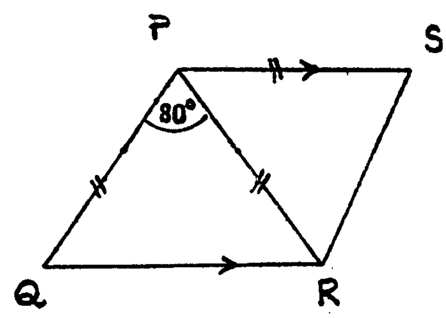
A. 6^{12}
B. 8^{12}
C. 8^{29}
D. 2^{11}
E. 2^{12}

2. Find the value of the following expression if $p = -2$ and $q = -1$:

$$\sqrt{4p^2 - 24pq + 36q^2}$$

A. 2
B. 3
C. 4
D. 10
E. 100

3. In quadrilateral PQRS, PS // QR and PQ = PR = PS. If $Q\hat{P}R = 80^\circ$, find $P\hat{R}S$.



A. 65°
B. 50°
C. 80°
D. 75°
E. 55°

4. Find the H.C.F. (Highest Common Factor) of $4x^2yz$, $6xy^2z$ and $8xyz^2$.

A. $24x^2y^2z^2$
B. $4x^2y^2z^2$
C. $4xyz$
D. $2xyz$
E. $2x^2y^2z^2$

5. Express as a single fraction:

$$\frac{2}{y} - \frac{3}{x} - \frac{x - 4}{xy}$$

A. $\frac{1}{xy}$

B. $\frac{x - 3y + 4}{xy}$

C. $\frac{x - 3y - 4}{xy}$

D. $\frac{x - 3y + 4}{x^2y^2}$

E. $\frac{x - 3y - 4}{x^2y^2}$

6. Simplify $(p^2q^4 \div pq^2)^2 + p^2q^4$

A. p^4q^8

B. $2p^4q^8$

C. $2p^2q^4$

D. $p^4q^4 + p^2q^4$

E. $2pq^2 + p^2q^4$

7. DB bisects $\hat{B}$ and DC bisects $\hat{C}$.
If $\hat{D} = 122^\circ$, then $\hat{A} = \ldots\ldots$

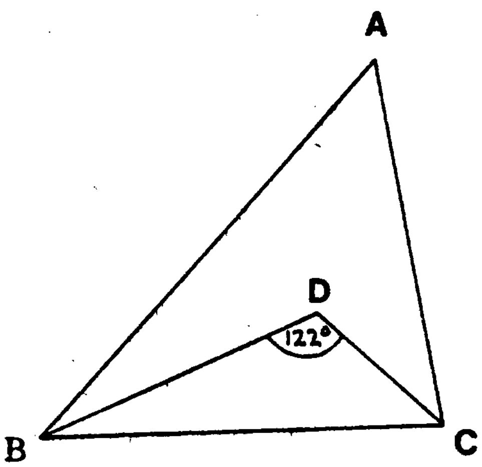
A. 58°

B. 61°

C. 62°

D. 64°

E. Insufficient information



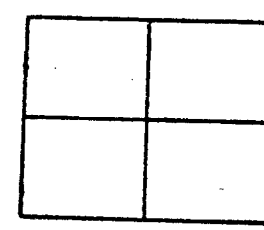
8. If x has a value between 4 and 8, and y has a value between 20 and 40, then $y \div x$ has a value between ....

A. $2\frac{1}{2}$ and 10
B. 4 and 40
C. 5 and 10
D. 8 and 20
E. $2\frac{1}{2}$ and 5

9. The equation $\frac{x+4}{4} - \frac{x+7}{4} = 0$ has ....

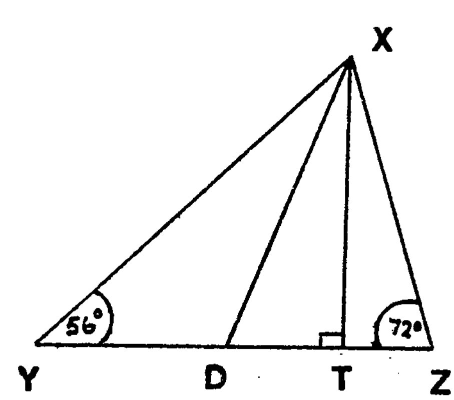
A. an infinity of solutions
B. a real irrational solution
C. one solution, $x = 0$
D. two solutions, $x = -4$, $x = -7$
E. no solution

10. You are given two white tiles and two black tiles with which to fill the rectangle. How many different patterns can you make ? (No broken tiles !)



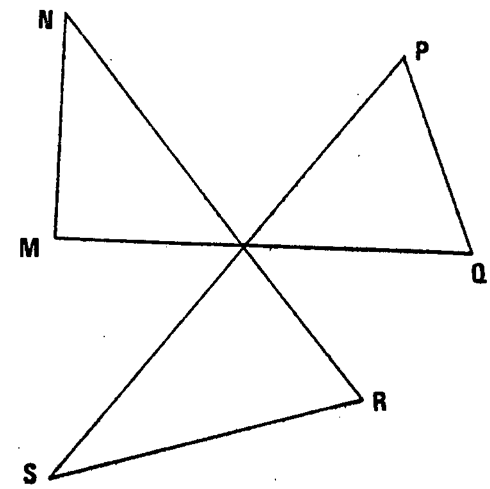
A. 4
B. 5
C. 6
D. 8
E. 12

11. If $XT \perp YZ$, $X\hat{Y}Z = 56^\circ$, $X\hat{Z}Y = 72^\circ$ and XD bisects $Y\hat{X}Z$, find $D\hat{X}T$.



A. 8°
B. 14°
C. 18°
D. 24°
E. 26°

12. What is the sum of the sizes of angles M, N, P, Q, R and S shown in the sketch ?

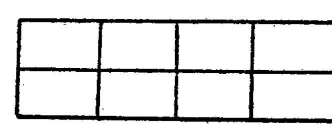


A. 180^o
B. 360^o
C. 540^o
D. 720^o
E. Insufficient information

13. A certain number is 5 more than half the number subtracted from 40. What is the number ?

A. 15
B. 30
C. 35
D. 70
E. 90

14. This is a sketch of part of Kit's circuitry. The length of the network is $2x + 5$ and the width is $x - 4$. The total length of wire, T, required to make this network is ....



A. $T = 4x^2 - 6x - 40$
B. $T = 2x^2 - 3x - 20$
C. $T = 3x - 1$
D. $T = 6x - 2$
E. $T = 11x - 5$

15. The interior angles of a quadrilateral are y, $2y$, $y+20^o$ and $2y+40^o$. Find the value of y.

A. 45^o
B. 40^o
C. 55^o
D. 50^o
E. Insufficient information

Examine carefully the algebraic expressions below. In each case select the statement which is appropriate to the expression.

16. $x < x + 2$

17. $x > 2x$

18. $x^2 > x$

A. True for all values of x
B. False for all values of x
C. False for all negative values of x
D. False for all positive values of x
E. False when $0 \leqslant x \leqslant 1$

19. Multiply $\frac{a + m}{x - y}$ by $\frac{y - x}{m + a}$

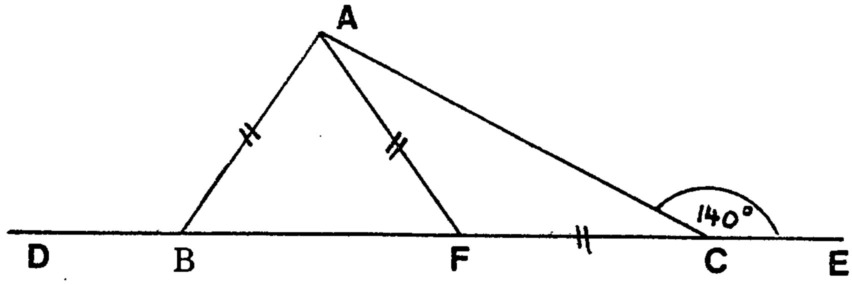
A. -1
B. 0
C. 1
D. $-xy$
E. $x - y$

20. If $x > 1$, which of the following will increase in size if x increases in size ?

I $x - \frac{1}{x}$ II $\frac{1}{x^2 - x}$ III $4x^3 - 2x^2$

A. I only
B. II only
C. III only
D. I and III
E. I, II and III

21. If $\hat{ACE} = 140^\circ$ and $AB = AF = FC$, find $\hat{ABD}$.

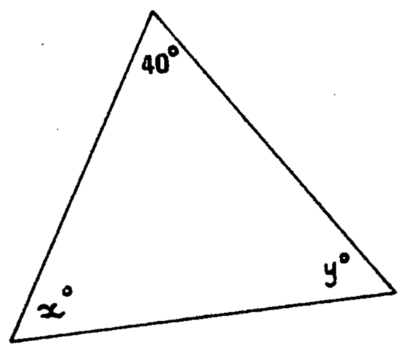


A. 80°
B. 100°
C. 120°
D. 140°
E. Impossible without more information

22. Multiply: $(-2a^3)^4 \times -4a^3$

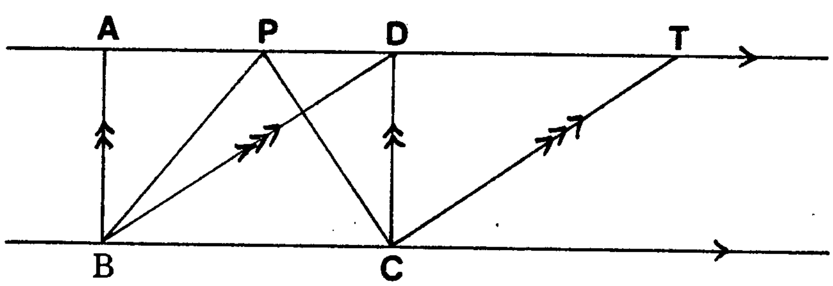
A. $-8a^{10}$
B. $-32a^{10}$
C. $-64a^{15}$
D. $+64a^{15}$
E. $+32a^{10}$

23. If $60 < y < 100$, then ....



A. $40 < x < 80$
B. $40 \leqslant x \leqslant 80$
C. $60 \leqslant x \leqslant 100$
D. $60 < x < 100$
E. $80 < x < 120$

24.



Which ONE of the following is NOT true ?

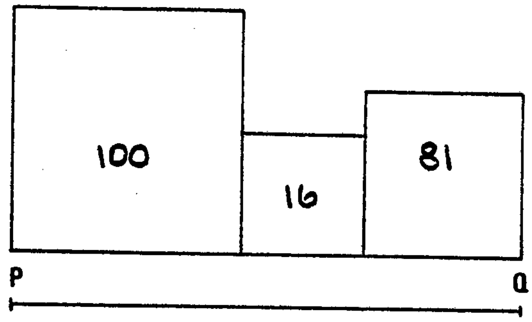
A. $\triangle DBC = \triangle DAB = \triangle PBC$ in area
B. $ABCD = \triangle DBC + \triangle DCT$ in area
C. $ABCD = DBCT$ in area
D. $\triangle PBC = \triangle CTD$ in area
E. $\triangle PBC = \triangle PTC$ in area

25. x and y are both integers. Both are divisible by 7. Which ONE of the following is NOT necessarily true ?

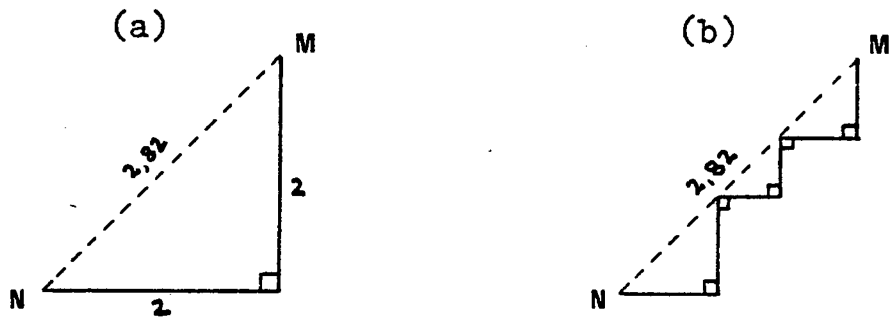
A. $x + y$ is divisible by 7
B. $x - y$ is divisible by 7
C. $x + y$ is divisible by 14
D. xy is divisible by 49
E. $x^2 + y^2$ is divisible by 49

26. Three squares, with areas of 100, 16 and 81, lie side by side as shown. By how much must the area of the middle square be reduced in order that the total length, PQ, of the resulting three squares be 21 ?

A. $\sqrt{2}$
B. 2
C. 4
D. 8
E. 12

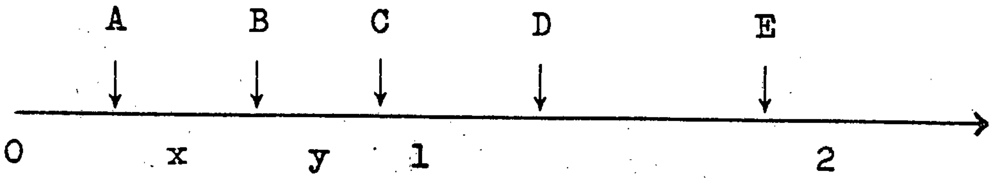


27. The solid lines in diagrams (a) and (b) show two different paths between M and N. What is the length of the path in diagram (b) ?



A. 4
B. 5
C. 6
D. 8
E. 12

28. x and y are numbers as indicated on the number line below. Which arrowhead (A, B, C, D or E) indicates a number that could represent the product of x and y ?



29. In this question, the symbol [x] means 'the largest integer which is less than or equal to x'.

eg $[4\tfrac{7}{8}] = 4$

Which ONE of the following statements is correct for all values of x greater than 1 ?

A. $x[x] = x^2$
B. $x \div [x] = 1$
C. $[x + 1] = [x] + 1$
D. $[x(x - 1)] = [x][x - 1]$
E. $[3x] = 3[x]$

30. The numbers a, b, c and d are such that $a > b > c > d$. Now study this reasoning:

$(a + b) > (c + d)$
A. $\therefore\ a > (c + d - b)$
B. $\therefore\ -a > (-c - d + b)$
C. $\therefore\ (d - a) > (b - c)$
D. $\therefore\ (d + c) > (b + a)$

If you think the reasoning is completely sound, circle the letter E on the answer sheet, otherwise, circle the letter corresponding to the first step in which an error occurs.

&&&&&&&&&&&&&&&&&&&&&&&&&&&&

MULTIPLE-CHOICE TEST ANSWER SHEET

NAME: .... STD: ....

Indicate the answer you think is correct by drawing a PENCIL RING around the LETTER representing that answer on this answer sheet.

1.	A	B	C	D	E	___	16.	A	B	C	D	E	___
2.	A	B	C	D	E	___	17.	A	B	C	D	E	___
3.	A	B	C	D	E	___	18.	A	B	C	D	E	___
4.	A	B	C	D	E	___	19.	A	B	C	D	E	___
5.	A	B	C	D	E	___	20.	A	B	C	D	E	___
6.	A	B	C	D	E	___	21.	A	B	C	D	E	___
7.	A	B	C	D	E	___	22.	A	B	C	D	E	___
8.	A	B	C	D	E	___	23.	A	B	C	D	E	___
9.	A	B	C	D	E	___	24.	A	B	C	D	E	___
10.	A	B	C	D	E	___	25.	A	B	C	D	E	___
11.	A	B	C	D	E	___	26.	A	B	C	D	E	___
12.	A	B	C	D	E	___	27.	A	B	C	D	E	___
13.	A	B	C	D	E	___	28.	A	B	C	D	E	___
14.	A	B	C	D	E	___	29.	A	B	C	D	E	___
15.	A	B	C	D	E	___	30.	A	B	C	D	E	___

Have you answered ALL of the questions ? If you have missed some, return to these and make an intelligent guess.

CODE: ....

MARK: ....

APPENDIX E

CHI-SQUARE CONTINGENCY TABLES FOR STANDARD 10 PUPILS OF SCHOOLS A, H AND E COMBINED

CHI-SQUARE CONTINGENCY TABLES FOR STANDARD 10 PUPILS OF SCHOOLS A, H AND E COMBINED

ITEM 1 ft 5

ITEM 2

37	21	58
31	23	54
68	45	112

Chi-Square = 0,24783
P = 0,6186

Cannot Reject Ho

ITEM 3

41	17	58
32	22	54
73	39	112

Chi-Square = 1,1456
P = 0,2845

Cannot Reject Ho

ITEM 4

40	18	58
25	29	54
65	47	112

Chi-Square = 5,0066
P = 0,0255

REJECT Ho at 0,05 Level

ITEM 5

44	14	58
30	24	54
74	38	112

Chi-Square = 4,278
P = 0,0386

REJECT Ho at 0,05 Level

ITEM 6

58	0	58
40	14	54
98	14	112

Chi-Square = 14,897
P = 0,0001

REJECT Ho at 0,02 Level

ITEM 7

35	23	58
15	38	53
50	61	111

Chi-Square = 6,4219
P = 0,0113

REJECT Ho at 0,02 Level

ITEM 8

46	12	58
35	19	54
81	31	112

Chi-Square = 2,2559
P = 0,1331

Cannot Reject Ho

ITEM 9

45	13	58
32	22	54
77	35	112

Chi-Square = 3,5604
P = 0,0592

Cannot Reject Ho

ITEM 10

48	10	58
42	12	54
90	22	112

Chi-Square = 0,18061
P = 0,6709

Cannot Reject Ho

ITEM 11

49	9	58
39	15	54
88	24	112

Chi-Square = 1,8216
P = 0,1776

Cannot Reject Ho

ITEM 12

36	22	58
23	31	54
59	53	112

Chi-Square = 3,5098
P = 0,0610

Cannot Reject Ho

ITEM 13

48	10	58
21	33	54
69	43	112

Chi-Square = 20,937
P = 0,0000

REJECT Ho at 0,02 Level

ITEM 14

50	8	58
36	18	54
86	26	112

Chi-Square = 4,9439
P = 0,0262

REJECT Ho at 0,05 Level

ITEM 15 ft 5

ITEM 16

51	7	58
48	6	54
99	13	112

Chi-Square = 0,018783
P = 0,8910

Cannot Reject Ho

ITEM 17

40	18	58
38	16	54
78	34	112

Chi-Square = 0,009417
P = 0,9649

Cannot Reject Ho

__ITEM 18__

52	6	58
31	23	54
83	29	112

Chi-Square = 13,521
P = 0,0002

REJECT Ho at 0,02 Level

__ITEM 19__

56	2	58
38	16	54
94	18	112

Chi-Square = 12,336
P = 0,0004

REJECT Ho at 0,02 Level

__ITEM 20__

52	6	58
37	17	54
89	23	112

Chi-Square = 6,4154
P = 0,0113

REJECT Ho at 0,02 Level

__ITEM 21__

53	5	58
46	8	54
99	13	112

Chi-Square = 0,52915
P = 0,467

Cannot Reject Ho

__ITEM 22__

49	9	58
35	19	54
84	28	112

Chi-Square = 4,768
P = 0,029

REJECT Ho at 0,05 Level

__ITEM 23__

47	11	58
36	18	54
83	29	112

Chi-Square = 2,3063
P = 0,1289

Cannot Reject Ho

__ITEM 24__

49	9	58
38	16	54
87	25	112

Chi-Square = 2,4497
P = 0,1175

Cannot Reject Ho

__ITEM 25__

49	9	58
30	24	54
79	33	112

Chi-Square = 9,9104
P = 0,0016

REJECT Ho at 0,02 Level

ITEM 26

37	21	58
29	25	54
66	46	112

Chi-Square = 0,79623
P = 0,3722

Cannot Reject Ho

ITEM 27

50	8	58
44	10	54
94	18	112

Chi-Square = 0,17888
P = 0,6723

Cannot Reject Ho

ITEM 28

56	2	58
34	20	54
90	22	112

Chi-Square = 17,916
P = 0,000

REJECT Ho at 0,02 Level

ITEM 29

27	31	58
9	45	54
36	76	112

Chi-Square = 10,122
P = 0,0015

REJECT Ho at 0,02 Level

ITEM 30

49	9	58
31	23	54
80	32	112

Chi-Square = 8,7621
P = 0,0031

REJECT Ho at 0,02 Level

APPENDIX F

REVIEW OF LITERATURE

SURVEY OF THE LITERATURE PERTAINING TO CHAPTER THREE

PREDICTION OF ACADEMIC PERFORMANCE:

Lavin (1965) presents three possible reasons for scholarly interest in the prediction of academic performance:

1. The tremendous increase in the student population matched with no commensurate increase in educational facilities resulting in the necessity of selecting those who are most likely to benefit from the existing resources and excluding those on whom these resources would ultimately be wasted.

2. The identification of the outstandingly talented in order to develop such talent in the scientific and technological spheres in the hope that these gifted individuals may take the lead in finding solutions to future national and international problems.

3. Development within the social sciences leading to the serious and concerted study of education.

Bereiter (1976) cites two reasons, namely, human resource development and individual development. Fox (1976) elaborates on these and sees prediction of academic performance as a necessary part of educational planning for the most economic use of resources and the optimum development of the individual's talents. Stanley (1976), Bereiter (1976) and Fox (1976) all sound warnings concerning the use of tests for the purposes of selection and prediction. They emphasise that test validity is never perfect and there is always the certainty that a number of "false positives" will be included and some "false negatives" will be excluded. To counter this Bereiter suggests that several tests should be administered, thereby reducing the possible error. Stanley contends that although test results can never capture all the talent nor exclude all those whose ability is slightly insufficient, nevertheless testing remains the fairest predictor, that is, testing produces the fewest errors. He stressed the point that any test employed should be of "appropriate difficulty", that is, that it should provide ample "ceiling" to enable the exceptionally talented to score above the marginally less talented. He added, however, that although high scores on standardised aptitude and achievement tests are probably the best single clue to high potential, such evidence should be corroborated and supplemented by further testing and perhaps by teacher recommendation.

Fox (1976) notes that when plans for the future education of an individual are to be made, it is not enough to have some knowledge of general ability, that is, it is not enough to know the student's I.Q. score. She contends that knowledge of a high score on an intelligence test may provide an estimate of higher learning potential, but reveals little about the individual's level of achievement, special abilities and interests. Fox asserts that the educational planning process is made more efficient when the student's interests, as well as his level of ability in particular subject areas are known. She warns those involved in educational planning for students that the pupil who shows no interest or enthusiasm in a subject or who is not motivated to learn that subject will not succeed simply on the basis of forcing by well-intentioned parents. Furthermore, poorly developed patterns of interests and the irresistable pull of peer pressure during the early teens may make pupils (especially girls) reject the study of certain subjects (especially mathematics) initially or cause a lack of interest or disenchantment with the subject. Such a reaction from a pupil in the South African system (particularly in mathematics) is fairly serious since it is not possible to "pick up" the subject at a later date and it is very difficult to regain lost ground in foundation work at the standard nine or ten level.

Stanley (1976), Fox (1976), Bereiter (1976) and Sherman (1979) all agree that prediction of academic performance is much too complex to be based on a single measure of general ability alone. Fox (1976) stressed the importance of the student's interest in or liking for the particular field of study as well as motivational factors, that is, powerful driving forces such as career or tertiary educational aspirations which may fuel the student's efforts in certain subjects.

Sherman (1979), in a study designed to establish cognitive and affective variables which might be predictive of success in Senior High School Geometry, Problem Solving and Theoretical Mathematics (Advanced Algebra, Trigonometry, Precalculus and Calculus) investigated the relationships between these and nine independent variables:

Cognitive: Ninth grade scores in (i) mathematics achievement
(ii) vocabulary
(iii) spatial visualization

Affective:
(i) confidence in learning mathematics
(ii) perceived attitudes of (a) pupil's mother, and (b) the pupil's father towards the study of mathematics
(iii) amount of encouragement from the teacher
(iv) the pupil's attitude towards success in mathematics
(v) perception of mathematics as a male domain
(vi) perceived usefulness of mathematics
(vii) effectance motivation in mathematics, that is, a joy in solving mathematical problems.

Sherman posed two questions, namely, would such cognitive and affective variables measured in the ninth grade predict mathematics achievement in grades ten through twelve, and would the relationships be the same for both boys and girls?

She found that in tenth grade ten of the eleven variables did, in fact, correlate significantly with female mathematics achievement, whereas for males only four of the eleven variables were significant. Correlations which were significantly higher for females than for males were the perceived usefulness of mathematics, the perception that mathematics is a male domain and mathematics achievement in the ninth grade. Results in the eleventh grade were different. For the females the affective variables made a better showing than the cognitive variables, particularly mathematics confidence, usefulness of mathematics and effectance motivation. For the males two variables were significantly correlated with grade eleven mathematics achievement, namely, ninth grade mathematics achievement and effectance motivation. For girls in the twelfth grade, three variables correlated positively and significantly with Theoretical Mathematics, ninth grade mathematics achievement, spatial visualization and effectance motivation; six of the eleven variables were significantly related to problem solving for twelfth grade girls, namely, ninth grade mathematics achievement, vocabulary, spatial visualization, mathematics confidence, attitudes towards mathematics of the parents and encouragement received from the teacher. Problem solving skill has been defined as the ability to solve a mathematical problem without possession of a standard solution or procedure as a guide - a skill thought to be particularly "problematic" for girls (Maccoby and Jacklin, 1974).

In general, Sherman (1979) found that ninth grade mathematics achievement showed significant correlation with the criterion variables in five out of the six analyses; spatial visualization, confidence in learning mathematics and effectance motivation showed significant relationships in four out of the six analyses. Furthermore, she found that relationships tended to be higher for females than for males and that there was evidence that girls with talent for mathematics fear success in mathematics to a greater extent in the ninth grade because it makes them seem peculiar or masculine. She concluded that the ninth grade data did successfully predict later performance to a greater or lesser extent, and that in general, the affective variables showed less predictive power than the cognitive variables.

These findings roughly substantiate the findings of Lindvall (1967) who stated that the best predictor of a student's success in a subject area is probably the record of his previous achievement in that sphere (or related sphere). On the subject of mathematics he had this to say:

> "The scores that a student makes on mathematics achievement tests covering his seventh and eighth grade work are a good predictor of success in high school algebra; since the same factors that have influenced past performance are likely to be the major determiners of future achievement. However, there are occasions when supplementary evidence is essential."

He explains that records of previous achievement in a subject area may present conflicting evidence owing to, for example, poor study habits. In such a case an aptitude test in that subject area would be useful in underscoring the individual's potential given that his study methods could be improved. Since efficient study methods are partly dependent upon the student's attitude towards the subject and his ability to persist with difficult tasks, improving study methods may involve some knowledge of the student's interests and attitudes to work. If the student happens to be a female her attitude towards mathematics may be coloured by the stereotyping of mathematics as a male preserve. It is this interaction among variables that makes the prediction of academic performance so complex an issue. Lavin (1965) points out numerous problems and difficulties in the area of academic prediction:

1. The traditional criterion for success has always been high scores in tests and examinations. Lavin cautions that before these are seen as inherently good, it should be determined exactly what kind of success is to be predicted, for example, fairly narrow success in future educational goals, future success in some career or future success as a well-adjusted adult human being. In the case of the latter two perhaps some measure of intellectual curiosity, critical-mindedness or creativity would be more useful than test scores.

2. Some variables in education are uncontrollable, which makes the sources of variation very difficult to determine.

3. There is the possibility that measures with similar conceptual labels are really quite dissimilar.

4. The converse problem is that very often variables with different labels are not independent of each other or are "tainted" by yet other variables, for example, "interest" in a subject may be a measure of a student's desire to "get on in life", or it may be related to his desire to please some significant other rather than fascination with the subject content.

5. Given some positive relationship between variables, how is this relationship to be interpreted? Can linearity be assumed? Correlation methods assume that the overall correlation is equally representative at all levels of intelligence, but one may find that ability measures are predictive in some segments of the range but not in others. McClelland (1958) poses the question,

> "...what evidence is there that intelligence is not a threshold type of variable, that once a person has a certain minimal level of intelligence his performance beyond that point is uncorrelated with his ability?"

Apart from the issue of threshold effects, the exclusive use of linear correlation methods may hinder the discovery of curvilinear relationships between predictors and academic performance, for example, the relationship between test taking anxiety and examination performance. It seems reasonable that very low levels of test taking anxiety might result in low scores; moderate levels of anxiety might result in high examination scores and very high levels of anxiety might result in low examination scores. If this were so, a linear correlation coefficient could not accurately reflect the actual relationship between the two variables.

6. The interpretation of very high and very low correlations creates certain difficulties. Should the low correlation variables be discarded? Do the high correlation variables indicate a causal relationship? Causal relationships can be extremely difficult to determine where sequence cannot be established - the cause precedes the effect in time. All too often the relationship is a feedback relationship where the two variables are interdependent and "feed upon each other", for example, high self-confidence and high performance. However, Lavin goes on to point out that it is frequently sufficient to know that a variable does predict performance and that the question of why it predicts is of lesser importance. This is known as the "actuarial approach".

Lavin lists four groups of basic correlates of academic performance - ability, sex, socio-economic status and personality factors. He urges researchers to separate the sexes in all analyses because although the variables may be the same for males and females, the relationships between the variables may very well be quite different. Likewise, he advises that social class should be controlled in all studies. He contends that socio-economic status consistently exhibits a positive association with academic performance. He points out that evidence suggests that socio-economic status summarises a variety of personality characteristics, that is, the social classes differ in terms of behavioural patterns such as child-rearing. These patterns, in turn, may be determinants of personality characteristics and values that are related to academic achievement.

He contends that although ability measures are the best single predictor of performance, they account for less than 50% of the variation in academic performance. Personality factors, although less consistent predictors, according to Lavin, nevertheless are of notable importance. He lists the following variables, while reminding us that many of them are interdependent:

Motivational factors - anxiety, achievement motivation, level of interest (this could mean liking for a subject, or it might reflect a need for affiliation or a desire for upward social mobility).

Personality style - degree of independence, impulse control (persistence), introversion.

Self-concept - this is related to ability and may be "tainted" by previous performance.

Study habits and attitudes toward study - a belief in the value of intellectual pursuit and education in general are positively related to academic performance.

SUMMARY:

The studies reviewed here represent a time span of more than two decades. Researchers have attacked the problem of prediction from different standpoints and in different ways but there is general agreement among them. All emphasise the complexity of the issue and warn against too glib interpretations from collected data. There seems to be consensus that the intellective factors, such as previous performance in the subject field, make the best single predictors, while not denying the existence and importance of motivational and personality variables which are often difficult to define accurately because of their interdependence.

SURVEY OF THE LITERATURE PERTAINING TO CHAPTER SIX

SEX DIFFERENCES IN MATHEMATICS:

Investigations consistently indicate that males outperform females on tests of mathematics achievement. Much research has been directed at explaining this phenomenon.

One avenue of investigation has been to examine mean differences between males and females in variables related to mathematical performance (Armstrong, 1981; Benbow and Stanley, 1980). Another avenue of research has been to investigate differences between males and females in the extent to which explanatory variables are correlated with mathematics achievement (Fennema and Sherman, 1977; Pallas and Alexander, 1983; Sherman, 1979).

Findings have often been contradictory and no consistent explanation for sex differences in mathematics achievement has been produced. The one clear conclusion from all the research is that the superiority of boys' performance in mathematics is not contingent upon some unitary trait or aptitude. Studies have been aimed at uncovering a multiplicity of explanatory variables, some biological, some psychological and some sociological. Most researchers come to rest on the conclusion that sex differences in mathematics performance are due to some complex interplay of sociological factors; however, the influence of one cognitive factor, namely, spatial ability, or, more precisely, spatial visualization, has been attracting the attention of many researchers.

It has been suggested that spatial ability is partly an inheritable trait since performance on <u>some</u> spatial tasks has been shown to be resistant to environmental influences. Furthermore, it was hypothesised that spatial ability may be carried by a recessive gene on the X-chromosome, and that whenever the recessive trait is present that it would be manifest in male behaviour. Since differences between the sexes in mathematical performance are not in evidence until adolescence, this theory does not seem to hold up, unless the development of spatial ability could, in some way, be affected by changes in levels of the sex hormones, androgen and estrogen during the adolescent years.

Spatial ability itself is difficult to define and different researchers investigating this construct may be examining different aspects of spatial ability. This certainly must contribute to the inconsistent results that are reported and makes the drawing of conclusions from these results an uncertain task. Sex differences have, most consistently, been discovered on tasks involving spatial visualization. This revolves around the visual imagery of objects, movements of the objects or changes in their properties, that is, the ability to manipulate objects mentally or in the "mind's eye". It has been found (Sherman, 1979; Maccoby and Jacklin, 1974) that from adolescence males have superior spatial visualization and, furthermore, that spatial visualization is related to mathematics achievement differently for males and females. Fennema and Sherman (1977) using the DAT Spatial Relations Test, found that the correlation between mathematics achievement and spatial visualization was approximately as high ($r = 0,5$) as the correlation between mathematics achievement and verbal ability. In addition, it seemed that spatial visualization scores were more highly predictive of mathematics performance for girls than for boys (Sherman, 1980). This finding has had some support from other studies (Fennema, 1981) which reported that spatial visualization tended to distinguish between girls who studied more advanced mathematics and those who did not. Fennema hypothesised that the critical relationship between mathematics and spatial visualization is indirect; that it involves the translation of words and/or symbols into a form where spatial visualising skills can be used. It has been suggested that the ability to make such translations allows for greater flexibility in problem solving and that some problems become much easier once the situation has been visualised. Thus boys, with their superior visualising skills have a clear advantage in geometry (with its obvious visual content) and greater latitude in dealing with many other problems which may be reduced in difficulty if they can be "pictured". Girls, with their early advantage in verbal ability, may be hindered in their problem solving by clinging to verbal strategies where a "visual attack" might have made the problem easier to handle. Werdelin (1958) found that spatial visualization related positively to mathematical reasoning but negatively to computation. This finding suggests that an emphasis on computation could interfere with the development of a spatial visualization ability and so hinder mathematical reasoning. Girls' superior performance on set procedures and computational work and poorer performance in other more highly cognitive mathematical tasks would appear to support Werdelin's findings.

Fennema and Sherman (1977) reject any suggestion that sex differences in spatial ability are due entirely to biological factors. They contend that spatial ability is strongly affected by learning and environmental influences. It is a fact that boys tend to gravitate towards or be steered towards more space-related activities both inside and outside school (wood-working and model-making from kits spring immediately to mind). Boys therefore gain more practice and experience in the manipulation of objects in space, and so tend to develop their spatial ability to a greater extent. Studies (Goldstein and Chance, 1965; Connor and Serbin, 1980; Vandenburg, 1975) have shown that girls' scores on tests of spatial ability improve after training and greater exposure to space-related activities.

The question of the relationship between spatial visualization and mathematics achievement, especially where girls are concerned, remains a vexed issue. Although there is some evidence that there exists a genetic bias in favour of males, there also exists fairly extensive evidence that whatever potential spatial ability there is may be developed by training and experience. Sherman (1978) concludes:

> "It has now become clear that an X-linked genetic factor is not responsible for differences sometimes observed between groups of males and females in their spatial or mathematical performance. These findings and the increasing weight of evidence pointing to the importance of spatial visualization to mathematics performance underscore the need for a better understanding of the development of spatial visualization and its relationship to education and female development."

Apart from spatial visualization, Fennema (1981) put forward a number of social and psychological factors which may interact to account for the poorer mathematical achievement of females from adolescence. Among these are the differential participation of females directly in mathematics but also in related subject areas; a general lack of self-confidence in their mathematical ability on the part of girls; differential treatment of boys and girls by teachers; the tendency of females to a greater extent than males to attribute success in mathematics to unstable factors but failure to stable factors and the perception that mathematics is a male preserve.

1. Differential Participation in Mathematics:

The higher up the educational ladder one climbs, the fewer females are found in mathematical pursuits. The reasons for this are possibly any combination of the previously-mentioned factors of lack of confidence, lack of support from significant others and social stereotyping (mathematics as a male domain), as well as a general lack of appreciation of the usefulness of mathematics among adolescent girls. The tendency among females to participate less in mathematics extends also to related fields of endeavour such as physics, mechanics, technical drawing and computer science. Practice of mathematical skills in these fields tends to reinforce problem solving, and since males participate more in these applied and higher level skill areas in classes other than mathematics (where females do not) it follows that their mathematical achievement would be further augmented.

Other researchers have supported the notion that reduced participation on the part of females has an effect on their achievement. Wise, Steel and MacDonald (1979) reported that when large sex differences in participation were controlled the difference in performance between males and females disappeared or, according to Pallas and Alexander (1983) were caused to shrink considerably. This contention, however, has not gone unchallenged. Armstrong (1981) found only slight differences in participation and Benbow and Stanley (1980, 1983) completely reject the hypothesis that differential participation accounts for sex differences in mathematics achievement.

Armstrong and Price (1982) attempted to identify the most important factors affecting women's participation in mathematics by interviewing a sample of students who had just completed or who would soon complete their high school mathematics careers. The questions raised were:

(a) attitudes towards mathematics (like/dislike)

(b) perceived usefulness of mathematics in respect of educational or career goals

(c) the positive influence of parents, teacher, counsellor and peers.

Perceived usefulness of mathematics was found to be an important predictor of participation for males and females as was liking for the subject. Teacher influence was found to be a more important factor for girls than for boys. The latter point is supported by Casserly (1980) who reported that many girls who have been accelerated in mathematics (that is, they are successful mathematicians) cite positive teacher influence as a cause of their success.

Armstrong concluded that any difference in participation could be ameliorated by first intervening in the attitude forming process by attempts on the teacher's part to decrease anxiety and increase confidence, and secondly, to ensure that parents, teachers and counsellors emphasise the importance of mathematics for further education and career aspirations.

2. Confidence in Mathematical Ability:

The influence on mathematics performance has been disputed. Some studies have concluded that sex differences in achievement could be attributed to the more positive attitudes towards mathematics by males (Sherman, 1980); whereas, other studies (Benbow and Stanley, 1982) found few sex differences in attitudes and, furthermore, little relationship between attitudes and achievement.

Possibly one of the most comprehensive studies of the influence of attitudes towards mathematics achievement was undertaken by Fennema and Sherman (Fennema and Sherman, 1977, 1978; Sherman and Fennema, 1977; Sherman, 1980). The investigation focussed on the components of attitudes towards mathematics thought to be related to performance, namely, self-confidence in learning mathematics, perceived usefulness of the subject, support and encouragement from parents and teacher, and the perception of mathematics as a male domain. This study was conducted among almost three thousand pupils from grades six to twelve, and perhaps the most striking fact to emerge was that girls were found to be significantly less self-confident in their ability to cope with mathematics even before any decline in their performance was evident. This finding tends to confirm the influence of this variable on mathematical achievement. Other findings of this study indicate that girls' denial that mathematics is a male domain tends to be positively related to the girls' relative mathematical success, and that, in general, boys perceive mathematics to be more useful to them than do girls. It is reasonable to expect that a girl, who firmly believes that mathematics is a subject for boys and that it holds no practical usefulness for the future, would not approach the learning of mathematics with any degree of enthusiasm or self-confidence.

3. Differential Treatment of Boys and Girls by Teachers

It has been suggested that some teachers tend to expect more of boys mathematically and, consequently, provide them with more opportunities to solve problems. Becker (1979) reported more teacher interaction of every kind with males than with females. Good, et al (1973) indicated that high achieving girls received significantly less attention in mathematics classes than high achieving boys. Becker (1981) reported that although girls initiated all types of teacher contact, boys still received the "lion's share" of teacher encouragement, persistence and individual help. This kind of unconscious, but nevertheless negatively biased treatment of girls by teachers, must surely reinforce their lack of confidence and perception of mathematics as a masculine pursuit. Although this would seem to be a logical conclusion, there seems to be no evidence that girls perceive teachers to be any less positive towards them (Fennema and Sherman, 1977).

4. Attribution Theory:

This theory has to do with the perceived causes of success and failure. Wiener (1974) proposed a model in which attributions of success and failure are categorised into the matrix in Figure 21, with the locus of causation being one dimension and stability the other.

Figure 21

WIENER'S MODEL OF ATTRIBUTION THEORY

	LOCUS OF CAUSATION	
	INTERNAL	EXTERNAL
STABLE	Ability	Task Difficulty
UNSTABLE	Effort	Luck

If success is attributed to internal causes, particularly ability, then future success can be expected and persistence will be worthwhile. However, if success is attributed to an external cause, future success is not assured and the task may be avoided. In contrast, failure attributed to an unstable cause leads the student to feel that future failure can be avoided and so persistence is encouraged; whereas, failure attributed to a stable cause will lead the student to the conclusion that failure cannot be avoided. Studies (Deaux, 1976; Bar-Tal and Frieze, 1977; Wolleat et al, 1980) show that females, to a greater extent than males, tend to attribute success in mathematics to unstable causes and failure in mathematics to stable causes. These patterns of attribution have been linked to a pattern of behaviour called "learned helplessness" - a condition in which failure is viewed as inevitable and insurmountable. Girls, more often caught in this pattern of behaviour than boys, may learn to avoid mathematics in order to preserve self-esteem (you can't fail if you don't take the course!) or may be less persistent in their attempts at problem solving. Repeated experiences of having to "give up" before the solution is found might be expected to have a negative effect on one's self-confidence, which in turn would dissolve any residual faith in one's ability.

In a study involving primary school children, Frieze and Snyder (1980) reported that younger pupils believe achievement situations are each controlled by their own specific causal mechanism, and furthermore, that these "beliefs are learned through experience" in specific situations, that is, causal attribution can be learned. If this is so it would seem to be an important task for the primary school teacher to instruct children in employing productive, beneficial and realistic causal schemata for all kinds of situations, especially those encountered in school. Such instruction would appear to have a special benefit for girls who generally tend to underestimate their ability (Deaux, 1976; Nicholls, 1979).

5. Social Stereotyping of Tasks

During the adolescent years boys and girls are initiated into the adult roles dictated by society. If society has deemed that certain tasks are appropriate for one gender and not for the other, it follows that the individual will be influenced by what is considered to be sexually appropriate. Sexually inappropriate tasks will be devalued and less time and effort will be expended on them in favour of the more valued sexually appropriate activities. Studies (Stein, Pohly and Mueller, 1971) have shown that the labelling of tasks as masculine or feminine influenced the importance attached to the task. Moreover, the pupil's perception of an activity as sex appropriate/inappropriate contributed more variance to those scores than actual biological sex, preference for a male or female sex role or even liking/disliking the task. Girls therefore, who perceive mathematics to be a male domain are placed in a situation of conflict about their achievement in mathematics. Girls may resolve this conflict either by avoiding mathematics or by avoiding success in mathematics by deliberately lowering their performance (Leder, 1982).

> "There are two potential sources for the negative consequences of success, i.e. loss of one's sense of femininity and self-esteem, regardless of whether anyone finds out about the success or not, and/or social rejection because of success."

As was stated earlier, it seems there is no easy answer to the inferiority of girls in mathematics achievement during the secondary school phase, and consequently their diminished representation in mathematical pursuits beyond school. The interplay of explanatory variables is highly complex making the drawing of consistent and valid conclusions virtually impossible. The one repeated implication appears to be that female self-confidence in mathematics needs to be boosted and that much has to be done in trying to make mathematics a sex-neutral zone if women (50% of the population) are to be enabled to make a valuable contribution.

CONCLUSION:

With regard to this experiment, that is, the writer's attempt to predict success in higher grade Senior Certificate mathematics at the Standard Seven level, it is clear that the sex of the pupil is a factor to be considered. The question remains at what point in the process of prediction should the gender of the student be taken into account? If the predictive device adequately reflects the type and quantity of spatial items of the final matriculation papers, then it seems reasonable that the predictive value of the correlation coefficients for girls would be just as accurate as they are for boys (spatial ability being the one cognitive variable possibly less well developed in girls than in boys). Since a test of mathematics cannot estimate what changes in interests pupils may experience in the future, it seems that the question of whether the sex of the pupil will have a bearing on their performance at the Senior Certificate level must be considered apart from any academic testing. When the possible problems of the increasing unattractiveness of mathematics for female students are known and appreciated by teachers it is surely possible to intervene in the process of feminine disenchantment with mathematics by bolstering self-confidence, presenting problems of interest to girls, emphasising career and educational openings for the mathematically astute, expecting as much from girls as from boys in the classroom and providing more space-related tasks for girls from an early age.

It is obvious from the abundant literature reporting on sex differences in mathematics that no investigation in mathematics education would be complete without the separation of the results of boys and girls. It is also fairly certain that girls have social difficulties with mathematics not experienced by boys. For these reasons boys' and girls' scores were handled separately throughout this investigation. However, it is unlikely that the nature of these difficulties would significantly affect the value of the correlation coefficients calculated from the data obtained from the various tests administered.

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