HYBRID SYSTEM MODELLING FOR THE *HYBRID DESIGNER* SOFTWARE TOOL

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Part 1: Hybrid system costing model

1 Introduction

The modelling of the hybrid system costing together with the system performance modelling make up the overall hybrid system model. This hybrid system design optimisation work aims to arrive at a solution for the hybrid system design problem. The solution is optimal only with respect to the accuracy of the model being employed. The model depicts the real system mathematically so that the model, rather than the physical system, can be manipulated. All models are necessarily abstractions, however, and the optimal solution with respect to the model may not be the optimal solution for an actual problem. If the model is well formulated, however, it is reasonable to expect that the resulting solution will provide a good approximation of the actual problem. Therefore emphasis has been placed on formulating the different ingredients of the hybrid system model as accurately as possible while trying to ensure that the model is still suitable for the problem-solving process.

The solution finding process to the model typically employs an algorithm. An algorithm is a set of procedures or rules that is followed in a step-by-step or iterative manner converging to result in the best solution for a given problem. Such an optimisation algorithm is usually programmed on a computer, which can then perform the calculations iteratively. It is important to note that a particular algorithm has a specific set of rules that apply to a specific problem formulation – therefore a large number of algorithms exist.

The model has an objective function that is optimised (minimised or maximised) subject to constraints that utilise decision variables (that is, the unknowns of the model) and parameters. The optimised set of decision variables prescribes the course of action that the decision maker/controller should select to achieve the defined objective.

The quantitative model used for describing the hybrid system contains the following elements:

Objective function. The objective function changes value as a result of changes in the decision variables. The objective function measures the desirability of the consequences of a decision. In this approach, the objective function describes the net present costing value of a hybrid system design in terms of a) initial equipment costs, and b) fuel and other operating costs discounted over the project planning period according to cost/benefit calculations (Marrison & Seeling-Hochmuth 1997).

Model. Based on the values of the decision variables the hybrid system model calculates the values of the:

- component sizes

at the beginning of each iteration, and at each time instant the values of

- component outputs
- transformation losses
- DC and AC bus inputs and outputs and the
- supply to the loads.

The resulting hybrid system performance over the simulation time period of, say, a year is extrapolated to the project life. Then the amount of load met as well as required fuel purchase, maintenance, overhaul and component replacements can be determined during the project life. This performance data yields the required operational costing data and penalty/benefit data to calculate the resulting value of the objective cost function for a certain system and operation choice.

Decision variables. Decision variables are the unknowns that are to be determined by solving the model equations. A specific decision is made when decision variables take on specific values. Decision variables concern component sizes, component numbers and installation settings and operating decisions such as amount of battery charge or discharge current (x_{bat}), diesel output power (x_{diesel}), position of switches (x_{SI}) and routing decisions (x_{RI}). The decision variables in the model are all labelled with an 'x' and an appropriate suffix. The above-mentioned operating decisions are time-dependent which can make a computer simulation very lengthy. If the time interval, over which the simulation is running, increases, then the number of operating decision variables is enlarged. Therefore, the methodology given in this research work links the operating decisions at each time instant to time-independent control settings that are then optimised during the algorithm's iterations.

Constraints. The constraints restrict the range of the decision variables as a result of technological, socio-economic, legal or physical constraints on the system. The constraints in the presented approach are given by technical characteristics of battery and diesel operation and by matching demand and supply. They can be incorporated in the model and in the objective function in form of a benefit description (benefit of meeting demand, other constraints) or in form of a penalty function (penalty on not meeting demand, other constraints).

In this design problem the requirement was to model the hybrid system with a high degree of accuracy while still being able to run an optimisation algorithm over the model. Therefore genetic algorithms were chosen as the optimisation tool. Genetic algorithms have the advantage of being able to optimise quite complex models, as they do not require the calculations of gradients. Their disadvantage is that they might require some time to converge, depending on the specific problem case.

The following sections explain the derived objective function description in terms of net present value (NPV) costing, the incorporation of the design constraints in the form of benefit and penalty descriptions, and the underlying system model with its decision variables. The shortcomings of rule-of-thumb or spreadsheet modelling in optimising a hybrid system design are highlighted and the characteristics of the developed optimisation algorithm for the hybrid system design are described.

2 Hybrid system life-cycle costs and net present value analysis

When designing a hybrid off-grid electricity supply system, the analysis comprises the comparison of mutually exclusive alternatives for the choice of system design that produce a similar techno-economic and social benefit for the chosen remote application. The evaluation of different hybrid and other off-grid design choices determines what type of system to use for a given application. This may also be done by comparing a hybrid system with electrification by grid extension, a local diesel generator, an SHS project, a central battery charging station or other options. Whichever technical solution is chosen, the benefits need to be determined. Such benefits can describe the technical reliability of the electricity supply or can also address socio-economic improvements induced through the provision of electricity service. The developed optimisation model is formulated such that after the optimisation algorithm's iterations, the type of system recommended offers the best combination of least-cost design and highest benefit choice.

In counting the cost of a project, it is customary to use net present value (NPV) analysis (Gowan 1985; Marrison & Seeling-Hochmuth 1997). The concept of net present value analysis is an extension to the principle of life-cycle costing. In both cases discounting of future costs is an important concept.

The present value of an asset/liability of value C held/incurred in n years from the present, where the value of C escalates by esc, is described as

$$PV = C \cdot R$$

Equation 1: Present value of an asset after n years

$$R = \frac{(1+esc)^n}{(1+r)^n}$$

Equation 2: Discount factor after n years

The present value of a cost C incurred every year for the next n years with a real escalation of esc per annum, is described as:

$$PV = C \cdot R_{yearly}$$

Equation 3: Present value of a cost incurred every year for n years

$$R_{yearly} = (\frac{1+esc}{r-esc}) \cdot (1-(\frac{1+esc}{1+r})^n)$$

Equation 4: Discount factor for a cost incurred every year for n years

where r is the discount rate. The discount rate represents an appropriate opportunity cost, with which future costs and benefits are discounted to their present value.

The opportunity cost of a scarce resource is defined as the benefit foregone by using the resource for one purpose instead of in its best possible alternative. The basis for this is that most resources have several potential uses. The direct opportunity cost of a person-day used for a rural electrification project is what this labourer would have produced, for example, by working on agricultural land had she not been taken away from her usual occupation to be employed in the rural electrification project.

In this analysis inflation is not included in any of the cash flows. 'r' is therefore the real discount rate. It is generally difficult to obtain the correct value of r because it depends on the riskiness of the cash flows (Marrison & Seeling-Hochmuth 1997). Davis & Horvei (1995) suggest that 8% is the appropriate rate to use for this type of project in the South African context and this is used for the base case.

In life-cycle costing equipment and operation costs are compiled and discounted over the assumed project life. The hybrid system life-cycle costs are comprised of initial investment costs and future discounted operation costs:

$$LCCs = InitialCosts + \sum_{i=1}^{NoOfComponents} Discounted\ OperationCosts_i$$

Equation 5: Life-cycle cost

$$Discounted\ OperationCosts_i = \sum_{year\ n}^{ProjectLife} \frac{OperationCosts_i(n)}{(1+r)^n}$$

Equation 6: Discounting of operation costs

In net present value analysis it is not only the cash flows for expenses that are considered, but also the discounted cash flows for income or benefits arising from providing electricity over the project life.

$$NPV = InitCosts + \sum_{i=1}^{NoOfComponents} Discounted\ OperationCosts_i - \sum_{j=1}^{NoOfIncomeSources} Discounted\ Income_j$$

Equation 7: Net present value

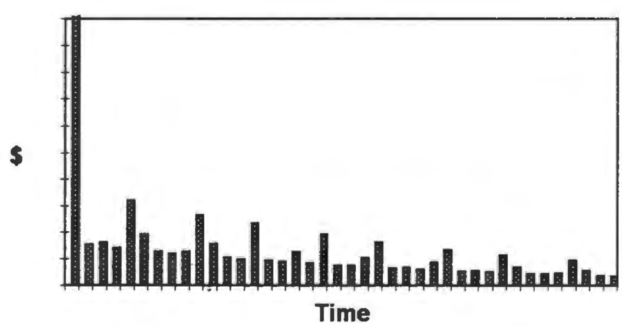
$$Discounted\ income_j = \sum_{year\ n}^{Projectlife} \frac{Income_j(n)}{(1+r)^n}$$

Equation 8: Discounting income

There are three broad types of expenditure cash flows: initial capital costs, costs that depend on the passage of time, and costs that depend on usage (Marrison & Seeling-Hochmuth 1997). The initial costs for a given design are typically well known and for a small project they will occur at t=0 with no discounting.

Costs that depend on the passage of time, for example monthly administration costs, are accumulated for a year or so and then discounted over the project life. Costs that depend on usage require information from the system performance, as can be estimated or, more accurately, simulated. Examples of costs that depend on usage are overhaul, diesel replacement, refuelling, and battery replacement. Figure 1 based on figures from Marrison & Seeling-Hochmuth (1997) illustrates capital, periodic, and usage cash flows. The initial peak represents the capital cost, the later large peaks represent diesel overhaul and replacement events.

Figure 1: Illustration of discounted cash flows (Marrison & Seeling-Hochmuth 1997)



3 Initial hybrid system costs

3.1 General

The initial costs are those incurred through purchasing equipment and hiring labour in order to install a hybrid system. A component purchase might also generate certain associated fixed costs for the user. For example, regardless of what kind of component size is purchased, a certain type of transport would always have to be paid for. When installing equipment certain costs arise due to installation, labour or required accessories. These costs depend on size and type of component and are often given as a percentage of individual or overall equipment purchase costs. All contributing initial component costs are included to give the overall initial costs (Equation 9 and Equation 10).

$$InitialCosts = (\sum_{\forall components} InitCosts) + \%of \cdot (\sum_{\forall components} InitCosts) + FixedCosts$$

Equation 9: Initial component costing

$$InitCosts(component\ i) = f(x_{size,type,i}, x_{number,type,i})$$

Equation 10: Dependence of initial costs on size and type

In general, the initial purchase costs of a component bank will depend on the size and type of the component, and on how many components are bought. For example, differently sized wind turbines are available at varying costs due to different material and labour costs of producing them. In addition, similarly sized wind turbines from different manufacturers might be priced differently due to the use of particular designs, materials, quality standards and mark-ups.

The component size and type are subject to optimising system design criteria. They are therefore selected as decision variables to be optimised in the developed hybrid system design model. The initial cost of the component is multiplied by the required number of components to be installed in the system in series ($ni,_{series}$) and in parallel ($xi,_{parallel}$). The number of components to be installed in series is often straightforward and determined by the nominal operating voltages of the system and the components. However, the number of components to be installed in parallel is subject to system design and its optimisation and is therefore labelled with an 'x', as is the size of a component type.

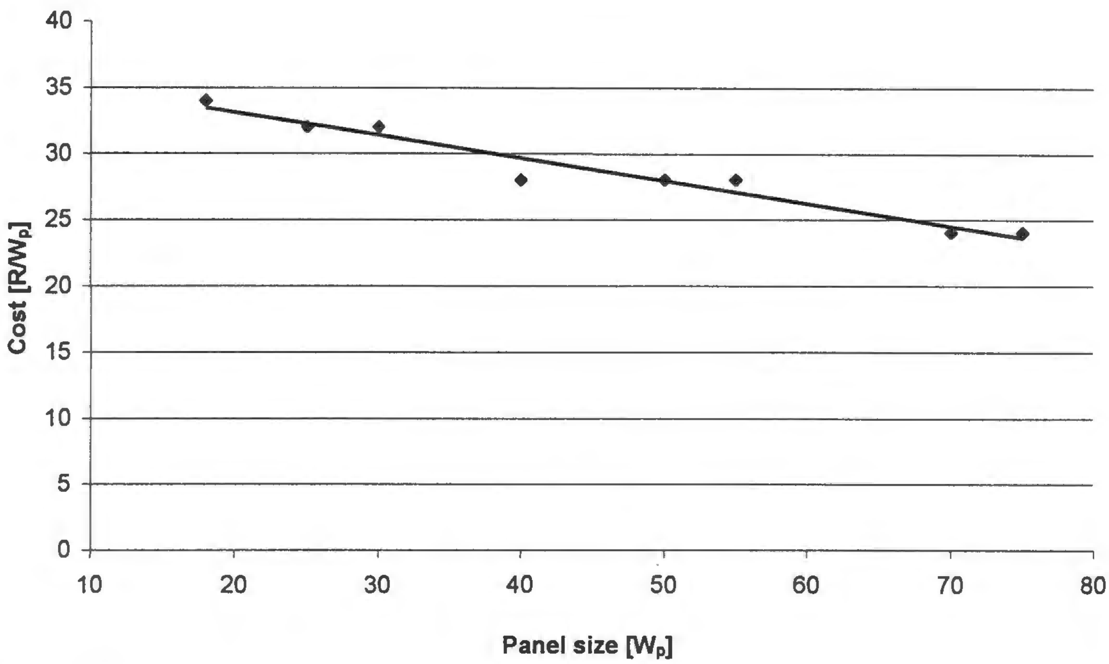
Therefore, in the initial cost modelling, the size and type of a component and the required number of parallel connected components are taken to be decision variables to be optimised in the developed hybrid system model.

In the following, the initial costs of different component types are discussed.

3.2 Initial costs of the PV array

Photovoltaic (PV) panels are available in different sizes, which are characterised by peak wattage (W_p) that stands for the maximum power a PV panel can produce under standard test conditions STC; 1 000W/m^2, AM of 1.5, 25°C and perpendicular insolation). The most common sizes are 40W_p - 80W_p. 18W_p and 300W_p are more rare and used specifically for very low or very high PV power requirements respectively. An economy of scale in panel prices occurs at between 18W_p and 75W_p (see Figure 2). There are, however, increased installation costs for smaller panels.

Figure 2: Initial PV panel costs in R/W_p



The economy of scale in PV panel costs becomes apparent if different combinations of different panel sizes can be used to produce the required PV power. As a general rule the designer will use the largest possible panel size. 350W, for example, can be made up of seven panels of $50W_p$ for R9 800 or five panels of $70W_p$ for an overall cost of R8 400. However, in some cases it might be more economic to consider a smaller panel size. For example, if a design was carried out for a certain site and the PV power requirement was calculated to be $6kW_p$, then with a 48V nominal voltage bus and panel voltages of 12V, 120 panels of 50Wp are needed to meet the designed power requirement, amounting to R168 000. Or alternatively, 88 panels of 70Wp would be required, costing R172 480, a difference of R4 480. However, even though the costs would have increased in the system design using the $70W_p$ panel size, the installed kW_p would have been slightly increased to $6.16kW_p$, higher than specified as required, but nevertheless available in case of slightly higher demand levels. As can be seen, design often involves considering the trade-offs of different possible design and cost solutions.

If PV panels with 24V nominal voltage are available on the market then the $70W_p$ panels would become cost-competitive with the $50W_p$ panel sizes in meeting a design requirement of $6kW_p$. Figure 3 and Figure 4 illustrate for 12V and 24V PV panels that from a certain amount of required kW_p onwards, the most economic panel size to meet the power requirements is the largest available size with the highest possible panel voltage.

Figure 3: Best 12V PV panel size for lowest PV array costs

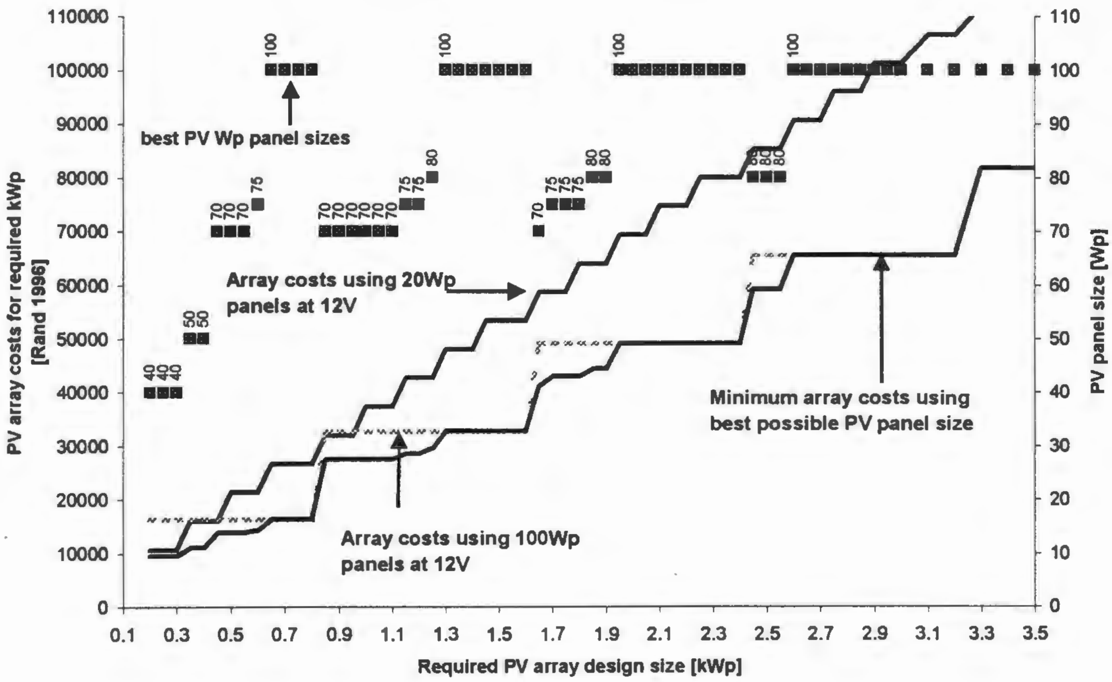
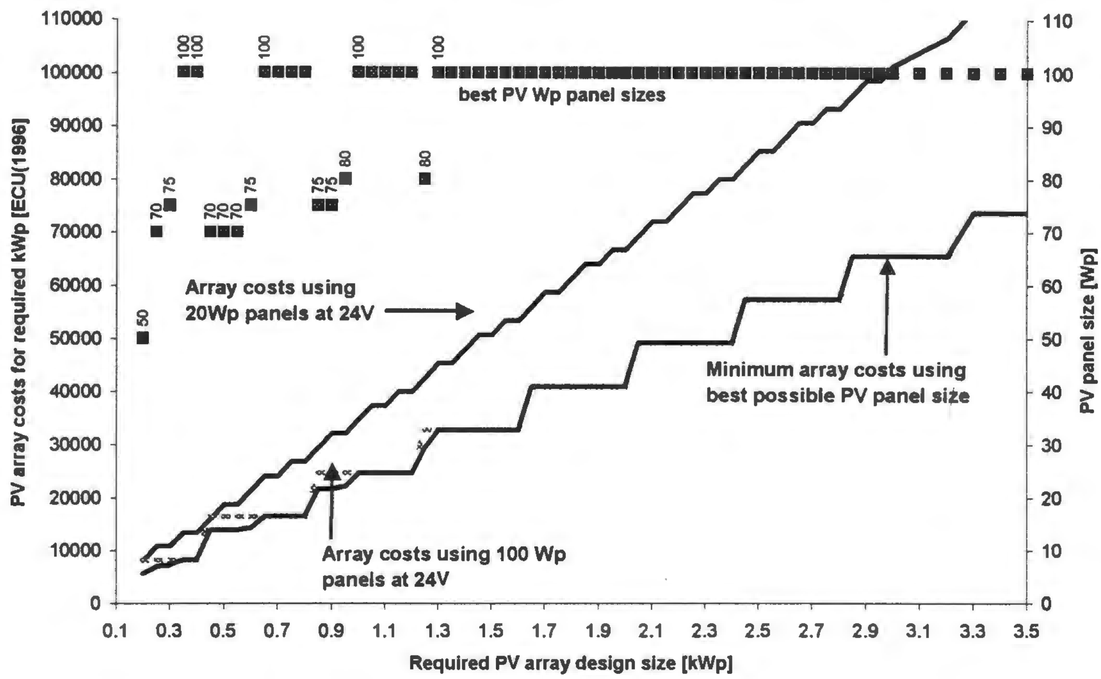


Figure 4: Best 24V PV panel size for lowest PV array costs



It can be seen that for PV power requirements of up to 2.6kW$_p$, using 12V PV panels, the use of PV panels with less than 75W$_p$ or a 100W$_p$ can in some cases be more economical.

Using 24V PV panels, it is in some cases more economical to use small W$_p$ PV panels for power requirements of up to 1.3kW$_p$. The prices per W$_p$ vary with manufacturers and are also country-specific. For this reason, Figure 2 gives only an indication of the price variations.

The required number of panels to be installed in series in order to obtain the required nominal bus voltage is given as

$$n_{series,PV} = \frac{U_{DCbus,Nom}}{U_{PVpanel,Nom}}$$

Equation 11: Number of PV panels required in series connection

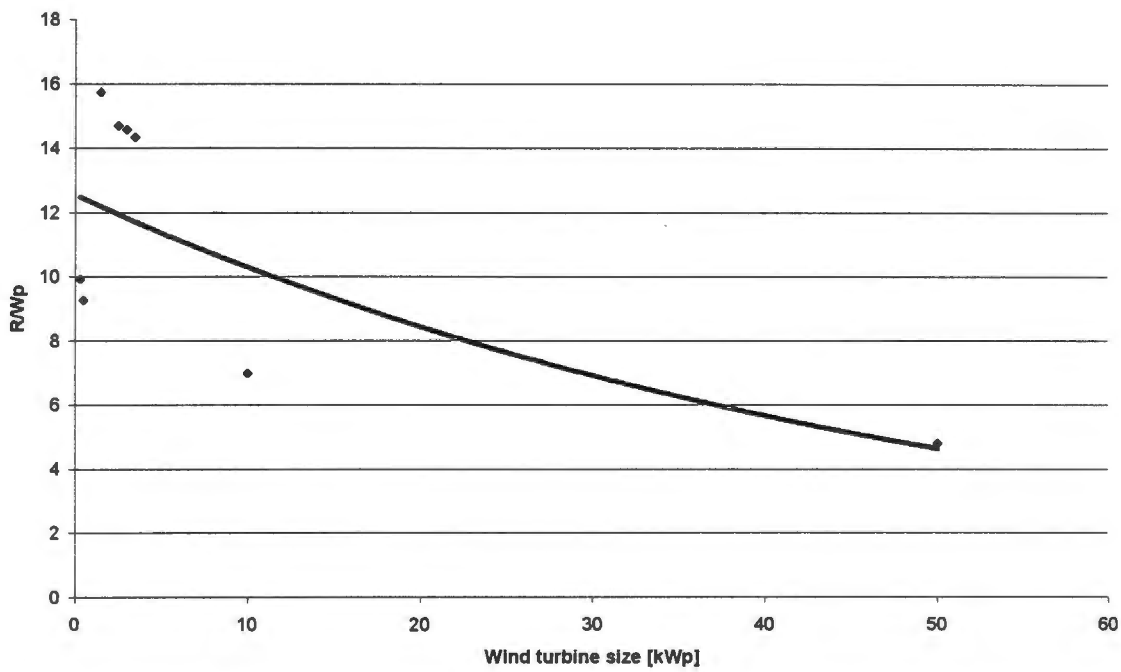
The number of PV panel strings required in parallel to produce the desired PV output energy, needs to be optimised and is roughly given by the required system Ah current from the PV array, divided by the nominal PV panel current times the number of average sunshine hours. The amount of energy desired from the PV array as percentage of overall system energy generation needs to be determined by a design or optimisation procedure.

The values for installation costs, balance of system (BoS) costs and fixed costs can differ, however, for different applications and projects. For example, Davis & Horvei (1995) indicate that the installation costs for PV systems ranged from 20% of PV array costs for a 400Wh/day domestic system and up to 55% to 67% of PV array costs for very small systems or very critical load systems. In some cases, the installation and BoS costs are added to the overall system equipment costs. In order to summarise all required equipment costs, namely PV, batteries and BoS parts, the installation costs would amount to 16% of capital costs for the domestic system, 30–35% for the small load systems, 20% for higher load systems, and 26% for the critical load system.

3.3 Wind turbine generators: initial costs

The initial wind turbine generator costs vary profoundly (see Figure 5). This is partly due to the wide range of wind turbines designed for differing applications and regions using diverse quality standards, materials, and production methods. Wind turbines require tower constructions and often difficult installation procedures. The costs of the tower and control equipment and installation contribute a large percentage to wind turbine capital costs. An advantage of wind turbines when compared to PV panels is that the nominal output voltages of the DC and AC turbines can often be adjusted to the corresponding bus voltages. Adding wind turbines is therefore achieved by connecting each additional turbine in parallel.

Figure 5: Wind turbine initial costs in R/kW_p

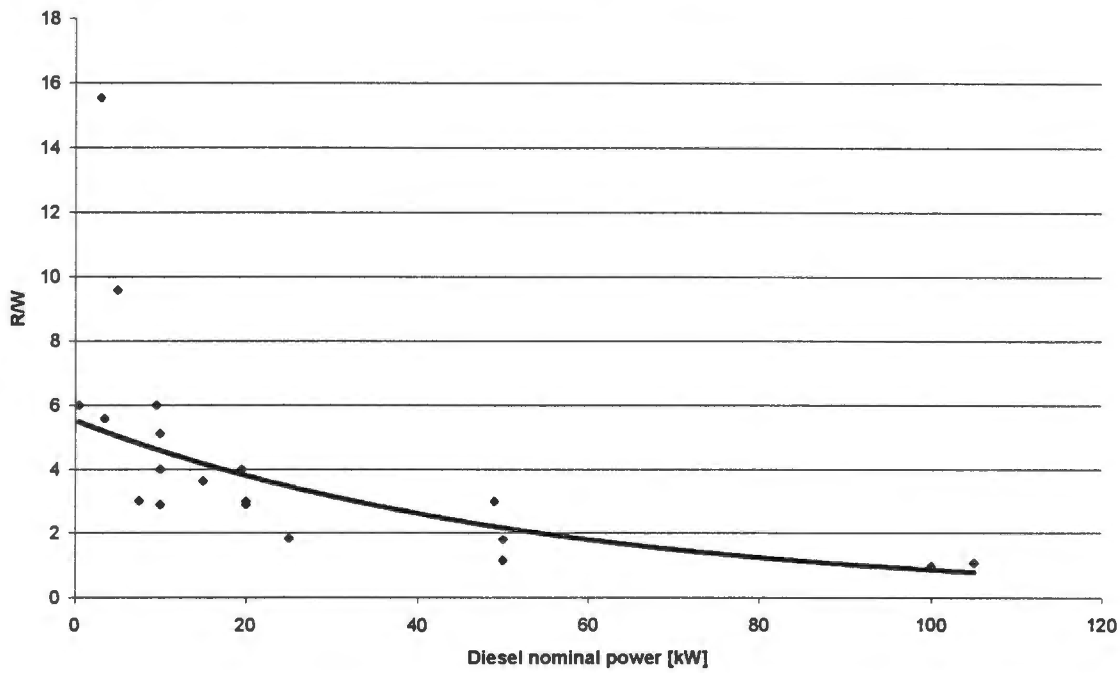


The balance-of-system costs are quite high for a wind turbine due to the tower and control equipment required. These costs can be expressed as a percentage of the wind turbine capital costs or the overall system equipment costs. The installation costs are also accounted for in many cases as a percentage of wind turbine capital costs or overall system equipment costs.

3.4 Initial costs of diesel generators

The diesel generator initial costs vary with size, model and design as can be seen in Figure 6.

Figure 6: Diesel generator initial costs in R/kW nominal capacity



Although diesel generators can come with AC or DC outputs, the AC diesel generator is the more common one. In general, the nominal output voltage will correspond with the bus voltage. Therefore each additional diesel generator can be connected to the system in parallel. Installation costs often include transport costs and intensive labour costs. A diesel generator will need a well-designed shelter with good ventilation. A tank is also needed to store the required fuel. These items can add to the installation and balance-of-system costs or are sometimes classified as fixed costs.

BoS costs and diesel generator installation costs as a percentage of diesel generator capital costs are given in Figure 7 and Figure 8 respectively (Davis & Horvei 1995). The share of labour costs in these expenditures is country-specific and might be higher for countries with higher labour costs.

Figure 7: Diesel generator installation costs as percentage of diesel generator capital costs

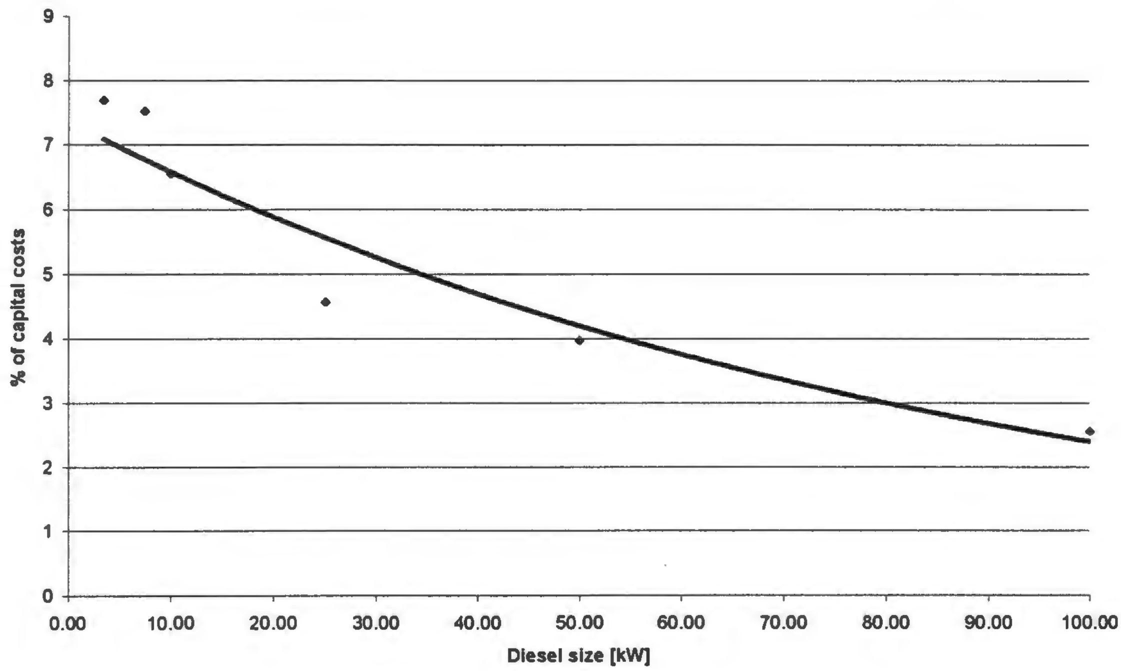
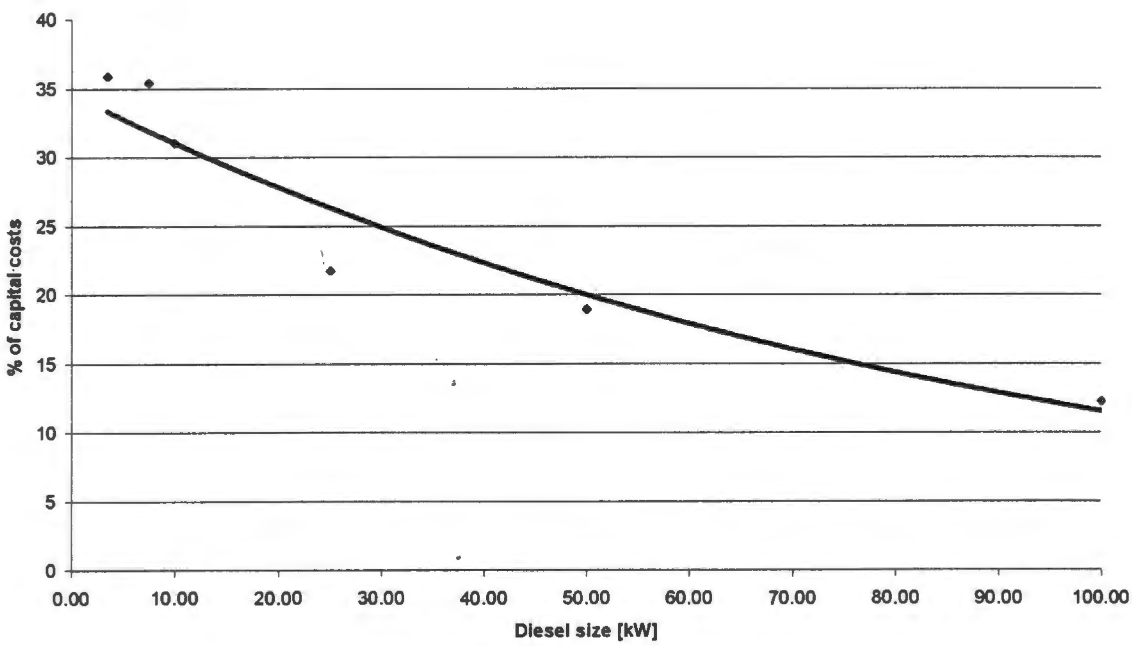


Figure 8: Diesel generator BoS costs as percentage of diesel generator capital costs



3.5 Initial costs of batteries

Various battery types exist, with different nominal voltage ratings. The most common ones in a hybrid system are the 2V cells with different Ah ratings. The initial costs of flat plate batteries are lower than those of tubular batteries, for example. However, flat plate batteries do not last as long as tubular batteries in terms of the estimated number of cycles during the battery's lifetime.

The batteries are connected in series to give the desired nominal DC operating voltage and are connected in parallel to yield a desired Ah system storage capacity. The batteries need a suitable battery shelter and a battery controller. The battery shelter can be accounted for in the fixed costs or the costs proportional to capital costs. Installation is often measured as a percentage of capital costs and the controller expense is often accounted for under BoS costs.

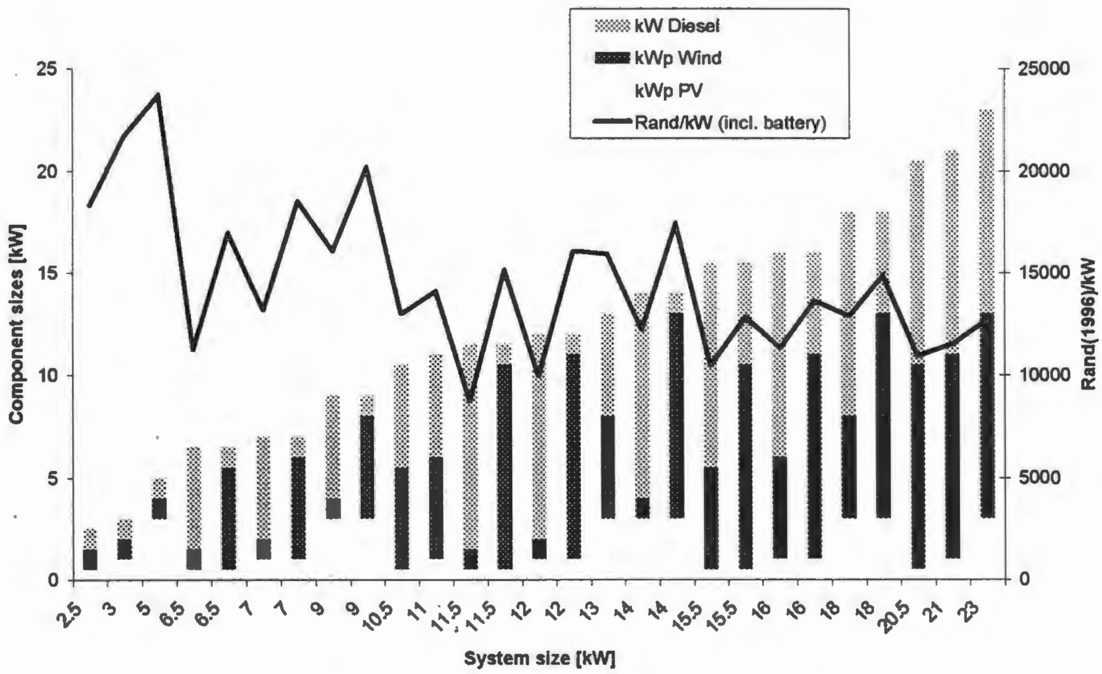
3.6 Balance of system (BoS) initial costs

The balance-of-system costs can include costs for components like system control, inverters, battery chargers, converters, wiring, etc and can in some cases be up to 30–35% of overall system investment costs.

3.7 Overall initial costing

The initial costs in the objective function description need to be determined carefully and a variety of component types and cost figures exist. As can be seen in Figure 9, the initial costs do not vary linearly with system size. Therefore the initial system costing already introduces non-linearity into the hybrid system design optimisation.

Figure 9: Initial costs of a PV/wind/diesel hybrid system versus system size in R/kW



4 Hybrid system operation costs

4.1 General

The operation costs describe costs incurred after installation in order to run the system for a certain number of years, the so-called 'project life'. The project life is an important parameter for system designers as it helps to benchmark different life-cycle costs or net present value costs for different designs. The operation costs include expenses for fuel, maintenance, component overhaul and component replacement. As operation costs occur in the near and distant future and are only estimates, they are more difficult to determine than initial costs. It is also difficult to estimate component degradation as part of the replacement and maintenance costing. It is stated in the literature that the relationship between operation and component degradation has so far not been well understood.

In the developed model the predicted timing for maintenance, overhaul and replacements is based on the number of hours or operational cycles a component has been operating which is influenced by the system size and system operation.

Operation costs can be split into a number of contributing expenditures, mainly costs for maintenance, refuelling, component overhaul, component replacement and administration. In many projects a maintenance person is employed to look after the system or several systems. This person's monthly salary or part of it will therefore occur in the maintenance costs. Often systems will need some kind of administrative support, and monthly administrative costs will be included in the operation costs. The operation costs for either a component or for the overall system can contain fixed costs and costs that are counted as a percentage of initial capital expenditure. Equation 12 summarises the different shares of operation costs.

$$OpCosts_i(year\ n) = no_{i,series} \cdot x_{i,parallel} \cdot \begin{bmatrix} MaintenanceCosts_i(year\ n) + \%ofInitCostsOC_i(year\ n) + FixedOC_i\ (year\ n) + \\ + FuelCosts_i(year\ n) + HoursOn\_CyclingOC_i(year\ n) + Admin(year\ n) + \\ + ReplacementCosts(year\ n) + OverhaulCosts(year\ n) \end{bmatrix}$$

Equation 12: Operation costs in a hybrid system

As is implicitly reflected in Equation 12, component sizes impact on the operation costs. The operation costs are determined yearly or at any other suitable time interval and are then discounted for the project life. The annual operation costs are sometimes obtained differently for the different contributions. If a diesel generator is part of a hybrid system then the incurred fuel costs are summed up for a year and can then be easily discounted over the whole project life. Each occurrence of maintenance, overhaul and replacement of a component in every year during the project life is determined, costed and then discounted corresponding to the number of years from the beginning of the project.

Every contribution to the operation costs can have fixed costs such as transport or labour. For example, maintenance costs per year are often given as a percentage of the initial component or system capital costs.

$$MaintenanceCosts_i(year\ n) = FixedMaintCosts(year\ n) + \%ofInitCosts(year\ n) + \\ + CostHoursComponentOn(year\ n)$$

Equation 13: Maintenance cost components

In some cases, a certain estimated percentage of cost is added to the overall operation costs, for example, 10% may account for oil costs, transport and labour (Morris 1994). The overall operation costs incurred over a project life are then given by Equation 14.

$$OperationCosts = \sum_{\forall i\ components} OpCosts_i + \%of_2 \cdot \left(\sum_{\forall i\ components} OpCosts_i \right)$$

Equation 14: Added operation cost margin

The following sections will describe operating costs of individual components.

4.2 PV operation costs

PV maintenance costs are often collected in monthly payments. These payments can cover system inspection by a maintenance person. The PV panels are in many cases assumed to have a lifetime of 20 years. So in this case, if a project life of up to 20 years is chosen, no PV panel replacement costs occur. Some of the factors influencing operating costs are reduced panel lifetimes (due to vandalism, corrosion and other factors) and reduced panel output power (due to dusty panel surfaces, corrosion, shading by trees and other factors). Monthly maintenance costs in South Africa can range from R5 (SHS)–R150 per month (clinic, school systems) or more, depending on system sizes. Some maintenance costs can be assumed to increase with the PV array size.

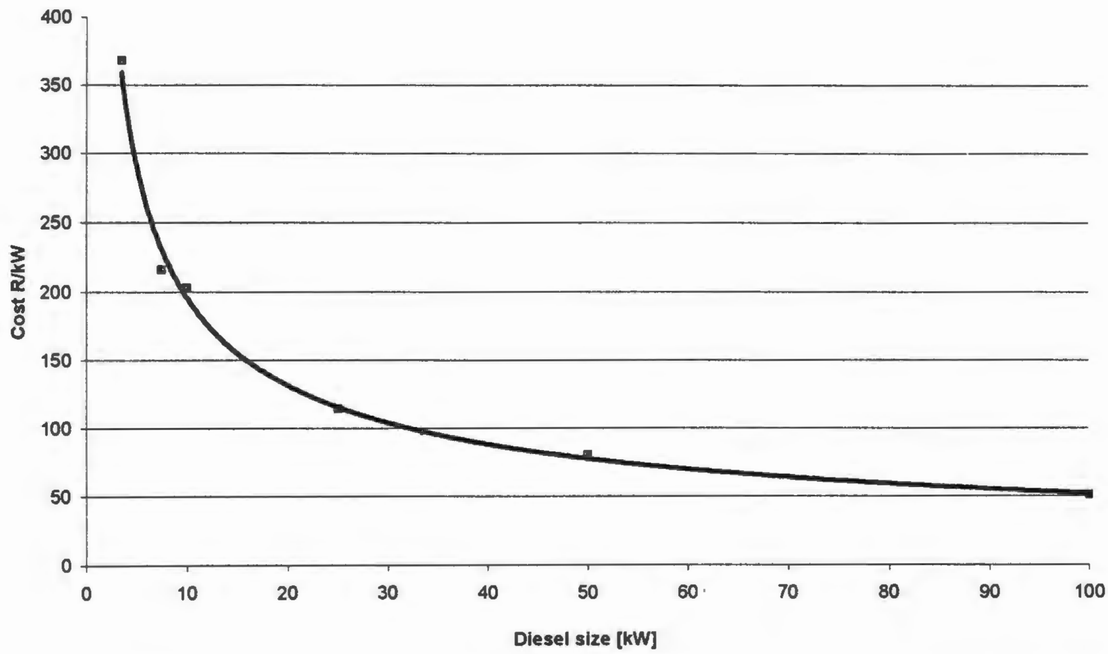
4.3 Wind turbine operation costs

Wind turbine operating costs comprise maintenance and replacement costs. The wind turbine life is often assumed to be more than 20 years, and therefore in many life-cycle costings no wind turbine replacement will take place. Maintenance is carried out after a certain amount of run time. Some larger wind turbines are inspected every six months. Maintenance costs for wind turbines can vary depending on the application, type of maintenance and wind turbine sizes. For smaller wind turbine sizes, they are sometimes assumed to be roughly in the range of PV maintenance costs.

4.4 Diesel generator operation costs

The diesel generator operating costs comprise fuel costs, and maintenance, overhaul and replacement costs. The fuel costs occur whenever the fuel storage tanks are refilled. Overhaul is assumed half way through the diesel lifetime. Replacement occurs after a certain number of run-time hours. The effective lifetime of a diesel generator is defined by the time until a mechanical overhaul becomes uneconomic (that is, when overhaul costs exceed 60% of the replacement costs). Factors that affect lifetime include the quality and regularity of maintenance, the average capacity factor and the number of start-ups. An air-cooled machine is likely to have a shorter life than a water-cooled unit that can keep the operating temperature down. Maintenance costs include a lubrication service around every 250 hours, an oil change and a tune-up every 500 hours, and a decoking service every 2 000 hours. Maintenance costs depend partly on the diesel generator size. Figure 10 depicts a maintenance estimate for different diesel sizes. The lower curve depicts the costing per 500 hours of maintenance intervals into which all other maintenance services have been averaged.

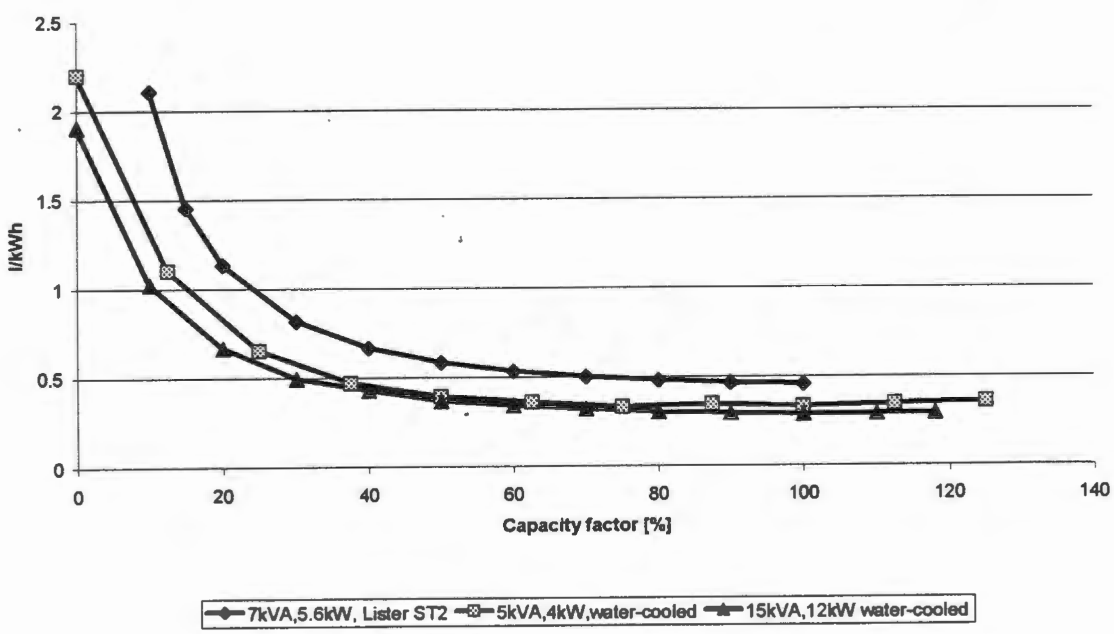
Figure 10: Diesel generator maintenance costs in R/kW nominal capacity for 500hours maintenance intervals



The lifetime of a diesel generator varies between roughly 20 000 to 40 000 hours of run time. On average 30 000 hours can be assumed. Sometimes the total hours of run time are assumed to increase with larger diesel sizes (Davis 1994). Some studies suggest that the diesel generator lifetime decreases with increased number of diesel generator cold starts, while other findings suggest that this might not have a strong impact.

The diesel fuel consumption varies according to generator size and loading factor (Figure 11), and is non-linear.

Figure 11: Diesel generator fuel consumption in l/kWh as function of capacity factor

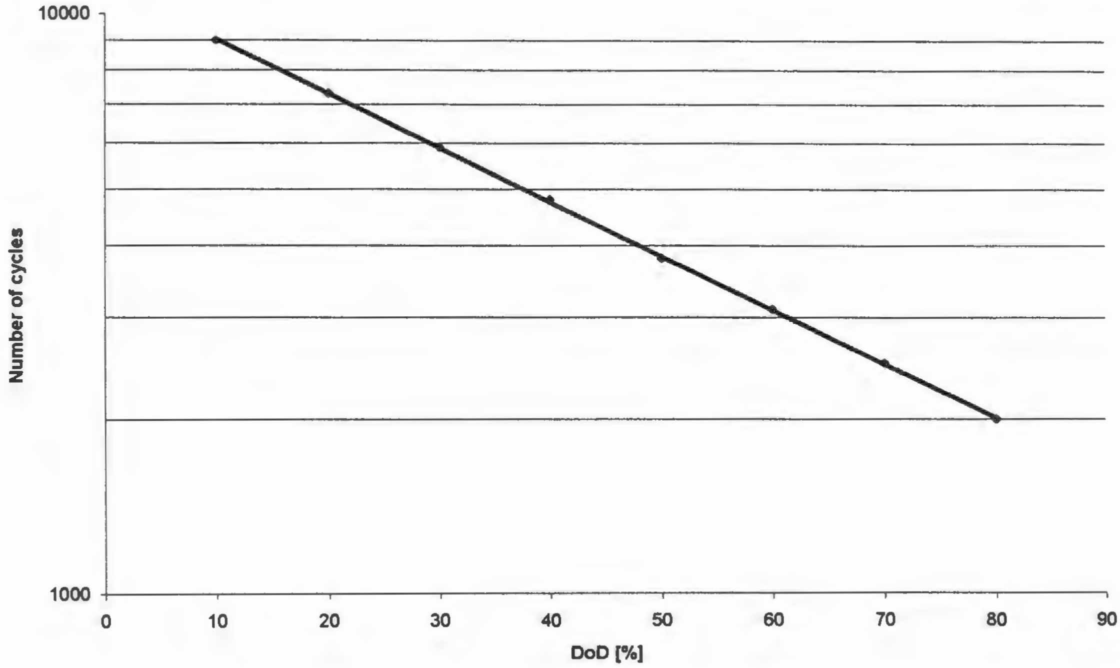


4.5 Battery operation costs

Battery operation costs comprise expenses for maintenance and replacement. Maintenance includes checking the battery electrolyte levels (Purcell 1991). Battery maintenance costs are often included in maintenance costs of the overall system or individual electricity generators.

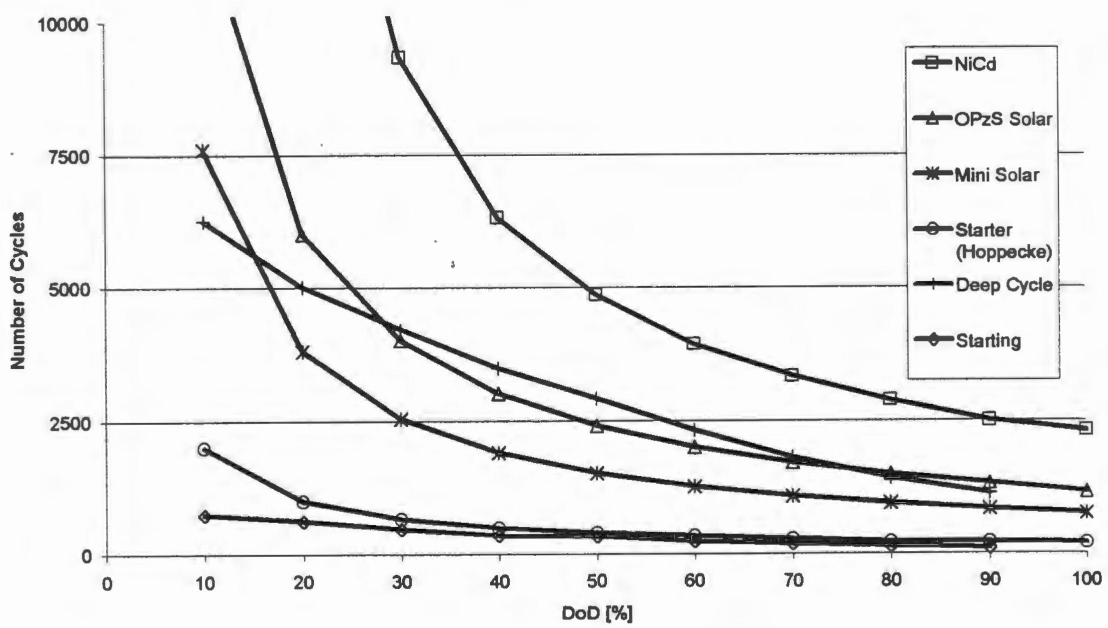
Battery lifetime is rated on the basis of the charge-discharge cycles obtained during laboratory bench tests (Purcell 1991). Most conventions specify cycle life as the number of cycles a battery attains at the specified depth of discharge (DoD) before its capacity is reduced by wear and ageing to 80% of the rated value. Some manufactures, however, specify cycle life as the number of cycles to 50% loss of capacity. A typical battery life-cycle curve is shown in Figure 12 as a function of the average DoD assumed during system operation.

Figure 12: Typical diagram on number of battery full cycles versus depth of discharge



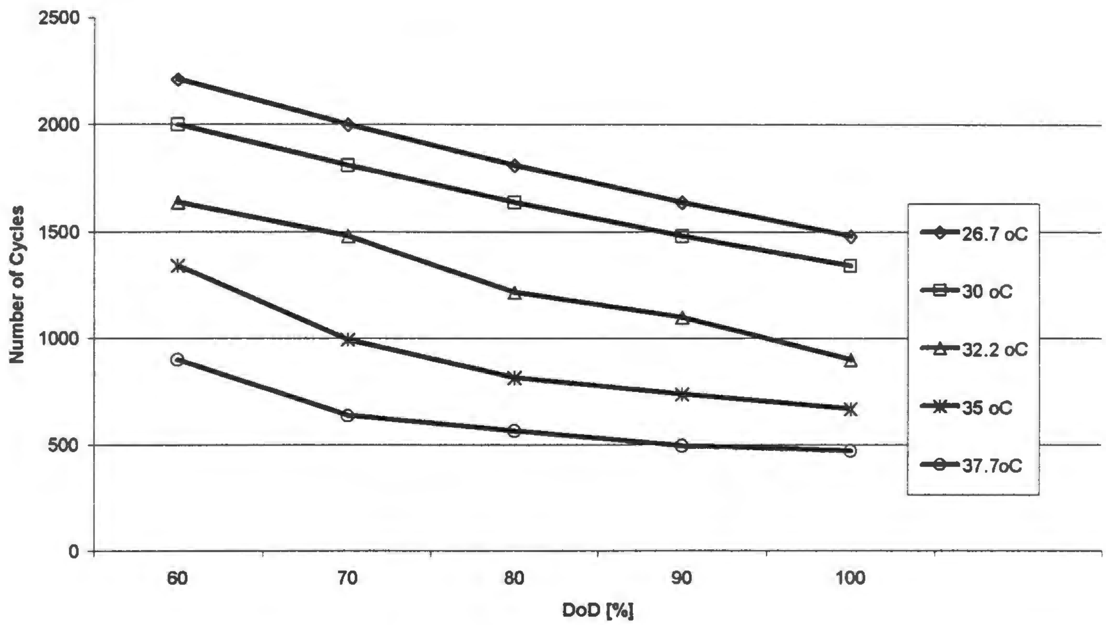
Battery cycle life as a function of depth of cycling is most often portrayed as a straight line on a semi-logarithmic graph (Purcell 1991). Curves for various lead-acid types tend to conform to this assumption, as is shown in Figure 13 (Barley et al 1995). The cycle life also depends on temperature (Figure 14).

Figure 13: Number of full cycles versus depth of discharge for various battery types



The actual battery life may be longer or shorter than the laboratory tests due to operation conditions which cannot be modelled with the laboratory test conditions. In laboratory tests the battery is usually operated for the whole of its lifetime such that it is fully charged and then fully discharged. During real system operations, batteries are to a large extent partially cycled.

Figure 14: Effect of temperature on number of full battery cycles at a certain DoD



It is difficult to account for the wear in partial cycling. Purcell (1991) indicates that the incremental wear from partial cycling between DoD1 and DoD2 can be modelled as

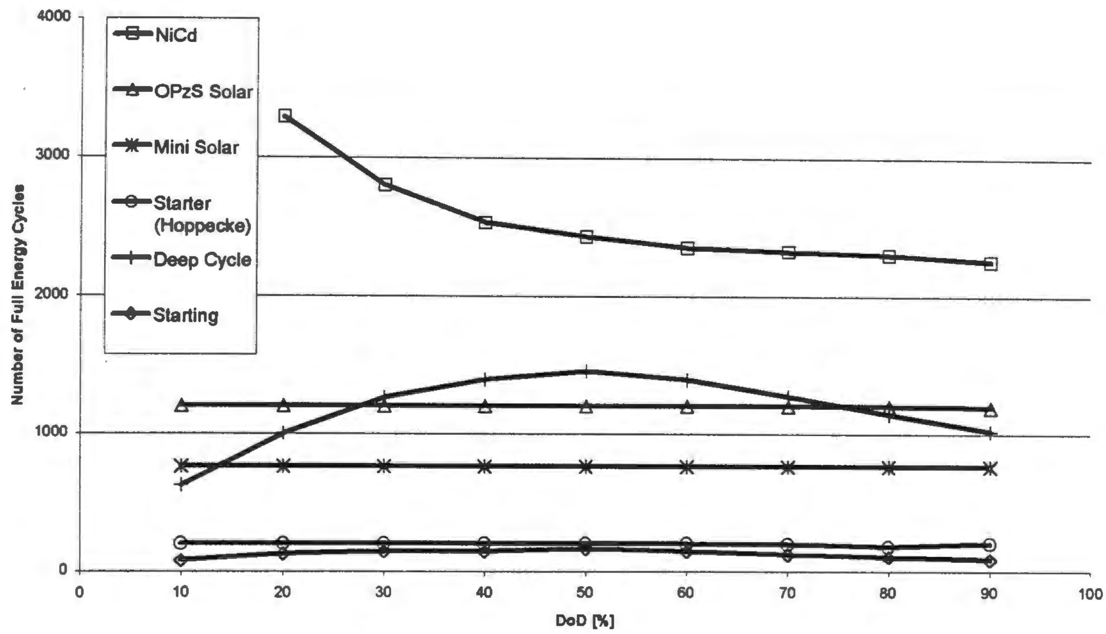
$$\Delta W = \frac{\left(\frac{1}{L_{DoD,1}} + \frac{1}{L_{DoD,2}}\right)}{2}$$

Equation 16: Estimate of wear in partial cycling

where $L_{dod,i}$ is the expected cycles life at a certain DoD. This approach accounts for both cycling wear and a penalty for partial state of charge (SOC) operation, though the weighting seems to be arbitrary (Purcell 1991).

In general, estimating battery lifetime according to its operation is difficult. Some approaches attempt to construct the number of full battery cycles by adding the energy cycled during partial battery cycles. Here it is useful that on average the energy that is taken out of the battery during its lifetime is almost constant and independent of the depth of discharge between 20% and 80% DoD. This can be seen in Figure 15 which shows the number of full energy cycles versus the depth of discharge.

Figure 15: Full cycles of energy versus battery depth of discharge



Therefore the amount of Ah or Wh that can be stored and taken out of a battery during its lifetime is obtained by multiplying the averaged full cycles of energy with the nominal battery capacity rated in Ah or Wh.

$$Ah_{BatteryLife} = C_{nom} \cdot \overline{\sum_{i=20\%}^{80\%} DoD(i) \cdot NoCycles(i)}$$

Equation 23: Average battery energy that can be taken out during its lifetime

The advantage is that this expression is nearly independent of the system depth of discharge. This is useful as many different depths of discharge are encountered in hybrid system operation. However, the effect of partial cycling on the amount of energy that can be taken out during the battery's lifetime is not explicitly addressed.

If the discharge taken out of the battery over a year is equal to or less than the charge put into the battery, then the battery life time in years is obtained as

$$BatteryLifeInYears = \frac{Ah_{BatteryLife}}{\sum_{1\ year} I_{discharge}(t) \cdot \Delta t}$$

Equation 24: Battery life in years

Equation 24 calculates the number of times the required yearly discharge can be obtained out of the estimated energy available from the battery.

To ease the following calculations an abbreviation for the discharge and charge taken out of a battery over a year is introduced in Equation 25.

$$Disch\arg e = \sum_{1\,year} I_{disch\arg e}(t) \cdot \Delta t$$

$$Ch\arg e = \sum_{1\,year} I_{ch\arg e}(t) \cdot \eta_{Bat,Ch\arg e}(t) \cdot \Delta t$$

Equation 25: Charge and discharge during one year of battery operation

If the discharge taken out of the battery during a year of system operation is greater than the charge put into the battery, then this level of discharge can only be sustained for the number of years given by $BatteryLifeInYears_{Dh\geq Ch}$ in Equation 26.

The calculation of $BatteryLifeInYears_{Dh\geq Ch}$ derives from the fact that the system design is based on unchanged PV, wind, and diesel energy contributions and unchanged demand requirements in following years. Therefore the system with its specified sizing and operation strategy relies on the yearly amount of battery discharge $\Sigma I_{Discharge}(t)\cdot\Delta t$. Otherwise the demand will not be met as it was in the initial year, and therefore the system reliability measure will be lower. Alternatively, the life-cycle costing will change because more energy from other resources needs to be made available.

$$BatteryLifeInYears_{Dh\geq Ch} = \frac{SOC(t_0) - SOC_{min}}{\sum_{1year} I_{Disch\arg e}(t)\cdot\Delta t - \sum_{1year} I_{Ch\arg e}(t)\cdot\eta_{Bat,Ch\arg e}(t)\cdot\Delta t}$$

Equation 26: Battery life in years, if the yearly discharge is greater than the yearly charge

Therefore Equation 26 equals number of years in which the full required discharges at the level $\Sigma I_{Discharge}(t)\cdot\Delta t$ are possible. Theoretically the battery needs to be replaced after $BatteryLifeInYears_{Dh\geq Ch}$,, in order to cover the demand as in previous years, because the PV, wind, diesel resources will not be enough to cover it in a system design that was based on discharging the battery to this level. That means if Equation 24 and Equation 26 are implemented in an optimisation simulation, the resulting costs will be very high, and the corresponding solution might be a very bad one and will be thrown out when iterating the developed algorithm.

If the battery is not replaced in year $BatteryLifeInYears_{Dh\geq Ch}$,, and the system energy sources and demand requirements stay the same, then the battery will last $BatteryLifeInYears_{Dh\geq Ch',Unreplaced}$, with the effect, however, that demand will be increasingly less met.

The year after $BatteryLifeInYears_{Dh\geq Ch}$,, the battery can only deliver part of the required discharge, namely

$$Disch\arg e_{InBetween} = [(BatteryLifeInYears_{Dh\geq Ch} - \lfloor BatteryLifeInYears_{Dh\geq Ch} \rfloor) \cdot (Disch\arg e - Ch\arg e)] + Ch\arg e$$

Equation 27: Discharge during the year in which the yearly discharge level cannot be met

After this, the battery discharges to the amount that it has been charged with as is described in Equation 28.

$$Discharge_{AfterThat} = \sum_{1year} I_{Charge}(t) \cdot \eta_{Bat,Charge}(t) \cdot \Delta t$$
$$= Charge$$

Equation 28: Discharge determined by charge levels

The overall discharge during the battery's lifetime will therefore be as given in Equation 29.

$$OverallDischargeDuringBatLife = BatteryLifeInYears_{Dh \geq Ch} \cdot Discharge +$$
$$+ Discharge_{InBetween} + y \cdot Discharge_{AfterThat}$$

Equation 29: Overall discharge during the battery life

Whereby y, the number of years the battery is discharged at its charging level before its lifetime ends, is determined by computing how many intervals it takes until the battery is depleted.

$$y = \frac{Ah_{BatteryLife} - (BatteryLifeInYears_{Dh \geq Ch} \cdot Discharge + Discharge_{InBetween})}{Charge}$$

Equation 30: Number of years discharge equals the charge level

Therefore in the case of continuing to use the battery in the hybrid system after $BatteryLifeInYears_{Dh \geq Ch}$ years, demand will not be satisfactorily met and replacement occurs after $BatteryLifeInYear_{Dh \geq Ch,Unreplaced}$ years.

$$BatteryLifeInYears_{Dh>Ch,Unreplaced} = BatteryLifeInYears_{Dh \geq Ch} + 1 + y$$

Equation 31: Battery lifetime in number of years if the yearly discharge is greater than the yearly charge and battery is used until its average available energy is used up

In case a shorter time horizon than one year is chosen in Equations 26–31, say one winter month, adjustments can be made to account for increased charging during the summer months or when running the diesel generator a few extra times to charge the batteries.

Piller (1997) attempts to trace battery degradation in detail by describing the impacts of battery operation on degradation with fuzzy intervals and then superimposing the different fuzzyfied degradation contributions according to rules derived from collected battery operation experiences. Other models use a $/kWh cost description for energy taken out of the battery during system simulation, and avoid the charge-discharge ratio problem.

4.6 Balance of system operation costs

Other operating costs such as wear and tear and replacement of inverter, battery chargers, controllers and other BoS parts should be taken into account as well. Some designers calculate the hourly costs of these components based on the invested component costs. Others include these expenditures as fixed or proportional costs in the component costing or overall system costing.

4.7 Overall life-cycle costs

The hybrid system operation costs are in general non-linear. As they depend on future operations, they can only be estimated roughly. The operation costs depend largely on component size and type, and the way the system is operated.

In order to demonstrate the high level of non-linearity of operation costs in general, which make an optimisation algorithm necessary, and the unwieldiness of spreadsheet methods, a few examples of overall life cycle costing of hybrid systems using specially created spreadsheet calculations are presented in Figure 16 and Figure 17. Thereby, operation costs had to be expressed through fitted equations that were specifically developed for that purpose. However, using fitted equations does not give accurate operation costs for a component, as the fit might have been very bad for a particular component, or not all available component types are included making the estimated fit unreliable.

Figure 16 and Figure 17 clearly show the impacts of non-linear initial and operation costs, which result in non-linear objective function (i.e. life cycle costing) shapes. The life cycle costs and estimated energy supply were calculated for nearly hundred different hybrid system component combinations based on three differently sized PV arrays and four differently sized wind turbines and diesel generators. The required battery storage was estimated based on these combined component sizes. The shortcomings of such spreadsheet methods are clearly highlighted through these examples:

- the spreadsheet calculations involve a large number of cell calculations, that can become difficult to track
- the different scenarios can only be calculated using weather and demand data for an average day
- fixed estimates of diesel generator runtime hours per day are necessary
- the lifetime of the battery can only be estimated but not be made dependent on actual operation
- only an extremely limited number of combinations of weather conditions, component sizes, daily generator runtimes and average capacity factors can be analysed in the spreadsheet model due to the complexity involved in tracking calculations

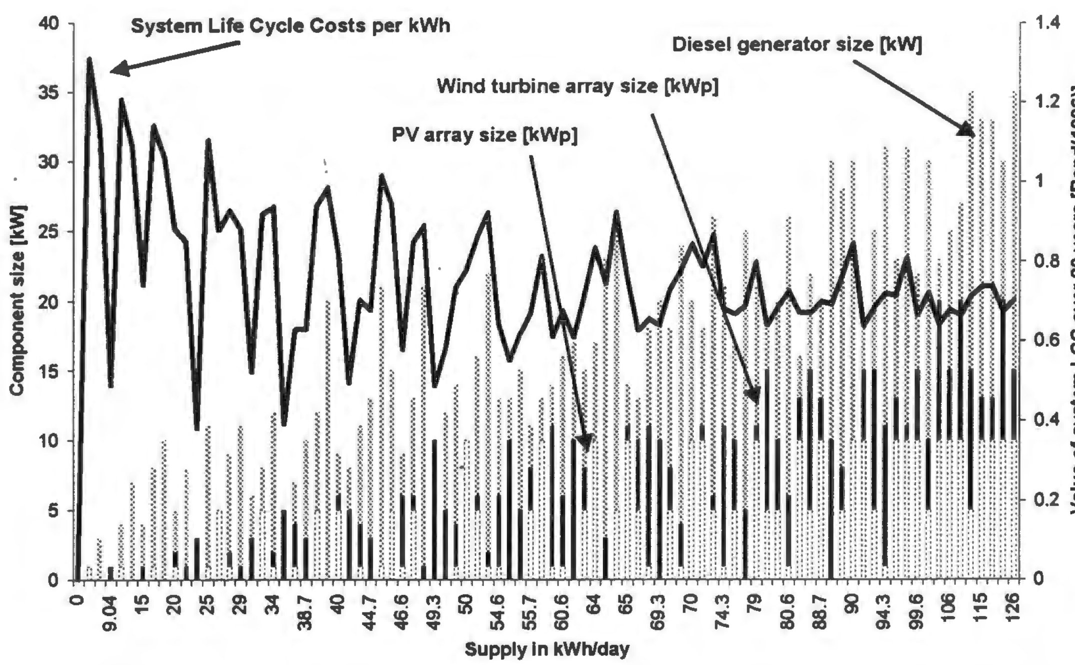


Figure 16: Life-cycle costs of single source and PV/wind/diesel hybrid systems, average diesel generator runtimes 2hrs/day

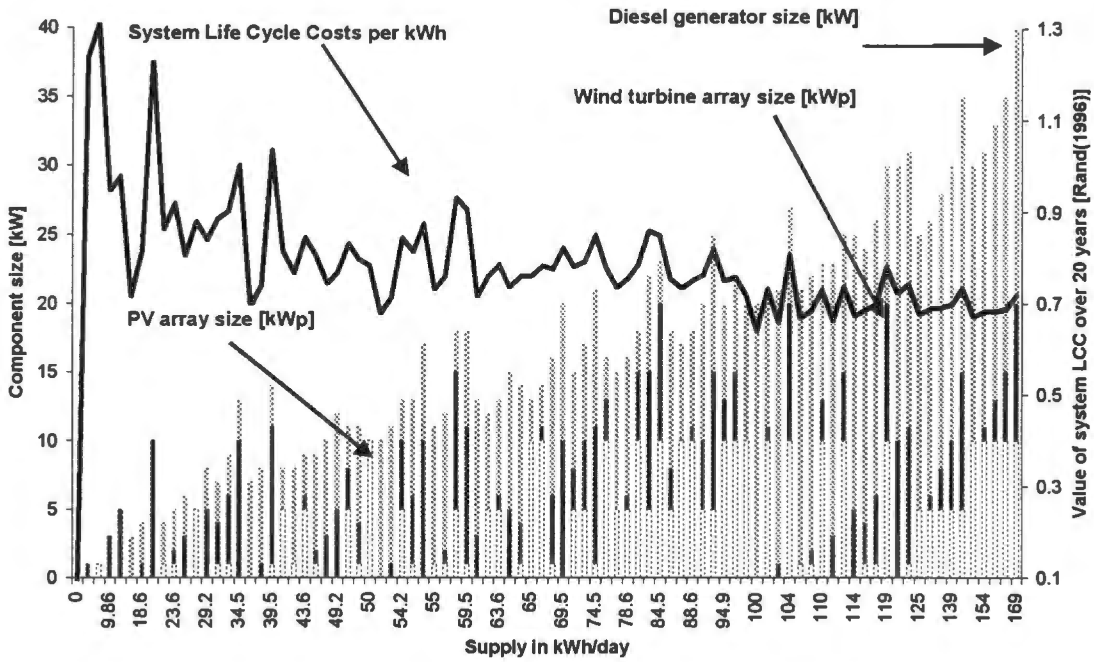


Figure 17: Life-cycle costs of single source and PV/wind/diesel hybrid systems, average diesel generator runtimes 5 hrs/day

It can be seen that a computer-based optimisation algorithm is necessary to sufficiently investigate different sizing and operation design choices. In the developed algorithm, a computer program carries out all the required iterative tasks and the component costs are entered for each component specifically instead of using fitted equations. Thereby a component database is built up with specific characteristics for operation and costing of each entered component.

5 Quantification of benefits

A benefit can be defined in a number of different ways. It can range from purely technical quantification such as the reliability of electricity supply, to detailed socio-economic descriptions such as how the introduction of electricity improves the degree of education and overall living standards.

Benefits in this design work are quantified as the percentage of demand covered by the electricity generated from the hybrid system. The benefits are an important tool enabling the evaluation of the trade-off between lowest cost against highest benefit designs (Marrison & Seeling-Hochmuth 1997). In this regard over-sizing a system ensures a low probability of loss-of-power but causes an excessive capital cost, while under-sizing a system minimises the capital cost but causes frequent loss-of-power. By quantifying benefits for the given application, it is possible to match the benefit of reliable power with the cost of avoiding a loss and thereby find the optimal design for each application.

As with costs, the total benefit can be defined in terms of a net present value.

$$NPVBenefits = \sum_{i}^{NOofBenefitTypes} Discounted\ Benefit_i$$

Equation 32: Net present value of benefits

$$Discouned\ Benefit_i = \sum_{year\ n}^{ProjectLife} \frac{Benefit_i(n)}{(1+r)^n}$$

Equation 33: Discounting benefits

For commercial applications, such as telecommunications relay stations, or a refrigeration plant, the financial benefit of powering the application can be obtained by considering the resulting operating revenue. Similarly, loss of benefit caused by system failures can be quantified in terms of lost revenue, damage caused, or the cost of making alternative arrangements. In this case, an event in the simulation which causes a loss of power will generate a change in cash flow in the NPV as given by Equation 7.

More complex functions can also be implemented. The loss of power at one time step, for example, may reduce revenues at all other time steps because an unreliable service is less valuable.

The benefits in domestic or non-revenue generating applications are more difficult to quantify. The reliability of the electricity supply for a domestic application and its value to the user cannot in most cases be measured in terms of revenues lost and costs incurred due to power losses.

One way to obtain a measure of supply reliability can be to introduce a weight in the cost function that is inversely proportional to the level of supply reliability. Therefore low reliability would have a high weight factor resulting in high costs and high reliability would have a low weight factor resulting in low costs. This weight function is often called "penalty function", as it penalises undesirable outcomes and unmet conditions, such as constraints placed on the system, with high costs.

Figure 18 and Figure 19 show how the penalty function works for the two previously discussed spreadsheet examples (Figure 16 and Figure 17). The system configurations are displayed along the kWh supplied. In Figure 18 the average diesel generator runtime per day is much lower while the renewable energy sources are higher than in Figure 19. Therefore the same system is supplying different levels of kWh/day in Figure 18 and Figure 19. Assuming a daily design demand requirement of 40kWh/day for both examples, the penalty function has a very high value for every system that cannot supply at least 40kWh/day. When adding this penalty value to the life cycle costs of various systems representing numerous levels of supply, a design estimate can be obtained through minimisation of the combined penalty and life cycle cost function as shown in Figure 20 (see also next section). Again the drawbacks of using spreadsheet methods can be easily identified. In order to truly optimise the system design for a given demand requirement, it is necessary to analyse many more combinations of component sizes and operating possibilities, especially around the supply level that matches the demand requirement. This can only be achieved through simulations given the amount of combinations that can produce supply levels that match the required design demand, and therefore the amount of iterations involved.

Figure 18: Penalty function for different single source and hybrid systems, average diesel generator runtime 2hrs/day, demand requirement of 40kWh/day

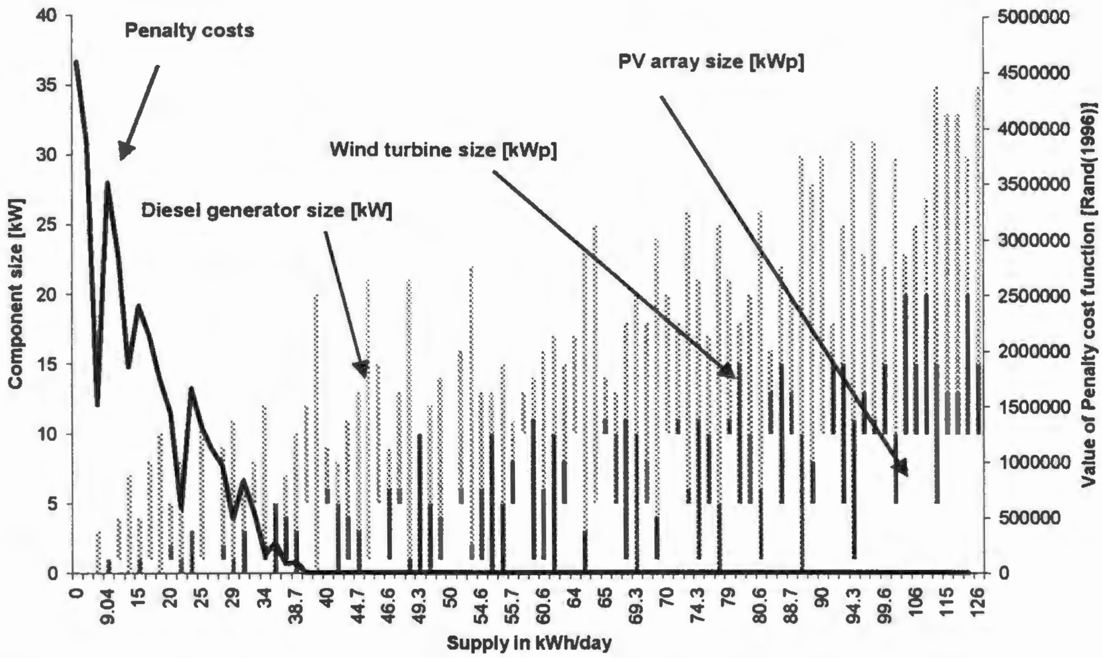


Figure 19: Penalty function for different single source and hybrid systems, average diesel generator runtime 5hrs/day, demand requirement of 40kWh/day

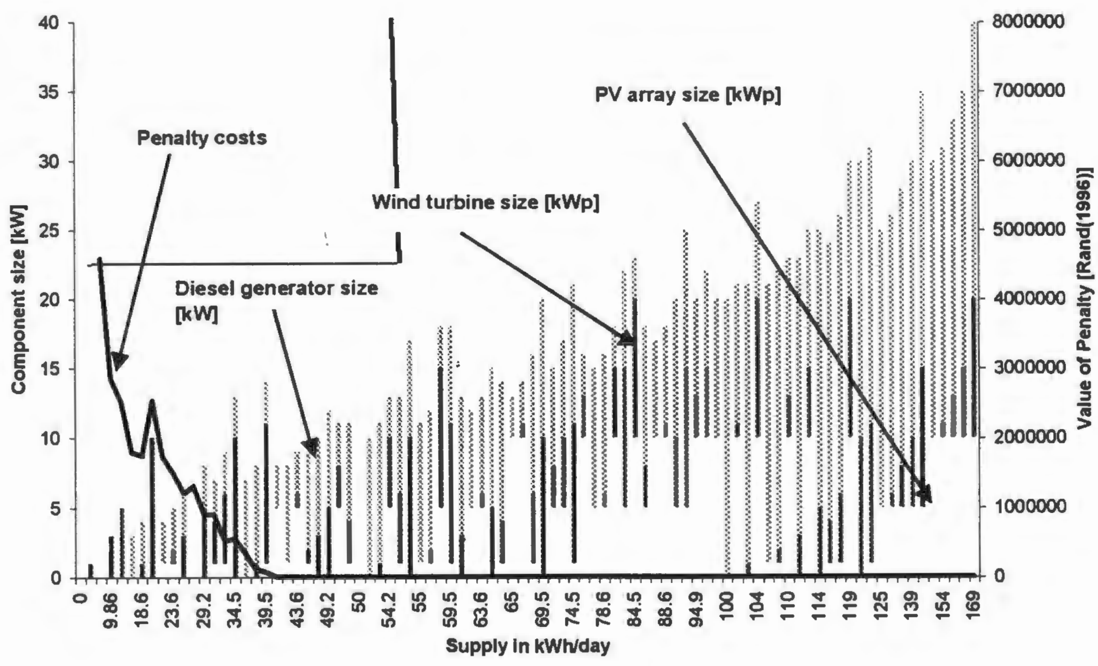
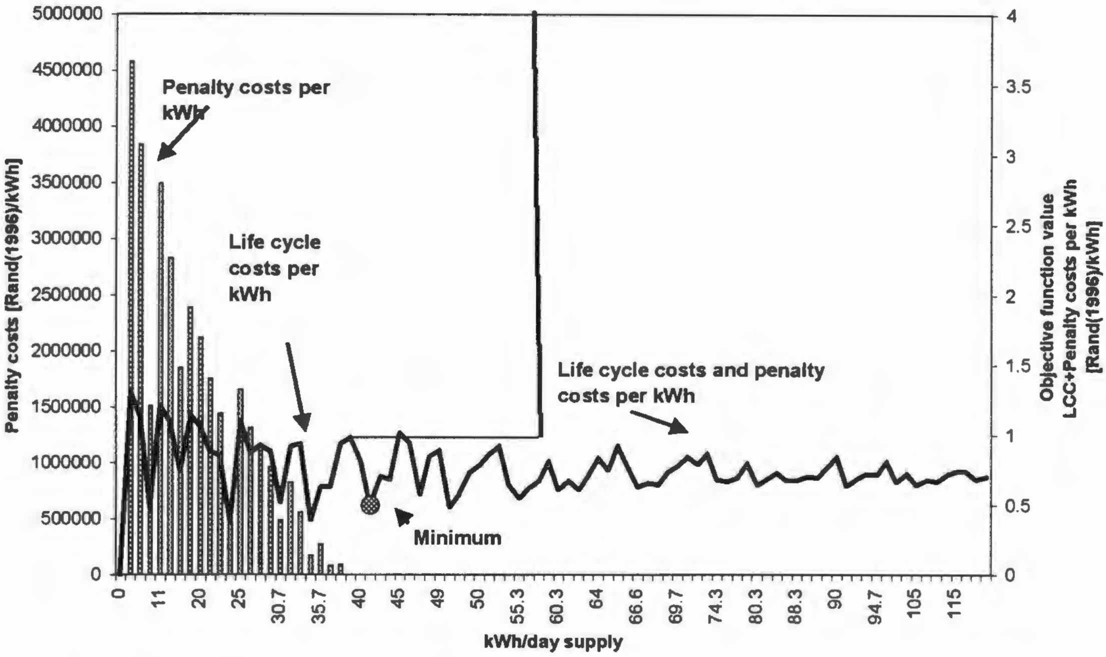


Figure 20: Penalty function scenario



In general, the penalty approach gives easily obtainable thresholds on the desirability of a design. However, it cannot readily be fine-tuned to account for more complex factors in the design, such as what the electricity is worth to a user, the environment, and the living standards of a community. These factors can be better accommodated in a more complex benefit analysis.

The benefit formulation is derived as follows: the marginal benefit of supplying power decreases as the supply increases because the initial supply is used for the most important services (for example, lighting) and further supply is used for services that could also be supplied by other sources (for example, heating). According to Marrison & Seeling-Hochmuth (1997), the marginal benefit of reliable supply can be approximated by Equation 34.

$$MarginalBenefit(t)_{Reliable} = BenefitperUnit \cdot Demand(t) \cdot \left(\frac{Supply(t)}{Demand(t)}\right)^{-W1}$$

Equation 34: Marginal benefit of meeting demand reliably

where BenefitperUnit is the marginal benefit per kWh supplied to the demand Demand(t). The variable BenefitperUnit can account purely for the technical reliability or also for the socio-economic worth of supplying a given demand. W_1 is a weight between zero and one. The total benefit of supply is the integral of Equation 34:

$$Benefit(t)_{Reliable} = BenefitperUnit \cdot Supply(t) \cdot \frac{1}{-W1} \cdot \left(\frac{Supply(t)}{Demand(t)}\right)^{-W1}$$

Equation 35: Benefit of reliable supply

Equation 35 quantifies the benefit derived from a reliable supply. If there is the possibility of a loss of power, this benefit will be reduced due to the unmet demand or the inconvenience in using back-up sources. In the case of a grey-out, that is, a supply less than demand, the following form can be considered:

$$Benefit(t) = Benefit(t)_{Reliable} \cdot \left(\min \left[1, \frac{Supply(t)}{Demand(t)} \right] \right)^{W2}$$

Equation 36: Overall benefit formulation

W_2 is a weight greater than zero and dictates the seriousness of a power loss. The minimum formulation in Equation 36 guarantees that the oversupply of electricity will not be counted as a benefit.

A power loss can be annoying or catastrophic depending on the application. For some applications such as irrigation the hourly reliability is not important and the weight W_2 will be close to zero. For sensitive applications, such as a hospital, any power loss will be serious and W_2 will be large.

Figure 21 and Figure 22 show again the two case scenarios of Figure 18 and 20, this time using benefit functions with different values for W_1 and W_2. Again, there are different average diesel generator runtimes per day, but the same demand and weather conditions are assumed. It can be seen that the shape of the benefit function depends largely on the values of W_1 and W_2. The value of the benefit function is highest or stays highest, depending on whether the system can supply the required demand level. In this example this is 40kWh/day.

Figure 21: Benefit functions with different values for W_1 and W_2 for given demand requirement of 40kWh/day, weather conditions and diesel runtime of 2 hours per day

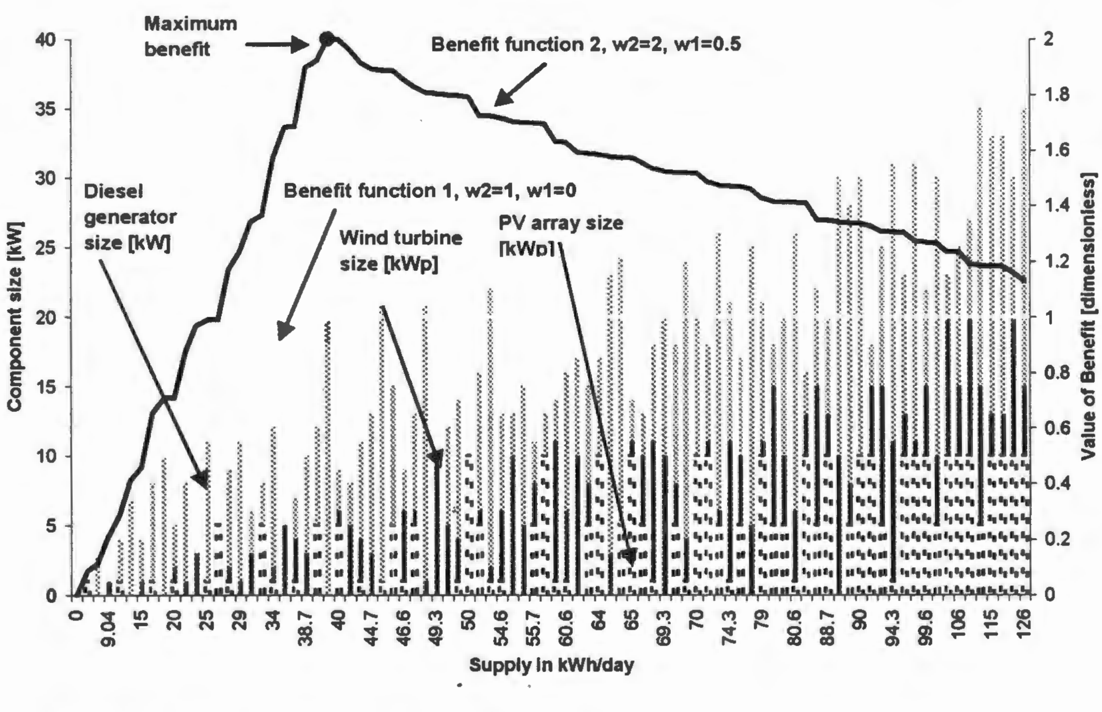
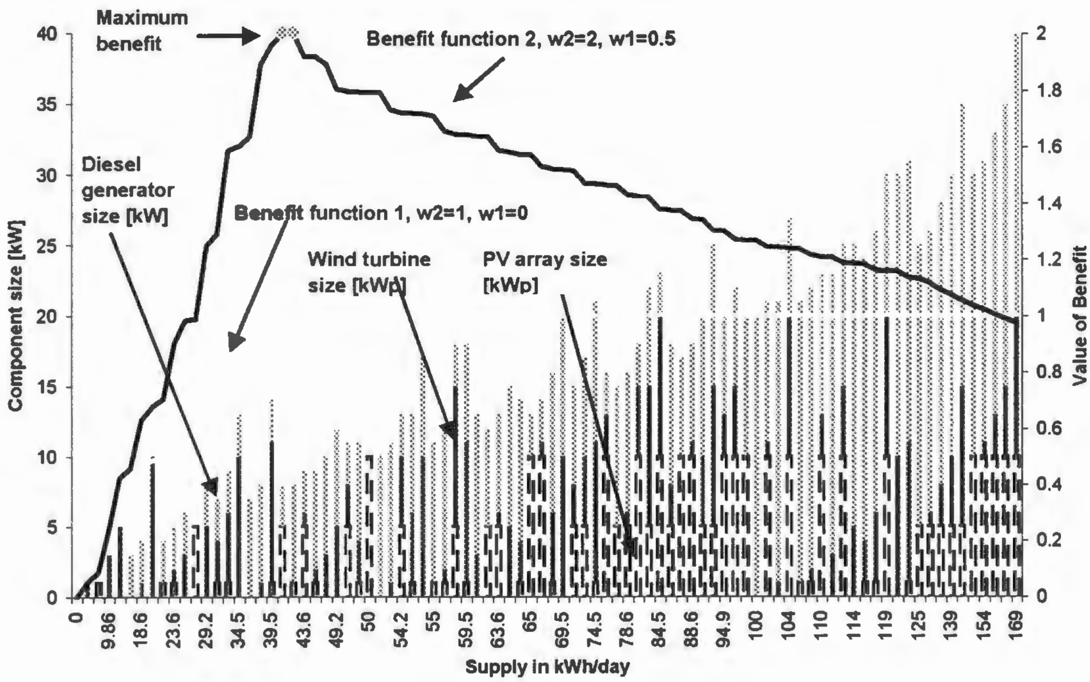
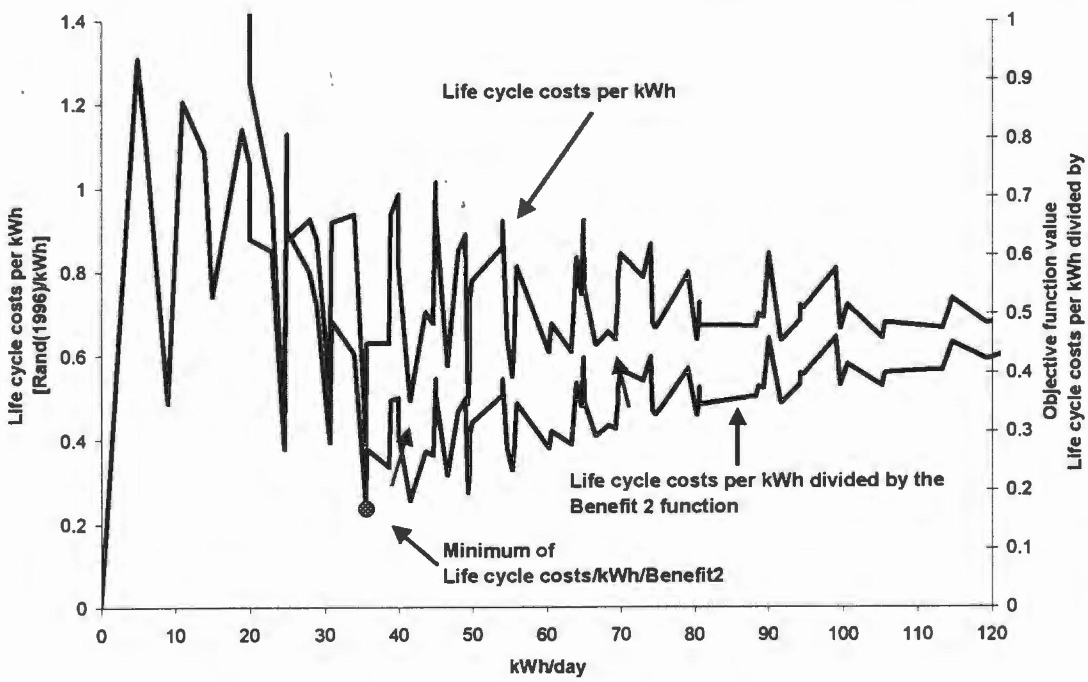


Figure 22: Benefit functions with different values for W_1 and W_2 for given demand requirement of 40kWh/day, weather conditions and diesel runtime of 5 hours per day



If the life-cycle-cost/benefit ratio is formed, the minimum of this function can indicate a recommendable system design (see also next section). Figures 24 and 25 show such estimated design results for the case scenarios depicted in Figure 21 and 23, using benefit functions shaped by the values of 0.5 and 2 for W_1 and W_2 respectively. The design recommended in Figure 23 does not meet the demand requirements, whereas in Figure 24 it does. As can be seen the benefit function can be fine-tuned to potentially accommodate different conflicting needs in the design. For example, the benefit function could recommend a system that was under-sized as in Figure 23, if another factor than meeting the load was more important.

Figure 23: Life-cycle cost-benefit description for 40kWh/d demand, weather conditions and diesel average runtime of 2hrs/day



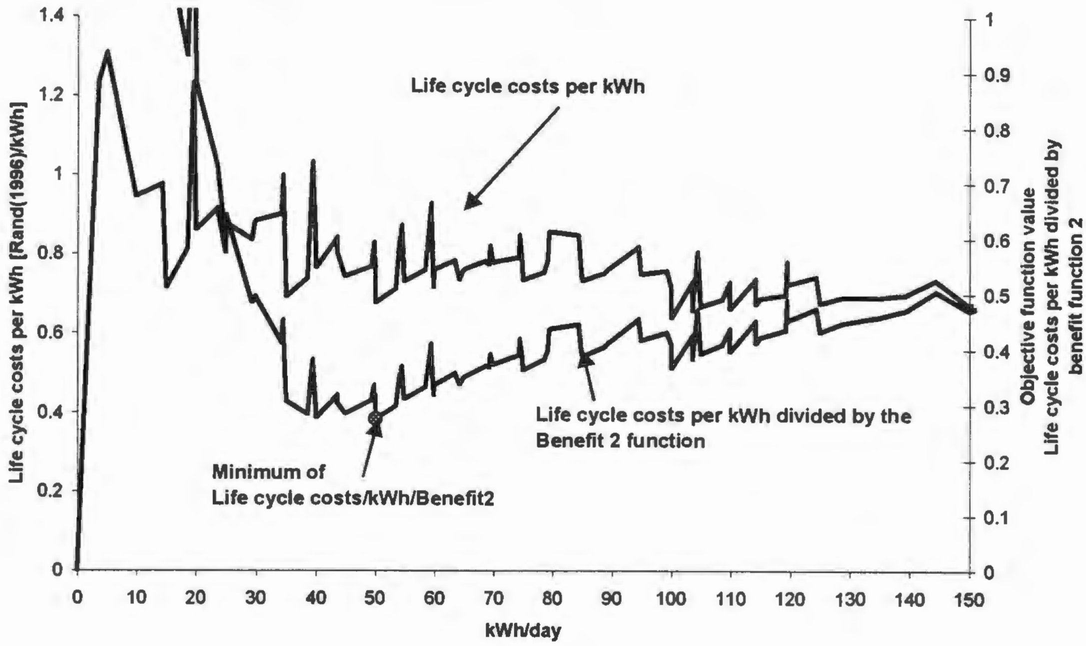


Figure 24: Life-cycle cost-benefit description for 40kWh/d demand, weather conditions and diesel average runtime of 5hrs/day

The developed algorithm uses the penalty function description to measure the reliability of electricity supply achieved with a hybrid system design. However, if more benefit measures are desired than just the technical reliability of the system, for example in assessing macro-economic and socio-economic impacts in an economic cost-benefit analysis, the described benefit methodology needs to be employed.

6 The objective function formulation

The objective function collects the different initial costs and discounted operation costs, which are incurred by a certain system design, in the system life-cycle costs (LCCs) as in Equation 5. In case income is generated by the application, the discounted flow of income during the project life can be subtracted from the incurred LCCs, giving the NPV in Equation 7. It should be noted that the discounted costs can also be subtracted from the discounted income, thereby changing the sign of the NPV value.

As discussed in the previous section, a measure that indicates the reliability of supply and other socio-economic benefits needs to be introduced. This can be done using the benefits or penalties as described.

The costing and penalty or benefit descriptions can be combined into a single measure of worth either by subtracting the benefits from the NPV (J_1) or dividing the NPV by the benefits (J_2), or by adding penalties to the NPV (J_3).

$$J_1 = NPV - NPVBenefits$$

Equation 37: Objective function formulation as difference between the NPV of design and NPV of corresponding benefits

$$J_2 = \frac{NPV}{NPVBenefits}$$

Equation 38: Objective function formulation as division of the NPV of the design by the NPV of corresponding benefits

$$J_3 = NPV + Penalty$$

Equation 39: Objective function formulation as adding a penalty description to the NPV of the design

The objective is to minimise the overall costs. If income is positive and expenditure is negative than the objective would be to maximise the overall objective function. Davis & Horvei (1995) point out that J_2 should be used if there are multiple projects under consideration. J_2 also has the advantage that only the relative shapes of the cost and benefit functions are important (Marrison & Seeling-Hochmuth 1997). When comparing two designs, the absolute value of the benefits is not important. This is a useful feature given the difficulty in accurately quantifying the benefits.

Figure 23 and Figure 24 show cost/benefit scenarios as obtained by using J_2.

The overall objective function for J_2 can be given as

$$J_2(size_{componts}, number_{components}, operation_{components}) = \frac{1}{\sum Supply(size_{comp}, operation_{comp}, weather_{site})} \cdot$$

$$\frac{\sum_{\forall components\ i} (InitialCosts_i(size_{Comp_i}) + OpCosts_i(size_{Comp_i}, characterticts_{Comp_i}, operation_{Comp_i})) + \sum Income}{BenefitperUnit \cdot (\max[1, (\frac{\sum Demand}{\sum Supply(size_{comp}, operation_{comp}, weather_{site})})^{(W_1 - W_2)}])}$$

Equation 40: Full formulation of objective cost-benefit function J_2

The maximum formulation in Equation 40 again guarantees that the over-supply of electricity is not counted as a benefit. It is up to the designer to include this restriction on assigning a benefit to oversupply or not.

The penalty description in the objective function as in J_3 was demonstrated in Figure 20. J_3 is given as

$$\begin{aligned} J_3(size_{componts}, number_{components}, operation_{components}) = \\ \sum_{\forall components\ i} (InitialCosts_i(size_{Comp_i}) + OpCosts_i(size_{Comp_i}, characterticts_{Comp_i}, operation_{Comp_i}) - \\ - \sum_{\forall incomesouces\ j} Income_j + Penalty(unmet\ demand) \end{aligned}$$

Equation 41: Full formulation of cost-penalty function J_3

As mentioned before, J_3 will be employed in this approach as only the technical reliability and not additional socio-economic benefits are analysed. Therefore the cruder but more easily implementable penalty measure is used.

As can be seen the objective function, whether J_1, J_2 or J_3, depends, apart from demand and weather conditions, on the component sizes, the operation employed and the often intricate characteristics of the components.

The objective function is optimised if optimum values for the component sizing and operation strategy can be found that minimise the objective cost function without violating any system constraints.

This optimisation problem and the constraints placed on the system are in almost all cases non-linear. In addition, there is often the need to solve a sub-optimisation problem to find the right operating strategies for a chosen component configuration. Therefore the task of optimising the hybrid system design is difficult to solve and requires intensive computation. The performance modelling section will describe the developed hybrid system model on which the calculation of the values of the sizing and operation costs in the objective function are based.

7 Summary

The hybrid system costing section developed the net present value costing as an objective function description in dependence of decision variables to be optimised by an optimisation algorithm as developed in this design work. The net present value formulation is highly non-linear and complex, and cannot satisfactorily be solved by spreadsheet evaluation methods. The optimisation aims to choose a least cost alternative with lowest present value of total financial or economic costs, when discounted at an appropriate opportunity cost, and greatest benefit design with the highest socio-economic impact.

The initial costs in the objective function description need to be determined carefully and a variety of component types and cost figures exist. It is important to notice that for the majority of components an economy of scale exists, not only for the capital component costs but also for costs of installation and accessories. The actual non-linear relationship between component sizes and their costing is difficult to determine and can only be approximated. Very different experiences which are often country-dependent, complicate the costing of expenses of installation and system accessories.

Hybrid system operation costs are complex and highly non-linear, and can only be roughly estimated.

It has been shown that the operational costing of components depends largely on their size, their type and brand, and the way they are operated. Using the fitted equations will not give accurate operation costs for a component, as the fit might have been very bad for a particular component, or not all available component type are included making the estimated fit not universally reliable.

Therefore, in the developed algorithm, the component costs are entered for each component specifically instead of using fitted equations. A component database is thereby built up with specific characteristics for operation and costing of each entered component.

It can be seen that the diagrams derived from the rough spreadsheet analysis (Figures 10, 17-25) give some rough design and cost estimates. The spreadsheet calculations are an improvement over rough rules-of–thumb methods, but will still be inaccurate due to

- Weather and demand estimates based on one typical day only
- Fixed "diesel-run-time" per day regardless of weather, needs, variations
- Assumption of one single average diesel capacity factor
- Assumption of average efficiencies of conversion elements and batteries
- Assumption of component replacement after a fixed number of years for the battery independent of actual use
- Only an extremely limited number of combinations of weather conditions, component sizes, daily generator runtimes and average capacity factors can be analysed in the spreadsheet model due to the complexity involved in tracking calculations.

In addition, the spreadsheet analysis is cumbersome and can get out of control due to the high number of parameters that can be varied in the design.

The optimisation algorithm as developed and described in this research work improves this situation. This is due to the fact that:

- The demand requirements and local weather can be simulated for a period of time
- Components operate at various capacity levels to meet the demand based on 10 minutes to 1-hour intervals

- The different component efficiencies encountered in inverter losses, battery charging losses, fuel efficiency, renewable energy production are taken into account during the simulation
- Component maintenance, overhaul and replacement needs can be determined more accurately based on actual operation
- The impact of different control strategies can be evaluated
- The overall life cycle costing is more accurate.

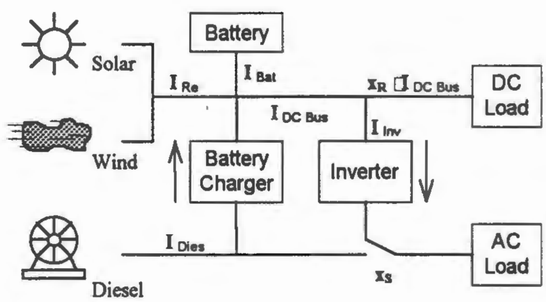
Part 2: Hybrid system performance modelling

1 General

The hybrid system model described in the following sections is the core of the simulation. Apart from correct costing and optimisation, the quality and accuracy of the model and its implementation in the algorithm, greatly determines the usefulness of the simulation results.

The system model is based on a description of current flows through the system which depend on the system design decision variables to be optimised and include efficiency losses and other descriptive design parameters (see Figure 1).

Figure 1: Basic hybrid system set-up



The decision variables are component sizes, number of components, and operation control settings which then determine the amount of diesel output power and amount of battery output. The decision variables are optimised by the algorithm in such a way that minimum life-cycle costs are achieved subject to satisfying demand.

The estimated power consumption of the appliances utilised should be given in one hour to 15 minutes intervals, for the length of a year, possibly with estimations of likely demand changes.

2 System component models

2.1 Renewable energy components

The simulation determines how many renewable energy components are needed in parallel and in series, their current outputs and power outputs as well as the component initial, operation and replacement costs.

With weather data from the site, component output characteristics and installation details like angle and height, the energy output can be calculated based on the models explained in Schuhmacher (1993) and Dijk (1996).

2.2 PV module model

The model of the PV module consists of two parts: an electrical model and a thermal model based on an energy balance. The model is described in Appendix A: A1.

Manufacturers give the characteristic curves for their PV modules as I-U curves with irradiation and temperature as parameters, and for wind turbines as power output versus wind speed respectively. In the design tool the energy output and current output of each renewable component for each time instant is then computed, based on the local weather conditions.

2.2.1 PV sizing variables

The PV sizing variables to be optimised are the size of a PV panel and the number of strings in a PV array. The necessary number of PV panels to be connected in series is derived by the number of panels needed to match the bus operating voltage ($U_{Bus,Nom}$). In most cases DC PV panels will be used. The operating voltage will therefore be the one of the DC bus[1].

$$n_{PV,series} = U_{Bus,nom} / U_{panel,nom}$$

Equation 1: Number of required PV panels in series

where $U_{Panel,Nom}$ is the PV panel voltage and $n_{PV,series}$ is the number of PV panels in series.

The number of PV panels to be installed in series is therefore not subject to optimisation but is a straightforward calculation. When matching the current requirements of the system, several PV strings, each consisting of $n_{PV,series}$ panels connected in series, need to be installed in parallel. The number of parallel PV strings is a design variable that needs optimisation.

In the simulation the number of parallel PV strings is therefore handled as a variable, $x_{PV,parallel}$ to be found through iterating the optimisation algorithm. By changing the value of $x_{PV,parallel}$ in the simulation, the amount of available output current from the overall PV array changes. Through simulating the system with a certain value for $x_{PV,parallel}$ (and also certain values for other design variables) the quality of a system design can be assessed. The current output of a PV array is related to the number of parallel strings as follows:

$$I_{PV,Array}(t) = I_{PVpanel}(t, x_{Size,Type,PV}) \cdot x_{PV,parallel} \cdot f_{MM}$$

Equation 2: PV array current output

where $x_{Size,Type,PV}$ is PV panel size of a certain PV panel type, $I_{PV,panel}(t, x_{Size,Type,PV})$ is the PV panel current output at time t depending on panel type, and f_{MM} is the mismatch factor for different PV panel current outputs.

$x_{PV,parallel}$ is to be optimised in order to cover the system energy requirements. The value of $x_{PV,parallel}$ can range from 0 to the highest number of parallel PV strings needed when a stand-alone PV system is used to cover the energy requirements. For a PV stand-alone system, the number of PV strings in parallel is equivalent to the average daily energy requirements divided by: energy losses through cables and inverter; the expected wattage of a PV string based on local irradiation conditions; and the average number of hours of sunshine per day.

$$x_{PV,parallel} \in [0, \overline{Demand}_{Wh/day} / (\eta_{losses} \cdot W_{expected,PVpanel}(x_{Size,Type,PV}) \cdot n_{PV,series} \cdot \overline{Hours}_{sunshine/day})]$$

Equation 3: Boundaries on the PV array sizing

where η_{losses} are efficiency losses due to conversion losses, wire losses, battery cycling losses, $W_{expected,PVpanel}$ is the expected PV panel output power, and $Hours_{sunshine/day}$ is the average number of estimated sunshine hours per day.

The current output of a PV panel, $I_{PV}(t)$, is determined by the external irradiation and temperature. The power output of the PV array is calculated by multiplying the panel wattage with the number of panels in series and the number of PV strings:

$$Power_{PV,Array}(t) = I_{PV,panel}(t, x_{Size,Type,PV}) \cdot U_{PVpanel,nom} \cdot n_{PV,series} \cdot x_{PV,parallel} \cdot f_{MM}$$

Equation 4: PV array power output

[1] There are also AC PV panels, that is, DC PV panels with integrated inverters (Schmid 1997)

The energy produced by the array is obtained through multiplying the array output power with the time interval over which the power is produced.

2.2.2 PV costing variables

The initial costs of the PV array are

$$InitCost_{PV} = n_{PV,series} \cdot x_{PV,parallel} \cdot Cost_{PV}(x_{Size,Type,PV}) \cdot (1 + \%ofCC_{PV}) + FixedCosts_{PV}$$

Equation 5: Initial costs of the PV array

where $Cost_{PV}$, $\%ofCC_{PV}$, $FixedCosts_{PV}$ are the PV panel costs according to the size of the PV panel type, the percentage of capital costs added for installation and BOS parts, and added fixed costs accounting for installation and BOS parts respectively.

Operation and maintenance of PV arrays can be described with monthly fixed costs and yearly costs as a percentage of capital costs. Replacement events of PV arrays are assumed to occur every 20 years, so for a project life of 20 years or less, there will be no PV replacement costs.

$$OPcost_{PV}(n\ years) = (FixedCosts_{perYear,PV} + OPas\%ofCC_{perYear,PV}) \cdot R(n\ years)$$

Equation 6: PV operation costs after n years

where $Opcost_{PV}(n)$ are the overall PV operation costs after n years, $FixedCosts_{perYear,PV}$ are the fixed operation costs arising during PV operation each year, $Opas\%ofCC_{perYear}$,PV is the percentage of capital costs arising as PV operation cost each year, and R is the discount factor for maintenance and operation costs which occur regularly every year and r is the discount rate assumed for the project life.

$$replacement\,costs_{PV} = InitCost_{PV} \cdot \frac{1}{(1+r)^{Replacement\,year,PV}}$$

Equation 7: PV replacement costs

where $replacementcosts_{PV}$ are overall PV replacement costs, $InitCost_{PV}$ are PV initial costs, and $Replacement_{year,PV}$ is the lifetime of the PV panels in number of years.

2.2.3 Wind turbine sizing variables

A similar listing of relations as for the PV array can be obtained for wind turbine components. Wind turbines are usually connected in parallel, not in series.

Therefore the number of wind turbines in series, $n_{Wtseries}$, will be equal to one. Several wind turbines can be connected in parallel to match the system current requirements. This can be done with parallel strings of the same wind turbine type or with strings of a different wind turbine type. It is assumed here that at most two different turbine types are used at the same time in one system. The current output of the wind turbine array of an AC or DC bus k is then obtained as follows:

$$I_{WT,Array,Bus,k}(t) = \sum_{i=1}^{NOofWTtypes} I_{WT,i,k}(t, x_{Size,Type,WT,i,k}) \cdot x_{WT,i,parallel,k} \qquad , NOofWTtypes \le 2$$

Equation 8: Wind turbine array current output where NOofWTtypes is the number of different wind turbine types available for the optimisation from a pool of wind turbines. $x_{Size,Type,WT,i,k}$ is the size of wind turbine type i on bus k. $x_{WTi,parallel}$ is the number of strings in parallel for wind turbine type i and $I_{WT,i}$ is the wind turbine current output of wind turbine type i.

$x_{WTi,parallel}$ is the number of strings in parallel for wind turbine type i and $I_{WT,i}$ is the wind turbine current output of wind turbine type i. $I_{WT,i}$ depends mainly on the local wind speed. Again, $x_{WTi,parallel}$ and the type of wind turbines used in the design are design variables to be optimised by iterating the developed algorithm.

$x_{WT,i,parallel} \in [0, WT\_kWrating = f(Rotordiameter)]$,

$$Rotordiameter = \sqrt{\frac{Demand_{kWh/year,Av}}{\eta_{WTEnergyConversion} \cdot hrs/year \cdot PowerDensity(WindSpeed_{yearlyAv}, location, heigth) \cdot \pi \cdot \frac{1}{4}}}]$$

Equation 9: Defining wind turbine sizing boundaries

The power output of the wind turbine array is then:

$$Power_{WT,Array}(t) = \sum_{i=1}^{NOofWTtypes} I_{WT,i}(t, x_{Size,Type,WT,,i}) \cdot U_{WT,i,nom} \cdot x_{WT,i,parallel}$$

Equation 10: Wind turbine array power output, DC or AC

where $U_{WT,i,Nom}$ is the nominal voltage of wind turbine type i, and NOofWTtypes is the number of different wind turbine types available for the optimisation from a pool of wind turbines.

2.2.4 Wind turbine costing variables

The initial costs of the wind turbine array are similarly derived as for the PV array:

$$InitCost_{WT} = \sum_{i=1}^{NOofWTtypes} x_{WT,i,parallel} \cdot Cost_{WT}(x_{Size,Type,WT,i}) \cdot (1 + \%ofCC_{WT,type,i}) + FixedCosts_{WT,type,i}$$

Equation 11: Initial costs of wind turbine array, DC or AC

where $Cost_{WT}$ is the wind turbine cost according to the size of hte wind turbine type, $\%ofCC_{WT}$ is the percentage of capital costs added for installation and BOS parts, and $FixedCosts_{WT}$ are the corresponding added fixed costs.

$$OP\,cost_{WT}(n\ years) = \sum_{i=}^{NOofWTtypes} (FixedCosts_{perYear,WT,i} + OPas\%ofCC_{perYear,WT,i}) \cdot R(n\ years)$$

Equation 12: Wind turbine operation costs

where $Opas\%ofCC_{perYear,WT,i}$ is the percentage of capital costs arising as operation costs of wind turbine type i, $FixedCosts_{perYear,WT,i}$ are the fixed operation costs arising during operation of wind turbine i each year.

$$replacement\ costs_{WT} = \sum_{i=1}^{NOofWTtyps} InitCost_{WT,type,i} \cdot \frac{1}{(1+r)^{Replacement\ yearWT,i}}$$

Equation 1: Wind turbine replacement costs where $InitCost_{WT,type,i}$ are the initial costs of wind turbine type i, and $Replacement_{year,WT,i}$ is the lifetime of the wind turbine type i in number of years.

For both PV and wind turbines, arising installation costs and balance of system costs are included in the % of CC (percentage of component capital costs) and the fixed costs (FixedCosts).

2.2.5 Overall renewable costing and output

Other energy sources and their performance, such as micro-hydro generators, AC wind turbines and PV panels with AC output can be modelled and included. The costs and output currents are calculated similarly to the calculated costs and current outputs of the DC PV array and the DC wind turbine array. The additional outputs are added to the DC and AC bus current and power equations.

The overall renewable DC current output is then given as:

$$I_{RE-DC}(t) = I_{PV,Array-DC}(t) + I_{WT,Array-DC}(t) + I_{Other\,Re\,Sources-DC}(t)$$

Equation 14: DC renewable current

The AC renewable current then becomes

$$I_{RE-AC}(t) = I_{PV,Array-AC}(t) + I_{WT,Array-AC}(t) + I_{Other\,Re\,Sources-AC}(t)$$

Equation 15: AC wind turbine array current output

2.3 Diesel generator

2.3.1 Diesel generator sizing and operation variables

The nominal voltage of the diesel generator in most cases matches the AC bus or DC bus nominal voltage. Several diesel generators can run in parallel, so as to be able to output different load levels at good capacity factors. It is assumed here that the number of diesels in parallel does not exceed five, a reasonable assumption.

The diesel output current at each time instant is an operation decision variable. Therefore it is modelled as the product of the maximum nominal diesel current $I_{dieselmax}$ and the diesel output decision variable $x_{Diesel}(t)$ which can be a number between 0 (no diesel output current) and 1 (maximum diesel output current) at each time instant:

$$x_{Diesel,i} \in [0,1]$$

Equation 16: Limits of the diesel generator output decision variable

where $x_{Diesel,i}(t)$ is the output of diesel generator type i at time t as percentage of maximum possible nominal output power in W.

The maximum nominal diesel output current depends on the size of the diesel generator, x_{sizeD}, used in the system:

$$x_{sizeD,i} = I_{Diesel\,max,i} \cdot U_{Bus,Nom}$$

Equation 17: Diesel nominal power sizes

where $x_{sizeD,i}$ is the nominal output power in W of diesel generator type i, and $I_{Dieselmax,i}$ is the maximum possible output current of diesel generator type i.

The $x_{sizeD,i}$ are optimised through iterating the design algorithm. The size of the diesel generator is limited between 0 (no diesel generator) and the size for a diesel single source system to cover the given application reliably.

Therefore

$$x_{sizeD,i} \in [0, PeakDemandPower]$$

Equation 18: Boundaries on diesel generator sizing

It follows that the diesel output current of the diesel generator array is obtained as follows:

$$I_{Diesel,Array,Bus,k}(t) = \sum_{i=1}^{NOofDieselTypes} I_{Dieselmax,i,Bus,k} \cdot x_{Diesel,i,parallel} \cdot x_{Diesel,i}(t) \qquad , NOofDieselTypes \leq 5$$

Equation 19: Diesel current contributions from different diesel generators at time t

where $x_{Diesel,i,parallel}$ is the number of diesel generators of type i installed in parallel. $x_{Diesel,i}(t)$ is the output of diesel generator type i at time t as percentage of maximum possible nominal output power in W. NoofDieselTypes is the number of different diesel generator types available for the optimisation from the diesel generator pool. And $I_{Dieselmax,i,Bus,k}$ is the maximum possible output current of diesel generator type i. Thereby k is an index indicating to which bus the diesel generator is connected. The bus can be DC or AC. If diesel generators with DC outputs are used, their outputs and costs are similarly calculated and their output needs are added onto the DC bus.

The diesel output current at each time instant depends on the size of the diesel and the operation decision about the output level of the diesel generator at that time instant. The sizing variable $x_{sizeD,i}$ and the types of diesel generators used are optimised directly in the design algorithm. The operation decision variables contained in $x_{Diesel,i}(t)$ are arrived at through optimising the system operational control settings. The found control settings then form an operation strategy.

2.3.2 Fuel consumption and fuel costs

In general, the fuel consumption of a diesel generator is related non-linearly to the diesel output power and the diesel run time length (Morris 1988). Fuel consumption levels versus the power running level of the diesel generator can be entered by the user as data points for each diesel generator type. The arising fuel consumption costs during operation are calculated as follows:

$$\begin{aligned} FuelCosts &= {}^{Fuel\ Cost}\!/_{Litre} \cdot LitresUsed \\ &= {}^{Fuel\ Cost}\!/_{Litre} \cdot \sum_{k=1}^{NOofBusTypes} \sum_{t=t_0}^{T} Litres(I_{Diesel,Array,Bus,k}(t) \cdot U_{Bus,k,Nom}) \cdot corr_{Factor} \\ &= {}^{Fuel\ Cost}\!/_{Litre} \cdot \sum_{i=1}^{NOofDieselTypes} \sum_{t=t_0}^{T} Litres(x_{sizeD,i} \cdot x_{Diesel,i,parallel} \cdot x_{Diesel,i}(t)) \cdot corr_{Factor} \end{aligned}$$

Equation 20: Overall fuel consumption and fuel costs

where NOofBusTypes is the number of different DC and AC busses in the system, Fuel Cost/ Litre is the cost of fuel in ECU/litre and LitresUsed is the fuel used during the simulation time interval T. Litres(:) is a function relating the diesle generator output power to its fuel consumption. $corr_{Factor}$ is a correction factor accounting for increases in fuel needs during start-up.

Running the diesel with a low capacity factor increases relative fuel consumption and wear. The fuel consumption is higher than normal during a cold start of the diesel, especially under low capacity factors. Many such cold-starts over a short period contribute to increased diesel wear. In cases where the diesel is running at no loading a certain amount of fuel will still be consumed.

2.3.3 Diesel generator costing variables

The initial costs of the diesel gensets are

$$InitCost_{Diesel} = \sum_{i=1}^{NOofDiesTypes} n_{Diesel,i,series} \cdot x_{Diesel,i,parallel} \cdot Cost_{Dies}(x_{sizeD,i}) \cdot (1 + \%ofCC_{D,size,i}) + FixedCosts_{Diesel,type,i}$$
$$NOofDiesTypes \leq 5$$

Equation 21: Initial equipment costs of the diesel generator

where $Cost_{Dies}$ is the diesel generator cost according to the size of the diesel generator type. $\%ofCC_{D,size,i}$ is the percentage of capital costs added for installation and BOS parts for diesel generator type i and $FixedCosts_{Diesel,type,i}$ are the corresponding added fixed costs.

$$OPcost_{Diesel}(n\ years) = [\sum_{i=1}^{NOofDieselTypes} (FixedCosts_{perYear,Dies,i} + OPas\%ofCC_{perYear,Dies,i} +$$
$$+ OPcostPerRunTime_{PerYear,Dies,i}) + FuelCosts(T = 1\ year)\ ] \cdot R(n\ years)$$

Equation 22: Operation costs of the diesel generator

where $Opas\%ofCC_{perYear,Dies,i}$ is the percentage of capital costs arising as diesel generator operation.

$$Replacementcosts_{Diesel} = \sum_{i=1}^{NOofDieselTypes} \sum_{j=1}^{NOofReplDiesType,i} InitCost_{Diesel,type,i} \cdot \frac{1}{(1+r)^{j \cdot (Repl\ yearDies,i)}}$$

Equation 23: Diesel generator replacement costs

where $Repl_{year,Dies,i}$ is the lifetime of the diesel generator type i in number of years.

The installation costs, balance of system costs, fuel tank and shelter costs are included in the fixed costs or as a percentage of initial costs. The lifetime of the diesel generator in number of years is usually obtained through noting when the operating hours of the diesel generator equal the number of hours given as the operating lifetime by the manufacturers.

2.4 Battery

2.4.1 Battery design variables

The battery model described by Schuhmacher (1993) and according to Shepard (1965) is explained in detail in appendix A2.

Batteries in a hybrid system are connected in series to yield the appropriate nominal bus voltage. Therefore the number of batteries connected in series for the same type of battery in a battery bank is

$$n_{Bat,series,Bank,k} = U_{Bus,Nom} / U_{Bat,Nom,Bank,k}$$

Equation 24: Number of batteries required in series

The hybrid system can have several battery banks, which typically consist of different battery types. For example, the second battery bank can consist of batteries which are smaller in size than the ones of the first battery bank to produce better battery cycling patterns in case of very diverse demand levels.

Each battery bank therefore has a certain number of batteries of the corresponding battery type connected in series to match the bus nominal operating voltage. In addition, each battery bank may have several strings of serial connected batteries so as to increase the Ampere hours (Ah) available to the system.

It is assumed here that the number of battery banks (Figure 2) is limited to two, a realistic presumption.

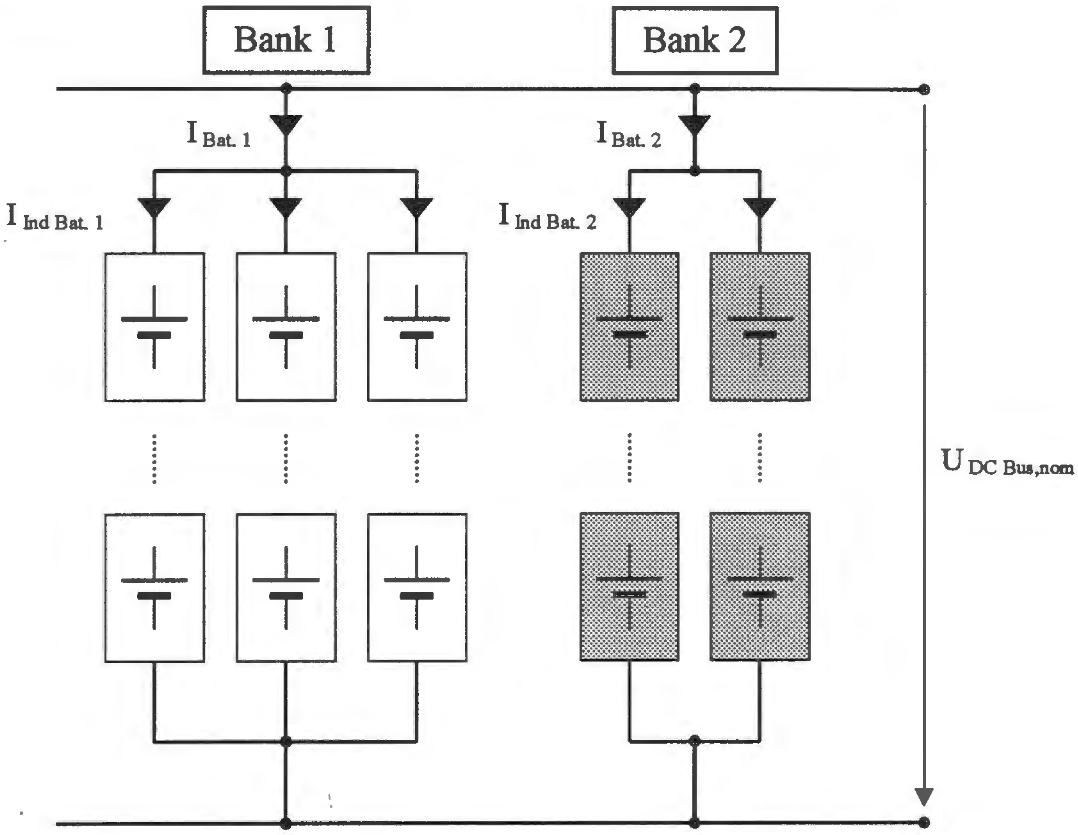


Figure 2: Several battery banks

It follows that the system battery state of charge, SOC_{Nom}, in Ah, is compiled by adding the SOC available from each battery bank i, $SOC_{Nom,Bank,i}$. The size of the individual batteries, $x_{size,Bat,Bank,i}$ in each battery bank i is a design variable to be optimised by the algorithm. This size corresponds to the nominal battery capacity in Ah.

$$SOC_{Nom} = \sum_{i=1}^{NOofBatBanks} SOC_{Nom,Bank,i} =$$

$$= \sum_{i=1}^{NOofBatBanks} SOC_{Nom,Bat,Bank,i} \cdot x_{Bat,parallel,Bank,i}$$

$$= \sum_{i=1}^{NOofBatBanks} x_{size,Bat,Bank,i} \cdot x_{Bat,parallel,Bank,i}$$

Equation 25: Battery nominal capacities

$x_{Bat,parallel,Bank,i}$ is the number of parallel battery strings in a battery bank i.

The system battery state of charge, SOC_{Nom} is limited to a value between 0 (no batteries used) and five days of storage, a big enough size for many even single-source systems requiring typical system reliability.

$$SOC_{Nom} \in [0;5 \text{ days of storage in Ah}]$$
$$\Leftrightarrow SOC_{Nom} \in [0;5 \text{ days} \cdot \sum_{t=1h}^{24h} I_{Demand,Daily}(t) \cdot \Delta t]$$

Equation 26: Battery sizing boundaries

The battery's state of charge is limited between maximum and minimum state of charge (SOC_{max} and SOC_{min}). The minimum state of charge is often set to 50% nominal capacity, or even 20% nominal capacity, depending on the following factors: type of battery used, outside temperature, battery age and battery condition. The maximum state of charge is often set to 100% nominal capacity.

The system maximum or minimum state of charge $SOC_{Max(Min)}$ is obtained by adding the maximum and minimum state of charge of each battery bank, which are made up of the maximum (minimum) state of charge of the individual battery times the number of parallel strings in a battery bank i. The maximum (minimum) state of charge of a battery is often expressed as percentage of the nominal capacity of the battery.

$$SOC_{Max(Min)} = \sum_{i=1}^{NOofBatBanks} SOC_{Max(Min),Bank,i} =$$
$$= \sum_{i=1}^{NOofBatBanks} SOC_{Max(Min),Bat,Bank,i} \cdot x_{Bat,parallel,Bank,i}$$
$$= \sum_{i=1}^{NOofBatBanks} \%_{Max(Min)} \cdot x_{size,Bat,Bank,i} \cdot x_{Bat,parallel,Bank,i}$$

Equation 27: Maximum and minimum battery state of charge

The state of charge of a battery bank at time t is calculated by adding the charge current (positive sign) or discharge current (negative sign) to the battery bank state of charge at the previous time instant. When adding the battery current to the battery state of charge, self discharge losses and battery charging losses (see appendix A2) need to be taken into account.

$$SOC(t+1) = \sum_{i=1}^{NOofBatBanks} SOC_{Bank,i}(t+1)$$
$$= \sum_{i=1}^{NOofBatBanks} SOC_{Bank,i}(t) \cdot \sigma_i + I_{Bat,Bank,i}(t) \cdot \Delta t \cdot \eta_i(I_{Bat,Bank,i}(t))$$
$$= \sum_{i=1}^{NOofBatBanks} [SOC_{IndBat,i}(t) \cdot \sigma_i + I_{IndBat,i}(t) \cdot \Delta t \cdot \eta_i(I_{IndBat,i}(t))] \cdot x_{Bat,parallel,i}$$

Equation 28: System battery state of charge at time t

σ is the self-discharge rate, η the charging efficiency and I_{bat} the charge/discharge current. During discharge, η is assumed to be 1. When charging, η is 0.85 to 0.65, depending on the charging current. When gassing starts at a critical state of charge $SOC_{crit}(I_{bat})$, η drops to 0.3 to 0.01 according to Schuhmacher (1993). The battery efficiency also depends on battery cycling and its history (Degner et al 1994).

The maximum charge current that the battery can receive from the system is determined by the amount the battery banks can accept for charge. This amount is proportional to SOC_{max} - SOC(t). However, the amount the battery can accept for charge at any one time instant cannot exceed the maximum allowed charging rate of an individual battery given by the manufacturer, $I_{Max,Ch,IndBat}$.

The maximum discharge current the system can obtain from the battery is determined by the amount the battery banks can deliver as discharge, which is proportional to SOC(t) - SOCmin. However, the amount the battery can deliver as discharge at any one time instant cannot exceed the maximum allowed discharging rate of an individual battery given by the manufacturer, $I_{Max,Ch,IndBat}$.

The maximum possible overall battery charging (discharging) current $I_{bat,max}(t)$ at time t consists of the sum of the maximum possible battery bank currents at time t. The maximum battery bank current in turn is composed of the maximum current of the individual battery bank multiplied by the number of strings in the corresponding battery bank i.

$$I_{Bat,\max}(t)=\sum_{i=1}^{NOofBatBanks} I_{Bat,\max,i}(t)=\sum_{i=1}^{NOofBatBanks} I_{Bat,\max,IndBat,i}(t)\cdot x_{Bat,parallel,i}$$

$$=\sum_{i=1}^{NOofBatBanks} Max[0,Min[c_i\cdot I_{Max,Ch,i}+(1-c_i)\cdot I_{Max,Dh,i},\ \frac{c_i\cdot(SOC_{Max,i}-SOC_i(t))+(1-c_i)\cdot(SOC_i(t)-SOC_{Min,i})}{\Delta t}]$$

$$=\sum_{i=1}^{NOofBatBanks} Max[0,Min[(c_i\cdot I_{Max,Ch,IndBat,i}+(1-c_i)\cdot I_{Max,Dh,IndBat,i})\cdot x_{Bat,parallel,i},\ \frac{c_i\cdot(SOC_{Max,IndBat,i}-SOC_{IndBat,i}(t))+(1-c_i)\cdot(SOC_{IndBat,i}(t)-SOC_{Min,IndBat,i})}{\Delta t}\cdot x_{Bat,parallel,i}]$$

$$=\sum_{i=1}^{NOofBatBanks} Max[0,Min[(c_i\cdot I_{Max,Ch,IndBat,i}+(1-c_i)\cdot I_{Max,Dh,IndBat,i})\cdot x_{Bat,parallel,i},\ x_{Bat,parallel,i}\cdot \frac{c_i\cdot(\%_{Max,IndBat,i}\cdot x_{size,Bat,Bank,i}-SOC_{IndBat,i}(t))+(1-c_i)\cdot(SOC_{IndBat,i}(t)-\%_{Min,IndBat,i}\cdot x_{size,Bat,Bank,i})}{\Delta t}$$

Equation 29: Maximum possible DC battery current at time t

Therefore the amount of the maximum, time-dependent current the battery banks can discharge to the system or charge from the system, $I_{Bat,max}(t)$, depends on the system battery state of charge at each time instant and the maximum battery charging/discharging rates allowed by the manufacturer. The maximum charge (discharge) rate, $I_{maxCh(Dh)}\cdot\delta t$ is often given by manufacturers as around 20% of the value of nominal capacity. The charge/discharge indicator, c, is zero during discharge and 1 during charge.

The battery current is an operation decision variable to be optimised by the algorithm as part of the overall system operation strategy.

The actual battery current at time t, $I_{Bat}(t)$, is a percentage of the maximum possible battery current, $I_{Bat,max}$, at that instant. This percentage, x_{bat}, is determined by the optimisation algorithm, where $x_{bat}(t) \in [0,1]$.

$$I_{Bat}(t)=I_{Bat,\max}(t)\cdot x_{Bat}(t)$$

Equation 30: Actual battery current at time t

The system battery current is the sum of the individual battery bank currents which in turn are composed of the individual battery currents multiplied by the number of battery strings in a battery bank i. The individual battery currents of battery bank i, $I_{IndBat,i}$, are determined by the percentage, $x_{Bat,i}$, of the maximum possible charge or discharge current, $I_{Bat,max,IndBat,i}$, which can be utilised at time instant t in battery bank i.

$$I_{Bat}(t) = \sum_{i=1}^{NOofBatBanks} I_{Bat,i}(t) = \sum_{i=1}^{NOofBatBanks} I_{IndBat,i}(t) \cdot x_{Bat,parallel,i}$$
$$= \sum_{i=1}^{NOofBatBanks} I_{Bat,max,IndBat,i}(t) \cdot x_{Bat,i}(t) \cdot x_{Bat,parallel,i} \qquad , I \leq 2$$

Equation 31: Battery current contributions from different batteries

The derived equations are implemented in the developed design optimisation algorithm. Before executing the developed algorithm, the initial battery state of charge needs to be known. The initial state of charge of a battery bank at time 0 will be a percentage of $SOC_{nom,bank,i}$. The user of the algorithm also enters the maximum charge and discharge current rates, maximum/minimum possible battery state of charge, the battery efficiency versus state of charge, and the number of cycles a battery lasts versus the average depth of discharge. The latter are important to determine the battery operational costs.

2.4.2 Battery costing variables

The initial battery costs are

$$InitCost_{Bat} = \sum_{i=1}^{NOofBatBanks} n_{Bat,i,series} \cdot x_{Bat,i,parallel} \cdot Cost_{Bat}(x_{sizeBat,i}) \cdot (1 + \%ofCC_{Bat,i}) + FixedCosts_{Bat,Bank,i}$$
$$NOofBatBanks \leq 2$$

Equation 32: Initial battery costs

Installation costs and balance of system costs will be accounted for in the fixed costs or as a percentage of initial battery capital costs. The battery operation costs depend on the battery cycling during system operation and include fixed costs and costs that occur at regular intervals such as maintenance costs.

$$OP\cos t_{Bat}(n\ years) = \sum_{i=1}^{NOofBatBanks} (FixedCosts_{perYear,Bat,i} + OPas\%ofCC_{perYear,Bat,i} +$$
$$+ OP\cos tPerCycling_{PerYear,Bat,i}) \cdot R(n\ years)$$

Equation 33: Battery operation costs

Replacement costs occur whenever the battery needs to be exchanged with a new or newer one.

$$Re\,placement\cos ts_{Battery} = \sum_{i=1}^{NOofBatBanks} \sum_{j=1}^{NOofReplBatType,i} InitCost_{Bat,Bank,i} \cdot \frac{1}{(1+r)^{j \cdot (Re\,pl\,yearBat,i)}}$$

Equation 34: Battery replacement costs

For the calculation of the battery replacement year see the previous sections.

2.5 Inverter

2.5.1 Inverter design variables

The size of the inverter can be defined in terms of its AC output power.

$$x_{size,Inv} = P_{Inv-o/p}$$

Equation 35: Inverter power rating as sizing decision variable

The inverter size can be optimised by the design algorithm or can be determined according to some rules of thumb, for example, by choosing the inverter size in the range of the peak power demand, or below, depending on the technical design.

The inverter DC to AC power transformation is accompanied by conversion losses that depend on the inverter characteristics. The influence of these losses on the power flow from the energy sources to the load needs to be taken account of when determining how best to match demand and supply.

The inverter characteristics can be described by the inverter input-output relationship. Some of the power going into the inverter will be lost due to transformation losses. This is accounted for by the inverter efficiency losses, named eff_{Inv}.

$$P_{Inv-i/p} \cdot eff_{Inv} = P_{Inv-o/p} \quad , \qquad eff_{Inv} = f(P_{Inv-o/p})$$

Equation 36: Inverter power transformation equation

$$I_{Inv-i/p} \cdot eff_{Inv} \cdot U_{DCBus,Nom} = I_{Inv-o/p} \cdot U_{ACBus,Nom}$$

Equation 37: Inverter current transformation equation

Manufacturers give the inverter efficiency eff_{Inv} over the inverter output power. The characteristic inverter curves are usually non-linear, that is, the efficiency of the power transformation is non-linearly related to the obtained inverter output power. If the inverter is a parallel inverter, then efficiency losses also occur when transforming AC power back into DC power. This transformation can have higher losses and be less efficient than the other way round due to internal power electronics.

In some cases more than one inverter is installed in a hybrid system. Sometimes the second inverter is used as a back up in case the main inverter fails. In other cases, several inverters can be used to transform different DC bus power levels more efficiently into AC power using the inverter with the best conversion rate for a particular power demand level. In this case the DC bus input current to the set of parallel-connected inverters is split and/or routed through the most appropriate inverter or set of inverters (Figure 3). It is assumed that the number of parallel-connected inverters does not exceed three, a realistic assumption.

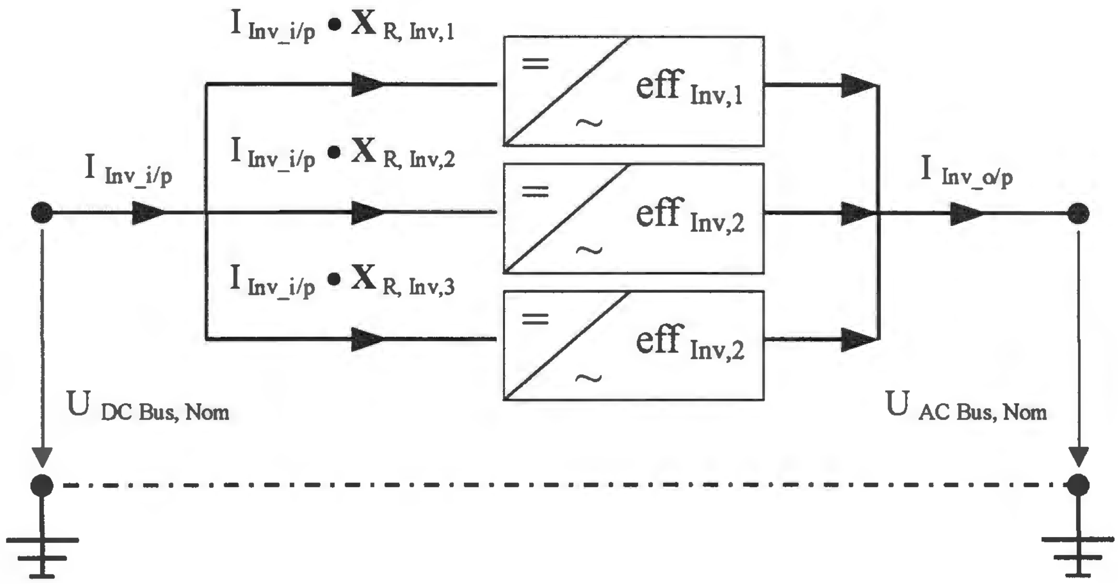


Figure 3: Current and efficiency relations for several parallel inverters

The percentage of current routed through inverter number i, xR,Inv,i , is an operation decision variable to be optimised by the design algorithm.

$$I_{Inv-i/p} = \sum_{j=1}^{NOofInv} I_{Inv-i/p,j} = I_{Inv-i/p} \cdot \sum_{j=1}^{NOofInv} x_{R,Inv,j} \quad , \sum_{j=1}^{NOofInv} x_{R,Inv,j} = 1 \text{ , } NOofInv \le 3$$

Equation 38: Inverter current contributions

The inverter input-output relationship for the case of several inverters becomes:

$$I_{Inv-i/p} \cdot eff_{Inv} \cdot U_{DCBus,Nom} = I_{Inv-o/p} \cdot U_{ACBus,Nom}$$

$$\Leftrightarrow \quad \sum_{j=1}^{NOofInv} I_{Inv-i/p,j} \cdot eff_{Inv,j} \cdot U_{DCBus,Nom} = I_{Inv-o/p} \cdot U_{ACBus,Nom}$$

$$\Leftrightarrow \quad I_{Inv-i/p} \cdot \sum_{j=1}^{NOofInv} x_{R,Inv,j} \cdot eff_{Inv,j} \cdot U_{DCBus,Nom} = I_{Inv-o/p} \cdot U_{ACBus,Nom}$$

$$\Rightarrow \quad eff_{Inv} = \sum_{j=1}^{NOofInv} x_{R,Inv,j} \cdot eff_{Inv,j}$$

Equation 39: Inverter efficiency relationships for several parallel inverter

The overall transformation efficiency is the weighted sum of the efficiencies of the individual inverter power transformations.

2.5.2 Inverter costing variables

The initial inverter costs are:

$$InitCost_{Inv} = \sum_{i=1}^{NOofInv} n_{Inv,i,series} \cdot x_{Inv,i,parallel} \cdot Cost_{Inv}(x_{sizeInv,i}) \cdot (1 + \%ofCC_{Inv,i}) + FixedCosts_{Inv,i}$$

$$NOofInv \le 3$$

Equation 40: Initial costs inverters

Operation costs of the inverter can be included in overall system maintenance or operation costs. The discounted replacement costs depend on the inverter lifetime and the number of its replacements during the assumed project life.

$$Re\,placement\,cos\,ts_{Inv} = \sum_{i=1}^{NOofInv} \sum_{j=1}^{NOof\,Re\,plInv,i} InitCost_{Inv,i} \cdot \frac{1}{(1+r)^{j \cdot (Re\,pl\,yearInv,i)}}$$

Equation 41: Inverter replacement costs

Usually the inverter lifetime is as long or longer than the assumed project lifetime.

2.6 Battery charger

2.6.1 Battery charger design variables

The size of the battery charger can be defined in terms of its DC output power.

$$x_{size,BC} = P_{BC-o/p}$$

Equation 42: Battery charger rating as sizing variable

This battery charger size can be optimised by the design algorithm or can be determined according to some rules of thumb. For example, to choose the battery charger size in the range of the peak power diesel output power or the maximum allowed or required battery charging current.

It is necessary to describe the inverter AC to DC power transformation equation since the battery charger characteristics influence the decision how to best match supply and demand.

The battery charger characteristics can be described by the battery charger input-output relationship. Some of the power going through the battery charger will be lost due to transformation losses. This is accounted for by the battery charger efficiency eff_{BC}.

$$P_{BC-i/p} \cdot eff_{BC} = P_{BC-o/p} \quad , \quad eff_{BC} = f(P_{BC-o/p})$$

Equation 43: Battery charger power transformation equation

The power transformation equation can also be expressed in terms of the input and output current and bus voltage descriptions:

$$I_{BC-i/p} \cdot eff_{BC} \cdot U_{ACBus,Nom} = I_{BC-o/p} \cdot U_{DCBus,Nom}$$

Equation 44: Battery charger current transformation equation

The output power $P_{o/p}$ of the battery charger equals the input power $P_{i/p}$ multiplied with the efficiency losses eff_{BC} during the energy conversion.

Manufacturers give the characteristic curves of battery chargers as efficiency losses eff_{BC} versus output power. In such a curve eff_{BC} depends non-linearly on the DC output power, and therefore non-linearly on the DC output current of the battery charger $I_{BC-o/p}$. Again, the user of the developed design optimisation algorithm can enter the data points of the characteristic curve. Sometimes the battery charger function can be incorporated in a so-called tri-mode inverter which can also operate in parallel with a diesel generator, and can in addition function in reverse as a battery charger.

In some cases more than one battery charger could be installed in a hybrid system. The different battery chargers can be used to transform different AC bus power levels more efficiently into DC power using the battery charger with the best conversion rate, for example, for a certain diesel generator output power level to be converted into DC power. In this case the AC bus input current to the set of parallel-connected battery chargers is split and/or routed through the most appropriate battery charger or set of battery chargers (Figure 4). It is assumed that the number of parallel-connected battery chargers does not exceed three, a realistic assumption.

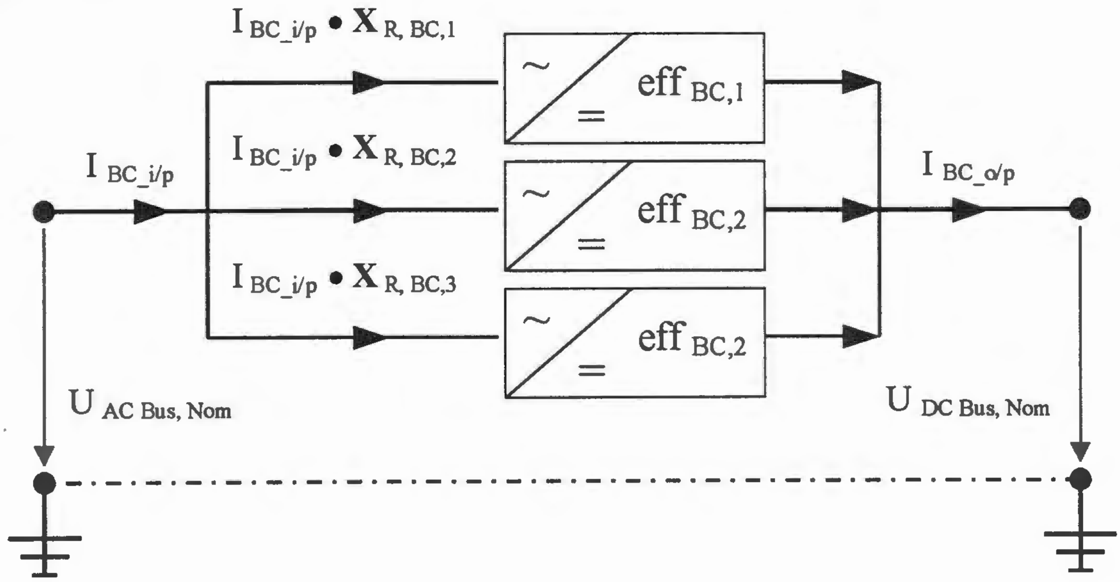


Figure 4: Current and efficiency relations for several parallel battery chargers

The percentage of current routed through battery charger number i, $x_{RBC,i}$, is an operation decision variable to be optimised by the design algorithm.

$$I_{BC-i/p} = \sum_{j=1}^{NOofBC} I_{BC-i/p,j} = I_{BC-i/p} \cdot \sum_{j=1}^{NOofBC} x_{R,BC,j} \quad , \sum_{j=1}^{NOofBC} x_{R,BC,j} = 1 \, , NOofBC \leq 3$$

Equation 45: Battery charger current contributions

The battery charger input-output relationship for the case of several battery charger becomes:

$$I_{BC-i/p} \cdot eff_{BC} \cdot U_{ACBus,Nom} = I_{BC-o/p} \cdot U_{DCBus,Nom}$$

$$\Leftrightarrow \quad \sum_{j=1}^{NOofBC} I_{BC-i/p,j} \cdot eff_{BC,j} \cdot U_{ACBus,Nom} = I_{BC-o/p} \cdot U_{DCBus,Nom}$$

$$\Leftrightarrow \quad I_{BC-i/p} \cdot \sum_{j=1}^{NOofBC} x_{R,BC,j} \cdot eff_{BC,j} \cdot U_{ACBus,Nom} = I_{BC-o/p} \cdot U_{DCBus,Nom}$$

$$\Rightarrow \quad eff_{BC} = \sum_{j=1}^{NOofBC} x_{R,BC,j} \cdot eff_{BC,j}$$

Equation 46: Battery charger efficiency for several parallel battery charger

The overall transformation efficiency is then the weighted sum of the efficiencies of the individual inverter power transformations. The operation decision variables xR,BC,j are the percentage of current routed through battery charger number i.

2.6.2 Battery charger costing variables

The initial battery charger costs are:

$$InitCost_{BC} = \sum_{i=1}^{NOofBC} n_{BC,i,series} \cdot x_{BC,i,parallel} \cdot Cost_{BC}(x_{sizeBC,i}) \cdot (1 + \%ofCC_{BC,i}) + FixedCosts_{BC,i}$$

$$NOofBC \leq 3$$

Equation 47: Initial costs battery charger

Operation costs of the battery charger can be included in overall system maintenance or operation costs. The replacement costs depend on the battery charger lifetime and the number of its replacements during the assumed project life.

$$Re\,placement\,cos\,ts_{BC} = \sum_{i=1}^{NOofBC} \sum_{j=1}^{NOof\,Re\,plBC,i} InitCost_{BC,i} \cdot \frac{1}{(1+r)^{j \cdot (Re\,pl\,yearBC,i)}}$$

Equation 48: Battery charger replacement costs

In general, the battery charger will last as long or longer than the assumed project life.

2.7 Transfer switches

The transfer switch is located between the inverter output and the diesel generator output. In many hybrid systems the functions of this switch are implemented electronically. In quite a few hybrid systems, the functions of the transfer switch are carried out manually by the hybrid system operator, switching the diesel on and off when deemed necessary. In this approach, $x_S(t)$ models the switch positions between 0 (inverter off – all AC load can be supplied by the diesel generator, but not through the inverter) and 1 (inverter on – all AC load can be supplied through the inverter, but not through the diesel generator). If a parallel inverter is used, then $x_s \in [0,1]$ and both the diesel and the inverter can supply the AC load at the same time.

2.8 Loads

Most common loads are 12V, 24V, 36V, 48V DC appliances or 220V AC appliances. The estimated power consumption should be given in intervals of hours or minutes, for the length of a week, month or year.

If both a DC and AC system bus exist, some of the DC bus energy can be routed through the inverter to the AC loads. The loads can possess different priorities in terms of when they need to be met by the electricity supplied. There are optional loads ($I_{load,OPT}$), which can either be supplied at the specified time instant but do not have to. They are suitable as dump loads. Then there are deferrable loads ($I_{load,DEF}$), which do not have to be supplied at the specified time instant, but need to be covered within a certain time interval. Other high priority loads ($I_{load,HP}$) need to be run at the given time instant.

$$I_{Load}(t) = I_{Load,HP}(t) + x_{Load,OPT}(t) \cdot I_{Load,OPT}(t) + x_{Load,DEF}(t) \cdot I_{Load,DEF}(t) +$$
$$+ \sum_{\tau} [x_{Load,DEF2}(t,\tau) \cdot I_{Load,DEF}(\tau) \cdot (1 - x_{Load,DEF}(\tau)) \cdot \sum_{\zeta} (1 - x_{Load,DEF2}(\zeta,\tau))]$$

Equation 49: Load constituents: high priority, optional, deferrable

where

$\tau = 0 \ldots t\text{-}1$
$\zeta = \tau+1 \ldots\ldots \min[t\text{-}1, T_{stop}(\tau)\text{-}\tau]$
$T_{stop}(\tau)$ = amount of time load from time τ can be deferred.
$x_{Load,OPT} = 1$ or 0, $x_{Load,DEF} = 1$ or 0, $x_{Load,DEF2} = 1$ or 0
The $x_{load,opt}$, $x_{load,def}$, $x_{load,def2}$ can be considered for optimisation.

Instead of dividing the load group at each time instant into high priority, deferrable and optional, this can also be done for individual appliances. However, this is computationally intensive in terms of optimising the different load priorities.

3 Power flow

3.1 Overview

The system model is based on a description of current flows through the system including efficiency losses. The described power flow can be traced in Figure 1.

The aim of the power flow description is to trace the power arriving at the DC and AC loads. The energy flow is followed from its start at the generating power sources, considering losses and the influence of the operating decisions along the way up to the loads.

It can then be assessed whether the currents being supplied to the AC and DC loads match the load requirements. If not, a difference results between electricity supplied and demanded. This difference is positive, if there is an oversupply of electricity; it will be negative if there is an under-supply of electricity. This difference between demand and supply, in form of an under-supply or over-supply of electricity, will then be attributed a penalty or cost/benefit description in the overall system cost function. The overall cost function, or so-called objective function, serves as a figure of merit to assess the quality of a certain system sizing and operation control design.

3.2 Constraints on operation

The design objective is to provide energy for the lowest possible costs. Without the additional specification that the demand has to be covered with a predefined reliability, the designed system may not possess any system components at all resulting in zero costs. No energy, however, would be generated or supplied. Through the constraint '*satisfy the demand allowing for certain tasks to be postponed or cancelled when necessary but observe the servicing of high priority tasks*' the model is forced to minimise system energy prices, and is also compelled to meet the demand needs with a sufficient amount of electricity. The electricity produced from PV, wind, diesel and other generator sources and discharged from the battery must add up to cover the demand requirements as economically and efficiently as possible, taking a long-term perspective. The operational constraints can therefore be formulated as saying that the current supply arriving at the AC and DC loads needs to equal the required AC and DC demand levels.

$$I_{DCSupply} \overset{!}{\equiv} I_{DCLoad}$$

Equation 50: DC current supply equals demanded DC current

$$I_{ACSupply} \overset{!}{\equiv} I_{ACLoad}$$

Equation 51: AC current supply equals demanded AC current

The following sections derive the current description for the AC and DC supply currents arriving at the AC and DC loads respectively.

3.3 AC load supply

At the AC load, the arriving current flow can come from two sources: from the DC bus via the inverter or from the AC bus supplied by the AC electricity generators. The aim of the design is to match the demanded load and the supplied electricity as best as possible (Equation 51). This can be described as follows:

$$I_{ACSupply} = I_{Inv-o/p} + I_{ACBus,o/p}$$

Equation 52: AC current demand covered through the inverter and by the diesel

The amount of AC load that is supplied from the DC bus through the inverter is described by $I_{ACload} \cdot x_s$ (Equation 53, Figure 5). If the inverter has bi-directional operation characteristics, the value of x_s will range between 0 and 1. In case the installed inverter is not bi-directional, the value of x_s will be either 1 or 0. A value of 1 for x_s indicates that all AC load needs are required to be covered by the inverter transformed DC bus output. A value of 0 for x_s indicates that all AC load needs are required to be covered by the AC sources only.

$$I_{Inv-o/p} \overset{!}{\equiv} I_{ACLoad} \cdot x_S$$

Equation 53: Amount of demand to be supplied by the inverter

The amount of AC load that is supplied from the AC bus is described by $I_{ACload} \cdot (1-x_s)$:

$$I_{ACBus,o/p} \overset{!}{\equiv} I_{ACLoad} \cdot (1 - x_S)$$

Equation 54: Amount of demand to be supplied by the AC bus

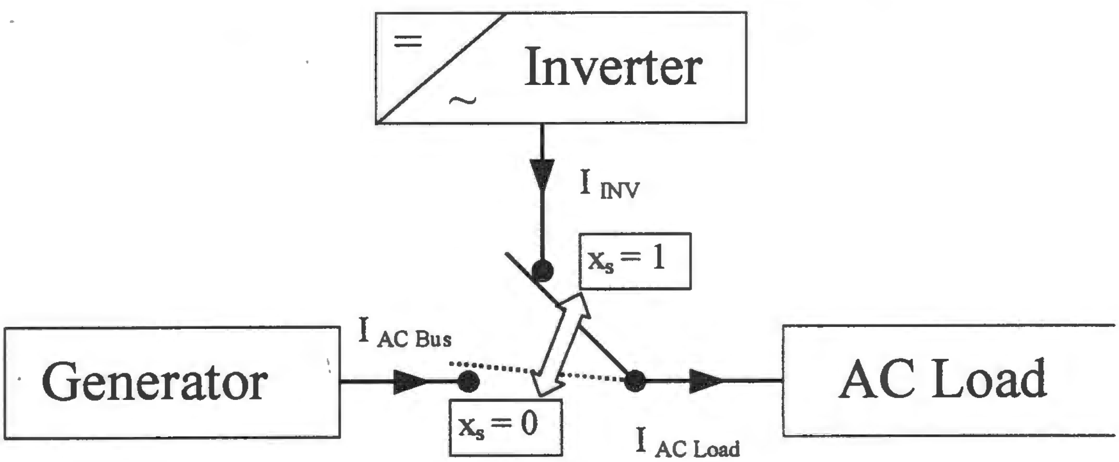


Figure 5: AC load supply

The AC load is supplied with electricity from the AC and DC bus and both contributions are supposed to cover the AC load requirements:

$$I_{ACSupply} = I_{Inv-o/p} + I_{ACBus,o/p} \overset{!}{\equiv} I_{ACLoad} \cdot x_S + I_{ACLoad} \cdot (1 - x_S) = I_{ACLoad}$$

Equation 55: AC load, x_s (100% supplied by the inverter, $(1-x_s)$ (100% supplied from the AC bus

The AC supply coming from the DC bus through the inverter is composed as follows:

$$I_{Inv-o/p} = I_{Inv-i/p} \cdot eff_{Inv} \cdot \frac{U_{DCBus,Nom}}{U_{ACBus,Nom}} \overset{!}{\equiv} I_{ACLoad} \cdot x_S$$

Equation 56: Inverter output equated to xs 100% of AC demand

The AC bus current, from which the AC load is partly or fully supplied, is produced by different AC generating sources such as AC diesel generators, AC renewable energy generators and other AC generators:

$$I_{ACBus} = I_{RE-AC} + I_{Diesel-AC} + I_{o/pOther-AC}$$

Equation 57: Definition of AC bus current

A share of the AC bus current can be routed to the battery charger, and the complementing share can be routed to the AC load. Therefore the decision variable x_{RD} is introduced which reflects the percentage of the AC bus current which goes to the battery charger. (1-x_{RD}) is routed directly to the AC load. The decision variable x_{RD} influences the operation strategy to be adopted for the hybrid system operation. It will be optimised implicitly through optimising the system's control setting and thereby defining an optimum system operation strategy.

The part of the AC load that is supplied by the AC bus contribution is then obtained as follows:

$$I_{ACBus-o/p} = (1 - x_{RD}) \cdot I_{ACBus}$$
$$= (1 - x_{RD}) \cdot (I_{RE-AC} + I_{Diesel-AC} + I_{o/pOther-AC}) \overset{!}{\equiv} I_{ACLoad} \cdot (1 - x_S)$$

Equation 58: Direct AC supply equated to (1– x_s) 100% of AC demand

Therefore by combining Equation 56 and Equation 58, the current supplied to the AC load is obtained:

$$I_{ACSupply} = I_{Inv-i/p} \cdot eff_{Inv} \cdot \frac{U_{DCBus,Nom}}{U_{ACBus,Nom}} + (1 - x_{RD}) \cdot (I_{RE-AC} + I_{Diesel-AC} + I_{o/pOther-AC}) \overset{!}{\equiv} I_{ACLoad}$$

Equation 59: AC current supplied to AC load

3.4 DC load supply

The arriving current description at the DC load is composed of renewable DC current, battery current, diesel DC current, DC currents from other sources and AC currents routed from the AC bus through the battery charger. It is the task of the optimisation algorithm to match the DC supply current as best as possible to the DC load requirements (Equation 50). Several DC sources (DC renewable energy sources, DC diesel generators and other possible DC sources) and the DC output of the battery charger generate the DC bus currents (Figure 6).

$$I_{DCBus} = I_{BC-o/p} + I_{DCSources} - I_{Bat}$$

Equation 60: Definition of DC bus current

The battery charger output and the DC source current can be described as

$$I_{BC-o/p} = x_{RD} \cdot (I_{Diesel-AC} + I_{RE-AC} + I_{OtherSources-AC}) \cdot eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}}$$
$$I_{DCSources} = I_{RE-DC} + I_{Diesel-DC} + I_{OtherSources-DC}$$

Equation 61: Battery charger output current and definition of DC sources current

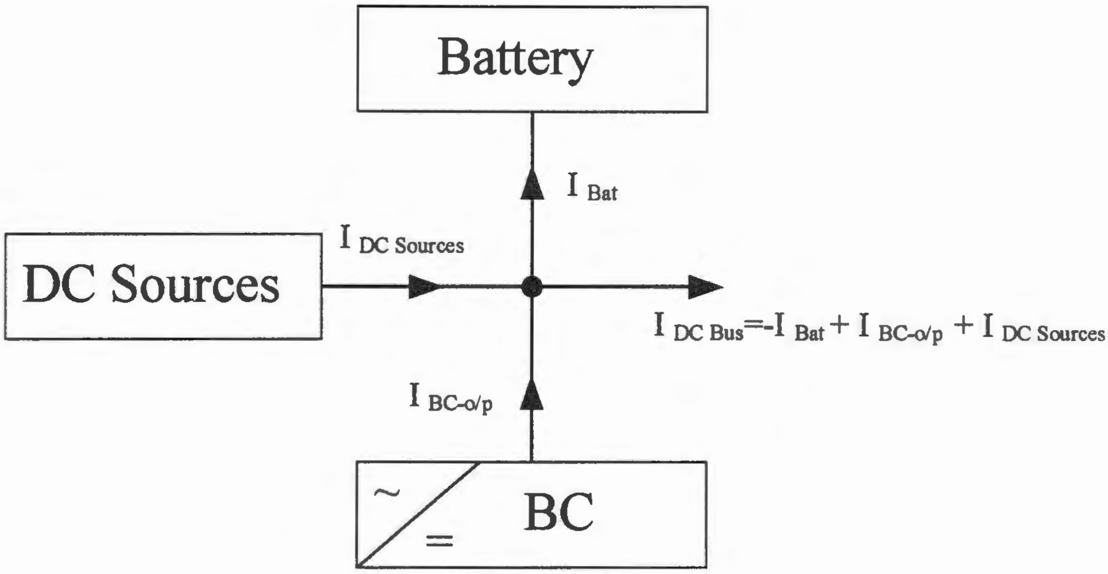


Figure 6: DC bus currents

The DC bus supplies DC current to the DC loads on the bus and to the inverter. The design algorithm needs to ensure that this supply meets the requirements of the loads, that is, that it supplies the DC loads sufficiently ($I_{DCSupply}$) and contributes to the AC loads through the inverter suitably (with $I_{Inv-i/p}$).

$$I_{DCBus} = I_{DCSupply} + I_{Inv-i/p}$$

Equation 62: DC bus current is split into DC supply current and inverter input current

A share of the DC bus current can be routed to the inverter, and the complementing share can be routed to the DC load. Therefore the decision variable x_R is introduced which reflects the percentage of the DC bus current which goes to the DC load. $(1- x_R)$ is routed directly to the inverter. The decision variable x_R influences the operation strategy to be adopted for the hybrid system operation. It will be optimised implicitly through optimising the systems control setting and thereby defining an optimum system operation strategy.

The DC load placed on the DC bus obtains the share x_R of the DC bus currents (Figure 7).

$$I_{DCSupply} = x_R \cdot I_{DCBus}$$
$$I_{Inv-i/p} = (1 - x_R) \cdot I_{DCBus}$$
$$x_R \in [0,1]$$

Equation 63: Amount of DC bus current going to the inverter and to the supply of the DC load

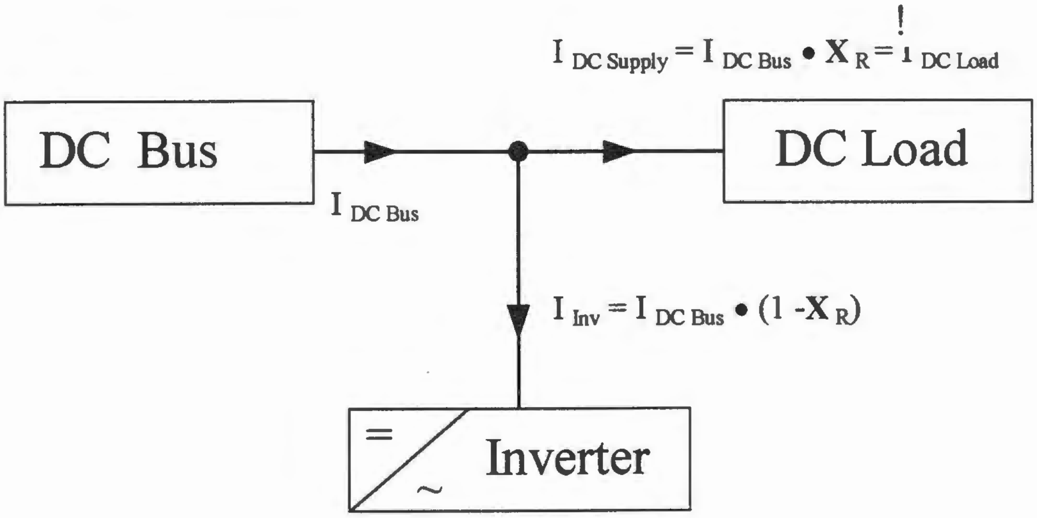


Figure 7: DC bus current routing

The task of the design algorithm is to achieve equal DC demand and DC supply currents.

$$I_{DCSupply} = x_R \cdot [x_{RD} \cdot (I_{ACBus}) \cdot eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} + I_{DCSources} - I_{Bat}] \overset{!}{\equiv} I_{DCLoad}$$

Equation 64: DC current supplied to DC load

As a reminder, all these currents depend on the component sizes and the control operational settings.

The design task also includes finding the right amount of DC bus current to go through the inverter to the AC load in order to optimise the overall load supply. The DC bus current going through the inverter is $I_{Inv-i/p}$:

$$I_{Inv-i/p} = (1 - x_R) \cdot [x_{RD} \cdot (I_{ACBus}) \cdot eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} + I_{DCSources} - I_{Bat}]$$
$$\overset{!}{\equiv} I_{ACLoad} \cdot x_S \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}}$$

Equation 65: DC current input to inverter

It is of advantage to reduce the number of design variables that have to be optimised. Therefore xR is going to be substituted as the amount of AC load that is desired to be supplied through the inverter.

With Equation 64 and Equation 65 we obtain

$$x_R \overset{!}{\equiv} - I_{ACLoad} \cdot x_S \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}} \cdot \frac{1}{I_{DCBus}} + 1$$

Equation 66: Determining x_R, the decision on routing percentages between inverter and DC load

Therefore $I_{DCSupply}$ and $I_{Inv-i/p}$ are becoming

$$I_{DCSupply} = I_{DCBus} - I_{ACLoad} \cdot x_S \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}} =$$

$$= [x_{RD} \cdot (I_{ACBus}) \cdot eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} + I_{DCSources} - I_{Bat}] - I_{ACLoad} \cdot x_S \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}} \overset{!}{\equiv} I_{DCLoad}$$

Equation 67: DC supply relation to AC load and DC bus current

$$I_{Inv-i/p} \overset{!}{\equiv} I_{ACLoad} \cdot x_S \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{I}{eff_{Inv}}$$

Equation 68: Inverter input and AC load relation

3.5 Load balance equations

The load balance equations is the difference between the demand supplied and the demand required for both the DC and AC loads. The aim of the design algorithm is to set this load balance to zero or to a user-defined allowed percentage of unmet or over-supplied load, while minimising system costs.

If the load balance equation is not zero it means that either the demand is over-supplied or under-supplied. In this case of under-supply or over-supply a penalty or cost/benefit description of the excess supply or undersupply is added to the system objective function description which directs the design optimisation process.

Therefore the arriving load currents at the DC load should equal the DC demand requirements. If not there is an imbalance which is weighted with a cost/benefit description or penalty description.

$$I_{DCLoad} - I_{DCSupply} \equiv 0$$

Equation 69: Definition DC bus balance equation

$$Imbalance_{DC} = I_{DCLoad} - I_{DCSupply} \neq 0$$

Equation 70: Definition DC over- (<0) or under-supply (>0)

$$\left.\begin{matrix} I_{DCLoad} - \{[x_{RD} \cdot I_{ACBus} \cdot eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} + \\ + I_{DCSources} - I_{Bat}] - I_{ACLoad} \cdot x_S \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}}\} \end{matrix}\right\} = \begin{Bmatrix} <0\ Excess \\ 0\ \ Balance \\ >0\ Unmet \end{Bmatrix}$$

Equation 71: DC Imbalance equation

$$x_S = \frac{I_{DCBus} - I_{DCLoad}}{I_{ACLoad} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}}}$$

$$\Leftrightarrow x_S = \frac{1}{I_{ACLoad} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}}} \cdot \cdot \left(\left[[x_{RD} \cdot I_{ACBus} \cdot eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} + I_{DCSources} - I_{Bat}] \right] - I_{DCLoad} \right)$$

Equation 72: Determining x_S, the decision on output share between inverter and AC bus

The arriving current flow description at the AC load is the amount of inverter output current and diesel current on the AC bus routed to the AC load (Equation 52) and should equal the amount of AC load demanded (Equation 51: AC current supply equals demanded AC current):

$$I_{ACLoad} - I_{ACSupply} \equiv 0$$

Equation 73: Definition AC bus balance equation

$$Im\,balane_{AC} = I_{ACLoad} - I_{ACSupply} \neq 0$$

Equation 74: Definition AC over- (<0) or under-supply (>0)

$$\left. \begin{array}{l} I_{ACLoad} - \{(1 - x_{RD}) \cdot I_{ACBus} + \\ + [I_{DCSources} - I_{Bat} - I_{DCLoad} + eff_{BC} \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot x_{RD} \cdot I_{ACBus}] \cdot eff_{Inv} \frac{U_{DCBus,Nom}}{U_{ACBus,Nom}} \} \end{array} \right\} = \left\{ \begin{array}{l} < 0\ Excess \\ 0\ \ Balance \\ > 0\ Unmet \end{array} \right\}$$

Equation 75: AC Imbalance equation

Equation 76: Determining of x_{RD}, the routing decision for the diesel generator output

Then it follows that

$$x_S = \frac{(I_{DCSources} - I_{Bat} - I_{DCLoad}) \cdot \frac{1}{1 - eff_{Inv} \cdot eff_{BC}} + (I_{ACBus} - I_{ACLoad}) \cdot \frac{eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}}}{1 - eff_{BC} \cdot eff_{Inv}}}{I_{ACLoad} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot \frac{1}{eff_{Inv}}}$$

Equation 77: x_S in terms of DC and AC load, and DC and AC resources

Plugging x_{RD} and x_S into the equations for $I_{ACSupply}$ and $I_{DCSupply}$ yields I_{ACLoad} and I_{DCLoad} respectively. However, x_{RD} and x_S can only take on certain values between 0 and 1 therefore it can happen that for values of x_{RD} and x_S outside this range the DC and AC loads are not covered exactly, but an imbalance occurs through over- or under-supply of electricity.

In order to determine the overall unmet or dumped load in a system, it is useful to be able to express the overall current imbalance and overall required load current in either AC or DC.

The AC and DC imbalance equations can be converted into each other by

$$Imbalance_{DC} = Imbalance_{AC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom} \cdot eff_{Inv}}$$

Equation 78: DC/AC external conversion

The overall load in AC and DC can be expressed as:

$$Load_{AC} = I_{ACLoad} + I_{DCLoad} \cdot eff_{Inv} \cdot \frac{U_{DCBus,Nom}}{U_{ACBus,Nom}} = Load_{DC} \cdot eff_{Inv} \cdot \frac{U_{DCBus,Nom}}{U_{ACBus,Nom}}$$

Equation 79: Overall load placed on the system expressed in DC and AC current

3.6 Operation strategy formulation

3.6.1 From operation decisions at each time instant to control strategies

In the previous sections the flow of currents in the hybrid system has been described and modelled. The current flow depends on the size and number of devices as well as the operation decisions at each time instant. The operation decisions identified were routing and switching decisions as well as battery and diesel output current decisions. As shown in the previous sections, the routing and switching decisions could be derived from solving the DC and AC balance equations at different nodes in the hybrid system network.

Therefore, the remaining operation decisions at each time instant concern the battery and diesel output current levels. The operation decisions can be optimised for each time step. However, this demands much computing time as optimising the output current levels at each time instant requires a large number of decision variables to be optimised if the simulations are carried out over a large number of time intervals. In addition, adjusting the battery and diesel current output levels at each time instant in a field-operated hybrid system in a remote area is difficult as it would require reliable weather and demand estimates to be available to the system control. The system control would have to be very sophisticated and well tested. The system operation would be vulnerable to any malfunctioning or failures of the control electronics or to the reliability of the data estimates upon which the control strategy was based.

Therefore the development of optimised system control strategies, rather than optimised operating decisions at each time instant, has been undertaken. A control strategy consists of certain predetermined control settings that are set when installing the system. Such settings concern the timing of when to switch on the diesel or not, based on certain values representing the system state, such as the battery state of charge and demand placed on the system. The general time-independent controller settings are modelled and optimised in the developed design algorithm instead of optimising every decision variable at each time instant.

In order to develop operation strategies, the battery and diesel output levels need to be linked to adjustable control settings for a field-operated hybrid system. The battery and diesel output levels are contained in the diesel and battery current equations (Equation 80; Equation 83) derived from the DC and AC balance equations.

$$I_{Bat} = ((1 - x_{RD}) \cdot I_{ACBus} - (1 - x_S) \cdot I_{ACLoad}) \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom} \cdot eff_{Inv}} +$$

$$+ I_{DCSources} - I_{DCLoad} + eff_{BC} \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom}} \cdot I_{ACBus} \cdot x_{RD}$$

$$I_{ACBus} = I_{Diesel-AC} + I_{RE-AC} + I_{OtherSources-AC}$$

$$I_{DCSources} = I_{RE-DC} + I_{Diesel-DC} + I_{OtherSources-DC}$$

Equation 80: Battery current I_{bat} based on DC balance equation

The battery can accept as charge at time t:

$$I_{Bat,ch}(t) = Max[\ 0, Min\ [(\frac{SOC_{max} - SOC(t)}{\Delta t}), I_{Bat\,maxCh}, I_{BatsysCh}\ ]],$$

Equation 81: Battery charging current

where $I_{batmaxch}$ is the maximum allowed charging current. $I_{batsysch}$ is the charge the system wants to supply to the battery.

The battery can discharge at time t:

$$-I_{Bat,Dh}(t) = Max[\ 0, Min\ [(\frac{SOC(t) - SOC_{Min}}{\Delta t}), I_{Bat\,max\,Dh}, -I_{BatsysDh}\ ]],$$

Equation 82: Battery discharging current

where $I_{batmaxdisch}$ is the maximum allowed discharging current

$I_{batsysdisch}$ is the discharge the system needs from the battery

$$I_{Diesel-AC} = (I_{DCLoad} + I_{ACLoad} \cdot x_S \cdot \frac{U_{ACBus,Nom}}{U_{DCBus,Nom} \cdot eff_{Inv}} + I_{Bat} - I_{DCSources}) \cdot \frac{U_{DCBus,Nom}}{U_{ACBus,Nom} \cdot eff_{BC}} +$$

$$+ I_{ACLoad} \cdot (1 - x_S) - I_{RE-AC} - I_{OthSources-AC}$$

Equation 83: Diesel current based on AC balance equation

All these variables are time-dependent.

Both the diesel output current and battery current equations can be converted into each other. Once either the battery or diesel current output level is determined, the other one follows automatically.

3.6.1.1 Prefer AC diesel generator bank

If the AC diesel current output is determined first, then it is realistic to assume that the AC diesel array output level should be as high as possible in order to obtain a high capacity loading level and a high output/cost ratio. Therefore the AC diesel generator bank will cover as much as possible of the AC load share not covered by any AC renewable energy sources, the maximum possible battery charge acceptance, and perhaps any remaining uncovered DC demand.

3.6.1.1.1 AC demand not coverable by AC diesel bank

If the diesel generator cannot fully supply the AC demand that was not covered by any available AC renewable energy sources, then the AC demand could be additionally covered by the inverter output (bi-directional inverter) or covered by the inverter output instead of the AC bus output (normal inverter).

For the latter case, the decision whether to direct the AC diesel output to the AC load, or to the DC bus and use the inverter output to cover the AC demand, depends on which option yields the lowest unmet AC demand.

3.6.1.1.2 Determining DC output levels

After having determined the AC diesel generator output level through this process, the DC battery output level and DC diesel generator output levels can be derived for the case that further DC currents on the DC bus are needed. Here again a decision might have to be made whether the DC diesel output is determined before the battery output level.

3.6.1.1.3 Prefer DC diesel array

If the DC diesel generator output is determined first, then again the highest possible loading factor for the DC diesel is desired, and the battery covers any additional DC current requirements.

3.6.1.1.4 Prefer battery bank

In case the battery output level is determined first before the DC generator output level for covering any additional energy requirements which the AC diesel generator was not able to cover, the battery discharges as much as possible to meet the required DC current levels. If the battery cannot supply the required DC currents, the DC diesel generator is used instead and its output is determined, again to yield highest possible load levels and any remaining DC current requirements are tried to be covered by the battery bank.

3.6.1.2 Prefer battery bank

In case the battery output level is determined before the AC diesel generator output level, then if the battery bank can cover the AC and DC loads together with renewable and other energy sources, the battery is discharged with the appropriate level.

In case the battery plus renewable energy sources cannot achieve demand coverage, the 'prefer AC diesel array' option is executed.

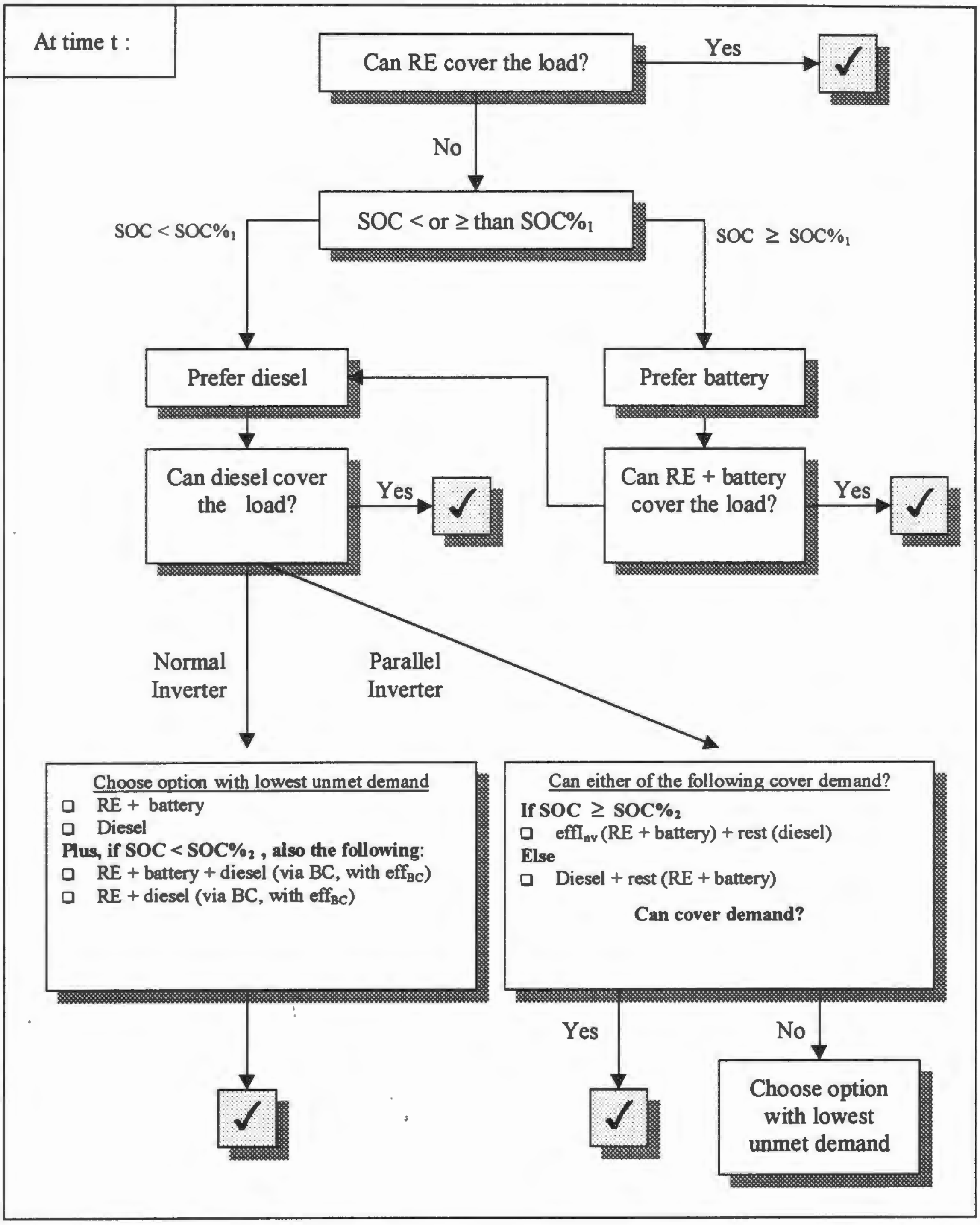


Figure 8: Overview over the decision strategy for system operation

3.6.2 Control settings linked to system state

The previous sections developed the operation decision equations from the DC and AC bus balance equations and identified the decision-making for the operation decisions at each time instant assuming certain 'preference' settings. The decision-making is demonstrated in Figure 8 showing the different possible operation decisions at a time instant depending on the 'preference' settings.

As can be seen the decision-making depends on the amount of demand that could not be covered by renewable energy sources, and the 'preference' setting regarding component usage. This means the only actual 'outside' or controller decision-making occurs when determining which components to prefer, namely, the battery bank, AC diesel generator or DC diesel generator, to satisfy the AC and DC load requirements.

If several battery banks and several diesel generators exist, the decision making indicates the order in which to prefer the different diesel generators and battery banks in the system to cover the AC and DC loads. This decision depends on the efficiency with which the different diesel generators and batteries can cover the loads. Therefore in a case where several diesel generators are available to cover the load, then the generator or the combination of generators is chosen which have together the closest nominal power rating to the desired load level. If additional other DC or AC energy sources are available, the decision making needs to be extended to include those as well in the preference settings.

In order to optimise the component preference settings, these have to be linked to system state indicators. The main system state indicators are the state of charge of a battery bank, the diesel capacitor factors, the achieved system reliability and the diesel runtimes.

The preference settings are encoded when installing the system controller. During system operation, the system is operated according to the answers 'yes' or 'no' to the preference questions, that is, whether the values of the battery state of charges and the diesel capacity factors are in the range as predefined in the controller settings. The number of times the batteries have reached full state of charge or have been in the minimum state of charge, as well as the number of diesel runtime hours can also be included.

The boundaries of these decision ranges can then be optimised by the algorithm which is a big advantage over optimising each operation decision at each time instant because the boundaries are valid for the whole simulation interval and do not change at every time instant.

To illustrate this the following example is given:

Prefer diesel generator to battery	*Prefer the battery to the diesel generator*
If the SOC of the battery is between s_{A1}% and s_{A2}%.	If the SOC of the battery is between s_{B1}% and s_{B2}%.
AND	*AND*
If the capacity factor of the diesel generator is between c_{A1}% and c_{A2} %.	If the capacity factor of the diesel generator is between c_{B1}% and c_{B2} %.

Table 1: Example of component preference decision making

The ranges c_{Ai} and c_{Bi}, as well as s_{Ai} and s_{Bi} are optimised. The more components used, the more control setting decision ranges need to be optimised.

The output of minimising the objective functions is the desired decision variables for operation control settings and sizing. They give a system design which, when operated according to the operational control settings, will yield satisfactory electricity supply at lowest life cycle costs.

3.6.3 Formulation of system performance indicators

An indication of the overall system performance of the designs is given in terms of the ratios between supplied and generated energy, and the average ratios for battery cycling and diesel loading.

$$Index_{Performance} = \frac{SuppliedEnergy}{GeneratedEnergy} = \frac{P_{DCSupply} + P_{ACSupply}}{P_{DCSources\backslash Bat} + P_{ACSources}}$$

Equation 84: Overall system efficiency as performance index

$$Ratio_{BatteryCycling} = \frac{\sum_{t=1}^{T} SOC(t)}{SOC_{Max} \cdot T}$$

Equation 85: Average battery cycling ratio during a time interval

$$Ratio_{DieselLoading} = \frac{\sum_{t=1}^{T} Power_{Diesel}(t)}{Power_{Diesel,Max} \cdot T}$$

Equation 86: Average diesel generator capacity loading during a time interval

These ratios are useful when comparing different designs for the same or similar application.

4 Summary

The developed hybrid system performance model has been presented here with the component models, the power flow between energy sources and demand and the modelling of the operation strategy.

The sizing variables, that is, sizes of component types and their number to be installed and optimised were collected in a vector x_{size}. The operation decision variables to be optimised represent routing and operation decisions that are based on the power flow modelled for the hybrid system. The hybrid system model needs to be optimised with respect to the decision variables x_{size} and $x_{OpStrat}$ such that a minimum of the life cycle costs is achieved. Table 2 gives an overview over the main decision variables used in the optimisation.

As it is computationally intensive to optimise each individual decision variable for the operation decisions at each time instant, an alternative was to formulate, within the model and based on the operation decisions, possible operating strategies that can be implemented in the field with available controllers. The optimisation then does not have to find the optimal value for each $x_i(t)$, but will find the optimal values for the time-independent controller settings, which requires less computation.

The identified decision-making for the operation decisions at each time instant depends on the amount of demand that could not be covered by renewable energy sources, and the 'preference' settings regarding which components to prefer to satisfy the AC and DC load requirements. The decision which component to prefer is made between the battery bank, AC diesel generator or DC diesel generator.

The component preference settings are linked to system state indicators such as the state of charge of a battery bank, the diesel capacitor factors, the achieved system reliability and the diesel runtimes.

During system operation, the system can be operated according to whether the values of the battery state of charges and the diesel capacity factors are in the range as predefined in the controller settings. Because only the time-independent boundaries of these decision ranges need to be optimised by the developed algorithm, this is a big advantage over optimising each operation decision at each time instant.

Table 2: Overview of the decision variables in the hybrid system performance model

Modelling steps	*Decision variables x*
Sizing	$x_{Size} = [x_{PV,parallel}, x_{WT,parallel}, x_{sizeD}, x_{bat,parallel}, x_{sizeBat}, inverter_{type}, batch\arg er_{type}, x_{sizeOtherSources}$ **Equation 2:** Sizing optimisation variables
Operation decisions	$x_{OpDec}(t) = [x_{bat}, x_{diesel}, x_S, x_R, x_{RD}, x_{RInv}, x_{RBC}, x_{load}](t)$ **Equation 3:** Operation decisions to be optimised at time t
Operation strategy	$x_{OpStrat} = [x_{SOC\%1}, x_{SOC\%2}, x_{UnmetLoa}]$ **Equation 4:** Time independent operation strategy
Constraints	$I_{DCSupply}(t, x_{Size}, x_{OpStrat}) \overset{!}{\equiv} I_{DCLoad}(t), \quad I_{ACSupply}(t, x_{Size}, x_{OpStrat}) \overset{!}{\equiv} I_{ACLoad}(t)$ **Equation 5:** Constraints on operation
Objective function	$LCC = CaptialCosts(x_{Size}, x_{OpStrat}) + Discounted\ OperationCosts(x_{Size}, x_{OpStrat})$ **Equation 6:** Life cycle costs as dependent on sizing and operation strategy

Appendix A: Detailed PV and battery model

A1: PV model

The model of the PV module consists of two parts: an electrical model and a thermal model based on an energy balance (Schuhmacher 1993).

In the electrical model the relationship between voltage U_c [V] of a solar cell and current density j [Am^{-2}] is given by a two-diode model

$$j = j_{ph} - j_{01} \cdot (exp^{e_0 \cdot \frac{U_c + j \cdot r_S}{\alpha_k \cdot T}} - 1) - j_{02} \cdot (exp^{e_0 \cdot \frac{U_c + j \cdot r_S}{\beta_k \cdot T}} - 1) - \frac{U_c + j \cdot r_S}{r_{sh}}$$

Equation 1: PV current density model (Schuhmacher 1993)

where

r_s series resistance parameter of the cell [Wm^2]

r_{sh} shunt resistance parameter [Wm^2]

α diode parameter (should be set to 1 in case of the standard two diode model)

β diode parameter (should be set to 2 in case of the standard two diode model)

T cell temperature [K]

e_0 charge of an electron ($1.6021 \cdot 10^{-19}$ As)

k Boltzmann Constant ($1.3854 \cdot 10^{-23}$ JK^{-1})

Hence, the operating point of the PV generator is given by

$$U = U_c \cdot N_s$$
$$I = j \cdot A_c \cdot N_p$$

Equation 2: PV operating points (Schuhmacher 1993)

where

A_c area of a single cell [m^2]

N_s number of cells in series (whole generator)

N_p number of cells in parallel (whole generator)

The light-generated current density j_{ph} [Am^{-2}] is proportional to the global radiation G [Wm^{-2}] on the generator plane and is assumed to be linearly dependent on the cell temperature T [K]

$$j_{ph} = (C_0 + C_1 \cdot T) \cdot G$$

Equation 3: Light-generated PV current density (Schuhmacher 1993)

where

C_0 coefficient of light-generated current density [V^{-1}]

C_1 temperature coefficient of light-generated current density [$V^{-1}K^{-1}$]

The dependence of the saturation current densities j_{01} and j_{02} on temperature is given by

$$j_{01} = C_{01} \cdot T^3 \cdot exp^{-\frac{e_0 \cdot U_{gap}}{k \cdot T}}$$

$$j_{02} = C_{02} \cdot T^{\frac{5}{2}} \cdot exp^{-\frac{e_0 \cdot U_{gap}}{2 \cdot k \cdot T}}$$

Equation 4: PV saturation current densities (Schuhmacher 1993)

where

C_{01} coefficient of saturation current density $[Am^{-2}K^{-3}]$

C_{02} coefficient of saturation current density $[Am^{-2}K^{-5/2}]$

U_{gap} band gap (set constant to 1.12V).

The dependence of the band gap on cell temperature is neglected.

The thermal model is based on an energy balance

$$m_{mod} \cdot N_{mod} \cdot c_{mod} \cdot \frac{dT}{dt} + P_{el} = \dot{Q}_G - \dot{Q}_r - \dot{Q}_c$$

Equation 5: PV thermal model

where

m_{mod} mass of a PV module [kg]

c_{mod} specific heat of a PV module $[Jkg^{-1}K^{-1}]$

P_{el} electrical power output of the generator [W]

$\dot{Q}_G$ insolation [W] on the whole generator.

$\dot{Q}_r$ losses through radiation [W]

$\dot{Q}_c$ losses through convection [W]

The absorbed insolation is modelled by

$$\dot{Q}_G = a \cdot G(t) \cdot A_{mod} \cdot N_{mod}$$

Equation 6: PV module absorbed insolation

The coefficient a of absorption is assumed to be constant. Losses through radiation are modelled by

$$\dot{Q}_r = 2 \cdot e \cdot A_{mod} \cdot N_{mod} \cdot \sigma \cdot (T^4 - T_a^{\,4})$$

Equation 7: Thermal losses

where

e emission factor

σ Stefan-Boltzmann Constant ($5.6697 \times 10^{-8} Wm^{-2}K^{-4}$)

T_a ambient temperature [K]

Losses through convection are given by

$$\dot{Q}_c = 2 \cdot \gamma \cdot A_{mod} \cdot N_{mod} \cdot (T - T_a)$$

Equation 8: Convection losses

In case of free convection the heat loss coefficient g is set to

$$\gamma \overset{!}{\equiv} \gamma_f = 1.78 \cdot (T - T_a)^{\frac{1}{3}}$$

Equation 9: Free convection

Forced convection is modelled through

$$\gamma_w = \frac{4.77 \cdot v_w \cdot 0.8 \cdot t_{mod} \cdot N_{mod} - 0.2}{1 - 0.17 \cdot v_w - 0.1 \cdot t_{mod} \cdot N_{mod} - 0.1}$$

Equation 10: Forced convection

with wind speed v_w. In case of mixed convection γ is set to

$$\gamma = \sqrt[3]{\gamma_f^{\,3} + \gamma_w^{\,3}}$$

Equation 11: Mixed convection

A 2: Battery model

The model of the battery is based on the model of Shepherd (1965) and is explained in Schuhmacher (1993). The relation between voltage U [V] and current I [A] for discharging (I <0) the battery is

$$U = U_{od} - g_d \cdot H + r_d \cdot \frac{I}{Q_n} \cdot (1 + M_d \cdot \frac{H}{C_d - H})$$

Equation 12

where

U_{od} open circuit voltage (discharge) [V]

g_d electrolyte coefficient (discharge) [V]

r_d inner resistance parameter (discharge) [W Ah]

M_d battery type coeffiecient (discharge)

C_d capacity coefficient (discharge)

H normalised depth of discharge (1 – Q/Q_n)

For charging the following equation is used

$$U = U_{oc} - g_c \cdot H + r_c \cdot \frac{I}{Q_n} \cdot (1 + M_c \cdot \frac{F}{C_c - F})$$

Equation 13

where

U_{oc} open circuit voltage (charge) [V]

g_c electrolyte coefficient (charge) [V]

r_c inner resistance parameter (charge) [Vh]

M_c battery type coefficient (charge)

C_c capacity coefficient (charge)

F normalised state of charge Q/Q_n

In case of charging an ampere hour efficiency h after Wood/Crutcher is used. The current at which gassing effects occur is defined through

$$I_m = \begin{cases} b_w \cdot Q_n \cdot (1-F) & \text{if } F < F_l \\ b_w \cdot Q_n \cdot (1-F_l) & \text{else} \end{cases}$$

Equation 14

where

b_w Wood parameter

F_l limiting state of charge [V]

F normalised state of charge Q/Q_n

Q_n nominal capacity [Ah]

Depending on the actual current I, the Wood-efficiency is defined as

$$\eta = \begin{cases} a_w & \text{if } I \le I_m \\ a_w \cdot \frac{I_m}{I} & \text{else} \end{cases}$$

Equation 15

where

a_w Wood parameter

Using the described battery efficiency, the battery state of charge SOC at the next time step (t+Δt) is calculated as

$$SOC(t+\Delta t) = SOC(t) + (\eta(t) \cdot I_{Bat}(t) - \frac{q_s \cdot SOC(t)}{100 \cdot t_s}) \cdot \Delta t$$

Equation 16

Energy losses occur when charging a battery. The battery efficiency drops further when the battery is ageing or is not operated correctly.