

A Multiple Factor Analysis of the Relationship  
Between Musicality, General Intelligence,  
and Literary Ability.

By

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## Table of Contents.

Acknowledgements.....	ii.
Foreword.....	iv.
I. Introduction.....	1.
II. Literature Related to This Study.....	3.
III. The Nature of the Problem.....	10.
IV. Apparatus, Materials, and Procedure.....	12.
V. Results: (i) Statistical Analysis.....	27.
(ii) Psychological Considerations.....	47.
VI. Conclusions.....	75.
VII. Summary.....	110.
Appendix.....	113.
Bibliography.....	115.

## Foreword.

The judgement 'that person is musical' is commonplace in our daily lives. The present writer has grown up in an intellectual environment coloured to a marked extent by interest in music and the related arts. To him it had always been a very natural phenomenon that a person should be regarded as 'musical' or 'not-musical', and it seems justifiable to state that his attitude was but typical of his fellow-men.

There appears to have prevailed for some time the idea that musicality represented some 'innate permeating spirit' which distinguished the possessor of the 'musical spark' from his brethren of more common clay. This implied that certain human beings had some general musical capacity or 'turn of mind' which was not possessed by others. Is there any justification for such an assumption? It was a question such as this which first stimulated the writer's interest towards a scientific investigation of the problem of musicality. This interest was refreshed almost continuously by a consideration of a number of further questions inevitably bound up with the consequences of the initial problem: Of the musical individuals themselves, was it true to say that their musicality was evidenced homogeneously either in degree or in quality throughout different musical operations? Was, for example, the music student who was keenest at discrimination of pitch also best at the memorizing of his music? Could anything definite be stipulated, either on a priori grounds or on the basis of experience, about the relationship existing between one's musicality and one's general intelligence? And what of the so-called allied arts; did literary ability, for instance, likewise involve a special quality, and was this ichor the same as was to explain musicality?

It was as an attempt to provide a scientifically-investigated answer to such and similar questions that this analysis was originally undertaken. At the time of writing the field of the problem was entirely virgin soil; and though the writer would like to feel that this work does take an appreciable step towards clearing the obstructing growths of ignorance on the subject, yet he is obliged to point out the severely limited nature of the scope of this thesis, owing partly to the brevity of time at his disposal and partly to the amount of wasted labour attendant upon all "pioneer" endeavour.

Any value in a study such as this will lie, surely, in its effectiveness in dispelling illusions of existing ideology and pointing the road for further workers. The results of the analysis, it is hoped, will not be regarded as altogether uninteresting or uninteresting, but they are far from fully or categorically providing a solution to the many questions about musicality with which the intelligent thinker must be confronted. It is hoped, however, that perusal of these pages with reference to musicality will show the shallowness of deductive logic arm-chair conclusions about the nature of mental traits. It is becoming increasingly clear that the only scientifically valid hypotheses are those based upon data which have been experimentally treated to a vigorous analysis. So, if the general verdict passed on this work may justifiably be written as: *Disciplina vitae scipio*: then it does not seem over-much to claim that this first delving into research of the present writer will not be considered altogether devoid of scientific interest both to the general reader and the future musical-research worker.

The primary object of this treatise was to apply the methods of Mental Factor Analysis as an aid in answering the question: What is musicality?

Historically, the analysis of any intellectual capacity has been shown to be a task of the greatest complexity attended by theories and objections, hypotheses and counter-hypotheses. Thus we had mental abstractions provided carte blanche by the Monarchic doctrine, the Oligarchic or Faculty psychology theory, and the Anarchic or 'pooling' method of the treating of human abilities. Each of these approaches has generally been condemned officially; yet many of the points raised by these various doctrines have survived in modern-day theories. Perhaps the two best-known and most widely accepted of contemporary methods of analyzing human ability are Spearman's Two Factor theory (29) and Thurstone's theory of Multiple Factor Analysis (35). It is with the methods and principles implied in two-factor and multiple factor techniques that we shall be concerned here.

To enable it to be treated by such methods, musicality had to be approached in a special way: An attempt was made to list the basic operations of individuals in which one might reasonably conclude evidence of musicality to be manifested; where possible, there were devised or adopted tests which would in addition admit of the grading of individual performances on the tasks prescribed. A difficulty here was immediately apparent. There being no satisfactory definition of musicality, how could one tell in which operations musicality was manifested? As will be described in the next section, recourse was had to current hypotheses and what experimental evidence already existed in the field. This procedure resulted in a final selection of tests of musicality. It must be stressed here that these tests are by no means exhaustive. The shortness of the experimental period, the difficulty in persuading a sufficiently large number of subjects to spend more than a few hours for testing purposes, and the restricted range of suitable apparatus, are but a few of the reasons why the selection of the tests must be regarded as eclectic only. But it does seem feasible to contend that the selection is such that representative tests of all the major divisions of musicality are here included. Reference to this point will be made again when the topic of 'relevant literature' is discussed in the next section.

A tantalizing issue has always been the debated interdependence of musicality and intelligence. It was hoped that one of the relatively definite results of this analysis would be the providing of an estimation of any such relationship; and it may fairly be held that, in so far as the intelligence and music

batteries of tests were valid for their purpose, an answer to this question has been arrived at. It will perhaps be generally agreed that music and poetry are commonly thought to be closely allied arts; So at some future date there will no doubt be research made into the existence of a possible common 'artistic' element in the human mental equipment. As a first step towards such an end, it was thought of use to attempt a rough estimation of the psychological connections between musicality and literary ability. Hence what appeared to be three valid tests of literary ability were included among the total tests. Probably it will be said that such tests are greatly inadequate for their purpose, since the attempt to gauge literary ability of individuals must necessarily be lengthy and complicated. On the other hand, while admitting that the tests of this ability given here make no pretence to exhaustiveness, we may point out that they will, however, be sufficient for our present purposes as, through them, we can obtain an approximate index to the individual's knowledge of, and facility in the use of verbal ideology.

We must make one apology. During the investigation the analysis of the intelligence and literary tests shed light on some unexpectedly interesting features of these tests, and, consequently, these tests were for a time analysed and discusses per se, for their own worth. In the end various conclusions about general intelligence and literary ability were drawn although such conclusions had no direct bearing on the problem of musicality.

It is hoped that the results of the analysis of a battery of tests such as this, including, as it does, tests of musicality, general intelligence, and literary ability will throw sufficient light on the problem of musicality for that little-understood aspect of the make-up of the human mind to be amenable to more precise and exhaustive research.

## 11. A Survey of Literature Related to this Study.

In this country music is an art, but not yet a science. The musical world restricts itself to the training of musical performance, music teaching, composition and the enjoyment of music listening. But not a word has been written about the scientific side of musicality, about what it is and to what else in our intellectual lives it may be related. For this reason, all the literature pertaining to our present purpose will be seen to have been written beyond our native shores, and that principally in different journals by various writers, and two weighty tomes by C.E. Seashore (25,27) who has done a tremendous amount in the purely experimental field but has not applied the technique of factor analysis to his results. We will deal first with the literature relevant to music testing, then that relevant to the relationship of musicality and intelligence, and finally that relating musicality to literary ability.

A surprisingly large amount of literature may be obtained on the subject of music testing, yet very little of this is relevant to the present problem. At the time of writing Seashore's two books (25,27) remain the outstanding general works. Although only one of the tests used in the present investigation was taken 'in toto' from this source, much useful advice was gleaned from a study of the contents of these works. Revez (22) proposes eight measures of musical capacities based on his study of the prodigy Nyiregehazi, and insists that musicality is unitary rather than atomic. This is in line with a point of view such as the previously mentioned Oligarchic doctrine and implies that various operations in which musicality is manifested are all governed by the quality of the individual's 'music faculty'.

Possible music tests may be devised in great numbers, and for the experimenter who has to rely on just a few on which to base his calculations, the selection of a round dozen or so of these represents an embarrassing task. The problem is then: which kinds of tasks would test the basic aspects of musicality? The schematization of the tests finally adopted was based upon Seashore's theory\*. The relevant quotation is as follows:-  
The normal musical mind is first of all a normal mind. What makes it musical is the possession, in a serviceable degree, of those capacities which are essential for the hearing, the feeling, the understanding, and, ordinarily, for some form of expression of music with a resulting drive or urge toward Music". From this he derives his 'chart' of the musical mind, the regions being:-

- (i) The Sensory Capacities;
- (ii) Musical Imagery, Imagination, and Memory;
- (iii) Musical Feeling.

\* Psychology of Music. p.2.

(i) The Sensory Capacities are evidenced in the appreciation of the psychological attributes of sound, namely, pitch, intensity, duration, and timbre. These are the four ultimate tonal attributes, basic for all musical effects. These he regards as arising from the physical properties of the sound wave (i.e. frequency, amplitude, duration, and form) to which he insists we must return for any scientific account of musical phenomena.

As early as 1904, however, Meyer (13) had advanced the view that these are not all independent attributes, but in some cases are separable for convenience only. Today we find significant divergencies from Seashore's position among various workers e.g. Rich (23), and in particular an insistence upon the importance of tonal volume.

For our present purpose it is sufficient that we pass over such controversial points and use sufficient tests for the defining of the sensory capacities in question. We regard this branch of musicality as having four aspects: the tonal, the dynamic, the temporal, and the qualitative. Seashore regards each of these as being the 'main trunk of a type'. Concerned with the tonal type (peculiarly sensitive to pitch and timbre) there were tests of discrimination of pitch (Test K), absolute pitch (Test L) and discrimination of intervals (Test M); with the dynamic type (fine acuity of hearing and sense of loudness) discrimination of intensity (Test S); with the temporal type (peculiarly sensitive to time, tempo, and rhythm) discrimination of time (Test O) and discrimination of rhythm (Test P); with the qualitative type (sensitive to timbre and harmonic constitution of the tone) the test for discrimination of consonance (Test N).

The division of the tests according to Seashore's typology is, from our point of view, purely descriptive. Seashore finds these sensory capacities largely inborn and functioning from early childhood. He goes on to say, that, after a comparatively early age, they do not vary with intelligence, with training, or with increasing age, and hence diagnosis of musical talent is possible before training is begun and the way pointed out for certain very definite principles of musical education. Furthermore, the apparently complex forms of sensory capacities also tend to be elemental to a considerable degree; that is, a young child has these capacities long before he begins to sing or know anything about music. It is the meaning, and not the capacity of these forms of impression which are trained and which matures with age in proportion to the degree of intelligence and emotional drive. We have thus seven tests of musical sensory capacities; we proceed here to outline the literature relevant to the features of the more specific individual tests.

## (a) Discrimination of Pitch:

Köhler (12) suggests that too much attention has been paid to pitch and urges the importance of tonal body. From the Iowa laboratory come a number of studies dealing with pitch, and the measurement of its differential limen. Seashore (26) shows that for this purpose the tones should be rich, constant in intensity and loud enough to be easily heard. Smith (28) showed that pitch discrimination gives a meager response to training. Seashore (27) finds that the physiological limit for the sense of pitch does not vary significantly with intelligence. He says: 'the moron may have as keen a sense of pitch as the philosopher.' Likewise he can discover no concomitant variation between sense of pitch and age. He supports Smith's findings with regard to the effect of training on the sense of pitch, pointing out that training, like maturation, results in the conscious recognition of the nature of pitch, its meaning, and the development of habits of use in musical operations, but that it does not modify the capacity of the sense organ any more than the playing of a good violin may improve the quality of its tone. For Seashore, since pitch is the fundamental character of a tone, and pitch discrimination is a measure of the capacity of this sense, "it ordinarily may be regarded as the most basic measure of musical capacity that we have. It determines not only what we shall hear, but fundamentally what we shall remember.....and what emotional reaction we shall have".

## (b) Absolute Pitch:

Previous work on absolute pitch serves to emphasize the complexity of the phenomenon both in its psychological basis and as an index of musicality. Köhler (12) insists that it does not arise exclusively from the sense of pitch; Gough (8) likewise recognizes the significance of non-pitch factors in the introspections of her subjects; Seashore (27) thinks that the identification is not only in terms of pitch but also in part in terms of timbre or tone quality, particularly in relation to a similar instrument; Gough further shows at some length that it can be produced by training, a conclusion emphatically substantiated by Moll (17). Revez (22) suggests that it indicates unusual musicality.

## (c) Discrimination of Interval:

There has been very little work done on discrimination of interval as the ability to judge the spatial distance between two notes sounded either simultaneously or successively. All in all, there seems to be a tendency to regard interval as a configurational phenomenon depending on many factors, and unstable alike in its outlines and its affective values. Moran and Pratt (16), among others, have shown that small variations in size of intervals do not change their essential character as thirds, fifths, etc., a result which

Ogden (18) explains as due to the differential limen for volume not coinciding with that for pitch. Farmsworth (7) shows that with increase of musical training there goes an increasing tendency to judge the pitch of a combination of tones as the pitch of the upper tone.

(d) Discrimination of Consonance:

This is a more complex musical-mental phenomenon. Of the older work reference must be made to the classic view of Helmholtz which explains dissonance as due to beating partials, and to Stumpf's (32) explanation in terms of tonal fusion. More recently the multiplicity of criteria is seen in the work of Guernsey (9) who has shown that our reaction to intervals is governed by fusion, smoothness, and unpleasantness and that these are more or less independent. Concerning the influence of learning, experience and tradition, Valentine (37) shows that practice is a potent factor in evaluating discords; Ogden (19) postulates a factor of racial adaptation; and Peterson (21) explains the system of interval preferences largely in terms of musical tradition and use. Doubt as to antimony between consonance and dissonance is cast by Moore (15) and Heinlein (10) whose work indicates that no absolute distinction is possible between consonance and dissonance. Bugg (3) points out that consonance is affected by a plurality of conditions, and advocates the correlation of sets of conditions with types of discriminations obtained. Seashore (27) combines the three consonance criteria, blending, smoothness, and purity, and arrives at a final Norm for Rank of Consonance and Dissonance. This last was employed in the present investigation. Seashore finds discrimination of consonance to be relatively independent of age, intelligence, or training. For him, consonance is an affective process.

(e) Discrimination of Intensity:

Most of the significant work on intensity has been carried out in the Iowa laboratory. Seashore (27) points out the following results: That there are two fundamental aspects of the hearing of intensity: first the hearing acuity measured in terms of the faintest audible sound; and, second, the ability to hear differences in the intensity of sounds, which is measured in terms of the least perceptible difference and is usually called intensity discrimination (the aspect that was tested here); that there is no evidence to show that intensity discrimination varies with age, intelligence or training.

(f) Discrimination of Time:

Investigation into the temporal aspect of musicality has mainly been into the problem of rhythm, and of the little that has been done on the sense of time, Seashore's (25)

contribution alone is significant. He regards time as a basic condition of the perception of rhythm, but also stipulates that precision in the sense of time is not essential to the feeling of rhythm. For him, the sense of time, unlike the sense of pitch or intensity which depend on the sensory mechanism of the ear, is capable of large variation for different individuals, and he explains it by a motor theory of time which is bound up with a time imagery complex. Individual differences are explained, therefore, in terms of a characteristic predisposition of the individual, resulting in a particular degree of favourableness for the grasping of time intervals in terms of his own action. He finds little correlation between sense of time and age; training has but small effect on this capacity; intelligence appears to vary independently of the sense of time. He finds a correlation of .17 (p.e. .04) between sense of time and sense of pitch.

(g) Discrimination of Rhythm:

We are dealing here with musical rhythm, a mental function whose significance is only partly exhausted by its musical manifestations. Stetson (31) formulates an ingenious and convincing motor theory of rhythm. Tuthill (36) shows that the presence of rhythmic factors cause time deflections in the reactions of highly trained musicians. A correlation of .71 is found between rhythmic value of successive intervals and judgements of pleasantness by Sterzinger (30). Woodrow (38) produces results which are, however in conflict with those of Sterzinger. Seashore (25) regards rhythm, the instinctive disposition to group recurrent sense impressions, as being more complex than attributes of sensation, like time and intensity. Elsewhere\* he postulates a similarity between rhythm in music and rhythm in poetry based on the 'attention wave'.

(ii)

Following Seashore's schema of the elements of the musical mind we pass on to the next 'region' which he entitles Musical Imagery, Imagination, and Memory. We quote: "Granting the presence of sensory capacities in adequate degree, success or failure in music depends upon capacity for living in a tonal world through productive and reproductive imagination". He elaborates upon the related topics of sensations of a motor and kinaesthetic nature, and upon the mental imagery imagination all associated with the possession of the elusive 'musical temperament'. Unfortunately the conditions of the present investigation did not allow tests of such a nature to be used. But if the affective side of musicality is to be analysed in addition to the intellectual aspect here treated such tests will be both relevant and necessary.

The other section of this branch of the field was 'musical memory'. For Seashore, "while retentive and serviceable memory is a great asset to a musical person, it is not at all an essential condition for musical mindedness. A person may have

naturally very poor memory of all kinds and get along well in music, just as an absent-minded philosopher may get on very well in his field. A reproductive and an unretentive mind may be equally musical."

It was possible for our purpose to obtain two, or if absolute pitch (memory for isolated tones) be included, three, measures of this capacity. These were the tests on Musical Memory and Musical Retentiveness. Seashore (25) regards musical memory as being capable of much cultivation. The work on musical learning and memory is meager. In passing we note merely that the Musical Memory test was constructed to measure memory for melody itself, while the Musical Retentiveness test measured the more elemental factors of melody.

(iii) The literature relevant to nine of the ten music tests employed has now been dealt with. The tenth test, 'Emotional Sensitivity to Music', applies to yet another 'region' of Seashore's scheme, viz., Musical Feeling.

Beaunis (1) discusses the derivation of musical emotion from the factors of rhythm, duration, intensity, pitch, and timbre, and emphasizes the importance of mental adaptation to music. Hevner (11) postulates that rhythm, tempo and pitch, but not melody, harmony, or modality, give the best guide in interpretation of the affective tone of music. Seashore (25) contends that it is possible to classify persons into characteristic types of listeners in terms of affective responsiveness. The response to a musical situation, in his view, is dependent on and will vary with the acuity of the sensory capacities; further, the limits and characteristics of the affective life are set largely by the limits of intelligence and natural aptness in motor skills.

This concludes the discussion of the literature on the possible consequences and implications of the music tests. Before passing on, we point out various other fields in which work will have to be done before an exhaustive inventory of the capacities of the musical mind can be said to have been completed. Such fields are: Musical Imagery and Imagination, Musical Performance and Creativity, and sensory capacities like the sense of timbre. The literature dealt with here is by no means indicative of the amount written on musicality, but it is representative of and relevant to the work done on those aspects of musicality which are analysed in the present study. The literature discussed here is restricted to points upon which some light may conceivably be thrown by this analysis.

The inclusion of the intelligence and literary tests in the battery was, as already explained, the result of a more or less original interest in and theory as to the relationship between these two sub-groups and the musical sub-group. The literature describing experimental evidence on any such relationship is extremely scanty:-

Miller (14) made a study of the school achievements of 826 students from eighteen to twenty one years old, to ascertain the relation of musical ability to other capacities, and confirms other students in finding that musical talent is ordinarily accompanied by good all-round ability. Seashore (25) postulates a musical intellect based on the view that though a person may be poor in mathematics or other sciences he may be a good thinker in music. His general position\* is that "on account of the emotional bent and the necessary activities in art, musicians live largely in the world of feeling and as a class have sacrificed much in intellectual pursuits for their goal. Their learning is more contingent upon the feeling of appreciation and emotional action than upon facts and reasoning. Therefore, the cultivation of scientific and abstract thinking has been generally neglected in musical education. On the other hand, the musical profession makes as high a demand upon the intelligence as any other profession. Rating on intelligence as a supplement to measurement of musical talent is one of the best indices for the prediction of success in musical education or a musical career....."

The correlation between abstract intelligence and art appreciation was shown positive but low as a result of an investigation by Carroll and Eurlich (4), from which was concluded "only that when abstract intelligence appears in excess, it precludes the possibility of an unusually low score in art appreciation; when there is a deficit, as in the case of the near feeble-minded, it does not preclude the possibility of a comparatively high score in art appreciation.

Spearman (29) estimates the ratio of general intelligence to musical ability as 1:4. A systematic investigation into the correlations between music, poetry and intelligence is described by Rigg (24). Unfortunately the statistical population in this case was only 65 in number. He finds that the correlation between discrimination in music and discrimination in poetry is low, and is lower still if intelligence is partialled out. Doubt is cast on the existence of a specific aesthetic discrimination factor.

\* C.E.Seashore (27) p.177.

111.

The Nature of the Problem.

## (i) Statement of the Problem

The purpose of this investigation is, primarily, to determine what capacities, be they sensory, intellectual, affective, or what you will, are needed for their possessor to be dubbed 'musical'. At a time when the advent of the factorial analysis of human ability has given quite a new complexion to the face of psychological research, experimental data based on human operations are being ladled in a wholesale fashion into the mathematical melting-pot and the resulting mental ore is thereby moulded by the experienced tools of the psychologist into finished components of human abilities. To the best of the writer's knowledge, no such work on musicality has been attempted, and it seemed to him that both the time and the subject were ripe for treatment. Accordingly the question was asked: In what human operations might musicality be supposed to be manifested? Ten such operations were produced as a result of this enquiry.

To these were added were added six more tests in the hope that a solution might be obtained to the "question which has been a bogey of the musical profession and the butt for scurrilous remark" ever since music became a serious art: 'Does musicality imply a low intelligence'?

These sixteen tests, finally had their number augmented by three further tasks all of an undoubted literary nature. As these last three tests were deemed by competent authorities to be quite a fair measure of an individual's literary ability it was hoped that their inclusion in the total battery would at least provide an embryonic pointer to the existence of an aesthetic or artistic factor which would show itself in operations dependent upon more than one of the Nine Arts. At the same time it was thought that it might be interesting to test for a relationship between literary ability and intelligence.

The exposition of our thesis, then, is: 'Musicality: what is it; through which mental factors is manifested; how is it related to innate intelligence; is it explained by the same mental components explaining literary ability? How is literary ability itself related to intelligence'?

## (ii) Need for Its Investigation.

An analysis of musicality, systematically and scientifically carried out, must yield results of great service to musical educators. Apart from the intrinsic value of such a work per se, academic and scholastic fields of music are sorely in need of some absolute standards round which to

systematize their tenets of education. There is also a social need for such a dissection of the body of musical education. The fact that contemporary music is of the lowest quality in centuries; the fact that numerous children in every little town are put to musical training because 'their parents like music' or 'think it would be so nice if their children could play the piano or the violin'; the fact that many young men and women are studying music teaching as a means of livelihood because as children they were thought to be musical, and who are now compelled to finish their training although they have come to detest the drudgery associated with part of their education; and the fact that general public interest in music is waning for want of a better type of music performer, composer, and teacher, are but some of the reasons why the services of science are called for in the task of keeping music as an art in a scientific age.

### (iii) Its Practical Value.

If we can analyse musicality and if we can isolate the mental factors which the musical person possesses, then we can perform a definite service to the musical world and to society in general. Knowing the primary musical abilities, it represents but a step to the formation of prognosis tests. And if, as seems likely, musical abilities are evidenced at an early age, then long before the maturing child is required to be set to the learning of its task in life, such prognosis will reveal its capacities. The civic order of the highest efficiency must be that in which, as in Plato's Republic, all do that task which by nature they are best fitted to perform. It seems not over-much to anticipate that when we have students of music who have been proved (as far as psychological methods can do so) to be musical in high degree, then the world of music is due for a rejuvenation. The state around the musical world, being untroubled by any fortuitous 'bohemian' section of the population, will be better organized and more efficient; the typical individual within the musical world happier and more competent.

This picture is one of the future. If this analysis helps to bring that future but one step nearer the present, that is more than recompense for the work that went into it.

## 1. Subjects:

The tests were given to students drawn from three departments of the University, there being in all 32 from the Music department, 23 from the English department, 15 from the Speech-training department, 107 from the Psychology department, and, in addition, 30 from the Music department, Stellenbosch University.

The restriction of the subjects to students was motivated by a desire to maintain a relatively similar cultural and intellectual background from the population tested. The diversity of the departments from which the students were drawn was due to:

(i) There being no department with an enrolment of students sufficiently large for the purposes of the present analysis i.e. to keep the probable error of fallible correlations within reasonable limits.

(ii) It was thought that as the investigation was concerned with the properties of musicality, general intelligence and literary ability it would be of use to number among the subjects representatives of those specific departments whose training pertained to the development of the abilities under consideration. Thus the isolation of the mental factors of the total battery would be facilitated by the proportionately heavier loads settled upon any such factors by the effect of superior training of the various groups of students in regard to abilities required in their departmental work. Hence a possible literary factor bespoke students of English language and literature; music students would then emphasize any musical factor; psychology students were to provide the 'leavening' effect of too much specialization and help to act as material for the intelligence testing.

(iii) The inclusion of the Afrikaans speaking students from the Conservatorium of Music, Stellenbosch University, arose primarily from an insufficiency of music students at this University. The language difference presented no difficulty here, since the main body of the music tests were such as to be independent of the influence of language, a 'x', a 'o', or actual musical notation being all that was required to answer any question. The scores on the literary tests, certainly, and perhaps the scores on the intelligence test might be supposed to be adversely affected by a relative unfamiliarity with the language in which the tests were given. This obstacle was overcome, as far as possible, by treatment which will be described in the section on 'Methods of Scoring'.

There were in all 206 subjects, but as the tests required a total testing time of  $5\frac{1}{2}$  hours, many of the subjects omitted one or more portions of the total tests. The number of subjects performing each test is indicated in the description of that test.

The subjects were evenly drawn from both sexes, their being perhaps a slight preponderance of the female element.

The ages of the subjects showed little variation, 80% of the students being between the ages of 17 years and 21 years.

It was assumed that, apart from the actual tests included in the battery, the three major influences upon test performance for the

purposes of this study, were age, cultural and intellectual background, and specific training in musical or literary subjects. 'Age' variation was deemed too slight to warrant correction - a conclusion supported by the current psychological view that maturity of mental abilities do not substantially, and, at all events, rapidly change after the age of 16 years.<sup>x</sup> This inference is especially emphasized in the realms of music, where the earliness of appearance of talent and ability in the maturity cycle of the individual is too well known to bear further emphasis. Therefore age was not included as a variable in the test battery. As previously stated, the cultural and intellectual background of the subjects was considered sufficiently uniform to preclude setting this feature as a separate variable in the battery.

The third influence, that of specific training, differed from group to group, but this very difference was desirable for reasons already given. This difference, in a way, constituted a variable element, but was not included as such in the battery as it would already be operative in magnifying the loads of the factors likely to be extracted. Hence, all other things having been made equal, it seemed legitimate to conclude that the results of the tests could be affected only by the capacities of the subjects themselves; and, in this statistical population, may be regarded as having a bearing on definite and relatively pure abilities as analysed by the centroid process.

## 2. Tasks.

The choice of tasks was determined on the basis of several criteria:-

- (i) To have as varied a sample of material for each suspected factor as possible,
- (ii) to use, where possible, standardized tests with predetermined high reliabilities,
- (iii) to grade the difficulty of each test, so as to have, at one end of the scale, certain questions which everybody could answer, and at the other, questions which nobody, even with specialized training in the mental processes involved in the test, could answer altogether satisfactorily. This, it was hoped, would yield precisely graduated coefficients of efficiency, and
- (iv) to employ tests which would be suitable for as extensive a range of marking as feasible in order to obtain a true dispersion of the subjects' abilities.

## 3. Selection of Material and Methods of Scoring.

The tests of the entire battery fell naturally into three sub-systems:

- (i) Intelligence.
- (ii) Literary.
- (iii) Musical.

### (i) Intelligence.

An index to the general intellectual capabilities of each individual was arrived at by the use of the Cattell Intelligence Test. About half the subjects completed Scale 111: Form B. and the other half Scale 111; Form A.

The 6 parts of the test were completed during two different periods separated in most cases by 24 hours, but in some few cases by a week. There were six parts to the test, all of which were of the multiple choice type i.e. for each question or problem, the subject had to detect the correct answer from among four incorrect answers. The parts of the test were all of the 'speed' variety, strict regard being paid to the imposed time-limit for the performance of each task. Each part was graded, starting with easy problems and ending with difficult ones. Furthermore, the test as a whole became increasingly more difficult as each part succeeded the previous one. Various points arising from the results of this standardized intelligence test raised doubts about the purity of the 'g' factor it was held to isolate, and consequently the six parts of the test were treated as independent variables in the ensuing analysis. The various parts of the test were:-

(A) Synonyms Test.

There were 20 items requiring the choosing of an appropriate synonym of a given word. One mark was awarded for each correct answer. 180 subjects completed this test.

(B) Classification Test.

This consisted of 25 items; of these 21 required the picking out of a word which represented an element foreign to the rest of the words in the item. The remaining 4 items involved similar selection, only this time geometrical figures were employed. One mark was awarded for each correct answer. 180 subjects completed this test.

(C) Opposites Test.

This was composed of 20 items, in each of which the opposite of a given word had to be chosen from the list of possible antonyms. One mark was awarded for each correct answer. 180 subjects completed this test.

(D) Analogies Test.

In each of 21 items the subject was required to choose from a selection, words related to a given word in such a way as to make the relationship analogous to a previously stated relationship, e.g.

'Palace is to King', as 'Kennel' is to (man, animal etc.). In each of 4 further items the same principle had to be applied to geometrical figures. One mark was awarded for each correct answer. 179 subjects performed this test.

(E) Completion Test.

Four short continuous passages are given, each with 5 or 8 gaps breaking the continuity of the passage. Each gap must be filled in with a word chosen from the given alternatives. One mark was awarded for each correct word chosen. 171 subjects completed this test.

(F) Inferences Test.

This consisted of 12 items each presenting a problem requiring a good deal of reasoning power for its solution. Each

problem had between 4 and 8 given solutions; the subject was required to choose the correct one or ones, as the case demanded. Three marks were awarded for each problem correctly solved. The test was attempted by 169 subjects.

(G) This niche in the battery was kept vacant to be used as a composite variable representing the total scores of individuals in the complete intelligence test. As the mathematical analysis proceeded it was seen that this variable could perform no function not already operating through the combined effects of the individual parts of the test, and so 'G' was discarded as a variable. In view of possible confusion between variable 'G' and Spearman's 'g', it was decided to name the next test 'H'. Hence 'G' does not exist as a test variable at all in this work.

(ii) Literary.

Three sets of tests of literary ability were used. None of these tests were of the 'speed' type i.e. no time limit was imposed.

(H) Reading Comprehension Test.

This was a standardized test taken from the 'Iowa Silent Reading Tests'. Subjects read through a passage of verse of 31 lines in length, various phrases of which were under-lined and numbered. Adjacent to the passage was a number of questions relating to the full understanding of the literary content and each question had to be answered by filling in the number of the under-lined phrase best answering the question. One mark was awarded for each correct answer. 135 subjects completed this test.

(I) Vocabulary Test.

This was composed of 100 words, each of which had attached to it 5 words or phrases. The subject was required to put a ring round the word or phrase equivalent in meaning in each case. The first 50 words were taken from the Terman test; the second 50 words were originally selected in conjunction with the head of the psychology department.<sup>x</sup>

The words were graded in difficulty; there being a good number that everybody would be likely to know, and a sprinkling with which nobody would be likely to be acquainted.

Due regard was paid to the avoidance of technical, archaic and colloquial words. The criterion for the inclusion of a doubtful word was: Would an intelligent student accustomed to indulge in a reasonably high standard and extensive range of reading be likely ever to encounter the word in question?

One mark was awarded for each correct meaning selected. 145 subjects performed this test.

(J) Poetical Appreciation.

The Abbot and Trabue 'X' and 'Y' series of poetry tests were given here. Each of the two series had thirteen parts. Each part consisted of a verse written by a poet of renown, and accompanied by three malversions of this verse in which the poetic quality is lowered by introducing a 'sentimental' version (insincere feelings,

'gushy emotions etc.), a 'prosaic' version (reducing the poet's imagery to a more commonplace level), and a 'metrical version' (making the movement awkward).

The subjects were required to state which of the four versions of each poem they considered best as poetry.

There were thus 26 tests in all. Again the tests were graded in difficulty, the first of each series being the easiest, and the last the most difficult in which to discriminate between version and mal-version. Each poem had been pronounced superior to its variants by a group of experts including poets, literary editors, critics, and professors of literature.

According to the Pearson Formula  $r = \frac{\sum xy}{\sqrt{\sum x^2} \sqrt{\sum y^2}}$  the reliability coefficient between the two series 'x' and 'y' was .723. Hence the system of scoring was one mark for each correct response throughout the 26 tests.

(iii) Musical.

The music tests comprised half the total tests, and were of the 'goodness of response' type, no attention being paid to speed of response. The tests were ten in number:-

(K) Pitch Discrimination Test.

Ten items were given, the procedure for each item being:- a standard tuning-fork of 435 vibrations per second, was struck sideways against the tines with a felt hammer, allowed to sound for two seconds, and was then immediately damped down. Another tuning-fork of (435+30) vibrations per second was similarly sounded and damped down. The subjects were required to state which of the two sounds was the higher in pitch. For each item there were 4 such comparisons. The procedure was such as to require two responses each of 'higher' and 'lower' for each item. The order of appearance of 'higher or lower' in each item was entirely random. The next item had the comparison tuning-fork 435, and (435+23) vibrations per second. Discrimination became increasingly difficult as the items progressed, the order of vibrations per second being, (435+35), (435+23), (435+17), (435+12), (435+8), (435+5), (435+3), (435+2), (435+1) and (435+.5). In all there were 40 responses and one mark was awarded for each correct response. 135 subjects performed this test.

(L) Absolute Pitch Test.

This consisted of 7 items. In each item a note in the middle range of the key-board was struck, once only. The subjects had been previously instructed to memorize the pitch of the given sound. After a lapse of about 5 seconds a part of the next test (M) was given and answered appropriately; a second note in the main test was now struck and it was required of the subjects that they stipulate whether the pitch of the sound of this latter note was, or was not identical with that of the former. A similar procedure was adopted with the 6 other items. One mark was awarded for each correct response. 134 subjects completed the test.

(M) Interval Discrimination Test.

This was based on a standardized test<sup>1</sup> devised with due respect to validity and reliability. The test required the comparison of two musical intervals played on the piano to determine whether the last interval was larger or smaller in range of pitch than the first. There were 20 pairs of intervals to be compared. The first interval of each trial always began in the region of the octave below middle C on the piano; the second interval was always given above the first interval to avoid over-lapping between the two. The intervals all proceeded in an upward direction. Trials 10 to 20 were the reverse orders of trials 1. to 10, with the interval in each trial also reversed. Difficulty was increased in the first half of the test, whereas the last half became easier. The difficulty was based on the ratio of the difference in length of two intervals. Thus in trial 1, C to B contains 11 semi-tones; its companion interval, F to A has 4 semi-tones. Dividing 11 by 4 gives 2.75, which was taken as a measure of the difficulty, expressed as a ratio between the two intervals. One mark was awarded for each correct discrimination. 146 subjects completed the test.

(N) Consonance Discrimination Test.

This test was modelled upon the results of a specific study of the conditions influencing consonance discrimination carried out elsewhere<sup>2</sup>. The test was given in two groups:-

(i) 15 pairs of intervals were presented on the piano each interval being damped down before its companion is sounded.

The subjects were instructed to state which of the two intervals of each pair was the smoother (i.e. relatively freer from beats and roughness). This is taken to be the more consonant.

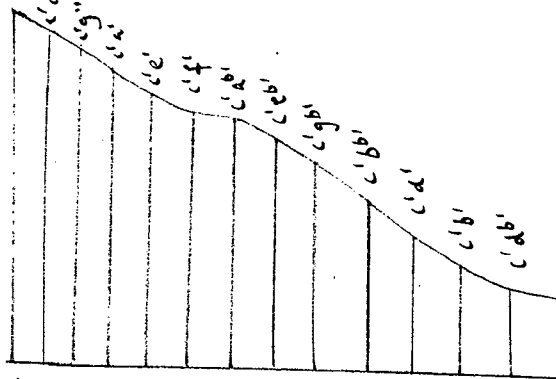
One mark was awarded for each correct response.

(ii) 15 pairs of intervals, different from the previous group, were now similarly presented. Here the subjects were to state which of the intervals in each pair had its two component notes blending better (i.e. whether the two tones agreed or belonged together). This again represented a test of consonance. One mark was awarded for each correct answer. 138 subjects completed both parts of this test.

The problem of scoring in this connection represented a well-known difficulty in musical tests of consonance, owing to the subjective methods of preferences for different tonal combinations. The method of scoring finally adopted was that given by the most competent authority on the question known to the present writer.<sup>3</sup>

1. R.N.Drake (6) Wesleyan College U.S.A.
2. E.G.Bugg Vanderbilt University U.S.A. (3)
3. C.E.Seashore Chap V11 page 152 (27)

The characteristic basis of judgment is indicated in the following graph:



(O) Rhythm Test.

It would have been convenient to have used the Seashore phonographic tests for this and the following tests, but as the opinion of both the writer and an expert on musical topics  $\phi$  was that the tests were of too difficult a nature for the average student not musically trained, it was decided to leave the devising of the rhythm and the time test which follows to a member of the music staff competent and experienced in aural testing under local conditions.<sup>x</sup>

These tests were 16 in number and ranged from very easy tests to tests which few, even of the music students, could answer correctly. Each test consisted of a short melodic phrase played on the piano with reasonable accent on the 'strong' beats. The subject had to decide whether the phrase was in 2, 3, 4 or 6 time. The significance of these last tempi was 4, 4, 4, 8. The significance of these last tempi was amply illustrated with examples, and the test proper was not begun until all present had signified their complete understanding of what was required.

One mark was awarded for each correct answer. 139 subjects attempted this test.

(P) Time Test.

As in the previous test, a practice experiment was performed and each subject had explained to him the significance of the relative time-values, minim ( $\text{♩}$ ), crotchet ( $\text{♪}$ ), quaver ( $\text{♫}$ ), semi-quaver ( $\text{♬}$ ), and combinations of these values. Illustrations of the explanations were given until each subject professed adequate understanding of the material to be used. The subjects

$\phi$  Dr. W. Bell. Ex-Dean of Music Faculty, U.C.T.  
 x Mr. J. Andrews, Vocal & Aural Dept. Music Faculty,  
 U.C.T.

were informed that no notes other than the above 4 would be used.

Twelve melodic phrases, chosen from the 16 phrases of the previous test in order of difficulty were then played, and the subjects were required to write down the symbols equivalent to the phrase played.

e.g. the phrase  would be written as 

As in this test it was obviously not a case of an answer being wholly right or wholly wrong, 4 marks were allowed each item, and each response scored according to how much of the question was answered in a manner showing comprehension of the principles involved. In this way a better dispersion of the 'sense of time' abilities of individuals could be obtained than by awarding one mark or **none** to an answer. 139 subjects attempted this test.

#### (Q) Musical Memory Test.

Both this and the next test were suggested by ingenious tests devised elsewhere.\* This test follows the practice of having the material as near like the function to be tested as possible. The test consists of 12 trials for which there are as many original 2 to 4 bar melodies. All of the melodies were originally composed to obviate any possibility of familiarity with the material. Each one of these 12 melodies serve as a standard of comparison with four possible variables: change of key; change of time; change of note; or the same melody repeated. Thus in trial 7 the original melody or standard is:



to which each of the following are to be compared and identified (the original melody is not repeated for each comparison):

(i)  (ii)

(ii)  (iv)

The first variable is the same melody as the standard except that one of the notes has been altered; the second variable has the same notes as the standard, but the time values of some of the notes are altered; the third variable is the standard melody played in another key, and the last variable is identical with the standard melody.

Each variable is identified by the first letter of the four possible variables. Thus 'S' stands for same as the standard, K stands for change of key, T stands for change of time, and N. stands for a change of one or more notes. Only one kind of change is made in any one variable. All that need be recorded for trial 7 would be N.T.K.T. which are placed in a specially prepared form as shown in Fig. I.

Fig. I, showing the correct responses to all 12 trials, indicates how with each two trials there is one additional variable requiring to be indentified. The order of the variables in random generally, but care was taken both to avoid stereo-typed presentation and to have each variable occurring about the same number of times during the entire experiment.

Figure I.

1. S. N.
2. K. T.
3. N. T. S.
4. T. K. N.
5. N. K. T. S.
6. T. N. K. T.
7. N. T. K. T. S.
8. K. S. N. T. T.
9. N. T. N. K. T. N.
10. T. K. N. S. N. T.
11. T. N. N. K. T. S. T.
12. N. T. N. K. S. S. N.

The standards begin with two-bar melodies and increase slightly in length and complexity from one trial to the next. Here again, the practice experiment of thoroughly explaining what is implied in S,K,T,N, was performed and illustrated and only when all present quite understood the nature of the test, was the test proper commenced.

One mark is awarded for each letter correctly put down. 134 subjects performed this test.

(R) Retentivity Test.

This test was constructed to measure memory for the more elemental factors of melody than for melody itself as was the purpose in Musical Memory. It was, therefore, different in character. There were 10 trials each containing two sections. The first section is composed of three items which must be remembered to compare with the three corresponding items, arranged in the same order, in the second section. The items are:-

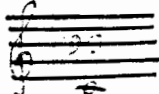
- (1) An interval played on the piano.
- (2) A rate of time given by a metronome.
- (3) A chord of three notes played on the piano.

The three items are given in a certain order. In the second section, different items are given in the same order. Where there is an interval it is to be compared (larger or smaller, L or S) with the corresponding interval in the first section; where there is a rate of time, it is to be compared (faster or slower, F or S) with the rate given in the first section; and where one note is given it is to be compared with, or identified as different from (D), the first (1), second (2), or third (3) of the three notes given in the first section. The same order of presentation of items as in the above test was followed.


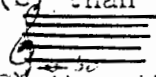
The metronome rate of 78 beats per minute was taken as the standard to be compared with the "rate of time" variables ranging from 164 to 88 beats per minute. The standard comes first in 5 trials and last in the other 5 trials arranged in chance order. The metronome is allowed to beat for a short random length of time in each trial (if the same length of time had been used for all cases it would have been possible merely to count the number of beats in each section and get the correct answer without paying any attention to the rate of beats.)

To illustrate what the test requires with trial V:-

There is first given a chord of 3 notes on the piano; this is followed by the metronome beating at the rate of 110 beats per minute; and then a musical interval is given



These are the three items in the first section, all of which must be kept in mind to compare with the respective items in section two, viz:-

one note  is presented on the piano and has to be identified as D, 1, 2 or 3, of the three notes presented in the first section; then a rate of time is given on the metronome of 78 beats per minute which must be compared to the rate given in the first section to determine if it is faster (F) or slower (S) than that in the first section; finally an interval is given, and the testee must determine if this interval is larger (L) or smaller (S) than the interval given in section one. 

In addition, for the benefit of students who could place the single note as being part of the general harmony of the previously given chord, but who could not be expected, owing to lack of training, to know which one of the three notes it would be, provision was made for such subjects to simply place an 'X' instead of 1, 2 or 3.

One mark is awarded for each correct response. 134 subjects performed the test.

(S) Intensity Discrimination Test.

For this purpose the Seashore phonographic test was used. A record is played upon which are sounded 50 pairs of isolated notes. In each of the 50 cases the subject is required to decide whether the second note of each pair is louder or softer than the first.

One mark is awarded for each correct answer. 132 subjects completed the test.

(T) Emotional Sensitivity Test.

This was a test of emotional appreciation of extracts from musical works played upon gramophone records. The work were selected in accordance with expert advice.<sup>x</sup> regard being paid to:

(i) choosing works all of an orchestral nature to preclude any guide being given to the emotional character of the piece by the nuances and timbre of solo artist performance (especially vocalists);

(ii) choosing works little known to the average music public, to obviate responses of a conventional nature, and

(iii) choosing extracts of works which, in the unanimous opinion of members of an experienced and critical advisory board, <sup>o</sup> typified the emotions in question beyond doubt. Some of the works were redolent with their specific emotion, while others were more subtle and controlled, and required greater power of appreciation.

Each record was accompanied by a list of 8 adjectives; the subjects' task was to choose the 3 words in each case which best described the character or emotional mood of the music played. The adjectives were chosen by the aforementioned advisory board. Of the 8 words in each case there were 4 which described the mood of the work with varying degrees of accuracy. Of these the choice of the best word was awarded 4 marks, the next best 3 marks, then 2, and finally one mark. The other 4 adjectives were irrelevant to the task and carried no marks.

There were 12 records, and each was played for about two minutes. The works played were extracts from:-

1. March of the Caucassian Chief - Ipoletov;
2. Prelude to Fervaal - d'Indy;
3. Tristan Prelude - Wagner;
4. Overture to Casse-Noisette Suite-Tschaikowsky;
5. Marche Joyeuse - Chabrier;
6. Meistersinger Overture - Wagner;
7. Symphony No. 4, 2nd Mov. Tschaikowsky;
8. Danse Chinois - Tschaikowsky;
9. Traume - Wagner;
10. Parsifal Prelude - Wagner;
11. Siegfried Funeral March - Gotterdammerung - Wagner;
12. Tristan Prelude Act III - Wagner.

x Dr. W. Bell

<sup>o</sup> Members of the Staff, Music Dept. U.C.T.

132 Subjects performed the test.

### Stellenbosch Scores.

If the scores of the Stellenbosch students were to be considered relatively lower all round for any test owing to unfamiliarity with the English language, then certain statistical results are to be expected and allowance can be made for such constant errors.

The procedure adopted was:

(i) To calculate first the arithmetic Mean of the combined scores of the Cape Town Music students on each of the tests, and then the Mean for the scores of the Stellenbosch students on the same tests. i.e.

$$\frac{\sum x}{n} \text{ and } \frac{\sum y}{n} \text{ [where } x \text{ \& } y \text{ are raw scores]}$$

(ii) From the Mean of each test for each of the two groups the Standard Deviations was worked out, i.e. S.D. =  $\sqrt{\frac{\sum x^2}{n}}$

$$\text{and } \sqrt{\frac{\sum y^2}{n}}$$

(where  $x \text{ \& } y$  are deviations from the Mean)

(iii) The probable error of each Mean is then calculated i.e.

$$E_1 = \frac{.6745 \times S.D.}{\sqrt{n}} \text{ and } E_2 = \frac{.6745 \times S.D.}{\sqrt{n}}$$

(iv) The Probable Error of the difference between the two Means for each test is found. This is given by

$$\sqrt{E_1^2 + E_2^2}$$

(v) If the difference between the two Means concerned is equal to or greater than 5 times the Probable Error of the difference between the Means, i.e. if  $\frac{\sum x}{n} - \frac{\sum y}{n} \geq 5 \text{ p.e.}$

then it was assumed that a chance factor was insufficient to account for such a difference, and the Stellenbosch scores were raised in the proportion of the Cape Town Mean to the Stellenbosch Mean.

To illustrate this method, an example is cited of what was done in the case of the vocabulary test:-

Cape Town: Mean = 49

" S. D. = 15.91

P.E. of Mean = 2.683

Stellenbosch: Mean = 26

S.D. = 9.198

P.E. of Mean = 1.423

$$\begin{aligned} \text{P.E. of difference between the Means} &= \sqrt{(1.423)^2 + (2.683)^2} \\ &= 3.037 \end{aligned}$$

This difference,  $349-26$ , = 7 P.E.  
 =  $> 5$  P.E.

Each Stellenbosch student's score in the vocabulary test was multiplied by  $\frac{49}{26}$ . This corrected score was used for

computation of subsequent correlations.

Other tests requiring this correction were those on Reading Comprehension, Poetry Appreciation and the first five parts of the Intelligence Test.

#### General Music-test Scores.

It is natural to suppose that the Mean scores of the music students as a group would be higher than those of the students lacking a specific musical training. But, as the correlational technique is largely unaffected by such differences, raw scores for both music and non-music students were taken.

#### 4. The Experimental Procedure.

All the experiments, with but a single exception<sup>x</sup> were carried out under group test conditions. The stress of the experimental period would have been greatly lightened had it been possible to organize the subjects into about three different groups; in this way the testing might feasibly have been completed in a week. As it was, owing to harassing difficulties for any would-be experimenter, the subjects had to be tested in small groups, and so the testing was not finally completed until seven weeks had passed. In retrospect these difficulties can be seen to have advantages. No group was composed of more than 15 subjects and consequently many random factors attendant upon group testing could be eliminated: there was no possibility of subjects copying the answers of their neighbour; with the watchful gaze of the experimenter constantly upon them, subjects could not 'fool', become disinterested, or otherwise give irrelevant responses; in addition to subjects being encouraged to ask questions freely about difficulties in the comprehension of the task in hand, the experimenter was able, on several occasions, to notice puzzlement among various subjects who were of too retiring a disposition to ask these questions. In such cases, the task was re-explained until complete comprehension of all was evidenced.

The intelligence tests, alone, were given to groups of 30 subjects or more. The English and Music students did the test in groups of about 30, while the Psychology students did theirs in groups of 80, 15 and 15. In all tests there were quite a number of isolated cases of subjects being unable to join any of the groups, and having to be tested for the entire battery individually.

The conditions under which the tests were given were, with the exception of the case of the music students, conditions of quietness, and free from disturbances. The music student had to be tested at the College of Music in a room into which muffled sounds of music-playing penetrated almost constantly. A certain interference with the attempt at concentration by the students here might be contended, but, against this distraction should be recorded the fact that the music students involved were accustomed to do all their work under exactly similar, if not worse, conditions, and hence a certain immunity might be supposed. It was finally decided to leave the scores of the music students unchanged.

The intelligence tests were given under regulation conditions according to the instructions laid down by the deviser of the test.<sup>x</sup> The giving of literary tests presented no difficulty. The music tests were given as described in the previous section, stress being laid on familiarizing non-music students with the tasks by the use of ample illustration.

The test for discrimination of intensity presented a good deal of difficulty which was to a certain extent overcome by use of the technical advice of an electrical engineering expert.<sup>ii</sup> Without going into the question of acoustical dynamics, it may be pointed out that:

- (i) persons seated at different parts of the room with respect to the sound-box of the gramophone, would not be subjected to exactly the same amount of auditory stimulation.
- (ii) the reception of an auditory stimulus by a subject would depend both upon his distance from the sound-box in the frontal-parallel plane, and, if the sound box is parallel to the back wall of the room, his distance away from the perpendicular line joining the sound-box to the wall. In the diagram



Subject A receives more stimulation than B; B is more favourably placed than C for sound reception; and D is in the least favourable position for sound reception.

x' R.B. Cattell.

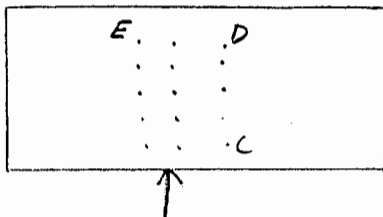
x'' Dr. Gulke.

(iii) the distortion of pure sound in a closed room which has not been acoustically-treated really makes it desirable that tests of this nature should be performed in the open air.

These difficulties were partly surmounted by the following procedure:

The subjects, of whom there were never more than 15 at a time in the experiment (often there were only 6 or 7), were placed close together, three on a first bench, then three exactly behind on the second bench, and the rest similarly in the next benches.

The position was:



The distance C to D was about 8 feet which would cause a very slight difference between sound reception in the first row and in the last row. The distance D to E was about 3 feet, and as the horn of the gramophone was 3 feet in diameter at its opening we can presume that the three subjects in each row would be subjected to equal intensities of auditory stimulation.

This was the final procedure adopted. The objection that the Seashore test employed was not a test of pure sounds was deemed to be of negligible importance in this experiment. The validity of the test seemed proved by the reasonably wide dispersion of the results obtained.

The intelligence tests were given in two periods of three-quarters of an hour each; the literary tests were done in one period for the comprehension and vocabulary tests, and another period for the poetry tests. The music tests were given to about one half of the subjects in one testing period of  $2\frac{1}{2}$  hours; the remaining subjects spread the performance of the test over three hour periods.

## Results.

### (i) Statistical Analysis.

Both the raw scores of the 206 subjects on all tests, and the scatter-diagrams of the 171 inter-correlations between the tests are given under a separate cover. As the number of subjects performing each test varied appreciably, the constants of distribution had to be recalculated individually for each one of the 171 correlations. A fair idea of the actual number of the statistical population upon which the different correlations are based may be gained from a diagram:

	INTELLIGENCE	LITERARY	MUSIC
INTELLIGENCE	173	130	113
LITERARY	130	116	110
MUSIC	113	110	127

We begin here with the first table of the raw coefficients of correlation. It should be noted that the entire analysis is based on correlation between sets of fallible data. No attempt has been made to correct the correlations either for effects of attenuation or for uniqueness. The correlations may therefore be regarded as being, at the least, as low as they could be. It may be pointed out that the errors attached to fallible scores help to make up the raw <sup>SCORES</sup> and yet are uncorrelated; they make up a part of the factors which the two tests do not have in common and thus they reduce or attenuate the apparent degree of relation. The true correlation between two variables can only be arrived at by making some correction for this attenuation in the raw coefficient.

The correlations were computed by Pearson's Product-Moment formula, viz.,

$$r = \frac{\sum xy - (V_{ix} V_{iy})}{\sigma_x \sigma_y}$$

where  $x$  and  $y$  are deviations from the mean (i.e.  $x - \bar{x}$  and  $y - \bar{y}$ ) and  $V_{ix}$  and  $V_{iy}$  are corrections for the adoption of guessed averages.

The entire battery was composed of 19 tests. In the correlational table these were arranged, after the Least Squares method of Burt, in an order which was predisposed to an a priori grouping of tests involving possible similar factors. Thus,

+M	+N	+O	+P	+Q	+R	+S	+T	.	.	Σ <sub>0</sub>	D
-114	-110	+135	-091	+088	+161	+062	+066	.	.	+3.929	
-055	+028	+204	-042	+057	+059	+235	+016	.	.	+3.728	
-049	-130	+094	+069	+067	+016	-079	+127	.	.	+3.808	
-059	-061	+163	+130	+096	+018	+007	+169	.	.	+4.050	
-025	-020	+094	-124	+023	+124	-079	+152	.	.	+3.714	
073	-034	+206	-014	-036	+131	-082	+080	.	.	+3.484	
+020	-034	+044	+034	-060	+141	-120	+168	.	.	+3.785	
-025	-022	+281	-064	+137	-054	-002	+324	.	.	+4.953	
+051	-025	+095	+004	+049	+029	-105	+285	.	.	+4.125	
+343	+061	+110	+359	+269	+227	-009	+176	.	.	+1.691	
+122	+001	+072	+203	+250	+262	+093	+032	.	.	+1.526	
+(439)	+095	+210	+439	+267	+389	+067	+091	.	.	+2.163	
+085	+(191)	+110	+062	+191	-011	+059	-005	.	.	+ .316	
+210	+110	+(331)	+228	+331	+322	+002	+218	.	.	+3.257	
+439	+062	+228	+(439)	+582	+342	+056	+180	.	.	+2.792	
+267	+191	+331	+582	+(582)	+510	-025	+236	.	.	+3.614	
+389	-011	+322	+342	+510	+(510)	-056	+231	.	.	+3.321	
+067	+059	+002	+056	-025	-056	+(231)	-023	.	.	+ .236	
+091	-005	+218	+180	+326	+231	-023	+(324)	.	.	+2.847	
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
+2.163	+ .316	+ 3.257	+2.792	+3.614	+3.321	+ .236	+2.847	.	.	57.339	1/4
+ .236	+ .042	+ .430	+ .369	+ .477	+ .438	+ .031	+ .376	→ ΣE=7.570	7.572	7.572	√1/4
+ .236	+ .042	+ .430	+ .369	+ .477	+ .438	+ .031	+ .376	.	.	-1320	1/√4

TABLE I

	+A	+B	+C	+D	+E	+F	+H	+I	+J	+K	+L
+A	+(504)	+512	+409	+232	+346	+299	+391	+564	+396	-056	+072
+B	+512	+(511)	+319	+334	+269	+356	+264	+279	+353	-068	+096
+C	+409	+319	+(493)	+450	+411	+391	+285	+493	+400	+025	+013
+D	+232	+334	+450	+(493)	+413	+493	+336	+476	+429	-052	-017
+E	+346	+269	+411	+413	+(502)	+333	+437	+502	+430	-096	+022
+F	+299	+356	+391	+493	+333	+(493)	+265	+451	+354	-099	-030
+H	+391	+264	+285	+336	+437	+265	+(476)	+476	+432	+171	+059
+I	+564	+279	+493	+476	+502	+451	+476	+(564)	+562	-005	+026
+J	+396	+353	+400	+429	+430	+354	+432	+562	+(562)	-084	-022
+K	-056	-068	+025	-052	-096	-099	+171	-005	-084	+(359)	+060
+L	+092	+096	+013	-017	+022	-030	+059	+026	-072	+060	+(262)
+M	-114	-055	-049	-059	-025	-073	+020	-025	+081	+343	+122
+N	-110	+028	-130	-061	-020	-034	-034	-002	-075	+061	+001
+O	+138	+204	+098	+163	+094	+206	+044	+281	+095	+110	+072
+P	-091	-042	+069	+130	-124	-014	+034	-064	+004	+359	+203
+Q	+088	+057	+067	+096	+023	-036	-060	+137	+049	+269	+250
+R	+161	+059	+016	+018	+124	+131	+141	-084	+029	+227	+262
+S	+062	+235	-079	+007	-079	-082	-120	-002	-105	-009	+093
+T	+066	+016	+127	+169	+152	+080	+168	+324	+285	+176	+032
.	.	.	.	.	.	.	.	.	.	.	.
E	.	.	.	.	.	.	.	.	.	.	.
D	+3.929	+3.725	+3.305	+4.050	+3.714	+3.414	+3.785	+4.453	+4.025	+1.691	+1.5
E	+5.18	+4.42	+5.03	+5.35	+4.690	+4.460	+5.000	+6.54	+5.45	+2.23	+2.20
K	+5.18	+4.42	+5.03	+5.35	+4.690	+4.460	+5.000	+6.54	+5.45	+2.23	+2.01

tests A to F were the six parts of the Cattell Intelligence Test; H to J were the three literary tests, while K to T represented the tests of musical ability. Owing to the size of the tabulated correlational data involved it was deemed advisable to give each test an alphabetical connotation for economy of space. The letters used are repeated here:-

Letter	Test	Type of Test
A	Synonyms	} Intelligence
B	Classification	
C	Opposites	
D	Analogies	
E	Completion of Sentences	
F	Inferences	
G	omitted	
H	Reading Comprehension	} Literary
I	Vocabulary	
J	Poetical Appreciation	
K	Pitch Discrimination	} Music
L	Absolute Pitch	
M	Interval Discrimination	
N	Consonance Discrimination	
O	Rhythm	
P	Time	
Q	Musical Memory	
R	Retentivity	
S	Intensity Discrimination	
T	Emotional Sensitivity	

Given an initial table (Matrix  $R_0$ ) of the raw correlation coefficients\*, the procedure was to use Thurstone's Centroid method for the extraction of the first factor involved in the data. This table (1) is reproduced below. In the table:

$D = \sum_{k=A}^T r_{i,k} = \sum_{k=A}^T r_{i,k}$  = Sum of the individual columnar correlations with respect to sign.

$Y_+ = \sum_{j=A}^T \sum_{k=A}^T r_{j,k} =$  total sum of correlations of the table.

$E = \frac{D}{\sqrt{Y_+}} = \frac{\sum r_{i,k}}{\sqrt{Y_+}}$  = factor load (saturation coefficient) of each test on 1st. factor.

$\sum E \text{ should} = \sqrt{Y_+}$ . This is a check on the arithmetic of computing the saturation coefficients of the tests.

$K = E$  with respect to alteration of signs due to reflection of the individual tests.

The communalities, in view of the number of variables, are estimated as being equivalent to the largest correlation coefficients in their

\* all results were calculated to 4 decimal places and are given correct to 3 decimal places.

TABLE II

FIRST-FACTOR

E		+518	+492	+503	+535	+490	+460	+500	+654	+545	+223	+201
		+A	+B	+C	+D	+E	+F	+H	+I	+J	+K	+L
		+296	+270	+240	+207	+262	+241	+226	+136	+265	+309	+222
+518	+A	+252	+257	+145	-045	+092	+061	+132	+225	+114	-172	-012
+492	+B	+257	+255	+072	+071	+028	+130	+015	-043	+085	-178	-003
+503	+C	+145	+072	+204	+181	+165	+159	+033	+164	+126	-087	-088
+535	+D	-045	+071	+181	+267	+151	+247	+068	+126	+137	-171	-125
+490	+E	+092	+028	+165	+151	+305	+108	+192	+182	+163	-205	-077
+460	+F	+061	+130	+159	+247	+108	+255	+035	+150	+103	-202	-122
+500	+H	+132	+015	+033	+068	+192	+055	+299	+149	+179	+059	-042
+654	+I	+225	-043	+164	+126	+182	+150	+149	+370	+206	-151	-105
+545	+J	+114	+085	+126	+137	+163	+103	+159	+206	+211	-206	-152
+223	+K	-172	+178	+087	+171	+205	+202	-059	+151	+206	+279	+015
+201	+L	+012	+003	+008	+125	+077	+122	+042	+105	+182	+015	+182
+286	+M	+262	+196	+193	+242	+165	+205	+123	+212	+025	+279	+065
+042	+N	+132	+007	+151	+083	+041	+053	+055	+029	+098	+052	+007
+430	+O	+085	+008	+118	+067	+117	-008	+171	+000	+139	+014	+014
+369	+P	+282	+224	+117	+067	+305	+184	+151	+305	+197	+277	+129
+427	+Q	+159	+178	+173	+159	+211	+255	+299	+175	+211	+163	+154
+438	+R	+066	+156	+204	+216	+091	+070	+078	+370	+210	+129	+176
+031	+S	+046	+220	+095	+009	+094	+096	+136	+022	+122	+016	+087
+376	+T	+129	+169	+062	+032	+032	+093	+020	+078	+050	+092	+044
2570												
	Σo	.004	.003	.000	.002	.004	.002	.004	.004	.002	.001	.004
	D	+2.539	+1.760	+2.540	+2.324	+2.724	+2.520	+2.101	+2.820	+2.664	+2.597	+1.517
	E	+3.835	+2.659	+3.844	+3.351	+4.111	+3.851	+3.18	+4.46	+4.02	+3.92	+2.29
	K	+3.835	+2.659	+3.844	+3.351	+4.111	+3.851	+3.18	+4.46	+4.02	-3.92	-2.29



## REFLECTION TABLE

A	B	C	D	E	F	H	I	J	K	L	M	N	O	P	Q	R	S	T	CHECK	K <sub>i</sub>
0	9	10	11	10	9	9	9	9	9	12	10	11	9	9	10	11	12	11	190	L
9	8	9	10	9	8	8	8	8	10	6	11	10	8	10	11	12	13	10	178	S
10	9	8	9	8	7	7	7	7	9	5	12	11	7	11	10	11	5	9	162	M
11	8	7	8	7	6	6	6	6	10	4	6	12	8	12	11	12	4	8	150	N
12	9	6	7	6	5	5	5	5	11	5	5	6	9	13	12	11	3	7	138	P
13	8	5	6	5	4	4	4	4	12	4	4	5	10	5	13	12	2	8	122	Q
14	7	4	5	4	3	3	3	3	13	3	3	4	11	4	5	13	3	9	106	R
15	6	3	4	3	2	2	2	2	14	2	2	5	12	3	4	5	4	10	90	K
16	5	2	3	2	1	3	1	1	4	1	1	4	13	2	3	6	3	11	70	O
17	4	1	2	1	2	2	2	0	3	2	0	3	5	1	2	5	4	12	54	T
18	3	0	1	0	1	1	3	1	2	3	1	4	4	0	1	4	5	6	42	
19	2	3	0	1	0	1	3	1	2	3	1	4	4	0	1	4	5	6		

RESIDUALS

MATRIX  $\gamma_{3,3}$

+492	+189	+215	+129	+158	+410	+091	+112			
$\bar{M}$	$\bar{N}$	$\bar{O}$	$\bar{P}$	$\bar{Q}$	$\bar{R}$	$\bar{S}$	$\bar{T}$			
+091	+135	+125	+082	+095	+167	+212	+112	.	$\Sigma_0$	D
+074	+060	+003	+064	-158	-106	-051	+056	.	.004	+ .941
+065	+057	-049	+073	+030	-036	-244	+139	.	.001	+1.499
+004	+078	+035	-101	-041	+031	+060	+019	.	.002	+ .355
+059	+017	-008	-133	-037	+018	-023	-007	.	.001	+ .368
-037	-037	+029	+071	-018	-094	+057	-014	.	.004	+ .225
+018	-019	-090	-033	+042	-101	+061	+050	.	.001	+ .258
-033	-005	+103	-030	+122	-065	+107	-106	.	.004	+ .794
+002	-052	-092	+063	-063	+178	-017	-126	.	.001	+ .354
-123	+022	+053	-032	-013	+029	+085	-125	.	.003	+ .767
+086	-022	-070	+054	-056	-047	-052	+048	.	.001	+ .982
-048	-050	-063	-001	+026	+071	+066	+070	.	.001	+ .507
+ (144)	-010	-019	+053	-144	+043	+013	-072	.	.002	+ .681
-010	+ (114)	+051	-061	+066	-114	+041	-042	.	.001	+ .282
-019	+051	+ (103)	+053	+007	+037	-031	+032	.	.000	+ .470
+053	-061	-053	+ (133)	+088	-076	-007	-023	.	.002	+ .593
-144	+066	+007	+088	+ (144)	+050	-091	-005	.	.003	+ .252
+043	-114	+037	-076	+050	+ (178)	-111	+016	.	.002	+ .483
+013	+041	-031	-007	-091	-111	+ (244)	-045	.	.000	+ .742
-072	-042	+032	-076	-005	+016	-045	+ (139)	.	.001	+ .750
.	.	.	.	.	.	.	.	.	.	.
.002	.001	.000	.002	.003	.002	.000	.001	.	.	.
+ .681	+ .242	+ .470	+ .593	+ .252	+ .483	+ .742	+ .750	.	.	11.736 $\sqrt{}$
+ .202	+ .084	+ .140	+ .176	+ .075	+ .143	+ .220	+ .223	3.767	.	3.3669 $\sqrt{}$
+ .202	- .084	- .140	+ .176	- .075	- .143	- .220	+ .223	.	.	-2970 $\frac{1}{54}$

## The Magnitude of Residual Coefficients.

The theory underlying this criterion is, briefly:-

The raw correlations represent fallible data and are thus partly constituted of chance error elements. When all the systematic material in the correlations has been exhausted by extraction of sufficient factors there will be residue a varying quantity composed solely of error due to sampling. Hence, when the analysis has reduced the correlational data to proportions entirely explicable by chance, no further factors need be extracted. But then we have the enquiry: when can the residue be considered as explained entirely by chance? There are two answers provided here:

(i)\* When the median residual coefficient is equal to or less than the probable error of the median raw coefficient in the original correlational table multiplied by  $(1-\frac{r}{n})^r$ , where 'n' is the number of tests, and 'r' the number of factors hithertoe extracted. i.e.  $\tilde{r} \leq (1-\frac{r}{n})^r \cdot E_r$

It is probable that this formula will allow the minimum amount of attenuation of error likely.

(ii)† When the median residual coefficient is equal to or less than the probable error of the median raw coefficient in the original correlational table multiplied by  $(1-\frac{r}{n}+\frac{r}{n})^r$ , where, again, 'n' is the number of tests, and 'r' the number of factors hithertoe extracted. i.e.  $\tilde{r} \leq (1-\frac{r}{n}+\frac{r}{n})^r \cdot E_r$

Such a formula permits a maximum amount of attenuation of error.

Both the above formulae agree in principle that the median residual coefficient should not be of greater magnitude than the probable error of the median original correlation coefficient. In both cases the comparison is not with the probable error as such, but with the probable error decreased by an amount allowing for the amount of error that it, so to speak, removed from each correlation coefficient by each factor extraction. The second formula  $(1-\frac{r}{n}+\frac{r}{n})^r$  decreases  $E_r$  by a greater amount than the first, in that it assumes that more of the error incorporated in the co-variance is taken out with each factor extraction than is assumed by the first; hence it allows more of the correlation coefficient to be treated as systematic material than does the first. For Thurstone's view of the matter see Appendix 3.

## The Dispersion of Residual Coefficients.

Criteria based on the dispersion of residual coefficients represent the theoretical application of various characteristics of the normal curve to the problem in hand.

\* Prof. H.A. Reyburn.

† Mr. J.G. Taylor.

The argument here is: although there seems no reason why the distribution of fallible correlations should follow any known mathematical law, it may justifiably be supposed that the errors incorporated in the co-variance of the test correlations will be normally distributed. Hence, when the systematic material in the correlation table is completely exhausted, there will remain residuals smaller in magnitude than the errors of the original correlation co-efficients (smaller, since each factor removes with its loading some of the error.) The distribution of these final residuals should be normal.

The problem is thus one of normality of dispersion, and may conveniently be treated in two sections:

(i) Pearson\* gives the equations of different types of curves and has developed criteria which enable a given distribution to be defined as belonging to one of these types. The type of curve depends upon the value of two constants,  $\beta_1$  &  $\beta_2$ , derived from the 2nd., 3rd., and 4th. moments about the mean. The 1st. moment about a given point is  $\sum \kappa = 0$  where  $\kappa$  = deviations from a given point, which is normally the working origin corrected for arbitrariness. (i.e.  $\mu_1$ )  
 The 2nd. moment about a given point is  $\frac{\sum \kappa^2}{n}$  ( $\dots \mu_2$ )  
 The 3rd. moment about a given point is  $\frac{\sum \kappa^3}{n}$  ( $\dots \mu_3$ )

It is the 2nd. moment, therefore, which is used for calculating S.D. ( $\sigma$ ).

(a) Let us suppose, first of all, that we are dealing with half of a normal curve i.e. that the curve is folded back from its centre. Corresponding points on the negative portion of the curve will then completely coincide with those on the positive portion. In such a normal curve  $\mu_3 = 0$  ( $\mu_3$  &  $\mu_4$  are found from deviations equally on either side of the mean; hence positive and negative values cancel each other out)

$\beta_1$  is given by  $\frac{(\mu_3)^2}{(\mu_2)^3}$  (in such a normal curve this = 0)

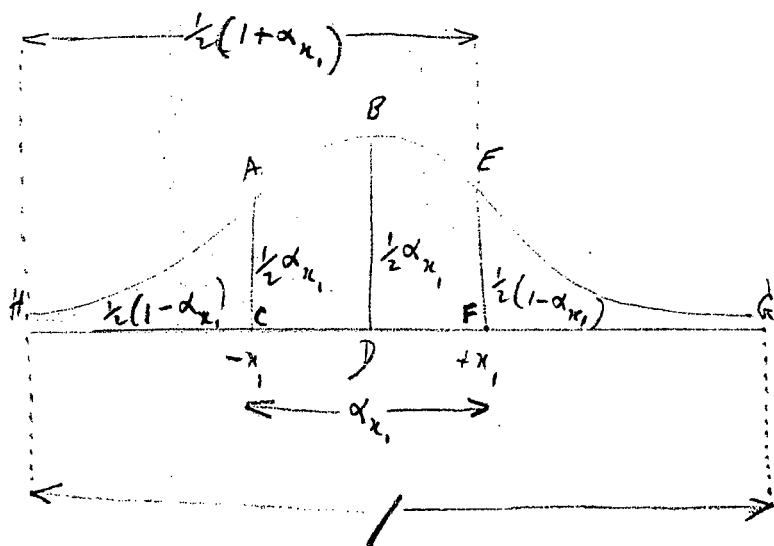
$\beta_2$  is given by  $\frac{\mu_4}{(\mu_2)^2}$

In the Gaussian or normal curve, folded over,

$$\beta_1 = 0 \quad \text{and} \quad \beta_2 = 3$$

If the dispersion of the residual coefficients is such that  $\beta_1 \neq 0$  and  $\beta_2 \neq 3$  then we have a skew curve; the given distribution does not fit its theoretical distribution and it is therefore evident that something more than error is incorporated in the material and, hence, a further factor should be extracted. On the other hand, if it is evident that  $\beta_1 = 0$ , and  $\beta_2 = 3$ , that fact alone could not certify that the systematic material is exhausted; it would simply mean that a normal curve would give the best fit to the data but this best fit may be a bad fit in comparison with other kinds of curves.

\* Karl Pearson. 'Tables for Statisticians and Biometricians'. Part 1.



The successive values of the probability integral recorded represent the shaded-in portion . i.e. records the successive values of  $E$  as  $F$  varies.

[ Case of the entire curve, not folded.]

It would thus be necessary to test the goodness of fit, as described below.

In actual practice  $\beta_1$  would not be exactly zero, nor  $\beta_2$  exactly 3; both these constants are subject to probable errors depending on the size of the population and given by Tables 37 and 38 in Pearson Part 1. These values must be multiplied by  $\frac{.6745}{\sigma_n}$

(b) In the case where the curve is not folded over,  $\beta_1 \neq 0$ . The values of  $\beta_1$  and  $\beta_2$  must therefore be calculated from  $\mu_2, \mu_3$ , and  $\mu_4$  as follows:

If  $V_1 = \frac{\sum x}{n}$  where  $x$  is the deviation from the mean,

$$V_2 = \frac{\sum x^2}{n} \quad \text{and} \quad V_3 = \frac{\sum x^3}{n}$$

then  $\mu_2 = V_2 - V_1^2$

$$\mu_3 = V_3 - 3V_1V_2 + 2V_1^3$$

$$\mu_4 = V_4 - 4V_1V_3 + 6V_1^2V_2 - 3V_1^4 - \frac{1}{2}(V_2 - V_1^2)^2 + \frac{7}{24}V_1^4$$

Again,  $\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$  and  $\beta_2 = \frac{\mu_4}{(\mu_2)^2}$

In all other respects, what applies to the folded curve applies to the entire curve.

(ii) Pearson likewise gives tables for the calculation of the Probability Integral and for the Test of Goodness of Fit, and these tables may be used to apply further characteristics of the normal curve to the solution of the present problem. The argument here, again, is simply that if the residual coefficients are due to chance error, their calculated distribution should tally with a corresponding theoretical distribution. The method of testing such correspondence may be briefly outlined as follows:-

(a) The case where only a half of the given and half of the theoretic curves are compared:  
The residuals are strung out according to magnitude, irrespective of sign (since equally positive and negative values coincide when the curve is doubled) and the deviations of their class bounding ordinates expressed in terms of S.D. as unit. Record the values of the probability integral corresponding to these S.D. deviation units, beginning with the value of the integral for deviation 0.  
(The geometrical picture of the concept of the probability integral may be represented by the diagram facing this sheet. The successive values of the probability integral recorded represent the shaded-in portion i.e. records the successive values of E as F varies. (This is the case of an entire curve, not folded))

Successive differences between these values are found. These give the proportions of the area in each of the classes, which must now be multiplied by  $2n$  to give the theoretical frequency of each class. To test for the goodness of fit, find the difference between the theoretical and observed frequencies. Square the differences and divide by the theoretical frequencies. This gives a value  $\chi^2.P$  (goodness of fit) is found from Pearson Table 12., by entering the table in column 'n' (no.

TABLE A.

$$\sigma = 3.4137$$

$$\frac{1}{\sigma} = .29294$$

<u>Units.</u>	<u>Areas</u>	<u>Diffs.</u>	<u>Y</u>	<u>Y'</u>	<u><math>\delta</math></u>	<u><math>\delta^2</math></u>	<u><math>\frac{\delta^2}{N}</math></u>
0	.5	.1152	39.40	38	1.4	1.96	.05
.2924	.6152	.1059	36.22	34	2.22	4.93	.14
.5858	.7211	.0892	30.51	36	5.49	30.14	1.00
.8782	.8103	.0692	23.63	28	4.37	19.10	.81
1.17176	.8794	.0491	16.79	14	2.79	7.78	.46
1.46470	.9285	.0321	10.98	8	2.98	8.88	.81
1.75764	.9606	.0193	6.60	4	.6	.36	.05
2.05058	.9799	.0106	3.63	3	.63	.40	.11
2.34352	.9905	.0053	1.81	2	.19	.04	.02
2.63646	.9958	.0025	.86	1	.14	.02	.02
2.92940	.9983	.0011	.38	0			
3.22234	.9994	.0004	.14	0	.62	.38	.66
3.51528	.9998	.0001	.03	1			
3.80822	.9999	.0001	.03	0			
			<u>171.01</u>				

$$\chi^2 = 4.13$$

Reduced  $\chi^2 = 11$

$$P = .947 \rightarrow$$

of cells in distribution) and taking  $\chi^2$  as argument.

(b) Where the entire observed and theoretic curves are compared the procedure is similar except for the following points:-

Regard is paid to the sign of the residual coefficients; hence zero is in the middle of the class distribution with a resulting greatest frequency at the mean and a tapering off at either end. At zero the probability integral for  $\left\{ \frac{(+1 \text{ class}) + (-1 \text{ class})}{2} \right\}$  is taken. The differences in areas cut off are multiplied by  $n$  (not  $2n$ ). The observed frequencies are clubbed together at the tails owing to the limited number of cases. Therefore Table 12 is entered at column 'n' where 'n' is the number of cells in the distribution with respect to the 'clubbing' together of the tails of the curve.

This somewhat lengthy exposition of the arguments upon which these criteria are based is given because at some stage or other of the analysis all were employed, and because, as a result of the present work, their theoretical significance will be farther discussed by us. At each stage of the analysis one or more of these criteria were employed to test for residual systematic material. Only one worked-out example of each type of criterion will be given; in all other cases the results only, and not the working out of these criteria will be recorded. The residual coefficients in Table 4 are surveyed and two criteria are employed to judge whether the residue is systematic material or not:-

- (i) Median Residual Coefficient = .048.  
Median p.e. of original coefficients = .062.  
if the residue co-variance is exhausted of

systematic material, then

$$\begin{aligned} .048 \text{ should } &\leq \left(1 - \frac{1}{2}\right)^2 \times .062 \\ &\leq \left(1 - \frac{1}{4}\right)^2 \times .062 \\ &\leq (.75)^2 \times .062 \\ &\leq .056 \end{aligned}$$

Hence, according to this criterion the residue in the table can be satisfactorily explained by chance.

- (ii) Prepare a folded over curve of the dispersion of the residual coefficients and discover  $P$  (Table A)

The fit here is excellent; again, it must be concluded that random errors of sampling suffice to explain the residue. Had the curve not been folded over the fit would not have been so extremely high, but nevertheless, it would have quite high enough for a similar conclusion to be arrived at.

E'		+279	+445	+106	+109	+067	+056	+236	+105	+228	+292	+151	+202	+
	- A	+ B	+ C	+ D	+ E	+ F	+ H	+ I	+ J	+ K	- L	+ M	-	+
		+102	+046	+090	+167	+090	+106	+128	+167	+073	+099	+067	+103	+
+279	- A	(149)	-031	031	+149	+046	+109	-076	+091	+024	-059	+014	+08	+
+445	+ B	-031	(146)	+017	+027	-051	-009	+038	+109	-079	+056	+009	+025	+
+106	+ C	-031	+017	(114)	+034	+000	+022	-114	+011	+052	+033	-016	-026	-
+109	+ D	+149	+027	+034	(149)	+000	+122	-070	+035	+029	-065	+029	-061	-
+067	+ E	+046	-051	+000	+000	(300)	-043	+045	-000	+017	-064	-027	+023	+
+056	- F	+109	-009	+022	+122	-043	(122)	-066	+003	+030	-028	+048	-001	+
+236	+ H	-076	+038	-114	-070	+045	-066	(115)	+011	+023	+115	-067	-015	+
+105	+ I	+091	+109	+011	+035	-000	+003	+011	(163)	+011	+015	+009	+023	-
+228	- J	+024	-079	+052	+029	+017	+030	+023	+011	(115)	+115	-056	-077	-
+292	+ K	-059	+056	+033	-065	-064	-028	+115	+015	+115	(115)	+031	+027	+
+151	- L	+014	+009	-016	+029	-027	+045	-067	+009	-056	+031	(084)	+017	+
+202	+ M	+018	+025	-025	-061	+023	-001	-015	+023	-077	+027	+017	(129)	+
+084	- N	+083	+020	-069	-008	+043	+012	+025	-061	+003	+003	+063	+007	+
+140	- O	-042	+013	+020	-023	+020	-078	+070	+107	-021	+029	-084	-009	-
+176	+ P	+015	+005	+082	+114	-083	+048	-012	+081	+008	+003	-026	+017	-
+075	- Q	+031	+063	-049	-045	-023	+045	+104	+071	+030	+034	+015	+129	-
+143	- R	+066	+100	+016	+042	-104	-089	-099	-163	+004	+005	+049	-072	+
+220	- S	-021	+146	-037	+047	-042	-080	-055	-040	+035	+012	-033	+057	+
+223	+ T	-024	+040	+043	+017	+001	+031	+037	+103	+074	+017	-036	+117	+
	Σ <sub>0</sub>	+002	+002	+000	+002	-001	-001	003	-001	-001	-002	-002	-001	.
	D	+0511	+0644	+0103	+0522	+0505	+0201	+0009	+0579	+0337	+0394	+023	+0329	-
	E	+0197	+0248	+040	+0201	+022	+078	+003	+0223	+0130	+0152	+009	+0127	?
	K	-0197	-0248	-040	-0201	-022	-078	-003	-0223	-0130	-0152	-009	-0127	-

RESIDUAL COEFFICIENTS.

MATRIX  $\gamma_{4,jk}$

+140	+176	+075	+143	+220	+223			
-0	+P	-Q	-R	+S	+T			
+083	+102	+138	+158	+195	+087		$\Sigma_0$	D
-042	+015	+071	+066	-021	-024		.002	+511
+013	+005	+063	+100	+146	+040		.002	+644
+020	+082	-049	+016	-037	+043		.000	+103
-023	+114	-045	+042	+047	+017		.002	+522
+020	-087	-023	+04	-042	+001		.001	+058
-078	+048	+048	-089	-080	+071		.007	+207
+070	-012	+104	-099	-055	+037		.003	+009
+107	+087	+071	-163	-040	+103		.001	+579
-021	+008	+030	+004	+035	+074		.007	+337
+029	+003	+034	+005	+012	+017		.002	+394
-084	-026	+015	+049	-033	-036		.002	+023
-009	+017	+129	-072	+057	+117		.001	+327
-039	-046	-060	+126	+023	+023		.001	+274
+(107)	+028	-004	+017	+062	+063		.002	+236
+028	+014	-102	+051	+032	+062		.000	+391
04	-102	+(129)	+039	+108	+012		.003	+530
017	+051	+039	+(163)	+142	+048		.000	+341
062	+032	+108	+142	+(146)	-004		.002	+498
067	+062	+012	+048	-004	+(117)		.001	+741
.002	.000	.003	.000	.002	.001			
+236	+391	+530	+341	+498	+741		6.721	$\gamma_T$
+091	+151	+204	+132	+192	+286	2.592	2.5925	$\sqrt{\gamma_T}$
-091	+151	-204	-132	+192	-286		.3857	$\frac{1}{\sqrt{\gamma_T}}$

TABLE 6.

	<u>I</u>	<u>II</u>	<u>III</u>
A.	.518	.383	-.279
B.	.492	.266	-.445
C.	.503	.384	.106
D.	.535	.351	.109
E.	.490	.411	.067
F.	.460	.381	-.086
H.	.500	.318	.236
I.	.654	.426	.105
J.	.545	.402	.228
K.	.223	-.392	-.292
L.	.201	-.229	-.151
M.	.286	-.492	-.202
N.	.042	-.189	.084
O.	.430	-.215	.140
P.	.369	-.569	-.176
Q.	.477	-.558	.075
R.	.438	-.450	.143
S.	.031	-.091	.220
T.	.376	-.112	-.223

TABLE 8.

$V_1 = .3859$

$\sigma = 1.91413$

$\frac{y}{\sigma} = .5224.$

$\frac{V-a}{x}$	$\frac{x}{\sigma}$	$\frac{1}{2}(1+d)$	$\Delta$	$y$	$y'$	$\delta^4$	$\delta^2$	$\frac{\delta^2}{y}$
4.1141	2.1492	.9842	-.0158	2.7018	2	.7018	.4925	-.18
3.1141	1.6268	.9484	-.0358	6.1218	8	1.8782	3.5276	-.58
2.1141	1.1044	.8643	-.0841	14.3811	13	1.3811	1.9074	-.13
1.1141	.5820	.7206	-.1437	24.5727	23	1.5727	2.4734	.10
.1141	-.0596	.5454	-.1752	25.9592	35	9.0408	81.7361	3.15
.0000	.	.	.2238	38.2698	38	.2698	.0728	—
-.8859	-.4628	.6784	-.1583	27.0693	25	2.0693	4.2820	-.16
1.8859	-.9852	.8367	-.0978	16.7238	15	1.7238	2.9715	-.18
2.8859	1.5076	.9345	-.0443	7.5753	$\left. \begin{matrix} 8 \\ 3 \\ 1 \\ 0 \end{matrix} \right\} \begin{matrix} 11 \\ 2008 \end{matrix}$	-	-	-
3.8859	2.0300	.9788	-.0158	2.7018				
4.8859	2.5524	.9946	-.0043	-.7353				
5.8859	3.0748	.9989	-.0011	-.1881				
				170.8652				
						.7995	.6392	.06

$\chi^2 = 4.54.$

Reduced  $\chi^2 = 9$

$P = .81$   
 $\longrightarrow$

Nevertheless, in spite of this apparently strong evidence, it was decided to extract a third factor, which could, if necessary, be rejected at will. The 3rd. factor loads are given by row K of Table 4 and are written down as column three in the Factorial matrix in Table 6.

We are now left with a table of 3rd. factor residuals. These are given in Table 5.

To test for systematic material in this table a normal curve was fitted (without folding) and calculated again. (Table B)

The fit again is quite good enough, and hence the analysis of the battery of 19 tests we deemed complete. This result was confirmed by a test used here only and not described elsewhere:-

$$E_{\gamma} = .062.$$

Correcting this for maximum attention we have

$$E_{\gamma} = .044.$$

Of the 171 correlations in the table, 72 are greater than .044 and 99 less than .044. Of these 5 correlations are greater than  $3 \text{ h.c. } (.132)$

According to the Table of the Probability Integral in which the argument is expressed in terms of the probable error, 4 correlations above  $3 \text{ h.c.}$  are to be expected by chance. The 5 obtained is deemed a close enough approximation. Hence 3 factors seem to be sufficient. At this point, the conclusion was that the co-variance of the entire battery of tests could be adequately explained by 3 factors. The factorial matrix thus derived is given in Table 6

### (ii) Psychological Considerations

The above result is, a priori, unsatisfactory. The make-up of the battery is so varied that an explanation of its complexity in terms of three factors does not seem likely. It was felt that more should be known about the constitution of the factorial components before the above result was accepted as valid and final. It was proposed, therefore, to split up the battery into its obvious groups:-

(i) Intelligence group

(ii) Music group.

(iii) As there were only 3 literary tests, these could not be treated analytically\* by themselves, and hence the literary tests were combined with the intelligence tests to form a third sub-battery.

Each of these sub-batteries was independently treated to a factorial analysis. By this method it was hoped that if any factors had been 'swamped' in the entire battery and rendered, so to speak, specific, they would be brought to light in the smaller analyses.

	+K	+L	+M	+N	+O	+P	+Q	+R	+S	+T		
											<del>+</del> L	D
	+359	+060	+343	+061	+110	+359	+269	+227	-009	+176	.	+1.955
	+060	+262	+122	+001	+072	+203	+250	+262	+093	+032	.	+1.357
	+343	+122	+439	+095	+210	+439	+267	+389	+067	+091	.	+2.462
	+061	+001	+095	+191	+110	+062	+191	-011	+059	-005	.	+ .754
	+110	+072	+210	+110	+331	+228	+331	+322	+002	+218	.	+1.934
	+359	+203	+439	+062	+228	+582	+582	+342	+056	+180	.	+3.043
	+269	+250	+267	+191	+331	+582	+582	+570	-025	+236	.	+3.193
	+227	+262	+389	-011	+322	+342	+570	+570	-056	+231	.	+2.726
	-009	+093	+067	+059	+002	+056	-025	-056	+093	-023	.	+ .257
	+176	+032	+091	-005	+218	+180	+236	+231	-023	+236	.	+1.372
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	+1.955	+1.357	+2.462	+ .754	+1.934	+3.043	+3.193	+2.726	+ .257	+1.372	.	19.053
	+448	+311	+524	+173	+443	+697	+732	+625	+059	+314	.	4.365
	+448	+311	+584	+173	+443	+697	+732	+625	+059	+314	.	.2291

$\frac{1}{\sqrt{4}}$   
 $\frac{1}{\sqrt{4}}$   
 $\frac{1}{\sqrt{4}}$

	+448	+311	+564	+173	+443	+697	+732	+625	+059	+314			
	<u>K</u>	<u>L</u>	<u>M</u>	<u>N</u>	<u>O</u>	<u>P</u>	<u>Q</u>	<u>R</u>	<u>S</u>	<u>T</u>			
	+158	+165	+121	+161	+135	+096	+046	+119	+090	+137	.	$\Sigma$	
K	+(188)	+079	+090	+017	+088	+047	+059	+053	+035	-035	.	.001	+ .561
L	+079	+(169)	+053	-053	-066	+014	+022	+068	-075	-066	.	.001	+ .145
M	+090	+053	+(231)	+003	+040	+046	+146	-037	+034	+086	.	.003	+ .692
N	+017	-053	+003	+(146)	+033	+059	+064	-119	-049	-059	.	.003	+ .042
O	+088	-066	+080	+033	+(151)	+081	+007	+045	+024	+079	.	.000	+ .482
P	+047	+014	+046	+059	+081	+(169)	-072	+094	+015	+039	.	.001	+ .492
Q	+059	+022	+146	+064	+007	-072	+(221)	+052	+068	+006	.	.003	+ .580
R	+053	+068	-037	-119	+045	+094	+052	+(149)	+093	+035	.	.003	+ .473
S	-035	-075	+034	-049	+024	+015	+068	+093	+(103)	+041	.	.002	+ .219
T	-035	-066	+086	-059	+079	+039	+006	+035	+041	+(174)	.	.001	+ .300
	.	.	.	.	.	.	.	.	.	.	.	.	.
	.001	.001	.003	.003	.000	.001	.003	.003	.002	.001	.	.	.
	+ .561	+ .145	+ .692	+ .042	+ .482	+ .492	+ .580	+ .473	+ .219	+ .300	.	.	3.986
	+ .281	+ .073	+ .347	+ .021	+ .241	+ .246	+ .291	+ .237	+ .110	+ .150	.	1.996	1.9965
	- .281	+ .073	- .347	+ .021	+ .241	- .246	+ .291	+ .237	- .110	+ .150	.	.	.50087

$\gamma_+$   
 $\sqrt{\gamma_+}$   
 $\frac{1}{\sqrt{\gamma_+}}$

51	+093	+347	+021	+241	+246	+291	+237	+110	+170			
K	$\pm L$	$\mp M$	$\pm N$	+0	-P	$\mp Q$	+R	-S	+T			
9	+164	+111	+146	+093	+108	+143	+133	+091	+151	.	$\Sigma_0$	D
77)	+058	-008	+011	-020	+022	-023	+014	+066	+077	.	.002	+ .274
058	+(084)	+028	-055	+084	+004	+001	-051	+083	+077	.	.001	+ .313
008	+028	+(119)	-004	+044	+039	+045	+119	+004	-034	.	.000	+ .352
011	-055	-004	+(124)	-028	-054	+058	+124	+051	+062	.	.001	+ .289
20	+084	+044	-028	+(084)	+022	+063	-012	-003	+043	.	.000	+ .277
22	+004	+039	-054	+022	+(144)	+144	+036	-012	+002	.	.001	+ .347
23	+001	+045	+058	+063	+144	+(144)	+017	-036	+038	.	.002	+ .451
014	-051	+119	+124	-012	+036	+017	+(124)	+067	-001	.	.000	+ .437
066	+083	+004	+051	-003	-012	-036	+067	+(083)	+024	.	.001	+ .327
077	+077	-034	+062	+043	+002	+038	-001	+024	+(077)	.	.001	+ .365
	.	.	.	.	.	.	.	.	.	.	.	.
002	.001	.000	.001	.000	.001	.002	.000	.001	.001	.	.	.
274	+ .313	+ .352	+ .289	+ .277	+ .347	+ .451	+ .437	+ .327	+ .365	.	.	3.432
148	+ .169	+ .190	+ .156	+ .150	+ .187	+ .243	+ .236	+ .177	+ .197	→	1.853	1.8526
48	- .169	+ .190	- .156	+ .150	- .187	- .243	+ .236	- .177	+ .197	.	.	- .5398

$\sqrt{x}$   
 $\sqrt[3]{x}$

TABLE C.

$$\bar{e} = -\frac{.374}{45} = -.008 \text{ (uncorrected)}$$

$$\bar{e} = -.008 - (.0667 \times .03) = -.010$$

$$V_1 = -\frac{3}{45} = -.0667$$

$$V_2 = .0044$$

$$V_3 = -.0003$$

$$V_4 = \frac{147}{45} = 3.2667$$

$$\mu_2 = V_2 - V_1^2 - \frac{1}{12} = 3.1790$$

$$\sigma = \sqrt{\mu_2} = 1.783$$

$$V_3 = 2.7333$$

$$\mu_3 = V_3 - 3V_1V_2 + 2V_1^3 = 3.3864$$

$$V_4 = 35.5333$$

$$\mu_4 = V_4 - 4V_1V_3 + 6V_1^2V_2 - 3V_1^4 - \frac{1}{2}(V_2 - V_1)^2 + \frac{7}{240} = 34.7741$$

$$\mu_2^2 = 10.1060$$

$$\mu_2^3 = 32.1271$$

$$\mu_3^2 = 11.4677$$

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = .3569$$

$$\beta_2 = \frac{(\mu_4)}{(\mu_2)^2} = 3.4409$$

$$E_{\beta_1} = 3.30 \times .10055 = \frac{.330}{.3167}$$

$$E_{\beta_2} = .9964$$

The music battery was the first to be analyzed. This constituted a battery of 10 tests. The table of raw correlations is given in Table 7. There being only ten variables, another method of estimating the communalities had to be devised. Finally, Spearman's formula for this was adopted:-

$$i.e. r_{19} = \frac{r_{27} \cdot r_{12}}{r_{23}}$$

In this case, the product of the two

highest correlations in the column were divided by their inter-correlation. The theoretical justification for this method of computing the communalities is outlined by Thurstone (p. 87. 'VECTORS') as follows:- If the test battery is so constructed that each postulated ability is represented by several tests, it can be expected that the 3 tests will be represented by test vectors with relatively small angular separations. Hence, the 3 vectors can be represented approximately by their projections on a common centroid factor..... The formula here may be written as  $h^2 = \frac{r_{1K} r_{jL}}{r_{KL}}$  The communalities

were calculated afresh at each stage of the analysis.

Row K becomes the first column of the factorial matrix in Table 12. The residual coefficients are given in Table 8. One factor is thus extracted. Does it, by itself account for the inter-correlations in the table?

(i) A comparison of the median residual coefficient and median correlation coefficient corrected for minimum attenuation gives: .053 ~~≠~~ .052.

(ii) Fitting half a curve to the data, we have  $P = .369$ .

Both these criteria postulate the advisability of extracting a second factor.

Row K in Table 8 represents the second factor loadings duly corrected for reflection, and are written down as the 2nd. column of the factorial matrix in Table 12. The residual coefficients are given in Table 10.

Do these two factors sufficiently explain the correlations?

(i)  $\bar{r} = .038$ ;  $E\bar{r}$  (corrected for MINIMUM ATTENUATION) = .045.  $\therefore .038 < .045$ .

According to this standard of judgement, the systematic material has been exhausted.

(ii) Testing to find whether the  $\beta_1, \beta_2$  constants of the equation of the normal curve are satisfied we have Table C.

$\beta_1, \beta_2$  are not exactly 0 and 3 respectively. The error permitted in  $\beta_1$  is 3.30 (from table of  $\beta_1, \beta_2$ , Pearson)  $\times$  .10055 (S.D. from Table 5, Pearson) = .330.

53

TABLE 12.

	<u>I</u>	<u>II</u>	<u>III</u>
K.	.448	-.281	-.148
L.	.311	.073	-.169
M.	.564	-.347	-.190
N.	.173	.021	-.156
O.	.443	.241	.150
P.	.697	-.246	.187
Q.	.732	.291	-.243
R.	.625	.237	.236
S.	.059	-.110	.177
T.	.314	.150	.197



$E'$		$+A$	$+B$	$+C$	$+D$	$+E$	$+F$	.	$\Sigma_0$	$D$
$+A$	$(651)$	$+512$	$+409$	$+232$	$+346$	$+299$	.	.	.	$+2.454$
$+B$	$+512$	$(610)$	$+319$	$+334$	$+269$	$+352$	.	.	.	$+2.400$
$+C$	$+409$	$+319$	$(444)$	$+450$	$+411$	$+391$	.	.	.	$+2.428$
$+D$	$+232$	$+334$	$+450$	$(387)$	$+413$	$+493$	.	.	.	$+2.489$
$+E$	$+346$	$+269$	$+411$	$+413$	$(377)$	$+333$	.	.	.	$+2.149$
$+F$	$+299$	$+352$	$+391$	$+493$	$+333$	$(424)$	.	.	.	$+2.300$
$\Sigma_0$	.	.	.	.	.	.	.	.	.	.
$D$	$+2.454$	$+2.400$	$+2.428$	$+2.489$	$+2.149$	$+2.300$	.	.	.	$+14.220$
$E$	$+651$	$+636$	$+644$	$+660$	$+570$	$+610$	$\rightarrow$	$3.7697$	.	$+3.77094$
$K$	$+651$	$+636$	$+644$	$+660$	$+570$	$+610$	.	.	.	$+2651$

$\checkmark$

$\checkmark$

$\frac{1}{\checkmark}$



		+431	+449	+157	+357	+166	+178			
		+A	+B	-C	-D	-E	-F			
		+137	+178	+079	+058	+076	+073		$\Sigma_0$	D
+431	+A	( )	-096	-058	+043	-048	+020		.002	
+449	+B	-096	( )	+021	-074	+018	-048		.001	
+157	-C	-058	+021	( )	-031	+018	-029		.000	
+357	-D	+043	-074	-031	( )	-022	+027		.001	
+166	-E	-048	+018	+018	-022	( )	-044		.002	
+178	-F	+020	-048	-029	+027	-044	( )		.001	
		.	.	.	.	.	.		.	.
	$\Sigma_0$	.002	.001	.000	.001	.002	.001		.	.
	D	.	.	.	.	.	.		.	.
	E	.	.	.	.	.	.		.	.
	K	.	.	.	.	.	.		.	.

The .3569 obtained is reasonably close to the theoretical .330. Similarly the error for  $\beta_2 = .996$ . The observed deviation = .441, which is within the permitted deviation.

Altogether the position is doubtful.  $\beta_1/\beta_2$  may represent a normal curve. Is this a good curve? Applying the test of goodness of fit to an unfolded curve, we have  $\rho = .245$ . Manifestly, this criterion indicates a third factor.

Here we have a dilemma. Different criteria yield different solutions to the problem. It was eventually decided to proceed with the extraction of a third factor.

The table of 2nd factor residual coefficients is given in Table 10. Row K represents the third factor loads and becomes the 3rd column in the factorial matrix in Table 12. The residual coefficients are given in Table 11. Applying tests for systematic residue to the table after the 3rd. factor extraction, we notice:-

(i) Comparing the median residual coefficient with the median probable error corrected for maximum attenuation.

.034 ~~≠~~ .029. This indicates the existence of systematic residue.

(ii) A similar comparison with  $\mu.c.$  corrected for minimum attenuation yields:

.034  $\approx$  .040. The assertion here is that there is no systematic material left.

There being no conclusive method of deciding the issue, it was decided to leave the analysis of the music battery as resulting in 3 factors only. It was hoped that as the analysis progressed there would be thrown further light on the justifiableness of such a procedure.

The factorial matrix thus derived is given in Table 12.

The intelligence battery was the next to be treated to an analysis. This was composed of the 6 parts of the Cattell test,

In view of the smallness in number of the variables, the communalities were again estimated by the Spearman formula.

The first factor loadings extracted from the intelligence battery are given by row K of Table 13, and are written down as column one of the factorial matrix in Table 16. The residual coefficients are contained in Table 14. If the intelligence test is a pure test of intelligence there should be only this one general factor present in the battery. Is this the case?:-

$$(i) \bar{r} = .086 ; E_{\bar{r}} = .045$$

$$.045 \text{ should } \geq .086 \times (1 - \frac{1}{n})$$

$$\text{i.e. } .045 \text{ shall } \geq .070$$

Obviously, one factor is insufficient to account for the residue.

TABLE 16

	<u>I</u>	<u>II</u>
A.	.6506	.431
B.	.6362	.449
C.	.6437	— .157
D.	.6598	— .357
E.	.5897	— .166
F.	.6097	— .178

E														
	+A	+B	+C	+D	+E	+F	+H	+I	+J					
													$E_0$	D
+A	+564	+512	+409	+232	+346	+299	+391	+564	+396					3.713
+B	+512	+(512)	+319	+334	+269	+356	+264	+279	+353					3.198
+C	+409	+319	+(493)	+450	+411	+391	+285	+493	+400					3.651
+D	+232	+334	+450	+(493)	+413	+493	+336	+476	+429					3.656
+E	+346	+269	+411	+413	+(502)	+333	+437	+502	+430					3.643
+F	+299	+356	+391	+493	+333	+(493)	+265	+451	+354					3.435
+H	+391	+264	+285	+336	+437	+265	+(476)	+476	+432					3.362
+I	+564	+279	+493	+476	+502	+451	+476	+(584)	+562					4.367
+J	+396	+353	+400	+429	+430	+354	+432	+562	+(562)					3.718
.	.	.	.	.	.	.	.	.	.					.
.	.	.	.	.	.	.	.	.	.					.
$E_0$	.	.	.	.	.	.	.	.	.					.
D	+3.713	+3.198	+3.651	+3.656	+3.643	+3.435	+3.362	+4.367	+3.718					32.943 $\frac{1}{4}$
E	+647	+557	+636	+637	+635	+598	+586	+761	+683					5.7396 $\sqrt{\frac{1}{4}}$
K	+647	+557	+636	+637	+635	+598	+586	+761	+683					17423 $\frac{1}{\sqrt{4}}$

E'		+647	+557	+636	+637	+635	+598	+586	+761	+683				
		+A	+B	+C	+D	+E	+F	+H	+I	+J				
		+146	+202	+089	+087	+099	+135	+133	+005	+096			$\Sigma_0$	D
+647	+A	+(150)	-152	+002	+180	+065	+088	+012	+072	-046			.001	+ .401
+557	+B	-152	+(152)	-035	-021	-085	+023	+062	+145	+027			.002	+ .116
+636	+C	+002	-035	+(188)	+045	+007	+011	+088	-009	+034			.002	+ .331
+637	+D	+180	-021	+045	+(180)	+009	+112	+037	+009	+006			.000	+ .557
+635	+E	+065	-085	+007	+009	+(185)	-047	-065	-019	+004			.002	+ .054
+598	+F	+088	+023	+011	+112	-047	+(112)	+085	+004	+054			.002	+ .442
+586	+H	+012	+062	+088	+037	-065	+085	+(188)	+030	+032			.000	+ .469
+761	+I	+072	+145	-009	+009	-019	+004	+030	+(145)	+042			.001	+ .419
+683	+J	-046	+027	+034	+006	+004	+054	+032	+042	+(154)			.001	+ .307
	$\Sigma_0$	.001	.002	.002	.000	.002	.002	.000	.000	.001				
	D	+ .401	+ .116	+ .331	+ .557	+ .054	+ .442	+ .469	+ .419	+ .307				3.096 $\frac{1}{4}$
	E	+ .228	+ .066	+ .188	+ .317	+ .031	+ .251	+ .267	+ .238	+ .174				1.759 $\frac{1}{\sqrt{4}}$
	K	- .228	+ .066	+ .188	+ .317	+ .031	+ .251	- .267	- .238	- .174				.5684 $\frac{1}{\sqrt{4}}$

TABLE IV  
FACTORS RESIDUALS

E'		+228	+066	+188	+317	+031	+251	+267	+238	+174				
		<del>A</del> +A	+B	+C	+D	+E	+F	-H	-I	-J				
		+128	+148	+153	+080	+184	+049	+187	+088	+124		$\Sigma_0$	D	
+228	+A	+(167)	+167	+041	+108	+058	-031	+049	-018	+086			.000	+ .627
+066	+B	+167	+(167)	-047	+042	+087	+006	+044	+129	+016			.000	+ .611
+188	+C	+041	-047	+(154)	+015	-001	-036	+038	-084	+001			.000	+ .111
+317	+D	+108	+042	+015	+(108)	-001	-032	+048	+066	+049			.001	+ .403
+031	+E	+058	+087	-001	-001	+(137)	+055	+073	+026	+001			.000	+ .435
+251	+F	-031	+006	-036	-032	+058	+(106)	+018	-086	+010			.001	+ .040
+267	-H	+049	+044	+038	+048	+073	+018	+(123)	-034	-014			.001	+ .345
+238	-I	-018	+129	-054	+066	+026	-086	-034	+(129)	+001			.000	+ .189
+174	-J	+086	+016	+001	+049	+001	+010	-014	+001	+(126)			.000	+ .286
	$\Sigma_0$	.000	.000	.000	.001	.000	.001	.001	.000	.000				
	D	+ .627	+ .611	+ .111	+ .403	+ .435	+ .040	+ .345	+ .189	+ .286				3.047 $\frac{1}{4}$
	E	+ .359	+ .350	+ .064	+ .231	+ .249	+ .023	+ .198	+ .108	+ .164				1.7456 $\frac{1}{4}$
	K	+ .359	+ .350	+ .064	- .231	- .249	+ .023	- .198	- .108	- .164				.5729 $\frac{1}{4}$

We have now the results of four distinct analyses of parts of the same battery, producing factorial matrices for the entire battery, the music battery, the intelligence and literary battery, and the intelligence battery. How far are the factors thus isolated valid? It has been shown that the criteria for determining the point at which to stop extracting factors are speculative and giving far from coincident verdicts. It might therefore be reasonably supposed that the 'core' of each analysis is vouched for but that the factors with lighter loads on the fringe of the analysis, as it were, are uncertain. Is there any way in which we can judge the validity of all the factors from a fixed and decisive point of view, or are the factors vague entities relative to the specific battery from which they are isolated?

Thurstone's answer to this question implies that factors extracted must have a certain degree of objectivity. The factors must be such that, in another context, the test in which they are manifested will reproduce very similar loads on the same factors. He writes <sup>x</sup>: "one of the important restrictions that must be satisfied by any acceptable solution to the factor problem is that the factorial description of a trait or test must be invariant when it is moved from one battery to another. No form of uniqueness can be scientifically meaningful which violates this principle." Again we quote from the 'Vectors': 'The factorial description of a trait must remain invariant when the trait is moved from one battery to another which involves <sup>the</sup> same common factors or abilities.'

From our point of view, there can be no doubt that the sub-battery does involve some of the same common factors concerned in the entire battery. Such a procedure for the testing of factorial validity requires that the test in question should be transported into a different battery and the analytic process gone through again. Such a method was impossible in the present case and so the matter seemed to rest there. Fortunately, however, during a discussion of this point, which the writer had with a member of the staff of the department <sup>who</sup> he had suggested to him a solution to the problem which seemed not only neat and convenient but also decisive: If the factors explaining the tests in the entire battery were objectively valid, then when the tests were treated independently in the sub-batteries, surely they should yield a factorial composition similar to that of the tests in the entire battery?

In place of the conventional procedure of comparing one battery with another, we here proposed to compare a battery with part of itself. The principle underlying this theory appeared sound and a technique was devised to test for satisfaction of the theory in the present instance. If the factorial make-up of certain tests in the entire battery is to be the same as its make-up in the sub-battery, then it ought to be possible, by means of an orthogonal transformation, to convert the orthogonal reference frame of the one factorial

x 'Vectors of Mind' Preface p.viii.  
 ø Mr. J.G. Taylor.

matrix into the orthogonal reference frame of the other. The issue, therefore, reduces to discovering the necessary transformation matrix.

Since the entire battery yielded three factors, we have, in matrix notation:

$$\|A\| \cdot \|L\| = \|X\|$$

where A represents the coefficients of the unknown direction-cosines, L the required direction-cosines, and X the constants of the equations. This gives three equations in three unknowns and hence the requisite direction-cosines can be found.

In this manner there were worked out two transformation matrices, one for rotating the intelligence + literary test loads in the entire battery into the loads of the intelligence + literary tests in the sub-battery, and the other for a similar purpose with the music tests in the entire and sub-battery. Both these transformation matrices, if they are orthogonal, should have direction-cosines such that the sum of their squares should be equal to unity. This condition was not fulfilled for either of the transformation matrices thus obtained. Hence, it can be concluded that the analysis of the entire battery is not equivalent to that of its 2 parts. Before this equivalence was finally obtained it proved necessary to extract three more factors from the entire battery. As an example of the method employed, there is given below Table 21 by which it will be seen that the extraction of 5 factors renders the analysis of the entire battery equivalent to the analysis of the intelligence + literary battery.

Table 21.

Given the factorial matrices:

F. Entire Battery					F. Intelligence + Literary Battery.			
	I	II	III	IV	V	I	II	III
A	.518	.383	-.279	.197	-.314	.647	-.228	.359
B	.492	.266	-.445	.248	.153	.357	.066	.350
C	.503	.384	.106	.040	.086	.636	.188	.064
D	.535	.351	.109	.201	-.175	.637	.317	-.231
E	.490	.411	.067	.022	-.171	.635	.031	-.249

We can write down the necessary equations:-

( Let  $\lambda_1, \dots, \lambda_r$  be the required direction-cosines of the required transformation matrix.)

$$\begin{aligned}
 A: & \cdot 518 d_1 + \cdot 383 d_2 - \cdot 279 d_3 + \cdot 197 d_4 - \cdot 314 d_5 = \cdot 687 \\
 B: & \cdot 492 d_1 + \cdot 266 d_2 - \cdot 445 d_3 - \cdot 248 d_4 + \cdot 153 d_5 = \cdot 157 \\
 C: & \cdot 103 d_1 + \cdot 384 d_2 + \cdot 106 d_3 + \cdot 040 d_4 + \cdot 086 d_5 = \cdot 636 \\
 D: & \cdot 535 d_1 + \cdot 351 d_2 + \cdot 109 d_3 + \cdot 201 d_4 - \cdot 175 d_5 = \cdot 637 \\
 E: & \cdot 490 d_1 + \cdot 411 d_2 + \cdot 067 d_3 + \cdot 022 d_4 - \cdot 171 d_5 = \cdot 635
 \end{aligned}$$

The solution of these 5 simultaneous linear equations will give the 1st column of the transformation matrix. By substituting the 2nd. column of factor loads of the intelligence+ literary factorial matrix in the place of the constants on the right hand of the above equations, the second column of the transformation matrix may be derived. The third column of the transformation matrix will be such as fulfills the condition that the sum of the squares of the direction-cosines for each axis will be unity. By this method the first two columns of the transformation matrix are seen to be:

$$\begin{aligned}
 \lambda_1 &= \cdot 721 & \cdot 953 \\
 \lambda_2 &= \cdot 714 & -\cdot 932 \\
 \lambda_3 &= \cdot 000 & 1\cdot 011 \\
 \lambda_4 &= \cdot 000 & -\cdot 329 \\
 \lambda_5 &= \cdot 000 & \cdot 057
 \end{aligned}$$

The sum of their cross products should be reasonably close to zero. i.e.

$$\begin{aligned}
 \cdot 721 \times \cdot 953 &= \cdot 6871 \\
 \cdot 714 \times -\cdot 932 &= -\cdot 6654 \\
 \cdot 000 \times 1\cdot 011 &= \cdot 000 \\
 \cdot 000 \times -\cdot 329 &= \cdot 000 \\
 \cdot 000 \times \cdot 057 &= \cdot 000
 \end{aligned}$$

$$\therefore \sum_{d=1}^5 \lambda = \cdot 0217 = A$$

This gives a sufficiently close approximation.

It should be noted that the direction-cosines here calculated are not the real direction-cosines but cosines in proportion to the real ones. Hence, although the first condition of orthogonality of the transformation matrix is satisfied, it cannot be expected that the second (sum of squares = 1) should be directly satisfied. It is necessary to normalize the cosines obtained in this found before this latter condition can be satisfied. If the two axes implied in the direction cosines are to form part of an orthogonal frame they must be mutually perpendicular. Normalizing the cosines we find:

		+197	+248	+040	+201	+022	+078	+003	+223	+130	+152	+009	+127	+106
		$\bar{F}A$	+B	+C	$\pm D$	$\pm E$	$\pm F$	+H	-I	-J	+K	$\bar{L}$	$\bar{M}$	$\bar{N}$
		-110	+054	+112	+109	+300	+116	+115	+113	+098	+092	+084	+113	+115
+197	$\bar{F}A$	+(109)	+050	+039	+109	+042	+094	+077	-047	+002	+089	+012	-007	+062
+248	+B	+050	+(111)	+007	+023	+056	+018	+037	+054	-111	+018	-007	+006	+086
+040	+C	+039	+007	+(114)	-026	+001	-019	-114	+002	+047	+027	+016	+030	+073
+201	$\pm D$	+109	+023	-026	+(109)	-002	+106	+071	+010	+003	+096	+027	-087	-019
+022	$\pm E$	+042	+056	+001	-002	+(107)	-045	-045	+005	+014	+067	-027	+020	+041
+078	$\pm F$	+094	+018	-019	+106	-045	+(106)	+066	+014	-020	+045	+047	-011	+004
+003	+H	+077	+037	-114	+071	-045	+066	+(117)	+010	+023	+115	+067	+015	-025
+223	-I	-047	+054	+002	+010	+005	+014	+010	+(192)	-018	-019	-007	+005	+085
+130	-J	+022	-111	+047	+003	-014	-020	+023	-018	+(111)	+095	+057	+053	+011
+152	+K	+089	+018	+027	+096	+067	+040	+115	-019	+095	+(115)	-030	-008	+013
+009	$\bar{L}$	+012	-007	+016	+027	-027	+047	+067	-007	+057	-030	+(085)	+016	+062
+127	$\bar{M}$	-007	+006	+030	-087	+020	$\frac{3}{5} 011$	+015	+005	+093	-008	+(016)	+(103)	-006
+106	$\bar{N}$	+062	+006	+073	-029	+041	+004	-025	+035	+011	+013	+062	-006	+(112)
+091	-O	+060	-010	+016	+041	-018	+085	+070	+087	-033	+015	+085	+021	+044
+151	+P	+015	-032	+076	-084	+056	-036	-012	+047	-012	-020	+027	+002	+062
+204	-Q	+009	+012	-059	+046	+027	-032	+103	+026	+003	+003	-013	-103	+082
+132	-R	-040	+067	+001	-015	+107	+099	-099	-192	-013	-015	-048	+089	-112
+192	+S	+057	+098	$\frac{3}{5} 005$	-008	+046	+095	-054	-083	+010	-017	+035	-033	-003
+266	-T	+080	-031	+032	+040	+005	-009	+036	+039	+037	-027	+039	-081	+007
$\Sigma_0$		+002	+001	000	-002	+002	000	-003	+001	+000	+003	000	+002	-001
D		+044	+0412	+0230	+0470	+0457	+0612	+0456	+020	+0271	+0557	+0443	+0060	+0494
E		+0314	+0153	+0086	+0175	+0171	+0228	+0170	+0078	+0101	+0207	+0165	+0022	+0184
K		+0314	+0153	+0086	+0175	+0171	+0228	+0170	+0078	+0101	+0207	+0165	+0022	+0184

RESIDUAL COEFFICIENTS

MATRIX  $T_{5JK}$

+091	+151	+204	+132	+192	+256			
-0	+P	-Q	-R	+S	-T			
099	+091	+087	+146	+109	+035		$\Sigma$	D
+060	+015	+009	-040	+059	+080		+002	+844
-010	-032	+012	+067	+058	-031		+001	+412
+016	+076	-059	+011	-045	+032		000	+230
+041	+084	+086	-015	-008	+040		+002	+470
-018	+086	+027	+07	+046	+005		+002	+459
+085	-036	-032	+099	+095	-009		000	+612
+070	-012	+103	-099	-054	+036		-003	+456
+087	+047	+026	-192	-083	+039		+001	+210
-033	-012	+003	-013	+010	+037		+000	+271
+015	-020	+003	-015	-017	-027		+003	+557
+055	+027	-013	-048	+035	+039		000	+443
+021	+002	-103	+089	-033	-081		+002	+060
+049	+062	+082	-112	-003	+007		-001	+494
+087	+014	-023	+005	+045	+037		+001	+633
+014	+133	-133	+031	+003	+019		+001	+186
-023	-133	+133	+012	+069	-046		+000	+158
+005	+031	+012	+117	+117	+010		000	+131
+045	+003	+069	+117	+117	-059		+002	+392
+037	+019	-046	+010	-059	+081		+001	+209
+001	+001	000	000	+002	+001			
+633	+186	+158	+131	+392	+209		7227	4
+235	+069	+059	+049	+146	+078		26883	$\sqrt{4}$
-235	+069	-059	-049	+146	-078		37198	$\frac{1}{\sqrt{4}}$

TABLE 23.

5<sup>th</sup> FACTOR.

		+314	+153	+086	+176	+171	+228	+170	+078	+101	+207	+165	+0
		± A	+ B	± C	± D	- E	± F	± H	- I	± J	± K	± L	
		+010	+088	+107	+078	+078	+054	+086	+186	+101	+072	+058	
314	± A	(071)	-032	+0.2	+054	+012	+022	-024	+071	-030	+024	+040	
153	+ B	-032	(126)	+006	+004	+030	+007	+011	+042	+126	+014	+032	
086	± C	+054	+006	(150)	-041	+014	-039	-129	+005	+038	+009	+002	
175	+ D	+012	+004	-041	(096)	+032	+066	+041	+004	-015	+060	-002	
171	- E	+022	+030	+014	+032	+099	+084	+075	+008	+031	-032	+055	
228	± F	+024	+007	-039	+066	+084	(088)	+027	+004	-043	-007	+009	
170	± H	+071	-011	-129	+041	+075	+027	(129)	+003	+006	+080	+039	
078	- I	-030	+042	+005	+004	-008	+004	+003	(188)	+026	+035	+030	
101	+ J	+024	+126	+038	-015	+031	-043	+006	+026	(126)	+074	+040	
207	± K	+043	+014	+009	+060	-032	-007	+080	+035	+074	(080)	-064	
165	± L	+014	+032	+002	-002	+055	+009	+039	+030	+040	-064	(064)	
022	- M	-024	+003	-028	+091	+016	+016	-011	+003	-091	+013	-012	
184	- N	-014	-022	-057	+061	+010	+038	+056	+071	+007	+025	-032	
235	+ O	+007	+046	-004	+000	+058	+031	+030	-069	-057	-034	+026	
069	+ P	+010	-043	-070	+096	+074	+052	+024	+042	+019	+034	-016	
059	- Q	+055	+003	+062	-076	+017	+045	-093	+021	+003	+009	+023	
049	- R	-050	+060	-007	+024	+099	-088	+107	-188	+018	+025	+056	
146	+ S	-013	+076	+058	+034	+021	-062	-079	-094	+005	+047	-011	
678	+ T	+056	+043	+025	+026	+008	-027	+023	-033	+029	-043	+026	
20		-002	-002	-002	-002	-000	-002	-001	-001	-001	-001	-001	
	D	+379	+422	+006	+555	+695	+223	+499	+153	+312	+349	+325	
	E	+153	+170	+002	+224	+280	+090	+201	+062	+126	+141	+131	
	K	-153	+170	-002	+224	-280	+090	-261	-062	+126	-141	-131	

# RESIDUALS

# MATRIX $r_{jk}$

+184	+235	+069	+059	+049	+146	+078			
-N	-O	+P	-Q	-R	+S	-T			
+078	+032	+128	+150	+115	+076	+075	$\Sigma_0$	D	
-004	-017	+007	+010	+055	-012	+056	.002	+379	
-022	+046	-043	+003	+060	+076	+043	.002	+422	
-057	-004	-070	+062	-007	+058	+025	.002	+006	
+061	+000	+096	-076	-024	+034	+026	.002	+555	
+010	+058	+074	+017	+077	+021	+009	.000	+675	
+038	+031	+052	+045	-087	-062	-027	.002	+223	
+056	+030	+024	-093	+107	+079	+023	.001	+499	
+071	-069	+042	+021	-188	-094	-033	.001	+153	
+007	-057	+019	+003	+018	+005	+029	.001	+312	
+025	-034	+034	+009	+025	+047	-043	.001	+347	
-032	+046	-016	+023	+056	-011	+026	.001	+325	
-010	-016	+000	-102	+088	-036	+091	.002	+131	
+(121)	-066	+049	+071	-121	-030	+007	.001	+234	
-006	+(069)	+002	+037	+007	-011	+019	.001	+134	
+049	+002	+(131)	-051	+028	-007	-014	.000	+277	
+071	+037	-131	+(137)	+009	+060	+051	.001	+150	
-121	+007	+028	+009	+(488)	+110	-006	.001	+164	
-030	-011	-007	+060	+110	+(110)	+070	.000	+106	
+007	+019	-014	+051	-006	+070	+(671)	.000	+142	
.001	.001	.000	.001	.001	.000	.000			
+234	+134	+277	+150	+164	+100	+412			
+094	+054	+112	+060	+177	+164	+178			
-094	+054	+112	-060	-177	+164	+178			

$$\sum_{\lambda=1}^5 \lambda_1^2 + \sum_{\lambda=1}^5 \lambda_2^2 = 2.9965$$

$$\sqrt{2.9965} = 1.731$$

$$1.731 \times A = 1.731 \times .0217$$

$$= .0376$$

$\therefore$  the angle between the axes =  $\cos^{-1} .0376$

$$= 87^{\circ} 50'$$

This is very close to  $90^{\circ}$ . Perfect normality will be obtained only when 'n' factors are extracted - a futile position.

Hence we see that the analysis of the entire battery is equivalent to the analysis of the sub-battery; the factors extracted in either case have an objective reality. The theoretic implications of the this method would seem to be of some significance. Where the routine testing of factors is to place the tests concerned in another battery, we can point out that the splitting up of a battery provides an alternative and far more convenient method of testing for the validity of the factors. At the same time, a check upon the number of factors that may justifiably be extracted from the table is indicated. In view of the speculative and unsatisfactory nature of the criteria for determining the existence of systematic residue such a check is doubly of value. From the psychological point of view it would appear of more use than any of the more mathematical devices.

The three tables from which the 4th., 5th., and 6th. factors are calculated are Table 5, Table 22, and Table 23.

In the case of having to find the transformation matrix of a factorial matrix of 6 dimensions, the arithmetical labour involved in calculating d-cosines is prohibitive and can hardly be reckoned in mere hours. The shortest method for solving for 6 unknowns appears to be the method of calculating the inverse or reciprocal of the given matrix of coefficients.

$$\llcorner \text{ if } \|A\| \cdot \|L\| = \|X\|$$

$$\text{then the required } \|L\| = \frac{\|X\|}{\|A\|}$$

If we can calculate the cells of the matrix which is the inverse of the matrix of the coefficients of the unknowns, the problem is solved. Two methods of achieving this solution are generally used:

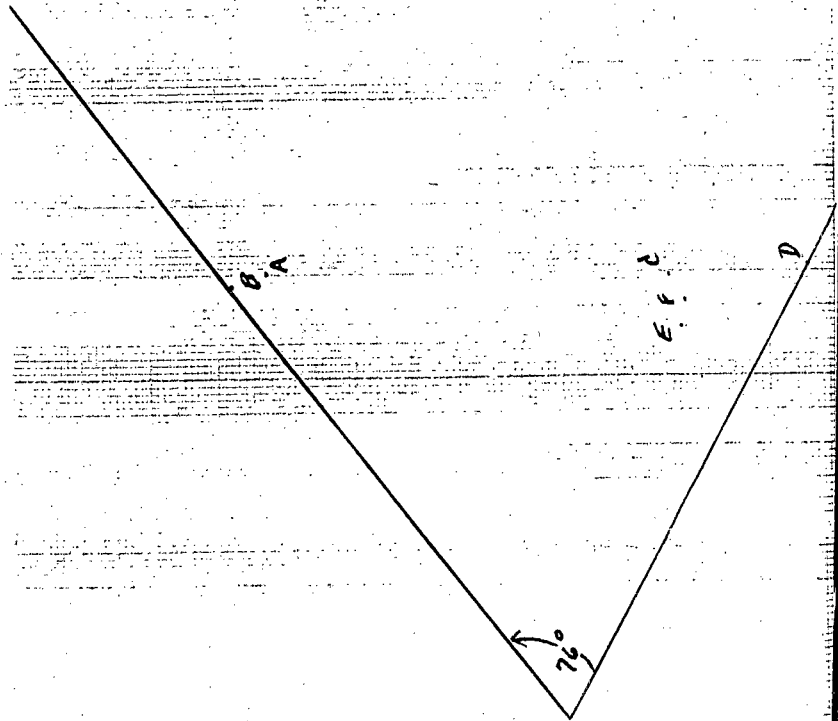
(i) Calculate the adjoint of A. This gives the inverse  $A^{-1}$  with elements  $f_{ij} = \frac{f_{ji}}{|A|}$

φ  
 (11) If  $R_{AA}$  is the given matrix, then by Aitken's Method of Pivotal Condensation,  $R_{AA}^{-1}$  can be found, such that

$$R_{AA} \cdot R_{AA}^{-1} = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} = I$$

If there is any impression that the extraction of six factors from the entire battery seems overmuch it will perhaps be corrected by reference to the original correlation coefficients in Table 1. It will be noticed that the correlations between the music tests and either the intelligence or literary tests is small. The median correlation here is only slightly above zero. From Pearson Table 5 it can be shown that for 113 cases, as we have here, a zero correlation has a probable error of .063. Among the correlations concerned roughly half the correlations are less than .063. This makes it reasonable to suppose that the three factors involved in the music battery are independent of the three factors involved in the intelligence and literary tests. Hence an analysis yielding six factors in the entire battery is not unexpected.

FIGURE 1



## CONCLUSIONS.

The analysis of the entire battery yields six Factors. Each of the two main sub-batteries is explained by 3 Factors. Inspection of the entire correlational matrix shows a practically zero correlation between the music tests, on the one hand, and the intelligence and literary tests on the other. This lack of significant correlation between the two sub-batteries tends to show that the 3 factors in the music battery are probably distinct from the 3 factors in the intelligence and literary battery. It might therefore be deduced that the 6 factors in the entire battery comprise no more or less than the two lots of 3 factors of each of its half-batteries.

This assumption as to the general nature of the six factors in the entire battery appears reasonable, and so it was decided to examine the two groups of 3 factors produced by each sub-battery before deciding upon the necessity of attempting to give meaning to the 6 factors of the entire battery.

The first group of factors to be examined was that in the combined intelligence and literary battery. As a preliminary to this examination it was thought of use to examine the 3 factors of the intelligence battery alone. It might be that the 3 factors of the combined intelligence and literary battery were equivalent to the 2 factors of the intelligence battery and a third factor introduced by some additional element in the 3 literary tests.

The important result emerging from the analysis of the Cattell Intelligence Test was the establishing of a definite second factor essential for success at the test. The test was devised as a pure measure of 'g'. According to the present evidence, the test fails to achieve this purity of measure of 'g'. The loads of each of the variables are treated as co-ordinates and a bi-dimensional graph is plotted of their location in the co-ordinate field. The graph obtained is shown in Fig. I.

It will be noticed that all the points plotted fall within an angle of  $76^{\circ}$ , hence whatever interpretation is given to the factors, the existence of a general factor is indisputable. This means that there is  $14^{\circ}$  latitude for the swinging of the co-ordinate frame. The problem of how far to rotate the axis depends upon the validity of the hypothesis of the moment.

Various hypotheses were tried out and found wanting in some important respect. For example, if we assume 2 general factors i.e. that as the tests are all concerned with words, each test may depend both upon 'g' and a verbal factor for its covariance. In that case the axis can only be rotated less than  $14^{\circ}$  past B or D. If we rotate

76

FIGURE II

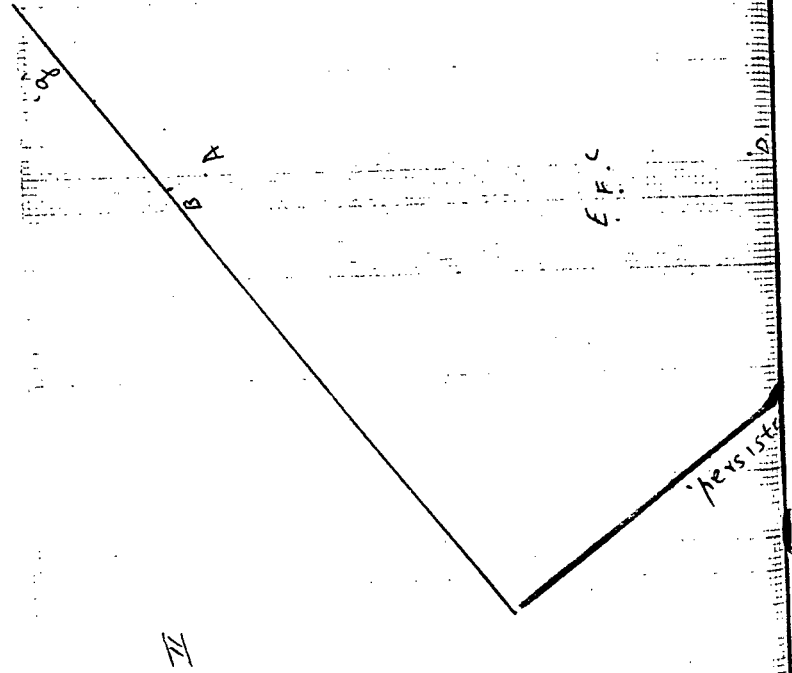
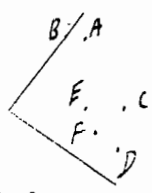
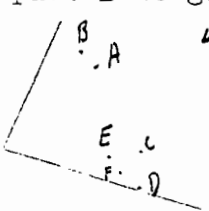


TABLE 25.

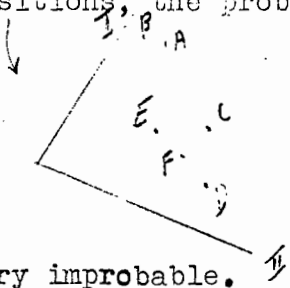
INTELLIGENCE BATTERY.

	<u>I</u> [g]	<u>II</u> [PERSISTENCE]
A	.776	.025
B	.775	.000
C	.438	.495
D	.334	.670
E	.367	.470
F	.395	.494

past B we get i.e. with test D having a negligible load on either 'g' or 'verbal' factor. Similarly with Here tests



tests B have negligible loads on either 'g' or verbal factor. By rotating the axis about 70° as a compromise between the above 2 positions, the problem is still unsolved. In such a plot,



if axis I is 'g' then tests D & F (both tests of logical relationships) must be taken to indicate general intelligence to a far less degree than test A (synonyms). This is a highly unlikely state of affairs. If, on the other hand, axis II is 'g', then tests B & A have almost zero loads on intelligence - again,

The hypothesis finally adopted was that success in the intelligence tests was dependent partly on 'g', but also quite substantially on a factor of persistence.

The graph indicating the factor loads on 'g' and persistence is given in Fig 2, and was obtained by the following reasoning process:

By passing an axis through point B it is clear:

- i) That all the loads are positive. In point of fact there is no reason to suppose that any of the tests contain negative loads on any factor likely to be contained in these tests.
- ii) that all the tests have substantial loads on this axis. It seems legitimate, therefore to conclude that this factor is 'g'.
- iii) that if another axis is drawn at right angles to the first we have a factor which is not represented in the co-variance of test B, hardly represented in A, and mostly represented in D. A & B are the first two tests of the intelligence battery and D was the first test to be given in the first session or experimental period. This points strongly to the influence of a factor of 'persistence'. The detailed results are given in Table 25.

The centroid loads are post-multiplied by an orthogonal transforma-

tion to give the final factorial matrix.

As the test period progressed subjects with greater persistence would tend to maintain their interest and energy in the test situation, while others would lose interest and give a poorer quality of response, deteriorating as a direct function of the length of time of the testing situation. Had all 6 parts of the battery been completed at one sitting, tests E & F would probably have had even higher loads than D on persistence. As it was, tests E & F were done at a second period, when interest in the tests for all subjects was partly reawakened; even here, tests E & F have not the attraction of novelty enjoyed by A & B. Hence we might expect tests E & F to have moderate loads on persistence.

It would appear, then, that success at the Cattell Intelligence Test Scale III, depends upon the individual's innate 'g', but also to an appreciable extent upon his powers of persistence, i.e. his ability to maintain interest in a test situation for any length of time.

The I.F. as given by this test, would appear to be dependent to some extent upon whether the test was given at one sitting or not. More persistent people rate relatively higher on the test than non-persistent subjects, when the testing period is long enough for the latter subjects to begin to experience waning interest in the test and put forth concentration progressively falling off from the optimal conditions. The inference here seems to be either that the Cattell Intelligence test is too long, or that it should be split up and given in, say, two <sup>or more</sup> different experimental periods. It is assumed that 'g' and persistence are statistically independent, as in the graph, and so the measure of 'g' should not be influenced by the effect of persistence.

The rotation of the two axes of the intelligence battery showed the two factors 'g' and 'persistence'. What is the effect of analyzing a battery composed of the same 6 intelligence tests, with the addition of the 3 literary tests? The centroid analysis gave clear evidence of 3 factors in this combined battery. It was thought possible that of these 3 factors two were 'g' and 'persistence', with a third factor, yet to be identified, brought in by the literary tests. We have thus 3 axes. It was assumed that two factors were similar to the two in the intelligence battery; therefore the first factor loads were so extracted as to be identical with the 1st factor loads of the intelligence battery. The method employed was to pair off the 6 intelligence test loads in the combined intelligence and literary battery according to similarity of position on the graph, average their co-ordinates and equate them to constants represented by their loads on the 1st factor in the analysis of the intelligence battery. The relevant figures are given below.

Rotated analysis of Intelligence Battery.

	I ('g')	II ('Persistence')			
A	.776	.025	}	I	II
B	.775	.000			
C	.458	.495	}	.775	.013
D	.334	.670			
E	.367	.470	}	.386	.583
F	.395	.494			

Centroid analysis of Intelligence plus Literary Battery  
(Averaged)

	I	II	II	
A } B }	.502	-.081	.355	I
C } D }	.636	.252	-.084	II
E } F }	.616	.141	-.113	III

Equating the I, II, III of this analysis to the I, II and III of previous analysis respectively:-

$$\begin{aligned}
 .602 \lambda_1 - .081 \lambda_2 + .355 \lambda_3 &= .775 && \underline{\underline{I}} \\
 .636 \lambda_1 + .252 \lambda_2 - .084 \lambda_3 &= .386 && \underline{\underline{II}} \\
 .616 \lambda_1 + .141 \lambda_2 - .113 \lambda_3 &= .381 && \underline{\underline{III}}
 \end{aligned}$$

from which,

$$\begin{aligned}
 \lambda_1 &= .727 \\
 \lambda_2 &= -.293 \\
 \lambda_3 &= .819
 \end{aligned}$$

These direction-cosines are not the true d-cosines of this 1st axis, but are proportional to the true ones. Therefore, the true d-cosines may be derived by normalizing the above co-ordinates. These are:

$$\begin{aligned}
 \lambda_1 &= .6413 \\
 \lambda_2 &= -.2585 \\
 \lambda_3 &= .7225
 \end{aligned}$$

Pre-multiplying this column of the transformation matrix by the centroid matrix we have first factor loads of the tests in the combined battery equivalent to the loads on the 1st factor in the analysis of the intelligence battery.

The remaining two columns of the transformation matrix may be rapidly and conveniently computed by a method devised in this department. The principle of this method may be illustrated by an artificial illustration:-

1. Mr. J.G. Taylor.



The product of 2 orthogonal matrices is itself an orthogonal matrix, i.e.

$$L_1 \times L_2 = L_3$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & h & -z \\ 0 & z & h \end{vmatrix} \cdot \begin{vmatrix} x & -y & 0 \\ y & x & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} x & -z & 0 \\ hy & hx & -z \\ zy & zx & h \end{vmatrix}$$

From which follows,

$$\begin{aligned} x^2 + z^2(h^2 + z^2) &= 1 \\ y^2 + x^2(h^2 + z^2) &= 1 \\ -xy + h^2xz + z^2xz &= 0 \end{aligned}$$

$$\begin{aligned} \text{Put } x &= d_1 \\ hy &= d_2 \\ z &= d_3 \end{aligned}$$

$$\begin{aligned} \text{Then, } y &= \frac{\sqrt{1-d_1^2}}{h} \\ h &= \frac{d_2}{y} \\ z &= \frac{d_3}{y} \end{aligned}$$

Hence, by substituting the obtained values of  $d_1, d_2, x, d_3$  in the 1st column of  $L_3$ , it is possible to calculate the remaining columns by using the equations given above. Each set of direction-cosines thus obtained must, of course, be normalized as usual.

The transformation matrix thus derived is:

$$\begin{vmatrix} .6413 & -.2820 & 0 \\ -.12585 & -.3273 & -.9317 \\ .7225 & .9147 & -.3632 \end{vmatrix}$$

This transformation gives the loads on the remaining 2 factors.

Plotting the variables graphically and taking the axes in pairs, we have first of all Fig. 3 obtained by plotting axis I against axis II.

This plot appears to indicate a 'g' factor and a verbal factor quite reasonably, except that test F has rather a higher load on the verbal factor than one might expect.

83

I

A

B

F. 2

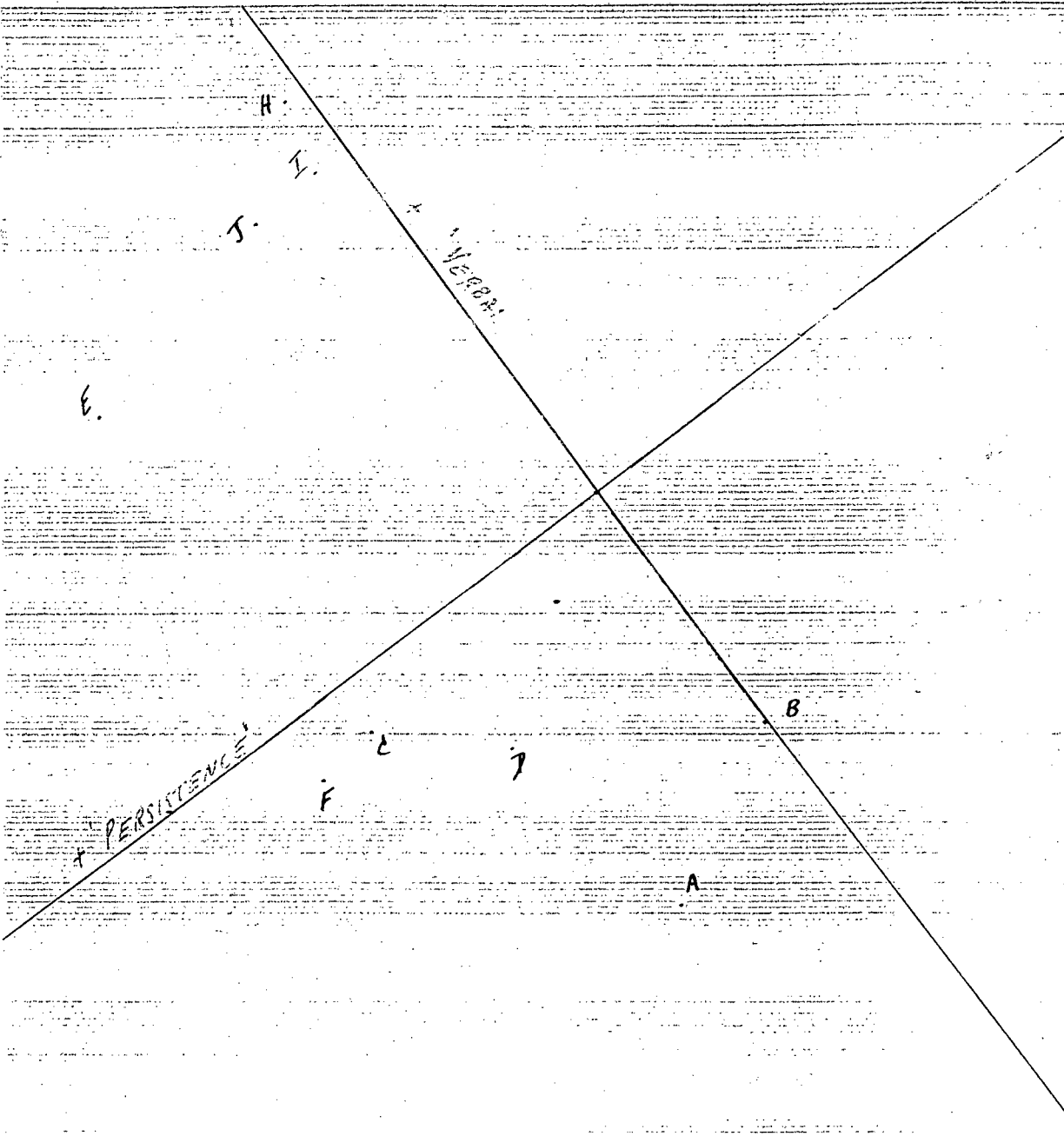
J

H

E

D

G



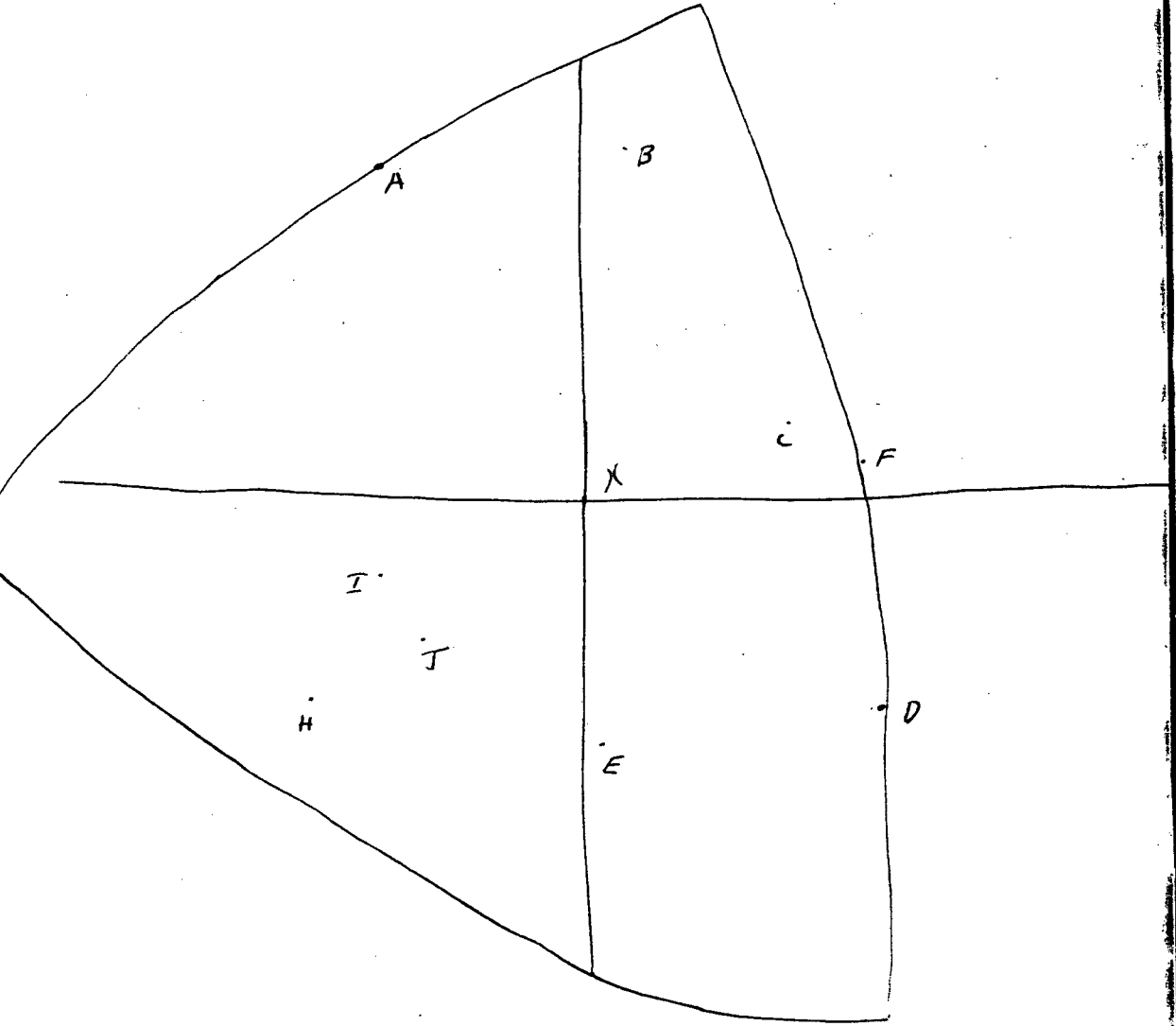
FIGURE

TABLE 2

	<u>I</u>	<u>II</u>	<u>III</u>
A.	.836	-.294	.464
B.	.855	.101	.538
C	.955	.282	.096
D	.852	.423	-.304
E	.930	.045	-.365
F	.921	.387	+.035
H	.870	-.396	-.294
I	.946	-.296	-.134
J	.944	-.240	-.227

FIGURE 6.

SPHERICAL PLOT



But, plotting axis I against axis III gives a graph which appears to lack psychological meaning. This is Fig. 4.

No suitable rotation could be determined for this plot.

Finally, plotting axis II against axis III (as in Fig. 5) we again encounter a situation difficult to explain. Axis II might be a somewhat poor fit to the theory of a persistence factor, but the loads on axis III is quite unintelligible if that axis is any sort of factor connected with verbal usage. Tests A and B can hardly be opposed to H, I and J, since both, and especially A, are intrinsically verbal. The attempt to graph the variable points in two dimensions by taking the axes two a time was therefore abandoned.

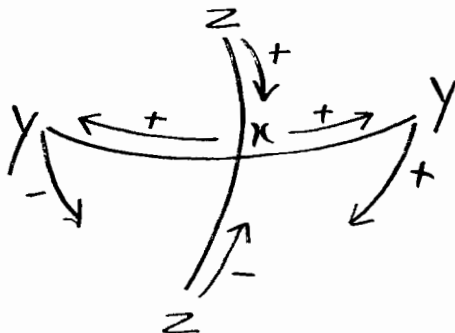
The next step was to locate the termini of the test vectors in a tri-dimensional space, and to view the dispersion of the test vector termini directly. The most convenient approach to the problem is the plotting of the test vector termini on the surface of a sphere. The centroid loads, as they stand, have communality vectors of different lengths and hence buried at different depths in the sphere. To bring these vector endings to the surface of the sphere where their dispersion may be noted it is necessary to extend the vectors by different amounts. This is done by normalizing the co-ordinates of each test in the centroid matrix. For example,

Test A in the centroid matrix, has factor loads of .647, -.228, .359; its communality is .599474; hence, the length of the vector is  $\sqrt{.599474}$  which is .77426. The reciprocal of .77426 is 1.29156. Multiplying each of .647, -.228, .359 by .29156 we have the normalized co-ordinates for test A of (.836, -.294, .1464) which defines a point on the surface of the sphere. In this way all the other vectors are augmented to unit length.

The augmented loads thus obtained are given in Table  $\alpha$ .

The plot of these points is given in Fig. 6.

The parts of the 4 quadrants into which all the points fell are positive for the X axis and either positive or negative for the Y and Z axes. The directions are:



[ X negative on  
the hemisphere  
away from the  
one shown ]

It is seen that some sort of simple structure prevails in the plot. The hypothesis finally tested satisfactorily was obtained by minimizing the loads of tests F and D on one factor, and A on another. The axis to which the plane DF is normal is taken to represent a verbal factor; that to which the plane containing A is normal, a persistence factor; the 3rd factor is g.

The method of finding the direction cosines of the orthogonal co-ordinate frame contained in the plot is as follows:

The axis of the Verbal Factor.

Lines D and F both lie in the plane  $\perp$  ~~A~~  
required axis:

$$D: .637 \lambda_1 + .317 \lambda_2 - .231 \lambda_3 = 0$$

$$F: .598 \lambda_1 + .251 \lambda_2 + .023 \lambda_3 = 0$$

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1$$

Hence,

$$\lambda_1 = .3867$$

$$\lambda_2 = -.9053$$

$$\lambda_3 = -.1760$$

The axis of the Persistence Factor.

Again, since A and the terminus of the verbal factor axis just found line in the plane  $\perp$  required axis -

$$\lambda: \lambda_1 \lambda'_1 + \lambda_2 \lambda'_2 + \lambda_3 \lambda'_3 = 0$$

$$A: .687 \lambda'_1 - .224 \lambda'_2 + .359 \lambda'_3 = 0$$

$$(\lambda'_1)^2 + (\lambda'_2)^2 + (\lambda'_3)^2 = 0$$

Substituting for

$\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ , and solving:-

$$\lambda'_1 = .5475$$

$$\lambda'_2 = .3789$$

$$\lambda'_3 = -.7461$$

TABLE B

	<u>I</u> [g]	<u>II</u> [PERSISTENCE]	<u>III</u> [VERBAL]
A.	.657	.116	.393
B.	.653	.046	.094
C.	.606	-.270	.064
D.	.491	-.564	.000
E.	.407	-.482	.261
F.	.570	-.311	.000
H.	.317	-.317	.503
I.	.514	-.323	.529
J.	.437	-.360	.451

The axis of the General Factor.

Both the end of the persistence factor axis and that of the verbal factor axis lie in the plane normal to the general factor axis, hence:-

$$\lambda_1 \lambda_1'' + \lambda_2 \lambda_2'' + \lambda_3 \lambda_3'' = 0$$

$$\lambda_1' \lambda_1'' + \lambda_2' \lambda_2'' + \lambda_3' \lambda_3'' = 0$$

$$(\lambda_1'')^2 + (\lambda_2'')^2 + (\lambda_3'')^2 = 1$$

From which,

$$\lambda_1'' = .7421$$

$$\lambda_2'' = .1921$$

$$\lambda_3'' = .6422$$

Multiplying the centroid loads by this 3 columned transformation matrix we have the following final loads for the tests in the combined literary and intelligence test battery: (Table  $\beta$  ).

It will be observed that the test communalities of this factorial matrix are identical with those of the centroid matrix. This equivalence is a check upon the orthogonality of the transformation.

We have therefore substantiated our initial hypothesis, viz: that the 9 test battery can be explained by the same 2 factors as explained the co-variance of the 9 tests, with the addition of a further factor introduced by the 3 literary tests, i.e. a verbal factor.

At this point it seems in place to discuss the factorial make-up of each of the 3 literary tests. Their loads on the 3 factors are repeated here:

		I (g)	II (Persistence)	III (verbal)
Reading Com- prehension	H	.256	.367	.503
Vocabulary	I	.450	.407	.529
Poetical	J	.368	.430	.451

Averaging the saturation coefficients we notice that the loads of literary ability as a whole on the several factors are:-

I	II	III
.358	.401	.494

The inference to be drawn from this result is that in constructing prognosis tests for literary ability regard should be paid to:

- (i) testing the candidates familiarity with verbal usage, as, for example, in understanding the meaning of the matter read, having an extensive vocabulary and the desire to widen its scope, and an appreciation of the rhythm and nuances of poetry.
- (ii) estimating the amount of general persistence or perseverance possessed by the candidate. The study of literature calls for a high degree of this quality; the reading of a novel or a play, for example, is very apt to become an interminable task unless the reader's perseverance is such as to allow of no falling off of concentration from beginning to end of the operation;
- (iii) showing that the candidate has at least a reasonable general intelligence. The correlations between the literary tests and the general intelligence tests, ranging from .264 to .564, are amongst the highest in the entire battery; hence, *ceteris paribus*, the greater the general intelligence the more promising the literary ability.

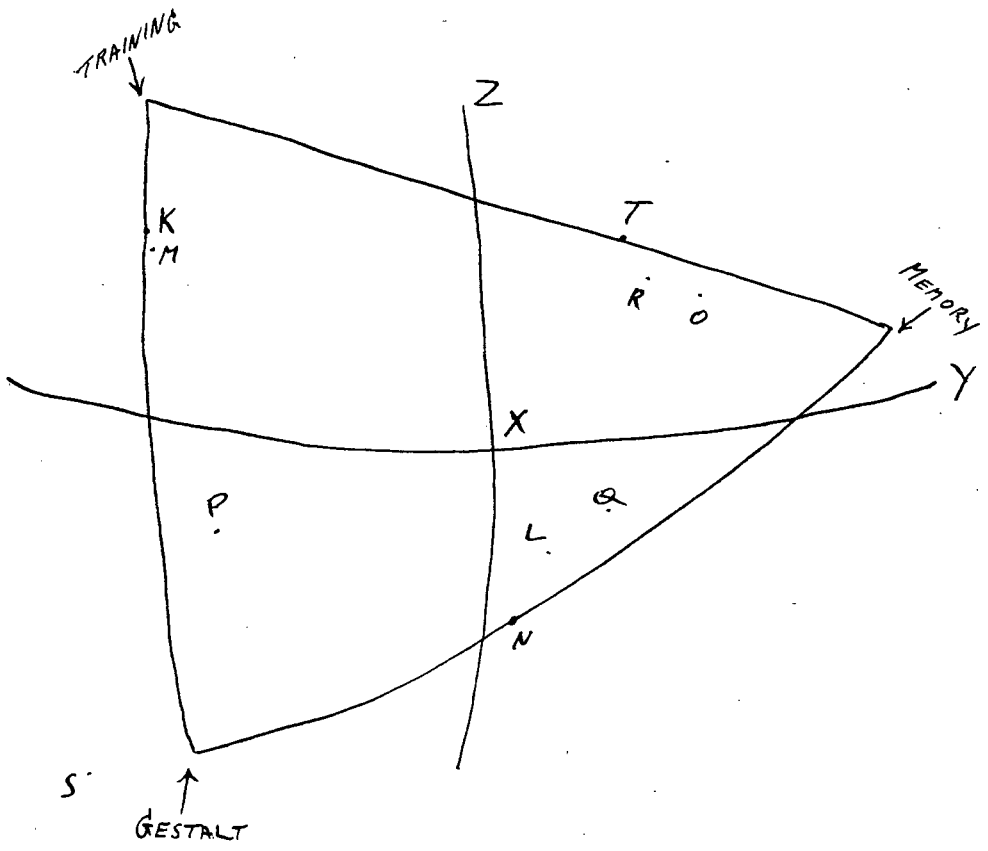
Such a conclusion seems quite in order when it is remembered that a vocabulary test has long been considered a good test of general intelligence. In fact, were it necessary to decide within the space of a single interview the amount of literary ability possessed by a candidate, probably the best method of doing so would be to use a lengthy intelligence test like the Cattell test. This would, of course, give his I.Q.; it would also show his persistence quite decisively; finally, it would yield quite a fair index to his familiarity with verbal usage.

All in all, our conclusions about the nature of intelligence and literary ability are rather in conflict with those of Carrol and Eurlich (p. 9) who found low correlations between intelligence and art appreciation. What they meant by 'art appreciation' is something wider than the literary aspect of it; but as far as the literary side of art appreciation is concerned, we definitely find a high correlation between literary appreciation and general intelligence.

When we commenced analyzing the combined literary and intelligence battery we had a preconceived theory as to the nature of at least one or perhaps two, of the three factors. Turning now to a consideration of the music battery we find that our approach was not the same as in the previous battery. There was no obvious factor to be looked for or any immediate clues as to any factor. Even the ubiquitous 'g' could offer no help. A glance at the correlation table of the entire battery (Table I) shows that of the 60 inter-correlations between the music and intelligence tests, only 17 coefficients were as high as .1; the average correlation being about .05. There could, therefore, be no

	<u>I</u>	<u>II</u>	<u>III</u>
K.	.8157	-.5117	.2695
L.	.8605	.2020	-.4676
M.	.8186	-.5037	.2758
N.	.7396	-.0898	-.6669
O.	.8420	-.4580	.2851
P.	.9142	-.3227	-.2453
Q.	.8840	.3530	-.2948
R.	.8817	-.3343	.3329
S.	-.2724	-.5079	-.8172
T.	.7852	.3751	.4926

FIGURE 7



general intelligence factor present in the covariance of the music battery. Similarly, the correlation between the music tests and the literary tests is so low that no literary factor can be assumed to account for any appreciable portion of the co-variance of the music battery.

The operation of defining the 3 factors in the music battery began with obtaining a perspective of the vector termini on the surface of a sphere. Accordingly the co-ordinates of the centroid matrix were normalized and a factorial matrix of the augmented loads obtained. This is shown in Table 26.

The variable points are plotted on the sphere, as in Fig. 7. Here again, a species of simple structure appears indicated. Test S is rather an anomaly. Its communality in the music battery is almost negligible (.047); hence, although in the plot it appears to lie without the spherical triangle showing the simple structure, in reality its vector is so short as to bring it close to the centroid of the sphere. From the point of view of locating the axes of reference for the required factorial matrix the point S may be neglected for the time being.

The position for the spherical triangle in Fig. 7 was, after a good deal of experimentation, determined in this wise:

Test N (consonance) seemed most likely to be dependent upon the physiological mechanism of the ear and hence least subject to training influence. The axis normal to the plane passing through N was therefore taken to represent a training factor. A second axis, memory, was passed through K (pitch discrimination) as being the test least likely to be dependent upon memory for success at its operation.

The third factor plane was observed to pass through T. The axis normal to this plane seemed to be a sort of "gestalt" factor, the ability to perceive images, whether its nature be visual, auditory or kinaesthetic.

The transformation matrix necessary to obtain these rotated loads was worked out in the usual method:-

From Point N and the end points of the spherical triangle we have:-

$$.7369\lambda_1 + .0898\lambda_2 - .6667\lambda_3 = 0$$

$$.5550\lambda_1 + .8387\lambda_2 + .1045\lambda_3 = 0$$

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1$$

Whence,

$$\lambda_1 = .6208$$

$$\lambda_2 = -.4738$$

$$\lambda_3 = .6247$$

TABLE 27.

	<u>I</u> [TRAINING]	<u>II</u> [GESTALT]	<u>III</u> [MEMORY]
K.	.504	.209	.124
L.	.053	.298	.198
M.	.633	.271	.020
N.	.000	.218	.127
O.	.255	.091	.451
P.	.433	.616	.122
Q.	.165	.554	.588
R.	.423	.135	.533
S.	-.022	.197	-.087
T.	.247	.000	.315

Since the second plane passes through 'T' and the extremity of the plane through N, we have 3 further equations, giving:

$$\lambda_1' = .5946$$

$$\lambda_2' = .2348$$

$$\lambda_3' = .7690$$

Finally, the third plane passes through the extremities of the triangular sides of both the previous planes, resulting in;

$$\lambda_1'' = .5111$$

$$\lambda_2'' = .8487$$

$$\lambda_3'' = .1359$$

Postmultiplying the centroid matrix by this transformation matrix we obtain the loads in Table 27.

That the centroid and the rotated analyses are equivalent is evident from the completeness of agreement as to communalities.

Unfortunately this rotated factorial matrix shows up several incompatibilities of loads which were partly obscured in the spherical frame owing to the augmentation of the co-ordinates to unit length. The position of a test variable point on the surface of a sphere must always be modified by a consideration of the communality of a test. To illustrate some of the above incompatibilities:-

- (i) Test K (pitch discrim.) shows a heavy load on the training factor. All the current evidence (v. page 5) is against such a conclusion.
- (ii) Test O (rhythm) has a negligible load on the gestalt factor, while P (time) has a very high load on the same factor. The first is far too low, and the second too high. Perception of rhythm obviously involves a marked gestalt, since judgments of motor effects must depend on a gestalt of a standard motor effect, probably largely kinaesthetic in nature.

We are confronted then with an analysis which appears to satisfy various psychological considerations in the main, but which is contradicted by the 'perverse' saturation coefficients of some of the tests. The position here adopted appeared to be the best possible position for the spherical triangle on the sphere. Is one justified in leaving the matter here, Before answering this query in the affirmative it was decided to try one more method. Perhaps, it was argued, the lengthening of the test vectors to unit length/

TABLE 28.

	<u>I</u>	<u>II</u>	<u>III</u>		<u>I</u>	<u>II</u>	<u>III</u>		
K:	.448	-.281	.148	.8157	.5783	0	.549	.000	.000
L:	.311	.073	-.169	.5117	.7218	.4660	.171	.297	-.116
M:	.564	-.347	.190	.2695	-.3801	.8848	.689	.004	.006
N:	.173	-.021	-.156				.088	-.275	-.128
O:	.443	.241	.150				.279	.373	.245
P:	.697	-.246	-.187				.644	.297	-.280
Q:	.732	.291	-.243				.383	.726	-.079
R:	.625	.237	.236				.452	.443	.319
S:	.059	-.110	-.177				.057	.022	-.20
T:	.314	.150	.197				.232	.215	.24

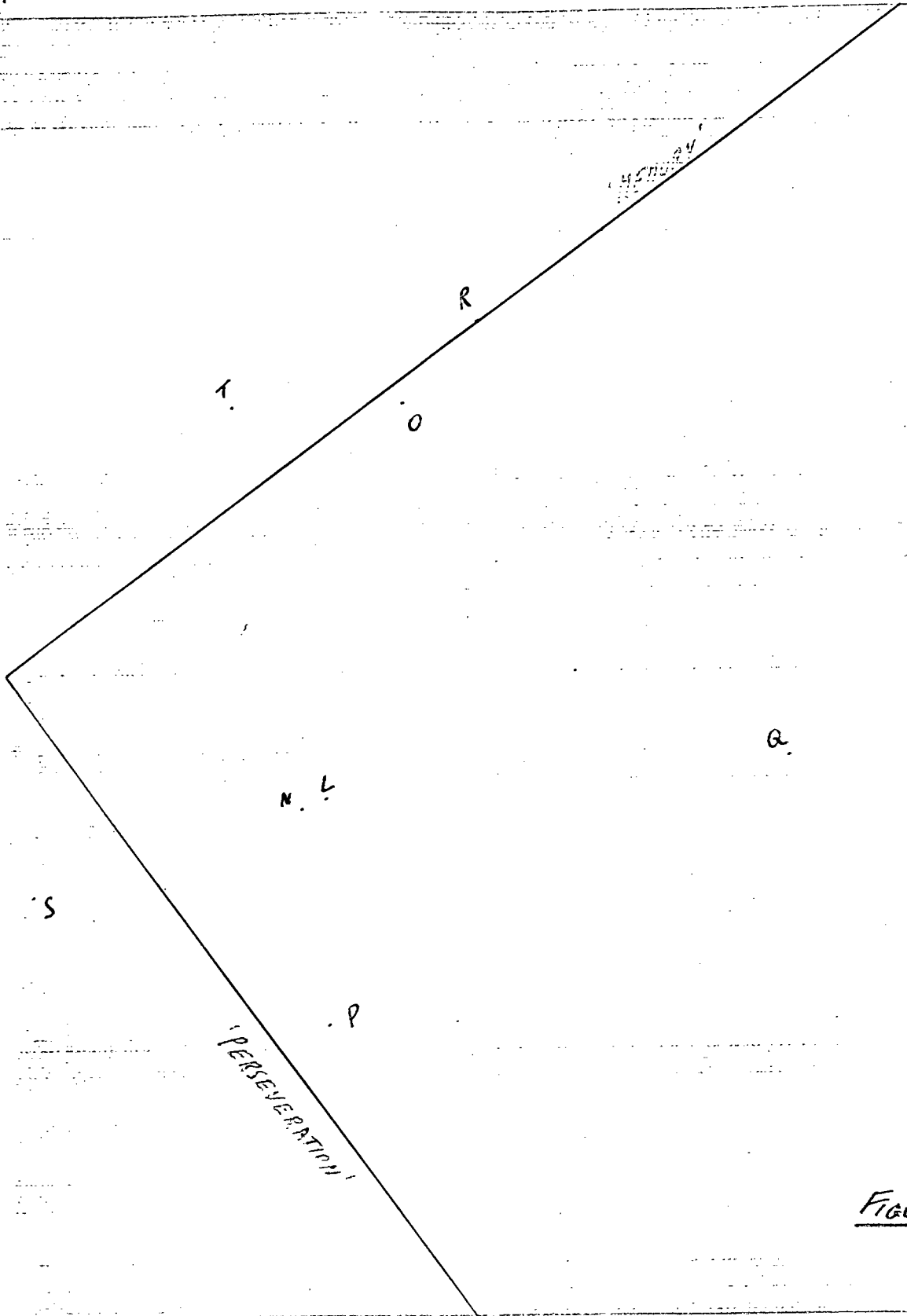


FIGURE 8

length had rather distorted the true factorial picture.

Recourse was had again to the spherical plot. Test K (pitch discrim.) seemed as simple a test as any. What if some such pitch factor were the sine qua non of musical ability? One can scarcely imagine any musical ability where insensitivity to ~~pitch~~ pitch would not lower the quality of successful performance at that ability. It seems at least feasible to postulate the existence of a general factor of the nature of pitch sensitivity.

An axis was therefore passed through K, the assumption being that the covariance of test K was entirely explained as being due to this general factor, with zero loads on both the other axes. The direction cosines of this first axis prove to be

$$\begin{aligned} \lambda_1 &= .9157 \\ \lambda_2 &= .5117 \\ \lambda_3 &= .2695 \end{aligned}$$

By means of the arbitrary transformation matrix described previously it is possible to find the 2nd and 3rd columns of the transformation matrix. Converting the centroid loads to the new loads determined by the axis through test K, we have Table 28.

In the rotated factorial matrix the loads on factor I (pitch factor) all lie in the 1st co-ordinate plane and are therefore determined. The test vectors may now be rotated in two dimensions (their radial position in the 3rd dimension being fixed). Plotting axis 2 against axis 3 we have Fig. 8.

The interpretation of this plot offered difficulty for a long time. It was obvious that axis II represented a memory factor, the two highest factor loads being those of the memory tests. But the 3rd factor had to be such as distinguished between what appeared at first sight to be pairs of tests of a similar nature, viz., Tests C (rhythm) and P (time), and R (Retentivity) and Q (musical memory). It seemed definite that no cognitive factor could thus separate success at the above pairs of tests. A solution was finally found in the functions of the factor 'perseveration' i.e. the tendency for mental processes to have a certain lag or inertia and in this meaning to persevere. Spearman has shown that perseveration is independent of 'g'; we have shown here that musical ability is independent of 'g'; hence there appears no objection to assuming that perseveration is associated with musical ability.

Perseveration here is opposed to memory although both processes are concerned with the retaining of sensations in consciousness. Perseveration may be regarded as showing the more or less momentary physiological and reflex after-effect of an isolated stimulation of the periphery. Memory, however, is concerned not with the immediate after-effects of single impressions but with a conscious endeavour, centrally aroused, to retain in consciousness over as long a period as possible, a group or succession of stimuli in the

TABLE 29.

1<sup>st</sup> COLUMN REMAINS SAME AS IN FIGURE 28.

2<sup>nd</sup> } COLUMNS OF FIGURE 28 MULTIPLIED BY THE  
3<sup>rd</sup> }

TRANSFORMATION MATRIX  $\begin{vmatrix} .811 & .584 \\ .584 & -.811 \end{vmatrix}$

	<u>I</u> [PITCH]	<u>II</u> [MEMORY]	<u>III</u> [PERSEVERATION]
K.	.549	.000	.000
L.	.171	.173	.003
M.	.689	.007	.003
N.	.088	.148	.265
O.	.279	.446	.019
P.	.644	.077	.400
Q.	.383	.543	.488
R.	.452	.585	.000
S.	.057	-.103	.182
T.	.232	.36	-.072

form of a gestalt. Thus, the difference in factorial make-up between tests apparently as alike as O and P, and R and Q, is more understandable, if we argue somewhat in this strain.

In test O (rhythm) a good deal of successful performance depends on the conscious effort to discern the pulse or beat of the music which is immanent in the music but not obvious. In test P (time) the subject will tend to hear the phrase played, and will be able, on account of its shortness, to retain it in memory with little conscious effort, the perseverating effect of the stimulus being sufficient for the purposes of symbolization.

Again, test R (Retentivity) was complex, the subject being required to memorize three different sets of stimuli presented in close succession. Here the perseverating effect of each stimulus, in turn, would be obliterated by the effect of the one succeeding, and, consequently, success at this test requires a maximum of conscious effort at memorizing.

Test Q (musical memory) was the discriminating between a musical phrase as it stood and a slightly distorted version of it. A fair amount of success at this operation obviously depended on conscious memorizing; but, in addition, the stimulus was a single short phrase, and hence perseveration might be expected to play an important part in such discrimination, as, for example, when a subject would 'feel' that the phrase played was not identical with the correct phrase, but is not certain in what respect it differs. The after-effect left in the periphery is different, although the subject did not consciously distinguish in what respects it was different.

The transformation matrix for the two-dimensional rotation was thus calculated and the final loads of the 2nd and 3rd factors on the tests computed. The process is given in Table 29.

The correlational or trait configuration of the music battery has thus been analyzed in terms of its orthogonal reference vectors.

It is necessary now to turn to a consideration of the complexity of each test, i.e. a more precise factorial description of the reference vectors that are involved in the linear description of the trait.

#### Pitch Discrimination (Test K).

The covariance of this trait is attributable entirely to a pitch sensitivity factor. The evidence here is that this is an innate quality and may be equally developed among music and non-music students.

We agree with Smith, (p. 5) in saying that pitch discrimination gives a meager response to training; with Seashore (p. 5), that the physiological limit for the sense of pitch does not vary significantly with intelligence, the inter-correlation we found being .06 (p.e. .06). Finally, our postulate that pitch sensitivity is a general musical factor finds strong corroboration in the view of Seashore (P. 5) who states that since pitch is the fundamental character of a tone, and

character of a tone, and pitch discrimination is a measure of the capacity of this sense, it ordinarily may be regarded as the most basic measure of musical capacity that we have.

#### Absolute Pitch (Test L).

According to the present analysis the conventional view that this trait is complex is fully substantiated. Its communality is only .131, indicating that many more factors than are present in the battery will be required to give a factorial description of its nature. It has positive, but low, loads on the three reference traits; its inter-correlations in the battery are mostly very low, the exceptions being a slight correlation with interval discrimination and a significant correlation with retentivity ( $r = .262$  with a load of .173 on the memory factor) and memory ( $r = .250$  with a load of .267 on perseveration). Since absolute pitch is the memory for isolated tones, these latter correlations are not unexpected.

We agree, therefore: with Köhler (p. 5) that it does not arise exclusively from the sense of pitch; with Gough (p. 5) that it can be produced by training, just as ordinary memory can be trained. We cannot, however, substantiate Révész' (p. 5) contention that it indicates unusual musicality, as success at this operation was by no means found to be predominant among music students.

#### Interval discrimination.

Like pitch discrimination, this trait is almost entirely explained by the general pitch factor. The correlation with discrimination of pitch is naturally significant (.343). There is, however, also significant correlation with the memory tests which is not borne out in the factorial composition. It is probable here that the angle of rotation for the axes should have been slightly different to account for a load on the memory factor, as it seems almost certain that this trait is capable of some development by training. It is significant that its correlation with sense of consonance is negligible (.095). The ability to judge the spatial distance between two notes sounded simultaneously is probably a cognitive function, whereas the ability to judge the fusional effect of two simultaneously presented tones seems largely affective in nature.

#### Sense of Consonance (Test M).

The traditional complexity of this trait is hardly lessened by the present analysis. It has no appreciable relationship with the basic pitch factor, its correlations with the other pitch tests being .061, .001 and .095.

Its communality is .057, indicating an extreme specificity. Its highest correlation is with memory (.191), giving it some sort

of loads on the memory and perseveration factors.

The slight correlation with memory is probably due to the rapid, almost unconscious, comparison of the auditory stimulus to certain relatively well-defined standards of affective judgment of such tonal effects. We must agree with the general view that consonance is due to innate properties of the tympanic structure; in this connection we find Valentine's (p. 6) assertion that the process of evaluating discords changes with practice rather surprising. Seashore's (6) findings of an independence of consonance and intelligence, and consonance and training is more in line with the present results. The attitude that consonance is a direct product of social factors like learning, experience, and tradition seems only partly justified. The results here indicate clearly that certain types of fusional effects are almost invariably judged as consonant or dissonant. The social forces appear to exert an influence chiefly on the fusional effects of the middle class where consonance or dissonance is not emphasized to the normal ear.

#### Intensity Discrimination (Test S)

Of all the tests in the battery, least light is thrown on the nature of this trait. Its communality is .047; as far as the factors tapped in this analysis are concerned this trait shows itself to be an almost entirely specific element. Its saturation with perseveration alone can be taken to be significant.

This last inference is quite intelligible in view of the nature of the test. One tone is produced and is very closely succeeded by another. The perseverating influence of the first upon the judgment of the intensity of the second tone can scarcely be doubted. The trait has not a single correlation as high as .100 with the remaining music tests.

Like Seashore (p. 6) we can assert that this trait varies independently of intelligence or training. Like sense of consonance, it is probably a function depending upon the inherent properties of organic structure.

#### Sense of Rhythm (Test O).

This trait has an appreciable load on the general pitch factor (.279). Although this trait is usually thought of as involving a large motor factor its successful operation does vary with the pitch factor. The discrimination of rhythmic patterns of monotonies is not the same as that of musical notes. Music is so structured as to be compatible with and emphasizing rhythm and 'swing' in a way not found, for example, in the beating of a drum.

Seashore (p. 7) finds the correlation between this trait and pitch to be .17.

When we come to consider the 2nd and 3rd factor loads of this variable, we find ourselves faced by a problem. Does the ability to listen to a phrase and discern its rhythmic pattern tend to be a cognitive or a motor function? Our answer assumes that

some kinaesthesia is involved, but that the relating of the amorphous motor effects depends largely upon the cognitive aspect of discrimination. There must be a conscious effort to remember and resolve the kinaesthetic tone established. The load on the cognitive memory factor is therefore large (.446) and the load on the perseveration factor negligible (.019).

We agree with Woodrow (p.7) when he questions the validity of Sterzinger's (p. 7) finding of a correlation of .71 between rhythm and consonance. The present correlation corresponding is .110. Again, we are in agreement with Seashore (p. 7) when he postulates communalities are .277 and .581. We cannot, however, find the basis for his extension of a similar hypothesis for time and sensation.

The conventional view that musical rhythm is only part of a wider phenomenon is expressed by Seashore (p. 7) in finding a similarity between rhythm in music and rhythm in poetry. According to our results, however, the correlation between rhythm in music and poetical appreciation (in which verbal rhythm played an important part) is almost negligible (.095 with p.e. .065).

Rhythm seems to be capable of being trained, and there seems to be a strong tendency for a somewhat low positive correlation between sense of rhythm and intelligence.

#### Sense of Time (Test P).

The surprising feature of this trait is the high load it has on the general pitch factor. It seems feasible to suppose that time values are better established when added significance is given them by melodic continuity, but it seems hardly likely that the influence of pitch is so marked in the perception of time. This would appear to be another weak link in the factorial chain. On the other hand, it should be noted that sense of time shows correlations of .353, .203 and .439 with other pitch tests. Whether these correlations are high enough to justify a load of .644 on the pitch factor is a matter for individual judgment. The fact that time has a load of only .077 on memory and .400 on perseveration indicates that it is opposed to rhythm in this respect. According to our way of thinking, the reproduction of symbolic time values is possible with an almost minimum amount of conscious effort. The impression of the temporal pattern as left by a short 3-bar phrase tends to persevere long enough for the reproduction of the time values directly from the kinaesthetic tensions set up. Viewed in this light, the writing down of the symbolic time values is simply a resolving of the kinaesthetic tensions and requires the minimum of centrally aroused direction.

This hypothesis is substantiated by the correlation of .582 which this trait has with the test involving the greatest amount of perseveration, i.e. musical memory (Test Q).

that the complexity and covariance of this trait is largely explained in terms of the three factors here isolated is evident from the highness of the communality (.582).

Seashore (p. 7) thinks that time and rhythm have something in common. With this we are quite in agreement. Both the traits involve appreciable loads on the pitch factor; the inter-correlation of the two tests is significant (.228). Like him, we will contend that this trait and intelligence vary independently. Since the two highest factor loads for this test are on pitch and perseveration neither of which alters appreciably with training, we can further corroborate his point that training has but small effect on this trait.

#### Musical Memory (Test Q)

The cases of musical memory and retentivity have already been discussed in some detail in connection with the general difference between memory and perseveration. We add here a few more observations.

The load on the pitch factor is evident by virtue of part of the memory test being the recognition of pitch changes in melodic phrases. This trait appears to have the most general and representative loads on all the factors, and there is a temptation to treat it as being a very good index of general musical ability. Yet Seashore (p. 7) states specifically that retentive and serviceable memory, while a very great asset to a musical person, is not an essential condition for musical mindedness. In the present case we are inclined to form our own opinion in view of the number of cases upon which the results are based.

#### Retentivity (Test R).

There is little that has not yet been said about this trait. Its similarity to musical memory stops short of any load on perseveration for reasons already outlined. The magnitude of its communality (.503) indicates that the 3 reference trait vectors explain much of its covariance. It is probable that this test measures memory for the more elemental factors of melody than melody itself, as in musical memory.

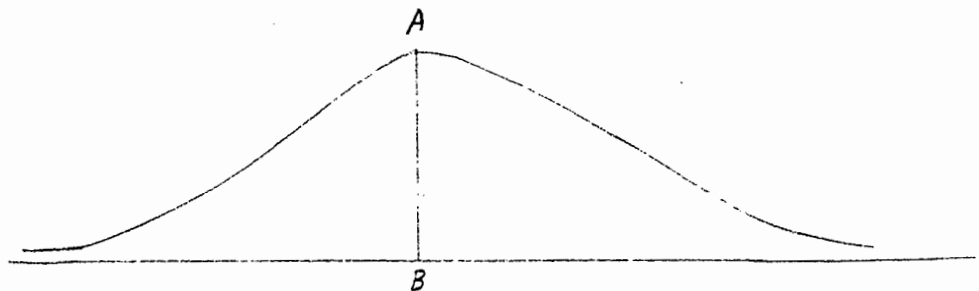
#### Emotional Sensitivity (Test T)

This appears to be a highly complex phenomenon. Its communality here is a mere .160. It has a load of .232 on the general pitch factor with a correlation of .176 with pitch discrimination; on the memory factor it has a load of .316, a result well supported by its correlation with rhythm, (.213) and with retentivity (.231); its factor loading on perseveration is small and negative, and since the appreciation of lengthy or continuous passages of music does not appear to have any connection with lag of auditory stimuli, this last saturation coefficient is not at all surprising.

As regards this trait we can offer proof to Beaunis' (p. 8) contention that musical emotion derives from rhythm, duration and pitch, but must differ from him when he would add to this list intensity. The correlation of this trait with intensity is  $-.023$ . Hevner's (p. 8) finding that the requisite qualities are rhythm, tempo and pitch is fully substantiated by the present results.

In reviewing the general position we notice that the intelligence and literary tests were included in one battery, and the music tests in another. Both these batteries were analyzed with respect to the general covariance of the battery and the specific covariance of each test. The final question may be asked: what would happen if the covariance of the entire battery of 19 tests were analyzed? The statistical analysis showed the production of 6 factors in such a case. It was stated previously that these 6 factors were the combined groups of 3 factors extracted from the sub-batteries. If such an assertion is true, then the general covariance of the music battery should have little to do with that of the intelligence and literary battery, i.e. the average correlation between the 2 batteries should be approximately zero. A cursory glance at the table shows this to be a correct reflection of the true state of affairs.

Testing the matter more precisely, we have the following procedure:-



Let the above curve represent the dispersion of the correlations between the 2 batteries. If the average correlation is zero it would fall on AB. In that case?

$$\Sigma \gamma_0 = \frac{.5745}{\sqrt{n}} \quad (\text{since } r=0) \text{ where } n \text{ is size of sample.}$$

But since we are dealing with fallible data, the average will not fall precisely on the line AB. i.e.

$$E_{A_1} = \frac{\Sigma \gamma_0}{\sqrt{n}}$$

Where  $n'$  is the number of coefficients.

$$= \frac{-.5745}{\sqrt{2} \sqrt{n}}$$

If the correlation between the two batteries is zero, the average correlation should not exceed 3 p.c. of the theoretic zero correlation.

$$1. \bar{r} \text{ should } \leq 3 \times \frac{.6745}{\sqrt{2}}$$

$$2. \frac{2.333}{90} \text{ " } \leq \frac{3 \times .6745}{\sqrt{1700}}$$

$$3. .024 \leq .019$$

$$\text{But, } 3 \text{ p.c.} < .024 < 4 \text{ p.c.}$$

This means that the average correlation is .005 slightly too big to be entirely explained as a zero correlation of fallible data. The discrepancy is so small as to make it of negligible importance. Hence we may assume that the two batteries have nothing in common, and that the covariance of each of the 19 tests is as fully explained in an analysis of its own battery as it would be in an analysis of the entire battery. The 6 factors of the entire battery may therefore be taken to be the 6 factors together isolated by the 2 sub-battery analyses. It would be possible to establish a 6-dimensional transformation matrix which would rotate the entire battery in such a way as to make the respective 3 factor loads of the intelligence and literary tests the same as those in the intelligence and literary sub-battery. The loads on the 4th, 5th and 6th factors would be zero. Conversely, for the music tests in the entire battery the 4th, 5th and 6th factor loads would be equivalent to the 3 factor loads computed in the music sub-battery, while the tests would have zero loads on the 1st, 2nd and 3rd factors.

The labour of calculating the direction-cosines of the axes of the transformation, as we pointed out previously, is prohibitive. We are compelled to leave the matter while, at the same time, drawing attention to the fact that although the task has not been attempted it was proved that the 6 factor loads in the entire battery represented an analysis equivalent to that of the 2 sub-batteries.

What conclusions may be drawn from the results of this analysis concerning the nature of musicality, What we have really accomplished in this investigation is an answer to the question of which are the basic factors underlying musical ability. Are 'musical ability' and 'musicality' synonymous terms? Generally, the reply must be in the negative. But it may fairly be contended that there are certain traits which are basic both for musicality and musical ability; such traits, we suggest, are partly measured by the tests employed in this analysis. No matter whether musicality or musical ability is meant, we cannot see either of these having any valid meaning unless a fair measure of the 3 factors extracted from the Music tests is included in the individual's mental make-up. We may say definitely then

that the present music battery does measure musicality, but that it does not measure all aspects of it. Many more tests, upon the lines suggested previously (p.8) must be analyzed factorially before an adequate account of musicality can be given. Therefore, it should be remembered that our final judgments about the nature of musicality will apply only within the limitations imposed by the number of tests used in the analysis.

The question remains: When is an individual musical? (1) Wundt (39) considered anyone musical if he were able to recognize and remember an octave, a fifth, and a third, even if only for the time of the experiment. Stumpf (2) (32) postulated sense of pitch or sense of consonance. Crzellitser (3) demands a sense of pitch and moderate memory capacity; Billroth<sup>4</sup> (2) believes one should have a talent for rhythm, pitch and intensity. There are other views of the nature of musicality, but those given above alone appear to be based upon any sort of experimental evidence. It will be seen that all the above conceptions could be explained in terms of the 3 basis reference traits or factors that were isolated during the present analysis. If this analysis has been built up on logical grounds, then we give the following ultimate conclusions:-

Musicality will probably not be entirely defined in terms of the trio of basic mental factors isolated in this analysis. But of one thing we may be confident: nobody can be musical unless they have:

- (i) an inherent sensitivity to pitch;
- (ii) a power of retaining peripherally-aroused impressions in consciousness for varying lengths of time; and
- (iii) a normal perseverative reaction of the bodily mechanism to auditory stimuli.

Before closing this section of the study it would perhaps be advisable to relate the consequences of our more general findings to the points raised in the discussion of the literature on musicality.

Révéc's (p.3) insistence upon musicality being unitary rather than atomic is by no means compatible with present results. Musicality is complex, there being at least three statistically independent factors which are essential to the individual possessing marked musicality.

We must emphatically contradict Miller (p.19) who finds that musical talent is ordinarily accompanied by good all-round ability. The relation of musicality to literary ability or to general intelligence has been found here to be well-nigh zero. We would find even the low dependence of music ability upon intelligence postulated by Spearman (p.9) not borne out by the facts. Carrol and Eurich's (9) result that in the main intelligence and art appreciation are not appreciably related, is more in line with our findings. Similarly Rigg (p.9) can discover no significant correlation between discrimination in music and discrimination in poetry. With him we must cast doubt on the existence of a general aesthetic discrimination factor.

We conclude this discussion by relating our position to that of Seashore (p.3). For him the musical mind is a normal mind which possesses, over and above the normal quota of these qualities the capacities essential for the hearing, the feeling, the understanding, and the expression of music. Such a definition, in the present writer's opinion, merely points out the obvious. The classification here is 'a priori' and does not really touch the fundamental aspects of the problem. It describes but does not explain. What are the essential and least number of reference traits that will account for the difference between the musical and the non-musical normal mind? We suggest that underlying Seashore's classification of the musical functions there are at least the three mental factors, sensitivity to pitch, memory, and perseveration. There will, no doubt, be further factors required before we can state definitely why it is that, to be musical, one must be more than ordinarily capable of "hearing, feeling, understanding and expressing" music. But, in the meantime, the three factors we have isolated in this analysis may be considered to supply three fractional parts of the solution to the problem. How many more will be needed to make up the integer remains for further investigators to decide.

What, finally, can be said about the part played by general intelligence in the phenomenon of musicality? The correlation between the two has been shown negligible. Does this imply that musicians are unintelligent? We think not. Consider Seashore's (p.21) attitude: "a person may be a poor thinker in science but a good thinker in music .....musicians, by the nature of their art, sacrifice intellectual pursuits for a life in the world of feeling.....intelligence is one of the best indices for prediction of success in a musical career." For us, the inference to be drawn from the present analysis does not lie in any invidious comparison between a person's musicality and intelligence; if intelligence and musicality were opposed we would expect a significant negative correlation between the two. This is most decidedly not the case. Assuming that 'g' or general intelligence can be identified with mental energy, as Spearman would have it, then it becomes a most difficult task to determine which type of operation involves the greatest amount of 'g'. According to the Law of Universal Mental Competition, the more one does along one line, the less can one do along another. The problem seems really to be: does a musical career demand for its success more or less than the normal amount of 'g' required in other professions? There appears to be no adequate reason why music should not involve as much of the mental energy, as say, mathematics.

From this point of view Seashore must be judged as legitimately postulating intelligence as being an index to musicality. On the other hand, if 'g' or general intelligence be not the vague, amorphous mental energy, but specifically, the intellectual aspect of it, as Thurston appears to think, then we cannot pass verdict on Seashore's ruling. It would seem likely that as musicians work more by feeling than by cognition, far less of this intellectual

general factor is necessary for their successful application to music than in spheres like science where feeling is entirely subservient to intellect.

One definite result does emerge from the present investigation. Persons musically trained have normally less of the general intellectual factor than persons trained in scientific, philosophic and literary subjects. The important word is 'normally'. The distribution of the mental processes of music students must be deemed subject to the laws of normal dispersion. The inference is that the average music student may possess greater or less musical ability quite independently of his stock of general intelligence. Normally, the more the individual comes to 'think musically', the less adapted to the general environmental demands will his thoughts become.

The inference to be drawn is reversible in its application: if a student be generally intelligent, possesses an interest in music, and is not lacking in the three factors isolated as a result of this analysis, viz., pitch sensitivity, memory, and perseveration, then there seems to be no obstacle to predicting undoubted musicality in his case; on the other hand, if the student possesses the 3rd and 2nd qualifications, but not the 1st (general intelligence) we still have no guarantee that he will not possess a fair degree of musicality.

All in all, general intelligence is not indispensable for musicality, and normally is not associated with it. Musicality appears to depend directly to a far greater extent upon such factors as pitch sensitivity, memory and perseveration than upon the individual's stock of general intelligence. Any further conclusion is not justified by the present evidence.

## VII. SUMMARY.

1. The object of the present study was to analyse the phenomenon of musicality as displayed in 10 tests of musical ability. It was hoped that further light would be thrown on the phenomenon by relating it to general intelligence and literary ability.
2. The bibliography, though fairly extensive, offered no direct guidance in the application of the method of this analysis, since the latter was only possible as a result of psychological technique developed in very recent years.
3. It was felt that should any significant evidence be adduced from this investigation, the task of prognosis of musicality would be greatly facilitated.
4. There were in all 206 subjects drawn from both sexes of whom 60 were undergoing a full-time musical training. The remaining 146 were samples of a homogeneous university population.
5. Nineteen tasks were selected on the basis of the four criteria mentioned in the text. The tasks were: 6 intelligence tests (synonyms, classification, opposites, analogies, completion of sentences, inferences), 3 literary tests (reading comprehension, vocabulary, poetical appreciation), and 10 music tests (discrimination of pitch, interval, consonance, time, rhythm, intensity, absolute pitch, musical memory, retentivity, emotional sensitivity). These tests were given in five different periods: the intelligence tests in two sessions of  $\frac{3}{4}$  hour each; the literary tests in two  $\frac{3}{4}$  hour sessions, and the music tests in one 2 hour session.
6. The entire battery yielded 171 inter-correlations. The analysis of the correlation table was carried out by the centroid method.
7. Three factors had to be extracted from the entire battery before the various criteria for indicating the unreliability of residual correlational material were satisfied.
8. It was noticed that the correlations between the music tests and the rest of the battery were extremely small, and it was decided that it might be advisable to split the battery in two, the one half comprising the intelligence and literary tests and the other the music tests. In addition, doubt had been cast on the purity of the measure of 'g' which the intelligence battery was held to undertake. The 6 intelligence tests were, therefore, analyzed in a separate analysis.
9. Analysis of the intelligence battery proved decisively

that there was a second factor being measured by the tests. This second factor appeared to be 'persistence'.

10. Analysis (in this and in all cases by the centroid method) of the intelligence and literary battery yielded 3 factors before the relevant criteria for sufficient factors were satisfied.

11. Analysis of the music battery similarly resulted in the extraction of 3 factors.

12. Rotation of the axes in the 1st battery showed the presence of g, persistence, and a verbal factor.

13. Rotation of the axes of the 2nd battery pointed to the existence of 3 factors, pitch sensitivity, memory, and perseveration.

14. No attempt was made to give meaning to the 6 factors isolated in the entire battery, since it appeared that these were merely the 6 factors already described. It was shown possible to rotate the loads of the tests in the entire battery on the six factors into their corresponding loads in the sub-batteries.

15. The process of analyzing different parts of the same battery brought out a point of some importance to the theory of factorial analysis, viz., that when the covariance of a test is analyzed first when the test is part of an entire battery, it should resolve into the same factorial make-up as that obtained by analyzing the same test in a sub-battery. Such an equivalence indicates the degree of stability of the factors isolated.

16. Success at the Cattell Intelligence Test was shown to depend not only on the innate 'g' but also quite appreciably on the possession of 'persistence'. The inference is that the test is too lengthy or, preferably, that it should be given in about three sessions.

17. According to the present results, a good prognosis test of literary ability will include tests of a verbal factor, persistence, and 'g'.

18. Music ability is not necessarily dependent upon 'g'; it appears to be bound up far more closely with such factors as pitch sensitivity, memory and perseveration.

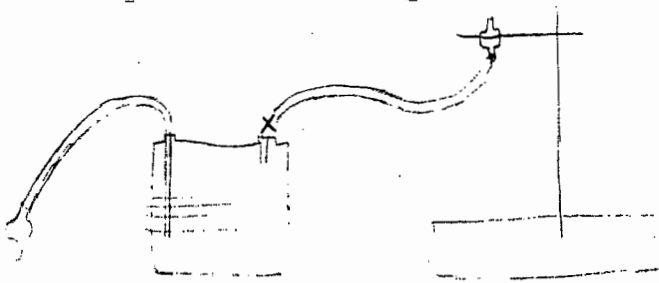
19. The relation of musicality to literary ability is negligible, thus casting doubt upon the existence of any general 'aesthetic' or 'artistic' factor.

20. The tests of musicality used here are by no means exhaustive.

It seems probable that musicality is not entirely explained in terms of the three factors here isolated , and that further tests of the nature described in the text will be needed to isolate any further factors underlying the manifestations of musicality.

APPENDIX.

1. It was originally intended to include in the test battery a variable test of the upper range of audibility. The apparatus employed was the Galton whistle which was clamped in a stand. To the nozzle of the whistle was connected a long piece of rubber tubing of narrow bore. This tubing, in turn was joined to the mouth of a wash-bottle, and air was pumped into the wash-bottle by means of a rubber bulb, not unlike the everyday scent-spray. The wash-bottle had attached to it a glass tap which could be turned on or off, thus conserving or allowing through a constant and steady stream of air into the tubing and thereby sounding the whistle. Some idea of the experiment can be gained from the sketch below.



The test was actually performed on all the subjects, but, as the results proved too variable owing to the shortcomings of the Galton whistle, the test was omitted from the battery.

For the benefit of any future experimenters on this auditory feature, it can be suggested that a turbine machine, electrically-driven, might yield more reliable results.

2. Another test dropped from the battery was that testing physiological and quantitative emotional response to the hearing of a musical work. The subject is placed by himself in a darkened room, seated comfortably on a chair with his left hand resting naturally on a table.

The experimenter is seated in a neighbouring room and by talking into a microphone is able to issue instructions to his assistant (in the darkened room) who is wearing ear-phones, without danger of the subject over-hearing the words. The assistant ensures the comfort of the testee and fastens an electrode to his resting palm, instructing him to relax and enjoy the musical performance.

The experimenter, next-door, meanwhile measures the electrical resistance of the subject on the Wheatstone-bridge and adjusts the spot-light from the psycho-galvanometer to register a zero value on the indicator.

The music is now played, and the deflections of the spot-light are noted. Unfortunately, the test requiring individual testing, could only be tried on about 40 subjects before the end of the academic term, and this relative paucity of material resulted in the omission of the test from the battery.

The results, however, were interesting and instructive. In all cases, without any exception at all, where a subject registered a high initial resistance, the resulting changes in resistance during the hearing of the music were slight. Furthermore such cases coincided with each subject's introspective description of himself or herself as being a 'steady' sort and little given to emotional variability without substantial cause. It may, therefore, be concluded that this group represented an emotionally phlegmatic type, to whom the hearing of music is relatively an impersonal sort of affair.

There was, likewise, another clearly-defined group with low initial resistances, marked changes during the progress of the music, and introspectively described as 'emotional' and 'changeable'. This type of person is easily moved by emotional stimulation as expressed through a musical medium.

This tests appears to indicate a promising line of enquiry for future research-workers.

3. Thurstone, in the 'Vectors of Mind' is by no means definite about instructions as to when to stop extracting factors in an analysis of fallible data. His general attitude is to extract factors until the correlation residuals are so small as to be negligible. He does not, however, define 'negligibility'.

In his 'Primary Mental Abilities' he takes the mean value of the residuals at any one stage and divides them by their mean value at the previous stage. This quotient, he says, should reach a limiting value given by  $(1 - \frac{1}{2} + \frac{1}{4})$

This is exactly the same formula as the one described on page 37, [ $\bar{r} \approx (1 - \frac{1}{2} + \frac{1}{4})^n \cdot E_T$ ]

because, if the systematic material is exhausted, then according to the formula, the next stage will result in the residuals being reduced in proportion to  $(1 - \frac{1}{2} + \frac{1}{4})$

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