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STUDENT MATHEMATICAL WRITING, PEDAGOGIC PRACTICE AND QUALITY LEARNING: A COMPARATIVE ANALYSIS OF STUDENT PRODUCTIONS AT TWO SCHOOLS

A DISSERTATION PRESENTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF EDUCATION
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Chapter 1: Introduction

1.1 The research question

This study was initiated by a larger research project, the Learner Progress and Achievement Study (LPAS) which was based at the School of Education at the University of Cape Town. In the initial stages of the LPAS, their focus was on "the quality of learning", and it is this concern which underpinned my study. During this stage of the LPAS teachers were not comfortable with having lessons observed or taped, although they were happy for researchers to interview them and have access to students' writing. This then meant that interviews with the staff and students at the school and the student productions became the main source from which data could be generated. This situation begged the question: What, if anything, can student productions tell us about the quality of learning? It is this question which forms the basis of this study. Since this study is a partial requirement for the fulfillment of a master's degree specialising in mathematics education, the focus of the question is on the student productions in the Standard 10 / Grade 12[1] mathematics classes, and the productions focused upon are limited to those in the students notebooks.

The two schools chosen for this study were those used for the LPAS. At the time that this study was initiated, LPAS was focussing on the Standard 10 / Grade 12 classes, which is why this study focuses on the matric mathematics classes at the two high schools. The focus of the research was then confined to: What, if anything, can the student productions captured in the notebooks of the Standard 10/Grade 12 mathematics classes at two schools tell us about the quality of mathematics education? From this more focused question one might be able to develop an orientation towards the broader question: What, if anything, can student productions tell us about the quality of learning?

In order to answer the question: "What, if anything, can student productions tell us about the quality of learning?" I begin by trying to come to a coherent definition of the concept of quality. The aim is to be able to state the parameters of the problem behind the question more clearly. It becomes clear that this term is used differently by different people both in relation to education and more broadly, in ways that masked considerable differences. This prompts me to begin by asking why and how the notion of "quality" has become an important concept in education. I then adopt the approach that quality is that which is valued. This, in turn, leads me to discuss the most commonly accepted views on what is currently valued in education before coming to a position that what I find of value in mathematical education is the induction of students into mathematical discourse.

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[1] At the time that this research was initiated the final year of school was commonly referred to as Standard 10 or matric, but it has subsequently, with the phasing in of Curriculum 2005 become more common for this year to be called Grade 12. Because this term has changed over the period of the study I use both terms.

Once I have established a definition of quality, I then ask sub-questions which attempt to probe ways in which the student productions might give clues to pedagogic practice at the two schools and potentially some ways in which quality of learning may be inferred from them. These sub-questions are grouped in sections 1.1.1 – 1.1.3 below.

1.1.1 Linking patterns in student productions to the form of pedagogy privileged at each school

- Are there similarities between the student productions within each school? If so, what, if anything, can be inferred from these patterns of similarities about the form of pedagogy privileged within the mathematics classes within each school?
- Are there differences between the student productions at each school? If so, what, if anything, can be inferred from these patterns of differences about the form of pedagogy privileged at each school? How, if at all, can these differences be related to the different levels of achievement of students from the two schools in the final matriculation examination?
- To what extent do the patterns in the students' mathematics notebooks resonate with information gleaned, from other sources, about the form of pedagogy privileged in the Standard 10/Grade 12 classes at each school?

1.1.2 Linking student productions, pedagogic form and quality

- Do the student productions captured in the mathematics notebooks provide enough of a record of the pedagogy privileged by the two teachers, to enable one to assess the extent to which students are inducted into mathematical discourse? If not, what other forms or records of pedagogy (for example other student records, observations of forms of interactions in the classrooms) are needed or would be useful before describing and evaluating the pedagogic practices at each school?
- What, if anything, do the student productions that are captured in their mathematics notebooks display of the extent and forms of evaluation in the different classrooms? How might these forms of evaluation be related to other forms of evaluation that are not displayed in the notebooks, both in form and extent? What is the relationship between evaluation in general and the induction of students into mathematical discourse? Is the proportion of evaluation that is captured in the notebooks sufficient to use as a basis to discuss the differential potential for students at each school to be inducted into mathematical discourse?

1.1.3 Linking patterns in the student productions within each school to student achievement and the form of privileged pedagogy

- Are there differences between the productions of the more successful and less successful students within each of the schools? If so, are there any patterns in these differences and how might these patterns relate to the form of pedagogy privileged at each school?
- What, if anything, is the relationship between student achievement and quality of education; or in student success in examinations and the extent to which they are inducted into mathematical discourse?

To begin then with the final set of sub-questions, what is the relationship between success in examinations and the quality of learning? The two schools focussed on within this study were chosen because they are situated close to one another and the students at the two schools tend to achieve different results in the final examination[2]. In this sense it is clear that the student achievement levels differ. However, whether one relates achievement in the final examination to quality of learning or not depends on what one understands by quality learning. Quality education, to rehearse a commonplace platitude, means different things to different people. This is partly because both the terms "quality" and "education" (and hence the related term "learning") can be described as floating[3] signifiers. The terms signify different things to different people and even different things to the same people in different contexts. In the sections 1.2 and 1.3 below I describe how the concept of "quality" came to be so centrally inscribed into educational discourse, and trace some of the influences that have shaped the use of this term. I then (in section 1.4) discuss a range of contemporary views on what is valuable in mathematics education; it is these differing views which underpin the differences in the ways in which quality of mathematics education is currently viewed). In this section I both assert that it is important to separate out the underlying conceptual differences in the different ways that quality in education is perceived and also present a critique of the decontextualised constructions of dominant views on quality in mathematics education. Finally in section 1.5 I present a definition of how the issue of quality in mathematics education will be viewed in this dissertation.

Although it is uncommon in the first chapter of a minor dissertation to trace a history of the insertion of a term into the discourse, or to present contemporary literature on how the term is used in the discourse, this is what is presented in the remainder of the chapter. The reason for this is the high level of ambiguity that is presently inscribed into the usage of the term in educational (and possibly other discourses). I believe that a description of how the term 'quality' has been recontextualised from

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[2] In 1997 the matriculation pass rate at one of these schools was 86%, whilst it was only 27% at the other study school. The national matriculation pass rate in 1997 was 47,1% (LPAS Report 1, 1998: 28)

[3] Harris (1992) attributes the notion of floating or slipping signifier to Barthes. I follow Harris' definition of it as a process of signification whereby meaning is produced "by signifiers relating to other signifiers and not to some fixed 'object' or social practice." (Harris 1992: 32)

industrial management contexts gives insights into some of the ways in which the terms is used in educational discourse. A description of the influences on the understanding of the term quality in education helps to produce an account of how the term has changed signification and meaning. I briefly trace the insertion of the term quality into educational discourse, prior to offering a critique of the decontextualised "commonsensical' ways in which quality education provision is generally viewed. A description of the values often inscribed into the term 'quality' in mathematics education serves to inform and position the definition of quality in mathematics education that concludes this chapter. The aim is to arrive at an informed definition of quality in education for the purposes of this study.

1.2 From where does the notion of quality in schooling arise?

> 'Quality' is one of the key terms in current educational debates: 'quality schooling', 'quality management', 'quality teaching and learning' and 'quality assurance' are all themes that exercise the attention and draw the criticism of policy-makers, administrators and practitioners widely across the international arena. (Aspin, Chapman with Wilkinson, 1994: xiii)

Much research and practice in contemporary education throughout the world is concerned with improving the quality of education (Aspin, Chapman with Wilkinson, 1994; Botha, 2000; Govinda and Vargese, 1993; Heneveld, 1994; Morley and Rassool, 1999; Heyns, 2000; Schmelkes *et. al* 1996, Winch 1996, West and Hopkins, 1996). It has become commonsensical to equate quality with educational worth: quality becomes a synonym for that which is valued. The difficulty with assessing quality in education is that the extensive use of this term often masks the different values that underpin different conceptions of quality.

Where does this interest in quality come from? Winch (1996) asserts that the term "quality" was transferred from the economic sector to the service sector. He examines ways, in which the economic metaphor of quality has and can be transferred to education and identifies four ways in which industry has approached quality (Winch, 1994: 9): (1) quality as equated with excellence; (2) quality as a measurable attribute of the product; (3) quality as defined as meeting the users needs; (4) quality as meaning value for money. He argues that the shift in industry and also in education has been away from the first two of these interpretations towards the latter two. Kissack and Meyer (1995) cite Harvey and Green's (1993) five conceptions of quality in education. These are similar to Winch's four conceptions, but they add a fifth way of perceiving quality as "transformative or empowering (value added)." (Kissack & Meyer, 1995: 234)

Winch (1994) describes a movement away from education being the concern of the producers (which he reports has come to be interpreted as teachers[4]) to being the concern of the user. Winch then discusses ways in which "the user" in state schools can be conceptualised[5]. In response to what he identifies as dominant neo-liberal notions, Winch (1994: 4) suggests that "the user" should not be thought of only directly as the student or the parents, or even the local community, but as the taxpayer in general, since it is they who are responsible for the funding of national school systems. For Winch (1994) it is the responsibility of governments to reconcile the interest of various groups of taxpayers. He adds

> it is not often, if ever, reasonable to expect people to negotiate away their values, it is reasonable and often necessary that they be prepared to compromise about the degree to which those values are implemented, particularly in the public sphere. This opens the way to a consideration of the desirability of diverse but clearly articulated aims for education, emerging from political debate. (Winch, 1994:7)

Here Winch appears to be approaching society and social institutions merely as the outcome of a process of consensus reached by a collection of rationally debating individuals or groups. This position ignores issues of power and control, how these become inscribed into social institutions, and community and individual identities over time and the conservative tendency of social institutions and identities to incorporate and project the past influences of power and control well into the future. As Giddens (1979:70) writes "all action exists in continuity with the past." Just as social analysis needs to take cognisance of how issues of power and control are inscribed into institutions over time, so too the planning around and resourcing of social institutions requires more than a snap vote on values and beliefs.

Even in Winch's own terms, he appears to ignore the fact that the notion of the consumer cannot exist without a producer, and that teachers do not act only in terms of any government policy, but are also influenced by current and past debates on education and by their own education and training. Similarly, schools operate within a more complex social milieu than industry, since in industry the factors that affect the final product are more controllable than the factors that influence the development of students and the process of learning It can be argued that to reduce the complexity of social institutions to the input–output systems of manufacturing will mean simplifying the issues and debates to the point where the analysis fails to be helpful. Since, as Angus (1993: 33) argues, to define

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[4] However, teachers do not act in isolation to produce lessons, they act within parameters prescribed by syllabi, principals, governing bodies, local, provincial and national educational authorities and within parameters inscribed into both current and past educational ideology and discourses. They are influenced not only by the constraints and opportunities of their teaching contexts but also their own educational and teacher training experiences, as well as past teaching contexts. I would argue that it is not possible to isolate an educational producer within a single person at a single time.

[5] Defining "the user" has become one of the major debates linked to how to provide quality education within the school effectiveness / school improvement movement (see for example Govinda and Vargese (1993); Heneveld (1994) Hopkins and West (1996); Maged (1998); Winch (1994)).

the unit of analysis as the input-output of the school in isolation "disconnects specific practices from social and cultural constructions and from political and economic interest". It leaves no room to consider how power and control are inscribed in the macro and micro structuring of society. As Morely and Rassool (1999) write, the notion of quality arose as part of a discourse in which

> [e]ducational success has been reduced to factors that can be measured. Explanatory variables identified in research findings have constantly shifted between psychosocial focus on the family on one hand and organizational factors on the other. The current skew is towards the organizational, with a belief in the capacity of bureaucratic co-ordination to deliver predictable outputs (Morely and Rassool, 1999: 3).

1.2.1 How did this managerial term become so prevalent in educational description? A case study from British education

Morely and Rassool (1999: 1) argue that "quality" as a term has become naturalized within the school effectiveness/school improvement discourse "which has both been enabled and forged in the West by the enterprize culture of the late 1980s and 1990s". However, as Morely and Rassool (1999:17) assert, key words, concepts and discourses do not have static definitions. Rather, different significations arise within different social and ideological mileux, in a way that does not exclude the interaction between different past and present discourses. Morely and Rassool (1999) trace how the term quality has risen to dominance within changing social contexts in Britain over a long period in which school improvement and school effectiveness debates gained importance. They argue, that disparate and often contradictory educational theory has been used to justify and lay the basis for the ways in which socio-economic changes have led to changes in educational policies and in particular to policies around schooling. A brief description of how the notion of "quality" has become inscribed into educational discourse reveals how analyses and critiques from within one school of thought can be drawn upon to publicly justify policies that bear no relation to the concerns voiced in the original theories and debates.

In the 1960s, the modernisation theory of national development started to become dominant, as did Vaizey's (1961, 1958, 1962, 1967) human capital theory[6]. The assumption was that investment in human capital, especially in education was a necessary precursor to economic success[7]. Within this context, the launch of Sputnik by the Soviets caused panic in the West. Morely and Rassool (1999) argue that, during the 1960s, the dominant view on education was that the provision of mass education

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[6] The notion that education was an important precursor to economic development was not invented by Vaizey: for example Ernest (1991) cites Williams (1961) who cites Foster (1870) as writing, "Upon the speedy provision of elementary education depends our industrial prosperity". However, it assumed dominance in the 1960s and this notion that the provision of high quality education is a precursor to economic development has circulated in a privileged position in ever since.

[7] Countries are positioned within global economic, social and political influences that cannot simply be altered by changing the quality of their national education systems. Lyster (1992) notes that illiteracy and poor education results are a feature or function of poverty, an individual's and a nation's position within a national and global socio-economic situation rather than the cause of it.

would provide all children with the opportunity for social mobility, that education could assist with social improvement and that increased public spending (on education and other social institutions) would counter social inequality and facilitate economic development. During the 1960s in Britain, the dominant view was that it was important to increase public spending on education.

Morely and Rassool (1999) assert that threads from disparate discourses were interwoven into public arguments during the 70s, 80s and 90s in ways which challenged the view that it was necessary for the state to maintain high levels of spending on education. One influence was educational research. From the late 60s, research from a variety of epistemological bases started to challenge the notion of schooling providing social mobility. Bernstein (1971, 1977), Bourdieu (1977), Bowles and Gintis (1976), Willis (1977), and Foucault (1979), in different ways showed that school was a mechanism for distributing students and that working class children were less likely to succeed at school[8]. Over the same period critiques of progressivist methodology began to circulate. Simultaneously oil costs increased, Britain's productivity declined, the nature of manufacturing industry changed, unemployment increased considerably and a range of other socio-economic crises led to political shifts. Aspects of human capital theory, elements from the disparate sociological studies mentioned before, and elements of a range of critiques of progressivism were evoked in ways which ignored and even ran counter to their original concerns and presented as if they were all a seamless single critique of the provision of education in Britain. The combining of aspects of these discourses allowed for the generation of a space in which different ways of centralising control in education could be asserted.

Morley and Rassool (1999) state that school effectiveness gained ascendancy with the New Right school reforms in Britain in the 1980s and 1990s. A modified version of human capital theory was evoked to link concerns with the pressures of global competition in the 80s and 90s with a manufactured panic over falling educational standards. This allowed some justification for education reforms, which were constructed around the concept of the market in education, as managerialism and enterprize culture started to gain currency. Whilst the increasing dominance of neo-liberal politics and economics was probably the driving force behind the reduction of public spending on education and formed the backdrop to state policies which supported the efficiency rhetoric and managerial focus of the effect-school movement, Morely and Rassool (1999) attempt to show that a convergence in aspects of other educational and economic theory and critiques allowed these concepts to penetrate educational discourse including teacher training and research. The shift that occurred over this period was to delink macro-social concerns from education and to privilege micro-organisation concerns. Schools began to be approached as separate units and viewed as the concern only of each school's

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[8] Although this point was made far more indirectly by Foucault.

community. As Morely and Rassool (1999) describe, much of the focus on education shifted from analysing the link between education and society to school development[9].

> In the 1990s school effectiveness researchers have moved away from the consideration of sociological factors, and towards a quest for student achievement indicators.... The core of research is to identify and isolate variables, techniques and procedures that can be transferred to any school and to any management system. (Morely and Rassool, 1999: 12)

1.2.2 Quality and managerialism in education beyond Britain

Similar trends within schooling are described within other countries: for example Angus (1993) traces how this trend developed in the USA; Aspin, Chapman and Wilkinson (1994) trace similar developments in New Zealand. There are obvious differences in the processes with which notions of quality via the school effectiveness / school improvement movement gained importance in other countries: for example Muller and Roberts (2000: 4) describe how in the USA the school effectiveness movement developed from Dewey's notions of progressivism and teacher professionalism, despite the claims that the Coleman Report (1966) was their launching pad, rather than the British and European sociological studies.

Morely and Rassool (1999) describe how this managerial approach to education and the school effectiveness movement have borrowed from industry the concept of quality and naturalised this concept within educational discourse. This has allowed for debate in industry around "quality control versus quality assurance", "quality systems" and " continuous quality improvement", "Total quality Management" (an American industrial model from the 50s and 60s), and "Total Quality Concern" (from Japanese industry), to spill over into education. Morely and Rassool's (1999: 135) concern is that this form of new managerialism focusses on what works rather than whose interests are being served: it is a form of pragmatism, marketed predominantly as efficiency, that conceals not only is own construction but also the power issues inscribed within it.

1.2.3 Quality and school effectiveness as dominant descriptions of education internationally

Concerns with quality and managerial systems are not isolated to Britain, USA and New Zealand, but have assumed an international importance. This is probably linked to the increasing dominance of neo-liberal politics and economics. Muller and Roberts (2000) argue that the focus on quality and managerial systems has entered educational discourse primarily though the influence of international aid agencies. Morely and Rassool (1999) also argue that the influence of the World Bank, UNESCO and IMF (where the focus has shifted from provision of education to all, to a focus on cost effective,

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[9] Within the school effectiveness discourse, debate about the link between schools and society has become reduced to views on the best ways to consult and include the views the various stakeholders in the school community (see Heneveld, 1994; Gilmour 1997; Govinda & Vargese, 1993; Maget 1999).

quality–provision) are influential. They add that the role played by international education consultants and tertiary level courses offered to educationalists from developing countries has also been important.

However, educational policy and research is not only influenced by international aid agencies, their consultants and tertiary level courses. The focus on quality and managerial systems has filtered through to other places and forms: for example many state educational departments, business-funded projects operating both inside and alongside non-governmental organisations, journals and research. Much contemporary education discourse is shot through with notions of measuring and constructing input-output systems that promote quality and other managerial concerns around organisational issues.

Notions of school effectiveness and concerns with quality at school level have a hypnotic effect: it would seem almost unnatural to be unconcerned with how to improve the cost effectiveness and quality of provision. At school level it might seem that there is little that can be done about the school's, community's or nation's socio-economic status, so that the only ethical position is to focus on quality at a local level. However, the issue is that school effectiveness and managerial[10] concerns with quality have become part of a dominant international discourse that tends to downplay or ignore concerns with how education is linked into social structures at a macro level and how this impacts on the micro structuring of individual schools and their particular practices.

1. 3 Measuring Quality and constructing input-output models

One way to measure the quality of education is by measuring student success. This is commonly done by schools being rated publicly on the performance of students in external examinations, for example the matriculation examination (at a national level), and international studies such as the Third

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[10] However, one should not assume that this managerial role of the teacher and education systems which assumes such dominance is a product only of the last half of the 20-Century. It has roots that go far back into the history of mass schooling. Foucault writes (1979: 147)

> the organisation of serial space was one of the great technical mutations of elementary education. It made it possible to supersede the traditional system (a pupil working with the master, while the rest of the heterogeneous group remained idle and unattended). By assigning individual places it made possible the supervision of each individual and the simultaneous work of all. It organised a new economy of the time of apprenticeship. It made the educational space function like a learning machine, but also as a machine for supervising, hierachising, rewarding.

The spatial and temporal organisation of mass schooling has strong resonances with the organisation of factories. It is all too easy to say that schools were set up to produce factory workers and the organisation of the school day and space was to prepare working class children for factory life. As Hamilton (1989) suggests, education could have been provided for more people by simply creating more one person schools: the multi-classroom, multi-teacher institution that developed resonated with 19th century assumptions about hierarchical management. Foucault (1979) discusses how the forms of power and control operating in both work places and educational institutions (and a good many other institutions) draw on a long history of organisation and formation of both the church/monasteries and the military/army. He suggests that these forms of organisations are refracted instances of dominant, but fluid, forms of social power and control, which necessarily display some level of continuity with past social forms.

International Mathematics and Science Study (TIMSS). This assumes that the way in which students are evaluated matches with one's understanding of quality education, that is, the student outputs in the tests, are an adequate measure of student competence. On the basis of these tests students and learning institutions are publicly differentiated and distributed in terms of their levels of success and by implication there is a social evaluation of the quality of the learning. At the top of the range institutions are held up as "getting it right" while those lower down the scale are pathologised. What the tests in themselves are not able to indicate is what produces these differences. Further investigation is often then focussed on which of the inputs in the education processes are responsible for the successful or unsuccessful outputs.

There are a multitude of studies attempting to ascertain which of the inputs are responsible for successful outputs: for example, Creemers (1996); Fuller & Clarke (1994); Darling Hammond & Ball (1999); Govinda & Vargese (1993); Hanushek (1995); Heneveld (1994); Hopkins & & Lagerweij (1996); Lloyd, Mensch, & Clark, (2000); Maged (1998); Reynolds (1998); Verwimp (1999); and West & Hopkins (1996). There are also plenty of studies that summarise collections of these studies about the impact of various educational inputs, for example Stringfield and Slavin (1992); Hanushek (1995); Creemers (1996); and Darling Hammond and Ball (1999). One of the problems with this type of quantitative overview of other research, which is acknowledged by Hanushek (1995), is that most of the categories, for example school facilities, are defined differently in different research studies. Nevertheless, Muller and Roberts conclude that these studies concur that the important factors are:

> Textbooks (availability of textbooks and supplementary materials); teacher quality (as measured by the kind and amount of pre-service teacher training); and time (as measured by the time and work demands placed on students) (Muller and Roberts 2000: 8) [11]

Angus (1993:335) criticises these attempts to isolate schools as units of analysis detached from social contexts for the evaluation of educational outputs in an attempt to develop techniques of evaluation of inputs-outputs correlations which "can be applied directly in any educational or management situation". Of interest here is the fact that Fuller and Clarke's (1994) synopsis finds that the quality of education is improved by a different set of school effects in developing and developed countries. Similarly the World Bank (1995) reports that the level of resourcing of schools has more of an impact in developing countries. These reports indicate that in different contexts different input variables are important in influencing student achievement. Although this is hardly surprising, it is perhaps a superficial example of how educational quality is context dependent, and how inappropriate it is to follow business models that attempt to isolate socially decontextualised variables that apply across contexts. It indicates one reason why attempting to develop key indicators or variables of educational

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[11] What is of interest here is that the research talks about the quality of teachers as defined by the amount of pre-service training, which presumably means professional teacher training and not education in the discipline that they are teaching.

quality (or even of student success) which can be applied regardless of context is an inappropriate approach to educational research.

In section 1.2 above I have discussed how the term "quality" has come to be a commonsensical term in education generally. Since quality can be understood as that which is valued, the term is now applied in a variety of ways in education. The term quality then comes to amplify whatever is thought of as valuable in education. This in turn depends on the how the aims of education are viewed.

This study was driven by an attempt to read the students' productions in their mathematical notebooks as an index of the quality of their mathematical teaching and learning. In the next sections I discuss dominant views on what is considered valuable in mathematics education, in order to come to refine my own definition of "quality" as I attempt to answer the question: "What, if anything, can the student productions captured in the notebooks of the matric mathematics classes at two schools tell us about the quality of mathematics education?" Initially I describe the construction of dominant contemporary views on quality in mathematics education. Later I criticise these views for not taking cognisance of the context of learning.

1.4 Constructing quality of mathematics teaching and learning

> In discussions and debates about 'quality' what matters and what helps us begin dimly to discern some sort of 'truth' in the irregular flow and the shifting interplay [... are] such factors as: the intellectual backgrounds and traditions, the disciplinary upbringings and the individual and collective intentions of the participants in the debates about 'quality'; the contexts in which such debates take place; the outcomes aimed at and the purposes held in mind; the considerations that make certain criteria important and certain moves decisive. These will give us far more the flavour, direction and sense of the particular ways in which 'quality' has significance in such discussions (Aspin, Chapman with Wilkinson, 1994:1)

Quality as a term denotes what is valued and as such carries an element of evaluation. What is valued in education is often constructed as the aims of education. Ernest (1991: 122) argues that one cannot assume universal agreement over the aims of education, in general, or mathematics education, in particular. He adds that different social groups will have different interests and ideologies and that these will influence what they value in education: that is, their educational aims.

Ernest (1991: 127- 216) draws on the work of Cosin (1972) and Golby (1982) to adapt Williams (1961) analysis of three distinctive groups that have influenced education to produce a typography of educational ideologies (see Table 1.1) that pertain to mathematics education.

Table 1.1: Overview of five Educational Ideologies from Ernest (1991)

Social group	**Industrial Trainer**	**Technological; Pragmatist**	**Old humanist**	**Progressive Educator**	**Public educator**
Political Ideology	Radical right, 'New Right"	Meritocratic, conservative	Conservative/liberal	Liberal	Democratic socialist
View of Mathematics	Set of Truths, and Rules	Unquestioned body of useful knowledge	Body of structured pure knowledge	Process view: personalized maths	Social constructivism
Moral Values	Authoritarian 'Victorian' values, choice, effort, self-help, work, moral weakness, us-good, them-bad	Utilitarian, pragmatism, expediency, 'wealth creation', technological development	'Blind justice, objectivity, rule-centred structure, hierarchy, paternalistic 'classical' view	Person-centred, caring, empathy, human values, nurturing, maternalistic, 'romantic' view	Social justice, liberty, equality, fraternity, social awareness, engagement and citizenship
Theory of Society	Rigid hierarchy Marketplace	Meritocratic hierarchy	Elitist, class stratified	Soft hierarchy, welfare state	Inequitable hierarchy needing reform
Theory of the Child	Elementary school tradition: Child 'fallen angel' and 'empty vessel'	Child 'empty vessel' and 'blunt tool', future worker or manager	Dilute elementary school view, character building, culture tames	Child-centred, progressive view, child 'growing flower' and 'innocent savage'	Social conditions view: 'clay moulded by environment' and 'sleeping giant'
Theory of Ability	Fixed and inherited Realized by effort	Inherited ability	Inherited cast of mind	Varies, but needs cherishing	Cultural product: not fixed.
Mathematical Aims	'Back-to-basics': numeracy and social training in obedience	Useful maths to appropriate level and Certification (industry-centred)	Transmit body of mathematical knowledge (maths-centred)	Creativity, self-realization through mathematics (child-centred)	Critical awareness and democratic citizenship via mathematics.
Theory of Learning	Hard work, effort, practice, rote	Skill acquisition, practical experience	Understanding and application	Activity, play, exploration	Questioning, decision making, negotiation
Theory of Teaching mathematics	Authoritarian transmission, drill, no 'frills'	Skill instructor, motivate through work relevance	Explain, motivate, pass on structure	Facilitate personal exploration, prevent failure	Discussion, conflict, question of content and pedagogy.
Theory of Resources	Chalk-and-talk only, anti-calculator	Hands-on and micro-computers	Visual aids to motivate	Rich environment to explore	Socially relevant, authentic
Theory of Assessment in Maths	External testing of simple basics	Avoid cheating, external tests and certification, skill profiling	External examinations based on hierarchy	Teacher-led internal assessment, avoid failure	Various modes, use of social issues and content
Theory of Social Diversity	Differentiated schooling by class, crypto-racist, mono-culturalist	Vary curriculum by future occupations	Vary curriculum by ability only (maths neutral)	Humanize neutral maths for all: use local culture	Accommodation of social and cultural diversity a necessity

Ernest's contention is that different social groups have influenced and continue to influence education curricula in different ways. However, these influences are often most easy to see at times of transition. One example of this is the influence of the ideological positions of different groupings in the amalgamation of MASA (The Mathematics Association of Southern Africa) with the Mathematics Commission of the NECC (National Education Crisis Committee) to form AMESA (The Association for Mathematics Education of South Africa). During the formation process of AMESA negotiations ensued between people who articulated various forms of the old humanist tradition, progressive educator traditions and public educator positions. Within AMESA ideological differences still occur, but a common curriculum language has developed which allows people to often use the same expressions and activities to connote quality education, even where these may at times denote different practices to different people. AMESA has had a strong influence both on curriculum construction and on how best practice (or quality education) gets interpreted. In many ways the ambiguities in the

Mathematical Literacy, Mathematics and Mathematical Science Learning Areas of Curriculum 2005 resonate with the differences contained within the common language developed by AMESA.

Both Ernest and Williams acknowledge that any social group's aims tend to be mixed and not pure as represented in the Table 1.1. Further, Ernest (1991) argues that one cannot merely consider the aims of education, because, as education is a complex social process one also needs to consider "the means of attaining these aims [....] For the values embodied in the educational aims should determine, or at least constrain, the means of attaining them" (Ernest, 1991: 123). Elsewhere, Ernest (1989) writes of how teacher beliefs[12] influence teachers' espoused models of teaching and learning but that social contexts always impact on and modify enacted models of teaching and learning mathematics. Davis (1995: 37), commenting on Ernest's typology, argues that any transmitter could draw selectively on the categories of belief and practice and that this will be influenced both by their experience and the local context in which teaching and learning occur. Ensor (1998) argues that context should not only be understood as something that constrains how teachers implement their idealised beliefs, but that context will also modify teachers' espoused beliefs. She writes that,

> Human subjects are inserted into different social activities and that what they "believe" is contingent and not necessarily stable across these. While human subjects do acquire repertoires of knowledge and skills and more or less loosely structured webs of dispositions or commitments to act, which one might call "beliefs", these are foregrounded according to the context in which subjects operate .... Each context is an invitation to subjects to position themselves in relation to each other, and to recruit or recontextualise linguistic and somatic resources in order to achieve this. The evoking context in each case foregrounds and backgrounds subjectivities, repertoires and positions and in this way motivates the selective recruitment of resources. (Ensor 1998: 282 - 3)

In mathematics education, both in South Africa and internationally, constructivism in various forms has become a dominant discourse (Brodie, 1998; Davis, 1995; Ernest, 1993; Muller and Taylor, 1995; Parker, 1994). The influences of constructivism are manifest in school mathematics curricula, teacher training, textbook construction, professional and research journals. Constructivism forms the backdrop against which much of mathematics education is evaluated. For this reason any discussion of quality in (mathematics) education needs to take account of the disparate values inscribed into the various forms of constructivism. At the heart of constructivism is the notion that students construct their own knowledge. This has been reinterpreted variously to make suggestions for how teaching and learning ought to happen and the quality of mathematics education is generally assessed against this. Varieties of constructivism have circulated in South African education departments during the 1980s and early 1990s, first through the influence (albeit an uneven influence across different schools, towns and

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[12] There is a wide range of literature on how teachers' beliefs influence their practice; for example, Thompson (1992) does a review of this literature.

provinces) of the Problem-Centred Approach in House of Assembly[13] schools and later through the House of Representative and House of Delegates school syllabi and teacher training/re-orientation workshops. It has also circulated through a range of Non-Governmental Organisations (NGOs) which offered in-service training (INSET) for example COUNT, RUMEUS, MCPT, RUMEP, MEP and through the teacher training offered at most tertiary[14] level institutions. Brodie (1998) argues that constructivism has been influenced by progressivist notions of child- or learner - centredness, and that it is inscribed into the Curriculum 2005 via these same notions of learner- centredness.

There is a range of versions of constructivism (Muller and Taylor, 1995; Phillips, 1995; Ernest, 1993). Each of these versions will present a different yardstick against which to measure quality teaching, learning and schooling. Ernest (1993) divides constructivism into three categories: information processing constructivism, radical constructivism and social constructivism.

	Absolutist Epistemology	**Fallibilist Epistemology**
Individual Focus Alone	Information-processing Constructivism	Radical Constructivism
Individual and Social Focus		Social Constructivism

Table 1.2 Classification by epistemology and focus (Ernest, 1993: 171)

Ernest suggests that radical constructivism tends to draw selectively on Piagetian learning theories, in ways that suggest that learning appears to happen relatively independent of social context[15]. He suggests that social constructivism tends to draw more on Vygotskian learning theories in which language and consciousness develop differently in different social contexts.

Within the discourse of radical constructivism, which forms the basis of the Problem-Centred Approach that has strongly influenced school mathematical curricula in South Africa, much is said and written about social interactions. However, generally the notion of the social is restricted to consideration of the student's classmates and much of the discussion of social interaction is restricted to group work. Ernest (1993) criticises this form of constructivism by saying, "it is a distraction from the social and political goals in education [.... since it fails] to engage with issues of values, power and politics" (1993: 171 – 173). However, it is not just that these forms of constructivism do not

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[13] In the 1980s the South African government, as part of their Tricameral parliament, instituted the apparently parallel "houses" to deal with the governance and administration of different racial groups: the House of Assembly dealt with affairs of "white" citizens, the House of Representatives with the affairs of "coloured" citizens and the House of Delegates, the affairs of "indian" citizens. Africans were denied citizenship of South Africa, and instead offered citizenship of a Bantustan.

[14] I am using tertiary institutions to include teacher-training colleges, although they were technically classified as part of the secondary educational sector until the latter half of the 1990s.

[15] This belief that Piaget's work suggests that cognitive development, whilst dependent on language, is independent of social context has become a common misreading that underpins certain forms of constructivism.

acknowledge issues of power in the broader society, but that they do not consider how "mathematical activity must be consistent with the social organisation of its site of elaboration" (Dowling, 1990: 94).

1.4.1 Everyday relevance and learner discovery methods

The Problem Centred Approach and Curriculum 2005[16] suggest, albeit in different configurations, that quality teaching and learning should entail students working with relevant everyday problems and discovering the mathematics embedded in the problems. Teachers are implored to let students construct knowledge for themselves and not to teach decontexualised mathematical skills. This in turn implies that invisible forms of pedagogy[17] constitute better teaching practices. It allows for the construction of an ideal teacher as being someone who displays only masked forms of authority. This ideal is constructed without cognisance of the effect of different teaching contexts on the construction of appropriate and possible pedagogy.

However, not all research supports the dominant constructivist position that there is one ideal teaching practice regardless of contexts. For example, Hasan's (1995) research suggests that a classroom practice, which privileges invisible pedagogy and masked forms of authority, might only make learning easier for limited sectors of the population[18]. Hasan (1995) focused on the communication patterns between mothers and children, in an exploration of whether participation in everyday talk established ways of learning. Hasan reports that higher autonomy professional (HAP) mothers "control their children's behaviour by giving indirect or suggestive commands" while lower autonomy professional (LAP) mother's "authority is flagrantly obvious" (Hasan, 1995: 98, 99). Hasan concludes that initial patterns of communication and the ways in which power relations appear to be inscribed into these communicative patterns have an impact on the child's identity formation:

> the HAP child is thus socio-semiotically produced as an individual with his own unique subjectivity, the sharing of which is in his personal discretion.

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[16] The student productions under consideration were produced by students in their matric year. Whilst neither the Problem-Centred Approach nor Curriculum 2005 were implemented in the Further Education Phase of schooling, their dominance in educational circles has meant that much teaching and learning was evaluated against the values of the pedagogy that they promoted.

[17] Bernstein (1990, 1996) distinguishes between visible and invisible forms of pedagogy. He defines these in the following way

> I called the practice *visible* when the hierarchical relations between teacher and pupils, the rules of organisations (sequence, pace) and the criteria were explicit and so known to the pupils. In the case of *invisible* pedagogic practice the hierarchical rules, the rules of organisations and the criteria were implicit and so not known to the pupils [....] In the case of invisible pedagogy it is **as if** the pupil is author of the practice and even the authority, whereas in the case of visible practices **it is clearly** the teacher who is author and authority. (Bernstein, 1996: 112: italics in the original, emphasis added.)

[18] One cannot assume that differing communication and local pedagogic patterns in Australia translate into similarly differing patterns in South Africa. However, similar research has emerged from North America (Delpit 1988, Ellsworth 1989, Heath 1993), and Britain (Bernstein 1971a, cited in Bernstein 1996).

> This granting of unique individuality, the masking of maternal power, and the granting of discretion combine to produce a sense of the world under one's control, where the external control on a child's actions is rendered invisible, its motivation being presented as either above human manipulation (reasons are 'logical', guided by 'unavoidable' rational principles') or as self regulated (the child has discretion and own judgement). The LAP child is socio-semiotically produced as someone whose experience of the collective is an aspect of his subjectivity, the sharing of which does not depend on his personal discretion; the control on the child's action is quite visibly external. The power that controls his actions is derived from the speaker's social position vis-a-vis the addressee...The child's action is quite clearly under the control of this greater power. (Hasan, 1995: 98, 99)

The forms of pedagogy privileged by radical constructivism tend to ignore how "inequalities in the distribution of power and in the principles of control necessarily implicate forms of interaction, and so forms of consciousness" (Hasan, 1995: 189). Hasan (1993:87) argues that "the meaning system of human language is not arbitrary; it is functionally specialised with respect to context." Picking up on this in a later article, she (Hasan 1995) criticises frameworks that do not theorise the use of language in social context and do not incorporate issues relating to the social positioning of subjects and how this impacts on social mediation:

> [t]he non-theorisation of social context [...] has the consequence of simplifying the concept of social interaction. The speaker and the listener become culturally non-specific (Bernstein, 1990:48); the range of verbal and practical activities lack contextual specialisation as if anyone can say or do anything anywhere anytime [....and] no concept of text develops. (Hasan 1995:188)

The radical constructivist assumption of a singular idealisable teaching practice does not allow a consideration of the regulations on, the possibilities provided by, or the constraints imposed by the local contexts. The impact of different contexts on the construction of potential subjectivities and intersubjectivities is not taken into account. Similarly the impact of recontextualisations on texts or knowledge productions is ignored. There is no consideration of how

> class relations generate, distribute, reproduce and legitimate distinctive forms of communication, which transmit dominant and dominated codes, ...[and how] subjects are differently positioned by these codes in the process of acquiring them.... Class regulated codes position subjects with respect to dominant and dominated forms of communication and to the relationship between them. (Bernstein 1990: 13)

The notions of good and bad teaching are constructed as if anyone can say or do anything anywhere anytime. Parker dismisses these idealist notions as an "utopian image, a wishing to say where class, culture, politics and power cease to exist, and everybody desires to be an intellectual." (Parker, 1995: 16)

Whilst various forms of constructivism ignore the interdependence of context and communicative practices, many forms of constructivism assert that concepts should be contextualised within situations familiar to students. Behind the notion of everyday contexts assisting the transition into mathematical activity is the concept that students are already familiar with these contexts and so (relatively) competent in completing tasks within these contexts. However, as Dowling (1995: 218) writes,

> the criteria for successful completion of the tasks within the mathematical context are, therefore, different to and so not available within [for example] shopping practices. There is, in other words, no basis for the reader's competence.

Just as the strategies that are available in mathematics classrooms are not necessarily available in other everyday contexts, so the strategies available and that work best in everyday contexts are not necessarily available in mathematics classrooms, and if they are, they might not be evaluated favourably. Mathematical activities, even where these are designed to be embedded in apparently familiar everyday activities are evaluated in particular ways, according to mathematical principles. Dowling (1995) argues that there are different regulations on the potential practices in different fields of discursive activity. This means that in different activities there are different regulations on who can say and do and mean what to whom and how this can be done. This is why everyday or work examples in school mathematics discourse are always constructed in particular ways, which privilege the mathematical regulations on the activities. The mathematical gaze distorts the apparently everyday problem. It is for this reason that Dowling (1995) questions the value of invisible forms of pedagogy in mathematics. He argues (1995: 218) that since there is no pedagogising of method it has been assumed that students need to be already competent in order to solve the problems.

Dowling (1995) further dismisses the related notion that socially contextualised school tasks are more likely to help students in their everyday lives. He argues that because the regulations on school tasks are different, doing mathematical tasks disguised as everyday tasks will not assist the students with their everyday tasks. Dowling argues that the belief that school knowledge can be made directly relevant to everyday and working practices is mythical.

1.5 Towards a socially contextualised definition of quality in education

The discussion of how the term quality came to be applied to education, via the school effectiveness / school improvement movement, attempted to show how schooling is often thought of as a generic activity. Similarly I have argued that whilst many forms of constructivism suggest that learning tasks should be cast within contexts familiar to students, they do not acknowledge the relationship between communicative contexts and linguistic patterning, and in this way dominant pedagogical practices also present schooling as a generic activity.

It is not only that cognisance ought to be taken of how dynamics and contexts vary across and within schools, but also that schools as an institution impose certain regulations on activities. It is this latter point which Ivic (1989) stresses:

> [School] as an institution and quite apart from the content of its teaching, ... implies certain structuring of time and space and is based on a system of social relations (between pupils and teacher, between the pupils themselves, between the school and its surroundings, and so on) (Ivic (1989: 434) cited in Daniels, 1995: 518)

Bernstein (1990, 1996) concurs with Ivic's (1989) position in writing that pedagogic discourse specialises text, time and space[19]. Morphet (1997) draws on Bernstein's notions of the specialising of text, time and space in order to comment on what he calls "the elusive issue of 'quality'" (1997: 270). He argues that the more schools attempt to reproduce forms of everyday knowledge and activities, the less likely they are to induct students into disciplinary knowledge. He suggests that attempts to democratise schools and increase students' access by reducing the differences between school and everyday life is, in effect, reducing the extent to which schools specialise time, text and space, and that this will reduce the chances of students acquiring the disciplinary knowledge and so will disadvantage the very students it attempts to advantage. Morphet adds that when considering the issue of quality one needs to consider the extent to which students acquire school-based, subject specific or disciplinary knowledge and that, "judgements of quality [...] have to be exercised on educational [...] grounds [...] they need to be about the acquisition of vertical rather than horizontal dicourses - about principled and propositional knowledges – rather than local and strategic knowledges" (1997: 271).

Dowling similarly suggests that education should "open up the availability of academic discourses to all" since,

> [t]he acquisition of such vertical practices necessarily involves subjugation to the evaluative principles of these discourses. Subjectivity of necessity entails subjugation. Academic subjectivity entails subjugation to a discipline. Only in this way can the DS⁺ [high discursive saturation] practices be made available as structured resources for the interrogation of everyday practices by the practitioners themselves; discipline and then, and only then, mathematise" (1995: 223).

I base my definition on quality in education on those of Morphet (1997) and Dowling (1995), both derived in part from Bernstein, in that I interpret quality education, at a school level, as that which provides students with access to school discourses. 'Quality' is an elusive term, largely because the values or moralities inscribed into it are not fixed by tend to shift. Morphet's, Dowling's and my own concern with the learning of disciplinary knowledge may be interpreted as backgrounding the moral order in the notion of quality. In Chapter 3, I discuss in more detail how Bernstein describes the instructional discourse being embedded in the moral order, which he also refers to as the regulative discourse. I see it as impossible to learn disciplinary knowledge in the absence of values; and although Morphet's and Dowling's position may appear to background the moral order, it is very much

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[19] Although there are strong similarities in the ways that all school specialise time, text and space this is not to deny that both within schools and across any particular school the specialising of text, time and space differs. These differences in specialisation are not idiosyncratic but related to and (re)productive of contextual differences.

inscribed into their and my own privileging of the learning of disciplinary knowledge.

In order to assess the quality of education provided to the two Standard 10/Grade 12 mathematics classes I intend to analyse how pedagogic practices in these classes may be inferred from potential patterns in the students' productions, before assessing the extent to which these pedagogic practices induct students into the discourse of school mathematics.

1.6 The structure of this dissertation

This study aims at using the analytic tools developed by Basil Bernstein to discuss the student productions as an instantiation of communication which captures differing pedagogic practices and in so doing can be read as distributing students along the continuum of quality education.

In this study, I intend to use Bernstein's model of the pedagogic device to redescribe the issue of quality in terms of the differing nature of the pedagogic practices in the Standard 10/Grade 12 mathematics classes at the two schools. The intention is to analyse the student notebooks at the two schools in order to generate a description of the pedagogic practices of each school which, in turn, can be related to both the form of social organisation and to the students' differing levels of success at each school.

Bernstein's model allows me to understand how context both constitutes and is constituted by pedagogic practice. It provides a "grammar" that one can use to infer differences in regulative and instructional discourses[20] at the schools from the students' productions. The model allows me to infer, from the differences in these students' productions, what the differing rules of cultural communication in the two matric classes are, and thereafter, to read these as an index of their differing micro-interactions and the classroom based structuring of the social division of labour, without losing the broader picture of how these are related to the macro-structuring of society.

This first chapter presents a brief history of how the notion of quality entered educational discourse. This history gives some clues as to why the notion of quality is generally read in a decontextualised manner. It then discusses some contemporary approaches to the quality of mathematical teaching and learning before arriving at a definition of quality mathematics education as that which inducts students into the discourse of mathematics.

Chapter 2 briefly outlines issues in the research around analysis of students' mathematical productions. It notes the paucity of research around student's mathematical productions that focuses

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[20] This is discussed in detail in Chapter 3.

longitudinally on students' note taking and routine calculations within a framework and which considers the impacts of macro and micro social structuring of classroom contexts.

In Chapter 3 I discuss the theoretical resources of Basil Bernstein on which I draw and the methodological approach I use in the process of data production and analysis.

In Chapter 4 I present an analysis of differences in the student notebooks. These differences indicate that different kinds of pedagogy are privileged in the matric mathematics classes at each of the schools. These differences resonate with differences in the regulative discourse or the structuring of time and space at the two schools.

Bernstein (1990, 1996) argues that evaluation is the key to pedagogy. In Chapter 5 I discuss what forms of evaluation are evident from an analysis of the student productions. I hypothesize about the relative importance of the evaluation captured in the notebooks compared with the other forms of evaluation that are assumed to operate in the two classrooms. I discuss the relationship between the forms and frequency of evaluation reflected in the notebooks and the way in which students develop criteria for the construction of texts. It is this process of developing the criteria for the construction of text that captures the process of the students' potential induction into the discourse of school mathematics. It is, in turn, the degree of the students' induction into the discourse of school mathematics that I argue represents the quality of education.

In Chapter 6 I discuss the extent to which student productions, captured in their notebooks, are a good measure of students' induction into the discourse of school mathematics, and what this therefore suggests about the quality of their education. I consider whether the theoretical framework and analytic methods chosen for the research project have been adequate in addressing the research question. I present some possibilities for extending the scope of research presented in this dissertation.

Chapter 2: Literature survey on students' written productions

In this chapter I discuss the research literature on students' written productions in mathematics and comparative school studies that include a focus on students' written productions. Much of this literature is grounded in cognitive and developmental psychology and does not consider sociological factors. In particular much of it ignores how social regulations impact on the structuring of communicative texts. I have argued in Chapter 1 that language becomes specialised within contexts and that since school is not a generic activity cognisance needs to be taken of the context in which the teaching and learning occurs. This in turn suggests that student productions be read as a form of cultural communication which is influenced by a broader reading of the social context within which the communication occurs. It is this latter concern that is absent from much of the literature on student productions.

2.1 Analyses of student productions generated in mathematics classrooms

2.1.1 Dominant interests in student productions in mathematics lessons

Much contemporary work on students' productions in mathematics classes does not focus on all the writing that students do in class: it is concerned neither with the use of writing as a tool to aid calculation nor note taking as an aid to memory, nor yet with analysing students notes as an index of the privileged pedagogic communication of different classrooms. In fact much of this research and professional writing (Borasi and Rose (1989), Burns (1988), Clarke, Wayward and Stephens (1993), Davison and Pearce (1988), Hoffman and Powell (1989); Johnson and Galant (1997), Miller (1990), Nahrang and Petersen (1986), Powell and Lopez (1989), Rose (1989), Stempien and Borasi (1985), Swinson (1992), van den Brink, (1987), Venne (1989)) is dismissive of these forms of writing and offers alternative forms of writing along with the suggestion that the proposed alternatives will assist student learning. These alternative forms and foci of student writing include: students reflecting through journals on their emotional response to events in the mathematics class; students written records of the development of their own expressions to explain mathematical phenomenon, and their written explanations of how they derived these expressions (Borasi and Rose (1989), Burns (1988), Nahrang and Petersen (1986)); the writing that students do for school mathematics where they are explaining what they understand about certain topics (Johnson and Galant (1997), Powell and Lopez (1989), Shield and Swinson (1994)); constructing their own calculations or stories from given numbers or number sentences or an extension of this where students devise, write and illustrate problems which are presented to other students in the form of a textbook (Frankenstein (1993), Hoffman and Powell (1989)). These foci of student productions can be understood in the terms in which some constructivists separate out so-called "rote learning" and "real understanding" or in Skemp's (1978) language, instrumental and relational understanding. Just as these constructivists privilege what Skemp referred to as relational understanding, so they might tend to undervalue students' note-taking of the teachers expository writing and exemplars and also students' routine calculations. For example, Shield and Galbraith describe "writing tasks" in a way that indicates the predominant lack of interest in analysing students' day to day note taking and calculating of routine exercises.

> The writing tasks that have been used in mathematics classrooms have generally been categorised as journal writing or expository writing [....] Journal writing [...] normally invites students to reflect on their learning by expressing their thoughts and feelings about the mathematics they are learning. Expository writing is intended to describe and explain. (Shield and Galbraith, 1998:30)

The focus of this dissertation is on students' daily routine calculations and notes, whilst the approach taken is that these may well be revealing of the pedagogic practice that students experience. The sorts of studies outlined above were not found to concur either with the focus (a quick scan of the students notebooks, revealed that students did not engage journal writing or other written forms of reflection in their mathematics lessons) or with the approach taken. Also as I have indicated in Chapter 1, I do not read school as a generic activity (as is the case in much of constructivism) but as an activity which is contextually inscribed. I assert that the social regulations operating within any context will impact on the forms of communication and instruction. For these reasons, I believe that the way in which student notes are organised will provide some level of understanding of the pedagogic practice they experience and also of student's positioning within that practice.

2.1.2 Coding systems for analysis student productions in school mathematics

Van Dormolen (1985) develops a coding system for analysing types of text in school mathematical textbooks. Shield and Galbraith (1998) draw on van Dormolen's (1985) coding system to analyse student writing, in an attempt to assess the extent to which narrative and reflexive "journal-type" writing enhances learning in mathematics. They conclude that most of the student productions are linguistically structured in a way that is very similar to textbooks. Their criticism appears to be that students do not show a fluency with paraphrasing concepts in forms different from those used in school mathematics textbooks, and hence do not display a deep understanding of the topics.

Students' ability to paraphrase the concepts is one way of showing their expertise in a discourse. However, understanding the discourse itself requires a fluency with the genre, which includes knowing the permitted units of texts and the rules for their combination. These grammars and forms of discourses are social conventions. Students are highly unlikely to discover them for themselves. Press (1999) draws on Hodge and Kress (1988) to analyse how a mathematics textbook forms a logonomic system. Hodge and Kress (1988) discuss how social semiotics allows one to describe the potential ways in which discourses position individuals within social systems. They describe meanings and messages as socially constructed and state that

> [e]ach producer of a message relies on its recipients for it to function as intended. This requires the recipients to have knowledge of a set of messages on another level. Messages that provide specific information about how to read the message (Hodge and Kress, 1988: 4).

It is these messages about messages that constitute the logonomic system, which provides not only the parameters within which to interpret but also the parameters within which to construct messages and meanings.

> A logonomic system is a set of rules prescribing the conditions for the production and reception of meanings; which specify who can claim to initiate (produce, communicate) or to know (receive, understand) meanings about what topics under what circumstances and with what modalities (how, when, why). Logonomic systems prescribe social semiotic behaviours at points of production and reception (Hodge and Kress, 1998: 4).

Teachers and textbooks are the most likely sources from which students will learn school discourses: they provide models of potential ways for students to present content. Most students will not have many other sources of logonomic systems upon which to base their attempts to construct legitimate mathematics texts other than those provided by textbooks and by teachers. The fact that students' work reflects a structuring which is influenced by the format of textbooks is then hardly surprising. Using the language and format of the textbook may be part of process of student's induction into the discourse. If students' writing displays language or formatting similar to that of a textbook, it does not necessarily indicate that their level of understanding of the discourse is very weak nor does it necessarily reflect poorly on the quality of their learning.

Within constructivism the notion that rote learning does not lead to a deep understanding has developed. Whilst rote learning does not necessarily lead to a deep understanding of how concepts are interrelated, there are students who do develop such understanding from rote learning and/or from doing many routine practice examples in mathematics. On the other hand, both Powell and Lopez (1989) and Shield and Galbraith (1998) conclude that there is little evidence that student narrative and expository forms of writing, which are assumed to "promote a personalised and constructive approach to learning" (Shield and Galbraith, 1998: 30) enhance learning in mathematics. Shield and Galbraith (1989) do not consider that by coding student writing into categories designed to analyse and mark out sections of textbooks, they may have developed a reading which maximises the potential for finding these same elements in the student texts. They also do not consider how the ways that teachers and students select, sequence and pace the topics and problems prescribed in the syllabus and presented in textbooks are revealing of the forms of cultural communication privileged in a classroom. Nor do Shield and Galbraith (1989) consider how the privileged forms of cultural communication may be related to issues of forms of social organisation privileged at a school or in a community. For this reason they do not consider that students' written productions, even when they are structured like a school mathematics textbook, can be indicators of the form of pedagogy privileged in a classroom.

However, for the purposes of this dissertation all student productions are analysed as a form of cultural communication. The different foci and methodologies of this dissertation and both those of Shield and Galbraith's (1998) and van Dormolen's (1985), has meant that much of the detail of their categories of coding are not appropriate for this study. In this dissertation, however, use is made of their categories of expository writing (writing intended to describe and explain mathematical ideas), exemplars (which model an algorithm or how to do a calculation) and a third category is included: routine practice calculations. I, however, find it difficult to distinguish between exemplars and expository writing, since the level of student (and often also teacher) apprenticeship into mathematics, means that at times they do not have sufficient mathematical language to allow for exposition to happen in any other way, except by exemplars. In fact, a feature of South African school mathematics textbooks is that series of exemplars take the place of mathematically constructed exposition: success in school mathematics[1] does not appear to require access to the 'general principles'[2].

2.1.3 Situated Learning: an attempt to consider contextualising features

Meira (1995a, 1995b, 1997, 1998) analyses student productions in combination with their dialogues with peers and responses to an interviewer while working with physical apparatuses. He is concerned with how students make sense of apparatuses, written mathematics forms and mathematical concepts. The focus of his research is on how students ascribe mathematical meaning to empirical phenomena by drawing both on what they have been taught to expect about forms of mathematical representation and on the knowledge of available experts. He challenges the constructivist notions that assign an automatic match between apparatus and the mathematics that is read into them by mathematicians or mathematics educators. He criticises the notion that

> the transparency of a display is directly proportional to the quality of the links between the tangible features of an object and a target knowledge domain as understood by experts (Meira, 1998: 124)

Meira (1995a, 1995b, 1997, 1998) draws on both Vygotsky and Lave in the ways that he considers both thought and communication to develop within specific cultural practices. Meira provides individual or pairs of students with apparatuses, which might signify the embodiment of function relationships to some mathematicians and mathematics educators. He (1995a, 1995b, 1997, 1998) documents how students create the data. Meira links this data production with the students' experience of tables of number patterns and to the questions posed by the interviewer. His own data is generated by analysis of both what the students say and do during the filmed interactions and the

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[1] This has particularly been the case in the Standard Grade stream of South African school mathematics curriculum

[2] This is not to deny that some students do construct elements of the 'general principles' from particular examples.

analysis of what they write. Meira's study resonates with mine in that it does not dismiss the analysis of what students write in the process of solving problems (as do many of the studies that focus on and promote the use of student journals) and in that it does consider past learning to be an important component of the way students structure both their written texts and the way that they structure their interactions with other learning texts (whether these be apparatuses or conversations with significant others). Meira (1998) writes:

> what the relation between people and instrumental artifacts becomes depends ultimately on their participation in specific practices, and on the practical and discursive activities in which artifacts are made to signify (1995a: 1-110)

While he does acknowledge the importance of social organisation in the structuring of interactions, communication and learning, he does not develop an in-depth analysis of the specific interactions or the social organisation of either the students classrooms, school or communities. For example, although Meira describes how students' expectations of the results of experimenting with the apparatus are overlaid by the structuring of their previous work with tabulating number patterns, he does not consider the ways in which the research context has similarities to and differences from the students' classroom context. He assumes that they will regard both situations as parts of the same practice of mathematical learning, and hence transport the cultural communication patterns from the latter situation to the former. Meira criticises educational researchers and practitioners for

> Not paying enough attention to the contexts, [...] the perspectives and histories of [the students] (1995a, 1-110)

However, this criticism may well be aimed at him too, since his analysis of the students learning with the apparatus does reflect back on a detailed analysis of the forms of cultural communication privileged in the students classrooms, school or homes. Meira does not allow space for how patterns of communication may differ in different classrooms, schools or homes and the extent to which patterns of communication formed in one context are simply transferred or transformed from context to context or how communication forms from one context may influence another context. While he does consider some of students' past learning histories and practices, this is done in quite a superficial way. This is despite the fact that he spent three weeks observing the students in their mathematics classes before engaging with the "research activities" themselves (Meira, 1998:125). Meira appears to regard the practice of school mathematics as a single seamless discourse that does not show strong differences across learning contexts.

One difference between this dissertation and Meira's study is that in the data collection for this dissertation there was no access to teaching and learning interactions. This necessitated an analysis of pedagogical forms that was dependent on student's written productions without information about interactions. The focus of this dissertation is on quality of learning (read as the degree to which different forms of pedagogy induct students into school mathematics). Induction into any practice or discourse occurs over time, for this reason I need to take a more long-term view of learning. The analysis of the notebooks over most of the year potentially provides an opportunity to track this progress. This dissertation aims to compare pedagogical forms in the Std. 10 / Grade 12 Mathematics classes at two schools in ways which explore how learning, as a form of cultural communication, is affected by the forms that social organisation takes in different contexts. My study requires a model of how social context impacts on pedagogy, which is absent in Meira's work.

2.1.4 Developmental Semiotics: another attempt to consider social contexts of learning

Vile and Lerman (1996), and Vile (1998) present what they call "developmental semiotics" as an approach to understanding mathematical texts and meaning making. They suggest an approach that combines Pierce's notion of diachronic semiotics, which established a hierarchy of forms of symbolic representation, with Vygotsky's notion of a hierarchy of mental functions, which are developed within specific social formations, is useful in analysing student texts. Their concern is to ensure that meaning making is viewed as a process that occurs over time through the interplay of intra- and inter-subjectivities.

> There are two main elements of the process of developmental semiotics. The first, from semiotics, describes the way that a cognising subject is in possession of a network of experiences into which any semiotic act will be inserted, as result of which is the production of an interpretant (and hence a meaning). This network of experiences is built up over time with each new experience affecting the whole network. The second derived from Vygotsky (1977) puts forward the idea of a developmental process in which the external social factors affect the level of mental functioning and force the transition from elementary to higher mental functions. (Vile, 1998:22)
>
> Central to this process is the insertion of the act of semiosis into a **network of experiences**; in order to address the question concerning the creation of this network of experiences, which could be classed as the content of the intersubjective, one must consider semiosis to act diasynchronically, and it is at this point that the notion of development is brought into the frame. Lev Vygotsky (1977, 1978) presents a psychology with semiosis, activity and development as central to the genesis of higher mental functions. In fact his notion of "scientific" concepts (which act on higher mental functioning) as concepts that make meaning through links with other concepts, can be equated with the notion of interpretant, which first acts in semiosis and then becomes a sign. We want to suggest that the network of experiences is a web of interconnected interpretants (acting as signs) that have been internalised diachronically through acts of semiosis" (Vile and Lerman: 1996: 4 – 397; emphasis in the original)

Although Vile (1998) and Vile and Lerman (1996) present developmental semiotics as an analytical tool to analyse meaning making as a sociocultural activity that occurs over time, the examples of analysis they provide seem to focus on instantiations of student productions with only scant reference to the sequencing and pacing of the development of key skills within that particular mathematical topic. They suggest that

> Research carried out within a developmental semiotic perspective will concentrate on the nature of the sign in specific acts of semiosis. This focuses the need for a research

> methodology aimed at eliciting meanings made in those contexts. (Vile and Lerman, 1996: 400 –401)

However, the examples of textual analysis, which they provide, show no evidence of consideration or analysis of the social context. It appears that their understanding of the social is limited to an acknowledgement of school mathematics as a practice, but does not allow for the consideration of how the forms of cultural communication vary from school to school and that students' home forms of cultural communication also differ. Vile acknowledges that

> Semiotic analysis is not confined to the mathematics aspects of the mathematics classroom. One of the key elements in semiotics is the experiences that are brought to each semiotic act. Students bring experiences from home, from the playground, from their encounters with television and so forth into the classroom. Those experiences will have an effect on the way in which they interpret and act with the signs all around them (even the mathematical ones as Walkerdine (1989) has shown). (Vile 1998:26)

However, this is not revealed in their sample analyses of student productions. Even Vile (1998) acknowledges that the sample analysis ignores key elements of the "diachronic nature of Becky's learning". Vile and Lerman (1996) recommend that research within this paradigm requires a concentration on

> The evaluation of unstructured interview[s] and mathematical writing as methods of data capture suitable for providing data for semiotic analysis. (Vile and Lerman, 1996: 4-401)

It is here that the backgrounding of the social is revealed since analysis of small sections of student productions and unstructured interviews do not necessarily facilitate the acquisition of knowledge of the cultural communication patterns that are represented in the locational and interactional features of a context, nor yet are they the most suitable ways to collect diachronic information about the development and perpetuation of such patterns at each site. When examining how students are apprenticed into a discourse one would ideally like to track the entire apprenticeship process. However, the cost of this both in terms of time and of money takes it beyond the scope of this and possibly most studies. The researcher has to make a decision about what "slice" of time is reasonable to show key aspects of this apprenticeship. Since my own study takes a sociological bias I wish to discuss how various aspects of the social impact on the forms of cultural communication privileged by the school and modulate the transmitter and acquirer texts. This may only serve to restrict one's focus to those years of apprenticeship that fall within the student's career at any particular school. However, this study was limited by the LPAS to the Standard 10/Grade 12 year. Although this may appear to be an arbitrary delineation of time, I outline in Chapter 4 how at one of the study schools (as is possibly the case in other schools) the social organisation of the Standard 10/Grade 12s has some differences with the rest of the school. For this reason, it is possible to argue that extending the study to include students' school leaving year with other years may have led to a less focussed understanding of the cultural context[3]. My own study sets out to assess the extent to which analysing students' productions over the period of a year will enable one to evaluate the forms of pedagogy into which students are inducted, and the degree to which students successfully master those forms of pedagogy.

2.1.5 The impact of social organisation on the forms of cultural communication

Daniels (1989, 1995) analyses student texts as instantiations of forms of cultural communication which are related to the specific social organisation of different schools. He criticises those forms of post-Vygotskian research that fail to acknowledge schools as "organised institutions" and do not acknowledge how this impacts on structuring and privileging different forms of cultural communication.

> Clearly schooling constitutes a form of collective social activity with specific forms of interpersonal communication. Furthermore within schools and between schools there are differences in the content, structure and function of interpersonal communication. However, a good deal of post-Vygotskian research conducted in the West has focussed exclusively on the effects of interaction at the interpersonal level, with insufficient attention paid to the interrelations between interpersonal and socio-cultural levels. Additionally and perhaps as a consequence of this, schooling is often thought of as a generic activity, as if it were a social institution, which is uniform in its psychological effects. (Daniels, 1995: 517)

Daniels (1989, 1995) discusses and analyses how differences in the structuring of schools are of "social, semiotic and psychological" significance and this can result in different emphases in genres being privileged at different schools. In order to analyse the differences in the structuring of schools and how this impacts on the differences in the privileged communication pattern and to students' different understanding of different discourse at different schools, Daniels (1989, 1995) draws on Basil Bernstein's model of cultural transmission. Daniels (1989) compares and analyses both locational features (in particular the wall displays of students' work) of two school and the key features of classroom interactions. This provides him with data on how the cultural patterns of communication are regulated, what Bernstein (1990, 1996) refers to as the regulative discourse, and how instruction is structured, what Bernstein refers to as the instructional discourse. Bernstein (1990, 1996) argues that the instructional discourse is always embedded in the regulative discourse. This acknowledgement of how social organisation is linked to the structuring of learning is one aspect of why Bernstein's model is useful in doing comparative studies of schools. It allows for the macro structuring of power and control relations in society to be linked (not necessarily directly) to micro interactions at a classroom level, and their resultant impact on the structuring of individual student productions and understanding of various discourses. Bernstein's model of the pedagogic device, which captures the analytic categories outlined above, is further elaborated in Chapter 3.

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[3] Apart from the possibility of the final year of schooling having a slightly different context to other years at any particular school, I also follow Domingos' (1989) view (discussed later in the chapter) that the communicative style privileged by individual teachers is not only impacted upon by the cultural context but is also productive of the communicative context. This makes it useful to focus on the student writing that is produced within the time that they are taught by any particular teacher. Students are often taught by a different teacher each year.

Daniels' model of analysis has provided a potential model for my study of how to link visual and other features of location with interviews and analyses of students' productions in ways which allow one to infer key features of the pedagogic practices privileged at each school.

2.2 Studies that draw on students' written productions in their comparisons of schools, classes or pedagogic modalities

2. 2.1 Linking pedagogic modality with effective learning

In a study which appears to be motivated to counter the British educational call to go back to basics, Boaler (1997, 1998a, 1998b) researches the effectiveness of different teaching methodologies in the mathematics classes at two working class schools in England. Three hundred students were followed over three years using classroom observations, student and teacher interviews, questionnaires to staff and students, analysing as range of student assessments. Boaler (1997) identifies and constructs two teaching styles, one of which dominates at each school: namely "progressive" (project work, mixed ability classes, group problem solving, exposition of new concepts only occurring when students can not solve a problem without new contents) and "traditional" (exposition by the teacher on the blackboard prior to student working through routine practice examples, teaching the content in unconnected small fragmented sections). She raises questions about each teaching style: Which is most effective in preparing students for the demands outside of the classroom? Which produces the best examination results? What impact does each have on the differential success of girls and boys? What is the impact of students learning in mixed ability or setted classes?

Boaler briefly mentions the broad differences in the types of students' productions at each school. Whilst she does describe fleeting conversations with students about their written productions, she does not use the opportunity to do in-depth analyses of their written productions. Boaler concludes that the students taught in the "progressive teaching style" do better in the final examination and are better able to use the mathematics that they learned outside of school than the students taught in the "traditional" approach. However, she does not relate the fact that at the school with the "progressive" teaching style, students spend much their final school year being prepared to answer examination questions (Boaler, 1997: 149) to the level of success that these students attained in the examination. Similarly she does not problematise students' responses about their ability to transfer mathematical skills to extra school activities: she does not consider the extent to which this may have formed a subtext of their mathematics learning. Boaler draws on Lave's (1988) concept of situated learning to argue that the reason that the students who learned within the "progressive" style are able to transfer these skills to out of school context is that their learning happened predominantly through projects incorporating "realistic" out of school contexts. Boaler (1997, 1998a, 1998b) appears not to have considered the fact that out of school contexts are always recontextualised within the context of school mathematical activity: the practice that students engage in remains school mathematics. She does not engage with the studies, for example Walkerdine (1988, 1994), that argue that different contexts such as a classroom or a shop provide students with different reservoirs of strategies for solving problems.

The categories "progressive" and "traditional" are loose categories that can each describe a range of potentially overlapping pedagogic modalities. Boaler also fails to integrate the descriptions of the social organisation with the school and the forms of pedagogy privileged at each school in an analytic way. She makes mention of Bernstein's (1990) categories of visible and invisible pedagogy, as well as his notions of classification and framing. However, these are not well integrated into her analyses of the two pedagogic modes, for example she neither uses the categories of classification and framing to describe the finer differences in pedagogic modality, nor uses the related concepts of message and voice to discuss the development of potential student identities. Nor does she, as Edwards (1998) argues, work with Bernstein's categories of recognition and realisation rules or in any other way discuss students path in developing criteria for the construction of legitimate texts. One of the advantages that a longitudinal study such as this offers is to track student's development over time; I suggest that there was the potential within Boaler's study to more closely track students apprenticeship into forms of pedagogy privileged in the mathematical classes at each of the schools. Nor does Boaler use the opportunity for doing in depth study of students written productions. Although both this and Boaler's study examine pedagogical forms at two working class schools in close proximity, Boaler's study was not found to resonate with the analytic form being developed for this dissertation.

2.2.2 Relating achievement, social class and pedagogic modalities

Domingos and others (Domingos, 1989; Morais & Antunes, 1994; Morais, Fontinhas & Neves, 1992) as part of the Project ESSA, Sociological Studies of the Classroom, attempt to analyse children's level of achievement in science classes from a sociological perspective. Each of these studies has a different focus.

Domingos (1989) relates teachers' differential pedagogic practices to both the context of their present and past teaching and to the nature and context of their teacher training. Domingos' (1989) description of context is limited to the class context (predominantly middle class or working class population) and to site (urban or rural), there is no attempt to describe the more subtle nor the interactive aspects of context. She concentrates only on macro-social divisions and ignores micro-social structuring of both the social division of labour and time, text and space. I argue that one may need to focus on both macro and micro structuring of the social, which needs to consider the forms in which time, text and space are specialised, in order to understand how different forms of pedagogy have the potential to induct students into the discourse of school mathematics.

The studies by Morais & Antunes (1994) and Morais, Fontinhas & Neves (1992) are conducted around the same teacher, who is asked to teach in three different teaching styles each with different strengths of classification and framing. Morais, Fontinhas & Neves (1992) investigated the effect of class and gender on students' ability to answer questions that required different levels of cognitive ability. This study assessed both students' display of the recognition or the realisation rules to produce texts, and related this to their class backgrounds. They found that working class and black children had more difficulty in answering cognitively demanding questions and but that within the pedagogic form where students were explicitly told the differences between cognitively demanding and cognitively undemanding tasks, they fared much better. The Morais, Fontinhas & Neves (1992) study differs from this study in that it does not analyse the production of student texts over time, and so is unable to postulate on how students develop the criteria for the production of the text.

Morais & Antunes (1994) built on the Morais, Fontinhas & Neves (1992) study by focusing on students ability to understand and display the socio-affective disposition required by different pedagogic modalities (classes which have difference in the social and moral regulation). They selected six students, whose performance they claim, differed from the "norm" for their social class: one child from a middle class background and one from a working class background from each of the three classes in the standard. They describe how differences in children's home background (stated mostly in terms of framing values) from the norm for that class lead to the children breaking the expected class based pattern of achievement at school. They argue that similarities and differences between the communicative contexts of students' homes and schools impact on their ability to develop the required social dispositions and the recognition and realisation rules at school. They state that although one can generalise about social groups orientations to communicative contexts, there are differences within these patterns. It appears that the importance of the context of the school asserted in the earlier study (Domingos, 1989) is not considered in the later studies (Morais & Antunes, 1994; Morais, Fontinhas & Neves, 1992). Neither of the later studies consider how students experience of the structuring of the school organisation and the communicative forms privileged at a school level may impact on the performances in classrooms which may be regulated in a way that contrasts with the dominant from of regulation at the school. For these reasons they were of limited value to this dissertation.

2.2.3 Linking social organisation or dominant form of community with the privileged form of pedagogy

Dowling and Brown (1997) examine three schools in Cape Town, South Africa, in different geographical areas. An ex-model C[4] school: that is ex-HOA, which they call Mont Clair, an ex-HOR school, which they call Protea, an ex-DET school, which they call Siyafunda. They argue that the racial differences in student composition at these schools, are indexes of class differences. They track a Std 7[5] class in each school for one day and interview various staff members. They discuss ways in which the dominant forms of pedagogy displayed at each school resonates with the dominant forms of community established at each school, which they in turn relate to dominant socio-economic class of the communities from which the majority of students in each school are drawn. Whilst they do make unresearched generalisations about the communities that each school serves, they also acknowledge that these communities are structurally divided and temporally fragmented. They also acknowledge that in all schools there was a range of pedagogic relations, but state that their analysis draws on the most commonly observed relations at each school. They suggest that this work is tentative and provide it as a springboard for further empirical data collection and theoretical work around how the relations established at school are intertwined with the form of pedagogy privileged at each site of learning.

From their work one can infer a notion of subjectivity as being constituted by practices and activities. This is elaborated elsewhere by Dowling

> the subject is inscribed within, constructed by social practices and relations. Therefore subjectivity is produced within pedagogic action. (Dowling 1987:87)

Dowling and Brown (1997) draw on Durkheim in their categorisation of the dominant tendencies in the social relations at each school. They describe Mont Clair as displaying a "virtual" community in which the social relations have the greatest tendency amongst the three schools towards a complex division of labour, in which the "cohesion of the social is affected by interdependence within differentiation"(Dowling and Brown, 1997:12). Mont Clair most strongly represents a form of organic solidarity. The strong communal relations at Siyafunda lead them to describe this school as displaying a strong form of mechanical solidarity. The social relations at Protea they describe as falling between the stark forms of organic and mechanical solidarity displayed at Mont Clair and Siyafunda. Dowling and Brown use the categories vertical (hierarchical) and horizontal (non-hierarchical) in an attempt to

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[4] Under the 1980's Tricameral Legislation, the House of Assembly (HOA) dealt with affairs of "white" citizens, the House of Representatives (HOR) with the affairs of "coloured" citizens The majority of Africans were denied citizenship of South Africa, and instead required to take up citizenship in a Bantustan. Schools for those Africans outside of the Bantustans were administered by the Department of Education and Training (DET). In the early 1990's many schools previously administered by the House of Assembly chose to change their form of governance to a model named "model C".

[5] At the time that Brown and Dowling gathered the information for this piece of research, Grade 9 classes were still referred to as Standard 7 classes.

analyse the relations between transmitters and acquirers, including the production of transmission and acquisition texts. However, probably because of the rapid way in which information was gathered and the speed with which the data were constructed, they do not apply these categories in ways which differentiate the data, (the relations of transmission at all three schools are described as vertical and the implication is that at all three schools the relations of acquisition are horizontal) but they rather fall back onto descriptive categories of the degree of communality developed in the classrooms.

Whilst the Brown and Dowling descriptions do show ways in which further research (including my own study) may look for links between the dominant social class of the students at a school, the form of social organisation privileged at that school and the forms of pedagogy privileged in the majority of classes at the school, its level of analysis remains unsophisticated. In order to be more valuable, studies of this nature will need to make stronger analytic links between the forms of social organisation and the forms of pedagogy, rather than to rely on generalised descriptions. For example Basil Bernstein's (1990, 1996) categories of classification and framing and his notion of how the instructional discourse is embedded in the regulative discourse may be more helpful in categorising the different mechanisms at work in the relations of transmission and acquisition. These analytic categories of Basil Bernstein are further elaborated in Chapter 3, and used as tools of analysis in Chapter 4.

It may also be more useful to consider more carefully the dominant ways in which transmission and acquisition texts are developed over time, rather than to describe the patterns observed on a single day, since it is only over time that one can clearly establish how transmitters and acquirers develop their understanding of how to construct the criteria for the production of legitimate text. Observing lessons over a long period of time is time consuming and costly, but analysis of students' written productions may provide an alternative source of the development of text over time.

2.3 Conclusion

This engagement with the literature about students' written productions in mathematics classes has revealed a number of key weaknesses in the field, namely that much of research does not consider there to be any value in analysing students' records of teacher's exposition and worked examples nor of students' records of their calculation of routine problems. There is also little research that analyses patterns in students' written productions over time in order to evaluate how they develop criteria for the production of legitimate text. Further, little of the research links how the structuring of the social organisation of the school relates to the predominant patterns in the structuring of the students' written productions. In the next chapter I outline how Basil Bernstein's (1990, 1996) theory of the pedagogic device allows me to link the social organisation of the school with the structuring of the students' written productions and how these features can be read in conjunction to give clues to some of the key features of the form of pedagogy privileged at each school.

Chapter 3: Theory and methodology

In this chapter I give an account of the lens through which I have read the student productions and that I have used to produce the data for analysis and discussion. This lens has drawn substantially on Basil Bernstein's sociological analysis of pedagogy.

In the second part of this chapter I describe the methodology I employed in setting up my analysis.

This study aims at using the analytic tools developed by Basil Bernstein to discuss the student productions as instantiations of cultural communication. It is asserted that the student productions give insights into the differing pedagogic practices at each school and also into the differential success rate of student's construction of the criteria for the production of the privileged text. It is argued that the pedagogic practices privileged at different schools or classrooms provide a different potential to induct students into school discourses. It is this potential to induct students into a particular discourse that I construct as an indicator of the quality of education.

I begin with an overview of Bernstein's work and a description of the notion of the pedagogic device and I then focus, in particular, on his construction of the notion of pedagogic discourse.

3.1 Bernstein's theory of cultural transmission and pedagogy

Bernstein states that his concern is with "theory about symbolic control and change, which in turn is part of a theory of culture and knowledges [...]" (1998: 29). He adds that his work is about

> understanding and describing the agencies, contexts and practices through which we are both constructed and constructing ourselves and others. This involves understanding how power and control enter into these constructions to include or exclude, to privilege or marginalise [....] The theory attempts to show both the limiting power of forms of regulation and their possibilities so that we are better able to choose the forms we create rather than the forms be created for us. (Bernstein and Solomon 1998: 25)

Basil Bernstein attempts to show, at a theoretical level, the relationship between a particular symbolic order and forms of cultural communication, which are always about the selective organisation, transmission and evaluation of knowledge.

> How a society selects, classifies, distributes, transmits and evaluates the educational knowledge that it considers to be public, reflects both the distribution of power and the principles of control. (Bernstein, 1975: 85)

Bernstein addresses three interrelated problems

> First, how does a dominating distribution of power and principles of control generate, distribute, reproduce and legitimise dominating and dominated principles of communication?
> Second, how does such a distribution of principles of communication regulate relations within and between social groups?
> Third, how do these principles of communication produce a distribution of forms of pedagogic consciousness? (Bernstein, 1996: 18)

For Bernstein formal school education is only one form of pedagogy, but as such it represents instantiations of forms of cultural communication, in which issues of power and social control are manifest.

Bernstein's theory is a useful tool to construct and analyse that data from the students' notebooks, since it allows me, on the one hand, to relate patterns in their productions, to patterns in the form of pedagogy at each school and also to the students' degree of apprenticeship into the discourse of school mathematics, which is how I have defined quality in education. On the other, it allows me to relate the pedagogic patterns privileged at each school to the dominant forms of pedagogy and cultural communication privileged in society and hence to the potential success rate of cohorts of students at each school.

A key issue in Bernstein's work is that he aims to be able to show how interactions on a micro level carry relations of power and control that can be identified at a macro level. He develops a grammar[1] for discussing micro interactions, for example pedagogic practice, in a way which is able to capture the macro relations of power and symbolic control in society and in so doing relate various pedagogic modalities with the development of consciousness. Bernstein is critical of many sociological analyses in education, since he feels that they are only concerned with the students' position in relation to the privileged text. Much discussion in the sociology of education focuses only on the subject's relations to the privileging text. It describes how class, race, gender and age position the subject differentially with regard to a privileging text, and so distribute potential access to it differentially. The distance between one's local pedagogic practice (for example, pedagogic practice within the family, peer group, community) and the official pedagogic practice for example schooling impacts on the ease with which one is able to appropriate the privileging text. It is potentially easier for those who have received in the

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[1] Grammar here, following Bernstein, is used metaphorically. Just as in language grammar provides the rules for the permitted combination of words, so the "grammar" for the structuring of society provides rules of the permitted combination of texts.

home a version of the official pedagogic practice to access the privileging text. However, for Bernstein the differential distance between a student's local pedagogic practice and the pedagogic practice privileged within institutions, for example in a school, is only part of how institutions distribute subjects differentially. Bernstein (1990, 1996) criticises approaches that do not pay attention to the rules of the construction of pedagogic discourse. For Bernstein what is missing and the gap he attempts to describe is about the **relations within** the privileging text

> 'Relations within' refer to the rules whereby the 'privileging text' has been internally constructed.... The rules whereby the text has been constituted, which makes the text as it is, which gives it its distinctive features, its distinctive relations, its mode of transmission and contextualisation. (1990: 176)

It is important to "focus on the underlying rules shaping the constructions of pedagogic discourse and its various practices" (Bernstein, 1996:17). These rules constitute what Bernstein refers to as the pedagogic device.

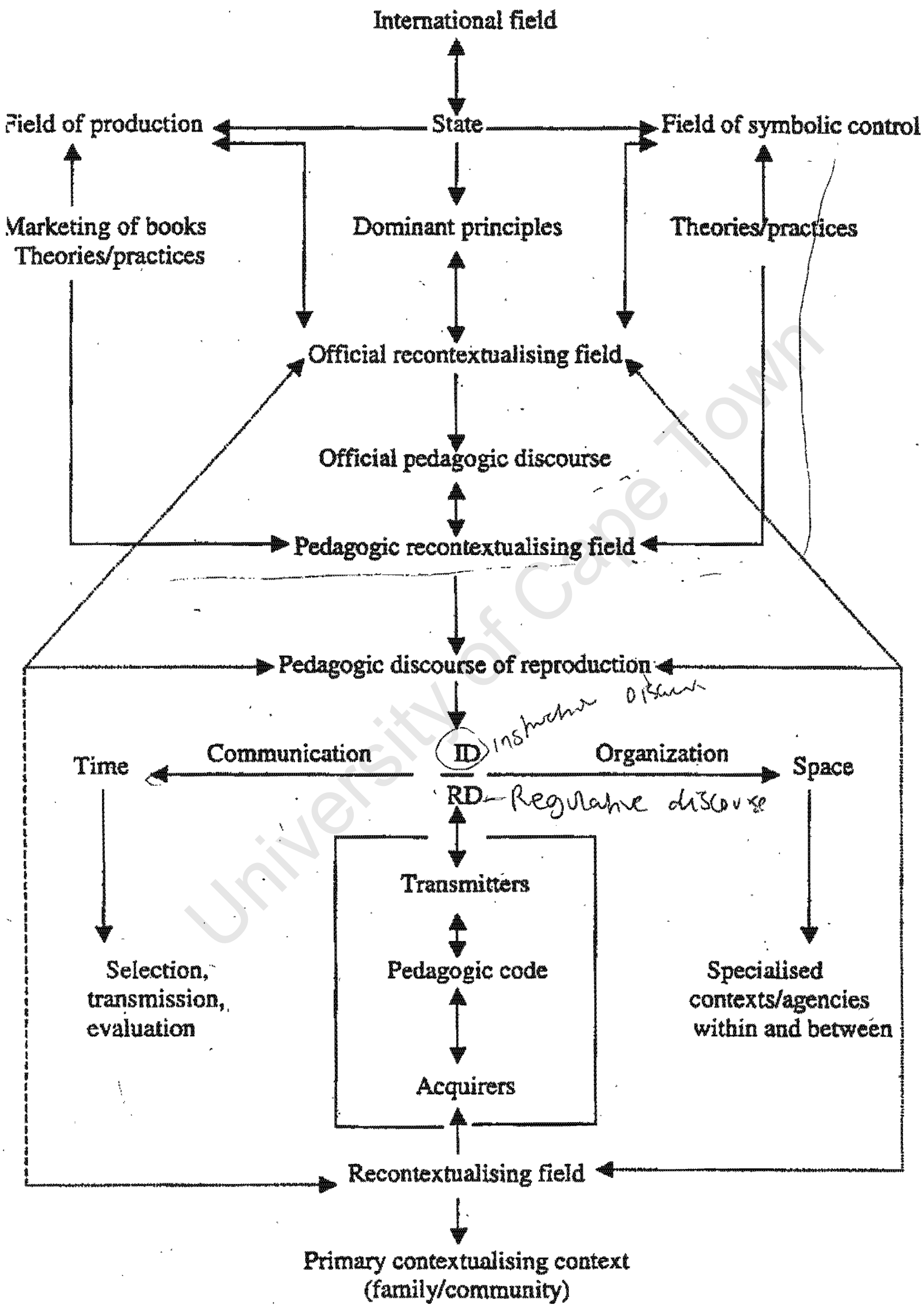
> [T]he pedagogic device is ... the symbolic ruler of consciousness in its selective creation, positioning, and oppositioning of pedagogic subjects. It is the condition for the production, reproduction and transformation of culture. (Bernstein,1990: 189)
>
> The pedagogic device makes possible the transformation of power (that is, its basis in social relations and their generating sites) into differently specialised consciousness (subjects) through the device's regulation and distribution of knowledges and of the discourse such knowledges presuppose. (op.cit. 1990: 210)

The formal education system is only one institution through which the pedagogic device operates. This in turn means that the education system is only one institution through which culture is reproduced by the processes of specialising knowledge and consciousness, but that social power and control operate throughout this process and that forms of communication are central to this process.

Bernstein (1990: 197) presents a model of how the pedagogic device operates through the formal education system. The model summarises how the macro-structuring of society impacts on the micro-structuring of pedagogic institutions, including their forms of interactions and underlying rules of cultural communication (see Figure 3.1 on p38).

Figure 3.1 from Bernstein (1990: 107)

The Social Construction of Discourse



3.2 Bernstein's rules of the pedagogic device

Bernstein describes the pedagogic device as the symbolic ruler of consciousness. He defines it as the "distributive, recontextualising and evaluative rules for specialising forms of consciousness" (Bernstein, 1990:181). Bernstein describes the pedagogic device as consisting of three hierarchically organised rules, which constitute its internal grammar.

> First, the function of the distributive rule is to regulate the relationships between power, social groups, forms of consciousness and practice. Distributive rules specialise forms of knowledge, forms of consciousness and forms of practice to social groups. Distributive rules distribute forms of consciousness through distributing different forms of knowledge.
> Second, recontextualising rules regulate the formation of specific pedagogic discourse.
> Third, evaluative rules constitute any pedagogic practice. Any specific pedagogic practice is there for one purpose: to transmit criteria. (1996: 42 - 43)

3.2.1 Pedagogic Discourse

Bernstein describes pedagogic discourse as

> a principle for appropriating other discourses and bringing them into a special relation with each other for the purposes of selective transmission and acquisition [....] It is a recontextualising principle which selectively appropriates, relocates, refocuses and relates other discourses to constitute its own order and ordering. (1990: 183 - 184)

According to Bernstein the dominant principles of society have an impact on what is available for selection and recontextualisation. This, however, does not happen in any straightforward or deterministic way. Bernstein describes how both the official recontextualising field (ORF), which is largely the state educational bureaucracy, and the pedagogic recontextualising field (PRF), which includes academics, teacher organisations, educational media, teachers, have an influence both on what knowledge is recontextualised and how it is recontextualised. The balance of power between the ORF and PRF will change from time to time and society to society, and this is likely to have some effect on the form in which the recontextualising rules are realised.

Bernstein (1990) describes the function of the recontextualising rules in the following way:

> The recontextualising rules regulate not only selection, sequence, pace and relations with other subjects but also the theory of instruction from which the transmission rules are derived. (op.cit.: 185).

He adds that the

> The theory of instruction is a crucial recontextualized discourse, as it regulates the orderings of pedagogic practice, constructs the model of the pedagogic subjects (the acquirer), the model of the transmitter, the model of the pedagogic context, and the model of the communicative pedagogic competence (op.cit.: 189).

This recontextualising principle, which constitutes pedagogic discourse has two parts, which are always intertwined: the regulative discourse (RD), which is about social order, and the instructional discourse, which is about skills or competences. The regulative discourse is the dominant discourse and sets the parameters of possibility on the instructional discourse (ID).

> Instructional discourse regulates the rules which constitute the legitimate variety, internal and relational features of specialized competencies. This discourse is embedded in a regulative discourse, the rules of which regulate what counts as legitimate order between and within transmitters, acquirers, competencies and contexts. (op. cit.: 188)
>
> the recontextualising principle which selectively creates an instructional discourse embedded in a regulative discourse produces a specialisation of time, text (or its metaphoric equivalent), and space and the conditions for their interrelation. (op. cit.:185)

However, despite the fact that the dominant social regulations set the parameters for the forms of communication practices that are reproduced in institutions, this does not happen in an unmediated way. Local regulations and practices are able to transect the dominant social regulations and become part of the local process of recontextualisation. This impact of the local on the dominant social relations is part of how differentiation and specialisation operates.

> [A] given ID/RD creates specific communicative practices (time) and organisational practices (space) to constitute the code to be acquired.... However, what is acquired in schools may itself be subject to recontextualising principles arising out of the specific context of a given school and the effectiveness of external control over the reproduction of official pedagogic discourse. Further, what is reproduced may be affected by the power relationships of the recontextualizing field between the school and the primary cultural context of the acquirer (family/community/peer relations). (op.cit.: 199)

In this thesis I attempt to infer from the differences in student's productions and the locational features of the school, the differences in pedagogic practice at each school. This I intend to do by attempting to interpret the differences in terms of the instructional and regulative discourses at the two schools. This in turn, will assist in the generation of a description of the hypothesised privileged pedagogic practice at each school; I will attempt to use this as a backdrop against which to comment on students' level of induction into mathematical discourse.

3.2.2 Pedagogic Practice

For Bernstein (1996: 17 –18) pedagogic practice is a fundamental social context through which cultural production – reproduction takes place, and forms of communication are central in this process. Bernstein (1990: 189) identifies two modalities of pedagogic practice, one that privileges "graded performances of the pedagogic discourse" and another that privileges "shared competence of the acquirer". He draws on Durkheim's categorisation of the intensity of the division of labour in society to elaborate this.

> If the agency is producing a range of shared competences, then the social relations created are of 'similar to' [...] relations of relatively low level of specialisation, creating a simple division of labour, celebrating, and controlled by, mechanical solidarity [...]
> Where the output of a pedagogic agency is specialized performances. Such a pedagogic agency is concerned to bring about specialised difference between individuals: differences in their performances. Such an output points to gradings, not only within but also between specialisms. Such an agency is [...] designed [...] to produce [...] different from relations, and points to a complex division of labour, that is organic solidarity. (op. cit.: 207 - 208)

All contemporary schools are modern institutions, and as such if one strictly applies Durkheim's categories, they all display a relatively complex division of labour. However, Bernstein uses these terms metaphorically to describe comparative degrees of specialisation within and between different institutions. The function of modern schooling and of examinations, including the final matriculation examination is to distribute students according to results. Any form of pedagogic discourse, which has a strong tendency towards mechanical solidarity, may miss opportunities to prepare students for the final examination. Students' success in the final examination is implicated in one of the dominant social ways in which quality of learning is evaluated.

As mentioned above Bernstein (op.cit.: 186) sees evaluation as the key to pedagogic practice. For him the purpose of evaluation is to assist the student in developing the criteria for the construction of the privileged pedagogic text. However, he adds that

> pedagogic practice does not necessarily reproduce pedagogic discourse and what is acquired is not necessarily what is transmitted. (op.cit.:187)

The distance between the privileged pedagogic discourse and that relayed in any school will impact on student success rates, and on the way in which society evaluates the quality of educational provision at that school. However, I have suggested that the quality of learning is better defined as the extent to which students are able to construct the criteria for the reproduction of the privileging text as this is an index of the extent to which they have been inducted into any discourse.

3.3 Classification and Framing

Bernstein introduces two further analytic categories which allow for descriptions of the differences in pedagogic modalities. The concepts of classification and framing allow for the analysis of the way in which power and control are manifest in pedagogic contexts.

Classification is about the strength of the boundary or the degree of insulation between categories (Bernstein, 1975, 1990, 1996) such as school and community; different subjects; genders and so on.

These differences in boundary strength vary across time and space or context. Since category distinctions are about the social division of labour, classification is a manifestation of power relations. Where classification values are strong, there are higher degrees of specialisation; where classification values are weak, one finds greater tendencies towards the integration of discourses or practices.

> Framing refers to the principle regulating the communicative practices of the social relations within the reproduction of the discursive resources, that is between transmitters and acquirers. (Bernstein, 1990: 36)

Framing is then about the locus of control: if the locus of control lies predominantly within the transmitter, the framing is said to be strong; if the locus of control lies within the acquirer, the framing is said to be weak. As Bernstein (ibid.) suggests, these are differences in the form of control, that is the differences are often only apparent differences rather than real differences in degree of control.

To refine the analytic categories, Bernstein distinguishes between internal classification (C^i) and external classification (C^e) and also between internal framing (F^i) and external framing (F^e). For example, external can come to stand for the classification and framing between a school and the community, and internal can come to stand for classification and framing within a school or classroom or other equivalent communicative context.

> If we consider a school where the F^e is strong, then the transmitter regulates what features of non-school communication and practice may be realised within the school's pedagogic context [....]
>
> In a classroom, for example, the locational position of the pupils, teacher, desks, cupboard, wall ordering are a feature of the internal classification (C^i) together with the distribution of task among pupils [....] (Bernstein, 1990: 37)

Just as power and control can be distinguished as separate analytic categories, but are empirically embedded, so too classification and framing are separate analytic categories, but they function in an interrelated way. This does not however, mean that internal and external values of classification and framing cannot vary independently of each other. As with all of Bernstein's concepts these are not absolute descriptive categories but are at their most useful when used to compare social institutions: the categories are relative, which is why they are so useful to analyse the different pedagogic modality at School A and School B in this study, as presented in Chapter 4.

In summary then,

> curriculum defines what counts as valid knowledge, pedagogy defines what counts as valid transmission of knowledge, evaluation defines what counts as a valid realisation of this knowledge on the part of the taught. (Bernstein, 1975: 85)

Bernstein's model allows one to analyse the extent to which student's productions display a fluency with the recognition and realisation rules of the privileged pedagogic discourse. Bernstein (1996) writes that pedagogic practice exists to transmit criteria for the production of texts and that it is evaluation which guides students' construction of the criteria for the production of texts. Within the context of pedagogic practice evaluation condenses the pedagogic device. This allows one to look for indicators of evaluation as pointers to the modality of pedagogic practice that is privileged. Bernstein's model of the pedagogic device allows me to infer, from the differences in the student productions, what the differing rules of cultural communication in the matric mathematical classes at the two school are - to read these rules of cultural communication as an index of the differing micro-interactions and the classroom based structuring of the social division of labour, without losing the broader picture of how these are related to the macro-structuring of society.

3. 4 Methodology

In the initial stages of the LPAS research, while teachers' confidence was still being won, teachers were not keen for researchers to observe lessons. For this reason it was decided to use student notebooks as a secondary record of cultural communication from which to make inferences about pedagogic practices.

3.4.1 Study population

As mentioned in Chapter 1, the two schools were pre-selected as the study schools by the LPAS project. The schools were chosen because although they have many similarities (they were both previously administered by the Department of Education and Training; they are in close proximity to each other; their student populations, for the most part, speak the same home language (Xhosa), and the same medium of instruction, English, is used at both schools; and the students at both schools are predominantly working class), the success rates of students in their final school leaving examination is very different[2]. This allows one to investigate various micro-structuring differences in the schools, while the most of the important macro-structural issues that regulate them are constant. At the time that this study was initiated, the LPAS project was focussing on Standard 10/Grade 12 students. For this reason I have analysed the student productions captured in the notebooks of the Standard 10/Grade 12 classes at the two school to assess to what extent it is possible to ascertain issues around the differences in pedagogic modalities at the two schools.

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[2] A more in depth discussion of the two schools is presented in Chapter 4, since it forms part of the analysis of the impact of the contextual features on the communicative features, of which notebooks are one form.

3.4.2 Case Studies

This study focuses on eight student notebooks produced in Standard 10 / Grade 12 mathematics classes at two schools. The aim is both to investigate whether student productions can reveal anything useful about the forms of pedagogy practiced at those sites and from this to assess whether this provides a basis also to be able to judge the quality of learning at those sites. Since the scope of the study is limited to two sites, this study may be viewed as a case study. Brown and Dowling (1998: 30) write that "all research is case study research insofar as it makes claims about one or more specific cases of or in relation to a broader field of instances and phenomena". They do, however, criticise research that constructs case studies as detailing the nature of a single object under natural conditions. Their concerns are both that all objects occur within particular contexts (the extent to which these constitute the objects needs to be acknowledged) and that any description acts selectively and productively on the object.

This study takes a sociological bias in its analysis of the student productions, in ways which attempt both to acknowledge the impact of context and the bias imposed by the theoretical model that informed the construction of the data. The extent to which this study is generalisable would rely, in part, on the extent to which further research focused on how the differences in contexts would impact on other students' productions. In the section 3.4.4 I describe the process of developing the linkages between information and theory in the production of data for this study.

3.4.3 Collecting and selecting the notebooks

I arranged with the matric mathematics teachers at each of the school that I needed a selection of matric notebooks that would include both male and female student achievers and non-achievers, across all of their matric classes. I obtained class lists which included student details (gender, family socio-economic status[3], results from the final exam in Std 9/Grade 11) from colleagues on the LPAS. My intention was to use the information from the LPAS schedules to make further selections, so that I would be able to analyse the notebooks of a male and female achiever and non-achiever at each school. It was predicted that each student's productions would represent a refracted image of the patterns of cultural communication in their class. Notes have the potential to appear to be a highly idiosyncratic representation of communication, and in a similar way different students may appear to have highly idiosyncratic ways of representing their ability to calculate or solve mathematical problems. The motivation for choosing students who were differently distributed on the continuum of success and with gender differences, was both to analyse if there were any patterns in the way students' positioning impacted on their productions as refracted the patterns of cultural communication, and to assess the notebooks for similarities in patterning within and across sites. Another reason for choosing differently positioned students was to generate a single multi-dimensional picture of each school's Standard 10/ Grade 12 mathematics lessons, in a similar way to which a stereoscope assists in interpreting a three dimensional image from more than one two-dimensional photograph.

The collection, selection and copying of notebooks proved more complicated than I had imagined. The fluidity of the school day, meant that often teachers were not available at the scheduled times of meeting. There is substantial pressure on matric students and their teachers. Teachers were concerned that students should not be without their books for longer than one afternoon. This meant that teachers did not select or collect books, nor even discuss with students, that I had requested to borrow and copy a selection of their notebooks, prior to my visits. It also meant that at School A, the teacher simply selected books from one class of students. The books that he selected did not cover a full range of student results but consisted of one high achieving student and three whose results were close to the average marks of the standard. These students are labeled A1 (the top student), A2, A3 & A4 (the other students with the middle range results). At School B the teacher collected four books from each of the four matric classes. This meant that I could select four from the sixteen notebooks, to ensure a greater range of results and take sample books from both high and low achieving male and female students. The students are labeled B1 – B4 in order of their examination results (the students B1 and B2 were the top achievers, student B3 results were at the lower end of the range and student B4 achieved the lowest marks of all the matrics). Student B2 and student B3 were in the same class.

Because of the pressure of time on the copying of the notebooks, I did not spend sufficient time considering the effect of comparing notebooks of students from only one class (the top stream) at one school with notebooks from students of different classes at another school. In Chapter 4, I shall show that this was not as problematic as I initially assumed, because of the different regulative discourses operating at each of the schools.

3.4.4 School visits: observations and interviews

I visited each school on three occasions to talk with the mathematics teachers and to collect student notebooks. I took notes about the location and visible features of each schools organisation. Bernstein discusses ways in which context both constitutes and is constituted by pedagogic practice. Locational features and institutional patterns of interaction and communication are key aspects of context. As Daniels (1989: 124) writes,

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[3] The parents' employment status and forms of work were used as an index of socio-economic status.

> [i]n different schools (or cultures) actions and objects signify different meanings. Indeed at a general level it is possible to conceive as cultures or schools as worlds of signs and signs about signs (Hawkes 1977) .... That which is taken to signify competence in one culture may signify incompetence in another or irrelevance in a third.

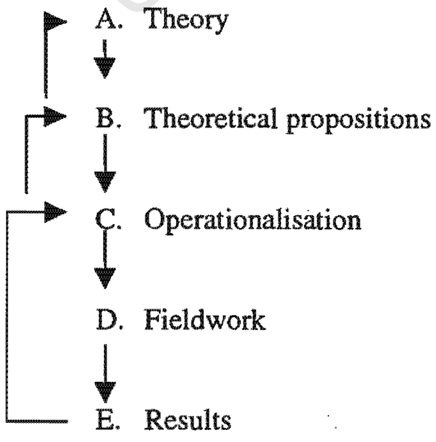
The visits to the schools provided opportunities to be able to understand features of the regulative discourse operating at each site. Interviews with the Standard 10 / Grade 12 mathematics teacher at each school, provided further information about aspects of both the instructional and regulative discourses (see Appendix 1, pxx, for the interview schedule).

3.4.5 Learner Progress and Achievement Study research

Other sources of information and data were the research reports and data collected by the LPAS as well as discussion with staff of this study programme. This information was particularly useful in confirming and extending information gathered about the locational features of the schools, the student populations and the regulative discourses operating at the schools.

3.4.6 The process of producing data: linkages between information and theory

In attempting to answer, from a sociological perspective, the question: What, if anything, can student productions tell one about pedagogic modalities and the extent to which students are inducted into the discourse of schooling, I was aware that the sociological theories of Paul Dowling and Basil Bernstein would provide useful analytical tools. However, at each stage of the analysis the patterns that were revealed in the construction of data had an influence on the aspects and ways in which the theory was used to generate data. This is a form of the research process is schematically represented by Rose (1992: 14) in the following way:



Similarly Dowling (1998) and Brown and Dowling (1998) suggest that the production and analysis of data consists of continual movement, through the processes of induction and deduction between theory and the established body of literature on the one hand, and the information on the empirical setting on the other.

3.4.7 Initial observation of patterns

I initially analysed one notebook of a high achieving student from each of the schools. I read through the notebooks to look for any textual patterns, any textual structuring which might give clues / access towards the forms of cultural communication practiced and privileged in each classroom. I produced a set of low level descriptors of both the form and the content of what was recorded on each page, for example how each section of work was separated out or marked out from other sections of work (date, heading, numbering of exercises), the sequencing of mathematical topics and sub-topics, the types of algorithms used to solve problems, the recording of laws, definitions, formulae and other forms of notes. This gave some indications of similarities and differences in the structuring of the teaching and learning. Even at this stage it became clear that at the one school students were taking down more detailed explanatory notes about each section of work while at the other school there was a greater focus on mathematical practice exercises.

3.4.7 Patterns in student productions within each school

I then wrote similar notes on the categorisation of each of the four student's productions within each of the two schools, to assess whether the initial differences were idiosyncratic or systematic across each school. Drawing on Bernstein's (1990, 1996) work, I compared which mathematical topics were selected and how different mathematical topics, and sub-topics were sequenced and paced. In addition I examined how students set out and structured their notes and work. I compared how students marked out different sections of work, for example, I looked for whether students had all done the same pieces of work, on the same days, in the same order and whether exercises were set out in the same way. Once again the similarities of the student productions within each school were notable. Although the same work was often marked out as being done on different days.

I then spent more time reading about ways in which cultural communication and pedagogy are structured. Then I re–examined the notebooks again, focussing on each school separately. I examined both my own notes and the four notebooks from the one school. Then I repeated this with the notebooks from the other school. Here I looked again for patterns of similarities and differences. Since Bernstein refers to evaluation[4] as the basis of pedagogic practice and the key to students' construction of the legitimate text, I also noted the frequency of when the books were marked, and whether student errors had an impact on how exemplars, exposition and algorithms were presented to the class. This would have been reflected as a change in the form of the exemplars, exposition and algorithms in subsequent work. Notes were taken on how often the teacher signed work without correcting it (and how often this happened when student calculations were correct or incorrect), how often the teacher marked work as correct or incorrect (and how often this was an accurate reflection on the accuracy of the work), how often the students marked, redid, or wrote out "corrections" of their own work and whether this always happened on the same day and with the same examples. My interest was to be able to discuss how evaluation was related to different student's development of the recognition and realisation rules. This in turn, I intended to relate to both the degree of students' success and the quality of learning at each school. Although this was done quantitatively, it became apparent that one should not so much look at how often the books were signed or marked but rather at whose books were signed or marked and the particular times when books were most frequently marked or signed.

The patterns that became apparent in the notebooks within each of the schools, helped to focus both how I drew on the theory and what features to analyse for when comparing the notebooks across the two schools.

3.4.7.1: Prominent patterns of evaluation at School B

Students at School B started each section of work with a detailed narrative in a mixture of written language and mathematical symbols and diagrams. These notes were practically identical, despite the fact that the students were in different classes. At first I thought that the students were writing out the teacher's verbal explanations. However, I soon realised language was quite formal and had a tone similar to a textbook. I was able to examine the textbooks that the teacher at this school had mentioned using and found that these notes replicated the contents of particular pages of these textbooks. This is discussed further in Chapter 4. This very marked pattern in the student productions at School B, however, appeared to shift towards the end of each of the student productions. On re-examining this, it appeared that the shift occurred after the June examinations. The evaluative effect of the June examination on the students' construction of the text at School B is discussed in Chapter 5.

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[4] Evaluation can occur in a variety of forms including repetition, non- repetition, or other forms of silences, changes in tones of voice, changes in gesture, facial expression or other forms of body language (Bernstein, 1990). Since the focus of this study is on student productions, the only data available on evaluation was that reflected in the student notebooks. This is not in an attempt to undervalue their importance nor yet the role they would have played in the development of student consciousness, but simply because they were beyond to scope of this study.

3.4.7.2 Prominent patterns of evaluation at School A

At each stage of the analysis patterns and peculiarities were thrown up and I went back to the notebooks to check on these in detail. An example of this is the way the students were drawing trigonometric curves at school A. I initially noticed that many of these curves were not presented in the standard form. Then I did detailed analysis of the sequencing of each student's productions. This started to point to the fact that the minimum extent of evaluation of the students productions (both in terms of how often their work was marked and in terms of how even when their books were signed by the teacher, the misrepresentations were not corrected) had an influence on how students constructed the curves. A re-reading of Bernstein's notion of evaluation as condensing the meaning of the pedagogic device (1996: 50) helped me to shift my focus from the presences and absences of marking in isolation to examine this in conjunction both with the selection, sequencing and pacing of their work and with how the regulative discourse was impacting on the instructional discourse in that class.

Then I compared each of the student productions of trigonometric curves across the books to assess the extent to which features that appeared in individual notebooks were consistent across the notebooks. This pointed to the fact that the selection and privileging of material, methodology, and procedures, the pacing and sequencing of the work had steered students towards developing their own set of criteria as a basis for representations, which were at odds with the behaviour of each of the trigonometric functions. The effect of these aspects of evaluation, which go beyond correcting written texts, on students' construction of the criteria for the production of texts and how this relates to Bernstein's (1990, 1996) theory of the pedagogic device is discussed in Chapter 5.

3.4.7.3 Gender issues

Another feature was that at both schools the top male students tended to write out notes, definitions and mathematical exercises in considerably less detail than the other students did. They were also less inclined to mark out different pieces of work by noting the date or giving the piece of work a title. They were also less inclined to copy down the initial question statement, but rather tended to merely note whatever part of the calculation they felt useful toward the solution. The sample of books was, however, too small to analyse gender differences thoroughly.

3.4.7.4 Re(de)fining the focus

The initial intention of this study was to examine the notebooks for clues to the individual student's construction of recognition and realisation rules. However, as the analysis progressed it became clear that analysing student's written productions alone did not lend itself to a strong focus on recognition and realisation rules. Other studies that have focused on recognition and realisation rules (for example Daniels (1989) and Morais, Fontinhas & Neves (1992)) combined an analysis of students' written productions with student interviews which helped to tease out the differences between recognition and realisation rules. It became clear that the sources of information and approach that I was using lent itself more to an analysis of the impact of the regulative discourse on the instructional discourse which would include an analysis of the pedagogic modalities by considering issues of classification and framing; the specialisation of time, text and space; the selection, sequencing and pacing of work and students' construction of the criteria for the production of text.

3.4.8 Comparisons of the student productions at the two schools

The notes on the similarities between student productions within each school were used as a basis for comparing the student productions across each school. Notebooks from the two students who took the most detailed notes were used to do the comparative analysis (these were not the same two notebooks that were used in the initial comparison). Comparisons were made in several ways.

First the selection and sequencing of mathematical topics and subtopics were noted. Then the amount of time (in days) and space (in terms of pages of their notebooks and numbers of exercises) were used for different forms of work: the frequency of expository notes and exemplars[5], routine practice examples and were noted.

In Chapter 4, I focus on both the contextual differences at the two schools and textual differences in the student productions at the two schools. The dominant regulative discourse at each school is constructed from information in the Learner Progress and Achievement reports, from interviews with teachers and observations of the schools. This allows me to discuss how the differing regulative discourse at the school resonates with the differing instructional discourses at each of the schools. The different pedagogic modalities are described using Bernstein's concepts of classification and framing. Bernstein's model also allows me to compare how the different regulative discourses at the schools are differentially positioned

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[5] As mentioned in Chapter 2, unlike Shield and Galbraith (1998), I have grouped exposition and exemplars as a single category. This is because the teachers often give the students exemplars as a means of explaining a topic to them. One assumes that at times students do not have sufficient mathematical language to allow for exposition to proceed in terms of universal statements, this means that exposition can only be realised in particular exemplars.

with regard to the socially dominant regulative discourse and similarly how the instructional discourses at each of the schools are differently positioned in relation to what is evaluated in the final examination and what is privileged at tertiary institutions.

Bernstein writes that evaluation is the key to pedagogic practice. In Chapter 5, I focus on examples of the evaluative rules at each of the school, since this related to issues around pedagogic practice. First I compare the extent to which evaluation is reflected in the student productions at each school. I compare the forms and frequency of the evaluation that is visible in the student notebooks. Then I discuss how this related to pedagogical differences at the two schools. Then I hypothesize about how key features and changes in features in the students' text related to potential evaluation. I focus in particular on an example from each school. I suggest that changes in the patterns of student productions at school B can be related to the evaluative function of the June examination at School B, which results in the students beginning to develop new rules for the construction of texts. I discuss how the evaluative function of the exam weakens the form of both the regulative discourse and the related instructional discourse, resulting in a changed form of pedagogy at School B. Then I examine how, at School A, forms of evaluation are related to the way that student construct the criteria for the construction of trigonometric curves.

Chapter 4. Comparing the student productions at the two schools

The intention of this analysis is to generate a description of student notebooks at the two schools, which can provide a secondary mirror off which to read the pedagogic practices of each school. The aim is to relate the pedagogic practices at each school to both the forms of social organisation and to the differing levels of success in public examinations, which is an important way in which society evaluates the quality of educational provision at schools.

In order to work towards and answer to the question: "What, if anything, can student productions tell us about the quality of learning?", this chapter attempts to answer the following sub-questions begged by the former question:

- Are there similarities between the student productions within each school? If so, what, if anything, can be inferred from these patterns of similarities about the form of pedagogy privileged within the mathematics classes within each school?
- Are there differences between the student productions at each school? If so, what, if anything, can be inferred from these patterns of differences about the form of pedagogy privileged at each school? How, if at all, can these differences be related to the different levels of achievement of students from the two schools in the final matriculation examination?
- To what extent do the patterns in the students mathematics notebooks resonate with information gleaned, from other sources, about the form of pedagogy privileged in the Standard 10/Grade 12 classes at each school?

This study focuses on student productions in their notebooks and since no class observations were carried out, there is no direct focus on the interactional. However, the schools were visited to collect the notebooks and interview the teachers. On these occasions locational features of the schools and classrooms were noted, and through the interviews, issues around the privileged forms of interactions at each school were also noted. Further information about the student population and contextual features of the schools were obtained from the Learner Progress and Achievement Reports and project staff. Brief descriptions of each school provide clues to the context of each site of pedagogic practice, and provide clues to their dominant regulative discourses. The descriptions are further visual evidence of the selection and combination of cultural elements that is privileged and can be used as indicators of pedagogic modalities. These different approaches are differentially distant from the privileged form of education in society and come to position the school differently not only in terms of the level of success in the students results in public examinations but also in terms of the quality of educational provision.

> Differences in the structure of pedagogic practices constitute differences in contexts, which are of semiotic significance. Bernstein both theorises the semiotics of the transmission and provides a language with which differences in structure can be brought to the focus of empirical studies of individual acquisition. (Daniels, 1989: 530)

Bernstein (1990, 1996) argues that pedagogic discourse specialises text, time and space. Both within schools and across schools the specialising of text, time and space differs. This is not idiosyncratic but related to and (re)productive of contextual differences.

4.1. The two schools

Both schools are located in Khayelitsha on the Cape Flats. The official medium of instruction at both schools is English. Both schools previously were under the administration of the Department of Education and Training. School A is positioned within a formal housing settlement. School B is located closer to area of shack dwellings, in an area, which can be described as formalising. One of the main differences between the two schools is the success rates of students in their final school leaving examination. For example, in the 1997 matriculation examinations School A had an 86 % pass rate and School B had a 27 % pass rate.

> The national 1997 matriculation pass rate was 47,1 % and the Western Cape pass rate was 76,3 %. Thus the pass rate at School B was well below both the national and the provincial pass rates, and conversely the pass rates at School A was well above these rates. (LPAS Report 1, 1998: 28)

Students' mathematics results were the weakest of all the student results in this examination at both schools: with 49 % of students passing mathematics at School A and only 24 % passing mathematics at School B.

4.1.1 The student population: a function of the recruitment and enrolment procedures

School A is regarded as a prestigious school in the area. This is largely based on successful results of its matric class over a number of years. The community served by the school view it as offering a high quality of education based on the good results achieved by the Standard 10/ Grade 12 students annually. This view of quality education being housed in institutions where students achieve well in public examinations is widely held.

Since the community tends to perceive of School A as a better quality school, it is has more students applying to enroll and is thus able to be more selective about its intake of student population. In 1997, there were approximately 300 more students at School A than at School B. At the end of 1997 School A had about 4 500 applicants of which 400 could be accommodated. Much of the literature around school choice assumes that parents choose freely which schools to send their children to. Hoadley (1999) discusses how this is not the case in many South African contexts. On the one hand she outlines ways in which schools select students as much as students select schools, and on the other hand outlines the extent to which students are constrained in their choices. Hoadley reports (1999: 15) that

> [t]here are strategies at School A to draw high achievers from the surrounding primary schools. Ten application forms for School A are sent to principals of the primary schools to be given to the top ten achievers in grade 7. Several students reported having received these forms.

The increased number of applicants not only allows the school to be more selective of students, but also means that it can complete its registration process earlier and start the academic year sooner. Hoadley writes that

> [b]y December 1997, the end of the school year, school A had a notice posted on the front door saying "1998 Standard 6 full". Whereas School B was sending letters to a number of primary schools in the near community stating that places were still available for Standard 6 students. (1999:15)

At School B registration, for all standards, in 1998 continued into the second week of the first term (1999:15). Only once registration was complete could the timetable be finalised. The timetable at School A is finalised by the first day of school (matric mathematics teacher, personal communication, July 1998).

According to the LPAS Report 1 (1998: 16) the students at School A appear to be of a slightly higher socio-economic level than the students at School B [1]. Another social difference in the student population described by Hoadley (1999: 8) is that

> a large number of students at School B come from the Eastern Cape. There is a large influx of students particularly at the grade 10 level from the Eastern Cape....The majority of students at School A have received most of their schooling in the Western Cape.

Many of the migrant, Xhosa speaking students from the Eastern Cape do not have the required level of Afrikaans, which is a compulsory subject, to pass, and students from many rural areas of the Eastern Cape have had far less exposure to English both in and out of school. Many of these students tend to move between the two areas, their family and other support networks are generally dispersed and vulnerable. Hoadley (1999) adds that this leads to a larger proportion of educationally and emotionally "at risk" students at School B.

According to Hoadley most students at School A indicated that it was their first choice of school (1999:11). Their decision was based on the school's reputation in the community. More than half the

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[1] This is based on interviews around the personal histories of a sample of high and low achieving students at each school.

students at School B indicated that this was not their first choice of school, with many of these students indicating that it was the only school at which they could find a place (Hoadley, 1999: 12). This was especially true for the students from the Eastern Cape who arrived after the start of the school year.

4.1.2 Locational and visual features

The exterior of School A looks like a typical DET school building built in the 1980s. It has several blocks of rooms. The first block is single story and houses the administrative functions of the school. This is the only part of the school to have internal passages. The other blocks contain classrooms with windows opening out onto the school grounds on the one side and doors and windows opening out onto to external passages or verandahs on the other side. Similar external passages and stairwells link the blocks.

The one wall of the foyer of the administration block, at School A, was covered in posters detailing a range of activities like computer training, music festivals, various government campaigns like COLTS[2] and YEAST[3]. The posters appeared to be placed for both the students' and teachers' benefit. On the other walls were examples of students' artwork, each piece of artwork was signed and mounted on cardboard. Much of the artwork was neither mounted straight nor hung straight. Two secretaries were working in a room, which opened onto the foyer through a glass-fronted hatch. Leading off from one side of the foyer was a furnished staff room, in which the walls were decorated with framed art reproductions depicting township scenes. A cleaner was busy cleaning the reception area. Leading off from the other side of the foyer was an internal passage which lead to a suite of rooms with the following signs on the respective doors: secretaries' office, principal's office, computer laboratory (I was told that the computers were for teachers to use to keep records, type up tests and so on), conference room (which was apparently also for the use of teachers). On the occasions that I visited the school, teachers were working on the computers in the computer room and with individual or small groups of students in the conference room. Just inside the main entrance was an open staff register on a small table. Teachers were required to sign it and note their time of arrival. On the mornings that I went to the school few teachers arrived late. Before the teaching day started the teachers and the headmaster met in the staff room. The headmaster spoke of "learners" and "educators", using the language of Curriculum 2005, although this was prior to implementation of curriculum change in the senior or further education phase[4]. This resonated with the matric class being

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[2] Culture of Learning and Teaching in Schools, is run by the National Department of Education. It is an umbrella term for a range of strategies since 1995, aimed at improving learning and teaching in schools.

[3] Year of Science and Technology, a campaign run in 1998 by the Department of Arts, Culture, Science and Technology.

[4] Grade 8 implementation of Curriculum 2005 is scheduled for January 2001.

referred to as grade 12 on both the mark schedules[5] and the student notebooks. By the time school started, there were few teachers in the administration block and very few students were visible out of class. By then several of the staff had enquired whether they could assist me.

Hoadley (1999: 12) writes that schools are assessed by the community according to the results of the final matric examinations, which are published in the press and also spread verbally through informal community networks. However, she adds that local communities also develop opinions on schools based on visible appearances. In this regard, School A (1999: 14) was assessed by the community as a good quality school on the basis of only a few students being seen outside of class during school hours and that most of students were seen to be wearing uniforms.

School B has a new school building. The building does not have the usual township school design with separate blocks of rooms. Internal passages have classrooms leading off on either side. According to the matric mathematics teacher, the students "own" the classrooms: each class of students are allocated a classroom in which they remain for most of the day, whilst the teachers move around to give lessons in different classrooms. The names of the classes were scribbled on the doors in large, rough handwriting in chalk, for example 10 B, 10 A. I saw no media up in the classrooms, the desks appeared to be arranged randomly, but with a separate space and table for the teacher at the front of the classrooms. The building has a number of rooms set aside for specialised functions, like laboratories and a library, but these were as yet unfurnished and unutilized. It has a much greater surface of the walls plastered and painted. However, there was nothing on the walls either for information or for decoration.

The secretaries worked in an office which opened onto the foyer with an open a counter top. Their office had a computer for each secretary, telephones (whose use they monitored strictly), a photocopy machine and a duplicating machine, both of which the secretaries had difficulty working. On one occasion one of the secretaries was typing a Standard 9 Science test with her screen visible for any student or staff member to read as she typed. The secretaries did not appear to assist either staff or students who crowded around the hatch opening for assistance. On either side of the foyer were passages leading to a number of rooms which included a staff room, the principals office and a number of other office, which were not marked out in any way.

There appeared to be large numbers of student and staff members in the administration section of the school on all the occasions (which spanned different times on different days) I visited the school. The perception of students being out of class, at School B, is one that appears to be widely held. Hoadley

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[5] Mark schedules revealed that the students, who were in grade 12.1 in 1998, had almost all been in standard 9.1 in 1987.

(1999: 14) reports that one of the ways in which a school is judged is on the number of students that are seen to be out of class. As one student wrote in the school magazine,

> [t]he children who come late are putting our school in shame and disgrace because the community is watching them and saying bad words about the school. (Hoadley, 1999: 14)

There also appeared to be a greater number of students in a greater variety of permutations of school uniform and other everyday clothes than at School A. None of the staff seemed to take any interest in why there would be a stranger at their school.

4. 1.3. The organisation of the Standard 10/Grade 12 classes and teachers

The mathematics teacher at School A reported that only students who have completed standard 9 at the school are allowed into Standard 10/Grade 12: no new students are permitted into the Standard 10/Grade 12 class and matrics who fail are not allowed to reregister at the school.[6]

The Standard 10/Grade 12 students at School A were easily distinguishable from the rest of the students, since they wore a tracksuits which were cut from a different cloth and with a different pattern to the tracksuit tops worn by the rest of the school. On the back of the tops was embroidered "Matric School A 1998". On the front each top was embroidered the name of the student to whom it belonged.

The Standard 10/Grade 12 mathematics teacher reported "we treat our matrics special." All Standard 10/Grade 12 students had a mathematics textbook, but only about half the rest of the students had textbooks, since the other students were required to pay for their textbooks, and many of them could not afford them.

According to my informant, the most dedicated and experienced teachers at School A, are allocated the Standard 10/Grade 12 classes to teach. Matrics are strongly encouraged to attend Saturday classes, organised by a range of projects such as PROTEC (Programme for Technological Careers), SAILI (Scientific and Industrial Leadership Initiative), SAIRR (South African Institute of Race Relations), and only a few of them do not attend some of these classes. Three times during the matric year the principal and the matric teachers meet with the matric students to motivate them. He spoke of how at these meeting students are encouraged to "uphold the good results of the school" and that it is impressed upon them that they "don't want to be known as the group who lets the standards drop".

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[6] According to Hoadley (1999: 15) both schools have a policy of not enrolling students who have not previously been at their school into matric. However, in practice new students are enrolled in the matric classes at School B.

Students are given advice about how much time to set aside to study, how to draw up and implement study timetables. Each student's study timetable is checked. The principal and staff also meet with the parents of the matrics several times over the year and discuss issues such as "attendance, the importance of homework, how much time students should spend studying, relieving students of domestic chores" and so forth.

At School B the Standard 10/Grade 12 mathematics teacher was in her first year of teaching. She arrived in the school in April. She felt that Standard 10/Grade 12 students ought to be taught by a more experienced teacher (at many schools, including School A, she would not have been allocated Standard 10/Grade 12 classes). She added that the 1998 standard 9 class only got a mathematics teacher in August and said that few of the matrics attend Saturday classes; she estimated that less than 25% attend Saturday classes.

4.1.4 Differentiation and specialisation

As the sections (4.1.1; 4.1.2 & 4.1.3) above indicate, the time, space, people and their interactions are structured differently at the two schools. At School A there is a much greater specialisation of space and time. Similarly there is a greater differentiation of students and teachers within the school. There is a high level of selection of which students are permitted to enroll at the school. Not only do school uniforms distinguish School A students from other members of the community, but the matric tracksuits differentiate these students from the rest of the school and each student is regarded as special enough to have their own name embroidered on their tracksuit top. Similarly, the signed student artwork hanging in the foyer indicates a level of individualisation of identity. Specialisation of identity is visually apparent at a number of levels in school A. As discussed in Chapter 2, Daniels (1989, 1995) argues that both locational features and student texts are revealing of the privileged pedagogy at a school. He writes,

> [r]esearch suggests that the grammar of the pedagogic practice of the school is both revealed and relayed indirectly by the visual representations of significant texts. (Daniels, 1989: 123)

It can be argued that at School A the matric students themselves are presented as "exemplary texts": as models towards which the other students can aspire.

Not only are students' time and roles specialised within school time at School A, but as the discussion of the meetings with students and parents indicates, students and parents are advised how to and cajoled into specialising the students time and roles out of school time.

In comparing the two schools, the structuring of time and space at school A displays a more intense specialisation and division of labour: there is greater scope for the development of individual identities. School A can be said to display a relatively higher degree of organic solidarity. At School B there is a lower degree of specialisation of time and space and the forms of interaction displayed are a relatively more intense tendency towards mechanical solidarity.

4.1.5 School results: an index of differential degrees of specialisation

The marked differences in the students' matriculation results have already been stated. This marked difference is less apparent when results across all the grades are reflected.

Table 4.1 Percentages pass rate of students, adapted from LPAS Report 1 (1998: 19).

Pass rates in 1997		
	School B	School A
Standard 6 / Grade 8	69 %	59 %
Standard 7/ Grade 9	78 %	59 %
Standard 8/ Grade 10	57 %	63 %
Standard 9/ Grade 11	65 %	73 %
Standard 10/ Grade 12	27 %	86 %

In the lower standards the pass rate at School B was higher than at School A. The pass rate at School B dropped in Standard 8/ Grade 10. This has been explained as being due to an influx of students from the Eastern Cape. The pass rate at School B dropped even more dramatically in the final examination. In contrast, the pass rates at School A increased steadily. It is possible that at School A more of the school's resources and expertise were focussed on the higher standards (LPAS Report 1, 1998: 20), which would concur with what the mathematics teacher said about the matric teachers and the matrics having textbooks. This could work in combination with "a sorting process at the lower levels in order to optimise the performance of students at the higher levels" (LPAS Report 1, 1998: 20). Another interpretation might be that the standard at School B is below what it ought to be and that this only becomes apparent in the final matriculation examination. For example, the mathematics teacher at School B remarked that the Standard 10/Grade 12 June exam was based mostly on Std 9/Grade 11 work. In terms of the model of organic and mechanical solidarity, one could also say that at School B there is less of an attempt to distribute the students. Since a stronger form of mechanical solidarity prevails at the school, more students are allowed to pass, rather than disrupting the social bonds by more widely distributing the students according to marks. At School A there is a stronger tendency towards organic solidarity, which is less disrupted by students having a wider distribution of results.

4.1.6 Subject choice: limitations on specialisation

At both schools students decide, from the subjects offered, on their own subject combinations. According to the mathematics teachers, students are placed in classes according to subject choice.

The mathematics teacher at School A reported that the class lists are not in good enough order for the teachers to be able to influence or advise students on which subjects they should do. He explained that the students themselves choose which subject combinations they would like to do. He reported that the students stand next to their friends on registration day. They choose subject combinations based on what their friends choose. Once the class is full they have to choose different subject combinations.

This contrasts strongly with the degree of preparation that goes into the streaming of students in more middle class schools. For example, many schools in middle class areas have strict school policies on subject choice. Careful streaming of students into subject choices is largely aimed at enabling them to achieve good results in their final school leaving examinations. Since these examination results are public, successful schools generally have more applicants for enrolment and are thus able to select students who are more likely to achieve well and a spiral of success is established. The most commonly used mechanism for differentiating between students are subject based exams and test results which act as a mechanism for the dispersal of students across a range of marks. Many schools subject students to aptitude tests, which together with exam and test results become translated into hierarchies of intelligence. At many schools there is almost constant surveillance of achievement, which works to specialise consciousness. This is then linked in with school policies on subject choice.

At both of the study schools there are no school policies about subject choice. There appears to be no attempt by the schools to dovetail the distribution of students according to subject specific exam results with a distribution of students according to subject choice. This can be read as an attempt by teachers not to differentiate students, which in turn can be read as attempts to not disturb the strong communal bonds between students. Although School A social relations show a greater tendency towards organic solidarity than those at School B, these categories are used only relative to each other here. They are not static and essential categories. In relation to more middle class schools, one could say that both schools display a stronger tendency towards mechanical solidarity. In general middle class institutions have a greater tendency towards organic solidarity and working class institutions a greater tendency toward mechanical solidarity. However, at both schools the teachers advise the students whether to do subjects on the higher, standard or lower grade. For example at School A, students are only allowed to do subjects on the higher grade if their total aggregate is more than 720.

It might seem surprising that students are placed into subject grades on the basis of their aggregates, and not on the basis of the subject specific results. Once again this is interpreted as indicating that at School A the relations of organic solidarity are relatively weak, and that too strong a differentiation between students and too strong a specialisation of consciousness is more likely to disrupt the communal relations than the dispersal of the aggregated self. Subject performance based distribution might do more violence to the communal relations than overall performance based distribution. To insert a weak disruption of the communal identity is more likely to minimise resistance.

At both schools the number of students who enroll for mathematics on the higher grade is very small. So all mathematics students attend the standard grade lessons and those doing higher grade are expected to attend extra lessons in the afternoons. At each of the schools there had initially been a small group of higher grade students but the mathematics teachers reported that these students had stopped coming to the afternoon classes and so all the students would register for their final mathematics exam on the standard grade.

4.2 The notebooks

4.2.1 The potential role of the notebooks as a technology for the induction of students into school discourses

Before embarking on an analysis of the notebooks, one might ask what are the potential roles that notebooks play in the induction of students into schooling practices and discourses. In school mathematics, notebooks are an aid to memory in at least two different ways. Notebooks allow students to set down externally the steps in a calculation. This frees the students from holding the previous sequences of the transformation in memory and allows them to concentrate on the present transformation of the calculation. Notebooks also serve as a record of problem types and a toolkit of solutions to which students can refer in preparation for the examination. In this sense they signify, for the student, an instantiation of the parameters of the discourse at a particular stage. As an alternative technology to notebooks, the use of a slate made much more sense in periods where students had direct access to a master. In these forms of organisation of education the privileged discourse was embodied in the master and the apprentice had direct access to the person and hence the discourse over time. This required large amounts of contact time between the apprentice and the master. However, with mass schooling this contact time is reduced and is shared by large numbers of students. Strategies to deal with this reduction in contact time included a range of management and surveillance techniques as well as the development of particular specialisations of space and time (Foucault, 1979; Bernstein, 1990, 1996). Other developments included the provision of textbooks to students. These acted as an artificial replacement of the master, with the role of the teacher then changing to include the responsibility to skill students in accessing knowledge from the textbook. One of these strategies is to develop the notebook into a proto-type reference, which contains an evaluated record of the student's attempts to produce the privileged performance. This allows students to learn to reference their own notebooks as an initial step to gain access to the privileged discourse. This role of notebooks becomes even more important where there is a lack of textbooks or a history of a lack of textbooks, since students are either physically unable to have access to textbooks, or the students and teachers have not been inducted into a practice of referencing books. Where teachers' authority is under threat, as for example it has been, albeit in different ways, in the majority of South African schools for the past 50 years, teachers may tend towards asserting themselves as the embodiment of the privileged discourse and may not induct students into a practice of referring to textbooks. In these situations the role of the notebook as the representation of the legitimate text is often strengthened. On the other hand notebooks represent a physical form of the student's performance which allows for the evaluation of the performance. They also serve as a record of the student's attempt to construct the criteria for the production of legitimate text. It is this record of how the student's attempt to construct the criteria for the production of legitimate text (this record over time of students attempts to develop the required recognition and realisation rules) that makes notebooks valuable objects of analysis, since they provide form of recording of the privileged forms of pedagogy in any particular context.

4.2.2 Macro-structural differences in the notebooks

At School B all the students took down notes (which were primarily in the form of exposition and exemplars) at the back of their books, working from the last page forwards. They did practice examples in the front of their books. It was only the occasional features like the "mini – test" where there was some variation in placement, however, all students wrote out the memoranda to these at the back of their books.

At School A there was more variation in where students wrote up their work. However, most students appeared to write only the section on Analytic Geometry at the back of their books and wrote all other sections (Logs, Sequences and Series, Trigonometry, Calculus) at the front of their books.

At both schools students punctuated their work into sections with dates, although often different students appeared to do the same sections of work on different days. Most of the students (the high achieving male students were often the exception) also marked out different sections of work with various headings such as the name of the sub-topic, for example "straight lines", "point of intersection"; "circle – centre origin"; "parallel and perpendicular lines", and the type of work, for example "examples", "classwork", "homework", "corrections".

4.2.3 Categorising and quantifying the differences

In the following section I do a quantitative comparison of notebooks of one student from each school. From each school I have selected the student who appears to take down the most detailed notes from the teacher and who writes out solutions in the most detail, to compare the amounts of different types of text in the student productions. I have used the following categories and ways of quantifying the text[7]s: number of days spent on the topic; exposition and exemplars; number of practice examples done[8]; and number of pages of "other" types of work. For the purposes of comparison I have listed the various mathematical topics in the same sequence, although, as discussed elsewhere, the sequencing of work was not the same at the two schools.

Table 4.2: Selection and pacing differences at School A and School B, as reflected in the student notebooks [9]

Mathematical topic	**Total number of days spent**		**Total number of pages**		**Pages of exposition and exemplars**		**Number of practice examples**		**Other pages**	
	School **B**	School **A**	School **B**	School **A**	School **B**	School **A**	School **B**	School **A**	School **B**	School **A**
Logarithms	28	14	30	12	19	< 1	13	43	3 : Assign-ments and minitest	2 Tricks and tips, Test memo
Sequences and Series	18	20	27	10	20	½ initially + 1 halfway through	20	54		
Analytic Geometry	10	20	20	23	12	2 initially + 1 page summary towards the end of the section	27	50		
Calculus	10	7	23	11	12	3: 1 page of formulae, 1 page of exposition, 1 page of exemplars	28	23		

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[7] Bernstein defines text as including "curriculum, pedagogic practice, but also any pedagogic representation, spoken, written, visual, postural, sartorial, spatial". (1990: 175). However, this study limits itself only to a focus of student productions, not because this is the only aspect of text but because it is one aspect of pedagogic text that is useful to analyse. I use the term "student productions" to indicate that this is only one aspect of text.

[8] This was, initially analyzed in terms of pages covered; it became apparent that this was not a reliable method of quantifying the work. The students sometimes wrote out questions in one place in their notebooks and wrote out the calculations in other places. Counting pages also does not take account of where students write out "corrections". The students often left spaces for unfinished work in one place and would repeat this work at a later date. Also one of the four students did some sections of work twice both at the front and the back of her book. It appears as if the calculations at the back of her book were done first, since they follow the exposition, and are labeled homework, also a few examples at the back of the book are incorrect, but are correct at the front of the book.

[9] The sequencing of topics of work at the two schools was different. For the purposes of comparison Table 4.2 shows the work in the order done at School B. The topic trigonometry is not reflected as this was only done at School A: it is discussed further in section 5.3.

4.2.4 Comments on Logarithms

At School B they spent twice as much time on Logarithms, yet students at School A did three times the number of practice examples. The more dramatic differences are, however, shown in the number of pages spent on the combined topics of exposition and exemplars and "other": School B students write 22 pages of "notes" (exposition, exemplars, assignments, mini-tests and their memoranda) while at School A they write less than 3 pages. Students at School B wrote more than seven times more "notes" in this section of work. The bulk of this was in the form of exposition and exemplars used to introduce each sub-topic. At School A 'tricks and tips' and the test memorandum were done at the end of the topic.

4.2.5 Comments on Analytic Geometry

At School B students only spent half the time on Analytic Geometry as they did at School A. Similar subtopics were covered, except that at School A the students also dealt with how to calculate equations of a tangent to the circle, which was included in all their general revision problems at the end of the topic. At School B students did a little over half the number of practice examples that were done at School A. However, the last examples done at School A were of a composite nature requiring many sub-calculations. The students at School B did not do this sort of composite calculation. This has the effect of masking the fact that the students at School A did many more calculations, since each of the many sub-calculations were similar to one calculation done by the School B students. Although at both schools the students used similar numbers of pages, at School B about three times as much space was dedicated to exposition and exemplars, which were used to introduce the sub-topics.

4.2.6 Comments on Calculus

Both schools covered similar sub-topics, namely limits, average gradients, derivatives, laws of differentiation, but at School A they also dealt with cubic graphs. School B spent about 1,4 times as much time on this topic, and did 1,2 times as many practice examples. However, the pattern of more time and space being spent on exposition and exemplars is still evident, with School B spending 3 times as much space on exposition and exemplars. Again, at School B this was done as the start of each subsection, whereas with School A more than 1/3 of this was in the form of a summary towards the end of the topic.

4.2.7 Trigonometry

The students at School B did not do any Trigonometry. In Chapter 5 (section 5.3) I analyse the Trigonometry notes, in particular the Trigonometry graphs of the students at School A.

4.2.8 Detailed analysis of student productions on Sequences and Series

Similar amounts of time were spent covering this topic at each school. However, this more detailed analysis aims to show that different types of text are privileged at each of the schools. The bulk of each student's productions at School B consist of exposition and exemplars (20 out of 27 pages or 75 %), whilst the bulk of each students productions at School A consist of practice examples (8 ½ out of 10 pages or 85 %). At School B students do 20 practice examples, while at School A they did 54. The students at School A did 2,7 times the number of practice examples that the students at School B did.

In Appendix 2 (pages 125 - 134) I have included the productions on this topic by 4 students. Comparing the student productions with each other, it can be seen that except for the occasional errors and random words, they are identical, despite the students being in 3 different classes.

The mathematics teacher at School B reported that she would like the students to have their own copies of *Understanding Mathematics.* In Appendix 3 (pages 135 - 137) I have included a copy of the initial section of the sub-topic Arithmetic Sequences from *Understanding Mathematics,* which consists of exposition and exemplars and the first set of exercises questions, to allow the reader to compare the content of this section of the textbook with the student productions.

Comparing the student productions with the textbook, it can be seen that they are almost identical. The greatest differences (apart from the layout of the pages) are the additions of the following in the student productions: the heading "definition", in the initial paragraph, the two line note about finite numbers and the heading "how to solve a problem if the sequence is not given" above exemplar 6, two lines of extra calculation in exemplar 6; and the omission of the explanation of the symbol T, and exemplar 7. The student productions include the entire exposition and all the exemplars, (except exemplar number 7 and exemplar 4 is not fully worked out for them). Apart from the exposition and exemplar, the student productions contain the first 4 questions of Exercise 3.1. These calculations, together with exemplar 4, are reproduced in the front of all 4 students' notebooks. The student productions then reflect some revision of Logarithms and Exponents and Test 1, which also deals with logarithms and exponents, before proceeding to the next topic.

All 5 subtopics on sequences and series follow a similar pattern. The bulk of the student productions consist of the exposition and exemplars, almost word perfect from *Understanding Mathematics*, followed by between 2 and 5 practice examples for the students to solve.

In all other sections of work covered, except for the first topic, which was logarithms, similar patterns can be discerned in the student notebooks. The extensive sections of exposition and exemplars come directly from *Understanding Mathematics*. In similar ways, students copy down the entire section of exposition and most of the exemplars. They then calculate only a few practice examples[10] (see Appendix 5 (page 140) for a list of the sections of exemplars and exposition from *Understanding Mathematics* reproduced in the student notebooks).

The mathematics teacher reported that once a week she gives the students a tutorial based on past papers. This was obviously an ideal that she would have liked to have implemented, since there is little evidence in the student notebooks of such regular tutorials. She added that the biggest problem that they have is time. She suggested that this was because of her own inexperience, but did also indicate that student attendance was poor. She added that she used a variety of books to teach from including the following: *Classroom Mathematics, Understanding Mathematics, Dynamic Maths, Just mathematics* and the compilation of exam questions, *Pythagorus* and its teacher guides. However, as discussed above there is little evidence in the student notebooks that she used any resource other than *Understanding Mathematics.*

In the section above it can be seen how the teacher at School B draws heavily on the textbook *Understanding Mathematics*. However, although the teacher draws so heavily on the textbook, she, like all teachers, makes selections from the textbook and she sequences and paces the work differently from the textbook. For example the textbook presents the subtopics in the following order: Arithmetic Sequences, Arithmetic Series, Geometric Sequences, Geometric Series, (the sum to infinity of a geometric sequence: HG only), Sigma Notation (more problems on sequences and series: HG only). The teacher presents the sub topics in the following order: Arithmetic Sequences, Geometric Sequences, Arithmetic Series, Geometric Series, Sigma Notation. Table 4.3 presents a breakdown and quantification of the types of work in *Understanding Mathematics* on the topic sequences and series compared with those done in the student productions in the notebooks.

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[10] The student B1, whose productions were part of the basis of Table 4.3, tends to have more practice examples than other student productions at School B. This is especially the case after the June examination, as discussed in section 5.2.

Table 4.3: Pacing differences in the topic Sequences and Series between *Understanding Mathematics* and the student productions in their notebooks at School B.

Sub topic	**Pages of exposition**		**Number of Exemplars**		**Exercise questions / practice examples**	
	Textbook[11]	School B	Textbook	School B	Textbook	School B
Arithmetic Sequences	1	1,5	7	6	21	5
Arithmetic Series	¾	1,5	4	4	28	3
Geometric Sequences	½	1	4	4	20	2
Geometric Series	½	1	4	1	22	5
Sigma Notation	¼	0,5	4	2	13 Standard Grade + 5 Higher Grade	5
Total	3	5,5	23	17	104 SG + 5 HG	20

In Table 4.3 it is meaningless to compare the number of pages spent on exposition in the textbook and the notebooks for any topic, since the size of the font in the textbook and the size of the student handwriting are so different. However, these numbers are recorded to allow for proportional comparison of the ratio of exposition: exemplars: practice questions within any one sub-topic across the student notebooks and the textbook.

In total there are approximately 4,5 times as many practice questions or exercise examples as there are exemplars in *Understanding Mathematics*. At School B students copied down 17 exemplars and calculated 20 practice questions: an almost equal number of exemplars and practice questions. Although the teacher at School B, relies extensively on the expository notes and exemplars from *Understanding Mathematics* for inducting the students into each section of work, she disrupts the pacing of the textbook by re-organising the ratio of expository material and exemplars to practice examples: she vastly reduces the proportion of practice examples that is given to students.

Appendix 4 (pages 138 & 139) contains a copy of one students' productions on the topic Arithmetic Sequences from School A. This contrasts strongly with the student productions at School B in that much less space is dedicated to exposition and exemplars and students do many more practice examples. The extensive narrative exposition that occurs in the student productions at School B is missing. This difference is even more marked when comparing the entire sections of sequences and

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[11] The textbook here is *Understanding Mathematics*.

series. As mentioned above, except for writing out the various formulae for geometric sequences, arithmetic series, geometric series and a brief explanation of sigma notion, there is only one further page of exposition, which comes halfway through this topic of work.

4.2.9 Comparing the selection, sequencing and pacing of mathematics at the two schools

At School A, students tend rather to be given a few laws or definitions to use as resources for the production of legitimate texts. The notes, summaries and tricks and tips, are given either halfway into the topic or more commonly just before they end a section of work. This can be interpreted as the teacher letting the students learn by developing a toolkit of examples and then drawing this knowledge together before allowing them to do the final more "applied examples" (which are sometimes examples from past exam papers or equivalent types of problems). In the initial stages of any topic and sub-topic, their work is broken up into small sections and subtopics, when they do the last sections of the topic (exam type questions) they are expected to draw on their knowledge of all the sections. The teacher explained that he used past exam papers to teach from. However, the day to day organisation in the student productions, which are generally broken down into separate sub-topics for the bulk of the notes, tends to resonate with the structuring of sub-topics of a textbook or a syllabus document rather than the way that exam questions tend to integrate selections of these sub-topics. It is only in the last section in each topic at this school, that students calculate composite questions, which are more similar to exam type questions. The student productions indicate that perhaps the teacher would like to, or feels that he ought to teach matrics by letting them work through past exam papers, but that the students spend a fair proportion of their time first having their learning structured by the textbook. Although, as discussed later, the selection, sequencing and pacing of the practice examples from the textbook appear to be done within the context of the requirements of the final school examination. One can infer that learning by doing calculations is privileged at School A.

In contrast the students at School B spent a lot of time (and space in their notebooks) initially in each section and subsection of work writing out exposition and exemplars. The teacher appears to perceive the students as not yet ready to understand these laws without noting substantial explanatory narratives and being led through many instantiations of these as exemplars. The School B matric mathematics teacher appears to defer students' moment of readiness and spends far more time inducting the students into the topic through narrative exposition and exemplars. There is practically no paraphrasis of the exposition and exemplars in the textbook and students have little chance to apply what they have learnt through other practice examples. The teacher at School B indicated in the interview that she felt a little underprepared to teach the matrics. She feels that it would be better if they had an experienced teacher who could "summarise what they need to do and make it shorter and simpler". She felt that her explanations were "too complicated". This mathematics teacher also felt that *Classroom Mathematics* was also "too complicated." She felt that it would be better if they had

Understanding Mathematics. Recall that the teacher is in her first year of teaching. She indicated that she felt insecure about her own competence. In much of the discourse of contemporary teacher training much emphasis is placed on developing students understanding, while teaching students to implement a variety of algorithms is devalued. The teacher is deferring her authority to that of the "expert" narrative developed in the textbook, in order to develop the students understanding of the various mathematical topics.

In order to simulate productivity, the teacher at School B reproduces particular aspects of the textbook on the board. She is using the exposition and exemplars from *Understanding Mathematics* as a prosthetic (a tool which can be transported from situation to situation without changing it, which is believed to compensate for perceived absences or disabilities). It appears that she reproduces the exposition and exemplars from *Understanding Mathematics* on the board, for students to transcribe into their notebooks. It is these sections of the textbook, which constitute the basis of the privileged text. Both Eco (1990) and Dowling (1998) draw on Lotman to differentiate between a grammar oriented and a text oriented approach to learning. The teacher at School B appears to be privileging a text oriented approach.

> Lotman introduces, even within the same culture, a difference between grammatical learning and textual learning. Cultures can be governed by a system of rules or a repertoire of texts imposing models of behaviour. In the former category, texts are generated by combinations of discrete units and are judged correct or incorrect according to their conformity of the combinatorial rules. In the latter category, society generates texts, which constitute macro-units from which rules can eventually be inferred, but which initially and most importantly propose models to be followed and imitated. A grammar-oriented culture depends on "Handbooks" while a text-oriented society depends on "The Book". A handbook is a text, generated by an as-yet-unknown rule which, once analyzed and reduced to a handbook from, can suggest new ways of producing further texts. (Eco, 1990: xi)[12]

The process of students reproducing this privileged text in their notebooks appears to represent, to the teacher at School B, the process of them acquiring the text. In this way a strong text orientation is developed. The students are not given much opportunity to develop a fluency in the manipulation of the techniques, which is what is assessed in the examination. Nor is it likely that they would develop a deep understanding of the mathematical topics.

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[12] Dowling (1998: 90, 91) suggests that Lotman sees text and grammar orientations as evolutionary forms of society. However, Eco(1990) writes that "Lotman was and is aware of the fact that no historical period has a sole cultural code ... and that in any culture there exists simultaneously various codes." (Eco, 1990: x)

At both schools there is a strong text orientation, although at School A there is a relative tendency towards a grammar orientation. This does not only happen within the matric mathematics classroom. One way to deal with a strong text orientation to produce its opposite is to hold up exemplary texts as ideals to which the other students can aim. The display of mounted student artwork is one such an example of exemplary texts, the matrics themselves at School A become constructed as another such text. The matric uniform becomes a form of display, a sign of the achievement to which the other students can aim. They are marked out as a form of ideal or exemplary text.

At School A the students plunge straight into calculations within each sub-topic. At School A students do many examples, they appear to be expected to solve the bulk of the problems themselves once they have some sort of rule, law or definition. They appear to be generating a universe of tokens, or a toolkit of techniques. This does not necessarily prepare them for the type of problem met in the exam, nor does it necessarily induct them into mathematical discourse. However, there may well be some students who are able to generalise from the examples and apply versions of the practiced techniques to the problems met in the examination, and perhaps even some who are able to develop an understanding of the discourse from the typology of problems. In the final matriculation mathematics examinations, more especially when students enroll for the examinations on the Standard Grade, there is little that demands access to the grammar of the discourse: students are seldom asked to explain how a technique was generated, and they are seldom asked to generate new techniques. What is evaluated is students' fluency with certain techniques. After giving the students plenty of practice with techniques within each sub-topic, the teacher summarised the techniques along with the main definitions and laws to constitute general rules and in so doing does attempt to draw students into a privileged discourse rather than merely presenting them with a privileged text. The final sets of more applied exam type questions give students the opportunity to check their ability to make translations within this discourse.

As discussed earlier, School A, has a strong orientation towards the final examination. It is possible for Standard 10/Grade 12 students, especially standard grade Standard 10/Grade 12 students to pass and do well in the examination if they know enough techniques to be able to solve many of the examination questions. This teacher appears to see his task as helping as many of the students as possible to pass and achieve reasonable results in the examination. For him, it appears that student performance in the examination forms the major criterion against which both he and the students will be evaluated. The examination provides the framework for how he teaches in class. Although he claimed that he teaches from past exam papers this is not done in isolation. One might surmise that his assessment of students predisposes him to first make them work through examples from the textbook. Once students have developed fluency with the basic algorithms and techniques, he summarises the work before ending each with composite exam type problems. This indicates that he is attempting to develop a toolkit of techniques for students to reproduce in the examination, although the structuring of a textbook is apparent in the foreground of the student productions: work is broken up into small sub-topics and the way in which students number their work indicates that practice questions are taken from a textbook. It appears that this teacher's selection, sequencing and pacing of the work is driven by the requirements of the final examination. This teacher did his teacher training in an Education Faculty at an English medium university, with a predominantly liberal influence, which would have privileged a form of constructivism. However, this would not necessarily have resonated with his own school learning experience. He has been teaching matric students, primarily standard grade students, for several years which would mean that there would be some pressure on him to structure his teaching according to the requirements of the examination. Moreover, the strong influence of constructivism had, not yet in 1998, made itself felt on the teaching and learning of mathematics in standards 8 – 10 at the majority of schools. His own learning and teaching experiences resonate with the form of regulative discourse dominant at the school, and it is this that probably sets the parameters for his selection, sequencing and pacing of the work and hence the development of the instructional discourse in his classes.

In contrast with developing the students' fluency with techniques, the teacher at School B appears to want to develop student's understanding of the various mathematical topics. One could interpret her focus on developing students' understanding as being a result of her recent teacher training: she only qualified in 1997. Much of the focus of her training is likely to have been framed within a broadly constructivist approach, which would have emphasized that it is important to develop students understanding, and that getting the calculations right is not enough. Morais (1989) suggests that teacher experience is partially productive of learning context. She suggests that teacher experience within one context can amplify or dampen other contextual factors at a site of learning. At school B, the teacher appears to be concerned that students understand the meaning behind the calculations/algorithms. For her the object of mathematics must be clear to the students. This provides an example of how elements of the pedagogic recontextualising field, through a reinterpretation of the theory of instruction, can have an impact on local pedagogic practice. The teacher's approach at School B, of reproducing elements of the textbook, may appear to conflict with current dominant notions of student self discovery and active participation. However, earlier it was suggested that School B displays a stronger tendency towards mechanical solidarity. In a situation of mechanical solidarity where the division of labour is less strongly marked out, a text orientation is less likely to be disruptive of the strong communal relations. Reproducing "The Book" reduces the focus on individual performance and minimizes the differentiation of students. In a situation in which the division of labour resonates more strongly with organic solidarity, a stronger tendency towards a grammar orientation is less likely to be disruptive of strong communal social relations. In this way the form of pedagogic practice that develops contains elements of the teachers experiences (potentially her own school learning experience) which are recontextualised within context of learning. As Bernstein (1990, 1996) suggests the regulative discourse dominates the instructional discourse. The selection, sequencing and pacing of the work at School B appears to be driven by both the strong relations of mechanical solidarity and by the teacher's desire to develop students' understanding of the mathematics. Since she feels insecure about herself as the expert authority, she defers to the expert authority represented by the textbook. However, her belief in the importance of students' understanding means that she, as all teacher's will, makes a further selection and changes the sequencing and pacing developed in the textbook. The driving force behind her selection, sequencing and pacing of the learning is her objective for students to learn by understanding rather than by practice and she uses elements of the textbook to achieve this.

4.3 Describing the differences in pedagogic modalities using the analytic concepts of classification and framing

When comparing the two schools in terms of classification and framing values it must be stressed that these values and terms are used to describe relative differences, so for example, where one might say that School A shows relatively stronger external values of classification, this is only in contrast to School B. If School A were to be contrasted to, for example, a private school in another area, one might say that relatively its values of external classification were weaker. Similarly one needs to bear in mind those parameters of state schooling which are prescribed by the curriculum, for example the following features of what were in 1997 defined as "high/secondary" schools were set: minimum school hours; the wearing of uniforms; the minimum number of subjects to be taken by students; the minimum number of subjects students are required to sit for in the final matriculation examination; the syllabus to be covered in each examinable subject in each year of high/secondary school. These features certainly impact on the way in which state schools are internally and externally classified. But, as Bernstein remarks:

> what is reproduced in schools may itself be subject to recontextualising principles arising out of the specific context of a given school and the effectiveness of external control over the reproduction of official pedagogic discourse. Further, what is reproduced may be affected by the power relationships of the recontextualizing field between the school and the primary cultural context of the acquirer (family/community/peer relations). (1990: 199)

Although certain features of state schools may be prescribed by the curriculum, the manner in which these are implemented differs from school to school. Variations in how issues of power and control are manifested in the prescribed and non-prescribed features of school give rise to different forms of pedagogic modality. In this section I redescribe the differences in pedagogic modality of the two schools in terms of internal and external classification and framing variables.

It has been discussed that, at School A the students spend more time in class (students and teachers are more likely to arrive on time and stay at school for the full school day), students are under greater pressure (not necessarily in a coercive sense), to wear uniforms (which clearly marks them out as particular members of the community, at least during school hours) than the students at School B. The categories of school and community are more clearly differentiated at School A than at School B. In this sense one can say that the external values of classification are stronger at School A.

The architectural features of the two school buildings are designed to specialise space to different degrees. At School B the building is designed to allow for a greater specialisation of space and function than at School A: there are more rooms designed for special functions such as the indoor "quad" which doubles as a school hall, the technology room, library. The differences in architecture are a manifestation of the different dominant social regulations in the mid 1980s and late 1990s. The state is a macro agency of symbolic control: the change in governments in the early 1990s had an impact on educational policy from the late 1990s[13], this is reflected not only in syllabus changes but in a range of other policy changes including the design of school buildings. The design of the building at School B should allow for a more strongly pronounced internal classification. However, in practice, the internal spaces at School A are more clearly differentiated from each other: the arrangement of furniture, door signs and wall displays (such as artwork and posters) mark out the internal usage of different spaces far more clearly at School A than at School B. Even the matric students are marked out as different to the other students: they represent an ideal to aim towards. At School A the teacher's "own" the classroom, that is each teacher is allocated a room, and when the students are to be taught by that particular teacher, they are required to move to his/her classroom. This can be read as allowing for a greater internal classification. Not only is the teacher identified with the subject but so too is the classroom. Whereas at School B, it is only the teacher who acts as the marker between one subject and the next, all subjects get taught within the same classroom: students stay in the same class and teachers move.

In order to explain why the internal and external classification values at School A are much greater, one can examine the framing values. At School A the external framing values are much higher than at School B. The school management asserts its control in maintaining not only the time, space and text of the school identity as different to, non-school time, text and space, but it also establishes the school as different to other schools in a powerful way.

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[13] Previously policy and curricula for black South Africans were designed to limit the degree of specialisation relative to the education designed for white South Africans. After the change of government in the 1990s changes in educational policy and curricula were aimed at all school students receiving the same opportunities to specialise.

The recruitment of successful students, the non-acceptance of students from outside the school into the matric year are all indications of a strong form of external framing. The attempt by the principal and matric teachers to command students' out of school time for studying is a further example of how much stronger the external framing is at School A than at School B. This results in a stronger external and internal classification, for example one assumes that most of the posters that are displayed at School A are also sent to School B. However, they are not displayed at School B. In this way a control function of the transmitters is not taken up and in this process of weakening the framing values, the internal classification values are weakened.

The fact that the teachers 'own' the classrooms at School A might set up possibility for greater internal framing, in terms of locationary factors, allowing the teacher to have greater control over the organisation of the space within the classrooms. Similarly the fact that the students 'own' the classroom at School B, might allow for the diminishing of internal framing, allowing the students more control of the furniture and space within the classroom. Although the desks did appear to be placed randomly within the classrooms at School B, there was no evidence of them being placed in rows or groups, I do not have comparative information about the organisation of the classrooms at School A.

On the other hand the student productions at School A show a greater variation of where different aspects of work are carried out in the notebooks, than the students at School B. Here students seem to have greater control over their notebooks, and as described in Chapter 5, the work in their notebooks is corrected less often than the students' work at School B. This points to greater internal framing values at School B. However, no data were collected on direct interactions between students and teachers at the two schools, so that this indirect data should be read as tentative. On the whole there are stronger internal and external classification values at School A than at School B, the external framing values at School A are also stronger. The indirect evidence gleaned of the internal framing values is insufficient to compare across the two schools without direct data on teaching and other transmitter / acquirer interaction.

4.4 Conclusion

The schools have been described as having different levels of specialisation on time and space and these resonate with differences in the student productions and also with the results students achieve in their final examination. Recall that the community interprets the success of the students in the matriculation examination as an indication of the quality of education offered at the school. There are differences in the regulative discourses at the two schools and these differences result in differences in the student productions, which can be read as an index of the transmission texts generated.

A reading of the student productions from the two schools indicates that they display different forms of pedagogic communication which have differential distance from the form of pedagogic communication evaluated in the matric exam or privileged at the university.

In summary, from the differences in the student productions one can infer differences in pedagogic modalities at the two schools. How then, does this relate to the issue of quality of learning? One can recast the question of how one assesses the quality of learning as how one judges different pedagogic practices. Some might say that the extent to which the criteria being transmitted in any institution matches with those of the dominant social organs will constitute the extent to which one can mark out quality, but perhaps this is only a measure of achievement and success. The relatively stronger form of grammar orientation developed at School A is likely to position its students more favourably in terms of the final matriculation examination, whereas the stronger form of text orientation is unlikely to prepare the students at School B to achieve well in the examination. The dominance of the middle class in contemporary society can be read as a dominance of forms of organic solidarity in society. Education, in modern society, serves to distribute people socially and so students who are more deeply inducted into the identities developed by relations of organic solidarity might be said to have a better quality of education, since such forms of solidarity present fewer threats to dominant groups and so greater access to social goods is a reward.

In the next chapter I shall examine what the student productions reveal of how students construct the criteria for the production of texts, and the extent to which these texts can be read to indicate the degree to which students are being inducted into the discourse of mathematics. The extent to which students are being inducted into mathematical discourse marks out the quality of their learning of mathematics.

Chapter 5. The effect of evaluation on students' construction of the criteria for the production of text

As discussed in Chapter 3, Bernstein regards continuous evaluation[1] as key to pedagogic practice, since it functions to assist students in the construction of the criteria for the production of legitimate text. In this chapter I discuss some key aspects of evaluation that are reflected in the student productions at each school. I also discuss how this evaluation is an essential component of the modality of pedagogy at each school and is central in the student's construction of criteria for the production of text. An analysis of evaluation is thus key to understanding the students' development of the construction of criteria for the production of legitimate text. Since student productions are one form of the texts produced in classroom, analysing the evaluation that is revealed in the student productions thus becomes an important aspect of answering the question: "to what extent are students being apprenticed into the discourse?" which is how I have recast the question "what can student productions tell us about the quality of learning?" In order to answer both of these questions, this chapter attempts to provide answers to the sub-questions outlined below:

- Do the student productions captured in the mathematics notebooks provide enough of a record of the pedagogy privileged by the two teachers, to enable one to assess the extent to which students are inducted into mathematical discourse? If not, what other forms or records of pedagogy are needed or would be useful before describing and evaluating the pedagogic practices at each school?
- What, if anything, do the student productions that are captured in their mathematics notebooks display of the extent and forms of evaluation in the classrooms? How might these forms of evaluation be related to other forms of evaluation that are not displayed in the notebooks, both in form and extent? What is the relationship between evaluation in general and the induction of students into mathematical discourse? Is the proportion of evaluation that is captured in the notebooks sufficient to use as a basis to discuss the differential potential for students at each school to be inducted into mathematical discourse?
- Are there differences between the productions of the more successful and less successful students within each of the schools? If so, are there any patterns in these differences and how might these patterns relate to the form of pedagogy privileged at each school?

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[1] "Continuous evaluation" has become a common term in contemporary South African pedagogy. However, my use differs from the common usage, which could possibly be better termed "continual assessment". As mentioned in Chapter 3, evaluation can occur in a variety of forms including repetition, non-repetition, or other forms of silences, changes in tones of voice, changes in gesture, facial expression or other forms of body language (Bernstein, 1990). Evaluation in the sense that I use it, following Bernstein, is thus continuous: there can be no pedagogy without evaluation.

- What, if anything, is the relationship between student achievement and quality of education: or in student success in examinations and the extent to which they are inducted into mathematical discourse?

Schmidt *et al.* distinguish between the "intended" curriculum, the "implemented" curriculum and the "attained" curriculum (Schmidt, W.H., McKnight, C.C., Valverde, G.A., Houang, R.T. & Wiley, D. E. (1997) cited in Taylor, N., 1999). Bernstein writes that it is through the processes of selection, sequencing and pacing that discourses are recontextualised in the production of the "intended" curricula. The "implemented" curriculum can be thought of as the result of a further selection, sequencing and pacing by the teacher influenced by each classroom context. The "attained" curriculum can be redescribed as student acquisition texts, which are constructed by students as a result of the process of evaluation. While recontextualising and evaluation are constructed as separate analytic categories, in practice they act both in parallel and often simultaneously.

The extent to which students are able to construct legitimate texts in external examinations will impact on their success and on how society assesses the quality of education provided by their institution. Public examination results are a common way in which institutions are evaluated. As I have argued, another way to assess quality of learning is to define quality in terms of the extent to which students are inducted into any particular discourse.

5.1 Forms of evaluation

Evaluation occurs in a number of ways, which work in conjunction and often simultaneously with each other: through verbal feedback (which can occur as correction, questioning, repetition, or non-repetition, through changes in intonation or emphasis, or even silence), it can be through body language (such as gestures or changes in facial expression), or it can be through written feedback (in the form of ticks, crosses, question marks or other punctuation or editorial marks, or in the form of written evaluative comments). Evaluation can occur between teachers and students or between students and students. Any formal assessment such as a test, examination project or presentation provides multiple opportunities for evaluation.

In this study the only forms of evaluation that were accessible were those represented in the student notebooks. Here evaluation can take the form of the teacher signing the notebooks; commenting or not commenting on work done; marking[2] the practice examples and either writing out the correct answer or an appropriate algorithm or sketch. Potentially much of the teacher's evaluation on students' productions in their notebooks is not directly reflected in the notebooks themselves, but might occur through interaction with individual students or interaction with the whole class. Much evaluation might be indirectly reflected either through the students marking their own work, and potentially "writing out corrections", in response to work being generated on the blackboard, or students changing a form of presentation or approach or algorithm used to solve problems or to work out practice examples, in response to feedback from the teacher. Another form of evaluation is student-student evaluation. This is more difficult to ascertain simply through a reading of the student productions. It may need direct observation of students to be discussed with any certainty. However, particular patterns and changes in patterns in the student productions may index some level of student-student evaluation.

5.1.1 Forms of written evaluation reflected in the students' notebooks

In this section I reflect the most obvious forms of written evaluation in the student notebooks and discuss what one might infer from these about the pedagogic practices at the two schools.

School B

Table 5.1 sets out some of the written forms of evaluation that are reflected in the student productions at school B. The numbers refer to the number of days of work that is number of mathematics lesson units.

At School B it can be seen that the teacher appears to engage with different student's productions to a different extent. Recall that the students are labeled B1 – B4 in order of their examination results (the students B1 and B2 are the top achievers, student B3 results were at the lower end of the range and student B4 achieved the lowest marks of all the matrics). What is most striking is that student B4 has never had his book signed, nor did he mark his own work on any occasion. The number of times the teacher signs each student's notebooks appears to be proportional to the student results. It is unclear whether it is the teacher who prefers to engage directly with the more successful students' work, or whether the students who are less confident about their work prefer to keep their books away from her, or even whether it relates to some extent to the amount of work that students do, or their level of attendance. In all likelihood all these factors are at play and probably several others. In a similar way the number of times that students mark their own work also decreases as their results decrease.

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[2] By marking I refer to the convention of the placement of a tick or a cross alongside work to indicate whether or not the solution is correct and potentially whether the use of symbols, algorithms and other mathematical conventions are appropriate.

Table 5.1: Days of work captured in student notebooks at School B which show evidence of evaluation.

	Indication of teacher's engagement with student's productions in notebooks									Total
	Direct engagement in notebooks				Total	Indirect engagement in productions			Total	
	Signing only	Marking /correcting	Student reworking			Student marking	Student reworking			
Student			With marking	Without marking			With marking	Without marking		
B1	2	17[3]	5[4]	0	**19**	10[5]	1	1	**11**	**30**
B2	2	12[6]	3	3	**17**	7	2	4	**11**	**28**
B3	1	4	1	0	**5**	2	0	1	**3**	**8**
B4	0	0	0	0	**0**	0	0	0	**0**	**0**

If one considers evaluation to be the key to the development of the criteria for the construction of legitimate text, it is not surprising that the level of written evaluation in students' books decreases as their level of achievement decreases. At School B, where the teacher signs the books, she usually corrects the students' work at the same time. It is, however, worth noting that student B2, who was regarded as the top student by the teacher (although student B1's results were slightly better), does correct (rather than just mark) his work more often that any of the other students. He shows a preference for rewriting the correct work, rather than getting involved in the more bureaucratic approach of ticking and crossing. His motivation seems to be to have a correct version of each problem. In this way he is generating for himself a record of the toolkit of problem types and practising how to reproduce these problem types.

Both the female students (B1 & B3) at School B, at times, write the questions out in pen and do the calculations or sketched the solutions in pencil. Student B3 tends to work this way for at least half of the work that she does. One might hypothesise that she is doing this in order to change the calculations when she gets access to the correct version, which, one might surmise, is often generated on the classroom blackboard the following day. And there are indeed only two occasions where she appears to redo her work. However, in the work that is not marked most of it is either incomplete or has parts that are incorrect. Potentially then one may assume that she is using the officially sanctioned classroom version of

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[3] Includes three days of work in which only the first few examples were marked and two days of work which include incorrect work being marked correct.

[4] Includes one example of writing out "corrections" when the original calculation was correct.

[5] Includes four days of work in which only the first few examples were marked.

[6] Includes three days of work where only the first few examples were marked.

the corrected problems not only to correct but also to complete her record of the practice exercises. If this practice of copying work (either from the board, or from a friend) is common, then it will interfere with patterns in the teacher's or the student's written reflection of marking and correcting of their work, and complicates any attempt at analysis. If one compares the practice of student B2 with students B1 & B3, then one might say that students B1 & B3 are more concerned with having an orderly sequence of correct problem types (without any display of error) than they are about maximising their opportunities to practice generating those problem types. This differs slightly from the approach of student B2 described above. In a sense one could say that the approach of students B1 and B3 shows a stronger tendency towards a text orientation.

Table 5.2 shows the number of days on which there was no evidence of written evaluation of student's productions in their notebooks, and the extent to which this work contained errors or was incomplete.

Table 5.2: Days of work captured in student notebooks at School B, which show no evidence of evaluation.

Student	**Number of days of work in notebooks that show no evidence of evaluation**	**Comments**
B1	11	3 days contained some incorrect work
B2	13	4 days contain incomplete work, on: one of these days all the calculations are incomplete, 2 days contain incorrect calculations, 2 days symbols are used unconventionally
B3	11	7 days contain all or some of the work incorrect or incomplete
B4	10	2 days contain all or some of the work incomplete or incorrect

At School B all students have their roughly similar numbers of days of work unmarked by both the teacher and the student. However, what Table 5.2 and Table 5.1 read in conjunction show is that whilst students write practically identical notes on each topic (see Chapter 4), the amount of practice examples they do varies. This variation is linked with their level of achievement and as discussed in Section 5.2: the variation is more extreme after the June examinations. Table 5.3 indicates that whilst the number of days of unmarked work in the student productions of School B is roughly similar, the percentage of days of unmarked work of the total number of days work declines dramatically as the students' level of success declines.

Table 5.3: Percentage of days of work captured in student notebooks at School B which show no evidence of evaluation

Student	Days of work that reflect evaluation	Days of work that do not reflect evaluation	Percentage of days that do not reflect evaluation
B1	30	11	27 %
B2	28	13	32 %
B3	8	11	58 %
B4	0	10	100 %

At School B, the two top achieving students have their notebooks marked on many more days than they do not have their books marked. However, it should be noted that when their books are marked, it does not mean that every example of the work done on that day was evaluated. On several occasions only the first few examples are evaluated. There are also examples of incorrect work of students being marked correct. This weakens the evaluative role of the marking. In Section 5.3 I discuss further how a weak expression of evaluation of student productions can limit the induction of students into mathematical discourse.

Table 5.4 sets out some of the written forms of evaluation that are reflected in the student productions at school A. The numbers refer to the number of days of work, that is the number of mathematics lesson units.

Table 5.4: Days of work captured in student notebooks at School A, which show evidence of evaluation.

	Indication of teacher's engagement with student's productions									**Total**
	Direct engagement in notebooks				Total	Indirect engagement in productions			Total	
	Signing only	Marking /correcting	Student reworking			Student marking	Student reworking			
Student			With marking	Without marking			With marking	Without marking		
A1[7]	1	1	1	0	**2**	5	1	1	**6**	**8**
A2	2	6	0	0	**8**	5	0	1	**5**	**13**
A3	1	2	0	0	**3**	1	1	0	**2**	**5**
A4	0	8	0	0	**8**	10	1	2	**11**	**19**

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[7] This notebook of this student only started in May, he must have used another notebook before then, for this reason, the number of occurrences of his book being marked or not marked is less than the other students.

At School A, there is a less clear relationship between the student's results and the level of written evaluation reflected in their notebooks. This may be because at this school the sample of students had a less clearly defined distribution of results (student A1 was the top student and students A2, A3, & A4 achieved similar results, which were in the middle range of the standards results). It is also affected by the fact that the notebook of student A1 only had work from May onwards (the work that he did from January to April must have been in a separate notebook). The bulk of written evaluation in the other notebooks occurred in the first section of work that the students did on logarithms.

At School A the teacher often signs the books, but does not check whether students have done the work correctly, nor if they have done all the work. The only places where he comments that students have not done work is where they write out the question but do not do the calculation. One might surmise that his signature appears to be much more of an indication that he has fulfilled his responsibility for generating work for students to do. This can be explained in terms of the pressures that teachers are under both in terms of time (there are potentially too many students and too little learning time, for him to spend time correcting individual productions in the notebooks) and in terms of the multiple pressures on teachers outside the classroom, to prove that they are doing their job. One might conjecture that this teacher sees his evaluative role as the production of the correct version of the calculations on the board, and that it is the students' responsibility to correct their own work. However, the evidence shows that these students do not mark their own notebooks frequently. Table 5.5 indicates that the notebooks of all the sample students at School A had many more days of work without any evidence of written evaluation, than days of work with evidence of written evaluation.

Table 5.5 Percentage of days of work that show no written evidence of marking in the student productions at School A.

Student	Days of work that reflect evaluation	Days of work that do not reflect evaluation	Percentage of days that do not reflect evaluation
A1	8	13	62 %
A2	13	35	73 %
A3	5	31	86 %
A4	19	31	62 %

Whilst it may seem surprising that there is so little evidence of evaluation of the student productions at School A, it is important to note that on quite a few of the days when work was unmarked, students' work was either incomplete or contained errors. This is captured in the Table 5.6.

Table 5.6: Number of days of unmarked work in the student productions at School A.

Student	No evidence of engagement with student productions in notebooks	Comments
A1	13	On 5 days calculations are incorrect/incomplete or mathematical conventions are not adhered to
A2	35	On 8 days calculations are incorrect/incomplete or mathematical conventions are not adhered to
A3	31	On 19 days calculations are incorrect/incomplete or mathematical conventions are not adhered to
A4	31	On 9 days calculations are incorrect/incomplete or mathematical conventions are not adhered to

Table 5.6 indicates how frequently unmarked student work at School A is incorrect, incomplete or breaks with key mathematical conventions or forms of representation. This issue of student work breaking with mathematical conventions is discussed further in Section 5.3.

As discussed in Chapter 4, at School A there is a stronger tendency towards organic solidarity and a grammar orientation than at School B. Both within School A, as a whole, and within the Standard 10/Grade 12 classes, in particular, there are greater opportunities for the development of individual identity, for the distribution of students and the specialisation of consciousness in a way that enables them to deal with the matric examination. The entire school, and particularly the Standard 10/Grade 12 mathematics teacher, gears learning towards the final matriculation examination. It is likely that much of the evaluation of students occurs outside of what has been reflected above. For example, at many schools students themselves are asked to generate the correct solutions on the blackboard. Their success at being able to do this becomes an evaluation in itself. At many schools that exhibit a strong grammar orientation, teachers do not engage with student's notebooks at all. Other forms of evaluation could include frequent tests and assignments. A weakness with this study is that I was unable to get access to the exam scripts and memoranda, and did not request details of other formal forms of testing and assessment. Since I did not have access to classroom interactions and practice, it was difficult to pinpoint what the other forms of evaluation were.

As discussed in Chapter 4, at School B, on the other hand, there is a strong tendency towards a form of mechanical solidarity and a stronger text orientation is developed than at School A. There appears to be little ongoing evaluation and distribution of students both in terms of the dominant regulation in the school and the classroom at School B. The specialisation of student consciousness and distribution of students is kept to a minimum. For example, the mathematics teacher prepares students for tests and exams by working through similar model assessments before and the memoranda afterwards. The incident of the secretary typing a Science test with her screen visible for any student or staff member to read was discussed in Chapter 4. One could argue that the notebooks come to form the basis of the privileged text. In this sense the notebooks themselves come to represent a major opportunity for evaluation of students. This might explain why there is a greater level of evaluation of the notebooks at School B than at School A.

5.2 The effect of the examination on the construction of criteria for the production of legitimate texts at School B[8]

Just as evaluation is key to the construction of criteria for the production of legitimate texts, so too can one infer from the nature of texts (in this study limited to student productions) key elements of the criteria that students are using in their production and link these with potential evaluative moments.

In Chapter 4, it was discussed that School B shows a stronger text orientation than School A. It was mentioned that the effect of transcribing large sections of the exposition and exemplars for the teacher could be described as simulating productivity, for the students it develops both a sense of community and a false sense of all students being on top of their work. This strong form of text orientation does not prepare students for the final examination, which has an individualising and distributive effect on students. As described in Chapter 4, students do not have much opportunity to develop fluency with techniques, which is what is assessed in the examination. Nor does this form of text orientation practised at School B induct students in to the discourse of school mathematics.

5.2.1 Constructing criteria from the teacher

Curriculum authority in schooling is embodied in an interplay of various institutions, functionaries and individuals, for example, the national or provincial syllabi, the external exam, the education department officials, school principals, governing bodies, teacher trainers, textbooks and teachers. For most of the time in students' school careers, the teacher is a very prominent source of authorisation of the curriculum.

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[8] There is less distance between the form of pedagogy privileged at School A and the requirements of the final examination, than between the form of pedagogy privileged at School B and the requirements of the final examination. Because of this the evaluative effect of the examination on pedagogic practice is more starkly represented in the students' productions at School B. This is why this particular aspect of evaluation is only discussed at School B.

Those students who have access to professional academic assistance outside of their school day, which could take different forms such as extra lessons, libraries, study guides and other books, computer and television programs, are more likely to have a range of sources of curriculum authority. The less that students encounter schooling outside of their school day and building, the more important the role of the teacher is. Perhaps one can say that the curriculum is embodied in the teacher for many working class students for most of their school careers. The students look towards the teacher for guidance on acquisition on the important signifiers that index access to the discourse. The teacher is a primary resource for the simulation of the existence of knowledge. However, external examinations can collapse this. Section 5.2.2 details how the student notebooks indicate that this was the experience of the students at School B.

5.2.2 Constructing criteria from the examination

All examinations distribute students differentially. External public examinations make this distribution of students public. The degree of similarity or difference between the requirement of external public examinations, such as the matriculation examination and the criteria of previous internal school examinations varies from school to school. In the June and September exam that precede the final school leaving examination, teachers are under considerable pressure not merely to examine what they have taught but also to set an examination which more closely resembles that final external examination. Partly this requires teachers to demonstrate that they have examined the full syllabus. They are also under pressure to discontinue practices of priming students about the content of the examinations[9] in order to prepare students for an examination in which the teacher is unlikely to have access to the examination questions before hand.

Students on their part will read their achievement in the June and September examinations as an indication of what they might achieve in the final examination. Where students have been learning under conditions of strong mechanical solidarity, these examinations become a disruptive mechanism for evaluating and distributing students. The communal bonds are weakened and students are distributed in terms of success in the examination. They get new feedback in terms of what is required from them to meet the nationally or provincially constructed criterion. Students may come to realise that the symbolic authority of the discourse as it is realised in the examination is a "higher" authority than that of the teacher.

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[9] For example, the teacher at school B provided matric students with mini-tests and their memoranda prior to some of the big class tests.

In distributing students differently, the examination also disturbs their pedagogic identities. What I hope to show in this next section is that, at school B, after the June examination has disrupted the bonds of mechanical solidarity, both the students and the teacher begin to weaken the text orientation.

5.2.3 Differences in student productions before and after the June examination

Table 5.7 shows the number of pages each of the four sample students spend on exposition and exemplars and the number of practice questions that they did on each topic.

Table 5.7: Breakdown of types of work done by sample students at School B.

Topic	Type of work	Student B1[10]	Student B2	Student B3	Student B4
Logarithms and exponents	Pages of exposition	19	15	12	8
	Number of practice questions[11]	13	14	12	7
Sequences and Series	Pages of exposition	20	21	17	12
	Number of practice questions	20	17	16	6
Analytic Geometry	Pages of exposition	12	15	6	4
	Number of practice questions	27	26	11	0
Calculus	Pages of exposition	12	10	6	4
	Number of practice questions	28	20	3	0

The June examination occurred when students were busy with the section on Analytic Geometry. From Table 5.7, one can read that for the sections on logarithms and sequences and series, students B1, B2 and B3 wrote out more or less the same number of pages of exposition and exemplars: as discussed in Chapter 4, these sections are practically identical (some of the differences in page numbers can be accounted for by considering the differences in handwriting). They also did more or less the same number of practice

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[10] Recall that the students are labeled B1 – B4 in order of their examination results (the students B1 and B2 are the top achievers, student B3 results were at the lower end of the range and student B4 achieved the lowest marks of all the matrics). Student B2 and student B3 are in the same class.

[11] As discussed on p 63, initially the routine practice exemplars were analyzed in terms of pages covered; it became apparent that this was not a reliable method of quantifying the work. The students sometimes wrote out questions in one place in their notebooks and wrote out the calculations in other places. Counting pages also does not take account of where students write out "corrections". The students often left spaces for unfinished work in one place and would repeat this work at a later date. On the other hand, exposition was quantified in terms of pages since this contained not only exemplars but also quite large amounts of narrative exposition.

questions. Student B4 wrote approximately half of the number of pages on exposition and exemplars as student B2 for both topics, and did about half the number of practice examples for logarithms and about a third the number for sequences and series.

For the topics Analytic Geometry and Calculus, student B1 and student B2 continue to do very similar amounts of work. The amounts written by students B3 and B4 drop considerably.

It is not only the amounts of work that students do that change after the June examination, but also the type of work that each of the students do. After the June exam it appears that the teacher no longer always gives identical work to each of the classes, although the work is similar and comparable.

5.2.4 Comparing student productions on the topic calculus[12]

In the sub-topic on "Average Gradients" the teacher does not take the exposition and exemplars from *Understanding Mathematics*. Only students B1 and B2 have the notes (which are different from each other) on "Average Gradient" (see Appendix 7, pp. 158 - 160). It is unclear whether this is because the teacher gave the classes slightly different work or whether it is because the students each noted different aspects of what was done in the classroom or whether it was a combination of these two factors.

In other sub-sections of the topic calculus, the exposition is taken from *Understanding Mathematics* but unlike with other topics, the teacher tends to select from the exemplars and not give all of the exemplars in the textbook to the students.

The students also start to write down different things from each other. Student B4 wrote out the exposition and exemplars for only the sub-topics "Derivative of a Function". For the sub-topic "Rules for differentiating" he only wrote out only the exposition and none of the exemplars. He did not write out any exemplars or other form of exposition for any of the other sub-topics. For the whole section of calculus student B4 did not do any of the practice examples. It appears that his exam results have demotivated him.

Student B3 appears to write out less of the narrative exposition although she does still write out the exemplars. She does very few of the practice questions. She also appears to be demotivated after the

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[12] It was possible to pinpoint the exact date of the mathematics examination to 17 June 1998. However, I thought that the full effect of the examination would only be apparent sometime after the students both got their examination results back and went through the examination memorandum. This date was more difficult to discern. For this reason I have focussed on the topic calculus because this entire topic was done some time after the examination.

examination, but this results in a different strategy on her part. Student B2 on the other hand tends to write more of the exposition than other students do. He writes down exemplars but not all of them. He does many practice questions but not all of the questions done by either student B1 or student B3. Perhaps he has realised that he can pass without noting all the exemplars and doing all the practice examples. Appendix 8 (pages 164 - 167) shows how different the notebooks of student B2 and student B3 (students in the same class) are for the sections "Limits", "Average Gradients" and "Rules for Differentiating".

Student B1 also starts to write out less of the exposition, for example, in the section "The gradient of a curve at a point" she does not write out any of the exposition but only does the exemplar. However, unlike student B3, student B1 wrote down a lot of practice exemplars. She appears to have realised that what is required by the examination is a fluency with the techniques of calculating, rather than narrative descriptions of the sub-topics.

5.2.5 The effect of the examination – disrupting the relations of mechanical solidarity at school B

It appears that the June examination considerably weakened the relations of mechanical solidarity. The effect of distributing the students in terms of mathematical ability appears to have disrupted the pedagogic practice and related pedagogic identities at school B. The students have been positioned differentially in terms of the pedagogic practice that is privileged nationally or provincially, and they can no longer participate in the same way in a practice which positions them in relations of sameness towards text privileged in the classroom. Their differences in performance in the examination appear to have impacted on the specialisation of their consciousness.

Students at school B appear to have realised that that there is some dissonance between the modality of the pedagogy practised in the classroom and that privileged in both the examinations they have been through and the ones that they are about to face. The student productions suggest that each of the four students whose productions were analysed begin to select and participate in different aspects of the pedagogy privileged in the classroom. The more successful students are able to read new criteria for the construction of legitimate texts from the examination. The less successful students appear to understand that they need to use new criteria for the construction of legitimate text, but are unable to construct criteria that will help them in the final examination. They appear to have understood that the form of the recognition rules has changed, but seem unable from the examination, as a single event, to construct forms of realisation rules that will assist them to succeed in the final examination. The feedback from the examination also appears to modify the form of pedagogy practice privileged by the teacher.

School is one institution through which the dominant principles of society are reproduced. It functions to specialise consciousness and to distribute discourse and knowledge differentially. As discussed in Chapter 4, the dominance of the middle class in contemporary society is associated with a dominance of forms of organic solidarity. As such it is more likely that at schools that display a strong form of organic solidarity students will be more successful in their matriculation examinations, whilst the students at schools that display a stronger form of mechanical solidarity are likely not to achieve as highly in their final school examinations. At School B, the form of mechanical solidarity has been weakened by the June examination. However, the students are left with only about two months of schooling in which to catch up with the form of specialisation that has occurred at many other schools. They have only a short time in which to learn new recognition and realisation rules for the construction of the privileged text. This is likely to jeopardise their chances of success in the final examination. It remains the social norm for the quality of both students and schools to be assessed in terms of their success in terms of the final examination. The extent to which the texts constructed by students matches with the requirements of the privileging text will mark out how students are distributed in terms of results and will be read as an index of the quality of their education.

In Chapter 4 I discussed how the regulative discourses at the two schools differ and that this forms the parameters for the instructional discourse developed in the matric mathematics classes at each school. Since the pedagogic practice at School A appears to be driven by the requirements of the final mathematics examinations, the June examination is unlikely to produce any shifts in the pedagogic practice and indeed no changes after June are apparent in the student notebooks.

5.3 The example of sketching trigonometric curves at school A[13]

In section 5.1 I discussed how a weak expression of evaluation of student productions could limit the induction of students into mathematical discourse. The student's representations of trigonometric curves at school A do not comply with the mathematical conventions and for the most part do not represent the behaviour of the trigonometric functions (see Appendix 6, pages 148 - 157, for copies of the student productions of trigonometric curves at School A). In this section I analyse both what the student productions on the topic trigonometric curves indicate about the selection, pacing and sequencing of this sub-topic and how this combines with the effects of evaluation on the criteria that students develop for the construction of texts.

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[13] I focus only on the trigonometric curves at School A since the topic of trigonometry was not covered by students at school B.

Teachers have a fair amount of choice in how issues in any topic are selected, paced and sequenced. However, what is in the background are the parameters influenced by the provincial syllabus, past examination papers, the structuring of textbooks, relations and regulations at a school and community level. In this section of work the Western Cape Education Department Syllabus of Mathematics Higher and Standard Grade (implementation date January 1996) places a range of restrictions on what students enrolled for the matric on the Standard Grade need to know. One such restriction is that students only need to work in the domain 0° to 360°. The assumption behind this is that it will assist students who will not manage with more complex conceptualisations of the topic. Dowling (1998) shows that ability is a social construct and develops a language of description for how this is manifest in texts and subjects. He argues that much of the fragmenting of topics into sub-topics which are then simplified into procedures does not facilitate students access into mathematics, but it in fact prevents students' apprenticeship into the discourse and positions them as alienated subjects.

The way in which a topic is introduced, and the way the parameters of the topic are initially outlined can set limitations on the way that students come to understand the topic. At School A students first did calculations and equations with trigonometric identities and reciprocal relationships, before starting any work on the trigonometric curves[14]. Since students first approached trigonometry from a more algebraic and a less graphical form, it appears that this has had an effect on the way in which students conceptualisation of trigonometry developed. Students appear to understand trigonometry primarily as a collection of algebraic identities. This appears to be partially responsible for the way that students tend to construct trigonometric curves algebraically by plotting the critical points. If the graphical representation had been incorporated into the initial induction into trigonometry, students might have developed different criteria for the construction of the curves and might have understood the behaviour of the trigonometric functions in different ways.

The students at school A deal with the trigonometric curves as a separate and final sub-topic of trigonometry (see Appendix 6 for copies of each student's productions on trigonometric curves). The students appeared to have spent only three days on this subtopic. However, not all the students dated their work. In particular, student A1, who was the top student, seldom dated his work and often did not write down the questions that students were required to calculate, or the definitions, laws, notes or exemplars

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[14] Only one of the four students, viz. Student A3 did rough sketches of sine, cosine and tangent curves in the initial notes on trigonometry. However, there is no clear reference back to these curves while students were busy doing the sections on trigonometric identities and relationships.

provided by the teacher. He generally only wrote down what was useful to himself during the process of calculating or solving of problems.

In addition students sometimes have different headings (both date and type of work) on the same piece of work[15]. For example, most of the work the student A3 does in this section has different dates and headings to the other students and is sequenced in an inconsistent manner. The work that other students dated 20 July, she dated 21 July and gave it the title "Homework" although it consists of exemplars as a form of exposition. The next piece of work is "classwork" and was dated 22 July. On the page after the classwork of 22 July, she again wrote "Homework" which was again dated 21 July. This was followed by "homework" which was dated 23 July. One explanation for this is that she copied the work from another student on 21 July, which might explain why the teacher did not mark her work when he marked the other books and why the dates in her headings are so inconsistent. She often includes "Grade 12.1" as part of her headings. This appears to have been copied down off the blackboard, since it might be something that the teacher would write, whereas students both know what class they are in and already have this written on the outside of their notebooks.

It appears as if student A1 understands that the key to evaluation in school mathematics is the production of certain transformations. He understands enough of the discourse to be able to note only the essential signifiers and those aspects of the transformations that will help him calculate the answers to the questions. Student A3, who is amongst the least successful students in the class, on the other hand, appears to write down everything that is put up on the blackboard. This can be described as a form of radical nominalism. She appears to be outside of the discourse to the extent that she is unable to distinguish which are the essential signifiers, and is compelled to note everything to ensure that she does not miss out on any of them.

5.3.1 Initial notes on trigonometry curves

Although it is unusual at this school for students to start a section of work with exemplars, all four students copy down examples of $y = \sin x$, $y = \cos x$, $y = \tan x$ curves, for the domain 0° to 360° (see Plates 1 – 4). Most of the students date these notes as "20/07/98" and write their notes at the back of their notebooks, which is another unusual practice for this class.

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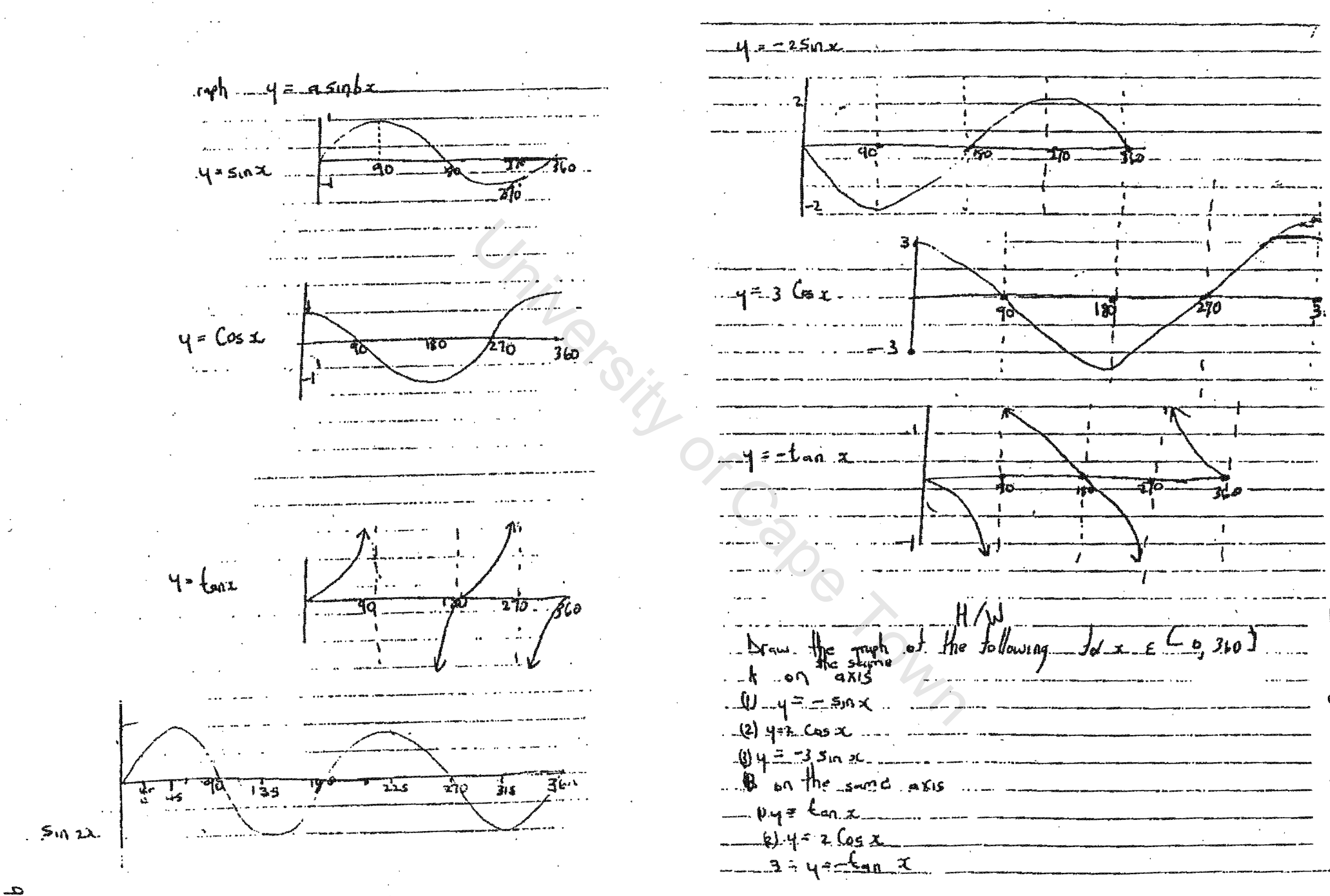
[15] Recall that the notebooks analysed from School A belonged to students in the same class.

Plate 1 : Student A1
Initial Trigonometry notes



H/W

Draw the graph of the following for x ε [0, 360]
& on the same axis
(1) y = – sin x
(2) y = 2 Cos x
(3) y = –3 sin x
B on the same axis
1) y = tan x
2) y = 2 Cos x
3 ; y = –tan x

Plate 3 : Student A3
Initial Trigonometry
notes

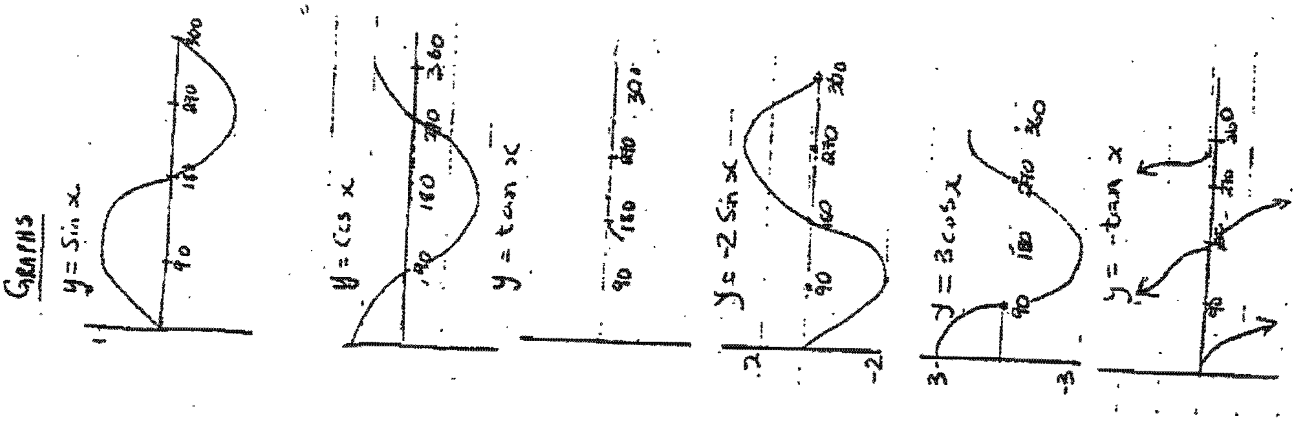


Plate 2 : Student A2
Initial Trigonometry
notes

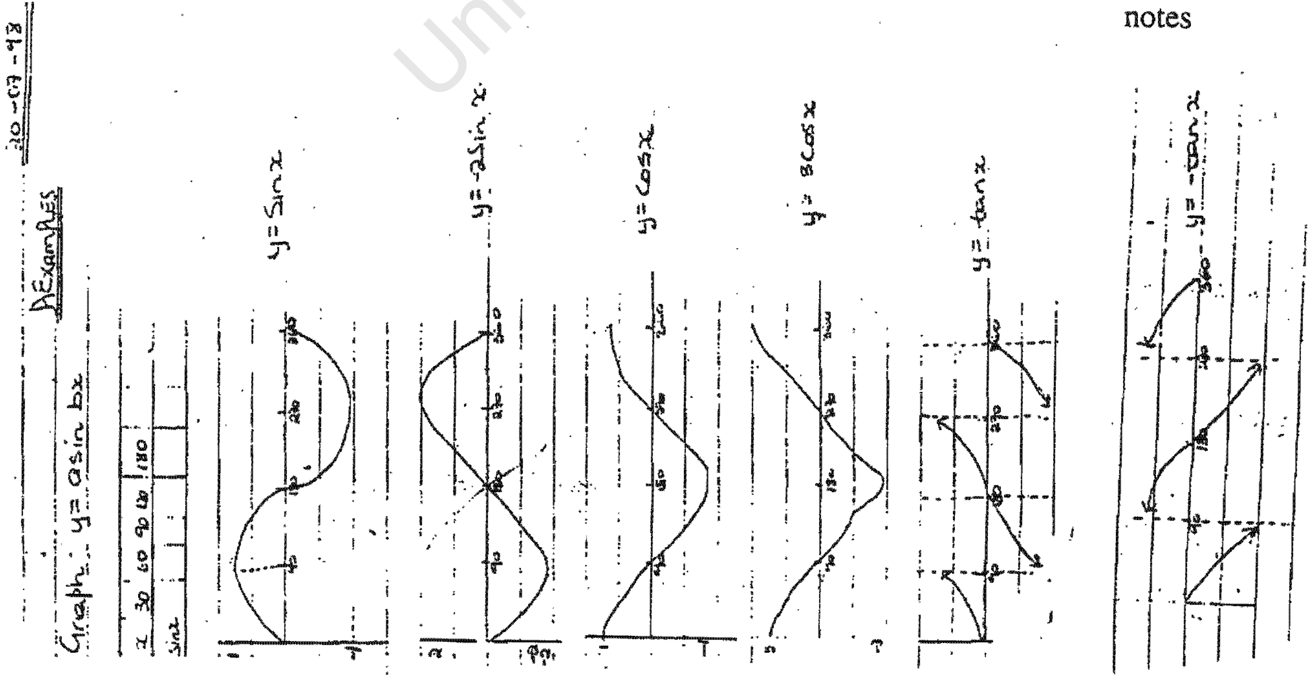
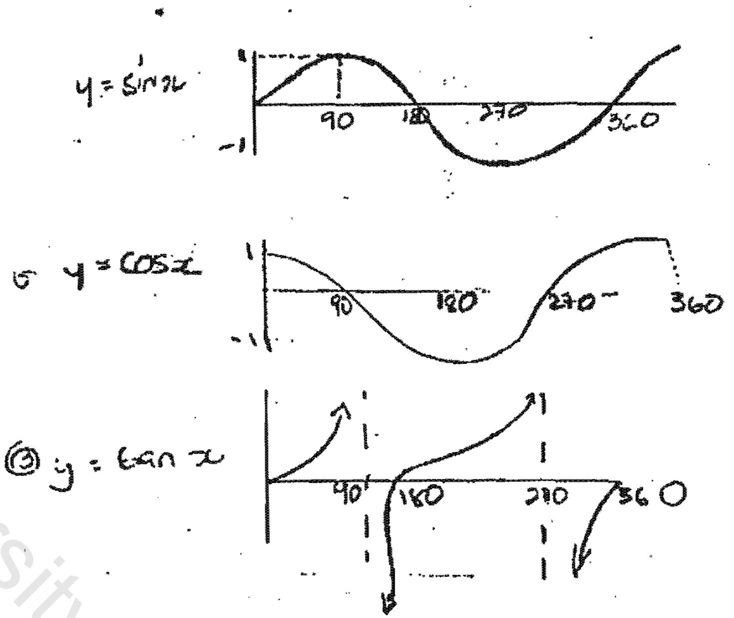


Plate 4 : Student A4
Initial Trigonometry
notes



x	0	30	60	90	180		360
sin x	0	0,5	987	1	0,09	0,5	—

H/w

[illegible] the graph of the following for x = (0,360)

on the same graph

y = -sin x

y = 2 cos x

y = -3 sin x

on the same axis

y = tan x

y = 2 cos x

y = -tan x

Each curve is sketched on its own set of axes. Students all label each curve either alongside the axes or above them. In subsequent work where students sketch several curves on the same sets of axes, they often do not label the individual curves. It appears from this that students might not understand that in the exemplar the teacher is labelling each individual curve. Students may be assuming that the label needs to be as a heading alongside the set of axes, rather than as a label on each individual curve.

The sketches do approximate the behaviour of the functions over this range. Student A2 (Plate 2) wrote out a table of y-values for x = 30°, 60°, 90°, 120°, 180°, 240°, 270° & 360°, before sketching any of the curves student A4, wrote out a table of y-values, where $y = \sin 2x$ for $x = 30°$, 60°, 90°, 120°, 180°, 240°, 270° & 360° (despite this she did not sketch the y = sin $2x$ curve). This tends to indicate that the teacher was privileging the algebraic calculation of critical points from the outset. Although there is no indication initially of him plotting these points as the basis of the construction of the curves.

Three of the four students: student A1 (Plate 1), student A2 (Plate 2) and student A3 (Plate 3) also draw curves for $y = 2 \sin x$, $y = - 2 \sin x$, $y = 3 \cos x$ and $y = -\tan x$. None of these students correctly represent the inflection of the $y = - \tan x$ curve around the x – axis. The two high achieving students, student A1 and student A2, draw the $y = 3 \cos x$ curve in an unsteady way and have one of the turning points more sharp than it should be. In the latter three curves the student A1 and student A3 mark 90°, 180°, 270° & 360° with large dots along the x-axis, student A2 indicates these positions with small lines that cross the x- axis. Plotting the points first makes it more difficult to draw a smooth curve. Compared with the curves drawn on subsequent days, these sketches are reasonable representations of the behaviour of the functions.

5.3.2 First piece of homework

Most students label this work "homework 20 July". Students write down an instruction to sketch the following curves for $x \in [0°, 360°]$. On the first set [A] of axes they are asked to sketch:

1) $y = - \sin x$
2) $y = 2 \cos x$
3) $y = -3 \sin x$

On the second set [B] of axes they are asked to draw

1) $y = \tan x$
2) $y = 2 \cos x$
3) $y = -\tan x$

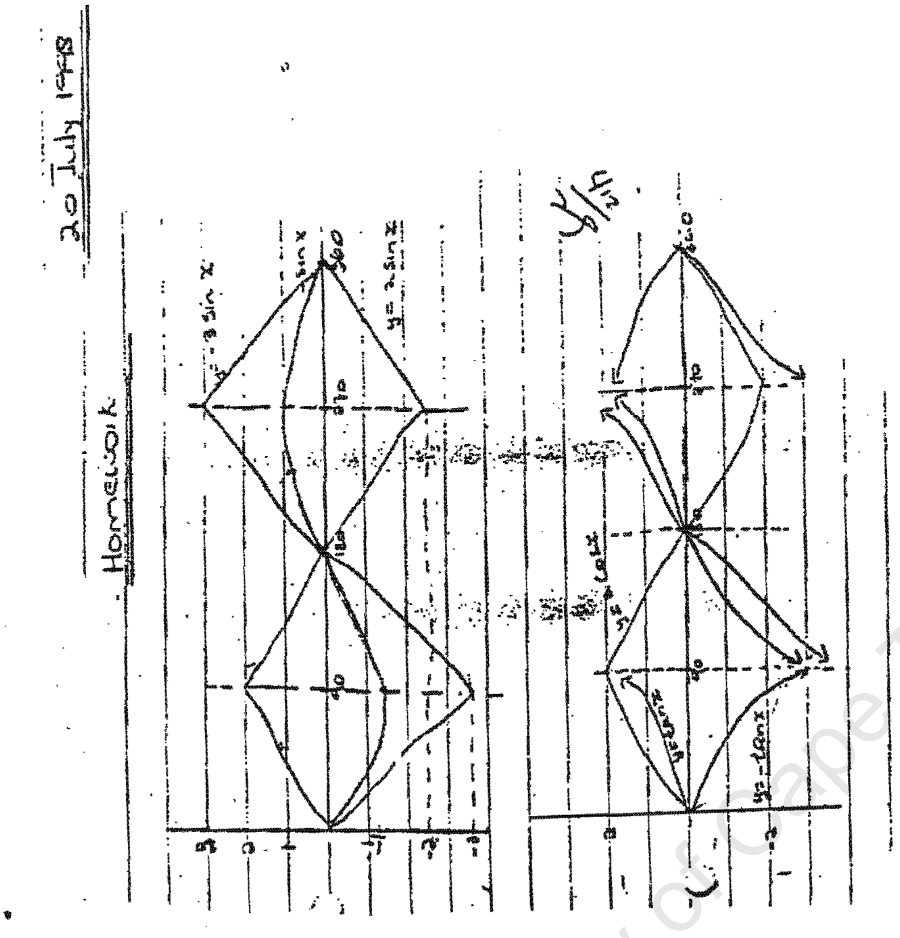


Plate 6 : Student A2
First piece of
Trigonometry homework

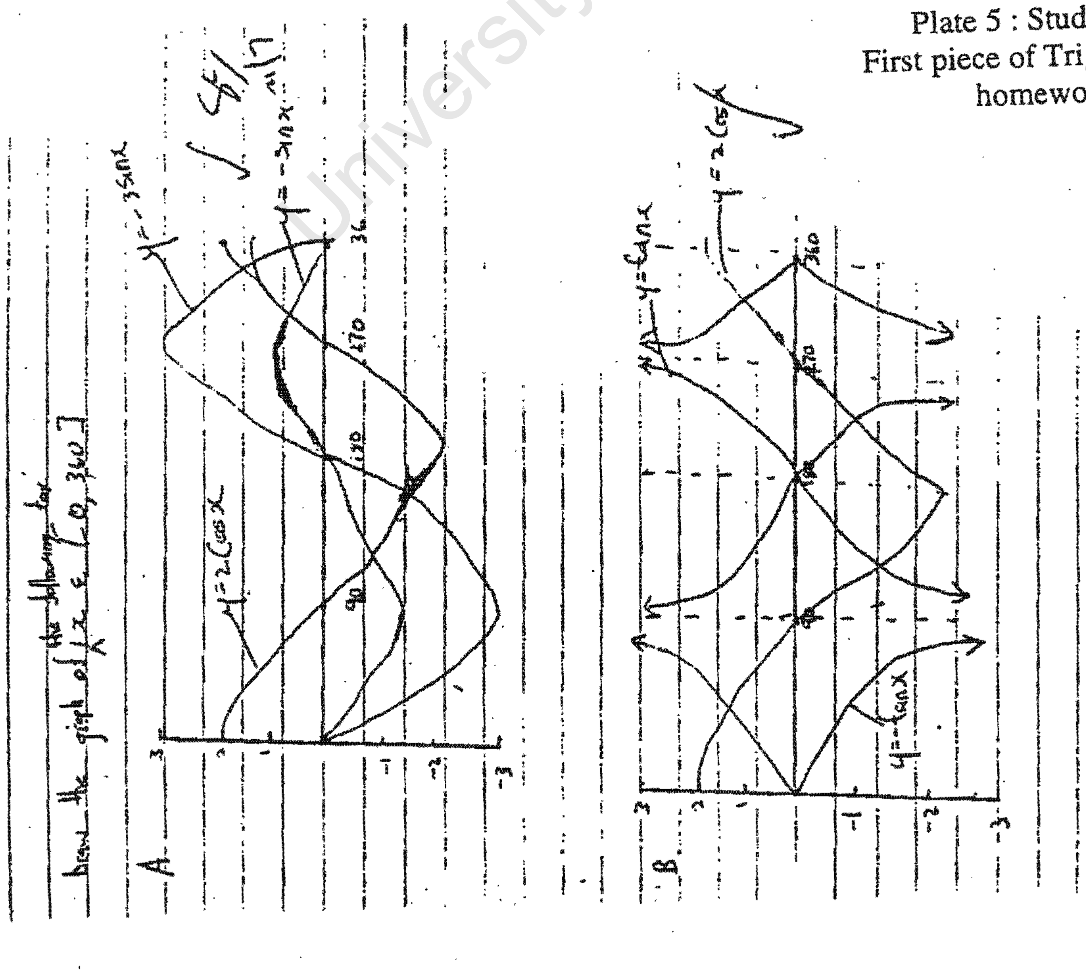


Plate 5 : Student A1
First piece of Trigonometry
homework

Student A1 (Plate 5) labels all three curves on both sets of axes. He shows correctly where the various curves pass through the critical points. The positions where the curves cut the x – axis are shown in the form of large black dots, similarly to the way the last three exemplars were represented. The other critical points are not marked out in any way. The curves are sketched very roughly (the lines are unsteady, at times he draws double lines, and both cosine curves are a bit sharp about the bottom turning point) but their behaviour does approximate the behaviour of sine, cosine and tangent curves. He is the only student who correctly indicates the behaviour of tangent curves as they approach the asymptotes. The teacher signs his book, on "21/7" but does not comment on the work. Student A2 (Plate 6) sketches and labels each of the six curves. She indicates in the correct places where the curves cut the axes. She correctly shows the turning points of the sine and cosine curves. However, most of the curves that she draws appear to look much more like repeated absolute value curves and do not represent the behaviour of sine and cosine curves. Only one of the four sine curves approximates the behaviour of the sine function. Similarly she shows correctly where the tangent curves cuts the y-axis but she does not show correctly the behaviour of the curves as they approach the asymptotes. The $y = -\tan x$ curve is incorrect between 90° degrees and 270° degrees. The teacher also signs her book, on "21/7" but neither corrects nor comments on her work.

Student A4 (Plate 7) sketches six functions. She does not indicate which of the curves represent which of the functions. She does not have the correct period for five of the six curves. The amplitudes of the sine and cosine curves vary for each curve. She labels the y-axis incorrectly: she places – 3 above –2. She also shows the $y = -3 \sin x$ curve cutting the y-axis at –3 and not at zero. She does not distinguish between the behaviour of the $y = \tan x$ and $y = -\tan x$ curves. The way in which these two curves are drawn does not represent the functions in any way. Her work is neither signed by the teacher nor corrected by the student herself.

In the first piece of homework student A3 (Plate 8) correctly indicates the value of the sine and cosine functions at x = 90°, 180°, 270°, 360° by drawing large black dots. Other students only start to mark these points with large black dots from the homework of 21 July. Her sketches of the sine and cosine curves do approximate the behaviours of those functions. She correctly labels each of the curves. On another set of axes she draws two tangent curves and a cosine curve. She correctly labels each of the curves. The cosine curve correctly indicates the behaviour of the function. Her sketch of the tangent curve correctly indicates where it crosses the x – axis, but it does not approximate the behaviour of the curve as it approaches the asymptotes.

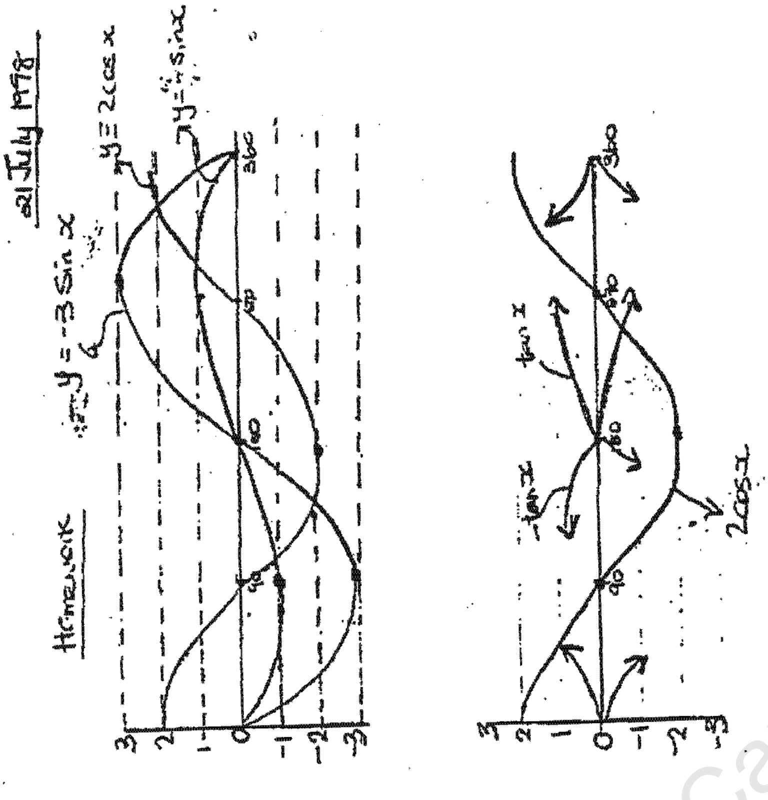
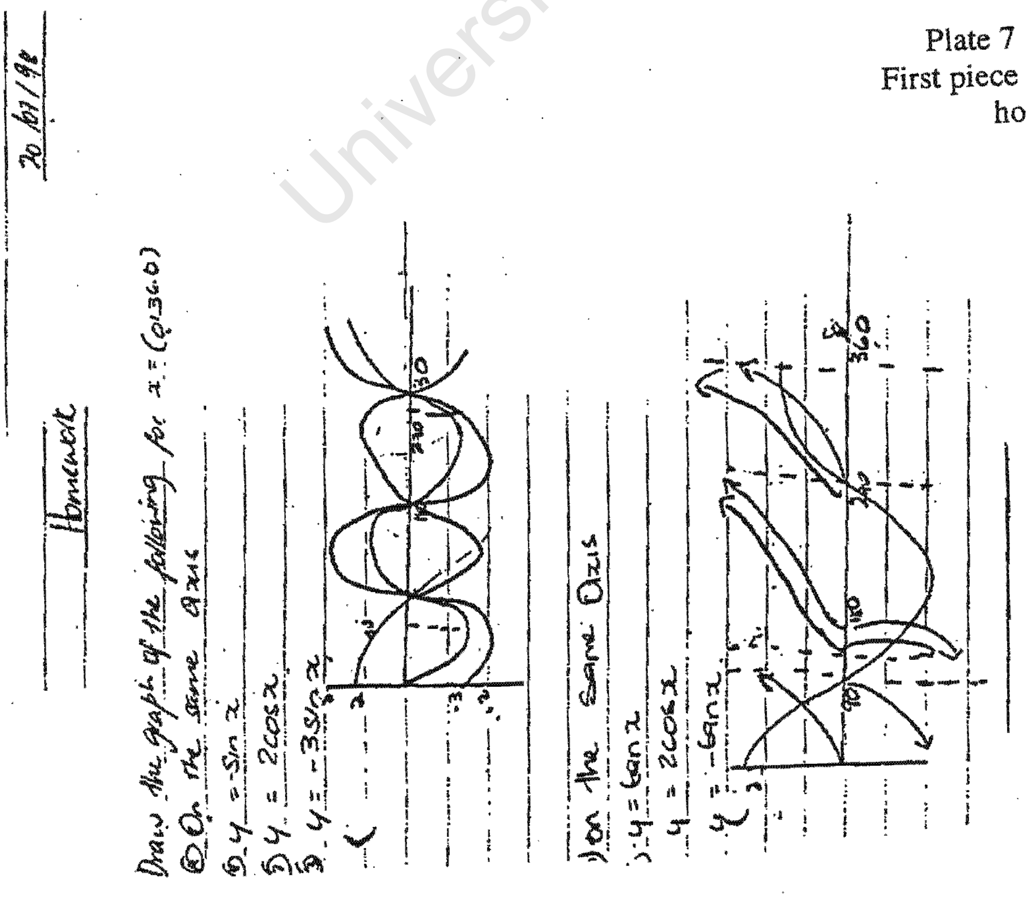


Plate 8 : Student A3
First piece of Trigonometry
homework


Homework

20/07/98

Draw the graph of the following for x ∈ (0;360)

On the same axis

y = sin x

y = 2cos x

y = -3sin x

On the same axis

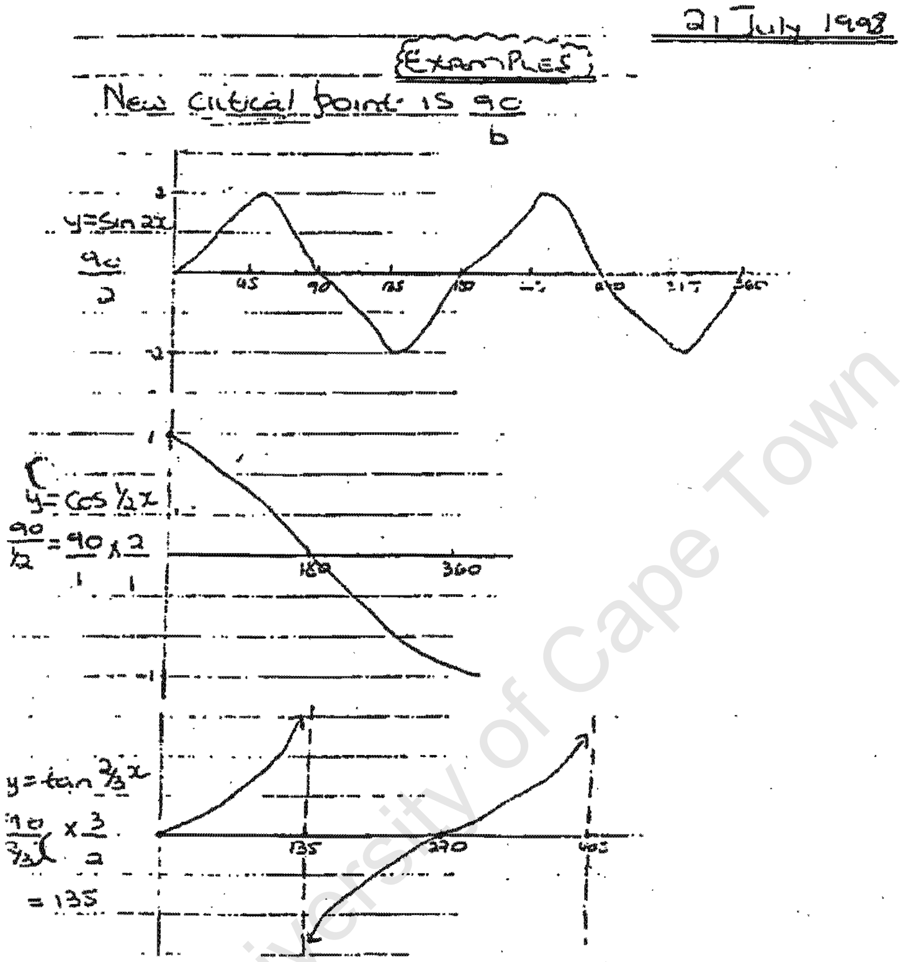
y = tan x

y = 2cos x

y = -tan x

Plate 7 : Student A 4
First piece of Trigonometry
homework

Plate 9 : Student A2
Second set of Trigonometry notes



21 July 1998

Examples

New critical point is $\frac{90}{b}$

5.3.3 Second set of notes

Only student A2 writes out the exemplars given on 21 July (see Plate 9). She, characteristically, gives these the title "Examples". These exemplars are introduced with the note: " new critical point is 90/b". However, the focus of these exemplars appears to be on how to calculate the distance between critical points. This is corroborated by the notes towards the end of the section of work on trigonometric curves, where all the students have noted:

$$\text{Period of }\left\}\begin{array}{l} y = \sin bx \\ y = \cos bx \\ y = \tan bx \end{array}\right. \qquad \text{is } 360/b$$

Distance between critical points is 90/b

Alongside each of student A2's curves are the name of each function and calculations of 90/b. The cosine and tangent curves are reasonable representations of the behaviour of those functions. However, the sine curve is drawn with sharp turning points and in an unsteady way.

5.3.4 Second piece of classwork

In these productions students sketch the curves of the functions $y = 3 \sin 3/2\ x$ and $y = -\cos 3x$. Student A1 shows some working out of the distance between critical points (see Plate 10). He draws and labels the curves. Here the critical points are marked out with large black dots, some of the turning points are sharp and some are more rounded.

Student A2 labels this work "classwork, 21 July" (see Plate 11). She shows calculations of the distance between critical points for the functions $y = 3 \sin 3/2\ x$ and $y = -\cos 3x$. The critical points of the cosine curve are marked with dots. She only draws the cosine curve, which, like student A1, has some turning points pointed and some curved.

Student A3 writes out the equations of the functions and draws the axes (see Plate 12). She does not attempt to draw the curves, nor yet does she calculate the distance between the critical points.

Student A4 does not do this work at all.

Plate 10 : Student A1 Second piece of classwork

Draw on the same set of axis

1) $y = 3 \sin \frac{3}{2}x$ . . . sin $3 \times \frac{2}{3} = \frac{180}{3} = 60$

2) $y = -\cos 3x$ . . . $\frac{90}{3} = 30.$

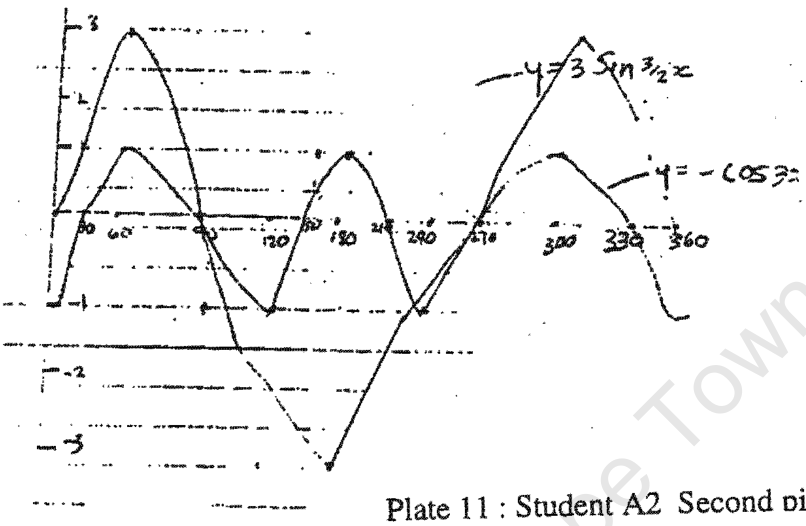


Plate 11 : Student A2 Second piece of classwork

21 July 1998

~~Homework~~ Classwork

(a) $y = 3 \sin(\frac{3}{2}x)$ & $y = -\cos 3x$

$\frac{90}{P} = \frac{90}{1} \times \frac{2}{3}$ $\frac{90}{3} = 30$

$= \frac{90(180)}{3} = 60$

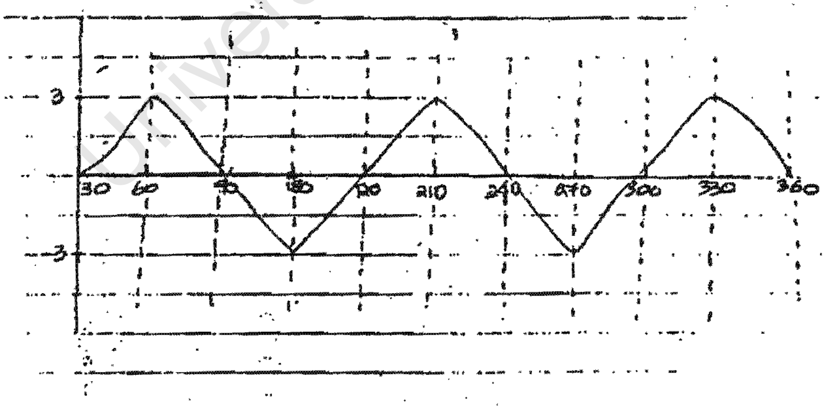
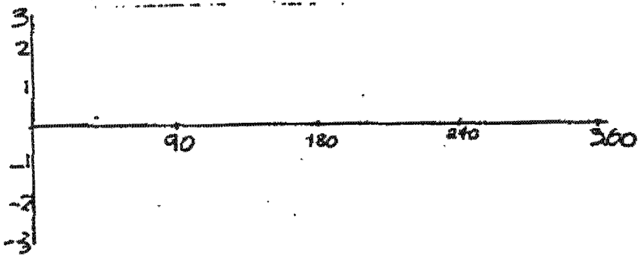


Plate 12 : Student A3 : Second piece of classwork

Classwork 22 July 1998

1) $y = 3 \sin \frac{3}{2}x$

2) $y = -\cos 3x$



5.3.5 Second piece of homework

Here students are required to sketch the following three curves on the same set of axes:

$y = 2 \sin 3x$

$y = -3 \cos 3/2\, x$

$y = -\tan x$

In the student productions of this second piece of homework the effect of the privileging of the calculation of the distance between critical points is clearly displayed (see Plates 13 – 16).

In the work that other students have labelled as "homework 21 July", student A1 shows not only the x-intercepts, but also all critical points as large black dots (see Plate 13). He continues to label each of the curves. However, these curves are drawn to look more like absolute value curves than sine or cosine curves. Over time the curves that student A1 sketches increasingly misrepresent the behaviour of the functions. What is interesting here is that this comes after the worked examples, written down by student A2, in which the teacher shows how to calculate the distance between critical points. It strengthens the assumption that they way the teacher is privileging these critical points is inadvertently encouraging students to draw the curves in a way which does not represent the behaviour of the function. This is an example of how evaluation goes beyond marking things correctly or incorrectly but includes what is and is not privileged in the interaction and what is and is not noted or considered worthy of comment. Evaluation works as much in its absences as in its presences.

In the homework of 21 July, student A2 once more starts by calculating the distance between critical points for each of the curves (see Plate 14). Once more the tangent curves do not correctly show the behaviour as they approach the asymptotes, although this behaviour is better approximated at some asymptotes than at others. A small cross within a circle marks out the critical points. From this date onwards student A2 draws the critical points as very distinctive circles with crosses inside them, until the last day's work where the critical points are shown as large black dots. The sine curve is drawn with smooth curves, but the cosine curve looks like a repeated absolute value curve. Student A2 does not label the curves. This is despite the fact that the three curves all appear on the same set of axes. The y-axis is incorrectly labelled with -3 appearing above -2. The amplitude of the sine and cosine curves is shown as the same, instead of the amplitude of the sine curve being 2 and the cosine curve being 3. What is notable about student A2's errors is that the teacher signs the book but does not point out the error. Student A2 does write out "corrections" of the homework of 21 July. This she does after the classwork of 22 July. In these "corrections" the axes are correctly labelled and the critical points are correctly placed, the curves do have the correct amplitude, but the curves do not approximate the behaviour of the functions: they appear even more like repeated absolute value curves.

Student A4 labels what the other students call homework of 21 July as classwork dated 21 July (see Plate 15). In her curves the critical points are also marked by large black dots and she attempts to calculate the distances between critical points. She does not do this calculation for the tangent function. Surprisingly, this is done beneath the curves, which might indicate that it was calculated after the curves were drawn and not before they were drawn, and even possibly that she copied down the work from another student. Here the spacing of the numbers on the y –axis is inconsistent. This may partially explain why her y = -3 cos 3/2 x curve shows the turning point at 2 and not at 3: she might have been attempting to draw the curve symmetrically and since 2 is at the same distance from the origin as -3 the curve turns at these two points and not at 3 and -3. It may also explain why the amplitude of the sine and cosine curves is shown as the same. The tangent curves in this example are incorrect and inconsistent: the distances from y-axis to the asymptote have different lengths in different places. The reader is left questioning whether she understands tangent curves at all.

Student A3 does this work under the heading "Grade 12.1 Homework 21 July" (see Plate 16). Student A3 draws large black dots to correctly mark the position of the critical points of the sin and cosine curves, and she labels each curve. However, her curves look like repeated absolute value curves. She does not show the tangent curve, but draws a miniature version of what part of it might look like slightly away from the axes.

5.3.6 Third set of exemplars

In the exemplars given to the students on 22 July, students copy sine and cosine curves and are shown how to read the critical points off the curve to complete the function equations (see Plates 17 - 20). Student A2 (see Plate 17), student A4 (see Plate 18) and student A3 (see Plate 19) sketch these curves in a more appropriate way than they did previously. This is probably both because they are copying the curves of the blackboard and because they are not sketching the curves by joining up the critical points. They also calculate the critical points. Student A1 draws these curves and several other curves, and calculates the critical points from the curve (see Plate 20). However, since he does not always label the curves or the axes, it may be unclear to another reader what he is calculating. Student A1 characteristically notes down what he needs to as an intermediary step towards answering the questions: once he has the critical points he also does not complete the function equations.

Plate 13 : Student A1 Second piece of homework

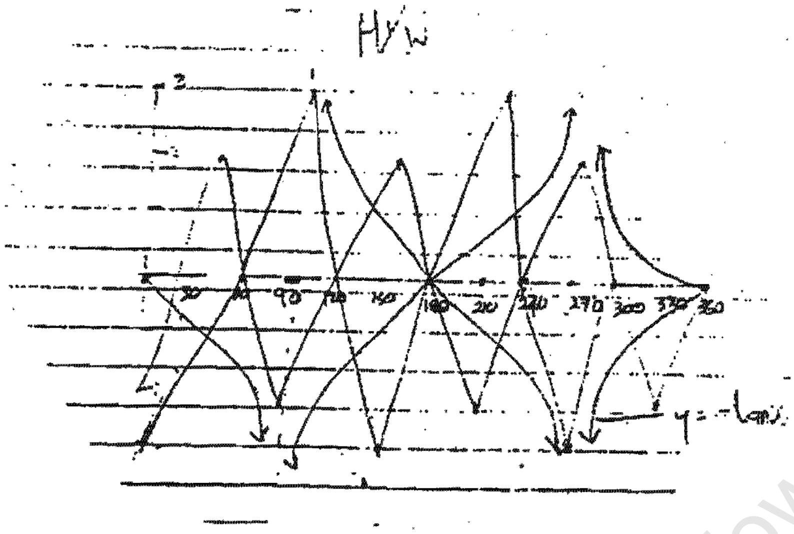


Plate 14 Student A2 Second piece of homework

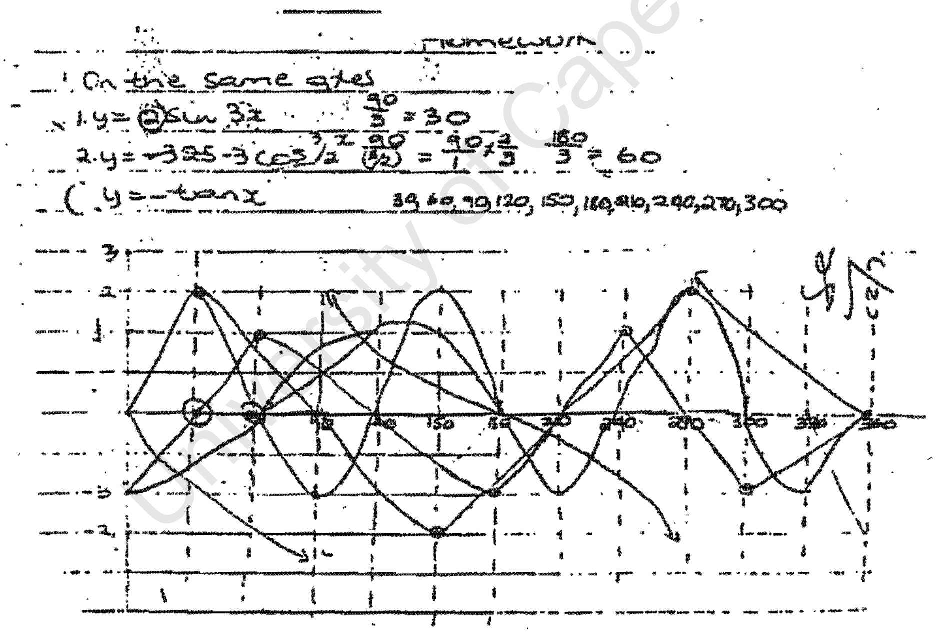


Plate 15 : Student A4 Second piece of homework

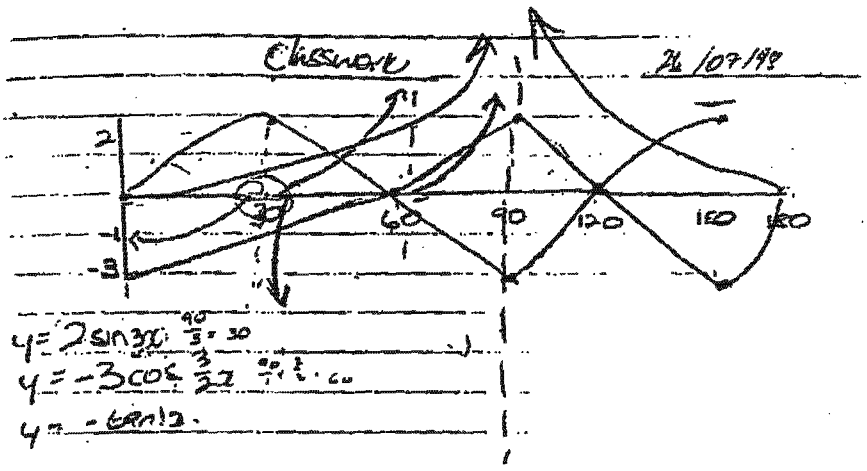


Plate 16 : Student A3 Second piece of homework

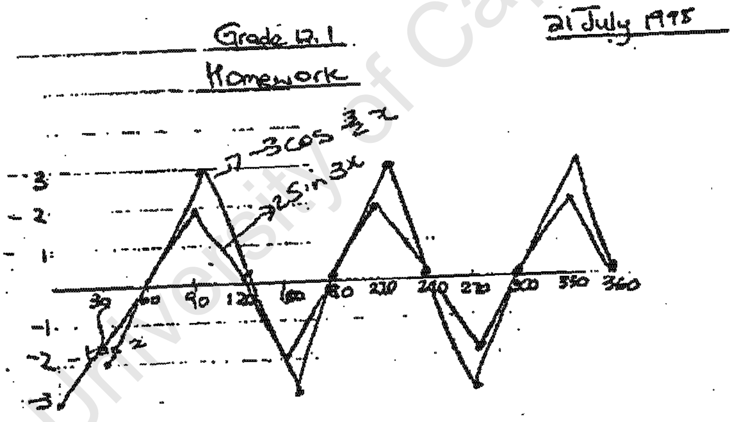
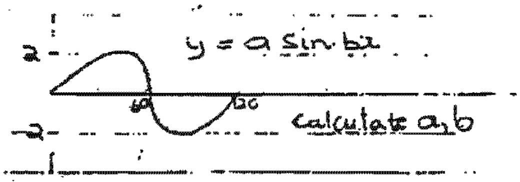


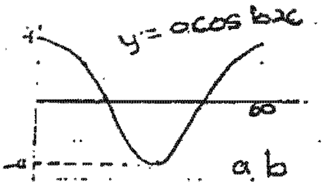
Plate 17 : Student A2 Third set of exemplars

22 July 1993

EXAM [illegible]

From the following functions Calculate



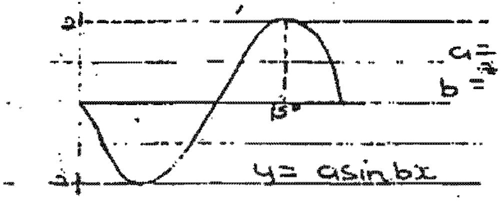


a = 2

$b = \frac{180}{60} = 3$

a = 4

$b = \frac{360}{60} = 6$



a = -2

$b = \frac{270}{15} = 18$

Corrections

y = 2sin 3x ! y = -3 Cos $\frac{3}{2}x$

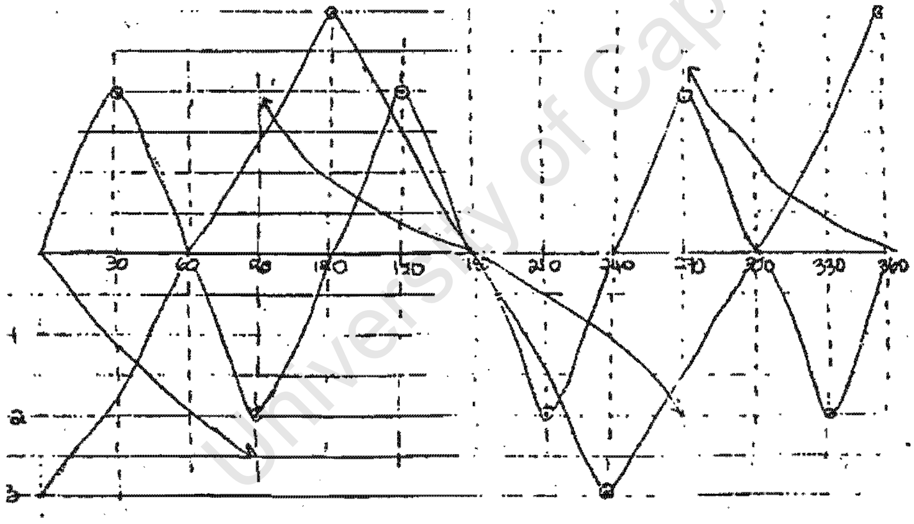


Plate 19 : Student A3 Third set of exemplars

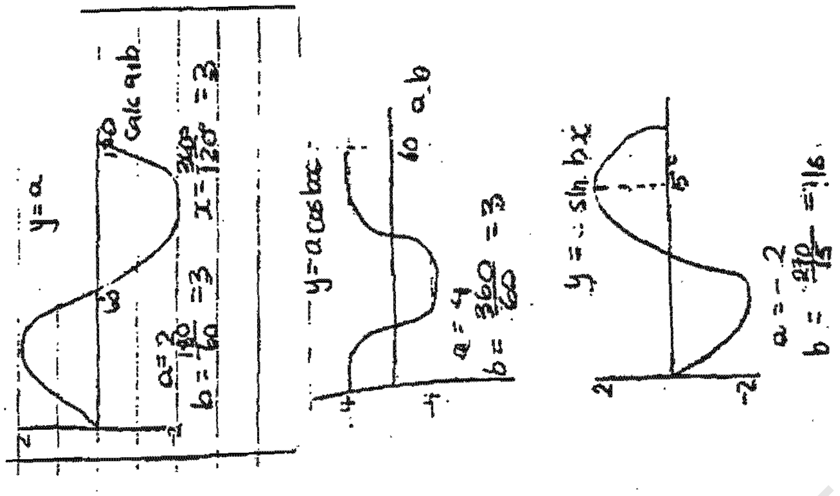


Plate 18 : Student A4 Third set of exemplars

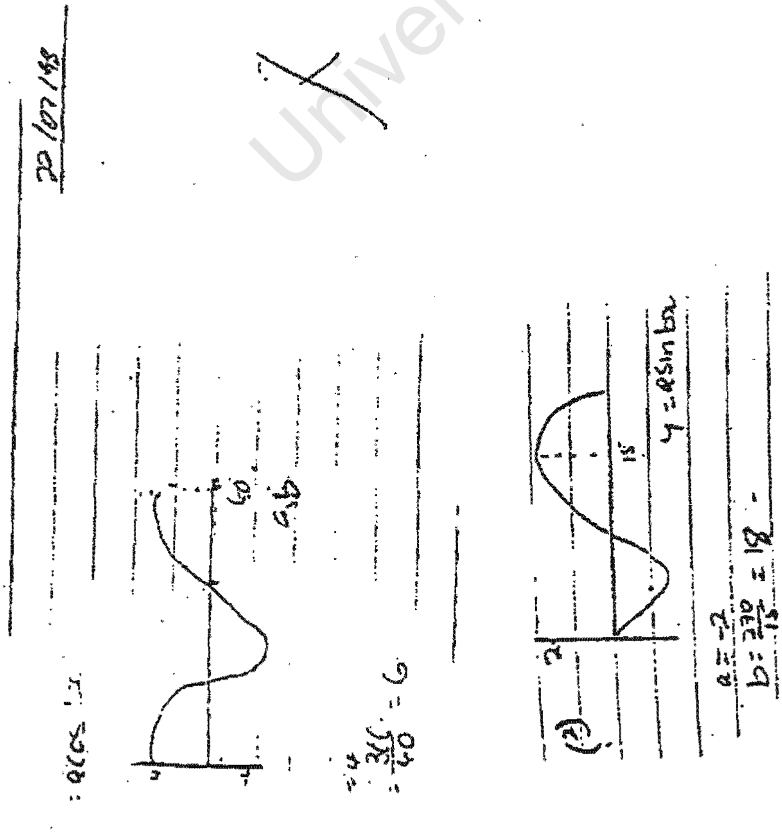
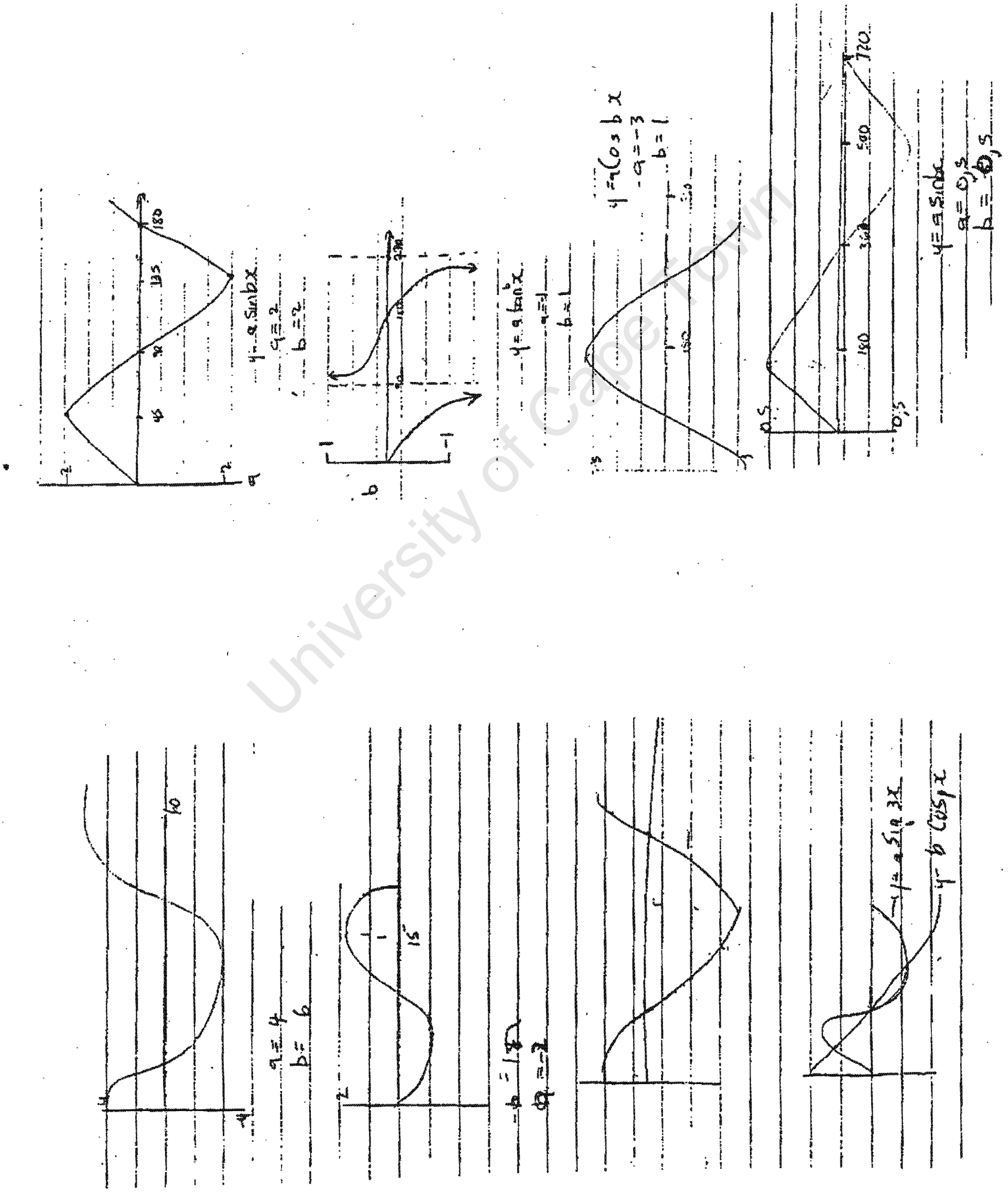


Plate 20 : Student A1 Third set of exemplars



5.3.7 Final set of notes

In the final set of notes all students write out definitions for amplitude, period and critical points (see Plates 21 - 24). They note how to find out the amplitude, period and distance between critical points. This is in keeping with the general style of pedagogy in this class, where students start with definitions, do lots of practice examples, get some form of summary notes before doing the final section of work. Most students write this out at the back of their books, but student A2 places it amongst her other work.

5.3.8 Third set of classwork

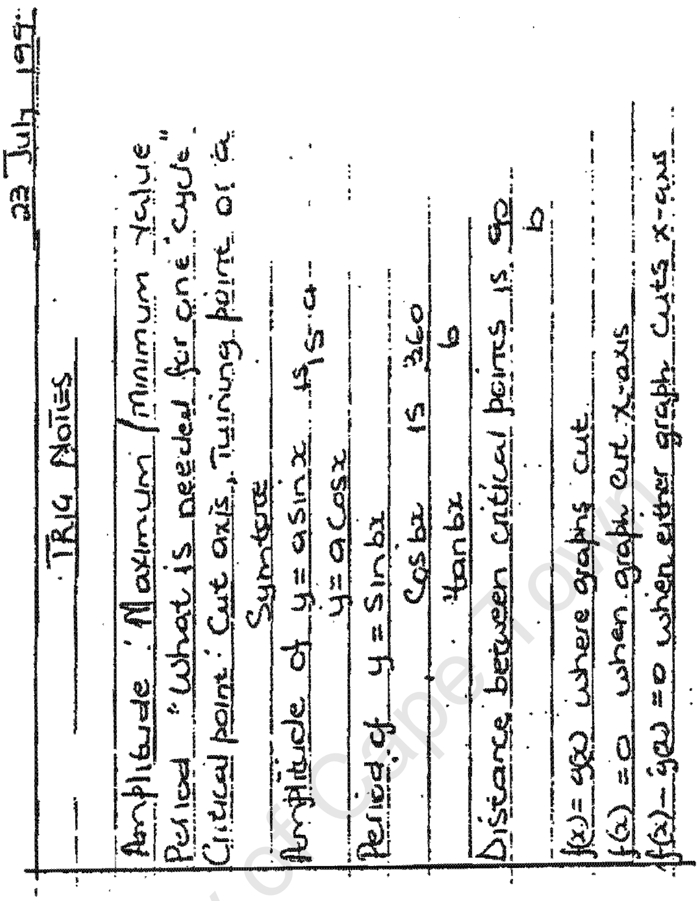
In this practice exercise students copy down a given sine and tangent curve and are asked to calculate specific points on the curves. Here student A2 (see Plate 25) and student A4 (see Plate 26) copy down the curves correctly (student A4 calls this piece of work "Homework"). Student A1 merely does a rough sketch in which the sine curve is rather uneven. Only student A2 notes the points on the curve that they are asked to calculate. All three students calculate the points correctly.

Student A1 (see Plate 27) and student A2 then draw another axis with both a tangent curve and a cosine curve. They calculate various points on the two curves. Only student A2 marks these points on the curves, student A1 does not note these points nor does he write out any questions about this. These are the types of questions that students are likely to have to solve in the final examination. Neither of the other two students attempts to do this work.

5. 3. 9 Written evaluation

In this section of work the teacher only signs two of the students notebooks. This he does on two occasions in each of the notebooks. On none of these occasions does he correct any of their work. He does not use the opportunity to advise students on how the curves ought to look, and so misses the opportunity to teach students the appropriate form of representation. Neither do the students' misrepresentations of the functions appear to impact on his further teaching of the topic. One of these students writes out "corrections" after one occasion in which the teacher signed the book, but only on the following day after taking down new notes from the board. However, although aspects of these "corrected" curves are corrected, they remain an inaccurate reflection of the behaviour of the functions. One student places a cross alongside one piece of work. This might be read as an indication of an error. However, the work is correct.

Plate 22 : Student A2 Final Trigonometry notes



23 July 199

TRIG NOTES

Amplitude : Maximum / Minimum value

Period : "What is needed for one 'cycle'"

Critical point : Cut axis, turning point or a Symtote

Amplitude of $y = a\sin x$ is a

$y = a\cos x$ is a

Period of $y = \sin bx$

$\cos bx$ is $\frac{360}{b}$

$\tan bx$

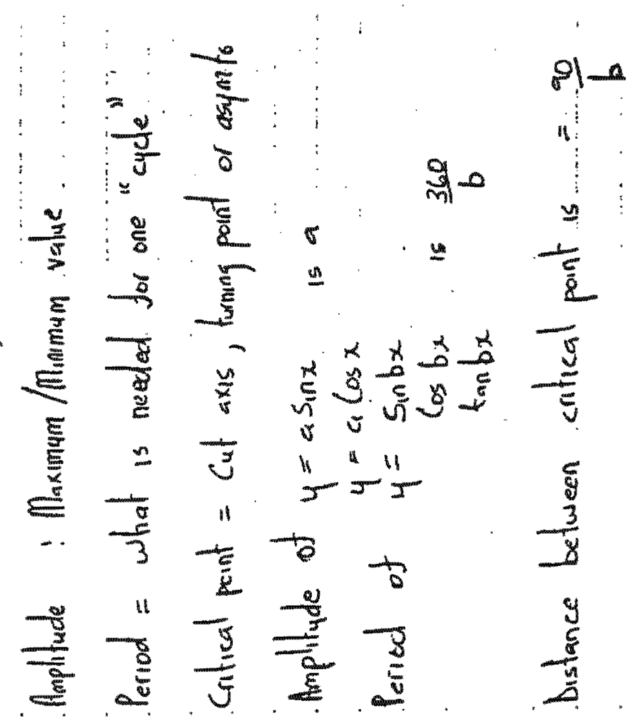
Distance between critical points is $\frac{90}{b}$

$f(x) = g(x)$ where graphs cut

$f(x) = 0$ when graph cut x-axis

$f(x) - g(x) = 0$ when either graph cuts x-axis

Plate 21 : Student A1 Final Trigonometry notes



Trig Notes

Amplitude : Maximum / Minimum value

Period = what is needed for one "cycle"

Critical point = Cut axis, turning point or asymptote

Amplitude of $y = a\sin x$ is a

$y = a\cos x$

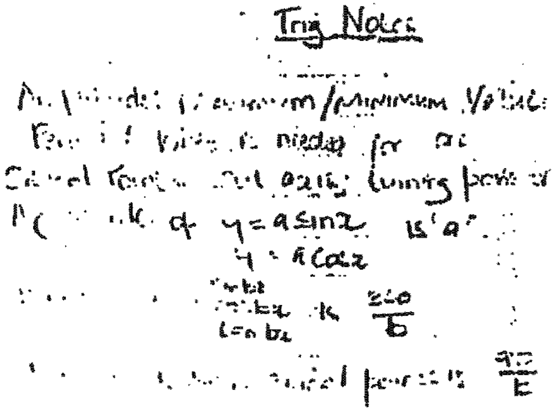
Period of $y = \sin bx$

$\cos bx$ is $\frac{360}{b}$

$\tan bx$

Distance between critical point is = $\frac{90}{b}$

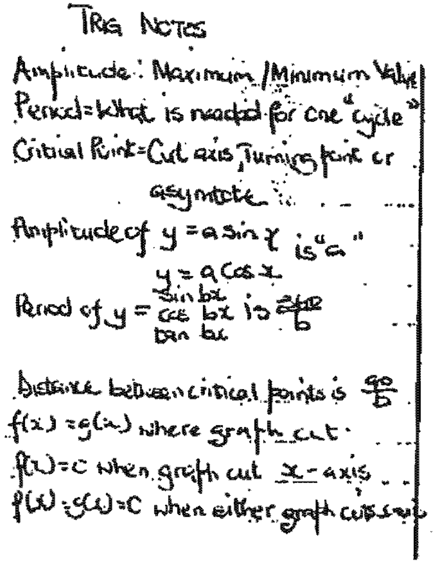
Plate 24 : Student A4 Final Trigonometry notes



Trig Notes

Amplitude: Maximum/Minimum Value
Period: [illegible] needed for one
Critical Point: cut axis, turning point or
Amplitude of $y = a\sin x$ is 'a'
$y = a\cos x$
[illegible] $\sin bx$ $\cos bx$ $\tan bx$ is $\frac{360}{b}$
[illegible] critical points is $\frac{90}{b}$

Plate 23 : Student A3 Final Trigonometry notes

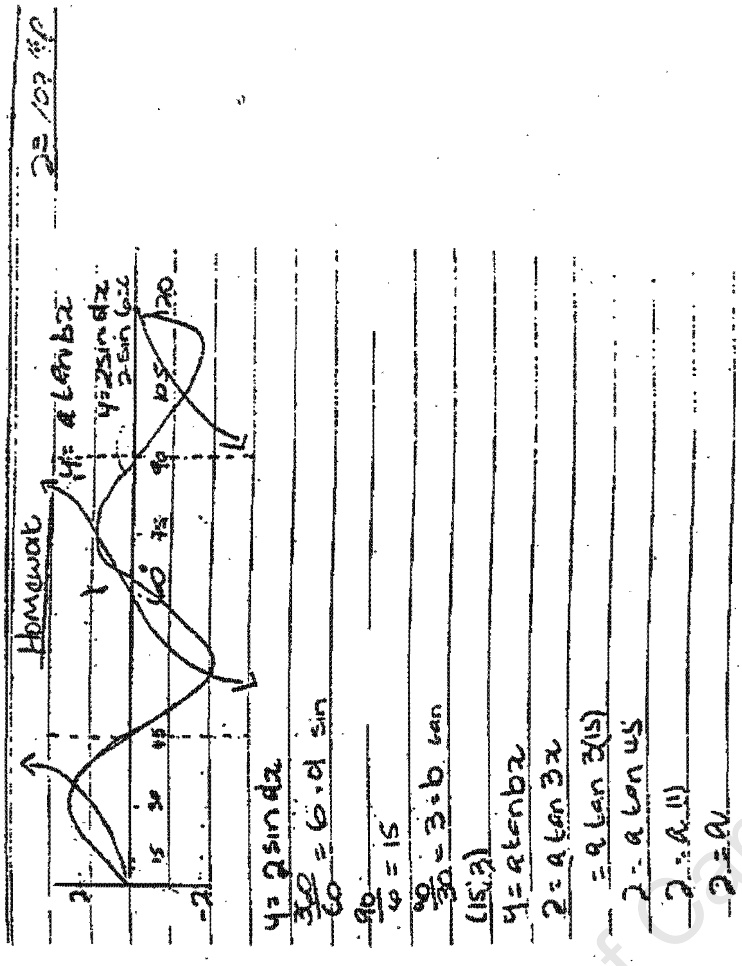


TRIG NOTES

Amplitude: Maximum/Minimum Value
Period = What is needed for one "cycle"
Critical Point = Cut axis, Turning point or asymptote
Amplitude of $y = a\sin x$ is "a"
$y = a\cos x$
Period of $y = \sin bx$, $\cos bx$, $\tan bx$ is $\frac{360}{b}$

Distance between critical points is $\frac{90}{b}$
$f(x) = g(x)$ where graph cut
$f(x) = c$ when graph cut x-axis
$f(x) = g(x) = c$ when either graph cuts [illegible]

Plate 26 : Student A4 Third set of classwork



Homewok

23/07/98

y = 2sin dx

$\frac{360}{60} = 6 : d$ sin

$\frac{90}{6} = 15$

$\frac{90}{30} = 3 : b$ tan

(15;2)

y = a tan bx

2 = a tan 3x

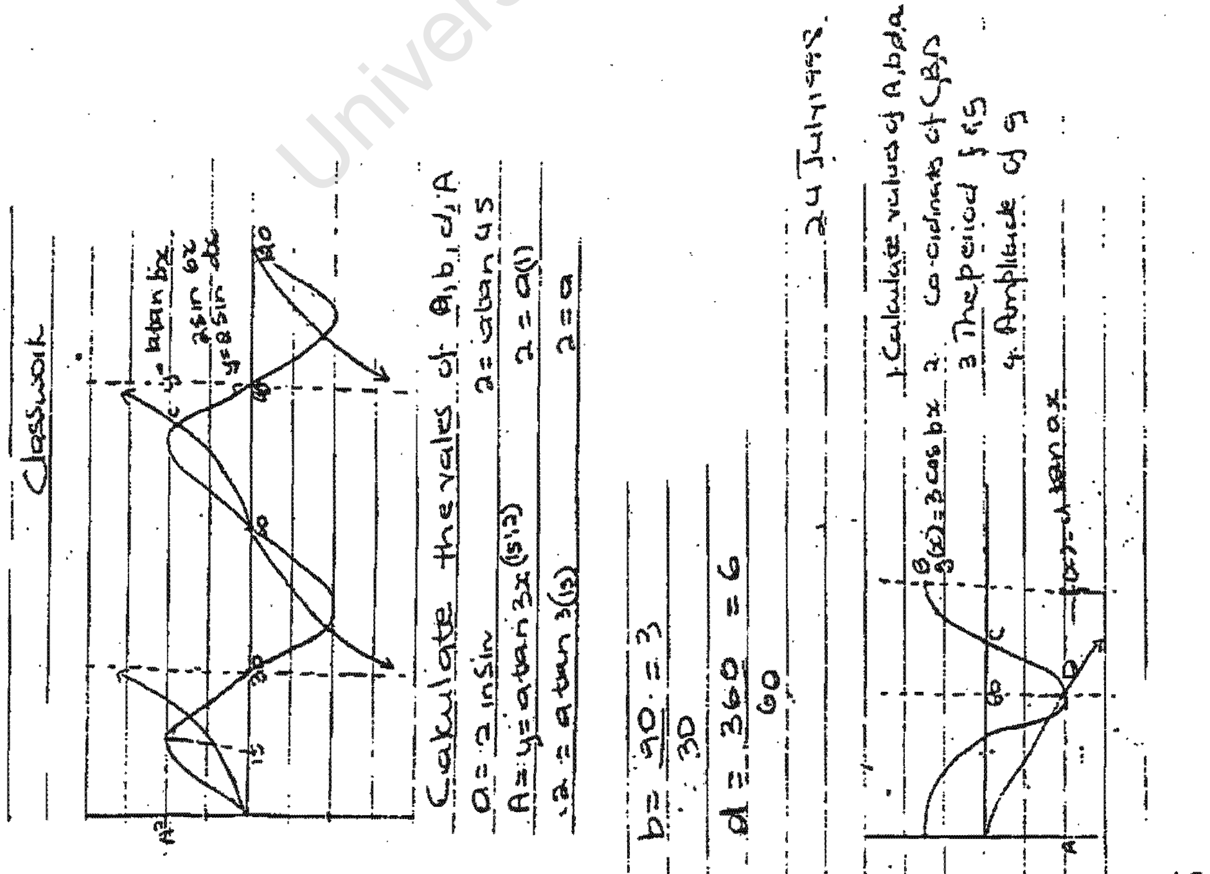
= a tan 3(15)

2 = a tan 45

2 = a(1)

2 = a

Plate 25 : Student A2 Third set of classwork



Classwork

Calculate the vales of a,b,d,A

a = 2 insin

A = y = a tan 3x (15;2)

2 = a tan 3(15)

2 = a tan 45

2 = a(1)

2 = a

$b = \frac{90}{30} = 3$

$d = \frac{360}{60} = 6$

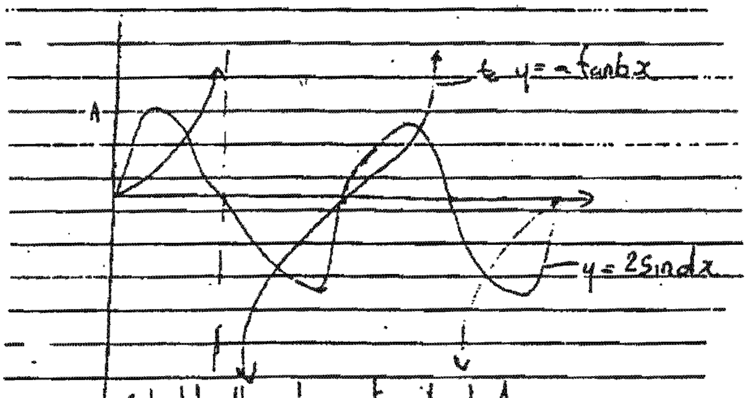
24 July 1998

1. Calculate values of a,b,d,a
2. Co-ordinates of C,B,D
3. The period of g
4. Amplitude of g

g(x) = 3 cos bx

f(x) = a tan ax

Plate 27 : Student A1 Third piece of classwork



Calculate the values of a, b, d, A

(i) a = 1

b = 1 d = 2

A = 2

Calculate the co-ordinates

(ii) $c = \frac{360}{60}$ = C (75°, 2)

B = (30, 0)

A = (105°, -2)

What is the period f & g

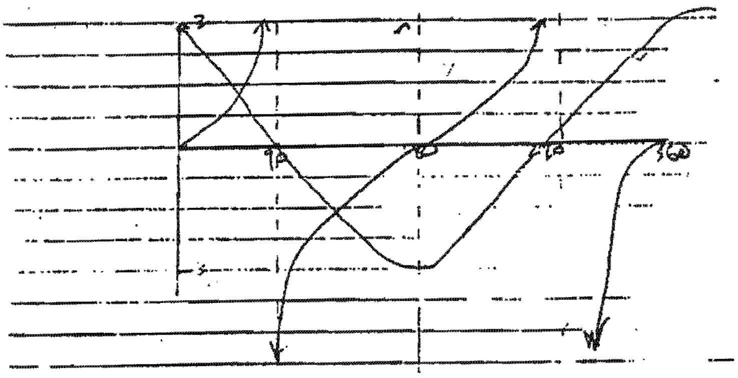
What is the amplitude of g

Solve for x

a) f(x) · g(x) = 0

b f(x) = g(x)

c) g(x) > f(x)



1 7

2. 720

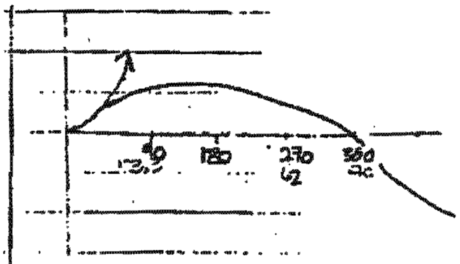
3 3

4.

4

6

Plate 28 : Student A3 Third piece of classwork



f = tan a x

g = b sin c x

Calculate the values of a, b and c

$a = \frac{360}{90} = 4$

$c = \frac{180}{90} = 2$

b = 2

CP = $\frac{90}{4}$ = 22,5

CP = [illegible]

A = [illegible]

1.4 What is the domain of f

[illegible]

1.5. Range of g (y-values)

[illegible]

= 2

5.3.10 Constructing criteria

Students correctly copy down the form of the sine and cosine curves when the teacher puts these on the board. However, from the second day that students spend on this topic, their curves start to increasingly misrepresent the functions. It appears that the combined effect of first doing a lot to trigonometric calculations, and the effect of the teacher privileging the calculation of critical points means that the students are only concerned to link up these critical points in the most direct way, they are not aware of how the curve should behave at any place between these critical points.

In general, the students' sketches of the tangent curves misrepresented the behaviour of the tangent function. Student A1 was the only student who on occasions had a fairly good approximation of the behaviour of the tangent curves as they approached the asymptotes. However, even many of his tangent curves did not represent the behaviour of the function at the points of inflection. One might infer that the problem was not with the teacher's sketching of the curves, but that he probably did not explicitly draw students' attention to the key features of the behaviour of these curves at the point of inflection and the asymptotes and stressed the importance of the critical points to the detriment of their understanding of the function's behaviour.

One could argue that if the teacher evaluated the students' sketches of these curves, he would be able to see where they are making errors, to point these out to the students so that they could learn how the curves should be drawn. Even where the teacher does sign and tick the books, it appears that he is not correcting the students' work either in the notebooks or in any other form of interaction with them and so they are not given access to the conventions and meanings of these curves.

The selection, pacing and sequencing of the material combined with the low level of written feedback by the teacher on the students' work, impacts on the criteria which students acquire about how to construct a legitimate text. In this particular example, students are constructing criteria about which features are important in the construction of trigonometric curves. The criteria that students develop result in the production of curves that do not capture the behaviour of the functions. The effect of this is to prevent students from being able to read data off the curves, and to prevent students from developing an understanding that one function of graphical representation is to be able to more easily ascertain information that is backgrounded by the algebraic form of the function.

In his analysis of pedagogical texts Dowling (1998) distinguishes between three modes of signification[16]: icon, index and symbol. For Dowling

> An icon signifies the virtual, physical presence of the viewing reader....
> An index incorporates visual or spatial codes, but does not assert the physical presence of the reader...
> A symbol is alphanumeric and is visual only in linear terms. (Dowling, 1998: 153 - 154)

Students are not given assistance to develop the criteria for the production of what Dowling calls the indexical form of the trigonometric functions. This prevents them from being able to translate between the indexical and symbolic form. Translating between forms of representation is one way of displaying fluency in any discourse. The fact that they are not taught to sketch the curves correctly prevents students from developing a coherent understanding of the nature of trigonometric functions and in this way their access into mathematics in a principled manner is hindered.

The teacher at School A appears to be giving students a procedure that relies on calculating and plotting critical points for the construction of trigonometric functions. The specific criteria transmitted by the teacher disrupt the potential for students to deduce the general behaviour of the functions or even its conventional representation.

Dowling (1998) discusses how different productions and forms of text have the potential to distribute the student in different ways. The transmission texts generated on this topic could be described in his language as texts which are localising (limiting in range and proceduralising in discourse). They do not have the potential to apprentice the student into mathematical discourse and have only the potential to produce a dependent or an alienated student. This provides an example of how the extent of evaluation and modality of pedagogy can impede students' induction into the discourse and this has implications for the quality of the student's learning.

5.4 Written evaluation in the student productions and its relationship with pedagogic practice and the quality of learning

In this chapter I have argued that the written evaluations that appear in the student productions are a very small component of evaluation. I have discussed how the student productions reveal aspects of the pedagogic discourse that is dominant at each of the schools. I suggest that changes in the student

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[16] In distinguishing these three modes of signification, Dowling draws on Pierce, however, his categories do not coincide with those of Pierce: in particular, forms of signification that Pierce would classify as iconic, Dowling classifies as indexical.

productions after the June examination at School B indicate how the evaluative effect of the examination has changed the way in which students construct the criteria for the production of legitimate text. In this way a form of evaluation that is not itself present in the notebooks makes an impact and becomes present in the student productions. However, formal assessments such as tests and examinations are themselves only isolated examples of evaluation. To be able to assess how evaluation impacts on the student's development of recognition and realisation rules one needs access to at least some of the classroom interaction. Without a more detailed record of the pedagogic practice one can only examine certain limited features that impact on quality.

The discussion of the student's representations of trigonometric graphs provided another example of how one might infer elements of evaluation and pedagogic practice from what is present and absent in the student productions. Student productions in their notebooks provide a partial record of the pedagogic modality in any classroom. They contain limited elements of evaluation, which are potentially outweighed by elements of evaluation that are not reflected as written interventions in the notebooks.

Student productions are certainly useful as one record of the cultural communication that transpires in a classroom. However, they remain only a partial record and one needs access to other aspects of the communication in order to evaluate successfully the quality of learning.

Chapter 6: Conclusion

In this final chapter of the dissertation I revisit the research question, posed by initially by the Learner Progress and Achievement Study (of which this dissertation formed a part), namely: "what if anything can student productions tell us about the quality of learning?" Although I came to a definition of quality education as one that successfully inducts students into a discourse, I maintain that quality is a floating signifier and that it is more productive to examine issues around pedagogy. That is why this dissertation has tended to focus more strongly around a set of sub-questions (outlined in Chapter 1) that linked patterns in students' written productions and written evaluations to forms of pedagogy, and then assessed the potential for different pedagogical forms to induct students into the discourse of school mathematics.

I examine the extent to which the dissertation has met the challenges implicit in the research question. Then I outline the potential value in analysing students' production and discuss some limitations of this study and potential ways of extending the framework used for the present analysis so that it may better address the question: "what if anything can student productions tell us about the quality of learning?"

6.1 Summary of this dissertation

In Chapter 1 I explained the reason why this study was limited to the analysis of students' written productions. I also examined how the notion of quality became embedded in educational discourse and traced historically how the notion of quality was transferred from economic discourses. Bernstein (1993: *xx*) discusses broadly how schooling discourses have been influenced by economic discourses:

> [t]he present [education] climate's emphasis upon.... effective means of acquisition under conditions which optimize their economy and accountability, have spectacular implications for pedagogic practice.... Such a policy leads to change in those responsible for its implementation. Thus, we now have new forms of management and managers....

It was discussed in Chapter 1, how managerial models of quality have been transferred from economic activities onto schooling and other educational activities. When terms or concepts are recontextualised from one discourse to another there is likely to be a shift in the way the term is conceived. However, there may well be strong continuities between the concept in the former and the latter discourses. In much policy development and research internationally, schooling in general, and concerns around quality schooling, in particular, have been delinked from their social contexts and assessed in terms of managerial or organisational models. This has served to shift a reading of schooling away from a linkage with social development to a more decontextualised reading of schooling that relied on the identification of a ubiquitously applicable set of indicators of quality. However, what counts as quality depends not only on the social context but also upon the values of the evaluator. In this chapter I outlined various contemporary views on the quality of mathematics teaching and learning. I conclude that the quality of mathematics teaching and learning depends on the extent to which students are inducted into the discourse of school mathematics. I argue that forms of pedagogy are always influenced by macro and micro social features and that any assessment of pedagogic modality needs to take account of both the macro and micro social contexts. I draw on Basil Bernstein's (1990, 1996) theory to argue that pedagogy is a form of cultural communication, which is specialised with respect to context.

In Chapter 2 I examined the literature on the analysis of students' productions in mathematics classes, most of which does not focus on either the notes that students write of the teacher's exposition or the on students' routine calculations. There is also little literature that focuses on student productions over time and in this way the issues of how students develop the criteria for the production of text is not analysed. Most of this literature also ignores the impact of social context in the structuring of texts. Where the literature does allude to social contexts this is generally done in a superficial manner. This gap in the literature means that there is little evidence of students' productions being read as a record of the particular forms of cultural communication privileged in any classroom and how this may relate to the social organisation of the class and the school. I believe that the analysis chapters of this thesis, summarised below have shown that it is productive to examine students' productions, within an analysis of the social organisation of the class and school, as a record of the particular forms of cultural communication privileged in any classroom.

In Chapter 3 I outlined how Basil Bernstein's model of the pedagogic device allows me to redescribe the issue of quality in terms of an analysis of the differing nature of pedagogic practices. I described how Bernstein's model of the pedagogic device, especially the concept of how the instructional discourse is embedded in the regulative discourse, allowed me to link the structuring of the pedagogic practices with the dominant social organisation of the school. I discussed how Basil Bernstein's model allowed me to analyse the student productions, as an instantiation of communication, which capture differing pedagogic practices. I further discussed how this model will potentially allow me to infer, from the differences in the student productions, what the differing rules of cultural communication in the two Standard 10/Grade 12 classes are - to read these as an index of their differing micro-interactions and the classroom based structuring of the social division of labour, without losing the broader picture of how these are related to the macro-structuring of society. I described how Bernstein's (1990, 1996) model of the pedagogic device and his notions of classification and framing (1977, 1990, 1996) were found to provide potential tools to structure and interpret the data constructed from the engagement with the notebooks, interviews with teachers, observations of the locational features of the two schools and research of LPAS, in ways which would prevent falling into the pitfalls of much of the research outlined in Chapters 1 and 2.

In Chapter 4 I related the social organisation of each school
the students' notebooks at each school. I argued that it is
notebooks at School A, which shows a relatively stronger form of
tendency towards a grammar orientation and the students' notebooks
stronger tendency towards a mechanical solidarity, show a stronger
orientation. As mentioned in Chapter 2, Domingos (1989) relates teacher pe
context within which they teach.

> The social context of a school ... is a powerful factor influencing the teac gic practice. [1] (Domingos, 1989: 361)

It is the differences in orientation that tends to position the students at School A more favourably in terms of the final matriculation examination, since grammar orientation is more privileged in our society than a text orientation.

In Chapter 5 I discussed the effect of evaluation on students' construction of the criteria for the production of text. I discuss how analysis of notebooks allows one to read the students attempts to construct the criteria for the production of legitimate text over a time period. Evaluation works over time to guide the student in his/her construction of the criteria of the legitimate text. The student productions are analysed in terms of what their construction of the criteria for the production of texts indicates of their level of induction into the discourse of school mathematics. I used the successful induction of students into the discourse of school mathematics as the criterion for marking out quality. My analysis of the students' representations of trigonometric functions at School A, is an example that is used to indicate how a low level of evaluation of students' text impedes their induction into the discourse and impacts on the quality of their learning.

I also attempted to relate the extent to which students' productions show evidence of evaluation with their level of success at each school. While there appeared to be a matching of students results and level of visible evaluation at School B, this is not obvious at School A. This I argued is because at School B, the students' notebooks come to represent the privileged text, whereas at School A other forms of evaluation may be more important. It is here that a key weakness of this study is revealed. More direct data on classroom interactions and practices, for example video footage or observations, would have allowed the structuring of the notebooks to be read within the context of these interactions and would have provided a better picture of the forms of pedagogy. One situation where the effect of

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[1] Domingos does, however, caution that teachers' types and contexts of training and the previous contexts of their teaching also influence the present teaching.

another form of evaluation is revealed in the students notebooks at School B, is the marked differences that occur in the students' structuring of the productions after the June examinations.

6.2 Potential for further research

This dissertation has linked patterns in the Standard 10 / Grade 12 students' written productions in their mathematics notebooks, with the dominant regulative discourse at each of the two schools. This has further allowed for the generation of descriptions of aspects of the pedagogic modalities at each of the schools. From the nature of the patterns displayed in particular students' written production in their Standard 10/ Grade 12 mathematics notebooks key elements of the criteria students were using in their construction of texts were found. From this I was able to infer aspects of how they attempt to construct recognition and realisation rules. The students written productions provided a window through which to view their level of induction into mathematical discourse. I submit that this form of analysis is generalisable beyond this study and the contexts on which it focussed.

Assessing what students know has been a key concern in education for a long time. As described in Chapter 1, public examinations, such as the Matriculation Examination and other tests of students' knowledge such as Third International Mathematics and Science Study (TIMMS) come to stand for assessments of the quality of education. What these forms of testing show is what students can reproduce of what they know, under particular conditions but only within the parameters of what they are asked. These are valid forms of assessment. However, analyses of students written productions allow one to analyse how different students (under either the same or different contexts) construct the criteria for the production of legitimate texts over a period of time. This I am arguing is an equally valid way of analysing student's knowledge. I have argued that it is the extent to which students are inducted into the discourse that marks out the quality of education. Apprenticeship into a discourse is not an instantaneous event; it is a process that occurs over time. Analyses of student notebooks are a useful source of information from which to reconstruct this process. This study, I believe, provides evidence of the value of using student's written productions as a source of information about the extent of their induction into mathematics. I suggest that this form of analysis can usefully complement other approaches, such as testing, determining students' knowledge and understanding of particular discourse.

As mentioned previously evaluation occurs in many forms such as repetition, non-repetition, or other forms of silences, changes in tones of voice, changes in gesture, facial expression or other forms of body language (Bernstein, 1990). Evaluation can occur between students and teachers or students and students. Since evaluation is key to the construction of the criteria for the production of the legitimate text, one needs access to more than the student notebooks in order to analyse how patterns of evaluation have directed students in their attempts to construct the realisation and recognition rules or the extent to which they have been inducted into the discourse of school mathematics. It is the extent to which students have grasped the recognition and realisation rules that marks out the quality of their learning. One needs access to other classroom practices in order to obtain a clearer picture of how evaluation impacts on the construction of transmission and acquisition texts.

This dissertation has described the notebooks as an instantiation of the cultural communication that occurs within classrooms and schools. Key elements of the pedagogic modalities at the two schools were inferred from the patterns represented in the notebooks. Many studies that focus on teaching and learning generate data from small-scale observations of classroom interactions. The pedagogic texts that form the basis of these small-scale observations represent a snapshot of the pedagogic practice, which develops over time. I maintain that a clearer picture of pedagogic practice can be obtained if one is able to track development over time. However, longitudinal studies such as Boaler (1997) are costly both in terms of time and money. Analyses of students' notebooks may provide a more cost-effective mechanism to track the development of pedagogy over time. I suggest that analysis of notebooks, if it is complemented by analysis of contexts (as is done in this study) and also some classroom observations, can generate a better description of the pedagogic practice than snapshot analyses of classroom interaction or analyses of notebooks on their own. Whilst analyses of student notebooks in conjunction with testing can provide a more coherent analysis of students' knowledge, than is possible through testing alone.

Although some attempt was made to relate the analysis not only to students differing level of success across and within each of the two schools, no attempt was made to link analyses differences in the notebooks with different genders. Some initial surface patterns were observed and noted in Chapter 3, however, an in-depth analysis of this would require a larger sample of student notebooks.

Another way in which this study could be extended is to link the patterning displayed in the students' notebooks with the development of their identity and their positioning in the class. One might be able to extend the brief descriptions of the pedagogic differences using Bernstein's notions of classification and framing to include a coding of the students' production in terms of message and voice and hence to comment on the construction of their pedagogic identities, or choose to draw on other theory.

This dissertation has attempted to show that analyses of students' written productions in conjunction with analyses of the contexts within which they are generated (which should include some observations of class-based teaching) are able to generate complex descriptions of both pedagogic modalities and students' induction into particular discourses. The form of analysis suggested here is able to provide descriptions of how the regulative discourses set the parameters within which the instructional discourses develop and these in combination form the communicative contexts within which students attempt to generate criteria for the production of legitimate texts.

While this dissertation focuses on two schools in Khayelitsha, I suggest that this form of analysis is generalisable beyond this context and the scope of this study and may provide a useful tool in other analyses of either or both pedagogic practice and/ or students' induction into a variety of discourses.

Appendix 1: Interview Questions

Students / learners are placed in different classes.
Is this according to subject choice?

Do the students decide what subjects they want to do?
Are there any restrictions on what subjects they do? For example results, subject combinations are combination of these.

Do students decide what on what grade to take mathematics, or is this jointly decided by teachers and students? Are there any guidelines e.g. results. On what are these based?

Do students get taught according to register class, or are they split into grades?

Does the same mathematics teacher teach all the matrics, or are there different teachers?

Do the students use a textbook, if so which textbook?

Do you, as the teacher, use textbooks as resources, if so what book or which books. What has influenced your selection of textbook/s or other resources?

How long are classes?
How many classes each week?
Is it the same for every matric class?

Do the students or the teachers move between classrooms?

How many of this years matric students that you teach do you think were at this school last year? Are many of the students repeating matric or most of them from Std. 9?

At many schools students do not attend regularly. This obviously affects what will or won't be written into students' books. How is attendance in the matric mathematics classes?
Do you have copies of attendance records?

How do you assess the students over the year e.g. do they have a series of tests over the year. Could you give me copies of test results, exam results and question papers: April & June

How are the June and September exams set?

Do the June and September exams cover the same work as the final examination or only the work that the students have completed?

It is important to the students and the school that they get good final results.
What mechanisms do you set in place to achieve this?

e) i) Simplify without using a calculator

$$\frac{\log 16 - \log 4}{\log 16 + \log 4}$$

(ii) Solve

$$\log(x+6) + \log(x-3) = 1$$

SEQUENCE AND SERIES

DEFINITION

Sequence is an orderd collection of terms where any term except the 1st can be found from the previous term by applying a rule.
This means that the term in a sequence have a pattern so it is possible to work out the next term of the ~~es~~ sequence if the rule is known.

In short A sequence is a set of ordered red number
eg 1; 3; 5; 7; 9; ... } These infinite
2, 4; 8; 16; 32; ...

Finite numbers are random sequences
eg. 14, 19, 27, 144

Arithmetic sequence is when the difference between 2 successive term is CONSTANT

5 5 5 common difference

eg 3; 8; 13; 18 ...

In this AS Common difference is 5. 5 is added to get from 1 term to the other and the difference between any term before it is 5

e.g. 8 - 3 = 5 ; 13 - 8 = 5 ; 18 - 13 = 5

T_1 T_2 T_3 T_4 ...... T_n
3 8 13 18 ... P

$3+0(5); 3+1(5); 3+2(5); 3+3(5) \; 3+(n-1)5 = T_n$
$a; a+d; a+2d; a+3d \; a+(n-1)d = T_n$

a = first term
d = Common difference
* To get the 3rd term add 2xd to the 1st term
* To get the nth term add (n-1) times 'd' to the first term
~~F~~ $a; d \in R$; and $n \in \mathbb{R}$, then $a, a+d, a+2d \ldots a+(n-1)d$ is an Arithmetic sequence with n terms where
a = first term
d = common difference
$= T_n - T_{n-1}$
T_n = nth term
$= a + (n-1)d$

eg. Find the tenth term of the sequence
7, 3; -1 ...
3-7 = -4 and -1-3 = -4
∴ This is an A.S with a = 7, d = -4 ~~and~~ n = 10
$T_n = a + (n-1)d$
$= 7 + (10-1) \cdot -4$
$= 7 + (9) - 4$
[illegible]
$= -29$

2. Find the eighth term of the A.S

$-4; -1\frac{3}{4}; \frac{1}{2} \ldots$ $T_8 = a+(n-1)d$

$d = -1\frac{3}{4} - (-4)$ $4\times1+3$ $= -4+(8-1)\frac{9}{4}$

$= -\frac{7}{4}+4$ $= -4+(7)\frac{9}{4}$

$= \frac{-7+16}{4}$ $= \frac{-4+63}{4}$

$= \frac{9}{4}$ $= \frac{-16+63}{4}$

$= \frac{47}{4}$

3. Which term of the sequence

4; 9; 14 … will be 34

7 - 4 = 5 ; ~~14~~ 14 - 9 = 6

∴ This is an A.S with $a = 4$; $d = 6$

$T_n = 34$

We have to find n

$T_n = a+(n-1)d$

$34 = 4+(n-1)5$

$34 = 4+5n-6$

$34 = 5n-1$

$5n-1 = 34$

$5n = 34+1$

$n = \frac{35}{5}$

$= 7$

∴ The 7nth term has a value of 34.

H/w 04-05-98

4. Determine how many terms there are in the seq

$45; 43\frac{3}{4}; 42\frac{1}{2} \ldots 33\frac{3}{4}$

$43\frac{3}{4} - 45 = -1\frac{1}{4}$

$42\frac{1}{2} - 43\frac{3}{4} = -1\frac{1}{4}$

∴ This is an A.S. with $a = 45$ $d = -1\frac{1}{4}$ $T_n = 33\frac{3}{4}$

* Find n   H/w 04-05-98

use formula

$T_n = a+(n-1)d$

The ans. is $n = 10$

5. P; 2p+2; 5p+3 are the 1st 3 terms of an A.S.
Find p and hence write down the values of the 1st 3 terms.

$d = 2p+2-p$ and $d = 5p+3-(2p+2)$

$d = p+2$ and $d = 5p+3-2p-2$

$= 3p+1$

$p+2 = 3p+1$

$p-3p = -2+1$

$-2p = -1$

$p = \frac{-1}{-2}$

$= \frac{1}{2}$

1st three terms are

$P; 2p+2; 5p+3$

$\frac{1}{2}; 2(\frac{1}{2})+2; 5(\frac{1}{2})+3$ simplify further

Ans: $\frac{1}{2}; 3; 5\frac{1}{2}$

HOW TO SOLVE A PROBLEM IF THE SEQUENCE IS NOT GIVEN

Take note of the example below

6. The twelth term of an arithmetic seq is -3 and the ~~tenth~~ twentieth term is 7, Find the 7th term

We know that this is an arithmetic seq. but we don't know the values of a or d. However we are given 2 sets of information

$\therefore$ Construct 2 equation and solve simataneously.

$T_{12} = -5$ and $T_{20} = 7$

$a + 11d = -5 \ldots ①$

$a + 19d = 7 \ldots ②$

$d = ② - ①$

$a + 19d - (a + 11d) = 7 - (-5)$

$a + 19d - a - 11d = 12$

$8d = 12$

$d = \frac{12}{8}$

$= \frac{3}{2}$

Substitude ① $a + 11(\frac{3}{2}) = -5$

$a + \frac{33}{2} = -5$

$2a + 33 = -10$

$2a = -43$

$a = -\frac{43}{2}$

$= -21\frac{1}{2}$

$T_n = a + 6d$

$= -21\frac{1}{2} + 6(\frac{3}{2})$

$= -21\frac{1}{2} + 9$

$= \frac{-43}{2} + 9$

$= \frac{-43 + 18}{2}$

$= \frac{-25}{2}$

$= -12\frac{1}{2}$

05-05-98

7/6 = Classwork

1. Find the seventeeth term of the sequence
   $-2, 5, 8 \ldots$
2. How many terms are there in the sequence
   $3; 7; 11; \ldots ; 147$?
3. In an arithmetic seq the seventh term is 30 and the fourth term is 18. Find the first term and the common deffence
4. Find the twenbeth term of the sequence $1, 5, 9 \ldots$

Homework

1.1 Write $27^{\frac{1}{2a}}$ as a power of 3

1.2 write $(\frac{1}{8})^{a+1}$ as a power of 2

1.3 Write 72^{a} as a product of powers with prime bases

2. Simplify without using a calculator

2.1 $(0{,}125)^{2/3}$

2.2 $(2.3)^{-2} + 2.3^{-2}$

2.3 $\frac{27^{-\frac{1}{3}} \times 9^{2}}{81^{\frac{1}{4}}}$

2.4 $(2^{-1} + 3^{-1})^{2}$

2.5 $\frac{5^{-1}}{3^{2}} \times \frac{40}{27^{2/3}} \div 15^{-2}$

3. Simplify

3.1 $4^{a} - 2^{2a}$

3.2 $\frac{9^{a-2} \cdot 10^{a-2}}{6^{a-4} \cdot 15^{a}}$

Appendix 2: Student B2

(v) $\log x = \log_x \left(\frac{x^3}{100}\right)$

Sequence & Series
Definition:
Sequence is an ordered collection of terms where any term except the 1st can be found from the previous term by apply the rule
this means: that the term is a sequence have a pattern so it is possible to work out the next term of the in sequence if the rule is known
Or Sequence is a set of real number
e.g. 1, 3, 5, 7, 9, 11, 13 ...
2, 4, 6, 8, ...
2, 4, 8, 16 } these are infinite

The finite numbers are called random sequence e.g. [illegible]

Arithmetic Sequence,

Is when the difference between 2 successive terms is constant

$$\text{e.g. } \overset{5}{\frown}\ \overset{5}{\frown}\ \overset{5}{\frown} \quad 3, 8, 13, 18$$

In this Arith-seq the common difference is 5. 5 is added to get from 1 term and the term below

Is the other & the difference between any term and the term before it's 5

e.g. 8 − 5 = 3, 13 − 5 = 8, 18 − 13 = 5

T_1 T_2 T_3 T_4 T_n
3 8 13 18 ?

$3+0(5)$; $3+1(5)$; $3+2(5)$; $3+3(5)$, $3+(n-1)$
∴ $3+(n-1)5$
a; $a+d$; $a+2d$; $a+3d$ ... $a+(n-1)d$

a − first Term
d − Common difference

To get the third term add $2 \times d$ to the first term
To get the n^{th} term add $(n-1) \times d$ to the first term

If $a, d \in R$ & $r \in R$ then $a, a+d, \ldots, a+(n-1)d$ is an Arith Seq. with n term
where ~~a first~~
where a − first Term
d − Common Term
$= T_n - T_{n-1}$
$T_n = n^{th}$ term
$= a+(n-1)d$

E.g. find the seventh Term of sequence
7 7 ,

$3 - 7 = -4$; a.d $-1-3 = -4$
This is an Ar Seq with $a = 7$,
$d = -4$ $n = 10$

$T_n = a+(n-1)d$
$= 7+(10-1)-4$
$= 7+(9)-4$
$= 7-36$
$= -29$

Find the eighth term of the Arith Seq
$-4; -1\frac{3}{4}; \frac{1}{2} \ldots$

$-1\frac{3}{4} - (-4)$ $\frac{4+3}{4}$
$-\frac{7}{4} + 4$ $\frac{4.4}{4 \quad 4\times1}$
$\frac{-7+16}{4}$

$d = \frac{9}{4}$

$T_n = a+(n-1)d$
$= -4+(8-1)\frac{9}{4}$
$= -4+(7)\frac{9}{4}$
$= -4 + \frac{63}{4}$
$= \frac{16+63}{4}$
$T_8 = \frac{47}{4} \Rightarrow 11(\frac{3}{4})$

3. Which Term of sequence 4; 9; 14 ... 34
$9 - 4 = 5$ and $14 - 9 = 5$

* Find n

$T_n = a+(n-1)d$
$34 = 4+(n-1)5$
$34 = 4+(5n-5$
$34 = 4+5n-5$
$34-4+5 = 5n$
$n = \frac{35}{5}$
$n = 7$

The 7th term has a value of 34
You can do this by inspection

4. Determine how many terms there are in the ff sequence

$45, 43\frac{3}{4}, 42\frac{1}{2} \ldots 33\frac{3}{4}$

$d = 43\frac{3}{4} - 45$ and $42\frac{1}{2} - 43\frac{3}{4}$
$= -1\frac{1}{4}$ and $-1\frac{1}{4}$
This is a A/Seq with $a = 45$
$d = -1\frac{1}{4}$
$T_n = 33\frac{3}{4}$

Find ...

Find n

$T_n = a + (n-1)d$

$33\frac{3}{4} = 45 + (n-1)\frac{-5}{4}$

$33\frac{3}{4} = 45 - (\frac{5}{4}n + \frac{5}{4})$

$33\frac{3}{4} = 45 - \frac{5n}{4} + \frac{5}{4}$

$33\frac{3}{4} = 45 + \frac{5}{4} - \frac{5}{4}n$

$33\frac{3}{4} = 45 + \frac{5}{4} - \frac{5}{4}n$

$33\frac{3}{4} = \frac{185}{4} - \frac{5n}{4}$

$37\frac{3}{4} \pm 33\frac{3}{4} - \frac{185}{4} = -\frac{5}{4}n$

$-\frac{50}{4} = -\frac{5}{4}n$

$-\frac{50}{4} \times -\frac{4}{5} = n$

$n = 10$

So, p, 2p+2; 5p+3 are the 1st 3 terms of an arithmetic sequence find p and hence write down the values of the 1st 3 terms

$d = 2p + 2 - p$ and $d = 5p + 3 - (2p + 2)$

$d = p + 2$ and $5p + 3 - 2p - 2$

$= 3p + 1$

$p + 2 = 3p + 1$

$-2p = -1$

$p = \frac{1}{2}$

1st 3 Term p; 2p + 2; 5p + 3

$\frac{1}{2}$; $2(\frac{1}{2}) + 2$; $5(\frac{1}{2}) + 3$

Simplify for your self

Ans $\frac{1}{2}$, 3, $5\frac{1}{2}$

How to solve a problem of the sequence is not given

Take note of the example below

e.g. The twelfth term of an A.S. is -5 and the twentieth term is 7 find the 7th term

We know that this is an A.S. but we don't know values of a and d. However, we are given 2 sets of information to construct 2 equations and solve simultaneously

$T_{12} = -5$ and $T_{20} = 7$

$a + 11d = -5$ ...(1) and $a + 19d = 7$ (2)

② - ①

$a + 19d - (a + 11d) = 7 - (-5)$

$a + 19d - a - 11d = 12$

$8d = 12$

$d = \frac{12}{8}$

$d = \frac{3}{2}$

Substitute equation 1: $a + 11d = -5$

$a + (\frac{3}{2})11 = -5$

$a + \frac{33}{2} = -5$

$2a + 33 = -10$

$2a = -43$

$a = -\frac{43}{2}$

$a = 21\frac{1}{2}$

Appendix 2 : Student B3

(iii) $2\log x + 3\log x = 10$
(iv) $\log x = \log_x \left(\frac{x^3}{1000}\right.$

Sequence & Series

Definition is an ordered collection of items where any term ~~or~~ except the 1st can be found from the previous term by applying the rule

This means that the term in a sequence have a pattern so it is possible 2o workout the next term of the sequence if the rule is known or sequence is a set of ordered real numbers. e.g. 1; 3; 5; 7; 9 ...

2; 4; 6; 8 .... } These are infinite

2; 4; 6; 14 ......

The finite number are called random sequence

e.g. 14; 19; 114 ......

Permanent Sequence

Is when the different between 2 successive term is constant

e.g. $\overset{\frown}{3 \quad 8} \quad \overset{\frown}{13 \quad 18}$ ....

In this AS the common difference is 5. 5 is added to get from 1 term to the other & the difference between any term and term before it is 5

e.g $8-3=5$; $13-8=5$; $18-13=5$

$T_1 \quad T_2 \quad T_3 \quad T_4 \;----\; T_n$

13 8 18 ?

$3+0(5)$; $3+1(5)$; $3+2(5)$; $3+3(5)$ ----- $3+(n-1)5$

a ; $a+d$; $a+2d$; $a+3d$ --- $a+(n-1)d$

a - first term

d = Common difference

To get the 3rd term add 2×d to the 1st term

To get nth term add $(n-1)\times d$ to the 1st term

If $a; d \in R$ & $n \in R$

$a+d$; $a+2d$; --- ; $a+(n-1)d$ is

An A.S with n terms, where

a - first term

d - Common different

T_n = nth term

$= a+(n-1)d$

eg Find the ~~te~~ tenth term of the sequence 7; 3; -1

$3-7=-4$ and $-1-3=-4$

This is an A.S with $a=7$; $d=-4$; $n=10$

$T_n = a+(n-1)d$

$= 7+(10-1)-4$

$= 7+(9)-4$

$= 7-36$

$= -29$

2. Find the eight term of the A.S.

-4; $-1\frac{3}{4}$; $\frac{1}{2}$ ......

$-1\frac{3}{4} - (-4)$

$-\frac{7}{4} + 4$

$\frac{-7+16}{4}$

$d = \frac{9}{4}$

$T_n = a+(n-1)d$

$= -4+(8-1)\frac{9}{4}$

$= -4+(7)\frac{9}{4}$

$= -4+\frac{63}{4}$

$\frac{-16+63}{4} = \frac{47}{4} \rightarrow 11\frac{3}{4}$

3 Which term of the sequence 4; 9; 14 --- will be 34?

$a - 4 = 5$ and $14 - 9 = 5$

Find n

$T_n = a + (n-1)d$

$34 = 4 + (n-1)5$

$34 - 4 + 5 = 5n$

$35 = 5n$

$n = 35/5$

$n = 7$

∴ The 7th term has a value of 34 you can do this by inspection

Determine how many terms there are in the ff sequence

$45, 43\frac{3}{4}, 42\frac{1}{2} ---- 33\frac{3}{4}$

$d = 43\frac{3}{4} - 45$ and $42\frac{1}{2} - 43\frac{3}{4}$

$= -1\frac{1}{4}$ and $1\frac{1}{4}$

∴ This is an AS

5 1; $2p+2$; $5p+3$ are the 1st 3 terms of an arithmetic sequence

Find P and hence write down the values of the 1st 3 terms

$d = 2p + 2 - p$ and $d = 5p + 3 - (2p+2)$

$d = p + 2$ and $5p + 3 - 2p - 2 = 3p + 1$

$P + 2 = 3p + 1$

$-2p = -1$

$p = \frac{1}{2}$

1st 3 terms p; $2p+2$; $5p+3$

$\frac{1}{2}$; $2(\frac{1}{2})+2$; $5(\frac{1}{2})+3$

Simplify for yourself

Ans $\frac{1}{2}$; 3; $5\frac{1}{2}$

HOW TO SOLVE A PROBLEM IF THE SEQUENCE IS NOT GIVEN

Take note of the example below

6 The twelfth term of an AS is -5 and the twentieth term is 7

Find the 7th term

We know that this is an arithmetic sequence but we don't

know values of a and d. However we are given 2 sets of

information ∴ Construct 2 equation and solve simultaneously

$T_{12} = -5$ and $T_{20} = 7$

$a + 11d = -5$ ---- ① $a + 19d = 7$ ---- ②

② - ①

$a + 19d - (a + 11d = 7 - (-5)$

$a + 19d - a - 11d = 12$

$8d = 12$

$d = \frac{12}{8}$

$= \frac{3}{2}$

Substitute equation :- $a + 11d = -5$

$$a + 11\left(\frac{3}{2}\right) = -5$$

$$a + \frac{33}{2} = -5$$

$$2a + 33 = -10$$

$$2a = -43$$

$$a = -\frac{43}{2}$$

$$x = 21\tfrac{1}{2}$$

$$T_7 = a + 6d$$

$$= \frac{-43}{2} + 6\left(\frac{3}{2}\right)$$

$$= \frac{-43}{2} + 9$$

$$= \frac{-43 + 18}{2}$$

$$= \frac{25}{2}$$

$$= 12\tfrac{1}{2}$$

Solve for x

LAWS OF indices

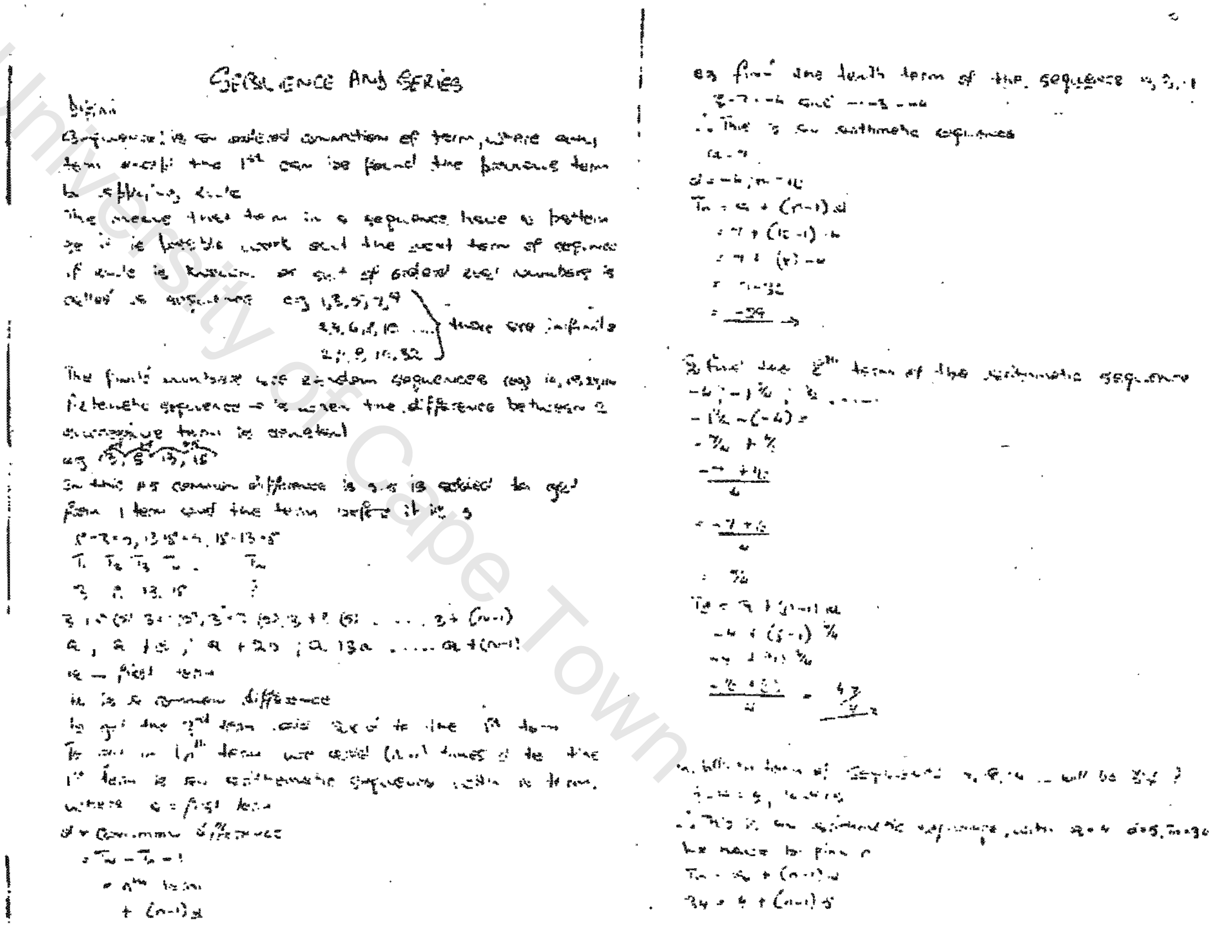
1. $a^m . a^n = a^{m+n}$ e.g ⓐ $a^6 . a^3 = a^{6+3} = a^7$

ⓑ $2^3 . 2^4 = 2^7$

ⓒ $2^{2n+3} = 2^{2n} . 3^3$

2. $\frac{a^m}{a^n} = a^{m-n}$ e.g ⓐ $\frac{a^6}{a^2} = a^{6-2} = a^4$

ⓑ $\underline{3^4} = 3^{4-3} = 3$



SEQUENCE AND SERIES

Defini

Sequence: is an ordered connection of term, where any term except the 1st can be found the previous term by applying rule
The means that term in a sequence have a pattern so it is possible work out the next term of sequence if rule is known. or set of ordered real numbers is called a sequence eg 1, 3, 5, 7, 9 / 2, 4, 6, 8, 10 ..... / 2, 4, 8, 16, 32 } these are infinite

The finite number are random sequences (eg) 16, 18, 24
Arithmetic sequence → is when the difference between 2 successive term is constant
eg 3, 8, 13, 18
In this the common difference is 5 is added to get from 1 term and the term before it is 5
8−3=5, 13−8=5, 18−13=5
T_1 T_2 T_3 T_4 ... T_n
3 8 13 18 ?
3 ; 3+(5) ; 3+(2)(5) ; 3+(3)(5) ..... 3+(n−1)
a ; a + d ; a + 2d ; a + 3d ..... a + (n−1)
a — first term
d is a common difference
to get the 2nd term add d to the 1st term
To get the nth term we add (n−1) times d to the 1st term in an arithmetic sequence with n term,
where a = first term
d = common difference
$= T_n - T_{n-1}$
= nth term
+ (n−1)d

eg find the tenth term of the sequence 7, 3, −1
3−7 = −4 and −1−3 = −4
∴ This is an arithmetic sequence
a = 7
d = −4 ; n = 15
$T_n = a + (n-1)d$
= 7 + (15 − 1) −4
= 7 + (14) −4
= 7 − 56
= −49 →

3 find the 5th term of the arithmetic sequence
−4 ; −1¾ ; ½ .....
−1¾ − (−4) =
−7/4 + 4
$\frac{-7 + 16}{4}$
$= \frac{-7 + 16}{4}$
= 9/4
$T_5 = a + (n-1)d$
−4 + (5 − 1) 9/4
−4 + (4) 9/4
$\frac{-16 + 36}{4}$ = [illegible]

In which term of sequence 4, 9, 14 ... will be 34 ?
9 − 4 = 5, 14 − 9 = 5
∴ This is an arithmetic sequence, with a = 4 d = 5, Tn = 34
we have to find n
$T_n = a + (n-1)d$
34 = 4 + (n − 1) 5

$34 = 4 + 5n - 5$	The n^{th} term has a value of 34
$5n = 5n - 5$	(4) Determine how many terms there are
$5n - 1 = 34$	in the ff. sequence......
$5n = 35$	$45, 43\frac{3}{4}, 42\frac{1}{2}, \ldots\ldots 33\frac{3}{4}$
$n = \frac{35}{5}$	$43\frac{3}{4} - 45 = -1\frac{1}{4}$ & $42\frac{1}{2} - 43\frac{3}{4} = 1\frac{1}{4}$
$n = 7$	This is an arithmetic sequence with $a = 45$;
	$d = 1\frac{1}{4}$ $T_n = 33\frac{3}{4}$

Find n:

$T_n = a + (n-1)d$

$33\frac{3}{4} = 45 + (n-1) - 1\frac{1}{4}$

$33\frac{3}{4} = 45 - \frac{5}{4}n + \frac{5}{4}$

$33\frac{3}{4} = 45 + \frac{5}{4} - \frac{5}{4}n$

$33\frac{3}{4} = \frac{180+5}{4} - \frac{5}{4}n$

$\frac{135}{4} - \frac{185}{4} = -\frac{5}{4}n \quad 135 - 185 = -5n$

$50 = -5n$

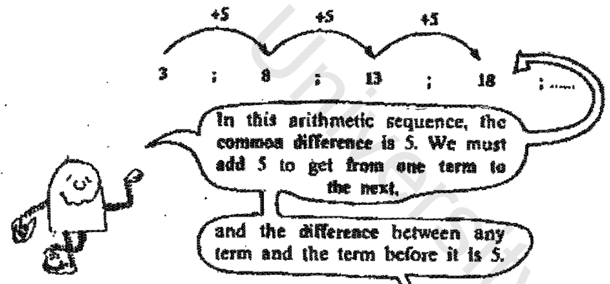
$\frac{135-185}{4} = -\frac{5}{4}n \qquad n = 10$

3 Sequences and series

TOPIC 3.1: ARITHMETIC SEQUENCES

A set of ordered real numbers is called a sequence. For example:
1; 3; 5; 7; 9; …
2; 4; 8; 16; 32; …

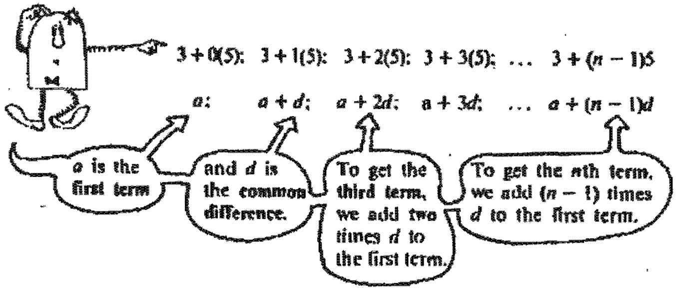
In an arithmetic sequence, the difference between successive terms is constant.

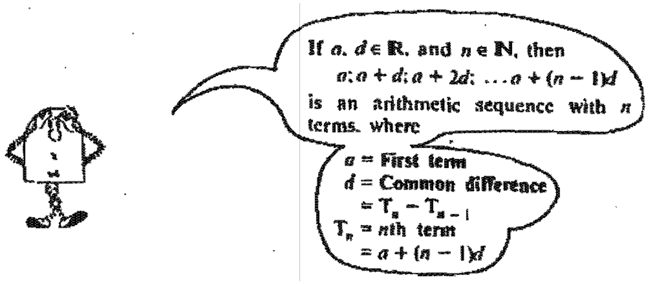


8 − 3 = 5; 13 − 8 = 5; 18 − 13 = 5

Let's label the terms T_1 for the first term, etc.

T_1	T_2	T_3	T_4	…	T_n
3;	8;	13;	18;	…	?





Examples:

(1) Find the tenth term of the sequence 7; 3; − 1; …

$3 - 7 = -4$ *and* $-1 - 3 = -4$

∴ this is an arithmetic sequence, with $a = 7$, $d = -4$, $n = 10$
We have to find T_{10}.

$$\begin{aligned} T_n &= a + (n-1)d \\ \therefore\ T_{10} &= 7 + (10-1)(-4) \\ &= 7 + 9(-4) \\ &= 7 - 36 \\ &= -29 \end{aligned}$$

(2) Find the eighth term of the arithmetic sequence $-4;\ -1\frac{3}{4};\ \frac{1}{2};\ \ldots$

$a = -4$ *and*
$$\begin{aligned} d &= -1\tfrac{3}{4} - (-4) \\ &= -\tfrac{7}{4} + \tfrac{4}{1} \\ &= -\tfrac{7}{4} + \tfrac{16}{4} \\ &= \tfrac{9}{4} \end{aligned}$$

$$\begin{aligned} T_8 &= -4 + 7(\tfrac{9}{4}) \\ &= -\tfrac{16}{4} + \tfrac{63}{4} \\ &= \tfrac{47}{4} \end{aligned}$$

(3) Which term of the sequence 4; 9; 14; … will be 34?

$9 - 4 = 5$ *and* $14 - 9 = 5$

∴ this is an arithmetic sequence, with $a = 4$, $d = 5$, $T_n = 34$
We have to find n.

$$\begin{aligned} T_n &= a + (n-1)d \\ 34 &= 4 + (n-1)5 \\ &= 4 + 5n - 5 \\ 35 &= 5n \\ 7 &= n \end{aligned}$$

The seventh term has a value of 34.

(4) Determine how many terms there are in the following sequence:

$45;\ 43\frac{3}{4},\ 42\frac{1}{2};\ \ldots 33\frac{3}{4}$

$43\frac{3}{4} - 45 = -1\frac{1}{4}$ *and* $42\frac{1}{2} - 43\frac{3}{4} = -1\frac{1}{4}$

$\therefore$ this is an arithmetic sequence, with $a = 45$, $d = -1\frac{1}{4}$

$T_n = 33\frac{3}{4}$

We must find n.

$$\begin{aligned}
T_n &= a + (n-1)d \\
33\tfrac{3}{4} &= 45 + (n-1)(-1\tfrac{1}{4}) \\
\tfrac{135}{4} &= 45 - \tfrac{5n}{4} + \tfrac{5}{4} \\
135 &= 180 - 5n + 5 \\
5n &= 50 \\
n &= 10
\end{aligned}$$

(5) $p;\ 2p+2;\ 5p+3$ are the first three terms of an arithmetic sequence. Find p, and hence write down the values of the first three terms.

$d = 2p + 2 - p$ *and* $d = 5p + 3 - (2p + 2)$

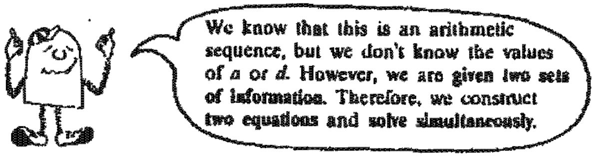
$$\begin{aligned}
\therefore\quad 2p + 2 - p &= 5p + 3 - (2p + 2) \\
p + 2 &= 5p + 3 - 2p - 2 \\
-2p &= -1 \\
\therefore\quad p &= \tfrac{1}{2}
\end{aligned}$$

First 3 terms: $p;\ 2p + 2\ ;\ 5p + 3$

$\frac{1}{2};\ 2(\frac{1}{2}) + 2;\ 5(\frac{1}{2}) + 3$

$\frac{1}{2};\ 3;\ 5\frac{1}{2}$

(6) The twelfth term of an arithmetic sequence is -5, and the twentieth tern is 7. Find the seventh term.



$T_{12} = -5$ *and* $T_{20} = 7$

$a + 11d = -5 \ldots (1)$ $\qquad a + 19d = 7 \ldots (2$

$(2) - (1)$: $8d = 12$

$d = \frac{3}{2}$

Substitute in (1): $a + 11(\frac{3}{2}) = -5$

$$\begin{aligned}
a + \tfrac{33}{2} &= -5 \\
2a + 33 &= -10 \\
2a &= -43 \\
a &= -21\tfrac{1}{2} \\
T_7 &= a + 6d \\
&= -21\tfrac{1}{2} + 6(\tfrac{3}{2}) \\
&= -12\tfrac{1}{2}
\end{aligned}$$

(7) The seventh term of an arithmetic sequence is 20 and the thirteenth term is 38. Find the first three terms.

$T_7 = 20$ *and* $T_{13} = 38$

$a + 6d = 20 \ldots\ldots (1)$ $\qquad a + 12d = 38 \ldots\ldots (2)$

$(2) - (1)$: $6d = 18$

$d = 3$

Substitute in (1): $a + 6(3) = 20$

$a = 2$

First three terms: 2; 5; 8

EXERCISE 3.1

1. Find the seventeenth term of the sequence 2; 5; 8; ...
2. How many terms are there in the sequence 3; 7; 11; ...; 147?
3. Which term of the arithmetic sequence 10; 7; ... will be − 5?
4. In an arithmetic sequence the seventh term is 30 and the fourth term is 18. Find the first term and the common difference.
5. Find the twentieth term of an arithmetic sequence if the fifth term is 11 and the twelfth term is 25.
6. The fourth term of an arithmetic sequence is 14 and the sixteenth term is 50. Which term has a value of 77?
7. Find the 9th term of the sequence − 1; 3; 7; ....
8. An arithmetic sequence has 19 terms. What is the common difference if the first term is 3 and the last term is 39?
9. Given the sequence 39; 34; 29; ...; − 1, find the number of terms.
10. The eighth term of an arithmetic sequence is 50 and the 50th term is 176.
    (a) Find the first term.
    (b) Find the common difference.
    (c) Write down the first three terms of the sequence.

11. Given the sequence x; $\frac{x+y}{2}$; y; . . .:
    (a) Write down the 4th term in terms of x and y.
    (b) Write down the 13th term in terms of x and y.
    (c) If the last term is $10y - 9x$, determine the number of terms, in terms of x and y.

12. Given the sequence 20; $23\frac{1}{2}$; 27; . . .:
    (a) What is the 50th term?
    (b) Which term in the sequence is equal to 1 063?

13. The 5th term of an arithmetic sequence is 18 and the 15th term is -2.
    (a) Write down the first three terms of the sequence.
    (b) What is the 19th term?
    (c) Which term is equal to -26?

14. The rth term of a sequence is given by $5r + 2$.
    (a) Write this formula in the form $a + (r - 1)d$ to show that it is an arithmetic sequence.
    (b) Find the 47th term.
    (c) Which term is equal to 497?

15. Given that $\frac{1}{2}$; x; 4; . . . is an arithmetic sequence:
    (a) Which term is equal to $42\frac{1}{2}$?
    (b) What is the 13th term?
    (c) If the last term is 32, how many terms does the sequence have?

16. In an arithmetic sequence, the sum of the 33rd term and the 43rd term is 100 and the difference between them is 40, where $T_{43} > T_{33}$.
    (a) If the specification $T_{43} > T_{33}$ was omitted, would it have made any difference to the results? Explain your answer.
    (b) What is the 13th term?
    (c) Which term is equal to 158?

17. The 10th and 13th terms of an arithmetic sequence are $10(x + y)$ and $13(x + y)$ respectively:
    (a) What is the common difference?
    (b) What is the first term?
    (c) Find the sum of the 50th and 59th terms.
    (d) Find the difference between the 22nd and 19th terms.
    (e) Find $T_{18} \div T_{40}$.
    (f) Find $T_2 \times T_7$.

18. An arithmetic sequence has $2n + 2$ terms. The middle term is 13, the third term is 7 and the common difference is 2. Find the last term.

19. Find the 2nd term of an arithmetic sequence which has $T_1 = a$ and $T_3 = b$.

20. Given the sequence $x - y$; $2x$; $3x + y$; . . .:
    (a) Show that the sequence is arithmetic.
    (b) Find the 13th term.
    (c) Give an expression for the nth term.

21. You are given two arithmetic sequences:
    a; $a + d_1$; $a + 2d_1$; . . . T_7; . . . and
    a; $a + d_2$; $a + 2d_2$; . . . . . . T_9; . . .
    where $T_7 = T_9 = 41$.
    Write down the first three terms of each sequence if $d_1 = -4$.

TOPIC 3.2: ARITHMETIC SERIES

The sum of the terms of a sequence is called a series.

Arithmetic sequence: 3; 7; 11; . . .
Arithmetic series: $3 + 7 + 11 + \ldots$

Derivation of the formula for the sum to n terms of an arithmetic sequence

Consider an arithmetic series of n terms with first term a, common difference d, nth term ℓ and sum to n terms S_n.

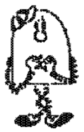
$$S_n = a + (a + d) + (a + 2d) + \ldots + (\ell - 2d) + (\ell - d) + \ell \;\ldots\ldots\ldots (1)$$
$$S_n = \ell + (\ell - d) + (\ell - 2d) + \ldots + (a + 2d) + (a + d) + a \;\ldots\ldots\ldots (2)$$

Add (1) and (2)

$$2S_n = (a + \ell) + (a + \ell) + (a + \ell) \ldots + (a + \ell) + (a + \ell) + (a + \ell)$$
$$S_n = \frac{n}{2}(a + \ell) \ldots\ldots (3)$$

But $T_n = \ell = a + (n - 1)d$

Substitute in (3): $S_n = \frac{n}{2}[a + a + (n - 1)d]$
$= \frac{n}{2}[2a + (n - 1)d] \ldots\ldots (4)$



Note:
(1) In equation (2) of the derivation we write the first line backwards.
(2) The equation for $2S_n$ is obtained by adding the first two equations.
(3) We use (3) when we have the last (nth) term. We use (4) when we have a, n and d, but not ℓ

Appendix 4: Student A4

SEQUENCES AND SERIES

FEB 1998

-A Sequence IS a ~~t~~ String of numbers

1, 2, 3, 4, 5, 6, 7, 8, ...

x 3, 6, 9, 12, 15, 18, 21 . . . .

1, 4, 9, 16, 25, 36, 49, 36, 49, 64, 81, 100, 121, ...

x2 3, 6, 12, 24, 48, 96, 192, 384, 468, ...

+d 7 4, 11, 18, 25, 32, 39, 46, 53, 60, 67, 74, 81, 88

Arithmetic Sequence has the same difference Between Consecutive terms

eg Say which is AS.

u7 346x2 1) 5, 7, 9, 11, 13 - Arithmetic because it has the same difference wi...

2) 5, 16, 28, 39, 50 - Not Arithmetic

3) 2, 4, 8, 16, 32 - Arithmetic

4) 3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, $T_{20} = 3+19\times6$

$= 117$

$T_{347} = 3+346\times6$

$= 2079$

$T_k = 5+(k-1)\times2$

---

26 FEB 1998

Classwork

In each of the following find

1 Tenth term

2 T_{101}

a) 3, 7, 11, 15, 19, 23, 27,

---

26 FEB 199

Classwork

In each of the following find

1. Tenth term

2. T_{101}

General Term of AS

$T_n = a+(n-1)d$

ⓐ 3, 7, 11, . . .

$T_{10} = 4\times9+3$

$= 39$ ✓

$T_{101} = 100^{-4}$

$= 4\times100+3$

$= 403$ ✓

ⓑ 45, 41, 37, 33, . .

$T_{10} = -4\times9+45$

$= -36+45$

$= 9$ ✓

$T_{101} = 100\times4+45$

$= -400+45$

$= -355$ ✓

ⓒ -131, -126 -121, . . .

$T_{10} = 5\times9 - 131$

$= -86$ ✓

$T_{101} = 5\times100 - 131$

$= 500 - 131$

$= 369$ ✓

ⓓ 3, $4\frac{1}{2}$, 6, . . .

$T_{10} = 1{,}5\times9+3$

$= 16{,}5$ ✓

$T_{101} = 1{,}5\times100+3$

$= 150+3$

$= 153$ ✓

ⓔ 10, 16, 22, . . .

$T_{10} = 6 \times 9 + 10$

$= 64$

$T_{101} = 6 \times 100 + 10$

$= 600 + 10$

$= 610$

f) $\log x, \log x^3, \log x^5 \ldots$

$T_{10} = \log x + (9 \times \log x^2)$ $\log x \div \log x^{8}$

$= 1 + 18$

$= 19 \log x$

$T_{101} = 1 + 100 \times 2$ $(\log x \div 100 \log x^2$

$= 200 + 1$

$= 201$

ⓖ $\sin x, 2\sin x, 3\sin x$

$T_{10} = \sin x(9 \times \sin x)$

$= 10 \sin x$

$T_{101} = \sin x + (100 \sin x)$

$= \sin x + 100 \sin x$

$= 101 \sin x$

ⓗ $x^2, 2x^2, 3x^2$

$T_{10} = x^2, 9x$

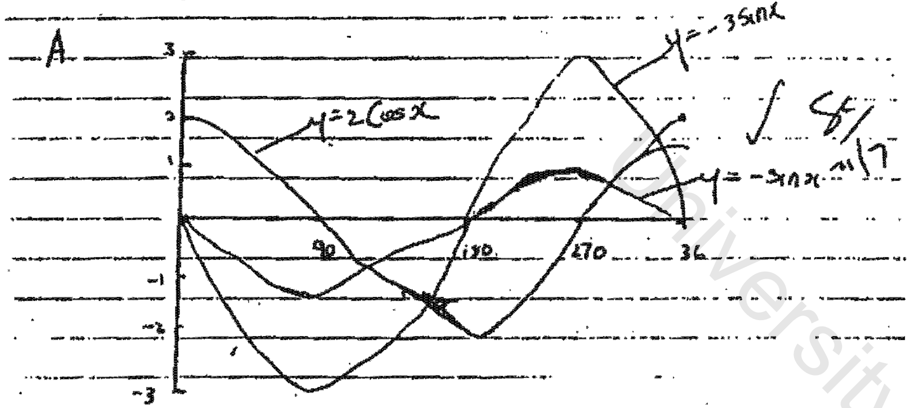
Appendix 5:

List of exposition and exemplars in the student productions at school B and the corresponding pages of exposition and exemplars in *Understanding Mathematics*.

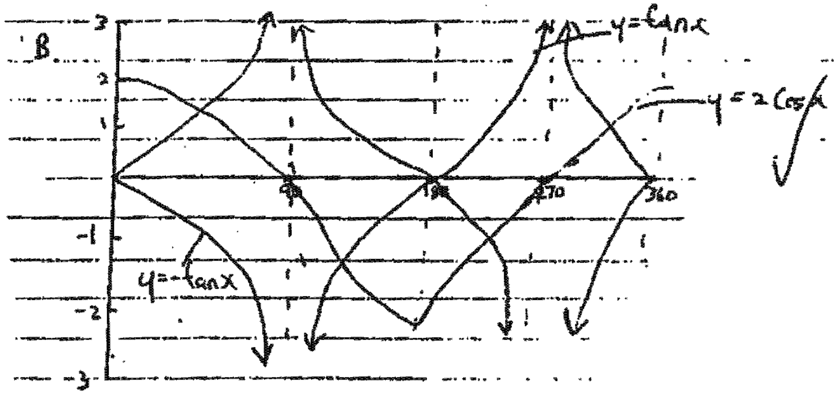
Topic	Sub-topic	Exposition and exemplars in students notebooks from the following pages of *Understanding Mathematics*	Sub-topic number in *Understanding Mathematics*
Logarithms and Exponents		Not taken from *Understanding Mathematics*	
Sequences and Series	Arithmetic Sequences	P 44, 45, 46	Topic 3.1
	Geometric Sequences	P 54, 55, 56	Topic 3.3
	Arithmetic Series	P 49, 50, 51	Topic 3.2
	Geometric Series	P 58	Topic 3.4
	Sigma Notation	P68	Topic 3.6
Analytic Geometry	Distance between two points	P 215, 216	Topic 9.1
	Midpoint of a line segment	P 218, 219	Topic 9.2
	Gradient of a line	P 221	Topic 9.3
	Equation of a straight line	P 226, 222	Topic 9.4, exemplar from topic 9.3
Calculus	Limits	P 92, 93	Topic 4.3
	Average gradient	Not directly from *Understanding Mathematics*	
	Gradient of a curve at a point	P100	Topic 4.5
	Derivative of a function	P 103, 105	Topic 4.6
	Rules for differentiation	P 107, 110, 111	Topic 4.7

Draw the graph of the following for $x \in [0, 360]$

A



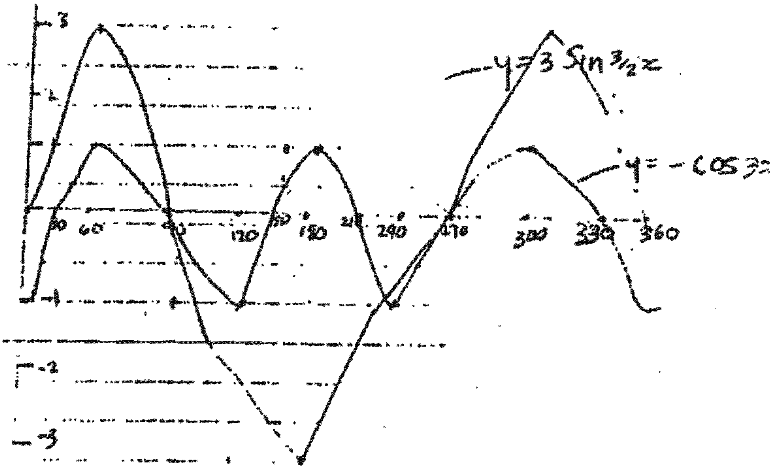
B



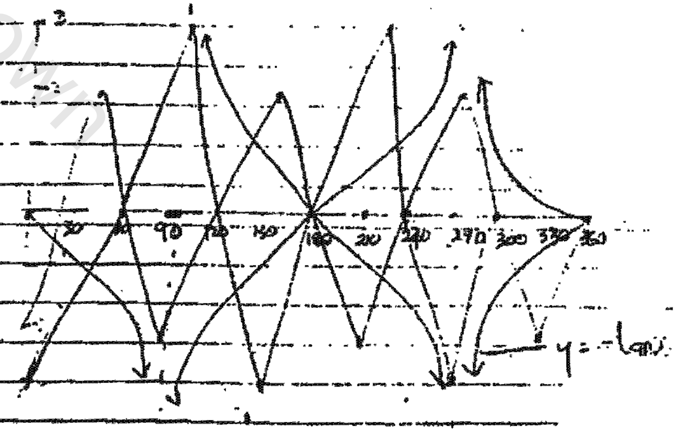
Draw on the same set of axis

1) $y = 3 \sin \frac{3}{2}x$ sin $3 \times \frac{3}{3} = \frac{180}{3} = 60$

2) $y = -\cos \frac{3}{2}x$ $\frac{90}{3} = 30.3$



H/W



$k^2 - 64k + 64 - 16k^2 + 256 \geq 0$

$-64k \geq -320$

$k \leq 5$

4\. [illegible]

$3mx^2$ $f(2-3x)$ $m = 2mx$ [illegible]

$3mx^2 + 2m - 3mx - 2mx = 0$

$3mx^2 - 5mx + 2 = 0$

$\Delta = (-5m)^2 - 4(3m)(2) = 0$

$25m^2 - (12m)(2) = 0$

$25m^2 - 24m = 0$

$m(25m - 24) = 0$

$m = \frac{24}{25}$

5 $x^2 - px + p = 2$ [illegible]

$x^2 - px + p - 2 \not< 0$

$\Delta = (-p)^2 - 4(1)(p-2) \not< 0$

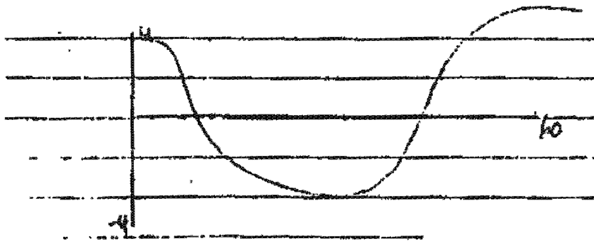
$p^2 - 4p + 8 \not< 0$ No solution

1\. $y = 2 \sin 3x$

2 $y = -3 \cos 3x$

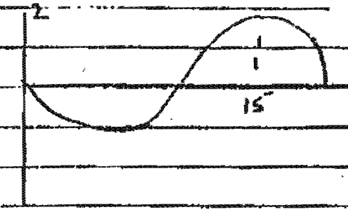
3 $y = -\tan x$

Pg 311 No 5 a, c, d, f.



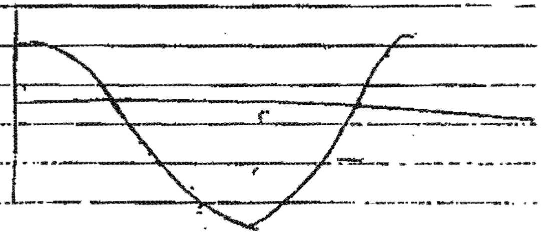
$a = 4$

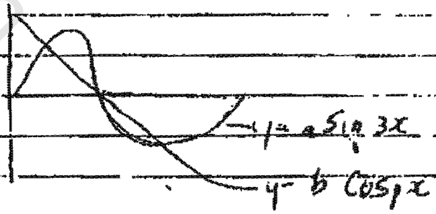
$b = 6$

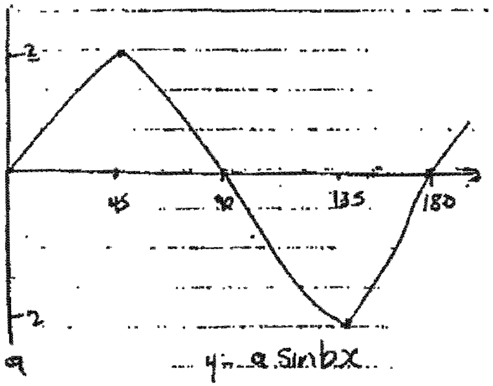


$-b = 180$

$a = -2$



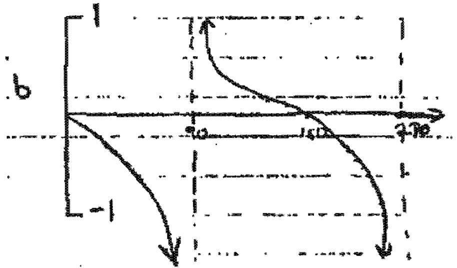




y = a sin bx

a = 2

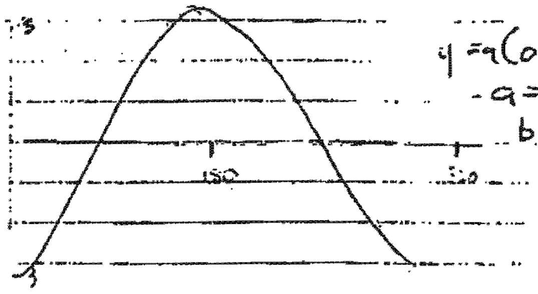
b = 2



$y = a \tan^{b} x$

a = 1

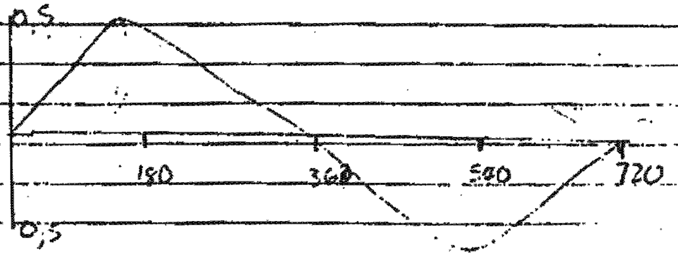
b = 1



y = a cos b x

- a = -3

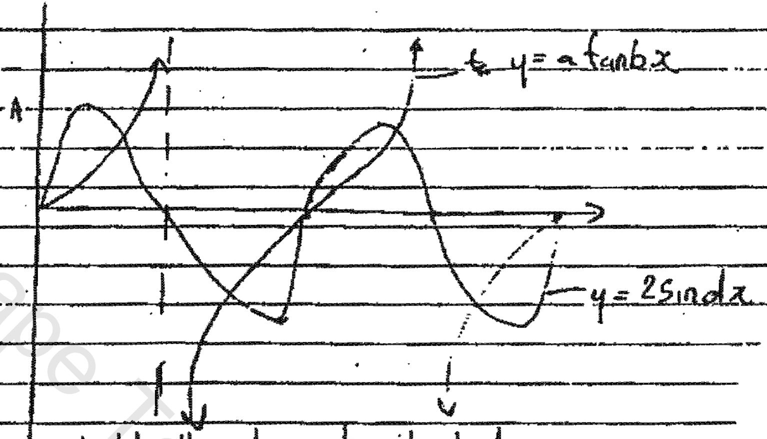
b = 1



y = a sin bx

a = 0,5

b = 0,5



Calculate the values of a, b, d, A

(i) a = 1

b = 1 d = 2

A = 2

Calculate the co-ordinates

(ii) $C = \frac{360}{60}$ = C (75°; 2)

B = (30; 0)

A = (105°; -2)

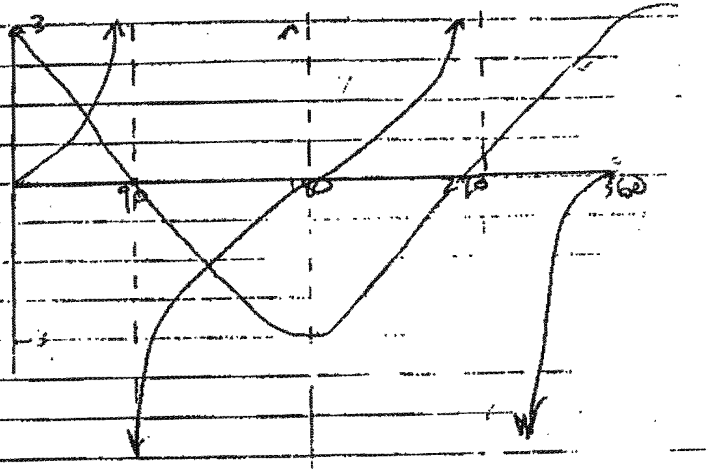
What is the period f & g
f(

What is the amplitude of g

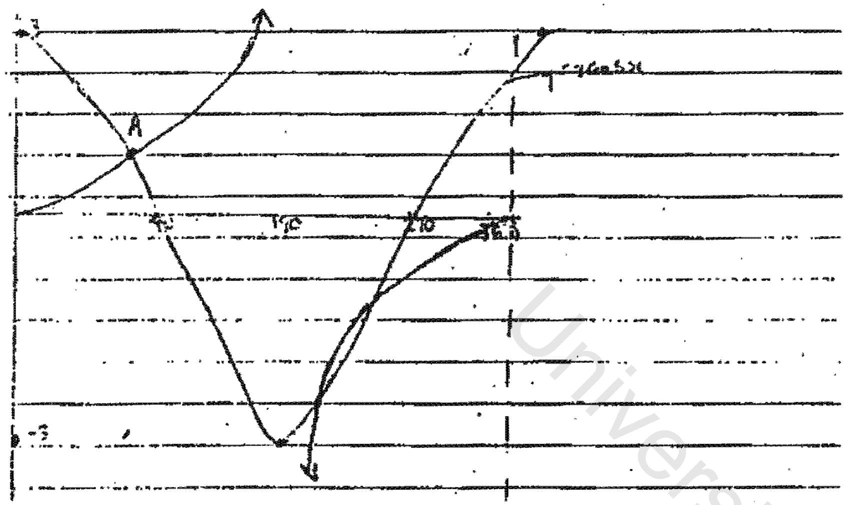
Solve for x
a) $f(x) \cdot g(x) = 0$

b $f(x) = g(x)$

c) $g(x) > f(x)$



1 7
2. 720
3. 3
[illegible]
[illegible]
6

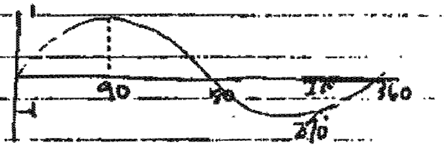


1. 360°
2. 3
3. x = (180°, x = 360°
4
5.
6.

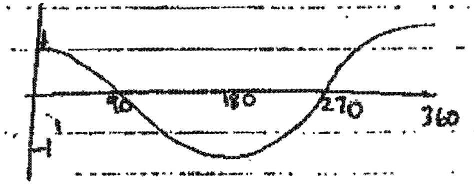
2.
(1) a = 3
1 b = 2
2.
3. x = -80 , x = 60 , x = 120
4
5
6. 30 ≤ x ≤ 120
7. 10,7

raph y = a sin bx

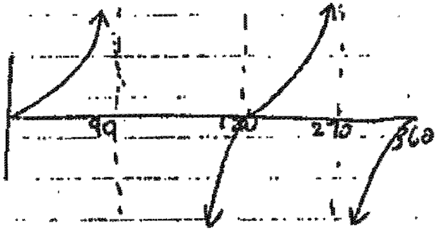
y = sin x



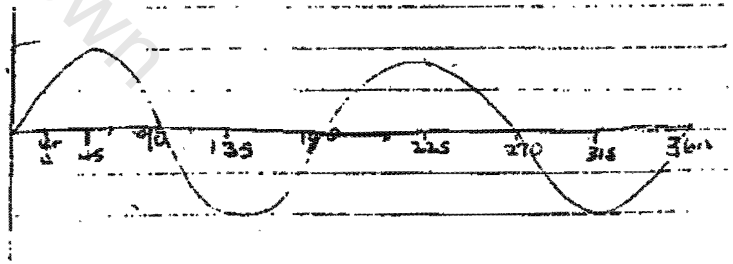
y = Cos x



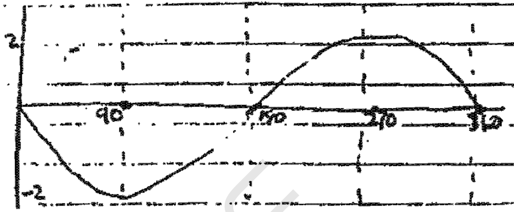
y = tan x



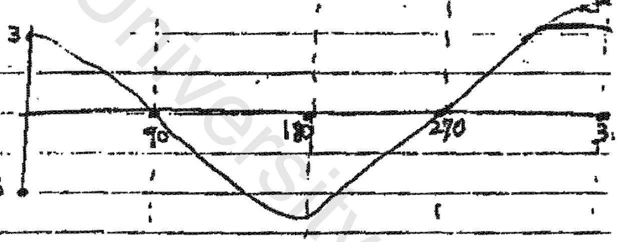
Sin 2x



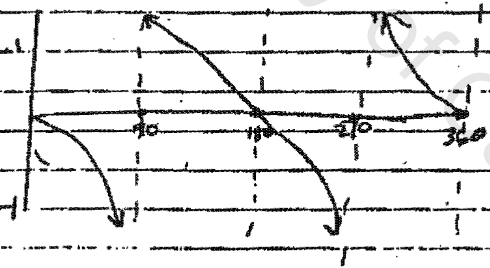
y = -2sin x



y = 3 Cos x



y = -tan x



H/W

Draw the graph of the following for x ε [0, 360]

k on the same axis

(1) y = - sin x

(2) y = 2 Cos x

(3) y = -3 sin x

B on the same axis

(1) y = tan x

(2) y = 2 Cos x

3 y = -tan x

Nature of roots

Δ < 0 no solution

Δ = one real root

Δ ≥ real root

Δ > real unequal

Δ = perfect square rational

6 (1) a = 1

(b = 4

(12,5°), -22,5), (67,5), (67,5)

6 (3) -67,5 ≤ x ≤ 60

$\frac{360}{90} = 4$

y = [illegible]

y = [illegible]

Trig Notes

Amplitude : Maximum / Minimum value

Period = what is needed for one "cycle"

Critical point = Cut axis, turning point or asymto

Amplitude of $y = a\,Sinx$ is a

$y = a\,Cos\,x$

Period of $y = Sin\,bx$

$Cos\,bx$ is $\frac{360}{b}$

$tan\,bx$

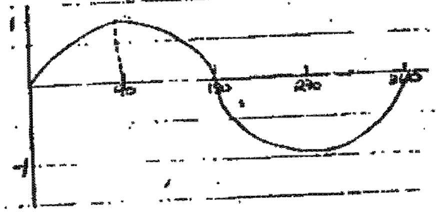
Distance between critical point is $= \frac{90}{b}$

20-07-98

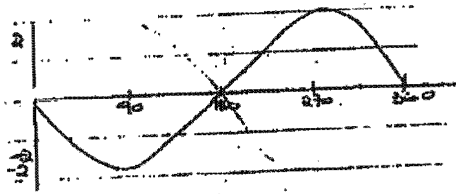
Examples

Graph: y = a sin bx

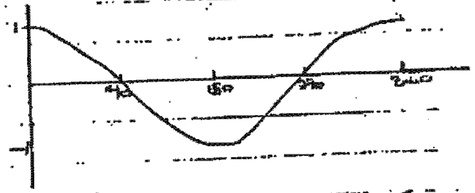
x	30	60	90	120	180
sin x					



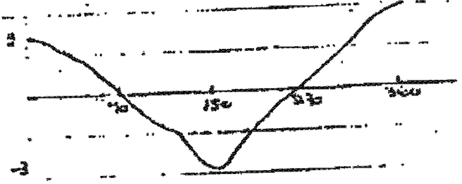
y = sin x



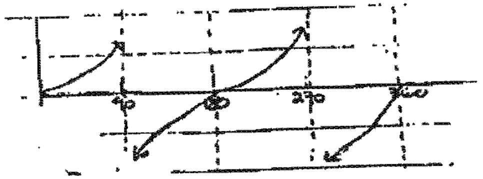
y = -2sin x



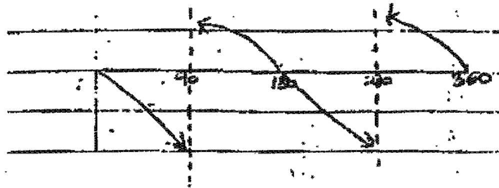
y = cos x



y = 3cos x



y = tan x



y = -tan x

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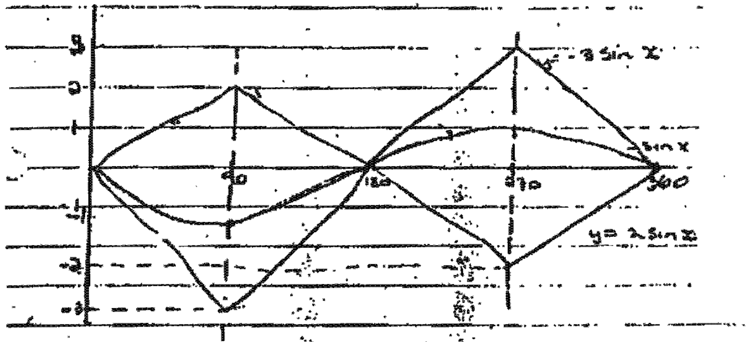
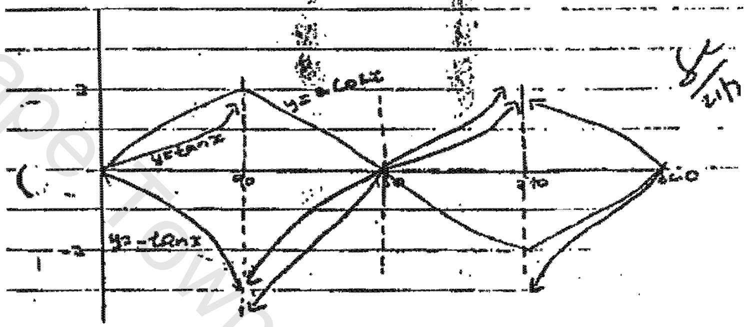
Homework





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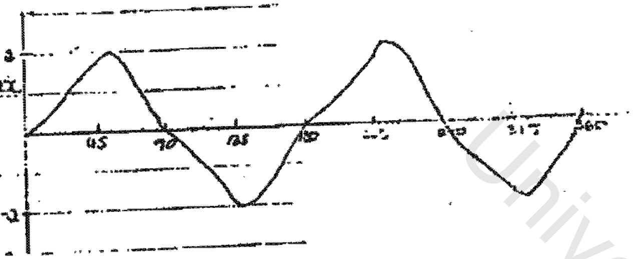
EXAMPLES

New critical point is $\frac{90}{b}$

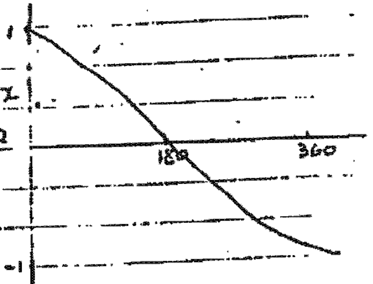
$y=\sin 2x$

$\frac{90}{2}$



$y=\cos \frac{1}{2}x$

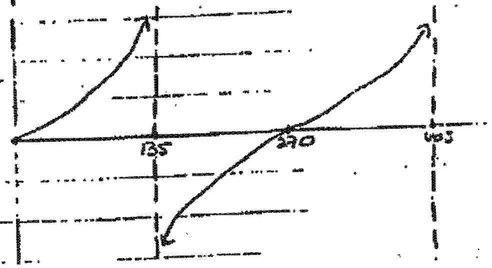
$\frac{90}{\frac{1}{2}}=\frac{90}{1}\times\frac{2}{1}$



$y=\tan \frac{2}{3}x$

$\frac{90}{\frac{2}{3}}\times\frac{3}{2}$

$=135$



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~~Homework~~ Classwork

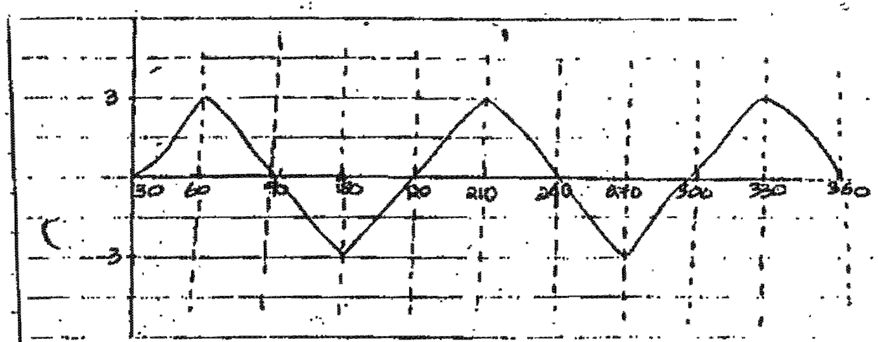
(a) $y=3\sin(\frac{3}{2}x$

$\frac{90}{b}=\frac{90}{1}\times\frac{2}{3}$

$=\frac{90(30)}{2}=60$

(b) $y=-\cos 3x$

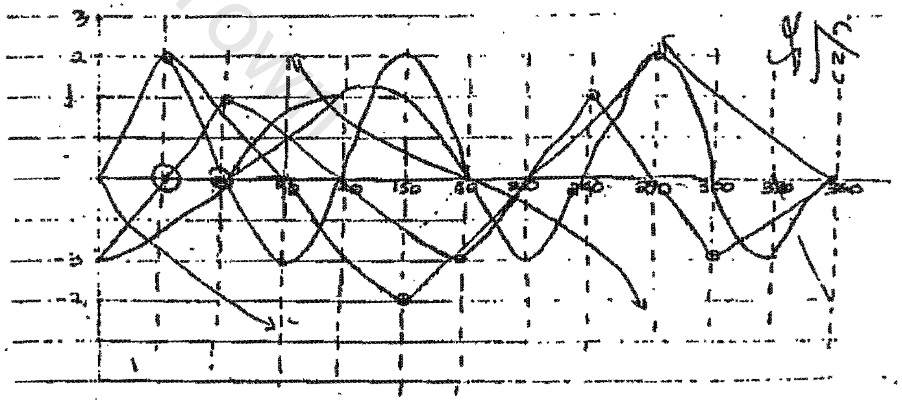
$\frac{90}{3}=30$



Homework

On the same axes

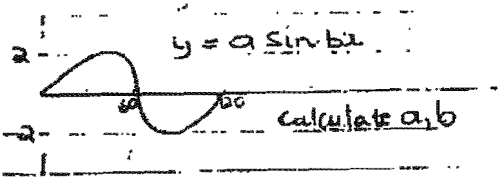
1. $y=2\sin 3x$ $\frac{90}{3}=30$
2. $y=-3\cos-3\cos\frac{3}{2}x$ $\frac{90}{(\frac{3}{2})}=\frac{90}{1}\times\frac{2}{3}$ $\frac{180}{3}=60$
3. $y=-\tan x$ 30, 60, 90, 120, 150, 180, 210, 240, 270, 300



22 July 1993

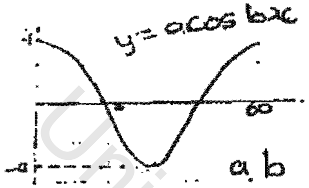
EXAMPLE

From the following functions Calculate



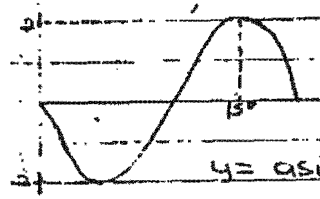
calculate a, b

a = 2
$b = \frac{180}{60} = 3$



a, b

a = 4
$b = \frac{360}{60} = 6$

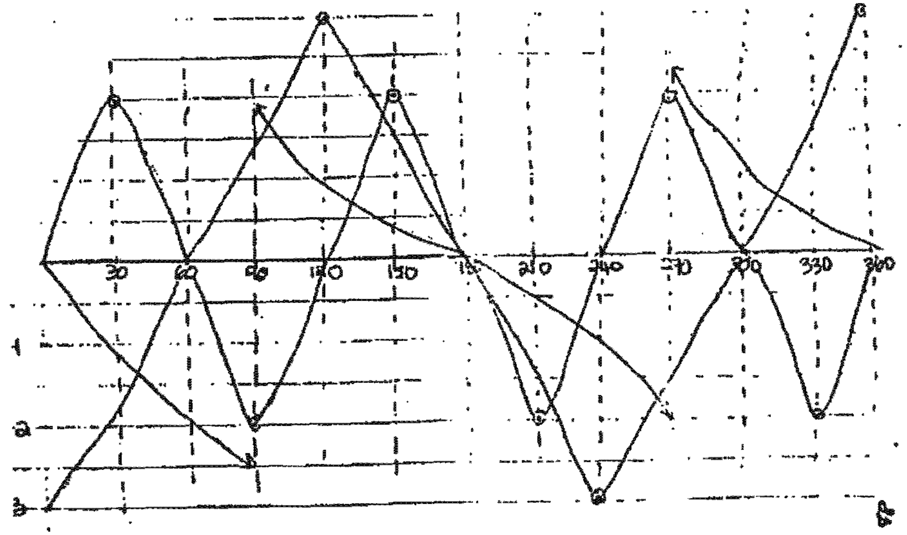


a = -2
$b = \frac{270}{15} = 18$

y = a sin bx

Corrections

y = 2 sin 3x ! y = -3 Cos $\frac{3}{12}x$



23 July 199

TRIG NOTES

Amplitude : Maximum / Minimum value

Period : "What is needed for one "cycle"

Critical point : Cut axis, Turning point or a Symtote

Amplitude of y = a sin x is a
y = a Cos x

Period of y = Sin bx
Cos bx is $\frac{360}{b}$
tan bx

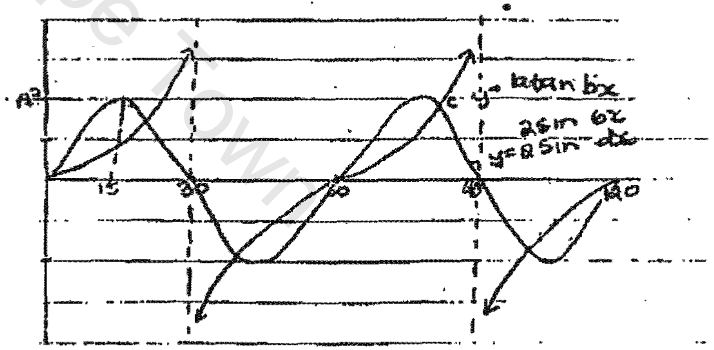
Distance between critical points is $\frac{90}{b}$

f(x) = g(x) where graphs cut.

f(x) = 0 when graph cut x-axis

f(x) - g(x) = 0 when either graph cuts x-axis

Classwork



Calculate the vales of a, b, c, d, A

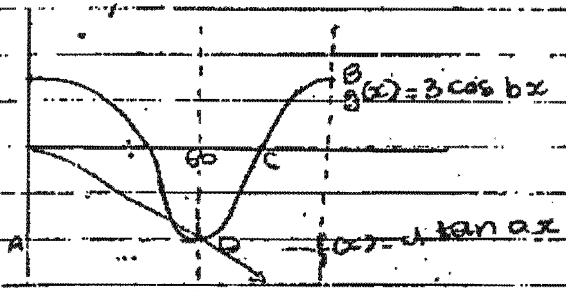
a = 2 in Sin
A = y = a tan 3x (15; 2)
2 = a tan 3(15)

2 = a tan 45
2 = a(1)
2 = a

$b = \frac{90}{30} = 3$

$d = \frac{360}{60} = 6$

24 July1998.



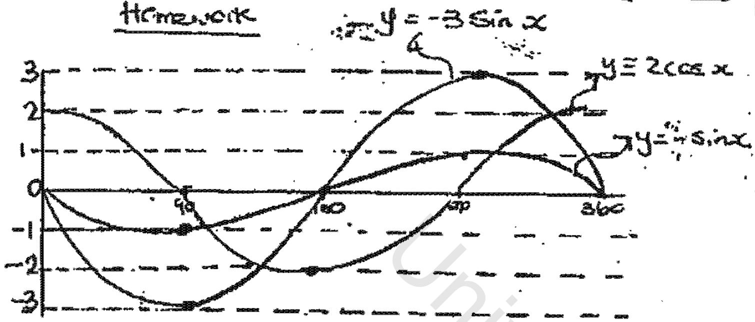
1. Calculate values of A,b,d,a
2. Co-ordinates of C,B,D
3. The period f & g
4. Amplitude of g

Appendix 6: Student A3

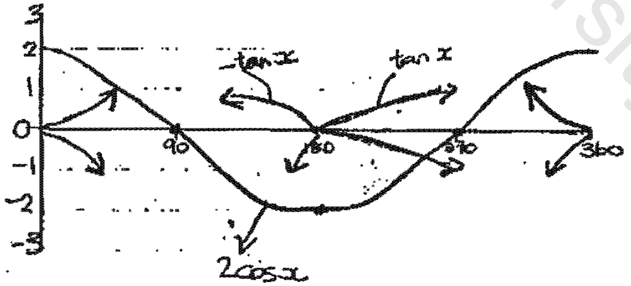
21 July 1998

Homework

1.



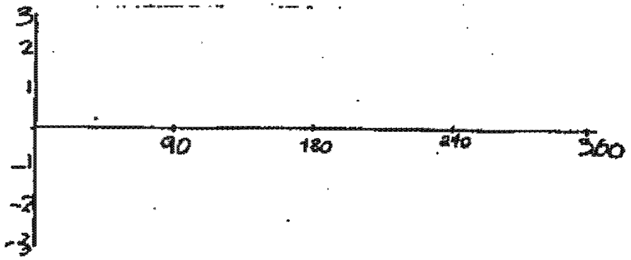
2.



Classwork 22 July 1998

1) $y = 3 \sin \frac{3}{2}x$

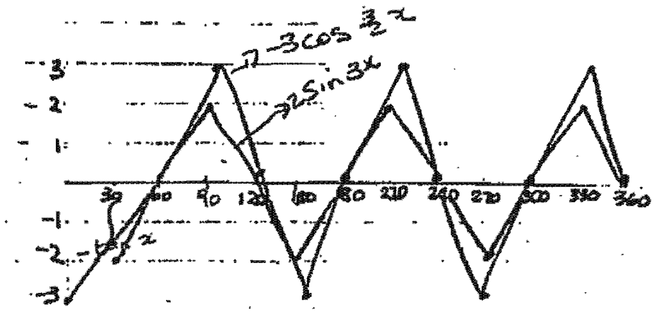
2) $y = -\cos 3x$



Grade 12.1

Homework

21 July 1998



Grade 12.1

Homework

23 July 1998

1. $x^2 + 2kx + k^2 = 1 - x$ has equal roots

$x^2 + k^2 + (2kx + x) - 1 = 0$

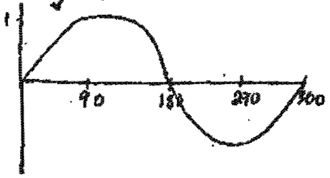
$x^2 + k^2 + x$ [illegible] $- 1 = 0$

$\Delta =$ [illegible] $= 0$

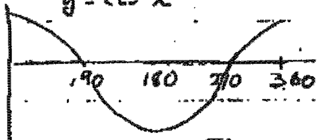
[illegible]

GRAPHS

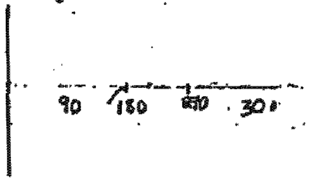
y = Sin x



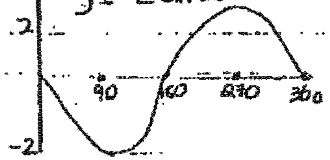
y = Cos x



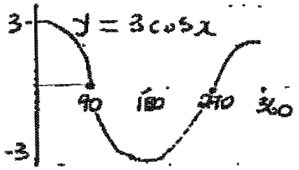
y = tan x



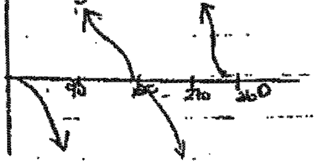
y = -2 Sin x



y = 3 cos x



y = -tan x



Prove 21 July 19

$x + 2 + 2k = \frac{9}{x} + kx$ has equal ro

Calculate k

$x(x) + 2(x) + 2k(x) = \frac{9x}{x} + kx(x)$

$x^2 + 2x + 2kx = 9 + kx^2$

$(x^2 - kx^2) + (2x + 2kx) - 9 = 0$

$x^2(1-k) + x(2+2k) - 9 = 0$

$\Delta = b^2 - 4ac = 0$

$(2+2k)^2 - 4(1-k)(-9) = 0$

$(4 + 8k + 4k^2) + 36(1-k) = 0$

$4 + 8k + 4k^2 + 36 - 36k = 0$

$4k^2 - 28k + 40 = 0$

$4(k^2 - 7k + 10) = 0$

$(k-2)(k-5) = 0$

k = 2 or k = 5

2.2 Prove $x^2 + x + a = 0$ if $a > \frac{1}{4}$ has no r

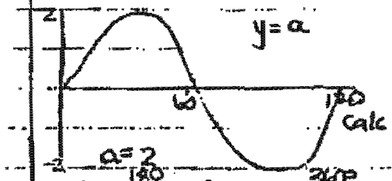
$b^2 - 4ac < 0$

$1^2 - 4(1)a < 0$

$1 - 4a < 0$

$\frac{-4a}{-4} < \frac{-1}{-4}$

$a > \frac{1}{4}$

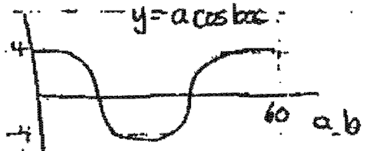


Calc a+b

$a = 2$

$b = \frac{180}{60} = 3$ $x = \frac{360}{120} = 3$

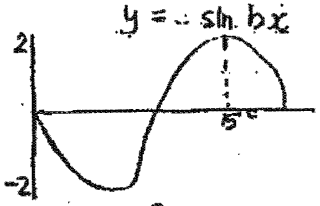
y = a cos bx



$a = 4$

$b = \frac{360}{60} = 3$

y = sin bx



$a = -2$

$b = \frac{270}{15} = 18$

TRIG NOTES

Amplitude: Maximum/Minimum Value

Period = What is needed for one "cycle"

Critical Point = Cut axis, Turning point or asymptote

Amplitude of y = a sin x is "a"
y = a cos x

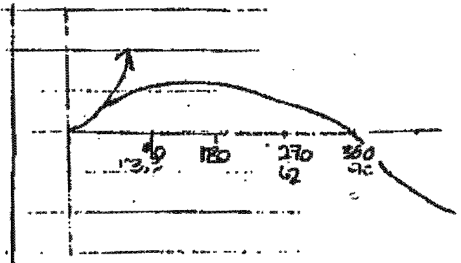
Period of y = sin bx, cos bx, tan bx is $\frac{360}{b}$

Distance between critical points is $\frac{90}{b}$

f(x) = g(x) where graph cut

f(x) = c when graph cut x-axis

f(x) = g(x) = c when either graph cuts



f = tan ax

g = b sin ax

Calculate the values of a, b and c

$a = \frac{360}{90} = 4$

$c = \frac{180}{90} = 2$

$b = 2$

CP = [illegible] = 22,4

CP = [illegible] = 1,5

A = ([illegible] (13,2)

[illegible]

1.4 What is the domain of f

[illegible]

[illegible]

[illegible]

1.5. Range of g (y-values)

[illegible]

[illegible]

[illegible]

= 2

5 2.3) $\cdot$ Log$_2(x-2)+\log_2(x-3)=2$

20/07/98

Homework

Draw the graph of the following for $x=(0;360)$

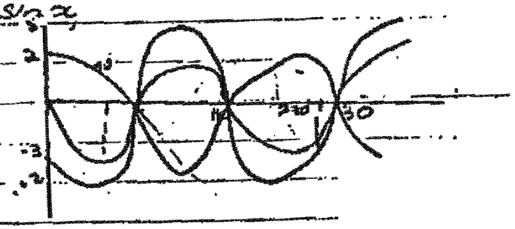
① On the same axis

① $y=-\sin x$

② $y=2\cos x$

③ $y=-3\sin x$

(



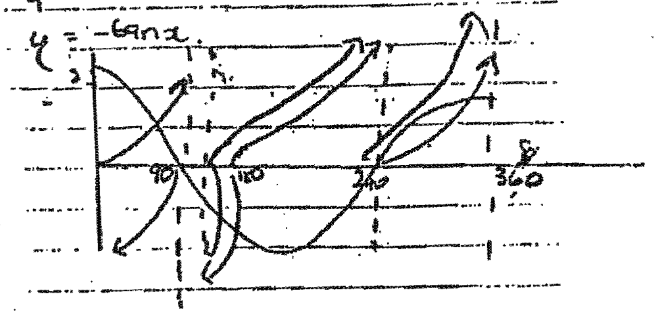
) on the same axis

) $y=\tan x$

$y=2\cos x$

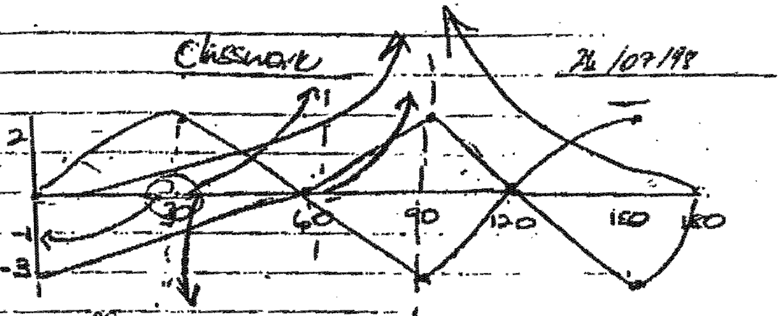
$y=-\tan x$

(



Classwork

21/07/98



$y=2\sin 3x \quad \frac{90}{3}=30$

$y=-3\cos\frac{3}{2}x \quad \frac{90}{1}\cdot\frac{2}{3}=60$

$y=-\tan\frac{1}{2}x$

(

22/07/98

Example

$x^2+x+a=0 \quad$ if $a>\frac{1}{4}$

non real $b^2-4ac<0$

$1^2-4(1)a<0$

$1-4a<0$

$\frac{-4a}{4}<\frac{-1}{4}$

$a>\frac{1}{4}$

23/07/98

Homework

$x^2+2kx+k^2=1-x$ equal roots

$x(x+1)+k=x$ non real roots

$x^2-4kx+3x+k^2-16=0$ real roots

roots of

$3mx^2+(2-3n)n=2mx$ are rational

$\alpha+\beta=?$ ac

$x^2 + x - 2$

$x^2 + 2kx + k^2 = 1 - x$

$(x + [illegible] k) = 1 - x$

$x^2 + 2kx + k^2 = 1 - x^{1x}$

$x^3 + 2kx [illegible] = 1$ $x^2 + 2kx + k^2 - 1 + x = 0$

[illegible] $+ k^2) = 1$ $x^2 + (2k + 1)x + k^2 - 1 = 0$

[illegible] $+ k)$ $a = 1$ $b = 2k + 1$ $c = k^2 - 1$

$x = -k$ [illegible] $\Delta = 0$

(4) $3mx^2 + (2 - 3x)n = 2mx$

$3mx^2 + [illegible] = 2mx$

3) $2x(x - 1) + k = x$

$b^2 - 4ac < 0$

$2x^2 - 2x + k = x - x$

$2x^2 - 3x + kx = 0$

$(2)^2 - 4(2)(k) < 0$

$4 - 8k < 0$

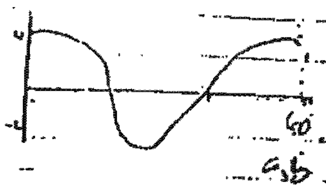
$k < \frac{-4}{-8}$

$k = \frac{1}{2}$

$4x^2 - [illegible] kx + 8x + k^2 - 16 = 0$

22/07/98

$= a \cos bx$

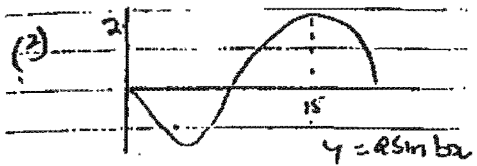


a, b

$= 4$

$= \frac{360}{60} = 6$

(2)



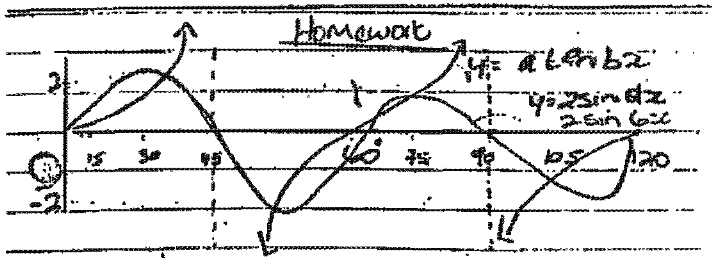
$y = a \sin bx$

$a = -2$

$b = \frac{270}{15} = 18$

23/07/98

Homework



$y = 2 \sin dx$

$\frac{360}{60} = 6 \cdot d$ sin

$\frac{90}{6} = 15$

$\frac{90}{30} = 3 = b$ tan

$(15; 3)$

$y = a \tan bx$

$y = a \tan 3x$

$2 = a \tan 3(15)$

$2 = a \tan 45$

$2 = a(1)$

$2 = a$

y = [illegible] x

Time table

Monday: 8 Time physics
Tuesday 9 Time Xhosa II
Wed 10 Time 8'30
12 A[illegible] II [illegible]
15 Xh[illegible] III
17 Maths
18

Time Table

Monday	8	Time	Physics	8:30
[illegible]	9	Time	Xhosa II	8:30
Friday	12	Time	Afr II	[illegible]
[illegible]	10	Time	Ec	[illegible]
[illegible]	15	Time	Xhosa II	
	17	Time	Maths	

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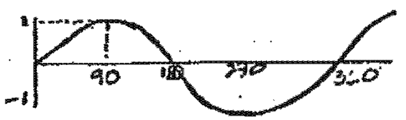
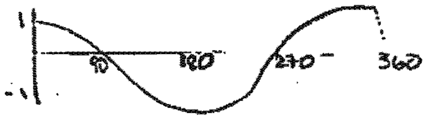
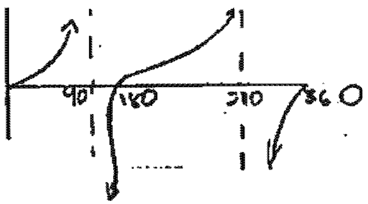
(1) y = sin x



y = cos x



(3) y = tan x



x	0	30	60	90	150		360
[illegible]	0	0,5	987	1	0,09	015	—

H/w

D[illegible] the graph of the following for x = (0,360).

[illegible] same graph

y = −sin x

y = 2 cos x

y = −2 sin x

[illegible] same axis

[illegible]

y = 2 cos x

[illegible] − cos x

Trig Notes

Amplitude: [illegible] maximum/minimum Value

[illegible] is needed for all

[illegible] axis; turning point of

[illegible] of y = a sin x is 'a'.

y = a cos x

[illegible] is $\frac{360}{b}$

[illegible] period is $\frac{90}{b}$

5.2.3) $\log_2(x-2)+\log_2(x-3)=2$

20/07/98

Homework

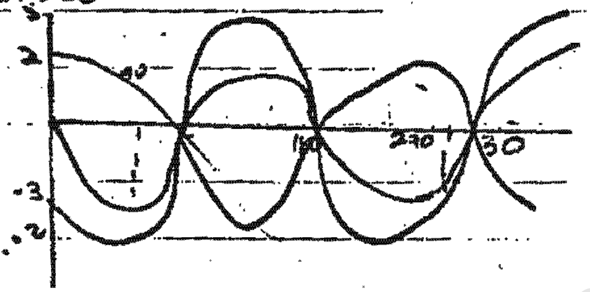
Draw the graph of the following for $x = (0;360)$

ⓐ On the same axis

1) $y = -\sin x$

2) $y = 2\cos x$

3) $y = -3\sin x$

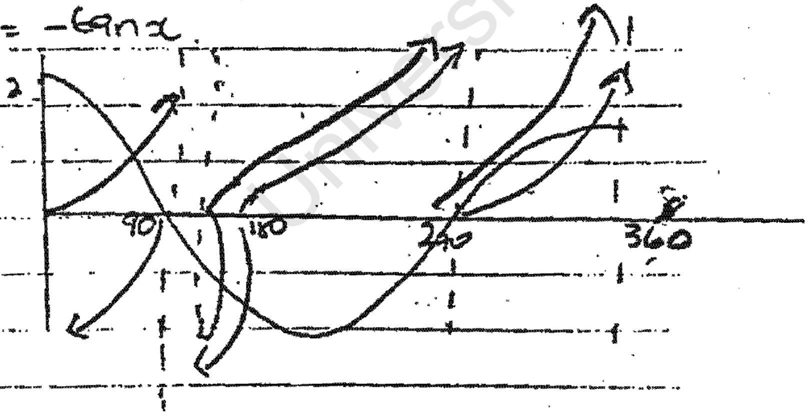


b) On the same axis

1) $y = \tan x$

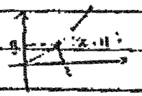
$y = 2\cos x$

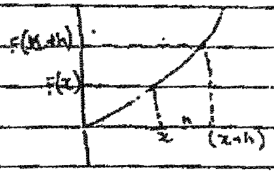
$y = -\tan x$



AVERAGE GRADIENT

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y-y}{x-x} = \frac{0}{0}$$ undif inerr





$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{\cancel{x}+h-\cancel{x}} = m$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2 - 3x + 2$$

$$f(x+h) = (x+h)^2 - 3(x+h) + 2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) + 2 - (x^2 - 3x + 2)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 3)}{h}$$

$$= \lim_{h \to 0} 2x + h - 3 = 2x + 0 - 3$$

$$= 2x - 3$$

2. Differentiate from first principles: $3x^2 + 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 + 5 - (3x^2 + 5)}{h}$$

[illegible]

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 5 - 3x^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - \cancel{9x^2}\ 3x^2}{h}$$

$-3(-5)^2$

$$= \lim_{h \to 0} \frac{\cancel{6x^2} + 6xh + 3h^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 + 5 - (3x^2 + 5)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 5 - 3x^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$$

$$= \lim_{h \to 0} \frac{3h(2x + h)}{h}$$

$$= \lim_{h \to 0} 3(2x + h)$$

$$= 6x + 0$$

$$6x$$ ✓

ⓑ $5x^2 - 3x + 2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5(x+h)^2 - 3(x+h) + 2 - (5x^2 - 3x + 2)}{h}$$

$$= \lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) - 3x - 3h + 2 - 5x^2 + 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{5x^3 + 10xh + 5h^2 - 3h - 5x^3}{h}$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(10x + 5h - 3)}{\cancel{h}}$$

$$= \lim_{h \to 0} 10x + 5h - 3$$

$$= 10x + 5(0) - 3$$

$$= 10x - 3$$

ⓒ $5x^4$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5(x+h)^4 - 5x^4}{h}$$

d) $3x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - (3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3x^3 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(6x + 3h)}{h}$$

$$= \lim_{h \to 0} 6x + 3h$$

$$= 6x + 3(0)$$

$$= 6x$$

$(x+h)(x+h)$

$x^2 + xh + xh + h^2$

$(x^2 + 2xh + h^2)(x^2 + 2xh + h^2)$

$x^4 + 2x^3h + x^2h^2 + 2x^3h + 4x^2h^2 + 2xh^3 + h^2x^2 + 2xh^3 + h^4$

$x^4 + 4xh^3 + 4x^3h + 2x^2h^2 + h^4$

$$\frac{5x^4 + 20xh^3 + 20x^3h + 10x^2h^2 + 4h^4 - 5x^4}{h}$$

$$\frac{h(20xh^2 + 20x^3 + 10x^2h + 4h^3)}{xh}$$

$20xh^2 + 20x^3 + 10x^2h + 4h^3$

$20x(0)^2 + 20x^3 + 10x^2(0) + 4(0)^3$

$20x^3$

Appendix 7 : Student B2

(b) $\lim_{x \to 2} \frac{x^2 - 4}{x - \cancel{4}2}$

$$\lim_{x \to 2} f(2) = \frac{2^2 - 4}{2 - 2}$$

$$= \frac{0}{0}$$

$$\underline{= 0} \rightarrow$$

(b) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

$$= x + 2$$

$$\lim_{x \to 2} f(x) = 2 + 2$$

$$= 4 \checkmark$$

Average Gradient

$$m = \frac{f(x+h) - f(x)}{x + h - x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

At a point

$$x' = \lim_{\substack{h \to 0 \\ \cancel{h \to 0}}} \cancel{f'} = \frac{f(x+h) - f(x)}{h}$$

Appendix 8: Student B2

Calculus

Example

$$\lim_{x \to -\frac{1}{2}} \quad f(x), f(-\tfrac{1}{2}) = 2(\tfrac{1}{2}) - 1$$

$$= -1 - 1$$

$$= \cancel{0} - 2$$

Let us Examine what happens to f(x) when, we get close to point $x \to 4$ from the right

When $x = 5$

$x = 4.5$

$x = 4.1$

$x = 4.01$

$- + + = -$

$- \times - = +$

$f(x) = 2x^2 - x - 5$

Find the following limits if they exist

a, $\lim_{x \to 3} 3x^2 - 2x + 4$

$$\lim_{x \to 3} f(3) = 3(3)^2 - 2(3) + 4$$

$$= 27 - 6 + 4$$

$$= 25 \; \checkmark$$

(b) $\lim_{x \to 2} \dfrac{x^2 - 4}{x - \cancel{4}2}$

$$\lim_{x \to 2} f(2) = \frac{2^2 - 4}{2 - 2}$$

$$= \frac{0}{0}$$

$$= 0$$

(b) $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

$$= x + 2$$

$$\lim_{x \to 2} f(x) = 2 + 2$$

$$= 4 \; \checkmark$$

Average Gradient

$$m = \frac{f(x+h) - f(x)}{x + h - x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

At a point

$$f'(x) = \lim_{\substack{h \to 0 \\ \cancel{h \to x}}} f' = \frac{f(x+h) - f(x)}{h}$$

Example using derivatives

a) $f(x) = -2x^2$ use first principles to find

$f'(x) = \lim_{h \to 0} \frac{-2(x+h)^2 - 2x^2}{h}$

$= 42$

b Now find the value of the derivative

if $x = -3$

f

RULE 1

The derivative of a constant is zero

If $f(x) = 2$ find $f'(x)$ 0

R

$f(x) = x^3$

$f(x+h) = (x+h)^3$

$\lim_{h \to 0} \frac{(x+h)^3 - (x^3)}{h}$

$\lim_{h \to 0} \frac{(x+h)^2(x+h) - x^3}{h}$

$\lim_{h \to 0} (x^2 + 2hx + h^2)(x+h) - x^3$

$\frac{x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3 - x^3}{h}$

$\lim_{h \to 0} \frac{h^2x + hx^2 + h^3}{h}$

$\lim_{h \to 0} \frac{x(hx + x^2 + h^2)}{h}$

$f(x) = x^3$

$f(x+h) = (x+h)^3$

$\lim_{h \to 0} (x+h)^2(x+h)(x+h)$

$= 3x^2$

$f(x) = x^3$ ← derivative

$= 3 \times 1 \times x^2$

$= 3x^{3-1}$

$= 3x^2$

$(x-2)^3$

$\frac{3x^2 \cdot 1x}{2}$

Rule 3

The derivative of the product of constant and a function

$D_x[kf(x)] = kD_x[f(x)]$

e.g. $D_x(-2x^2)$

$D_x = -2D_x(x^2)$

$= -2 \times 2 \times 1 \times x$

$= -2 \times 2x$

$= -4x$

Using the formula to find

$f(x) = x^3$

D

$D_x(2x^2)$

$= 2D_x(x^2)$

$= 2 \times 2x$

$= 4x$

2) $D_x\left(\frac{x^3}{2}\right)$

$= \frac{1}{2}D_x(x^3)$

$= \frac{1}{2} \times 3 \times x^{3-1}$

$= \frac{1}{2} \times 3x^2$

$= \frac{3x^2}{2}$ ✓

Rule 4

The derivative of the sum of two or more functions

1) $D_x[f(x) + g(x)] = D_x[f(x)] + D_x[g$

eg

$D_x(3x^2 + 2x)$

$= D_x(3x^2) + D_x(2x)$

$= 3D_x(x^2) + 1 \times 2D_x(x)$

$= 3 \times 2x^{2-1} + 2x^{1-1}$

$= 3 \times 2x^1 + 2x^0$

$= 6x + 2$

Rule 5: The derivative of Difference between two fu

$D_x(f(x) - g(x)) = D_x[f(x)] - D_x[g(x)]$

$D_x(2x^2 - 3x)$

$= D_x(2x^2) - D_x(3x)$

$= 2D_x(x^2) - 3D_x(x)$

$= 2 \times 2x^{2-1} - 3x^{1-1}$

$= 4x - 3$

Rule 6 The derivative of the product of two or more function

* first multiply the function and then differentiate

$D_x(x-2)(x+3)$

$= D_x(x^2 + x - 6)$

$= D_x(x^2) + D_x(x) - D_x(0)$

$= 1D_x(x^2) + 1D_x(x) - 0$

$= 1 \times 2x^{2-1} + x^{1-1}$

$= 2x + 1$ ✓

Rule 7

The derivative of the Quotient of two functions

* first divide the functions then differentiate

$D_x\left(\frac{x^2-4}{(x-2)}\right)$

$= D_x\left(\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}\right)$

$= D(x+2)$

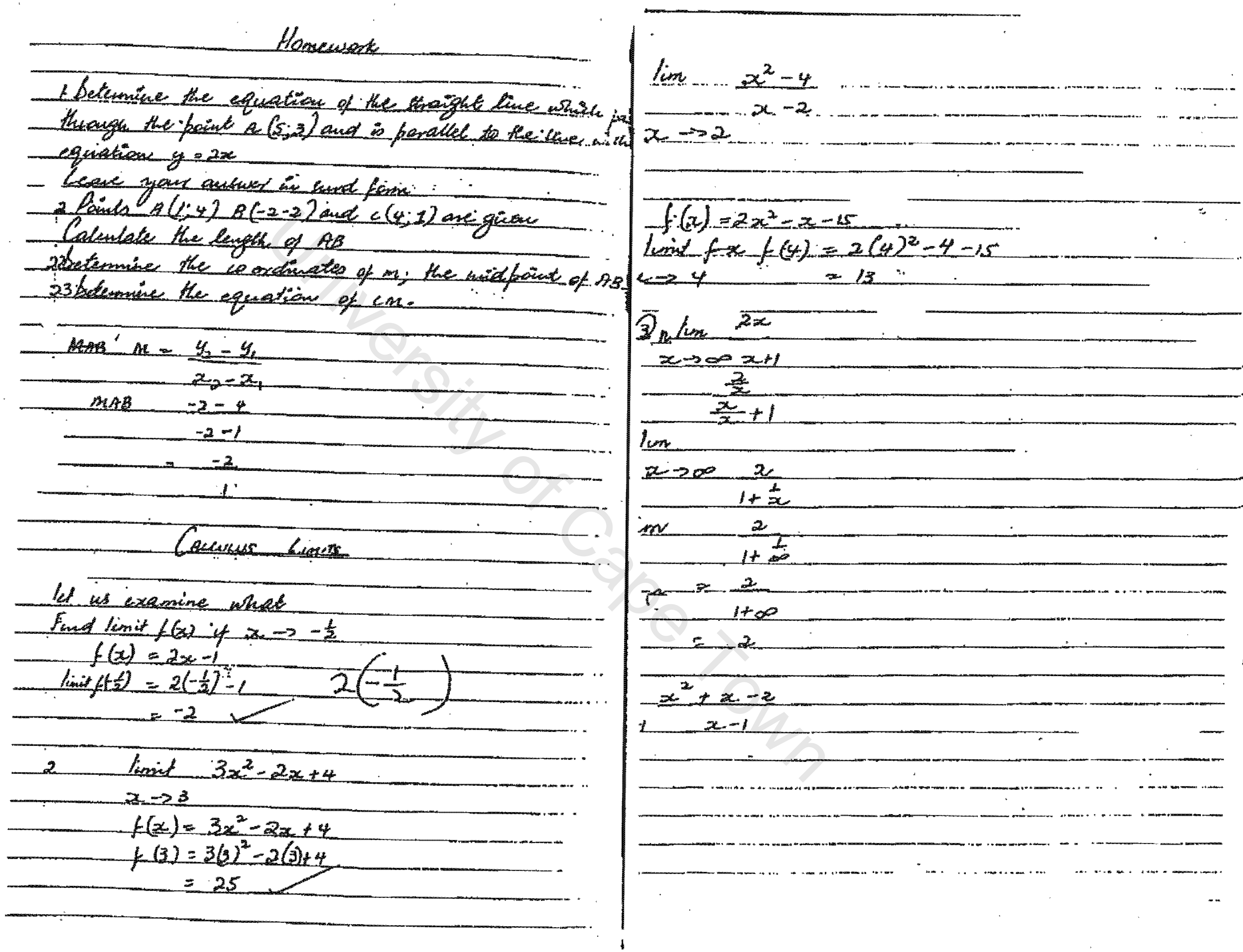
$= 1D_x(x) + D_x(2)$

$= 1 \times x^{1-1}) + D_x(0)$

x^0

$= 1$ ✓

Appendix 8: Student B3



Homework

1 Determine the equation of the straight line which pa
through the point A (5;3) and is parallel to the line with
equation $y = 2x$
Leave your answer in surd form
2 Points A(1;4) B(-2-2) and C(4;1) are given
Calculate the length of AB
22 Determine the coordinates of m; the midpoint of AB
23 Determine the equation of cm.

MAB' $m = \frac{y_2 - y_1}{x_2 - x_1}$

MAB $\frac{-2-4}{-2-1}$

$= \frac{-2}{1}$

Calculus Limits

let us examine what
Find limit $f(x)$ if $x \to -\frac{1}{2}$
$f(x) = 2x - 1$
$\text{limit} f(-\frac{1}{2}) = 2(-\frac{1}{2}) - 1$
$= -2$ ✓ $2\left(-\frac{1}{2}\right)$

2 $\lim_{x \to 3} 3x^2 - 2x + 4$
$f(x) = 3x^2 - 2x + 4$
$f(3) = 3(3)^2 - 2(3) + 4$
$= 25$ ✓

$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

$f(x) = 2x^2 - x - 15$
$\lim_{x \to 4} fx \quad f(4) = 2(4)^2 - 4 - 15$
$= 13$

③ $\lim_{x \to \infty} \frac{2x}{x+1}$

$\frac{\frac{2}{x}}{\frac{x}{x}+1}$

$\lim_{x \to \infty} \frac{2}{1 + \frac{1}{x}}$

$\frac{2}{1 + \frac{1}{\infty}}$

$= \frac{2}{1 + \infty}$

$= 2$

$\frac{x^2 + x - 2}{x - 1}$

Derivative of a functions

The function for finding the gradient of a function at any point is called the derivative function or the derivative

The derivative of $f(x)$ is usually $f'(x)$. But it may also be denoted

$$D_x\,[f(x)]$$

or $$\frac{d}{dx}\,f(x)$$

or $$\frac{d f(x)}{dx}$$

If the function is written in the form $y = ax^n$, then the derivative is $\frac{dy}{dx}$

Examples using derivatives.

If $f(x) = -2x^2$ use first principles to find $f'(x)$

$$f'x = \lim_{h \to 0} \frac{-2(x+h)^2 - 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) - (-2x^2)}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 2x^2}{h}$$

$$\lim_{h \to} \frac{-4xh - 2h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-4x - 2h)}{h}$$

$$= \lim_{h \to 0} 4x - 2h$$

$$\lim_{h \to 0} -4x - 2(0) =$$

② Now find the value of the deriative

① $x = -3$

$f'x = -4x$

$= -4(-3)$

$= 12$

Example

$f'(x)$ if $f(x) = 3$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{3-3}{h}$

$\lim_{h \to 0} \frac{0}{h}$

$= 0$

Rule 1

The derivative of a constant in zero

xample If $f(x) = 2$ find $f'(x)$.

ule 2 : A formula for the derivative

$f'x$ if $fx = x^3$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{\cancel{h}^3 - x^3}{h} (x+h)(x+h)(x+h)$

Rule :

The derivative of the sum of two or more functions

① $D_x[f(x) + g(x)] = D_x[f(x)] + D_x[g(x)]$

e.g. $D_x(3x^2 + 2x)$

ⓐ $f(-2)$
ⓑ $f'(x)$

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Rule 3

2 $Dx\left(\frac{x^3}{2}\right) = Dx\left(\frac{1}{2}x^3\right)$
~~Dx~~ $\frac{1}{2} Dx\,(x^3)$
$= \frac{1}{2} x\, 3x^{3-1}$
$= \frac{1}{2} x\, 3x^2$
$= \frac{3}{2} x^2$

① Rule 4
$Dx\,(3x^2)$
$3\,Dx\,(x^2)$
$3 x\, 2x^{2-1}$
$3 x\, 2x$
$= 6x$

⑪ $Dx\,(2x)$
$2\,Dx\,(x)$
$2x\, x^{1-1}$
$= 2x$

$Dx\,(3x^2+2x)$
$3\,Dx\,(x^2)+2\,Dx\,(x)$
$3 \times 2 \times 1 + 2 \times x^{1-1}$
$6+2x$

$2\,Dx\,(x^2) - 3\,Dx\,(x)$
$2 \times 2 \times 1\, x^2 - 3 \times 1$

=

Rule 6
$Dx\,(x-2)(x+3)$
$Dx\,(x^2-3x-2x-6)$
$x^2 -$ ~~6~~ $-$

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