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INDUCTION MOTORS WITH MODULATED

TERMINAL VOLTAGE

by

Colin David Schauder

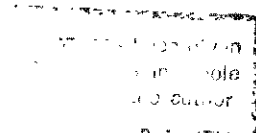
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ABSTRACT

Following the development of high speed solid state switches for the forced commutation of current in AC power circuits it has become feasible to apply forced commutation techniques to the frequency and voltage control of induction motors as variable speed drives.

The availability of these methods raises questions regarding their effect on the performance of the motor and the generation of harmonic currents in the electrical supply. After the initial use of numerical simulation in the study of these systems it became apparent that this approach was not suitable for the evaluation of steady state performance factors. This thesis sets out to establish an analysis of the induction motor under these conditions which leads to the algebraic formulation of the performance of the system in the steady state. This approach can be used in any situation where the electrical supply to an induction motor is modulated by switching techniques.

The analysis has been applied to chopper modulation of the supply voltage and also to a phase-stepping frequency convertor which incorporates chopper modulation. Experimental results are included to illustrate the validity of the analytical method.

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LIST OF SYMBOLS

MATRICES

$[u], [u_1], [u_2]$	Instantaneous voltage vectors in different reference frames
$[i], [i_1], [i_2]$	Instantaneous current vectors in different reference frames
$[Z], [Z_1], [Z_2]$	Impedance matrix of motor in different reference frames
$[C_a], [C_b]$	Connection matrices for the transformation of motor equations to the instantaneous symmetrical components reference frame

SYMBOLS

u_s^+, u_s^-	Instantaneous positive, negative sequence components of reference phase voltage in stator	
i_s^+, i_s^-	Instantaneous positive, negative sequence components of reference phase current in stator	
i_r^+, i_r^-	As above for rotor current	
u_s^a, u_s^b, u_s^c	Real instantaneous stator phase voltages	
i_s^a, i_s^b, i_s^c	Real instantaneous stator phase currents	
i_r^a, i_r^b, i_r^c	As above for rotor currents	
$i_s^\beta, i_s^\gamma, i_s^\delta$	Transformed stator currents	
$i_r^\beta, i_r^\gamma, i_r^\delta$	As above for rotor currents	
V_s^+	$= V_s^+ \angle 0 = \frac{1}{2} V_s \angle 0$ $I_s^+ = I_s^+ \angle \theta_s^+ = \frac{1}{2} I_s \angle \theta_s$	Phasors denoting instantaneous positive sequence stator voltage and current with sinusoidal excitation
I_s^+		
I_r^+	As above for rotor current	
i_s'	$= i_s' \angle \theta_s' = \frac{1}{2} i' \angle \theta'$ $= i_s'' \angle \theta_s'' = \frac{1}{2} i'' \angle \theta''$	Initial values of transient stator current components
i_s''		
i_r', i_r''	As above for rotor currents	
W_{term}, W_{out} $W_{loss}, W_{mag.}$	Power components (See page 27)	
$B_{max}, H_{max.}$ $V_{max}, I_{max.}$	Amplitudes of space distribution of flux density, field strength, voltage, current density	
L_s, L_r	Stator, rotor self-inductance per phase	
L_1, L_2	Effective self-inductance of stator, rotor per phase with three-phase excitation (no zero sequence component)	

R_s, R_r	Stator, rotor resistance per phase
M	Stator to rotor maximum coupling coefficient between phases
M'	Effective mutual inductance with three-phase excitation
k_s, k_r	Coupling coefficient between adjacent phases on stator, rotor
P	Pole pair number
ω	Angular frequency
ω_e	Angular frequency of supply voltage
ω'	Electrical angular velocity of shaft
ω_r^+	$= (\omega_e - \omega')$. Angular slip frequency relative to positive phasor sequence excitation
ω_r^-	$= (\omega_e + \omega')$. Angular slip frequency relative to negative phasor sequence excitation
ω_{out}	Fundamental angular frequency of convertor output voltage
$\omega_{chop}, \omega_{step}$	$= 2\pi f_{chop}, 2\pi f_{step}$
f	Frequency
f_e	Frequency of supply voltage
f_{chop}, f_{step}	Chopping, phase-stepping frequency
F', F''	$= (\alpha' - j\omega_e), (\alpha'' - j\omega_e)$
α', α''	Complex pole frequencies of motor equations
p', p'', ν', ν''	$\alpha' = (-p' + j\nu')$; $\alpha'' = (-p'' + j\nu'')$
σ	Leakage coefficient
ρ_1, ρ_2	$= \frac{R_s}{\sigma L_1}, \frac{R_r}{\sigma L_2}$
C', C''	Complex constants relating stator and rotor transient current components
E_{mag}	Energy stored in the magnetic field
Φ	Flux linkage phasor
θ_m	Motor voltage and current modulation angle
θ_{supply}	Supply current modulation angle
Q	Ordinal denoting successive modulation cycles

N	Number of stages in the modulation cycle
k	Ordinal denoting successive stages of a modulation cycle
i'_{sk}, i''_{sk}	Values of transient current components immediately after the commutation ending the k th. stage of a modulation cycle
t	Time
T	Period of modulation cycle
t_k	Duration of the k th. stage of a modulation cycle
T_a	Duration of short-circuit interval in a chopper modulation system
T_b	Duration of ON-interval in a chopper modulation system
$T(on)/T(total)$	$= \left(\frac{T_b}{T} \right)$. Ratio of mark to total time in a chopper modulated system
m, n	Integers
$ I_1 $	Amplitude of reference phase fundamental supply current
θ_1	Displacement angle of fundamental supply current
Z	Number of chops per phase step in a phase-stepping frequency convertor
D	Total conductors in a distributed three-phase winding
k_{w1}	Fundamental winding factor
g	Width of airgap
r	Radius of airgap
l	Length of core lamination stack
μ_0	Approximate permeability of air

SUFFIXES

s, r	Denote stator, rotor variables
$+, -$	Positive, negative sequence components
$0+, 0-$	Denote variable evaluated at $t=0+$, $t=0-$
a, b, c	Denote A, B, C-phases
β, γ, δ	Designation of transformed current variables
$', ''$	Denote the two transient components of current

$*$	Complex conjugate
t	Transpose of matrix
-1	Inverse of matrix

OPERATORS AND FUNCTIONS

$p.$	Time derivative operator, d/dt
λ	$e^{j\frac{2\pi}{3}}$
$Re.()$	Real part of complex variable
$Im.()$	Imaginary part of complex variable
$\delta(\omega)$	Kronecker delta function
$g(\omega, \rho)$	$= \frac{1}{(\rho + j\omega)} \left\{ 1 - e^{-\rho t_k} \cdot e^{-j\omega t_k} \right\}$

CHAPTER ONE

INTRODUCTION

Induction motors are the most commonly used industrial drives because of their lower capital cost, lower maintenance costs and especially their compatibility with the AC power supply. These advantages make the induction motor preferable to its D.C. counterpart in most applications where only a single operating speed is required. However, where continuous control of the speed over a wide range is required it is necessary to either make fundamental internal changes to the induction motor or supply it through external frequency changing equipment. The need for such costly measures ensured that until recently D.C. motors were preferred to induction motors in these applications. The development of semiconductor power devices, however, has stimulated a new generation of static frequency convertors which have now begun to make induction motor drives viable in speed control applications which were formerly the preserve of D.C. motors.

Induction motors have themselves been the subject of extensive research throughout this century, and during this time they have evolved to the modern machine which has a much higher power to frame size ratio than its predecessors and which operates within much closer electrical and mechanical tolerances.

The motor is traditionally designed for operation at constant speed with a constant frequency balanced sinusoidal supply voltage. Such standard production motors are now commonly being used in conjunction with static convertors with very little consideration of the reduced motor efficiency, the exact nature of the motor output and the distortion of currents throughout the system. While these factors may not be critical in many of the small industrial drives, they certainly are important in the case of larger installations. For a proper assessment of the cost effectiveness and general viability of a particular system it is essential to understand the response of the motor to the convertor output under all possible conditions.

In this thesis it is intended to present a technique for the analysis of the motor performance which may be applied to a wide range of convertor systems. This method will be used to investigate a particular system which is currently of interest.

1.1 MODULATION IN POWER SYSTEMS:

Any static convertor used to interface a machine with the electrical supply has the effect of modulating the supply voltage before it is applied to the motor terminals. If amplitude modulation is interpreted as generating the instantaneous product of the carrier or supply voltage with a modulating function, then this is the general category into which most power modulators fall. More particularly, we find that the modulation function in such convertors cannot in practice assume values other than $+1, 0, -1$ except for the briefest intervals. The load is thus always either connected directly to the supply, reverse connected or short circuited depending on the state of the modulation function. If the modulation function were to assume other values it would imply that the modulator itself was acting as a source or sink of power which is not feasible in practice. For the same reason, changes of state which are dictated by instantaneous changes in the modulation function must be effected as rapidly as possible in order to minimize the power which is exchanged with the modulator. These changes involve the switching of the supply connection to the load and, as far as possible, are achieved without any discontinuity of load current. The load current is in fact steered from one supply circuit to the next by "natural" or "forced" operation of the switching elements in the modulator. In the course of this work we will assume that power modulators achieve ideal commutation which implies that current is instantaneously steered to the new circuit with no loss.

If several supply voltages are available, as in the case of a polyphase system, the modulation function assumes a new dimension since it must stipulate the particular supply voltage to which the load is connected. Depending on the available supply and the requirements for the system, each convertor has its own modulation function. However the action is similar in every case insofar as it synthesizes an output voltage which consists of sequential samples of the supply voltages. In

general the effect of the modulator should be seen as twofold:-

- (a) The motor voltage is the modulated supply voltage.
- (b) The supply current is the modulated motor current. (This should more correctly be termed the de-modulated motor current. The supply current is constituted from consecutive samples of the motor currents where the modulation function is the inverse of the voltage modulation function).

Since only a limited number of supply voltages are usually available, the following common features may be discerned in most convertors:-

- (a) *The modulation function is cyclic. A modulation cycle consists of a sequence of supply connections which is exactly repeated as a function of time.*
- (b) *The modulation functions pertaining to different phases of the motor are similar so that a balanced voltage set is applied to the machine at all times.*

Although modulating functions which do not adhere to these rules are quite conceivable, they will not be dealt with in this thesis. This exclusion is not a serious loss of generality since a wide range of convertors do fall under restrictions (a) and (b) above.

1.2 BACKGROUND TO THE ANALYSIS:

The generalized machine theory is well understood, and the solutions to the induction motor equations in several different reference frames have been applied to a wide range of problems. Much work (Refs (1)) has been published on the effects of rapidly switching the supply to an induction motor, which corresponds to a modulating function that is a single step. Ward et al.⁽⁹⁾ realized that this theory might be

extended to provide an algebraic formulation of the induction motor response to more complicated modulation functions. They considered the particular case of a transistorized inverter and published solutions for the instantaneous motor current obtained in a 2-axis reference frame. More recently Bellini⁽¹⁰⁾ has shown how the motor equations can be used to determine the behaviour of a pulse width modulated inverter drive. The present work serves to both generalize and simplify the use of the motor equations in calculating the response of the machine to a broad range of modulating functions, irrespective of whether the supply voltage is AC or DC.

The success of this analysis is largely attributable to the use of instantaneous symmetrical components which are a special form of the general symmetrical components transformation developed by Fortescue. W.V. Lyon⁽¹¹⁾ first used the instantaneous components extensively in his work on transient analysis of AC machines. Rudenberg⁽¹⁴⁾ showed how conveniently the transient solutions obtained in this reference frame might be interpreted in terms of the revolving field theory of polyphase machines. Using this approach it is possible to express the state of the motor in terms of a single current variable for the stator and one for the rotor, which makes the problem more tenable.

The analysis technique used here was originally developed for the purpose of investigating the motor performance with the following supply modulations:-

- (a) Chopper modulation of the sinusoidal supply voltage.
- (b) AC - AC frequency conversion by phase-stepping, with chopper modulation of the output voltage.

These investigations were not merely intended as an academic exercise, but were needed to shed light on a modulation form which has only recently been realized in practice and which holds great promise. The work which was subsequently done in this regard is included as an integral part of this thesis since it provides a clear illustration of the applications of the analysis.

1.3 ALTERNATIVE ANALYSIS TECHNIQUES:

The approach which will be used here enables the performance of the system to be algebraically formulated in terms of the general solution to the motor equations. The advantages of this are seen as follows:-

- (a) The instantaneous values of the motor variables are explicitly defined as functions of time.
- (b) The de-modulated supply currents are similarly defined since they are comprised of segments of the motor currents.
- (c) Defined in this way the currents lend themselves to the algebraic calculation of R.M.S. and spectral content which in turn enables all relevant performance factors to be evaluated.
- (d) A better appreciation of the instantaneous physical situation in the machine is afforded.

Before proceeding, however, it will be pertinent to mention the two alternative methods by which these solutions could be obtained:-

- 1.3.1 If the output voltage of the convertor is analyzed and the steady state sinusoidal response to each of the infinite series of harmonic components is determined, then the spectrum of the nett motor current may be obtained. This must necessarily involve the truncation of the harmonic series at some point. The average output torque can be evaluated as the sum of the torques produced by each harmonic component, but the instantaneous waveform of the torque is not readily revealed unless the harmonic current components are first assembled in the time domain. An accurate assessment of the supply current implies that the de-modulation function must be performed on every harmonic component of the

motor current spectrum and the results superimposed, which constitutes a prohibitive task. Naturally a greater or lesser degree of simplification may be introduced by means of approximation, but this is the very thing which we seek to avoid here since it clouds the issue of the losses which the convertor produces.

- 1.3.2 Numerical simulation techniques are widely used to obtain time domain solutions for motor response with any terminal conditions, and indeed in some cases, it is the only practical way in which these solutions can be obtained. Simulation does not employ a formulated solution, using instead a model of the system which is represented by the equations. When the appropriate terminal conditions are applied to such a model, the response for that particular excitation may be recorded. It is comparatively simple to obtain solutions in this way for any of the voltage waveforms generated by static convertors and even the dynamic response of the electro-mechanical system can be simulated.

The author has used numerical simulation extensively for induction motor analysis and has found two main disadvantages which disqualify the method for use in the present application:-

- (a) Accurate numerical simulation is slow as a result of the following features:-
 - (i) The accuracy depends on the shortness of the integration interval which is used. Since errors are cumulative during the course of a simulation, a very short interval must be used where the voltage waveform is in any way discontinuous.

- (ii) A full cycle of calculation must be performed for every integration interval that passes. Since suitable initial conditions are seldom available, this means that many superfluous calculation cycles are performed before the model settles into the required steady state, which is itself often difficult to identify.
 - (iii) It is usually necessary to perform the simulation on an interactive basis with a computer. This means that the computer is required to output information to the operator about the state of the model during the course of the simulation. This is very wasteful of time.
- (b) Since only real instantaneous values of variables are calculated by simulation it is necessary to perform further analysis on the generated waveforms in order to determine their R.M.S. or average values and the composition of their spectra. In the case of the R.M.S. and average values it is possible to perform numerical integration on the system variables as they are generated, but it is frequently difficult to determine over exactly what interval such integration should be performed if the waveforms are not cyclic in time. (Note that a cyclic modulation function does not imply periodic voltage and current waveforms). The spectral content of the waveforms can only be obtained by performing a Fourier transform algorithm on the generated data which involves considerable computation.

While the usefulness of numerical simulation must not be underestimated, it is nevertheless unsuitable as a method of obtaining performance factors for convertor-fed induction motors. This is especially true if it is desired to observe trends in these factors as the modulation parameters are varied, since a large number of individual simulations would be required. In this thesis numerical simulation has been

used to illustrate many of the time domain waveforms and has also provided an invaluable means of cross-checking results obtained from the algebraic formulations.

CHAPTER TWO

BASIC ANALYSIS OF THE INDUCTION MOTOR

Because of the many physical limitations which are encountered, the manufactured induction motor inevitably has a number of features which detract from its envisaged operation. Some of these are unavoidable, being set by the properties of the best materials which are available or the nature of the supply prescribed for the motor or mechanical tolerances. Others again may be controlled if not eliminated, by careful design procedure. In general with good design procedure and the improved materials which are constantly being developed, these parasitic effects may be reduced to a level where they are small in comparison with the fundamental or desirable operation of the machine. At its simplest the idealised three-phase motor still comprises six coupled circuits with variable coupling coefficients. This in itself presents a complex analysis problem and although theoretical work by Kron and others has given equations and equivalent circuits which encompass almost every possible effect, it is seldom possible to incorporate parasitic phenomena in practical calculations. In particular when the response to unfamiliar excitation waveforms is being considered, it is wise to consider first the idealised response which may then be compared with waveforms obtained in practice thus indicating to what extent further refinements of the analysis are required for acceptable accuracy. In the present work an idealised model of the motor has been used in this way and it has also been assumed that the applied excitation has ideal characteristics. In Chapter seven it will be shown how the results obtained compare with measurements on an available system.

The following simplifications are made:

- (a) No losses due to hysteresis or eddy currents.
- (b) Sinusoidally distributed radial mmf due to the excitation of each phase winding.

- (c) Smooth airgap and homogeneous magnetic circuit such that the radial flux density distribution in the airgap is linearly related to the mmf distribution. It is assumed that saturation of the magnetic circuit does not affect this linearity.
- (d) In a wound rotor motor the rotor circuits are clearly defined. This is not the case in the cage rotor which is of primary interest in this work. For the purpose of analysis it is assumed that the cage rotor comprises a three phase winding with a unity turns ratio between stator and rotor. It is also assumed that the rotor "phase" currents produce only a sinusoidal mmf distribution of the fundamental pole number.

Arising from these assumptions a very general mathematical model of the motor may be stated. In this formulation it is presumed that there are three clearly defined circuits on the stator and a similar arrangement on the rotor:-

$$[V] = [Z] [i]$$

$$[Z] = \begin{bmatrix} (R_s + p.L_s) & p.k_s L_s & p.k_s L_s & f(0) & f(\frac{2\pi}{3}) & f(-\frac{2\pi}{3}) \\ p.k_s L_s & (R_s + p.L_s) & p.k_s L_s & f(-\frac{2\pi}{3}) & f(0) & f(\frac{2\pi}{3}) \\ p.k_s L_s & p.k_s L_s & (R_s + p.L_s) & f(\frac{2\pi}{3}) & f(-\frac{2\pi}{3}) & f(0) \\ f(0) & f(\frac{2\pi}{3}) & f(-\frac{2\pi}{3}) & (R_r + p.L_r) & p.k_r L_r & p.k_r L_r \\ f(-\frac{2\pi}{3}) & f(0) & f(\frac{2\pi}{3}) & p.k_r L_r & (R_r + p.L_r) & p.k_r L_r \\ f(\frac{2\pi}{3}) & f(-\frac{2\pi}{3}) & f(0) & p.k_r L_r & p.k_r L_r & (R_r + p.L_r) \end{bmatrix}$$

$$[V]_t = [V_s^a \ V_s^b \ V_s^c \ V_r^a \ V_r^b \ V_r^c]$$

$$[i]_t = [i_s^a \ i_s^b \ i_s^c \ i_r^a \ i_r^b \ i_r^c]$$

where $f(\phi) = p.M \cos(P\theta + \phi)$
 θ = Mechanical angle between corresponding stator and rotor circuit axes.

These equations could be solved directly, but the solution is

complicated by the dependence of the stator to rotor coupling coefficients on the angular position of the rotor. As a result the equations are usually transformed to a reference frame where the rotor circuits are effectively held stationary relative to the stator while producing the same magnetic field distribution. A further simplification is achieved if it may be assumed that no zero sequence components of current can exist in the windings, in which case two redundant variables are eliminated.

i.e.

$$i_s^b = -(i_s^a + i_s^c)$$

$$i_r^b = -(i_r^a + i_r^c)$$

In Appendix 1 the connection matrix is given for the nett power invariant transformation. If we set the rotor voltages to zero, then this transformation yields the following equations:-

$$[u_i] = [Z_i][i_i]$$

where

$$[u_i]_t = \begin{bmatrix} (u_s^a - u_s^b) & (u_s^c - u_s^b) & 0 & 0 \end{bmatrix}$$

$$[i_i]_t = \begin{bmatrix} i_s^a & i_s^b & i_r^a & i_r^b \end{bmatrix}$$

$$[Z_i] = [R] + [L]p.$$

where

$$[R] = \begin{bmatrix} 2R_s & R_s & 0 & 0 \\ R_s & 2R_s & 0 & 0 \\ 0 & -\sqrt{3}M'\omega' & 2R_r & (R_r - \sqrt{3}L_2\omega') \\ \sqrt{3}M'\omega' & 0 & (R_r + \sqrt{3}L_2\omega') & 2R_r \end{bmatrix}; [L] = \begin{bmatrix} 2L_1 & L_1 & 2M' & M' \\ L_1 & 2L_1 & M' & 2M' \\ 2M' & M' & 2L_2 & L_2 \\ M' & 2M' & L_2 & 2L_2 \end{bmatrix}$$

where

$$M' = \frac{3}{2}M$$

$$L_1 = (1 - k_s)L_s$$

$$L_2 = (1 - k_r)L_r$$

If we now apply the appropriate instantaneous symmetrical component transformation to the new variables as described in APPENDIX 2, we obtain the following:-

$$[u_2] = [Z_2] [i_2]$$

where

$$[u_2]_t = \begin{bmatrix} (u_s^a + \lambda u_s^b + \lambda^2 u_s^c) & (u_s^a + \lambda^2 u_s^b + \lambda u_s^c) & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3u_s^+ & 3u_s^- & 0 & 0 \end{bmatrix}$$

$$[i_2]_t = \begin{bmatrix} i_s^+ & i_s^- & i_r^+ & i_r^- \end{bmatrix}$$

$$[Z_2] = \begin{bmatrix} (3R_s + 3L_1 p) & 0 & 3M' p & 0 \\ 0 & (3R_s + 3L_1 p) & 0 & 3M' p \\ [3M'(p - j\omega)] & 0 & [3R_r + 3L_2(p - j\omega)] & 0 \\ 0 & [3M'(p + j\omega)] & 0 & [3R_r + 3L_2(p + j\omega)] \end{bmatrix}$$

By definition the instantaneous negative sequence components are the complex conjugates of their positive sequence counterparts. In the above equations rows 1 and 3 comprise terms which are the complex conjugates of the terms in rows 2 and 4 respectively. Unless a further current constraint is imposed there is no interdependence between the variables which occur in each of these pairs of equations. It is therefore only necessary to obtain the solution for one pair, since the other solution is always the complex conjugate of this. The machine equations can thus be written in reduced form as two simultaneous equations:-

$$3u_s^+ = (3R_s + 3L_1 p) i_s^+ + 3M' p i_r^+$$

$$0 = 3M'(p - j\omega) i_s^+ + [3R_r + 3L_2(p - j\omega)] i_r^+$$

The only loss of generality in this reduction of the equations has been the neglect of zero sequence components. A proper treatment of the zero sequence components could be included with little difficulty, but they are irrelevant to the present theme.

Note that the reduced equations are non-linear unless the assumption is made that the *shaft rotation speed is a constant*. This is an essential condition if it is desired to obtain algebraic solutions as in the present work. It does not detract in any way from the value of the solution in calculating steady state performance factors for convertor-fed motors, since shaft speed is in fact constant under these conditions. (The equations could be solved without the constant speed constraint by the use of numerical simulation techniques if the equations governing the mechanical system are incorporated.)

At this stage it will be useful to re-iterate the relationship between the variables i_s^+ , i_r^+ and the actual instantaneous values of phase current flowing in the machine.

$$\begin{aligned} i_s^a &= 2 \operatorname{Re.} (i_s^+) \\ i_s^b &= 2 \operatorname{Re.} (i_s^+ e^{-j\frac{2\pi}{3}}) \\ i_s^c &= 2 \operatorname{Re.} (i_s^+ e^{j\frac{2\pi}{3}}) \end{aligned}$$

In squirrel cage motors it will not usually be possible to assess the rotor current correctly since the current paths are not clearly defined. In wound rotors we have

$$\begin{aligned} i_r^\beta &= 2 \operatorname{Re.} (i_r^+) \\ i_r^\gamma &= 2 \operatorname{Re.} (i_r^+ e^{-j\frac{2\pi}{3}}) \\ i_r^\delta &= 2 \operatorname{Re.} (i_r^+ e^{j\frac{2\pi}{3}}) \end{aligned}$$

where

$$\begin{bmatrix} i_r^a \\ i_r^b \\ i_r^c \end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix} \cos(P\theta + \frac{\pi}{6}) & -\sin(P\theta) \\ -\sin(P\theta + \frac{\pi}{3}) & \sin(P\theta - \frac{\pi}{3}) \\ \sin(P\theta) & \cos(P\theta - \frac{\pi}{6}) \end{bmatrix} \begin{bmatrix} i_r^\beta \\ i_r^\gamma \\ i_r^\delta \end{bmatrix}$$

which is a rotor-position-dependent relation.

and

$$i_r^\gamma = -(i_r^\beta + i_r^\delta)$$

2.1 STEADY-STATE SINUSOIDAL EXCITATION

It may be shown that if the excitation of the three phases is sinusoidal with each voltage instantaneously of the form $v = (V e^{j\omega_e t} + V^* e^{-j\omega_e t})$ where $2V = 2|V|e^{j\phi}$ is the usual phasor representation of the voltage, then the instantaneous positive sequence symmetrical component of the reference phase voltage is $v_s^+ = (V_s^+ e^{j\omega_e t} + V_s^{-*} e^{-j\omega_e t})$ where $2V_s^+$ and $2V_s^-$ are the usual phasor symmetrical components of the reference phase voltage.

Using the principle of superposition it is in order to evaluate separately the response to each of the terms comprising v_s^+ . Consider firstly the term $V_s^+ e^{j\omega_e t}$ and assume response of the form $I_s^+ e^{j\omega_e t}$ and $I_r^+ e^{j\omega_e t}$ since the equations are linear. Thus the instantaneous positive sequence equations become:-

$$\begin{aligned} 3V_s^+ e^{j\omega_e t} &= (3R_s + j3\omega_e L_1) I_s^+ e^{j\omega_e t} + j3\omega_e M' I_r^+ e^{j\omega_e t} \\ 0 &= j\omega_r^+ 3M' I_s^+ e^{j\omega_e t} + (3R_r + j\omega_r^+ 3L_2) I_r^+ e^{j\omega_e t} \end{aligned}$$

where $\omega_r^+ = (\omega_e - \omega')$

and for the conjugate excitation term the response is of the form $I_s^{-*} e^{-j\omega_e t}$ and $I_r^{-*} e^{-j\omega_e t}$ so that the instantaneous positive sequence equations become:-

$$\begin{aligned} 3V_s^{-*} e^{-j\omega_e t} &= (3R_s - j3\omega_e L_1) I_s^{-*} e^{-j\omega_e t} - j3\omega_e M' I_r^{-*} e^{-j\omega_e t} \\ 0 &= -j\omega_r^- 3M' I_s^{-*} e^{-j\omega_e t} + (3R_r - j\omega_r^- 3L_2) I_r^{-*} e^{-j\omega_e t} \end{aligned}$$

where $\omega_r^- = (\omega_e + \omega')$

and, solving these equations:-

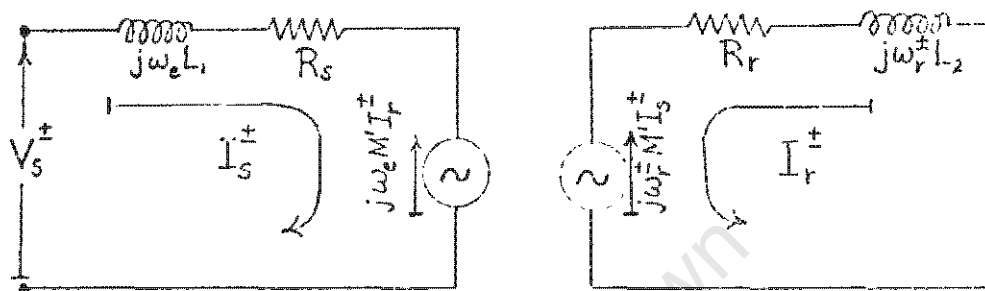
$$I_s^+ = \frac{V_s^+ (R_r + j\omega_r^+ L_2)}{(R_s + j\omega_e L_1)(R_r + j\omega_r^+ L_2) + \omega_e \omega_r^+ M'^2}$$

$$\text{and } I_r^+ = \frac{V_s^+ (-j\omega_r^+ M')}{(R_s + j\omega_e L_1)(R_r + j\omega_r^+ L_2) + \omega_e \omega_r^+ M'^2}$$

Thus the particular integral or steady state solution for sinusoidal excitation of the motor is given by

$$i_s^+ = (I_s^+ e^{j\omega_e t} + I_s^{-*} e^{-j\omega_e t})$$

This solution can conveniently be represented in terms of the following equivalent circuit:-



Note that in the present work it is assumed that only balanced positive phasor sequence voltages are applied to the motor. The instantaneous positive sequence particular integral is

$$i_s^+ = I_s^+ e^{j\omega_e t} \quad \text{since } V_s^- = 0$$

$$i_r^+ = I_r^+ e^{j\omega_e t}$$

2.2 TRANSIENT SOLUTION:-

The positive sequence motor equations are 1st order simultaneous linear differential equations. The complete solution is given by the Particular Integral (developed above for sinusoidal excitation) plus the Complementary Function which expresses the transient response of the system. The complementary function is the solution to the equations with the excitation terms suppressed and may be assumed to have the general form:-

$$i_s^+ = i_s e^{\alpha t} \quad ; \quad i_r^+ = i_r e^{\alpha t}$$

where α satisfies the equations:-

$$\left. \begin{aligned} 0 &= (R_s + L_1 \alpha) i_s + M' \alpha i_r \\ 0 &= (\alpha - j\omega') M' i_s + (R_r + L_2 (\alpha - j\omega')) i_r \end{aligned} \right\} \quad (1)$$

Hence we obtain:-

$$\frac{i_s}{i_r} = \frac{-M'\alpha}{(R_s + \alpha L_1)} = \frac{-(R_r + (\alpha - j\omega')L_2)}{M'(\alpha - j\omega')} = C$$

Thus i_s, i_r may be eliminated from (1) to yield the characteristic equation:

$$M'^2 \alpha (\alpha - j\omega') - (R_s + L_1 \alpha)(R_r + L_2 (\alpha - j\omega'))$$

$$\text{ie. } \alpha (\alpha - j\omega')(L_1 L_2 - M'^2) + R_s R_r (\alpha - j\omega') + \alpha L_1 R_r = 0$$

$$\text{Let } \sigma = \left\{ 1 - \frac{M'^2}{L_1 L_2} \right\} ; \rho_1 = \frac{R_s}{\sigma L_1} ; \rho_2 = \frac{R_r}{\sigma L_2}$$

$$\text{Then: } \alpha(\alpha - j\omega')\sigma + \rho_1 \rho_2 \sigma^2 + \sigma \rho_1 (\alpha - j\omega') + \alpha \sigma \rho_2 = 0$$

$$\text{ie. } \alpha^2(\sigma) + \alpha(-j\omega'\sigma + \sigma \rho_1 + \sigma \rho_2) + (\rho_1 \rho_2 \sigma^2 - j\omega'\sigma \rho_1) = 0$$

$$\alpha = \frac{j\omega' - (\rho_1 + \rho_2)}{2} \pm \sqrt{\frac{((\rho_1 + \rho_2) - j\omega')^2}{4} - (\rho_1 \rho_2 \sigma - j\omega' \rho_1)}$$

There are thus two roots for α so the full complementary function for the unconstrained motor may be written in the following form:-

$$\begin{aligned} i_s^+ &= i_s' e^{\alpha' t} + i_s'' e^{\alpha'' t} \\ i_r^+ &= i_r' e^{\alpha' t} + i_r'' e^{\alpha'' t} \end{aligned}$$

The complete solution to the motor equations thus contains four transient current terms. The two complex frequencies associated with these components are entirely defined by the motor parameters and the angular velocity of the shaft which, is assumed to be a constant. In Fig(1) α' and α'' are plotted as functions of speed for the test motor, and in Fig (2) C' and C'' are similarly plotted. The initial amplitudes of the transient current components are given by the four complex quantities i_s', i_s'', i_r', i_r'' which may be evaluated for a particular situation from four complex equations.

Let us assume that a sudden change of state in the motor is dictated at $t=0$ by an instantaneous change in the steady state excitation. Since the phase currents must be continuous at $t=0$ we can develop a pair of equations in terms of the instantaneous symmetrical components to express this continuity.

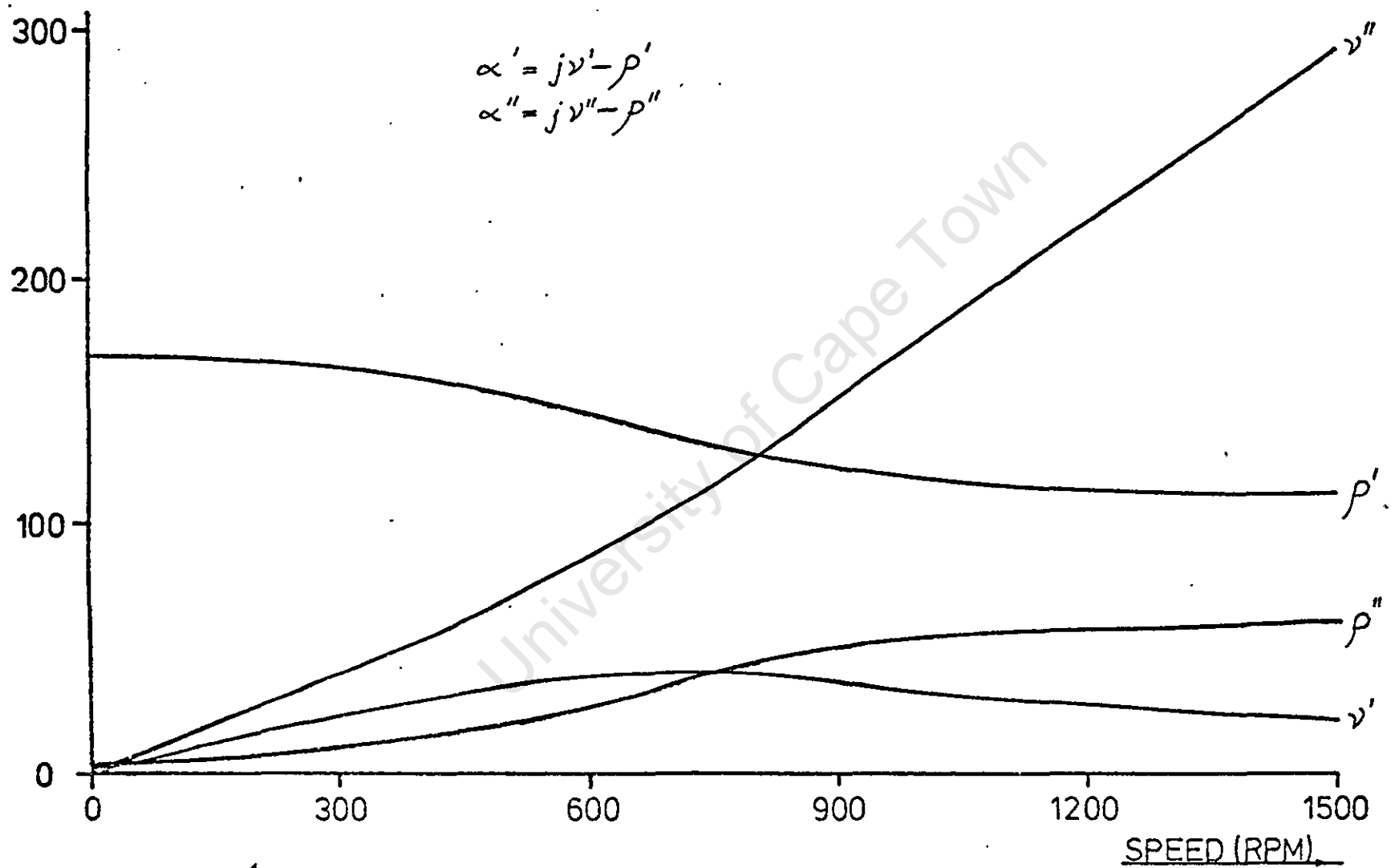


Fig. 1 Roots of the motor characteristic equation evaluated for the case of the test motor. ($M'=0.447H$, $L_1=L_2=0.473H$, $R_s=5.5$ Ohm, $R_r=3.28$ Ohm, $P=2$)

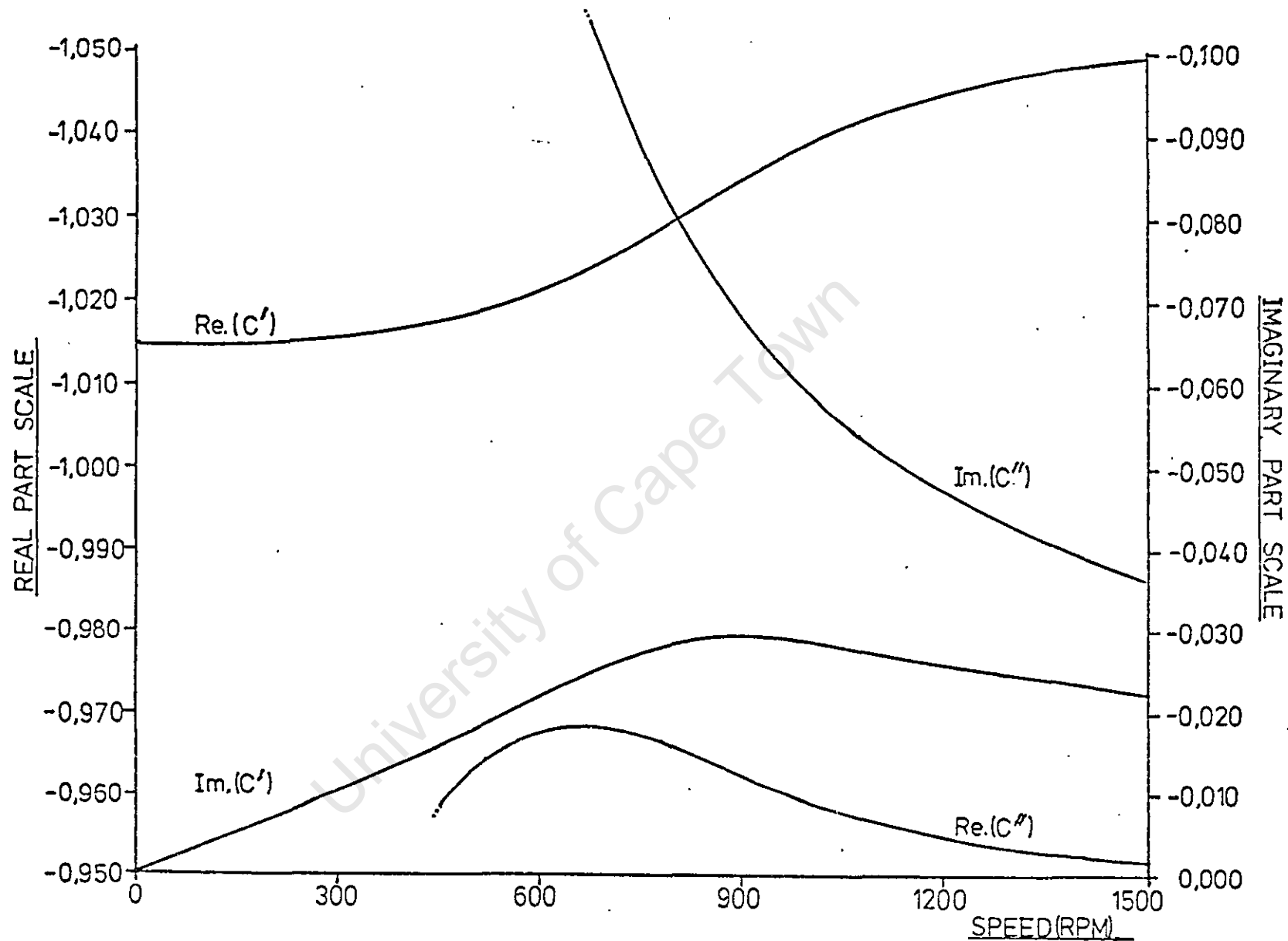


Fig. 2 Evaluation of the complex constants relating stator and rotor transient current components in the test motor. ($\sigma' = 0.447$, $L_1 = L_2 = 0.475$, $R_s = 5.5$, $R_r = 3.28$, $P = 2$)

If the instantaneous positive sequence components of the steady state currents at $t=0_-$ are expressed as I_{s0-} , I_{r0-} and the steady state response after $t=0$ is evaluated at $t=0_+$ as I_{s0+} , I_{r0+} then

$$\begin{aligned} I_{s0-} &= I_{s0+} + i'_s + i''_s \\ I_{r0-} &= I_{r0+} + i'_r + i''_r \end{aligned}$$

If the situation in the motor prior to $t=0$ is comprised not only of I_{s0-} and I_{r0-} but also the residual transient currents following some previous change of state, then the above equations can be more generally written:-

$$\begin{aligned} I_{s0-} + i'_{s0-} + i''_{s0-} &= I_{s0+} + i'_s + i''_s \\ I_{r0-} + i'_{r0-} + i''_{r0-} &= I_{r0+} + i'_r + i''_r \end{aligned}$$

Let

$$\begin{aligned} I_{s0} &= I_{s0-} - I_{s0+} + i'_{s0-} + i''_{s0-} \\ I_{r0} &= I_{r0-} - I_{r0+} + i'_{r0-} + i''_{r0-} \end{aligned}$$

Then

$$\left. \begin{aligned} i'_s + i''_s &= I_{s0} \\ i'_r + i''_r &= I_{r0} \end{aligned} \right\}$$

and from before

$$\left. \begin{aligned} \frac{i'_s}{i'_r} &= C' \\ \frac{i''_s}{i''_r} &= C'' \end{aligned} \right\}$$

Solving these equations:-

$$\boxed{\begin{aligned} i'_s &= \frac{C'(I_{s0} - C''I_{r0})}{C' - C''} \\ i''_s &= \frac{C''(C'I_{r0} - I_{s0})}{C' - C''} \end{aligned}}$$

And the full solution for $t \geq 0$ can be written as:-

$$\begin{aligned} i_s^+ &= I_{s0+} e^{j\omega_e t} + \left[I_{s0+}^{-*} e^{-j\omega_e t} \right] + i'_s e^{\alpha' t} + i''_s e^{\alpha'' t} \\ i_r^+ &= I_{r0+} e^{j\omega_e t} + \left[I_{r0+}^{-*} e^{-j\omega_e t} \right] + i'_r e^{\alpha' t} + i''_r e^{\alpha'' t} \end{aligned}$$

Bracketed terms zero with balanced excitation.

2.3 NOTES RELATING TO THE TRANSIENT SOLUTION

- (a) The poles of the positive sequence equations occur at the complex frequencies α', α'' which are determined solely by the parameters of the machine and the angular velocity of the shaft.
- (b) The initial amplitudes of the transient current components i_s', i_s'', i_r', i_r'' are determined only by the phasor differences between steady state currents evaluated at $\psi=0_-$ and $t=0_+$ if there are no residual transients at $t=0_-$.

Thus the previous history and future nature of the steady state response does not directly affect the characteristics of the transients. The transients are not directly influenced by either the frequency of the steady state excitation or the presence of phasor negative sequence components in the excitation.

- (c) If we write $\alpha' = j\gamma' - \rho'$ and $\alpha'' = j\gamma'' - \rho''$ and refer to the expression for α it is apparent that

$$\begin{aligned}\gamma' + \gamma'' &= \omega' \\ \rho' + \rho'' &= \rho_1 + \rho_2\end{aligned}$$

2.4 INTERPRETATION OF SOLUTION IN THE ROTATING FIELD REFERENCE FRAME

The symmetrical components approach is a mathematical technique which enables us to write time-domain functions to express the instantaneous values of the currents in the motor phase windings under specified conditions. It is, however, extremely useful to relate these solutions to the physical situation within the motor, especially under transient conditions. It is thus fortunate that the solutions lend themselves to interpretation in terms of the "rotating field" concept, which relates balanced sinusoidal currents in the phase windings to the spatial distributions of electrical quantities which they cause in the motor. Without this concept it would be difficult to visualize meaningfully the situation which is represented by the mathematical solution.

The transformation from phase quantities to the rotating field reference frame is formulated in APPENDIX(3). It is seen that a balanced sinusoidal set of currents in the phase windings produces space distributions of m.m.f., current density and flux density which each have a fundamental component corresponding to the pole number of the machine. In accordance with the initial assumptions we may neglect the higher order harmonics of these distributions which are caused by irregularities of the electric and magnetic circuits. The periphery of the airgap is regarded as a continuous conduction sheet on both stator and rotor and carries the axially-flowing current density waves. All distributions associated with a given phase current set rotate relative to the stator with an electrical angular velocity equal to the angular frequency of the set.

It has been shown that the complete solution to the instantaneous positive sequence equations is comprised of the steady state solution plus the complementary function. If the excitation is sinusoidal and zero sequence components of current are excluded then it is always possible to resolve the steady state current vectors into balanced vector sets of either positive or negative sequence. The steady state currents may therefore give rise to one or more superimposed rotating field systems in the motor, each related to one of the balanced component sets. The positive sequence systems rotate "forwards" at the synchronous velocity corresponding to the frequency of the phase current and the negative sequence systems rotate "backwards" at their synchronous speeds.

Irrespective of the form of the steady state excitation, the stator and rotor transient currents may each be resolved into two balanced sinusoidal sets with angular frequencies γ', γ'' . As shown previously these sets are not sustained but decay exponentially in time with decrement factors ρ', ρ'' . Each of

the two transient current sets may therefore be interpreted as part of a rotating field system in which the various distributed quantities decay in amplitude as the system rotates.

The electrical response of the motor to any form of excitation may thus be resolved into a collection of superimposed synchronously rotating systems in the motor, each of these comprising stator and rotor current density waves and the associated flux density distribution. If there are F distinct frequencies present in the excitation at some stage, then the maximum number of rotating systems which may be present is $(2F + 2)$. It is important to emphasize that all such distributed systems in a given motor have the same pole number. Since they rotate at different speed, however, they move past each other causing time variation in the cross-product terms of power and torque. Since only the fundamental pole number occurs, the instantaneous superposition of the rotating components can yield only nett distributions of this pole number. The presence of transient and unbalanced current components in the motor cannot therefore serve to modulate the amplitude of the spatial distribution of electrical quantities around the airgap. However, although these distributions cannot be amplitude modulated in space, they may be deeply modulated as a function of time under transient or unbalanced conditions. In practice these conditions may cause the state of saturation in the magnetic circuit to be rapidly varied which can cause a variety of problems.

The instantaneous positive sequence solution may conveniently be translated in terms of the rotating field concept by the use of a graphical technique. The instantaneous values of the various terms in the solution may be plotted on a complex plane for successive instants of time. If an A-phase axis is drawn in place of the axis of reals and B- and C- phase axes are constructed at angles of $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ to the A-phase axis, then (with due regard for scaling) the actual phase current at each instant is simply the sum of the projections

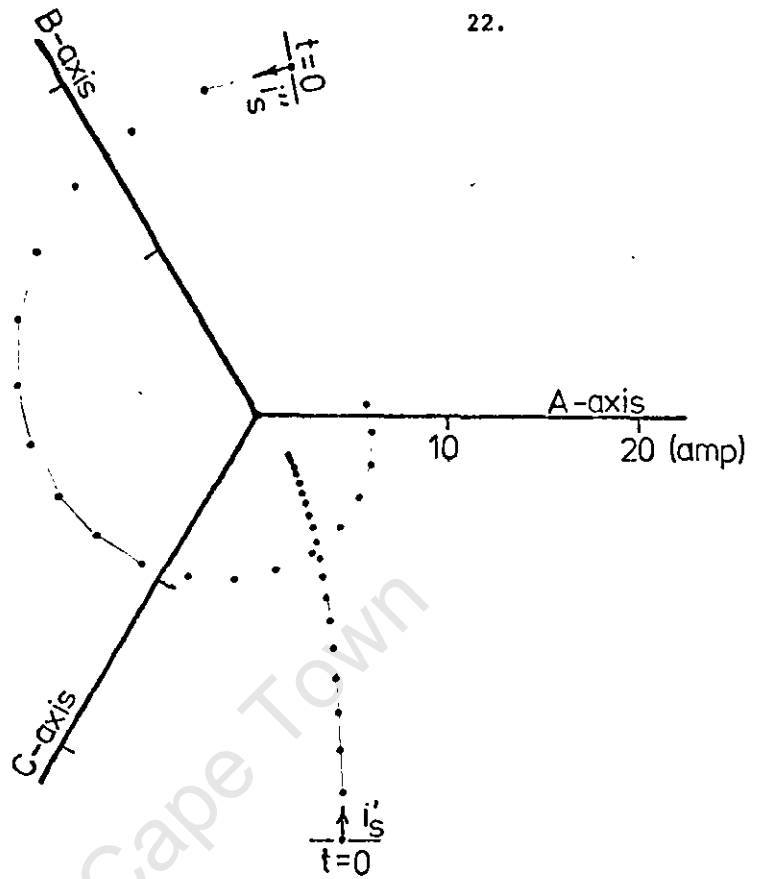


Fig. 3 a) SHORT-CIRCUIT OF ENERGISED MACHINE. POINTS PLOTTED AT 1 MILLISEC. SLIP=10% , TERMINAL VOLTAGE INITIALLY 220 VOLTS RMS 50 HZ.

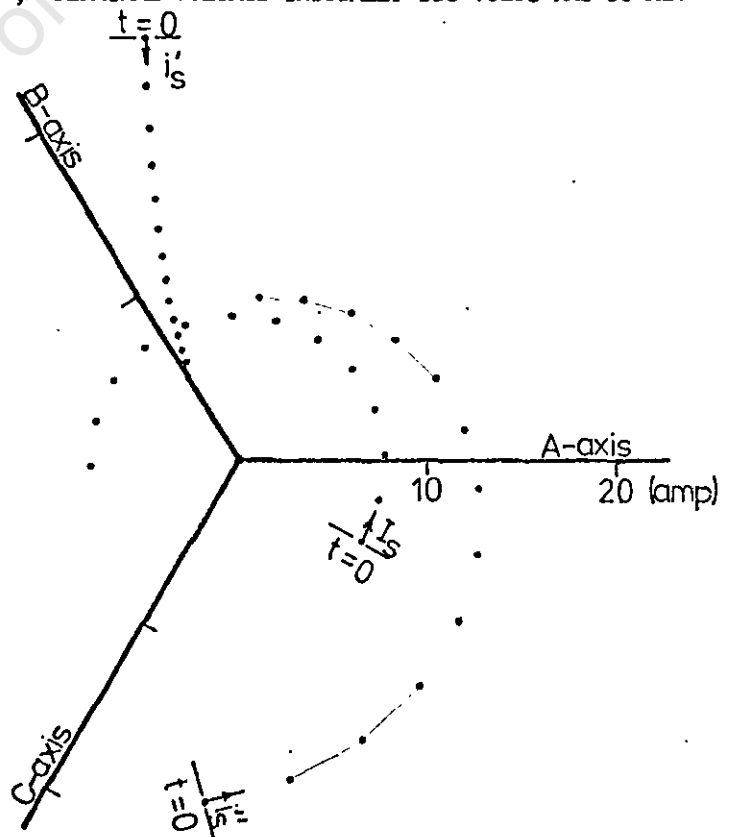


Fig.3 b) SWITCH-ON OF DE-ENERGISED MACHINE. POINTS PLOTTED AT 1 MILLISEC. I SLIP =10% , TERMINAL VOLTAGE 220 VOLTS RMS 50 HZ.

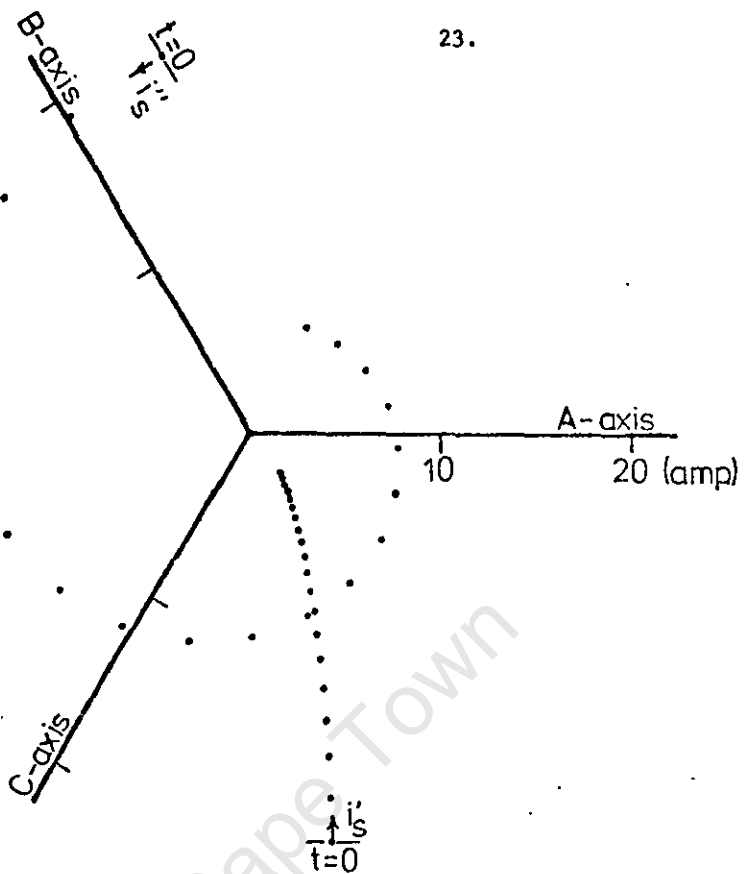


Fig.4 a) SHORT-CIRCUIT OF ENERGISED MACHINE. POINTS PLOTTED AT 1 MILLISEC. INTERVALS. SLIP=0, TERMINAL VOLTAGE INITIALLY 220 VOLTS RMS 50 HZ.

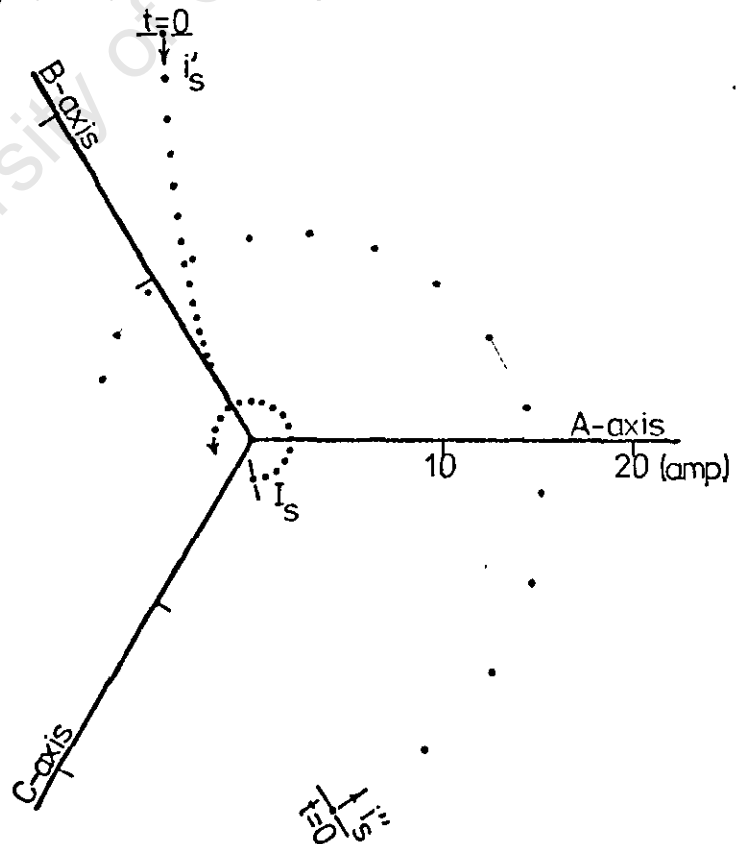


Fig.4 b) SWITCH-ON OF DE-ENERGISED MACHINE. POINTS PLOTTED AT 1 MILLISEC. INTERVALS. SLIP=0, TERMINAL VOLTAGE 220 VOLT RMS 50 HZ.

of the plotted points onto the appropriate phase axis. At the same time each plotted point represents in angle the spatial location of the axis of the associated current density wave, and in magnitude denotes the instantaneous amplitude of the wave. The loci of these plotted points clearly illustrate the movement and decay of the various component current distributions in the airgap while allowing the phase currents to be estimated at a glance. The method provides a concise illustration of a complex situation. (Note that rotation in the diagram is naturally in terms of electrical angle). The graphical method has been used in Figs (3) and (4) to illustrate several transient situations for the case of the test motor. Note that in general the one transient rotating system rotates slowly with respect to the stator (being sometimes called "stator-adherent") and the other rotates at the same low velocity negatively with respect to the rotor (rotor-adherent).

CHAPTER THREE

ENERGY CONVERSION IN THE INDUCTION MOTOR

As mentioned previously the present work arises out of a need for a more complete and fundamental understanding of the response of an induction motor to a modulated power supply, and it is hoped to provide this in terms of the time domain solutions to the motor equations. In particular it is intended to treat in detail the response to chopper modulation of the AC supply voltage and assess the feasibility of this method of voltage control in chopper modulated frequency convertors. In Chapter Two, we established the full algebraic solution to the motor equations in the instantaneous symmetrical components reference frame and related this solution to the concept of rotating current and magnetic flux density waves within the motor. Having thus treated the induction motor as a network problem it will now be of interest to consider closely the process of energy conversion in the machine under both

steady state and transient conditions. A full appreciation of power flow in the device is relevant to the interpretation of the analytic solutions which will be derived in the subsequent chapters.

3.1 Energy sources and storage:

Within our present experience it is axiomatic that no form of electromechanical energy convertor can function without the storage of energy in at least one element of both the electrical and mechanical systems. In the induction motor the internal or electrical energy is held in the magnetic flux produced by the current distributions and, as a good approximation, it may be assumed that this energy is wholly contained in the volume of the airgap. Three "pools" of energy may thus be distinguished in the idealised system which we are considering:

- (a) The electrical supply system connected to the windings is regarded as an infinite pool, capable of sourcing or sinking unlimited amounts of electrical energy with no change in electrical potential (voltage).
- (b) The mechanical system connected to the shaft is similarly viewed as an infinite pool which can source or sink unlimited energy without any change in nett mechanical (kinetic) energy. This is a consequence of the assumption that the shaft velocity is constant.
- (c) The magnetic field represents stored energy. This energy may be yielded to the system, may remain static or may increase as a result of energy supplied by the other pools. In theory there is no limit to the energy which may be stored in this way, but in practice, the flux levels are limited by saturation of the

magnetic circuit and the resistive dissipation in the electrical circuit.

Under various operating conditions it is possible for the nett power drawn from the electrical supply to be positive, negative or zero while the nett power supplied to the mechanical pool is simultaneously positive negative or zero. The overall power flow may therefore be in either direction between electrical and mechanical pools or simply into the machine from one or both of these.

The energy stored in the magnetic field cannot be changed instantaneously. Since any change in the electrical state of the motor is unavoidably associated with changes in the internally stores energy it follows that such changes of state cannot be achieved instantaneously. However, it will be shown later that it is possible to minimise the delay which is involved. These considerations are important when formulating the optimum control strategy for solid state convertor drives.

3.2 The Power Balance Equation.

In the previous chapter the motor equations were expressed in the instantaneous symmetrical components reference frame in the form:-

$$[u_2] = \{ [R] + [G]\omega' \} [i_2] + [L] p. [i_2]$$

$$[R] = \begin{bmatrix} 3R_s & 0 & 0 & 0 \\ 0 & 3R_s & 0 & 0 \\ 0 & 0 & 3R_r & 0 \\ 0 & 0 & 0 & 3R_r \end{bmatrix} ; \quad [L] = \begin{bmatrix} 3L_1 & 0 & 3M' & 0 \\ 0 & 3L_1 & 0 & 3M' \\ 3M' & 0 & 3L_2 & 0 \\ 0 & 3M' & 0 & 3L_2 \end{bmatrix}$$

$$\omega' [G] = \omega' \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (-3jM') & 0 & (-3jL_2) & 0 \\ 0 & (3jM') & 0 & (3jL_2) \end{bmatrix}$$

$$[u_2]_t = [3u_s^+ \quad 3u_s^- \quad 0 \quad 0]$$

$$[i_2]_t = [i_s^+ \quad i_s^- \quad i_r^+ \quad i_r^-]$$

By considering the conservation of energy we may write a general equation:-

$$W_{\text{term.}} = W_{\text{out}} + W_{\text{loss}} + W_{\text{mag.}}$$

where

$W_{\text{term.}}$ = Power from the electrical supply

W_{out} = Power to the mechanical system

W_{loss} = Power dissipated in the machine

$W_{\text{mag.}}$ = Power to the magnetic field storage

also,

$$\begin{aligned} W_{\text{term.}} &= [i]_t^* [w] \\ &= [i]_t^* \left[\{ [R] + [G]\omega' \} [i] + [L]p.[i] \right] \\ &= \underbrace{[i]_t^* [R] [i]}_{W_{\text{loss}}} + \underbrace{\omega' [i]_t^* [G] [i]}_{W_{\text{out}}} + \underbrace{[i]_t^* [L] p.[i]}_{W_{\text{mag.}}} \end{aligned}$$

Expanding this expression, the power balance equation can be written in terms of the instantaneous positive sequence variables:

$$\begin{aligned} \underbrace{6 \operatorname{Re}(i_s^+ i_s^{+*})}_{W_{\text{term.}}} &= \underbrace{6 M' \omega' \operatorname{Im}(i_s^+ i_r^{+*})}_{W_{\text{out}}} \\ &+ \underbrace{6 \{ |i_s^+|^2 R_s + |i_r^+|^2 R_r \}}_{W_{\text{loss}}} \\ &+ \underbrace{6 \left\{ \frac{1}{2} L_1 p. |i_s^+|^2 + \frac{1}{2} L_2 p. |i_r^+|^2 + M' \operatorname{Re} \{ p. (i_s^+ i_r^{+*}) \} \right\}}_{W_{\text{mag.}}} \end{aligned}$$

This equation is quite general and holds for the instantaneous values of the variables, whatever their waveform in time. From it we can write:-

$$\begin{aligned} \text{Torque} &= \frac{P W_{\text{out}}}{\omega} = 6 M' P \operatorname{Im}(i_s^+ i_r^{+*}) \\ \text{and } E_{\text{mag.}} &= \text{energy stored in the magnetic field} \\ &= \int W_{\text{mag.}} dt. \\ &= 6 \left\{ \frac{1}{2} L_1 |i_s^+|^2 + \frac{1}{2} L_2 |i_r^+|^2 + M' \operatorname{Re}(i_s^+ i_r^{+*}) \right\} \end{aligned}$$

In terms of the rotating field concept, it will be appreciated that the expression for torque corresponds to $\frac{\partial E_{mag}}{\partial \theta}$ where θ is the mechanical angular displacement between the stator current distribution axis and the corresponding rotor axis. If we neglect leakage and define the instantaneous component flux density distribution due to the stator current alone as having maximum density B_{sm} and similarly B_{rm} for the rotor and angle θ between the two field axes, then the energy stored in the airgap is obtained by integration of $\frac{B_{sm}^2}{2\mu_0}$ over the volume of the airgap.

Then,
$$E_{mag} = K \left\{ \frac{1}{2} B_{sm}^2 + \frac{1}{2} B_{rm}^2 + B_{sm} B_{rm} \cos(P\theta) \right\}$$
 where $K = \{ (\text{Volume of airgap}) / 2\mu_0 \}$

Although this expression is deficient insofar as energy stored in the leakage fields is neglected, the term by term equivalence with the previous expression for E_{mag} is clearly seen. Furthermore the leakage field energy is not a function of θ in our model so the torque may be correctly represented by:

$$\text{Torque} = \frac{\partial E_{mag}}{\partial \theta} = -K P B_{sm} B_{rm} \sin(P\theta)$$

The negative sign indicates that the direction of the torque exerted on the rotor is such as to reduce the angle θ . Once again the equivalence with the earlier expression in terms of symmetrical components is clearly recognised.

3.3 Power Flow in the Steady State:

Under steady state conditions with balanced positive phasor sequence sinusoidal excitation we have:

$$\begin{aligned} i_s^+ &= I_s^+ e^{j\omega_e t} & v_s^+ &= V_s^+ e^{j\omega_e t} \\ i_r^+ &= I_r^+ e^{j\omega_e t} \end{aligned}$$

Then the instantaneous phase values are:

$$\begin{aligned} v_s^a &= 2 \operatorname{Re}. (V_s^+ e^{j\omega_e t}) = |V_s| \cos(\omega_e t) \\ i_s^a &= 2 \operatorname{Re}. (I_s^+ e^{j\omega_e t}) = |I_s| \cos(\omega_e t + \theta_s) \end{aligned}$$

Thus the instantaneous a-phase terminal power is:

$$W_{\text{term. A-phase}} = \frac{|V_s| |I_s|}{2} \left[\cos(\theta_s) \{ 1 + \cos(2\omega_e t) \} - \sin(\theta_s) \sin(2\omega_e t) \right]$$

The first term in this expression oscillates between $|V_s|/|I_s| \cos(\theta_s)$ and zero with average value $\frac{|V_s| |I_s|}{2} \cos(\theta_s)$ and may be identified as the "real" component of power supplied to the phase. It is the product of the phase voltage with only the in-phase component of the phase current. The remaining term has mean value zero and oscillates between $\frac{|V_s| |I_s|}{2} \sin(\theta_s)$ and $-\frac{|V_s| |I_s|}{2} \sin(\theta_s)$, being the product of the voltage with the quadrature current component. This represents a quantum of energy being alternately supplied to and removed from the motor phase.

If the positive sequence currents for this condition are substituted in the power balance equation it is clear that $W_{mag} = 0$ which indicates that the overall energy stored in the magnetic field of the machine is constant. By integration with respect to time of the reactive phase power we find that the energy stored in the field of a single phase varies as:-

$$\frac{\frac{|V_s| |I_s|}{2} \sin(\theta_s)}{2 \omega_e} [1 - \cos(2\omega_e t)]$$

And, summing over the three phases, we can express the nett energy stored as

$$E_{mag} = \frac{3 \times (\text{VAR per phase})}{2 \omega_e}$$

Thus although the nett magnetic energy is constant, it is held alternately in the fields of the different phases. It is possible to estimate E_{mag} by a standard terminal measurement with no detailed knowledge of the system. In order to illustrate the typical levels of energy stored in a small motor, Fig. 5 shows the total stored energy as a function of speed in the test machine and indicates how this is allocated to the main and leakage fields respectively.

3.4 Transient Power Flow:

Since the stored field energy is non-zero and constant during normal steady state operation, it follows that this energy is injected during transient conditions only. Thus not only do the transient currents in the machine preserve the continuity of current between two different steady states dictated by the

terminal conditions, but in so doing they also serve to negotiate the change of stored energy required for the new steady state condition.

Consider a transient condition initiated at time $t=0$ by the sudden transition from one balanced positive phasor sequence sinusoidal excitation to another. Then it has been shown that the currents during the subsequent time may be written as:

$$i_s^+ = I_s^+ e^{j\omega_e t} + i_s'^+ e^{(-\rho' + j\nu')t} + i_s''^+ e^{(-\rho'' + j\nu'')t}$$

$$i_r^+ = I_r^+ e^{j\omega_e t} + \frac{i_s'^+}{C} e^{(-\rho' + j\nu')t} + \frac{i_s''^+}{C} e^{(-\rho'' + j\nu'')t}$$

If these expressions are substituted into the power balance equation, the nature of each power components may be determined. It is apparent that the power balance equation comprises terms of six distinct complex frequencies under these conditions. These may be listed as:-

TABLE 1.

Component number	Angular frequency	Logarithmic decay rate
1	0	0
2	0	$2\rho'$
3	0	$2\rho''$
4	$(\omega_e - \nu')$	ρ'
5	$(\omega_e - \nu'')$	ρ''
6	$(\nu' - \nu'')$	$(\rho' + \rho'')$

All these components occur in the expressions for output power, loss and power supplied to the magnetic field, but the expression for terminal power contains only components corresponding to nos. 1, 4 and 5 in the above table. The expanded terms are as follows:-

(a) Terminal Power

$$W_{term} = 6 \operatorname{Re} \left\{ V_s^+ e^{j\omega_e t} \cdot \left(I_s^+ e^{j\omega_e t} + i_s'^+ e^{\alpha' t} + i_s''^+ e^{\alpha'' t} \right)^* \right\}$$

$$= 6 \operatorname{Re} \left\{ V_s^+ I_s^{+*} + V_s^+ i_s'^+ e^{(\rho' + j(\omega_e - \nu'))t} + V_s^+ i_s''^+ e^{(\rho'' + j(\omega_e - \nu''))t} \right\}$$

(b) Output Power

$$W_{out} = 6M'\omega' \operatorname{Im} \left[I_s^+ I_r^{+*} + \frac{|i_s'|^2}{C'^*} e^{-2p't} + \frac{|i_s''|^2}{C''^*} e^{-2p''t} \right. \\
+ \left\{ I_s^+ \frac{i_s''^*}{C''^*} - i_s''^* I_r^+ \right\} e^{(-p''+j(\omega_e-\gamma''))t} \\
+ \left\{ I_s^+ \frac{i_s'^*}{C'^*} - i_s'^* I_r^+ \right\} e^{(-p'+j(\omega_e-\gamma'))t} \\
+ \left. \left\{ \frac{i_s' i_s''^*}{C'^*} - \frac{i_s''^* i_s'}{C'} \right\} e^{(-(p'+p'')+j(\gamma'-\gamma''))t} \right]$$

(c) Power Loss

$$\begin{aligned} \text{Stator Loss} &= 6 R_s |i_s^+|^2 \\ &= 6 R_s \left[|I_s^+|^2 + |i_s'|^2 e^{-2p't} + |i_s''|^2 e^{-2p''t} \right. \\ &\quad + 2 \operatorname{Re} \left\{ I_s^+ i_s'^* e^{(-p'+j(\omega_e-\gamma'))t} \right\} \\ &\quad + 2 \operatorname{Re} \left\{ I_s^+ i_s''^* e^{(-p''+j(\omega_e-\gamma''))t} \right\} \\ &\quad + \left. 2 \operatorname{Re} \left\{ i_s' i_s''^* e^{(-(p'+p'')+j(\gamma'-\gamma''))t} \right\} \right] \end{aligned}$$

$$\begin{aligned} \text{Rotor Loss} &= 6 R_r |i_r^+|^2 \\ &= 6 R_r \left[|I_r^+|^2 + \frac{|i_s'|^2}{|C'|^2} e^{-2p't} + \frac{|i_s''|^2}{|C''|^2} e^{-2p''t} \right. \\ &\quad + 2 \operatorname{Re} \left\{ I_r^+ \frac{i_s'^*}{C'^*} e^{(-p'+j(\omega_e-\gamma'))t} \right\} \\ &\quad + 2 \operatorname{Re} \left\{ I_r^+ \frac{i_s''^*}{C''^*} e^{(-p''+j(\omega_e-\gamma''))t} \right\} \\ &\quad + \left. 2 \operatorname{Re} \left\{ \frac{i_s'}{C'} \cdot \frac{i_s''^*}{C''^*} e^{(-(p'+p'')+j(\gamma'-\gamma''))t} \right\} \right] \end{aligned}$$

(d) Power Supplied to the Magnetic Field

$$W_{mag.} = \left[\frac{1}{2} \cdot \frac{L_1}{R_s} \cdot p \cdot (W_{loss \text{ stator}}) \right. \\
+ \frac{1}{2} \cdot \frac{L_2}{R_r} \cdot p \cdot (W_{loss \text{ rotor}}) \\
+ \left. 6 M' \operatorname{Re} \left\{ p \cdot (i_s^+ i_r^{+*}) \right\} \right]$$

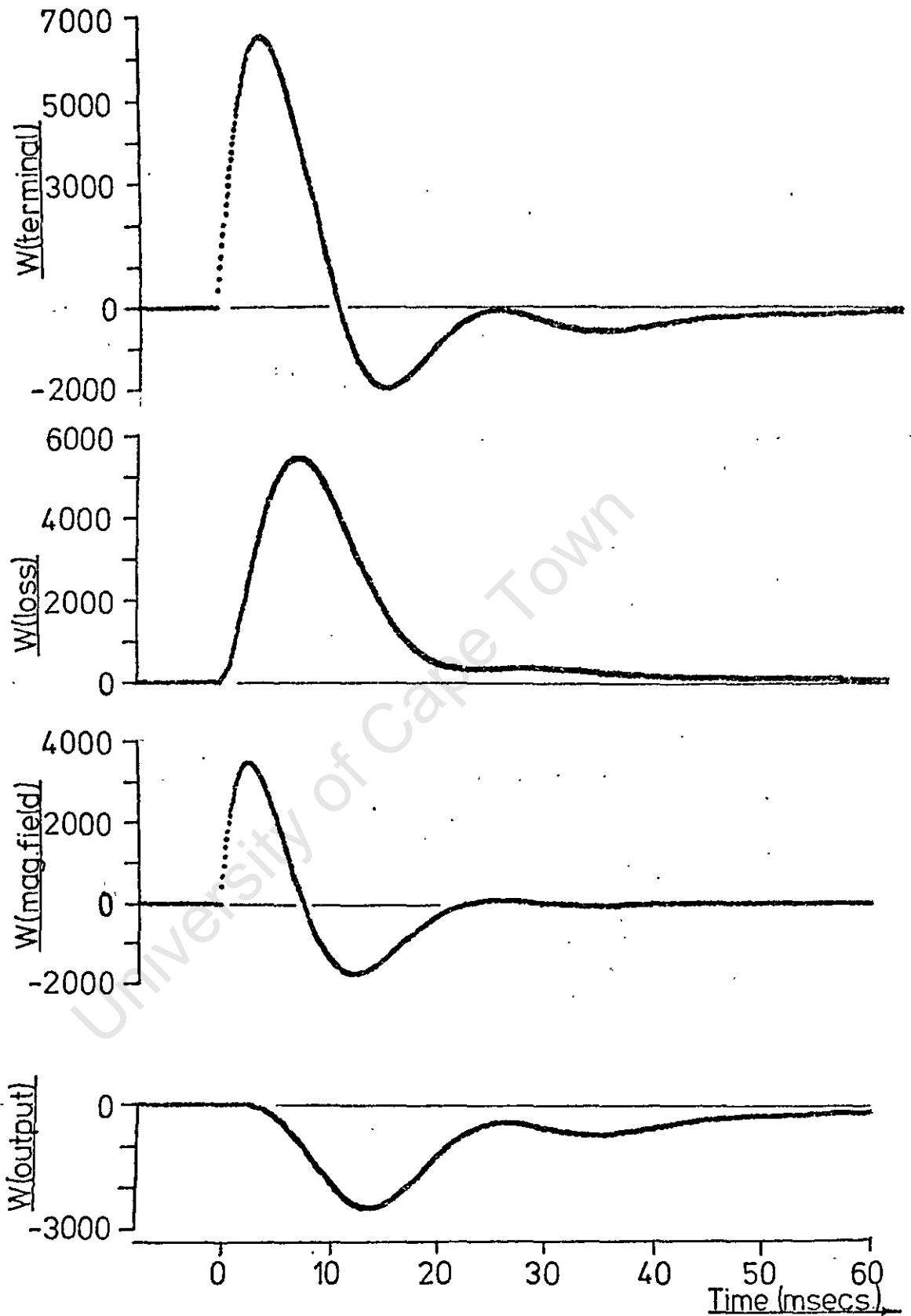
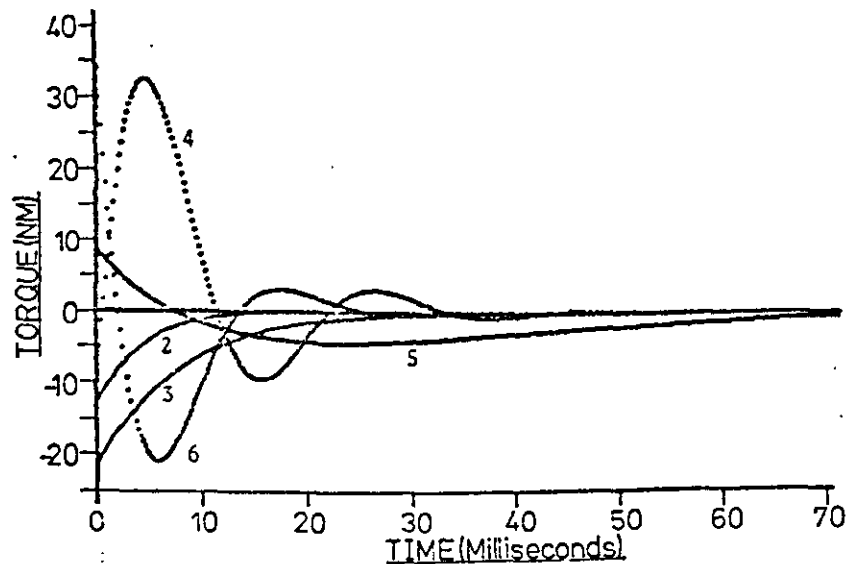


Fig.6 Power components during switch-on of an initially de-energised motor at $T = 0$. Slip = 0, $V_s = 220$ Volt RMS 50 Hz.

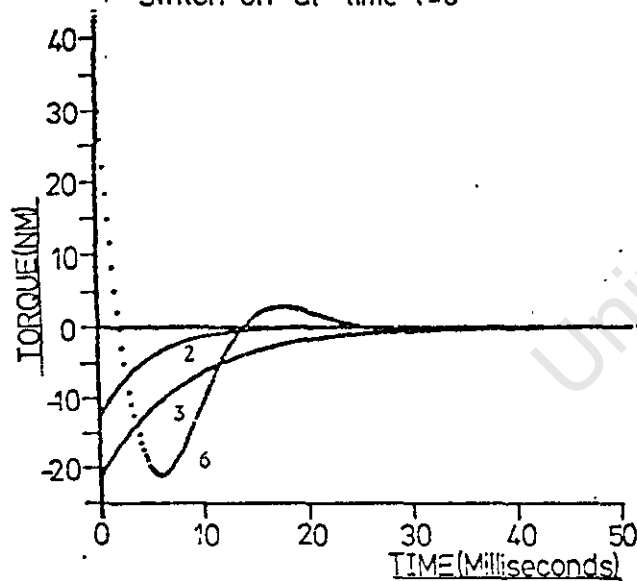
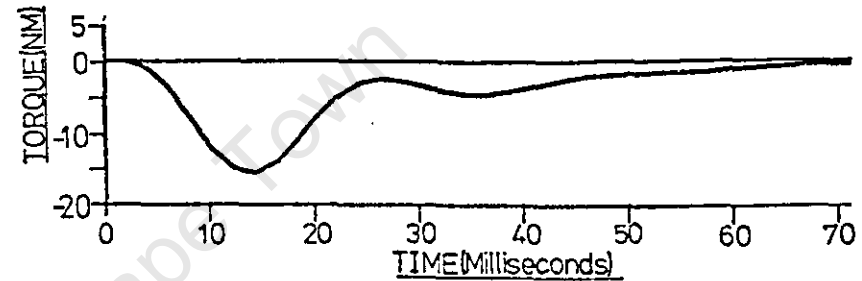
Calculated for the test motor.

($M' = 0.447$, $L_1 = L_2 = 0.473$, $R_s = 5.5$, $R_r = 3.28$, $P = 2$)



$V_s = 220$ VOLT RMS 50 HZ SLIP=0

Switch-on at time $t=0$



$V_s = 220$ VOLT RMS 50 HZ SLIP=0

Short-circuit at time $t=0$

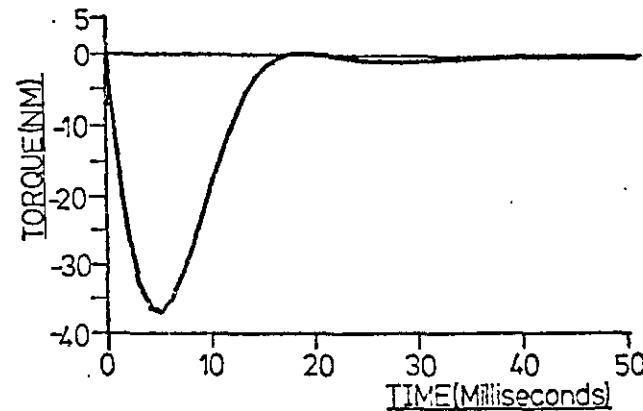
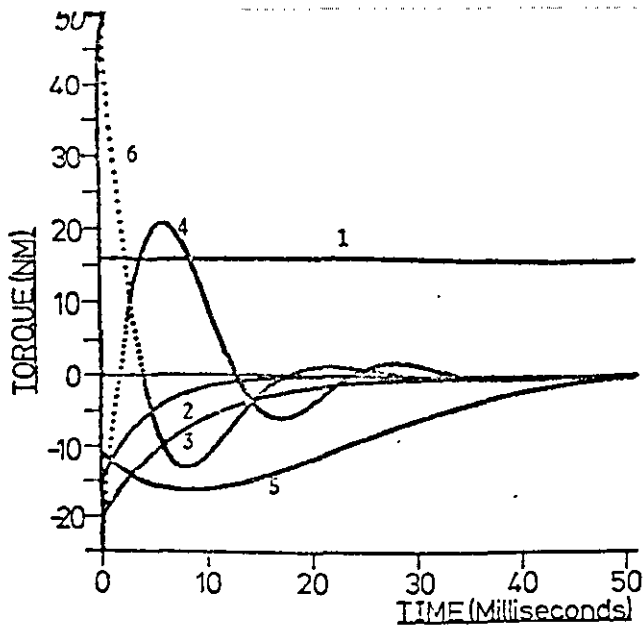
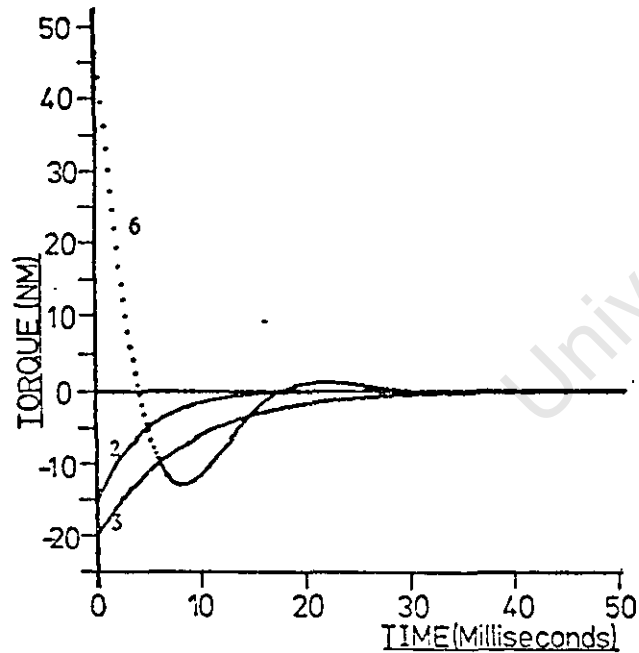
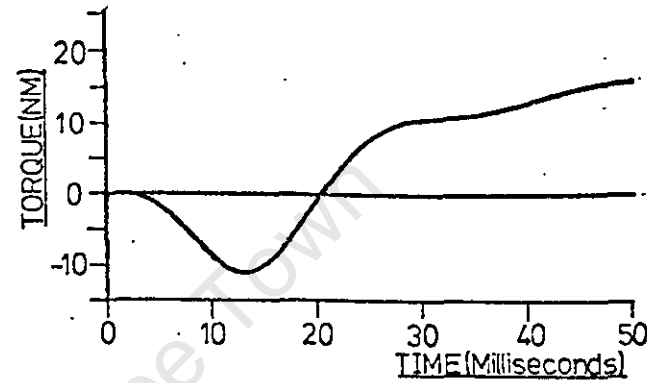


Fig. 7 Transient torque components and their combined values calculated for switch-on and short-circuit of the test motor. ($M' = 0.447$, $L_1 = L_2 = 0.473$, $R_s = 5.5$, $R_r = 3.28$, $P = 2$)



$V_s = 220$ VOLT RMS 50 HZ. SLIP=10%

Switch-on at time $t=0$



$V_s = 220$ VOLT RMS 50 HZ. SLIP=10%

Short-circuit at time $t=0$

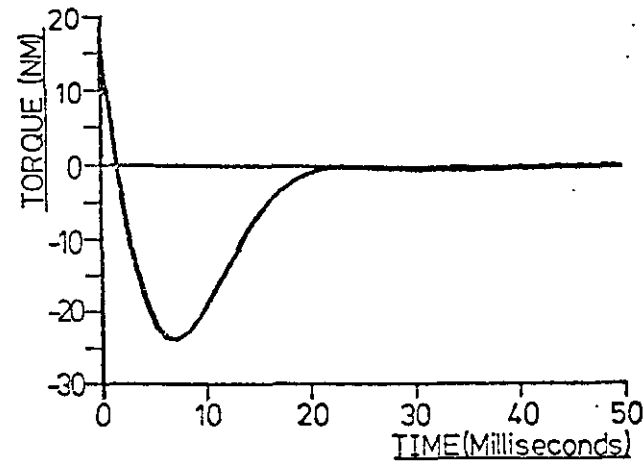


Fig. 8 Transient torque components and their combined values calculated for switch-on and short-circuit of the test motor. ($M'=0.447$, $L_1=L_2=0.473$, $R_c=5.5$, $R_r=3.28$, $P=2$)

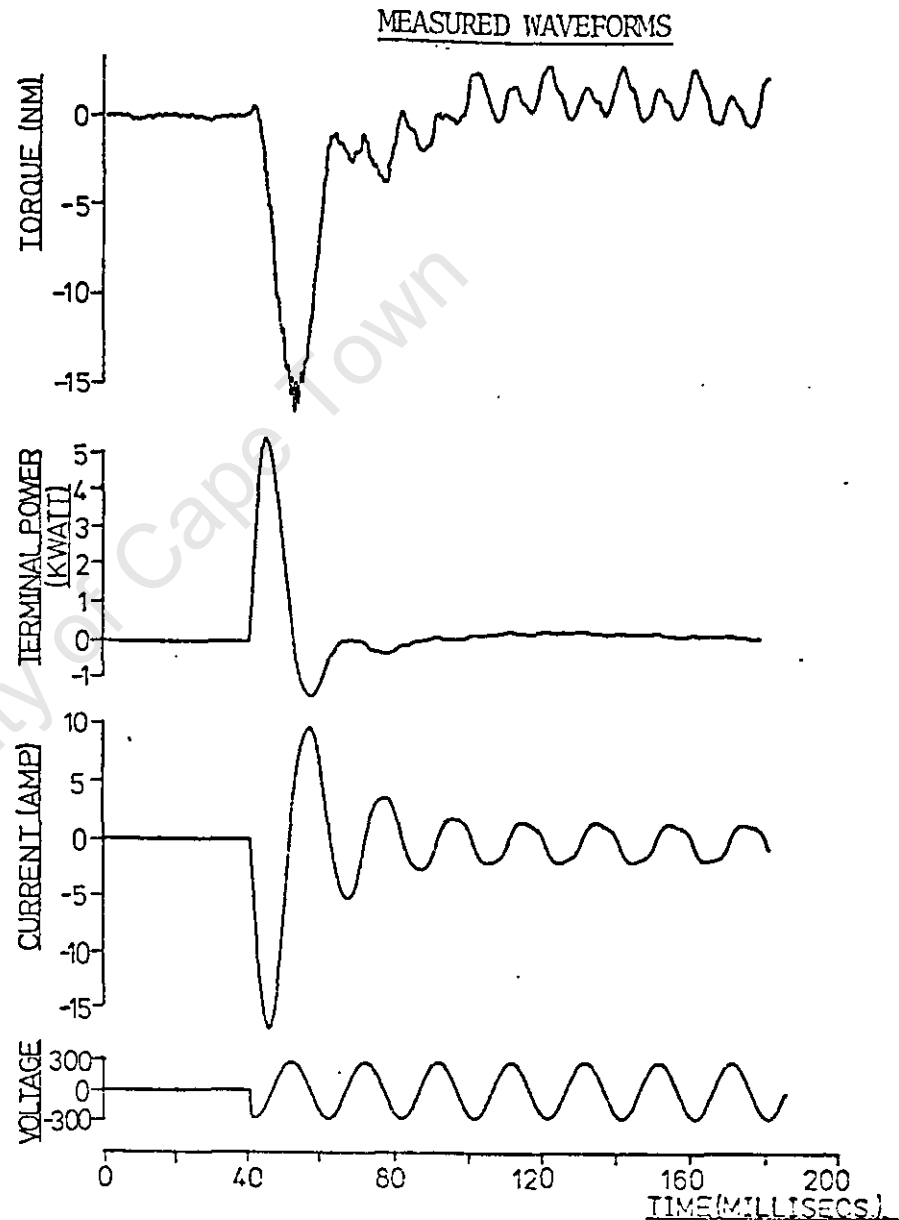
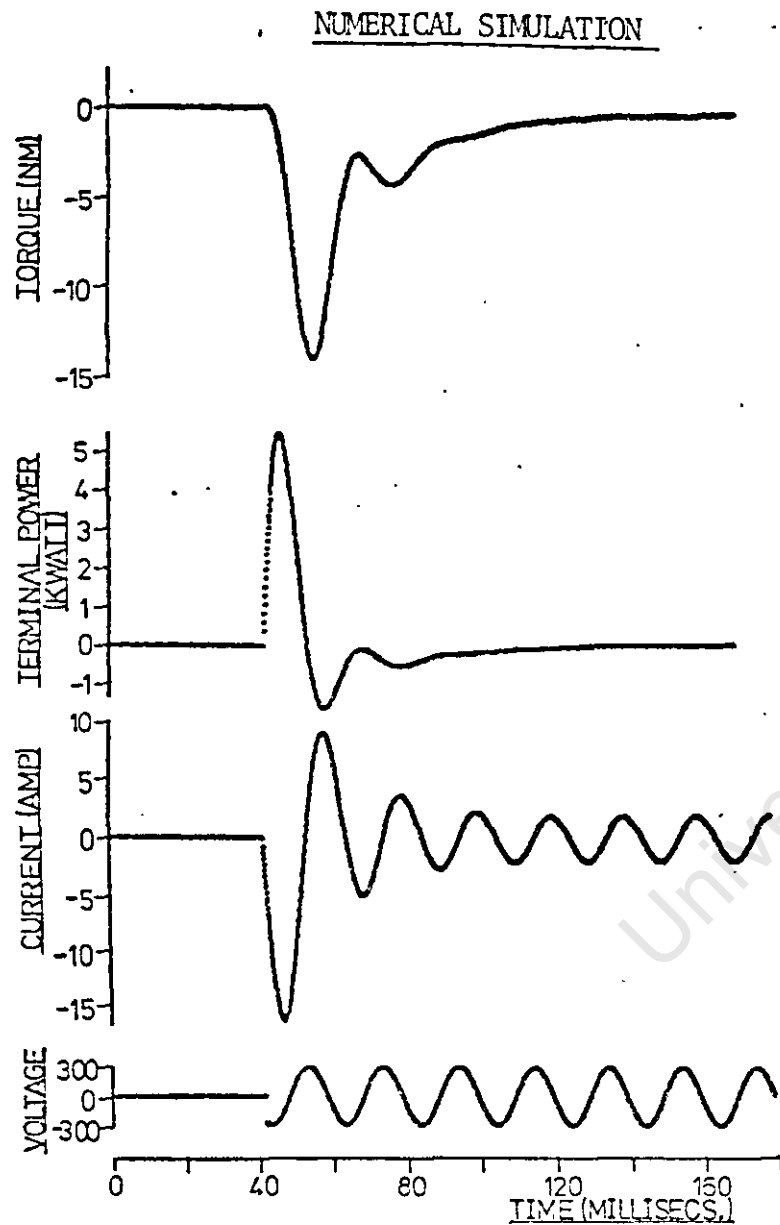


Fig.9 Measured and simulated waveforms for the case of the test motor switched on suddenly while running at zero slip, initially de-energised. $V_s = 200$ volt RMS 50 Hz.

Expanding the third term:-

$$6M'p. \operatorname{Re} \left[I_s^+ I_r^{+*} + \frac{|i_s'|^2}{c'^*} e^{-2\rho't} + \frac{|i_s''|^2}{c''*} e^{-2\rho''t} \right. \\
+ \left\{ I_s^+ \frac{i_s''^*}{c''*} + I_r^+ i_s''^* \right\} e^{(-\rho'' + j(\omega_e - \nu''))t} \\
+ \left\{ I_s^+ \frac{i_s'^*}{c'^*} + I_r^+ i_s'^* \right\} e^{(-\rho' + j(\omega_e - \nu'))t} \\
\left. + \left\{ \frac{i_s' i_s''^*}{c''*} + \frac{i_s' i_s''^*}{c'} \right\} e^{(-(\rho' + \rho'') + j(\nu' - \nu''))t} \right]$$

Fig. 6 illustrates how the power components vary under typical transient conditions. Figs. 7 and 8 show the 6 complex frequency terms separated under similar conditions and Fig. 9 shows calculated characteristics compared with actual measured curves from the test motor.

3.5 Withdrawal of Stored Magnetic Energy

If the excitation frequency is held constant then the terminal voltage of an induction motor is closely related to the nett magnetic flux linking the phase windings. Any method which effectively controls the terminal voltage of the machine must therefore produce an effective variation of the magnetic flux. There are two distinct ways in which switching action can be applied in the supply circuit to reduce the flux in the machine. These actions are fundamentally different, though either could form the basis of a modulator for voltage control. It is thus important to consider each action closely and determine which is best suited to the purpose:-

- (a) Open-circuit of the stator windings
- (b) Short-circuit of the stator terminals

3.6 SYMMETRICAL OPEN-CIRCUIT OF STATOR

In a system of electromagnetically coupled circuits, each circuit usually has a component of flux linking it which does not link any other circuit in the system. Since this leakage flux is supported by the current flowing in the circuit, there is no way in which this current can be reduced to zero without the dissipation of the energy stored in the leakage field. In regard to the remaining flux linkage component which is mutually coupled to other circuits, the action of reducing the first current tends to induce additional currents in these coupled circuits which, as far as possible, maintain continuity of flux linkages throughout the system. In practice an inductive current-carrying circuit cannot be instantaneously interrupted and the open-circuiting is usually a complex phenomenon during which the leakage energy is dissipated in arc and circuit resistances. In order to illustrate more clearly the effect of a symmetrical open-circuit on an induction motor, we may consider the hypothetical limit where the stator currents are instantaneously reduced to zero. In this instance we assume that the energy stored in the leakage field of the stator is "instantaneously dissipated", while the rotor circuits remain intact. Since the constant flux linkage theorem implies that the flux linking the rotor must be constant during this process, it follows that the rotor current is discontinuous at the instant of opening. The nett effect is as though a component of the stator current has "hopped across" to the rotor windings.

Consider an open-circuit at time $t=0$ where the steady state currents at $t=0_-$ are:-

$$i_s^+ = I_s^+ \quad ; \quad i_r^+ = I_r^+$$

Then we can denote the instantaneous positive sequence rotor flux linkages at $t = 0_-$ by

$$\Phi_{r0-}^+ = jL_2 I_r^+ + jM' I_s^+$$

If the rotor current at $t = 0_+$ is $i_r^+ = I_r^{+'}$ then

$$\Phi_{r0+}^+ = jL_2 I_r^{+'}$$

Thus since $\Phi_{r0-}^+ = \Phi_{r0+}^+$,

$$I_r^{+'} = I_r^+ + \frac{M'}{L_2} I_s^+$$

At $t = 0_+$ then

$$\begin{aligned} E_{mag.} &= 3L_2 \left\{ \left| I_r^+ + \frac{M'}{L_2} I_s^+ \right|^2 \right\} \\ &= 3L_2 \left\{ |I_r^+|^2 + \frac{M'^2}{L_2^2} |I_s^+|^2 + \frac{2M'}{L_2} \text{Re.}(I_s^+ I_r^{+*}) \right\} \end{aligned}$$

The overall loss of stored energy at $t=0$ is thus

$$\begin{aligned} &3L_1 |I_s^+|^2 - 3 \frac{M'^2}{L_2} |I_s^+|^2 \\ &= 3L_1 |I_s^+|^2 \left\{ 1 - \frac{M'^2}{L_1 L_2} \right\} = \underline{3L_1 |I_s^+|^2 \sigma} \end{aligned}$$

Subsequent to $t=0$, $i_s^+ = 0$ and the solution to the constrained positive sequence equations is

$$i_r^+ = I_r^{+'} e^{(j\omega' - \frac{R_r}{L_2})t}$$

Under these conditions the rotor current density distribution is stationary with respect to the rotor itself, and rotates relative to the stator at electrical angular velocity equal to ω' , while decaying with the DC time constant of the rotor. This phenomenon is vividly illustrated by measurement of the stator terminal voltage of the test motor when the supply lines are suddenly opened while the motor is running at $slip = 0$. As seen from Fig. 10 the back e.m.f. persists for many cycles after the stator circuits are opened. Since the rotor current was initially zero, this would not be possible without the transfer of current to the rotor during opening. Open-circuits of the energized induction motor have been investigated previously; however the phenomenon has

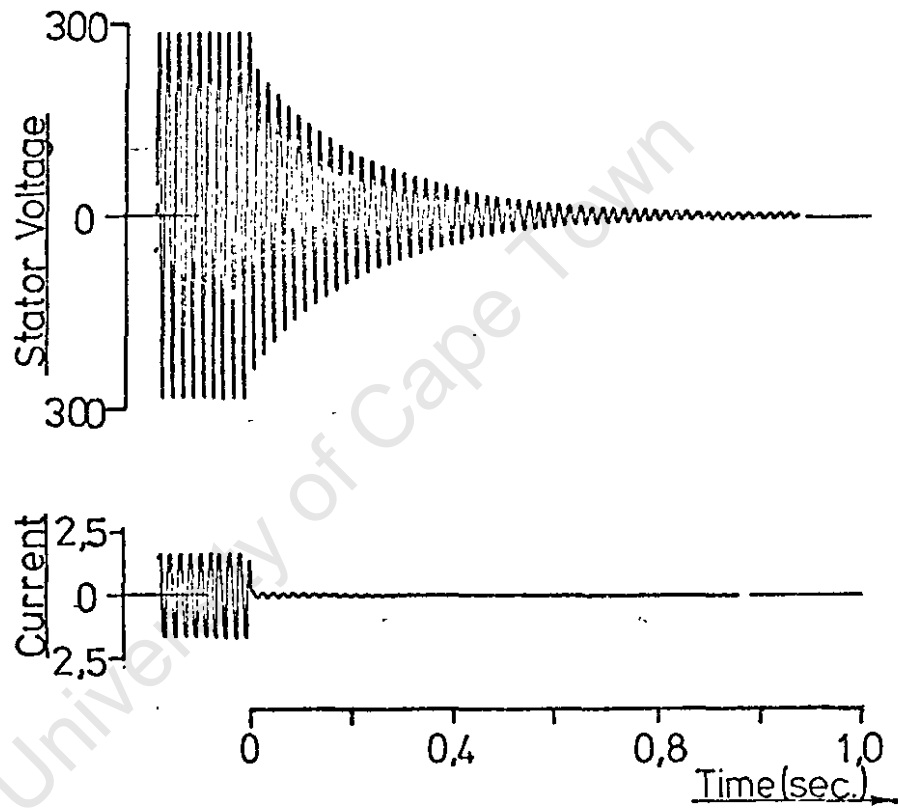
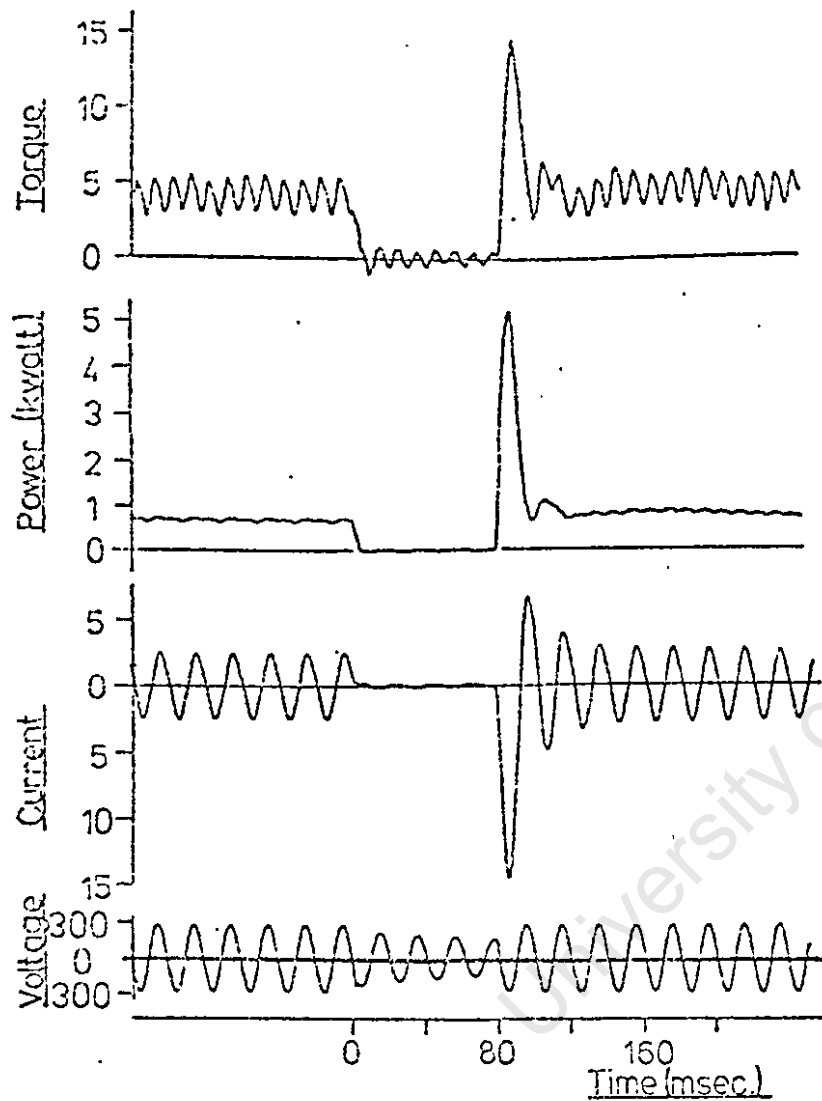


Fig. 10 Sudden open-circuit of the stator windings of energised induction motor at $T=0$. Slip=0, $V_s=200$ Volt rms 50 Hz. Measured on the test motor.



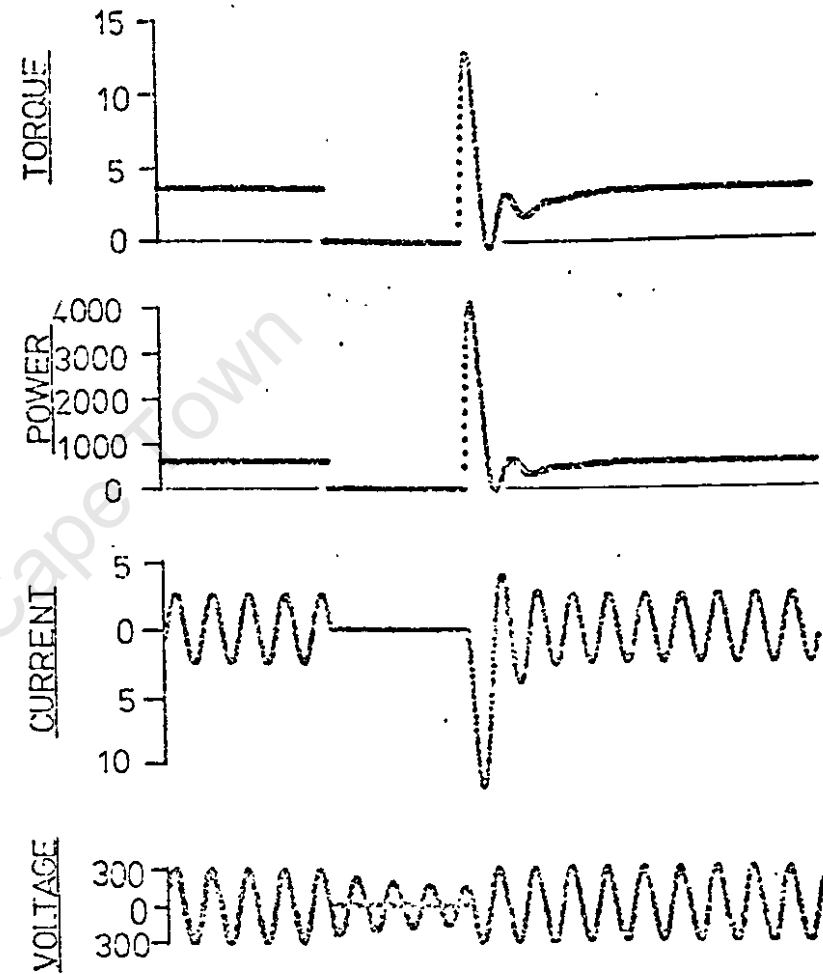
MEASURED WAVEFORMS.

Fig. 11

Open-circuit of initially excited induction motor at time $T=0$, followed by reconnection after a short interval.

$V_s = 200$ volt RMS 50 Hz. Slip = 2%.

Test motor ($M'=0.447$, $L_1=0.473=L_2$, $R_s=5.5$, $R_r=3.28$, $P=2$)



CALCULATED WAVEFORMS.

clearly been misunderstood in many instances where the continuity of flux linkages is neglected at the instant of opening.

Fig. 11 shows the test motor opened for a short period before being reconnected to the supply while running at 2% slip. The measured waveforms are seen to compare favourably with the results obtained by calculation for the same situation.

While the stator circuits are open and the trapped rotor currents are decaying, no torque is produced, and naturally there is no flow of terminal power. There is also no stator copper loss so the power balance equation reduces to:-

$$0 = 3L_2 p. |i_r^+|^2 + 6 |i_r^+|^2 R_r$$

(Note:- $\frac{L_2}{R_r}$ is measured for the test motor as 144 msecs.)

The action of suddenly open-circuiting the stator thus dissipates a component of the stored magnetic energy immediately in the arc and circuit resistance and dissipates the remainder internally at a comparatively slow rate. As a method of reducing the flux in the motor, open-circuiting is uncontrolled and may also be extremely hazardous. Extremely high self-induced voltages arise in the stator circuits as they are interrupted and these may puncture the motor insulation or destroy the switching device. As seen from the examples, the open-circuiting can also be a rather slow method of de-fluxing the motor.

3.7 SYMMETRICAL SHORT-CIRCUIT OF STATOR WINDINGS.

Short-circuit of an inductive circuit can be achieved virtually instantaneously. The circuit current remains continuous during this action while it is steered into an alternative supply path which contains no impedance and no voltage sources. Through the instant of short-circuiting the flux linkages in the system containing the circuit remain continuous and consequently

the currents in other circuits which are magnetically coupled to the first are also continuous. If the stator terminals of an energized induction motor are suddenly short-circuited at time $t=0$, then the steady state component in the subsequent motor current is zero. During this period therefore the current has the form:-

$$i_s^+ = i_s' e^{(-\rho' + j\nu')t} + i_s'' e^{(-\rho'' + j\nu'')t}$$

It is clear from Fig. (1) that the time constants and angular frequencies associated with each of the two current components are dependant on the speed of rotation and on the electrical parameters of the motor. As speed increases beyond a certain point we find that $\frac{1}{\rho'} \rightarrow \frac{\sigma L_s}{R_s}$ (9,2 msecs. for the test motor) and $\frac{1}{\rho''} \rightarrow \frac{\sigma L_r}{R_r}$ (15,4 msecs. for the test motor). These are the ideal leakage time constants of the motor. Similarly at these higher speeds $\nu' \rightarrow 0$ and $\nu'' \rightarrow \omega'$. However, depending on the winding resistances, the values of ν and $\frac{1}{\rho}$ deviate considerably from these limits as the machine operates at lower shaft speeds. In particular the time constant of the "rotor-adherent" i_s'' component becomes comparatively large at the lowest speeds. In the case of the test motor at standstill $\frac{1}{\rho''} = 224,3 \text{ msec.}$ Consequently the exact nature of the current and power transients following a sudden short-circuit vary considerably depending on the operating speed.

In the case of an ideal short-circuit no energy is dissipated outside the machine so that there is continuity of output power, internal losses and stored magnetic energy. After $t=0$ the terminal power is zero so that the power balance equation reduces to

$$W_{mag} + W_{loss} + W_{out} = 0$$

This represents a nett flow of energy in either direction between the magnetic and mechanical storage pools. Since W_{loss} is always positive, it follows that if either W_{mag} or W_{out} is positive then the other one is negative, which indicates that any increase in the energy of either pool is supplied from the other. It is also clear that if W_{term} had a certain value at $t=0$ then exactly this power flows from the magnetic field at $t=0$.

in order to accommodate the continuity of the other two power components. Thus

$$W_{mag}(t=0_+) = -W_{term}(t=0_-)$$

Since the steady state component of current is zero after $t=0$ each power component comprises only 3 distinct terms. As seen previously, these are related to the real or imaginary parts of complex exponentials with exponents equal to $(-2\rho't)$,

$(-2\rho''t)$ and $[-(\rho'+\rho'') + j(\nu'-\nu'')]t$ respectively.

These terms correspond to numbers 2, 3 and 6 in Table (1). The first two terms simply decay exponentially. They always contribute negatively to both W_{mag} and W_{out} , thus representing a nett drain of energy from both pools to supply losses in the motor. The third term oscillates as it decays with angular frequency $(\nu'-\nu'')$ where $|\nu'-\nu''| \leq |\omega|$ (See Figs.(12), (13))

Immediately after a motor is short-circuited, therefore, we find that

- (a) if $W_{out} > 0$ at $t=0$, the motor continues to deliver useful output power to the shaft for a short time.
- (b) if $W_{term} < 0$ at $t=0$, the load continues to supply power to the motor, but instead of regenerating into the electrical supply, the field energy increases for a short period.

Since the stored energy is usually low it is apparent that the motor can only supply useful power output for a very short time following the short circuit in case (a) above. During this period the stored field energy drops rapidly as does the output power. After a time the output becomes negative and the stored energy in the field tends to rise again. The output power passes through a negative peak as a result of the oscillating component term. This peak represents a sudden braking torque which may be greater than twice $T(\max)$ in magnitude. In Fig (14), we

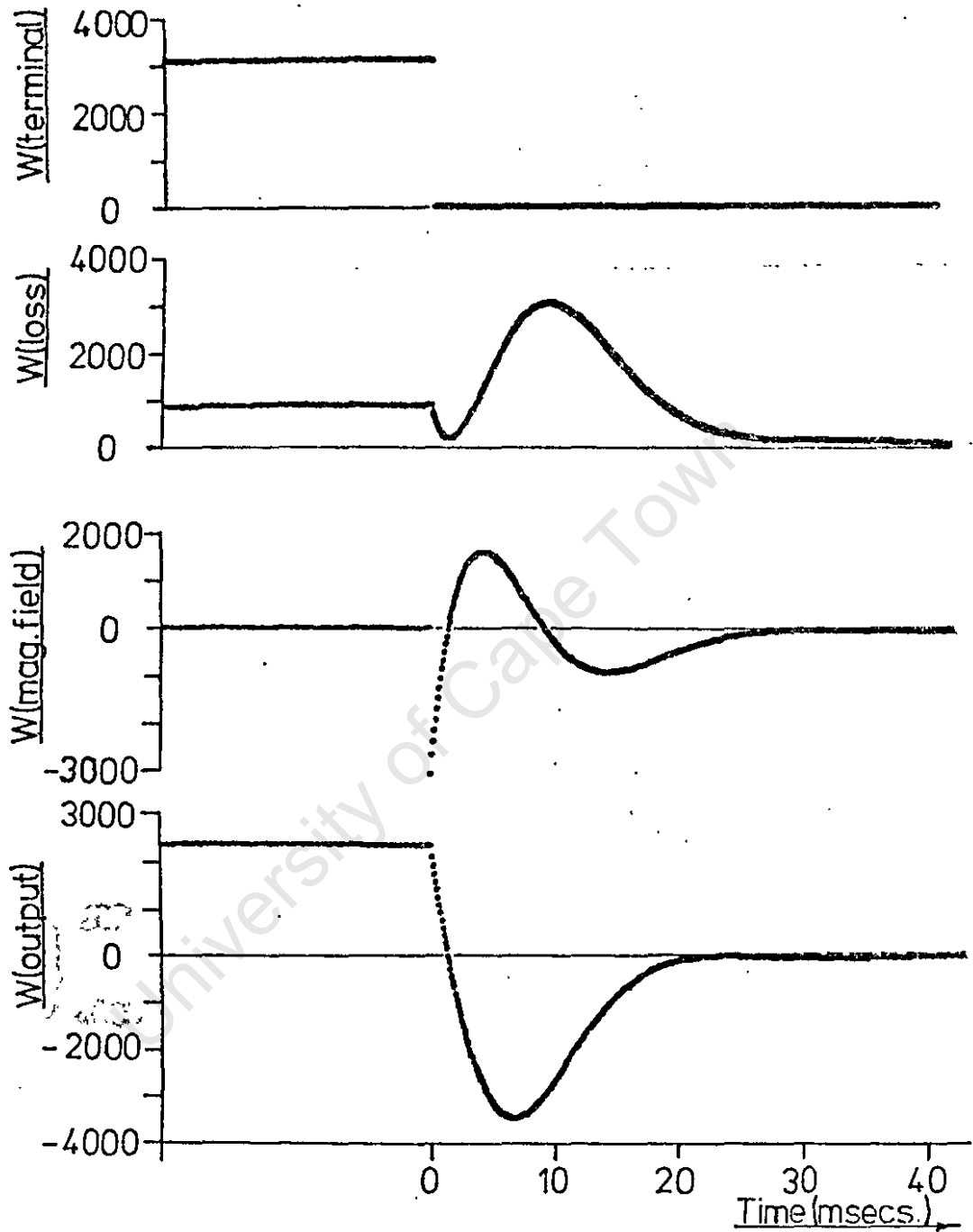


Fig. / 2 Power components during short circuit of the stator windings of an initially energised motor. Slip = 10%, $V_s = 220$ Volt RMS 50 Hz.
Calculated for the test motor ($M' = 0.447$, $L_1 = L_2 = 0.473$, $R_s = 5.5$, $R_r = 3.28$)

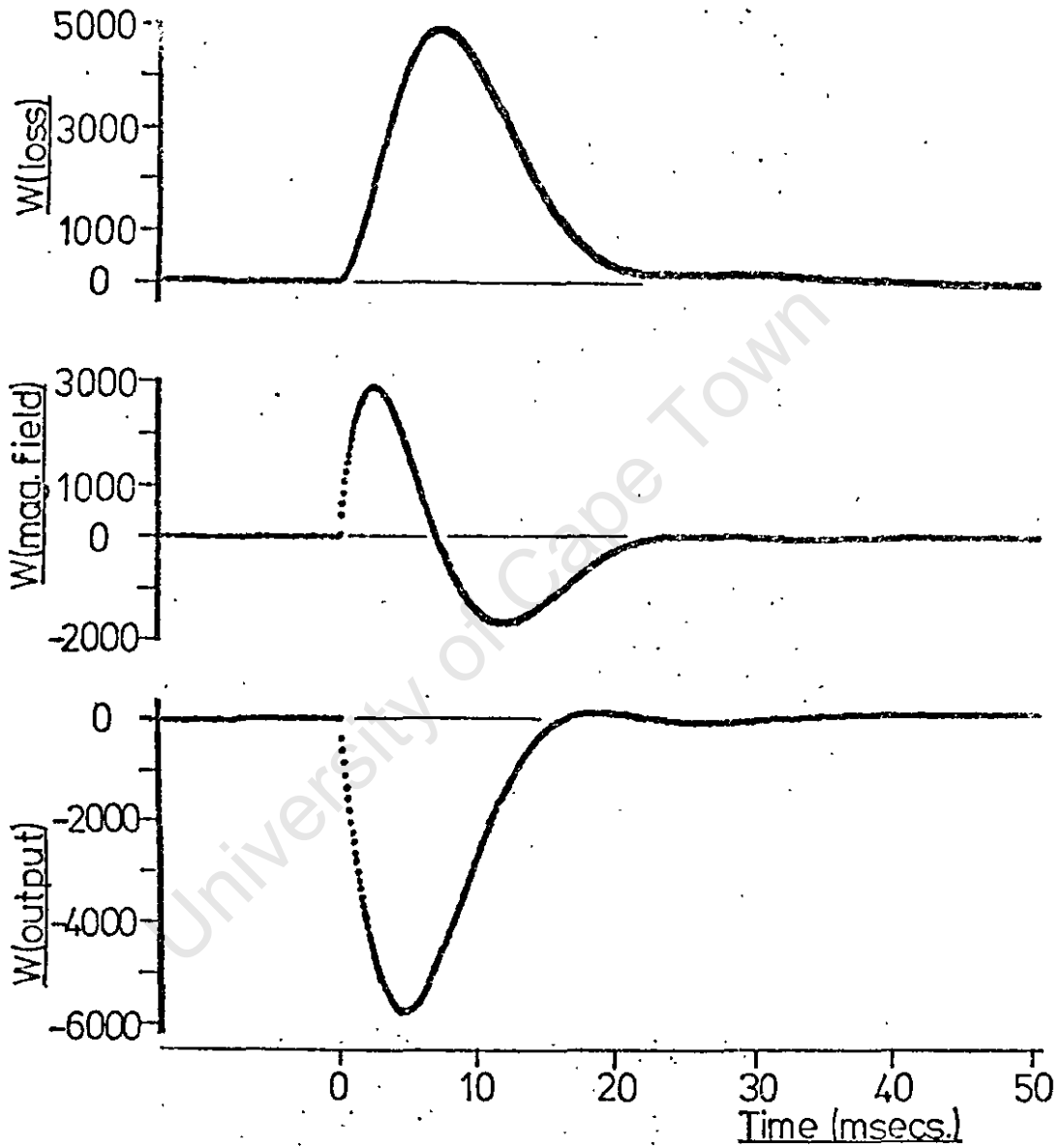


Fig. 13 Power components during short-circuit of the stator windings of an initially energised motor. Slip=0, $V_s = 220$ Volt RMS 50 Hz. Calculated for the test motor ($M' = 0.447$, $L_1 = L_2 = 0.473$, $R_s = 5.5$, $R_r = 3.28$)

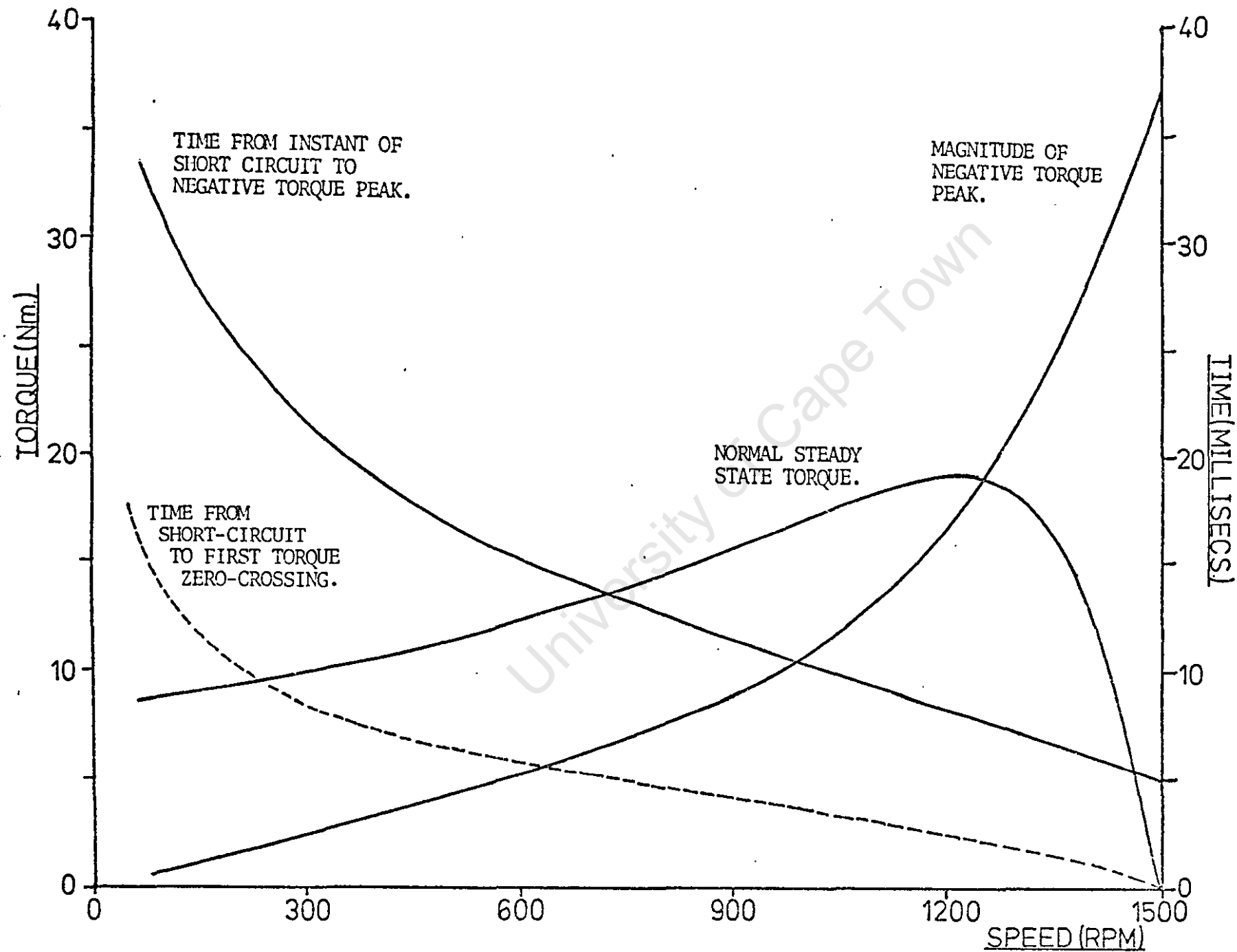


Fig.14

Values calculated for the test motor.
 $V_s = 220$ Volt RMS 50 Hz.
 $(L_1 = L_2 = 0.473, M' = 0.447,$
 $R_s = 5.5, R_r = 3.28, P = 2)$

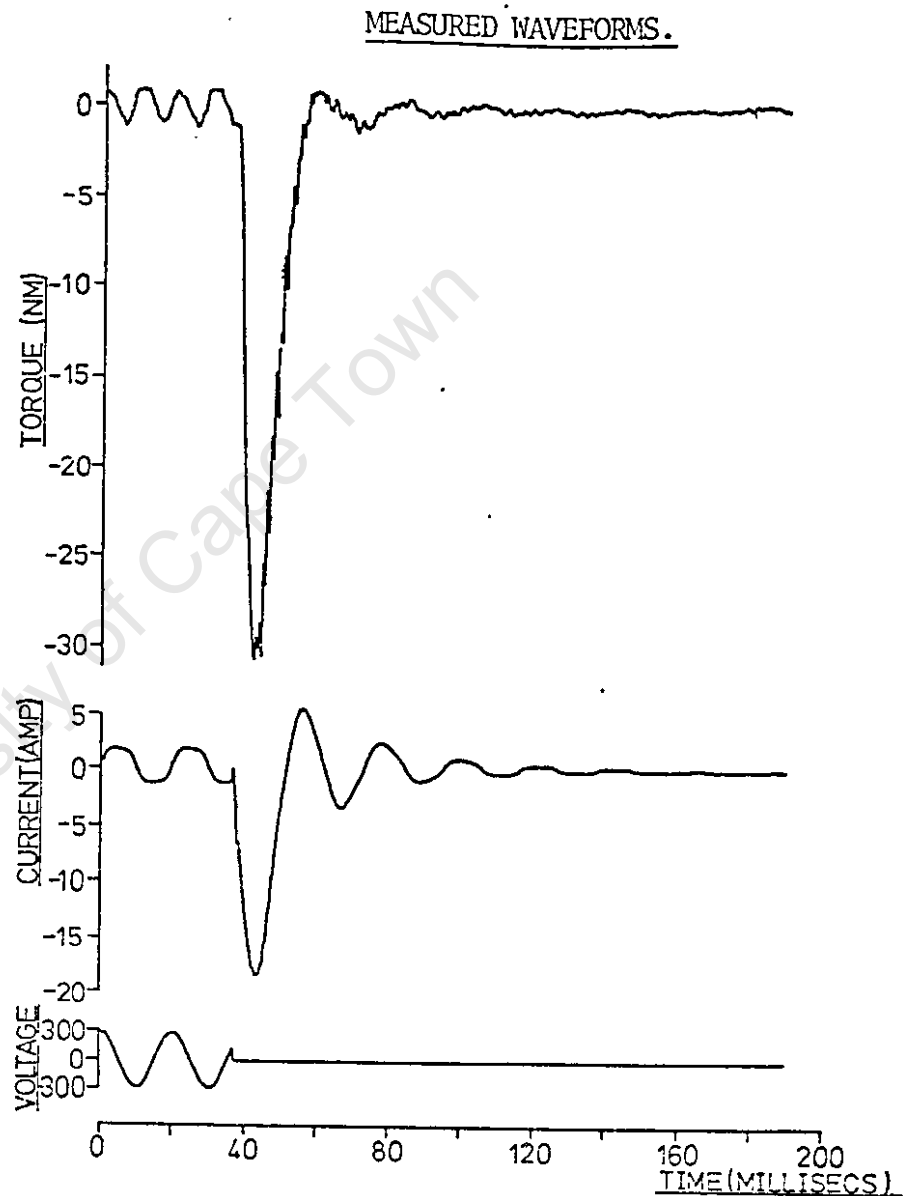
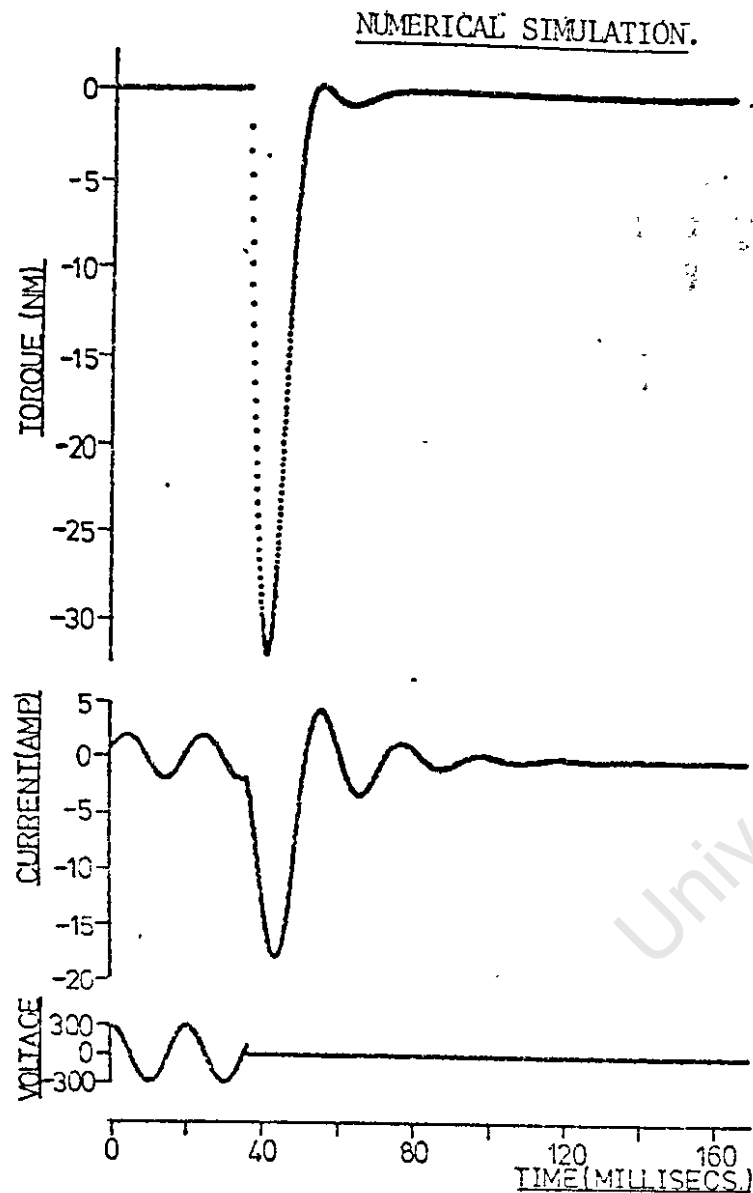


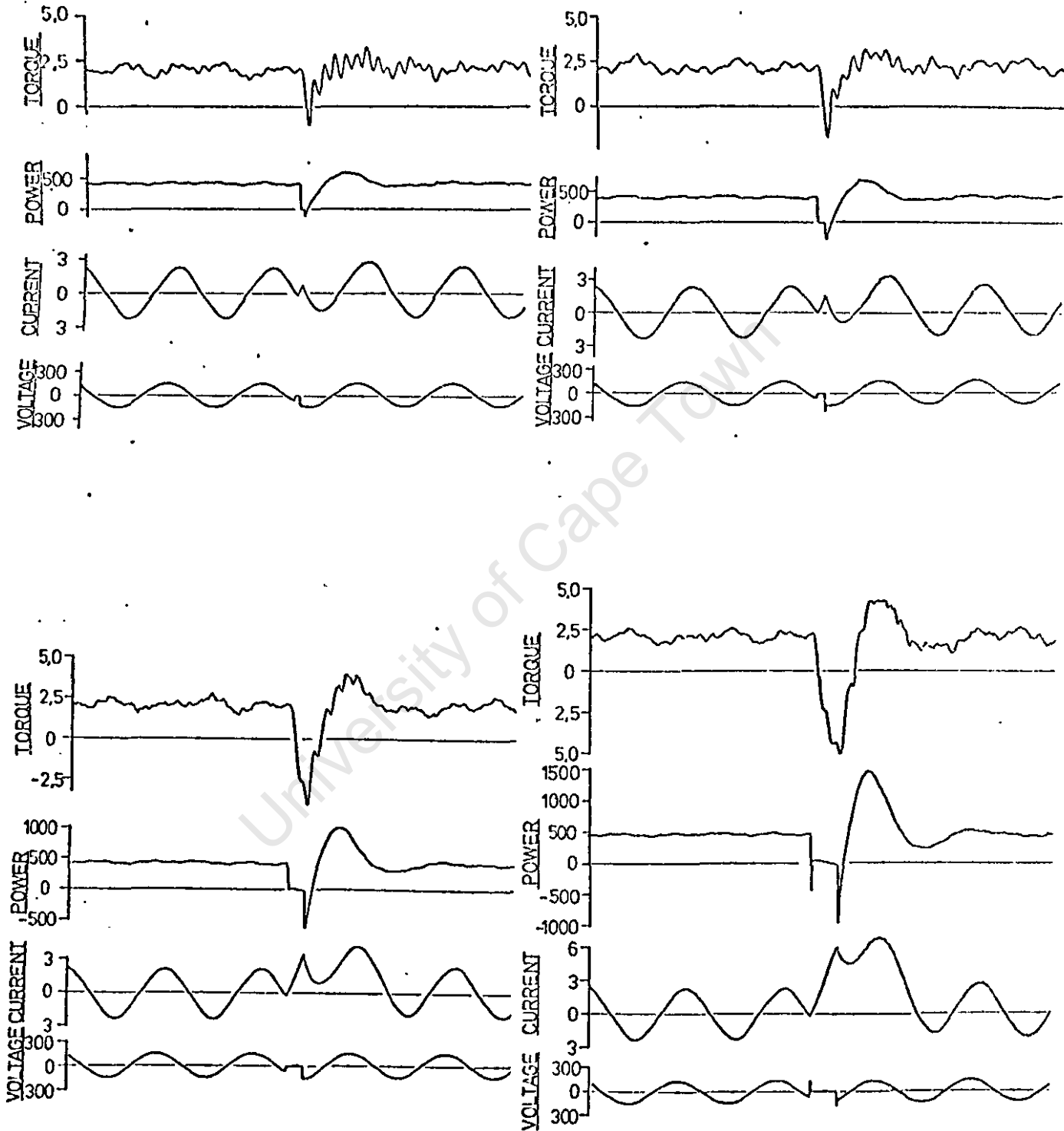
Fig.15

Measured and calculated waveforms for the case of the test motor suddenly short circuited while running at zero slip, initially energised.
 $V_s = 200$ volt RMS 50 Hz.

illustrate the characteristics of this torque transient as a function of shaft speed for the case of the test motor. It will be seen that the negative torque peak is larger and (see Fig.15) occurs much sooner as the shaft speed is increased to the 1500 RPM synchronous speed. This is mainly due to:-

- (a) Increasing frequency of the oscillating term in W_{out} , which thus reaches its negative peak with less drop in amplitude.
- (b) Increasing magnitude of rotor-adherent transient currents which peak at synchronous speed.

If periodic short-circuiting of the stator is to be used as a method of effectively reducing the flux in the machine it is clear that the short circuit should not be allowed to persist beyond the initial interval during which the power flow components are approximately linear functions of time. Within this period the stored energy changes approximately linearly, decreasing in the motoring region and increasing in the regenerative mode. As we have seen the length of this safe interval is dependant on the speed of the shaft, being longer for low speeds. Since chopper modulation is of particular interest in later chapters, Figs. (16) and (17) show the measured and calculated results obtained during an experiment where the test motor was subjected to isolated short-circuits which were allowed to persist for various intervals. This was achieved by means of electronic power commutating switches described later in relation to work on chopper modulation. It is observed that a virtually linear run-down and run-up of output torque is achieved if the shorted period is kept short. This is a controllable action and is in all ways preferable to open circuiting as a form of modulation for voltage control, provided that it can be realized in practice.



Short-circuit of energised induction motor, persisting for various intervals.
 100 volt RMS. 1,6 amp RMS. 405 watt. 2,1 Nm. Slip= 5%.
 Supply frequency = 50 Hz.

MEASURED WAVEFORMS.

Fig.(16)

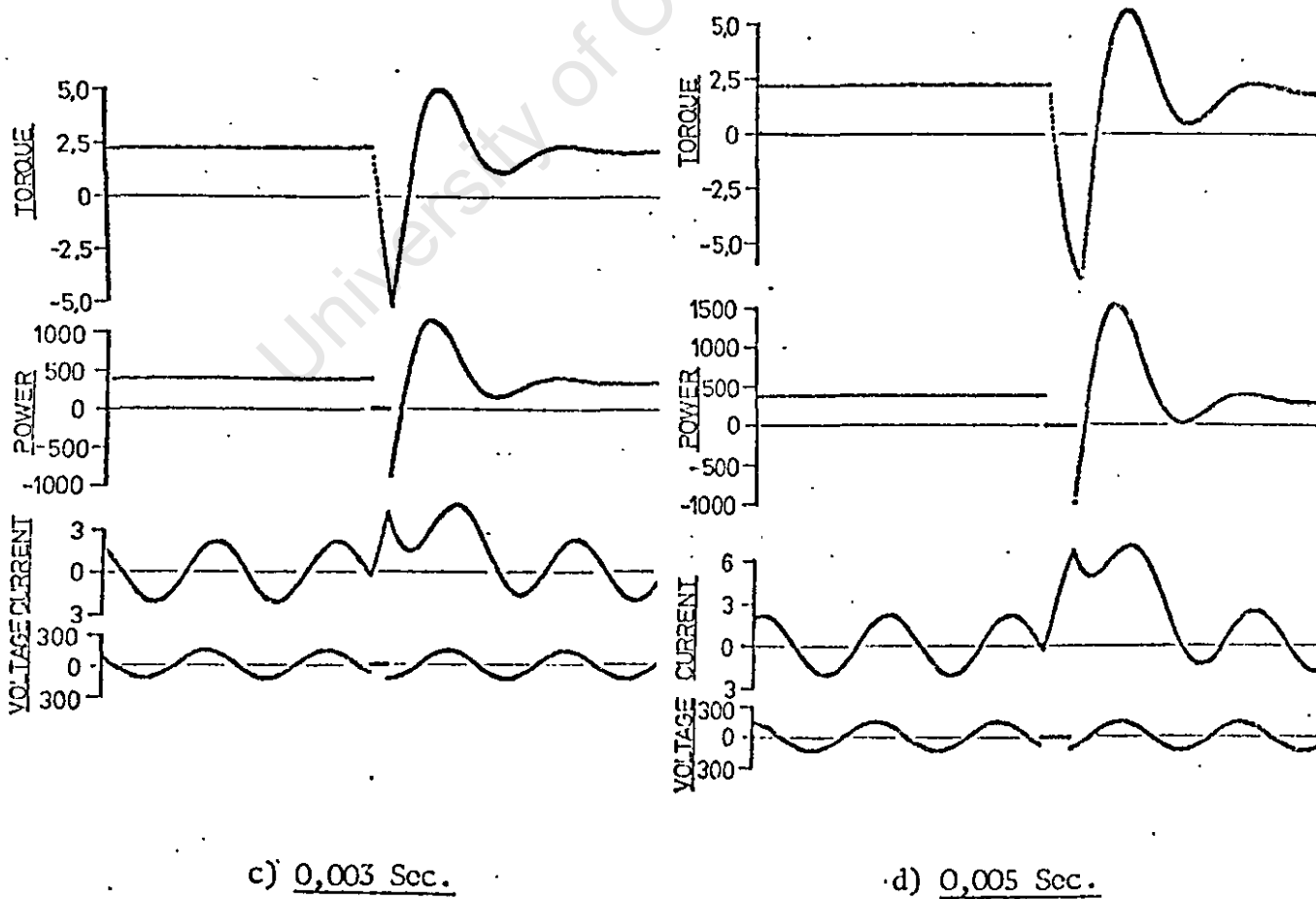
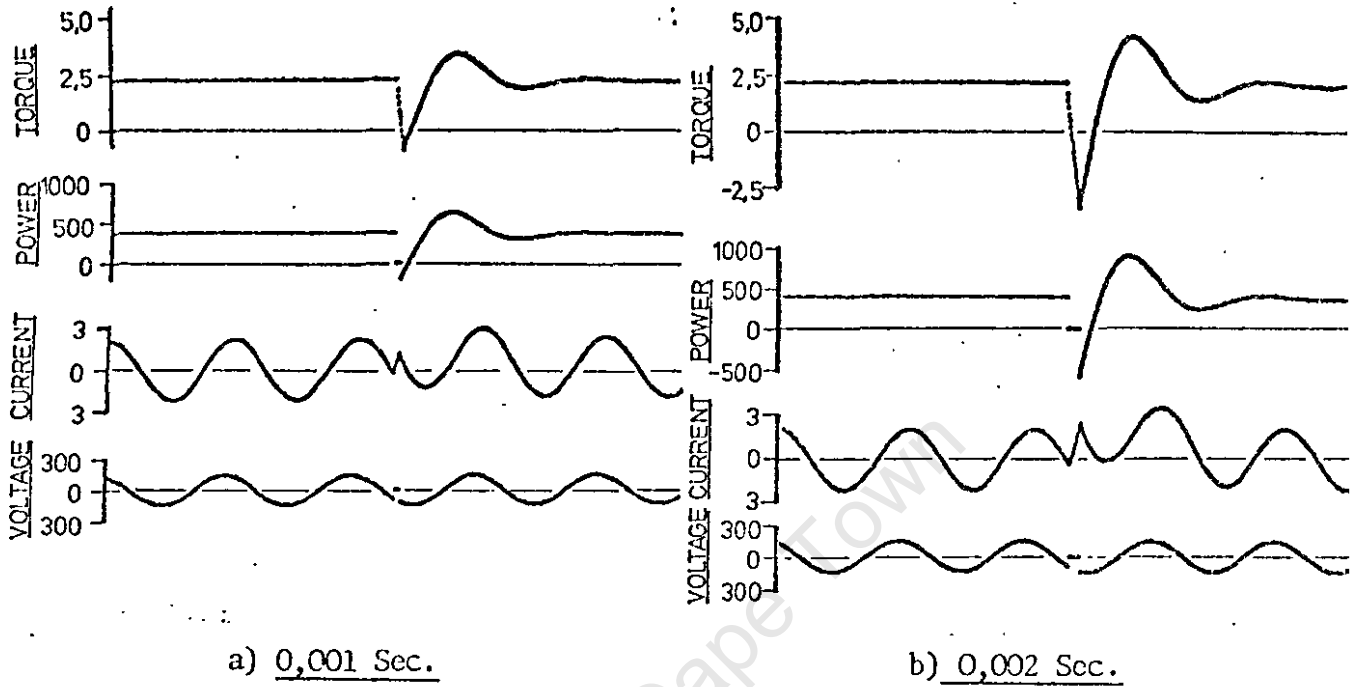


Fig.(17) CALCULATED WAVEFORMS showing short-circuits of similar duration to those measured. 100 volt 50Hz. Slip = 5%.

CHAPTER FOUR

INDUCTION MOTOR RESPONSE WITH CYCLICALLY MODULATED TERMINAL VOLTAGE

In the previous chapters we have developed the full algebraic solution to the induction motor equations, comprising the particular integral or steady state solution and the complementary function. The complementary function has two component terms which are exponentially decaying functions of time and which are excited by any sudden change in the terminal voltage waveform. Such transients are initiated by every switching action performed by a modulator which interfaces the motor and the electrical supply. If the modulating frequency is high the transients cannot die away completely in the interval before the subsequent switching action occurs. As a result there are at all times decaying transient components in the machine's response.

Consider a sudden change in terminal conditions such as might typically occur during normal convertor operation at a particular commutation instant. Let the instantaneous positive sequence currents at $t=0_-$ be

$$\begin{aligned} i_s^+ &= I_{s0-}^+ + i_{s0-}' + i_{s0-}'' \\ i_r^+ &= I_{r0-}^+ + \frac{i_{s0-}'}{C'} + \frac{i_{s0-}''}{C''} \end{aligned}$$

where i_{s0-}' and i_{s0-}'' are the residual transient current components in the stator. Also at $t=0_+$ we can define these same currents as

$$\begin{aligned} i_s^+ &= I_{s0+}^+ + i_s' + i_s'' \\ i_r^+ &= I_{r0+}^+ + \frac{i_s'}{C'} + \frac{i_s''}{C''} \end{aligned}$$

The steady state step is thus

$$I_{s0}^+ = (I_{s0-}^+ - I_{s0+}^+) \quad \text{for the stator}$$

and

$$I_{r0}^+ = (I_{r0-}^+ - I_{r0+}^+) \quad \text{for the rotor.}$$

From Chapter Two we may thus write,

$$i_s' = \frac{C'(I_{so}^+ - C''I_{ro}^+)}{(C' - C'')} + i_{so-}'$$

$$i_s'' = \frac{C''(C'I_{ro}^+ - I_{so}^+)}{(C' - C'')} + i_{so-}''$$

We thus observe that each of the transient current components immediately after the terminal voltage is switched is comprised of the corresponding residual transient prior to switching plus a function of the steady state step at the instant of switching.

In the present work we are confining our attention to convertors where

- (a) the modulation function is cyclic and
- (b) the three phases are switched simultaneously

so that balanced voltage is continuously applied to the motor during the intervals between commutation instants. Under these conditions, if the switching action is maintained and the motor speed is held constant, the system settles into a state which will be termed the QUASI-STEADY state. This is the state which prevails under normal continuous operating conditions and in regard to which we intend to establish performance factors.

4.1 THE QUASI-STEADY STATE

At the start of this work it was not clear what constraints characterize the quasi-steady state. Using zero initial conditions the complete instantaneous positive sequence solution was evaluated stage by stage for several cyclic modulation functions. In each case it was not possible to discern any pattern in the results while the calculations settled. Once settled into the quasi-steady state however, the following common features were observed:-

- (a) *The nett input and output power waveforms are periodic, being repeated during every modulation cycle.*

- (b) *The instantaneous positive sequence current at any point during a modulation cycle is the same as at the corresponding point in the previous cycle except for a constant phase angle.*

This phase angle is dependent on the supply frequency and the particular modulation function and is equal to the phase shift between a given point on the terminal voltage waveform and the corresponding point during the following modulation cycle. This angle may be defined as the MODULATION ANGLE. In general, if the supply voltages have angular frequency ω_e and the length of the modulation cycle is τ seconds then the modulation angle is

$$\theta = \omega_e \tau$$

We can thus state the basic constraint which typifies the quasi-steady state operation:-

$$i_s^+(t) = i_s^+(t - \tau) \cdot e^{j\theta}$$

and, considering the steady state and transient component currents separately we obtain:-

$$I_s^+(t) = I_s^+(t - \tau) \cdot e^{j\theta}$$

$$i_s'(t) = i_s'(t - \tau) \cdot e^{j\theta}$$

$$i_s''(t) = i_s''(t - \tau) \cdot e^{j\theta}$$

This result is illustrated by means of the polar plot for a specific case in Fig. (18)

With the aid of this constraint the instantaneous positive sequence solution for the motor currents may be determined for quasi-steady state conditions.

4.2 THE MODULATION CYCLE

As discussed previously, the modulator performs a series of supply connections to the motor. This series is cyclic insofar as a certain sequence of supply connections is exactly repeated. During consecutive sequences, the applied voltage is not identical but phase shifted by an angle $\omega_e \tau$. It may happen, however, that a more basic cycle is discernible

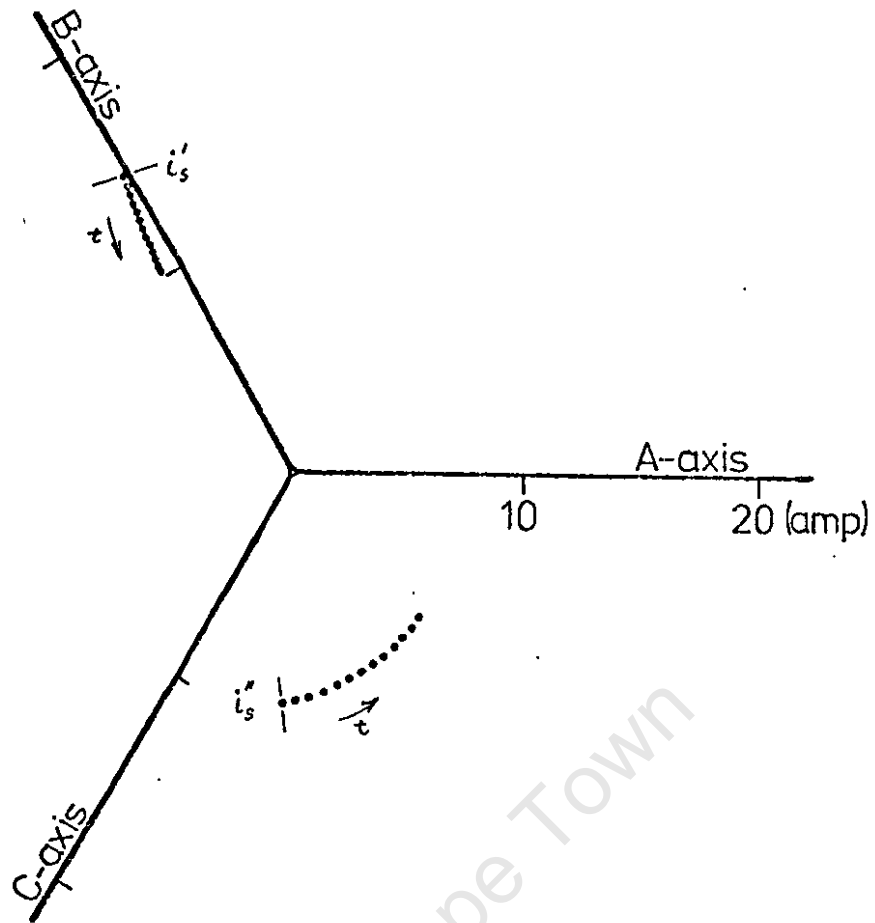


Fig.18

- a) CHOPPER MODULATED INDUCTION MOTOR, SLIP = 10%, $T_{on} = 0,5 \times T_{total} = 3$ MILLISEC.
 $V_s = 220$ VOLT RMS 50 HZ.

CURRENT COMPONENTS DURING A SINGLE SHORT-CIRCUIT INTERVAL IN THE QUASI-STEADY STATE. POINTS PLOTTED AT $\frac{1}{4}$ MILLISEC. INTERVALS.

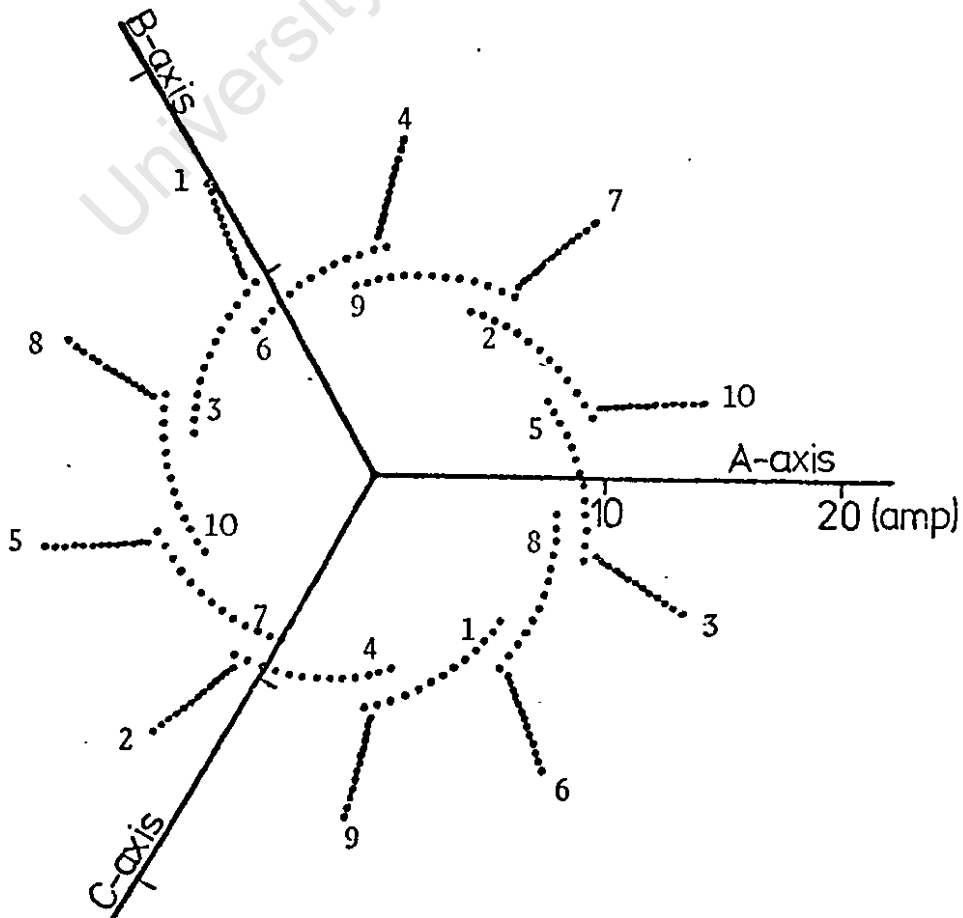


Fig.18

- b) LOCUS OF CURRENT COMPONENTS DURING 10 SUCCESSIVE S.C.-INTERVALS AS IN a).

within the full switching sequence of the modulator. For example, the applied voltage during each stage of the sequence may be indistinguishable from the voltage during the previous stage except for a constant phase-shift. If this holds true throughout the waveform, then the single stage should be identified as the modulation cycle, and the modulation angle should be re-defined as the phase shift from stage to stage. As a general rule the shortest modulation cycle should always be identified in this way rather than simply adopting the full switching sequence without further thought. The two approaches are equally valid but a shorter modulation cycle always involves less calculation for solution. If there is any doubt in identifying the basic cycle of a particular modulator, then a quick examination of the measured or simulated torque or terminal power waveforms is recommended, since these waveforms are periodic with the shortest period equal to that of the basic modulation cycle.

Fig. (19) illustrates several typical output voltage waveforms and indicates their basic modulation cycles.

It is important to note that the voltage waveforms do not in general repeat themselves during consecutive cycles although this does occur in certain circumstances which will be treated later as special cases. If θ_m is the basic modulation angle for the motor terminal voltage and the length of the basic modulation cycle is τ' seconds then, as before, the following constraint applies to the motor current

$$i_s^+(t) = i_s^+(t - \tau') e^{j\theta_m}$$

So that θ_m is likewise the basic modulation angle of the motor current. However, when the positive sequence supply current is deduced in terms of the modulating function it is always found to have a basic modulation cycle of length τ' secs. with basic modulation angle $\theta_{supply} = \omega_e \tau'$.

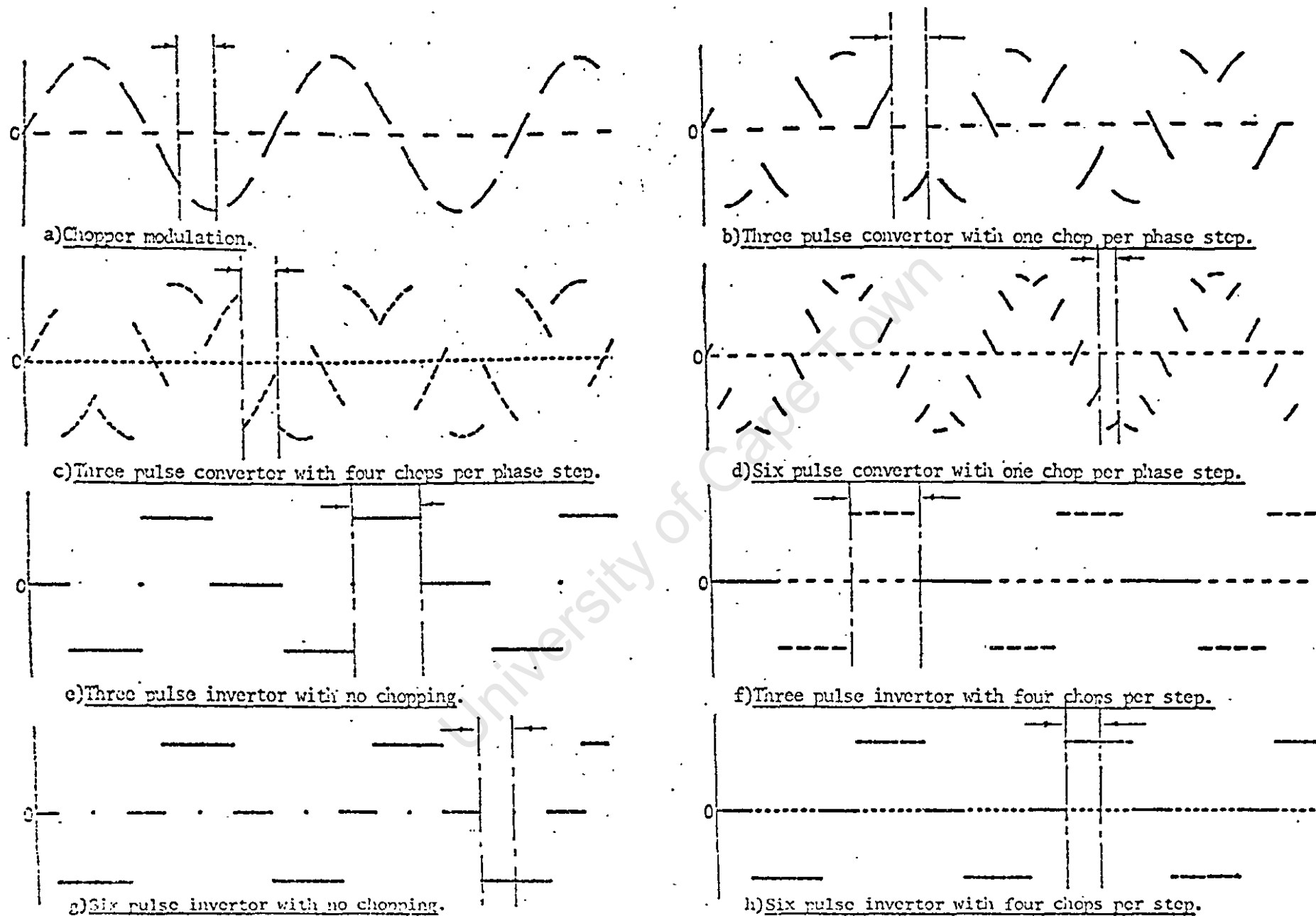


Fig.19 Illustration of several typical converter output voltage waveforms.

Thus

$$i_{supply}^+(t) = i_{supply}^+(t - \tau') e^{j\omega_e \tau'}$$

In many instances we find $\theta_{supply} = \theta_m$ where the basic cycle is in fact the full switching sequence. If $\theta_m \neq \theta_{supply}$ it is an indication that the modulation produces a frequency conversion. This will be more clearly seen in the following chapter.

In the following work the terms "modulation cycle" and "modulation angle" will be understood to refer to the basic cycle and the basic angle respectively.

Once the instantaneous positive sequence solution is determined over a single modulation cycle it is fully defined in terms of the quasi-steady state constraint, and may be used to deduce the required performance factors. In order to illustrate the scope of this technique, we will firstly consider a very broadly defined modulation cycle. Subsequently the solution will be simplified to include only a certain class of modulators which is of particular interest.

4.3 Solutions for a general modulation cycle

Consider a case where the motor is successively connected to N supply configurations labelled by the ordinal $k = 1, 2, \dots, N$ and remains connected to each for a period τ_k . Let the k th. commutation instant be the one which ends the k th. stage and let $k = 0$ denote the commutation instant at the start of the first stage. Let the modulation angle be θ . Then for each commutation instant define the following :-

$$i_k' = \frac{C' \{ I_{sk}^+ - C'' I_{rk}^+ \}}{(C' - C'')}$$

$$i_k'' = \frac{C'' \{ C' I_{rk}^+ - I_{sk}^+ \}}{(C' - C'')}$$

where

$$\left. \begin{aligned} I_{sk}^+ &= I_{sk-}^+ - I_{sk+}^+ \\ I_{rk}^+ &= I_{rk-}^+ - I_{rk+}^+ \end{aligned} \right\} \quad \begin{array}{l} \text{Steady state step at the } k\text{th.} \\ \text{commutation instant.} \end{array}$$

and let the instantaneous positive sequence current immediately after the k th. commutation instant be

$$i_{sk}^+ = I_{sk+}^+ + i_{sk}' + i_{sk}''$$

Then in accordance with our earlier observations we can write :-

$$\begin{aligned} i_{sN}' &= \left\{ i_{s0}' e^{\alpha'(t_1+t_2+\dots+t_N)} + i_1' e^{\alpha'(t_2+t_3+\dots+t_N)} + \dots + i_{(N-1)}' e^{\alpha' t_N} + i_N' \right\} \\ i_{sN}'' &= \left\{ i_{s0}'' e^{\alpha''(t_1+t_2+\dots+t_N)} + i_1'' e^{\alpha''(t_2+t_3+\dots+t_N)} + \dots + i_{(N-1)}'' e^{\alpha'' t_N} + i_N'' \right\} \end{aligned}$$

From the quasi-steady state constraint we have

$$\left. \begin{aligned} i_{sN}' &= i_{s0}' e^{j\theta} \\ i_{sN}'' &= i_{s0}'' e^{j\theta} \end{aligned} \right\}$$

If we now define $T_k = [t_1 + t_2 + \dots + t_k]$ then the above equations become :-

$$i_{s0}' = \left[\frac{\sum_{k=1}^N i_k' \cdot e^{[\alpha'(T_N - T_k) - j\theta]}}{(1 - e^{[\alpha' T_N - j\theta]})} \right]$$

$$i_{s0}'' = \left[\frac{\sum_{k=1}^N i_k'' \cdot e^{[\alpha''(T_N - T_k) - j\theta]}}{(1 - e^{[\alpha'' T_N - j\theta]})} \right]$$

In the same way, for the k th commutation instant we can write :-

$$i'_{sk} = \frac{\left\{ \sum_{m=1}^k i'_m \cdot e^{\alpha'(T_k - T_m)} \right\} + \left\{ e^{(\alpha'T_k - j\theta)} \cdot \sum_{m=(k+1)}^N i'_m \cdot e^{\alpha'(T_N - T_m)} \right\}}{\left\{ 1 - e^{[\alpha'T_N - j\theta]} \right\}}$$

and similarly for i''_{sk} where the primed variables in the above expression are replaced with double primes.

This solution is completely general and the format is necessarily broad in order to accommodate any specified modulation. If $t = 0$ at the start of the k th stage of this cycle, the complete solution for the instantaneous positive sequence stator current during the k th stage is

$$i_s^+ = I_{sk}^+ e^{j\omega_e t} + i'_{sk} e^{\alpha't} + i''_{sk} e^{\alpha''t}$$

We may now apply this general solution to some specific forms of modulation.

4.4 Solution for a specific class of modulators

The general modulation cycle discussed above may be more closely specified to incorporate only a specific class of modulation which still includes a wide range of convertors as well as the particular modulation at which this work was originally directed.

The modulation cycle in question comprises N stages where N is an even number. For even values of k , $t_k = \tau_a$ and for odd values of k , $t_k = \tau_b$. During those intervals where k is odd, the same supply connection prevails. During the stages where k is even the applied voltage is zero.

In the N th commutation instant at the end of the cycle a discrete phase shift of $-\phi$ radians is introduced so that the modulation angle is

$$\theta_m = \left[\frac{\omega_e N \tau}{2} - \phi \right] \quad \text{where } \tau = (\tau_a + \tau_b)$$

In effect therefore, the modulator alternately makes a certain supply connection and then short circuits the machine. This action is exactly repeated N times after which a new supply connection is made where the voltages are shifted by $-\phi$ radians in phase relative to the previous supply connection.

This basic specification of the modulation cycle is still comparatively flexible. For example if no chopping action was required in a particular instance, the short-circuits could be eliminated by setting $\tau_a = 0$ in the solution which is obtained; or if no phase jump is required, $\phi = 0$ and $N = 2$. Since the supply voltages may have any frequency including zero and any number of phases, it is clear that a considerable number of practical convertor modulation functions can be defined within the scope of this modulation cycle. In general these would be expected to be forced commutation systems in practice.

The output voltage waveforms shown in Fig (19) are all generated by this class of modulation.

If we note that

$$i'_k = \left[i'_0 \cdot e^{j\omega_e k \tau} \right] \quad \text{For even values of } k$$

$$\text{and } i'_k = \left[i'_0 \cdot e^{j\omega_e \left\{ \left(\frac{k-1}{2} \right) \tau + \tau_b \right\}} \right] \quad \text{For odd values of } k$$

and also define

$$f' = (\alpha' - j\omega_e)$$

$$f'' = (\alpha'' - j\omega_e)$$

then using the general result obtained previously we may formulate a quasi-steady state solution for this class of modulation :-

(a) For odd values of k : (i.e. as supply is restored)

$$i'_{sk} = i'_k \frac{(1 - e^{f'v_b}) \left[\left\{ \sum_{m=0}^{(\frac{k-1}{2})} e^{mf'v} \right\} + \left\{ e^{j\phi} \sum_{m=(\frac{k+1}{2})}^{(\frac{N-2}{2})} e^{mf'v} \right\} \right]}{[1 - e^{(f'\frac{N}{2}v + j\phi)]}$$

and similarly for i''_{sk}

(b) For even values of k : (i.e. at start of short-circuit)

$$i'_{sk} = i'_k \frac{(1 - e^{f'v_a}) \left[\left\{ \sum_{m=0}^{(\frac{k}{2}-1)} e^{mf'v} \right\} + \left\{ e^{j\phi} \sum_{m=(\frac{k}{2}+1)}^{(\frac{N-2}{2})} e^{mf'v} \right\} \right] + (1 - e^{(f'v_a + j\phi)}) e^{\frac{k}{2}f'v}}{[1 - e^{(f'\frac{N}{2}v + j\phi)]}$$

and similarly for i''_{sk}

It is thus seen that the transient current components immediately after a commutation instant are given by i'_k and i''_k , each modified by a complex factor which is a function of the parameters of the modulation and of the machine. The functions i'_k and i''_k , as defined earlier, represent the transient stator current components which would be present immediately after the commutation if there were absolutely no residual transients prior to commutating. It is clear therefore that for odd values of k , the complex factor approaches 1 as v_a (and hence v) becomes large. Similarly for even values of k , the complex factor approaches 1 as v_b (and hence v) becomes large.

4.5 Particular examples

The solution obtained above is still broadly defined and it will now be of interest to show how this result may be applied in specific cases.

4.5.1. Frequency converter with no chopper modulation

This constitutes the simplest conceivable modulation cycle.

It consists of only a single stage during which a particular supply connection prevails. At the end of each stage the supply is immediately reconnected to provide a discrete phase step of $-\phi$ radians in the applied voltage. (This action is the basis of the frequency converter which is considered in detail later). The cycle may be fitted to the general result by setting :-

$$\left. \begin{aligned} N &= 2 \\ \tau_a &= 0 \\ \tau &= \tau_b \end{aligned} \right\}$$

Hence

$$i'_{so} = i'_o \left[\frac{1}{1 - e^{(f'\tau_b + j\phi)}} \right]$$

and similarly for i''_{so}

If $t = 0$ at the start of a cycle then during that cycle the instantaneous positive sequence stator current is

$$i_s^+ = I_{so}^+ e^{j\omega_e t} + i'_{so} e^{\alpha' t} + i''_{so} e^{\alpha'' t}$$

Note that i'_{s1} , i''_{s1} have no meaning since we have stipulated $\tau_a = 0$. Also $i'_{so} \rightarrow i'_o$ as τ_b increases.

4.5.2 Chopper modulation of the normal supply voltage

In this case no phase step is required so that we can set

$$\left. \begin{aligned} N &= 2 \\ \phi &= 0 \end{aligned} \right\} \quad \text{in the general result.}$$

Hence

$$i'_{s0} = i'_0 \left[\frac{(1 - e^{F'x_a})}{(1 - e^{F'\tau})} \right]$$

and similarly for i''_{s0}

and

$$i'_{s1} = i'_1 \left[\frac{(1 - e^{F'x_b})}{(1 - e^{F'\tau})} \right]$$

and similarly for i''_{s1}

If $t = 0$ at the start of each stage, the positive sequence stator current solution can be written as follows :-

(a) during the ON-stage :-

$$i_s^+ = I_{s0}^+ e^{j\omega_e t} + i'_{s0} e^{\alpha' t} + i''_{s0} e^{\alpha'' t}$$

(b) during the short-circuit :-

$$i_s^+ = i'_{s1} e^{\alpha' t} + i''_{s1} e^{\alpha'' t}$$

4.5.3 A frequency convertor with two chops per phase step

This is an example of a multistage modulation cycle. The motor is connected to the supply and then shorted. The action is repeated once and subsequently the motor is connected to a new supply voltage phase shifted by $-\phi$ rad.

Thus $N = 4$

Hence

$$i'_{s0} = i'_0 \cdot \left[\frac{1 - e^{(F'\tau_a + j\phi)} + e^{(F'\tau + j\phi)} - e^{(F'(\tau + \tau_a) + j\phi)}}{(1 - e^{(2F'\tau + j\phi)})} \right]$$

$$i'_{s1} = i'_1 \cdot \left[\frac{1 - e^{F'\tau_b} + e^{(F'\tau + j\phi)} - e^{(F'(\tau + \tau_b) + j\phi)}}{(1 - e^{(2F'\tau + j\phi)})} \right]$$

$$i'_{s2} = i'_2 \cdot \left[\frac{1 - e^{F'\tau_a} + e^{F'\tau} - e^{(F'(\tau + \tau_a) + j\phi)}}{(1 - e^{(2F'\tau + j\phi)})} \right]$$

$$i'_{s3} = i'_3 \cdot \left[\frac{1 - e^{F'\tau_b} + e^{F'\tau} - e^{F'(\tau + \tau_b)}}{(1 - e^{(2F'\tau + j\phi)})} \right]$$

and similarly for i''_{s0} , i''_{s1} , i''_{s2} , i''_{s3} .

CHAPTER FIVE

INTERPRETATION OF SYMMETRICAL COMPONENT SOLUTION FOR MODULATOR-FED INDUCTION MOTOR

In order to assess the viability of any convertor-fed induction motor it will generally be necessary to evaluate certain established performance factors for quasi-steady state operation. Before this can be done it is essential to interpret the instantaneous-positive sequence current solutions which have been obtained in such a way as to find :-

- (a) RMS of current waveforms.
- (b) Frequency spectrum of current waveforms.
- (c) Average input and output power.
- (d) Frequency spectrum of torque waveform.

In this chapter we will again consider a very broadly defined modulation and formulate the technique of obtaining the above information from the symmetrical component solution. Attention here will be drawn only to very general features of the results, and in subsequent chapters attention will be focussed on particular modulators and their specific characteristics.

5.1 The Electrical Supply

Although the motor being considered is a three-phase device, the electrical supply might in principle comprise any number of alternative paths each having a different e.m.f. It is reasonable to assume, however, that the supply consists of a number of balanced three-phase voltage sets, each of which has the same angular frequency and phase sequence, and three distinct current paths associated with it. If the voltage is zero in a particular current path, then it constitutes a short-circuit when it is connected to the motor. Where the supply is simply a balanced set of $3n$ -phase voltages (where $n = 2, 3 \dots$) it is viewed as n balanced 3-phase supplies which are phase-shifted by $\frac{2\pi}{n}$ radians relative to each other.

NOTE : If the supply frequency is zero then a balanced three-phase voltage set would correspond to

$$|V_s| \cos(\psi) \quad ; \quad |V_s| \cos\left(\psi - \frac{2\pi}{3}\right) \quad ; \quad |V_s| \cos\left(\psi + \frac{2\pi}{3}\right)$$

If $\psi = \frac{\pi}{2}$ for example these would be D.C.

rails of level 0 , $\frac{+|V_s|}{2}$ and $\frac{-|V_s|}{2}$ respectively.

5.2 Current in a modulated induction motor system

The instantaneous positive sequence motor current solution during one modulation cycle is obtained as discussed in Chapter 4. By virtue of the quasi-steady state constraint the complete waveform is thus a fully defined function of time.

In general, only one three-phase supply path may be connected to the motor during a particular stage of the modulation cycle. There are only six possible ways of making such a connection, and only three ways of making it so as to produce the same phase sequence excitation. If the positive sequence motor current during this stage is $i_s^+(t)$ and the instantaneous positive sequence component of current in the three-phase supply path is $i_{supply}^+(t)$ then

$$i_{supply}^+(t) = i_s^+(t) \cdot e^{j\psi}$$

where $\psi = 0, \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$ depending on the connection.

More specifically, $\psi = 0$ if the reference phase of the motor is connected to the reference phase of the supply path.

$\psi = -\frac{2\pi}{3}$ if B motor phase $\rightarrow$ reference supply phase.

$\psi = \frac{2\pi}{3}$ if C motor phase $\rightarrow$ reference supply phase.

During the subsequent stage of the modulation cycle we may find that either

- (a) the supply considered above remains connected to the motor but the connection is switched, thus changing the value of ψ ,

or

- (b) the motor current is commutated away into an alternative 3-phase supply path, in which case $i_{supply}^+(t) = 0$

In a particular three-phase supply path we thus see that the current waveforms are comprised of stages between commutation instants which have the same duration as the stages of the motor current and voltage waveforms. The current during these stages is conveniently defined in terms of the instantaneous positive sequence motor current and the logic of the modulator action. In the quasi-steady state we find that the instantaneous positive sequence current in each three phase supply path repeats itself with a set phase shift during every modulation cycle. This is a result of the cyclic constraint governing the motor current in this state. However, the modulation angle may not be the same for the supply current as for the motor current. We can write the constraint as

$$i_{supply}^+(t) = i_{supply}^+(t - \tau) \cdot e^{j\theta_{supply}}$$

where τ is the duration of the modulation cycle and θ_{supply} is the particular supply modulation angle.

NOTE : It will be seen from the spectral analysis later that if the supply angular frequency is ω_e then in order to draw nett average power the modulation angle of the supply current must be $\theta_{supply} = \omega_e \tau$. Otherwise there would be no supply frequency components in the supply current waveform.

It must be emphasized that this approach to the analysis enables the various supply currents to be investigated with particular ease in comparison with other methods.

Since the various currents in the system are similarly constituted, it will not be necessary to specify for the purpose of analysis whether a *motor* or *supply* current waveform is being considered. Instead a general current waveform will be assumed, having N stages of total duration τ and with the cyclic constraint :-

$$i_s^+(t) = i_s^+(t - \tau) \cdot e^{j\theta}$$

5.3 Real Instantaneous Phase Currents

Consider a particular stage of the modulation cycle where $t = 0$ at the start of the stage. Then if

$$i_s^+(t) = I_s^+ e^{j\omega_e t} + i_s' e^{\alpha' t} + i_s'' e^{\alpha'' t}$$

during this stage, the reference phase current is given by

$$i_s^a = 2 \operatorname{Re} (i_s^+) = \left\{ \begin{array}{l} |I_s| \cos(\omega_e t + \theta_s) \\ + |i'| e^{-\rho' t} \cos(\nu' t + \theta') \\ + |i''| e^{-\rho'' t} \cos(\nu'' t + \theta'') \end{array} \right\}$$

To illustrate how the phase current is constituted as a function of time we have chosen the case of the test motor with chopper modulated terminal voltage. This is shown in Fig (20) for the case where the modulator short-circuits the motor for 3 m.secs. then restores the usual supply for 3 m. secs. alternately, under quasi-steady state conditions. Note that this example is identical to that for which a polar plot of i_s^+ was shown in the previous chapter, 4.1.

i_a = "Steady state" component of a-phase current.
 i'_a = "Transient" component of a-phase current.
 i''_a = "Transient" component of a-phase current.

Fig. 20

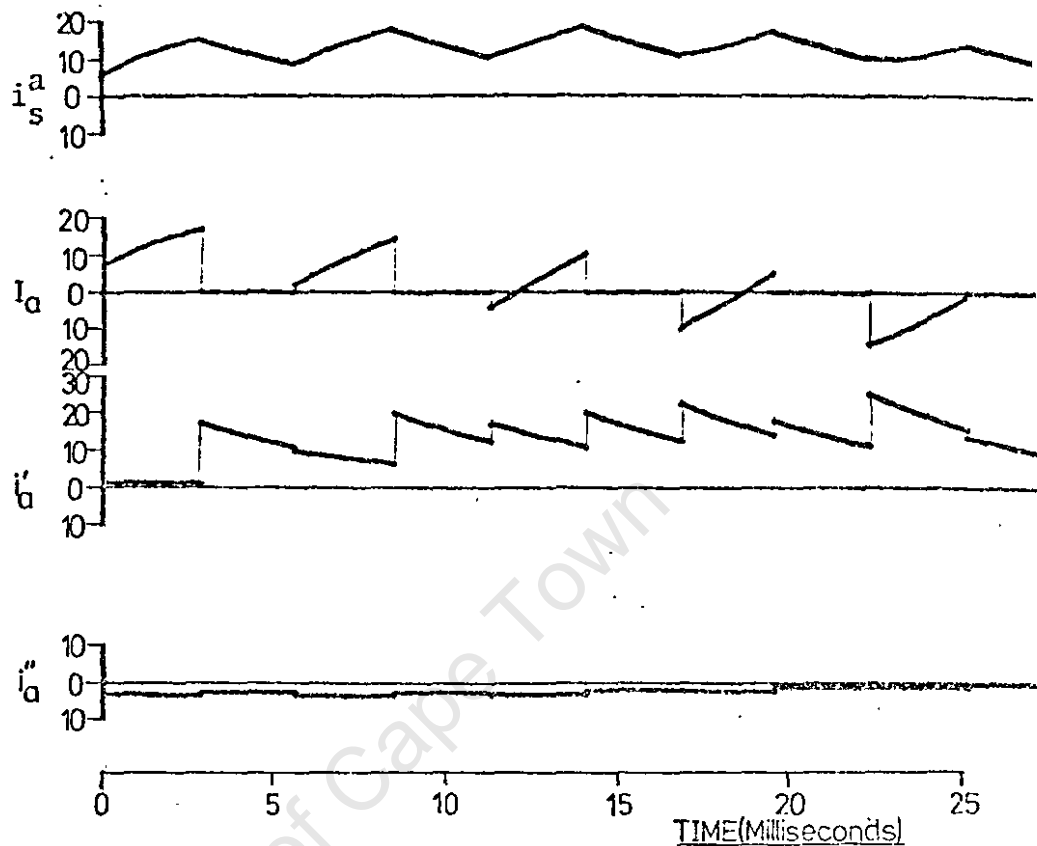


Fig. 21

Supply voltage= 220 volt RMS 50 Hz. Slip frequency= 2,5 Hz.
 $T(on)/T(total)=0,5$ Phase stepping frequency=180 Hz. $f_{out}=10$ Hz.

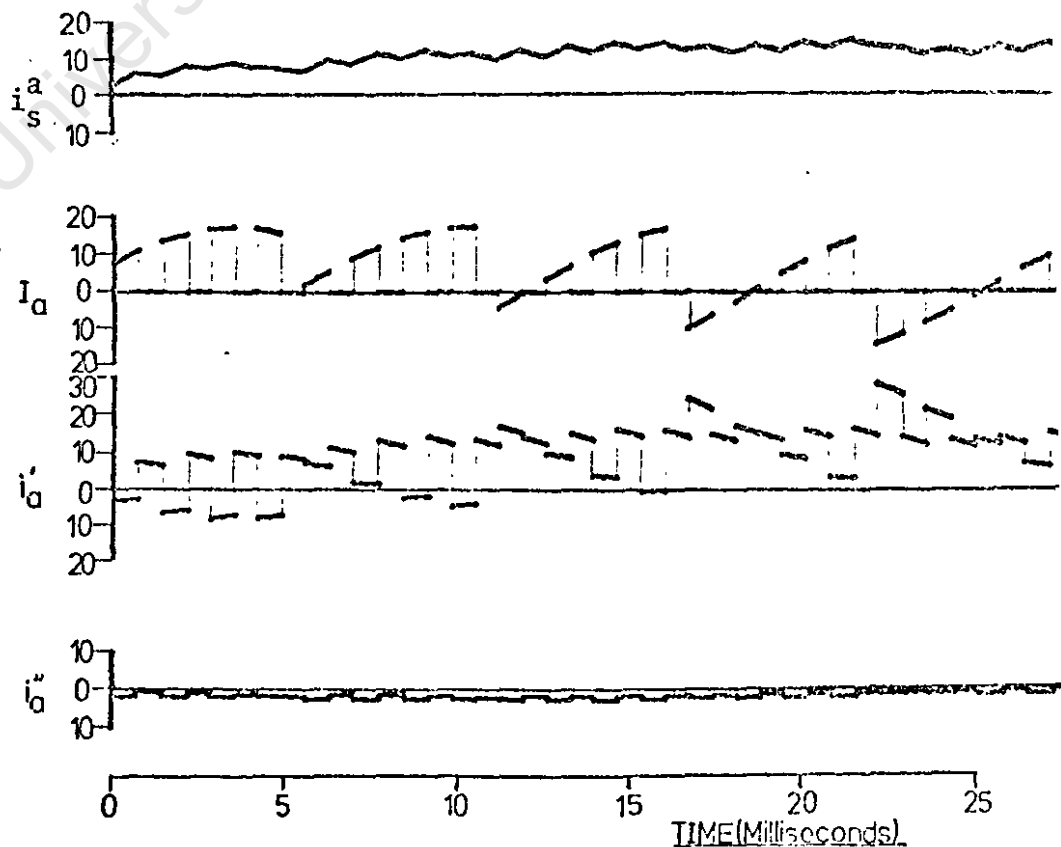


Fig. 22

Supply voltage= 220 volt RMS 50 Hz. Slip frequency= 2,5 Hz.
 $T(on)/T(total)=0,5$ Phase stepping frequency=180 Hz. (4 chops/step)

In Figs. (21) and (22) we show the components of the phase current in the same motor supplied through two different forms of the phase stepping frequency convertor analysed in the previous chapter, 4.5.1 .

5.4 RMS value of phase current waveforms

Consider a general current waveform and let $t = 0$ at the start of the 0th modulation cycle where successive modulation cycles are denoted by the ordinal $Q = 0, 1, 2, \dots$. Let the k th stage of each cycle have length t_k secs.

Then if we define $t'_{Qk} = (t - Q\tau)$ (where τ = period of cycle)

$$\text{and } \psi_Q = Q \cdot \theta$$

the phase current in the reference phase during the k th stage of the Q th modulation cycle only can be written as :-

$$i_{Qk} = \left\{ \begin{aligned} &|I_s| \cos(\omega_e t'_{Qk} + \psi_Q + \theta_s) \\ &+ |i'| \cdot e^{-p' t'_{Qk}} \cos(\gamma' t'_{Qk} + \psi_Q + \theta') \\ &+ |i''| \cdot e^{-p'' t'_{Qk}} \cos(\gamma'' t'_{Qk} + \psi_Q + \theta'') \end{aligned} \right\}$$

For convenience we will assume here that the angle θ is such that $K\theta = G \cdot 2\pi$ where K and G are positive integers. This implies that the waveforms repeat themselves exactly after K cycles of the modulation.

NOTE : That $K = 1$ and $K = 2$ are special cases which are treated later. (see 5.7)

It will be shown that this assumption is not a fundamental limitation on the analysis and the waveforms need not be repetitive at all.

In order to obtain the overall RMS of the current waveform, we must evaluate :-

$$i_s^{\alpha} (RMS) = \sqrt{\frac{1}{K \cdot \tau} \int_0^{K \cdot \tau} (i_s^{\alpha})^2 \cdot dt}$$

The instantaneous square of the current during the k th. stage of the Q th. cycle is :-

$$\begin{aligned}
 i_{Qk}^2 = & |I_s|^2 \cos^2(\omega_e t'_{Q1} + \psi_Q + \theta_s) \\
 & + |i'|^2 \cdot e^{-2\rho' t'_{Q1}} \cos^2(\nu' t'_{Q1} + \psi_Q + \theta') \\
 & + |i''|^2 \cdot e^{-2\rho'' t'_{Q1}} \cos^2(\nu'' t'_{Q1} + \psi_Q + \theta'') \\
 & + 2|I_s| \cdot |i'| \cdot e^{-\rho' t'_{Q1}} \cos(\omega_e t'_{Q1} + \psi_Q + \theta_s) \cdot \cos(\nu' t'_{Q1} + \psi_Q + \theta') \\
 & + 2|I_s| \cdot |i''| \cdot e^{-\rho'' t'_{Q1}} \cos(\omega_e t'_{Q1} + \psi_Q + \theta_s) \cdot \cos(\nu'' t'_{Q1} + \psi_Q + \theta'') \\
 & + 2|i'| \cdot |i''| \cdot e^{-(\rho' + \rho'') t'_{Q1}} \cos(\nu' t'_{Q1} + \psi_Q + \theta') \cdot \cos(\nu'' t'_{Q1} + \psi_Q + \theta'')
 \end{aligned}$$

Then,

$$i_{S(RMS)}^2 = \sqrt{\frac{1}{K \cdot N} \sum_{k=1}^N \sum_{Q=0}^{(K-1)} \int_0^{t_k} i_{Qk}^2 \cdot dt'_{Q1}}$$

consider first the function,

$$\sum_{Q=0}^{(K-1)} \int_0^{t_k} i_{Qk}^2 \cdot dt'_{Q1}$$

In this case it is permissible to reverse the order of integration and summation so that the same value is obtained if we consider,

$$\int_0^{t_k} \sum_{Q=0}^{(K-1)} i_{Qk}^2 \cdot dt'_{Q1}$$

Except where $K = 1$ or 2 (treated later as special cases (5.7)) we can write :-

$$\sum_{Q=0}^{(K-1)} i_{Qk}^2 = \left[\frac{K |I_s|^2}{2} + \frac{K |i'|^2}{2} e^{-2\rho' t'_{Q1}} + \frac{K |i''|^2}{2} e^{-2\rho'' t'_{Q1}} + K |i'| |i''| e^{-(\rho' + \rho'') t'_{Q1}} \cos((\nu' - \nu'') t'_{Q1} + (\theta'_s - \theta''_s)) \right. \\ \left. + K |I_s| |i'| e^{-\rho' t'_{Q1}} \cos((\nu' - \omega_c) t'_{Q1} + (\theta'_s - \theta_s)) \right. \\ \left. + K |I_s| |i''| e^{-\rho'' t'_{Q1}} \cos((\nu'' - \omega_c) t'_{Q1} + (\theta''_s - \theta_s)) \right]$$

If we now use the following identity:

$$\int e^{-\rho t} \cos(at) dt = \frac{\rho e^{-\rho t}}{\rho^2 + a^2} \left\{ \frac{a}{\rho} \sin(at) - \cos(at) \right\}$$

Then:

$$\int_0^{t_k} \sum_{Q=0}^{(K-1)} i_{Qk}^2 dt'_{Q1} = \left[\frac{K |I_s|^2}{2} t_k + \frac{K |i'|^2}{4\rho'} \left\{ 1 - e^{-2\rho' t_k} \right\} + \frac{K |i''|^2}{4\rho''} \left\{ 1 - e^{-2\rho'' t_k} \right\} \right. \\ \left. + \frac{K |i'| |i''| (\rho' + \rho'')}{((\rho' + \rho'')^2 + (\nu' - \nu'')^2)} \cdot \left\{ e^{-(\rho' + \rho'') t_k} \left\{ \frac{(\nu' - \nu'')}{(\rho' + \rho'')} \sin((\nu' - \nu'') t_k + (\theta'_s - \theta''_s)) - \cos((\nu' - \nu'') t_k + (\theta'_s - \theta''_s)) \right\} \right. \right. \\ \left. \left. - \left\{ \frac{(\nu' - \nu'')}{(\rho' + \rho'')} \sin(\theta'_s - \theta''_s) - \cos(\theta'_s - \theta''_s) \right\} \right\} \right. \\ \left. + \frac{K |I_s| |i'| \rho'}{(\rho'^2 + (\nu' - \omega_c)^2)} \cdot \left\{ e^{-\rho' t_k} \cdot \left\{ \frac{(\nu' - \omega_c)}{\rho'} \sin((\nu' - \omega_c) t_k + (\theta'_s - \theta_s)) - \cos((\nu' - \omega_c) t_k + (\theta'_s - \theta_s)) \right\} \right. \right. \\ \left. \left. - \left\{ \frac{(\nu' - \omega_c)}{\rho'} \sin(\theta'_s - \theta_s) - \cos(\theta'_s - \theta_s) \right\} \right\} \right. \\ \left. + \frac{K |I_s| |i''| \rho''}{(\rho''^2 + (\nu'' - \omega_c)^2)} \cdot \left\{ e^{-\rho'' t_k} \cdot \left\{ \frac{(\nu'' - \omega_c)}{\rho''} \sin((\nu'' - \omega_c) t_k + (\theta''_s - \theta_s)) - \cos((\nu'' - \omega_c) t_k + (\theta''_s - \theta_s)) \right\} \right. \right. \\ \left. \left. - \left\{ \frac{(\nu'' - \omega_c)}{\rho''} \sin(\theta''_s - \theta_s) - \cos(\theta''_s - \theta_s) \right\} \right\} \right]$$

If we consider

$$\overline{i_k^2} = \frac{1}{K\pi} \int_0^{z_k^{(K-1)}} \sum_{Q=0} i_{Qk}^2 dt_{Q1}$$

we obtain the nett contribution of all the k th. stages to the mean square of the overall waveform. Note that K cancels out of this expression.

The overall RMS of the waveform is then :-

$$i_{S(RMS)} = \sqrt{\sum_{k=1}^N \overline{i_k^2}}$$

This analysis has been thoroughly checked against values of RMS current obtained by means of a numerical simulation technique, and has been found to be correct in all cases. Although the formulation of the result is lengthy, the arithmetic involved is trivial and lends itself to computer methods. Typically, the computation time required is fractional in contrast to the time which would be involved in obtaining the same result by accurate numerical simulation.

Note that the RMS of the three phase currents are identical although the waveforms will usually be different. This is only true for $K > 2$ and the situation corresponding to $K = 1$ and $K = 2$ will be treated later as special cases. (5.7)

5.5 Spectrum Analysis of Current Waveforms

As in the previous section, consider the k th. stage of the Q th. modulation cycle, during which

$$i_{Qk} = \left\{ \begin{aligned} &|I_s| \cos(\omega_e(t - Q\tau) + \theta_s + Q\theta) \\ &+ |i'| \cdot e^{-\rho'(t - Q\tau)} \cos(\nu'(t - Q\tau) + \theta' + Q\theta) \\ &+ |i''| \cdot e^{-\rho''(t - Q\tau)} \cos(\nu''(t - Q\tau) + \theta'' + Q\theta) \end{aligned} \right\}$$

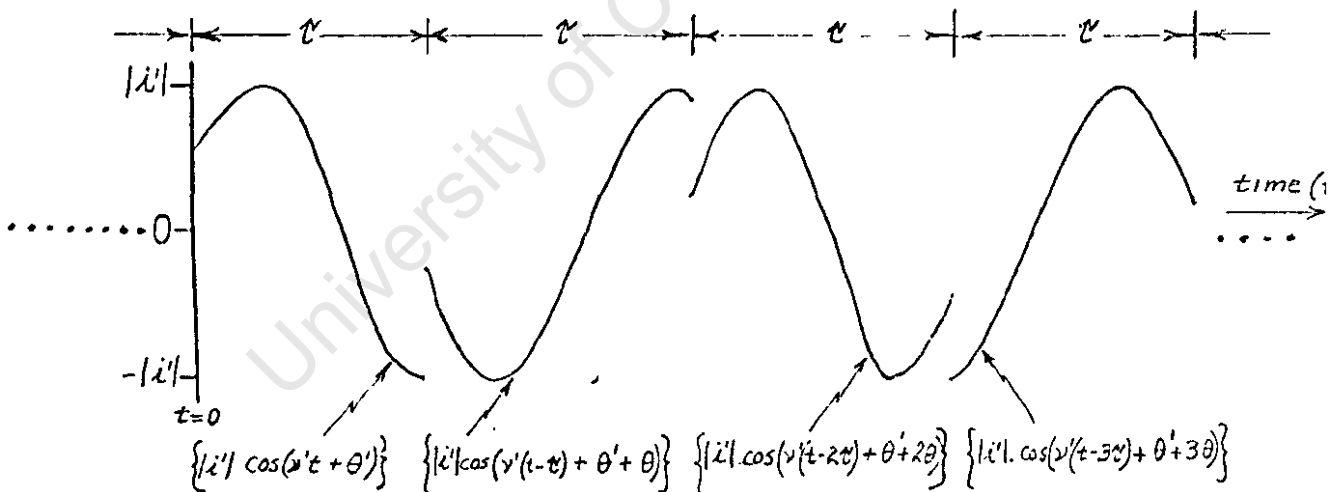
The k th. stages of successive modulation cycles are thus similar, but are each shifted in time by τ seconds and in phase by θ radians relative to the preceding k th. stage. Consider first the component waveform which is constituted by the k th. stage of all the modulation cycles alone and zero during the rest of time. If the spectrum of this component waveform is established, then the overall spectrum may be found by superposition of the spectra of the components corresponding to $k = 1$ to N .

The expression for i_{Qk} above comprises 3 terms. Consider the term

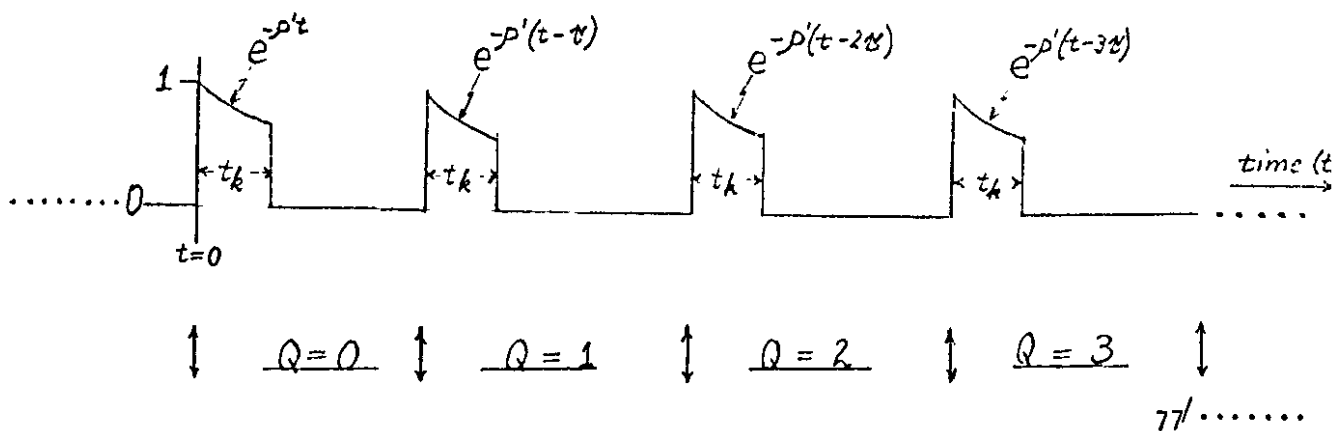
$$|i'| e^{-\rho'(t-Q\tau)} \cos(\nu'(t-Q\tau) + \theta' + Q\theta)$$

This may be regarded as the instantaneous product of the following two functions :-

(a)



(b)



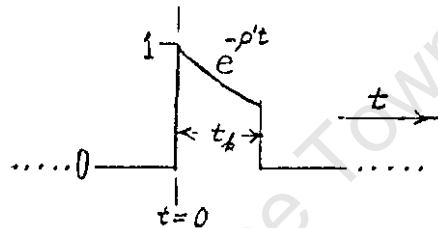
Consider the spectrum of the $Q = 0$ segment of this waveform alone. This can again be regarded as the instantaneous product of a continuous sinusoid,

$$|i'| \cos(\nu't + \theta')$$

[with amplitude density spectrum,

$$|i'| \pi \left\{ \delta(\omega + \nu') e^{-j\theta'} + \delta(\omega - \nu') e^{j\theta'} \right\}$$

and a decaying unit pulse,



[with spectrum,

$$g(\omega, p') = \frac{1}{(p' + j\omega)} \left\{ 1 - e^{-p't_k} e^{-j\omega t_k} \right\}$$

If the two spectra are convolved, then we obtain

$$G_0(\omega, p') = \frac{|i'|}{2} \left\{ g((\omega + \nu'), p') e^{-j\theta'} + g((\omega - \nu'), p') e^{j\theta'} \right\}$$

and in general the Q th segment of the waveform alone has the spectrum :-

$$G_Q(\omega, p') = \frac{|i'|}{2} \left\{ g((\omega + \nu'), p') e^{-j(\theta' + Q\theta)} + g((\omega - \nu'), p') e^{j(\theta' + Q\theta)} \right\} e^{-jQ\omega\tau}$$

The overall spectrum of this term is thus :-

$$G(\omega, \rho') = \sum_{Q=-\infty}^{Q=\infty} G_Q(\omega, \rho')$$

$$= \frac{|i'|}{2} \left\{ \sum_{Q=-\infty}^{\infty} g((\omega+\nu'), \rho') e^{-j(\theta'+Q\theta)} e^{-jQ\omega\tau} \right\} + \left\{ \sum_{Q=-\infty}^{\infty} g((\omega-\nu'), \rho') e^{j(\theta'+Q\theta)} e^{-jQ\omega\tau} \right\}$$

This expression is only non-zero at certain discrete angular frequencies which satisfy the relations :-

$$-(\theta + \omega\tau) = 2\pi m \quad \Rightarrow \quad \omega = \left(-\frac{\theta}{\tau} - m \frac{2\pi}{\tau}\right)$$

OR

$$(\theta - \omega\tau) = 2\pi m \quad \Rightarrow \quad \omega = \left(\frac{\theta}{\tau} - m \frac{2\pi}{\tau}\right)$$

where m is any integer, and the amplitude density spectrum thus takes the form of weighted delta functions at these angular frequencies.

The spectrum may be evaluated at these particular frequencies by the expressions

$$\frac{1}{\tau} \cdot \frac{|i'|}{2} \left\{ g((\omega+\nu'), \rho') \right\} e^{-j\theta'} \quad \text{for} \quad \omega = \left(-\frac{\theta}{\tau} - m \frac{2\pi}{\tau}\right)$$

and

$$\frac{1}{\tau} \cdot \frac{|i'|}{2} \left\{ g((\omega-\nu'), \rho') \right\} e^{j\theta'} \quad \text{for} \quad \omega = \left(\frac{\theta}{\tau} - m \frac{2\pi}{\tau}\right)$$

Since the "negative" frequency domain comprises conjugate values to those found for the corresponding positive frequencies, we may plot an amplitude and phase spectrum with discrete lines at $\omega = \left| \frac{\theta}{\tau} - m \frac{2\pi}{\tau} \right|$. The amplitude and phase are then just the modulus and argument of :-

$$\left\{ \frac{2}{\tau} \cdot \frac{|i'|}{2} \cdot g((\omega - \nu'), \rho') e^{j\theta'} \right\} \text{ for } \omega = \left(\frac{\theta}{\tau} - m \frac{2\pi}{\tau} \right) > 0$$

and

$$\left\{ \frac{2}{\tau} \cdot \frac{|i'|}{2} \cdot g^*((\omega - \nu'), \rho') e^{-j\theta'} \right\} \text{ for } \omega = \left(\frac{\theta}{\tau} - m \frac{2\pi}{\tau} \right) < 0$$

An exactly similar result may be obtained for the second term in i_{Qk} ,

i.e.

$$\left(|i''| e^{-\rho''(t-Q\tau)} \cdot \cos(\nu''(t-Q\tau) + \theta'' + Q\theta) \right)$$

and, putting $\rho' = 0$; $\nu' = \omega_e$; $\theta' = \theta_s$ and $|i'| = |I_s|$ the same result applies to the "steady state" term,

i.e.

$$\left(|I_s| e^{-0(t-Q\tau)} \cdot \cos(\omega_e(t-Q\tau) + \theta_s + Q\theta) \right)$$

NOTE : In evaluating the spectrum of the term above, it is seen that if ($m = 0$) and ($\frac{\theta}{\tau} = \omega_e$) it will be necessary to evaluate $g(0,0)$. In fact, in the limit, we find $g(0,0) = \tau k$.

By superposition of the spectra of each of the three terms of i_{Qk} , we find the nett contribution of the k th. stage component to the spectrum of the overall current waveform. It is interesting to note that only angular frequencies

$$\omega = \left| \frac{\theta}{\tau} - \frac{2\pi m}{\tau} \right|$$

occur, and neither ω_e , ν' or ν'' specifically (although in many instances $\frac{\theta}{\tau} = \omega_e$).

Now let the nett spectrum of the k th. stage component waveform be

$$G_k \quad \text{for } k = 1, 2, \dots, N$$

In order to obtain the overall spectrum, these N component spectra must be superimposed, taking account of the fact that they are time-shifted relative to each other. The overall spectrum is then

$$G = \sum_{k=1}^N G_k e^{-j\omega T_{(k-1)}}$$

where

$$T_{(k-1)} = (t_1 + t_2 + \dots + t_{(k-1)})$$

Once again we see that the only angular frequencies which can occur are

$$\omega = \left| \frac{\theta}{\tau} - \frac{2\pi m}{\tau} \right| \quad \text{for all integer } m.$$

This is true in general for any modulation cycle within the scope of this thesis.

5.6 Observations Based on the Spectral Analysis

Arising from this analysis, two interesting observations can be made :-

- 5.6.1 The waveform which we have analysed has arbitrarily been assigned to the A-phase and is obtained as $2 \cdot \text{Re.}(i_s^+)$. Corresponding to this waveform are the remaining two phase currents, being

$$i_s^b = 2 \text{Re.}(i_s^+ e^{j(-\frac{2\pi}{3})})$$

$$i_s^c = 2 \text{Re.}(i_s^+ e^{j(\frac{2\pi}{3})})$$

If the same analysis is now performed on these two waveforms, similar results are obtained to the A-phase result, with the exception of a phase shift in each spectral component. In fact, each component of the A-phase spectrum is matched by two of the same amplitude and frequency in the other phases, which constitute a balanced phasor set.

We observe the following interesting fact in regard to the phase sequence of these phasor sets :-

If the angular frequency corresponds to $\left(\frac{\theta}{T} - m\frac{2\pi}{T}\right) < 0$
 $\implies$ negative phase sequence

If the angular frequency corresponds to $\left(\frac{\theta}{T} - m\frac{2\pi}{T}\right) > 0$
 $\implies$ positive phase sequence

This observation will be seen to have a bearing on the average torque produced by an induction motor with modulated voltage.

5.6.2 In general, the waveforms of the three phase currents are different, although each may be periodic if

$K\theta = G.2\pi$, in which case each waveform repeats after $K\tau$ seconds which corresponds to K modulation cycles.

However, if $\theta = \frac{2\pi}{3n}$ where n is an integer, then the spectrum of the A-phase waveform has components

corresponding to $\omega = \frac{1}{T}\left(\frac{2\pi}{3n} - 2\pi m\right)$

In this case a time shift of $n\tau$ seconds introduces a phase shift of $-n\tau\omega = \left(-\frac{2\pi}{3} + 2m\pi\right)$ into the m th.

component. Each value corresponding to positive ω

is thus phase shifted by $-\frac{2\pi}{3}$ radians and those corresponding to negative ω are shifted by $\frac{2\pi}{3}$ radians. This

is exactly the spectrum of the B-phase waveform by definition.

Thus when $\theta = \frac{2\pi}{3n}$ the phase currents are identical but time-shifted by an amount $n\tau$ seconds relative to each other.

5.7 Special Cases

As mentioned previously, there are certain instances where the analysis of current waveforms in the modulated induction motor warrants special consideration. These cases occur whenever $\theta = n\pi$ (where n is any integer).

5.7.1 $n = \text{even integer}$

Here the current waveform repeats itself during each successive cycle of the modulation. The spectrum has lines at $\omega = \left| \frac{n\pi}{\tau} - m \cdot \frac{2\pi}{\tau} \right|$ which means that there is a D.C. component and lines spaced by $\frac{2\pi}{\tau}$ throughout the spectrum. Under these conditions, corresponding frequency components in the three phases do not form a balanced three phase set. Furthermore the RMS value of the waveforms is different in each phase and cannot be calculated as was shown in 5.4.

5.7.2 $n = \text{odd integer}$

These cases are very similar to the situation discussed above, with the exception that there is no D.C. component. The spectrum has lines at $\omega = \frac{\pi}{\tau}, \frac{3\pi}{\tau}, \frac{5\pi}{\tau}, \dots$. The waveform thus repeats itself after every second modulation cycle, and during each modulation cycle the waveform is the mirror image of the previous cycle and the following cycle. In the same way as for case 5.7.1 the corresponding components in the three phases form unbalanced sets and the RMS of the three waveforms are different.

In any convertor the situation where $\theta = n\pi$ must be regarded as a hazard condition, more serious in some cases than in others. Particular examples will be considered later where appropriate, and the analysis of RMS in these cases is discussed in Appendix (4).

5.8 Average Terminal Power

The current drawn through a supply phase is constituted from segments of the motor phase currents and the exact nature of a supply current will be determined by the logic of the convertor action.

The supply current waveforms are thus comprised of stages of the same duration as those in the motor currents and occurring at the same time. In general, however, the modulation angle θ which represents phase shift between corresponding points in successive modulation cycles may be different for the supply current waveform, usually being $\theta = \omega_e \tau$ where ω_e is the angular frequency of the supply voltage. Once the composition of a supply current waveform is determined, exactly similar analysis may be used to calculate the RMS and the harmonic content. The spectrum of the supply current is invariably comprised of lines at

$$\omega = \left| \omega_e - m \frac{2\pi}{\tau} \right|$$

and only the component of frequency ω_e is able to negotiate non-zero average power from the supply. If the supply voltage is

$$|V_s| \cos(\omega_e t)$$

and the corresponding current component is

$$|I_s| \cos(\omega_e t + \theta_s)$$

then the average power negotiated through this supply line is

$\frac{|V_s| \cdot |I_s| \cos(\theta_s)}{2}$ and the nett terminal power is the sum of the average power on all the supply lines.

Since there are only three motor phases, only three supply phases will be conducting during a particular stage of the cycle, and in accordance with our earlier assumption the voltages in these three phases form a balanced 3-phase set, being spaced in phase by $\frac{2\pi}{3}$ radians. It was observed at the outset that the nett terminal power and output power waveforms repeat themselves during every modulation cycle. An alternative expression for average terminal power may thus be obtained by evaluating

$$W_{av.} = \frac{1}{\tau} \int_0^{\tau} W(t) dt$$

Where $W(t)$ = instantaneous total power

Let the current flowing in the conducting phases during the k th, stage of a given cycle be

$$i_{J\text{-phase}} = \begin{cases} |I_s| \cos(\omega_e t + \theta_s + \phi_J) \\ + |i'| e^{-p't} \cos(\nu't + \theta' + \phi_J) \\ + |i''| e^{-p''t} \cos(\nu''t + \theta'' + \phi_J) \end{cases}$$

where $t = 0$ at the start of the stage and

$$\phi_J = 0, \frac{2\pi}{3}, \frac{4\pi}{3} \quad \text{respectively}$$

Then, if the corresponding phase voltages are

$$v_{J\text{-phase}} = |V_s| \cos(\omega_e t + \phi_J)$$

the instantaneous phase power in the J-phase is

$$W_{\text{inst J-phase}} = \begin{cases} |V_s| |I_s| \cos(\omega_e t + \theta_s + \phi_J) \cos(\omega_e t + \phi_J) \\ + |V_s| |i'| e^{-p't} \cos(\nu't + \theta' + \phi_J) \cos(\omega_e t + \phi_J) \\ + |V_s| |i''| e^{-p''t} \cos(\nu''t + \theta'' + \phi_J) \cos(\omega_e t + \phi_J) \end{cases}$$

expanding this expression and summing over the three phases we obtain the total instantaneous power during this stage :-

$$W_{\text{inst. total}} = \begin{cases} \frac{3}{2} |V_s| |I_s| \cos(\theta_s) \\ + \frac{3}{2} |V_s| |i'| e^{-p't} \cos((\nu' - \omega_e)t + \theta') \\ + \frac{3}{2} |V_s| |i''| e^{-p''t} \cos((\nu'' - \omega_e)t + \theta'') \end{cases}$$

Then the nett energy drawn during this stage of the cycle is :-

$$E_k = \int_0^{t_k} W_{inst. total} dt. =$$

$$\left[\begin{aligned} & \{t_k \cdot |I_s| \cos(\theta_s)\} \\ & + \frac{|i'| \rho'}{(\rho'^2 + (\gamma' - \omega_e)^2)} \left\{ e^{-\rho' t_k} \left\{ \frac{(\gamma' - \omega_e)}{\rho'} \left(\sin((\gamma' - \omega_e)t_k + \theta') - \cos((\gamma' - \omega_e)t_k + \theta') \right) \right\} \right. \right. \\ & \quad \left. \left. - \left\{ \frac{(\gamma' - \omega_e)}{\rho'} \sin(\theta') - \cos(\theta') \right\} \right\} \right. \\ & \left. + \frac{|i''| \rho''}{(\rho''^2 + (\gamma'' - \omega_e)^2)} \left\{ e^{-\rho'' t_k} \left\{ \frac{(\gamma'' - \omega_e)}{\rho''} \left(\sin((\gamma'' - \omega_e)t_k + \theta'') - \cos((\gamma'' - \omega_e)t_k + \theta'') \right) \right\} \right. \right. \\ & \quad \left. \left. - \left\{ \frac{(\gamma'' - \omega_e)}{\rho''} \sin(\theta'') - \cos(\theta'') \right\} \right\} \right] \end{aligned} \right]$$

Similar expressions can be obtained for each of the N stages of the cycle, and the average terminal power is thus

$$W_{av.} = \frac{1}{\tau} \sum_{k=1}^N E_k$$

5.9 Output Power

Similarly to the instantaneous nett electrical terminal power, the mechanical output power waveform repeats itself during every modulation cycle. Since shaft speed is assumed constant, the output power is directly related to the instantaneous torque. We have observed previously that the modulated current waveforms in the motor have spectra comprised of lines at $\omega = \left| \frac{\theta}{\tau} - \frac{2\pi \cdot m}{\tau} \right|$

Except for $\theta = n\pi$, therefore, the spectrum has a "fundamental" component at $\omega = \left| \frac{\partial \theta}{\partial t} \right|$ and pairs of "sidebands" separated by $\frac{2\theta}{\tau}$ throughout. As we have seen, each line in a phase-waveform spectrum is matched by two others in the other phases which form a balanced set, and the two sets belonging to a given pair of sidebands have opposite phase sequence. Furthermore it will be seen that if the rotor runs synchronously with the field produced by the fundamental current set, then it has exactly the same slip frequency relative to each one of any sideband pair set, except with opposite phase rotation. For small fundamental slip therefore, the tendency is for the two sets forming each sideband pair to produce contributions to the average torque which act in opposite directions. Since the set of currents corresponding to the fundamental is the only one which does not have a matching set acting against it, it is usually responsible for most of the average torque produced. A quantitative assessment of this effect will be presented in the following chapters.

For a rigorous treatment of the output power produced by a modulator-fed motor, the following two aspects should be considered :-

- (a) average torque
- (b) harmonic analysis of the torque waveform.

If the instantaneous positive sequence stator current during the k th. stage of a particular cycle is

$$i_s^+ = I_s^+ e^{j\omega_e t} + i_s^{+'} e^{\alpha' t} + i_s^{+''} e^{\alpha'' t}$$

and in the rotor

$$i_r^+ = I_r^+ e^{j\omega_e t} + i_r^{+'} e^{\alpha' t} + i_r^{+''} e^{\alpha'' t}$$

where $i_s^{+'} = \frac{i_s^{+'}}{C}$; $i_r^{+'} = \frac{i_s^{+'}}{C}$ and $t=0$ at the start of this stage, then, in accordance with the result in Chapter Three, the instantaneous torque is

$$\begin{aligned}
T_{inst.} &= 6M'P \operatorname{Im}(i_s^+ i_r^{+*}) \\
&= 6M'P \operatorname{Im} \left\{ (I_s^+ e^{j\omega_e t} + i_s' e^{\alpha' t} + i_s'' e^{\alpha'' t}) (I_r^{+*} e^{-j\omega_e t} + i_r'^* e^{\alpha'^* t} + i_r''^* e^{\alpha''^* t}) \right\} \\
&= 6M'P \left[\begin{aligned}
&\{ |I_s^+| |I_r^+| \sin(\theta_s^+ - \theta_r^+) \} + \{ |i_s'| |i_r'| e^{-\rho' t} \sin(\theta_s' - \theta_r') \} + \{ |i_s''| |i_r''| e^{-\rho'' t} \sin(\theta_s'' - \theta_r'') \} \\
&+ \{ |I_s^+| |i_r'| e^{-\rho' t} \sin(\theta_s^+ - \theta_r' + (\omega_e - \nu')t) \} \\
&+ \{ |I_s^+| |i_r''| e^{-\rho'' t} \sin(\theta_s^+ - \theta_r'' + (\omega_e - \nu'')t) \} \\
&+ \{ |i_s'| |I_r^+| e^{-\rho' t} \sin(\theta_s' - \theta_r^+ + (\nu' - \omega_e)t) \} \\
&+ \{ |i_s'| |i_r''| e^{-(\rho' + \rho'')t} \sin(\theta_s' - \theta_r'' + (\nu' - \nu'')t) \} \\
&+ \{ |i_s''| |I_r^+| e^{-\rho'' t} \sin(\theta_s'' - \theta_r^+ + (\nu'' - \omega_e)t) \} \\
&+ \{ |i_s''| |i_r'| e^{-(\rho' + \rho'')t} \sin(\theta_s'' - \theta_r' + (\nu'' - \nu')t) \}
\end{aligned} \right]
\end{aligned}$$

5.9.1 The average value of torque during this k th stage is

$$T_{av.k} = \frac{1}{t_k} \int_0^{t_k} T_{inst.} dt.$$

This is easily evaluated, since each term in the expression for $T_{av.k}$ has the general form

$$\int_0^{t_k} A e^{-Rt} \sin(Wt + \psi) dt.$$

$$= \frac{A}{(R^2 + W^2)} \left[\begin{aligned}
&e^{-Rt_k} (-R \sin(Wt_k + \psi) - W \cos(Wt_k + \psi)) \\
&+ (R \sin(\psi) + W \cos(\psi))
\end{aligned} \right]$$

The overall average torque is then

$$T_{av} = \frac{1}{\tau} \sum_{k=1}^N T_{av.k} \cdot t_k$$

5.9.2 Harmonic analysis of the torque waveform

We have noted that the torque waveform is repetitive with repetition frequency $\frac{1}{\tau}$ Hz. The spectrum may thus be expected to contain discrete lines at

$$\omega = 0, \frac{2\pi}{\tau}, \frac{4\pi}{\tau}, \dots \text{ etc.}$$

The torque waveform segment during the k th. stage defined above has 9 terms of the form

$$A e^{-Rt} \sin(Wt + \psi)$$

Each of these has continuous spectrum

$$f(\omega, R) = \frac{A}{2} \left\{ g((\omega+W), R) \frac{e^{-j\psi}}{-j} + g((\omega-W), R) \frac{e^{j\psi}}{j} \right\}$$

and each is repeated in time every τ seconds. This repetitive component waveform has spectrum comprised of lines at $\omega = m \frac{2\pi}{\tau}$ where m is a positive integer

For $m > 0$ these are evaluated as

$$\frac{A}{\tau} \left\{ g((\omega-W), R) \frac{e^{j\psi}}{j} + g^*(-(\omega-W), R) \frac{e^{-j\psi}}{-j} \right\}$$

and for $m=0$ the value is one-half of the above expression and real-valued, representing the average value.

If the 9 spectra of the repeated terms of the k th. torque segment are added, we obtain the contribution of the k th. stages of the successive modulation cycles to the spectrum of the overall waveform,

$$\text{i.e.} \quad F_k(\omega)$$

The overall spectrum is obtained by adding the spectra for each of the N stages of the cycle, with due regard for the fact that successive stages are shifted in time.

$$F(\omega) = \sum_{k=1}^N F_k(\omega) \cdot e^{-j\omega T_{(k-1)}}$$

$$\text{where } T_{(k-1)} = (t_1 + t_2 + \dots + t_{(k-1)})$$

5.10 Performance Criteria

In assessing the performance of any modulator-fed induction motor the following factors will generally be required in order to establish a measure of the "goodness" of the system relative to any alternatives :-

(a) Efficiency This is defined as:

$$(i) \quad \frac{\text{Average mechanical output}}{\text{Average electrical input}} \times 100\% \text{ when motoring}$$

$$(ii) \quad \frac{\text{Average electrical output}}{\text{Average mechanical input}} \times 100\% \text{ when regenerating}$$

(iii) Zero when both electrical and mechanical energy sources are supplying power to the machine.

$$(b) \quad \text{Distortion factor} = \frac{\text{RMS fundamental current}}{\text{RMS total current waveform}}$$

$$(c) \quad \text{Displacement factor} = \cos(\phi)$$

where ϕ is the angle between a sinusoidal voltage and the corresponding current component.

- (d) Power factor $= \frac{\text{Average power}}{\text{RMS volt} \times \text{RMS amps.}}$
- $= (\text{distortion factor}) \times (\text{displacement factor})$
if voltage is sinusoidal.
- (e) RMS of current waveforms , gives an indication of the reading which would be obtained on standard measurement apparatus and enables distortion and power factors to be calculated. Also enables a quick assessment of the copper loss in the circuit.
- (f) Spectral content of current waveforms . Perhaps the most topical aspect of modulator technology is the effect on supply networks of these non-linear loads. The analysis, aside from providing the necessary information regarding the fundamental current, may be used to obtain an idea of the nature of the harmonic distortion. Minimization of harmonic content in supply current is likely to be a prime cost factor in the future, and the analysis which has been detailed here may advantageously be used in such studies.
- (g) Spectrum of torque waveform . The information obtained regarding A.C. torque components is likely to apply to larger drives especially. It may happen that these alternating components can excite resonances in the stator structure or rotor-cum-load under certain circumstances. It is thus as well to be aware of the dominant components which may be expected in a given drive.

CHAPTER SIX

THEORETICAL PERFORMANCE OF AN INDUCTION MOTOR WITH CHOPPER MODULATED TERMINAL VOLTAGE

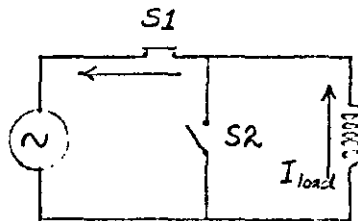
6.1 Chopper Modulation for Voltage Control

Chopper modulation of the A.C. supply voltage is a new technique for voltage control in power circuits. This form of modulation has been widely used in DC applications where it may be applied with comparatively simple hardware, but until recently no equivalent hardware had been developed for use in AC power circuits, where special problems are encountered. In 1970 K. Nohara⁽¹⁶⁾ discussed an AC power modulator which was able to reverse the polarity of the supply voltage at high frequency. In 1976 Mozdzer⁽¹⁷⁾ and Bose demonstrated how a pure chopping action might be achieved with the use of transistorized circuitry, and tested their prototype to a few kilowatts with resistive and induction motor loading. This apparatus was unable to produce the chopping action when the instantaneous voltage and current waveforms had opposite polarity. Thus although Mozdzer and Bose were able to demonstrate the feasibility of such a modulator, they were not entirely successful in producing the ideal chopping action. At the same time, Enslin⁽¹⁸⁾ et al. at the University of Cape Town had developed a versatile high speed electronic commutating device which enabled a realistic chopper to be assembled. Previous work by Hurley⁽¹⁹⁾ indicated that such chopping action might be advantageously incorporated into the action of a force-commutated static frequency converter for purposes of voltage control. It was realized, however, that the use of chopper modulation on induction motors had not been carefully considered with regard to its effect on efficiency and overall performance. Although chopper modulation obviously enables control of the RMS voltage, it must be remembered that the induction motor is sensitive to both the amplitude and frequency of distortion components which are introduced in this way.

Mozdzer and Bose paid little attention to the effect of their modulator on the performance factors of their motor. Thus, although simple static voltage control has limited application in 3-phase induction motors, it was thought advisable to study the use of chopper modulation for this purpose, preliminary to using the technique in a frequency convertor.

6.2 The Chopper Modulator

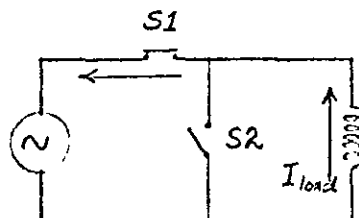
Chopper modulation is commonly used for speed control in DC traction applications and is used to provide effective control of the output voltage in DC-link invertors. The chopping action is achieved in various ways, but in all cases the principle is similar. The D.C. supply voltage is connected to the load via a series switch. When this is opened, the load current is not interrupted (unless power factor = 1) but is smoothly commutated into a "free-wheeling" path which is provided by a strategically located diode circuit. Provided the load current does not tend to reverse, the diodes can effectively hold the load voltage to zero during the subsequent time. When the D.C. supply connection is again enabled, the diode path is reverse biased and the load current is commutated into the supply circuit, so that the load voltage is equal to the supply voltage. Because the supply polarity is never reversed it is possible to leave the freewheeling diodes permanently connected and the commutation is performed by opening a single switch. This is not possible when the supply is sinusoidal since the freewheel diodes would short-circuit the supply voltage. In order to chopper modulate an AC supply voltage, it is therefore necessary to have a controlled switch both in series with the supply connection to the load and in parallel with the load. The parallel switch thus replaces the diode circuit in providing a freewheeling path.



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As switch S1 is opened, S2 is simultaneously closed in such a way that the load current remains continuous as it is commutated into S2 and the supply is not short-circuited. It will be appreciated that this requires very accurate synchronization between the actions of the two switches. To date this has been achieved with circuits using power transistors, and it seems likely that success can also be achieved with thyristor circuitry. Naturally, the operation of a practical chopper will always fall short of the ideal. However, in this chapter we will consider the performance of the induction motor fed through an ideal convertor of this sort. We thus assume:-

- (i) No losses in the convertor.
- (ii) Instantaneous commutation.
- (iii) Zero supply impedance.

6.3 The Chopper Modulation Cycle

6.3.1

The Modulated Voltage.

The chopping action envisaged here holds the terminal voltage on the three motor phases to zero for a short-circuit interval of duration τ_a , and then restores the 3-phase supply connection for an ON-interval of duration τ_b . This cycle is repeated with frequency $\frac{1}{\tau}$ Hz, where $\tau = \tau_a + \tau_b$. Fig (23) illustrates the A-phase terminal voltage produced by this modulation.

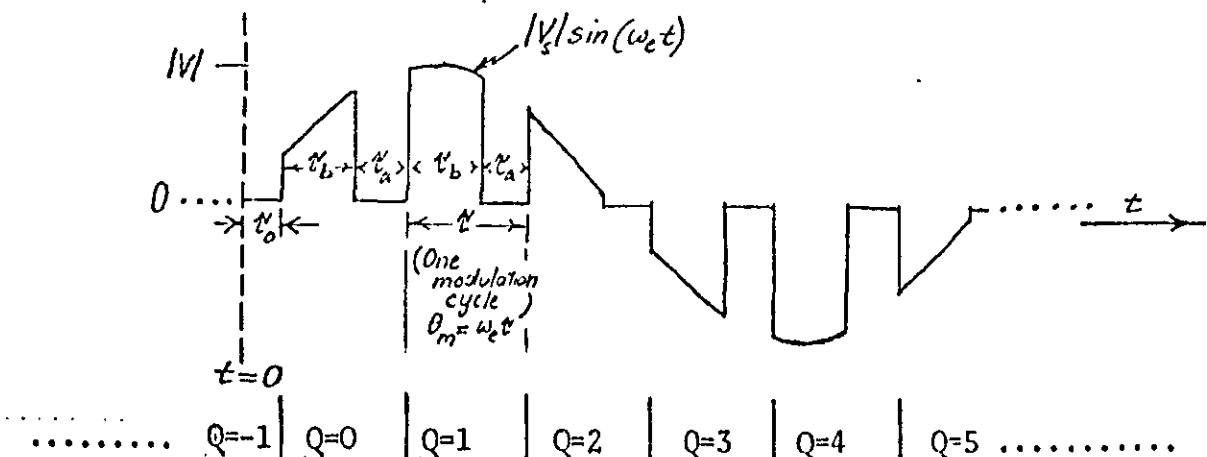


Fig. 23

6.3.2 R.M.S. Value:-

It can be shown that the value of $\int (v_s^a)^2 dt$ over the Q th. ON-interval is

$$\int_Q (v_s^a)^2 dt = |V_s|^2 \left[\frac{\tau_b}{2} - \frac{1}{4\omega_e} \sin\{2\omega_e(\tau_0 + \tau_b)\} + \frac{1}{4\omega_e} \sin\{2\omega_e\tau_0 + 2\omega_e Q\tau\} \right]$$

and the RMS voltage,

$$V_{s \text{ RMS}}^a = \sqrt{\lim_{k \rightarrow \infty} \left\{ \frac{1}{(|2k|+1)\tau} \sum_{Q=-k}^{Q=k} \int_Q (v_s^a)^2 dt \right\}}$$

If $\omega_e \tau \neq n\pi$

where n is an integer,

then

$$V_{s \text{ RMS}}^a = \frac{|V_s|}{\sqrt{2}} \cdot \sqrt{\frac{\tau_b}{\tau}}$$

and in the special cases where $\omega_e \tau = n\pi$

then

$$V_{s \text{ RMS}}^a = \frac{|V_s|}{\sqrt{2\tau}} \cdot \sqrt{\tau_b - \frac{\sin(2\omega_e(\tau_0 + \tau_b))}{2\omega_e} + \frac{\sin(2\omega_e\tau_0)}{2\omega_e}}$$

Thus, if $\omega_e \tau \neq n\pi$ the same RMS voltage is produced on the three phases by the simultaneous chopping action although the three phase voltages are chopped at different points-on-wave. However, when $\omega_e \tau = n\pi$ the RMS is different on the three phases.

6.3.3 Distortion Factor:-

The fundamental frequency component of the chopped voltage in Fig.(23) is

$$V_s^a (\text{Fund. RMS}) = \frac{|V_s|}{\sqrt{2}} \cdot \frac{\tau_b}{\tau}$$

and is in-phase with the original sinewave. The output voltage distortion factor is thus

$$\mu = \sqrt{\frac{\tau_b}{\tau}} \quad (\text{provided } \omega_e \tau \neq n\pi)$$

6.4

The Instantaneous Positive Sequence
Solution for Chopped Excitation.

The solution was obtained in Chapter 4.5.2 and is illustrated in Fig. (24). During an ON-interval the instantaneous positive sequence stator current is:-

$$i_s^+ = I_s^+ e^{j\omega_e t} + i_{s0}' e^{\alpha' t} + i_{s0}'' e^{\alpha'' t} \quad \text{where } t=0 \text{ at the start of the interval}$$

and in the subsequent shorted interval

$$i_s^+ = i_{s1}' e^{\alpha'(t-\tau_b)} + i_{s1}'' e^{\alpha''(t-\tau_b)}$$

since there is no steady state response.

The modulation angle is $\theta_m = \omega_e \tau$ so that the motor current has a spectrum comprised of discrete lines at

$$\omega = \left| \omega_e - m \cdot \frac{2\pi}{\tau} \right| \quad \text{where } m \text{ is any integer.}$$

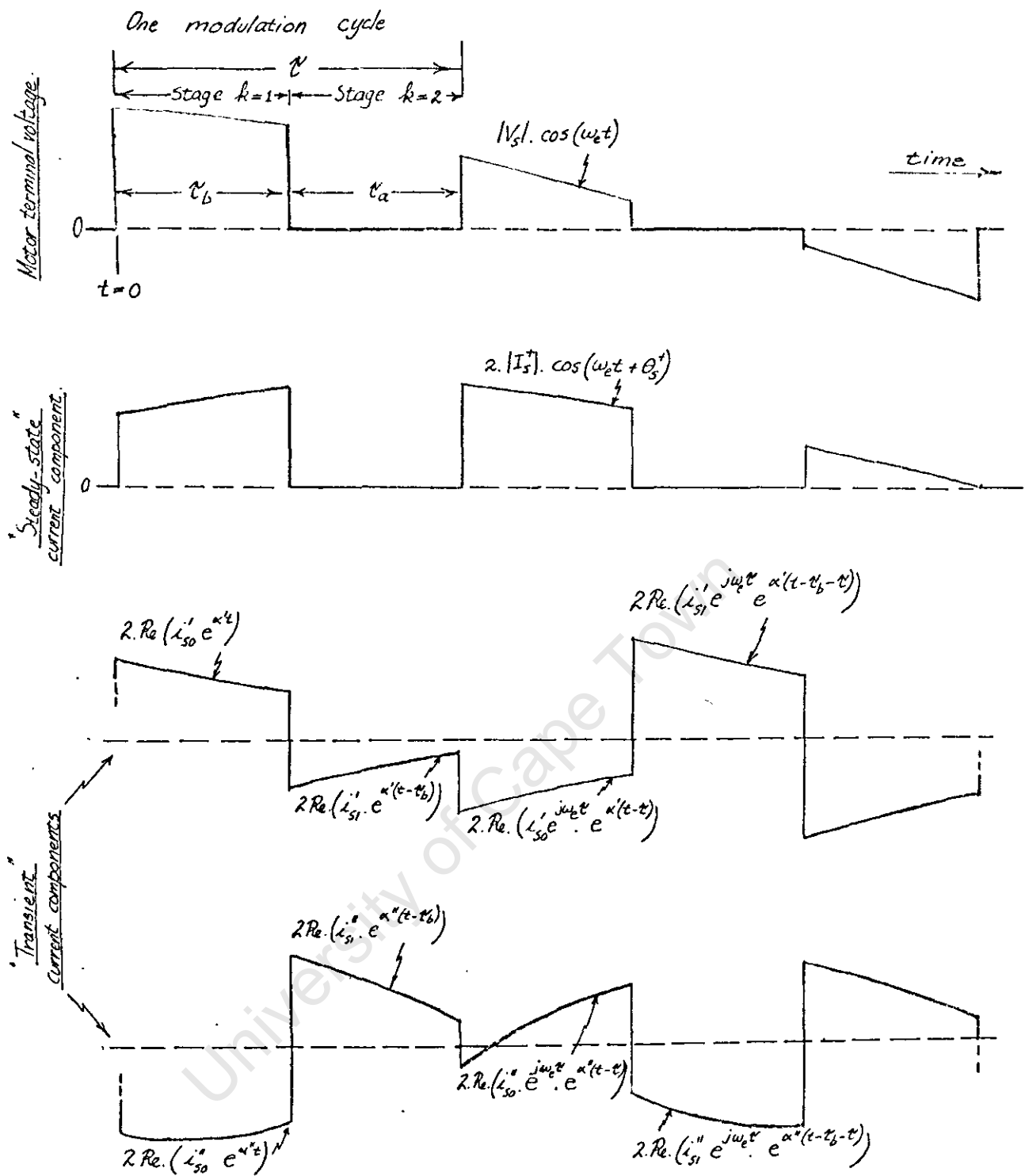
The amplitude and phase of these components as well as the overall RMS may be evaluated as detailed in Chapter Five. (Note that when $\tau_a = \tau_b = \tau/2$ only components at values of ω corresponding to $m=0$ and $m = \text{odd integers}$, are non-zero).

6.5 Mains supply and short circuit loop currents:

There are two distinct supply circuits:-

- (i) Three short-circuit loops containing zero e.m.f.
- (ii) Three supply phases with supply phase voltage.

These supply paths are alternately connected, so that -



Define

$$i_o' = \frac{C'(-I_s^+ + C''I_r^+)}{(C' - C'')} \quad ; \quad i_o'' = \frac{C''(-C'I_r^+ + I_s^+)}{(C' - C'')}$$

$$i_1' = -i_o' e^{j\omega_e \tau_b} \quad ; \quad i_1'' = -i_o'' e^{j\omega_e \tau_b}$$

Then

$$i_{s0}' = i_o' \left\{ \frac{1 - e^{F'\tau_a}}{1 - e^{F'\tau}} \right\} \quad ; \quad i_{s1}' = i_1' \left\{ \frac{1 - e^{F'\tau_b}}{1 - e^{F'\tau}} \right\}$$

and similarly for i_{s0}'' and i_{s1}''

Fig. 24

- (i) The mains supply current is equal to the motor phase current during ON-intervals, and otherwise zero.
- (ii) The short circuit loop current is equal to the motor phase current during short-circuit intervals, and otherwise zero.

(Note that only the A-phase supply and short-cct. loops carry the A-phase motor current, and similarly for the other phases.)

It is thus very simple to calculate the R.M.S. and spectral content of either of these supply waveforms, since each is non-zero during only one stage of the modulation cycle. Since $\theta_{supply} = \omega_e t$ in each waveform, both supply currents have components at

$$\omega = \left| \omega_e - m \cdot \frac{2\pi}{T} \right| \quad \text{where } m \text{ is any integer.}$$

(Note that although the situation where $\alpha_a = \alpha_b = \pi/2$ is a special case in regard to the motor current spectrum it is not a special case in regard to either of the supply currents.)

Furthermore it is useful to note that

$$i_s^a \text{ RMS} = \sqrt{\left(i_{supply}^a \text{ RMS} \right)^2 + \left(i_{short-cct.}^a \text{ RMS} \right)^2}$$

6.6 Average Terminal Power

The average terminal power may be calculated from the fundamental mains supply current component as

$$W_{av} = \frac{3}{2} \cdot |V_s| \cdot |I_1| \cdot \cos(\theta_1)$$

where $\cos(\theta_1) =$ the fundamental displacement factor.

Alternatively, the average terminal power may be written as:

$$W_{av} = 6 \cdot \frac{|V_s|}{\pi} \left[\begin{aligned} & \left\{ r_b \cdot |I_{s1}^+| \cos(\theta_{s1}) \right\} \\ & + \frac{|i_{s1}'| \cdot p'}{(p'^2 + (\gamma' - \omega_e)^2)} \left\{ e^{-p' r_b} \left\{ \frac{(\gamma' - \omega_e)}{p'} (\sin((\gamma' - \omega_e) r_b + \theta_{s1}') - \cos((\gamma' - \omega_e) r_b + \theta_{s1}')) \right\} \right. \right. \\ & \quad \left. \left. - \left\{ \frac{(\gamma' - \omega_e)}{p'} \sin(\theta_{s1}') - \cos(\theta_{s1}') \right\} \right\} \right. \\ & + \frac{|i_{s1}''| \cdot p''}{(p''^2 + (\gamma'' - \omega_e)^2)} \left\{ e^{-p'' r_b} \left\{ \frac{(\gamma'' - \omega_e)}{p''} (\sin((\gamma'' - \omega_e) r_b + \theta_{s1}'') - \cos((\gamma'' - \omega_e) r_b + \theta_{s1}'')) \right\} \right. \\ & \quad \left. \left. - \left\{ \frac{(\gamma'' - \omega_e)}{p''} \sin(\theta_{s1}'') - \cos(\theta_{s1}'') \right\} \right\} \right] \end{aligned} \right.$$

Once again, these results do not apply whenever $\theta_{supply} = n\pi$ as explained previously. This happens whenever the chopping frequency, $f_{chop} = \frac{1}{T}$ is such that $f_{chop} = \frac{2f_e}{m}$ where m is a positive integer > 0

Under these conditions the average terminal powers on the three supply phases are different.

6.7 Torque:

The general expression for instantaneous torque during the ON-interval can be written as detailed in Chapter 5.9. During the short circuit interval a similar expression applies, but with $I_{s1}^+ = I_{r1}^+ = 0$

6.7.1

Limits between which the torque alternates

It has been shown that the harmonic analysis of the torque waveform can be performed to yield the amplitude and phase of the oscillating torque components of frequency,

$$\omega = m \cdot \frac{2\pi}{T}$$

where m is positive integer > 0

In many instances, however, it will be useful to know the upper and lower limits between which the torque waveform oscillates. Although not true in general, the following assumption has been found valid in all cases where $2\pi f_{chop}$ is large in relation to $(\nu'' - \nu')$ i.e. where the chopping cycle is sufficiently short. It is assumed that the peak of the torque wave occurs at the start of each short circuit interval and the minimum occurs at the end of each short-cct. interval. Thus,

$$T_{max.inst.} = 6M'P. \left\{ \begin{aligned} & |i'_{s1}| |i'_{r1}| \sin(\theta'_{s1} - \theta'_{r1}) + |i''_{s1}| |i''_{r1}| \sin(\theta''_{s1} - \theta''_{r1}) \\ & + |i'_{s1}| |i''_{r1}| \sin(\theta'_{s1} - \theta''_{r1}) \\ & + |i''_{s1}| |i'_{r1}| \sin(\theta''_{s1} - \theta'_{r1}) \end{aligned} \right\}$$

$$T_{min.inst.} = 6M'P. \left\{ \begin{aligned} & |i'_{s1}| |i'_{r1}| e^{-2p'a} \sin(\theta'_{s1} - \theta'_{r1}) + |i''_{s1}| |i''_{r1}| e^{-2p''a} \sin(\theta''_{s1} - \theta''_{r1}) \\ & + |i'_{s1}| |i''_{r1}| e^{-(p'+p'')a} \sin((\theta'_{s1} - \theta''_{r1}) + (\nu' - \nu'')a) \\ & + |i''_{s1}| |i'_{r1}| e^{-(p'+p'')a} \sin((\theta''_{s1} - \theta'_{r1}) + (\nu'' - \nu')a) \end{aligned} \right\}$$

The torque wave oscillates between these limits with frequency f_{chop} Hz.

6.7.2 Average Torque

The expression for instantaneous torque during a particular stage of a modulation cycle may be rephrased as -

$$T_{inst.} = G.M.P. g_m \left\{ \begin{aligned} & I_s^+ I_r^{+*} + i_s' i_r'^* e^{(\alpha' - \alpha'^*)t} + i_s'' i_r''^* e^{(\alpha'' - \alpha''^*)t} \\ & + \{ I_s^+ i_r''^* - i_s''^* I_r^+ \} e^{(j\omega_e - \alpha''^*)t} \\ & + \{ i_s' I_r^{+*} - I_s^{+*} i_r' \} e^{(\alpha' - j\omega_e)t} \\ & + \{ i_s'' i_r'^* - i_s'^* i_r'' \} e^{(\alpha'' - \alpha'^*)t} \end{aligned} \right.$$

The average value of the torque, as explained in Chapter 5.9.1 is obtained as

$$T_{av} = \frac{1}{T} \left\{ \int_0^{T_a} T_{inst.} (short\ ckt.) dt + \int_0^{T_b} T_{inst.} (ON) dt \right\}$$

The indefinite integral of $T_{inst.}$ is,

$$\int T_{inst} dt = G.M.P. g_m \left[\begin{aligned} & I_s^+ I_r^{+*} t + \frac{i_s' i_r'^* e^{(\alpha' - \alpha'^*)t}}{(\alpha' - \alpha'^*)} + \frac{i_s'' i_r''^* e^{(\alpha'' - \alpha''^*)t}}{(\alpha'' - \alpha''^*)} \\ & + \frac{(I_s^+ i_r''^* - i_s''^* I_r^+)}{(j\omega_e - \alpha''^*)} e^{(j\omega_e - \alpha''^*)t} \\ & + \frac{(i_s' I_r^{+*} - I_s^{+*} i_r')}{(\alpha' - j\omega_e)} e^{(\alpha' - j\omega_e)t} \\ & + \frac{(i_s'' i_r'^* - i_s'^* i_r'')}{(\alpha'' - \alpha'^*)} e^{(\alpha'' - \alpha'^*)t} \end{aligned} \right]$$

When this expression is inserted in the equation for T_{av} and the algebraic values of i_s' , i_r' , I_s^+ etc., are substituted, an immensely complicated expression results. However, if we make a series approximation of the form

$$e^{(\alpha - j\omega_e)t} = 1 + (\alpha - j\omega_e)t + \dots$$

and neglect the higher order terms, then it can be demonstrated that this expression collapses to yield -

$$T_{av} = \frac{6M'P}{\tau} \cdot \Im_m \left\{ \frac{\tau_b^2}{\tau} \cdot I_s^+ I_r^{+*} \right\}$$

$$\text{i.e.} \quad \underline{\underline{T_{av} = \left(\frac{\tau_b}{\tau} \right)^2 \times T_s}}$$

where T_s is the torque which would be produced if no chopping action was present.

Having obtained this result it is interesting to consider it with regard to the spectral content of the voltage being applied to the motor.

The fundamental motor voltage has amplitude $|V_s| \left(\frac{\tau_b}{\tau} \right)$ and produces torque $\left(\frac{\tau_b}{\tau} \right)^2 \cdot T_s$ which alone accounts for the nett average torque calculated above. The harmonic components occur in "sideband" pairs of frequency

$$\omega = \omega_e \pm m \cdot \frac{2\pi}{\tau}$$

and for each value of m the two components have the same amplitude, but constitute balanced 3-phase sets of opposing phase sequence in the 3-phases. It can then be shown that if the stator resistance is negligible at the harmonic frequencies, and the fundamental slip is small compared with these, then the torque produced by one of a particular sideband pair is just equal and opposite to that produced by the other, resulting in *no contribution to the nett average torque*. (This does not hold if $f_{chop} \leq f_e$ in which case some of the sideband pairs have the same phase sequence. Also, as $f_{chop} \rightarrow f_e$ from above, the approximations above no longer apply).

Exhaustive calculations in the case of the test motor have indicated that the approximate expression for average torque yields a value which is extremely close to the correct value calculated from the full formula. Indeed, the errors incurred are so slight that they do not warrant a graphical illustration, being only a fraction of a percent in most instances. The largest of these slight errors occurs as the lower chopping frequencies are approached and also tend larger for small values of τ_b .

6.8 Calculated Performance of the Test Motor with Chopped Excitation:

The theoretical techniques which have been developed may now be used to illustrate the characteristics of a typical small squirrel cage motor with chopper modulated terminal voltage. The results illustrated here are calculated for an ideal convertor-motor system. In Chapter Seven we will include measurements made on a working laboratory system, and these may then be viewed in the light of the theoretical predictions. This form of presentation has been chosen to avoid any confusion between the behaviour of a particular non-ideal system and the general trends which we wish to identify from the theoretical investigation.

It should be noted that the general analysis of the phase-stepping modulation cycle with N chops per step, as detailed in Chapter 4.4, has been arranged as a general purpose computer calculation program listed in Appendix (7). This has been used to generate the data which is presented here, by specifying a phase-step of zero degrees at the end of each modulation cycle.

The motor considered here is the cage motor which has been used in the previous examples. The results would be unrealistic if no account was taken of the effective reduction of the fundamental flux density distribution which is brought about by chopper modulation. Consequently, the inductances used in generating the following data have unless otherwise stated been adjusted in accordance with the measured saturation characteristic of the motor. Thus different values of self- and mutual-inductance have been used for each value of $\frac{\tau_b}{\tau}$ which is considered.

The parameter values are listed in Appendix (6).

6.9 Observations Based on the Calculated Performance of the Induction Motor with Chopper Modulated Supply.

The calculated results presented in Figs. (25) to (45) represent a detailed investigation of the response of the test motor with chopper modulated supply voltage. Unless otherwise stated, these results have been obtained by means of the algebraic formulation which has been established in the preceeding chapters. Because of the number of motor and modulator parameters which influence the system in this mode it is difficult to present a cross-section of results which highlight the most important characteristics. In compiling these results attention has been given to the effects of three main parameters:

- (a) Chopping frequency.
- (b) Slip frequency.
- (c) Mark-to-total time ratio. (T_{on}/T)

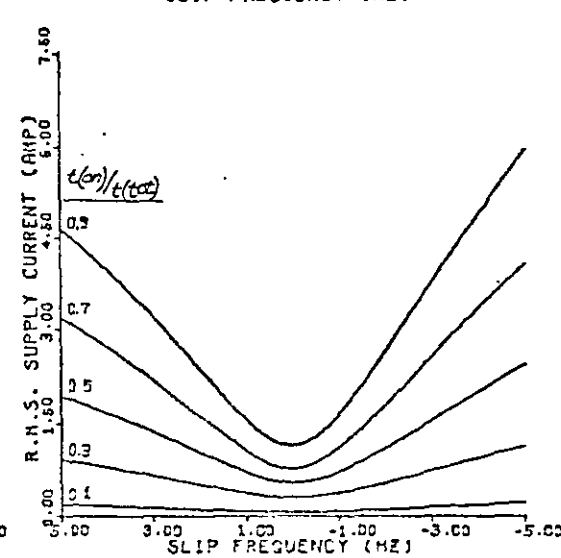
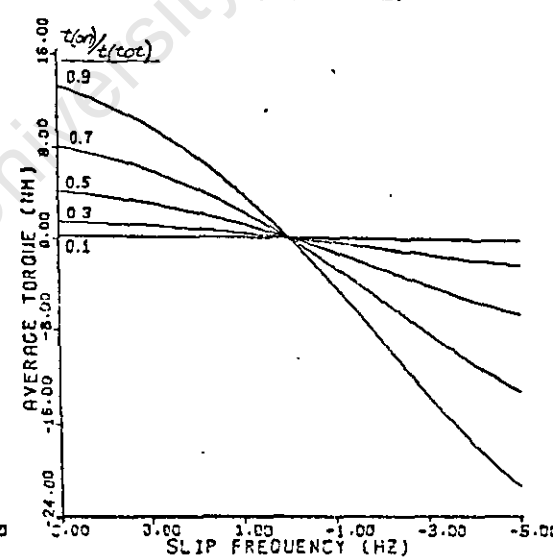
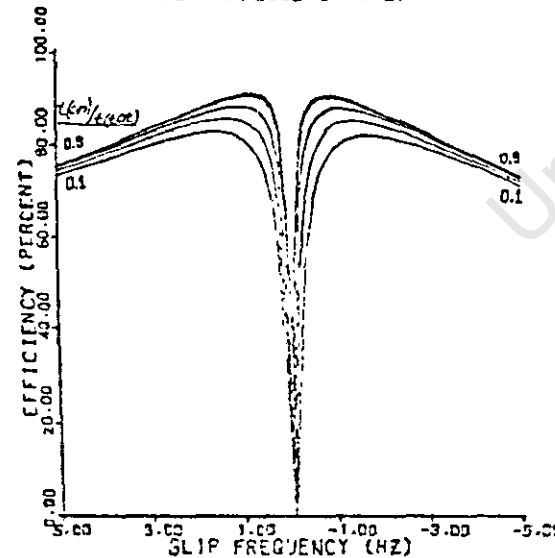
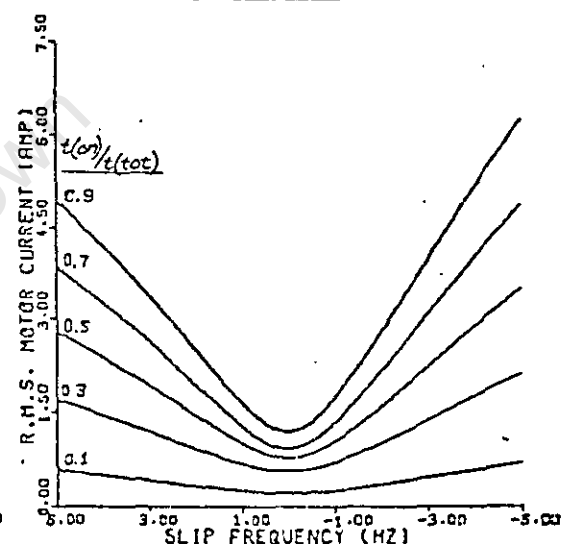
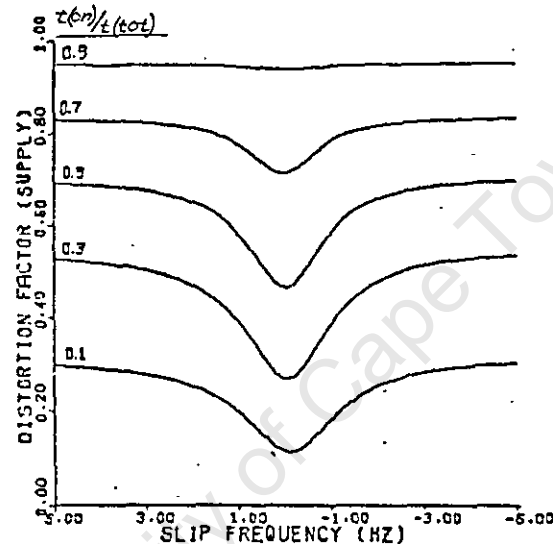
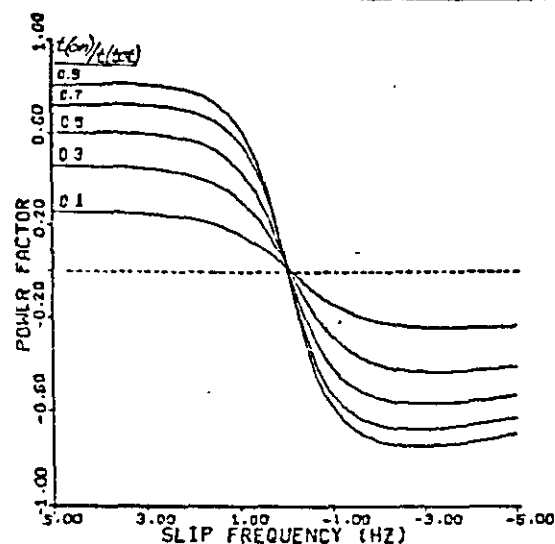
In a few cases, results have also been calculated for different supply frequencies although in most instances, the rated motor frequency of 50 Hz. has been used. The graphical illustrations can be categorized as follows:-

- (a) Figs. (25) to (28) each show calculated performance as a function of slip frequency for several values of mark-to-total time ratio (T_{on}/T_{total}). Each of these figures is for a different set value of chopping frequency.
- (b) Figs. (29) to (34) each show calculated performance as a function of chopping frequency for several values of T_{on}/T_{total} . Each of these figures is for a different set slip frequency.

Figs. (35) to (36) and Figs. (37) to (38) are similar, but for different supply frequencies.

- (c) Figs. (39) to (43) show calculated performance as a function of $T(\text{on})/T(\text{total})$ for several chopping frequencies and for each of two slip frequencies. (In these particular calculations no account has been taken of the effect of saturation on the inductive parameters of the motor).
- (d) Figs (44) to (48) are a separate illustration of the torque pulsation amplitudes which have been calculated for the test motor. These are displayed as functions of slip frequency, $T(\text{on})/T(\text{total})$ and chopping frequency.
- (e) Figs. (49) to (53) illustrate the first five components of the spectra of the motor and supply currents calculated from the algebraic solution and shown as functions of the main parameters.
- (f) Figs. (54) to (63) present spectra of motor and supply currents and torque which are algebraically derived. These are displayed adjacent to the corresponding time domain waveforms which were obtained by numerical simulation. Figs (54) to (57) represent different chopping frequencies for $T(\text{on})/T(\text{total}) = 0.8$ and slip frequency = 2.5 Hz. Figs. (58) to (62) are similar but with $T(\text{on})/T(\text{total}) = 0.5$. Fig (63) is a single illustration at reduced supply frequency.
- (g) Figs. (64) to (65) show the spectra of motor and supply current and torque calculated for a set value of negative slip (regeneration).

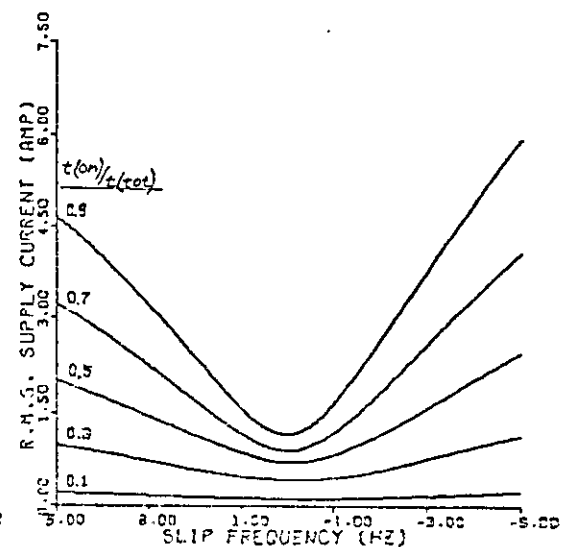
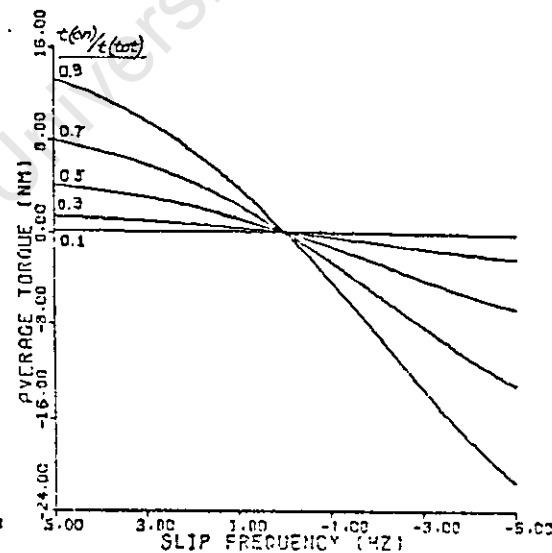
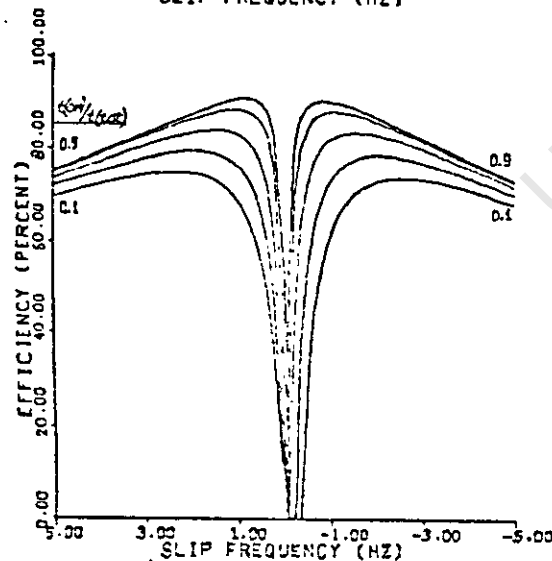
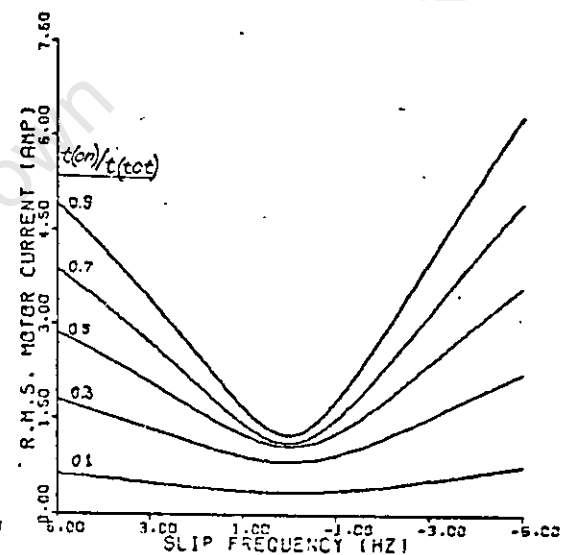
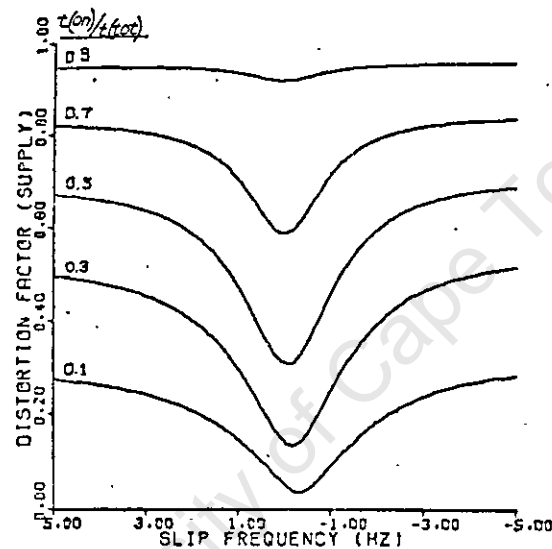
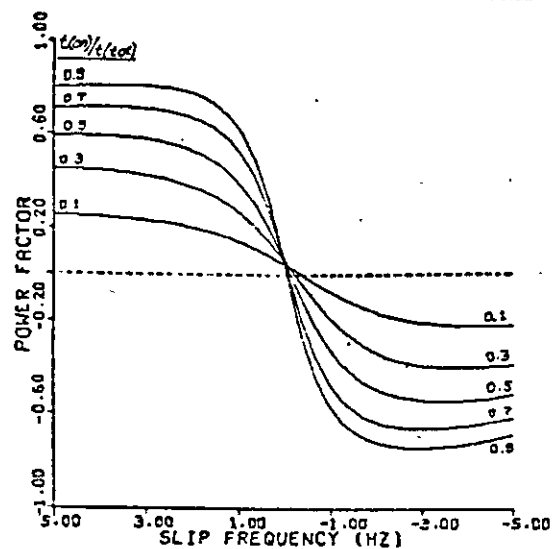
CHOPPER MODULATION
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION OF THE MOTOR EQUATIONS.



CONVERTOR STEPS -0.0 DEGREES, CHOPS PER STEP=1, STEPPING FREQUENCY (HZ)=550.0

SUPPLY VOLTAGE = 220 VOLT RMS 50 HZ.

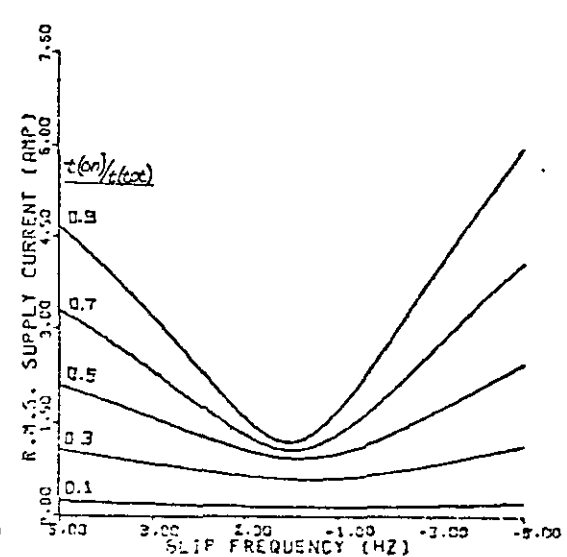
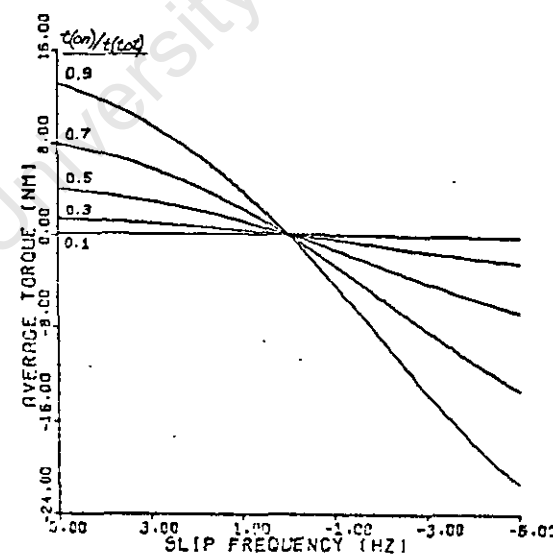
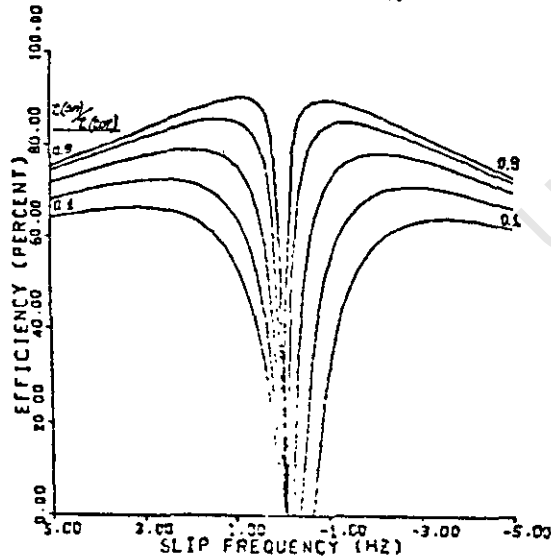
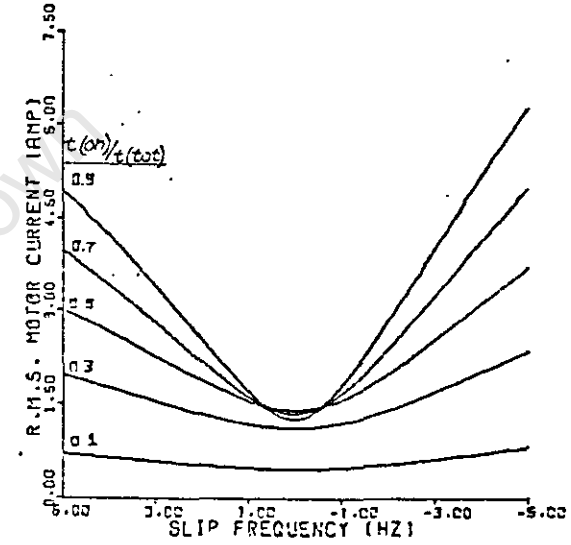
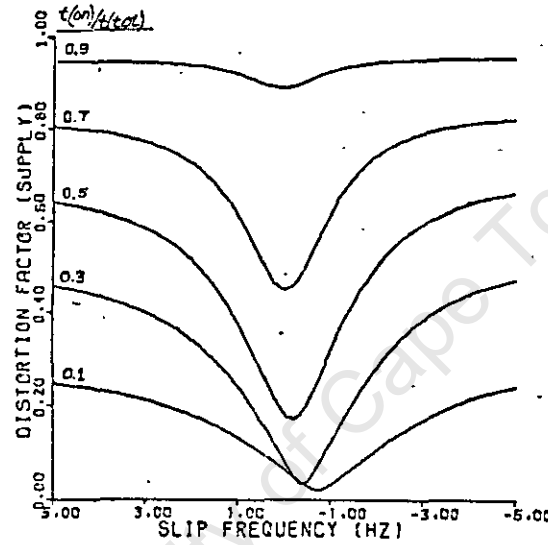
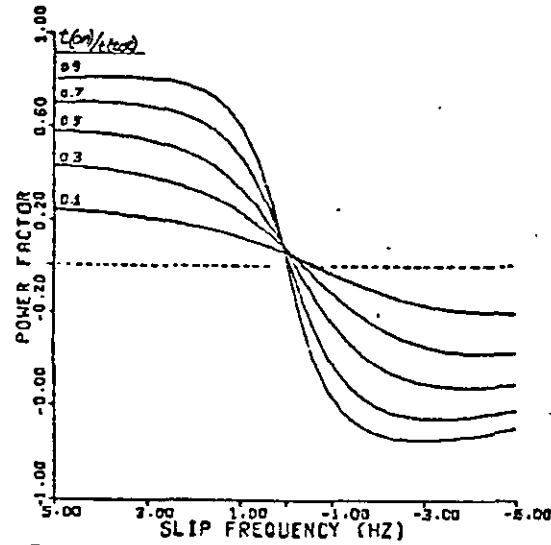
CHOPPER MODULATION
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION OF THE MOTOR EQUATIONS.



CONVERTOR STEPS -0.0 DEGREES, CHOPS PER STEP=1., STEPPING FREQUENCY (HZ)=350.0

SUPPLY VOLTAGE = 220 VOLT RMS 50 HZ.

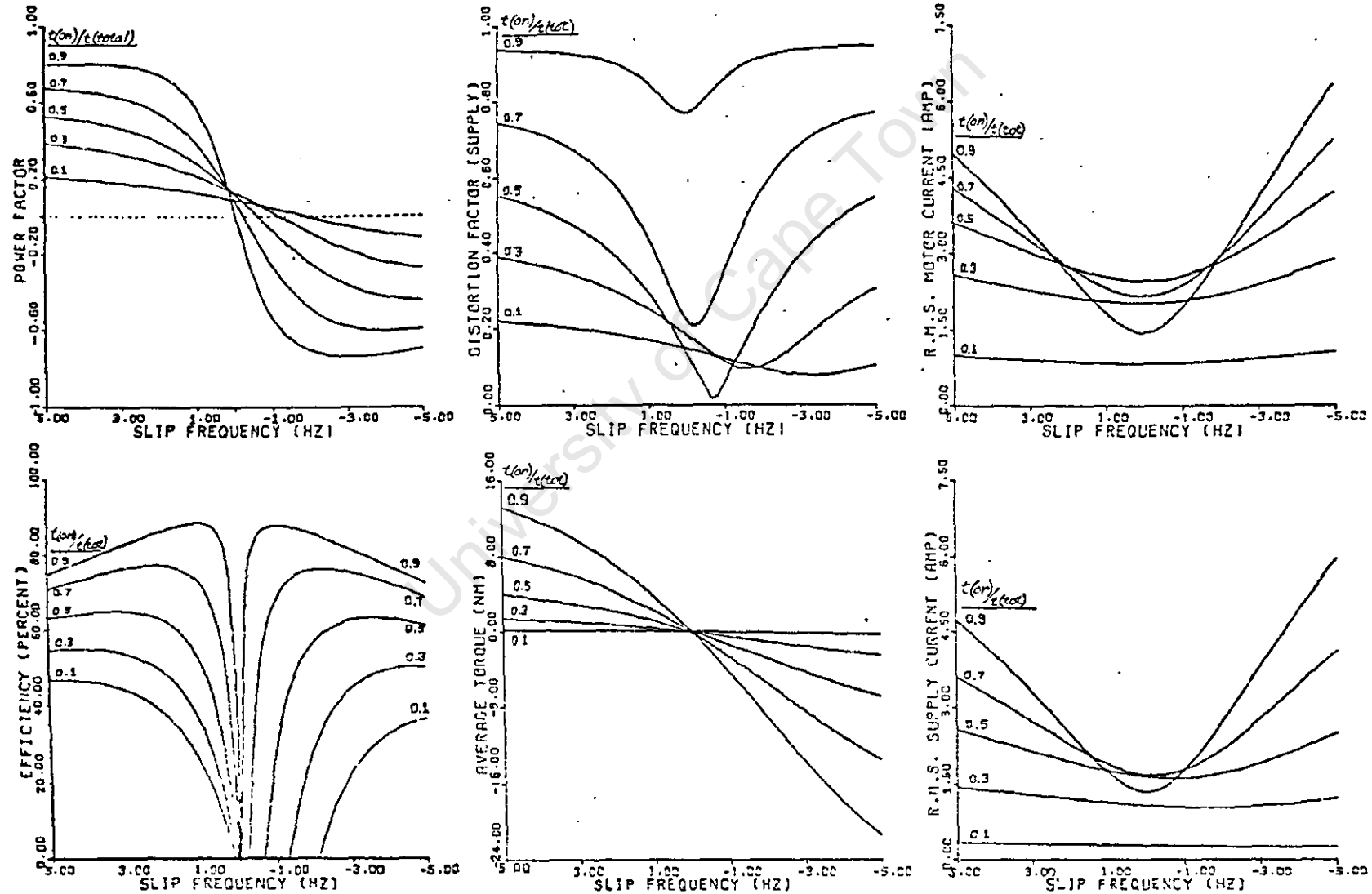
CHOPPER MODULATION
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION OF THE MOTOR EQUATIONS.



CONVERTOR STEPS -0.0 DEGREES , CHOPS PER STEP=1. , STEPPING FREQUENCY (HZ)=250.0

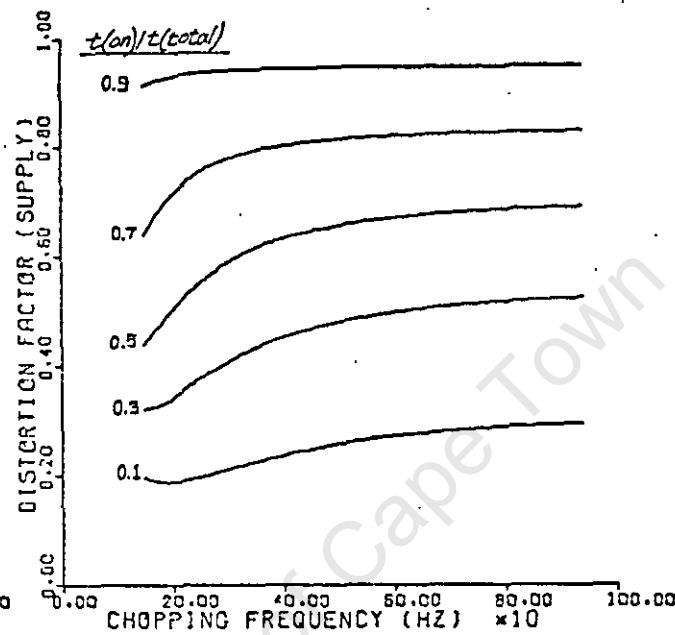
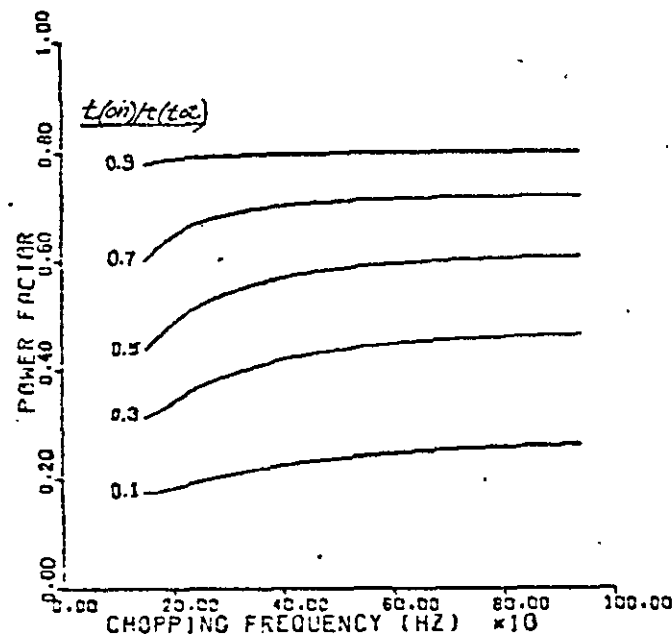
SUPPLY VOLTAGE = 220 Volt RMS 50 Hz.

CHOPPER MODULATION
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION OF THE MOTOR EQUATIONS.



CONVERTOR STEPS -0.0 DEGREES , CHOPS PER STEP=1. . STEPPING FREQUENCY (HZ)=150.0

SUPPLY VOLTAGE = 220 V/L RMS 50 HZ.



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$V_s = 220$ VOLT 50 HZ.

SLIP FREQUENCY = 2.5 HZ.

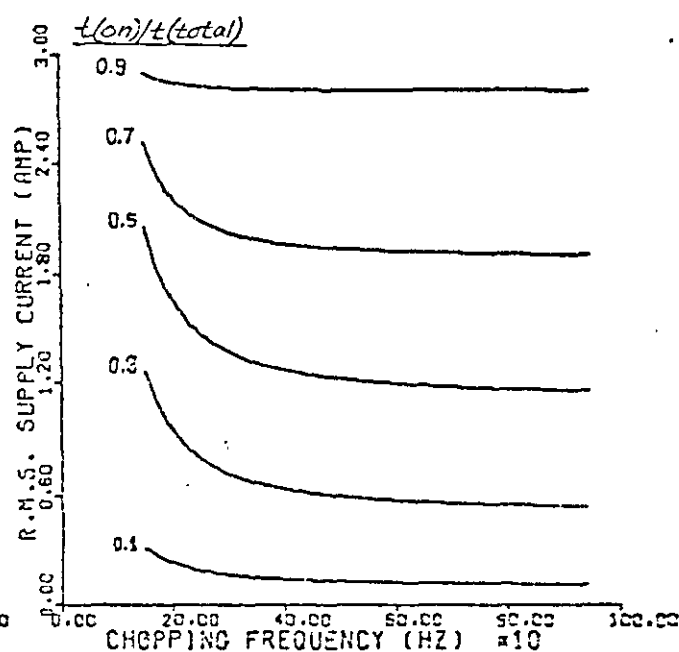
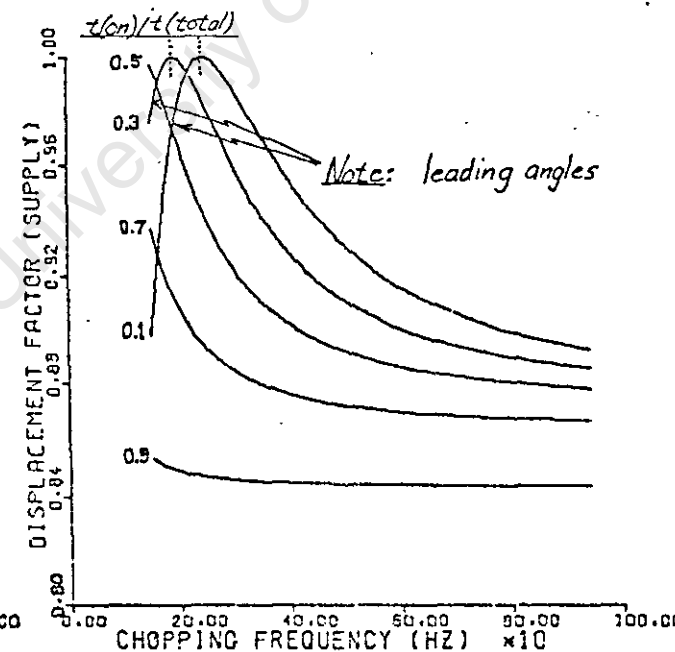
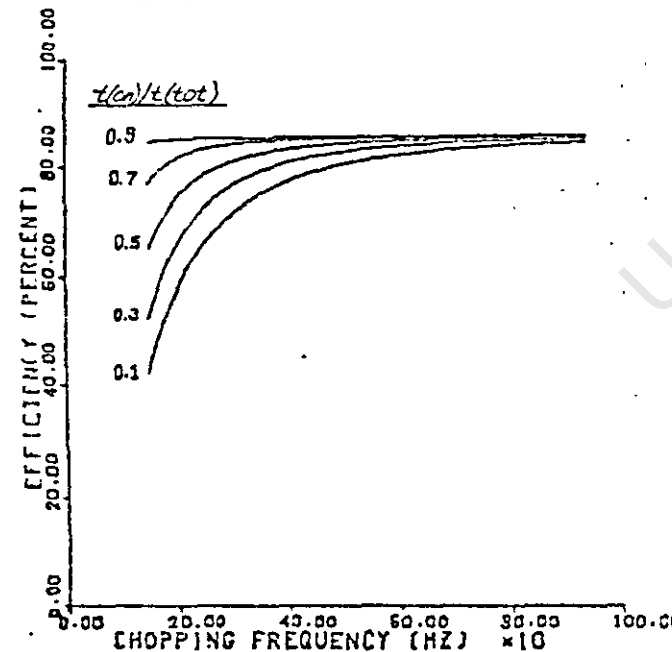
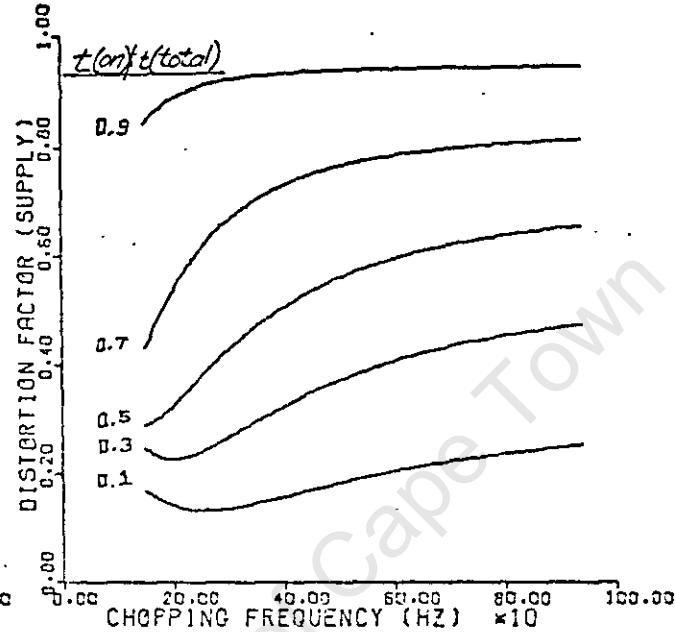
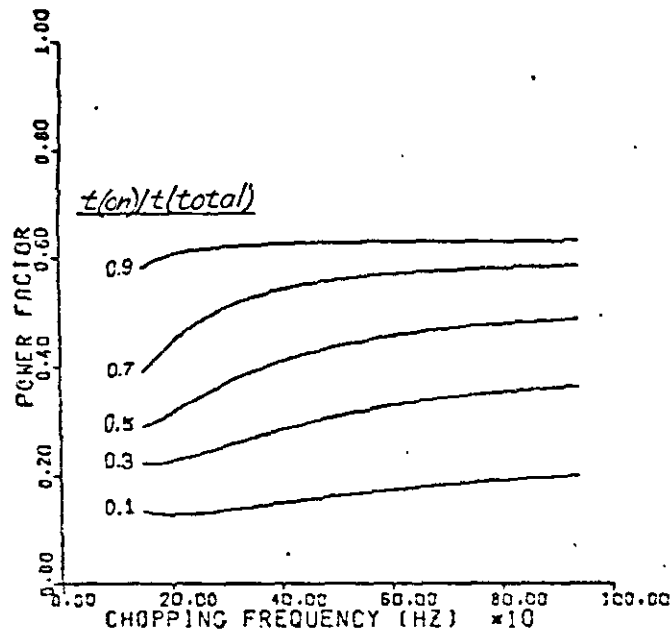


Fig. 30



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$V_s = 220$ VOLT RMS 50 HZ.

SLIP FREQUENCY = 1 HZ.

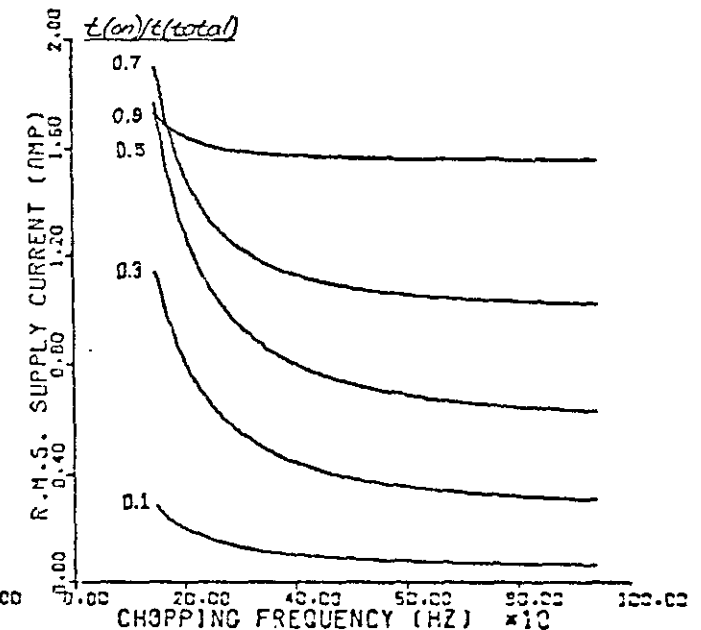
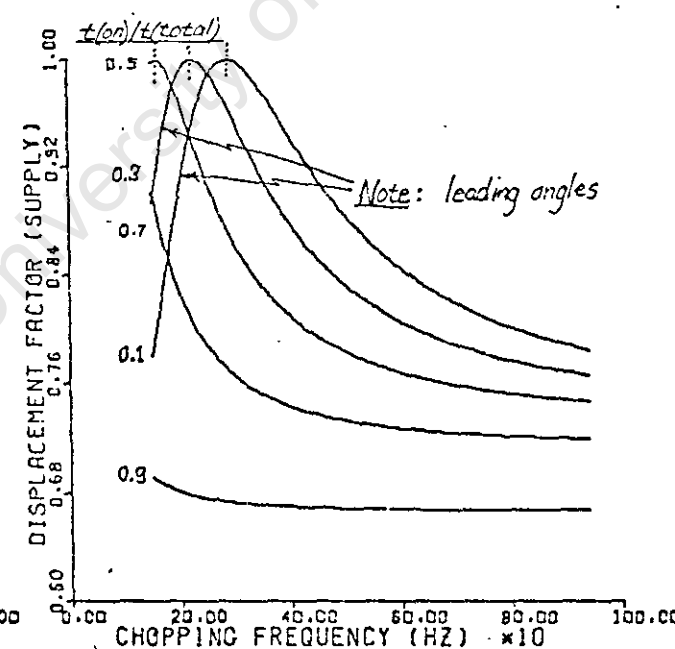
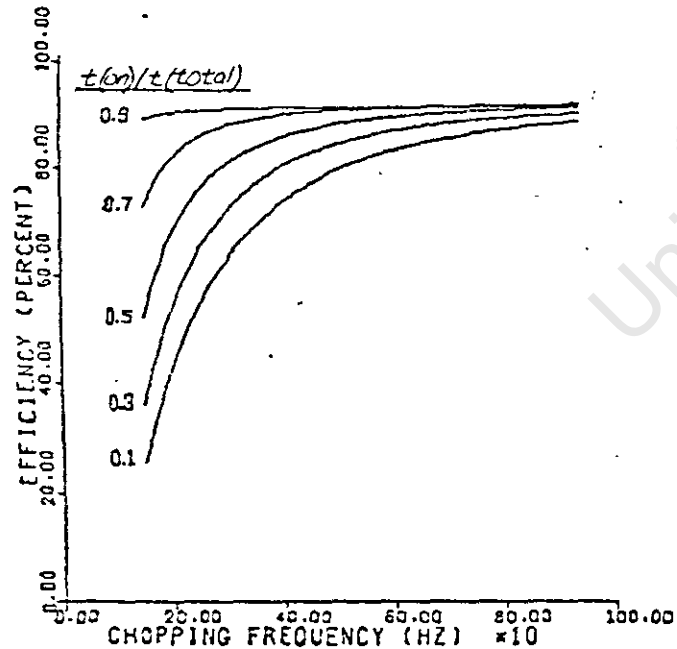
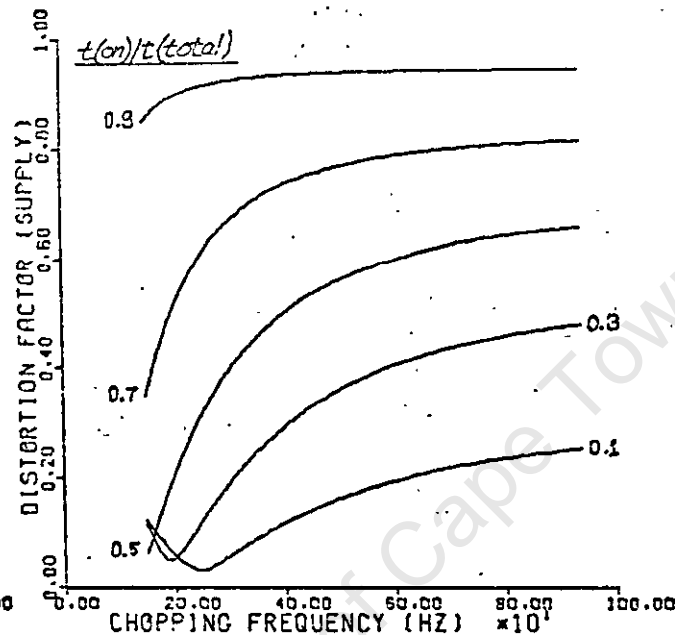
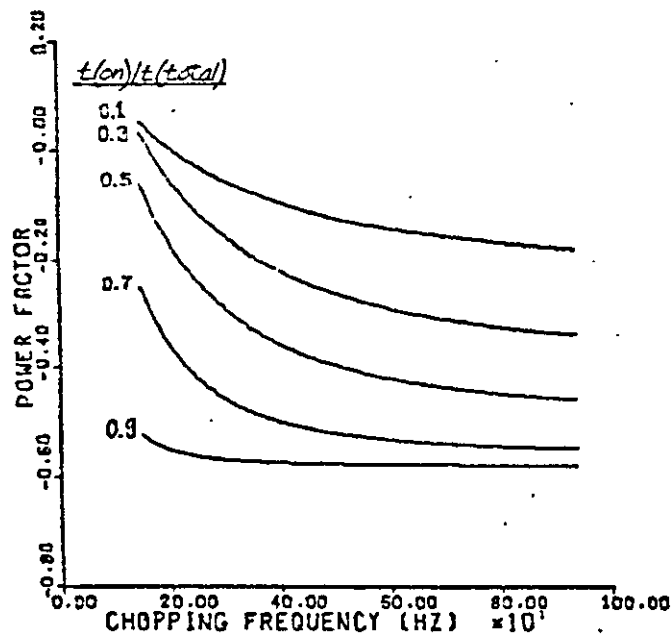


Fig.31



CHOPPER MODULATION.
 PERFORMANCE FACTORS FOR THE TEST MOTOR
 CALCULATED FROM THE ALGEBRAIC SOLUTION
 OF THE MOTOR EQUATIONS.

$V_s = 220$ VOLT RMS 50 HZ.

SLIP FREQUENCY = -1 HZ.

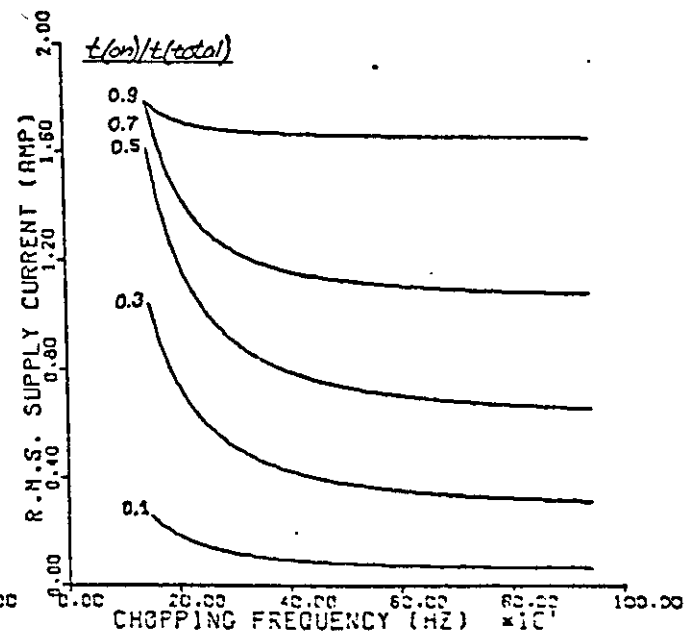
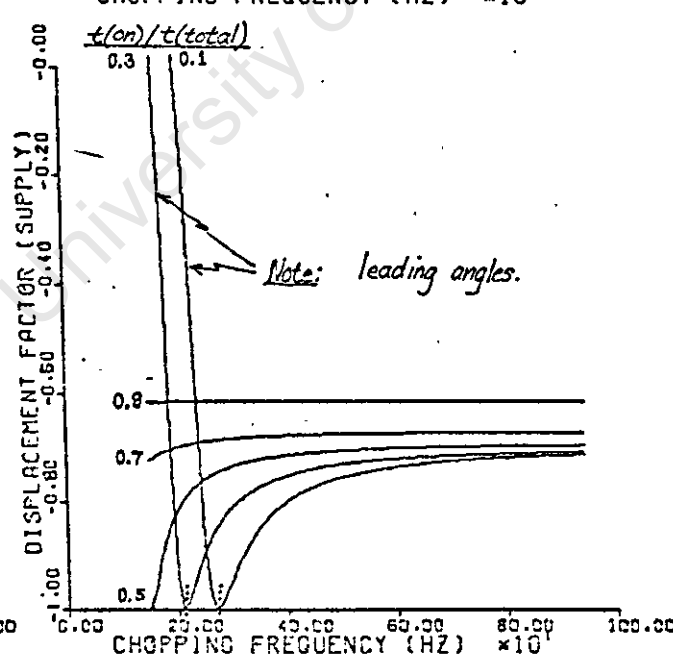
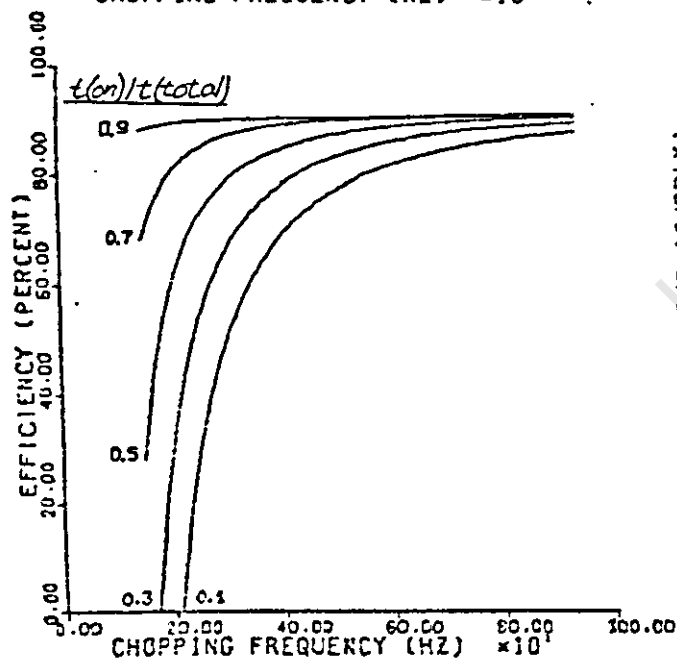
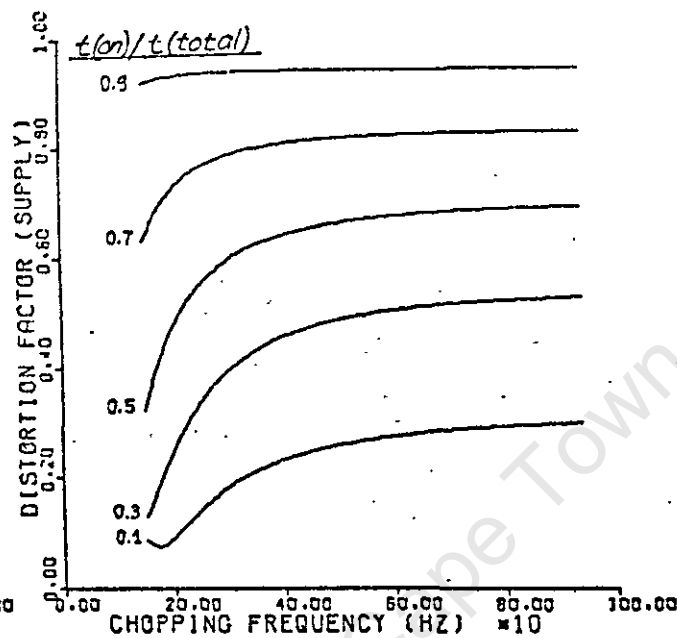
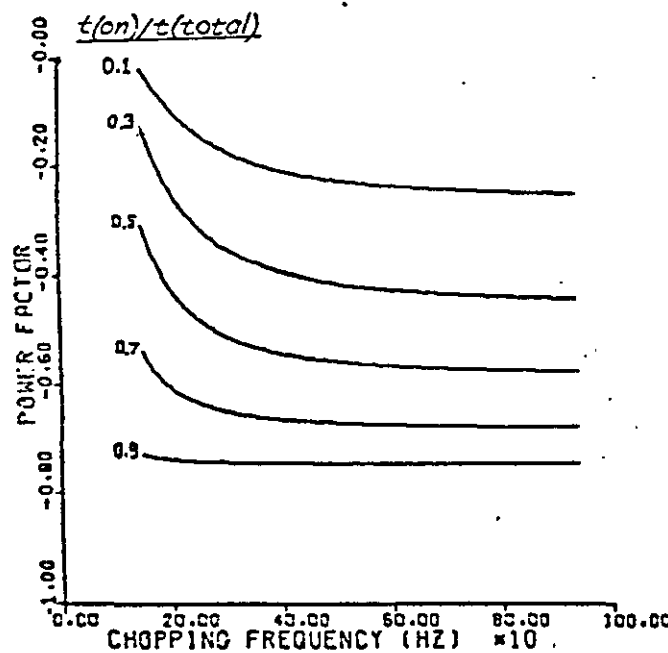


Fig. 32



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$V_s = 220$ VOLT RMS 50 HZ.

SLIP FREQUENCY = -2.5 HZ.

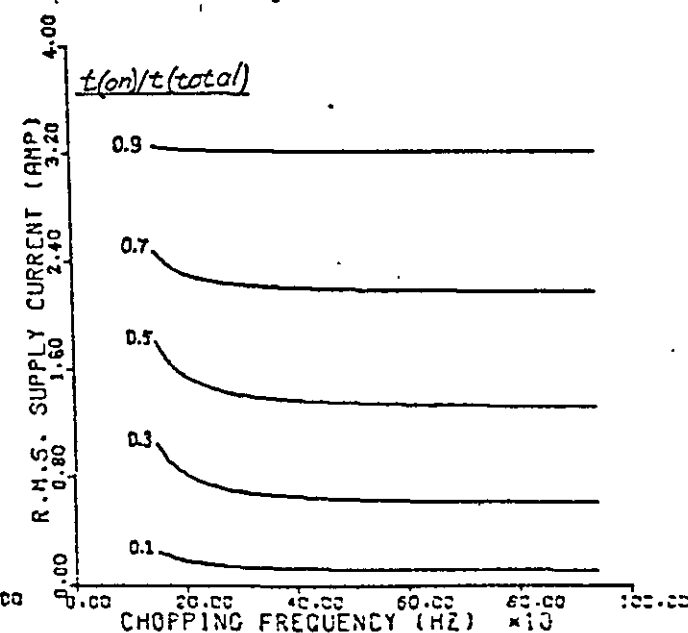
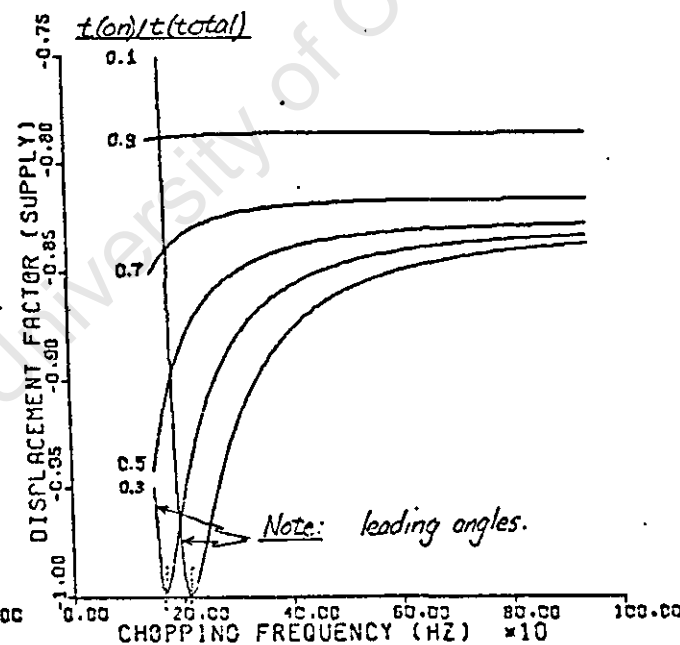
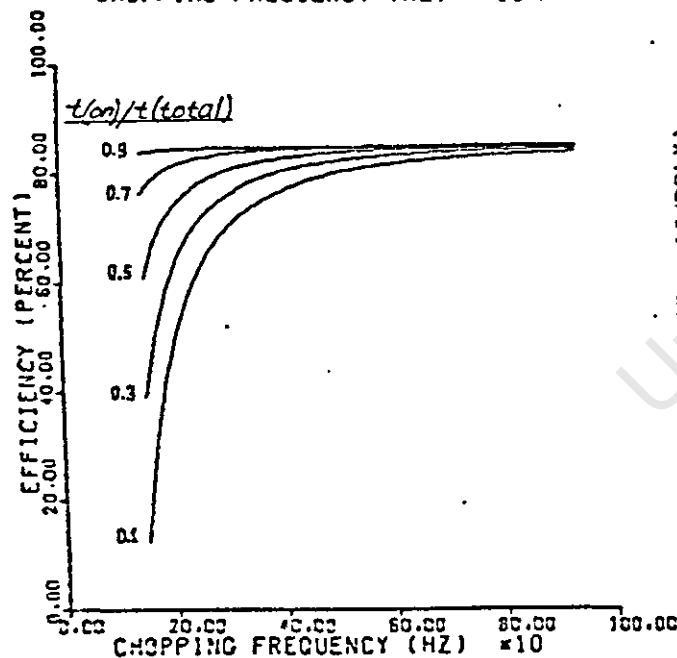
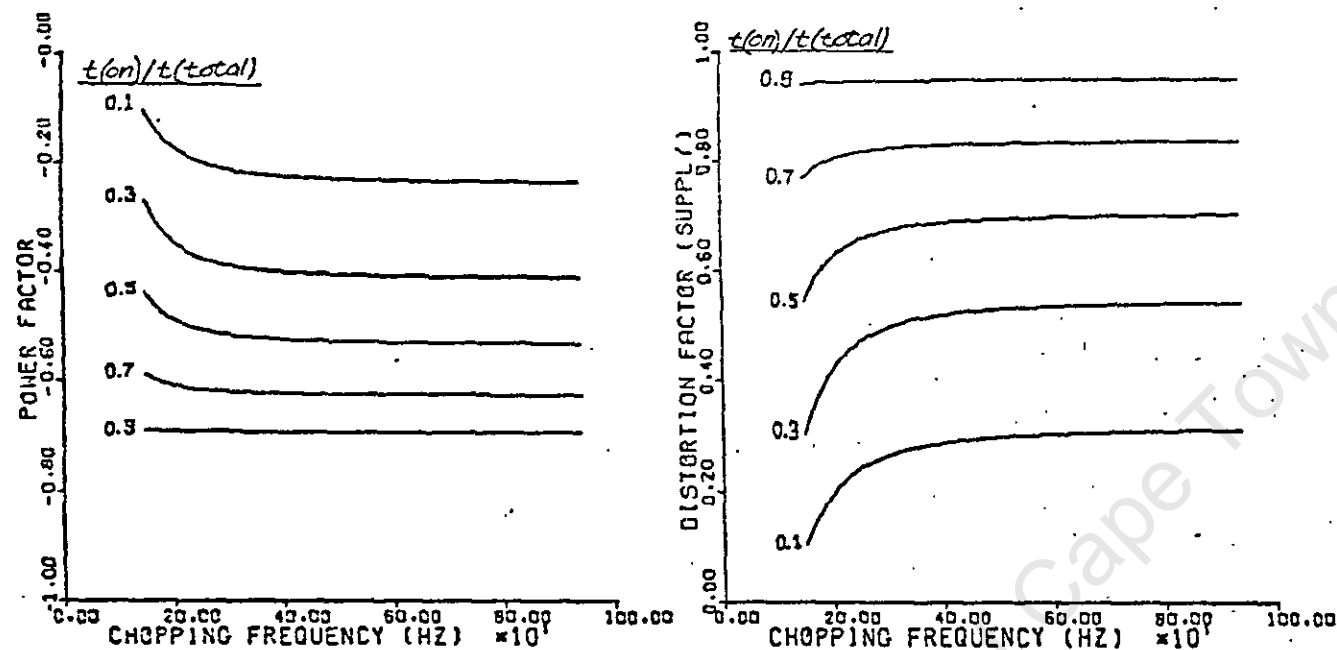


Fig. 33



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$V_s = 220$ VOLT RMS 50 HZ.

SLIP FREQUENCY = -5 HZ.

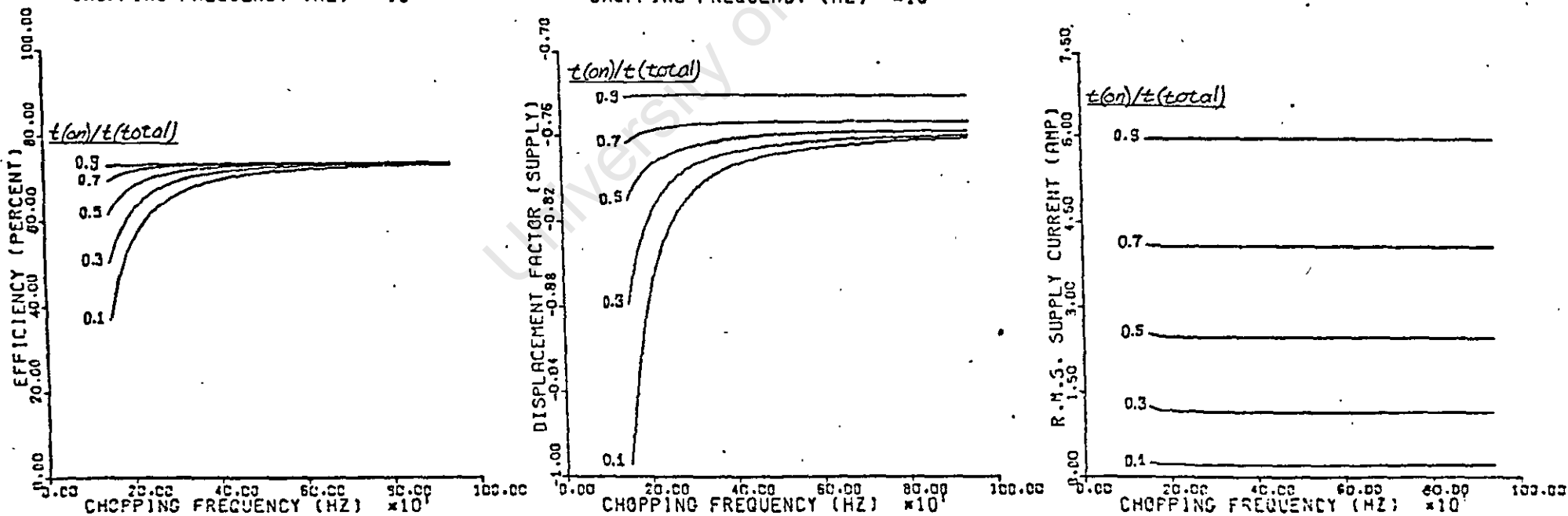
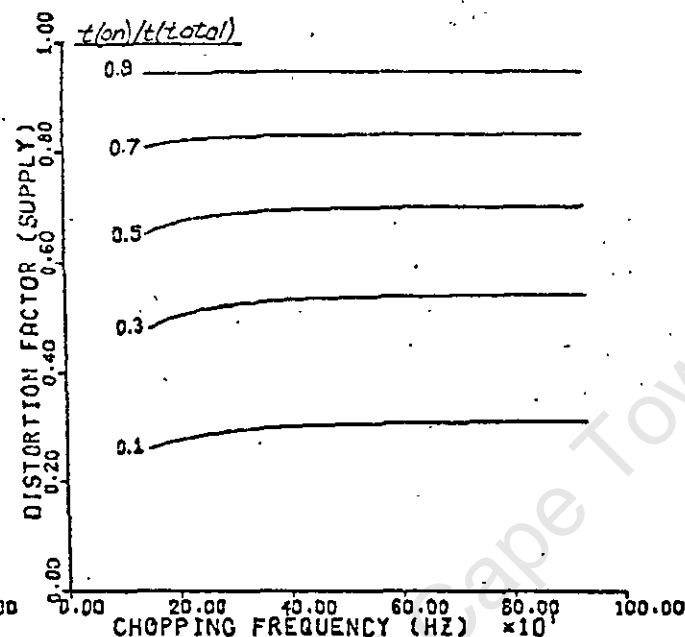
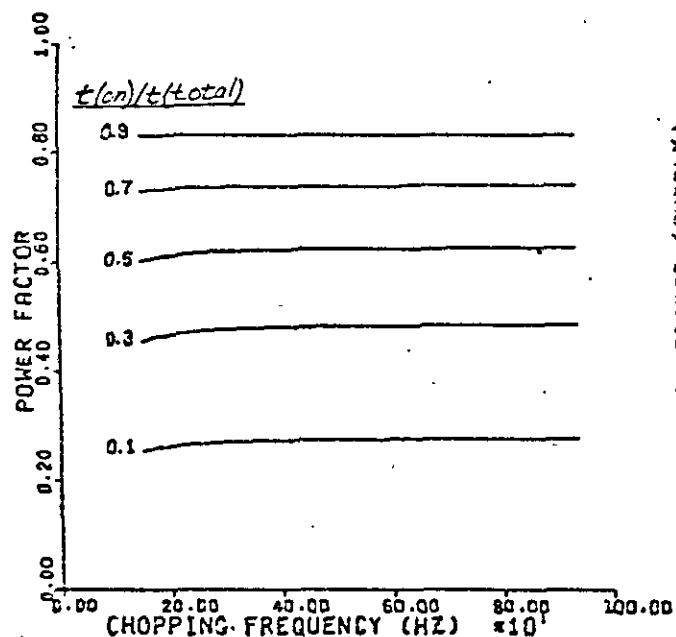


Fig. 34



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$$V_s = 110 \text{ VOLT RMS } 25 \text{ HZ.}$$

$$\text{SLIP FREQUENCY} = 5 \text{ HZ.}$$

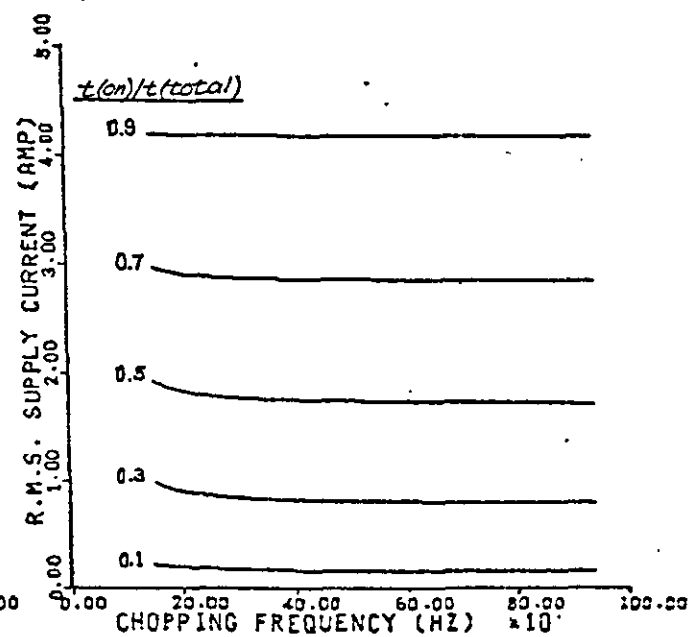
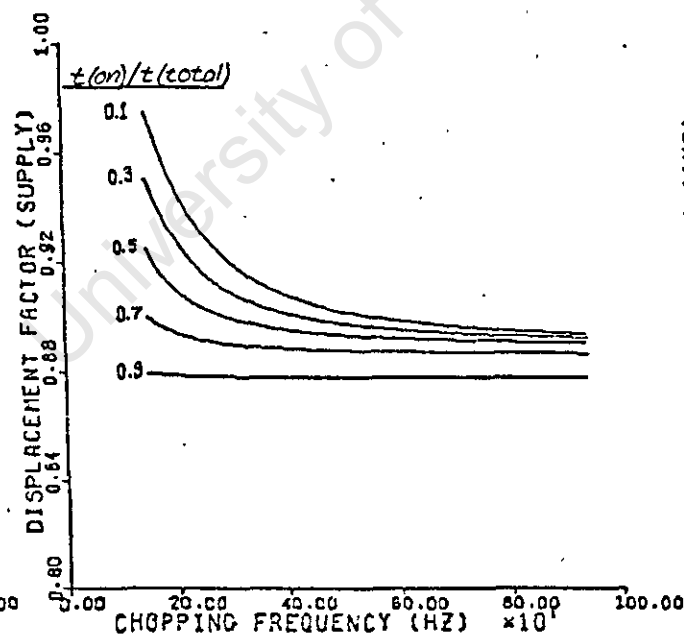
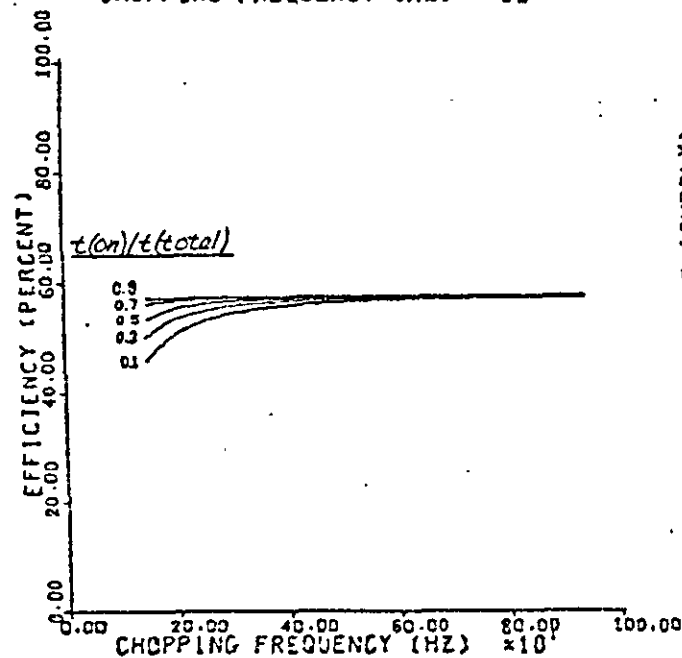
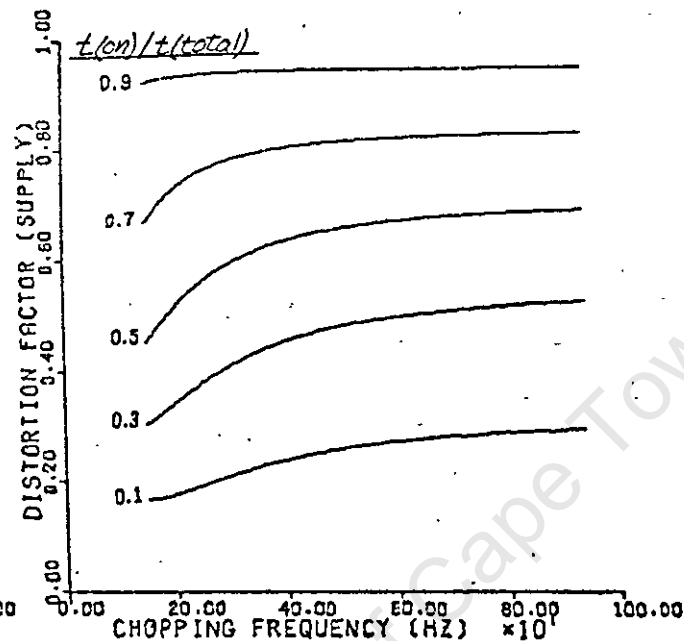
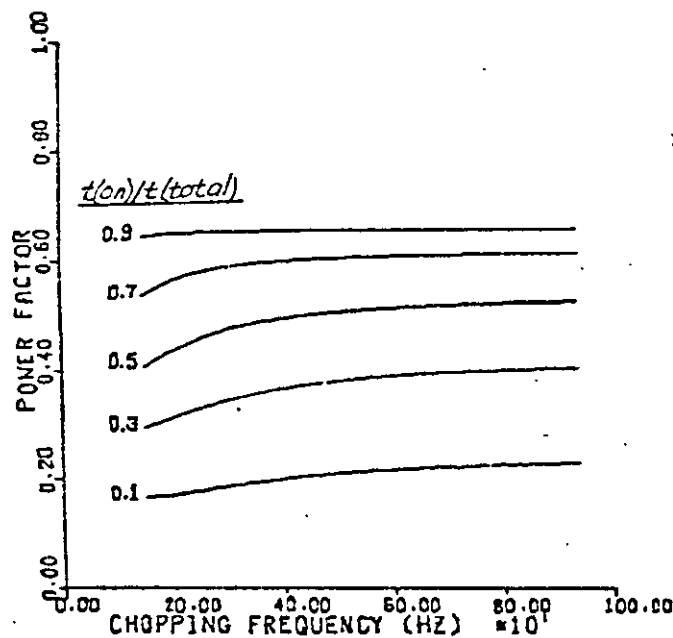


Fig. 35



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$V_s = 110$ VOLT RMS 25 HZ.

SLIP FREQUENCY = 1 HZ.

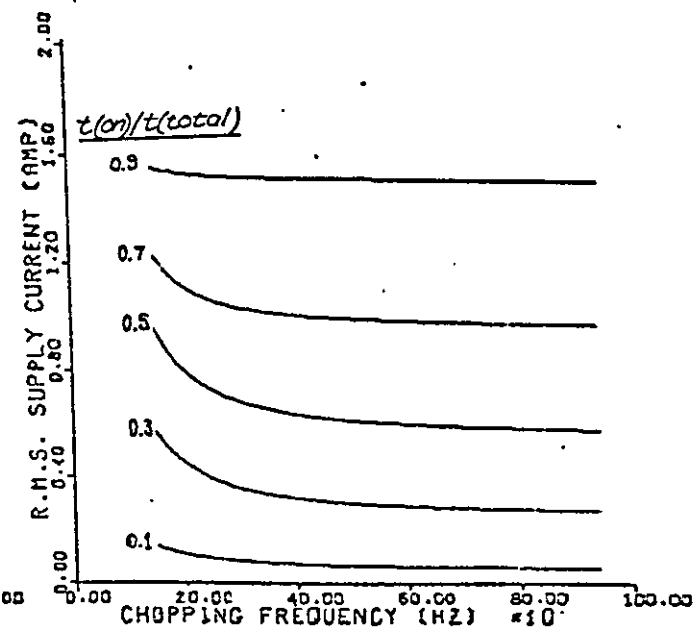
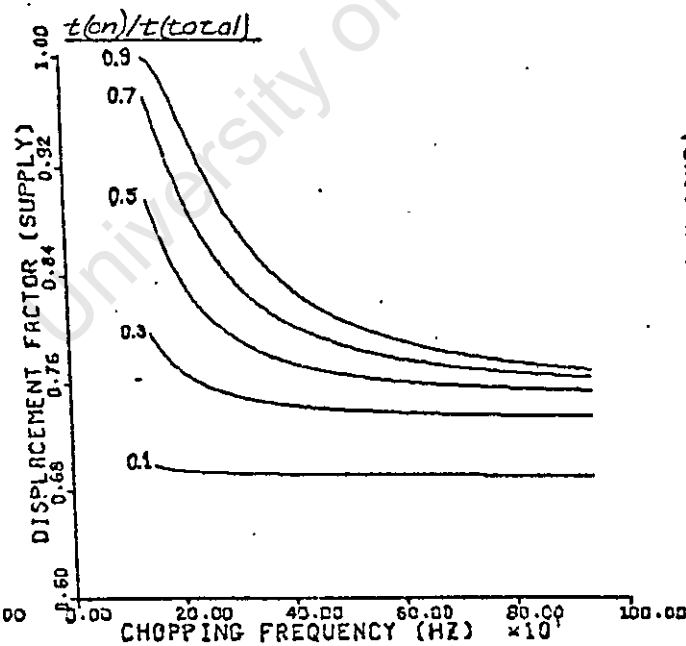
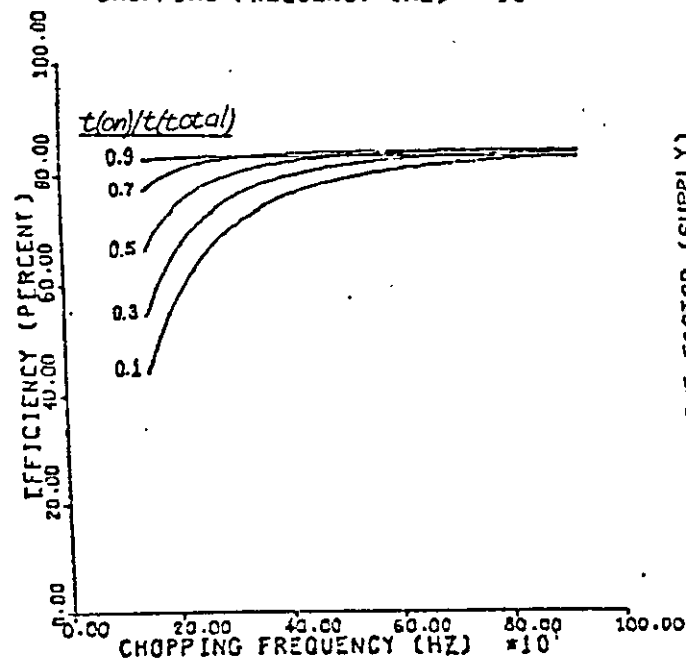
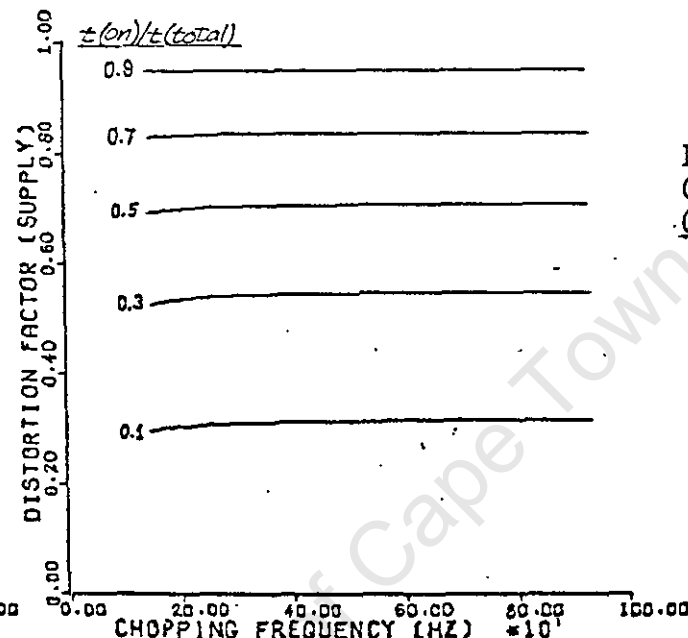
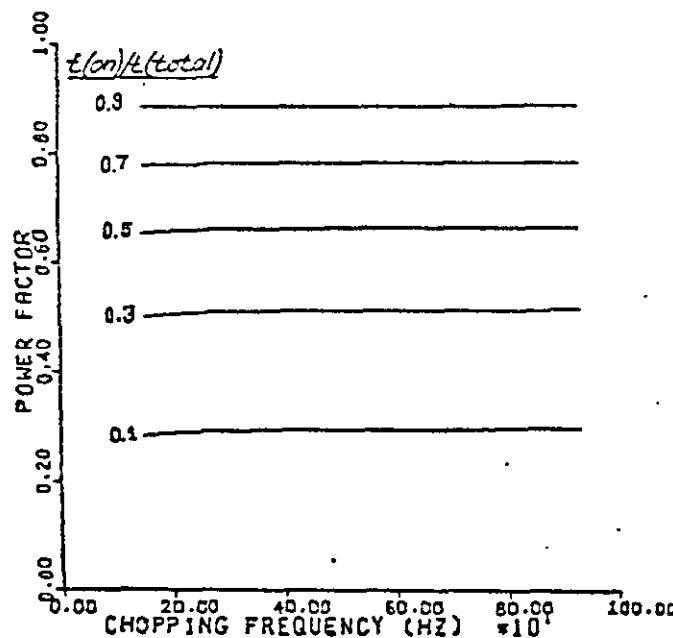


Fig. 36



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$V_s = 44$ VOLT RMS 10 HZ.

SLIP FREQUENCY = 5 HZ.

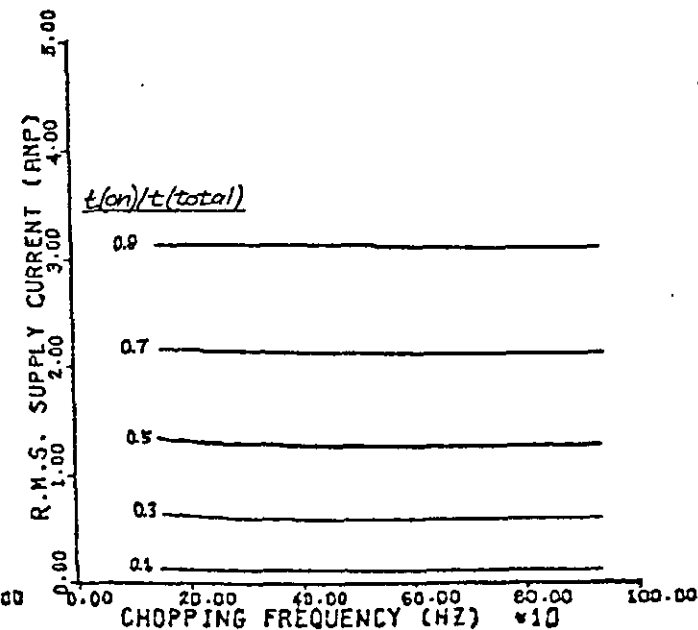
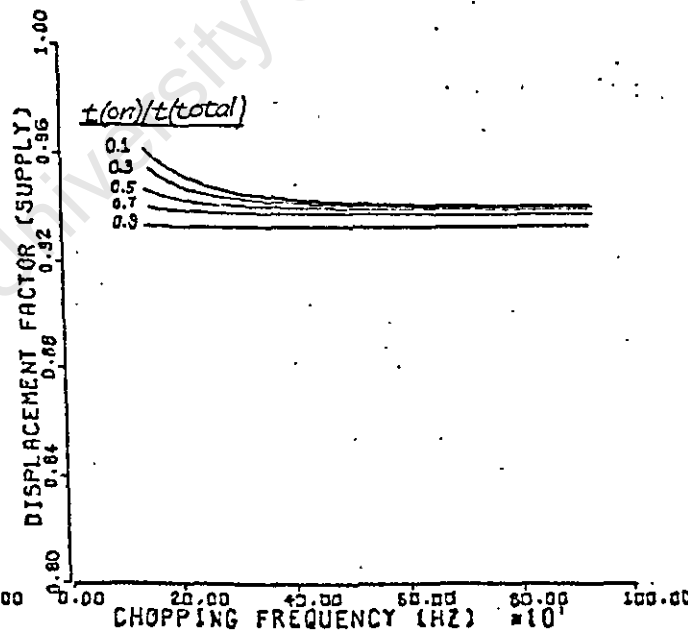
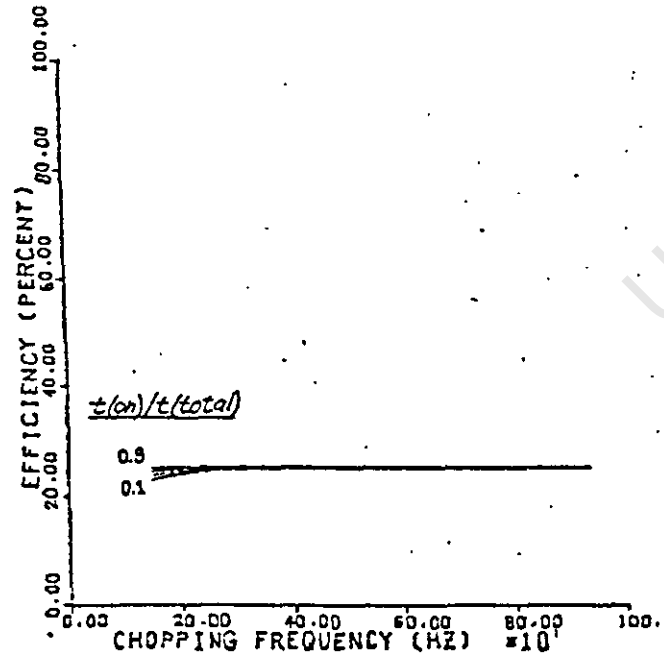
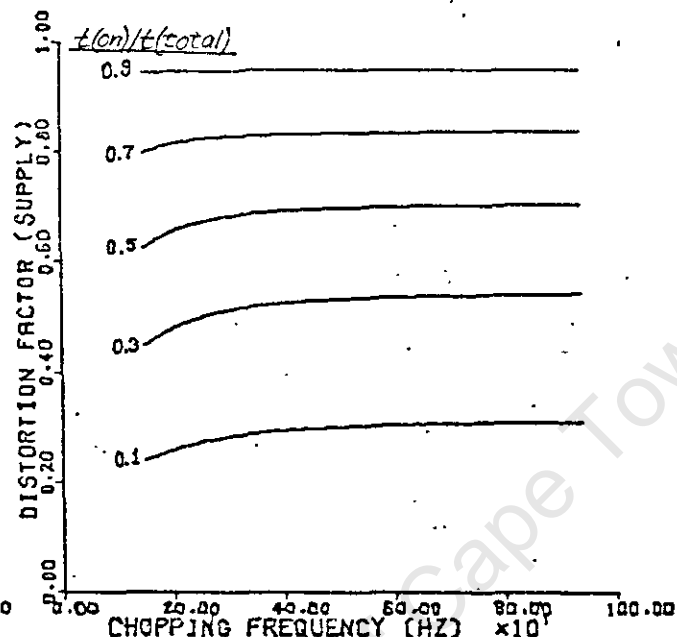
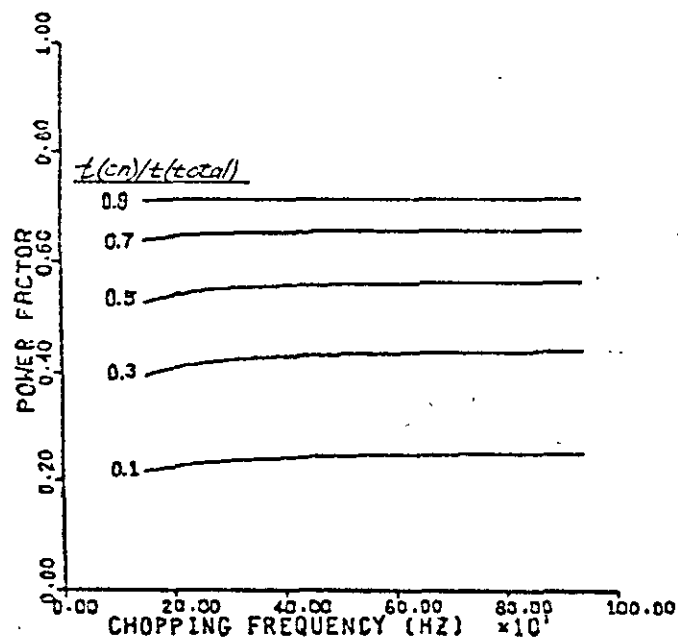


Fig. 37



CHOPPER MODULATION.

PERFORMANCE FACTORS FOR THE TEST MOTOR
CALCULATED FROM THE ALGEBRAIC SOLUTION
OF THE MOTOR EQUATIONS.

$V_s = 44$ VOLT RMS 10 HZ.

SLIP FREQUENCY = 1 HZ.

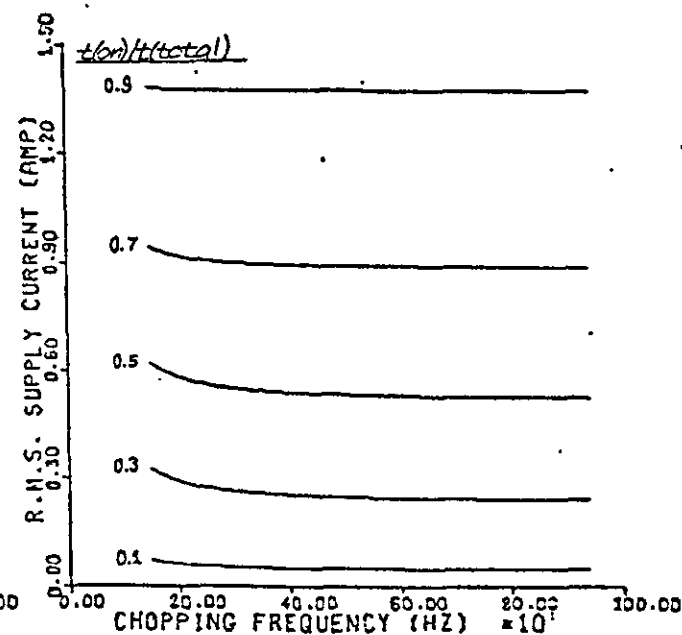
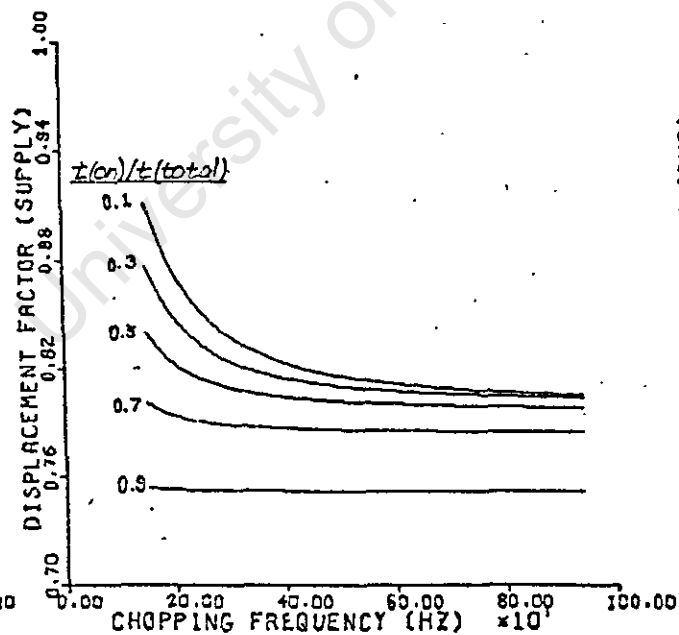
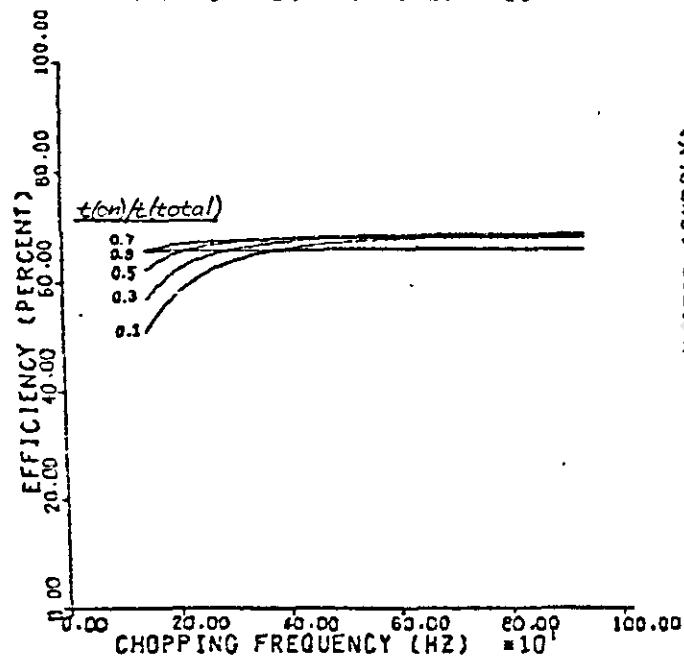


Fig. 38

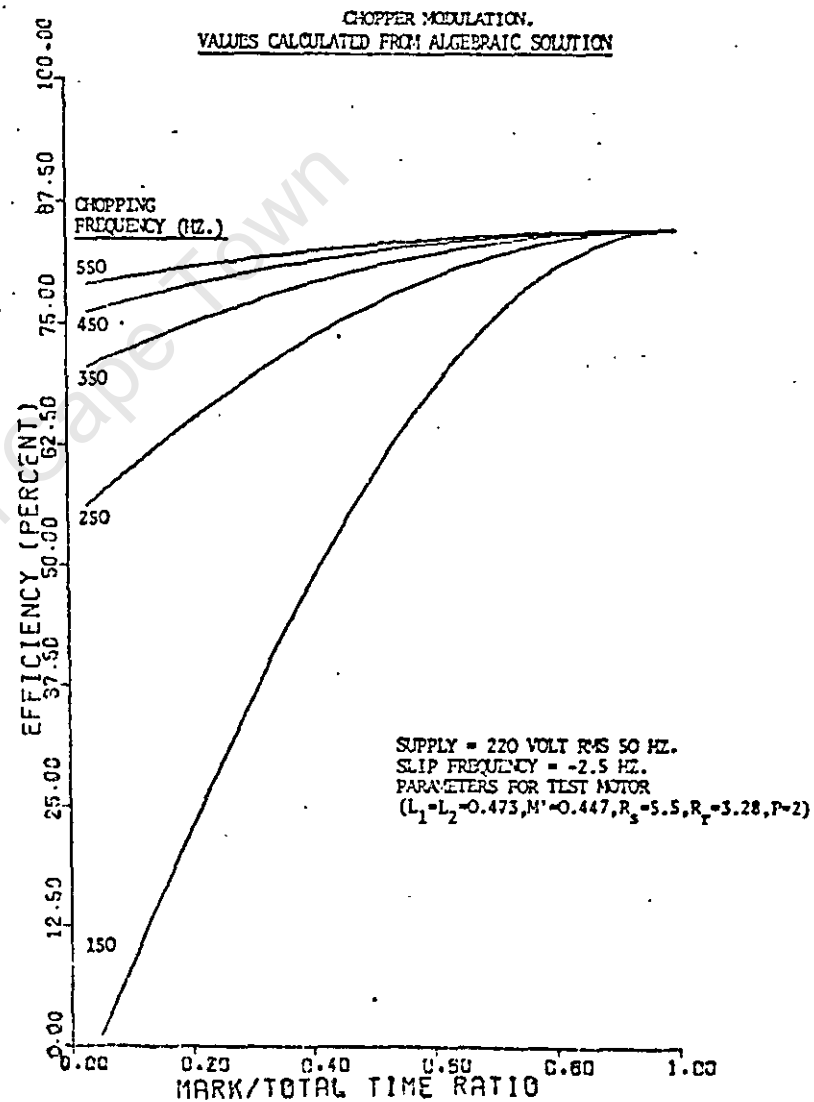
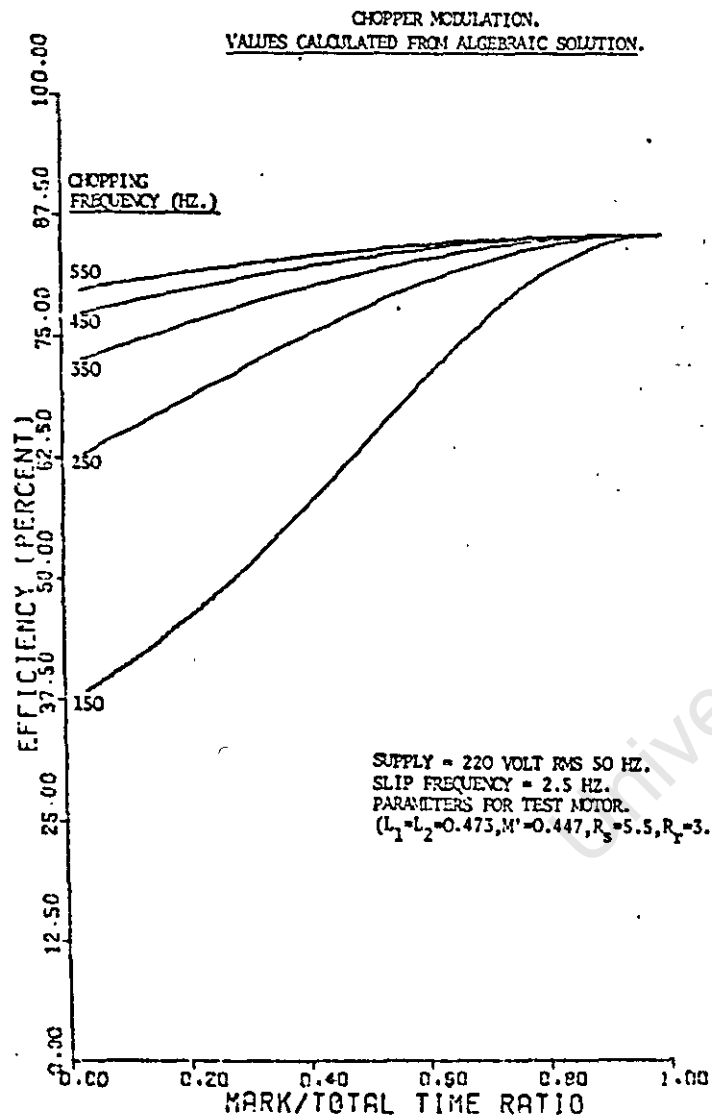


Fig. 39

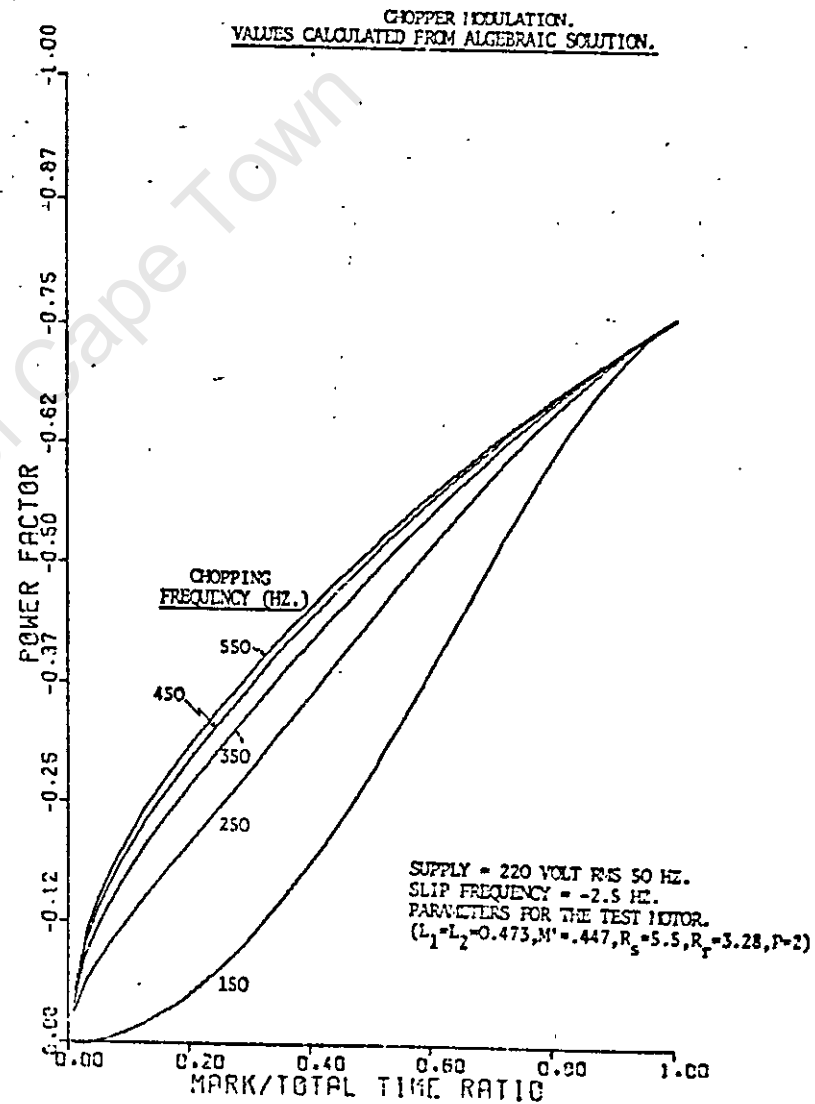
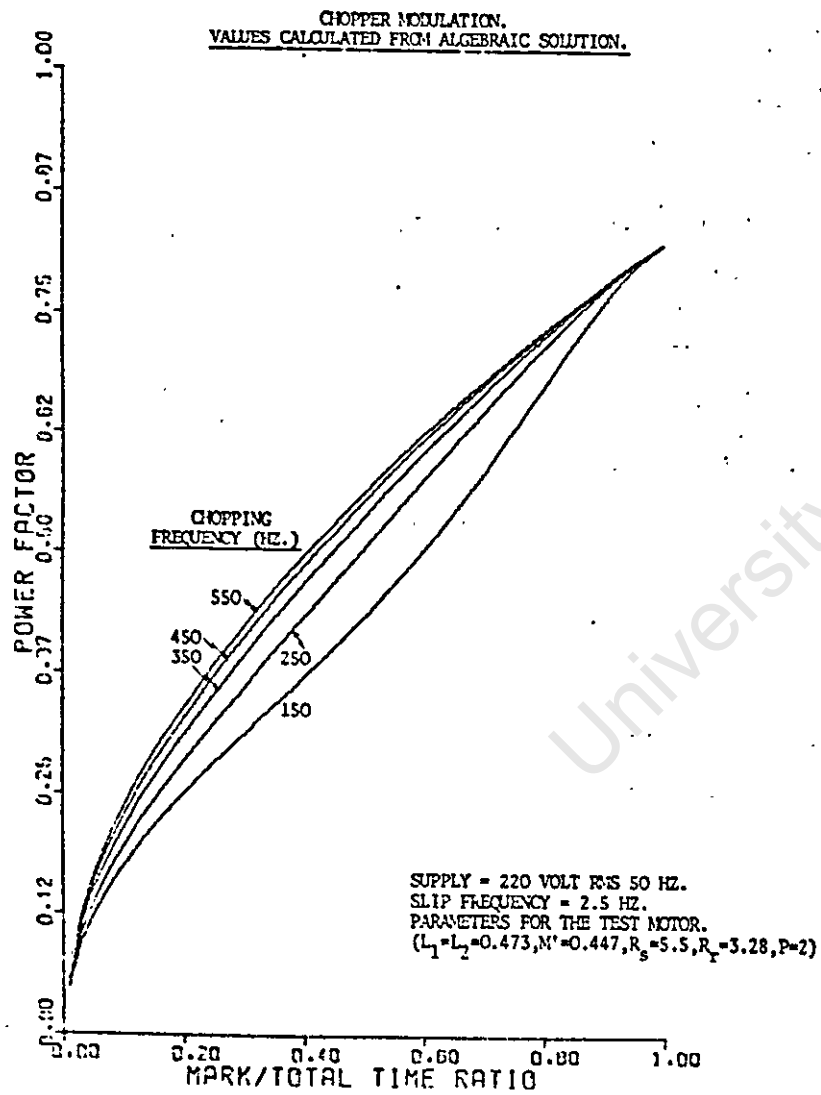


Fig. 40

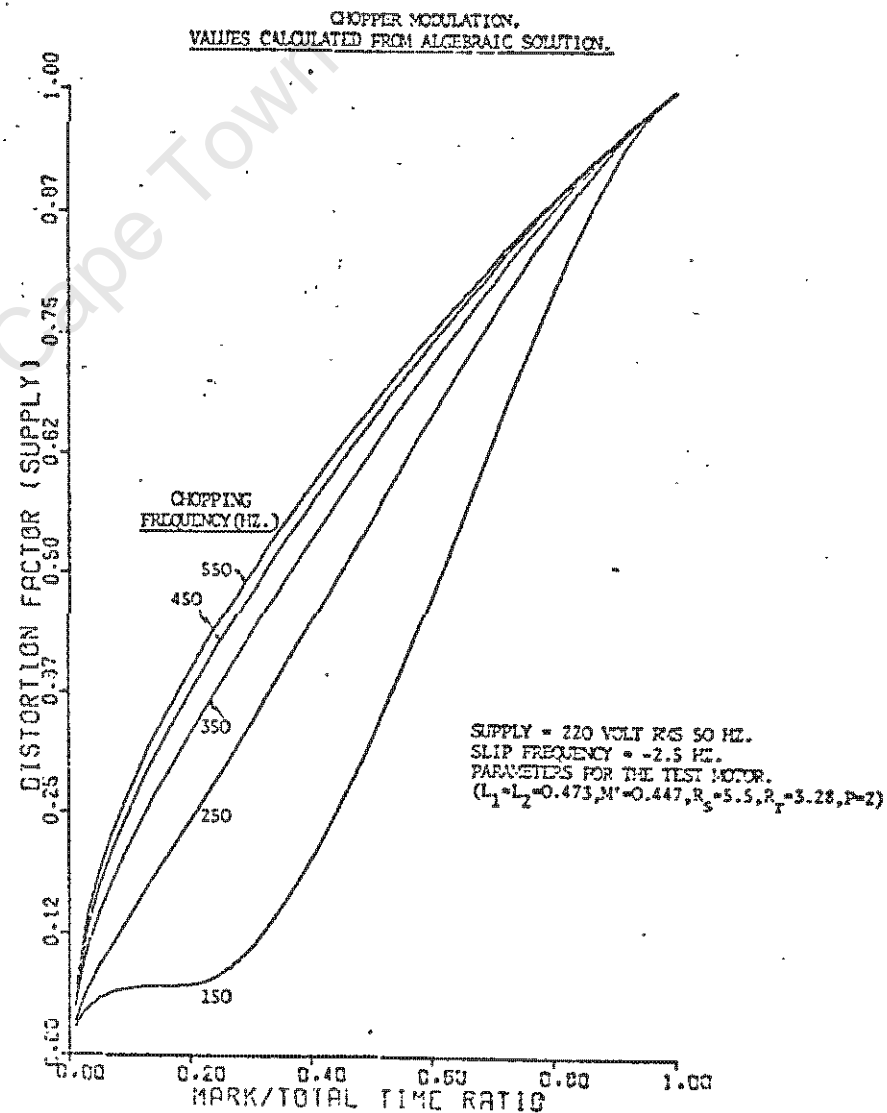
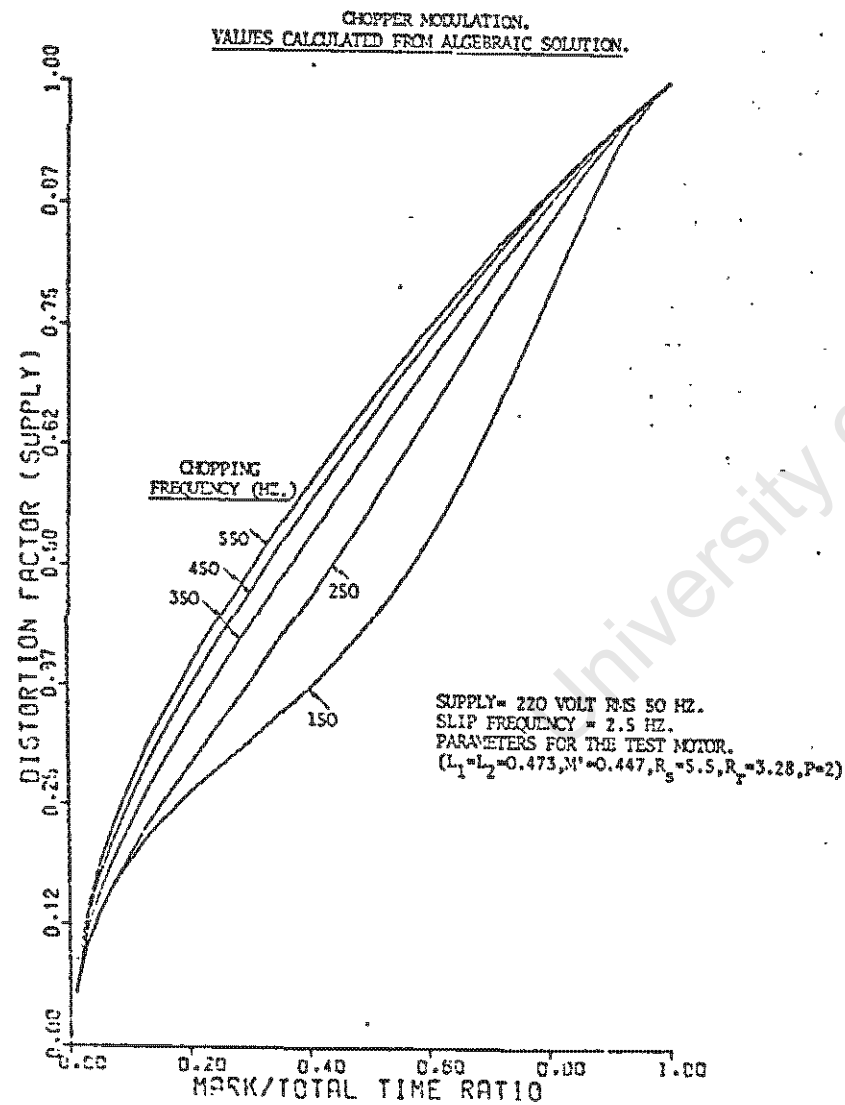


Fig. 4/

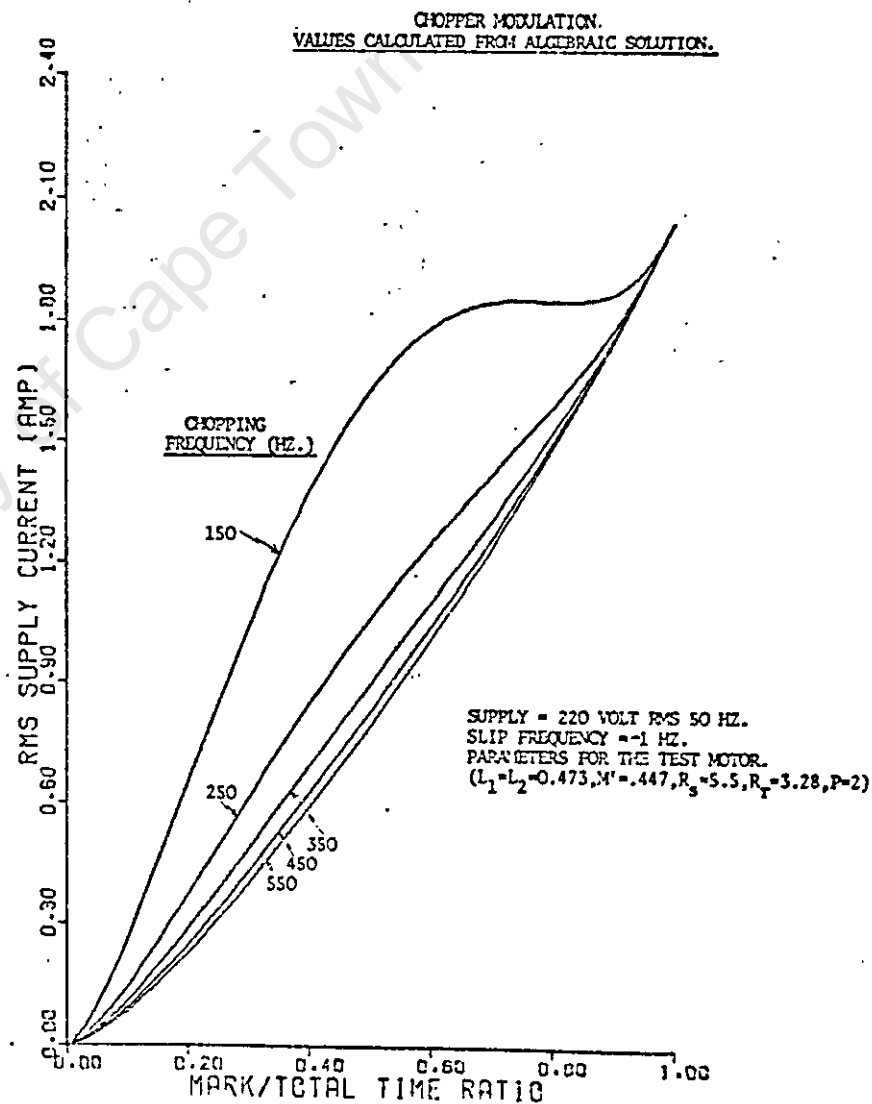
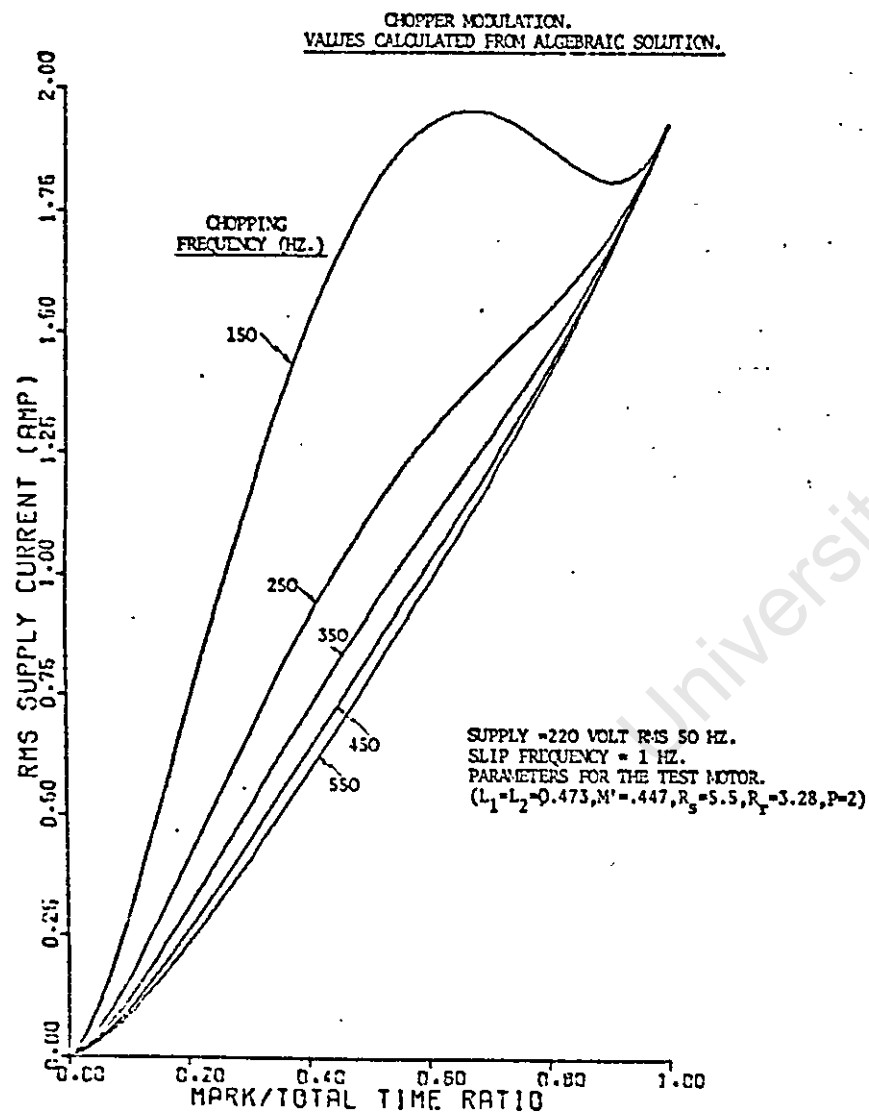


Fig. 42

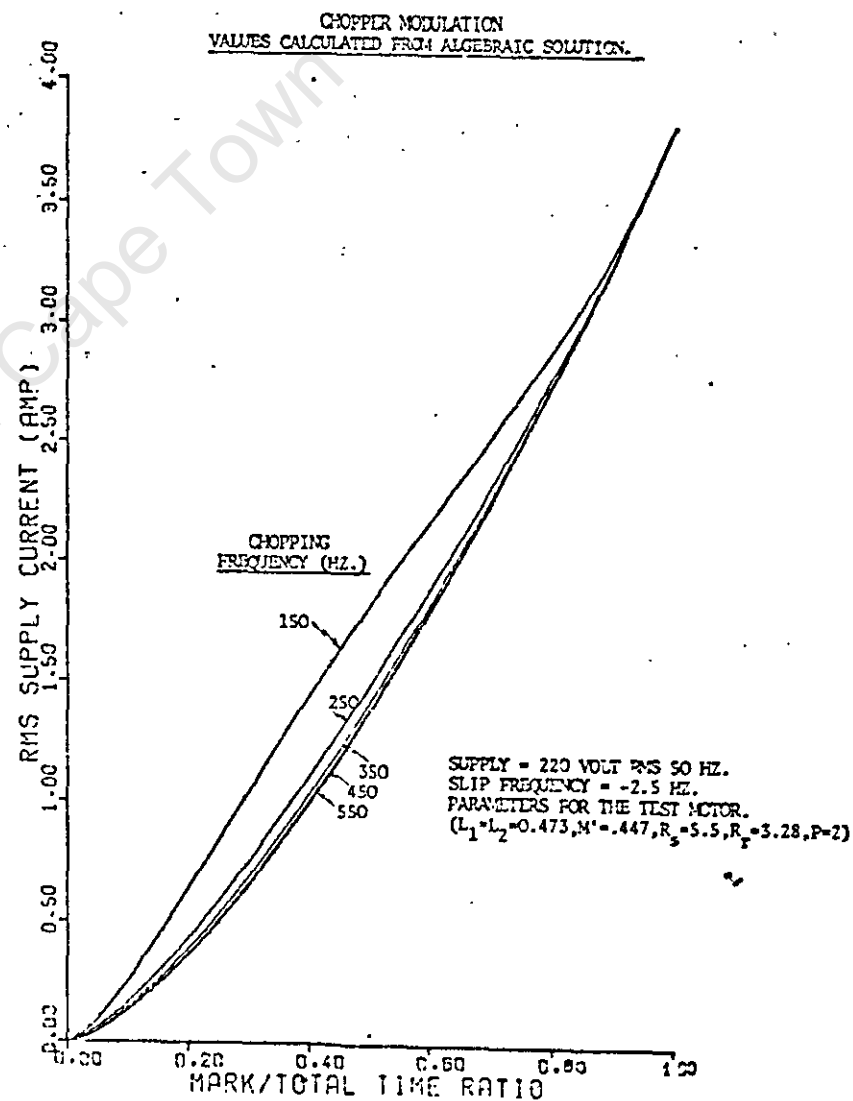
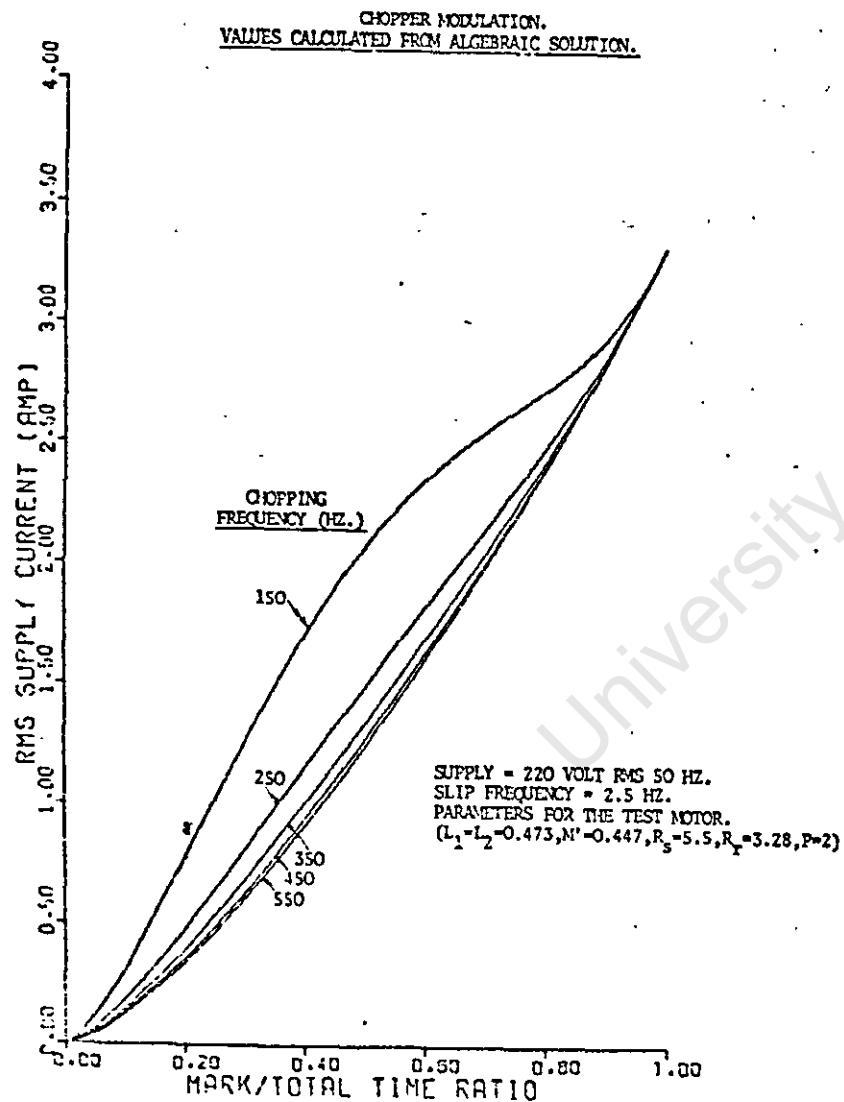


Fig.4-3

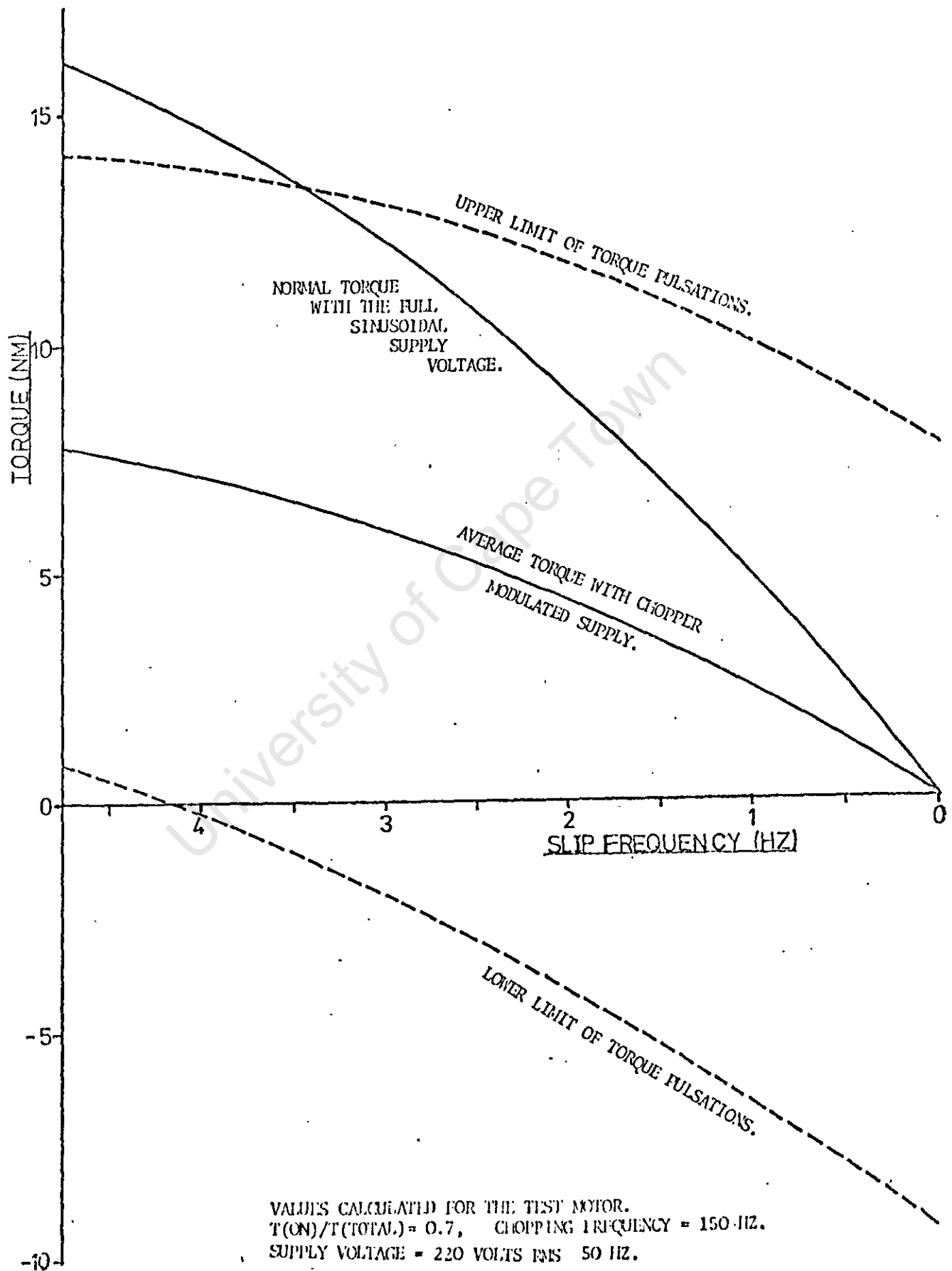


Fig. 44

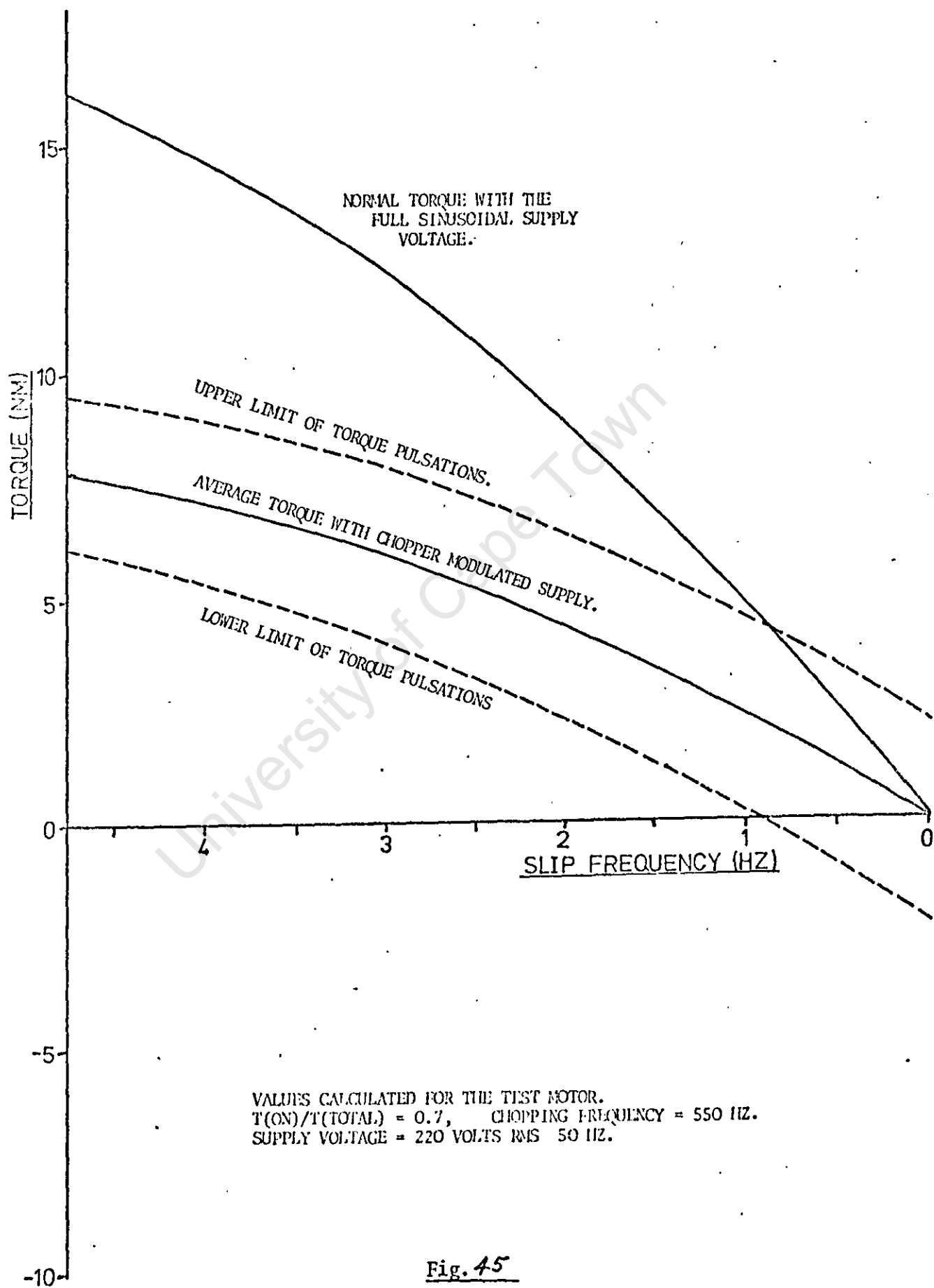
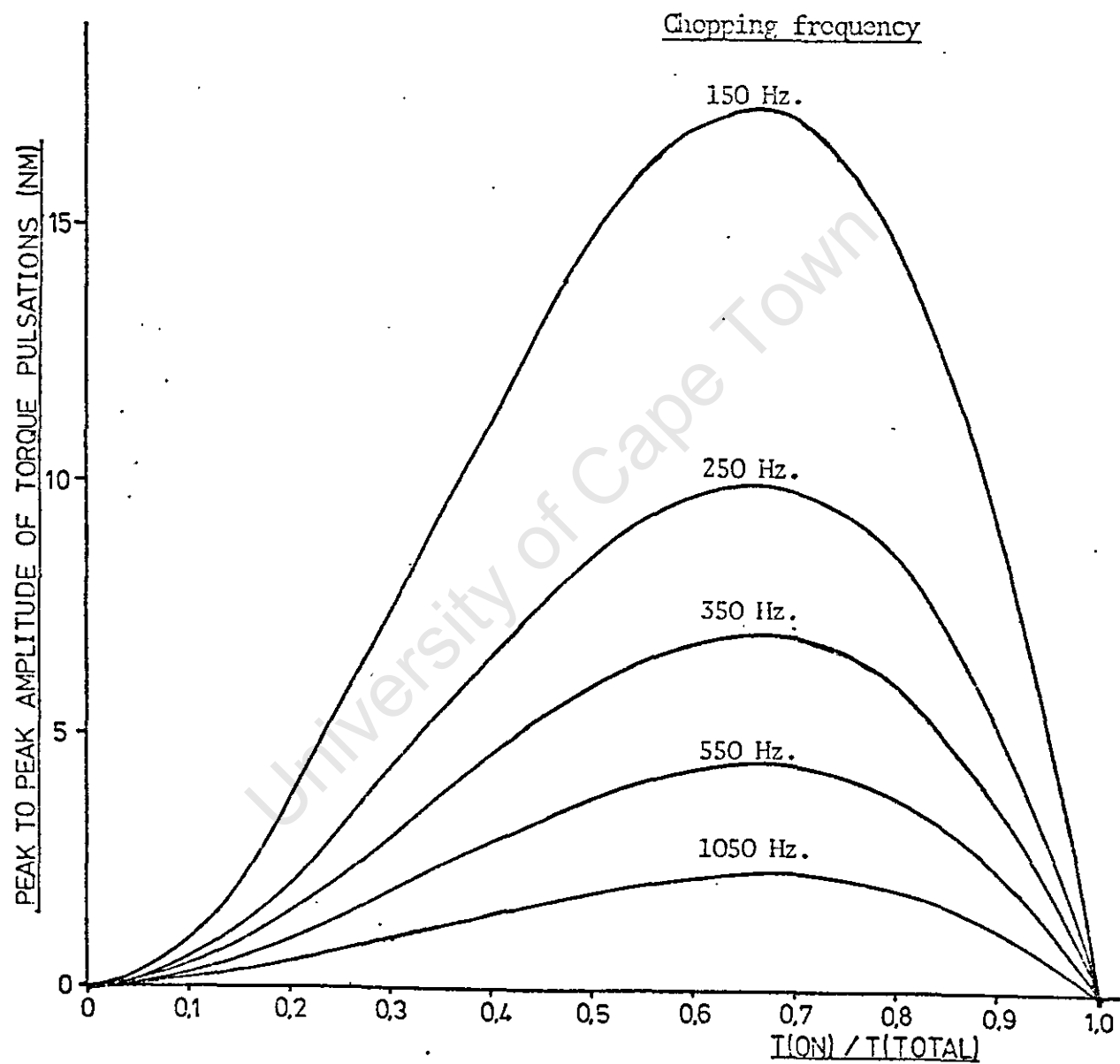
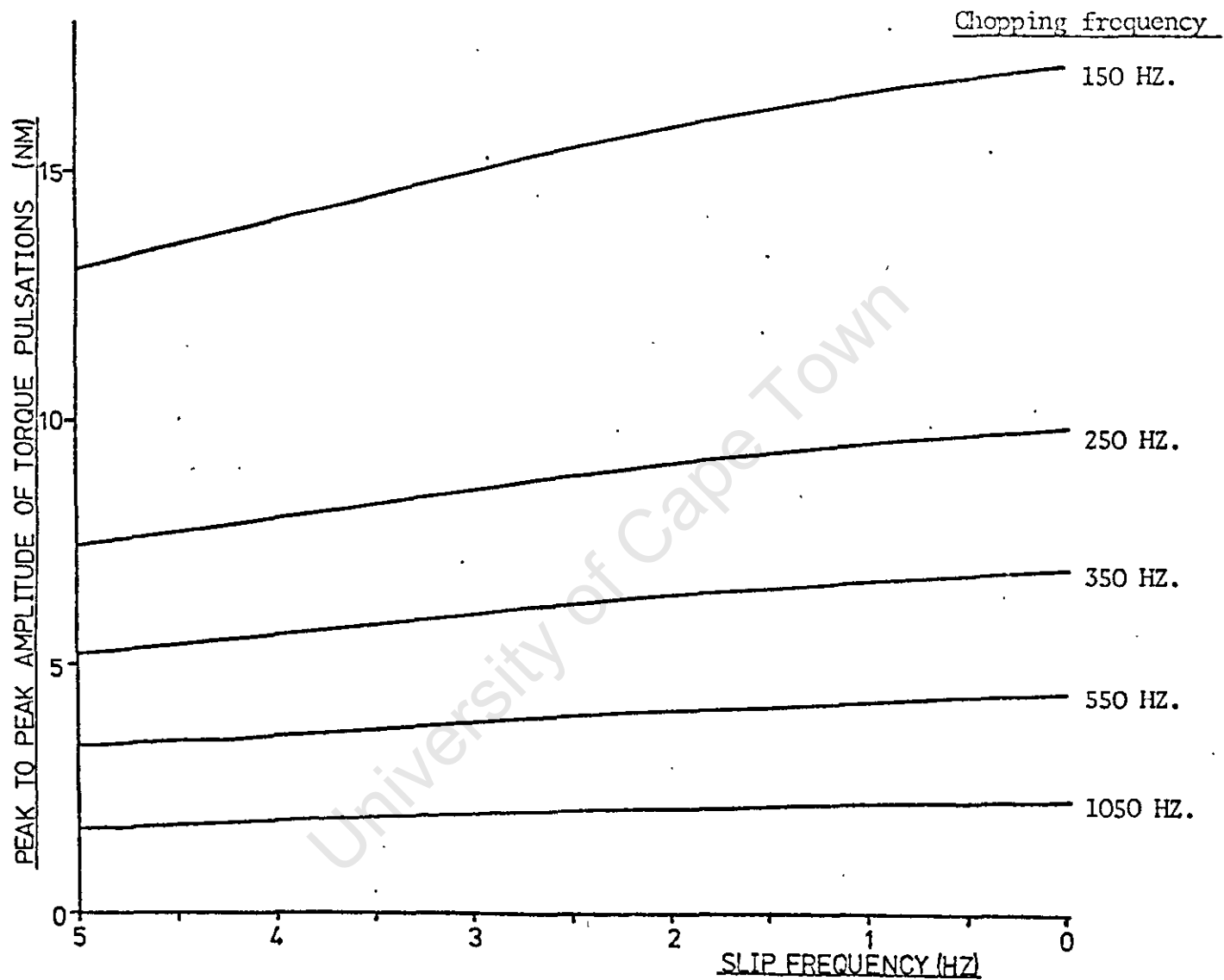


Fig. 45



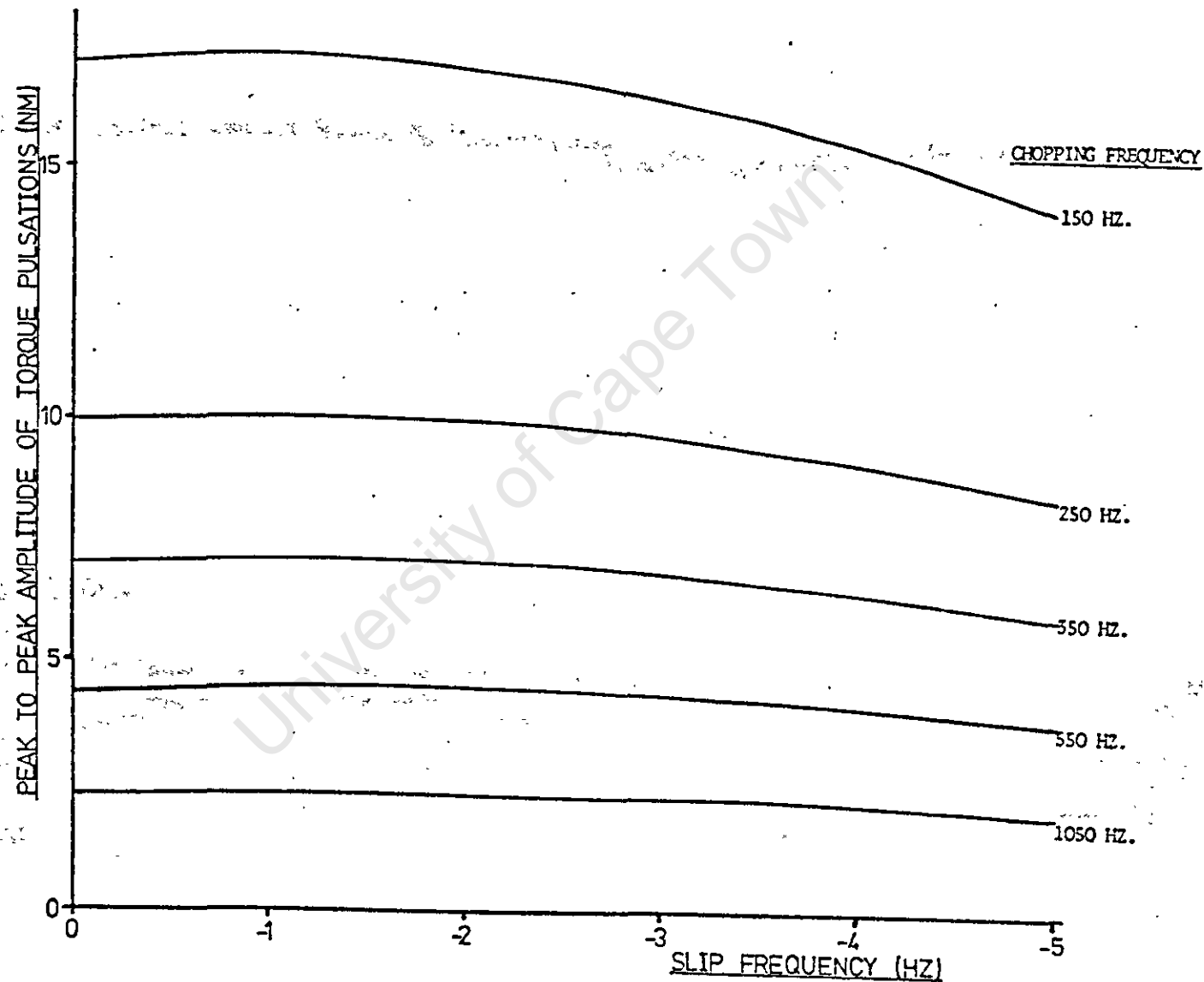
Values calculated for the test motor. Supply voltage = 220 Volt RMS 50 Hz. Slip = 0.

Fig. 46



Values calculated for the test motor. Supply voltage = 220 Volt RMS 50 Hz. $T(\text{on})/T(\text{total}) = 0.7$

Fig. 47



Values calculated for the test motor. Supply voltage = 220 Volt RMS 50 Hz. $T(\text{on})/T(\text{total}) = 0.7$

Fig. 48

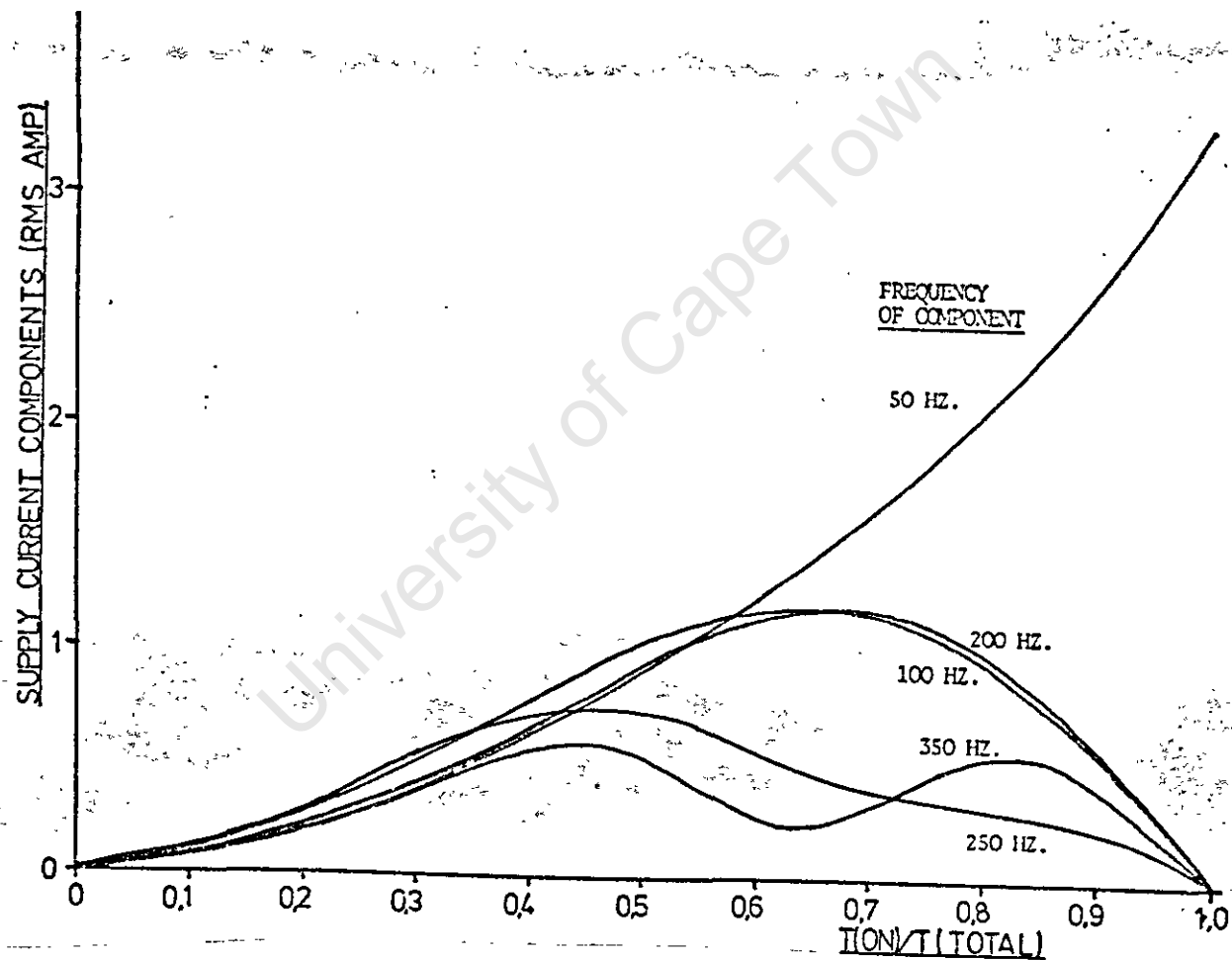
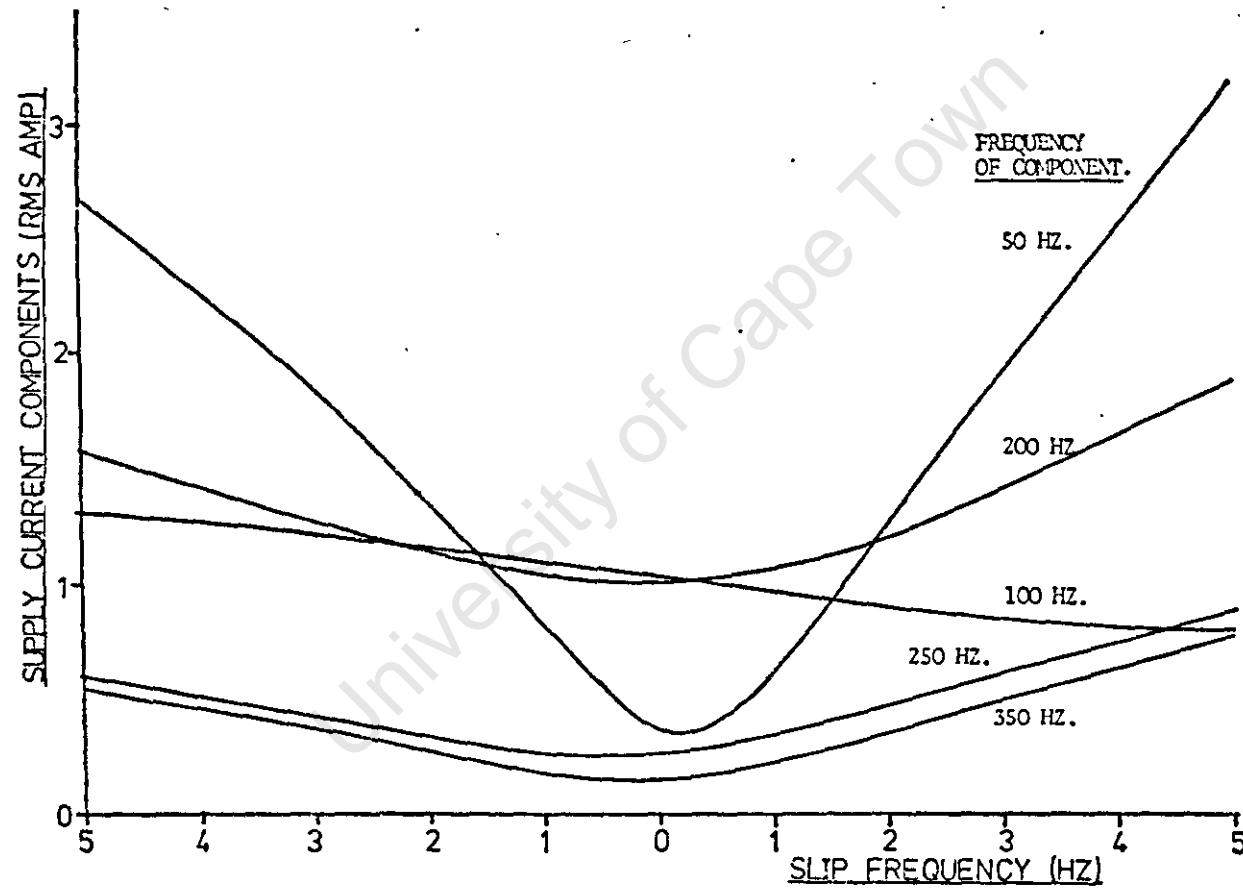
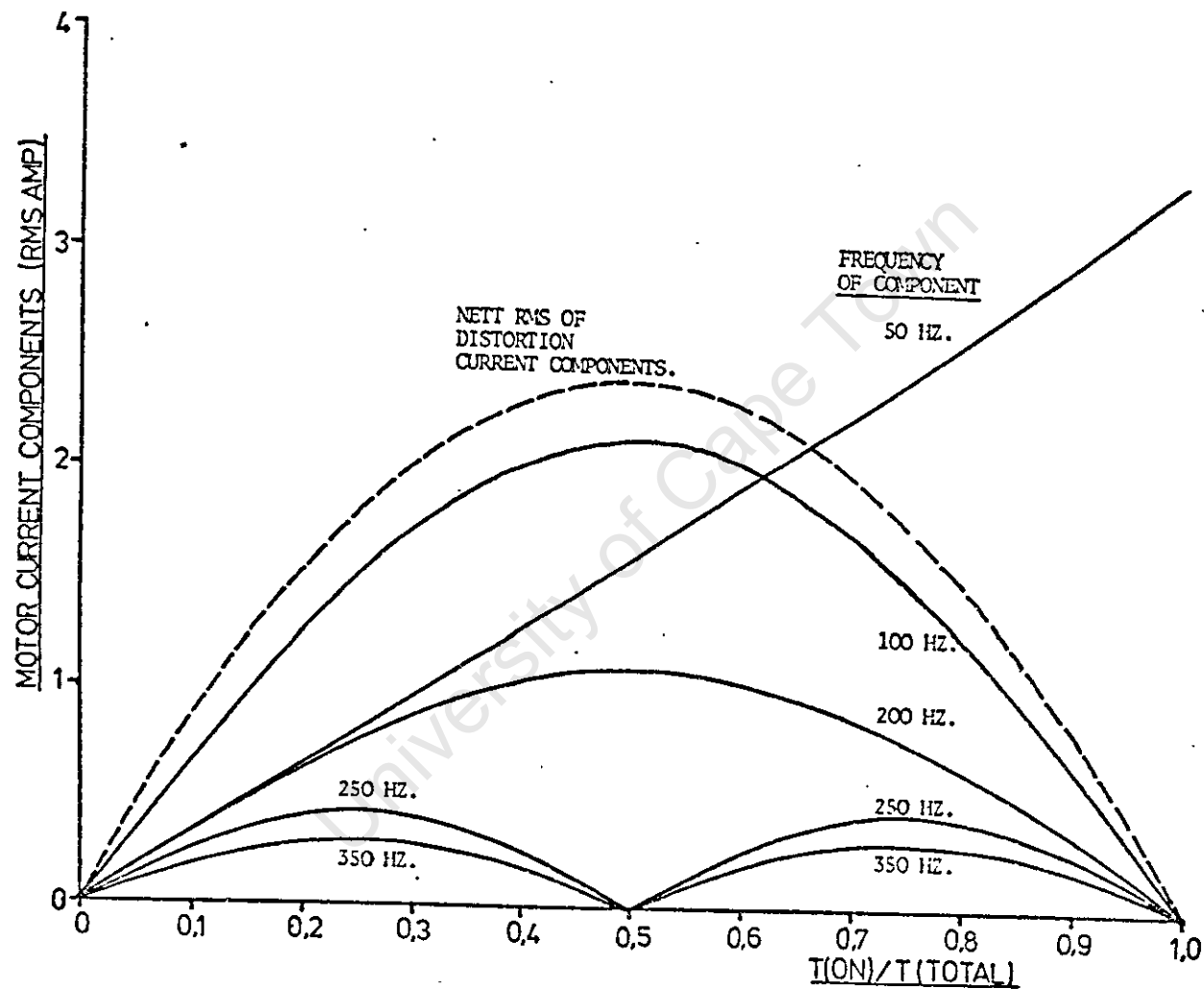


Fig. 49 Values calculated for the test motor. $V_s = 220$ volt RMS 50 Hz., Slip = 2.5 Hz., Chopping frequency = 150 Hz.



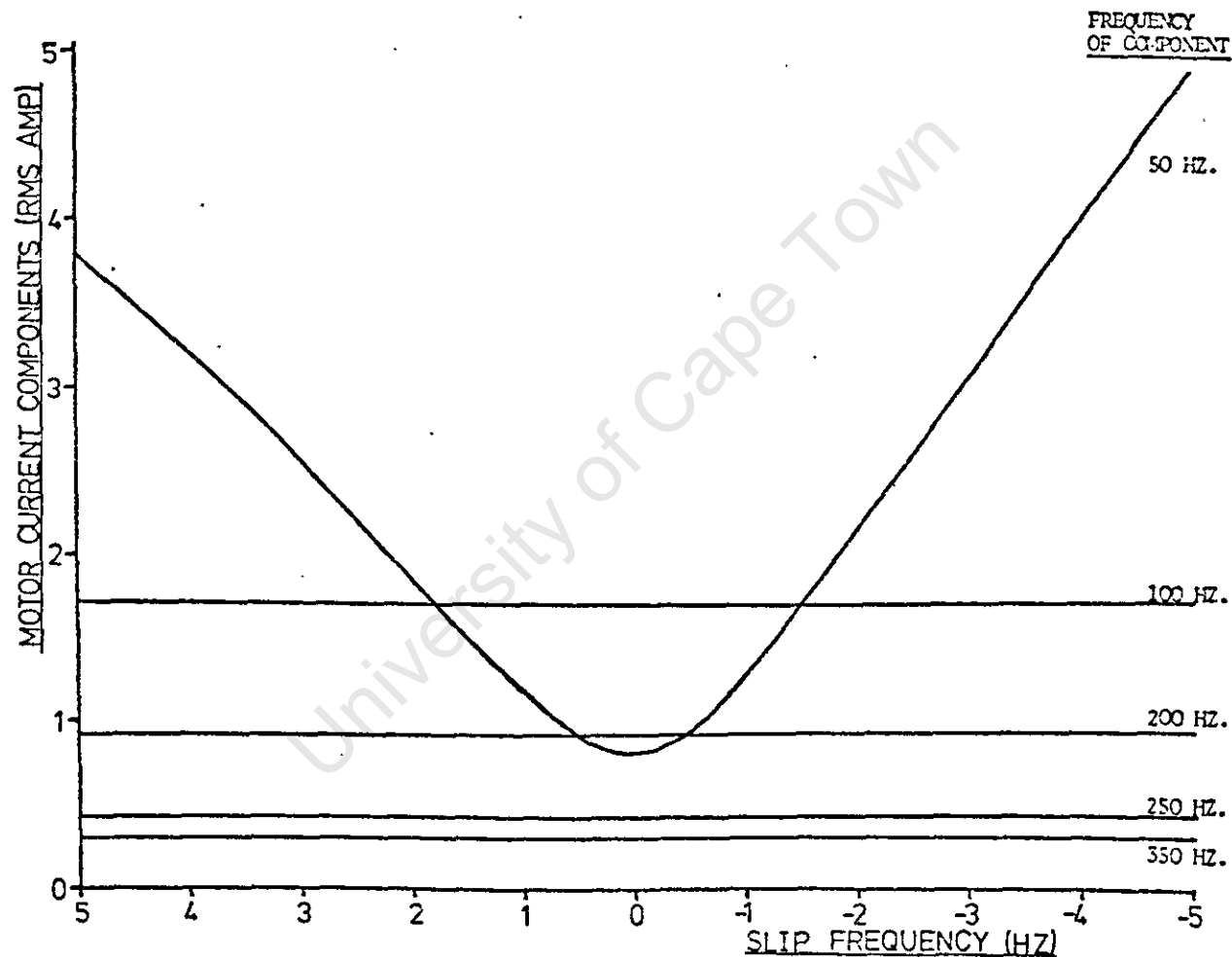
Values calculated for the test motor. $V_s = 220$ Volt RMS 50 Hz., $T(\text{on})/T(\text{total}) = 0.7$,
Chopping frequency = 150 Hz.

Fig. 50



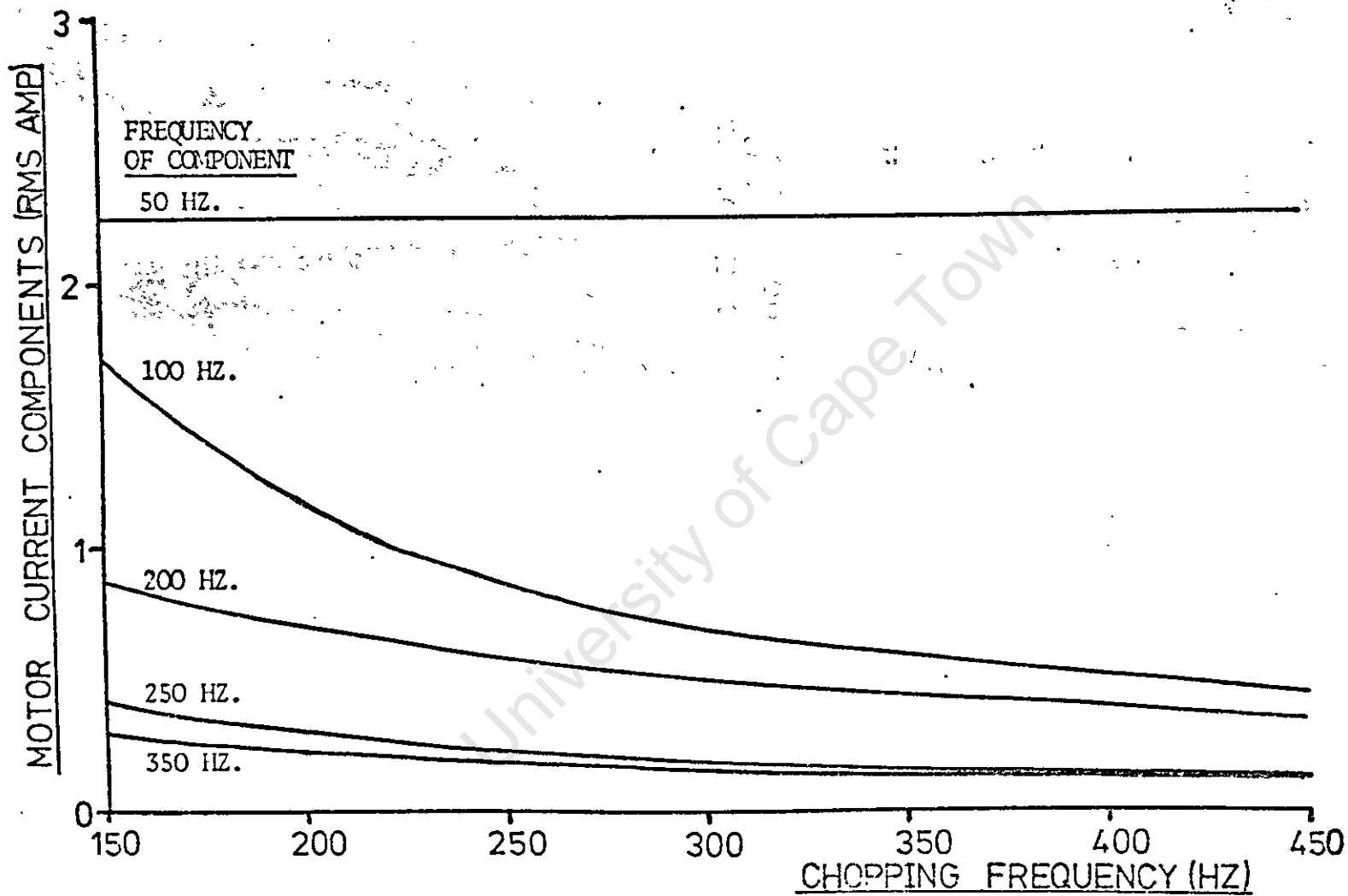
Values calculated for the test motor. $V_s = 220$ volt RMS 50 Hz., Slip = 2.5 Hz.,
Chopping frequency = 150 Hz.

Fig. 51



Values calculated for the test motor. $V_s = 220$ volt RMS 50 Hz., $T(\text{on})/T(\text{total}) = 0.7$
 Chopping frequency = 150 Hz.

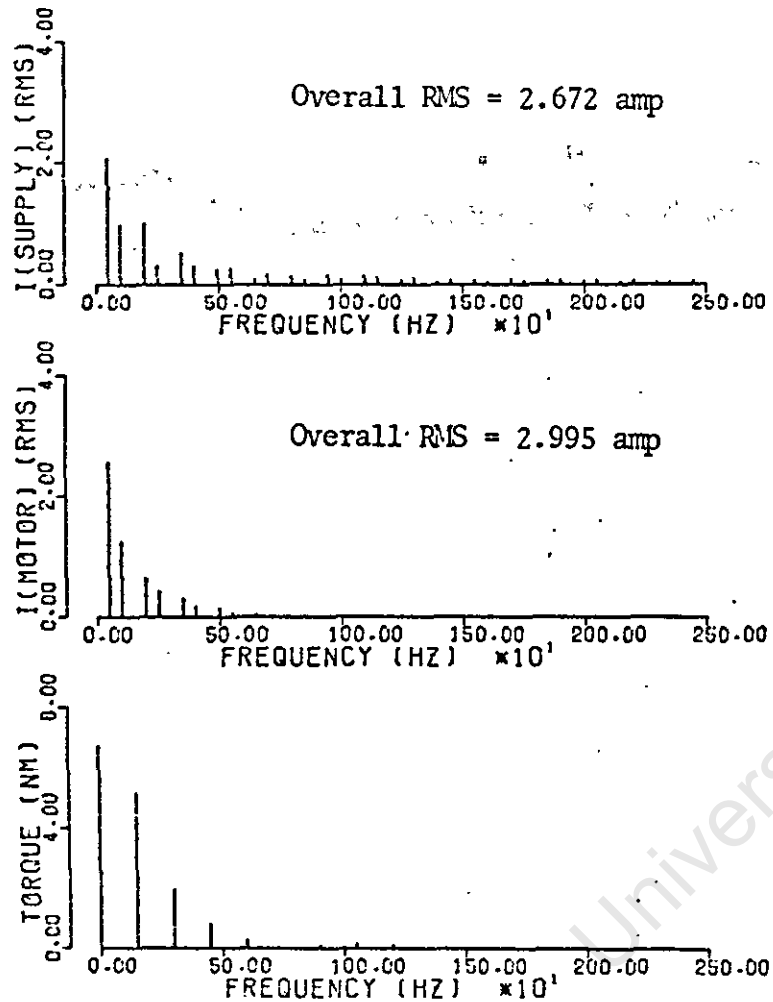
Fig. 52



Values calculated for the test motor. $V_s = 220$ Volt RMS 50 Hz., Slip = 2.5 Hz., $T(\text{on})/T(\text{total}) = 0.7$

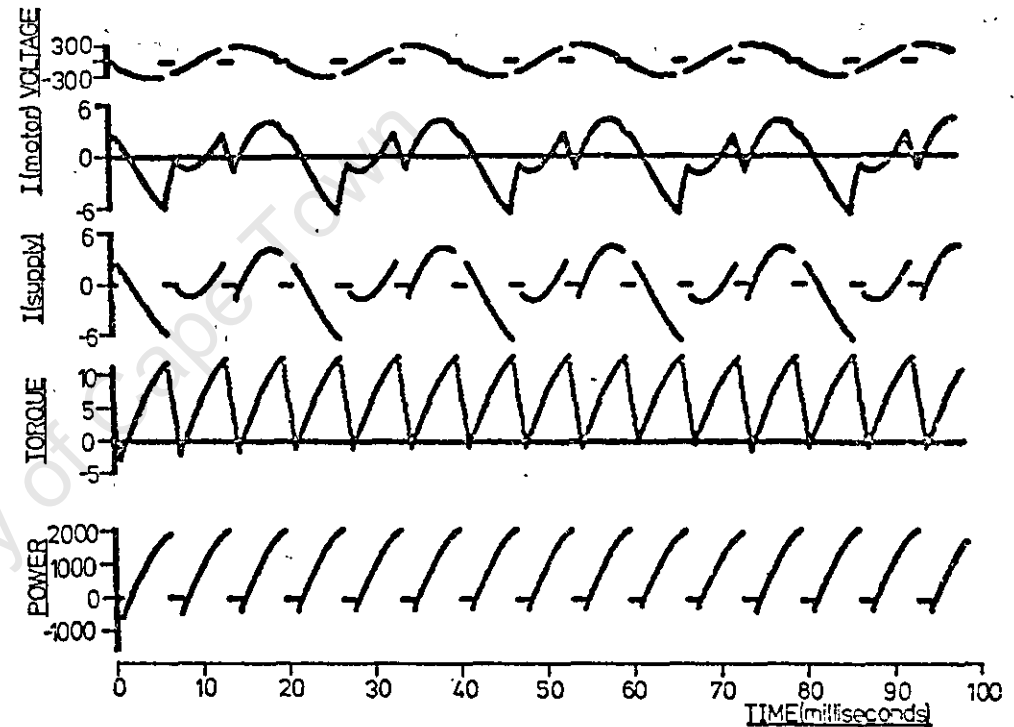
Fig. 53

ALGEBRAIC SOLUTION



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =0.0 DEGREES , F(STEP)=150.0 HZ. , CHOPS/STEP=1.
 T(ON)/T(TOTAL)=0.800 , SPEED=1425.0 RPM
 M=0.551 , L1=L2=0.577 , R1=5.50 , R2=3.28

NUMERICAL SIMULATION

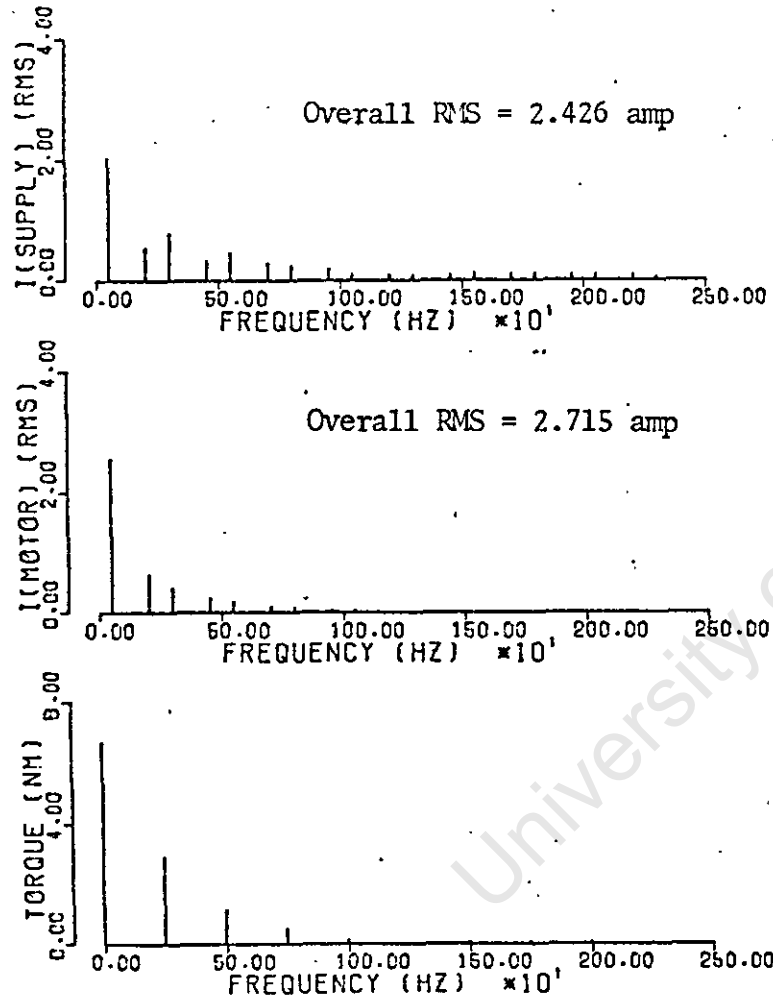


CALCULATED RESULTS.

CHOPPING FREQUENCY = 150 HZ.
 $T(\text{ON})/T(\text{TOTAL}) = 0.8$
 SLIP FREQUENCY = 2.5 HZ.

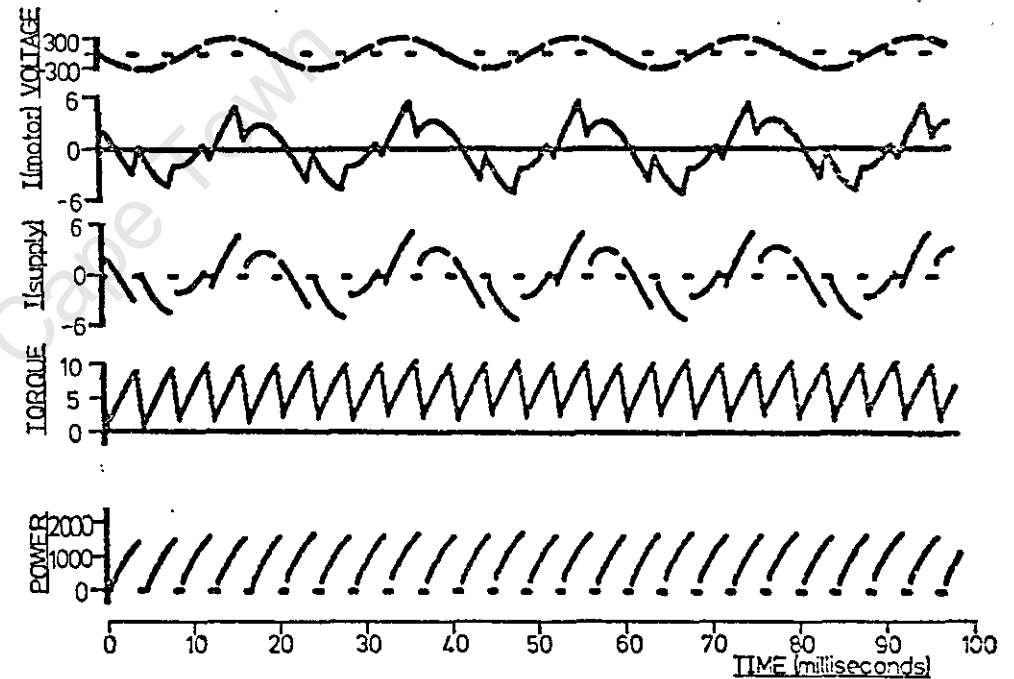
Fig. 54

ALGEBRAIC SOLUTION.



SUPPLY: 220.0 VOLT RMS, 50.0 HZ.
 CONVERTER STEPS: 0.0 DEGREES, F(STEP): 250.0 HZ., CHOPS/STEP: 1.
 TICH1/TOT1: 0.800, SPEED: 1425.0 RPM
 M: 0.55, L1: 12.0, R1: 5.50, R2: 3.20

NUMERICAL SIMULATION.

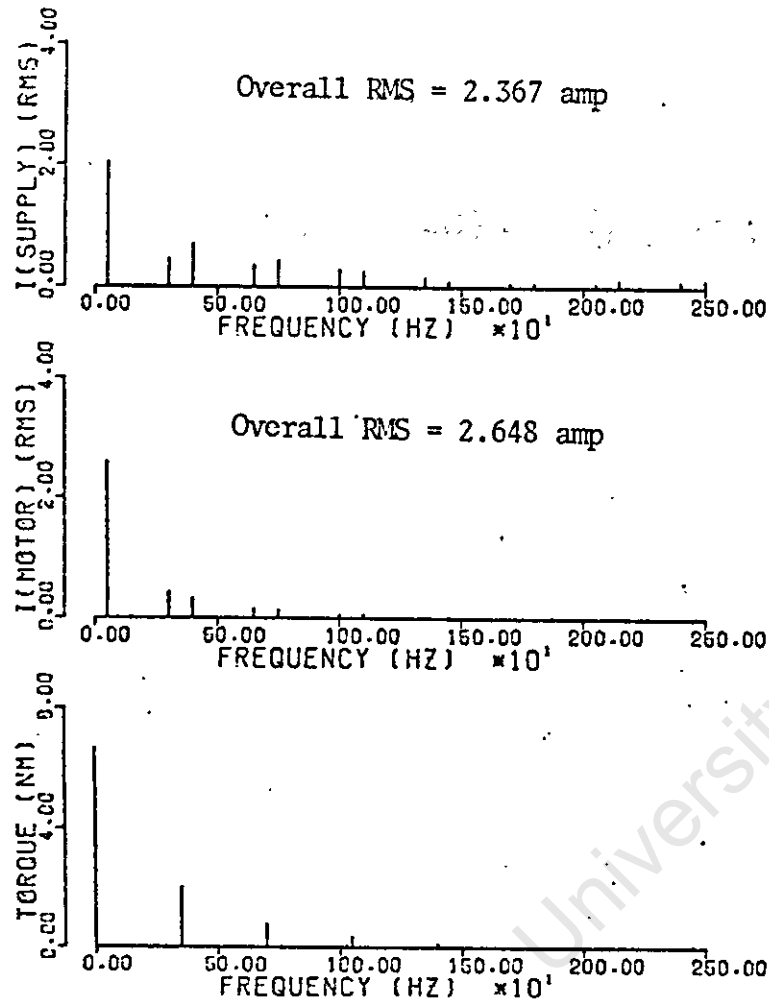


CALCULATED RESULTS

CHOPPING FREQUENCY = 250 HZ.
 $T(\text{ON})/T(\text{TOTAL}) = 0.8$
 SLIP FREQUENCY = 2.5 HZ.

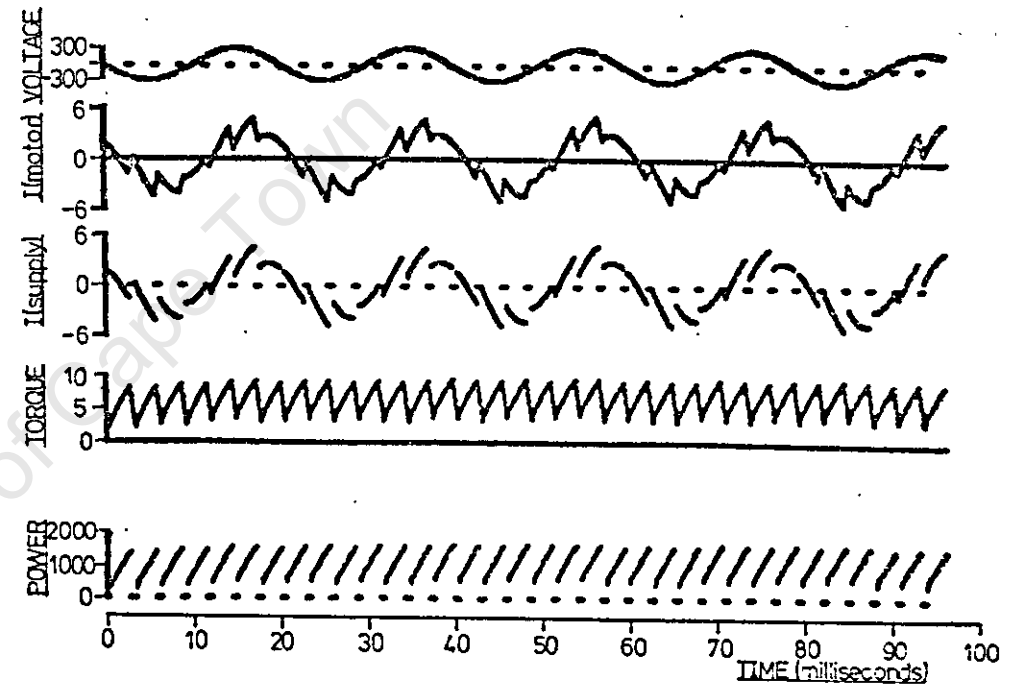
Fig. 55

ALGEBRAIC SOLUTION



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS -0.0 DEGREES , F(STEP)=350.0 HZ. , CHOPS/STEP=1.
 T(CN)/T(TOTAL)=0.800 , SPEED=1425.0 RPM
 M=0.551 , L1=1.2=0.577 , R1=5.50 , R2=3.28

NUMERICAL SIMULATION.

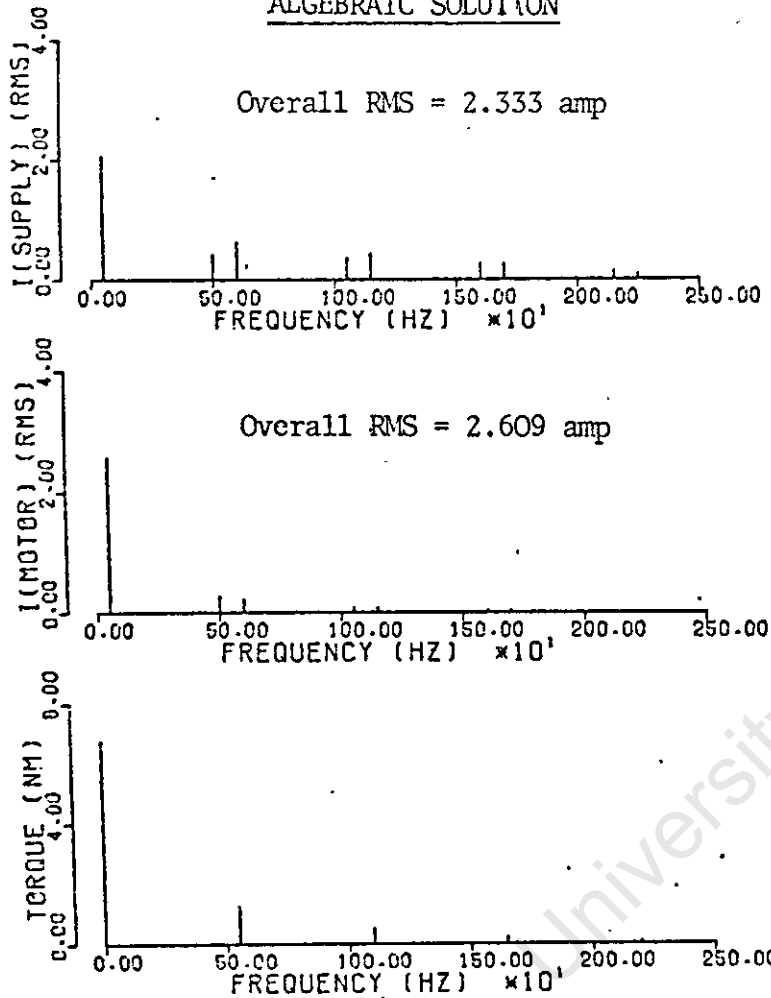


CALCULATED RESULTS

CHOPPING FREQUENCY = 350 HZ.
 T(ON)/T(TOTAL) = 0.8
 SLIP FREQUENCY = 2.5 HZ.

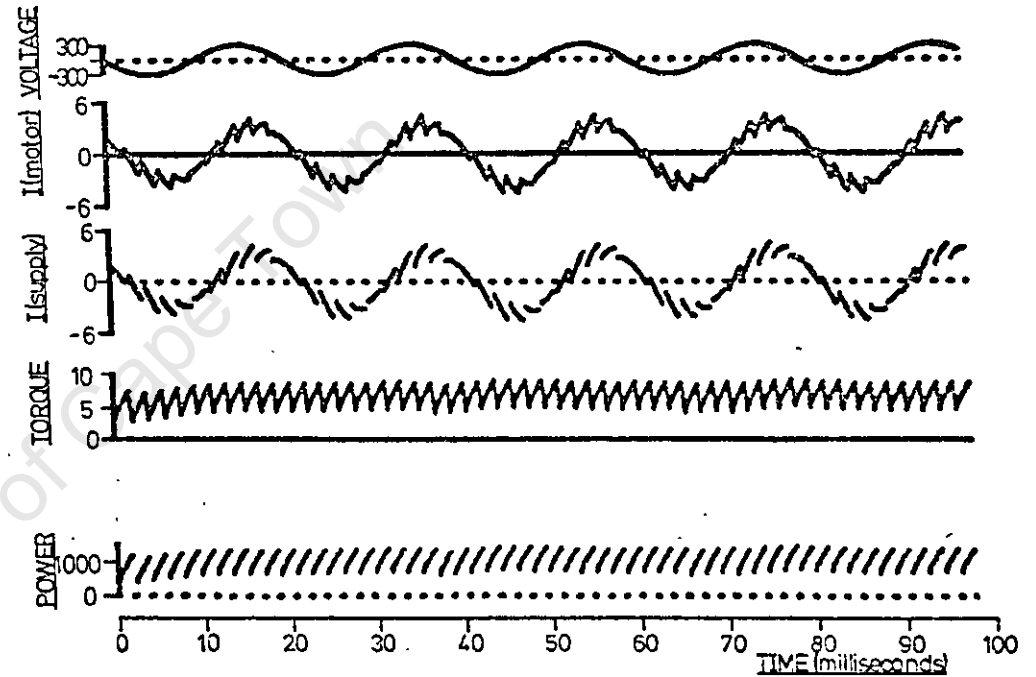
Fig. 56

ALGEBRAIC SOLUTION



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =0.0 DEGREES , F(STEP)=550.0 HZ. , CHOPS/STEP=1.
 T(ON)/T(TOTAL)=0.800 , SPEED=1425.0 RPM
 M=0.551 , L1=12.0,577 , R1=5.50 , R2=3.20

NUMERICAL SIMULATION

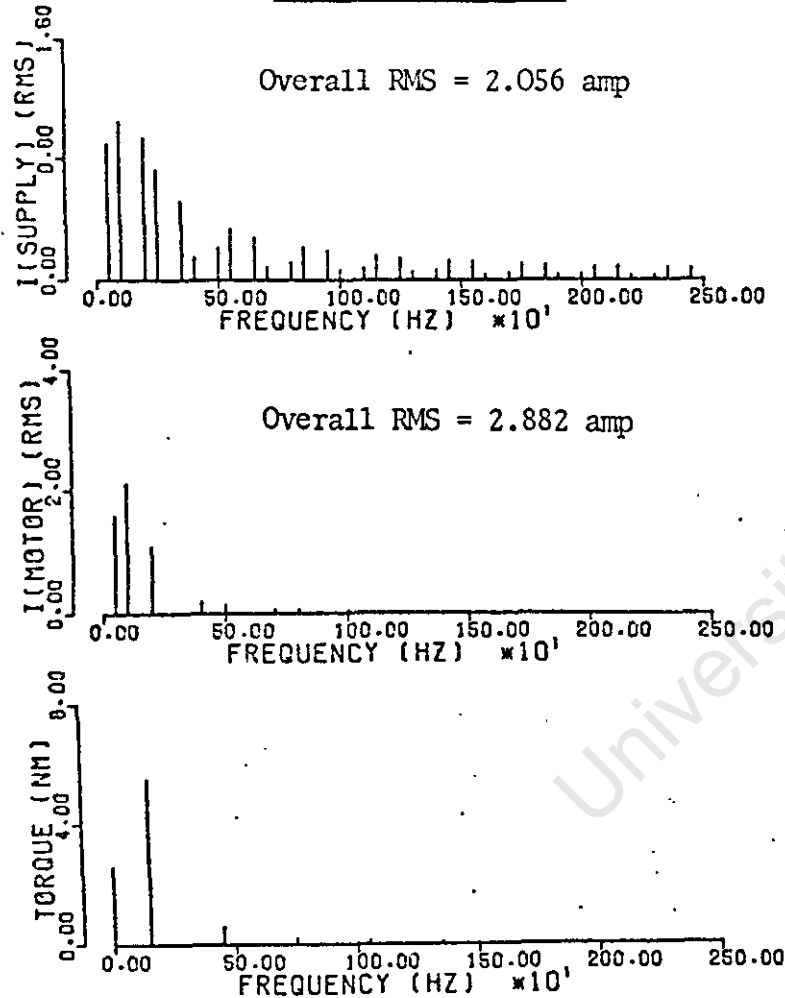


CALCULATED RESULTS

CHOPPING FREQUENCY = 550 HZ.
 T(ON)/T(TOTAL) = 0.8
 SLIP FREQUENCY = 2.5 HZ.

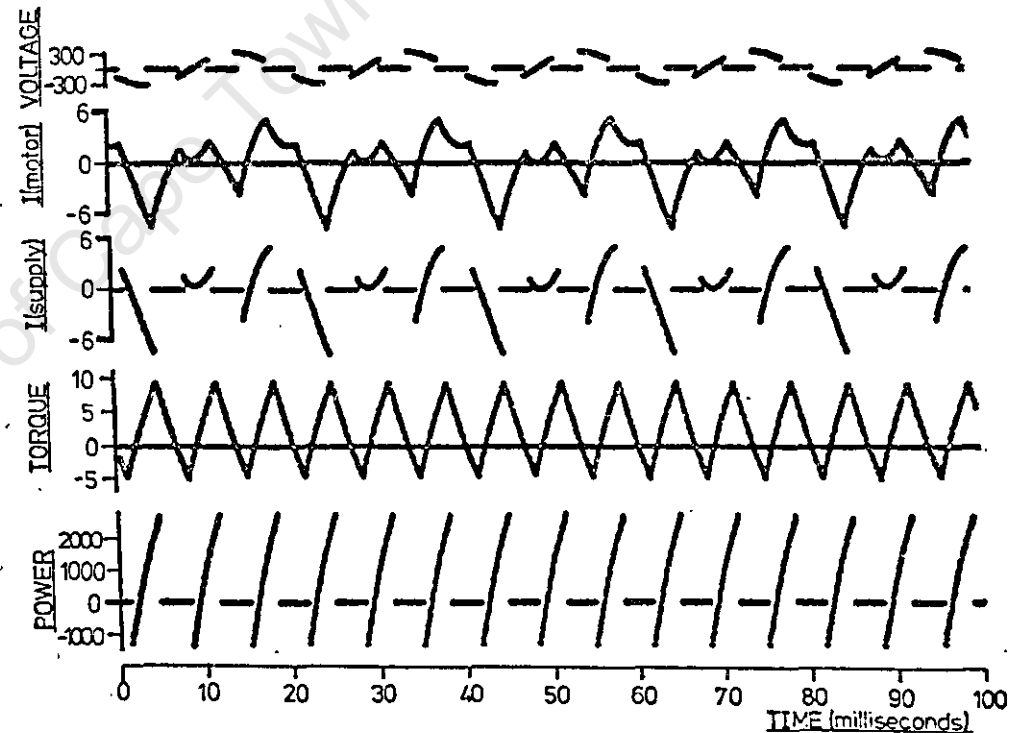
Fig. 57

ALGEBRAIC SOLUTION.



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS -0.0 DEGREES , F(STEP)=150.0 HZ. , CHOPS/STEP=1.
 T(ON)/T(TOTAL)=0.500 , SPEED=1425.0 RPM
 R=0.619,L1=L2=0.545,R1=5.50,R2=3.28

NUMERICAL SIMULATION.

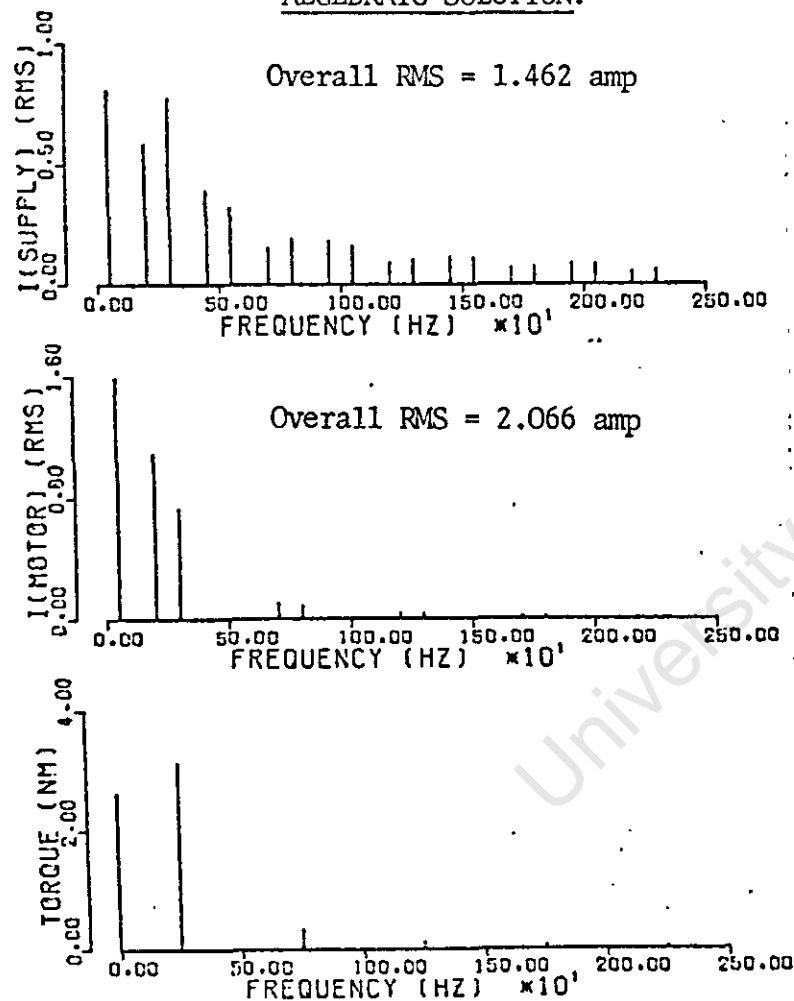


CALCULATED RESULTS

CHOPPING FREQUENCY = 150 HZ.
 T(ON)/T(TOTAL) = 0.5
 SLIP FREQUENCY = 2.5 HZ.

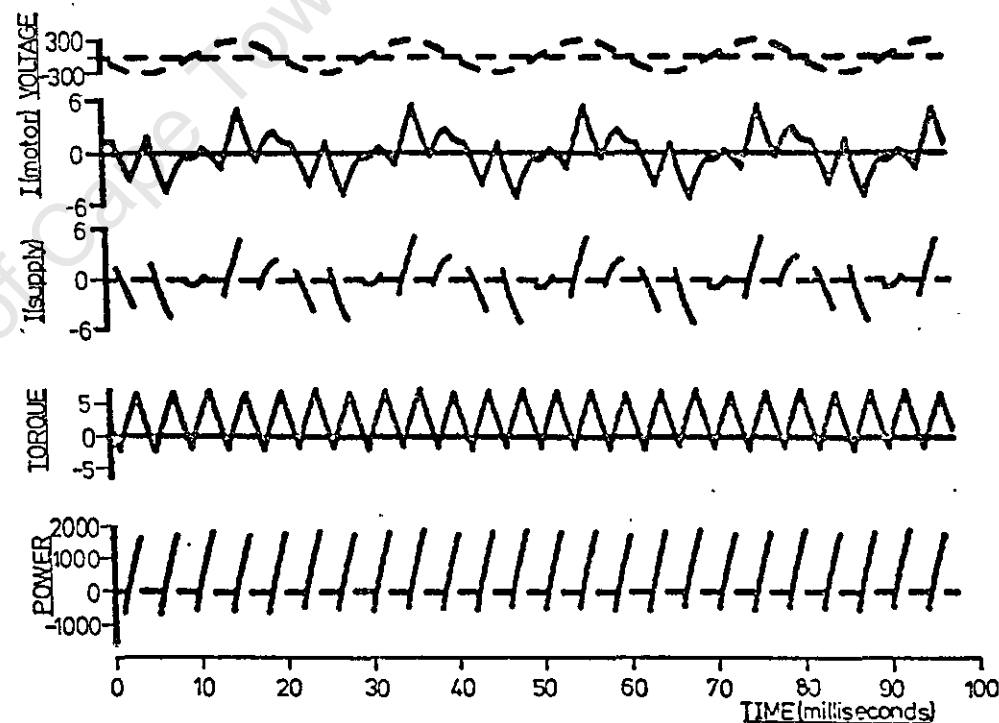
Fig. 58

ALGEBRAIC SOLUTION.



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS =0.0 DEGREES , F(STEP)=250.0 HZ. , CHOPS/STEP=1.
 T(ON)/T(TOTAL)=0.500 , SPEED=1425.0 RPM
 R=0.619,L1=L2=0.645,R1=5.50,R2=3.20

NUMERICAL SIMULATION.

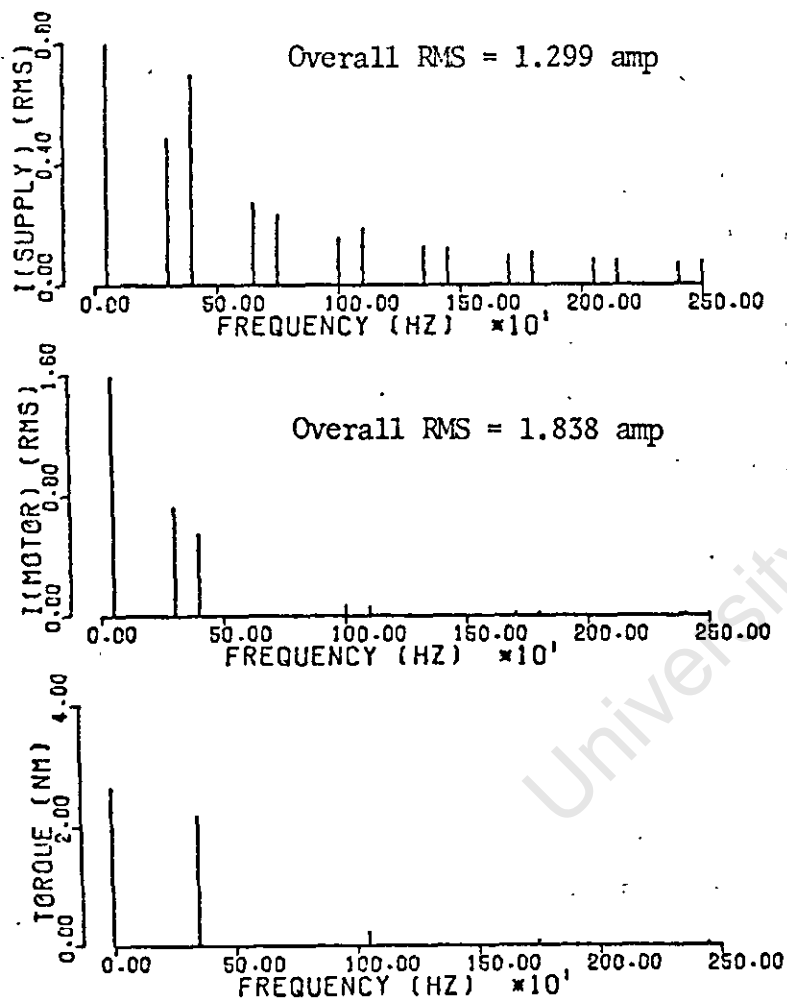


CALCULATED RESULTS

CHOPPING FREQUENCY = 250 HZ.
 T(ON)/T(TOTAL) = 0.5
 SLIP FREQUENCY = 2.5 HZ.

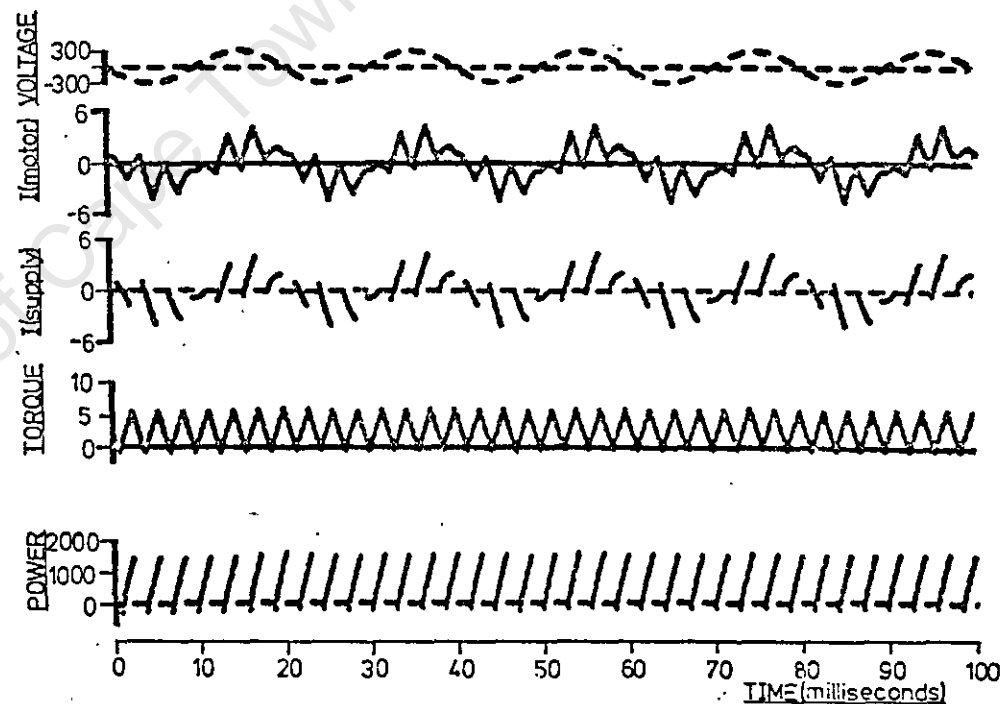
Fig. 59

ALGEBRAIC SOLUTION



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS -0.0 DEGREES , F(STEP)=350.0 HZ. , CHOPS/STEP=1.
 T(ON)/T(TOTAL)=0.500 , SPEED=1425.0 RPM
 M=0.619,L1=L2=0.645,R1=5.50,R2=3.28

NUMERICAL SIMULATION.

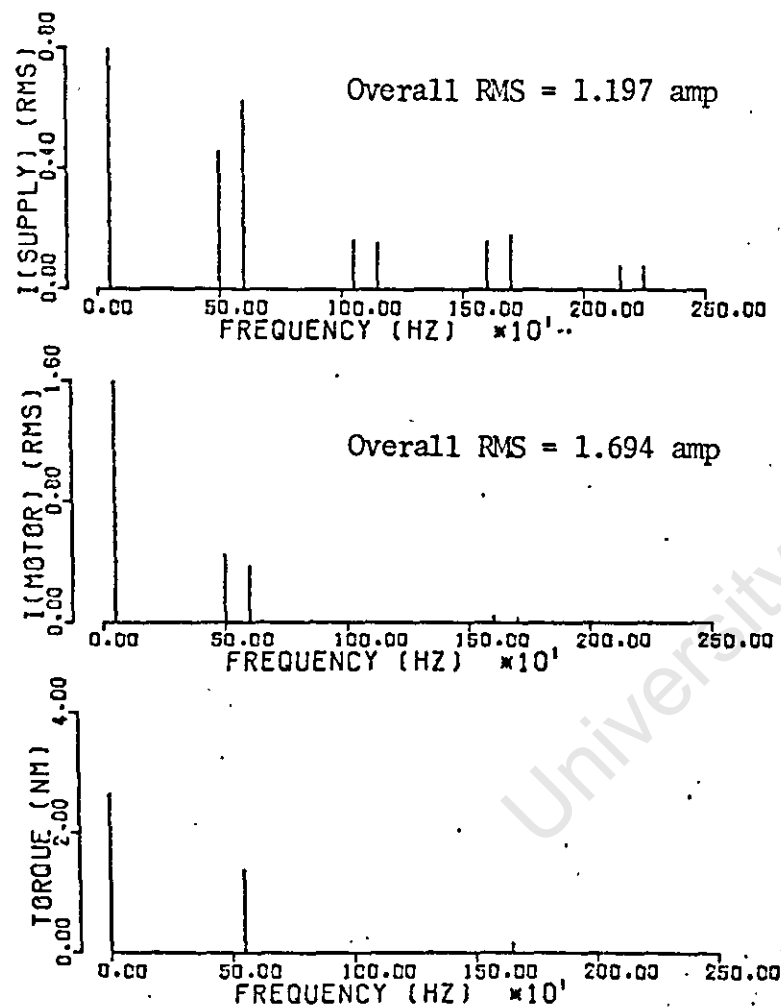


CALCULATED RESULTS.

CHOPPING FREQUENCY = 350 HZ.
 T(ON)/T(TOTAL) = 0.5
 SLIP FREQUENCY = 2.5 HZ.

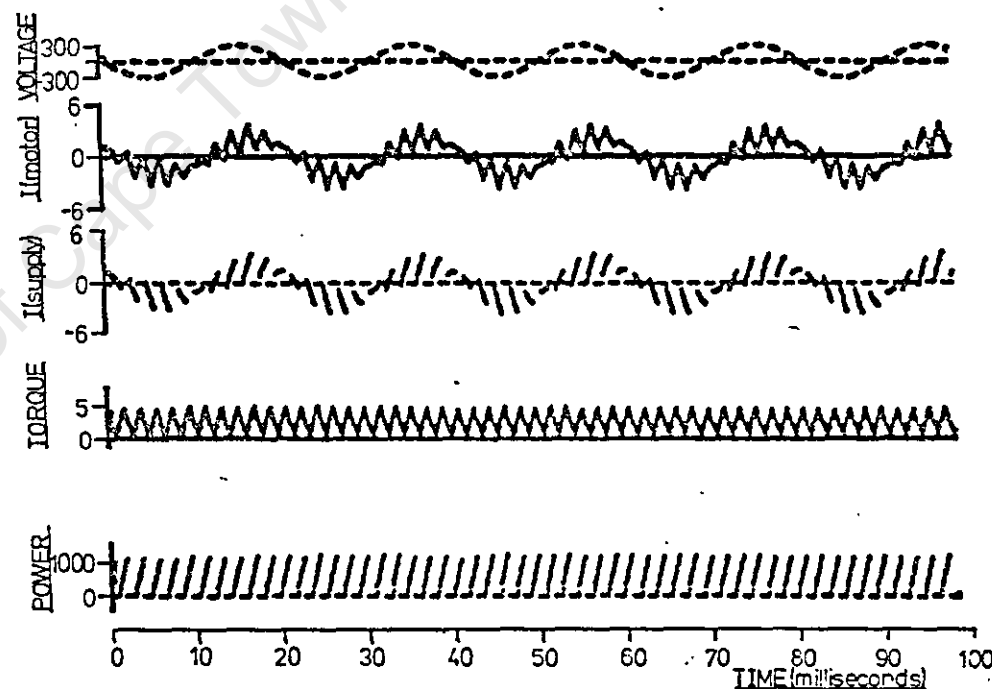
Fig. 60

ALGEBRAIC SOLUTION



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =0.0 DEGREES , F(STEP)=550.0 HZ. , CHOP5/STEP=1.
 T(CN)/T(TOTAL)=0.500 , SPEED=1425.0 RPM
 M=0.619,L1=L2=0.645,R1=5.50,R2=3.20

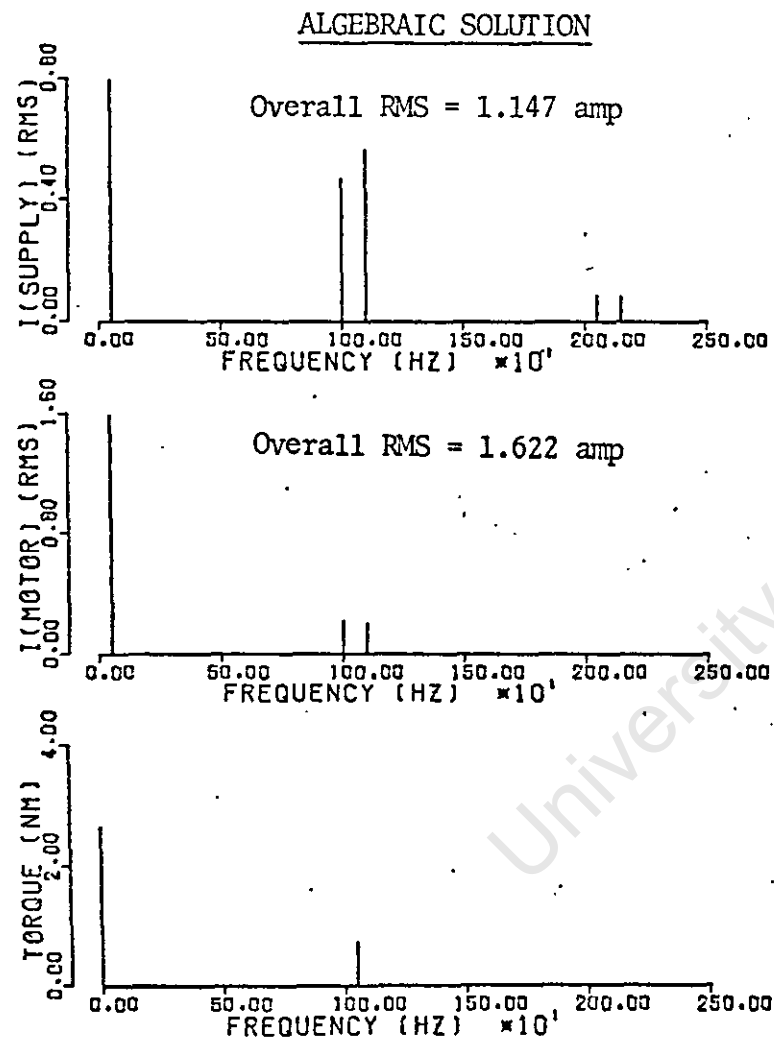
NUMERICAL SIMULATION



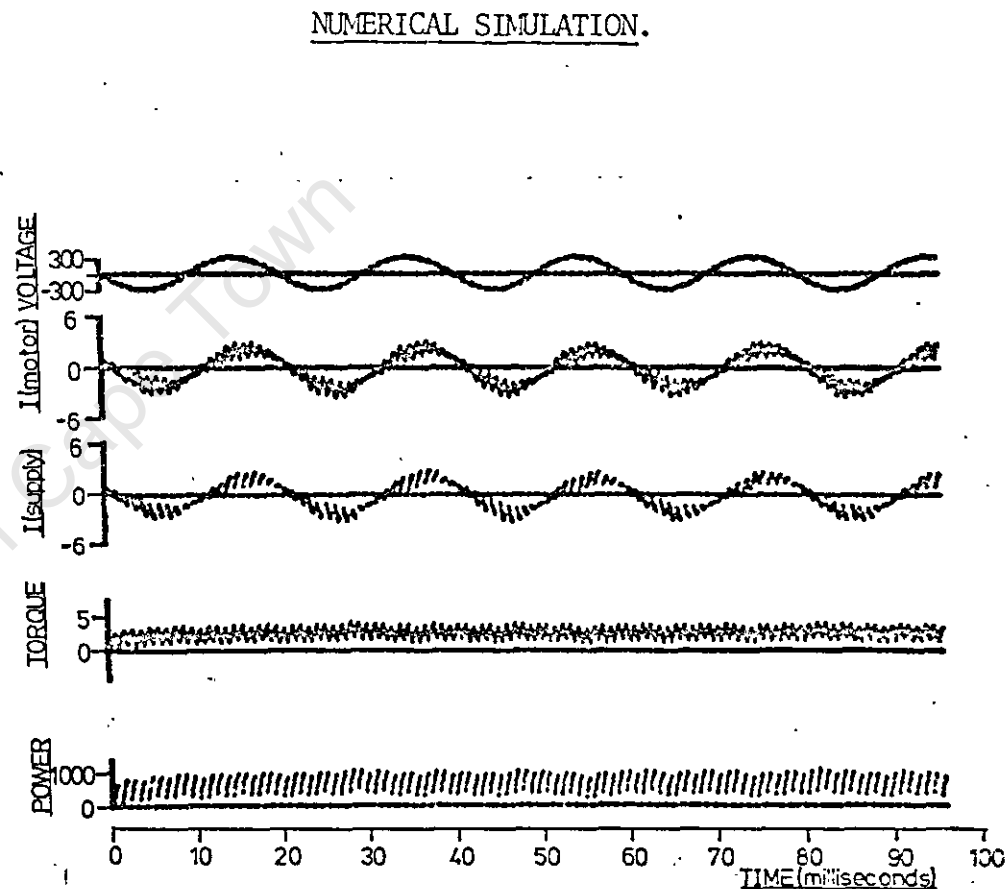
CALCULATED RESULTS.

CHOPPING FREQUENCY = 550 HZ.
 T(ON)/T(TOTAL) = 0.5
 SLIP FREQUENCY = 2.5 HZ.

Fig. 61



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS -0.0 DEGREES , F(STEP)=1050.0 HZ. , CHOPS/STEP=1.
 T(ON)/T(TOTAL)=0.500 , SPEED=1425.0 RPM
 M=0.619,L1=L2=0.645,R1=5.50,R2=3.28



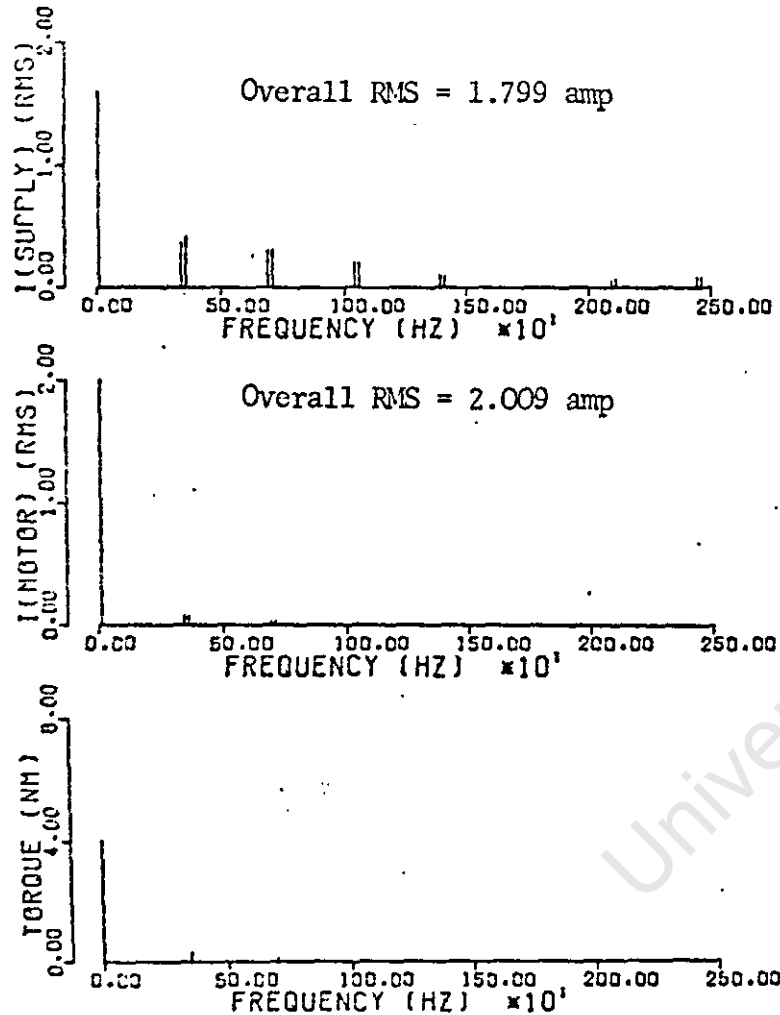
CALCULATED RESULTS

CHOPPING FREQUENCY = 1050 HZ.
 T(ON)/T(TOTAL) = 0.5
 SLIP FREQUENCY = 2.5 HZ.

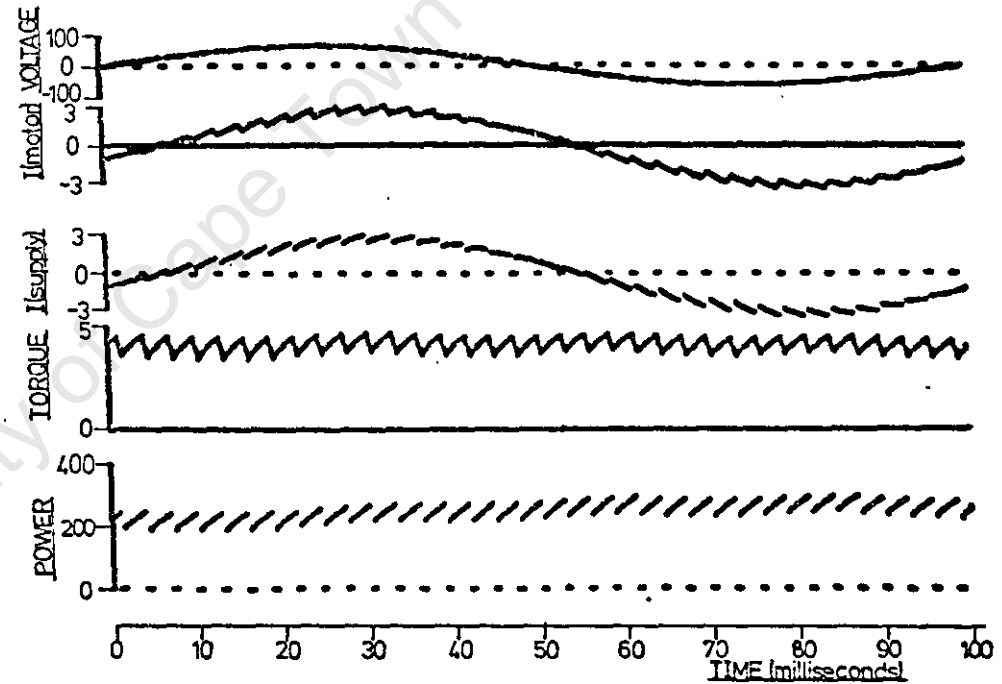
Fig. 62

ALGEBRAIC SOLUTION.

NUMERICAL SIMULATION



SUPPLY=44.0 VOLT RMS , 10.HZ.
 CONVERTOR STEPS =0.0 DEGREES , F(STEP)=350.0 HZ. , CHOPS/STEP=1.
 T(ON)/T(TOTAL)=0.800 , SPEED=225.0 RPM
 M=0.551,L1=L2=0.577,R1=5.50,R2=3.28

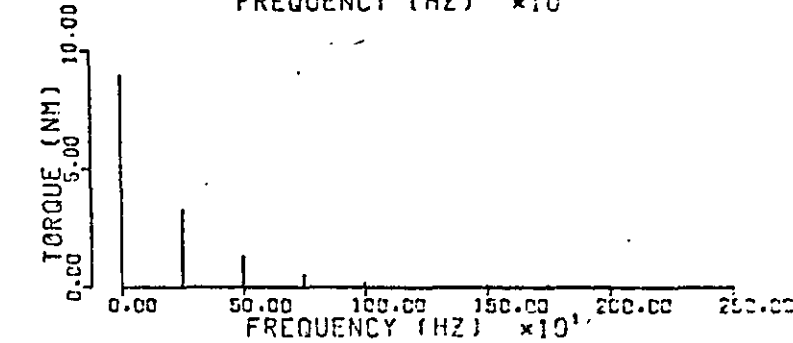
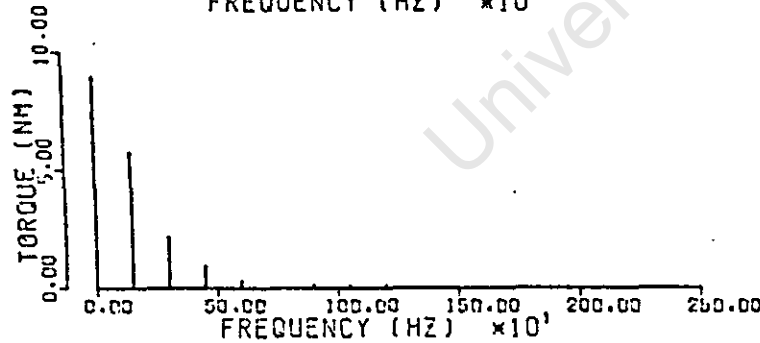
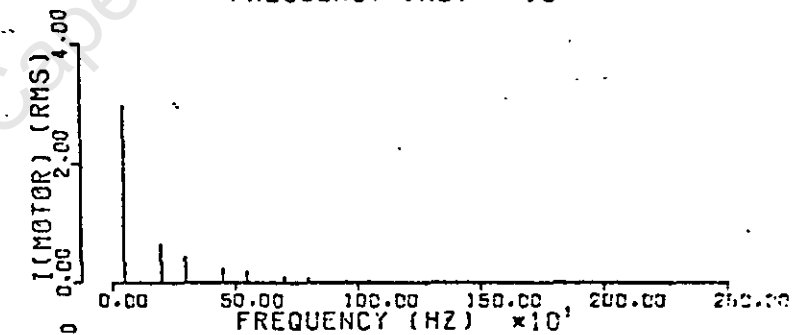
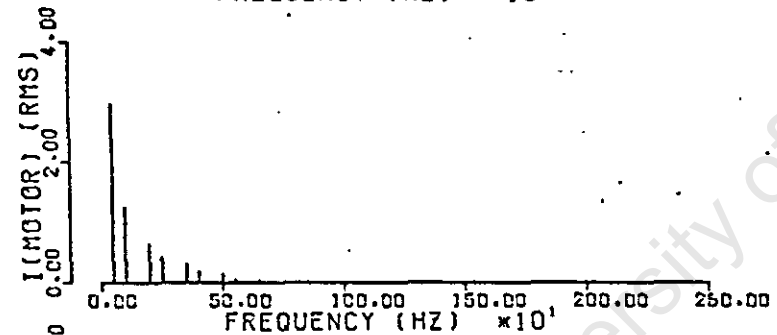
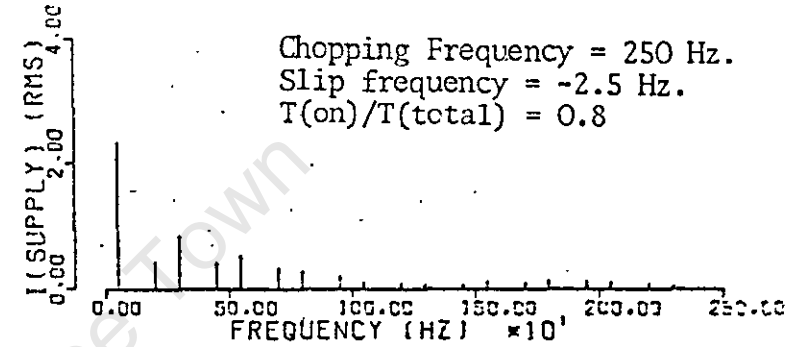
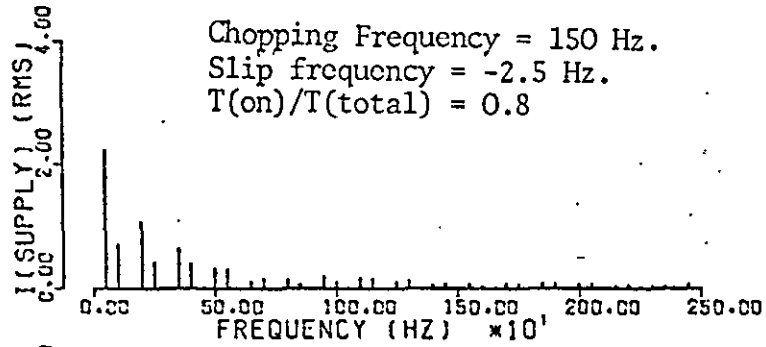


CALCULATED RESULTS

CHOPPING FREQUENCY = 350 HZ.
 T(ON)/T(TOTAL) = 0.8
 SLIP FREQUENCY = 2.5 HZ.
 SUPPLY = 44 volt RMS 10 HZ.

Fig. 63

CALCULATED FOR TEST MOTOR FROM THE ALGEBRAIC SOLUTION TO THE MOTOR EQUATIONS.

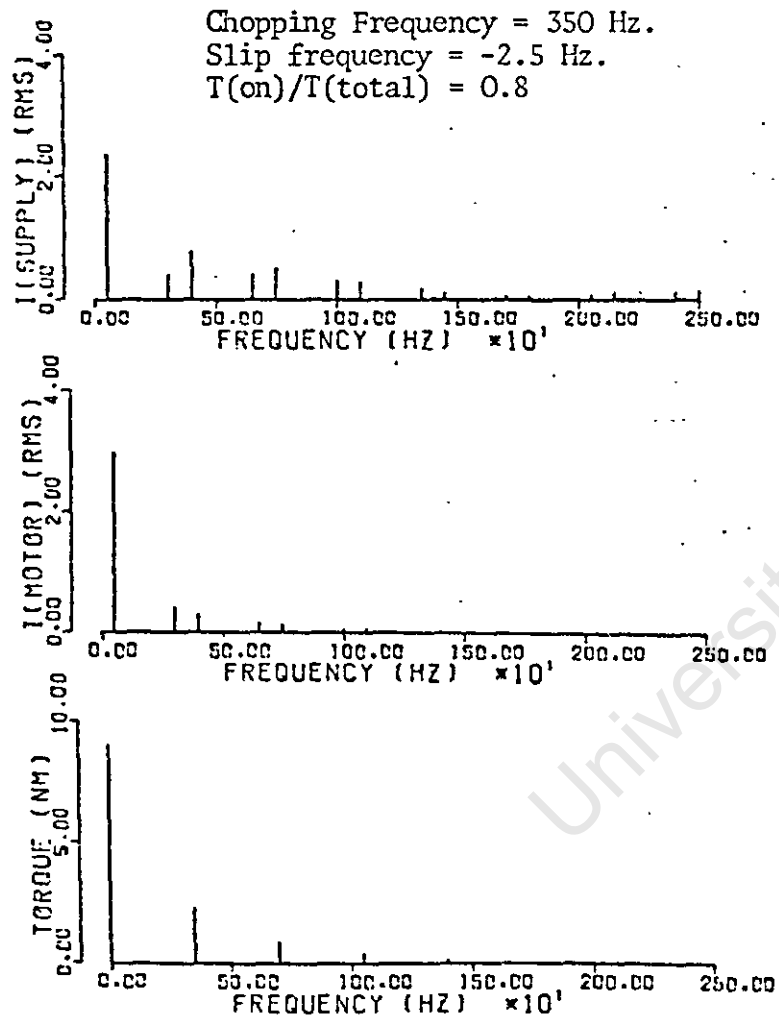


SUPPLY=220.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS -0.0 DEGREES , F(STEP)=150.0 HZ. , CHOPS/STEP=1.
 $T(\text{ON})/T(\text{TOTAL})=0.800$. SPEED=1575.0 RPM
 $M=0.551, L1=L2=0.577, R1=5.50, R2=3.28$

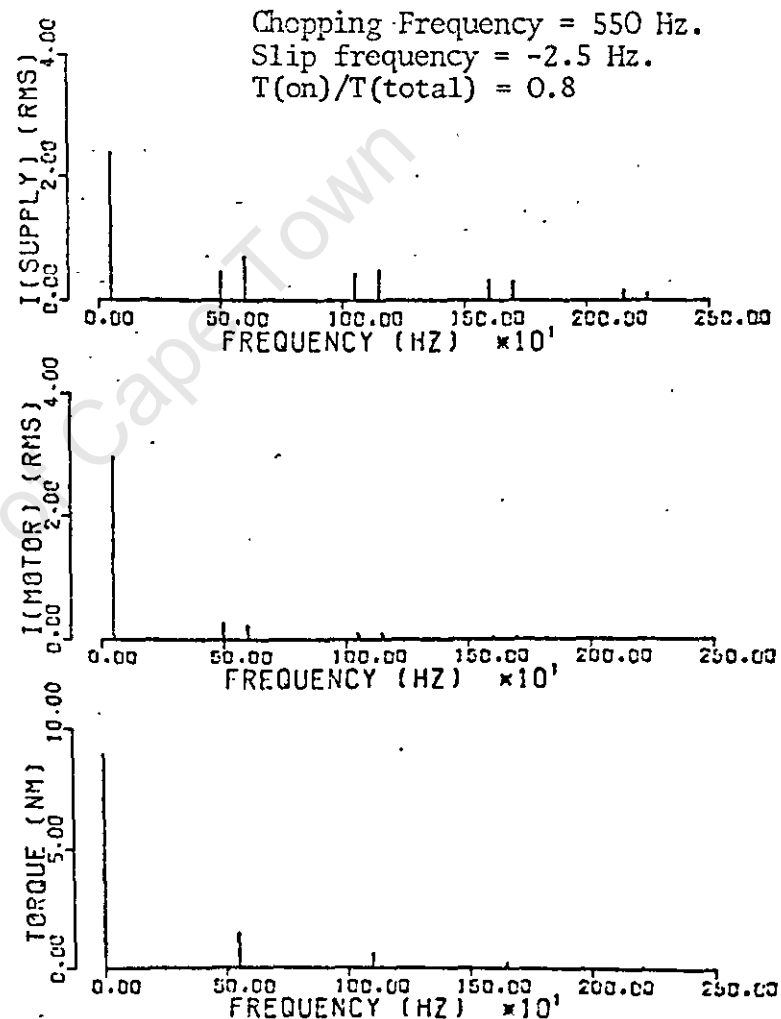
SUPPLY=220.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS -0.0 DEGREES , F(STEP)=250.0 HZ. , CHOPS/STEP=1.
 $T(\text{ON})/T(\text{TOTAL})=0.800$. SPEED=1575.0 RPM
 $M=0.551, L1=L2=0.577, R1=5.50, R2=3.28$

Fig. 64 Spectra of system waveforms in the regenerating mode.

CALCULATED FOR THE TEST MOTOR FROM THE ALGEBRAIC SOLUTION TO THE MOTOR EQUATIONS.



SUPPLY=220.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS -0.0 DEGREES , F1STEP1=350.0 HZ. , CHOPS/STEP=1.
 $T(\text{ON})/T(\text{TOTAL})=0.800$, SPEED=1575.0 RPM
 $M=0.551$, $L1=L2=0.577$, $R1=5.50$, $R2=3.28$



SUPPLY=220.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS -0.0 DEGREES , F1STEP1=550.0 HZ. , CHOPS/STEP=1.
 $T(\text{ON})/T(\text{TOTAL})=0.800$, SPEED=1575.0 RPM
 $M=0.551$, $L1=L2=0.577$, $R1=5.50$, $R2=3.28$

Fig. 65 Spectra of system waveforms in the regenerating mode.

The following observations are related to the theoretical investigation. It will be convenient to comment separately in regard to:-

- (i) Torque.
- (ii) Efficiency
- (iii) Supply current fundamental displacement factor.
- (iv) Current Distortion.

6.9.1 Torque

Chopper modulation of the supply voltage is an effective method of producing a controlled reduction of the average torque developed by the motor. As an accurate approximation this torque is $\left(\frac{v_b}{v_s}\right)^2 T_s$ where T_s is the normal torque with the full supply voltage. The average torque variation is as it would be if the terminal voltage was pure sinewave and varied in amplitude linearly with $\left(\frac{v_b}{v_s}\right)$. From Figs. (25) to (28) it is seen that the average torque is *not a function of chopping frequency*. As shown theoretically, this is true provided the fundamental slip frequency is small in relation to the chopping frequency and also provided the chopping frequency is greater than the supply frequency.

6.9.2 Efficiency

It has been shown that the distortion current components in the motor contribute a negligible amount to the average output power. As a result the copper losses which they cause are provided from the electrical supply. If the chopping frequency is greater than the supply frequency by an appreciable amount (say 100 Hz. for the test motor) then the amplitude of the motor distortion currents is effectively constant over the normal range of operating speeds (say approximately 15 Hz. slip frequency). This is clearly seen in Fig (52). These currents have frequencies corresponding to $|F_e - mf_{chop}|, m=1,2,3$. From Fig (51) we see that if m is even they have equal peak values at $\left(\frac{v_b}{v_s}\right) = 0.25$ and 0.75 and zeros at $\left(\frac{v_b}{v_s}\right) = 0, 0.5$ and 1.0 . For odd values of m the distortion currents peak at $\left(\frac{v_b}{v_s}\right) = 0.5$ with zeros at $\left(\frac{v_b}{v_s}\right) = 0, 1.0$. The actual magnitude of each distortion current component at a particular value of $\left(\frac{v_b}{v_s}\right)$ is a function of chopping frequency as illustrated in Fig. (53).

In particular, as the chopping frequency is reduced to the 150 Hz. minimum considered here the size of the distortion currents increases rapidly. This is a simple consequence of the observations in Chapter 3.7 where it was concluded that the absolute length of the shorted interval should be kept below a certain value. For a given chopping frequency and (t_b/t) value therefore, the distortion components produce a constant loss in the motor which is unaffected by fundamental slip (Fig. (52)). The fundamental motor current however has the usual slip characteristic with a minimum at fundamental synchronous speed. If W_s is the terminal power when the full sinusoidal supply voltage is applied to the motor and W_{out} is the output power under these conditions, then we can separate the terminal power as

$$W_{term.} = \left(\frac{t_b}{t}\right)^2 W_s + \left[\begin{array}{l} \text{losses due to} \\ \text{distortion} \\ \text{components} \end{array} \right]$$

and the per unit efficiency is then

$$\eta = \frac{\left(\frac{t_b}{t}\right)^2 W_{out}}{\left(\frac{t_b}{t}\right)^2 W_s + \left[\text{losses} \right]}$$

Note:- for positive values only, and inverted if this expression > 1

As seen from the calculated results of Figs. (25) to (28) and Figs. (29) to (39) the effect of this additional loss is to reduce the nett motor efficiency compared with the value for sinewave excitation. This reduction is worst for low chopping frequencies, small values of $T(\text{on})/T(\text{total})$ and small fundamental slip. It should also be noted that in certain cases where each of the above conditions is bad (see Fig. (27) and Fig(28)) no nett regeneration can be achieved until a particular negative slip value is reached. Up to this point the power returned from the shaft is insufficient to account for the losses due to the distortion currents, so there is no nett return to the supply (although braking torque is developed). There are definite advantages to be gained in regard to efficiency by the use of increased chopping frequencies (see Fig. (39)) but beyond a certain

Slip frequency (Hz)	$\frac{T(\text{on})}{T(\text{total})}$	Efficiency (%) at different chopping frequencies			Efficiency (%) with pure sinewave using the same parameters for calculation.
		$F_c = 150 \text{ Hz}$	$F_c = 550 \text{ Hz}$	$F_c = 1050 \text{ Hz}$	
+5	0.1	46.73	73.28	75.16	75.86
	0.5	63.24	75.01	75.61	75.83
	0.9	74.90	75.43	75.45	75.46
+2.5	0.1	41.98	81.56	84.97	86.36
	0.5	64.77	84.77	85.89	86.32
	0.9	84.76	85.75	85.79	85.81
+1.0	0.1	25.66	81.51	89.25	92.52
	0.5	51.99	88.73	91.40	92.43
	0.9	88.83	91.17	91.28	91.33
-1.0	0.1	0	79.90	88.91	92.23
	0.5	28.18	88.30	91.10	92.13
	0.9	88.29	90.72	90.83	90.88
-2.5	0.1	12.76	81.12	84.50	85.71
	0.5	61.20	84.23	85.27	85.66
	0.9	84.07	84.98	85.02	85.04
-5.0	0.1	36.91	71.66	73.34	73.97
	0.5	61.67	73.21	73.73	73.92
	0.9	72.92	73.37	73.39	73.40

Table 2

Values calculated for the test motor. $V_s = 220 \text{ Volt RMS } 50 \text{ Hz}$.

value (approximately 500 Hz. for the test motor) the improvement to be gained by further increases is slight. In practice this improvement would be offset by increased switching losses in the chopper modulator.

It is interesting to note that the reduction in efficiency, as seen in Figs. (35) and (37) is less severe in the usual slip region with *low supply frequency*. This is a result of the increased discrimination of the motor against the harmonic currents at these reduced supply frequencies. For example, if the supply frequency is reduced from 50 Hz. to 25 Hz. while chopping at 150 Hz. with the slip frequency held constant, then the fundamental impedance is reduced by a factor of approximately one-half whereas the impedance to the first distortion component is increased by a quarter. It is thus acceptable to use lower chopping frequencies when the supply frequency is low. The improved situation under reduced frequency conditions is clearly illustrated by the time-domain waveforms in Fig. (63).

Table 2 provides a quick summary of the information which has been presented graphically in regard to efficiency in the case of the test motor with $V_s = 220$ Volt RMS 50 Hz.

Note that successful regeneration, as seen from these results, is quite possible in theory with a chopper modulated supply. The physical realization of this depends on the capability of the modulator to perform its function irrespective of the instantaneous polarity of the voltage and current waveforms in a particular phase. The modulator used for the experimental work described in the following chapter achieves this and is fully regenerative.

6.9.3 Supply Current Fundamental Displacement Factor:

The fundamental motor current is reflected into the supply to produce a fundamental supply current component with the usual displacement angle of the motor at the given supply frequency. If this were the only constituent of the fundamental supply current, the supply current displacement factor would be the same as the power factor for full sinusoidal excitation.

SLIP FREQUENCY (HZ)	T(ON)/T(TOTAL)	Fundamental Supply Current Displacement angle for Different chopping Frequencies			Displacement angle with full sinewave excitation, using the same motor parameters.
		$F_C = 150 \text{ Hz}$	$F_C = 550 \text{ Hz}$	$F_C = 1050 \text{ Hz}$	
+5	0.1	+11.00°	-26.93°	-29.56°	-30.60°
	0.5	-15.07°	-29.62°	-30.42°	-30.71°
	0.9	-31.68°	-32.31°	-32.31°	-32.31°
+2.5	0.1	+26.07°	-22.75°	-27.27°	-28.93°
	0.5	-4.18°	-27.33°	-28.71°	-29.28°
	0.9	-31.40°	-32.37°	-32.43°	-32.49°
+1.0	0.1	+38.79°	-30.37°	-39.13°	-42.34°
	0.5	+4.47°	-39.30°	-41.94°	-42.86°
	0.9	-46.24°	-48.01°	-48.07°	-48.128°
-1.0	0.1	+65.09°	-138.03°	-135.16°	-134.24°
	0.5	+177.85°	-134.65°	-133.90°	-133.67°
	0.9	-127.94°	-127.77°	-127.77°	-127.77°
-2.5	0.1	+102.67°	-148.63°	-146.62°	-145.93°
	0.5	-160.37°	-146.33°	-145.76°	-145.59°
	0.9	-142.04°	-141.75°	-141.75°	-141.75°
-5.0	0.1	-172.52°	-140.32°	-139.52°	-139.23°
	0.5	-143.64°	-139.34°	-139.11°	-139.06°
	0.9	-137.05°	-136.94°	-136.94°	-136.94°

Table 3.

Values calculated for the test motor. $V_s = 220 \text{ Volt RMS } 50 \text{ Hz}$.

However, each distortion component in the motor current, when de-modulated, contributes a further component to the fundamental supply current. The addition of these components produces a nett fundamental supply current with a different displacement factor. The extent to which this factor differs from the sinewave case depends on the magnitude of the distortion components in the motor current. As we have seen the motor current distortion factor is worst with low chopping frequencies, low (ω_b/ω_r) and small fundamental slip values. In these cases the supply displacement angle is largely influenced by the distortion currents (see Figs. (29) to (33)). In particular we note that decreasing chopping frequency or (ω_b/ω_r) at a given slip tends to reduce the lagging displacement angle if it is less than 90° (motoring) and to increase it if greater than 90° (regenerating). This effect is summarized for the test motor in Table 3. Of special interest is the fact that in the extremes these angles pass through 0° and 180° respectively to become *leading* displacement angles (See Figs (29) to (33)). This emphasizes the fact that the modulator performs a non-linear transformation function between the supply and the linear load, and the current response as seen in the supply lines does not reflect the sinusoidal impedance of the load as normally understood. It will be seen later that leading supply current fundamental displacement factors are commonly encountered in the phase-stepping frequency convertor.

6.9.4 Supply Current Distortion:

Although the current distortion components in the supply have the same frequencies as those in the motor, they differ in magnitude and phase for the same reasons as in the case of the fundamental component. As a result, these distortion currents are not constant for different slip frequencies as they are in the motor, although the variations are not large (see Fig. (50)). From Fig (49) we note that the maximum nett RMS of the distortion currents in the supply occurs at approximately $\omega_b/\omega_r = 0.65$ (compared with $(\omega_b/\omega_r) = 0.5$ for the motor current). In general the distortion factor is worse for the supply current than for the motor current. Table 4 summarizes the results and contrasts motor and supply current distortion factors.

SLIP FREQUENCY (HZ.)	T (ON) / T (TOTAL)	Distortion factors for different chopping frequencies					
		$F_c = 150 \text{ Hz}$		$F_c = 550 \text{ Hz}$		$F_c = 1050 \text{ Hz}$	
		supply current	motor current	supply current	motor current	supply current	motor current
+5	0.1	0.219	0.542	0.296	0.934	0.310	0.982
	0.5	0.552	0.748	0.692	0.978	0.703	0.994
	0.9	0.935	0.986	0.948	0.999	0.949	1.000
+2.5	0.1	0.195	0.354	0.266	0.839	0.299	0.946
	0.5	0.441	0.553	0.666	0.942	0.695	0.983
	0.9	0.913	0.961	0.946	0.998	0.948	1.000
+1.0	0.1	0.170	0.193	0.195	0.625	0.264	0.838
	0.5	0.292	0.328	0.581	0.824	0.666	0.942
	0.9	0.844	0.889	0.941	0.992	0.947	0.998
-1.0	0.1	0.122	0.205	0.181	0.652	0.265	0.854
	0.5	0.061	0.346	0.510	0.688	0.669	0.948
	0.9	0.850	0.900	0.942	0.993	0.947	0.998
-2.5	0.1	0.086	0.401	0.271	0.872	0.303	0.958
	0.5	0.324	0.609	0.674	0.955	0.698	0.988
	0.9	0.921	0.971	0.947	0.999	0.948	1.000
-5.0	0.1	0.105	0.637	0.302	0.959	0.313	0.989
	0.5	0.544	0.823	0.698	0.987	0.705	0.997
	0.9	0.940	0.991	0.948	1.000	0.949	1.000

Table 4

Values calculated for the test motor. $V_s = 220 \text{ Volt RMS } 50 \text{ Hz}$.

6.9.5 Torque Pulsations:

The peak-to-peak torque pulsation amplitude tends to have a maximum value at fundamental synchronous speed, and the pulsations are reduced in size by increasing the chopping frequency. (See Figs. (44) to (48)). In Chapter 3.7 it was seen that the torque following a short-circuit goes rapidly from the initial value towards a negative maximum. The magnitude of this excursion in a given time is largest at synchronous speed which is in accordance with the above observation. For a certain slip and chopping frequency, maximum torque pulsations are found at $(t/t_s) = 0.7$ approximately (see Fig. (46)).

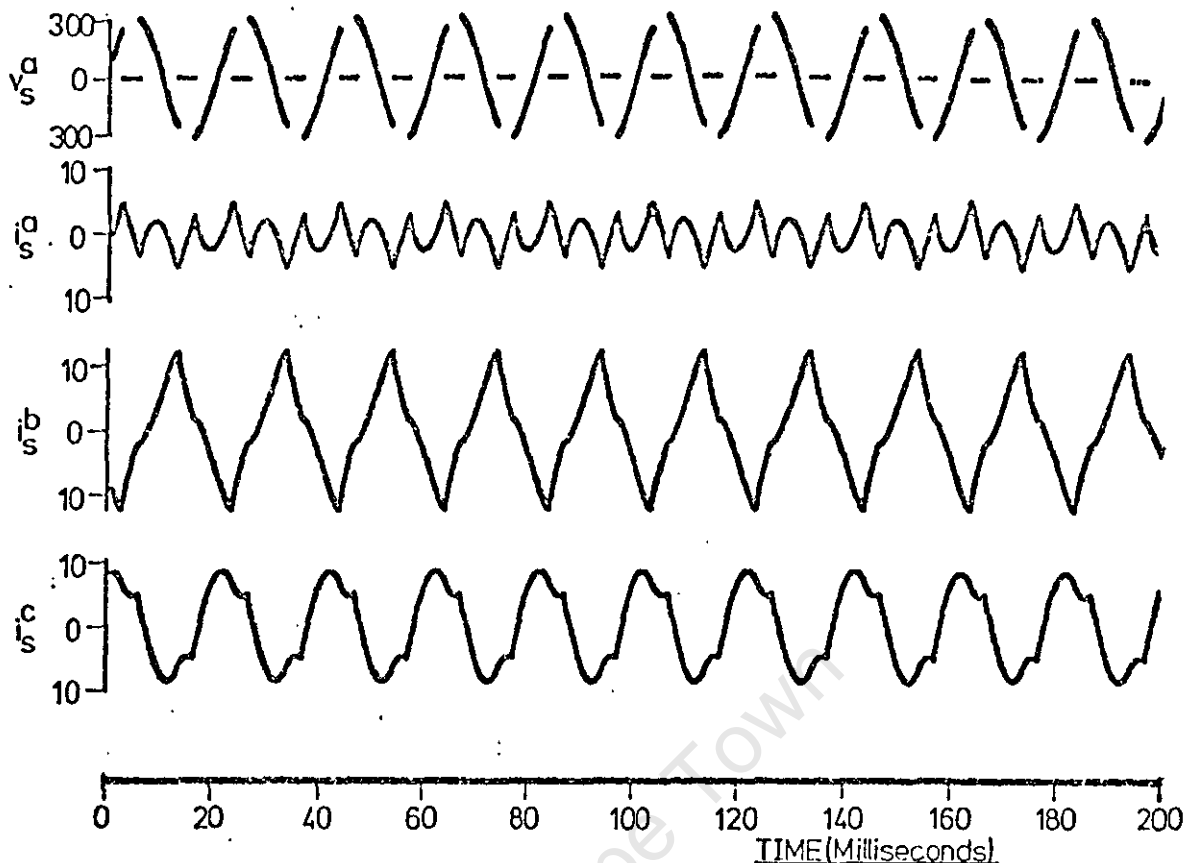
6.10 Low Chopping Frequencies and Special Considerations:

As observed in the general analysis of Chapter 5, cases where the modulation angle $\theta = n\pi$ require special consideration. In the chopper modulation, we have $\theta = \omega t$ so that the special cases occur when

$$f_{chop} = \frac{2 \cdot f_e}{n} \quad n = 1, 2, 3, \dots$$

In the small motors which have been the subject of this analysis, operation in this region (i.e. with chopping frequency less than or equal to 2x supply frequency) is precluded on the grounds of poor efficiency and large RMS currents, as we have seen. It is nevertheless important to include this region in the work for the sake of completeness and because it may be necessary to use low modulation frequencies under certain conditions.

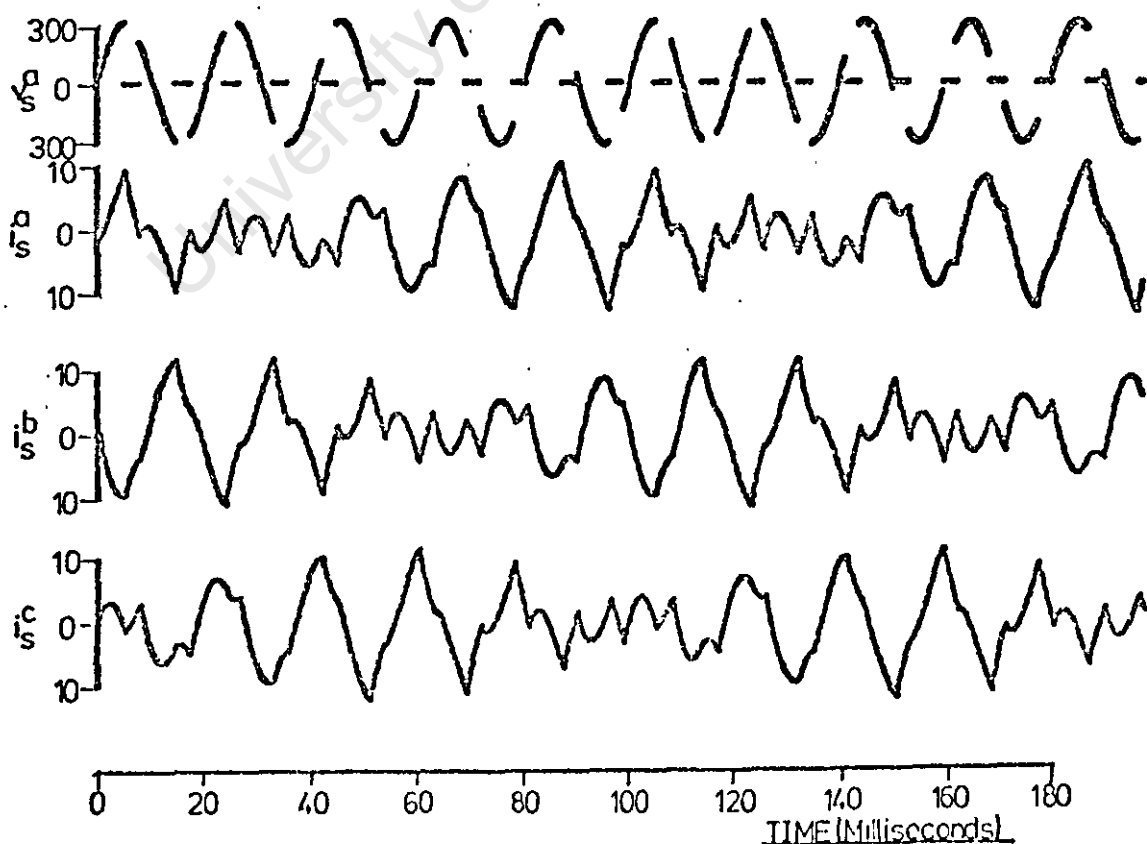
In each case where $f_{chop} = \frac{2f_e}{n}$, the RMS phase currents differ from each other both in the motor and the supply and the three-phase sets corresponding to each frequency component (including the fundamental) are unbalanced. When n is odd there is no D.C. component in either the supply or motor currents, but where n is even, each phase current has a D.C. component. These situations are illustrated for $n = 1$ and $n = 2$ in Figs. (68) and (66). If $f_{chop} < \frac{2f_e}{n}$ subharmonics are generated. If f_{chop} approaches $\frac{2f_e}{n}$ for any n , the waveforms have identical RMS and the phase spectra

Numerical Simulation.

Supply voltage = 220 volt RMS 50 Hz. Slip = 10%

Chopping frequency = 100 Hz. $T(\text{on})/T(\text{total}) = 0,7$

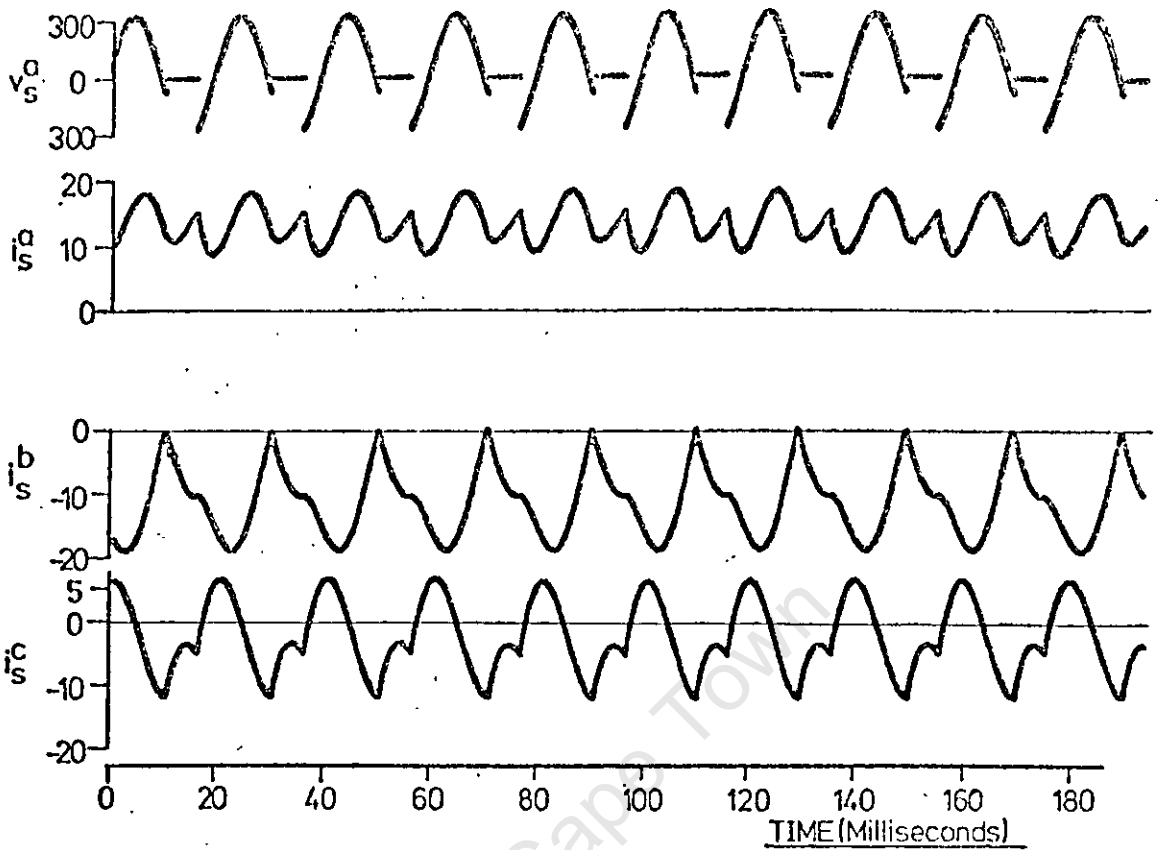
Fig. 66 Illustration of unbalanced currents in the test motor.



Supply voltage = 220 volt RMS 50 Hz. Slip = 10%

Chopping frequency = 110 Hz. $T(\text{on})/T(\text{total}) = 0,7$

Fig. 67 Illustration of balanced phase currents with a beat frequency amplitude modulation as the chopping frequency approaches one of the 'special' values.

Numerical Simulation.

Supply voltage = 220 volt RMS 50 Hz. Slip = 10%
 Chopping frequency = 50 Hz. $T(\text{on})/T(\text{total}) = 0.7$

Fig. 68 Illustration of D.C. components and unbalanced phase currents in the test motor

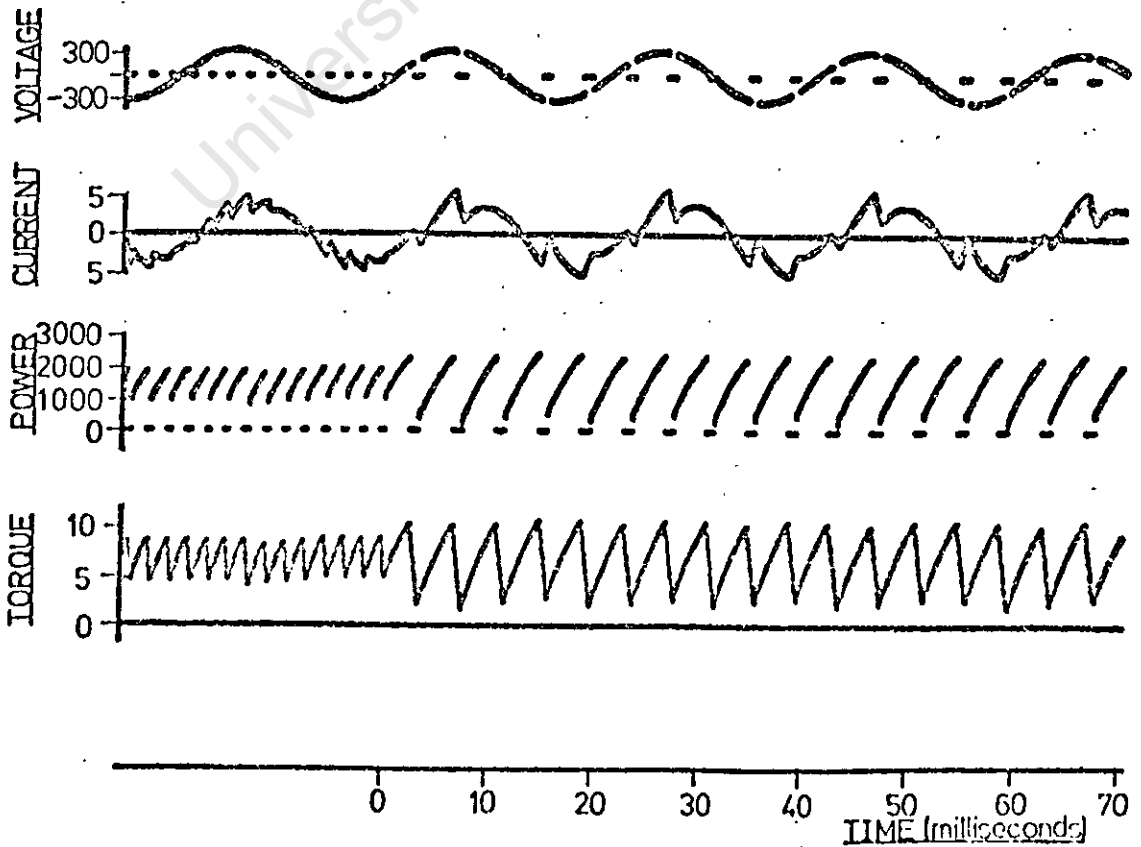


Fig. 69 Illustration of a sudden change in chopping frequency.
 $V_s = 220$ Volt RMS 50 Hz., $T(\text{on})/T(\text{total}) = 0.8$, slip = 5%
 Chopping frequency: 550 Hz $\rightarrow$ 250 Hz

form balanced sets, but the waveform is amplitude modulated by the beat frequency which is the difference between f_{chop} and $\frac{2f_e}{n}$. This is illustrated in Fig. (67).

If the chopping frequency is any integral multiple greater than 2 times the supply frequency then the individual phase waveforms are periodic in both motor and supply. If this multiple is divisible by three then the waveforms in the three phases are identical but time-shifted relative to each other. If there is no integral relationship between f_{chop} and f_e then the waveforms do not repeat themselves exactly. These situations are illustrated in Figs. (70), (71).

6.11 Transients Produced by Sudden Changes in the Parameters of the Modulation.

We have shown that chopper modulation of the terminal voltage applied to the induction motor effectively reduces the fundamental (time) flux level in the machine by the factor (τ_b/τ) and this is independent of chopping frequency. If the factor (τ_b/τ) is changed suddenly while the motor is operating in the quasi-steady state then the system moves through the transition to the new quasi-steady state with transients which are similar to those generated by a sudden change in the level of an applied sinewave voltage. This is clearly seen from the simulated waveforms in Figs. (72), (73), (74). If, however, a sudden change of chopping frequency occurs with no change in the value of (τ_b/τ) then there is point-on-wave continuity of the fundamental frequency flux distribution and virtually no transients in the envelopes of the various waveforms occur. This is shown in Fig. (69).

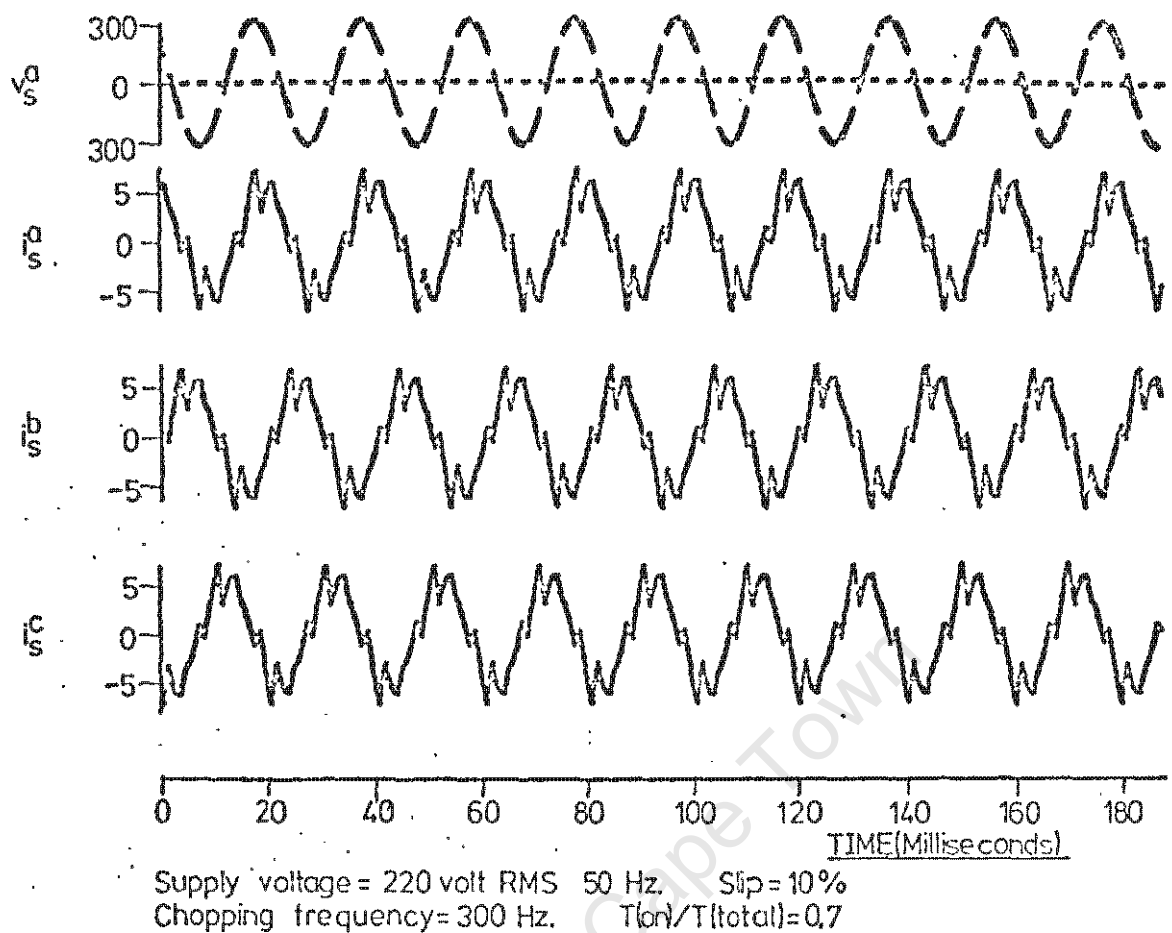
Numerical Simulation.

Fig. 70 Illustration of similar but time-shifted phase current waveforms in the test motor with chopper modulated supply voltage.

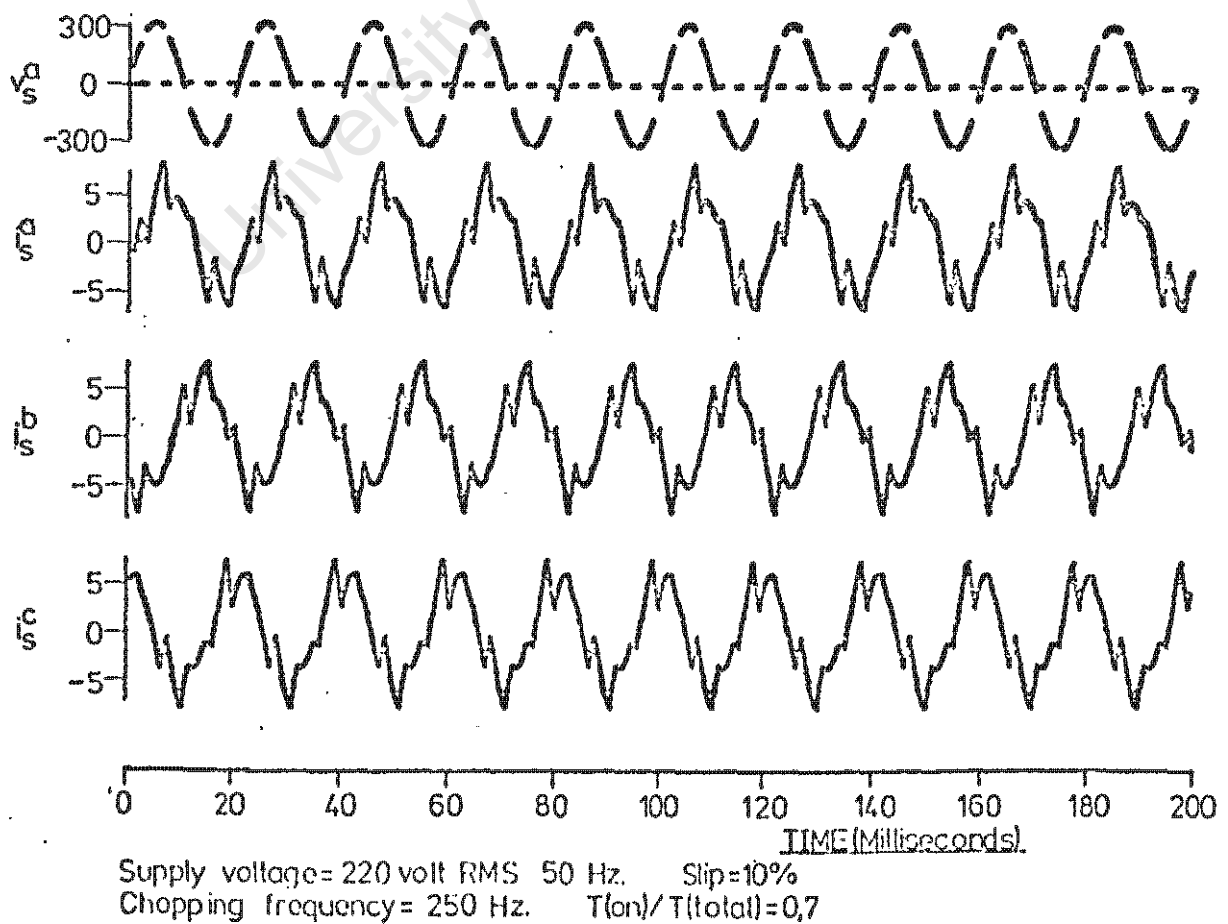
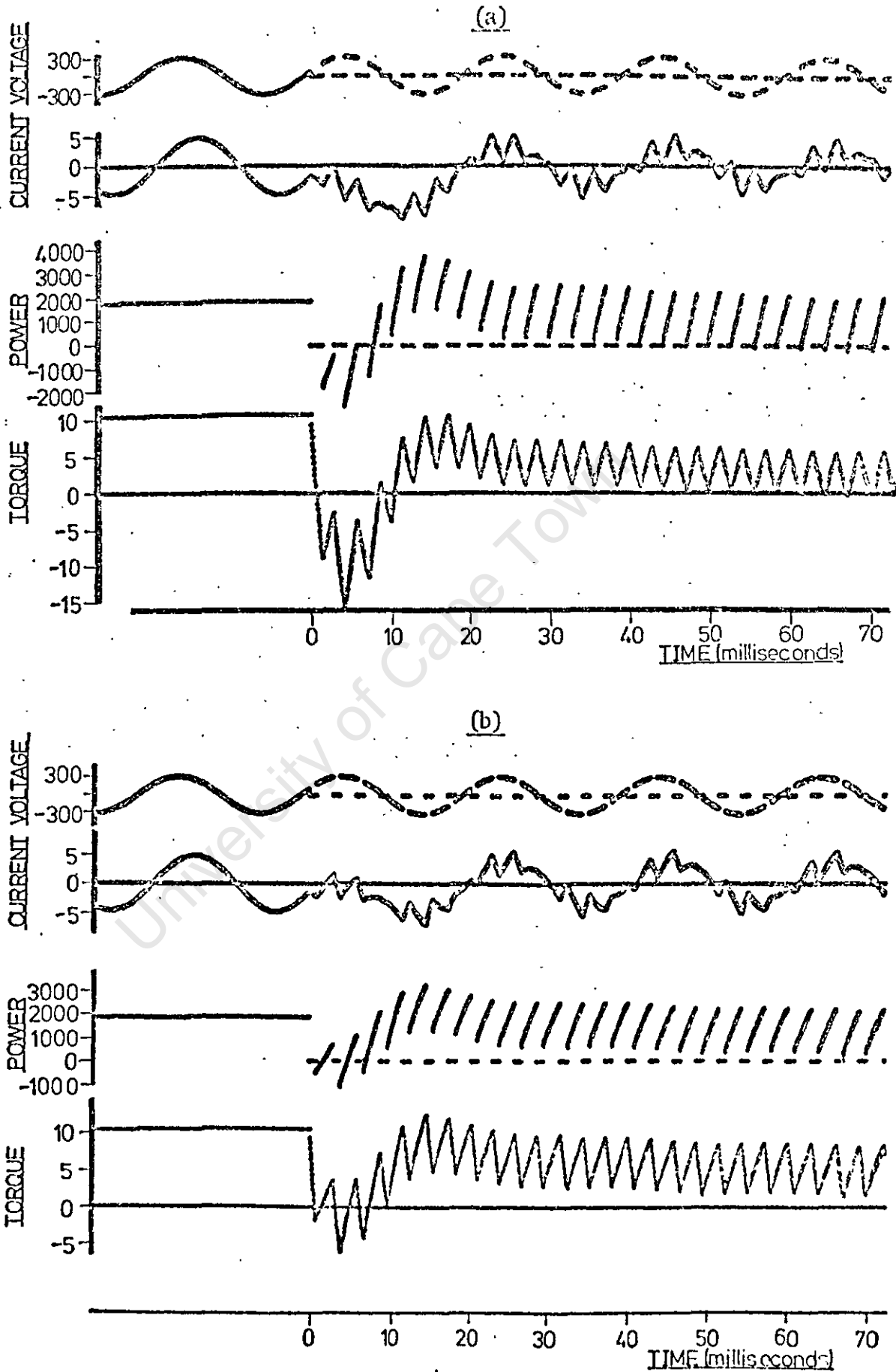


Fig. 71 Illustration of differing phase current waveforms in the test motor.

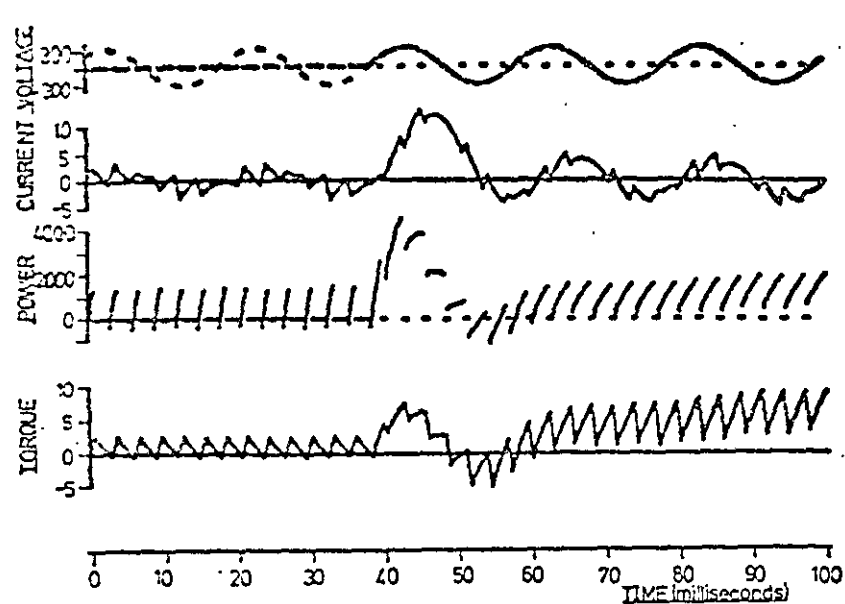
Numerical Simulation.Fig. 72

Sudden initiation of chopping in the case of the test motor. $V_s = 220$ Volt RMS 50 Hz., Slip = 5%

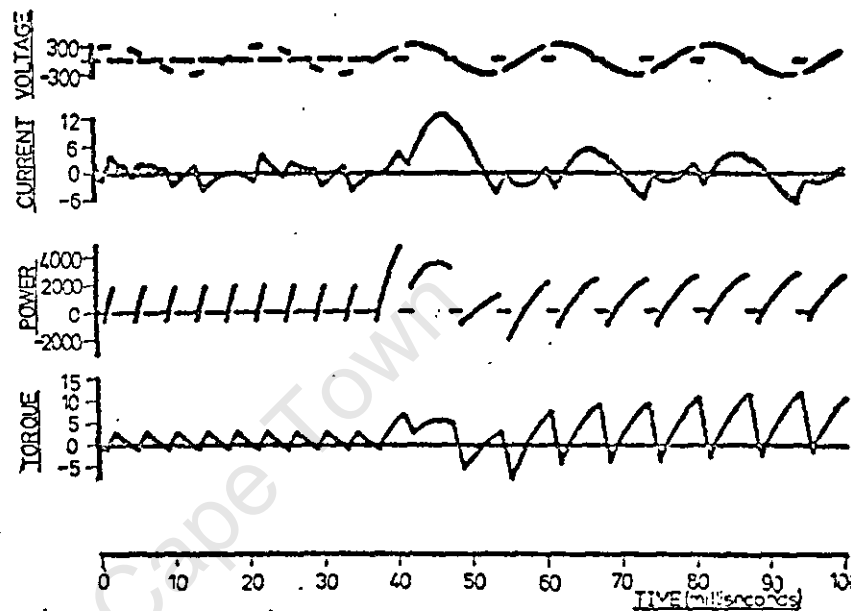
Chopping frequency = 350 Hz.

a) $T(\text{on})/T(\text{total}) = 0.5$

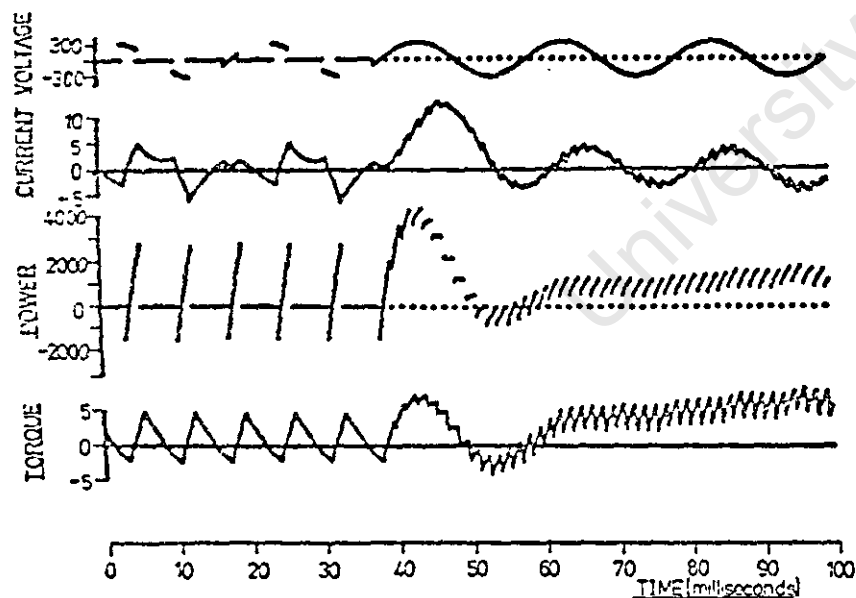
b) $T(\text{on})/T(\text{total}) = 0.7$



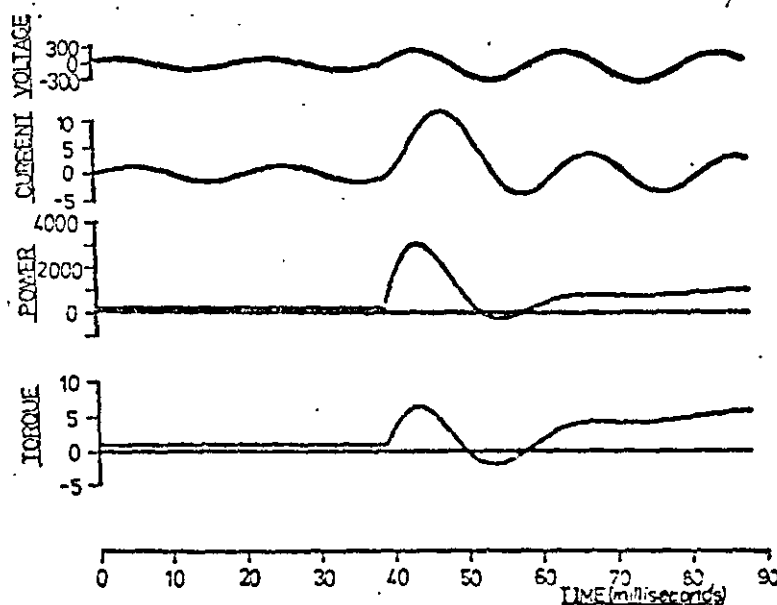
a) $T(\text{on})/T(\text{total}): 0.3 \rightarrow 0.8$
Chopping frequency = 350 Hz. $V_s = 220$ Volt RMS 50 Hz. $Slip = 5\%$



c) $T(\text{on})/T(\text{total}): 0.3 \rightarrow 0.8$
Chopping frequency: 250 $\rightarrow$ 150 Hz. $V_s = 220$ Volt RMS 50 Hz. $Slip = 5\%$



b) $T(\text{on})/T(\text{total}): 0.3 \rightarrow 0.8$
Chopping frequency: 150 $\rightarrow$ 700 Hz. $V_s = 220$ Volt RMS 50 Hz. $Slip = 5\%$



d) Sinewave excitation: 132 $\rightarrow$ 176 Volt RMS, 50 Hz. $Slip = 5\%$

Fig. 73 Illustration of sudden changes in mark-to-total time ratio.

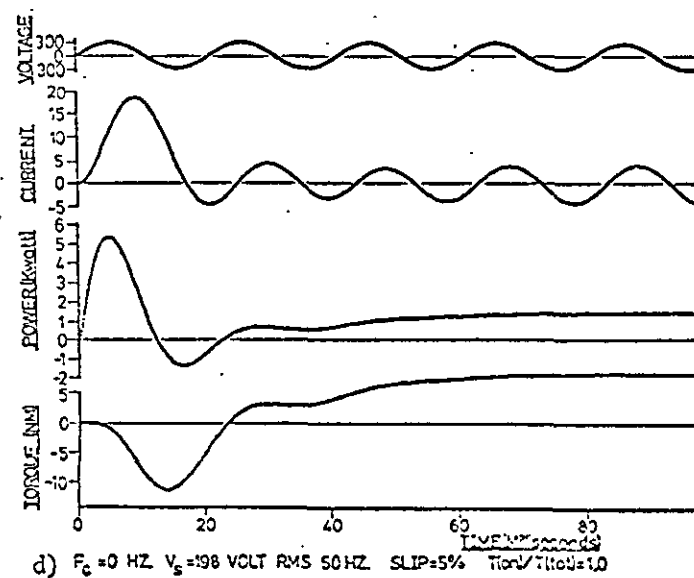
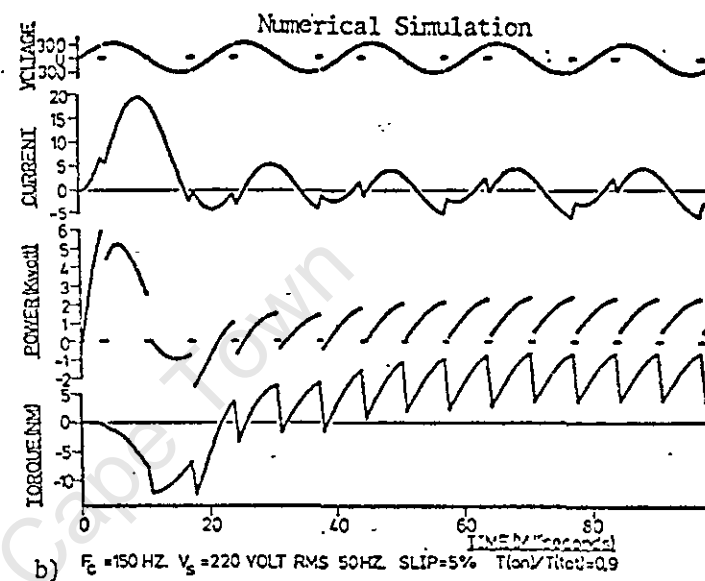
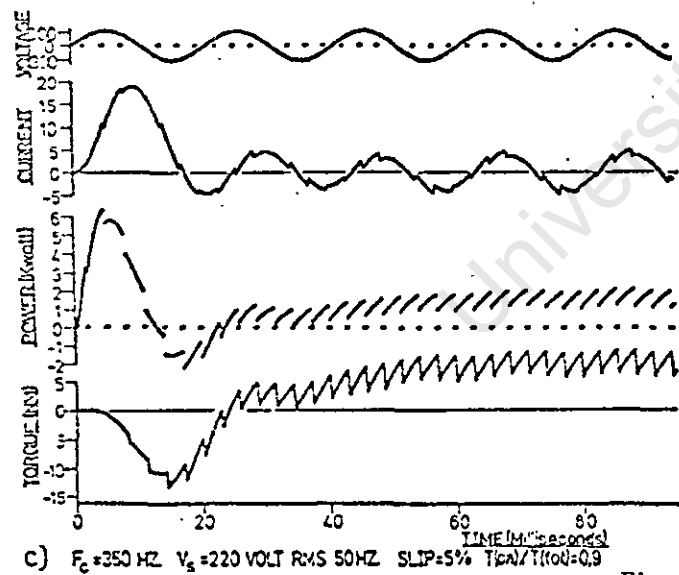
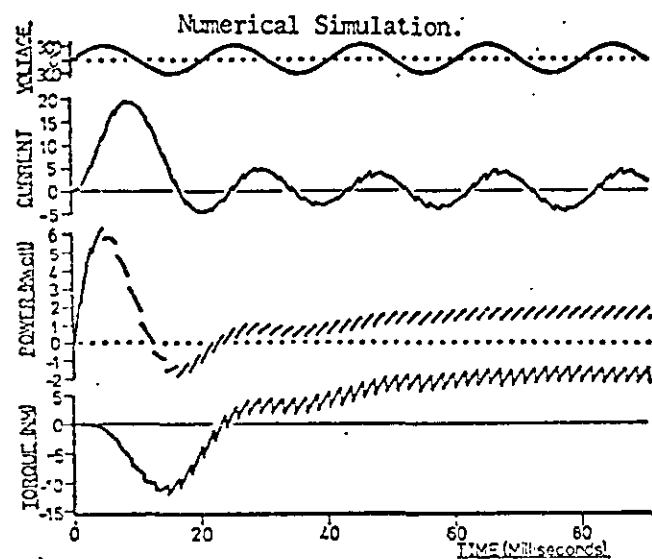


Fig. 74

Switch-on of de-energised test motor.

CHAPTER SEVEN

EXPERIMENTAL INVESTIGATION OF AN INDUCTION MOTOR WITH CHOPPER MODULATED SUPPLY VOLTAGE

In the preceding chapters a general analysis of the response of induction motors to modulated voltage waveforms has been presented, and this approach has been used to evaluate the theoretical characteristics of a chopper modulated motor. This formulation is based on an idealized coupled circuit model of the motor and also assumes that the chopping action is performed by an ideal modulator. The calculated results thus overlook the many parasitic mechanisms which influence the overall behaviour of an actual motor, as well as the inevitable switching losses which arise in the chopper. In order to estimate to what extent this theory correlates with the performance of an actual system, a chopper was assembled using novel high speed commutating switches and the response of the small test motor was investigated in the laboratory.

The physical description of the test motor is contained in Appendix (6), and in Appendix (5) we discuss some of the measurement problems which were encountered in the course of the experimental work.

7.1 Verification of Motor Parameters for Sinusoidal Excitation:

Before considering unfamiliar modulated excitation, the performance of the test motor with sinusoidal excitation was recorded. Since the fundamental flux density is reduced by chopper modulation, it was considered necessary to repeat the measurements for several different levels of sinusoidal voltage corresponding to the fundamental output voltages which would be produced by the chopper with different values of (τ_b/τ) .

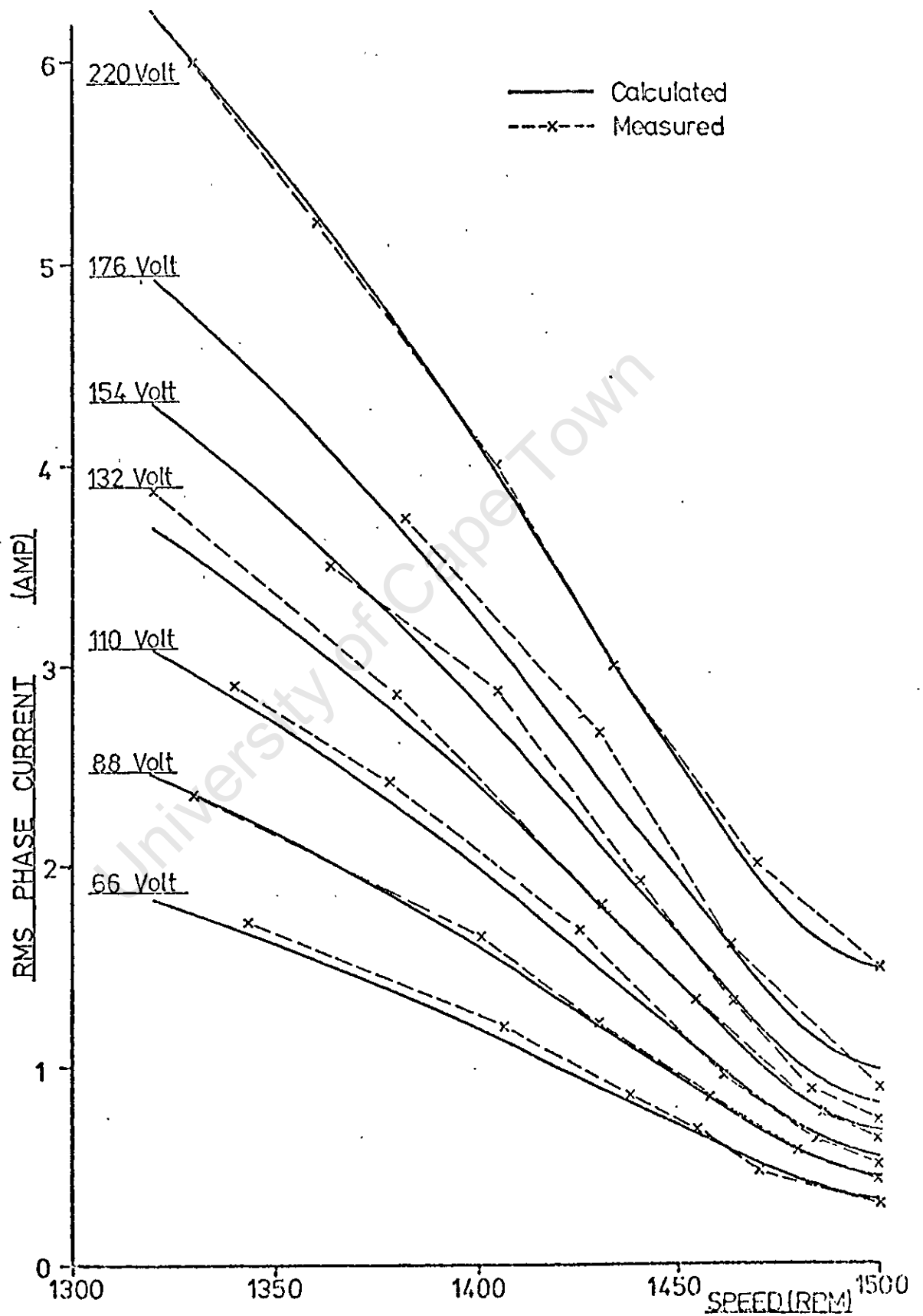


Fig. 75 Measured and calculated characteristics of the test motor with sinusoidal supply voltage.
Supply frequency = 50 Hz.

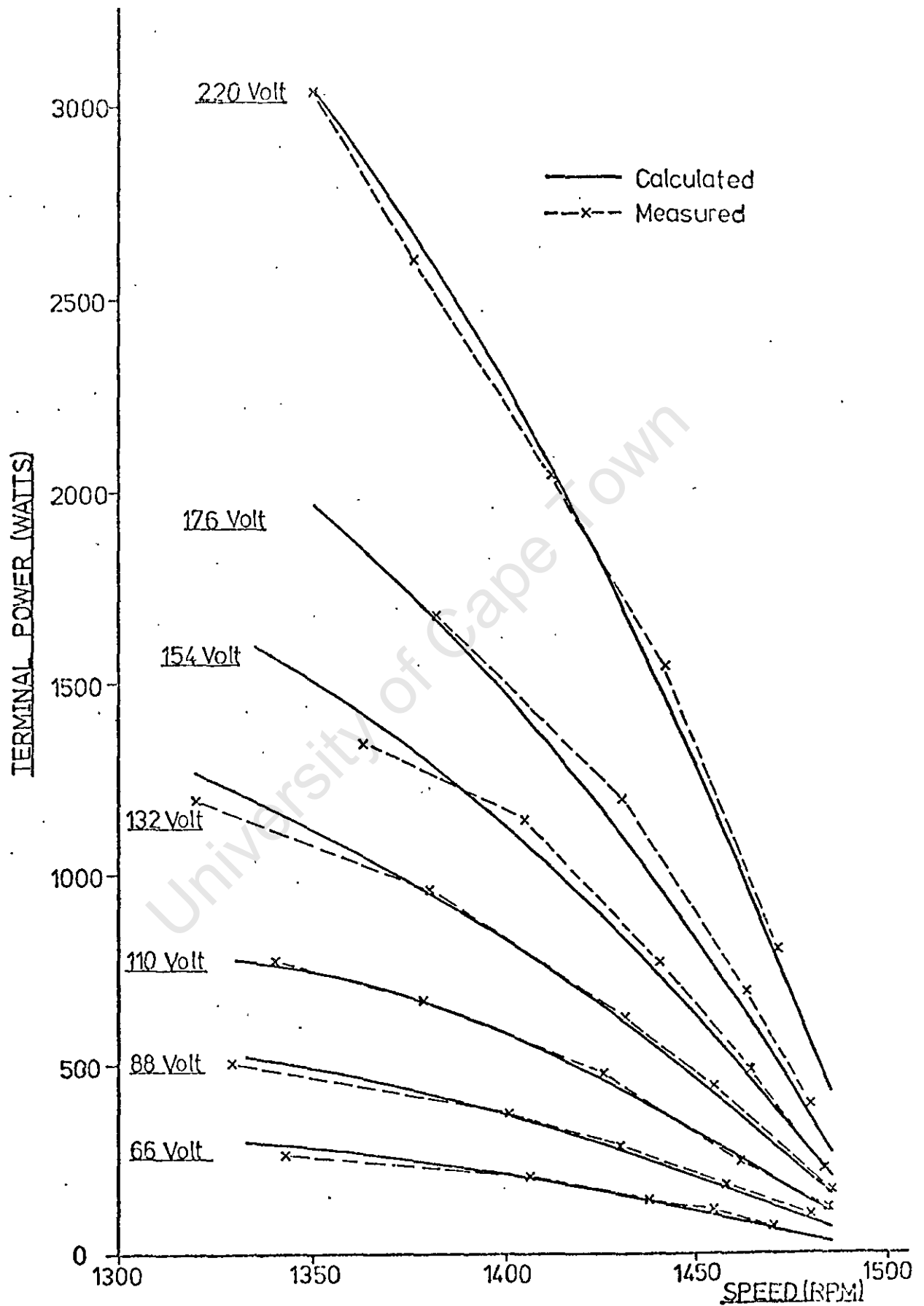


Fig. 76 Measured and calculated characteristics of the test motor with sinusoidal supply voltage.
Supply frequency = 50 Hz.

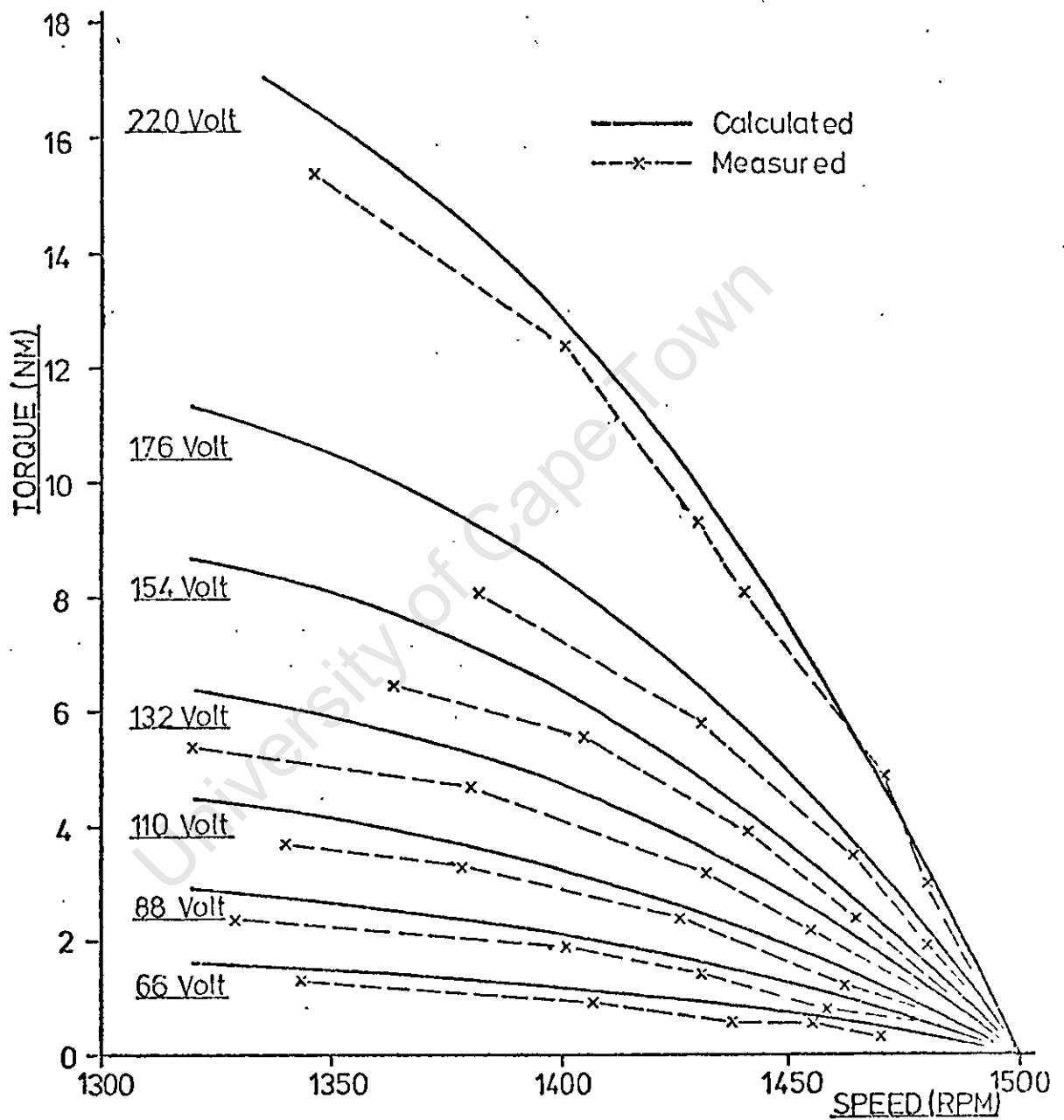


Fig. 77

Measured and calculated characteristics of the test motor with sinusoidal supply voltage.
Supply frequency = 50 Hz.

The measured characteristics are plotted in Figs. (75), (76), (77) together with the calculated characteristics. For each value of applied voltage the appropriate inductive parameters have been used in accordance with the discussion in Appendix (6).

7.2 The Chopper

The chopper modulation was achieved by the use of prototype transistorized switches which are capable of rapidly commutating the current in a load from one supply circuit to another. These switches are described elsewhere⁽²¹⁾ and will not be treated in detail here. It is necessary however to discuss briefly how the commutation action is achieved, since this has a bearing on the correlation between measured and calculated results.

As we have mentioned, the chopping action requires the opening or closing of a series-connected switch while a switch in parallel with the load performs the opposite function simultaneously. This requires careful synchronization to avoid either short-circuiting the supply or open-circuiting the load, either of which might be dangerous. In the case of the switches which were used, the commutation of load current is achieved during a "window" interval which has a maximum value of $30\mu\text{sec}$. Each commutation is initiated by instructing the conducting switch to open. As this starts to happen, the load voltage rises extremely rapidly, depending on the inductance and the actual instantaneous value of the current. The rising load voltage is detected in the control circuit of the other switch as soon as it exceeds an established threshold level, and the closure of this switch is immediately initiated. This feedback technique emulates in principle the mechanism by which the freewheel path is forward-biased when the series switch is opened in a D.C. chopper system. The use of this strategy implies the following:-

- (a) In order to generate the feedback voltage, a small change in the motor current must be instituted during commutation.

- (b) High voltages equal to the reference level plus the instantaneous supply voltage exist across the load for a short time during every commutation (where $i_{load} \neq 0$). These voltages do not exceed 700 volts in the prototype and last for less than $30 \mu\text{sec}$. The polarity of each voltage spike is such as to maintain the flow of current through the loop being opened. Thus if the power factor is less than unity the spikes all have the opposite polarity to the motor voltage while forward power is being supplied and the same polarity as the motor voltage during the regenerative part of the cycle. In effect, these spikes represent a departure from the ideal terminal waveforms upon which the theory has been based. In the motoring region they subtract from the effective mark time and in the regenerative mode they add to the mark, but in a manner which cannot easily be incorporated into the theoretical treatment. In practice the effect of the spikes (together with the forward voltage drop across the switches) is seen as regulation of the RMS motor voltage by comparison with expected values. This effect is worst at high chopping frequency.

The operation of the switches is illustrated in the oscillograms of Figs (78) and (79).

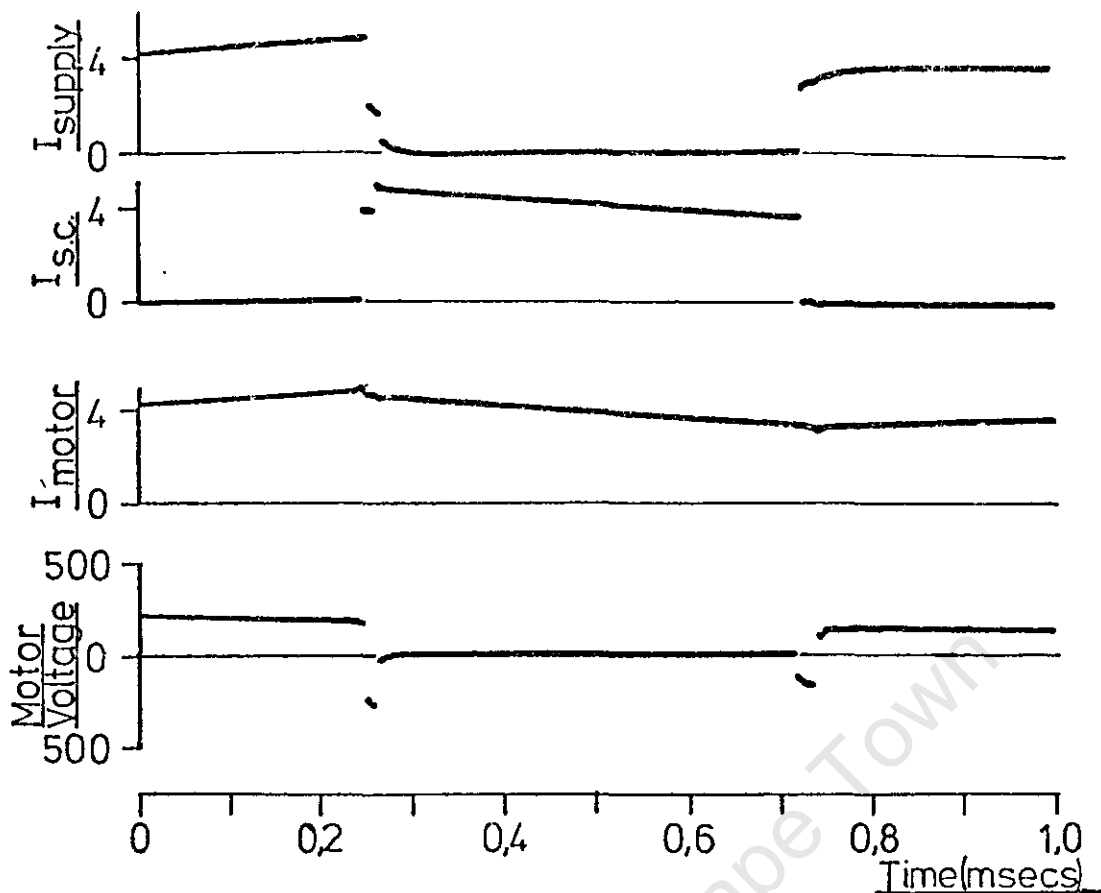


Fig. 78 Illustration of waveforms in the test motor with chopper modulated supply voltage. This oscillogram indicates to what extent the ideal chopping action was realised in the experimental work.

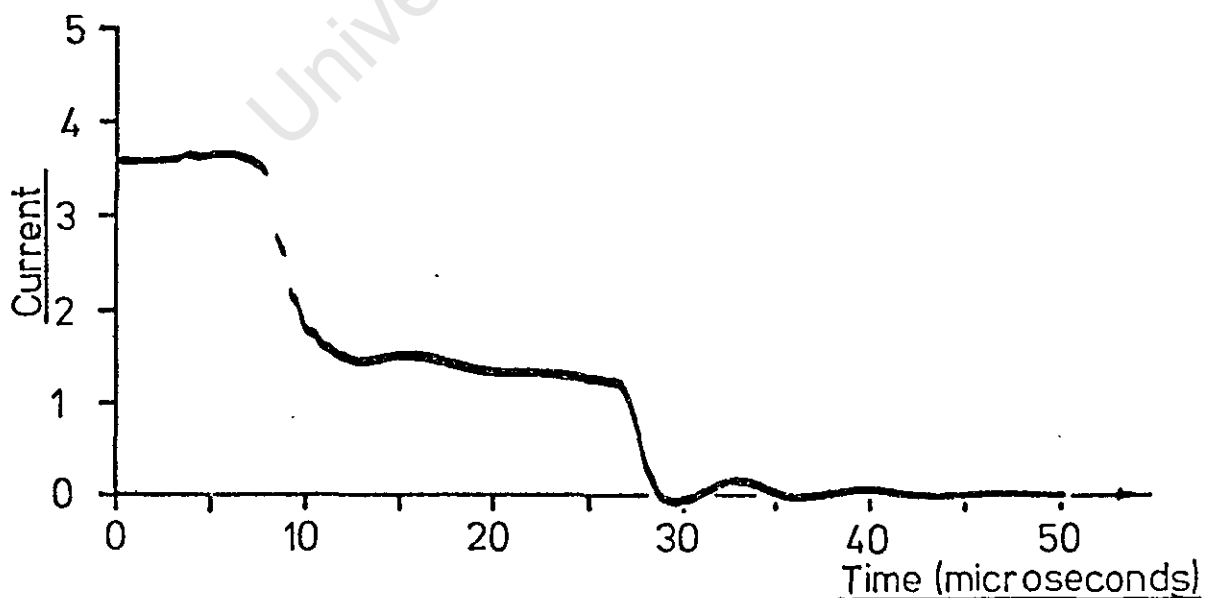
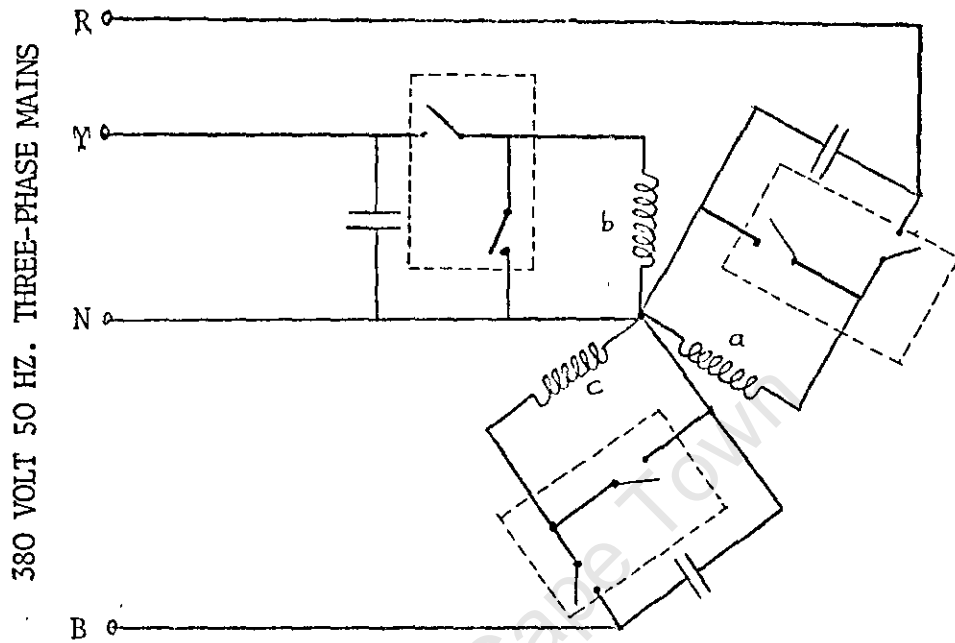


Fig. 79 Oscillogram showing the supply current during a typical forced commutation. Initially the supply line carries the full load current which is then commutated into the short circuit path in less than $30\mu\text{sec}$. The calculations in the preceding chapters assume this action is instantaneous.

7.3 The Experimental Circuit



It was found essential to include the capacitors in parallel with the supply to provide a path for high frequency commutation currents. The control signal for the operation of the choppers was generated in synchronism with the mains with the aid of a phase-lock loop and thus only multiples of the mains frequency were used for chopping frequency. Continuous variation of mark-to-total time ratio was possible between 0.2 and 0.8. The mark time was set visually against an expanded oscilloscope trace. Fig (80) is a photograph of the experimental area in the laboratory during these tests and Fig. (81) shows the switches.



Fig.80 The experimental work area during tests performed on the chopper modulator.

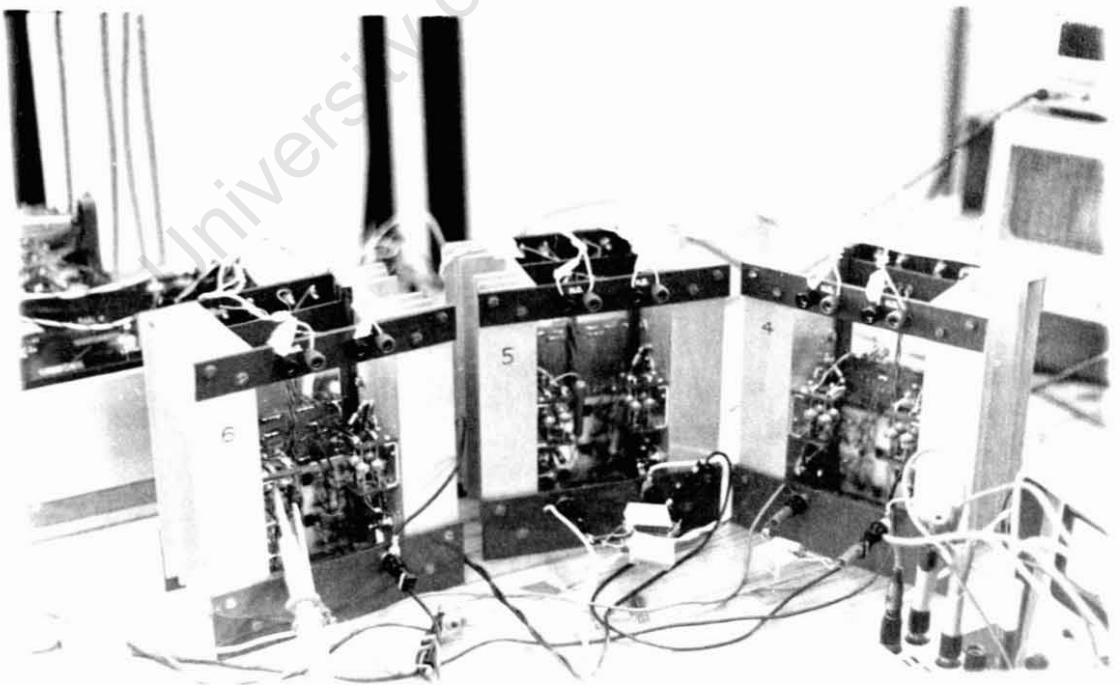


Fig.81 The power commutating switches.

7.4 Quasi-steady State Measurements

The measured characteristics of the test motor with chopper modulated excitation are illustrated in Figs. (82) to (88) for quasi-steady state operation.

- (i) Figs (82) to (84) are presented in groups of five labelled (a) to (e) which can be categorized as follows:-
- (a) Measured RMS motor and supply currents as a function of motor speed for several values of $T(\text{on})/T(\text{total})$.
 - (b) Measured average torque and power as a function of motor speed for several values of $T(\text{on})/T(\text{total})$.
 - (c) Measured spectra of motor and supply current waveforms for $T(\text{on})/T(\text{total}) = 0.5$ and 0.8 . Note that the fundamental frequency component has been suppressed in certain cases as indicated in order to display the distortion components clearly.
 - (d) Oscillograms showing waveforms of motor voltage and current and supply current for four different slip values with $T(\text{on})/T(\text{total}) = 0.5$.
 - (e) As in (d) but with $T(\text{on})/T(\text{total}) = 0.8$.

Each group of illustrations as above were recorded with a constant value of chopping frequency. The calculated results which are displayed in each case were compensated for the observed voltage regulation in the convertor so as to provide a basis for comparison. If the observed RMS motor voltage was V_{obs} for example and that expected from an ideal modulator under the

CHOPPER MODULATION.

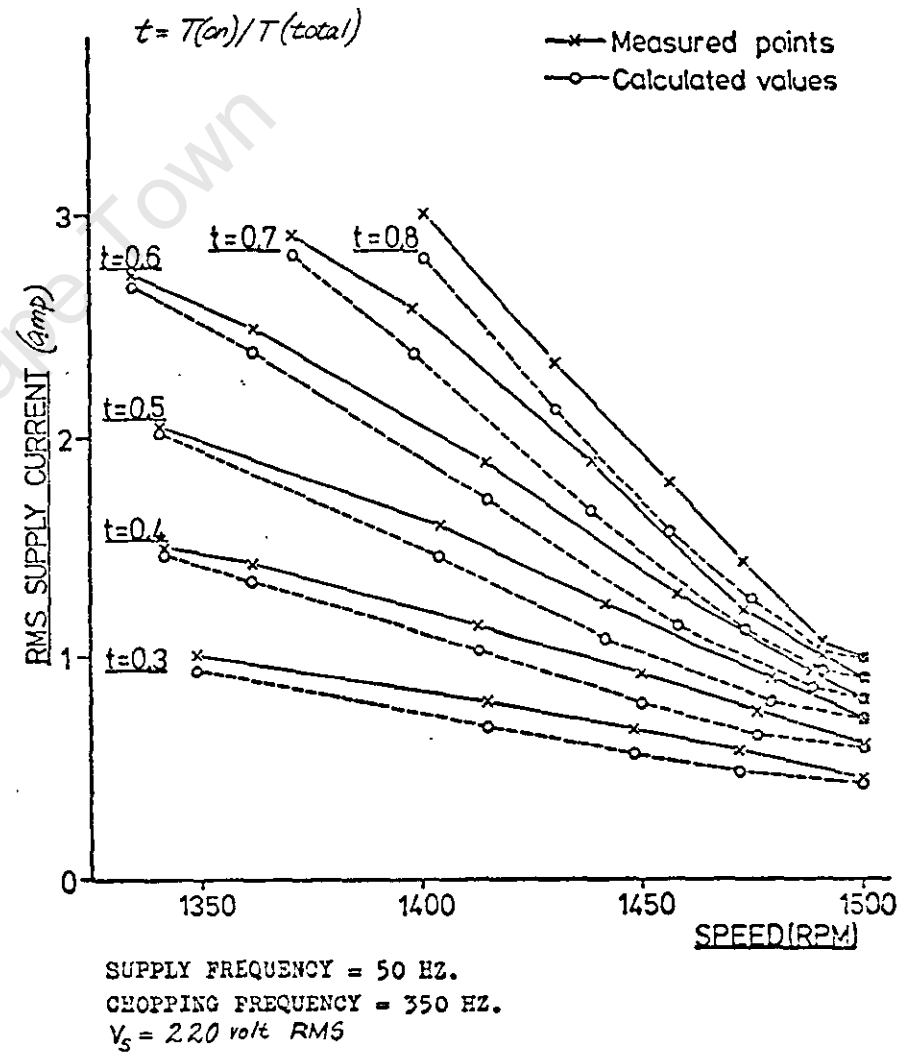
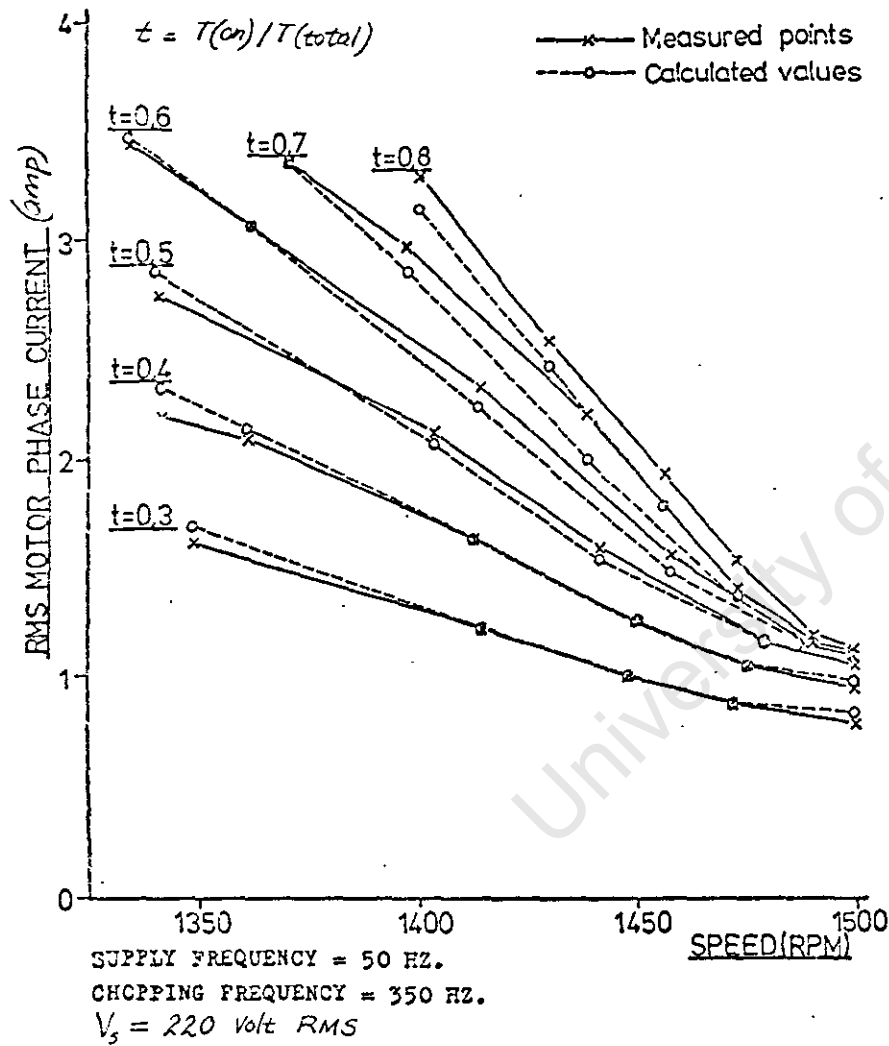


Fig. 82 a

CHOPPER MODULATION.

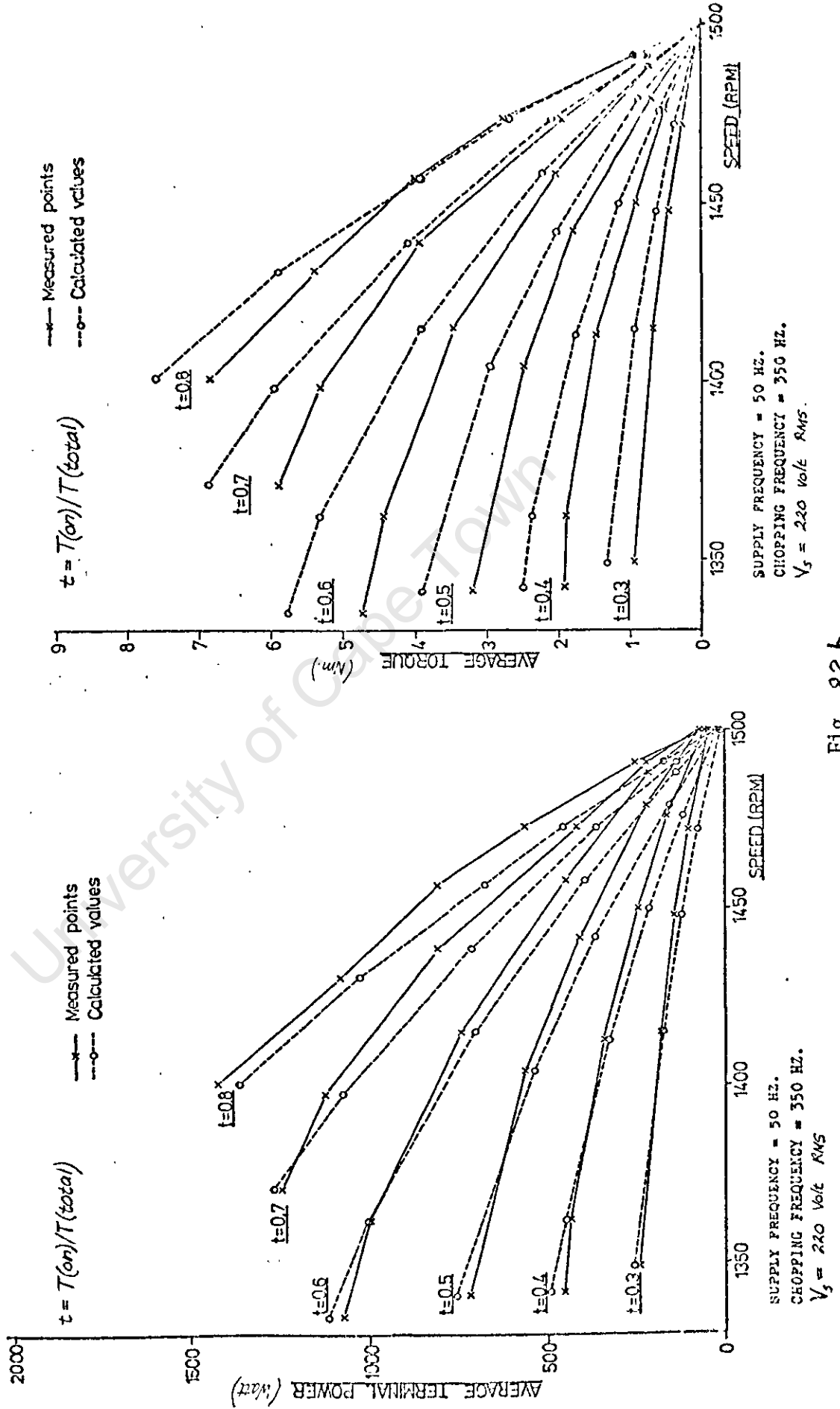
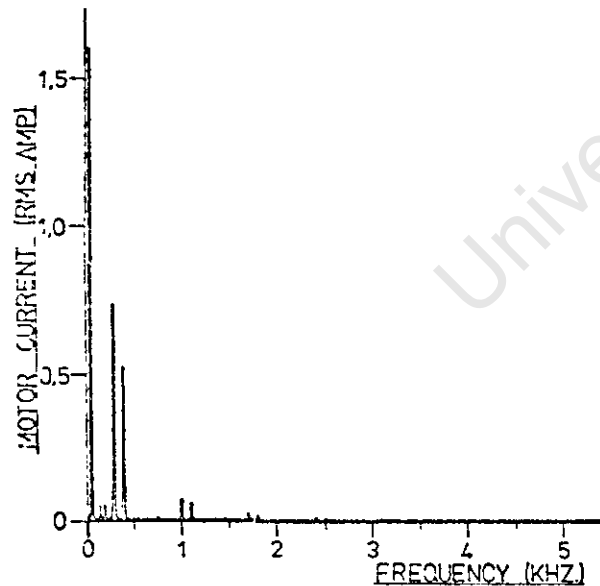
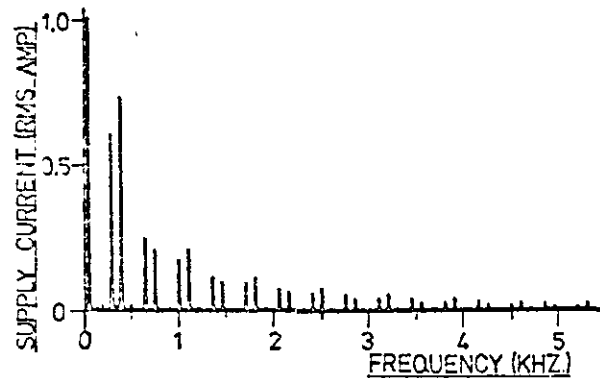
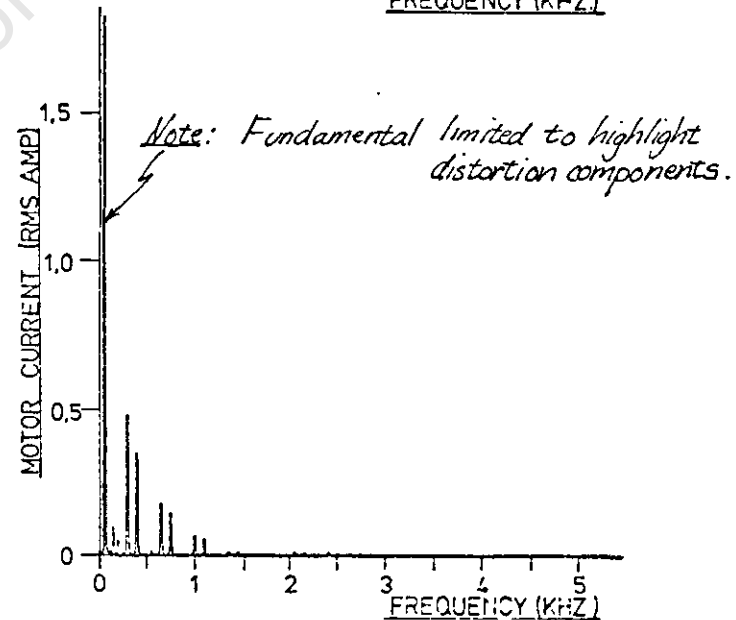
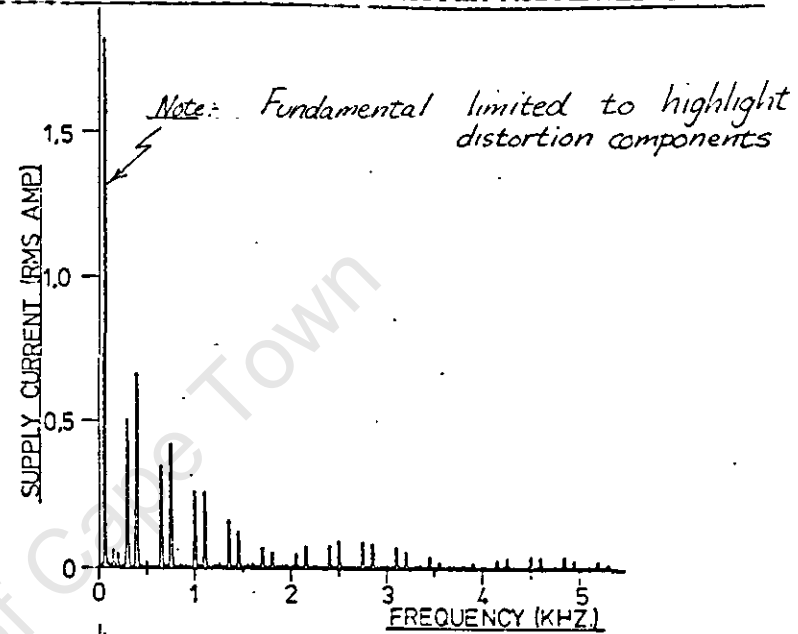


Fig. 82b

MEASURED SPECTRA OF SYSTEM CURRENTS IN THE TEST MOTOR WITH CHOPPER MODULATED SUPPLY.

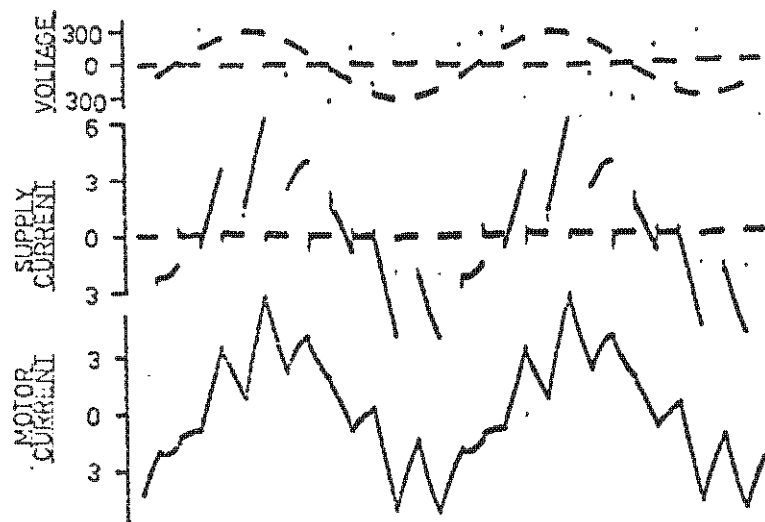


$F_c = 350$ HZ. SLIP = 5% $T_{(on)}/T_{(tot)} = 0.5$
 $V_s = 220$ VOLT RMS. 50 HZ.

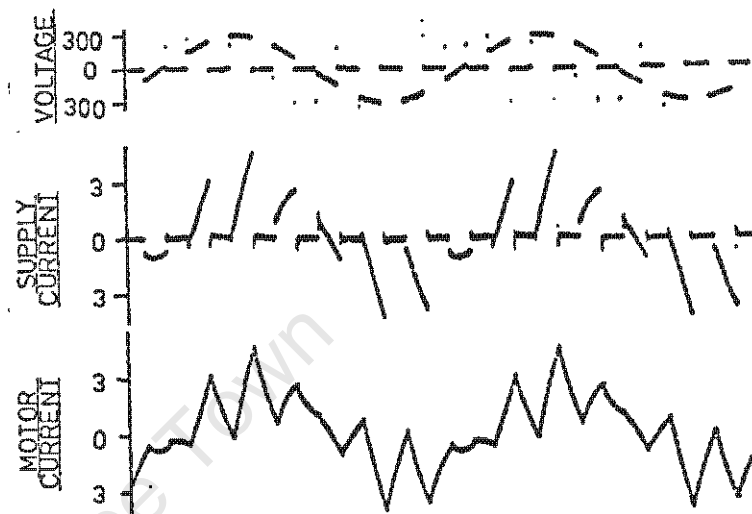


$F_c = 350$ HZ. SLIP = 5% $T_{(on)}/T_{(tot)} = 0.8$
 $V_s = 220$ VOLT RMS 50 HZ.

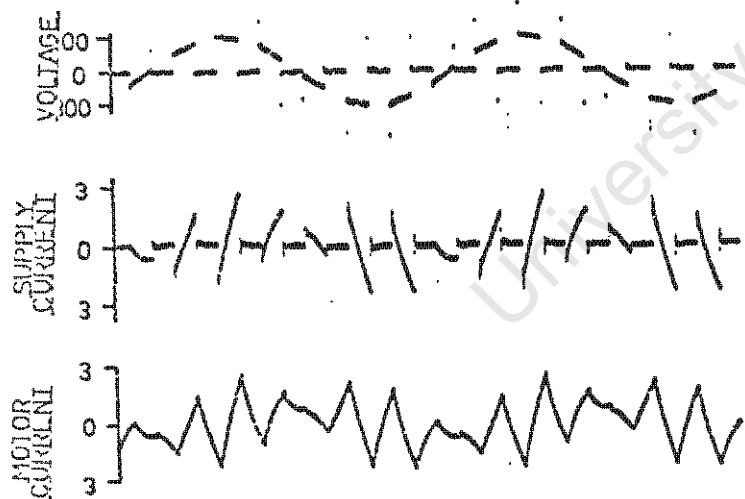
Fig. 82c



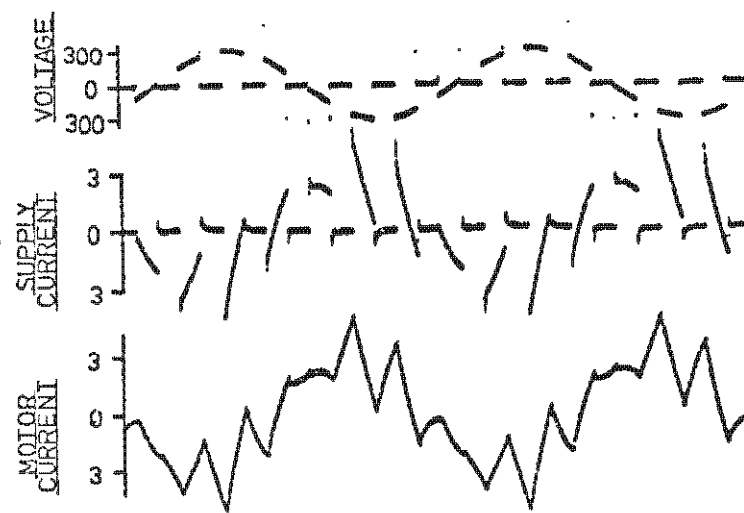
Slip = 10%



Slip = 5%



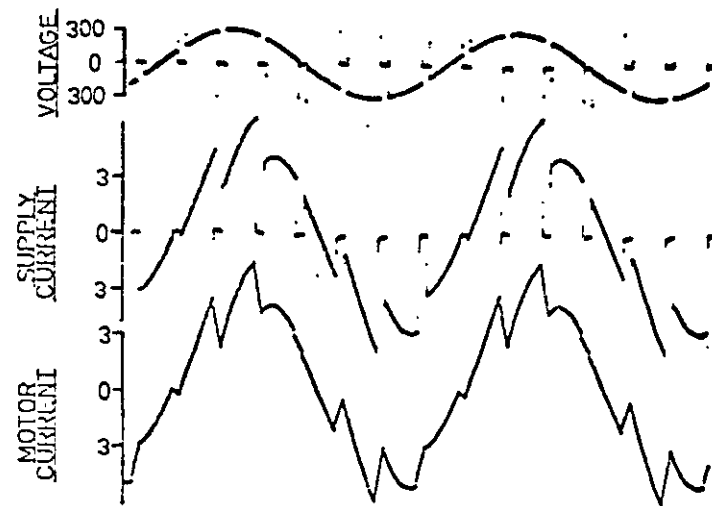
Slip = 0



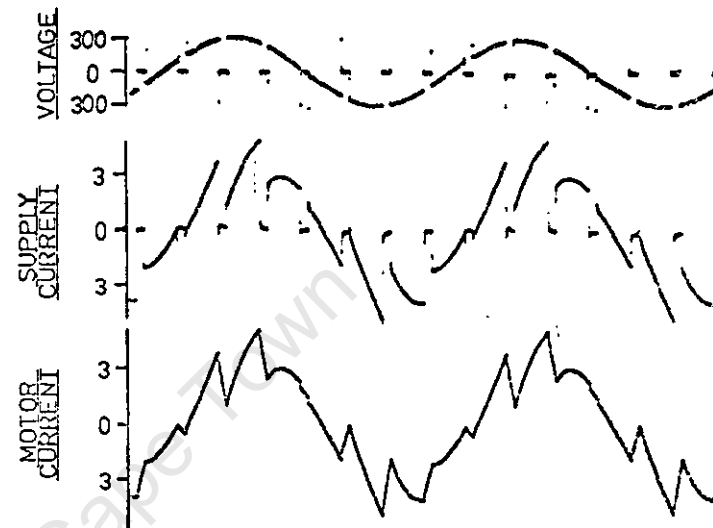
Slip = -5%

MEASURED WAVEFORMS

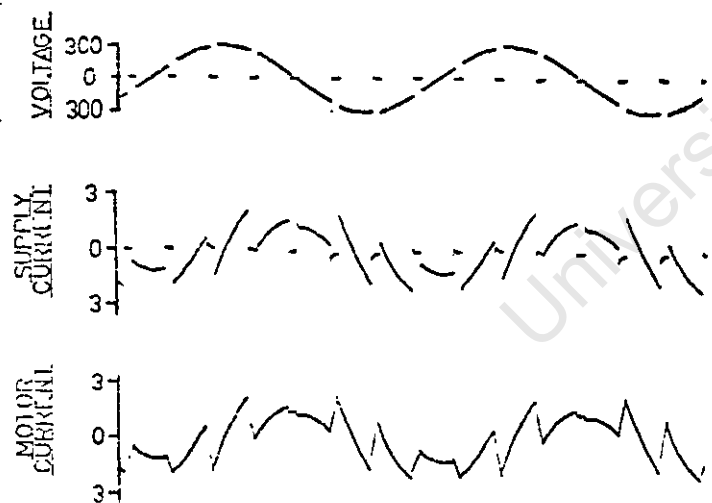
Fig. 82d Chopping frequency = 350 Hz., $T(\text{on})/T(\text{total}) = .5$, $V_s = 220$ Volt RMS 50 Hz.



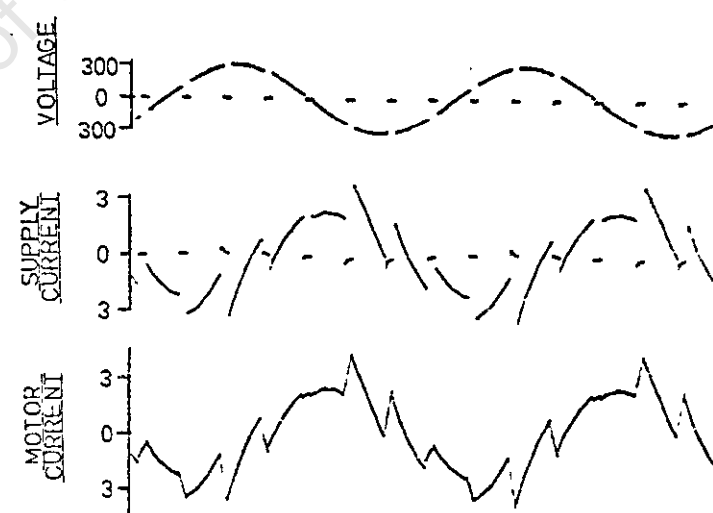
Slip = 7.5 %



Slip = 5%



Slip = 0



Slip = -2.5%

MEASURED WAVEFORMS.

Fig. 82e Chopping frequency = 350 Hz., $T(\text{on})/T(\text{total}) = 0.8$, $V_s = 220$ Volt RMS 50 Hz.

Chopper modulation

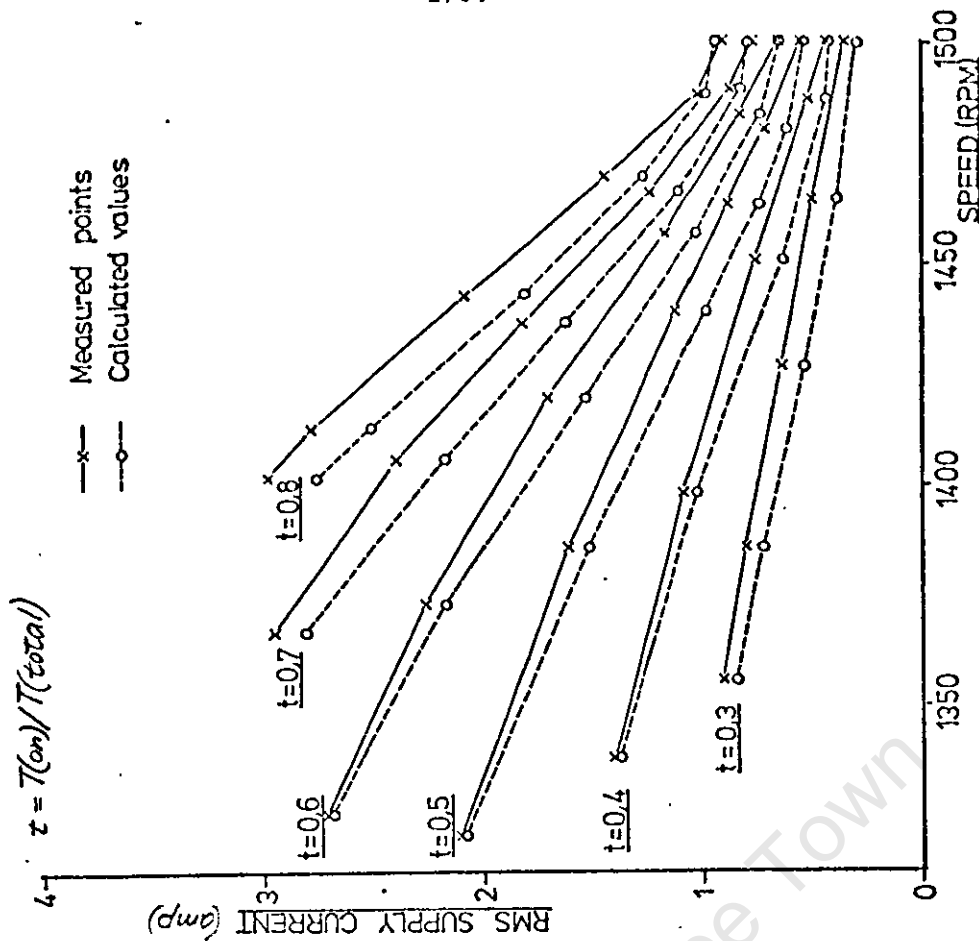
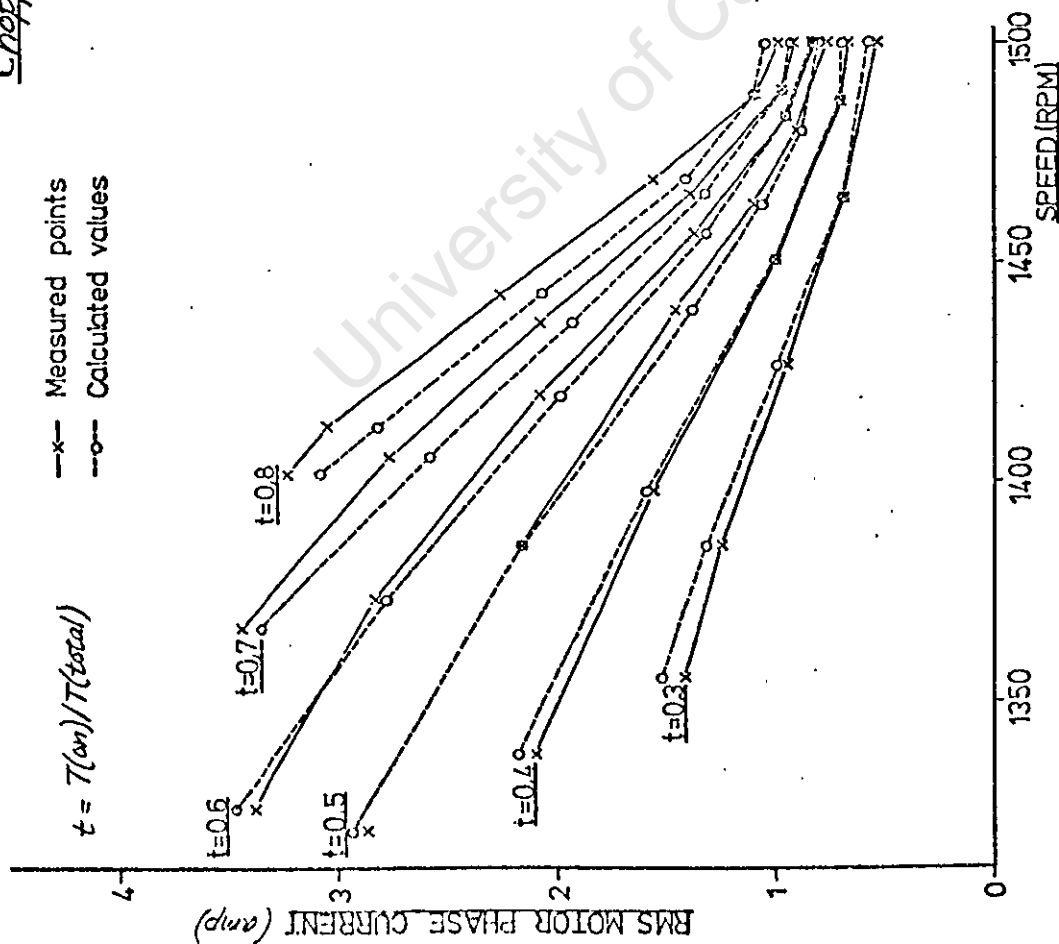
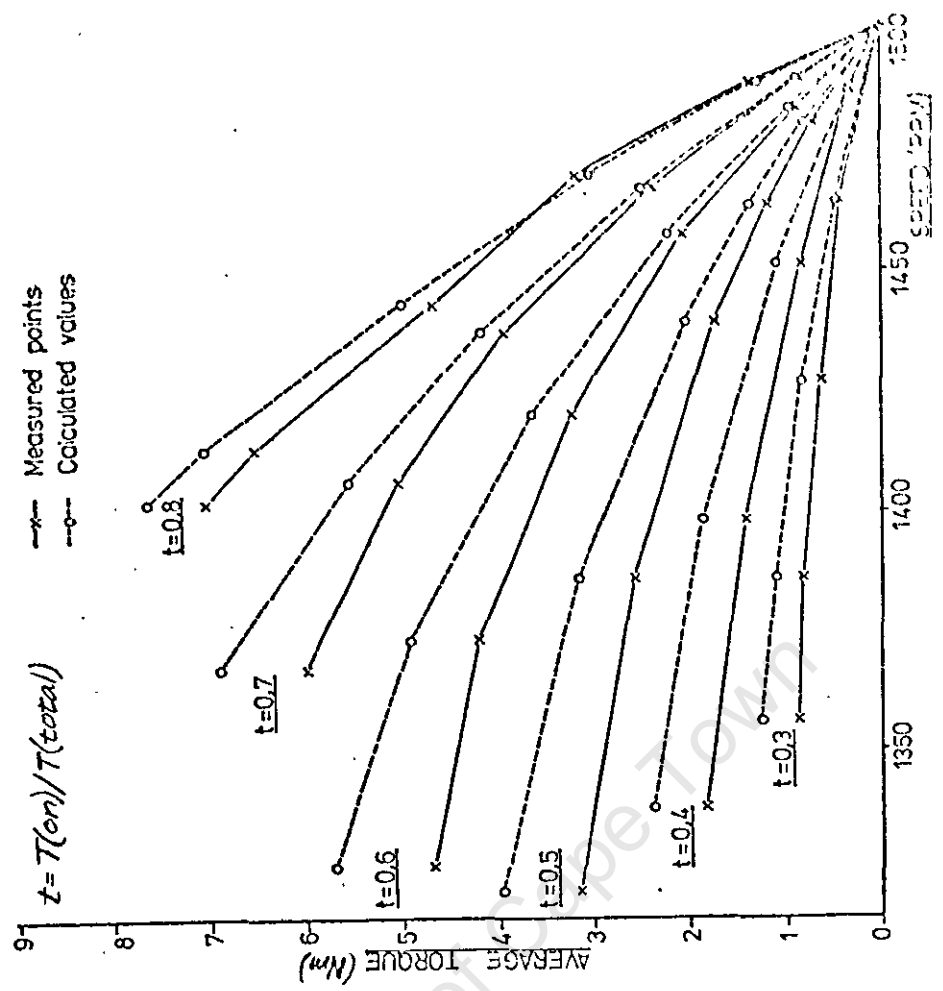
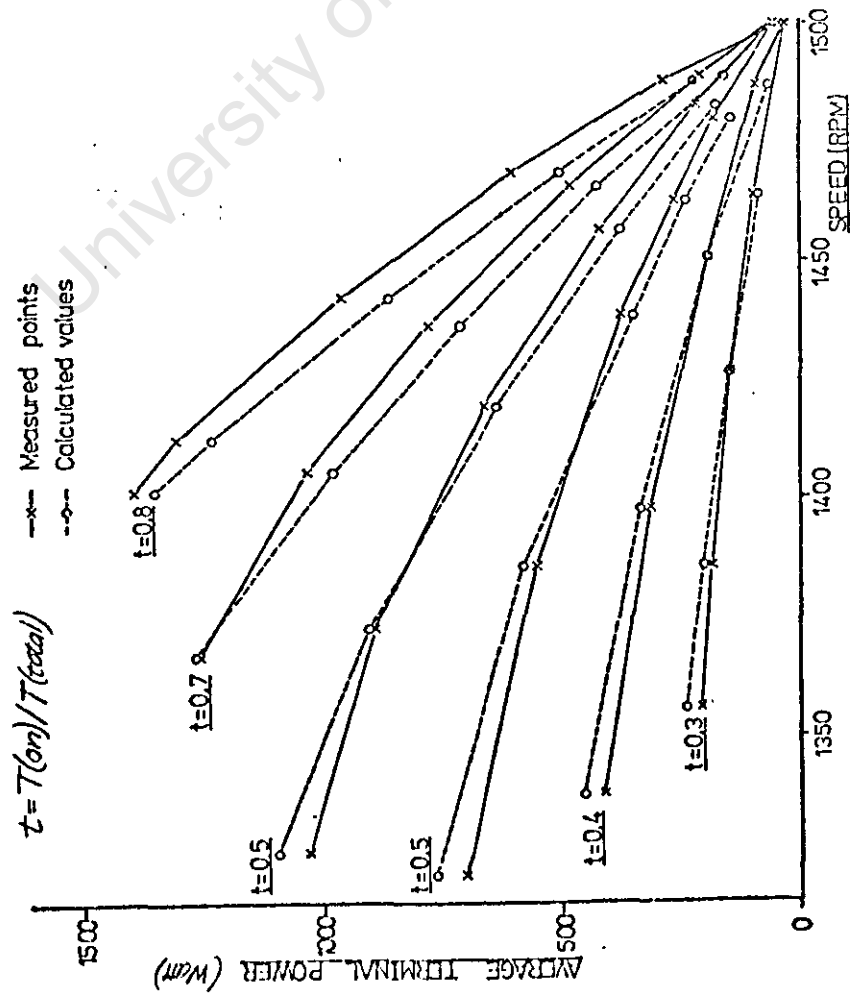


Fig. 83a

Chopper modulation



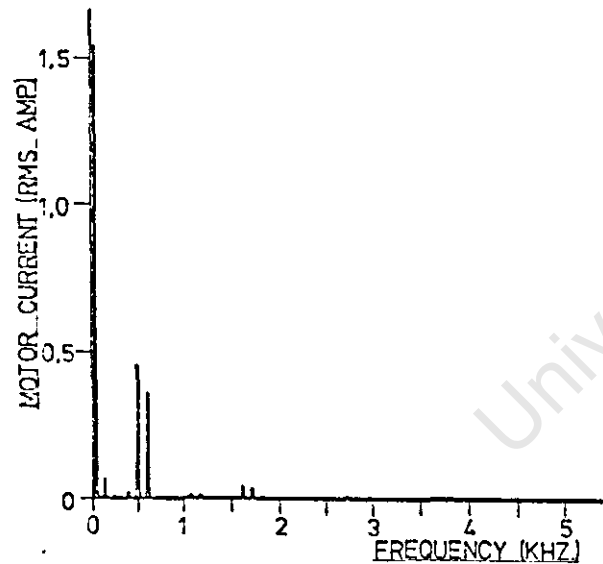
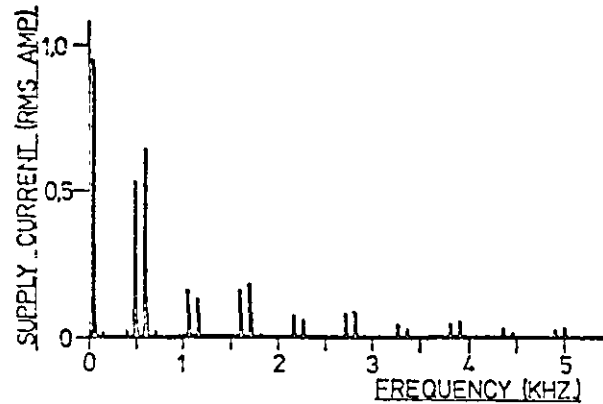
SUPPLY FREQUENCY = 50 HZ.
CHOPPING FREQUENCY = 550 HZ.
 $V_s = 220$ Volt RMS.



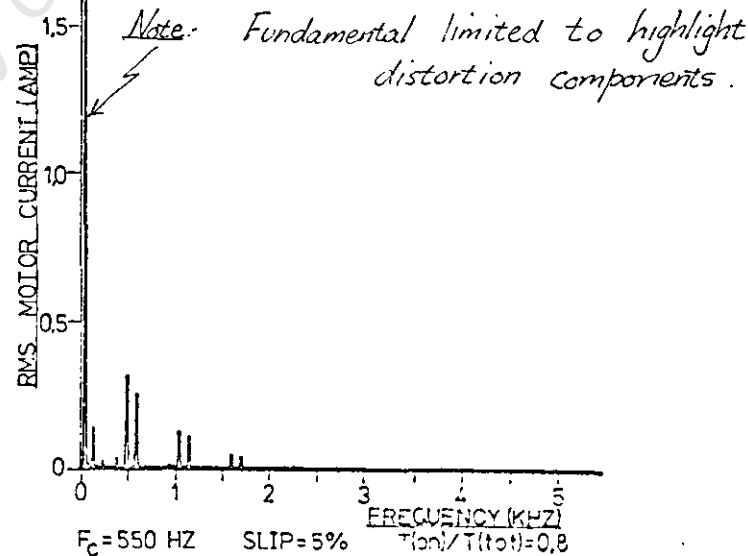
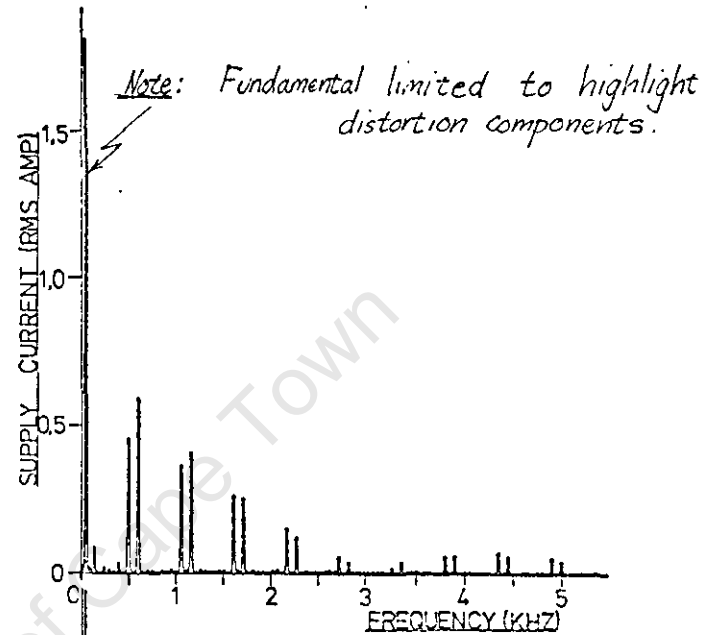
SUPPLY FREQUENCY = 50 HZ.
CHOPPING FREQUENCY = 550 HZ.
 $V_s = 220$ Volt RMS

Fig. 8.3b

MEASURED SPECTRA OF SYSTEM CURRENTS IN THE TEST MOTOR WITH CHOPPER MODULATED SUPPLY.

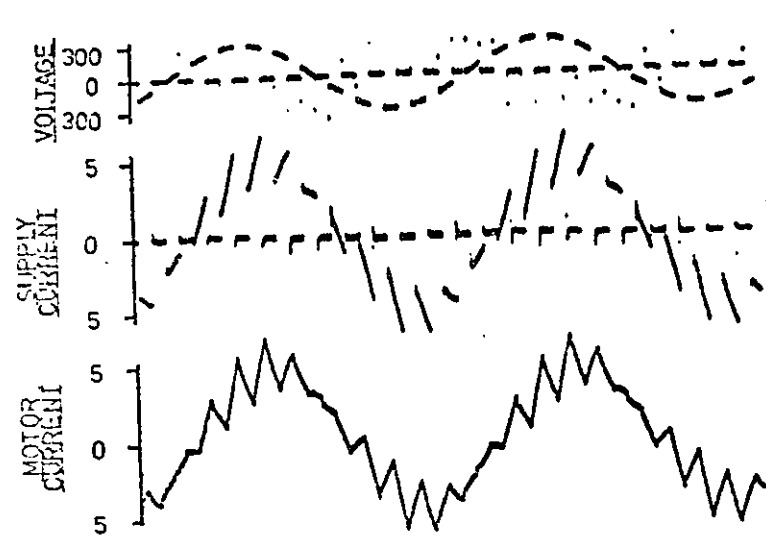


$F_c = 550 \text{ HZ}$ SLIP=5% $T(\text{on})/T(\text{tot})=0.5$
 $V_s = 220 \text{ VOLT RMS } 50 \text{ HZ}$

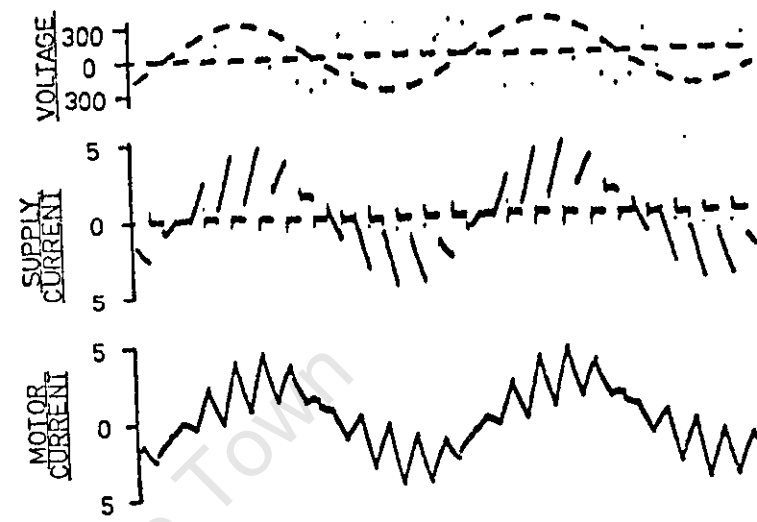


$F_c = 550 \text{ HZ}$ SLIP=5% $T(\text{on})/T(\text{tot})=0.8$

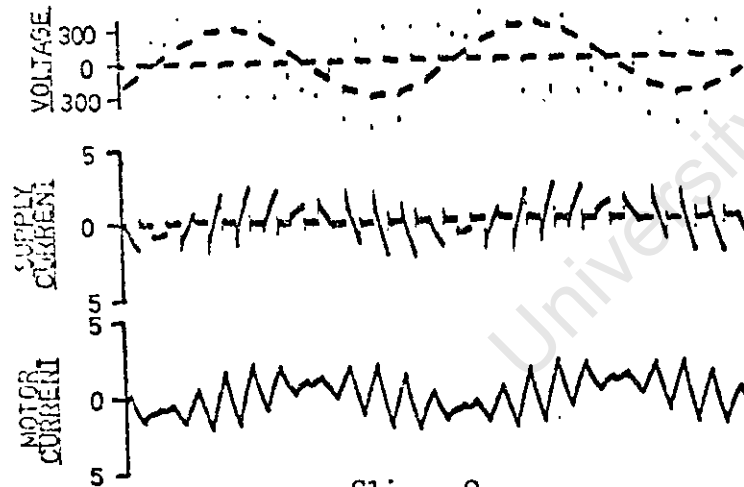
Fig. 83c



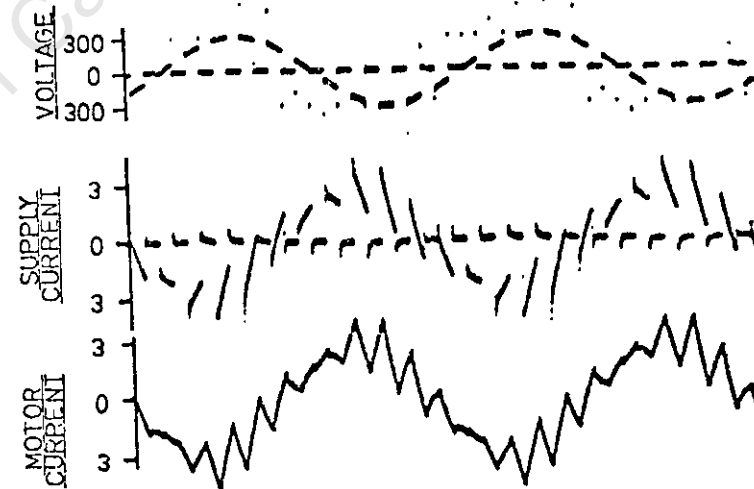
Slip = 10%



Slip = 5%



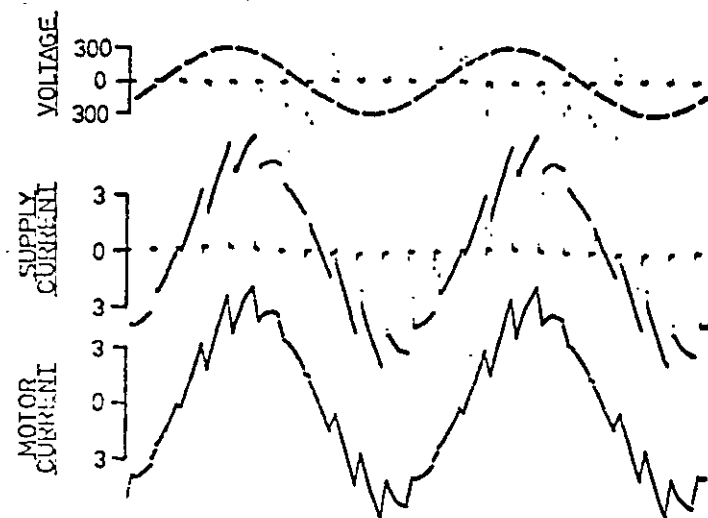
Slip = 0



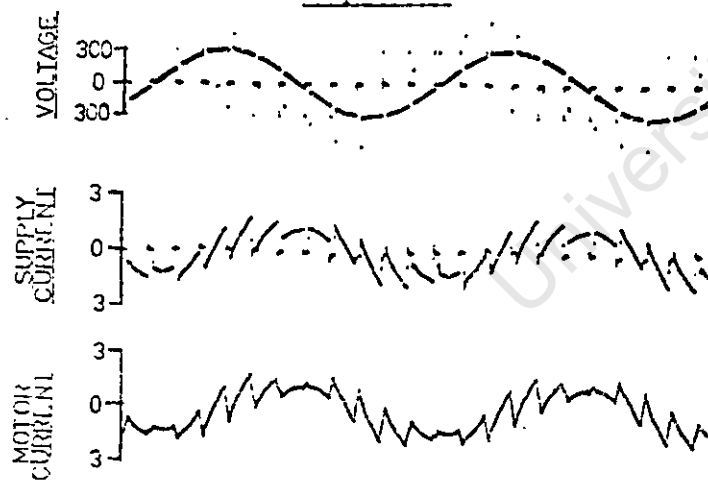
Slip = -5%

MEASURED WAVEFORMS

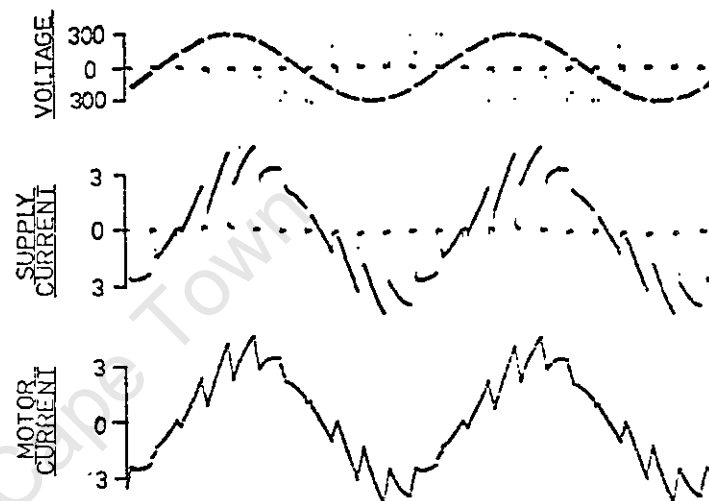
Fig. 83d Chopping frequency = 550 Hz., $T(\text{on})/T(\text{total}) = 0.5$, $V_s = 220 \text{ Volt RMS } 50 \text{ Hz.}$



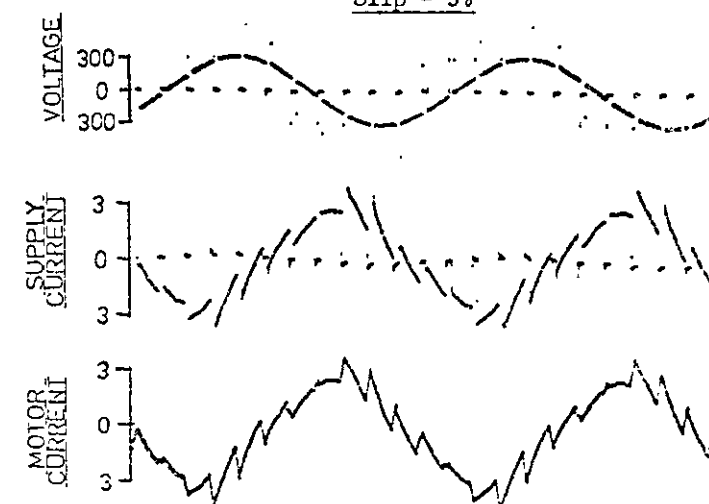
Slip = 7.5%



Slip = 0



Slip = 5%



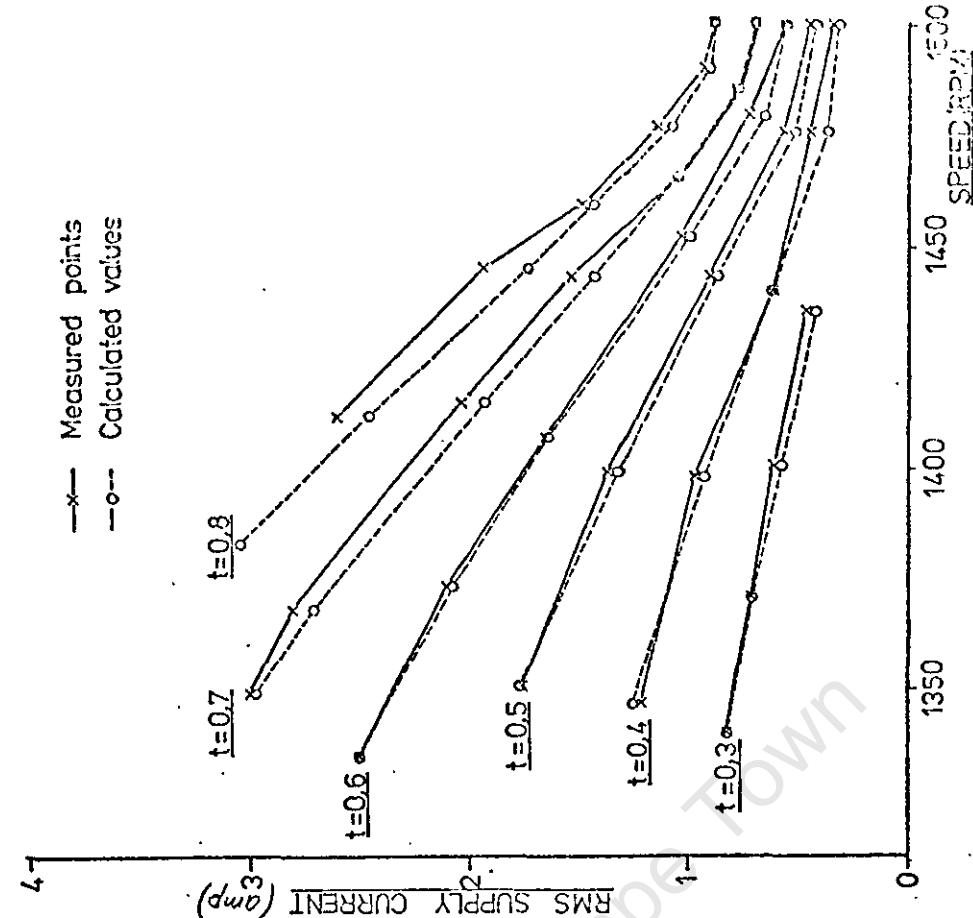
Slip = -2.5%

MEASURED WAVEFORMS.

Fig. 83e

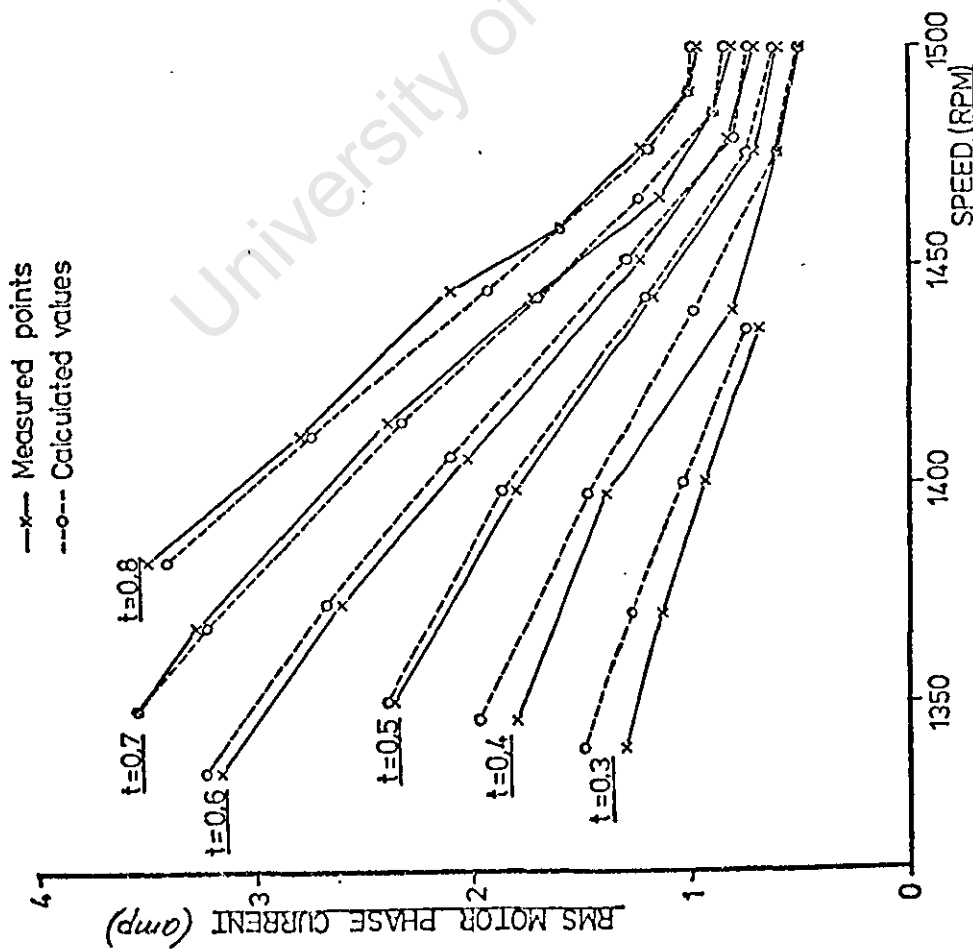
Chopping frequency = 550 Hz., $T(\text{on})/T(\text{total}) = 0.8$, $V_s = 220$ Volt RMS 50 Hz.

$$t = T_{(on)} / T_{(total)}$$



SUPPLY FREQUENCY = 50 HZ.
CHOPPING FREQUENCY = 1050 HZ.
 $V_s = 220$ Volt RMS

$$t = T_{(on)} / T_{(total)}$$



SUPPLY FREQUENCY = 50 HZ.
CHOPPING FREQUENCY = 1050 HZ.
 $V_s = 220$ Volt RMS

Fig. 84 a

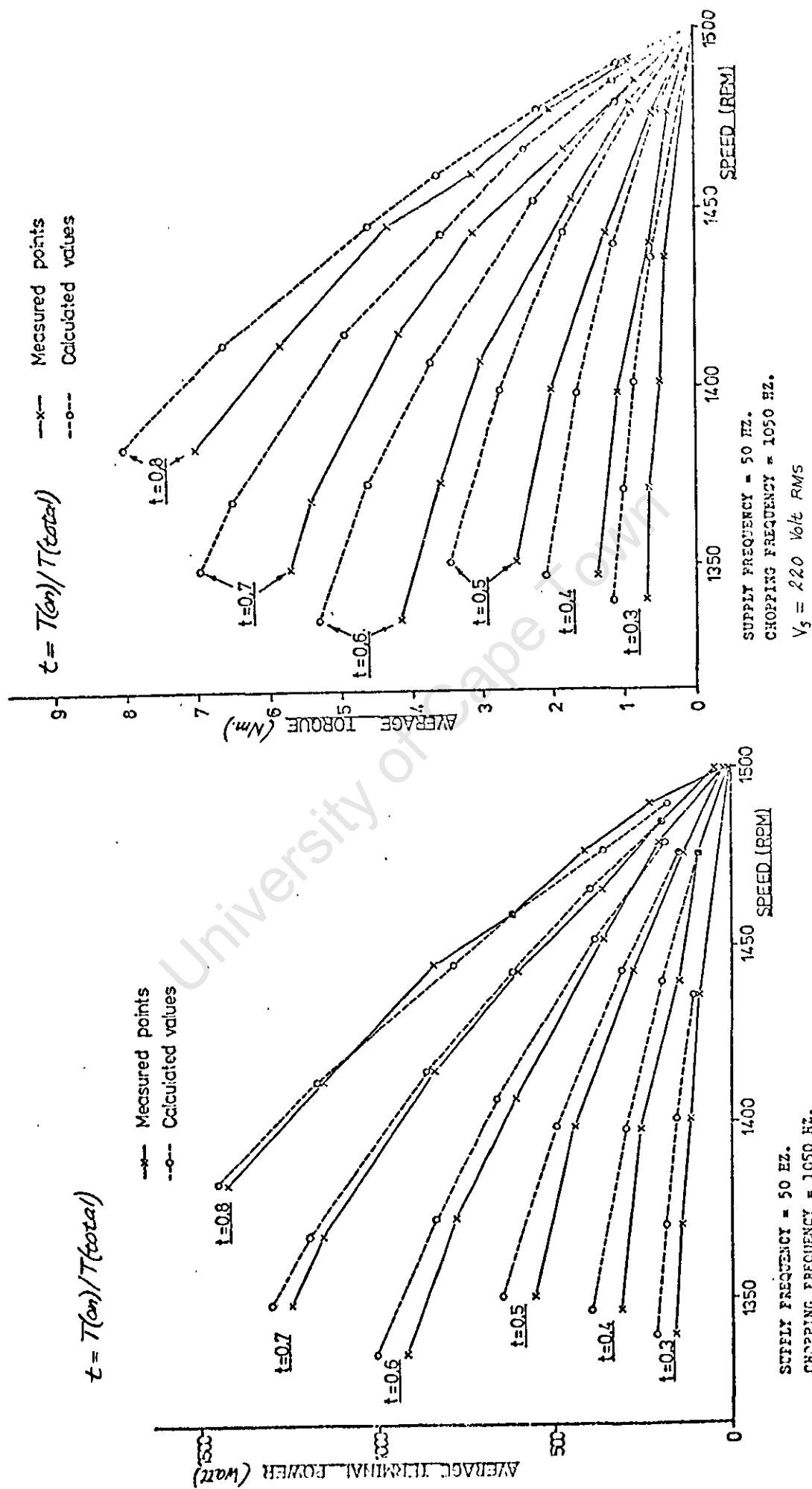
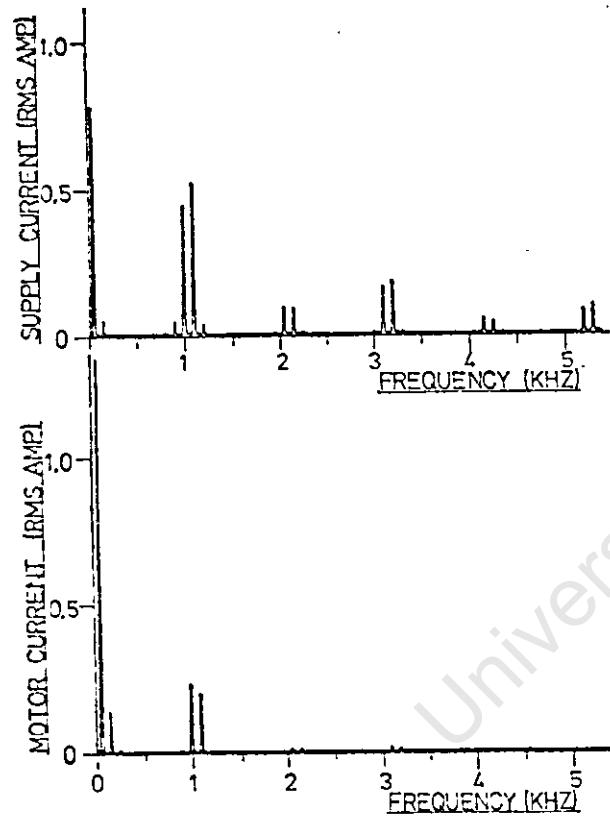
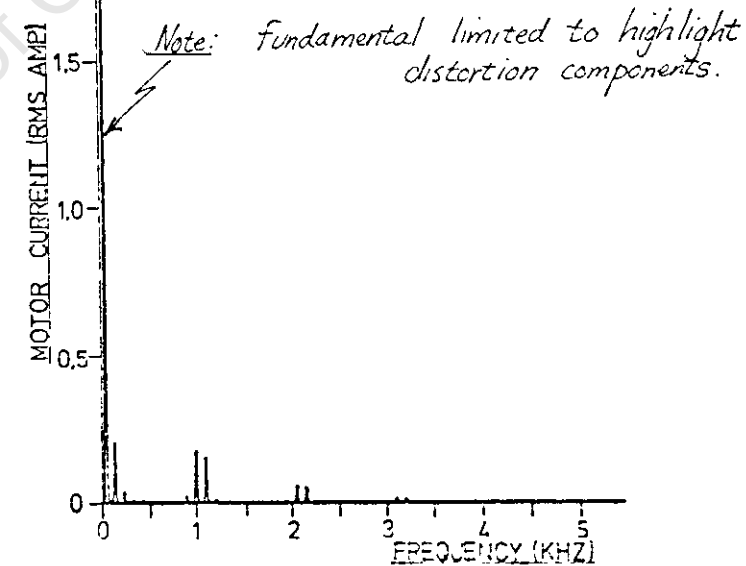
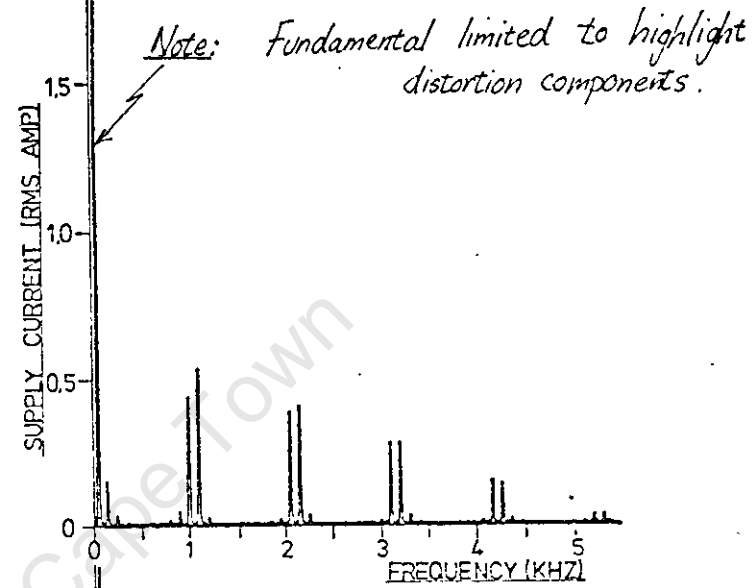


Fig. 84b.

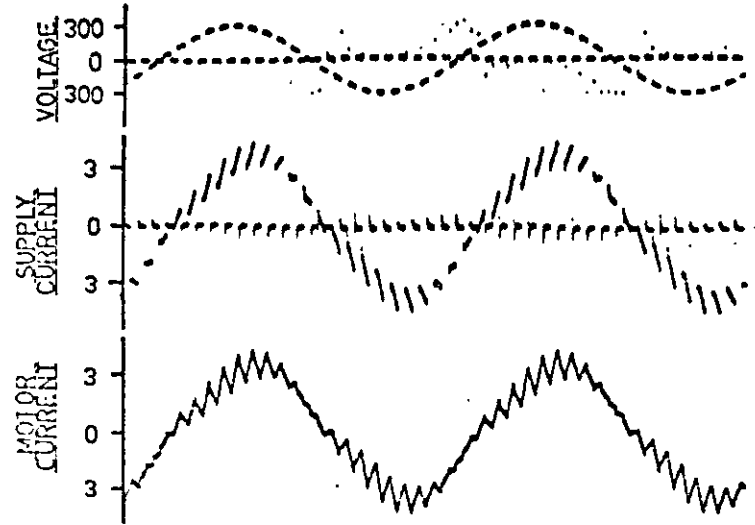


$F_c = 1050 \text{ HZ.}$ $SLIP = 5\%$ $T_{(on)}/T_{(tot)} = 0.5$
 $V_s = 220 \text{ VOLT RMS}$ 50 HZ.

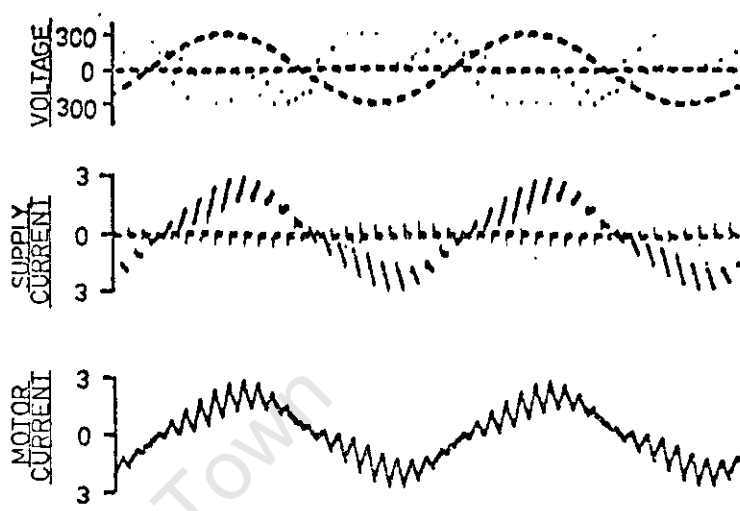


$F_c = 1050 \text{ HZ.}$ $SLIP = 5\%$ $T_{(on)}/T_{(tot)} = 0.8$

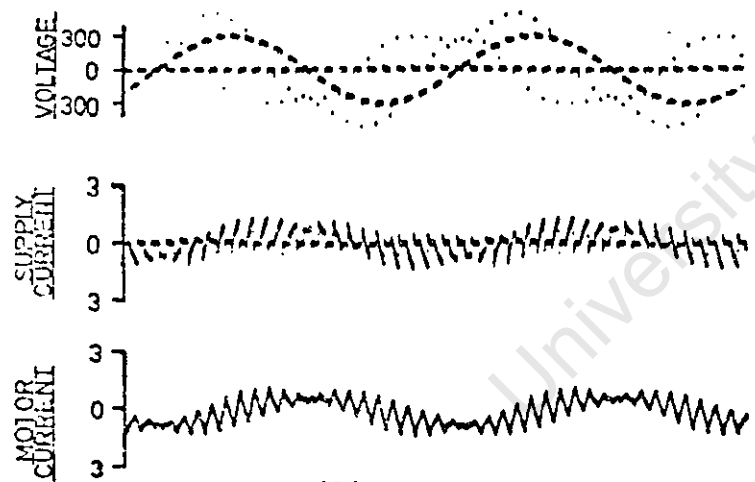
Fig. 84c



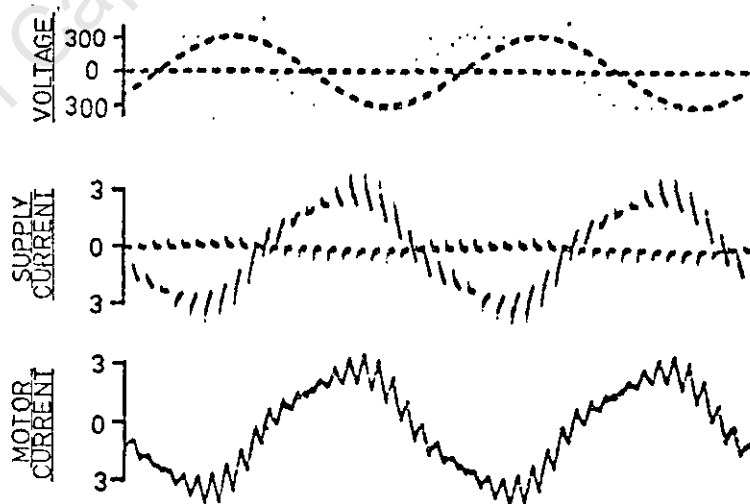
Slip = 10%



Slip = 5%



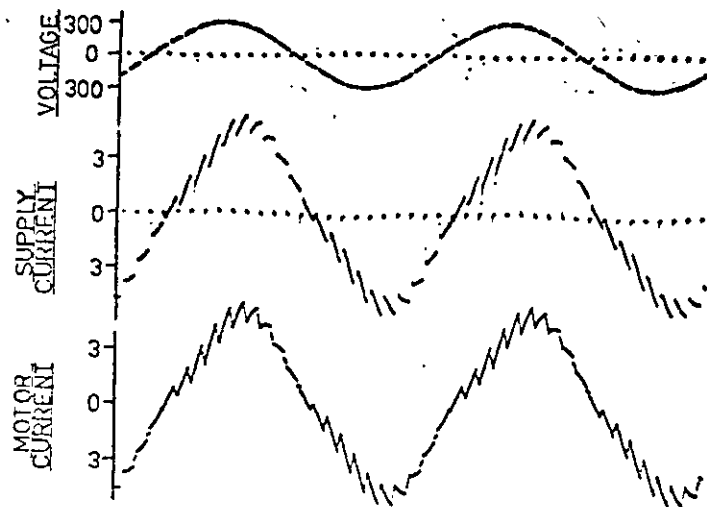
Slip = 0



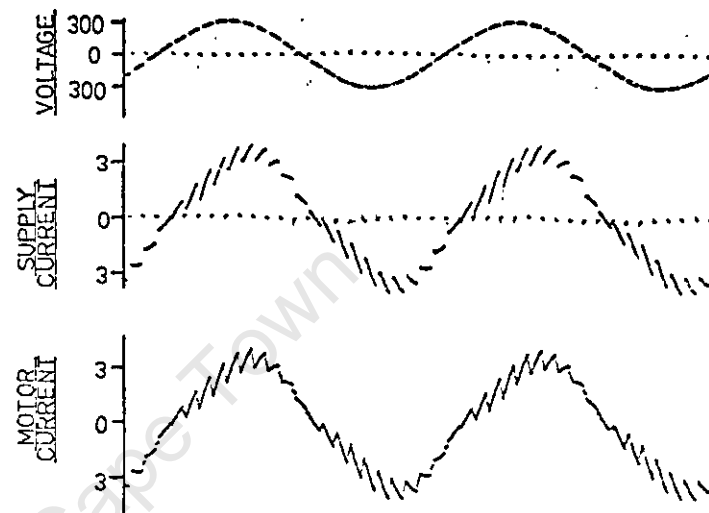
Slip = -5%

MEASURED WAVEFORMS.

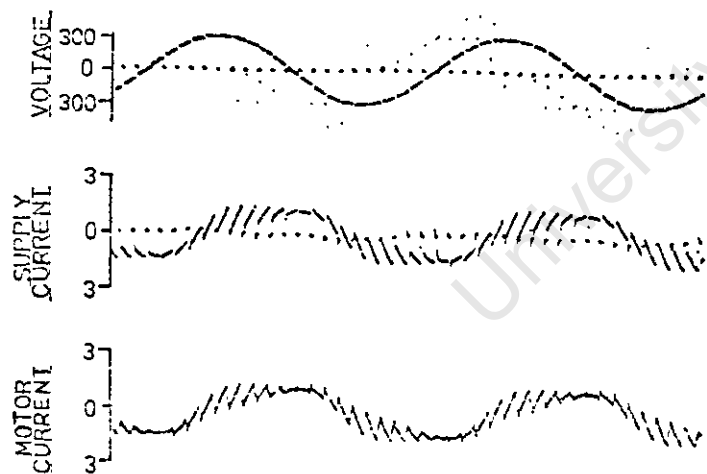
Fig.84d. Chopping frequency = 1050 Hz., $T(\text{on})/T(\text{total}) = 0.5$, $V_s = 220$ Volt RMS 50 Hz.



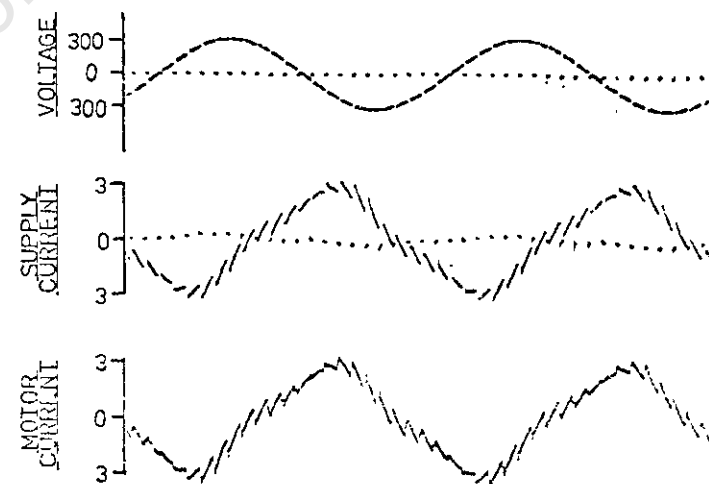
Slip = 7.5 %



Slip = 5 %



Slip = 0



Slip = -2.5%

MEASURED WAVEFORMS

Fig. 24e. Chopping frequency = 1050 Hz, $T(\text{on})/T(\text{total}) = 0.8$, $V_s = 220$ Volt RMS 50 Hz.

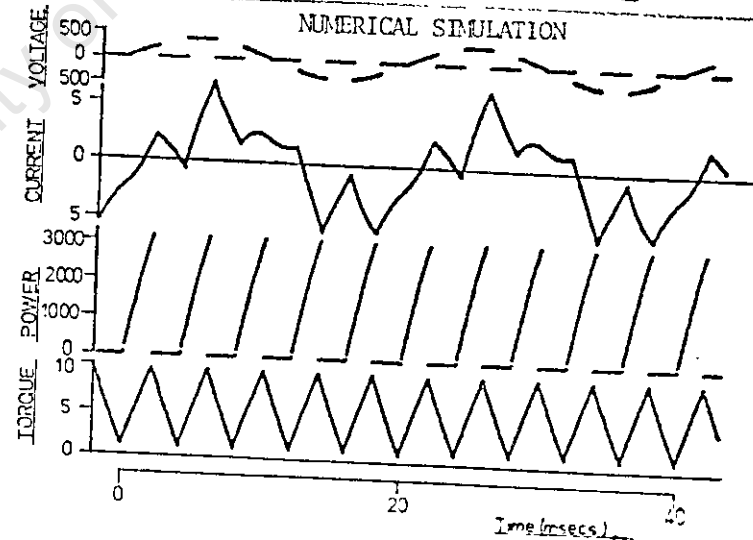
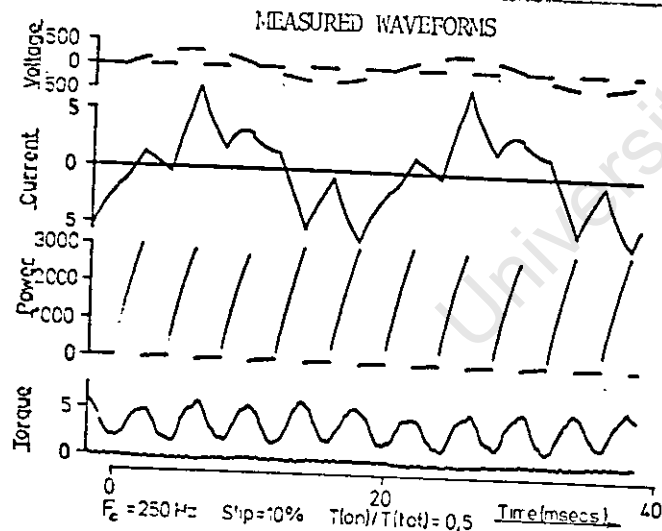
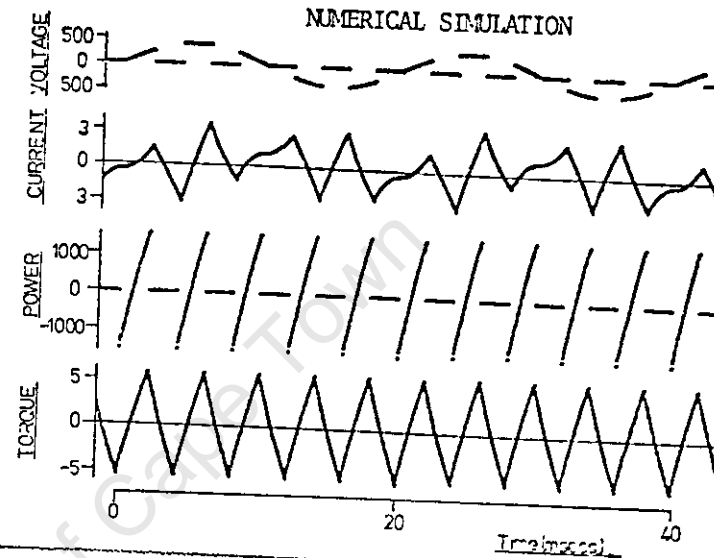
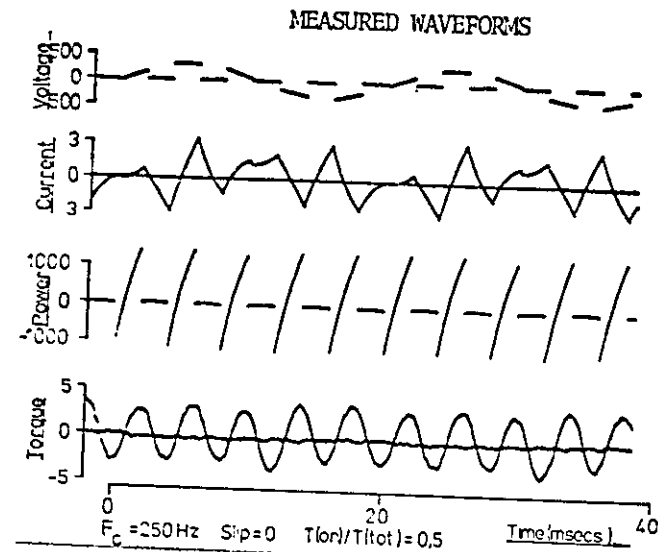


Fig. 85 Measured and simulated waveforms for the test motor with chopper modulated supply. $V_s = 220$ Volt RMS 50 Hz.

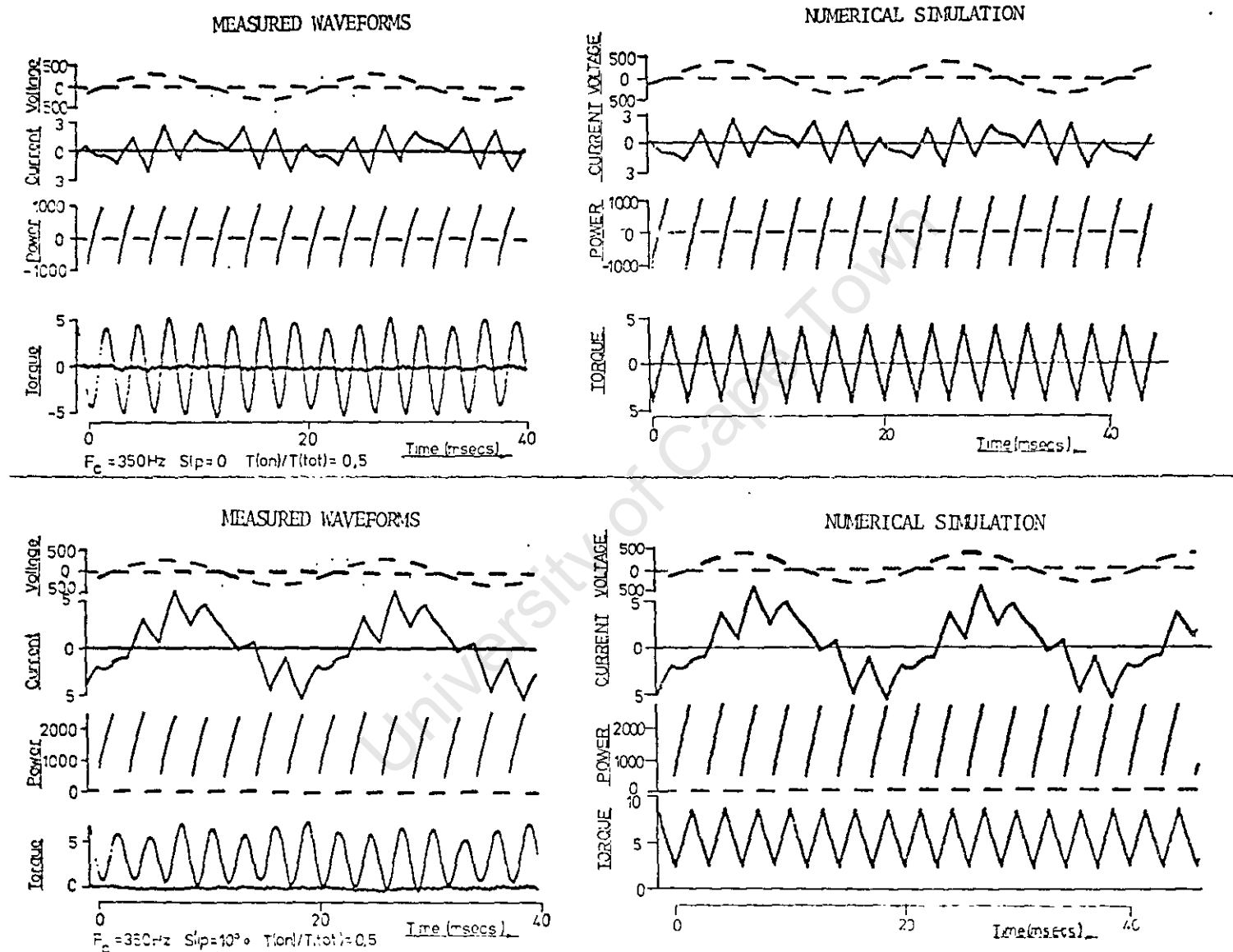


Fig. 86 Measured and simulated waveforms for the test motor with chopper modulated supply. $V_s = 220\text{ Volt RMS } 50\text{ Hz}$.

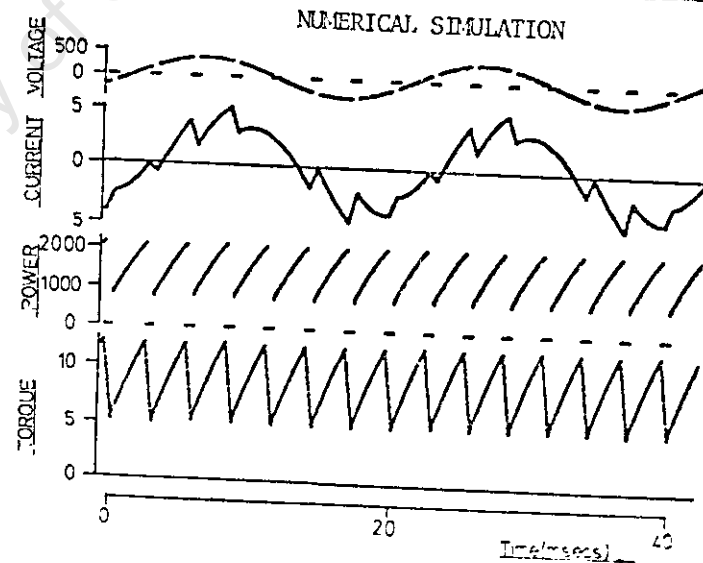
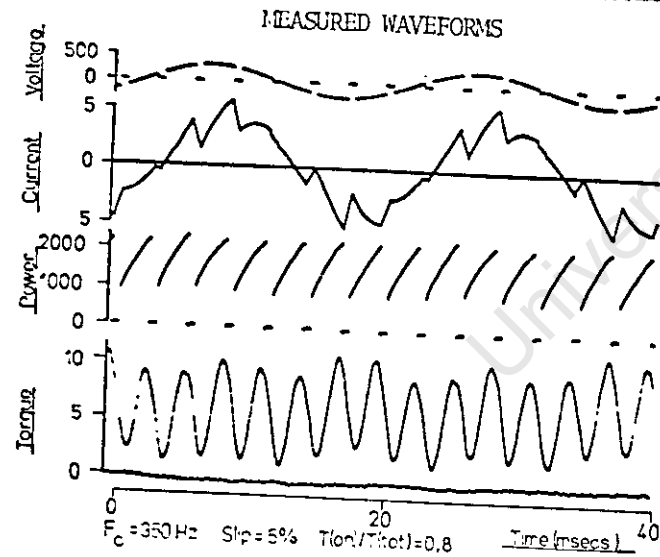
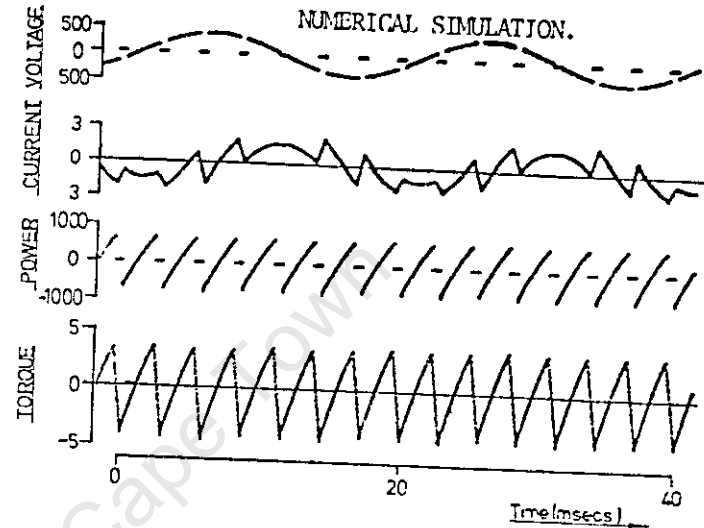
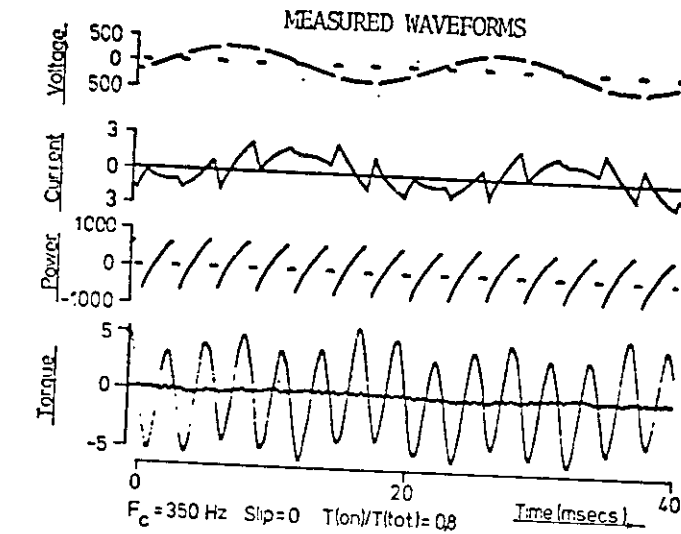
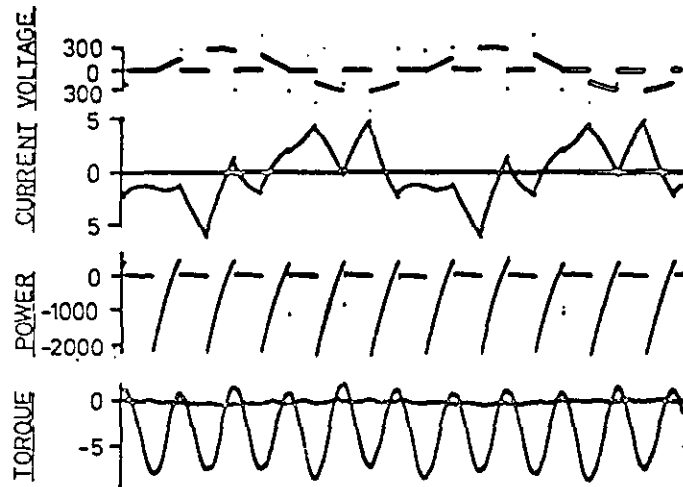
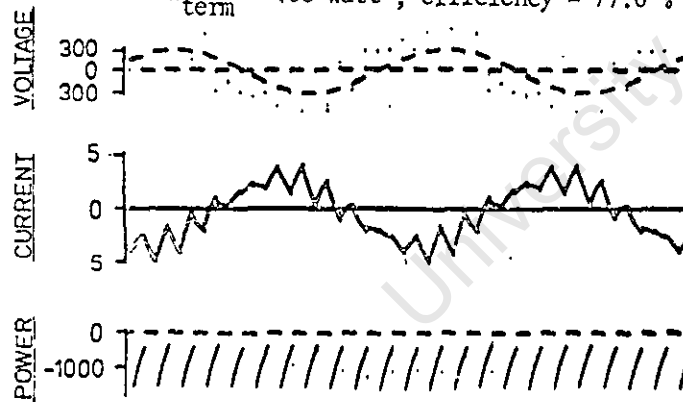


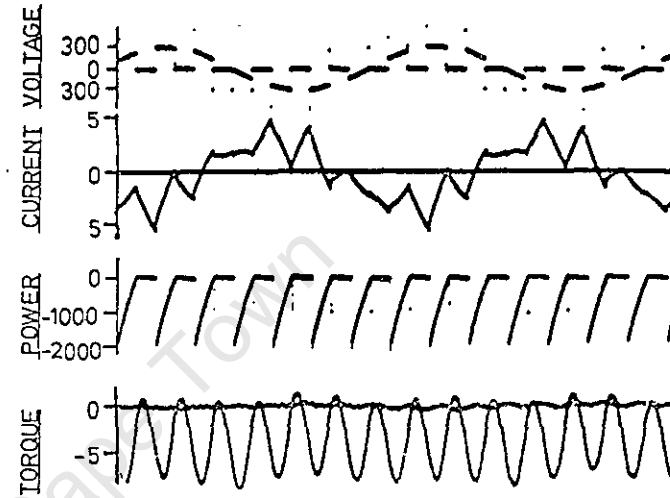
Fig. 87 Measured and simulated waveforms for the test motor with chopper modulated supply. $V_s = 220 \text{ Volt RMS } 50 \text{ Hz}$.



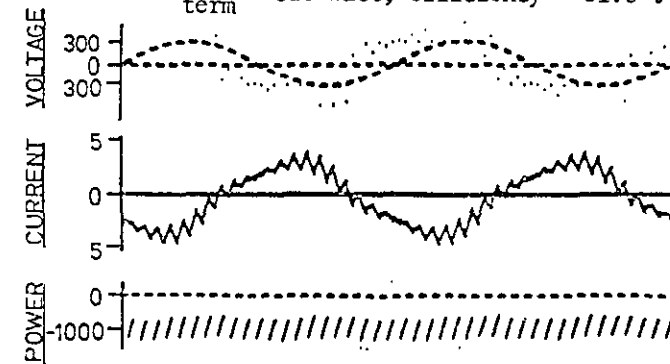
a) $F_c = 250 \text{ Hz.}; V_s = 220 \text{ Volt } 50 \text{ Hz.}; S = -5\%$
Motor: 2.4 amp RMS, 156 volt RMS, -3.75 Nm.
 $W_{\text{term}} = -480 \text{ watt}$, efficiency = 77.6 %



c) $F_c = 550 \text{ Hz.}; V_s = 220 \text{ Volt } 50 \text{ Hz.}; S = -5\%$
Motor: 2.15 amp RMS, 159 volt RMS, -3.89 Nm.,
 $W_{\text{term}} = -540 \text{ watt}$, efficiency = 84.2 %



b) $F_c = 350 \text{ Hz.}; V_s = 220 \text{ Volt } 50 \text{ Hz.}; S = -5\%$
Motor: 2.25 amp RMS, 158 volt RMS, -3.79 Nm.,
 $W_{\text{term}} = -510 \text{ watt}$, efficiency = 81.6 %



d) $F_c = 1050 \text{ Hz.}; V_s = 220 \text{ Volt } 50 \text{ Hz.}; S = -5\%$
Motor: 2.26 amp RMS, 161 volt RMS, -4.4 Nm.,
 $W_{\text{term}} = -600 \text{ watt}$, efficiency = 81.8 %

Fig. 88 Illustration of measured test motor characteristics while regenerating with chopper modulated supply voltage. ($T(\text{ch})/T(\text{total}) = 0.5$)

circumstances was V_{exp} , then the calculated results were obtained using a supply voltage modified by the factor (V_{obs}/V_{exp}) .

- (ii) Figs. (85) to (87) show measured system waveforms for different conditions contrasted in each case against the corresponding numerical simulation.
- (iii) Fig. (88) illustrates operation in the regenerating mode with four different chopping frequencies.

The measured values of RMS current and average terminal power differ by less than 10% from the calculated values in most cases. This is considered to be a very satisfactory correlation and indicates that the theory can be used to predict these quantities without further refinements. The estimation of torque is always a problem because of the complex loss mechanisms which influence torque production in the motor. From Fig. (77) it may be seen that the discrepancies are appreciable even in the case of sinusoidal excitation. With chopped excitation the error is larger as seen from Figs. (82b), (83b) and (84b). It is clear that efficiencies estimated from the theory in this way will tend to be rather higher than observed in practice.

The correlation in respect of waveform shown in Figs. (85) to (87) is particularly pleasing. It will be appreciated that the frequency response of the torque transducer is limited and the measured torque waveforms should be viewed in the light of the response curve shown in Appendix 5, Fig. 128.

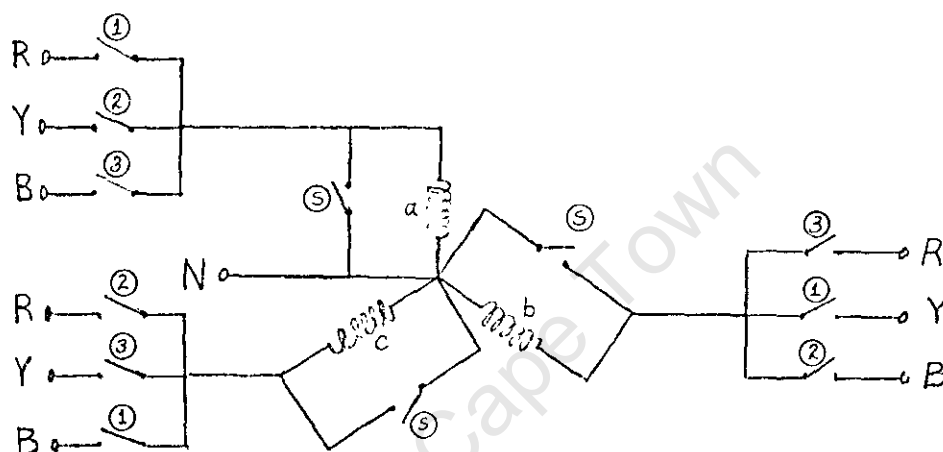
CHAPTER EIGHTCHOPPER MODULATION FOR VOLTAGE CONTROL
OF AN INDUCTION MOTOR SUPPLIED THROUGH
A STATIC FREQUENCY
CONVERTOR

An immediate practical application of the chopper modulation which has been investigated in the previous chapters is for the control of the fundamental output voltage which is generated by an AC to AC static frequency convertor. In previous work, Hurley (19) has considered different forms of such a convertor and was able to demonstrate the advantages of a convertor with a constant phase-stepping frequency over the more familiar cycloconvertor configuration. Hurley termed the action of this convertor an "asynchronous modulation" of the supply voltage, insofar as the stepping rate is controlled by a free-running master oscillator. In its basic form such a convertor lacks the inherent facility to control the output voltage level by means of simple changes in the switching logic. Hurley showed that this problem could be overcome by using forced-commutation techniques to hold the load voltage to zero for short "freewheel" periods. This not only constituted effective voltage control but, if done at the stepping-rate of the convertor, provided a useful interval for reconnection of the supply circuit in order to provide the subsequent phase-step. In the course of his work Hurley was not able to enter deeply into the influence of this convertor on an induction motor load. While his analysis of the voltage-waveforms is rigorous, the treatment of the system currents is approximate. There is no indication of the nett effect of the convertor on efficiency in the quasi-steady state and no treatment of the effect of mark/total time on the overall performance.

The particular modulation cycle used in the action of this convertor is ideally suited to analysis in terms of the techniques which have been developed. It is intended to demonstrate this here by presenting calculated characteristics

of the test motor supplied through such a convertor. It is hoped that these results will serve to complement present knowledge about the convertor, which is currently of particular interest.

8.1 Action of the Convertor:



Switches labelled in the above schematic with the same symbol are opened or closed simultaneously and held in the same state. The basic convertor action is achieved by keeping switches ⑤ open and commutating the supply currents sequentially through switches ①, ②, ③, ①, ... at a fixed rate. This is a three-pulse system and each motor phase sees a discrete $-\frac{2\pi}{3}$ radian phase shift in its voltage every $\frac{1}{f}$ seconds where $\frac{1}{f}$ is the phase stepping frequency. In this form there is no chopping action and the fundamental output voltage is constant. The chopping may be introduced by closing switches ⑤ each time a set of series switches is opened. These switches are held closed for a suitable interval, then they are opened while the subsequent supply connection is closed.

If this supply connection is different to that prevailing before the short-circuit, then the modulator produces one chop per

phase step. An alternative operation is also possible where the chopping action is repeated several times in relation to one particular supply connection before a new supply connection is made.

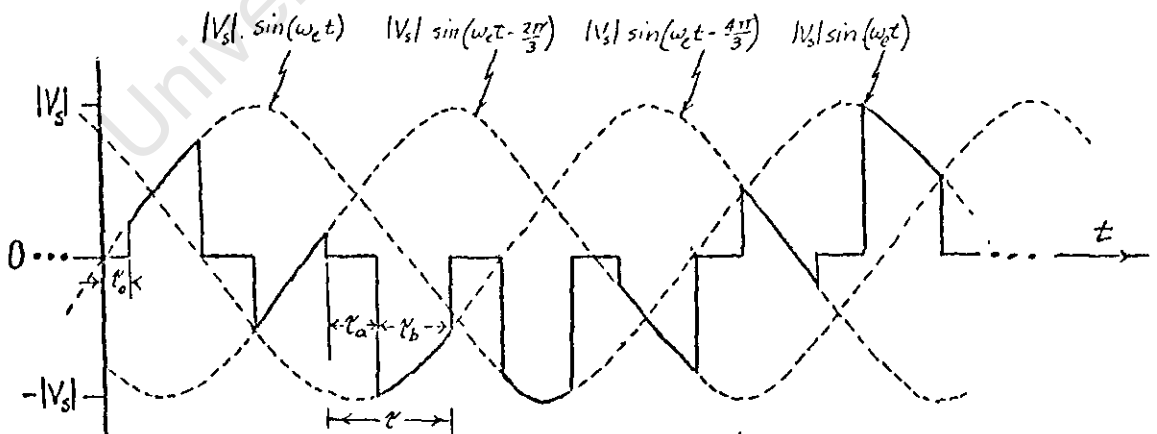
The convertor action can be realized in practice with the power commutating switches discussed earlier. Twelve of these are required for the power circuit, and the control logic is elementary.

8.2 Output Voltage

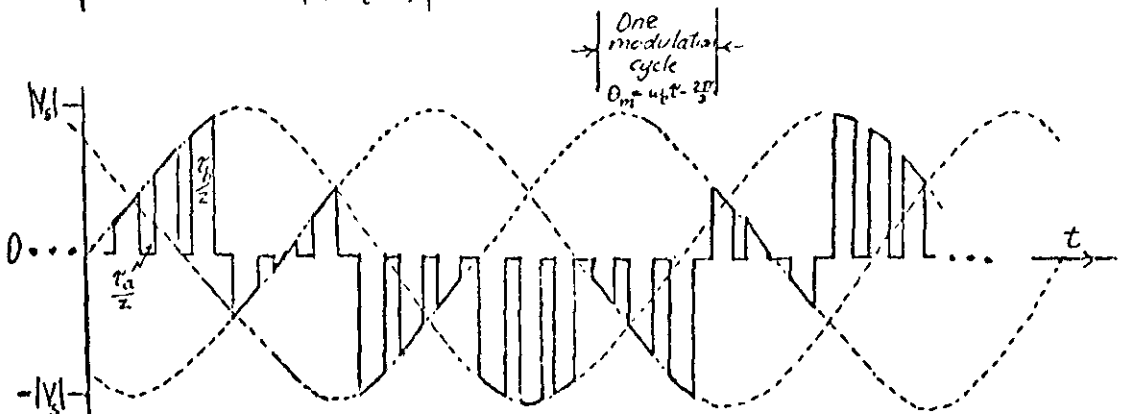
The output voltage produced by the convertor is illustrated below with -

- (a) One chop per phase step ($Z = 1$)
- (b) 3 chops per phase step ($Z = 3$)

(a)



(b)



It may be shown that in each case the RMS output voltage is given by

$$U_{S \text{ RMS}}^a = \frac{|V_s|}{\sqrt{2}} \cdot \sqrt{\frac{r_b}{r}}$$

which is the same value as for a chopped sinewave with no phase-stepping.

It can also be shown that the "fundamental" output voltage component has amplitude which is given by

$$U_{S \text{ fund.}}^a = \left\{ \frac{3\sqrt{3}}{2\sqrt{2}} \cdot \frac{|V_s|}{\pi} \cdot \frac{\sin\left(\frac{\pi \cdot \frac{r_b}{r}}{3Z}\right)}{\sin\left(\frac{\pi}{3Z}\right)} \right\} \quad (\text{RMS})$$

with fundamental frequency,

$$\begin{aligned} f_{\text{out}} &= \left| f_e - \frac{f_{\text{step}}}{3} \right| \quad \text{where } f_{\text{step}} = \frac{1}{Z} \\ &= \left| f_e - \frac{f_{\text{chop}}}{3Z} \right| \end{aligned}$$

and the associated three-phase set has positive phase sequence if

$$(f_e - \frac{f_{\text{step}}}{3}) > 0$$

and negative phase sequence if

$$(f_e - \frac{f_{\text{step}}}{3}) < 0$$

The amplitude of the fundamental component of voltage in this mode is constant, irrespective of the stepping frequency, so that the output voltage distortion factor is a constant.

i.e.

$$\mu = \frac{3\sqrt{3}}{2\pi} \cdot \sqrt{\frac{r}{r_b}} \left\{ \frac{\left(\sin\left(\frac{\pi \cdot \frac{r_b}{r}}{3Z}\right) \right)}{\left(\sin\left(\frac{\pi}{3Z}\right) \right)} \right\}$$

Note that the amplitude of the fundamental output voltage is not linear with $T(\text{on})/T(\text{total})$ although the deviations from linearity are not large. As the number of chops per step, Z , is increased above one this relationship approaches linearity closely.

In this form of modulator, "special" cases occur whenever

$$\omega_e \tau - \frac{2\pi}{3} = m\pi$$

which corresponds to a stepping frequency

$$f_{\text{step}} = \frac{6}{(3m+2)} \cdot f_e \quad (m = 0, 1, 2, \dots)$$

In each of these cases the phase voltages are unbalanced as discussed previously. This situation will be discussed separately. (8.8)

8.3 Quasi-steady State Operation - Motor Current Solution with no Chopping:

Consider firstly the case where no chopping action is present. This is an instance where the basic modulation cycle consists of only one stage and the modulation angle is

$$\theta_m = \left(\omega_e \tau_b - \frac{2\pi}{3} \right)$$

The solution for this case was obtained in Chapter 4.5.1 from the solution for the general modulation cycle, and will be repeated here for convenience .

Consider a commutation instant at time $t=0$, and following the notation of Chapter 4.3, define

$$i_o' = \frac{C'(I_{s0}^+ - C''I_{r0}^+)}{(C' - C'')} \quad ; \quad i_o'' = \frac{C''(C'I_{r0}^+ - I_{s0}^+)}{(C' - C'')}$$

which are functions of the step in Particular Integral or steady-state solution at $t=0$

Hence define:

$$i_{s0}' = i_o' \left\{ \frac{1}{1 - e^{(F'\tau_b + j\frac{2\pi}{3})}} \right\}$$

and

$$i_{s0}'' = i_o'' \left\{ \frac{1}{1 - e^{(F''\tau_b + j\frac{2\pi}{3})}} \right\}$$

In the interval after $t=0$ we thus have

$$i_s^+(t) = (I_s^+ e^{j\omega_e t} + i_{s0}' e^{\alpha' t} + i_{s0}'' e^{\alpha'' t})$$

and, in accordance with the quasi-steady state constraint, the instantaneous positive sequence motor current in the subsequent interval is

$$i_s^+(t + \tau_b) = i_s^+(t) e^{j\theta_m}$$

where $\theta_m = (\omega_e \tau_b - \frac{2\pi}{3})$

8.4 Supply Current Solution:

Only one three-phase supply path is used in this mode of

operation. During the first interval considered above the reference phases of supply and motor are connected, so during this stage

$$i_{supply}^+(t) = i_s^+(t)$$

During the subsequent stage the reference supply phase is connected to the C-phase of the motor so that during this stage

$$i_{supply}^+(t + \tau_b) = i_s^+(t + \tau_b) \cdot e^{j\frac{2\pi}{3}}$$

in accordance with the discussion in Chapter 5.2

Substituting for $i_s^+(t + \tau_b)$ we obtain

$$\begin{aligned} i_{supply}^+(t + \tau_b) &= (i_s^+(t) \cdot e^{j\theta_m}) \cdot e^{j\frac{2\pi}{3}} \\ &= i_s^+(t) \cdot e^{j\omega_e \tau_b} \\ &= i_{supply}^+(t) \cdot e^{j\omega_e \tau_b} \end{aligned}$$

So that the modulation angle for the supply current is, as might be expected,

$$\theta_{supply} = \omega_e \tau_b$$

Thus, given the values of i_{s0}^I, i_{s0}^{II} as defined at $t=0$, The motor and supply current waveforms are completely and conveniently determined in a form which lends itself to the use of the expressions for RMS and spectral analysis which were developed in Chapter Five.

8.5 RMS Current

Since the modulation cycle has only one stage it is particularly convenient to calculate the RMS values as detailed in Chapter 5.4

Note that

$$i_{supply}^a(RMS) = i_s^a(RMS)$$

8.6 Current Spectra

This analysis is also achieved as shown in Chapter 5.5

Note that
$$\theta_m = \left(\omega_e \tau_b - \frac{2\pi}{3} \right)$$

$$\theta_{supply} = \omega_e \tau_b$$

So that the supply current has components of angular frequency,

$$\omega = \left| \omega_e - m\omega_{step} \right|$$

and the motor current has components at

$$\omega = \left| \omega_{out} - m\omega_{step} \right|$$

In each case the phase sequence of the set associated with a particular component is dependant on the sign of

$$(\omega_e - m\omega_{step}) \quad \text{for the supply current}$$

$$(\omega_{out} - m\omega_{step}) \quad \text{for the motor current.}$$

8.7 Convertor with one or more chops per phase step

In principle the analysis of the chopped convertor waveforms is the same as for the case considered above once the motor current over one cycle of the modulation has been determined.

Naturally the computation of the motor current becomes more complicated as the number of stages in the modulation cycle is increased, since the initial values of the transient current components must be evaluated for the start of each stage. Also the evaluation of the RMS values and spectral content becomes more lengthy. However, since the analysis is computer-oriented the treatment of multistage modulation cycles is not regarded as a formidable task, and is undoubtedly more easily done in this way than by the use of simulation.

In Chapter 4.5.3 a phase-stepping cycle with two chops per phase step was considered and the current components at the

start of each stage of the cycle were defined.

The motor current and supply line currents each have the same modulation angle as when no chopping action was present. It is thus clear that the same frequency components are found in the system currents irrespective of the number of chops per step.

The short circuit loop currents are simply equal to the motor currents during the shorted intervals and zero otherwise. The same frequency components thus occur in both motor and short circuit loop currents (since they have the same modulation angle). Also we have

$$i_{s(RMS)}^a = \sqrt{\left(i_{supply(RMS)}^a\right)^2 + \left(i_{short-cct(RMS)}^a\right)^2}$$

Note: In accordance with earlier discussion (Chapter 5.6.2) the phase currents are similar but time-shifted

$$\begin{aligned} \text{(a) in the supply if} \quad f_{step} &= 3nf_e \quad (n=1,2,\dots) \\ \text{(b) in the motor if} \quad f_{step} &= \left(\frac{3n}{n+1}\right) \cdot f_e \quad (n=1,2,\dots) \end{aligned}$$

8.8 Special Cases:-

The special cases which occur when $\theta = n\pi$ correspond to

$$f_{step} = \frac{2 \cdot f_e}{n} \quad \text{in the case of the supply currents where } n=1,2,3 \dots$$

and to

$$f_{step} = \frac{6f_e}{(3n+2)} \quad \text{in the case of the motor currents where } n=0,1,2,3 \dots$$

When the first situation occurs then the supply currents are unbalanced and have different RMS values, but the motor currents are balanced. The RMS and spectrum of the motor currents may be calculated in the usual way, but the supply currents must be specially treated. D.C. values in the supply currents occur in the case of n even.

Similarly, in the second situation, the supply currents are balanced but the motor currents are unbalanced.

As these particular stepping frequencies are approached, the relevant current waveforms are noticeably amplitude modulated by the beat frequency, although they are still balanced.

8.9 Torque

Because of the tendency of the positive phase sequence harmonic motor currents to cancel the average torque produced by the negative phase sequence harmonic currents, we find that the average torque produced by the motor is largely accounted for by the fundamental motor current which is the only unpaired component. The average torque is thus closely approximated by the torque which would be produced by a balanced sinusoidal terminal voltage set of amplitude (RMS)

$$V_s^a \text{ Fund.} = \frac{3\sqrt{3}}{2\sqrt{2}} \frac{|V_s|}{\pi} \cdot \left\{ \frac{\sin\left(\frac{\pi \frac{M}{p}}{3Z}\right)}{\sin\left(\frac{\pi}{3Z}\right)} \right\}$$

and frequency = $\left| f_e - \frac{f_{step}}{3} \right|$

The torque acts in a direction which is appropriate to the phase sequence of the fundamental and the slip relative to the fundamental.

It is not valid to make the assumptions made previously that the instantaneous peaks and minima occur at a particular point in the modulation cycle and it is therefore necessary to use the formulated analysis of the torque waveform

in order to obtain details of the torque pulsations. In every case the torque is comprised of a D.C. average component plus oscillating components at multiples of the phase-stepping frequency. If there are more than one chops per step the same torque frequencies occur, but the components which correspond to multiples of the chopping frequency are accentuated.

8.10 Calculated Performance of the Induction Motor Supplied through a Phase-stepping frequency Convertor

The calculation program listed in Appendix (7), has been used to perform a wide range of calculations for the case of the test motor supplied through a phase-stepping frequency convertor. As in the case of the chopper modulation the presentation of these results is complicated by the number of parameters which influence the system. In each instance the parameters for the motor which correspond to the fully fluxed condition are used, since with both voltage and frequency control an induction motor will usually be kept in this state (see Chapter Nine). The effects of the following parameters on the system have been closely considered.

- (a) Phase-stepping frequency.
- (b) Number of chops per phase step.
- (c) Mark-to-total time ratio $T(\text{on})/T(\text{total})$.
- (d) Slip frequency.

The calculated results may be categorized as follows:-

- (i) Figs. (89) to (95) each display the four main performance factors as functions of slip frequency. These figures are grouped in sets of three, being labelled (a), (b) and (c) where -

- (a) shows the case with no chopping action

- (b) shows one chop per step for different values of $T(\text{on})/T(\text{total})$
- (c) shows four chops per step for different values of $T(\text{on})/T(\text{total})$

Each set of three illustrations corresponds to a particular phase-stepping frequency. The stepping frequencies considered range from 90 to 300 Hz.

- (ii) Figs (96) to (102) illustrate the performance factors as a function of $T(\text{on})/T(\text{total})$. In each case the slip frequency is kept constant and the results are displayed for several stepping frequencies and for 1 and 4 chops per phase-step.
- (iii) Figs. (103) to (109) show the algebraically calculated spectra of the system waveforms displayed adjacent to the corresponding time-domain waveforms obtained from numerical simulation. These figures are grouped in pairs, labelled (a) and (b), where -
 - (a) shows the results with one chop per step
 - (b) shows the results with four chops per step.
- (iv) Figs (110) and (111) show the effect on the spectra of the system waveforms of progressively increasing the number of chops per step to 8.

8.11 Observations related to the Calculated Results

While it is not feasible to discuss all aspects of the convertor operation in great detail here, the more striking

FREQUENCY CONVERTOR WITH NO CHOPPING - PERFORMANCE FACTORS FROM ALGEBRAIC SOLUTION.

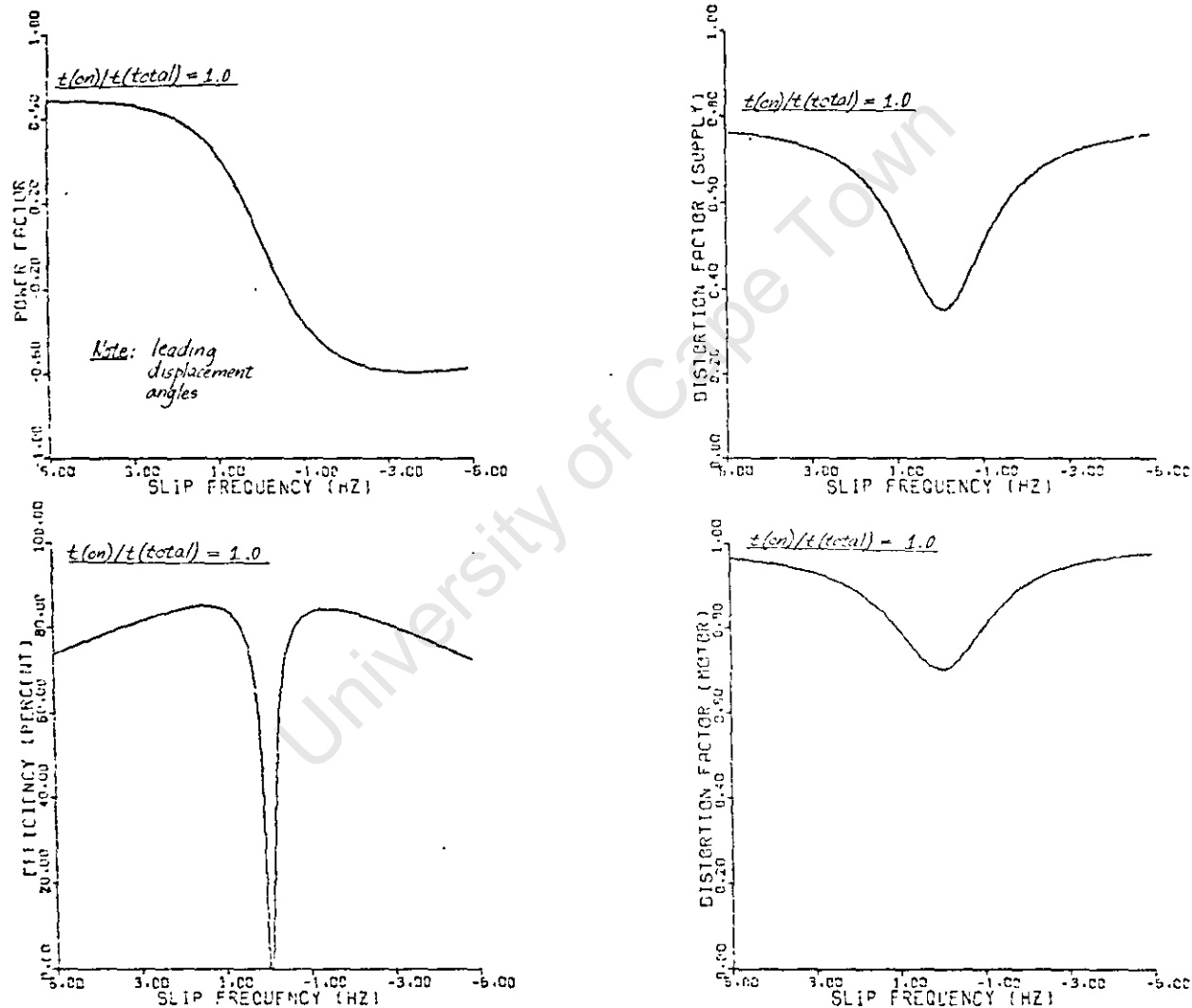


Fig.89a CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=1. . STEPPING FREQUENCY (HZ)=300.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447,L1=L2=0.473,R1=5.00,R2=3.25

FREQUENCY CONVERTOR WITH ONE CHOP PER PHASE STEP
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

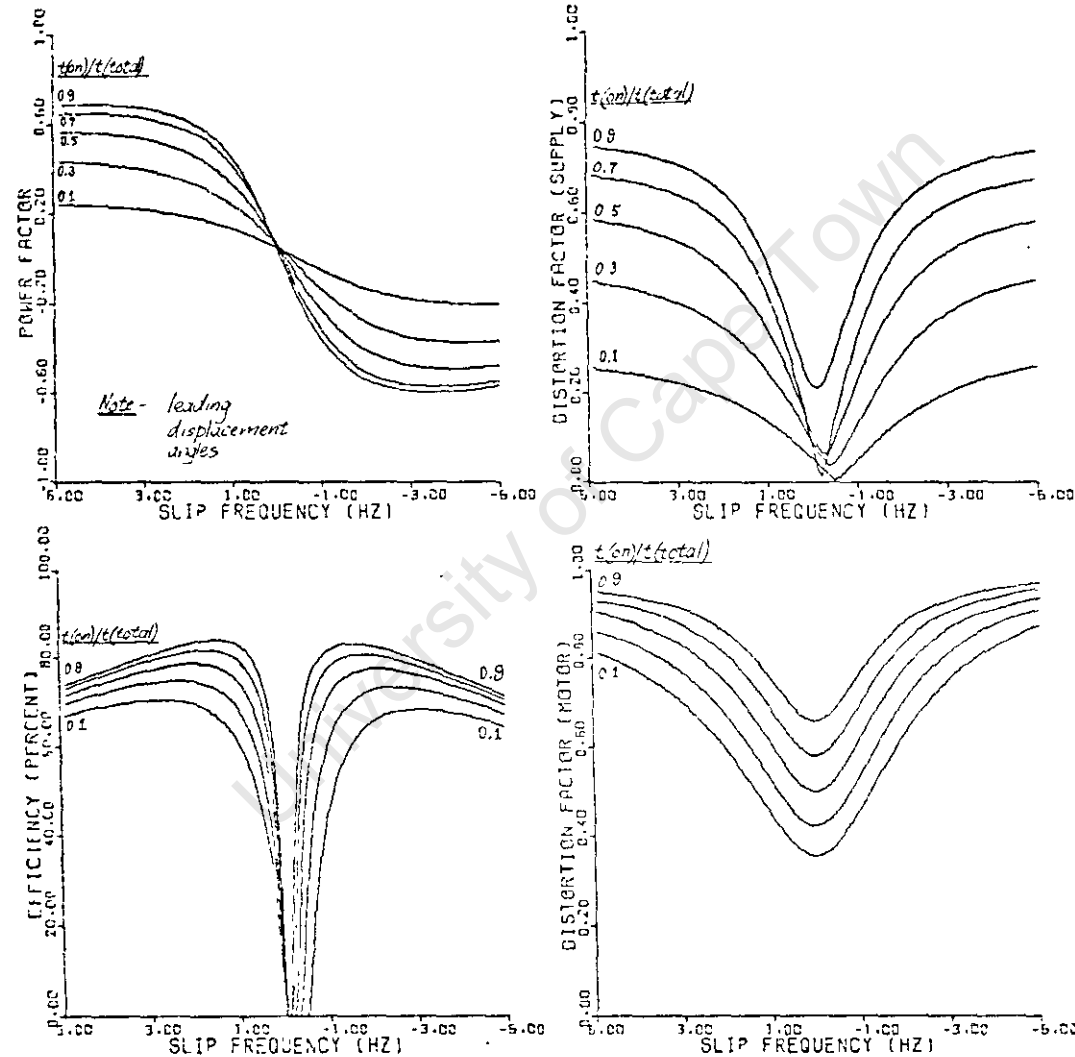


Fig.89b

CONVERTOR STEPS -120.0 DEGREES . CHOPS PER STEP=1 . STEPPING FREQUENCY (HZ)=300.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447.L1=L2=0.473.R1=5.50.R2=3.29

FREQUENCY CONVERTOR WITH FOUR CHOPS PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

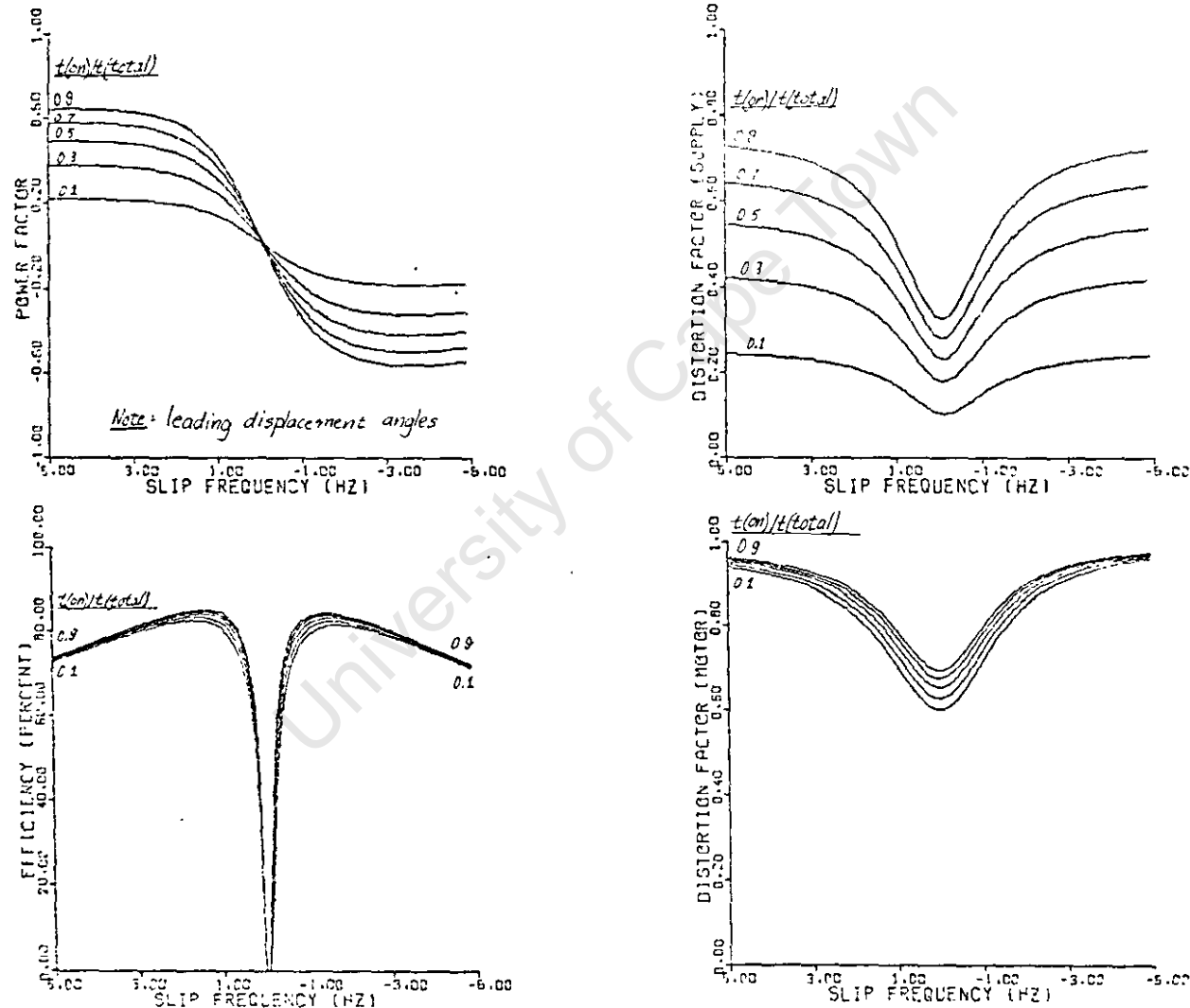


Fig. 89c CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=4., STEPPING FREQUENCY (HZ)=300.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447, L1=L2=0.473, R1=5.50, R2=3.29

FREQUENCY CONVERTOR WITH ONE CHOP PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

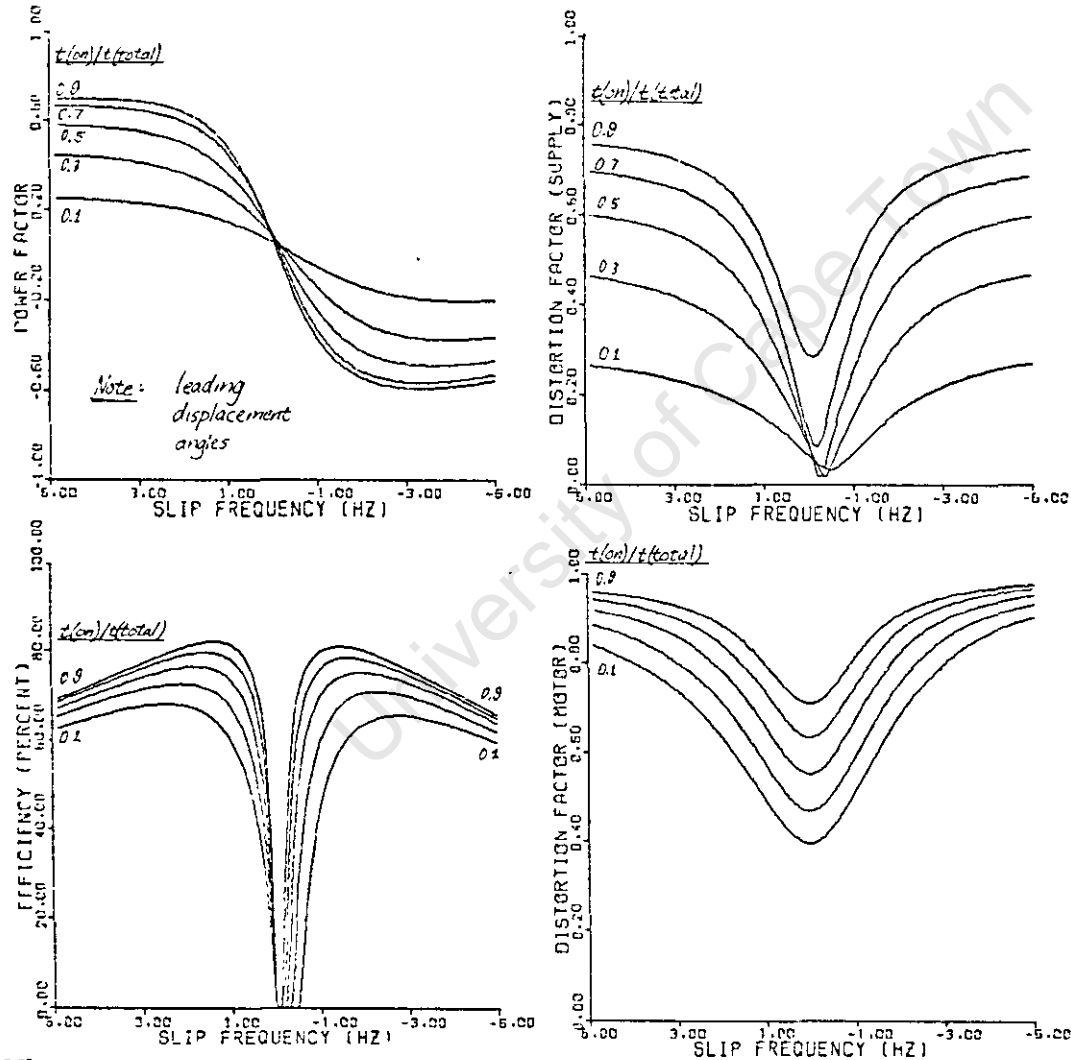


Fig. 90b CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=1., STEPPING FREQUENCY (HZ)=270.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) $M=0.447, L1=L2=0.473, R1=5.50, R2=3.28$

FREQUENCY CONVERTOR WITH FOUR CHOPS PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

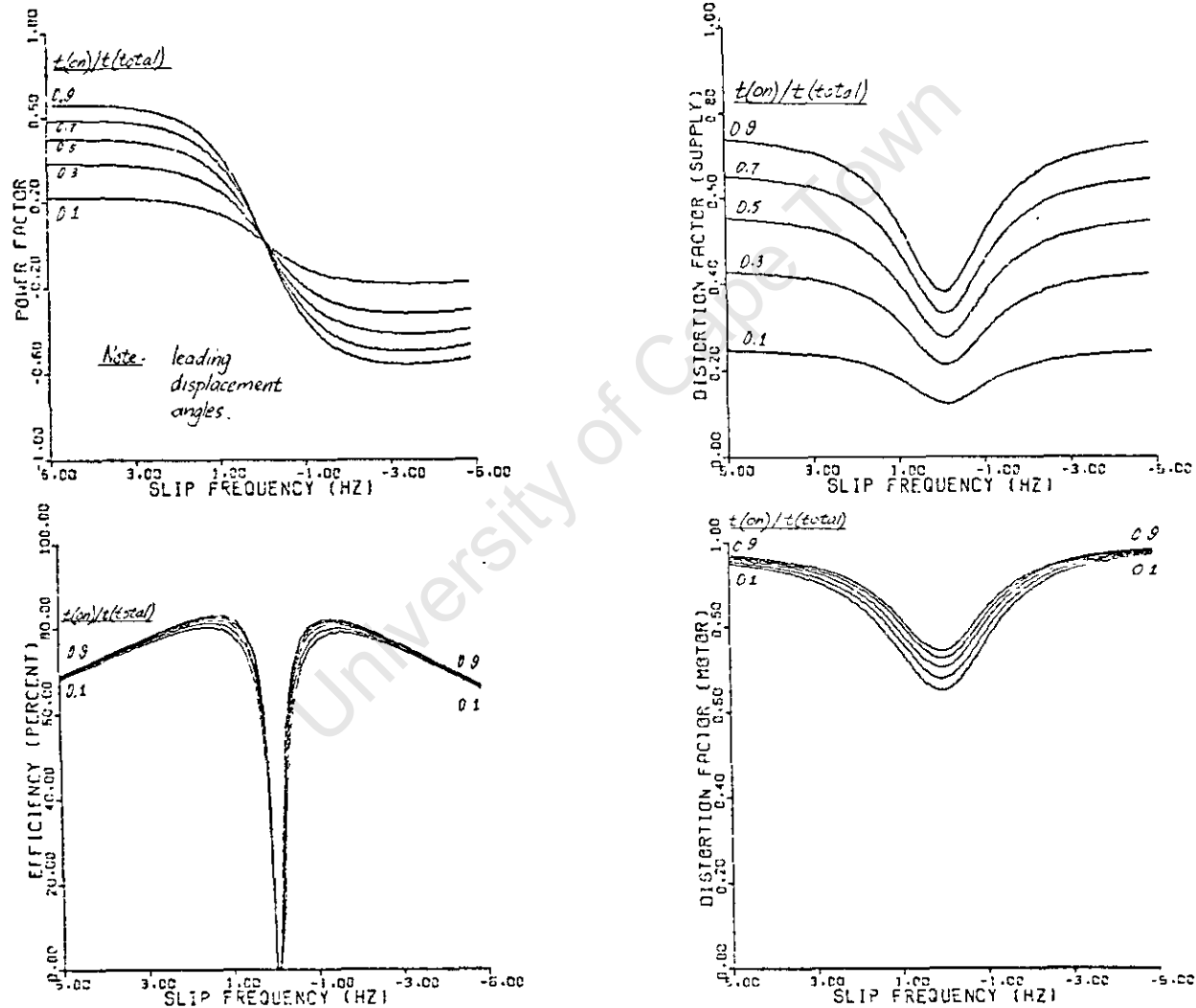


Fig.90c

CONVERTOR STEPS -120.0 DEGREES . CHOPS PER STEP=4. . STEPPING FREQUENCY (HZ)=270.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) $M=0.447, L1=L2=0.473, R1=5.50, R2=3.28$

FREQUENCY CONVERTOR WITH NO CHOPPING - PERFORMANCE FACTORS FROM ALGEBRAIC SOLUTION.

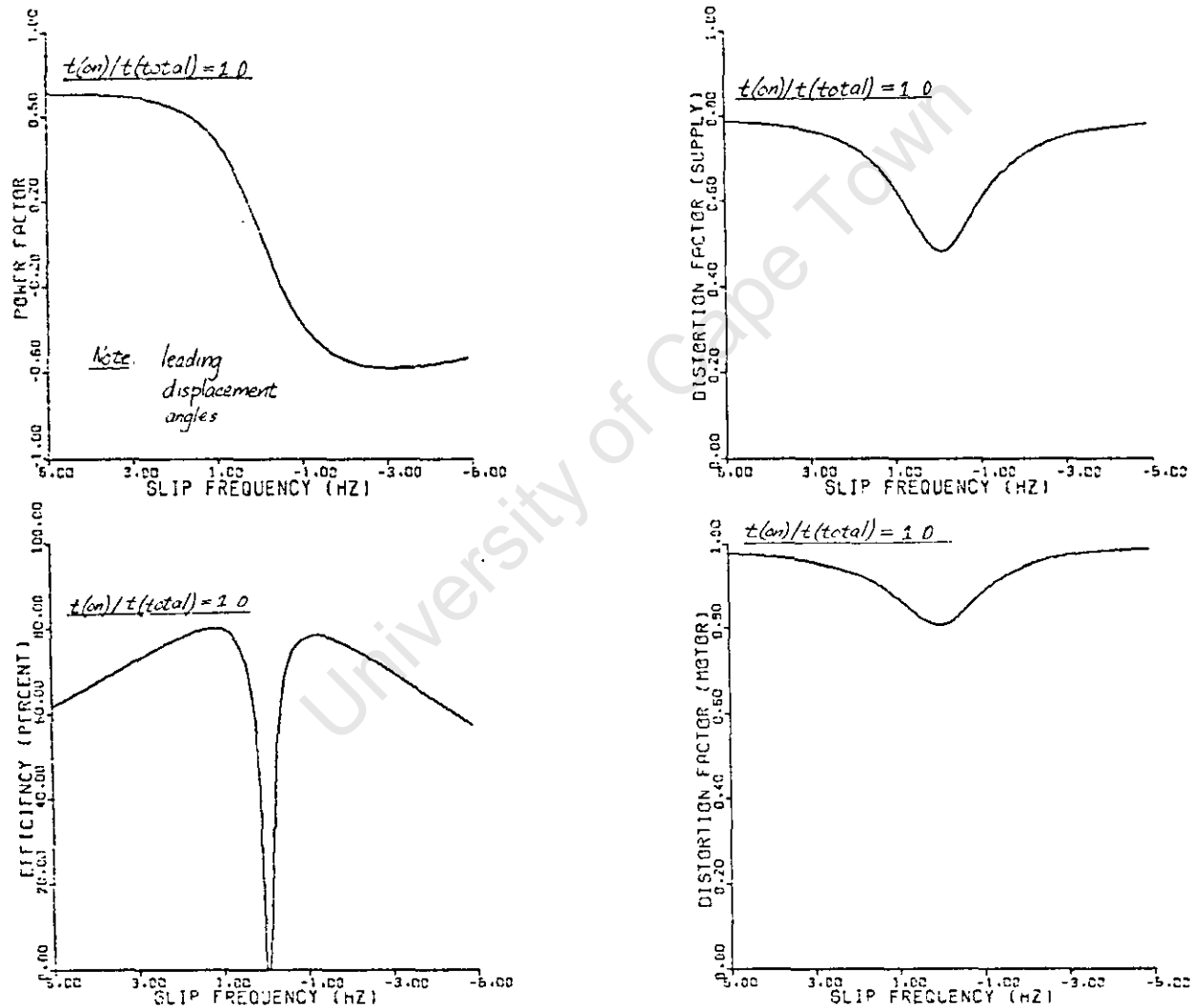


Fig. 91a

CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=1., STEPPING FREQUENCY (HZ)=240.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447, L1=L2=0.473, R1=5.50, R2=3.29

FREQUENCY CONVERTOR WITH ONE CHOP PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

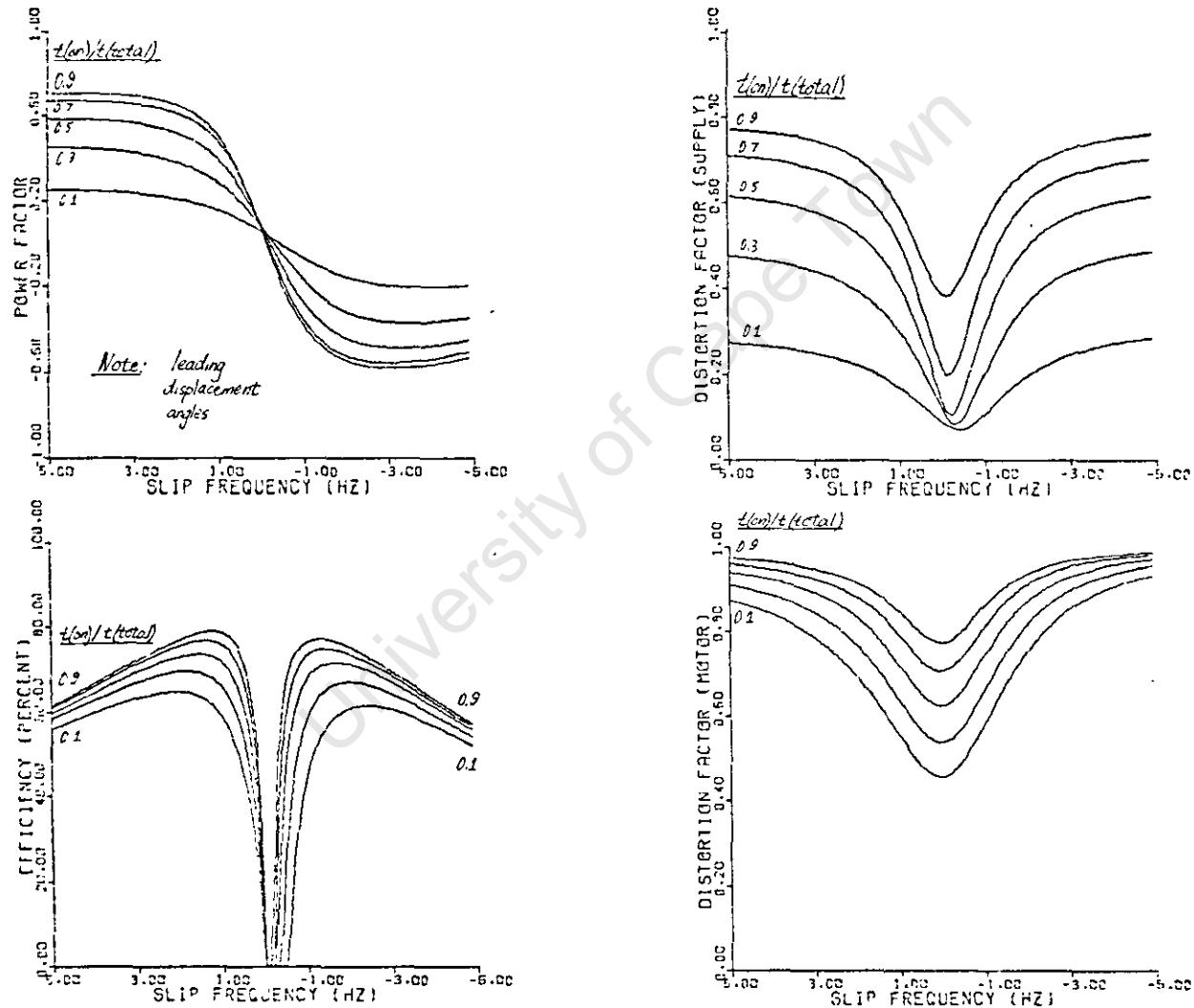


Fig. 91b CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=1. . STEPPING FREQUENCY (HZ)=240.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447.L1=L2=0.473.R1=5.30.R2=3.29

FREQUENCY CONVERTOR WITH FOUR CHOPS PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

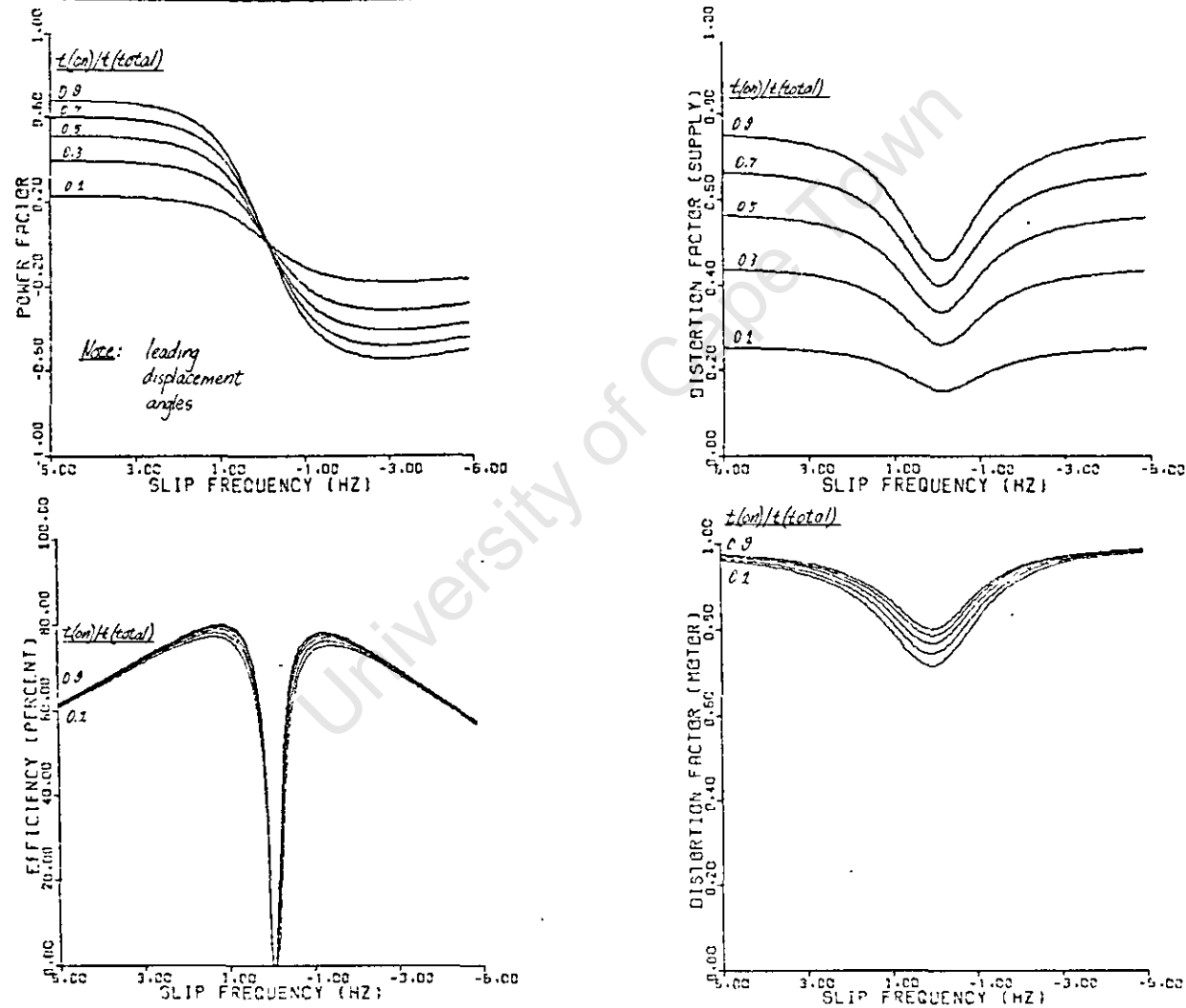


Fig.91c.

CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=4. . STEPPING FREQUENCY (HZ)=240.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) $M=0.447, L_1=L_2=0.473, R_1=5.50, R_2=3.28$

FREQUENCY CONVERTOR WITH NO CHOPPING - PERFORMANCE FACTORS FROM ALGEBRAIC SOLUTION.

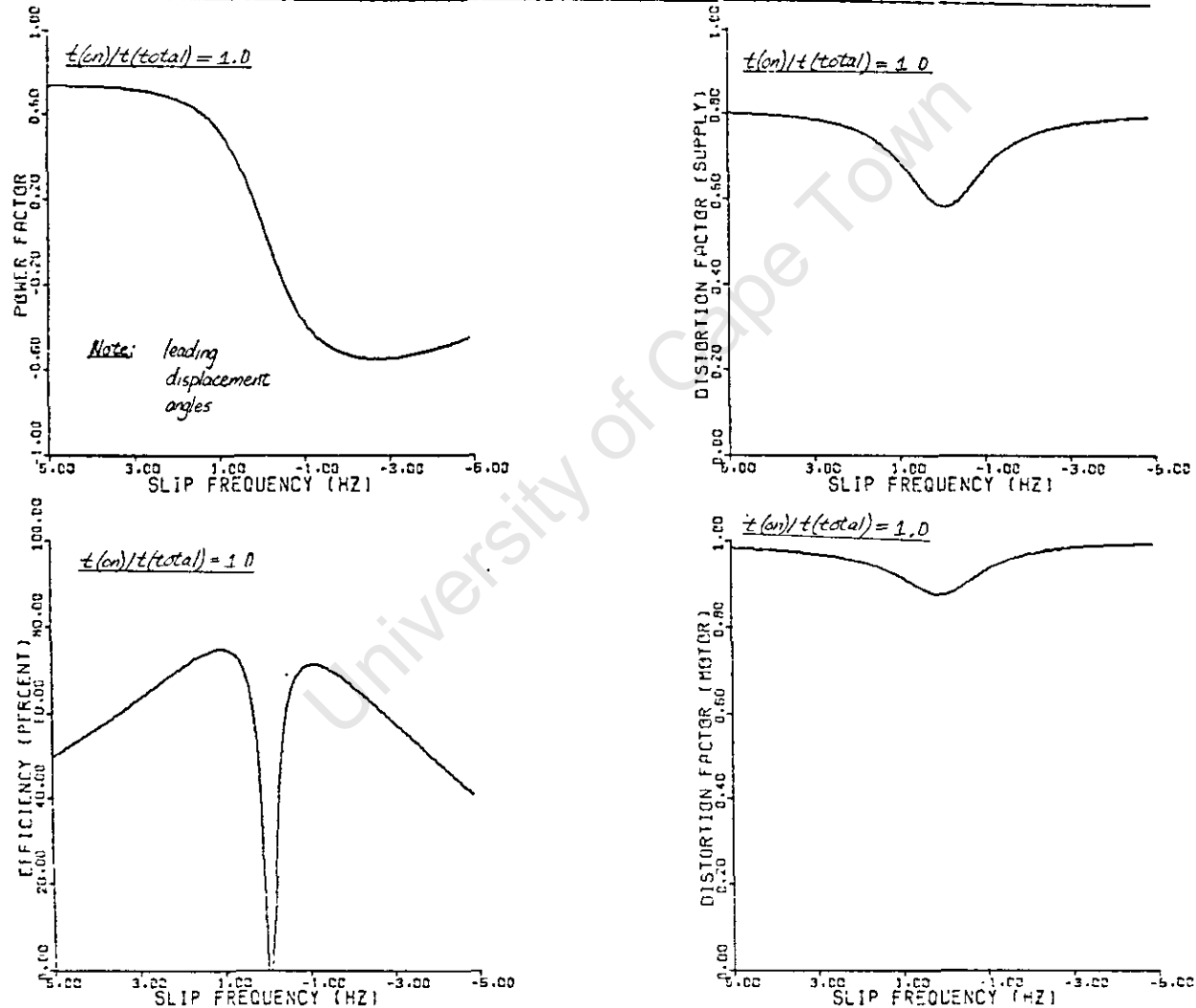


Fig 92a

CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=1. . STEPPING FREQUENCY (HZ)=210.0
 SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447, L1=12.0, R1=5.50, R2=3.28

FREQUENCY CONVERTOR WITH ONE CHOP PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

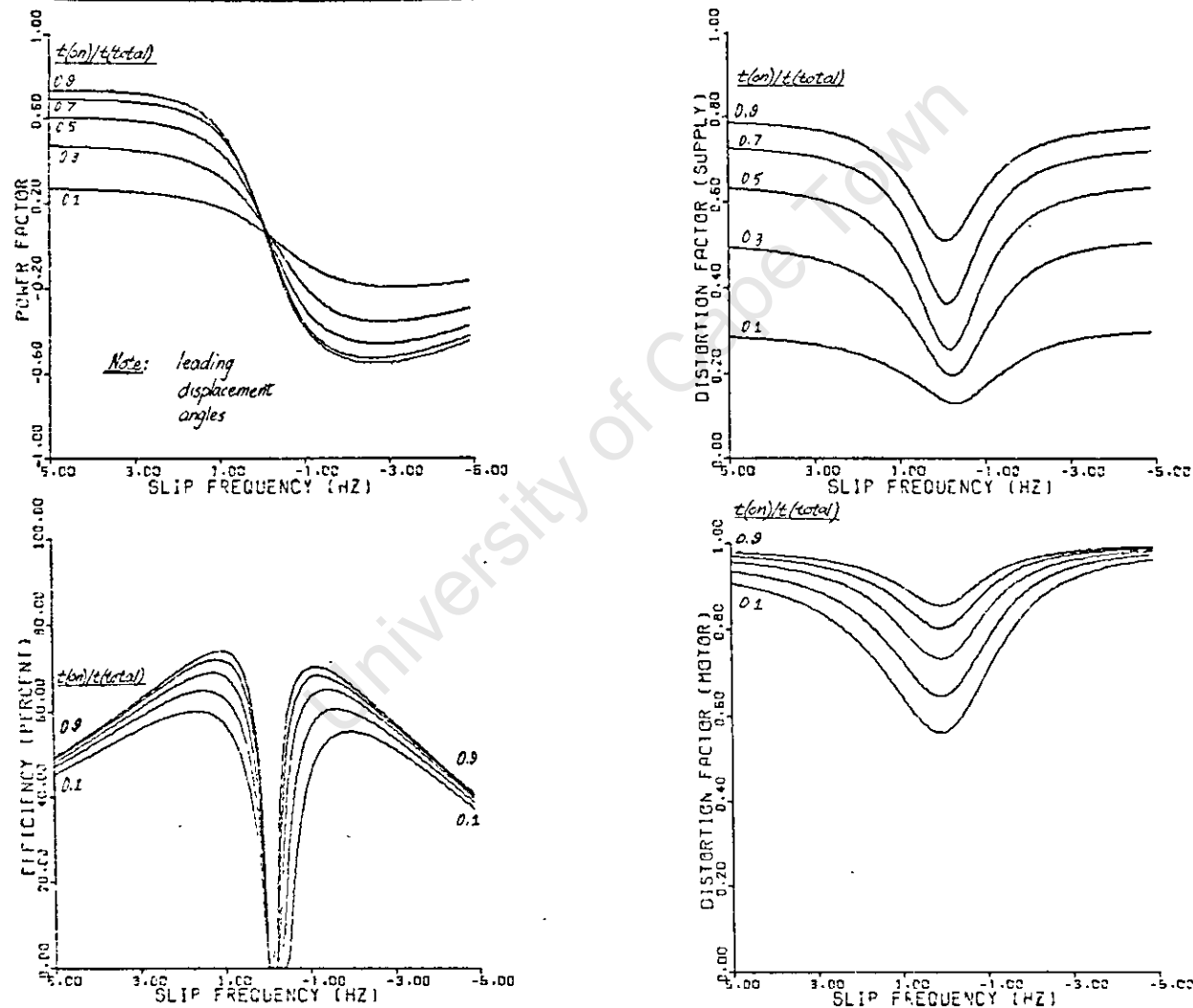


Fig. 92b

CONVERTOR STEPS -120.0 DEGREES . CHOPS PER STEP=1 . . STEPPING FREQUENCY (HZ)=210.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447, L1=L2=0.473, R1=5.50, R2=3.29

FREQUENCY CONVERTOR WITH FOUR CHOPS PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

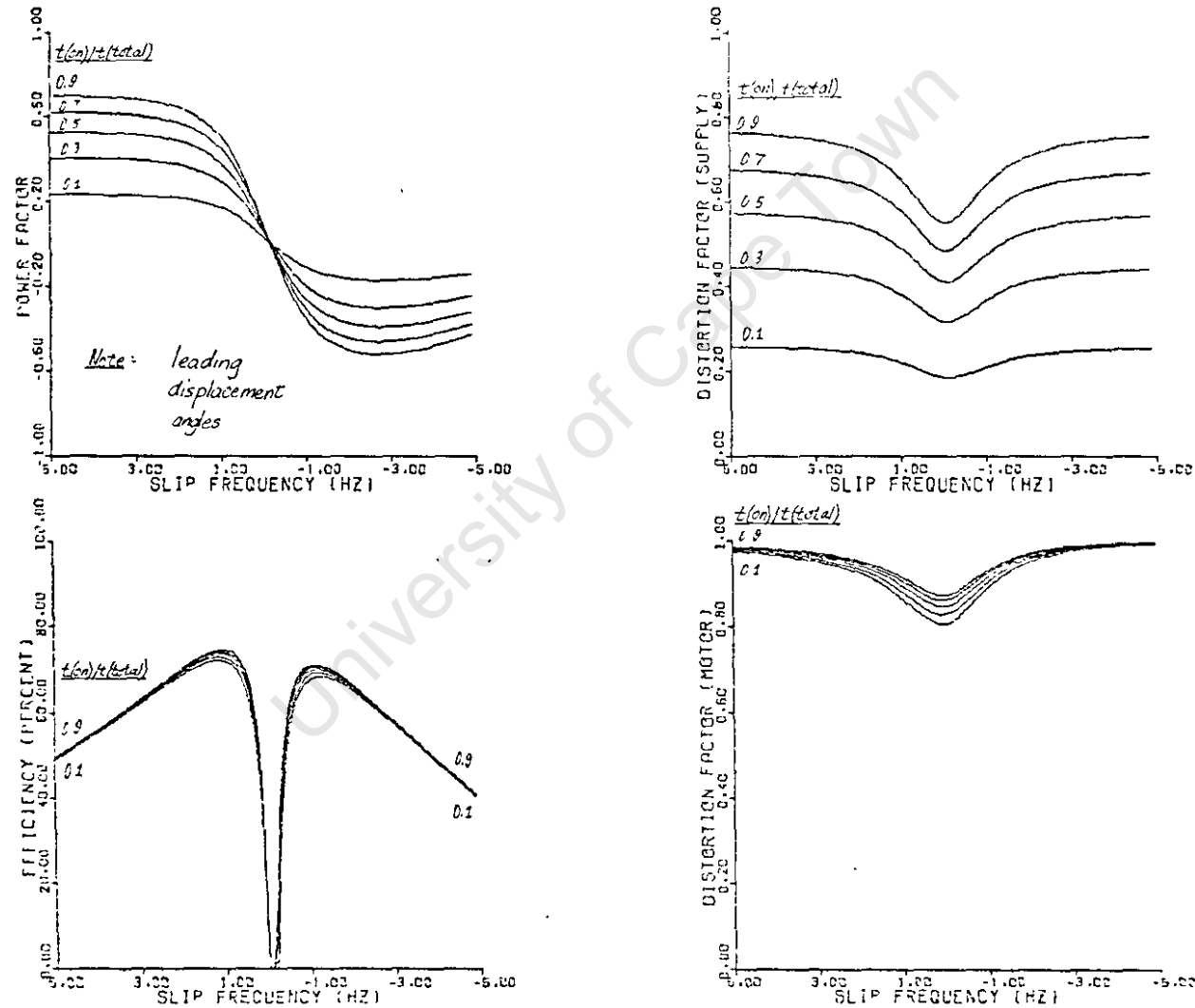


Fig. 92c

CONVERTER STEPS -120.0 DEGREES, CHOPS PER STEP=4., STEPPING FREQUENCY (HZ)=210.0
SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447, L1=[2.0.473, R1=3.60, R2=3.29

FREQUENCY CONVERTOR WITH NO CHOPPING - PERFORMANCE FACTORS FROM ALGEBRAIC SOLUTION.

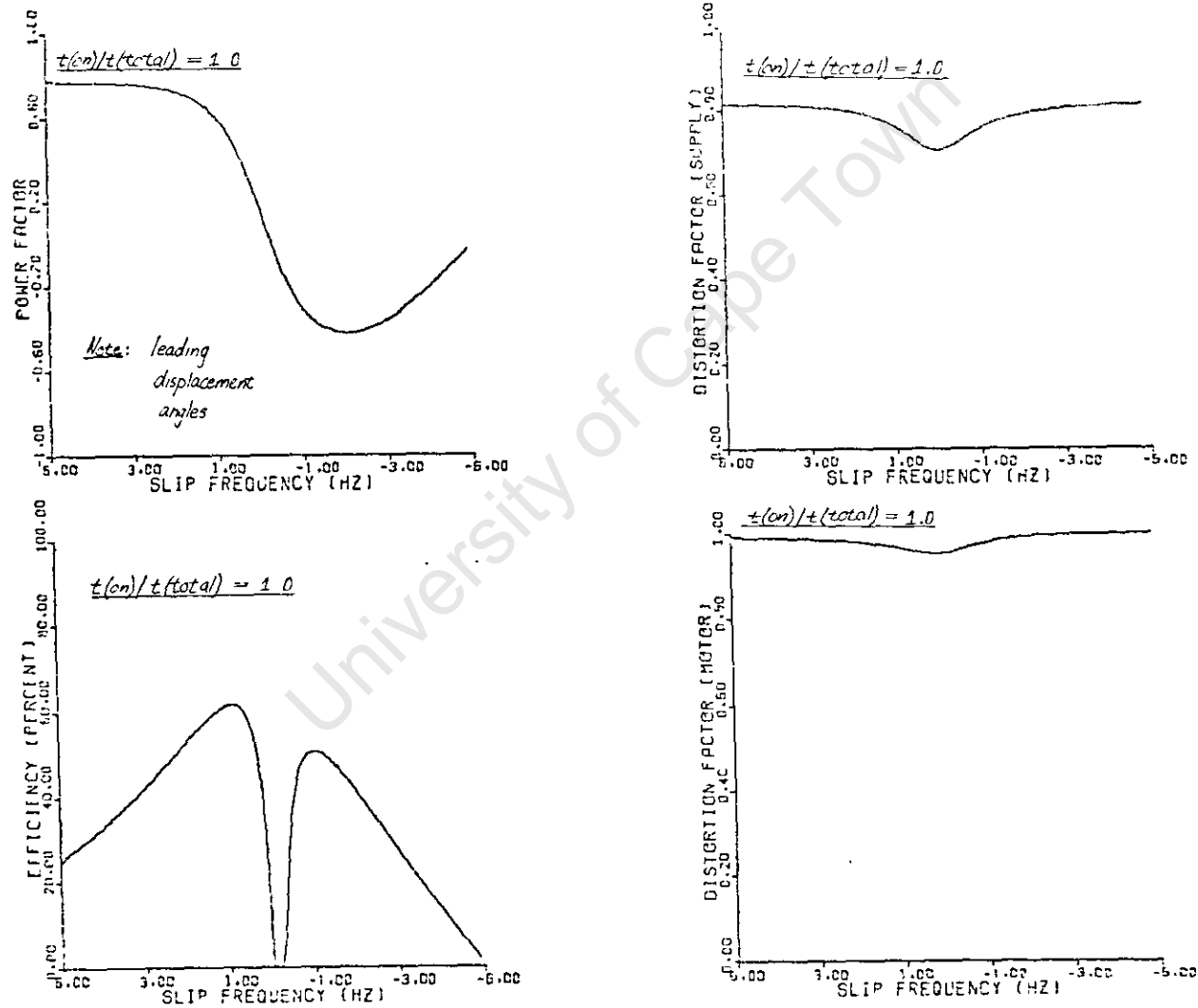


Fig.93a

CONVERTER STEPS -120.0 DEGREES, CHOPS PER STEP=1., STEPPING FREQUENCY (HZ)=120.0
 SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447.L1=L2=0.473.R1=5.50.R2=3.29

FREQUENCY CONVERTOR WITH ONE CHOP PER PHASE STEP.
 PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

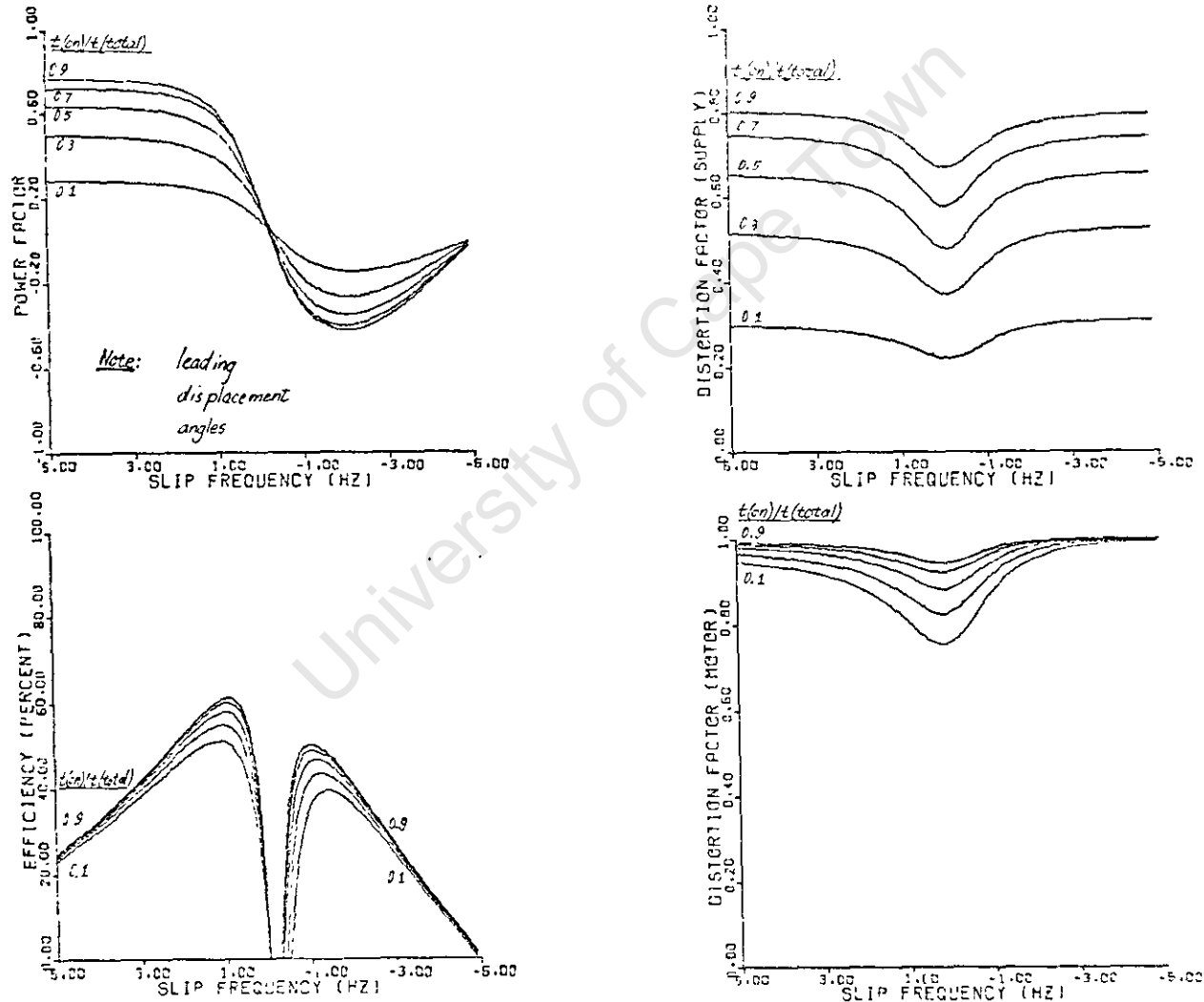


Fig. 93b

CONVERTOR STEPS -120.0 DEGREES. CHOPS PER STEP=1. STEPPING FREQUENCY (HZ)=180.0
 SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447, L1=L2=0.473, R1=5.50, R2=3.29

FREQUENCY CONVERTOR WITH FOUR CHOPS PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

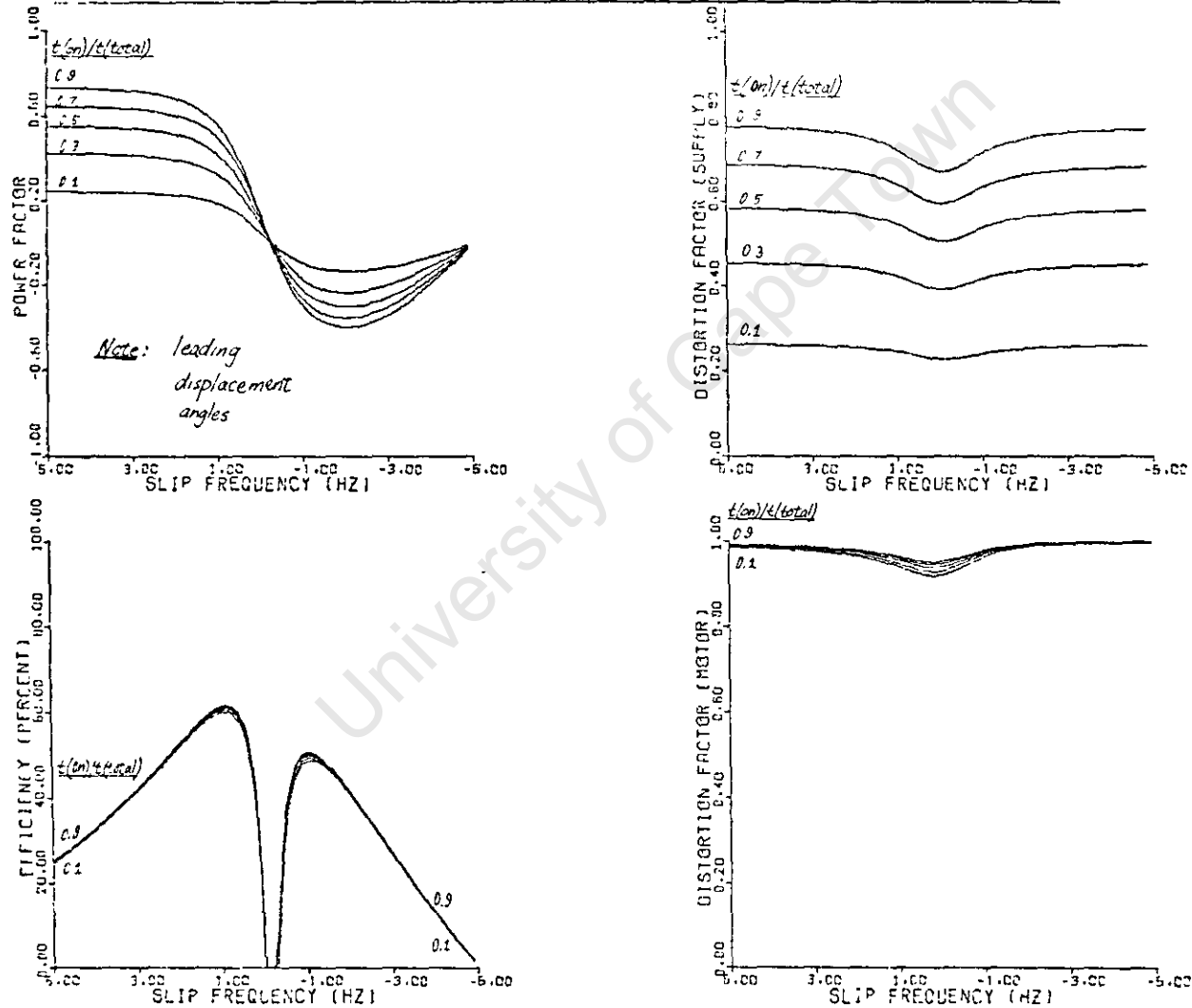


Fig. 93c

CONVERTOR STEPS = 120.0 DEGREES, CHOPS PER STEP = 4, STEPPING FREQUENCY (HZ) = 180.0
SUPPLY VOLTAGE = 220.0 VOLT RMS 50.0 (HZ) $M = 0.447, L_1 = 12.0, R_1 = 5.50, R_2 = 3.25$

FREQUENCY CONVERTOR WITH NO CHOPPING - PERFORMANCE FACTORS FROM ALGEBRAIC SOLUTION.

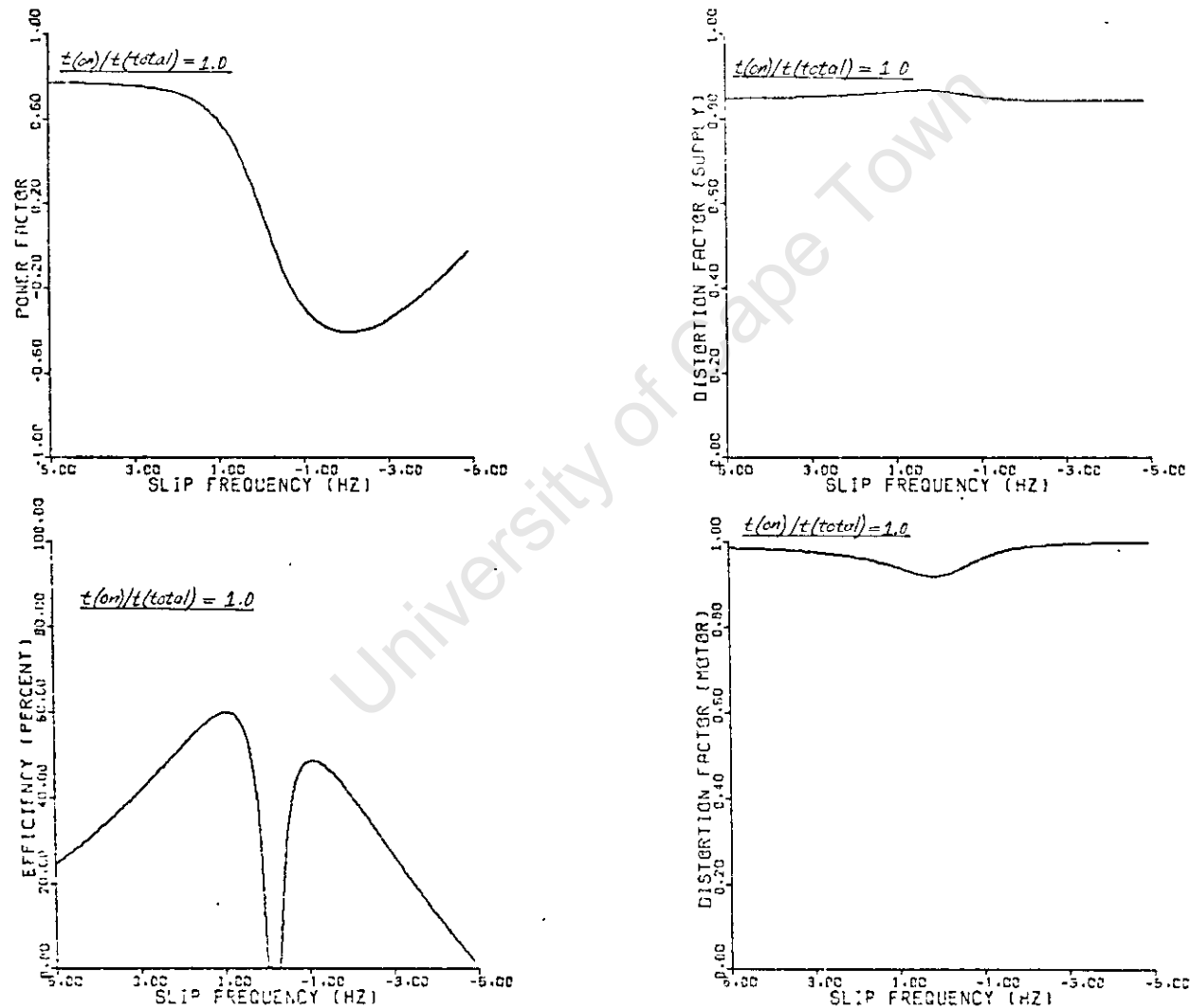


Fig. 94a

CONVERTOR STEPS -120.0 DEGREES , CHOPS PER STEP=1. , STEPPING FREQUENCY (HZ)=120.0
 SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447,L1=L2=0.473,R1=5.50,R2=3.28

FREQUENCY CONVERTOR WITH NO CHOPPING - PERFORMANCE FACTORS FROM ALGEBRAIC SOLUTION.

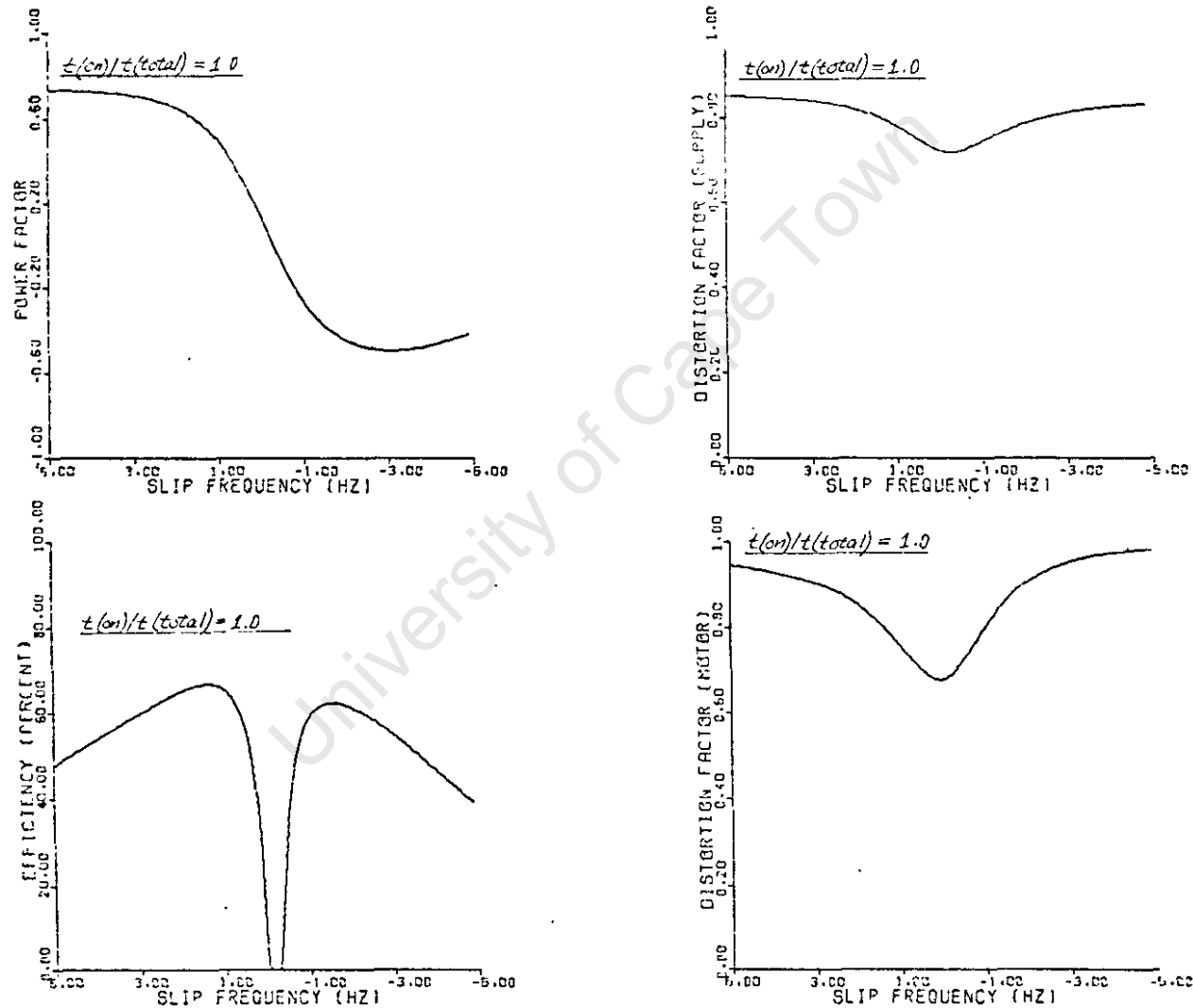


Fig 95a

CONVERTOR STEPS -120.0 DEGREES , CHOPS PER STEP=1, STEPPING FREQUENCY (HZ)=90.0
 SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) M=0.447,L1=L2=0.473,R1=5.50,R2=3.28

FREQUENCY CONVERTOR WITH ONE CHOP PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

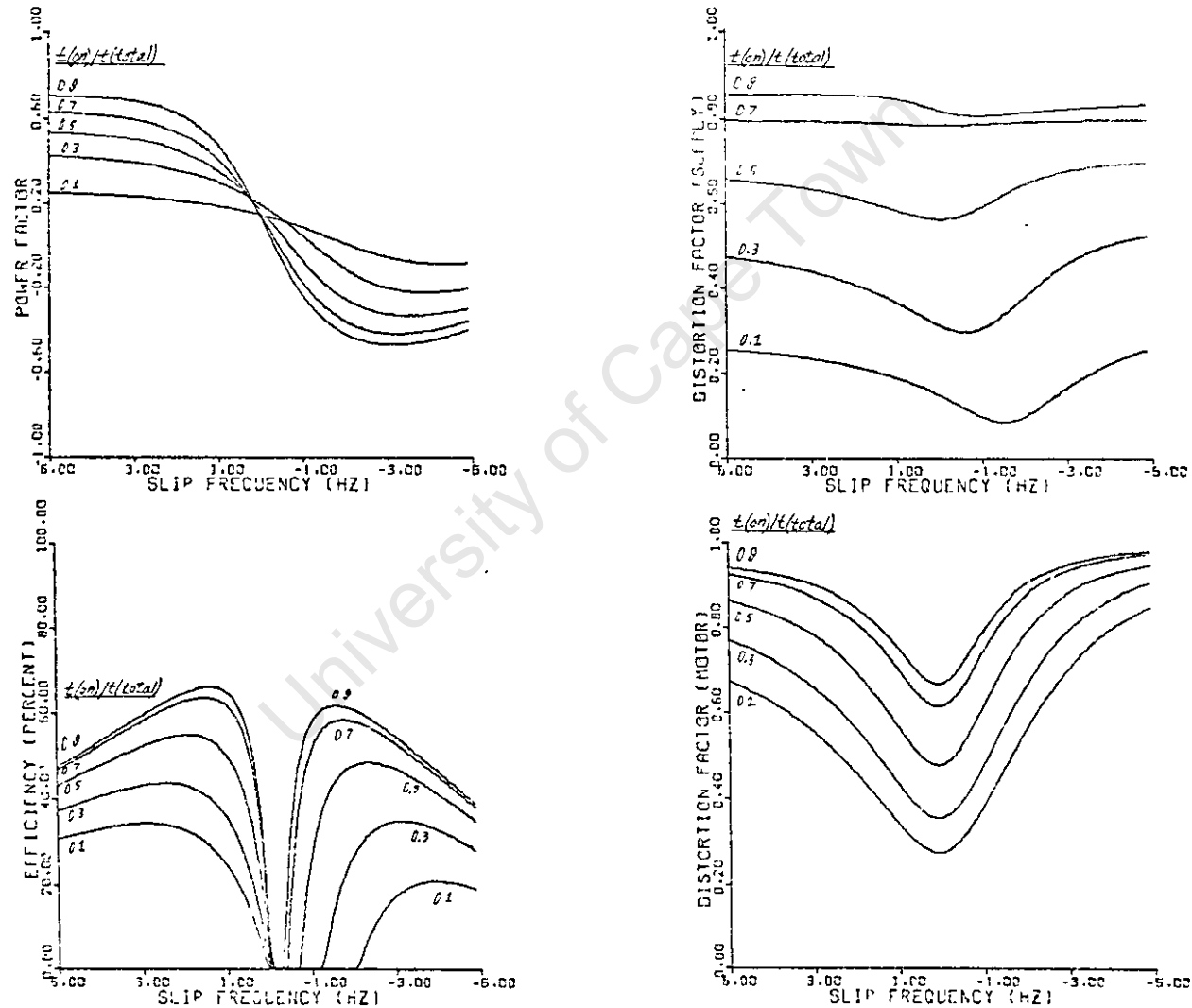


Fig.95b

CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=1., STEPPING FREQUENCY (HZ)=90.0
 SUPPLY VOLTAGE=220.0 VOLT RMS 50.0 (HZ) $M=0.447, L1=L2=0.473, R1=5.50, R2=3.25$

FREQUENCY CONVERTOR WITH FOUR CHOPS PER PHASE STEP.
PERFORMANCE FACTORS FOR THE TEST MOTOR CALCULATED FROM THE ALGEBRAIC SOLUTION.

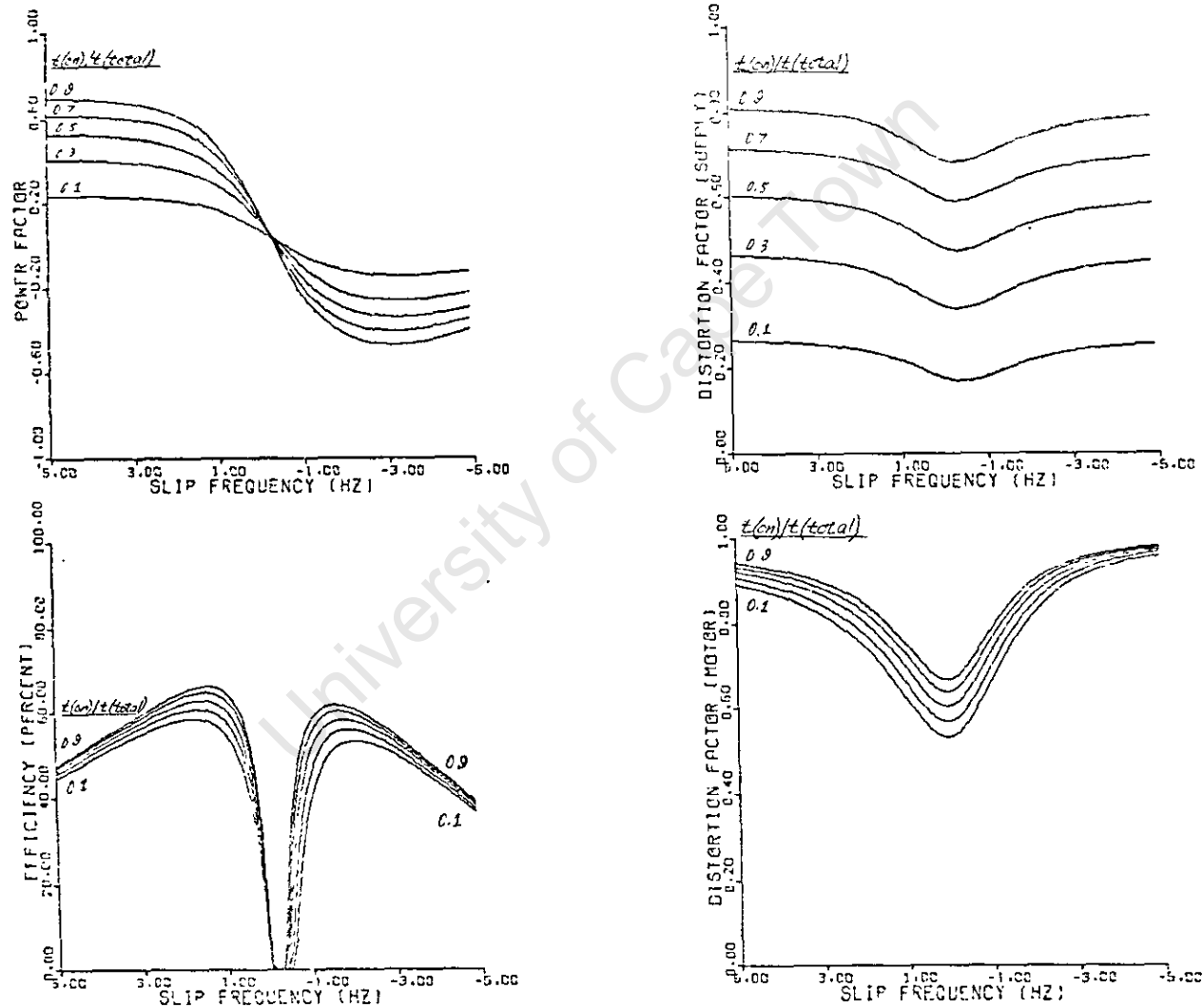


Fig. 95c

CONVERTER STEPS = 120.0 DEGREES, CHOPS PER STEP = 4., STEPPING FREQUENCY (HZ) = 90.0
SUPPLY VOLTAGE = 220.0 VOLT RMS 50.0 (HZ) M = 0.447, L1 = 2.0, 0.479, R1 = 5.50, R2 = 3.28

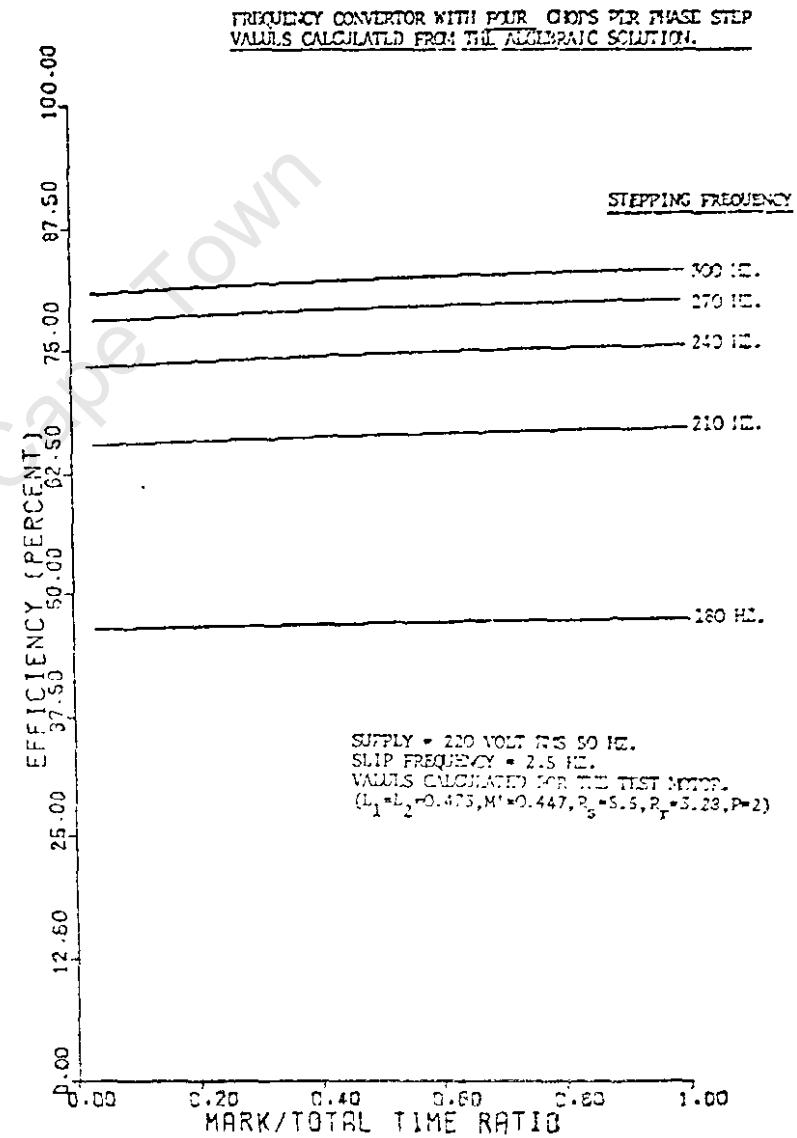
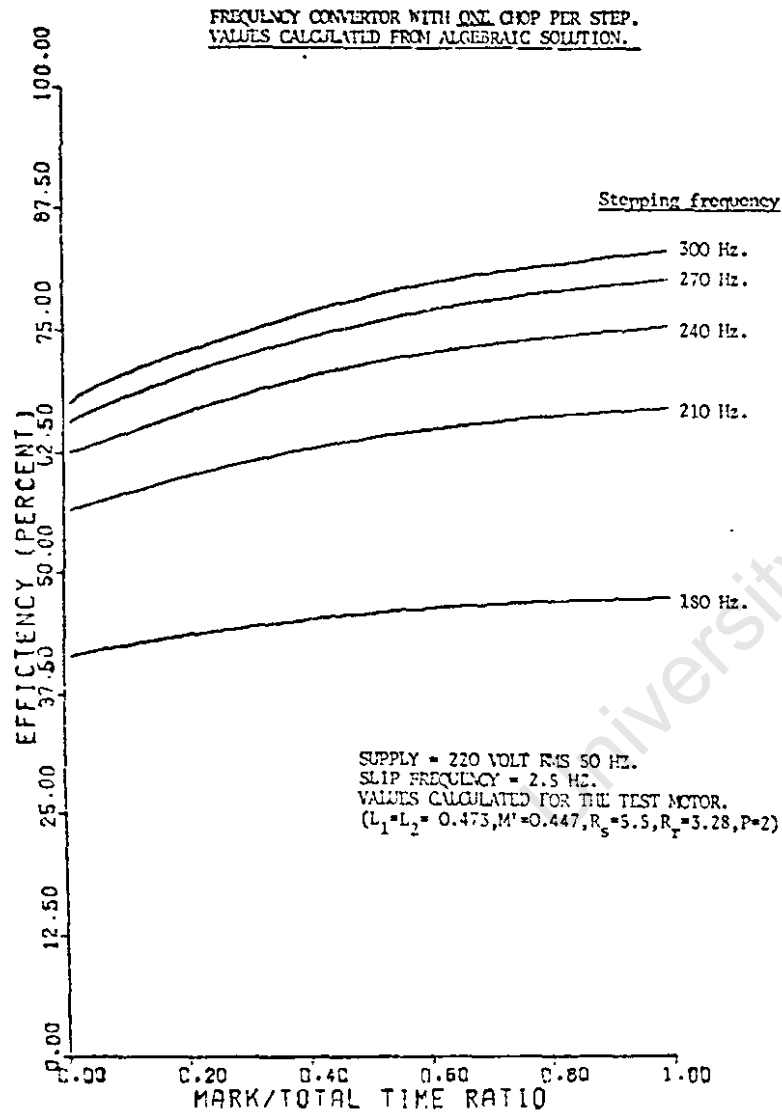


Fig. 96

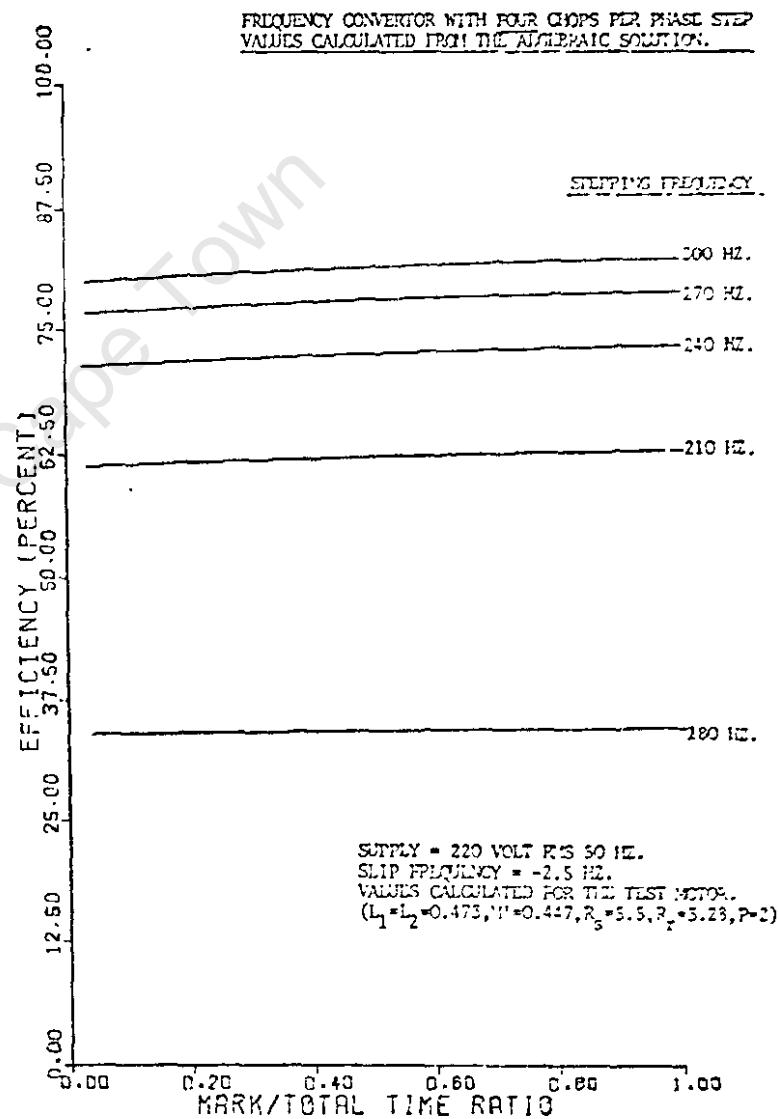
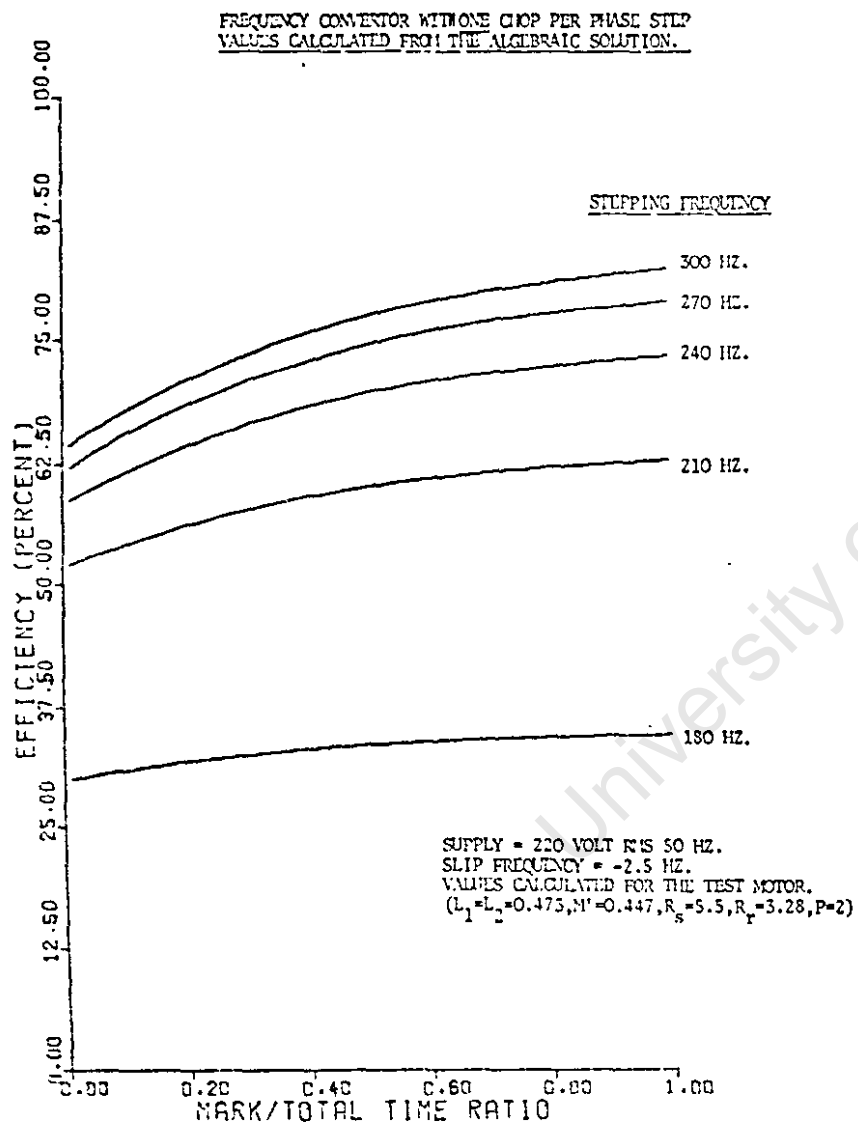


Fig. 97

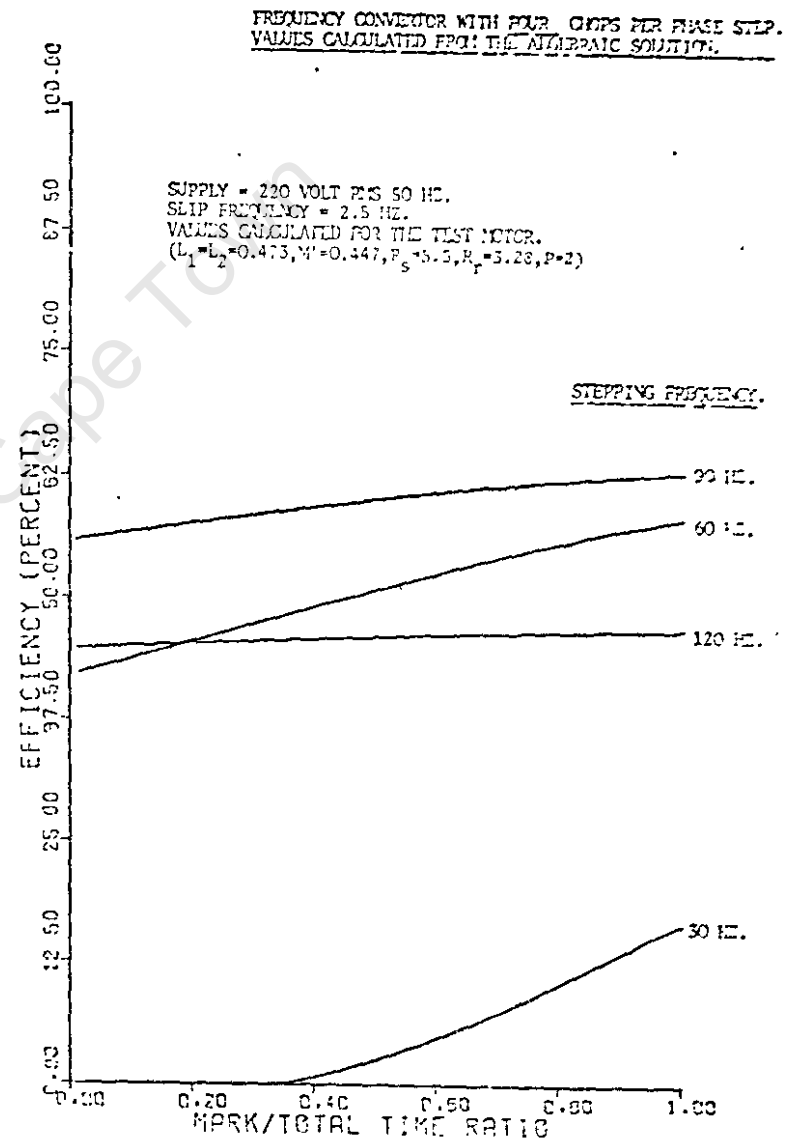
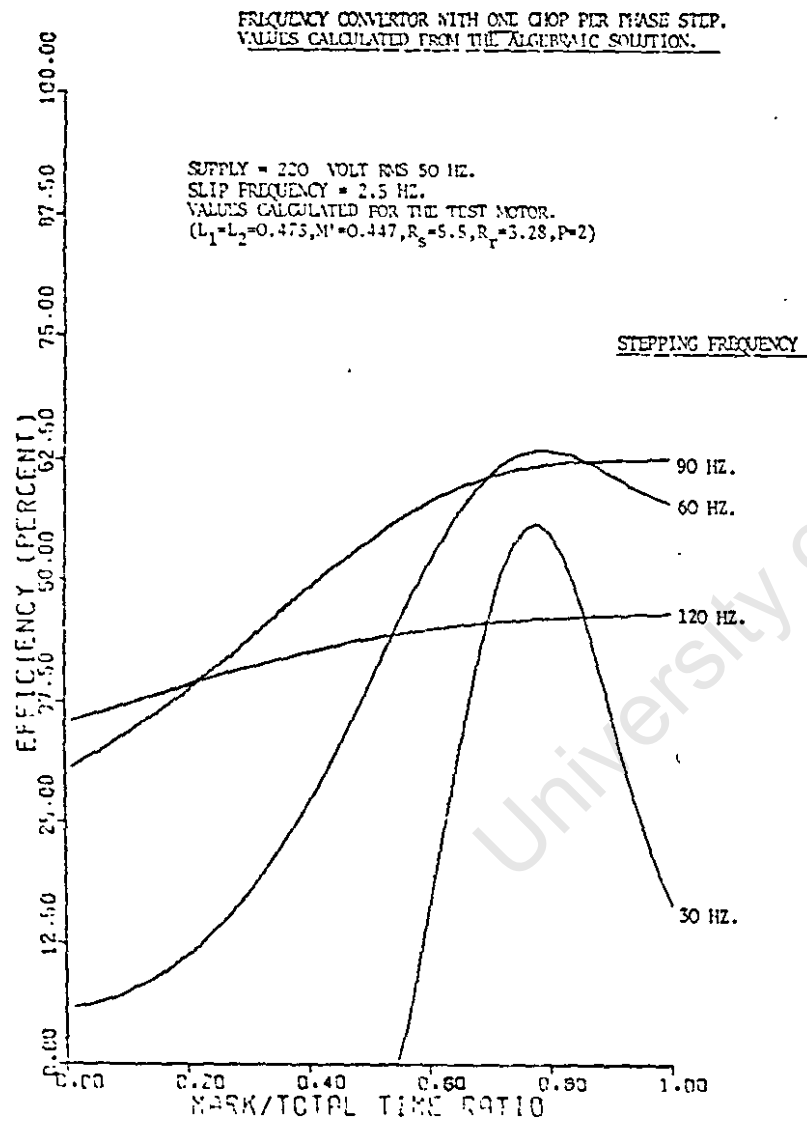


Fig. 22

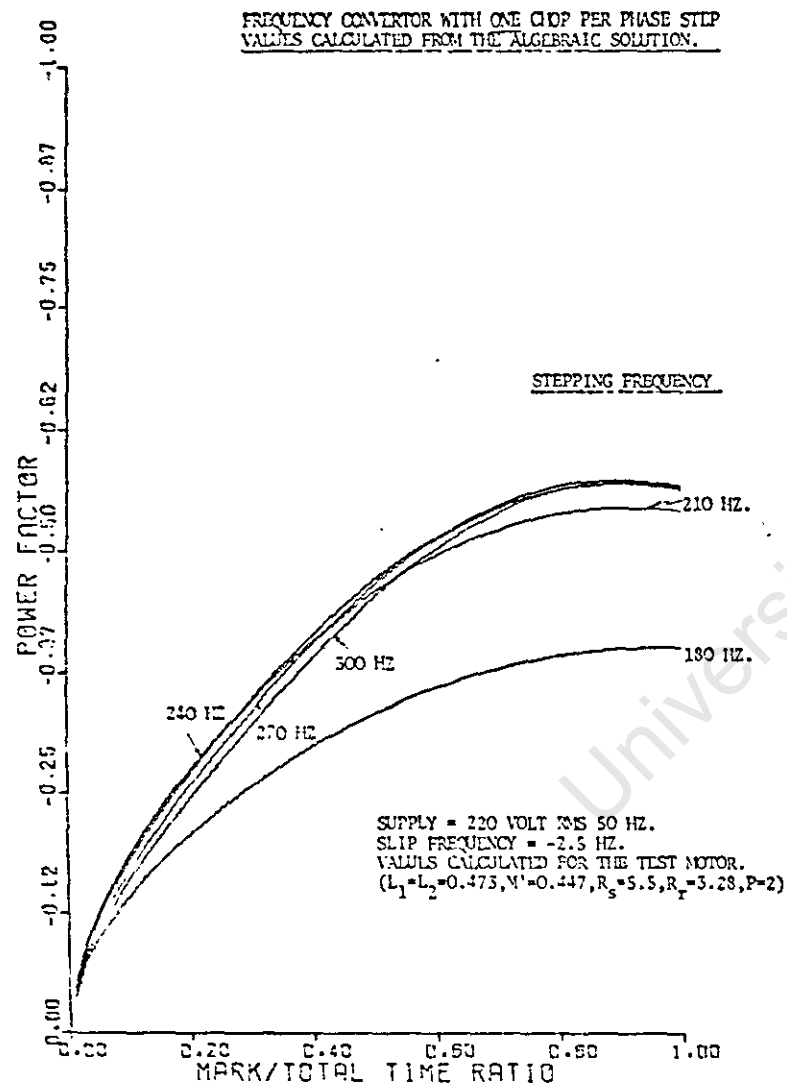
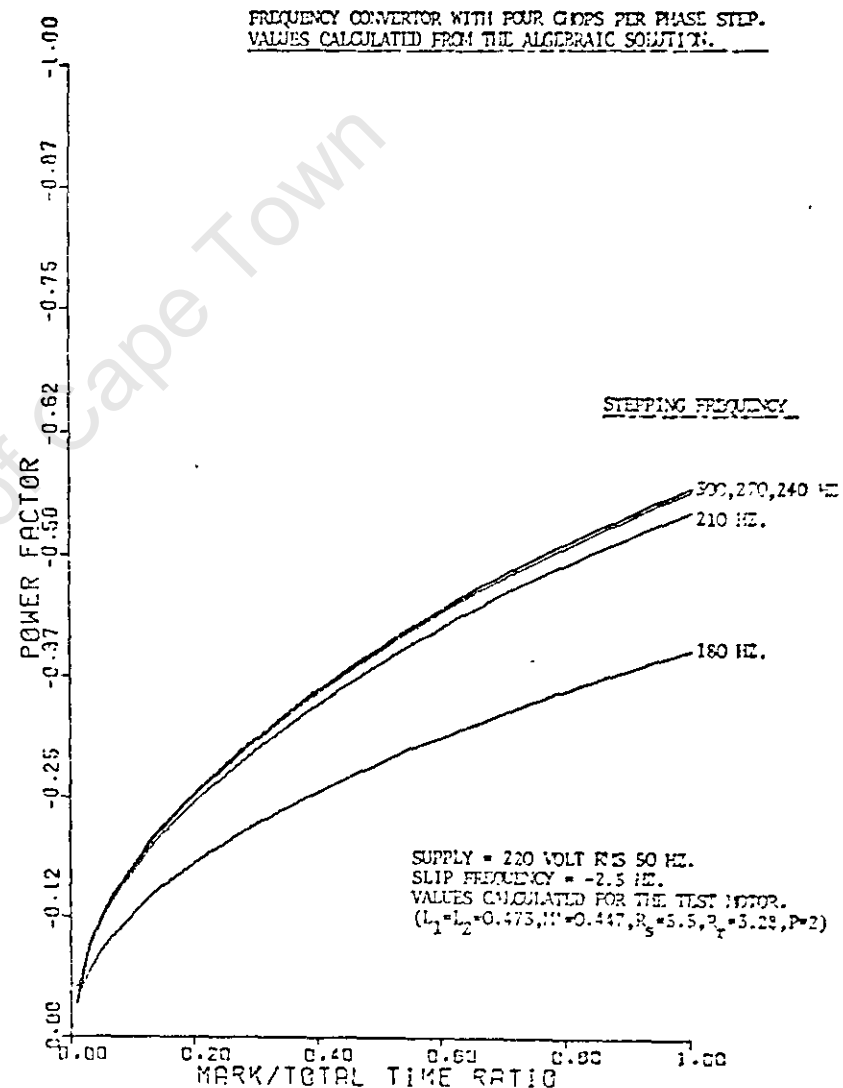


Fig. 99



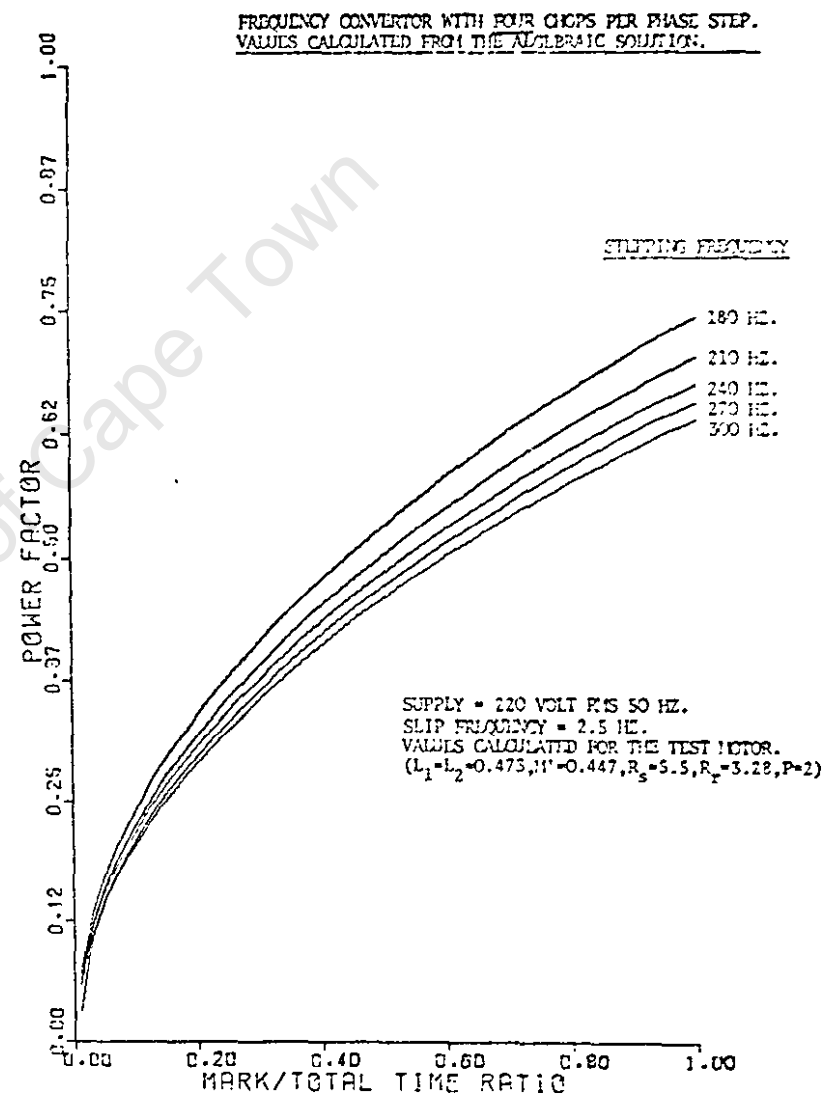
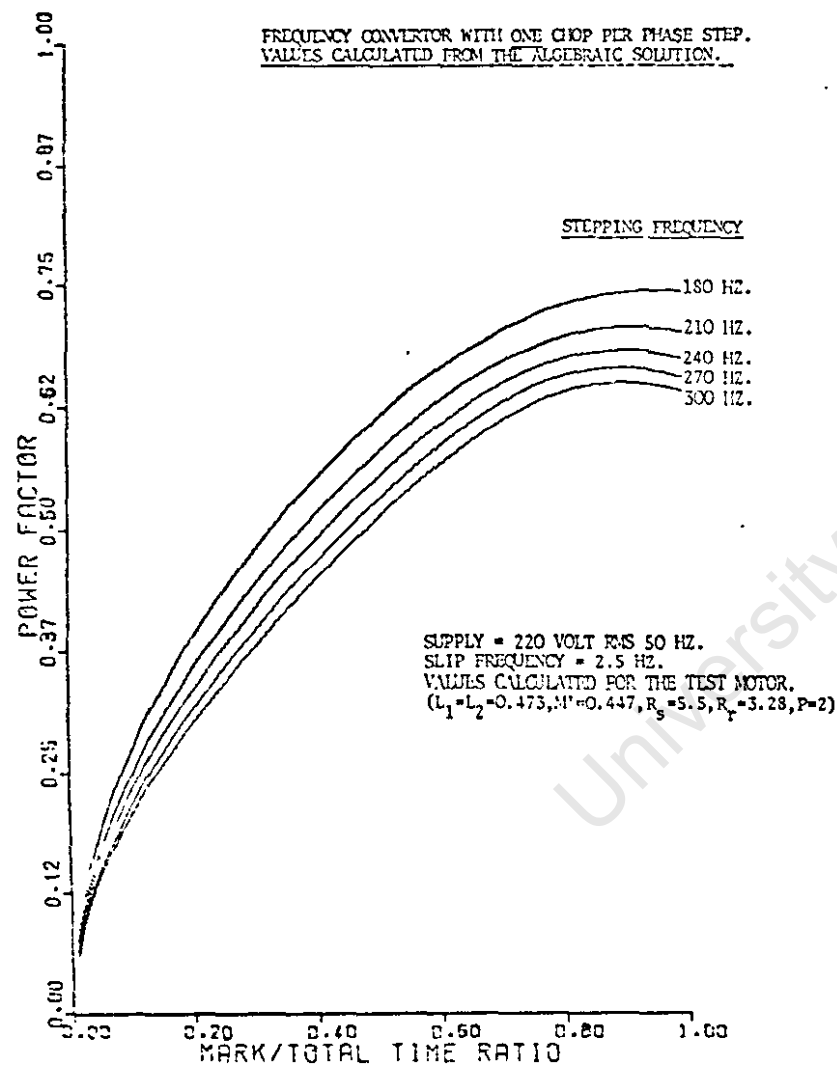


Fig. 100

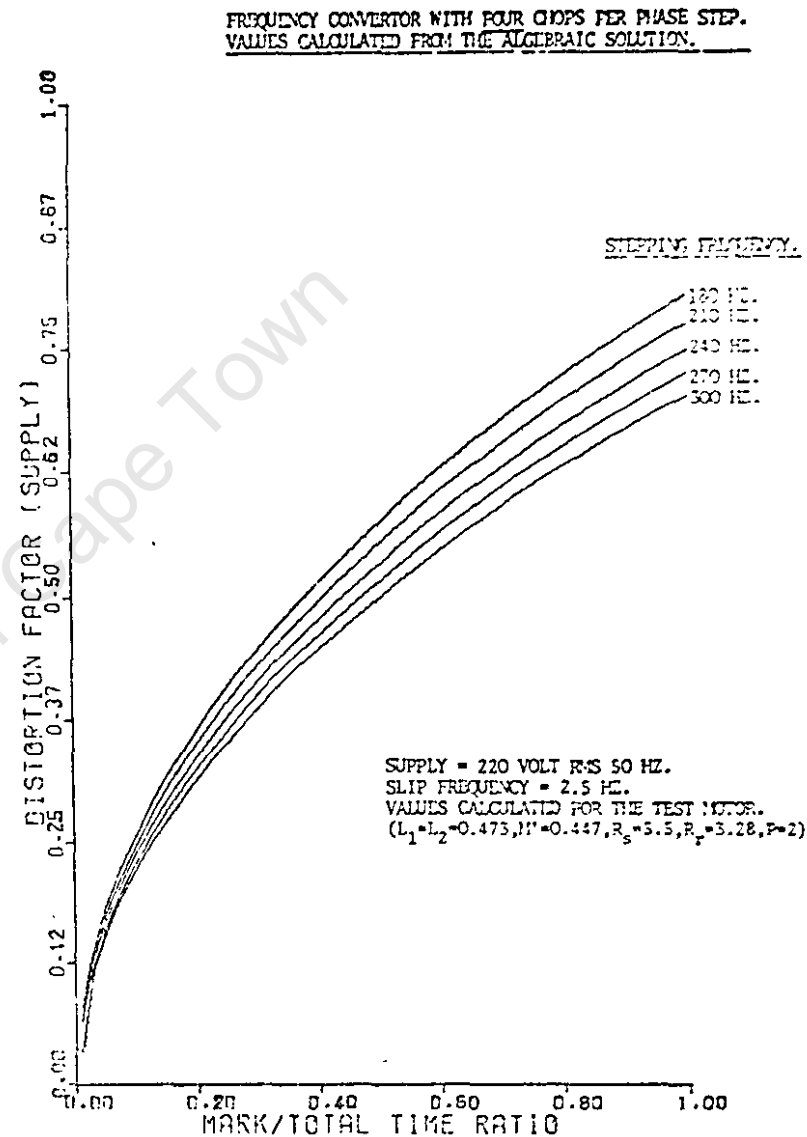
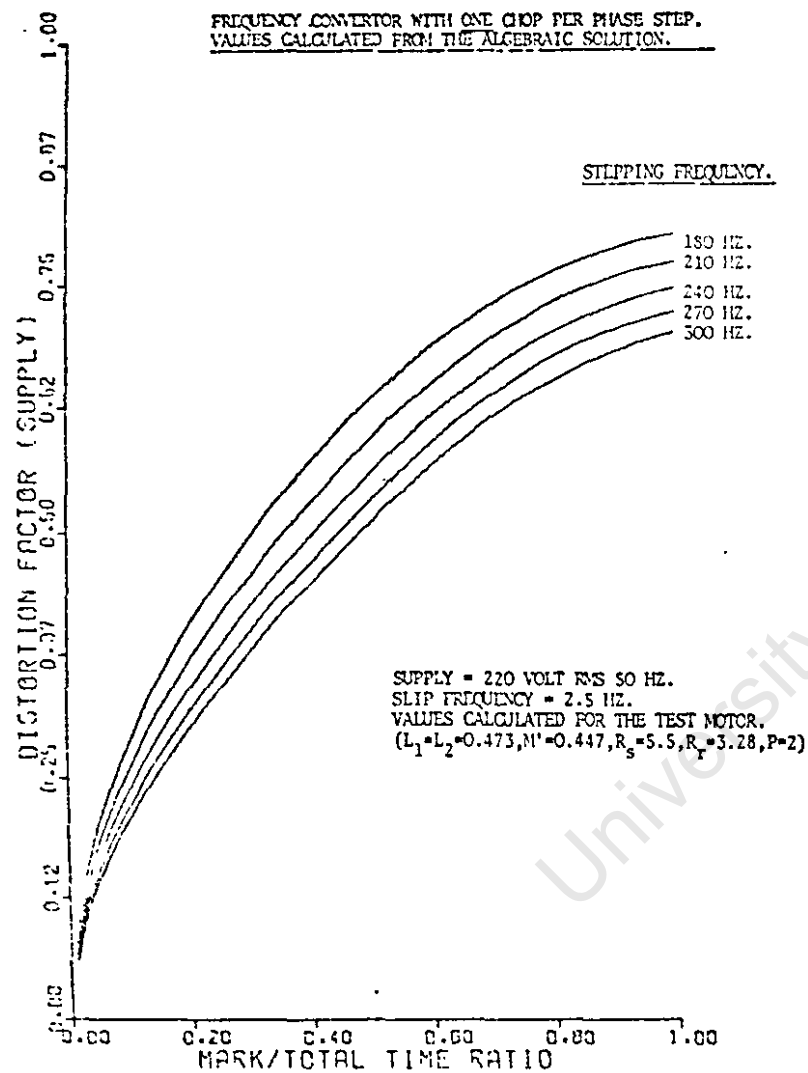
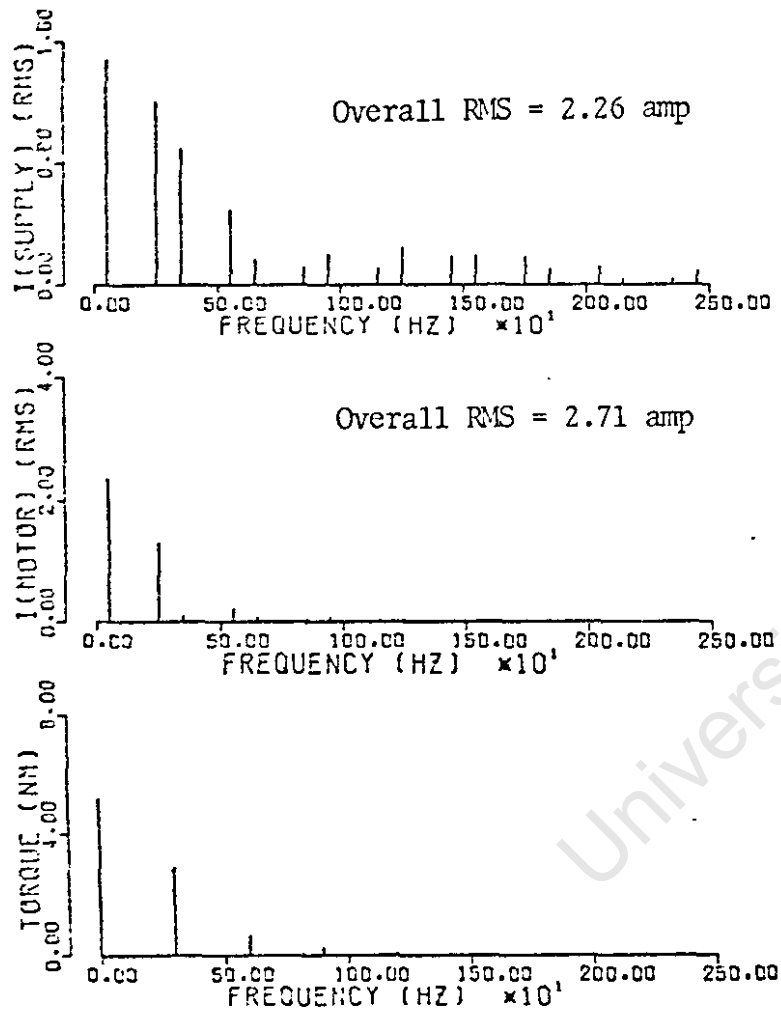


Fig. 102

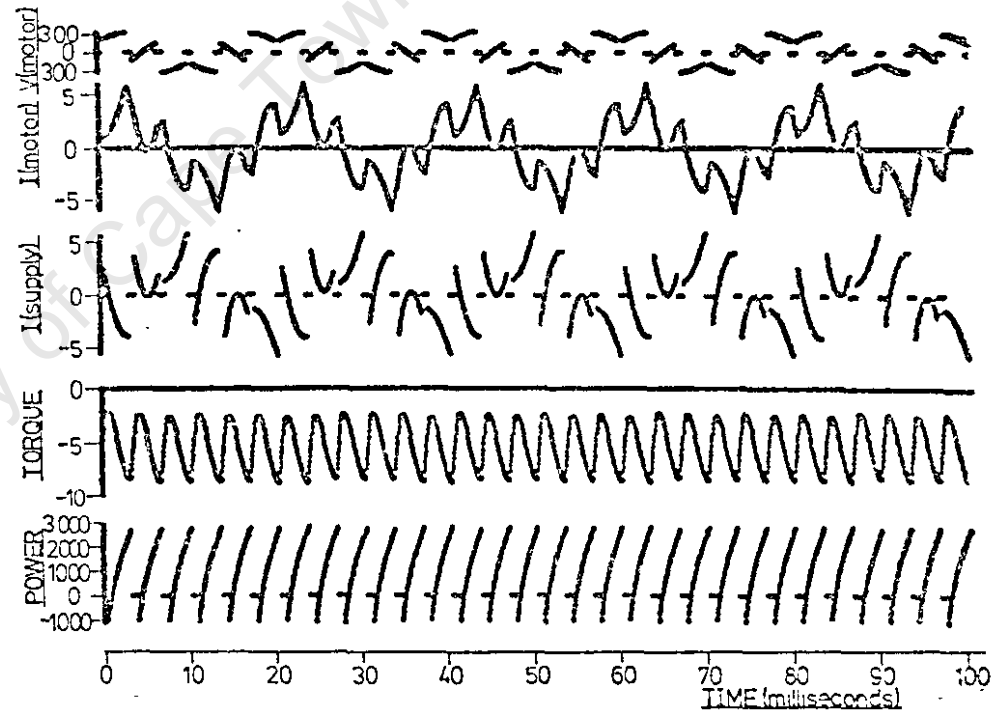
ALGEBRAIC SOLUTION.



SUPPLY=220.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =120.0 DEGREES , F(STEP)=300.0 HZ. , CHOPS/STEP=1.
 T(CN)/T(TOTAL)=0.800 , SPEED=-1425.0 RPM
 R=C.447,L1=L2=0.473,R1=5.50,R2=3.28

Fig. 103 a

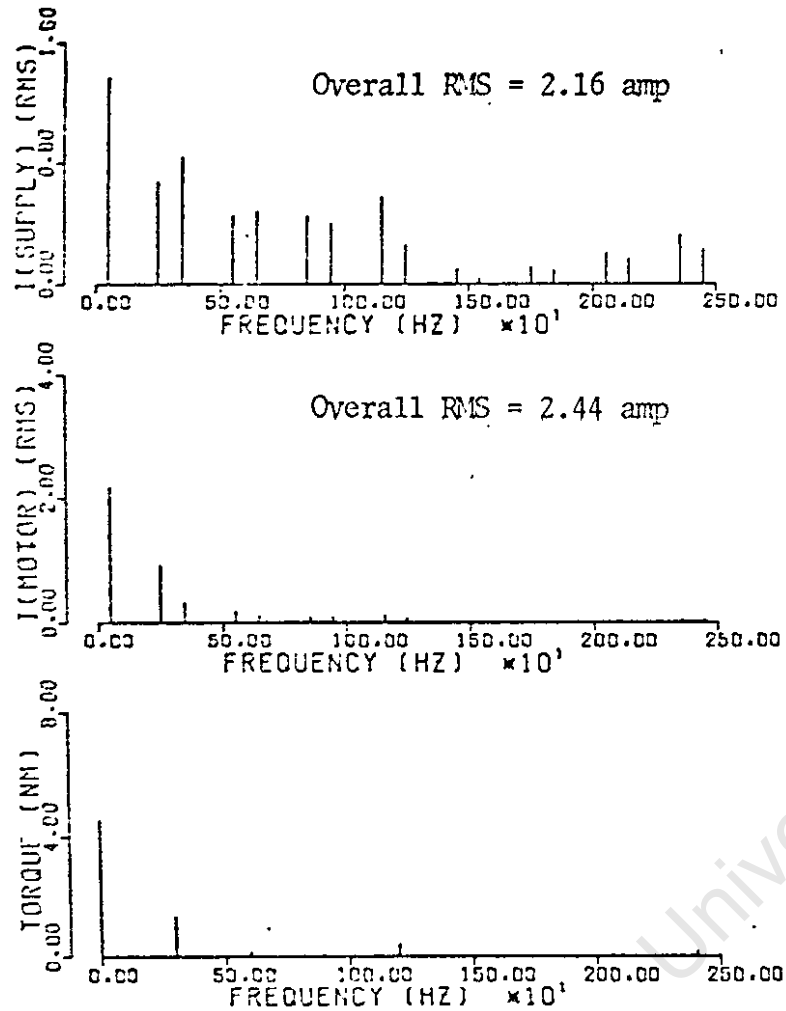
NUMERICAL SIMULATION



Calculated results.

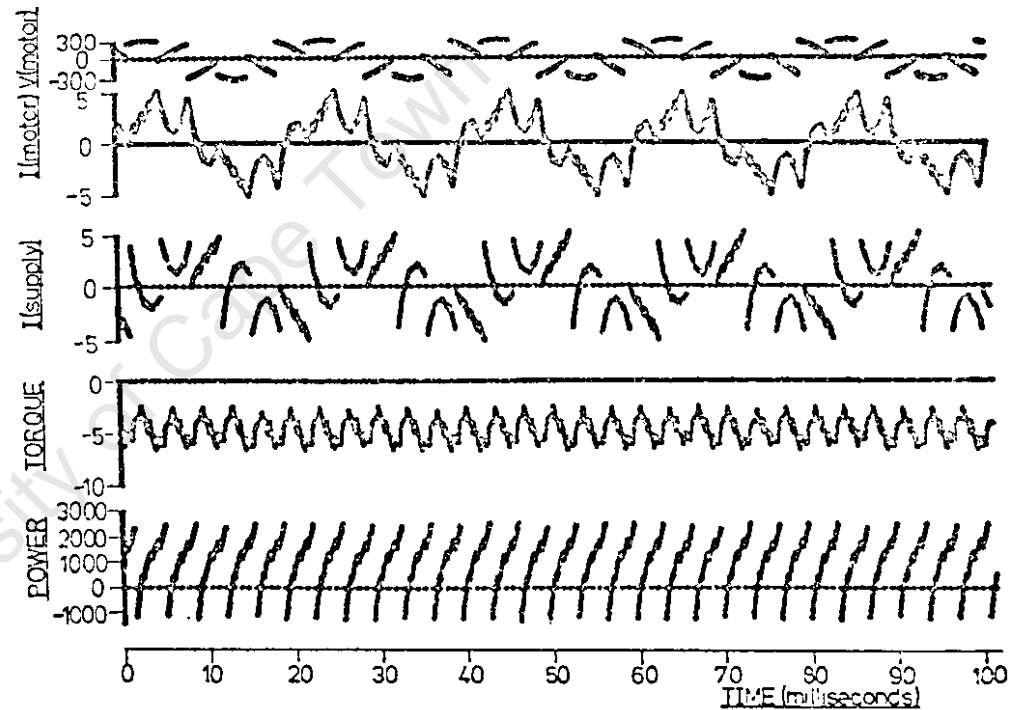
Fundamental convertor output frequency= 50 HZ.

ALGEBRAIC SOLUTION.



SUPPLY:220.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS -120.0 DEGREES , F(STEP)=300.0 HZ. , CHOPS/STEP=4.
T1251/T11TOTAL=0.800 , SPEED=-1425.0 RPM
M=0.447,L1=L2=0.473,R1=5.50,R2=3.28

NUMERICAL SIMULATION.

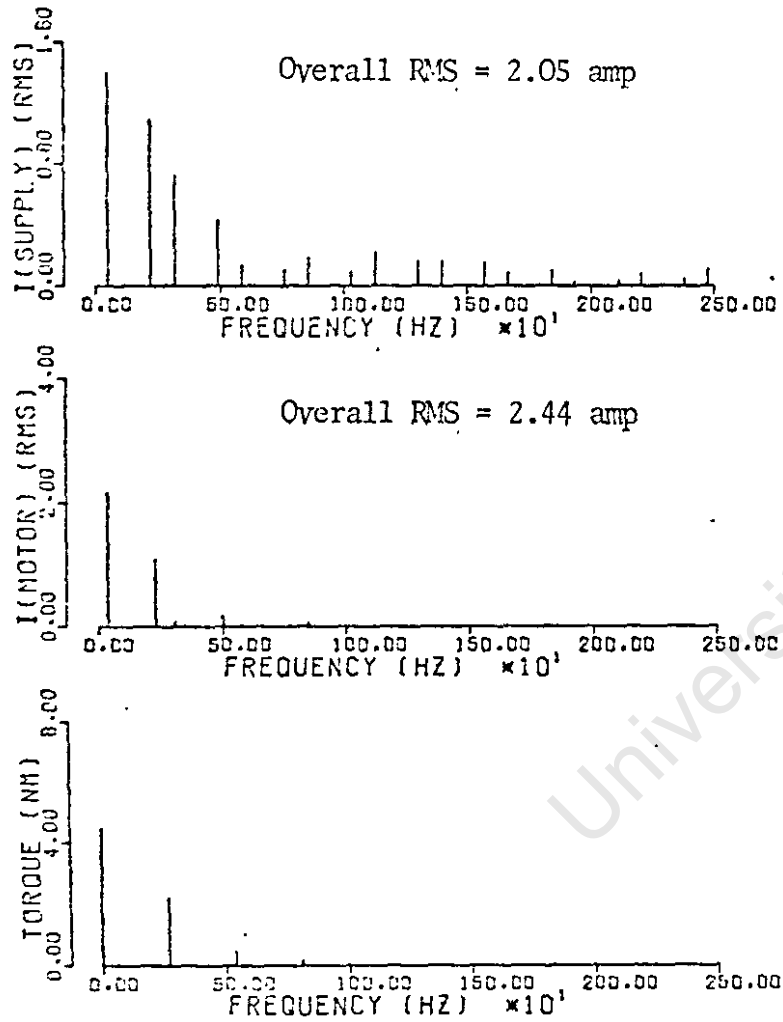


Calculated results.

Fundamental convertor output frequency = 50 HZ.

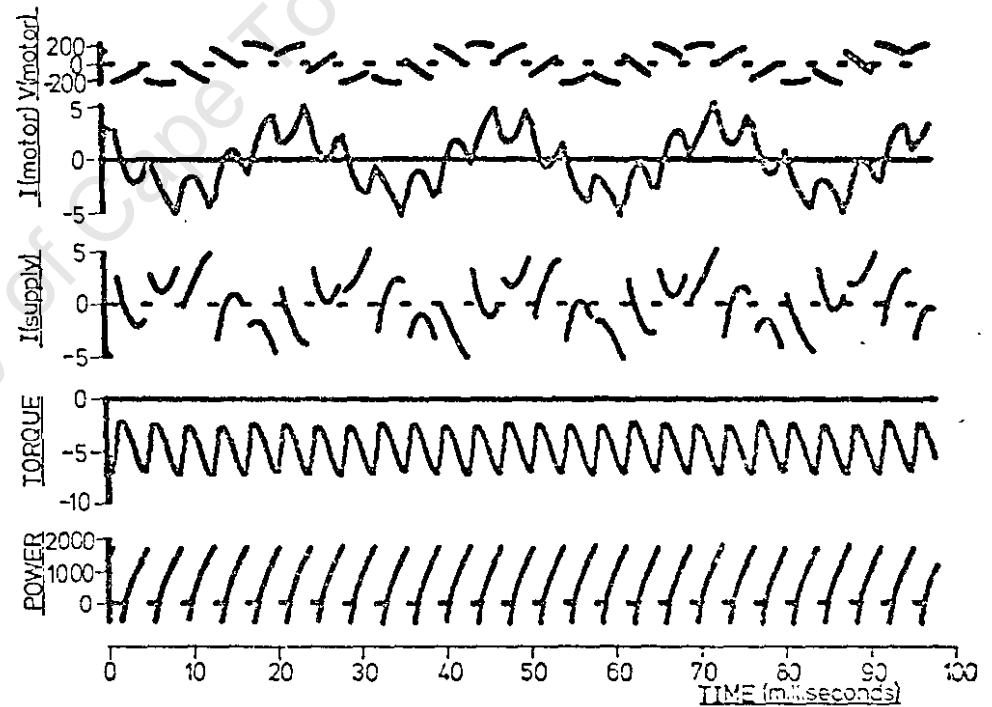
Fig. 103b.

ALGEBRAIC SOLUTION



SUPPLY=166.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =120.0 DEGREES , F(STEP)=270.0 HZ. , CHOP/STEP=1.
 THETA/TOTAL=0.000 , SPEED=-1125.0 RPM
 M=0.447,L1=L2=0.473,P1=0.50,R2=3.28

NUMERICAL SIMULATION

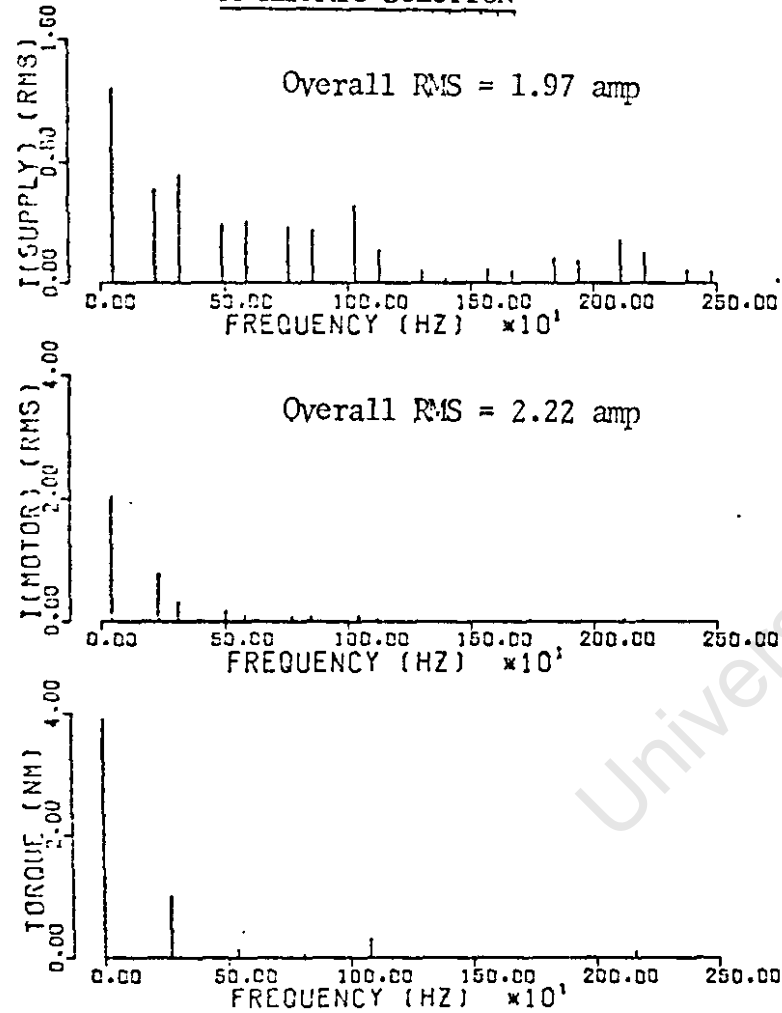


Calculated results.

Fundamental converter output frequency = 40 HZ.

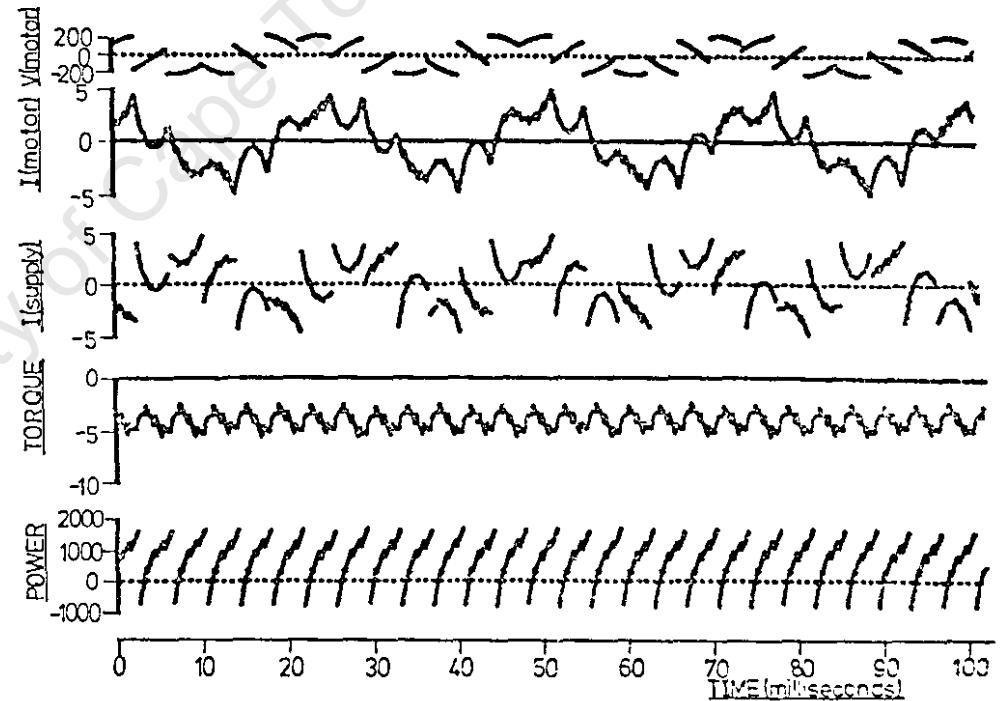
Fig.104a

ALGEBRAIC SOLUTION



SUPPLY=155.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =120.0 DEGREES , F(STEP)=270.0 HZ. , CHOPS/STEP=4.
 T1(24)/T1(TOTAL)=0.000 , SPEED=-1125.0 RPM
 R=0.447,L1=L2=0.473,R1=5.50,R2=3.28

NUMERICAL SIMULATION.



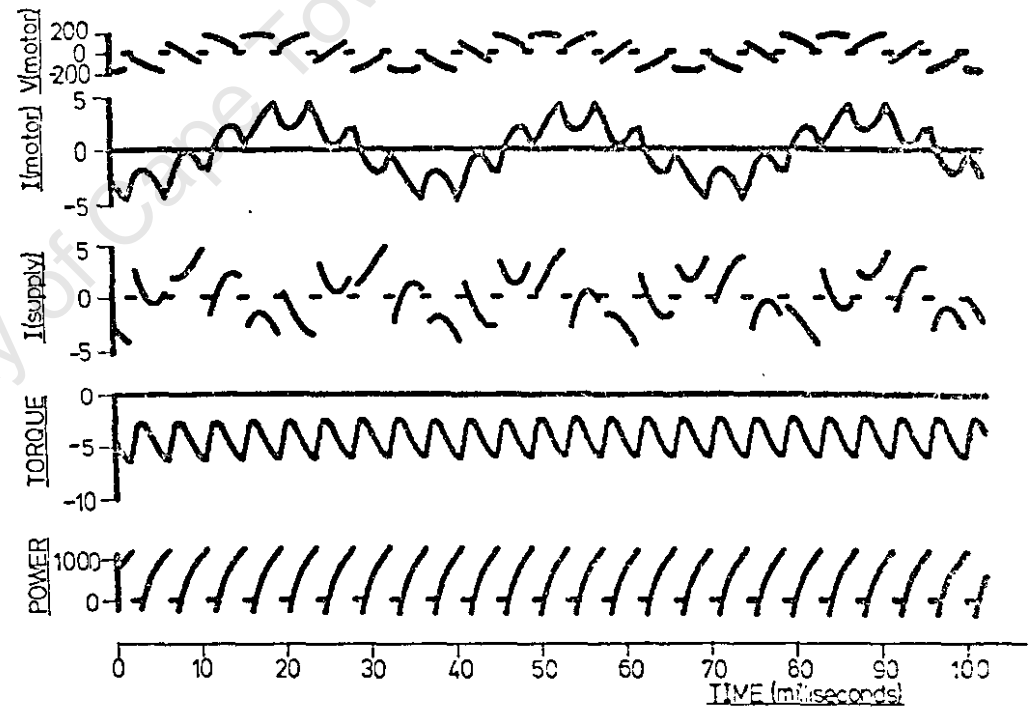
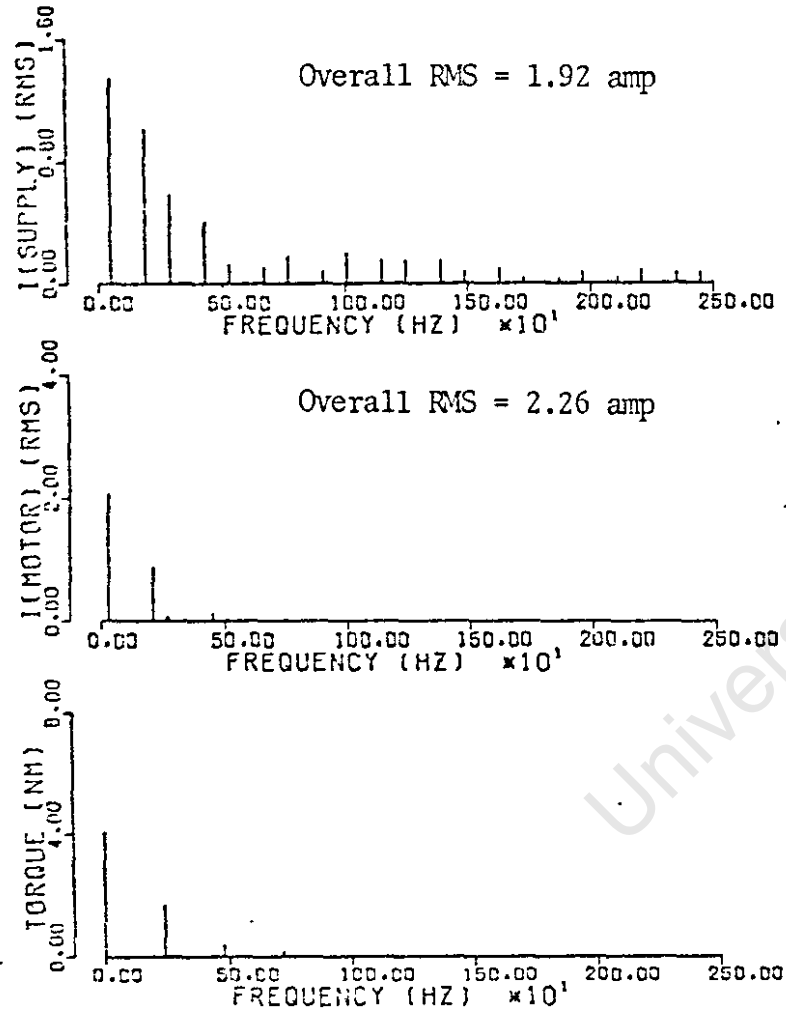
Calculated results.

Fundamental convertor output frequency = 40 HZ.

Fig.104b

ALGEBRAIC SOLUTION

NUMERICAL SIMULATION.



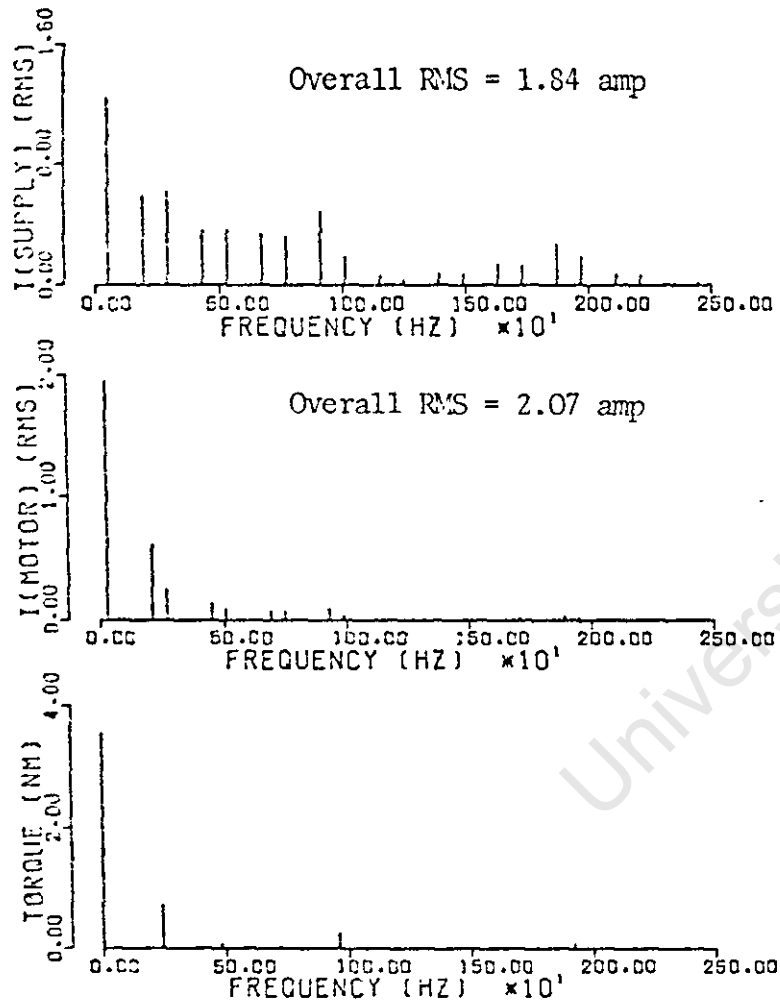
SUPPLY=122.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS -120.0 DEGREES , F(STEP)=240.0 HZ. , CHOPS/STEP=1.
 T(CN)/T(TOTAL)=0.800 , SPEED=-825.0 RPM
 M=0.447,L1=L2=0.475,R1=5.50,R2=3.28

Calculated results.

Fundamental converter output frequency = 30 HZ.

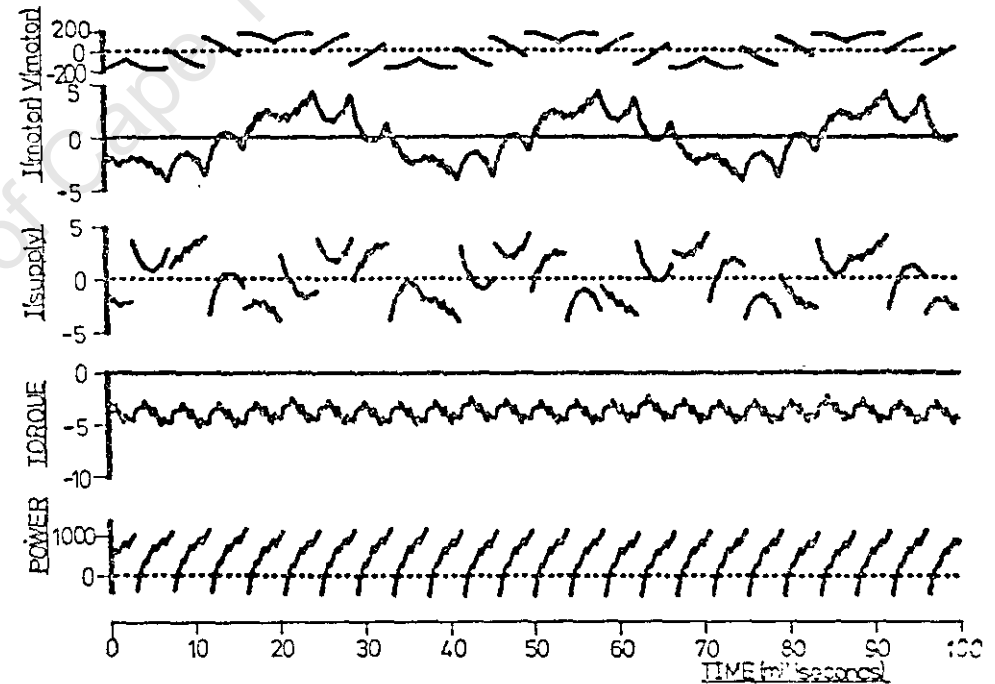
Fig.105a

ALGEBRAIC SOLUTION.



SUPPLY=122.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS =120.0 DEGREES , F(STEP)=240.0 HZ. , CHOPS/STEP=4.
 T1CN1/T1T2PL1=0.900 , SPEED=825.0 RPM
 R=0.447,L1=0.473,R1=5.50,R2=3.28

NUMERICAL SIMULATION.



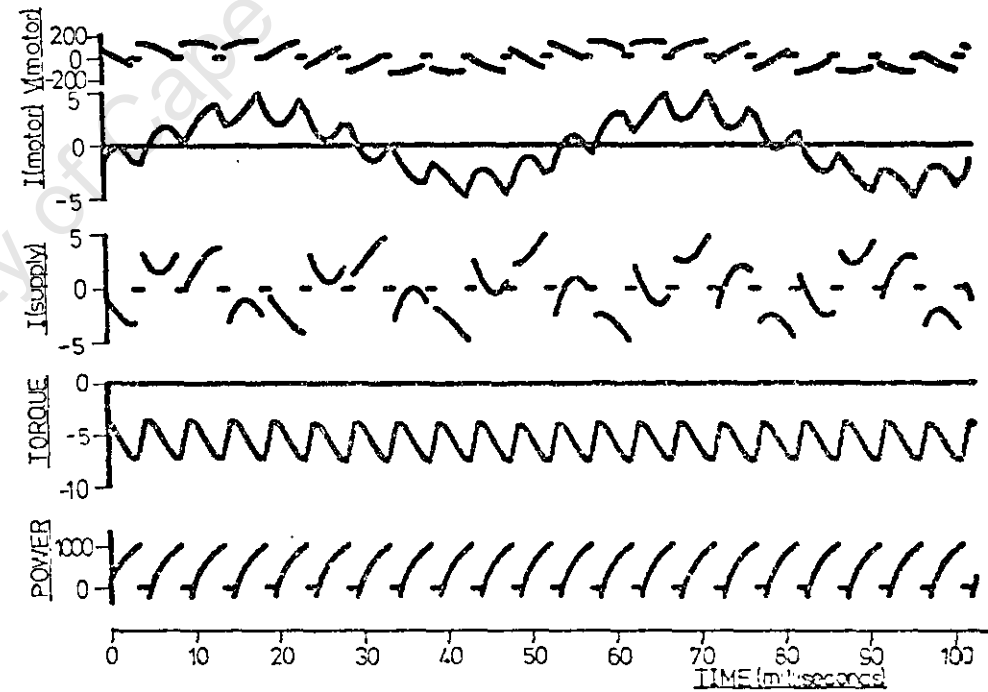
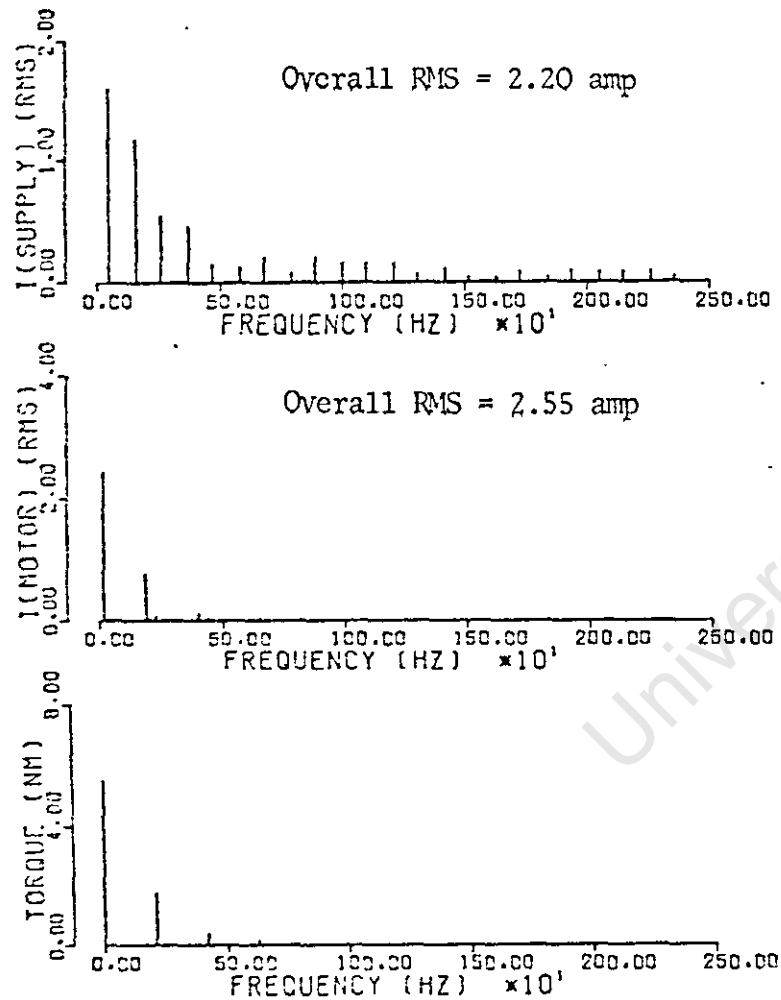
Calculated results.

Fundamental convertor output frequency = 30 HZ.

Fig.105b

ALGEBRAIC SOLUTION.

NUMERICAL SIMULATION.



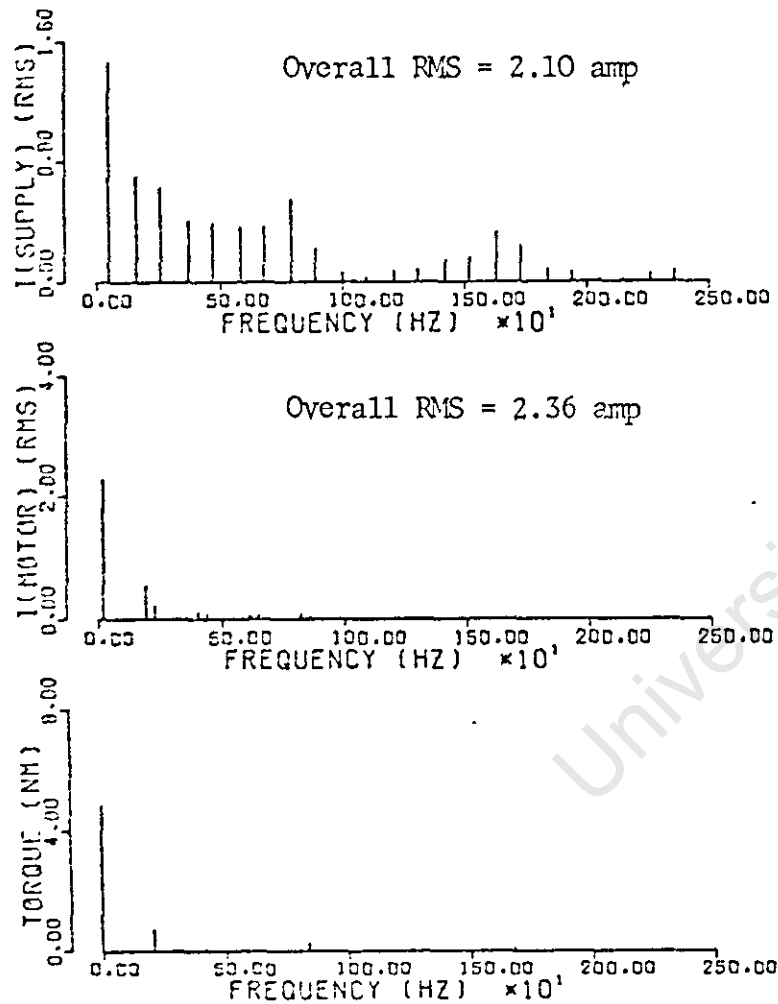
SUPPLY:120.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS -120.0 DEGREES , F1STEP1=210.0 HZ. , CHOPS/STEP=1.
 T1ON/T1TOTAL=0.800 , SPEED1=525.0 RPM
 M=0.447,L1=120.473,R1=5.50,R2=3.28

Calculated results.

Fundamental convertor output frequency = 20 HZ.

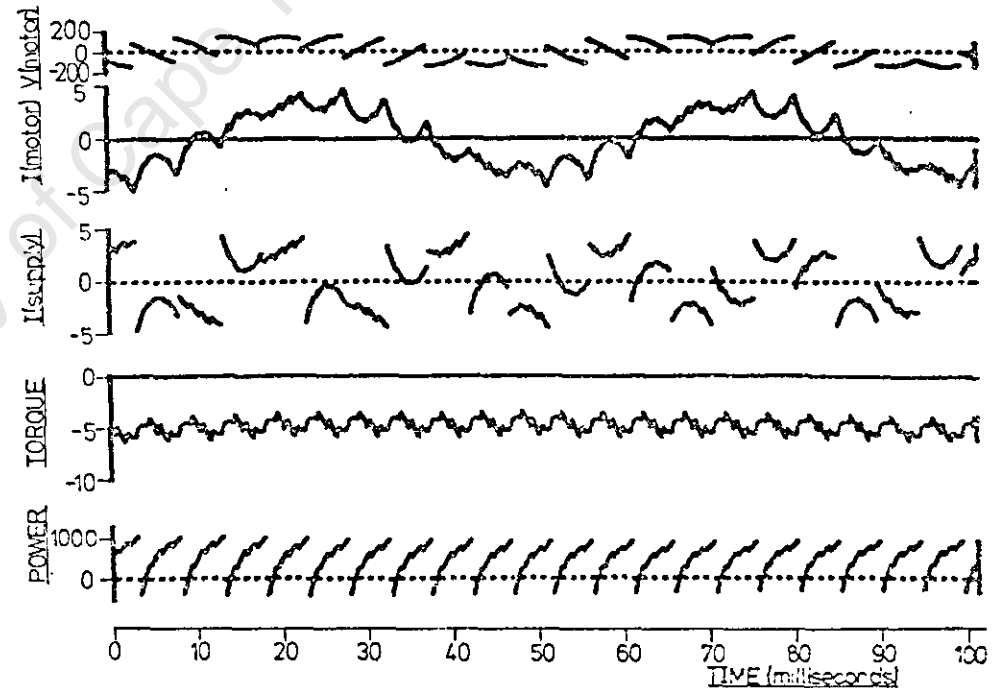
Fig.106a

ALGEBRAIC SOLUTION.



SUPPLY=100.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS =120.0 DEGREES , F(STEP)=210.0 HZ. , CHOPS/STEP=4.
 T1CN1/T1(TOTAL)=0.900 , SPEED=-525.0 RPM
 R=0.447,L1=12.0,473.R1=5.50,R2=3.28

NUMERICAL SIMULATION.

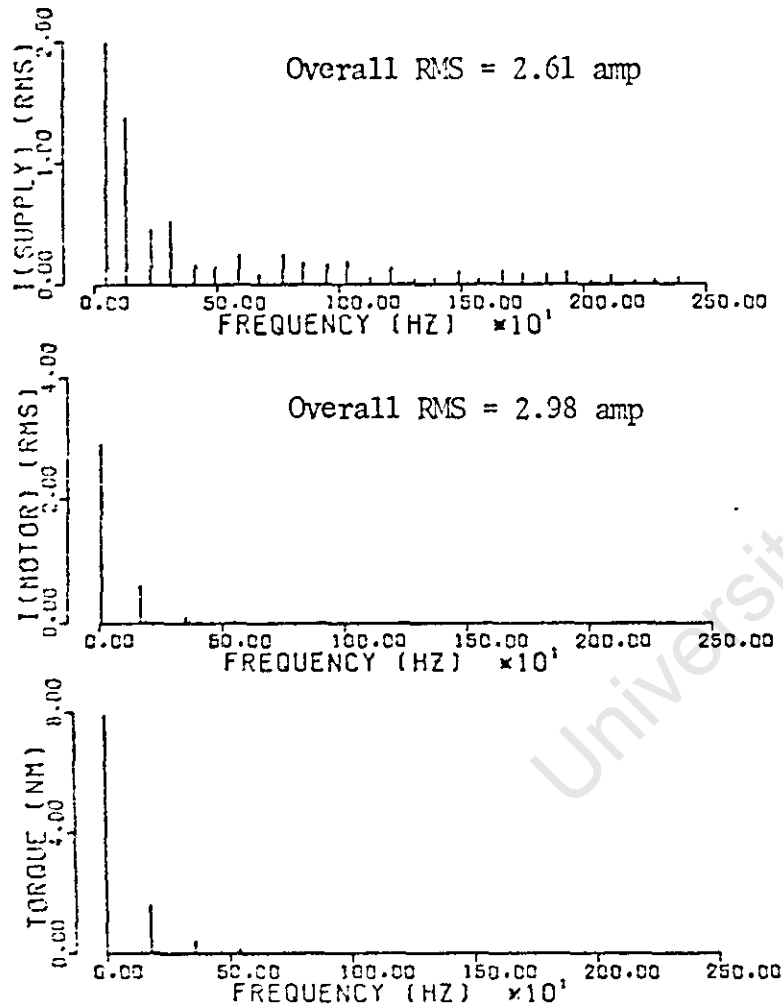


Calculated results.

Fundamental convertor output frequency = 20 HZ.

Fig. 106b

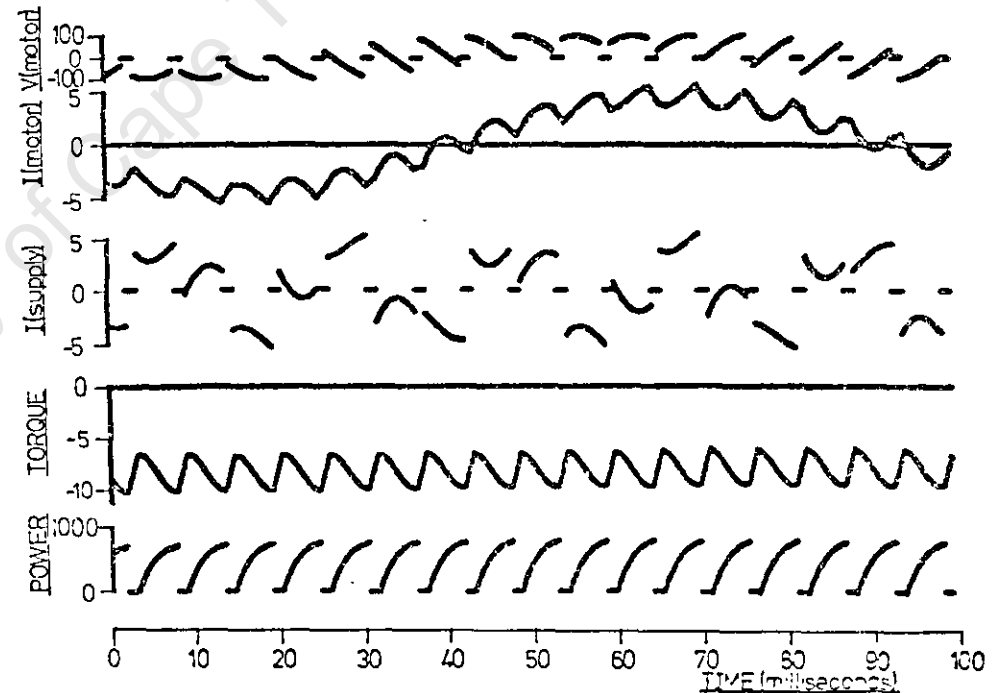
ALGEBRAIC SOLUTION.



SUPPLY=70.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS =120.0 DEGREES , F(STEP)=100.0 HZ. , CHOPS/STEP=1.
 T1CN1/T1TOTAL=0.800 , CPFLD=-225.0 RPM
 M=0.447,L1=L2=0.473,R1=5.50,R2=3.28

Fig.107a

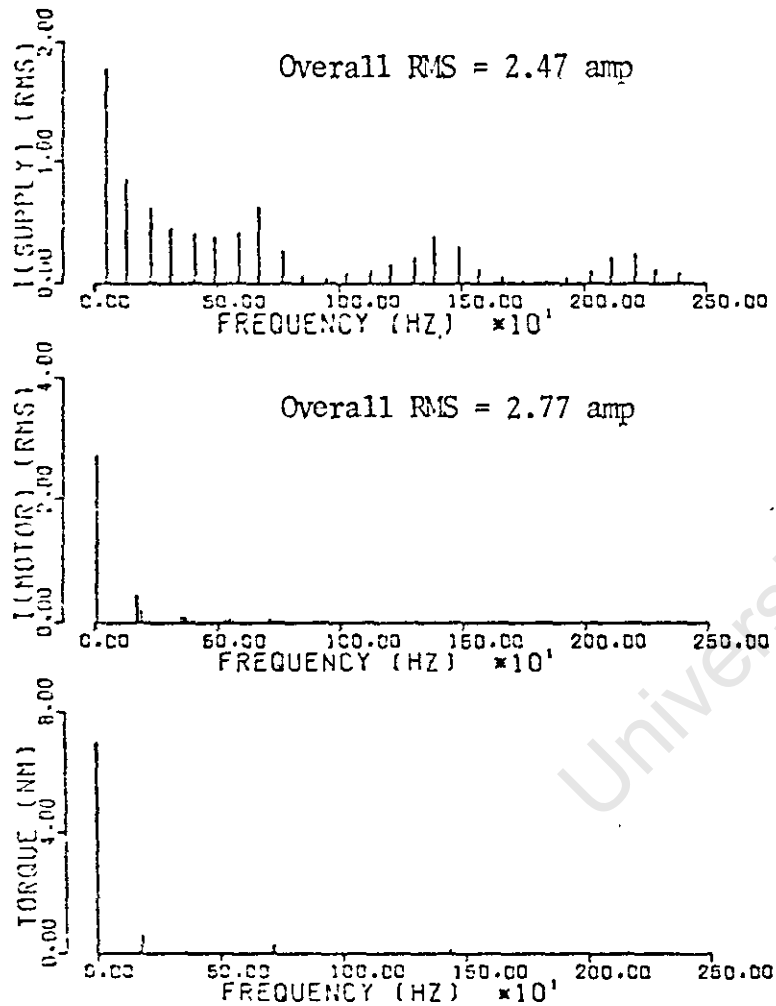
NUMERICAL SIMULATION.



Calculated results.

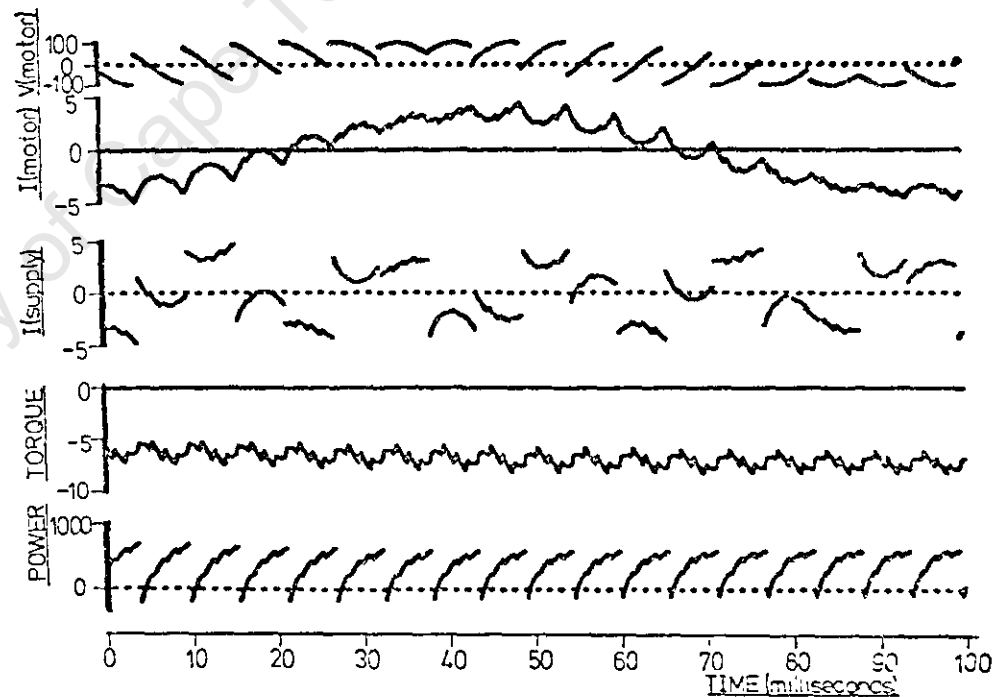
Fundamental converter output frequency = 10 HZ.

ALGEBRAIC SOLUTION.



SUPPLY=70.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =120.0 DEGREES , F(STEP)=180.0 HZ. , CHOPS/STEP=4.
 T1CH1/T1TOTAL=0.800 , SPEED=-225.0 RPM
 R1=0.447,L1=L2=0.473,R2=5.50,R3=3.28

NUMERICAL SIMULATION.

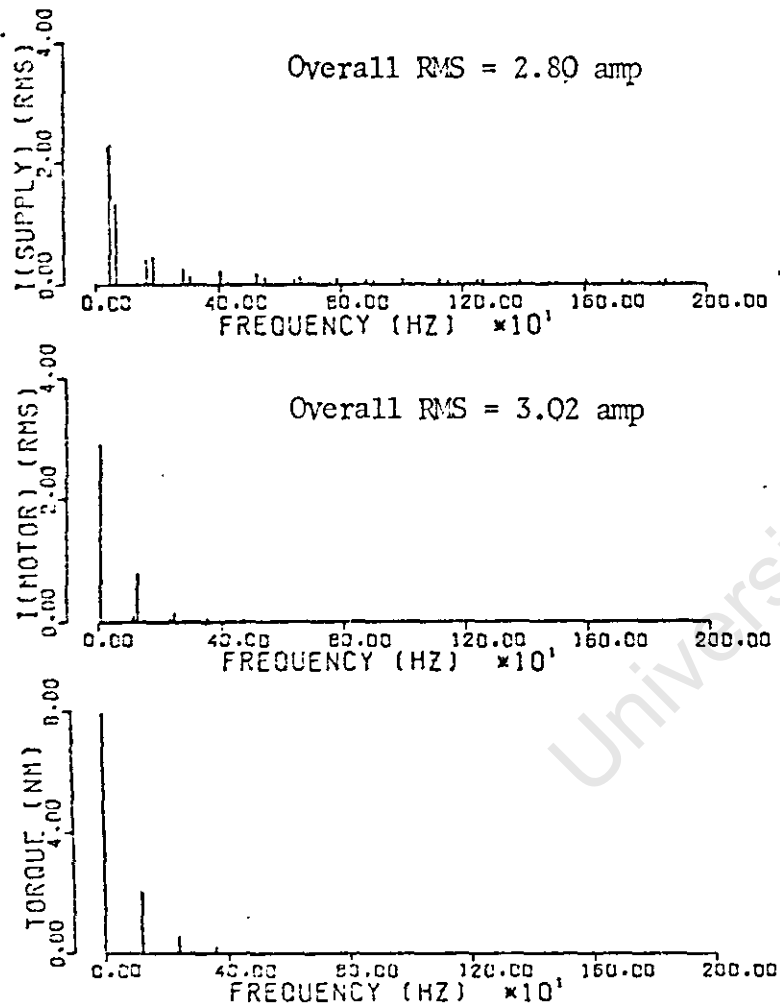


Calculated results.

Fundamental convertor output frequency = 10 HZ.

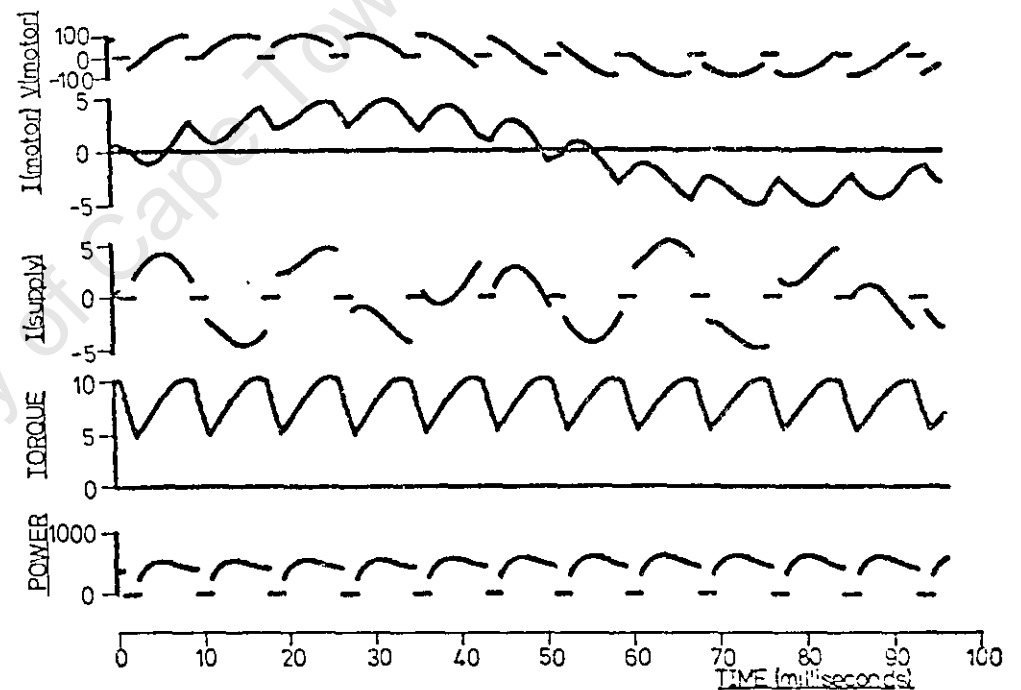
Fig.107b

ALGEBRAIC SOLUTION.



SUPPLY=70.0 VOLT RMS , 50.HZ.
 CONVERTOR STEPS =120.0 DEGREES , F1STEP1=120.0 HZ. , CHOPS/STEP=1.
 T1C11/T1:TOTAL=0.600 , SPEED=225.0 RPM
 M=0.447,L1=12.0,R1=5.50,R2=3.28

NUMERICAL SIMULATION.

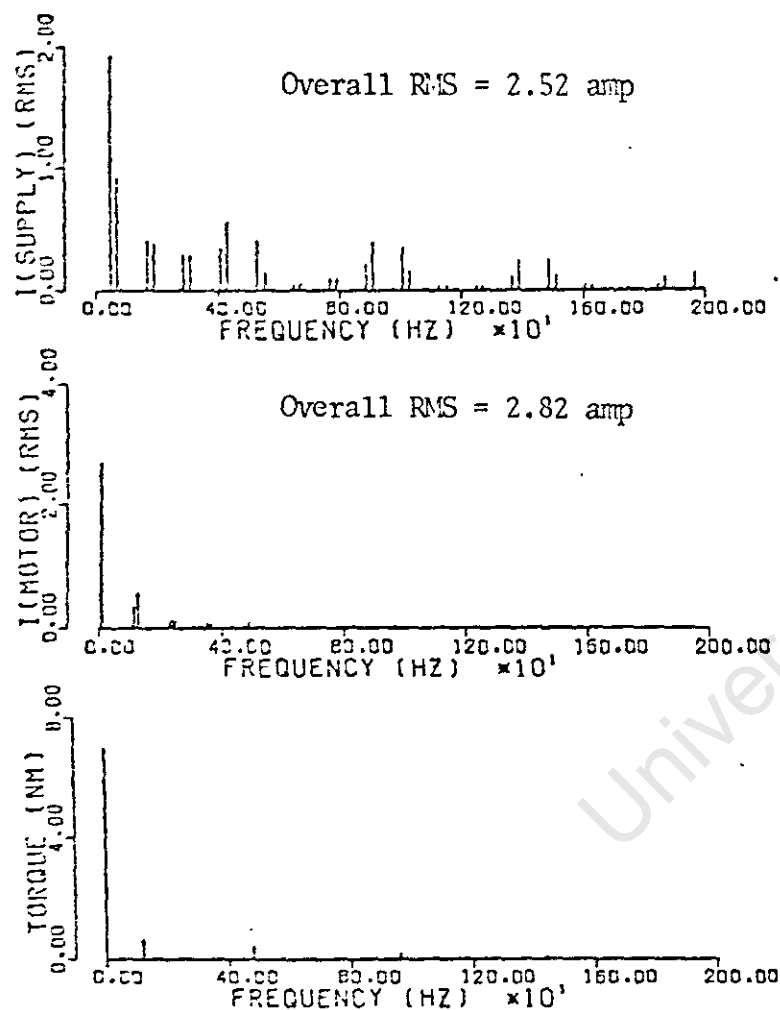


Calculated results.

Fundamental convertor output frequency = 10 HZ.

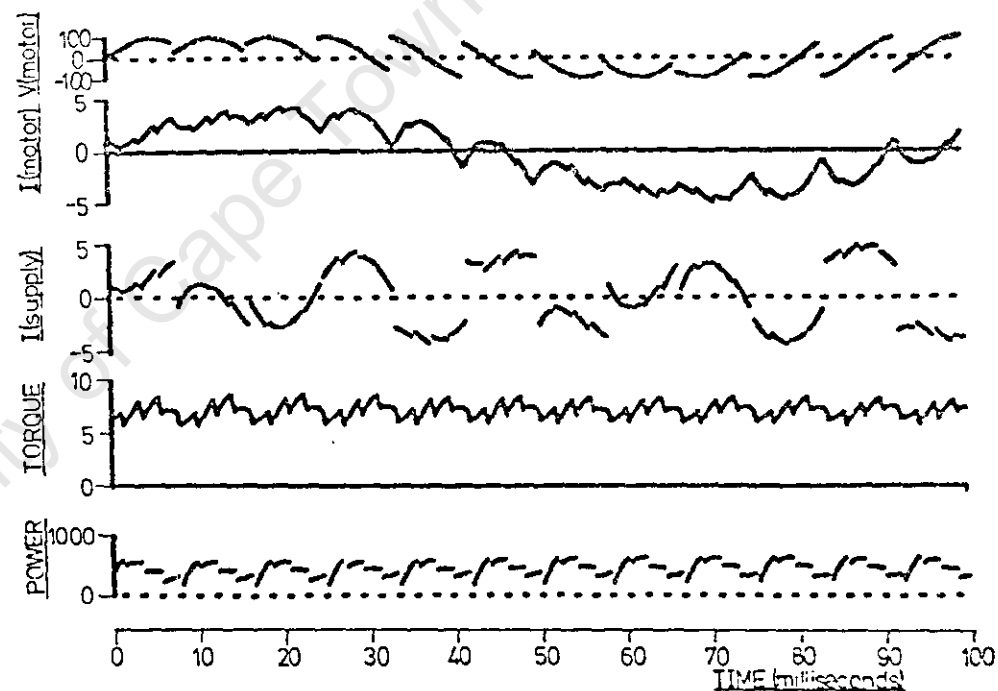
Fig.108a

ALGEBRAIC SOLUTION.



SUPPLY=70.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS =120.0 DEGREES , F(STEP)=120.0 HZ. , CHOP5/STEP=4.
TIC41/T(TOTAL)=0.900 , SPEED=225.0 RPM
R=0.447,L1=L2=0.473,R1=5.50,R2=3.28

NUMERICAL SIMULATION.

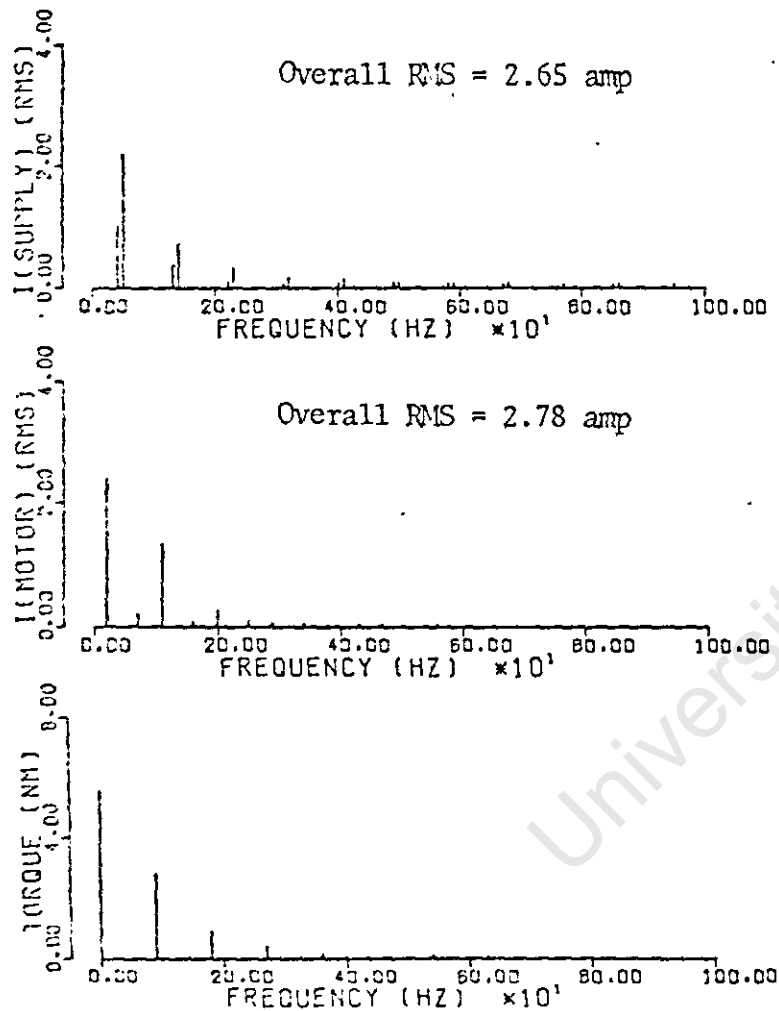


Calculated results.

Fundamental convertor output frequency = 10 HZ.

Fig.108b

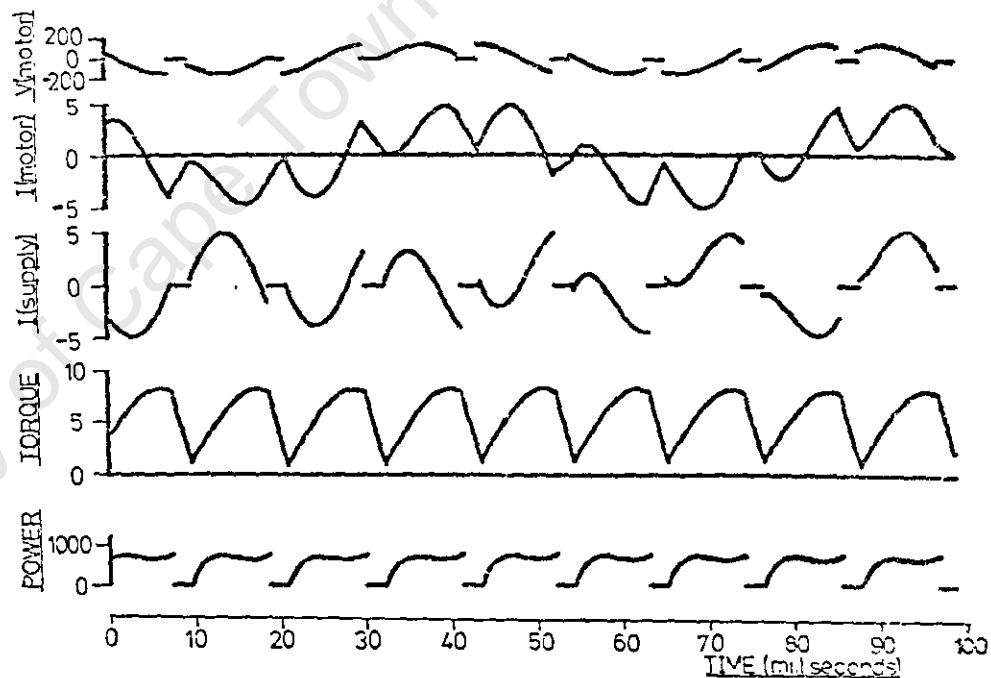
ALGEBRAIC SOLUTION.



SUPPLY=100.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =120.0 DEGREES , FSTEP=50.0 HZ. , CHOPS/STEP=1.
 T1ON/T1OFF=1.0/0.000 , SPEED=525.0 RPM
 R=0.447,L1=12.0,473,R1=5.50,R2=3.28

Fig.103a

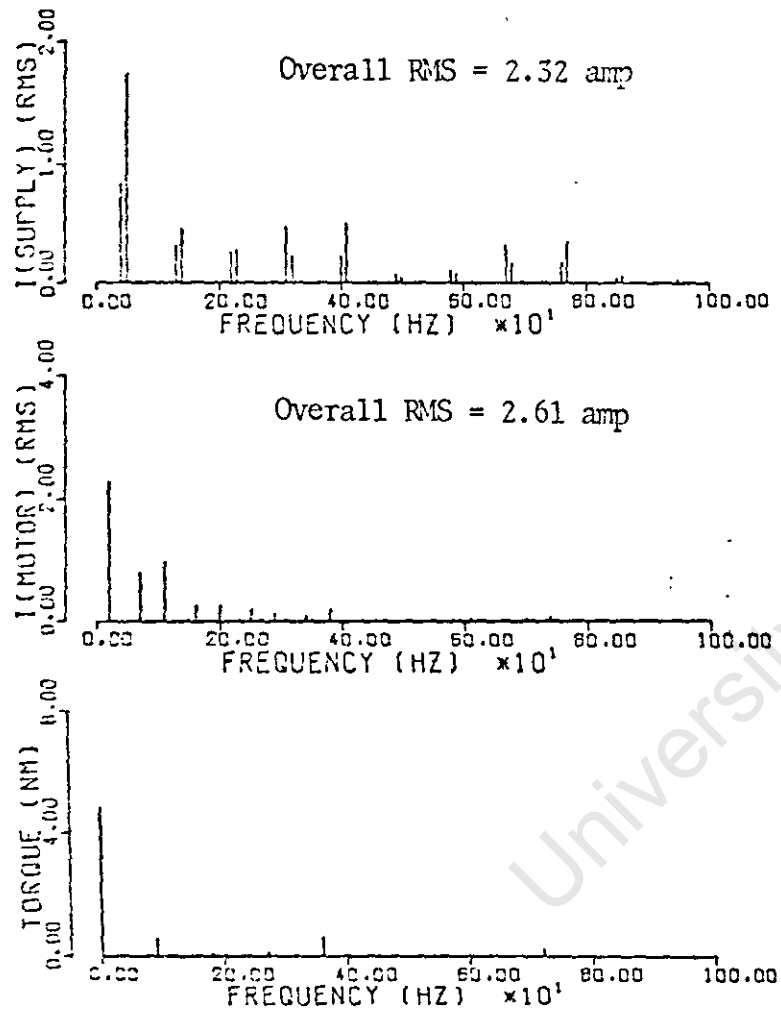
NUMERICAL SIMULATION.



Calculated results.

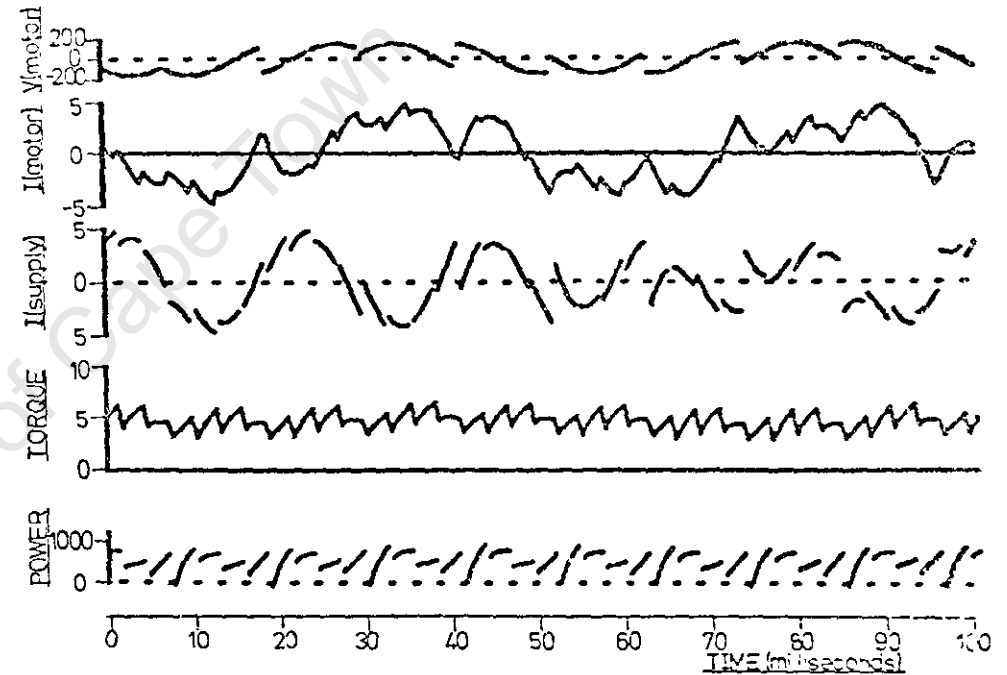
Fundamental convertor output frequency = 20 HZ.

ALGEBRAIC SOLUTION.



SUPPLY=100.0 VOLT RMS , 50.HZ.
 CONVERTER STEPS =120.0 DEGREES , F(STEP)=90.0 HZ. , CHOPS/STEP=4.
 T(DN)/T(TOTAL)=0.000 , SPEED=525.0 RPM
 X=2.447,L1=L2=0.473,R1=5.50,R2=3.28

NUMERICAL SIMULATION.

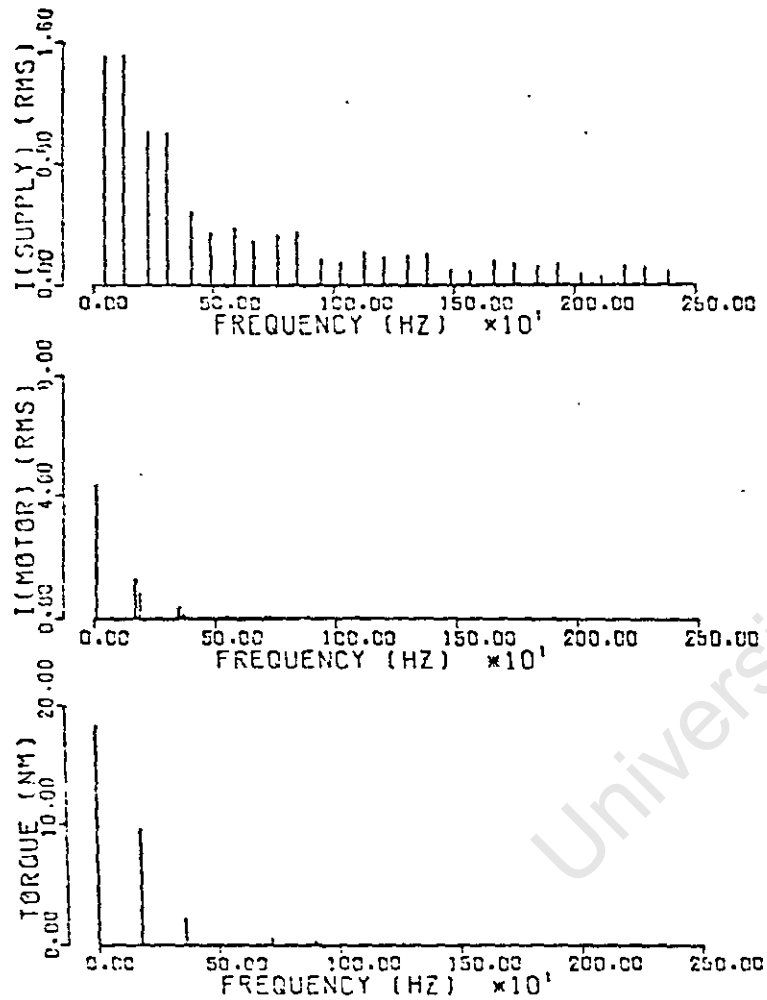


Calculated results.

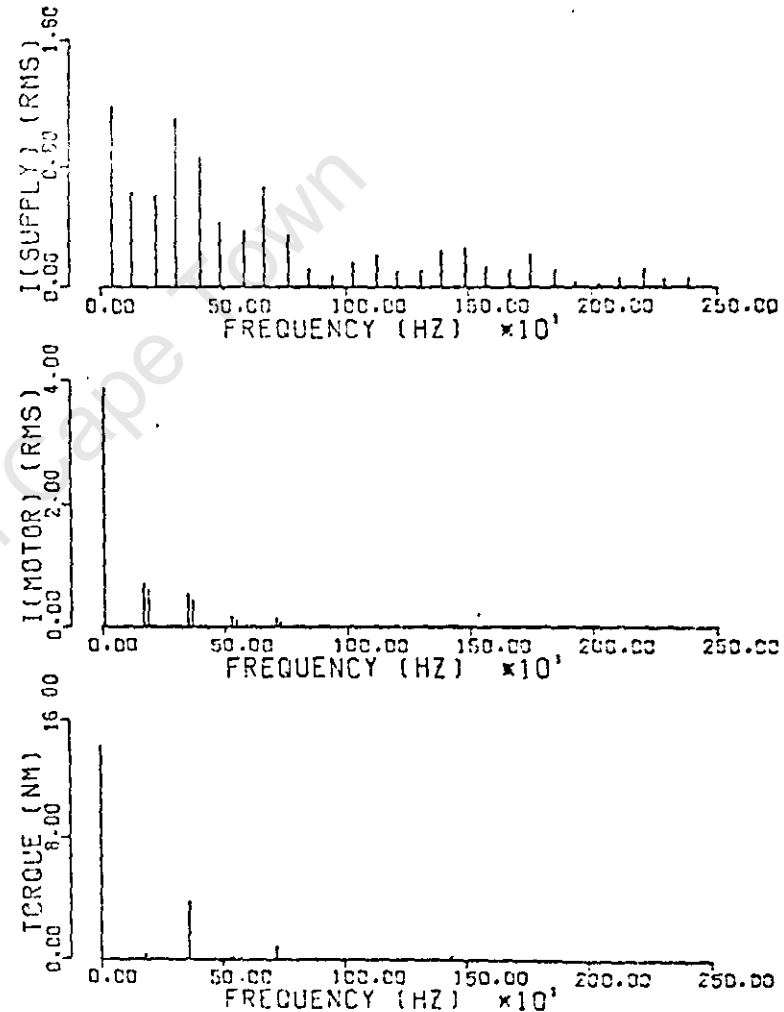
Fundamental convertor output frequency = 20 HZ.

Fig.103b

SPECTRA CALCULATED FROM THE ALGEBRAIC FORMULATION OF THE MOTOR EQUATION SOLUTION.
ILLUSTRATION OF THE EFFECT OF INCREASING THE NUMBER OF CHOPS PER STEP
IN THE FREQUENCY CONVERTOR.



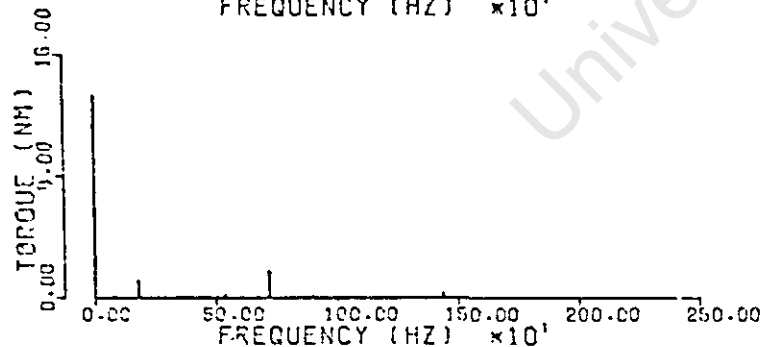
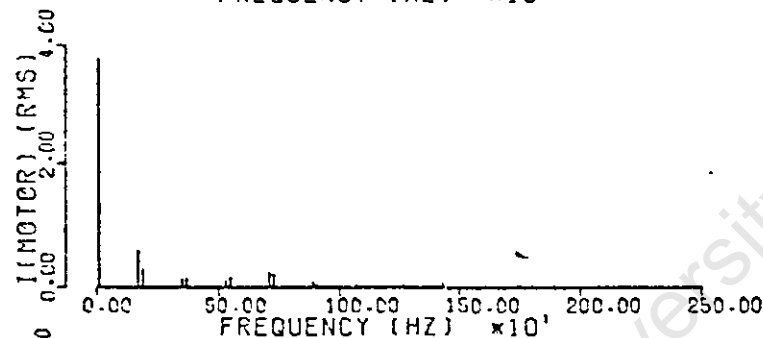
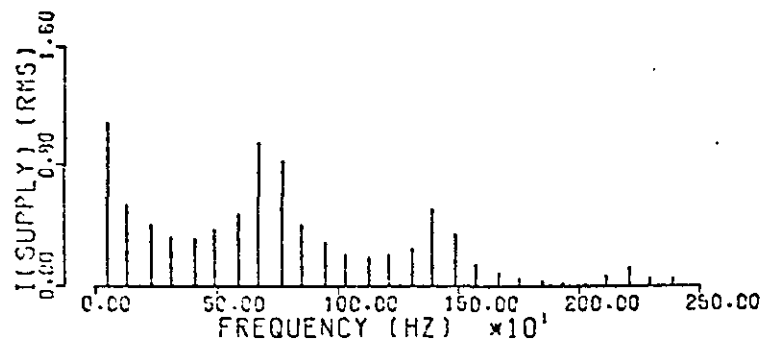
SUPPLY=220.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS -120.0 DEGREES , F(STEP)=180.0 HZ. , CHOPS/STEP=1.
T(ON)/T(TOTAL)=0.350 , SPEED=-225.0 RPM
M=0.447,L1=L2=0.473,R1=5.50,R2=3.28



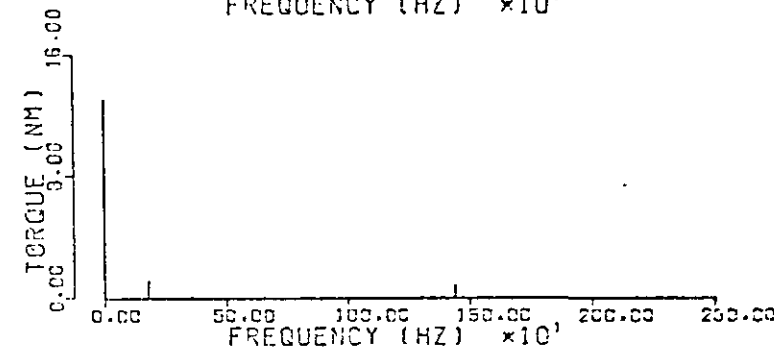
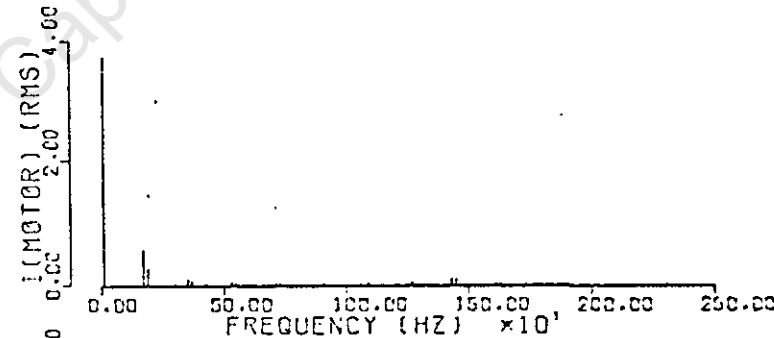
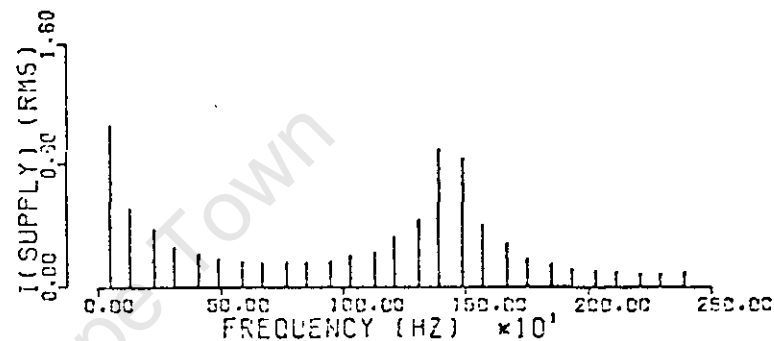
SUPPLY=220.0 VOLT RMS , 50.HZ.
CONVERTOR STEPS -120.0 DEGREES , F(STEP)=180.0 HZ. , CHOPS/STEP=2.
T(ON)/T(TOTAL)=0.350 , SPEED=-225.0 RPM
M=0.447,L1=L2=0.473,R1=5.50,R2=3.28

Fig.110

SPECTRA CALCULATED FROM THE ALGEBRAIC FORMULATION OF THE MOTOR EQUATION SOLUTION.
ILLUSTRATION OF THE EFFECT OF INCREASING THE NUMBER OF CHOPS PER STEP IN
THE FREQUENCY CONVERTOR.



SUPPLY=220.0 VOLT RMS . 50.HZ.
CONVERTOR STEPS -120.0 DEGREES . F(STEP)=180.0 HZ. . CHOPS/STEP=4.
T(ON)/T(TOTAL)=0.350 . SPEED=-225.0 RPM
M=0.447, L1=L2=0.473, R1=5.50, R2=3.28



SUPPLY=220.0 VOLT RMS . 50.HZ.
CONVERTOR STEPS -120.0 DEGREES . F(STEP)=180.0 HZ. . CHOPS/STEP=8.
T(ON)/T(TOTAL)=0.350 . SPEED=-225.0 RPM
M=0.447, L1=L2=0.473, R1=5.50, R2=3.28

Fig.!!!

Slip Frequency (Hz)	Phase Stepping Frequency (Hz)	I(or)/I(total)	Displacement angle with two different values of Z		Displacement angle with sine wave of frequency equal to output of converter
			Z = 1 <i>chop/phase step</i>	Z = 4 <i>chops/phase step</i>	
+5.0 (relative to fundamental output of converter)	120 (10 Hz output)	0.1	-18.33°	-22.46°	-21.72°
		0.5	-27.79°	-23.72°	
		0.9	-26.07°	-24.01°	
	180 (10 Hz output)	0.1	+16.50°	+19.19°	-21.72°
		0.5	+14.04°	+18.91°	
		0.9	+17.30°	+18.96°	
	300 (50 Hz output)	0.1	+16.56°	+26.99°	-33.40°
		0.5	+14.95°	+26.81°	
		0.9	+23.61°	+27.22°	
+1.0 (relative to fundamental output of converter)	120 (10 Hz output)	0.1	-31.51°	-42.81°	-43.49°
		0.5	-49.96°	-45.38°	
		0.9	-49.05°	-46.18°	
	180 (10 Hz output)	0.1	+30.71°	+38.10°	-43.49°
		0.5	+27.27°	+37.82°	
		0.9	+34.84°	+38.16°	
	300 (50 Hz output)	0.1	+0.34°	+33.12°	-51.11°
		0.5	-9.22°	+32.72°	
		0.9	+21.83°	+34.26°	
-2.5 (relative to fundamental output of converter)	120 (10 Hz output)	0.1	-113.04°	-117.69°	-119.86°
		0.5	-114.94°	-117.80°	
		0.9	-117.11°	-117.97°	
	180 (10 Hz output)	0.1	+118.26°	+119.98°	-119.86°
		0.5	+121.58°	+120.32°	
		0.9	+121.00°	+120.44°	
	300 (50 Hz output)	0.1	+159.59°	+145.70°	-139.23°
		0.5	+164.04°	+146.56°	
		0.9	+151.43°	+146.28°	

Table 5.

VALUES CALCULATED FOR THE TEST MOTOR. $V_s = 220$ VOLT RMS 50 HZ.

illustrate the combined effect at low output frequencies of decreased losses and low basic efficiency.

8.11.3 Note that regeneration is quite feasible and full four quadrant operation can be realized (i.e. forward and reverse, motoring and regenerating) which can be an advantage in control applications. Note also that the modulating frequencies which produce low output frequencies are high (around 3 times supply frequency). As a result control is continuous through D.C. output with low harmonic distortion.

8.11.4 Hurley's assertion that the modulator should always be operated in the "fast-stepping" mode (i.e. with stepping-frequency greater than 3 times supply frequency) is confirmed. This is clearly seen from Fig. (98) which shows the poor efficiencies in the "slow-stepping" mode where the distortion currents have frequencies not far removed from that of the fundamental.

8.11.5 The use of more than one chop per phase step is indicated since this reduces the length of the short-circuit interval in accordance with the earlier conclusions in Chapter Three. In the results of Figs (89) to (109) we observe that increasing the number of chops per step from 1 to 4 has the following general effects:

- (i) Generally improved efficiency
- (ii) Greatly improved distortion factors for the motor current
- (iii) Supply current distortion improved at small slip frequencies but worse at larger slips.
- (iv) Lower power factor.

The reduced power factor is a result of the lower fundamental output voltage of the convertor as Z is increased from one to four.

In most instances the reduction of power factor is less significant than the increased efficiency.

8.11.6 Since we have seen that lower modulation frequencies are tolerable when the fundamental output frequency is reduced, it may be concluded that it is suitable to use a fixed value of Z chops per step provided operation is in the "fast-stepping" mode. In this mode the fundamental output frequency is decreased by decreasing the modulating frequency. In the "slow-stepping" region however a larger value of Z would be required as larger output frequencies are produced.

8.11.7 Hurley reported that exceptionally clean waveforms were obtained when the harmonic currents coincided with the fundamental output frequency. While this is true, it has also been shown here that this situation happens under the special conditions when the modulation angle equals $n\pi$ radians which results in unbalanced phase currents. Figs. (112) and (113) illustrate simulated waveforms in the test motor for two cases where

$$f_{step} = \left(\frac{6 \cdot f_e}{3n + 2} \right)$$

Note also that these conditions are never encountered in the "fast-stepping" mode.

8.12 Experimental Operation of the Test Motor with a Frequency Convertor

Extensive measurements were not performed on the test motor with this form of modulation. However, a working system was assembled using the power commutating switches which were

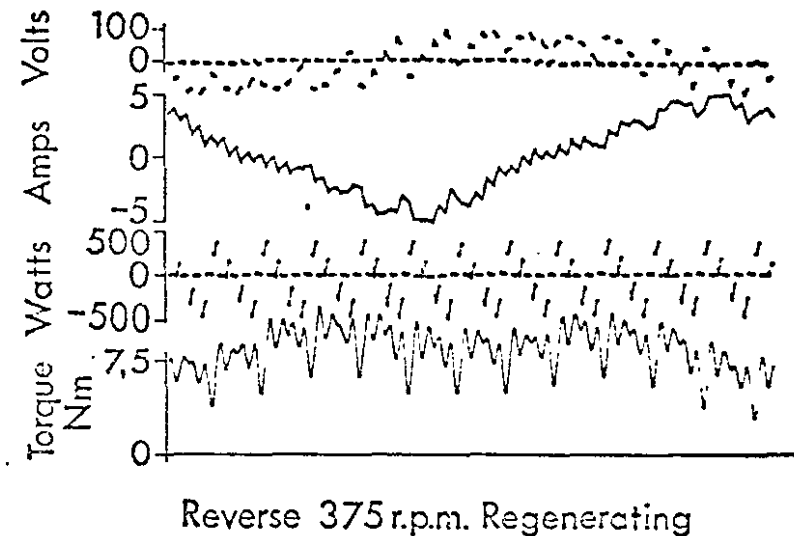
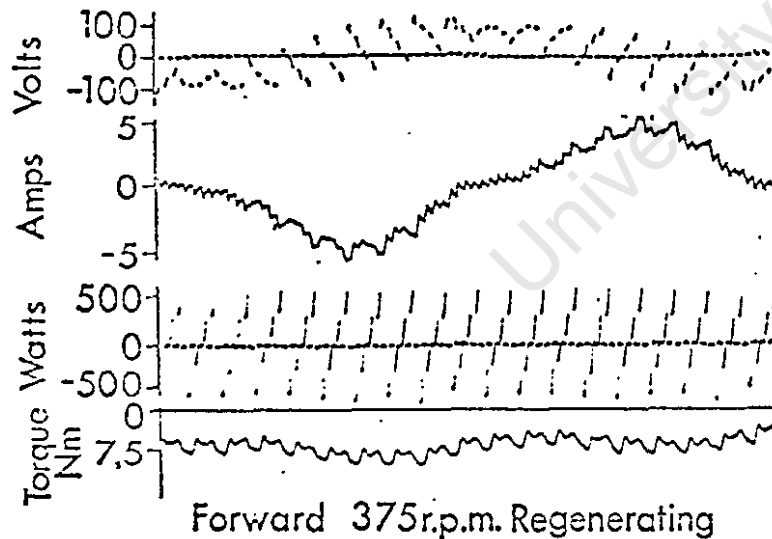
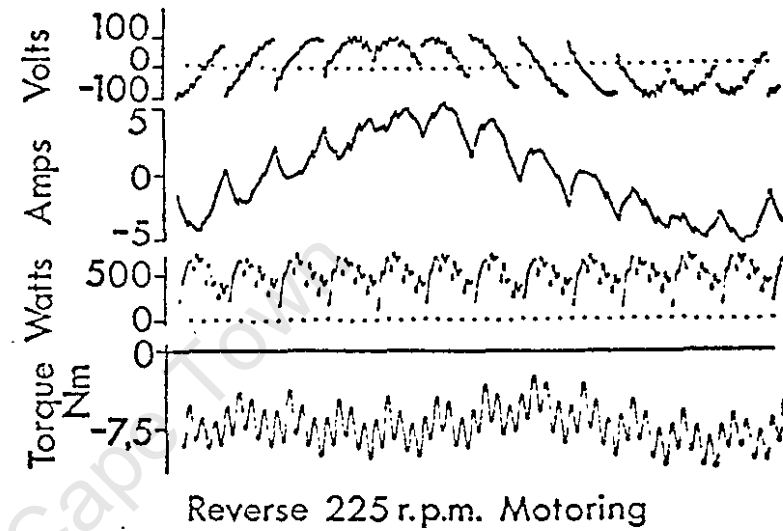
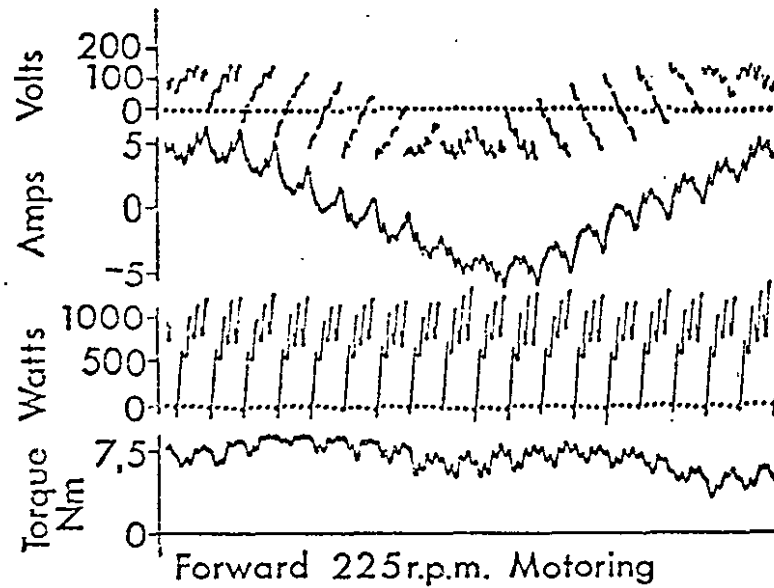
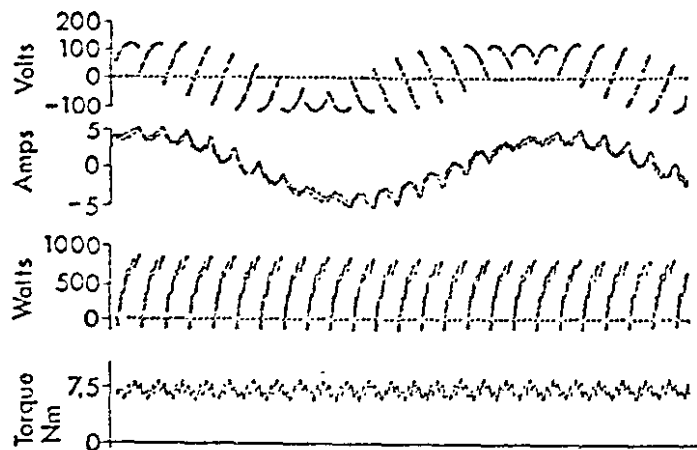
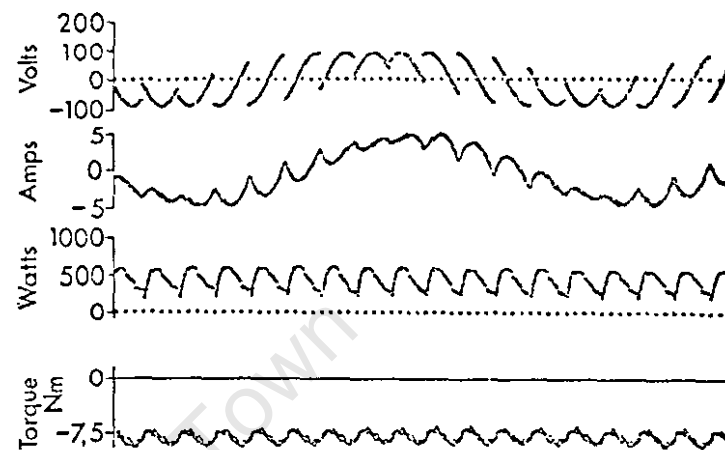


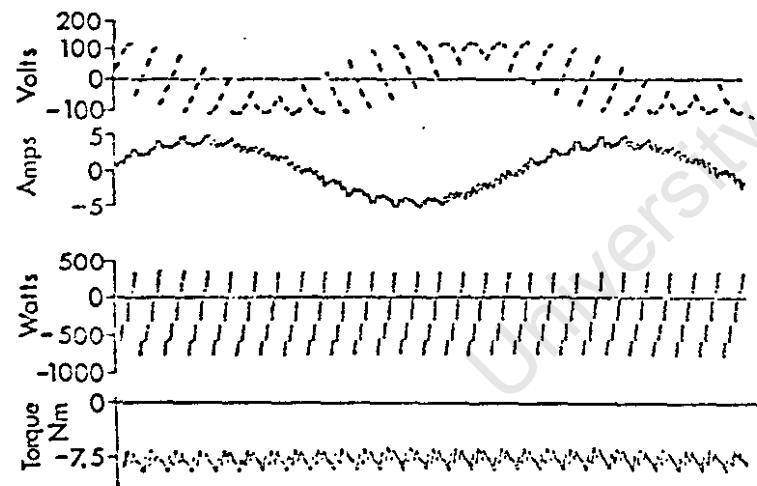
Fig.114 Measured waveforms for the case of the test motor fed by a phase-stepping frequency convertor with four chops per phase step. See Table 6 for summary of operating conditions.



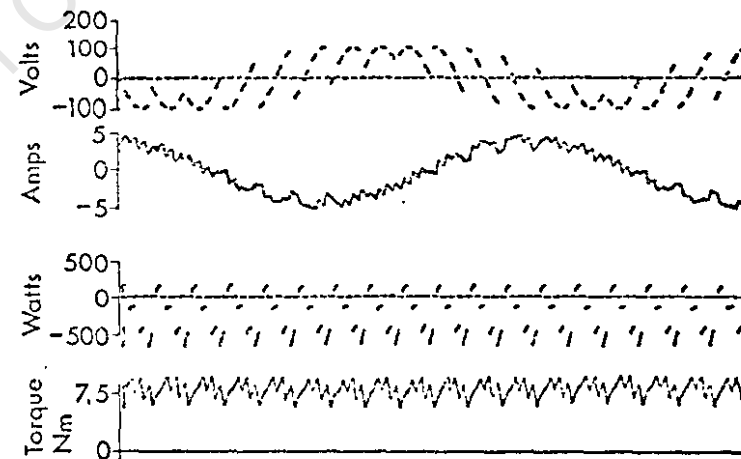
Forward 225 r.p.m. Motoring



Reverse 225 r.p.m. Motoring



Forward 375 r.p.m. Regenerating



Reverse 375 r.p.m. Regenerating

Fig.115

Waveforms predicted by numerical simulation of the operating conditions for which measured waveforms are presented in Fig.114.
See Table 6 for summary of operating conditions.

available.

Fig. (//4) shows measured waveforms in the test motor for four-quadrant operation of this convertor which was set for 4 chops per phase step.

Fig. (//5) shows the corresponding numerical simulation.

TABLE 6

TABLE OF MEASURED AND CALCULATED RESULTS

CALCULATED VALUES IN BRACKETS

MOTOR DRIVEN BY CONVERTER	MOTOR DRIVEN BY ALTERNATOR	MOTOR DRIVEN BY CONVERTER
F_c chopping frequency = 720 Hz F_p phase stepping frequency = 180 Hz F_0 output fundamental frequency = 10 Hz	Fundamental frequency = 10 Hz	F_c chopping frequency = 480 Hz F_p phase stepping frequency = 120 Hz F_0 output fundamental frequency = 10 Hz
Forward at 225 r.p.m. and motoring	Forward at 225 r.p.m. and motoring	Reverse at -225 r.p.m. and motoring
Phase voltage 80,4 (66,25) Phase current 3,02 (2,76) Total Watts 405 (357) Watts/(RMS Volts x RMS Amps) 0,56 (0,65) Mean Torque Nm. 7,5 (7,12) Efficiency 43,6% (47,0%) Mark to Mark + Space Ratio 0,72 (0,72) Supply Voltage 110 (78,4)	Phase voltage 49,6 (49,6) Phase current 2,86 (2,91) Total Watts 396 (396) Watts/(RMS Volts x RMS Amps) 0,93 (0,896) Mean Torque Nm. 7,5 (7,92) Efficiency 44,6% (48,1%)	Phase voltage 68,8 (60,0) Phase current 3,10 (2,83) Total Watts 396 (380,5) Watts/(RMS Volts x RMS Amps) 0,62 (0,73) Mean Torque Nm. -7,5 (-7,58) Efficiency 44,6% (46,9%) Mark to Mark + Space Ratio 0,94 (0,94) Supply Voltage 80 (62,1)
Forward at 375 r.p.m. and regenerating	Forward at 375 r.p.m. and regenerating	Reverse at -375 r.p.m. and regenerating
Phase voltage 48,4 (46,57) Phase current 2,86 (2,75) Total Watts -109,8 (-99,9) Watts/(RMS Volts x RMS Amps) 0,26 (0,26) Mean Torque Nm. -7,5 (-7,17) Efficiency 39,0% (35,46%) Mark to Mark + Space Ratio 0,38 (0,38) Supply Voltage 80 (76,3)	Phase voltage 25,0 (25,1) Phase current 2,72 (2,94) Total Watts -114 (-108) Watts/(RMS Volts x RMS Amps) 0,56 (0,488) Mean Torque Nm. -7,51 (-7,87) Efficiency 40,5% (34,9%)	Phase voltage 45,4 (45,81) Phase current 2,90 (2,86) Total Watts -108 (-103,9) Watts/(RMS Volts x RMS Amps) 0,27 (0,267) Mean Torque Nm. 7,5 (7,78) Efficiency 38,4% (33,0%) Mark to Mark + Space Ratio 0,42 (0,42) Supply Voltage 70 (70,3)

CHAPTER NINESPEED CONTROL OF THE INDUCTION MOTOR

This thesis is ultimately directed at the analysis of a method of speed control of induction motors. In the previous chapters, we have closely considered various forms of modulation which may be applied to the supply in order to achieve effective control of voltage level and frequency. In conclusion, it is appropriate to discuss briefly how the modulation parameters should be set in order to achieve the best performance of the motor under different operating conditions.

In previous work (22) the author has investigated the behaviour of induction motors with variable frequency sinusoidal supply voltage. It was shown that with constant voltage excitation the level of magnetic flux in the machine varies as a function of both supply frequency and slip frequency. In a control application it is important to keep this flux at an optimum level under all operating conditions in order to achieve maximum torque per ampere and uniform performance characteristics as functions of slip frequency. The flux level which is held constant could be:-

- (a) Flux in the airgap.
- (b) Rotor flux linkages.
- (c) Stator flux linkages.

Holding each of these quantities constant theoretically produces different slip characteristics. It is not usually feasible to measure (a) and (b) above, but it is not difficult to monitor stator flux linkages in a practical system. If the stator phase voltage is given by

$$V = R_s I + j\omega_e \Phi$$

where V and I are the usual phasor representations of the

sinusoidal motor phase quantities with angular frequency ω_e and $\bar{\Phi}$ is the phasor representation of flux linking a stator phase,

then for constant ψ

$$V = j \bar{\Phi} \left\{ \omega_e + \frac{R_s}{jL_1} + \frac{R_s M'^2 \omega_r^+}{L_1^2 (R_r + j\omega_r^+ \sigma L_2)} \right\}$$

This expression defines the voltage level which should be applied for constant stator flux linkages as a function of ω_e and ω_r^+

In Chapter 8.2 the fundamental output voltage of the phase-stepping frequency convertor was shown to have amplitude

$$|V| = \frac{3\sqrt{3}}{2} \frac{|V_s|}{\pi} \cdot \left\{ \frac{\sin\left(\frac{\pi \frac{\alpha_b}{Z}}{3Z}\right)}{\sin\left(\frac{\pi}{3Z}\right)} \right\}$$

where $|V_s|$ is the amplitude of the mains supply voltage.

If $Z > 1$ this is closely approximated by

$$|V| = \frac{3\sqrt{3}}{2} \cdot \frac{|V_s|}{\pi} \cdot \left(\frac{\alpha_b}{Z} \right)$$

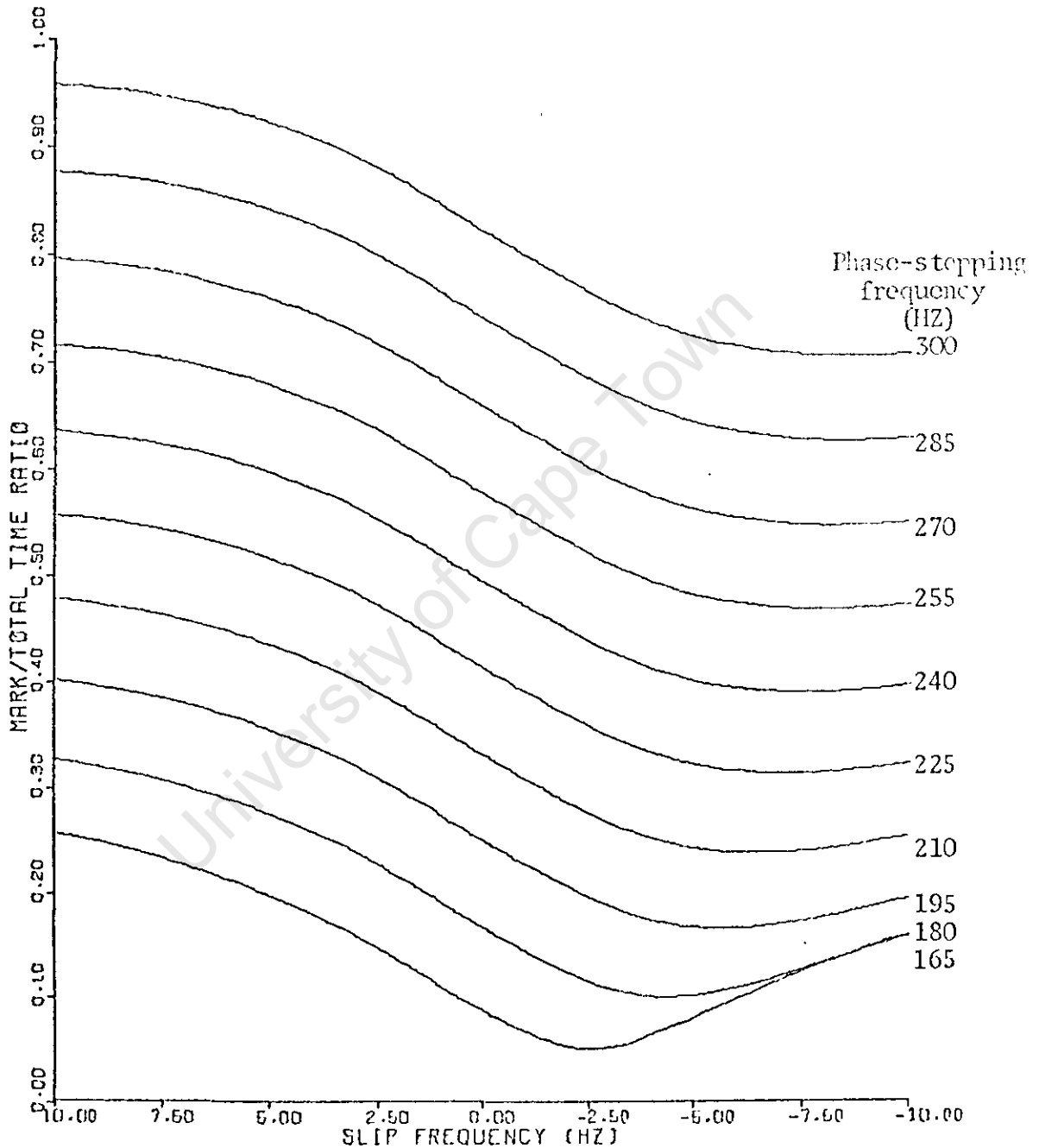
In order to maintain constant fundamental flux linkages in the stator while supplying the machine via such a convertor it is necessary to set

$$\left(\frac{\alpha_b}{Z} \right) = \frac{2\pi}{3\sqrt{3} |V_s|} \left| \bar{\Phi} \cdot \left\{ \omega_{out} + \frac{R_s}{jL_1} + \frac{R_s M'^2 \omega_r^+}{L_1^2 (R_r + j\omega_r^+ \sigma L_2)} \right\} \right|$$

where $\omega_{out} = |2\pi(f_e - \frac{f_{step}}{3})|$

and ω_r^+ = angular slip frequency relative to ω_{out}

These values have been calculated in the case of the test motor for several phase-stepping frequencies as a function of slip frequency, with $Z = 4$ (See Fig. (116)). The value

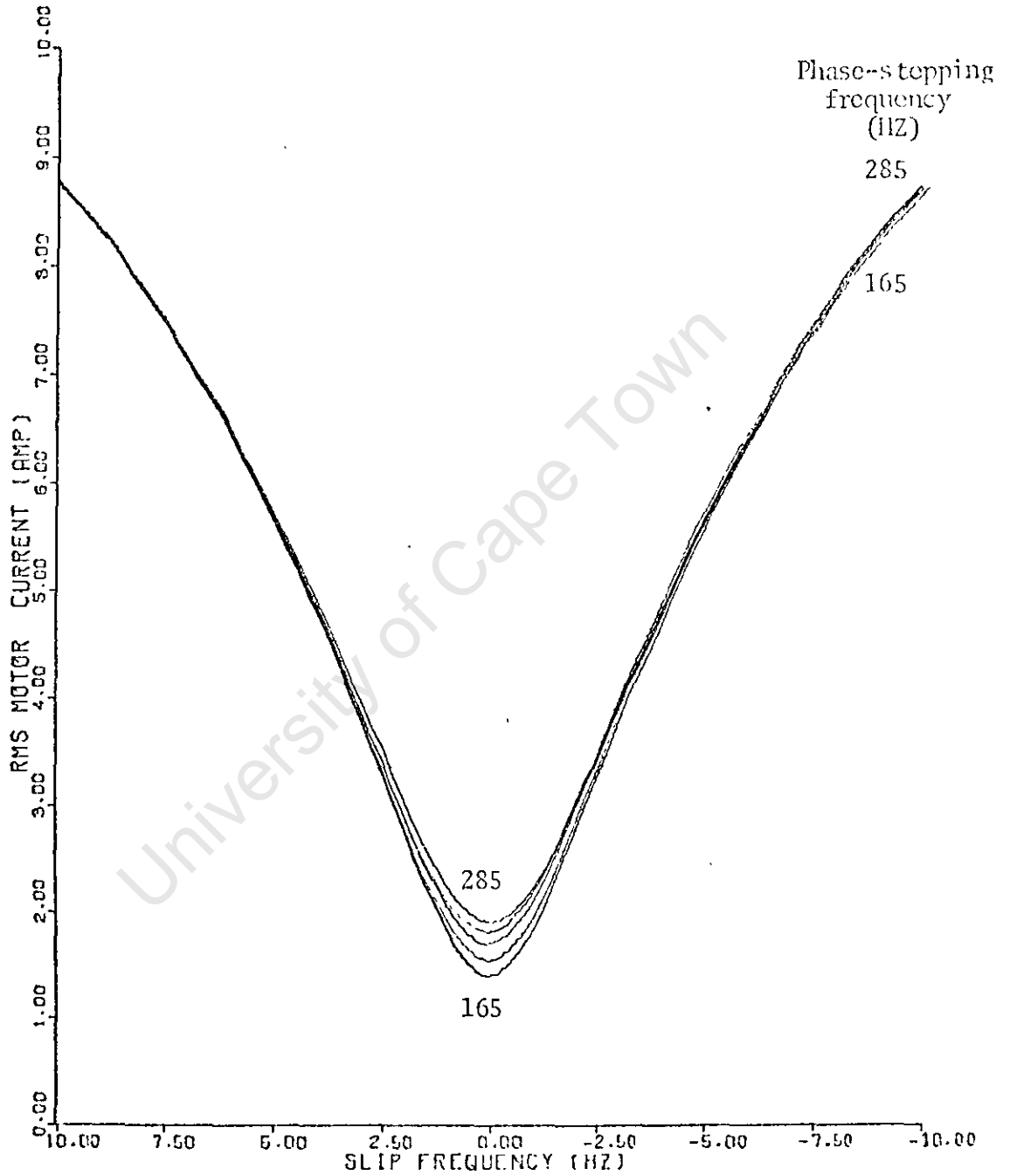


CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP -4.

CONST. FLUX $\Phi(I_{RMS}) = 0.6226$, SUPPLY VOLTAGE - 288.7 VOLTS RMS 50.0 HZ

Fig. 116 Mark/total time ratios to maintain constant fundamental stator flux linkages for different slip frequencies at several different convertor phase-stepping frequencies.

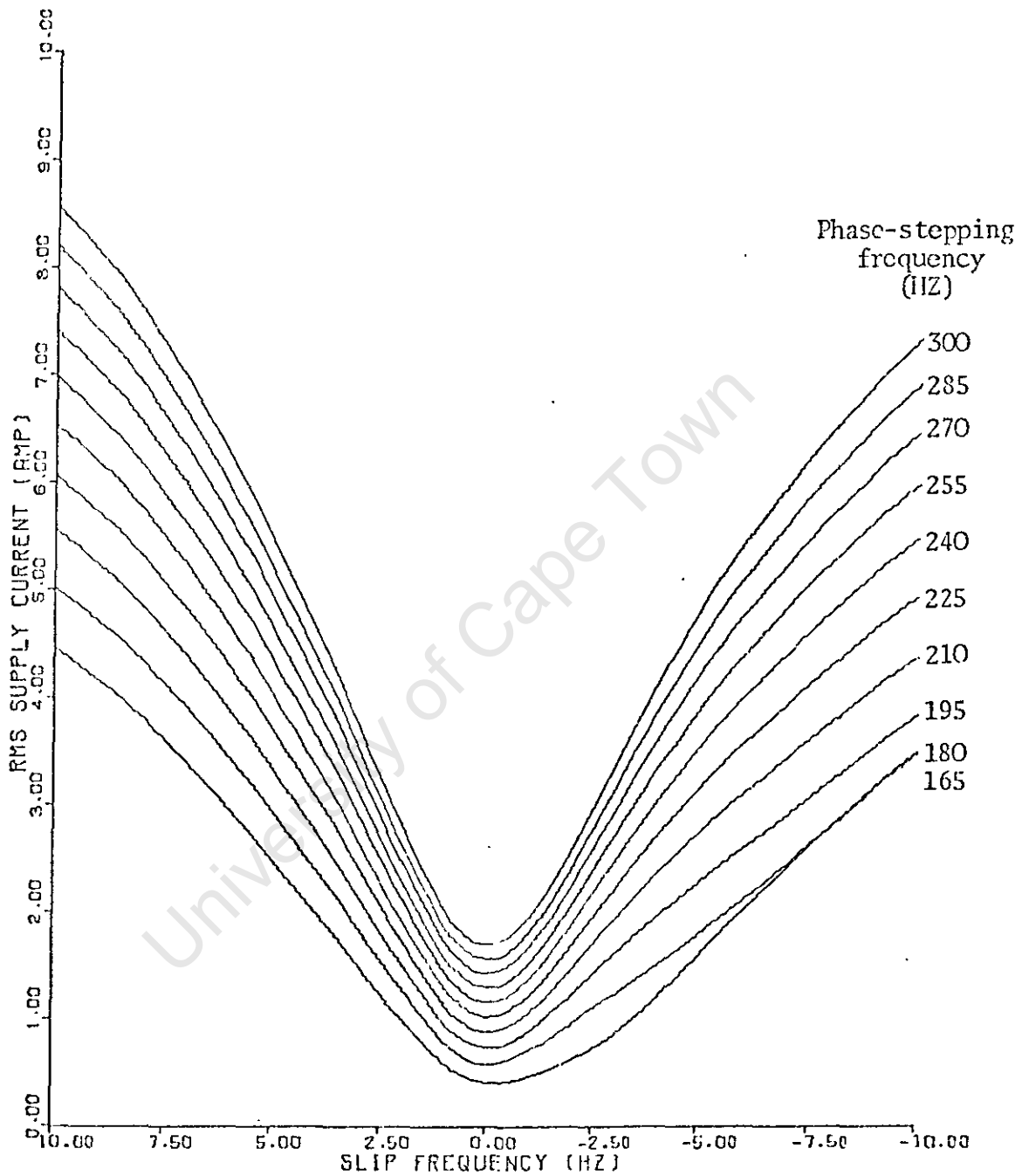
Note: Slip frequency taken relative to synchronous speed which corresponds to the fundamental convertor output frequency.



CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=4.

CONST.FLUX PHIRMS=0.6226, SUPPLY VOLTAGE=298.7 VOLTS RMS 50.0 HZ

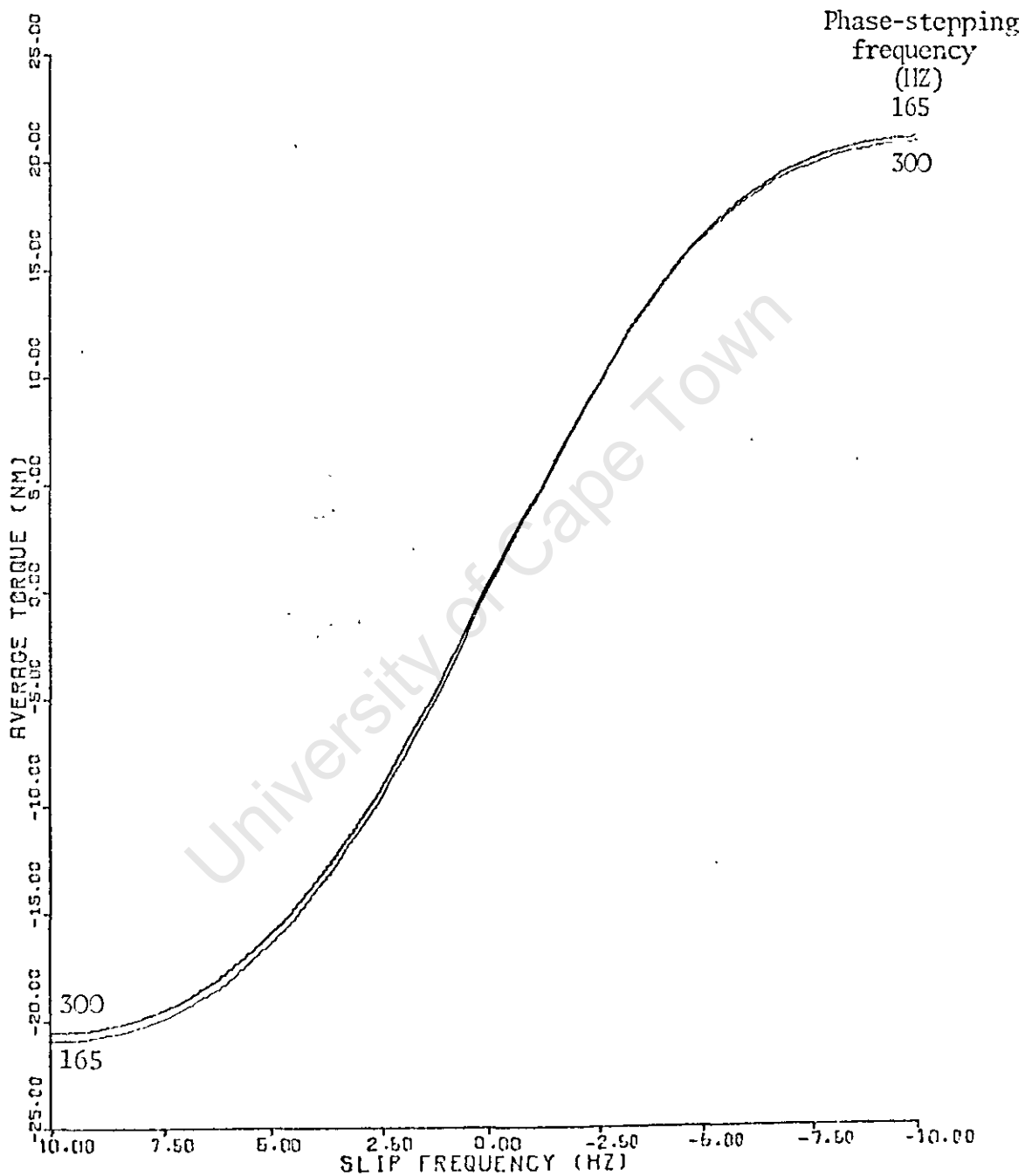
Fig.117 RMS motor current values with mark/total time ratios as shown in Fig. 116



CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=4.

CONST. FLUX $\Phi(I_{RMS})=0.6226$, SUPPLY VOLTAGE=288.7 VOLTS RMS 50.0 HZ

Fig. 118 RMS supply current values corresponding to the mark/total time ratios for constant stator flux linkages as shown in Fig. 116



CONVERTOR STEPS -120.0 DEGREES, CHOPS PER STEP=4.

CONST.FLUX Φ (RMS)=0.6226, SUPPLY VOLTAGE=286.7 VOLTS RMS 50.0 HZ

Fig.119 Torque characteristics corresponding to the constant flux excitation illustrated in Fig.116

of $\bar{\Phi}$ which has been used for this illustration corresponds to the flux level in the motor with 220 volt 50 Hz sinusoidal supply at 5 Hz slip frequency. Fig. (117) shows the RMS motor current characteristics corresponding to the prescribed settings of $\left(\frac{c_b}{c_v}\right)$, illustrating the uniformity which is achieved. Fig. (118) shows the corresponding values of RMS supply current. As might be expected, the uniform motor current characteristics are not reflected when the current is de-modulated into the supply lines. Fig. (119) illustrates the corresponding average torque characteristics. Note that the phase-stepping frequencies which are illustrated are all greater than $(3.F_e)$ so that the fundamental output of the convertor has negative phase sequence. Hence, keeping our convention, negative torque indicates that the machine is motoring in this mode.

It is clear that the motor can be kept fully fluxed and developing constant torque if

- (a) the slip frequency is held constant - this is not difficult to achieve since ω_{sl} is controlled by logic instructions which govern the stepping rate of the convertor, and
- (b) the RMS motor current is held at a predetermined level by varying $\frac{c_b}{c_v}$. This strategy forms the basis of the so-called "current-source/slip-regulated" induction motor control systems and may readily be achieved using the phase-stepping convertor with chopper modulation. The convenient control of voltage by the chopping action is a particular advantage in this respect.

Although a complete treatment is not feasible here, it is interesting to consider the rate at which changes of state may be effected in the motor during speed-controlled operation. Fig. (120) illustrates the numerical simulation of a sudden change of state dictated by the convertor at time $t=0$. In

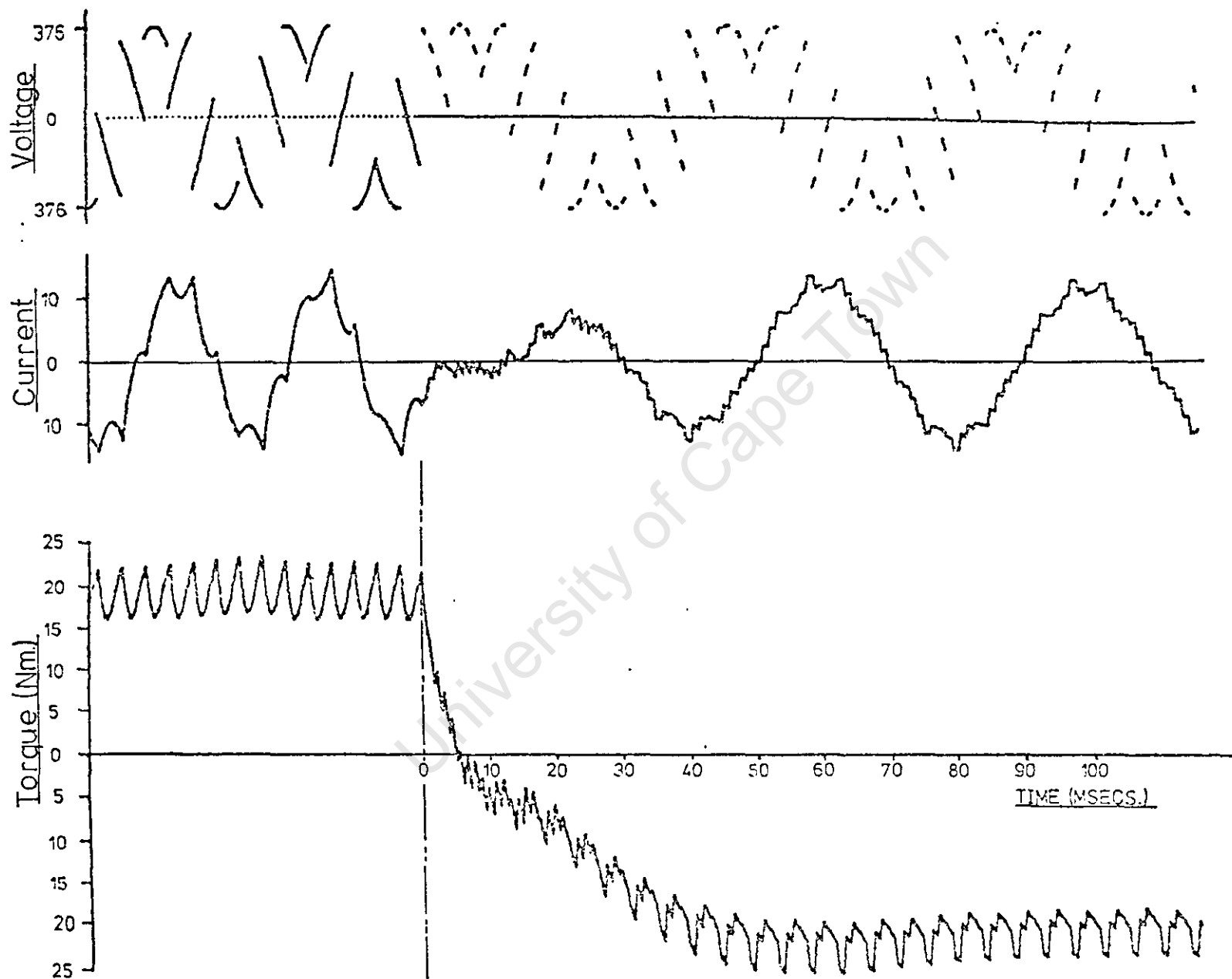


Fig.120

Sudden change of state brought about by changing the modulator parameters in a phase stepping frequency converter feeding the test motor. (See note on Page 263 for details)

both the initial and final states, the modulator has been set for the same stator flux linkages according to the preceding discussion. Note that there is inherent point-on-wave continuity of the convertor fundamental output voltages at $t=0$. This has the advantage of producing minimal transient currents in the motor under such conditions. This simulation assumes constant shaft speed - a constraint which has been retained throughout this work. Dynamic response of the electro-mechanical system can be studied by numerical simulation methods and will no doubt form the basis of further work in regard to the phase-stepping convertor.

Note relating to Fig.120

Numerical simulation illustrating a sudden change in operating conditions at time $t=0$ dictated by a phase-stepping frequency convertor (4 chops per phase step) feeding the test motor. In both the initial and final states the excitation produces the same fundamental level of stator flux linkages ($\bar{\Phi} = 0.623$ RMS) as was considered in Figs.(116) to (119). The motor speed is held constant at 1050 RPM while the modulation parameters are changed from initially producing peak accelerating torque to finally developing peak decelerating torque. The following values were used:

$$V_s = 266 \text{ Volt RMS per phase, } 50 \text{ Hz.}$$

<u>INITIAL STATE</u>	<u>FINAL STATE</u>
$T(\text{on})/T(\text{total}) = 0.958$	$T(\text{on})/T(\text{total}) = 0.346$
$F_{\text{step}} = 285.9 \text{ Hz.}$	$F_{\text{step}} = 224 \text{ Hz.}$
$F_{\text{out}} = 24.7 \text{ Hz.}$	$F_{\text{out}} = 45.3 \text{ Hz.}$
Slip frequency = +10.3 Hz.	Slip frequency = -10.3 Hz.

CHAPTER TENCONCLUSION

In the course of this thesis a technique has been formulated for obtaining algebraic solutions to the induction motor equations with modulated excitation. This method has been found to have definite advantages over numerical simulation when it is necessary to establish performance factors for a particular modulation system. The approach which has been described should find application in regard to a wide range of static convertor systems which already exist and should aid designers in their implementation of new modulation forms for induction motor drives. The correlation which has been demonstrated between calculated and measured performance is sufficient to justify confidence in the practical usefulness of the solutions which are obtained in this way.

The application of the analysis to chopper modulated systems in the latter half of the thesis constitutes a research area in itself. The use of chopper modulation in AC motors had not previously been established. This work has shown that, provided the modulation is executed within certain limits, chopper modulation provides effective control of the applied voltage with only slight loss of efficiency in the motor.

The frequency convertor system which has been studied has two major advantages:

- (a) It is fully regenerative and also has the ability to change continuously from positive to negative phase rotation of the fundamental output voltage. As a result "four-quadrant" control of the motor may be achieved.
- (b) The system can operate with any available supply frequency using only a single power conversion stage.

When commutating switches of higher power rating are developed, it

seems likely that the AC to AC frequency convertor with chopper modulation will be a viable proposition for induction motor speed control in preference to other systems. It has been shown that under constant flux conditions the convertor-fed induction motor can respond to control instructions with minimal transient effects. With the added advantages of good damping and low inertia rotors it is probable that when operated in this way the induction motor system will surpass the response of DC motors in control applications.

The modulation forms which have been considered should be regarded as basic strategies. Perhaps the most striking feature of these is the severely distorted current which they draw from the supply network. There is great scope for other methods of applying modulation techniques, and no doubt as better and cheaper forced commutation switches become available configurations will be possible which will overcome the problems of harmonic generation with respect to both the motor and supply. It seems inevitable that static switching techniques achieved with mass produced components will be applied to an increasing number of power conversion requirements as the production of labour intensive devices such as transformers becomes more costly.

Power electronics has indeed provided intriguing possibilities for the future use of induction motors. It is hoped that this thesis will aid future work in this area and provide a means for evolving better modulation strategies.

TRANSFORMATION OF MOTOR EQUATIONS TO REMOVE
POSITION DEPENDENT COUPLING COEFFICIENTS

The power invariant transformation of the coupled circuit motor equations to a reference frame where the rotor circuits are effectively held stationary, is accomplished without changing the stator variables to equivalent d-q axis values. This method was suggested by Robertson and Hebbar⁽¹⁵⁾ and has been found very useful by the author, since solution of the equations in this form yields stator phase quantities directly without further transformation. The connection equation is as follows :-

$$[i] = [C_a] [i_1]$$

$$[i]_t = [i_s^a \ i_s^b \ i_s^c \ i_r^a \ i_r^b \ i_r^c]$$

$$[i_1]_t = [i_s^\beta \ i_s^\delta \ i_r^\beta \ i_r^\delta]$$

$$[C_a] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{\sqrt{3}} \cos(P\theta + \frac{\pi}{6}) & -\frac{2}{\sqrt{3}} \sin(P\theta) \\ 0 & 0 & -\frac{2}{\sqrt{3}} \sin(P\theta + \frac{\pi}{3}) & \frac{2}{\sqrt{3}} \sin(P\theta - \frac{\pi}{3}) \\ 0 & 0 & \frac{2}{\sqrt{3}} \sin(P\theta) & \frac{2}{\sqrt{3}} \cos(P\theta - \frac{\pi}{6}) \end{bmatrix}$$

Then, in the usual way for power invariant transformation :

$$[Z_1] = [C_a]_t^* [Z] [C_a]$$

and

$$[v_1] = [C_a]_t^* [v]$$

APPENDIX 2

TRANSFORMATION TO INSTANTANEOUS
SYMMETRICAL COMPONENTS

$$[i_1] = [C_b][i_2]$$

$$[i_1]_t = [i_s^\beta \ i_s^\delta \ i_r^\beta \ i_r^\delta]$$

$$[i_2]_t = [i_s^+ \ i_s^- \ i_r^+ \ i_r^-]$$

$$[C_b] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ \lambda & \lambda^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & \lambda & \lambda^2 \end{bmatrix}$$

where $\lambda = e^{j\frac{2\pi}{3}}$

Note that the inverse of the connection matrix is :-

$$[C_b]^{-1} = \frac{1}{3} \begin{bmatrix} (\frac{3}{2} - j\frac{\sqrt{3}}{2}) & (-j\sqrt{3}) & 0 & 0 \\ (\frac{3}{2} + j\frac{\sqrt{3}}{2}) & (j\sqrt{3}) & 0 & 0 \\ 0 & 0 & (\frac{3}{2} - j\frac{\sqrt{3}}{2}) & (-j\sqrt{3}) \\ 0 & 0 & (\frac{3}{2} + j\frac{\sqrt{3}}{2}) & (j\sqrt{3}) \end{bmatrix}$$

and since $[i_2] = [C_b]^{-1}[i_1]$

and $[i_1]$ is real-valued,

it follows that

$$\underline{i_s^+ = (i_s^-)^* \text{ and } i_r^+ = (i_r^-)^*}$$

also,
$$[v_2] = [C_b]_t^* [v_1]$$

$$= [3v_s^+ \ 3v_s^- \ 0 \ 0]$$

where $v_s^+ = \frac{1}{3}(v_s^a + \lambda v_s^b + \lambda^2 v_s^c)$; $v_s^- = \frac{1}{3}(v_s^a + \lambda^2 v_s^b + \lambda v_s^c)$

APPENDIX 3TRANSFORMATION OF PHASE QUANTITIES TO
OBTAIN THE AMPLITUDES OF DISTRIBUTED
QUANTITIES IN THE ROTATING FIELD
REFERENCE FRAME

In Chapter 2 a coupled circuit model of the induction motor was developed. This model uses lumped electrical parameters to formulate Kirchoff's loop law for each of the circuits which are defined. In this way the values of phase current are conveniently calculated and hence terminal power and torque. Because of the number of circuits, however, it is conceptually difficult to interpret phase currents in terms of the fields they produce in the motor unless use is made of the rotating field approach.

If the three-phase voltages and currents in a distributed motor winding constitute balanced sinusoidal sets then they can be defined by their peak values, their angular frequencies in time and the phase angle of the reference phase quantity relative to some datum. Each such balanced phase set gives rise to a space distribution around the winding with the fundamental pole number (neglecting space harmonic distortion). The amplitude of each space-distribution is related to the peak value of the corresponding phase quantity in terms of the physical constants of the machine. The orientation of the distributions in space depends on the corresponding time-phase angles of phase quantities, so that sinusoidal variation of a phase quantity causes rotation of the space distribution. There may be many such superimposed rotating systems contributing to a particular state of the motor. The instantaneous symmetrical components have particular significance in this regard. At each instant in time, irrespective of the form of the excitation, the instantaneous positive sequence component is a complex number which defines,

- (a) by its magnitude, the instantaneous amplitude of the corresponding nett space distribution, and
- (b) by its angle, the instantaneous orientation of the nett space distribution.

The constant relating the instantaneous symmetrical components to the space distributions can be deduced as follows. Consider, for example, a series connected winding with $\mathcal{U}_S^+ = |\mathcal{U}|$ and $i_S^+ = |\mathcal{I}| e^{-j\phi}$ being the instantaneous symmetrical components of the reference phase voltage and current.

Then from standard winding analysis we can write the maximum value of the fundamental radial field distribution as

$$H_{max} = \frac{2}{\pi} \cdot \frac{D}{4P} \cdot k_{w1} \cdot (2|i|)$$

where D = Total conductors in winding
 k_{w1} = Fundamental winding factor.

The field distribution is thus $H_{max} \sin(P\theta)$ and is in space quadrature with the current density distribution, $I_{max} \sin(P\theta - \frac{\pi}{2})$ where

$$H_{max} = \int_{\frac{\pi}{2P}}^{\frac{\pi}{P} + \frac{\pi}{2P}} I_{max} \sin(P\theta - \frac{\pi}{2}) \cdot r \cdot d\theta.$$

i.e.

$$\underline{I_{max} = \frac{P}{2r} \cdot H_{max} = \left\{ \frac{D}{4\pi r} \cdot k_{w1} (2|i|) \right\}} \quad \begin{array}{l} \text{amps per} \\ \text{unit length} \\ \text{of airgap} \\ \text{circumference.} \end{array}$$

g = length of airgap

r = radius

l = length of core

θ = mechanical angle round circumference

Assuming an idealized magnetic path, we can assume that the only reluctance is encountered in the airgap of the machine so that the field distribution can be shown to produce a flux density distribution $B_{max} \sin(P\theta)$ where

$$\begin{aligned} B_{max} &= \frac{\mu_0}{g} \cdot H_{max} \\ &= \frac{\mu_0}{g} \cdot \frac{D}{2\pi P} \cdot k_{w1} \cdot (2|i|) \end{aligned}$$

With due regard for the power invariance of the transformation it is also possible to define a voltage distribution as $V_{max} \sin(p\theta - \frac{\pi}{2} + \phi)$ where

$$V_{max} = \frac{(2 |w|)}{\frac{1}{2} \cdot \frac{D}{3} \cdot k_{w1} \cdot l} \quad \text{volts per unit axial length.}$$

The nett terminal power of the winding can then be expressed as

$$W_{term} = \int_0^{2\pi} (V_{max} \sin(p\theta - \frac{\pi}{2} + \phi)) (I_{max} \sin(p\theta - \frac{\pi}{2})) \cdot r l d\theta.$$

$$= 6 |w| i l \cos(\phi) \quad \text{as before.}$$

Finally consider a winding of radius r with a flux density distribution $(B_{max} \sin(p\theta))$ impinging on it, and carrying a current density distribution $(I_{max} \sin(p\theta - \psi))$. The tangential force on an elemental area $(r l d\theta)$ of the active area is thus

$$dF = (B_{max} \sin(p\theta)) (I_{max} \sin(p\theta - \psi)) \cdot l r d\theta$$

Thus the force per pole = $\int_{\theta=0}^{\frac{\pi}{p}} dF.$

and the total torque on the winding = $(B_{max} I_{max} l r^2 \pi \cdot \cos(\psi))$

The instantaneous symmetrical components thus represent a concise expression of the nett situation in the machine under both transient and steady state conditions and, in the present work, provide an invaluable interpretation of an otherwise confusing mathematical solution.

CALCULATION OF RMS CURRENT WHEN THE MODULATION
ANGLE IS AN INTEGRAL MULTIPLE OF π RADIANS.

As discussed in Chapter 5.7 there are certain modulating frequencies where the calculation of the RMS of current waveforms differs from the more general case. These conditions occur in a waveform whenever the modulation angle, θ , is an integer multiple of π radians. The situation when this happens is characterized by unbalanced phase currents including D.C. components if θ is an even multiple of π . This situation although usually avoided, may be encountered during the normal operation of a modulator if -

- (a) The desirable output component of the modulator is D.C., in which case the motor currents have different RMS values,
- (b) the supply is D.C. in which case the supply currents have different RMS values.

Using the same notation as before, we have in these special cases only

$$i_{S RMS}^2 = \sqrt{\frac{1}{t_1} \int_0^{t_1} i_{Q1}^2 dt}$$

where.

$$i_{Q1}^2 = \left[\begin{aligned} & |I_s|^2 \left\{ \frac{1}{2} + \frac{1}{2} \cos(2\omega_e t + 2\theta_s) \right\} \\ & + |i'|^2 e^{-2\rho' t} \left\{ \frac{1}{2} + \frac{1}{2} \cos(2\nu' t + 2\theta') \right\} \\ & + |i''|^2 e^{-2\rho'' t} \left\{ \frac{1}{2} + \frac{1}{2} \cos(2\nu'' t + 2\theta'') \right\} \\ & + |i'| |i''| e^{-(\rho' + \rho'') t} \left\{ \begin{aligned} & \cos((\nu' - \nu'') t + (\theta_s - \theta'')) \\ & + \cos((\nu' + \nu'') t + (\theta_s + \theta'')) \end{aligned} \right\} \\ & + |I_s| |i'| e^{-\rho' t} \left\{ \begin{aligned} & \cos((\nu' - \omega_e) t + (\theta' - \theta_s)) \\ & + \cos((\nu' + \omega_e) t + (\theta' + \theta_s)) \end{aligned} \right\} \\ & + |I_s| |i''| e^{-\rho'' t} \left\{ \begin{aligned} & \cos((\nu'' - \omega_e) t + (\theta'' - \theta_s)) \\ & + \cos((\nu'' + \omega_e) t + (\theta'' + \theta_s)) \end{aligned} \right\} \end{aligned} \right]$$

which is different for each of the three phases.

APPENDIX 5
MEASUREMENT IN AN INDUCTION MOTOR SUPPLIED
WITH MODULATED TERMINAL VOLTAGE

Following a theoretical study of the response of an induction motor to modulated supply voltage it was necessary to investigate the correlation between predicted and experimental performance of the machine in this mode of operation. For this purpose, transistorized A.C. power commutating switches were available. These switches were developed at the University of Cape Town and are presently rated at 7 amp. pk. 320 volt pk. per phase. The test motor used for measurement was built for the purpose using standard production laminations. It is rated at 1,5 Kwatt, 380 volt, 3,3 amp., 1425 r.p.m., 50 Hz.

In the present work it was necessary to measure the following quantities :-

- (a) Current Instantaneous waveform; overall RMS.
- (b) Terminal Power Instantaneous waveform of nett three-phase power and the average value.
- (c) Torque Instantaneous value and average.

In general, the same measurement problems are encountered in any power modulator system. These measurements are complicated by two factors :-

- (a) Electrical noise generated by the modulation system. This noise is chiefly due to the very high rates of change of current in the power transistors during the forced commutation process.
- (b) Wideband signals (by power standards) require special techniques in order to record the instantaneous waveforms. The current waveforms typically contain frequency components at $f = |f_0 - m.f_{mod}|$ where f_0 is the desirable or fundamental component, f_{mod} is the modulation frequency and m is an integer. The nett torque and input power waveforms have a D.C. component and harmonics at integral multiples of the modulating frequency.

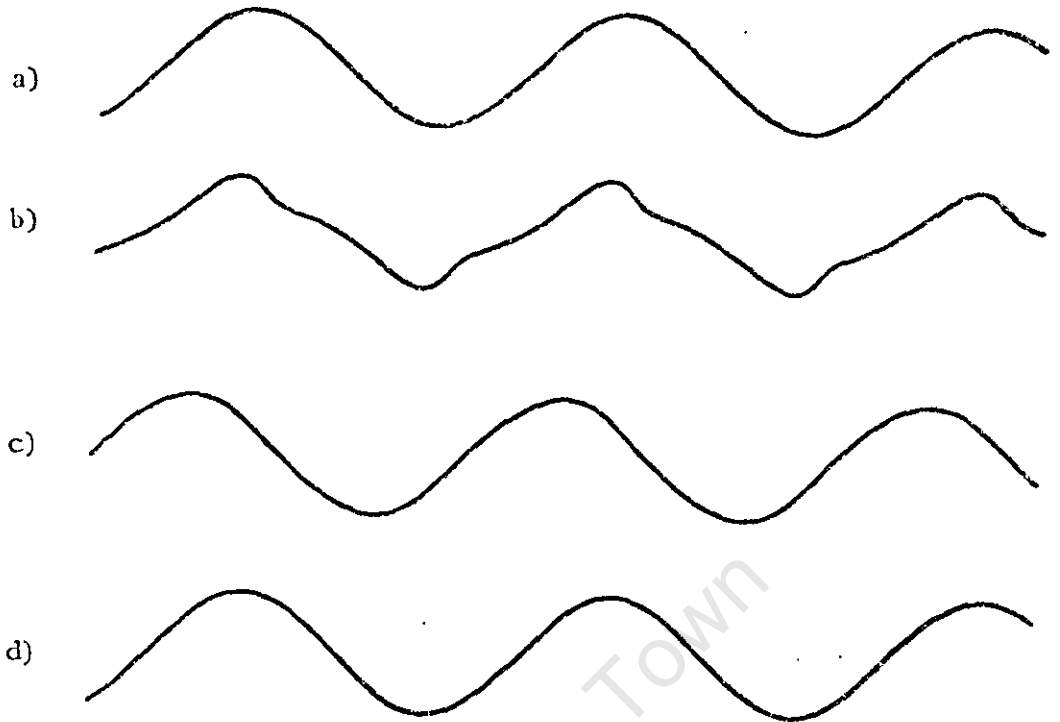
The measurement and recording systems are thus required to have a high noise immunity and a flat response over the appropriate bandwidth.

A) Measurement of Instantaneous Current

A.C. current measurements are normally made with the use of resistive shunts or current transformers.

(i) Although shunts may be carefully designed to minimize their reactance it is difficult to obtain a shunt which is entirely resistive. In many instances the ratio of reactive to resistive voltage dropped across a shunt may be small so that the relationship between nett voltage and current is sufficiently linear. In the case of a modulator using the available power switches, however, currents as high as 7 amps. are encountered with rise and fall times of only a few microseconds so that if the resistance of the shunt is low (as it must be) then the reactive voltages across it during commutation have peak values far in excess of the resistive voltages. In these systems, therefore, such a shunt is not acceptable. Fig. (122) illustrates the use of a standard shunt, although it has not been possible to record the high commutation spikes which occur in this oscillogram.

(ii) A well-designed current transformer with specified burden will accurately reproduce in its secondary winding a sinusoidal primary current of a specified frequency provided that the magnetic characteristic of the core may be assumed linear over the operating range. In practice the core is never perfect so that the magnetizing current component must to some extent be non-sinusoidal in order to maintain sinusoidal flux variation. This distortion is naturally not seen in the nett primary current but is reflected into the secondary current. As frequency is reduced for a constant amplitude of primary current the core flux must rise in order to maintain the nearly constant voltage dropped across a resistive burden. The magnetizing current component thus increases in magnitude and is also further distorted as the core is driven into saturation. This manifests itself in the secondary current as increasing magnitude and phase error and harmonic distortion, usually with a dominant third harmonic component. On the other hand, at very high frequencies hysteresis losses become a problem calling for special attention to the core. The current transformer is thus not always suitable for accurate measurement of multi-frequency signals.

Fig.121

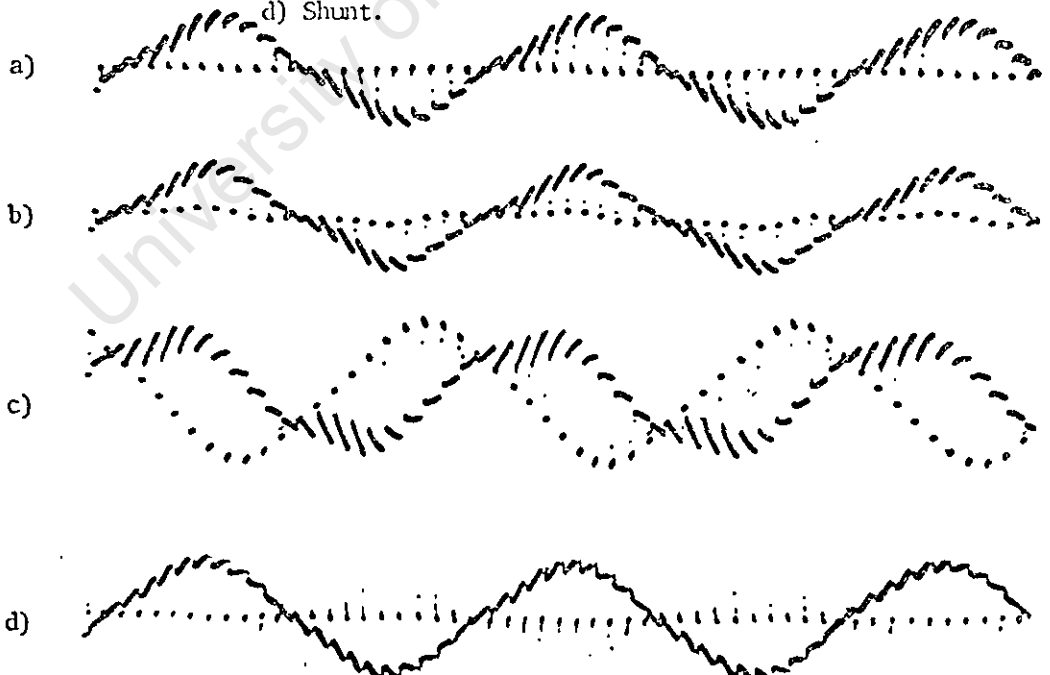
Measurement of a 50 Hz. 3 amp RMS sinusoidal current waveform using four separate current transducers to illustrate the low frequency distortion which occurs in high frequency C.T. probes.

a) Hall effect current transducer.

b) Tektronix high frequency clip-on C.T. and amplifier.

c) Hewlett-Packard clip-on C.T. with 1 Ohm resistive burden.

d) Shunt.

Fig.122

Measurement of supply current to an induction motor running under load with chopper modulated terminal voltage. The measurements were made simultaneously using the four transducers illustrated in Fig.(121)

Supply frequency= 50 Hz.

Chopping frequency= 1050 Hz.; $T(\text{on})/T(\text{total}) = 0,8$

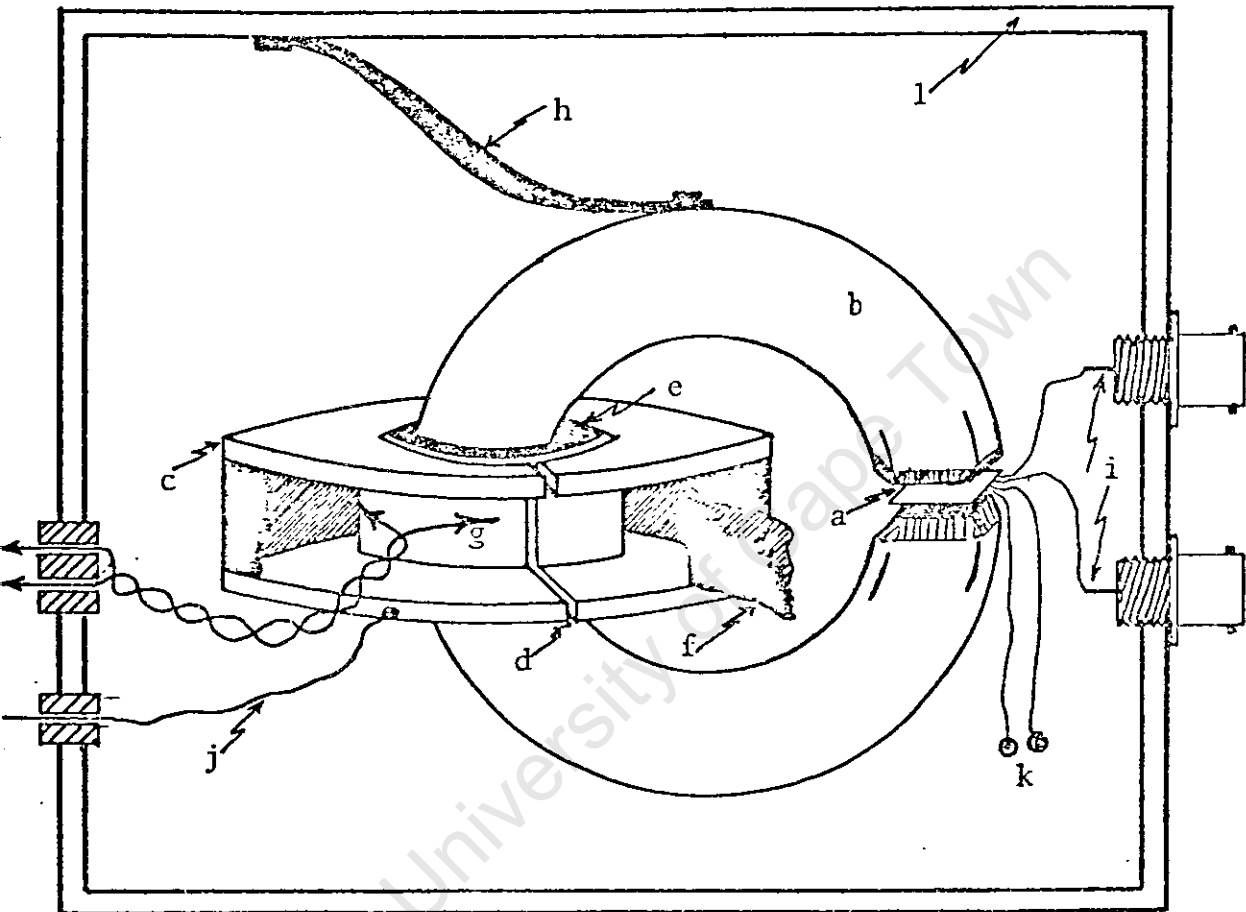
Slip = 5%

In particular at the lowest frequencies the results obtained from any current transformer will be unacceptable. The lowest frequency at which a given CT can operate will be largely governed by the size of the core. Thus a CT which is rated at the supply voltage frequency cannot be used if it is expected that the modulator will draw sub-harmonic or D.C. currents through the system. Figs. (121b) and (121c) illustrate low frequency distortion in the case of two different high frequency CTs. In each case the CTs are specified as having low amplitude and phase error between approximately 1 KHz and 40 MHz, but are hopelessly saturated with 50 Hz excitation. Figs. (122b) and (122c) show measurements made with these current probes of a chopped current in the supply to the test motor, illustrating how low frequency distortion distorts the overall waveform.

Although the 50 Hz distortion of these high frequency CTs is an extreme example of the low frequency deficiencies of the device, it was nevertheless decided that the CT was inherently unsuitable for measurement of many of the modulated currents of interest.

(iii) The Hall effect has been used for wideband multiplication in a variety of applications. If the control current is arranged to be a constant D.C. level and the magnetic flux density normal to the Hall plate is linearly related to the current in the power circuit, then ideally the Hall voltage is also linearly related to this current. This effect has successfully been used in the experiments on the chopper modulator, and Fig. (123) illustrates the physical construction of the transducers which have been built. Special attention has been given to the screening of the device, without which the signal to noise ratio obtained is unacceptable. Tested against a standard ammeter at 50 Hz and 500 Hz the Hall transducer deviates from the true value by less than 3% from 1 amp to 6 amps RMS. Although these transducers do not yield an exceptionally precise measurement, their major advantage is that the error is largely unaffected by frequency and the low frequency response extends to D.C. in this way.

SCREENED HALL EFFECT MULTIPLIER FOR USE AS A TRANSDUCER OF INSTANTANEOUS POWER OR CURRENT IN CHOPPER MODULATED POWER CIRCUITS.



- a) Hall plate (electrically insulated from the magnetic core)
- b) Magnetic core
- c) Aluminium bobbin to carry and screen the exciter coil
- d) Slot in bobbin to prevent transformer currents flowing.
- e) Electrical insulation between bobbin and magnetic core
- f) Copper screen to seal bobbin (cut away to view)
- g) 10 turn coil in series with the power circuit (totally screened and insulated)
- h) Earthing strap connecting magnetic core to steel outer screen which is also the measurement circuit earth.
- i) Output Hall voltage leads fed to high impedance differential measurement amplifier through two co-axial leads with screens tied to measurement circuit earth
- j) Lead for earthing the electrostatic screening around the exciter coil (usually tied to the neutral of the power system if the exciter coil is at high potential to neutral)
- k) Leads to source of bias current for Hall plate
- l) Steel magnetic and electrostatic outer screen (also measurement circuit earth)

Fig.123

The major source of error in the Hall generator is the misalignment potential produced between the Hall electrodes owing to their not being positioned accurately on an equipotential contour of the crystal in the absence of a magnetic field. The size of this potential is affected by Joule heating within the crystal as well as the magneto-resistive effect when the magnetic field is non-zero. It is possible to compensate for misalignment potential in a variety of ways provided that extra Hall electrodes are present on the crystal. However, such Hall plates were not available and those used were uncompensated. It has also been observed that a voltage is generated on the Hall electrodes in the absence of control current when alternating magnetic flux cuts the plate. This is presumably another form of misalignment potential resulting from voltage induced in the plate. Figs. (121) and (122) illustrate measurements made with the Hall transducer in comparison with the other devices which have been mentioned. It is thought that the principle employed in the Hall effect measurement is in fact the correct one for current measurement in convertor circuits, i.e. measurement of the magnetic flux produced by the current in an unsaturated core.

(B) Measurement of instantaneous power

The required waveform is the sum of the instantaneous volt-amp products in the three phases. In 50 Hz applications this can be achieved by any one of the many methods of multiplication. The voltage signal is usually isolated and reduced to workable levels by means of high resistance in series with a milliamp CT, and the current signal is isolated through a conventional CT. There is no doubt that these electronic methods could be employed in the chopper modulator provided that the signals could be isolated without distorting them, which is unlikely, and also providing that due consideration is given to the processing of the high frequency components which will be present. The Hall effect was thought to be eminently suitable for this application especially since a number of Hall elements were available. The physical construction of the power transducer is identical to that of the current transducer described previously and the magnetic field is similarly excited by a small coil in series with the power circuit.

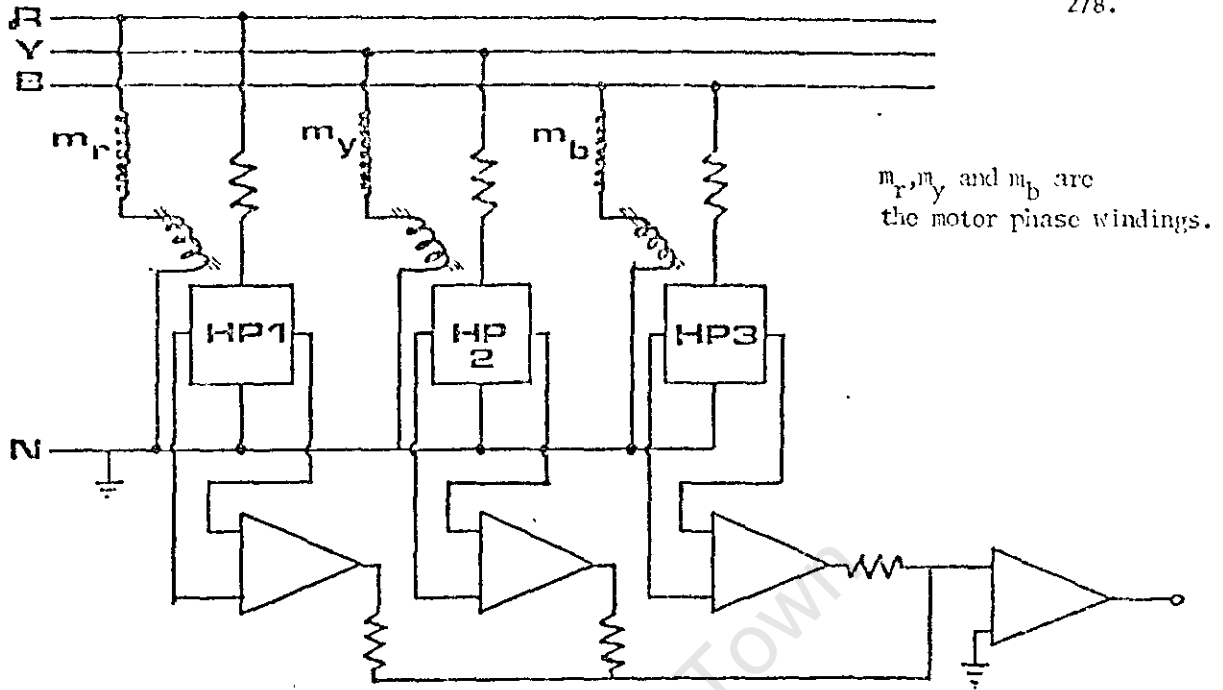


Fig.124 Circuit for measurement of instantaneous three phase power using Hall effect power transducers.

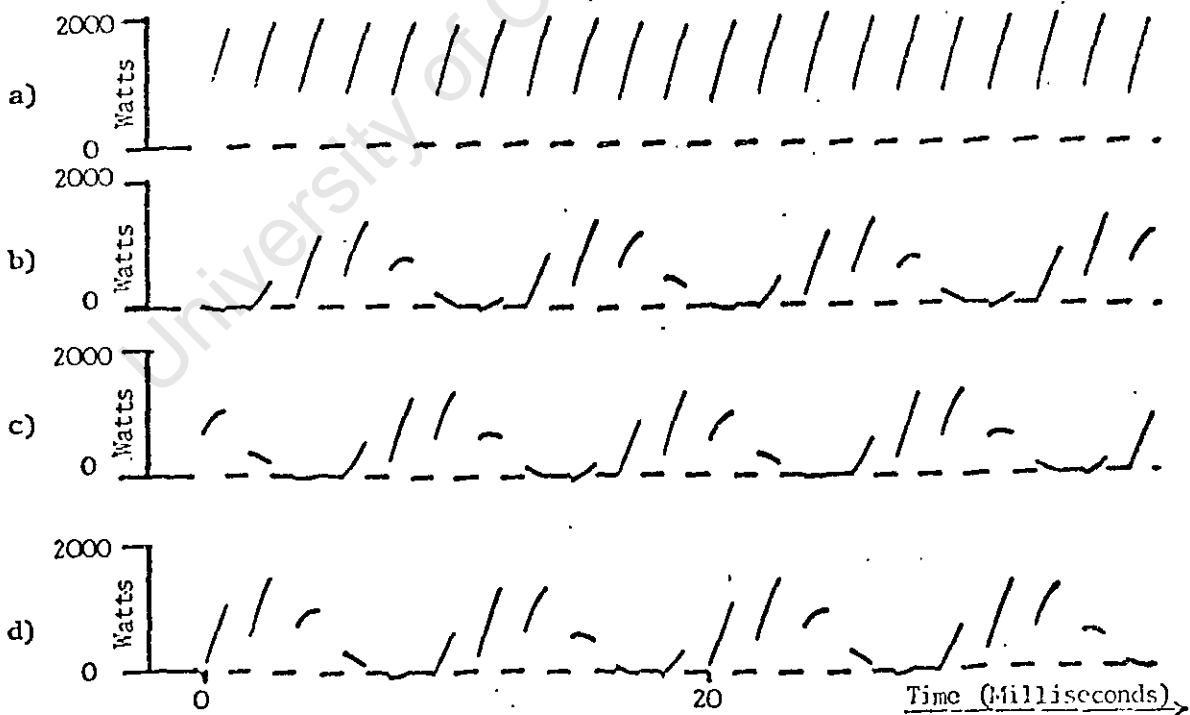


Fig. 125 Measurement of instantaneous power supplied to an induction motor running under load with chopper modulated terminal voltage.
 b), c), and d) are the phase powers and a) is the nett terminal power.
 Supply = 220 volt R.M.S. 50 Hz.
 Chopping frequency = 550 Hz.
 $T(\text{on})/T(\text{total}) = 0,5$
 Slip = 10%

The control current is derived from the phase voltage by direct connection to the plate via series resistance. The circuit for three-phase measurement is shown in Fig. (124). Note that there is an isolation problem in regard to the summation of the three Hall voltages which prevents them from simply being terminated and connected in series. Each plate in fact feeds a differential amplifier with high input impedance to ground and the resulting signals are summed to give a single-ended output proportional to instantaneous three-phase power. Fig. (125) illustrates a typical measurement in the chopper modulator made using this arrangement.

A major drawback of the present system is the amount of power which is dissipated in the divider resistors in the voltage circuit. At 220 volt this is approximately 10 watts per phase so that the resistance is effectively governed by the RMS phase voltage and the wattmeters must be differently calibrated for different RMS applied voltages.

(C) Measurement of instantaneous torque

With modulated terminal voltage the induction motor generates torque which has a D.C. level and alternating components at integral multiples of the modulating frequency. The measurement of instantaneous torque has always presented problems and can basically be achieved in two ways :-

(i) By accurately measuring the instantaneous acceleration of the shaft the acceleration torque is obtained if the inertia is a known quantity. This method has the drawback of being unable to detect torque if the shaft speed is really being held constant. Furthermore the shaft is continually subjected to slight forces through the action of ball or roller bearings in small machines and these will all constitute noise in such a torque measurement.

(ii) Torque may also be obtained by measuring the strain of the stator structure due to the reaction torque. This is the method which was used in the present work. In order to facilitate measurement it is necessary to ensure that some part of the stator will strain approximately linearly with generated torque.

This was achieved by suspending the stator on four symmetrically located steel cantilevers as illustrated in Fig. (126). Strain is detected in the cantilevers by means of two identically tuned microwave cavities which are coupled to the structure by rigid rods. Strain of the cantilevers distorts the cavities thus producing a change in the detected microwave output. The outputs of the two cavities are arranged in bridge configuration to provide better discrimination against lateral thrusts, amplitude variations from the microwave source and thermal expansion in the structure. Despite the bridge configuration, two major problems are encountered in the use of this technique:-

(a) The stator has a torsional mode resonance at a frequency which is determined by the stiffness of the cantilevers and the moment of inertia of the stator. In the test machine the cantilevers are 1" long by 2" broad by 1/8" thick mild steel and the stator has a mass of 13,7 Kg and moment of inertia of 0,114 Kg.m². The impulse response of the system in the torsional mode was recorded as in Fig. (127) and the continuous spectrum of this was obtained as in Fig. (128). It is seen that a reasonably flat response is achieved for torques up to about 300 Hz beyond which the resonance is strongly excited. This is something of a limitation in regard to modulator measurements, and as a result we have been careful to include instantaneous torque measurements in these results only where the critical frequencies are avoided.

(b) To minimize the mass of the stator, the laminations are held in a steel collar less than 1/4" thick. The flux in the airgap of the motor produces very large local stresses which act in a radial direction on the stator. If the whole structure is ideally rigid and symmetrical then these forces cancel to produce no nett lateral thrust. However, if the structure is relatively soft, as in this case, the stator is radially strained in areas of high local flux density. Since the flux distribution rotates, this effect manifests itself as a rotating strain wave which is detected by the strain gauges. As a result it will be observed that many of the instantaneous torque oscillograms show a noise component at twice the supply frequency.

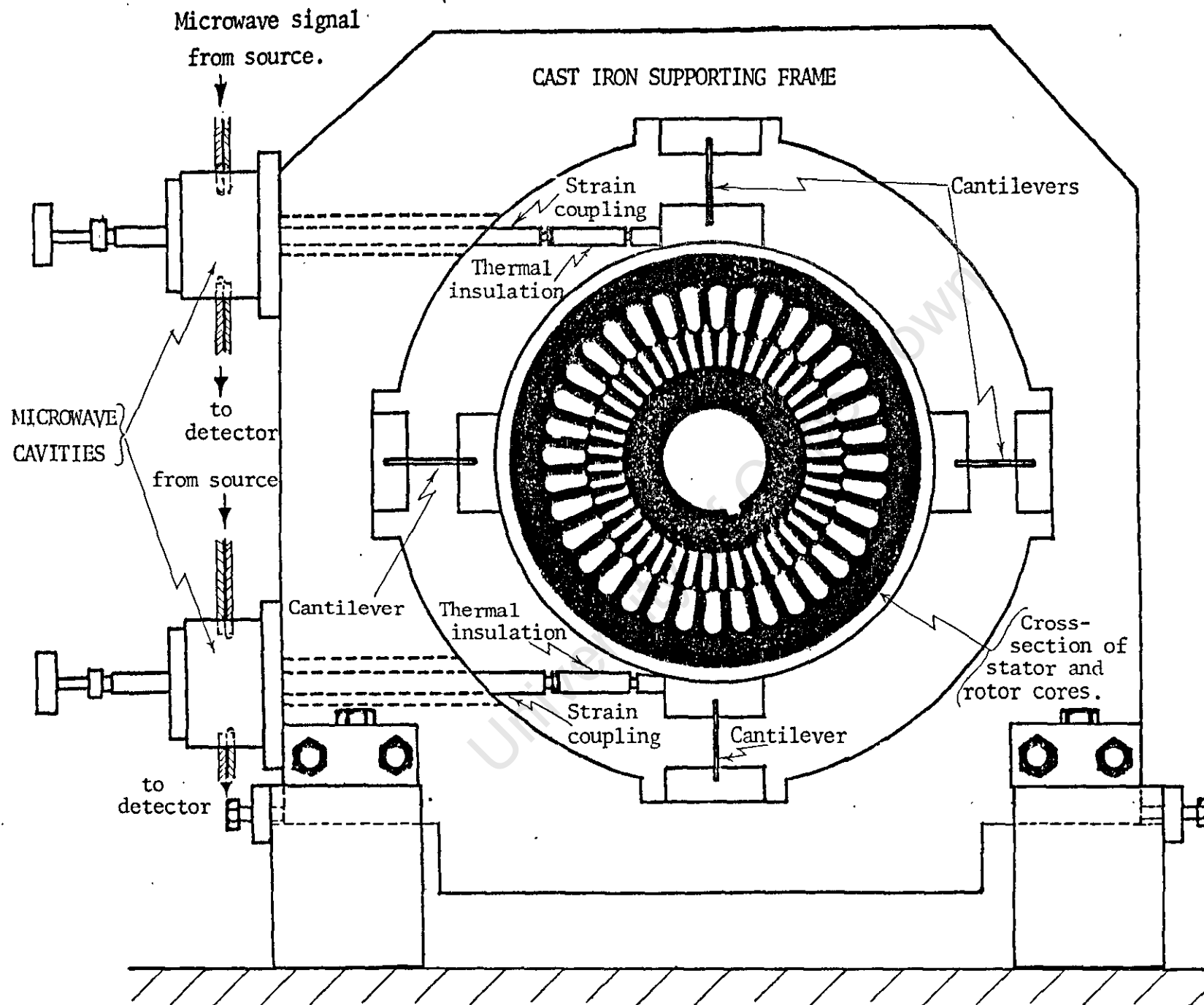


Fig.126

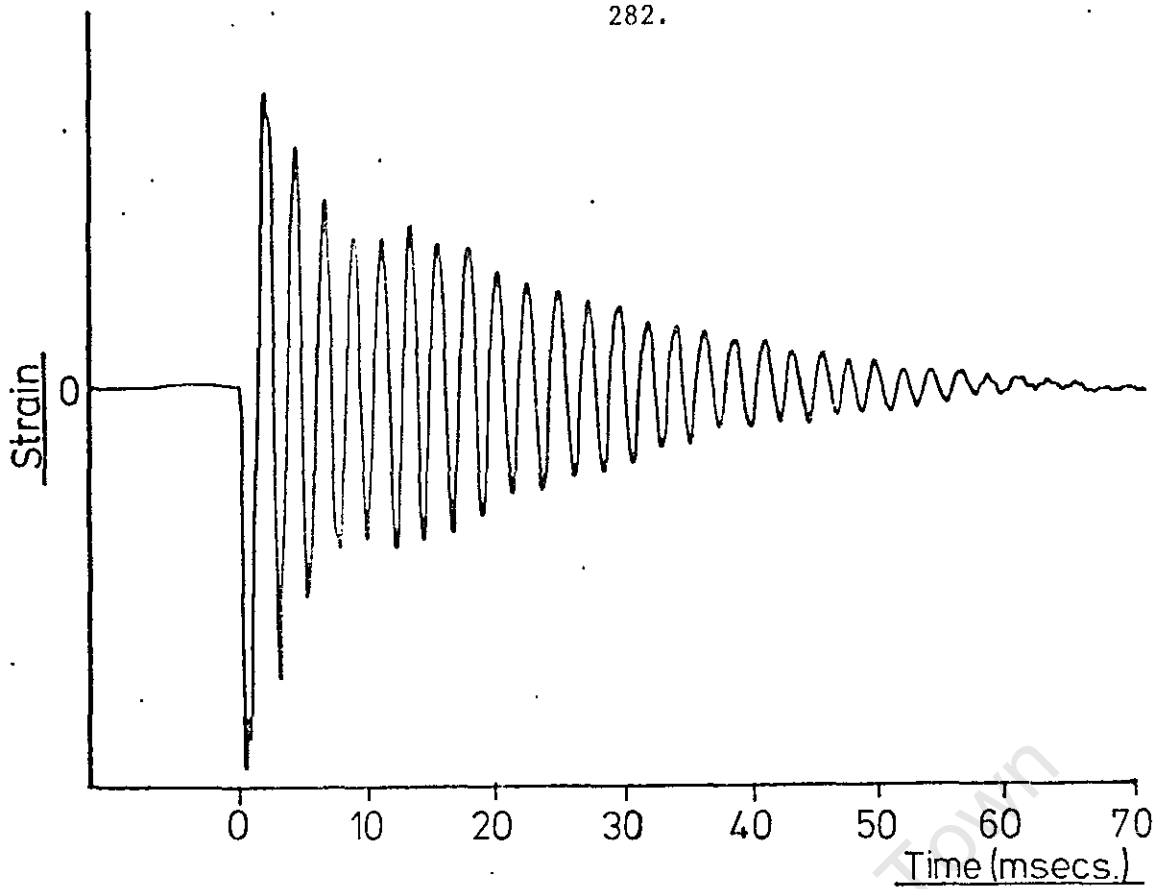


Fig. 127 Impulse response of the torque measurement system. The impulse was obtained by striking the stator sharply so as to excite the torsional mode.

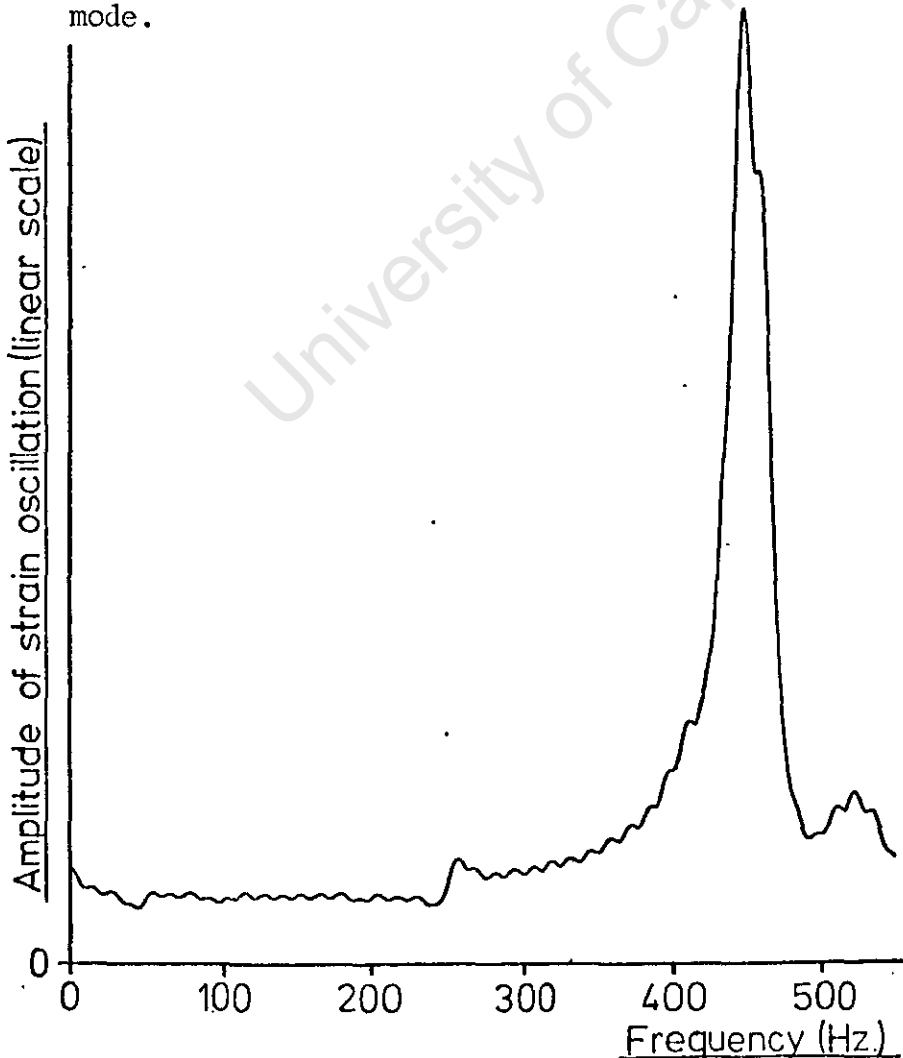


Fig. 128 Continuous spectrum of the impulse response showing resonance at approximately 440 Hz.

It must be mentioned that a similar arrangement has been used in previous work⁽²³⁾ where the principles of the strain measurement are discussed more fully. Average torque was measured with a D.C. dynamometer and spring balance.

(D) Measurement of RMS and average values

In the course of this work no special equipment was available for the accurate measurement of RMS voltage and current and average terminal power in the modulated system. Voltage and current were measured using moving-iron meters rated at class 0,5% accuracy 40 ... 50 ... 60 ... 400 Hz. Average power was obtained using an instrument with electro-dynamic movement and the same accuracy rating. Since no information is available regarding the accuracy of these instruments at frequencies higher than 400 Hz, it is difficult to assess the accuracy of the measurements made on the modulated signals. If the modulating frequency is above 400 Hz, then the entire harmonic content of the current waveforms falls within the dubious range and the accuracy will be related to the distortion factor in each case. Where the modulating frequency is below 400 Hz, then at least the three lowest frequency components of the current waveforms fall within the 1,0% accuracy specification. Since these components are usually the dominant contributors to the overall RMS, the accuracy should certainly be within 5% in these cases.

(E) Recording of instantaneous waveforms

Many of the waveforms shown in this thesis were recorded with the aid of a 4-channel transient recorder and subsequently drawn on an X-Y plotter. Because of the limited sampling rate and the slow rate of the plotter this method is not suitable for accurately recording waveforms with high rates of change. For these signals, a 4-channel oscilloscope and camera were used. The exposed film was reverse processed to yield the desirable black trace on white background. The author has spent considerable effort in pursuit of a high-speed recording technique which will yield results suitable for publication and has found these reversed oscillograms to be best. The power measurements in Fig. (25) are an illustration of this technique.

APPENDIX 6DETAILS OF THE TEST MOTOR

The test motor used in experimental work for this thesis was constructed using standard laminations. The stator and rotor laminations as well as the details of the stator suspension may be seen in Fig. (126). The stator bore is 5" and the core length is 2". The stator has 36 slots carrying a full-pitched 4-pole double layer winding with 80 turns per coil, 3 coils per phaseband. The rotor is a squirrel cage with deep copper bars, skewed by 1 stator slot pitch. The airgap is 0,015".

As seen from Fig. (129) the self-inductance of the stator windings with three-phase excitation depends on the state of saturation of the magnetic circuit and it is necessary to use different values of the inductive parameters for different applied voltages with a given fundamental frequency. Thus in the calculations which were performed for the case of the chopper modulator, different inductive parameters were used for each value of mark to total time ratio since the fundamental component of the output voltage varies linearly with this quantity. These values are listed in Table 7. However, in the calculations involving the chopper modulated frequency converter, the values which correspond to the fully fluxed condition were used throughout since it was assumed that with effective control of voltage and frequency the magnetic flux in the motor would always be held at a constant optimum value as discussed in Chapter 9.

In squirrel cage machines it is customary to evaluate leakage reactance and rotor resistance from the results of standard no-load and short-circuit tests. This is simply a curve-fitting technique which makes use of the following assumptions in most cases :-

- (a) Unity turns ratio assumed between stator and rotor "winding".
- (b) Stator and rotor self-inductances equal.
- (c) Stator and rotor leakage-inductances equal.

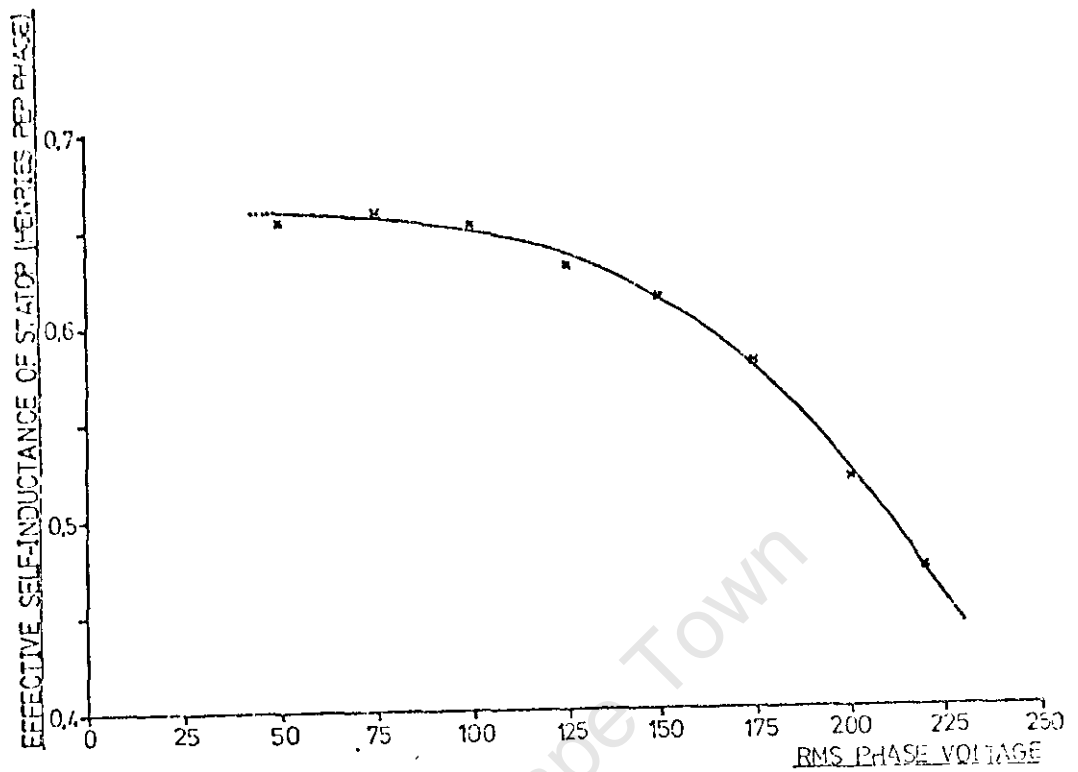


Fig. 129 Measured variation in stator inductance as a function of the terminal voltage (sinewave).

T(on)/T(total)	R ₁ Ohms	R ₂ Ohms	L ₁ =L ₂ Henries	M Henries
1.0	5.5	3.28	0.473	0.447
0.9			0.530	0.504
0.8			0.577	0.551
0.7			0.610	0.584
0.6			0.632	0.606
0.5			0.645	0.619
0.4			0.654	0.628
0.3 0.2 0.1			0.658	0.632

Table 7 Test motor parameters used for calculations on the chopper modulated system with supply voltage 220 volt 50 Hz. For calculations on the chopper modulated frequency converter values corresponding to T(on)/T(total)=1.0 were used throughout.

For the purposes of this thesis it was decided to use a similar technique but rather to fit the parameters to give the correct current and power values as measured at a slip of 10% with 50 Hz excitation. Using an iterative method the following values were determined :

$$R_s = 5.5 \Omega \quad ; \quad R_r = 3.28 \Omega$$

$$(L_1 - M') = 25.86 \text{ mH.}$$

These values are assumed constant throughout the calculations and their use is justified by the fact that they enable the calculation of terminal power and stator current to within 5% of the measured characteristics with 50 Hz sinusoidal excitation. See Fig(75) to (77). (The error in predicted torque is always greater owing to stray losses which are not taken into account by the idealized theory). It should be noted that any estimation of leakage inductance and resistance is inevitably a compromise since these parameters are subject to wide variations as a function of temperature and load.

APPENDIX 7

HOPPER*FCONVERTOR(1).NCHOPA

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1      C *  FORTRAN V PROGRAM FOR THE CALCULATION OF
2      C *  PERFORMANCE FACTORS FOR A THREE PHASE INDUCTION
3      C *  MOTOR SUPPLIED WITH MODULATED TERMINAL VOLTAGE.
4      C *  THE PROGRAM MAY BE APPLIED TO CASES WHERE THE
5      C *  MODULATION IS CYCLIC AND HAS THE GENERAL
6      C *  FORM (CHOP-RESTORE-CHOP-RESTORE-....-RESTORE WITH
7      C *  PHASE SHIFT). THIS MODULATION CYCLE IS DISCUSSED IN
8      C *  CHAPTER 4 OF >INDUCTION MOTOR RESPONSE WITH MODULATED
9      C *  TERMINAL VOLTAGE> - PH.D. THESIS UNIVERSITY OF CAPE TOWN,
10     C *  MARCH 1978.
11     C *  THE PROGRAM REQUIRES INPUT OF MOTOR PARAMETERS, DETAILS
12     C *  OF THE SUPPLY VOLTAGE, NUMBER OF CHOPS PER MODULATION
13     C *  CYCLE AND THE PHASE STEP FROM CYCLE TO CYCLE, AS WELL
14     C *  AS DETAILS OF THE RANGE OF OPERATING CONDITIONS OVER
15     C *  WHICH PERFORMANCE CALCULATIONS ARE REQUIRED.
16     C *  THE SOLUTIONS ARE BASED ENTIRELY ON THE ALGEBRAIC SOLUTIONS
17     C *  OF THE MOTOR EQUATIONS AS DISCUSSED IN THE ABOVE THESIS.
18     C *****
19     IMPLICIT REAL(L,M,I)
20     IMPLICIT COMPLEX(Z)
21     DIMENSION R(2),F(2),ZALPH(3),ZIS(2),ZIR(2),ZISS(3),ZIRR(3),ZC(2)
22     DIMENSION O(2),TI(20,2),Q(10,4),S(2),IS(2),IR(2),PS(2),PR(2)
23     DIMENSION ZU(2)
24     READ(8,10)M,L1,L2,R1,R2,N1
25     READ(8,10)V,NFS
26     READ(8,10)PSTEP,N
27     READ(8,10)SR1,SRINC,KSR
28     READ(8,10)SF1,SFINC,KSF
29     READ(8,10)MS1,MSINC,KM
30     10  FORMAT (I)
31     AZ=0
32     HZ=.0001
33     WRITE(5,15)M,L1,L2,R1,R2,N1,V,NFS,PSTEP,N
34     15  FORMAT (1H1,2HM=,F5.3,2X,3HL1=,F5.3,2X,
35     13HL2=,F5.3,2X,3HR1=,F6.3,2X,3HR2=,F6.3,2X,
36     11H1POLE PAIRS=,I2,/,16H SUPPLY VOLTAGE=,F6.2,
37     11X,8HVOLT RMS,2X,I3,2HHZ,/,18H CONVERTOR STEPS -,
38     1F7.4,1X,8HRAD WITH,1X,I2,1X,14HCHOPS PER STEP)
39     WRITE(5,20)
40     20  FORMAT (1H ,2X,3HRPM,6X,6HIMFUND,3X,5HTON/T,
41     14X,5HIMRMS,4X,6HISCRMS,3X,5HISRMS,4X,6HISFUND,
42     13X,6HISFANG,2X,6HDOPLFAC,2X,6HDISFAC,2X,
43     16HPOWFAC,2X,6HAVTORK,1X,7HAVPOWIN,1X,
44     18HAVPOWOUT,2X,4HEFF%)
45     DO 990 KSR=0,KSR
46     FS=2*3.14159*NFS
47     FSTEP=SR1+(KSR*SRINC)
48     WRITE(5,30)FSTEP
49     30  FORMAT (1H ,19HSTEPPING FREQUENCY=,F6.1,1X,2HHZ)
50     WSYNC=FS-(FSTEP*PSTEP)
51     FOUT=WSYNC/(2*3.14159)
52     DO 980 KMA=0,KM
53     MS=MS1+(KMA*MSINC)
54     DO 970 KSFA=0,KSF
55     FSLIP=(SF1+(KSFA*SFINC))
56     WSLIP=(2*3.14159*FSLIP)
57     W=SIGN(1,WSYNC)*(ABS(WSYNC)-WSLIP)
58     SPEED=(W*60.)/(2*3.14159*N1)
59     C*****CALC. ST.STATE STATOR CURRENT*****

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60      WR=FS=W
61      A=V*R2
62      B=V*WR*L2
63      Z1=CMPLX(A,B)
64      A=(R1*R2)-(FS*WR*L1*L2)+(FS*WR*(M**2))
65      B=(R1*L2*WR)+(FS*L1*R2)
66      Z2=CMPLX(A,B)
67      ZI1=Z1/Z2
68      ZISS(3)=ZI1
69      IF(ABS(REAL(ZISS(3))).LT.BZ) AZ=BZ
70      P1=ATAN2(AIMAG(ZISS(3)),REAL(ZISS(3))+AZ)
71      AZ=0
72      C*****CALC. ST.STATE ROTOR CURRENT*****
73      B=-WR*M
74      Z1=CMPLX(0,B)
75      B=WR*L2
76      Z2=CMPLX(R2,B)
77      ZI2=(Z1*ZI1)/Z2
78      ZIRR(3)=ZI2
79      IF(ABS(REAL(ZIRR(3))).LT.BZ) AZ=BZ
80      P2=ATAN2(AIMAG(ZIRR(3)),REAL(ZIRR(3))+AZ)
81      AZ=0
82      C*****CALC. MOTOR POLE FREQUENCIES*****
83      SIGMA=1-((M**2)/(L1*L2))
84      R01=R1/(SIGMA*L1)
85      R02=R2/(SIGMA*L2)
86      A=-(R01+R02)/2
87      B=W/2
88      Z1=CMPLX(A,B)
89      A=-R01*R02*SIGMA
90      B=W*R01
91      Z2=CMPLX(A,B)
92      Z3=((Z1**2)+Z2)
93      A=CABS(Z3)
94      B=ATAN2(AIMAG(Z3),REAL(Z3))
95      IF(B.LT.0)B=B+6.28318
96      A=SQRT(A)
97      B=B/2
98      Z3=CMPLX((A*COS(B)), (A*SIN(B)))
99      ZALPH(1)=Z1+Z3
100     ZALPH(2)=Z1-Z3
101     ZALPH(3)=CMPLX(0,FS)
102     C*****SOLVE INIT. VALS. OF SC. TRANS. CURRENTS**
103     DO 100 J=1,2
104     Z1=-M*ZALPH(J)
105     Z2=L1*ZALPH(J)
106     ZC=CMPLX((REAL(Z2)+R1),AIMAG(Z2))
107     100 ZC(J)=Z1/Z2
108     Z1=ZC(1)-ZC(2)
109     ZIS(1)=ZC(1)*(ZI1-(ZC(2)*ZI2))/Z1
110     ZIS(1)=-ZIS(1)
111     ZIS(2)=ZC(2)*((ZC(1)*ZI2)-ZI1)/Z1
112     ZIS(2)=-ZIS(2)
113     DO 200 J=1,2
114     200 ZIR(J)=ZIS(J)/ZC(J)
115     C*****INITIALIZE MAIN LOOPS*****
116     C*****KH=1 FOR ON-INTERVALS*****
117     C*****KH=2 FOR SHORTED INTERVALS*****
118     CAK IS ORDINAL FOR SUCCESSIVE INTERVALS**
119     C*****O(1),O(2) ACCUMULATE INTEG.(I**2)*

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120 C*****U ACCUMULATES TORQUE INTEGRAL***
121 C*****ZU(1) ACCUMULATES I(SUP) FUNDAM.***
122 C*****ZU(2) ACCUMULATES I(MOT) FUNDAM.***
123 C*****
124 O(1)=0
125 O(2)=0
126 U=0
127 ZU(1)=CMPLX(0,0)
128 ZU(2)=CMPLX(0,0)
129 DO 460 KH=1,2
130 DO 450 K=1,N
131 IF (KH.EQ.1) GOTO250
132 M4=1-M5
133 ZISS(3)=CMPLX(0,0)
134 ZIRR(3)=CMPLX(0,0)
135 GOTO260
136 250 M4=M5
137 260 T=1/(FSTEP*N)
138 I9=M4*T
139 T8=T9
140 T0=T-T9
141 C*****
142 C***CALC.MODIFIED VALUES OF TRANSIENTS**
143 C***FOR THE PARTICULAR CONVERTOR CYCLE**
144 C*****
145 DO320 J=1,2
146 ZF=ZALPH(J)
147 ZF=CMPLX(REAL(ZF),(AIMAG(ZF)-FS))
148 ZF2=(ZF*N*T)
149 ZF2=CMPLX(REAL(ZF2),(AIMAG(ZF2)+PSTEP))
150 Z1=-CEXP(ZF2)
151 Z1=CMPLX((REAL(Z1)+1),AIMAG(Z1))
152 DO280 J1=1,N
153 J2=(2*J1)-1
154 TT(J2,1)=(J1-1)*T
155 280 TT(J2+1,1)=(J1*T0)+((J1-1)*T9)
156 J0=(2*K)+(KH-2)
157 DO290 J1=1,J0
158 290 TT(J1,2)=0
159 J9=(2*K)+(KH-1)
160 J0=(2*N)
161 DO300 J1=J9,J0
162 300 TT(J1,2)=PSTEP
163 Z2=CMPLX(0,0)
164 DO310 J1=1,J0
165 ZF1=ZF*TT(J1,1)
166 ZF1=CMPLX(REAL(ZF1),(AIMAG(ZF1)+TT(J1,2)))
167 310 Z2=Z2+(((1)*((J1+1))*CEXP(ZF1))
168 Z1=Z2/Z1
169 ZISS(J)=ZISS(J)*Z1
170 ZIRR(J)=ZIRR(J)*Z1
171 IS(J)=CAHS(ZISS(J))
172 IR(J)=CAHS(ZIRR(J))
173 IF(ABS(REAL(ZISS(J))).LT.BZ) AZ=BZ
174 PS(J)=ATAN2(AIMAG(ZISS(J)),REAL(ZISS(J))+AZ)
175 AZ=0
176 IF(ABS(REAL(ZIRR(J))).LT.BZ) AZ=BZ
177 PR(J)=ATAN2(AIMAG(ZIRR(J)),REAL(ZIRR(J))+AZ)
178 320 AZ=0
179 I1=CAHS(ZISS(3))

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180          I2=CABS(ZIRR(3))
181          C*****
182          C**CALC. INTEGRAL(I**2) OVER 1 INTERVAL*
183          C*****
184          DO 330 J=1,2
185              R(J)=-REAL(ZALPH(J))
186          330      F(J)=AIMAG(ZALPH(J))
187              S1=(I1**2)*T9/2
188          DO 340 J=1,2
189          340      S(J)=((IS(J)**2)/(4*R(J)))*(1-EXP(-2*R(J)*T9))
190              S1=S1+S(1)+S(2)
191          DO 360 J=1,2
192              S2=(IS(1)*IS(2)*(R(1)+R(2)))/(((R(1)+R(2))**2)+((F(1)-F
193              S3=((F(1)-F(2))*T9)+(PS(1)-PS(2))
194              S4=SIN(S3)
195              S5=COS(S3)
196              S6=(F(1)-F(2))/(R(1)+R(2))
197              S7=(S6*S4)-S5
198              S8=S7*EXP(-T9*(R(1)+R(2)))
199              Q(3,J)=S8*S2
200          DO 350 J1=1,2
201              S2=(I1*IS(J1)*R(J1))/((R(J1)**2)+((F(J1)-FS)**2))
202              S3=(T9*(F(J1)-FS))+(PS(J1)-P1)
203              S4=SIN(S3)
204              S5=COS(S3)
205              S6=(F(J1)-FS)/R(J1)
206              S7=(S6*S4)-S5
207              S8=S7*EXP(-T9*R(J1))
208          350      Q(J1,J)=S8*S2
209          360      T9=0
210              T9=T8
211              Q(KH)=Q(KH)+Q(3,1)+Q(1,1)+Q(2,1)-Q(3,2)-Q(1,2)-Q(2,2)
212              Q(KH)=Q(KH)+S1
213          C*****
214          C**CALC. INTEGRAL(TORQUE) OVER 1 INTERVAL*
215          C*****
216          J2=0
217          DO 380 J=1,3
218          DO 380 J1=1,3
219              J2=J2+1
220              Z1=ZISS(J)*CONJG(ZIRR(J1))
221              Z2=ZALPH(J)+CONJG(ZALPH(J1))
222              Q(J2,1)=CABS(Z1)
223              Q(J2,2)=-REAL(Z2)
224              Q(J2,3)=AIMAG(Z2)
225              IF(ABS(REAL(Z1)).LT.BZ) AZ=BZ
226              Q(J2,4)=ATAN2(AIMAG(Z1),REAL(Z1)+AZ)
227          380      AZ=0
228          DO 390 J=1,8
229              S1=Q(J,1)/((Q(J,2)**2)+(Q(J,3)**2))
230              S2=-Q(J,2)*SIN((Q(J,3)*T9)+Q(J,4))
231              S3=-Q(J,3)*COS((Q(J,3)*T9)+Q(J,4))
232              S4=(S2+S3)*EXP(-T9*Q(J,2))
233              S5=-(Q(J,2)*SIN(Q(J,4)))+(-Q(J,3)*COS(Q(J,4)))
234              S6=S4-S5
235              S7=S1*S6
236          390      U=U+S7
237          C*****
238          C**CALCULATE FUNDAMENTAL I(SUPPLY) AND I(MOTOR)*****
239          C*****

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240      T=1/FSTEP
241      DO 450 JJ=1,2
242      IF(JJ.EQ.2)GO TO 395
243      IF(KH.EQ.2)GO TO 450
244      395 DO 440 J=1,3
245      IF(J.LT.3)GO TO 400
246      IF(PSTEP.EQ.0)GO TO 397
247      IF(JJ.EQ.2)GO TO 400
248      397 Z1=ZISS(J)*T9/T
249      GO TO 430
250      400 W1=FS-((JJ-1)*PSTEP*FSTEP)
251      W0=W1-AIMAG(ZALPH(J))
252      Z1=CMPLX(-REAL(ZALPH(J)),W0)
253      Z2=CEXP(-Z1*T9)
254      Z3=CMPLX(1,0)
255      Z1=(Z3-Z2)*ZISS(J)/(Z1*T)
256      430 THETA=(FS-W1)*(((K-1)*(T/N))+((KH-1)*T0))
257      Z2=((-1)**(KH+1))*CEXP(CMPLX(0,THETA))
258      Z1=Z1*Z2
259      IF(W1.LT.0)Z1=CONJG(Z1)
260      ZU(JJ)=ZU(JJ)+Z1
261      440 CONTINUE
262      450 CONTINUE
263      460 CONTINUE
264      C*****
265      C***CALC.PERFORMANCE FACTORS*****
266      C*****
267      T=1/FSTEP
268      X1=T0*N*AIMAG(ZI1*CONJG(ZI2))
269      X1=(X1+U)*3*M*N1/T
270      X2=CABS(ZU(1))
271      X20=CABS(ZU(2))
272      X3=ATAN2(AIMAG(ZU(1)),REAL(ZU(1)))
273      X4=COS(X3)
274      X5=3*V*X2*X4
275      X6=1.414*SQRT((O(1)+O(2))/T)
276      X7=1.414*SQRT(O(1)/T)
277      X8=1.414*SQRT(O(2)/T)
278      X9=X2/X7
279      X10=X9*X4
280      X11=X1*W/N1
281      C=X11/X5
282      A=SIGN(1,X5)
283      B=SIGN(1,X11)
284      IF(A.LT.0.) C=1/C
285      IF((A*B).LT.0.) C=0.
286      X12=100*C
287      WRITE(5,900)SPEED,X20,M5,X6,X8,X7,X2,X3,X4,X9,X10,X1
288      1,X5,X11,X12
289      900 FORMAT (1H ,F9.2,F8.3,F7.3,5(F9.3),3F8.3,F9.3,2F9.2,F7.2)
290      970 CONTINUE
291      980 CONTINUE
292      990 CONTINUE
293      STOP
294      END

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