



J/ψ Suppression in Ultra-Relativistic Heavy Ion Collisions

Robert Alexander Ritchie

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Abstract

A review is undertaken of methods to account for the suppression in the number of J/ψ produced in central collisions of ultra-relativistic heavy ions performed at the CERN SPS, concentrating on models for quark gluon plasma suppression and hadronic gas absorption of J/ψ . The aim in mind is to establish whether, from data collected by the NA38 group on J/ψ suppression, it can be concluded that evidence exists for the formation of a quark gluon plasma during these collisions.

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Chapter 1

Introduction

The development of Quantum Chromo-Dynamics (Q.C.D.) as a theory of strong interactions has led to a general understanding of hadronic phenomenology and a good quantitative description of small distance phenomenology [1]. Q.C.D. appears to predict the confinement of quarks and gluons to within colourless hadrons, in agreement with experiment, and asymptotic freedom on the short distance scale. To conclude that this theory is the correct one, it is necessary to test it with experiment at varying scales of energy. It is particularly relevant to test it at high energy, where perturbative solutions of the theory exist which can be compared with experiment.

In addition from Lattice studies of Q.C.D. [2] a deconfinement phase transition is predicted, where quarks and gluons ordinarily confined to colourless hadrons become deconfined in a macroscopic region where-in they move freely forming a quark gluon plasma. In fig(1.1) a

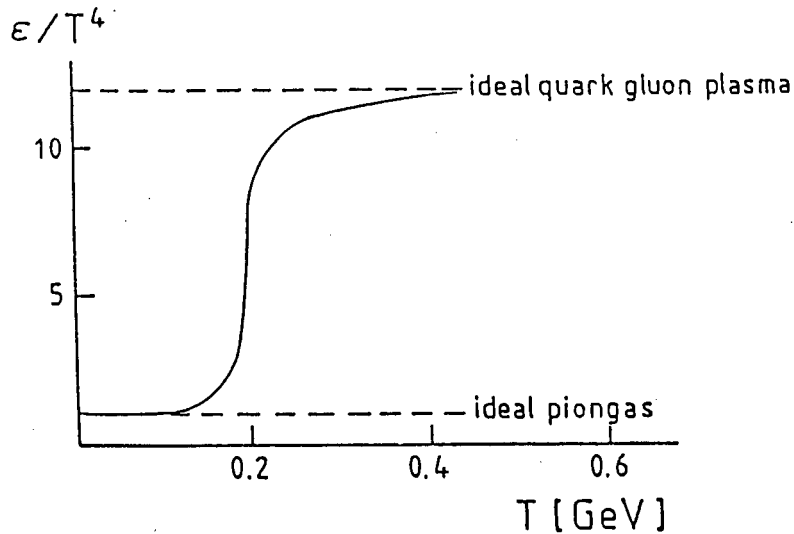


Figure 1.1: Schematic behaviour of the energy density ϵ of strongly interacting matter, as a function of the temperature T [3].

sketch of the energy density as a function of temperature is given, where the phase transition

is evident by the sudden change in slope of the energy density corresponding to an increase in the available degrees of freedom. It is important to estimate the conditions required for, and the nature of, the phase transition in order to search experimentally for verification of it, and to discover perhaps new physics when quarks and gluons become deconfined. Unfortunately the estimates arising from Lattice Q.C.D. have large uncertainties involved at present which complicate the prediction.

Three situations are now discussed where quark gluon plasma formation should be possible. For times of $t \sim 10^{-5}$ sec the early universe was extremely hot and hence was in a deconfined state, then as it cooled hadronisation occurred. It therefore should be possible to search for effects arising from this phase transition; one such effect could be the presence of quark gluon plasma droplets which still remain [4]. An alternative possibility is that the cores of massive neutron stars are sufficiently dense that hadronic matter becomes deconfined. As with the early universe, this deconfined state is not immediately observable, which is a major obstacle in the way of confirmation of a phase transition. A further alternative, is the possibility of the collision of high energy heavy ions, to create a region of sufficiently high energy density that the deconfinement transition is undergone. The present ultra-relativistic heavy ion collision program at CERN where 200 GeV/A oxygen ions are collided on large atomic mass targets such as uranium is a typical example of such an attempt to produce a quark gluon plasma experimentally.

A diagram illustrating conditions required for the phase transition to quark gluon plasma is sketched in fig(1.2) where the three situations discussed above are shown. The early

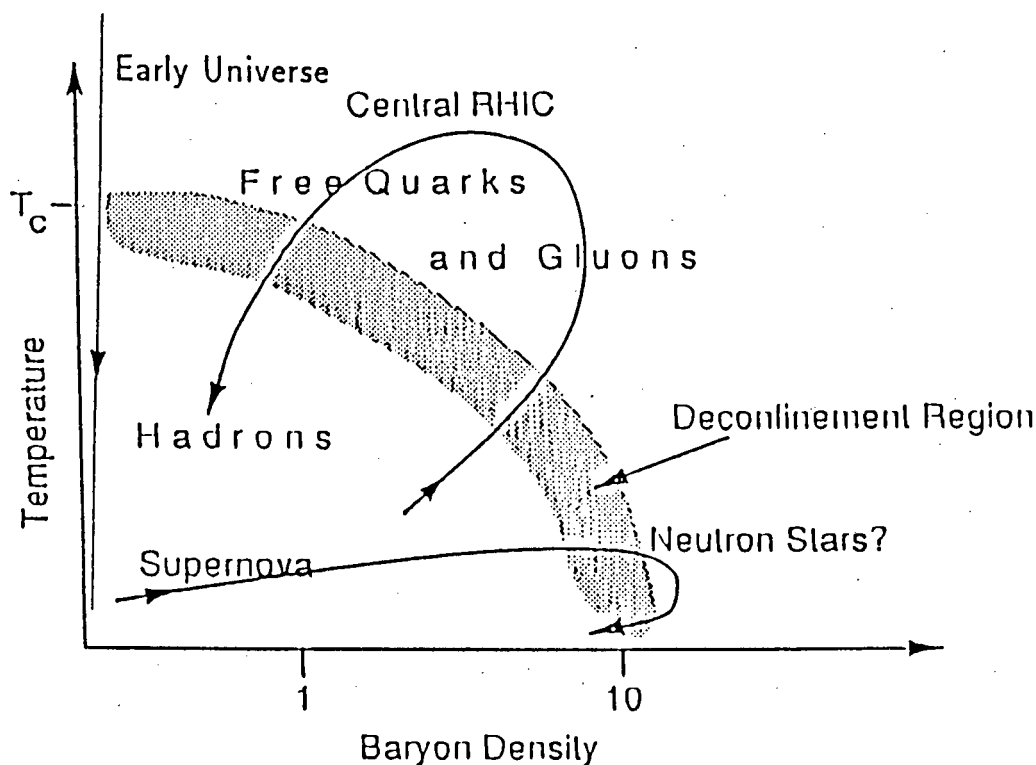


Figure 1.2: Expected phase diagram for nuclear matter [5].

universe is believed to have had high temperature and low baryon number density, whilst

the neutron star is cool but has a high density. The heavy ion collision scenario occurs between these two limiting situations. A typical estimate for the critical temperature of deconfinement for the formation of quark gluon plasma in heavy ion collisions is found from Lattice calculations to be of the order of 160-200 MeV, with a corresponding energy density of the order of $\varepsilon = 2 \text{ GeV/fm}^3$ [6]. This work concentrates only on the heavy ion collision scenario. It should be realised that an important feature is present for these collisions even if quark gluon plasma formation does not occur: the time-scales involved in the production of secondaries are extremely short, of the order of the hadronic formation time, which already allows the possibility of new physics to enter when collisions occur between partially formed hadrons.

A major problem with the quark gluon plasma produced in heavy ion collisions is that it is expected to be very short lived, of the order of 10 fm/c. Thus by the time the collision is observed, the plasma has disappeared, and one is left looking for indirect evidence of its formation. Unfortunately, the lack of direct experimental evidence complicates the situation, as confidence in the existence of the deconfined phase of matter rests on the need for several independent sets of indirect evidence which are consistent with plasma formation and are not easily explainable within the framework of known hadronic physics.

It is the need for such evidence that has led to the proposal for J/ψ suppression in ultra-relativistic heavy ion collisions by Matsui and Satz [7]. This thesis reviews the J/ψ suppression scenario, especially with the aim in mind of establishing whether, from data collected by the NA38 group¹ on suppression arising in the production of J/ψ from heavy ion collisions, it can be concluded that evidence exists that a quark gluon plasma has formed during the collision.

Other such possibilities of indirect evidence for quark gluon plasma formation should be mentioned at this moment. These are not considered in any detail within the work which follows but are listed below:

- Thermal production of dileptons from the plasma as a measure of the temperature of the central region [8].
- Excess strangeness production from quark gluon plasma has been predicted [9,10]. However recent calculations within the framework of a hadronic gas appear to be able to reproduce the available data [11].
- Pion and photon interferometry may provide an indication of the size of the system after hadronisation which is dependent on the space-time evolution of the plasma or hadronic gas [12].
- The correlation between p_T and dN/dy depends on the equation of state and hence could indicate plasma production [8].

1.1 Overview of the Material Covered

The basic development of the following chapters is as follows: the next chapter describes the hydrodynamics expected to be valid for plasma production in the central region, which forms for head-on ultra-relativistic heavy ion collisions. In particular, conditions valid for heavy ion collisions occurring at the CERN SPS are discussed with reference to the simple model

¹for a list of references see section 3.2

introduced earlier in the chapter. It should be realised that the hydrodynamics is based on a simple ideal relativistic gas model valid whether a quark gluon plasma forms or just an extremely dense hadronic system.

Chapter 3 introduces the prediction by Matsui and Satz [7] of J/ψ suppression by quark gluon plasma and the predicted momentum dependence of this suppression, to produce a qualitative picture of J/ψ suppression in heavy ion collisions. Also discussed in this chapter is the NA38 data on J/ψ suppression. The fourth chapter deals with more advanced models of plasma suppression of J/ψ in an attempt to quantitatively reproduce the NA38 data.

In chapter 5, alternative scenarios for the suppression of J/ψ are discussed, especially with respect to the possibility of reproducing the NA38 data within the realm of hadronic physics. Here consideration is made of the destruction of J/ψ by its collision with hadrons from either part of the nucleus in which it is created, or by the hadronic gas of secondaries which forms from the collision. Other possibilities to account for J/ψ suppression, such as $c\bar{c}$ formation processes, are also referred to in this chapter. This is followed by a summary of the conclusions which can be drawn from the NA38 J/ψ data and the models which have been discussed, including a suggestion of where future experiments might be able to decide between models of absorption and plasma suppression of J/ψ .

The units chosen are such that $c = \hbar = 1$ unless otherwise stated.

Chapter 2

Hydrodynamic Description

2.1 Introduction

The high temperatures and densities attained in central ultra-relativistic heavy ion collisions produce a system with a short mean free path between inter-particle collisions. This will allow for local thermal equilibrium to be attained during the collision, if there is sufficient time available. The high number of secondaries makes it desirable to average over particle trajectories, describing this averaged motion in terms of a relativistic hydrodynamical model of the system, where thermal equilibrium is maintained on a local scale. This is particularly relevant for quark gluon plasma formation where, the number of degrees of freedom of the system will increase considerably further and a hadronisation phase transition has to be dealt with.

However for the use of hydrodynamics to be valid certain criteria should be satisfied. It is the purpose of this chapter to discuss these deciding if the hydrodynamical picture is relevant, and then to develop a relatively simple hydrodynamical model for the expansion of the hot region assuming that plasma formation has occurred. The hadronisation phase transition is not considered in any great detail.

2.2 The Validity of Hydrodynamics

For a hydrodynamical description of quark gluon plasma to be valid certain criteria must be satisfied. These are:

- (a) A high number of inter-particle collisions must exist, corresponding to a short mean free path length λ for the quarks inside the plasma. Some knowledge of λ must therefore be obtained to check if the collision rate is high; in the discussion which follows, λ will be determined from an estimate of the energy density ε obtained initially in the collision.
- (b) Statistical (thermal) equilibrium of the plasma must be reached, and maintained on a local scale (globally equilibrium will not be attained as the heated region will expand rapidly at velocities comparable to that of the speed of light). This is aided by a high collision rate required in (a) above, but most certainly requires some finite time t_0 to set in.

An estimate of the energy density ε can be made from the naive picture of the collision of two highly energetic Lorentz contracted pan-cake like protons which completely stop each other,

so that the entire kinetic energy of the collision is released in a flat pancake “fireball”. This is the picture discussed by Landau in [13], where an estimate of:

$$\varepsilon = \frac{2E_{\text{cm}}}{V_p * m_p/E_{\text{cm}}} = 620 \text{ GeV/fm}^3$$

is made for the initial energy density, in a collision with center of mass (c.o.m.) energy $E_{\text{cm}} = 25 \text{ GeV}$ and rest frame proton volume $V_p = \frac{4}{3} \times \pi(0.8)^3 \text{ fm}^3$ [14].

The above estimate needs correction for:

- a limiting minimum thickness of $\sim 1 \text{ fm}$ which is predicted for the incident protons, due to the fact that nucleons are composed of partons, some of which will always be at rest when the proton is viewed from an arbitrary frame. Thus an increase occurs in the volume into which the energy is released, with a corresponding drop in energy density to a much lower value of 23 GeV/fm^3 [14].
- the inside-outside cascade, where the faster moving projectile hadronises outside the nucleus must be taken into account. This effectively yields the nuclei partially transparent when they collide. The degree of transparency may be measured in terms of the transverse energy flow relative to the total energy in the center of mass — the degree of asymmetry observed for differing beam energies, is an indication of the transparency [15] in the collision, which is quite dependent on the collision energies. The region created between the projectile and the target as they move apart contains the debris of the collision and is referred to as the “central region” of the collision, and is the region with small rapidity (in the c.o.m. frame of the collision). Typically the central region has almost zero net baryon number, as this is concentrated in the projectile and target.
- Finally, it should be realised that after the initial collision has occurred, a finite time t_0 should be allowed so that the resulting system can approach statistical equilibrium on a local scale to satisfy (b) above. This allows for some time during which the volume expands yet further, reducing the energy density. In proton-proton collisions this is a problem, because the region of high energy density may disappear before there is sufficient time for thermalisation to occur on a local scale.

The Bjorken model [16] provides an alternative estimate for the energy density which includes both finite size effects of the colliding projectile and the transparency of the collision. Further, nucleus-nucleus collisions are considered, where significantly more particles are formed, creating a much larger region of high density. These have a correspondingly longer life-time which may be sufficient to allow local thermal equilibrium to be attained and for plasma formation to occur. The assumptions involved in the model are:

- The nuclei must be quite transparent (if they are not then this model will underestimate the energy density; for complete stopping the above estimate by Landau with the corrected projectile volume is a better estimate). A simplification then exists for the model as the central region mentioned above forms, in which the net baryon number is approximately zero; it is for this region that the energy density will be calculated. This model is valid only in the region of very energetic central collisions of at least $\sim 200 \text{ GeV/A}$ in the laboratory (lab) frame [15]. It is in this region that the high energy density remains the longest, as the partons (in the c.o.m. frame of the collision) have less energy concentrated in the longitudinal motion and hence the increase in volume is less. A schematic diagram for the above picture is displayed in fig(2.1)

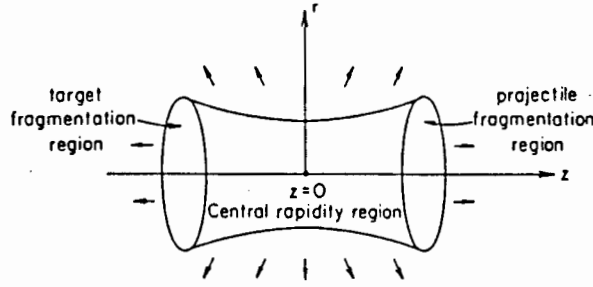


Figure 2.1: Schematic view of a central collision of two highly Lorentz contracted ultra-relativistic heavy ions in the c.o.m. frame, showing the target and projectile fragmentation regions as well as the central rapidity region [19].

- the longitudinal component of the plasma velocity is assumed to scale as $v = z/t$ which includes a boost invariance for the central region. In proton-proton collisions a central rapidity plateau is observed [17]. If one assumes that the nuclei behave merely as a collection of nucleons for the time and energy scales involved, a similar plateau is to be expected for the case of two colliding nuclei (experimental data from the HELIOS collaboration [18] is available more recently on the transverse energy distribution as a function of rapidity from O-W collisions in the CERN heavy ion program which does not show this rapidity plateau. However for more energetic collisions it should emerge). Thus one would expect that the measured quantities from the central region will be boost invariant. By introducing the above scaling the boost invariance is automatically built into the model, simplifying the hydrodynamical equations. The effect of the transverse hydrodynamical flow is to remove the scale invariance at radial distances of the order of the nuclear radius; this effect will be dealt with later.

An estimate of ε , assuming most of the energy goes into the production of pions can be made for nucleus-nucleus collisions using the data from proton-proton scattering, where a value for the charged pion production

$$\frac{dN_{ch}}{dy} \approx 3$$

is obtained. A value of $\langle E_\pi \rangle \sim 400$ MeV is used in the model which is assumed independent of the colliding projectile and target, and $N_\pi/N_{ch} \sim 0.5$ for pions, from which a value of

$$\frac{d\langle E_\pi \rangle}{dy} = 3 \times 0.4 \times 1.5 = 1.8 \text{ GeV}$$

is obtained for the central region. The energy for a thin slab around the origin, in the c.o.m. frame can then be found.

$$E = N_0 \frac{dN}{dy} \langle E_\pi \rangle \Delta y$$

where N_0 is the number of initial collisions. Assuming no transverse expansion

$$dt dz d^2x = \tau d\tau dy dz d^2x \quad (2.1)$$

it follows that $dy = dz/t_0$ at the origin. Hence

$$E = N_0 \frac{dN}{dy} \langle E_\pi \rangle \frac{\Delta z}{t_0}$$

and an estimate for the central energy density ε is obtained.

$$\varepsilon = \frac{E}{V} = \frac{N_0}{A_T} \frac{d\langle E_\pi \rangle}{dy} \frac{1}{t_0} \quad (2.2)$$

where A_T is the transverse area of the plasma. At least a time of ~ 1 fm should be allowed for the system to form. For the cases:

1. p-p with $A_T \sim 4 \text{ fm}^2$

$$\varepsilon = \frac{1}{4} \frac{1.8}{1} \approx 0.5 \text{ GeV/fm}^3$$

2. U-U $A_T \sim 4 \text{ fm}^2$

$$\varepsilon = \frac{238}{173} \frac{1.8}{1} \approx 2.5 \text{ GeV/fm}^3$$

From the boost symmetry, ε may be found for $z \neq 0$ if t_0 is replaced by the proper time τ_0 at z .

Using ε , an estimate for the mean free path length $\lambda = 1/\rho\sigma$ can be obtained. As a rough estimate the energy per quark ~ 300 MeV (as in the case when they are confined to nucleons at rest), and the q-q cross-section $\sigma \sim 5$ mb (from p-p scattering), can be used to estimate λ :

1. for p-p $\lambda \sim 1$ fm with a plasma radial distance of ~ 1 fm
2. for U-U $\lambda \sim 0.25$ fm with a plasma radial distance of ~ 7 fm

Thus the use of the hydrodynamical description is not really justified for p-p collisions, but is reasonable for U-U collisions. For the CERN heavy ion collisions at 200 GeV/A where partial stopping occurs, an even higher energy density will exist than would be predicted by the Bjorken model and hence a hydrodynamical approach should be valid, though the projectile (oxygen or sulphur) has considerably fewer nucleons, which may result in too small a collision volume for local thermal equilibrium to be established.

2.3 Hydrodynamic Equations

A set of hydrodynamic equations describing the time evolution of the expansion of the quark gluon plasma produced in heavy ion collisions from t_0 , the time for local thermal equilibrium to be attained, till the hadronisation phase transition and free streaming starts, is now derived. A more complete derivation of the results which follow below can be found in [14] by Cleymans et al.

2.3.1 The Equations

The starting point is the energy momentum tensor $T^{\mu\nu}$ for a perfect gas

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - g^{\mu\nu} P \quad (2.3)$$

where $\varepsilon(x)$ is the energy density, $P(x)$ is the pressure and $u^\mu(x)$ is the four-velocity flow of the gas, at x^μ . The convention used is that $u^\mu u_\mu = 1$, γ will be the boost matrix; and the units are chosen such that $c = \hbar = 1$. The energy momentum conservation law is:

$$\partial_\mu T^{\mu\nu} = 0 \quad (2.4)$$

this corresponds to

$$\frac{\partial \varepsilon}{\partial t} = 0, \quad \frac{\partial P}{\partial x^i} = 0$$

in the rest frame of the plasma. (i.e. no change in energy density with time, but changes allowed spacially, and no local pressure gradients, though time fluctuations in P are allowed.) A good derivation of this form of the energy momentum tensor for the plasma is found in [14]. Only central collisions will be considered in this treatment as the inherent cylindrical symmetry simplifies the solutions; the chosen spacial coordinates are then:

1. z -axis, corresponding to the beam axis
2. r coordinate, which will measure the radius out from the beam axis
3. ϕ coordinate, which will not appear in the equations due to the symmetry, will be the rotation around the z -axis

Eqn(2.4) contains four conservation equations, but the symmetry implies that effectively only three of them are independent. The first is found via the projection of the four velocity u^μ on eqn(2.4), whilst the other two follow, by finding the normal to the four velocity $g^{\mu\nu} - u^\mu u^\nu$ and then projecting it against eqn(2.4). Thus:

$$u_\nu \partial_\mu T^{\mu\nu} = 0 \quad (2.5)$$

$$(g_{\mu\nu} - u_\mu u_\nu) \partial_\lambda T^{\lambda\mu} = 0 \quad (2.6)$$

There is also the conservation of baryon number density $n_B(x)$ which should be included as there are no outside sources.

$$\partial_\mu (n_B u^\mu) = 0 \quad (2.7)$$

The expanded form of these equations, obtained by substituting for $T^{\mu\nu}$ from eqn(2.3) and rearranging the terms, yields

$$u^\mu \partial_\mu \varepsilon + (\varepsilon + P) \partial_\mu u^\mu = 0 \quad (2.8)$$

$$u^\mu (\varepsilon + P) \partial_\mu u^\lambda - \partial^\lambda P + u^\lambda \frac{d}{d\tau} P = 0 \quad (2.9)$$

$$u^\mu \partial_\mu n_B + n_B \partial_\mu u^\mu = 0 \quad (2.10)$$

From thermodynamics:

$$dU = -P dV + T dS + \mu dN \quad (2.11)$$

where U is the internal energy and P is the pressure, for a free gas. For quarks and gluons it is necessary to consider the effect of the vacuum. If one considers the M.I.T. bag model, the internal energy should be corrected for by a constant B , to take into account the energy associated with the vacuum [14]. Therefore the energy density of the plasma becomes:

$$\varepsilon = \varepsilon_{\text{th}} + B$$

for the pressure a reduction occurs, given by:

$$P = P_{\text{th}} - B$$

By examining the differentials in eqn(2.11) one concludes that $U = U(V, S, N)$; from this functional dependence, after finding the differential form

$$dU = \frac{\partial U}{\partial V} dV + \frac{\partial U}{\partial S} dS + \frac{\partial U}{\partial N} dN$$

and comparing this with eqn(2.11), the following relations hold

$$P = -\frac{\partial U}{\partial V}, \quad T = \frac{\partial U}{\partial S} \quad \text{and} \quad \mu = \frac{\partial U}{\partial N} \quad (2.12)$$

If one scales the volume up by a factor of λ then, provided no phase transitions are occurring and no external fields act, the entropy S , particle number N , and internal energy U , all scale by the same factor. That is

$$U\lambda = U(V\lambda, S\lambda, N\lambda)$$

For a small change in λ one arrives at

$$d\lambda U = d\lambda \left(\frac{\partial U}{\partial V} V + \frac{\partial U}{\partial S} S + \frac{\partial U}{\partial N} N \right)$$

Substituting for the partial derivatives from eqns(2.12) yields the equation

$$U = -PV + TS + \mu N$$

from which a differential form can be found

$$dU = -P dV - V dP + T dS + S dT + \mu dN + N d\mu$$

comparing this with the eqn(2.11) one concludes that

$$V dP = S dT + N d\mu$$

or in terms of densities

$$dP = s dT + n d\mu \quad (2.13)$$

and eqn(2.11) becomes

$$\varepsilon_{\text{th}} = -P + sT + n\mu \quad (2.14)$$

(where ε_{th} is the thermodynamic energy density, s is the entropy density and n is the number density) This has the differential form

$$d\varepsilon_{\text{th}} = -dP + T ds + n d\mu$$

which, on substitution of eqn(2.13), yields an equation in terms of the entropy density and baryon number

$$d\varepsilon_{\text{th}} = T ds + \mu dn \quad (2.15)$$

To show that entropy conservation is contained in the hydrodynamic equations substitute eqns(2.14,2.15) into eqn(2.8), realising that B is a constant, so it does not influence the differential in eqn(2.14) and that it cancels out from eqn(2.15) when used in eqn(2.8), with the result that ε_{th} and P_{th} may be used in eqn(2.8)

$$T(u^\mu \partial_\mu s + s \partial_\mu u^\mu) + \mu(u^\mu \partial_\mu n_B + n_B \partial_\mu u^\mu) = 0$$

which becomes under rearrangement of the terms

$$T \partial_\mu (s u^\mu) + \mu \partial_\mu (n_B u^\mu) = 0$$

From the baryon number conservation of eqn(2.7), it then follows that the entropy is conserved. (i.e. $\partial_\mu (s u^\mu) = 0$)

2.3.2 One Dimensional Solution

As a first approximation, under the observation that the radial velocity v_r is usually much less than the longitudinal v_z , a one dimensional solution will be found, where

$$u^\mu = (\cosh(y_p), 0, 0, \sinh(y_p)) \quad (2.16)$$

With this form of the velocity in mind, one defines the spacial rapidity y_s as

$$y_s = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right) \quad (2.17)$$

and the momentum rapidity, y_p by

$$y_p = \frac{1}{2} \ln \left(\frac{E+p_z}{E-p_z} \right) \quad (2.18)$$

It should be noted that the 1-spacial, 1-time form of the four-velocity is indeed given by eqn(2.16) with y_p , being the momentum rapidity. Next the spacial proper-time is introduced via

$$\tau_s = \sqrt{t^2 - z^2} \quad (2.19)$$

With these definitions of spacial-rapidity and proper-time one can find the inverse transformations

$$t = \tau_s \cosh(y_s), \quad z = \tau_s \sinh(y_s) \quad (2.20)$$

The transformation of the derivatives is

$$\begin{pmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \cosh(y_s) & -\sinh(y_s) \\ -\sinh(y_s) & \cosh(y_s) \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial \tau_s} \\ \frac{1}{\tau_s} \frac{\partial}{\partial y_s} \end{pmatrix} \quad (2.21)$$

Using the transformation defined above on the conservation eqns(2.8,2.9,2.10), realising that the simplification to only one spacial dimension reduces eqn(2.9), which is in fact two equations, to one equation (i.e. a total of only two equations from energy-momentum conservation), it can be shown that

$$\tau_s \frac{\partial \varepsilon}{\partial \tau_s} + \tanh(y_p - y_s) \frac{\partial \varepsilon}{\partial y_s} + (\varepsilon + P) \left[\frac{\partial y_p}{\partial y_s} + \tanh(y_p - y_s) \tau_s \frac{\partial y_p}{\partial \tau_s} \right] = 0 \quad (2.22)$$

$$\frac{\partial P}{\partial y_s} + \tanh(y_p - y_s) \tau_s \frac{\partial P}{\partial \tau_s} + (\varepsilon + P) \left[\tau_s \frac{\partial y_p}{\partial \tau_s} + \tanh(y_p - y_s) \frac{\partial y_p}{\partial y_s} \right] = 0 \quad (2.23)$$

$$\tau \frac{\partial n_B}{\partial \tau_s} + n_B \frac{\partial y_p}{\partial y_s} + \tanh(y_p - y_s) \left(\frac{\partial n_B}{\partial y_s} + n_B \tau \frac{\partial y_p}{\partial \tau_s} \right) = 0 \quad (2.24)$$

As a further approximation, the scaling $v = z/t$ discussed earlier can be introduced, which one should remember came in as a result of the observed rapidity independence of the measured particle production (the radial motion will modify the solution but this is ignored in the first approximation).

The introduction of the scaling will be shown to imply that the spacial and momentum rapidities are in fact equal. As scaling holds eqn(2.17) may be rewritten

$$y_s = \frac{1}{2} \ln \left(\frac{1+v}{1-v} \right)$$

Multiplying through by $m\gamma/m\gamma$ where γ is the boost matrix, and m is the invariant mass, yields

$$y_s = \frac{1}{2} \ln \left(\frac{\gamma m + \gamma m v}{\gamma m - \gamma m v} \right)$$

but, $E = \gamma m$ and $p = m\gamma v$ so that

$$y_s = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) = y_p$$

It may also be shown that the spacial proper-time defined in eqn(2.19) is in fact just the proper-time of the plasma. The proper-time is defined by

$$d\tau^2 = dt^2 - dz^2$$

with scaling this becomes

$$d\tau^2 = dt^2(1 - v^2)$$

where v is a constant over time and space for some portion of the plasma. Thus one may integrate the equation which becomes

$$\tau^2 = t^2 - z^2$$

Having just shown that scaling implies $y_s = y_p$ it should be noted that considerable simplification of eqns(2.22,2.23,2.24) occurs

$$\tau \frac{\partial \varepsilon}{\partial \tau} + \varepsilon + P = 0 \quad (2.25)$$

$$\frac{\partial P}{\partial y} = 0 \quad (2.26)$$

$$\tau \frac{\partial n_B}{\partial \tau} + n_B = 0 \quad (2.27)$$

The first two were also derived by Bjorken [16]. The analytic solutions of these equations are readily obtainable (unlike the case for the radial solutions).

The entropy conservation equation simplifies as well

$$\tau \frac{\partial s}{\partial \tau} + s = 0$$

This may be integrated to obtain

$$s(\tau) = \frac{s_0 \tau_0}{\tau}$$

To understand the meaning of this equation in terms of entropy, it should be evaluated at $y_s = 0$, which implies that the proper-time may be replaced by the time; then from rapidity invariance, the solution can be applied for differing rapidities, by going back to the proper-time. Hence

$$ts(t) = s_0 t_0 = \text{constant} \quad (2.28)$$

From the computation of the entropy, via the entropy density

$$\int dt S = \int d^4x s(x)$$

using eqn(2.1) and the resulting simplification at $y_s = 0$, it follows that

$$\left[\frac{dS}{dy_s} \right]_{y_s=0} = \int d^2x ts(x) \quad (2.29)$$

But the model is one-spacial dimensional hence the integral is just some cross-sectional area. Thus one concludes that

$$\frac{dS}{dy_s} = \text{constant}$$

If one takes the entropy density conservation during the expansion, then with knowledge of $\frac{dS}{dy}$ at the end of the expansion, one may infer that this must have been similar at some earlier stage of the plasma, before expansion. Pion multiplicity (per unit rapidity) will be proportional to $\frac{dS}{dy}$ and hence should reveal some information about the initial entropy density.

Until this point, no equation of state has been assumed, but to solve eqn(2.25), some knowledge of how ϵ and P are related must be used. For an ideal relativistic gas the equation of state is $\epsilon = 3P$. To be more general, an equation of state may be found via the definition

$$\left(\frac{\partial P_{th}}{\partial \epsilon_{th}} \right)_S = c_s^2 \quad (2.30)$$

where c_s is the speed of sound. Unfortunately it is expected that c_s will be quite strongly temperature dependent around T_c (hadronic phase transition temperature), though at temperatures above T_c the dependence is not as strong. Thus, for a first approximation c_s will be assumed to be temperature independent, with the interpretation of this approximation being that the solution is found for temperatures much higher than T_c . Integrating eqn(2.30) with constant c_s one obtains

$$P_{th} = c_s^2 \epsilon_{th} \quad (2.31)$$

with c_s entering as a parameter.

The integration of eqn(2.25) follows immediately to yield

$$\varepsilon_{\text{th}} = \varepsilon_{0\text{th}} \left(\frac{\tau_0}{\tau} \right)^{1+c_s^2}$$

and hence

$$(\varepsilon - B) = (\varepsilon_0 - B) \left(\frac{\tau_0}{\tau} \right)^{1+c_s^2} \quad (2.32)$$

This should be compared with the case where the volume expands in proper-time resulting in an energy density decrease

$$(\varepsilon - B) = (\varepsilon_0 - B) \frac{\tau_0}{\tau}$$

The energy density drops off faster in the Bjorken model, the difference being attributed to the pressure [14]. In the central region the nett baryon number density is expected to be small due to the high density of particles created initially and the absence of the two nuclei from the central region. Thus to a good approximation $\mu = 0$ can be used, and the temperature dependence may be found, using eqn(2.14) together with eqn(2.31) to obtain

$$\frac{sT}{\tau} = \frac{s}{c_s^2} \frac{dT}{d\tau}$$

which integrates to

$$T = T_0 \left(\frac{\tau_0}{\tau} \right)^{c_s^2} \quad (2.33)$$

Combining eqn(2.28) with eqn(2.33) yields the temperature dependence for the entropy

$$s = s_0 \left(\frac{T}{T_0} \right)^{1/c_s^2} \quad (2.34)$$

The stability of the solutions under the assumption of scaling has been checked by taking a first order correction [14]. Eqn(2.25) is found to be unchanged by the correction whilst eqn(2.26) becomes

$$\frac{\partial P}{\partial y_s} + y'_{ps}(\varepsilon + P) = 0$$

where $y'_{ps} = y_p - y_s$. This allows for pressure variations in the rapidity. Using the equation of state it may be solved to yield

$$(\varepsilon - B) = (\varepsilon_0 - B) e^{-\left(\frac{1+c_s^2}{c_s^2} \right) y'_{ps}(y_s - y_{s0})}$$

This implies some kind of energy density cut-off for large rapidity, confining the solution to the central region, as well as allowing the presence of pressure variations with rapidity (note this does not contradict the fact that no local pressure variations are allowed in the rest frame of the plasma). Having solved the one-dimensional case, the next problem is the solution of the equations in 3-dimensions in the cylindrically symmetric case. Unfortunately no analytical solutions are known at this stage for the three-dimensional case.

2.3.3 Three-Dimensional Solutions

Most solutions for the 3-dimensional case assume Bjorken scaling. This simplifies the solution by reducing the dimension to solve by one (the solution at $z = 0$ is found and then boosted to find the solution elsewhere, by replacing the time by the associated proper-time); the effect is to decouple the transverse expansion from the longitudinal one. First-off an intuitive solution is examined — the transverse expansion should result in an outflow radially of the plasma, thus cooling the system at larger radius, with hadronisation occurring as the plasma temperature approaches T_c . The transition will move radially inwards at some velocity, related to c_s , until it reaches the beam axis. Outside of this the hadron gas continues to expand till it becomes so rarified that thermal equilibrium is no longer maintainable (the mean free path length has increased to a scale much larger than that of the gas) leading to free streaming of the hadrons. The inclusion of the longitudinal motion however, is expected to cause greater cooling of the plasma in comparison to the transverse effect, reducing the time as a function of the radius at which the phase transition occurs; thus in comparison to the Landau model of [13], the plasma will be cooler at the same radius. The radial solution should be similar to the two dimensional Landau case and the expected approach would be to form Riemann similarity solutions due to the scale invariant nature of the radial expansion.

The solution is found at $z = 0$. Here the four-velocity is given by

$$u^\mu = (\cosh \alpha, \sinh \alpha, 0, 0)$$

Conservation of entropy, and of energy-momentum (see eqn(2.9)), are considered first. In cylindrical coordinates about the beam axis these equations become

$$\partial_0 u^0 + \partial_r u^r + \frac{u^r}{r} + \partial_z u^z + \frac{u^0 \partial_0}{s}(s) + \frac{u^r \partial_r}{s}(s) = 0$$

$$(\varepsilon + P)(u^0 \partial_0 u^0 + u^r \partial_r u_0) - \partial_0 P + u^0 u_0 \partial^0 P + U^0 u_0 \partial^r P = 0$$

To write these equations in terms of the temperature, with the Baym solution [19] in mind, the thermodynamic equations given by eqn(2.13) and eqn(2.14), with $\mu = 0$ in the energy-momentum conservation equation are used; the equation of state for an ideal gas with constant c_s , is introduced for the entropy conservation equation to relate the entropy density to the temperature. This enables one to write

$$c_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{s}{T} \frac{\partial T}{\partial s}$$

from which it follows that

$$\partial s = \frac{T}{c_s^2 s} \partial T$$

and hence the equations become

$$\partial_0 u^0 + \partial_r u^r + \frac{u^r}{r} + \frac{\partial_z u^z + u^0 \partial_0 \ln T}{c_s^2} + \frac{u^r \partial_r \ln T}{c_s^2} = 0$$

$$(u^0 \partial_0 u^0 + u^r \partial_r u^0) - \partial_0 \ln T + u^0 u_0 \partial^0 \ln T + u^0 u_r \partial^r \ln T = 0$$

Inserting the explicit form for the 4-velocity gives the equations

$$\frac{1}{c_s} \frac{\partial \ln T}{\partial t} + v_r c_s \frac{\partial \alpha}{\partial t} + \frac{v_r}{c_s} \frac{\partial \ln T}{\partial r} + \frac{v_r c_s}{r} + \frac{c_s}{t} = 0$$

$$\frac{\partial \alpha}{\partial t} + v_r \frac{\partial \alpha}{\partial r} + v_r \frac{\partial \ln T}{\partial t} + \frac{\partial \ln T}{\partial r} = 0$$

Adding and subtracting these equations yields

$$\frac{\partial}{\partial t} \left(\frac{1}{c_s} \ln T \pm \alpha \right) + \frac{v_r \pm c_s}{1 \pm v_r c_s} \frac{\partial}{\partial r} \left(\frac{1}{c_s} \ln T \pm \alpha \right) + \left(\frac{v_r}{r} + \frac{1}{t} \right) \frac{c_s}{(1 \pm v_r c_s)} = 0 \quad (2.35)$$

This equation has a driving term which prevents $(1/c_s \ln T \pm \alpha)$ from being constant in either r or t at any stage of in the expansion.

The above equation has been solved [19]. The solution is built up from a study of eqn(2.35) in one dimension. Here the driving terms do not appear which makes the problem easier to solve. Similarity flow is assumed which reduces the partial d.e. in two variables to a full differential in one variable. The solution is similar to the non-relativistic Riemann solution, and is compared below to the Bjorken scaling solution

$$T_R = T_0 \left(\frac{t - X + R}{t + X - R} \frac{1 - c_s}{1 + c_s} \right)^{\frac{c_s}{2}} \quad (\text{Riemann solution}) \quad (2.36)$$

$$T = T_0 \left(\frac{t_0}{t} \right)^{c_s^2} \quad (\text{Bjorken solution})$$

for initial conditions of, $T = T_0$ and the velocity of $v = 0$, on the range $-\infty < 0 < R$ where R corresponds to some initial radius of the plasma. A rarefaction wave is expected to propagate inwards, and once it reaches the axis, the solution is expected to change over to the scaling type. It is found that the Riemann solution can admit solutions containing discontinuities in entropy, under certain conditions

$$\frac{d}{dT} \left(\frac{sc_s}{T} \right) < 0$$

which are expected to be possible when the hadronisation transition occurs, thus allowing shock fronts to be present.

In the 3-dimensional case, the immediate effect of the longitudinal scaling is to introduce cooling, which serverely reduces the time duration of the plasma phase. The effect is expected to be strong enough to prevent the rarefaction wave from ever reaching the axis whilst the matter is in a plasma state, thus the radial expansion is expected to be dominated by the Riemann solution. In fact for a reasonably large T_c , the transverse expansion does not really get sufficient time to have much effect and an approximate solution is

$$T = T_R \left(\frac{t}{t_0} \right)^{c_s^2}$$

predicted for the central region, where T_R is given by eqn(2.36). In fig(2.2) the numerical solution for the temperature distribution as a function of radius is displayed for various times, showing how the rarefaction wave propagates inwards, whilst the longitudinal expansion cools the plasma. Fig(2.3) displays the velocity distribution as a function of the radius. In fig(2.4) the time evolution of the isotherms is displayed as a function of radius. Of particular importance is the sharp change in radius as a function of time for the higher isotherms, which will be relevant for the J/ψ suppression picture later.

Some immediate criticisms can be levelled at this solution. The first and most important being the fact that the equation of state used does not contain a phase transition anywhere —

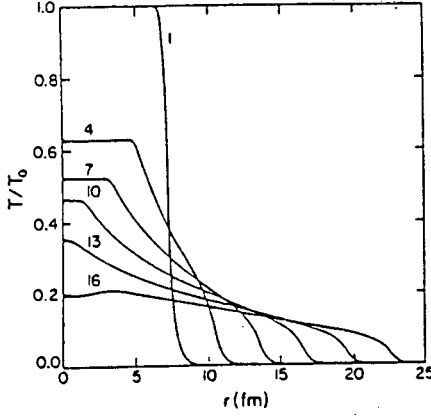


Figure 2.2: Temperature distributions as functions of r . Each curve is labelled by the time (in fm) that has elapsed since the collision ($t_0 = 1$ fm) [19].

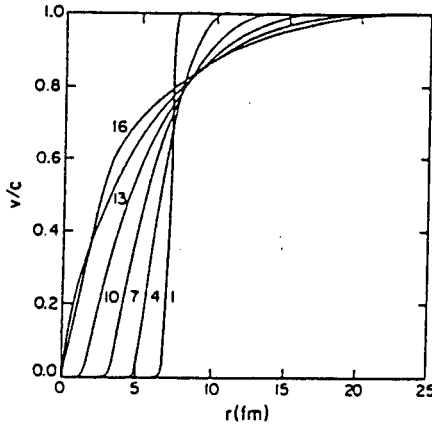


Figure 2.3: Velocity distributions for cylindrical expansion, labelled as in fig(2.2) [19].

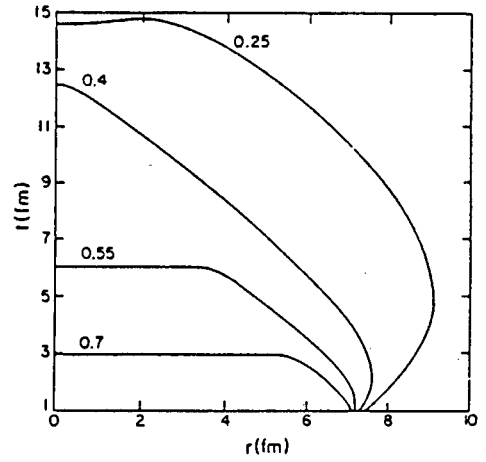


Figure 2.4: Isotherms in the r, t plane for $t_0 = 1$ fm. Isotherms are labelled by T/T_0 where T_0 is the initial temperature [19].

this is added in by hand. With the improvement of Lattice Q.C.D. it is possible to model the phase transition and so improve on the model quite considerably. This has been done in [20]. The phase transition is assumed to occur on a timescale much faster than the expansion, which is shown to yield the solution reasonably independent of the nature of the phase transition. The most essential point of the improved model is that the transverse expansion is much stronger than that of [19], and experimentally, larger transverse energy flows would be measured. The basic equation of state used is the ideal gas equation of state but in the transition region it is the bag-model, which includes effects such as the vacuum energy and degrees of freedom. The phase transition used is of first order. Similar calculations have also been done [21] but with a different numerical method, producing comparable results with [20] and much larger transverse momentum is found in comparison to [19]. The effect of the choice of the equation of state has been discussed in [22] and has been shown to have little effect on the rapidity plateau. However it does have an effect on the relation between $\langle p_T \rangle$ and $\frac{dN}{dy}$. Hence the input of the expected transverse momentum correlation with multiplicity

is expected to yield information about the equation of state. However [22] do not manage to fit the experimental data to the model at this point. Another approach used [23] is to examine the similarity solutions in a similar manner to that of [19], but now to improve the model by the inclusion of a phase transition given via the bag equation of state. A similarity wave followed by the shock front of [19] is again predicted, with the time for the wave to reach the beam axis of the order of 12 fm/c.

A model has been proposed [25] where a distribution of particles is assumed initially in terms of rapidity, which is different to the Bjorken scaling used by most of the models. The particle distribution at the end of the expansion becomes Gaussian-like, as opposed to the plateau predicted initially by Bjorken. The rapidity enters linearly, and the effect of the increase in magnitude of the dependence is found to flatten the observed distribution for large rapidities. The effect on the equation of state is also discussed.

In order to use the above models, some degree of comparison with experimental results must be made. At present energies of 200 GeV/A in the lab. frame the nuclear transparency is not altogether obvious. In fact for the lower energies ~ 30 GeV/A complete stopping of the nuclei is to be expected [15]. Thus the assumption of the validity of the Bjorken scaling is not valid at the lower energies, and in fact one is back to the Landau picture again. For the beam energies of ~ 200 GeV/A there is definitely some degree of transparency but the observed multiplicity distributions [26,27] do not display the flat plateau assumed in the Bjorken model. The observed distribution at these energies is best fitted by a Gaussian. It follows therefore, that in order to model collisions effectively at present beam energies, Bjorken scaling should not be used. Models where complete stopping occurs, [28] for example, use the Landau type solution where spherical symmetry is rather used. However this symmetry is clearly broken at beam energies of 200 GeV/fm³ or higher where partial transparency occurs, and so one should rather maintain cylindrical symmetry in the solutions, as has been done in [25]. Further problems which are expected to occur include the non-zero net baryon density due to the presence of the nuclei in the central region.

However, partial transparency does imply that the expected initial energy densities are more optimistic than the Bjorken estimate so that the chance of a quark gluon plasma phase transition is improved. As suggested by [29] the degree of transparency, and in fact the final particle multiplicity needs to be measured more accurately from experiment in order to have more precise knowledge on the degree of transparency found in the collisions and so improve on theoretical models of the hydrodynamical expansion. A more complete review of the hydrodynamics of nucleus-nucleus collisions is given by Kataja in [30].

Chapter 3

An Introduction to J/ψ Suppression

3.1 Introduction

To relate the background behind the prediction of plasma suppression of J/ψ production in ultra-relativistic heavy ion collisions, a brief historical review is given, of earlier models which consider the use of charmonium resonances to detect the effects of the presence of a deconfining phase transition to quark gluon plasma [31], predicted to arise in high energy nucleus-nucleus collisions. The conclusions arising from these models form an important part of the basis upon which the prediction of J/ψ suppression by quark gluon plasma in ultra-relativistic heavy ion collisions is made.

3.1.1 Historical Review

In ultra-relativistic heavy ion collisions it is important to know the degree of thermal equilibrium reached and the corresponding temperatures attained, in order to estimate the validity of the deconfining plasma description. This is particularly relevant with respect to the limiting Hagedorn temperature $T \sim 150$ MeV found for hadronic collisions [32] — a significant increase above this temperature would signal the presence of new physics. It was for this purpose that some of the early studies on charm production by a quark-gluon plasma and the resulting J/ψ resonances were made [33].

Calculations of the production of charmed quarks by a plasma formed from the collision of relativistic hadrons have been made by Shuryak [34] for the experiments [35] at the ISR energies. In order to find this production some estimate of the temperature is necessary. Two main approaches are used by Shuryak [36] to predict the temperature:

1. In the Feynman expansion picture [37], which corresponds to the partons passing through each other creating a hot central region, the initial temperature is determined from the average parton transverse momentum via $T_i \approx \langle p_T \rangle / 2$ which for the ISR data with an averaged initial parton momentum value of $\langle p_T \rangle = 1 - 1.4$ GeV [35] corresponds to a temperature of $T_i = 0.5 - 0.7$ GeV. A similar value for the initial temperature has been obtained by Feinberg [157] from a study of the p_T distribution of high p_T hadrons. Within the framework of the model considered by Feinberg, high p_T hadrons are interpreted to have escaped from the central region during early stages of the expansion

when the temperature is high, and to have a momentum distribution characteristic of that stage; once they escape the central region they free-stream with few interactions thus the momentum distribution remains frozen at the thermal distribution in the central region at the time they left the central region. In the case of pions at high p_T with a Boltzmann distribution of the form

$$\frac{dN}{dp_T^2} \sim \exp \left\{ -\frac{\sqrt{p_T^2 + m_\pi^2}}{T_{\text{eff}}} \right\}$$

where m_π refers to the pion mass, an effective temperature compatible with 0.4 GeV is found.

2. In the Landau model [39] the case where complete stopping of the projectile occurs resulting in the creation of a central fireball is considered. Here, the complete centre-of-mass energy is available, and hence it is expected that the temperature will be correspondingly higher. An estimate of the initial temperature which is approximately given by

$$T_i = 0.12 \left(\frac{s}{4m^2} \right)^{\frac{1}{4}}$$

where s is the centre-of-mass energy squared of the collision, and m the hadron mass [36] is found to yield a value of $T_i \approx .5$ GeV. Feinberg [38] obtains an estimate of $T_i \approx 1.1$ GeV for the Landau model.

Both these estimates are consistent with an initial temperature $T_i \gtrsim 0.5$ GeV. This rather high estimate for the temperature led to the conclusion that the production of charm quarks in the plasma would dominate over the hard initial collision. A corresponding energy density of ~ 23 GeV/fm³ has been estimated by [14] for the case of complete stopping of the colliding particles which corresponds to the Landau picture, but with limiting thickness for the proton after boosting, for $p\bar{p}$ collisions at the ISR with centre of mass energy 25 GeV. In the case of heavy ions at extremely high energy some degree of transparency is expected corresponding more to the Feynman expansion picture. A new calculation based on complete transparency for heavy ion collisions has been discussed by [16] to generate an entirely different estimate for the resulting central region energy density of ~ 3 GeV/fm³ [14], with a corresponding temperature of 200 MeV which is considerably lower than that used by Shuryak. This leads to a correspondingly significant decrease in the expected number of charmed quarks produced by the plasma, due to the statistical thermal factor for the production of charmed quarks $e^{-2m_c/T}$.

In [40] the production of charm quarks by a quark gluon plasma at $T \sim 200$ MeV is considered with respect to the time required for chemical equilibrium to occur. It is concluded that this time $\sim 10^2$ fm is much larger than the expected plasma life-time of ~ 10 fm [16]. As the hadronic cross-section of the charmed quarks at normal densities is relatively small [41], the plasma (as well as the surrounding hadrons) is interpreted to be fairly transparent, so that the charm quarks will not thermalise with the surrounding plasma. The plasma will cool as it expands leading to a corresponding drop in the production rate of charmed quarks at later stages, thus the charm quarks will carry information about the early stages of the plasma evolution. This idea has been continued further in [42] to suggest the thermal production of J/ψ distribution expected from heavy ion collisions as a function of rapidity and temperature,

including some estimate of target and projectile mass as well as collision energy dependence. The discussion has been reformulated in [43], to include the central region discussed by [16] where it is concluded that the plasma production of charmed particles as a result will be much smaller than that found for the lower invariant masses, for example by a factor

$$\frac{dN(J/\psi)}{dN(\phi)} \approx e^{\frac{m_\phi - m_\psi}{T}} \frac{\Gamma_{\text{tot}}(J/\psi)}{\Gamma_{\text{tot}}(\phi)}$$

in the case where the comparison is made between the ϕ and J/ψ resonances, which at the temperature $T = 200$ MeV, corresponds to a reduction of about 10^{-7} in the production of J/ψ over ϕ . A more complete discussion has been made by [44], where the excited states of the $c\bar{c}$ resonances are also included, the basic result is that ~ 3 times larger production of J/ψ is found. Further comments have been made by [10] where it is pointed out that the plasma production of charmed quarks should not be significantly large enough to draw any conclusions about the presence of a quark-gluon plasma.

3.1.2 Prediction of J/ψ Suppression in Heavy Ion Collisions

A strong prediction has been made by Matsui and Satz [7,45], that the presence of a plasma will lead to a directly observable decrease in the number of J/ψ produced in high energy heavy ion collisions – this prediction is based on the screening properties of the QCD plasma and will now be discussed.

The basic method of deconfinement is assumed to arise from Debye screening of the quark colour charge [46], where the corresponding Debye screening radius r_D is a measure of the screening strength. It follows that when r_D is smaller than the binding radius r_H of the quark system, the confining force no longer holds the quarks together and the resonance may become unbound. Some knowledge of r_D must be gained in order to estimate the effect of the screening on the quark resonances; this is obtained from a study of the $SU(2)$ [47] and $SU(3)$ [48] gauge theories using the Lattice approach. The temperature dependence of the screening radius has been calculated [48], for interactions with a static quark-antiquark system, in a purely gluonic thermal environment. The absence of dynamical quarks changes the screening quite considerably [47], however in the high temperature region the quark interaction is dominated by one gluon exchange, made massive by gluonic colour screening, thus the approximation is not too drastic; the presence of dynamical quarks will lead to an increase in the density of colour constituents and thus strengthen the overall screening.

The correlation function $\Gamma(r, T)$, where r is the distance of separation of the static $q\bar{q}$ system and T the temperature of the gluonic heat-bath, is used to parameterise the interaction. For large r , $\Gamma(r, T)$ decreases as

$$\Gamma(r, T) \approx e^{-\frac{r}{\xi(T)}}$$

where $\xi(T)$ is the correlation length – this is estimated from Lattice calculations. In order to interpret the calculations, it is necessary to fix the temperature scale – this is chosen so that the critical temperature is $T_c = 200$ MeV; the dependence is shown in fig(3.1). For $T \sim T_c$, ξ drops off steeply, till around $T/T_c = 1.5$ where it starts flattening, having a value of the order $0.2 - 0.3$ fm. This is an upper bound to the screening, due to the lack of dynamical quarks in the model – the presence of dynamical quarks will lead to screening even at $T = T_c$.

In hadronic collisions the dominant production of the J/ψ is from hard parton-parton interactions which produce $c\bar{c}$ pairs, leading to the J/ψ resonance formation. If the production

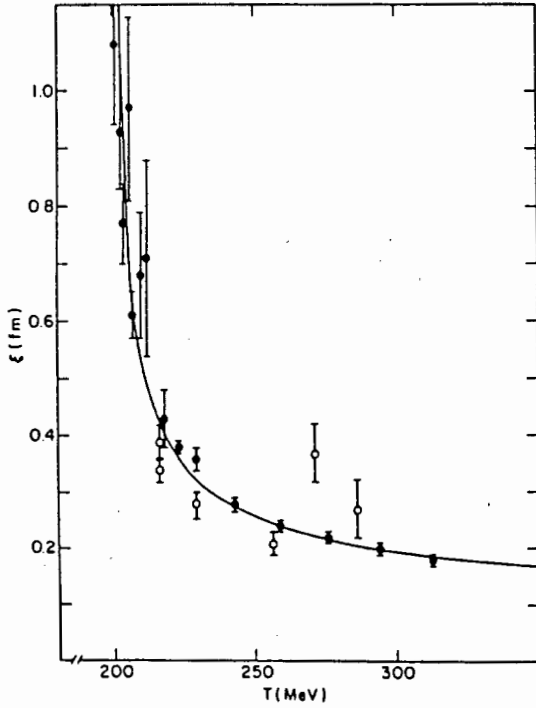


Figure 3.1: Temperature dependence of the correlation length, as obtained in SU(2) gauge theory (solid dots, from [47]) and in SU(3) gauge theory (open circles, from [48]); here $T_c = 200$ MeV was used to fix the scale [7].

occurs from a high energy nucleus-nucleus collision and if a quark gluon plasma has formed, then the deconfining environment will influence the resonance formation – if r_D is less than the J/ψ binding radius it may be that the J/ψ resonance is no longer a bound state, the charm quarks separating instead, until the confinement transition where they pair rather with the more abundant u and d quarks, leading to “open charm” meson production instead – it is important to note that the J/ψ outside the deconfining plasma are inherently stable against this mode of decay, not being massive enough to spontaneously decay into two charmed mesons.

A closer examination is necessary to decide if the effect will be readily observable:

1. The time for plasma formation and thermal equilibrium to start occurring is expected to be of the order of 1 fm [49]; this is expected to be larger than the time for the hard production of the $c\bar{c}$ pairs but to be of the order of the formation time for the J/ψ resonance (a rough estimate is the time required for the quarks to travel to the J/ψ radius) if the J/ψ radius is not unusually small on the hadronic scale. Even once the resonance has formed, it still has to travel out of the plasma – the average distance to travel is about $A^{1/3}$ fm (approximately the nuclear radius). Thus one concludes that there is sufficient time for the screening to effect the resonance formation.
2. It is important to estimate the relevant temperatures required for a significant plasma effect on the J/ψ resonance. To do this it is necessary to introduce some kind of model for the J/ψ ; since the J/ψ is a heavy-quark resonance, it is expected that to a good approximation the bound state may be described by a non-relativistic phenomenological model, the binding being reproduced by a potential. Such a model has been considered for the resonance at zero temperature [50] with reasonable success at reproducing the

hadronic spectra for low $c\bar{c}$ resonances. The potential used is of the form

$$V(r) = \sigma r - \frac{\alpha_{\text{eff}}}{r}$$

with the constants $\sigma = 0.16 \text{ GeV}^2$ the string tension and $\alpha_{\text{eff}} = \frac{1}{2}$ the Coulombic interaction coupling. An estimate of the bound state energy and radius may be obtained with use of the uncertainty relation

$$\Delta p \Delta r \geq 1$$

in units where $\hbar = c = 1$. This places a lower limit on the size of the J/ψ for some given momenta range. By substituting for the radial momenta and noting that the J/ψ has zero angular momentum, the relation

$$E(r) \geq 2m + \frac{1}{2mr^2} + V(r) \quad (3.1)$$

occurs. This may be varied with respect to r to find the energy minimum, yielding

$$\frac{1}{mr_{J/\psi}^3} - \frac{\alpha_{\text{eff}}}{r_{J/\psi}^2} - \sigma = 0 \quad (3.2)$$

and hence, with the knowledge that the J/ψ has a mass $\sim 3.1 \text{ GeV}$, the two equations in the two unknowns, charm-quark mass m_c and J/ψ radius $r_{J/\psi}$, can be solved to yield $m_c \sim 1.56 \text{ GeV}$ and corresponding radius $r_{J/\psi} = 0.2 \text{ fm}$. Other values for the parameters may be considered (a smaller α_{eff} will lead to slightly smaller mass but larger $r_{J/\psi}$), the range $0.2 \leq r_{J/\psi} \leq 0.5 \text{ fm}$ is quite typical for the radius [7] – the values agree reasonably well with those obtained from the quantum mechanical treatment by [51]. If the parameter $\alpha_{\text{eff}} = 0$ then eqn(3.2) becomes $r_{J/\psi} = (m\sigma)^{-\frac{1}{3}}$ and the values do not change much. Thus it is possible to conclude that at $T = 0$, even though the J/ψ is smaller than the lighter hadrons, the J/ψ binding radius is still largely determined by the confining part of the potential. At finite temperature, the nature of the resonance is determined by the temperature dependence of the parameters σ and α_{eff} . The confinement term σ decreases with increasing temperature till at $T = T_c$ where it vanishes, corresponding to the deconfinement transition; here it still may be possible to have bound resonances (even above T_c) due to the coulomb part of the potential, which should be multiplied by the Debye screening term to simulate the screening of the colour charge. Hence eqn(3.1) becomes

$$E(r) = 2m + \frac{1}{2mr^2} - \frac{\alpha_{\text{eff}}}{r} e^{-\frac{r}{r_D(T)}}$$

and the energy minimization with respect to r yields

$$\frac{1}{m\alpha_{\text{eff}}r_D} = \tilde{r}(1 + \tilde{r})e^{-\tilde{r}}$$

where $\tilde{r} = r/r_D$. The righthand side has a finite maximum upper-limit value at some radius, thus an estimate for the smallest screening radius where a minimum in the energy occurs can be found. This corresponds to

$$\frac{1}{m\alpha_{\text{eff}}r_D} \leq 0.84 \quad \text{with a value of } \tilde{r} = 1.61 \quad (3.3)$$

Using the value for $m_c = 1.56$ and $\alpha_{\text{eff}} = 0.5$ an estimate of $r_D \geq 0.30$ fm can be obtained. However this assumes α_{eff} is independent of the temperature — in fact α_{eff} depends weakly on temperature as seen from lattice calculations [47] at $\frac{T}{T_c} = 1.5$ where the value of $\alpha_{\text{eff}} = 0.2$ is found to be consistent. This effectively ensures that the transition to unbound states occurs at a much larger value of $r_D = 0.76$ fm. The J/ψ radius will become quite large above T_c — with use of eqn(3.3) the range of $r_{J/\psi}$

$$0.5 \leq r_{J/\psi} \leq 1.22 \text{ fm}$$

can be estimated.

Comparison is now made to the correlation length $\xi(T)$ which at $\frac{T}{T_c} = 1.5$, corresponds to $\xi \sim 0.2$ fm — where no J/ψ resonance formation is possible. It is thus fairly conclusive that, for the range of parameters studied above, the J/ψ resonance will be unbound when T is still quite close to T_c (i.e. $\frac{T}{T_c} \sim 1.2$ or less), so that in consideration with the first point, suppression in the number of J/ψ produced should occur if a deconfining plasma is present. In fact the suppression should be fairly general, in the sense that the heavier $c\bar{c}$ resonances presumably being slightly larger, will be more screened by the plasma than the J/ψ resonance with a corresponding lower production rate (i.e. more suppression should be observed for the higher energy resonances).

3. In order to infer the existence of a deconfining plasma from any observed J/ψ suppression, it is necessary to examine other possible suppression mechanisms. Essentially, the J/ψ may be absorbed by either the nuclear matter of the collision participants (nuclear suppression) or the hadronic matter produced by the collision (hadronic suppression). Both of these suppression mechanisms are expected to be relatively small due to the small measured J/ψ nucleon cross-section of $\sigma \sim 1 - 3$ mb [52,53,41,54] in comparison to that for example for pp of $\sigma \sim 40$ mb. These suppression mechanisms have been predicted by [7] not to be significant, but have been discussed in much detail in the literature more recently especially with regard to hadronic secondaries which may be sufficiently dense to counter the effect of a small J/ψ absorption cross-section; this possibility is discussed in much more detail in chapter 4.

It is important to consider the possibility that the plasma may produce sufficient $c\bar{c}$ pairs to spoil the expected suppression. This has been discussed earlier by [40], where the time for significant $c\bar{c}$ production to occur is found to be much larger than the plasma lifetime — this should be compared with the prediction by Shuryak [34] which is for a significantly higher plasma temperature. A more recent review of the plasma production has been made by [55], which is discussed later in the chapter on alternate mechanisms of J/ψ suppression.

The production of J/ψ during and after hadronisation should be considered as well — the large mass of the charm quark will most likely make it energetically unfavourable to form $c\bar{c}$ pairs thus not effecting the suppression, this argument is strengthened by the observation that hadronisation occurs after the hydrodynamical expansion has cooled the central region.

4. As a final comment, the detectability of the J/ψ dimuon decay signal over that of any background should be mentioned. The $c\bar{c}$ pairs produced near outer regions of the collision will not be surrounded by a deconfining plasma and thus can produce

J/ψ resonances – a J/ψ resonance peak is thus expected to appear. The relative production from this outer region in comparison to that expected from the interior will vary as the ratio of surface area to volume of the colliding ions and the size of the plasma produced, thus the lighter projectiles such as oxygen, and grazing collisions with little to no plasma formation, should show less screening than the heavier ions such as sulphur, and head-on collisions.

Background production of dilepton pairs will come mostly from either Drell-Yan (hard $q\bar{q}$ annihilation) or from the thermal radiation by the plasma. The hadronic dilepton production will be dominated by the ρ , ω , ϕ resonances in the low mass region. The Drell-Yan production mechanism for pp collisions has the form [56]

$$\left(\frac{d^2\sigma}{dM^2 dy} \right)_{pp}^{DY} = \left(\frac{1}{M^4} \right) f \left(\frac{M}{\sqrt{s}} \right) [\text{mb GeV}^{-2}] \quad (3.4)$$

which should be scaled by a factor A^{-1} in the case of pA collisions, and in the central region ($y \sim 0$)

$$f \left(\frac{M}{\sqrt{s}} \right) \approx C e^{-15 \frac{M}{\sqrt{s}}}$$

is suggested for the function f [56]. The form fits the data well, up to the constant multiple, which does not match the theoretical calculations due to the absence of correction terms [57], thus the experimentally determined constant from [56] is rather used. The average lepton pair yield per inelastic pp collision, given by

$$\left(\frac{d^2N}{dM^2 dy} \right)_{pp}^{DY} = \frac{1}{\sigma_{in}^{pp}} \left(\frac{d^2\sigma}{dM^2 dy} \right)_{pp}^{DY} \quad (3.5)$$

where σ_{in}^{pp} is the inelastic pp cross-section, may be calculated with use of eqn(3.4). Generalisation of this relation to nucleus-nucleus collisions requires the calculation of ν , the number of effective nucleon-nucleon collisions; in the case $A > B$

$$\nu(A,B) = \left(\frac{2R_A}{\lambda_{in}} \right) B = 2R_A \rho_0 \sigma_{in}^{pp} B$$

where λ_{in} is the inelastic mean-free path given by $\lambda_{in} = (\sigma_{in}^{pp} \rho_0)^{-1}$. The product of ν with eqn(3.5) yields (in the case where $A \sim B$ an additional geometrical factor must be found — for $A = B$ a factor $\frac{3}{4}$ is found, by averaging over path lengths through a spherical target, weighted by the spherical distribution of the projectile nucleons)

$$\left(\frac{d^2N}{dM^2 dy} \right)_{AB}^{DY} = \frac{3}{4} 2r_0 \rho_0 C \frac{A^{\frac{1}{3}} B}{M^4} e^{-15 \frac{M}{\sqrt{s}}}$$

The thermal production mechanism is only present when a deconfinement transition has occurred and so is not experimentally known. The theoretical expression for the production rate of lepton pairs by a quark gluon plasma undergoing the longitudinal scaling hydrodynamic expansion of [16] is given by

$$\left(\frac{d^2N}{dM^2 dy} \right)_{pp}^{TH} \approx B^{\frac{2}{3}} \left(\frac{25\alpha^2}{\pi^2} \right) \sqrt{\frac{\pi}{2}} \left(\frac{T_0}{M} \right)^{\frac{1}{2}} e^{-\frac{M}{T_0}} [\text{GeV}^2]$$

where T_0 is the initial temperature of the plasma at the time of its formation and $\alpha = \frac{1}{137}$, derived from the result of [58] by use of the relation $\tau_0 T_0 = 1$ with $M \gg T_0 > T_c$. The comparison is made in fig(3.2) between the production for Drell-Yan and thermal

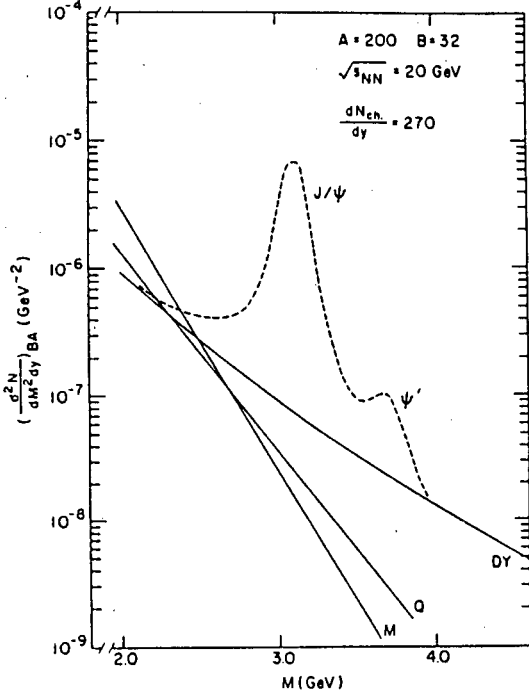


Figure 3.2: Lepton pair production by Drell-Yan mechanism (DY), thermal emission from a plasma at $T_0 = 300$ MeV (Q) and thermal emission from the transition region at $T_c = 200$ MeV (M). The dashed line indicates the unsuppressed, scaled-up J/ψ and ψ' signal. The collision parameters are chosen for the CERN SPS heavy ion program [7].

mechanisms; from these it is evident that the Drell-Yan should dominate in the region suggested.

Thus Matsui and Satz conclude that the suppression effect of the plasma on J/ψ resonance production should be observable in ultra-relativistic heavy ion collisions. The J/ψ peak should appear above a smooth Drell Yan background which is not expected to completely swamp the J/ψ signal and the plasma will not produce sufficient $c\bar{c}$ pairs to destroy the suppression.

3.1.3 The Momentum Dependence of the Suppression

Karsch and Petronzio [59] predict that the suppression discussed by Matsui and Satz [7] should be observable by the NA38 group (the goal of the NA38 group is to study dimuons produced in nucleus-nucleus collisions with a 200 GeV/A projectile of oxygen, and later sulphur, on a target of high atomic mass (uranium or copper) [60]) and then go further, to predict the momentum dependence of the suppression. The argument used goes as follows:

The formation of a J/ψ has two typical timescales. The creation of a $c\bar{c}$ pair is a short distance phenomenon which occurs at a distance of the order of $\frac{1}{M} \sim 0.05$ fm, where M is the J/ψ mass; relativistic effects on this distance are ignored due to the large mass of the J/ψ resonance when compared to the expected momentum of the pair. Once created, the quarks must separate to a binding radius of ≥ 0.2 fm typical of the charmonium radius, as mentioned by Satz and Matsui [7]. The relative motion of the pair is to first approximation

non-relativistic, with relative momentum $\vec{p}_{\text{rel}} \sim 300$ MeV and charm quark mass ~ 1.5 GeV (the parameters for the momentum and mass for the charm quark pairs in the J/ψ resonance have been estimated by [51] using a potential model of the form Coulomb + linear). An estimate for the separation time τ_ψ in the centre of mass (c.m.) frame of the pair is given by

$$\tau_\psi = \frac{E}{|\vec{p}_{\text{rel}}|} (0.2 - 0.05) \approx 0.7 \text{ fm}$$

which is to be interpreted as the time for a $c\bar{c}$ pair to form a J/ψ resonance. This is of the order of the thermalisation time [33,16] expected for the plasma. After this time, the $c\bar{c}$ pair with momentum $\vec{p}$ and energy E will have traversed a displacement $\vec{r}$ in the plasma rest frame with the γ factor given by $\gamma = E/M_\psi$, of

$$\vec{r} = \frac{\vec{p}}{E} \gamma \tau = \frac{\vec{p}}{M_\psi} \tau_\psi \quad (3.6)$$

where τ_ψ is the proper time of the pair and where $\vec{p}$ and M_ψ refer to the J/ψ resonance which has formed outside, or is to be destroyed inside, some region which will be spatially determined by the plasma (this may not however necessarily correspond physically to the entire plasma region). It is important to realise that $c\bar{c}$ pairs have a small separation initially and hence will not feel the effect of a surrounding plasma; as the pair moves apart the surrounding plasma environment begins to screen the charges and hence effects the resonance formation. In order to quantify this effect, a simplification is made: the effect of plasma screening on a $c\bar{c}$ pair is ignored until it forms a J/ψ , when the screening is suddenly turned on. This is the basis of the argument which follows. To create a more tractable model, a fixed plasma disk is considered with the effect of plasma screening on the formation of J/ψ resonances significantly simplified to the condition that: for a cylindrical symmetric static plasma, if a $c\bar{c}$ pair at $\tau \geq 0.7$ fm is within some radius of the beam axis determined by the plasma called the hot region, the pair are not sufficiently bound due to the plasma screening to form a J/ψ resonance and continue to separate till the hadronization phase transition when they pair with the more abundant lighter quarks to form charmed mesons (for example $D\bar{D}$), whilst $c\bar{c}$ pairs outside of the hot region can form J/ψ which escape completely (even from the hadronisation phase transition); the hot region must be distinguished from the interaction region, which refers to the initial collision where hard collisions produce the $c\bar{c}$ pairs, and is spacially dependent on the radius of the smaller colliding ion ($r \sim A^{1/3}$ where A is the atomic mass number) and the degree of centrality of the collision, though only central collisions will be considered. The hot region which forms after the initial collision, is smaller in spatial dimension with radius given by $r = hA^{1/3}$ with $0 \leq h \leq 1$ some parameter of the model. The length of the interaction region is estimated to be ~ 1 fm from the Lorentz contraction of the colliding nuclei and this is carried over to the hot region — the longitudinal expansion of the plasma is completely ignored, and only the low rapidity regions (in the c.m. frame) are considered.

An acceptance function $\mathfrak{R}_\psi(p_T, p_z)$ can then be estimated from the fraction of $c\bar{c}$ pairs created in the interaction region with momentum (p_T, p_z) which escape the plasma screening effect. The fraction is given by

$$\mathfrak{R}_\psi(p_T, p_z) = \frac{\#c\bar{c} \text{ pairs with } p = (p_T, p_z) \text{ outside the hot region at } t = \gamma\tau_\psi}{\#c\bar{c} \text{ pairs with } p = (p_T, p_z) \text{ created in the interaction region}}$$

which may be rewritten

$$\mathfrak{R}_\psi(p_T, p_z) = \frac{dN_{c\bar{c}}(t)/dp_T^2 dp_z}{dN_{c\bar{c}}(0)/dp_T^2 dp_z} \quad (3.7)$$

and lies between the limits of 0 for complete suppression and 1 for no suppression. An estimate of the spacial distribution of the $c\bar{c}$ pairs, created in the interaction region, is determined from the collision geometry. The probability that a $c\bar{c}$ pair is created at the point (x, y, z) inside the interaction region is determined by taking a sphere of constant density and Lorentz contracting it to a pancake, approximated by a cylinder of the same radius as the sphere, but with length corresponding to the thickness of the pancake. This generates a radial probability distribution of

$$P(x, y) = C(A^{\frac{2}{3}} - r_T^2)^{\frac{1}{2}} \quad (3.8)$$

where $r_T^2 = x^2 + y^2$ and the normalisation factor $C^{-1} = \frac{2}{3}\pi A$ is determined via

$$1 = \int_{\text{interaction region}} dx dy P(x, y)$$

A uniform longitudinal distribution is chosen.

The ratio $\mathfrak{R}_\psi(p_T, 0)$, which is the production for zero rapidity, has been calculated for different values of the parameter h , for the values $A = 16$ (oxygen) and $A = 200$ (heavy nucleus), see figs(3.3,3.4). The case for $\mathfrak{R}_\psi(0, 0)$ corresponding to static $c\bar{c}$ pairs in the plasma

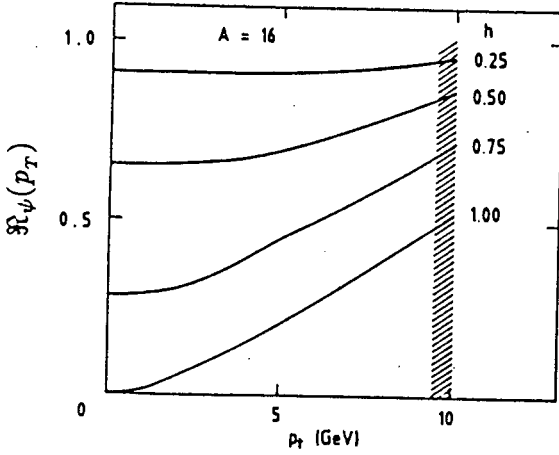


Figure 3.3: The ratio $\mathfrak{R}_\psi(p_T, 0)$ versus p_T for $A = 16$ and various sizes of the hot region, $h = 0, 0.25, 0.5, 0.75$ and 1.0 [59].

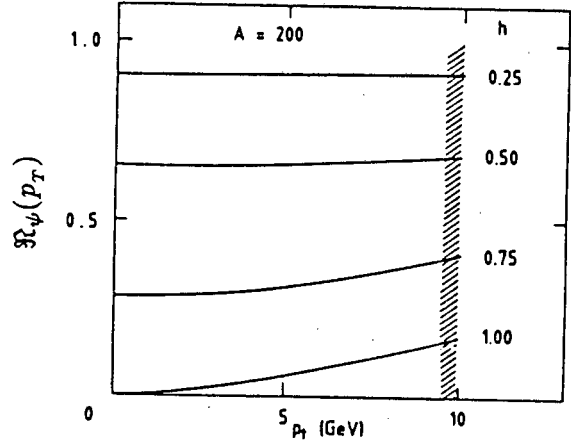


Figure 3.4: Same as for fig(3.3) but for $A = 200$ [59].

is readily found from the integral of the distribution function, and is given by

$$\mathfrak{R}_\psi(0, 0) = (1 - h^2)^{\frac{3}{2}}$$

which is independent of A and depends only on the size of the hot region. As p_T increases the ratio becomes dependent on A — compare fig(3.3) ($A=16$) to fig(3.4) ($A=200$) where more suppression for a corresponding h is observed for the larger nucleus. In fact the p_T dependence

of $\mathcal{R}_\psi$ will be accentuated by the inclusion of a finite lifetime for the plasma of ~ 5 fm [30]. The large p_T $c\bar{c}$ pairs will be observed to take longer ($t \sim 3\tau_\psi$ for $p_T \sim 10$ GeV/c) to form bound states — this may exceed the lifetime of the plasma leading to no suppression. The shadowed region in figs(3.3,3.4) corresponds to the region where this effect must be included.

A limit on h for the detectability (30% suppression) of this suppression effect can be read from fig(3.3) corresponding to $h \geq 0.5$ in the case $A = 16$ and $p_T < 5$ GeV/c (for the NA38 group experiments).

An estimate for the longitudinal dependence of the suppression resulting from the finite height of the plasma disk may be determined, by allowing $c\bar{c}$ pairs with sufficient $p_{||}$ momentum to escape through the flat sides of the hot region. This will allow J/ψ resonances with zero p_T to form, even for a hot region the same size as the interaction region. However, the assumption that the plasma does not expand longitudinally is somewhat drastic and thus the longitudinal dependence should not be too realistic.

Thus the finite size of the plasma is sufficient to induce both a transverse and longitudinal dependence in the suppression of J/ψ resonance production. To be more realistic it is necessary to include the finite lifetime of the plasma expected from the longitudinal expansion of the plasma.

3.2 Evidence for the Suppression ?

In order to verify the predictions of Matsui and Satz [7] some experimental evidence for the J/ψ suppression should be seen. The NA38 group has the aim to look for quark gluon plasma in high energy nucleus-nucleus collisions by studying the production of dimuons and the A-dependence of J/ψ production as a function of the transverse electromagnetic energy [76,60,61]. Initially the motivation was the detection of the thermal production of dimuons from a quark gluon plasma, but with the Matsui and Satz prediction interest was focused on the detection of J/ψ suppression [72]. The NA38 group has performed experiments in late 1986 to 1987 at the CERN SPS collider [60,63] with 200 GeV/A beams of: oxygen on active uranium and copper targets; protons and then later sulphur on active uranium targets. The experiment recorded dimuons particularly in the J/ψ mass region, together with a measure of the neutral transverse energy flow from the corresponding collision, to yield the results amenable to the analysis for J/ψ suppression suggested by Matsui and Satz. The measured transverse energy flow is estimated to be approximately 50% of the total transverse energy flow [72]. Preliminary results have been made available (see references [62]–[76]) with strong emphasis towards the observation of J/ψ suppression.

The experimental fit used for the dimuon spectrum consists of a double gaussian, corresponding to the J/ψ and ψ' resonances from which a check on the J/ψ mass M_ψ may be obtained, superimposed on a Drell-Yan type continuum,

$$\frac{dN}{dM} = N_c \frac{e^{-M/M_c}}{M^3} + N_1 e^{-(M-M_\psi)^2/2\sigma_\psi^2} + N_2 e^{-(M-M_{\psi'})^2/2\sigma_{\psi'}^2}, \quad (3.9)$$

where M_c determines the shape of the continuum, and σ refers to the widths of the gaussian peaks (related to the mass resolution of the spectrometer). This type of fit is found to realistically reconstruct the data [75], with the J/ψ peak clearly observable above the continuum but with the ψ' peak poorly defined. To check for J/ψ suppression, fits are made for different corresponding transverse energy (E_T) intervals, especially for the high and low E_T intervals.

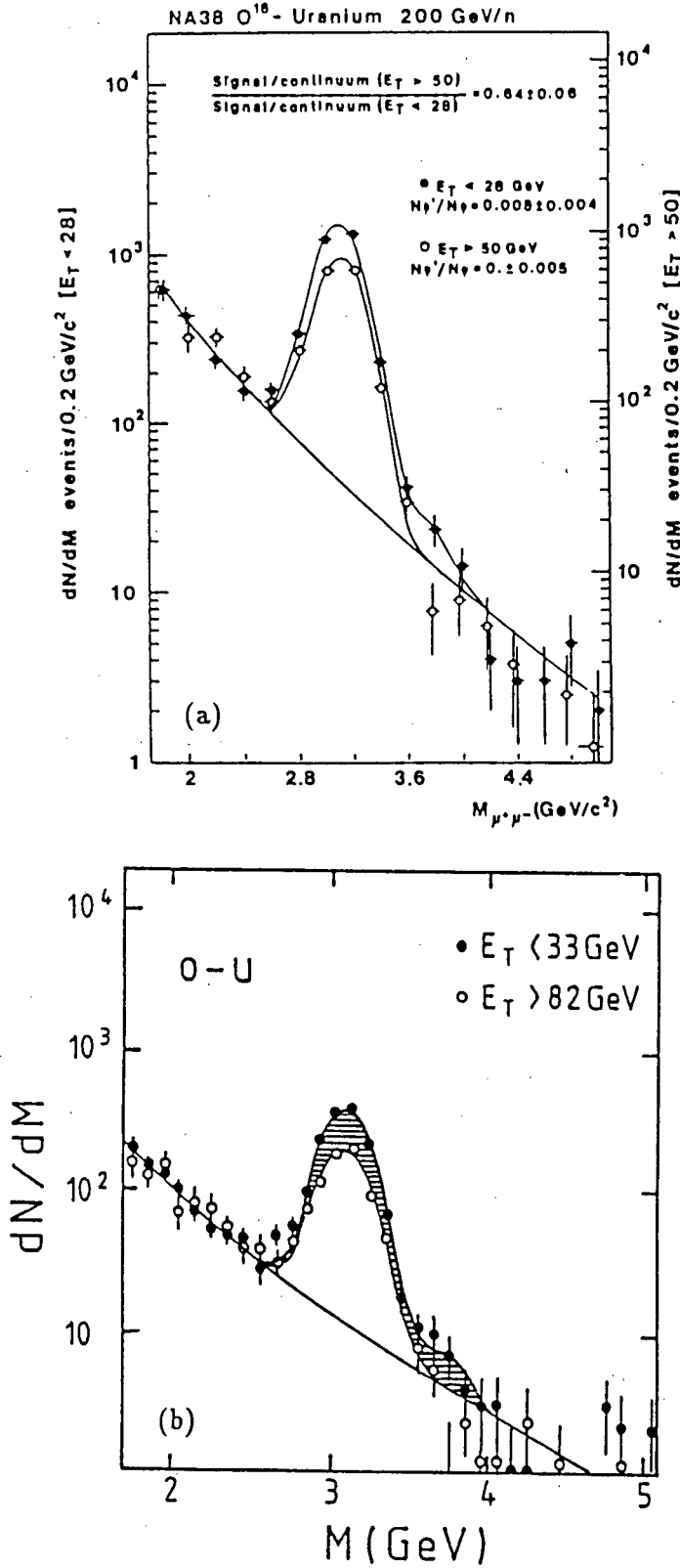


Figure 3.5: (a) The $\mu^+\mu^-$ mass distribution in 200 GeV/A O-U collisions for $E_T^{\text{high}} > 50 \text{ GeV}$ and $E_T^{\text{low}} < 28 \text{ GeV}$ [63]. (b) the same as for (a) but for $E_T^{\text{high}} > 82 \text{ GeV}$ and $E_T^{\text{low}} < 33 \text{ GeV}$ (data is from [69]) [77].

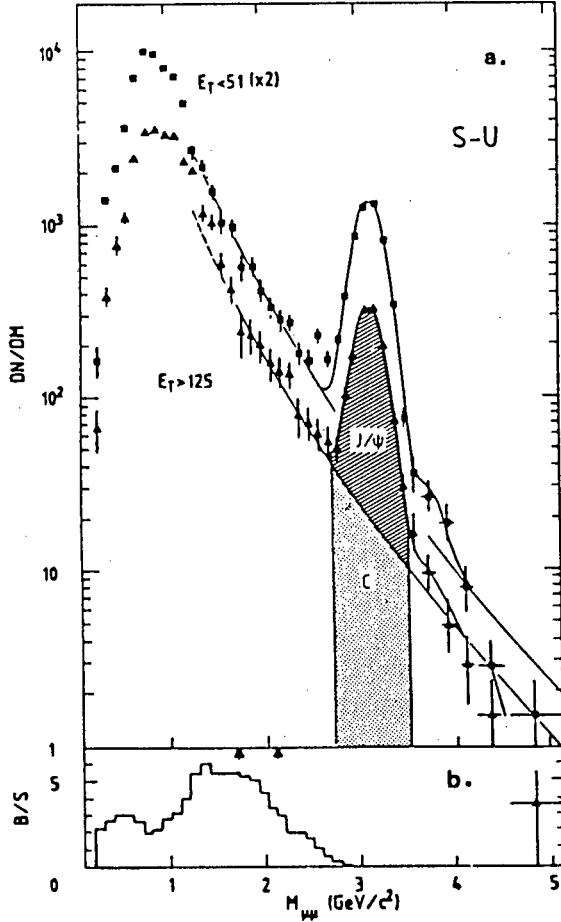


Figure 3.6: (a) Dimuon mass spectra observed in S-U collisions for $E_T^{\text{high}} > 125 \text{ GeV}$ and $E_T^{\text{low}} > 51 \text{ GeV}$. (b) The background-to-signal ratio for S-U collisions [72].

A comparison can then be made between the high and low E_T fits see figs(3.5,3.6) for the plots in the case of O-U and S-U combinations. Suppression in the J/ψ signal for the high E_T events over the low E_T events relative to the continuum is observed.

The E_T dependence of the J/ψ suppression may be more clearly demonstrated by a plot of the ratio R_c^ψ of the number of J/ψ , to continuum events as a function of E_T . This ratio is more precisely defined as the number of J/ψ events within some invariant mass range, divided by the number of continuum events within the same range, demonstrated in fig(3.6) where a mass range of $2.7 \leq M \leq 3.5 \text{ GeV}$ is chosen (shaded area) with the appropriate J/ψ and continuum areas marked accordingly. The above procedure has been done for each of the beam-target combinations chosen and is displayed in fig(3.7) where a decrease in the ratio is found in all three plots. (There is found to be almost no correlation in the p-U data between the number of participating nucleons and E_T [72]). The data may be combined into a single plot in terms of the energy density ϵ attained in each of the collisions. Unfortunately there is no model independent method to obtain the energy density in terms of the measured E_T for the collision. Two plots exist for the data: For the first, which has the continuum mass cut for fitting eqn(3.9) at $M \geq 1.7 \text{ GeV}$, the Bjorken energy estimate is used with the effective number of nucleons A^{inc} given by the mass number of the smaller ion [70]. The second plot has the cut at $M \geq 2.1 \text{ GeV}$ and applies the HIJET model to find the effective number

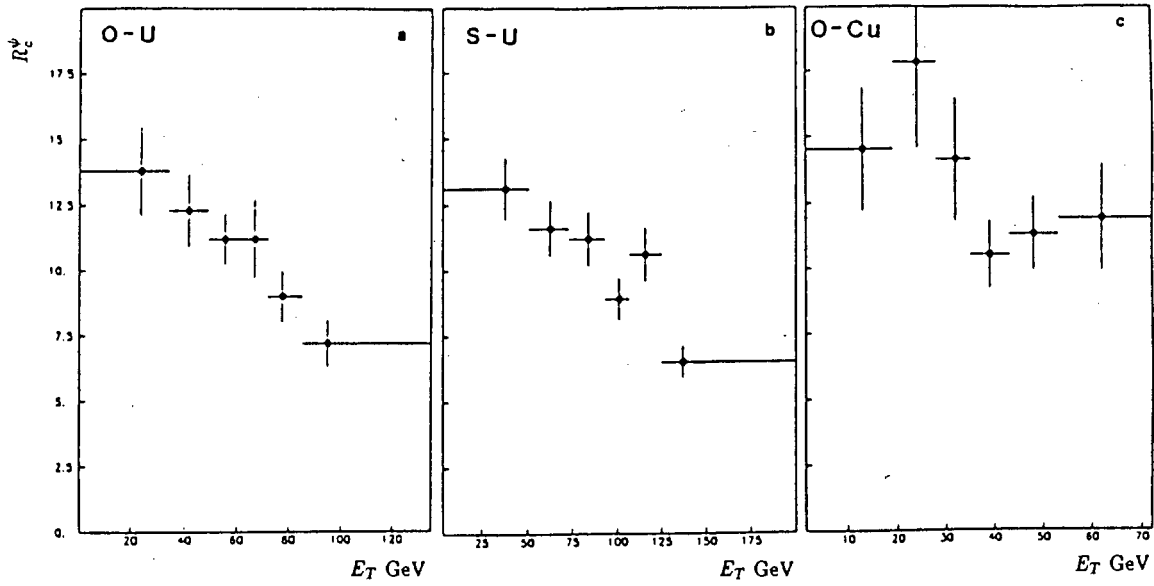


Figure 3.7: The ratio R_c^ψ as a function of E_T for (a) O-U, (b) S-U (c) O-Cu collisions [71]

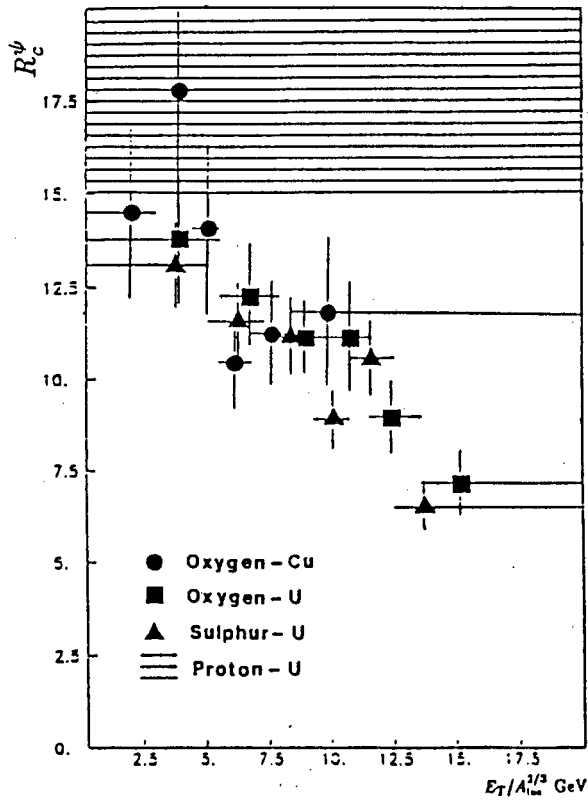


Figure 3.8: The ratio R_c^ψ as a function of the energy density estimated by $E_T/A_{inc}^{2/3}$. The horizontal strip corresponds to the p-U data from the same experiment for which no E_T dependence is observed [71].

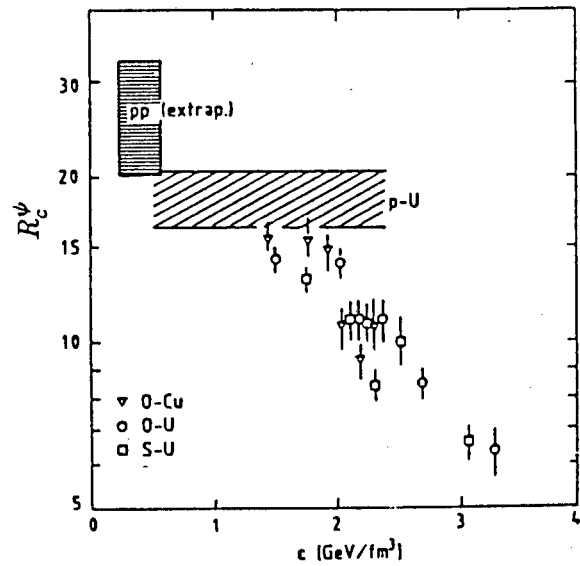


Figure 3.9: J/ψ to continuum ratios versus ϵ . Bjorken's energy density [16] for O-Cu, O-U and S-U and p-U collisions, together with extrapolated p-p data. (Data is from fits for $M > 2.1\text{GeV}$) [72].

of nucleons to use in the Bjorken energy density estimate [72]. These plots are shown in figs(3.8,3.9), where the p-U data is represented by a continuous band as the E_T dependence is weak. The data appear to lie in a single straight band meeting the p-U band — no obvious kink is observed which might signal the start of a deconfinement phase transition [76], though this may be lost in the statistics. In order to draw any firm conclusions, the plot must be made more complete in the lower energy density region. These may be obtained from lower energy collisions or, especially from the use of lighter nuclei.

It is necessary to question if the observed suppression is a feature of the experimental technique:

1. the observed suppression is shown [72] to be relatively independent of the cut-off mass M used for fitting the dimuon spectrum, and is thus not a feature of the fitting procedure or of the background subtraction.
2. The shape of the continuum, as determined by M_c in the dimuon fits, is not dependent on E_T (see fig(3.10a)).

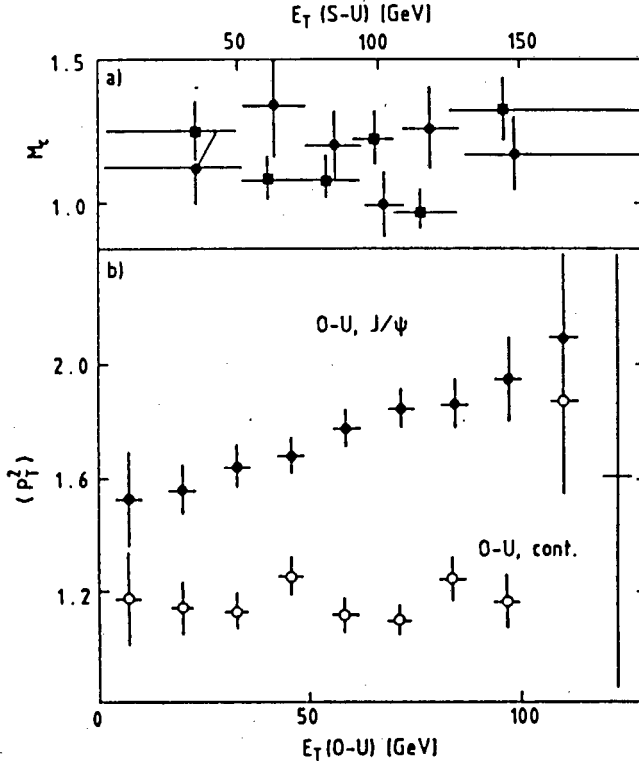


Figure 3.10: E_T independence of the continuum parameters (a) M_c for O-U and S-U collisions and (b) $\langle p_T^2 \rangle$ for O-U collisions is observed, while in (b) $\langle p_T^2 \rangle$ for J/ψ increases with E_T . (Top scale for S-U, bottom scale for O-U) [72].

3. The results must still include corrections for acceptance but this is not expected to influence the observed suppression by much [75].

Thus suppression in the J/ψ signal as predicted by Matsui and Satz [59] is indeed seen. A further feature of the data, is that the p-U and Drell-Yan background display weak E_T dependence in agreement with [59].

Perhaps the most important feature of the J/ψ suppression is the momentum dependence, displayed via the ratio $\mathcal{R}_\psi(p_T)$ which is the number of J/ψ events at high E_T , normalised by the integrated Drell-Yan continuum, versus low E_T , again normalised by the appropriate Drell-Yan continuum, in some momentum range.

$$\mathcal{R}_\psi^{\text{exp}}(p_T) = \frac{N_{J/\psi}^{\text{high}E_T}(p_T)}{N_c^{\text{high}E_T}} \bigg/ \frac{N_{J/\psi}^{\text{low}E_T}(p_T)}{N_c^{\text{low}E_T}} \quad (3.10)$$

This ratio is similar to eqn(3.7) suggested by Karsch and Petronzio. Plots of $\mathcal{R}_\psi^{\text{exp}}(p_T)$ have been made for: oxygen on uranium [65,70] (see fig(3.11a), with a slightly differing mass range for fig(3.11b)); oxygen on a lighter target, namely copper [69,73] (see fig(3.12)); a heavier projectile namely sulphur on uranium [71,73] (see fig(3.13)) and protons on uranium [65,69] (see fig(3.14)). A similar ratio to eqn(3.10) has been constructed for the continuum events outside the J/ψ mass region, with the corresponding plot available for the O-U data [65,68,70,71] (see fig(3.15) where two different mass ranges are used).

Dependence on p_T is observed for the O-U, O-Cu, and S-U data with a tendency toward no suppression at high p_T . The data for the Cu target is much flatter and has less suppression in comparison to the O-U data. A possible explanation may be found in terms of the quark-gluon plasma argument; less plasma is expected to form for the lower mass target, and hence less suppression will result. The more difficult data to explain is the S-U which is again flatter than the O-U data, though having more suppression. This may be explained in terms of the plasma volume being large enough in the S-U case so that even at the largest measured $p_T = 2.4$ GeV, suppression is still active, whilst for the O-U data at $p_T = 2.4$ GeV significantly more J/ψ escape the plasma effects. Little p_T dependence is observed for the p-U data, where no plasma formation is expected. The continuum shows no detectable p_T dependence, though the errors are too large to draw any conclusions. Thus the data may be qualitatively explained in terms of the Karsch-Petronzio model. It should be noticed in figs(3.11,3.15), where differing mass ranges are used, that the data is relatively insensitive to the mass range.

Is the observed p_T dependence merely due to a relative continuum increase at low p_T ? This is perhaps partially answered by the observation that the average p_T distribution of J/ψ widens at higher E_T values whilst the continuum does not show any E_T dependence. This may be seen in fig(3.10b).

Some very preliminary data is available for the $p_{||}$ dependence. This has been presented by Salmeron [76], see fig(3.16); the values of $p_{||}$ are found to range from zero to 4 GeV. This data should be included in a model which is to describe J/ψ suppression data.

Thus the conclusions that may be drawn from the NA38 data are: that suppression of the J/ψ over the continuum signal for central collisions is indeed seen in relativistic heavy ion collisions, being least effective for the smaller target mass namely Cu. The data appear to lie approximately in a straight line when plotted in terms of the expected energy density for the collision. No dependence on transverse energy is seen for the continuum slope parameter M_c . Little p_T dependence for the p-U data and no observable p_T dependence in the continuum is seen, though it should be noted that the experimental errors are large for the continuum p_T dependence. Finally longitudinal momentum dependence for the production of the J/ψ is present, peaked around 1.7 GeV.

The momentum dependence of the suppression is in qualitative agreement with the Karsch-Petronzio [59] prediction — this includes the dependence on target and projectile

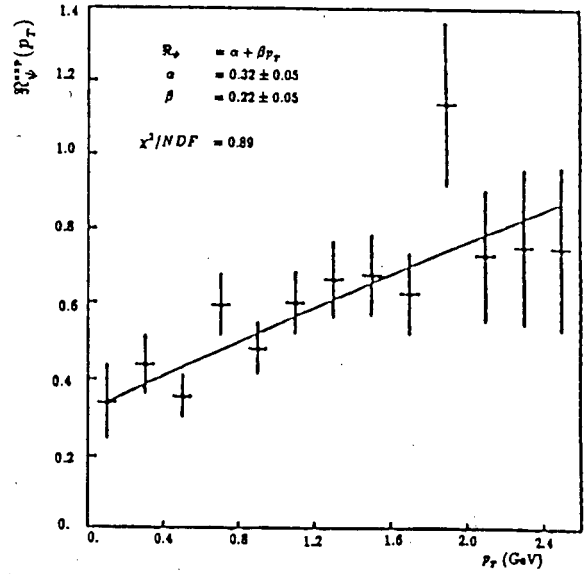
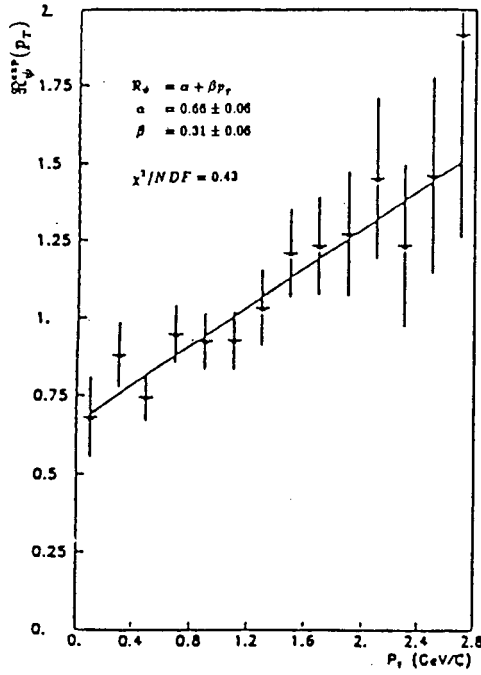


Figure 3.11: The ratio $R_{\psi}^{\text{exp}}(p_T)$ for O-U collisions as a function of p_T . The mass range used is (a) $2.5 \leq M_{\mu^+\mu^-} \leq 3.5$ GeV [65] and (b) $2.7 \leq M_{\mu^+\mu^-} \leq 3.5$ GeV [69].

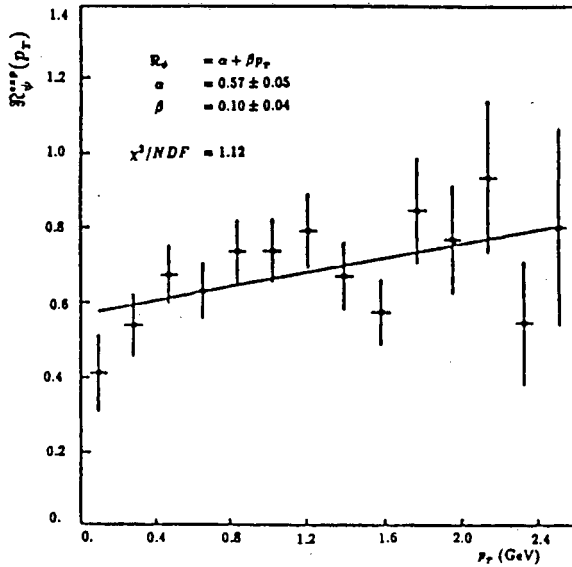


Figure 3.12: Same as fig(3.11a) but for O-Cu collisions [69].

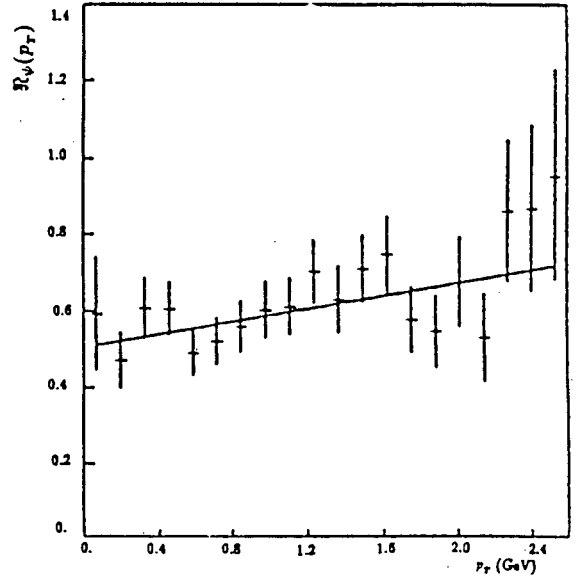


Figure 3.13: Same as fig(3.11a) but for S-U collisions [69].

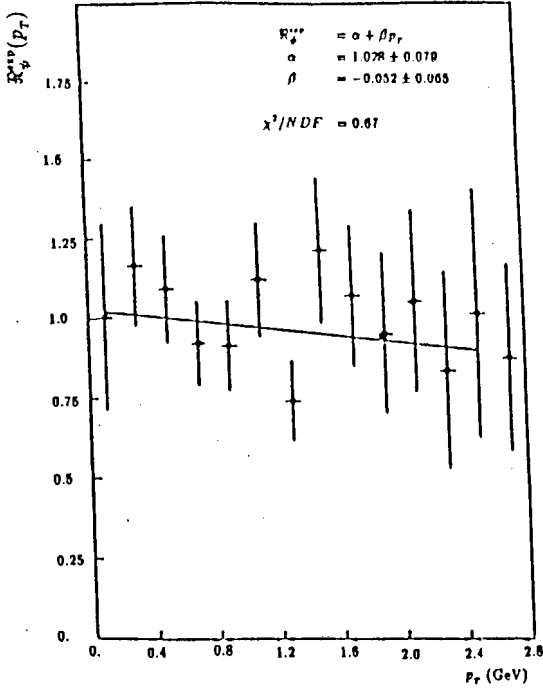


Figure 3.14: The ratio R_{ψ}^{exp} of the number of high E_T to low E_T J/ψ , as a function of p_T for p-U collisions [69].

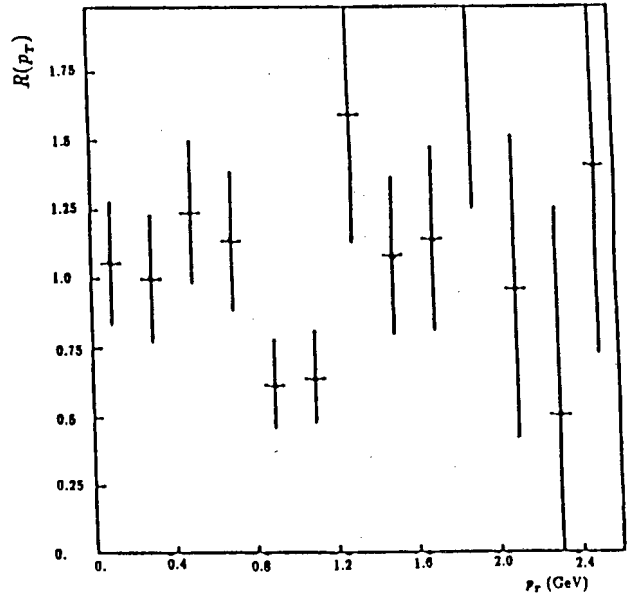
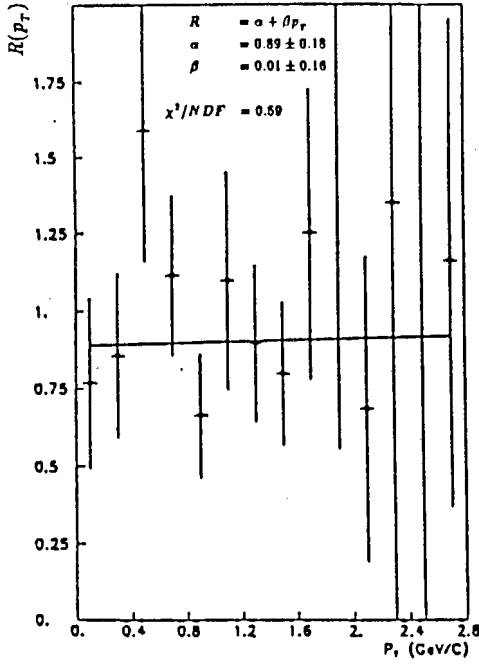


Figure 3.15: The ratio $R(p_T)$ of the number of continuum events with high E_T versus low E_T as a function of p_T , for the continuum events with mass range (excluding the J/ψ region) of: (a) $M_{\mu^+\mu^-} > 1.6$ GeV [65] and (b) $1.4 \leq M_{\mu^+\mu^-} \leq 10.0$ GeV [69]

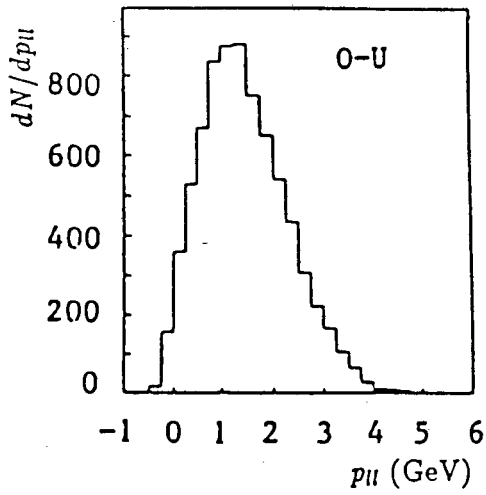


Figure 3.16: J/ψ p_T distribution of O-U events in the NA38 experiment [76].

mass. To enable the use of the data as evidence for a deconfining quark-gluon plasma it is essential to improve the model to a level of quantitative agreement with the data, and to examine possible alternatives to plasma suppression of the J/ψ .

Chapter 4

Plasma Suppression Models

The NA38 experiments led to the need for a more sophisticated model to account for the observed J/ψ suppression pattern, in particular the E_T and p_T dependence of this suppression. The reproduction of the p_T dependence of the NA38 data has been particularly concentrated upon, and has led to numerous models being proposed. In this chapter, models which account for J/ψ suppression in terms of the presence of a quark gluon plasma are discussed. As a first approach, the formation of J/ψ is treated classically, occurring after a distinct time τ_ψ , in a similar manner to the model by Karsch and Petronzio [59] discussed earlier, leading to a prediction for the p_T dependence of J/ψ suppression. More advanced models also incorporate a prediction for the E_T dependence of J/ψ suppression. However, some ambiguity is found to arise in the determination of τ_ψ ; this has led to the introduction of models which attempt to clarify this ambiguity by introducing a quantum mechanical approach to J/ψ formation. These are discussed later in the chapter.

4.1 Classical Models for the p_T Dependence of the Suppression

These models essentially build on the ideas suggested by Karsch and Petronzio in their model [59], where a typical formation time for the J/ψ resonance is introduced. This was obtained from a rough estimate of the time required for the c and $\bar{c}$ quarks to reach the J/ψ binding radius, after which if the J/ψ finds itself still within the plasma, it is destroyed. The extensions to the model basically include the finite lifetime of the dynamically expanding plasma, and the production of J/ψ from excited state $c\bar{c} \gamma$ decay. Later the effect of non-central collisions is included. In order to make the models more predictive, an improved estimate for the finite temperature effects on the heavy quark resonances has to be introduced.

4.1.1 Finite Temperature Effects on the Heavy Quark Resonances

In [79] Karsch, Mehr and Satz discuss the influence of a deconfining plasma thermal environment on the heavy quark $c\bar{c}$ and $b\bar{b}$ resonances. Due to the large quark mass, it is possible to parameterise the quark-antiquark potential binding in terms of a non-relativistic potential, of the Cornell form [83]

$$V(r, T = 0) = \sigma r - \frac{\alpha}{r} \quad (4.1)$$

with $\sigma = 0.192 \text{ GeV}^2$ and $\alpha = 0.471$ determined in [84]. The quark masses used are $m_c = 1.320 \text{ GeV}$ and $m_b = 4.746 \text{ GeV}$. The presence of the linear term in the potential implies that an infinite amount of energy is required to separate the quarks; thus some kind of quark confinement exists for the model. The model is quantised in terms of the time independent Schrödinger equation, and the wave-function computed numerically to yield the energy spectrum and associated mean square radius for the first few bound states. The J/ψ is identified as the $n = 1$ ground state; the ψ' as the $n = 2, \ell = 0$ state; and the χ as the $n = 2, \ell = 1$ bound state. Comparison with the experimental spectrum is possible, though it must be realised that the spin effects have been ignored, so that the $n = 1$ ground state corresponds to a mixture of η_c and J/ψ resonances; reasonable agreement is found for the first few resonances. An important feature of the model is the non-relativistic approximation used which should be checked — for the first few resonances, the quark masses dominate the contribution to the energy, and thus the quark momentum is relatively small, whilst the higher energy bound states must be treated relativistically. The approximation is better for the heavier $b\bar{b}$ quark system than the $c\bar{c}$. This model is particularly useful, as it serves to provide an estimate of the mean c to $\bar{c}$ separation distance and relative momentum, from which the J/ψ formation time is estimated in the Karsch-Petronzio model of J/ψ suppression [59] discussed earlier. In fact this is generalised to include formation times for the ψ' and χ resonances as well [82].

At finite temperature, the potential of

$$V(r, T) = \frac{\sigma}{\mu(T)}(1 - e^{-\mu(T)r}) - \frac{\alpha}{r}e^{-\mu(T)r}$$

is proposed, where $\mu(T)$ enters as a Debye screening mass term in the Coulomb part of the potential. The linear term of the potential is suggested by the Schwinger Model [85]. The effect on the potential, is to remove the confining form, as can be seen from the finite nature of the potential at large r . As a first approximation use of the Heisenberg uncertainty relation $\langle p^2 \rangle \langle r^2 \rangle \sim 1$ can be made to estimate a lower bound for the kinetic energy. This leads to a form of the hamiltonian, in the $\ell = 0$ case, given by

$$E(r, \mu) = 2m + \frac{c}{mr^2} + V(r, \mu)$$

where c is an arbitrary constant to be determined later. By minimizing the energy with respect to the radius r , the radius r_0 corresponding to the most tightly bound state can be determined. For the case $\mu = 0$ the energy can be compared to the experimental value, spin averaged over the J/ψ and η_c , and a reasonable estimate for c obtained; a comparison can then be made between r_0 and the value obtained from the quantised model at $T = 0$ — the value for r_0 is found to be approximately 15% smaller than the corresponding radius. The effect of a thermal environment can be determined by increasing $\mu(T)$ and performing the energy minimisation procedure. A minimum is still seen for sufficiently small μ , with r_0 increasing slightly, and the binding energy decreasing, till some critical value μ_c is reached, where no minimum exists — the quark-antiquark system is no longer bound, see fig(4.1), (the radius does not diverge as $\mu \rightarrow \mu_c$ see fig(4.2)).

A similar procedure to solving the quantised $T = 0$ case may be followed for the finite temperature case. Following the discussion for the semi-classical model, the existence of bound states at finite temperature is expected; in particular it may be possible for the heavy quark resonances such as the J/ψ which are small on the hadronic scale to persist even above the temperature where the lighter quark resonances have become deconfined, till the screening

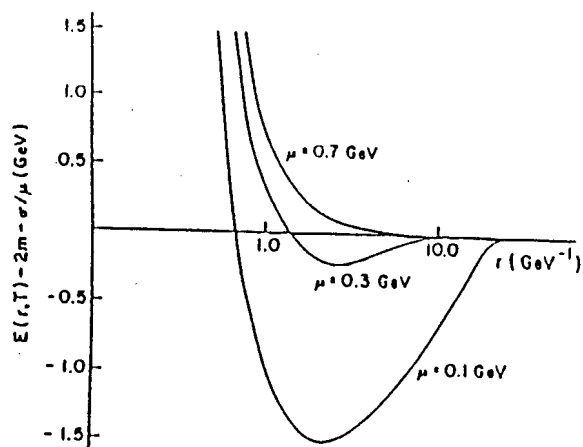


Figure 4.1: Effective binding potential in the semi-classical approximation [79].

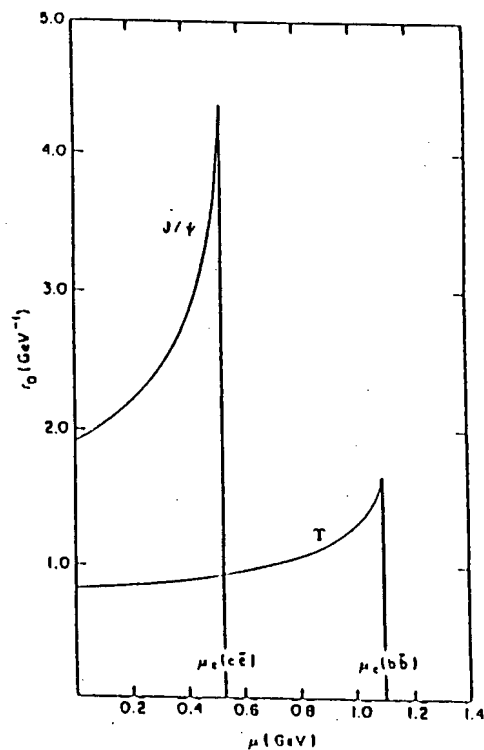


Figure 4.2: Radii of the lowest $c\bar{c}$ and $b\bar{b}$ bound states in the semi-classical approximation [79].

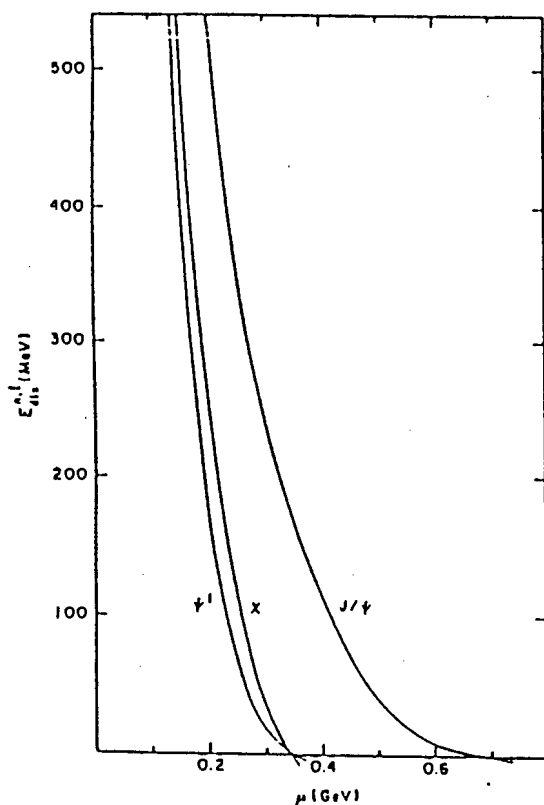


Figure 4.3: Dissociation energies for $c\bar{c}$ bound states [79].

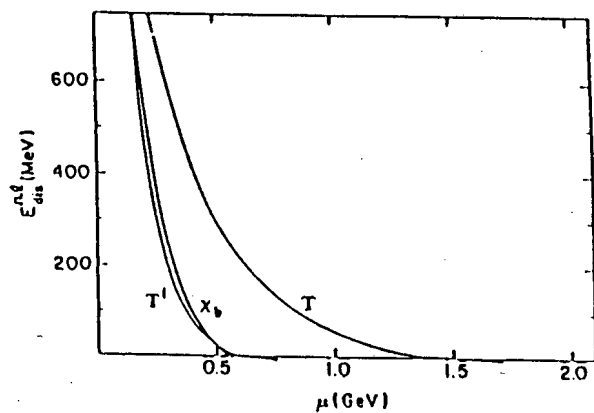


Figure 4.4: Dissociation energies for $b\bar{b}$ bound states [79].

radius becomes comparable to their own radius. This would correspond to the case where the Debye screening mass $\mu > \mu_c$ when the solutions will go over to the continuum. A useful quantity to use for examining the shift from bound states to continuum is the dissociation energy $E_{dis}^{n,\ell}$ defined as

$$E_{dis}^{n,\ell} = 2m + \frac{\sigma}{\mu} - E_{n,\ell}(\mu)$$

This can be understood with the realisation that the potential maximum is $\frac{\sigma}{\mu}$, so any states with energy above this will not bind — the factor of $2m$ must be added to account for the additional energy of the quark masses. Bound states are only possible for $E_{dis}^{n,\ell}(\mu) > 0$. In figs(4.3,4.4) the dissociation energy is plotted as a function of μ . The χ and ψ' resonances are observed to become unbound for a $\mu(T) \sim .4$ whilst the J/ψ remains until $\mu(T) \sim .6$, thus excited states will become unbound first. The radius plotted as a function of μ in fig(4.5) is observed to diverge as $\mu \rightarrow \mu_c$, unlike the previous semi-classical treatment. Little

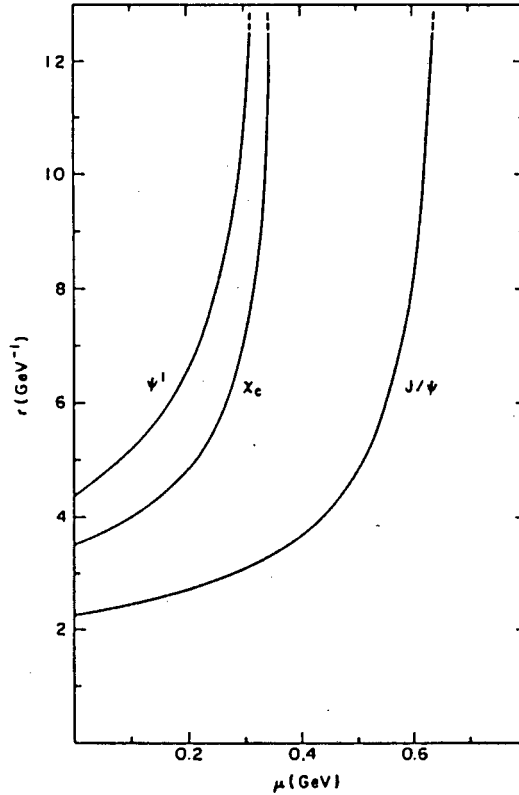


Figure 4.5: Radii for $c\bar{c}$ bound states [79].

change occurs in the mass of the resonances for $\mu \rightarrow \mu_c$, which is partly explained by the large contribution to the resonance mass by the heavy quark mass. Values for the critical Debye screening mass and corresponding resonance mass are tabulated in table(4.1); a noticeable trend is that the resonance mass is lighter for $\mu \rightarrow \mu_c$ except for the Υ which is heavier for μ_c .

The physical situation is expected to lead to the breaking up of the resonances before μ_c as

Table 4.1: Parameters for $c\bar{c}$ and $b\bar{b}$ bound states at $\mu = 0$ and $\mu = \mu_c$ [79].

		$c\bar{c}$	$b\bar{b}$
$n = 1, \ell = 0$ $(J/\psi, \Upsilon)$	$\mu[\text{GeV}]$	0	0
	$r[\text{GeV}^{-1}]$	2.263	1.130
	$r[\text{fm}]$	0.453	0.226
	$M[\text{GeV}]$	3.070	9.445
	$\mu_c[\text{GeV}]$	0.699	1.565
	$M(\mu_c)[\text{GeV}]$	2.915	9.615
$n = 2, \ell = 0$ (ψ', Υ')	$\mu[\text{GeV}]$	0	0
	$r[\text{GeV}^{-1}]$	4.373	2.546
	$r[\text{fm}]$	0.875	0.509
	$M[\text{GeV}]$	3.698	10.004
	$\mu_c[\text{GeV}]$	0.357	0.671
	$M(\mu_c)[\text{GeV}]$	3.177	9.778
$n = 2, \ell = 1$ (χ_c, χ_b)	$\mu[\text{GeV}]$	0	0
	$r[\text{GeV}^{-1}]$	3.478	2.039
	$r[\text{fm}]$	0.696	0.408
	$M[\text{GeV}]$	3.500	9.897
	$\mu_c[\text{GeV}]$	0.342	0.558
	$M(\mu_c)[\text{GeV}]$	3.198	9.829

the thermal motion of the environment has not been considered. Another effect which must be considered, is that for $T = 0$, μ is not expected to be zero. This is due to the vacuum screening effect which allows for the creation of light quark pairs when the binding energy exceeds the energy required to do this. As a result a more meaningful value of $\mu \approx 1 \text{ fm}^{-1}$ is expected.

The main conclusions reached are:

1. Bound states exist at finite temperature for $\mu < \mu_c$, even possibly when the lighter resonances are deconfined.
2. After some critical μ_c , the states are no longer bound.
3. The excited states become unbound first.
4. The mass of the resonances does not change much as a function of $\mu < \mu_c$.

5. A “static” thermal environment has been considered, in which the motion of the constituents is ignored. In reality the thermal motion of the constituents would lead to an earlier dissociation.

In order to make quantitative predictions from this discussion, the dependence of the screening mass on temperature should be known, as the temperature is the input for heavy ion collisions. The temperature dependence is however not well understood. Estimates from Lattice Q.C.D. calculations for the heavy quark potential in a thermal heatbath are in agreement with the form of the potential [86]; however an alternate formulation of the potential for $T > T_c$ which also agrees with Lattice calculations [87,88,89] is the Martin potential

$$\frac{V(r, T)}{T} = \frac{\alpha(T)}{(rT)^d} e^{-\mu r}$$

with $d \sim 1.2$. The results indicate that the ψ' and χ_c resonances essentially disappear at T_c . Indications are that the J/ψ will disappear for $T \approx 1.3T_c$ where the screening mass is of the order of 700 MeV [87].

4.1.2 The Ruuskanen and Satz Model

In [80] Ruuskanen and Satz comment on the plasma disk introduced by Karsch and Petronzio [59] which was discussed in section 3.1.3. For the collision energies present in the NA38 experiment, the nuclei are expected to be partially transparent [15], creating a central region in which the physics is essentially unchanged under a longitudinal Lorentz boost. In high energy proton-proton collisions the transparency gives rise to the plateau for particle production as a function of rapidity [17]. This has been discussed in more detail in section 2.2.

The plasma disk used by Karsch and Petronzio [59] to derive the longitudinal momentum dependence of the J/ψ suppression has a fixed length. This however totally neglects the longitudinal expansion of the plasma (it is the longitudinal expansion of the plasma which creates the boost invariant property of the central region) thus the boost symmetry of the central region is violated [80]. The picture described by Karsch and Petronzio will be more valid in the case of more stopping of the ions, present for lower collision energies and for heavier ions. To explain the longitudinal dependence correctly, the boost invariance of the central region should be built into the model.

A more realistic model of the central region consists of a boost invariant central region which evolves according to the proper time. The plasma is determined by the region in which the energy density is above some critical energy density ϵ_c required for plasma formation, thus the real plasma region will have a finite cutoff in rapidity y_c corresponding to the boundary where ϵ_c is reached. Further, due to the longitudinal expansion of the plasma, the energy density will drop till ϵ_c is reached, whereupon the pure plasma phase will no longer exist. This introduces a finite lifetime to the plasma. In the rest frame of some arbitrary plasma volume in the central region, the time evolution of that volume will be characterised by two times: the plasma will take approximately 1 fm to form (this was discussed in section 2.2), and a hadronisation time t_h when the energy density ϵ_c is reached. This is displayed in fig(4.6) where the plasma region is shown in terms of the more general proper time evolution of each volume, and the rapidity cutoff (this is assumed to be time independent for the purpose of the diagram).

The spacial extension of the original nuclei is not expected to strongly influence the picture — even though the soft secondaries can be emitted in a region of approximately 1 fm

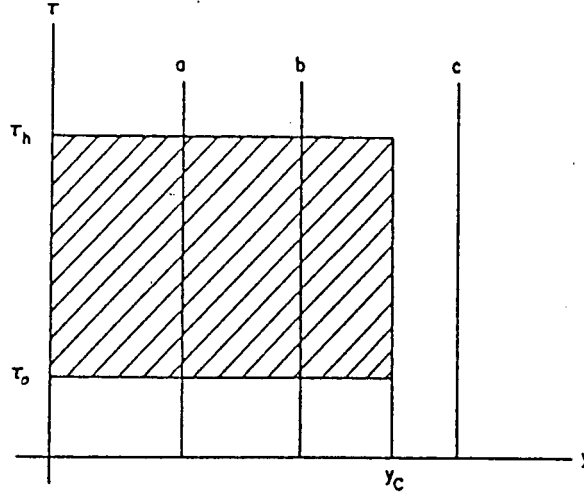


Figure 4.6: Schematic development of plasma in terms of proper time and c.m.s. rapidity [80].

[49], the boost invariance of the central region in high energy collisions requires that the faster particles should originate from a smaller z region, and hence the interaction is still well approximated by a pointlike picture.

An estimate for the rapidity cutoff can be gained by considering the total energy of the central region. The energy density of the central region is given by [16,49]

$$\varepsilon = \frac{m_T}{\pi \tau_0^2 A^{2/3}} \left(\frac{dN}{dy} \right)_{AA}$$

where m_T is the average transverse mass of the produced secondaries, $\tau_0 = 1.15$ fm and $(dN/dy)_{AA}$ refers to the number of secondaries per rapidity. This is similar to the form discussed in section 2.2. A parameterisation is defined

$$\left(\frac{dN}{dy} \right)_{AA} = c A^\alpha \left(\frac{dN}{dy} \right)_{pp}$$

to estimate the A dependence of the produced secondaries in terms of that for pp collisions. The exponent α is estimated to be in the range $1 \lesssim \alpha \lesssim 4/3$. The factor c is of the order of unity for average events but increases linearly with the multiplicity of the nucleus-nucleus collisions. A critical value of $\varepsilon \gtrsim \varepsilon_c \approx 2.5$ GeV/fm³ is chosen for the critical energy density, which is estimated from lattice simulations [81], required for plasma formation. A lower limit given by $\tau_h \geq \tau_\psi$ with $\tau_\psi \approx 0.89$ fm exists for the plasma lifetime if J/ψ suppression is to occur. From these limits a lower bound for the mass number A arises, given by

$$A \gtrsim \left(\frac{\pi \tau_0^2 \varepsilon_c \tau_h}{c m_T} \left(\frac{dN}{dy} \right)_{pp}^{-1} \right)^{\frac{3}{3\alpha-2}}$$

With the choice of $m_T = 0.5$ GeV, $\alpha = 1$, $c = 2$ and $(\frac{dN}{dy})_{pp} = 2.5$ a value of $A \gtrsim 51$ arises. However this value is very sensitive to small changes in the parameters due to the overall cubic power; for instance a value of $c = 3$ allows plasma production for $A = 15$.

If the energy density is assumed independent of the rapidity for $y \leq y_c$, then the total energy is given by

$$E(y_c, \varepsilon) = 2\pi r_0^2 A^{2/3} \tau \varepsilon \sinh y_c$$

As the energy is bounded by the collision energy, this yields an upper bound for the rapidity cutoff y_c as a function of τ for a given minimum energy density for the plasma. This is expected to be an upper bound, for the reasons that: the energy density does not immediately vanish for $y > y_c$ which allows for a region not included in the energy above, and the energy density is expected to decrease slightly as a function of increasing rapidity. Thus estimates for the CERN data with $E = A\sqrt{s} = 20$ GeV obtain a value of $y_c = 1.7$ and 2.6 for $A = 16$ and 240 respectively. It is pointed out that the decrease in rapidity values for smaller nuclei, which is due to the requirement that more stopping must occur to create the necessary energy density for plasma formation, need not necessarily hold. In fact it is more likely that the lighter ions will be more transparent as they have a limited thickness. Thus the results may have to be left to the case of a statistical fluctuation where more than average stopping occurred for the lighter ion.

4.1.3 The Blaizot-Ollitrault model

This model [78] builds upon the earlier Karsch-Petronzio model [59] by introducing dynamic effects of an expanding plasma, which provides a finite lifetime for the plasma — this is a crucial feature of the model as the predicted p_T suppression pattern is found to depend strongly on finite lifetime effects and not much on finite size effects in the range of p_T relevant to the NA38 data. The main features of the model include:

- A characteristic formation time for the J/ψ given by τ_ψ — only $c\bar{c}$ pairs outside the plasma region at this time (in the frame of the $c\bar{c}$ pair) produce J/ψ resonances.
- Dynamic longitudinally expanding plasma, with Bjorken scaling under longitudinal Lorentz boosts. The transverse expansion is small during the early stages of the plasma evolution associated with the formation of J/ψ , and thus to first approximation is ignored. Further, due to the scaling of the plasma under Lorentz boosts, it is possible to boost into a frame where the $c\bar{c}$ longitudinal momentum vanishes, and thus the first approach deals with only transverse moving J/ψ . The resulting prediction is that the suppression should not depend strongly on the longitudinal momentum.
- At a temperature T_1 the screening effect of the plasma is sufficiently strong that the $c\bar{c}$ pairs are effectively unbound and J/ψ resonance formation is impossible.
- The transverse dependence of the entropy density is expected to be similar to the density of the colliding nuclei [21], this is of the same form for the nuclear density that is used by Karsch-Petronzio given by eqn(3.8), which yields the equation

$$s(r, t_0) = s_0 \left(1 - \frac{r^2}{R_0^2}\right)^a \quad (4.2)$$

where R_0 corresponds to the plasma radius determined by the smaller nuclear radius, t_0 is the plasma formation time and a is a free parameter determined by the collision geometry. As the transverse expansion is expected to be small this equation will be valid at later times as well. The parameter s_0 is defined as $s_0 = s(r = 0, t = t_0)$. In

a longitudinally expanding plasma with Bjorken scaling and entropy density current conservation, the time evolution of the entropy density is given by eqn(2.28).

- Only central collisions are considered at first.
- It is pointed out that the model is of course not valid for slow moving $c\bar{c}$ pairs which form J/ψ before the plasma has had time to thermalise (this depends strongly on the thermalisation time).

In order to estimate the dependence of the suppression it is necessary to have some input for the initial parameters s_0 , t_0 and R_0 . For an ideal relativistic boson gas, the energy density ε and number density n are given by

$$\varepsilon = g \int d^3p \frac{E}{e^{E/T} - 1} \quad \text{and} \quad n = g \int d^3p \frac{1}{e^{E/T} - 1}$$

where g is the degeneracy factor [14]. The above integrals may be performed to yield the ratio

$$\frac{\varepsilon}{n} = \frac{\Gamma(4)\zeta(4)}{\Gamma(3)\zeta(3)} T = \frac{\pi^4 T}{30\zeta(3)}$$

where $\Gamma(n)$ is the Gamma Function and $\zeta(n)$ the Riemann Zeta Function. The ratio of entropy density to number density may be determined with the aid of eqns(2.14,2.31), where $c_s^2 = \frac{1}{3}$ for an ideal gas and $\mu = 0$ for bosons,

$$\frac{s}{n} = \frac{4}{3} \frac{\pi^4}{30\zeta(3)} \quad (4.3)$$

Integration of eqn(4.3) via a similar argument to that used in the derivation of eqn(2.29) yields the entropy per unit rapidity given by

$$\frac{dS}{dy} = \int d^2x \, ts = \frac{4}{3} \frac{\pi^4}{30\zeta(3)} \frac{dN}{dy}$$

Use of eqn(2.28) may be made to rewrite the integrand in terms of the initial entropy density and time (this is equivalent to stating that the entropy per rapidity bin is constant during the expansion). Thus

$$\frac{dS}{dy} = \int d^2x \, t_0 s_0 = \frac{2}{45} \frac{\pi^4}{\zeta(3)} \frac{dN}{dy}$$

The integration may be performed, under the assumption that the thermalisation time t_0 is a constant and that the entropy distribution is given by eqn(4.2), to yield

$$\frac{dS}{dy} = \frac{t_0 s_0 \pi R_0^2}{(a+1)} = \frac{2}{45} \frac{\pi^4}{\zeta(3)} \frac{dN}{dy}$$

Thus a condition for t_0 , s_0 , R_0 is that they satisfy

$$\frac{t_0 s_0}{R_0} = \frac{2\pi^3}{45\zeta(3)} \frac{(a+1)}{r_0^3} \frac{1}{A} \frac{dN}{dy}$$

where R_0 is interpreted as the nuclear radius $R_0 = r_0 A^{1/3}$. For $a = \frac{1}{2}$ and $r_0 = 1.2$ fm the equation may be simplified to

$$\frac{t_0 s_0}{R_0} \simeq \frac{1}{A} \frac{dN}{dy} \text{ fm}^{-3} \quad (4.4)$$

where $\frac{dN}{dy}$ is the final particle distribution per rapidity bin.

Following the argument of Karsch-Petronzio [59] a hot region is defined wherein J/ψ resonances cannot form. However, the border of this region is to be determined by the temperature T_1 isotherm, found a radial distance $R_1(t)$ from the collision axis which will move towards the collision axis as the plasma expands and cools; thus the hot region is not constant during the plasma expansion. The time dependence of the isotherm is obtained by associating an entropy density to each temperature (the details of the relation are not important at this stage); then it is possible using entropy current conservation for the longitudinal expansion, which leads to eqn(2.28), to find the time dependence for the radial isotherms. With increasing time eqn(2.28) leads to a smaller entropy density along the collision axis, and correspondingly the temperature isotherms move towards the collision axis till some time t_1 , when the isotherm determined by temperature T_1 reaches the collision axis and the hot region disappears. Use of eqn(2.28) is then made to relate the entropy density for times $t \leq t_1$ to the entropy density s_1 at t_1 , given by

$$s(r=0, t) = s_1 \frac{t_1}{t} \quad (4.5)$$

The radial dependence for the entropy density given by eqn(4.2) determines the radius $R_1(t)$ where the entropy density s_1 (temperature T_1) for times $t \leq t_1$ is to be found

$$s_1 = s(r, t) = s(r=0, t) \left(1 - \left(\frac{R_1(t)}{R_0} \right)^2 \right)^a \quad (4.6)$$

With use of eqn(4.5) the entropy density $s(r=0, t)$ along the collision axis, in eqn(4.6) can be eliminated to yield an equation for the radial isotherm $R_1(t)$ given by

$$\frac{t}{t_1} = \left(1 - \left(\frac{R_1(t)}{R_0} \right)^2 \right)^a$$

from which the isotherm $R_1(t)$ can be found

$$R_1(t) = R_0 \left(1 - \left(\frac{t}{t_1} \right)^{\frac{1}{a}} \right)^{\frac{1}{2}} \quad (4.7)$$

It is necessary to determine the parameter t_1 in order to make use of eqn(4.7). A condition required for t_0 , s_0 , R_0 to satisfy has been established by eqn(4.4) which, with use of eqn(4.5) becomes

$$\frac{t_1 s_1}{R_0} = \frac{1}{A} \frac{dN}{dy} \quad (4.8)$$

The temperature dependence for the entropy density must now be established, and is given by eqn(2.34), where c_s^2 is chosen to be the speed of sound for an ideal relativistic gas ($c_s^2 = \frac{1}{3}$) from which it follows that

$$\frac{s}{s_c} = \left(\frac{T}{T_c} \right)^3 \quad (4.9)$$

where T_c and s_c refer to the deconfinement transition temperature and associated entropy density. It is important to realise that the speed of sound c_s is not constant near a phase transition so that it is not completely justified to use $c_s^2 = \frac{1}{3}$ close to the phase transition temperature. Combining eqn(4.7) with eqn(4.8) yields

$$\frac{t_1}{R_0} = \frac{1}{A s_c} \left(\frac{T_c}{T_1} \right)^3 \frac{dN}{dy} \quad (4.10)$$

A necessary condition for the approximation that the transverse expansion may be ignored, is that the cooling effect of the longitudinal expansion is sufficiently fast to destroy the hot region in a time t_1 , much shorter than the time for the transverse expansion effects, propagating inwards from the plasma radius R_0 , to reach the beam axis (see [21]). That is $t_1/R_0 \ll 1$ [78]. To check this an estimate for s_c must be obtained: from [21,14] one obtains, for an ideal relativistic gas of quarks and gluons with zero net baryon number,

$$\varepsilon_{th} = \frac{\pi^2}{60} \{7N_c N_f + 4N_g\} T^4 \quad (4.11)$$

Now from eqn(2.14) with $\mu = 0$ and eqn(2.31) for an ideal relativistic gas with $c_s^2 = \frac{1}{3}$, the two equations

$$s = \frac{\varepsilon_{th} + P_{th}}{T} \quad \text{and} \quad P_{th} = c_s^2 \varepsilon_{th}$$

become, with the Bag constant cancellation upon adding ε and P ,

$$s = \frac{4}{3} \frac{\varepsilon}{T} = \frac{4}{3} \frac{\pi^2}{60} \{7N_c N_f + 4N_g\} T^3$$

thus for $N_c = 3$, $N_f = 2$ and $N_g = 8$

$$s = 37 \frac{2\pi^2}{45} T^3$$

so for $T_c = 170$ Mev, $s_c \approx 10 \text{ fm}^{-3}$. Thus, with the approximation $T_1 \approx T_c$, eqn(4.10) becomes

$$\frac{t_1}{R_0} = \frac{0.1}{A} \frac{dN}{dy}$$

which for an oxygen projectile satisfies $t_s/R_0 \ll 1$.

The initial $c\bar{c}$ distribution is determined in a similar manner to that used in the Karsch and Petronzio model earlier given by eqn(3.8). That is

$$P(x, y) = C \left(1 - \left(\frac{r_T^2}{R_0^2} \right)^2 \right)^b \quad (4.12)$$

where b is a free parameter to be determined by the collision geometry; the value of $b = \frac{1}{2}$ is chosen which agrees with that used earlier in the Karsch-Petronzio model. The distribution is assumed to be independent of the transverse momentum $\vec{p}_T$ of the pairs.

The ratio the ratio $\mathfrak{R}_\psi$ corresponding to eqn(3.7) is defined as: the number of $c\bar{c}$ pairs which, at proper time τ_ψ for each pair, are outside the hot region and can form J/ψ resonances; normalised by dividing by the total number of $c\bar{c}$ pairs produced in the initial collision.

A relation must now be found to determine which $c\bar{c}$ pairs will form the J/ψ resonances. If a $c\bar{c}$ pair is created at a position $\vec{r}_0$ from the collision axis with transverse momentum $\vec{p}_T$, using eqn(3.6) it then follows that the $c\bar{c}$ pair is later at a radial distance

$$r = \left| \vec{r}_0 + \vec{p}_T \frac{\tau_\psi}{M_\psi} \right|$$

where τ_ψ is the proper time of the pair. A $c\bar{c}$ pair can form a J/ψ resonance if they are outside the hot region which corresponds to the inequality

$$R_1(t_\psi) \leq \left| \vec{r}_0 + \vec{p}_T \frac{\tau_\psi}{M_\psi} \right|$$

With the cylindrical symmetry of the plasma it is sufficient just to define the angle θ between $\vec{r}_0$ and $\vec{p}_T$. From squaring the inequality it follows that

$$R_1^2(t_\psi) \leq r_0^2 + \left(p_T \frac{\tau_\psi}{M_\psi} \right)^2 + 2\vec{r}_0 \cdot \vec{p}_T \frac{\tau_\psi}{M_\psi} \cos \theta$$

and a limit for the allowed θ range can then be determined

$$1 \geq \cos \theta \geq \frac{R_0^2 \left(1 - \left(\frac{\tau_\psi}{t_1} \sqrt{1 + \left(\frac{p_T}{M_\psi} \right)^2} \right)^{\frac{1}{a}} \right) - r_0^2 - \left(\frac{p_T \tau_\psi}{M_\psi} \right)^2}{2 r_0 p_T \frac{\tau_\psi}{M_\psi}} \quad (4.13)$$

The $c\bar{c}$ pairs which satisfy the above inequality can form J/ψ . Therefore, a summation of all $c\bar{c}$ pairs according to the given density distribution eqn(4.12) with some given p_T , over $0 \leq r_0 \leq R_0$ and $0 \leq \theta \leq 2\pi$, which satisfy the above condition eqn(4.13) will yield the number of $c\bar{c}$ pairs which form J/ψ . The ratio $\mathfrak{R}_\psi(p_T)$ corresponding to eqn(3.10) for the NA38 data can then be computed by dividing by the total number of $c\bar{c}$ pairs produced. The parameters a and b are chosen to be $\frac{1}{2}$ for the purposes of calculation in agreement with the value used by Karsch and Petronzio [59] discussed in section 3.1.3. The main parameters still entering in the sum are, the ratio τ_ψ/t_1 and R_0 , which should be determined from the NA38 data.

The summation however in general is difficult to perform and it is easier to look at the end points of the curve. For the case $p_T = 0$ where the proper time of the $c\bar{c}$ pairs is the same as the plasma time, that is $\tau_\psi = t_\psi$, only $c\bar{c}$ pairs whose initial position is outside the hot region determined by $R_1(t_\psi)$ form J/ψ . The summation becomes an integral of the density of the $c\bar{c}$ pairs over the corresponding radial values, to yield the ratio

$$\mathfrak{R}_\psi(p_T = 0) = \frac{\int_{R_1(t_\psi)}^{R_0} dr r (1 - r^2/R_0^2)^b}{\int_0^{R_0} dr r (1 - r^2/R_0^2)^b}$$

The integration may be performed, and substitution for $R_1(t_\psi)$ made from eqn(4.7) to yield the ratio

$$\mathfrak{R}_\psi(0) = \left(\frac{\tau_\psi}{t_1} \right)^{\frac{b+1}{a}} \quad (4.14)$$

Thus the intercept at $p_T = 0$ is determined by the ratio τ_ψ/t_1 and is independent of R_0 . An extension of eqn(4.14) to finite p_T can be made by replacing the J/ψ formation time τ_ψ by the formation time t_ψ in the frame of the plasma where

$$t_\psi = \tau_\psi \sqrt{1 + \left(\frac{p_T}{M} \right)^2} \quad (4.15)$$

The motivation of this extension is discussed later in the model by Chu and Matsui. It should be noticed that this extension reproduces the high p_T limit of zero suppression discussed below.

An important limiting case exists for large p_T , where the Lorentz dilated time t_ψ is large enough so that the hot region has disappeared (i.e. $t_\psi \geq t_1$) before the $c\bar{c}$ pairs will have evolved to proper-time τ_ψ and thus can form J/ψ resonances. Therefore a limit of $\mathfrak{R}_\psi(p_T) = 1$ is attained for

$$t_1 \leq \tau_\psi \sqrt{1 + \left(\frac{p_T}{M_\psi}\right)^2} \quad \text{corresponding to} \quad p_T \geq M_\psi \sqrt{\left(\frac{t_1}{\tau_\psi}\right)^2 - 1} \quad (4.16)$$

Thus the prediction is made that for some sufficiently large p_T value, no suppression occurs. It is thus possible also to fix the ratio τ_ψ/t_1 from this point as well. For the NA38 data the ratio $\mathfrak{R}_\psi(p_T) \approx 1$ at $p_T \sim 3$ GeV. The ratio $\mathfrak{R}_\psi(p_T)$ for arbitrary p_T , found by numerically summing for each p_T , results in the curve fig(4.7), with the ratio τ_ψ/t_1 fixed from the high

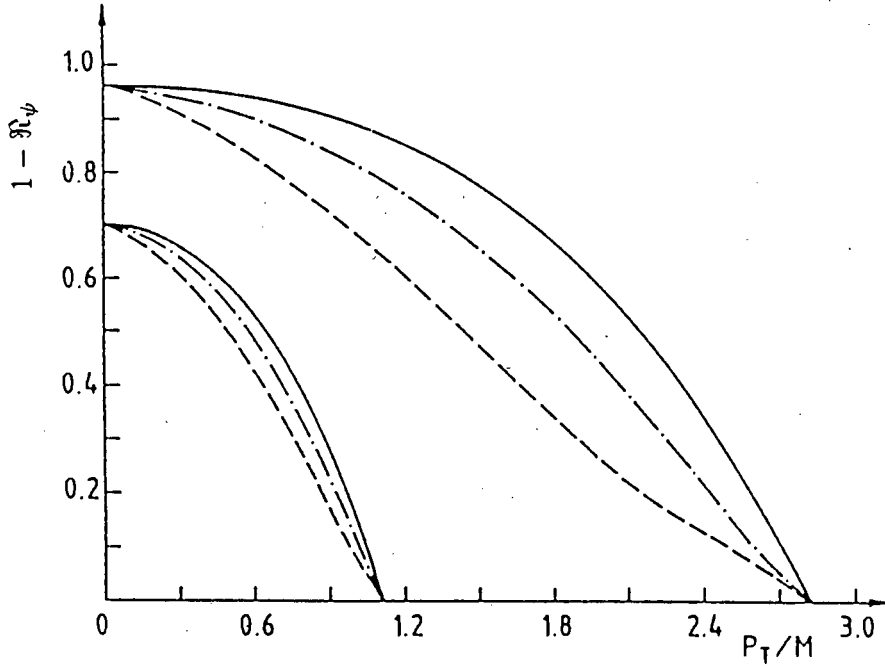


Figure 4.7: Variation of the suppression rate $\mathfrak{R}_\psi$ as a function of p_T/M for several values of t_1/τ_ψ and R_0 . The parameters a and b in eqns(4.6,4.12) have been set equal to $\frac{1}{2}$. The formation time τ_ψ is fixed at 0.7 fm. Full line: analytical extension of eqn(4.14) by replacing τ_ψ the formation time of J/ψ by t_ψ defined by eqn(4.15) in the c.m.s. frame of the collision. Dashed line: $R_0 = 3$ fm. Dash-dotted line: $R_0 = 7.1$ fm. The two sets of curves correspond respectively to values 1.5 and 3 of t_1/τ_ψ [78].

p_T limit.

In comparison to the NA38 data the model fails in the low p_T region. This may be accounted for by the effect of non-central collisions, which are expected to form less plasma,

and hence have less suppression of the J/ψ resonance formation, particularly for the $c\bar{c}$ pairs with low p_T . Thus $\mathcal{R}_\psi(p_T)$ is expected to increase with the inclusion of non-central collisions, for low p_T , whilst the high p_T limit will remain unchanged. This effect will be discussed in more detail later.

4.1.4 The Improved Karsch-Petronzio Model

In [82] an improvement to the initial prediction by Karsch and Petronzio [59] for the momentum dependence of J/ψ suppression is made. Essentially, this is achieved by including the plasma finite lifetime effects discussed by Blaizot and Ollitrault [78], for a plasma which is expanding. A feature which is to be inherited from the previous model by Karsch and Petronzio [59] is the finite size of the region in which the $c\bar{c}$ pairs are produced. This is to be a flat disk of height 1 fm and radius essentially determined by the smaller projectile nuclear radius.

In order to describe the J/ψ suppression successfully it is important to realise that the measured J/ψ decays are not all necessarily coming from directly produced J/ψ — the decay of excited $c\bar{c}$ resonances to J/ψ first is possible. The experimental result [90] is that $\sim 40\%$ of the J/ψ originate from the radiative decay of the $\chi_{c1}(3510)$ and $\chi_{c2}(3555)$ states via $\chi \rightarrow J/\psi + \gamma$; approximately 5% of the J/ψ are coming from the decay of the ψ' as well. At higher energies it is expected that the contribution from these and higher excited states will become even higher. The NA38 experiment cannot detect the decays so it is important for the models to incorporate this feature [76].

It therefore must be pointed out that the Blaizot-Ollitrault treatment [78] does not include the χ resonance, which will be included in this model. A prediction immediately arises: in section 4.1.1 it was observed that the χ resonance is expected to be destroyed at a lower plasma temperature than the J/ψ ; thus the possibility arises, with increased statistics, that a threshold effect might be observed where χ resonances are still suppressed, but the directly produced J/ψ are not.

The dependence of the J/ψ suppression on the longitudinal momentum fraction x_F is also considered. This is different to the treatment in section 4.1.3 as it does not assume Bjorken scaling in the production of J/ψ ; this is based upon the fact that scaling is not completely evident at the present experimental energies. The NA38 data for the p_T dependence of the J/ψ suppression given as the ratio in eqn(3.10), for the data [65], needs to be rescaled by a factor of 0.67 to obtain the “absolute” normalisation [82]. The data is rescaled with use of a straight line fit for each of the x_F bins considered, so that they all coincide with the value of $\mathcal{R}_\psi = 1$ at $P_T = 3$ GeV. This yields the experimental values of

$$\begin{aligned} \mathcal{R}_\psi(0) &= 0.52 \pm 0.07 & \text{all } x_F \\ \mathcal{R}_\psi(0) &= 0.65 \pm 0.09 & x_f > 0.15 \\ \mathcal{R}_\psi(0) &= 0.44 \pm 0.11 & x_f < 0.15 \end{aligned} \tag{4.17}$$

where the average of the two lowest p_T bins has been used with the error estimated from the spread of values in these bins.

The experimental x_F distribution of J/ψ is found to be peaked around 0.15 with an upper limit of 0.4 [65] as discussed in section 3.2 and in fig(3.16). To match this a parameterization

for the x_F dependence of the form

$$\begin{aligned} \frac{dN}{dx_F} &= \frac{100}{3} x_F & 0 < x_F < 0.15 \\ \frac{dN}{dx_F} &= 8 - 20x_F & 0.15 < x_F < 0.4 \end{aligned} \quad (4.18)$$

is chosen. This corresponds to roughly 50% of the $c\bar{c}$ pairs with momenta larger than 1.5 GeV — the scale is determined from the total available energy in the nucleon-nucleon centre of mass frame of about 20 GeV.

Following the method outlined in section 3.1.3, the typical formation times for the J/ψ , χ , and ψ' resonances are calculated from the mean radius and momentum values determined from [79] (discussed in section 4.1.1), and are given in table(4.2). The χ has a longer formation

Table 4.2: Mean-square radius (r), mean-square radial momentum (p), formation time (τ) and the critical screening mass μ_c for charmonium and bottomonium bound states. The numbers have been obtained from a solution of the non-relativistic Schrödinger equation ([79,82] and section 4.1.1) [82].

Resonance	r [fm]	p [GeV]	τ [fm]	μ_c [GeV]
J/ψ	0.453	0.672	0.89	0.699
ψ'	0.875	0.768	1.50	0.357
χ_c	0.696	0.456	2.01	0.342
Υ	0.226	1.408	0.76	1.565
Υ'	0.509	1.271	1.90	0.671
χ_b	0.408	0.744	2.60	0.558

time than the J/ψ , so that whilst it is suppressed still at a lower temperature than the J/ψ , it has longer to escape the plasma region. The two effects may work against each other to produce a momentum dependence which is not too different to that for the J/ψ . The discussion of the time dependence for the isotherms, in section 4.1.3 from [78] for a plasma with Bjorken scaling, is carried over to this model (it is the $c\bar{c}$ pairs which do not scale). The limiting p_T value corresponding to $\mathfrak{R}_\psi = 1$, where the Lorentz dilation on the formation time is sufficiently large that the hot region has disappeared, is again chosen as 3 GeV for the J/ψ . However since the present data do not show any threshold effects, with the lack of information to determine where this should in fact be, and the expected conspiracy of the longer formation time versus longer lifetime, the same value for the complete escape momentum of $p_T = 3$ GeV is chosen. It should be noted that at least some χ suppression does occur since the J/ψ suppression in the zero x_F bin is of the order of 70%. Use can be made of eqn(4.16) to estimate the time when the corresponding hot region disappears — if the form for the time dependence of the temperature is chosen as $T = ct^\alpha$ then it follows that

the ratio of the corresponding critical temperatures is

$$\frac{T_{c,\psi}}{T_{c,\chi}} = \left(\frac{t_\psi}{t_\chi} \right)^a$$

If the assumption is made that the entropy density time dependence is of the form $t_1 s_1 = t_2 s_2$ as it would be, for instance in a plasma with Bjorken scaling given by eqn(2.28), and with the form for the entropy density temperature dependence used previously in eqn(4.9) the value for a can be determined, so that

$$\frac{T_{c,\psi}}{T_{c,\chi}} = \left(\frac{t_\psi}{t_\chi} \right)^{-\frac{1}{3}} \quad (4.19)$$

which is the same dependence as is found for the Bjorken scaling given by eqn(2.33) with $c_s^2 = \frac{1}{3}$. If use is made of perturbation theory, with the screening mass given by [91]

$$\mu_c = \left(1 + \frac{n_f}{6} \right)^{\frac{1}{2}} g(T) T$$

where $g(T)$ is the temperature dependent running coupling constant and n_f the number of light quark flavours, which is supported by non-perturbative Lattice calculations [82], then the ratio of screening masses may be estimated. The values of the critical times t_ψ and t_χ for the momentum choice of 3 GeV are

$$t_\psi = 1.24 \text{ fm} \quad t_\chi = 2.65 \text{ fm}$$

The χ is suppressed for a considerable larger time than the J/ψ , but this is due to the non-linear relation between temperature and time in eqn(4.19). The times are within reasonable bounds, as the plasma lifetime is expected to be of the order of a few fermi. The ration of the times yields the ratio for the corresponding temperatures, in comparison to the ratio of the critical masses obtained from table(4.1), of

$$\frac{T_\psi}{T_\chi} = 1.3 \quad \frac{\mu_{c,\psi}}{\mu_{c,\chi}} = 2.0$$

which are of similar perhaps justifying the choice of the critical momentum.

The computation of the p_T dependence of the J/ψ suppression then proceeds similalry to Blaizot-Ollitrault [78] discussed in section 4.1.3; the x_F dependence is however included, and the choice of the parameters a and b in eqns(4.2,4.12) are chosen to be equal and estimated to be approximately $\frac{1}{3.3}$ from fitting the x_F averaged suppression ratio for $p_T = 0$. For simplicity a value of $b = \frac{1}{3}$ is chosen to examine the x_F dependence. The x_F dependence for $p_T = 0$ will be found in the Lorentz dilation of the formation time for the resonances, thus the $c\bar{c}$ with a higher x_F will take longer to form resonances in the plasma frame. This will be evident in the finite lifetime effect of the plasma which will have cooled more due to the expansion, with the critical isotherm moving inwards, and a greater number of $c\bar{c}$ pairs will be outside of it when they form resonances. The finite longitudinal extension of the plasma is not important since the $c\bar{c}$ pairs have a maximum $p_{||}$ value of 4 GeV so that at the formation time the maximum distance travelled by a $c\bar{c}$ pair is given by

$$z = p_{||} \frac{\tau_\psi}{M_\psi}$$

which has the value of 1.2 fm for the J/ψ and 2.3 fm for χ resonance. This is much less than larger nuclear radius which is of the order of 5 fm so that the plasma is essentially infinite in the longitudinal direction. Calculations at $p_T = 0$ with the chosen x_F distribution given by eqn(4.18) averaged over the given x_F ranges yield

$$\begin{aligned} \mathcal{R}_\psi(0) &= 0.57 & 0 \leq x_F \leq 0.4 \\ \mathcal{R}_\psi(0) &= 0.69 & 1.5 \leq x_F \leq 0.4 \\ \mathcal{R}_\psi(0) &= 0.37 & 0 \leq x_F \leq 1.5 \end{aligned} \quad (4.20)$$

which compare favourably with the experimental values given in eqn(4.18). It should be noted that the experimental data [65] for the x_F dependence is very preliminary.

The calculation of the x_F averaged p_T dependence of the suppression is then done; the dependence is presented together with the experimental data in fig(4.8). Reasonably good

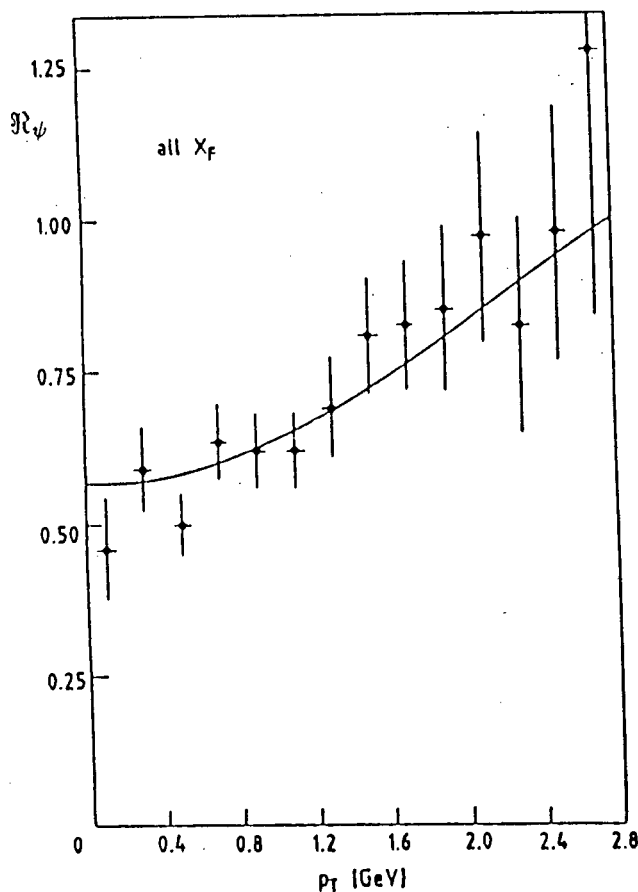


Figure 4.8: J/ψ suppression as a function of p_T . Shown is a comparison of the model with the NA38 data [65]. The vertical scale has been rescaled with $x = 0.67$ [82]

agreement is obtained within the given experimental errors. The dependence on x_F over all p_T is given in fig(4.9).

At present the statistics are not sufficient for use of the experimental data for the ψ' resonance. The only parameter which is not known for the ψ' is the time $t_{\psi'}$ for the plasma to be below the critical temperature for destroying the ψ' resonances. From table(4.2) the values of the critical screening masses for the ψ' and χ are found to be similar, thus the time $t_{\psi'} \approx t_\chi$. From this it follows with use of eqn(4.16) that the critical momentum required is 5.4 GeV which explains why the signal is not present within the accuracy of the data. The p_T dependence has been calculated and is given in fig(4.10).

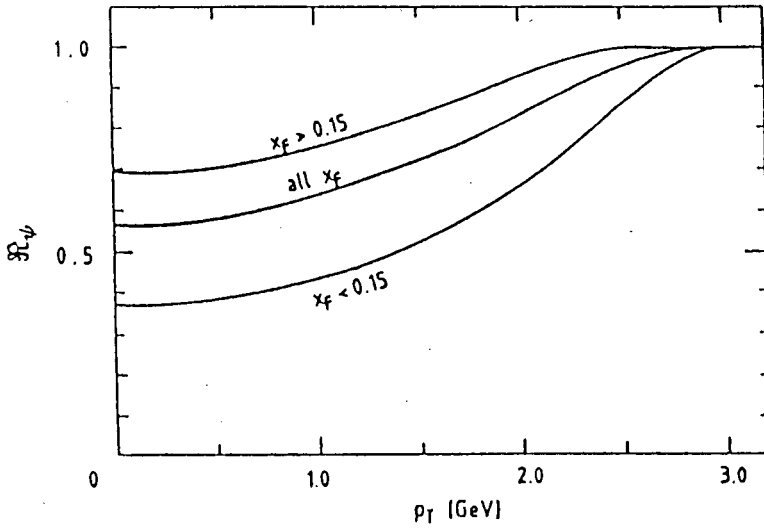


Figure 4.9: J/ψ suppression pattern as a function of p_T for the x_F integrated case for two different x_F bins [82].

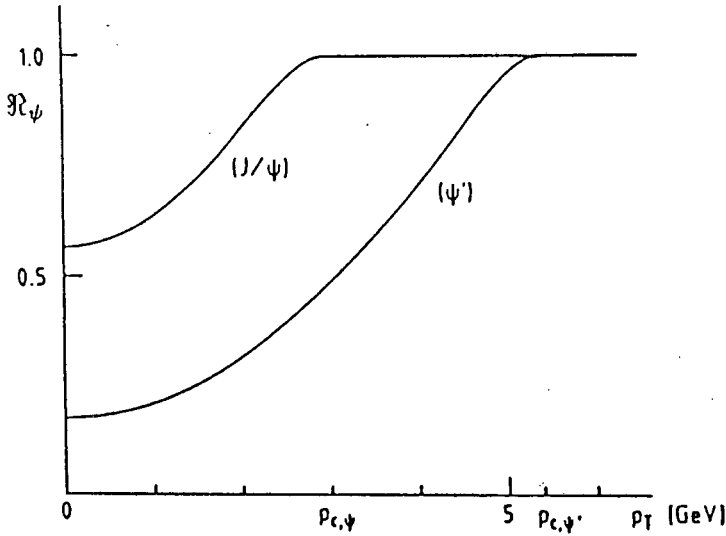


Figure 4.10: Comparison of the p_T dependence of ψ' and J/ψ suppression rates [82].

Finally, a prediction for the preliminary sulphur data for the NA38 experiment [71] is made. The plasma lifetime is expected to be longer, which will result in a larger value for critical time for the plasma to cool. This in turn will require a larger p_T value, which is estimated to be approximately 4 GeV. The production region is also expected to be larger, corresponding to the larger nuclear radius. The zero p_T suppression, assuming the same x_F dependence as for the J/ψ , is expected to significantly drop from 50% to 80% suppression.

4.1.5 The Chu Matsui Model

This model [92,93] is similar to the Blaizot and Ollitrault model [78], discussed in section 4.1.3, with the major difference being that the model uses energy density instead of temperature to describe the time evolution. This is advantageous in that it is simpler to relate the energy density with the transverse energy dependence than the temperature. Thus an energy density ε_1 is identified as the critical energy density of the plasma, in parallel to the temperature T_1 used by Blaizot and Ollitrault, above which the screening is sufficient to destroy the J/ψ resonance. The model assumes that the colliding energy is sufficiently high that the Bjorken scaling picture [16] mentioned in section 2.2 is valid for the central region of the plasma. This is to be true also for the $c\bar{c}$ longitudinal distribution which is clearly different to the production “disk” used by Karsch and Petronzio [59,82] as discussed in section 4.1.4. The longitudinal dependence is thus not present in this model — instead the boost invariant characteristic of the plasma is carried through to the $c\bar{c}$ pairs. Hence it is sufficient to develop the model for $c\bar{c}$ pairs with zero longitudinal momentum; for an arbitrary $c\bar{c}$ pair it is always possible to boost to a frame where the pair has zero longitudinal momentum without changing the physics due to the boost invariant property of the plasma. The transverse expansion is assumed to be small and is ignored in the model, and initially only central collisions will be considered.

The time dependence for the energy density of the plasma is given by eqn(2.32), but the Bag term is to be ignored and the solution is to be found for the $z = 0$ plane so that the proper time is just replaced by the time, thus

$$\varepsilon(r = 0, t) = \varepsilon(0, 0) \left(\frac{t_0}{t} \right)^{1+c_s^2} \quad (4.21)$$

The radial dependence of the energy density is based upon the density of the colliding nuclei, discussed in section 3.1.3. In parallel with eqn(4.2) in the Blaizot and Ollitrault model [78] the radial dependence is given by

$$\varepsilon(r, t = 0) = \varepsilon(0, 0) \left(1 - \frac{r^2}{R_0^2} \right)^\alpha \quad (4.22)$$

with R_0 determined by the smaller nuclear radius and where α is a free parameter. This equation is assumed valid at later times, as the transverse expansion is small for the J/ψ formation timescale and has in fact been ignored in the model. The radial distance of the ε_1 constant energy circles can then be determined following a similar derivation in section 4.1.3 to yield the equation

$$R_1(t) = R_0 \left(1 - \left(\frac{t}{t_1} \right)^{\frac{c_1}{\alpha}} \right)^{\frac{1}{2}}$$

where the parameters are

$$t_1 = t_0 \left(\frac{\varepsilon(0, 0)}{\varepsilon_1} \right)^{\frac{1}{c_1}} \quad c_1 = c_s^2 + 1$$

In particular t_1 is the time when the critical energy density ε_1 reaches the beam axis — this is essentially the same quantity used by Blaizot and Ollitrault [78] discussed in section 4.1.3. From a comparison between eqn(4.7) and eqn(4.22) the relation $a = \alpha/c_1$ identifies the link

between the parameters used in the two models. For purposes of later using the radial dependence of the energy density to compute the transverse energy flow, it is useful to generalise t_1 in terms of the time $t_1(r)$ when the critical energy density is reached at radius r , given by

$$t_1(r) = t_0 \left(\frac{\varepsilon(0,0)}{\varepsilon_1} \right)^{\frac{1}{c_1}} \left(1 - \left(\frac{r}{R_0} \right)^2 \right)^{\frac{\alpha}{c_1}} \quad (4.23)$$

which can be obtained by generalising the definition of t_1 to arbitrary radius (replacing $\varepsilon(0,0)$ with $\varepsilon(r,0)$) and then making use of eqn(4.22).

The spacial distribution of the $c\bar{c}$ pairs is based on the distribution used by Karsch and Petronzio [59] discussed in section 3.1.3, which was later generalised by Blaizot and Ollitrault [78], is given by eqn(4.12). The calculation of the p_T dependence of the J/ψ suppression given by $\mathfrak{R}_\psi(p_T)$ then follows in a similar way to that discussed for the Blaizot and Ollitrault Model [78] in section 4.1.3. The parameters are chosen to be: $c_s^2 = \frac{1}{3}$ for a free relativistic gas, $b = \frac{1}{2}$ which is the same in the Karsch and Petronzio Model [59] and $\alpha = \frac{1}{3}$. This choice for α corresponds to $a = \frac{1}{4}$ in the Blaizot and Ollitrault Model [78]. A particular value for τ_ψ is not chosen at this stage, but rather the ratio τ_ψ/t_1 is fixed by the limiting p_T value when the suppression observed for the NA38 data discussed in section 3.2 vanishes. This is similar to the procedure outlined by Blaizot and Ollitrault [59] given by eqn(4.16).

The small p_T end point is then found via a Taylor series expansion of $\mathfrak{R}_\psi(p_T)$ around $p_T = 0$. First order contributions are found to cancel, so to second order in p_T the ratio $\mathfrak{R}_\psi$ for small p_T is given by

$$\mathfrak{R}_\psi(p_T) = \left(\frac{\tau_\psi}{t_1} \right)^{\frac{c_1}{\alpha} b} \left(1 + \kappa \left(\frac{p_T}{M_\psi} \right)^2 + O(p_T^3) \right)$$

where κ is given by

$$\kappa = b(b+1) \left(\frac{\tau_\psi}{t_1} \right)^{-2\frac{c_1}{\alpha}} \left(1 - \left(\frac{\tau_\psi}{t_1} \right)^{\frac{c_1}{\alpha}} \right) \left(\frac{\tau_\psi}{R_0} \right)^2 + \frac{c_1(b+1)}{2\alpha}$$

It then follows that the slope of $\mathfrak{R}_\psi$ at $p_T = 0$ is zero as there is no contribution from the first order term.

In order to compare the theoretical calculation with the NA38 data, discussed in section 3.2 with the corresponding references, it is necessary to estimate the freeze-out time t_f when, at the collision axis, the energy density corresponds to the energy density ε_f at which freeze-out occurs. This is equivalent to the time when T_c is reached at the collision axis. With use of eqn(2.1) the corresponding relation to eqn(2.29) can be determined, given by

$$\left[\frac{dE_T}{dy} \right]_{y=0} = 2\pi \int dr r \varepsilon_f t_f(r)$$

where the angular integration has been performed. If the substitution for $t_f(r)$ is made from eqn(4.23), where the equation has been applied to ε_f instead of ε_1 , the integration can be performed to yield

$$\frac{dE_T}{dy} = \pi \varepsilon_f \frac{t_0}{R_0^2} \left(\frac{\varepsilon(0,0)}{\varepsilon_f} \right)^{\frac{1}{c_1}} \frac{c_1}{\alpha + c_1}$$

With use of eqn(4.21) from the Bjorken scaling solution for the plasma eqn(2.32), t_f can be found in terms of ε_f

$$t_f(r=0) = \frac{dE_T}{dy} \left(\frac{\alpha + c_1}{c_1} \right) \frac{1}{\pi R_0^2 \varepsilon_f} \quad (4.24)$$

The transverse energy for the central region is expected to form a plateau when plotted against the rapidity (this is the basis for the Bjorken Model [16], and underlies the definition of the central region) so that the differential may be approximated by the measured transverse energy per rapidity bin for the central region. However, the NA38 experiment measures the “neutral” transverse energy E_T^0 which is expected to be approximately half of the total transverse energy [92], as was mentioned in section 3.2, therefore $E_T = 2E_T^0$. Typical values for E_T^0 correspond to 50 GeV for approximately central collisions, and 28 GeV for grazing collisions — the ratio given by eqn(3.10) is determined with $E_T^{\text{high}} \geq 50$ GeV and $E_T^{\text{low}} \leq 28$ GeV. For a given input ε_f , the freeze-out time can then be determined which, together with the use of eqn(4.21), allows for an estimate of the energy density at any time during the pure plasma phase to be made; the time evolution is plotted in [92] corresponding to three cases: for each of the two extremes of the equation of state, corresponding to $c_s^2 = \frac{1}{3}$ where an ideal relativistic gas approximation is valid, $c_s^2 = 0$ where free streaming is occurring, and some mixture of the two with the Bag Model equation of state for each of the phases, where the ideal gas value of c_s^2 is used for both the pure hadronic $\varepsilon \leq \varepsilon_h = 0.2$ GeV/fm³ and plasma $\varepsilon \geq \varepsilon_q \geq 2$ GeV/fm³ phases, and the mixed phase has $c_s^2 = 0$. The true evolution of the energy density is expected to lie within the two extreme cases for c_s^2 .

A plot has been produced by Chu and Matsui [92] for the time evolution of the energy density corresponding to each of the three cases. The parameters chosen are: $\varepsilon_f = 0.1$ GeV/fm³ for the freeze-out energy density, $E_T^0 = 50$ GeV for the transverse energy, $\alpha = \frac{1}{3}$ as before, and the case for an oxygen projectile $A = 16$ is considered; the plot is displayed in fig(4.11). A time scale factor of $t_0 = 1$ fm is used which corresponds roughly to the plasma formation time [16]; for the log plot in fig(4.11) this corresponds to the case $s = 0$.

In order to make use of the parameter τ_ψ/t_1 some input is required for the J/ψ formation time. The estimates of Karsch and Petronzio [59,82], and Blaizot and Ollitrault [78], are obtained from the time it requires for a $c\bar{c}$ pair, with a momentum corresponding to that of a quark in a J/ψ resonance, to move apart the typical J/ψ radius. Kapusta [94] has a different estimate: use is made of the Bohr model which quantises the angular momentum, $\ell = n\hbar$ where n is the principal quantum number. The J/ψ will correspond to the lowest angular momentum ground state $\ell = 1\hbar$. The formation time τ_ψ is then the time for the quarks to orbit once, which is given by

$$\tau_\psi = 2\pi \frac{r}{v} \quad \text{with} \quad \ell = \frac{m_c}{2} v r \quad (4.25)$$

An estimate for the dependence of r on v is obtained by putting $\alpha = 0$ in eqn(4.1) and then balancing the forces; this equation can then be combined with eqn(4.25) to yield $r = 0.4n^{2/3}$ (in the case $n = 1$, r is just the Bohr radius). The formation time can then be computed and is found to be $\tau_\psi = 3.9$ fm. In comparison to the values used by Karsch and Petronzio [82] displayed in table(4.2) the radius is similar, but the formation time is found to be much larger. The formation time in the Chu and Matsui Model [92] is allowed to vary between the values proposed by the two models, thus $0.7 \leq \tau_\psi \leq 4$ fm. The range of values for τ_ψ sets a limit on the maximum value of t_1 which is displayed in fig(4.11) as the vertical dashed line. The corresponding energy density ε_1 can then be read off from the intersection with the

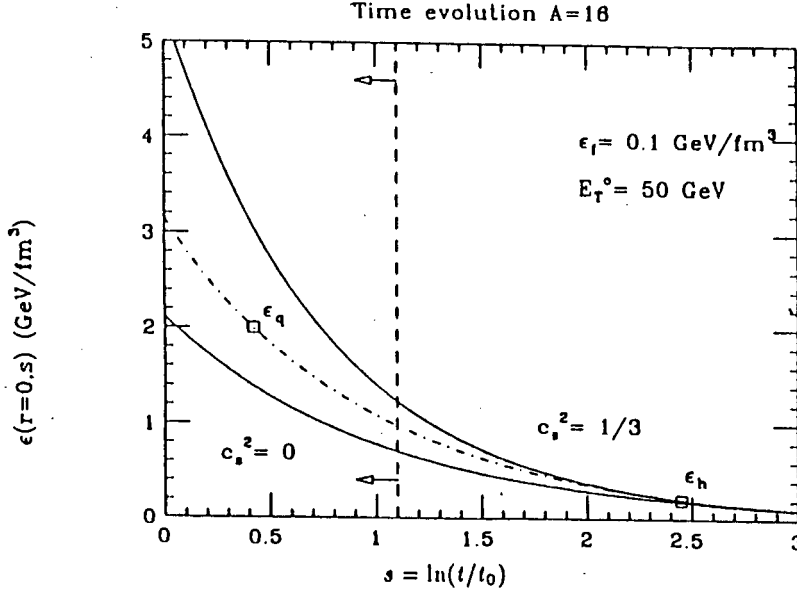


Figure 4.11: Energy density at $r = 0$ in the central rapidity region as a function of $s \equiv \ln(t/t_0)$ for central collisions of oxygen ions with total transverse “neutral” energy $E_T^0 = 50$ GeV. The freeze-out energy density ϵ_f is fixed at 0.1 GeV/fm³. The two solid curves correspond to two limiting equations of state: $c_s^2 = 0$ and $\frac{1}{3}$, respectively, while the dotted-dashed line depicts the expansion with a bag model equation of state (pure quark phase for $\epsilon \geq \epsilon_q$, pure hadron gas phase for $\epsilon \leq \epsilon_h$, and a mixed phase in between; ϵ_h and ϵ_q are chosen to be 0.2 and 2.0 GeV/fm³ respectively). The allowed range of the screening duration t_1 is indicated by the dashed line and arrows. The points where the dashed line crosses with the energy density curves give the lower bounds on the screening energy density ϵ_1 : $\epsilon_1 \geq 0.7 - 1.3$ GeV/fm³ depending on the equation of state [92].

energy density evolution curves; the lower limit of ϵ_1 is determined to be, for each of the cases, within the range $\epsilon_1 \geq 0.7 - 1.3$ GeV/fm³. Thus if the p_T dependence observed by the NA38 group is explained in this manner, then the prediction is made that high energy densities of the order of 0.7 GeV/fm³ or greater have been attained in central collisions during the NA38 experiment.

In central collisions, the models of Karsch and Petronzio [59,82] and Blaizot and Ollitrault [78] use the limiting p_T value when $\mathcal{R}_\psi(p_T) = 1$ to fix the time when the temperature T_1 (or corresponding energy density ϵ_1) reaches the collision axis, namely t_1 as was discussed earlier. The Chu and Matsui argument is similar at this level. However, Chu and Matsui go further to predict t_1 for non-central collisions as well. The value of ϵ_1 has been fixed from consideration of the energy density evolution — this is not the case with the models mentioned previously, where no definite prediction of the deconfining temperature T_1 is made. For non-central collisions with transverse energy flow E_T^0 , use can still be made of eqn(4.24)

with the input for ε_f , to fix t_f . Given the value of ε_1 , eqn(4.21) can be used to predict the time t_1 for the corresponding transverse energy flow E_T^0 . From a knowledge of the parameter t_1/τ_ψ , and the plasma size (corresponding to R_0) together with the choice of parameters for b and α , the range of theta specified in eqn(4.13) of section 4.1.3 can then be determined as a function of r and p_T , which enables the sum over all possible r values to be made and the p_T dependence for each E_T^0 to be determined. Let this dependence be given by the function $A(p_T, E_T^0)$. A function $W(E_T^0)$ is to be the normalised observed E_T^0 distribution in the J/ψ mass range (2.7 – 3.5 GeV). The function is found to decrease rapidly for $E_T^0 \geq 60$ GeV so that a cutoff of $E_{\max} = 80$ GeV is realistic. The p_T dependence can then be computed as the integral over the E_T^0 values between some range ($E_{\min}, E_{\max}$) and is given by

$$\mathcal{A}(p_T, E_{\min}, E_{\max}) = \int_{E_{\min}}^{E_{\max}} dE_T^0 W(E_T^0) A(p_T, E_T^0)$$

The ratio can then be constructed in terms of different values for the limits of the integral in comparison to the NA38 plots, thus

$$\mathcal{R}_\psi(p_T) = \frac{\mathcal{A}(p_T, E_{\min}, E_{\max} = 80 \text{ GeV})}{\mathcal{A}(p_T, E_{\min} = 0, E_{\max} = 28 \text{ GeV})} \quad (4.26)$$

where no suppression is assumed for $E_T \leq 28$ GeV. This enables a more realistic comparison with the NA38 p_T suppression data which contains the whole range of impact parameters, from completely central collisions, to just grazing collisions, the range of which is determined by the values of $E_{\min}$ chosen for the ratio in eqn(3.10).

The ratio $\mathcal{R}_\psi(p_T)$ thus defined, is presented in fig(4.12) for the values of $E_{\min}$ of 40, 50 and 60 GeV with $\tau_\psi = 2$ fm and the case for a static plasma, together with the NA38 data. Agreement with the experimental data is found for $E_{\min} = 50$ GeV whilst the static plasma does not contain sufficient p_T dependence.

The prediction for the p_T dependence of the suppression for the sulphur data can be made. A larger transverse energy flow is expected, which will lead to an increased t_f and correspondingly more suppression. The prediction for $E_{\min}$ with values 80, 100, and 120 GeV has been calculated and is presented in fig(4.13). The dependence of t_f on A through eqn(4.24) is via $A^{-2/3}$ so that if the E_T^0 dependence is scaled by this factor, the function $A(p_T, E_T^0/A^{2/3})$ is independent of the projectile size, and hence $\mathcal{A}(p_T)$ is the same for all ions for fixed $E_T^0/A^{2/3}$.

The inclusion of the χ resonances can be made by repeating the above discussion with the typical χ formation time. The formation times chosen for the model when both J/ψ and χ resonances are included are, $\tau_\psi = 1.7$ fm and $\tau_\chi = 2.6$ fm. The fractions $\mathcal{A}$ are added

$$\mathcal{A}(p_T, E_{\min}, E_{\max}) = 0.6 \mathcal{A}(p_T, E_{\min}, E_{\max}; \tau_\psi) + 0.4 \mathcal{A}(p_T, E_{\min}, E_{\max}; \tau_\chi)$$

corresponding to a 40% contribution of the J/ψ from the χ resonances. No comparison with the data is made with this improved model. It should be noted the transverse energy dependence is only approximate, and may differ by up to 50% to the NA38 scale [76]. This could be improved by the inclusion of a more detailed model for the transverse energy flow.

4.2 Improvements to Earlier Models

Improvements have been made to some models to include non-central collisions from which a prediction for the transverse energy dependence of J/ψ suppression can be made.

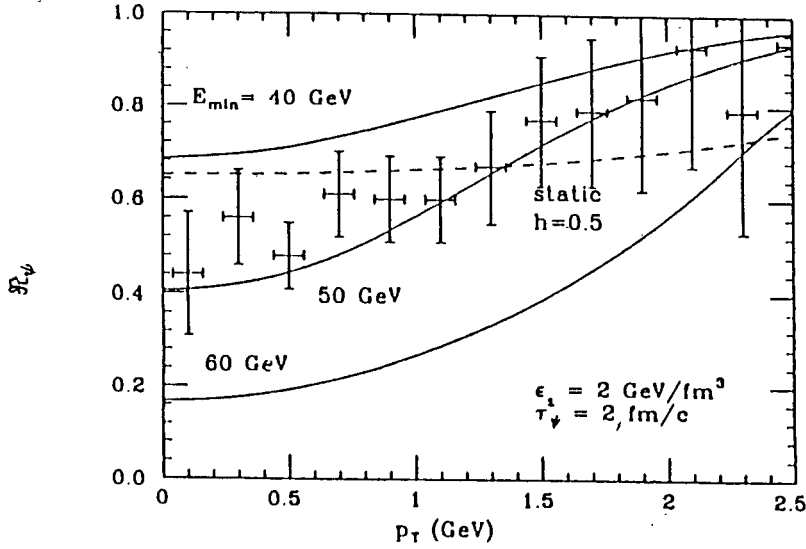


Figure 4.12: Transverse momentum dependence of $\mathcal{R}_\psi$ from eqn(4.26) for a 200 GeV/A oxygen beam compared to the NA38 data [65]. The values of J/ψ formation time $\tau_\psi = 2$ fm, screening energy density $\epsilon_1 = 2$ GeV, with the parameters $\alpha = \frac{1}{2}$, $b = \frac{1}{3}$ and $c_s^2 = \frac{1}{3}$ are used. The three solid curves correspond to $E_{\min} = 40, 50, 60$ GeV respectively. The dashed curve corresponds to a static plasma with radius $0.5R_0$ [92].

This in turn leads to a better estimate for the p_T dependence, as the concept of high E_T and low ET is better understood within the framework of the model, and hence the ratio for the p_T dependence becomes more definitive.

Blaizot Ollitrault Model

The model discussed by Blaizot and Ollitrault [78] has been improved in [95,96] to include the effects of non-central collisions. A very much simplified version of the p_T dependence of the J/ψ suppression model discussed in section 4.1.3 is used in which the finite size effects of the plasma are completely ignored, leading to the generalisation of eqn(4.14)

$$\mathcal{R}_\psi(p_T) = \left(\frac{\tau_\psi}{t_1} \sqrt{1 + \left(\frac{p_T}{M_\psi} \right)^2} \right)^{c_1} \quad (4.27)$$

where $c_1 = \frac{b+1}{a}$ has been used to avoid confusion between the parameter b used in the p_T dependence models, and the impact parameter b which will enter the model.

To introduce the dependence on impact parameter, the differential cross-section for the production of J/ψ is written as

$$\frac{d\sigma_{AB \rightarrow \psi}}{dE_T dp_T^2} = \frac{d\sigma_{NN \rightarrow \psi}}{dp_T^2} T_{AB}(b) \mathcal{N}(b, p_T) \frac{2\pi b db}{dE_T} \quad (4.28)$$

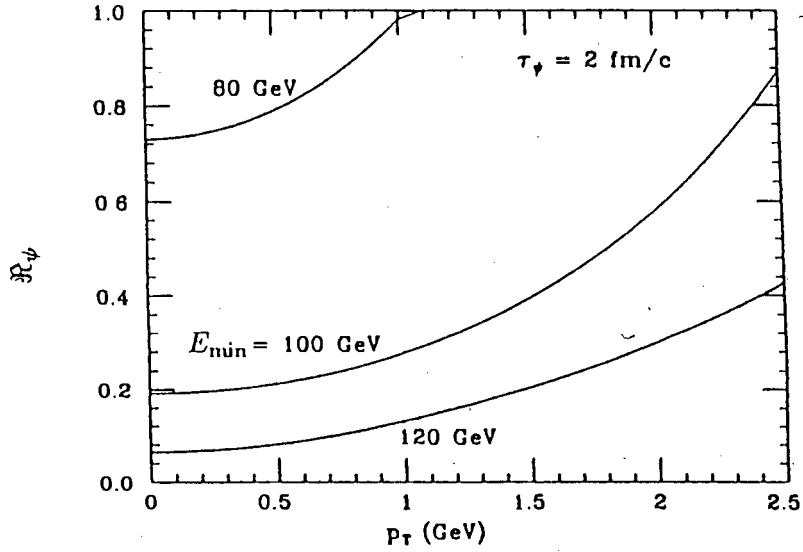


Figure 4.13: Same as fig(4.12) except for a sulphur beam.
 $E_{\min} = 80, 100, 200$ GeV [92].

where:

- $\frac{d\sigma_{AB \rightarrow \psi}}{dp_T^2}$ is the nucleon-nucleon production cross-section of J/ψ
- $T_{AB}(b)$ is the effective number of nucleon-nucleon collisions for a nucleus-nucleus collision at impact parameter b . The product of $T_{AB}(b)$ with the nucleon-nucleon production cross-section will yield a value proportional to the number of J/ψ produced initially in a collision of impact parameter b .
- $\mathcal{N}(b, p_T)$ refers to the effects of the plasma, formed in a collision with impact parameter b , on the final number of J/ψ produced from the collision — it is in this term that the p_T dependence of the plasma suppression lies.
- the last term relates the transverse energy to the impact parameter.

The evaluation of these terms is particularly difficult, especially the dependence of the transverse energy flow on the impact parameter, which at present is very model dependent.

The function $T_A(\vec{s})$ is defined as

$$T_A(\vec{s}) = \int dz \rho(\vec{s}, z)$$

where $\rho(\vec{s}, z)$ is the density of nucleons for nucleus A, and $\vec{s}$ refers to the transverse coordinates. A similar integral can be formed for nucleus B, but with the radial $\vec{s}$ shifted by the impact parameter b . The integral of the product of $T_A(\vec{s})$ and $T_B(\vec{s} + \vec{b})$ will be proportional to the number of nucleon-nucleon collisions, and leads to the definition

$$T_{AB}(b) = \int d^2s T_A(\vec{s}) T_B(\vec{s} + \vec{b})$$

The estimation of the dependence of transverse energy flow on the impact parameter is based upon the approximation that the transverse energy flow is proportional to the number of participating nucleons [97]. Thus the relation

$$E_T = E_0 N(b)$$

follows where E_0 is the energy flow per nucleon and $N(b)$ is the number of participating nucleons for an impact parameter b . The differential is then given by

$$\frac{d^2 b}{dE_T} = \frac{d^2 b}{E_0 dN(b)} \quad (4.29)$$

The evaluation of the number of participating nucleons is just the integral over the collision area

$$N(b) = \int d^2 s T_A(\vec{s}) T_B(\vec{s} + \vec{b})$$

from which eqn(4.29) can be evaluated.

The effect of non-zero impact parameter on $\mathcal{N}(b, p_T)$ will be to reduce the size and duration of the plasma screening effect — the effect of the decreased production of $c\bar{c}$ pairs is taken into account already by the T_{AB} term. To examine the effect of the impact parameter, the finite size effects of the plasma are to a first approximation ignored. This is motivated by the observation that the p_T for complete escape from a fixed plasma is much larger than the experimentally observed value (the fixed plasma case has been discussed in section 3.1.3). The argument goes as follows: The average time to escape a fixed plasma of radius R_0 is

$$\bar{t} = \frac{2R_0}{\pi} \sqrt{1 + \left(\frac{M_\psi}{p_T}\right)^2} \int_0^1 du u \int_0^\pi d\theta \sqrt{1 - u^2 \sin^2 \theta} - u \cos \theta$$

where

$$u = \frac{r}{R_0}$$

The integration can be performed numerically to yield a factor $\sim \frac{4}{3}$ so that the average time is given by

$$\bar{t} = \frac{8R_0}{3\pi} \sqrt{1 + \left(\frac{M_\psi}{p_T}\right)^2}$$

The corresponding p_T where on average most $c\bar{c}$ form J/ψ , corresponding to $\bar{t} = t_\psi$, can then be found

$$p_T = M_\psi \frac{8}{3\pi} \frac{R_0}{\tau_\psi} \approx 9 \text{ GeV}$$

This is much larger than the experimentally determined value of 3 GeV for zero suppression, so to first approximation the finite size effects are ignored. This is equivalent to a model in which the typical distances travelled by $c\bar{c}$ pairs whilst forming J/ψ are much less than the transverse dimension of the plasma, and where the velocity of the isotherms of the plasma as the critical temperature is approached are also correspondingly much larger than the velocities of $c\bar{c}$ pairs. Thus the primary effect of non-zero impact parameter on $\mathcal{N}(b, p_T)$ will be in the reduction of the plasma lifetime. Now with use of eqn(2.28) and eqn(4.3) the equation for the number density time evolution

$$n(t) = n_0 \frac{t}{t_0} \quad (4.30)$$

arises, hence the plasma lifetime can be expressed in terms of a function of the initial number density n_0 and the two parameters t_0 , which can be estimated from the thermalization time for the plasma, and n_1 determined from the temperature T_1 estimated from lattice simulations (however a great deal of uncertainty lies in this value). The conservation of entropy per rapidity bin for the longitudinal expansion (assumed true even through the hadronisation transition) which came from eqn(2.29) can be carried through to the particle number yielding the equation

$$\int d^2s n(\vec{s}, t_0) t_0 = \frac{dN}{dy} \quad (4.31)$$

The assumption is made that the multiplicity distribution of particles produced in the central region is proportional to the number of participants [98], thus if

$$\frac{dN_0}{dy} \approx 2.3$$

is the particle multiplicity distribution per participant [98], then eqn(4.31) becomes upon differentiation with respect to $\vec{s}$

$$n(\vec{s}, t_0) t_0 = \frac{dN_0}{dy} T_A(\vec{s}) + T_B(\vec{s} - \vec{b})$$

where the dependence is now in terms of the particle distribution in the colliding nuclei. Thus the initial number density n_0 , including the r dependence, can be estimated from the above equation. To a first approximation, the spacial dependence of $n(\vec{s}, t_0)$ can be averaged by integrating over the collision cross-section and then dividing by the collision cross-section. From this value for n_0 the plasma lifetime can be found and the dependence of $\mathcal{N}(b, p_T)$ on impact parameter can then be estimated. The p_T dependence of $\mathcal{N}$ is shown in fig(4.14) for two impact parameters.

The p_T dependence of the suppression ratio relevant to the NA38 data can then be obtained by integrating eqn(4.28) over the corresponding transverse energy ranges in a similar manner to that used by Chu and Matsui [92]. Reasonable agreement is found with the experimental data.

The Transverse Energy Dependence

Karsch and Petronzio [99,100,101] have developed a plasma model to describe the transverse energy dependence of the J/ψ suppression observed in the NA38 data [75]. The model is based upon the plasma description for the p_T dependence of the J/ψ suppression discussed in section 4.1.4, but without the longitudinal dependence. The parameters $a = \frac{1}{3}$ and $b = \frac{1}{2}$ are assumed to remain unchanged from those which fitted the data. In the first approach only central collisions are considered; by this statement it is meant that whilst the total energy content of the plasma may change, the basic radial dependence for the energy density is to remain unchanged. This assumption is based upon the fact that it is the finite lifetime of the plasma which is the important feature — see the previous discussion of the Blaizot and Ollitrault model for the impact parameter dependence [95] where a similar assumption is made. Initially, following the original discussion of Karsch and Petronzio [59], the plasma is a disk of length $l \approx 1$ fm and radius $R_c \leq R_0$, where R_0 is the projectile nuclear radius $1.2A^{1/3}$. A central temperature of T_0 exists for the plasma at the time of thermalisation $t_0 \approx 1$ fm [16] (this temperature will be allowed to vary corresponding to nearly central collisions).

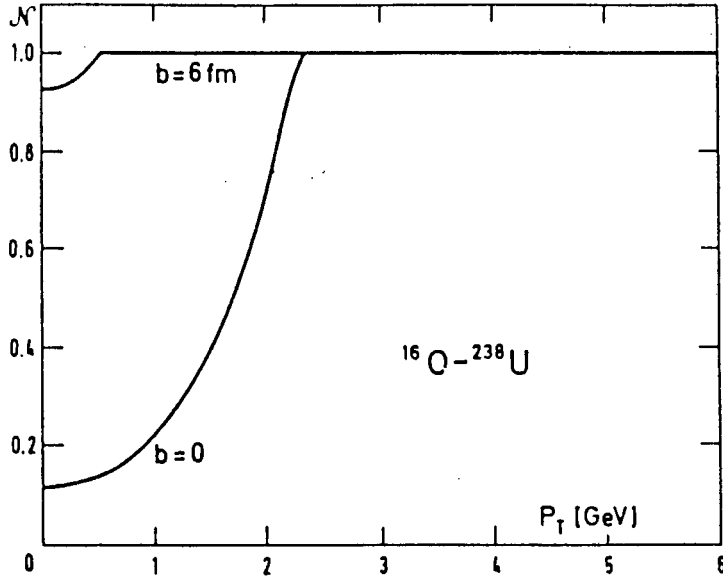


Figure 4.14: p_T dependence of J/ψ suppression by a quark gluon plasma in an oxygen-uranium collision, for two values of the impact parameter $b = 0$ (central collisions) and $b = 6$ fm (peripheral collisions). A value of $n_1 \tau_\psi = 5.5 \text{ fm}^{-2}$ is used for product of the critical density n_1 and J/ψ formation time [95].

The transverse energy which radiates from the collision is assumed to be proportional to the total energy of the plasma, which will be estimated from the integral of the energy density over the volume of the plasma. The underlying radial dependence for the entropy density used by Blaizot and Ollitrault [78] in the model discussed in section 4.1.3 is again assumed, though eqn(4.2) is written in terms of the temperature (substitution for the entropy can be made with use of eqn(2.34) where $c_s^2 = \frac{1}{3}$), thus

$$T(r) = T(0) \left(1 - \left(\frac{r}{R_0} \right)^2 \right)^{\frac{2}{3}} \quad (4.32)$$

Then eqn(4.32) can be re-expressed in terms of R_c to yield an estimate for the radial dimension of the plasma in terms of the initial temperature and projectile radius, given by

$$R_c = R_0 \sqrt{1 - \left(\frac{T_c}{T_0} \right)^{\frac{3}{2}}}$$

The assumption is then made, based upon lattice simulations [102], that the ideal quark-gluon gas picture is valid for the energy density, even for temperatures close to T_c . This is a similar picture to that assumed by Blaizot and Ollitrault [78] which is discussed in section 4.1.3 to obtain limits for the parameters of the plasma. From eqn(4.11)

$$\varepsilon(r) = \frac{1}{15} \nu \pi^2 T^4(r)$$

follows, where ν is the effective number of degrees of freedom, which for $N_c = 3$, $N_f = 2$ and $N_g = 8$ has a value $\nu = \frac{37}{2}$. The initial energy of the plasma can be obtained from the integration of the energy density over the plasma region described above, to yield

$$E = \frac{\pi^3}{15} \frac{3\nu}{4a+3} l R_0^2 T_0^4 \left(1 - \left(\frac{T_c}{T_0} \right)^{3c_1} \right)$$

where $c_1 = \frac{4a+3}{3a}$. This equation can be expressed in terms of the plasma lifetime with use of eqn(2.33) valid for the scaling picture of [16] which underlies the plasma evolution, thus

$$\frac{E}{A^{2/3}} = \frac{1}{15} \pi^3 \frac{3\nu}{4a+3} l 1.2^2 T_c^4 \left(\frac{t_f}{t_0} \right)^{\frac{4}{3}} \left(1 - \left(\frac{t_0}{t_f} \right)^{c_1} \right) \quad (4.33)$$

where $c_1^2 = \frac{1}{3}$. This equation can be used to estimate the plasma lifetime as a function of plasma energy, and formation time t_0 . Further, the lifetime is independent of projectile radius for constant $E/A^{2/3}$. The finite lifetime of the plasma largely determines the J/ψ suppression pattern given by $\mathfrak{R}_\psi(p_T)$, thus if it is noted that the suppression in the case $p_T = 0$ is given by eqn(4.14) which is relatively independent of the plasma size, a realistic estimate of the dependence of the suppression ratio $\mathfrak{R}_\psi(p_T = 0)$ on the energy can be determined and a comparison made with the relevant NA38 data.

The plasma length l and critical temperature T_c are not well known as inputs, in particular the dependence on critical temperature is very sensitive due to the large power to which it is raised, so an arbitrary scale parameter x is defined

$$x = \left(\frac{l}{1 \text{ fm}} \right) \left(\frac{T_c}{200 \text{ MeV}} \right)^4$$

which will be allowed to vary as a free parameter to fit the data. With the choice of parameters above, eqn(4.33) then becomes

$$\frac{E}{A^{2/3}} = 7.6x \left(\frac{t_f}{t_0} \right)^{4/3} \left(1 - \left(\frac{t_0}{t_f} \right)^{13/3} \right) \text{ GeV} \quad (4.34)$$

The dependence of $\mathfrak{R}_\psi(p_T = 0)$ on the energy $E/A^{2/3}$ is displayed in fig(4.15) for the values $x = 1.0, 1.5$ and 2.0 . Below some critical energy E_c no suppression occurs, this corresponds to the plasma lifetime becoming shorter than the formation time of the χ resonance so that the suppression disappears. The χ formation time is chosen because it is assumed, following the discussion in section 4.1.1, that the J/ψ resonances are suppressed for $T \geq 1.3T_c$ which corresponds to a shorter plasma lifetime and is assumed to outweigh the shorter formation time for the J/ψ so that the J/ψ stopped being suppressed before the χ resonances. A further point worth noticing, is that the phase transition temperature T_c is also used as an estimate of when the plasma screening effect disappears for the $c\bar{c}$ resonances — from the above discussion it follows that the temperature limit on the screening is really only valid for the χ resonances. A typical value for E_c is given by eqn(4.34) when the plasma lifetime is just the χ formation time, thus

$$\begin{aligned} \frac{E}{A^{2/3}} &= 7.6x \left(\frac{t_\chi}{t_0} \right)^{4/3} \left(1 - \left(\frac{t_0}{t_\chi} \right)^{13/3} \right) \text{ GeV} \\ &= 18.1x \text{ GeV} \end{aligned}$$

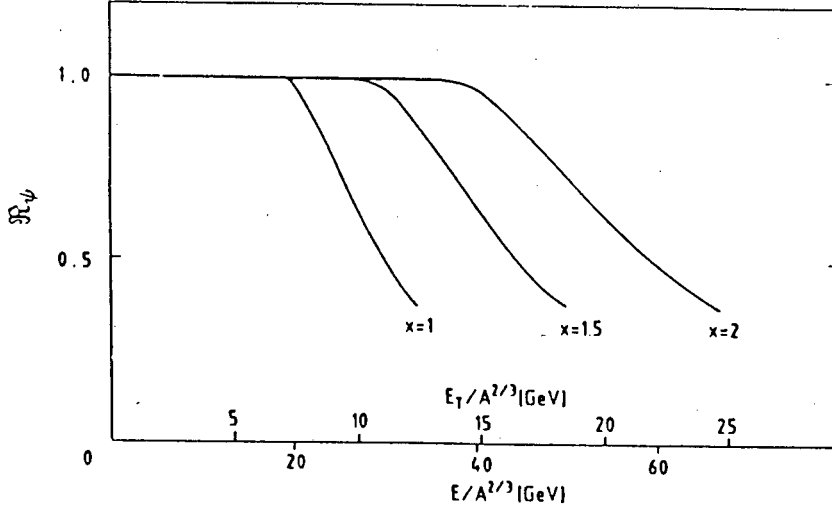


Figure 4.15: Survival probability of J/ψ with $p_T = 0$ versus $E/A^{2/3}$ for three different values of x . Also shown is the “experimental” transverse energy scale as defined in eqn(4.35) [99].

To compare the energy with the NA38 E_T^0 values the transverse energy flow must be determined, and further, as the transverse energy flow measured in the NA38 experiment consists of the neutral energy and approximately one third of the charged energy [65], this must be corrected for. The ratio of charged to neutral energy, from the ratio of the number of neutral to charged pions, is expected to be approximately one half, thus the true transverse energy flow E_T is $E_T \approx \frac{9}{5} E_T^0$. As a rough approximation, if the energy is radiated off spherically, then the transverse energy flow is given by $E_T = \frac{2}{3} E$, where E is the total energy radiated. Thus in terms of the transverse energy flow measured by the NA38 group, eqn(4.34) becomes

$$\frac{E_T^0}{A^{2/3}} = 2.8x \left(\frac{t_f}{t_0} \right)^{4/3} \left(1 - \left(\frac{t_0}{t_f} \right)^{13/3} \right) \text{ GeV} \quad (4.35)$$

The dependence of $\mathcal{R}_\psi(p_T = 0)$ on $E/A^{2/3}$ can then be compared to experiment. This is done in fig(4.16) for $x = 1.2$, where both the oxygen and sulphur experimental data [70] have been normalised so that for the lowest E_T^0 bin $\mathcal{R}_\psi^{\text{exp}}(p_T = 0) = 1$. Reasonable agreement is found for large $E_T^0/A^{2/3}$, but deviations occur for smaller values.

The critical E_T^0 value below which no suppression occurs corresponds to $E_T^0 = 8.2$ GeV, which for an oxygen (sulphur) projectile yields the value $E_T^0 = 52$ (82) GeV for the critical transverse energy. The plasma length corresponding to the region from which the transverse energy is measured is estimated in [99] to be approximately 2 fm, which for $x = 1.2$ yields a critical temperature of $T_c \approx 180$ GeV in reasonable agreement with the current estimates from Lattice simulations [102].

The input of the plasma lifetime, as a function of transverse energy, into the model discussed in section 4.1.4 is now possible, to obtain the complete prediction of the suppression of J/ψ resonances over all p_T values. Previously the plasma lifetime was determined from

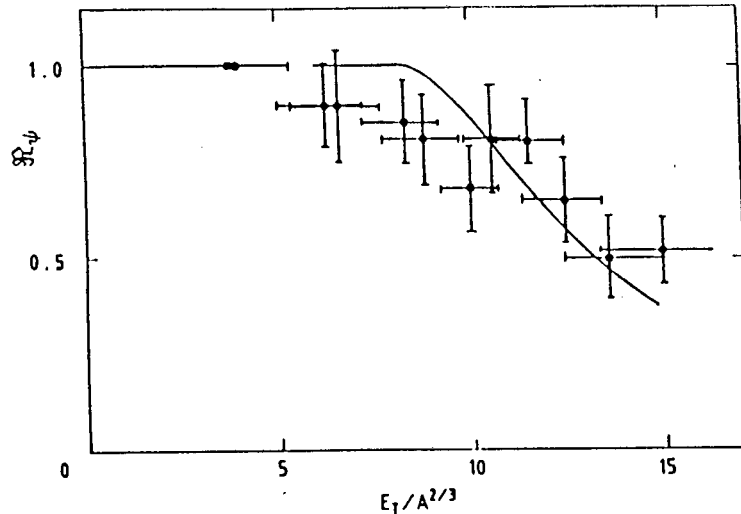


Figure 4.16: Survival probability for J/ψ with $p_T = 0$ versus $E_T/A^{2/3}$ for $x = 1.2$. Also shown are the experimental data for the p_T integrated survival probability for oxygen-uranium ($\circ$) and sulphur-uranium ($\blacksquare$). Data are taken from [70] and normalised to the lowest E_T bin [99].

the p_T limit of no suppression. The calculated p_T dependence when compared with the corresponding NA38 data agrees well — in fig(4.17) the p_T dependence of the suppression is displayed against the corresponding NA38 data with reasonable agreement.

One important point to mention, is that the present model is in fact describing only the central collision picture, but with varying plasma energy. However, to be more realistic the reduction in the plasma size due to a non-central collision should be considered. The model is still expected to be valid for the central collision region corresponding to high E_T . The overall effect of the inclusion of the finite size of the plasma is the expected flattening of the curve in fig(4.16) for low E_T so that it is in better agreement with the data.

4.3 Quantum Models for the J/ψ Formation Time

Models have been produced to introduce a quantum approach to estimating the formation time for J/ψ resonances arising in heavy ion collisions.

4.3.1 Cleymans and Thews Model

One particular feature of the models for J/ψ plasma suppression [59,78,82,92] which have been discussed thus far is that they rely strongly on the following argument: from the instant a $c\bar{c}$ pair is formed by a hard process in the initial highly energetic collision they move apart freely, without mutual influence from the interquark potential, following a classical trajectory determined by their initial momenta. The momenta for the pair may be characterised by a mutual centre of mass motion, and a relative momentum between the c and $\bar{c}$; as a rough estimate this relative momentum is chosen to equal that for the bound quarks

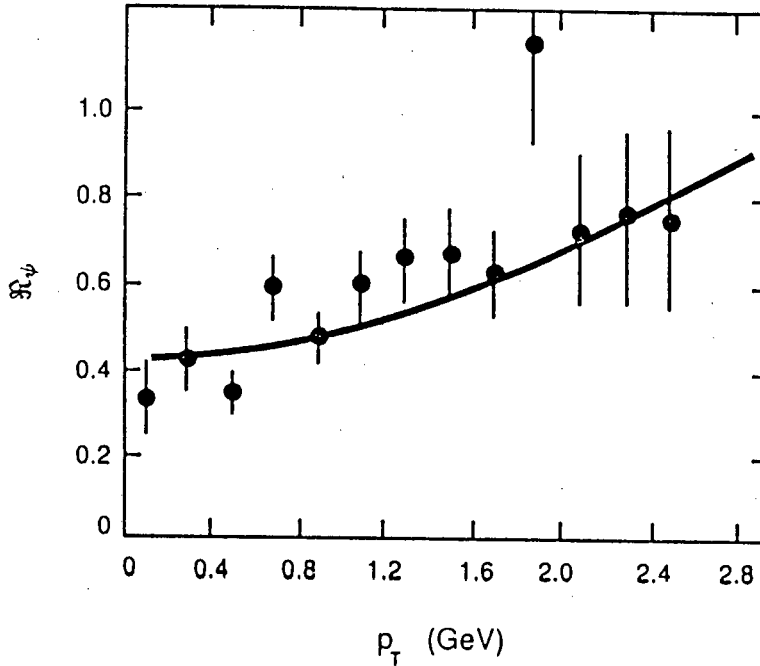


Figure 4.17: Survival probability for J/ψ versus p_T for oxygen-uranium collision events with $E_T/A^{2/3} \geq 14.2$ GeV divided by those with $E_T/A^{2/3} \leq 8$ GeV [101].

in the J/ψ resonance. The relative momentum determines the time τ_ψ when the interquark separation is typical of that for the J/ψ resonance — if the pair is outside the plasma region at this moment, then the interquark potential is switched on and a J/ψ resonance is said to form; else the potential is said to have been screened by the plasma so that quarks do not feel the potential and continue to move apart, till at the hadronisation phase transition when they pair with the more numerous light quarks to form open charm resonances, thereby reducing the number of J/ψ resonances observed.

However, this is not the only prescription discussed thus far — in section 4.1.5 where a model proposed by Chu and Matsui [92] has been discussed, mention is made of an alternative picture. Here use is made of the formation time discussed by Kapusta [94], which refers to the typical time for a single orbit by the quarks comprising a J/ψ resonance to be completed, yielding a description of the $c\bar{c}$ pair which incorporates the interquark potential from the moment of creation of the $c\bar{c}$ pair. The formation time in this model is substantially larger than the formation time suggested above, which naturally leads to ambiguity as to which model is more realistic.

It has been suggested by Cleymans and Thews [103] that the notion of the formation time for a J/ψ resonance is not really valid in the above description — the models are ignorant of the initial state into which a $c\bar{c}$ pair is created. It is further suggested that the quark-quark potential is operative immediately after the $c\bar{c}$ pair has formed, though in a somewhat modified form which should include the formation of a screening environment. The classical trajectory of the $c\bar{c}$ pair is then completely determined from these conditions, and the resulting suppression can be found.

A further point raised by Cleymans and Thews [103] is that the use of quantum mechanics in the above model is obligated by the dimension of the J/ψ resonance. This is the basis of the discussion used to model the J/ψ suppression by a quark gluon plasma. The classical model which they introduced is quantised in terms of nonrelativistic quantum mechanics. This is justified by the small relative momenta of the quarks compared to the charm quark mass as was discussed in section 4.1.1.

The quantised model goes as follows: The creation of a $c\bar{c}$ pair is a short distance phenomenon of the order $\sim 1/m_c$ which is ~ 0.1 fm, due to the large mass of the charmed quarks. This is considerably smaller than the interquark potential scale, and thus will not be influenced by the subsequent interactions. The quarks are created in a state which will be characterised by an initial wave function. At some time t_0 the interquark potential is introduced. The projection of the initial wave function on each bound state of this potential at t_0 , will determine the probability that the $c\bar{c}$ pair is created in this state. The influence of the plasma is introduced by the space-time variance of this potential as the plasma evolves. To find the probability that a J/ψ is formed, the time evolution of the bound states must be followed till the plasma decays and then the averaging must be done over all the initial states produced from the hard interaction. The appropriate ratio to compare with the NA38 data can then be constructed by taking the ratio of this probability with that of the case where the above treatment is repeated, but without the presence of the plasma.

The above discussion has been illustrated by Cleymans and Thews [104] in terms of a simple model in which the quark-quark potential is simplified to the Coulomb potential, and the action of the plasma is to switch off this potential. Both the classical and quantum treatments are given:

In the classical case, an initial bound state is specified in which the quarks perform a prescribed circular orbit, with kinetic energy $E_T = \frac{1}{2}mv_0^2$ and potential energy $V(r) = -k/r_0$; thereafter the potential is turned off and the quarks fly apart till some time t when the potential is again turned on. At this time, the quarks still have the same kinetic energy, but the potential energy will have been reduced to

$$V(r) = \frac{-k}{\sqrt{r_0^2 + (v_0 t)^2}}$$

due to the larger quark-quark separation. In fact it is even possible for the quarks to no longer be in a bound state; this is the case for a time

$$t \geq \sqrt{3} \frac{r_0}{v_0}$$

Thus the transition from bound to unbound states is sharply defined, in a similar manner to the formation time suggested for the J/ψ in the previous models.

The quantum mechanical case is treated for the lowest $\ell = 0$ bound state $\phi_0(r)$. At some time t_i the Coulomb potential is switched off, the system evolves in terms of plane waves, till time t_f when the Coulomb potential is again switched on. The probability that the system is still in the lowest $\ell = 0$ state is given by

$$P(t_f - t_i) = | \langle \phi_0, t_f | \phi_0, t_i \rangle |^2$$

The time evolution of the wavefunction is given by

$$| \phi_0, t_f \rangle = U(t, t_i) | \phi_0, t_i \rangle$$

To evaluate $U(t_f, t_i)$ in general is non-trivial, but for a time independent potential, the unitary time evolution operator can be written in terms of the Hamiltonian of the system in the following form,

$$|\psi, t\rangle = e^{-i\hat{H}t}|\psi, 0\rangle \quad (4.36)$$

This is the case in the present model, where the potential is indeed time independent for, the Coulomb period $t < t_i$ and $t_f < t$ with the time-independent Coulomb solution $\phi_0(r)$, and the period when the potential is switched off $t_i < t < t_f$ which has plane wave solutions. In each of these regions the full time dependent solution is obtained by the addition of a phase factor with use of eqn(4.36). The switch off of the Coulomb is instantaneous, so that the wavefunction remains unchanged across the potential jumps. It is thus possible to write

$$|\phi_0, t_f\rangle = e^{-i\hat{H}(t_f-t_i)}|\phi_0, t_i\rangle$$

where $\hat{H}$ refers to the free hamiltonian. To set the notation, the time t_{fi} is to be given by $t_{fi} = t_f - t_i$, and the time independent wave function is chosen so that

$$|\phi_0, t_i\rangle = |\phi_0\rangle$$

The probability $P(t_{fi})$ is then given by

$$P(t_{fi}) = |\langle \phi_0 | e^{-i\hat{H}t_{fi}} | \phi_0 \rangle|^2 \quad (4.37)$$

where $\hat{H}$ refers to the free Hamiltonian. To evaluate this probability, the wavefunction $|\phi_0\rangle$ must be expressed in terms of the plane waves which are solutions to the free Hamiltonian. The wavefunction for the system is just given by the sum of the projections of $\phi_0(r)$ onto each of these plane wave solutions, thus

$$|\phi, r\rangle = \int d^3k \langle \vec{k} \cdot \vec{r} | \phi_0(r) \rangle |\vec{k} \cdot \vec{r}\rangle$$

The expansion of a plane wave in terms of spherical coordinates is given by [105]

$$e^{ikz} = \sum_{\ell} (2\ell + 1) i^{\ell} j_{\ell}(kr) \mathcal{P}_{\ell}(\cos\theta)$$

If the coordinate axes are chosen so that $\vec{k}$ is along the z-axis, then the projection is given by

$$\langle \vec{k} \cdot \vec{r} | \phi_0(r) \rangle = \int_0^{\infty} dr r^2 \phi_0(r) j_0(kr)$$

where the angular integration has been performed, with the sum reduced to just $\ell = 0$ due to the orthogonality of the Legendre polynomials. To simplify the equations the function $g(\vec{k})$ is chosen to represent the projection, then

$$g(\vec{k}) = \int_0^{\infty} dr r^2 \phi_0(r) j_0(kr) \quad (4.38)$$

It should be pointed out that integral is independent of the direction of $\vec{k}$ so that it is sufficient to write $g(\vec{k})$ as a function of $|\vec{k}|$ only. The Coulomb wavefunction is given by

$$|\phi_0, r\rangle = \int d^3k g(k) |\vec{k} \cdot \vec{r}\rangle$$

The plane wave solutions are eigen-functions of the free Hamiltonian, thus

$$e^{i\hat{H}t_{fi}}|\vec{k} \cdot \vec{r}\rangle = e^{i(k^2/2\mu)t_{fi}}|\vec{k} \cdot \vec{r}\rangle$$

where μ is the reduced quark mass. Then eqn(4.37) becomes

$$P(t_{fi}) = \left| \int d^3k \langle \phi_0, r | \vec{k} \cdot \vec{r} \rangle g(k) e^{-i(k^2/2\mu)t_{fi}} \right|^2 \quad (4.39)$$

Substitution can be made for the projection of ϕ_0 on the plane wave states, with use of eqn(4.38), thus it follows that the probability is given by

$$P(t_{fi}) = \left| \int d^3k |g(k)|^2 e^{i(k^2/2\mu)t_{fi}} \right|^2$$

The exact form of $g(k)$ is given by

$$g(k) = \frac{4\sqrt{4\pi}}{(2\pi)^3} \left(\frac{Z}{a_0} \right)^{\frac{5}{2}} \frac{1}{[k^2 + (Z/a_0)^2]^2}$$

where Z is the quark charge and a_0 corresponds to the Bohr radius. A parameter η which is given by

$$\eta = \frac{Z^2 t_{fi}}{2\mu a_0^2}$$

turns out to be relevant for studying the quantum mechanical case [104]. The probability has been calculated numerically in terms of this parameter, and is given in fig(4.18), in comparison to the sharp transition from bound to unbound states expected in the classical

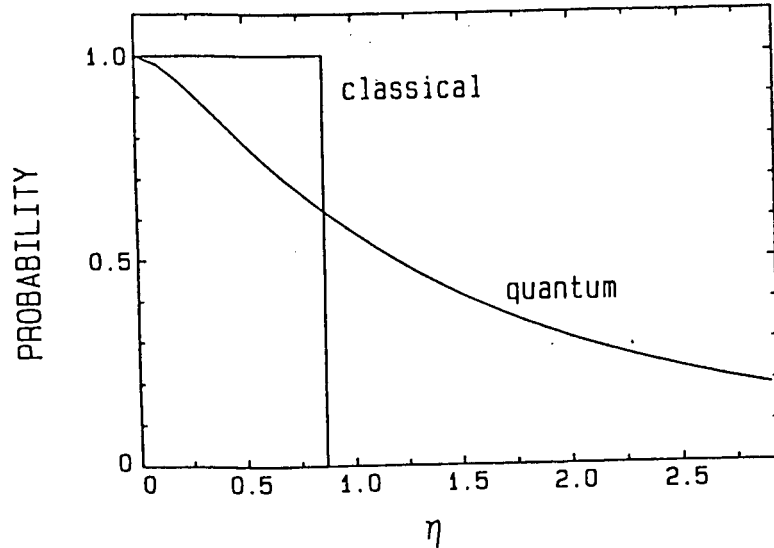


Figure 4.18: Comparison showing the probability that the system is still in the lowest state for the classical and quantum pictures of Coulomb binding [104].

case. Unlike the classical formation time, the quantum probability to remain in the ground state is observed to drop smoothly.

In [103] Cleymans and Thews propose a simple model to describe the effects of the plasma on the formation of J/ψ resonances in heavy ion collisions. The model is based upon the use of an infinite square well to simulate the confining potential of the J/ψ resonance. The width of this well is chosen to be of the order of a few fermis which is of the order of $c\bar{c}$ binding radius in a J/ψ . The effect of the plasma deconfining environment is accounted for by allowing the width to increase to much larger values. The initial wavefunction for the charmed quarks is to be given by a wave-packet of width $2a$, with momentum p

$$\phi_p(x) = \frac{1}{\sqrt{2a}} e^{ipx} \quad \text{for } |x| < a \quad (4.40)$$

with $a \approx 1/m_c$ determined from the uncertainty principle. For the combined $c\bar{c}$ system the coordinates are chosen as

$$\text{centre of mass} \quad R = \frac{x_1 + x_2}{2}, \quad \text{relative separation} \quad r = x_1 - x_2$$

The initial wavefunction for the pair in terms of these coordinates is given by

$$\psi(r, R) = \frac{1}{2a} e^{i(p_1 + p_2)R} e^{i(p_1 - p_2)\frac{r}{2}} \quad , \quad |R \pm \frac{r}{2}| < a \quad (4.41)$$

The motion of the centre of mass will determine the spacial position of the $c\bar{c}$ pair, which in turn determines the effect of the plasma on the potential (for instance the $c\bar{c}$ pair may exit the plasma); it is thus not possible to separate out the centre of mass motion from the problem. As a first approach, the case where a $c\bar{c}$ pair approaches the plasma boundary will be ignored. The eigen-functions are given by plane waves for the centre of mass motion

$$\Gamma_p(R) = \frac{1}{\sqrt{2\pi}} e^{ipR}$$

and even bound state solutions for the infinite square-well potential of width $L(t)$

$$\chi_n(r, t) = \frac{1}{\sqrt{L}} \cos \left[\left(n + \frac{1}{2} \right) \frac{\pi r}{L(t)} \right]$$

The wavefunction for the $c\bar{c}$ in the plasma is then given by

$$\psi(r, R, t) = \int dp \sum_n c_{pn}(t) \Gamma_p(R) \chi_n(r, t) \quad (4.42)$$

where the probability amplitudes at $t = 0$ are given by

$$\begin{aligned} c_{pn}(t=0) &= \int_0^a dR \Gamma_p^*(R) \int_{-2(a-R)}^{2(a-R)} dr \chi_n^*(r) \phi_1(x_1) \phi_2(x_2) \\ &+ \int_{-a}^0 dR \Gamma_p^*(R) \int_{-2(a+R)}^{2(a+R)} dr \chi_n^*(r) \phi_1(x_1) \phi_2(x_2) \end{aligned}$$

The integrals can be performed to yield

$$\begin{aligned} c_{pn}(t=0) &= \frac{2}{a\sqrt{2\pi L}} \left[\frac{\cos[(p_1 - p_2 + 2k_n)a] - \cos(qa)}{q^2 - (p_1 - p_2 + 2k_n)^2} \right] \\ &+ \left[\frac{2}{a\sqrt{2\pi L}} \frac{\cos[(p_2 - p_1 + 2k_n)a] - \cos(qa)}{q^2 - (p_2 - p_1 + 2k_n)^2} \right] \end{aligned}$$

where $k_n = (n + \frac{1}{2})\frac{\pi}{L}$ are the momentum of the bound state eigen-functions, and $q = p_1 + p_2 - p$ is the deviation of the wave packet total momentum from that of the centre of mass. This has a maximum for the case that the $c\bar{c}$ relative and total momenta match the bound state eigen-function momenta, and where the mismatch of the total momentum is compensated by an equivalent mismatch between the bound state momentum and the relative momentum.

The time evolution of the amplitude is difficult in general to follow, due to the time dependence of the potential term. The general equation requires the solving of a coupled differential equation, which can be derived from substituting eqn(4.42) into the Schrödinger equation and performing the differentiation on the bound state wavefunction $\chi_n(r, t)$. This leads to

$$\frac{dc_{pn}}{dt} = \sum_m c_{pm}(t) \left[-iE_{pm}(t)\delta_{mn} + \tau \frac{(m + \frac{1}{2})(n + \frac{1}{2})(-1)^{m+n}}{(m + n + 1)(m - n)} \right]$$

where

$$\tau = \frac{2}{L(t)} \frac{dL(t)}{dt}$$

is the rate at which the plasma expands per unit plasma length.

The case is studied where the plasma is introduced as a sudden expansion of the well width, which is then constant for the lifetime of the plasma. This introduces the simplification that the potential is time independent during the plasma phase, so that the approach used in the Coulomb model above can be repeated for this model. To illustrate the procedure a model has been proposed [104] with the following basic assumptions

- The quark confining potential is simulated by an infinite square well potential with a radius R_0 corresponding to the binding radius of the $c\bar{c}$ quarks forming a J/ψ resonance.
- A $c\bar{c}$ pair is assumed to form a J/ψ resonance which corresponds to the groundstate, before the plasma forms (this is the only initial state considered).
- The plasma screening starts suddenly at time t_0 , which is simulated by a sudden increase in the well width L . The plasma decay at time t_f is also assumed to be sudden, so that the well width is given by

$$\begin{aligned} R &= R_0 & t < t_0 \text{ and } t > t_f \\ R &= R_p & t_0 < t < t_f \end{aligned}$$

The probability for a J/ψ to remain after the plasma screening is given by

$$P(t_f - t_i = t_{fi}) = | \langle J/\psi, t_f | J/\psi, t_i \rangle |^2$$

The time dependence of the potential is essentially of the form required in the derivation of eqn(4.37), so that the argument can be repeated, to yield

$$P(t_{fi}) = | \langle J/\psi | e^{-i\hat{H}_p t_{fi}} | J/\psi \rangle |^2 \quad (4.43)$$

where $\hat{H}_p$ is the Hamiltonian which has the screened potential term. Let the eigen-states of $\hat{H}_p$ be given by

$$\hat{H}_p |n\rangle = E_n |n\rangle$$

then eqn(4.43) becomes

$$P(t_{fi}) = \left| \sum_n | \langle J/\psi | n \rangle |^2 e^{-iE_n t_{fi}} \right|^2 \quad (4.44)$$

which when expanded, and dropping the subscripts on t , yields

$$P(t) = \sum_{nn'} | \langle J/\psi | n \rangle |^2 | \langle J/\psi | n' \rangle |^2 \cos((E_n - E_{n'})t) \quad (4.45)$$

This is similar to eqn(4.39) for the Coulomb potential. The sum can typically be truncated at some sufficiently high n where the projection of the J/ψ state onto the higher energy states is small.

The evaluation of eqn(4.44) has been made for both the one-dimensional [104] and three dimensional [106] cases where in the three dimensional case, the J/ψ corresponds to an $\ell = 0$ state, so that from conservation of angular momentum, only the $\ell = 0$ states are considered. The three dimensional bound states for an infinite potential well are given by

$$\psi_k(r) = \sqrt{\frac{2}{R^3}} \frac{1}{j_1(kR)} j_0(kr) Y_{00}(\Omega)$$

where $j_\ell(kr)$ are the Spherical Bessel Functions with $k^2/2\mu = E$ restricted by the boundary conditions, to the momenta

$$k = \frac{n\pi}{R}$$

and R corresponds to the relevant well radius. The J/ψ wavefunction is given by the ground-state $n = 1$ with $R = R_0$. The projections are then given by

$$\langle J/\psi | n \rangle = \frac{R_0^2 k_0 2}{\sqrt{R_0^3 R_p^3}} \frac{j_0(k_n R_0)}{j_1(k_n R_p) (k_n^2 - k_0^2)}$$

Using this, eqn(4.45) becomes

$$P(t) = \left(\frac{4r_{0p}}{\pi^2} \right)^2 \sum_{nn'} \left| \frac{\sin(r_{0p} n \pi) \sin(r_{0p} n' \pi)}{(r_{0p}^2 n^2 - 1)(r_{0p}^2 n'^2 - 1)} \right|^2 \cos \left(\frac{t}{t_s} (n^2 - n'^2) \right) \quad (4.46)$$

with the parameters r_{0p} , t_s given by

$$r_{0p} = \frac{R_0}{R_p}, \quad t_s = \frac{2\mu R_p^2}{\pi^2}$$

where t_s determines the timescale and r_{0p} gives the strength of the screening with a typical plasma lifetime of the order of $t_1 \lesssim 15$ fm. From the form of eqn(4.46) oscillations of $P(t)$ are possible, with a fundamental period of $2\pi t_s$.

In fig(4.19) the probability of survival is plotted, for the choice of parameters, $r_{0p} = 0.5$ corresponding to $t_s \sim 10$ fm and $r_{0p} = 0.1$ for $t_s \sim 200$ fm. Oscillations are possible for the smaller t_s whilst the timescale is too large for these in the case of the larger t_s . The probability that a J/ψ remains after the plasma has decayed falls off rapidly with increasing t , for the higher t_s value.

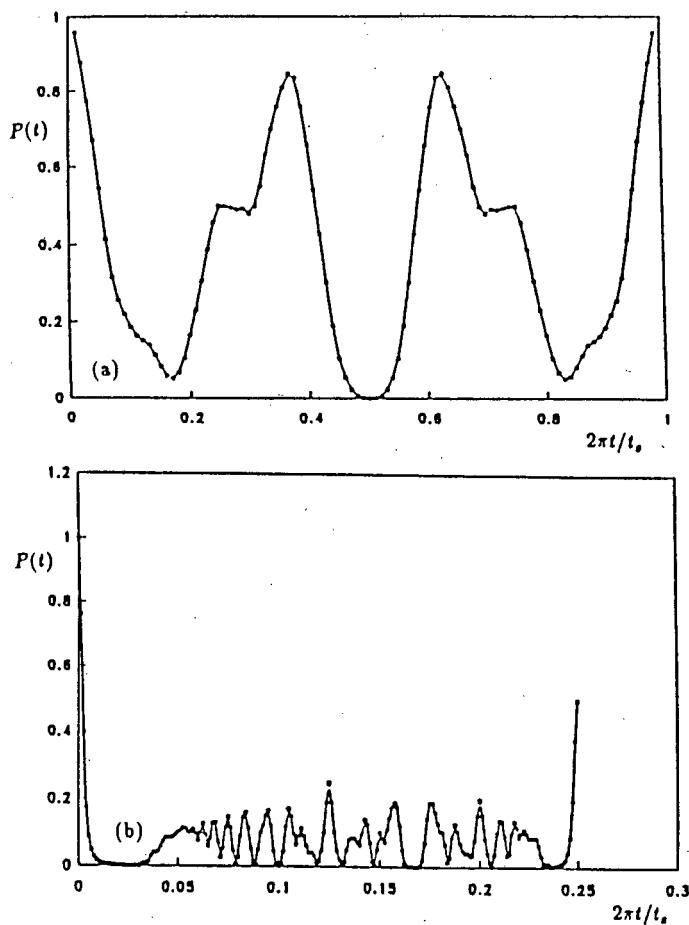


Figure 4.19: (a) The probability of survival $P(t)$ after a plasma with a size corresponding to $R_p = 2R_0$ has been present ($r_{0p} = 0.5$). (b) The probability of survival $P(t)$ after a plasma with a size corresponding to $R_p = 10R_0$ has been present, for $t \leq 15 \text{ fm} \sim$ the lifetime of the plasma ($r_{0p} = 0.1$) [106].

To compare this data with experiment, the motion of the centre of mass through the plasma must be considered. The finite size effects are again ignored, but the Lorentz time dilation will reduce the proptime that the $c\bar{c}$ pair is in the plasma, thus leading to less suppression. The one-dimensional case has been studied [104], which leads to non-trivial momentum dependence. This is displayed in fig(4.20) where L_{plasma} refers to the plasma dimension, L_{normal} refers to the size of the J/ψ , with a the size of the initial wave packet defined in eqn(4.40). Due to the harsh assumptions involved in the model, no comparison with experiment has been made.

Some criticisms of the model include

- The use of more realistic potential should be made, for instance, of the Cornell [83] type. However, if use is made of this type of potential with the finite temperature extension suggested in section 4.1.1 there may in fact be no bound states in the plasma phase,

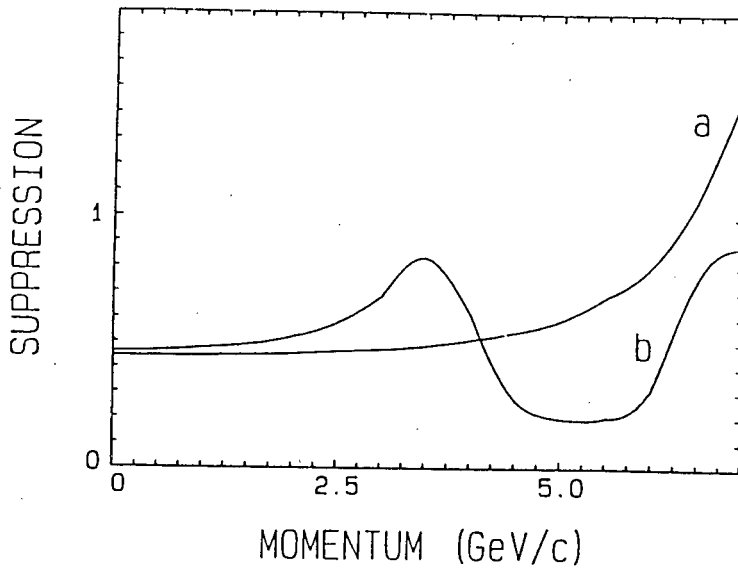


Figure 4.20: Suppression factor versus momentum for different choices of the parameters: Curve (a) is for an initial wavepacket width $a = 0.13$ fm, plasma dimension $L_{\text{plasma}} = 0.9$ fm and J/ψ dimension $L_{\text{normal}} = 0.3$ fm. Curve (b) uses $a = 0.25$ fm, $L_{\text{plasma}} = 1.5$ fm and $L_{\text{normal}} = 0.5$ fm [103].

but rather the confinement of the plasma boundary is the dominant feature, which will be similar to the infinite well confinement.

- A more realistic model should include all the initial states. This problem is treated in the more complete model which follows.
- The sudden approximation should be replaced by the full time evolution of the plasma. This is difficult to do and is not done in the models of [103,104,106].

4.3.2 Horváth et al. Model

A similar model to that proposed by Cleymans and Thews [103] has been proposed by Horváth et al. [107,108]. The direct production of J/ψ is studied as a first approximation, but it is important to generalise this later to include the χ resonances, which can cascade electromagnetically to form J/ψ and have been found to contribute approximately 40% to the number of J/ψ produced from hadronic collisions [90]. A more realistic potential for the J/ψ than that used by Thews and Cleymans [103] is used, given by eqn(4.1), but with the finite temperature extrapolation

$$V_D(r, T) = -\frac{\alpha(T)}{r} e^{-r/r_D} \quad (4.47)$$

where r_D is the Debye screening radius.

A first approach is considered in [107] corresponding to the simplification that the J/ψ is immersed in a plasma. The probability that the J/ψ survives inside this environment is then

calculated — this will essentially yield the time it takes for a J/ψ to dissolve in the plasma. An approximation is made that the J/ψ wavefunction $\phi_\psi(r)$ is given by the Gaussian distribution

$$\psi(r, t = 0) = \phi_\psi(r) = (2\pi\sigma^2)^{-\frac{1}{4}} e^{-r^2/4\sigma^2}$$

with σ chosen so that the realistic mean square radius $\langle r^2 \rangle^{\frac{1}{2}} = 0.5$ fm is obtained, and the wavefunction of the $c\bar{c}$ pair is given by $\psi(r, t)$ which is equal to $\phi_\psi(r)$ immediately before the plasma forms. Use is made of Galerkin's method [109] to solve the numerical time evolution of this function when the screened potential V_D is active for $t \neq 0$. The probability that a J/ψ is still present after a time t in the plasma is then given by

$$P(t) = \left| \int d^3r \phi_\psi^*(r) \psi(r, t) \right|^2 \quad (4.48)$$

The probability depends on the strength of the Debye screening, with the dependence given in fig(4.21) where plots have been made of $P(t)$ for values of $r_D = 5.0, 0.2, 0$ fm. A typical

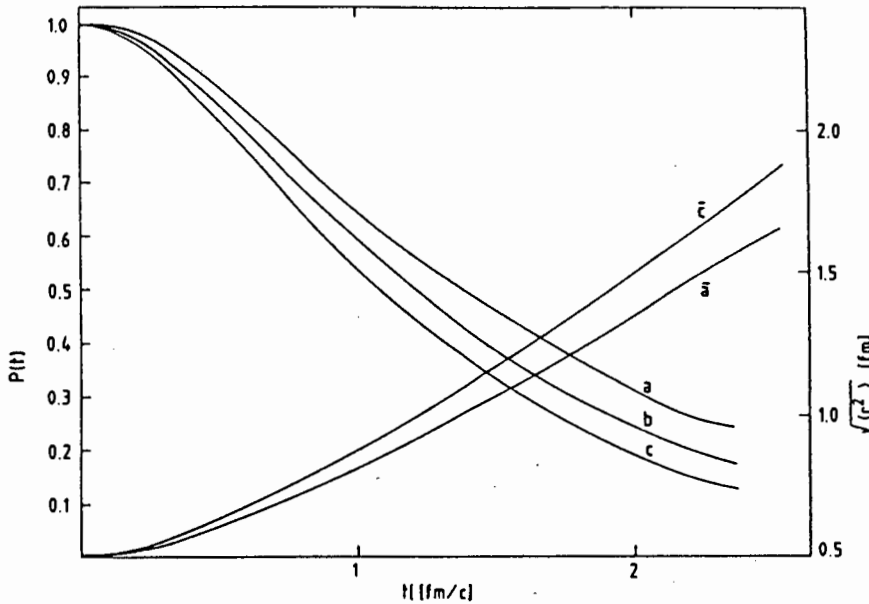


Figure 4.21: The survival probability $P(t)$ (left-hand vertical scale) for J/ψ in quark gluon plasma. The potential for $c\bar{c}$ pairs within the plasma is given by eqn(4.47) with $\alpha_{\text{eff}} = 0.18$ and (a) $r_D = 5.0$ fm, (b) $r_D = 0.2$ fm, (c) $r_D = 0$ fm (free particle wave packet spreading). For cases (a) and (c) the dependence of $\langle r^2(t) \rangle^{1/2}$ versus t is shown to illustrate how fast the corresponding wave packets are spreading (right-hand vertical scale). The curves $\bar{a}$ and $\bar{c}$ correspond to cases (a) and (c) [107].

time of 1 – 1.5 fm is required to dissolve 0.5 of the J/ψ , in the case where $T \approx 1.5T_c$. The dependence on the Debye screening radius is observed to be fairly weak, with a shorter dissolution time for smaller r_D .

To check the validity of ignoring the plasma boundary, $\langle r^2(t) \rangle^{\frac{1}{2}}$ the expectation value of the mean square radius should be found, and compared to the plasma radius; the corresponding plot is also given in fig(4.21) where, for a time of ~ 2 fm the radius increases from 0.5 to 1.5 fm. This corresponds to a radial expansion velocity of the order of .5, which compares favourably with the classical discussions such as [59] which expect a radial expansion velocity of $2 < p > / m_c \approx 0.5$. For times greater than ~ 2 fm the radial size approaches the plasma dimension and the model is expected to break down.

Finally, note is made of the fact that the time dependence of the plasma should be used, especially with reference to a decay time when the screening disappears. The suggested ratio to take for comparison with the NA38 data is given by

$$\Re_{\psi}(t_2, t_1) = P(t_2)/P(t_1)$$

where the plasma forms at t_1 , and decays at t_2 , which is valid provided V_D does not change quickly when compared to that of the J/ψ system.

In [108] Černý et al. a more general situation than in [107] is allowed for, though the "sudden approximation" discussed by Cleymans and Thews [103] where the screening switches on suddenly is still assumed. This approximation implies that the time evolution is much easier to find as it is simply given by $\exp(i\hat{H}t)$ where the H corresponds to the relevant Hamiltonian. In particular three possible situations are considered:

1. At the beginning, the intermediate state, though it has not had time to thermalise, might still provide a screening environment into which a $c\bar{c}$ pair is created. The initial state of the $c\bar{c}$ pair at the start of plasma screening is thus not a J/ψ , but rather in the form of a narrow wave packet. This case is considered in [108] to be the most realistic for heavy ion collisions.
2. The plasma screening may only start after some time t_0 , sufficiently large that $c\bar{c}$ pairs have formed J/ψ (the other resonances are not considered). The initial state when screening begins is then a J/ψ resonance. This is similar to the simplified situation discussed above in [107] and the model by Thews and Cleymans [104], but probably the screening effect of the plasma sets in too early for this situation to be correct.
3. The plasma screening does not set in till some time t_0 , before which $c\bar{c}$ pairs are in a confining environment which will be described by a phenomenological screening potential model.

The probability that a J/ψ survives the plasma is to be the same form as eqn(4.48) above, with a half-time for the J/ψ suppression defined as

$$\frac{P_{\psi}(t_{1/2})}{P_{\psi}(t=0)} = \frac{1}{2}$$

where the time evolution of $\psi(\vec{r}, t)$ is determined by the Schrödinger equation.

In the absence of screening the Hamiltonian H_c is simply that for the case of confinement with the potential given by eqn(4.1). The probability for survival is then simply given by

$$P(t) = \left| \int d^3r \phi_{\psi}^*(r) e^{-i\hat{H}_c t} \psi(r, t=0) \right|^2$$

where ψ consists of a sum over orthogonal eigen-functions of H_c such that the projection of ϕ_ψ onto these states is zero for all states but ϕ_ψ thus

$$P(t) = \left| e^{-iE_\psi t} \int d^3r \phi_\psi^* \psi(r, 0) \right|^2 \quad (4.49)$$

which is independent of time.

The Hamiltonian and initial conditions for the cases studied are specified as:

1. the Hamiltonian H_D is that resulting from a screened potential of eqn(4.47) with an initial condition for the wavefunction given by a narrow Gaussian

$$\psi(\vec{r}, t) = \frac{1}{(2\pi\sigma^2)^{3/4}} \exp\left(-\frac{\vec{r}^2}{4\sigma^2}\right) \quad (4.50)$$

where σ is chosen such that

$$\langle r^2 \rangle^{1/2} = \frac{1}{2m_c} = 0.075 \text{ fm}$$

for a quark mass $m_c = 1.32 \text{ GeV}$.

2. For case two, after some initial time $t_0 \approx 1 \text{ fm}$ screening sets in and the Hamiltonian is H_D , the initial condition $\psi(t_0)$ is that of corresponding to J/ψ with wavefunction given by ϕ_ψ .
3. Case three has the confined Hamiltonian H_c for $t \leq t_0$ whilst the plasma screening effect forms, but after t_0 the Hamiltonian is then the screened Hamiltonian H_D .

$P(t)$ has been computed numerically for each of the situations and is given graphically: a comparison is made between situation one and two in fig(4.22) where the first more narrow wavefunction expands faster than the that for the J/ψ wavefunction. In fig(4.23) the result for situation three is displayed for varying starting times t_0 , where the slope of $P(t)$ is found not to be dependent on t_0 . The dependence on the parameters r_D , α and m_c is found not to be strong.

The prediction for the p_T dependence of the J/ψ plasma suppression is based upon the Blaizot-Ollitrault model [78] which was discussed in section 4.1.3, where the classical suppression is given by

$$\mathfrak{R}_\psi = \frac{\int_0^{R_0} dr \int_0^\pi d\theta \rho(r) \Theta(\tau_\psi - \tau_1)}{2\pi \int_0^{R_0} dr r \rho(r)}$$

where τ_1 is the lifetime of the plasma in the frame of the $c\bar{c}$ pair. In the quantum case the sharp theta function is replaced by the smoothly varying probability of survival $P(\tau_1)$. The parameters used in the Blaizot model are carried through to the quantum model. The integral has been computed numerically and is given in fig(4.24) with the classical result as well. The quantum model is observed to lack the strong p_T dependence present in the classical case.

To obtain an estimate for the classical J/ψ formation, comparison with the half-time for $P(t)$ can be made, for which a value of $t_{1/2} \approx 0.2 \text{ fm}$ is obtained. This is similar to the value which follows for a narrow Gaussian distribution of width $\langle r^2(0) \rangle \approx 1/2m_c = 0.075 \text{ fm}$ to expand (complete suppression) to the J/ψ radius ($\approx 0.5 \text{ fm}$) with the equation given by

$$\langle r^2(t) \rangle = \langle r^2(0) \rangle + \frac{\langle p^2 \rangle}{\mu^2} t^2$$

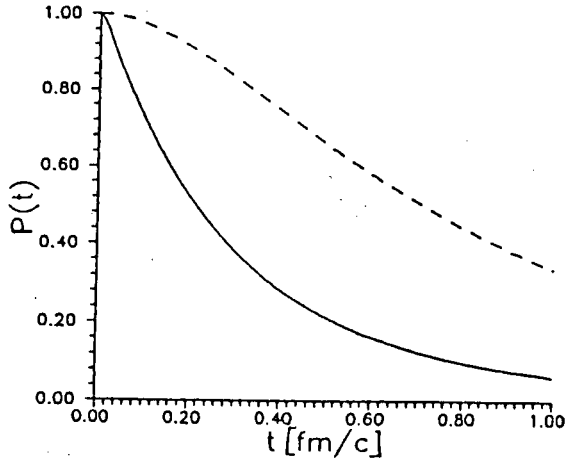


Figure 4.22: J/ψ survival probability $P(t)$ for (1) a realistic case of an initial gaussian wave packet with $\langle r^2 \rangle^{1/2} \simeq 1/2m_c$ — solid line, and (2) for the wave packet equal at $t = 0$ to the J/ψ wavefunction — dashed line. In both cases the survival probability is normalised to 1 at $t = 0$. Parameters of the deconfining potential H_D are: $\alpha = 0.18$ and $r_D = 1$ GeV [108].

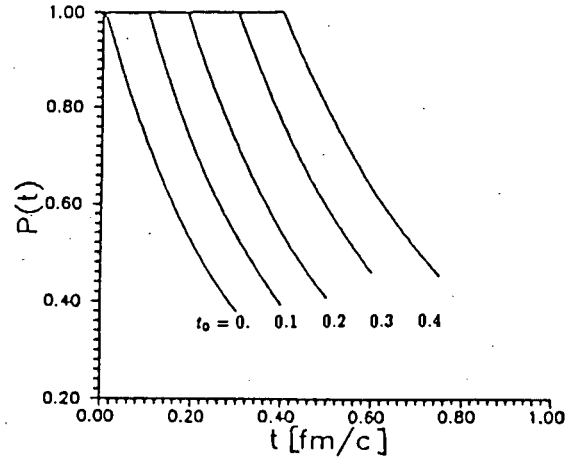


Figure 4.23: J/ψ survival probability $P(t)$ for a situation with $H = H_c$ for $t < t_0$ and $H = H_D$ for $t > t_0$. The wave function at $t = 0$ is a narrow wave packet with $\langle r^2 \rangle^{1/2} = 1/2m_c$. Survival probabilities $P(t)$ are constant and equal to one for $t < t_0$ as follows from the argument which leads to eqn(4.49) [108].

where $\langle p^2 \rangle$ the expectation value of the momentum squared is unaffected by the expansion

$$\langle p^2 \rangle = \frac{9}{4} \langle r^2(0) \rangle^{-1}$$

and μ is the reduced mass. A value of $t_{1/2} = 3.8$ fm is obtained from this approach which is comparable to that of the numerical result.

In particular in [108], the cutoff of momentum at 3 GeV observed for the CERN data [70] is examined as a possible interpretation as a finite size effect — where the $c\bar{c}$ has sufficient momentum to escape the plasma region completely. To do this, the plasma radius must be of the order

$$r = t_{1/2} \frac{p_T}{M_\psi} \approx 0.25 \text{ fm}$$

This is probably too small for the CERN data, where [108] predict a plasma radius of the order of 1 fm. Hence the finite size effect appears incapable of producing the required momentum dependence.

4.3.3 Matsui Model

In [110] Matsui has presented a very detailed model to predict the suppression of J/ψ in a QCD deconfining plasma. The argument is based upon the production of J/ψ from the electromagnetic cascade of χ resonances, which are produced in the initial collision. The initial production of $c\bar{c}$ pairs is a hard process for which the coupling constant is expected to be sufficiently small so that the use of perturbation theory is justified. However, resonance

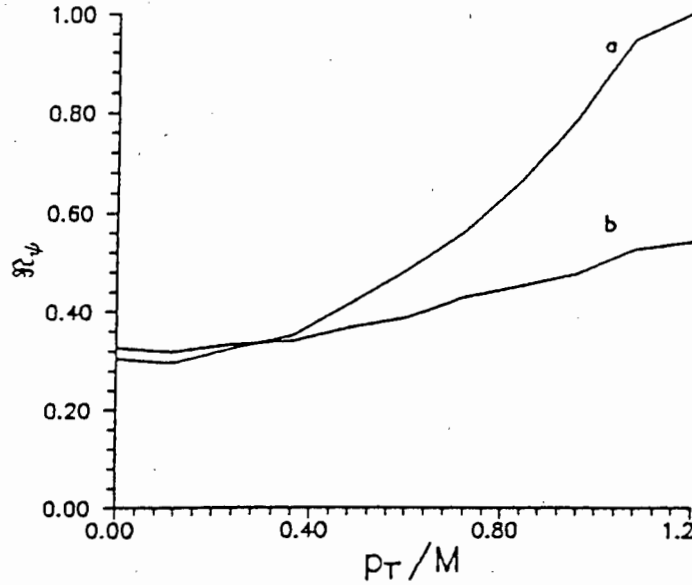


Figure 4.24: Comparison of the semi-classical and quantum pictures of J/ψ suppression: (a) p_T dependence of J/ψ suppression in the simple model by Blaizot and Ollitrault [78], (b) the same model but with the semi-classical picture replaced by the quantum one [108].

formation is not within the regime of perturbation theory and must be approximated within the framework of some model, which to a good approximation, can be described in terms of non-relativistic physics due to the large mass of a charm quark in a similar manner to that used in section 4.1.1.

To introduce the approach, a calculation is first made of the hadronic production of J/ψ based upon the gluon fusion model of charmonium production [111,112,113]. The dominant process of charmonium production in these models is via the two vertex perturbative diagram of the fusion of a gluon from each hadron, to form spin even states $J = 0, 2$. Dominance of gluon production is based upon the idea that the gluon content of a proton is sufficiently large enough to account for some 50% of the total proton momentum, and that this diagram is of lower order than that for instance of $q\bar{q}$ production. However quantum number considerations of the J/ψ ($J^{PC} = 1^{--}$) forbid direct gluon-gluon fusion into J/ψ , thus the addition of another vertex is necessary to form the J/ψ state; for instance this could be a third radiated gluon after the $c\bar{c}$ pair creation occurs, or $q\bar{q}$ annihilation. Direct production of J/ψ is thus expected to be comparatively small against that for example of η_c [111], involving an additional factor of α_s [112]. Further, the production of the χ ($^3P_1$) state is expected to be similarly suppressed against the that of the even J states [116]. This is established experimentally via the observation that the contribution of the spin odd χ state to J/ψ production is in fact quite small [90] relative to the even spin states despite the smaller branching ratio of the spin even states into J/ψ .

An important mechanism of J/ψ hadronic production is from indirect electromagnetic cascade of the χ mesons, which can be produced by gluon-gluon fusion. The χ mesons are

too light to decay into charmed mesons and are thus sufficiently stable against strong decay to produce significant numbers of J/ψ by electromagnetic cascade ($\chi \rightarrow J/\psi\gamma$); this is evident from the relatively large branching ratio of the χ decay into J/ψ . The electromagnetic cascade of χ states into J/ψ is found experimentally to account for at least 40% of the J/ψ produced hadronically [90,114]. This is an important prediction from the gluon fusion models [111,112,113]. The situation at high transverse momentum p_T is somewhat more complicated — direct production of J/ψ to lowest order from gluon-gluon fusion requires that a third gluon be radiated off, which can impart an additional p_T to the J/ψ not present in the lowest order two-gluon production of χ states, thus increasing the number at high p_T . This effect is however offset by the third order contributions coming from the production of χ states [115]. The transverse momentum dependence has been treated in [115,116,117].

An assumption is thus made that the production of J/ψ is in two separate distinct stages: the first involves the formation of a χ resonance from gluon-gluon fusion, whilst the second involves the cascade of the electromagnetic E1 transition of a χ into a J/ψ resonance. The plasma is expected to be only present for the first stage, and will in no way directly affect the final J/ψ production process.

The first stage is divided into two processes: the first involves the gluon-gluon fusion into a $c\bar{c}$ pair in a colour singlet ($J = 0, 2$) state, which may be described perturbatively. This process occurs at the time of the collision and hence will not be influenced by the plasma which will later form. The second process involves the formation of a χ resonance which is described by a non-relativistic potential model, which is expected to feel the influence of a plasma if it is present.

In the absence of plasma, the cross-section for the hadronic production of χ resonances is given by an incoherent sum of the subprocess cross-section [111]

$$\sigma(pp \rightarrow \chi + X) = \sum_{\substack{\text{pol.} \\ \text{colour}}} \int dx_1 f_g(x_1) \int dx_2 f_g(x_2) \bar{\sigma}_0(gg \rightarrow \chi)$$

where $f_g(x)$ is the probability of finding a gluon with longitudinal momentum fraction x in one of the colliding hadrons and $\bar{\sigma}_0(gg \rightarrow \chi)$ is the cross-section for the process $gg \rightarrow \chi$ averaged over the colour and polarization of the gluons, which can be found in terms of the decay width of a χ into two gluons given by

$$\bar{\sigma}_0(gg \rightarrow \chi) = \frac{8\pi^2}{16 \cdot 16 M_\chi} \Gamma(\chi \rightarrow 2g) \delta(x_1 x_2 s - M_\chi^2) \quad (4.51)$$

where $16 = 2.8$ is the spin and colour degeneracy factor for the gluons, M_χ is the χ mass, and $x_1 x_2 s$ is the square of the centre of mass energy of two gluons. The addition of the second stage ($\chi \rightarrow J/\psi\gamma$) is made by multiplying the χ cross-section by the corresponding branching ratio, yielding

$$\sigma(pp \rightarrow J/\psi + X) = \sigma(pp \rightarrow \chi + X) \frac{\Gamma(\chi \rightarrow J/\psi + \gamma)}{\Gamma_{\text{tot}}(\chi)}$$

However, the introduction of the plasma environment will lead to a modification to this form. Matsui produces a generalised form in which the final state resonance production of the χ may be distorted by the temporary presence of a plasma.

The Hamiltonian is given by the sum

$$H = H_{2g} + H_{c\bar{c}} + H_{2g-c\bar{c}}$$

where H_{2g} and $H_{c\bar{c}}$ describe the relative motion of the gluons and $c\bar{c}$ respectively, in the centre of mass frame of the $c\bar{c}$ pair, and where $H_{2g-c\bar{c}}$ is the interaction which couples the gluons to the $c\bar{c}$. If $|2g; \vec{k}_0\rangle$ describes the initial gluon-gluon state with momentum $\vec{k}$, and the $|c\bar{c}; \chi\rangle$ ket describes the $c\bar{c}$ pair state once a resonance has formed, then the transition probability in terms of the overlap is given by

$$P(2g \rightarrow \chi) = |\langle c\bar{c}; \chi | U(t, t_i) | 2g; \vec{k}_0 \rangle|^2$$

where $U(t, t_i)$ is the time evolution operator.

Matsui proves, with use of the Dyson series expansion that the overlap becomes

$$-i \int_{t_i}^t dt' \langle c\bar{c}; \chi | U_0(t, t') H_{2g-c\bar{c}}(t') U_0(t', t_i) | 2g; \vec{k}_0 \rangle \quad (4.52)$$

where U_0 refers to the time evolution under the uncoupled Hamiltonian $H_{2g} + H_{c\bar{c}}$. The interpretation of this equation is that the two gluons propagate according to H_{2g} from a time t_i till t' when they are converted into a $c\bar{c}$ pair by $H_{2g-c\bar{c}}$; the time evolution operator U_0 is simply given by a phase factor $e^{-i2k_0(t'-t_i)}$ during this period, as H_{2g} can be taken to be time independent (the two gluons will not interact initially as they come from two different hadrons). The $c\bar{c}$ pair then propagates under $H_{c\bar{c}}$ till time t , when the overlap with $\langle c\bar{c}; \chi |$ is taken. If t is sufficiently large, after some time t_0 the hamiltonian will have reached its asymptotic form, and is then time independent so that the time-evolution U_0 is simply given by a phase factor. The integrand in eqn(4.52) then has a simple phase factor multiplying

$$\langle c\bar{c}; \chi | U_{c\bar{c}}(t, t') H_{2g-c\bar{c}}(t') | 2g; \vec{k}_0 \rangle$$

The product $U_{c\bar{c}}(t, t') H_{2g-c\bar{c}}(t')$ is assumed to be independent of t' and is thus written in the form

$$U_{c\bar{c}}(t, t') H_{2g-c\bar{c}}(t') = U_c(t_0) H_{2g-c\bar{c}}$$

The integration for the overlap is then fairly trivial to perform, which then allows the computation of the transition probability, given by

$$P(2g \rightarrow \chi) = \frac{4 \sin^2[(M_\chi - 2k_0)(t - t_i)/2]}{(M_\chi - 2k_0)^2} |\langle c\bar{c}; \chi | U_c(t_0) H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle|^2$$

Taking the limit as $t - t_i$ gets large yields the famous Golden Rule for the production rate of χ

$$w(2g \rightarrow \chi) = 2\pi |\langle c\bar{c}; \chi | U_c(t_0) H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle|^2 \delta(M_\chi - 2k_0)$$

from which the cross-section

$$\sigma(2g \rightarrow \chi) = \pi V |\langle c\bar{c}; \chi | U_c(t_0) H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle|^2 \delta(M_\chi - 2k_0) \quad (4.53)$$

follows. This differs from eqn(4.51) in that it allows for the distortion of the $c\bar{c}$ propagation from the asymptotic value form, within the time t_0 . If t_0 is set to zero, then this distortion is not present, and the $c\bar{c}$ pair becomes a χ state immediately after the Hamiltonian $H_{2g-c\bar{c}}$ operates. The effect of adding a distortion can be illustrated by writing eqn(4.53) in the form

$$\sigma(2g \rightarrow \chi) = R(t_0) \sigma_0(2g \rightarrow \chi)$$

where

$$\sigma_0(2g \rightarrow \chi) = \pi V | \langle c\bar{c}; \chi | H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle |^2 \delta(M_\chi - 2k_0)$$

with $R(t_0)$ given by

$$R(t_0) = \frac{| \langle c\bar{c}; \chi | U_c(t_0) H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle |^2}{| \langle c\bar{c}; \chi | H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle |^2} \quad (4.54)$$

It is this ratio which measures the time dependence of the suppression due to plasma screening.

The evaluation of the corresponding matrix elements must then be made in order to estimate $R(t_0)$. First the calculation is made in the case that no plasma forms ($t_0 = 0$). The first stage (the production of a χ resonance from gluon fusion) is split into two subprocesses, the formation of the $c\bar{c}$ pair, and then its time evolution to a χ resonance. The matrix element can be expanded in terms of the intermediate momentum states $|c\bar{c}; \vec{p} \rangle$ into which the $c\bar{c}$ pairs are formed from the gluon-gluon fusion. The matrix element then can be written as

$$\langle c\bar{c}; \chi | H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle = \frac{V}{(2\pi)^3} \int d^3p \langle c\bar{c}; \chi | c\bar{c}; \vec{p} \rangle \langle c\bar{c}; \vec{p} | H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle \quad (4.55)$$

where the matrix element $\langle c\bar{c}; \vec{p} | H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle$ is evaluated perturbatively in a non-relativistic expansion to first order in (p/m) . The overlap with the χ states is computed against the wavefunctions ϕ_ℓ arising from the bound state potential model for charmonium resonances. The matrix element can then be computed, to yield the decay widths

$$\Gamma(\chi_0 \rightarrow 2g) = \left(\frac{2}{3}\right) \alpha_s^2 \frac{9}{m^4} |\phi_1'(0)|^2$$

$$\Gamma(\chi_1 \rightarrow 2g) = 0$$

$$\Gamma(\chi_2 \rightarrow 2g) = \left(\frac{2}{3}\right) \alpha_s^2 \frac{12}{5m^4} |\phi_1'(0)|^2$$

The calculation is similar to that performed in [112].

A simplification has been assumed in that immediately the $c\bar{c}$ pairs form they become a bound state; this follows the basic principle introduced earlier by Cleymans and Thews in [103]. No time dependence is expected in the above solution due to this simplification, as the $c\bar{c}$ pair will be in the asymptotic state immediately it is produced, and an effective infinite amount of time is available for the formation of a $c\bar{c}$ pair.

The introduction of a screening plasma environment is expected to cause a distortion in the formation of the bound state resonances from which it follows that the $c\bar{c}$ pair will not be in the asymptotic state immediately after $c\bar{c}$ creation. Matsui has chosen a simplifying model in which the effect of distortion can be calculated in closed form. A coordinate space representation is chosen for the computation, where the matrix element becomes

$$\begin{aligned} \langle c\bar{c}; \chi | U_c(t_0) H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle = & \frac{1}{V^2} \int d^3x \int d^3x' \langle c\bar{c}; \chi | c\bar{c}; \vec{x} \rangle \\ & \times U_c(\vec{x}, \vec{x}'; t_0) \langle c\bar{c}; \vec{x}' | H_{2g-c\bar{c}} | 2g; \vec{k} \rangle \end{aligned} \quad (4.56)$$

where

$$U_c(\vec{x}, \vec{x}'; t_0) = \langle c\bar{c}; \vec{x} | U_c(t_0) | c\bar{c}; \vec{x}' \rangle$$

describes the distorted propagation of the $c\bar{c}$ pair, where $\vec{x}$ and $\vec{x}'$ both refer to the relative coordinates of the $c\bar{c}$ pair.

The propagator must satisfy the time dependent Schrödinger equation

$$i \frac{\partial U_c(\vec{x}, \vec{x}'; t)}{\partial t} = H_{c\bar{c}}(\vec{x}, t) U_c(\vec{x}, \vec{x}'; t)$$

with the initial condition

$$U_c(\vec{x}, \vec{x}'; 0) = V \delta^3(\vec{x} - \vec{x}')$$

The general solution to this equation is written in the form of the Feynman path integral

$$U_c(\vec{x}, \vec{x}'; t_0) = \int_{\vec{y}(0)=\vec{x}'}^{\vec{y}(t_0)=\vec{x}} \mathcal{D}[\vec{y}(t)] \exp \left[i \int_0^{t_0} dt \mathcal{L}_c(\vec{y}, \dot{\vec{y}}, t) \right]$$

where $\mathcal{L}_c(\vec{y}, \dot{\vec{y}}, t)$ is the classical Lagrangian for the distorted Hamiltonian $H_{c\bar{c}}(t)$.

To enable a solution in closed form to be found, an approximation is made for the Hamiltonian: the asymptotic states are chosen to be eigen solutions to the harmonic oscillator Hamiltonian

$$H_\psi = \frac{\vec{p}^2}{2\mu} + \frac{1}{2} \mu \omega_\psi^2 \vec{x}^2$$

where $\mu = m_c/2$ is the reduced mass, with the wavefunctions

$$\phi_\chi(r) = 2 \left(\frac{2}{3\sqrt{\pi} r_\psi} \right)^{\frac{3}{2}} r e^{-\frac{1}{2} \left(\frac{r}{r_\psi} \right)^2}$$

where the radius parameter is related to the oscillator frequency via

$$r_\psi = \frac{1}{\sqrt{\mu \omega_\psi}} \quad (4.57)$$

The distortion from the plasma environment is introduced by allowing ω to vary: for complete deconfinement $\omega = 0$, whilst the case exists for "partial deconfinement" where $\omega < \omega_\psi$

The solution for U_c in the above case has been found analytically [118] and is given by

$$U_c(\vec{x}, \vec{x}'; t_0) = V \left(\frac{\mu \omega}{2\pi i \sin(\omega t_0)} \right)^{\frac{2}{3}} \exp \left(\frac{i \mu \omega}{2 \sin(\omega t_0)} [(\vec{x}^2 + \vec{x}'^2) \cos(\omega t_0) - 2 \vec{x} \cdot \vec{x}'] \right) \quad (4.58)$$

The spin and angular momentum are assumed to remain unaffected by the distortion due to the isotropic nature of the plasma for this solution.

In order to evaluate the transition matrix element a set of momentum states $|c\bar{c}; \vec{p}\rangle$ is introduced with the projection against the coordinates states given by

$$\langle c\bar{c}; \vec{x} | c\bar{c}; \vec{p} \rangle = e^{i \vec{p} \cdot \vec{x}}$$

thus the transition matrix element becomes

$$\langle c\bar{c}; \vec{x} | H_{2g-c\bar{c}} | 2g; \vec{k} \rangle = \frac{V}{(2\pi)^3} \int d^3 p e^{i \vec{p} \cdot \vec{x}'} \langle c\bar{c}; \vec{p} | H_{2g-c\bar{c}} | 2g; \vec{k} \rangle \quad (4.59)$$

where the matrix element $\langle c\bar{c}; \vec{p} | 2g; \vec{k} \rangle$ is the same as that in eqn(4.55), which was evaluated with use of perturbative theory. It thus follows that the coordinate representation of the transition matrix element can be found.

Finally, the matrix element $\langle c\bar{c}; \chi | c\bar{c}; \vec{x} \rangle$ must be found. This can be also expanded in terms of the momentum states in a similar manner to that used to find the transition matrix element, to yield

$$\langle c\bar{c}; \chi | c\bar{c}; \vec{x} \rangle = \frac{V}{(2\pi)^3} \int e^{-i\vec{p}\cdot\vec{x}} \langle c\bar{c}; \chi | c\bar{c}; \vec{p} \rangle \quad (4.60)$$

Substituting eqns(4.58–4.60) into eqn(4.56) enables the integration of eqn(4.56) to be performed. Matsui [110] has performed the integral for two cases, namely the production of η_c and χ . In the case of the η_c

$$\langle c\bar{c}; \eta_c | U_c(t_0) H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle = \frac{\langle c\bar{c}; \eta_c | H_{2g-c\bar{c}} | 2g; \vec{k}_0 \rangle}{[\cos(\omega t_0) + i(\omega_\psi/\omega) \sin(\omega t_0)]^{3/2}} \quad (4.61)$$

The ratio $R_{\eta_c}(t_0, \omega)$ of eqn(4.54) can then be computed to yield

$$R_{\eta_c}(t_0, \omega) = \left[\cos^2(\omega t_0) + i \left(\frac{\omega_\psi}{\omega} \right)^2 \sin^2(\omega t_0) \right]^{-\frac{3}{2}} \quad (4.62)$$

where the common factor in the numerator of eqn(4.61) cancels out with that arising from the zero plasma case. For $\omega = 0$ which corresponds to complete deconfinement eqn(4.62) simplifies to

$$R_{\eta_c}(t_0; 0) = (1 + \omega_\psi^2 t_0^2)^{-\frac{3}{2}}$$

A similar procedure is followed to compute R_χ for the χ resonances. The matrix element in eqn(4.56) becomes

$$\langle c\bar{c}; \chi_{JM} | U_c(t_0) H_{2g-c\bar{c}} | 2g; \vec{k} \rangle = \frac{\langle c\bar{c}; \chi_{JM} | H_{2g-c\bar{c}} | 2g; k_0 \rangle}{[\cos(\omega t_0) + i(\omega_\psi/\omega) \sin(\omega t_0)]^{5/2}}$$

from which it can be observed that the suppression of χ resonances is independent of the angular momentum state. The ratio R_χ is then given by

$$R_\chi(t_0, \omega) = \left[\cos^2(\omega t_0) + \left(\frac{\omega_\psi}{\omega} \right)^2 \sin^2(\omega t_0) \right]^{-\frac{5}{2}}$$

which in the case of complete deconfinement simplifies to

$$R_\chi(t_0, 0) = [1 + \omega_\psi^2 t_0^2]^{-\frac{5}{2}}$$

A comparison is then made with the semi-classical picture. In the case of complete deconfinement, the ratio of the charmonium formation probability $R(t_0)$ is monotonically decreasing with increasing plasma lifetime t_0 . A characteristic timescale for this decrease is given by the inverse frequency

$$\tau_\psi = \frac{1}{\omega_\psi} = \frac{1}{2} m r_\psi^2$$

which coincides with the classical formation time of Karsch and Petronzio discussed in section 3.1.3 because r_ψ can be related approximately to the quark momentum with use of

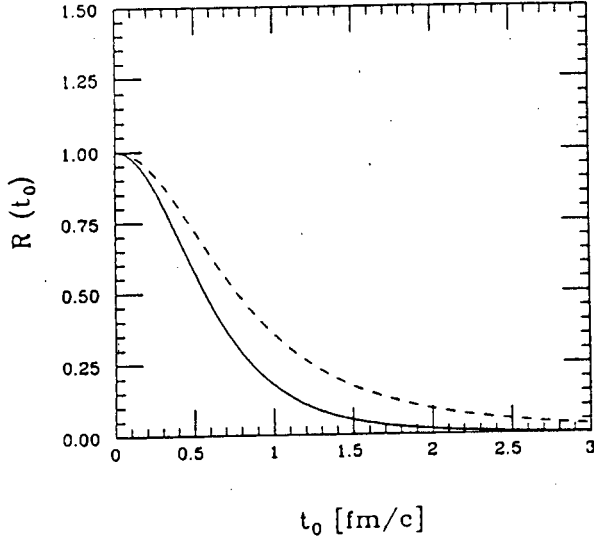


Figure 4.25: Suppression factor $R(t_0, 0)$ which decreases monotonically as t_0 (the duration of the system in the “completely deconfined phase”) increases. The physical scales are set by $r_\psi = 0.5$ fm and $m_c = 1.5$ GeV. The characteristic timescale $1/\omega_0 = \frac{1}{2}mr_0^2$ for the attenuation of R coincides with the classical estimate of the “charmionium formation” time. The curves correspond to: solid line χ and dashed line η_c [110].

Heisenberg’s uncertainty relation in a similar manner to that used in section 4.1.1, were it was demonstrated that this is not too harsh an approximation. The dependence of $R(t_0)$ is displayed in fig(4.25) for both the η_c and χ resonances, which indeed displays the suppression of χ resonance formation for $t_0 > \tau_\psi$. Unlike the classical prescription, this suppression is not a sharp feature. This can be further demonstrated in the case that the deconfinement is not complete; for instance in the case where r_ψ and r , defined analogously to eqn(4.57) as $r = (\mu\omega)^{-1/2}$, have the values 0.50 and 0.60 fm respectively, then oscillations in $R(t_0)$, displayed in fig(4.26), are observed. The dependence on the parameter ω is shown in fig(4.27)

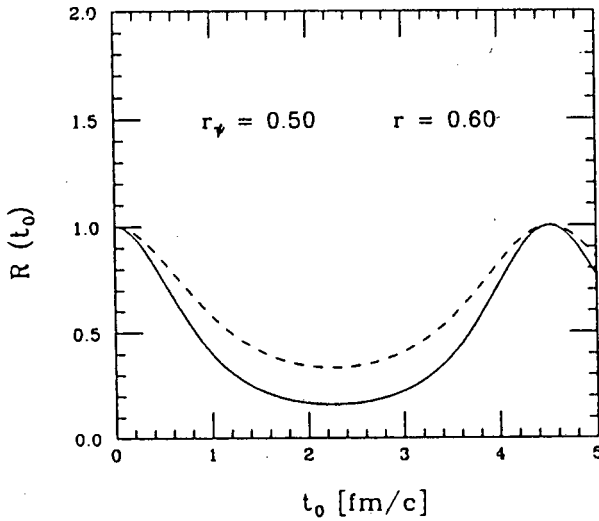


Figure 4.26: Same as fig(4.25) but for non-vanishing ω with $r = 1/\sqrt{\mu\omega} = 0.6$ fm. In this case $R(t_0, \omega)$ oscillates with the frequency ω . The first minimum appears at $t_0 = \pi\omega/2$, where $R_{\eta_c} = (\omega/\omega_\psi)^3$ and $R_\chi = (\omega/\omega_\psi)^5$. The solid line is for χ and dashed line for η_c [110].

and is observed to have a strong influence on $R(t_0)$.

The absence of a threshold effect for the formation time will influence predictions for the p_T dependence of J/ψ plasma suppression. To include the effects of the transverse momen-

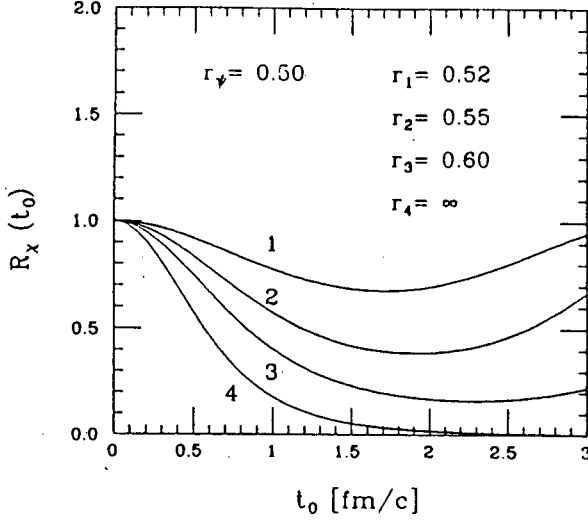


Figure 4.27: ω dependence of the suppression factor $R(t_0, \omega)$ for the P -wave state. The size of the corresponding ground state wave functions ($r = (\mu\omega)^{-1/2}$) is as indicated in the figure [110].

tum, the argument of $R(t_0)$ should be replaced by τ_0 which is given by

$$\tau_0 = \frac{t_0 M_\chi}{\sqrt{M_\chi^2 + p_T^2}}$$

where τ_0 refers to the time when the plasma effect vanishes in the frame of the $c\bar{c}$ pair. The parameter τ_0 is similar to that of τ_1 , which appears in the classical models discussed earlier, in that it is an estimation of the time when the screening effect of the plasma disappears. A geometrical averaging is then done to include finite size effects of the plasma, with the result presented in the form of a graph in fig(4.28). Superimposed on the graph is the

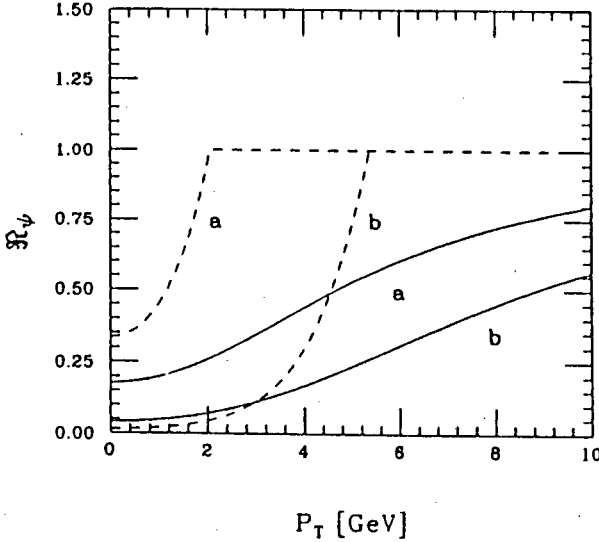


Figure 4.28: p_T dependence of J/ψ suppression predicted by the Matsui model [110]. The solid lines are obtained by taking a geometrical average of $R(t_0, \omega)$ to include finite size effects, whilst the dashed lines are for the semi-classical model of Blaizot and Ollitrault [95] based on eqn(4.27), averaged under the same geometrical conditions. The curves correspond to a plasma screening life-time of (a) $t_0 = t_1 = 1.2$ fm and (b) $t_0 = t_1 = 2$ fm [110]

classical prediction with a threshold effect for the formation time. The simplified model by Blaizot and Ollitrault [95] given by eqn(4.27) which ignores finite size effects, is used for the classical model, but with the same geometrical averaging of the quantum model repeated for

the classical model. The quantum model is not observed to have the strong p_T dependence associated with the classical models.

The transverse momentum dependence of the suppression has been evaluated with use of the Bjorken result [16] for the energy density

$$t_f = \frac{1}{\pi R^2 \epsilon_f} \frac{dE_T}{dy}$$

and is given in fig(4.29) with the classical prediction superimposed. The threshold effect

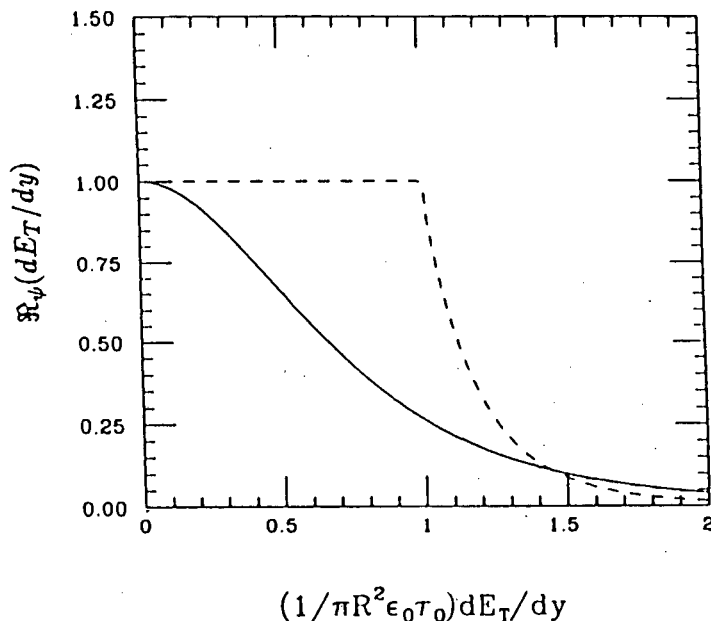


Figure 4.29: E_T dependence of suppression at $p_T = 0$. The dashed lines correspond to the semi-classical model discussed in fig(4.28) [110].

appearing in the classical case below which no suppression occurs is absent in the quantum case, but a strong dependence is still present.

No attempt is made to fit the experimental data, and improvements must be made to the model to include direct production of J/ψ . Of particular importance is the dominance of other diagrams in the high p_T region which have not been considered here, and will lead to an increase in the observed momentum dependence.

The main feature arising out of the study of the quantum models is the flatness of the predicted p_T dependence which is not in agreement with the CERN data of [70]. The threshold effect at $p_T \approx 3$ GeV for the oxygen data, arising out of the classical models where the suppression disappears is completely lacking. Hence if the plasma is to cause the suppression of J/ψ resonances observed for the NA38 data, then there must be another mechanism dominating the p_T dependence. One possibility is the existence of initial state scattering which at present can explain part of the increased p_T dependence observed for the hadron-nucleus collisions over hadron-hadron collisions. This effect will be discussed later in more detail.

Some improvements should still be made to the quantum models: the effect of the plasma boundary should be studied. The potential which the $c\bar{c}$ pair feels is dependent on the relative position of the pair to the plasma boundaries, which in turn is dependent on the motion of the pair. A more complete quantum description should include a study of plasma boundary effects as well. The motion of the $c\bar{c}$ pair through the plasma will also influence the screening effect of the plasma. This effect has been discussed by Chu and Matsui in [119], but must still be included in the above models. Further, the inclusion of all the main production processes leading to the formation of J/ψ resonances should be done.

4.4 Conclusions from Plasma Suppression Models

Within the framework of classical plasma suppression models the NA38 data for the p_T and E_T dependence of J/ψ suppression can be described in a quantitative manner. However, these models require a sharp time τ_ψ for J/ψ formation, before which $c\bar{c}$ pairs are not significantly influenced by the presence of a quark gluon plasma; this time is ambiguous, as the resonance formation is essentially a quantum phenomenon and has led to the need for quantum models to describe the suppression.

For the quantum models the sharp formation time is absent, and instead there is some probability that a transition to J/ψ has occurred. This leads to a much flatter p_T dependence for the suppression, underestimating the data quite severely. A remedy does exist for the p_T dependence in the form of initial state interactions, which are discussed in the next chapter. It is still premature to discuss the E_T dependence arising in the quantum models quantitatively, especially with the influence of boundary effects at small E_T , though the predictions can be considered at a qualitative level. In the quantum model by Matsui [110] the E_T dependence is flatter than the classical models; this may pose a problem in a more detailed model where comparison with the experimental E_T dependence is made.

The quantum models thus have difficulty matching the NA38 data p_T dependence on a quantitative level without the presence of other contributing factors such as initial state interactions, and appear to underestimate the E_T dependence.

The possibility of explaining the data within known hadronic physics should be examined as an alternative method of accounting for J/ψ suppression; this is discussed in the next chapter.

Chapter 5

Alternate Models

5.1 Introduction

The observation by the NA38 group [75] of suppression in the number of J/ψ found in collisions with a high versus low transverse energy distribution, in qualitative agreement with the prediction by Matsui and Satz [7], has strengthened the argument for formation of quark-gluon plasma during heavy ion collisions. However, before it is possible to draw any firm conclusions from the suppression data, a more conventional explanation should be sought. This ought especially to be done since the NA34 and NA35 Collaboration's transverse energy distribution data [141,120] can be described by incoherent sums of individual nucleons [139], which casts doubt on the formation of a quark gluon plasma during the collision. Further, the energy density attained for central collisions in the NA38 experiment is expected to be close to the limiting energy density for quark-gluon plasma formation which might not quite be reached. There is also the problem arising from a study of quantum models of plasma suppression of J/ψ (see section 4.3.3) where the strong p dependence initially predicted by earlier semi-classical treatments disappears.

Whilst it may be possible to explain the data by a single alternative mechanism, it should be borne in mind that it might be necessary to use some combination to reproduce the observed data. Some of the suggested methods to account for the observed suppression include:

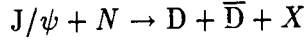
- Nuclear absorption effects [121]–[125]
- Hadronic gas suppression [95],[135]–[138]
- Initial state scattering [154]–[156]
- Charmonium production mechanisms [117],[158]–[161]

Two features of the NA38 data should in particular be explained by these methods, namely the dependence of the suppression on the transverse energy flow from the collision, and the observed relative suppression of the number of J/ψ which occur at low versus high transverse momenta.

Some of the models rely on the absorption of J/ψ to account for the suppression pattern. This absorption arises both from nuclear effects (whilst the J/ψ forms) and later from secondaries arising from the collision, which form a hadronic gas. Nuclear absorption is discussed first below, followed by a description of hadronic gas absorption.

5.2 Nuclear Absorption Models

These models [121]–[125] try to account for the observed J/ψ suppression pattern in terms of the absorption of J/ψ by surrounding nucleons. The models follow a similar pattern, where first the case for hadron-nucleus collisions is considered, and later this is generalised to nucleus-nucleus collisions. The suppression of the number of J/ψ arises from their collision with surrounding nucleons from the corresponding target or projectile nucleus which are participating in the collision, to produce open charm mesons via the reaction



5.2.1 Capella et al. Model

In [121,122] Capella et al., use the Glauber probabilistic model [127] to find the dependence of the J/ψ production cross-section $\sigma^{pA \rightarrow \psi}$ for hadron-nucleus collisions in terms of the hadron-hadron production cross-section and target mass. A parameterisation of the form

$$\sigma^{pA \rightarrow \psi} = A^\alpha \sigma^{pp \rightarrow \psi} \quad (5.1)$$

follows from the experimental results, where the parameter α is dependent on the momentum fraction x_f which the J/ψ carries. Experimentally, a value of $\alpha = 0.97$ is found when $x_f = 0$ decreasing to $\alpha = 0.7$ as x_f approaches unity. For Drell-Yan production, no such decrease is observed [128]. In the Glauber model, a sum is performed over the possible collision configurations, including a varied number of collisions between the incident hadron and the target nucleons. The model assumes that the energy of the incident hadron is sufficiently high that inelastic collisions with the target nucleons which produce J/ψ do not significantly reduce the projectile energy to below the threshold for charm production, hence a constant cross-section for J/ψ hadro-production can be used which implies that multiple J/ψ production can occur in the case where several inelastic collisions of the projectile with the target nucleons occurs. The absorption of J/ψ into open charm mesons by the surrounding nucleons is considered within this sum. An equation of the form

$$\sigma^{pA \rightarrow \psi} = \frac{A \sigma^{pN \rightarrow \psi}}{\sigma_{\text{abs}}^{\psi N} (A + 1)} \int d^2b \{1 - [1 - \sigma_{\text{abs}}^{\psi N} T_A(b)]^{A+1}\} \quad (5.2)$$

arises from the Glauber model for small $\sigma_{\text{abs}}^{\psi N}$, where $\sigma^{pN \rightarrow \psi}$ is the hadronic J/ψ production cross-section which from experiment [128] is of the order of a few nb, $\sigma_{\text{abs}}^{\psi N}$ is the J/ψ absorption cross-section for nuclear matter, and

$$T_A(b) = \int_{-\infty}^{\infty} \rho_A(\vec{b}, z) dz$$

where $\rho_A(\vec{b}, z)$ is the nuclear density for which a Woods-Saxon parameterisation has been chosen. For a value of $\sigma_{\text{abs}}^{\psi N} = 4$ mb, an estimate of $\alpha = 0.95$ for small x_f is obtained, in reasonable agreement with experiment.

The generalisation to nucleus-nucleus collisions is then made by including the possibility of multiple collisions for each target target nucleon with the projectile nucleons in a similar manner. The dependence on impact parameter is determined simply by the total number of nucleons

which are involved in the collision, which is further related via the Dual Parton Model [129] to the measured transverse energy flow for the corresponding impact parameter.

The Drell-Yan background is treated in a similar fashion, except that final state absorption is absent, thus setting $\sigma_{\text{abs}}^{\psi N} = 0$ for Drell-Yan production. A ratio of J/ψ to Drell-Yan background continuum as a function of impact parameter can then be determined; to compare this ratio with the NA38 data, an average over the corresponding transverse energy flows must be done, which is a model dependent procedure. In [121] the Dual Parton Model [129] has been used. However, in comparison with the experimental data for oxygen-uranium collisions [75], insufficient suppression is found to account for the observed data.

One possible solution is the inclusion of the strongly absorbed part of the cross-section, which dominates the cross-section in the large x_f region. This is introduced by the energy-momentum constraint that only one J/ψ per nucleon is allowed to be produced in the initial collision. This reproduces a value of $\alpha = 0.7$ for the hadron-nucleus case at large x_f , and results in a reduction in the relative number of J/ψ produced; however to reproduce the NA38 results require a contribution of the order of 50% from the strongly absorbed part, which is unlikely as the data is in the small x_f region. Hence it is not possible to account for the NA38 data using only this model to account for the observed suppression.

5.2.2 Gerschel and Hüfner Nuclear Absorption Model

A similar approach has been used by Gerschel and Hüfner in [123], where the approach of a linear expansion of eqn(5.2) in terms of the mean free path-length

$$\lambda = \frac{1}{\rho(\vec{b}, z) \sigma_{\text{abs}}^{\psi N}}$$

is made which is found to yield the ratio

$$\frac{\sigma^{pA \rightarrow \psi}}{\sigma^{pN \rightarrow \psi}} = A \left[1 - \frac{1}{2} \left(\frac{\bar{L}}{\lambda} \right) + \frac{8}{45} \left(\frac{\bar{L}}{\lambda} \right)^2 + \dots \right]$$

where the parameter $\bar{L}/2$ is the mean length of nuclear matter encountered; in the case of hadron-nucleus collisions, $\bar{L} = \frac{3}{2}R$ where R is the nuclear radius. Comparison with the experimental form given in eqn(5.1) yields a value for λ in first order approximation of

$$\lambda = \frac{3}{4} \frac{r_0}{(1 - \alpha)}$$

where r_0 is defined by $R = r_0 A^{1/3}$. In the case of Drell-Yan $\alpha \approx 1$ which implies an extremely large mean free path length, and the above expansion terminates at the zero'th term in λ . This reproduces a self consistent value for α .

A similar procedure to the above model is now followed for the case of nucleus-nucleus collisions. A generalisation of eqn(5.2) is now introduced, but without the linear expansion in $\sigma_{\text{abs}}^{\psi N}$ being made, to yield

$$\begin{aligned} \frac{\sigma^{ATA \rightarrow \psi}}{\sigma^{NN \rightarrow \psi}} &= \int d^2b \int d^2s \, dz_1 \, dz_2 \, \rho_T(\vec{s}, z_1) \rho_p(\vec{b} - \vec{s}, z_2) \\ &\times \exp \left\{ - \int_z^{-\infty} dz' \sigma_{\text{abs}}^{\psi N} \rho_p(\vec{s}, z') \right\} \exp \left\{ - \int_z^{\infty} dz' \sigma_{\text{abs}}^{\psi N} \rho_T(\vec{b} - \vec{s}, z') \right\} \end{aligned} \quad (5.3)$$

where $\vec{s}$ refers to the position of the interacting nucleons in a collision with impact parameter b . The exponentials in eqn(5.3) can be linearised in the case where the mean free path length is large. Integration over all impact parameters has been done to yield the simplified equation

$$\frac{\sigma^{ATA \rightarrow \psi}}{\sigma^{NN \rightarrow \psi}} = \left(\frac{\sigma^{NA \rightarrow \psi}}{\sigma^{NN \rightarrow \psi}} \right) \left(\frac{\sigma^{NAT \rightarrow \psi}}{\sigma^{NN \rightarrow \psi}} \right) \quad (5.4)$$

in terms of the hadron-nucleus production cross-sections. A similar derivation can be done for the case of Drell-Yan production, but with $\sigma_{\text{abs}}^{\psi N} = 0$.

To compare this model with the NA38 data, the dependence on transverse energy flow must be found. This is done by including a distribution function $W(b)$ in eqn(5.3) and integrating over the relevant impact parameter range, determined from the transverse energy limits via the HIJET model. A ratio of 0.91 is found for the number of J/ψ emerging for collisions with transverse energy $E_T > 50$ GeV versus $E_T < 28$ GeV. This is insufficient to account for the NA38 data, which has a value of 0.64 [75]. The model has been generalised to include χ and ψ' resonances which decay to J/ψ outside the nuclear environment, but this introduces only a small change in the ratio. Thus this model is also unable to account for the NA38 data.

A further attempt by Efremov et al. [130] to reproduce the J/ψ suppression pattern with use of a dependence of the E_T flow on the ratio of gluon to quark structure functions of the colliding nuclei has been made using a cluster model for nuclear structure functions but this gives only 7% suppression [126].

A suggestion has been made by Brodsky [126] that a momentum dependence for the production of J/ψ in nucleus-nucleus collisions can arise due to the presence of a final-state interaction effect where, after the creation of a $c\bar{c}$ pair, the charm quarks interact strongly with a co-moving quark or gluon spectator which has low p_T to form open charm instead of J/ψ . The $c\bar{c}$ pairs with small p_T will be comoving with the spectators, and hence will be most likely to form open charm. This has been further discussed in [124] to account for the reduction in J/ψ production at large x_f .

A model [125] has been proposed by Blaizot and Ollitrault to try and remedy the inability of nuclear absorption models to account for the E_T dependence of the NA38 data [70]. This model is based upon arguments arising from hadronic gas absorption [95] discussed later in this section which for greater clarity should be first read.

5.2.3 Blaizot Ollitrault Model

In [125] Blaizot and Ollitrault examine a model for J/ψ absorption due to the surrounding nuclear matter, following a similar approach to previous models [122,123] discussed above, which have been unable to account for the observed NA38 data for J/ψ suppression. Absorption due to any secondaries created by the collision is ignored. The main problem with previous nuclear absorption models is that the absorption, as a function of decreasing impact parameter, sets in rapidly and then quickly saturates, failing to reproduce the observed E_T dependence. These models assume that immediately after a $c\bar{c}$ pair is created, it forms a J/ψ , and hence the absorption cross-section is approximately constant from the moment the $c\bar{c}$ is created.

Blaizot and Ollitrault in [125] follow the idea discussed by Brodsky and Mueller [124] that the J/ψ does not form immediately after the creation of a $c\bar{c}$ pair, but rather requires some formation time τ_ψ ; some intermediate state exists between the creation of $c\bar{c}$ and the

J/ψ state which possibly has a different cross-section to the J/ψ . This cross-section is not known at present and hence a degree of arbitrariness is involved. It is most likely that the intermediate state cross-section will evolve in time as the $c\bar{c}$ system expands from that of a $c\bar{c}$ pair to a J/ψ . In [95] Blaizot and Ollitrault have argued that since the photo- and hadro-production experiments [128,132,133] are performed at high energy, $c\bar{c}$ pairs most likely form J/ψ outside of the nuclear environment, so that it is the intermediate state production cross-section that is measured. The intermediate state cross-section is thus not negligible which is assumed in some of the hadronic gas suppression models. In their hadronic gas suppression model [95] Blaizot and Ollitrault assume as a first approximation that the intermediate state has the same cross-section as J/ψ . This is reasonable as hadronic suppression occurs once the $c\bar{c}$ pairs have left the nuclear environment and hence have had time to evolve to J/ψ . In contrast, the situation arising in [125] is different as Blaizot and Ollitrault consider nuclear absorption which can occur at much earlier times, thus it may be necessary to incorporate a cross-section which evolves in time. A fairly arbitrary parameterisation of the form

$$\sigma_{\text{abs}}^{\psi N} = \sigma_0^{\text{abs}} f(\tau)$$

is chosen for the cross-section, where τ is the time in the frame of the $c\bar{c}$ pair. A typical form for $f(\tau)$ is displayed in fig(5.1), with a time τ_ψ corresponding to the typical time when $c\bar{c}$

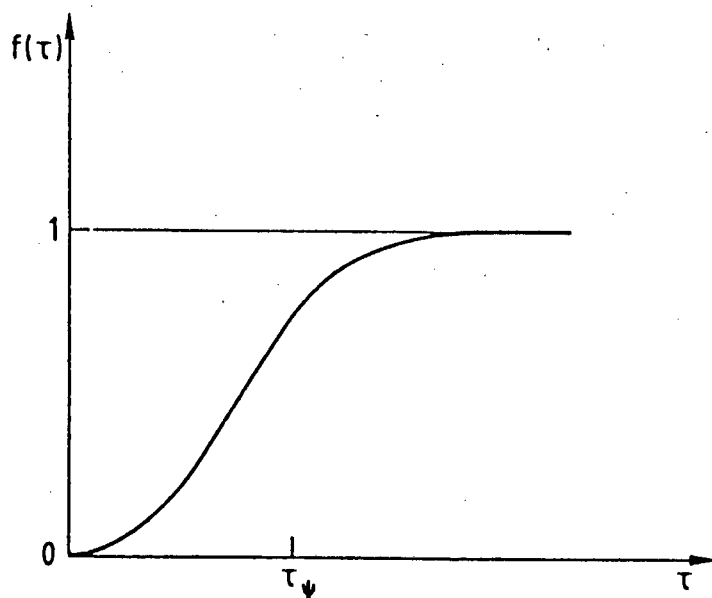


Figure 5.1: Schematic form of the function $f(\tau)$ used in the parameterization of the cross-section. τ_ψ is the “formation time” [125].

pairs evolve to J/ψ . In particular $f(\tau)$ is chosen to be

$$f(\tau) = \left(\frac{\tau}{\tau_\psi} \right)^{c_\beta} \quad (5.5)$$

for $\tau < \tau_\psi$, where c_β is some parameter. The colour transparency discussed by Brodsky and Mueller [124] is in agreement with this kind of parameterisation.

To check the approach of the model, first hadro-production of J/ψ is considered, where the J/ψ production cross-section for hadron collisions on a nucleus of atomic mass A is given by

$$\sigma_{hA \rightarrow \psi} = \sigma_{hN \rightarrow \psi} \int d^2b dz \rho_A(\vec{b}, z) \exp \left\{ - \int_z^\infty \rho_A(\vec{b}, z') \sigma_{\text{abs}}^{\psi N} dz' \right\} \quad (5.6)$$

which is similar to eqn(5.2), where a linearisation of the above equation has been made (later an equivalent linearisation is also made in this model). The parameters are: $\sigma_{hN \rightarrow \psi}$ the J/ψ production cross-section for hadron-nucleon collisions, ρ the nuclear density, and b the impact parameter. The major difference to the previous models [122,123] is that the absorption cross-section $\sigma_{\text{abs}}^{\psi N}$ is not constant, but grows with the distance $|z' - z|$ travelled by a $c\bar{c}$ pair from the point of creation z which, for the form given by eqn(5.5), is given by

$$\sigma_{\text{abs}}^{\psi N} = \frac{\sigma_0^{\text{abs}}}{(\gamma_A v_A \tau_\psi)^{c\beta}} |z' - z|^{c\beta} \quad (5.7)$$

where v_A is the velocity of the J/ψ in the rest frame of the nucleus.

The impact parameter dependence of the cross-section is parameterised in the form

$$d\sigma_{hA \rightarrow \psi}(b) = \sigma_{hN \rightarrow \psi} T(b) N(b) d^2b \quad (5.8)$$

where the effective cross-sectional density $T_A(b)$ is given by

$$T_A(b) = \int_{-\infty}^{\infty} dz \rho_A(b, z) \quad (5.9)$$

By rearranging eqn(5.8) $N(b)$ is given by

$$N(b) = \frac{d\sigma_{hA \rightarrow \psi}}{d^2b} \frac{1}{T_A(b) \sigma_{hN \rightarrow \psi}}$$

A linearisation of eqn(5.6) expanded to first order in $\sigma_{hA \rightarrow \psi}$ then yields

$$N_{\text{lin}}(b) = 1 - \frac{1}{T(b)} \int_{-\infty}^{\infty} dz \rho_A(b, z) \int_z^\infty \rho_A(b, z') \sigma_{\text{abs}}^{\psi N} dz' \quad (5.10)$$

The absorption of J/ψ is defined via

$$\mathcal{A}_{\text{lin}}(b) = 1 - N_{\text{lin}}$$

which can be written

$$\mathcal{A}_{\text{lin}}(b) = \sigma_0^{\text{abs}} I_A(b) \quad (5.11)$$

where after substitution of $\sigma_{\text{abs}}^{\psi N}$ from eqn(5.7) and for a uniform nuclear density within a sphere of radius $R_A = r_0 A^{1/3}$ the integration in eqn(5.10) has been performed to yield

$$I_A(b) = \frac{1}{(\gamma_A v_A \tau_\psi)^{c\beta}} \frac{2^{c\beta+1} \rho_0}{(c\beta + 1)(c\beta + 2)} (R_A^2 - b^2)^{(c\beta+1)/2} \quad (5.12)$$

A problem arises, especially in the nucleus-nucleus absorption case where the observed suppression can be of the order of 50%; with this value for the absorption, the approximation that the absorption is small is no longer valid. A remedy is proposed both in this model [125]

and in the hadronic gas model [95] where the resulting absorption is exponentiated, which amounts to averaging over the arguments of the exponential instead of over the exponentials. The expression for absorption cross-section averaged over impact parameter then becomes

$$\begin{aligned}\sigma_{hA \rightarrow \psi} &= A\sigma_{hN \rightarrow \psi} \exp \left\{ -\frac{1}{A} \int d^2b \mathcal{A}_{\text{lin}}(b) T_A(b) \right\} \\ &\equiv A\sigma_{hN \rightarrow \psi} e^{-\bar{\mathcal{A}}_{\text{lin}}}\end{aligned}\quad (5.13)$$

An idea of the validity of the approximation can be found by considering absorption through a slab of nuclear matter of constant density and thickness. With $x = a\rho\sigma_{\text{abs}}^{\psi N}$ and for constant c_β the values for N obtained are

$$N_{\text{exact}} = \frac{1 - e^{-x}}{x} \quad N_{\text{app}} = e^{-x/2}$$

The impact parameter integration in eqn(5.13) can be carried out with use of eqn(5.11), to yield

$$\bar{\mathcal{A}}_{\text{lin}} = \frac{9\sigma_0^{\text{abs}}}{4\pi r_0^2} \left(\frac{r_0}{\gamma_A v_A \tau_\psi} \right)^{c_\beta} \frac{2^{c_\beta+1}}{(c_\beta+1)(c_\beta+2)(c_\beta+4)} A^{(c_\beta+1)/3} \quad (5.14)$$

In the case of a constant cross-section $c_\beta = 0$ the standard result

$$\sigma_{hA \rightarrow \psi} = A^\alpha \sigma_{hN \rightarrow \psi}$$

is found with

$$1 - \alpha = \frac{9\sigma_0^{\text{abs}}}{16\pi r_0^2}$$

and where a purely numerical approximation

$$A^{1/3} \approx \ln A \quad \text{valid to within 5\% in the range} \quad 54 \leq A \leq 141 \quad (5.15)$$

is made. Experimental data [41,128,131] suggest that $\alpha \approx 0.94$ which corresponds to a cross-section $\sigma_0^{\text{abs}} = 4.5$ mb. It is important to realise that the approximation given by eqn(5.15) is only valid over a limited range of A considered in these experiments. The FNAL data [133] suggest a different A dependence which can be fitted by eqn(5.14) for $c_\beta = 2$, yielding the equation

$$\sigma_{hA \rightarrow \psi} \approx A\sigma_\psi \exp \left\{ -\frac{\sigma_0^{\text{abs}}}{4\pi(\gamma_A v_A \tau_\psi)^2} A \right\} \quad (5.16)$$

which from a fit to the FNAL data yields

$$\frac{\sigma_0^{\text{abs}}}{4\pi(\gamma_A v_A \tau_\psi)^2} \approx 1.7 \times 10^{-3} \quad (5.17)$$

for the exponent. For J/ψ produced with small rapidity in the centre of mass frame of the hadron-nucleon collision with $v_A \sim 1$ corresponding to $\gamma_A \sim 8$, with a formation time $\tau_\psi \sim 0.7$ fm a cross-section of $\sigma_0^{\text{abs}} \sim 7$ mb is found, which is much larger than the standard result for the total cross-section above. However, this value is not too certain due to the second power dependence on τ_ψ which is not well known. It should be further pointed out that as v_A depends on x_f , the value $x_f = 0$ used in the model should be generalised to an

average over $x_f > 0$ corresponding to the measured data. This model does not reproduce the observed dependence at large x_f which implies that other effects are present, not included in this model.

The above procedure for hadron-nucleus collisions at small x_f can be generalised to nucleus-nucleus collisions. A parameterisation of the cross-section in terms of the impact parameter is done in a similar way to eqn(5.8) to yield

$$d\sigma_{AB \rightarrow \psi} = \sigma_{NN \rightarrow \psi} T_{AB}(b) N(b) d^2b$$

where

$$T_{AB}(b) = \int d^2s T_A(\vec{s}) T_B(\vec{s} - \vec{b})$$

is proportional to the number of nucleon-nucleon collisions, for colliding nuclei A and B at an impact parameter $\vec{b}$. The factor $N(b)$ is calculated within the same kind of approximation used in the hadron-nucleus case

$$N(b) \approx e^{-A_{\text{lin}}(b)} \quad (5.18)$$

with A_{lin} given by

$$A_{\text{lin}}(b) = \frac{\sigma_0^{\text{abs}}}{T_{AB}(b)} \int d^2s [T_A(\vec{s} - \vec{b}) I_B(\vec{s}) + I_A(\vec{s} - \vec{b}) T_B(\vec{s})] \quad (5.19)$$

where the integrals T_A and I_A are those defined by eqns(5.9,5.12). The evaluation of A_{lin} requires numerical integration. This has been done for the ratio of O-U collisions to the hadron-nucleus, with the assumption that the J/ψ have small rapidity and that the gamma factors are similar for both the O-U and p-U collisions, so that they cancel leaving a function of A , B and c_β . The result is plotted in fig(5.2) as a function of the number of participants n defined as

$$n = \int_{S_{\text{eff}}} d^2s [T_A(\vec{s}) + T_B(\vec{s} - \vec{b})] \quad (5.20)$$

where S_{eff} is the transverse area over which the two ions overlap. A strong dependence on c_β is observed with the quick saturation with $c_\beta = 0$ no longer present for $c_\beta = 2$, where the peripheral collisions do not last long enough for the $c\bar{c}$ pair cross-section to become large. One further comment should be made regarding the limits of validity of this model: in eqn(5.7) the $c\bar{c}$ cross-section continues to grow even once a J/ψ has formed. Essentially this model lacks a cut-off for the cross-section growth; this may be a problem in central collisions of large ions, where the $c\bar{c}$ pair can travel a considerable distance through nuclear matter forming a J/ψ which then continues with a fixed cross-section. This is an explanation for the increase in absorption seen for $c_\beta = 1, 2$ at high numbers of participants not seen for $c_\beta = 0$. More rigorously, the model is valid for the condition that the formation length $d = \gamma v \tau_\psi$ is larger than the distance travelled by the J/ψ through nuclear matter ~ 10 fm, for a J/ψ formation time of 1 fm this requires a γ factor of ~ 10 . If eqn(5.17) is assumed valid for nucleus-nucleus collisions then a rather large value of $\sigma_0^{\text{abs}} = 20$ mb is found for the cross-section.

A comparison with the NA38 data has been made, where use has been made of the J/ψ - nucleon cross-section found from the J/ψ hadro-production model. Use is made of the rather simplified relation

$$E_T = n E_0 \quad (5.21)$$

to relate the number of participants in the collision to the measured transverse energy flow. A value of $E_0 = 1.1$ GeV corresponds to about half the value required to account for the total

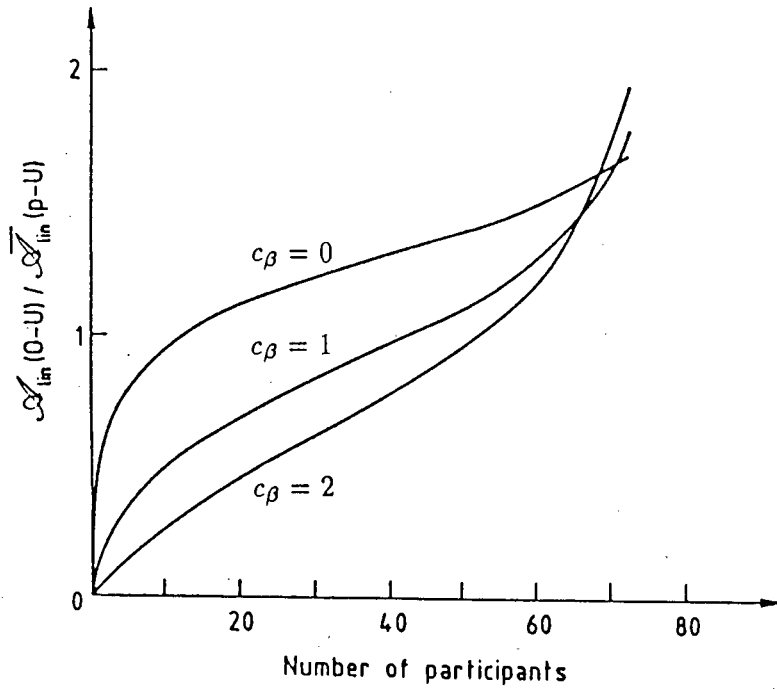


Figure 5.2: The ratio of $\mathcal{A}_{lin}(b)$ given by eqn(5.19) for a nucleus-nucleus collision, to $\mathcal{A}_{lin}(b)$ given by eqn(5.14) for proton-nucleus collisions, as a function of the number of participants n in the nucleus-nucleus collision. n is defined in eqn(5.20) [125].

transverse energy measured by the WA80 group [134], which is consistent as the NA38 group measures approximately 50% of the total transverse energy flow. The experimental NA38 data [70] is presented in fig(5.3) in the form of the measured J/ψ to continuum ratio as a function of the number of participants normalised so that the first point coincides with the model. The model with $c_\beta = 2$ appears to be able to reproduce the observed E_T dependence. As a further check the ratio of the two extreme energy bins for the data has been compared to that arising from the model. The results are displayed in table(5.1), where the NA38 data [65] produces a result of 0.64 ± 0.06 comparable to the model value of 0.73, but much less than the $c_\beta = 0$ result of 0.86.

Thus the model appears capable of reproducing the shape of the E_T dependence of the NA38 data, but no attempt has been made to explain the p_T dependence. The model has a problem with using eqn(5.21) to relate the transverse energy flow to the number of participants as the NA38 data appears to have more than the maximum number of participants (70) found in central collisions, hence the model cannot account for the last data point in fig(5.3) at ~ 88 participants, and thus appears to underestimate the absorption for the last energy bin.

Finally, an attempt is made to estimate the contribution to J/ψ absorption which might occur from other sources. This is done by comparing the p-U data to the O-U peripheral data, where it is found by the NA38 group [70] that for p-U collisions, the J/ψ over continuum

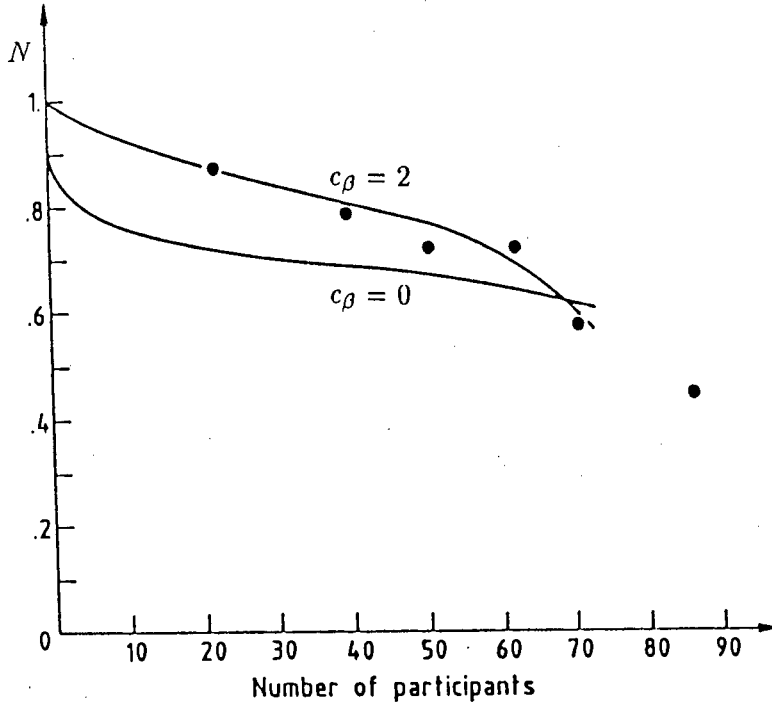


Figure 5.3: The factor N as a function of the number of participants n , for values of the parameter $c_\beta = 0$ and 2 . In each case the absorption has been adjusted to yield 25% absorption in p-U collisions. The large dots correspond to the measured ratios of J/ψ over continuum dimuons, scaled down by a constant factor so that the point at $n = 23$ matches the calculated value [125].

ratio is considerably larger than even the peripheral O-U collision ratio. If eqn(5.16) is parameterised

$$\frac{N_\psi}{N_c} = ce^{-aA}$$

where a value of $a \sim 1.7 \times 10^{-3}$ is found, then from a comparison to the data on p-U collisions [70] a value of $c = 26 \pm 4$ arises. If a similar form is assumed in nucleus-nucleus collisions

$$\frac{N_\psi}{N_c} = cN(b)$$

with $N(b)$ defined by eqn(5.18) and the same value assumed for c , then a value for the ratio N_ψ/N_c of 21.5 ± 3.5 is found for the lowest energy bin, compared to the experimental value of 13.8 ± 1.7 , which leaves open the possibility of another source of suppression to account for the difference, and perhaps the p_T dependence of the NA38 data [70].

In summary, it can be concluded that this model can account for the E_T dependence of the data. However, the assumption of a cross-section for $c\bar{c}$ pairs which evolves in time allows a large degree of model dependence to enter as it has not been verified experimentally. There is also a problem with large x_f production of J/ψ coming through from the hadronic part of the model. No attempt is made to reproduce the p_T dependence of the data.

Table 5.1: The factor N for an O-U collision calculated from eqn(5.18) and averaged in the four energy bins corresponding approximately to those used by the NA38 Collaboration [65,125].

O-U		Bin 1	Bin 2	Bin 3	Bin 4
$\beta = 0$	N	0.69	0.64	0.61	0.59
	N_i/N_1	1.00	0.93	0.89	0.85
$\beta = 2$	N	0.82	0.73	0.66	0.60
	N_i/N_1	1.00	0.89	0.81	0.73
experiment [65]	N_i/N_1	1.00	0.85	0.80	0.64

5.2.4 Conclusions

The presence of nuclear absorption effects in nucleus-nucleus collisions is sufficient to reproduce some of the suppression observed in the NA38 data [70] but requires a finite formation time for J/ψ of the order of 1 fm. There is also the suggestion of there being further suppression which is as yet unaccounted for, whilst no attempt has been made to account for the p_T dependence of the NA38 data. However, the abundant presence of secondaries expected to form a dense hadron gas can produce a significant suppression in the number of J/ψ produced in collisions. This has been ignored in the above models, and the effect of this environment should now be considered.

5.3 Hadronic Gas Suppression

In heavy ion collisions at energies attained for the NA38 experiment, a dense hadronic medium is expected to form. Significant suppression in the number of J/ψ produced from these collisions may occur, despite the relatively small J/ψ cross-section, due to the high density of the hadronic medium. Inelastic scattering with hadronic gas could even be present in the case of quark-gluon plasma formation adding a further contribution to the observed suppression. It is therefore important to examine suppression of J/ψ by a dense hadronic gas.

A summary by Salmeron [76] of several models [95],[135]–[137], upon which the discussion below is based, highlights similarities in the approach used. A discussion involving hadronic suppression of J/ψ invariably relies on an equation relating the number of J/ψ which interact, to the densities of both the hadronic medium and J/ψ , given by

$$dN = n_1 n_2 |v_1 - v_2| \sigma dV dt$$

where the densities n_i are usually functions of particle momentum, position and time, with a volume dV , time interval dt , and where $|v_1 - v_2|$ is the relative velocity between a colliding J/ψ with a hadron, with σ the cross-section a function of the relative velocity. However, the

above quantities are not always well known. In particular, this is the case for the cross-section which will be discussed later. The hadronic medium arising in heavy ion collisions is also not well understood, thus it is necessary to make a series of assumptions about dN .

Among hadronic suppression models [95],[135]–[138] common features are evident, these are:

1. They all assume that the J/ψ has only transverse momentum p_T , in the frame of the nucleus-nucleus collision. Thus no attempt is made to predict a longitudinal momentum dependence of J/ψ suppression.
2. The $c\bar{c}$ pairs which form J/ψ are produced inside the nucleus. However, some models [138] do not consider that a J/ψ forms immediately but rather at some time τ_ψ in the frame of the $c\bar{c}$ pair.
3. Use is made of the value $\sigma(\text{nucleon} + J/\psi) \approx (1 - 3) \text{ mb}$ obtained from the photo-production experiments [41],[52]–[54]. A great deal of ambiguity arises in the use of this cross-section as it can be argued [138] that some J/ψ formation takes place after the $c\bar{c}$ pair leaves the target, and that it is not the J/ψ cross-section that is measured, but rather some smaller intermediate state. The small J/ψ elastic cross-section simplifies the physics, as J/ψ essentially free-stream until they are absorbed, thus the momentum distribution of J/ψ is independent of time, apart from any absorption effects.
4. Models have a form for the probability of survival of J/ψ which can arise from

$$P = \exp \left\{ - \int_{t_0}^{t_f} \nu [\vec{p}, \vec{r}_1(t), t] dt \right\}$$

where t_0 is the initial time when J/ψ absorption starts, t_f is the final time when absorption stops, ν is the collision frequency per unit time, and r_1 is the J/ψ position at time t . A limiting form of

$$P \approx \left(\frac{t_0}{t_f} \right)^\beta$$

is found, where β , the absorption parameter is equal to the ratio of ν , the collision rate, to the expansion rate. It is shown in the models that β is given by

$$\beta = \bar{\sigma} v_{\text{rel}} n_0 t_0 \quad (5.22)$$

where σ is the averaged cross-section, v_{rel} is the meson- J/ψ collision velocity averaged over the hadronic medium of initial density n_0 at time t_0 .

5. The Bjorken estimate [16] given by eqn(2.2) can be used to estimate the hadronic density if the energy per hadron is divided out, and use is further made of eqn(4.30) with the realisation that equations arising in the case of a plasma modeled by an ideal gas, will be valid in the case of an ideal hadronic gas, thus

$$n_0 t_0 = nt = \frac{1}{\pi R_A^2} \frac{dN_h}{dy} \quad (5.23)$$

can be used to estimate the particle density from the measured final particle distribution in rapidity, where R_A is the radius of the incident nucleus. It then follows from combining eqn(5.22) with eqn(5.23) that

$$\beta = \bar{\sigma} v_{\text{rel}} \frac{1}{\pi R_A^2} \frac{dN_h}{dy}$$

It is thus possible to determine n_0 from eqn(5.23) and β for some given thermalisation time t_0 , for oxygen $R_A \approx 3$ fm and from the NA35 Collaboration [98] measured $dN_h/dy \approx 120$. This leads to an estimate of $n_0 \approx 4$ hadrons/fm³ for t_0 of the order of a fermi which is significantly higher than that of normal nuclear matter $n \approx 0.17$ nucleons/fm³. It is due to this high density that the relatively small cross-section for J/ψ may contribute significantly to the observed J/ψ suppression. However, it is not at all obvious what the nature of matter is at about 25 times that of normal hadrons, and especially how the cross-section will behave.

The specific processes studied by models which will be considered are listed in table(5.2) based on the table produced by Salmeron in [76].

5.3.1 Ftáčnik, Lichard and Pišút Hadronic Gas Absorption Model

In [135] the composition of the hadronic gas is simplified to include only π with a density of 5 pions/fm³ in the central region, where the energy density is estimated in [135] to be $\varepsilon = 2.5$ GeV/fm³, with an average energy density of 0.5 GeV per pion. To illustrate the model a first case is considered where the hadronic gas is static, but with some finite lifetime Δt after which the gas no longer suppresses J/ψ . As little is known about the cross-section of pions on J/ψ it is necessary to estimate the cross-section from nucleon+ J/ψ interactions. From photo- and hadro-production of J/ψ [52,53,54] a total nucleon- J/ψ cross-section of 1 – 3 mb is found. For pions the reaction

$$\pi + J/\psi \rightarrow \pi + D + \bar{D}$$

has a minimum threshold in momentum, below which the cross-section vanishes; above this threshold a cross-section of $\sigma = 1 - 2$ mb is used in the model. A spatial density for pions of the form

$$\rho(r) = \frac{3}{2} \sqrt{1 - \frac{r^2}{R^2}} \frac{150}{\pi R^2 l} \quad (5.24)$$

is chosen corresponding to the density of initial nucleons involved in the collision, where 150 refers to the number of pions per rapidity interval calculated for a collision with $E_{0T} \sim 50$ GeV over a rapidity range of 2 in the NA38 experiment, where the measured transverse energy corresponds to approximately one third of the total, and the average energy per pion is 0.5 GeV. A Boltzmann probability distribution in energy $\mathcal{P}(E)$ at some temperature T_0 is assumed

$$\mathcal{P}(E) \sim \exp\left\{-\frac{E}{T_0}\right\}$$

for the pions.

For J/ψ with vanishing momentum in a pion gas at energy E_π the rate of J/ψ absorption per J/ψ is given by

$$R_{\text{abs}} = \rho \mathcal{P}(E_\pi > E_0) \sigma_{\text{abs}}^{\pi\psi}$$

Table 5.2: Comparison of processes leading to J/ψ absorption [76].

Model	Hadron Gas Composition	Reactions
Ftáčnik, Lichard and Pišút [135]	π	$\pi\psi \rightarrow D\bar{D}$
Ftáčnik, Lichard Pišútová and Pišút [138]	π K ρ	$\pi \begin{pmatrix} \psi \\ x \end{pmatrix} \rightarrow D\bar{D}$ $\pi\psi \rightarrow \eta_c\pi$ $K \begin{pmatrix} \psi \\ x \end{pmatrix} \rightarrow D_s\bar{D}$ $\rho \begin{pmatrix} \psi \\ x \end{pmatrix} \rightarrow D\bar{D}$
Gavin, Gyulassy and Jackson [136]	π ρ ω η	$\pi\psi \rightarrow \eta_c\pi\pi$ $\rightarrow D\bar{D}$ $\rho\psi \rightarrow \eta_c\pi$ $\omega\psi \rightarrow \eta_c\pi\pi$ $\eta\psi \rightarrow \eta_c\pi\pi\pi$
Vogt, Prakash, Koch and Hansson [137]	$\pi \left. \begin{array}{c} \\ \\ \end{array} \right] M, M'$ $\rho \left. \begin{array}{c} \\ \\ \end{array} \right]$	$M\psi \rightarrow D\bar{D}$ $M\psi \rightarrow \begin{pmatrix} \eta_c \\ x \end{pmatrix} M'$ $M \begin{pmatrix} \eta_c \\ x \end{pmatrix} \rightarrow D\bar{D}$ $M \begin{pmatrix} \eta_c \\ x \end{pmatrix} \rightarrow \psi M'$
Blaizot and Ollitrault [95]	π	$\pi\psi \rightarrow D\bar{D}$

which can be integrated to give the probability of survival

$$P = e^{-W}$$

where

$$W = -\rho_\pi \Delta t \mathcal{P}(E_\pi > E_0) \sigma_{\text{abs}}^{\pi\psi}$$

and where E_0 refers to the threshold energy required for the reaction $\pi + J/\psi \rightarrow \pi + D + \bar{D}$ to occur. A value

$$\mathcal{P}(E_\pi > E_0) = 0.2$$

is found, which together with an estimation of a lifetime of the pion gas $\Delta t = 3$ fm yields a value of $P = 0.73$ for J/ψ to survive.

The inclusion of χ resonances is necessary as these can account for some 40% of the observed J/ψ resonances; this is done by repeating the above calculation for χ resonances. The cross-section is estimated from that of a J/ψ using a purely geometric argument by taking the ratio of radii to give

$$\sigma(\pi + \chi) = \left(\frac{r_\chi}{r_{J/\psi}} \right)^2 \sigma(\pi + J/\psi)$$

which is larger than that for the J/ψ and hence can lead to more suppression. The total number of J/ψ produced is given by

$$P = 0.6e^{-W_\psi} + 0.4e^{-W_\chi}$$

which is expected to be somewhat smaller than the case without the inclusion of χ resonances.

In general W is given by the equation

$$W = \frac{1}{2p_\psi} \int dl \int d^3p_\pi \rho(\vec{p}_\pi, \vec{r}, t) \frac{1}{E_\pi} \sqrt{(s - s_a)(s - s_b)} \sigma^* \quad (5.25)$$

where $s_a = (m_\psi + m_\pi)^2$, $s_b = (m_\psi - m_\pi)^2$, $\vec{r} = \vec{r}_\psi + \vec{v}_\psi t$, $dl = |d\vec{r}|$, the integral is over the path of the J/ψ , σ^* is the boost invariant value found for collinear collisions non-vanishing above the threshold energy and $\rho(\vec{p}_2, \vec{r})$ is the pion density given by $\mathcal{P}(\vec{p}_2) \rho(\vec{r})$. Eqn(5.25) is the integrated absorption rate for J/ψ with arbitrary momentum, with the main difference to the static J/ψ absorption case arising from the need to write the cross-section in a form which is Lorentz invariant under non-collinear boosts. A detailed derivation of the above equation can be found in the appendix of [138].

One important input is the formation time of J/ψ which is estimated in a similar manner to the classical plasma suppression models. The $c\bar{c}$ pairs are assumed to interact with the hadron gas only after the J/ψ has formed. This will introduce some p_T dependence, as J/ψ with higher p_T will be observed to form later in the hadron gas, and hence will suffer less suppression. This effect is however countered to some extent by the threshold required for absorption by pions, thus it is possible to get either J/ψ enhancement or J/ψ suppression as a function of p_T . The enhancement at very low versus high p_T due to the threshold limit is not found for the heavier χ resonances, whose production is expected to monotonically increase with increasing p_T .

The calculation for W can then be made, and an average over initial positions of the J/ψ can be taken. This has been done in [135] for a gas lifetime $t_1 = 4$ fm and is displayed in

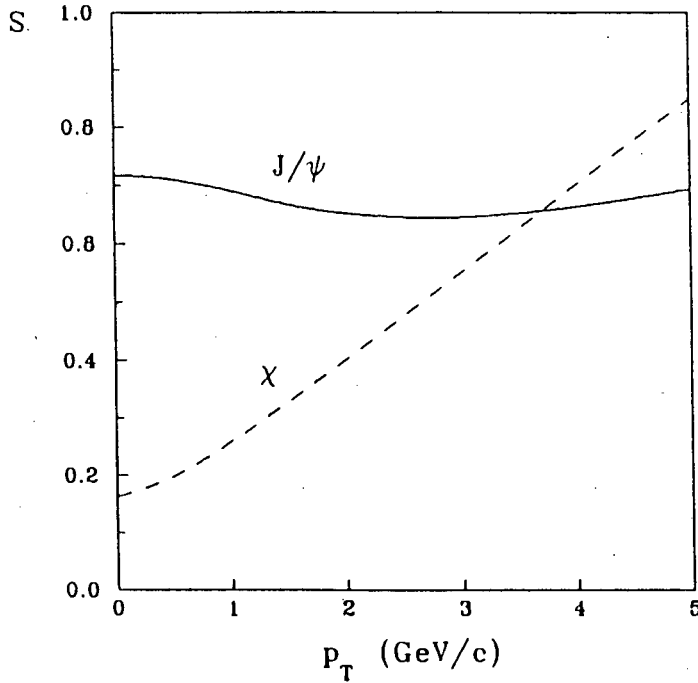


Figure 5.4: The p_T dependence of directly produced J/ψ and χ suppression. $\sigma_{\text{abs}}(\pi + J/\psi) = 1$ mb, $\sigma_{\text{abs}}(\pi + \chi) = 2.36$ mb and $t_1 = 4$ fm [135].

fig(5.4). The increase in the number of resonances which escape is observed for both J/ψ and χ resonances, whilst the effect of a threshold for J/ψ absorption by pions is clearly observable. The inclusion of heavier hadrons into the gas will counter this threshold effect.

However, there is insufficient p_T dependence coming from this model to account for the NA38 data, even with a considerable contribution from χ resonances; though it is possible that some of the E_T dependence could be accounted for in this model. An improvement to the model to try to remedy this problem is suggested. The alteration from a static to a dynamic finite sized gas should be made, which will have the effect of allowing faster J/ψ to escape from the absorbing region, hence possibly enhancing the high p_T production region, leading to a stronger p_T dependence in closer agreement with the data.

This model has been improved upon considerably in [138] where a very detailed calculation of hadronic suppression of J/ψ is given. A smaller input is made for the pion multiplicity for the central region, estimated from the E_T flow measured in the NA38 experiment. It is important to realise that the measured E_T flow is for a rapidity range of 2.2, so that to find the flow per unit rapidity, the measured flow must be divided by 2.2, whilst correction for the fact that the measured E_T flow does not include all the charged particle flow and in fact corresponds approximately to 0.58 of the total energy flow should be made. This leads to a value of $dN_h/dy = 90$ in comparison to the value of 150 used previously. If the assumption is again made that the pions each contribute 0.5 GeV to the total energy, then a density of 3 fm^{-3} is found for the central region.

An important feature of this model is the inclusion of a “dressing-up time”. This is

the time for the $c\bar{c}$ pair to build up clouds of soft partons, and hence can be different to the J/ψ formation time introduced by Karsch and Petronzio [82] discussed in section 4.1.4. In [138] Ftáčnik et al., suggest that the dressing-up time should be relatively similar for all hadrons and should manifest itself clearly in cascading nuclei. A recent study by Ranft [139] of cascading in the target fragmentation region of ^{16}O -Pb collisions has shown that a most likely formation time is 2 ± 1 fm. A value of $\tau_\psi = 2 - 3$ fm is suggested as the effective formation time of J/ψ , which is much larger than the formation time used by Karsch and Petronzio [59] in their model of plasma suppression of J/ψ . The effect of increasing τ_ψ , is to increase the p_T dependence sufficiently strongly that J/ψ with p_T of the order of a few GeV do not get absorbed. The effect of increasing τ_ψ is displayed in fig(5.5). It is now possible

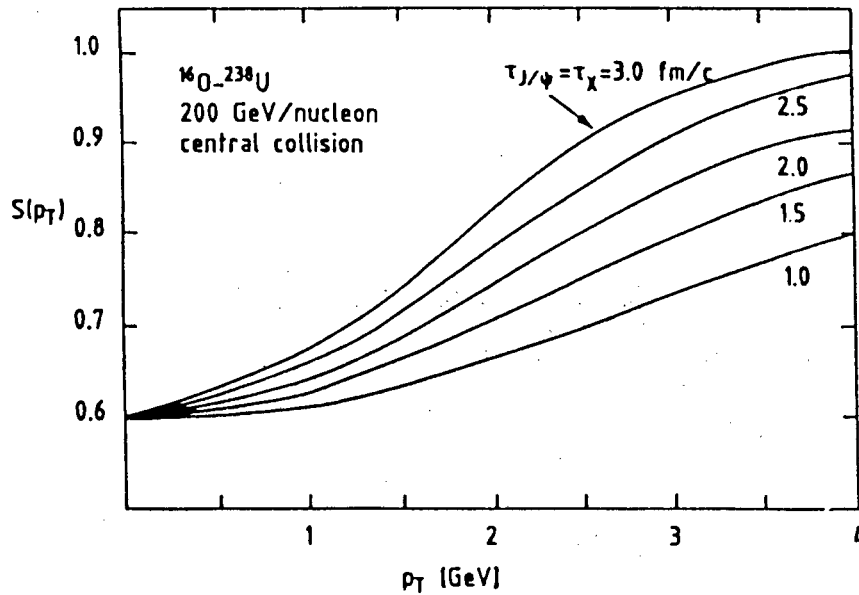


Figure 5.5: J/ψ survival probability as a function of p_T for a set of values of the “dressing up time” $\tau_\psi = \tau_\chi$ [138].

to reproduce the p_T dependence of the NA38 data which was a major defect of the previous model.

A further improvement to the previous model is the inclusion of a dynamical hadronic gas, which expands rapidly longitudinally, whilst the transverse time dependence is introduced by allowing for a rarefaction wave which propagates radial inwards at c_s , the speed of sound in the hadronic gas. If the hadronic gas is treated as an ideal gas, then the hydrodynamic results found for the case where the quark-gluon plasma was treated as an ideal gas are valid for the hadronic gas as well, but with a different number of degrees of freedom. Thus from entropy flow conservation, it follows that the temperature is just a function of the proper time of the gas see eqn(2.33), which for the $z = 0$ slice is given by

$$T = T_0 \left(\frac{t_0}{t} \right)^{c_s^2}$$

An ideal gas of massless particles has a speed of sound corresponding to $c_s^2 = \frac{1}{3}$, but for a

hadron gas it is found that a lower value of 0.2 is more realistic [140]. This gives a lifetime for the hadron gas of 5 fm in comparison to the value of 3 fm used in the previous model. The time evolution of the hadron gas density follows from eqn(5.23), and is given by

$$\rho(\vec{r}, t) = \rho(\vec{r}, t_0) \frac{t_0}{t} \theta(R_0 - c_s(t - t_0) - r) \quad (5.26)$$

where the theta function constrains the gas to lie within the region determined by the rarefaction wave. The time dependence of the temperature is to a first approximation ignored as it varies slowly, so that a constant temperature T_0 is used.

The separation of ρ into

$$\rho(\vec{r}, \vec{p}, t) = \rho(\vec{r}, t) \mathcal{P}(\vec{p})$$

is assumed where $\mathcal{P}$ is given by the Boltzmann distribution

$$\mathcal{P}(\vec{p}) \sim \exp \left\{ -\frac{(\vec{p}^2 + m_\pi^2)^{1/2}}{T_0} \right\}$$

with an estimate of $T_0 = 200$ MeV where the temperature corresponds to the transverse momentum distributions in multiparticle production and may thus be considered as an average temperature of the expanding hadron gas [138].

This model extends the previous one to include other hadronic resonances. The hadron densities used at $t = t_0$ are

$$\begin{aligned} \langle \rho_\pi(t_0) \rangle &= 2.5 \text{ fm}^{-3} \\ \langle \rho_K(t_0) \rangle &= \langle \rho_\rho(t_0) \rangle = 1.25 \text{ fm}^{-3} \end{aligned} \quad (5.27)$$

whose ratios are the same as those found for the produced particles in multiparticle production. The radial dependence given in eqn(5.24) is generalised to include the case where non-identical nuclei are used, which in the case of pions is given by

$$\rho_\pi(r, t_0) = (2.5 \text{ fm}^{-3}) \frac{3}{2} \frac{1}{2R_A} \frac{1}{2R_B} 2\sqrt{R_A^2 - r^2} 2\sqrt{R_B^2 - r^2} \quad (5.28)$$

where the factors

$$t_A(r) \equiv 2\sqrt{R_A^2 - r^2} \quad \text{and} \quad t_B(r) \equiv 2\sqrt{R_B^2 - r^2} \quad (5.29)$$

represent the longitudinal dimension of nuclei A and B at a given value of r , for central A-B collisions. The generalisation to a non-central collision at impact parameter b is made by replacing r with $\vec{r} - \vec{b}$ in $t_B(r)$, where the z dependence has been ignored. The generalisation of eqn(5.28) to include the K and ρ resonances can be made by simply replacing the average pion density by the relevant averaged density. Thus an equation describing spacial and time dependence of the hadron gas density has been found. A generalisation of eqn(5.26) to non-central and non-identical nuclei is given by

$$\rho(\vec{r}, t) = \rho(\vec{r}, t_0) \theta(R_A - c_s(t - t_0) - r) \theta(R_B - c_s(t - t_0) - |\vec{b} - \vec{r}|)$$

For non-central collisions the rarefaction wave will reach the centre of the hadron gas in a time

$$t = \frac{R_A + R_B - b}{2}$$

In [138] a minimum thickness of the lens-like region in non-central collisions of 1 fm is allowed before the gas is assumed to have reached a density below which no significant J/ψ absorption can occur, as the mean free path is J/ψ will then be of the order of the dimension of the hadronic gas.

It is now necessary to generalise the cross-sections to include K and ρ resonances. To estimate these it is necessary to make a series of approximations: the ρ cross-section is chosen to be the same as for pions, whilst for K it is chosen to be half the value of the π cross-section. The procedure to generalise to χ resonances used in the previous model is carried through. The reaction $\pi + J/\psi \rightarrow \eta_c + \pi$ which is exothermic is not considered as the cross-section is not known at all. It is also possible to regenerate J/ψ via the hadronic scattering of the χ resonances into the J/ψ state which is also not considered in this approach. The appropriate cross-section and corresponding threshold energy squared s_0 for each reaction are given in table(5.3).

Table 5.3: The disintegration cross-sections, where the ratio of r_χ^2/r_ψ^2 is taken for $r_\psi = 0.45$ fm and $r_\chi = 0.70$ fm, and threshold energies squared [138].

Collision	Disintegration cross-section [mb]	Threshold s_0 [GeV ²]
$\pi + J/\psi$	2.0	15.054
$\pi + \chi$	4.67	13.988
$K + J/\psi$	1.0	15.840
$K + \chi$	2.4	16.128
$\rho + J/\psi$	2.0	15.031
$\rho + \chi$	4.67	18.404

The survival J/ψ survival probability is then given by the generalised form

$$S(p_T) = \langle \exp\{-T_\pi - T_K - T_\rho\} \rangle$$

where the averaging is done over position and directions of the momentum of J/ψ , with the 40% contribution from χ cascades included. This has been done in [138] for the case of central collisions with a formation time $\tau_\psi = \tau_\chi = 2$ fm for the J/ψ and χ resonances. Comparison can be made for high and low projectile mass for J/ψ with $p_T = 3$ GeV where the ratio of

$$\frac{S_{O+U}}{S_{Pb+U}} = \frac{0.75}{0.13}$$

is obtained.

The next step is to generalise to non-central collisions. One of the problems, is to link the observed transverse energy flow E_T^0 to the impact parameter b of the collision. An analysis of the NA35 data [141,142] shows that the shape of $d\sigma/dE_T$ can be well described on geometrical grounds, by performing the appropriate convolution of the transverse energy distribution from

p-U collisions for the impact parameter. In central O-U collisions this would correspond to a 16-fold convolution, yielding the maximum E_T value.

The transverse energy is essentially assumed to be proportional to the number of nucleon-nucleon collisions at impact parameter b , given by

$$N_{AB}(b) = \frac{1}{\sigma_{nn}} \int d^2r [\sigma_{nn}\rho_A t_A(\vec{r})][\sigma_{nn}\rho_B t_B(\vec{b} - \vec{r})]$$

where t_A and t_B are the lengths defined earlier in eqn(5.29), σ_{nn} is the inelastic nucleon-nucleon cross-section and ρ_A and ρ_B are the corresponding nuclear densities.

The effect of the exclusion of the rarefaction wave has been considered, but not much change is noted in the shape of the p_T dependence of hadronic absorption, though a net increase in the strength of the absorption is found. This is expected as the increased size of the hadronic gas allows for more absorption.

The transverse energy dependence has been examined, and is given in fig(5.6) for the case of zero p_T J/ψ production, the case where $p_T = 2$ GeV is shown in fig(5.7), where the parameter x is given by

$$x = \frac{E_T}{E_T^{\max}}$$

In order to compare the model with NA38 data it is necessary to fix some of the parameters:

- The product $\rho\sigma\mathcal{P}(s > s_0)$ should be estimated. The value of $\rho\sigma\mathcal{P}(s > s_0)\Delta t$ can be found from the $p_T = 0$ value of the suppression, and $\Delta t \approx R/c_s$ is the life-time of the hadronic gas absorption. The value for c_s , the speed of sound, is important for determining the E_T dependence, and hence a reasonable value for it can be estimated from comparison with the experimental E_T distribution. An indication of ρ can be obtained via the Bjorken estimate of the energy density, though the formation time of the hadron gas is ambiguous.
- an estimation of c_s also arises from a study of the hydrodynamics of the expansion of the gas [19].
- The dressing-up time for J/ψ and χ resonances can be estimated from intra-nuclear cascading.

One question which is raised in [138] is whether the lowest E_T bin has suppression or not. This approach predicts suppression in the lowest E_T bin in nucleus-nucleus collisions, but none for the p-U data. Comparison can be made with the NA38 data and is displayed in fig(5.8) for the E_T dependence and fig(5.9) for the p_T dependence. Reasonable agreement is found for both.

Comments made about this model are:

- It depends crucially on the dress-up time for J/ψ and χ resonances in order to reproduce the p_T dependence. A suggestion is made that the predicted cross-section from photo-production [52,53,54] of J/ψ should be re-analysed to include a finite resonance formation time. This has been also mentioned by Brodsky and Mueller [124]. An increase in the J/ψ cross-section is thus expected which should enhance hadronic gas absorption for lower hadron densities.

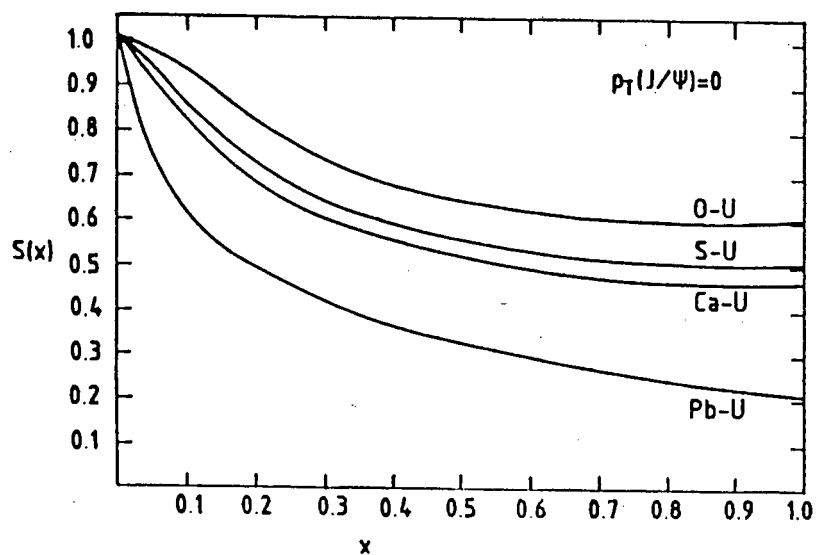


Figure 5.6: Dependence of J/ψ suppression on $x = E_T/E_T^{\max}$ for $p_T = 0$ O-U, S-U, Ca-U and Pb-U collisions. Formation times for both J/ψ and χ are 2 fm, $c_s^2 = 0.2$. Other parameters are specified in table(5.3) and by eqn(5.27) [138].

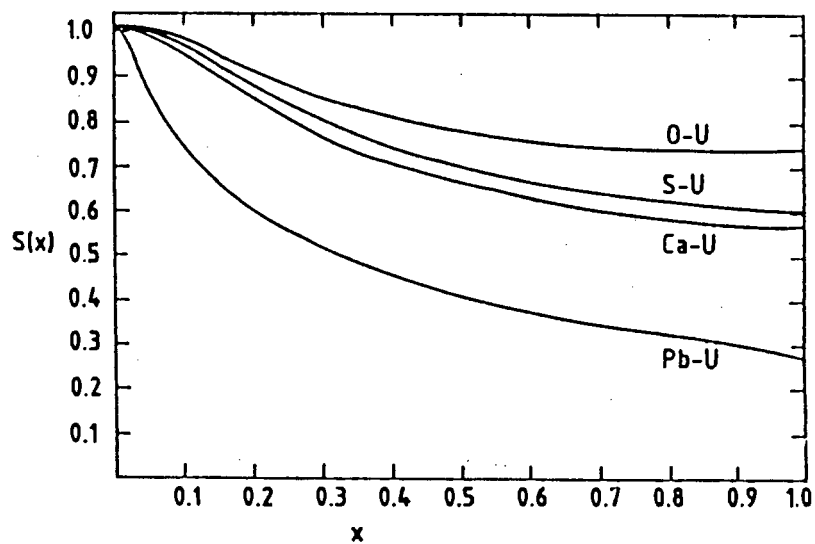


Figure 5.7: The same as for fig(5.6) but with $p_T = 2$ GeV

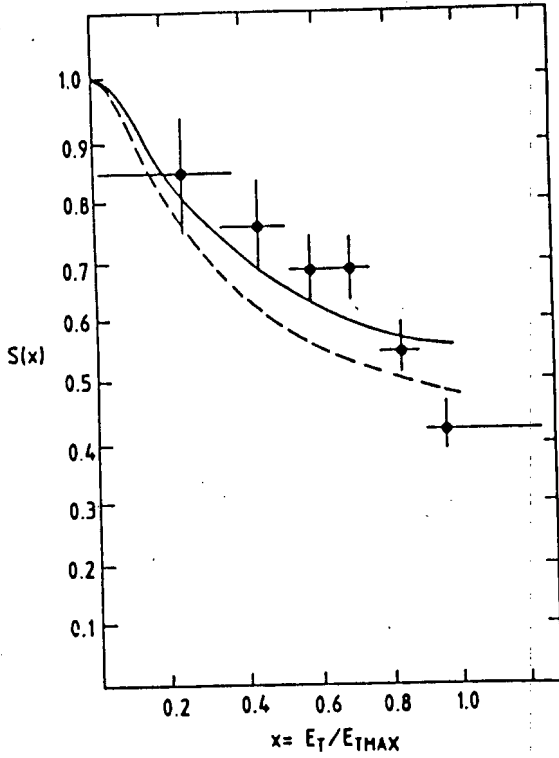


Figure 5.8: Comparison of NA38 preliminary data obtained from [71] under the the assumption of no J/ψ suppression in p-U collisions. The solid curve corresponds to the results from the model with $\tau_\psi = \tau_\chi$ and $c_s^2 = 0.2$, other parameters are specified in table(5.3) and by eqn(5.27). The dashed curve is from the model with the cross-section in table(5.3) multiplied by 1.36 to obtain the same suppression as the data at $x = 1$. E_T^{max} is fixed at 100 GeV. The J/ψ suppression in the lowest bin is obtained from p-U data on J/ψ /continuum data from [71]. [138]

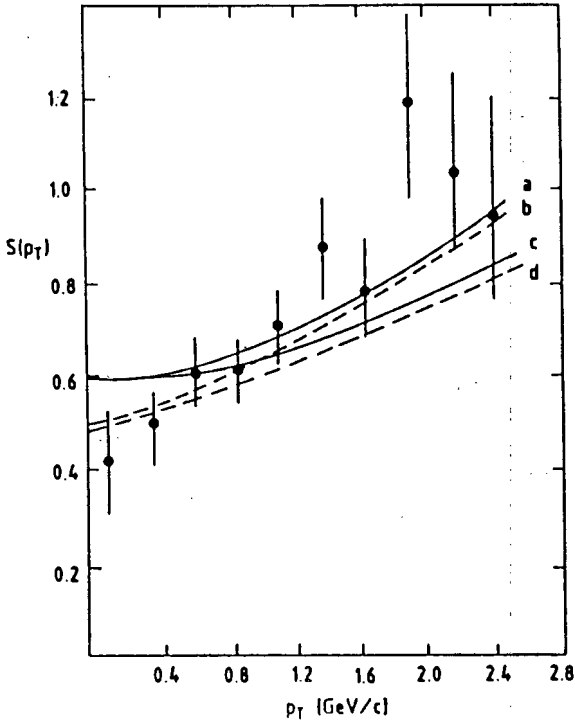


Figure 5.9: Comparison with the NA38 data [71] on the p_T dependence of J/ψ suppression with the model. Curves (a,b,c,d) have a "dressing up time" of both τ_ψ and τ_χ corresponding to for (a,b) 3 fm, and for (c,d) 2 fm. For (c) the cross-sections are taken from table(5.3) and eqn(5.27), whilst for the other cases somewhat higher cross-sections and initial densities are used to account for increased suppression at $p_T = 0$. It should be noted that the results from the model correspond to the highest E_T bin whilst the data have the ratio of high to low E_T bins [138].

- That absorption of J/ψ by hadronic gas must occur in the lowest E_T bin for the NA38 data. If this was shown otherwise the prediction from hadronic gas suppression will be

incorrect.

- The time dependence of the temperature has been ignored. The effect of this assumption should be examined.
- The model predicts a very high hadronic density some 10 times above normal nuclear densities. The nature of matter at this density is not well known which should particularly effect the J/ψ cross-section which has been assumed to be independent of hadronic density.
- Use could be made of a more sophisticated model to relate the measured transverse energy flow to the impact parameter for the collision.
- Inclusion of nuclear effects can be made. This would most probably not alter the shape p_T dependence of the absorption, but may affect the overall E_T dependence.

5.3.2 Gavin, Gyulassy and Jackson Hadronic Absorption Model

A somewhat simplified approach is taken in [136] to find the p_T dependence of J/ψ suppression by hadronic absorption. In contrast to the earlier approach of Ftáčnik et al. [135], the inclusion of heavier hadronic resonances, namely $\rho(770)$, $\eta(549)$ and $\omega(783)$ is made. These heavier resonances have a lower threshold for reaction with J/ψ , and in particular the reaction can be exothermic. This is similar to the later model of Ftáčnik et al. [138] which was discussed above. The reaction threshold for pion absorption may be masked by the absorption by heavier resonances; this is particularly the case when the initial densities of the heavier resonance are quite high.

The hadron gas density is found from the measured particle multiplicity distribution with use of eqn(5.23) which is valid in the central region where the particle multiplicity forms a plateau. In particular the case for O+Au collisions studied by the NA35 group [143] is considered. An initial thermalisation time of 1 fm for the hadron gas is assumed. This does not take into account the finite size of the collision area suggested in the previous model. The gas is assumed to a first approximation not to expand transversely, and to have a radius determined by the smaller nucleus of $R_A \approx 3.0$ fm. For a measured particle multiplicity of $dN/dy \sim 41$ observed by the NA35 collaboration [143], realising that this measures approximately one third of the total multiplicity taken in a rapidity interval of ~ 2.5 , a density of $n(t_0) \sim 4 \text{ fm}^{-3}$ is found.

The initial distribution of the hadronic densities is not known, as the heavier resonances decay to pions as the gas cools, thus some model must be used to estimate the distribution. In [136] the LUND FRITIOF/JETSET 6.3 string fragmentation model [144,145] is used. The composition of produced hadrons is found to be

$$f_\pi \approx 0.25, \quad f_{\eta+\eta'} \approx 0.07, \quad f_\rho \approx 0.21$$

$$f_\omega \approx 0.07, \quad f_N \approx 0.15, \quad f_{K+K^*} \approx 0.15$$

The measured pion rapidity density dN_π/dy , which is increased from the original density by the decay products of the heavier resonances, can be related to the initial hadron density via

$$\frac{dN_H}{dy} \approx \frac{f_\pi + f_{\eta+\eta'} + f_\rho + f_\omega}{f_\pi + 2(f_{\eta+\eta'}) + 3f_\omega} \frac{dN_\pi}{dy} \approx 0.6 \frac{dN_\pi}{dy}$$

where the factors of 2 and 3 in the denominator arise from decay multiplicities into pions of the heavier resonances.

The J/ψ absorption cross-section is estimated from $\sigma_{\psi N} \approx 2.2$ mb the J/ψ nucleon total cross-section found from photoproduction experiments [41], the inclusive scattering cross-section is found as well, but is much smaller than the total cross-section, hence the absorption cross-section dominates. The primary absorption process is thought to be

$$\psi + N \rightarrow D\bar{D} + X$$

In [136] a mean cross-section for hadronic absorption of the from

$$\bar{\sigma} = \frac{2}{3} \sigma_{\psi N \rightarrow D\bar{D}+X} \approx 1 - 2 \text{ mb}$$

is used. This is thought to be an underestimate, as the photoproduction calculations do not take into account the finite formation time of J/ψ , discussed in the previous model [138].

If a J/ψ is formed by a hard collision at $\vec{r}_{T0}$ at time $t_{0\psi}$ then the probability that it still survives at a time t_f , noting that the elastic cross-section for J/ψ scattering off the hadron gas is much smaller than the total cross-section [54] and hence that elastic scattering of J/ψ can be ignored, is given by

$$P(\vec{p}_T, \vec{r}_{T0}) = e^{-N} \quad (5.30)$$

where N the mean number of collisions suffered by the J/ψ is given by

$$N = \int_{t_{0\psi}}^{t_f} dt \nu(\vec{p}_T, \vec{r}_T(t), t) \quad (5.31)$$

where $\vec{r}_T$ describes the J/ψ trajectory through the gas and ν is the collision frequency determined from

$$\nu = n\sigma v_{\text{rel}} \approx \bar{\sigma} v_{\text{rel}}$$

where v_{rel} is the average meson- J/ψ relative velocity. The hadron gas density n is assumed to be isotropic radially thus removing the dependence of the collision frequency ν on the J/ψ trajectory through the gas; the time dependence of the hadronic density is assumed to be of the form given in eqn(5.23) valid when the central region is boost invariant. Thus to a first approximation the time dependence is situated only in the hadron density (average quantities are used for v_{rel} and σ) so that the integral in eqn(5.31) can be immediately performed to yield the equation

$$N = \beta \log \left(\frac{t_f}{t_{0\psi}} \right)$$

where β the absorption parameter is given by

$$\beta \equiv \bar{\sigma} v_{\text{rel}} t_0 n(t_0) = \nu \left(\frac{1}{n} \frac{dn}{dt} \right)^{-1}$$

The conditions for t_f the time when the absorption stops, particularly its dependence on J/ψ momentum and position, must now be determined. At this point, it is also important to note that a J/ψ formation time $t_{\psi 0}$ has been introduced as a simplification to account for the dilation time for the expanding $c\bar{c}$ pair to reach the J/ψ radius. This type of simplification is similar to that which was introduced in the classical plasma suppression models by Karsch

and Petronzio [59], which allowed for a maximum momentum where screening can occur. A similar feature is found for the hadronic gas absorption with the above assumption for the formation time of J/ψ . Thus eqn(4.16) for the plasma models can be applied for this model, where τ_ψ corresponds to the J/ψ formation time and t_1 is the lifetime of the hadronic gas. The finite size effect should also be considered where J/ψ with sufficiently large momenta, a limit in the case of centrally created J/ψ is determined by

$$(p_T)_{\min} = M_\psi \frac{R_A}{t_f} \left[1 - \left(\frac{R_A}{t_f} \right)^2 \right]^{-\frac{1}{2}} \quad (5.32)$$

escape out of gas. Thus three cases arise:

1. Slow moving J/ψ with $p_T < (p_T)_{\min}$ do not escape from the absorption region and hence t_f is just the life-time of the hadronic gas
2. For $(p_T)_{\min} < p_T < (p_T)_{\max}$ where $(p_T)_{\max}$ is the limiting momentum for which absorption disappears, t_f is determined by the time at which the J/ψ leaves the hadron-gas
3. J/ψ with $p_T > (p_T)_{\max}$ do not form before the gas has thinned sufficiently that the J/ψ mean free path in the gas is of the order of the transverse size of the gas.

The survival probability given by eqn(5.30) becomes

$$P(\vec{p}_T, \vec{r}_{T0}) = \left(\frac{t_{0\psi}}{t_f} \right)^\beta \quad (5.33)$$

which can then be averaged over the initial creation positions of the J/ψ , where $t_{0\psi}$ is the formation time of the J/ψ which may be different to the formation time of the hadronic gas and t_f is the time at which the absorption stops. With use of eqn(5.23) for n_0 , β can be expressed in terms of the hadron multiplicity

$$\beta = \bar{\sigma} v_{\text{rel}} (\pi R_A^2)^{-1} \frac{dN}{dy}$$

The assumption is made that J/ψ are mostly removed by ρ and ω scattering, and that they have $\langle p_T \rangle \approx 0.5$ GeV, then $v_{\text{rel}} \approx 0.6$. The inclusion of pions is expected to increase this estimate. A value of $(\pi R_A^2)^{-1} dN/dy \approx 2 - 4$ is estimated from the NA35 data [143]. The generalisation of the NA35 data to arbitrary target and projectile is made, realising that the particle production is a soft process, with use of the equation

$$A^{-2/3} \frac{dN}{dy} \propto B^{1/3}$$

which leads to the form for β of

$$\beta = (0.13 - 0.26) \left(\frac{\bar{\sigma}}{1 \text{ mb}} \right) \left(\frac{B}{197} \right)^{1/3}$$

which is relatively independent of the projectile mass. The freeze-out time is estimated from the time that a rarefaction wave from the edge of the gas would reach the origin $t_f = R_A/c_s$, where the ideal gas value is used for c_s . The probability of survival is then given by

$$P = \left(c_s \frac{t_{0\psi}}{R_A} \right)^\beta$$

A thermalisation time of 1 fm is assumed, with the J/ψ formation time $t_{0\psi}$ also given by 1 fm. The use of $c_s = 0.45$ which is a more realistic value is not expected to change the results significantly. The probability of survival has been averaged over and the result is displayed in fig(5.10). The kink in the curve should be noticed — this is where the momentum of

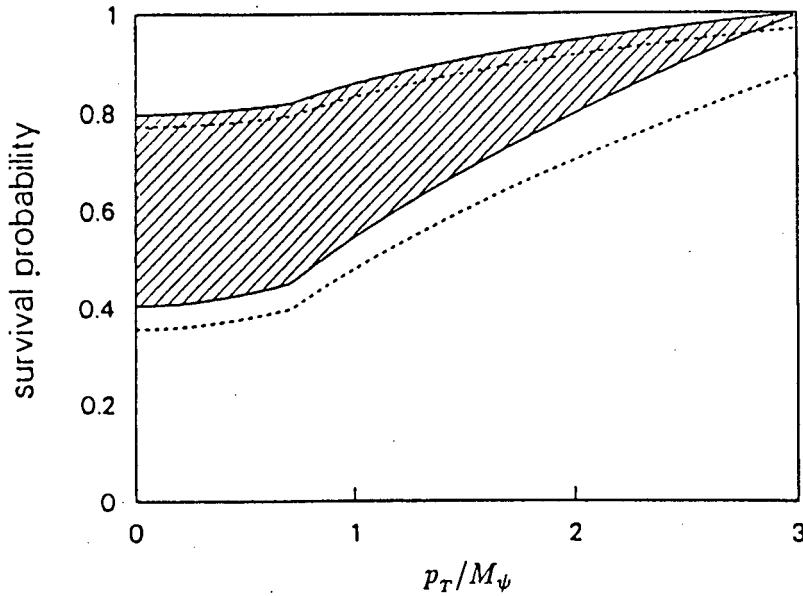


Figure 5.10: The survival probability of a J/ψ , eqn(5.33) is shown for central $^{16}\text{O}+\text{U}$ (solid curves), and for $^{32}\text{S}+\text{U}$ (dashed curves) at 200 GeV per nucleon. The shaded region indicates the uncertainty in the oxygen result that arises through uncertainty in $\beta \sim 0.14 - 0.55$, and is bounded below by the $\beta = 0.55$ curve. The lower dashed line is the sulphur prediction for $\beta = 0.55$ and should be compared with the lower solid curve. The transition between low and high p_T behaviour occurs at $(p_T)_{\min}$ which is given by eqn(5.32) [136].

the J/ψ is sufficient that it can escape from the gas. The E_T dependence is introduced by assuming that dN/dy scales with E_T . Hence comparison with the NA38 data can be made with the survival probability given by

$$\mathfrak{R}_\psi(p_T) \sim P^{[1-(E_T)_{\text{low}}/(E_T)_{\text{high}}]} \sim 0.6 - 0.9$$

In conclusion, insufficient p_T dependence is found to account for the NA38 data. It should be mentioned however, that the inclusion of initial state scattering could change this dependence. This will be discussed later. The main result of this model, is that a high hadronic density is necessary to account for the strength of the observed suppression.

5.3.3 Vogt, Prakash, Koch and Hansson Model

Absorption of J/ψ by a dense hadronic gas formed from the secondaries created in a highly energetic heavy ion collision is considered in this model [137]. A finite J/ψ formation time

$\tau_\psi \approx 1$ fm is required for the J/ψ to form before absorption by the surrounding hadron gas is allowed to set in. The hadronic gas is assumed to expand isentropically longitudinally such that the Bjorken formula given in eqn(5.23) is valid. The transverse expansion is ignored, but a finite lifetime, when the J/ψ mean free path through the gas is of the order of the transverse dimension of the gas, is kept. An ideal gas Boltzmann distribution is assumed for the momentum dependence of the hadron gas. Two cases are considered which differ in the constituent meson resonances of the hadronic gas: the first corresponds to a hadronic gas consisting of just pions, with the absorption reaction

$$J/\psi + \pi \rightarrow D\bar{D} + X$$

For the second case, of which the first is just a simplification, the gas contains both π and ρ mesons. Inclusion is also made of other charmonium $c\bar{c}$ states which lead to the possibility of regeneration of J/ψ from $c\bar{c}$ resonances; the χ and η_c are considered specifically by this model, leading to the reaction $\chi + \pi \rightarrow J/\psi + X$. Assuming that the η_c is produced at least as strongly as χ then the ratio R

$$R = \frac{N_{\eta_c} + N_\chi}{N} \approx 3 - 4 \quad (5.34)$$

is expected for experiments at the CERN SPS, whilst in pp collisions at ISR energies, R increases to 10 [115,146]. The predominance of χ and η_c over J/ψ can be understood in terms of the gluon fusion model [115] which was discussed earlier in section 4.3.3, where it is found that the χ and η_c states couple to two gluons whilst the J/ψ couples to three gluons and hence is relatively suppressed. This is especially the case for higher energies.

The cross-section for J/ψ absorption is taken directly from the photo-production results where a total cross-section of 1.3 mb [147] is found together with the elastic cross-section which is considerably smaller; thus the absorption cross-section is approximately the same as the total cross-section. This also implies that the $c\bar{c}$ resonances do not scatter much and hence do not thermalise with the surrounding hadron gas. No allowance is made in finding these cross-sections for a finite formation time of J/ψ resonances, discussed by Ftáčnik et al. [138] as mentioned earlier; if this is included the cross-section may be larger. The two cases are treated as follows

1. For a hadron gas composed of only pions, the absorption cross-section is dominated by

$$J/\psi + \pi \rightarrow D\bar{D} + X$$

which is estimated to be in the range $\sigma_I = 1 - 3$ mb, for energies above some threshold required for the reaction to proceed.

2. The second case where the hadron gas consists of ρ mesons as well, the threshold for pions will not be as important, whilst the inclusion of χ and η_c states allows for the regeneration of J/ψ . The following reactions are considered, where M stands for either π or ρ resonances, and C stands for either χ or η_c :

- The removal of the charm resonances via

$$M + \begin{pmatrix} J/\psi \\ C \end{pmatrix} \rightarrow D\bar{D}$$

- The redistribution of the C resonances

$$M + C \rightarrow J/\psi + M'$$

The cross-sections for the above processes are assumed to be similar. The second process will rearrange the distribution of χ , η and J/ψ to tend towards some equilibrium. The first process essentially removes J/ψ via four possible reactions, hence the cross-section will be $\frac{1}{4}$ the value arising in the first case above. Vogt et al. [137] assume a drop by approximately 50% in the cross-section in going from high energies to the threshold limit. Hence a value for the cross-section of $\sigma_{II} = \frac{1}{8}\sigma_I$ is chosen.

The first case is now almost identical to the treatment given by Gavin et al [136] discussed above. The space-time evolution of J/ψ is given by

$$\frac{dN}{dx^4} = -\frac{\langle\sigma v\rangle}{V^2} N_\pi N \quad (5.35)$$

where V is the interaction volume and v is the relative velocity between the J/ψ and π mesons. The mean interaction rate $\langle\sigma v\rangle$ is determined by averaging over the momentum of π and over energies of collision, with the condition that the collision energy is above the threshold energy, to give the equation

$$\langle\sigma v\rangle = \frac{1}{2E_0} \int_{S_0}^{\infty} dS \sigma(S) \lambda^{1/2}(S) \int \frac{d^3p}{2E_\pi} f_\pi(p_\pi) \delta[S - (p_0 + p_\pi)^2]$$

where p_0 and E_0 are the four-momenta and energy of the J/ψ respectively, λ is the incoming flux factor and f_π is the Boltzmann distribution function normalised to one. The threshold for the absorption of J/ψ by pions is $\sqrt{S_0} = 2m_D$, with

$$\sigma(S) = \sigma_I \theta(S - S_0)$$

The temperature dependence of $\langle\sigma v\rangle$ is not found to be very strong in the range 140 – 200 MeV so that it is averaged out. Two subcases are considered, where first the dependence of $\langle\sigma v\rangle$ on J/ψ momentum is averaged out, whilst in the second it is kept. Introducing the time dependence of N_π from the Bjorken model via eqn(5.23), allows the integration of eqn(5.35) for N to be done yielding the equation

$$\mathfrak{R}_\psi(p_T) = \frac{N(t_f)}{N(t_i)} = \left(\frac{t_i}{t_f} \right)^\beta$$

which is the same as eqn(5.33), with a similar corresponding equation for β . The parameters t_i and t_f refer to the times when absorption starts and stops, in the rest frame of the gas. Since the time when absorption starts is dependent on the J/ψ formation time τ_ψ (measured in the frame of the J/ψ) the dilated formation time

$$t_i = \gamma \tau_\psi$$

at which absorption begins depends on p_T , the J/ψ transverse momentum. A J/ψ formation time, of the order of the time that the hadron gas forms of 1 fm is used in parallel to [136],

with t_f determined by the gas lifetime of $t_1 = 5$ fm. The major difference is in the inclusion of an initial distribution of J/ψ determined by the density of colliding nucleons given by

$$N(r, t_0) = N_0 \left(1 - \left(\frac{r}{R} \right)^2 \right)^{1/2}$$

where R is the radius of the smaller colliding nucleus. The averaging over initial positions is then done, with the resulting plot shown in fig(5.11a). A further difference is that in

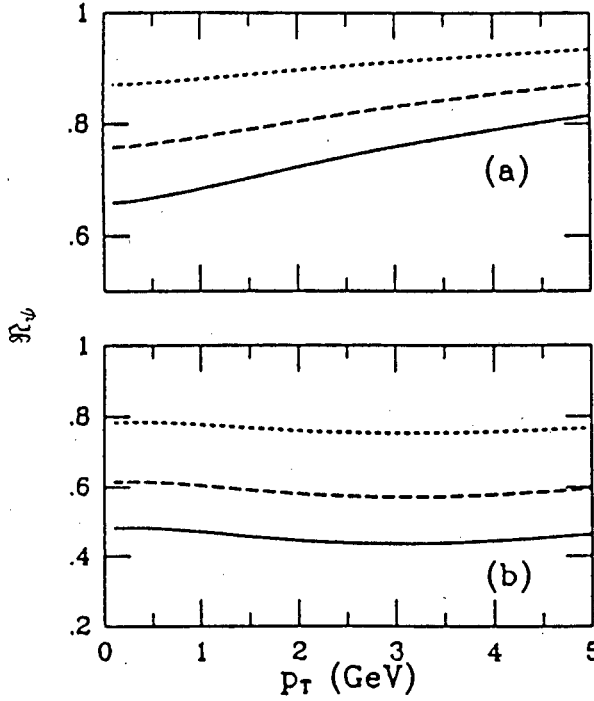


Figure 5.11: J/ψ suppression versus p_T from model I. In (a) the rates used are at an average p_T . The solid, long-dashed and short-dashed curves correspond to $\langle\sigma_I v\rangle = 0.6, 0.4$ and 0.2 mb respectively. In (b) the full p_T dependence of the rates is retained. The curve designations are the same as in (a) with $\sigma_I = 3, 2$ and 1 mb respectively [75].

the second subcase no averaging is done for $\langle\sigma v\rangle$ over J/ψ momenta. The second subcase is displayed in fig(5.11b) for varying values of σ_I . The inclusion of momentum dependence is seen to dampen the overall p_T dependence significantly. Variation in the gas lifetime and J/ψ formation time has also been studied, but whilst these can change the overall magnitude of the absorption, no change is observed in the shape of the p_T dependence.

The second case introduces the possibility of J/ψ regeneration. The mixture of ρ to π is an important input. The ratio r defined by

$$r_\rho = N_\rho/N_\pi$$

is assumed to vary between $r_\rho^{-1} = 6.6$ for $T = 140$ MeV and 2.2 for $T = 200$ MeV. Defining $N_M = (1+r_\rho)N_\pi$ as the average meson density, two coupled equations describing the evolution of J/ψ and χ arise

$$\begin{aligned} \frac{dN}{dx^4} &= -\frac{\langle\sigma v\rangle}{V^2} N_M [(1+d)N - N_\chi] \\ \frac{dN_\chi}{dx^4} &= -\frac{\langle\sigma v\rangle}{V^2} N_M (-dN + 2N_\chi) \end{aligned}$$

where N_χ refers to the number of non- J/ψ $c\bar{c}$ resonances and d is the number of effective such states ($d = 4$ corresponds to η_c and three of the χ states). The equations have been

integrated to give

$$\mathfrak{R}_\psi = \frac{1+R}{1+d} \left(\frac{t_i}{t_f} \right)^{\beta_{II}} + \frac{d-R}{1+d} \left(\frac{t_i}{t_f} \right)^{(2+d)\beta_{II}}$$

$$\mathfrak{R}_\chi = \frac{d}{R} \frac{1+R}{1+d} \left(\frac{t_i}{t_f} \right)^{\beta_{II}} - \frac{1}{R} \frac{d-R}{1+d} \left(\frac{t_i}{t_f} \right)^{(2+d)\beta_{II}}$$

where R is the ratio given in eqn(5.34). If ($R = d$) then the solution is the same as the first case, but with the smaller cross-section σ_{II} . A mismatch between R and d then allows either the regeneration or further depletion of J/ψ to occur, though the dominant feature is still absorption of $c\bar{c}$ resonances by the hadronic gas. The resulting plot showing the effect of varying R and d is displayed in fig(5.12). Significant suppression is still found, but

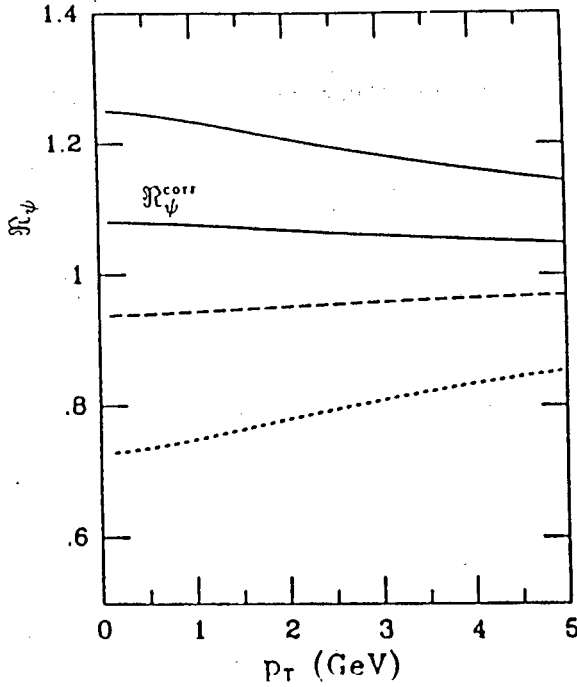


Figure 5.12: J/ψ abundance versus p_T from model II, using $\sigma_{II} = 1/8\sigma_I$ with $\sigma_I = 1$ mb. The solid lines are results for $R = 10$, $d = 4$ and show J/ψ regeneration. The curve marked $\mathfrak{R}_\psi^{corr}$ includes the radiative decays of χ for $R = 10$ and $d = 4$. The long-dashed are for $R = d = 4$ and the short-dashed lines for $R = 0, d = 4$; here J/ψ suppression occurs [137].

insufficient p_T dependence to account for the observed NA38 data. No attempt is made to fit this model to the observed E_T and p_T dependence of the NA38 data. A comment is made that the inclusion of electromagnetic radiative decays of χ to J/ψ can be made, but this is not observed to greatly change the overall p_T dependence of hadronic gas absorption.

5.3.4 Blaizot Ollitrault Hadronic Gas Absorption Model

A detailed model illustrating the E_T and p_T dependence of J/ψ suppression arising from hadronic gas absorption of J/ψ is discussed in [95] with specific disregard to nuclear effects such as nuclear absorption in order to highlight hadronic gas absorption effects. A comparison is also made between the absorption model and predictions arising from plasma suppression models. In order to simplify the model, the hadron gas is assumed to consist of only pions. The presence of other $c\bar{c}$ resonances is not discussed in this model, in particular with respect to the regeneration of J/ψ mentioned in the previous model. The change in cross-section of

$c\bar{c}$ pairs as they evolve to J/ψ is not considered, instead a simplification is made where the cross-section remains unchanged as $c\bar{c}$ pairs evolve to J/ψ , thus the absorption cross-section will just be the total cross-section found from the photo-production experiments.

Initially a simplified picture is considered, where a central collision occurs between two identical mass nuclei. There exists a great deal of ambiguity as to exactly when the collision starts, especially with regard to the formation of a hadron gas, because the nuclei have finite thickness and hence take a finite time to cross each other. The formation of a $c\bar{c}$ is a hard process with a time of the order of $\sim 1/2m_c$ which will most likely occur in the initial collision. In [95] the initial collision is said to start when the nuclei have maximum overlap, the creation of $c\bar{c}$ pairs is said to also take place at this time. The model considers as a first approach only J/ψ with zero longitudinal momentum along the beam axis, formed in the $z = 0$ slice perpendicular to the collision axis. It is recognised that J/ψ have a finite formation time τ_ψ , but that $c\bar{c}$ pairs can be absorbed even before forming J/ψ . This is made from the observation that in the experiments [128] a finite cross-section is still measured despite the fact that J/ψ formation might take place outside the nucleus; thus in [95] the case is considered where absorption starts immediately $c\bar{c}$ pairs form, which neglects any change in cross-section as $c\bar{c}$ pairs evolve with time. The elastic channel is small, as was discussed in previous models, thus the absorption cross-section is simply given by the total cross-section. Coherent effects arising in the cross-section are ignored in this first approach.

The time t_0 is the time when products produced in the initial collision begin to form the “central region” discussed in the Bjorken model [16]. It is even possible at this stage that these products have not as yet formed hadrons and some intermediate absorption occurs before a hadron gas forms. In this model Blaizot and Ollitrault consider the simplified case where hadrons have formed at t_0 and in fact sufficient time has passed for thermalisation of the hadronic gas to have occurred. The parameter t_0 has a natural lower limit of half the crossing time of the nuclei, if the initial collision occurs when the nuclei have maximum overlap.

The distribution for both $c\bar{c}$ pairs in an intermediate state and J/ψ is given by $f(\vec{r}, \vec{p}_T, t)$ where $\vec{r}$ refers to the transverse dimension only; similarly $f_\pi(\vec{r}, \vec{p}_T, t)$ is the distribution of pions. The distribution functions satisfy the kinetic equation

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f = -f \int \frac{d^3 p'}{(2\pi)^3} f_\pi(\vec{r}, \vec{p}', t) \sigma_{\pi\psi} v_{\text{rel}} \frac{p \cdot p'}{EE'} \quad (5.36)$$

with v_{rel} the relative velocity defined as

$$v_{\text{rel}} = \left[1 - \left(\frac{m_\pi M_\psi}{p \cdot p'} \right)^2 \right]^{\frac{1}{2}}$$

where the left-hand side of eqn(5.36) is just the time derivative of f along the J/ψ trajectory which is, as discussed earlier, a straightline. The J/ψ , π have four momentum p, p' and energy E, E' respectively; $\vec{v}$ is the velocity of the J/ψ and $\sigma_{\pi\psi}$ is the invariant total cross-section for $\pi - \psi$ scattering. No J/ψ generation term is present in eqn(5.36) as it is assumed that J/ψ creation occurs only in the initial collision; the possibility of J/ψ regeneration discussed in the previous model [137] by Vogt et al. is not included in this model.

If $f_0 = f(t = 0)$ is the J/ψ initial distribution, then eqn(5.36) has a solution

$$f(\vec{r}, \vec{p}_T, t) = f_0(\vec{r} - \vec{v}t, \vec{p}_T) \exp \left\{ - \int_0^t dt' \int \frac{d^3 p'}{(2\pi)^3} f_\pi(\vec{r} - \vec{v}(t - t'), \vec{p}', t') \sigma_{\pi\psi} v_{\text{rel}} \frac{p \cdot p'}{EE'} \right\} \quad (5.37)$$

The probability of survival $N(\vec{p}_T)$ is then given by averaging f over all initial positions of J/ψ in the $z = 0$ slice,

$$N(\vec{p}_T) = \frac{\int d^2r f(\vec{r}, \vec{p}_T, t = +\infty)}{\int d^2r f_0(\vec{r}, \vec{p}_T)} \quad (5.38)$$

The assumption is made that $f_0(\vec{r}, \vec{p}_T)$ factorises into a spacial and momentum dependence given by $f_0(\vec{r})g(\vec{p}_T)$. The pionic distribution is assumed to quickly reach local thermal equilibrium, and its expansion is assumed to be well described by the hydrodynamics of an ideal fluid, where $u^\mu(\vec{r}, t)$ is the fluid four-velocity and $T(\vec{r}, t)$ the local temperature. The pion distribution is then simply the Bose-Einstein distribution for non-interacting particles

$$f_\pi(\vec{r}, \vec{p}_T, t) = \frac{3}{\exp\{p'_\mu u^\mu/T\} - 1}$$

where the factor 3 is the degeneracy factor for pions. In the case that the pions are assumed massless eqn(5.36) simplifies, particularly the relative velocity which is just $v_{\text{rel}} = 1$, to yield

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f = -f \sigma_{\pi\psi} n (1 - \vec{v}_f \cdot \vec{v}) \quad (5.39)$$

where n is the fluid density

$$n(\vec{r}, t) = \int \frac{d^3p'}{(2\pi)^3} f_\pi(\vec{r}, \vec{p}', t) \quad (5.40)$$

and $\vec{v}_f$ is the fluid velocity

$$\vec{v}_f(\vec{r}, t) = \frac{1}{n(\vec{r}, t)} \int \frac{d^3p'}{(2\pi)^3} f_\pi(\vec{r}, \vec{p}', t) \frac{\vec{p}'}{E'} \quad (5.41)$$

The cross-section has assumed to be constant in deriving eqn(5.39) which corresponds to ignoring the energy threshold required for J/ψ absorption via the reaction

$$\pi + J/\psi \rightarrow \pi + D\bar{D}$$

to occur. The effect of including an energy threshold is considered in the discussion of a static pion gas below. The hydrodynamical model developed in [16,19], which is discussed in the chapter on hydrodynamics, is used in this model. In order to gain some understanding of the various parameters of the model, a three step approach is taken. The first step is to consider a static isotropic hadron gas, then later to add in the longitudinal expansion, and finally deal with the transverse expansion and finite size effects.

Static pion gas

The isotropic nature of the hadron gas which has no fluid flow simplifies the hadron distribution function to

$$f_\pi(E) = \frac{3}{\exp(E/T) - 1}$$

Then eqn(5.37) becomes

$$f(\vec{r}, \vec{p}_T, t) = f_0(\vec{r} - \vec{v}t, \vec{p}_T) e^{-t/\theta}$$

where $\vec{v}$ is the velocity of the J/ψ relative to the frame in which the pion gas is static and $1/\theta = \nu$ is the collision frequency given by

$$\frac{1}{\theta} = \int \frac{d^3p}{(2\pi)^3} f_\pi(E') \sigma_{\pi\psi} v_{\text{rel}} \frac{p \cdot p'}{EE'} \quad (5.42)$$

The integral in eqn(5.42) has been evaluated in [95], with the result displayed in fig(5.13) showing the p_T dependence and temperature dependence of θ for two particular cases. The

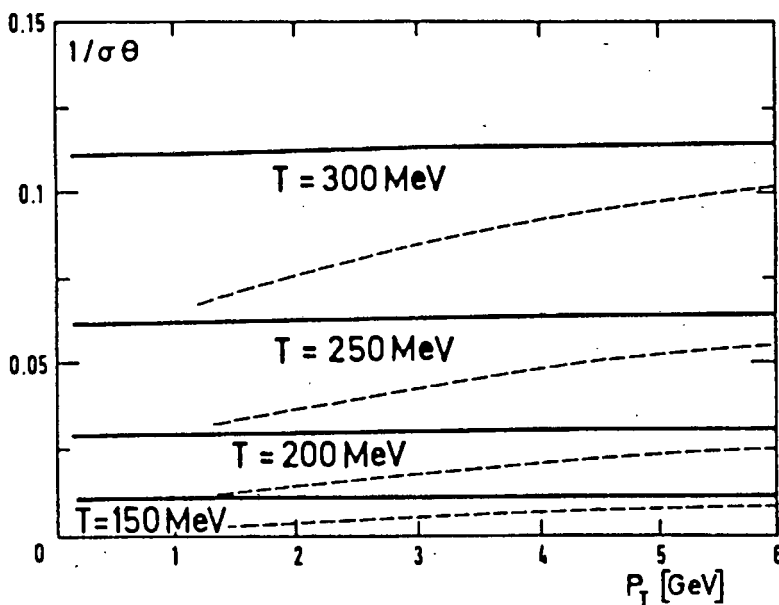


Figure 5.13: The quantity $1/\sigma\theta$ as a function of momentum p_T of J/ψ , for a pion gas in thermal equilibrium. σ is the absorption cross-section $\sigma_{\pi\psi}$ in mb, θ is the collision time given by eqn(5.42) in fm. The solid line corresponds to a constant cross-section, whilst the dashed line includes the effect of the threshold for the reaction $\psi\pi \rightarrow D\bar{D}$ [95].

first, corresponding to the solid lines is where the threshold for the reaction of J/ψ with pions is ignored; the p_T dependence is observed to be somewhat flat. In the second case, corresponding to the dashed lines, a threshold of 498 MeV for the reaction $\psi + \pi \rightarrow D\bar{D}$ has been used, which results in a stronger p_T dependence. It should be further noticed that the shape of the p_T dependence is relatively independent of the temperature of the hadronic gas, but the magnitude is quite temperature dependent. The large p_T limit for J/ψ absorption in the case of a reaction threshold is observed in fig(5.13) to be given by the J/ψ absorption curve when the threshold is absent. The effect of using a massless pion gas approximation has also been considered. This is perhaps most easily done for the $p_T = 0$ limit, where a simplification of θ arises to yield

$$\frac{1}{\theta} = \sigma_{\pi\psi} n(T) \langle v \rangle_T$$

where $n(T)$ is defined by eqn(5.40) and $\langle v \rangle_T$ is the average thermal velocity of the pion. Typical values for the average thermal velocity are for $T = 100(300)$ MeV $\langle v \rangle_T = 0.862(0.968)$,

thus the approximation is not too bad, whilst for J/ψ with non-zero p_T the approximation is expected to be even better, especially in comparison to the threshold effect. The case of J/ψ with infinite p_T , where $v_{\text{rel}} = 1$, has been solved by [95] and is found to yield

$$\frac{1}{\theta} = \sigma_{\pi\psi} n(T) \quad (5.43)$$

thus it is possible to attribute the slow p_T dependence observed for the case with no reaction threshold, to non-zero mass effects of the pion, which would disappear in the zero-mass limit. If heavier resonances such as ρ are introduced, the massless approximation will not be as good and the mass-dependence should then be kept, whilst the reaction threshold becomes less important and may perhaps be ignored.

Expanding pion gas

The next step is to include the longitudinal expansion of the gas, which has the effect of introducing cooling, whilst at the same time maintaining the isotropic distribution in the transverse plane. The model considers only the $y = 0$ slice of rapidity. The time dependence of the temperature is brought in via the entropy conservation equation given by eqn(2.28) valid for an ideal gas which remains unchanged under small shifts of rapidity y thus

$$s(T)t = s_0 t_0$$

determines the temperature as a function of time, for the $y = 0$ slice. Substitution can then be made for time in terms of temperature, so that the evolution of the system is now described in terms of a decreasing temperature instead of the time, which is now implicit. This is completely different to the approach used by Ftáčnik et al. [138] who use a time averaged temperature, and prefer to maintain the explicit time dependence. The result given in eqn(4.3) for massless bosons links the entropy density to the number density thus

$$s = \frac{4}{3} \frac{\pi^4}{30\zeta(3)} n \approx 3.6n$$

Then the time integration in eqn(5.37) becomes

$$\ln \left[\frac{N(T)}{N(T_0)} \right] = -s_0 t_0 \int_{T_0}^T d \left[\frac{1}{S(T)} \right] \frac{1}{\theta(T)}$$

where $N(T)$, the fraction of J/ψ which survive, when the temperature is at T , is proportional to f ; with θ defined in eqn(5.42). The cross-section with corresponding threshold is the same as that used earlier for the static pion gas. In fig(5.14a) the temperature dependence of N is displayed for a few values of p_T , where T_0 is fixed at 300 MeV, and the dotted line corresponds to the infinite p_T limit. Quite a strong shift is seen in the temperature dependence for p_T in the range 1 – 6 GeV, approaching the infinite p_T limit for $p_T = 6$ GeV. The p_T dependence is attributed to the presence of the reaction threshold, where J/ψ with low p_T are more likely to be below the threshold for absorption, and hence do not get absorbed as easily. A simplified form for the infinite p_T limit can be found, where θ is simply given by eqn(5.43), thus

$$\ln \left[\frac{N(T)}{N(T_0)} \right] = 3\sigma_{\pi\psi} n(T_0) t_0 \ln \left[\frac{T}{T_0} \right] \quad (5.44)$$

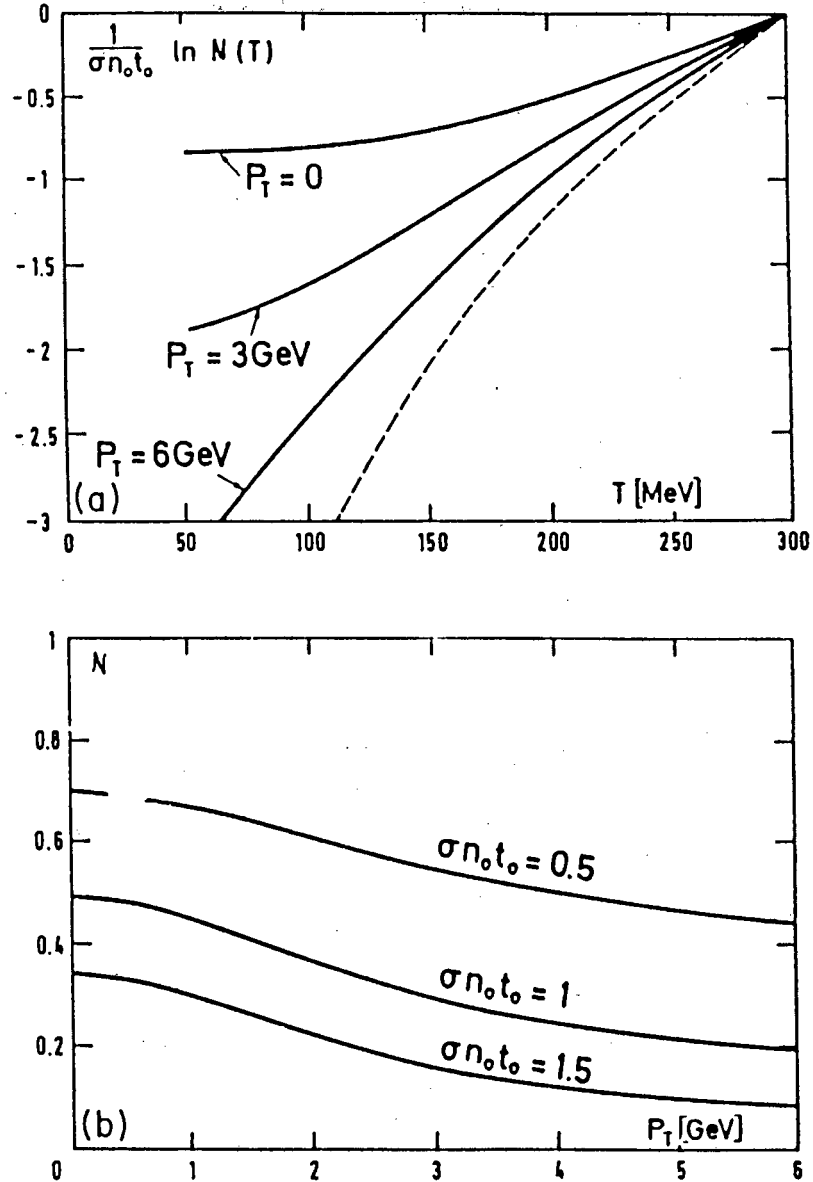


Figure 5.14: In (a) the quantity $(1/\sigma n_0 t_0) \ln N(T)$ is shown as a function of the temperature T where $\sigma = \sigma_{\pi\psi}$. $N(T)$ is chosen to be 1 when $T = 300 \text{ MeV}$. The dashed curve corresponds to the zero mass pion limit (see eqn(5.44)). In (b) the quantity $N(T = 100 \text{ MeV})$ is plotted as a function of p_T of J/ψ for several values of the dimensionless parameter $\sigma n_0 t_0$. As in (a), $N(T = 300 \text{ MeV}) = 1$ [95].

In fig(5.14b) the p_T dependence of the survival probability N is plotted for a constant temperature but with varying values of $\sigma_{\pi\psi} n_0 t_0$. The decrease of N with p_T is again apparent; for increasing $\sigma_{\pi\psi} n_0 t_0$, which corresponds essentially to starting the absorption at earlier times or with a denser pion gas with J/ψ having a larger cross-section, results in a decrease

in the J/ψ survival probability, approximately decreasing logarithmically.

It is stated in [95] that at this stage the resulting p_T dependence is apparently opposite to the observed NA38 data [65]. This may be premature as it is the ratio of slopes in fig(5.14b) which is relevant to the NA38 data. The reaction threshold is extremely ambiguous in this model, as collisions involving states which have not quite formed hadron resonances are possible initially. Thus Blaizot and Ollitrault in [95] drop the dependence on reaction threshold and assume the pions are massless. In the current model it should be noted that there is no cut-off time when the hadron gas density becomes sufficiently small that further significant J/ψ absorption by the gas requires unrealistic times. The current model also does not include the possibility that J/ψ with high p_T escape out of the hadron gas; the finite size effects of the hadron gas should be examined.

Finite-size effects and transverse expansion

A model is examined where the cross-section $\sigma_{\pi\psi}$ is constant (no reaction threshold) in the massless pion limit. The J/ψ distribution is thus simplified to the conditions required for eqn(5.39) whose solution is given by

$$f(\vec{r}, \vec{p}_T, t) = f_0(\vec{r} - \vec{v}t, \vec{p}_T) \exp \left\{ -\sigma_{\pi\psi} \int_0^t n(\vec{x}', t') [1 - \vec{v} \cdot \vec{v}_f(\vec{x}, t')] dt' \right\} \quad (5.45)$$

where $\vec{x} = \vec{r} - \vec{v}(t - t')$. It should be noticed that the $\vec{p}_T$ dependence appears both in the geometrical translation $\vec{v}(t - t')$ and in the relativistic factor $(1 - \vec{v} \cdot \vec{v}_f)$.

An assumption that the absorption is small is made to simplify the integration. This is not too realistic however, as the NA38 data suggest up to 50% absorption of J/ψ can occur, but a remedy will be introduced later. The linearisation of eqn(5.45) results in the simplified equation for the probability of survival eqn(5.38) which becomes

$$\mathcal{A}_{\text{lin}}(p_T) = 1 - N_{\text{lin}}(\vec{p}_T) = \frac{\sigma_{\pi\psi} \int d^2r f_0(\vec{r}, \vec{p}_T) \int_0^\infty dt n(\vec{x}, t) [1 - \vec{v} \cdot \vec{v}_f(\vec{x}, t)]}{\int d^2r f_0(\vec{r}, \vec{p}_T)} \quad (5.46)$$

where $\vec{x} = \vec{r} + \vec{v}t$. For central collisions, cylindrical symmetry about the beam axis is present for the hydrodynamical flow, and since the co-ordinate positions of J/ψ are averaged over, $N_{\text{lin}}(\vec{p}_T)$ depends only on the magnitude of p_T . A radial distribution based on similar arguments used to derive eqn(3.8) in terms of the number of colliding nucleons at radius r for the initial distribution of J/ψ is given by

$$f_0(r, p_T) = \frac{b+1}{\pi R_0^2} \left[1 - \left(\frac{r}{R_0} \right)^2 \right]^b g(p_T)$$

where b is a free parameter with values $b = \frac{1}{2}$ to 1 determined by the collision geometry. A more complete motivation for this is given in the appendix of [95]. A similar distribution is chosen for the initial pion gas density

$$n(r, t_0) = (a+1)n_0 \left[1 - \left(\frac{r}{R_0} \right)^2 \right]^a$$

with no initial fluid flow $v_f(r, t_0) = 0$ and where R_0 in central collisions is just the nuclear radius.

It is observed in [95] by scale changes in T_0 , and later equivalent scale changes in R_0 and t_0 , that N in eqn(5.46) depends linearly on the product $\sigma_{\pi\psi} n_0 t_0$ and on some function of the ratio of t_0 to R_0 and p_T . The numeric solution to eqn(5.46) has been found in [95] and is displayed in fig(5.15) as a function of p_T for various values of t_0/R_0 . The function drops off for

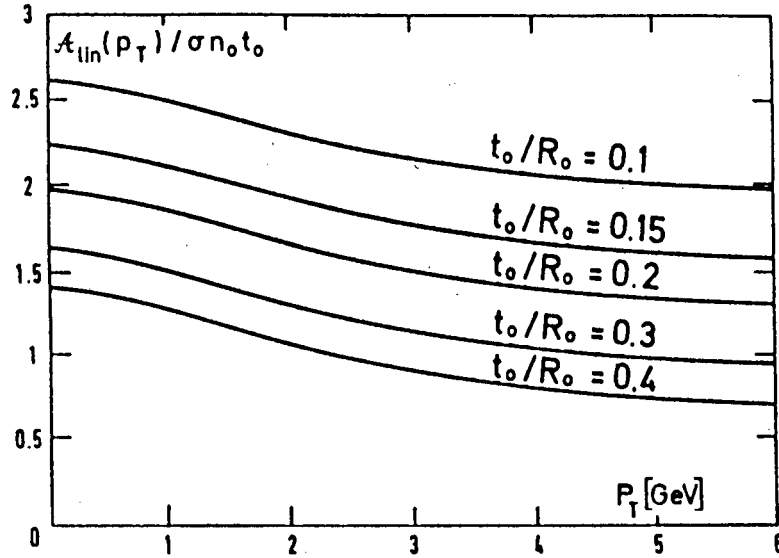


Figure 5.15: The Absorption factor in the linear approximation $\mathcal{A}_{\text{lin}}(p_T)$ in units of the dimensionless parameter $\sigma n_0 t_0$ as a function of J/ψ momentum p_T , and for various values of t_0/R_0 . t_0 is the time at which matter appears in the central region and starts to expand; it is also the time at which absorption starts [95].

p_T below 3 GeV tending towards some asymptotic limit, thus suppression occurs for all values of p_T . The curves are found to be approximately parallel, with a logarithmic dependence on t_0/R_0 . This is motivated by the observation that as hydrodynamic flows are similar for different $t_0 \ll R_0$ provided $n_0 t_0$ is held fixed, a different value of t_0 corresponds to starting the model off at an earlier time with a higher initial density of pions. More time is thus available for J/ψ absorption starting at a smaller t_0 and hence a smaller survival probability is found. This effect can be found, by considering the extra absorption that occurs for a smaller t_0 curve and some larger t'_0 curve. As this absorption occurs at an early stage, the fluid flow is negligible and as the time interval can be assumed small, the J/ψ will not travel far removing the need to examine finite size effects. The difference is then simply given in terms of a model including only longitudinal expansion. The integration over time from t_0 to t'_0 has been carried out with use of the relation

$$t = \frac{t_0 n_0}{n(r, t)}$$

to yield

$$\mathcal{A}_{\text{lin}} = \sigma_{\pi\psi} \bar{n}_0 t_0 \ln \frac{t'_0}{t_0} \quad (5.47)$$

which has a logarithmic dependence, where $\bar{n}$, the mean value for the pion density, is given by

$$\bar{n}_0 = \frac{\int d^2r f_0(\vec{r}) n(\vec{r}, t_0)}{\int d^2r f_0(\vec{r})} \quad (5.48)$$

It can thus be concluded from the logarithmic dependence in eqn(5.47) that the strength of the p_T dependence is not very sensitive to the early stage of absorption, but rather to the late stage where finite size effects and transverse expansion are important.

To remedy the error arising in the linearisation of eqn(5.45), the exponential of $\mathcal{A}_{\text{lin}}$ is taken. This amounts to performing an average of the argument of the exponential instead of an average of the exponential. The suppression arising from this approximation will then be an underestimate of the actual suppression. In [95] the validity of this approximation has been checked for the $p_T = 0$ case and is found to be good within a few percent error. A similar procedure has been carried out by Blaizot and Ollitrault for the model describing nuclear absorption of J/ψ [125].

Use is now made of the approximation derived in eqn(5.47) for the early time absorption

$$\mathcal{A}'_{\text{lin}}(p_T) = \sigma_{\pi\psi} \bar{n}_0 \ln \frac{t_f}{t_0} \quad (5.49)$$

but where t_f is to be found when $\mathcal{A}'_{\text{lin}}$ has the same value as the exact solution for the weak absorption case. A plot of t_f/R_0 against t_0/R_0 is displayed in fig(5.16) for various values of

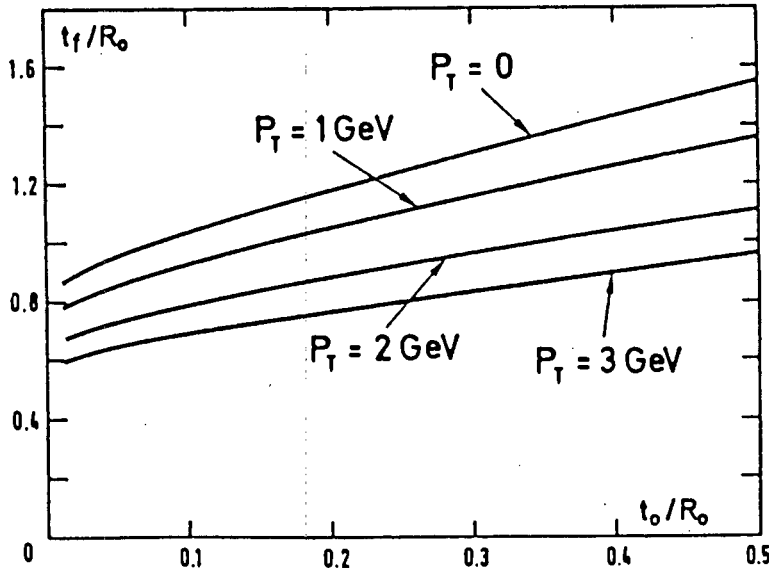


Figure 5.16: The freeze-out time t_f for various values of the J/ψ transverse momentum as a function of the initial time t_0 at which matter is formed in the central region and absorption starts (see eqn(5.50)) [95].

p_T . Exponentiation of eqn(5.49) then leads to

$$N(p_T) = \left[\frac{t_0}{t_f} \right]^{\sigma_{\pi\psi} \bar{n}_0 t_0} \quad (5.50)$$

where t_f has the same p_T dependence as the weak absorption case above.

In order to compare the model with the NA38 data, it is necessary to consider non-central collisions of unlike ions, and relate the resulting measured transverse energy flow to the impact parameter b of the colliding ions. A model for the transverse energy flow dependence on the number of participating nucleons is introduced where the transverse energy and multiplicity density in the central region are taken as proportional to the average number of participating nucleons [98]. The relation between the number of participating nucleons and impact parameter has been found in [95] in terms of a simple geometric argument. The differential J/ψ cross-section is given by

$$\frac{d\sigma_{AB \rightarrow \psi}}{dE_T} = \sigma_{NN \rightarrow \psi} T_{AB}(b) N(b) \frac{2\pi b db}{dE_T}$$

where $N(b)$ is the J/ψ survival probability for a general collision at impact parameter b and $T_{AB}(b)$ is proportional to the total number of nucleon-nucleon collisions. The ratio corresponding to the NA38 data is then given by

$$\mathcal{R}_\psi = \frac{\int_{E_2}^{E'_2} \frac{d\sigma_\psi}{dE_T} dE_T}{\int_{E_1}^{E'_1} \frac{d\sigma_\psi}{dE_T} dE_T}$$

where E'_2, E_2 refer to the high transverse energy bin, and E'_1, E_1 to the low rapidity bin.

Use is made of the eqn(5.50) to find N for an initial time t_0 estimated as half the time for the nuclei to cross each other given by

$$t_0 = \frac{1}{2\gamma_{c.m.}} [(R_A + R_B)^2 - b^2]^{\frac{1}{2}}$$

where $\gamma_{c.m.}$ is the gamma factor of the nucleons in the centre of mass frame of the collision. An estimate for R_0 is given by the smaller nuclear radius of the colliding ions. The parameter t_f which contains the finite size effects must be estimated; two possible situations are examined: the first corresponds to nearly central collisions with high E_T , for a hadron gas lifetime of the order of the smaller ion radius, where the central collision value is taken; whilst the second case refers to grazing collisions where the hadron gas life-time is of the order

$$t = \frac{(R_A + R_B - b)}{2}$$

where t_f is taken to be given by

$$t_f = t_{fc} \frac{(R_A + R_B - b)}{2r_B}$$

where B is chosen to be the smaller ion, and t_{fc} is the central collision time. The average pion density $\bar{n}$ given by eqn(5.48) has been found in [95] by considering simple geometric arguments in terms of the number of colliding nucleons as a function of radius and impact parameter.

The p_T dependence of N is displayed in fig(5.17) for two impact parameters $b = 0, 6$ fm. Insufficient p_T dependence is observed to account for the NA38 data.

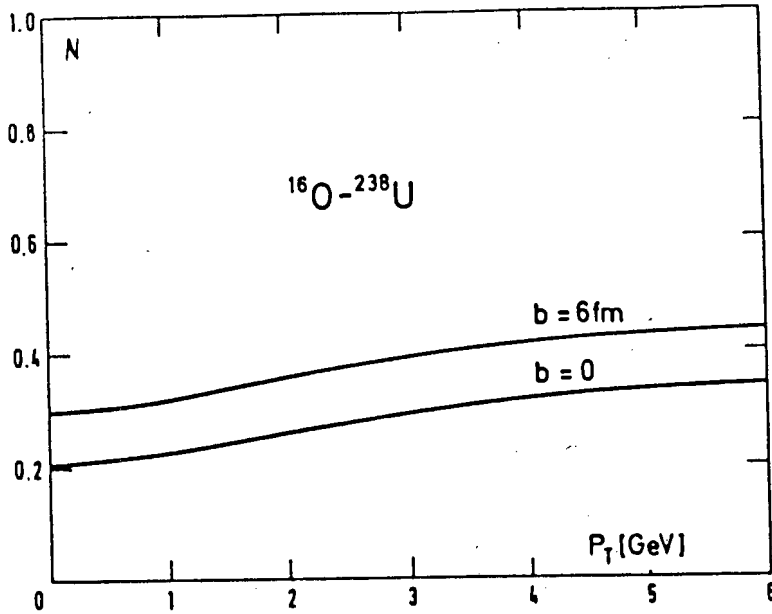


Figure 5.17: The p_T dependence of J/ψ absorption by a pion gas, in the case of oxygen-uranium collisions, for two values of the impact parameter: $b = 0$ for a central collision with correspondingly maximum suppression; $b = 6$ fm is a peripheral collision. $\sigma_{\pi\psi} = 1.5$ mb [95].

5.3.5 Conclusions of the Hadronic Gas Models

The absorption of J/ψ by hadronic gas cannot alone account for the observed NA38 data. The main defect is the inability to reproduce the strength of the observed p_T dependence. A remedy has been proposed to counter this problem: the solution lies in adding another source of p_T dependence to either the hadronic or nuclear absorption pictures, namely the inclusion of initial state effects. A discussion of these effects follows below. Some comment should be made on the strength of hadronic absorption, and especially on the E_T dependence arising. Hadronic absorption has sufficient strength to account for the observed data, particularly the E_T dependence and even the shape of the E_T dependence as for instance in the model by Ftáčnik et al. [138] displayed in fig(5.8). However, to produce such strong absorption requires the existence of an extremely dense medium of approximately 20 times the ordinary nuclear density, whose very nature is rather ambiguous, as this corresponds to the case where several mesons must be squeezed in to a single meson volume.

5.4 Initial State Interactions

First a review of hadron-nucleus collisions is made to determine the effect of initial state interactions on the transverse momentum distribution of particles arising from the collision

5.4.1 Review for Hadron-Nucleus Collisions

For hadron-hadron collisions producing Drell-Yan dileptons, the transverse momentum (p_T) of the dileptons comes almost entirely from the initial state fermi motion of the partons [148].

Thus the average transverse momentum $\langle p_T^2 \rangle$ of the dileptons is expected to be of hadronic magnitude, and independent of the incident energy of the colliding hadrons.

In hadron-nucleus collisions it is also possible for partons in the projectile hadron to collide elastically with partons of the target nucleus before colliding inelastically to produce a dilepton pair. Each elastic collision can increase the p_T of the parton, with the cumulative effect being displayed in a higher $\langle p_T^2 \rangle$ observed for the dilepton production. The Drell-Yan production mechanism is particularly suited for the observation of this p_T generation, due to the expected lack of final state interactions of the dileptons.

The NA10 group [149] have produced results for pion beams on deuterium (D) and tungsten (W) targets at 140 and 280 GeV. The p_T dependence can be observed in the ratio

$$R(p_T) = \frac{d\sigma_{\pi W \rightarrow \mu^+ \mu^-}}{d\sigma_{\pi D \rightarrow \mu^+ \mu^-}}$$

This ratio is displayed in fig(5.18) where, for p_T greater than about 2 GeV, an increasing value for R is evident. The steeper increase at the lower energy is due to the p_T range at 2 GeV $\ll$ 140 GeV being much lower than for the higher energy at 2 GeV $\ll$ 286 GeV [149].

An estimate of the change in the average p_T^2

$$\langle \Delta p_T^2 \rangle \equiv \langle p_T^2 \rangle_W - \langle p_T^2 \rangle_D$$

yields

$$\langle \Delta p_T^2 \rangle = \begin{cases} 0.15 \pm 0.06 \text{ GeV}^2 & \text{at 286 GeV} \\ 0.16 \pm 0.06 \text{ GeV}^2 & \text{at 140 GeV} \end{cases} \quad (5.51)$$

The effect of scattering off other nuclei is found to be small, hence it is inferred that the increase in the observed p_T is due to the elastic scattering of the partons inside the nucleus in which the inelastic Drell-Yan production takes place. This is supported by the fact that no dependence on longitudinal momentum or dimuon mass is observed for $\langle p_T^2 \rangle_W - \langle p_T^2 \rangle_D$, see fig(5.19).

The integrated cross-section over p_T , $\sigma_{pA} = \int d^2 p_T \sigma(p_T)$ still scales as $\sigma_{pA} = A^\alpha \sigma_{pp}$ with $\alpha \sim 1$ (volume effect).

Observations for the p_T dependence of J/ψ production from the same experiment were also made, producing a value

$$\langle p_T^2 \rangle_{\psi, W} - \langle p_T^2 \rangle_{\psi, D} = 0.29 \pm 0.02 \text{ GeV}^2 \quad (5.52)$$

in agreement with the NA3 result [128].

The NA3 group has performed a similar experiment but with $\bar{p}$ and π^- beams on proton and platinum targets, and measured the p_T distribution of the J/ψ produced in the collision. The p_T distribution is expected to be influenced by final state interactions as the J/ψ moved through the nucleus in which it is formed – these interactions are predicted to be of the same order as those of the initial interactions observed for the Drell-Yan measurements, this follows from the observation that elastic scattering of charmed quarks off partons comprising the surrounding nucleons should be similar to elastic scattering of the initial projectile partons off partons of the nucleons; thus the overall effect is to produce approximately double the value of the Drell-Yan $\langle p_T^2 \rangle$. The p_T dependence has been measured by the NA3 group [128] and is displayed in fig(5.20) where a shift in the average p_T^2 of

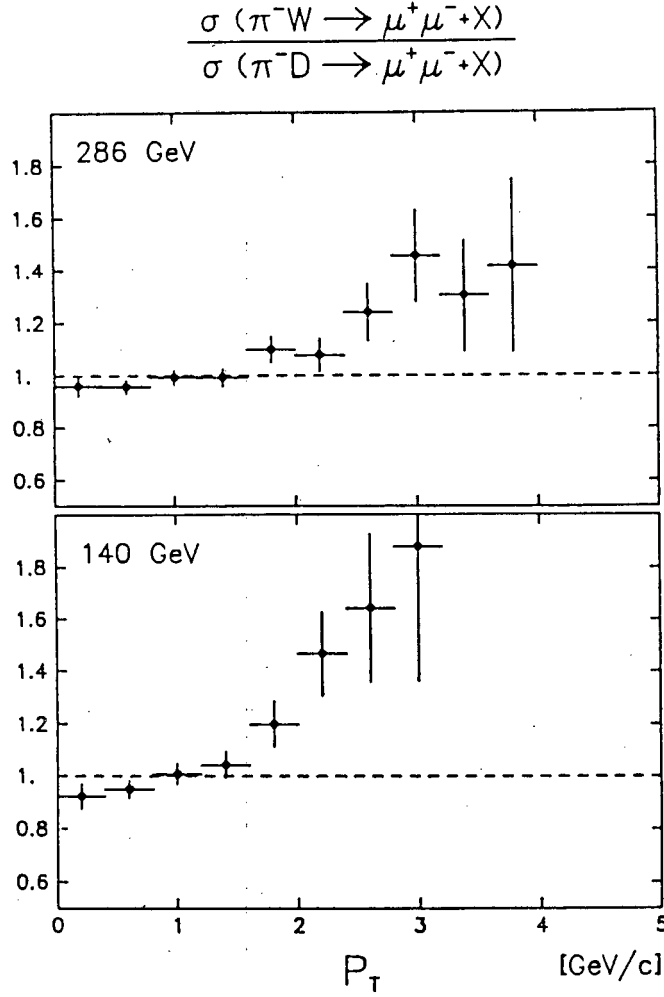


Figure 5.18: Ratios of the p_T distributions of dimuons produced from W and D targets. The W and D samples are normalised to the same number of events. Only statistical uncertainties are shown [149].

$$\langle p_T^2 \rangle_{\bar{p}P_t}^{\psi} - \langle p_T^2 \rangle_{\bar{p}p}^{\psi} = 0.34 \pm 0.06 \text{ GeV}^2 \quad (5.53)$$

is found. This can be compared for consistency with the NA10 result displayed in eqn(5.52) above, and with twice the Drell-Yan result appearing in eqn(5.51) where reasonable agreement is found.

It has been predicted in [150] that a large proportion of J/ψ production occurs via gg fusion, with only approximately 10% arising from qq scattering; this has been tested by the NA3 group and is found to be consistent with the data; a comparison is shown in fig(5.21). It should be further mentioned, as discussed previously in section 4.3.3, that a relatively large fraction of the $J/\psi \sim 40\%$ is found to come from the electromagnetic cascades of excited state $c\bar{c}$ resonances such as (χ_1, χ_2, ψ') [90].

A similar experiment to [128] producing consistent results has been carried out at FNAL [133], using $\bar{p}, \pi^-$ beams at 125 GeV on Be, Cu and W targets. A drop in the J/ψ cross-

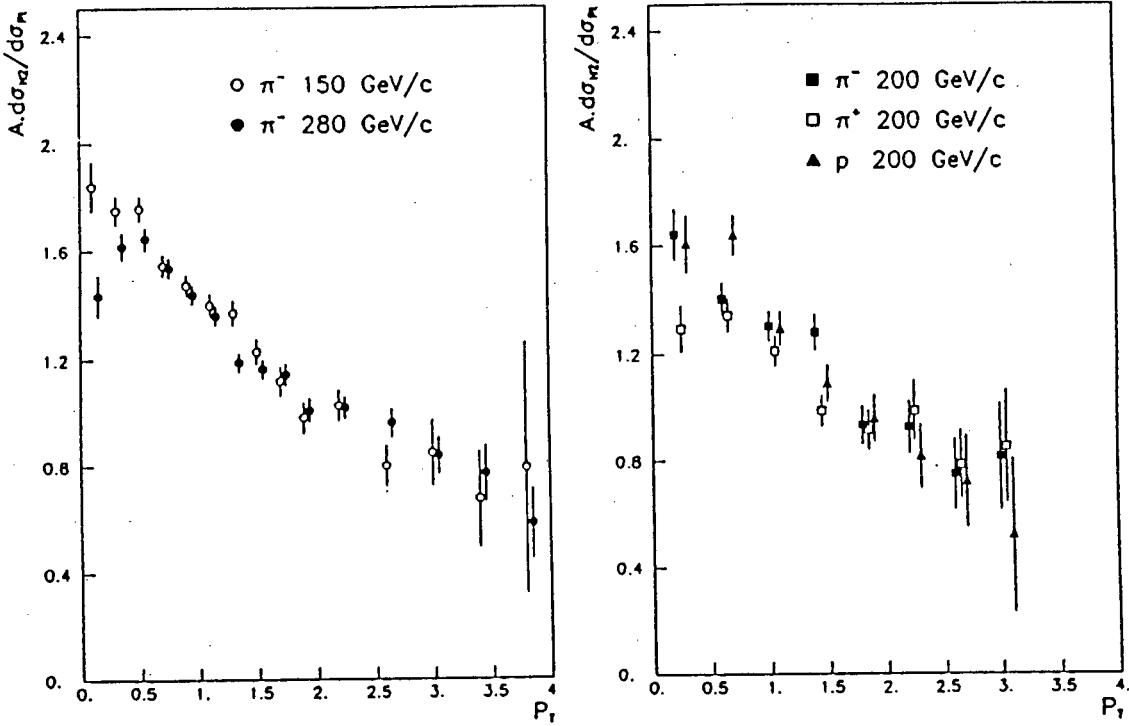


Figure 5.20: Ratio of J/ψ cross-sections for hydrogen and platinum targets with $R(p_T) = A \frac{d\sigma(H_2)}{dp_T} / \frac{d\sigma(Pt)}{dp_T}$ for various π^+ , π^- , p data from the NA3 experiment [128].

immediately the $c\bar{c}$ pair is created, which is a reasonable approximation as it is the initial state effects before $c\bar{c}$ pairs are created which are being considered; later the effect of final state interactions can be added together with a corresponding finite J/ψ formation time.

5.4.3 Gavin and Gyulassy Model for Initial State Interactions

This model [154] begins by parameterising the overall p_T dependence of J/ψ suppression into two components arising from initial and final state interactions. The ratio $\mathcal{R}_\psi(p_T)$ of the number of J/ψ at high versus low E_T is written,

$$\mathcal{R}_\psi(p_T) = \frac{(\mathcal{N}_c^{-1} dN_\psi/dp_T^2)_{E_T^{\text{high}}}}{(\mathcal{N}_c^{-1} dN_\psi/dp_T^2)_{E_T^{\text{low}}}}$$

with $\mathcal{N}_c$ the integrated continuum dimuon yield. The momentum dependence is expanded into three terms

$$\frac{dN_\psi}{dp_T^2} = \mathcal{N}_\psi F_\psi(p_T) S(p_T)$$

where

- $\mathcal{N}_\psi F_\psi(p_T)$ is the formation distribution for the J/ψ before final state interactions arise. In an analogous manner the formation distribution for the continuum is $\mathcal{N}_c F_c(p_T)$. The ratio $\mathcal{N}_\psi/\mathcal{N}_c$ is assumed to be independent of E_T , as the hard processes involved in the

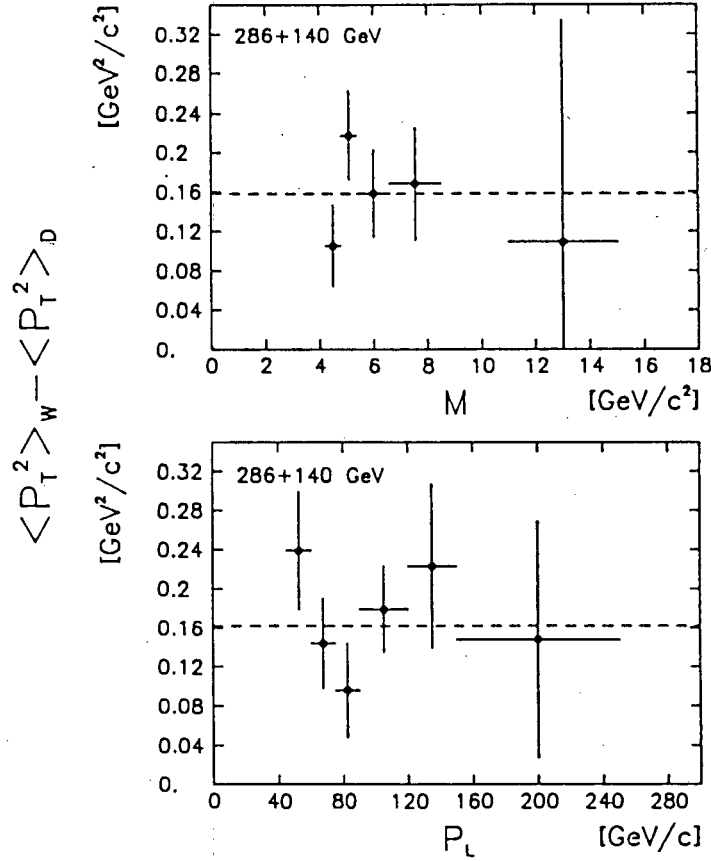


Figure 5.19: $\langle p_T^2 \rangle_W - \langle p_T^2 \rangle_D$ versus dimuon mass M and longitudinal momentum p_L . The horizontal line represents the weighted average of data points. Only statistical uncertainties are shown [149].

section is noticed (by [128] as well) for large longitudinal momenta (p_z) at small p_T which is not easily explained by the above discussion, thus it is best to exclude the large p_z J/ψ from models, in the first approach.

5.4.2 Nucleus-Nucleus Collisions

Having described the relevance of initial state scattering in hadron-nucleus collisions, a generalisation to nucleus-nucleus collisions is now made, in an attempt to provide a source for the p_T dependence observed in the NA38 data [73] from heavy ion collisions at CERN. Models have been produced [154,155,156] using hadronic absorption to account for the observed J/ψ suppression as discussed in the previous section, but with the p_T dependence of J/ψ suppression, attributed to the presence of initial state interactions before the production of the $c\bar{c}$ resonances occurs. These models follow a similar method to discover the p_T dependence, which will be presented by discussing the Gavin and Gyulassy model [154]. Two other models will be presented, but in a more summarised form, based on this approach. In these models, the formation time of J/ψ is ignored, and the J/ψ is treated as if it forms

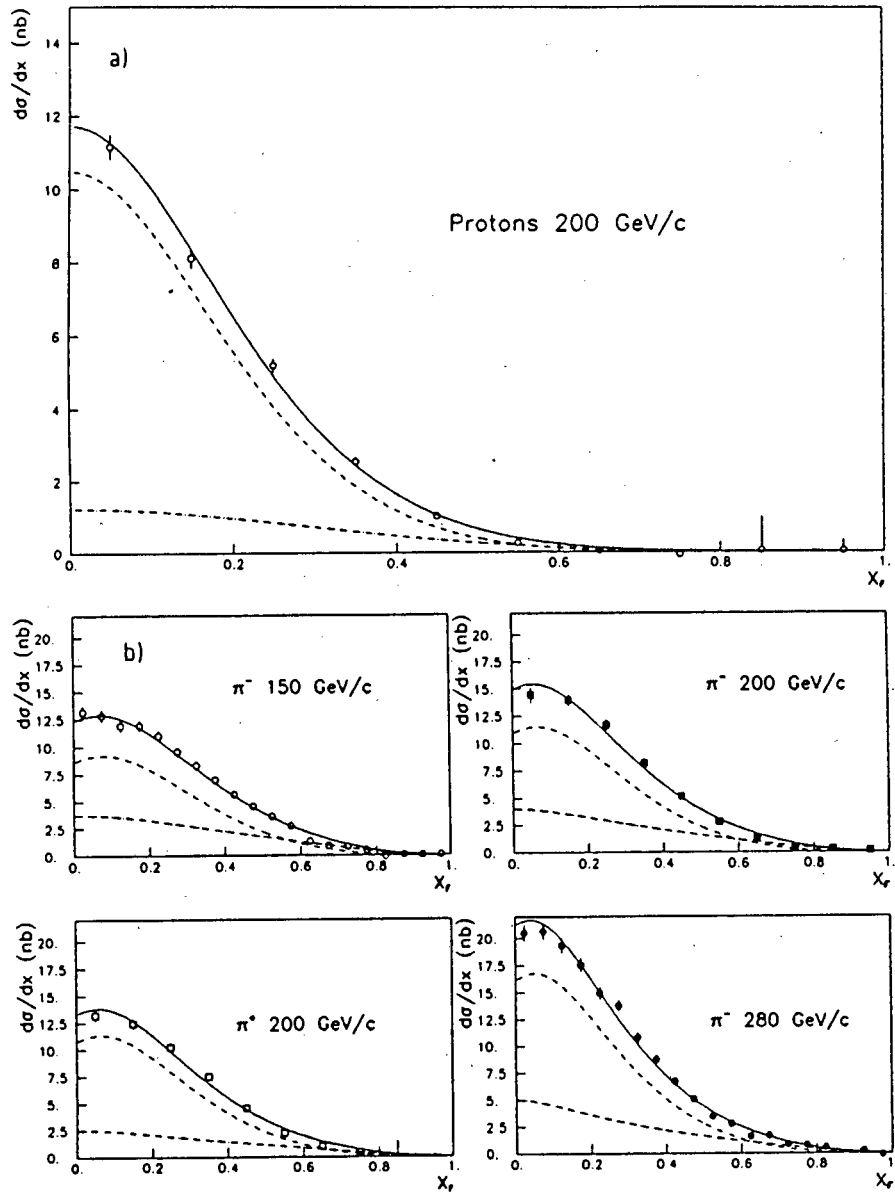


Figure 5.21: Hard component $d\sigma_h/dx_f$ for (a) incident protons and (b) pions. Dashed line: gluon-gluon fusion; dash-dotted line: $q\bar{q}$ fusion; full line: total [128].

production of J/ψ and the Drell-Yan dileptons have a similar dependence on E_T . The function $F_\psi(p_T)$ displays the p_T dependence from the initial state interactions, with the normalisation

$$\int d^2p_T F(p_T) = 1$$

- $S(p_T)$ is the momentum distribution due to final state interactions. For J/ψ , the hadronic absorption models suggest that $\leq 20\%$ of the observed variance in $\Re_\psi(p_T)$, for p_T in the range 0 – 3 GeV [135]–[138], is due to final state interactions; whilst in the case of the Drell-Yan continuum, final state interactions will not be present, thus $S_c(p_T)$ is expected to be independent of p_T . The assumption is made that the dominant p_T dependence for J/ψ distribution will not come from the final state interactions, but rather from the initial state interactions. It should be pointed out that in pA type collisions this interpretation leads to a stronger p_T contribution for the gg scattering than for qq scattering (J/ψ come mostly from gg scattering, as discussed earlier, and the observed $\langle p_T^2 \rangle$ is larger for J/ψ than for Drell-Yan production [154]), rather than the increase in the observed p_T dependence being interpreted as final state $c\bar{c}$ scattering off the partons, stated by the NA10 group [149].

A parameterisation of the form

$$F(p_T) = Ae^{-\alpha m_T} \quad \text{where} \quad m_T = \sqrt{p_T^2 + m^2}$$

is chosen, which in the case of pA type collisions fits the data reasonably well (see fig(5.22a))—it is important to check $F(p_T)$ for the Drell-Yan data, as no final state interactions are expected to be present to influence the observed p_T dependence. The coefficient α is not well fixed due to the relatively limited range of the data, thus it is rather found via

$$\langle p_T^2 \rangle = \int d^2 p_T F(p_T) p_T^2$$

where $\langle p_T^2 \rangle$ must be found in terms of E_T . To do this, the dependence of $\langle p_T^2 \rangle$ on the average number of collisions should be found.

First consider a pA type collision. In this collision, let a type i parton in the target proton collide with n nucleons; in the m 'th one, let it suffer a hard interaction to produce a $c\bar{c}$ pair. If the probability of the parton suffering an elastic collision is P_{iN} , where P_{iN} is assumed independent of momentum and the number of collisions but proportional to an effective parton-nucleon cross-section, then the average number of elastic collisions per parton is

$$\langle n_i \rangle = \frac{1}{n} \sum_{m=1}^n P_{iN}(m-1) = \frac{1}{2} P_{iN}(n-1)$$

If in each of the elastic collisions, the parton acquires random transverse momentum $\delta \vec{p}_T$, then an average

$$p_{Ti}^2(n) = p_{T0i}^2 + \langle n_i \rangle \delta p_{Ti}^2$$

is found, with p_{T0i}^2 from the intrinsic (fermi) motion of the partons inside the nucleon and, δp_{Ti}^2 , the momentum dispersion of the parton i ; thus

$$p_{Ti}^2(n) = p_{T0i}^2 + \frac{1}{2} P_{iN} \delta p_{Ti}^2 (n-1)$$

One obtains from experiment

$$\langle p_{T0g}^2 \rangle \approx \langle p_T^2 \rangle_{pp \rightarrow \psi + x} = 1.23 \pm 0.05 \text{ GeV}^2$$

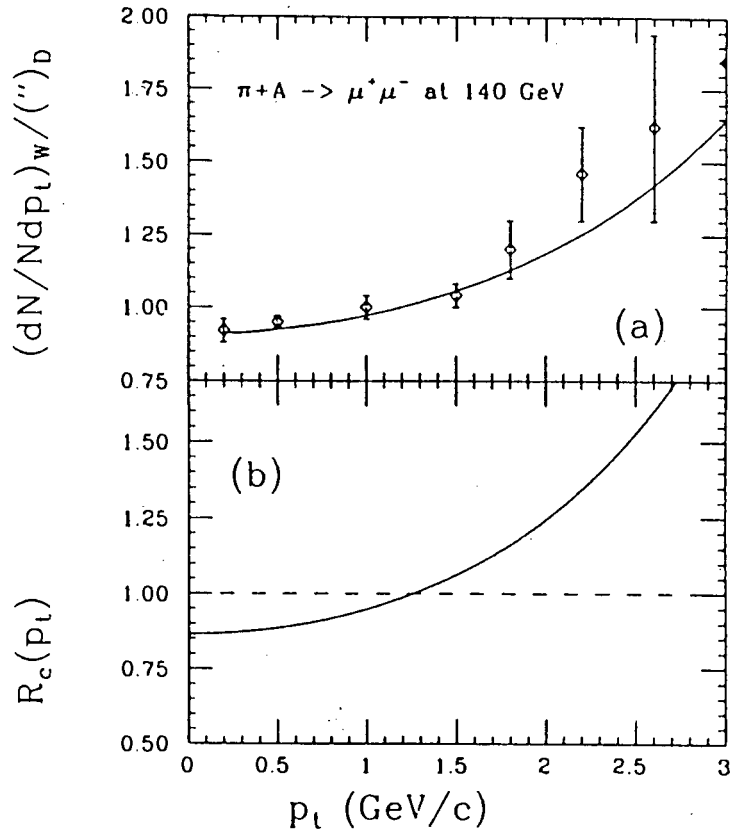


Figure 5.22: (a) Measured ratio of Drell-Yan yields as a function of p_T for a 140 GeV π^- beam on ^{184}W and deuterium targets compared to a gaussian fit (solid curve) with $\langle p_T^2 \rangle$ taken from [149,157]. (b) Shown is the expected p_T variation of the Drell-Yan continuum ratio for $E_T^{\text{high}} > 85$ GeV over $E_T^{\text{low}} < 34$ GeV [154].

from [128] and

$$\langle p_{T0q}^2 \rangle \approx \langle p_T^2 \rangle_{pp \rightarrow \mu^+ \mu^- + x} = 1.38 \pm 0.07 \text{ GeV}^2$$

from [149]. Averaging over impact parameter

$$\langle p_T^2 \rangle \approx p_{T0i}^2 + \lambda_i^2 (\bar{n} - 1)$$

where $\lambda_i = (P_{iN} \delta p_{Ti}^2 \frac{1}{2})^{\frac{1}{2}}$ is the effective momentum transfer and $\bar{n}$ is the number of elastic collisions averaged over impact parameters.

It is then stated

$$\bar{n}_{pPt} \approx \bar{n}_{pW} \approx 0.77 A^{0.3} \approx 3.7$$

Using the NA3 result displayed in eqn(5.53)

$$\langle p_T^2 \rangle_{pPt \rightarrow \psi} - \langle p_T^2 \rangle_{pp \rightarrow \psi} = 0.34 \pm 0.06 \text{ GeV}^2$$

which is mostly due to gg scattering, yields

$$\lambda_g^2 = 0.12 \pm 0.02 \text{ GeV}^2$$

Similarly, with the use of the NA10 result eqn(5.52) for quark-parton scattering

$$\lambda_q^2 = 0.056 \pm 0.022 \text{ GeV}^2$$

The λ_i are found to be independent of the target hadron due to the fact that the p_T shift for the J/ψ is the same for the $\pi^\pm$ and p beams [149,128,133].

To consider nucleus-nucleus collisions, the scattering of the target partons by the beam partons must be considered as well. Thus

$$\langle p_T^2 \rangle_{AB} \approx p_{T0i}^2 + \lambda_i^2(\bar{n}_A + \bar{n}_B - 2)$$

where n_A, n_B refer to the average number of inelastic collisions suffered by the projectile, target.

Some estimate of how $\bar{n}$ depends on E_T must now be found. Use of the ATTILA version [145] of the LUND Fritiof/Jetset 6.3 string fragmentation model is made to estimate

$$\bar{n}_O + \bar{n}_U \approx 2.5 + \frac{E_T}{(11 \text{ GeV})}$$

with the initial hadronic resonance density per rapidity interval

$$\frac{dN_H}{dy} \approx \frac{E_T}{(1 \text{ GeV})}$$

However as the coefficients are extremely sensitive to the definition of E_T and the fact that the ATTILA/LUND model under-estimates the NA38 E_T distribution by as much as 50% in central collisions, no real quantitative estimates can be made, leading to a fair degree of arbitrariness in the way the scaling of E_T is chosen; the values predicted by the ATTILA/LUND model for E_T are used, in order to keep the model consistent. It may also be that the final state interactions will modify the observed E_T distribution. The E_T values chosen are: $E_T^{\text{high}} = 60 \text{ GeV}$ and $E_T^{\text{low}} = 15 \text{ GeV}$. Combining the previously calculated estimate for $S(p_T)$ from the absorption models [135]–[138] yields a more realistic p_T dependence displayed in fig(5.23).

An attempt is made to reproduce the S+U data produced by the NA38 group [73], which exhibits a weaker p_T dependence than that of O+U. An increase in $\bar{n}_U$ is expected, and taking the E_T calculated from the ATTILA/LUND model, it is found that R is indeed less dependent on p_T .

The initial state interactions will effect the p_T dependence observed in the Drell-Yan continuum as well. This dependence has been calculated by [154] and is shown in fig(5.22b). A summary based on [76] will now be given for the other two models.

5.4.4 Hüfner, Kurihara and Pirner Model

The model [155] begins with a look at hadron-nucleus collisions and then generalises to nucleus-nucleus collisions. To begin with pA type collisions are considered, in order to test

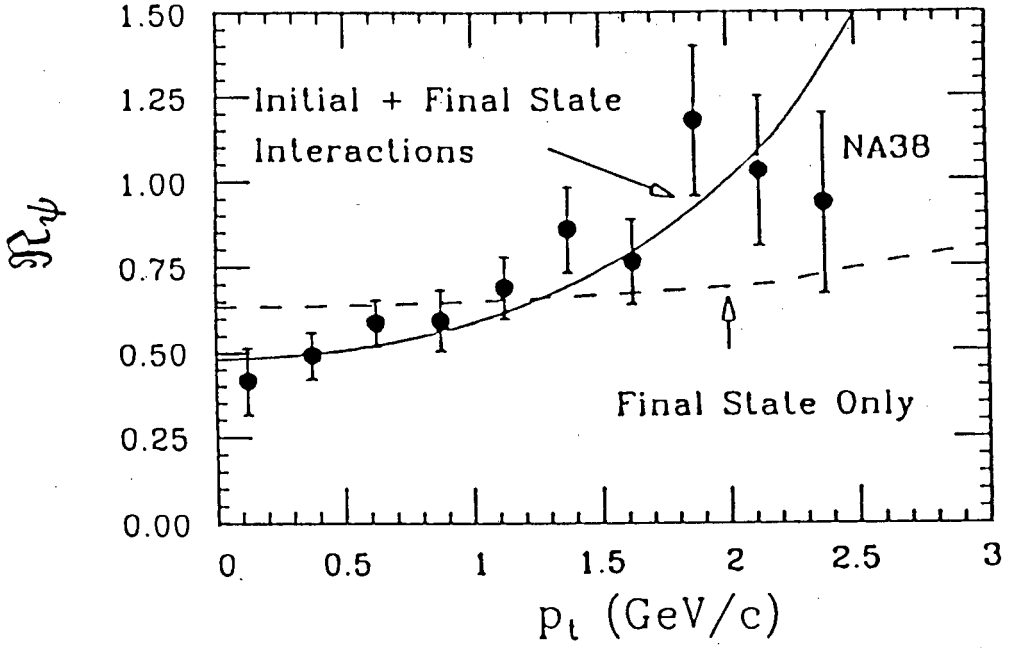


Figure 5.23: The preliminary NA38 data [70] on the relative J/ψ to continuum yield with $E_T^{\text{high}} > 85$ GeV and $E_T^{\text{low}} < 34$ GeV are compared to the prediction of the model, for the analogous LUND model values of $E_T = 60$ and 15 GeV, including both quasi-elastic and inelastic initial state scattering and final state absorption for $\bar{\sigma} = 5$ mb. The dashed curve shows the suppression factor in the absence of initial state interactions [154].

the model and fix the parameters; with the added assumption that the production of the J/ψ occurs inside the nucleus. The p_T dependence enters via

$$\langle p_T^2 \rangle_{pA} = \langle p_T^2 \rangle_{pp} + \sigma_{gN} \langle p_T^2 \rangle_{gN} \mathcal{I}$$

where

- $\langle p_T^2 \rangle_{gN}$ is the average p_T^2 gained by a gluon per gluon-nucleon scattering
- σ_{gN} is the gluon-nucleon elastic scattering cross-section
- $\mathcal{I}$ is the integral over the impact parameter, of the nuclear density in the nucleus, thus containing the averaged number of nucleons in a collision.

From the NA3 results [128] the value of

$$\sigma_{gN} \langle p_T^2 \rangle_{gN} = (0.39 \pm 0.08) \text{ fm}^2 (\text{GeV}^2)$$

is fixed. A Gaussian parameterisation, which after the integration, yields a ratio of the form

$$r(p_T) \propto A e^{b p_T^2 / \langle p_T^2 \rangle_{pp}^\psi}$$

is chosen for the initial state interaction p_T dependence, where the parameter b is related to the gluon trajectory length L , and the parameter $\sigma_{gN}\langle p_T^2 \rangle_{gN}$ found above; with $\langle p_T^2 \rangle_{pp}^\psi$ from the intrinsic motion of the partons.

A strong assumption is made that the average length of the gluon trajectory before the fusion which produces the $c\bar{c}$ pair is equal to the average length of the J/ψ trajectory inside the nucleus after formation. This length is estimated to be $\sim \frac{3}{4}R$ where R is the nuclear radius. With this choice for the parameters, the shape of the NA3 data [128] can be reproduced, with a discrepancy at high $p_T \sim 3$ GeV possibly due to the Gaussian parameterisation chosen.

In A+B type collisions, the trajectory length is modified to include the scattering of both the target and projectile, thus the total scattering length is given by twice that for hadron-nucleus collisions. The length is taken from the absorption model [123]

$$L(E_T) = \frac{3}{4}R_0 + 4 \text{ fm} \quad E_T = E_T^{low}$$

$$L(E_T) = \frac{3}{4}R_0 + 6.5 \text{ fm} \quad E_T = E_T^{high}$$

With the inclusion of the final state hadronic absorption from [123], the NA38 p_T data can be reproduced, though it must be noted that [123] underestimates the E_T suppression, thus only the shape of the curve is relevant (see fig(5.24). In the case of Sulphur, the radius of

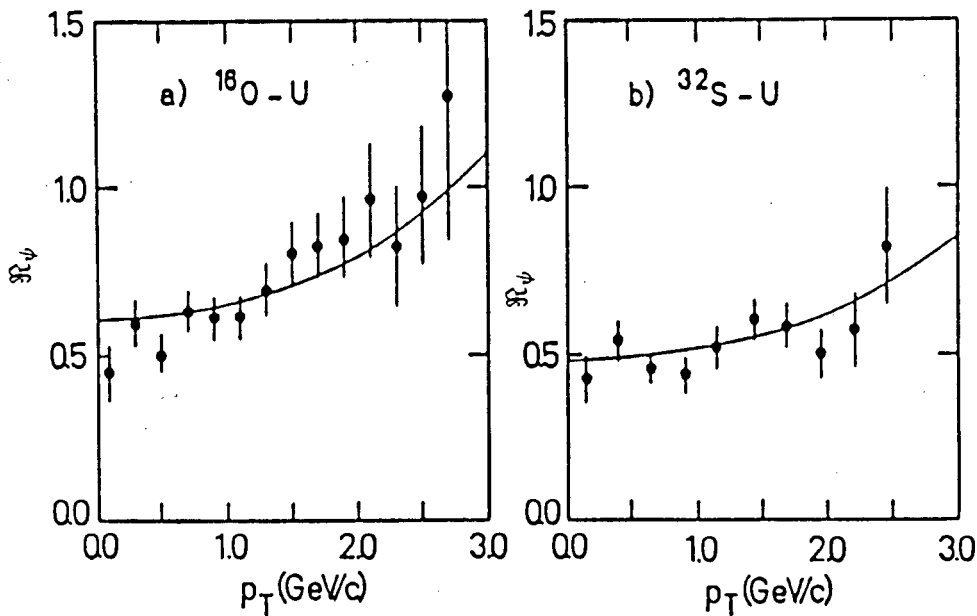


Figure 5.24: The ratio R_ψ of the number of events with large E_T to the number of events with low E_T for muon pairs in the mass range $2.6 \leq M_{\mu^+\mu^-} \leq 3.5$ GeV as a function of the transverse momentum for O-U and S-U collisions at 200 GeV/A. The data are from [65], the solid line is the model with an overall factor being adjusted at $p_T = 0$ [155].

oxygen, R_O , is replaced by that for sulphur. The flatter distribution observed for sulphur by the NA38 group is then also reproduced.

5.4.5 Blaizot and Ollitrault Model of Initial State Interactions

This model [156] considers only J/ψ with small p_z (in the nucleon-nucleon centre of momentum frame) to avoid the complicating physical effects observed for large p_z by [133]. The parameterisation of

$$\frac{dN}{dE_T dp_T^2} = N_{\text{int}}(p_T, b) N_{\text{abs}}(p_T, b) \frac{d^2b}{dE_T} T_{AB}(b)$$

is used, where:

- $N_{\text{int}}(p_T, b)$ is the distribution in p_T due to the initial state, and $N_{\text{abs}}(p_T, b)$ for the final state, interactions.
- $\frac{d^2b}{dE_T}$ links the impact parameter with the transverse energy flow.
- $T_{AB}(b)$ is an integral of the product of the nuclear densities at impact parameter b .

The simplified case of pA type collisions is first considered. A parameterisation of the form

$$\langle p_T^2 \rangle_r = \langle p_T^2 \rangle_0 + \beta \mathcal{I}$$

is used, where

- $\mathcal{I}$ is an integral of the nuclear densities at the impact parameter b
- β is some parameter to be related below to the experimental data from the NA3 experiment [128].

The average is taken at some transverse r , the z averaging already contained in the nuclear density integrals. By using an exponential form

$$f(p_T, r) \propto p_T e^{-2 \frac{p_T}{\langle p_T \rangle_r}}$$

for the distribution over p_T of the $c\bar{c}$ pairs where r is the radial distance, in the integral, the parameterisation

$$\langle p_T^2 \rangle = \langle p_T^2 \rangle_0 + \frac{9\beta}{8\pi r_0^2} (A^{\frac{1}{3}} - 1)$$

is obtained. When compared to the NA3 data [128] a value of

$$\frac{9\beta}{8\pi r_0^2} = 0.07 \text{ GeV}^2$$

is obtained, thus fixing β .

A linear form $R(p_T) \approx 0.63 + 0.13p_T$ is suggested to reproduce the FNAL data [133], which agrees well for $p_T \leq 2 \text{ GeV}$.

For nucleus-nucleus collisions, the dependence of $\langle p_T^2 \rangle$ on the impact parameter is related to the number of participants in the collision, which can be related further to E_T . A linear dependence of N_{int} on p_T is again predicted by using the exponential distribution in p_T , to obtain

$$\Re_{\psi}^{\text{int}}(p_T) = \frac{N_{\text{int}}^{\text{high}E_T}}{N_{\text{int}}^{\text{low}E_T}} \approx 0.87 + 0.13p_T$$

The contribution to the final ratio by the final state interactions can be evaluated, assuming no p_T dependence in the final state interactions, by taking the ratio

$$\mathcal{R}_\psi^{\text{abs}} = \frac{N_{\text{abs}}(E_T^{\text{high}})}{N_{\text{abs}}(E_T^{\text{low}})} = 0.64$$

The theoretical ratio can then be compared to the experimental ratio (see fig(5.25)). There is

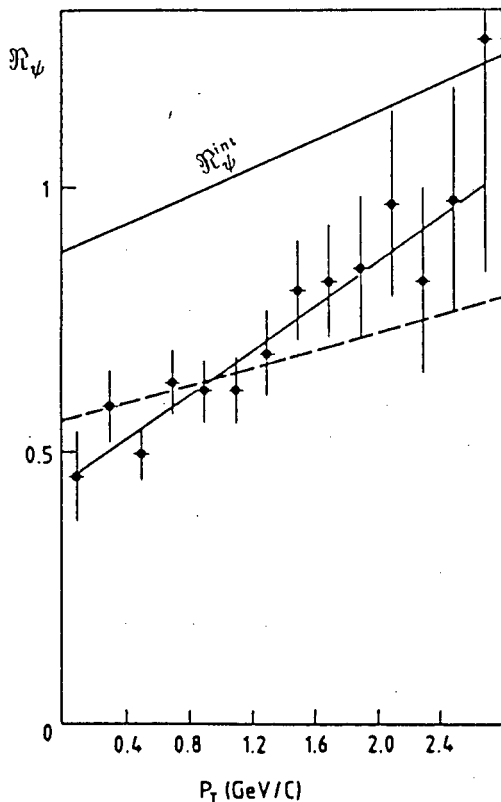


Figure 5.25: The ratio $\mathcal{R}_\psi^{\text{int}}$ calculated for the two ranges of impact parameters which correspond approximately to the two extreme E_T bins used in the NA38 analysis [65]. The dashed line represents $\mathcal{R}_\psi(p_T) = \mathcal{R}_\psi^{\text{int}}(p_T)\mathcal{R}_\psi^{\text{abs}}$ with $\mathcal{R}_\psi^{\text{abs}} = 0.64$. A normalisation corresponding to the number of continuum events in the bin is used above, whilst the data in [65] are normalised to the number of events in the bin. A linear fit to the data is given by $\mathcal{R}_\psi^{\text{exp}}(p_T) = 0.44 + 0.21p_T$, represented by the straight line going through the data points [156].

insufficient p_T dependence to account for the observed data, thus implying some p_T dependence arising from the final state interactions.

5.4.6 Conclusions From the Models

It is thus possible to explain the NA38 data with the combination of hadronic absorption and initial state interactions, noting especially that the integrated form over p_T of the initial state interaction is not expected to influence the results already calculated for absorption as a function of E_T , thus maintaining the E_T suppression of the absorption models. The major problems with respect to the initial state interactions still to be resolved are:

1. The method to relate E_T to the impact parameter/number of parton collisions is ambiguous – various model dependent assumptions have to be made, hence the scale of E_T is unknown.
2. None of the models include the excited states of $c\bar{c}$ resonances, which contribute approximately 40% to the observed J/ψ [149].

The effect of these initial state interactions, if strong enough, could overdo the p_T dependence for the plasma models, though for the full quantum mechanical treatment given by Matsui [110] discussed in section 4.3.3 the p_T dependence is insufficient to account for the NA38 data and hence the added initial state effects will help improve the model. It is thus important to gain an estimate of the strength of the dependence from initial state interactions in order to realistically compare the models with the NA38 data. Some information on the extent of the initial state interaction can in principle be gained from observations of the p_T dependence in the Drell-Yan continuum, which is not influenced by the final state interactions. However the available data (see fig(3.15)), though not showing any dependence, has experimental errors too large to draw any conclusions from [153].

At high p_T and E_T the models predict that the J/ψ to continuum ratio can exceed 1 due to the initial state gg scattering having a stronger p_T production than qq , the primary production of large p_T J/ψ being gluon fusion, whilst that of Drell-Yan is that of quark fusion [128]. For a plasma, in which initial state scattering has been ignored, it is not possible to exceed one. Unfortunately, again experimental errors are too large to say anything conclusive [153].

The major contrast between the absorption models with initial state interactions, and the plasma models, is that for large $p_T > 3$ GeV it is expected that R_ψ may exceed one due to the greater amount of scattering required, whilst in pure plasma models, it approaches one. The experimental data at present admit either possibility.

5.5 Production Mechanisms

Gavai and Gupta have suggested [158,159] that the production mechanisms of $c\bar{c}$ pairs in heavy ion collisions may produce a significant contribution to the observed J/ψ suppression. A basic approach is taken, where the production mechanisms of $c\bar{c}$ are considered to have two intrinsic timescales. The first involves the creation of $c\bar{c}$ pairs which is a hard process occurring in a typical time of the order of $1/m_c \sim 0.07$ fm. The evolution of these pairs into charmonium resonances (hadronisation) requires a time of the order of the separation time for $c\bar{c}$ quarks to reach the bound state radius of the order of 1 fm and is in general a soft process. Gavai and Gupta study the initial $c\bar{c}$ creation mechanisms, for which perturbation theory is valid. A simple model for the hadronisation of the $c\bar{c}$ pairs is used, where the J/ψ cross-section is assumed proportional to the $c\bar{c}$ production cross-section, with the condition that the centre-of-mass energy of $c\bar{c}$ pairs is restricted between a lower limit determined by the charm quark mass ($2m_c$) and an upper limit determined by the dissociation energy required to form open charmed mesons ($2m_D$). The phenomenological proportionality constant relating pair creation cross-sections to the J/ψ production cross-section is thus assumed to be independent of the initial creation processes of the $c\bar{c}$ pairs.

The J/ψ cross-section is then given by

$$\sigma_\psi = C_{\text{had}} \int dx_1 dx_2 \sum f_i(x_1) f_j(x_2) \sigma_{ij} \quad (5.54)$$

where f_i are the quark, anti-quark and gluon distribution functions as a function of x , the parton momentum fraction; σ_{ij} is the cross-section for the ij subprocess, found from the corresponding QCD diagrams where the i and j each refer to either a quark, anti-quark or gluon in the production diagram; to lowest order, the relevant diagrams involve either $q\bar{q}$ or gg fusion [158]. The hadronisation constant is given by C_{had} .

It has been established by the European Muon Collaboration (EMC) [162] that the presence of a nuclear environment changes the parton distribution functions of nucleons away from those found for an isolated nucleon. Thus the parton distribution functions which appear in eqn(5.54) will be dependent upon the nature of the projectile and target. In particular Gavai and Gupta choose to examine the ratio of J/ψ cross-section in nucleus-nucleus collisions to proton-proton collisions given by

$$R = \sigma_{AB}^{\psi} / \sigma_{pp}^{\psi} \quad (5.55)$$

The proportionality constant in eqn(5.54) cancels out in the above ratio, which is equivalent to ignoring any final state interactions. The model thus describes a background process which is predicted by Gavai and Gupta [159] to be present in either plasma or hadronic/nuclear suppression mechanisms of J/ψ production.

It is necessary in this model to know the parton distribution functions in a nuclear environment: the quark and anti-quark distribution functions can essentially be estimated from a fit to the EMC data; however the gluon distribution has not been determined experimentally, hence it is necessary to use of a model to approximate the distribution. Three models are considered:

1. The rescaling model [163] assumes that the parton distributions in nuclei at a given Q^2 are obtained from those in a free nucleon at the same Q^2 by evolving the free nucleon density through the Altarelli-Parisi equations to a scale of $\xi_1 Q^2$, where the parameter ξ_1 is obtained from data on deep inelastic scattering and can be chosen to be $(\frac{1}{2}A)^{2/3}$ [159].
2. The nucleon gas model [164] assumes that the partial deconfinement of quarks in nucleons occurs resulting in a two component model of a gas of free nucleons, and a gas of deconfined quarks and gluons at a temperature T . There is a probability w for each nucleon of a nucleus to contribute to the gas [158].
3. The 6-quark model [165] also consists of two components, the first is just free nucleons, whilst the second is baryon-number 2 objects [159].

If the model is studied for conditions arising in the the NA38 data [75], which examines a region of rapidity $-1 \leq y \leq 1$ then the momentum fractions are restricted to remain within the region covered by the EMC data. The model dependence will thus enter mostly via the gluon distribution, which may soon be known from experiment [117]. The Drell-Yan production cross-section has also been computed by Gavai and Gupta [159] using a similar approach.

The values for the ratio R given by eqn(5.55) are displayed in table(5.4) for the various models, corresponding to O, S and U beams on a U target, and for two different charm quark masses. The results are found to be quite strongly model dependent, with a most optimistic value for the suppression in O-U collisions of the order of 40%. An equivalent set of results for the Drell-Yan continuum is displayed in table(5.5) where little to no dependence on projectile mass is observed in contrast to the J/ψ results. It is mentioned in [159] that the inclusion of the six-quark model is mostly to elucidate the model dependence arising from the estimation of the nuclear parton distributions. The dependence on charm quark mass is found to be small.

A similar model for hadron-nucleus collisions [166] has been discussed by Gavai and Gupta, with particular use of the nucleon gas model [164] discussed above. The model

Table 5.4: R_ψ computed to lowest order in perturbative QCD. The target nucleus is always U^{238} . The result of using different models is shown for two values of m_c . The errors displayed are Monte Carlo errors [160].

Beam	Model	$R_\psi(m_c = 1.5 \text{ GeV})$	$R_\psi(m_c = 1.3 \text{ GeV})$
O	Rescaling	0.59 ± 0.02	0.65 ± 0.02
	gas	0.53 ± 0.03	0.77 ± 0.02
	6 quark	1.28 ± 0.04	1.28 ± 0.03
S	Rescaling	0.56 ± 0.02	0.62 ± 0.02
	gas	0.53 ± 0.02	0.77 ± 0.02
	6 quark	1.30 ± 0.04	1.31 ± 0.03
U	Rescaling	0.47 ± 0.02	0.53 ± 0.02
	gas	0.48 ± 0.02	0.70 ± 0.02
	6 quark	1.33 ± 0.04	1.33 ± 0.04

Table 5.5: R_{DY} computed to lowest order in perturbative Q.C.D. The notation is the same as table(5.4) [160].

Beam	Model	$R_\psi(m_c = 1.5 \text{ GeV})$	$R_\psi(m_c = 1.3 \text{ GeV})$
O	Rescaling	0.59 ± 0.03	0.63 ± 0.02
	gas	0.72 ± 0.06	0.72 ± 0.02
	6 quark	1.00 ± 0.05	0.99 ± 0.03
S	Rescaling	0.57 ± 0.03	0.60 ± 0.02
	gas	0.70 ± 0.05	0.70 ± 0.02
	6 quark	1.00 ± 0.05	1.00 ± 0.03
U	Rescaling	0.49 ± 0.03	0.52 ± 0.02
	gas	0.70 ± 0.05	0.69 ± 0.02
	6 quark	1.00 ± 0.05	0.99 ± 0.04

predicts both an x_f and p_T dependence for the J/ψ production cross-section which is found to agree reasonably well with experiment.

In [117] Gavai, Gupta and Sridhar calculate a p_T dependence for the ratio given in

eqn(5.55) determined by the equation

$$R_\psi(p_T) = \left(\frac{d\sigma}{dp_T} \right)_{AA \rightarrow J/\psi} / \left(\frac{d\sigma}{dp_T} \right)_{pp \rightarrow J/\psi} \quad (5.56)$$

within the framework of the above model. However, the calculation of high p_T production subprocesses involves the inclusion of next to leading order diagrams, in which the radiation of a third particle is possible. Subprocesses which can lead to large p_T for $c\bar{c}$ pairs involve

$$q\bar{q} \rightarrow c\bar{c}g, \quad gq(\bar{q}) \rightarrow c\bar{c}q(\bar{q}), \quad gg \rightarrow c\bar{c}g$$

where the third particle forms a jet, this is absent in the lowest order diagrams which therefore do not significantly contribute to high p_T J/ψ production. Use is made of the nucleon gas model [164] discussed above to estimate the gluon densities in the presence of a nucleus. The results are presented in fig(5.26a) for the ratio given by eqn(5.56). Suppression is observed in

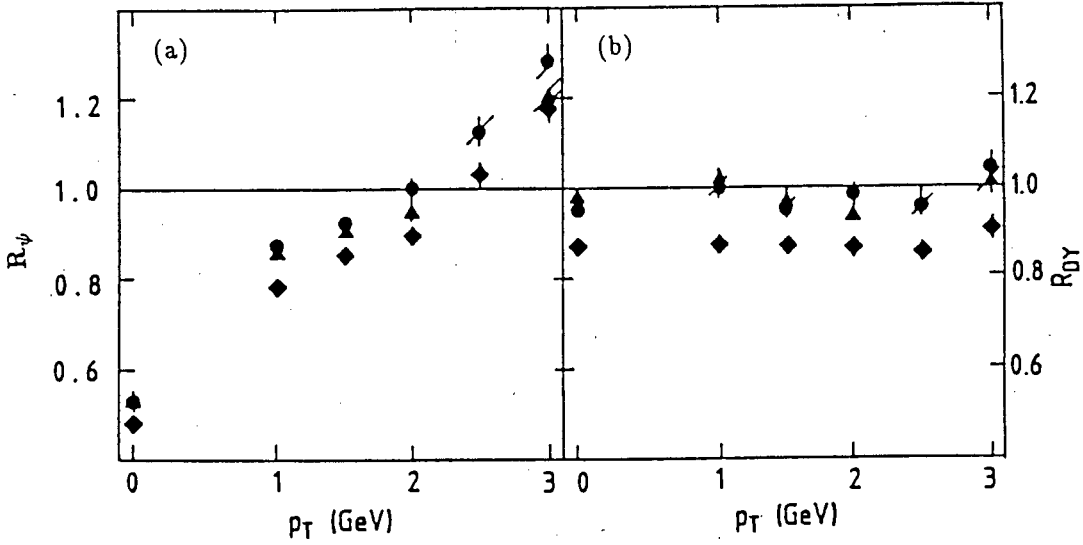


Figure 5.26: The ratios (a) R_ψ defined in eqn(5.56) and (b) R_{DY} a similar ratio for the continuum, as a function of p_T , with $m_c = 1.5$ GeV. Circles denote the ratio for O-U collisions, triangles for S-U, and diamonds for U-U [117].

the number of J/ψ produced in central nucleus-nucleus collisions over pp collisions at low p_T , in agreement with the earlier calculations [159] where the lowest order diagrams dominate. At large p_T this suppression becomes enhancement, where the next-to-lowest order diagrams become important. The major contribution at high p_T is found to come from $qg \rightarrow c\bar{c}q$, which depends on the quark distribution function. The ratio

$$\rho_{Ap}(x) = \frac{q^A(x)}{q^p(x)} \quad (5.57)$$

of the nuclear to free quark distribution functions is found to decrease with x within the range $0.2 \leq x \leq 0.6$ as displayed in fig(5.27), and then increase rapidly. For J/ψ with low p_T the range of x is approximately $0.057 \leq x \leq 0.6$, whilst for $p_T = 3$ GeV $x \geq 0.1$ is expected,

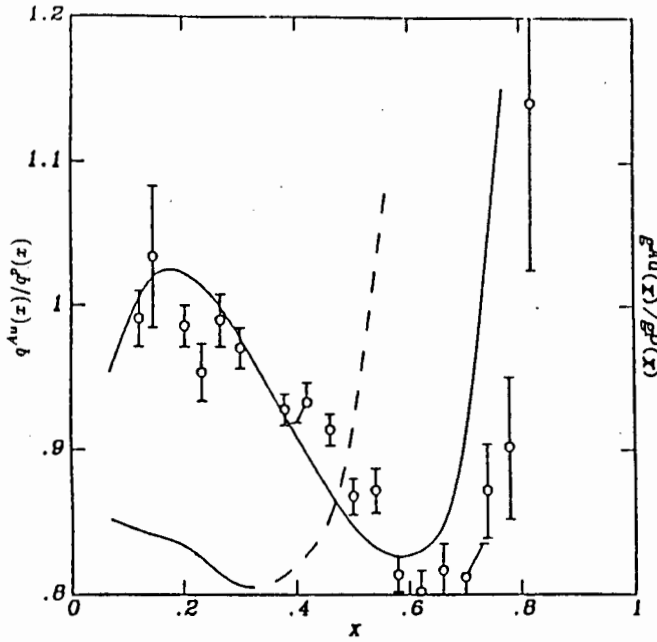


Figure 5.27: The ratio of quark (q) to gluon (g) densities for gold and a free nucleon in the gas model. While the former is a fit to the data, the latter is a prediction of the model. The solid line indicates the region where present work checks it against existing J/ψ production [166].

which produces a larger value for an averaged ρ_{Ap} , weighted by the corresponding cross-sections [117]. High p_T production in nucleus-nucleus collisions thus requires higher values for x in comparison to low p_T production with correspondingly higher quark-distributions and hence enhancement in the production of J/ψ in comparison to the free nucleon case. The important diagram for low p_T $c\bar{c}$ pair production is $gg \rightarrow c\bar{c}$, which for gluons in the range $0.057 \leq x \leq 0.6$ leads to a net suppression in the production of J/ψ in comparison to pp production.

The p_T dependence of $c\bar{c}$ production has also been calculated for Drell-Yan production, and is displayed in fig(5.26b). The strong p_T dependence evident for $c\bar{c}$ production is absent. This is partially explained by the smaller range in x required for Drell-Yan production at high p_T [117], and a smaller value for an averaged ρ_{Ap} determined from eqn(5.57) when weighted by the relevant cross-sections. A realistic comparison with NA38 data is not possible at this stage, because the model compares the production cross-section of J/ψ in nucleus-nucleus and proton-proton collisions, whilst the NA38 data is for J/ψ production from high and low transverse energy nucleus-nucleus collisions.

The dependence on A the atomic mass of the projectile and m_c is once again found to be small. Gupta in [161] has suggested an E_T dependence for the above model. A simple random walk model is introduced to illustrate the procedure. The perturbative series for hard-scattering is considered: to lowest order the p_T of a pair is zero, and the hadronic transverse energy flow is simply E_T^{np} which is non-perturbative arising from hadronisation. The next-to-lowest order generates a perturbative contribution to p_T and E_T , which is continued to higher order, giving the equations

$$|p_T| = \left| \sum_i p_T^i \right| \quad E_T = \sum_i |p_T^i| + E_T^{np}$$

For small E_T^{np} this sum corresponds to a random walk where the displacement is determined by the first sum, whilst the actual distance travelled corresponds to the second sum. QCD

will give the distribution of step sizes and the average length of the walk. Large distance walks will tend to give larger fluctuations away from zero final displacement, which corresponds to larger E_T producing a greater number of high p_T and fewer low p_T on average. The results from this rather simplified model, in which the step size is assumed constant, appears to be consistent with the NA38 data. However to get quantitative estimates, a much more detailed analysis is required, especially with the inclusion of the dependence on transverse energy flow from the collision, to enable a realistic comparison with the NA38 data to be made.

5.6 Other Models

A few of the alternate methods to account for J/ψ suppression not discussed above are now presented in a more summarised form.

1. A string model for J/ψ suppression [167] has been discussed by Neubauer et al. in terms of a survival time for J/ψ in a hadron gas. No attempt is made to reproduce the NA38 data.
2. The thermal production of dileptons is discussed by [168] Kajantie and Ruuskanen, who suggest the possibility that thermal dilepton production from quark gluon plasma might account for an enhanced background in central collisions and hence an observed suppression in the relative number of J/ψ produced in central collisions.
3. A higher twist effect is discussed by Alonso et al. [169]. The next-to-leading twist contributions are expected to be important for low p_T J/ψ ; this is due to the requirement of three-gluon fusion production to produce the correct quantum numbers in the leading twist contribution. The next-to-leading twist contribution allows for two-gluon fusion which is important at low p_T . The contribution is found to be negative and hence leads to a reduction in the J/ψ production cross-section at low p_T in qualitative agreement with the NA38 data.
4. Adami et al. [170] discuss charmonium disintegration by field-ionisation. Projectile nucleons interact with at least one target nucleon via one-gluon exchange, forming a gluonic flux tube as the nucleons separate. If one-gluon exchange occurs coherently, the colour flux tubes can add up to yield a macroscopic colour-electric field which, if it is of the order of the string tension, can essentially tear apart charmonium resonances, and lead to a reduction in J/ψ resonance production. No attempt is made to reproduce qualitative comparison to the NA38 data.

Two models have also been suggested with regard to the influence of quark gluon plasma on $c\bar{c}$ resonances. Hansson et al. [171] discuss the possibility that J/ψ remain bound state resonances considerably above the phase transition temperature. If the charm quark mass is not considered heavy for temperatures above T_c , then it would still feel the confining momentum dependent forces. This is based upon a quark-gluon plasma which is dynamically confining at the magnetic mass scale determined by $m_{\text{mag}}^{-1} \sim (g^2 T)^{-1}$ as conjectured originally by DeTar [172], which appears to be supported by numerical simulations [172,173] and perturbative theory [174]. The existence of bound J/ψ above T_c leaves open the possibility of the dissociation of J/ψ into $D\bar{D}$ states in this thermal environment, which may increase the width of the observed J/ψ peak and hence be detectable.

Shor [55] discusses the possibility of $c\bar{c}$ production by quark-gluon plasma which is at a high temperature, of the order of 0.5 GeV; this may possibly be obtained in CERN heavy ion experiments in the event of almost complete nuclear stopping. The production of $c\bar{c}$ pairs implies an increase in the formation of $D\bar{D}$ mesons, whose decay products would complicate the background under the J/ψ peak.

Chapter 6

Conclusions from the Review

NA38 data shows J/ψ suppression for central collisions in the CERN experiments in qualitative agreement with the predictions of Matsui and Satz [7] and Karsch and Petronzio [59]. However, it is still premature to conclude that the formation of quark gluon plasma has occurred. The predictions arising from the quantum models of J/ψ suppression discussed in the latter half of chapter 4 appear to underestimate the p_T dependence of J/ψ suppression quite severely. The main difference between the quantum models as opposed to classical models is the use of the J/ψ formation time which is not rigid in the quantum models but forms rather some probability of formation weakening the strong momentum dependence found in the classical models. However, a large fraction of the p_T dependence could be coming from the initial state interactions discussed in the previous chapter, possibly providing the remedy. The existence of alternative hadronic gas absorption models which can explain the data is also worth taking note of before any conclusions of plasma formation are made.

However, for these absorption models the required hadronic density to account for the observed J/ψ suppression features is enormously high; the physics where hadrons are forced to strongly overlap is not understood at present. The J/ψ formation time is also a problem in the nuclear models of absorption, because it is of the same order as the time-scale of the absorption, introducing ambiguity in the form of the size evolution of the J/ψ absorption cross-section. This has been mentioned by Blaizot and Ollitrault [156] in their model discussed in chapter 5 and it may therefore be worthwhile to re-examine the situation of J/ψ production in high energy p-A collisions in terms of a finite formation time for J/ψ , hence adding a time evolution to the J/ψ cross-section.

Future experimental evidence may help solve the problem of the contribution of initial state scattering to the p_T dependence of J/ψ suppression. Improved statistics for the p_T dependence of the continuum is especially important in this regard because the continuum is not influenced by final state interactions, either by quark gluon plasma or hadronic gas and hence has a final p_T distribution determined by initial state interactions.

Further experimental evidence in the form of a wider spread of E_T values will help decide if there exists a “kink” in the E_T dependence of J/ψ suppression. Such a “kink” would almost certainly indicate a phase transition, and it is a problem that at present this feature is not seen in the data (see fig(3.9)); however the statistics of the data are not sufficiently precise at present to completely rule out such a kink.

Finally, from the study of J/ψ suppression in ultra-relativistic heavy ion collisions it becomes clear that a region of high density is almost certainly created, whether it be quark-

gluon plasma or hadron gas. Experimentally a new region of physics can now be explored, namely that of matter at extreme conditions of high energy and density which provides a testing ground for Q.C.D.

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