Interaction Effects Within Factor Investing in a South African Context

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

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Abstract

The notion of portfolio tilting towards fundamental factors has been the subject of many empirical studies over the last few decades. With this being said, there is limited literature on the interaction effects between these individual factors. This dissertation focuses specifically on quality, value, low volatility and momentum and determines which factors have the largest impact on portfolio return. In addition to testing these single factor portfolios, the various interaction effects between the individual factors are investigated. This framework is divided into two parts. The first, is an empirical study on the JSE Top 100 over the 15 year period beginning September 2001 and ending September 2016. Quarterly and monthly rebalancing as well as transaction costs of 100 basis points (per trade) have been employed to mimic realistic investment management. Much of the framework used to incorporate these factors is adapted from Bender and Wang (2016) who tested these interactions on the S&P 500. The second, involves the construction of a controlled market model in an attempt to provide mathematical justification to the framework. The controlled model simulates stock price paths, in a Mil'shtein (1974) fashion, using Geometric Brownian Motion with a stochastic alpha component added to the drift. Factors are simulated randomly using correlated uniform distributions. The controlled model uses realistic market parameters and constructs the factor portfolio in the same manner as the empirical study.

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Chapter 1

Introduction

There has been ongoing debate around the topic of active management and whether or not consistent alpha returns can be achieved. Alpha and stock returns are positively correlated and are in a way, synchronised. Jensen (1967) was amongst the first to coin the term alpha and described it as the excess abnormal return of a portfolio over and above the expected market level. Thus, the higher the alpha of a portfolio, the higher the portfolio return. Large collections of literature have provided evidence justifying the claims of active management. Portfolio construction as a means to achieve market alpha became the subject of many rigorous empirical studies during the 1990s and 2000s. Of such literature, there exists a new notion of stylised investing which attempts to incorporate the fundamental factors of securities in order to outperform the market. Factor investing has formed the base of many empirical studies over recent years with some of the founding fathers of modern finance dedicating much time towards this avenue of research. The idea of abnormal market return is of great importance to this study as factor investing has been highlighted as a method of capturing alpha. The evidence of fundamental factors driving stock returns date back to the 1970s with Ross (1976) and his Arbitrage Pricing Theory. He was among the first to attribute stock returns to various macroeconomic factors without explicitly stating what these factors may be.

The factors under consideration for the purposes of this study are quality, value, low volatility and momentum. These specific factors are the most frequent and widely used in modern literature. In addition, Bender and Wang (2016) investigate these factors in a US context and keeping these factors consistent will allow their claims to be validated. These factors form a vital component of portfolio return and in a very unique but profound way, capture the core fundamentals of a business. In order to incorporate these fundamental factors, methods of portfolio tilting have been explored by Bender *et al.* (2014) and Bender and Wang (2016), of which forms the base of my research. Factor tilting is merely a method of portfolio construction whereby a portfolio captures the defining characteristics of a certain factor. Depending on investor expectations, a portfolio can be tilted towards these factors (if believed that they will help capture alpha return) and tilted against the factors believed to cause negative return. As a result, combining the factors in different ways can produce different results and some portfolios may outperform while others may underperform. Bender and Wang (2016) draw attention to interaction effects between the individual factors. They conclude that when combined, the interaction effects can produce returns in excess, over and above when these factors are combined independently. The core focus of this study is to ascertain which factor produces the highest return as well as investigate these interaction effects in a South African context using the JSE Top 100.

Following on, chapter 2 will give a comprehensive literature review of factor investing and its validity in practice. A succinct account of portfolio construction methods, relating to ranking and portfolio tilting, is also given. Chapter 3 gives an outline of the framework that will be used in this empirical study. The chapter specifies the details of the dataset and methodology used to test the claims of Bender and Wang (2016). It describes an overview of the techniques to be implemented in this study and discusses the portfolio construction method of choice. Any empirical study is heavily reliant on the underlying data and period of investigation. Thus, in order to provide some justification for this investigation, a controlled mathematical market will be created and tested. Towards the end of chapter 3, the details of this Black and Scholes (1973) type mathematical market are set out. This includes stock price simulation, methods of discretisation and market parameters used in this model. Chapter 4 groups the empirical results by two different stock starting weights: equal weights and market capitalisation weights. Within each group, monthly and quarterly results are given and the various interaction effects are broken down into their component parts and investigated further. In addition, volatilities, excess returns, Sharpe ratios and annualised returns, across all portfolios, are computed. Finally, chapter 5 brings all of this information, surrounding interaction effects, together and delivers conclusions about the existence and power of these interaction effects in practice.

Chapter 2

Literature Review

2.1 Background

The notion of predicting stock performance and determining what drives return has been one of the pillars of modern finance (Bender *et al.*, 2013). Dating back to the CAPM Model, first put forward by Treynor (1961), and further developed by Sharpe (1964) and many others, it is easy to see that factor modelling has a rich history. Over the years, many macroeconomic and fundamental factors have emerged in attempt to decrypt the dynamics underlying stock performance. Of these many factors, the most widely used are: quality, value, size, low volatility and momentum (Bender *et al.*, 2013). This study focuses on all the above mentioned factors with the exception of size in order to follow the framework set by Bender and Wang (2016). These factors are defined in the table below (Bender *et al.*, 2013) .

Tab. 2.1: Factors Commonly Used in Academic Literature

Factor	Description
Quality	Stocks with low debt and stable growth reflect higher quality
Value	Stocks that possess low value compared to their underlying fundamental value
Low Volatility	Captures the excess return on securities attributable to lower volatility, specific risk and beta
Momentum	Stocks with better recent performance tend to achieve excess return

Included below is a table summarising criteria that attempt to capture a particular factor (Bender *et al.*, 2013).

Tab. 2.2: Factors and the Criteria That Capture Them

Factor	Proxy
Quality	Return on equity, Return on assets, Debt to equity, Earnings growth and Stability
Value	Price earnings ratio, Price to book, Price to cashflow, Net profit, Sales
Low Volatility	Standard deviation (ranging time periods), Beta
Momentum	6 month/12 month return, Historical alpha

2.2 Alpha Return

Alpha return is defined as the abnormal excess return of a portfolio over and above what is generally expected. This notion of alpha was first developed by Jensen (1967) whereby the CAPM model was extended. Jensen's alpha can refer to excess return on any security whether it be bonds, derivatives or stocks. Jensen's alpha can be computed using many statistical models and when the underlying model is CAPM, the calculation is as follows,

$$\alpha_J = R_i - [R_f + \beta_i(R_m - R_f)]$$

where α_J is Jensen's alpha, β_i is the volatility of security i, R_i is the return of security i, R_m is the market return and R_f is the risk-free rate of return.

Active management and alpha return are closely linked. Active management refers to the process of frequently trading and rebalancing a portfolio in order to attempt to achieve alpha returns (Sharpe, 1991). The downfall of this management style is the high cost implications that come with it. After all transaction costs are taken into consideration, it can be difficult to outperform the market. A measure of active management is necessary when separating manager skill from manager luck. The Information ratio is a ratio of average excess return (over a certain benchmark) to active volatility (Goodwin, 1998). The higher the portfolio return and the more consistent the active return, the higher the Information ratio. The ratio is defined below,

$$\Omega = \frac{E[R_p - R_b]}{\sqrt{var[R_p - R_b]}}$$

where Ω is the Information ratio, R_p is the portfolio return and R_b is the benchmark return.

2.3 Evidence of Factor Investing

2.3.1 Global Evidence

On a global scale, over a range of diverse markets and periods, factor investing has been at the forefront of many empirical studies covering investment strategy. For the purposes of this investigation, a concise account of quality, value, low volatility and momentum is given below.

Quality

Despite the vast amount of literature outlining ideas around quality, for the purpose of this paper, only the key authors and conclusions are discussed here. Sloan (1996) was among the initial authors to attribute the excess returns to high quality earnings stocks. More recently, Asness *et al.* (2013) showed that quality criteria such as profitability, high payout ratio and stable growth have generated above market return after accounting for risk. Asness *et al.* (2013) implemented a long high quality, short low quality strategy on a US dataset and found compelling evidence of quality being a driver of stock return. Novy-Marx (2013) reinforced the claims of quality as source of outperformance.

Value

The history of value investing dates back to ideas generated by Graham and Dodd (1934) where they hypothesise that firms achieving large growth are unable to sustain these levels in the long term. Although the idea of value was born in this era, it was not until the 1980s where factors such as value were measured and explained (Bird and Whitaker, 2003). Rosenberg *et al.* (1985) used price to book, Chan *et al.* (1991) used cashflow to price and many other authors evaluated a combination of these criteria in order to explain value investing. All these authors have consistent beliefs that value investing is a profitable strategy (Bird and Whitaker, 2003).

Fama and French (1993) developed their world renowned Small Minus Big Capitalisation (SMB) and High [book to market ratio] Minus Low (HML) factors that captured size and value respectively. They studied size and value in a US context over the period July 1962 - December 1990. They found that their size portfolio and their value portfolio achieved monthly excess returns of 57 basis points and 101 basis points respectively (Fama and French, 1993). Fama and French (1993) also developed a three factor extension to the Sharpe (1964) CAPM model whereby they regressed these SMB and HML factors against return,

$$r = R_f + \beta_0(R_m - R_f) + \beta_1 SMB + \beta_2 HML + \epsilon \tag{2.1}$$

where r is the return on a stock, β_0 is synonymous to the CAPM β, but is slightly different as there are now two additional factors to take into consideration. R_f is the risk free rate of return, R_m is the market return, β_1 and β_2 are the regression coefficients of the SMB and HML factors defined above. ϵ is the random error term in the linear regression.

This three factor model developed by Fama and French (1993) explained 90% of return versus the 70% explained by the CAPM.

Low Volatility

Low volatility has been proven to provide excess returns at a lower risk when compared to traditional market capitalisation indices (Chow *et al.*, 2013). Chow *et al.* (2013) confirm the effects of low volatility over an extensive period as well as across a variety of markets. Ang *et al.* (2006) also provide persuasive empirical evidence on the low volatility effect. Low volatility is synonymous with a low beta input into the CAPM model. In other words, stocks with lower beta, exhibit lower levels of volatility. The literature also suggests a low volatility anomaly, termed the low volatility puzzle, which provides some robust evidence that low beta stocks outperform high beta stocks (Haugen and Heins, 1975). Frazzini and Pedersen (2011) uncover an important fact about this low volatility puzzle. Frazzini and Pedersen (2011) created a zero-beta factor portfolio formally termed as the Betting Against Beta (BAB) portfolio (long low beta stocks, short high beta stocks). They concluded that the low beta outperformance is more holistically explained by this BAB factor.

Momentum

Momentum investing is a more modern investment strategy that contravenes the notion of mean-reversion techniques. DeBondt and Thaler (1985, 1987) found that when stocks outperform over 3-5 years, they tend to exhibit underperformance in the years to follow and revert back to the long term mean. This idea ties in with value as discussed earlier. Although these findings suggest that mean-reversion exists, the more modern work focuses on medium term investing (Bird and Whitaker, 2003). Jegadeesh and Titman (1993) found that US stock returns over the next 3-12 months were highly correlated to the return over the previous 3-12 months and thus, reinforced the notion of momentum.

A thorough investigation into the validity of the momentum effect is given in Novy-Marx (2012). Stocks that exhibit superior performance in the previous 12 months, tend to keep the same performance trend over the short term future (Novy-Marx, 2012). In addition, Novy-Marx (2012) show (on US Data) that using 7-12 month return as a criterion, is a better predictor for momentum than 2-6 month return. This momentum effect is again confirmed in Fama and French (2012) whereby they find strong momentum returns in North America, Europe, and Asia Pacific markets. Asness *et al.* (2013) found evidence of significant momentum returns in eight different markets including the US, UK, Japan and Europe. Carhart (1997) extended the three factor model defined by Fama and French (1993) and included an additional explanatory momentum variable to help explain return,

$$r = R_f + \beta_0(R_m - R_f) + \beta_1 SMB + \beta_2 HML + \beta_3 PR1YR + \epsilon \quad (2.2)$$

where r is now defined as the excess return of the one month T-bill (proxy for the excess return above risk free rate). $PR1YR$ is defined as the previous one year momentum. All other parameters and explanatory variables are defined as in (2.1) above.

2.3.2 South African Evidence

In addition to this wealth of global research, there is an abundance of South African literature spanning over two decades. The importance of fundamental factors become evident not only in a global setting, but also in a local economy.

The value effect in South Africa has arguably the richest body of literature. Page and Palmer (1993) hypothesised and supported higher levels of risk-adjusted return in stocks exhibiting higher earnings yields and thus lower price to earnings ratios. Following on this research, Page (1996) measured the value effect by price to earnings ratio and that this effect was robust to risk adjustment in line with an Arbitrage Pricing Theory model with five factors. Van Rensburg (2001) confirmed these value effects and attempted to forecast future returns using price to earnings ratios, market capitalisation (size) and previous 12 month return (momentum). This was again supported by Van Rensburg and Robertson (2003b) who constructed a multifactor model to describe the cross-section of returns on the JSE. The significance levels of the individual factors as well as various combinations were tested. Ultimately, Van Rensburg and Robertson (2003b) found that a two factor model based on size and value best described returns. The regression model used market capitalisation and price to earnings ratios, respectively, as measuring criteria. Strugnell *et al.* (2011) confirm the positive effects of size and value, however they also draw attention to a possible deterioration of the size effect over time.

Van Rensburg and Robertson (2003a) focus on the relationship between beta and returns. Beta can be regarded as a measure of volatility. Van Rensburg and Robertson (2003a) uncover that there is surprisingly a negative correlation between beta and returns. Their research is not sample specific as tests were conducted across a lengthy period and beta estimates were calculated by observing the 12 - 30 month rolling variation. This reinforces the effect of the low volatility factor in a South African context.

Muller and Ward (2013) conducted a study of price to cashflow, return on capital and momentum in a South African setting, and found that a portfolio titled in favour of these factors outperformed the market from 1986 to 2011. From Muller and Ward (2013), evidence of the momentum effect and quality effect become apparent. Van Rensburg (2001) also found evidence of 12 month return explaining price movement on the JSE. Thus, the effect of momentum is clear over a long term period in South Africa.

2.4 Evidence of Interaction Effects

There has been limited literature around the topic of factor interaction effects. Asness (1997) evaluated the interaction effects between value and momentum and found that due to the large negative correlation between the factors, expensive stocks seemed to exhibit higher momentum and visa versa. Asness (1997) found no conclusive evidence of an extra marginal effect of including these two factors. Bender and Wang (2015, 2016) conducted a comprehensive empirical study on US data and found the interaction effects to have a positive effect unanimously across the spectrum of factors, namely: quality, value, low volatility and momentum. The interesting point to note is that Bender and Wang (2016) found interaction effects to add excess return even when the factors in question were negatively correlated.

2.5 Portfolio Construction

Over time, there have been many techniques/methods proposed to construct portfolios. Portfolio construction can vary in level of complexity. There are methods that simply tilt portfolios towards various fundamental characteristics and there are methods that revolve around solving complex optimisation problems. The former category encompasses the Simple Multiplier approach developed by Bender and Wang (2015, 2016). The latter category encompasses anything from basic modern portfolio theory to Black and Litterman (1991) optimisation or Meucci (2005) optimisation extensions. This section outlines the aforementioned approaches and gives a brief discussion of rank based optimisation.

2.5.1 Simple Multiplier Method

This basic approach of portfolio tilting towards specific factors is given in Bender and Wang (2016). Brandhorst (2013) also implements a very similar technique but with slight differences in the method of ranking. Through a long only, simplistic strategy of applying multipliers to prior preliminary weightings (possibly after an optimisation has occurred), portfolios can be tilted towards a factor in a succinct and effective manner. Bender and Wang (2016) recognise that complex portfolio construction procedures add little benefit in some empirical studies. The approach is outlined below,

$$w_i = w_{i,marketcap}\lambda_i \tag{2.3}$$

where w_i are the adjusted weights, the $w_{i,marketcap}$ are the weightings of the stocks by market capitalisation and λ_i's are the scores of the stocks according to the factor used (Bender and Wang, 2016). This method could cause the weightings to be greater than 100% and thus should be re-weighted using the following equation:

$$w_i^* = \frac{w_i}{\sum_{i=1}^{N} w_i} \tag{2.4}$$

where w_i^* are the final weights and N is the total number of stocks.

The $w_{i,marketcap}$ in (2.3) need not be the market cap and could be replaced by equal weightings, $w_{i,equal}$. In other words the $w_{i,marketcap}$ could be replaced by $1/N$.

2.5.2 Black Litterman Model

This model is an extended mean-variance type optimisation problem whereby the expected return vector can take into account investor views. Advantages of this approach over the generic Markowitz (1952) Modern Portfolio Theory approach, include the fact that the expected returns are now more accurate and less random. Final weightings under the Markowitz (1952) model also often blow up to massive long and short positions which are extremely unrealistic. The Black and Litterman (1991) model attempts to overcome these problems. The basic approach is outlined below. N is the number of assets and K is the number of views for the remainder of this Black and Litterman (1991) outline.

Black and Litterman (1991) use neutral equilibrium returns as a starting point and this neutral return can be computed as follows,

$$\Pi = \varphi \Sigma W_{marketcap}$$

where Π is the implied equilibrium return vector $(N \times 1)$, φ is a risk aversion coefficient, Σ is the covariance matrix of returns $(N \times N)$ and $W_{marketcap}$ are the market capitalisation weightings.

The famous Black and Litterman (1991) formula is,

$$E[R] = [(\psi\Sigma)^{-1} P' \Lambda^{-1} P]^{-1} [(\psi\Sigma)^{-1} \Pi P' \Lambda^{-1} Q]$$

where $E[R]$ is the posterior combined $(N \times 1)$ return vector, ψ is a scalar, P is an identifier matrix that recognises the views $(K \times N)$ matrix, Λ is the $(K \times K)$ diagonal covariance matrix of the error terms of the views and Q is the view vector $(K \times 1)$. All other parameters are defined as above.

Examples of two types of views are included for the purposes of understanding:

1. *Global bonds will have an excess return of 3% (Confidence level = 30%)*
2. *Financial Equities will outperform Resources Equities by 30 basis points (Confidence level = 25%)*

2.5.3 Rank Based Optimisation

Similar to Modern Portfolio Theory, this rank based approach optimises portfolio weights according to highest rank whilst simultaneously minimising rank variance. Satchell and Scowcroft (2003) developed this method of stock rank mean-variance optimisation.

Suppose there are N assets ranked according to a specific metric and each asset is assigned to an M-ranked portfolio (with $2 \leq M \leq N$). Note that assets are ranked in descending order. Let $j = 1, ..., M$ and let R_t be $(N \times 1)$ rank vector at time t (where $t = 1, ..., T$). Now denote the $(N \times 1)$ expected rank vector and the $(N \times N)$ rank covariance matrix by μ and Σ respectively.

The sample mean vector, $\hat{\mu}$ can be readily calculated by,

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} R_t$$

The sample covariance matrix, $\hat{\Sigma}$ can be readily calculated by,

$$\hat{\Sigma} = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \hat{\mu})^2$$

The rank covariance matrix, Σ, is singular as having $(N-1)$ ranks means that the last rank is known by default (Satchell and Scowcroft, 2003). This important property means that the dimension of the null space is 1 and the matrix cannot be inverted. There are various simple methods that can be used to overcome this problem. The method chosen is to randomly choose $(N-1)$ assets. This ensures that it is impossible for the optimal portfolio to fall on the efficient frontier but when the number of assets becomes large, this difference becomes negligible (Satchell and Scowcroft, 2003). Now define Σ_{N-1} to be the $(N-1) \times (N-1)$ covariance matrix and define μ_{N-1} to be the $(N-1) \times 1$ expected rank vector. Finally, define W_{N-1} to be the $(N-1)$ weight vector. The optimisation problem then becomes,

$$\begin{aligned} \underset{w}{\text{minimise}} \quad & W_{N-1}^{'}\Sigma_{N-1}W_{N-1} \\ \text{subject to} \quad & \mu^P = W_{N-1}^{'}\mu_{N-1}, \quad W_{N-1}^{'}e = 1 \end{aligned}$$

where μ^P is the expected rank of the optimal portfolio and e is a $(N-1) \times 1$ vector of ones. The Lagrangian multiplier is given by,

$$W_{N-1}^P = \delta\Sigma_{N-1}^{-1}e + \epsilon\Sigma_{N-1}^{-1}\mu_{N-1}$$

where

$$\delta = \frac{D\mu^p - E}{DF - E^2}$$
$$\epsilon = \frac{F - E\mu^p}{DF - E^2}$$

and

$$D = e^{'}\Sigma_{N-1}e$$
$$E = e^{'}\Sigma_{N-1}\mu_{N-1}$$
$$F = \mu_{N-1}^{'}\Sigma_{N-1}\mu_{N-1}$$

2.6 Rank Correlation

In this investigation, numerous instances of correlation and interaction exist. Correlation, for the purposes of this study, will be based on rank/order as opposed to quantity. Thus, general correlation cannot be implemented and different measures of calculating correlation need to be explored. There are various methods that can be used to compute rank correlation but this study focuses on the most common.

2.6.1 Spearman's Rank Correlation Coefficient

Spearman (1904) developed the following method of computing rank correlation.

Let n be the sample size and let the ith raw scores be denoted X_i and Y_i. Define the ith ranked scores as X_i^r and Y_i^r.

$$\rho = \frac{cov(X^r, Y^r)}{\sigma_x^r \sigma_y^r}$$

where cov(. , .) is the covariance between the two random variables. X^r and Y^r are defined as above and σ_x^r and σ_y^r are the standard deviations of the rank variables X^r and Y^r respectively.

When the ranks are distinct integers,

$$\rho = 1 - \frac{6\Sigma_{i=1}^n d_i^2}{n(n^2-1)} \tag{2.5}$$

where $d_i = X_i^r - Y_i^r$.

Chapter 3

Outline of Empirical and Mathematical Framework

3.1 Data and Methodology

3.1.1 Dataset

The JSE Top 100 was used as the base stock screening universe in order to capture as high a level of diversification as possible. Data was acquired from *Bloomberg* and *Thomson Reuters* terminals and combined to form a complete set of fundamental data. The two datasets were compiled using identical data specifications and the numerical values were verified via comparison and overlap. There were many concerns over the quality and validity of the data. *Bloomberg* and *Thomson Reuters* possessed missing entries and thus by incorporating both datasets, the number of null entries was minimised. Even after these datasets has been merged, there was still around a 5% error rate. With these missing pieces of data, the most recent known value was used. In the cases where errors prevailed and there was no known value, the stock was omitted from the dataset for that date. The period of investigation begins on 30th September 2001 and goes through to the 30th September 2016. The data was extracted monthly for the 15 year period under consideration. This period, over which the investigation took place, incorporates many tail-end events that the market has experienced such as the Financial Crisis of 2007/2008 as well as periods of excessive performance. For this reason, this period was chosen to accurately reflect adverse market events and volatility.

Before the methodology is discussed in detail, the table below highlights the data captured for each factor used in this study.

The JSE All Share Index will be used as a benchmark for performance as well as an indicator for market return. Total return values are used for all stock and indices with transaction costs being charged at 100 basis points.

Tab. 3.1: Factor Criteria Used in this Study

Factor	Criterion
Quality	Return on equity
Value	Price to book
Low Volatility	Standard deviation (3 year)
Momentum	12 month return less most recent month's return

3.1.2 Methodology

The methods devised to test this framework are largely based on Bender and Wang (2016). The reasons for this level of similarity include tractability and comparability between the studies.

The portfolios under investigation are those titled towards quality, value, low volatility and momentum. Of course, these portfolios will then be combined to form the Combination portfolio (which serves as an independent portfolio, eliminating all correlation effects between the factors themselves) and a Bottom-Up portfolio (comprising of all factors averaged out from the outset). This Bottom-Up portfolio will thus incorporate interaction effects between all four factors. These portfolios will be defined in a more rigorous manner in the following sections.

In order to keep in line with the realistic nature of portfolio construction, the portfolios will be rebalanced on a monthly and quarterly basis. The reason for this is once again investment sensibility. The rebalancing procedure is simple; every period the stocks are ranked according to a specified fundamental factor and the portfolio weights are recalculated to encompass this new information. This tilting and re-weighting process is given in equation (2.3) and (2.4). The Top 20 stocks for each factor are used to construct all the portfolios. It is important to note that the JSE Top 100 stocks are re-screened at every rebalancing point in order to avoid a survivorship bias. Finally, one of the downfalls of active management is the high cost associated with it and thus transaction costs are charged at 100 basis points.

3.2 Choice of Portfolio Construction Method

The problem of interest is not one of optimisation but rather a problem of performance due to portfolio tilting. As a result, implementing these complex optimisation procedures will not be of benefit and a simpler, more tractable method will be utilised. The question of how to incorporate fundamental factors into a portfolio now arises. The preferred approach of such a task is the long only multiplier method outlined in (2.3) and (2.4). For the purposes of this study, the multiplier in (2.3) will be the ranking based on factor score. When assigning scores to securities, one must tread with care as scores can become negative. Thus, in order to overcome this problem, securities need to be ranked according to score. But one might ask the question, why even calculate scores if they are synonymous to direct rank? The answer to this question lies in the methods that will be implemented to capture various interaction effects between the factors themselves and will be discussed further in the next section. Scores are calculated in the following manner,

$$S_i = \frac{x_i - \bar{x}}{\eta} \tag{3.1}$$

where S_i is the score of the security, x_i is the value of the factor, $\bar{x}$ is the mean of the factor values and η is the standard deviation of the factor values.

3.3 Interaction Effects

In order to unbundle the interaction effects of the individual factors themselves, two portfolios have been devised (Bender and Wang, 2016). These portfolios are constructed specifically to test whether interaction effects do indeed have an impact on portfolio return.

3.3.1 Combination Approach

The Combination approach broadly refers to constructing independent portfolios based on each individual factor and then independently combining the various portfolios together. This portfolio contains no interaction effects. The stock universe is screened and each stock is ranked according to a specific fundamental factor such as return on equity (quality). The higher the return on equity, the higher the weight allocated to a certain stock.

Now that the single factor portfolio has been constructed, we can combine other single factor portfolios based on different factors by simply weighting each of the portfolios.

$$R^{**} = \kappa_1 r^Q + \kappa_2 r^V + \kappa_3 r^{LV} + \kappa_4 r^M. \tag{3.2}$$

where R^{**} is the final return of the Combination portfolio, the κ_i's are merely weights with $\sum_i \kappa_i = 1$ and r^Q is the portfolio return of the quality portfolio and r^V the value portfolio and so on.

3.3.2 Bottom-Up Approach

The Bottom-Up approach refers to the construction of a correlated portfolio based on multiple factors simultaneously (Bender and Wang, 2016) and incorporates the interaction effects,

$$\gamma_i = \kappa_1 S_i^Q + \kappa_2 S_i^V + \kappa_3 S_i^{LV} + \kappa_4 S_i^V \tag{3.3}$$

and $\lambda_i = rank(\gamma_i)$ where the λ_i is the same λ_i in (2.3) and S_i's are the scores of each respective factor (defined synonymously to (3.1) above). Note that κ in (3.2) and (3.3) will be the same for comparability between approaches.

3.3.3 Interaction Pairs

It is difficult to attribute the outperformance of either portfolio to single factor pair interaction as all four factors are being combined and tested in the Bottom-Up and Combination frameworks. Therefore, in order to extract the individual interaction effects, several factor pairs will be evaluated. Keeping in line the methodology in Bender and Wang (2016), the following six pairs will be evaluated:

- Quality - Value
- Quality - Low volatility
- Momentum - Quality
- Momentum - Low volatility
- Momentum - Value
- Value - Low volatility

To begin, the distributions of the various pairs need to be analysed. In order to obtain the distributions, each factor needs to be considered in isolation. The distributions will be obtained through a simple ranking procedure whereby the current stock universe will be divided into quartiles based on the rank of each factor individually. The stocks that rank in the highest quartile receive a rank of 4, second quartile receive a rank of 3 and so on. Now, by summing up the rank of each stock over the factor pair being tested, we can observe a distribution starting to take place. Clearly the highest rank for each factor pair will be 8 and the lowest will be 2. For example, if we are evaluating the quality - value pair, and the stock achieved a score of 4 for quality and 1 for value, the stock will be allocated a rank of 5.

These distributional differences will help contextualise the reasons for divergence between the Bottom-Up and Combination portfolios. They will also enable us to infer reasons for possible outperformance. Finally, we will be able to hypothesise which pairs will perform better based on their interaction effects. The dataset used to derive the distributions of the various factor pairs is based on the JSE Top 200 as at September 2016 to increase accuracy and relevance.

3.4 Mathematical Market Model

3.4.1 Introduction

Any empirical study is heavily reliant on its underlying dataset, and results can differ drastically from period to period. Some studies may deem a particular fundamental factor paramount in driving stock return whilst others may conclude the opposite. This volume of empirical research in the field of finance lacks the much needed rigour. Thus, some mathematical justification is necessary before we commence such a study. It is for this reason that a controlled market model will be constructed in order to truly discover how these fundamental factors interact with one another. Real-life data can be emulated in this mathematical model and stock price paths can be simulated and made to follow a particular underlying statistical distribution. Once realistic data has been reproduced, the empirical methodology can be followed to determine the impact of the factor interactions. Rebalancing in this controlled model will be ignored as change in market factors are a consequence of unexplained market noise which can be modelled as a stochastic process in itself. However, for the purpose of this simulation, factors are assumed to be constant throughout the 15 year period of investigation.

3.4.2 Simulation

This mathematical model will require simulation in two parts. The first being the simulation of stock price paths and the second, simulation of the factor portfolios with the desired correlation structures.

Part 1: Stock Price Paths

Stock price paths will develop stochastically and follow a Geometric Brownian Motion (GBM) process. GBM offers significant benefits such as independence of stock return to stock price but does not come without drawbacks, including constant volatility and exclusion of stock jumps. Although GBM does not offer a perfect map to reality, its simplicity and tractability warrant such a process. The methodology will deviate slightly from standard GBM by incorporating an additional stochastic drift component to emulate the random alpha that can be achieved by active management. This process is given by the following dynamics,

$$d\vec{S}_t = D[\vec{S}_t]((\vec{\alpha}_t + \mu_0)dt + \sigma d\vec{W}_t^1) \quad (3.4)$$

$$d\vec{\alpha}_t = \vec{a}dt + bd\vec{W}_t^2 \quad (3.5)$$

$\vec{S}_t$ is a stock price vector and $D[.]$ denotes a matrix with the enveloped vector down the diagonal and zeros elsewhere. μ_0 is the market drift or beta return, σ is the stock volatility matrix, $\vec{a}$ is the α_t drift vector and b is the α_t volatility matrix. $\vec{W}_t^1$ and $\vec{W}_t^2$ are correlated standard brownian motion vectors.

Through an application of Ito's Lemma,

$$\alpha_{t_i} \sim N(\alpha_0 + a_i t, b_{ii}^2 t) \quad (3.6)$$

and

$$ln(S_t)_i \sim N(ln(S_0)_i + (\mu_0 + \alpha_0 + a_i t - \frac{1}{2}\sigma_{ii}^2)t, b_{ii}^2 t^3 + \sigma_{ii}^2 t) \quad (3.7)$$

where S_0 is the initial stock price, t is time, α_0 is the initial value of the α process and all other parameters are defined as above.

Equations (3.6) and (3.7) can be easily extended to a multi-dimensional setting.

Finally, in reality, stocks in similar industries or exposed to similar macro factors will be correlated. Thus, through the use of σ and b, a pre-specified correlation structure can be transferred into the stocks. In addition to correlation between individual stocks, there is also a correlation between each stock and the corresponding α. Correlation matrices are simulated randomly but must be forced to be positive definite for a Cholesky decomposition to be used.

Part 2: Factor Portfolios

In order to accurately reflect market data, realistic correlation structures between the individual factors will need to be transferred into the simulation. After observing JSE market data, it becomes apparent that quality, value, low volatility and momentum all follow uniform distributions (see chapter 4). The ultimate goal is to generate four correlated uniform random variables with the desired Spearman correlation structure. The first problem lies in the method of generating correlated uniform distributions. This is more complex and less efficient than generating correlated normal random variables and then transforming the distribution. Thus, these four factors are simulated by generating four independent standard normal random variables. The random variables are then fitted with the desired Spearman correlation structure (so as to reflect the South African market) using a Cholesky decomposition. The problem with directly transforming these normal variates into uniform variates, is that there is no guarantee that the desired Spearman correlation structure will be preserved. Hotelling and Pabst (1936) provide a two step process that requires an adjustment from Spearman correlation into Pearson correlation, followed by a transformation back into Spearman correlation. The method is outlined below,

$$\Sigma^P = 2sin(\frac{\pi}{6}\Sigma^S)$$

where Σ is the desired Spearman correlation matrix and Σ_P is the adjusted Spearman correlation matrix (or Pearson correlation matrix) that preserves the correlation structure under the transform.

The adjustment converts the Spearman correlation into Pearson correlation between the normal variates. The normal variates can now simply be transformed into uniform variates using the inverse probability integral transform relationship i.e. $F(Z) = U$. Z is a correlated normal random variable, $F(.)$ is the cumulative distribution function of the enclosed random variable and U is the correlated uniform random variable. Intuitively, it may seem that we are transferring the Pearson correlation structure into the uniform variates, but actually the inverse probability integral transform gives rise to the relationship below.

$$\Sigma^S_{X,Y} = \Sigma^P_{F(X),F(Y)} \tag{3.8}$$

where $\Sigma^S_{X,Y}$ is the Spearman correlation matrix between random variables X and Y and $\Sigma^P_{F(X),F(Y)}$ is the Pearson correlation matrix between $F(X)$ and $F(Y)$. Thus, we can see that, from (3.8), a Pearson correlation between the normal variates is a Spearman correlation between the uniform variates (Hotelling and Pabst, 1936).

3.4.3 Discretisation Method

The underlying Ito dynamics and distributions have been determined and all that remains, is to efficiently discretise the process. A Mil'shtein (1974) method will be used to propagate the stock price process through time and calculate returns. Mathematically, the Mil'shtein (1974) method is given as,

$$\hat{S}_i = \begin{cases} S_0 \text{ if } i = 0 \\ \hat{S_{i-1}} \exp((\mu_0 + \hat{\alpha}_i - \frac{1}{2}\sigma^2_{ii})\Delta t + (b_{ii}\Delta t^{1.5} + \sigma_{ii}\sqrt{\Delta t})Z_{1,i} \text{ if } i > 0 \end{cases}$$

where

$$\hat{\alpha_i} = \begin{cases} \alpha_0 \text{ if } i = 0 \\ \hat{\alpha_{i-1}} + a_i \Delta t + b_{ii}\sqrt{\Delta t} Z_{2,i} \text{ if } i > 0 \end{cases}$$

Δt is the incremental change in time and $Z_{1,i}$ and $Z_{2,i}$ are the standard normal random variables generated with a correlation. All other parameters are defined as above.

3.4.4 Market Parameters

The market model will be constructed in a manner similar to that provided by Black and Scholes (1973). The simplifying assumptions that will be used are:

1. Taxes and transaction costs will be disregarded.
2. Interest rates and volatilities will remain constant.

Stock return can be decomposed into parts, namely market return (μ_0 in (3.4)) and excess return or alpha (α_t in (3.5)). Non-stochastic drift and volatility components will be assumed constant and will be quantified using market data. The stocks are then ranked according to their factor scores, which will inherently characterise the λ multipliers in (2.3), and according to these rankings, the alpha component will be chosen. Assuming the expected returns of a stock increase with rank, the alpha process will be constrained to the following,

$$a_i \sim \mathcal{U}\left(\frac{R_i^Q}{1000} - 0.003, 0.002 + \frac{R_i^Q}{1000}\right) \tag{3.9}$$

where R_i^Q is the ranking with respect to quality. The Q can be replaced by value, low volatility, momentum and the Bottom-Up ranking.

This alpha drift in (3.9) above, is a function of the ranking given to the factor. This is because the higher the ranking according to a specific factor, the higher the expected alpha drift. Rankings will go from 1 - number of stocks simulated.

Table 3.2 contains the values of the remaining parameters needed to create the mathematical market. All parameters are chosen so that they reasonably reflect real market data.

Tab. 3.2: Model Parameters

Parameter	Description	Value
S_0	Starting stock price	100
α_0	Initial value for α_t process	0
n_i	Number of shares issued	$\mathcal{U}(5_e6; 5_e8)$
b_{ii}	Alpha process volatility	$\mathcal{U}(0.001; 0.006)$
μ_0	Market return	0.018 (0.15 Annualised)
σ_{ii}	Diagonal of covariance matrix	$\mathcal{U}(0.01; 0.06)$
ρ_{ij}	Correlation matrix entries	$\mathcal{U}(-1; 1)$

where 5_e6 refers to 5×10^6 and so on. All values are given from a monthly perspective (where applicable) due to the monthly computation.

3.4.5 Results

The controlled market model was simulated 1000 times to allow for convergence to its true result. The market was constructed with a portfolio consisting of 100 correlated stocks. No rebalancing is employed in this controlled model as the reason these factors are randomly changing is due to market noise. The model was tested to see how many simulations out of 1000 would result in the Bottom-Up portfolio outperforming the Combination portfolio and the result clearly converges to 50%. This result persisted across all factor correlations tested. Whether a correlation of 0.9 or -0.9 was used between factors, the result converged towards 50%. Thus, this indicates that these interaction effects either do not exist or do exist but do not have an effect on portfolio return.

Chapter 4

Empirical Results of Backtest

This empirical study aims to uncover whether these interaction effects exist. Intuition says that there should be no difference in the Combination and Bottom-Up approaches as long as the same weighting and ranking procedure is used. This is shown to be incorrect by Bender and Wang (2016). This difference in portfolio construction approach is attributed to this interaction effect between the individual factors. This arises from the correlations and joint distributions between the individual factors (Bender and Wang, 2016). The empirical tests, in this study, are set out to evaluate the claims made by Bender and Wang (2016). The distributions of the individual factors as well as the factor pairs are given in the first section. Market capitalisation weights are used as starting weights in the second section in order to resemble the weightings of the JSE All Share Index constituents. In the third section, equal starting weights are used.
For the rest of this study, Spearman rank correlation is merely referred to as correlation. The dates given on the figures and plots begin at 30 September 2001. They run from 0 - 180 (in the case of monthly rebalancing) depending on the number of months thereafter. The dates run from 0 - 60 (in the case of quarterly rebalancing) depending on the number of quarters thereafter.

4.1 Factor Distributions

Figure 4.1 illustrates the distributions of the individual factors. It is clear that the factors are uniformly distributed and from Table 4.1, we see that the factors are correlated. Figure 4.2 illustrates the joint distributions of the factor pairs. From these distributional histograms, it looks as if quality-value and momentum-value have the largest distributional differences and thus should cause the largest disparity between the Bottom-Up and Combination portfolios. Similarly, Bender and Wang (2016) conclude that the quality-value and momentum-value pair exhibit the greatest differences between the Combination and Bottom-Up portfolios.

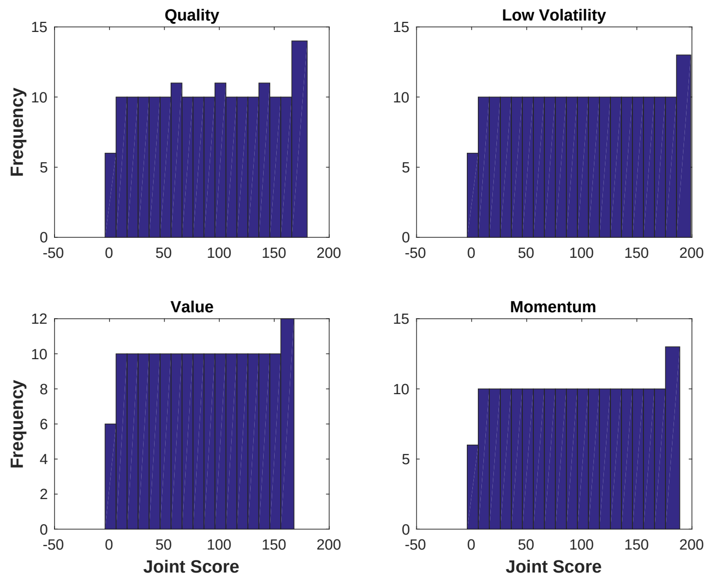

Fig. 4.1: Individual Factor Distributions

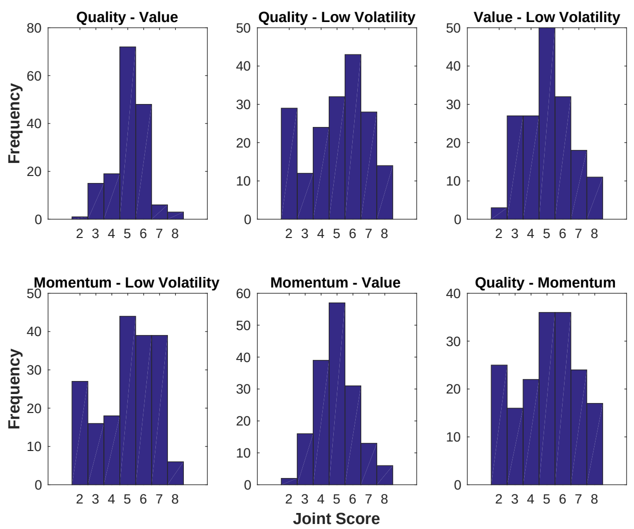

Fig. 4.2: Joint Distributions

Tab. 4.1: Spearman Rank Correlation Between Factors

	Quality	Value	Low Volatility	Momentum
Quality	1.0000	-0.6050	0.3634	0.4463
Value	-0.6050	1.0000	-0.1492	-0.3604
Low Volatility	0.3634	-0.1492	1.0000	0.2965
Momentum	0.4463	-0.3604	0.2965	1.0000

4.2 Market Capitalisation Weights

The problem with this approach lies in the ability of certain heavily weighted stocks to drive the return. When it comes to resource stocks, they are extremely cyclical and could distort certain results. With that being said, it is useful to test this framework using market capitalisation starting weights as this best resembles investment decisions in reality. Below in Figure 4.3 and 4.4 and Table 4.2, we can see that the low volatility portfolio achieves the highest return and experiences the lowest volatility, thus leading to the highest Sharpe ratio. This seems to be a slight anomaly and it could be due to a single heavily weighted stock that is driving the process. The Bottom-Up portfolio achieves the lowest annualised return in the monthly rebalancing scheme but seems to have outperformed the Combination portfolio for the majority of the period under investigation. The Combination portfolio does experience a lower volatility and thus, from Table 4.2, the Sharpe ratio is lower. In the quarterly rebalancing scheme (Figure 4.4), the Bottom-Up portfolio outperforms the Combination portfolio across the entire period of investigation. Furthermore, the Combination portfolio achieves a larger Sharpe ratio over both rebalancing frequencies. Quarterly rebalancing outperforms monthly rebalancing across the board.

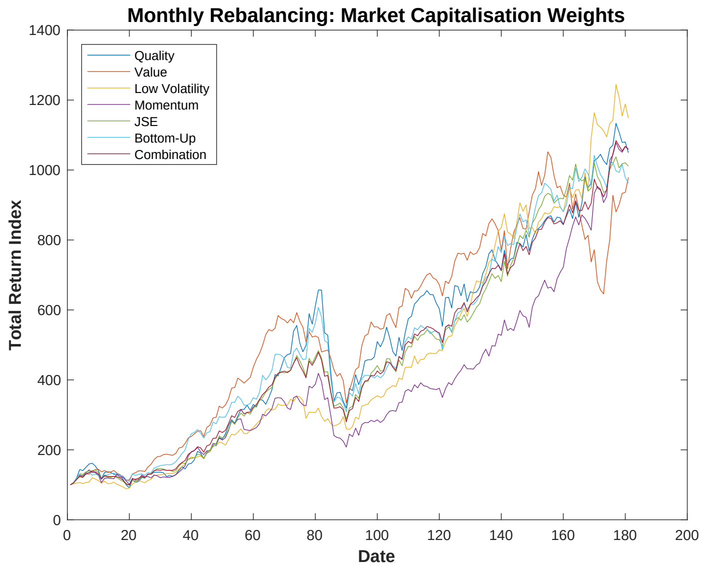

Fig. 4.3: Factor Portfolios: Monthly Rebalancing

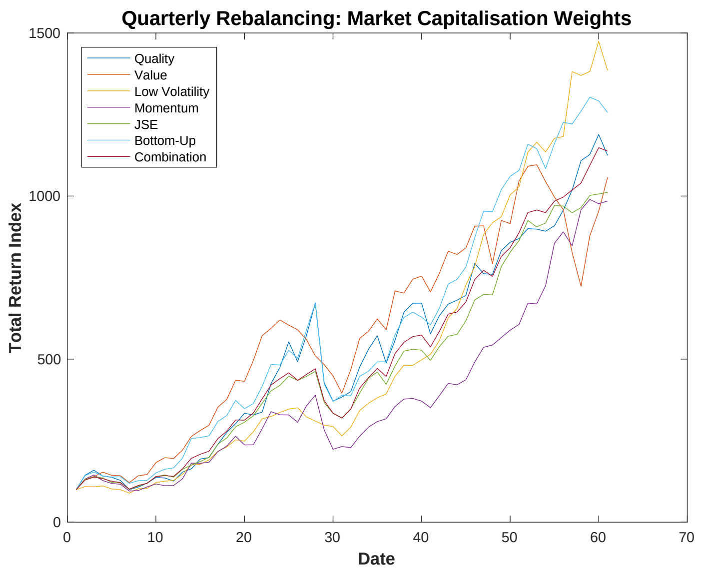

Fig. 4.4: Factor Portfolios: Quarterly Rebalancing

Appendix A contains the 12 month rolling returns of the Bottom-Up and Com-

Tab. 4.2: Summary of Monthly and Quarterly Rebalancing: Market Capitalisation Weights

	Quality		Value		Low Volatility		Momentum		Combination		Bottom-Up	
	M	Q	M	Q	M	Q	M	Q	M	Q	M	Q
Annualised Return	0.1697	0.1751	0.1643	0.1703	0.1767	0.1915	0.1704	0.1647	0.1704	0.1760	0.1629	0.1838
Annualised Volatility	0.2366	0.2451	0.1789	0.2098	0.1528	0.1415	0.2034	0.2205	0.1583	0.1682	0.1951	0.2184
Excess Return	0.0029	0.0083	-0.0025	0.0035	0.0099	0.0247	0.0036	-0.0021	0.0036	0.0092	-0.0039	0.0170
Sharpe Ratio	0.4212	0.4287	0.5270	0.4780	0.6983	0.8586	0.4937	0.4295	0.6339	0.6302	0.4762	0.5209

bination portfolios. Figure A.1 illustrates the rolling returns of the monthly rebalancing scheme. In this plot, the Bottom-Up portfolio seems to outperform the Combination portfolio over the first 4 years. The Bottom-Up portfolio seems to be more volatile than the Combination portfolio and is more sensitive to market events. The 2007/2008 Financial Crisis is clearly depicted in Figure A.1 (around the 90 mark) and it appears that the Bottom-Up portfolio reacts more severely during this time. This aligns with the notion that the Bottom-Up portfolio is far more responsive to periods of recession. The Combination portfolio outperforms the Bottom-Up portfolio quite significantly in the last year of investigation. This could also be a result of the Bottom-Up portfolio overreacting to market fluctuations.

Figure A.2 portrays the rolling returns of the quarterly rebalancing scheme. The pattern in Figure A.2 largely follows that of Figure A.1 and similar observations can be made. Overall, it can be observed that the portfolio rebalancing makes little difference to the performance of the Bottom-Up and Combination portfolios.

4.2.1 Interaction Pairs

The interaction pairs discussed in the previous chapter are evaluated in Figure 4.5 and 4.6. It is worth analysing these figures in conjunction with Table 4.1 in order to infer relationships between the correlations and the interaction effects. From Table 4.1 the correlation between quality and value is -0.6050, but the Bottom-Up portfolio outperforms the Combination portfolio in the quality-value plot which seems counter-intuitive. The Combination portfolio outperforms the Bottom-Up portfolio in the momentum-value plot with the pair exhibiting a correlation of -0.3604. Now, referring to Figure 4.6, it is clear that the portfolios are very similar to when the monthly rebalancing scheme is employed. At this point, the correlations seem random and there is no evidence that these correlations are having any effect on the interaction effects. The Bottom-Up portfolio does, however, outperform the Combination portfolio in most pairs. Figure 4.5 and 4.6 confirm the hypothesis made from Figure 4.2 that quality-value and momentum-value do have the largest differences in performance.

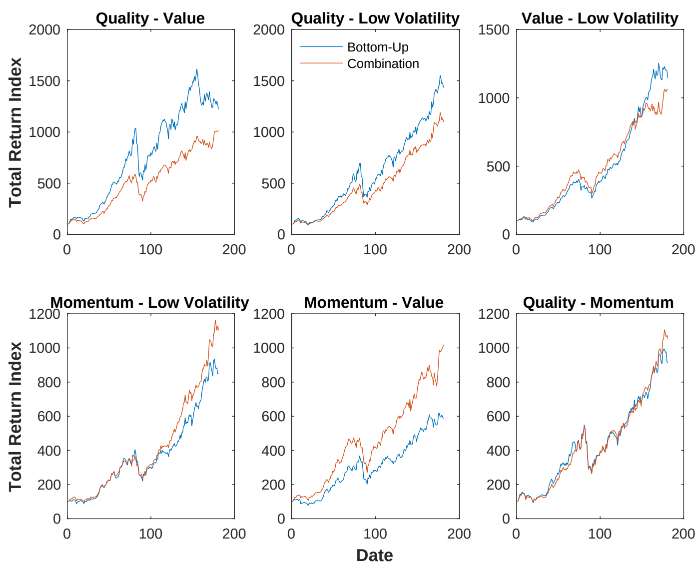

Fig. 4.5: Interaction Pairs: Monthly Rebalancing

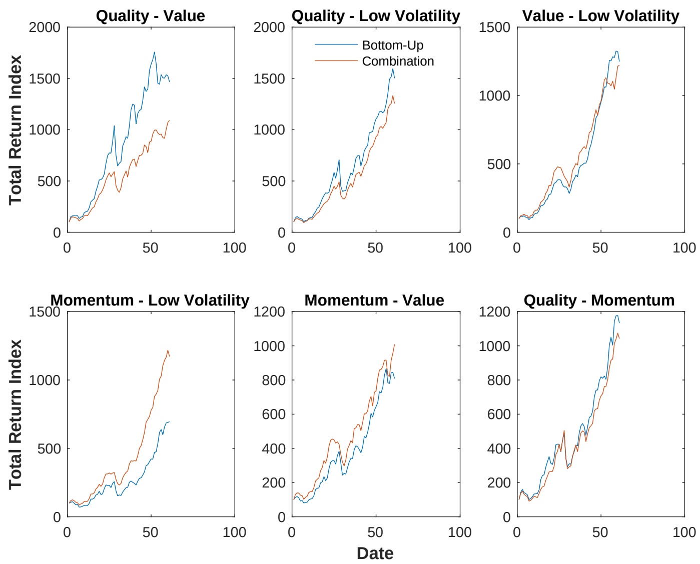

Fig. 4.6: Interaction Pairs: Quarterly Rebalancing

4.3 Equal Weights

The size effect, discussed in Fama and French (1993), is also omitted as there is a concern that it may be present when using market capitalisation weights as starting weights. For this reason, this section uses equal stock weightings as a starting reference point in order to eliminate all traces of the size effect. This method of starting with equal weights is not practical in reality, however for the purpose of this study, it does eliminate this confounding effect of size mentioned above. Thus, although not realistic, the equal weights environment may give some valuable insight into these factor interactions.

Now, when looking at Figure 4.7, one cannot help but notice the stability of returns (with the exception of value). The Bottom-Up portfolio has the highest performance while the low volatility portfolio exhibits the lowest returns. This confirms that the reason the low volatility portfolio attained such high excess return, when using market capitalisation weights, was due to one highly weighted stock driving return. Low volatility still achieves the largest Sharpe ratio unanimously under monthly rebalancing due to such a low annualised volatility. It is also worth noticing the portfolio dips during the 2007/2008 Financial Crisis. All portfolios follow the same pattern and dip around the 2007/2008 mark. The Bottom-Up portfolio exhibits the largest dip during this period and the low volatility portfolio changes

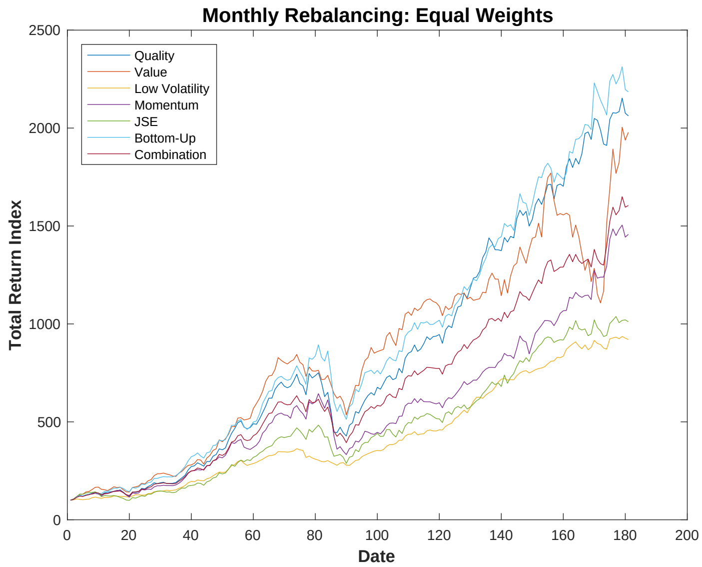

Fig. 4.7: Factor Portfolios: Monthly Rebalancing

Tab. 4.3: Summary of Monthly and Quarterly Rebalancing: Equal Weights

	Quality		Value		Low Volatility		Momentum		Combination		Bottom-Up	
	M	Q	M	Q	M	Q	M	Q	M	Q	M	Q
Annualised Return	0.2235	0.2228	0.2202	0.2188	0.1595	0.1632	0.1956	0.1999	0.2033	0.2042	0.2283	0.2106
Annualised Volatility	0.1745	0.1614	0.1942	0.2125	0.0915	0.1068	0.1767	0.1845	0.1426	0.1524	0.1659	0.1574
Excess Return	0.0568	0.0561	0.0534	0.0520	-0.0073	-0.0036	0.0288	0.0332	0.0365	0.0374	0.0615	0.0438
Sharpe Ratio	0.8799	0.9471	0.7732	0.7003	0.9781	0.8726	0.7105	0.7044	0.9343	0.8804	0.9543	0.8934

minimally. This gives an indication of the sensitivity of the Bottom-Up portfolio to market events.

The quarterly rebalancing employed in Figure 4.8 alters the portfolios total return minimally, but decreases the performance of the Bottom-Up portfolio relative to the others. The Sharpe ratios, in Table 4.3, of the Bottom-Up portfolio are more attractive than the Combination portfolio across all rebalancing frequencies.

Appendix A also contains the 12 month rolling returns of the Bottom-Up and Combination portfolios when starting with equal weights. Figure A.3 and Figure A.4 illustrate the monthly and quarterly rebalancing schemes respectively. The Bottom-Up portfolio, in Figure A.3, outperforms the Combination portfolio for the majority of the period of investigation. Figure A.4, however, shows a massive increase in performance of the Combination portfolio across the board. This seems contradictory to the notion that quarterly rebalancing improves the performance of the Bottom-Up portfolio.

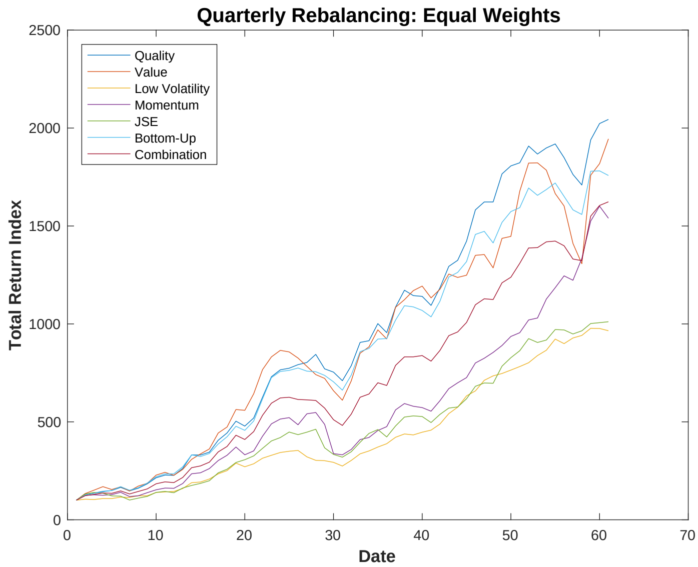

Fig. 4.8: Factor Portfolios: Quarterly Rebalancing

4.3.1 Interaction Pairs

The individual factor pairs are now evaluated to determine areas in which the interaction effects are strongest. Figure 4.9 and 4.10 below, both exhibit improved performance in the Bottom-Up portfolio relative to the Combination portfolio. The quality-low volatility pair has the biggest disparity between the Bottom-Up and Combination portfolios and these factors have a correlation of 0.3634 which could explain some of the positive interaction. In the quarterly results, little changes between the pairs, with the exception of all Bottom-Up portfolios achieving higher returns across all pairs. Quality-momentum also has large disparity between the Bottom-Up and Combination portfolios when compared to Figure 4.5 where the two portfolios barely diverged.

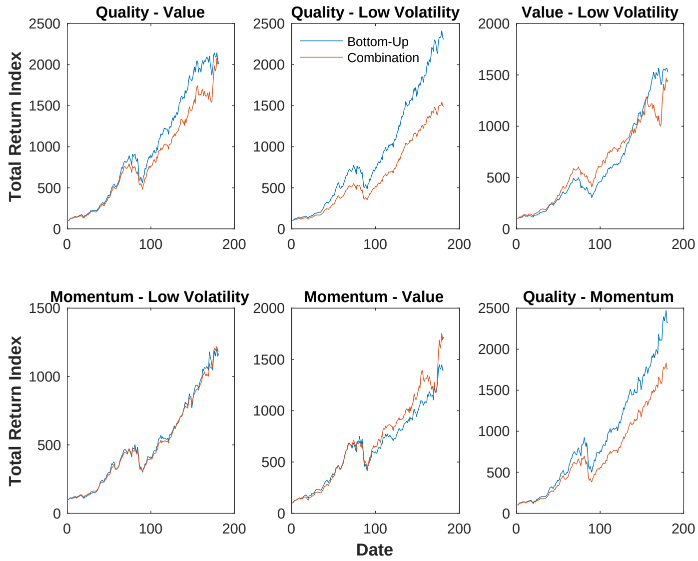

Fig. 4.9: Interaction Pairs: Monthly Rebalancing

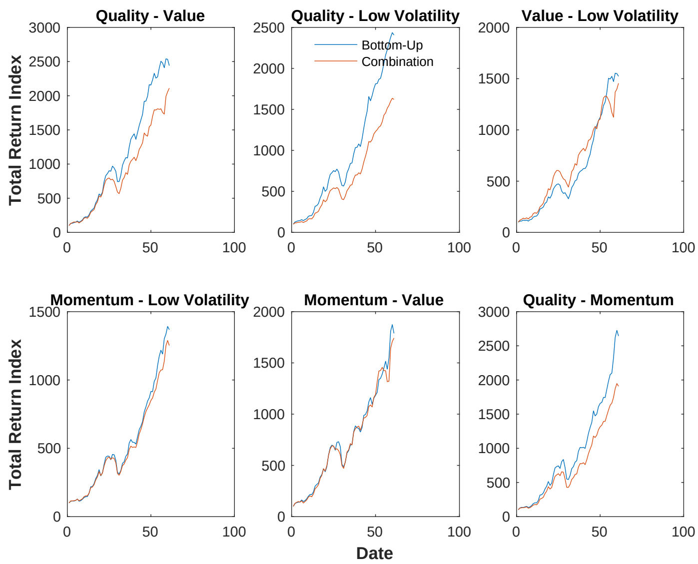

Fig. 4.10: Interaction Pairs: Quarterly Rebalancing

Chapter 5

Conclusion

Factor investing undoubtedly has a massive volume of literature proving its ability to outperform the market. Quality, value, low volatility and momentum have been evaluated by many renowned authors and the results provided are consistent in that investing based on these factors, make for profitable strategies. This study successfully fills a gap in the literature by focusing on interaction effects in a South African context and investigating them in a controlled mathematical model.

Bender and Wang (2016) developed a framework that constructs two portfolios. The first, incorporating the interaction effects between factors and the second, an independent portfolio termed the Bottom-Up portfolio and Combination portfolio respectively. This framework was implemented in a South African setting on the JSE Top 100 and certain additions, such as transaction costs, were included to make the study more realistic.

The distributions of the individual factors were found to be correlated uniform random variables. The joint distributions of quality-value and momentum-value did present the largest disparities between the Bottom-Up and Combination portfolios which is exactly in line with the findings of Bender and Wang (2016). The effect of correlation on interaction effects was sometimes counter-intuitive and no definitive results can be given about this relationship. This requires further investigation. When equal starting weights were used, the differences between the Bottom-Up and Combination portfolios became far more distinct and were in favour of the Bottom-Up portfolio. This may indicate that the size effect was confounding the results of the investigation into market capitalisation starting weights. Bender and Wang (2016) found that the Bottom-Up portfolio unanimously outperforms the Combination approach, not only in annualised return but also in risk adjusted return. The evidence found in this study does point towards the Bottom-Up portfolio in most cases but is not conclusive enough to make definitive statements.

An additional result stemming from this study is that the Bottom-Up portfolio is far more volatile than the Combination portfolio. This effect could be a result of the extra diversification benefits that are present in the Combination portfolio. The Bottom-Up portfolio does not diversify by factor in the same way as the Combination portfolio. The Bottom-Up portfolio focuses more on the diversification of specific factor noise whereas the Combination portfolio works in the same manner as a portfolio attempting to diversify away stock specific noise. From all the empirical results given, the most conclusive was the notion that quarterly rebalancing tends to improve the Bottom-Up portfolio relative to the Combination portfolio.

Any empirical study is only as powerful as its underlying dataset and thus a mathematical model was developed to test the existence of the interaction effects. Geometric Brownian Motion with an added stochastic drift was used to simulate stock paths which were then discretised according to a Mil'shtein (1974) method. Random factors were simulated uniformly with a desired correlation structure. In this mathematical market model, the Bottom-Up and Combination portfolios both exhibited outperformance that converged to 50%. Therefore, the Bottom-Up and Combination portfolios outperformed 50% of the time. This would lead to the conclusion that interaction effects do not exist. However, with this being said, it is not a perfect model of reality and the interactions between the factors could be far more complex than accounted for in the model.

In conclusion, there was an insufficient amount of evidence to determine whether these interaction effects are beneficial to portfolio performance. More research into these interaction effects needs to be conducted in order to fully understand them and whether they have a substantial effect in factor investing.

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Appendix A

12 Month Rolling Returns

A.1 Market Capitalisation Weights

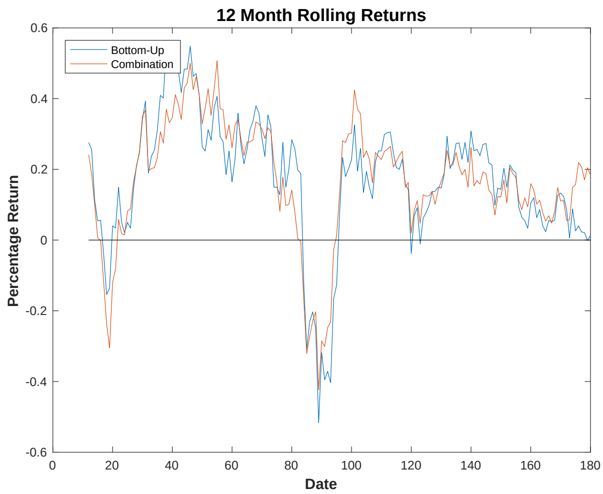

Fig. A.1: Rolling Return: Monthly Rebalancing

Fig. A.2: Rolling Return: Quarterly Rebalancing

A.2 Equal Weights

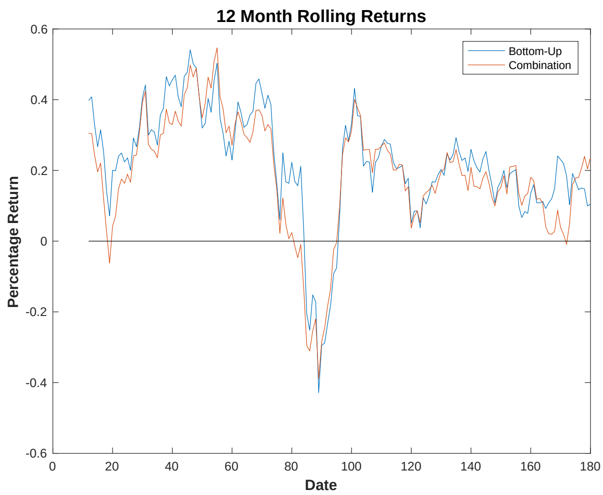

Fig. A.3: Rolling Return: Monthly Rebalancing

Fig. A.4: Rolling Return: Quarterly Rebalancing