An exploration into the sparse representation of Spectra

Linda Mthembu

A dissertation submitted to the Department of Electrical Engineering, University of Cape Town, in partial fulfillment of the requirements for the degree of Master of Science in Engineering

Cape Town, South Africa

February 2007

Declaration

I know the meaning of plagiarism and declare that all the work in the dissertation,save for that which is properly acknowledged,is my own. It is being submitted for the degree of Master of Science in Engineering in the University of Cape Town, Rondebosch, Cape Town. It has not been submitted before for any degree or examination in any other university.

Signature of Author ....

Signed on the 13th of February 2007, Cape Town

Abstract

This thesis describes an exploration in achieving sparse representations of objects,with special focus on spectral data. Given a database of objects one would like to know the actual aspects of each class that distinguish it from any other class in the database. We explore the hypothesis that simple abstractions (descriptions) that humans normally make, especially based on the visual phenomenology or physics on the problem, can be helpful in extracting and formulating useful sparse representations of the observed objects. In this thesis we focus on the discovery of such underlying features,employing a number of recent methods from machine learning.

Firstly we find that an approach to automatic feature discovery recently proposed in the literature (Non Negative Matrix Factorization) is not as it seems. We show the limitations of this approach and demonstrate a more efficient method on a synthetic problem. Secondly we explore a more empirical approach to extracting visually attractive features of spectra from which we formulate simple re-representation of spectral data and show that the identification and discovery of certain intuitive features at various scales can be sufficient to describe a spectrum profile. Finally we explore a more traditional and principled automatic method of analyzing a spectrum at different resolutions (Wavelets). We find that certain classes of spectra can easily be discriminated between by a simple approximation of the spectrum profile while in other cases only the finer profile details are important.

Throughout this thesis we employ a measure called the separability index as our measure of how easy it is to discriminate objects in a database with the proposed representations.

Author : Linda Mthembu

Thesis Supervisor : John Greene, Associate Professor in the Electrical

Engineering Department, UCT

Acknowledgments

I would like to thank my parents, brother and sister for their support throughout my academic career. I would like to thank my supervisor, Professor John Greene for his ideas, suggestions, guidance, support and encouragements throughout this thesis and Professor G. De Jager for his assistance during this project. Special thanks to Patrik Hoyer for making the NMF software package available at his home page.

I would also like to thank the following organizations for their generous financial support throughout this project:

The National Research Foundation

DeBeers Technologies

Thank you

Table of Contents

Chapter 3

Chapter 4

Appendix

A - Separability Index of combined classes.

B - Effect of changing representation from Reflectance to Absorption representation of the separability index.

C - Effect of changing representation from Absorption to On or Off representation of the separability index -with no sensitivity parameter.

D - Effect of changing from absorption to On or Off slope 2 representation of the separability index – No sensitivity parameter.

E - Percentage separability difference between using Reflectance & On or Off representations at different sensitivity parameter (SP) settings.

F - Sparse features with wavelets on On or Off representations.

G - Sparse features with wavelets on Reflectance representation.

List of Figures

List of Tables

Notation and Abbreviations

n - Number of features or dimensions

m - Number of instances/observations/objects.

r - Number of basis

FOI - Features of interest

ICA - Independent Component Analysis.

NMF - Non-Negative Matrix Factorization

MRA - Multi Resolution Analysis

ANN – Artificial Neural Networks

MLP – Multi layer perceptrons

SI - Separability Index

SP- Sensitivity Parameter

TC – Target Class

NTC -Non Target Class

Chapter 1

Introduction

1.1 The need for sparse representation

The field of computational intelligence has come a long way over the past two to three decades producing such focus areas as Neural networks,Bayesian analysis and Support vector machines. In most of these architectures an object or a situation is represented in a vector form;for example the pixels in images, the wavelength intensities in spectrum and player's current positions on a board game. But it is well known that *most* of the sensory measurements are not relevant when one needs to describe an object especially in relation to other objects and naive vector representations tend to be highly redundant. A human,having sensed the whole object, would subsequently try and find only a small subset of salient features to describe the object by possibly asking the following questions:

- what is interesting(striking/appealing etc) about this object
- what could be the governing or the basic underlying structure

especially when given other objects in the domain. The agent would then try and use these features to form a concise re-description of the object. We propose that in machine-based pattern recognition it is worth the researcher's time to explore such underlying-structure-seeking descriptors or representations. This might improve the interpretability of the processing in the above architectures.

Inductive logic systems (as those mentioned above) have little to say about the underlying structure of the objects they are dealing with; what makes these objects different or whether the representations we used are the best for the particular problem. These types of advanced inductive algorithms have yielded improved results but often not significantly more than simpler systems [13]. Indeed algorithms such as k-means, Decision trees, Case Based Reasoning etc, that are based on simple geometric reasoning are still very common and are still often considered to be fundamentally the most significant algorithms in machine learning. Researchers have built systems with many tuning parameters that will tune the algorithm to produce slightly better results but still do not reveal much about the underlying patterns, relationships or important features of the data objects.

This we hypothesize is what a learning agent initially should try to discover in any situation or problem: it should try to compress the situation/object information and maybe store it as little "factoids" for future reference. The hypothesis explored in this thesis is that better results,understanding and design of simple learning systems could be achieved if they were based on the *explicit* discovery,capturing and re-representation of intuitive notions of objects. Most modern techniques in machine learning have not really focused on this aspect of dealing with data.

Some recent research has tended in this direction [12,19,21,32], where parts or features of the given object set are discovered and used to distinguish objects between each other [11,12 25,27]. This approach has the added advantage of discovering the parts that make an object and also revealing the intrinsic differences between the features that discriminate the objects. In this thesis we explore a number of ways of discovering and using these features to form alternative descriptions or representations of our original objects and investigate their potential value in machine-based inductive learning.

1.1.1 Sensor representation of spectra

Spectra are usually described by a number of sensory measurements (for example the intensity at specific wavelengths) or variables called features. This description of an object is normally in vector form, thus an *m*-feature object can be taken as a point in *m*-dimensional space. The feature space dimensionality can be very high, as for example in images, where a 32x32 pixel sized image will be represented as a point in 1024 dimensional space. The 1024 dimension representation is often achieved by simply stringing together the pixels into one long vector, row by row.

Intuitively, of the 1024 'new' features, only a *small fraction* can be deemed to capture the essence of the actual object and the rest can be discarded. Indeed this problem is encountered in many instances of high dimensional data e.g. biotechnology (genome data), Internet text documents, movies, satellite imagery etc. This problem has been termed the curse of dimensionality [6]. Similarly, in spectral data only a few aspects/parts of the spectrum profile are important in isolating a particular spectrum from a group of spectra.

These parts are often announced by various visually obvious characteristic features,for example peaks and dips as well as the general trend of the profile. Thus by representing each spectrum measurement as a point in vector space one does not focus on the actual important discriminatory features but 'hides' them in the feature space. In images the *intrinsic structure* is also lost, for example the pixels that formed an eye in the 32x32 representation do not necessarily need to be next to each other (in the m =1024 representation) but could be intermingled with the nose pixels which could be next to background pixels and the resultant vector-string would still be considered an image from which the system is to determine coherent object-structure from.

Vector representation for the above domains has a major disadvantage when it is used in recent architectures such as MLP. The importance of the *intrinsic shape* of the object *is lost* – the connectivity between a series of successive sensory measurements in a spectrum is lost in the feature space representation. This is shown in figure 1.1 below where the order of the sensory inputs(λ) is randomized in the "unstructured spectrum" as compared to the measured "structured spectrum".

The order of the inputs is not important in these architectures but in the actual physics of the problem it is! In a classifier using vector represented spectra. classification performance would be completely unaffected by an arbitrary permutation applied to all the data objects – a transformation which would totally destroy all the features on which a human expert would rely on. The same observation applies to an image classification system in which images are represented by vectors formed by row-concatenation. In this case all the information in the vertical relations between rows would be lost. The immunity of machine classifiers to transformations which would be disastrous for human classifiers may point to a severe deficiency in this form of representation. This loss of spatial connectivity [30] can have a number of repercussions:

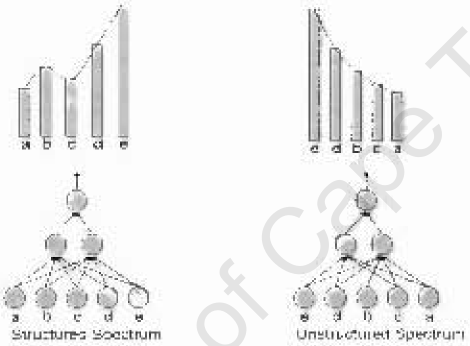


Figure 1.1 Loss of spatial connectivity in MLP type architectures

- *interpretability* of the learning process and the end results may be lost.
- The clustering/classification algorithms-(many of which rely on the statistics of the data) are trying to learn from a 'meaningless' representation. This can unnecessarily increase the classifier *complexity*, which will be trying to find the structure which was lost in the original representation itself.

- Processing *speed* is reduced in learning and searching from information from which relevant information has been lost

It appears that an alternative, more intuitive, object description could be advantageous. We would prefer a method or methods that can capture the underlying structure of an object while making as few assumptions as possible. In spectral data we would like to discover intuitive patterns for example: a spectrum's peaks and other irregularities, and their locations. This is analogous to a method that can discover the eyes, mouths and hair in facial images.

1.2 Finding structure

The above sections can be seen as proposing a focus on finding structure whilst staying more true to the physics or phenomenology of the problem. This means we explore how one may be able to find the underlying structure (parts) such that with this new compressed and intuitive form of spectrum representation one will be able to reveal the physical *causal* structure of the different spectral classes or objects. We explore the applicability of three different methods that can be used to discover intrinsic structure in data via completely different approaches.

We explore the applicability of the recent Non negative Matrix Factorization (NMF) [19],[20], that has provided promising results in images [14,19,21], to spectral data. Of particular interest is its non-negativity constraints on the way we approximate the input objects. This constraint is most significant to the interpretability of the method's discovered structural parts. It allows us to find a purely *additive* combination of positive matrix elements,in this way the positive elements sum to form a whole object. In the second section we explore simple representations of spectra based on salient features in the spectrum. These methods do not, per se, discover the underlying structure of a spectrum but expose what could be sufficient features to describe such objects. In the third section we consider the slightly older Wavelet [5], [16] method which is capable of representing objects to varying degrees of refinement. This refinement should lead to the identification and location of potential discriminatory parts of spectra through stochastic search using a genetic algorithm. The main difference between the above approaches is that in NMF one is trying to determine the spectrum structure in *relation* to other different spectra in the input database while in the other two approaches one analyzes an *individual* spectrum to gain insights to its underlying structure. By re-representing these objects by their underlying features/parts we hope to obtain better discrimination ability as measured by a simple separability index [9] .The basic idea of this measure is to calculate the average number of data objects that have a similarly labeled nearest neighbor. The closer this value is to one, the easily separable is the data. The separability index has been shown to be closely related to the ease and/or difficulty of data classification [39,9].

1.3 Chapter 1 Summary

Unstructured vector representation of objects (for example movie clips, pictures and curves) is highly inadequate. An alternative method of representing these objects is to discover their underlying structure thereby describing them in terms of it. A recent line of research approaches is the representation of objects by their parts. We have motivated the reason for this in a number of domains where this might be useful or advantageous.

1.4 Thesis Layout

Chapter 1 We motivate the need for a sparse representation of data in general and give tentative ideas on the type of features that can be used to efficiently represent objects in two different domains.

Chapter 2 Covers the background to this work. We discuss related methods as well as the differences between our work and these. We introduce the Non Negative Matrix factorization concept. We formally describe this approach to the sparse representation of data and provide a motivating example for its application to spectral type data.

Chapter 3 We experiment with NMF on real spectral data, analyze the results obtained by the method and discuss the limitations of the algorithm. To explain the limitations implied by the results, we do a detailed analysis of NMF in the context in which it is introduced - i.e image processing. This reveals some subtleties and complexities not evident at first sight and cast doubts on a naive interpretation of NMF as a simple decomposition into parts. This is in fact realized only in rather special(and contrived) circumstances.

Chapter 4 We explore two simple intuitive representations of spectral data that are formulated from the physics of the problem (spectrum sensory measurements and resultant profile). We choose features that might perhaps be adequate to explain a spectrum. We model the transition between these features in two logically consistent ways.

Chapter 5 We introduce the concept of Wavelets and genetic algorithms and motivate their combined application in this problem. The motivation for using wavelets is two fold; we would like to experiment with multiresolutional analysis in selecting appropriate features for discriminating spectra in different representations and to compare the performance of wavelet functions given two slightly different descriptions of the same object.

Chapter 6 We consolidate all the spectrum sparsity approaches presented in the thesis. We highlight each method's advantages and disadvantages. Our contributions are noted and we conclude with possible future work.

Chapter 2

2. Background

In this chapter we review some of the most common approaches to the sparse representation of spectral data. The Non negative matrix factorization algorithm and its recent advances is introduced. The database to be used throughout the thesis is presented.

2.1 Alternative Approaches to sparse spectrum representation

Most of the common approaches to the re-representation of spectral data can be said to address the following questions:

- What *makes* two spectra from different classes to be classified differently? And having found these features/parts;

- Can we *use only* these features to describe spectra in a more efficient way? Closely linked to this question is the question -How can we model spectra with this information?

The new representation is further often required to have the following characteristics over the original representation of an object: it should be

- More intuitive - *Simple* and should capture what a human would easily look for.

- More Informative - reveal more of the discriminative features of the spectrum profile.

- More Condensed - it has to explain the spectrum in a *concise* and direct way.

2.1.1 Feature selection

The first obvious approach to the sparse representation of objects is feature selection where we select the features or attributes that when combined in a new representation maximize or minimize some criterion function. Substantial research has been done on different ways to achieve the best features; these vary from randomly selecting features from the initial feature set to structured methods e.g. Sequential forward/backward searching to exhaustive search [22,31] this can be done via the filter or wrapper approaches [22,31]. In the filter approach the criterion function is not linked to the classifier as in the wrapper approach.

In [26] a wrapper approach to feature selection is performed after a number of sub-intervals (parts) of a spectrum have been selected. The spectra were divided into *x* sub-intervals of equal sizes and a number of search strategies were employed to discover the most discriminating intervals. The authors concluded that a parts-based representation does produce better results There are a number of known limitations of feature selection [31,22]. One of these we will just mention: the disregard for correlation and or mutual occurrence of features. This would be most pertinent in images and spectrum. In spectra consecutive sensory measurements are not random but lead to a specific profile (with particular transients) for each class of spectra thus a blind selection of features can destroy this interrelation between features.

This is also significant in images where each feature (pixel) has meaning only in relation to other pixels. Feature selection is usually not designed to discover the structural parts of objects but the lowest dimensional space that can represent a particular object for a particular goal (for example to visualize object class distribution) . The consensus in the literature seems to be that “there is no universally optimal method of feature selection”, every problem is different [22].

2.1.2 Functional Models

Spectral data has been studied for a long time especially in remote-sensing and spectroscopic imaging [17,35]. A common approach in trying to discover the structure in such data has been to use a linear mixture model [17,28,29]. This approach assumes a given spectrum object is made up of known pure substances whose spectra is known. This is an unsupervised approach [29]. The pure substances are often called *endmembers*. The linear model is expressed as follows:

$$Y = E \times A + N \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where for one object, Y (n x 1) is the unknown spectrum substance, the E (n x p) matrix consists of the assumed p-endmembers, A (p x l) is the abundance/quantities of the endmembers and N (n x1) is the noise model matrix. By approximating the unknown spectrum using spectra of known pure spectra, it is assumed one can identify and determine the proportions of each endmember [28,29].A number of disadvantages of this modeling approach are known

- A huge library of potential pure spectra has to exist for each unknown input spectrum.
- The spectrum measurement range of the endmembers has to be the same as the measurements of the unknown spectra.
- The application scenario must be at the same measuring angles, atmospheric conditions as the library endmember spectra were.

A recent modification to the linear model approach has been to constrain the model by having statistically independent endmembers [28]. This is the Independent Component Analysis [15,36] paradigm. The only constraint placed on the components to be discovered is that they are to be statistically independent, thus they can be negative [15]. This means we can have subtractive 'additions' of elements which is not an intuitive concept and is inconsistent with parts-based representations. Furthermore as in the above model, the endmembers are assumed to be known. The objective of the method is to calculate the A matrix (often called the mixing matrix as in (1)) from the given observation matrix Y. In this thesis we do not assume we have such a database but instead use the actual corpus of spectra to discover and identify prominent and characteristic parts. We use the actual spectra to tell us how they are made up.

In [1] a non linear functional approximation of a spectrum is proposed. Each spectrum is modeled by what the authors call a peak parameter representation (PPR); where a spectrum is described by just three features of a peak: its height (H), location (L) and width (W). A non-linear combination of *n* Gaussian or Lorentzian functions is assumed, where *n* is the number of predetermined significant peaks. We thus have:

$$\text{spectrum}_k = \sum_{j}^{n} peak_j(x) \quad ,$$

where each peak_k *(x)* is any function we can associate with a concept of a peak. Thus a full spectrum can be described by a parameter vector p_k = [$(H_1,L_1,W_1),(H_2,L_2,W_2),\ldots(H_n,L_n,W_n)$]. This form of spectrum representation has the advantage of providing a highly compressed representation of spectra (in this case using only three features per peak). The *interpretation* of the resulting description can be very useful in understanding the interrelation amongst the features [1] and possibly between classes of spectra.

Different software packages exist for approximating such data. In [1] MATLAB$^{(R)}$'s curve fitting toolbox using Levenberg & Marquardt approximation algorithm was used and the user is required to estimate/specify:

- the number of peaks in a spectrum (a separate script was written to identify these)
- the modeling function to be used (Spline/Gaussian/Lorentzian etc)
- the starting parameters[H,L,W] of the model

Our approaches to be presented in chapter 4 follow a similar format of identifying the features of interest,modeling them and also incorporating a sensitivity parameter. The difference between this approach and ours is that we follow a very simple,explicit and intuitive analogy making paradigm that a non-expert human might make in describing a spectrum.

2.1.3 Projection Approaches

Other approaches that somewhat estimate an object's underlying structure are the following: Principal Component Analysis (PCA) [40], Local Linear Embedding [34], Isomap [18], Manifold Embedding [3]. Most of these methods approach the problem by finding the lowest projection of the high dimensional data while preserving the relational structure of the objects in question. One of the concerns of this thesis is the physical interpretability of proposed representations. This is usually lost in these methods, mainly due to the projection techniques (e.g. PCA) employed which result in the resultant projected vectors losing their interpretability since for example *no non negative* constraints are set on the projection operators (e.g. in Eigenanalysis). This means data that was originally positive (be it financial,text,musical pitch measurements) can take on negative values. The positive objects that one could be working with do not have any intelligible meaning after such projections.

Indeed the features that are obtained by the PCA method tend to be distorted holistic images of the original objects [19,40] and not the intuitive parts one might expect. What these methods offer is dimension reduction but not an intelligible interpretation of the same object in this lower dimensional space. These geometric techniques do not make available the actual parts/features that capture/represent this lower dimension. To address the shortcomings of some of these approaches a method, Non negative matrix factorization (NMF), based on introducing non-negativity constraints on the features to be found has recently been proposed [19,20]. Section 2.3 presents this method. In order to facilitate a clearer picture of the features we are interested in in spectra, section 2.2 introduces a change in spectrum representation.

NMF was introduced primarily for its evident applicability to image processing and interpretation, and it is in this context that we will first study it, in the hope of clearly elucidating some of its more subtle features and conclusions. We will then consider its application to spectra (which are really a one dimensional image).

2.2 Database and experiment setup

The spectral database we use has 42 classes consisting of 7657 spectra each with 1024 sensory measurements. We combined the classes as follows: Class 1 = [Class 1 and class 2], Class 2 = [3 to12] and class 3 = [13 to 14]. The non target classes were [15-42] now relabeled as classes [4-31][1]. We selected the class pair comparisons shown in table 2.1 as class 1's non target classes we selected classes that had a separability index less than 98% and kept two easily separable classes (26 and 31) as 'controls'. Class 2 and 3's non target classes have a separability index less than 99% each with one 'control' (see table 2.1 below). The controls would be used to determine if this clear separability can be maintained by the new proposed representations.

TC	NTC	SI
1	5	0.972
	15	0.939
	16	0.962
	17	0.974
	20	0.949
	21	0.958
	26	1.000
	31	1.000

TC	NTC	SI
2	10	0.979
	12	0.981
	13	0.984
	18	0.987
	21	0.998
	22	0.989
	30	0.986
	31	1.000

TC	NTC	SI
3	5	0.987
	7	0.999
	14	0.989
	16	0.983
	25	0.983
	26	1.000
	28	0.965

Table 2.1 Selected Target and Non target classes for reflectance representation

We initially reformulate the spectrum representation by removing the convex hull. This is a common starting point in the processing of spectral data in spectroscopy research [17,38]. This makes it easier to obtain potentially interesting parts of each spectrum and specifically reveals the absorption peaks, an attractive conceptual category in terms of which humans experts can discriminate spectra. However it needs to be established whether this (nonlinear) transformation assists or impairs the categorization of the spectra. To determine how this new formulation changes the separability of the classes as compared to the given measuring instrument formulation a comparison between the two is shown in table 2.2 (see Appendix B for a full tabulation). Each class is split into 5 'training sets' , the separability index calculated

[1] The separability index between all the class comparisons is shown in appendix A

and an average of these five runs is used as the final S.I value in the table. In the rest of the thesis the original representation (figure 2.1) will be known as the "Reflectance" spectrum and figure 2.2 (with the convex hull removed) will be known as the "Absorption" spectrum.

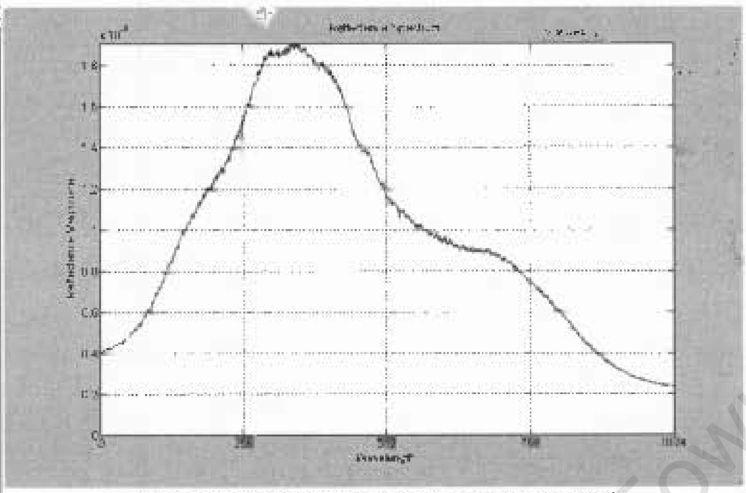


Figure 2.1 "Reflectance Spectrum"

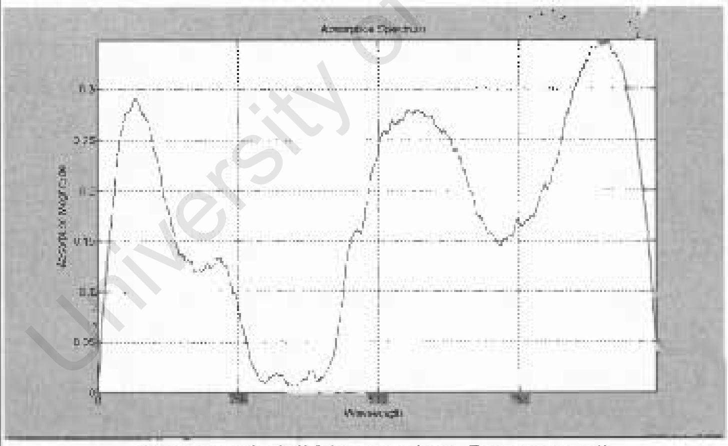


Figure 2.2 "Absorption Spectrum"

It is clear from the above figures that a much more visually appealing information is available in figure 2.2 after the convex hull removal. As can be seen from table 2.2 and appendix B the change in representation from reflectance to absorption does not significantly lower the separability indices between classes but often improves it.

Reflectance Data			**Reflectance Data**			**Reflectance Data**		
TC	*NTC*	*S.I*	*TC*	*NTC*	*S.I*	*TC*	*NTC*	*S.I*
1	5	0.972	2	10	0.9786	3	16	0.9834
1	15	0.9389	2	12	0.9814	3	18	0.9868
1	26	1	2	31	1	3	26	1
Absorption Data			**Absorption Data**			**Absorption Data**		
TC	*NTC*	*S.I*	*TC*	*NTC*	*S.I*	*TC*	*NTC*	*S.I*
1	5	0.971	2	10	0.981	3	16	0.9829
1	15	0.967	2	12	0.9809	3	18	0.9889
1	26	1	2	31	0.9997	3	26	1

Table 2.2 Effects of changing from reflectance to absorption representation on Separability index(S.I)between target (TC) and non target classes (NTC)[2]

This is the form of spectrum formulation that will be used in the Non Negative Matrix Factorization (NMF) analysis. In the next section we introduce the NMF theory and devise a motivation example for the applicability of NMF to the discovery of constituent parts of a spectra database. In this way we hope to discover which parts (if possible, distinct parts) can be attributed for the different spectral class. This would then allow us to formulate a new sparse representation of all the spectral classes.

2.3 Non Negative Matrix Factorization

2.3.1 NMF Theory

Non negative matrix factorization (NMF) is a recently introduced concept [19,20]. The basic idea of NMF is that given a positive matrix V we approximate it by a product of two non negative matrices W and H. The non negativity constraint on the approximation matrices is the core of the algorithm. It leads to only additive combinations of the matrix elements since there will be no negative elements.

[2] A separability index of 1 means the classes are clearly separable.

Formally: Given a non negative $n \times m$ matrix V (m vectors, each with n elements/features), NMF finds a non negative $n \times r$ matrix W and a non negative $r \times m$ matrix H such that V ≈ WH. This leads to the interpretation of the constant W columns as being the general features present in all the input vectors (m) but they occur in different amounts (accounted for by the elements of H). The columns of W are called the basis or parts and the row elements of the H are the abundances or coefficients.

Since we do not know how many general parts the input matrix can be minimally factorized into we have a user specified parameter r which also controls how much dimensional reduction we can achieve, it is limited by:

$$r < \frac{(n \times m)}{(n + m)}$$ see [19,20].

To determine the approximation accuracy of the decomposition an objective function has to be defined, the Euclidian or the Divergence objective functions have been used in the literature (see Section 2.33). The problem is then simplified to:

Minimizing the objective function under the non-negativity constraints.

The original NMF algorithm [20] used the multiplicative update learning rule in contrast to the gradient descent updating technique normally employed in function minimization. The Euclidian objective function is minimized via the following update scheme:

$$W_{ic} \leftarrow W_{ic} \frac{(VH^T)_{ic}}{(WHH^T)_{ic}}$$

$$H_{cj} \leftarrow H_{cj} \frac{(W^T V)_{cj}}{(W^T WH)_{cj}}$$

Where the i refers to a particular row element, the c to a column element in W whereas the j refers to a column element in H. The c is common to both matrices since the number of rows in H is the same as the number of columns in W. The multiplication and divisions in the above and subsequent equations are performed element by element. In the updating of W's element at position (2,3) for example, the current value of W at (2,3) on the right hand side is multiplied by the current quality of decomposition matrix factor [VH' / WHH'] at the corresponding positions, (2,3) of this matrix. The closer the WH approximation is to V the more this factor approaches unity. A similar explaination can be given for the updating of H's elements. Subsequnt to [19,20] a number of alternative updating schemes and objective function have been presented (see Section 2.3.3). Most of this work on NMF has been mainly on facial images [10,14,21,32]. The results obtained in this domain have been encouraging but little research in other domains has been done. Thus we explore the applicability of NMF in spectral data!

2.3.2 The promise of NMF

In this section we present a simple synthetic example to motivate the potential applicability of non negative matrix factorization for the decomposition of spectral data into intuitive parts. We specifically show that NMF can discover the underlying parts,with approximately correct locations and profiles, of a number of spectral profiles. We setup two sets of exponential Gaussian functions;

$$e^{-(x-x_0)^2/2\sigma^2}$$

The first has a sigma (σ) or width parameter of 0.8 centred (x_0) at 45 degree intervals and the second with a σ value of 0.2 centred at 60 degree intervals. We randomly combine these basis Gaussian functions, sampled at 10 degree intervals in the range from 0 to 360 degrees, to form *m =10* composite spectra. Our spectra input matrix is thus V, (37x10). For factorization we can determine the upper bound for *r*, which is given in [19],[20] by:

$$r < \frac{(n \times m)}{(n+m)}, \quad r < \frac{(37 \times 10)}{(37+10)}, \quad r < 7.87 \,.$$

This means the maximum number of basis we can search for is 7. So the number of basis/parts that reasonably constitute the input objects is smaller than 7.

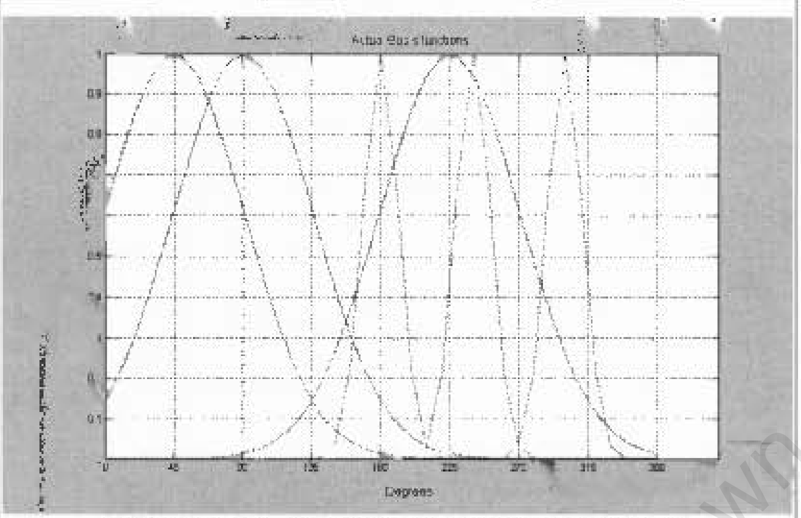


Figure 2.3.1 Six Gaussian basis Functions

Note the widths of the basis functions in figure 2.3.1 are not the same. For clarity and simplicity we equated the amplitudes of the exponential functions. Figure 2.3.2 shows the L2 normalized input composite functions formed by randomly combining the basis functions in figure 2.3.1. We applied all the algorithms given in Section 2.33 and the typical results of the Non negative matrix factorization with sparsness constraint (NMFSC) algorithm by P.Hoyer [14] are shown below in figure 2.3.3

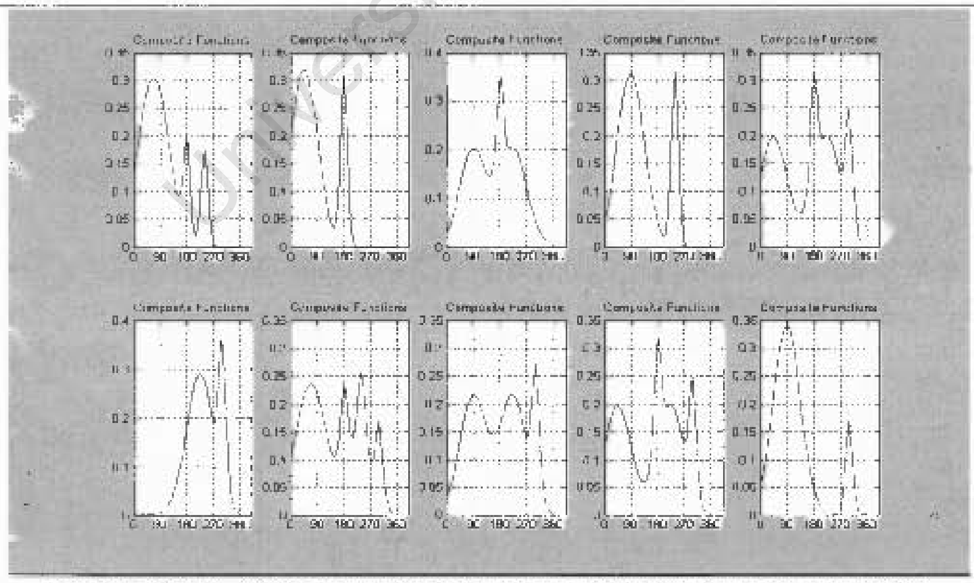


Figure 2.3.2 10 Composite Functions of Basis from Figure 2.3.1

We set the number of basis vectors to be discovered to $r=6$ since we know that 6 basis were used in the formation of the input dataset. In a practical problem setting we would have to search for a reasonably good looking basis set by experimenting with different values of r less than 7 as explained above.

The sW and sH specify the amount of sparsity each of the matrices should have. A high value (values vary between 0 and 1) indicates that we want the corresponding matrix to have few components active and vice versa. In this setting we would want a few bases (gaussian functions) to be active with an unconstrained or slightly sparse (meaning many active) coefficient elements to account for the abundances or effects of the basis contribution in the input matrix. After a number of combinations of these parameters were explored the results presented in figure 2.3.3 presented the best[3] basis.

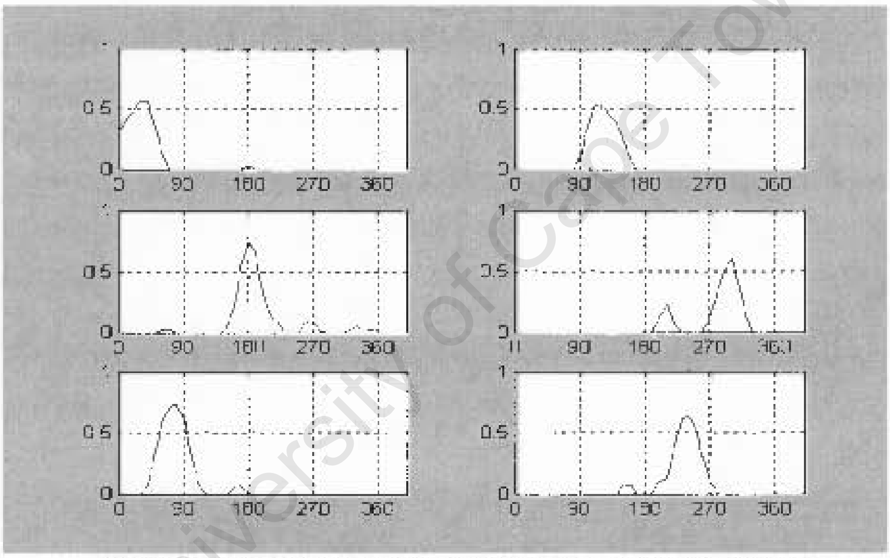


Figure 2.3.3 NMFSC results at sW=0.70 & sH=0.10

Most of the basis found by NMF in this example are not well defined in terms of the correct profiles and locations. Some of the basis found look somewhat like composite functions.

[3] The quality of bases discovered is initially a subjective matter. Obviously classification (an objective form of evaluation) etc can be applied at a later stage. But to narrow the search for a good bases set it seems user subjectivity is required.

In general the results found by NMF on this example are not as we expected from the encouraging though imperfect results in facial images in the literature (see section 3.2). Besides the lack of high decomposition accuracy of the method some form of differences (especially the profiles) is notable in the bases found but one is not clear whether this is as a result of the factorization approximation or it is a definite 'intended' solution. But the method seems to exhibit potential and to further explore its capability we explore its applicability on real datasets of spectral data. We first introduce the different variations of the non negative matrix factorization algorithm before experimenting with real datasets.

2.3.3 Recent NMF advances

In this section we introduce as well as describe the differences between recent popular non-negative matrix factorization algorithms these are:

a) Local NMF by Li.S et al. [21]

b) Sparse NMF by Liu W et al [23]

c) NMF with sparseness constraint by P.Hoyer [14]

a) Local Non-Negative Matrix Factorization (LNMF)

LNMF was introduced to mitigate the limitations of the applicability of the original NMF[19] algorithm on some databases. When the original NMF algorithm was applied to a different database, of the same domain, which was not as heavily pre-processed as the one in [19] NMF discovered holistic basis. It was noted in [21] that the pre-processing (e.g alignment and rotations) of input objects can drastically influence the results obtained by NMF. This algorithm offers a much better set of bases in both datasets as compared to the NMF algorithm from [19].

This method uses a multi-constrained Divergence objective function:

$$\mathrm{D}(V\|WH)\ \sum(V\log\frac{V}{WH}-V+WH)+\ \alpha\sum W^{T}W-\beta\sum H^{T}H$$

with α and β being user specified parameters and where the $\alpha\sum W^T W$ and the $\beta\sum H^T H$ components of the objective function constrains the basis to be:

- mostly non-zero.
- as different as possible,that is, be orthogonal.
- and the bases are to be as expressive as possible

This method implements the multiplicative approach in finding the minimum of the objective function. This results in the following update strategy:

$$H_{al} \leftarrow \sqrt{H_{al} \sum_i W_{ia}{}^T \frac{V_{il}}{(WH)_{il}}}$$

$$W_{ia} \leftarrow W_{ia} \sum \frac{V_{il}}{(WH)_{il}} H_{al}{}^T$$

Then the bases are normalized to have unit length

$$W_{ia} \leftarrow \frac{W_{ia}}{\sum_l W_{la}}$$

Where again the [.] $_{ia}$ indicate the element by element multiplications and divisions of the matrices.

b) Sparse Non Negative Matrix Factorization (SNMF)

Another method introduced to provide sparsity in the basis to be discovered is SNMF. This method uses the following divergence objective function:

$$\mathrm{D}(V\|WH) \sum (V \log \frac{V}{WH} - V + WH) + \alpha \sum H$$

with the multiplicative updating of the coefficients and basis.

$$W_{ia} \leftarrow \frac{W_{ia} \sum_{l} \frac{V_{il}}{(WH)_{il}} H_{al}^{T}}{\sum_{l} H_{al}^{T}}$$

$$H_{au} \leftarrow \frac{H_{au} \sum_{a} W_{ia}^{T} \frac{V_{iu}}{(WH)_{iu}}}{(1+\alpha)}$$

There is also a basis normalization step

$$w_{ia} = \frac{w_{ia}}{\sum_{j} w_{ja}}$$

and α is a use specified parameter. The main difference between these methods is the way they incorporate the sparsity factor in their objective functions.

c) Non Negative Matrix Factorization with sparseness constraint (NMFSC)

A more powerful and general method of NMF is NMFSC. This algorithm incorporates open parameters to control the amount of sparsity one may deem necessary in both the W and the H matrices. This is important as one may have situations where prior information has a bearing on what needs to be sparse. An example would be in audio recordings one may know that only a certain few notes dominate and that these notes occupy most of the music segments. Here you would want to have the W matrix sparse to find those few notes but the H matrix unconstrained as the abundances of the notes is quite high in the time segments taken.

This method uses the Euclidean objective function

$$\frac{1}{2}\|V - WH\|^{2}$$

that is optimized via the gradient descent approach. *Updating W*: If the sparsity of *W* is specified

$$W \leftarrow W - \mu(WH - V)H^T$$

where μ is the gradient step size, then you implement the projection step (see projection step below). If the sparsity of the basis is not specified then the algorithm uses the standard [19] multiplicative update rule

$$W_{ic} \leftarrow W_{ic} \frac{(VH^T)_{ic}}{(WHH^T)_{ic} + eps}$$

Updating H

If the sparsity of *H* is given, the following update rule is used

$$H \leftarrow H - \eta(W^T(WH - V))$$

where η is the gradient step size then the projection step below is implemented and if the sparsity of the coefficients is not specified, the multiplicative update is used

$$H_{cj} \leftarrow H_{cj} \frac{(W^T V)_{cj}}{(W^T WH)_{cj} + eps}$$

where $eps = 1x10^{-9}$. Normalize the rows of *H* on a unit sphere and multiply each element in the i^{th} column of each basis by the normalized, i^{th} row of *H*. The updating of the *W* & *H* components are such that a constantly decreasing objective function is achieved. The sparsity is incorporated in the updating step of the basis and the coefficient matrices. This is done via a projection step [14].

Projection function

When the sparsity of the basis is specified the projection step updates the basis by restricting the L1-norm (columns) of the W to achieve the specified sparsity. The L2-norm is to be unchanged. If the sparsity of the rows of H is specified, then the L2 norm is restricted to a unit sphere and the L1-norm (rows of H) is set to achieve the specified sparsity. In both cases the problem is then to solve for the equivalence of the projected L1 norm and the L2 norm, i.e. the sum constraint plane and the unit sphere.

This therefore requires the user to experiment with sW and sH according to what one wants to sparsify, do we want as few as possible parts/basis or do we need their abundances (coefficient matrix elements) to be maximized?

2.3.4 User Parameters

There are several parameters that a user needs to specify in the NMF algorithms these are:

- the number of columns *r* or basis to be discovered
- the algorithm termination criterion and
- in the case of NMFSC and LNMF the sparsity factors have to be specified before hand. These factors are generally considered classic model selection parameters in that there are no definite rules for approximating them.

2.4 Chapter 2 Discussion

Non negative matrix factorization promises to discover the parts that constitute the input vectors *without* the user specifying *what* these parts are or what to look for! The parts discovered can be understood as the underlying structure or constituents of the input objects. The various modifications to the original non-negative matrix factorization algorithm have added sparsity control parameters. These in effect allow the user to constrain the algorithm to certain types of bases e.g. in NMFSC [14] the sparsity parameters control the size and resolution of the bases discovered when the number of bases is a given. The synthetic example has somewhat motivated the applicability of the method to functional type data. The sparsity parameters have proven to be of critical importance in achieving any reasonable bases set .

In the next chapter we experiment on real spectra with some of the above algorithms. Our main focus will be on the application of the different sparsity factors and determining their results on our real spectral dataset. We analyze the behavior of the algorithm at different parameter settings to get a clearer interpretation of the basis and coefficient matrices.

Chapter 3

3. Non negative matrix analysis

3.1 Non-Negative Matrix Factorization on real spectra

The aim of the experiments in this section is to determine whether the NMF algorithm can factorize an input database of real spectra into an interpretable bases set. As motivated in section 2.3.2 NMF does seem to discover parts of synthetic spectrum, see figure 2.3.3 when given input like in figure 2.3.2. In the first experiment we select a random set of 10 spectra from each class, as described in section 2.2 and form an input matrix V. The maximum rank of the NMF decomposition value (*r*) is given by

$$r < \frac{(n \times m)}{(n + m)},$$

in this case $r < (1024 \text{x} 10)/(1024+10) < 9.9$. We normalize the input matrix V by the L2 norm. The first target class is class 1 and its non target classes are15,16,17,20,21,25,31. Figure 3.1 shows each classes' 10 spectra thus we have 80 input spectra in the V matrix. Figure 3.2 illustrates the NMF input matrix V for these classes and target class 1. The bases found by NMF for r =6 are displayed in figure 3.3 at a basis sparsity factor (sW) of 75%, with no sparsity set for the coefficient matrix (sH). It is not clear how one should interpret these highly discrete bases. Figure 3.4 illustrates another bases set discovered by NMF on the same input matrix V when sW=0.55 and sH=0.3[4]

[4] All the experiments in this section used the NMFSC algorithm which we terminated when the objective function started stabilizing.

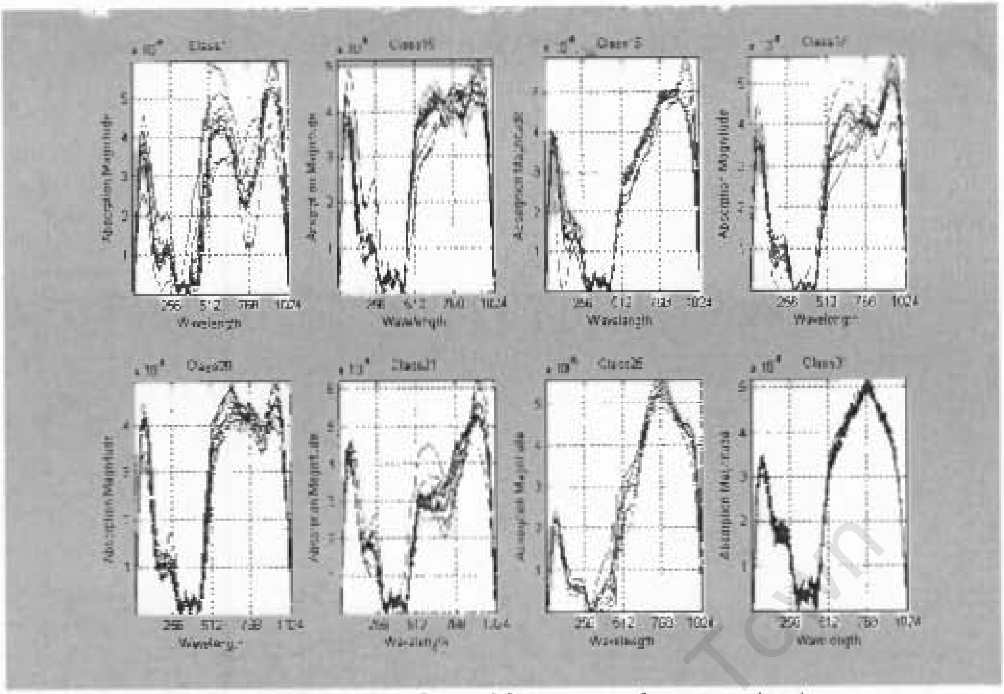


Figure 3.1 Each of the 10 spectra from each class

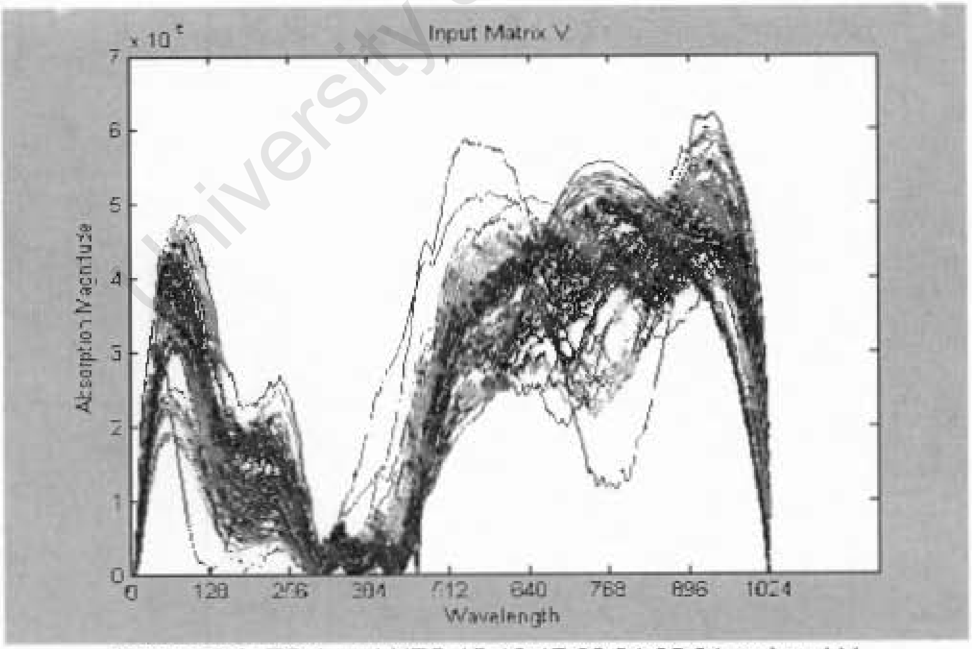


Figure 3.2 TC 1 and NTC 15,16,17,20,21,25,31 as input V

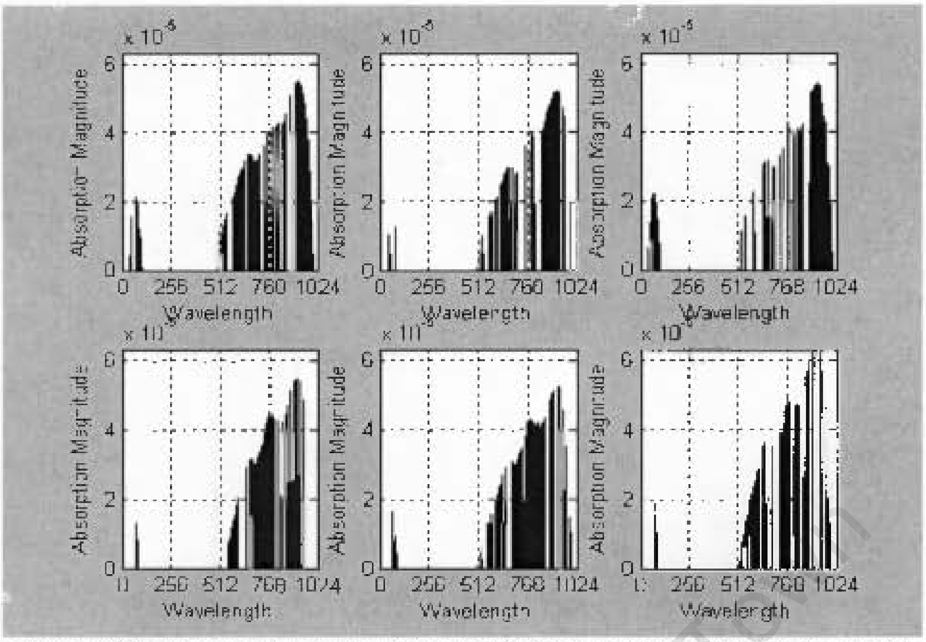


Figure 3.3 Bases discovered by NMFSC for r=6 at sW=0.75 sH=[];

When the sparsity of the basis matrix (sW) is decreased the algorithm tends to factorize the input matrix into bases that occupy certain sharp subregions of the full spectrum measurement range. Some of these bases seem to overlap and considering the additive nature of the factorization it is not exactly clear what the individual discovered bases really mean but some sort of underlying structure is beginning to become visible.

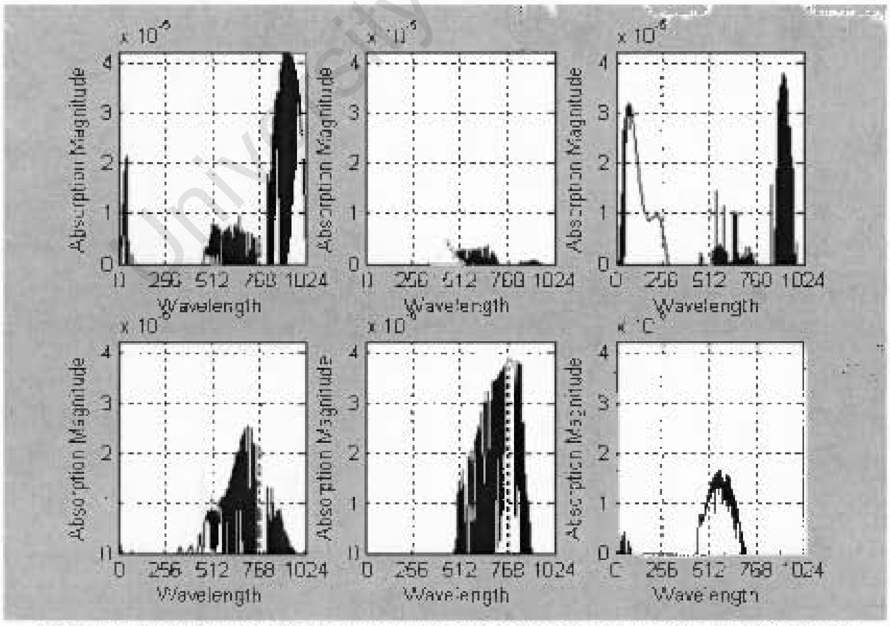


Figure 3.4 Basis discovered by NMFSC for r=6 at sW=0.55 sH=[];

The coefficient matrix of the bases in figure 3.4 is shown in figure 3.5 It reveals that the discovered bases occur in most of the input spectra except for the second basis which does not seem to occur in a number of input spectra. The interpretation of this basis is not simple as other bases found also occupy the same wavelength range. The fact that most of the bases occur in almost all the input spectra means the sparsity settings are not optimal.

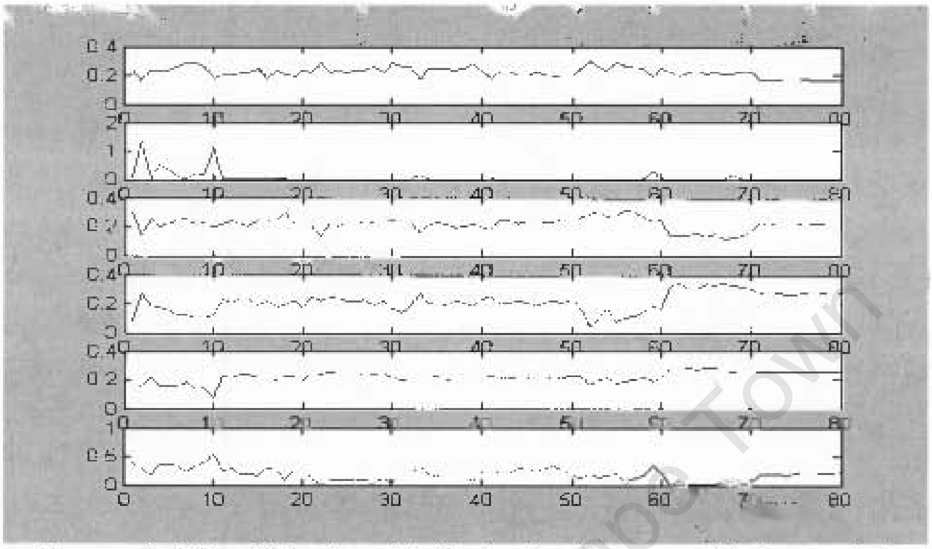


Figure 3.5 Coefficient matrix for basis discovered in figure 3.4

When the basis sparsity is furthermore decreased as in figure 3.6 spectra segments are found. The bases vary from sharp discrete segments to respectable short-segmented ones. The coefficient matrix. figure 3.7, somewhat better explains how to interpret figure 3.6 . The coefficients in figure 3.7 signify the presence or abundances of the basis thus the bottom middle basis in figure 3.6 occurs in the input spectra from *some* of class 26's spectra, spectra 60 -70 in the input matrix and *constantly* in class 31 (spectra 70-80 in the input V).

This basis occurs with highest abundance in class 26 as indicated by the higher values in the 5th row of figure 3.7. It does not occur anywhere else in the input matrix. The first (left hand corner of figure 3.6), third and last basis look like full spectra. The coefficient matrix does not illustrate their occurrence in classes 26 and 31 One would at least expect the peak between 0 - 256 wavelengths to be found in these spectra as it occurs in the class input figures 3.1. We increased the number of basis to be discovered in this example but segment (0 - 250) of the spectrum was not identified as a unique basis. This introduces some uncertainty in the meaning of NMF results.

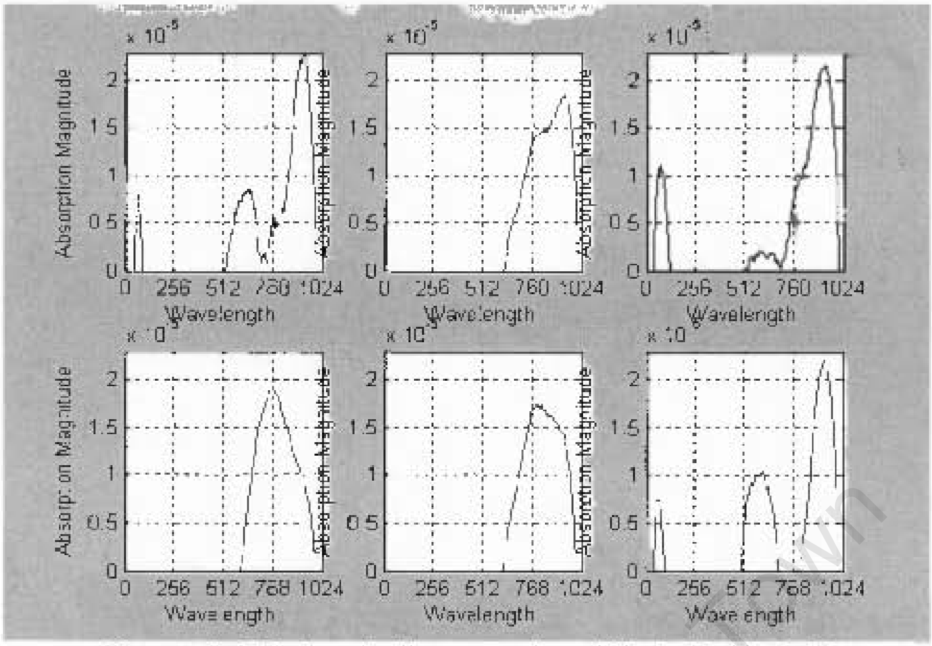


Figure 3.6 Six basis discovered at sW=0.45 sH=0.75

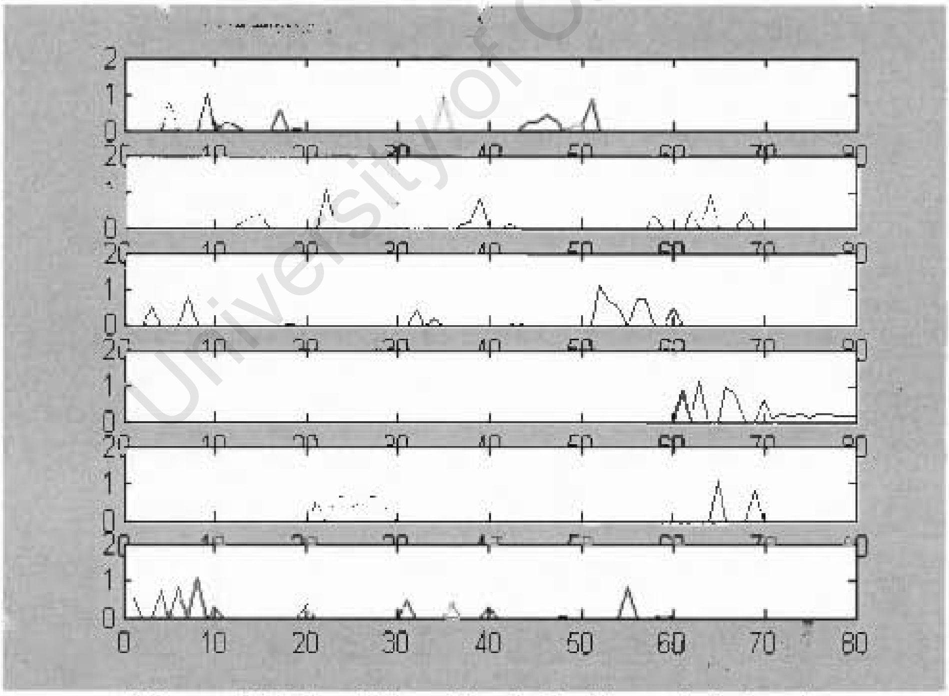


Figure 3.7 Coefficient Matrix for figure 3.6's basis

The above experiments have highlighted a number of questions that need to be addressed in order to understand and better interpret NMF results.

- When does one know that the discovered basis set is optimal?
- What does it mean if a potential (intuitive) part is not identified as a basis?
- Why do some basis appear holistic, for example basis 1, 3 and 6 in figure 3.7and not as unique segments?
- What input regularities does NMF actually exploit in data?

We address the above questions by experimenting with the algorithm in a more intuitive and familiar domain, that of images. This we believe is easier to interpret and critical insights can be gleaned from such explorations.

3.2 The limitations of NMF

To gain better understanding into how NMF discovers the parts it does we explore the algorithm with different input settings with objects from the same domain. We make use of another algorithm (Local NMF) provided in the NMF package by [14] and introduced in [21] for result comparison.

3.2.1 What NMF cannot do

- Non-Negative Matrix Factorization cannot decompose one object's multiple occurrences in the input. When we are given a number of copies of the same object as input NMF does not decompose these into constituent bases. The bases decomposition achieved by NMF algorithms for such an input is holistic since the same parts are present at the same positions in all the input objects.

This is shown below in figure 3.8 (input) and LNMF and NMFSC results in figure 3.9 for a large number of iterations are similar.

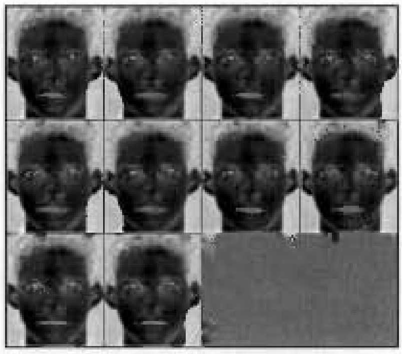


Figure 3.8 One individual's same ten images as input V

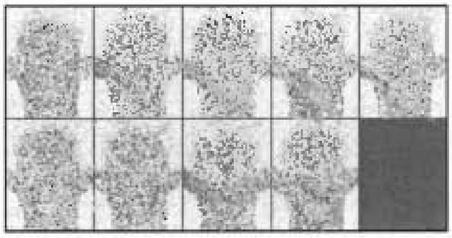


Figure 3.9LNMF result at r=9 or less after 1500 iterations

- NMF could possibly be a method for factorizing multiples of one object's two different images. NMF is not able to find convincing parts when given essentially similar objects as in figure 3.10 but note that as soon as we start to have a slight variation in the input images (figure 3.10) then some forms of parts are evident as basis, figure 3.11.

Figure 3.10 One individual's two different images

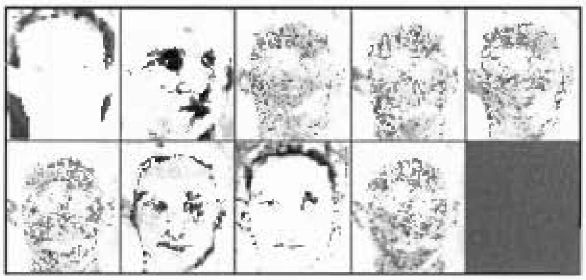


Figure 3.11 LNMF result at r=9 after 1500 iterations

These are the parts that differ significantly between the input objects. This then raises the question - under what conditions will NMF provide us with the ideal basis, that is eyes,ears and mouths etc as separate and possibly unique bases?.

3.3 Ideal basis & Limitations of NMF

In the literature non-negative matrix factorization has been presented as a method that discovers object parts. It would seem the interpretability of the parts has not been a major concern for researchers but rather the sparsity and classification results that are obtainable after the factorization. Indeed the majority of subsequent papers (14,21) to the original NMF[19] all focused on the sparsity of the bases to be discovered via changing the factorization constrains. In light of the above results of NMF analysis (sections 3.1 and 3.2) the meaning and achievability of the ideal NMF basis is questionable!

A number of recent papers have started to highlight similar concerns and have in fact questioned the interpretation of the basis discovered [4,7]. [4] gives an example of NMF applied to a database of eyes images, from this paper it is clear that the interpretation of NMF can be very difficult even for a simple object like this. In [7] the conditions for the ideal performance of NMF is presented. The basic outcome is that the input database has to have a specific pattern for NMF to produce clearly interpretable and optimum parts

3.4 Extension to interpretation of NMF

In this section we seek to extend on the interpretation of NMF factorization ability as hinted in [4] and [7].We create a database with characteristcs similar to the swimmer database in [7], which we call the sticks database in figure 3.12 below. This dataset will allow us to explore why and perhaps what the NMF algorithms discovers to be parts of an input set. This is a hypothetical input set made up of 16 (4 rows x 14 columns) objects in different poses. There are actually two sticks that vary their positions from the initial two sides of a triangle to the fully open triangle position on the last object. This input like the swimmer database in [7] consists of all the combinations of how these two sticks can be oriented between images 1 and 16. Note that there is a non position varying part (at the bottom of each object) in the database.

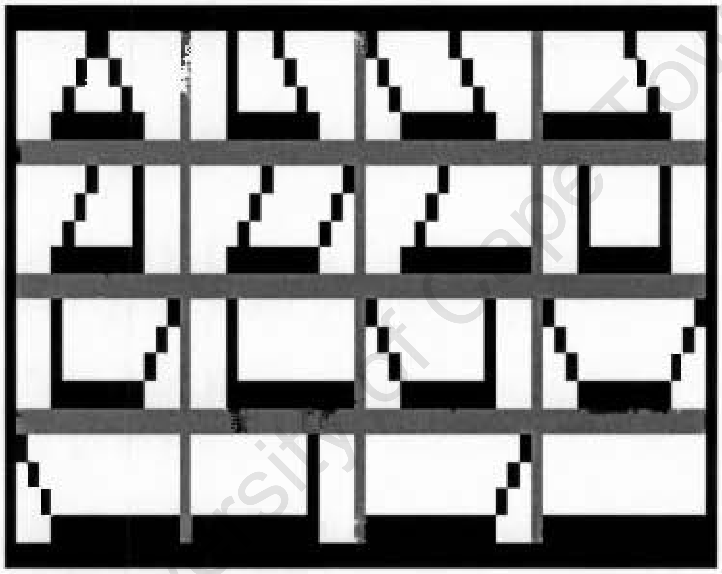


Figure 3.12 The sticks database

Often one cannot look through a typical input database to find the parts that can be described as making-up the data. We thus seek an automatic method of finding the number of parts that constitute all or most of the input. When the standard NMF algorithm is applied one must empirically guess a reasonabe value of *r* to factorize the input to (see esction 2.3.1). If we arbitrarily choose *r* = 2, 5, 8 the NMF algorithm obtains figures 3.13 3.14 and 3.15 below respectively.

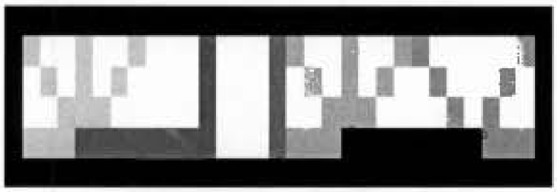


Figure 3.13 NMF r=2

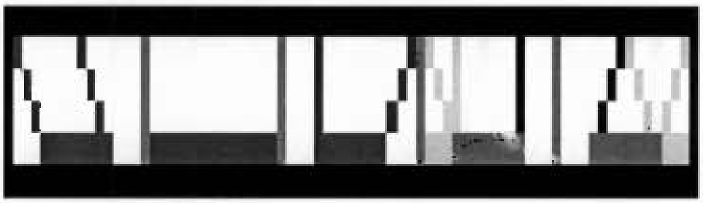


Figure 3.14 NMF r=5

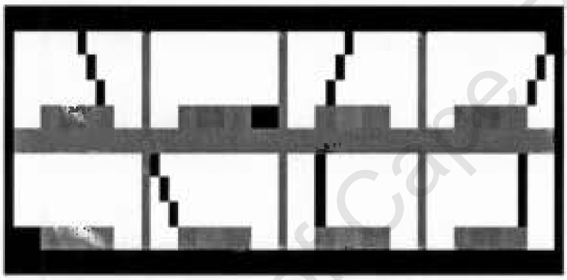


Figure 3.15 NMF r =8 Best Decomposition

Note in figure 3.15, NMF discovers 8 bases/parts that are really 2 parts at different positions. The bottom of the object is common in all bases. Figure 3.15 shows us that NMF does not really discover the fundamental parts of a database (Here actually 2) but the parts that show the most variation (that are the most different amongst the objects) in the database. This then explains why NMF cannot factorize one or multiples of an object as posed in section 3.2. This explains why [7] refers to the fact that better results are possible with a larger database. We should add that the database should consist of as many slightly differently posed objects as possible for any reasonable bases to be found!

3.5 Approximating the best *r*

In the above type of problem setting it is possible to eliminate the guessing of the best rank (*r*) for the factorization. One can compare each object to all the other objects and note the differences. Then count the differences that have been noted the most from each object, as compared to the other objects. The pseudocode of the above argument is presented below:

Given data V = [*n* x *m*] where *n* is the number of pixels in each object and *m* is the number of objects, 16 in this case.

1. Compare each object in the database with every other object.

2. Note the *differences*[5] for each object.

3. Extract only the differences that occur the *most* for each object. Call these max_occur

4. compare each object's max_occur with every other object's max_occur.

4.1 Store only max_occur s that are unique.

4.2 Count the number of max_occurs obtained.

Figure 3.16 below shows the results discovered by this algorithm.

Figure 3.16 Pseudo code results for **different** parts

[5] We can look at the *differences* or the number of *equal* 'parts' in the objects. *See* Fig.3.16 & 3.17.

Note that the bottom of the triangle does not appear in figure 3.16 due to step 2 (*differences*) of the algorithm. The above result shows the parts that differ the most. This is why we argue that the presentation of NMF as a technique for discovering parts of objects is somewhat misleading.

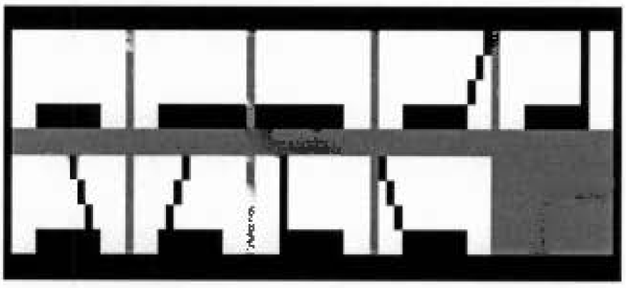


Figure 3.17 Pseudo code for **equal** parts

Figure 3.17 shows the results obtained by changing step 2 in the pseudocode to the number of *equal* aspects. Note that the common part from the input is discovered as a separate basis as compared to figure 3.16's results. This is a more ideal result as compared to figure 3.16 where we would never know that there is a constant common feature in all the objects in the database.

3.6 Interpretation of the Coefficient Matrix

The synthetic problem modeled above shows well how one can interprete the coefficient matrix. This simple setting shows when or where in the input matrix a particular part observed in the basis matrix occurs. The first row in figure 3.18 models the first basis function discovered in fig 3.15 NMF's best decomposition, subsequent rows model the corresponding bases in figure 3.15. As it can be seen the first basis/part found by NMF occurs in all the first four input objects (figure 3.12) of the sticks database in the same position. This is the right arm at that particular position.

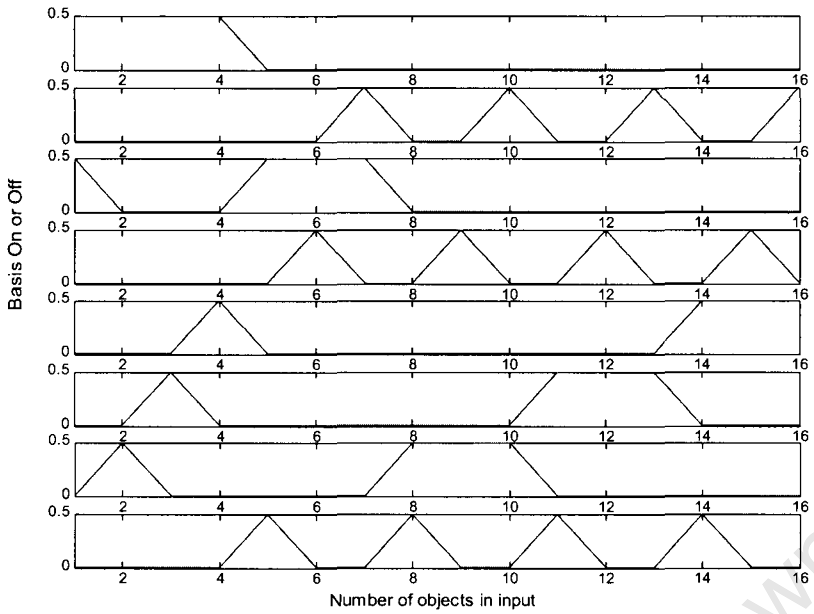


Figure 3.18 Coefficient Matrix *r* =8

The seventh row of figure 3.18 shows where the 7th basis occurs in the same position in the sticks database;input objects 2,8,9,10. This is the left arm in a vertical position. This part in this particular position does not occur anywhere else in the sticks input. Note that in this synthetic problem a basis is either present or not, that is why we do not have fractions in the coefficient matrix. It is either on or off.

3.7 Relation to spectral data

The above argument somewhat explains why in the motivation example (see section 2.3.2) NMF tentatively decomposed the input synthetic spectra into the correct basis. *This was an example of a functional form of the sticks database!.* As explained in section 3.4; there are essentially six parts that are combined in some fashion, some composite input spectra have certain bases some do not, to produce the input. Thus by applying the above proposed algorithm (see section 3.5) one would also obtain the required basis[6]. This analysis leads us

[6] Note: The algorithm would have to be modified slightly to accommodate functional data.

to conclude that if spectra from different classes have common parts but these parts occur in certain arrangements NMF might discover them as bases. This means if most spectral classes have the *same* part in the *same location* and different parts which have to be similar in some spectra, occur in the same range of the profile, NMF could discover them. Then the common parts would be identified as bases since they occur in the same location and the other parts will form the other bases since some of them will be common to other spectra.

The observation that NMF factorization can result in holistic spectrum basis is also explained by what the algorithm searches for in the input. In figure 3.6 the first, third and sixth basis are almost holistic, this means in each of these bases the peak spectrum segments (0 -256 and the 512-1024 segments in those forms) always occur together, NMF cannot separate them to form unique basis out of them. This is similar to the bottom, non varying part in the sticks dataset in figure 3.12 as explained in sections 3.3 & 3.4.

As a further step if one could discover or explain the relationships between the parts (the fact that two of the parts in figure 3.12 have moved) we could reveal the fact that there are only three fundamental parts in the sticks dataset.

3.8 Chapter 3 Discussion

This chapter has explored the applicability of non negative matrix factorization (NMF) in the factorization of real spectral data into constituent parts and we have subsequently noted fundamental limitations of NMF paradigm in general. Real object parts through NMF seem to be only attainable if the input data is arranged in a specific pattern. The input objects firstly have to be generally similar but will also have to have certain parts occurring at the same locations in a number of objects and have some other parts that occur in other objects not occur in the first set of objects. Such a permutation of object parts is unlikely to occur in real spectral datasets. A few domains such as music often exhibit such behavior where certain musical notes are played in a certain segment and are not present in others [37].

Whereas chapter 3 focused on the experimentation with the sparsity factor of non negative matrix factorization algorithms in the next chapter we formulate another approach to discovering a sparse expression of spectra. This approach is based on the few aspects of spectra that seem to be intuitively appealing or attractive to the human eye. We use this information to device a new sparse description of spectra. Unlike NMF the following chapters' approaches to attaining sparsity do not analyze the spectra as one input but as individual spectrum.

Where we hypothesize that simple,intuitive, sparse empirical features of spectrum can be sufficient to capture the physics of sensory representations of spectra

Chapter 4

4. Proposed Representations

4.1 Motivation for Alternative representation

In solving or explaining a problem humans can normally consider different perspectives, those that include details and those that don't. If one particular view-point can be further simplified then usually that new perspective is preferred. The new perspective has a strict constraint: it still has to capture the core/essence of the problem at hand. By identifying what seems to be the bare essentials of a problem one would like to wrap these up in a more simple and intuitive package - "Saying more with less complexity".

Humans can quickly identify certain aspects or features that can be deemed to be discriminatory just by looking at objects (e.g. Spectra). The general profiles of a number of spectra might be similar but there are at least certain features of the spectra that are different thus leading to different classes of spectra. One would thus like to capture and characterize these aspects as possible sufficient *descriptors* of whole spectra.

In [30] Paclik et al. have proposed *describing* a particular spectrum by its similarity (dissimilarity) distances between it and every other spectrum in the dataset. This description of a spectrum captures well what seems to be of interest when one needs to cluster or characterize objects but this only occurs if we treat spectra as points in some high dimensional space. Another relatively simple sparse representation (discussed in section 2.1.2) of spectra is proposed in [1]. Here a particular feature (peaks) was found to be visually prominent in spectra. This feature's location, height and width form the basis of the compressed representation of the spectrum. But unlike in [1] we do not try to discover the possible *causes* of the profile but we capture what is *visually* perceptible and form a sparse representation from it. The main purpose of exploring these representations then is to determine if such sparse features will be sufficient to capture the basic *structure* of the spectrum profiles. This was argued (see section 1.1.1) to be important if we want to stay true to the physics of the problem.

This chapter is arranged as follows: in the next section we explain how we capture the features we deem to be of intuitive interest in spectra. We formulate two compact descriptions of spectrum based on these features. To convey the new formulations clearer we graphically show how they would appear in function form. We are not, per se, interested in the visualization of the new descriptions but in their ability to capture the whole spectrum profile via a few features whilst being able to discriminate different classes of spectra as in the original representations.

4.2 Capturing the general profile

Spectra in the same class, let alone in different classes do not necessarily have the same number of features of interest (FOI) nor do these necessarily occur at the same exact *location* in the profile. Figure 4.1 shows an example where we consider the troughs as our features of interest. If a new representation using only troughs is required we have a problem, the number of FOI is not equal! We cannot deal with the resultant vector description in the normal way for example using Euclidean distances since the representations are not in a common vector space.

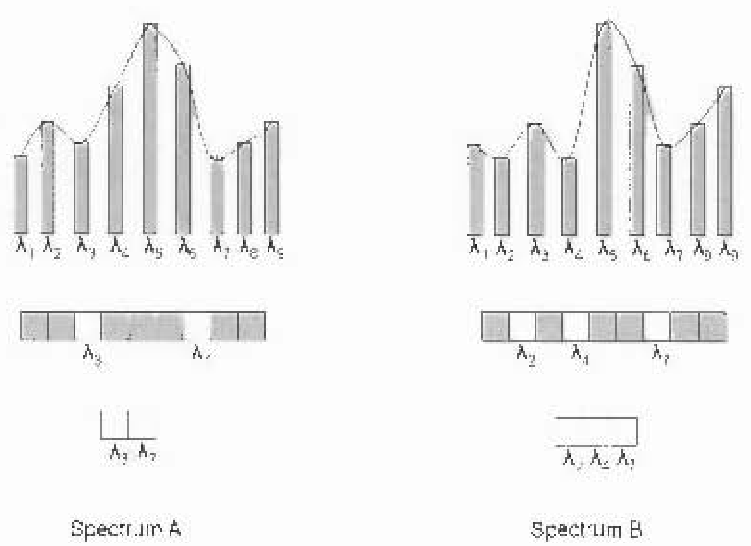


Figure 4.1 Unequal number of features of interest(troughs) in this vector representation

Little research in machine learning is concerned with unequal sized vectors – one measure that can deal with unequal sized vectors is the Earth movers distance (EMD) commonly use in imaging [33]. When one needs to compare or document two or more images in some order it is often easier to compare their color histograms [33]. EMD has been shown to work well in such domains. The measure's basic hypothesis is to calculate the minimum amount of work required to change one image's color spectrum (often in histogram form) into the another even if the number of bins is not equal.

We do not use this measure in this thesis. If we are to use the normal vector representation (where Euclidean calculations are valid) for each spectrum; we have a number of options for this problem :

- Choose a vector length that is just more than all the classes' FOI and add zeros at the end to equate all the spectra vectors.

- Retain the original vector length by :
  - Listing all the FOI of a spectrum in the beginning of the vector and add zeros until the original given vector length.
  - Listing all the FOI at the positions they were found and inserting zeros in between

But if we are to emphasize the physics of spectrum it is not enough just to identify the FOI but to also capture what happens between them! All the above options neglect to model the transition between the features of interest. As mentioned in section 3.8 it important to discover the parts that explain a set of objects but their configurations has a fundamental bearing on each object's description or structure. In section 4.2.2 and 4.2.3 we explore two different ways of modeling the transitions. But firstly we discuss what we consider salient features of spectra.

4.2.1 Features of Interest

We identify *peaks* and *troughs* as important visual discriminatory features for our proposed representations of spectra. We use the change in profile slope to identify features as peaks or troughs. Figure 4.2 illustrates the case where peaks are identified as features of interest. Note how *every* change in the sign of the slope defines a feature of interest (in this case a peak). We also incorporate a sensitivity parameter to define how significant the change has to be before a potential feature is considered interesting. Figure 4.3 illustrates a particular set of features that were found at a certain sensitivity setting.

The sensitivity parameter is a particular constant fraction (lets say q) of each spectrum's maximum value. In order to capture more prominent peaks rather than every change in the sign of the slope as a peak we require only those peaks that are a particular magnitude (q in this case) more than the previous trough. If this magnitude is less than q then the peak is not considered to be significant. This way the smaller the threshold fraction the more profile 'noise' we capture as in figure 4.2.

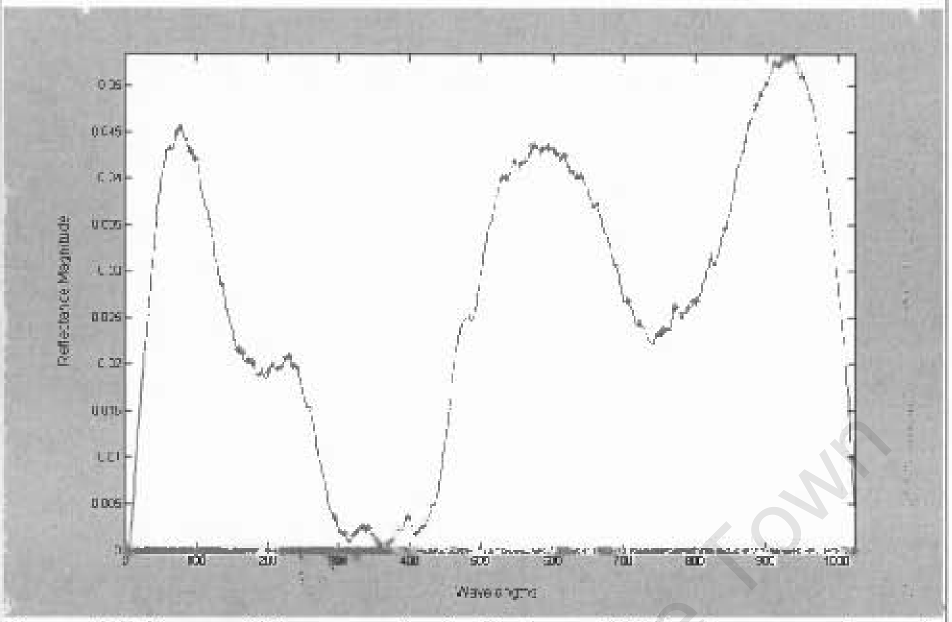


Figure 4.2 No sensitivity parameter for Peaks as FOI Tracing every change in slope.

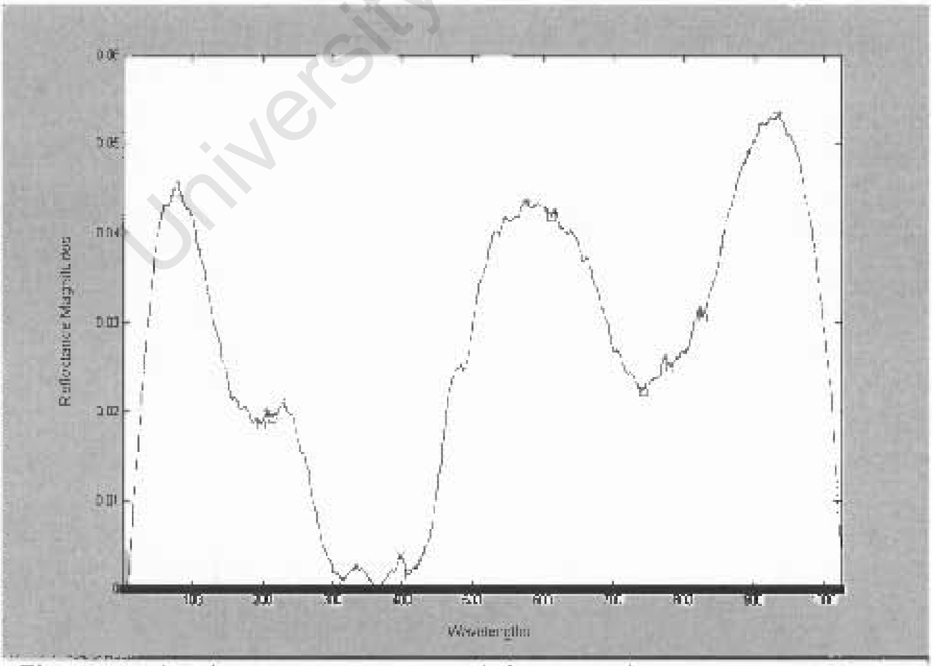


Figure 4.3 An absorption spectrum with features of interest identified at a particular sensitivity setting The stars capture the peaks and the squares the troughs

The higher the sensitivity parameter the the more profile noise is traced and the smaller it is, the more coarse an approximation of the profile we should be able to regenerate. In this chapter we present two slightly different approaches to using features thus found to describe whole spectrum profiles. The differences in the proposed representations is the modeling of the transitions between these features of interest. Figure 4.4 illustrates the process we used to formulate the new spectrum representations using our features of interest.

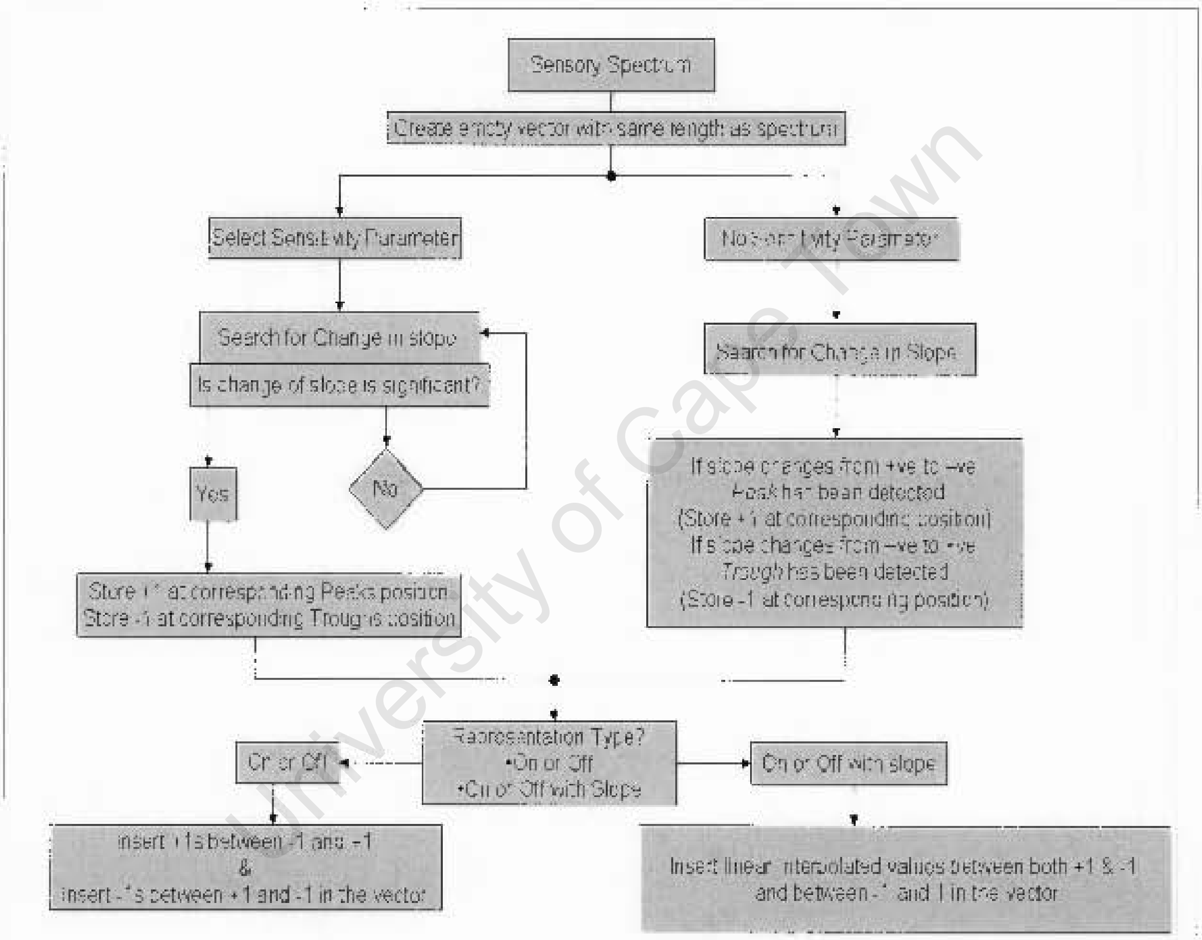


Figure 4.4 Flow chart for new representation formulation using features of interest

4.2.2 On or Off representation-slope1

At a low level of abstraction humans can be said to look primarily only at the *general* variations of the spectrum profile and where these changes occur i.e. is the profile rising or descending and where do this occur? The magnitudes of the features are only of secondary concern if and only if the overall general profiles between the spectra are not different. A further consideration might be to focus on the minor changes in the profile if primary cues do not seem to reveal obvious differences in objects. Having identified the peaks and troughs as our features of interest for the new description we neglect[7] the magnitudes of the foi and only consider the fact that the spectrum is either increasing towards a peak and decreasing towards a trough. We use an "on (+1) or off (-1)" analogy to capture the positive and negative gradients of the spectrum's profile. This type to representation eliminates the problem of unequal sized vectors expressed in the beginning of section 4.2 This then leads to the following sparse description of any spectrum profile S(x):

$$S(x) = \begin{cases} +1 \ \textit{if} \ \nabla \ \textit{is increasing} \\ -1 \ \textit{if} \ \nabla \ \textit{is decreasing} \end{cases}$$

$$\textit{where} \ \nabla \ \textit{is the slope}$$

The slope changes are captured by the change in signs between the 1s in the function vector. When the gradient of 1 sign changes from positive to negative then we know the slope is decreasing and vice versa. If we plot this function we will have a positive constant value of 1 to indicate the positive slope and a constant value of -1 to indicate where the slope is decreasing in the spectrum. Figure 4.5 illustrates the general concept on how we change from the sensory to the On or Off vector representation. Note that in figure 4.5 there are actually four features of interest,from a fourteen element long signal, that *anchor/structure* the new representation, this means we strictly use (4/14)x100 (=29%) of the original features!

[7]Within one class reflectance or absorption magnitudes might be different, the magnitudes only capture the abundance or percentage of the elements/substances in the object .

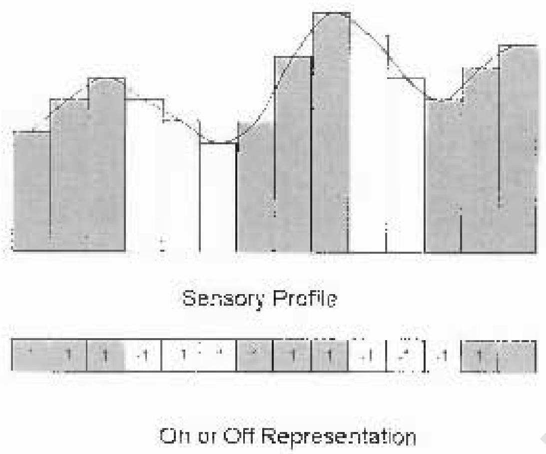


Figure 4.5 Illustration of change from sensory to On or Off Representation

The regions of rapid slope variations in the reflectance representation are captured by the close vertical lines in the On or Off representation as shown in figure 4.6a. The sensitivity parameter will, as discussed, determine how significant we regard these variations. Figure 4.6a illustrates the graphical form of this type of representation when no sensitivity parameter is incorporated. This figure is the result of changing figure 4.2's representation (with troughs included as FOI) into an On or Off representation.

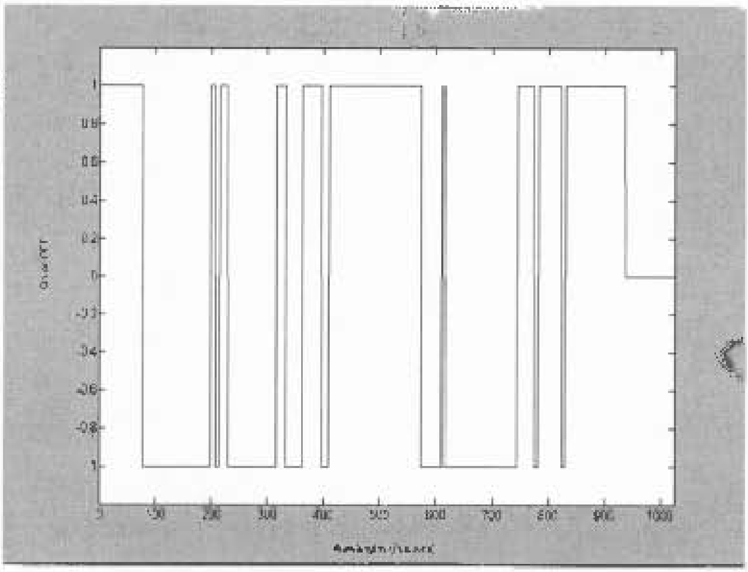


Figure 4.6a On or Off slope 1 Representation

4.2.3 On or Off slope2 representation

In this representation we also consider the peaks and troughs as our features of interest. The main difference is that we explicitly express the slope and neglect the magnitudes of the different features in the new spectrum description. The sparse formulation for every spectrum is expressed by:

$$S(x) = \begin{cases} yt + ((yp - yt)/(xp - xt))\,x_i \text{ for positive slope} \\ yp + ((yt - yp)/(xt - xp))\,x_i \text{ for negative slope} \end{cases}$$

where where x_i indicates a particular point along the wavelength, (xp-xt) and (xt-xp) are the distances between the peak and trough in the current spectrum segment. *yt* and *yp* are the values of the trough and peak respectively. Note that each *yp* is represented by a +1 and each *yt* by a -1. The transition between the features of interest is modeled by a linear interpolation between +1 and -1 or -1 and +1. Figure 4.6b shows the plot of this description of spectrum when the sensitivity parameter is not used. As mentioned before these figures are only included for general formulation clarity. In both the above descriptions the choice of using +1 or -1 is arbitrary, we could have used +1 and 0 respectively. The advantage of using the former will be evident in chapter 5.

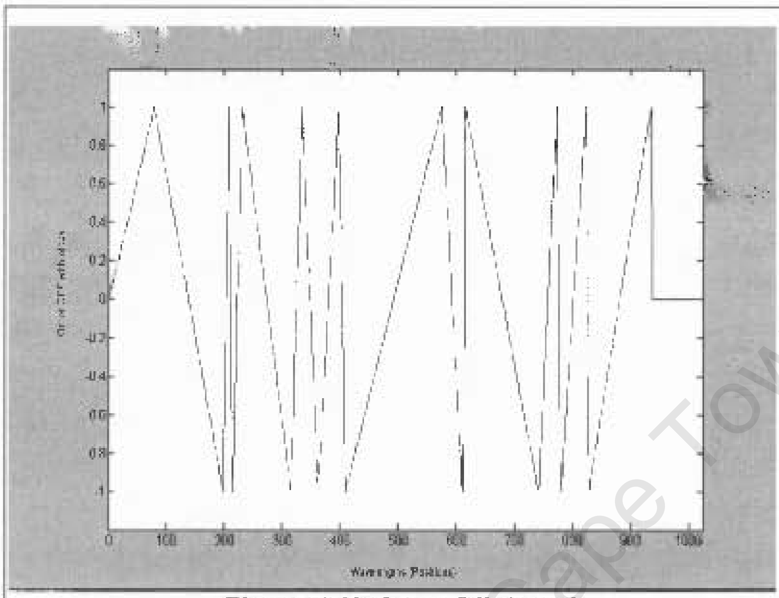


Figure 4.6b On or Off slope 2

4.3 Experimenting with sparse spectrum descriptions

In order to determine how well these new sparse descriptions of spectrum are able to capture the information implicit in the original sensory representation we compared their ability to separate classes via the separability index. Similar to section 2.2 each class comparison is split into five "training" sets and the average separability index over the five splits is then calculated[8]. Figures 4.7a-c show the separability percentage difference between using the 'Reflectance' and On of Off representations (without the sensitivity parameter)[9]. In figure 4.7a class 1 (target class) vs 5 (non target class,NTC) for example, the percentage separability

[8] See Appendix C&D for the effects of changing from the Absorption to the two On or Off representations when no sensitivity parameter is used.
[9] See Appendix E for the percentage separability differences between using the On or Off and reflectance representations at different sensitivity settings.

difference between the Reflectance and On or Off slope 1 representations is 0.8% and between the Reflectance and the On or off slope 2 is 9.76%. In this case this means it is better to use the Reflectance representation. The percentage separability differences in figures 4.7a-c are generally low, significantly less than 6% except in a few classes considering the number of features used to capture the spectrum structure.

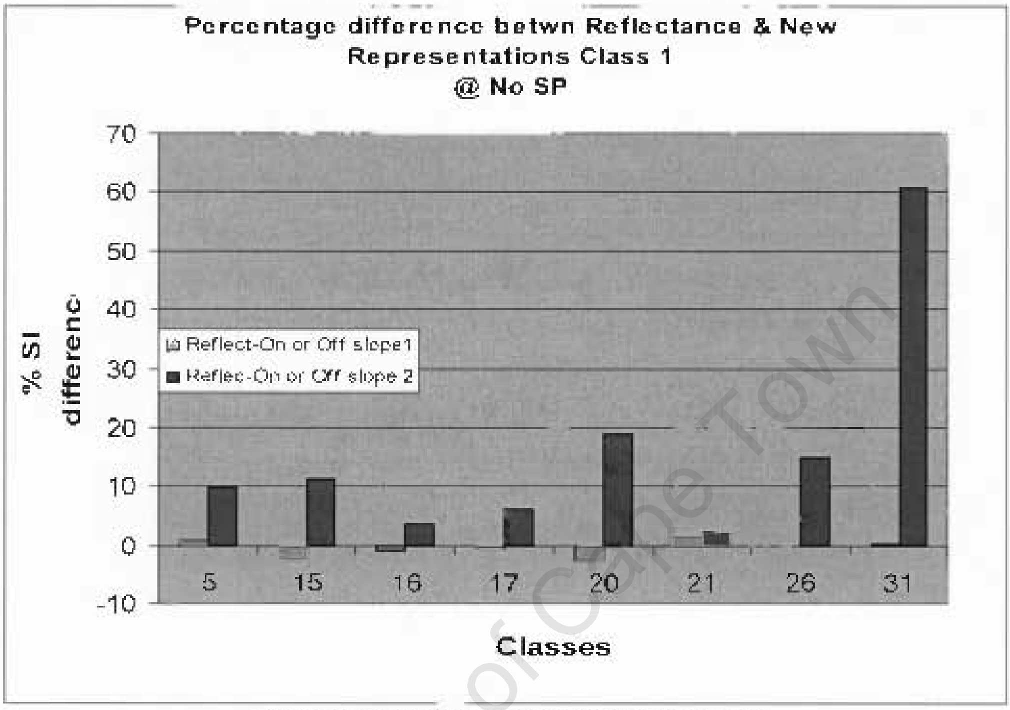


Figure 4.7a Class 1 vs Non target classes

These results indicate that peaks and trough do capture discriminatory information of spectrum profiles. Figure 4.8 shows the average percentage of features used (as explained in section 4.2.2 but with a 1024 element long signal) in each class for all representations at different sensitivity parameter settings.

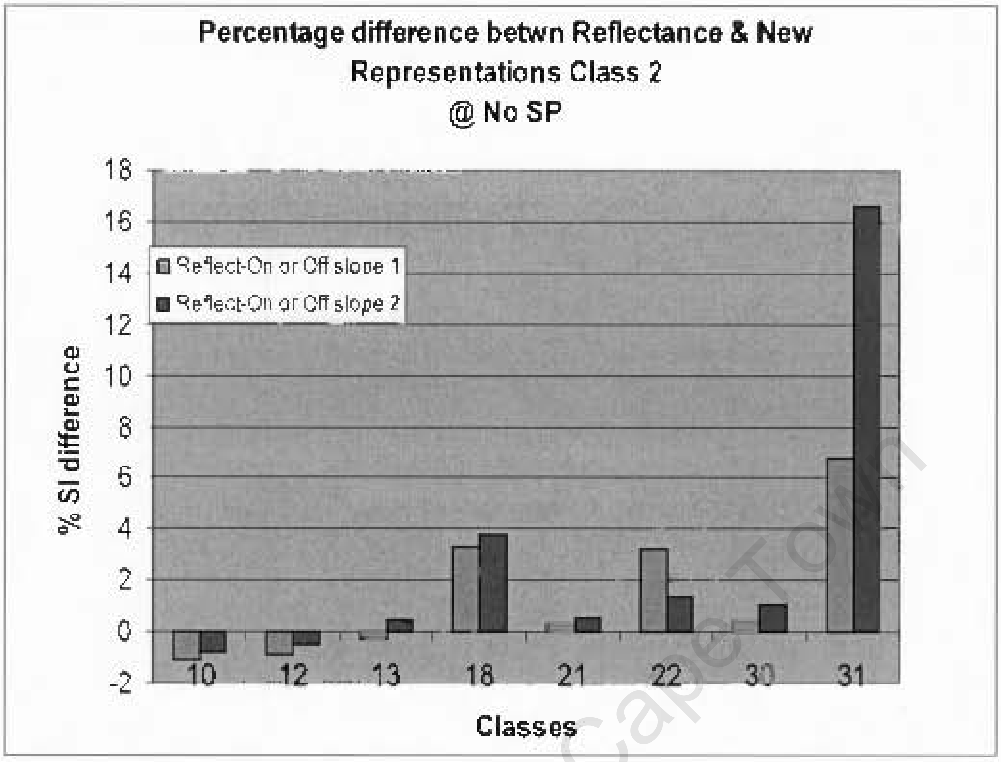


Figure 4.7b Class 2 vs Non Target Classes

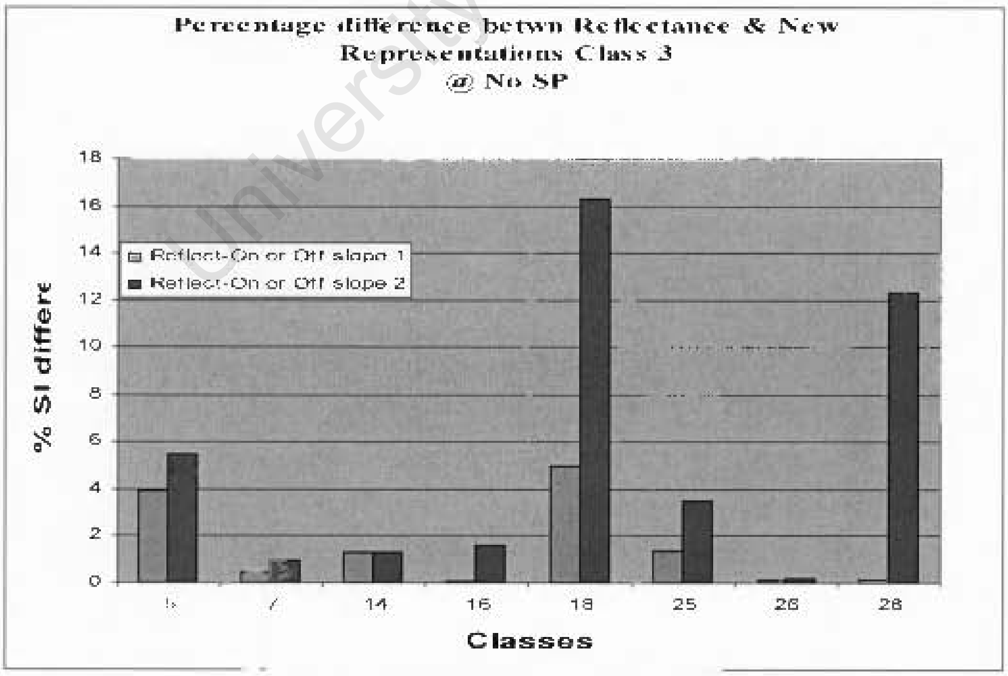


Figure 4.7c Class 3 vs Non Target Classes

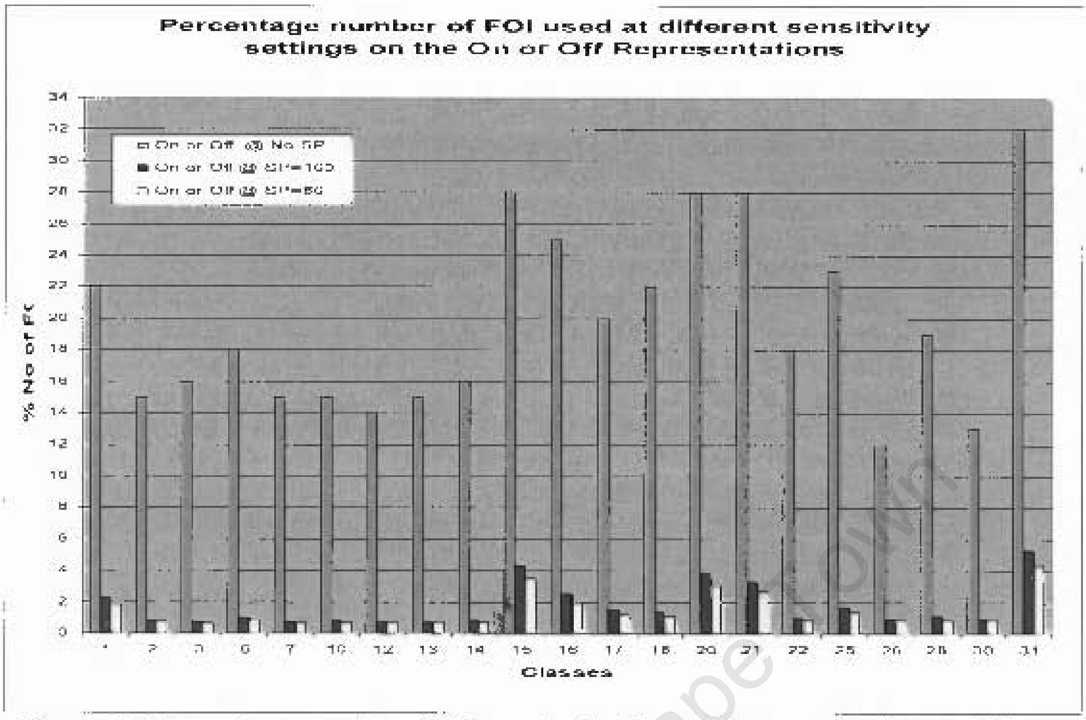


Figure 4.8 Percentage number of FOI used in the On or Off representations at different sensitivity parameter values

As can be seen in figure 4.8 the representations without the sensitivity parameter (No SP) identified the most number of features of interest. This is because in this representation we are interested in every change of slope sign that occurs in the spectrum. Figure 4.8 thus reveals which classes have the most 'noise' or fluctuations in their profiles; classes 31,15,20 & 21 etc. As expected there are fewer features of interest identified as the sensitivity parameter decreases.

Chapter 4 Discussion

We have to some extent confirmed our hypothesis that very simple sparse intuitive methods can achieve comparable performances to more complex (detailed-sensory) representations. This leads one to conclude that perhaps one can often find simple low level descriptions of objects that are normally 'clouded' in the more complex descriptions. All that is required is to identify the features of interest and re-represent our objects with these. The identification of features of interest using the sensitivity parameter can numerically reveal interesting characteristics of spectrum profiles such as how much variation there is in the profile. The results in table 2.2 and Appendix B,C & D show that just by using peaks and troughs with a certain modeling of transitions between these, we can achieve comparably similar separability results between classes as with full sensory representations. Some classes are better represented by slope 1 and others by slope 2 representations. We do not currently have an explanation for this.

We have achieved sparsity in the sense that we have used few features to describe the whole spectrum with not too much loss of classification accuracy. Thus instead and or in addition to the normal preprocessing that is often performed in pattern recognition, it would be to the researchers interest to find simpler descriptions of the given objects or situations. This would be useful not only in the latter steps in the analysis of such data e.g. classification or clustering etc but could give more insight into the causes of the differences between given objects thus making the analysis easier and more meaningful.

In general the performances of the two new representations is similar, there is no formulation that is outright better than the other. A definitive conclusion on all the classes is not possible as the non target class (NTC) are being compared to different target classes.

In the next chapter we explore another approach to obtaining a sparse representation of spectra. This approach holistically looks at an individual spectrum and determines if the general profile or the finer details of a spectrum is sufficient to differentiate it from other spectra. This information should also allow us to re-represent a spectrum with only salient information. The difference from the approaches outlined in this chapter is that the method to be investigated is a principled multiresolution approach in that the optimal resolution scales are determined automatically rather than in a naive intuitive fashion.

Where we consider the coarse approximations and finer details of a spectrum profile as a way to discriminate spectra

Chapter 5

5. Wavelets

In this chapter we introduce and motivate the use of wavelets combined with a genetic algorithm in the discrimination of spectra. We also compare the choice of two different wavelet functions when one is given two different representations of the same object.

5.1 Motivation for Wavelets

Another approach to a sparse representation of objects could possibly be attained via a wavelet decomposition. Wavelets are a technique that is used to approximate an object (often called a signal) to particular levels of detail or resolution. This is done via a wavelet transformation process explained in section 5.2. This type of decomposition allows one to visualize the coarse and detailed decompositions of the signal. By treating these different signal resolutions as 'features' of the signal we can perform the traditional feature selection on them and use the selected 'features' to reconstruct the signal.

This will allow one to determine whether the differences between two signals can be explained by the general signal profiles or the finer details of the signals. In general we would expect signals that have completely different profiles to be easily separable by their coarse approximations and those with similar profiles to be separable by detailed approximations.

5.2 Theory

5.2.1 Wavelets

The most common wavelet transformation representation of a 1 dimensional signal is Mallet's so-called cascade/tree/pyramid algorithm [16] as shown in figure 5.1 below. A signal (S) is approximated by two faculties - the scaling function (Φ) and the wavelet function (Ψ). The scaling function acts as a low frequency filter and the wavelet function can be interpreted as the high pass filter. This means the scaling function is responsible for the rough approximation of the signal whilst the wavelet function is responsible for the details in the signal. As an example in figure 5.1 we initially have $\boldsymbol{s_j}$ an 8 element signal that we want to approximate using as few elements as possible. In the first step of the wavelet transformation analysis the signal is approximated by the scaling and wavelet functions. Thus the signal is now decomposed into two segments $\boldsymbol{s_{j-1}}$ and $\boldsymbol{d_{j-1}}$, each half the length of the original signal. The scaling function produces $\boldsymbol{s_{j-1}}$ (the average of the signal) and the wavelet function produces $\boldsymbol{d_{j-1}}$ (the small details in the signal – actually the small changes in the signal).

In the second step of the wavelet transformation the input signal is now $\boldsymbol{s_{j-1}}$ from the previous step. This new signal is further transformed as before into two further segments ($\boldsymbol{s_{j-2}}$ and $\boldsymbol{d_{j-2}}$) that are each half it's length. This process is repeated until a single scaling value is reached. Note: The length of the input signal sets the limit on how many times (levels) one can transform it. The resultant components of the signal in the final step of the decomposition [s_{j-3}, d_{j-3} d_{j-2} d_{j-1}] are called wavelet coefficients. Once we have obtained these one can modify them individually according to one's needs, we can set a threshold on the elements and any element below such a value be set to zero. This is akin to removing noise. The 'new' decomposed signal can then be reconstructed by the same filters as in the analysis to form a sparse signal.

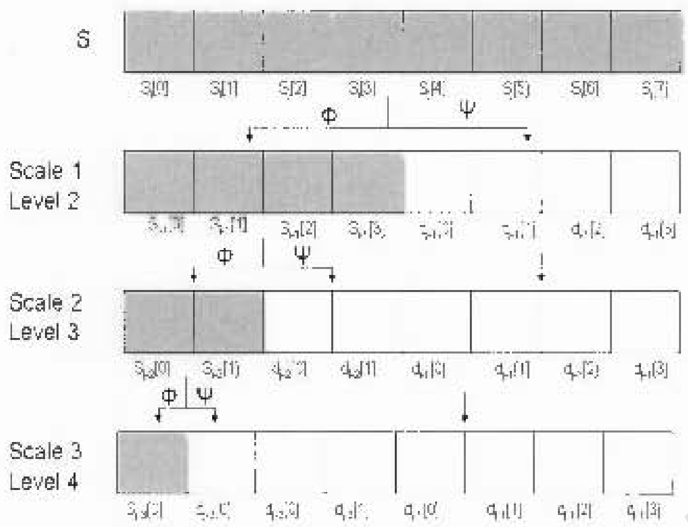


Figure 5.1 Cascade Tree

There are a number of approaches to how one can approximate a signal. This leads to the different types of wavelets. The main difference between them is the way they predict and update their signal approximation steps. In general some wavelets approximate the signal by assuming the signal elements are almost (i) equal/constant (Haar), (ii) linearly related (CDF) or have some other functional relationship.for further details refer to [5,16]. To further explain the interpretation of wavelet coefficients we plot figure 5.2. This figure illustrates the reconstruction of different wavelet coefficients of a signal (S) in what is referred to as a Multi-resolution plot.

The signal S here has been decomposed via the Daubechies - 4 wavelet for 3 scales (4 levels). The a5 reconstruction is a good coarse approximation of the input signal and the d1 reconstruction captures the finer details in the signal. By stochastically searching for an optimal set of coefficients to sparsely represent our input spectrum we are essentially selecting different levels of spectrum detail while maintaining the signal structure. Sparsity is attained by the use of as few of these coefficients as possible.

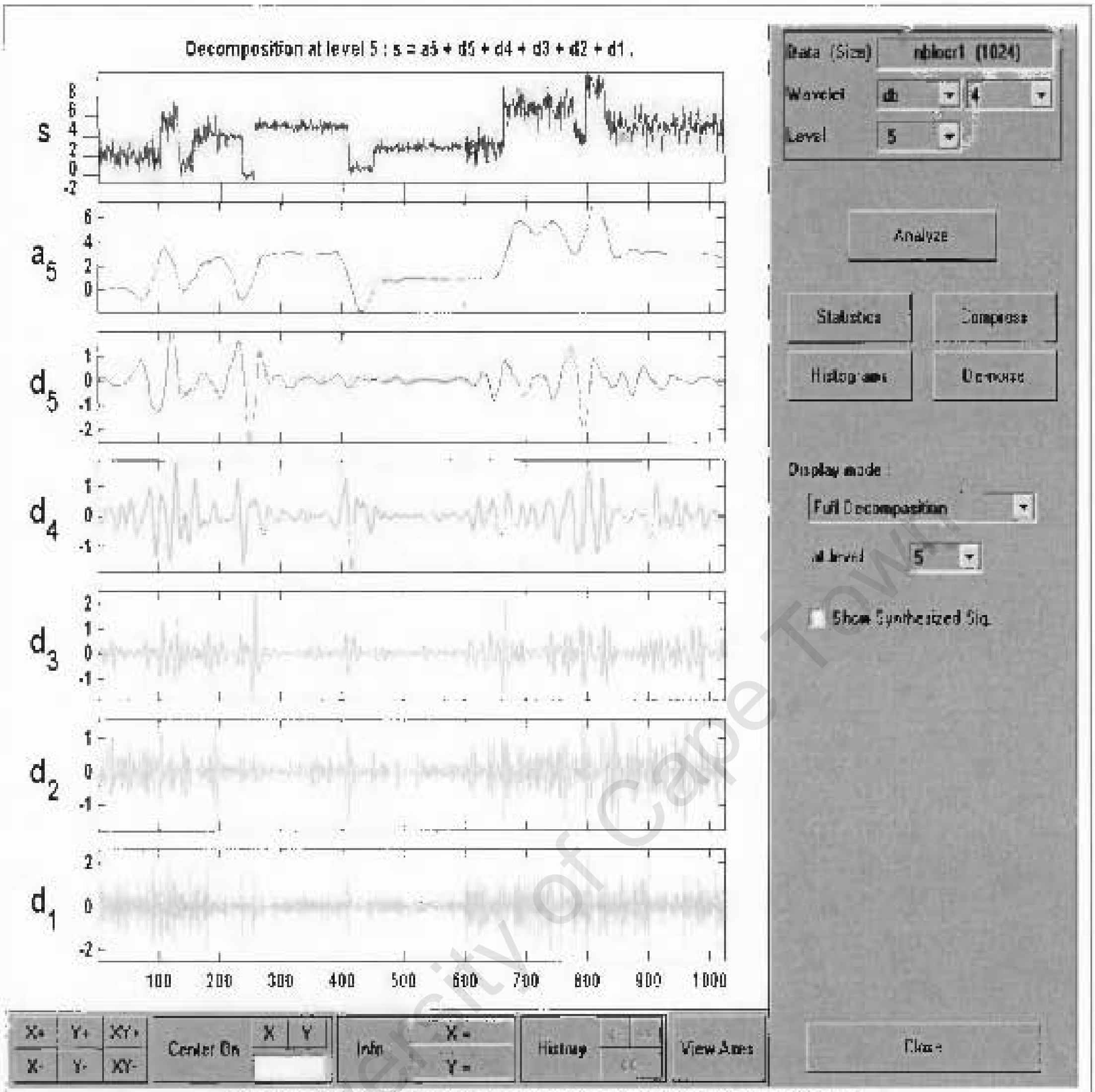


Figure 5.2 Multiresolution decomposition of a signal S.

a5 -Low pass (Coarse) approximation, d1-High pass details of Signal

5.2.2 Genetic Algorithms

Computational genetic algorithms are based on a crude model of natural evolution. As an example a trial population of individuals in a particular generation combine to form a new set of individuals made from the genes of the former. This combination of individuals repeats itself for a number of trial populations and each time better and better individuals are produced. To prevent 'run away elitism' an element of random mutation is injected in the formation of the new set of individuals. Most of the 'stronger' individuals survive and a few weaker ones are retained. After each generation of trial populations the best individuals from that generation are used to initialize the next generation.

The cycle repeats itself again for a number of trials within that generation and all the time better sets of individuals are produced than in the previous populations [24]. By using this genetics analogy computational algorithms that efficiently search a huge space of potentially good candidate solutions to a problem have been developed. In the present problem each trial generates a population of possible wavelet coefficients and using the separability index as our goodness criterion we can select the better coefficients for each population. The best set of coefficients is then kept for the next trial generation. Since our optimization algorithm included the decomposition and reconstruction of the signal thus computationally demanding, we constrained our algorithm to only 5 generations with each having 10 different trial populations of possible candidate coefficients.

Figure 5.3 below illustrates the strategy we follow in selecting the wavelet coefficients that result in the best separability between the classes. We make use of a variation of a genetic algorithm called, Population Based Incremental Learning – PBIL [2,8], to stochastically search for the best wavelets coefficients that produce the reconstruction that best separates the classes.

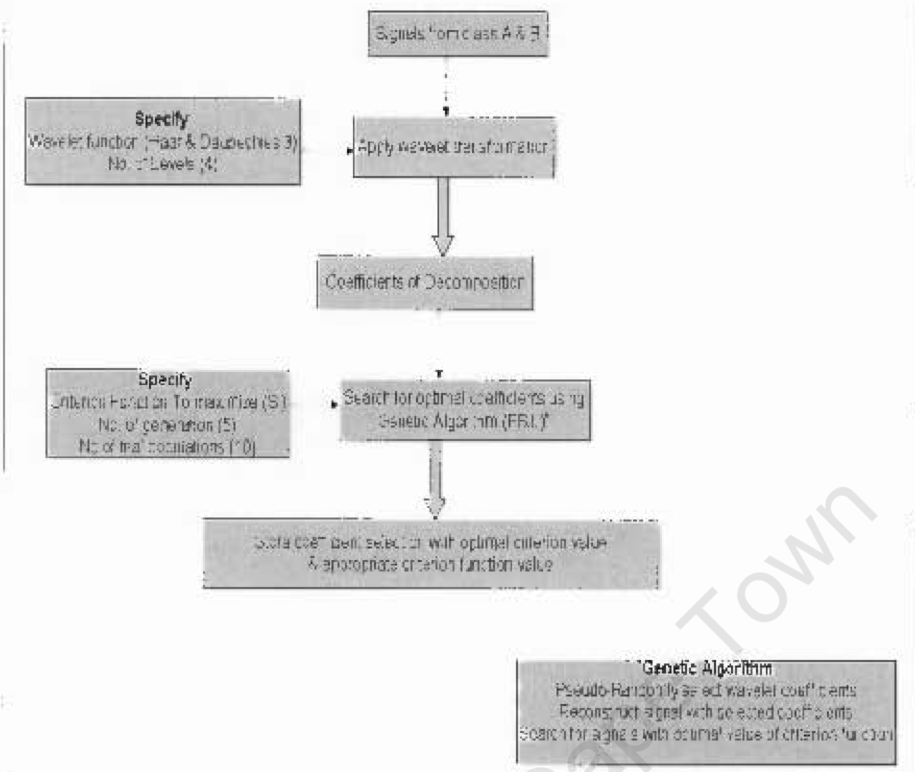


Figure 5.3 Wavelet coefficient selection flow chart

5.3 Choice of Wavelet function

In this dissertation we chose to use the Haar and the Daubechies 3 wavelets because:

- They somewhat resemble the spectra 'profiles' we are dealing with (the On or Off slope 1 and On or Off slope 2 representations of the spectra formulated in chapter 4). This is the advantage of using the +1 and -1, for the features of interest, as the wavelets functions cross the origin.

- They will give insight into whether the choice in wavelet function can be related to the signal we are trying to approximate.

We expect the Haar wavelet will produce the best spectrum reconstruction results on the On or Off slope 1 spectrum signal developed in chapter 4 as it closely resembles it. Figure 5.4 show the haar scaling and wavelet functions. Figure 5.5 illustrates the Daubechies scaling and wavelet functions.

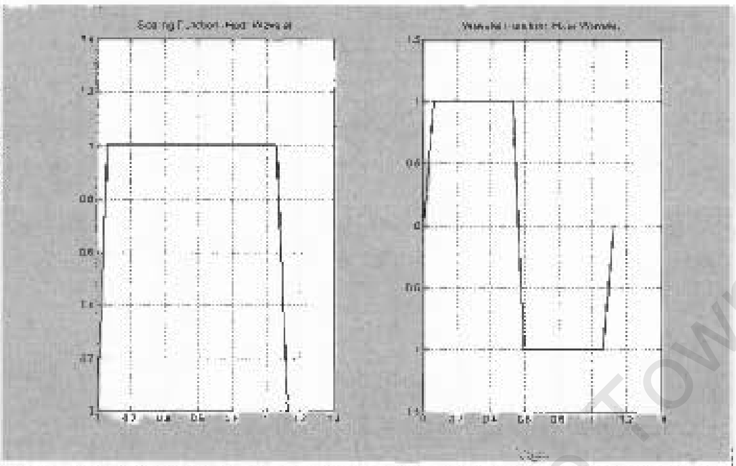


Figure 5.4 Haar Scaling and Wavelet Functions respectively

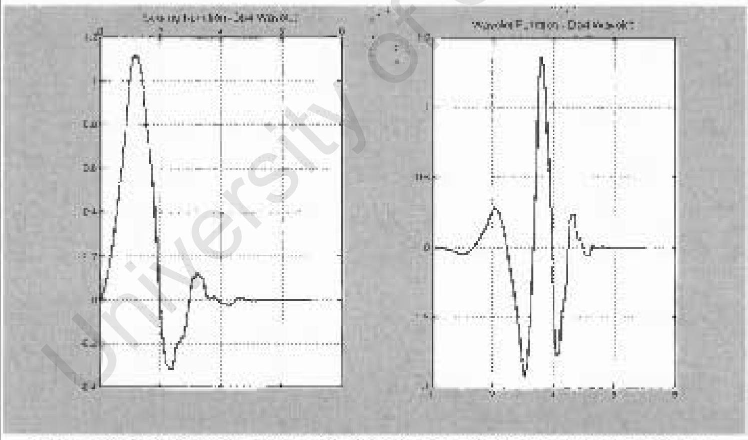


Figure 5.5 Daubechies 3 Scaling and Wavelet functions

5.4 Sparse features using Wavelets

It can be shown [16] that the signal length - 2^{10} sensory measurements in our case - determines the number of possible coefficients (10+1 for our spectra). To minimize the computational effort required in the wavelet and genetic algorithm coefficient optimization in this space we constrain the number of wavelet decomposition levels to four. We tested the wavelets optimization algorithm (with Haar and Daubechies wavelet function) on all the different spectrum representations, the reflectance and both On or Off formulations. Better separability results were obtained for the absorption representation (Appendix G) as compared with the results of On or off representations in appendix F and the original reflectance representation.

The algorithm reveals that some classes are easily separable just by their general profiles. This is indicated in cases where only the scaling coefficient is selected. An example of such a case is shown in figure 5.6a. Appendix F tabulates the full results of the wavelet coefficient optimization approach to the On or Off sparse representation for all target classes and different sensitivity settings.

5.4.1 Wavelet function relation with signal representation

In general, across all class comparisons, the haar wavelet produces better separability results when it is used on the On or Off slope 1 representation. This is to be expected as the haar wavelet functions have a closer resemblance to the slope 1 function. This result is also shown in figure 5.6a-b below. The Daubechies - 3 wavelet function performed surprisingly better on the On or Off slope 1 than on the slope 2 formulation perhaps indicating that it has a closer resemblance to former formulation than the latter. In figure 5.6a the shaded in columns show where only the general coarse approximation to the spectrum profile would be sufficient to discriminate the two classes.

Haar at scale of 3

On or Off Representation Slope 1 @ Rough Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	1	0	1	0.93
		1	0	0	0	0.93
		1	0	0	0	0.95
		1	1	0	0	0.94
		1	0	0	0	0.93
					Average	0.94

Daub 3 at scale of 3

On or Off Representation Slope 1 @ Rough Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	1	0	0.96
		1	0	0	1	0.96
		1	0	0	0	0.99
		1	0	0	0	0.96
		1	0	0	1	0.96
					Average	0.96

Haar at scale of 3

On or Off Representation Slope 1 @ 100 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	0	0	0.9
		1	0	1	0	0.9
		1	0	0	0	0.9
		1	0	1	0	0.9
		1	1	0	0	0.95
					Average	0.9

Daub 3 at scale of 3

On or Off Representation Slope 1 @ 100 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	0	0	0.9
		1	1	1	1	0.9
		1	0	0	0	0.89
		1	0	1	1	0.9
		1	1	1	1	0.86
					Average	0.9

Haar at scale of 3

On or Off Representation Slope 1 @ 50 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	0	0	0.9
		1	0	1	0	0.9
		1	0	0	1	0.9
		1	0	0	0	0.9
		1	0	0	0	0.85
					Average	0.9

Daub 3 at scale of 3

On or Off Representation Slope 1 @ 50 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	0	0	0.9
		1	0	1	0	0.9
		1	1	1	1	0.9
		1	0	0	0	0.9
		1	1	1	1	0.9
					Average	0.9

Figure 5.6a Class 3 vs 5 On or Off slope 1 representation at no ***SP*** for Haar & Daub 3 wavelets

Haar at scale of 3

On or Off Representation Slope 2 @ Rough Setting

TC	NTC	a3	d1	d2	d3	Separation
		1	0	1	0	0.93
		1	0	0	0	0.94
		1	0	1	0	0.95
		1	0	0	0	0.95
		1	0	1	0	0.94
					Average	0.94

Daub 3 at scale of 3

On or Off Representation Slope 2 @ Rough Setting

TC	NTC	a3	d1	d2	d3	Separation
		1	1	1	0	0.95
		1	0	1	0	0.95
		1	1	0	0	0.9
		1	1	1	0	0.95
		1	1	1	0	0.95
					Average	0.95

Haar at scale of 3

On or Off Representation Slope 2 @ 100 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	1	1	0.89
		1	1	0	0	0.89
		1	0	1	0	0.89
		1	0	0	0	0.9
		1	0	1	0	0.9
					Average	0.89

Daub 3 at scale of 3

On or Off Representation Slope 2 @ 100 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	1	1	0.88
		1	0	0	1	0.88
		1	1	1	0	0.89
		1	0	0	1	0.9
		1	0	1	0	0.9
					Average	0.89

Haar at scale of 3

On or Off Representation Slope 2 @ 50 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	1	0	0	0.89
		1	1	0	0	0.90
		1	0	1	0	0.90
		1	0	1	0	0.9
		1	0	1	0	0.9
					Average	0.89

Daub 3 at scale of 3

On or Off Representation Slope 2 @ 50 Setting

TC	NTC	a3	d1	d2	d3	Separation
3	5	1	0	1	1	0.89
		1	0	1	1	0.89
		1	1	0	0	0.89
		1	0	0	1	0.9
		1	0	1	0	0.9
					Average	0.89

Figure 5.6b Class 3 vs 5 On or Off **slope 2** representation at no ***SP*** for Haar & Daub 3 wavelets

In a few cases the general spectrum profile was not important and only the finer details were sufficient to separate the classes well. This mostly occurred with the absorption representation, see Appendix G. The two wavelet functions selected different coefficients for the same class comparisons under the same and different representations. The wavelet coefficient search that selects the fewest coefficients to form the spectrum signal would be the preferred choice.

5.5 Wavelet vs non-wavelet based On or Off representations

We can consolidate meaningful performance comparisons of these approaches to sparse representation by looking at their behavior for each target class. From here on we will refer to the On or Off with wavelets and genetic algorithms representation as wavelet - based representation and with no wavelets as non - wavelet based.

5.5.1 Target class 1

Table 5.1 below shows the performance of each method as the sensitivity parameter is varied[10]. The wavelet-based approach generally achieves slightly better (highlighted values) class separabilities than non - wavelet based representations (NW), this is true in all target classes. The behavior of all the slope 1 representation's separability indices (the two On or Off slope 1 wavelet based and 1 non-wavelet-based) are similar. They generally start high at no SP setting, drop at SP of 100 and increase again at an SP value of 80. The On or Off slope 2 representations do not exhibit the same behavior as the slope 1 representations, the wavelet-based approaches agree on the effect the sensitivity parameter (SP) has on the separability index (SI) but they do not exhibit the same behavior as the non-wavelet based approach.

[10]To get an idea on the achievable separability indices of the wavelet-based representations, we averaged across the 5 runs (as shown for example in figures 5.6a-b) for each representation even though the features selected were different.

On or Off Slope 1

		At No SP			At SP =100			At SP=80		
TC	NTC	Separability Index			Separability Index			Separability Index		
		NW	Haar	Daub 3	NW	Haar	Daub 3	NW	Haar	Daub 3
1	5	0.9640	0.978	0.978	0.944	[illegible]	[illegible]	0.948	0.956	[illegible]
	15	0.9640	0.980	0.973	[illegible]	[illegible]	[illegible]	0.941	0.987	[illegible]
	16	0.9721	0.984	0.984	0.960	0.968	0.958	0.981	[illegible]	0.971
	17	0.9801	0.988	0.990	0.947	0.965	0.953	0.954	[illegible]	[illegible]
	20	0.9778	0.978	0.963	[illegible]	0.980	0.982	0.974	0.979	0.983
	21	0.9484	0.956	0.948	[illegible]	0.945	0.947	0.939	[illegible]	[illegible]
	26	1.0000	1.000	1.000	[illegible]	0.996	0.997	0.998	[illegible]	[illegible]
	31	0.9982	0.990	0.990	[illegible]	1.000	1.000	0.999	1.000	0.999

On or Off Slope 2

		At No SP			At SP =100			At SP=80		
TC	NTC	Separability Index			Separability Index			Separability Index		
1		NW	Haar	Daub 3	NW	Haar	Daub 3	NW	Haar	Daub 3
	5	0.874	[illegible]	[illegible]	0.942	0.941	0.941	0.954	0.956	0.953
	15	0.827	[illegible]	0.859	[illegible]	0.922	[illegible]	[illegible]	0.927	0.921
	16	[illegible]	0.947	0.940	0.994	[illegible]	[illegible]	0.987	0.961	0.959
	17	0.914	0.965	0.969	0.949	[illegible]	[illegible]	0.957	0.963	0.960
	20	0.760	0.865	0.870	[illegible]	[illegible]	[illegible]	[illegible]	0.938	0.941
	21	[illegible]	[illegible]	[illegible]	0.942	0.947	0.944	0.944	0.953	0.950
	26	[illegible]	1.000	1.000	[illegible]	0.998	0.998	1.000	1.000	1.000
	31	[illegible]	0.906	0.931	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.991

Table 5.1 Separability Index comparison between wavelets (Haar & Daub) and No wavelets(NW) for TC =1at different SP values

5.5.2 Target class 2

The two On or Off slope 1 wavelet - based representations also project consistent SI behavior as the SP is varied for each non - target class. The On or Off Slope 2 wavelet - based representations exhibit similar results for the change in SP value but this behavior is not consistent with the non-wavelet based method.

On or Off Slope 1

		At No SP			At SP =100			At SP=80		
TC	NTC	Separability Index			Separability Index			Separability Index		
2		NW	Haar	Daub 3	NW	Haar	Daub 3	NW	Haar	Daub 3
	10	0.985	0.992	0.992	[illegible]	0.986	0.989	0.963	0.985	0.986
	12	[illegible]	0.992	0.992	0.983	0.983	0.976	0.987	0.986	[illegible]
	13	[illegible]	0.989	0.988	0.984	0.983	0.985	[illegible]	0.988	0.980
	16	0.984	0.984	[illegible]	[illegible]	0.966	0.969	0.958	0.967	0.968
	21	[illegible]	0.999	0.999	0.994	0.996	0.996	0.994	0.998	0.998
	28	[illegible]	0.987	[illegible]	0.973	0.974	0.976	[illegible]	0.973	0.973
	30	0.981	0.956	0.956	[illegible]	0.981	0.980	0.977	0.989	0.978
	31	[illegible]	1.000	1.000	0.847	0.975	0.968	[illegible]	0.960	0.962

On or Off Slope 2

		At No SP			At SP =100			At SP=80		
TC	NTC	Separability Index			Separability Index			Separability Index		
2		NW	Haar	Daub 3	NW	Haar	Daub 3	NW	Haar	Daub 3
	10	0.99	0.978	0.980	[illegible]	0.984	0.984	0.978	0.980	0.980
	12	0.99	0.984	0.984	0.981	0.982	0.981	0.980	0.980	0.981
	13	[illegible]	0.984	0.980	[illegible]	0.982	0.983	0.975	0.980	0.976
	18	[illegible]	[illegible]	[illegible]	0.964	0.965	0.965	0.966	0.966	0.966
	21	[illegible]	0.993	0.994	[illegible]	0.997	0.996	0.996	0.998	0.996
	28	[illegible]	0.982	[illegible]	[illegible]	0.969	0.969	0.969	0.967	0.966
	30	[illegible]	0.982	[illegible]	[illegible]	0.978	0.978	0.976	0.976	0.976
	31	[illegible]	0.992	[illegible]	[illegible]	0.996	0.994	0.978	0.993	[illegible]

Table 5.2 Separability Index comparison between wavelets (Haar & Daub) and non wavelet-based representations(NW) for TC =2 at different SP values

5.5.3 Target class 3

In table 5.3 the wavelet - based representations agree on the influence of the sensitivity parameter on the separability index i.e similar behavior is observed when the sensitivity parameter is varied. The non - wavelet based representation does not quiet exhibit similar behavior.

On or Off Slope 1										
		At No SP			At SP =100			At SP=80		
TC	NTC	Separability Index			Separability Index			Separability Index		
3		NW	Haar	Daub 3	NW	Haar	Daub 3	NW	Haar	Daub 3
	5	0.946	[illegible]	0.983	0.897	0.898	0.898	0.897	0.905	0.906
	7	0.995	0.998	0.996	0.994	0.994	0.993	0.994	0.995	0.994
	14	0.976	0.994	0.992	0.922	0.908	0.907	0.905	0.922	0.925
	16	0.983	0.989	0.991	0.978	0.980	0.981	0.978	[illegible]	[illegible]
	18	0.937	0.983	0.980	0.947	0.944	0.942	0.941	[illegible]	[illegible]
	[illegible]	0.969	0.982	0.981	0.965	0.972	0.971	0.965	0.977	0.974
	26	0.999	1.000	1.000	0.996	0.972	0.997	0.997	0.996	0.996
	28	0.964	[illegible]	0.976	0.910	0.915	0.917	0.917	0.915	0.917

On or Off Slope 2										
		At No SP			At SP =100			At SP=80		
TC	NTC	Separability Index			Separability Index			Separability Index		
3		NW	Haar	Daub 3	NW	Haar	Daub 3	NW	Haar	Daub 3
	5	0.933	0.942	0.954	0.897	0.892	0.893	0.887	0.892	0.893
	7	0.990	0.993	0.994	0.981	0.989	0.986	0.982	0.989	0.985
	14	0.876	0.975	0.874	0.901	0.899	0.897	0.891	0.899	0.893
	16	0.967	0.968	0.970	0.980	0.974	0.974	0.973	0.974	0.974
	18	0.824	0.917	0.928	0.943	0.945	0.944	0.941	0.945	0.944
	[illegible]	0.948	[illegible]	[illegible]	0.961	0.970	0.967	0.964	0.970	0.967
	26	0.998	1.000	1.000	0.996	0.997	0.995	0.995	0.997	0.998
	28	0.842	0.852	0.957	0.925	0.926	0.924	0.921	0.926	0.924

Table 5.3 Separability Index comparison between wavelet (Haar & Daub) and non wavelet-based representations(NW) for TC =3 at different SP values

In all target classes the discrepancy in the results between the wavelet - based and non-wavelet based representations is difficult to dissect. This could be due to the different features which the methods are focused on at different sensitivity parameter settings;

- the non – wavelet based approach of the On or Off representation only uses the peaks and troughs as the features thus it does not have a global picture of the spectrum but only these features while
- the wavelet - based representation considers the peaks and troughs as perhaps one 'feature' among other features such as the general change of the whole spectrum. This provides the wavelet - based approach with an upper hand when considering different combinations of these features at different resolutions.

5.6 Chapter 5 Discussion

In this chapter we have broadened the concept of "parts of a spectrum" to be the coarse and finer details across the whole spectrum measurement range. We have explored the use of wavelets combined with a genetic algorithm to formulate sparse representations of spectrum. This paradigm has revealed that certain classes can be efficiently discriminated between by using just their profile approximations others require full high pass details, that is, most of the details that are present in the sensory signal (see Appendix G class 2 and 3).

We explored the influence of the wavelet function choice on the reconstruction of a signal when given different representations of the same signal under the above mentioned setting. The signal representation frequently had a significant influence on the performance of wavelet function used. The Haar wavelet function has a visually closer resemblance to the On or Off slope 1 signal and it provided better signal reconstruction in terms of the resultant class separability. In general the wavelet approach to a sparse representation produces slightly better separability indices between classes than the sensory representation. This can be attributed to the use of global stochastic search for the best combination of wavelet coefficients to reconstruct a signal. Unfortunately this approach also presented an inconsistent feature set which makes it difficult to determine which coefficient features are optimal.

Where we summaries the main ideas of thesis and describe each chapters contribution

Chapter 6

6. Conclusion

6.1 Discussion

The main hypothesis of the thesis is that objects dealt with in machine learning could be better represented using simple, human - interpretable descriptions. These descriptions should automatically capture the intuitively salient underlying structure of the objects in question and this we hypothesize will provide the researcher with better insights into not only the data but the designing of interpretable classifiers and clustering algorithms. Common paradigms (for example 'blind' feature selection, data projections(PCA) etc) to this goal do not explicitly reveal the underlying structure of objects but tend to provide more computationally efficient models of representing objects which often do not have a physical interpretability. We observed that human classifiers of images and spectra extract meaningful features when describing such objects and hypothesize that this might also be a helpful strategy for inductive machine classifiers. In this regard we considered three paradigms of meaningful feature discovery.

We reviewed and analyzed a method called Non Negative matrix factorization (NMF), on account of its ability to automatically discover intuitive parts of objects, as a plausible way of extracting such features. Careful investigation of NMF on synthetic problems in the original 'image' context showed that the reality is more complex than this and in general independent object parts are not found. We investigated the relevance of NMF on real spectra. In chapter 4 we explored a more empirical approach to extracting sparse, intuitive visual features from spectra. The features of interests thereby extracted were used to re-describe spectra and this new formulation's efficacy was investigated in terms of class separation of spectra. The final chapter explored wavelets as a more traditional and more principled approach to multiresolution feature extraction and investigated its effect on class separability.

6.2 Section conclusions

6.2.1 Sparse spectrum representation using NMF

We explored the applicability of a method called Non negative matrix factorization (NMF) in finding parts of spectra when one is given a database of spectra. This was motivated by its applicability in a number of domains (especially images). Unfortunately this method was found not to work well with spectrum type data. The parts found were somewhat holistic, in the sense that it was difficult to distinguish the parts discovered by the method from whole spectra. This limitation of NMF was explained by a synthetic example that revealed what type of problems NMF would be better suited for. This was our novel contribution in this section of the thesis.

6.2.2 Sparse re-representation of spectrum using features of interest

By identifying sparse salient features of a spectrum profile we were able to compress the description of the spectrum into simple mathematical models. We chose peaks and troughs as our salient features. We modeled the transition between the features in two simple ways; in one model if the slope of the spectrum profile increased we had a constant positive value and if it decreased we had a constant negative value. The other model interpolated between the peaks and troughs. This type of re-description for spectra produces class separabilities that are in general comparable to the original(sensory) description of spectrum even though they used few 'features'.

This goes some way to show that simple, sparse features can capture important information latent in full sensory formulations of spectrum. A similar argument of using sparse features to describe detailed objects in other domains (facial images) can be drawn by identifying the discriminatory features and forming new descriptions of such images with them one can for example determine how different facial images vary. Two main advantages of this formulation are that is a *simple* and *intuitive* approach to capturing the structure of the objects while giving comparable results to more complex representations. Our novel contribution in this section is the re-representation of spectra using simple mathematical formulations that are intuitively attractive and practical.

6.2.3 Sparse representation using wavelets

In this chapter we explored the use of wavelets to determine if the spectrum classes could be better discriminated by coarse approximations or detailed formulations of spectra. By using the coarse and detailed approximations of the wavelet transform as features we used a genetic algorithm, PBIL, to search for the best sparse representation in terms of class separability. Since the class separabilities were initially high with the original reflectance spectrum only a slight improvement was noticeable is some classes especially with the absorption formulation.

On average the wavelet formulation improved the separability indices between the classes *but* this was a computationally expensive process. It consisted of the following processes: wavelet decomposition of signal, representation of wavelet coefficients as features and class separability optimization with these features using genetic algorithm (see figure 5.3) as is frequently the case in feature selection. The method was often not consistent in the selected optimal features for any of the representations over the trial generations (see Appendix F & G). One of the primary motivations of this study is to discover simple interpretable features to describe spectra such that they can assist us in later processing, we would thus prefer simple and quick guides to how we can achieve this. The class separability gains achieved with the wavelet approach do not justify the computational costs incurred over the non wavelet approach presented in chapter 4.

6.3 Representation Conclusion

In general, for the approaches considered in this thesis, there is no clearly 'overall best' approach to achieving a sparse spectrum representation. This can be attribute to the following factors:

- The target classes are different and
- Most of the non-target classes are not common to all the target classes and this makes it difficult to succinctly generate a good characterization of the classes.
- The different approaches to sparsity achieve quite similar separability results.

But in terms of providing *slightly* better class separabilities wavelet representation is better. This is shown at the end of chapter 5. What is clear is that the results of the different approaches are quite similar and the main differences are in implementation practicality and result presentation. Perhaps then we can only evaluate these approaches by considering their strengths and weaknesses:

Advantages of wavelets with genetic algorithm approach

- Slightly better class separability which can be attributed to the genetic algorithm's optimal search of the huge solution space.
- Ability to automatically reveal which features are is important in object discrimination.
- Method allows global visualization of the optimal features using multiresolution analysis.

Disadvantages of wavelet with genetic algorithm approach

- Huge computational cost due to a number of processes: signal decomposition, stochastic search and signal reconstruction.
- Too many free (user specified) parameters: Number of decomposition levels, wavelet function to use, number of trials and generations in the genetic algorithm.
- Non consistent optimal feature set.

Advantages of non-wavelet-based approach

- Single user parameter (the sensitivity parameter) – This results in quick and simple experiments.

Disadvantages of non-wavelet-based approach

- User defined features of interest – this limits the method to exploit *only* these.

Taking computational and method - result-presentation into account we would not recommend the wavelet approach as a simple, initial preprocessing, exercise to explore dataset structures. This can be better achieved by simpler and quicker methods such as those presented in chapter 4. If a significant amount of knowledge about data objects is required for long term projects then wavelet based simulations could be worth the computational burden.

6.4 Open problems/Future work

A possible extension to this work could be having identified the important discriminatory features in objects to actually determine how they are *inter-related*. Just as in the synthetic Sticks problem or facial images; if the features (sticks,eyes,nose etc) are identified and their possible positions in the images are determined, one could have an overall description of how complete objects in the images are formed. This we believe could lead one to formulate a powerful new representation of objects.

References

[1] B.K Alsberg,M.K Winson,D.B Kell. **Improving the interpretation of multivariate and rule in duction models by using a peak parameter representation**. Chemometrics and Intelligent Laboratory Systems, pages 95-109,Vol. 36.

[2] S. Baluja. **Population-Based incremental Learning: A method of integrating genetic search based function optimization and competitive learning.** Tech. Rep. No. CMU-CS-94-163, Carnegie Mellon University, Pittsburgh,PA,(1994).

[3] M.Belkin. **Problems of learning on Manifolds**. PhD thesis Dept. of Mathematics, University of Chicago, Chicago, Illinois 2003.

[4] M.Chu, F.Diele, R.Plemmons and S.Ragni. **Optimality, computation and interpretation of Non negative matrix factorizations**. Unpublished Report, October 2004. Available at http://www.wfu.edu/~plemmons/papers.htm

[5] I.Daubechies. **Ten Lectures on Wavelets**. CBMS-NSF Regional Conference Series in Applied Mathematics. SIAM 1992.

[6] D.L Donoho. **High dimensional data analysis: The curses and blessings of Dimensionality.** 2000. American Math. Society "Math challenges of the 21st Century" (2000). Available at http://www-stat.stanford.edu/~donoho/lectures.html

[7] D. Donoho and V.Stodden. **When does non-negative matrix factorization give a correct Decomposition into parts?** In Proc. Of Advances in Neural Information Processing 16 (NIPS 2003).MIT Press,2004.

[8] T. Gosling, N.Jin and E.Tsang. **Population-Based Incremental Learning versus Genetic Algorithms: Iterated Prisoners Dilemma**. Tech. Rep. CSM-401,University of Essex,England (March 2004).

[9] J. Greene. **Feature subset selection using Thornton's separability index and its applicability to a number of sparse proximity-based classifiers**. Sixteenth Annual Symposium of the Pattern recognition association of South Africa (PRASA), 2001.

[10] D. Guillamet,J. Vitria. **Classifying faces with Non Negative matrix factorization**. Accepted CCIA 2002, Castelló de la Plana, Spain, 2002.

[11]B.Heisele,T.Serre,M.Pontil,T.Poggio. **Component based face detection**. In:Proceedings of 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR2001), IEEE Computer Society Press, Kauai, Hawaii, pages 657-662, Vol. 1, December 2001.

[12] B.Heisele,P.Ho,J.Wu and T.Poggio. **Face Recognition: component -based versus global approaches.** Computer Vision and Image Understanding, pages 6-21,Vol. 91,2003.

[13] R.Holte. **Very Simpleclassification rules perform well on most commonly used datasets**. Machine Learning, pages 63-91,Vol. 3.1993.

[14] P.Hoyer. **Non Negative Matrix Factorization with sparseness constraints**. Journal of Machine Learning Research, pages 1457-1469,Vol. 5, 2004. http://www.cs.helsinki.fi/patrik.hoyer/

[15] A.Hyvarinen. Survey on Independent Component Analysis. Available at http://www.cis.hut.fi/`aapo/

[16] A. Jensen and A. la Cour-Harbo. **Ripples in Mathematics; The discrete Wavelet Transform**. Springer Verlag Berlin Heidelberg. 2001.

[17] R.F Kokaly,R.N Clark. **Spectroscopic determination of leaf biochemistry using band-depth analysis of absorption features and stepwise multiple linear regression**. Remote Sensing and Environment, pages 267-287, Vol. 67 Issue 3,1999.

[18] D. Kulpinski. **LLE and Isomap analysis of spectra and colour images**. . MSc thesis Department of Computer Science, University of Simon Fraser 2002.

[19] D.D Lee and H.S. Seung. **Learning the parts of objects by non-negative matrix factorization**. Nature, pages 788-791, Vol. 401, October 1999.

[20] D.D Lee and H.S. Seung. **Algorithms for non negative matrix factorization**. In Proceedings of Advances in Neural Information Processing Systems (NIPS), pages 556-562,Vol. 13, 2001.

[21] S.Z. Li, W.Hou and H.J Zhang. **Learning spatially localized, parts-based representation**. In 2001 Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), pages 207-212, Vol. 1, 2001.

[22] H.Liu,H Motoda. **Feature selection, extraction and construction**. In The Handbook of Data Mining, Lawrence Erlbaum Associates, Inc. Publishers. Editor: N. Ye, pages 409 – 423, 2003.

[23] W.Liu, N Zheng and X.Lu. **Non negative matrix factorization for visual coding**. In Proceedings to the IEEE International Conference on Acoustic, Speech and Signal Processing (ICASSP 2003), 2003.

[24] T.Mitchell. Chapter 9, **Machine Learning**.McGraw-Hill International Edition.ISBN 0-07-115467-1,1997.

[25] A.Mohan,C.Papageorgiou,T.Poggio. **Example-based object detection in images by components**. IEEE Transactions on Pattern Analysis and Machien Intelligence,pages Vol 23, No. 4, 2001.

[26] J. Moody, R. Silva, J. Vanderwaart. **Data filtering for automatic classification of rocks from Reflectance spectra**. 7TH ACM SIGKDD International Conference of Knowledge Discovery and Data Mining. San Francisco, California USA 2001.

[27] M.Oren, C.Papageorgiou, T.Poggio. **A trainable system for people detection**. International Journal of Computer Vision, 38, 1, 15-33, 2000.

[28] L. Parra, C. Spence, P. Sajda, A. Ziehe, K-R Muller. **Unmixing Hyperspectral Data**. Advances in Neural Information Processing Systems 12, MIT Press, pp. 942-948, 2000.

[29] J. Piper, V.P Pauca, R.J.Plemmons, M.Giffin. **Object characterization from Spectral data using Non negative Factorization and information Theory**. AMOS Technical Conference, Maui, HI, September, 2004

[30] P. Paclik,R.W Duin. **Dissimilarity-based representation of spectra: computational issues**. Real-Time Imaging Vol 9 pages 237-244. 2003.

[31] P.Pudil, P,Somol. **An overview of feature selection techniques in statistical pattern recognition**. Tutorial paper in Proceedings of the 16th Annual Symposium of the Pattern Recognition association of South Africa pages1-14 November 2005.

[32] D.A. Ross. **Learning parts-based representation of Data**. MSc thesis Department of Computer Science ,University of Toronto, Toronto, Canada 2003.

[33] Y. Rubner, C.Tomasi, L.J.Guibas. **A metric for Distributions with Applications to image databases.** In Proceedings of the IEEE International Conference on Computer Vision.1998.

[34] L.K Saul, S.T Roweis. **Non linear dimensionality reduction by local linear embedding.** Science 290.pages 2323-2336,2000.

[35] P. Sajda, S.Du, L.Parra. **Recovery of constituent spectra using non-negative matrix Factorization**. Invited submission, SPIE Wavelets X, San Diego, CA, 2003.

[36] P. Smaragdis and J.C Brown. **Independent component analysis for automatic note extraction from musical thrills**. In Journal of Acoustical Society of America. No.115, Vol5, May 2004.

[37] P.Smaragdis. **Discovering auditory objects through non-negativity constraints**. Statistical and Perceptual Audio Processing (SAPA), October 2004, SAPA 2004. http://www.merl.com/publications/TR2004-094/

[38]available at http://speclab.cr.usgs.gov/PAPERS.refl-mrs/refl4.html#section2.0 Section 6.4.

[39] C. Thornton. **Truth from Trash**,How learning makes sense. MIT Press, 2000.

[40] M. A Turk and A. P. Pentland. **Face Recognition using eigenfaces**. In Proc. IEEE Conference on Computer Vision and Pattern Recognition(CVPR '91), pages 586-591 Maui, Hawaii, 1991

Appendix

Appendix A

Separability Index of combined classes on Reflectance data

1=[1 to 2], 2=[3to12], 3 =[13 to 14] and [15-42] relabeled as classes [4-31].

Amanda1 Combined		
TC	NTC	SI
1	2	0.997
1	3	0.997
1	4	0.985
1	5	0.972
1	6	0.998
1	7	0.998
1	8	0.995
1	9	0.996
1	10	0.996
1	11	0.994
1	12	0.994
1	13	1.000
1	14	0.989
1	15	0.939
1	16	0.962
1	17	0.974
1	18	0.982
1	19	0.996
1	20	0.943
1	21	0.958
1	22	0.991
1	23	0.993
1	24	0.986
1	25	0.986
1	26	1.000
1	27	0.997
1	28	0.968
1	29	0.978
1	30	0.993
1	31	1.000

Amanda 1 Combined		
TC	NTC	SI
2	3	0.996
2	4	0.995
2	5	0.983
2	6	0.982
2	7	0.982
2	8	0.993
2	9	0.992
2	10	0.979
2	11	0.992
2	12	0.981
2	13	0.984
2	14	0.991
2	15	0.998
2	16	0.995
2	17	0.997
2	18	0.987
2	19	0.996
2	20	0.999
2	21	0.998
2	22	0.989
2	23	0.994
2	24	0.968
2	25	0.992
2	26	0.997
2	27	0.993
2	28	0.992
2	29	0.990
2	30	0.966
2	31	1.000

Amanda 1 Combined		
TC	NTC	SI
3	4	0.996
3	5	0.987
3	6	0.998
3	7	0.999
3	8	0.996
3	9	0.997
3	10	0.994
3	11	0.995
3	12	0.997
3	13	0.998
3	14	0.989
3	15	0.994
3	16	0.983
3	17	0.996
3	18	0.987
3	19	0.996
3	20	0.996
3	21	0.995
3	22	0.990
3	23	0.991
3	24	0.990
3	25	0.983
3	26	1.000
3	27	0.998
3	28	0.965
3	29	0.998
3	30	0.993
3	31	1.000

TC :-Target Class

NTC :- Non Target Class

SI :-Separability Index

Appendix B

Effect of Changing Representation from Reflectance to Absorption Representation on the Separability index

Reflectance Representation				*Reflectance Representation*				*Reflectance Representation*		
TC	***NTC***	***S.I***		***TC***	***NTC***	***S.I***		***TC***	***NTC***	***S.I***
1	5	0.972		2	10	0.9786		3	5	0.9872
1	15	0.9389		2	12	0.9814		3	7	0.9988
1	16	0.9615		2	13	0.9843		3	14	0.9889
1	17	0.9737		2	18	0.987		3	16	0.9834
1	20	0.9491		2	21	0.9982		3	18	0.9868
1	21	0.9577		2	22	0.9894		3	25	0.9832
1	26	1		2	30	0.9858		3	26	1
1	31	1		2	31	1		3	28	0.9653
Absorption Representation				*Absorption Representation*				*Absorption Representation*		
TC	***NTC***	***S.I***		***TC***	***NTC***	***S.I***		***TC***	***NTC***	***S.I***
1	5	0.9713		2	10	0.981		3	5	0.9849
1	15	0.9674		2	12	0.9809		3	7	0.9968
1	16	0.9683		2	13	0.9838		3	14	0.9894
1	17	0.9827		2	18	0.9877		3	16	0.9829
1	20	0.9749		2	21	0.9981		3	18	0.9889
1	21	0.9385		2	22	0.9896		3	25	0.9827
1	26	1		2	30	0.9885		3	26	1
1	31	1		2	31	0.9997		3	28	0.9599

Note:
Reflectance Representation : = Original Given Data.
Absorption Representation : = Original Data with Convex Hull Removed.
Sepindex (S.I) : = Average Separability index over 5 'fold cross validation'
TC : = Target Class
NTC : = Non Target Class

Appendix C

Effect of Changing Representation from Absorption to On or Off slope1 Representation on the separability index -with no sensitivity parameter

Reflectance Representation			*Reflectance Representation*			*Reflectance Representation*		
TC	***NTC***	***S.I***	***TC***	***NTC***	***S.I***	***TC***	***NTC***	***S.I***
1	5	0.972	2	10	0.9786	3	5	0.9872
1	15	0.9389	2	12	0.9814	3	7	0.9988
1	16	0.9615	2	13	0.9843	3	14	0.9889
1	17	0.9737	2	18	0.987	3	16	0.9834
1	20	0.9491	2	21	0.9982	3	18	0.9868
1	21	0.9577	2	22	0.9894	3	25	0.9832
1	26	1	2	30	0.9858	3	26	1
1	31	1	2	31	1	3	28	0.9653
Absorption Data			*Absorption Data*			*Absorption Data*		
TC	***NTC***	***S.I***	***TC***	***NTC***	***S.I***	***TC***	***NTC***	***S.I***
1	5	0.9713	2	10	0.981	3	5	0.9849
1	15	0.9674	2	12	0.9809	3	7	0.9968
1	16	0.9683	2	13	0.9838	3	14	0.9894
1	17	0.9827	2	18	0.9877	3	16	0.9829
1	20	0.9749	2	21	0.9981	3	18	0.9889
1	21	0.9385	2	22	0.9896	3	25	0.9827
1	26	1	2	30	0.9885	3	26	1
1	31	1	2	31	0.9997	3	28	0.9599
On or Off slope1			*On or Off slope1*			*On or Off slope1*		
TC	***NTC***	***S.I***	***TC***	***NTC***	***S.I***	***TC***	***NTC***	***S.I***
1	5	0.964	2	10	0.9894	3	5	0.9478
1	15	0.964	2	12	0.99	3	7	0.9948
1	16	0.9721	2	13	0.9871	3	14	0.9762
1	17	0.9801	2	18	0.9539	3	16	0.9829
1	20	0.977	2	21	0.9951	3	18	0.937
1	21	0.9404	2	22	0.9577	3	25	0.9694
1	26	1	2	30	0.9814	3	26	0.9987
1	31	0.9982	2	31	0.9319	3	28	0.9637

Note:

Reflectance Representation: = Original Given Data.
Absorption Representation : = Original Data with Convex Hull Removed.
On or Off Representation : = Spectrum defined by peaks (+1),troughs(-1) and transition modeled by constant positive (slope increasing) or negative value (slope decreasing)
Sepindex (S.I) : = Average Separability index over 5 'fold cross validation'
TC : = Target Class & NTC : = Non Target Class

Appendix D

Effect of changing from Absorption to On or Off slope 2 Representation on the separability index – No sensitivity parameter

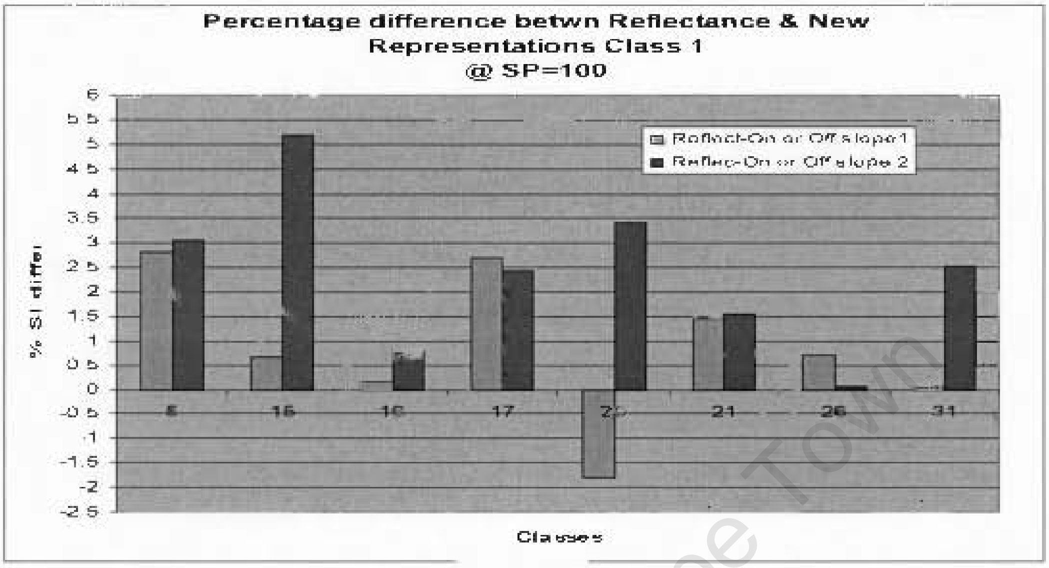
Reflectance Representation				*Reflectance Representation*				*Reflectance Representation*		
TC	***NTC***	***S.I***		***TC***	***NTC***	***S.I***		***TC***	***NTC***	***S.I***
1	5	0.972		2	10	0.9786		3	5	0.9872
1	15	0.9389		2	12	0.9814		3	7	0.9988
1	16	0.9615		2	13	0.9843		3	14	0.9889
1	17	0.9737		2	18	0.987		3	16	0.9834
1	20	0.9491		2	21	0.9982		3	18	0.9868
1	21	0.9577		2	22	0.9894		3	25	0.9832
1	26	1		2	30	0.9858		3	26	1
1	31	1		2	31	1		3	28	0.9653
Absorption Data				*Absorption Data*				*Absorption Data*		
TC	***NTC***	***S.I***		***TC***	***NTC***	***S.I***		***TC***	***NTC***	***S.I***
1	5	0.9713		2	10	0.981		3	5	0.9849
1	15	0.9674		2	12	0.9809		3	7	0.9968
1	16	0.9683		2	13	0.9838		3	14	0.9894
1	17	0.9827		2	18	0.9877		3	16	0.9829
1	20	0.9749		2	21	0.9981		3	18	0.9889
1	21	0.9385		2	22	0.9896		3	25	0.9827
1	26	1		2	30	0.9885		3	26	1
1	31	1		2	31	0.9997		3	28	0.9599
On or Off Slope 2				*On or Off Slope 2*				*On or Off Slope 2*		
TC	***NTC***	***S.I***		***TC***	***NTC***	***S.I***		***TC***	***NTC***	***S.I***
1	5	0.8744		2	10	0.9865		3	5	0.9327
1	15	0.8268		2	12	0.9864		3	7	0.9896
1	16	0.926		2	13	0.9796		3	14	0.9762
1	17	0.9135		2	18	0.9493		3	16	0.9673
1	20	0.7596		2	21	0.9929		3	18	0.8239
1	21	0.9375		2	22	0.9763		3	25	0.9481
1	26	0.8509		2	30	0.9756		3	26	0.9983
1	31	0.3921		2	31	0.8342		3	28	0.8422

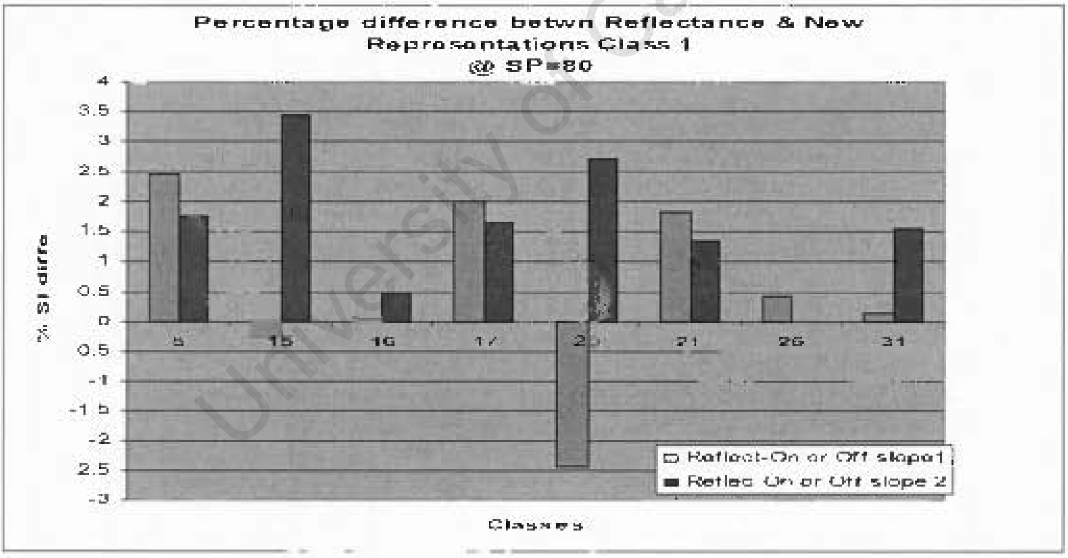
Note:

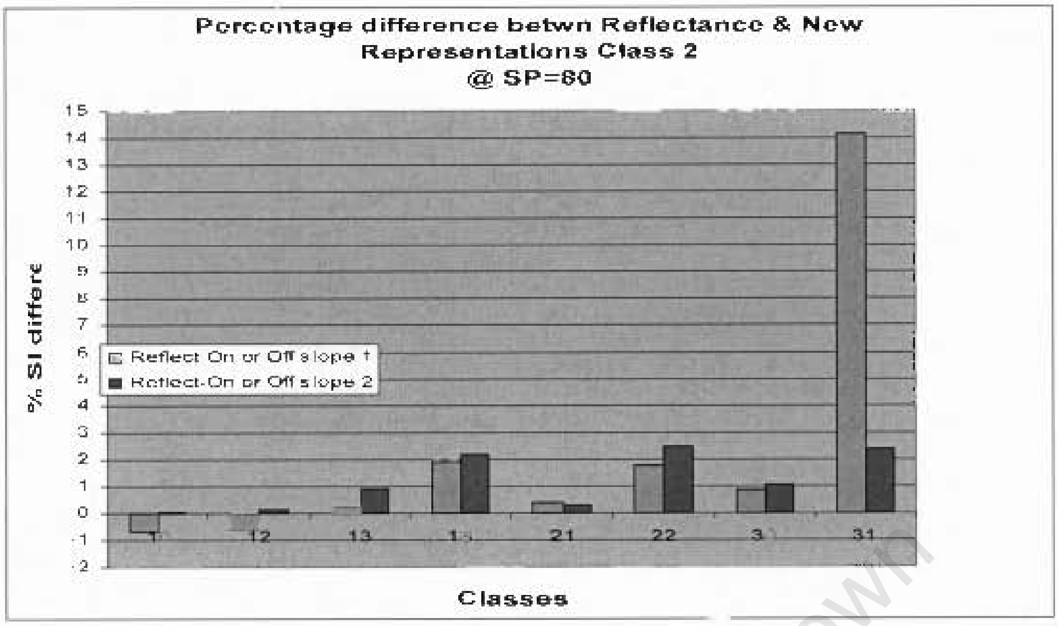
Reflectance Representation : = Original Given Data.
Absorption Representation : = Original Data with Convex Hull Removed.
On or Off slope 2 Representation : = Spectrum defined by peaks and troughs and transition between these modeled by linear interpolation
Sepindex (S.I) : = Average Separability index over 5 'fold cross validation'.
TC : = Target Class and NTC : = Non Target Class

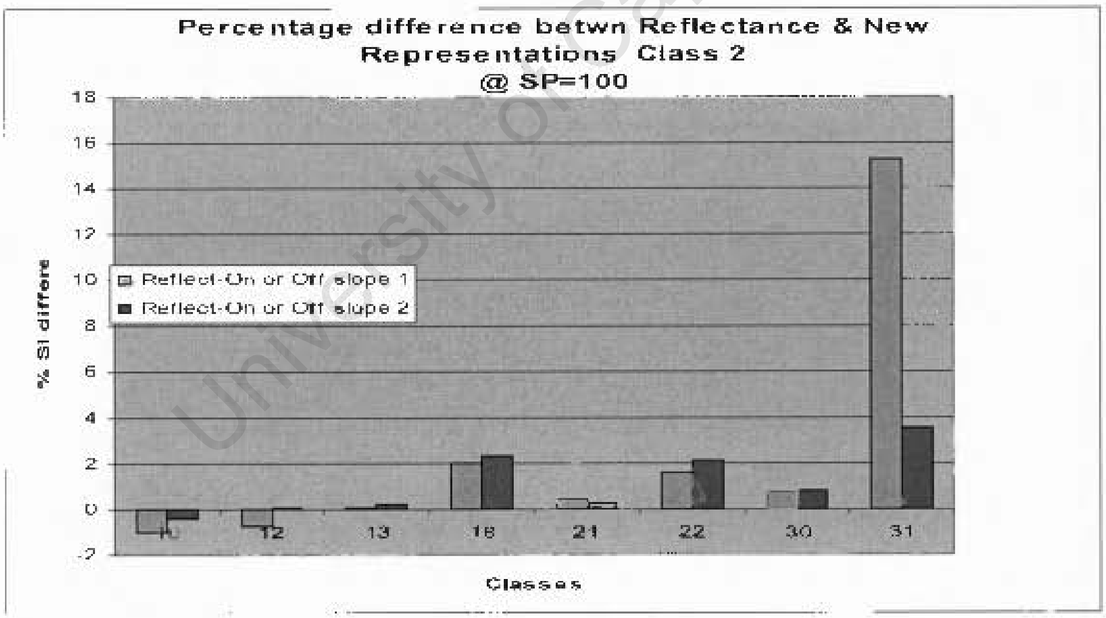
Appendix E

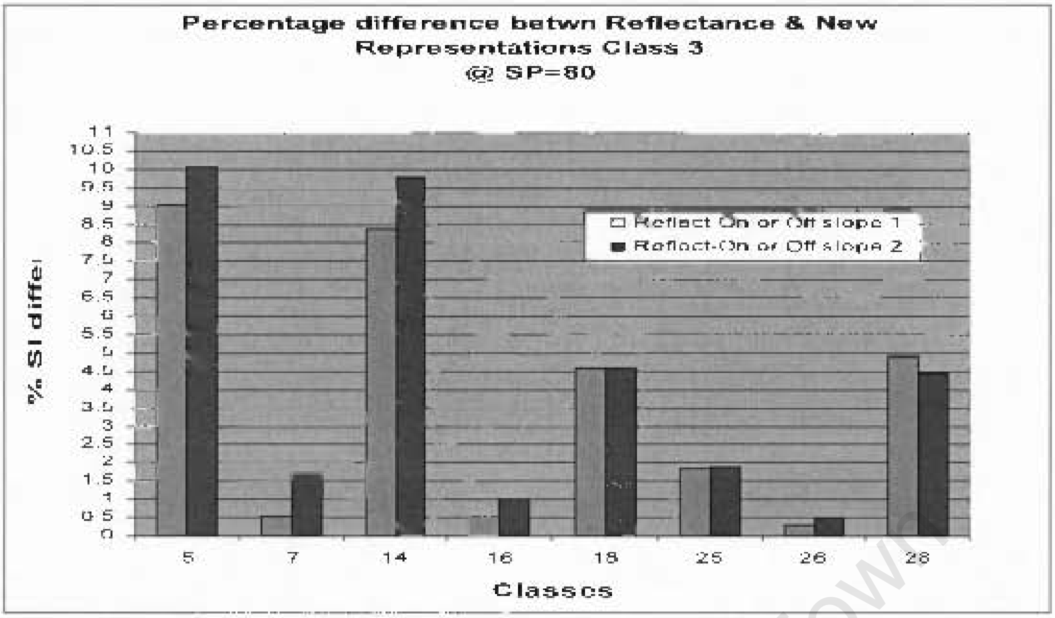
Percentage Separability difference between using Reflectance & On or Off Representations at different Sensitivity Parameter (SP) settings

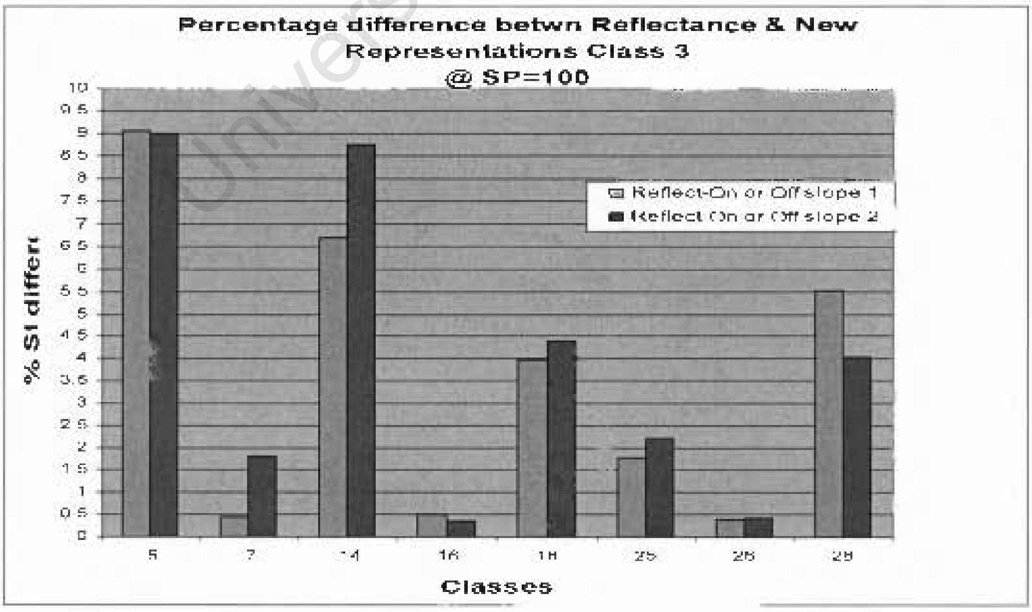





Percentage difference betwn Reflectance & New Representations Class 2
@ SP=80
% SI differe
Reflect On or Off slope 1
Reflect-On or Off slope 2
Classes


Percentage difference betwn Reflectance & New Representations Class 2
@ SP=100
% SI differe
Reflect-On or Off slope 1
Reflect-On or Off slope 2
Classes


Percentage difference betwn Reflectance & New Representations Class 3 @ SP=80
% SI differ
Reflect On or Off slope 1
Reflect-On or Off slope 2
Classes
5
7
14
16
18
25
26
28


Percentage difference betwn Reflectance & New Representations Class 3 @ SP=100
% SI differ
Reflect-On or Off slope 1
Reflect On or Off slope 2
Classes

Appendix F

Sparse features with Wavelets on On or Off representations

Target Class= 3 Non Target Class =5

Haar at scale of 2

On or Off Representation Slope 1 @ No SP

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
3	5	[illegible]	1	0	1	[illegible]
		[illegible]	0	0	0	[illegible]
		[illegible]	0	0	0	[illegible]
		[illegible]	1	0	[illegible]	[illegible]
		1	0	0	[illegible]	[illegible]
					Average	[illegible]

Daub 3 at scale of 2

On or Off Representation Slope 1 @ No SP

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
3	[illegible]	1	0	[illegible]	[illegible]	0.55
		1	0	[illegible]	1	0.55
		1	0	[illegible]	[illegible]	0.55
		1	0	[illegible]	[illegible]	0.55
		1	0	[illegible]	1	0.56
					Average	[illegible]

Haar at scale of 2

On or Off Representation Slope 1 @ SP=100

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
3	5	1	0	[illegible]	[illegible]	[illegible]
		1	0	[illegible]	[illegible]	[illegible]
		1	0	[illegible]	[illegible]	[illegible]
		1	0	[illegible]	[illegible]	[illegible]
		1	[illegible]	[illegible]	[illegible]	0.66
					Average	[illegible]

Daub 3 at scale of 2

On or Off Representation Slope 1 @ SP=100

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
3	5	1	[illegible]	[illegible]	0	[illegible]
		[illegible]	[illegible]	1	1	0.5
		[illegible]	[illegible]	[illegible]	0	[illegible]
		[illegible]	[illegible]	1	1	[illegible]
		[illegible]	1	1	1	[illegible]
					Average	[illegible]

Haar at scale of 2

On or Off Representation Slope 1 @ SP=[illegible]

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
[illegible]	5	1	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	1	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	1	[illegible]
		[illegible]	[illegible]	[illegible]	0	[illegible]
		[illegible]	[illegible]	[illegible]	0	[illegible]
					Average	[illegible]

Daub 3 at scale of 2

On or Off Representation Slope 1 @ SP=[illegible]

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
3	5	[illegible]	[illegible]	0	[illegible]	[illegible]
			[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	1	[illegible]	[illegible]	[illegible]
		1	[illegible]	[illegible]	[illegible]	[illegible]
		1	1	[illegible]	[illegible]	[illegible]
					Average	0.91

On or Off **slope 1** at different SP values

Haar at scale of 2

On or Off Representation Slope 2 @ No SP

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
		1	0	1	[illegible]	[illegible]
		1	0	0	0	0.94
		1	0	1	[illegible]	[illegible]
		1	[illegible]	0	0	0.95
		1	0	1	[illegible]	0.94
					Average	[illegible]

Daub 3 at scale of 2

On or Off Representation Slope 2 @ No SP

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
		[illegible]	1	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	1	[illegible]	[illegible]	[illegible]
		[illegible]	1	[illegible]	[illegible]	[illegible]
		[illegible]	1	[illegible]	[illegible]	[illegible]
					Average	[illegible]

Haar at scale of 2

[illegible] @ SP=100

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
[illegible]	5	[illegible]	[illegible]	1	[illegible]	0.85
		[illegible]	1	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	0.85
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					Average	[illegible]

Daub 3 at scale of 2

On or Off Representation Slope 2 @ SP=100

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
[illegible]	5	1	0	[illegible]	1	[illegible]
		1	0	[illegible]	1	[illegible]
		1	1	1	0	[illegible]
		1	0	[illegible]	1	[illegible]
		1	0	1	0	[illegible]
					Average	[illegible]

Haar at scale of 2

[illegible] @ SP=[illegible]

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
[illegible]	5	[illegible]	1	[illegible]	[illegible]	[illegible]
		[illegible]	1	[illegible]	[illegible]	[illegible]
		[illegible]	0	[illegible]	[illegible]	[illegible]
		1	0	[illegible]	[illegible]	[illegible]
		1	0	[illegible]	0	[illegible]
					Average	[illegible]

Daub 3 at scale of 2

On or Off Representation Slope 2 @ 30 Sorting

TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Sparsity
[illegible]	5	1	[illegible]	1	1	[illegible]
		1	[illegible]	[illegible]	[illegible]	[illegible]
		1	[illegible]	0	[illegible]	[illegible]
		1	[illegible]	[illegible]	[illegible]	[illegible]
		1	[illegible]	1	[illegible]	[illegible]
					Average	[illegible]

On or Off **slope 2** at different SP values

Target Class= 3 Non Target Class =7
Haar & Daubechies 3 Wavelets

Haar at scale of 3 — On or Off Representation, Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	1.000	0.000	0.000	0.998
		1.000	0.000	0.000	0.000	1.000
		1.000	1.000	0.000	1.000	0.994
		1.000	0.000	0.000	0.000	0.998
		1.000	1.000	0.000	1.000	0.996
					Average	0.998

Daub 3 at scale of 3 — On or Off Representation, Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	0.000	1.000	0.000	0.990
		1.000	1.000	1.000	1.000	0.998
		1.000	1.000	0.000	0.000	0.998
		1.000	0.000	0.000	0.000	0.990
		1.000	1.000	0.000	0.000	0.996
					Average	0.998

Haar at scale of 3 — On or Off Representation, Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	0.000	1.000	1.000	0.994
		1.000	0.000	1.000	1.000	0.997
		1.000	0.000	0.000	1.000	0.994
		1.000	1.000	0.000	0.000	0.992
		1.000	0.000	0.000	1.000	0.993
					Average	0.994

Daub 3 at scale of 3 — On or Off Representation, Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	1.000	1.000	0.000	0.994
		1.000	0.000	1.000	0.000	0.997
		1.000	1.000	0.000	0.000	0.996
		1.000	0.000	0.000	0.000	0.993
		1.000	0.000	0.000	0.000	0.996
					Average	0.993

Haar at scale of 3 — On or Off Representation, Slope 1 @ SP=50

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	1.000	0.000	0.000	0.994
		1.000	0.000	1.000	1.000	0.996
		1.000	1.000	1.000	0.000	0.994
		1.000	1.000	0.000	0.000	0.994
		1.000	1.000	1.000	0.000	0.998
					Average	0.996

Daub 3 at scale of 3 — On or Off Representation, Slope 1 @ SP=50

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	0.000	0.000	0.000	0.962
		1.000	0.000	0.000	0.000	0.966
		1.000	1.000	1.000	1.000	0.925
		1.000	1.000	1.000	1.000	0.952
		1.000	1.000	0.000	0.000	[illegible]
					Average	0.934

On or Off **slope 1** at different SP values

Haar at scale of 3 — On or Off Representation, Slope 2 @ no SP

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	0.000	1.000	0.000	0.990
		1.000	1.000	0.000	0.000	0.994
		1.000	1.000	1.000	0.000	0.990
		1.000	0.000	1.000	0.000	0.994
		1.000	1.000	1.000	0.000	0.990
					Average	[illegible]

Daub 3 at scale of 3 — On or Off Representation, Slope 2 @ no SP

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	1.000	0.000	1.000	0.998
		1.000	0.000	0.000	1.000	0.994
		1.000	1.000	0.000	1.000	0.994
		1.000	0.000	0.000	1.000	0.963
		1.000	1.000	0.000	0.000	0.970
					Average	0.954

Haar at scale of 3 — On or Off Representation, Slope 2 @ 100 Setting

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	0.000	1.000	1.000	0.000	0.990
		0.000	1.000	1.000	1.000	0.994
		0.000	1.000	1.000	1.000	0.992
		0.000	0.000	0.000	1.000	0.996
		0.000	1.000	1.000	1.000	0.994
					Average	0.989

Daub 3 at scale of 3 — On or Off Representation, Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	1.000	1.000	1.000	0.958
		1.000	1.000	0.000	0.000	0.970
		0.000	1.000	1.000	1.000	0.992
		1.000	1.000	1.000	1.000	0.930
		0.000	1.000	1.000	1.000	0.932
					Average	0.956

Haar at scale of 3 — On or Off Representation, Slope 2 @ SP=50

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	0.000	1.000	1.000	0.000	0.992
		0.000	0.000	1.000	1.000	0.984
		0.000	1.000	1.000	1.000	0.992
		0.000	0.000	0.000	1.000	0.956
		0.000	1.000	1.000	0.000	0.984
					Average	0.989

Daub 3 at scale of 3 — On or Off Representation, Slope 2 @ SP=50

TC	NTC	a1	d1	d2	d3	Sep index
3.000	7.000	1.000	1.000	1.000	1.000	0.988
		1.000	0.000	0.000	0.000	0.975
		0.000	1.000	1.000	1.000	0.990
		1.000	0.000	0.000	0.000	0.975
		0.000	1.000	1.000	1.000	0.972
					Average	0.986

On or Off **slope 2** at different SP values

Target Class= 3 Non Target Class =14
Haar & Daubechies 3 Wavelets

Haar at scale of 3

On or Off Representation Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	0.000	0.000	[illegible]	0.992
		1.000	0.000	0.000	[illegible]	0.994
		1.000	0.500	0.000	[illegible]	0.996
		1.000	0.500	0.000	[illegible]	[illegible]
		1.000	0.500	0.000	[illegible]	[illegible]
					Average	[illegible]

Daub3 at scale of 3

On or Off Representation Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	0.500	0.000	0.000	[illegible]
		1.000	0.000	0.000	0.000	0.994
		1.000	1.000	0.000	0.000	0.994
		1.000	0.500	0.000	1.000	[illegible]
		1.000	1.000	0.000	1.000	[illegible]
					Average	0.992

On or Off Representation Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	0.000	0.000	0.000	0.992
		1.000	0.000	0.000	1.000	0.904
		1.000	1.000	1.000	0.000	0.900
		1.000	0.000	0.000	0.000	0.917
		1.000	0.000	0.000	1.000	0.906
					Average	0.908

Daub3 at scale of 3

On or Off Representation Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	0.500	1.000	0.000	0.904
		1.000	0.500	0.000	0.000	0.907
		1.000	0.500	0.000	1.000	[illegible]
		1.000	1.500	1.000	1.000	[illegible]
		1.000	1.500	0.000	0.000	[illegible]
					Average	0.907

Haar at scale of 3

On or Off Representation Slope 1 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
[illegible]	14.000	[illegible]	0.000	[illegible]	[illegible]	[illegible]
		[illegible]	0.000	[illegible]	[illegible]	[illegible]
		[illegible]	1.000	[illegible]	[illegible]	[illegible]
		[illegible]	0.000	[illegible]	[illegible]	[illegible]
		[illegible]	0.000	[illegible]	[illegible]	[illegible]
					Average	[illegible]

Daub3 at scale of 3

On or Off Representation Slope 1 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	0.500	0.000	0.000	[illegible]
		1.000	0.500	0.000	0.000	[illegible]
		1.000	1.500	0.000	0.000	[illegible]
		1.000	0.500	0.000	1.000	[illegible]
		1.000	1.500	0.000	0.000	[illegible]
					Average	[illegible]

On or Off **slope 1** at different SP values

Haar at scale of 3

On or Off Representation Slope 2 @ Rough Setting no SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	[illegible]	1.000	[illegible]	0.968
		1.000	[illegible]	0.000	[illegible]	0.975
		1.000	[illegible]	1.000	[illegible]	0.981
		1.000	[illegible]	0.000	[illegible]	0.963
		1.000	[illegible]	1.000	[illegible]	0.965
					Average	0.975

Daub3 at scale of 3

On or Off Representation Slope 2 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	1.000	1.000	0.000	0.984
		1.000	0.000	1.000	0.000	0.977
		1.000	1.000	0.000	0.000	0.958
		1.000	1.000	0.000	0.000	0.977
		1.000	1.000	0.000	0.000	0.967
					Average	0.974

Haar at scale of 3

On or Off Representation Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	1.000	0.000	0.000	0.848
		0.000	1.000	1.000	[illegible]	0.902
		1.000	0.000	0.000	0.000	[illegible]
		1.000	0.000	1.000	1.000	0.890
		1.000	0.000	0.000	0.000	0.839
					Average	0.899

Daub3 at scale of 3

On or Off Representation Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	0.000	0.000	0.000	0.900
		1.000	0.000	0.000	1.000	0.872
		1.000	1.000	0.000	0.000	0.917
		1.000	0.000	0.000	1.000	0.896
		1.000	0.000	0.000	0.000	0.894
					Average	0.897

Haar at scale of 3

On or Off Representation Slope 2 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	1.000	0.000	0.000	0.898
		0.000	1.000	1.000	0.000	0.902
		1.000	0.000	0.000	0.000	0.901
		1.000	0.000	1.000	1.000	0.896
		1.000	0.000	0.000	0.000	0.899
					Average	0.899

Daub3 at scale of 3

On or Off Representation Slope 2 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	14.000	1.000	0.000	1.000	1.000	0.896
		1.000	0.000	0.000	1.000	0.894
		1.000	1.000	0.000	0.000	0.892
		1.000	0.000	0.000	1.000	0.896
		1.000	0.000	1.000	0.000	0.899
					Average	0.898

On or Off **slope 2** at different SP values

Target Class= 3 Non Target Class =16
Haar & Daubechies 3 Wavelets

Haar at scale of 3

On or Off Representation Slope 1 @ no SP						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	1.000	1.000	0.990
		1.000	0.000	0.000	1.000	0.987
		1.000	1.000	0.000	0.000	0.987
		1.000	0.000	1.000	0.000	0.990
		1.000	1.000	0.000	1.000	0.986
					Average	0.989

Daub 3 at scale of 3

On or Off Representation Slope 1 @ no SP						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	0.000	1.000	0.990
		1.000	0.000	0.000	1.000	0.992
		1.000	1.000	0.000	0.000	0.990
		1.000	0.000	0.000	1.000	0.994
		1.000	1.000	0.000	0.000	0.990
					Average	0.991

Haar at scale of 3

On or Off Representation Slope 1 @ SP=100						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	0.000	0.000	0.902
		1.000	0.000	0.000	1.000	0.904
		1.000	1.000	1.000	0.000	0.906
		1.000	0.000	0.000	0.000	0.917
		1.000	0.000	0.000	1.000	0.909
					Average	0.908

Daub 3 at scale of 3

On or Off Representation Slope 1 @ SP=100						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	0.000	0.000	0.982
		1.000	0.000	1.000	0.000	0.980
		1.000	0.000	1.000	1.000	0.975
		1.000	0.000	0.000	0.000	0.985
		1.000	0.000	0.000	1.000	0.982
					Average	0.981

Haar at scale of 3

On or Off Representation Slope 1 @ 80 Setting						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	0.000	0.000	0.965
		1.000	0.000	0.000	1.000	0.967
		1.000	0.000	0.000	0.000	0.960
		1.000	0.000	0.000	0.000	0.967
		1.000	0.000	0.000	0.000	0.962
					Average	0.964

Daub 3 at scale of 3

On or Off Representation Slope 1 @ SP= 80						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	0.000	0.000	0.988
		1.000	0.000	0.000	0.000	0.985
		1.000	0.000	0.000	0.000	0.980
		1.000	0.000	1.000	0.000	0.963
		1.000	0.000	1.000	0.000	0.982
					Average	0.850

On or Off **slope 1** at different SP values

Haar at scale of 3

On or Off Representation Slope 2 @ no SP						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	0.000	1.000	0.962
		0.000	0.000	1.000	1.000	0.967
		1.000	1.000	1.000	0.000	0.970
		1.000	0.000	0.000	1.000	0.970
		0.000	1.000	1.000	1.000	0.967
					Average	0.966

Daub 3 at scale of 3

On or Off Representation Slope 2 @ no SP						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	0.000	0.000	1.000	0.000	0.970
		0.000	1.000	1.000	1.000	0.967
		1.000	0.000	1.000	0.000	0.972
		1.000	1.000	1.000	0.000	0.972
		1.000	0.000	1.000	0.000	0.972
					Average	0.970

Haar at scale of 3

On or Off Representation Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	1.000	0.000	0.972
		1.000	1.000	1.000	0.000	0.972
		1.000	0.000	0.000	0.000	0.975
		1.000	1.000	1.000	1.000	0.976
		1.000	1.000	1.000	0.000	0.975
					Average	0.974

Daub 3 at scale of 3

On or Off Representation Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	[illegible]	0.000	[illegible]	[illegible]
		1.000	1.000	0.000	1.000	[illegible]
		1.000	1.000	1.000	0.000	[illegible]
		1.000	1.000	1.000	0.000	[illegible]
		1.000	1.000	1.000	1.000	[illegible]
					Average	[illegible]

Haar at scale of 3

On or Off Representation Slope 2 @ SP=50						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	0.000	0.000	1.000	0.972
		1.000	0.000	1.000	0.000	0.972
		1.000	0.000	0.000	1.000	0.975
		1.000	0.000	0.000	1.000	0.976
		1.000	0.000	1.000	1.000	0.975
					Average	0.974

Daub 3 at scale of 3

On or Off Representation Slope 2 @ SP=50						
TC	NTC	a1	d1	d2	d3	Seconds
3.000	16.000	1.000	1.000	1.000	0.000	0.972
		1.000	1.000	0.000	1.000	0.977
		1.000	1.000	1.000	0.000	0.977
		1.000	0.000	1.000	1.000	0.977
		1.000	1.000	1.000	0.000	[illegible]
					Average	0.974

On or Off **slope 2** at different SP values

Target Class= 3 Non Target Class =18
Haar & Daubechies 3 Wavelets

Haar at scale of 3

On or Off Representation Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	18.000	1.000	0.000	1.000	0.000	0.976
		1.000	0.000	0.000	0.000	0.954
		1.000	0.000	0.000	0.000	0.992
		1.000	0.000	1.000	0.000	0.977
		1.000	0.000	0.000	0.000	0.951
					Average	0.953

Daub 3 at scale of 3

On or Off Representation Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	0.000	0.000	0.000	0.971
		1.000	0.000	0.000	0.000	0.978
		1.000	0.000	0.000	0.000	0.992
		1.000	0.000	0.000	0.000	0.973
		1.000	0.000	0.000	0.000	0.986
					Average	0.980

Haar at scale of 3

On or Off Representation Slope 1 @ 100 Setting

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	0.000	1.000	0.000	0.928
		1.000	0.000	0.000	0.000	0.947
		1.000	0.000	0.000	0.000	0.953
		1.000	0.000	0.000	0.000	0.940
		1.000	0.000	0.000	0.000	0.952
					Average	0.944

Daub 3 at scale of 3

On or Off Representation Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	18.000	1.000	1.000	0.000	0.000	0.936
		1.000	1.000	1.000	1.000	0.951
		1.000	0.000	0.000	1.000	0.940
		1.000	1.000	1.000	1.000	0.936
		1.000	1.000	0.000	1.000	0.916
					Average	0.942

Haar at scale of 3

On or Off Representation Slope 1 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	0.000	0.000	1.000	0.952
		1.000	0.000	0.000	0.000	0.969
		1.000	0.000	0.000	0.000	0.952
		1.000	0.000	0.000	0.000	0.956
		1.000	0.000	0.000	0.000	0.959
					Average	0.958

Daub 3 at scale of 3

On or Off Representation Slope 1 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
[illegible]	[illegible]	1.000	0.000	0.000	0.000	0.956
		1.000	0.000	1.000	0.000	0.966
		1.000	0.000	0.000	1.000	0.951
		1.000	0.000	0.000	0.000	0.953
		1.000	1.000	0.000	1.000	0.959
					Average	0.967

On or Off **slope 1** at different SP values

Haar at scale of 3

On or Off Representation Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	0.000	1.000	0.000	0.922
		1.000	0.000	1.000	0.000	0.907
		1.000	0.000	1.000	0.000	0.910
		1.000	0.000	1.000	0.000	0.909
		1.000	1.000	1.000	0.000	0.928
					Average	0.917

Daub 3 at scale of 3

On or Off Representation Slope 2 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
[illegible]	[illegible]	1.000	1.000	1.000	0.000	0.928
		1.000	1.000	1.000	0.000	[illegible]
		1.000	1.000	1.000	0.000	0.914
		1.000	0.000	1.000	0.000	[illegible]
		1.000	1.000	1.000	0.000	0.944
					Average	[illegible]

Haar at scale of 3

On or Off Representation Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	0.000	0.000	1.000	0.944
		1.000	0.000	0.000	0.000	0.934
		1.000	0.000	1.000	0.000	0.945
		1.000	0.000	0.000	1.000	0.967
		1.000	0.000	0.000	0.000	0.940
					Average	0.945

Daub 3 at scale of 3

On or Off Representation Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	1.000	0.000	0.000	0.940
		1.000	1.000	0.000	0.000	0.934
		1.000	0.000	0.000	0.000	0.953
		1.000	0.000	1.000	1.000	0.951
		1.000	0.000	1.000	1.000	0.942
					Average	0.944

Haar at scale of 3

On or Off Representation Slope 2 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	0.000	0.000	1.000	0.944
		1.000	0.000	0.000	0.000	0.934
		1.000	0.000	1.000	0.000	0.949
		1.000	0.000	0.000	1.000	0.967
		1.000	0.000	1.000	1.000	0.940
					Average	0.945

Daub 3 at scale of 3

On or Off Representation Slope 2 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	15.000	1.000	1.000	0.000	0.000	0.940
		1.000	1.000	0.000	0.000	0.934
		1.000	0.000	0.000	0.000	0.950
		1.000	0.000	1.000	0.000	0.967
		1.000	0.000	1.000	1.000	0.942
					Average	0.944

On or Off slope 2 at different SP values

Target Class= 3 Non Target Class =25
Haar & Daubechies 3 Wavelets

Haar at scale of 3						
On or Off Representation Slope 1 @ no SP						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	1.000	1.000	1.000	0.980
		1.000	0.000	0.000	0.000	0.981
		1.000	0.000	1.000	0.000	[illegible]
		1.000	0.000	1.000	0.000	0.985
		1.000	1.000	0.000	1.000	[illegible]
					Average	0.982

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ no SP						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	1.000	1.000	1.000	[illegible]
		1.000	0.000	0.000	1.000	0.980
		1.000	1.000	0.000	0.000	0.980
		1.000	1.000	0.000	0.000	0.980
		1.000	0.000	0.000	0.000	0.980
					Average	0.981

Haar at scale of 3						
On or Off Representation Slope 1 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	0.000	0.000	0.000	0.960
		1.000	0.000	0.000	0.000	0.975
		1.000	1.000	0.000	0.000	0.973
		1.000	0.000	0.000	0.000	0.978
		1.000	0.000	0.000	1.000	0.956
					Average	0.972

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ 100 Setting						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	0.000	1.000	0.000	0.970
		1.000	0.000	1.000	1.000	0.973
		1.000	1.000	0.000	0.000	0.970
		1.000	0.000	1.000	0.000	0.978
		1.000	0.000	0.000	0.000	0.966
					Average	0.971

Haar at scale of 3						
On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	0.000	0.000	0.000	0.973
		1.000	0.000	0.000	0.000	0.981
		1.000	1.000	0.000	1.000	0.975
		1.000	0.000	0.000	0.000	0.983
		1.000	0.000	0.000	0.000	0.970
					Average	0.977

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	0.000	0.000	0.000	0.976
		1.000	0.000	0.000	1.000	0.978
		1.000	0.000	0.000	0.000	0.973
		1.000	0.000	0.000	0.000	0.978
		1.000	0.000	0.000	0.000	0.968
					Average	0.974

On or Off slope 1 at different SP values

Haar at scale of 3						
On or Off Representation Slope 2 @ no SP						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	0.000	0.000	0.000	[illegible]
		1.000	0.000	1.000	0.000	0.963
		1.000	0.000	0.000	0.000	0.956
		1.000	0.000	0.000	0.000	0.965
		1.000	1.000	1.000	0.000	0.963
					Average	[illegible]

Daub 3 at scale of 3						
On or Off Representation Slope 2 @ Rough Setting						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	1.000	0.000	0.000	0.958
		1.000	1.000	1.000	0.000	0.960
		1.000	1.000	0.000	0.000	0.963
		1.000	1.000	1.000	0.000	0.965
		1.000	1.000	0.000	0.000	0.958
					Average	0.969

Haar at scale of 3						
On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	1.000	0.000	0.000	0.970
		1.000	1.000	1.000	1.000	0.970
		1.000	0.000	1.000	1.000	0.973
		1.000	1.000	0.000	1.000	0.974
		0.000	1.000	1.000	0.000	0.963
					Average	0.970

Daub 3 at scale of 3						
On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	1.000	0.000	0.000	0.961
		1.000	1.000	1.000	0.000	0.968
		1.000	0.000	0.000	1.000	0.975
		1.000	1.000	0.000	0.000	0.978
		0.000	1.000	0.000	1.000	0.956
					Average	0.967

Haar at scale of 3						
On or Off Representation Slope 2 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	1.000	1.000	[illegible]	[illegible]
		1.000	1.000	1.000	1.000	0.970
		1.000	[illegible]	1.000	1.000	[illegible]
		1.000	0.000	1.000	[illegible]	[illegible]
		[illegible]	1.000	1.000	1.000	0.959
					Average	[illegible]

Daub 3 at scale of 3						
On or Off Representation Slope 2 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sepindex
3.000	25.000	1.000	1.000	1.000	0.000	0.961
		1.000	1.000	1.000	1.000	0.965
		1.000	0.000	0.000	0.000	0.975
		1.000	1.000	0.000	0.000	0.976
		1.000	0.000	0.000	1.000	0.956
					Average	0.967

On or Off **slope 2** at different SP values

Target Class= 3 Non Target Class =26
Haar & Daubechies 3 Wavelets

Haar at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 1 @ no SP						
3.000	26.000	1.000	0.000	1.000	0.000	1.000
		1.000	1.000	1.000	1.000	1.000
		1.000	0.000	0.000	0.000	1.000
		1.000	1.000	0.000	1.000	1.000
		1.000	0.000	0.000	0.000	1.000
					Average	1.000

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 1 @ no SP						
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.000	1.000
		[illegible]	1.000	1.000	1.000	1.000
		1.000	[illegible]	[illegible]	[illegible]	1.000
		1.000	1.000	0.000	1.000	1.000
		[illegible]	[illegible]	[illegible]	[illegible]	1.000
					Average	1.000

Haar at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 1 @ SP=100						
3.000	26.000	1.000	1.000	1.000	1.000	0.995
		1.000	1.000	1.000	0.000	1.000
		1.000	1.000	0.000	0.000	0.996
		1.000	1.000	1.000	0.000	0.995
		1.000	1.000	0.000	0.000	0.995
					Average	0.997

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 1 @ SP=100						
3.000	26.000	1.000	0.000	1.000	0.000	0.996
		1.000	1.000	1.000	0.000	1.000
		1.000	1.000	1.000	1.000	0.996
		1.000	0.000	1.000	1.000	0.998
		1.000	1.000	0.000	1.000	0.998
					Average	0.997

Haar at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 1 @ SP=60						
3.000	26.000	1.000	1.000	0.000	1.000	0.995
		1.000	1.000	1.000	1.000	0.995
		1.000	1.000	0.000	1.000	0.996
		1.000	1.000	0.000	1.000	0.996
		1.000	0.000	1.000	1.000	0.996
					Average	0.996

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 1 @ SP=60						
3.000	26.000	1.000	0.000	1.000	0.000	0.996
		1.000	1.000	1.000	1.000	0.998
		1.000	1.000	1.000	0.000	0.996
		1.000	1.000	0.000	0.000	0.996
		1.000	1.000	1.000	0.000	0.996
					Average	0.996

On or Off **slope 1** at different SP values

Haar at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 2 @ No SP						
3.000	26.000	1.000	0.000	0.000	0.000	1.000
		1.000	2.000	1.000	0.000	1.000
		1.000	0.000	0.000	0.000	1.000
		1.000	0.000	0.000	0.000	1.000
		1.000	1.000	1.000	0.000	1.000
					Average	1.000

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 2 @ No SP						
3.000	26.000	1.000	0.000	1.000	1.000	1.000
		1.000	0.000	0.000	0.000	1.000
		1.000	1.000	1.000	0.000	1.000
		1.000	0.000	1.000	1.000	1.000
		1.000	0.000	0.000	1.000	1.000
					Average	1.000

Haar at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 2 @ SP=100						
3.000	26.000	0.000	1.000	1.000	1.000	0.995
		1.000	0.000	1.000	1.000	0.995
		1.000	1.000	0.000	0.000	0.995
		0.000	1.000	1.000	0.000	0.994
		1.000	1.000	0.000	0.000	0.996
					Average	0.997

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 2 @ SP=100						
3.000	26.000	1.000	1.000	0.000	1.000	0.994
		1.000	1.000	1.000	1.000	0.996
		1.000	1.000	0.000	1.000	0.998
		1.000	1.000	0.000	1.000	0.997
		1.000	0.000	1.000	1.000	0.998
					Average	0.995

Haar at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 2 @ SP=50						
3.000	26.000	0.000	1.000	1.000	0.000	0.994
		1.000	1.000	0.000	0.000	0.996
		1.000	1.000	1.000	1.000	0.994
		0.000	1.000	1.000	1.000	0.994
		1.000	1.000	0.000	1.000	0.998
					Average	0.997

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separindex
On or Off Representation Slope 2 @ SP=50						
3.000	26.000	1.000	0.000	1.000	0.000	0.994
		1.000	1.000	1.000	1.000	0.996
		1.000	0.000	0.000	0.000	0.998
		1.000	1.000	0.000	1.000	0.992
		1.000	1.000	0.000	1.000	0.998
					Average	0.995

On or Off **slope 2** at different SP values

Target Class= 3 Non Target Class =28
Haar & Daubechies 3 Wavelets

Haar at scale of 3 — On or Off Representation Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	0.000	[illegible]	0.000	[illegible]
		1.000	0.000	0.000	[illegible]	[illegible]
		1.000	0.000	0.000	1.000	[illegible]
		1.000	0.000	0.000	0.000	[illegible]
		1.000	0.000	0.000	0.000	[illegible]
					Average	[illegible]

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	0.000	0.000	[illegible]	0.958
		1.000	0.000	0.000	0.000	0.971
		1.000	0.000	0.000	0.000	0.980
		1.000	0.000	0.000	0.000	0.971
		1.000	0.000	0.000	0.000	0.980
					Average	0.976

Haar at scale of 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	0.000	1.000	[illegible]	[illegible]
		1.000	0.000	1.000	0.000	[illegible]
		1.000	[illegible]	1.000	[illegible]	[illegible]
		1.000	0.000	0.000	0.000	[illegible]
		1.000	1.000	0.000	1.000	[illegible]
					Average	0.916

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	1.000	1.000	0.000	0.919
		1.000	1.000	1.000	1.000	0.924
		1.000	0.000	1.000	0.000	0.933
		1.000	0.000	1.000	0.000	0.922
		1.000	0.000	1.000	0.000	0.927
					Average	0.917

Haar at scale of 3 — On or Off Representation Slope 1 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	0.000	1.000	1.000	0.920
		1.000	0.000	1.000	0.000	0.896
		1.000	0.000	0.000	0.000	0.933
		1.000	0.000	1.000	0.000	[illegible]
		1.000	1.000	0.000	1.000	0.923
					Average	0.915

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	1.000	0.000	0.000	0.927
		1.000	1.000	1.000	1.000	0.900
		1.000	1.000	0.000	0.000	[illegible]
		1.000	1.000	0.000	0.000	0.922
		1.000	1.000	1.000	0.000	0.919
					Average	0.917

On or Off **slope 1** at different SP values

Haar at scale of 3 — On or Off Representation Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	1.000	1.000	0.000	0.869
		1.000	1.000	1.000	0.000	0.875
		1.000	0.000	1.000	0.000	0.821
		1.000	0.000	1.000	0.000	[illegible]
		1.000	1.000	1.000	0.000	0.825
					Average	0.852

Daub 3 at scale of 3 — On or Off Representation Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	0.000	1.000	0.000	0.896
		1.000	0.000	1.000	0.000	0.867
		1.000	1.000	1.000	0.000	0.900
		1.000	1.000	1.000	0.000	0.841
		1.000	0.000	1.000	0.000	0.896
					Average	0.867

Haar at scale of 3 — On or Off Representation Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	1.000	1.000	1.000	0.927
		1.000	0.000	1.000	0.000	0.921
		1.000	1.000	1.000	0.000	0.940
		1.000	0.000	0.000	0.000	0.923
		1.000	0.000	1.000	0.000	0.919
					Average	0.926

Daub 3 at scale of 3 — On or Off Representation Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
3.000	28.000	1.000	1.000	1.000	0.000	0.923
		1.000	0.000	0.000	1.000	0.921
		1.000	0.000	0.000	0.000	0.903
		1.000	0.000	0.000	0.000	0.927
		1.000	0.000	0.000	1.000	[illegible]
					Average	0.924

Haar at scale of 3 — On or Off Representation Slope 2 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					Average	0.926

Daub 3 at scale of 3 — On or Off Representation Slope 2 @ SP=50

TC	NTC	a1	d1	d2	d3	Sepindex
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	0.000	0.000	0.000	0.927
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					Average	0.924

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =5
Haar & Daubechies 3 Wavelets

Haar at scale of 3
On or Off Representation Slope 1 @ Rough Setting

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	1.000	0.000	1.000	0.979
		1.000	0.000	1.000	0.000	0.976
		1.000	1.000	0.000	0.000	0.982
		1.000	0.000	1.000	0.000	0.976
		1.000	1.000	1.000	1.000	0.952
					Average	0.976

Daub 3 at scale of 3
On or Off Representation Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	1.000	0.000	1.000	0.979
		1.000	[illegible]	0.500	1.000	[illegible]
		1.000	[illegible]	0.500	0.000	[illegible]
		1.000	[illegible]	1.000	0.000	[illegible]
		1.000	0.000	1.000	0.000	0.979
					Average	[illegible]

Haar at scale of 3
On or Off Representation Slope 1 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	0.000	0.000	1.000	0.951
		1.000	0.000	1.000	1.000	0.970
		1.000	0.000	1.000	0.000	0.959
		1.000	0.000	1.000	1.000	0.951
		1.000	0.000	0.000	0.500	0.963
					Average	0.958

Daub 3 at scale of 3
On or Off Representation Slope 1 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	0.000	0.000	1.000	0.949
		1.000	0.000	1.000	1.000	0.970
		1.000	0.000	0.000	1.000	0.935
		1.000	0.000	1.000	0.000	0.951
		1.000	0.000	0.000	0.000	0.963
					Average	0.954

Haar at scale of 3
On or Off Representation Slope 1 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	0.000	0.000	0.500	0.951
		1.000	1.000	0.000	1.500	0.967
		1.000	0.000	0.000	1.000	0.951
		1.000	0.000	1.000	0.000	0.945
		1.000	0.000	0.000	1.000	0.961
					Average	0.956

Daub 3 at scale of 3
On or Off Representation Slope 1 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	0.000	0.000	0.000	0.951
		1.000	1.000	0.000	1.000	0.967
		1.000	0.000	0.000	0.000	0.951
		1.000	0.000	1.000	0.000	0.945
		1.000	0.000	0.000	0.000	0.960
					Average	0.956

On or Off **slope 1** at different SP values

Haar at scale of 3
On or Off Representation Slope [illegible] no SP

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	0.000	1.000	0.000	0.929
		1.000	0.000	1.000	0.500	0.927
		1.000	1.000	1.000	0.500	0.927
		1.000	0.000	1.000	0.000	0.927
		1.000	1.000	1.000	0.000	0.930
					Average	0.928

Daub 3 at scale of 3
On or Off Representation Slope 1 @ no SP

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.500	0.500	0.500	1.000	0.918
		1.500	0.500	0.500	1.000	0.930
		1.500	1.500	1.500	1.000	0.908
		1.500	0.500	1.500	0.000	0.945
		1.500	0.500	1.500	1.000	0.927
					Average	0.926

Haar at scale of 3
On or Off Representation Slope 2 @ 100 Setting SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	1.000	1.000	0.000	0.947
		1.000	0.000	0.000	0.000	0.951
		1.000	0.000	0.000	0.500	0.947
		1.000	0.000	0.000	1.500	0.930
		1.000	0.000	1.000	0.500	0.933
					Average	0.941

Daub 3 at scale of 3
On or Off Representation Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.500	1.500	0.500	1.000	0.939
		1.500	1.500	1.500	1.000	0.951
		1.500	0.500	1.500	1.000	0.939
		1.500	1.500	1.000	0.000	0.939
		1.500	0.500	0.500	1.000	0.936
					Average	0.941

Haar at scale of 3
On or Off Representation Slope 2 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.000	1.000	1.500	0.000	0.954
		1.000	1.000	1.500	1.500	0.951
		1.000	1.000	1.500	1.000	0.951
		1.000	0.000	1.500	1.000	0.945
		1.000	0.500	1.500	1.000	0.970
					Average	0.956

Daub 3 at scale of 3
On or Off Representation Slope 2 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sepindex
1.000	5.000	1.500	0.000	0.000	0.000	0.958
		1.500	0.500	1.000	1.000	0.945
		1.500	1.000	0.000	1.000	0.951
		1.500	1.000	1.000	0.000	0.948
		1.500	1.000	1.000	1.000	0.967
					Average	0.953

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =16
Haar & Daubechies 3 Wavelets

Haar at scale of 3 — On or Off Representation Slope 1 @ No SP

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	1.000	1.000	0.000	[illegible]
		1.000	0.000	1.000	1.000	0.981
		1.000	0.000	[illegible]	1.000	[illegible]
		1.000	0.000	1.000	0.000	[illegible]
		[illegible]	0.000	1.000	0.000	0.971
					Average	[illegible]

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ no SP

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	1.000	0.000	1.000	0.986
		1.000	1.000	0.000	1.000	[illegible]
		1.000	1.000	0.000	1.000	0.981
		1.000	1.000	1.000	0.000	[illegible]
		1.000	0.000	0.000	1.000	[illegible]
					Average	[illegible]

Haar at scale of 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	1.000	0.000	0.966
		1.000	0.000	1.000	1.000	0.976
		1.000	0.000	1.000	0.000	0.962
		1.000	0.000	1.000	0.000	0.971
		1.000	0.000	1.000	0.000	0.966
					Average	0.968

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	1.000	0.000	0.962
		1.000	1.000	1.000	0.000	0.976
		1.000	0.000	1.000	0.000	0.962
		1.000	0.000	1.000	1.000	0.971
		1.000	0.000	0.000	1.000	0.971
					Average	0.968

Haar at scale of 3 — On or Off Representation Slope 1 @ SP=80

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	0.000	0.000	0.967
		1.000	0.000	0.000	1.000	0.971
		1.000	0.000	1.000	0.000	0.966
		1.000	0.000	1.000	1.000	[illegible]
		1.000	1.000	0.000	1.000	0.966
					Average	0.974

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ SP=80

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	1.000	0.000	0.971
		1.000	1.000	0.000	0.000	0.971
		1.000	0.000	0.000	0.000	0.966
		1.000	1.000	1.000	1.000	0.981
		1.000	0.000	1.000	0.000	0.966
					Average	0.971

On or Off **slope 1** at different SP values

Haar at scale of 3 — On or Off Representation Slope 1 @ no SP

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	1.000	1.000	0.947
		1.000	0.000	0.000	1.000	0.938
		1.000	1.000	1.000	0.000	[illegible]
		1.000	0.000	1.000	0.000	0.947
		1.000	0.000	1.000	1.000	0.952
					Average	[illegible]

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ no SP

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	1.000	0.000	1.000	0.938
		0.000	1.000	0.000	1.000	0.938
		0.000	0.000	1.000	1.000	0.938
		1.000	1.000	1.000	1.000	0.947
		0.000	0.000	1.000	1.000	0.942
					Average	0.940

Haar at scale of 3 — On or Off Representation Slope 2 @ SP=100

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	0.000	1.000	0.976
		1.000	0.000	0.000	1.000	0.957
		1.000	0.000	0.000	0.000	0.952
		1.000	1.000	1.000	0.000	0.957
		0.000	0.000	1.000	0.000	0.947
					Average	0.957

Daub 3 at scale of 3 — On or Off Representation Slope 2 @ SP=100

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	0.000	0.000	[illegible]
		1.000	1.000	0.000	1.000	[illegible]
		1.000	1.000	1.000	0.000	0.947
		1.000	1.000	1.000	0.000	0.957
		0.000	1.000	0.000	0.000	0.952
					Average	0.965

Haar at scale of 3 — On or Off Representation Slope 2 @ SP=80

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	1.000	1.000	0.956
		1.000	1.000	1.000	1.000	0.966
		1.000	1.000	0.000	0.000	0.957
		1.000	1.000	0.000	0.000	0.952
		1.000	1.000	0.000	0.000	0.962
					Average	0.961

Daub 3 at scale of 3 — On or Off Representation Slope 2 @ SP=80

TC	NTC	a3	d1	d2	d3	Separation
1.000	16.000	1.000	0.000	1.000	0.000	0.962
		1.000	0.000	0.000	0.000	0.966
		1.000	1.000	1.000	1.000	0.957
		1.000	0.000	1.000	0.000	0.947
		1.000	0.000	0.000	0.000	0.962
					Average	0.959

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =17
Haar & Daubechies 3 Wavelets

Haar scale of 1

TC	NTC	d1	d1	d2	d3	Separability
1.000	17.000	1.000	0.000	0.000	[illegible]	[illegible]
		1.000	1.000	1.000	[illegible]	[illegible]
		1.000	0.000	0.000	[illegible]	[illegible]
		1.000	0.000	0.000	[illegible]	[illegible]
		1.000	0.000	0.000	0.000	[illegible]
					Average	[illegible]

Daub 3 at scale of 3

TC	NTC	[illegible]	d1	d2	d3	Separability
1.000	17.000	1.000	1.000	0.000	[illegible]	[illegible]
		1.000	1.000	0.000	1.000	0.984
		1.000	1.000	0.000	0.000	0.984
		1.000	1.000	0.000	0.000	0.000
		1.000	2.500	0.000	0.000	[illegible]
					Average	0.990

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	1.000	1.000	1.000	0.96[illegible]
		1.000	[illegible]	0.000	1.000	0.967
		1.000	[illegible]	1.000	1.000	0.966
		1.000	[illegible]	1.000	0.000	0.964
		1.000	[illegible]	0.000	1.000	0.963
					Average	0.966

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	[illegible]	0.000	0.000	[illegible]
		1.000	0.000	1.000	0.000	0.962
		1.000	0.000	[illegible]	1.000	0.946
		1.000	0.000	[illegible]	1.000	0.952
		1.000	0.000	[illegible]	[illegible]	0.949
					Average	0.983

Haar at scale of 1

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	0.000	1.000	1.000	0.966
		1.000	1.000	[illegible]	1.000	0.950
		1.000	0.000	[illegible]	[illegible]	0.967
		1.000	1.000	0.000	[illegible]	0.962
		1.000	0.000	0.000	[illegible]	0.96[illegible]
					Average	0.962

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	1.000	0.000	[illegible]	[illegible]
		1.000	1.000	0.000	[illegible]	[illegible]
		1.000	0.000	0.000	1.000	[illegible]
		1.000	0.000	0.000	[illegible]	[illegible]
		1.000	0.000	0.000	[illegible]	[illegible]
					Average	0.963

On or Off **slope 1** at different SP values

Haar at scale of 1

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	0.000	[illegible]	0.000	0.955
		1.000	0.000	[illegible]	[illegible]	0.954
		1.000	0.000	[illegible]	[illegible]	0.957
		1.000	0.000	[illegible]	[illegible]	0.976
		1.000	1.000	[illegible]	[illegible]	0.952
					Average	[illegible]

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	0.000	[illegible]	0.000	0.768
		1.000	1.000	0.000	1.000	0.865
		1.000	2.500	0.000	[illegible]	0.865
		1.000	1.500	0.000	[illegible]	0.896
		1.000	1.000	0.000	[illegible]	[illegible]
					Average	[illegible]

Haar at scale of 1

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	1.000	1.000	0.000	[illegible]
		1.000	[illegible]	1.000	0.000	[illegible]
		1.000	[illegible]	1.000	1.000	[illegible]
		1.000	[illegible]	1.000	0.000	0.971
		1.000	1.000	0.000	1.000	[illegible]
					Average	0.962

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	0.000	1.000	1.000	[illegible]
		1.000	[illegible]	0.000	0.000	[illegible]
		1.000	0.000	1.000	1.000	[illegible]
		1.000	0.000	1.000	1.000	[illegible]
		1.000	0.000	0.000	0.000	0.954
					Average	0.958

Haar at scale of 1

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	0.000	1.000	1.000	0.962
		1.000	0.000	1.000	0.000	0.965
		1.000	1.000	[illegible]	0.000	0.952
		1.000	0.000	[illegible]	1.000	0.964
		1.000	1.000	[illegible]	0.000	0.964
					Average	0.963

Daub 3 at scale of 3

TC	NTC	a1	d1	d2	d3	Separability
1.000	17.000	1.000	1.000	[illegible]	1.000	0.954
		1.000	0.000	[illegible]	0.000	0.958
		1.000	1.000	[illegible]	1.000	0.962
		1.000	0.000	[illegible]	1.000	0.968
		1.000	1.000	[illegible]	0.000	0.962
					Average	[illegible]

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =20
Haar & Daubechies 3 Wavelets

Haar at scale of 3 — On or Off Representation Slope 1 @ No SP

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	0.000	1.000	1.000	[illegible]
		[illegible]	[illegible]	1.000	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	1.000	[illegible]	[illegible]	[illegible]
		[illegible]	1.000	[illegible]	[illegible]	[illegible]
					Average	[illegible]

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ No SP

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	1.000	0.000	0.000	0.976
		1.000	1.000	0.000	0.000	0.992
		1.000	1.000	0.000	0.000	0.973
		1.000	1.000	0.000	1.000	0.993
		1.000	1.000	0.000	1.000	0.976
					Average	0.983

Haar at scale of 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	0.000	0.000	0.000	0.937
		1.000	0.000	0.000	0.000	0.955
		1.000	0.000	1.000	1.000	0.979
		1.000	1.000	0.000	0.000	0.955
		1.000	0.000	0.000	0.000	0.957
					Average	0.933

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	0.000	0.000	0.000	0.990
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	1.000	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					Average	[illegible]

Haar at scale of 3 — On or Off Representation Slope 1 @ SP=40

TC	NTC	a3	d1	d2	d3	Sepindex
[illegible]	[illegible]	1.000	1.000	0.000	0.000	[illegible]
		1.000	[illegible]	[illegible]	1.000	[illegible]
		1.000	[illegible]	[illegible]	1.000	[illegible]
		[illegible]	[illegible]	[illegible]	1.000	[illegible]
		[illegible]	[illegible]	[illegible]	1.000	[illegible]
					Average	0.973

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ SP=40

TC	NTC	a3	d1	d2	d3	Sepindex
[illegible]	20.000	1.000	1.000	0.000	0.000	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	1.000	[illegible]
		[illegible]	1.000	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	0.993
					Average	[illegible]

On or Off **slope 1** at different SP values

Haar at scale of 3 — On or Off Representation Slope 1 @ no SP

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	0.000	0.000	0.000	0.982
		1.000	0.000	1.000	0.000	0.930
		1.000	0.000	1.000	0.000	0.954
		1.000	0.000	1.000	0.000	0.896
		1.000	0.000	1.000	0.000	0.864
					Average	0.885

Daub 3 at scale of 3 — On or Off Representation Slope 1 @ no SP

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	1.000	1.000	0.000	0.985
		1.000	1.000	0.000	0.000	0.987
		1.000	0.000	1.000	0.000	0.977
		1.000	1.000	1.000	0.000	0.965
		1.000	0.000	0.000	0.000	0.927
					Average	0.973

Haar at scale of 3 — On or Off Representation Slope 2 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	0.000	1.000	0.000	0.965
		1.000	0.000	0.000	0.000	0.944
		1.000	0.000	1.000	0.000	0.900
		1.000	0.000	1.000	0.000	0.951
		1.000	1.000	0.000	0.000	0.940
					Average	0.927

Daub 3 at scale of 3 — On or Off Representation Slope 2 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	0.000	0.000	1.000	0.969
		1.000	0.000	0.000	0.000	0.941
		1.000	0.000	0.000	1.000	0.920
		1.000	0.000	0.000	0.000	0.941
		1.000	0.000	1.000	1.000	0.951
					Average	0.933

Haar at scale of 3 — On or Off Representation Slope 2 @ SP=40

TC	NTC	a3	d1	d2	d3	Sepindex
1.000	20.000	1.000	0.000	0.000	1.000	0.941
		1.000	0.000	0.000	1.000	0.957
		1.000	0.000	0.000	0.000	0.930
		1.000	0.000	1.000	1.000	0.957
		1.000	0.000	0.000	1.000	0.948
					Average	0.938

Daub 3 at scale of 3 — On or Off Representation Slope 2 @ SP=40

TC	NTC	a3	d1	d2	d3	Sepindex
[illegible]	20.000	1.000	0.000	1.000	0.000	0.944
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	0.000	1.000	1.000	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	[illegible]	[illegible]	[illegible]	[illegible]
					Average	[illegible]

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =21
Haar & Daubechies 3 Wavelets

Haar at scale of 3						
On or Off Representation Slope 1 @ No SP						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	0.000	1.000	1.000	1.000	0.938
		1.000	0.000	0.000	1.000	0.966
		1.000	0.000	0.000	0.000	0.957
		1.000	0.000	0.000	0.000	0.957
		1.000	0.000	0.000	0.000	0.962
					Average	0.956

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ no SP						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	0.000	0.000	0.000	1.000	0.938
		1.000	0.000	0.000	1.000	0.962
		1.000	1.000	0.000	1.000	0.938
		1.000	0.000	0.000	0.000	0.947
		1.000	0.000	0.000	1.000	0.957
					Average	0.948

Haar at scale of 3						
On or Off Representation Slope 1 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	0.000	1.000	0.000	0.000	0.938
		1.000	0.000	1.000	0.000	0.952
		0.000	1.000	0.000	1.000	0.938
		1.000	1.000	0.000	0.000	0.938
		1.000	1.000	1.000	0.000	0.962
					Average	0.945

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	1.000	1.000	1.000	0.942
		1.000	1.000	0.000	0.000	0.957
		0.000	1.000	1.000	0.000	0.938
		1.000	1.000	1.000	0.000	0.942
		1.000	0.000	0.000	0.000	0.957
					Average	0.947

Haar at scale of 3						
On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	1.000	1.000	0.000	0.947
		0.000	1.000	0.000	0.000	0.942
		1.000	1.000	0.000	1.000	0.962
		0.000	1.000	1.000	1.000	0.938
		1.000	1.000	0.000	1.000	0.967
					Average	0.947

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	1.000	1.000	0.000	0.952
		1.000	1.000	1.000	0.000	0.933
		0.000	0.000	0.000	1.000	0.938
		1.000	1.000	1.000	1.000	0.942
		1.000	0.000	1.000	1.000	0.947
					Average	0.943

On or Off **slope 1** at different SP values

Haar at scale of 3						
On or Off Representation Slope 1 @ no SP						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	1.000	1.000	1.000	0.938
		1.000	1.000	1.000	1.000	0.938
		1.000	1.000	1.000	1.000	0.942
		0.000	1.000	1.000	1.000	0.938
		1.000	0.000	1.000	1.000	0.935
					Average	0.936

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ no SP						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	0.000	1.000	0.000	0.947
		1.000	0.000	1.000	1.000	0.938
		0.000	0.000	0.000	1.000	0.933
		1.000	1.000	0.000	1.000	0.942
		0.000	0.000	0.000	1.000	0.933
					Average	0.940

Haar at scale of 3						
On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	0.000	1.000	0.000	0.947
		1.000	1.000	1.000	1.000	0.967
		1.000	0.000	1.000	1.000	0.947
		1.000	0.000	1.000	0.000	0.947
		1.000	0.000	0.000	1.000	0.947
					Average	0.947

Daub 3 at scale of 3						
On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	1.000	1.000	0.000	0.947
		1.000	1.000	0.000	1.000	0.957
		1.000	0.000	1.000	1.000	0.942
		1.000	0.000	0.000	0.000	0.938
		1.000	0.000	0.000	1.000	0.938
					Average	0.944

Haar at scale of 3						
On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	0.000	1.000	0.000	0.947
		1.000	0.000	1.000	1.000	0.963
		1.000	0.000	1.000	1.000	0.947
		1.000	1.000	1.000	1.000	0.942
		1.000	0.000	0.000	0.000	0.962
					Average	0.953

Daub 3 at scale of 3						
On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Sep index
1.000	21.000	1.000	1.000	1.000	0.000	0.952
		1.000	1.000	0.000	1.000	0.952
		1.000	1.000	1.000	0.000	0.947
		1.000	1.000	1.000	0.000	0.942
		1.000	1.000	1.000	1.000	0.957
					Average	0.950

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =26
Haar & Daubechies 3 Wavelets

Haar at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ No SP						
1.000	26.000	1.000	0.000	1.000	0.000	1.000
		1.000	0.000	1.000	0.000	1.000
		1.000	1.000	0.000	1.000	1.000
		1.000	0.000	1.000	0.000	1.000
		1.000	1.000	0.000	1.000	1.000
					Average	1.000

Daub 3 at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ no SP						
1.000	26.000	1.000	0.000	1.000	0.000	1.000
		1.000	1.000	0.000	0.000	1.000
		1.000	0.000	0.000	0.000	1.000
		1.000	1.000	0.000	1.000	1.000
		1.000	1.000	0.000	1.000	1.000
					Average	1.000

Haar at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ SP=100						
1.000	26.000	1.000	1.000	0.000	1.000	0.997
		1.000	1.000	1.000	1.000	0.994
		1.000	1.000	0.000	1.000	0.997
		1.000	1.000	0.000	1.000	0.993
		1.000	0.000	1.000	1.000	0.997
					Average	0.996

Daub 3 at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ SP=100						
1.000	26.000	1.000	0.000	1.000	0.000	1.000
		1.000	0.000	1.000	1.000	0.997
		1.000	0.000	1.000	1.000	0.997
		1.000	1.000	1.000	0.000	0.997
		1.000	0.000	0.000	0.000	0.997
					Average	0.997

Haar at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ SP=50						
1.000	26.000	1.000	0.000	0.000	1.000	1.000
		1.000	0.000	1.000	1.000	0.996
		1.000	0.000	0.000	1.000	0.997
		1.000	0.000	0.000	1.000	0.997
		1.000	0.000	1.000	0.000	0.997
					Average	0.996

Daub 3 at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ SP=50						
1.000	26.000	1.000	0.000	1.000	1.000	1.000
		1.000	1.000	1.000	1.000	0.993
		1.000	0.000	0.000	0.000	0.997
		1.000	0.000	0.000	0.000	0.997
		1.000	1.000	1.000	0.000	0.997
					Average	0.996

On or Off **slope 1** at different SP values

Haar at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ no SP						
1.000	26.000	1.000	1.000	1.000	0.000	1.000
		1.000	1.000	0.000	0.000	1.000
		1.000	1.000	0.000	0.000	1.000
		1.000	0.000	1.000	0.000	1.000
		1.000	1.000	0.000	0.000	1.000
					Average	1.000

Daub 3 at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 1 @ no SP						
1.000	26.000	1.000	0.000	0.000	0.000	1.000
		1.000	1.000	0.000	0.000	1.000
		1.000	1.000	1.000	0.000	1.000
		1.000	0.000	0.000	1.000	1.000
		1.000	0.000	0.000	0.000	1.000
					Average	1.000

Haar at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 2 @ SP=100						
1.000	26.000	1.000	0.000	0.000	0.001	1.000
		1.000	1.001	0.000	0.001	1.001
		1.000	1.001	1.000	1.001	0.997
		1.000	0.001	0.000	1.000	0.997
		1.000	1.000	0.000	1.000	0.997
					Average	0.998

Daub 3 at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
On or Off Representation Slope 2 @ SP=100						
1.000	26.000	1.000	0.000	1.000	0.000	1.000
		1.000	1.000	1.000	1.000	1.000
		1.000	0.000	0.000	0.000	0.997
		1.000	1.000	0.000	1.000	0.997
		1.000	1.000	0.000	1.000	0.997
					Average	0.998

Haar at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
[illegible]						
1.000	26.000	1.000	1.000	0.000	0.000	1.000
		1.000	0.000	0.000	1.000	1.000
		[illegible]	[illegible]	[illegible]	0.000	1.000
		1.000	1.000	1.000	1.000	1.000
		1.000	1.000	0.000	0.000	1.000
					Average	1.000

Daub 3 at scale of 3

TC	NTC	a3	d1	d2	d3	Sepindex
[illegible]						
1.000	26.000	1.000	0.000	0.000	0.000	1.000
1.000	26.000	1.000	0.000	1.000	0.000	1.000
1.000	26.000	1.000	0.000	[illegible]	1.000	1.000
1.000	26.000	1.000	0.000	0.000	0.000	1.000
1.000	26.000	1.000	1.000	0.000	0.000	1.000
					Average	[illegible]

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =31
Haar & Daubechies 3 Wavelets

[illegible]

On or Off Representation Slope 1 @ No SP						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	0.000	1.000	0.000	0.995
		1.000	0.000	1.000	0.000	1.000
		1.000	0.000	1.000	0.000	0.998
		1.000	0.000	1.000	1.000	1.000
		1.000	0.000	1.000	0.000	0.998
					Average	[illegible]

[illegible]

On or Off Representation Slope 1 @ No SP						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	0.000	0.000	1.000	0.996
		1.000	1.000	0.000	1.000	0.996
		1.000	0.000	1.000	1.000	0.944
		1.000	1.000	0.000	1.000	1.000
		1.000	1.000	0.000	1.000	0.996
					Average	[illegible]

[illegible]

On or Off Representation Slope 1 @ SP=100						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	1.000	1.000	0.000	1.000
		1.000	0.000	1.000	1.000	1.000
		1.000	1.000	1.000	0.000	1.000
		1.000	0.000	1.000	0.000	1.000
		1.000	1.000	1.000	1.000	1.000
					Average	1.000

[illegible]

On or Off Representation Slope 1 @ SP=[illegible]						
TC	NTC	a3	d1	d2	d3	Separation
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.000
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	1.000	1.000	1.000	1.000
		1.000	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	1.000	1.000	[illegible]	1.000
					Average	[illegible]

[illegible]

On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	0.000	1.000	1.000	1.000
		1.000	1.000	1.000	0.000	1.000
		1.000	0.000	1.000	0.000	0.999
		1.000	0.000	1.000	1.000	1.000
		1.000	0.000	1.000	0.000	1.000
					Average	1.000

[illegible]

On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	0.000	0.000	1.000	1.000
		1.000	1.000	1.000	1.000	1.000
		1.000	1.000	0.000	1.000	0.996
		1.000	0.000	0.000	1.000	1.000
		1.000	1.000	1.000	1.000	1.000
					Average	0.999

On or Off **slope 1** at different SP values

[illegible]

On or Off Representation Slope 2 @ no SP						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	1.000	1.000	0.000	0.924
		1.001	1.000	1.000	0.000	0.880
		1.001	1.000	1.000	0.000	0.857
		1.000	1.000	1.000	0.000	0.849
		1.001	1.000	1.000	0.000	0.855
					Average	0.876

[illegible]

On or Off Representation Slope 2 @ no SP						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	0.000	1.001	1.000	0.928
		1.000	0.000	1.000	1.000	0.933
		1.000	0.000	1.001	1.000	0.930
		1.000	0.000	1.000	1.000	0.941
		1.000	0.000	1.000	1.000	0.917
					Average	0.931

[illegible]

On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	1.000	1.000	1.000	0.870
		1.001	1.000	0.000	0.000	0.985
		1.000	1.000	1.000	1.000	0.987
		1.001	1.000	1.000	0.000	0.984
		1.001	1.000	0.000	1.000	0.987
					Average	0.945

[illegible]

On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.000	0.000	0.000	1.000	0.969
		1.000	1.000	0.001	1.000	0.995
		1.000	0.000	1.001	1.000	0.980
		1.000	0.000	0.001	1.000	0.984
		1.000	1.000	1.001	1.000	0.987
					Average	0.985

[illegible]

On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Separation
1.000	31.000	1.001	1.000	1.000	1.000	[illegible]
		1.000	1.000	0.000	0.000	0.995
		1.001	1.000	1.000	1.000	0.985
		1.001	1.000	1.000	1.000	0.995
		1.000	1.000	0.000	1.000	0.941
					Average	0.992

[illegible]

On or Off Representation Slope 1 @ SP=50						
TC	NTC	a3	d1	d2	d3	Separation
1.001	31.000	1.000	0.000	1.001	1.000	0.991
		1.000	1.000	1.000	1.000	0.993
		1.000	0.000	1.000	1.000	0.987
		1.000	0.000	1.001	1.000	0.995
		1.000	1.000	0.001	1.000	0.987
					Average	0.991

On or Off **slope 2** at different SP values

Target Class= 1 Non Target Class =15
Haar & Daubechies 3 Wavelets

[illegible]						
On or Off Representation Slope 1 @ No SP						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	1.200	0.000	1.500	1.500	0.579
		[illegible]	1.200	0.300	0.200	0.567
		[illegible]	[illegible]	1.300	[illegible]	[illegible]
		[illegible]	[illegible]	1.300	2.500	[illegible]
		[illegible]	[illegible]	1.300	[illegible]	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 1 @ No SP						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	1.000	0.300	[illegible]	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 1 @ SP=[illegible]						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	1.000	[illegible]	[illegible]	[illegible]	0.946
		1.000	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	[illegible]	[illegible]	[illegible]	0.954
		1.000	[illegible]	[illegible]	[illegible]	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 1 @ SP=100						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	10.000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 1 @ SP=[illegible]						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 1 @ SP=50						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	1.000	0.300	0.000	[illegible]	0.964
		1.000	0.000	0.000	1.000	0.975
		1.000	0.000	1.000	[illegible]	0.971
		1.000	[illegible]	0.000	[illegible]	0.958
		1.000	[illegible]	0.000	1.000	[illegible]
					Average	[illegible]

On or Off **slope 1** at different SP values

[illegible]						
On or Off Representation Slope 2 @ No SP						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	1.000	0.000	0.300	0.300	[illegible]
		1.000	0.000	1.300	[illegible]	[illegible]
		1.000	1.000	0.300	1.300	[illegible]
		1.000	0.000	1.300	0.300	[illegible]
		1.000	0.000	1.300	2.500	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 2 @ No SP						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	1.300	[illegible]	1.000	[illegible]	[illegible]
		1.300	[illegible]	1.000	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	1.000	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 2 @ SP=100						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	1.300	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	1.000	[illegible]
		1.300	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	1.000	1.000	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope 2 @ SP=[illegible]						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	[illegible]	[illegible]	1.000	0.000	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	1.000	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		1.000	1.000	[illegible]	1.500	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope [illegible] @ SP=[illegible]						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	15.000	[illegible]	1.000	0.000	0.300	[illegible]
		1.000	0.000	0.000	[illegible]	[illegible]
		1.000	1.000	0.300	1.000	[illegible]
		1.000	0.000	0.300	[illegible]	[illegible]
		[illegible]	0.000	0.300	1.000	[illegible]
					Average	[illegible]

[illegible]						
On or Off Representation Slope [illegible] @ SP=50						
TC	NTC	[illegible]	[illegible]	[illegible]	[illegible]	Episodes
1.000	[illegible]	1.000	[illegible]	[illegible]	[illegible]	0.900
		1.000	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	[illegible]	[illegible]
		1.300	[illegible]	[illegible]	1.000	[illegible]
					Average	[illegible]

On or Off **slope 2** at different SP values

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =10

Haar at scale 3						
On or Off Representation Slope 1 @ No SP						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	0	0	0.9562
		1	0	0	1	1.9562
		1	0	0	0	2.9562
		1	0	0	0	3.9562
		1	0	0	1	4.9562
					Average	5.9562

Daub 3 at Scale of 3						
On or Off Representation Slope 1 @ No SP						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	0	0	0.9902
		1	0	0	1	0.9973
		1	0	0	0	0.9910
		1	0	0	0	0.9898
		1	0	0	0	0.9942
					Average	0.9917

Haar at scale 3						
On or Off Representation Slope 1 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	1	0	0.9678
		1	1	0	1	0.9631
		1	0	1	1	0.9652
		1	1	1	1	0.9652
		1	1	0	1	0.9610
					Average	0.9654

Daub 3 at Scale of 3						
On or Off Representation Slope 1 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	1	0	0.9818
		1	0	1	0	0.9621
		1	1	0	0	0.9800
		1	1	1	1	0.9588
		1	1	0	1	0.9590
					Average	0.9587

Haar at scale 3						
On or Off Representation Slope 1 @ SP=80						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	0	0	0.9580
		1	1	1	0	0.9520
		1	0	0	1	0.9614
		1	0	0	0	0.9514
		1	0	0	0	0.9507
					Average	0.9550

Daub 3 at Scale of 3						
On or Off Representation Slope 1 @ SP=80						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	0	1	0.9835
		1	1	1	0	0.9823
		1	0	0	0	0.9758
		1	1	1	1	0.9825
		1	0	0	1	0.9878
					Average	0.9854

Haar at Scale of 3						
On or Off Representation Slope 2 @ No SP						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	0	0	0.9729
		1	0	0	0	0.9846
		1	0	1	0	0.9779
		1	1	0	0	0.9755
		1	1	0	0	0.9819
					Average	0.9776

Daub 3 at Scale of 3						
On or Off Representation Slope 2 @ No SP						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10	1	1	0	0	0.9755
	10	1	0	0	0	0.9578
	10	1	0	0	0	0.9795
	10	1	0	0	0	0.9762
	10	1	0	0	0	0.9826
					Average	0.9759

Haar at Scale of 3						
On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	1	0	0.9787
		1	1	1	1	0.9915
		1	0	1	0	0.9803
		1	1	0	1	0.9808
		1	0	1	0	0.9872
					Average	0.9837

Daub 3 at Scale of 3						
On or Off Representation Slope 2 @ SP=100						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10	1	0	0	1	0.9806
		1	0	0	0	0.9894
		1	0	0	0	0.9793
		1	0	1	1	0.9803
		1	0	0	0	0.9878
					Average	0.9835

Haar at Scale of 3						
On or Off Representation Slope 2 @ SP=80						
TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	10.0000	1	0	1	1	0.6770
		1	1	1	1	0.5850
		1	0	1	0	0.6740
		1	0	1	0	0.6700
		1	0	0	1	0.5850

Daub 3 at Scale of 3						
On or Off Representation Slope 2 @ SP=80						
TC	NTC	a3	d1	d2	d3	Sepindex
2	10	1	1	0	0	0.97[illegible]
	10	1	0	0	0	0.9[illegible]
	10	1	0	0	0	0.9[illegible]
	10	1	0	0	0	0.9[illegible]
	10	1	0	0	0	[illegible]

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =12

Haar at scale 3

On or Off Representation Slope 1 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	1	0	0	0	0.9897
		1	0	0	0	0.9948
		1	0	0	0	0.9897
		1	0	0	0	0.9897
		1	0	0	0	0.9962
					Average	0.9920

Daub 3 at Scale of 3

On or Off Representation Slope 1 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	1	0	0	0	0.9897
		1	0	0	0	0.9940
		1	0	0	0	0.9897
		1	0	0	0	0.9897
		1	0	0	0	0.9962
					Average	0.9920

Haar at scale 3

On or Off Slope 1 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	1	1	0	0	0.9830
		1	1	0	1	0.9887
		1	1	0	1	0.9905
		1	0	1	1	0.9876
		1	1	0	1	0.9919
					Average	0.9877

Daub 3 at Scale of 3

On or Off Slope 1 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	1	1	1	1	0.9853
		1	1	0	1	0.9887
		1	1	0	1	0.9905
		1	0	1	1	0.9876
		1	1	0	1	0.9919
					Average	0.9877

Haar at scale 3

On or Off Slope 1 @SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	1	0	1	0	0.9527
		1	0	0	0	0.9870
		1	0	0	0	0.9836
		1	0	0	1	0.9860
		1	1	0	1	0.9903
					Average	0.9869

Daub 3 at Scale of 3

On or Off Slope 1 @ SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	1	0	0	1	0.9603
		1	0	0	0	0.9679
		1	0	0	1	0.9627
		1	0	1	1	0.9670
		1	1	0	1	0.9627
					Average	0.9649

Haar at Scale of 3

On or Off Slope 2 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	0	0	0	1	0.9784
		1	1	0	0	0.9887
		0	1	0	0	0.9902
		0	1	0	0	0.9816
		1	0	0	0	0.9870
					Average	0.9859

Daub 3 at Scale of 3

On or Off Slope 2 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12	1	1	1	1	0.9795
	12	1	1	0	0	0.9914
	12	0	1	1	0	0.9800
	12	1	1	0	0	0.9827
	12	1	0	0	0	0.9878
					Average	0.9842

Haar at Scale of 3

On or Off Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12.0000	1	0	0	1	[illegible]
		1	0	1	1	[illegible]
		0	1	1	0	[illegible]
		1	0	0	1	[illegible]
		1	1	0	1	[illegible]
					Average	[illegible]

Daub 3 at Scale of 3

On or Off Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	12	1	0	1	0	0.9768
		1	1	1	1	0.9811
		1	0	1	1	0.9757
		1	0	1	0	0.9802
		1	0	0	0	0.9892
					Average	0.9813

Haar at Scale of 3

On or Off Slope 2 @ SP=80						
TC	NTC					
2.0000	12.0000	1	0	1	1	0.9700
		1	0	0	0	0.9810
		1	0	1	0	0.9750
		1	0	1	0	0.9800
		1	1	1	1	0.9870
					Average	0.9802

Daub 3 at Scale of 3

On or Off Slope 2 @ SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2	12	1	1	1	1	0.9795
	12	1	1	0	0	0.9814
	12	0	1	1	0	0.9800
	12	1	1	0	0	0.9827
	12	1	0	0	0	0.9875
					Average	0.9842

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =13

Haar at scale 3 — On or OFF Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	0	0	0	0.9858
		1	0	0	0	0.9622
		1	0	0	0	0.9857
		1	0	0	1	0.9844
		1	0	0	0	0.9650
					Average	0.9875

Daub 3 at Scale of 3 — On or Off Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	0	0	0	0.9928
		1	0	0	0	0.9903
		1	0	0	0	0.9887
		1	0	0	1	0.9844
		1	0	0	0	0.9903
					Average	0.9876

Haar at scale 3 — On or Off Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	0	0	0	0.9790
		1	1	0	0	0.9801
		1	1	1	1	0.9871
		1	1	1	1	0.9817
		1	1	0	1	0.9587
					Average	0.9833

Daub 3 at Scale of 3 — On or Off Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	0	0	0	0.9790
		1	1	0	0	0.9801
		1	1	0	0	0.9871
		1	1	1	1	0.9817
		1	1	0	1	0.9857
					Average	0.9833

Haar at scale 3 — On or Off Slope 1 @ SP = 80

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	0	0	0	0.9785
		1	1	1	1	0.9779
		1	0	0	1	0.9817
		1	0	1	0	0.9785
		1	1	0	0	0.9808
					Average	0.9796

Daub 3 at Scale of 3 — On or Off Slope 1 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	0	1	0	0.9774
		1	1	1	0	0.9773
		1	1	1	1	0.9817
		1	0	1	0	0.9793
		1	0	1	1	0.9817
					Average	0.9796

Haar at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	1	0	0	0.9774
		1	0	0	1	0.9765
		1	0	0	0	0.9801
		1	0	0	1	0.9752
		1	0	0	0	0.9790
					Average	0.9894

Daub 3 at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13	1	0	1	0	0.9806
	13	1	0	0	0	0.9821
	13	1	0	0	1	0.9806
	13	1	1	0	0	0.9779
	13	1	0	0	0	0.9790
					Average	0.9796

Haar at Scale of 3 — On or Off Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13.0000	1	0	0	0	0.9822
		0	1	1	0	0.9812
		1	0	0	0	0.9871
		0	1	1	0	0.9790
		1	0	0	0	0.9582
					Average	0.9899

Daub 3 at Scale of 3 — On or On Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	13	1	1	0	0	0.9822
		1	1	0	1	0.9779
		1	1	1	1	0.9876
		1	1	1	0	0.9785
		1	0	1	1	0.9582
					Average	0.9829

Haar at Scale of 3 — On or Off Slope 2 @ SP=80

TC	NTC					
2.0000	13.0000	1	1	0	0	0.9790
		0	1	1	0	0.9730
		1	0	1	1	0.9820
		1	0	0	1	0.9752
		0	1	0	1	0.9500
					Average	0.9671

Daub 3 at Scale of 3 — On or Off Slope 2 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
2	13	1	0	1	0	0.9506
	13	1	0	0	0	0.9501
	13	1	0	0	1	0.9506
	13	1	1	0	0	0.9779
	13	1	0	0	0	0.9790
					Average	0.9794

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =18

Haar at scale 3						
On or Off slope 1 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18.0000	1	0	0	0	0.9865
		1	0	0	0	0.9833
		1	0	0	0	0.9730
		1	0	0	0	0.9830
		1	0	0	0	0.9822
					Average	0.9844

Daub 3 at Scale of 3						
On or Off Slope 1 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18.0000	1	0	0	0	0.9900
		1	0	0	0	0.9835
		1	0	0	0	0.9780
		1	0	0	0	0.9580
		1	0	0	0	0.9522
					Average	0.9844

Haar at scale 3						
On or Off Slope 1 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2	18	1	1	1	0	0.9744
		1	1	0	1	0.9629
		1	0	0	0	0.9609
		1	0	0	1	0.9623
		1	1	0	0	0.9613
					Average	0.9670

Daub 3 at Scale of 3						
On or Off Slope 1 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18.0000	1	1	0	1	0.9707
		1	1	0	0	0.9631
		1	1	0	1	0.9629
		1	0	1	0	0.9727
		1	0	0	1	0.9660
					Average	0.9679

Haar at scale 3						
On or Off Slope 1 @ SP=10						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18.0000	1	1	1	0	0.9746
		1	1	0	1	0.9629
		1	0	0	0	[illegible]
		1	0	0	1	0.9722
		1	1	0	0	[illegible]
					Average	[illegible]

Daub 3 at Scale of 3						
On or Off Slope 1 @ SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18.0000	1	0	1	0	0.9744
		1	1	0	1	0.9604
		1	1	0	1	0.9645
		1	0	1	0	0.9729
		1	0	0	1	0.9629
					Average	0.9678

Haar at Scale of 3						
On or Off Slope 2 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18.0000	1	0	1	0	0.9868
		1	0	0	1	0.9697
		1	0	1	0	0.9746
		1	0	1	0	0.9854
		1	1	0	0	0.9712
					Average	0.9774

Daub 3 at Scale of 3						
On or Off Slope 2 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18	1	0	1	0	0.9864
	18	1	0	1	0	0.9712
	18	1	0	1	0	0.9760
	18	1	0	1	0	0.9875
	18	1	0	1	0	0.9754
					Average	0.9797

Haar at Scale of 3						
On or Off Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18.0000	1	0	1	1	0.9671
		1	1	1	0	0.9587
		1	0	0	0	0.9097
		1	0	0	1	0.9652
		1	0	1	0	0.9655
					Average	0.9651

Daub 3 at Scale of 3						
On or Off Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	18	1	0	0	1	0.9671
		1	0	1	0	0.9582
		1	0	1	0	0.9602
		1	0	0	1	0.9690
		1	0	0	0	0.9643
					Average	0.9643

Haar at Scale of 3						
On or Off Slope 2 @ SP=80 Setting						
TC	NTC					
2.0000	18.0000	1	0	0	0	0.9760
		1	0	0	0	0.9530
		1	1	1	0	0.9630
		1	0	0	1	0.9710
		1	1	0	0	0.9610
					Average	0.9658

Daub 3 at Scale of 3						
On or Off Slope 2 @ SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2	18	1	0	1	0	0.9864
	18	1	0	1	0	0.9712
	18	1	0	1	0	0.9760
	18	1	0	1	0	0.9875
	18	1	0	1	0	0.9754
					Average	0.9797

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =21

Haar at scale 3 — On or Off Slope 1 @ No SP

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	1	0	0	0.9959
		1	0	0	0	0.9970
		1	0	0	0	0.9998
		1	0	0	1	1.0007
		1	0	0	0	0.9996
					Average	0.9986

Daub 3 at Scale of 3 — On or Off Representation Slope 1 @ No SP

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	0	0	0	0.9989
		1	0	0	0	0.9985
		1	1	0	0	0.9955
		1	0	0	0	1.0000
		1	1	0	0	0.9965
					Average	0.9995

Haar at scale 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
2	21	1	0	1	1	0.9986
		1	0	0	0	[illegible]
		1	0	0	0	0.9987
		1	0	0	1	[illegible]
		1	1	0	0	[illegible]
					Average	[illegible]

Daub 3 at Scale of 3 — On or Off Representation Slope 1 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	0	0	0	0.9967
		1	0	0	0	0.9951
		1	0	0	1	0.9956
		1	0	0	0	0.9956
		1	0	0	1	0.9962
					Average	0.9963

Haar at scale 3 — On or Off Representation Slope 1 @ SP=50

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	0	1	1	0.9964
		1	0	0	0	0.9967
		1	0	0	0	0.9967
		1	0	0	1	0.9948
		1	1	0	0	0.9950
					Average	0.9967

Daub 1 at Scale of 3 — On or Off Representation Slope 1 @ SP=50

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	0	0	1	[illegible]
		1	0	1	0	0.9956
		1	1	1	0	[illegible]
		1	0	1	1	0.9948
		1	1	0	0	[illegible]
					Average	0.9962

Haar at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	1	0	0	0.9884
		1	0	1	1	0.9920
		0	0	0	1	0.9929
		1	0	1	0	0.9943
		1	1	0	0	0.9943
					Average	0.9934

Daub 3 at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a3	d1	d2	d3	Sepindex
2	21	1	1	0	0	0.9950
	21	1	1	1	0	0.9934
	21	0	0	0	1	0.9939
	21	1	1	0	0	0.9956
	21	1	1	0	0	0.9945
					Average	0.9944

Haar at Scale of 3 — On or Off Slope 2 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	0	0	0	0.9967
		1	0	0	0	0.9967
		1	0	0	1	0.9970
		1	0	0	0	0.9962
		1	0	0	0	0.9967
					Average	0.9948

Daub 2 at Scale of 3 — On or Off Slope 2 @ SP=100

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21	1	0	0	1	0.9965
		1	0	0	0	0.9962
		1	0	0	1	0.9970
		1	0	0	1	0.9957
		1	0	1	0	0.9967
					Average	0.9963

Haar at Scale of 3 — On or Off Slope 2 @ SP=50

TC	NTC	a3	d1	d2	d3	Sepindex
2.0000	21.0000	1	0	1	1	0.9960
		1	0	0	0	0.9960
		1	0	0	0	[illegible]
		1	0	1	0	0.9940
		1	0	1	1	[illegible]
					Average	0.9964

Daub 3 at Scale of 3 — On or Off Slope 2 @ SP=50

TC	NTC	a3	d1	d2	d3	Sepindex
2	21	1	0	1	0	0.9967
		1	1	1	1	0.9946
		1	0	0	0	0.9970
		1	1	0	1	0.9940
		1	0	1	1	0.9977
					Average	0.9960

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =22

Haar at scale 3						
On or Off Slope 1 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	22.0000	1	0	0	0	0.9904
		1	0	0	0	0.9859
		1	0	0	0	0.9864
		1	0	0	0	0.9525
		1	0	0	0	0.9911
					Average	0.9867

Daub 3 at Scale of 3						
On or Off Slope 1 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	22.0000	1	0	0	0	0.9854
		1	0	0	0	0.9859
		1	0	0	0	0.9864
		1	2	0	0	0.9835
		1	[illegible]	0	0	0.9511
					Average	0.5867

Haar at scale 3						
On or Off Slope 1 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2	22	1	0	0	0	0.9726
		1	0	0	1	0.9715
		1	0	1	0	0.9703
		1	1	0	1	0.9713
		1	0	0	0	0.9729
					Average	0.9728

Daub 3 at Scale of 3						
On or Off Slope 1 @ SP=100						
[illegible]	[illegible]	a1	d1	d2	d3	Sepindex
2.0000	22.0000	1	1	1	1	[illegible]
		1	1	0	1	0.9759
		1	1	1	1	[illegible]
		1	1	0	0	0.9704
		1	0	0	1	[illegible]
					Average	[illegible]

Haar at scale 3						
On or Off Slope 1 @ SP = 80						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	22.0000	1	0	0	0	0.9726
		1	0	0	1	0.9710
		1	0	1	0	0.9703
		1	1	0	1	0.9713
		1	0	0	0	0.9729
					Average	0.9728

Daub 3 at Scale of 3						
On or Off Slope 1 @ SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2	22	1	1	0	0	0.9765
		1	1	0	1	0.9704
		1	1	0	0	0.9708
		1	1	0	0	0.9729
		1	0	1	0	0.9724
					Average	0.9732

Haar at Scale of 3						
On or Off Slope 2 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	22.0000	1	1	0	0	0.9854
		1	0	1	0	0.9766
		1	0	0	0	0.9785
		1	0	0	0	0.9849
		1	0	0	1	0.9623
					Average	0.9815

Daub 3 at Scale of 3						
On or Off Slope 2 @ No SP						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	22	1	1	[illegible]	0	0.9808
	22	1	[illegible]	1	0	0.9791
	22	1	0	[illegible]	0	0.9797
	22	1	1	0	0	0.9890
	22	1	1	1	0	0.9776
					Average	0.9818

Haar at Scale of 3						
On or Off Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
[illegible]	[illegible]	1	0	1	1	[illegible]
		1	1	0	1	[illegible]
		1	1	1	0	[illegible]
		1	1	0	0	0.9713
		1	0	0	0	[illegible]
					Average	0.9806

Daub 3 at Scale of 3						
On or Off Slope 2 @ SP=100						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	22	1	1	0	0	0.9744
		1	0	0	1	0.9571
		1	0	0	0	0.9674
		1	1	1	1	0.9703
		1	0	0	0	0.9607
					Average	0.9688

Haar at Scale of 3						
On or Off Slope 2 @ SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	22.0000	1	0	0	0	0.9710
		1	[illegible]	0	1	0.9650
		1	0	1	0	0.9620
		1	0	[illegible]	0	0.9670
		1	[illegible]	0	1	0.9570
					Average	0.9668

Daub 3 at Scale of 3						
On or Off Slope 2 @ SP=80						
TC	NTC	a1	d1	d2	d3	Sepindex
2	22	1	0	1	0	0.9703
		1	1	1	1	0.9579
		1	0	0	0	0.9615
		1	0	0	0	0.9567
		1	0	0	0	0.9650
					Average	0.9647

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =30

Haar at scale 3 — On or Off Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	0	0	0	0.9852
		1	0	0	0	0.9857
		1	0	0	0	0.9864
		1	0	0	1	0.9841
		1	0	0	0	0.9874
					Average	0.9862

Haar at scale 3 — On or Off Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sep.index
2	30	1	0	1	1	0.9871
		1	1	1	1	[illegible]
		1	0	1	0	0.9799
		1	0	1	0	[illegible]
		1	1	0	0	[illegible]
					Average	0.9799

Haar at scale 3 — On or Off Slope 1 @ SP=60

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	0	0	1	0.9815
		1	1	1	1	0.9794
		1	0	1	0	0.9799
		1	0	1	0	0.9728
		1	1	0	0	0.9769
					Average	0.9799

Daub 3 at Scale of 3 — On or Off Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	0	0	1	0.9852
		1	0	0	0	0.9857
		1	0	0	0	0.9844
		1	0	0	1	0.9841
		1	0	0	0	0.9876
					Average	0.9862

Daub 3 at Scale of 3 — On or Off Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	0	0	0	0.9826
		1	1	0	1	0.9758
		1	1	1	0	0.9809
		1	1	0	1	0.9790
		1	0	1	1	0.9751
					Average	0.9787

Daub 3 at Scale of 3 — On or Off Slope 1 @ SP=60

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	0	0	0	0.9815
		1	0	0	1	0.9757
		1	0	0	1	0.9762
		1	1	1	1	0.9772
		1	0	1	1	0.9757
					Average	0.9752

Haar at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	1	1	0	[illegible]
		1	0	1	0	0.9804
		1	1	1	0	[illegible]
		1	0	0	0	[illegible]
		1	1	0	0	[illegible]
					Average	0.9816

Haar at Scale of 3 — On or Off Slope 2 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	1	1	1	0.9794
		1	1	0	1	[illegible]
		1	1	1	0	[illegible]
		0	1	0	0	[illegible]
		1	0	1	1	0.9762
					Average	0.9784

Haar at Scale of 3 — On or Off Slope 2 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30.0000	1	0	1	0	[illegible]
		1	0	1	0	[illegible]
		1	0	1	1	[illegible]
		1	1	0	0	[illegible]
		1	1	0	1	0.9740
					Average	0.9762

Daub 3 at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30	1	0	1	0	0.9820
	30	1	1	0	0	0.9806
	30	1	0	0	1	0.9794
	30	1	1	0	0	0.9851
	30	1	1	0	0	0.9820
					Average	0.9820

Daub 3 at Scale of 3 — On or Off Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sep.index
2.0000	30	1	0	0	0	0.9782
		1	0	0	0	0.9772
		1	1	0	1	0.9794
		1	0	0	1	0.9778
		1	0	0	0	0.9751
					Average	0.9777

Daub 3 at Scale of 3 — On or Off Slope 2 @ SP=[illegible]

TC	NTC	a1	d1	d2	d3	Sep.index
2	30	1	1	1	1	0.9778
		1	0	1	0	0.9723
		1	1	0	1	0.9775
		1	1	0	1	0.9794
		1	1	0	1	0.9733
					Average	0.9761

Sparse features with Wavelets on On or Off representations

Target Class= 2 Non Target Class =31

Haar at Scale of 3 — On or Off Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31.0000	1	0	0	0	1.0000
		1	0	0	0	0.9995
		1	0	0	0	0.9995
		1	0	0	0	0.9995
		1	1	0	0	1.0000
					Average	0.9997

Daub 3 at Scale of 3 — On or Off Slope 1 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31.0000	1	0	0	0	1.0000
		1	0	0	0	0.9995
		1	0	0	0	0.9995
		1	0	0	0	0.9995
		1	1	0	0	1.0000
					Average	0.9997

Haar at Scale of 3 — On or Off Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2	31	1	0	0	0	0.9622
		1	0	0	0	0.9548
		1	0	0	0	0.9636
		1	0	0	0	0.9555
		1	0	0	0	0.9586
					Average	0.9597

Daub 3 at Scale of 3 — On or Off Slope 1 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31.0000	1	0	0	0	0.9710
		1	0	0	0	0.9636
		1	0	0	0	0.9742
		1	0	0	0	0.9656
		1	0	0	0	0.9654
					Average	0.9682

Haar at Scale of 3 — On or Off Slope 1 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31.0000	1	0	0	0	0.9622
		1	0	0	0	0.9645
		1	0	0	0	0.9606
		1	0	0	0	0.9585
		1	0	0	0	0.9595
					Average	0.9597

Daub 3 at Scale of 3 — On or Off Slope 1 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31.0000	1	0	0	0	0.9550
		1	0	0	0	0.9447
		1	0	0	0	0.9576
		1	0	0	0	0.9505
		1	0	0	0	0.9500
					Average	0.9516

Haar at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31.0000	1	0	1	0	0.9949
		1	0	1	0	0.9975
		1	0	1	0	0.9925
		1	0	1	0	0.9917
		1	0	1	0	0.9891
					Average	0.9920

Daub 3 at Scale of 3 — On or Off Slope 2 @ No SP

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31	1	0	1	0	0.9945
	31	1	0	1	0	0.9976
	31	1	0	1	0	0.9959
	31	1	0	1	0	0.9922
	31	1	0	1	0	0.9962
					Average	0.9948

Haar at Scale of 3 — On or Off Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31.0000	1	0	0	0	[illegible]
		1	0	0	0	0.9960
		1	0	0	0	[illegible]
		1	0	0	0	0.9960
		1	0	0	0	[illegible]
					Average	0.9960

Daub 3 at Scale of 3 — On or Off Slope 2 @ SP=100

TC	NTC	a1	d1	d2	d3	Sepindex
2.0000	31	1	0	0	0	0.9949
		1	0	0	0	0.9936
		1	0	0	0	0.9945
		1	0	0	0	0.9940
		1	0	0	1	0.9926
					Average	0.9939

Haar at Scale of 3 — On or Off Slope 2 @ SP = 80

TC	NTC	a1	d1	d2	d3	Sepindex
2	31	1	0	0	0	0.9926
		1	0	0	0	0.9917
		1	0	0	0	0.9940
		1	0	0	0	0.9940
		1	0	0	0	0.9912
					Average	0.9927

Daub 3 at Scale of 3 — On or Off Slope 2 @ SP=80

TC	NTC	a1	d1	d2	d3	Sepindex
2	31	1	0	0	0	0.9912
		1	0	0	0	0.9849
		1	0	0	0	0.9922
		1	0	0	0	0.9917
		1	0	0	0	0.9903
					Average	0.9903

Appendix G

Sparse features with Wavelets on Absorption representation

Target Class= 1 vs Non Target Classes
Haar & Daubechies 3 Wavelets

Haar at scale of 3 — Absorption Representation

TC	NTC	a1	d1	d2	d3	Sepindex
1	5	1	1	1	0	0.9634
		1	1	1	0	0.9605
		0	1	0	0	0.9758
		1	0	1	1	0.9720
		0	1	0	0	0.9726
					Average	0.9707

TC	NTC	a1	d1	d2	d3	Sepindex
1	15	0	1	0	1	0.9508
		1	1	0	0	0.9782
		1	0	0	1	0.9508
		0	1	1	0	0.9504
		0	0	0	1	0.9615
					Average	0.9719

TC	NTC	a1	d1	d2	d3	Sepindex
1	17	1	1	1	0	0.9960
		1	1	1	0	0.9546
		1	0	1	1	0.9572
		1	0	1	0	0.9484
		1	1	0	1	0.9640
					Average	0.9893

TC	NTC	a1	d1	d2	d3	Sepindex
1	20	1	1	1	1	0.9965
		1	0	0	1	0.9580
		1	0	1	1	0.9605
		1	0	1	1	0.9900
		1	1	0	1	0.9820
					Average	0.9902

TC	NTC	a1	d1	d2	d3	Sepindex
1	21	1	0	0	1	0.9513
		0	1	1	0	0.9615
		0	0	1	1	0.9471
		1	1	1	1	0.9423
		0	0	1	0	0.9760
					Average	0.9558

TC	NTC	a1	d1	d2	d3	Sepindex
1	26	0	0	1	1	1.0000
		1	0	1	0	1.0000
		0	1	0	1	1.0000
		0	1	1		1.0000
		1	0	1	1	1.0000
					Average	1.0000

TC	NTC	a1	d1	d2	d3	Sepindex
1	31	0	1	1	1	1.0000
		1	1	1	0	1.0000
		0	0	1	1	1.0000
		0	1	1	1	1.0000
		1	0	1	0	1.0000
					Average	1.0000

TC	NTC	a1	d1	d2	d3	Sepindex
1	15	1	1	1	0	0.9533
		1	1	1	1	0.9749
		1	0	0	0	0.9681
		1	0	0	1	0.9743
		1	1	0	1	0.9916
					Average	0.9816

Daub 3 at scale of 3 — Absorption Representation

TC	NTC	a1	d1	d2	d3	Sepindex
1.0000	5	1	0	1	0	0.9634
		1	0	1	1	0.9655
		1	0	0	0	0.9726
		1	1	0	0	0.9720
		1	1	0	1	0.9729
					Average	0.9701

TC	NTC	a1	d1	d2	d3	Sepindex
1	15	1	0	1	1	0.9663
		1	0	0	0	0.9760
		1	0	1	0	0.9808
		1	0	0	0	0.9808
		1	1	1	0	0.9567
					Average	0.9721

TC	NTC	a1	d1	d2	d3	Sepindex
1	17	1	1	1	0	0.9968
		1	0	0	0	0.9840
		1	0	1	0	0.9872
		1	1	0	1	0.9863
		1	1	0	0	0.9840
					Average	0.9898

TC	NTC	a1	d1	d2	d3	Sepindex
1	20	1	1	0	1	0.9965
		1	1	1	0	0.9895
		1	0	0	1	0.9925
		1	0	0	1	0.9933
		1	1	0	0	0.9820
					Average	0.9902

TC	NTC	a1	d1	d2	d3	Sepindex
1	21	1	0		1	0.9519
		1	0	0	1	0.9471
		1	1	1	0	0.9471
		1	0	0	1	0.9423
		1	0		1	0.9567
					Average	0.9495

TC	NTC	a1	d1	d2	d3	Sepindex
1	26	1	0	1	0	1.0000
		1	1	1	0	1.0000
		1	1	1	0	1.0000
		1	1	0	0	1.0000
		1	1	1	0	1.0000
					Average	1.0000

TC	NTC	a1	d1	d2	d3	Sepindex
1	31	1	1	0	1	1.0000
		1	0	1	1	1.0000
		1	1	0	0	1.0000
		1	0	0	0	1.0000
		1	0	0	0	1.0000
					Average	1.0000

TC	NTC	a1	d1	d2	d3	Sepindex
1	15	1	0	0	0	0.9835
		1	1	1	1	0.9749
		1	0	0	1	0.9893
		1	1	1	1	0.9743
		1	1	0	0	0.9815
					Average	0.9816

Target Class 1 vs Non Target Classes using the Absorption Representation

Target Class= 2 vs Non Target Classes
Absorption Representation
Haar & Daubechies 3 Wavelets

[illegible]

Absorption Representation

TC	NTC	a1	d1	d2	d3	Sepindex
2	10	0	0	1	0	0.9979
		0	0	1	1	0.9965
		0	0	1	0	0.9963
		0	0	1	0	0.9773
		0	1	0	1	0.9464
					Average	0.9878

TC	NTC	a1	d1	d2	d3	Sepindex
2	12	0	0	1	0	0.9951
		0	0	1	0	0.9978
		0	0	1	0	0.9962
		0	1	0	0	0.9957
		0	0	1	0	0.9954
					Average	0.9966

TC	NTC	a1	d1	d2	d3	Sepindex
2	13	0	0	1	0	0.9902
		0	0	1	0	0.9957
		0	1	0	0	0.9968
		0	0	1	0	0.9914
		0	1	1	1	0.9952
					Average	0.9951

TC	NTC	a1	d1	d2	d3	Sepindex
2	18	1	1	0	0	0.9952
		0	1	0	1	0.9927
		0	0	1	0	0.9837
		0	1	0	1	0.9863
		0	1	1	1	0.9927
					Average	0.9941

TC	NTC	a1	d1	d2	d3	Sepindex
2	21	1	0	1	1	0.9981
		1	1	1	1	0.9975
		1	0	0	0	0.9984
		1	0	0	0	0.9985
		1	0	0	1	0.9979
					Average	0.9954

TC	NTC	a1	d1	d2	d3	Sep index
2	22	0	0	1	1	0.9527
		0	0	1	0	0.9911
		0	0	1	0	0.9898
		0	0	1	0	0.9922
		0	0	1	0	0.9911
					Average	0.9516

TC	NTC	a1	d1	d2	d3	Sep index
2	30	0	1	0	1	0.9930
		0	1	0	0	0.9908
		0	1	0	0	0.9902
		0	0	1	0	0.9947
		0	1	0	0	0.9958
					Average	0.9954

TC	NTC	a1	d1	d2	d3	Sep index
2	31	1	1	1	1	1.0000
		1	0	0	1	0.9996
		1	0	0	1	0.9995
		1	0	0	0	1.0000
		1	1		0	0.9996
					Average	0.9957

[illegible]

Absorption Representation

TC	NTC	a1	d1	d2	d3	Sepindex
2	10	1	1	0	1	0.9910
		1	1	1	1	0.9981
		1	1	0	0	0.9951
		1	1	1	0	0.9967
		1	0	0	1	0.9572
					Average	0.9886

TC	NTC	a1	d1	d2	d3	Sepindex
2	12	0	1	1	0	0.9676
		0	1	0	0	0.9857
		0	1	0	0	0.9860
		0	1	0	0	0.9908
		0	1	0	0	0.9919
					Average	0.9882

TC	NTC	a1	d1	d2	d3	Sepindex
2	13	0	1	0	0	0.9852
		0	1	0	0	0.9908
		1	1	1	0	0.9914
		0	1	1	0	0.9434
		0	1	1	0	0.9541
					Average	0.9896

TC	NTC	a1	d1	d2	d3	Sepindex
2	18	1	1	1	0	0.9944
		1	1	0	1	0.9890
		1	1	0	1	0.9850
		1	0	1	1	0.9416
		1	0	0	0	0.9901
					Average	0.9907

TC	NTC	a1	d1	d2	d3	Sepindex
2	21	1	1	0	0	0.9604
		1	0	0	0	0.9976
		1	0	0	0	0.9984
		1	1	0	0	0.9985
		1	1		0	0.9975
					Average	0.9984

TC	NTC	a1	d1	d2	d3	Sep index
2	22	1	1	1	0	0.9901
		1	1	0	1	0.9654
		1	1	0	1	0.9550
		1	0	0	0	0.9950
		1	1	0	1	0.9575
					Average	0.9630

TC	NTC	a1	d1	d2	d3	Sep index
2	30	1	0	1	0	0.9634
		1	1	1	1	0.9854
		1	0	0	0	0.9829
		1	1	0	1	0.9904
		1	1	0	1	0.9868
					Average	0.9886

TC	NTC	a1	d1	d2	d3	Sep index
2	31	1	0	1	0	1.0000
		1	1	0	0	0.9995
		1	1	1	1	0.9995
		1	0	0	0	1.0000
		1	0	1	1	0.9996
					Average	0.9987

Target Class 2 vs Non Target Classes using the Absorption Representation

Target Class= 3 vs Non Target Classes
Absorption Representation
Haar & Daubechies 3 Wavelets

Haar at scale of 3

Absorption Representation

TC	NTC	a1	d1	d2	d3	SepIndex
3	5	0	0	1	1	0.955
		0	0	1	1	0.960
		0	0	0	1	0.958
		0	0	1	0	0.960
		0	0	1	1	0.968
					Average	0.964

TC	NTC	a1	d1	d2	d3	SepIndex
3	7	1	0	0	1	0.988
		2	1	1	1	0.996
		3	0	1	1	0.995
		4	0	0	0	0.995
		5	0	0	0	0.996
					Average	0.998

TC	NTC	a1	d1	d2	d3	SepIndex
3	14	0	1	1	1	0.994
		0	0	1	0	0.996
		1	1	1	1	0.996
		0	0	1	0	0.994
		0	0	1	0	0.996
					Average	0.995

TC	NTC	a1	d1	d2	d3	SepIndex
3	15	0	1	1	1	0.982
		1	0	1	0	0.990
		1	0	0	1	0.982
		0	1	0	0	0.985
		1	0	0	1	0.987
					Average	0.986

TC	NTC	a1	d1	d2	d3	SepIndex
3	16	0	1	1	0	0.965
		0	0	1	0	0.965
		0	1	1	1	0.968
		0	0	1	0	0.968
		0	0	1	0	0.968
					Average	0.991

TC	NTC	a1	d1	d2	d3	SepIndex
3	23	0	0	1	0	0.940
		1	0	1	1	0.978
		0	0	1	0	0.846
		1	0	1	0	0.981
		1	1	0	0	0.990
					Average	0.989

TC	NTC	a1	d1	d2	d3	SepIndex
3	26	1	0	0	0	1.000
		[illegible]	1	0	0	0.994
		1	1	1	1	1.000
		1	0	0	1	1.000
		1	1	0	1	1.000
					Average	1.000

TC	NTC	a1	d1	d2	d3	SepIndex
3	27	0	1	0	0	0.990
		0	1	1	0	0.977
		0	1	0	1	0.996
		0	1	0	0	0.981
		0	1	0	0	0.975
					Average	0.983

Daub 3 at scale of 3

Absorption Representation

TC	NTC	a1	d1	d2	d3	SepIndex
3	5	1	1	0	1	[illegible]
		1	1	1		0.980
		1	1	0	0	[illegible]
		1	1	1	0	[illegible]
		1	0	0	1	0.979
					Average	0.981

TC	NTC	a1	d1	d2	d3	SepIndex
3	7	1	1	1	1	0.998
		1	0	0	0	0.998
		1	0	1	1	0.993
		1	0	0	0	[illegible]
		1	1	0	1	0.995
					Average	0.998

TC	NTC	a1	d1	d2	d3	SepIndex
3	14	1	0	0	0	0.992
		1	1	0	1	0.994
		1	1	1	0	0.996
		1	1	1	[illegible]	0.992
		1	1	0	0	0.994
					Average	0.993

TC	NTC	a1	d1	d2	d3	SepIndex
3	15	1	1	1	0	0.982
		1	1	0	1	0.980
		1	1	0	1	0.982
		1	0	1	1	0.980
		1	0	0	0	0.987
					Average	0.984

TC	NTC	a1	d1	d2	d3	SepIndex
3	8	1	1	0	0	0.994
		1	0	0	0	0.992
		1	0	0	0	0.994
		1	1	0	0	0.989
		1	1	1	0	0.992
					Average	0.991

TC	NTC	a1	d1	d2	d3	SepIndex
3	23	1		1	0	0.940
		1	1	0	1	0.978
		1	1	0	1	0.851
		1	0	0	0	0.983
		1	1	0	1	0.990
					Average	0.986

TC	NTC	a1	d1	d2	d3	SepIndex
3	26	1	0	1	0	1.000
		1	1	1	[illegible]	0.994
		1	0	0	0	1.000
		1	1	0	1	1.000
		1	1	0	1	1.000
					Average	1.000

TC	NTC	a1	d1	d2	d3	SepIndex
3	28	1	0	1	0	0.903
		1	1	0	0	0.971
		1	1	1	1	0.985
		1	0	0	0	0.950
		1	0	1	1	0.965
					Average	0.975

Target Class 3 vs Non Target Classes using the Absorption Representation