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UNIVERSITY OF CAPE TOWN

DEPARTMENT OF EDUCATION

**AN INVESTIGATION INTO HOW GRADE 12
STUDENTS UNDERSTAND AND SOLVE
GEOMETRIC PROBLEMS.**

**A minor dissertation submitted in partial
fulfillment of the requirements for the
award of the degree of**

MASTER OF EDUCATION

by

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(Student number: CRNCOR001)

SEPTEMBER 2001

DECLARATION

I declare that this work has not been previously submitted in whole, or in part, for the award of any degree.

It is my own work. Each significant contribution to, and quotation in, this dissertation from the work, or works, of other people has been attributed, and has been cited and referenced.

Signature: _____

Date: _____

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This dissertation is dedicated to my mother who has always encouraged me to study.

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ABSTRACT

This study investigates how grade 12 students understand and solve geometric problems. A review of the literature on “how students learn and understand geometry” is used to develop a conceptual framework for discussion. The framework is used to assess students’ level of understanding and to analyse their difficulties in solving geometric problems.

The study was conducted at four low achieving schools in mathematics (based on student performance in the South African Senior Certificate Examination). It involved 267 students across the schools. The students’ level of understanding was assessed through the use of two tests. These tests were designed to cover 80% of the grade 11 syllabus and involved the testing of a terminology framework (test 1) and problem solving exercises (test 2). Test 1 included 10 items where students were asked to complete statements, as well as 9 items where students were asked to write down properties from given sketches. Test 2 included 16 items of true or false responses.

An in-depth analysis of 21 students, who produced a score of more than 70% in test 1, provided greater insight into how students learn and engage in problem solving activity. The 21 students were interviewed and questioned in connection with their performance in test 2.

The results of the study suggest that the majority of students do not possess a theoretical framework, consequently showing that they are unable to engage in problem solving. However, the fact that some students possessed a theoretical framework, did not necessarily mean a better performance in problem solving. The

findings suggest that students' lacked a strategy for applying their theory. They were looking for 'prototypes', as presented by their teacher and were unable to engage with the problems where the orientation of the diagrams was different.

The overall findings of the study revealed that at least 75% of the students had low levels of understanding geometry. There will be very little improvement in geometry performance, unless teaching practices change and teachers take into consideration the difficulties that students experience. The potential value of the results in this research for use in geometry classrooms is discussed.

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CHAPTER 1

INTRODUCTION

1.1. Background

“Sir, I hate geometry”

These were words so commonly expressed by my grade 10 students virtually throughout my 19 years of teaching senior mathematics. This expression, I believe, is also common to teachers in a vast number of schools. This hate for geometry manifested itself in many different ways, including, absolute disinterest in geometry, avoiding problem-solving activities and just poor results in geometry tests and examinations. As a sub-examiner for ten years and examiner for 3 years in the Senior Certificate Examination, I have witnessed the poor performance of students in the geometry section of the South African school-leaving matriculation examination. It was sad to see the errors that students were guilty of. Simply reading through the examiner’s reports at the end of each year provided insight into this problem.

Was the above related to poor teaching? Was it related to the way students learned or understood geometry? Was it a combination of the two? I discovered that the majority of my students came through a very traditional mode of teaching. They were undoubtedly passive learners and were not encouraged to engage in a ‘hands-on’ approach. The teacher, according to my students, would simply write on the board: Theorem 1, write down the statement, draw the sketch, write down what was given, what was required to prove and add any construction lines. This presentation was common in all textbooks (or should I rather say, reluctantly, most, since all the text

books we used at school had the same approach). After the third or fourth theorem was completed in exactly the same way, students who had already switched-off, were expected to apply those theorems to an exercise.

My response to this was to allow students to explore and investigate theorems through the use of tracing paper or cutting out angles. This had the desired effect in that students suddenly became more interested and more involved in their own learning and understanding of geometry. This hands-on approach really rekindled their love for geometry. However, despite their enthusiasm, knowledge of theorems and better results in tests, I realised that the students still battled with some problem-solving activities. Unscientifically, I looked at issues of strategies for problem solving, development of schemas (I may have used different terminology for this development) and effects of prototypes. I say unscientifically, simply because I would think of something and react or respond accordingly, purely based on my “gut-feeling” and not on any statistics. This kindled a desire to do some formal research in geometry, especially circle geometry, because of the value attached to it in the matriculation examination. I always asked myself the question, how do my students think, learn or understand geometry? Sometimes they responded so positively and seemed to understand concepts that I was taken aback by their poor responses in a test or examination.

I left the teaching profession with most of the above thoughts mulling around in my head and entered into a new domain by working predominantly with teachers in NGO project work. Immediately, I discovered that the teachers I worked with had enormous

problems with the teaching and learning of geometry. This really ignited my interest to go further than just think about doing some formal research in geometry.

1.2. Intention

The difficulties that the teachers and students experienced in my project work, based on improving matriculation results, was the injection that I needed to explore how students learn and understand geometry. It is my intention in this study to investigate possible underlying causes or reasons why some students actually hate geometry, which in my experience as a mathematics teacher is fairly common to many students in different contexts. I have already mentioned that students generally perform poorly in the geometry section of the matriculation syllabus and at some schools geometry is either totally neglected or in some cases not taught at all.

I was involved in a Secondary Schools Partnership Project located in the Schools Development Unit at the University of Cape Town. This was an 18-month pilot study (from July 1998 to the end of 1999) with its aim to develop a model for improving matriculation results at four low-achieving schools in mathematics and physical science. I have included the mathematics pass rates for the four schools to provide some baseline information regarding performances of grade 12 students for the period 1996 to 1999. Table 1.1 below provides information regarding the number of students and the pass rates per school for mathematics standard grade. The number of students taking mathematics on the higher-grade is minimal. Only 33 students wrote the higher-grade mathematics examination for this period across the four schools with limited success. Students were not encouraged to do mathematics on the higher-grade.

TABLE 1.1: Number of students and pass rates per school for 1996 – 1999

SCHOOL	Number of students					Pass Rates in %			
	96	97	98	99		96	97	98	99
PORTLAND	98	112	144	88		25.5	21.4	52	72.7
GLENDALE	30	62	50	32		13.3	4.8	34	40.6
ESANGWENI	59	49	61	40		11.8	8.1	15	12.5
SINETHEMBA	41	37	61	53		21.9	35.1	27	49

The schools were basically included in the Partnership Project because of the poor pass rates in 1996 and 1997. The above pass rates reflect the overall results for mathematics and includes paper 1 (Algebra, Calculus and Graphs) and paper 2 (Euclidean Geometry, Analytical Geometry and Trigonometry) results.

I have also included the following statistics merely to emphasise my concern for the teaching and learning of geometry. The table below (table 1.2) reflects the average marks for questions 6 and 7 (the two geometry questions) in the grade 12 September 1999 examination at the four schools that were included in this study. The four schools wrote a common examination paper for their trial (or mock) examination.

TABLE 1.2: Average Marks per question in geometry examination

school s	Average mark for question 6. Total mark (25)	Average Mark for question 7. Total .mark (25)	total (50)	number of students
Esangweni	2,7	1,2	3,9	28
Sinethemba	4,8	3,9	8,7	35
Portland	6,7	5	11,7	88
Glendale	5,1	0	5,1	55

Question 6 included the previous year's grade 11 geometry, which involved the following sections: subtended angles of a circle, tangents and cyclic quadrilaterals (sections that form the core of this investigation). One formal proof was included for 7 marks and two problem-solving activities for 18 marks. The average mark for this question across the four schools was 4.8 or 19%. This result raised alarm bells for me and I wanted to try to find reasons for these poor results.

Question 7 included the grade 12 geometry, namely, proportional theorems and similarity (sections not included in this investigation). One formal proof was included for 7 marks and two problem-solving activities for 18 marks.

It must be noted that the geometry component counts approximately 34% for standard grade and 35% for higher grade in the matriculation examination paper 2. The grade 11 geometry plays an important scaffolding role in the problem solving activities for grade 12. For this reason I decided that the focus of this investigation would be based on grade 11 geometry. More specifically it would be based on the sections mentioned earlier: subtended angles in a circle, cyclic quadrilaterals and tangents.

1.3. CONTEXT

I found the following statement by Clements and Battista very useful in terms of the difficulties in geometry in the United States. It made me realise that it is not only a problem in South Africa. Clements and Battista (1992, pg 420) claims that:

According to extensive evaluations of mathematics learning ,
elementary and middle school students in the United States are

failing basic geometric concepts and geometric problem solving ;
they are woefully underprepared for the study of more
sophisticated geometric concepts and proof.

There are similar trends in South Africa of students being woefully underprepared or in some cases wholly unprepared. This could be ascribed to a number of factors that include the following: overcrowded classrooms, a shortage of qualified teachers, home environment of the majority of our students, inadequate facilities, gangsterism or peer pressure, disruption of school time, student motivation and a lack of teaching materials. These factors, amongst others, contribute toward low levels of attainment in mathematics generally and geometry particularly. The fact that South African pupils were declared “worst in maths, science” (Cape Argus, 7 December 2000) and “dunces of Africa” (Sunday Times, 16 July 2000) places, to some extent, this study into context. Some teachers have rated geometry as the most difficult mathematical topic to teach and difficult for students to learn. An attempt to answer the following question:

Why is it that so many students struggle to learn geometry?

lies at the heart of this investigation.

A number of reasons have been cited for the low achievement in geometry, one of which refers to the teaching of geometry. Instruction is assumed to have an impact on students' learning, consequently to try to enhance conceptual understanding or improving achievement in geometry, we need to improve instructional practices. It has been claimed that an enhancement of teachers' content knowledge will influence instructional practice (Swafford, Jones, & Thornton, 1997).

Some mathematicians also claim that much learning of geometric concepts has been rote (Clements, & Battista, 1992). Mayberry (1983) suggests that some properties, class inclusions, relationships, and implications are often not perceived or understood by students. The South African geometry curriculum in lower grades is based on students recognising and naming geometric shapes, writing the proper symbolism for simple concepts, developing skill with measurement and construction tools such as a compass and protractor (in some cases these instruments are not even used) and using formulas. It seems like a 'hodgepodge' of unrelated concepts and could be the main reason for an impoverished geometry curriculum (Porter, 1989).

Porter (1989, pg. 11) also highlights, what has already been mentioned, an interesting phenomenon that some teachers spent "virtually no time teaching geometry" and geometry was merely taught for 'exposure'. I have encountered similar experiences with teachers who avoid teaching geometry. Students are only taught algebra, trigonometry and co-ordinate geometry. Many teachers rated the opportunity (in terms of time spent) to learn geometry much lower than the opportunity to learn any other topic and the traditional emphasis has been placed on formal proof, despite the inability or lack of readiness of students to deal with such proofs (Clements and Battista, 1992). Apart from the fact that some teachers are unqualified and have difficulty teaching geometry, they are unaware of the difficulties that students experience.

I was specifically interested in a research area that would in some way provide solutions that would assist in the teaching of geometry or provide a framework for further research in geometry classrooms.

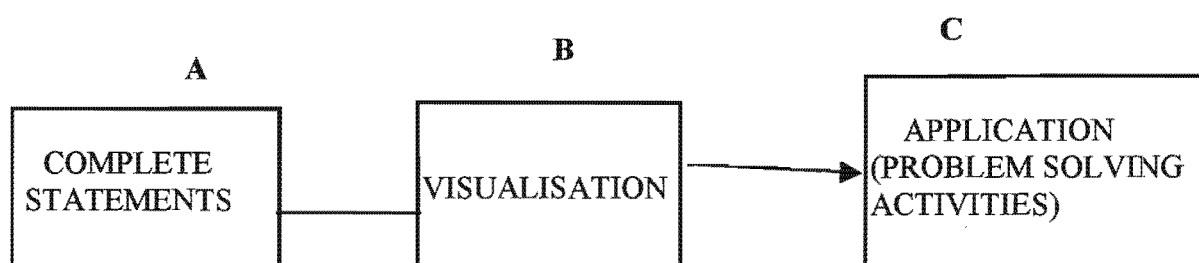
1.4 Rationale and Title

The focus of this study is on students and how they understand and solve geometric problems. It was difficult for me to ease the tension between skills-development and conceptual development. Should teachers spend more or less time on conceptual development? How do we then assess conceptual development? Is it the ability to articulate a concept? Is it the ability to apply the concept in a problem-solving activity? Does mere regurgitation of definitions or theorems reflect a lack of understanding?

It must be noted that the matriculation examination requires that students have a clear understanding of theorems and that they engage in abstract thought. It is expected that students know their definitions and theorems. They are required to write down the definitions and theorems in words. The problem-solving activities are based on a given sketch and students have to answer questions based on these sketches by applying their understanding of the theorems and definitions.

There are many teachers who believe that once these definitions and theorems are well known, then their students will do well in problem solving activities. It was precisely these thoughts that prompted the use of the following scheme as a model (table 1.3) in this investigation.

TABLE 1.3: Rationale



The above table represents the rationale for this investigation, which involves three crucial areas in the understanding and learning of geometry. The model presented in table 1.3 above is based on what grade 12 students are expected to do and highlights the following:

- Block A represents the ability of students to articulate their definitions and theorems in words, by completing given statements.
- Block B represents the ability of students to visualise the definitions or theorems from a given sketch.
- A and B combined provides a geometrical register from which students operate. By geometrical register I refer to a definitional or axiomatic framework. This register contains the terminology needed to solve problems. This register is assembled in two ways, namely, through rote (memorisation) or understanding or a mixture of both.
- Block C represents the ability of students to apply their geometrical register to problem solving activities.

The aim of this study was to examine and focus on the following assumption:

- If A and B are not in place (or if the geometric register is non-existent), then students will struggle with C, i.e. they will have difficulty in problem-solving activities.
- If A and B are in place (or the geometric register exists), then there exist at least two possibilities, which form the core of this investigation.

1. Students should perform well in C.

2. Students may struggle with C, because the orientation of the sketches are different to those that occur in B. It is at this point that the role and impact of prototypes may be the cause or attributing factor of a low-achievement in C. By prototypes I refer to sketches that are used to introduce a theorem and its that visual impression (i.e. a particular orientation) of the theorem that students use as the only indicator for the application of the theorem.

1.5. Title and Aim(s)

Using the framework presented in table 1.3 the aim(s) of the study can be formulated in the following way:

1. To look at student performances in geometry tests and to ascertain whether the students have a basic knowledge of circle geometry. This is an assessment of the geometric register and will provide an answer to the question: **Do students have the ability to complete statements in words and to visualise these statements through given sketches?**

2. To compare results of test items across the different tests, by noting differences between the visual and the writing down of statements.
3. To look at examples of items in students' geometrical register, by highlighting issues of language and misconceptions.
4. To look at the role of schemas and prototypes in the students' geometrical register and its impact on problem solving.

The focus of this study is to investigate how grade 12 students learn and understand circle geometry. I have used the following title for this study:

***AN INVESTIGATION INTO HOW GRADE 12 STUDENTS
UNDERSTAND AND SOLVE GEOMETRIC PROBLEMS.***

1.6. Outline of Contents

Chapter 1 provides the background to this study. I use my experience as a mathematics teacher, examiner and teacher trainer to convey issues around the teaching of geometry. The chapter also refers to the difficulties that teachers' experience in teaching mathematics, in general and geometry specifically. It includes aspects of mathematics teaching in the South African context. The rationale and aims are used to introduce the title of the study.

Chapter 2 provides an understanding of research in the teaching of geometry. Issues around the reasons for the development of an effective geometric register are noted. The impact of prototypical teaching is highlighted. The chapter also depicts the

development of geometric thinking by referring to Piaget and Inhelder, the van Hiele and others.

Chapter 3 describes the research design and defines the aims and focus of the study. It provides a detailed procedure for addressing the aims and explains the data collection techniques and process.

Chapter 4 provides an analysis of the tests and interviews. A detailed item analysis of test 1 and test 2 is given. There is a discussion on the impact of a geometrical register, or lack thereof, in problem solving. Striking features arising out of the interviews provide insight into how students go about solving problems and the difficulties that they experience.

Chapter 5 concludes with a summary of results stemming from the research. It then provides recommendations arising from the study and recommendations for future research.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

This chapter provides an overall perspective of the teaching and learning of geometry. The literature chosen helps to address issues which will inform this study and contextualise the relevance of this study. A special emphasis is placed on the geometric register, the impact of prototypes and the development of geometric thinking.

2.2. Why a Geometric Register?

In all school subjects, but especially mathematics, there is a dire need for teachers as well as students to see connections in different facets of life. Meaning emerges from context and connectedness. Without context, nothing makes sense. From a sociolinguistic perspective, the full meaning of the text is achieved only by understanding its context (Atweh et al, 1998).

Mathematics teaching poses many challenges for students and teachers. Mathematics education attempts to understand how mathematics is created, taught and learned effectively. To achieve this aim the teacher should pay attention to linguistics and should include the two essential ingredients of mathematics, namely, people and communication (Durkin, 1989). The role of language is crucial to the development of mathematics education, as noted by, “mathematics education begins and proceeds in

language, it advances and stumbles because of language, and its outcomes are often assessed in language” (Durkin, 1989).

An important function of language is to transmit meaning. A major problem in the use of language in mathematics education is that meanings conveyed are sometimes very complex and the words we use tend to have alternative meanings (also a problem in our multilingual classes). There are many mathematical words that have multiple meanings and research has provided evidence that students have difficulty interpreting the words as teachers intend them (Durkin, 1989). It is possible to list ambiguous words used in mathematical text. This raises an important question: Should mathematics be taught in ‘mother tongue’ languages? This is an on going debate in South Africa where Xhosa, Zulu and other Language speakers, especially at secondary level, receive mathematics instruction in English.

Confusion occurring as a result of differing linguistic interpretations is also highlighted by Pimm (1987), where teachers use a mathematical ‘dialect’ and students use non-mathematical meanings in a mathematical context. In this way we have a development of a mathematical register, which Halliday (1975, pg 65).describes as:

A set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. ... We can refer to a ‘mathematics register’, in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), ... We should not think of a mathematical register as consisting solely of terminology, or of the development of a register as simply a process of adding new words.

A subset of the mathematical register is the geometrical register. This 'geometrical' register would therefore include the use of technical terms, certain mathematical phrases, modes of arguing, coining of new terms, the use of specialist terms, as well as the visualisation of objects.

The aim of this register is to describe more precisely the relation between mathematics (geometry in particular) and English, having to do with the social usage of particular words and expressions and initiate ways of talking, but especially providing ways of meaning (Pimm, 1987). The register provides mechanisms or ways to create meaning, that is, ways in which teachers take seriously the function of transmitting this register and the degree of awareness of the existence of the register by teachers and students (Pimm, 1987). I see this register as the bridge to understanding especially in multilingual classrooms.

The creation of meaning within this register can occur in a number of different ways. In the area of proof, which emphasises the role of van Hiele's level 3, students develop reasoning ability, as well as the ability to understand formal definitions (de Villiers, 1987, 1994). De Villiers (1994) refers to the usefulness in the negotiation of linguistic meaning and functional meaning for hierarchical classification as depicted by the van Hiele model. He also found that students could classify hierarchically, but the students did not find it necessary to do so. This could be a consequence of the traditional approach to geometry teaching (de Villiers, 1994). This traditional approach places emphasis on skills and content development and hardly any time on conceptual development (Fuys & Liebov, 1997).

The involvement of students in the learning process cannot be underestimated. To increase “understanding of geometric definitions and the concepts to which they relate, teachers should involve students in the process of defining geometric concepts” (de Villiers, 1998, pg 2). De Villiers (1998) suggest that teachers have difficulty in coping with student responses or ideas in providing their own definitions, hence they avoid problematic interactions with students. De Villiers (1990) distinguished between the different functions of proof, by allowing students to record their own conclusions and conjectures. This approach led to the first stages of proof.

Within this geometric register, misconceptions can easily occur. Finzer and Bennet (1995) explored the misconception between drawing and construction when using Geometer’s sketchpad. They admitted that students needed a few hours to appreciate the importance of dynamic-geometry investigations, but by using this approach they claimed that the understanding of geometry concepts increased. Qing (1997) supports the use of computer software as an alternative approach to teaching geometry. Qing found that students struggled with paper, pencil, ruler and compass exercises and consequently were unable to draw a perfect equilateral triangle. Students needed to become familiar with most of the features and this familiarity was achieved by using computer software (Qing, 1997).

2.3. Prototypes

Presmeg (1992) refers to a prototype as a mental representation. Prototypes also exist or abound in the thinking of all children and these prototypes may be helpful (if constructed correctly) or unhelpful. Presmeg found that students had a prototypic image of right angled triangles and that students tended to rearrange the triangles to conform to their own image of a right angled triangle. These prototypic images are also found in the mathematical thinking of children at an early stage of schooling. This also supports the findings of Clements et al (1992) where pre-school children developed visual (prototype-based) descriptions.

The research findings of Schwarz and Hershkowitz (1999) provide valuable insight that could easily be used in the teaching of geometry, since there are similarities between learning the concept of function and learning geometrical concepts. They suggest that in 'concept learning' there are some examples that are more central to learning than others and these are called prototypes, which have been found to be important in conceptual judgement. If a verbal definition was presented for a geometrical concept without any example, students as well as teachers produced the same prototypical examples.

The findings of Vinner and Hershkowitz (1983) are valuable to this investigation and could provide a reason for poor performance in geometry. They found that very often children's concept images only include the prototypes, which indicates that prototypes are the examples of the concept that are constructed first. This provides a reason why students "tend to identify a concept with one or a few prototypical examples and why

the concept image often develops from one unique prototypical example” (Schwarz and Hershkowitz, 1999, pg 364). These prototypes lead to ‘prototypical judgement’ and serve as a frame of reference so that problems are judged by referring to the prototype.

Schwarz and Hershkowitz (1999) highlight two forms of prototypical judgement:

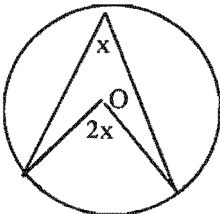
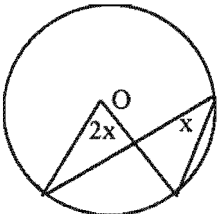
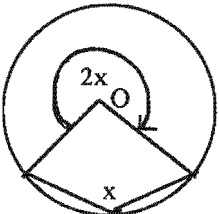
(a) visual - refers to the shape of the prototype which serves as criteria for judgement.

It is this kind of judgement, I believe, that leads to ER (Extreme Rubbish) responses and causes the downfall of many students.

(b) self-attributes of the prototype .

Concept learning is governed by prototypes, which can be beneficial or detrimental depending on how they are used as a frame of reference to solve problems. A typical example to represent the above would be the teaching of circle geometry. The theorem that states ‘The size of the angle, which an arc of a circle subtends at the centre, is double the size of an angle it subtends at any point on the circle’ is problematic because of the prototype example used. This aspect is highlighted in the findings of this study and explained in table 2.1 below.

TABLE 2.1: Prototypes

PROTOTYPE	NEGLECTED PROTOTYPES THAT CAUSE ERRORS	
		

In table 2.1, a particular prototype image is given (an image that textbooks and teachers tend to use to introduce that theorem). The other two images in table 2.1 are the neglected prototypes that are not often used to introduce that theorem. These neglected images, I contend, causes errors in the way students' solve geometric problems. These are also images that are often not found in the students' geometric register.

The prototypicality, according to Schwarz and Hershkowitz, is assessed through the examples that students invoke, the methods they use pertaining to specific examples, the links they make and the examples invoked to solve problems. They were able to ascertain "students' mental concept images" through qualitative and quantitative analysis of written solutions and justifications. In their analysis they grouped items according to prototypicality, part-whole reasoning, and attribute understanding. Their findings suggests that prototypes served as levers for concept learning within groups that had numerous experiences with function examples and transformations in their daily practice. It was interesting to trace these conceptual developments in the geometric domain.

Geometry is often referred to as the mathematics of space. For some teachers it is seen as a means of connecting mathematics to the real world. Geometry is an integral part of the mathematics curriculum and its concepts (perimeter, area, volume, etc) contribute to the students understanding of number and measurement. Recent studies emphasise key issues in learning geometry that could lead to greater understanding. These include spatial visualisation (Triadafillidis, 1995), levels of thinking (Clements, & Battista, 1992), concept formation (Schwarz, & Hershkowitz, 1999), reasoning

abilities (Clements, & Battista, 1992) and higher order thinking (Chinnappan, 1998). The teaching of geometry is also crucial to this development of understanding and research emphasises that teachers' content knowledge, confidence levels and attitude towards geometry teaching plays a vital role (Swafford, Jones, & Thornton, 1997). Most of the studies refer to either the theories of Piaget, based on space and geometry, or the van Hiele model, based on levels of geometric thinking and phases of instruction. It is important for me to look at the development of geometric thinking using Piaget and Inhelder, as well as the van Hiele models.

2.4. Development of Geometric Thinking

(a) Piaget and Inhelder: Space and Geometry

Geddes and Fortunato (1993) refer to three themes that characterise research on children's space concepts:

- The acquisition of spatial concepts and intellectual development. (Topological space)
- The building up of spatial representations through active manipulation of the spatial environment. (Projective space)
- The characterisation of acquiring spatial concepts according to the type of geometry involved. (Euclidean space)

Piaget discussed the child's ability to represent space. Clements and Battista 1992, pg 423) make the following observation on the development of space that is useful in later analysis:

Because making a drawing is an act of representation, not of perception, Piaget and Inhelder claim that inaccurate drawings reflect the inadequacy of mental tools for spatial representation.

Indeed, the inability of young children to draw a copy of even simple shapes is taken as an indication that co-ordination of actions, rather than passive perception, lies at the foundation of the conceptual development of space.

I do not want to engage in criticising or endorsing their work since this falls outside the scope of my investigation, but merely include a report that 2-3 year-old children are able to distinguish between curvilinear and rectilinear shapes, which is contrary to their theory (Lovell, 1959).

(b) The van Hiele: Levels of geometric thinking

They suggest that students' progress through levels of thought in geometry and these levels include:

- **Level 1: Visual:** students identify and operate on shapes and other geometric configurations according to their appearance. These are sometimes visual prototypes.
- **Level 2: Descriptive/Analytic:** students recognise and can characterise shapes by their properties. They see figures as wholes, but now as collections of properties rather than visual gestalts. These properties arise by observation, measuring, drawing and modelling.
- **Level 3: Abstract/Relational:** students are able to give abstract definitions, logical arguments in the geometric domain, classify figures hierarchically.
- **Level 4: Formal Deduction:** students are able to establish theorems within an axiomatic system when they reach this level. Able to construct original proofs and reason formally by logically interpreting geometric statements.
- **Level 5: Rigor? Metamathematical:** Students are now able to reason formally about mathematical systems.

Each van Hiele level is integrated by four key processes of thinking, namely, identification of an object, working with the definition, classifying objects into different families and proof of statements. For students to obtain certain levels, they must master all the processes of that level. My investigation also includes the use of van Hiele levels, but it integrates the visual (recognition of shapes) with the ability to articulate statements or theorems and then to make formal or informal deductions.

There are numerous studies that use the van Hiele levels to accurately describe students' geometric thinking or simply confirm the usefulness of the levels. These studies include, amongst others, the following:

- Usiskin (1982) found that 75% of secondary students fit the van Hiele model, noting that classification at each level varies and is dependent on the instrument and scoring scheme.
- Burger and Shaughnessy (1986) reported that the behaviour of students was generally consistent with the van Hiele's model.
- Clements and Battista (1992) in summarising declare that the levels appear to exist and describe students' geometric development. These were validated through interviews and written assessments.
- Swafford, Jones, & Thornton (1997) used the van Hiele model as a framework for discussing students' cognition and for describing how teachers' grew in geometric understanding.

There are also critical issues in using these levels of van Hiele that should be flagged. Briefly, these issues revolve around interpretation of results as a support for its validity, the meaning of students 'at a level', the meaning of thinking of shapes in terms of their properties and the observed lack of 'discontinuities in learning'.

A common trend in all the studies is the realisation that students are failing to learn basic geometric concepts and geometric problem solving. I concur with the belief that in teaching for understanding one should build on a child's existing ideas. There is common support for the idea that "one's representation of space is not a perceptual 'reading off' of the spatial environment but is built up from prior active manipulation of that environment" (Clements, Swaminathan, Hannibal, & Sarama, 1999, pg. 192).

In their investigation (Clements, et al, 1999) based on the criteria that young children (ages 3,5 to 6,9) use to distinguish geometric shapes common in our social-cultural environment, there was evidence that indicated that the children matched the shapes to a visual prototype.

Clements, et al (1999) also propose that children at this level are starting to form schemas, which refers to a network of relationships connecting geometric concepts and processes in specific patterns. It is the unconscious schemas that lead to pattern matching. These schemas widen to accept more forms to such an extent that it must be constrained. This theory is supported by evidence "that the largest internal consistency was for those shapes with less visual and property variance within the class (circles and squares); the more variance (from rectangles to triangles), the less internal consistency we found" (1999, pg.207). There are similar problems with older children.

Pre-school children exhibit working knowledge of simple geometric forms and as teachers we should build on this knowledge and move beyond it. There are claims that students fail to reach the descriptive level of geometry in part because they are not

exposed to it in their early years. This period of geometric inactivity, I believe, leads to 'geometrically deprived' children.

Chinnappan (1998) investigated students' ability at accessing and making flexible use of previously learnt knowledge by examining the potential links between mental models constructed by students, the organisational quality of students' prior geometric knowledge and the use of that knowledge during problem solving. I found this investigation useful in that some questions concerning students' ability to remember or recall prior knowledge and even the way in which they 'file' or 'order' their knowledge has been answered to some extent. Chinnappan (1998, pg. 201) notes that:

Studies of this nature that target knowledge building and application in mathematics are essential if we are to understand the problem of student failure to apply previously learnt mathematical knowledge to the solution of new problems.

It is essential for us as educators to create student interest and student participation in classroom learning activities in geometry. Chinnappan's research shows that student failure to activate and apply prior knowledge "can be largely attributed to the poor quality of their mathematical knowledge base"(pg. 202). This knowledge base is similar to my reference to the geometrical register. The way in which students organise and assemble their vast geometric information (including definitions, axioms, theorems, etc) has a strong influence on how this information is activated. There is a need to focus on the 'quality of the geometric knowledge structures' of students'. This could be achieved through the notions of 'schema and mental models', which according to Chinnappan "serve as powerful constructs in our investigation of students' geometric knowledge base that drives the solution process" (pg. 202).

Chinnappan (1999) refers to schema as “a cluster of knowledge that contains information about core concepts, the relations between these concepts and knowledge about how and when to use these concepts”. (pg. 202). These schema’s will influence how students organise their knowledge to solve general problems. The organisation or visualisation of these schema involves looking at key concepts and diagrams that play a central role on which axioms are built. I argue that in each geometric register there exists different schema, for example, in circle geometry, you could have a schema for subtended angles, or cyclic quadrilaterals, or tangent theorems. The use of a combination of these schema forms a mental model which students use in a problem solving activity.

Chinnappan highlights two characteristics for understanding geometric schemas:

(a) organisation, which refers to the establishment of links between ideas.

(b) spread, refers to the extent of those links.

The high degree of organisation and spread determines the level of sophistication or complexity of the schema. To observe the way in which students’ integrate their schemas means an examination of their mental models during problem solving activities. These mental models provide a big picture of the students’ application ability during problem solving.

The aim of Chinnappan’s study was:

(a) to identify the schemas that students bring to a problem solving task,

(b) to determine the frequency with which these are activated,

(c) to generate a description about the nature of the mental models students utilise or construct.

Chinnappan (1999) concluded that the knowledge structures of the HA (High-achieving group of students in mathematics) group was qualitatively superior than those structures used by the LA (Low-achieving group of students in mathematics) group. The HA group could integrate their schemas with the demands of the problem in that they could “search their memory for the relevant geometric schemas” and bridge the gap. The LA group failed to see connections and focussed on “superficial aspects”. Chinnappan (1999, pg 213) ascribed this to the fact that:

... these students do not build models of the problem which draw out the links between their prior schematised knowledge and elements embedded in the problem.

It does seem to follow from the findings that those students who structure their prior geometrical knowledge into schemas also develop an understanding of how to use that knowledge productively during problem solving. As educators we need to pay attention to the conscious (in the case of low-achievers) organisation and development of a range of schemas and also to the activation of strategies (cannot be neglected) to solve problems.

Battista (1999) utilised psychological and sociological components to analyse the cognitive constructions that students made as they enumerated 3D arrays of cubes. He emphasised the mental processes of abstraction, reflection, perturbation, spatial structuring, co-ordination and especially social interaction within pairs, and how these processes brought about meaningful and powerful student learning.

Battista (1999) refers to the critical mental mechanism of ‘**abstraction**’, which is the process “by which the mind selects, co-ordinates, unifies, and registers in memory a collection of mental items or acts that appear in the attentional field” (pg. 418).

Understanding, according to Battista, needs more than just understanding, but requires **reflection**, which he sees as “the conscious process of re-presenting experiences, actions, or mental processes” (pg. 418). Meaningful learning occurs when a **perturbation**, seen as a “disturbance in mental equilibrium caused by an unexpected result or a realisation that something is amiss or does not work” (pg. 418), occurs. These perturbations arise as students interact with each other.

He also refers to mental models (that link strongly with Chinnappan’s ideas) as “non-verbal recall of experience”. He introduces the notion of a ‘scheme’, which he calls “an organised sequence of actions or operations that has been abstracted from experience and can be applied in response to similar circumstances” (pg 418). There is a strong overlap with Chinnappan’s notion of schemas. The terminology mentioned by Battista was useful in terms of providing some ideas about processes that takes place in the geometrical register.

Students had to work collaboratively in pairs to predict how many cubes could fit in a graphically represented box and then to check their predictions. The teacher monitored the interaction and encouraged communication and collaboration, and promoted individual sense making. Case studies were based on pairs of same sex students in both a high group and a low group.

It was found that differences between their predicted and actual answers resulted in perturbations that forced them to reflect on their enumeration strategies and their structuring of the cube arrays. Crucial to the study was the strategy of prediction, since students could establish the viability of their mental models and enumeration

schemes, and reflect on and refine their mental models. The perturbations acted as a stimulus and forced students to refine their strategy and this process could be used in our inquiry-based geometry classes. Personally it is a useful tool to employ in my own classroom. Working collaboratively also proved to be useful because students had to continually establish consensus about problem solutions. In some cases students consciously tried to make sense of their partners responses, while some did not understand their partners explanations, but “their final conceptualisations are influenced by things their partners say and do” (Battista, 1999, pg. 443). However, personality traits of students (e.g. overbearing students) cannot be underestimated and this could hinder the collaboration between students. The aspect of miscommunication, where students interpret their partners’ understandings only in terms of their own conceptual structures. Battista claims that students generally made sense of their partner’s communications. This idea of making sense of your partner’s communications is vital for effective collaboration and even developing the geometrical register. The success of the study, despite the small sample, was based on the willingness of students to complete assigned tasks, working collaboratively and attempting to make sense of solutions. It would be ideal for our students to regularly engage in active participation, where students reflect on and analyse ideas, communicate ideas to others and most of all to make sense of what is communicated to them.

A very important aspect of the study is the acknowledgement that theory building is guided by teachers and also instructional materials. Teachers, have the power to establish the culture of inquiry, where students could be encouraged to construct more sophisticated structurings and enumeration procedures. The findings support the

notion that direct teacher intervention is sometimes critical to students' progress and achievements. Within problem-centred inquiry-based learning, we need to be aware that the "development of viable theories and concepts is often confused, irregular, and untidy" (Battista, 1999, pg. 447) and students regularly make mistakes and struggle to make sense of ideas. The crucial question to teachers is simply when to intervene and what this intervention entails. Students need repeated and varied opportunities to properly construct difficult and complex concepts, as well as an ability to check the viability of their developing mental models in which well organised schemas exist.

Triadafillidis (1995) suggests that students, at secondary level, are unable to identify and name shapes like kite, rhombus, trapezium, etc. Their understanding of general properties of these shapes is poor. Perhaps, there is a problem with the teaching of these shapes. Triadafillidis (1995) claims that learning in lower secondary school mathematics has become didactic and analytic, and a 'hands on' involvement of students occurs sporadically. He is concerned with the idea that the learning of these shapes may be supplemented by active teaching.

Triadafillidis accepts that there is no generalised diagram for a given shape, and suggest that there is a tendency to rely on certain prototypes. His ideas connect strongly with earlier thoughts on prototypes. The 'visual limitation' is emphasised by the way concepts are introduced in teaching geometry. If concepts are introduced by some "prototypical examples, students' knowledge of a concept is constrained to the cases that are more often addressed in a textbook or by a teacher, neglecting uncommon cases" (Triadafillidis, 1995, pg. 226). He suggests that to build a concept around certain diagrams intensify the misleading potential of prototypical examples.

Perhaps the development of a schema incorporating a variety of these prototypical examples could form a basis for geometry teaching.

The representation of a geometrical object should involve the use of diagrams, along with words, apparatus and activities. Triadafillidis claims that a stimulus does not have to be figural to be processed visually. He refers to visual processing, which places emphasis on the process of the stimulus, as a means of overcoming geometry's visual limitations "through the recognition and discrimination of a concept's critical attributes and the dismissal of the additive ones" (Triadafillidis, 1995, pg. 227). These visual processing skills are also trainable to some degree. This training should include perceptual stimuli referring to the concept with other senses apart from vision and therefore could include touching, manipulating, drawing, etc. This will provide a 'hands on' experience for students, which Triadafillidis refer to as **haptic** exploration. Haptic exploration includes visualisation and action, and could circumvent some visual limitations in geometry teaching. These exploration strategies are regulated by developmental changes in children. At first objects are simply grouped (age 3-4), then systematic explorations (age 5-6) and methodical (at 6). Strategies are more efficient as children start to use their fingers. These visual and tactile experiences provide students with an external frame of reference. Triadafillidis claims that by using the van Hiele's model there is a correlation between the development of geometric thought and the strategies employed in haptic exploration of shapes.

Jerry (1994) pleads for the defence of geometry, which means handling and feeling shapes, both two and three-dimensional shapes. Jerry, like Triadafillidis, appeals for a practical hands-on approach to geometry, which could be a starting point for new

ideas and investigations. Students should develop a feeling for beauty and symmetry for underlying patterns (Jerry, 1994).

A limitation to the visual aspects of geometry is that only one hemisphere of the brain is used when teachers provide these visual aspects of geometry (Hoffer, 1981). For understanding it seems as if the visual aspects does not provide enough stimulation, since we need to involve the full brain (Hoffer, 1981).

The aspect of proof or the nature of proof has already been mentioned, but Durrel (1976) suggest that theorems were ordered completely thematically and that constructions were treated separately from theorems. Durrel emphasised the use of 'subtle' proofs. However, he acknowledges that it is problematic to confront students in the early stages of geometry with proof. Consequently, he provided proofs in the appendix and not in the text.

The potential to pick out misunderstandings is useful and valuable as a teaching aid and as educators we "should aim to provide experiences and support in order to promote understanding and build positive attitudes towards mathematics and learning in general" (Triadafillidis, 1995, pg. 234). One way of achieving this positive attitude is to allow students the space for their own productions and to allow them to evaluate the work of the fellow students to clear up misapprehensions within a students own theory (Van den Brink, 1995).

We should allow a tacit interaction among our students and to ensure that students are able to deliberate the various definitions. The students should be allowed to decide on

important characteristics, which are essential or incorrect and must consider their own productions as important (Van den Brink, 1995).

University of Cape Town

CHAPTER 3

RESEARCH METHODOLOGY

3.1. Introduction

This chapter reviews the procedures used in the study to investigate how grade 12 students understand and solve geometric problems. It includes a description of the research design, population and sample, and data collection.

3.2. Research Design

3.2.1. Introduction

Since the focus of this study is based on how grade 12 students understand and solve geometric problems, the research design involves tests (based on the grade 11 syllabus) and interviews with students based on their performance in the tests. The aim of the interviews is to focus on the student's responses in the tests, to look at procedures and structures, as well as difficulties in their understanding of the geometric concepts. The interviews will also be used to analyse their ability to reason carefully and to look at their efficient or inefficient use of their geometric register.

Through this design I will attempt to begin to form an opinion concerning the following questions:

How grade 12 students perform in tests based on the grade 11 syllabus?

Do the majority (say 60%) of students have an efficient geometric register? Is this register intact (are definitions, axioms and theorems known?)

Are students able to complete statements and visualise geometric theorems from sketches?

Are those students with an intact geometric register able to solve problems?

What are the difficulties around problem solving?

3.2.2. Population and Sample

The study was undertaken at four senior secondary schools during the third term in 2000. These schools (as already mentioned in chapter 1) were involved in an 18-month pilot project called the Secondary Schools Partnership Project (SSPP) initiated by the University of Cape Town which ran from July 1998 to 1999, but extended to 2000. All grade 12 mathematics students at each of the schools were included in the study. The following table 3.1 shows the names of the participating schools, the areas in which the schools are located, the number of students involved in the study, the number of teachers teaching grade 12 mathematics and the medium of instruction.

TABLE 3.1: Population and Sample

SCHOOLS	AREA	STUDENTS	TEACHERS	MEDIUM OF INSTRUCTION
GLENDALE	MITCHELL'S PLAIN	24	2	ENGLISH AND AFRIKAANS
PORTLAND	MITCHELL'S PLAIN	71	1	ENGLISH AND AFRIKAANS

SINETHEMBA	PHILIPPI – CROSSROADS	93	2	ENGLISH AND XHOSA
ESANGWENI	KHYALITSHA	79	1	ENGLISH AND XHOSA

The majority of the students at the schools were from a lower socio-economic background. All the schools are co-educational. A sample of 267 students was included in the study. This sample included those students who failed grade 12 in 1999. The schools were also chosen for the study, because geometry was taught at the schools and the grade 11 geometry, in particular, was completed in 1999. Some grade 11 geometry revision also took place, through a series of tutorials after school hours, in preparation for the June examination in 2000. The majority, if not all, of students were tested in examinations in June 1999, November 1999 and June 2000. In between the examinations class tests (control tests for continuous assessment) were used to assess geometry as well as algebra.

For the above reasons students were expected, as a minimum standard, to at least know their bookwork or in terms of this study have an intact geometric register.

Testing was a common means of assessment in the four schools. Hence, the reason for using tests as an instrument.

3.2.3. The Instrument

Two tests were designed. The first aimed to test the geometric register of students (their bookwork knowledge or theoretical framework), and the second to test their

ability to apply their geometric register to problems. These tests were designed in the following way:

(a) Test 1

The matriculation (grade 12) geometry syllabus includes, amongst others, the following core sections: subtended angles in a circle, cyclic quadrilaterals and tangents. Test 1 (See appendix 1), basically a bookwork test, consisted of two parts:

- Part 1 was designed to test the student's definitional framework as well as their ability to complete the statements of most of the relevant theorems based on the three core sections. Part 1 consisted of 10 items.
- Part 2 was designed to test whether students were able to provide a definition or theorem statement from a given sketch or simply to test their visualisation skills based on the three core sections. Part 2 consisted of 9 items.

The aims of test 1 were the following:

- To see how many students had their geometrical register in place in terms of a basic geometrical vocabulary.
- To see whether they were able to recognise or link a diagram (sketch) to a theorem or definition.
- To look at student performances in the tests by categorising their responses.

(b) Test 2

Test 2 consisted of 16 items. The test was designed to test students' ability to apply their understandings of test 1, through problem solving activities. This test was based on true or false responses. Students were given a sketch in which they had to solve for x , where x was a given angle. However, they were also given a solution to each of the problems. They had to decide whether the given solution was true or false. If false, students' had to give a reason and then find the correct value for x . The items were selected in such a way that all three core sections of the syllabus were tested. Both tests were submitted to three senior mathematics teachers and my supervisor for comment and refinement.

3.2.4. The Procedure

Students wrote the tests in the week ending 28 July 2000. Dates and times were finalised with each school. Students were allocated 15 minutes for test 1 and 45 minutes for test 2, without a break between tests. Outside invigilators were used to administer the tests at each school. A meeting was held with the invigilators to explain their role in administering the tests. In this way instructions to students were uniformed and standardised. Invigilators were not allowed to help or assist students, after the instructions were given. Teachers were not allowed to enter the test venues. There were no problems or difficulties in administering the tests. All the schools, especially the mathematics teachers and those teachers who gave their periods for the tests, as well as the students, co-operated very well with the invigilators. The researcher supervised the administration of the tests.

Two experienced mathematics teachers formed a marking panel, while the researcher acted as the moderator. A detailed training session was held with the markers to ensure correct application of the memorandum. The following coding system was used to mark the tests:

(a) TEST 1: 0 marks for no attempt by the student.

1 mark for the wrong answer.

2 marks for the correct answer.

(b) TEST 2: 0 marks for no attempt by the student.

1 mark for wrong response true or false

2 marks for correct response, i.e. false with correct reason.

3 marks for correct solution, i.e. true.

4 marks for full answer, i.e. false, with correct reason and correct value for x .

The purpose of the coding system was to provide the researcher with information regarding ways in which students answered the questions, that is, how many students did not attempt a question or how many simply said true or false.

The results of the tests were returned to the schools within a week. Teachers did not receive the coded system of marks, because they (and their students) were merely interested in how their students performed in the tests. So, for test 1, the marking team only added the solutions with 2 marks. For test 2, the marking team only added the solutions with 2, 3 and 4 marks. In other words, the schools only received the name of the student and their mark for the tests combined. The teachers at each of the schools were very interested in the results, because they felt that the tests were well designed.

Some teachers even used the results of the tests as a control test. It must be noted that this arrangement was not part of the research design agreement, except that schools would receive the results as soon as it was available.

3.2.5. Interviews

As already mentioned in the rationale for the investigation, it was pointed out that some students may or may not have their geometrical register in place. Both situations would impact on the performance of individual students. However, for the purpose of this investigation, it was decided that only those students who performed very well (at least 90%) in test 1 would be interviewed. The rationale for this decision was based on the notion that this group of students would be expected to perform well in test 2 since they knew their theory. This particular group was also chosen because I wanted to see how they would be able to use their register to solve problems in test 2.

However, this was not the only criterion for being selected as an interviewee. I also looked for students who obtained a mark of 70% in test 1, but produced interesting responses in test 2. It was hoped that students chosen to be interviewed would have used their register and in this way guessing could be eliminated from the problem solving activity in test 2.

Only a sample of the students who did well in test 1 was interviewed. As a focus question for the interviews the following was highlighted.

If the geometrical register is in place does this lead to better performances in problem-solving?

Through the interviews it was hoped that students would reflect on the difficulties and their struggles or challenges in problem-solving.

After the tests 24 students (six from each school) were selected and basically included those who performed well in test 1, as well as those who produced interesting solutions in test 2. The marking team identified these students. A series of 21 interviews took place. At Portland and Sinethemba 6 students were interviewed. At Esangweni only 5 students were interviewed, because one of the students did not arrive for the interview. At Glendale 6 students were interviewed, but only 4 interviews were transcribed. The transcriptions of the two students would not add value to the investigation because of (a) the attitude of one student who only answered in monosyllables and showed no interest in the interviews, (b) the home language of the second student meant that his English was too poor to understand.

Before the start of the interviews a profile of each student was produced, based on the students' performance in the tests. The profile consisted of two parts:

Part 1 reflected the performance of the student in the test.

Part 2 provided the researcher with possible lead questions for the interviews.

However, these questions were not strictly adhered to and did not hinder the flexibility of the researcher's interview questions.

The following represents an example of a student profile used in the interviews.

INTERVIEW SCHEDULE (PART 1 and PART 2)

NAME: BRONWYN DUNCAN

SCHOOL: PORTLAND

ANALYSIS OF TEST RESULTS (PART 1)

TEST	STUDENTS RESPONSE	POSSIBLE INTERVIEW QUEST
TEST 1.1 - 1.10	FULL MARKS	
2.1 - 2.9	FULL MARKS except 2.7	
TEST 2	4, 5, 10, 12 FULL MARKS	1, 2, 3, 6, 7, 8, 9,11, 13, 14, 15, 16

SEMI-STRUCTURED INTERVIEW QUESTIONS (PART 2)

Refer Q2.7. Explain your answer?

Refer Q1: Why do you think this question is true?

Are you familiar with this figure?

Refer Q2: Explain your reasoning in this problem?

How did your teacher explain this?

Refer Q3 How did you go about answering this problem?

Are you familiar with this figure?

Refer Q6 How did you go about answering this problem?

Are you familiar with this figure?

Refer Q7: Why do you think this question is true?

Did your teacher emphasise this definition?

Refer Q8: How did you go about answering this question?

How did your teacher explain this?

Refer Q9: Explain your reasoning in this problem?

Refer Q 10. How did you go about answering this question?

Are you familiar with this type of figure?

Refer Q11. Why do you think this question is false?

Are you familiar with this figure?

Refer Q12. How did you go about answering this question?

Why did you choose that strategy?

Refer Q13. Explain your reasoning for this question?

Are you familiar with this figure?

Refer Q14 How did you go about answering this question?

How did your teacher explain this?

Refer Q15 Explain your reasoning for this question?

Why did you choose this strategy?

Refer Q16. How did you go about answering this question?

What do you mean by that response?

The researcher made a careful study of the learner's response in the test and used the profile to guide his questions, while the student received his script and used it as a frame of reference.

All interviews were conducted at the individual schools. Generally students were keen to respond to questions and showed enthusiasm for the interviews and re-solving or explaining the problems.

CHAPTER 4

DATA ANALYSIS AND DISCUSSION

4.1. Introduction

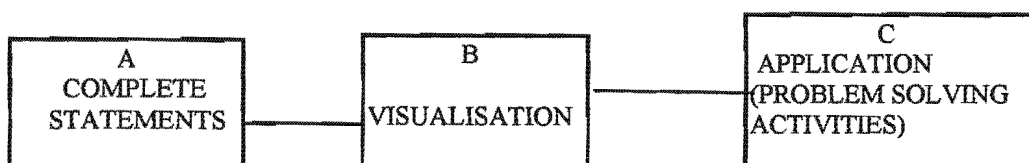
This chapter presents analysis of data obtained from:

1. Item analysis of tests 1 and test 2 (a quantitative response).
2. Impact of Geometrical Register in problem-solving
3. Transcription of interviews.

The aims of this investigation as highlighted in chapter 1 are:

1. To look at student performances in the geometry tests and to ascertain whether the students have a basic knowledge about circle geometry.
2. To compare results of test items in test 1 with items in test 2.
3. To look at examples of items in students' geometrical register.
4. To look at the role of prototypes in the students' geometrical register.

It is useful at this point to look at the rationale framework (Table 1.3, redrawn below) as presented in chapter 1, since each of the above aims are analysed and interpreted through the data collected and refer back to this framework. Blocks A and B are covered in test 1, while block C is covered in test 2.



4.2. Item Analysis of Results for Test 1

The focus of this section is to raise issues relating to the geometrical register of students. It should provide a quantitative analysis of what students know, what really exists in their register and which statement/theorems and visualisation of the diagrams causes the most problems in circle geometry as required for grade 11.

The following table was extracted from appendix 2 (the summary analysis for test 1 and test 2). The table represents a comparison between question 1 (completing of statements) and question 2 (the visualisation of statements) on the same item in test 1. For example (see appendix 1):

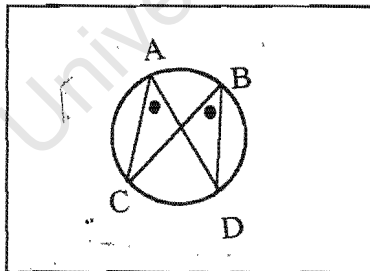
Test 1: Question 1. Complete the following theorem or definition:

1.1. Angles subtended by the same arc or chord

Question 2. Write down the theorem or definition for each of the diagrams:

2.2. Diagram

Theorem or Definition



.....

The table indicates that there were 142 correct responses for question 1.1 and 152 for question 2, a difference of 3.8 % favouring question 2.

TABLE 4.1: Comparing test 1 items in question 1 and question 2

QUE 1	QUE 2	STATEMENT OR THEOREM	CR1	CR2	%1	%2	%D
1.1	2.2	Angles subtended by the same chord/arc are equal	142	152	53.1	56.9	3.8
1.2	2.6	Angle at the centre = 2 angle at circumference	165	167	61.7	62.5	0.8

1.3	2.1	A cyclic quad is a quad with the four vertices on the circumference	81	160	30.3	59.9	29.6
1.4	2.4	Opposite angles of a cyclic quad are supplementary	152	123	56.9	46.0	10.9
1.5	2.3	Exterior angle of a cyclic quad = the interior angle	172	159	64.4	59.5	4.9
1.6	none	A tangent is a line that cuts the circle at one point	92	None	34.4	none	none
1.7	2.7	Angle between tangent and chord = angle in alternate segment	109	120	40.8	44.9	4.1
1.8	2.5	Radius is perpendicular to the tangent	87	104	32.5	38.9	6.4
1.9	2.9	Tangents from a common point are =	143	117	53.5	43.8	9.7
1.10	2.8	Angle subtended by diameter = 90 degrees	90	69	33.7	25.8	7.9

Coding of table:

QUE 1 and QUE 2 represents the test items from test 1

Statement or theorem represents the item tested.

CR1 represents the number of correct responses in question 1. (sample 267).

CR2 represents the number of correct responses in question 2. (sample 267).

%1 represents CR1 expressed as a percentage.

%2 represents CR2 expressed as a percentage.

%D represents the difference between %1 and %2

Definition items

Test 1 covered only two definitions that are important for circle geometry, namely, defining a cyclic quadrilateral and a tangent.

Item 1.6 tested a definition and the aim of including this item was to see how many students knew the definition of a tangent. Surprisingly, only 34.4% could define it.

Item 1.3 tested the definition of a cyclic quadrilateral and only 30.3% got it correct in terms of completing the statement. However, 59.9% identified a cyclic quad through a given sketch, but most of the students used it as a property and did not say ABCD is a cyclic quadrilateral. Instead, they wrote, opposite angles of a cyclic quadrilateral are supplementary. The marking team insisted that because they identified the cyclic

quadrilateral it had to be marked correct. Hence, a reason for the disparity, in terms of percentage, between completing the definition and visualising the figure.

Theorem Items

Each theorem/statement (excluding corollaries) relevant to circle geometry for grade 11 was tested in terms of completing statements and visualising these statements from given sketches. It must be noted that only a few items yielded above 50% correct responses. The best known theorem in terms of completing statements and visualisation was *the angle at the centre of the circle equal to twice the angle at circumference subtended by the same arc/chord*. The percentage difference was 0.8.

The biggest discrepancy between completing statements and the visualisation was for *opposite angles of a cyclic quadrilateral are supplementary*. The percentage difference was 10.9. The majority of students had difficulty distinguishing between the definition and a property of the cyclic quadrilateral. There is evidence that students, who generally completed the statements correctly, could also visualise these statements/theorems from a given sketch. Students who performed poorly in the completing of statements showed a slight improvement in the visualisation category. Based on the performances in test 1 less than 50 % of students had some kind of geometric register in place. In Appendix 2a we note that out of 19 items there were only 9 items with a correct response greater than 50% and the range of correct responses from 69 to 172 out of a possible 267.

The above statistics (Appendix 2a) indicates that the majority of students do not possess an effective geometric register, which is necessary for problem solving activities in test 2.

4.3. Categories of Responses

The quality of responses by students provides an insight into the difficulties that students have in expressing themselves. I have highlighted some categories of responses, with examples from scripts, based on responses in test 1. These categories will be re-visited when the interview is discussed in 4.5.

(a) *Bad use of language*

:

Examples based on the completing of statements:

Q1.1. Angles subtended by the same arc or chord ...

- *equals to an angle of other segment*
- *angle or the centre are equal*

Q1.4. The opposite angles of a cyclic quadrilateral ...

- *because supplimental*

(b) *Poor understanding of Basic Concepts*

Q1.1 Angles subtended by the same arc or chord ...

- *equilateral angles which are subtended by the same chord*
- *are called supplementary angle*
- *are twice the size of a inscribed angle*
- *equal each other add up to 180°*

Q1.2 The angle at the centre of the circle is ...

- *the angle at the centre which is zero*
- *equal to 30 degree*
- *perpendicular to the circle*

Q1.3 A cyclic quadrilateral is a quadrilateral with ...

- *four equal sides*
- *two equal opposite angles*
- *a arc or chord*

- *a circle*

Q1.4 The opposite angles of a cyclic quadrilateral ...

- *are equal*

Q1.6 A tangent to a circle is a line that ...

- *divides the circle into two equal halves*
- *touches the centre and the circumference*
- *is joining the midpoint of the chord to the circle*

Q1.7 The angle between a tangent and a chord is equal to ...

- *180°*
- *one another*
- *90°*

Q1.8 The radius of a circle and a tangent are ...

- *180°*
- *equal*
- *is the centre of the circumference*
- *both equal to 180°*

Q1.9 Two tangents drawn from a common point ...

- *is the radius*

Q1.10 The angle subtended by the diameter ...

- *equals to 180°*
- *equal 30°*
- *is twice the size of the angle*
- *is the radius*
- *is equal to 360°*
- *drawn from the midpoint is a right angle*
- *then the sides are equal*

**(c) Random use of terms or concepts without reference to their meanings
(statements that does not make sense)**

Q1.1 Angles subtended by the same arc or chord ...

- *is the right angle*
- *cyclic quad*
- *is perpendicular to the radius*

Q1.2 The angle at the centre of the circle is ...

- *quadratic equation*
- *radius*
- *chord*
- *point*
- *perpendicular to the tangent of a chord.*

Q1.3 A cyclic quadrilateral is a quadrilateral with ...

- *a tangent*

- *the Δ of the angles*
- *perpendicular*
- *4 points joined to one chord of a circle.*
- *The same tangent*
- *A extra line drawn to the circle*

Q1.4 The opposite angles of a cyclic quadrilateral ...

- *triangle*
- *is the circle or angle which is a circle and has opp. Sides*

Q1.5 The exterior angle of a cyclic quadrilateral ...

- *is the angle that are extended*
- *tangent angle*
- *vertical line*

Q1.6 A tangent to a circle is a line that ...

- *pass through the circle and has an outside angle*
- *cuts the middle of an angle*
- *are formed*
- *is attached to the circle that it is subtended by the same chord*
- *subtends the circle at both sides*

Q1.7 The angle between a tangent and a chord is equal to ...

- *the midpoint of a circle*
- *the bisect*

Q1.8 The radius of a circle and a tangent are ...

- *straight line*
- *the same as quadrilateral*
- *normally cut the radius is from centre to the tangent*
- *equal to the interior angles of a triangle*

Q1.9 Two tangents drawn from a common point ...

- *diametre*
- *are common distance*
- *opposite angles*
- *rhombus triangle*

Q1.10 The angle subtended by the diameter ...

- *is made of 360° of the angle*
- *is called a tangent because it is subtended by the diameter*
- *is equal to the exterior angle of the chord*
- *is a tangent*
- *is equal*

It is possible to extract a few other categories, but for the purpose of this study it suffices to highlight some of the responses that impact on the problem solving

capabilities of students. It also indicates a confused state of mind of a number of students, since the quotations have been extracted from a cross-section of the sample.

4.4. Impact of Geometrical Register in Problem-solving in test 2

The researcher's perspective

Appendix 2b represents the mark distribution for test 2 and highlights a number of issues concerning the responses of students, which relate to their performance in problem solving. The following table (table 4.2) represents a comparison between test 1 (bookwork- theory) and test 2 (application - practical). It also indicates how students responded in terms of true or false. The first part of the table represents exactly the same information as in 4.1, the second part represents (from appendix 2b) results of student responses in test 2. It should also be noted that some items are repeated in different theorem categories, because of the nature of the question. These include question [Q8], [Q14], [Q15] and [Q16]. The questions that required a **false** response are indicated by: [Q?], followed vertically by two numbers. The first number indicates the correct response, but no value for x or the incorrect value for x. The second number indicates the full solution, namely the correct response and the correct value for x. A true response is indicated by: [Q?], followed by one number representing the number of correct responses.

The coding is exactly the same as table 4.1 (coding repeated below table 4.2), to facilitate comparisons between the two tests, and the extra part of the table below includes the test 2 summary. This summary indicates the question in test 2 (denoted

by [], the correct response (in the false category, does not include the value of x) and the correct solution (in the false category, includes correct value for x).

TABLE 4.2: Analysis of test 2 compared with test 1

TEST 1 Analysis

QUE 1	QUE 2	STATEMENT OR THEOREM	CR1	CR2	TEST 2		
					[Question]	* Correct response (False)	** Solved for x
1.1 Row 1	2.2	Angles subtended by the same chord/arc are equal	142	152	[Q3] 9 * 16 **	[Q4] 228	[Q6] 224
1.2 Row 2	2.6	Angle at the centre = 2 angle at circumference	165	167	[Q1] 4 * 37 **	[Q2] 185	[Q10] 176
1.3 Row 3	2.1	A cyclic quad is a quad with the four vertices on the circumference	81	160	[Q7] 8 * 14 **		
1.4 Row 4	2.4	Opposite angles of a cyclic quad are supplementary	152	123	[Q14] 1 * 1 **	[Q15] 218	
1.5 Row 5	2.3	Exterior angle of a cyclic quad = the interior angle	172	159	[Q14] 1 * 1 **	[Q15] 218	[Q16] 1 * 9 **
1.6 Row 6	none	A tangent is a line that cuts the circle at one point	92	None			
1.7 Row 7	2.7	Angle between tangent and chord = angle in alternate segment	109	120	[Q8] 3 * 5 ** [Q12] 197	[Q9] 3 * 5 ** [Q16] 1 * 9 **	[Q11] 6 * 1 **
1.8 Row 8	2.5	Radius is perpendicular to the tangent	87	104	[Q8] 3 * 5 **		
1.9 Row 9	2.9	Tangents from a common point are =	143	117	[13] 6 * 6 **		
1.10 Row10	2.8	Angle subtended by diameter = 90 degrees	90	69	[Q5] 8 * 77 **		

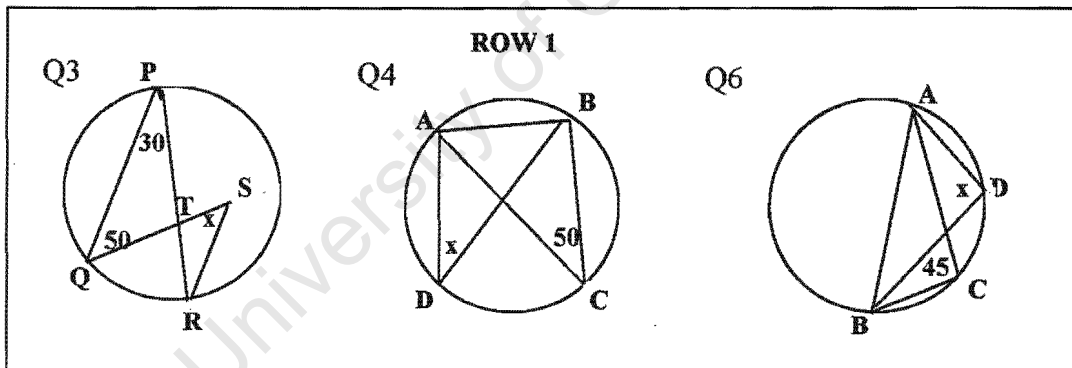
Coding of table:

QUE 1 and QUE 2 represents the test items.
 Statement or theorem represents the item tested.
 CR1 represents the number of correct responses in question 1. (sample 267).
 CR2 represents the number of correct responses in question 2. (sample 267).

A quick overview of the above table across rows revealed an interesting pattern of responses, for example: (the statistics for each row is extracted from table 4.2).

1.1	2.2	Angles subtended by the same chord/arc are equal	142	152	[Q3] 9 *	[Q4] 228	[Q6] 224
Row 1					16 **		

(1) In row 1, the first application in test 2 based on the theorem [Angles subtended by the same chord/arc are equal] only 25 students responded correctly, but only 16 solved for x , while in question 4 and question 6, 228 and 224 students respectively responded correctly.

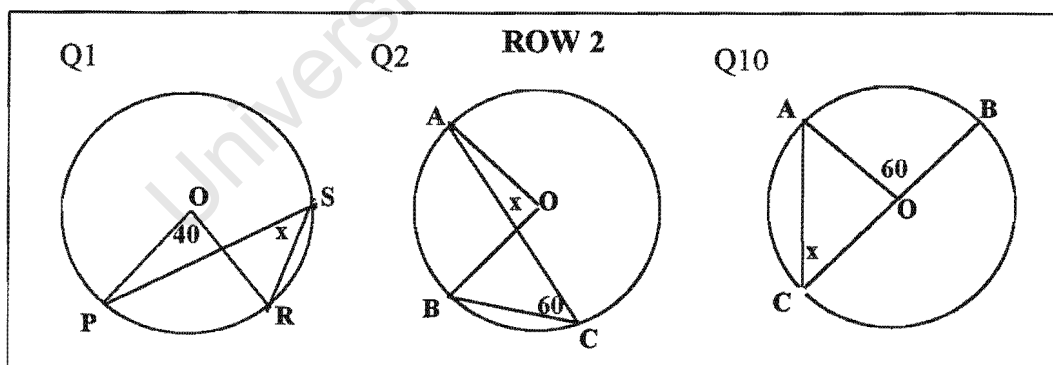


The orientation of the sketches in both question 4 and 6 were different (see appendix 1 test 2), and over 220 of the 267 students responded correctly. This contrasts to the 142 and 152 students who responded correctly in completing statements and visualising tests respectively in test 1. It is evident from the scripts that some students may have guessed the correct response, because they performed poorly in test 1 and their geometric register was not in place. However, another possibility was that the sketches may have triggered a familiar look about it (teacher's may have done a similar problem in the classroom) and they remembered what the teacher said "angles

subtended by the same chord are equal” a statement that I repeated very often in my teaching and so do many other teachers in my observations of classroom teaching. This same strategy was also applied to question 3 in test 2 where 150 students responded incorrectly. The orientation of this figure was similar to the one used in test 1, but one of the subtended angles was not at the circumference. Only 25 students realised that the answer was false and only 16 out of the 25 managed to give the correct value for x . The fact that the vertex S was not on the circumference served as a strong distracter and the majority of students inappropriately applied the subtended angle theorem.

1.2	2.6	Angle at the centre = 2 angle at circumference	165	167	[Q1] 4 * 37 **	[Q2] 185	[Q10] 176
Row 2							

(2) In row 2, 185 and 176 students responded correctly to question 2 and 10 in test 2, respectively.



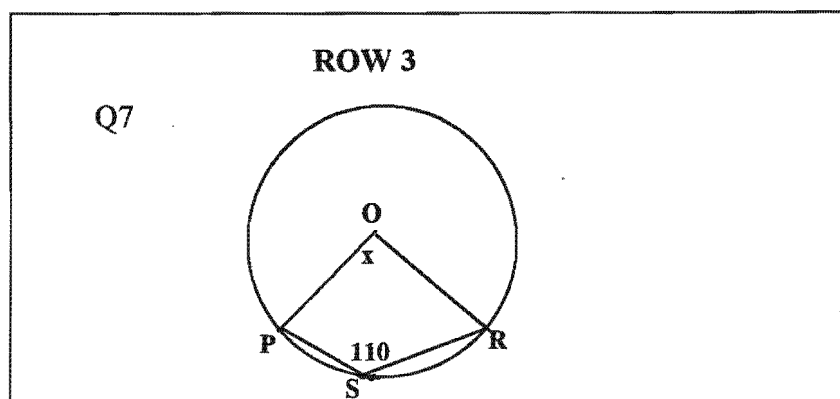
The orientation of both sketches was very different and a similar strategy, as noted in row 1, was used. However, the fact that the correct statement was given may also have strongly influenced this correct response from students. The **angle at the centre** also triggered a familiar understanding of the sketch. This notion was

strengthened by the students response in question 1, where only 41 students responded correctly, in terms of false, and 37 out of the 41 responded with the correct value for x. The orientation of this sketch was different to the one in test 1, where 165 and 167 students responded correctly in the completing of statements and visualising tests respectively.

Due to the nature of test 2 it also seems as if students, especially those students with a poor geometric register, can spot the correct answer (assuming they did not guess) if its given, hence the good response to all the true statements. This, however, does not eliminate the use of good guessing by a number of students. This is a common observation across all the 'true' items in test 2. The questions that yielded false responses, but looked very familiar despite orientation changes, were generally very poorly done as reflected in table 4.2. Despite the fact that the sketches in Q1 and Q2, in particular, were based on the same theorem, admittedly with slightly different orientations, the difference in responses was very striking.

1.3	2.1	A cyclic quad is a quad with the four vertices on the circumference	81	160		[Q7] 8 * 14 **		
Row 3								

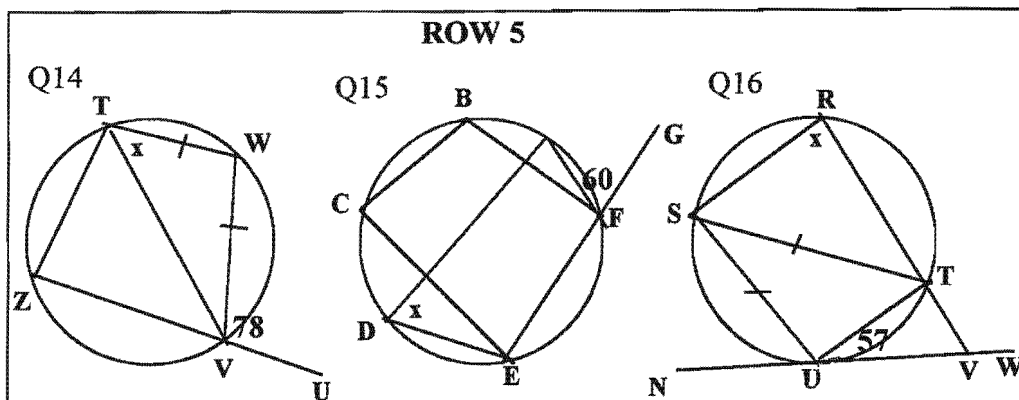
(3) In row 3, only 22 students realised that the given sketch in question 7 in test 2 (drawn below), was not a cyclic quadrilateral, despite the fact that 160 students correctly visualised what a cyclic quadrilateral looked like.



The orientation of this sketch was similar to the one in test 1, but had one vertex of the quadrilateral at the centre of the circle. This proved to be a great distracter for students. Of the 22 students that gave the correct response, only 14 managed to solve for x . In other words only 5.2% solved question 7 correctly, despite the fact that 59.9% could identify a cyclic quadrilateral in test 1.

1.5	2.3	Exterior angle of a cyclic quad = the interior angle	172	159		[Q14] 1 * 1 **	[Q15] 218	[Q16] 1 * 9 **
Row 5								

(4) In row 5, the theorem (**the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle**) involved in question 14, 15 and 16 yielded the best response in terms of the highest percentage of students got this statement correct, namely 172, while 159 students visualised the theorem correctly from a given sketch.

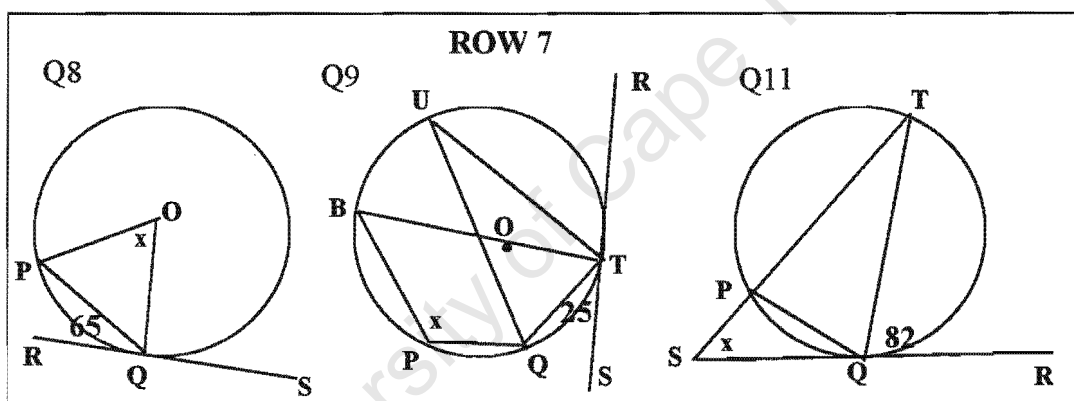


Despite these good statistics in test 1 (suggesting that a number of students knew their theory) only 1 student responded correctly to question 14 (drawn above) in test 2 and said that x could not be calculated. There was insufficient information. Perhaps the fact that there was not enough information was something very new for the students. The orientation of this sketch was similar to the one in test 1, but included a diagonal, which served as a distracter. This really distracted the students in that 179 said the given statement in question 14 in test 2 was true. Students were simply unable to give the correct interior opposite angle.

However, in question 15 (drawn above), 218 students responded correctly. But, once again question 15 was a true statement. In question 16, 188 students said the given statement was true, an incorrect response. Only 10 students responded correctly and 9 of them managed to solve for x . In question 14, distinguishing the interior angle was a problem. While in question 16, they failed to recognise that the exterior angle of the cyclic quadrilateral suggested as the reason was in fact, not the exterior angle. In other words angle TUV was not the exterior angle.

1.7	2.7	Angle between tangent and chord = angle in alternate segment	109	120	[Q8] 3 * 5 **	[Q9] 3 * 5 **	[Q11] 6 * 1 **
Row 7					[Q12] 197	[Q16] 1 * 9 **	

(5) In row 7, we note that 109 and 120 students responded correctly in the two categories, completing the statements and visualising respectively, based on the theorem **the angle between the tangent and the chord is equal to the angle in the alternate segment.**



However, in applying this theorem students were unable to see the incorrectness of the given statements in each of the following: (all questions from test 2 and drawn above).

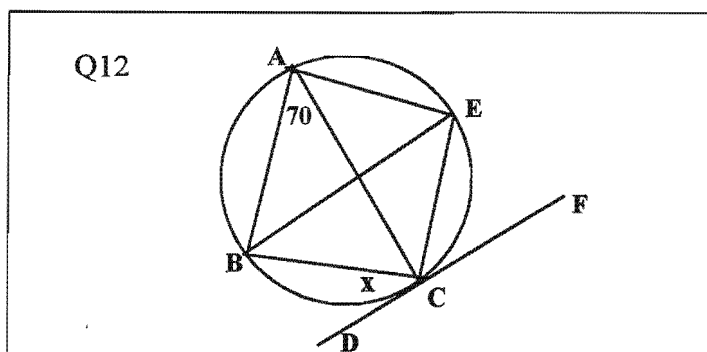
- Question 8: 8 students responded correctly, while 196 said the given statement was correct. The orientation of this sketch was similar to the one in test 1, but did not have an angle in the other segment or the point was at the centre of the circle and not at the circumference. Only 5 out the 8 students managed to solve for x . Only 1.8% solved question 8 correctly, despite the fact that 44.9% correctly visualised the theorem from a given

sketch. The fact that the angle was at the centre and not at the circumference proved to be a distracter. The majority of students fell into this trap.

- Question 9: Once again only 8 students responded correctly, while 152 thought the given statement was correct. Students failed to choose the correct angle in the alternate segment, that is, they accepted that the given angle x was the angle in the alternate segment. Only 5 students were able to solve for x . It is clear from the responses that virtually all the students did not have a strategy or a technique for finding the angle in the other segment. Consequently, only 1.8% solved question 9 correctly.
- Question 11: 7 students responded correctly and only one of them correctly noted that x could not be determined. Once again 153 students agreed with the given statement. This time they did not see that the two chords meeting at the circumference forming the angle in the alternate segment was incorrect. The given point was outside of the circle, but this made no difference to their response.

In all three questions above the fact that the angle in the alternate segment was (1) inside the circle (2) outside the circle and (3) not subtended by the same chord, made absolutely no difference to the students' application of the theorem.

- Question 12 (drawn below): 197 students correctly responded true. This



highlights the fact that students with or without a good geometric register respond in a number of different ways. As a summary these ways would include:

- simply guessing.
- look at familiarity of sketches. (chiefly due to nature of test).
- governed by prototypes (where the sketches in test 1 serves as prototypes) in terms of not being able to apply theorems when the orientation of the sketch is different.
- lack confidence in solving for x . (only a few fully correct solutions, see appendix 4 or the table below).

The table 4.3 below represents the questions in test 2 that contained all the false solutions and the number of students that were able to solve for x .

TABLE 4.3: False responses

QUESTIONS	FALSE A	FALSE B	CORRECT
1	43	4	37
3	84	9	16
5	43	8	77
7	74	8	14
8	53	3	5
9	91	3	5
11	93	6	1
13	79	6	6
14	76	1	1
16	59	1	9
TOTAL	695	49	171 (out of 2670)

False A: students respond with false but no reason or an incorrect reason.

False B: students respond with false and correct reason.

Correct: students solve for x .

The above table has been included to give an indication of how many students out of the sample 267 were able to solve the questions with false responses. The false responses were chosen because it provided answers with respect to reasons for the students' response as well as an indication of their ability to solve the problem. The

true responses, as already mentioned, caused problems in terms of an inability to eliminate or ascertain the degree of guessing or to eliminate the influence of the correct answer provided in the question.

Only 1.8% were able to say false with a correct reason, while 6.4% of the students were able to solve for x . This poor percentage included both categories of students, namely, the group with the geometric register in place (meaning students' theoretical framework is in place), and the group without a geometric register in place (meaning students' have a minimal or no theoretical framework). The formation of this geometric register in the first category of students may be due, as already mentioned, to a good memory, rote learning or a sound understanding of circle geometry.

To focus on this investigation "into how grade 12 students understand and solve geometric problems", the group of students with an intact geometric register played an important role. The underlying assumption is that students with a good theoretical basis of geometric principles should be able to solve basic geometric problems. However, only 6.4% (171 out of 2670 possible correct responses) of students were able to solve for the unknown x in 10 problems. The following section provides an indication of the thought processes of those students with a geometric register in place.

4.5. Interviews

Students Perspective

In the previous section I presented the researcher's perspective, based on the general responses of students in the two tests, as well as an analysis of the statistics based on the performance of students in the tests. In this section I present how the students analysed or solved the problems based on their response in a series of interviews. Firstly, I highlight the difficulties in conducting the interviews at each school. Secondly, I suggest a rationale for the interviews. Finally, a look at the striking features through a series of focus questions.

4.5.1. Difficulties in conducting the interviews and transcribing of tapes

(a) Venues

At Esangweni (computer room), Glendale (science laboratory) and Portland (science laboratory) the venues were not suitable for interviews. The rooms allocated were easily accessible by other students and staff members resulting in the interruption of many interviews. Unfortunately, there were no other venues available and the interview sessions continued. At Sinethemba the mathematics HOD's office provided the best venue for the interviews.

(b) Noise

Added to the venue problem was the excessive noise in the background. The noise came from students who were not in their classroom during lessons, from students who enjoyed their interval, from knocks on the door and sirens depicting the end of

periods or beginning and ending of interval. These were all picked up on the recording tapes, resulting in transcription problems

(c) Indistinct speech of students

A number of students were slightly indistinct during the interview sessions and this made transcriptions very difficult. Some students were slightly indistinct due to their inability to pronounce a word, sometimes they were swallowing their words, some spoke very fast, some engaged in mumbling especially when thinking aloud and some due to their lack of confidence in answering questions.

(d) Interview technicalities

Due to the nature of the interviews, students basically had to explain their solutions and give reasons for their answers, a major stumbling block was transcribing when students were merely gesturing and pointing to sketches, especially when angles were not named. Some students had difficulty in expressing themselves in words and used actions instead. Unfortunately, some of these actions were lost during transcribing.

(e) Personality of students

The majority of students were eager and willing to participate in the interviews. Some were confident in answering the questions promptly (even when wrong) and those students spoke clearly. There seems to be a correlation between confidence and clarity of responses. Most of the students remained very positive throughout the interview despite the time taken and questions (sometimes it seemed like an interrogation) asked.

4.5.2. Rationale for interview

In chapter two, the following notion was mooted that if the geometric register exists then there are two possibilities, which forms the core of this investigation:

1. Students should perform well in problem solving (application of their theory).
2. Students may struggle with problem solving, because the orientation of sketches was different to those in test 1 question 2. The role of prototypes may be an attributing factor of low-achievement in problem solving.

The interviews with 21 students have provided a wealth of information in terms of how students understand their geometric definitions and theorems, how they apply this understanding, the emotions encountered during problem-solving and the errors they make in applying their theory. The following section will focus on the responses of students in the following categories: *subtended angles, cyclic quadrilaterals and tangents*. These responses only provide a partial picture about possible interactions between the geometric register and the ability to solve routine problems, especially how they access that knowledge during problem solving. How they access this knowledge will determine how to improve current levels of understanding.

At the end of each interview the students were asked to indicate which sketch or sketches (see appendix 3) were used by their teacher to introduce a particular theorem. They were asked to rank the top two sketches. Students had no difficulty in choosing the top two sketches from the given list. This activity, together with the visualisation of test 1, provided a framework from which students responded. The following table

represent the responses of students to the four questions based on: *subtended angles, cyclic quadrilaterals and tangents*: (See Appendix 4).

Table 4.4 Most familiar sketches - response

sketches are ranked in order of priority:

STUDENT	QUESTION 1	QUESTION 2	QUESTION 3	QUESTION 4
RAFULA	3 THEN 4	2 THEN 5	1 THEN 2	2 THEN 4
MANGANI	4 THEN 1	2 THEN 1	1 THEN 2	4 THEN 2
VUYOLWATHU	3 THEN 1	2 THEN 3	1 THEN 2	1 THEN 4
ZAMEKA	3 THEN 4	2 THEN 1	1 THEN 2	4 THEN 2
NDZILA	3 THEN 1	2 THEN 5	1 THEN 2	4 THEN 2
RUSHANA	1 THEN 3	1 ONLY	1 THEN 2	1 THEN 2
LANCE	3 THEN 2	2 THEN 3	1 THEN 2	4 THEN 1
REPHWAAI	3 THEN 4	2 THEN 3	1 ONLY	2 THEN 1
BRONWYN	3 THEN 2	2 THEN 4	1 ONLY	4 ONLY
SHAKIER	3 THEN 4	2 THEN 5	1 THEN 2	4 THEN 2
SHERIZAAAN	3 THEN 4	2 THEN 3	1 THEN 2	4 THEN 1
RASMEN	ALL FAMILIAR	2 THEN 4	1 ONLY	1 THEN 2
LUVUYO	3 THEN 4	1 THEN 3	1 THEN 3	2 THEN 1
MZOXOLO	1 ONLY	2 THEN 3	1 THEN 4	4 THEN 1
GCOBISA	3 ONLY	2 THEN 5	1 THEN 2	4 ONLY
SPHOKAZI	3 ONLY	2 THEN 3	1 ONLY	2 THEN 4
CIKIZWA	3 ONLY	2 THEN 4	1 THEN 2	1 THEN 2
ETIENNE	3 ONLY	2 THEN 5	1 ONLY	4 ONLY
PAUL	3 THEN 4	2 THEN 5	1 THEN 2	4 THEN 1
WARDA	3 ONLY	2 THEN 5	1 THEN 3	4 THEN 2
SONIA	3 THEN 1	2 THEN 5	1 THEN 4	4 THEN 2
MOST POPULAR RESPONSE	3	2	1	4

The information obtained from the above table will be incorporated into the following analysis of the interviews, which will be divided into the three categories already mentioned, with 21 students across the four schools.

4.5.3. Striking Features

During the interviews the students' response to researcher's questions, based on their performance in test 2, provided some interesting insights into how students understand their theorems, insights into their misunderstandings and insights into how

they utilised their geometric register. In many cases they reinforced some of the researcher's perspectives as presented and in some cases provided some strange solutions based on their own perspective. The aim of this section is to draw on those parts of interviews that were striking for me as researcher. In each category there is a focus question followed by student responses in selected sub-divisions. The responses are in the form of extracts from the interviews. Each category is then summarised by the researcher's observations.

The following key will be used to analyse the students' response. These responses are based on their performance in test 2.

- 1) True or false answers are indicated as a correct or incorrect response depending on the question. For example:
 - Question 1 is false. If the student says true, then it is an incorrect response.
 - Question 2 is true. If the student says false, then it is an incorrect response.
- 2) If correct response is false, but students provide a wrong reason or cannot solve for x then I refer to an incorrect solution.
- 3) If correct response is false then I refer to a correct or incorrect application.

CATEGORY 1: Subtended Angles

Focus Question: Why did the majority of students perform so poorly in this category, despite the fact that they knew their theory based on subtended angles.

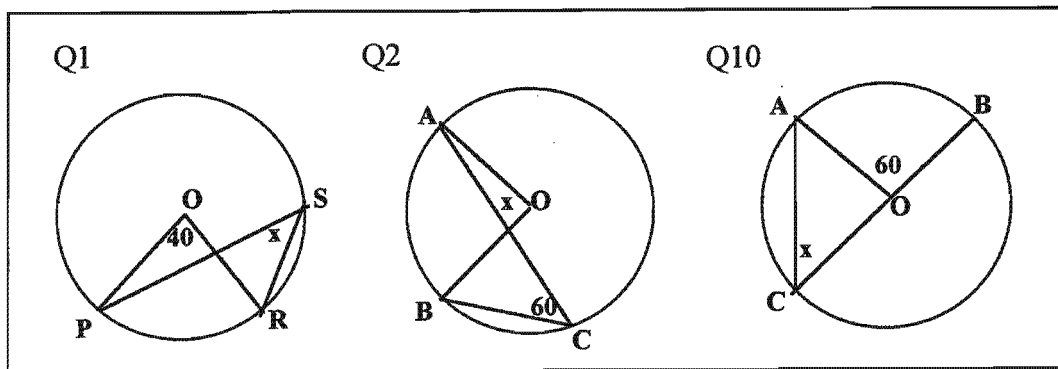
The following table represents the performance of the interviewees in test 1 and provides information regarding the completing of statements (question 1) and visualising from sketches (question 2), as well as their performance in test 2.

TABLE 4.5: Interviewees performance in test 1 and test 2.

STUDENT	TEST 1		TEST 2	
	QUESTION 1 (% Correct)	QUESTION 2 (% Correct)	QUESTIONS WITH FULL MARKS	% FULL MARKS
RAFULA	70	88.8	6,10,12,15	25
MANGANI	100	100	4,5,6,12,15	31.2
VUYOLWATHU	100	100	2,4,5,6,10,12,15	43.7
ZAMEKA	100	100	1,2,3,4,5,6,10,12,15	56.2
NDZILA	100	100	1,2,4,5,6,10,12	43.7
RUSHANA	100	100	1,2,3,4,5,6,10,12,15	56.2
LANCE	100	88.8	2,4,5,6,10,12, 13,15	50
REPHWAAI	100	100	2,4,5,6,10,12,15	43.7
BRONWYN	100	88.8	4,5,10,12	25
SHAKIER	100	100	1,2,3,4,5,6,9,12,15	56.2
SHERIZAN	90	100	1,2,4,5,6,10,12,15	50
RASMEN	100	100	4,5,6,10,12,15	37.5
LOVUYO	100	100	2,4,5,6,7,10,12,15, 16	56.2
MZOXOLO	90	77.7	10,12,15	18.7
GCOBISA	90	88.8	2,4,6,10,12,15	37.5
SPHOKAZI	100	100	2,4,5,6,10,12,15	43.7
CIKIZWA	100	100	1,2,3,4,5,6,10,12,15	56.2
ETIENNE	100	100	2,4,5,6, 8,10,12,13,15,16	62.5
PAUL	70	55.5	2,4,5,6,10,12,15	43.7
WARDA	80	66.6	2,4,5,6,10,12,15	43.7
SONIA	100	100	1,2,4,5,6,10,12,15	50

From the above table, the majority of students achieved full marks for test 1. Yet their performance in test 2 varies considerably. I have highlighted the questions that produced full marks for the false responses. This once again raises the problems concerning the true responses.

For simplification of explanation, illustration and reference, I have reproduced the sketches of question 1 and 2 from Test 2. (The same grouping as in Row 2 used earlier)



Student Responses

The students were asked by the researcher to provide a reason for their ‘incorrect’ response in test 2, based on their true or false answers. However, students were not aware or did not realise or know that their answers in the test were wrong.

Consequently, they responded in a number of different ways:

(a) **Confirmation of incorrect response:** Some were convinced that their answer to question 1 was correct and provided reasons for it. Take Mangani, for example, who responded in the interview by defending her answer in the test with “I think it true because ... for the theorem said that angles subtended by the same chord are equal” (App 6, pp 6). This response from Mangani was incorrect because she used the wrong theorem.

(b) **Immediately spotted their error.** As soon as Mangani confirmed her response to question 2, she realised her error and simply declares “Ja ...I don’t know why I said its false” (App 6, pp 6). Lance, found himself in a similar position in question 2 and responded “ Well, I guess I used the same method, which was incorrect, as I did with Q1 ... I guess. in fact, I think its true, sir” (App 6, pp 24). As soon as Mangani and Lance saw their answers in the test, they realised their error and corrected it.

(c) **Confused by their own explanation.** Lance trying to explain his reason became very confused by declaring “ ... I don’t really know” (App 6, pp 24), before actually solving the problem.

(d) **The Ah-Ha syndrome.** A few of the students were so engrossed in their wrong explanation and mid-sentence discovered their error. Mzoxolo highlighted this in “Of course x is ... umm ... no ... I can say x is 20 degrees here (refers to question 1)” (App 6, pp 58).

(e) **Difficulty in articulation.** Some students had extreme difficulty in expressing themselves, as depicted by Rafula’s response.

“It was because this (refers question 1) and the subtended are the same ... (long silence)” (App 6, pp 2). The student was trying to show that the angle at the centre of the circle was the same as the angle at the circumference. He continues with “ I think it is because ... there it is not stated ... When I look at this (makes shape of sketch with finger) I don’t see ... I think this reason is false because I don’t see ... here it is only 60 degrees (points to sketch) which is stated on the ...” (App 6, pp 2). The student articulates poorly, but tries to overcome this through a variety of gestures. Rafula once again is unable to see the difference between the angle at the centre and the angle at the circumference in question 2. This is what he refers to when he says “I don’t see”.

(f) **Absurd responses.** In question 2, Rafula says “ the correct value for x is ... will be 180 degrees” (App 6, pp 2). But, x is an angle in a triangle.

(g) **About the sketches.** The orientation of the sketches caused confusion as depicted by Rafula who noted “the sketches are the same but the way they put it, is not the same”. (App 6, pp 2). Here he refers to the sketches used by his teacher. Vuyolwathu concurs that there is no difference between question 1 and question 2. Rephwaai said

“I think the same thing, only it’s ... like at a different angle, here (referring to question 1 and 2)” (App 6, pp 25).

(h) **Confident responses.** Ndzila responded with confidence to the reason for question 1 being false “ it is false because this angle is touching the circumference and this one is the centre, so I must say the angle at the centre ...” (App 6, pp16).

(i) **Lack of confidence.** Gcobisa responded as many others did by sighing and saying “I’m not sure (she whispers)” (App 6, pp 64)

(j) **Wrong application of theorem.** Some of the students doubled the wrong angle as in the case of Ndzila who said angle S in question 1 is “x will be 80 degrees” (App 6, pp16), instead of 20 degrees.

Researcher’s observations

I have merely used snippets from individual students to highlight a number of similar responses. The majority of students that were interviewed responded incorrectly to question 1 and was confused (although they responded correctly) with the introduction to question 2. Most of them indicated that they were familiar with the sketches in question 1 and 2. Only 2 students indicated that they were not introduced to the theorem, the angle at the centre is double the angle at the circumference, by means of the prototype in test 1. Most of the students, despite prompting, were unable to pinpoint which theorem was being tested. There were two very obvious weaknesses in all their reasoning and this really stood out in the interviews:

- (1) Most of the students would clearly state as a reason “angles subtended by the same chord or arc are equal”, but failed to look at the position of the angle subtended by that chord or arc. It seems as if the emphasis was simply placed on

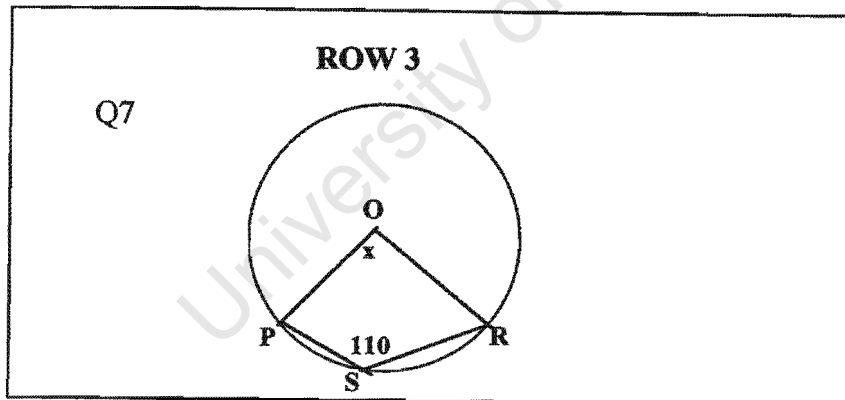
the same chord and not placed on the angle at centre and/or the angle at the circumference.

- (2) Analysing the response of students in test 1, in both completing statements and visualisation, revealed that virtually all students responded to the theorem “Angles subtended by the same chord/arc at the circumference are equal”, by leaving out the underlined bold italics part.

CATEGORY 2: Cyclic Quadrilaterals

Focus Question: Are students able to distinguish between the definition and the properties of cyclic quadrilaterals.

Question 7 from Test 2. (Same sketch used to represent Row 3)



Student Responses

A number of the students interviewed understood the difference between the definition and the property. As already highlighted the vast majority of students combined the definition and the property into one definition or statement. So in test 1 question 2.1, students simply said, “the opposite angles of the cyclic quadrilateral

ABCD are supplementary”. Responses varied greatly and for this section I will concentrate on what I thought were interesting responses.

a) Confused

Some students displayed confusion for different reasons, as shown below:

Sonia (App 6, pp 87) simply said, “I do not like that one”. Yet she knew that OPSR is not a cyclic quadrilateral. She did not like the problem because she could not apply the theorem that the reflex angle at the centre is also equal to the angle at the circumference.

Rasmen (App 6, pp 47) was in the same predicament, “Question 7 ... I think umm ... it was too difficult for me. I was guessing about it”. Despite his apparent confusion, with prompting, he managed to get the correct solution using the ‘angle at the centre’ theorem.

Sherizaan (App 6, pp 41) was really confused and struggled to articulate this confusion. She said

“... I think I have a problem with the cyclic quad, right? Now, I always think it has to go like that (indicates something on sketch, draws a true cyclic quad with her fingers) ... so when I see something like that (refers to the question7) ... I always get confused, because I always think, it can't be like that, man. So I forgot that ... somebody brought to my attention the other day in class, that umm ... it has to touch like four places ... So I was thinking it can be in the middle ... but it just doesn't seem right to me, like that ... that's why I said it ...”

b) Changed original responses immediately

A few students changed from a true answer in the test to a false one immediately after being asked the question in the interview. Bronwyn (App 6, pp35) responded in the following way:

Researcher: And the diameter subtends ... 90 degrees ... Okay that's fine ... What about question 7? You said its true ... would you still agree with that?

Bronwyn: No ... (barely audible).

Researcher: Why not?

Bronwyn: Its not touching all points of a circle (indicating on the sketch) and because in a cyclic quad its equal to 180 degrees (referring to the angles of a cyclic quad).

Researcher: Okay, and that ... (pointing to OPSR).

Bronwyn: ... that is not a cyclic quad.

Researcher: So you disagree with what you wrote there ... (pointing to her true answer in question 7 in her test script).

Bronwyn: ... (must have nodded affirmative)

In the test Bronwyn indicated that OPSR is a cyclic quadrilateral. In the interview she realised that OPSR is not a cyclic quadrilateral. Lance had similar problems in his interview.

c) Incorrect response: not clear understanding of definition

Warda agreed (wrongly) that OPSR in question 7 was a cyclic quadrilateral but really struggled to articulate her reason and responded with "Its umm ... its four corners ... say with a circle its four corners ... which ... its in the ... how can I say it ... a quadrilateral inside the circle ... four points that's on the circle" (App 6, pp 85). She did not distinguish clearly in her mind that a quadrilateral inside the circle and four points that's on the circle is not the same.

Gcobisa (App 6, pp 65) displayed similar problems in the following extract:

Researcher: Okay, and what is a cyclic quad?

Gcobisa: A cyclic quad is a four-sided figure.

Researcher: Just a four-sided figure?

Gcobisa: No ... with unequal ... (indistinct word).

Researcher: (turns to 2.1) ... is that a cyclic quad ... ABCD.

Gcobisa: Yes.

Researcher: Why is that a cyclic quad?

Gcobisa: The square is inside the circle.

Researcher: Okay, so you mean in or that the corners touch the circle.

Gcobisa: Ja.

By referring back to the theory part of the test helped Gcobisa's understanding of the

cyclic quadrilateral.

d) Confident correct response to true or false, but provides incorrect application

Paul was very confident in his response, when he said “First of all, its not a cyclic quad ... I know that if this (indicates to angle 110 degrees) angle here is twice that angle (referring to x), then this must be half it” (App 6, pp 81). His response that OPSR was not a cyclic quad was correct. He even used the correct theorem to solve the problem, namely, the angle at the centre is twice the size of the angle at the circumference, but the application of the theorem was incorrect. He did not use the reflex angle at the centre.

Similarly Cikizwa also, responded correctly by saying question 7 was false, but that x is equal to 220 degrees. She used the ‘angle at centre’ theorem, but applied it incorrectly by not using the reflex angle at the centre.

Etienne’s response produced a common wrong reason for question 7. He said false (correct response), but added that “To my understanding x is also supposed to be 110 degrees, because you can’t say its 70 degrees, its not a cyclic quad ... because they don’t say so ...” (App 6, pp77). He assumed that opposite angles of a quadrilateral are equal. In the rest of his response he contradicts himself as he discovers, incorrectly, that x must be 70 degrees.

Lance found himself in a similar position with the following response: “Okay, ... look, That’s the angle at the circumference (referring to 110 degrees, so we looking for the

angle at centre, but we can't use that angle (referring to x). So we have to use an exterior angle and using that is probably angles round a point, which is 360 degrees. So taking a look at that, that will be 55 degrees, the point around that angle (referring to O). So x won't be 55 degrees, its actually 360 minus 55". (App 6, pp 25). He applies the theorem based on the angle at the centre wrongly.

e) Incorrect response: prompts used by researcher?

A number of the students responded incorrectly, but through a series of questions from the researcher realised that they made a mistake. The following extract from Sphokazi's (App 6, pp 65) interview is an example of this interaction:

Researcher: Okay, lets look at question 7. You agreed with me. I said is equal to 70 degrees. The opposite angles of a cyclic quad are supplementary. Would you still agree with me?

Sphokazi: Yes, sir.

Researcher: Yes. Okay. Now in your bookwork, what's ABCD?

Sphokazi: A cyclic quad.

Researcher: Okay, what do you think is different from that one and that or are they the same? (refers to 2.1 and question 7).

Sphokazi: No it's not the same.

Researcher: What's different?

Sphokazi: ... because all the angles are in the circumference, and the other, only three in the circumference. (refers to 2.1 and question 7).

Researcher: Okay, so that's not a cyclic quad. So that means ... can we still apply that property to the cyclic quad?

Sphokazi: No ...

Researcher: No ... alright ... So how do you think you will calculate the value of x , there?

Sphokazi: I must found this angle ... (moves finger around on point O).

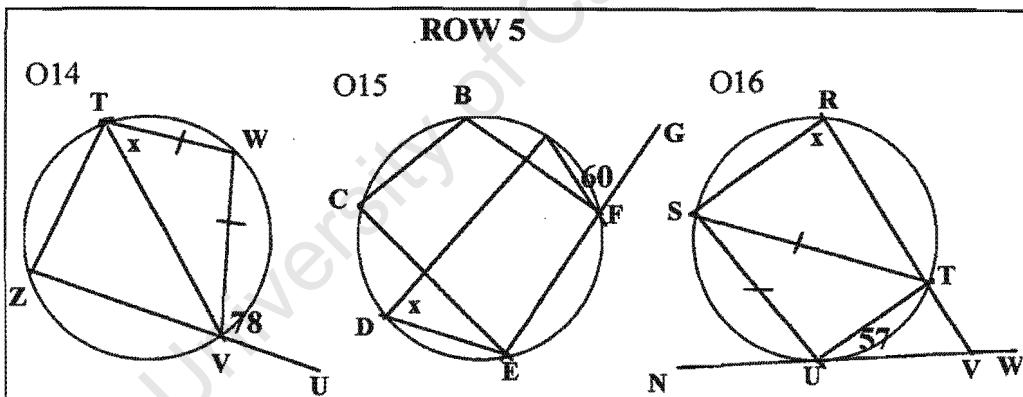
Researcher: And how do I get that angle?

Researcher's observation

A number of interviewees said, incorrectly, true to the question in the test. They applied the property, namely, opposite angles of a cyclic quad are supplementary, to a

quad that was not cyclic, but what they thought was cyclic. Some were convinced that their response was correct and were shocked to discover that they were wrong. Those who knew that the quad was not cyclic, struggled to find an alternative strategy to solve for x . Some used the correct theorem, but the implementation or application was poor. It was obvious that virtually all the students were unable or did not know how to use the reflex angle as the angle at the centre of the circle. This was despite the fact that 8 of the group indicated that it was the second figure used by their teacher to introduce that particular theorem and that they were familiar with the sketch.

I have looked at the application of cyclic quadrilaterals in question 7 and now will analyse question 14, 15 and 16 in test 2.



Student Responses

a) Assumption laws:

A number of the interviewees were guilty of the assumption law, that is, they simply assume that lines are parallel, because they look parallel and not because they are informed that the lines are parallel. In question 14, for example, students assumed that TV is a diameter. The following extract depicts Rafula's (App 6, pp5) reason for his answer to question 14:

Researcher: Okay, that's fine ... then lets look at no. 14 quickly. umm ... You said that one is true. How did you go about answering that one?

Rafula: I also thought of the two lines ... this ... I thought about this (pointing to line ZU) ... and it goes like this and it turns here and then this angle is equal to this one because that forms a Z.

Researcher: So it formed a Z again.

Rafula: Yes.

Researcher: So that's why they are equal? ...

Rafula: Yes.

Researcher: And the reason that we use?

Rafula: (Mentions reason) ... (a bit indistinct) {researcher's emphasis - student referred to alternate angles}

Researcher: So you agree with that theorem. (refers to reason in sketch).

Rafula: Yes, that theorem is correct, but, I don't know how to apply it ... this statement (refers to sketch) ... that is why I use that ... (refers to sketch ... the Z).

Researcher: Okay, did your teacher show you a mechanism for this (points to the sketch).

Rafula: Yes.

Researcher: Okay, can you remember what he showed you?

Rafula: No ... not now.

He assumes that the lines are parallel and uses the Z-strategy to show angles are alternate. However, he admits that he does not know how to apply the theorem, the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle, even though he correctly answered the theory questions.

b) Absurd responses

Rafula declares that the answer to question 16 is "there I said x is equal to 0" (App 6, pp 5). He refers to an angle that is equal to 0 degrees and in terms of the problem does not make sense.

c) Confused responses

Bronwyn admitted at the end of her attempts to explain her solution to question 14 that "And that's ... (indistinct) and then I became confused" (App 6, pp 37). However,

later in the interaction, with prompting she could confidently declare that “It will be the whole of angle T”, the correct response to question 14.

Mangani (App 6, pp 9) in answering question 16 displayed a confused response in the following extract:

Mangani: No. 16 ... umm ... LONG PAUSE

Researcher: You said its true.

Mangani: Okay, I said the exterior angle is equal (points to angle in the sketch) is equal to the interior opposite angles. (points to U and R). So its still the same thing as that ... (indicating parallel notations on sketch).

Researcher: Okay, so you agree with me that TUV is an exterior angle.

Mangani: Yes, or is an angle between a tangent and a chord.

He is confused about the exterior angle of a cyclic quad and an angle between a tangent and a chord. He clearly does not know the difference. This confusion is also highlighted in the interaction with Ndzila (App 6, pp 18 and 19). In question 14 and 15 the exterior angle of a cyclic quadrilateral was tested, yet Ndzila used the theorem in 14, but responded with the angle between tangent and chord theorem. She only realised her error when prompted by the question “is GE a tangent”. Her response was no, then she said “the exterior angle is equal to the interior opposite ...”.

Lance’s response to question 16 also suggests a level of absolute confusion in the following extract: (App 6, pp 28).

Lance: I mentioned that its true ... probably false, though its quite difficult to recognise the opposite angle ... its probably S or R ... obviously when I wrote the examinations I went for R ..., x is the opposite angle which is equal to 57 degrees, umm ...

d) Confident, despite incorrect solutions

A number students were convinced that their incorrect response was perfect as depicted by Rushana in answering question 16: “... this one is the exterior angle of a cyclic (refers to RSUT), so then it equals the interior opposite angle, that’s

why I got $x = 57$ degrees” (App 6, pp 23). Despite her confidence she did not realise that the given angle was not an exterior angle.

e) Correct responses

A few of the students were not able to articulate their correct responses very well, but knew what they were saying. Zameka displayed this in “Okay, sir, this x should be 78 degrees, but x is not 78 degrees, because this angle (refers to V), this exterior angle is equal to all angle T ” (App 6, pp 15). The first x referred to the whole angle T , while the second x referred to the x in the sketch.

f) Most interesting response to researcher

The interaction with Lance (App 6, pp 28) really provided an insight into how one student thinks in a problem situation. Follow the interaction in the following extract based on question 14:

Researcher: Okay, question 14, you also said its false ...

Lance: Ja, because, as my reason was here, although incorrect. I felt that the opposite angle of a cyclic quad is equal ... to the exterior angle of a cyclic quad is equal to the opposite interior angle.

Researcher: Yep ...

Lance: Then it will be the whole of angle T (referring to T on sketch 14) not T_1 or T_2 for that matter (breaking T into two adjacent angles).

Researcher: Okay, great.

Lance: So you couldn't go about solving for x using that method ... so the method I rather used I took, ... what ... I assumed that TV was a diameter. It meant that W was a 90 degree angle.

Researcher: Okay, so ...

Lance: Not interior angle of a cyclic quad, sorry, sum of the interior angle of a triangle. They give you that WT and WV are equal, so that means I just used the base angle, isosceles triangle for TVW .

Researcher: Okay ...

Lance: So we ... (very indistinct) ... 45 degrees.

Researcher: Okay, ja I'm happy with what you saying there, there's just the one thing ... you assumed that TV was the ... diameter, and do you think that assumption is correct? That's all ... we wont solve the problem.

Lance: Ja ... so okay ... probably one of the first lessons we learnt in geometry is never to assume, so that was a bit rash on my part.

Researcher: Alright, ... so that threw out the calculation, otherwise it would have been absolutely ...

Lance: correct, ja.

Researcher: Perfect.

Lance: it would just be on the basis if TV was actually a diameter.

Researcher: That's right ... okay ... okay, we're busy with Q16, lets see how you went about solving that one?

The above response is interesting in at least two ways:

- He realised that he could not solve the problem using his original observation, despite the fact that he used the correct theorem and also applied the theorem correctly. The correct solution was that x cannot be calculated. He opted for another solution.
- From an excellent explanation, he preferred to assume that TV was a diameter. Using this assumption he could get an answer, which was much better than his earlier attempt. He did acknowledge that his action was rash and not in line with geometrical thinking.

g) Prompting not enough

In some cases prompting did not help students as represented in the following extract based on question 16: (App 6, pp 32).

Researcher: Okay ... alright, lets go onto the last one. Question 16, you said its true. How would you go about that one?

Rephwaai: Yes, sir ... because of the exterior angle of a cyclic quad is equal to the interior opposite angle.

Researcher: Okay, so you would agree with exactly what's written, there. (refers to reason next to sketch).

Rephwaai: Ja.

Researcher: So, that is the exterior angle (refers to given angle 57 degrees).

Rephwaai: Ja.

Researcher: Now, coming back to your ... this one (refers question 2.3) ... you gave the statement which said, the exterior angle is equal to the interior opposite angle ... so is there any difference (referring to question 2.3) or is this exactly the same as what you've said there (referring to question 16).

Rephwaai: I don't see a major difference.

Researcher: So you would still agree that, that is the exterior angle (referring to given angle 57 degrees).

Rephwaai: (student must have nodded his agreement, no indication on video).

Researcher: Okay ... alright.

Despite efforts to highlight the comparison between the prototype suggested in the theory and the application in question 16, the student saw no difference.

h) Careless mistakes

Some students were guilty of careless mistakes in the form of wrong application of theorems. This is shown in the interaction with Sherizaan, where she applies 'the exterior angle of a cyclic quad is equal to the interior opposite angle' theorem incorrectly and then use the 'radius perpendicular to tangent' theorem, despite the fact that there is no radius.

The following extract represents this interaction: (App 6, pp 45).

Researcher: And, and you could also take me through your reasoning there (referring to response in question 16).

Sherizaan: (laughs) ... (points to sketch, says something to herself while working out answer) ... I said, okay, that the exterior angle of a cyclic quad, right. So S would be 57 degrees (refers to whole of angle S), and then the opposite angles of a cyclic quad is 180 degrees. So I got that ... umm ... I think the other theorem that I said about the ... tangent is perpendicular to your radius, so sir I made that 90 degrees (refers to angle SUV), and then this is angle (indistinct) ... so I draw a straight line and then probably that's how I got it.

Researcher: Okay, so you would say ... are you saying SU is a radius?

Sherizaan: No, it's a chord. SU is a chord.

Researcher: So if that's not the radius then how did you get 90 degrees, there (referring to angle SUV).

Sherizaan: (laughs).

Researcher: So again ... that's a tricky one.

RESEARCHER'S RESPONSE

Considering that all those interviewed had a theoretical understanding of the 'exterior angle' theorem, it was evident that students lacked a strategy for applying this theorem. Drawing the diagonal in question 14 proved to be a strong distracter and caused a great deal of confusion. The law of assumption was emphasised by students simply accepting that the diagonal bisected the angles of angle ZTW. They found lines that were parallel without indication or instruction that the lines were parallel.

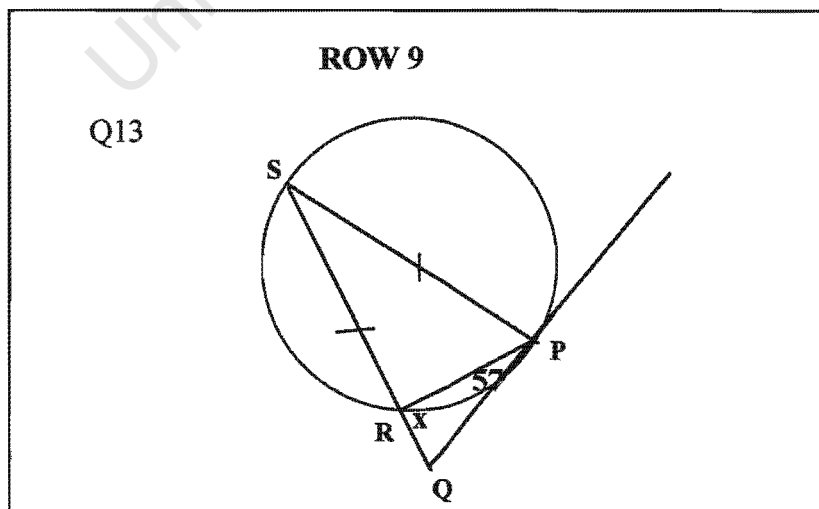
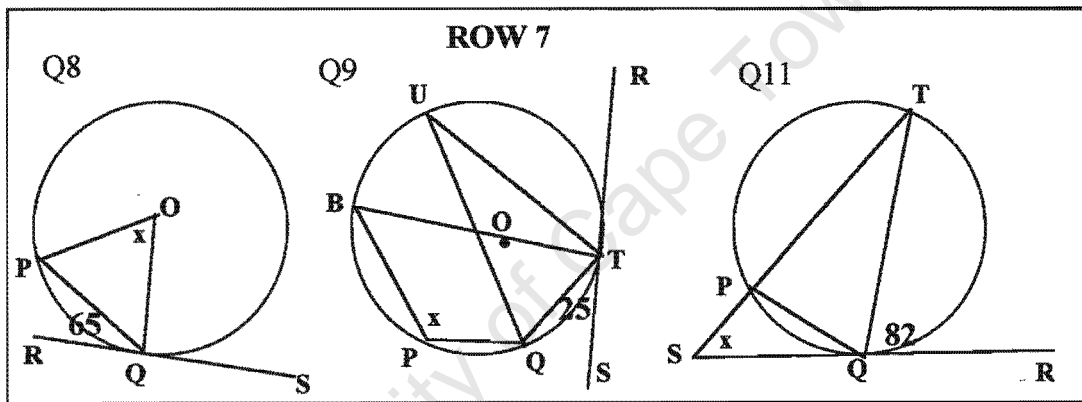
In question 16, a number of the students were confused with regards to the exterior angle. The fact that an exterior angle is formed when one side of a cyclic quad is produced, fell outside the understanding of those students. For the majority of students any line passing through a vertex of a cyclic quad is construed as forming an exterior angle with one of the sides of the quad. Despite the fact that students knew the theorem they lacked this crucial understanding which impacted on their problem solving ability.

In questions 14, 15 and 16 in test 2, students were able to pick out the cyclic quadrilaterals without much difficulty. The major problem in the questions was the inability of students to implement the property, **the exterior angle equals the interior opposite angle**, of a cyclic quadrilateral correctly.

CATEGORY 3: All about tangents

Focus Question: Do students possess a strategy for applying the tangent theorems and what are their difficulties in implementing the theorems in problem solving activities?

In this category I focus on question 8, 9, 11 and 13 from test 2. (same grouping as row 7 and row 9)



Student Responses

The majority of the students used the tangent/chord short cut to represent the following theorem: The angle between the tangent and a chord is equal to the angle in the other or alternate segment.

a) Confidence and Convincing

In most of the interviews the students were very confident in their solutions and were convinced that their efforts were correct. In the previous categories this aspect was already noted.

b) The Ah-Ha Experience

Rasmen displayed utter surprise and shock at his mistake and produced, for me, the best Ah-Ha experience in all the interviews. The following extract depicts this surprise that he was wrong, after a rather convincing argument that he was correct:

(App 6, pp 49)

Researcher: Sorry ... Tell me about it?

Rasmen: This angle is right ... is right, so I'm correct.

Researcher: So you happy with that?

Rasmen: Yes, I'm very happy.

Researcher: So you perfectly convinced that that is true?

Rasmen: Yes that is true.

Researcher: And you're applying that theorem (refers question 2.7), correctly?

Rasmen: Yes, I'm applying ... Wha! ... (he shouts because he sees something he probably didn't notice before) ... (he laughs).

Researcher: Okay ... tell me ... tell me. (researcher smiles).

Rasmen: These are two radius (meaning radii) ... it, it didn't touch there ... (points to the circumference) the circumference ... (smiles)

Researcher: And so, so what are you telling me? So you still agree with what you did there was true?

Rasmen: No. (shakes head vigorously). It was wrong.

c) Lots of prompting

Prompts were used to get students to think about their answers and to analyse and interrogate their theoretical framework, which according to their performance in test 1 was in place. Mzoxolo changed his incorrect response, after prompting and revisiting his theoretical framework. He said, in answer to the question what's different between the sketch in question 8 and the sketch in the theory section (question 2.7), that "In this triangle here, in question 8, only two sides which are in the circumference, but all its three (referring to 2.7) are touching the circumference. Here question 2.7. So it can't be true" (App 6, pp 60). In response to the use of the tangent/chord theorem to solve question 8, he said, "According to that question now ... I've just discovered it ... you can't use it" (App 6, pp 60).

This aspect was also highlighted in the following extract where Rasmén (App 6, pp 50) admitted that he could not solve for x in question 8, but managed to solve it after prompting:

Researcher: Ja ... let me take you back to ... (turns to page 2, diagrams of theorems), what you did over here. Tangent AB is perpendicular to radius OC (referring to question 2.5) ... You see that, the radius, and your statement, you got it correct as well.

Rasmén: Yes.

Researcher: So you know ... (turns back to sketches) from your bookwork, that the radius and the tangent are perpendicular?

Rasmén: Yerr ...

Researcher: Okay ... so now we can go back there (refers to question 8).

Rasmén: Okay, this radius is perpendicular to this tangent RS.

Researcher: Yes.

Rasmén: So when the radius is perpendicular, it give us 90 degrees, so this is ... okay ... 25 (refers to angle PQO), and then the two radius form the isosceles triangle, then this base is 25 degrees, so 25 plus 25 is 50 ... is 130 (points to x).

Researcher: Its 130 degrees ... okay, making sense now?

Rasmén: Yes, its making sense now [these last few lines took long to decipher, accent not too clear]

d) Accurate responses

There was also good accurate and well thought out solutions. Luvuyo's (App 6, pp 55) response to question 8 displayed a good understanding of her theory and she articulated this understanding very clearly in the following:

Luvuyo: So this is a radius (refers to OP) and this is a radius (refers to OQ). So they (base angles) must be equal. This one should be equal to that one. So now this is 25 degrees (refers to PQO) and I say that one is also 25 degrees (refers to P) because this is a isosceles triangle. So 25 plus 25 give me 50 degrees, so that's why it is 130 degrees (writes in 130 in angle POQ).

Researcher: Okay, great. Now ... just look at the reason that I gave. I said x is equal to 65 degrees. Angle between tangent RQ and chord PQ is equal to the angle in the opposite segment. Is that, ... do you think that reason is true?

Luvuyo: Its not true, because this triangle (refers to POQ) ... it is not a full circle triangle (means circumscribed circle).

Researcher: Okay, so O should be on the circumference?

Luvuyo: Should be on the circumference (repeats last part).

It was clear from the interaction with Luvuyo that she understood her theory and had a strategy for applying the tangent/chord theorem. Rushana (App 6, pp 22) answered with understanding because she had a strategy (used fingers and gestures) for finding the angle in the other segment:

Researcher: I'm just looking at ... a strategy. What strategy did you use?

Rushana: Oh! ... because this angle over here, angle x , is the angle between a tangent and a chord, so if you do this over here (her hand covers something, cannot see what), and then angle E will be ... (mumbles something indistinct, and continues) ... this two over here, (points to B and E) are angles in the same segment, that is why it equals ... 70 degrees. And then this one, here, is the angle between a tangent and a chord (referring to angle at C), that's why it will equal to angle E (points to E on Q12).

Researcher: Okay, so you had a good strategy for doing that one, that's fine ...

e) Law of Assumption

This is undoubtedly the most common error and caused problems in terms of solving for x . In question 9 Mzoxolo (App 6, pp 62) refers to the line SP as a diameter. The majority of students accepted that QR was a tangent in question 13, even though they

knew the definition of a tangent. A typical example is the interaction with Rushana

(App 6, pp 22):

Rushana: I have no idea how I worked out that one ... (giggles).

Researcher: But would you agree that it's true, or would you change your mind now?

Rushana: No, I don't think it may be true. (giggles)

Researcher: Why not?

Rushana: Umm ... PAUSE ... there's no reason for it (laughs) ... let me see now why they say (points to reason next to question 13), PQ equals QR ne.

Researcher: PQ equals QR ... so you agree with that?

Rushana: Umm ... (agrees).

Researcher: Alright, and the rest of the part where it says tangents drawn from a common point are equal? ... do you agree with that statement?

Rushana: Yes.

Researcher: Okay, now do you agree that RQ is a tangent?

Rushana: RQ?

Researcher: Is RQ a tangent?

Rushana: Yes.

Researcher: Okay, ... why is it a tangent, or what is a tangent?

Rushana: A line that touches a circle at only one point.

Researcher: And RQ, is it touching at one point?

Rushana: No ... (giggles).

f) Incorrect use of theorems

This was Warda's (App 6, pp 85) response to "how would you go about solving question 8":

Warda: Exterior angles is equal to the interior opposite angles.

Researcher: The exterior angle?

Warda: ... Is equal to the interior opposite angle.

Researcher: Of a cyclic quad, or ... of what ... Of the triangle?

Warda: Of a cyclic quad.

Warda's reason for solving question 8 depicts a lack of understanding on two levels:

(1) She did not know what was meant by the exterior angle.

(2) She was convinced that a three-sided figure was a cyclic quad.

She also insisted that the tangent/chord theorem could still apply to question 8 even though there was no circumscribed circle.

f) Absurd Responses

Rafula (App 6, pp 4) admitted that question 8 was familiar to him, but his solution did not make sense:

Rafula: Yes, I have seen it before.

Researcher: And did your teacher emphasise that sketch?

Rafula: Yes.

Researcher: Okay, question 8 ... again ... How did you go about answering that one?

Rafula: Q8, it is said that RS is a tangent, so we are told there that x is 65 degrees, and I said that is true. Why, because, I thought of this ... they formed a Z like this (points to sketch) ... so they are equal.

Researcher: Okay, they are equal because they formed a Z?

Rafula: Yes.

He was referring to the Z associated with alternate angles, which did not fit into this solution.

RESEARCHER'S RESPONSE

In question 8, 196 students responded with an incorrect response. They thought the solution was true. It was my intention, through the interviews, to encourage students to make conjectures of their own by asking probing questions and to challenge them to think deeply about their theoretical framework. Most of the students seemed to have a clear grasp of the theorem, but were confused about the application as depicted in the interviews. The nature of the application was missing. For example, there were no real problems with the angle between tangent and chord, but the angle in the other or alternate segment caused the problem. The fact that the angle in the other or alternate segment must be formed at a point on the circumference was not important or relevant to the student.

The distracters in questions 8, 9 and 11 were sufficient to confuse the students or simply just threw them off track and they were unable to spot that the theorem could not be used. The following distracters were used:

- (3) In question 8, the chord subtended an angle at the centre of the circle and not at the circumference.
- (4) In question 9, the angle at the circumference was not drawn from the same chord, that is, at the point of contact with the tangent.
- (5) In question 11, the angle subtended by the chord was drawn outside the circle.

From appendix 2 (b), there were 196, 152 and 153 incorrect responses for question 8, 9 and 11 respectively. The fact that the numbers included those students who received full marks for the related theory questions raises concerns around the teaching of geometry.

Most of the students interviewed only discovered their mistakes, in terms of understanding the nature of the theorem, through prompting. Those students who responded correctly (in terms of true or false) were unable to solve for x . They could not relate other theorems or properties to solving the problem. They did not have a network of relations in place and their efforts lacked understanding. This result is evident from the fact that only 5, 5 and 1 (out of 267) of the students solved for x in question 8, 9 and 11.

Question 13 also proved to be problematic. The majority of students knew that tangents drawn from a common point are equal, but did not realise that one of the lines was a secant. The fact that the two lines were drawn from a common point took

precedence over the fact that both lines needed to be tangents. The distracter of including a secant instead of the tangent revealed that students, especially those with a good theoretical framework, were careless in terms of their visualisation of the theorem. From the group of 79 students, who correctly spotted the secant, only 6 managed to solve for x .

Based on the answers obtained from the interviewees those students (not many of them) who understood the nature of the tangent theorems and how they were inter-related, were able to solve for x and overall did well in the two tests.

4.6. Summary of Results

The results (bearing in mind the limitations of the study) indicate that the majority of grade 12 students involved in the study were struggling with circle geometry. Most of the problems encountered by students relate back to a geometric register that is in a number of cases virtually non-existent, inefficient (in terms of a lack of conceptual understanding) or ineffective (a lack of strategy for application). The fact that some students with an intact geometric register were unable to solve basic geometric problems is disconcerting and in terms of the study raised a number of pedagogic issues relating to the teaching of geometry.

The majority of students were unable to distinguish between a definition and a property. Only a few of the tested items yielded above 50% correct responses. More specifically, the majority of students had difficulty distinguishing between the definition and a property of the cyclic quadrilateral. There is evidence from the data

that student's who generally completed the statements correctly, could also visualise these statements/theorems from a given sketch. While students who performed poorly in the completing of statements showed only a slight improvement in the visualisation category. Based on the performances in test 1 less than 50 % of students had some kind of geometric register in place. The statistics (Appendix 2a) indicates that the majority of students do not possess an effective geometric register, which is necessary for the problem solving activities in test 2.

The questions that yielded false responses, but looked very familiar despite orientation changes, were generally very poorly done as reflected in table 4.2. Despite the fact that the sketches in Q1 and Q2, in particular, were based on the same theorem, admittedly with slightly different orientations, the difference in responses was very striking.

The findings suggest that the change in sketch orientation, from those sketches in test 1, as well as the distracters per sketch, proved to be too much for the students to cope. The analysis of test 2 showed that the majority of students struggled with different orientations of sketches and were easily put off by the distracters in the form of extra lines. These distracters included the following: (From test 2)

Question 3: Vertex S was not on the circumference

Question 7: One vertex of the quadrilateral at the centre of the circle

Question 14: Included a diagonal TV.

Question 8: Did not have an angle in the other segment or the point was at the centre of the circle and not at the circumference.

Question 1: The angle was at the centre and not at the circumference.

Question 9: The angle at the circumference was not drawn from the same chord, that is, at the point of contact with the tangent.

Question 11: The angle subtended by the chord was drawn outside the circle.

Question 13: Including a secant instead of the tangent

Based on the data it is clear from the responses that virtually all the students did not have a strategy or a technique for solving problems in test 2. For example, there was no strategy for finding the angle in the other segment. Most of the students seemed to have a clear grasp of the theorems, but were confused about the application as depicted in the interviews. The distracters were sufficient to confuse the students or simply just threw them off track and they were unable to spot that the theorem could not be used.

The position of angles in circle geometry is very important, since angles are either placed on the circumference or at the centre to allow the smooth application of the theorems. Most of the students were guilty of not applying this idea. In the application of the tangent theorems the lack of a strategy proved the downfall of most of the students. The fact that the angle in the alternate segment was (1) inside the circle (2) outside the circle and (3) not subtended by the same chord, made absolutely no difference to the students' application of the theorem.

Most of the students interviewed only discovered their mistakes, in terms of understanding the nature of the theorem, through prompting. Those students who responded correctly (in terms of true or false) were unable to solve for x . They could

not relate other theorems or properties to solving the problem. They did not have a network of relations in place and their efforts lacked understanding.

Based on the answers obtained from the interviewees those students (not many of them) who understood the nature of the tangent theorems and how they were inter-related, were able to solve for x and overall did well in the two tests.

The striking features that arose during the interview stage provided insight into how students internalised the problem and how they used their geometric register to apply this theory. Each category produced a series of responses, which included amongst others, the following sub-categories: Ah-Ha experiences, absurd responses, confident responses, confused responses, wrong application of theorems, correct responses, confident correct responses with incorrect solutions, etc.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1. Summary of findings

This study was located in four low achieving schools in mathematics where the students were poor achievers in geometry. It was my intention to look at what was going on in terms of how students learn and understand geometry.

Firstly, I looked at the geometry register of the group of 267 students. This register included definitions (expressed in words) and properties (expressed in words and pictures). This register was tested in test 1. I used the following key to give me a sense of the types of registers that I was dealing with:

- Above 70% implied an excellent register.
- Between 50 and 70 % implied a good register.
- Below 50% implied a poor register.

Secondly, I placed student numbers to the above key.

- 75 students had an excellent register (above 70%)
- 42 students had a good register (between 50% and 70%)
- 150 students had a poor register (below 50%)

The fact that 150 students (more than 50% of the sample) had a poor register made me realise that we had a problem.

Thirdly, the following question needed to be answered. Does the fact that the students have a register in place mean they will be able to solve problems? To do this I needed to look at the performance of the students in test 2.

- 1) The sample with an excellent register performed as follows:
 - 39 out of the 75 scored below 50%.
 - 36 out of the 75 scored above 50%, but below 75%
- 2) The sample with a good register performed as follows:
 - 39 out of 42 scored below 50%.
 - 3 out of 42 scored above 50%, but less than 60%
- 3) The sample with a poor register performed as follows.
 - 150 out of 150 scored below 50%.

It must be noted that the scores above 50% included a number of true responses, which may have increased the likelihood of guessing and consequently impacted on the results in test 2. This investigation has unearthed that many students do not have a geometrical register in place and consequently cannot engage in problem solving activities.

Finally, I tried to answer the next question. What is the problem? I looked at a few individuals to see if I could discover what possible problems exist. I used a sample of the students (high achievers in test 1) with an excellent register in a series of interviews. They made a variety of mistakes and prototype thinking was dominant (see section 4.6 for detail).

Chinnappan (1998) suggested that high achievers would be more likely to activate more sophisticated schemas, access these more frequently and generate mental

models that show a high degree of understanding than the low-achieving group is congruent to my findings where the high achievers are classified as having their geometrical register in place and are able to access this register effectively. My findings suggest that not all of the high achievers could access this register effectively.

5.2. Aims of the study

Hans Freudenthal's criticism of the traditional practice of teaching geometry is useful to end this dissertation by linking the aims of the investigation to the analysis of the data:

... Good geometry instruction can mean much - learning to organise a subject matter and learning what is organizing, learning to conceptualize and what is conceptualizing, learning to define and what is a definition. It means leading pupils to understand why some organization, some concept, some definition is better than another. Traditional instruction is different. Rather than giving the child the opportunity to organize spatial experiences, the subject matter is offered as a preorganized structure. All concepts, definitions, and deductions are preconceived by the teacher, who knows what is its use in every detail - or rather by the textbook author who has carefully built all his secrets into this structure.

(Freudenthal, 1973, pp 417-418)

Considering the findings in this investigation it is clear that Freudenthal's expectation on students seem to be a bridge too far (the bridge representing teaching practice). The focus of this investigation was to look at 'How grade 12 students understand and solve geometric problems' and specifically to analyse student performance and

responses, in the two tests. Then, to interview a sample of participating students, and not to look at teaching practices across the four schools, which is a limitation of this study. I agree with Freudenthal that students should be led to understand that some organisation of geometrical definitions, concepts and deductions is necessary.

However, the findings of this investigation suggests that students lack the ability to organise and structure good working schemas (Chinnappan, 1999). The interviews, in particular, highlighted these deficiencies in student arguments. Despite the limitation of not looking at teaching practices, the response from students revealed much about teaching practices at the four schools. The analysis of this study, together with Freudenthal's view of good geometric practices seem to place an indictment on geometry teaching.

Research by Human and Nel (1977 - 1978) found that a large percentage of grade 10 students did not understand the nature of axioms. The findings of this research would concur with Human and Nel, but in this case is present at the grade 12 level. There is evidence that a large percentage of students merely memorised the definitions and statements of theorems and simply regurgitated them in the tests.

According to the Van Hiele theory students who are able to understand the logical inter-relationships between properties of a particular concept are functioning at the Second Level of thought. It is at this point that students should not have problems with the properties of a figure, but should be able to deduce from one another and develop a "network of relations". A large percentage of students in this investigation were unable to function at this Second Level. This finding concurs with research by

De Villiers & Njisane (1987) where only 45% of grade 12 students mastered this Second Level.

5.3. Limitations of the study

There are some possible limitations in this investigation in terms of the research methodology that could impact on the findings:

Firstly, as already mentioned, the fact that teaching practices at the four schools were not investigated does impact on making any conclusions about classroom practices, attitudes, language of instruction, group work, etc. Making comparisons across schools cannot be conclusively done. Strategies of teachers and the use of informal and/or formal activities cannot be measured. Inconsistencies in findings were therefore left to the interpretation, bias and subjectivity of the researcher.

Secondly, the choice of possible prototypes for test 1 part 2, is a limitation because it could indirectly influence responses from students. The prototypes were chosen based on how a number of textbooks introduced a concept.

Thirdly, the choice of true or false responses for test 2, may not be a fully reliable means of testing. The true responses, in particular, caused problems in terms of the difficulty in eliminating the aspect of guessing. The false responses seemed much more reliable and forced students to respond. Consequently, this study focussed predominantly on the false questions.

Fourthly, the selection of the sample group that was interviewed, may prejudice the results in terms of only the group with an intact (meaning a good theoretical framework) geometric register were interviewed. The assumption that an intact geometric register is essential for problem solving may be a faulty one.

Fifthly, the interview process occasionally lost some key responses in the form of words and gestures. Some students had difficulty in articulating themselves and this could be due to the presence of a video camera.

Lastly, there was no detailed information regarding the geometric history of the school or students. The fact that other non-grade 12 geometric axioms were required to solve for x may have impacted on the ability of students to solve problems. The lack of prior knowledge of grade 8 to 11 geometry was a limitation.

Though the limitations do exist and need to be accounted for, I believe, it does not discount the value of the findings. The study provides an insight into how students approach geometric problems or how they utilise their theoretical framework and in particular it highlights the difficulties in terms of how students understand their theoretical framework in circle geometry.

5.4. Implications for teaching geometry

Euclidean Geometry has played a key role in the South African curriculum and South Africa has been one of the few bastions of traditional congruency geometry (Human & Nel, 1977 - 1978). The new curriculum (curriculum 2005) following an outcomes

based approach will include circle geometry in the Further Education Training (FET) phase, despite numerous schools of thought to have it removed from the syllabus. This study supports the theory (Mayberry, 1981, pp 96) that geometric thought follows a sequence of levels (van Hiele levels in particular) and the inability to develop on one level prevents achievement on higher levels. However, this investigation question what is meant by the development in a level or how do we measure and assess this development. This investigation, despite the small sample, has shown that student's with a good definitional or theoretical background struggle with problem solving activities.

The model used in this study, with its limitations, will provide teachers with some insight into lesson planning by noting deficiencies in presenting a definition or theorem. There should be an awareness of the impact of distracters and prototypes on the understanding levels of students. This awareness moves across traditional methods, where student participation is minimal or non-existent, or informal methods, where students participate and even make choices about the appropriate wording of the definition or theorem. The findings of the study suggest that teachers should consciously, through formal or informal means, ensure that students make the correct connections in terms of applying their theory correctly.

Another contributing factor in this investigation is the importance of a strategy for applying a definition or a theorem. Teachers should use mechanisms, whether they use their fingers or dots or colours on the sketch, to show how to find, for example, 'the angle in the other segment' or 'the interior opposite angle. Even the 'high-level of

understanding' group had difficulty without a strategy for application. The use of a strategy will improve levels of understanding.

Lastly, the investigation revealed that students were unable to construct a logical explanation, whether in writing (as indicated in their scripts) or talking (as expressed in the interviews) and needed guidance in this regard. Some students were able to respond positively to prompts and lead questions.

5.5. Recommendations

In terms of research

Recommendations for future research are to include geometry teaching practices across schools, as well as across grades. In addition, to track the performance of students in geometry for five years in the different contexts and the exposure to different teaching methodologies and practices. In this way the geometric history of students' understanding can be assessed in relation to the teaching practices at that school. A similar model to the one used in this study could be used to track performances of students at lower grades.

The focus of this study was to see how grade 12 students understand and solve geometric problems, based on the grade 11 syllabus. The study did not take cognisance of the history and performance of students in geometry in lower grades, an aspect that the above recommendation can fulfil. It would also provide information on how the geometrical register of students has developed and how it has been utilised over the five years.

In terms of curriculum development

Curriculum 2005 places a great deal of emphasis on geometry, emphasising the utility of informal strategies. Traditional practices have been criticised by mathematicians and researchers who suggest that supplying ready made definitions is a vicious method (Griffiths & Howson, 1974, pp 216). We should avoid the direct provision of geometry definitions (Freudenthal, 1973, pp 417). It is not deductive enough, because it is imposed on the student (Freudenthal, 1973, 402), and characterised by the study of axioms, definitions, construction techniques and theorems in a fixed order, without any informal or practical introduction (Human & Nel, 1977-1978, pp 5). Teachers should be given the opportunity to be involved in a curriculum that is exciting, challenging and enriching, hence curriculum developers should ensure the implementation of key areas of geometry. Whether this includes Euclidean geometry or particularly circle geometry is a relevant question. Gerdes (1988) suggest that we broaden our understanding of what mathematics is, which is necessary, but not sufficient. We should in our endeavours “reconstruct mathematical traditions, when probably many of them have been, as a consequence of slavery, of colonialism, wiped out” (Gerdes, 1988, pp 140). Teachers should be conscious of the hidden mathematics and should also be aware of cultural, educational and scientific value of rediscovery and exploring hidden mathematics, particularly in geometry (Gerdes, 1988).

In terms of resources (text books)

In the light of the above it would seem as if many of our textbooks are outdated in terms of the informal approaches and a restructuring of this resource is vital to the demands of the new curriculum. De Villiers (1995, pp 1) was “appalled by how poorly definitions of quadrilaterals were handled in the high school textbooks our

students were using” My experience with textbooks and how circle geometry is introduced in them has been very depressing in terms of its direct teaching approach. There is almost no opportunity for creativity and it may well be that our textbooks may be the reason for such poor geometric understanding.

In terms of teacher training

On a more personal note, based on my work in teacher training and my teaching experience, the training of our teachers is essential and according to Swafford et al (1997) one way to try to enhance achievement in geometry is to improve instructional practice. The teachers, perhaps only a small sample that I work with have extreme difficulty in teaching geometry, because of their history. They hated geometry at school and now they are forced to teach it and many of them simply avoid teaching it.

Policies for teacher training must ensure that teachers are empowered with skills to teach geometry in the ‘new way’ and create courses for conceptual development to broaden their understanding of geometry and to raise their ability to teach higher cognitive skills to our students. If this is not achieved our students, as shown in this study, will continue to memorise definitions and geometry proofs, because their teachers did it at school and is encouraged by their teachers to do the same. For them it is the only way to learn and understand geometry.

Finally, I support the view of Qing (1997) that mathematics, geometry in particular should be useful, humanistic and a vibrant subject to be explored and understood.

CHAPTER 6

REFERENCES

- Atweh, B., Bleicher, R. & Cooper, J. (1998). The Construction of the Social Context of Mathematics Classrooms: A Sociolinguistic Analysis' *Journal for Research in Mathematics Education* 29(1). 30-46
- Battista, M. (1999). Fifth Graders' Enumeration of Cubes in 3D Arrays: Conceptual Progress in an Inquiry-Based Classroom. *Journal for Research in Mathematics Education*. 30, 4, 417-448.
- Burger, W.F. & Shaughnessy, J.M. (1986). Characterising the Van Hiele levels of development in Geometry. *Journal for Research in Mathematics Education*. 17, 1, 31-48
- Chinnappan, M. (1998). Schemas and Mental Models in Geometry Problem Solving. *Educational Studies in Mathematics* 36, 201-217.
- Clements, D., & Battista, M. (1992). Geometry and Spatial Reasoning. *Handbook of Research in Mathematics Teaching and Learning*. Macmillan Publishing Company. NY.
- Clements, D., Swaminathan, S., Hannibal, M., & Sarama, J. (1999). Young Children's Concepts of Shape. *Journal for Research in Mathematics Education*. 30, 2, 192-212
- De Villiers, M. D. & Njisane, R. M. (1987). The development of geometry thinking among black high school pupils in KwaZulu. *Proceedings of the 11th*

Conference of the International Group for Psychology of Mathematics Education, Montreal, Vol 3, 117-123.

De Villiers, M. D. (1987). Research evidence on hierarchical thinking, teaching strategies and the Van Hiele theory. Some critical comments. *RUMEUS report 10*, University of Stellenbosch.

De Villiers, M. D. (1990). The role and function of proof in mathematics. *Pythagoras*, 24, 17-24.

De Villiers, M. D. (1994). The role and function of a hierarchical classification of the quadrilaterals. *For the Learning of Mathematics*, 14, 1, 11-18.

De Villiers, M. D. (1995). The handling of geometry definitions in school textbooks. *Pythagoras*, 38, 3-4.

De Villiers, M. D. (1998). To teach definitions in geometry or teach to define? *Proceedings of the 22nd Conference of the International Group for Psychology of Mathematics Education*, RSA, 12-17.

Durell, C. V. (1976). *A New Geometry for Schools*. London: Bell & Sons. (First published 1939).

Durkin, K. (1989). 'Language in mathematical education: an introduction' in K. Durkin & B. Shire (eds.), *Language in Mathematics Education*, London: Open University Press.

Finzer, W., & Bennet, D. (1995). From Drawing to Construction with the Geometer's Sketchpad. *The Mathematics teacher*, 88, 5, 428-431.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Reidel.

Fuys, D. & Liebov, A. K. (1997). Concept Learning in Geometry. *Teaching Children*

Mathematics, 3, 5, 248-251.

- Geddes, D., & Fortunato, I. (1993) *Geometry: Research and Classroom Activities. Research Ideas for the Classroom Middle Grades Mathematics*. Douglas T. Owens. Macmillan Publishing Company. New York.
- Gerdes, P. (1988). On Culture, Geometrical Thinking and Mathematics Education. *Educational Studies in Mathematics* 19, 137-162.
- Griffiths, H.B. & Howson, A.G. (1974). *Mathematics: Society and Curriculum*. Cambridge: University Press.
- Halliday, M. A. K. (1975). *Learning how to mean: Explorations in the development of language*. London: Edward Arnold.
- Hoffer, A. (1981). Geometry is more than proof. *Mathematics Teacher*, 74, 11 - 18.
- Human, P., & Nel, J. (1977-1978). *Alternative Instructional Strategies For Geometry Education: A Theoretical and Empirical Study, Final Report of USEME – project: 1977–1978*.
- Lovell, K. (1959). A follow-up study of some aspects of the work of Piaget and Inhelder on the child's conception of space. *British Journal of Educational Psychology*, 29, 104-117.
- Mayberry, J. (1981). *An investigation of the Van Hiele levels of geometric thought in undergraduate pre-service teachers*. Unpublished Ed.D. Dissertation, University of Georgia. (University Microfilms No. DA 8123078).
- Mayberry, J. (1983). The van Hiele levels of geometric thought in understanding preservice teachers. *Journal for Research in Mathematics Education*, 14, 58-69.

- Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms*, New York, NY, Routledge and KeganPaul.
- Porter, A. (1989). A Curriculum out of balance: The Case of Elementary School Mathematics. *Educational Researcher*, 18, 9-15.
- Presmeg, N. (1992). Prototypes, Metaphors, Metonymies and Imaginative Rationality in High School Mathematics. *Educational Studies in Mathematics* 23, 595-610.
- Qing, L. (1997). Angles, Lines and Circles. *Mathematics in School*. 26. March 1997.
- Schwarz, B., & Hershkowitz, R. (1999). Prototypes: Brakes and Levers in Learning the Function Concept? The Role of Computer Tools. *Journal for Research in Mathematics Education*. 30, 4, 362-389.
- Swafford, J., Jones, G., & Thornton, C. (1997). Increased Knowledge in Geometry and Instructional Practice. *Journal for Research in Mathematics Education*. 28, 4, 467-483.
- Terry, S. (1994). Geometry, Please! *Mathematics Teaching*, No 149, 32-35.
- Triadafillidis, T. (1995). Circumventing Visual Limitations in Teaching the Geometry of shapes. *Educational Studies in Mathematics* 29, 225-235.
- Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (Final report of the Cognitive Development and Achievement in Secondary School Geometry Project). Chicago. IL: University of Chicago, Department of Education.
- Van den Brink, J. (1995). Geometry Education in the Midst of Theories. *For the Learning of Mathematics* 15, 21-28.

Vinner, S., & Hershkowitz, R. (1983). On Concept Formation in Geometry. *Zentralblatt für Didaktik der Mathematik* 15, 20 - 25.

ADDITIONAL READING THAT INFORMED RESEARCH

Choquet, G. (1969). *Geometry in a Modern Setting*. Paris: Hermann.

de Villiers, M. (1997) Approaching Geometry Theorems in Contexts: From History and Epistemology to Cognition. *Proceedings of 21st Conference of the International Group for the Psychology of Mathematics Education*. Vol. 1. 196–198.

Lang, B. & Ruane, P. (1981). Geometry in English Secondary Schools. *Educational Studies in Mathematics*, 12, 123-132.

Lo, J., Gaddis, K., & Henderson, D. (1996). Building Upon Student Experience in a College Geometry Course. *For the Learning of Mathematics* 16, 1

Nakahara, T. (1995). Children's Construction Process of the Concepts of Basic Quadrilaterals in Japan. *Psychology of Mathematics Education*. Vol. 3, 19–26.

Nasser, L. (1995). Long Term Effects of a Geometry Course Based on the Van Hiele Theory. *Psychology of Mathematics Education*, Vol. 1, 213-214.

Taplin, M., & Robertson, M. (1995). Spatial Patterning: A Pilot Study of Pattern Formation and Generalisation. *Psychology of Mathematics Education*, Vol. 3, 42-49.

APPENDIX 1

Test 1 and Test 2 Instruments

University of Cape Town

APPENDIX 1: Test 1 and Test 2 Instruments

NAME:

TIME: 15 MINUTES

CLASS:

SCHOOL:

TEST 1

ANSWER ALL THE QUESTIONS

1. COMPLETE EACH OF THE FOLLOWING THEOREMS OR DEFINITIONS:

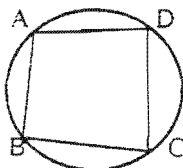
- 1.1. Angles subtended by the same arc or chord
- 1.2. The angle at the centre of the circle is
- 1.3. A cyclic quadrilateral is a quadrilateral with
- 1.4. The opposite angles of a cyclic quadrilateral
- 1.5. The exterior angle of a cyclic quadrilateral
- 1.6. A tangent to a circle is a line that
- 1.7. The angle between a tangent and a chord is equal to
- 1.8. The radius of a circle and a tangent are
- 1.9. Two tangents drawn from a common point
- 1.10. The angle subtended by the diameter

2. Write down the theorem or definition for each of the following diagrams: (O is the centre of the circle)

DIAGRAM

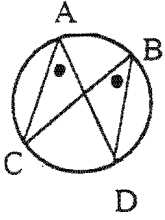
THEOREM OR DEFINITION

2.1.



.....
.....

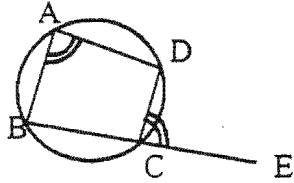
2.2.



.....

.....

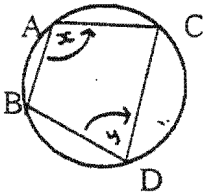
2.3.



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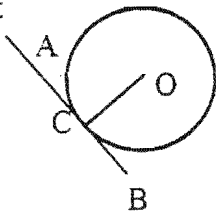
2.4.



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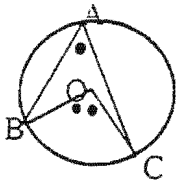
2.5. AB is a tangent



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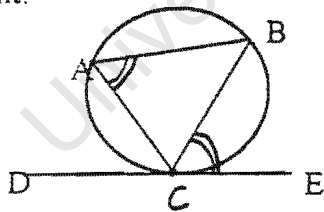
2.6.



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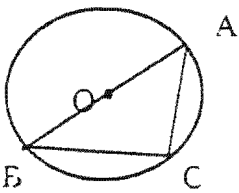
2.7. DE is a tangent.



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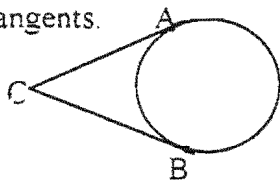
2.8.



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2.9. AC and BC are tangents.



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NAME : _____

SCHOOL : _____

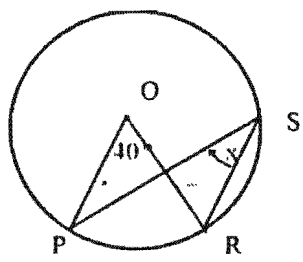
GRADE : _____

TEST 2

Some of the following exercises contain fundamental geometrical errors. In each of the following, decide whether the given value of x and the accompanying reason is TRUE or FALSE. If false, give the correct value of x and provide a reason for your answer. Show all your calculations where applicable.

NOTE: O is the centre of the circle.

NO	SKETCH	SOLUTION	TRUE OR FALSE
Q 1.		$x = 40^\circ$ REASON: Angles subtended by the same arc PR.	
Q 2.		$x = 120^\circ$ REASON: Angle at centre = 2 angle at circumference.	
Q 3.		$x = 30^\circ$ REASON: Angles subtended by the same arc QR.	
Q 4.		$x = 50^\circ$ REASON: Angle subtended by the same chord AB.	



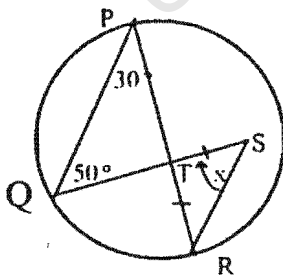
$x = 40^\circ$
 REASON: Angles subtended by the same arc PR.

Q 2.



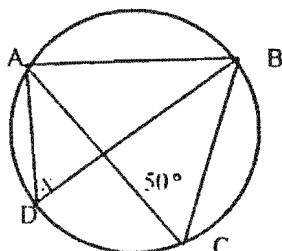
$x = 120^\circ$
 REASON: Angle at centre = 2 angle at circumference.

Q 3.



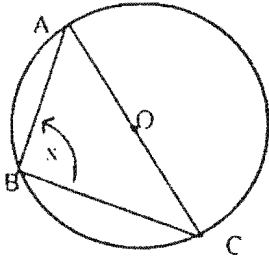
$x = 30^\circ$
 REASON: Angles subtended by the same arc QR.

Q 4.



$x = 50^\circ$
 REASON: Angle subtended by the same chord AB.

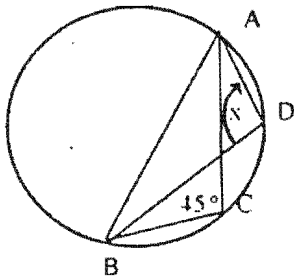
Q 5.



$$x = 60^\circ$$

REASON: Triangle ABC is an equilateral triangle.

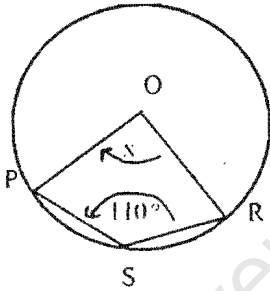
Q 6.



$$x = 45^\circ$$

REASON: Angle subtended by the same chord AB.

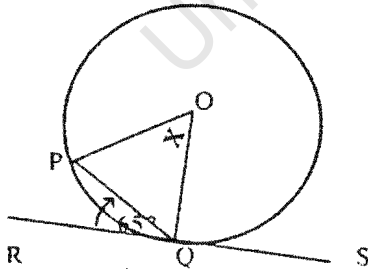
Q 7.



$$x = 70^\circ$$

REASON: Opposite angles of a cyclic quadrilateral are supplementary.

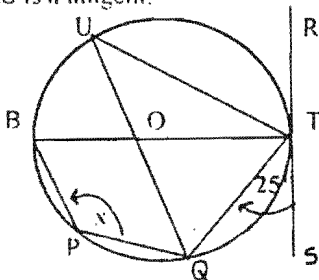
Q 8. RS is a tangent



$$x = 65^\circ$$

REASON: Angle between tangent RQ and chord PQ = angle in the opposite segment.

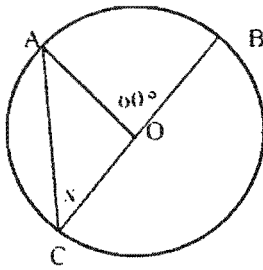
Q 9. RS is a tangent.



$$x = 25^\circ$$

REASON: Angle between tangent ST and chord TQ = angle in opposite segment.

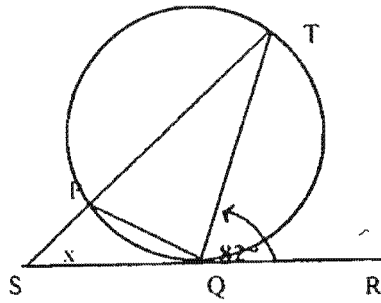
Q 10.



$$x = 30^\circ$$

REASON: Angle at centre =
2 angle at circumference

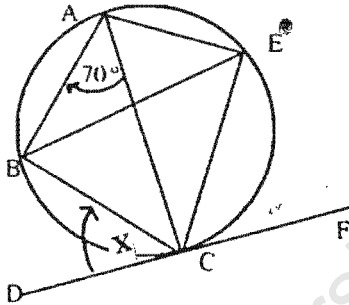
Q 11. SR is a tangent.



$$x = 82^\circ$$

REASON: Angle between
tangent QR and chord QT =
angle in opposite segment.

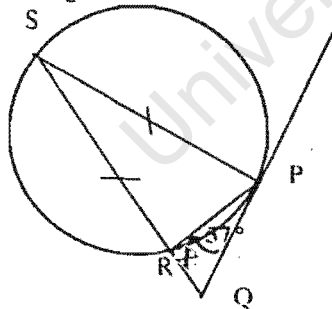
Q 12. DF is a tangent.



$$x = 70^\circ$$

REASON: Angle between
tangent DC and chord BC =
angle in opposite segment.

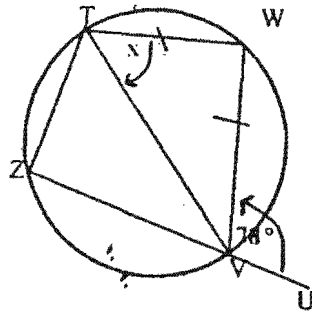
Q 13. PQ is a tangent.



$$x = 57^\circ$$

REASON: $PQ = QR$
(tangents drawn from a
common point are equal).

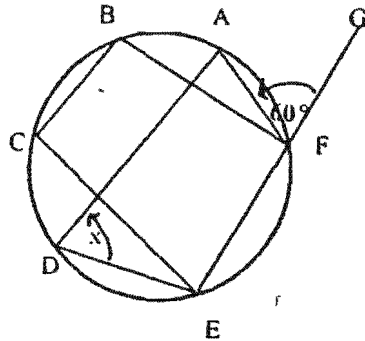
Q 14.



$$x = 78^\circ$$

REASON: Exterior angle of cyclic quad. = interior opposite angle.

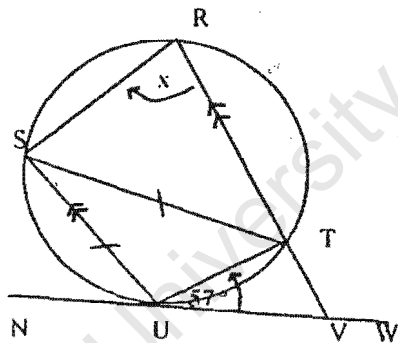
Q 15.



$$x = 60^\circ$$

REASON: Exterior angle of cyclic quadrilateral = interior opposite angle.

Q 16. NW is a tangent.
 $SU = ST$.
 $SU \parallel RT$.



$$x = 57^\circ$$

REASON: Exterior angle of cyclic quad. = interior opposite angle.

APPENDIX 2(a)

Summary Analysis of Items in Test 1

University of Cape Town

APPENDIX 2(b)

Summary Analysis of Items in Test 2

University of Cape Town

APPENDIX 2(b): Summary Analysis of Items in Test 2

NUMBER OF 0's

SCH	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	10	11	12	13	14	15	16
POR	0	0	2	1	0	2	1	1	7	4	4	1	7	3	2	3
GLE	0	0	2	0	1	0	1	1	1	1	0	0	1	0	0	0
SIN	3	4	4	4	4	3	3	5	6	7	7	7	8	6	6	6
ESA	0	2	0	1	6	0	0	3	2	1	3	0	1	1	2	1
TOT	3	6	8	6	11	5	5	10	16	13	14	8	17	10	10	10

NUMBER OF 1's

SCH	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	10	11	12	13	14	15	16
POR	39	22	42	7	18	8	41	62	31	23	37	11	33	35	2	45
GLE	20	4	16	2	10	2	17	14	19	7	16	6	18	19	4	19
SIN	57	24	47	7	50	15	58	64	65	20	54	18	59	65	16	67
ESA	64	26	45	17	50	12	50	56	37	26	46	26	49	60	15	57
TOT	180	76	150	33	128	37	166	196	152	76	153	61	159	179	37	188

NUMBER OF 2's

SCH	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	10	11	12	13	14	15	16
POR	7	0	12	0	12	0	20	5	27	1	28	1	24	32	2	16
GLE	1	0	6	0	2	0	6	8	3	0	8	0	4	5	0	3
SIN	24	0	34	0	20	0	25	21	21	0	27	0	25	21	0	19
ESA	11	0	32	0	9	1	23	19	40	1	30	0	26	18	0	21
TOT	43	0	84	0	43	1	74	53	91	2	93	1	79	76	2	59

NUMBER OF 3's

SCH	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	10	11	12	13	14	15	16
POR	0	49	4	63	1	61	2	1	2	43	2	58	2	1	65	1
GLE	1	20	0	22	3	22	0	0	0	16	0	18	0	0	20	0
SIN	2	65	5	82	4	75	4	2	1	66	4	68	1	0	71	0
ESA	1	51	0	61	0	66	2	0	0	51	0	53	3	0	62	0
TOT	4	185	9	228	8	224	8	3	3	176	6	197	6	1	218	1

NUMBER OF 4's

SCH	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	10	11	12	13	14	15	16
POR	25	0	11	0	40	0	7	2	4	0	0	0	5	0	0	6
GLE	2	0	0	0	8	0	0	1	1	0	0	0	1	0	0	2
SIN	7	0	3	0	15	0	3	1	0	0	1	0	0	1	0	1
ESA	3	0	2	0	14	0	4	1	0	0	0	0	0	0	0	0
TOT	37	0	16	0	77	0	14	5	5	0	1	0	6	1	0	9

GRAND TOTAL

SCH	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	10	11	12	13	14	15	16
No.0	3	6	8	6	11	5	5	10	16	13	14	8	17	10	10	10
No.1	180	76	150	33	128	37	166	196	152	76	153	61	159	179	37	188
No.2	43	0	84	0	43	1	74	53	91	2	93	1	79	76	2	59
No.3	4	185	9	228	8	224	8	3	3	176	6	197	6	1	218	1
No.4	37	0	16	0	77	0	14	5	5	0	1	0	6	1	0	9

APPENDIX 3

Item Analysis of Student Performance per school

University of Cape Town

APPENDIX 3: Item Analysis of Student Performance per school

PORTLAND'S GEOMETRY TEST SCORES

Student	First Name	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30	Q31	Q32	Q33	Q34	Q35	Q36	Q37	Q38	Q39	Q40	Q41	Q42	Q43	Q44	Q45	Q46	Q47	Q48	Q49	Q50	Q51	Q52	Q53	Q54	Q55	Q56	Q57	Q58	Q59	Q60	Q61	Q62	Q63	Q64	Q65	Q66	Q67	Q68	Q69	Q70	Q71	Q72	Q73	Q74	Q75	Q76	Q77	Q78	Q79	Q80	Q81	Q82	Q83	Q84	Q85	Q86	Q87	Q88	Q89	Q90	Q91	Q92	Q93	Q94	Q95	Q96	Q97	Q98	Q99	Q100	Q101	Q102	Q103	Q104	Q105	Q106	Q107	Q108	Q109	Q110	Q111	Q112	Q113	Q114	Q115	Q116	Q117	Q118	Q119	Q120	Q121	Q122	Q123	Q124	Q125	Q126	Q127	Q128	Q129	Q130	Q131	Q132	Q133	Q134	Q135	Q136	Q137	Q138	Q139	Q140	Q141	Q142	Q143	Q144	Q145	Q146	Q147	Q148	Q149	Q150	Q151	Q152	Q153	Q154	Q155	Q156	Q157	Q158	Q159	Q160	Q161	Q162	Q163	Q164	Q165	Q166	Q167	Q168	Q169	Q170	Q171	Q172	Q173	Q174	Q175	Q176	Q177	Q178	Q179	Q180	Q181	Q182	Q183	Q184	Q185	Q186	Q187	Q188	Q189	Q190	Q191	Q192	Q193	Q194	Q195	Q196	Q197	Q198	Q199	Q200	Q201	Q202	Q203	Q204	Q205	Q206	Q207	Q208	Q209	Q210	Q211	Q212	Q213	Q214	Q215	Q216	Q217	Q218	Q219	Q220	Q221	Q222	Q223	Q224	Q225	Q226	Q227	Q228	Q229	Q230	Q231	Q232	Q233	Q234	Q235	Q236	Q237	Q238	Q239	Q240	Q241	Q242	Q243	Q244	Q245	Q246	Q247	Q248	Q249	Q250	Q251	Q252	Q253	Q254	Q255	Q256	Q257	Q258	Q259	Q260	Q261	Q262	Q263	Q264	Q265	Q266	Q267	Q268	Q269	Q270	Q271	Q272	Q273	Q274	Q275	Q276	Q277	Q278	Q279	Q280	Q281	Q282	Q283	Q284	Q285	Q286	Q287	Q288	Q289	Q290	Q291	Q292	Q293	Q294	Q295	Q296	Q297	Q298	Q299	Q300	Q301	Q302	Q303	Q304	Q305	Q306	Q307	Q308	Q309	Q310	Q311	Q312	Q313	Q314	Q315	Q316	Q317	Q318	Q319	Q320	Q321	Q322	Q323	Q324	Q325	Q326	Q327	Q328	Q329	Q330	Q331	Q332	Q333	Q334	Q335	Q336	Q337	Q338	Q339	Q340	Q341	Q342	Q343	Q344	Q345	Q346	Q347	Q348	Q349	Q350	Q351	Q352	Q353	Q354	Q355	Q356	Q357	Q358	Q359	Q360	Q361	Q362	Q363	Q364	Q365	Q366	Q367	Q368	Q369	Q370	Q371	Q372	Q373	Q374	Q375	Q376	Q377	Q378	Q379	Q380	Q381	Q382	Q383	Q384	Q385	Q386	Q387	Q388	Q389	Q390	Q391	Q392	Q393	Q394	Q395	Q396	Q397	Q398	Q399	Q400	Q401	Q402	Q403	Q404	Q405	Q406	Q407	Q408	Q409	Q410	Q411	Q412	Q413	Q414	Q415	Q416	Q417	Q418	Q419	Q420	Q421	Q422	Q423	Q424	Q425	Q426	Q427	Q428	Q429	Q430	Q431	Q432	Q433	Q434	Q435	Q436	Q437	Q438	Q439	Q440	Q441	Q442	Q443	Q444	Q445	Q446	Q447	Q448	Q449	Q450	Q451	Q452	Q453	Q454	Q455	Q456	Q457	Q458	Q459	Q460	Q461	Q462	Q463	Q464	Q465	Q466	Q467	Q468	Q469	Q470	Q471	Q472	Q473	Q474	Q475	Q476	Q477	Q478	Q479	Q480	Q481	Q482	Q483	Q484	Q485	Q486	Q487	Q488	Q489	Q490	Q491	Q492	Q493	Q494	Q495	Q496	Q497	Q498	Q499	Q500	Q501	Q502	Q503	Q504	Q505	Q506	Q507	Q508	Q509	Q510	Q511	Q512	Q513	Q514	Q515	Q516	Q517	Q518	Q519	Q520	Q521	Q522	Q523	Q524	Q525	Q526	Q527	Q528	Q529	Q530	Q531	Q532	Q533	Q534	Q535	Q536	Q537	Q538	Q539	Q540	Q541	Q542	Q543	Q544	Q545	Q546	Q547	Q548	Q549	Q550	Q551	Q552	Q553	Q554	Q555	Q556	Q557	Q558	Q559	Q560	Q561	Q562	Q563	Q564	Q565	Q566	Q567	Q568	Q569	Q570	Q571	Q572	Q573	Q574	Q575	Q576	Q577	Q578	Q579	Q580	Q581	Q582	Q583	Q584	Q585	Q586	Q587	Q588	Q589	Q590	Q591	Q592	Q593	Q594	Q595	Q596	Q597	Q598	Q599	Q600	Q601	Q602	Q603	Q604	Q605	Q606	Q607	Q608	Q609	Q610	Q611	Q612	Q613	Q614	Q615	Q616	Q617	Q618	Q619	Q620	Q621	Q622	Q623	Q624	Q625	Q626	Q627	Q628	Q629	Q630	Q631	Q632	Q633	Q634	Q635	Q636	Q637	Q638	Q639	Q640	Q641	Q642	Q643	Q644	Q645	Q646	Q647	Q648	Q649	Q650	Q651	Q652	Q653	Q654	Q655	Q656	Q657	Q658	Q659	Q660	Q661	Q662	Q663	Q664	Q665	Q666	Q667	Q668	Q669	Q670	Q671	Q672	Q673	Q674	Q675	Q676	Q677	Q678	Q679	Q680	Q681	Q682	Q683	Q684	Q685	Q686	Q687	Q688	Q689	Q690	Q691	Q692	Q693	Q694	Q695	Q696	Q697	Q698	Q699	Q700	Q701	Q702	Q703	Q704	Q705	Q706	Q707	Q708	Q709	Q710	Q711	Q712	Q713	Q714	Q715	Q716	Q717	Q718	Q719	Q720	Q721	Q722	Q723	Q724	Q725	Q726	Q727	Q728	Q729	Q730	Q731	Q732	Q733	Q734	Q735	Q736	Q737	Q738	Q739	Q740	Q741	Q742	Q743	Q744	Q745	Q746	Q747	Q748	Q749	Q750	Q751	Q752	Q753	Q754	Q755	Q756	Q757	Q758	Q759	Q760	Q761	Q762	Q763	Q764	Q765	Q766	Q767	Q768	Q769	Q770	Q771	Q772	Q773	Q774	Q775	Q776	Q777	Q778	Q779	Q780	Q781	Q782	Q783	Q784	Q785	Q786	Q787	Q788	Q789	Q790	Q791	Q792	Q793	Q794	Q795	Q796	Q797	Q798	Q799	Q800	Q801	Q802	Q803	Q804	Q805	Q806	Q807	Q808	Q809	Q810	Q811	Q812	Q813	Q814	Q815	Q816	Q817	Q818	Q819	Q820	Q821	Q822	Q823	Q824	Q825	Q826	Q827	Q828	Q829	Q830	Q831	Q832	Q833	Q834	Q835	Q836	Q837	Q838	Q839	Q840	Q841	Q842	Q843	Q844	Q845	Q846	Q847	Q848	Q849	Q850	Q851	Q852	Q853	Q854	Q855	Q856	Q857	Q858	Q859	Q860	Q861	Q862	Q863	Q864	Q865	Q866	Q867	Q868	Q869	Q870	Q871	Q872	Q873	Q874	Q875	Q876	Q877	Q878	Q879	Q880	Q881	Q882	Q883	Q884	Q885	Q886	Q887	Q888	Q889	Q890	Q891	Q892	Q893	Q894	Q895	Q896	Q897	Q898	Q899	Q900	Q901	Q902	Q903	Q904	Q905	Q906	Q907	Q908	Q909	Q910	Q911	Q912	Q913	Q914	Q915	Q916	Q917	Q918	Q919	Q920	Q921	Q922	Q923	Q924	Q925	Q926	Q927	Q928	Q929	Q930	Q931	Q932	Q933	Q934	Q935	Q936	Q937	Q938	Q939	Q940	Q941	Q942	Q943	Q944	Q945	Q946	Q947	Q948	Q949	Q950	Q951	Q952	Q953	Q954	Q955	Q956	Q957	Q958	Q959	Q960	Q961	Q962	Q963	Q964	Q965	Q966	Q967	Q968	Q969	Q970	Q971	Q972	Q973	Q974	Q975	Q976	Q977	Q978	Q979	Q980	Q981	Q982	Q983	Q984	Q985	Q986	Q987	Q988	Q989	Q990	Q991	Q992	Q993	Q994	Q995	Q996	Q997	Q998	Q999	Q1000	Q1001	Q1002	Q1003	Q1004	Q1005	Q1006	Q1007	Q1008	Q1009	Q1010	Q1011	Q1012	Q1013	Q1014	Q1015	Q1016	Q1017	Q1018	Q1019	Q1020	Q1021	Q1022	Q1023	Q1024	Q1025	Q1026	Q1027	Q1028	Q1029	Q1030	Q1031	Q1032	Q1033	Q1034	Q1035	Q1036	Q1037	Q1038	Q1039	Q1040	Q1041	Q1042	Q1043	Q1044	Q1045	Q1046	Q1047	Q1048	Q1049	Q1050	Q1051	Q1052	Q1053	Q1054	Q1055	Q1056	Q1057	Q1058	Q1059	Q1060	Q1061	Q1062	Q1063	Q1064	Q1065	Q1066	Q1067	Q1068	Q1069	Q1070	Q1071	Q1072	Q1073	Q1074	Q1075	Q1076	Q1077	Q1078	Q1079	Q1080	Q1081	Q1082	Q1083	Q1084	Q1085	Q1086	Q1087	Q1088	Q1089	Q1090	Q1091	Q1092	Q1093	Q1094	Q1095	Q1096	Q1097	Q1098	Q1099	Q1100	Q1101	Q1102	Q1103	Q1104	Q1105	Q1106	Q1107	Q1108	Q1109	Q1110	Q1111	Q1112	Q1113	Q1114	Q1115	Q1116	Q1117	Q1118	Q1119	Q1120	Q1121	Q1122	Q1123	Q1124	Q1125	Q1126	Q1127	Q1128	Q1129	Q1130	Q1131	Q1132	Q1133	Q1134	Q1135	Q1136	Q1137	Q1138	Q1139	Q1140	Q1141	Q1142	Q1143	Q1144	Q1145	Q1146	Q1147	Q1148	Q1149	Q1150	Q1151	Q1152	Q1153	Q1154	Q1155	Q1156	Q1157	Q1158	Q1159	Q1160	Q1161	Q1162	Q1163	Q1164	Q1165	Q1166	Q1167	Q1168	Q1169	Q1170	Q1171	Q1172	Q1173	Q1174	Q1175	Q1176	Q1177	Q1178	Q1179	Q1180	Q1181	Q1182	Q1183	Q1184	Q1185	Q1186	Q1187	Q1188	Q1189	Q1190	Q1191	Q1192	Q1193	Q1194	Q1195	Q1196	Q1197	Q1198	Q1199	Q1200	Q1201	Q1202	Q1203	Q1204	Q1205	Q1206	Q1207	Q1208	Q1209	Q1210	Q1211	Q1212	Q1213	Q1214	Q1215	Q1216	Q1217	Q1218	Q1219	Q1220	Q1221	Q1222	Q1223	Q1224	Q1225	Q1226	Q1227	Q1228	Q1229	Q1230	Q1231	Q1232	Q1233	Q1234	Q1235	Q1236	Q1237	Q1238	Q1239	Q1240	Q1241	Q1242	Q1243	Q1244	Q1245	Q1246	Q1247	Q1248	Q1249	Q1250	Q1251	Q1252	Q1253	Q1254	Q1255	Q1256	Q1257	Q1258	Q1259	Q1260	Q1261	Q1262	Q1263	Q1264	Q1265	Q1266	Q1267	Q1268	Q1269	Q1270	Q1271	Q1272	Q1273	Q1274	Q1275	Q1276	Q1277	Q1278	Q1279	Q1280	Q1281	Q1282	Q1283	Q1284	Q1285	Q1286	Q1287	Q1288	Q1289	Q1290	Q1291	Q1292	Q1293	Q1294	Q1295	Q1296	Q1297	Q1298	Q1299	Q1300	Q1301	Q1302	Q1303	Q1304	Q1305	Q1306	Q1307	Q1308	Q1309	Q1310	Q1311	Q1312	Q1313	Q1314	Q1315	Q1316	Q1317	Q1318	Q1319	Q1320	Q1321	Q1322	Q1323	Q1324	Q1325	Q1326	Q1327	Q1328	Q1329	Q1330	Q1331	Q1332	Q1333	Q1334
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	Surname	First Name	Q1.1	Q1.2	Q1.3	Q1.4	Q1.5	Q1.6	Q1.7	Q1.8	Q1.9	Q1.10	Q1.11	Q1.12	Q1.13	Q1.14	Q1.15	Q1.16	Q1.17	Q1.18	Q1.19	Q1.20	Q1.21	Q1.22	Q1.23	Q1.24	Q1.25	Q1.26	Q1.27	Q1.28	Q1.29	Q1.30	Q1.31	Q1.32	Q1.33	Q1.34	Q1.35	Q1.36	Q1.37	Q1.38	Q1.39	Q1.40	Q1.41	Q1.42	Q1.43	Q1.44	Q1.45	Q1.46	Q1.47	Q1.48	Q1.49	Q1.50	Q1.51	Q1.52	Q1.53	Q1.54	Q1.55	Q1.56	Q1.57	Q1.58	Q1.59	Q1.60	Q1.61	Q1.62	Q1.63	Q1.64	Q1.65	Q1.66	Q1.67	Q1.68	Q1.69	Q1.70	Q1.71	Q1.72	Q1.73	Q1.74	Q1.75	Q1.76	Q1.77	Q1.78	Q1.79	Q1.80	Q1.81	Q1.82	Q1.83	Q1.84	Q1.85	Q1.86	Q1.87	Q1.88	Q1.89	Q1.90	Q1.91	Q1.92	Q1.93	Q1.94	Q1.95	Q1.96	Q1.97	Q1.98	Q1.99	Q1.100	Total
66	Twigg	Gaynor Joan	2	1	2	2	2	0	2	1	2	2	2	2	2	2	2	2	2	1	2	2	2	2	3	1	3	4	3	1	1	1	3	2	3	1	1	3	2	3	1	1	3	1	3	1	55																																																								
67	Van der Merwe	Riaan	2	1	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	3	1	3	4	3	2	1	1	3	3	3	0	2	3	1	67																																																															
68	Volkwyn	Eduard Emile	1	2	1	2	2	1	1	2	2	1	2	2	2	1	1	2	0	0	2	2	3	1	3	2	3	1	1	1	3	1	3	1	1	3	1	1	3	1	42																																																														
69	Williams	Naiema	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	1	3	1	3	2	3	2	1	2	3	2	3	1	1	3	1	62																																																																
70	Williams	Nicole Lesleigh	1	1	2	1	2	1	0	2	2	1	2	1	1	0	0	2	2	0	2	1	1	1	1	2	1	1	1	3	1	3	1	1	3	2	29																																																																		
71	Windvogel	Udele	2	2	2	2	2	0	1	0	0	0	2	2	2	2	1	2	2	0	0	1	3	2	3	4	1	2	1	0	1	2	1	1	3	2	43																																																																		
No. of 0s:			7	5	1	2	2	16	2	12	8	13	3	2	2	5	8	3	6	18	8	0	0	2	1	0	2	1	1	7	4	4	1	7	3	2	3																																																																		
No. of 1s:			17	18	31	16	5	22	29	23	6	17	10	19	10	21	19	4	20	16	7	39	22	42	7	18	8	41	62	31	23	37	11	33	35	2	45																																																																		
no. of 2s:			47	48	39	53	64	33	40	36	57	41	58	50	59	45	44	64	45	37	56	7	0	12	0	12	0	20	5	27	1	28	1	24	32	2	16																																																																		
No. of 3s:																						0	49	4	63	1	61	2	1	2	43	2	58	2	1	65	1																																																																		
No. of 4s:																						25	0	11	0	40	0	7	2	4	0	0	0	5	0	0	6																																																																		

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SINETHEMBA'S GEOMETRY TEST SCORES

Surname	First Name(s)	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30	Q31	Q32	Q33	Q34	Q35	Q36	Q37	Q38	Q39	Q40	Q41	Q42	Q43	Q44	Q45	Q46	Q47	Q48	Q49	Q50	Total	Rank				
1	Abibit	Sonwabale	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	22				
2	Adam	Asanda	0	2	0	2	2	0	2	0	0	2	2	2	2	0	2	1	0	0	0	4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24				
3	Apoli	Fezeka	2	2	1	2	2	1	2	2	2	1	2	2	1	2	2	1	0	1	4	3	1	3	3	1	1	1	1	3	2	3	1	1	1	3	2	3	1	1	3	2	3	1	1	3	1	1	3	1	1	3	1	51			
4	Apoli	Zodwa	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	18					
5	April	Nomandla	0	2	1	2	2	1	1	2	0	1	1	2	2	1	2	1	0	1	2	3	2	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	38			
6	Banayi	Veliswa	0	2	1	1	2	1	1	0	0	1	2	2	1	2	2	1	0	2	2	0	2	3	1	3	2	2	2	3	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	40		
7	Bani	Goobisa	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	2	3	1	3	1	2	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	25				
8	Booi	Zine Antoinette	1	2	1	1	2	1	1	2	1	2	1	2	1	2	1	2	1	2	1	3	1	3	2	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	31			
9	Butsheke	Nomveliso Mavis	2	2	0	2	2	0	2	2	2	1	2	2	1	1	0	2	2	0	2	3	3	2	3	2	1	3	2	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	53		
10	Butshingi	Sakhe Luzuko	2	2	2	2	0	2	2	2	0	2	2	2	2	2	2	1	1	1	1	3	1	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	2	41				
11	Dala	Thobile	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	3	2	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	1	21				
12	Dani	Ntobeko	1	1	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	2	3	2	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	23			
13	Dassie	Thamsanga	2	2	1	2	2	2	1	1	2	1	2	1	2	1	2	1	1	2	3	2	3	1	3	1	4	1	3	2	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	2	50			
14	Deyi	Mphumzi	1	2	1	0	0	2	2	0	2	1	2	2	1	0	2	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28				
15	Didi	Mongikazi	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	2	3	1	3	1	2	2	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	2	26				
16	Dlokweni	Fundiswa	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	20			
17	Fadana	Lumka	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	12					
18	Fati	Buyiselwa Gladys	2	2	1	2	2	2	2	2	2	1	2	2	2	2	2	2	1	2	1	3	1	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	50			
19	Gontsana	Ntomboxolo	2	2	1	2	2	1	2	1	2	2	2	2	2	2	1	2	1	1	1	3	2	3	1	3	1	2	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	44				
20	Gqabuza	Zuko Warren	2	2	1	2	2	2	1	1	1	2	2	2	2	2	2	2	0	2	2	3	3	3	1	3	3	1	1	3	3	1	3	3	1	3	3	1	3	3	1	3	3	1	3	3	1	3	3	1	3	3	1	57			
21	Gwangqqa	Phelisa	1	2	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	2	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	2	26				
22	Gxekwa	Mxolisi	1	1	1	1	1	2	1	1	2	1	2	1	2	1	2	1	1	2	1	3	2	3	1	3	2	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	34			
23	Iitua	Andiswa	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	2	3	1	3	1	2	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	24			
24	Jara	Thabisa	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	18					
25	Jwambi	Sindiswa	1	1	2	1	1	1	1	2	1	2	1	1	1	1	1	1	1	1	1	3	1	3	2	3	1	2	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	31			
26	Khusele	Luvuyo	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	3	1	3	4	3	4	2	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	4	72		
27	Koli	Mbonjeni	2	2	2	2	1	2	1	2	2	2	2	2	2	2	2	2	2	2	1	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	1	70	
28	Kula	Phathokazi	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	2	3	1	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	1	18				
29	Liwani	Nozuko	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	3	2	3	1	3	2	2	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	35			
30	Mabona	Bellina	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	3	1	3	1	1	1	1	3	2	3	1	1	3	2	3	1	1	3	2	3	1	1	3	2	3	1	1	3	2	3	1	11			
31	Madolo	Bulelani	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	1	1	2	1	3	2	3	1	1	3	2	3	1	1	3	2	3	1	1	3	2	3	1	1	3	2	3	1	15		
32	Malawu	Monde	1	2	1	1	2	1	1	0	1	0	2	1	2	1	1	1	1	1	1	2	3	1	3	2	1	1	3	2	3	1	3	2	3	1	3	2	3	1	3	2	3	1	3	2	3	1	3	2	3	1	3	1	31		
33	Maloyi	Sitha	2	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	1	3	3	4	3	1	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	62		
34	Manzi	M	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	1	3	1	3	4	3	2	1	2	3	2	3	1	3	2	3	1	3	2	3	1	3	2	3	1	3	2	3	1	3	2	3	1	3	1	62		
35	Maphoko	Nonceba	2	2	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	3	2	1	2	3	1	2	1	3	1	3	1	3	1																						

Surname	First Name(s)	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30	Q31	Q32	Q33	Q34	Q35	Q36	Q37	Q38	Q39	Q40	Total
68	Nyobole	Mkhusele	2	2	1	1	1	2	2	1	2	1	2	1	2	1	1	1	0	2	1	2	3	1	3	1	1	1	3	1	3	2	1	3	2	1	3	2	41			
69	Peter	Gcibisa	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	2	3	3	3	1	1	2	1	1	1	1	1	1	1	1	1	1	1	23	
70	Phama	Richard	2	2	1	2	2	2	2	1	2	2	2	1	2	2	1	2	1	2	1	3	2	3	4	3	1	1	3	1	3	1	1	1	3	1	1	1	3	1	52	
71	Qaqane	Vuyiswa	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	1	3	1	3	1	1	1	3	1	1	1	1	20	
72	Ranisi	Landiswa	1	2	1	2	2	1	2	1	0	2	2	2	1	1	1	0	1	2	3	1	3	1	3	1	1	1	2	0	0	0	0	0	0	0	0	0	0	0	26	
73	Ranyana	Momelezi	1	2	1	1	1	2	1	1	0	2	1	1	1	2	1	1	0	2	2	3	1	3	2	3	1	1	2	1	2	3	2	2	3	2	2	3	1	39		
74	Rasmen	Baxolele	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	1	1	3	4	3	1	1	1	3	1	3	1	1	3	1	1	3	1	1	3	1	55	
75	Sibango	Buzelwa	1	2	1	1	2	2	1	2	2	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	2	1	1	3	2	3	1	1	3	1	1	3	1	30		
76	Sibida	Luyolo	2	2	0	2	0	0	0	1	1	2	2	0	0	0	1	2	0	0	1	3	2	3	1	1	1	1	1	3	1	1	2	1	3	2	1	3	2	29		
77	Sinyanva	Khanvisa	2	2	1	2	2	1	0	2	2	2	2	2	1	2	0	0	0	3	3	2	3	1	3	2	1	1	3	3	1	1	1	1	1	1	1	1	1	46		
78	Siphika	Nhoko	1	2	1	2	1	1	1	1	2	1	1	2	1	2	1	0	1	2	3	1	1	1	3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	20		
79	Sixshe	Lusindiso	1	1	1	1	1	2	1	1	1	1	2	2	1	1	1	0	1	2	1	1	3	3	3	3	2	1	1	1	2	1	2	1	2	1	3	1	28			
80	Skolpati	Babalwa	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	2	1	3	3	3	4	3	3	1	1	1	3	3	3	1	3	1	3	1	3	1	65			
81	Somaguda	Nosiphiwo	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	3	1	3	1	3	1	1	1	3	1	3	1	1	3	1	1	3	1	20			
82	Somfongo	Amanda	2	2	0	1	1	0	1	0	1	0	1	0	0	0	0	2	0	0	0	2	3	1	0	1	1	0	0	3	1	3	2	2	1	1	1	1	21			
83	Sotapu	Phumza	2	2	1	1	1	1	2	1	1	1	1	1	1	1	1	2	2	1	1	3	1	3	1	3	1	1	2	3	2	3	1	1	3	1	1	3	1	30		
84	Stuurman	Loyiso	2	2	1	2	2	2	2	2	2	2	2	2	2	0	2	2	2	2	4	3	4	3	4	3	3	1	1	3	0	3	0	2	3	2	3	2	71			
85	Swelindevu	Lizeka	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	3	1	3	1	1	1	3	1	3	1	1	3	1	1	3	1	21			
86	Taho	Phumeza	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	2	3	2	3	1	1	3	1	3	1	1	3	1	1	3	1	21			
87	Tela	Noyise	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	1	2	3	2	3	1	3	1	1	1	3	1	3	1	1	3	1	54				
88	Tomose	Ziyanda	1	2	1	2	2	0	2	0	2	0	2	1	2	0	0	0	1	3	1	1	3	1	3	1	2	2	3	2	1	1	1	3	2	1	1	3	2	36		
89	Tshatani	Zanele	1	1	1	2	1	1	1	1	1	1	1	2	1	2	1	1	1	1	2	1	3	1	3	1	3	1	1	3	1	3	1	1	3	1	1	26				
90	Vokozela	Bongiswa	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	3	1	3	1	1	3	1	1	3	1	17			
91	Yenzekile	Mbulelo Ben	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	1	1	3	1	1	1	1	1	3	2	1	25				
92	Zamxaka	Thobeka Ethel	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	2	3	1	3	1	1	1	1	1	3	2	1	1	1	1	1	18				
93	Zazela	Mzoxolo	2	2	2	2	2	2	2	1	2	2	2	2	1	2	2	1	2	1	1	2	1	2	1	2	1	1	3	2	3	1	1	1	3	2	3	1	1	49		

No. of 0s:	5	0	6	1	3	3	5	8	7	7	2	3	5	4	7	0	4	27	8	3	4	4	4	4	3	3	5	6	7	7	7	7	8	6	6	6	6
No. of 1s:	41	35	68	46	44	52	54	62	45	53	48	41	42	47	61	40	51	49	51	57	24	47	7	50	15	58	64	65	20	54	18	59	65	16	67		
no. of 2s:	47	58	19	46	46	38	34	23	41	33	43	49	46	42	25	53	38	17	34	24	0	34	0	20	0	25	21	21	0	27	0	25	21	0	19		
No. of 3s:																				2	65	5	82	4	75	4	2	1	66	4	68	1	0	71	0		
No. of 4s:																				7	0	3	0	15	0	3	1	0	0	1	0	0	1	0	1	0	

APPENDIX 4

Most Familiar sketches Response Questionnaire

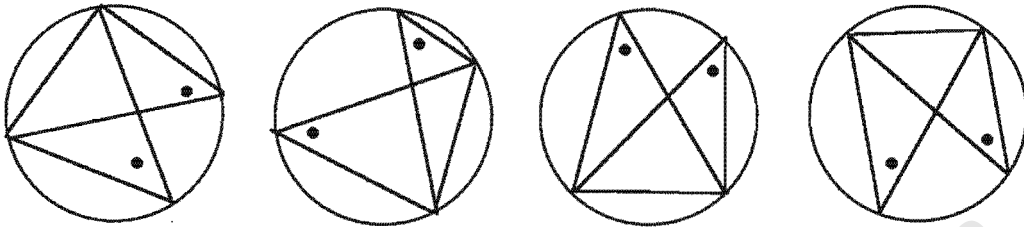
University of Cape Town

APPENDIX 4: Most Familiar Sketches Response Questionnaire

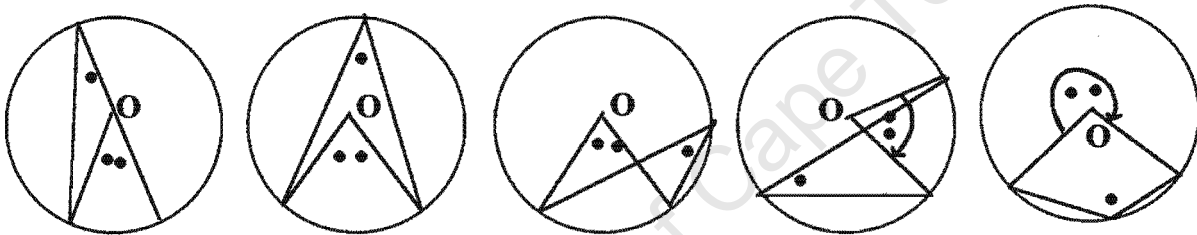
Name: School:

Interview Question: *Which of the following indicate the sketch that was used to introduce the theorem?*

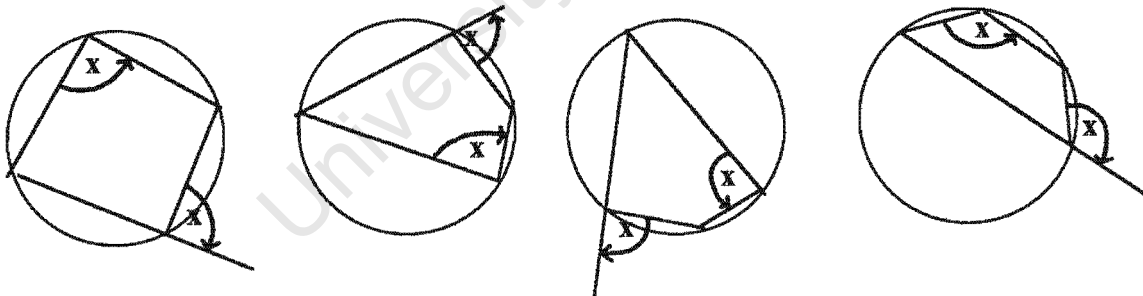
1. ANGLES SUBTENDED BY THE SAME ARC OR CHORD ARE EQUAL.



2. THE ANGLE AT THE CENTRE OF THE CIRCLE IS TWICE THE SIZE OF THE ANGLE AT THE CIRCUMFERENCE.

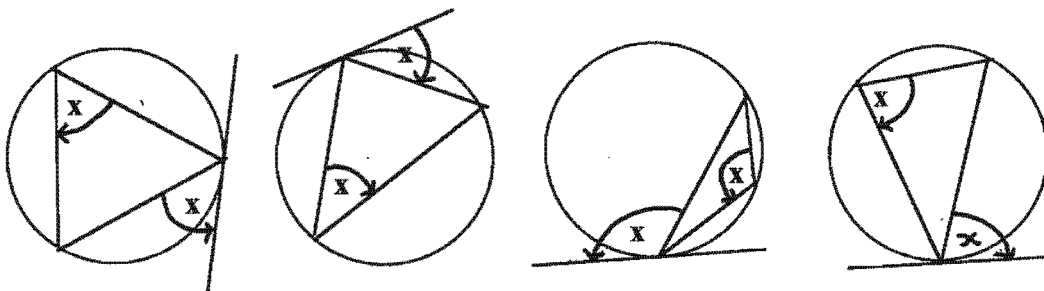


3. THE EXTERIOR ANGLE OF A CYCLIC QUADRILATERAL IS EQUAL TO THE INTERIOR OPPOSITE ANGLE.



THE INTERIOR OPPOSITE ANGLE.

4. THE ANGLE BETWEEN A TANGENT AND A CHORD IS EQUAL TO THE ANGLE IN THE OTHER SEGMENT.



APPENDIX 5

Summary of Interview Procedures

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APPENDIX 5: Summary of interview Procedures

The following represents a brief summary in terms of venues, noise, indistinct responses and transcription time for each interview:

RAFULA

- Esangweni venue very noisy.
- Rafula slightly indistinct.
- took approx. 3.5 hrs

MANGANI

- Not as noisy as when first interview took place
- Mangani spoke fairly clearly, except for when she could not pronounce a word, she would become very indistinct.
- took approx. 2.25 hrs.

VUYOLWATHU

- Lots of noise.
- Student very indistinct.
- Part of no.10 and the whole of no.11 - there was no sound
- Took very long with Q8.
- Took approx. 3.5 hrs.

ZAMEKA

- Student speaks quite clearly.
- Less questions dealt with.
- Lots of noise in background.
- Took approx. 1.15 hrs.

NDZILA

- Student indistinct, only now and again.
- Venue slightly noisy
- Took approx. 2.05 hrs.

RUSHANA

- Venue very quiet, hardly any background noise.
- Student speaks clearly, but giggles a lot and sometimes swallows words.
- Takes time to work out what is being asked for.
- Took approx. 3.5 hrs.

LANCE

- Background fairly quiet (some noise now and again).
- Student speaks alot while working out his answer.
- This interview was very long.
- Numerous disturbances eg. 5 phone calls.
- Student speaks fast and slight accent (sometimes indistinct).

- Took approx. 3.75 hrs.

REPHWAAI

- This interview went quicker than Lance's one.
- Very few interruptions.
- Student speaks clearly, but tends to mumble when thinking audibly.
- Less questions targeted.
- Took approx. 2.55 hrs.

BRONWYN

- A very long interview.
- Venue quite noisy, especially at interval.
- Student speaks very softly at times and very indistinct
- Took approx. 3.75 hrs

SHAKIER

- Venue fairly quiet.
- Student speaks too softly at times.
- Less questions handled.
- Last question, last two lines indistinct.
- Took approx. 1.5 hrs.

SHERIZAAN

- Venue noisy at interval.
- Student answers affirmative mainly by saying "umm", denoting ageeing sound.
- Speaks sometimes too fast, but was quite clear (audible).
- Took approx. 3.25 hrs.

RASMEN

- Venue very quiet, except for a particular period of noise which was short lived.
- Student very confident and quick to answer.
- Interview very long.
- Took approx. 3.75 hrs.

LUVUYO

- Venue fairly quiet.
- Student extremely eager, sometimes over enthusiastic, answers quickly and full of confidence.
- Speaks clearly, but fast and tends to mumble while working out answers.
- Interview shorter than previous one.
- Less questions handled.
- Took approx. 2.5 hrs.

MZOXOLO

- Venue quiet, some talking in the background.
- Student has very low, deep voice and one has to listen very carefully.
- A long interview.
- Took approx. 3.25 hrs

GCOBISA

- Venue quiet.
- Student speaks clearly at times and then again indistinct sometimes.
- She finds it difficult at times to express herself in words, so uses actions instead.
- Interview shortened because of time.
- Took approx. 1.85 hrs.

SPHOKAZI

- Venue fairly quiet (one can hear movement, voices and noise in background, at times, but not too bad.
- Student spoke clearly at first, but toward end became a bit indistinct, especially if she was not sure of her answer.
- Took approx. 2.25 hrs.

CIKIZWA

- Venue quiet except for certain short periods.
- Student spoke clearly, too fast at times.
- Attempted answers immediately. No long pauses.
- Took approx. 2.9 min.

ETIENNE

- Venue noisy at interval.
- Now and again student indistinct.
- Took approx. 3.15 hrs.

PAUL

- Venue fairly noisy.
- Student is very soft spoken and can hardly be heard at times, making the transcribing take longer than is necessary, because one has to listen to something a few times before it becomes more clear.
- Took approx. 3.45 hrs.

WARDA

- Venue noisy, but not as bad as previous student.
- Student quite clear, although at times indistinct.
- Shorter interview than others.
- Took approx. 2.5 hrs.

SONIA

- Venue very noisy
- Noise only lets up in last 10 minutes of interview.
- Student speaks clearly and confidently.
- She obviously knows her work.
- Answers promptly, reducing waiting time.
- Took approx. 2.1 hrs.

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APPENDIX 6

Interview Transcriptions

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APPENDIX 6: Interviews

The preamble for all the student interviews was very similar. The researcher congratulated the students for doing well in test 1, thanked them for their willingness to be interviewed and explained the reason(s) for the interview. Transcriptions start with the first question to the student.

INTERVIEW 1 (RAFULA)

Researcher: ... (refers to Q1.3). You said here, a cyclic quadrilateral is a quadrilateral ...

Rafula: Yes, sir.

Researcher: And you said: four opposite angles and four sides equal... is that what you understand by cyclic quad? or what do you understand by a cyclic quad?

Rafula: That is what I understood by cyclic quadrilateral.

Researcher: Okay ... that all the sides are equal?

Rafula: That it has four sides.

Researcher: So this is sort of only one case?

Rafula: Yes, Sir.

Researcher: That's good ... and then further on ... you know the theorem (refers to Q1.7) "the angle between a tangent and a chord is equal to ... the vertical opposite angle ... Now what do you mean by that?"

Rafula: The angle between the chord?

Researcher: Yes. [lots of noise in the background - something dropped loudly]

Rafula: It means that, it means that ... where (sounded like) the ending is, is in between the tangent, or it is equal to the ... that the angle is equal to the angle that is opposite it.

Researcher: Which is opposite it? ... Okay, because what we normally teach in terms of vertically opposite is this... (shows sketch with intersecting lines).

Rafula: Yes.

Researcher: So you don't mean that (pointing to the intersecting lines). [loud noise in background]

Rafula: No, I don't mean that.

Researcher: Okay, that's fine, that's fine ... The last one here (refers Q1.10). The angle subtended by the diameter is equal to the opposite angle on the same segment. So what do you mean by that one?

SILENCE..... HANDBELL RINGS IN BACKGROUND

Researcher: You know which theorem it is, hey! ... the angle subtended (researcher prompts ... student mumbles thoughts). You see, let me show you why ... Here you've got the angle subtended by the diameter is equal to the opposite (student recites with researcher) angle on the same segment. Okay, [BELL STILL RINGING] and then if you look at Q2.8 ... angles of a triangle are supplementary [LOTS OF NOISE]. So, okay, there they add up to ... 180°.

Rafula: ... 180°

Researcher: If this theorem is a problem "the angle subtended by the diameter ... What do you think the answer to that is?"

Rafula: Don't you want to go back, so that I can see it?

Researcher: (pages back) ... Okay, now ...

Rafula: Yes.

Researcher: And that one (refers Q1.10), it also links up to that one (points to Q2.8) and it also links up to that one (refers test 2, Q5) ... of the same theorem ... That's the one you having difficulty with?

Rafula: Yes.

Researcher: Okay, we're going to come back ... don't forget to ask me that one again at the end ... alright... but that's fine, so I know that that won't cause some difficulty.

Rafula: Yes.

Researcher: (referring to page 1) ... This I'm happy with, everything went well there. Now I'm going to ask some lovely questions based on your responses there (refers to test 2)... Now, in Q1, Why do you think that, that question is true ... You can see, hey?

Rafula: It was because this (refers Q1) and the subtended are the same ... (long silence)

Researcher: (points to Q1) because of the reason, there?

Rafula: Yes, it is because of the reason ... [NOISE SO LOUD THE REST OF THE RESPONSE IS DROWNED]

Researcher: Okay, so you agree with that. (pointing to students' response in Q1)

Rafula: Yes, I agree with that.

Researcher: Okay, [NOISE], then why would this one (refers Q2) be false? ... (long pause - Silence).

Rafula: (moves page and points to sketch) I think it is because ... there it is not stated ... When I look at this (makes shape of sketch with finger) I don't see ... I think this reason is false because I don't see ... here it is only 60 degrees (points to the sketch) which is stated on the ...

Researcher: Okay, so what would x be for you? (points to the angle). You said it's wrong, hey? ... Then you must say ... if that is wrong, then what would the correct value for x be?

SILENCE [SOUNDS OF CHILDREN PLAYING IN BACKGROUND] What do you think?

Rafula: The correct value for x is ... will be 180 degrees.

Researcher: Okay, and why?

Rafula: Because if I add 120 degrees to 60 degrees it will give me 180 degrees.

Researcher: Okay, right ... and these two sketches (refers Q1 and Q2) are they the same or do they differ?

Rafula: The sketches are the same but the way they put it, is not the same.

Researcher: Okay, okay, so that's why the one is true and the other one is false?

Rafula: Yes.

Researcher: Alright, that's fine ... what about Q3 ... How did you go about answering that one ... You said it is false. (SILENCE) ... By the way, are you familiar with these figures (refers Q1 & Q2) ... have you seen those figure before?

Rafula: Yes, I have seen ...

Researcher: Fine, Q3.

Rafula: Q3, the question says that x is equal to 30 degrees, which is false. x is equal to 50 degrees because x is equal to Q (refers to angle Q).

Researcher: Okay, what will your reason be?

Rafula: They are alternating.

Researcher: They are alternating, okay... and that's why you said 50 degrees there ... (pointing to the sketch).

Rafula: Yes, that is why.

Researcher: Okay, that's good ... and Q4 ... What strategy did you use to answer that one. [LOTS OF NOISE IN BACKGROUND]... How did you go about answering this question. You said it was false.

LONG PAUSE silence

Rafula: um ... (pointing to sketch). I think here, I made a mistake.

Researcher: Oh! you made a mistake... and what do you think your mistake is?

Rafula: I think that my mistake is that it is true, that x is 50 degrees.

Researcher: And, and your reason?

Rafula: This reason here (pointing to the sketch) is correct ... is correct.

Researcher: So, you're not happy with your answer?

Rafula: Yes.

Researcher: Okay, fine ... alright Q5 ... [VERY LOUD NOISE IN BACKGROUND].

How did you go about reasoning that one? [LOUD TALKING AND NOISE IN BACKGROUND].

Rafula: Here I just thought that [VERY LOUD NOISE] x is 60 degrees, so as I know that angles of a triangle measures 180 degrees, so I thought that ... that x is 60 degrees ... there are some values of other ... (rest of the response very indistinct).

Researcher: So, all the angles will be the same?

Rafula: No, not all the angles will be the same, but there are some values other than 60 degrees which will be given (slightly indistinct) ... to form a triangle.

Researcher: Okay, ... and tell me, the line AC, that line is called? (points to Q5).

Rafula: Yes, it is called a ... (pronounces 'radius' incorrectly).

Researcher: (researcher prompts) ... the radius? ... Okay, lets go to Q6. Okay, you said it was true ... how did you reason that one?

Rafula: My reason ... yes it is, this is true. X and C they are subtended by the same, by the same chord ... they are equal. C is 45 degrees and definitely x will be 45 degrees because of this.

Researcher: Okay, good ... I'm happy with that ... um Q7 ... Why do you think that, that question is true. [NOISE IN BACKGROUND].

Rafula: Because if I add, ... I thought that it is true if I add 70 degrees (points to the sketch) to 110 degrees, they make 180 degrees.

Researcher: Okay, and how do you know it must add up to 180 degrees.

Rafula: I know that by (pointing to the sketch) if by making a line here, it will be two triangles. One triangle is 180 degrees, (pointing to opposite side), one triangle is 180 degrees. (response indistinct) ... 180 degrees to make a triangle.

Researcher: And the reason there? (points to reason next to the sketch). Do you agree with that reason?

Rafula: Yes, I think that is ... I agree with that reason. Cyclic quadrilateral measures 360 degrees and 180 degrees is a half of it. If I cut it into two (makes dissecting gesture on sketch) it gives 180 degrees plus 180 degrees, which is 360 degrees.

Researcher: Okay, because earlier on you said here, ... that the opposite angles of a cyclic quad are supplementary.

Rafula: Yes ...

Researcher: What does supplementary mean?

Rafula: It means that they measure 180 degrees.

Researcher: Okay, so that's your reason ... so you'll agree ... that, that is a cyclic quad.

Rafula: Yes.

Researcher: Alright, lets go onto no. 8. How did you go about answering that question? ... By the way have you seen this sketch before? ... are you familiar with it?

Rafula: Yes, I have seen it before.

Researcher: And did your teacher emphasise that sketch?

Rafula: Yes.

Researcher: Okay, Q8 ... again ... How did you go about answering that one?

Rafula: Q8, it is said that RS is a tangent, so we are told there that x is 65 degrees, and I said that is true. Why, because, I thought of this ... they formed a Z like this (points to sketch) ... so they are equal.

Researcher: Okay, they are equal because they formed a Z ?

Rafula: Yes.

Researcher: Okay, ... Q9. Explain how you went about that one?

Rafula: (mumbles something about a tangent), I don't think x is 25 degrees. [LOUD TALKING AND LOTS OF NOISE IN BACKGROUND]

Researcher: Okay, and why do you say so? [VERY LOUD NOISE]

Rafula: Because X must be, must be two times (indistinct) ... so that (response drowned out by very loud talking in background) ... is a centre ... (indistinct). So I don't think this statement is true, that is why it is false.

[AT NO TIME DID RESEARCHER REQUEST LOUD TALKER/SPEAKER IN BACKGROUND TO LOWER VOICE OR BE QUIET].

Researcher: Okay, so you want to use the angle at the centre?

Rafula: Yes.

Researcher: Okay, right ... good! ... Lets see (turns the page) ... No. 10. How did you go about answering that question. [LOTS OF NOISE].

Rafula: When I read that statement and saw that x is 30 degrees, I say that is true.

Researcher: Okay, so you agree with that ... okay, and again you've seen that figure before?

Rafula: Yes, I have seen it before.

Researcher: And ... Q11. Why do you think that it is true?

Rafula: (mumbles something about x being 82 degrees ... Here I think this is a little problem, why because x cannot be equal to this angle. x is equal to ... I

think the problem is in here (points to the sketch). I must go to find ...
 (interrupted by researcher)

Researcher: So then which angle is equal to 82 degrees?

Rafula: The angle which may be 82 degrees, I think is T ... yes.

Researcher: Okay, so you think it is angle T ... and why would you say angle T?

Rafula: I would say that angle T is true, because of this statement ... and this between the angle and chord is equal to ... (indistinct ... interrupted).

Researcher: To the angle in the other segment ...

Rafula: Yes, I think so.

Researcher: Okay, we look at Q12. How did you go about that one?

Rafula: [LOUD SOUND OF DOOR BANGING] ... I said that x is 70 degrees because of, because the chord here, ... the angle A and x are subtended (not clear).

Researcher: Okay, and your happy with that reason?

Rafula: Yes, I agree with it.

Researcher: No. 13. Again you reason that its true ... how did ... how did you do that?
 Points to Q13)

Rafula: I thought that this reason is true because it says tangents drawn from common point are equal (points to sketch) because of this ...

Researcher: Okay, so which are the two tangents? ... Okay, if we go back (turns to Q1.9), you said here, two tangents drawn from a common point are ... equal. That's correct, and ... let me just check ... also you said a tangent to the circle is a line that touches the circle at one point.

Rafula: Yes, Sir.

Researcher: So would you still agree with what you said?

Rafula: No, I don't agree with what I said ... I don't agree (points to the sketch) ... here is a tangent (refers to line PQ).

Researcher: Okay, and how many tangents are there?

Rafula: Here are two tangents. ... I think the other one is this one (points to line RQ)

Researcher: Okay, so you would say that SQ is a tangent?

Rafula: Yes.

Researcher: Okay, that's fine ... then lets look at no. 14 quickly. umm ... You said that one is true. How did you go about answering that one?

Rafula: I also thought of the two lines ... this ... I thought about this (pointing to line ZU) ... and it goes like this and it turns here and then this angle is equal to this one because that forms a Z.

Researcher: So it formed a Z again.

Rafula: Yes.

Researcher: So that's why they are equal? ...

Rafula: Yes.

Researcher: And the reason that we use?

Rafula: (Mentions reason) ... (a bit indistinct) {researcher's emphasis - student referred to alternate angles}

Researcher: So you agree with that theorem. (refers to reason in sketch).

Rafula: Yes, that theorem is correct, but, I don't know how to apply it ... this statement (refers to sketch) ... that is why I use that ... (refers to sketch ... the Z).

Researcher: Okay, did your teacher show you a mechanism for this (points to the sketch).

Rafula: Yes.

Researcher: Okay, can you remember what he showed you?

Rafula: No ... not now.

Researcher: Okay, and what about no. 15. Again, just explain your reason there ... again you agreed that, that is true.

Rafula: I agreed because I have seen that. When they say that x is 60 degrees, x is opposite to ... to this 60 degrees (pointing to the given 60 degrees in the sketch). So it is equal.

Researcher: How many cyclic quads are there?

Rafula: There are 3 cyclic quads.

Researcher: Okay ... and just name it?

Rafula: Its ADEF and BFCE and BCEF.

Researcher: No ... that's the one you gave me... So it's only the two ... okay? But you'll agree with the reason there ... exterior angle is equal to the interior opposite angle.

Rafula: Yes

Researcher: Okay ... no. 16 is a very tricky one. How did you go about answering that one?

Rafula: (pointing to the sketch) There I said x is equal to 0 (zero).

Researcher: And you said angles at the centre is equal to two angles at circumference.

Rafula: Yes

Researcher: So you using the angle at the centre one ... there?

Rafula: Yes, I used the angle at the centre.

Researcher: Fine ... thank you very much.

INTERVIEW 2 (MANGANI)

Researcher: ... Now, the first one is ... why do you think that, that question is true (refers Q1).

Mangani: I think it true because ... for the theorem said that angles subtended by the same chord are equal.

Researcher: And no. 2.

Mangani: No. 2 is the same thing (points to the sketch) put it like this on this side (shows the sketch on no.1 to be the same as no.2 just in a different position).

Researcher: Okay, so in this case you said it is true (points to the answer in Q1) ... that theorem? (points to Q2's answer) ... you said its false, and the statement to the theorem Do you agree with that statement?

Mangani: Ja ... I don't know why I said its false.

Researcher: So you will agree that it should be true?

Mangani: Yes, definitely.

Researcher: Okay, and are you familiar with those two sketches? Have you seen it before?

Mangani: Yes.

Researcher: It has been highlighted to you? ... Okay, Q3. How did you go about answering that one?

Mangani: Ummmm ... I said if P is 30 degrees and then S is 30 degrees ... (points to the sketch) ... but they say when you say it is true normally (indistinct explanation) ...

Researcher: Ja, so you would agree with me that, that one is exactly the same (turns page) ... as that (points to the sketch on next page, refers Q2.2).

Mangani: Ja ...

Researcher: Okay ... and no. 4? What, what strategy did you use there, to say that it is true.

Mangani: (points to the sketch). They ... they didn't use the arc this time, they used the chord ... so its still the same thing.

Researcher: Okay, so it's exactly the same ... so you don't see a difference between no. 3 and no. 4?

Mangani: No

Researcher: Right ... umm ... Q5 ... Explain your reasoning for that one.

Mangani: Its theorem no. 3, they say the angle subtended by the diameter is 90 degrees and then here they say x is 60 degrees ... it's false.

Researcher: Okay, and do you know what an equilateral triangle is?

Mangani: (Shakes head and says) No.

Researcher: Alright ... no.6. Don't forget you can ask me later on what an equilateral triangle is ... Q6.

Mangani: You don't mind if you can tell me now?...

Researcher: Hey?

Mangani: You don't mind if you can tell me now what an equilateral triangle is?... (had difficulty pronouncing word).

Researcher: Oh, I can tell you now ... an equilateral triangle is a triangle where all the sides are equal. All the angles equal 60 degrees. Right... No. 6. Explain your reasoning there as well?

Mangani: No. 6 ... It's still the same as no.1 and up to 4, because it is the same theorem that angles subtended by the same chord are equal.

Researcher: Okay, so you agree that its based on 1 to 4... fine ... No. 7 ... Why do you think that that question is true?

Mangani: Because they say angles (indistinct mumbling) by the same chord. I think that in angles like this (points to O and S on sketch). When I was writing ... (indistinct) ... discovered that that is not a cyclic quad. (rest of response fades).

Researcher: Okay, okay ... so why isn't that a cyclic quad?

Mangani: A cyclic quad touches all those sides (points to the sketch) ... P, R and S.

Researcher: Okay, that's good ... umm ... how did you go about answering that one (moves to next sketch).

Mangani: Q8. They say its (indistinct word) ... I say its false, because when they say the angle between the tangent and a chord is equal to the angle at the circumference ... circumference means to touch the circle ... so that is not right.

Researcher: So that's false ... okay, good, because O is not at the circumference ... and Q9 ... You familiar with that sketch? ... Okay, and explain your reasoning for that question.

Mangani: Umm ... (mumbles indistinctly) ... LONG PAUSE ...

Researcher: You said that U (referring to sketch), ... U is equal to 25 degrees, okay? ... So why would that one be 25 degrees (refers to sketch).

Mangani: Because, umm ... its angle between the tangent and a chord is equal to the angle at the other side of the (indistinct word).

Researcher: Okay, and its not x?

Mangani: Yes, its not x ... x is P.

Researcher: Okay, so you can't use that theorem to find x?

Mangani: (Shakes head to answer No)

Researcher: Okay ... thank you. (turns page) ... See we're almost through ... No. 10. How did you go about answering that one?

Mangani: I ... I used the, I think, first or second theorem (points to sketch no. 10) ... They say a diameter is perpendicular to ... I ... I didn't know. I just answered it.

Researcher: Okay, so that is the diameter, and you said angles subtended by the diameter equals 90 degrees ... so which angle is 90 degrees?

Mangani: This angle by R and y.

Researcher: You mean A?

Mangani: A ... A (repeats A)

Researcher: Okay, so you would draw BA ... okay that's fine ... Q11 ... Why do you think that that one is false?

Mangani: Its ...LONG PAUSE... I think S and ... like angle Q are in a straight line (points to sketch) are 180 degrees ... are ... if I can say ... angle Q is equal to angle S and angle T ... exterior angle is equal to two opposite interior angles.

Researcher: Okay, so you saying ... as you said here ... exterior angles is equal to the two opposite interior angles ... So that would be S and T (points to relevant angles on sketch).

Mangani: (Mumbles something that sounded like, not x something, very indistinct). PLANE PASSING OVERHEAD.

Researcher: Okay, so the reason I use there ... the angle between tangent QR and chord QT is equal to the angle in the other segment ... its not true?

Mangani: It's not, because as there ... its not inside the circle.

Researcher: Okay, so if S had been there by P, then it would have been.

Mangani: Yes.

Researcher: Okay, no. 12 ... umm ... How did you go about answering that one.

Mangani: I think no. 12 ... when they say x is equal to ... I say myself x is equal to angle E ... why? ... (speaks fast and indistinct), because angle between tangent and chord and angle E is equal to angle A, so it is 30 degrees.

Researcher: Okay ... alright ... and Q13 ... explain your reason there? ... you also said that its false.

Mangani: I said its false ... umm ... angle 37 is equal to angle S then angle S ... (points to angle) ... that 37 and R are both base angles of the isosceles triangle.

Researcher: Okay.

Mangani: Then ... angle S is 37 degrees ... and the 37 degrees ... you can't write 37 degrees to make 180.

Researcher: Okay ... so you wouldn't agree with (points to reason next to sketch) ... (rest of the reasoning indistinct) ... in there.

Mangani: No.

Researcher: No. 14 ... How did you go about that one? You said its true.

Mangani: Yes, its true.

Researcher: So, exterior angles is equal to the opposite angles.

Mangani: Opposite angles ... yes.

Researcher: So, name the cyclic quad for me.

Mangani: TWU and Z. (indicates on sketch).

Researcher: Okay, and the exterior angle?

Mangani: Its W and U. (points to sketch).

Researcher: And the interior opposite angles?

Mangani: (points to T and x) Say T and x.

Researcher: Just, just that one there? (points to relevant angle) ...

Mangani: (Nods) Yes.

Researcher: Okay, and are you familiar with that sketch as well?

Mangani: Yes.

Researcher: Okay, what about no. 15. How would you go about that one?

Mangani: Umm ... its still the same thing as 14, because its, er ... er ... exterior angle ...

Researcher: Is equal to the ...

Mangani: Otherside of the cyclic quad.

Researcher: Alright, so its the same theorem, so there isn't a problem there ... And no. 16. How did you go about answering that question?

Mangani: No. 16 ... umm ... LONG PAUSE

Researcher: You said its true.

Mangani: Okay, I said the exterior angle is equal (points to angle in the sketch) is equal to the interior opposite angles. (points to U and R). So its still the same thing as that ... (indicating parallel notations on sketch).

Researcher: Okay, so you agree with me that TUV is an exterior angle.

Mangani: Yes, or is an angle between a tangent and a chord.

Researcher: Okay, great ... thank you very much ...

INTERVIEW 3 (VUYOLWATHU)

Researcher: ... So we can start off with Q1... Why do you think that ,that question is true? (Researcher moves page for student to see better) ... (repeats question)

Vuyolwathu: LONG PAUSE ... Because er... its because angle S is subtended by the arc PR.

Researcher: And you would agree that the angle subtended by the same arc are equal.

Vuyolwathu: Yes, yes.

Researcher: That (pointing to Q1) is exactly the same as Q2.2. Would you agree with that?

Vuyolwathu: Yes, sir.

Researcher: Okay, so there's no difference (turns page). Right ... question ... (turns to next page) ...and you are familiar with that sketch? You've seen it before? ... you've seen this figure before? (points to sketch no.1)

Vuyolwathu: Yes, sir.

Researcher: Alright, no.2 ... Explain your reasoning for that one. You also said it was true.

Vuyolwathu: PAUSE ... I think angle x is in the centre (very indistinct).

Researcher: Alright, ... that's why it's true?

Vuyolwathu: And x is twice the size the angle at the circumference ... If angle C is 60 degrees ... then angle x is 120 degrees.

Researcher: Okay, and is there a difference between no.1 and no.2?

Vuyolwathu: LONG PAUSE ...I wouldn't say there is a difference, sir.

Researcher: So, it's the same?

Vuyolwathu: Yes.

Researcher: Alright, no. 3 ... How did you go about answering that question.

Vuyolwathu: No. 3? ...First it's false.

Researcher: You said it's false?

Vuyolwathu: Yes ... umm ... I said P plus Q plus T is equal to 180 degrees ... reason ... (very indistinct response).

Researcher: Is 180 degrees?

Vuyolwathu: Yes.

Researcher: Okay ...

Vuyolwathu: Then I say, since angle Q is 50 degrees and angle P is 30 degrees, therefore angle T is equal to $180 - 50 - 30$, therefore angle T is equal to 100 degrees.

Researcher: 100 degrees ... okay.

Vuyolwathu: Angle T1 and T2 are equal, reason ... vertically opposite angles. (refers to sketch all the time while answering).

Researcher: Okay.

Vuyolwathu: Then if x is 30 degrees, therefore R is 30 degrees.

Researcher: So if x is 30 degrees, then R must be 30 degrees ... okay, so ... but then you agreeing with that statement, and you said it was false ... (turns page toward himself) ...Okay, now let me just ask this question ... I said $x = 30$... okay (pointing to the sketch no.3) ... that's the x, and the reason is ... angles subtended by the same arc QR ... would you agree with that statement?

Vuyolwathu: Yes, I agree.

Researcher: You agree ... so that would then be true?

Vuyolwathu: (Student moves page to see better) ... (mumbles something and is asked by researcher to speak a bit louder)...

Researcher: You must just talk a little bit louder, okay ... So you don't think that's true?

Vuyolwathu: Yes. NOISE IN BACKGROUND

Researcher: Okay, so you happy with your explanation there? (pointing to sketch).

Vuyolwathu: Yes (very softly)

Researcher: And that is still false to you?

Vuyolwathu: Yes.

Researcher: Okay, no problem ... okay, lets go onto no. 4. What strategy did you use here, to say that that's true?

Vuyolwathu: **NOISE IN BACKGROUND ...** You see, I use ... (mumbles something indistinct)

Researcher: Okay, we don't know the numbers, so you must give me the statement, ja.

Vuyolwathu: Angle subtended by the same chord? ... (indistinct last word).

Researcher: So you agree with that one?

Vuyolwathu: Yes.

Researcher: Okay, and what is the, the chord we said is? ... AB.

Vuyolwathu: (student repeats) ... AB

Researcher: And you're familiar with these sketches so far? (points to the completed sketches)

Vuyolwathu: Yes, sir.

Researcher: Alright, what about Q5.

Vuyolwathu: Q5 ... (student turn page toward him). First, angle ... (rest of response indistinct, but ends with) ... is 360 degrees.

Researcher: Okay.

Vuyolwathu: If we take one half this side, and one half this side (points to sketch no.5) ... this angle B is subtended by the arc AC (illustrates by gestures on sketch) ... so (indistinct word) ... which side O is subtended by the chord, ... I mean O subtended by the chord is 90 degrees. AC's a chord.

Researcher: Okay, okay, you used the word there ... AC's a chord.

Vuyolwathu: Diameter.

Researcher: You agree with that.

Vuyolwathu: Yes, sir. Yes, sir.

Researcher: Okay, so ... AC is a chord, but it's a special chord, that's why you called it the diameter.

Vuyolwathu: Yes, sir. Diameter.

Researcher: What about Q6 ... Explain your reasoning there? ... You also said its true (points to relevant sketch and reason)

Vuyolwathu: **LOTS OF NOISE IN BACKGROUND ... OTHERS TALKING LOUD IN THE SAME ROOM ...** Angle (indistinct) ... is an arc.

Researcher: Okay.

Vuyolwathu: And this arc (indistinct) ... angles subtended by the same arc are equal, so if angle A is 45 degrees then angle C is also 45 degrees ... (speaks fast and indistinct) ... (refers to sketch no.6, as he speaks).

Researcher: Okay, that's fine ... what about no.7 and the question again ... Why do you think that one is false.

Vuyolwathu: No. 7... **SOMEONE BEING LOUD IN BACKGROUND**

Researcher: Ja ...

Vuyolwathu: **PAUSE ...** I think this is not a cyclic quad. (refers to angles on Q7)

Researcher: That is not a cyclic quad... Okay, why isn't it a cyclic quad?

Vuyolwathu: Cause this angle doesn't touch the circumference. (refers to angle on sketch no. 7).

Researcher: Okay, fine ... Q8. How did you go about answering that one ... you said its true?

Vuyolwathu: I used the tan chord theorem.

Researcher: You used the tan chord theorem, so then you agree with that statement, there (refers to reason of sketch no.8).

Vuyolwathu: **PAUSE ... Yes (LOTS OF NOISE- NO ONE ASKING THAT NOISE STOP ...**

Researcher: Alright, and you would also agree that that sketch is the same as (turns page) ... that one ... (refers to Q 2.7).

Vuyolwathu: This one ... (agreeing with the researcher)

Researcher: Okay, so there isn't any difference?

Vuyolwathu: (Student turns pages to relevant sketches to check again) ... this sketch (asks about sketch 8 again).

Researcher: Ja, that sketch (refers no.8) ... and that one ... (refers to Q2.7)

BELL (HANDBELL) RINGS IN BACKGROUND _ SOUNDS AS IF ITS RIGHT OUTSIDE THE DOOR.
(VERY LOUD NOISE) ... (researcher has to once again point out the relevant/particular sketches) ... there's the one and here's this one ...

Vuyolwathu: They are different.

Researcher: They are different ... and, and what's different?

Vuyolwathu: This angle (refers Q8) which is subtended by this chord is not a ... (word indistinct)

Researcher: Okay, so you agree that, that that is ...

Vuyolwathu: True.

Researcher: You still agree that it's true?

Vuyolwathu: Ja, its true, yes.

Researcher: You're confused now because you realising that those are not the same things.

Vuyolwathu: Yes, sir. (speaks softly).

Researcher: Okay, that's fine ... What about Q9? **BELL STILL RINGING ...** How would you go about reasoning that one? You, you just said False. (refers to sketch) ... **LONG PAUSE ...** Why do you think its false?

Vuyolwathu: (Student mumbles to himself at first - indistinctly) ... I think the angle which is between the tangent and the chord, (points to no.9) doesn't subtend this angle (refers to P), it subtends this one ... angle U.

Researcher: Oh, so its U that is 25 degrees... (turns page) ... Q10 ... How did you go about answering that one?

Vuyolwathu: Well, I think, (indistinct) ... then if angle O is 60 degrees, therefore angle O is equal to twice the circumference ... **TAPE SOMEHOW GOES INAUDIBLE FROM THIS POINT TO Q12. LOTS OF GESTURING ON TAPE, BUT NO SOUND.**

Researcher: (Sound starts up again) ... Okay ... what about Q12, and again, you are familiar with that sketch ... you've seen it before?

Vuyolwathu: No ...

Researcher: No ... okay ... Q12. How did you go about answering that one?

LONG PAUSE ... LOTS OF NOISE IN BACKGROUND

Vuyolwathu: I think the angle which is in the alternate segment is angle A ... I used the theorem ... the angle between tan and chord subtend equal angle in opposite

segment, so A is in alternate segment. (THIS PART ALONE TOOK 10 MINUTES TO DECIPHER WITH THE HELP OF RESEARCHER. STUDENT WAS EXTREMELY INDISTINCT)

Researcher: Okay, so you will agree with this statement then?

Vuyolwathu: Yes.

Researcher: Okay, fine ... and you happy with the strategy you used ... the way you did it?

Vuyolwathu: Yes.

Researcher: Right ... Q13 ... Okay, explain your reasoning for saying that is false?

Vuyolwathu: LOTS OF NOISE AND TALKING ... I think QR is not a tangent.

Researcher: Is not ... so therefore the reason is not correct?

Vuyolwathu: Yes.

Researcher: Right ... What about Q14. How about answering that one. You said its true.

Vuyolwathu: (Student glance intently at page) ... LONG PAUSE ... STILL NOISE IN BACKGROUND ... SOUNDS OF YOUNG CHILDREN PLAYING... Angle V is a exterior angle.

Researcher: Okay, so that's the exterior angle (refers to no. 14) and where's the interior?

Vuyolwathu: Interior? ... this one (points to angle x on sketch)

Researcher: x ... So you agree that, that is correct. there's nothing, nothing wrong there?

Vuyolwathu: Yes.

Researcher: Okay ... and no. 15 ... How did your teacher explain that one to you?

Vuyolwathu: First, I use this theorem ... I use cyclic quad EDFA ... LOTS OF NOISE ... and if this is exterior angle (points to the given exterior angle) and this is the opposite angle (pointing to angle D) ... and the theorem says ... the exterior angle is equal to the opposite interior angle and D is the opposite interior angle, therefore angle D = 60 degrees.

Researcher: Okay, so you're happy with that one, and the way you answered that one (refers to no.15), is the same as this one. (refers to no.14).

Vuyolwathu: Yes.

Researcher: Okay ... umm ... Q16. Okay, how did you go about answering that one ... again you said, false.

Vuyolwathu: PAUSE ... umm ... 16 ... I think sir it is not the interior angle.

Researcher: Okay, which angle is between tangent and chord?

Vuyolwathu: Its, its this angle (refers to given angle 57 degrees).

Researcher: Okay ... sure?

Vuyolwathu: Yes, this angle is 57 degrees ... the angle at the alternate segment is x, therefore x is 57 degrees.

Researcher: So you agree with that statement? (refers to sketch).

Vuyolwathu: No, sir ... I do not agree with it.

Researcher: You don't agree with it, because here we said the exterior angle of a cyclic quad. is equal to the interior opposite angle.

Vuyolwathu: U is not the interior angle ... I mean the exterior angle.

Researcher: U is not the exterior angle ... fine ... great, that was good.

INTERVIEW 4 (ZAMEKA)

Researcher: ... Now the first one is ... you said this is false, and $x = 20$ degrees. Why do you think that was false (refers to Q1).

Zameka: NOISE IN BACKGROUND, LOUD VOICES ... Its because this angle, the angle this side (refers to 40 degrees on sketch) is twice the angle of the circumference, therefore x should be the half of that.

Researcher: Half of that ... Okay, and no. 2. You said its true, so you agree with the statement.

Zameka: Yes. (speaks softly).

Researcher: And are these two questions exactly the same?

Zameka: (turns page toward her) ... Yes.

Researcher: Okay, and you're familiar with those questions, I mean with those sketches ... you've seen it before?

Zameka: Yes, sir.

Researcher: Okay, umm ... this one you said was false (referring to Q3) ... that is true ... so there isn't a problem there (referring to Q4) ... lets go to Q7.

Zameka: (Student turns page toward her to see better).

Researcher: Okay, why do you think that question is false?

Zameka: They say the opposite angles of a cyclic quad are supplementary.

Researcher: Speak a little bit louder.

Zameka: Okay, sir ... this is not a cyclic quad, because this sides are not lying on the circumference.

Researcher: Okay, so that is false, because that's not a cyclic quad.. Perfect, I'm happy with that ... what about Q8 ... umm ... How did you go about answering that one.

NOISE AND TALKING IN BACKGROUND ...

Zameka: PAUSE ... (mumbling to herself)

Researcher: Cause you see, you just wrote false, so that's why I want to see why you said it was false.

LONG PAUSE AGAIN ... STILL LOUD TALKING IN BACKGROUND

Zameka: Okay, sir ... this angle 65 degrees there (pointing to the relevant angle in Q8) is between a tangent and a chord. That statement, this theorem says an angle between a tangent and a chord is equal to the angle in the alternate segment, but O is not an alternate segment.

Researcher: Is not on the ... circumference?

Zameka: (student merely repeats what researcher says)

Researcher: Okay, so that's why its false ... alright, good ... Q8 (researcher meant no. 9) ... How did you go about answering that one?

Zameka: (Student moves page to see better) ... PAUSE ... LOUD MALE VOICE IN BACKGROUND ... VERY LONG PAUSE ... VERY NOISY ... This angle B (pointing to angle on sketch 9) should be 25 degrees because ...

Researcher: (repeating) Angle B should be 25 degrees ... and why would angle B be 25 degrees?

Zameka: Because this angle (referring to T) should be equal to that one (referring to B).

Researcher: And what reason would you use?

Zameka: The angle between a tangent and a chord is equal to the alternate segment.

Researcher: Okay, if I had x by B, then that would be true? is that what you saying?

Zameka: Yes. ... **LOUD MALE VOICE IN BACKGROUND**

Researcher: Okay, right ... okay ... good ... **LOTS OF NOISE** ... (turns page) and you familiar with these sketches, you've seen it before (refers to sketches of test 2)

Zameka: Yes.

Researcher: Okay ... umm ... Q11 ... Why do you think that question is true?

Zameka: This one (points to Q11).

Researcher: (repeating) Ja ... why do you think it is true?

Zameka: (points to sketch) ... I say its false ... **VERY LOUD NOISE** ...

Researcher: Oh ... okay, false ... why do you say its false?

Zameka: Because x is not ... **LONG PAUSE** ... x is not a tangent ... **PAUSE** ... x , x is not a tangent, sir.

Researcher: S ... x is ...

Zameka: Is not a tangent ... is not an angle between a tangent and a chord.

Researcher: Okay, so where should x have been?

Zameka: **LOUD NOISE** ... (points to sketch. Here ... (referring to T).

NOISE IN BACKGROUND VERY LOUD.

Researcher: Because I see you put the 82 degrees over there (referring to P). So why do you put the 82 degrees over there?

Zameka: I was thinking, this 82 degrees (Q) should be the angle between the tangent and the chord, therefore equal to P.

Researcher: Okay, angle between tangent and chord should equal that one (refers to P) ... Okay ... and Q12 we said ... Okay and Q13 ... You said its false, lets see why.

Zameka: **LONG PAUSE** ... **LOTS OF NOISE IN BACKGROUND**... Here are not the tangents from the common point.

Researcher: What, what's not the tangent?

Zameka: PQ and QR.

Researcher: Are not tangents?

Zameka: Yes ... QR is not a tangent.

Researcher: QR, is not the tangent ... Okay, in Q14 you said it's false ... and your reason?

Zameka: **PAUSE** ... Okay, sir, this x should be 78 degrees, but x is not 78 degrees, because this angle (refers to V), this exterior angle is equal to all angle T.

Researcher: The whole angle T ... Okay ... alright ... the last one. No.16. How did you do that one?

Zameka: **PAUSE** ... (turns page toward her to see better) ... **SOUNDS OF TYPING AND CUPBOARD DOOR BANGING IN BACKGROUND** ... Angle U is not the ... is the tangent, therefore angle T should be equal to S1.

Researcher: Should be equal to S1 ... so the main thing is, that that angle is not the exterior angle (refers to the given angle).

Zameka: Yes, sir.

Researcher: Okay ... thank you very much ...

INTERVIEW 5 (NDZILA)

Researcher: Let me ask you this question here, the very first one (referring to Q1). Why do you think that one is false?

Ndzila: Umm ... it is false because this angle is touching the circumference and this one is the centre, so I must say the angle at the centre is twice the angle at the circumference, so this statement is false.

Researcher: Okay, good, now if that was the case, then what would x have been?

Ndzila: If this is the angle at the centre, umm ... it would be 80 degrees ... x (pointing to the centre of sketch).

Researcher: So, so the angle there ... (indicating S on the sketch) would be double that one (indicating 40 degrees on sketch), or is it the other way round?

Ndzila: It's double ...

Researcher: Which one is double?

Ndzila: (Student indicates 40 degrees)...

Researcher: That one is double S?

Ndzila: Yes.

Researcher: So then x would be what? Will be 80 degrees.

Ndzila: x will be 80 degrees.

Researcher: Okay ... we will come back to that again ... No. 2... you said its true. Alright, so if its true, then it means the angle at the centre is twice the size of the angle at the circumference ... okay ... Are those two sketches the same? (referring to Q1 and Q2).

Ndzila: Umm ... yes.

Researcher: Okay, so you don't have a problem, you've seen these figures before (referring to the sketches in 1 and 2).

Ndzila: Yes.

Researcher: Okay ... lets go onto the third one there, umm ... you said its false ... why do you say its false?

Ndzila: **LONG PAUSE** ... (student whispering to herself) ... umm ...

Researcher: You see, I'm also looking at the reason that you gave.

Ndzila: Okay ... **LONG PAUSE** ... (still trying to work out answer, points to sketch now and again).

Researcher: Or just tell me how do you think you went about it, ... I see a hundred there ... (pointing to 100 written in on centre of sketch).

Ndzila: Okay, this is a triangle (referring to PQT), so these are ... angles (response indistinct).

Researcher: Okay, and then?

Ndzila: Umm ... I don't know... **LOTS OF NOISE IN BACKGROUND** ... **PAUSE** ...

Researcher: Okay ... lets go onto this one (referring to Q4) ... No. 4. Why ... how did you go about solving that one (pointing to sketch no. 4) ... What strategy did you use?

Ndzila: Umm ... angles subtended by the same chord are equal angles.

Researcher: Okay, so you used that one ... are these two exactly the same? (referring to sketch 3 and 4).

Ndzila: No ...

Researcher: What's different?

Ndzila: Because here, referring to P on sketch 3), this line touches the circumference ... this line (referring to S in Q3) is just hanging ... so that's why.

Researcher: Okay, so that was the main reason why that one (referring to Q3) was false?

Ndzila: Yes.

Researcher: Okay ... now lets go onto Q7 ... you said false. Why do you think that is false?

Ndzila: Well ... this is not a cyclic quad. ... umm ... because a cyclic quad. ... must be on the circumference ... all four ... (indistinct) ... Here, there is only P, S and Q, O is hanging, so that's why.

Researcher: Okay ... so the main one is that O should be on the ... circumference?

Ndzila: Yes ... that's right.

Researcher: Okay, that's fine ... Q8. How did you go about answering that one ... you said its true, right?

Ndzila: (whispering to herself and then says loudly) ... Yes, it is ...there's the angle between the tangent and a chord. (referring to 65 degrees on Q8) ...

Researcher: Okay, so you'll agree with me that this one here (referring to Q8) and the one that you had correct in Q2.7 (referring to Q2.7) is the same? (pointing to the two relevant sketches).

Ndzila: Yes, it is.

Researcher: Okay, so there isn't a difference between the two?

Ndzila: No.

Researcher: Okay, ... that's fine ... What about question ... (pointing to 9 but not completing sentence) ... and you familiar with ... you've seen that sketch before? (pointing to Q7) and that one (pointing to Q8).

Ndzila: Yes.

Researcher: Okay ... and the others you are quite familiar with those as well?

Ndzila: Yes, sir.

Researcher: Alright, what about Q9. How did you answer that one?

Ndzila: Okay, this is the angle between the tan and a chord (referring to the 25 degrees) ... so it is equal to this ... an alternate segment (pointing to sketch)

Researcher: So, that is very much the same as this, that and that one (referring to Q8 and Q7, the other sketch R pointed to was out of the picture)

Ndzila: Yes, sir.

Researcher: Alright ... lets go onto no.10 (turning to the relevant page) ... umm ... at least no. 11... I am very interested just to see how you went about no.11.

Ndzila: Umm ... (turns page closer to her) ... PAUSES LONG ... (then whispers to herself and finally points to the sketch) ... Here 82 degrees is ... this is a triangle, so 82 degrees is outside the triangle. So this is an exterior angle ... so I've said, the exterior angle is equal to the opposite angle.

Researcher: (Researcher repeats) ... to the opposite angle ... So you don't agree with that statement? (referring to the reason next to the sketch no.11).

Ndzila: Yes ... (mumbles something incoherent).

Researcher: It says there the angle between the tangent and the chord ...

Ndzila: No ... I don't agree.

Researcher: Alright ... so you using the exterior angle of a triangle.

Ndzila: Yes.

Researcher: Okay, ... What about no.12. You said its ... you said it was true ... Now, what strategy did you use there, or how did you go about solving that problem?

Ndzila: PAUSE ... okay (points to sketch) ... this is a tangent (referring to DCF) ... now, this one ... (pointing to angle x) ... (indistinct) ... now, this one ... the chord is here (referring to CB) ... a tangent (pointing to CF) ... so (indistinct) ... this one (refers to centre) ... (SPOKE IN A STOP AND START MODE)

Researcher: And that's why you agree with it?

Ndzila: Yes.

Researcher: So the angle between the tangent and a chord is equal to that angle (points to the given angle on sketch 12).

Ndzila: Yes, sir.

Researcher: Okay, that's fine ... and Q13. What is your reasoning for that one ... you said false ... QR is not a tangent ... you agree that QR ... Why is QR not a tangent?

Ndzila: QR? ... is not a tangent ... a tangent must touch a circle at one point (referring to sketch 13) ... So this is just from here to here (referring to QR).

Researcher: Okay, so it's not a tangent ... that's fine ... Q14 ... How did you go about answering that one?

Ndzila: Err ... the exterior angle is equal to the interior opposite angle.

Researcher: Okay, ... and you've applied the theorem correctly?

Ndzila: Yes.

Researcher: Okay, so you, so you don't see anything wrong, there?

Ndzila: No.

Researcher: You happy with that?

Ndzila: Yes.

Researcher: Alright, ... no. 15. You said, false ... and then you said the reason ... is the angle between a tangent and a chord is equal to the angle in the alternate segment ... Now ... is GE a tangent?

Ndzila: No ... it's not a tangent.

Researcher: So that's not correct that you filled in there?

Ndzila: Yes.

Researcher: Okay, ... so, what do you think? ... So I've said that its 60 degrees, and my reason, would you agree, or not agree?

Ndzila: Ja ... the exterior angle is equal to the interior opposite.

Researcher: So you will agree that it is ... true?

Ndzila: Yes.

Researcher: Okay, ... and there isn't a ... is this exactly the same? ... (referring to Q14 and Q15) ... number 14 and 15?

Ndzila: No ... LONG PAUSE ...

Researcher: What do you think is different?

Ndzila: I must say this one ... (referring to T and x on sketch 14) is not really opposite to this angle ... (referring to given angle).

Researcher: Okay, so that wouldn't be true then ... (referring to the reason of Q14).

Ndzila: Yes.

Researcher: So it will be false?

Ndzila: I think so.

Researcher: That its false ... so you would disagree with your answer, there ... (referring to answer given in the sketch/ reason 14) as true?

Ndzila: That's the way I said it is true, but then its ... (response very indistinct).

Researcher: Okay, ... because you see, in the goodies ... (referring to test 1) you had the correct statement there, so if you look carefully at Q2.3 and sketch 14, are you applying it correctly?

Ndzila: I applied it correctly to this one. (refers to Q14)

Researcher: In that one, okay.

Ndzila: And now ...

Researcher: What, what did you do wrong there?

Ndzila: Well, this is a ... (indistinct) ... and these angles are equal ... and then what about this one? ... (refers to the 78 degrees).

Researcher: Okay, that's the 78 degrees.

Ndzila: Yes.

Researcher: Okay, now where's the interior opposite angle?

Ndzila: What about this one ... (pointing to Z) on sketch 14.

Researcher: Z

Ndzila: No, T.

Researcher: T, but not x or T and x?

Ndzila: x

Researcher: x?

Ndzila: Yes.

Researcher: So, so that's correct (pointing to the sketch) ... so you go back to your true statement.

Ndzila: LONG PAUSE ... Yes, that is true.

Researcher: Okay, no that is fine ... lets go to the last one. Q16... umm ... how did you go about answering that one? and you said vertically opposite angles? ... now you must explain. Tell me how did you think you were going to solve that one?

Ndzila: (Whispering to herself) ... (pointing to parallel signs on the lines) ... this sign here are telling me that these lines are parallel. So that is why I said false, I think ... if the lines are parallel, the corresponding ... angles are vertical, and so I just assume it is ... (indistinct last word).

Researcher: So, so you don't agree with what I did there ... (pointing to reason given) to say that that's the exterior angle of the cyclic quad.

Ndzila: I do, I do. (sounded uncertain).

Researcher: So, TUV is an exterior angle of a cyclic quad. You'll agree with that?

Ndzila: Yes.

Researcher: Okay, so then that will be similar to these ones? ... number 14 and 15?

Ndzila: Yes, yes.

Researcher: Okay, right, great.

INTERVIEW 6 (RUSHANA)

Researcher: ... I am going to start with Q7. There ... have a very good look at Q7. There ... why do you think that question is true?

Rushana: Cause its a cyclic quad, ... that's what I was thinking when I went ... (indistinct end of response).

Researcher: Okay, so you thought that was a cyclic quad?

Rushana: Mmm, opposite angles of a cyclic quad. Ja.

Researcher: Okay, and you obviously know what a cyclic quad is, because you got it right, you knew the definition, and you obviously picked up that, that figure was a cyclic quad.

Rushana: Cyclic quad, yes.

Researcher: And do you realise what went wrong there?

Rushana: Yes.

Researcher: Do you still agree with what you said?

Rushana: No (laughs) ... the points are not touching the circumference on the circle.

Researcher: So then it's not a ... cyclic quad. Fine ... what about question ... umm ... Did that sketch pop up in your teaching? Did your teacher show you something like that? Are you familiar with that sketch?

Rushana: With this one?

Researcher: With no.7 ... Have you seen a problem like that before?

Rushana: Yes ... (giggles).

Researcher: Okay ... and Q8 ... umm, ja, how did you go about answering that one? Again your response was true, hey.

Rushana: Yes, umm ... the angle between the chord and tangent equals the angle in the opposite segment.

Researcher: And you still agree with what you said, here?

Rushana: (giggle) ... I think that should also touch the circumference of the circle ... I'm not quite sure ...

Researcher: Which point?

Rushana: This one over here (points to the centre of sketch 8) ... point O.

Researcher: Okay, because again I think, if you check ... the statement you had correct, hey!

Rushana: Umm ...

Researcher: The angle between a tangent and a chord is equal to the angle in the other segment, (turns page) ... and DE is a tangent ... you see, you got that correct as well ... (points to Q2.7 on previous page), right ... and if you compare the two, then you can see, maybe, what went astray, and its exactly what you just said, now ... that, that point should be on the circumference.

Rushana: Yes. (giggle).

Researcher: So, it's those kinds of things that we need to pick up on. What I want to know from you again ... Was that emphasised in the class? Have you seen something like that?

Rushana: We don't get it that simple ... like only one figure with a lot of others.

Researcher: With a lot of lines and things in ...

Rushana: Umm ... (giggle).

Researcher: Okay, what we wanted to pick up here was just to find out, because obviously, if things like that go astray, then its obviously going to mess up the rest of the ... umm ... problem solving, alright?

Rushana: Yes.

Researcher: How about Q9, now you must take me through that, very slowly. You also said its true, just explain your reasoning?

Rushana: Cause this is the angle between a chord and a tangent, so it equals angle U in the opposite segment. (points to U on sketch 9).

Researcher: Okay, so it equals angle U. (pointing to U on sketch 9), but we want x, so would you agree that x is 25 degrees?

Rushana: PAUSE ... giggle ... No.

Researcher: Why not?

Rushana: (giggle) ... umm ... no that isn't in the opposite segment.

Researcher: Okay. So that's not the angle in the other segment?

Rushana: No.

Researcher: And if you should try and solve it, what do you think you'll do?

Rushana: (mumble to herself, probably trying to work out the answer).

Researcher: I hope I'm not putting you on a spot now?

Rushana: (laugh) ... No ... umm ... (laugh) ... I can't see this ... now. I think its the exterior angle of the cyclic quad, so then if ... let me just think ... this is the exterior angle (referring to the given 25 degrees) of the cyclic quad, ne, TQPB, okay?

Researcher: Okay.

Rushana: You have to look at the angle next to this one (points to given 25 degrees), then the one opposite that is equal, so that's why x will be 25 degrees.

Researcher: Okay, so you agree with that ... that's the exterior angle (pointing to same 25 degrees), which is equal to the interior opposite angle ... okay ... lets go on. (turns the page). Q11, you also said it was true ... can you explain that one to me?

Rushana: PAUSE ... I don't think its true because the triangle isn't in the circle.

Researcher: Okay, which triangle?

Rushana: Triangle PSQ.

Researcher: So S must be ... in the circle. Is that what you're saying?

Rushana: Umm ... (agreeing).

Researcher: Okay, and if S was on the circle, then that would be 82 degrees.

Rushana: Yes.

Researcher: Right, okay, fine and Q12, how did you go about that one ... you said true ...

Rushana: Umm ... (agreeing).

Researcher: I'm just looking at ... a strategy. What strategy did you use?

Rushana: Oh! ... because this angle over here, angle x, is the angle between a tangent and a chord, so if you do this over here (her hand covers something, cannot see what), and then angle E will be ... (mumbles something indistinct, and continues) ... this two over here, (points to B and E) are angles in the same segment, that is why it equals ... 70 degrees. And then this one, here, is the angle between a tangent and a chord (referring to angle at C), that's why it will equal to angle E (points to E on Q12).

Researcher: Okay, so you had a good strategy for doing that one, that's fine ... umm Q13, that one, you also responded with true.

Rushana: I have no idea how I worked out that one ... (giggles).

Researcher: But would you agree that it's true, or would you change your mind now?

Rushana: No, I don't think it may be true. (giggles)

Researcher: Why not?

Rushana: Umm ... PAUSE ... there's no reason for it (laughs) ... let me see now why they say (points to reason next to Q13), PQ equals QR ne.

Researcher: PQ equals QR ... so you agree with that?

Rushana: Umm ... (agrees).

Researcher: Alright, and the rest of the part where it says tangents drawn from a common point are equal? ... do you agree with that statement?

Rushana: Yes.

Researcher: Okay, now do you agree that RQ is a tangent?

Rushana: RQ?

Researcher: Is RQ a tangent?

Rushana: Yes.

Researcher: Okay, ... why is it a tangent, or what is a tangent?

Rushana: A line that touches a circle at only one point.

Researcher: And RQ, is it touching at one point?

Rushana: No ... (giggles).

Researcher: It cuts at? ... two points, at R and S (points to points on sketch 13) ... So now would you still agree with what you said?

Rushana: No ...

Researcher: Okay, and if you should try and solve it, what would you do?

Rushana: This angle over here, SQP ne? (points to SQP on sketch 13), that will also equal to 37 degrees.

Researcher: Okay ... and your reason?

Rushana: Because PQ equals QR. So it's an isosceles triangle.

Researcher: (repeats) ... because PQ equals QR.

Rushana: Umm ... (agrees). So that this means that this angles here are equal. (points to relevant angles).

Researcher: Because, then you're agreeing with the statement again. PQ and QR is equal (points to given reason).

Rushana: Ja ... I have no idea how to solve this one because the angles only starting from there (points to given 57 degrees), its not like that other one, you could see that its a isosceles triangle.

Researcher: Yes ... so what you could do ... is ... that's the angle (pointing to angle at P) between a tangent and a chord, so that would be equal to angle S. (points to S).

Rushana: Yes.

Researcher: Right ... using the tan/chord theorem, and then since this is an isosceles triangle, you can get the base angles, and then you can get to the supplement of that.

Rushana: Umm ... (agrees).

Researcher: Again, what I'm looking at is ... or what's missing here is the fact that you had the tangent correct. You knew what a tangent was by definition. You applied it there (points to sketch in Q2.7), but it just went astray over there (referring to Q13), so you just need to watch out for that ...

Rushana: Okay

Researcher: Umm ... and then the last one. Q16, can you explain how you would go about solving that one ... you ended up with 57 degrees, so I just want to see if you still agree with what you did there.

Rushana: Umm ... this one is the exterior angle of a cyclic (refers to RSUT), so then it equals the interior opposite angle, that's why I got $x = 57$ degrees.

Researcher: Okay, so it would be similar to the other one you mentioned earlier.

Rushana: Umm ... (agreeing)

Researcher: The exterior angle is equal to the interior opposite angle.

Rushana: (repeats the last part of sentence) ... to the interior opposite angle ...

Researcher: Okay, good ... nearly done.

AT END OF SKETCHES, RESEARCHER CAME BACK TO Q16. THE BELL RINGS FOR INTERVAL, VENUE BECOMES NOISY.

Researcher: Now, I'm going to come back to the last thing you gave me ... no.16 ... you said the exterior angle of a cyclic quad is equal to the interior opposite, and there you have a couple of sketches (pointing to Q14 and Q15). Do you think ... you're applying the theorem correctly? ... You don't have to solve it.

Rushana: No.

Researcher: Okay, why not.

Rushana: That is not the exterior angle (pointing to angle at U) ... This one over here is (pointing to angle at T).

Researcher: That's right, okay ... and you know why that one is?

Rushana: Because, umm ... ooh yes, a tangent (pointing to UW) ... (rest of response indistinct, Swallows words).

LOTS OF NOISE OUTSIDE

Researcher: So, so this would then be the angle between (pointing to given angle 57 degrees) ... the tangent and the chord.

Rushana: This one would be 57 degrees. (pointing to S).

Researcher: Yes ... okay, ... that's fine ... as long as you identify that that's not actually the exterior (pointing to the given angle) angle, if one side is not produced. Alright, Rushana, thank you very much (only now does the name of student get mentioned).

Rushana: You welcome.

INTERVIEW 7 (LANCE)

Researcher: ... I think the first one would be Q2.5. Can you explain that one to me?

Lance: (Turns page toward him). Well, basically the wording does'nt do right. Basically the theory states that the radius is perpendicular to the chord, (indistinct) ... to the tangent.

Researcher: ... to the tangent ... so you just misinterpreted it?

Lance: Ja.

Researcher: Okay, Q1. You said was true, would you agree with that still?

Lance: No, the ... does'nt touch the circumference (swallows or does not mention word)... (points to O). So it cannot be angles in the same setting there. The theory states that angles in the same segment ... but then it had to touch the circumference of the circle, sir.

Researcher: Okay, so you disagree with that, then ... So what would x be now? What would the value of x be?

Lance: PAUSE...

Researcher: Because you're disagreeing that x is 40 degrees.

Lance: Ja, I guess x would ... I don't really know ... Well, well then x would have to be 20 degrees, then ... (indistinct), ... angle at the circumference.

Researcher: Okay, so it's definitely not that theorem, but its the angle at the centre thats twice the size of that one. (points to S).

Lance: Yes.

Researcher: Alright, fine ... Q2, you said was true, right ... and is there a link between the two? What do you think ... are the two questions the same?

Lance: PAUSE ... err ... let me see ... Yes, it is, because (referring to sketch 2) AOB is the angle at the centre and ACB is the angle at the circumference.

Researcher: So you agree with that one (points to given reason).

Lance: Ja.

Researcher: Okay, fine and tell me, are you familiar with those sketches, you've seen it before in class?

Lance: Quite on numerous times.

Researcher: Okay ... now Q3, how did you go about answering that one?

Lance: Well, I guess I used the same method which was incorrect, as I did with Q1, because (referring to S on sketch 3) it did not touch the circumference of the circle, which was again my mistake, I guess. In fact, I think its true, sir.

Researcher: Okay, so if S had been on the circumference ...

Lance: Then it would have been 30 degrees ... ja.

Researcher: Okay, and if I should ask you what the value of x would be there?

Lance: x would ...

Researcher: You must calculate ...

Lance: 60 degrees, ... 60.

Researcher: Okay, how do you get to 60 degrees?

Lance: Well, (spoke fast and indistinct), ... is twice the angle at the circumference. Angle at the circumference equals 30 degrees. So the angle at the centre ... no, ...

but it can't be angle at the centre, could it, ... because the points of the centre of the circle ... (unfinished sentence)

Researcher: Only O is the centre of the circle.

Lance: Ja, so I think that's ... can't be that ...

Researcher: I mean, you understand why it can't be true?

Lance: It can't be true, ja.

Researcher: Okay, great ... what about question ... question 4, that went okay ... Q6

Lance: Q6 ...

Researcher: Did you have a strategy for getting to true there? How did you get to that one?

Lance: Ja, ... I think its the two dots method, that we learnt at camp ... umm ... where we would place two dots on the chord (pointing to the sketch) and then you would move it up to the circumference and the angle that you're left with at the circumference you would say, that would be the angle subtended by the same chord, ... I guess.

Researcher: Okay, ... Q6 and Q4, is it exactly the same?

Lance: Yes ... I think you could use the same method there.

Researcher: Okay, ... What about Q7, you did say it was false, and then you said $x = 55$ degrees, so what I want to explore is, how did you get to it?

Lance: Okay, ... look, that's the angle at the circumference (referring to 110 degrees, so we looking for the angle at centre, but we can't use that angle (referring to x). So we have to use an exterior angle and using that is probably angles round a point, which is 360 degrees. So taking a look at that, that will be 55 degrees, the point around that angle (referring to O). So x won't be 55 degrees, it's actually 360 minus 55.

Researcher: Okay, so you're saying this one here the x ... the reflex angle is 55 (referring to the angle at point O).

Lance: Ja, ja ... 55.

Researcher: And how did you get to 55 degrees?

Lance: Angle at centre is twice the size of the angle at circumference.

Researcher: Okay, so the angle at centre is twice the size of the angle at the circumference, which is this 110 degrees (referring to the given angle 110 degrees).

Lance: But, but ... then O should be 220 degrees.

Researcher: It should be 220. Okay, so you divided by two?

Lance: Ja ...

Researcher: Okay, that's fine ... Q8, you said true? Do you still agree with that?

Lance: No, its part of the main mistake again, that the point is not on the circumference.

Researcher: So O must be on the circumference. Are you familiar with those sketches?

Lance: Ja, ja ...

Researcher: Okay, Q9, you just said false, can you go a little bit further?

Lance: (laughs) ... Ja ... umm ... Well the reason they have here, is angle between tangent and chord equal to angle in opposite segment (says this fast and indistinct, almost speaking to himself). Well ... my reason for writing false in the exam was, because I felt that B was the opposite angle instead of P.

Researcher: So B would be the angle in the opposite segment?

Lance: Ja ...

Researcher: Okay.

Lance: Ja ... B, as for the value of x ... I don't know. My reason was false, but I'm not too sure what the value of x was.

Researcher: Okay, that's fine, what you could do to find the value of x though is ... do you remember the theorem I asked you ... that one ... the radius, where you said ...

Lance: Ja

Researcher: Where you said radius is perpendicular ... so that would be 25 degrees (referring to Q9) and you get that one (referring to the angle at T).

Lance: Ja, that would equal 90 degrees ... (response indistinct) ... plus something equals 90.

Researcher: Ja, and then you'll be able to get x .

Lance: Yes ... x ... opposite angles of a cyclic quad.

Researcher: Okay ... alright, so that's Q9 ... (turns the page). Okay, Q10, how did you go about that one. Cause I see you gave a nice long ...

Lance: Ja.

Researcher: Which I'm not saying is wrong. I just want to ...

Lance: Okay, I came to the conclusion first, that it was false, not realising that, using the theorem, it was actually true. And so I used another method and I got to the same answer as $x = 30$ degrees. So I basically used another method which was obviously much longer, than the method ... (indistinct) used in the paper (test) to get to ...

Researcher: Okay.

Lance: To get the same answer. The reason, the angle at the centre is twice the circumference. It was kind of hard to identify the angle at the centre, the angle at circumference ... that's why I probably thought it was false in the beginning.

Researcher: Okay ... So you went that route to get to the answer?

Lance: Ja ...

Researcher: If you now relook at the given answer, x is 30 degrees and the reason ... angle at centre is equal to 2 angle at circumference. Would you agree with that? or is it still confusing?

Lance: Well, in the exam I saw that it (rest of response indistinct) ... but we're going now to look at it immediately and see that its an angle at the centre theorem.

Researcher: Okay, ... and then you went for the other ... that's fine. What about Q11. Why do you think that question was false?

Lance: Q11? (talks to himself at first) ... Well, simply because I did'nt see angle S as in one of the other problems as being the angle in the opposite segment.

Researcher: Okay, what I'm interested in is your reason for saying false.

Lance: False?

Researcher: Ja ... can you explore that one for me?

Lance: As I said T is actually 82 degrees. I saw T as being the angle in the opposite segment, and so if T is the angle in the opposite segment and here we're saying that x is the angle (referring to reason no.11) ... I decided to go for false, because you obviously can't ...

Researcher: Alright, so you agree that angle T is the angle in the other segment.

Lance: Ja ...

Researcher: And would that be the same as in Q2.7?

Lance: Yes.

Researcher: Is it the same strategy?

Lance: Ja ... exactly the same.

Researcher: Exactly the same?

Lance: Ja.

Researcher: Okay, alright ... in 12. How did you go about getting to true there?

Lance: Okay ... x equals 57 degrees. Well you can see I obviously ... that this was false, that is printed in the paper here. (referring to Q13, although researcher asked about Q12).

Researcher: No, I'm talking about this one, here. (pointing to Q12).

Lance: Oh, you're talking about no.12. umm ... angle between tangent DC and chord BC is equal to angle in opposite segment ... True ... well, here you have your cyclic quad (referring to sketch) and your opposite angles ... the theorem states that the opposite angles of a cyclic quad is equal to the opposite interior angle, it will be angle A. Do they ask you what is x ... but then this whole angle should actually be equal to x (referring to sketch vaguely) because its obviously a cyclic quad.

Researcher: So you said that BCD is an exterior angle of a cyclic quad?

Lance: Ja ...

Researcher: You sure?

Lance: Or, but should'nt a tangent then run ... say on the line (referring to EC) ... that's where the exterior angle should be.

Researcher: Yes, okay ... so ... do you agree with that statement (referring to reason in Q12).

Lance: Ja ... that would be a more appropriate statement.

Researcher: Okay, so do you have a strategy? Did you apply some strategy to ...

Lance: Ja, basically the same ... you get your two dots on your chord, and then you move to the circumference, to get the angle ... and then that is your answer.

Researcher: Okay, now, can we go back to no.11? using that strategy, would you still agree with T being that angle?

Lance: Okay, if I use the same strategy, I'll put my fingers and move up to the circumference and I would get T (demonstrates on sketch with fingers).

Researcher: But, but what's the chord ... which is the chord that we referring to there?

Lance: PAUSE ... QP. I'm using the chord QP to get to T.

Researcher: Okay, but remember that you would get ...

Lance: So you actually saying we should use QT (indicates on sketch) ...

Researcher: And then you moving to ... P.

Lance: P ..., So P is actually the opposite.

Researcher: Okay, so a lot depends on which chord you use. Okay, no that's fine. I just want to make things ...

Lance: (completes the sentence) ... clear, ja ...

Researcher: You aware ... so that when you do it in an exam you can pick it up.

Lance: Ja.

Researcher: Okay, Q14, you also said its false ...

Lance: Ja, because, as my reason was here, although incorrect. I felt that the opposite angle of a cyclic quad is equal ... to the exterior angle of a cyclic quad is equal to the opposite interior angle.

Researcher: Yep ...

Lance: Then it will be the whole of angle T (referring to T on sketch 14) not T1 or T2 for that matter (breaking T into two adjacent angles).

Researcher: Okay, great.

Lance: So you could'nt go about solving for x using that method ... so the method I rather used I took, ... what ... I assumed that TV was a diameter. It mean't that W was a 90 degree angle.

Researcher: Okay, so ...

Lance: Not interior angle of a cyclic quad, sorry, sum of the interior angle of a triangle. They give you that WT and WV are equal, so that means I just used the base angle, isosceles triangle for TVW.

Researcher: Okay ...

Lance: So we ... (very indistinct) ... 45 degrees.

Researcher: Okay, ja I'm happy with what you saying there, there's just the one thing ... you assumed that TV was the ... diameter, and do you think that assumption is correct? That's all ... we wont solve the problem.

Lance: Ja ... so okay ... probably one of the first lessons we learnt in geometry is never to assume, so that was a bit rash on my part.

Researcher: Alright, ... so that threw out the calculation, otherwise it would have been absolutely ...

Lance: correct, ja.

Researcher: Perfect.

Lance: it would just be on the basis if TV was actually a diameter.

Researcher: That's right ... okay ... okay, we're busy with Q16, lets see how you went about solving that one?

Lance: I mentioned that its true ... probably false, though its quite difficult to recognise the opposite angle ... its probably S or R ... obviously when I wrote the examinations I went for R ..., x is the opposite angle which is equal to 57 degrees, umm ...

Researcher: So with hindsight now you realise that, that is not the exterior angle (referring to 57 degrees on sketch).

Lance: Oh the angle ... ja.

Researcher: But its actually the angle between the ...

Lance: Ja ...

Researcher: Tangent and a chord.

Lance: And a chord ... ja ... using the same method as I did, it will be actually S there because the chord moves up to the circumference ...

Researcher: Thats right ...

Lance: So if S is equal to 57 degrees, therefore ... ja, then x cannot be equal to 57 degrees ... using the mentioned reason.

Researcher: Ja ...

Lance: So I've obviously seen my mistake, there ...

Researcher: Okay, alright, great. Thank you very much.

INTERVIEW 8

Researcher: Lets start with Q1.

Rephwaai: (Student turns page toward him)

Researcher: Can you just ... or why do you think that one is true?

Rephwaai: Because the segments here (referring to sketch) ... they are in the same chord.

Researcher: So you would still agree with that answer ... true?

Rephwaai: That's the way I saw the ... (indistinct).

Researcher: Okay, ... Q2. How did you get to that one being true?

Rephwaai: (Student turns page back to him) ... I think the same thing, only it's ... like at a different angle, here (referring to Q1 and Q2).

Researcher: Okay, but now ... okay, here we said x is equal to 40 degrees. So it's exactly the same there (pointing to sketch 1) ... here we said x is equal to 120 degrees. You with me? ... So what I'm interested in is in the one test you said true ... in the other test you also said true, but if you compare the two sketches, are they exactly the same?

Rephwaai: No.

Researcher: Okay, what's different?

Rephwaai: (referring to O on Q1) ... here you have the angle in the centre but here (refers to Q2) you actually have to double the angle here.

Researcher: (pointing to O on Q2). So you spotted that that is the angle at the centre ... (pointing to O on sketch 1) and this one is also the angle at the centre ... You said at the theory part, the angle at the centre equals the ...

Rephwaai: ... equals the angle twice the circumference, subtended by the same chord.

Researcher: Okay ... lastly ... do you still agree with what you did there (pointing to Q1).

Rephwaai: Yes ...

Researcher: So you don't see anything wrong with that one?

Rephwaai: No.

Researcher: Alright, lets look at Q3 ... umm ... you also said true. Do you still agree with that?

Rephwaai: PAUSE ... (whispers) ... I don't think so...

Researcher: What's making you ...

Rephwaai: ... because there's not actually here, the chord there is not touching the ... (points to RS)

Researcher: Circumference ... Okay, so that would mean its not going to be necessarily 30 degrees.

Rephwaai: Yep.

Researcher: Okay ... alright, what about Q4.

Rephwaai: (student moves fingers over sketch while thinking) ... Its true.

Researcher: Its true ... and what strategy did you use there?

Rephwaai: It is the angles ... they are both on the circumference.

Researcher: And they come from the same chord.

Rephwaai: Umm ... (agrees)

Researcher: I can see you've got the strategy, because if you look at Q6, is it exactly ... is it the same question?

Rephwaai: It is similar ... to a degree.

Researcher: Alright, Q7, again you said true.

Rephwaai: PAUSE ... Ja ... yes I do, because the opposite angles of a cyclic quad are supplementary (points to Q7 ... not to anything in particular).

Researcher: So you agree that, that is a cyclic quad (refers to OPSR).

Rephwaai: Yes.

Researcher: Now, if you just check very quickly ... you gave me the statement, earlier ... about what a cyclic quad is ... and you said this (points to Q2.1), represents a cyclic quad.

Rephwaai: That's where I made an error (rest of response indistinct) ...

Researcher: Okay ... so that then would be false (referring to reason of Q7) ... and if I should ask you how would you then go about solving it, if its false?

Rephwaai: If its false ... then like you say, these angles around the chord here (pointing and indicating on diagram) ...

Researcher: And that would be?

Rephwaai: (mumbles to himself while working out answer) ... this point here 250 degrees (referring to reflex angle at O, moves finger in an arc) ...

Researcher: Okay ... why 250 degrees? How did you get to 250 degrees?

Rephwaai: Its 360 degrees here (moving finger over circle, indicating a revolution) ... minus that (refers to given angle 110 degrees on sketch) ... equals 250 degrees, therefore I think x shall also be 110 degrees.

Researcher: Okay, so you're saying that this one here (referring to reflex angle at O) is 250 degrees and that is 250 degrees because that one is 110 degrees (pointing to given angle 110 degrees), so what's the relationship between that one (refers to 110 degrees) and that angle (refers to angle POR).

Rephwaai: Must ... (indistinct) ... it ...

Researcher: Okay ... you see if you look at it there ... that's 110 degrees and you saying that this (hand obscuring the view) ... Plus, this, must add up to ... 360 degrees.

Rephwaai: (repeats) ...360 degrees.

Researcher: Okay ... but you not sure of the reason for that

Rephwaai: (can't make out student's response ... no answer to be heard)

Researcher: Okay ... lets go onto Q8, just to see how ... How would you go about solving that one? Do you agree with true again?

Rephwaai: PAUSE ... (noise in background) ... Ja

Researcher: So its still true, and the reason there? ... we said angle between ... (researcher does'nt wait for answer).

Rephwaai: ... tangent being RQ and chord PQ is angle in the opposite segment.

Researcher: Okay ... and coming back to ... you see, I like referring back ... because you had it correct here (refers to Q 2.8). Is this one (refers Q2.8), that's the theorem exactly the same as that (refers Q8).

Rephwaai: (student nods yes) ...

Researcher: You're sure? ... you don't see any difference between the two ... they're exactly the same.

Rephwaai: No.

Researcher: Okay, lets go onto Q9, you said true as well. Can you just explain, maybe, how you went about it?

Rephwaai: (student moves fingers over sketch without saying anything at first) ... I used the reason here (refers Q9) and I deduced that x is 25 degrees, because its the angle in the opposite segment.

VERY LOUD SOUND OF SIREN DISRUPTS EXPLANATION

Researcher: Okay, so you still applying the same theorem ...

Rephwaai: Yes.

Researcher: Right ... lets go onto Q11. Why do you think that one is true?

Rephwaai: PAUSE ... because I used the same theorem that I used for no.9.

Researcher: Alright ... and again if you look at that application? and that one (refers to Q11 and Q8) ... is it the same.

Rephwaai: No.

Researcher: What's different.

Rephwaai: This one here (refers Q8) ... actually it's the angle inside is actually inside the circle, and that one's on the outside (refers Q11)

Researcher: On the outside ... so you can't use that one ... alright so then that would have been (refers to reason in Q11) ... false.

Rephwaai: (repeats) ... false

Researcher: Okay, and those are the little things you need to be aware of in the examinations as well.

Rephwaai: ... be able to pick it up.

Researcher: Ja ... alright, what about Q12. How did you go about answering that one?

Rephwaai: PAUSES LONG ... I used this theorem (refers to Q11) and that's how I got the answer 70 degrees ... that is actually ...

Researcher: Okay, so you're using the same strategy ... alright, and tell me are you familiar with most of these problems?

Rephwaai: Ja.

Researcher: Have you seen it before?

Rephwaai: Ja, I have seen it before ... with a different ... (indistinct).

Researcher: Alright, Q13, umm ... again just your reasoning behind saying that its true, because if you're saying its true, the obviously you agreeing with the statement, there ...

Rephwaai: PAUSE ... (sounds of children enjoying interval)

Researcher: ... but I'm interested in what you said here as well?

Rephwaai: (indistinct response ends in) ... cyclic quad.

Researcher: Okay ...

Rephwaai: LONG PAUSE ... (while student moves fingers over sketch, thinking about his answer).

Researcher: So, so what I'm looking at is ... do you still agree with your statement there?

Rephwaai: no.

Researcher: Okay, why not?

Rephwaai: This is'nt really a cyclic quad. (points to the sketch).

Researcher: Okay, okay so you ... that's not correct, right? (referring to answer given to Q13) ... and that one? (referring to reason next to sketch 13).

Rephwaai: They are drawn from a common point and they are equal.

Researcher: And are they tangents?

Rephwaai: Tangents? ... I assume so ...

Researcher: Now remember you gave me the definition of a tangent ... Do you remember what's a tangent? ... (researcher pulls page toward himself to check something) ... let me just check ... the definition of a tangent (referring to Q1.6).

Rephwaai: ... it touches the circumference.

Researcher: ... it touches the ...

Rephwaai: circumference.

Researcher: Alright ... so will PQ be a tangent?

Rephwaai: PQ.

Researcher: It touches ...

Rephwaai: It does touch (pointing to sketch 13) at P, but not here (points at Q).

Researcher: Alright, ... but that's okay ... as long as it touches the ... circle. (points to P).

Rephwaai: (student agrees, mumbles affirmative)

Researcher: ... Now what about RQ? Does it only cut at the one point?

Rephwaai: Yes.

Researcher: Okay, but now you must remember the line continues (refers to QS) ... So it cuts there as well (points to point S).

Rephwaai: Yes.

Researcher: So would you still say RQ is a tangent?

Rephwaai: Yes.

Researcher: Okay ... alright, lets go onto the last one. Q16, you said its true. How would you go about that one?

Rephwaai: Yes, sir ... because of the exterior angle of a cyclic quad is equal to the interior opposite angle.

Researcher: Okay, so you would agree with exactly what's written, there. (refers to reason next to sketch).

Rephwaai: Ja.

Researcher: So, that is the exterior angle (refers to given angle 57 degrees).

Rephwaai: Ja.

Researcher: Now, coming back to your ... this one (refers Q2.3) ... you gave the statement which said, the exterior angle is equal to the interior opposite angle ... so is there any difference (referring to Q2.3) or is this exactly the same as what you've said there (referring to Q16).

Rephwaai: I don't see a major difference.

Researcher: So you would still agree that, that is the exterior angle (referring to given angle 57 degrees).

Rephwaai: (student must have nodded his agreement, no indication on video).

Researcher: Okay ... alright.

INTERVIEW 8 (BRONWYN)

Researcher: ... The first thing that interests me is Q1.3. Right, you said a cyclic quad is a quad with opposite angles equal to 180 degrees (referring to Q1.3). Would

that be your definition of a cyclic quad or is that the property of a cyclic quad.

Bronwyn: It's a property.

Researcher: (repeats) ... Its a property ... okay ... and the definition? What would you then say?

Bronwyn: Well, the theorem states that the opposite angles are equal (rest of response indistinct).

Researcher: Okay, and if I should ask you just ... what is a cyclic quad?

Bronwyn: A cyclic quad is ... umm ... can I?

Researcher: Yes.

Bronwyn: Its a circle with like a ... (points to Q2.1) ... (before student can complete the sentence, researcher interjects) ...

Researcher: Like that? Yes, like you got it right, there (referring to Q2.1). Okay, so its with the four points on the ... circumference. Okay, so you must be careful about the definition and a property, because that was the one thing that went astray, there. Okay, so now we can get onto the real questions. (turns the page). Q1, you said that that is true ... umm ... what I would like to know from you is just ... can you explain why you said its true?

Bronwyn: The angle ... its umm ... the angle at the centre (refers to O) is twice the angle at the ... no! ... (moves fingers over sketch while contemplating answer) ... ooh junne, no, I don't know ... umm ... but that is true, hey ... (points to reason next to sketch).

Researcher: So when we ... (word indistinct) ... that that is 40 degrees (refers to given angle 40 degrees) ... if that's 40 then that one must be 40 degrees (refers to x)... (student turns page toward her). So you agreeing with that?

Bronwyn: That supposed to be 80 degrees (referring to angle S) ... is'nt it?

Researcher: 80 degrees ... and your reason?

Bronwyn: (response too soft ... almost mumbling to herself, but researcher hears it)

Researcher: Okay, and what would your reason be?

Bronwyn: That the angle at the centre is twice ... No! it's supposed to be 20 degrees ... (moving finger onto point S). This angle at the circle ... (pointing to 40 degrees).

Researcher: At the centre ...

Bronwyn: At the centre is twice the angle at the circumference.

Researcher: Okay, so you've picked up that you've used the wrong theorem there. Okay, so we're busy with the angle at the centre. What about no.2? Okay ... again you said false, ... umm ... and you said x should be ... 60 degrees.

Bronwyn: PAUSE ... Its supposed to be 120 degrees.

Researcher: So then that one would be ... true ... (referring to $x = 120$) ... Okay and, tell me are you familiar with these two sketches? (pointing to Q1 and Q2).

Bronwyn: I get a bit confused.

Researcher: You get confused because they look almost as if they are on the same line? (referring to the position of the angles at centre and circumference).

Bronwyn: Yes ...

Researcher: Alright ... umm ... because what I find interesting is that a lot of students, ^ they've got the bookwork intact ... so you've said the angle at the centre is

twice ... and then a lot of students are having problems identifying it in the sketch. Okay ... What about no.3 ... Q3. How did you go about answering that one ... you said its true. Would you still agree that it's true?

Bronwyn: No ... (very softly)

Researcher: Why not?

Bronwyn: Umm ... PAUSE ... (points vaguely to the sketch) ... No!

Researcher: You, you agree that it's true?

Bronwyn: No ... it's not true.

Researcher: It's not true ... okay ... Now just why do you think so?

Bronwyn: Maybe its because that angle, is a bit smaller than that angle (referring to angles STR and TQP respectively) ... I'm not sure.

Researcher: You're not sure ... okay, and the reason that I've used there: I said angles subtended by the same arc QR. Do you think that's correct?

Bronwyn: LONG PAUSE ... (no visible answer).

Researcher: So QR subtends equal angles, that's the theorem that I'm ... (indistinct word).

Bronwyn: No, I don't.

Researcher: You don't agree?

Bronwyn: No, I don't agree.

Researcher: Okay, why don't you agree with that. okay, why not? Why don't you agree with it?

Bronwyn: Here's not a line drawn to QR ... (refers to the chord and not arc QR).

Researcher: Okay ... you're looking at the chord QR?

Bronwyn: Yes.

Researcher: Alright ... so if the chord is there? Now if I say alright, fine, if I draw the chord, there, then would it be true?

Bronwyn: No ... (laughs).

Researcher: Okay, so why do you say no?

Bronwyn: Isn't it equal to 50 degrees? (referring to value of x).

Researcher: And your reason?

Bronwyn: (moving fingers about on sketch) ... but they don't say it's ... (swallows last word)

Researcher: So it's confusing?

Bronwyn: Umm ... (agreeing).

Researcher: Okay, now lets look at Q4. You said its true. $x = 50$ degrees and the reason there ... angle subtended by the same chord ... AB ... okay ... if that's true, what strategy, what strategy did you use there, or how did you go about showing me that one is true?

Bronwyn: LONG PAUSE ...

Researcher: Difficult?

Bronwyn: Umm (agreeing and laughs)

Researcher: And ... alright ... lets look at Q2.2. (refers to the question), because that's the one you had correct. (uses another page with the same question). That's the theorem we are talking about. So ... if that's the ... Ja, angle subtended by the same chord or arc are ... equal. Now you gave the statement to that ... so do you think we're applying that theorem correctly in number 3?

Bronwyn: Yes.

Researcher: Okay, ... and in no. 4?

Bronwyn: LONG PAUSE ...

Researcher: is that the one that you're using there?

Bronwyn: Yes ... (softly, as if not sure).

Researcher: Okay ... now maybe just go look at Q6 quickly. Cause 3, 4 and 6 seem to go together. There you said it's false. (refers to Q6).

Bronwyn: I think I got confused here, so I dotted one in here (referring to pencil dotted line (chord CD) added in by herself on the sketch).

Researcher: Okay, and in Q6, why did you say $x = 90$ degrees?

Bronwyn: I made a mistake, because I normally ... umm ... a angle of a semi-circle is 90 degrees, but I, but there's no point to say ... that its (last words swallowed, indistinct).

Researcher: Okay, so that was a mistake there. So you thought AB was the diameter.

Bronwyn: Ja. (softly)

Researcher: And the diameter subtends ... 90 degrees ... Okay that's fine ... What about Q7? You said its true ... would you still agree with that?

Bronwyn: No ... (barely audible).

Researcher: Why not?

Bronwyn: Its not touching all points of a circle (indicating on the sketch) and because in a cyclic quad its equal to 180 degrees (referring to the angles of a cyclic quad).

Researcher: Okay, and that ... (pointing to OPSR).

Bronwyn: ... that is not a cyclic quad.

Researcher: So you disagree with what you wrote there ... (pointing to her true response in Q7).

Bronwyn: ... (must have nodded affirmative)

Researcher: Okay, good, what about Q8. How would you go about solving that one?

Bronwyn: I think I took 90 degrees minus 65 degrees and I got 25 degrees, and then I took it that that is base angles and its going to be equal ... and I took ... umm ... $180 - 25$ to get about 155 divide by 2.

Researcher: 155 degrees divide by two, okay, umm ... that's fine ... but what I wanted you to do ... look at the reason I've given there ... the angle between the tangent RQ and the chord PQ is equal to the angle in the other, or opposite segment. Can I use that theorem ... or can I use that statement? What do you think?

Bronwyn: ... (student must have nodded no, no audible or visible answer).

Researcher: No ... and the reason? And again you can check this is the one that you got there, hey! (referring to Q2.7).

Bronwyn: Ooh, yes.

Researcher: So, we sort of applying that particular theorem. So is that correct? So if I apply that one, there? ... is it exactly the same?

Bronwyn: No, it won't be equal to ...

Researcher: So you agree that what you said was true?

Bronwyn: Yes ... (very softly)

Researcher: Okay ... what about Q9. How did you go about solving that one? You said false, but I think it just went astray there, so I just want to explore that one a little bit.

Bronwyn: That's why I (indistinct) ... that one.

Researcher: You not sure, okay. Alright ... and umm ... maybe just looking at the reason. You don't agree with the reason that I've said there? Why do you disagree with the reason?

Bronwyn: Because it's not like that one (refers to Q8).

Researcher: Like that ... (responding to student who is pointing to somewhere). Okay, so its this one that we can't apply there ... (refers to Q2.7). Okay, so that's fine ... we're half way (turns the page). Q11. Also, I've said there that x is equal to 82 degrees and the reason is the angle between the tangent QR and chord QT is equal to the angle in the opposite segment ... Alright? Now you said its false, what I want to know is just why do you say its false?

Bronwyn: Because if x was at P, then it would have been opposite this one (refers to given angle 82 degrees).

Researcher: Then it would have been true. Okay, so ... S should be on the ... circumference. Fine ... what about Q13. You also said false and then you used interior angles of a triangle, which is fine, but I'm not sure which triangle you're talking about. But, what I'm interested in ... is the statement that I've given ... true. Do you disagree with it? and why, because that's why you said false.

Bronwyn: Okay ... tangents drawn from a common point are equal.

Researcher: Okay.

Bronwyn: (mumbles something inaudible)

Researcher: Alright, now you got this one correct ... (referring to Q2.9). AC and BC are tangents, and you said tangents from a common point are equal ... so you got that correct ... alright. Now is that what we're using there. (referring to Q13).

Bronwyn: But PR is not a tangent.

Researcher: PQ ... is PQ a tangent?

Bronwyn: Yes.

Researcher: Okay ... and is QR a tangent?

Bronwyn: Yes ... no! no! no!

Researcher: Why?

Bronwyn: Its going through the circle ... it's suppose to touch one point (pointing to cutting points on the circumference).

Researcher: Okay ... my statement ... true or false?

Bronwyn: (softly) ... It's false.

Researcher: It's false ... okay, because RQ is not a tangent. Okay ... good, we're almost there. Q14 ... You said false as well, and then you said x is 39 degrees. Can you explain?

LOTS OF NOISE IN THE BACKGROUND ... LOUD CHILDREN'S VOICES.

Bronwyn: No. (indistinct) ... x is supposed to be equal to ... (indistinct)... NOISE OF CHILDREN TOO LOUD ... the exterior angle is opposite here (referring to T on sketch 14) ... is equal to the opposite interior ...

Researcher: And the opposite interior angle is x ?

Bronwyn: (indistinct answer) ...

Researcher: So would you still agree that x is 39 degrees? You got x is 39 degrees. TV cuts equally, 78 divided by 2 equals 39 (researcher reading student's response in test).

Bronwyn: No, I don't.

Researcher: Okay, so what don't you agree with, from what you did there.

Bronwyn: That its equal to 39 degrees.

Researcher: okay.

Bronwyn: x is equal to 78 degrees.

Researcher: x ? ... but then you would go with what I said there. (referring to reason in Q14) ... because I said $x = 78$... the exterior angle of a cyclic quad is equal to the interior opposite angle ... So you agree with my statement?

Bronwyn: No, I don't agree with it.

Researcher: Okay, so its false ... but I'm still interested in 39 degrees or lets put it ... which angle is 78 degrees because you said 78 divide by 2?

Bronwyn: (points to 78 degrees on sketch).

Researcher: Okay, that one is 78 degrees, but that's not the one you're dividing by two, I'm sure?

Bronwyn: No ... umm ... the WVTW, was given that it has equal angles.

Researcher: Okay ...

Bronwyn: And that's ... (indistinct) and then I became confused.

Researcher: You're confused there ... okay lets go to 15. I said x is equal to 60 degrees and the reason: exterior angle of a cyclic quad is equal to interior opposite angle.

Bronwyn: I'd say its true.

Researcher: True? ... you agree with that. Now ... if that one is true just check back with no. 14. If I apply the same technique, then which angle is the interior opposite angle?

Bronwyn: It will be the whole of angle T. (indicates T on the sketch 14).

Researcher: The whole of angle T ... so that's the one that's 78 degrees?

Bronwyn: Umm (agrees).

Researcher: So what I think you probably meant here, is that, that 78 degrees ...

Bronwyn: And thats cut off (referring to line TV) ...

Researcher: Then the line there (refers to TV) and that's how you divided by two, but you didn't say that line bisects the angle, so you can't divide by two. but that's fine, as long as you spot the interior opposite angle. Okay now the last one. Q16. You said true ... just show me how did you get to true, because true means that you agreed with my statement, hey ... Exterior angle of the cyclic quad is equal to the interior opposite angle.

Bronwyn: SRTU is a cyclic quad, and the NW is a ...

Researcher: ... tangent.

Bronwyn: Tangent ... and that ... its true because they say it (points to reason).

Researcher: So TUV is a cyclic quad ... ag ... is an exterior angle. Is that what you're saying?

Bronwyn: Yes.

Researcher: Okay ... now, I like taking you back to what you had correct (refers to example on page 2) ... You said there (indicates to Q2.3) ... the exterior angle of a cyclic quad is equal to the interior opposite angle (points to Q2.3 again) ... So is that the one we're applying there? Is it exactly the same?

Bronwyn: No ... no, ... yes ... yes.

Researcher: Its the same ... okay, now, if that's the exterior angle of a cyclic quad. (refers to TUV) then its equal to the interior opposite, then that statement would be correct, but the problem is that this is in actual fact the angle between the tangent, which you see, and the chord (Points to NW and UT) so the angle between tangent and chord is equal to the angle in the other segment, which should be ... this one here. (refers to angle UST). So we using that ... (indistinct word) ... then you could say ... these are base, these are equal sides, so base angles are equal and then you get the opposite angles of a cyclic quad.

Bronwyn: Ohh!

Researcher: Alright, so its a bit of a problem-solving one ... alright, but you need to remember that this is not the exterior angle, because the exterior angle has one side produced. Okay, so that's the slight difference between the ... two. Okay, Bronwyn ... thank you very very much.

INTERVIEW 9 (SHAKIER)

Researcher: ... so the first one I want you to look at for me is Q7. Why do you think that question is true... Will you still agree with what you said there (referring to his response).

Shakier: No.

Researcher: Why not?

Shakier: The vertices of the quad must be on the circle (indicating toward O on the sketch)

Researcher: On the circle ... So that is not a ... cyclic quad. Okay ... so you can understand why that one went wrong there ... Okay, good, and tell me, are you familiar with that sketch?

Shakier: (Student must have nodded in the affirmative, no sound heard).

Researcher: Okay, ... what about Q8, you also said true. Can you just check the reason there? and then I need to know if you still agree with that.

Shakier: LONG PAUSE ... Because this point needs to be on the circle (referring to point O on the sketch).

Researcher: That must be on the ... circle. Okay ... so the application of the theorem ... is it correct or not?

Shakier: Incorrect.

Researcher: Incorrect ... okay ... its again those little things that you need to watch ... because I think you got full marks for most of them (referring to the

bookwork) ... and tell me ... are you familiar with that sketch as well?
You've seen it before in class?

Shakier: Yes (speaks very softly).

Researcher: Okay ... Q9 ... You did very well there. (turns the page) ... Alright, now you must tell me Q10. Why, why blank?

Shakier: Er ... I can't remember so far back, but I'll try.

Researcher: Ja ... just see what you think may have caused a problem there.

Shakier: Err ... yes, I think I know.

Researcher: Okay.

Shakier: Umm ... (referring to point O on the sketch) this is the radii (point to OA and OC), that is 120 degrees (points to angle AOC) ... and that means that, that will be 60 degrees divided by 2, so each angle (referring to base angles) will be 30 degrees because it (refers to triangle AOC) is an isosceles triangle.

Researcher: Okay, so x is equal to 30 degrees and tell me what do you think of the reason I've got there, would you agree or not? The angle at the centre is twice the size of the angle at the circumference.

Shakier: No, I wont (answers very softly).

Researcher: You don't agree with that one ... okay ... but you have found that x is equal to 30 degrees in a different way. Okay, so that's fine ... umm Q11 ... alright again you did'nt answer that one.

Shakier: LONG PAUSE ... x is not equal to 82 degrees.

Researcher: x is not equal to 82 degrees ... so that would then be false there (refers to given reason) ... now why do you think ...

Shakier: (agrees softly).

Researcher: Now why do you think it's not equal to 82 degrees.

Shakier: You see here ... here's a triangle (gestures with fingers on sketch, refers to SQT) ... and here's an exterior angle (referring to given angle 82 degrees) of the triangle, so x plus this angle (referring to T) must be equal to 82 degrees (points to given angle).

Researcher: Okay, ... alright that's good, and also, what do you think of the reason that I gave, the angle between tangent QR and chord QT is equal to the angle in the opposite segment. Would you agree with that?

Shakier: LONG PAUSE ... err ... with x equals 82 degrees? ... No.

Researcher: Okay, why not?

Shakier: Because x is not on the circle.

Researcher: (repeats) ... because x is not on the circle. Okay, great. So otherwise if it was there (pointing to angle TPQ), because I see you've indicated there, that that's the angle that is 82 degrees. So it means you've applied the theorem correctly. Okay ... lets look at Q13, ... also because you've left it blank there. What do you think ... what's the problem there?

Shakier: LONG PAUSE ... I'm not sure.

Researcher: Okay, and tell me, if you look at the reason, I said x is equal to 57 degrees. Alright ... and the reason I'm giving is that PQ is equal to QR ... and tangents from a common point are equal. So I want you to look at that and tell me if you think that's correct?

Shakier: (Softly says) No ... $PQ = QR$.

Researcher: PQ equals ...

Shakier: You said that PQ equals QR (refers to given reason).

Researcher: Okay, and you agree with that?

Shakier: No.

Researcher: Why not.

Shakier: Umm ... because PQ, umm ... is not one of these two sides that are equal to each other (refers to lines denoted as equal in sketch).

Researcher: Okay, lets take it ... lets go a little bit further ... is PQ a tangent?

Shakier: Yes.

Researcher: Why is PQ a tangent? or the line PQ a tangent?

Shakier: Because it touches the circle at one place.

Researcher: And that place is?

Shakier: At P (points to P on sketch).

Researcher: Umm ... what about RQ. Is that a tangent?

Shakier: Umm ... No.

Researcher: Why is'nt that a tangent?

Shakier: Because it bisects, cuts the circle. It does'nt touch the circle. It like runs through ... (indicating with finger on sketch)

Researcher: So it cuts the R and it cuts at ...

Shakier: At this point (referring to S) it touches it ...

Researcher: This is the line we're talking about, hey, QR. So the line touches the circle there (refers to R) and it touches at S (points to S on sketch). So, because it cuts at two places, ... can it still be a tangent?

Shakier: No.

Researcher: No, you said that QP just cuts at ... one point. Alright, and now, look at my reason. PQ is equal to QR because I've stated, tangents drawn from a common point ...

Shakier: Are false ...

Researcher: So that will be false, because the one line is not a tangent, and remember you said that two tangents drawn from a common point, are equal (referring to Q1.9). So I'm using that (refers to Q1.9) as a reason, the only difficulty or problem is that, that line is not a tangent, and so that's why it would be false. Now lets go back to no. 14.

LOTS OF NOISY CHILDRENS VOICES IN BACKGROUND

Shakier: It's false

Researcher: And the reason why it's false?

Shakier: Because, umm ... this exterior angle, equals to the whole of angle T.

Researcher: The whole of T. Okay and its not just x . Okay ... Q16. Can you explain that answer to me?

Shakier: LONG PAUSE ... No ... it's false.

Researcher: Its false ...

Shakier: Because this is the exterior angle of a cyclic quad. (referring to VUT)

Researcher: Umm ... name it?

Shakier: VTU

Researcher: VTU, that's the exterior angle. Okay, that is not the ...and, and then how would you go about solving it.

Shakier: For x. Umm ... this is a cyclic quad and umm ... (moves fingers, indicates SRTU on sketch).

Researcher: Okay ... **SOUNDS LIKE INTERVAL OUTSIDE** ... You see I'm just looking at what you said, you said x is equal to RSU and you got corresponding angles. Do you agree with that?

Shakier: (Mumbles something indistinct) Err ... No.

Researcher: No.

Shakier: Exterior ... (indistinct response)

Researcher: They (indistinct) together. Okay, fine, great. thank you.

INTERVIEW 11 (SHERIZAAAN)

Researcher: ... I think what I found very interesting was the first one here Q1.7, the angle between a tangent and a chord is equal to 90 degrees. Would you agree with that?

Sherizaan: I got really confused there (giggles). I read it first and I didn't understand. I know if you have a tangent like, and its drawn, then you know it has to form a 90 degree angle. Now I don't know if I got confused with that.

Researcher: Ja, because I think maybe if you look at Q1.8, the radius of a circle and a tangent are perpendicular to one another ... okay.

Sherizaan: So I think I got confused ... (last part indistinct).

Researcher: So those two ... alright (turns page). Like here, you sort of sorted it out here, because here you've picked it up ... tangent is perpendicular to the chord (refers to Q1.8) ... umm ... and where's the other one (refers to Q2.5)... okay, that's fine ... so there was just a little bit of confusion there? (turns page). Q3, you said it was true, would you still agree with that?

Sherizaan: **LONG PAUSE** ... err ... lets see ... I don't think so. I don't know how I got it in the first place. I know I didn't say its because its like angles in the same segment ... okay ... I know its not that ... I know it was'nt. No then it can't be that here. I don't think I ... but I didn't say that ... I didn't think that it would be, because that obviously didn't touch the circumference. (swallow last words).

Researcher: Okay, so you realise that S must be on the circumference?

Sherizaan: Yes, I did know that.

Researcher: Umm ... and tell me are you familiar with that sketch, you've seen it before?

Sherizaan: Umm ... (makes sound of agreement).

Researcher: Alright, lets go with Q7. Can you explain that one to me?

Sherizaan: (turns page to be able to see better)

Researcher: You also said true.

Sherizaan: (mumbles something indistinct) ... I think I have a problem with the cyclic quad, right? Now, I always think it has to go like that (indicates something on sketch, draws a true cyclic quad with her fingers) ... so when I see something like that (refers to the Q7) ... I always get confused, because I always think, it can't be like that, man. So I forgot that ... somebody

brought to my attention the other day in class, that umm ... it has to touch like four places ... So I was thinking it can be in the middle ... but it just doesn't seem right to me, like that ... that's why I said it ...

Researcher: Okay, remember, because you defined the cyclic quad (Refers to Q1.3) ...the four points touching the circumference of a circle, and that's correct (turns page), so if anything, if there's one point that doesn't touch, then obviously its not a cyclic quad.

Sherizaan: Yes.

Researcher: I think, that (refers to point), led to your response over there. So if I should push you a little bit and ask you, how you would then try and solve that particular problem, what would you do, or how would you go about doing that.

Sherizaan: (moves page to be able to see better) ... umm ... lets see ... if that is now 110 degrees (refers to angle S), and that is now an angle at the circumference, and this is now the angle at the centre (refers to O), okay ... and then the angles around the point is 360 degrees, then I get to this ...

Researcher: Okay, lets take it slowly ... so you said angle at the centre ... which is, ... which is the angle at the centre?

Sherizaan: Umm ...110 degrees is, ... I mean angle at the circumference is 110 degrees and obviously this here, O, that is obviously going to be the angle at the centre (refers to reflex POR) ...

Researcher: which is?

Sherizaan: 220 degrees

Researcher: 220 degrees. Good.

Sherizaan: So you say 360 minus 220 degrees.

Researcher: Minus 220 degrees, and okay ... then you would get your answer, okay?

Sherizaan: (laughs) ...

Researcher: Alright, so there you would use the angle at the centre. Fine, ... what about Q8. How did you go about getting to true, there?

Sherizaan: Umm ... I know that PQ was a chord, right? So I thought that the angle between the tangent and a chord is the one at the other ... the angle in the other segment.

Researcher: Alright ... and you would agree with that, that that's true?

Sherizaan: Umm ... (agrees)

Researcher: You still agree with that?

Sherizaan: Umm ... maybe not, because it didn't touch the circumference. (laughs a lot).

Researcher: Okay, so again the one that you had correct (turns the page to Q2.7), is that one hey! (refers to Q2.7).

Sherizaan: Umm (agrees).

Researcher: Are these two exactly the same (refers to Q2.7 and Q8).

Sherizaan: No, ... because that is touching the circumference, at angle A (refers to 2.7) and this one is'nt (refers to Q8).

Researcher: Okay, so you disagree with what you wrote there? (refers to response in Q8).

Sherizaan: Umm (agrees).

Researcher: So that one would be false, okay, because O is not on the circumference. That's fine ... Q9 ... Okay, explain your reasoning, there ... tell me, you are familiar with those sketches (refers to Q7 and Q8).

Sherizaan: Umm (makes sound of agreeing).

Researcher: You've seen it before?

Sherizaan: Umm ...

Researcher: Okay ...

Sherizaan: LONG PAUSE ... (mumbles to herself while working out answer). I can't remember how I got to that.

Researcher: You can't remember ... okay. Now I've given you here, that $x = 25$, and I said angle between tangent ST, and chord TQ is equal to the angle in the opposite segment. Would you agree?

Sherizaan: Umm (agrees).

Researcher: And you said that's true ... so would you still say its true?

Sherizaan: I know, so that is equal to that now (referring to angle at T and U on sketch 9)

Researcher: To which one?

Sherizaan: To U now ...

Researcher: Okay, so if you ... what, what strategy are you using?

Sherizaan: That the angle between the tangent and the chord (indistinct) ... angle in the other segment.

Researcher: Okay ...

Sherizaan: I was trying to look now (she indicates O and B on sketch 9, but speaks fast and indistinct, so response is lost) ... LONG PAUSE ...

Researcher: It's a tricky one ... but you do realise ... that's not true (refers to reason in Q9). Let me just show you what you could then do, is ... you know the theorem which you spoke about ... the radius is perpendicular to the tangent (points to line BT). So if that's 90 (refers to angle BTS) and this 25 (given angle), then this would have to be 65 degrees (refers to angle BTQ), and then this is your cyclic quad (refers to BPQT), then it would get you there.

Sherizaan: Now, that is what I don't see. I always think that it must always look like that. (laughs).

Researcher: Ja ... no, no. Because there are a lot of other lines that's distractors and you need to pick it up. Okay, but the main thing is for you to pick up now what the (indistinct word) states here ... LOUD SOUND OF SIREN DROWNS OUT WORD) ... (turns page) ... Q11 ... Again, why would you say that's true.

Sherizaan: PAUSE ... (pulls page toward her) ... I think that if its the angle between the tangent and the chord, right, then this is the chord over there ... (line QT) and it goes up like that, then it would be true.

Researcher: Right ... so that's your strategy that you're using?

Sherizaan: Umm (agrees).

Researcher: So you still agree that it's true?

Sherizaan: I think so ... (laughs).

Researcher: Your original one, which is the one you used (referring to Q2.7) ... is this exactly the same as that one? (refers to Q11)

Sherizaan: (laughs) ... No ... obviously not.

Researcher: Okay, what's different?

Sherizaan: See that is now inside the circle (refers to A in 2.7) and that is now outside of the circle (refers to Q11).

Researcher: What's outside of the circle?

Sherizaan: The x here ...

Researcher: The x ... so ... for that to be true, the x must be ... inside the circle.

Sherizaan: Like it had to be over here (pointing to P on Q11).

Researcher: Okay, so that would be ... false?

Sherizaan: False.

Researcher: Okay, that's fine ... the next question ... Q12 ... You said true ...

Sherizaan: Umm (agrees).

Researcher: And the strategy you're using?

Sherizaan: The same one like, umm ... the angle between the tangent and a chord is equal to the angle in the other segment.

Researcher: Okay, so you understand the difference between the two of them (refers to Q11 and Q12) ... Alright, Q13 ... what's your reasoning for that one?

Sherizaan: LONG PAUSE ... LOTS OF NOISE, CHILDREN PLAYING IN THE BACKGROUND ... I thought now because PQ's a tangent, so that means they are like drawn from a common point, and their angles would be equal to each other (indicates on the sketch, points to base angles).

Researcher: Okay, and what's the common point?

Sherizaan: It would be S ...

Researcher: S ... Okay, now ... you said QP is a tangent, right?

Sherizaan: Umm (agrees).

Researcher: Now, what about QR ... is that a tangent?

Sherizaan: (laughs and moves page toward her) ... I don't know ...

Researcher: Okay, what's your definition of a tangent?

Sherizaan: Well it cuts the ... not cuts the circle ... but it touches the circle at one point.

Researcher: At one point, okay! ... so QP?

Sherizaan: Is touching the circle at one point.

Researcher: So that's a tangent ... and what about QR? ... the line QR?

Sherizaan: (speaks softly, as if not sure). It is touching the angle ... I mean the circle at one point.

Researcher: Why at one place? ... remember its the line you're looking at, hey!

Sherizaan: Umm (agrees) ... the line is going that way (move finger over line from Q to S), but ...

Researcher: So you'll agree that QR is still a tangent?

Sherizaan: PAUSE ... Maybe its because I've never seen it drawn that way.

Researcher: Okay ...

Sherizaan: I've only seen it like that (pointing to sketch 12), and like that, or maybe on the side ...

Researcher: So that problem is unfamiliar to you?

Sherizaan: Umm (agrees).

Researcher: Alright, now just to answer that ... umm ... PQ is a tangent, but QR is not a tangent because it obviously cuts at R and at S, so my reason would then be false?

Sherizaan: Umm (agrees).

Researcher: Because I said PQ equals QR because they are two tangents (referring to Q13), and they're not ... alright?

Sherizaan: Umm (agrees).

Researcher: So that would then be false ... Lets look at Q16 ... you were well on your way to solving that one.

Sherizaan: (laughs).

Researcher: You said false, okay, so you did'nt agree with the statement that I had there?

Sherizaan: Umm (agrees).

Researcher: So how would you go about solving it?

Sherizaan: PAUSE ... (whispers to herself) ... STILL NOISE OF CHILDREN IN BACKGROUND.

Researcher: And, and you could also take me through your reasoning there (referring to response in Q16).

Sherizaan: (laughs) ... (points to sketch, says something to herself while working out answer) ... I said, okay, that the exterior angle of a cyclic quad, right. So S would be 57 degrees (refers to whole of angle S), and then the opposite angles of a cyclic quad is 180 degrees. So I got that ... umm ... I think the other theorem that I said about the ... tangent is perpendicular to your radius, so sir I made that 90 degrees (refers to angle SUV), and then this is angle (indistinct) ... so I draw a straight line and then probably that's how I got it.

Researcher: Okay, so you would say ... are you saying SU is a radius?

Sherizaan: No, it's a chord. SU is a chord.

Researcher: So if that's not the radius then how did you get 90 degrees, there (referring to angle SUV).

Sherizaan: (laughs).

Researcher: So again ... that's a tricky one.

Sherizaan: Umm (agrees)

Researcher: Okay, so now you could just check. If that 57 degrees is given as the angle between the tangent and the chord, right

Sherizaan: Umm.

Researcher: So then that would then equal angle UST. Would you agree with that, using your strategy ... that you used over there (refers to Q2.7).

Sherizaan: Umm (agrees).

Researcher: Angle between tan and chord is equal to the angle in the other segment.

Sherizaan: Umm ... umm (agrees).

Researcher: And then you could obviously get base angles that are equal because this (refers to triangle SUT) is an isosceles triangle and then opposite angles of a cyclic quad.

Sherizaan: Oh! So this is okay?

Researcher: Its fine ... but you did realise that that angle (refers to angle TUV) is not ... it is not the exterior angle?

Sherizaan: Umm (agrees).

Researcher: Okay, which is, I think, important.

INTERVIEW 12 (RASMEN)

Researcher: ... Alright, so lets look at Q1 and you need to talk a bit loud, so that he (the video operator) can pick it up over there. Alright, so the first question. If you look very carefully at that one, why do you think that question is true? Do you agree that it is true?

Rasmen: (turns page toward him) ... Its true.

Researcher: Its true ... okay.

Rasmen: It's true because they are subtended by the same arc. So angles subtended by the same arc are equal. (turns page this way, then that way).

Researcher: Okay.

Rasmen: I don't agree ... no!

Researcher: So you don't agree with what you're saying now. Why not?

Rasmen: Because this angle is not subtended by the arc ... is not touch the arc (refers to angle at O). This one is touching the arc (points to S on the sketch).

Researcher: Okay, you mean the circumference?

Rasmen: The circumference, yes. The angles are subtended by the same arc, but is not touching the circumference.

Researcher: So your first response was using that Refers to Q2.2) ... Okay. So would you agree that your answer's correct?

Rasmen: I got a wrong answer.

Researcher: It should have been ...

Rasmen: False.

Researcher: Okay, and if it was false, then what will the value of x be?

Rasmen: The value of x ... I think it will be 20 degrees.

Researcher: 20 degrees ... and your reason?

Rasmen: Angle at the centre is twice the angle at the circumference.

Researcher: Angle at the centre is twice the angle at the ... circumference ... okay, and are you familiar with this sketch. Have you seen that sketch before in the classroom?

Rasmen: Yes ... I didn't see.

Researcher: You not sure?

Rasmen: Yes, I'm not sure.

Researcher: Okay, now lets look at no. 2. Explain your reasoning for that one. You also said its false, okay!

Rasmen: I did like the first one (point to Q1). I say angle at the centre ... angle subtended by the same arc are equal ... are equal with the ... this angle over here (points to x) ... circumference. So that was my mistake.

Researcher: Okay, so you did the same thing there? (refers to Q1).

Rasmen: Yes, I did the same thing. the angle is 120 degrees now. (refers to x).

Researcher: Okay, so no.2 would be true?

Rasmen: Yes?

Researcher: Okay ... Alright. That's good. And tell me, these kinds of problems were they familiar to you. Did you do it in class?

Rasmen: I didn't quite ... I'm not sure.

Researcher: You're not sure ... Okay, but it's not totally new.

Rasmen: Its not new ... its not difficult, but if I did apply my ...

Researcher: Your toolbox?

Rasmen: My toolbox, yes, then I can do it.

Researcher: So you will remember that?

Rasmen: Yes.

Researcher: Okay, now lets look at Q3. How did you go about answering that one.

Rasmen: (turns page toward him, to see better). Well, I think I made the same mistake (referring first to Q3, then to Q2), but if I didn't count it now, PTQ is a triangle, so a triangle is equal to 180, so if I can say the triangle is 180 degrees, this angle would be 100 degrees (Refers to angle QTP), and then this one is opposite to this 100 degrees (refers to vertically opposite angles), so this is based on a chord, so see its 40 degrees (referring to x).

Researcher: So x will be 40 degrees. So then you won't agree with what you did there.

Rasmen: Yes ... it's false there.

Researcher: Okay, so that one should be false ... now let me ask you. I said x is equal to 30 degrees and I said as a reason, angles subtended by the arc QR. Can I still use that as an answer?

Rasmen: I don't think so.

Researcher: Why not?

Rasmen: Because it did not touch the circumference (referring to point S on the sketch).

Researcher: Okay, that's fine ... Now you notice that Q4 and Q6 are more or less the same.

Rasmen: Yes.

Researcher: Did you have a strategy ... what kind of strategy did you use to get to the answer there? or did you just ... umm ... How did you go about it?

Rasmen: I knew angles subtended by the same chord are equal ... So I use that theorem.

Researcher: Okay, that's fine. Now what about Q7?

Rasmen: Q7 ... I think umm ... it was too difficult for me. I was guessing about it.

Researcher: Oh, you were guessing ... and if you should look at it now, now when I say look what I've got there as an answer, $x = 70$ and my reason is opposite angles of a cyclic quad are supplementary. Would you agree with that?

Rasmen: Well, yes ... I'm not sure. It's the first time to see that (last words indistinct), question.

Researcher: Alright, now let me take your response here. Let me show you ... Here we go. Oh, you used a property. Opposite angles of a cyclic quad add up to ... 180 degrees (referring to Q2.1).

Rasmen: To 180 degrees.

Researcher: So you would agree that ADBC is a ...

Rasmen: Cyclic quad, because they touch the circumference.

Researcher: Okay ...

Rasmen: So this one (referring to Q7) is not a cyclic quad.

Researcher: Okay, so it's not a cyclic quad ... and your reason?

Rasmen: My reason ... cyclic quad touches the circumference on all four angles. So this one is three angles that touch the circumference. So these are two reason its not a cyclic quad.

Researcher: Okay, so that would be true or false?

Rasmen: I think its true. Can I say ... angle S plus angle O is 180 degrees, sir, but I don't know how can I (sounds like) name it, because its not a cyclic quad.

Researcher: But now the property ... what's the property of that cyclic quad? You gave it here (refers to 2.1).

Rasmen: Yes ... you see, all the angles of a cyclic quad are equal to 180 degrees.

Researcher: Add up to 180 degrees ... So if that's not a cyclic quad. Does that (refers to property) still apply? Can you still use that property?

Rasmen: Yes.

Researcher: You think so?

Rasmen: I think so ... I don't ... you can't use it.

Researcher: you can't use it?

Rasmen: Because (indistinct words) ... a cyclic quad.

Researcher: Alright, so if I should ask you. How would you find the value of x, there? How would you do it?

BELL RINGS IN DISTANCE, NO DISTURBANCE HOWEVER.

Rasmen: Can you do it for me ... I'm not sure.

Researcher: You're not sure. Okay, let me just show you. If I should find the value of x, here. This is an angle at the circumference (refers to sketch but hand obscures view).

Rasmen: Circumference, yes.

Researcher: So this angle here, ... POR (indicating angle on sketch), the reflex angle, that's the angle at the centre. So the angle at the centre is twice the size of the angle ...

Rasmen: At the circumference.

Researcher: At the circumference. So if this is 110 degrees, then this one will be?

Rasmen: 220 degrees

Researcher: 220 degrees. Alright ... and then what can you tell me about angles around a point?

Rasmen: Angle around a point add up to 360 degrees.

Researcher: Add up to ... 360 degrees. It's a revolution ... yes. So if that angle is 220 degrees, then what would x be?

Rasmen: Okay its 140 degrees.

Researcher: Its 140, yes. Alright, the other way is if you draw a imaginary cyclic quad like that (indicates on sketch a cyclic quad with a point above O on the circumference)

Rasmen: Yes.

Researcher: Then this is 110 degrees, then that one must be ... 70 degrees (referring to angle at top of circle).

Rasmen: 70 (repeats) ... 75 degrees, no 70 degrees.

Researcher: Yes, 70 degrees and then the angle at the centre is twice the size of that one (refers to 70), so 70 plus 70 is 140.

Rasmen: 140.
Researcher: Okay.
Rasmen: Yes, sir, that's a easy one.
Researcher: But you need to be careful that you spot that its not a cyclic quad, that's the main thing. Because if you say its a cyclic quad, then immediately it is wrong.
Rasmen: Yes
Researcher: Okay ... lets go onto Q8. You also said true there. Umm ... why? In other words you agree with my statement there?
Rasmen: Okay ... this is a tan/chord theorem.
Researcher: The tan/chord theorem ...
Rasmen: Yes. Angle between tangent RQ and chord PR equal to the opposite interior angle.
Researcher: Okay.
Rasmen: This one ... (refers to something on sketch, but hand obscures view)
Researcher: So x is 65 degrees. So you would agree with that.
Rasmen: Yes, sir. I agree with that. (turned page toward him to check).
Researcher: Okay, now, you got this one correct. (refers to Q2.7). This is your tan/chord theorem.
Rasmen: Yes ...
Researcher: Alright. Now are you applying this one Q2.7 correctly over there?
Rasmen: (says something but indistinct, because at the same time, the pen flies out of researcher's hand and hits table hard).
Researcher: Sorry ... Tell me about it?
Rasmen: This angle is right ... is right, so I'm correct.
Researcher: So you happy with that?
Rasmen: Yes, I'm very happy.
Researcher: So you perfectly convinced that that is true?
Rasmen: Yes, that is true.
Researcher: And you're applying that theorem (refers Q2.7), correctly?
Rasmen: Yes, I'm applying ... Wha! ... (he shouts because he sees something he probably didn't notice before) ... (he laughs).
Researcher: Okay ... tell me ... tell me. (researcher smiles).
Rasmen: These are two radius (meaning radii) ... it, it didn't touch there ... (points to the circumference) the circumference ... (smiles)
Researcher: And so, so what are you telling me? So you still agree with what you did there was true?
Rasmen: No. (shakes head vigorously). It was wrong.
Researcher: Okay, now why you wrong?
Rasmen: This was not a chord theorem. It did not touch the circumference. (indicates on sketch). So ... (almost talking to himself) ... What do I need to find the x?
Researcher: So you can't use ...
Rasmen: No.
Researcher: So that would be ... false?
Rasmen: False, yes.
Researcher: Now, how do you think you'll find the value of x?

Rasmen: I don't have a ... (probably says strategy, unclear).
Researcher: Too difficult. (researcher interjects before he can complete sentence).
Rasmen: Sort of ... Ja ... But ...
Researcher: Okay, now, you gave a theorem. What do you know about the radius and a tangent?
Rasmen: PAUSE ... Just let me think first ...
Researcher: Ja ... let me take you back to ... (turns to page 2, diagrams of theorems), what you did over here. Tangent AB is perpendicular to radius OC (referring to Q2.5) ... You see that, the radius, and your statement, you got it correct as well.
Rasmen: Yes.
Researcher: So you know ... (turns back to sketches) from your bookwork, that the radius and the tangent are perpendicular?
Rasmen: Yerr ...
Researcher: Okay ... so now we can go back there (refers to Q8).
Rasmen: Okay, this radius is perpendicular to this tangent RS.
Researcher: Yes.
Rasmen: So when the radius is perpendicular, it give us 90 degrees, so this is ... okay ... 25 (refers to angle PQO), and then the two radius form the isosceles triangle, then this base is 25 degrees, so 25 plus 25 is 50 ... is 130 (points to x).
Researcher: Its 130 degrees ... okay, making sense now?
Rasmen: Yes, its making sense now [these last few lines took long to decipher, accent not too clear]
Researcher: Alright, so you can see why that one was false, but again you need to spot that you cannot use your theorem, here ...
Rasmen: Yes.
Researcher: In that, way ... Alright, what about no.9. Q9. Again, how did you go about answering that question? (ROOM/BACKGROUND BECOMES NOISY). By the way are you familiar with these, have you seen this (refers to Q7 and Q8) ... did you do this in class?
Rasmen: (can't make out response).
Researcher: Not sure ... Doesn't ring a bell?
Rasmen: Yes ... This is a tan/chord theorem. I'm sure (refers to line RTS).
Researcher: Okay.
Rasmen: And this triangle TQU ... so ST ... wait! QT is a chord, so ST is a tangent, this angle (refers to 25 degrees) ... STQ is equal to ... TQU.
Researcher: TQU. Are you sure?
Rasmen: Angle between tangent and chord equal the opposite interior angle, That's Q ... and then ... the radius (indicates on sketch) and then this angle (adjacent to angle marked 25 degrees) ... is equal to 25 degrees.
Researcher: Why 25 degrees?
Rasmen: This angle ... two radius QO and TO.
Researcher: Okay.
Rasmen: A radius ... (says something indistinct).
Researcher: Okay, but you agree, that's not true (refers to reason given in Q9).

Rasmen: That's not true, yes, but I don't know how to ...

Researcher: Okay, you see what ...

AT THIS POINT THE SOUND GOES OFF (researcher indicates something on sketch) ... NO SOUND

Rasmen: ... That line if I can join BU.

Researcher: If you can join BU. ...

Rasmen: Yes.

Researcher: Okay, but its not joined.

Rasmen: Its not joined, yes.

Researcher: So, so can we apply this one (referring to Q2.7).

Rasmen: If its not joined then we can't apply it.

Researcher: You ... can't apply it ... so then that would be false (referring to Q9).

Rasmen; Yes, false.

Researcher: Alright, now how would you solve that? Again we said that the radius and the tangent is what?

Rasmen: The radius is perpendicular.

Researcher: Is perpendicular, yes ... Right?

Rasmen: So this angle plus this angle is (referring to angle marked 25 degrees and the one adjacent to it) is (indistinct) ... so this angle is 65 degrees.

Researcher: And what is this PBTQ?

Rasmen: I think ... a cyclic quad.

Researcher: Cyclic quad. And what is that property that you used earlier on?

Rasmen: Opposite angles is equal to 180 degrees.

Researcher: So if the one is 65 degrees, then what will x be?

Rasmen: 150 degrees.

Researcher: 150 degrees ... Okay lets look at Q11 (turns page). You said its true. Explain? Do you agree with my reason, there?

Rasmen: x is 82 degrees (speaks more to himself and indicates on sketch). This one is equal to T (referring to angle marked 82 degrees and then angle at point T) ... (he mumbles something indistinct, still trying to work out his answer).

Researcher: You see I'm looking at what you wrote in there. You wrote 82 degrees there (refers to response in sketch). Why did you write 82 degrees there?

Rasmen: The angle ... I used the tan/chord theorem. This is 82 degrees (refers to angle at point P).

Researcher: Okay ...

Rasmen: So 82 degrees plus this one is about 180 degrees ... so this one is ... must be 112 degrees. (adjacent to P).

Researcher: 112 degrees?

Rasmen: 102 degrees.

Researcher: Okay, so do you think my reason is correct? Angle between tangent and chord.

Rasmen: Yes ...

Researcher: But I'm saying x is that angle.

Rasmen: It's not correct.

Researcher: Okay ... and why am I not correct?

Rasmen: Sir, this one is outside the cyclic ...

Researcher: It's outside of the circle, so I can't use that particular theorem. Because all you need to do is ask yourself, am I applying this theorem (referring to 2.7) ... over there.

Rasmen: No ...

Researcher: And no ... and the reason? ... because, as you said, x is out there.

Rasmen: Yes.

Researcher: Alright, so that would again, have been false ... And again, have you seen that sketch during your class-work? or is it unfamiliar.

Rasmen: Umm ... I didn't do it.

Researcher: You didn't do that ... okay, lets look at Q13.

Rasmen: Q13 ... it's not a cyclic ... its outside the cyclic ... this one (probably means circle instead of cyclic). So we can't apply that rule of tan/chord theorem ... So ...

Researcher: But, but look at my reason. Look at my reason. Do you agree with that?

Rasmen: I don't agree with that.

Researcher: Now, why don't you agree?

Rasmen: Because tangent to the one point, outside the cyclic are equal, so it's not ... this is not a cyclic inside this triangle. So it is not a tangent, ... not a tangent.

Researcher: So, okay, lets take it slowly ... Is QP a tangent?

Rasmen: Yes, it is a tangent.

Researcher: And is QR a tangent?

Rasmen: Yes it is ... no its not a tangent.

Researcher: Why is it not a tangent?

Rasmen: PAUSE ... I don't know what I can say, there, but it's not a tangent.

Researcher: It's not a tangent. What is a tangent?

Rasmen: A tangent ... it touch the circle at one point.

Researcher: Okay, and that one. Is it touching at one point? QR?

Rasmen: I can say, this line here (referring to SQ), it start from S to Q that's why I can say its not a tangent.

Researcher: So it's not a tangent. So then I can't use that (referring to reason given in Q13).

Rasmen: Yes ...

Researcher: Because there I'm saying, both these lines are ... tangents. So that would have been ...

Rasmen: False.

Researcher: False, yes. Okay, that's fine. How about Q14? How would you explain that one to me? You said true. Again look at the reason I gave, because you said my reasoning is true.

Rasmen: PAUSE ... That's false.

Researcher: Why is it false?

Rasmen: This is not a tangent.(refers to line ZU)

Researcher: Okay, but now I'm not using a tangent (referring to reason for Q14) ... I'm saying the exterior angle of a cyclic quad is equal to the interior opposite angle.

Rasmen: Just checking this one ...

Researcher: Okay.

Rasmen: Umm ...
Researcher: So you agree that what you said there is correct?
Rasmen: That is not correct, but the statement is (indistinct word).
Researcher: Okay, now in your bookwork (refers to 2.3). Can you remember that one?
Rasmen: Yes, I can remember.
Researcher: That's your exterior angle ...
Rasmen: This angle is T ... is a whole ... but I think it's the half of T
Researcher: Is a half of T?
Rasmen: Yes ... that's why I was wrong.
Researcher: Okay, so that's ... so you don't agree that, that's true. So x can't be 78 degrees.
Rasmen: Yes.
Researcher: Okay, and what do you think x would be? Just show me what you would do.
Rasmen: (turns page this way and that to get the picture). Okay, this ... is a whole, plus WVU. I got 180 degrees. So I think that ZVW is 102 degrees.
Researcher: Okay, 102.
Rasmen: Yes, 102 plus, plus this angle (refers to angle ZTW) is equal to 180 degrees ... So this angle T as a whole is 78 degrees.
Researcher: So the whole angle T there is 180 minus ... is 78 degrees. That's fine, that's what I want to check, ... you aware which is the interior opposite angle.
Rasmen: Yes.
Researcher: Okay. Now the last one. Q16. How did you go about that one? You did quite a lot of work there. Can you take me through that?
Rasmen: This is what I did ... tan/chord theorem ... (hand obscures view of sketch and what he is indicating) ... Angle WUT is equal to UTS.
Researcher: Is equal to ... UTS.
Rasmen: UTS and then this angle UTS is a isosceles triangle ... base angles are equal.
Researcher: Okay, ... now just hold it there. You said this angle that's given TUV is equal to this angle at T.
Rasmen: Yes ... UTS.
Researcher: Okay, are you applying that one correctly. Because you're using that theorem (refers to 2.7)
Rasmen: I think it might be UST.
Researcher: It should be ... UST ... Okay, so that's why you can see the reasoning wasn't correct, there.
Rasmen: Yes.
Researcher: Okay, because of the way you started. But let me take you back. Just one question. I said x is equal to 57 degrees and my reason was, the exterior angle of the cyclic quad is equal to the interior opposite angle. Is that the exterior angle of a cyclic quad? (referring to the angle marked 57 degrees).
Rasmen: No, its the tan/chord theorem, not exterior.
Researcher: Okay, so it's the tan/chord theorem. Perfect. Alright we're almost done. I just want to get one more ...

INTERVIEW 13 (LUVUYO)

Researcher: ... Now, in no. 1, why do you think that that question is true?
Luvuyo: Its because, I know the theorem. I know that is angle double that one. So x is equal to 40 degrees. That's what I know.
Researcher: But you said its double.
Luvuyo: Okay, I made a mistake there. It's supposed to be 80 degrees, because the angle at the centre is twice the angle at the circumference.
Researcher: So the angle at the centre is double ...
Luvuyo: The angle at the circumference.
Researcher: So you're saying, that one must be 80 degrees?
Luvuyo: It must be 80 degrees.
Researcher: You're sure?
Luvuyo: I'm sure.
Researcher: Okay, you know ... are you using Q2.2?
Luvuyo: Yes.
Researcher: Is that the one or are you using 2.6?
Luvuyo: No, I'm using that one (refers to 2.6)
Researcher: Right ... So you don't agree with what you wrote there?
Luvuyo: Yes. That's supposed to be false.
Researcher: It's suppose to be ... false. Okay, that's fine. What about Q3. That one over there (referring to sketch no.3). How did you go about answering that question.
Luvuyo: Ja ... this one's true.
Researcher: You agree that it's true?
Luvuyo: Ja ...
Researcher: Okay, and do you agree with my statement? Angles subtended by the same arc QR. So you happy with that?
Luvuyo: This one's not right, because this line (refers to QS) is supposed to be ended here (pointing to the circumference).
Researcher: So S must be on the circumference?
Luvuyo: Ja, must be on the circumference.
Researcher: Okay.
Luvuyo: So x is not equal to 30 degrees.
Researcher: So what do you think x would be? Can you calculate x for me?
Luvuyo: (moves page closer to him and works out answer, speaking to himself). (writes values for angles PTQ and STR) ... So this must be 40 degrees.
Researcher: So it must be ... 40 degrees. Great. Okay. Q8. You said it was false.
Luvuyo: It was false ...
Researcher: Okay, can you just go through that one?
Luvuyo: It is false, because what I ... this line, this line (refers to OQ) is a radius and is perpendicular to the chord, so it form a 90 degree angle ... (angle RQO) ... so I know this one is 35 degrees (refers to PQO).
Researcher: Okay.
Luvuyo: So therefore 90 degrees.
Researcher: So 35 degrees and 65 degrees

Luvuyo: Form 90 degrees.

Researcher: You sure? 65 plus 35?

Luvuyo: Okay. It's not 35 degrees.

Researcher: It should be ...

Luvuyo: 25 degrees

Researcher: 25 degrees ... and if that's 25 degrees then what would x be ...

Luvuyo: So this is a radius (refers to OP) and this is a radius (refers to OQ). So they (base angles) must be equal. This one should be equal to that one. So now this is 25 degrees (refers to PQO) and I say that one is also 25 degrees (refers to P) because this is an isosceles triangle. So 25 plus 25 give me 50 degrees, so that's why it is 130 degrees (writes in 130 in angle POQ).

Researcher: Okay, great. Now ... just look at the reason that I gave. I said x is equal to 65 degrees. Angle between tangent RQ and chord PQ is equal to the angle in the opposite segment. Is that, ... do you think that reason is true?

Luvuyo: Its not true, because this triangle (refers to POQ) ... it is not a full circle triangle (means circumscribed circle).

Researcher: Okay, so O should be on the circumference?

Luvuyo: Should be on the circumference (repeats last part).

Researcher: Okay, great. What about Q9? How did you go about agreeing that that's true?

TALKING IN BACKGROUND.

Luvuyo: Okay, this is 25 degrees (refers to given angle), so this and this is a (indistinct, speaks fast, but writes in 65 for angle BTQ) ... So this must be also 25 degrees (write in the value at U).

Researcher: Can you ... you must just talk a little bit louder.

Luvuyo: okay, that ones 25 (refers to angle marked 25 degrees)... I want to find x , so what I know is that this angle must be 90 degrees (refers to angle BTS). So I must need 65 degrees ... this angle (refers to BTQ) must be 65 degrees.

Researcher: Okay.

Luvuyo: So, this isn't a cyclic quad. All the corners are at the circumference, so they form a cyclic quad (refers to BTQP). So this is 65 degrees (BTQ). So what I know is that the opposite angles of a cyclic quad add up to 180 degrees. So this one must be 125 degrees (refers to x).

Researcher: Okay, so that would have been false (refers to solution of Q9). That's fine. That's good. That is very good. Okay, Q12. I'm looking at what you did there. A lot of scratching out and you said eventually that its true. Do you still agree that x is equal to 82 degrees? (researcher probably means Q11).

Luvuyo: Okay, I must first try to look at it.

Researcher: Yep.

Luvuyo: This is 82 degrees (refers to marked angle), that's why I said $x = 82$. Okay, this is the exterior angle (refers to given angle), so this must be also equal to 82 degrees (refers to the angle at T) ... We can't say that's equal to 82 degrees (referring to x).

Researcher: Why not?

Luvuyo: Its because I found this ... is 82 degrees (marked angle 82). So this is a strange one (refers to adjacent angle to 82) ... this one must be 98 degrees because

Researcher: Okay, that's angle TQS, ja.

Luvuyo: So this is a tangent (line SR). So there is that theorem, if I want to prove that that is a tangent, so I must draw the line from the tangent to the circle (refers to line ST). So what I know is that this line (ST) must also be equal to that one (SR), according to the theorem that I want to prove that this is a tangent to the circle (indicating from S to T), so this one is 98 degrees (SQT), so this angle must equal to that angle because this line (ST) is going to be equal to that one (SR).

Researcher: So you saying because ST is equal to QR?

Luvuyo: Ja ...

Researcher: Okay, and now just let me ... you got this one correct in the bookwork. Is this the theorem that we're applying there? (refers to 2.7) ... Can we use this, there? ... umm ... Q2.7 ... can we use it ... to solve this?

Luvuyo: Yes, we can use it if the circle (refers to circle) was (indicates the circle passing through S, makes only actions, no words).

Researcher: Oh! If the circle passed through S?

Luvuyo: ja ...

Researcher: Okay that's fine ... Last one. Q13.

Luvuyo: (starts eagerly, working out his answer, all the while speaking to himself, what he says is fast and indistinct until he's ready to give an answer) ... so that's 75 degrees (refers to the given angle).

Researcher: 57 degrees, hey (researcher corrects him).

Luvuyo: Okay ... this line is equal to that one (refers to SR and SP). So this is a tangent (PQ), so this line (SP) is perpendicular to the tangent. So the angle that's going to be there should be 90 degrees (refers to angle SPQ).

Researcher: Okay, so how do you know that is the radius?

Luvuyo: Okay, that is the radius (refers to line PS).

Researcher: How do you know that?

Luvuyo: It because that line (PS) is perpendicular to the tangent. So it form an angle of 90 degrees ... so what I'm going to do is that if I want to prove that this (RQ) is a tangent, so this line (SP), it is supposed to be a centre in the middle of this line, the centre of the circle (points to the middle of line SP).

Researcher: Okay.

Luvuyo: So, if this line (SP) start from the centre and go to the circumference, so that's why I said this one (SP) is perpendicular to the tangent.

Researcher: Okay.

Luvuyo: So this angle is 57 degrees (the given angle) ... so this is 90 degrees (SPQ). So 57 ... you supposed to add 33 degrees in order to get 90 ... so this one (SPR) is 33 degrees ... so this angle is 90 degrees (SPQ) ... (turns page slightly the other way) ... so this line equal that line (refers to SP and SR). So I'm going to take angles SRP ... so this one, R will be equal to P (refers to base angles).

Researcher: Okay.

Luvuyo: So 33 plus 33 is equal to 66 degrees. So I subtract 66 degrees from 180 degrees to give me that (angle S) ... So its going to give me 124 degrees (writes in on sketch) ... (starts to mumble while working out answer). Okay I want 180 degrees ... (works out something again to himself) ... Its 116 degrees.

Researcher: Okay.

Luvuyo: So this is about 116 degrees (angle S), plus this one, plus that one (the other angles) ... triangle ... so this angle (SPR) ... I can take this angle as a part of the whole triangle. So I want to find x okay. this is 33 degrees (SRP).

Researcher: Ja.

Luvuyo: So this is a straight line.

Researcher: Okay.

Luvuyo: So I can say if that one is 33 degrees (SRP) ... so this one must be equal to ... (moves page but hand obscures view) ... must be equal to 143 degrees, so x is not equal to ... (fills in everything he works out on sketch)

Researcher: Okay, so that should be false there. Okay, now I just want to check with you. Is QP a tangent.

Luvuyo: Yes.

Researcher: Why?

Luvuyo: Because it touch the circle at one point.

Researcher: Okay ... is QR a tangent?

Luvuyo: It's not a tangent.

Researcher: Why not?

Luvuyo: Because it is a line that starts on the inside and extends to the outside and joins the tangents.

Researcher: Okay ... so could we use 2.9 to solve that?

Luvuyo: No.

Researcher: No, okay so that is why x can't be 57 degrees as well. Alright?

Luvuyo: Yes, sir. Can't be 57 degrees.

Researcher: Okay that's great. Thank you very much ...

INTERVIEW 14

The researcher interviews the student based on mistakes in the bookwork section, but we will start the transcription from question 1 in test 2.

Researcher: ... Lets go onto some of the other questions. (turns to test 2). Q1. you said the answer's true. Why do you think its true? Do you agree with my statement?

Mzoxolo: (asks if he could move the page toward him, to see better).

Researcher: Ja ...

Mzoxolo: (speaks to himself while working out the answer) ... I said its true, here.

Researcher: And you agree that it's true?

Mzoxolo: Umm ... Yes, sir.

Researcher: Okay. And look at no. 2, Q2. Explain your reasoning over there?

Mzoxolo: Okay ... (whispers to himself) ... I did take a look at 60 degrees. I ... to the statement. it says angle at the centre is equal to two angle at the circumference. So what I did ... I did look at C. C is 60 degrees (refers to given angle C). So if I times 60 degrees by two it gives me 120 degrees, and x is in the centre.

Researcher: Okay, so what would x be?

Mzoxolo: I think I said x is 120 degrees.

Researcher: Okay, but now you said it was false. So you disagree with what you said there?

Mzoxolo: I think I did make a mistake.

Researcher: You made a mistake, yes. So you sure x is equal to 120 degrees.

Mzoxolo: Ja.

Researcher: Alright, now are you familiar with sketches 1 and 2. Have you seen that before or is it something new to you?

Mzoxolo: No, I've seen it before.

Researcher: Okay, if you look again at number 1, Q1, you got this theorem correct (refers to 2.2), theorem/question 2.2. Is this the theorem that you're applying, there?

Mzoxolo: I think the theorem, the theorem of 2.2. I applied here at no.2, of course this one too (refers to Q1).

Researcher: So its 2.2 that you applied there?

Mzoxolo: Yes.

Researcher: So is this (Q1) exactly the same as that one? (sketch no.2). What do you think?

Mzoxolo: This ... no!

Researcher: What's different?

Mzoxolo: I think this one's got centre O.

Researcher: Because its got the ... centre O. So which one do you think we're applying there? This one 2.2 or 2.6?

Mzoxolo: I think it's 2.6.

Researcher: Okay, so why 2.6?

Mzoxolo: Because it consist of centre O and those dots represent the two angles which might be the half and those dots represent the angles that you should multiply ... the angle at the circumference by two ...

Researcher: Then you get that one ... okay, so now look at no.1 again. Would you still say your answer is true?

Mzoxolo: No ...

Researcher: What would x then be?

Mzoxolo: x ... umm ... (whispers to himself) ... Okay, I would say x is 80 degrees.

Researcher: 80 degrees, and your reason?

Mzoxolo: Of course x is ... umm ... no. I can say x is 20 degrees here. (refers to Q1, angle S) ... the reason why I said so, 40 degrees here is at the centre. So I think angle at centre is double the angle at the circumference.

Researcher: Okay, thats fine, great. What about Q3? How did you go about answering that one? You said it's false, but you just wrote false, okay. Why did you say false?

Mzoxolo: (works out answer all the while whispering to himself) ... of course, here it false.

Researcher: You said false, yes.

Mzoxolo: Umm ... this ... is can't be 30 degrees, here. (referring to solution given for Q3)

Researcher: Okay, why can't it be 30 degrees.

Mzoxolo: I think S is not subtended by the arc QR ...

Researcher: Okay, and why do you think S is not subtended by QR.

Mzoxolo: All I can say is, S is not touching the circumference.

Researcher: Okay, S is not on the circumference ... Okay that's fine. Look at Q4. You said that one was false, okay.

Mzoxolo: Okay ... x is 50 degrees here, because it is subtended by the same arc, this arc, which is DC here.

Researcher: So you're saying, the arc should have been, DC.

Mzoxolo: Yes, sir.

Researcher: Okay ... now look at Q6, you said that one was false as well.

Mzoxolo: Then we use the same reason here (reason no.4) ... to get the 45 degrees (referring to sketch no.6) ... angles subtended by the arc CD.

Researcher: And you looking at the arc CD ... Okay, that's fine. Q5. you said false. And then triangle ABC is a ...

Mzoxolo: Equilateral triangle.

Researcher: A semi-circle (referring to answer given by student in the test).

Mzoxolo: Or a semi-circle, yes.

Researcher: Now if its a semi-circle. What can you tell me?

Mzoxolo: I think triangle ABC ... its half the circle because there is not ... a chord there.

Researcher: Ja ... [NOISE FROM CHEERING STUDENTS OUTSIDE]

Mzoxolo: Which divides the circle in half, that's why I say triangle ABC is a semi-circle.

Researcher: Okay, so what will the value of x then be?

Mzoxolo: The value of x ... it will be ... of course 60 degrees. I think so.

Researcher: So you will agree with the 60 degrees.

Mzoxolo: Yes, I think.

Researcher: But not the reason?

Mzoxolo: But not the reason.

Researcher: Okay, do you know what an equilateral triangle is?

Mzoxolo: Yes.

Researcher: What's an equilateral triangle?

Mzoxolo: All the sides are equal.

Researcher: (repeats) ... All the sides are equal ... Okay, that's fine. Lets look at Q7.

Alright you gave false there, angles subtended by the same arc. Okay, can you explain to me?

Mzoxolo: Okay ... I think my statement ... I just don't ... I disagree with this statement.

Researcher: Okay, you disagree with your statement ... now, ...

Mzoxolo: Because x is 70 degrees, opposite angles of a cyclic quadrilateral are equal to 180.

Researcher: Okay, and that is a cyclic quad?

Mzoxolo: Ja, that is a cyclic quad, (refers to PORS).

Researcher: Okay, now in the bookwork, is this a cyclic quad (refers to 2.1). Is ABCD a cyclic quad?

Mzoxolo: I think so, yes.

Researcher: Why would that be a cyclic quad?

Mzoxolo: All the sides are ... all the angles are touching in the circle.

Researcher: Okay, so all the angles are attached to the circumference. So is that the same in Q7?

Mzoxolo: Umm ... no.

Researcher: Why not?

Mzoxolo: Because only, only two sides which are touching the circumference.

Researcher: So would that still be a cyclic?

Mzoxolo: No.

Researcher: No ... okay. Lets go onto Q8. You said its true, so you agree with my statement there, that it's the angle between the tangent and the chord. Okay.

Mzoxolo: (whispers to himself) ... Angle between the tangent and chord is equal to opposite angles ... okay ...

Researcher: So do you still agree with that?

Mzoxolo: Okay, I do agree with it.

Researcher: So, Q2.7. That's the theorem. That's the angle because you wrote it out. You said, its the angle between tangent and chord, is equal to the angle in the other segment. So are we applying that theorem to Q8?

Mzoxolo: No ...

Researcher: What's different?

Mzoxolo: In this triangle here, in Q8, only two sides which are in the circumference, but all its three (referring to 2.7) are touching the circumference. Here Q2.7. So it can't be true.

Researcher: So can we still use this theorem to solve that?

Mzoxolo: According to that question now ... I've just discovered it ... you can't use it.

Researcher: You can't use it, okay. So how would you then find the value of x. So that would then be false. Hey!

Mzoxolo: Ja ...

Researcher: Okay, so how would you then find the value of x?

Mzoxolo: (starts to work out, but speaks to himself) ... Okay ... I can say the exterior angle ... okay, okay I can say the angle between the tan and the chord, is equal to the interior angle ...

Researcher: Not so easy, hey! ... Now you remember the one theorem you had problems with was the fact that you said the radius is twice the diameter ... twice the tangent. Okay, now that's not correct, that should be the radius is perpendicular to the tangent. So perpendicular. You know what perpendicular mean?

Mzoxolo: Yes.

Researcher: What?

Mzoxolo: I think perpendicular ... cuts diameter into two equal lines.

Researcher: Into two equal lines and what about the angles?

Mzoxolo: I think the angles ... it is equals to ... if you talk about perpendicular. I think you talk about a straight line, as I can say RS is a straight line, and a

straight line is 180 degrees, so the perpendicular line will cut the straight line into 90/90 (gestures with his hands).

Researcher: Okay, so the radius is perpendicular to the tangent, so it means that angle (referring to angle BTS in sketch) must be 90 degrees, so if the whole angle is 90 degrees and that one is 65 degrees, what must that one be? (refers to angle BTQ).

Mzoxolo: 24 degrees.

Researcher: 24 degrees? You sure? 24 degrees and 65 degrees ...

Mzoxolo: No ... 25 degrees.

Researcher: 25 degrees ... so if that one is 25 degrees, can we calculate angle P?

Mzoxolo: I think so ...

Researcher: Okay, what would P be?

Mzoxolo: P would be 25 degrees.

Researcher: Also 25 degrees. Okay and why do you say that?

Mzoxolo: I got no reason for that.

Researcher: Okay, so OQ is called a radius and OP is also the radius. So this gives us what kind of triangle? OQP, what kind of triangle?

Mzoxolo: I would say an isosceles triangle.

Researcher: Isosceles triangle. So you've got 25 degrees and 25 degrees which is 50 degrees and 50 from 180 is equal to ...

Mzoxolo: 130 degrees.

Researcher: SO x is equal to 130 degrees, okay. Lets look at Q9 quickly. Again, look at my reason. I said $x = 25$ and I'm using that same theorem, angle between the tangent and the chord is equal to the angle in the other segment.

Mzoxolo: Its not.

Researcher: Its not, because you're applying, that one here ...

Mzoxolo: Theorem 2.7.

Researcher: Ja, Q2.7. So you established that x cannot be the angle in the other segment. So again how would you go about solving for x.

Mzoxolo: (speaks to himself for a while) ... I'm not sure ...

Researcher: Not sure ... Now remember what we said. See here, the radius and the tangent are ... what?

Mzoxolo: The radius and the tangent are perpendicular.

Researcher: So this angle here would be ... (referring to vicinity of 25 degrees).

Mzoxolo: (turns the page to see properly) ... it should be 90 degrees.

Researcher: So that means angle QTB is what?

Mzoxolo: Angle QTB ... (turns page and indicates on sketch ... is 90 degrees ...

Researcher: Minus?

Mzoxolo: Minus 25 degrees ...

Researcher: So that's 65 degrees and QTBP ... is what?

Mzoxolo: QTBP ... 360 degrees ... QTBP ...

Researcher: But what kind of quad is that?

Mzoxolo: I think, sir, quadrilateral ... QTBP. (still whispering or talking to himself while trying to work out the answer) ... or I would say ... is a semi cyclic ...

Researcher: Is a ... And you said over here (referring to page 1 of test 1). What is ... in 2.1 ... ABCD is what?

Mzoxolo: Cyclic quad.

Researcher: A cyclic quad ... so would that be a cyclic quad? (refers to QTBP).

Mzoxolo: No.

Researcher: Why not?

Mzoxolo: Cause ... it's suppose to be okay ... it might be a cyclic quad, because all these four angles are touching the circumference.

Researcher: Okay, and if its a cyclic quad ... then you gave one of the properties earlier on ... (turns page to 2.4). That property, Q2.4 is what? What can you tell me about x and y?

Mzoxolo: x and y are opposite angles of a cyclic quad which ... we find the sum of 180 degrees.

Researcher: Okay, so the opposite angles add up to 180 degrees. Now we've already said that (referring to angle BTQ) is 65 degrees, so what will x be?

Mzoxolo: Okay 65 degrees ... it will be 115 degrees.

Researcher: Okay 115 degrees ... so that's how we find the value of x. Okay ... let me just ask you a few more questions. ... umm ... Q13. Do you agree with my statement, because that's what you said there? That's true ... Can you explain why you agree with that statement?

Mzoxolo: Okay, we said $x = 57$ degrees, the reason, $PQ = QR$, tangents drawn ... okay. (turns page this way and that to get a look at sketch) ... There's a diameter (indicating with finger on S).

Researcher: Which is the diameter?

Mzoxolo: SP

Researcher: SP, a diameter. Why do you say that's the diameter?

Mzoxolo: ... the diameter is the line that cuts the circumference into two equal halves.

Researcher: And tell me, is QP a tangent?

Mzoxolo: QP ... yes.

Researcher: Why is it a tangent?

Mzoxolo: A tangent is a line, I think, that touches the circle at one point.

Researcher: Okay, good. And QR is that a tangent?

Mzoxolo: QR ... no!

Researcher: No ... and why's it not a tangent?

Mzoxolo: It does not touch the circle at one point.

Researcher: At how many points?

Mzoxolo: At two points.

Researcher: At two points. Okay. So now look at my reason.

Mzoxolo: Okay ... tangents drawn from a common point are equal.

Researcher: So is that still true?

Mzoxolo: No ...

Researcher: No ... because we said the RQ is not a tangent. So you can't use that as a reason. Okay, that's fine. Lets look at Q14. You said true. x is equal to 78 degrees and my reason was, exterior angle of a cyclic quad is equal to the interior opposite angle. And you agreed with that ... Alright? ... Would you still agree?

Mzoxolo: I think, sir, that it is exterior angle (referring to 78 degrees).

Researcher: So which is the opposite interior angle?

Mzoxolo: Opposite interior angle. I think its that (finger indicates somewhere between Z and T or maybe point Z).

Researcher: Z?

Mzoxolo: No its T, another half of T, here (indicates to T), because T is divided into two halves because of the diameter.

Researcher: Okay, so it would be the whole of T.

Mzoxolo: Yes, the whole T.

Researcher: So would that statement still be true?

Mzoxolo: No.

Researcher: Can x be 78 degrees.

Mzoxolo: No.

Researcher: Okay, so you disagree ... that that's not true, hey! Okay. The last one. Q16. You agreed with my statement that x is equal to 57 degrees, because it's the exterior angle of a cyclic quadrilateral. Is that the exterior angle?

Mzoxolo: Okay, x is 57 degrees, exterior angle of a cyclic quad is equal to the interior angle which is the opposite angle ... (indistinct) ... 57 degrees. x can be 57 degrees.

Researcher: And your reason?

Mzoxolo: According to the reason here. We've got a tangent which is NW here, and we've got an angle between the chord, the chord which is UT. And the angle between the chord and the tangent is 57 degrees.

Researcher: So which other angle would then be 57 degrees?

Mzoxolo: I think its S.

Researcher: The whole of S?

Mzoxolo: Yes, the whole of S is 57 degrees.

Researcher: The whole of S. So you're talking about angle USR. Will that angle be 57 degrees?

Mzoxolo: Yes.

Researcher: okay, were you given a strategy for finding this particular angle? (refers to 2.7). Did your teacher show you how to get to the angle in the other segment?

Mzoxolo: I think, yes, she did tell me.

Researcher: She did, okay. And you still agree that the angle in the other segment is USR.

Mzoxolo: Umm ... I think UST.

Researcher: It should be ... UST. Okay, that's fine. Okay. Thank you very much.

INTERVIEW 14 (GCOBISA)

The student admitted that she did not study and simply guessed the answers, especially the bookwork. That was the reason for not knowing some of the bookwork questions, namely, 1.7 and 2.4.

Researcher: ... Now in this one here, Q1. I gave a statement and you said my statement is true. Do you still agree with that? ... Angles subtended by the same arc PR. that is why they are equal. Would you agree with that?

Gcobisa: (sighs) ... I'm not sure (she whispers).

Researcher: You're not sure, okay ... and if I show you, you got this one correct (referring to 2.2). Can you remember the statement to that theorem?

Gcobisa: Umm ... Isn't it this one? (referring to statement of Q1).

Researcher: Okay, now is that the one that we're using there? Is it the same?

Gcobisa: Ja ...

Researcher: Is 2.2 the same as Q1, there.

Gcobisa: Umm ... (sounds like affirmative)

Researcher: Okay, so you don't see anything different?

Gcobisa: ... But this one ... umm ... is O at centre, but this one (2.2) here (points to A and B on the circumference, but does not seem to know how to express herself).

Researcher: So that's at the circumference, but that one (Q1) is at the centre. (referring first to Q2.2 and then Q1). And what about 2.6?

Gcobisa: Can I err ...

Researcher: Give me the statement.

Gcobisa: Okay. Angle at the centre is equal to two angle at circumference.

Researcher: Good. Now if you look at no.2, you said that was true. So x is equal to a20 degrees, angle at centre is equal to two angle at circumference.

Gcobisa: Ja ...

Researcher: So you agree with that? That is true?

Gcobisa: Ja.

Researcher: Now is 1 and 2 the same kind of question? Are they similar or are they different?

Gcobisa: They the same.

Researcher: They similar, okay ... and they deal with which one of these two, 2.2 or 2.6?

Gcobisa: (must have gestured something, nothing heard).

Researcher: 2.2. Okay, okay. Lets go onto Q3 ... Umm ... you said its true ... and the theorem we're using is 2.2, okay. And do you still agree with what you did?

Gcobisa: PAUSE ... No.

Researcher: Why not? ... What's different now?

Gcobisa: PAUSE ... Ja, its true.

Researcher: Its true. So you would agree that that theorem is right. So x is equal to 30 degrees.

Gcobisa: Ja.

Researcher: Okay, alright. Now Q7. Now if you check carefully, why would you say that one is true? Do you agree with my statement? ... The opposite angles of a cyclic quad are supplementary.

Gcobisa: (makes sound, but cannot decide if its yes or no) ...

Researcher: And that is why x is 70 degrees.

Gcobisa: Let me check something (turns pages)

Researcher: Yep, go ahead.

Gcobisa: (turns to page 2) ... I think I looked at this theorem (refers to 2.1), then I said that Q7 is true because of this.

Researcher: Okay, and what is a cyclic quad?

Gcobisa: A cyclic quad is a four-sided figure.

Researcher: Just a four-sided figure?

Gcobisa: No ... with unequal ... (indistinct word).

Researcher: (turns to 2.1) ... is that a cyclic quad ... ABCD.

Gcobisa: Yes.

Researcher: Why is that a cyclic quad?

Gcobisa: The square is inside the circle.

Researcher: Okay, so you mean in or that the corners touch the circle.

Gcobisa: Ja.

Researcher: Okay. Now would OPSR be a cyclic quad? (referring to Q7).

Gcobisa: OPSR ... No.

Researcher: Why not?

Gcobisa: I think because the (indistinct) ... of the ... (refers to O) ... (does not know how to express what she wants to say) does not touch the ... (uses her finger to make the shape of a circle) ... the circle.

Researcher: ... because it doesn't touch the circle. O, where must O be? For that to be a cyclic quad, where must O be?

Gcobisa: On here ... (touches the circle/circumference with her finger)

Researcher: On the ... circle. Okay, that's fine. Let me just check very quickly. Q13. I've said x is equal to 57 degrees, and the reason, $PQ = QR$. Tangents drawn from a common point are equal. And then you said its true. Would you agree with that?

Gcobisa: As I said, I was just ...

Researcher: Guessing? ... Okay, now in your guessing you gave me the statement here (referring to 2.9). Can you remember what's that statement?

Gcobisa: Umm ...

Researcher: 2.9. If AC and BC are tangents. What can you say about that? ... You forgot?

Gcobisa: Yes.

Researcher: Okay. Tangents from a common point? ... Can you remember? ... You forgot?

Gcobisa: Yes.

Researcher: Okay. So tangents from a common point, we know, is equal.

Gcobisa: Okay.

Researcher: Okay. Now let me ask you. Is QP a tangent?

Gcobisa: QP ... yes.

Researcher: Why is it a tangent?

Gcobisa: It touches at one point.

Researcher: At one point. And what about QR. Is QR a tangent?

Gcobisa: QR ... no.

Researcher: Why not?

Gcobisa: I don't know whether there can be two tangents in one circle. So I don't know.

Researcher: So you're not sure?

Gcobisa: I'm not sure.

Researcher: Okay. Now QR is not a tangent because it cuts the circle in R and in S. It can only be a tangent if it cuts at one point. Alright!

Gcobisa: Yes.

Researcher: So because that's not a tangent, it means that statement cannot be true. So that would have been false. Okay, thank you very, very much.

INTERVIEW 15 (SPHOKAZI)

Researcher: Okay, so lets look at Q1. Why do you think that question is true?

Sphokazi: (moves page toward her to see better). x is true, because if you do a chord, there (referring to imaginary line between P and R)

Researcher: Ja.

Sphokazi: P to R. This chord and to this (arm obscuring view of sketch) ... (rest of answer indistinct)

Researcher: Okay, and what about Q2?

Sphokazi: Q2.

Researcher: We said x is equal to 120 degrees, because the angle at the centre is twice the size of the angle at the ...

Sphokazi: ... circumference.

Researcher: And you said it was true. Would you still say its true?

Sphokazi: Yes ... (moves page side to side) ... probably no.

Researcher: And if you look at the two. Q1 and Q2. Are they similar? ... or are they different?

Sphokazi: They are the same.

Researcher: They similar.

Sphokazi: ... because if you connect A to B as a chord you will get x as 60 degrees.

Researcher: Okay, so if you go back to Q1, are you using that particular theorem (referring to Q2.2).

Sphokazi: (sounds like she said) ... Yes.

Researcher: Now look at number 1 and this theorem (2.2). Are they the same?

Sphokazi: No ...

Researcher: Whats different?

Sphokazi: Because this go to the circumference. (referring to line from C to A) ... circumference to the circumference, not the circumference to the centre.

Researcher: Okay. So would that answer still be 40 degrees?

Sphokazi: No ...

Researcher: What would x then be?

Sphokazi: x would be ... 20 degrees.

Researcher: 20?

Sphokazi: No ... I don't know, sir.

Researcher: Why did you say 20 degrees.

Sphokazi: ... because I said that fault is on the centre (points to O on sketch) so, the angle at the centre is twice the angle at the circumference, but I don't know.

Researcher: There we go. That's correct, isn't it. Because you said the angle at the centre is twice the size of the angle at the circumference. So that's why what I wrote there is not correct. Okay?

Sphokazi: Okay.

Researcher: And that is why these two are similar, because that is also the angle at the ... centre. So if you check very carefully (referring to 2.2) ... 2.2 both those angles are at the circumference, and 2.6, the one is at the centre and the one is at the circumference (referring to 2.2). So that is what you must look out for.

Sphokazi: Okay, sir.

Researcher: Okay, are you familiar with these sketches. Have you seen it in your application in the class?

Sphokazi: Yes, sir.

Researcher: Did you do it?

Sphokazi: Yes, sir.

Researcher: Okay, now lets look at Q3. You also said its true. Now why would you say its true?

Sphokazi: I used this theorem, here (referring to Q1).

Researcher: Okay, and is it correct, the way you've applied it?

Sphokazi: No ...

Researcher: What's not correct?

Sphokazi: Because this is not in that circumference (refers to S).

Researcher: So S is not on the circumference. Okay, and if I ask you to calculate the value of x , how would you do that?

Sphokazi: (answers softly).

Researcher: You're not sure. Okay, because ... (explains answer to the problem) ... So x should be 40 degrees.

Sphokazi: 40 degrees.

Researcher: Okay?

Sphokazi: Yes, sir.

Researcher: Okay, lets look at Q7. You agreed with me. I said is equal to 70 degrees. The opposite angles of a cyclic quad are supplementary. Would you still agree with me?

Sphokazi: Yes, sir.

Researcher: Yes. Okay. Now in your bookwork, What's ABCD?

Sphokazi: A cyclic quad.

Researcher: Okay, what do you think is different from that one and that or are they the same? (refers to 2.1 and Q7).

Sphokazi: No its not the same.

Researcher: What's different?

Sphokazi: ... because all the angles are in the circumference, and the other, only three in the circumference. (refers to 2.1 and Q7).

Researcher: Okay, so that's not a cyclic quad. So that means ... can we still apply that property to the cyclic quad?

Sphokazi: No ...

Researcher: No ... alright ... So how do you think you'll calculate the value of x , there?

Sphokazi: I must found this angle ... (moves finger around on point O).

Researcher: And how do I get that angle?

Sphokazi: This (angle 110) minus 360 degrees, I get this angle (point O). So this angle (O) I minus this to this (angle x) ... this angle O is the half of this (angle x).

Researcher: Okay, now lets look at what you said. This is a 110 degrees. Okay, you pointing at the reflex angle here (refers to reflex POR) and you're saying that is ... what?

Sphokazi: 360 minus 110.

Researcher: 360 minus 110. So you're saying this angle plus that one (110 degrees), must add up to 360 degrees.

Sphokazi: Yes.

Researcher: Why, have you got a reason?

Sphokazi: (makes some sound, but is indistinct)

Researcher: Okay, now this angle here, (refers to O) is the angle at the centre of the circle. The reflex angle at the centre. Okay!

Sphokazi: Yes.

Researcher: And the angle at the centre is always twice the size of the angle at the circumference. So the one at the circumference is ...

Sphokazi: ... 110 degrees.

Researcher: So this one (O) would be double that, and that will give you ... 220 degrees.

Sphokazi: 220 degrees.

Researcher: And now you subtract 220 degrees from 360 degrees, and that will give you the value of x , so 220 degrees from 360 degrees, equals 140.

Sphokazi: Okay.

Researcher: Okay, so that's how you would solve that one, but it's important for you to spot that that is not a cyclic quad. And if it's not a cyclic quad, then you cannot use the property. Alright? Now in Q8 you said it's false, because POQ did not touch the circumference. Okay, so that's perfect. That is correct, because then you are applying the tan/chord theorem correctly. (referring to 2.7), because the angle in the other segment must be at the circumference. Okay so that's fine. Now look at no.9. You said its true, so that means you agree that x is 25 degrees. So the angle between the tan and the chord there, is equal to x . Would you agree with that still?

Sphokazi: No ... (says it softly as if she's not sure)

Researcher: Why not?

Sphokazi: ... because angle between tangent and chord is opposite in ... is opposite interior angles ... in the circle. So its U.

Researcher: Its U, okay. And U is not equal to x ?

Sphokazi: No, its not equal to x .

Researcher: Okay, so do you know how to solve that one?
Sphokazi: x?
Researcher: Ja ... do you know how to find the value of x?
Sphokazi: I don't know ...
Researcher: Now, remember you gave me a statement here. What is this statement. Can you remember? (refers to 2.5).
Sphokazi: LONG PAUSE ...
Researcher: You forgot.
Sphokazi: I forgot.
Researcher: Okay, this is your radius. OC is a radius. And this is a ...
Sphokazi: tangent.
Researcher: Tangent (referring to AB in 2.5), and what is the relationship between the radius and the tangent?
Sphokazi: ... the radius is perpendicular
Researcher: They're perpendicular. And you know what we mean by perpendicular?
Sphokazi: Yes, sir, (rest of response indistinct, mumbles answer).
Researcher: So it's equal to ... 90 degrees. So using that here, ... There's your radius. There's your tangent (radius OT, tangent RS), so this angle here at T is ... 90 degrees.
Sphokazi: 90 degrees.
Researcher: So if that's 25 degrees (Refers to QTS) ... What must that one be? ... Angle BTQ.
Sphokazi: 90 degrees minus 25 degrees is 65 degrees
Researcher: And what is TBPQ called?
Sphokazi: It's a cyclic quad.
Researcher: Its a cyclic quad ... and that and x is what? (refers to angle BTQ and x). If that is 65 degrees (BTQ), then what must x be?
Sphokazi: Its 180 degrees minus 65 degrees is 130 degrees, 140 degrees ...
Researcher: One ...
Sphokazi: 125 degrees.
Researcher: 125 or 115
Sphokazi: 115 degrees.
Researcher: Okay, that's fine. Great. Now lets look at Q11. I've said that x is equal to 82 degrees and I'm using the same theorem. Angle between tangent and chord is equal to the tangent in the other ... segment, and you said its true. Would you still agree with that?
Sphokazi: No.
Researcher: Why?
Sphokazi: ... Because there's no angle equal to that 82 degrees, inside.
Researcher: Outside ... inside there?
Sphokazi: ... inside the circle?
Researcher: Okay, so x cannot be 82 degrees.
Sphokazi: Yes.
Researcher: In other words, if I apply this theorem, can I do it there? (shows the theorem, but it is not facing the camera).
Sphokazi: No.

Researcher: Why not?
Sphokazi: ... because x is not in the circle.
Researcher: (repeats) ... because x is not in the circle. Okay, that is fine. What about Q13. Now again look carefully. I said x is equal to 57 degrees. PQ is equal to QR, tangents drawn from a common point are equal. And you agreed with me. Alright! Do you still agree with me?
Sphokazi: I don't know this circle.
Researcher: Okay, now you gave me a statement here. Can you remember what this statement was for, 2.9? (refers to 2.9).
Sphokazi: (Says something unclear).
Researcher: Is what?
Sphokazi: (mumbles to herself) ... to the opposite interior angle. Is it that one?
Researcher: You forgot. Okay ... what is CA? CA is a tangent, and CB?
Sphokazi: Tangent.
Researcher: Okay, and tangents drawn from a common point?
Sphokazi: Are equal.
Researcher: Right, are equal. Now, is QP a tangent?
Sphokazi: Yes.
Researcher: Yes. Why is it a tangent?
Sphokazi: ... Because it touches the circle at one point.
Researcher: Okay ... and is QR a tangent?
Sphokazi: QR ... no.
Researcher: No. Okay, why not? You said no ... I'm just asking why not.
Sphokazi: ... PAUSE ... (mumble) ... It's not a tangent.
Researcher: It's not a tangent, okay! Does it cut the circle in one place? or in two places?
Sphokazi: In one place.
Researcher: In one place ... but if it cuts in one place, then it would be a tangent.
Sphokazi: I can say it's a tangent, because it touches one place ... I don't know.
Researcher: Okay, that is why it is not a tangent, so I cannot use that as a reason, because RQ is not a tangent. Okay. Lets take one more. Q14. You said its true. The exterior angle of a cyclic quad is equal to the interior opposite angle. Would you still agree with that?
Sphokazi: Umm ... No.
Researcher: Why not?
Sphokazi: ... Because ... (very indistinct) ... is equal to that.
Researcher: ... is equal to Z, okay.
Sphokazi: Yes ... (says something, once again indistinct).
Researcher: Now in your bookwork, you said here, the exterior angle is equal to the interior opposite angle ... (refers to 2.3) ... So if you apply that to this sketch (Q14), then what will it give you? or what would it tell you?
Sphokazi: It's not, its not this angle (refers to angle x) ... the whole angle is x. (indicating on sketch, the whole angle).
Researcher: x must be the whole angle. Is that what you're saying?
Sphokazi: (some hesitation here) ... Yes.
Researcher: So your interior opposite angle is which one?

Sphokazi: It's this angle T, the whole angle.

Researcher: The whole angle T ... So that's why it cannot be True. Okay!

Sphokazi: Yes.

Researcher: Alright. Thank you very, very much.

INTERVIEW 17 (CIKIZWA)

Researcher: Now the first one I want to ask you (referring to Q1). You said its true. Can you explain why you said its true?

Cikizwa: I think its true because angles subtended by the same chord are equal. So the arc PR subtends it.

Researcher: So that is 40 degrees. (referring to x).

Cikizwa: Yes.

Researcher: And number 2. Can you explain your reasoning there? You can turn it if you want to. (moves page closer to student to enable her to see better).

Cikizwa: Okay, I said number 2 is true because the angle at the centre is double the angle at the circumference. (indicates on sketch).

Researcher: Okay, so in that one, it's the angle at the centre, so that's why you agreed with that.

Cikizwa: Yes ... and this is the angle at the circumference, angle C.

Researcher: Okay, and if you look at one and two, are they the same kind of question or are they different?

Cikizwa: They are the same kind of question. They are not different.

Researcher: Alright, so now would you want to look at no. 1 again?

Cikizwa: Yes.

Researcher: And then what are you going to tell me?

Cikizwa: PAUSE ... I think x is 20 degrees.

Researcher: You think x is 20 degrees, and what would your reason be?

Cikizwa: The angle at the circumference equals this angle. (referring to angle marked x)

Researcher: So you will be using which one? 2.2 here or 2.6?

Cikizwa: 2.6.

Researcher: 2.6 (repeats) ... So you see where you went wrong?

Cikizwa: Ja, I see, sir

Researcher: Okay, what about Q3. You also said its true. Can you explain that to me?

Cikizwa: (speaks to herself while working out the answer). This angle is (refers to angle S) not on the circumference.

Researcher: Okay, ... you must just talk a little bit louder so that he (video-operator) can pick you up there.

Cikizwa: okay, sir.

Researcher: Okay, so S is not on the circumference. So can you then calculate the value of x for me ... Can you explain the value of x? Do you know how to get to the value of x?

Cikizwa: No I don't.

Researcher: Alright. Now we know that that is false because you said S is not on the circumference. Okay, now all you need to do is take 30 degrees and 50 degrees.

Cikizwa: Its 80 degrees.

Researcher: And ... what would that angle be, angle, angle PTQ.

Cikizwa: Its equal to ... 100 degrees.

Researcher: Its 100 degrees and then one is vertically opposite to angle STR and those two are equal (refers to angle S and R respectively). So if this one is 100 degrees ...

Cikizwa: ... because RTS is a isosceles triangle, so ... (indistinct words) ... are equal.

Researcher: Right, so x will be?

Cikizwa: 40 degrees.

Researcher: 40 ... Great! Okay, alright, so you've got that right. Lets look at Q7. Alright, you said there that it's false and x is equal to 220 degrees. The angle at the centre ... alright.

Cikizwa: Yes.

Researcher: ... is equal to twice the size of the angle at the circumference. Do you still agree with what you said there?

Cikizwa: Yes, I do.

Researcher: Okay, so you're saying x here is equal to 220 degrees and can I ask a question?

Cikizwa: Yes please.

Researcher: If x is equal to 70 degrees, can my reason be: opposite angles of a cyclic quad are supplementary.

Cikizwa: No ... it's wrong, because that is not a cyclic quad. O is not touching the circumference of the circle.

Researcher: So that is not a cyclic quad so that's why we can't use it.

Cikizwa: That's right, yes, we can't use it.

Researcher: Let me explain what you did. You more or less correct. The only problem is that this is the angle that's 220 degrees ... the reflex angle ... that is 220 degrees. So the angle at the centre is twice the size of the angle at the circumference. So its this angle (refers to reflex angle POR) that's 220 degrees. So what would x then be?

Cikizwa: 360. The revolution 220 subtracted by 360 degrees.

Researcher: And that will give you?

Cikizwa: It will give me 160.

Researcher: No ... 140 degrees.

Cikizwa: 140 degrees ... yes!

Researcher: But you know it's 360 minus 220. Okay.

Cikizwa: Yes.

Researcher: Okay, so that would have been correct, there. What about Q8. You said it was true, and the reason is: the angle between the tangent and the chord is equal to the angle in the other segment. You still agree with that?

Cikizwa: Yes I do.

Researcher: Okay, you got this one correct in the exam. Can you remember the statement for this?

Cikizwa: Yes I do.

Researcher: That's 2.7, alright (referring to 2.7)

Cikizwa: Ja ... the angle between the tangent and the chord is equal to the angle in the opposite segment.

Researcher: Okay, that's the one we're using there. Can we use this theorem to solve that? (refers to 2.7, to solve Q8)

Cikizwa: (answer indistinct)

Researcher: Okay, ... so what's different?

Cikizwa: A and B here (Q2.7) are see ... on the circumference, but here (Q8) its P and Q on the circumference. O is not on the circumference.

Researcher: Okay, so we can't use that one. That is why that would then be false. Okay, but now ... or can you calculate the value of x for me?

Cikizwa: Sorry sir, can I ask a question?

Researcher: Yes.

Cikizwa: Is O the centre of the circle?

Researcher: Yes, O is the centre of the circle.

Cikizwa: What I can say is if O is the centre of the circle, then OQ and OP are radii.

Researcher: That's correct.

Cikizwa: And then 65 degrees is the exterior of the triangle ... triangle POQ.

Researcher: Okay ... POQ. So POQ is that one (referring to Q8). Okay, and is that the exterior angle? I was asking what if I can say so?

Cikizwa: I was asking what if I can say so?

Researcher: Ja, but then, that's not the exterior angle. Okay, but you've got the statement to this ... 2.5 (refers to 2.5). Can you remember that particular theorem ... That's a radius and that is your tangent? (refers to radius OC and tangent AB) and what is the relationship between the two?

Cikizwa: The radius is perpendicular to the tangent.

Researcher: The radius is perpendicular to the tangent ... So there you've got the radius OQ (referring to Q7).

Cikizwa: ... And the tangent RS.

Researcher: This angle (angle marked 65), would then be ...

Cikizwa: 90 degrees.

Researcher: So if that one is 65 degrees, then OQP would be what?

Cikizwa: 25 degrees

Researcher: 25 degrees ... and then you said this is isosceles, so this one would also be (referring to P) ...

Cikizwa: 25 degrees.

Researcher: That's angle P, yes ... and therefore what is x?

Cikizwa: Its 140 degrees.

Researcher: One hundred and ...

Cikizwa: 130 degrees.

Researcher: Yes ... so you see how you can use your bookwork and solve there ... alright.

Cikizwa: Yes, I can see, sir.

Researcher: But it's also easy to make a mistake.

Cikizwa: Its confusing.

Researcher: (turns page) ... Okay, Q13. Look at the reason that I've given. I said PQ = QR: tangents drawn from a common point are equal, and that is why x must be 57 degrees. Would you agree with that? [NOISE IN THE BACKGROUND, SOMEONE SPEAKING LOUDLY].

Cikizwa: No, I don't agree with you.

Researcher: Why don't you agree with me?

Cikizwa: Because this is a tangent. (refers to PQ)

Researcher: Okay, which is a tangent? ... QP?

Cikizwa: QP's a tangent.

Researcher: Okay ... and what about QR?

Cikizwa: I don't think QR is a tangent.

Researcher: Why not?

Cikizwa: Because a tangent is the line which touches the circle at one point.

Researcher: Okay, so could we use the statement of this theorem to solve that? (referring to Q2.9) ... DO you know the statement to 2.9?

Cikizwa: Yes I do.

Researcher: What is it?

Cikizwa: Tangents drawn from common points are equal.

Researcher: Okay, so can we use it there?

Cikizwa: No, we cannot.

Researcher: No ... and that is why that would have been false. You see there? ... Because that's what I'm using ... Okay! Alright. Last one. Q16. You said true. Can you ... okay, let me take you first through my reason. I said $x = 57$: the exterior angle of a cyclic quad is equal to the interior opposite angle. And you said, you agree with that statement. Do you still agree?

Cikizwa: (talks to herself while working out answer) ... No, I don't.

Researcher: Okay. Why not?

Cikizwa: Because T is the exterior angle of the cyclic quad.

Researcher: Because T is the exterior angle ... okay. So how would you go about solving that?

Cikizwa: PAUSE ...

Researcher: And always remember when you are given an angle, you need to just look at the position ... alright? So you can see, the angle I'm giving you there is the angle between the tangent and the chord. (referring to sketch). So that's sort of a hint. So can you tell me which angle is in the other segment?

Cikizwa: Which is equal to this angle? (referring to angle marked 57).

Researcher: Ja ...

Cikizwa: I think its angle S.

Researcher: Angle S. (researcher is disturbed by someone who speaks to him). ... So which angle?

Cikizwa: I think angle S is 57 degrees.

Researcher: The whole angle S? RSU

Cikizwa: RSU ... no not the whole angle.

Researcher: So name it for me?

Cikizwa: Its angle UST.

Researcher: Angle UST ... okay. So that would then be 57 degrees and this is then a isosceles triangle UST. So you could easily get that, and that, the base angles (referring to angles at U and T).

Cikizwa: Base angles are equal ...

Researcher: Yes, and remember that TRSU is called what?

Cikizwa: A cyclic quadrilateral.

Researcher: And opposite angles? ... (referring to x and U).

Cikizwa: ... Are supplementary.

Researcher: Are supplementary. And that's how you would get the value of x. Okay ... So that's why that one was worth quite a few marks. Alright, thank you very much.

Cikizwa: Thank you sir.

INTERVIEW 18 (ETIENNE)

Researcher: ... Alright, lets fire away immediately with Q1. Umm ... why do you think that figure ... or why did you say that is true?

Etienne: Because the angle is subtended by the same chord.

Researcher: So you would agree with the statement there?

Etienne: Yep.

Researcher: And, you perfectly happy with that?

Etienne: Yes.

Researcher: Okay, if you have a second look at it, do you still go with the same response?

Etienne: I wasn't sure, 100% sure, because I normally know it as ... it supposed to be touching the circumference.

Researcher: What, what's supposed to be touching the circumference?

Etienne: This (indicating with finger to point O on the sketch).

Researcher: O ... Okay, and what about number 2. Explain your reasoning.

Etienne: I go with the same reason as I said over there.

Researcher: Alright, now if you look very, very carefully at the two, are they the same or are they different?

Etienne: They the same. [NOISY CHILDREN IN BACKGROUND]

Researcher: They the same ... Okay, in what way are they the same?

Etienne: Umm ... Its ... in both cases its to regulate and the answers probably the same ...

Researcher: Okay, now look at the reasoning here (referring to reason in Q2). We said x is equal to 120 degrees, and the reason: angle at the centre is equal to two angle at the circumference and you said its true, okay?

Etienne: Yes, this one (referring to Q2). I think, sir, it was ... (can't make out answer, went from loud to indistinct).

Researcher: Okay, so that you agree with that response ... Now my question again is ... Are those two things different or are they the same?

Etienne: The same.

Researcher: Just check no.1 again, and then tell me if you're still happy with true?

Etienne: Umm ... partially.

Researcher: What's confusing here?

Etienne: Its not confusing me ... I'm looking at it now and that was supposed to be the reason (refers to reason in Q1) ... that supposed to be over there.

Researcher: Okay, so would x still be 40 degrees?

Etienne: Yes ... no! x should be 20 degrees ...

Researcher: It will be ... 20 degrees

Etienne: 20 degrees.

Researcher: So ... then that's not correct (referring to solution in Q1).

Etienne: No ...

Researcher: Alright, because the one that you were looking at ... for the one at the circumference ... that's different ... okay ... so I think we've established that. That's fine ... what about Q3. How did you go about answering that question? You ended up with true, which means you probably agree with this statement?

Etienne: Again, over here I wasn't sure if I ... same feeling over here (referring to reason in Q1).

Researcher: So, so that's what I want to pick up. What's not making you're sure?

Etienne: The fact that it wasn't touching the circumference (referring to S).

Researcher: Okay, so that was the problem ... that it wasn't touching the ... circumference ... Because look, you answered that correct, hey! Q2.2. That's the theorem (referring to Q2.2), that we're talking about. Can you remember the statement?

Etienne: Angles subtended by the same arc ... should be equal.

Researcher: Yes, and if you check carefully ... A and B are at the circumference (refers to 2.2) ... Right! ... So if I should ask you ... okay ... so you're not saying that's true (Q3) or would you still say that's true?

Etienne: I wouldn't say its true.

Researcher: And if I should ask you how you would go about finding the value of x ... How would you do that?

Etienne: Umm ... I'll find that angle over there (pointing to T on sketch).

Researcher: Ja ...

Etienne: QTP ...

Researcher: QTP (repeats) ...

Etienne: It shall be 180 minus 80, the sum of that two (refers to Q and P), should 80, because the sum of the interior angles of a triangle should be 180 degrees, then that would be vertically opposite to that (refers to opposite angles at T), and it should be 100 and I'll divide by two which will give me 80 degrees divided by two is 40 degrees, 40 degrees (refers to S and R), because the two sides are the same. (refers to TS and TR).

Researcher: Okay, so x is equal to 40 degrees.

Etienne: ... is equal to 40 degrees.

Researcher: Okay, great. Alright, now if you look at Q7.

Etienne: Q7.

Researcher: Ja ... You said false, but you just stopped there. So I want to explore ... why did you think it was false?

Etienne: False ... umm ...

Researcher: Because by saying false, you obviously disagreed with my statement ... which is fine ...

Etienne: To my understanding x is also supposed to be 110 degrees, because you can't say its 70 degrees, its not a cyclic quad ... because they don't say so ...

Researcher: Okay, so you've established that it's not a cyclic quad ... Alright? ... how do you think you will go about solving that?

Etienne: I have a way ... I normally ... that is 110 degrees (referring to S), then to me that (referring to x) is also 110 degrees, because if I put another line over there (gesturing to a point above O on the circumference), then that would be half of that (refers the angle at the circumference). That's how I remember it. And if that is the half of that (refers to x), that would obviously make this whole thing a cyclic quad, then that and that (new angle at circumference and the given 110) will be 180 degrees.

Researcher: Oh ... so those two (O and S on sketch) would give you 180 degrees.

Etienne: (says something about a cyclic quad, a bit indistinct) ...

Researcher: Okay ... so if those two together will give me 180 degrees then what will ... then what will that one at the top be? (pointing to imaginary angle above O).

Etienne: Ja, I was talking about the one at the top. That one and 110 would give me 180 degrees. If it was a cyclic quad.

Researcher: Okay ...

Etienne: What I do is ... in my mind, I put two lines (indicates on sketch) ...

Researcher: That's fine ...

Etienne: Okay, and then ... umm ... if I see that's 110 there then that's 110 to me as well (refers to x), because then that's automatically 70 degrees (places finger near top of circumference), and 70 and 110 gives me 180 degrees.

Researcher: Alright ... now from that 70 to the x . How did you get 110 for x ? ... You see what I'm saying. I'm happy with what you're doing. I'm just going a little bit further, because you said 70 degrees, but you said x must also be 110 degrees ... if I remember correctly?

Etienne: LONG PAUSE ... SILENCE ...

Researcher: So you agree with x being 110 degrees.

Etienne: That's how I (indistinct word) ... 110 degrees.

Researcher: Then what would your reason be? If you say x is 110 degrees and remember you got that one up there which is 70 degrees.

Etienne: I'll try and redraw my structure on my question paper.

Researcher: Okay, so you would do something like ... (refers to same sketch on sample page and draws in dotted lines from point R to top of circumference and from P to top of circumference).

Etienne: ... But then it still wouldn't work.

Researcher: And you're saying that this is a cyclic quad (refers to lines drawn in on sketch, plus PS and SR) and you said that one (angle at top of circumference) would be 70 degrees.

Etienne: But you going to have to make that 140 degrees.

Researcher: So x would be 140 degrees (fills in 140 for x) ... Okay ... that's fine ... umm ... Okay, lets look at the next one Q8. How did you go about solving that one? You can follow your submission here, and just talk me through it.

Etienne: OQ was perpendicular to RS ... and that's obviously a 90 degree angle over there (referring to OQS) and if that's 65 over there, then 90 minus 65 will be 25 degrees, which is PQO .

Researcher: Okay ...

Etienne: PQO and that's 25 over there ... then that's also supposed to be 25 degrees (referring to point P and Q), because with the isosceles triangle the two radii are equal.

Researcher: Okay ...

Etienne: And ... umm ... together they give me 50 degrees and to get that one (refers to x), is 180 minus 50 will give me 130 degrees, because ... (his answer fades away because researcher interjects) ...

Researcher: Okay, I'm happy with that ... Now, if you look at what I said, x is 65 degrees and look at my reason. What's wrong with my reason?

Etienne: Tan and chord theorem ... umm ... the chord isn't touching the circumference of the circle.

Researcher: ... The chord.

Etienne: Ja, ... the ... the ...

Researcher: What's not touching?

Etienne: I just know that the ... (points to O) is supposed to touch the circumference.

Researcher: The point.

Etienne: Ja ...

Researcher: Okay, okay I'm with you ... Now ... Q9. You didn't answer ... What confused you there?

Etienne: LONG PAUSE ... The structure of the ...

Researcher: Because I see you filled in some things there ... can you talk me through that.

Etienne: Okay ... If that's 25 degrees there (referring to angle marked 25 degrees), then there's supposed to be 25 degrees (refers to angle B), because that's the tan/ chord theorem.

Researcher: So that would be ... you're applying that one (referring to Q2.7).

Etienne: (says something indistinct).

Researcher: Okay.

Etienne: And if that's 25 degrees, then that's supposed to be 65 degrees (refers to BTQ), because the radius to the tangent is perpendicular ... that will be 90 degrees over there (pointing to angle BTS) ... There's a number of (indistinct) ... that's the only thing I couldn't think it could be ... [NOISY CHILDREN IN BACKGROUND-OUTSIDE].

Researcher: Okay ... you're basically almost one step away from the answer with x , with what you've got there. [MUST BE INTERVAL OUTSIDE - NOISE VERY LOUD].

Etienne: (his words drowned out by the noise) ... angle over there, is it point O ... is that on point O .

Researcher: Yes, that's point O .

Etienne: So the angle over there ... 90 degrees ... that's 65 degrees ... that's 25 degrees and there ... that's as far as I got.

Researcher: And then ... Are we talking loud enough, John (refers to video-operator). Must we talk a bit louder? ... Okay. Alright ... so if you check carefully, there's a cyclic quad (illustrates on sketch) ... TBPO is a cyclic quad ...

Etienne: Yes ... a cyclic quad.

Researcher: And you've got that one which is 65 degrees, so 115 degrees is the value of x ... Alright ... Lets flip over (turns page). Okay ... Q10, you said it was true, you got x is equal to 30 degrees. That's fine ... umm ... In my reasoning I said x is 30 degrees: Angle at centre equals two angle at circumference ... umm ... Would you agree with what I got there?

Etienne: LONG PAUSE ... CHILDREN SHOUTING IN BACKGROUND ... Yes

Researcher: You agree with that ... Okay, so then that statement ... stays true ... Alright Q11. You just said false, and then you did some calculations on the sketch. Can you talk me through that. Explain to me what you're doing there?

Etienne: x is angle S over here (points to S on the sketch).

Researcher: Umm (agrees).

Etienne: You got 82 degrees over there ...

Researcher: Talk a bit louder.

Etienne: ... 82 degrees over there ... that was tan/chord theorem. Then I got $180 - 82$ which gave me 98. This angle's adjacent supplementary (pointing to P , where he pencilled in his own figures). And umm ... that was as far as I got ...

Researcher: That's as far as you got ... okay, now still, would x be 82 degrees. using my reason there?

Etienne: No ... it can't be 82 degrees. That's what I told myself, because if that was 82 degrees, then 82 and that 98, all that would do is give you 180. So that's also supposed to be a ... there (referring to Q), because its a triangle (last word is indistinct).

Researcher: Okay ... that's perfect. Alright ... no.16 ... Q16. You got 4 points, can you talk me through that one?

Etienne: Umm ... I started off here 57 degrees, took it up over there (refers to UST), that tan/chord theorem.

Researcher: Ja ...

Etienne: And umm ... that 57 degrees over there (hand still obscuring view of sketch) and STR must also be 57 degrees, because there are two parallel lines are apparently equal, and umm ... if that is 57 ... PAUSE ... and these two lines are equal over here (refers to ST and SU) ... which is making it an isosceles triangle. And that 57 degrees and that is 180 degrees minus 57 ... And the answer divided by two, which I got there, 61.5 ... SIREN GOES OFF LOUDLY ... and I said $180 - 61.5$ which gave me 118.5 ... SIREN STILL GOING ... because x is opposite angle in the cyclic quad.

Researcher: Okay ... What a noise! LONG PAUSE BECAUSE SIREN STILL GOING ON AND ON ... Does it always ring so long?

Etienne: (nods yes) ... for the others at the back.

Researcher: SIREN STOPS ... Okay, so I'm happy with that, and you obviously didn't agree with my reasoning, that exterior angle of a cyclic quad ... 57 degrees ... Is that an exterior angle?

Etienne: No, it can't be, umm ... because its supposed to be inside ... follow with the line (gestures on line RV) like that.

Researcher: Okay, great. Alright Etienne thank you very much.

INTERVIEW 19 (PAUL)

Some questions were given to the student to obtain some clarity based on the student's "bookwork" knowledge. These included question 1.4, 2.4 and 2.7.

Researcher: ... Q1. Why do you say that is true? or why do you think that's true. You agree with that?

Paul: I agree with it.

Researcher: And you happy with the reason? If x is equal to 40 degrees: angles subtended by the same arc PR .

Paul: Yes, I'm happy, because I look at it this way (makes a Z with finger on sketch).

Researcher: Okay, and then ... is that the same as 2.2? (refers to Q2.2. Researcher uses spare page 2 to indicate diagrams his speaking about).

Paul: No ...

Researcher: What's different?

Paul: That 2.2 ... that is touching the circumference.

Researcher: What is touching the circumference?

Paul: All four points of the ...

Researcher: All four points, okay. And in that one?

Paul: Only three points are touching.

Researcher: So are we using the same theorem?

Paul: No.

Researcher: No ... Okay, now if we go to Q2. Can you explain the reasoning for that one ... because there you said true as well, okay! ... But there the reason was $x = 120$: Angle at centre equals two angle at circumference.

Paul: The reason why I said true, I sort of ... umm ... the theorem that we have with the triangle and that we always do in class, that's why I said its true.

Researcher: Okay ... so this is familiar to you.

Paul: (nods yes)

Researcher: Okay ... What about Q3 ... You said false. Why did you say false? Because I'm not sure I understand what you did there?

Paul: LONG PAUSE ... STILL VERY NOISY IN BACKGROUND.

Researcher: I'm not sure why I said false.

Researcher: You're not sure? ... Okay. And tell me. Is this one the same as 2.2? (referring to sketch no.3).

Paul: No, it's not the same.

Researcher: Why ... what's different?

Paul: Because only three points is touching the circumference of the circle.

Researcher: So, could I still use that as a reason?
Paul: No, sir (softly)
Researcher: No. Okay ... umm ... Did you perhaps, when you said is equal to 50 degrees. You said interior angles. What did you mean by that?
Paul: I think I went wrong there, I ... (indistinct) ... subtend angles of a cyclic quad.
Researcher: Okay ... now, lets come to Q7, $x = 70$ and the reason: Opposite angles of a cyclic quad. And you said is false. Alright ... and then you just gave a value for x . So why do you say its false?
Paul: First of all, it's not a cyclic quad.
Researcher: So that's not a cyclic quad.
Paul: And then I said that ... I know that if this (indicates to angle 110 degrees) angle here is twice that angle (referring to x), then this must be half it.
Researcher: Okay, so this is the angle at the centre (refers to x), so it's half of the 110 degrees. Okay, and Q8. Why do you say that is true ... By the way was this thing (refers to sketch) emphasised in class? This particular sketch. Have you seen it before?
Paul: I did see something like that.
Researcher: Q8. Why do you say that's true? or ... do you agree with the statement.
Paul: I agree with the statement, yes.
Researcher: Okay ... Now if you agree with the statement, then we're using the angle between tangent and the chord ... So that would be the theorem that we ... apply. Alright ... and are you applying it correctly?
Paul: (answers too softly to make out what he's saying).
Researcher: So there's no difference between (refers to 2.7) that one and that sketch?
Paul: No ... there is a difference.
Researcher: Okay, what's the difference?
Paul: The triangle is ... the three parts of the triangle is touching the circumference, but here it's only two.
Researcher: Okay, so here we have three points touch the circumference, and there we only have two (indicates points A, B and C on Q2.7 and the latter part refers to sketch no.8).
Paul: Two points (repeats last few words).
Researcher: So can we still use the same reason?
Paul: No, sir.
Researcher: No, okay! ... Alright, Q9. You also said true. So that means you said that the angle between tangent ST (indicates on sketch) which is that ... and the chord TQ, so its that angle there (refers to given angle 25 degrees) ... is equal to the angle in the opposite segment. So we're still using that theorem. Okay, ... and is it the same? Are we using that one ...
SAME LOUD NOISE IN THE CLASSROOM/ LAB IN BACKGROUND
 ... (rest of sentence not clear).
Paul: Yes, we are using it.
Researcher: So you agree that if that angle is 25 degrees, then x must also be 25 degrees. Alright, and tell me did you get a strategy or mechanism to use to solve the problem?

SO MUCH BACKGROUND NOISE THAT STUDENT'S ANSWER IS HARDLY AUDIBLE.

Paul: No ... (rest of sentence indistinct).
Researcher: Alright, so you're just looking at it and seeing if that tries to work there?
Paul: Yes.
Researcher: Okay ... Lets flip over (turns page) ... Here we go ... in Q10. You said that the angle at the centre is twice the angle ... so you agree with that statement. So you don't have a problem with that.
Paul: No, sir.
Researcher: Its fine ... So is AOB the angle at the centre?
Paul: (answer indistinct, speaking too softly).
Researcher: Okay, okay lets go onto Q11. You just said false there, right, which means you don't agree with that reason. So why do you say it's false?
Paul: LONG PAUSE ...
Researcher: And that means again you don't agree: The angle between tangent and chord is equal to the angle in the opposite segment. (refers to reason of no.11). So again we're looking at the application of ... (refers to 2.7) ... Is it the same thing?
Paul: It is the same thing ... just there's another line coming from the tangent into ... these line (referring to line running from S to T).
Researcher: This line here ... (line ST). So okay ... so what's different between that one (Q11) and that one (Q2.7).
Paul: There's no difference there.
Researcher: They're exactly the same?
Paul: Yes. (softly).
Researcher: So I can use the theorem for this one (referring to sketch no.11).
Paul: Yes (very softly).
Researcher: But you said it was false.
Paul: ... PAUSE.
Researcher: So you still happy with it being false?
Paul: No ...
Researcher: I'm confusing you now?
Paul: ... (laughs).
Researcher: Okay, now if you go to Q12. You said that is true. Now, are we using the same strategy here, as we used in no.11.
Paul: They the same, sir.
Researcher: Okay, so the angle between the tangent and the chord is equal to the angle in the other segment. So the angle between tangent and chord is equal to the angle in the other segment ... (repeats this while applying the same theorem and indicating its application on first no.12 and then no.11). So you don't have a problem ... you happy.
Paul: No, sir ... Yes, I'm happy.
Researcher: Okay ... lets go onto Q13. Again you just said it was false. What I want to look at is why do you say it is false? Because if you say it is false then you obviously don't agree with my statement. So why don't you agree with the statement?

LONG PAUSE...

Paul: I'm not sure why ...
Researcher: Okay, is QP a tangent?
Paul: QP is a tangent.
Researcher: Why is it a tangent?
Paul: As its drawn to the circumference ...
Researcher: Because its draw? ...
Paul: ... to the circumference, sir.
Researcher: Okay ... will QR be a tangent?
Paul: Yes, it can be a tangent.
Researcher: Why?
Paul: Because there's umm ... because it's drawn to the circle.
Researcher: So it's drawn through the circle, okay?
Paul: Yes.
Researcher: And, and how do you define a tangent?
Paul: A tangent is a line that touches the circumference of a circle.
Researcher: That touches the circumference of a ... circle, so QR, does that line touch the circumference of a circle?
Paul: Yes, it touches.
Researcher: Okay, where does it touch.
Paul: ... (points to R).
Researcher: R ... only at R?
Paul: There at Q (points to Q)
Researcher: But is the circle passing through Q?
Paul: No ...
Researcher: Okay ... so there's still that confusion about what's meant by tangent. Okay, lets look at no.14. How did you go about deriving that's true. Can you show me or show me how your teacher would explain that.
Paul: I always remember that we used to do something like this: exterior angle (referring to 78 degrees) is equal to the opposite interior angle (referring to x).
Researcher: Okay, so the opposite interior angle is T, I mean, is x .
Paul: ... x , yes.
Researcher: Alright ... so is that why you agree that its ... (does not complete the sentence). Okay, now ... if you check your sketch, that's the theorem we're talking about. Is that exactly the same? (does not indicate here what he is referring to).
Paul: No ...
Researcher: What's different?
Paul: This line here ... line TV ...
Researcher: Okay, the line TV is making ... (last word indistinct) ... and if we take the line TV away, will that angle still be x ?
Paul: Yes, it will still be x .
Researcher: Just that part there (indicates x on sketch).
Paul: Yes.
Researcher: Okay, if we apply this: the exterior angle is equal to the interior opposite angle.(refers to 2.3). So you're happy with that? ... Now what about no.15?

Its the same thing, x is equal to 60 degrees and I've given the reason: exterior angle of the cyclic quad is equal to the interior opposite ... is that exactly the same as no.14.

Paul: Yes it is the same.
Researcher: Alright, so there's no problem with that?
Paul: No problem.
Researcher: Okay, and no.16, the exterior angle of a cyclic quad is also equal to the interior opposite angle. So you'd agree that this is the exterior angle of a cyclic quad (referring to angle 57 degrees on sketch)
Paul: ... (speaks softly, but probably agrees) ... Yes, I agree with it.
Researcher: No problems with it. And that's why you said it was true. Okay if we check (introduce end activity) ...

INTERVIEW 20 (WARDA)

Researcher: ... Alright, lets get onto Q1. You said Q1 is true, why do you think its true? Do you agree with the statement? ... PAUSE ... Would you still say its true?
Warda: No, I don't think so now.
Researcher: Okay, why not? ... Can you hear her? Must she talk louder? (refers to video-operator) ...
Warda: No, now it's the same, sir.
Researcher: It's the same?
Warda: Umm (agrees).
Researcher: Okay, and what about Q2? You said that's true as well. And here we said x is equal to 40 degrees (refers to solution in Q1), because there's 40 degrees (refers to angle 40 degrees in sketch), so x is equal to 40 degrees as well ... Here I said x is equal to 120 degrees (refers to Q2) and there it was 60 degrees (refers to 60 degrees in Q2) and the reasons are slightly different.
LONG SILENCE ... PAUSE FROM STUDENT ... So are these two sketches the same to you or are they different?
Warda: They're different.
Researcher: Okay, and why are they different?
Warda: That is umm ... angles, I think, of a triangle are the same, the segments are equal ... and that is the angle at the centre ... is the same as angle at the circumference.
Researcher: Okay, so number 2 is based on 2.6 (refers back to 2.6). Okay, that's the two theorems that you gave me. Okay, what about number 3. You said its true, so that means you agree with the statement: angles subtended by the same arc QR, okay! Are you happy with that? You said its true, so just checking.
LONG PAUSE ... And, is that the theorem we're applying (refers to 2.2)
Warda: That is the theorem ... I don't think so.
Researcher: What, what doesn't look right to you?
Warda: Because S don't touch the circumference.
Researcher: Because S is not on the circumference ... Okay, fine. Now what about no. 4? Q4, is it the same as Q3?
Warda: No, it is not the same.

Researcher: And are we using that one ... (refers to 2.2)

Warda: We're using that one.

Researcher: And did your teacher give you a strategy to find the angles that are equal? A way of finding it or do you just look and make a deduction?

Warda: Look and make a deduction. That's how we do it in class.

Researcher: And Q4 and Q6 are more or less the same?

Warda: Umm (agrees).

Researcher: Would you say that in Q1,2,3,4 and 6 we're using exactly the same theorem? or which ones?

Warda: I think this one is different (pointing to Q2).

Researcher: That one? only no.2 is different?

Warda: ... These are the same (points to Q1,3,6 and 4).

Researcher: So 1 and 6, 3 and 4 are the same. Alright now, Q7. How did you go about making that deduction. Do you agree that opposite angles of a cyclic quad are supplementary?

Warda: Yes.

Researcher: So that's why that one is 70 degrees. Okay ... what is a cyclic quad?

Warda: Its umm ... its four corners ... say with a circle its four corners ... which ... its in the ... how can I say it ... a quadrilateral inside the circle ... four points that's on the circle.

Researcher: The four points must be ... on the circle.

Warda: On the circle.

Researcher: Okay, so is that a quadrilateral with the four points on the circle?

Warda: No, it isn't ... (rest of sentence not clear, too much noise from other children in the background).

Researcher: Okay, so it's still a cyclic quad for you?

Warda: ... (must have nodded affirmative, cannot see or hear her answer).

Researcher: And if it is a cyclic quad then that is true ... okay ... but now remember you defined the cyclic quad as the four points must be on the circle, so if that point (refers to point O) is not on the circle then is that still a cyclic quad.

Warda: No ...

Researcher: And it can't be ... and that means you can't make use of the property. Okay ... umm ... Is there some way that you would try and calculate x? How would you try and calculate angle x there? ... Now that you know it's false ... What would x be?

Warda: I don't know.

Researcher: Not sure? ... Okay, that's fine. Lets go onto no. 8. How would you go about solving Q8?

Warda: Exterior angles is equal to the interior opposite angles.

Researcher: The exterior angle?

Warda: ... Is equal to the interior opposite angle.

Researcher: Of a cyclic quad, or ... of what ... Of the triangle?

Warda: Of a cyclic quad.

Researcher: Okay, of a cyclic quad. Now if you check with me. I've said that x is equal to 65 degrees. The angle between a tangent and a chord is equal to the angle in the other ... segment. Now you've already mentioned that theorem,

that one there ... (refers to Q2.7). Are we using this over there? (refers to Q8).

Warda: Yes.

Researcher: Are the two sketches exactly the same?

Warda: No.

Researcher: What's different?

Warda: The point is not on the circumference.

Researcher: ... Not on the circumference. Okay, so ... can we still apply that? (refers to 2.7).

Warda: Yes, sir.

Researcher: You can still apply that even though the point is not on the circumference. Okay ... and what about Q9. You said its true and I'm using exactly the same reason: Angle between tangent and chord, so that's the theorem there ... Can we use that theorem there?

Warda: Yes

Researcher: Okay, so we can still use it there, because that's why you said its true, hey! ... Okay, lets go onto Q10, quickly ... (researcher turns page). You said that was true, that's fine. Okay, Q11. Again its angle between the tangent and a chord, that's why $x = 82$ degrees. Would you agree with that, and then, are we still using that theorem, are we applying that one (refers to 2.7).

Warda: Yes ... (speaks softly).

Researcher: What do you think is the difference between this one (refers to 2.7), if there is, and Q11?

Warda: ... (points to sketch, but says nothing).

Researcher: ... the x?

Warda: ... the x is outside.

Researcher: the x is outside of ...

Warda: ... the circle.

Researcher: Of the ... where as here (refers to 2.7) everything is inside. So could we still use this theorem (refers to 2.7) to answer that one (refers to Q11).

Warda: No ...

Researcher: No, because x is outside. Okay, so obviously then it wouldn't be true. Okay ... umm ... what about Q13. What do you understand by a tangent. What is a tangent?

Warda: It's a line that touches the circle at a certain point.

Researcher: At one point ... So would PQ be considered a tangent?

Warda: Yes ... (speaks softly).

Researcher: And would RQ be a tangent?

Warda: No ...

Researcher: Why wouldn't it be a tangent?

Warda: Because it touches the circle at R and S.

Researcher: Right, because it touches the circle at R and S. Okay, so I said $x = 57$, that's x there (indicates on sketch) and my reason: is because $PQ = QR$ and ... what I'm doing ... I'm looking at this one. Can I use that theorem, there? (refers to 2.9) ... Because for this to apply I need two tangents?

Warda: No ...

Researcher: So that means you cannot use that particular theorem. Okay, Q16. You agree with that so you were saying that the exterior angle, which is TUV is equal to the interior opposite angle. Would you agree with it?

Warda: Yes.

Researcher: So there's no problem with that.

Warda: No.

Researcher: Okay, and is that exterior angle the same as no. 14's exterior angle?

Warda: No.

Researcher: Okay, no. What do you think is different?

Warda: This is only got one ... (indistinct) ... divided into two.

Researcher: Okay, so you divide by 2 there (refers to Q14, line TV). Okay, but that is a different exterior angle (refers to Q16, angle TUV), but you can still use it ... Alright, thank you very much.

INTERVIEW 21 (SONIA)

Researcher: ... So we'll start with Q3 ... Here we go ... Okay, you said it was true. That means you agree that angles subtended by the same arc or chord are equal. Do you still agree with that?

Sonia: No.

Researcher: Why not?

Sonia: Okay ... where's it ... This one here, are you asking me this one (points to Q3).

Researcher: Ja ...

Sonia: Because it's not touching the circumference (pointing to S).

Researcher: So S is not on the circumference. So can you use that theorem (refers to 2.2)

Sonia: No ... its not touching the circumference.

Researcher: No ... because it's not touching. So you won't agree with your answer there ... Okay, that's fine. Q4 and Q6 are more or less the same. Am I right?

Sonia: Right.

Researcher: Am I right ... And do you have a strategy for finding those answers? LOTS OF NOISE IN BACKGROUND.

Sonia: ... (says something indistinct but ends in) ... quickly.

Researcher: Umm ... I would say so. You get the ... your circle with the angles. Here we have its a ... circle (refers to Q4) and here we have the same ... (refers to Q6). Okay, but you don't have a problem with it.

Sonia: No.

Researcher: So that's fine. What about Q7?

Sonia: I do not like that one.

Researcher: Can you explain what you're trying to do there, or what you were doing there.

Sonia: I mistook it for this (pointing to sketch). I just saw this ... the centre ... angle at the centre ... So I took it ... angle at centre is twice the angle at the circumference.

Researcher: Okay, so that's why x is equal to 220 degrees ... Right! ... That's good ... Now I also said $x = 70$: opposite angles of a cyclic quad ... So do you agree with that?

Sonia: No.

Researcher: Why not?

Sonia: Cyclic quad ... it's not touching ... its not touching the circle line.

Researcher: Okay, so it's not a cyclic quad, so that's why you had to look for another method?

Sonia: Yes.

Researcher: Now, let me just explain to you, what should have happened. If that is 110 (refers to given angle), then this one here (refers to reflex POR) will be 220 degrees.

Sonia: Oh, the outside.

Researcher: Outside ... and if that is 220 degrees ... from 360 ... will give you x .

Sonia: Oh!

Researcher: Okay ... because you see, if this was the angle up there (indicates an angle above O on the circumference) ... then this (angle POR) will be the angle at the centre for that one, but since its here (refers to S) ... it must be on the outside.

Sonia: Oh, yes.

Researcher: Okay, so that's what went astray there. Umm ... Q8. You said its true, which means you agree with my statement, and since you've got all the other things correct (refers to test 1) ... that is the theorem that we're talking about (refers to 2.7) ... so are we applying this theorem over there?

Sonia: No ... (giggles)

Researcher: Why not?

Sonia: Yes, we are applying it, but ... once again its not touching the circumference (refers to Q8).

Researcher: So its not the angle in the other segment ... Can you talk me through Q9. You said false and then x ... and then something went astray ... You just gave up.

Sonia: Isn't there a clearer one (she had scribbled all over her sketch).

Researcher: You want this? (researcher hands her a clean sheet with sketch).

Sonia: Oh, I had the same one like this in the exams ... What did I say, sir ... It's false.

Researcher: You said false.

Sonia: Am I right?

Researcher: And just x ... Okay lets take you back here. I've said ... I said $x = 25$: Angle between tangent and chord is equal to the angle in the other segment. So what I'm saying is, x is 25 degrees because I'm using that (refers to 2.7) theorem ... And what you need to tell me is ... Am I doing that correctly? (pointing to 2.7) ... And you decided there ... false.

Sonia: Okay, I took it this way: if this is 25 degrees (refers to B) ... then how can this be 25 degrees (refers to x). It can't be.

Researcher: So it can't be?

Sonia: Umm ... I took it literally.

Researcher: But its confusing?

Sonia: Yes.

Researcher: ... (explains the solution to the student) ...

Sonia: Okay.

Researcher: Okay, Q11. You said its true. My reason was: Angle between tangent and chord is equal to angle in the opposite ... segment. So I'm using again that particular theorem. And you agreed with me, that's why you said true.

Sonia: I'm thinking now. Is it this one (pointing to 2.7) ... because there (refers to 2.7) it is not on the outside.

Researcher: Its not on the outside. Okay, so it would be false. So if x was inside there (refers Q11) and S was on the circumference, then it would be okay.

Sonia: Yes.

Researcher: Okay, fine. Q13. You said true. That means you agreed with my statement?
SIREN GOES IN BACKGROUND
... Can you hear us? (speaks to video-operator)

Sonia: I just thought, here was a common point (referring to Q) ... but its bisecting the circle ... (means cutting the circle)

Researcher: Okay, so RQ is not a tangent. Is that what you're saying?

Sonia: Umm ...

Researcher: Or let me ask. What is a tangent? What do you understand by a tangent?

Sonia: Its ... I want to draw it.

Researcher: Ja ... here ... (points to the page)

Sonia: (she draws) ... Here's the circle here. Then here's a line going (draws line under circle) ... only touching at one point.

Researcher: Okay ... so that's a tangent. So PQ will be a tangent?

Sonia: Yes.

Researcher: And QR ?

Sonia: No, I don't think so, because its not touching the ... (points to circumference of circle) ...

Researcher: That's right ... so then that would have been ...

Sonia: ... False

Researcher: False ... and the reason ... Because one is not a tangent. Okay, that's good. For Q14, you said: x is equal to 34 degrees. Can you explain to me? How did you go about solving that one?

Sonia: I took it that the exterior angle is equal to the opposite interior angle, so 78 degrees ... I just took it that that is equal to that inside, okay, then divided by two, because it can't just be equal to the one side.

Researcher: Okay, and you happy with the reason? The exterior angle is equal to the interior opposite. Right ... no. 16. You said true.

Sonia: That's an ugly one (giggles). Let me see now ...

Researcher: ... So by saying true, you obviously agreed with what I said there. $x = 57$ degrees: the exterior angle of a cyclic quad is equal to the interior opposite angle. Is that true?

Sonia: I think so (giggles) ... No, sir its not ...

Researcher: Try it?

Sonia: Umm ... if this was my exterior angle, wouldn't this be my opposite interior angle (refers to given 57 and angle S).

Researcher: Okay ... the question of course is ... is that the exterior angle of a cyclic quad?

Sonia: Ohh ... No!

Researcher: Why not?

Sonia: I would say, this is my exterior angle (refers to UTV), because ... or it could be an exterior angle, then it would be outside, here, or here (refers to angle at T and V) ...

Researcher: So you would agree that that's a headache.

Sonia: Yes.

Researcher: Okay, now let me just show you ... (researcher explains the solution) ...

Sonia: Okay.