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EVIDENCE OF RETURN PREDICTABILITY ON THE
JOHANNESBURG STOCK EXCHANGE

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Prepared under the supervision of Dr. Francois Toerien
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for the degree of

DOCTOR OF PHILOSOPHY

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Abstract

We investigate return predictability on the Johannesburg Stock Exchange (JSE) with a particular emphasis on (a) the incidence and nature of linear and nonlinear serial dependence underlying the return generation process and (b) the consistency of return predictability between a stable and market crisis period.

A data set comprising daily share and index returns is adjusted for thin trading and examined for both linear and nonlinear serial dependence. The range and scope of the tests applied are more comprehensive than has previously been conducted on the South African market and this study is the first to apply the battery of tests employed in the nonlinear examination of the JSE. We find that there is evidence of both linear and nonlinear serial dependence in share and index returns on the JSE but that these dependencies are episodic in nature. The return generation process is therefore characterized for the most part by extended periods of pure white noise interspersed with periods of return predictability. This may complicate the exploitation of the observed predictability in returns. The results suggest further that periods of nonlinear serial dependence arise more frequently than periods of linear serial dependence. We construct Autoregressive (AR) and Autoregressive Moving Average (ARMA) models and compare their forecasting ability to those of GARCH-in-mean and Markov Switching nonlinear models and find that the nonlinear models have lower forecast errors than the linear models during the stable market period. This suggests that there are potential forecasting gains to be made by enhancing linear models with nonlinear innovations. Over the market crisis period we find that the forecast performance of the nonlinear models is worse than that of the linear models, which suggests that the benefit of the nonlinear treatment of conditional volatility is diminished over this period.

We further examine the relationship between share returns and a range of firm characteristics. Ordinary Least Squares (OLS) univariate regressions of monthly stock returns against fundamental firm characteristics provide evidence that all anomalies identified in the prior literature on return predictability on the JSE are significant during the stable period of our sample. Over the market crisis period we find that only the cash flow-to-price variable remains a significant predictor of share returns. In order to assess whether the observed predictability in returns is practically exploitable we construct multivariate linear factor models using a stepwise procedure with varying criteria. We find that not only do portfolios formed on the basis of the predictions of these linear factor models outperform the All-Share Index (ALSI) during the stable market period, they also outperform during the out-of-sample market crisis period. We adjust the returns of the portfolios for transaction costs and find that they continue to outperform the ALSI over both periods at a transaction cost of 20bsp per (one-way) trade but underperform over the market crisis period at higher transaction costs.

Declaration

I, Ryan Kruger, hereby declare that the work on which this thesis is based is my original work (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, is being, or is to be submitted for another degree in this or any other University. I empower the University to reproduce for the purpose of research either the whole or any portion of the contents in any manner whatsoever.

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1. Introduction

The degree to which share returns can be predicted has long been the subject of intense academic debate. Early tests of share returns indicated that observed price changes are random, and the behaviour of share price series was therefore said to be a “random walk”¹. Due to this random behaviour, share returns should not be predictable. The so-called random walk theory formed the basis for the pioneering work of Fama (1965b) and Samuelson (1965), who extended the earlier unrecognized theories of Bachelier (1900) into a theory of pricing applicable to efficient markets. The Efficient Market Hypothesis (EMH) provides a framework for understanding the relationship between information and asset pricing, and describes whether it is possible to predict returns on investments given the state of information efficiency in the market. Where the information efficiency in a market is high (in other words, where share prices react quickly to new information), the hypothesis suggests that active management, the practice of selectively picking shares which are expected to outperform with the goal of beating the average market return, is a wasted exercise.

Recent developments in behavioural theory have suggested, however, that the assumptions of investor rationality which underpin the EMH may be invalid. In addition to this, while it is assumed that most developed markets are at least semi-strong efficient, there are numerous markets, particularly those of emerging economies, which are at best weak-form efficient. This would suggest that there are opportunities for earning abnormal returns in such markets on the basis of informational inefficiencies.

The EMH is closely related to the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and the Arbitrage Pricing Theory (APT) of Ross (1976). In an efficient market, both the CAPM and APT models suggest that assets should be fairly priced in equilibrium. It should therefore not be possible to earn abnormal returns on assets. Subsequent to the acceptance of the CAPM and APT models as sound models of asset pricing, however, a number of anomalies have been identified in the literature which demonstrate significant predictive relationships with returns and allow for abnormal returns to be earned. These anomalies may either indicate that markets may not be as efficient as previously believed or that there are risk factors that are not captured by the CAPM.

Proponents of the CAPM have argued that the potential abnormal returns documented in these tests are not large enough to provide profitable trading opportunities once adjustments have been made for transaction costs. Perhaps more importantly, however, they argue that such anomalies are unstable over time and may indeed be sample-specific. This view is supported by studies of structural change in markets which find that the power of a range of predictive variables is influenced by structural breaks in the data series, resulting from regime shifts in the market. Jensen (1978) has argued that unless it is possible to

¹ See Kendal (1953), Fama (1965) and Malkiel (1973)

exploit such anomalies profitably on a consistent basis, they do not represent a meaningful invalidation of the EMH.

There is evidence of the existence of such anomalies in South Africa, as documented by van Rensburg and Robertson (2003) and Auret and Sinclair (2006) amongst others. All prior tests in the South African market have been conducted over relatively stable market periods or have employed data sets including data both prior to and post market turmoil, such as the Asian crisis of 1997. In addition, the earliest tests were conducted on constrained samples due to liquidity concerns or limited availability of data. As yet, no studies have isolated periods of market upheaval and sought to establish whether the factors identified as significant in predicting returns in the prior literature are consistent over such periods. The recent period of market instability, precipitated by the subprime mortgage crisis in the U.S., provides an ideal opportunity for examining the predictability of returns in the South African market not only during a stable period prior to the crisis but also during a period of significant upheaval in the market.

More recent studies on return predictability have focused on non-linear relationships in share returns. There is strong evidence for the existence of such nonlinear relationships and it has been suggested that further research is required to better identify the kind of models that best capture these non-linear relationships. Appiah-Kusi and Menyah (2003) and Mlambo et al. (2007) have argued that emerging markets, and African markets in particular, may exhibit nonlinear behaviour given their onerous regulatory constraints, liquidity concerns and high transaction costs.

The goal of this study is to examine the predictability of returns in the Johannesburg Stock Exchange (JSE). The approach taken is analogous to testing for market efficiency, with both tests of serial dependence in returns (weak-form efficiency) and an examination of the predictability of returns on the basis of firm characteristics (semi-strong form efficiency) performed.

Firstly, we examine share returns on the JSE for evidence of nonlinear serial dependence as suggested by Appiah-Kusi and Menyah (2003) and Mlambo et al. (2007). To date the work that has been conducted in this regard on the South African market has been limited and prior studies of nonlinear dependence have traditionally focused on stochastic volatility using ARCH and GARCH models. In order to provide as comprehensive a study of the existence of nonlinear serial dependence on the JSE as possible, we employ the battery of tests proposed by Ashley and Patterson (2000). As a secondary goal of the analysis, we attempt to determine whether any evidence of serial dependence is consistent over the sample period or whether they are episodic in nature, using the methodology of Hinich and Patterson (2001). The former has implications for the kind of model to fit to the data while the latter provides an indication of whether such models are likely to perform consistently over time. To the author's knowledge, this is the first time that either methodology has been applied to the South African market. In order to support the findings

of the serial dependence tests we construct univariate linear and nonlinear models and compare their relative forecasting abilities over both a stable period and a market crisis period.

Next, we examine the so-called style anomalies identified in prior literature. It has been found that a number of firm characteristics demonstrate a significant relationship with share returns. We conduct multivariate linear tests in order to establish whether the anomalies identified in prior studies persist over our sample period. Our regression-based approach is similar to prior studies such as van Rensburg and Robertson (2003). However, we provide additional tests for the consistency of factor payoffs, given that Haugen and Baker (1996) suggest that this has implications for whether such anomalies can be exploited profitably over time. Of particular interest are those anomalies, if any, which persist even during the unstable market period as the identification of these factors would improve the robustness of predictive models over periods of market instability.

Missingness in data has serious implications for the results of regression analysis in particular and the majority of prior studies fail to account for this bias or, alternatively, correct their samples on an ad hoc basis using mean imputation or listwise deletion. As the data set employed in this study is not complete we employ a multiple imputation technique to examine the impact of missingness on the results and contrast these findings with those obtained after adjusting the data using listwise deletion.

In order to establish whether any anomalies identified in our multivariate testing allow for abnormal profits to be generated we construct multivariate linear factor models similar to those of Haugen and Baker (1996) and Frankish (2004). The models are optimized for a range of criteria using a stepwise procedure and compared to the All Share Index (ALSI) of the JSE over both the in-sample and out-of-sample periods. Frankish (2004) compared only the predictive power of the stepwise models constructed in this manner but did not examine their profitability. We extend this work in two ways. Firstly, we vary the estimation parameters of the models in order to assess their impact on the predictive power of the models constructed. Secondly, we assess the profitability of these models by constructing portfolios on the basis of model predictions in a manner similar to that of Haugen and Baker (1996). We vary the rebalancing period of the portfolios and also adjust their returns for transaction costs in order to assess the impact of portfolio turnover and the associated costs thereof on the practical returns achievable by these strategies.

The remaining chapters of the thesis are structured as follows. **Chapter 2** provides a review of the theory and prior literature on return predictability and asset pricing. **Chapter 3** discusses the data employed in the study as well as the transformations and adjustments made in order to correct for bias. **Chapter 4** provides an analysis of linear and nonlinear serial dependence in share returns on the JSE while **Chapter 5** compares the performance of constructed univariate linear and nonlinear models. **Chapter 6** conducts

multivariate linear regression analysis of potential anomalies on the exchange. **Chapter 7** details the construction of multivariate linear regression models which are used to construct investment portfolios whose returns are compared to the ALSI. **Chapter 8** concludes.

2. Market Efficiency and Return Prediction

This chapter introduces the concept of an efficient market and the theories of asset pricing that have originated from this early framework. We seek to provide perspective on the development of these theories from the early work of Fama (1965) and Samuelson (1965) through the recent literature on anomalies and evidence of market inefficiencies that have called these foundational models into question. It is the latter studies that are the focus of this thesis and we critique their findings in light of past and current financial theory and attempt to provide a background from which their relevance can be measured in the South African context.

2.1 Market Efficiency

The concept of an efficient market was first proposed by Bachelier (1900) whose work went largely unrecognised until it was returned to prominence in the 1960s through the efforts of Eugene Fama (1965a and 1965b) and Paul Samuelson (1965). Fama (1965b) argued in favour of the random walk model of stock prices and formalized the efficient market hypothesis (EMH), defining an efficient market as one in which prices fully reflect all available information. According to the random walk model, successive stock price changes are independent of each other, i.e. there should be no evidence of serial correlation in stock returns. Given that stock prices reflect all available information, stock price changes are therefore the result of the integration of new information into a stock's price. Such new information is, however, unpredictable and as such the resulting price changes must be both unpredictable and random. Evidence of unpredictability in stock price returns is therefore indicative of market efficiency. Within this framework the market price of a share should be an accurate reflection of its intrinsic value and all investments must have a zero expected net present value at the time they are made, i.e. there is no "free lunch" in an efficient market.

Fama (1970) defined three forms of market efficiency: weak-form, semi-strong and strong-form efficient. Weak form efficiency suggests that current stock prices reflect past prices and investment strategies based on historical information, such as technical analysis, are therefore futile. Semi-strong-form efficiency extends stock prices to reflect currently available public information in addition to historical data. At this level of efficiency, fundamental analysis is no longer exploitable. Finally, strong-form efficiency asserts that stock prices reflect all public and non-public information. It is generally accepted that strong-form efficiency is an extreme case with little empirical merit due to the potential for profiting from inside information and Fama (1970) suggests that its primary purpose is to serve as a benchmark for which to measure deviations away from an absolutely efficient state.

2.2 Asset Pricing

Asset pricing theory provides a framework for valuing assets which generate an uncertain stream of cash flows in an efficient market. The lynchpin of asset pricing theory has for many years been the Capital Asset Pricing Model (CAPM), first proposed by Sharpe (1964), which provides a framework for determining equilibrium expected returns on risky assets.

2.2.1 The Capital Asset Pricing Model (CAPM)

The development of the CAPM owes much to the work of Markowitz (1952) in portfolio theory and Tobin (1958) in developing the foundation of the so-called separation theorem.

While investors have always been intuitively aware of the benefits of holding a diverse range of assets², Markowitz (1952) was the first to quantify the relationship between risk and return, as it related to portfolios of assets. Markowitz provided a mathematically tractable model for measuring the impact of diversification on a portfolio of assets and, contrary to the prevailing thinking of the time, established that there is a limit to the reduction in portfolio risk achievable by means of diversification. Markowitz demonstrated that individual asset risk, as measured by the variance of the expected returns of the asset, was less important in the investment context than the contribution of each asset to the variance of the investor's portfolio as a whole. The latter is a function not only of individual asset risk but also of the covariances of the securities within a portfolio.

Markowitz (1952) proposed several assumptions regarding investor behaviour, the most important of which is that for a given risk level, investors prefer higher returns to lower returns and that for a given level of expected return, investors prefer lower risk to higher risk. On the basis of these assumptions, Markowitz suggested that an asset was efficient if no other asset offered the same return for a lower level of risk or had a lower risk for the same level of return. Markowitz suggested that investors were faced with a frontier of efficient assets, each of which minimized risk for given levels of return and were therefore superior to all other asset choices. Given the benefits of diversification in reducing risk relative to return, it was argued that this efficient frontier would be dominated by efficient portfolios rather than individual assets. Prior to this contribution to portfolio theory securities were evaluated in isolation on the basis of their relative risk and return functions.

Tobin (1958) extended the work of Markowitz (1952) by introducing a risk-free asset to the question of portfolio allocation and argued that market agents would all hold the same single efficient risky portfolio

² As early as 1738, Daniel Bernoulli argues that it is "...advisable to divide goods which are exposed to some small danger into several portions rather than to risk them all together".

of assets with the highest risk-to-return ratio regardless of risk tolerance. Investors could then adjust the risk of their portfolio to match their degree of tolerance by either lending (borrowing) the risk-free asset in the market and thus lower (increase) the risk of their portfolio. This has become known as the separation theorem. The separation theorem suggests that the portfolio allocation can be separated into an investment decision and a financing decision. The investment decision requires the identification of the efficient risky portfolio which would provide the highest possible return relative to risk. Once this portfolio has been identified (and it will be the same for all investors assuming they face the same opportunity set of assets) investors can adjust the level of risk in their portfolio by allocating their funds between this risky portfolio and the risk-free asset.

While Markowitz (1952) and Tobin (1958) had provided both market practitioners and academics with a framework for constructing efficient portfolios their model was impractical for large portfolios of assets as it required the calculation of the covariance of returns between every pair of assets in the portfolio. In addition, Markowitz (1952) failed to provide a model for expected returns that could be used to generate the efficient frontier of assets.

This dilemma was resolved by Sharpe (1964) who proposed the Capital Asset Pricing Model as a theoretical model of market equilibrium. Assuming a pure rate of interest that is common to all investors and that all investors have homogenous expectations, Sharpe proposed that all investors will hold the same optimal risky asset, the market portfolio, M . M is a portfolio of all risky assets in the market and is the point of tangency between the capital market line (the representation of all possible linear combinations of the optimal risky portfolio and the risk-free rate) and the efficient opportunity set faced by investors. As per Tobin (1958) investors will hold some combination of M and the risk-free rate depending on their risk aversion.

Importantly, Sharpe (1964) highlights that all assets in the market must be in the market portfolio, M , and that where one or more assets offers a higher or lower risk-return relationship than M , market forces will adjust the prices of those assets until they once more enter the market portfolio. What is important in measuring portfolio risk is therefore not the covariance of assets with one another but the covariance of these assets with M . This implies that it is possible to restate the required asset covariances for calculating portfolio standard deviation relative to a pre-specified market portfolio instead, dramatically reducing the number of covariance terms to be estimated. The ratio of the covariance of an asset with the variance of the market is now referred to as the beta of the asset.

Drawing on Markowitz's theories on the limits of diversification, Sharpe (1964) proposed that there are two elements of risk: specific or unsystematic risk which is unique to the asset in question and can thus be diversified away and non-specific or systematic risk which is common to all assets within the market. As

systematic risk is determined by general factors in the market over which the investor has no control, it is impossible to influence this aspect of asset risk. As the market portfolio, M , comprises all assets in the market it must be efficiently diversified and therefore represent purely systematic risk. As such, the CAPM suggests that the beta of an asset is a measure of systematic risk, in so far as it measures the risk of an asset relative to the systematic risk of the market portfolio.

The CAPM proposes that the expected returns on an asset are linearly related to the assets exposure to market risk, as measured by beta.

$$E(r_i) = r_f + \frac{\text{Cov}(r_i, r_m)}{\sigma_m^2} [E(r_m) - r_f] \quad (2.1)$$

Or

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f] \quad (2.2)$$

where $E(r_i)$ is the expected return on asset i , r_f is the return on the risk-free asset, β_i is the beta of asset i relative to the market portfolio, m , and $E(r_m)$ is the expected return on the market portfolio.

Equations (2.1) and (2.2) suggest that the expected return on an asset are based on the risk-free rate in the market as well as a premium for bearing risk. This premium is determined by the asset's relative exposure to systematic risk, as measured by β_i . Under conditions of market equilibrium all assets should be fairly priced according to equation (2.2). The relationship between risk and return for all assets should therefore be the same and as a consequence it should not be possible to outperform the market portfolio.

While the CAPM provides a parsimonious framework for determining expected asset returns it is flawed in some material respects. Firstly, it requires a set of simplifying assumptions that do not necessarily hold in practice. In particular, the CAPM assumes that investors have quadratic utility functions and that the distribution of asset prices is normal – the latter assumption contradicts empirical evidence that securities prices tend to have leptokurtic distributions. Secondly, it presupposes the existence of a measurable market portfolio that comprises all assets traded in the market, both public and private. While such a portfolio does exist it is not practically possible to either identify or measure its composition and proponents of the CAPM are forced to employ proxies, typically broad market indices for which the constituents are readily available, in an attempt to approximate the market portfolio. The use of such proxies in both the pricing of assets and testing of the CAPM results in the so-called joint hypothesis problem. Where it is found that the CAPM has failed to price assets correctly we cannot say with any certainty whether the fault lies with the specification of the model or the choice of proxy for the market portfolio.

2.2.2 Arbitrage-Pricing Theory (APT)

Contrary to the CAPM which develops a pricing mechanism for assets on the basis of the relationship between risk and return, the APT, proposed by Ross (1976), suggests that assets should be priced fairly due to the possibility of arbitrage. The model has fewer restrictive assumptions than the CAPM, and does not require the specification of a market portfolio, nor does it assume a quadratic utility function for investors or that asset returns are normally distributed.

The CAPM uses a risk-return dominance proposition to ensure fair pricing of assets. Where the equilibrium risk-return relationship is violated, each investor in the market takes a limited position in either the mispriced asset or the market portfolio depending on their risk aversion. Under the APT, mispricing requires only a small number of investors to restore equilibrium prices due the possibility of arbitrage enabling investors to take a riskless, costless position to take advantage of the mispricing.

This arbitrage mechanism ensures market equilibrium.

The APT model assumes that the stochastic process underlying the generation of asset returns can be modelled by a linear K-factor model of the form

$$R_i = E(R_i) + b_{i1}F_1 + \dots + b_{ik}F_k + \varepsilon_i \quad (2.3)$$

where R_i is the realised return on asset i , $E(R_i)$ is the expected return on asset i , b_{ik} is the factor loading for asset i on factor k , F_k is a common risk factor with a zero mean that influences the returns on all assets and ε_i is an unsystematic component of returns that is completely diversifiable (unsystematic) and has a mean of zero. It is assumed that these unsystematic components of return are uncorrelated across assets and uncorrelated with the risk factors.

The APT assumes that for portfolios of large enough size, the idiosyncratic element of return, ε_i , will be diversified away. Asset returns are therefore fully explained by their loadings on the common systematic risk factors. We can therefore restate the expected returns on an asset in the context of the risk premia associated with the risk factors such that

$$E(R_i) = r_f + b_{i1}\delta_1 + \dots + b_{ik}\delta_k \quad (2.4)$$

where $E(R_i)$ is the expected return on asset i , b_{ik} is the factor loading for asset i on factor k , δ_k is the risk premium on the common factor k and r_f is the return on the risk-free asset.

Although the APT provides an explanatory model of asset returns that is not constrained by the identification of a market portfolio it does not itself identify the priced risk factors that influence asset returns. The identification of these factors is also complicated by the fact that it is likely that the relevant factors change over time as market conditions evolve.

Most early empirical tests of the APT identified the number of priced risk factors by analysing the estimated covariance matrix of equity returns. Roll and Ross (1980) and Chen (1983) employed factor analysis in order to identify the number of priced factors in the return generation process for their respective data samples, while Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986) used principal component analysis. These early approaches were, however, unable to identify the nature of the priced factors identified.

More recent studies have attempted to identify not only the number of priced factors but also their nature. The most popular approach entails the researcher selecting a range of candidate factors on the basis of perceived suitability, estimating the factor loadings for this factor set and then determining which factors (if any) explain the cross-sectional variation in expected returns. Chen, Roll and Ross (1986) employ a set of financial and economic variables to serve as proxies for the factors underlying the return generation process on U.S. stocks. They find that the long and short interest rates, expected an unexpected inflation, industrial production and the spread between high- and low-grade bonds are all priced factors over their sample period covering 1953 to 1973. Shanken and Weinstein (2006) have suggested subsequently, however, that such models lack robustness because even small changes in the sample period or factors tested have a significant impact on the results.

There is a significant volume of prior literature related to the testing of the APT on the JSE. Campbell (1979), Gilbertson and Goldberg (1981), Page (1986) and Biger and Page (1993) all found evidence of multiple priced factors over different sample periods indicating the appropriateness of a multi-factor APT.

Subsequent to this, Page (1993) conducted simulation tests of artificially-generated one, three and five factor economies. His findings that neither principal factor analysis nor principal components analysis were able to distinguish between these economies in the presence of simulated thin trading and 'market microstructure effects' led him to conclude that commonly employed APT procedures lack power. Attempting to address this concern, van Rensburg and Slaney (1997) attempted to find observable macroeconomic proxies for the factors identified under their factor analysis. van Rensburg and Slaney (1997) applied principal factors, principal components and maximum likelihood factor analysis to both the correlation and covariance matrix of monthly index returns over a ten-year period. The results suggested that a two-factor APT model encompassing a 'gold/mining' and an 'industrial' factor was appropriate for the JSE.

In addition, following regression analysis of both market and two-factor models, van Rensburg and Slaney (1997) found strong evidence for the contention that the APT is more appropriate than the CAPM for pricing assets on the JSE. These findings were confirmed and updated in a subsequent study by van Rensburg (2002).

2.3 Tests of Weak-form Efficiency

As indicated in Section 2.1, weak-form efficiency posits that there is no relationship between historical and current asset returns thereby invalidating investment strategies that rely on patterns in price data. Tests of weak-form efficiency have therefore focused on testing the independence of returns over time and the success or failure of so-called trading rules or technical analysis. Our focus in this study is on tests of serial dependence.

2.3.1 Tests of Linear Serial Dependence

2.3.1.1 International Evidence

Kendall (1953) examined the price series of 22 UK stocks and commodities and found a near-zero serial correlation in the series. He concluded that the observed price changes were random and likened the behaviour of the data to a “wandering series”. This early empirical observation came to be labelled the ‘random walk model’. Kendall’s findings were subsequently supported by Roberts (1959) and Osborne (1959) who both found evidence of randomness in the price series of US stocks. Fama (1965) similarly found no evidence of serial dependence in the stock price changes of the 30 constituents of the Dow-Jones Industrial Average over the period January 1956 to September 1962.

In contrast, Fama and French (1988) examined serial correlation in US stock returns over various holding periods for a sample covering 1926 to 1985. They found weak evidence of autocorrelation for daily and weekly holding periods but found evidence of negative serial correlation in long holding-period returns for US stock returns. They also established that the predictable variation in returns is greater for small stocks than for larger stocks.

Lo and MacKinlay (1988) found evidence of positive short-run serial correlations in weekly US stock returns for the period September 1962 to December 1985. Although they rejected the random walk hypothesis they stressed that tests for independence do not themselves provide an indication of the stochastic process which best fits the data. Their findings were supported by Conrad and Kaul (1988).

Jegadeesh and Titman (1993) found evidence of short-term momentum effects in which short-term performance persists over time. Behavioural finance has suggested that such short-term momentum effects are consistent with investor underreaction to new information (see Barberis, Shleifer and Vishny (1998). Malkiel (2003) argued, however, that while statistically significant, such dependencies are not economically significant due to the significant trading costs associated with their exploitation. He proposed further that such dependencies are not stable over time and many disappear shortly after being brought to light, either due to the results being sample-specific or due to market correction in light of the new information.

In contrast, long-term negative serial dependence in stock returns has been documented in numerous studies³. DeBondt and Thaler (1985) argued that this is consistent with overreaction on the part of investors which causes stock prices to deviate significantly from intrinsic values. These deviations are subsequently corrected in the market, giving rise to mean reversion in returns. These findings were confirmed by Chopra, Lakonishok and Ritter (1992) even after adjusting for market risk and the size effect.

2.3.1.2 South African Evidence

Evidence of weak-form efficiency on the JSE has been mixed. Affleck-Graves and Money (1975) examined the weekly returns of a sample of 50 Industrial shares for evidence of serial dependence. Their findings indicated that the observed autocorrelations in return were insignificant and concluded that the JSE was at least weak-form efficient. In contrast, a subsequent study by Hadassin (1976) on the daily share returns of 33 industrial shares over the period January 1971 to December 1973 demonstrated significant evidence of serial correlation in returns, leading to the conclusion that the JSE is an inefficient market. Gilbertson and Roux (1977) critiqued both studies and suggested that their conclusions had been premature based on limitations in their methodologies. They concluded that while there is evidence of serial dependence in share returns on the JSE, these dependencies are too small to be exploited and there is therefore not sufficient evidence to reject the EMH.

Strebel (1977) suggested that market efficiency on the JSE is limited to actively traded shares and that the extensive thin trading which pervaded the JSE over the periods examined in the above studies rendered their findings inconclusive.

More recently, Mabhunu (2004) examined the weekly closing prices of Industrial shares as well as the All-Share index and Top 40 indices over the period January 1999 to July 2003 for evidence of autocorrelation. It was found that while a third of the sample exhibits significant autocorrelation, these are found only at

³ See Greene and Bruce (1977), Lo and MacKinlay (1988) and Poterba and Summers (1988) amongst others.

the first lag and no relationship is found between the characteristics of these firms. The author concluded that the JSE is an efficient market, due in large part to improvements in trading efficiency as a consequence of the introduction of the Johannesburg Equities Trading (JET) system.

2.3.2 Tests of Non-Linear Serial Dependence

Although the focus of most of these studies on return predictability was linear in nature there is a growing body of research that suggests that due to the interaction of noise and arbitrage traders, the return-generation process is in fact characterised, at least in part, by higher-order processes, necessitating different treatment for large and small stock returns (McMillan, 2002). Where stocks demonstrate such higher-order serial dependence they are said to exhibit nonlinear behaviour. Appiah-Kusi and Menyah (2003) argued that emerging markets in particular might exhibit nonlinear tendencies due to their propensity for thin trading, high transaction costs and regulatory constraints. This view is supported by Mlambo et al (2007) who suggest that linear models could lead to incorrect inferences in return modelling for African stock markets and that further testing of market efficiency and return predictability in these markets should incorporate nonlinear tests.

Tests for nonlinear dependencies in share returns employing the Brock, Dechert and Scheinkman (BDS) test found strong evidence of nonlinearities in both developed and emerging markets (see Hsieh, 1991, Abhyankar, Copeland and Wong, 1997, De Lima, 1998, and Mangani, 2007, amongst others). Hsieh (1991) applied the BDS test to a sample of U.S. weekly stock returns over a 24-year period. After filtering the data by autoregression the independent and identically distributed (i.i.d.) null hypothesis was rejected both for the overall returns as well as the decile portfolios. Importantly, Hsieh (1991) tested whether structural change or conditional heteroscedasticity is responsible for the rejection of the null hypothesis and found no evidence for the impact of the former. However, it was found that conditional heteroscedasticity is the primary cause of rejection of the hypothesis that the residuals of linear ARMA models are i.i.d. Hsieh (1991) thus concluded that, when one tests for nonlinearity in stock returns, the focus should be on conditional heteroscedasticity rather than conditional mean changes.

De Lima (1998) applied a recursive BDS statistic to two indices over a 10-year period. Results strongly rejected the i.i.d. null hypothesis and indicated the presence of nonlinearities in both data sets. Contrary to the findings of Hsieh (1991), however, tests of a subsample comprising the period leading up to the stock market crash of 1987 were unable to reject the null hypothesis, indicating that the crash represents a significant structural change in the data set. This led De Lima (1998) to conclude that such structural shifts might cause one to overstate the evidence for the impact of conditional heteroscedasticity on the findings.

Concerns have been raised that most tests for nonlinear dependence require large data sets which necessitates the use of data sets spanning periods of years, over which it is unlikely that the underlying structure remained stationary. Abhyankar, Copeland and Wong (1997) thus employed high-frequency real-time data sets comprising 10,000 observations that spanned only 3 months in testing for the presence of nonlinear dependence in real-time returns for the S&P 500, DAX, Nikkei 225 and FTSE-100 indices. Results from both BDS and Lee, White and Granger (LWG) neural-network based tests indicated persistent nonlinear dependence in all series investigated. They suggested that the rejection of the i.i.d null hypothesis might be explained in part by volatility clustering.

A study by Ashley and Patterson (2000) attempted to match a series of tests for nonlinear dependencies to nonlinear models of best fit for a range of data sets. Their hope was that in making such connections, tests which found significant nonlinearities in a given data set would point to the most appropriate nonlinear models to fit to the data. Their findings indicated that the BDS test provides the greatest overall power of the tests examined, being the best indicator for data generated by ARCH, Markov Switching, quadratic and cubic processes and demonstrating significant explanatory power for all other processes. The McLeod-Li and Engle LM tests demonstrated significant explanatory power for ARCH/GARCH processes as expected, while the Tsay test was superior for TAR processes. The Hinich and Patterson bicorrelation test demonstrated significant explanatory power for cubic tests but was poorly suited to nonlinearities generated by switching processes (TAR and Markov-switching models). The Hinich Bispectral test in contrast failed to demonstrate significant power for any of the nonlinear processes examined. The BDS test therefore appears to be most suited as a general test for the presence of a nonlinear data generation process but the findings also indicate that such a nonlinear process cannot easily be matched to any particular underlying model. Brooks and Hinich (2001) suggest that this is the reason for the lack of success in applying nonlinear forecasting models to economic data in recent years.

Recent tests for nonlinear dependencies in contrast have focused on the episodic nature of nonlinear dependencies in high frequency returns data, employing the methodology of Hinich and Patterson (2001). The data sample is separated into windows or frames to which bootstrapped portmanteau tests for linear and nonlinear dependencies referred to as the C and H statistics, respectively, are applied. The C test is a modified Box-Pierce portmanteau test statistic for linear dependence and the H test is derived from the sample bicorrelation function developed in Hinich and Patterson (1995). Studies utilizing this methodology (see Hinich and Patterson, 2001, Skaradzinski, 2003, Lim and Hinich, 2005, and Bonilla, Romero-Meza and Hinich, 2006) have established that the occurrence of nonlinear dependencies in share returns data are episodic in nature such that the data follows a switching process between long periods of white noise and occasional short-lived periods of either linear or nonlinear dependence. Lim and Hinich

(2005) suggest that failing to account for this sporadic nature of nonlinear behaviour may in fact be the cause of the failure of nonlinear modelling to date.

Skaradzinski (2003) extended the investigation of episodic nonlinear dependencies by examining the relationship between incidences of nonlinear dependencies and firm size, finding that large-cap US shares exhibit higher average H statistics, followed by mid-cap and small-cap shares. In addition, evidence of nonlinear dependence increased over the decade-long period examined, with the average H statistic increasing over time and a greater incidence of significant nonlinear windows found in later years. This agrees with the findings of Hinich and Patterson (2001) who found that, while the results of their battery of tests did not always agree (the BDS and Bispectral tests occasionally failed to reject the null), evidence of nonlinear dependence in S&P 500 returns increased steadily over the 35-year period they investigated. Skaradzinski (2003) also investigated the relationship between instances of nonlinear dependence and specific calendar periods. There was little evidence of a “calendar effect” for nonlinear dependencies, except for the month of December which demonstrated consistent significant nonlinear dependency for small-cap shares.

Finally, Mangani (2007) investigated the distributional properties of 42 randomly-sampled FTSE/JSE ALSI shares and tested for nonlinear dependencies using the BDS test. He found that the i.i.d assumption could not be accepted in 96% of tests conducted. To date this represents the only study of nonlinear dependencies within the South African market.

2.4 Tests of Semistrong-form Efficiency

The focus of this study is on the predictability of returns rather than market efficiency. While evidence of the former serves to invalidate the latter, the examination of market efficiency is not a direct goal of our analysis. Our review of the literature on semi-strong efficiency therefore focuses on tests of predictability in returns and we ignore those event-based studies which examine the market’s reaction to new price-relevant information.

While early tests of return predictability seemed to support theories of market inefficiency, Ferson and Harvey (1991), suggested that the predictability of asset returns is not due to inherent informational inefficiency within markets but due to the predictability of the variables which, in part, inform asset prices. Fama (1991) suggests that one of the primary avenues of research to be pursued in return predictability is the link between returns and business conditions, specifically consumption, investment and saving.

As indicated in Section 2.2.1, the CAPM posits a positive relationship between beta and asset returns. Recent evidence, however, has indicated that the CAPM beta either has no relationship with average share returns or is subsumed by various anomalies identified in the literature⁴. Indeed, a study by van Rensburg (2003) on the South African market found a negative relationship between beta and asset returns. Strugnell et al. (2010) subsequently corrected the sample employed by van Rensburg for thin trading bias and found that the previously observed relationship between beta and return is unchanged.

It has been well-established in the literature that there are certain firm characteristics (often referred to as styles or style anomalies) that appear to be proxies for risks not captured by the traditional CAPM model. The most prominent of these include firm size (see Banz 1981, Reinganum 1981, Fama and French, 1992, and Daniel and Titman 1997) dividend yield (see Litzemberger and Ramaswamy, 1979 and Fama and French, 1988), price-earnings ratios (Basu, 1977) and book-to-market ratio (see Rosenberg, Reid and Lanstein 1985 and Fama and French 1992). Many of these factors have been demonstrated as being significant across markets indicating that there is evidence of global commonality and consistency in these drivers of risk (see Haugen and Baker, 1996).

2.4.1 International Evidence

Extensive evidence can be found both internationally and locally for the positive relationship between returns and measures of firm value. Haugen and Baker (1996) define these measures as being indicators of the relative cheapness of stocks. Internationally, significant value anomalies have included price-to-earnings ratio, book-value-to-market value ratio, dividend yield, cash flow-to-price ratio and sales-to-price ratio.

Basu (1997) examined the relationship between price-earnings ratios and forecast stock returns for 1400 US stocks over the period 1956 to 1971 and observed that stocks with low price-earnings ratios outperformed those with high ratios.

Stattman (1980) and Rosenberg, Reid and Lanstein (1985) found a positive relationship between US stock returns and book-to-market ratio. Fama and French (1992) observed a similar relationship between size and the book-to-market ratio of US stock returns over the 1963 – 1990 period. Their results were robust to the inclusion of other anomalous characteristics and they concluded that size and book-to-market ratio subsume the leverage and earnings-to-price anomalies identified in earlier studies⁵.

⁴ See Fama and French (1992) for further reading.

⁵ See Bhandari (1988) for a discussion of the leverage effect and Basu (1977) for further reading on the E/P effect.

Fama and French (1993) constructed portfolios to mimic size and book-to-market factors and found that these factor-portfolios, when combined with a CAPM market factor, captured the cross-section of average returns for U.S. stocks. Subsequent research by Fama and French (1993 and 1996) found that when adjusting returns for risk using their 3-factor model, anomalies related to size, book-to-market ratio, dividend yield, earnings yield and cash flow to price were no longer significant.

Litzenberger and Ramaswamy (1979) demonstrated that dividend yield has a strong positive relationship with stock returns over long horizons. Their research is supported by Fama and French (1988) who found that the forecast power of dividend yields as measured by R^2 is negligible over short horizons but increases with the return horizon. They suggest that this is due to both high positive autocorrelation in returns and the time variation of expected returns giving rise to temporary components in stock prices.

Kirby (1997) disputed the conclusions of Fama and French (1988). He suggested that the overlapping returns in the data sets employed produced serially correlated errors that are compounded by small sample sizes. This led to difficulties in drawing conclusions as to the fit of the model. Lanne (2002) raised the further issue that most of the forecasting variables used in testing have both a large autoregressive root in their univariate representation and tend to be lagged endogenous variables which can lead to the slope variable being biased. Employing standard linear tests of predictability over the period 1928 to 1996, Lanne (2002) found predictability of returns using the log dividend-to-price ratio whereas the use of the KPSS test for stationarity indicated no predictability of returns using the same forecast variable.

The stability of the relationship between dividend yield and stock returns has also been questioned. Lettau and Ludvigsson (2001) and Goyal and Welch (2003) found that predictive models based on dividend yield were unstable during the bull run of the 1990s while Goetzmann and Jorion (1993) and Ang and Bekaert (2007) found that the predictability of returns using dividend yield is insignificant in the long term.

Evidence of an inverse relationship between stock returns and firm size as measured by market capitalization was first demonstrated in the US market by Banz (1981). He did not provide a theoretical foundation for the relationship and suggested that size might proxy for another, untested, factor that is correlated with firm size. Basu (1983) found that both earnings-to-price and firm size are significantly related to average firm returns in the US but argued that firm size is a proxy for the earnings-to-price effect.

2.4.2 South African Evidence

Evidence of asset pricing anomalies in the South African market has largely corresponded with those in international markets. Early tests of the size effect on the South African market failed to find evidence for

the anomaly (see De Villiers et al, 1986, Bradfield et al, 1988 and Page and Palmer, 1993). Van Rensburg (2003) suggested that this is due to the exclusion of small cap stocks from the samples in these early tests on the basis of thin trading adjustments. The subsequent improvement in liquidity on the JSE has, however, provided the opportunity to revisit these studies. To this end van Rensburg and Robertson (2003) employed both a univariate and paired factor multiple-regression methodology to determine the ability of 24 firm specific attributes to explain average monthly returns on the JSE over the period July 1990 to June 2000. They found that price-to-net asset value (NAV), dividend yield, price-to-earnings ratio, cash flow-to-price and size are all significant explanatory variables of returns. Following multi-factor regression analysis of these five factors they found a two-factor model comprising size and price-to-earnings ratio to be optimal when evaluating the regressions at a 5% significance level.

Auret and Sinclair (2006) selected the five most significant values from the van Rensburg and Robertson (2003) study and added the book-to-market ratio which the former study had neglected. They found that book-to-market ratio completely subsumes the effect of size and price-to-earnings ratio when included in the regression analysis but is not significant at a 5% level. The authors suggested that this is due to the variable's high correlation with the other variables in the analysis, all of which include price in their calculation. As such it was proposed that the size and price-to-earnings factors, which have a low correlation with each other, remain the ideal factors for such an explanatory model of returns.

Basiewicz and Auret (2009) examined a broader selection of firm characteristics for evidence of return predictability. Their methodology deviated extensively from prior research in that they attempted to correct for thin trading and transaction costs and employed independent rather than sequential sorts and both value- and market-weighted compositions in their portfolio tests. Interestingly, the authors found that both size and book-to-market were significant explanatory variables in their regressions, contradicting the finding of Auret and Sinclair (2006) that book-to-market subsumes the size effect, although book-to-market is the most significant of the factors examined. The authors suggested that this is due to the strength of the size factor over the period 2003 to 2005 which was not included in the latter study. The authors also found that the price-to-earnings factor was the weakest of the factors tested and the effect was all but insignificant once the analysis was adjusted for thin-trading and transaction costs. Indeed, including these adjustments reduced the explanatory power of most of the value factors considered.

2.5 Behavioural Explanations for Market Inefficiencies

In recent years the EMH has been criticized by both behavioural economists and empiricists who argue that its dependence on investor rationality is flawed. According to behavioural economists investor rationality presupposes that investors update their beliefs correctly when receiving new information and

make investment decisions that are normatively acceptable. Where one or both of these requirements is not met, it is possible for prices to deviate from their intrinsic value.

Grossman and Stiglitz (1980) go so far as to suggest that market efficiency is an impossibility. Market efficiency is pre-supposed upon the action of market participants in integrating new information into existing stock prices. Such action is not, however, rewarded by the market as efficiently-priced assets provide no possibility of abnormal returns above the fair value of the asset. There is therefore no incentive for market participants to facilitate the gathering and processing of information and the price-adjustment mechanism will fail as a consequence. There must therefore be some inefficiency in the market to incentivise participants to process new information.

The literature provides extensive theoretical cognitive biases that may constrain investor rationality, including overreaction (DeBondt and Thaler, 1985), overconfidence (Fischhoff et al., 1977 and Gervis and Odean, 2001), regret avoidance (Bell, 1982), mental accounting (Tversky and Kahneman, 1986) and herding (Huberman and Regev, 2001).

Proponents of the EMH have argued that the behaviour of irrational investors will be corrected by rational investors who exploit the resultant mispricing in the market using arbitrage. Behavioural theory suggests, however, that opportunities to correct mispricing are not always without cost and risk as the assumed arbitrage correction would suggest⁶.

While the theory of behavioural finance is well-developed, empirical evidence of these inefficiencies is hard to acquire given the limitations in testing cognitive biases in a manner which is both rigorous and quantitative.

2.6 The Impact of Structural Breaks on Return Predictability

It has been suggested that the inconsistency in the findings related to the predictive power of firm characteristics is due to the fact that such significant relationships are frequently artefacts of specific data sets or periods. Such relationships therefore tend to disappear when subsequent periods are tested⁷.

The literature has emphasised that the detection of structural change is of paramount importance in correctly specifying econometric relationships. Clements and Hendry (1998) go so far as to suggest that structural change is the primary source of forecast error.

⁶ These theories are collected under the broad category of “limits to arbitrage”. See Shleifer and Vishny (1997) and Gromb and Dimitri (2002) for further reading.

⁷ See Malkiel (2003) for a discussion on the inconsistency of style anomalies.

Empirical testing has provided significant evidence of instability across a range of predictive variables. Timmermann and Paye (2006) investigated the presence of factor instability in predictive models for a range of factors including lagged dividend yield, short interest rate, term spread and default premium. They established that there is evidence of structural breaks for most countries tested but these breaks are largely specific to each country. Importantly, there is evidence that the relationship between the factors and expected returns may change significantly following a break, indicating the need to accommodate such regime shifts into predictive models.

Ismail and Isa (2008) tested for the presence of structural breaks in Malaysian stock index data spanning 29 years. Bai-Perron and Andrew-Ploberger tests indicated the presence of multiple structural breaks in the data and a Markov Switching Autoregressive Model (MS-AR) was fitted in an attempt to capture the underlying regime shifts. The findings indicated that major economic events such as the 1974 oil shock, 1987 stock market crash and the 1997 financial crisis marked significant structural breaks in the data.

There is also evidence that a failure to accommodate structural breaks into econometric modelling may result in the non-rejection of the non-stationarity hypothesis (Perron 1989 and 1997). Valadkhani et al. (2005) employed the Lumsdaine and Papell (LP) test for the presence of structural breaks in Australian macroeconomic data. Initial Augmented Dickey and Fuller (ADF) tests indicated that none of the ten variables were stationary prior to adjusting for structural breaks. Once stationarity was examined in the presence of two structural breaks, however, four of the variables were found to be stationary. This confirmed the findings of Naryan and Smyth (2004) who also found that the unadjusted ADF test cannot reject the null hypothesis but adjusting for two breaks yields stationary variables for seven out of the sixteen macroeconomic factors examined.

3. Data

We employ two distinct but related data sets in answering the questions set out in Chapter 1. Our tests for linear and nonlinear serial dependence require high frequency data and we thus employ a data set of daily share returns for these tests. Our linear factor models, however, are rebalanced on a monthly basis and as such we employ a data set comprising monthly share returns for that purpose. All returns are collected for constituents of the Johannesburg Stock Exchange's FTSE/JSE Africa All-Share Index (ALSI) series subject to the restrictions set out in sections 3.2.1 and 3.2.2 below.

There are numerous methodological biases which may reduce the robustness of quantitative analysis of the nature to be undertaken in this study. This chapter therefore begins with a discussion of the methodological biases which may be present in the data as a consequence of the manner in which the data was captured. These potential biases are addressed in the next section which discusses the data sets, and the adjustments and corrections applied to each, in detail.

3.1 Methodological Biases

Where firms have been excluded from a data sample due to inactivity or delisting, average returns in the sample will tend to be overstated as those shares that remain will tend to have been stronger performers (for evidence of this see Kothari, Shanken and Sloan (1995) or Brown, Goetzmann and Ross, 1995). This is referred to as survivorship bias and is often encountered in studies of mutual fund returns where poor performing funds are excluded from data samples due to their subsequent failure. Chan, Jegadeesh and Lakonishok (1995) find that financial distress is only infrequently the reason for a share being removed from US data sets. Most removals are in fact due to mergers and acquisitions or firms failing to comply with exchange regulations and the impact of survivorship bias may therefore be overstated for firm samples. This finding is supported by Davis (1994) who examines a data set free of survivorship and look-ahead bias and finds that book-to-market, earnings-to-price and cashflow-to-price all have significant explanatory power over the period 1940-1993. He concludes that the significance of these style attributes is therefore not related to either data snooping or survivorship bias.

Lo and MacKinlay (1995) suggest that where a large range of factors is tested it is inevitable that spurious predictors will be identified. Such factors are unlikely to be significant in ensuing periods but will continue to persist in the literature where subsequent studies are conducted on similar data sets and employ the same factors found to be significant previously. Such data snooping is not always easily identified, however. The data snooping argument has been less robust in recent years as studies have increasingly found commonality in predictive style factors across international markets (see Haugen and Baker, 1996).

Lastly, many data sets are backfilled by data providers to reflect the true values of variables as at specific dates. The use of such data sets can lead to look-ahead bias where such data are employed as predictors even though the data might not have been available to market participants at that time. This is particularly prevalent for earnings figures which are only made available when financial reports are released a few months after the firm's financial year-end, but which may be backfilled to the actual year-end date by data providers. Use of such data will result in a spurious increase in predictive power.

3.2 Data for Tests for Linear and Nonlinear Serial Dependence

Daily share price data were obtained from Thompsons' Datastream for constituents of the Johannesburg Securities Exchange's All-Share Index (ALSI) over the period February 2000 to December 2009. The data was then divided into an in-sample period encompassing February 2000 to July 2007 (the stable period) and an out-of-sample period encompassing August 2007 to December 2009 (the market crisis period). In order to ensure comparability of shares when using the window procedure, shares not listed for the full period were excluded from the sample, yielding a final data set of 106 shares. These shares were further categorized according to size based on their inclusion in one of the JSE's Top 40, Mid Cap or Small Cap indices. The final sample comprised 34 Top 40 shares, 44 Mid Cap shares and 28 Small Cap shares.

In addition, daily total returns data was collected for the Top 40 Index, Mid Cap Index and Small Cap Index over the same periods.

Daily logarithmic total returns were calculated for each share and index as follows

$$R_t = \ln(P_t + D) - \ln(P_{t-1}) \quad (3.1)$$

where P_t is the price of the stock or index at time period t and D is the total dividend paid over the period.

3.2.1 Thin-trading Adjustments

Bowie (1994) investigated the impact of thin trading on the JSE and found that for thinly-traded shares absolute returns realised after long periods of inactivity were higher than those realised over periods of continuous trading. It is therefore necessary to adjust returns to account for price-age over periods of thin trading.

It has also been documented that thin trading can give rise to serial correlation in returns⁸. Observed dependencies in such data may therefore be spurious. We therefore adjust the data to account for these effects and consider the method of Atchison et al. (1987) and the adjusted time-to-trade approach of Mlambo et al. (2003) for this purpose. The Atchison technique equally weights the return after a period over which a stock is not traded, over the first subsequent day of trading as well as the prior non-trading days.

Given that the return after a period of none-trade, k , is given by

$$R_{kt} = \ln(P_t + D) - \ln(P_{t-k}) \quad (3.2)$$

the return on each of the days of zero trade, as well as the subsequent day of trade, are then adjusted to

$$\tilde{r}_t = \frac{1}{k} R_{kt} \quad (3.3)$$

While this method does adjust the data for price age and avoids a loss of observations from the series it fails to address the possibility of spurious autocorrelation within the series.

The method proposed by Mlambo et al. (2003) replaces all non-trade days as well as the return immediately following the period of thin trade by the single adjusted return figure computed in equation 3. All none-trade days are therefore eliminated from the sample. The greater the number of zero trade days in the series, the smaller the number of observations in the series after applying this method. The adjustment therefore controls for both price age and spurious autocorrelation within the series but also results in data series of varying length depending on the degree of thin trade for each share.

It should be noted that even after the above adjustments, a large number of true zero returns (the result of trades at unchanged prices) remain in the data. This is found to occur to a greater extent in smaller-cap shares than their large counterparts.

We quote all results in Chapter 4 for both adjustment techniques to determine whether the choice of thin trade adjustment has a material impact on the findings regarding serial dependence in the returns series.

⁸ See Dimson, 1979, Cohen et al., 1979 and Miller et al., 1994 amongst others.

3.3 Data for Linear Factor Model Construction

The data sample comprises all shares listed on the JSE's All-Share Index (ALSI) over the period February 2000 to December 2009, subject to the constraints discussed below. The data set is partially corrected for survivorship bias such that shares which delisted over the period are included in the sample during the period of their listing. Unfortunately, Datastream provides a limited history for the most thinly-traded delisted South African shares and only limited data could be sourced for these shares. From the available data, these shares demonstrated average turnover ratios of less than 0.001% per month indicating that they would have been removed from the sample in most months when applying the liquidity filter discussed below. It was thus decided to exclude these shares from the analysis as the degree of missingness in the available data was more substantial than the observed values and it was felt that they may skew the analysis as a result. All data are obtained from Thompsons Datastream and are free from look-ahead bias as Datastream records information only when it is made available to the market.

Total returns data are collected from Datastream for all shares in the sample abrogating the need to calculate these returns using the monthly price changes and dividends paid. For each share in the sample we collect monthly data for a range of 20 firm characteristics, hereinafter referred to as style attributes, attributes or factors. Each of these attributes is classified into a particular category which best describes the nature of the attribute, as indicated in Table 3.1. It should be noted that the analyst consensus recommendation data are collected from I-Net Bridge Real Time.

As certain attributes are calculated on a rolling 24-month basis, the first two years of the data are lost and the analysis is conducted over the period February 2002 to December 2009 with the period February 2002 to July 2007 comprising the in-sample period (also referred to as the "stable period") and August 2007 to December 2009 comprising the out-of-sample period (also referred to as the "market crisis period"). Shares which do not have at least 24 consecutive months of data are thus excluded from the data sample.

We apply a similar liquidity filter to that suggested by van Rensburg and Robertson (2003) and exclude all shares which have a turnover ratio of less than 0.001% from the sample in that month, where the turnover ratio is defined as the ratio of the average number of shares traded daily for the month relative to the number of ordinary shares outstanding at the end of the previous month. In order to limit the impact of outliers on the analysis we apply a trimming procedure to the sample which replaces values above the 1st percentile with the 1st percentile value and values below the 99th percentile with the 99th percentile value. Descriptive statistics for the data sample pre- and post-trimming is provided in Appendix A.1.

In selecting relevant factors for the analysis to follow we have attempted to include all factors previously found to be significant in tests of style anomalies. A list of the factors as well as their descriptions is provided in Table 3.1.

While the majority of the factors examined are self-explanatory as they are drawn from prior studies we discuss briefly the motivation for including the industry classification variable detailed in Table 3.1. While prior studies have examined industry classification as a possible predictor of returns they have typically employed multiple dummy variables to capture the effects across many sub-sectors. Van Rensburg (2002) and Kruger (2006) find that a two-factor APT model comprising the Financial-Industrials Index and the Resources Index provide greater explanatory power for share returns on the JSE than the CAPM Model. As a proxy for this apparent dichotomy in return generation characteristics for shares on the JSE we therefore create a dummy variable which is assigned a value of 0 for Resources shares and 1 for Financial and Industrial shares.

Table 3.1 List of firm-specific style attributes

We test a range of firm-specific style attributes in the linear and non-linear factor models investigated in this study. Attributes are classified into one of 10 categories on the basis of prior studies and financial application. A brief definition is provided for each factor.

FACTOR	CLASSIFICATION	DEFINITION
BVTM	VALUE	The firm's book to market ratio: Book value per share/Market value per share
CTP	VALUE	The firm's cash to price ratio: Cash flow per share/Price per share
DY	VALUE	The firm's dividend yield
EY	VALUE	The firm's earnings yield
C12MDPSP	GROWTH	The 12-month change in DPS relative to the current share price.
C12MEPSP	GROWTH	The 12-month change in EPS relative to the current share price.
C24MDPSP	GROWTH	The 24-month change in DPS relative to the current share price.
C24MEPSP	GROWTH	The 24-month change in EPS relative to the current share price.
POUT	GROWTH	The dividend payout ratio of the firm.
ROE	GROWTH	The firm's return on equity ratio: PBIT/Shareholder's Equity
MOM1	MOMENTUM	One-month momentum, i.e. The returns over the previous month
MOM3	MOMENTUM	Three-month momentum, i.e. The returns over the previous three months
MOM6	MOMENTUM	Six-month momentum, i.e. The returns over the previous six months
MOM12	MOMENTUM	Twelve-month momentum, i.e. The returns over the previous year
LMV	SIZE	The natural logarithm of the firm's market capitalization
BUY	ANALYST RECOMMENDATIONS	A dummy variable that is assigned a value of 1 where the consensus analyst recommendation is a buy and a value of 0 otherwise.
DOWNGRADE	ANALYST RECOMMENDATIONS	A dummy variable that is assigned a value of 1 where the consensus analyst forecast has been downgraded month-on-month for the share and a value of 0 otherwise.

Table 3.1 List of firm-specific style attributes (Continued)

We test a range of firm-specific style attributes in the linear and non-linear factor models investigated in this study. Attributes are classified into one of 10 categories on the basis of prior studies and financial application. A brief definition is provided for each factor.

FACTOR	CLASSIFICATION	DEFINITION
VT	LIQUIDITY	A measure of the firm's average monthly volume traded relative to its average issued share capital
DUMFINDI	CLASSIFICATION	A dummy variable that has a value of 1 for all Financials and Industrial shares and a value of 0 otherwise.
STDEV12	RISK	The standard deviation of the firm's prior 12 month's returns

3.3.2 Adjusting the data for missing observations

Appendix A.1 also provides a count of the number of observations for each factor over the full data period. After adjusting the sample for shares which listed or delisted partway through the sample we find that the maximum number of observations for each factor is 11671 during the in-sample period and 5014 during the out-of-sample period. Only 8 of the 20 factors have complete data over the full period – MOM1, DY, LMV, C12MDPSP, C24MDPSP, DUMFINDI, BUY and DOWNGRADE. 7.8% of observations are missing on average from those factors with missing observations during the in-sample period and 5.5% of observations missing on average during the out-of-sample period. This decrease in missingness over time was expected as there is some paucity in the data available for the South African market from Datastream prior to 2004.

Missing data is a common problem in quantitative analysis across a broad range of disciplines. The treatment of such missingness has serious implications for the robustness of analysis but in most cases the impact of missingness is often neglected entirely or, at best, adjusted for on an ad hoc basis which can significantly affect parameter estimates and standard errors (Schafer, 1997). The practice of listwise deletion, the excluding of entire cases with one or more missing observations from the data sample, has become standard in many statistical packages and, by extension, many studies. Such adjustment not only reduces the sample size, which may reduce the power of statistical tests applied to the data, but may also introduce bias to the sample where remaining cases are not representative of the full sample. In rare instances such adjustment can result in different magnitudes or signs of causal relationships in the data (Anderson et al, 1983). Mean imputation is an alternative technique which replaces missing data with the average for the observed values for that particular variable. This method has the effect of artificially reducing the variance of the variable in question as well as potentially impacting on its relationship with other variables in the analysis.

The treatment of missingness in data is informed by the cause of the missingness. Where the missingness of data is purely random and unrelated to the variables in the sample we refer to the data as missing completely at random (MCAR). Where the missingness is not random and may relate to the observed values in the sample but not to any unobserved values the data is said to be missing at random (MAR). Where missingness may be related to unobserved values the missing data is said to be not missing at random (NMAR)⁹. Graham and Donaldson (1993) refer to missing data mechanisms as being either “accessible” or “unaccessible”. An accessible mechanism is one where the cause of the missingness can be identified and measured and as such encompasses both MCAR and MAR. An unaccessible mechanism is one that cannot be identified and measured and is related to NMAR mechanisms where the cause is unobservable within the existing sample.

Assume that the data set is represented by D , a vector of p variables comprising both the dependent and independent variables of interest. D is partitioned into its observed and missing elements such that $D = \{D^{\text{obs}}, D^{\text{miss}}\}$. We represent the missingness in the data by means of an indicator matrix M , which corresponds in size to D , and for which values which are observed are assigned a value of 1 and missing values are assigned a value of 0. Assuming that the missing data is MCAR the missingness is completely unrelated to either the observed or missing observations and $p(M|D) = p(M)$. Where the missingness is MAR we can infer missing values on the basis of the observed values in D such that $p(M|D) = p(M|D^{\text{obs}})$. Finally, where the data is NMAR $p(M|D)$ cannot be simplified.

Multiple imputation is an increasingly common approach to dealing with missing data. The technique estimates or imputes missing data using a predictive model that incorporates both the available information and any prior knowledge of the data and the relationships between the variables. Multiple possible complete data sets are generated by the model and the expected value for any missing observation is the mean value of these multiple imputations, while the uncertainty of the predictive model applied is represented by the standard deviation of the imputed values for each observation. Each of these data sets can be analysed individually and the results combined using the approach of Rubin (1987). The use of multiple rather than single imputations for missing observations serves to provide a more robust standard error across the estimated values (Honaker and King, 2010).

We employ the multiple imputation technique of Honaker and King (2010) in order to adjust our data sample for missing data in order to assess the impact of missingness on the findings of our univariate tests in Chapter 6. The univariate tests are therefore conducted both on the unadjusted data set and the multiply imputed data sets.

⁹ Also sometimes referred to as nonignorable (NI). See King, et al (2001) for further discussion.

3.3.2.1 Methodology

Let D_i represent the vector of p variables comprising both the dependent and independent variables of interest, for all observations in the data, i . It is assumed that the data would be normally distributed in the absence of missingness with mean vector μ and variance matrix Σ . The likelihood function for the complete data is given by

$$L(\mu, \Sigma | D) \propto \prod_{i=1}^n N(D_i | \mu, \Sigma) \quad (3.4)$$

We assume that the data is MAR and, given that the marginal densities are normal based on our prior assumption of normality in the complete data, the observed data likelihood is

$$L(\mu, \Sigma | D_{obs}) \propto \prod_{i=1}^n N(D_{i,obs} | \mu_{i,obs}, \Sigma_{i,obs}) \quad (3.5)$$

where $D_{i,obs}$ represents the observed elements of row i of D , $\mu_{i,obs}$ is the subvector of μ and $\Sigma_{i,obs}$ is the submatrix of Σ . Given the assumption of normality, the missing data can be imputed using a linear regression function on random draws from the appropriate posterior, where

$$\tilde{D}_{ij} = D_{i,-j} \tilde{\beta} + \tilde{\epsilon}_i \quad (3.6)$$

and \sim indicates a random draw from the appropriate posterior. King et al (2001) suggest that the primary difficulty in applying multiple imputation is the computational difficulty in taking the required random draws from the posteriors of μ and Σ as the number of variables, p , increases. The primary techniques employed for this process have been the imputation-posterior approach (IP) which is a Markov-chain, Monte Carlo-based method and expectation maximization importance sampling (EMIS). Both techniques are computationally complex, however and require both extensive time and computing power for imputing larger data sets.

Honaker and King (2010) recommend a bootstrap method to resolve the computational difficulties in sampling the required posteriors. They apply the same Bayesian analysis as detailed in equations 3.4 to 3.6 but replace the sampling of the posteriors for μ and Σ with a bootstrapping algorithm. The Honaker and King (2010) multiple imputation technique draws m samples of size n with replacement from D . The expectation maximization (EM) algorithm is then used to produce point estimates of μ and Σ . The observed values of D are then used in conjunction with the estimates of μ and Σ to impute the missing values.

3.3.2.2 Results

We apply the multiple imputation technique of Honaker and King (2010) to our monthly data set. Rubin (1987) demonstrates that the efficiency of an estimate of m imputations is approximately

$$\left(1 + \frac{\gamma}{m}\right)^{-1} \quad (3.7)$$

where γ , the rate of missing information, measures how strongly the analysis is influenced by the missing data and m is the number of complete data sets imputed by the analysis. Note that γ is not the percentage of missing observations in the data set but rather a relative measure of the increase in the variance of the statistical quantity being measured due to the influence of missing data.

Table 3.2 presents a comparison of efficiencies of multiple imputation analysis given a set of m imputed data sets for a rate of missingness, γ , in the unadjusted data set.

Table 3.2 The Efficiency of Imputations for Given Levels of Missingness in Data

Rubins (1987) proposes the efficiency of m multiple imputation data sets in approximating the true complete data sets given a level of missingness in the data, γ . γ , the rate of missing information, measures how strongly the analysis is influenced by the missing. The table below demonstrates that in most cases efficiency may be maximized by as few as 3-10 imputations.

γ	m			
	3	5	10	20
0.1	97%	98%	99%	100%
0.5	86%	91%	95%	98%
0.9	77%	85%	92%	96%

Given the minor increase in efficiencies for $m > 10$, Rubin (1987) indicates that in most instances no more than three to ten imputations are required. Given the minimal degree of missingness in our data set and the limited improvements in efficiency for values of m greater than five we elect to generate five imputed data sets.

A comparison of the descriptive statistics for the observed and imputed data sets is provided in Appendix A2. In addition, comparative graphs are provided of the relative densities of the actual and mean imputed data sets for variables with missing observations in Appendix A.3.

It is evident from the results that the descriptive statistics of the imputed data sets differ insignificantly from the unadjusted data set and the process does not therefore seem to have materially changed the

distributions of the factors. The difference between the results of linear factor model testing on the unadjusted and imputed data sets will be investigated in Chapter 6.

4. Evidence of Linear and Nonlinear Serial Dependence in Share Returns

Early empirical tests of linear models demonstrated satisfactory goodness of fit, but such models fail to account for the empirically observed peculiarities of asset return distributions (Lye and Martin, 1994). In particular, assumptions of normality and independent and identically distributed (i.i.d) error terms cannot be reconciled with the empirical evidence of leptokurtic share return distributions and asymmetries in the returns data.

The importance of correctly identifying the underlying nature of the return-generation process cannot be overstated as this has implications for the appropriateness of the model fitted to the data set. Numerous studies over the past two decades have found evidence of significant nonlinearities in the underlying returns data sets for both developed and emerging markets yet linear models remain largely the de facto choice for modelling share returns. To date the work that has been conducted in this regard on the South African market has been limited. Mangani (2007) found significant evidence of nonlinear dependence on the returns of 42 listed JSE shares, using the Brock, Dechert and Scheinkman (BDS) test, as part of a more general examination of the distributional properties of JSE returns. He suggested that this implied profitable predictability in JSE share returns. Appiah-Kusi and Menyah (2003) tested for weak-form efficiency in 11 African markets, including South Africa, and found evidence of nonlinear behaviour in weekly index returns on the JSE over the period 1990 to 1995. Both studies examined only one of a myriad of possible tests for nonlinear behaviour. In the case of Mangani (2007) the sample size was limited and Appiah-Kusi and Menyah (2003) employed a sample dated prior to the significant changes to the regulatory structure and trading systems of the JSE. The latter is significant as Appiah-Kusi and Menyah (2003) argue that a possible reason for the observed nonlinear behaviour of African stock market returns is their propensity for imposing onerous regulatory constraints, thin trading issues and high transaction costs. Given the increase in liquidity on the JSE following the introduction of the Johannesburg Equities Trading (JET) automated screen trading system, further examination of the possible nonlinear behaviour of share returns is merited.

This chapter provides an investigation of the presence of both linear and nonlinear serial dependence in JSE share and index returns. In contrast to prior studies, we attempt to provide a more exhaustive examination of the presence and nature of nonlinear dependencies in the returns of JSE listed shares by broadening the range of tests applied. We attempt to answer two questions: (a) is there sufficient evidence of the existence of nonlinear dependencies in the return generation process on the JSE and (b) are these dependencies consistent over time or episodic in nature? The former question has implications for the

most appropriate approach to modelling the return generation process. The latter question will provide insight into whether it is possible to create a predictive model of returns that is consistent over time.

4.1 Methodology

4.1.1 Tests for Linear Serial Dependence in Returns

We test the data for both autocorrelations (AC) and partial autocorrelations (PAC) as evidence of linear serial dependence in the return series.

The autocorrelation coefficient, p_k , of a series Y at lag k is estimated by

$$p_k = \frac{\sum_{t=1}^{n-k} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{(Y_t - \bar{Y})^2} \quad (4.1)$$

We examine values of p_k up to lag $k = 10$ and test the hypothesis that $H_0: p_k = 0$ against the alternative hypothesis that $H_0: p_k \neq 0$. Tests are conducted at the 5% significance level. Where p_k is non-zero there is evidence of serial dependence in the data. Where p_k is zero, we note an absence of serial dependence in the data, which implies that no autoregressive linear function could be used to forecast future values of Y .

The partial autocorrelation coefficient is the autocorrelation between Y_t and Y_{t-k} after removing the correlation from lags 1 to $k-1$.

The partial autocorrelations, ϕ_k , are estimated recursively by

$$\phi_k = Y_k - \sum_{j=1}^{k-1} \phi_{k-1,j} Y_{k-j} \quad , k = 1 \quad (4.2)$$

$$\phi_k = 1 - \sum_{j=1}^{k-1} \phi_{k-1,j} Y_{k-j} \quad , k > 1 \quad (4.3)$$

As we did for the autocorrelations, we examine values for ϕ_k up to lag $k = 10$ and test the hypothesis that $H_0: \phi_k = 0$ against the alternative hypothesis that $H_0: \phi_k \neq 0$. Tests are again conducted at the 5% significance level.

4.1.2 Tests for Nonlinear Serial Dependence Returns

We first apply a battery of tests for nonlinear dependencies over the full period of our data sample. Following this a windowed testing procedure is applied to test over shorter window periods within the sample. The second battery of tests is run as it is possible that tests over the full period may overlook evidence of nonlinearities that occur only infrequently within the series. Conversely, it is possible that evidence of nonlinear activity over an extended period may be due to the activity of only a few short periods of nonlinear dependencies (Lim and Liew, 2004).

Concerns have been raised as to whether the asymptotic properties of the distributions for these tests are maintained for small samples (see Brock et al., 1996). De Lima (1998) also finds that the moment restrictions underlying the asymptotic properties of some nonlinear tests are not satisfied in the presence of leptokurtic data. As per Ashley and Patterson (2000), we therefore apply a bootstrap procedure by extracting 1000 N-size samples at random from the data. The bootstrap significance level is determined by the percentage of the N-samples for which the test statistic exceeds that in the actual data.

4.1.2.1 Full Period Tests

As per Ashley and Patterson (2000) we employ all of the following tests to detect the presence, if any, of any nonlinear dependencies in our sample over the full period: the Brock, Dechert and Scheinkman (1987) test, McLeod and Li (1983) test, Engle (1982) LM test, Tsay (1986) test and Hinich and Patterson bicoherence (1995) test. The BDS, McLeod-Li and Tsay tests focus on lower moments in the distributions of the data while the bicoherence test focuses on higher moments. Given the low explanatory power demonstrated for the Hinich Bispectral test by Ashley and Patterson (2000), we exclude this test from our analysis. For each of the tests we first remove any linear serial dependence from the data by fitting a prewhitening AR(p) model to the data. The appropriate order for the AR model is determined by minimizing the Schwartz information criterion (SIC) over a range of AR(p) models up to order 10. Both the prewhitening exercise and nonlinear tests are conducted using the Toolkit software developed by Hinich and Patterson.

Engle LM Test (1982)

Engle (1982) employs a LaGrange Multiplier test to detect ARCH disturbances based on the autocorrelation of the squared residuals of an OLS regression of the data. Given a set of sample residuals \hat{v}_t , the squared residuals are regressed on a constant with p lags such that

$$\hat{v}_t^2 = \alpha_0 + \alpha_1 \hat{v}_{t-1}^2 + \alpha_2 \hat{v}_{t-2}^2 + \dots + \alpha_i \hat{v}_{t-i}^2 + \varepsilon_t \quad (4.1)$$

Where the null hypothesis $\alpha_i = 0$ for all values of i holds, the distribution approximates chi square with p degrees of freedom.

$$NR^2 \sim \chi_p^2 \quad (4.2)$$

Mcleod-Li Test (1983)

The Mcleod-Li test determines whether the autocorrelation $\hat{r}(k)$ of the squared residuals of the prewhitened data is non-zero for some lag k, where

$$\hat{r}(k) = \frac{\sum_{t=k+1}^n (a_t^2 - \hat{\sigma}^2)(a_{t-k}^2 - \hat{\sigma}^2)}{\sum_{t=1}^n (a_t^2 - \hat{\sigma}^2)^2} \quad (4.3)$$

and $\hat{\sigma}^2 = \sum_{t=1}^n \frac{\hat{a}_t^2}{n}$.

Given that for a fixed M, $\sqrt{n}\hat{r} = (\hat{r}(1), \dots, \hat{r}(M))$ Mcleod and Li (1983) demonstrate that the Box-Ljung portmanteau statistic

$$Q^* = n(n+2) \sum_{i=1}^M \frac{\hat{r}^2(i)}{n-i} \quad (4.4)$$

is asymptotically $\chi^2(M)$ if α_t are independent.

Tsay Test (1986)

Tsay (1986) proposes a more powerful form of Keenan's (1985) time-domain variant of Tukey's (1949) 'one degree of freedom for nonadditivity' test. The test examines the data for quadratic serial dependence using quadratic terms lagged up to k periods.

$K = \frac{k(k+1)}{2}$ column vectors, $v_1 \dots v_K$, containing all the unique cross-products of the form $x_{t-i}x_{t-j}$ are extracted from the data. The column vectors are regressed on the k -lagged original data series to obtain a set of residuals $\hat{v}_{t,i}$.

The parameters $\alpha_1 \dots \alpha_K$ are then estimated for the following OLS regression

$$x_t = \alpha_0 + \sum_{i=1}^K \alpha_i \hat{v}_{t,i} + \varepsilon_t \quad (4.5)$$

The Tsay statistic is the F statistic for the null hypothesis that $\alpha_K = 0$ for all values of K holds.

Hinich and Patterson Bicorrelation (1995)

The Hinich and Patterson bicorrelation test is a generalization of the Box-Pierce portmanteau test which tests for serial independence of the residuals of the prewhitened data using the sample bicorrelations of the data.

Define the (r,s) sample bicorrelation as

$$C_3(r,s) = (N-s)^{-1} \sum_{t=1}^{N-s} x_t x_{t+r} x_{t+s} \quad (4.6)$$

where $C_3(r,s)$ is zero for zero-mean, serially i.i.d data and non-zero in the presence of higher moments.

The bicorrelation measure is then defined as

$$H_3 = \sum_{s=2}^l \sum_{r=1}^{s-1} [\sqrt{N-s} C_3(r,s)^2]$$

(4.7)

Hinich and Patterson (1995) demonstrate that under the null hypothesis of a serially i.i.d process, X_j is asymptotically distributed $\chi^2\left(\frac{l[l-1]}{2}\right)$ for $l = N^b$ where $0 \leq b \leq 0.5$. Hinich and Patterson (1995) suggest the use of $b = 0.4$ based on their testing.

BDS Test (Brock, Dechert and Scheinkman, 1987)

Brock, Dechert and Scheinkman (1987) develop the BDS test for serial dependence and nonlinear structure in time series. The test fits a linear model to the data sample in order to remove any linear dependence from the sample and then tests whether the estimated errors are i.i.d. Importantly, the first order asymptotic distribution of the BDS test statistic is independent of estimation error when applied to the residuals of linear models. (Crato and Lima, 1994). This includes both the ARMA and GARCH classes of models and thus has application in the modelling and testing of high frequency financial data (Brock and Potter, 1993).

The BDS test is based on the limiting value of the correlation integral of Grassberger and Pocaccia (1983) which measures the mean probability that two states u and v , embedded in m -dimensional space, are close within a stated tolerance ϵ .

$$C_{m,\epsilon} = \iint \chi_\epsilon \|u - v\| dF_m(u) dF_m(v) \quad (4.8)$$

where the process is i.i.d $C_{m,\epsilon} = C_{1,\epsilon^m}$ and this property gives rise to the BDS test statistic which converges to $N(0,1)$ as $N \rightarrow \infty$:

$$W_{m,\epsilon,n} = \frac{\sqrt{N}}{\sqrt{\sigma_{m,\epsilon}}} (C_{m,\epsilon} - C_{1,\epsilon})^m \quad (4.9)$$

Brock et al. (1996) suggest that the asymptotic distribution of the statistic does not require higher moments to exist which has implications for the modelling of leptokurtic returns data which typically complicate the use of other test statistics (see De Lima, 1998). Ashley and Patterson (2006) draw attention to the fact that the size of the BDS test for higher embedding dimensions is distorted in those instances

where the prewhitening procedure fails to remove all traces of linear dependence in the data series. All results in Chapter 4 are therefore reported for $m = 2$ only.

4.1.2.2 Episodic Tests

Hinich and Patterson (1995) suggest a windowed testing procedure for uncovering episodic nonlinear dependence in share returns. The data sample is divided into frames of equal length and the data in each frame are standardized. Brooks and Hinich (1998) suggest that the window length should be long enough to be statistically robust while also being sufficiently short to detect short-lived dependencies in the data series. For purposes of this study we have selected a window length of 35 trading days which yields a series of 54 non-overlapping windows for analysis over the in-sample period and 17 non-overlapping windows over the out-of-sample period. Todea and Zoicas-Ienciu (2008) have suggested that the use of non-overlapping windows may fail to accurately identify sub-periods with nonlinear dependence because the test results are dependent on the choice of the first day of the sample. This should be noted in evaluating the results of the analysis to follow.

Three variations of the Box-Pierce portmanteau test are then applied to each window. The C statistic tests for second order linear dependence, the H statistic corresponds to the Hinich and Patterson bicorrelation test (described by equation 4.10) and tests for third-order nonlinear serial dependence in the data and the H4 tricorrelation test statistic which tests for fourth-order nonlinear serial dependence. The H4 and C statistics are defined as follows

$$H_4 = \sum_{v=3}^L \sum_{s=2}^{v-1} \sum_{r=1}^{s-1} [\sqrt{N-s} C_4(r, s, v)^3] \quad (4.10)$$

where

$$C_4(r, s, v) = (N-v)^{-1} \sum_{t=1}^{N-v} Z_t Z_{t+r} Z_{t+s} Z_{t+v} \quad (4.11)$$

and

$$C = \sum_{r=1}^l (C_2(r))^2 \quad (4.12)$$

where

$$C_2(r) = (N - r)^{-1/2} \sum_{t=1}^{N-r} z_t z_{t+r} \quad (4.13)$$

The C statistic is calculated on the unadjusted data series obtained from the thin trading procedures discussed earlier. The H and H4 statistics are calculated after the data have been adjusted to remove linear dependencies by fitting an appropriate AR(p) model.

The tests conducted on each window yield a test statistic equalling one minus the p-value of the test. The greater the significance of the test statistic, the greater is the value of the statistic. For example, where the bivariate test has a p-value of 0.03 the H statistic would be quoted as 0.97. Average values for these statistics are used as an indication of the relative significance of that particular test for each share.

4.2 Descriptive Statistics

Table 4.1 presents a breakdown of the descriptive statistics for the shares according to the three size categories, both before and after applying the thin trading adjustments. It is clear from the results that the adjustment procedure does not unduly alter the distribution of the returns for any of the share categories and that all three categories demonstrate significant skewness and excess kurtosis both before and after the thin trading adjustments are applied. We note that these higher moments are greater on average for smaller cap stocks than for larger cap stocks as are the maximum and minimum values. While this may be indicative of outliers in the time series no further adjustments are made to the data at this stage.

An examination of the out-of-sample data yields similar results. We note that there is limited evidence for thin trading of shares across all three size categories. The thin trading adjustments alone therefore have little impact on the distributional properties of shares in the three categories.

None of the shares in the in- or out-of-sample periods demonstrate a probability greater than 5% that their Jarque-Bera statistic exceeds the observed value and we therefore reject the null hypothesis that the returns series are from a normal distribution. This finding is consistent with prior examinations of the distribution of share returns on the JSE (see Mlambo et al, 2003, and Mangani, 2007, amongst others).

We contrast the number of zero returns in the share returns series before and after adjusting for thin trading and present the relevant statistics in Table 4.2. It is evident that the average percentage of zero returns in the data over both the in-sample and out-of-sample periods increases as firm size decreases. Even after adjusting the data for thin trading we find a significant percentage of true zero returns

remaining in the data. These zero returns are likely to result in spurious autocorrelation in the data. Of interest is the limited evidence of thin trading over the out-of-sample period. Although there are a significant number of zero returns over the out-of-sample period they appear to be true zero returns.

Similarly, we examine the distributional properties of the Top 40 Index (Top 40), Mid Cap Index and Small Cap Index and present the results in Table 4.3. We observe the same positive relationship between size and skewness and kurtosis as we did for the individual share sample, over both the in- and out-of-sample periods.

Table 4.1 In-Sample Descriptive Statistics for Individual Shares

Descriptive statistics are calculated based on the daily total returns for the individual shares in the in-sample period both before and after adjusting for thin trading. These statistics are then averaged across the shares within each of the size categories.

In-Sample

Before Thin Trading Adjustments

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.136	-0.110	0.001	0.022	4.469	0.196
Mid-Cap	0.146	-0.153	0.001	0.021	9.220	-0.020
Small-Cap	0.230	-0.206	0.001	0.027	20.833	0.557

After The Thin Trading Adjustments of Atchison et al. (1987)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.134	-0.110	0.001	0.022	4.615	0.198
Mid-Cap	0.150	-0.145	0.001	0.021	9.691	0.072
Small-Cap	0.209	-0.204	0.001	0.027	14.854	0.289

After The Thin Trading Adjustments of Mlambo et al. (2003)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.136	-0.110	0.001	0.022	4.492	0.206
Mid-Cap	0.145	-0.139	0.001	0.021	8.718	0.110
Small-Cap	0.198	-0.183	0.001	0.026	13.650	0.484

Out-of-Sample

Before Thin Trading Adjustments

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.152	-0.122	0.000	0.029	4.985	0.386
Mid-Cap	0.133	-0.119	0.000	0.024	7.123	0.292
Small-Cap	0.267	-0.169	0.000	0.028	33.586	1.323

After The Thin Trading Adjustments of Atchison et al. (1987)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.152	-0.122	0.000	0.029	4.986	0.386
Mid-Cap	0.133	-0.119	0.000	0.024	7.117	0.297
Small-Cap	0.267	-0.169	0.000	0.028	33.673	1.325

After The Thin Trading Adjustments of Mlambo et al. (2003)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.152	-0.122	0.000	0.029	4.985	0.386
Mid-Cap	0.133	-0.119	0.000	0.024	7.115	0.298
Small-Cap	0.267	-0.169	0.000	0.028	33.333	1.318

Table 4.2 Percentage of Zero Returns for Individual Shares

The percentage of zero returns for shares in each size category are calculated for the in-sample and out-of-sample periods, both before and after adjusting for thin trading. These statistics are then averaged across the shares within each of the size categories.

In-Sample Period			
	Large-Cap	Mid-Cap	Small-Cap
Unadjusted Data	6.7%	23.2%	43.0%
After thin-trading adjustments	6.1%	17.5%	23.4%
Out-of-Sample Period			
	Large-Cap	Mid-Cap	Small-Cap
Unadjusted Data	2.5%	9.1%	27.2%
After thin-trading adjustments	2.5%	9.1%	27.1%

Table 4.3 In-Sample Descriptive Statistics for Index Returns

Descriptive statistics are calculated based on the daily total returns for the indices in the in-sample period both before and after adjusting for thin trading.

In-Sample						
Index	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Top 40 Index	0.068	-0.080	0.001	0.013	2.929	-0.087
Mid-Cap Index	0.037	-0.072	0.001	0.008	8.488	-1.083
Small-Cap Index	0.036	-0.057	0.001	0.006	9.328	-1.121
Out-of-Sample						
Index	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Top 40 Index	0.081	-0.075	0.000	0.019	1.899	0.095
Mid-Cap Index	0.048	-0.055	0.000	0.011	2.221	-0.297
Small-Cap Index	0.030	-0.045	0.000	0.007	4.866	-0.899

4.3 Results for Linear Serial Dependence Tests

Test results will be reported in aggregate according to share size while results for individual shares are presented in the appendices as indicated. In the former instance shares are sorted into three categories based on their inclusion in one of the JSE's Top 40 (referred to here as the "large cap" category), Mid Cap or Small Cap indices, and their statistics are then averaged over their samples.

Results for the autocorrelation and partial autocorrelation tests for the share sample are based on the share data which has been adjusted using the Mlambo et al. (2003) adjustment only. Thin trading introduces spurious autocorrelation in the data and both the raw data and the data adjusted using the Atchison et al. (1987) techniques are subject to this concern¹⁰.

We aggregate the in-sample results for the autocorrelation and partial autocorrelation tests in Table 4.4 and present the results for the individual shares in Appendix B1. The results suggest little pattern in the autocorrelation and partial autocorrelation coefficients for shares across the size categories with evidence of significance for at least one share in each size category at each of the lags tested. Most of the significant coefficients for both tests are found at the first lag in all three size categories. The sign of the autocorrelation and partial autocorrelation coefficients for the first lag are not consistent, although we observe that the majority of the shares in the large cap sample (71%) and mid cap sample (60%) have positive signs indicating positive inter-day momentum, while the majority of the shares in the small cap sample (73%) have negative signs indicating a short-term inter-day reversal effect for these shares. It is also found that the third lag yields a marginally higher number of significant autocorrelation coefficients for the large cap sample than does the first. This phenomenon is unique to the large cap sample. The sign for the autocorrelation coefficients for large cap shares at this lag is negative for 88% of the shares in that sample.

Table 4.4 Significant Autocorrelation and Partial Autocorrelation Coefficients for the In-Sample Period

The number and percentage of shares in each size category which demonstrate significant autocorrelation coefficients (AC) and partial autocorrelation coefficients (PAC) at the 5% level up to lag 10 are presented below. The share return series which were adjusted using the approach of Mlambo et al. (2003) are employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading.

	Large Cap		Mid Cap		Small Cap	
	AC	PAC	AC	PAC	AC	PAC
Lag 1	16 (47%)	16 (44%)	24 (55%)	24 (55%)	16 (62%)	16 (62%)
Lag 2	6 (18%)	9 (21%)	6 (14%)	7 (16%)	7 (27%)	8 (27%)
Lag 3	17 (50%)	16 (44%)	5 (11%)	4 (11%)	3 (12%)	4 (12%)
Lag 4	7 (21%)	5 (15%)	6 (14%)	7 (18%)	4 (15%)	4 (19%)
Lag 5	12 (35%)	12 (32%)	6 (14%)	6 (11%)	2 (8%)	3 (8%)
Lag 6	7 (21%)	7 (18%)	6 (14%)	8 (16%)	2 (8%)	2 (12%)
Lag 7	4 (12%)	3 (12%)	3 (7%)	3 (5%)	2 (8%)	1 (4%)
Lag 8	1 (3%)	2 (3%)	5 (11%)	6 (14%)	3 (12%)	2 (8%)
Lag 9	4 (12%)	4 (12%)	4 (9%)	4 (9%)	1 (4%)	1 (4%)
Lag 10	3 (9%)	3 (6%)	3 (7%)	3 (5%)	3 (12%)	3 (8%)

The out-of-sample results for the autocorrelation and partial autocorrelation tests are presented in Table 4.5 and present the results for the individual shares in Appendix B2. The results again indicate that the

¹⁰ The approach of Atchison et al. (1987) corrects for price age in the data due to thin trading but does not correct for spurious autocorrelation resulting from thin trading.

highest number of significant autocorrelation and partial autocorrelation coefficients for the mid cap and small cap samples is found at the first lag. The previously-observed positive relationship between size and the number of shares with significant coefficients is no longer obvious, with the mid cap sample displaying a greater percentage of shares with significant coefficients than the small cap sample. We also note that the percentage of significant coefficients at the first lag in the large cap sample has diminished significantly and the greatest percentage of significant coefficients for shares in this category now occur at the second lag.

Table 4.5 Significant Autocorrelation and Partial Autocorrelation Coefficients for the Out-of-Sample Period

The number and percentage of shares in each size category which demonstrate significant autocorrelation coefficients (AC) and partial autocorrelation coefficients (PAC) at the 5% level up to lag 10 are presented below. The share return series which were adjusted using the approach of Mlambo et al. (2003) are employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading.

	Large Cap		Mid Cap		Small Cap	
	AC	PAC	AC	PAC	AC	PAC
Lag 1	8 (24%)	8 (24%)	19 (43%)	19 (43%)	11 (39%)	11 (39%)
Lag 2	13 (38%)	16 (47%)	7 (16%)	10 (23%)	6 (21%)	7 (25%)
Lag 3	9 (26%)	9 (26%)	6 (14%)	6 (14%)	4 (14%)	5 (18%)
Lag 4	5 (15%)	9 (26%)	5 (11%)	4 (9%)	2 (7%)	1 (4%)
Lag 5	6 (18%)	5 (15%)	1 (2%)	0 (0%)	4 (14%)	5 (18%)
Lag 6	2 (6%)	4 (12%)	3 (7%)	4 (9%)	2 (7%)	1 (4%)
Lag 7	2 (6%)	2 (6%)	4 (9%)	3 (7%)	4 (14%)	3 (11%)
Lag 8	7 (21%)	5 (15%)	1 (2%)	2 (5%)	3 (11%)	3 (11%)
Lag 9	2 (6%)	1 (3%)	7 (16%)	5 (11%)	2 (7%)	1 (4%)
Lag 10	8 (24%)	6 (18%)	4 (9%)	4 (9%)	3 (11%)	5 (18%)

Next we examine the autocorrelation and partial autocorrelation coefficients for the index return series. Given the limited number of indices under examination we present the individual index results for the in-sample period in Table 4.6.

As was the case for the individual share returns we observe significant autocorrelation and partial autocorrelation coefficients at the first lag for all three indices over the in-sample period. Over the out-of-sample period we find that the Top 40 index demonstrates significant autocorrelation coefficients at the third lag only. Given the composition of the Top 40 index it may be concluded that this result arises due to the behaviour of large cap shares in both indices. This is consistent with the out-of-sample results of the share sample where we observe a greater incidence of significant autocorrelation and partial autocorrelation coefficients for large cap shares at the second and third lags than at the first lag. Unlike the share returns series, however, the index returns series do not demonstrate significant coefficients for higher lags except in the case of the Small Cap Index which has significant autocorrelation coefficients up to lag 5 over the in-sample period and up to lag 8 over the out-of-sample period. In addition it displays significant partial autocorrelation coefficients up to lag 4 over the in-sample period and at lags 1 and 3

over the out-of-sample period. This strong evidence of serial dependence in that series may be a consequence of thin trading in the shares underlying the index.

We note further that the signs of the autocorrelation coefficients are consistent with those observed for the share series for the Top 40 and Mid Cap indices at the first lag. The sign for the Small Cap Index is also positive, however, in contrast to the largely negative autocorrelations observed for the individual shares.

Table 4.6 Significant Autocorrelation and Partial Autocorrelation Coefficients for the Index Sample

The autocorrelation coefficients (AC) and partial autocorrelation coefficients (PAC) for each index in the sample are presented below. The analysis is conducted up a lag of 10 and coefficients which are significant at the 5% level are highlighted in bold.

In-Sample Period

Autocorrelation Coefficients (AC)

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
Top 40 Index	0.055	0.026	-0.040	-0.011	-0.046	-0.017	-0.003	0.010	-0.014	0.020
Mid Cap Index	0.189	0.045	0.013	0.092	-0.007	-0.013	0.027	0.052	0.052	0.054
Small Cap Index	0.245	0.111	0.109	0.096	0.060	0.047	0.047	0.075	0.080	0.071

Partial Autocorrelation Coefficients (PAC)

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
Top 40 Index	0.055	0.023	-0.043	-0.007	-0.043	-0.013	0.000	0.007	-0.016	0.019
Mid Cap Index	0.189	0.010	0.003	0.092	-0.043	-0.009	0.035	0.034	0.040	0.040
Small Cap Index	0.245	0.055	0.075	0.053	0.016	0.015	0.020	0.051	0.045	0.031

Out-of-Sample Period

Autocorrelation Coefficients (AC)

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
Top 40 Index	0.033	-0.012	-0.111	-0.060	-0.051	-0.009	0.057	0.013	0.010	-0.049
Mid Cap Index	0.192	0.026	-0.047	-0.036	0.019	-0.014	0.047	0.059	0.051	-0.048
Small Cap Index	0.252	0.109	0.105	0.103	0.083	0.080	0.109	0.076	0.042	0.024

Partial Autocorrelation Coefficients (PAC)

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
Top 40 Index	0.033	-0.013	-0.111	-0.054	-0.051	-0.020	0.044	-0.004	0.003	-0.043
Mid Cap Index	0.192	-0.012	-0.052	-0.018	0.032	-0.026	0.053	0.044	0.031	-0.066
Small Cap Index	0.252	0.048	0.071	0.061	0.037	0.040	0.070	0.019	-0.003	-0.010

4.4 Results for Nonlinear Serial Dependence Tests

4.4.1 Battery of tests for nonlinear dependencies

We apply the Brock, Dechert and Scheinkman (1987), McLeod and Li (1983), Engle LM (1982), Tsay (1986) and Hinich and Patterson bicorrelation (1995) tests to each of the shares in our sample over the in-sample period. Per share results are presented in Appendix B.3 and are aggregated in Tables 4.7 and 4.8 overleaf. The results vary little between the two thin trading adjustments. It is clear from the aggregate findings that all tests considered provide evidence of significant nonlinear dependencies across the sample. The BDS test in particular is significant for all shares considered. Only seven of the shares under investigation demonstrate significant nonlinear dependencies for fewer than three out of the five tests after the Atchison et al. (1987) adjustment. These are LON from the large cap sample, HCI and PAP from the mid-cap sample and ADR, KGM and OMN from the small-cap sample. For the Mlambo et al. (2003) method only the LON, PAP and MTA (from the small cap sample) demonstrate significant nonlinear dependencies for fewer than three out of the five tests.

While large-cap shares demonstrate significance for more tests than mid-cap and small-cap shares, there does not seem to be any relationship between significance levels and share size as a larger percentage of small-cap shares demonstrate significance than mid-cap shares.

Note that the results for the large-cap shares demonstrate little difference for the two thin trading adjustment techniques. This is likely due to the fact that the large-cap sample had few zero returns due to thin trading and the choice of technique therefore had little impact on the samples.

Table 4.7 Results for the Nonlinear Serial Dependence Tests of the Share Sample over the In-Sample Period (Atchison et al. (1987) adjustment)

All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications. Figures quoted for each test represent the number of shares which displayed significant evidence of nonlinear dependencies for that test, while the figure in brackets indicates the percentage of the category this figure represents. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\hat{l} = 1$. No significant difference in results was found for $\hat{l} = 0.5$ and $\hat{l} = 2$.

	McLeod-Li L = 24	Engle LM p = 5	BDS m = 2	Tsay k = 5	Bicovariance l = 8
Large-Cap	33 (97%)	33 (97%)	34 (100%)	21 (62%)	34 (100%)
Mid-Cap	42 (95%)	42 (95%)	44 (100%)	29 (66%)	41 (93%)
Small-Cap	22 (79%)	23 (82%)	28 (100%)	23 (82%)	26 (93%)

Table 4.8 Results for the Nonlinear Serial Dependence Tests of the Share Sample over the In-Sample Period (Mlambo et al. (2003) adjustment)

All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications. Figures quoted for each test represent the number of shares which displayed significant evidence of nonlinear dependencies for that test with the figure in brackets representing the percentage of the category this figure represents. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\hat{h} = 1$. No significant difference in results was found for $\hat{h} = 0.5$ and $\hat{h} = 2$.

	McLeod-Li L = 24	Engle LM p = 5	BDS m = 2	Tsay k = 5	Bicovariance l = 8
Large-Cap	33 (97%)	33 (97%)	34 (100%)	19 (62%)	34 (100%)
Mid-Cap	43 (98%)	42 (95%)	44 (100%)	29 (66%)	41 (93%)
Small-Cap	23 (83%)	25 (89%)	28 (100%)	20 (71%)	27 (96%)

Per share results for the out-of-sample period are presented in Appendix B.4 and are aggregated in tables 4.9 and 4.10. While the relative number of significant shares per category is similar between tests for both periods, the number of significant shares within each category is marginally lower during the out-of-sample period. This may be a result of the market instability over the out-of-sample period but even so, substantial evidence of nonlinear dependence remains even during this tumultuous period.

Table 4.9 Results for the Nonlinear Serial Dependence Tests of the Share Sample over the Out-of-Sample Period (Atchison et al. (1987) adjustment)

All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications. Figures quoted for each test represent the number of shares which displayed significant evidence of nonlinear dependencies for that test, while the figure in brackets indicates the percentage of the category this figure represents. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\hat{h} = 1$. No significant difference in results was found for $\hat{h} = 0.5$ and $\hat{h} = 2$.

	McLeod-Li L = 24	Engle LM p = 5	BDS m = 2	Tsay k = 5	Bicovariance l = 8
Large-Cap	31 (91%)	32 (94%)	34 (100%)	23 (68%)	33 (97%)
Mid-Cap	37 (84%)	36 (82%)	44 (100%)	23 (52%)	40 (91%)
Small-Cap	19 (68%)	23 (82%)	27 (96%)	19 (68%)	22 (79%)

Table 4.10 Results for the Nonlinear Serial Dependence Tests of the Share Sample over the Out-of-Sample Period (Mlambo et al. (2003) adjustment)

All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications. Figures quoted for each test represent the number of shares which displayed significant evidence of nonlinear dependencies for that test with the figure in brackets representing the percentage of the category this figure represents. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\hat{h} = 1$. No significant difference in results was found for $\hat{h} = 0.5$ and $\hat{h} = 2$.

	McLeod-Li L = 24	Engle LM p = 5	BDS m = 2	Tsay k = 5	Bicovariance l = 8
Large-Cap	31 (91%)	32 (94%)	34 (100%)	24 (71%)	33 (97%)
Mid-Cap	37 (84%)	36 (82%)	44 (100%)	23 (52%)	40 (91%)
Small-Cap	19 (68%)	23 (82%)	27 (96%)	20 (71%)	22 (79%)

We conduct a similar analysis for the index sample and present the results in Table 4.11 and Table 4.12. The results indicate that all four indices demonstrate significant evidence of nonlinear serial dependence for all tests considered at the 5% level over the in-sample period. Similar evidence of significant nonlinear serial dependence is observed over the in-sample period for all indices and tests apart from the Tsay test which is no longer significant for the Top 40 index.

These findings support those of the share analysis, providing strong evidence for the presence of nonlinear serial dependence in the return series at both share and index level and suggesting that linear models may not adequately describe the return generation process.

Table 4.11 Results for the Nonlinear Serial Dependence Tests of the Index Sample over the In-Sample Period

All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications. Figures quoted for each test represent the p-value for each test with a significant p-value indicating evidence of nonlinear serial dependence in the data. All tests are evaluated at the 5% significance level with significant p-values highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\hat{h} = 1$. No significant difference in results was found for $\hat{h} = 0.5$ and $\hat{h} = 2$.

	McLeod-Li L = 24	Engle LM p = 5	BDS m = 2	Tsay k = 5	Bicovariance l = 8
Top 40 Index	0.00	0.00	0.00	0.003	0.00
Mid Cap Index	0.00	0.00	0.00	0.000	0.00
Small Cap Index	0.00	0.00	0.00	0.000	0.00

Table 4.12 Results for the Nonlinear Serial Dependence Tests of the Index Sample over the Out-of-Sample Period

All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications. Figures quoted for each test represent the p-value for each test with a significant p-value indicating evidence of nonlinear serial dependence in the data. All tests are evaluated at the 5% significance level with significant p-values highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\hat{h} = 1$. No significant difference in results was found for $\hat{h} = 0.5$ and $\hat{h} = 2$.

	McLeod-Li L = 24	Engle LM p = 5	BDS m = 2	Tsay k = 5	Bicovariance l = 8
Top 40 Index	0.00	0.00	0.00	0.079	0.00
Mid Cap Index	0.00	0.00	0.00	0.000	0.00
Small Cap Index	0.00	0.00	0.00	0.000	0.00

4.4.2 Tests for Episodic Nonlinear Dependencies

We apply the Hinich-Patterson (1995) windowed procedure and compute the C, H and H4 statistics for each of the shares over the in-sample period. In addition we calculate the number of significant windows

for each statistic for each share, as defined in Section 4.1.2.2. All statistics are evaluated for significance at a 5% bootstrapped level obtained via 1000 replications and are then averaged to determine aggregated figures for each size category. The number of windows per share is 54 for all tests conducted on the Atchison et al. (1987)-adjusted sample but varies for each share depending on the extent of thin trading where the Mlambo et al. (2003) adjustment is used. The aggregate results for both methods are therefore presented as a percentage representing the average fraction of the number of significant windows for each test relative to the number of windows in the category. Per share results are presented in Appendix B.4 and the aggregated results can be found in Tables 4.13 and 4.14 below.

Note also that results for the C statistic are quoted only for the Mlambo et al. (2003)-adjusted sample. As the approach of Atchison et al. (1987) does not explicitly correct for the presence of spurious autocorrelation in the data due to thin trading, it is impossible to attribute significant C statistics to actual linear serial dependence within the data.

It is clear from the aggregate results that there is significant evidence of both linear and nonlinear dependence in the data but these occurrences are sporadic and much of the return generation process is therefore characterized by long periods of pure white noise. Given our decision to evaluate results at a five percent significant level we would expect on average to have five percent of each category exhibit significant test statistics but this is exceeded for all tests across the three size categories.

Table 4.13 Results for Episodic Nonlinearity Tests of the Share Sample Sorted by Size over the In-Sample Period (Atchison et al. (1987) adjustment)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample over the in-sample period. Average statistics are computed by determining the mean statistic value over the 54 windows per share and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	9.3%	0.396	8.6%	0.389
Mid-Cap	11.6%	0.398	11.5%	0.390
Small-Cap	14.6%	0.400	14.5%	0.370

Table 4.14 Results for Episodic Nonlinearity Tests of the Share Sample Sorted by Size over the In-Sample Period (Mlambo et al. (2003) adjustment)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample over the in-sample period. Average statistics are computed by determining the mean statistic value over the number of windows in each share series and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	8.0%	0.489	9.2%	0.395	8.4%	0.392
Mid-Cap	9.3%	0.499	10.6%	0.390	10.3%	0.375
Small-Cap	10.6%	0.515	10.6%	0.364	10.6%	0.342

The average number of significant windows and size of the test statistics is higher under the Atchison et al. (1987) adjustment than the Mlambo et al. (2003) adjustment for all categories except large-cap shares and is most noticeable for the small-cap sample. This may be a consequence of the greater number of zero returns in the small-cap series even after adjusting for thin trading which is exacerbated by the fact that the Atchison et al. (1987) technique fails to correct for spurious autocorrelation in the data.

We note that these differences are marginal, however, when examining the sample adjusted using the Mlambo et al. (2003) method and the average H and H4 windows and statistics for the three categories are not significantly different. This would seem to indicate that while there is clear evidence of nonlinear structure in the data, this structure is not obviously influenced by share size. This agrees with the findings of the battery of tests.

The comparison of linear and nonlinear statistics for the Mlambo et al. (2003) technique indicates that there are more non-linear than linear windows in the large-cap and mid-cap samples while the opposite is true for the small-cap sample. We note, however, that the average value of the C statistic is higher in all three size categories than the non-linear H and H4 statistics.

Results for the out-of-sample period are similar, as presented in Table 4.15 and 4.16. As per the results of the battery of linear tests we find that the average percentage of C, H and T4 statistics during the out-of-sample period is lower than that observed during the in-sample period, again indicating a marginal reduction in serial dependence (both linear and nonlinear) during the instable out-of-sample period. Even so, there is still clear and significant evidence of serial dependence over the out-of-sample period for all three statistics.

Table 4.15 Results for Episodic Nonlinearity Tests of the Share Sample Sorted by Size over the Out-of-Sample Period (Atchison et al. (1987) adjustment)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample over the out-of-sample period. Average statistics are computed by determining the mean statistic value over the 54 windows per share and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	8.8%	0.422	7.8%	0.403
Mid-Cap	10.7%	0.397	10.0%	0.375
Small-Cap	12.6%	0.379	11.8%	0.340

Table 4.16 Results for Episodic Nonlinearity Tests of the Share Sample Sorted by Size over the Out-of-Sample Period (Mlambo et al. (2003) adjustment)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample over the out-of-sample period. Average statistics are computed by determining the mean statistic value over the number of windows in each share series and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	7.40%	0.487	10.70%	0.487	7.80%	0.402
Mid-Cap	8.00%	0.512	11.80%	0.399	10.20%	0.375
Small-Cap	11.60%	0.496	12.00%	0.379	11.30%	0.340

We conduct a similar analysis for the index return series and present the results for the in-sample period in Tables 4.17. Overall, the results for the index sample are muted when compared to those of the share sample. This was expected, however, given the expectation that much of the characteristics which are specific to individual shares would have been averaged out in the process of constructing the index returns.

Table 4.17 Results for Episodic Nonlinearity Tests conducted on the Index Sample over the In-Sample Period
Figures quoted for the number of significant windows for each portfolio represent the number of windows with a significant test statistic per share in the sample over the in-sample period. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Top 40	2 (4.00%)	0.448	2 (4.00%)	0.366	3 (6.00%)	0.345
Mid Cap	2 (4.00%)	0.470	4 (8.00%)	0.412	7 (14.00%)	0.449
Small Cap	4 (8.00%)	0.583	3 (6.00%)	0.465	4 (8.00%)	0.371

The results for the Top 40 Index over the in-sample period are similar, as expected, given that the Top 40 Index is constructed on a market capitalization basis and is known for being a particularly concentrated index.

We observe the previously identified increase in the occurrence of significant nonlinear H and T4 windows for the smaller indices, although we find that the Mid Cap index yields a marginally higher number of such significant periods than does the Small Cap index. This contrasts with the earlier finding of increasing incidence of nonlinear serial dependence as share size decreased, observed at the share level, where shares in the mid cap category had a higher incidence of nonlinear windows than the large cap category, but a smaller incidence than the small cap category. The number of significant H windows exceeds the threshold of potentially spurious significant observations we would expect at the five percent level for only the Mid Cap and Small Cap indices. The five percent threshold is exceeded for all four indices for the T4 statistic, however. This indicates that there is stronger evidence of fourth-order nonlinear serial dependence than third-order nonlinear serial dependence in the data.

The out-of-sample results, presented in Table 4.18, suggest no evidence of episodic linear serial dependence in the indices with only the Small Cap index showing one significant C window over the period. This window could however be spurious given the five percent significance level. Evidence of episodic nonlinear serial dependence is slightly higher over this period for the Top 40 Index but lower for the Mid Cap and Small Cap Index. This contrasts with the finds on the share level which indicated a marginal decrease in nonlinear windows for all size categories over the out-of-sample period.

Table 4.18 Results for Episodic Nonlinearity Tests conducted on the Index Sample over the Out-of-Sample Period

Figures quoted for the number of significant windows for each portfolio represent the number of windows with a significant test statistic per share in the sample over the in-sample period. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Top 40	0 (0.00%)	0.406	2 (10.00%)	0.471	2 (10.00%)	0.406
Mid Cap	0 (0.00%)	0.510	2 (10.00%)	0.383	1 (5.00%)	0.352
Small Cap	1 (5.00%)	0.520	2 (10.00%)	0.441	0 (0.00%)	0.237

Given the extreme distributional properties noted in the data series in Table 4.1, we now examine the impact of trimming outliers from the series on the findings of the episodic tests. Wild et al. (2008) argue that it is possible for outliers within the data to give rise to evidence of nonlinear behaviour and suggest that trimming can be used to adjust for this and also determine whether any such nonlinear behaviour is representative of a “deep structure” within the data series. We therefore trim the upper and lower 1%

values of each return series by replacing values above the 1st percentile with the 1st percentile value and values below the 99th percentile with the 99th percentile value. The descriptive statistics for the trimmed series are presented in Table 4.19.

Table 4.19 Descriptive Statistics for the Share Sample after 1% Trimming Adjustment Applied

Descriptive statistics are calculated based on the price returns for the individual shares in the in-sample period after adjusting for thin trading and then trimmed at the 1st and 99th percentiles. These statistics are then averaged across the shares within each of the size categories.

In-Sample Period

After The Thin Trading Adjustments of Atchison et al. (1987)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.058	-0.055	0.001	0.020	0.704	0.064
Mid-Cap	0.059	-0.056	0.001	0.019	1.788	0.091
Small-Cap	0.083	-0.076	0.001	0.024	3.031	0.237

After The Thin Trading Adjustments of Mlambo et al. (2003)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.059	-0.055	0.001	0.021	0.702	0.072
Mid-Cap	0.061	-0.056	0.001	0.019	1.738	0.116
Small-Cap	0.093	-0.080	0.002	0.025	2.982	0.304

Out-of-Sample Period

After The Thin Trading Adjustments of Atchison et al. (1987)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.078	-0.072	0.000	0.027	0.722	0.121
Mid-Cap	0.065	-0.060	0.000	0.022	1.179	0.119
Small-Cap	0.075	-0.067	0.000	0.023	2.568	0.280

After The Thin Trading Adjustments of Mlambo et al. (2003)

	Max	Min	Mean	Std. Dev	Kurtosis	Skew
Large-Cap	0.078	-0.072	0.000	0.027	0.722	0.121
Mid-Cap	0.064	-0.060	0.000	0.021	1.275	0.116
Small-Cap	0.076	-0.069	0.000	0.023	2.698	0.304

As is evident from the table when compared to Table 4.1, the large disparity in the excess kurtosis figures between the small, mid-cap and large-cap stocks over the in-sample period have been reduced significantly by the trimming procedure. We also note that there is little difference between the distributions of shares for either the Atchison et al. (1987) or Mlambo et al. (2003) adjustments within the out-of-sample period due to the limited number of thin trading days observed during the period. We would therefore expect the results of any tests on the two adjusted samples to yield similar results.

An examination of the impact of the trimming procedure on the index returns yields similar results as presented in Table 4.20.

Table 4.20 Descriptive Statistics for the Index Sample after 1% Trimming Adjustment Applied

Descriptive statistics are calculated based on the daily total returns for the indices in the in-sample period both before and after adjusting for thin trading.

In-Sample Period						
Index	Max	Min	Mean	Std. Dev	Kurtosis	Skew
All-Share Index	0.030	-0.030	0.001	0.011	0.375	-0.064
Top 40 Index	0.033	-0.033	0.001	0.012	0.363	-0.029
Mid-Cap Index	0.019	-0.022	0.001	0.007	1.176	-0.409
Small-Cap Index	0.015	-0.017	0.001	0.005	1.022	-0.417
Out-of-Sample Period						
Index	Max	Min	Mean	Std. Dev	Kurtosis	Skew
All-Share Index	0.052	-0.045	0.000	0.017	0.851	0.086
Top 40 Index	0.058	-0.049	0.000	0.018	0.888	0.120
Mid-Cap Index	0.026	-0.032	0.000	0.011	0.734	-0.321
Small-Cap Index	0.015	-0.022	0.000	0.007	1.156	-0.573

We re-run the episodic test procedure again and present the results for the share sample over the in-sample period in Tables 4.21 and 4.22. Individual share results are available in Appendix B.5.

Table 4.21 In-Sample Results for Episodic Nonlinearity Tests Sorted by Size (Adjustment of Atchison et al. (1987) with 1% trimming)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample over the in-sample period. Average statistics are computed by determining the mean statistic value over the 54 windows per share and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	8.9%	0.408	8.1%	0.400
Mid-Cap	11.7%	0.418	12.4%	0.405
Small-Cap	14.9%	0.407	14.9%	0.406

Table 4.22 In-Sample Results for Episodic Nonlinearity Tests Sorted by Size (Adjustment of Mlambo, et al (2003) with 1% trimming)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample. Average statistics are computed by determining the mean statistic value over the number of windows in each share series and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	7.43%	0.493	8.97%	0.405	7.88%	0.403
Mid-Cap	9.03%	0.503	10.21%	0.411	10.31%	0.393
Small-Cap	10.10%	0.526	11.00%	0.381	10.04%	0.362

It is clear that the incidence of significant non-linear windows has decreased for all three categories under both thin-trading adjustments. All measures, however, remain above the five percent expectation given our significance level. The relationship between the test statistics for the three size categories remains largely the same with small-cap stocks still demonstrating slightly lower average test statistics even though they have a higher incidence of actual non-linear windows on average. It should also be noted that the mean value of all the average test statistics for both thin trading adjustments have increased from those measured prior to applying the trimming procedure.

Similar results are observed for the out-of-sample period as summarized in Table 4.23 and 4.24. We again note a decrease in the percentage of significant windows for all test statistics considered under both thin trading adjustments. As with the in-sample period, however, all statistics present a number of significant windows over the period in excess of the 5% expected from our significance level. Based on the results we therefore conclude that there is deep evidence of both linear and non-linear serial dependence in the data during both sample periods.

Table 4.23 Out-of-Sample Results for Episodic Nonlinearity Tests Sorted by Size (Adjustment of Atchison et al. (1987) with 1% trimming)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample over the out-of-sample period. Average statistics are computed by determining the mean statistic value over the 50 windows per share and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	10.73%	0.427	6.57%	0.414
Mid-Cap	10.96%	0.413	9.89%	0.390
Small-Cap	12.18%	0.384	11.97%	0.346

Table 4.24 Out-of-Sample Results for Episodic Nonlinearity Tests Sorted by Size (Adjustment of Mlambo, et al (2003) with 1% trimming)

Figures quoted for the number of significant windows for each portfolio represent the average number of windows with a significant test statistic per share in the sample. Average statistics are computed by determining the mean statistic value over the number of windows in each share series and then aggregating this over the number of shares in the sample. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
Large-Cap	7.96%	0.488	10.38%	0.422	6.06%	0.412
Mid-Cap	8.42%	0.513	11.23%	0.412	10.03%	0.390
Small-Cap	10.50%	0.495	11.55%	0.384	10.92%	0.348

In unreported results, a similar investigation of the relationship between industry classification and nonlinear structure found similarly inconclusive results suggesting that there is no significant difference in the nonlinear dependencies of shares from different industries. An examination of the incidence of serial dependence (both linear and nonlinear) relative to various firm characteristics was complicated by the differences in the periodicity of the windows for shares using the Mlambo et al. (2003) adjustment. As non-trading days were eliminated from the data samples for these shares their window periods did not necessarily correspond meaning that it was not possible to group shares on the basis of characteristics as these change over time and such groupings would therefore need to be rebalanced periodically.

Investigating the timing of significant linear and nonlinear windows also yielded no indication of key common calendar periods of dependence as there are significant windows for all tests and size categories within all 54 windows considered. While some windows are significant for a greater number of shares than others these windows still represent significance on average for less than 30% of the shares within each category or less than 20% of the shares within the sample as a whole. While it is thus clear that there are periods of linear and nonlinear predictability in share returns on the JSE the timing and incidence of these predictable periods is less obvious and may suggest an area of further study.

Finally, we re-examine the evidence of episode serial dependence for the index sample and present the results in Table 4.25. Over the in-sample period the Top 40 Index has fewer significant T4 windows indicating that the windows previously identified may be due to the presence of outliers in the data. The results for the Mid Cap and Small Cap indices are largely unchanged over the period as are the results for all four indices over the out-of-sample period.

Table 4.25 Results for Episodic Nonlinearity Tests conducted on the Index Sample over the In-Sample Period (1% Trimming)

Figures quoted for the number of significant windows for each portfolio represent the number of windows with a significant test statistic per share in the sample over the in-sample period. All test statistics are evaluated at the 5% level, obtained via 1000 bootstrap replications.

In-Sample

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
ALSI	2 (4.00%)	0.452	2 (4.00%)	0.397	2 (4.00%)	0.351
Top 40	2 (4.00%)	0.455	2 (4.00%)	0.383	2 (4.00%)	0.358
Mid Cap	3 (6.00%)	0.477	5 (10.00%)	0.412	7 (14.00%)	0.439
Small Cap	4 (8.00%)	0.594	4 (8.00%)	0.503	4 (8.00%)	0.369

Out-of-Sample

	Sig C Windows	Avg C Statistic	Sig H Windows	Avg H Statistic	Sig T4 Windows	Avg T4 Windows
ALSI	0 (0.00%)	0.386	2 (10.00%)	0.474	2 (10.00%)	0.406
Top 40	0 (0.00%)	0.400	2 (10.00%)	0.485	2 (10.00%)	0.409
Mid Cap	0 (0.00%)	0.503	2 (10.00%)	0.396	1 (5.00%)	0.369
Small Cap	1 (5.00%)	0.419	2 (10.00%)	0.464	1 (5.00%)	0.255

4.5 Conclusions

Tests for linear and nonlinear serial dependence in share returns on the JSE were performed on both individual shares and indices. The share samples were adjusted for thin trading by using the techniques of both Atchison et al. (1987) and Mlambo et al. (2003).

Evidence of linear serial dependence using autocorrelation and partial autocorrelation tests are inconclusive for the share sample with at least one share demonstrating a significant coefficient for each coefficient up to the tenth lag. We do note, however, that the majority of significant coefficients in each size category are found at the first lag and the incidence of significant coefficients at this lag increases as share size decreases. Given that the data is adjusted for thin trading, this would not appear to be a function of thin trading but it should be noted that a significant number of true zero returns remain in shares from both the mid and small cap categories and this may result in the greater incidence of observed significant coefficients.

Evidence of linear serial dependence is muted over the out-of-sample period and we find that while the mid cap and small categories continue to demonstrate greater incidences of significance at the first lag, the large cap stocks show greater evidence of linear serial dependence at the second and third lags. Similar findings were observed for the index returns sample.

Although the adjustment of Mlambo et al. (2003) yielded marginally greater evidence of nonlinear dependence in both the battery of tests and episodic tests the results were, for the most part, similar. The choice of thin-trading adjustment therefore had minimal impact on the results.

The findings of the episodic tests indicate that, although there is evidence of both significant nonlinear and linear dependence in share price returns on the JSE, these incidences are sporadic and, as such, return series are characterized for the most part by long periods of pure white noise. This implies that while there are periods of predictable serial dependence in returns they are not consistent over time which complicates the modelling of returns for predictive purposes. Although it was found that there were marginally lower incidences of linear and nonlinear windows in the returns in the out-of-sample period, which was characterized by significant market instability, there were still strong indications that both forms of serial dependence were present in the data during this period.

The findings for the index sample were similar for the most part but there were fewer episodes of both linear and nonlinear serial dependence observed for the index returns. This was expected as the aggregate nature of the index returns may obscure the characteristics of their individual constituent stocks.

An investigation of the relationship between nonlinear structure and share size, motivated by the findings of Skaradzinski (2003), was inconclusive. Even though small-cap shares demonstrated a marginally larger number of significant linear and nonlinear windows on average than larger-cap shares, their average test statistic was lower than that for the other size categories (except in the case of the linear statistic which was marginally higher).

It was not possible to link the periods of linear and nonlinear dependence identified to specific market events but results using the thin-trading adjustment of Mlambo et al. (2003) indicate that periods of nonlinear dependence arise more frequently than periods of linear dependence. This supports the theories of Appiah-Kusi and Menyah (2003) and Mlambo et al (2007) which suggest that nonlinear models may be more appropriate in modelling returns for African markets. These findings do not, however, provide any indication of what gives rise to these periods of linear and nonlinear dependence in the share returns data.

As indicated by Ashley and Patterson (2002) it is not possible to link a specific test for nonlinear serial dependence to an appropriate nonlinear model. The process is therefore one of trial-and-error and we attempt to fit a range of linear and nonlinear models to the data in the next chapter in order to compare the predictive power of both approaches.

5. A Comparison of Linear and Nonlinear Univariate Predictive Models

The results from Chapter 4 indicate the presence of both linear and nonlinear serial dependence in the market indices and individual shares considered, but suggest that a nonlinear data generation process may be better suited to modelling share returns than a linear process. This is consistent with findings in other markets¹¹.

The international literature on the perceived forecasting benefits of nonlinear models over linear models has however yielded mixed results. Clements and Henry (1988) demonstrate that a superior fit for a nonlinear model in-sample may not yield superior out-of-sample performance. Diebold and Nason (1990) argue that nonlinearities detected in data may result from outliers or structural breaks which cannot necessarily be exploited out-of-sample. They suggest further that nonlinearities related to conditional mean may not be significant enough to yield an improvement in forecasting.

Numerous studies have found that asset return series are heteroskedastic, exhibiting time-varying volatility clustering. The autoregressive conditional heteroskedastic (ARCH) model of Engle (1982) and the generalized (GARCH) variant of Bollerslev (1986) both attempt to account for these changes in volatility over time by relating the conditional variance of the series to past variances and observations in the data series. This family of models has attracted substantial interest in the literature. Numerous variations on the basic GARCH framework have been proposed subsequent to the work of Engle (1982) and Bollerslev (1986) but the majority of studies have found that they fail to provide a significant improvement on the basic GARCH(1,1) model and this specification is most often employed in share volatility forecasting models (Ashley and Patterson, 2009).

In asset pricing theory we derive a theoretical relationship between risk and return which is encapsulated in asset pricing models such as the Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT). Engle, Lilien and Robins (1985) propose that the yield on long-term bonds are dependent on their conditional variance and that, where the variance of the return generation process is not constant, the conditional expectation of the bond yields is a linear function of the conditional variance. This idea has been extended to share returns and models which provide for heteroscedasticity in the mean equation are referred to as GARCH-in-mean (GARCH-M) models.

Studies have also considered the impact of regime changes on the parameters of predictive models. Hamilton (1989) proposed the possibility of using hidden-layer Markov chains as a potential tool for fitting appropriate time series econometric models in the presence of regime changes. Much of the

¹¹ See Hinich and Patterson (2000), Bonilla, Romero-Meza and Hinich, 2006 and Skaradzinski (2003) amongst others.

application of so-called Markov-switching (MS) models has been in business cycle identification but these models have also found some use in the prediction of asset returns, particularly where the underlying regimes represent “bull” and “bear” markets.

This chapter constructs a range of linear and nonlinear models with the goal of assessing whether (a) nonlinear models provide a better model fit to the returns data than linear models and (b) nonlinear models provide a forecasting improvement over linear models. It should be noted that the analysis is conducted over both the relatively calm in-sample period and the tumultuous period of the market crisis and we expect forecast model performance to be impacted over the latter period. Our examination is therefore not only one of identifying the best model fit in-sample, but also of identifying whether any perceived improvements resulting from nonlinear specifications persist through the market crisis. Although numerous multivariate studies have been conducted using the GARCH and MS classes of models, Goyal and Welch (2007) argue that historical average excess share returns forecast future excess stock returns better than regressions of excess returns on exogenous predictor variables. We therefore restrict our analysis to the univariate case.

5.1 Methodology

We construct a range of linear and nonlinear models for forecasting the index return series examined in Chapter 4. All analysis is conducted using Oxmetrics 6.1. Models are constructed using a recursive procedure with an expanding window. The recursive procedure estimates the model over the in-sample period and then uses it to forecast the first observation in the out-of-sample period. This observation is then added to the estimation sample and the model is re-estimated and used to forecast the second observation in the out-of-sample period. This process is repeated until the final observation in the out-of-sample period has been forecast.

In order to assess the forecast accuracy of the models without the confounding influence of the market crisis we partition the in-sample period into two separate samples with the first period encompassing 1 February 2002 to 31 July 2005 and the second running from 1 August 2005 to 31 July 2007. The modified in-sample period for the analysis to follow is therefore the first period. We then use the models constructed over this period to forecast over the period 1 August 2005 to 31 July 2007 (the ‘stable’ period) and over the period 1 August 2007 to 31 December 2009 (the ‘crisis’ period) using the recursive expanding window procedure outlined above. This allows us to assess the relative performance of the linear and nonlinear models over both a stable and crisis period.

The section which follows provides a brief overview of the models employed in the analysis.

5.1.1 AR(p) and ARMA(p,q) Models

An autoregressive model relates the current value of a series to one or more past observations in that series. We define an autoregressive model of order p of a series Y by

$$Y_t = c + \sum_{i=1}^p \varphi Y_{t-i} + \varepsilon_t \quad (5.1)$$

where c is a constant, $\varphi_1 \dots \varphi_p$ are the parameters of the model and ε_t is an error term which is white noise.

An autoregressive moving average model, in contrast, provides for both an autoregressive and a moving average component in modelling the underlying data generation process, where the moving average component is defined by

$$Y_t = \mu + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (5.2)$$

where μ is the expectation of Y_t , $\theta_1 \dots \theta_q$ are the parameters of the model and ε_t is an error term which is white noise.

Given equations 5.1 and 5.2, an ARMA(p,q) model is then defined by

$$Y_t = c + \sum_{i=1}^p \varphi Y_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (5.3)$$

In determining the order of the autoregressive process (p) for our model we consider that the partial autocorrelation of an AR(p) process becomes zero at lag $p+1$ or greater. Similarly, the autocorrelation function of a MA(q) process becomes zero at lag $q+1$ or greater.

The autocorrelation and partial autocorrelation analysis in Chapter 4 indicates significant autocorrelation and partial autocorrelation coefficients for the Top 40 index at the first lag. The Mid Cap and Small Cap indices both demonstrate significant coefficients at higher lags indicating that higher order models may be

more appropriate for these indices. An examination of the AIC measure for a range of AR(p) and ARMA(p,q) models for the three indices demonstrates only a marginal improvement in the score at higher lags. We therefore consider the AR(1) and ARMA(1,1) models as a parsimonious model fit for these indices. We find further that the ARMA(1,1) model provides a poorer fit than the AR(1) model for the Top 40 and Mid Cap indices but provides an improved fit over the AR(1) model for the Small Cap Index. The analysis and discussion that follows therefore uses the AR(1) model as the baseline linear model for the Top 40 and Mid Cap indices and the ARMA(1,1) model specification the baseline linear model for the Small Cap index.

5.1.2 ARCH(q) and GARCH(p,q) Models

AR(p) and ARMA(p,q) models assume a constant variance in the time series (homoscedasticity) although empirical findings have indicated that share return series exhibit time-varying volatility. The ARCH(q) model framework proposed by Engle (1982) provides for heteroscedasticity in data series by relating the conditional variance of the error term at time t to the squared error term at time $t - 1$.

We define the ARCH(1) model for a series Y by

$$Y_t = \mu + \sqrt{h_t} \varepsilon_t \quad (5.4)$$

Where μ is the constant (conditional and unconditional) mean of Y , h_t is the conditional variance of ε_t , $\varepsilon_t = h_t z_t$ and z_t is a stochastic portion of the error which is i.i.d with an expected mean of zero and standard deviation of 1.

The conditional variance is then given by

$$h_t = \gamma_1 + \gamma_2 \varepsilon_{t-1}^2 \quad (5.5)$$

The conditional variance must be non-negative and therefore $\alpha_0 > 0$ and $\alpha_i \geq 0$. The ARCH(q) model therefore assumes an autoregressive process for the error variance.

In contrast, the basic GARCH(1,1) model of Bollerslev (1986) assumes an ARMA(1,1) model for the error variance and the conditional variance is therefore defined by

$$h_t = \gamma_1 + \gamma_2 h_{t-1} + \gamma_3 \varepsilon_{t-1}^2$$

(5.6)

where the conditional variance must again be non-negative and therefore $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$. It is assumed for both models that the conditional error distribution is normal.

In asset pricing theory we accept a theoretical relationship between risk and return which is encapsulated in asset pricing models such as the Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT). Engle, Lilien and Robins (1985) propose that the yield on long-term bonds are dependent on their conditional variance and that, where the variance of the return generation process is not constant, the conditional expectation of the bond yields is a linear function of the conditional variance. This idea has been extended to share returns and models which provide for heteroscedasticity in the mean equation are referred to as GARCH-in-mean (GARCH-M) models.

Assuming a GARCH-in-mean relationship in the data we can restate equation 5.4 as

$$Y_t = \mu + \delta\sqrt{h_t} + \sqrt{h_t}\varepsilon_t \quad (5.10)$$

where $\delta\sqrt{h_t}$ is the innovation in the conditional mean resulting from the conditional variance, and is sometimes specified as δh_t or $\delta \log h_t$.

We construct AR(1)-GARCH(1,1)-in-mean and ARMA(1,1)-GARCH(1,1)-in-mean models in order to observe the influence of allowing for heteroscedasticity in modelling the conditional mean in the return generation process, where the AR(1)-GARCH(1,1)-in-mean model is specified as

$$Y_t = c + \sum_{i=1}^p \varphi Y_{t-i} + \delta\sqrt{h_t} + \sqrt{h_t}\varepsilon_t \quad (5.11)$$

and the ARMA(1,1)-GARCH(1,1)-in-mean model is defined by

$$Y_t = c + \sum_{i=1}^p \varphi Y_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \delta\sqrt{h_t} + \sqrt{h_t}\varepsilon_t \quad (5.12)$$

One restriction associated with the GARCH model is that the response of the conditional variance is symmetrical with regards to volatility shocks. This implies that only the size of the shock, and not the sign, is relevant. However, there is extensive evidence for the existence of a leverage effect where negative returns shocks are correlated with larger increases in volatility than positive returns shocks¹².

In order to account for this shortcoming we also examine the GARCH model of Glosten, Jagannathan and Runkle (1993) (GJR-GARCH). The GJR-GARCH model is a threshold GARCH model similar to the TGARCH model of Rabemananjara and Zakoian (1993) but models the conditional variance rather than the conditional standard deviation.

We define the conditional variance for the GJR-GARCH model by

$$h_t = \gamma_1 + \gamma_2 h_{t-1} + \gamma_3 \varepsilon_{t-1}^2 + \gamma_4 I(\varepsilon_{t-1} < 0) \varepsilon_{t-1}^2 \quad (5.13)$$

where $I(\cdot)$ is an indicator function that takes a value of 1 when $\varepsilon_{t-1} < 0$ and 0 otherwise.

It has been noted that when the error terms in the GARCH model are assumed to be normally distributed, the leptokurtic nature of asset return series leads to a normally distributed conditional error distribution but a leptokurtic unconditional error distribution. Bollerslev (1987) suggests that the assumption of normally-distributed conditional errors may not account for this observed leptokurtic behaviour. Bollerslev (1987) therefore proposed a GARCH(p,q) model with conditionally student's t-distributed errors as an alternative. We compare the GARCH models constructed using both distributions.

5.1.3 Markov Switching Models

It has been observed empirically that financial and economic series are characterized by regimes which influence the behaviour the underlying data generation process. Such regimes are typically associated with significant recurring events such as market crises and changes in government policy. The implication of such regimes is that the data generation process may differ in one or more parameters depending on the regime into which the observations fall.

Markov-switching models were proposed in econometrics by Goldfeld and Quandt (1973) and extended to the time series context by Hamilton (1989). Consider a basic AR(1) model specification given by

¹² For the South African market, Samouilhan and Shannon (2008) observed the leverage effect in the volatility of returns on the Top 40 Index using the GJR-GARCH(1,1) model.

$$Y_t = c + \varphi Y_{t-1} + \varepsilon_t \quad (5.14)$$

Assume further that the data series is characterized by the influence of two regimes. We may expect under such conditions that the constant, c , autoregressive coefficient, φ , or the error variance, ε_t , may all vary depending on the state of the regime, s_t .

We may therefore restate equation 5.14 as

$$Y_t = c_{s_t} + \varphi_{s_t} Y_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \sim N[0, \sigma_{s_t}^2] \quad (5.15)$$

We assume that s_t is a realization of a first-order Markov chain where the probability of switching to state i between t and $t + 1$ given that the market is in state l is given by

$$\Pr(s_t = i | s_{t-1} = l) = p_{il} \quad (5.16)$$

s_t is never observed directly but instead inferred from the observed behaviour of Y_t . For purposes of our analysis we assume a two-state process which results in

$$Y_t = c_1 + \varphi_1 Y_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \sim N[0, \sigma_1^2] \quad (5.17)$$

and

$$Y_t = c_2 + \varphi_2 Y_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \sim N[0, \sigma_2^2] \quad (5.18)$$

and the probabilities for transferring between regimes is defined by

$$\begin{aligned} \Pr(s_t = 1 | s_{t-1} = 1) &= p_{11} & \Pr(s_t = 2 | s_{t-1} = 1) &= 1 - p_{11} \\ \Pr(s_t = 1 | s_{t-1} = 2) &= 1 - p_{22} & \Pr(s_t = 2 | s_{t-1} = 2) &= p_{22} \end{aligned}$$

We construct Markov-switching AR(1) and ARMA(1,1) models with varying constants and coefficients for the autoregressive and moving average terms, and provide for two specifications – one with homoscedasticity where the error variance is held constant and one with heteroscedasticity where the error variance is dependent on the regime state variable.

5.2 Model Evaluation

5.2.1 Assessment of In-Sample Model Fit

We assess the in-sample model fit for each of the constructed models using the Akaike Information Criterion (AIC). The AIC is a measure of entropy representing the relative information lost when using an approximate model to describe the true relationship between a set of variables. The metric attempts to find the model that best explains the data with a minimum of free parameters and thus discourages overfitting of the model.

The AIC is defined as follows

$$AIC = -2 \ln L + 2k \quad (5.18)$$

where L is the maximized value of the likelihood function for the estimated model and k is the number of parameters in the model. The first term in the equation therefore represents the model fit while the second is the penalty for overfitting.

5.2.2 Measuring Model Forecast Accuracy

Further to this we employ a range of tests to compare the out-of-sample forecasting accuracy of the models constructed. We define the forecast error for the observation at time t as

$$e_t = Y_t - F_t \quad (5.19)$$

where Y_t is the observed value of series Y at time t and F_t is the forecast point estimate of the model at time t .

These forecast errors are evaluated on the basis of the root mean square error (RMSE), mean absolute error (MAE) and success ratio (SR) measures. Due to the presence of a number of zero observations in the data series we omit the mean percentage error (MPE).

Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N e_t^2}$$
(5.20)

The RMSE indicates the absolute fit of the model to the data and is expressed in the same units of measurement as the dependent variable.

Mean Absolute Error (MAE)

$$MAE = \frac{1}{N} \sum_{t=1}^N |e_t|$$
(5.21)

The mean absolute error is an average of the absolute forecast errors of a model and measures the average magnitude of the errors without consideration for their sign. The MAE is generally less susceptible to the presence of outliers than the RMSE.

Success Ratio (SR)

$$SR = \frac{1}{N} \sum_{t=1}^N I_{\{Y_t F_t > 0\}}$$
(5.22)

where $I_{\{Y_t F_t > 0\}}$ is an indicator variable that takes a value of 1 when Y_t and F_t have the same sign. The success ratio measure the proportion of times for which the sign of the return series and the forecast series is correct. Guidolin, Hyde and McMillan (2007) argue that the forecast accuracy of the sign of daily returns is of greater importance than the accuracy of the magnitude of the forecast, given the relatively small size of the observations.

5.3 Results

A summary of the in-sample fit for the constructed models is presented in Table 5.1. It is clear from the AIC scores that the nonlinear specifications provide a significant improvement over the AR(1) and ARMA(1,1) models for all three indices. We note further that the GARCH models generally demonstrate higher log-likelihood and AIC values than the MS models, except for the MS heteroscedastic model constructed for the small cap index.

Table 5.1 Summary of Model Fit and Coefficients for the Top 40 Index over the In-Sample Period

We present the AIC statistics for each of the linear and nonlinear models constructed for the Top 40 Index data series. In addition, the coefficients and associated t-statistics for each model are provided, with t-values significant at the 5% level highlighted in bold.

	Model fit for the Top 40 and Mid Cap Indices with an AR(1) baseline model				Model fit for the Small Cap Index with an ARMA(1,1) baseline model		
	Top 40 Index		Mid Cap Index		Small Cap Index		
	AIC	Log-likelihood	AIC	Log-likelihood	AIC	Log-likelihood	
AR(1)	-8132.62	4069.31	-9601.90	4803.95	ARMA(1,1)	-10329.55	5168.78
AR(1)-GARCH (1,1)-in mean (gauss)	-8305.02	4158.51	-9942.04	4977.02	ARMA(1,1)-GARCH (1,1)-in mean (gauss)	-10501.96	5257.98
AR(1)-GARCH (1,1)-in mean (t dist)	-8327.30	4170.65	-10011.64	5012.82	ARMA(1,1)-GARCH (1,1)-in mean (t dist)	Did not converge	
AR(1)-GJR-GARCH (1,1)-in mean (gauss)	-8336.78	4175.39	-9984.74	4999.37	ARMA(1,1)-GJR-GARCH (1,1)-in mean (gauss)	-10529.70	5272.85
AR(1)-GJR-GARCH (1,1)-in mean (t dist)	-8351.68	4183.84	-10031.40	5023.70	ARMA(1,1)-GJR-GARCH (1,1)-in mean (t dist)	Did not converge	
MS AR(1) homoscedastic	-8171.85	4092.92	-9716.05	4865.02	MS ARMA(1,1) homoscedastic	-10503.38	5260.69
MS AR(1) heteroscedastic	-8290.06	4153.03	-9970.79	4993.39	MS ARMA(1,1) heteroscedastic	-10579.31	5299.65

Table 5.2 presents a summary of the performance of the forecasts generated by the models for the Top 40 Index over the stable and market crisis periods. We note that the forecast errors for the basic GARCH models over the stable period are lower than those for the GJR-GARCH models and the success ratios are also marginally higher. The use of the t-distribution for both the basic GARCH and GJR models also results in higher RMSE and MAE measures and lower success ratios. This seems to confirm the findings in the literature that extensions of the basic GARCH(1,1) model fail to outperform out-of-sample, even when they have fitted the data better in-sample (indeed, this may be due to such models overfitting the data in-sample).

The MS models achieve higher forecast errors and lower success ratios than both the GARCH models and the AR(1) model, suggesting that these models may either have over-fitted the data or may have been inappropriate for modelling the data generation process over the sample period. We note that the MS model provides a means to capture the influence of regime changes on the data generation process. The in-sample and stable periods both occurred over a period considered to be a ‘bull’ run and the probabilities attached to the “bull” and “bear” states for both models were significantly skewed in favour of the positive regime (the probability of the market being in an ‘up’ state was in excess of 96% for both MS models). On this basis it is suggested that the MS models may have performed better over a longer sample period, encompassing a greater number of regimes than is present in this study.

Table 5.2 Summary of Predictive Accuracy Measures for Top 40 Index

We construct a range of optimized linear and nonlinear models over the in-sample period using the AIC criterion. These models are then used to forecast over the out-of-sample period on the basis of recursive expanding window procedure. The root mean squared error (RMSE), mean absolute error (MAE) and success ratio (SR) are provided for each model.

Stable Period: 1 Feb 2005 to 31 July 2007

Model	RMSE	MAE	SR
AR(1)	1.330367	1.009067	55.289
AR(1)-GARCH (1,1)-in mean (gauss)	1.327972	1.005538	56.886
AR(1)-GARCH (1,1)-in mean (t dist)	1.329029	1.007197	56.287
AR(1)-GJR-GARCH (1,1)-in mean (gauss)	1.329220	1.009027	55.289
AR(1)-GJR-GARCH (1,1)-in mean (t dist)	1.329439	1.009432	55.289
MS AR(1) homoscedastic	1.339147	1.016274	52.295
MS AR(1) heteroscedastic	1.330357	1.009554	55.090

Crisis Period: 1 August 2007 to 31 December 2009

Model	RMSE	MAE	SR
AR(1)	2.038808	1.523201	52.893
AR(1)-GARCH (1,1)-in mean (gauss)	2.043272	1.526466	51.405
AR(1)-GARCH (1,1)-in mean (t dist)	2.043140	1.527296	50.744
AR(1)-GJR-GARCH (1,1)-in mean (gauss)	2.039704	1.523697	51.901
AR(1)-GJR-GARCH (1,1)-in mean (t dist)	2.040075	1.524312	50.909
MS AR(1) homoscedastic	2.037853	1.524918	50.083
MS AR(1) heteroscedastic	2.051164	1.527264	52.893

The results over the crisis period reflect the decrease in forecasting ability for all the models considered. The forecast error measures for all three models have increased significantly, while the success ratios have decreased. The improvement in forecast errors and the success ratio previously observed during the stable period are no longer present. The baseline AR(1) model therefore provides the lowest forecast errors over this period, although it should be emphasized that the forecast errors over the period are significant, indicating a decline in forecast accuracy for all models over the market crisis period. This highlights one of the shortcomings of nonlinear models. There is substantial evidence in the literature¹³ that such models tend to overfit the data in-sample and are sensitive to structural changes which may not persist out-of-sample.

Contrary to the findings for the stable period we find that the homoscedastic MS model outperforms the GARCH variants over the market crisis period, and indeed has the lowest RMSE over the period.

We tabulate the top three models by each of the metrics considered in Table 5.3 in order to compare relative model performance more readily. The table neatly summarizes our findings – the basic GARCH(1,1)-in-mean model with a Gaussian distribution for the error variance has the lowest forecast error over the stable period for both the RMSE and MAE and also demonstrates the highest success rate. The GARCH(1,1)-in-mean model with t-distributed errors ranks second, while the GJR-GARCH(1,1) model ranks third. Over the market crisis only the homoscedastic MS model provides a lower RMSE than the AR(1) model, while the latter model has the lowest MAE and highest success ratio of the models considered.

Table 5.3 Top 3 models by Forecast Performance Measure for the Top 40 Index

The table below provides a summary of the top 3 models by forecast performance measure for the Top 40 Index. Models are ranked in increasing order of RMSE and MAE and in decreasing order of SR. Due to space considerations, the order of the ARMA and GARCH terms is not specified in the table. All AR models are of the form AR(1), ARMA models are of the form ARMA(1,1) and all GARCH models are of the form GARCH(1,1).

Stable Period: 1 Feb 2005 to 31 July 2007

	1	2	3
RMSE	AR-GARCH-in mean (gauss)	AR-GARCH-in mean (t dist)	AR-GJR-GARCH-in mean (gauss)
MAE	AR-GARCH-in mean (gauss)	AR-GARCH-in mean (t dist)	AR-GJR-GARCH-in mean (gauss)
SR	AR-GARCH-in mean (gauss)	AR-GARCH-in mean (t dist)	Multiple

Crisis Period: 1 August 2007 to 31 December 2009

	1	2	3
RMSE	MS AR homoscedastic	AR	AR-GJR-GARCH-in mean (gauss)
MAE	AR	AR-GJR-GARCH-in mean (gauss)	AR-GJR-GARCH-in mean (t dist)
SR	AR and MS AR heteroscedastic		AR-GJR-GARCH-in mean (gauss)

¹³ See Diebold and Nason (1999)

We conduct a similar analysis for the Mid Cap Index and present a summary of the forecast accuracy measures for the models over the out-of-sample period in Table 5.4.

Table 5.4 Summary of Predictive Accuracy Measures for Mid Cap Index

We construct a range of optimized linear and nonlinear models over the in-sample period using the AIC criterion. These models are then used to forecast over the out-of-sample period on the basis of recursive expanding window procedure. The root mean squared error (RMSE), mean absolute error (MAE) and success ratio (SR) are provided for each model.

Stable Period: 1 Feb 2005 to 31 July 2007

Model	RMSE	MAE	SR
AR(1)	0.752946	0.553415	66.267
AR(1)-GARCH (1,1)-in mean (gauss)	0.750900	0.549502	65.669
AR(1)-GARCH (1,1)-in mean (t dist)	0.751412	0.550405	65.669
AR(1)-GJR-GARCH (1,1)-in mean (gauss)	0.756366	0.555584	64.870
AR(1)-GJR-GARCH (1,1)-in mean (t dist)	0.754892	0.554131	64.870
MS AR(1) homoscedastic	0.753489	0.553478	65.070
MS AR(1) heteroscedastic	0.757662	0.555432	63.673

Crisis Period: 1 August 2007 to 31 December 2009

Model	RMSE	MAE	SR
AR(1)	1.151033	0.85352	57.686
AR(1)-GARCH (1,1)-in mean (gauss)	1.152874	0.85639	57.686
AR(1)-GARCH (1,1)-in mean (t dist)	1.153794	0.85647	57.686
AR(1)-GJR-GARCH (1,1)-in mean (gauss)	1.156042	0.86112	58.182
AR(1)-GJR-GARCH (1,1)-in mean (t dist)	1.154149	0.85921	57.851
MS AR(1) homoscedastic	1.151717	0.85514	57.521
MS AR(1) heteroscedastic	1.154714	0.86243	55.702

The forecast errors for the models constructed using the Mid Cap Index sample are smaller over the stable period than those for the Top 40 Index sample, consistent with the lower average daily returns in the Mid Cap sample. We also note that the success ratios for both the linear and nonlinear models are significantly higher than those for the Top 40 Index which may be indicative of the greater linear and nonlinear serial dependence observed in the tests in Chapter 4. Our findings regarding the relative ranking of the models on the basis of lowest forecast error and highest success rate is consistent with those of the larger-cap sample. We again find that the basic GARCH(1,1) models with Gaussian errors outperform the GJR-GARCH(1,1) models and the GARCH models which assume a t-distribution for the errors.

We also observe that the MS models underperform relative to the basic GARCH(1,1) models with Gaussian errors but that the MS homoscedastic model demonstrates lower forecast errors than the GJR-GARCH(1,1) models.

Over the crisis period we note a similar phenomenon to that observed for the Top 40 Index. The forecast accuracy of the nonlinear models relative to the AR(1) model worsens with the result that AR(1) model demonstrates the lowest forecast errors over the period. Interestingly, the GJR-GARCH(1,1) models which underperformed over the stable period demonstrate the highest success ratios over the market crisis while the MS homoscedastic models provide lower forecast errors than the GARCH(1,1) variants.

We summarize the top three models in each category in Table 5.5. As indicated, over the stable period the GARCH(1,1) models with Gaussian errors demonstrates the lowest forecast errors, followed closely by the GARCH(1,1) models with t-distributed errors and the baseline AR(1) model. The AR(1) model also has the highest success ratio over the stable period.

Table 5.5 Top 3 models by Forecast Performance Measure for the Mid Cap Index

The table below provides a summary of the top 3 models by forecast performance measure for the Mid Cap Index. Models are ranked in increasing order of RMSE and MAE and in decreasing order of SR.

Stable Period: 1 Feb 2005 to 31 July 2007

	1	2	3
RMSE	AR-GARCH-in mean (gauss)	AR-GARCH-in mean (t dist)	AR
MAE	AR-GARCH-in mean (gauss)	AR-GARCH-in mean (t dist)	AR
SR	AR	AR-GARCH-in mean (gauss)	AR-GARCH-in mean (t dist)

Crisis Period: 1 August 2007 to 31 December 2009

	1	2	3
RMSE	AR	MS AR homoscedastic	AR-GARCH-in mean (gauss)
MAE	AR	MS AR homoscedastic	AR-GARCH-in mean (gauss)
SR	AR-GJR-GARCH-in mean (gauss)	AR-GJR-GARCH-in mean (t dist)	AR

Lastly, we examine the results of the out-of-sample forecasts for the Small Cap Index. As noted earlier, the ARMA(1,1) models provide a noticeable improvement on the in-sample model scores over the AR(1) models for this data series. As such we state the results for the ARMA(1,1) models in Table 5.6. The GARCH(1,1)-in-mean models which assumed a Student's t-distribution for the errors failed to converge and are omitted from the analysis as a result.

Table 5.6 Summary of Predictive Accuracy Measures for Small Cap Index

We construct a range of optimized linear and nonlinear models over the in-sample period using the AIC criterion. These models are then used to forecast over the out-of-sample period on the basis of recursive expanding window procedure. The root mean squared error (RMSE), mean absolute error (MAE) and success ratio (SR) are provided for each model.

Stable Period: 1 Feb 2005 to 31 July 2007

Model	RMSE	MAE	SR
ARMA(1,1)	0.5948	0.4207	66.068
ARMA(1,1)-GARCH (1,1)-in mean (gauss)	0.5933	0.4175	67.665
ARMA(1,1)-GJR-GARCH (1,1)-in mean (gauss)	0.5974	0.4222	66.667
MS ARMA(1,1) homoscedastic	0.6128	0.4331	65.469
MS ARMA(1,1) heteroscedastic	0.6142	0.4405	65.070

Crisis Period: 1 August 2007 to 31 December 2009

Model	RMSE	MAE	SR
ARMA(1,1)	0.7335	0.5041	61.28
ARMA(1,1)-GARCH (1,1)-in mean (gauss)	0.7404	0.5127	57.44
ARMA(1,1)-GJR-GARCH (1,1)-in mean (gauss)	0.7399	0.5118	58.24
MS ARMA(1,1) homoscedastic	0.7382	0.5278	60.17
MS ARMA(1,1) heteroscedastic	0.7677	0.5690	60.17

We observe that the success rates for all the models for the Small Cap Index are marginally higher than those observed for the Mid Cap Index while the forecast errors are smaller on average, again consistent with the smaller returns of the Small Cap Index relative to the Top 40 and Mid Cap indices. The results for the Small Cap Index mimic those of the Top 40 and Mid Cap indices. The basic GARCH(1,1) model again demonstrates lower forecast errors and higher success ratios than the GJR-GARCH(1,1) model over the stable period. In addition, the MS models again underperform relative to both the GARCH and the ARMA(1,1) models.

As expected, over the market crisis period the ARMA(1,1) model outperforms the nonlinear alternatives with the MS homoscedastic model again outperforming the GARCH alternatives over this period. Table 5.7 tabulates the top three models by forecast performance measure.

Table 5.7 Top 3 models by Forecast Performance Measure for the Small Cap Index

The table below provides a summary of the top 3 models by forecast performance measure for the Small Cap Index. Models are ranked in increasing order of RMSE and MAE and in decreasing order of SR.

Stable Period: 1 Feb 2005 to 31 July 2007

	1	2	3
RMSE	ARMA-GARCH-in mean (gauss)	ARMA	ARMA-GJR-GARCH-in mean (gauss)
MAE	ARMA-GARCH-in mean (gauss)	ARMA	ARMA-GJR-GARCH-in mean (gauss)
SR	ARMA-GARCH-in mean (gauss)	ARMA-GJR-GARCH-in mean (gauss)	ARMA

Table 5.7 Top 3 models by Forecast Performance Measure for the Small Cap Index (Continued)

The table below provides a summary of the top 3 models by forecast performance measure for the Small Cap Index. Models are ranked in increasing order of RMSE and MAE and in decreasing order of SR.

Crisis Period: 1 August 2007 to 31 December 2009

	1	2	3
RMSE	ARMA	MS ARMA homoscedastic	ARMA-GJR-GARCH-in mean (gauss)
MAE	ARMA	ARMA-GJR-GARCH-in mean (gauss)	ARMA-GARCH-in mean (gauss)
SR	ARMA	MS homoscedastic and heteroscedastic	

5.4 Conclusions

The results indicate that nonlinear model specifications provide a better fit to the data for all three index series over the in-sample period. We find that the GARCH-in-mean models fit the in-sample data better than the MS models and that the GJR-GARCH(1,1) model and the use of the Student's t-distribution of the error terms over the Gaussian distribution both improve the model fit. Although the MS models are found to have lower AIC scores than the linear models over the in-sample period, the GARCH variants are found to provide a better fit over the period.

Over the stable portion of the out-of-sample period we find that the basic GARCH(1,1)-in-mean model with normally-distributed errors have both the lowest forecast errors and highest success ratios of the models considered. This indicates that there is a definite benefit to accounting for potential nonlinear influences on share returns during stable market periods. In addition, the findings lend support to the use of the basic GARCH(1,1) model specification over the numerous extensions that pervade the literature. The results indicate that while alternative specifications of the GARCH(1,1) model may provide a better fit in-sample, they tend to overfit the data and do not necessarily provide similar improvement over the out-of-sample period.

Over the market crisis period we find that the nonlinear model specifications are outperformed by the baseline linear models – the AR(1) model in the case of the Top 40 and Mid Cap indices and the ARMA(1,1) model in the case of the Small Cap index. In addition, the MS models provide lower forecast errors than the GARCH models over this period although their success rates are generally lower. Interestingly, it is the homoscedastic MS models that outperform indicating the potential confounding effect of the market turmoil on conditional volatility forecasts in the GARCH and heteroscedastic MS models.

Our findings suggest that there are definite gains to be made in accounting for nonlinear influences on the return generation process on the JSE but also highlight the susceptibility of such models to overfitting the data.

6. Identification and Evaluation of Style Anomalies

As indicated in Chapter 2, research on return predictability in the South African market (see van Rensburg and Robertson, 2003 and Auret and Sinclair, 2006) has found evidence of a number of significant univariate relationships between realised returns and firm characteristics, including price-to-NAV, dividend yield, price-to-earnings ratio, cash flow-to-price and firm size.

The two primary approaches to investigating such so-called style anomalies are the portfolio sort and regression-based methods. In the portfolio sorting approach, shares are sorted monthly into fractiles on the basis of one or more firm characteristics. The differences between returns to the top and bottom fractiles represent the payoffs to the characteristics in question. The second approach is to fit a univariate regression of realized share returns against firm characteristics in order to establish whether certain characteristics are able to explain or predict share returns (Fama and Macbeth, 1973).

Achour, Harvey, Hopkins and Lang (1999) suggest that the regression approach fails to accurately reflect the time-varying nature of style coefficients, particularly where the analysis is limited to a single period. In addition, there are concerns that the noise that characterizes the returns for many emerging markets may distort the results of regression tests. The latter concern, however, has little merit in the increasingly efficient South African market.

The portfolio sorting approach is not without criticism. As the fractile returns are an average of the returns of the underlying firms in each fractile, it is possible for significant attributes to be overlooked because their impact has been averaged away (Roll, 1977). In addition, Berk (2000) argues that CAPM and APT predictions in the portfolio sorting approach will be less extreme, with lower variance, as part of the total variance occurs between fractiles and this is not captured by the approach. Fractiles therefore do not have to be extreme to be significant, making it easier to obtain significant results using this approach.

Based on the above criticisms we elect to employ the regression-based approach and conduct a univariate cross-sectional regression of the monthly realised firm returns against individual firm characteristics similar to Fama and Macbeth (1973). The time series of regression coefficients generated represents the time-varying factor payoffs to the characteristic tested (Fama and French, 1992).

While this is not a new area of research, prior South African studies have been conducted primarily during periods of sustained growth and stability in the market, and there has been no investigation of the consistency of these anomalies over time. As indicated by Malkiel (2003), evidence of style anomalies and return predictability is often sample-specific, meaning that such inefficiencies are impossible to exploit profitably over the long-term.

Our analysis therefore encompasses both the bull run over the period 2002 to 2007 and the period of market instability that followed the sub-prime mortgage crisis of 2007. Our goals are (a) to establish whether the anomalies identified in prior studies of style anomalies in the South African market are found to be of continued significance during the 2002 to 2007 period and (b) to determine which, if any, of the anomalies persist during the subsequent period of market instability.

6.1 Methodology

Each of the 20 share-specific attributes detailed in Chapter 3 is standardized to ensure comparability of the slopes estimated in the procedure¹⁴ and regressed over the full sample period against realised share returns as follows:

$$R_{i,t+1} = \gamma_{0,t+1} + \gamma_{1,t+1}A_{it} + \varepsilon_{i,t+1} \quad (6.1)$$

where the dependent variable $R_{i,t+1}$ is the return on share i for month $t + 1$, $\gamma_{1,t+1}$ is the cross-sectional slope coefficient and A_{it} is the standardized value of the attribute for share i in month t . The mean slope coefficient for each attribute is tested for significance using a t-test and evaluated at the 5% level of significance. The test procedure is then repeated for both the in-sample and out-of-sample periods.

We employ listwise deletion in our sample whereby we exclude shares with one or more missing observations from the sample in that month. This approach may result in a smaller sample size which has implications for the power of the statistical tests employed on the data and might also introduce bias where remaining cases are not representative of the full sample. This method is however standard in many studies of this nature (see van Rensburg and Robertson, 2003 and Auret and Sinclair, 2006). We provide an alternative to this method in Section 6.1.1 in order to assess whether our findings are robust to the missingness of the data set.

In order to ascertain whether significant factors are not simply proxies for systematic risk we risk-adjust the share returns using both a CAPM and two-factor APT approach. As per the CAPM model, share returns are given by

$$R_{i,t} = r_{f,t} + \beta_i(R_{m,t} - r_{f,t}) + \varepsilon_{i,t} \quad (6.2)$$

¹⁴ This approach is followed by van Rensburg and Robertson (2003) who find that the adjustment does not materially affect the t-statistics of the regressions.

where $R_{i,t}$ and $R_{m,t}$ are the realised returns in month t for share i and the market, m , respectively, $r_{f,t}$ is the return on the risk-free asset, β_i measures the exposure of share i to the market and $\varepsilon_{i,t}$ represents the residual return unexplained by the market.

Given that $\beta_i(R_{m,t} - r_{f,t})$ represents the systematic portion of the return, $r_{f,t} + \varepsilon_{i,t}$ represents the component of return not explained by systematic risk, hereinafter referred to as the abnormal return on share i .

We conduct OLS regressions of excess share returns against excess market returns across the full data sample where

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i(R_{m,t} - r_{f,t}) + \varepsilon_{i,t} \quad (6.3)$$

The JSE's All-Share Index (ALSI) is used as a proxy for the market portfolio, m , and the Short-term Fixed Interest (STEFI) index is used as the market risk-free rate. Rearranging the terms in equation 3 we isolate the abnormal return, $AR_{i,t}$, such that

$$AR_{i,t} = \alpha_i + \varepsilon_{i,t} = R_{i,t} - r_{f,t} - \beta_i(R_{m,t} - r_{f,t}) \quad (6.4)$$

Once these abnormal returns have been calculated, we repeat our cross-sectional univariate regression procedure replacing $R_{i,t+1}$ in equation 6.1 with $AR_{i,t+1}$ from equation 6.4.

Prior research into market risk decomposition on the JSE has demonstrated that a two-factor APT model of the underlying return-generation process is more applicable to the South African market than a single-factor market model using the ALSI as proxy (see van Rensburg, 2002, and Kruger, 2006). Based on the findings of these studies, we consider a second risk-adjusted OLS multi-factor regression where excess share returns are regressed against excess returns for the Financial-Industrials Index (J250 or FINDI) and the Resources Index (J258 or RESI) of the JSE.

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i(R_{FINDI,t} - r_{f,t}) + \beta_i(R_{RESI,t} - r_{f,t}) + \varepsilon_{i,t} \quad (6.5)$$

We again isolate the abnormal returns by rearranging equation 6.5 so that

$$AR_{i,t} = \alpha_i + \varepsilon_{i,t} = R_{i,t} - r_{f,t} - \beta_i(R_{FINDI,t} - r_{f,t}) - \beta_i(R_{RESI,t} - r_{f,t}) \quad (6.6)$$

and repeat our univariate regressions as before. The mean slope coefficient for each attribute is again tested for significance using Student's t-test and evaluated at the 5% level of significance.

6.1.1 Combining the Results

In addition to conducting the analysis on the original, unadjusted data set, the univariate methodology is applied to each of the multiply imputed data sets detailed in Chapter 3 and the results of the regressions are combined using the rules established by Rubin (1987).

Where the quantity of interest is a mean value we calculate a single combined mean, \bar{q} , which is a simple average of the estimated means for each of the multiply imputed data sets.

$$\bar{q} = \frac{1}{m} \sum_{j=1}^m q_j \quad (6.7)$$

The variance of \bar{q} is then a combination of the average variance, \bar{U} , of the variances, U_j , of the factor within each imputed data set plus the sample variance in the point estimates between the data sets, B , adjusted by a factor that corrects for bias as $m < \infty$. The total variance, T , is thus

$$T = \bar{U} + \left(1 + \frac{1}{m}\right) B \quad (6.8)$$

Where \bar{U} is defined as

$$\bar{U} = \frac{1}{m} \sum_{j=1}^m U_j \quad (6.9)$$

And B is defined as

$$B = \frac{1}{m-1} \sum_{j=1}^m (q_j - \bar{q})^2 \quad (6.10)$$

We then compute standard errors and t-statistics as usual.

6.1.2 Consistency of Factor Payoffs

Haugen and Baker (1996) suggest that it is important that the payoffs of significant factors should have consistent signs across independent examination periods for them to represent true priced anomalies. We examine the consistency of the significant factors identified in the univariate analysis using stacked graphs of the percentage of positive and negative payoffs for each factor over the full period, in-sample period and out-of-sample period. Factors that have consistently positive (negative) payoffs should be close to the upper (lower) boundary of these stacks.

We examine the significance of these ratios of positive to negative factor payoff percentages using nonparametric rank-based sign tests. The Wilcoxon signed ranks test compares the median of a series of values against some hypothetical median. For each observation not equal to the median, the difference is calculated between the observation and the median. These differences are then ranked from highest to lowest. The null hypothesis for the test is that the sum of the ranks above and below the median should be equal. If the null hypothesis is rejected it is likely that the observations in the data series are either more positive or negative. The van der Waerden test smooths the ranks of the Wilcoxon test by converting them to quantiles of the normal distribution (see Conover, 1980). We evaluate the Wilcoxon and van der Waerden tests against the null hypothesis that the median of the factor payoff series is equal to 0.

In addition to these measures, we graph the cumulative factor payoffs of the significant factors over the full period in order to examine graphically the consistency of the factor payoffs to the significant univariate factors over time.

6.2 Results

The attribute payoffs for the full sample period are presented in Table 6.1. We find eight significant variables at a 5% significance level for both the unadjusted data set and the multiple imputation data sets - cash flow-to-price, earnings yield and book-to-market from the value category, 12-month dividend growth-to-price and 12-month earnings growth-to-price from the growth category, 12-month momentum

and 6-month momentum from the momentum category and log of market value from the size category. The significance of book-to-market, earnings yield (the inverse of the price-earnings ratio), cash flow-to-price and log of market value are all consistent with the prior findings of van Rensburg and Robertson (2003a) and Auret and Sinclair (2006) who found these factors similarly significant on the JSE over the period of July 1990 to June 2000. This suggests long-term stability in these anomalies. Log of market value demonstrates a negative relationship with forward returns, confirming the size anomaly identified in prior international research (see Banz, 1981 and Fama and French, 1992) and for the South African market by van Rensburg and Robertson (2003a). The significance of 6-month momentum and 12-month momentum are consistent with international findings that there is either a lag in the incorporation of firm earnings reports into price or that the market underreacts initially to earnings surprises (see Jegadeesh and Titman, 1993). Van Rensburg and Robertson (2003b) found no evidence that one-month, 6-month or 12-month price momentum were significant over the period July 1990 to June 2000 for the South African market. Fraser and Page (2000) and van Rensburg (2001) did, however, observe significant momentum effects for samples of Industrial shares only, using portfolio sorts over the periods January 1973 to October 1997 and February 1983 to March 1999, respectively.

Earnings and dividend growth factors have been tested in prior South African studies but had previously not been identified as significant anomalies. Haugen and Baker (1996) suggest that profitable firms tend to grow faster and as such measures of growth should include not only measures of earnings and dividend growth but also measures of profitability, including return on equity and asset turnover. While return on equity was not found to be significant both 12-month measures of earnings and dividend growth relative to price were found to be significant over the in-sample period. While the findings suggest that value factors remain a fundamental driver of share returns on the JSE, 12-month growth in earnings and dividends are both significantly positively related to share returns.

The signs of the significant factor payoffs are all consistent with the literature. Haugen and Baker (1996) propose that value factors should have positive payoffs to reflect either their greater risk (see Chan and Chen, 1991 and Fama and French, 1992) or investor overreaction and subsequent correction (Lakonishok, Shleifer and Vishny, 1994 and Haugen, 1995). Similarly, positive growth factors are indicative of greater than average future growth in stock earnings and dividends and should have a positive relationship with returns (Haugen and Baker, 1996).

We adjust the share returns for risk using both the CAPM and APT techniques and we find little change in the significance of our factors for either the unadjusted or multiple imputation data sets. Given the similarities in the results we present only the unadjusted data set results in our tables. While some attributes demonstrate a slight decrease in significance, most in fact demonstrate an increase in t-statistic. This may be due to noise being filtered from the return series by means of the adjustments. One-month

momentum and payout ratio both become strongly significant after adjusting for risk. One-month momentum demonstrates a negative relationship with forward returns, similar to the short-term momentum reversal effect documented internationally by Jegadeesh (1990) and locally by van Rensburg (2001). The payout ratio factor may be a proxy for the dividend yield factor (which becomes significant after adjusting for the CAPM but not for the APT) with high payout ratio ratios generally being indicative of a higher dividend yield. This would agree with the positive relationship between dividend yield and stock returns established by Litzenberger and Ramaswamy (1979) and Fama and French (1988). 6-month momentum becomes insignificant when adjusting for risk using the CAPM while earnings yield becomes insignificant when adjusting for risk using the two-factor APT model.

Table 6.2 presents the results for the in-sample period. We find nine significant variables for both data sets over the in-sample period. The significant factors over this period differ from those found over the full period in that earnings yield is not significant over the shorter in-sample period while analyst downgrades and the 24-month dividend growth-to-price ratio is found to be significant. As expected, analyst downgrades has an inverse relationship with returns indicating that stocks which have their ratings downgraded tend subsequently to underperform. This supports the findings of Prayag and van Rensburg (2006) that changes in recommendations are more significant than absolute recommendations and that downgraded ratings provide greater predictability of returns on the JSE than other measures of analyst expectations.

All of the variables remain significant after both the CAPM and APT adjustments, while 24-month dividend growth-to-price, industry dummy and earnings yield become significant after adjusting for risk using the CAPM. One-month momentum and 24-month dividend growth-to-price become significant after the APT adjustment.

In contrast, the out-of-sample period yields only one significant variable, cash flow-to-price, which remains significant after the risk-adjustments. This makes intuitive sense. Given that the out-of-sample period corresponds with the market crash following the sub-prime mortgage crisis we would expect the market to place a premium on those firms generating strong cash flows relative to their price. The paucity of significant factors in the out-of-sample period was expected given the shift in market conditions following the market crash. Indeed, the mean absolute factor payoff of the significant factors identified in the in-sample period decreases from 0.00421 to 0.00182 following the transition to the out-of-sample period while the mean standard deviation of the factor payoffs increases from 0.00997 to 0.01381. This further emphasises both the reduction in efficacy of factors previously identified as significant during the in-sample period and the decrease in consistency of the factor payoffs.

Even though eight factors were found to be significant over the full period, it is clear that much of this significance derives from the in-sample period rather than the full sample period. This highlights the danger that factors shown to be significant over the entire data sample may be unstable due to their significance being attributable to strongly-significant sub-periods within the data set and not necessarily consistent over time. Of the factors examined, only cash flow-to-price is consistently significant over all three periods examined.

It should be noted that the above tests were conducted on a univariate basis and fail to account for the possibility that some factors have joint explanatory power when combined with others in a multi-factor model. In constructing our multi-factor models in Chapter 7 we therefore consider all available factors as candidate variables in the regression and not only those univariate factors found to be significant here.

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Table 6.1 Univariate Regression Results for the Full Sample Period

The mean monthly cross-sectional univariate regression results are presented for the full sample period February 2002 to December 2009. Results are ranked by absolute t-statistic value with results significant at the 5% level highlighted in bold. Results are presented for the unadjusted raw data and the data obtained via the multiple imputation procedure applied in Chapter 3. The same tests are then conducted for the raw data after the sample has been risk-adjusted using both a CAPM approach with the JSE's All-Share Index (ALSI) as market proxy and a two-factor APT approach employing the JSE's Financial Industrials (J250) and Resources (J258) indices.

	Unadjusted Raw Data			Multiple Imputation Data			CAPM-Adjusted*			APT-Adjusted*		
	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat
CTP	0.005	0.010	4.777	0.004	0.009	4.139	0.003	0.009	3.740	0.003	0.009	3.154
BVTM	0.004	0.011	3.748	0.004	0.011	3.619	0.004	0.010	3.763	0.003	0.008	3.736
C12MEPSP	0.003	0.010	2.595	0.003	0.010	3.147	0.003	0.010	3.026	0.003	0.010	2.673
LMV	-0.004	0.014	-2.467	-0.004	0.014	-2.469	-0.005	0.013	-3.764	-0.004	0.011	-3.387
MOM12	0.004	0.018	2.355	0.004	0.018	2.340	0.005	0.015	3.076	0.004	0.012	3.439
C12MDPSP	0.002	0.008	2.145	0.002	0.009	2.110	0.002	0.009	2.622	0.002	0.008	2.257
EY	0.002	0.012	2.032	0.002	0.011	2.021	0.003	0.011	2.322	0.002	0.010	1.977
MOM6	0.004	0.017	2.021	0.004	0.017	2.030	0.003	0.015	1.954	0.003	0.012	2.595
C24MEPSP	0.002	0.010	1.807	0.002	0.009	1.900	0.002	0.010	2.092	0.002	0.009	1.888
MOM1	-0.003	0.014	-1.785	-0.003	0.014	-1.785	-0.003	0.013	-2.105	-0.003	0.010	-2.666
POUT	0.002	0.011	1.771	0.002	0.011	1.405	0.002	0.010	2.429	0.002	0.009	2.305
C24MDPSP	0.001	0.008	1.251	0.001	0.009	1.244	0.002	0.009	2.136	0.001	0.008	1.659
DOWNGRADE	-0.001	0.006	-1.014	-0.001	0.007	-0.866	-0.001	0.006	-1.007	-0.001	0.006	-1.644
ROE	0.001	0.011	0.912	0.001	0.011	0.606	0.002	0.011	1.493	0.002	0.010	1.661
DY	0.001	0.014	0.864	0.001	0.014	0.880	0.003	0.013	2.052	0.002	0.011	1.572
BUY	0.000	0.007	0.669	0.001	0.007	0.698	0.000	0.007	0.495	0.000	0.007	0.340
VT	0.001	0.009	0.662	0.001	0.009	0.620	0.000	0.008	-0.469	-0.001	0.008	-0.657
DUMFINDI	0.001	0.021	0.536	0.001	0.022	0.518	0.004	0.019	1.846	0.001	0.011	0.917
MOM3	0.001	0.016	0.487	0.001	0.016	0.480	0.000	0.013	-0.357	0.000	0.010	-0.130
STDEV12	0.001	0.016	0.311	0.001	0.015	0.566	-0.001	0.013	-0.527	0.000	0.012	-0.023

Table 5.2 Univariate Regression Results for the In-Sample Period

The mean monthly cross-sectional univariate regression results are presented for the in-sample period February 2002 to July 2007. Results are ranked by absolute t-statistic value with results significant at the 5% level highlighted in bold. Results are presented for the unadjusted raw data and the data obtained via the multiple imputation procedure applied in Chapter 3. The same tests are then conducted for the raw data after the sample has been risk-adjusted using both a CAPM approach with the JSE's All-Share Index (ALSI) as market proxy and a two-factor APT approach employing the JSE's Financial Industrials (J250) and Resources (J258) indices.

	Unadjusted Raw Data			Multiple Imputation Data			CAPM-Adjusted*			APT-Adjusted*		
	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat
C12MEPSP	0.005	0.009	4.457	0.005	0.009	4.933	0.005	0.008	4.998	0.005	0.008	4.653
BVTM	0.006	0.010	4.304	0.005	0.010	4.123	0.006	0.010	4.499	0.005	0.008	4.604
CTP	0.004	0.008	4.203	0.003	0.008	3.416	0.003	0.008	2.928	0.002	0.007	2.251
LMV	-0.007	0.013	-4.071	-0.007	0.013	-4.076	-0.008	0.011	-6.237	-0.007	0.010	-5.855
MOM12	0.006	0.015	3.319	0.006	0.015	3.390	0.006	0.015	3.177	0.005	0.011	3.672
MOM6	0.005	0.014	2.942	0.005	0.014	3.026	0.005	0.013	2.719	0.005	0.010	3.529
DOWNGRADE	-0.002	0.005	-2.416	-0.002	0.006	-2.332	-0.002	0.006	-2.186	-0.002	0.006	-2.520
C12MDPSP	0.002	0.008	2.372	0.002	0.008	2.344	0.003	0.008	2.846	0.002	0.007	2.400
C24MDPSP	0.002	0.007	2.031	0.002	0.008	2.028	0.003	0.008	2.903	0.002	0.007	2.369
EY	0.003	0.012	1.878	0.003	0.012	1.815	0.004	0.012	2.340	0.002	0.010	1.844
C24MEPSP	0.002	0.011	1.548	0.002	0.010	1.531	0.002	0.011	1.603	0.002	0.010	1.391
MOM1	-0.002	0.014	-1.377	-0.002	0.014	-1.377	-0.003	0.013	-1.628	-0.003	0.010	-2.593
STDEV12	0.002	0.012	1.313	0.002	0.012	1.611	0.000	0.011	0.013	0.001	0.010	0.717
VT	0.001	0.008	0.581	0.001	0.008	0.564	0.000	0.007	0.031	0.000	0.006	-0.298
POUT	0.000	0.008	0.461	0.000	0.007	-0.118	0.002	0.007	1.853	0.001	0.007	1.563
DUMFINDI	0.001	0.020	0.458	0.001	0.020	0.446	0.005	0.019	2.213	0.001	0.012	0.905
ROE	0.001	0.012	0.451	0.000	0.011	0.113	0.001	0.011	0.881	0.001	0.010	0.609
MOM3	0.001	0.014	0.357	0.001	0.014	0.390	-0.001	0.012	-0.317	0.000	0.009	0.062
BUY	0.000	0.006	0.027	0.000	0.007	0.025	0.000	0.006	-0.102	-0.001	0.006	-0.777
DY	0.000	0.011	0.008	0.000	0.012	-0.008	0.003	0.011	1.867	0.001	0.010	0.986

Table 5.3 Univariate Regression Results for the Out-of-Sample Period

The mean monthly cross-sectional univariate regression results are presented for the out-of-sample period August 2007 to February 2009. Results are ranked by absolute t-statistic value with results significant at the 5% level highlighted in bold. Results are presented for the unadjusted raw data and the data obtained via the multiple imputation procedure applied in Chapter 3. The same tests are then conducted for the multiple imputation data after the sample has been risk-adjusted using both a CAPM approach with the JSE's All-Share Index (ALSI) as market proxy and a two-factor APT approach employing the JSE's Financial Industrials (J250) and Resources (J258) indices.

	Unadjusted Raw Data			Multiple Imputation Data			CAPM-Adjusted*			APT-Adjusted*		
	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat	Mean	Std Dev	T-Stat
CTP	0.007	0.013	2.697	0.006	0.012	2.698	0.004	0.011	2.359	0.004	0.010	2.169
POUT	0.006	0.017	1.868	0.005	0.016	1.955	0.004	0.013	1.692	0.004	0.013	1.724
DOWNGRADE	0.002	0.007	1.195	0.002	0.008	1.323	0.001	0.007	0.864	0.000	0.006	0.411
DY	0.004	0.019	1.152	0.004	0.019	1.314	0.003	0.016	1.073	0.002	0.012	1.240
MOM1	-0.003	0.015	-1.124	-0.003	0.015	-1.235	-0.003	0.011	-1.337	-0.002	0.010	-0.955
LMV	0.003	0.015	1.074	0.003	0.015	1.165	0.001	0.013	0.599	0.002	0.012	0.759
BUY	0.001	0.007	1.059	0.002	0.008	1.212	0.001	0.008	0.798	0.002	0.008	1.255
ROE	0.002	0.010	1.050	0.002	0.009	1.148	0.002	0.010	1.373	0.003	0.009	2.041
C12MEPSP	-0.002	0.012	-0.963	-0.001	0.012	-0.703	-0.001	0.012	-0.410	-0.001	0.012	-0.412
C24MEPSP	0.001	0.008	0.928	0.002	0.008	1.289	0.002	0.007	1.462	0.002	0.007	1.391
EY	0.002	0.011	0.815	0.002	0.010	0.970	0.001	0.010	0.636	0.001	0.009	0.799
STDEV12	-0.003	0.021	-0.740	-0.003	0.020	-0.760	-0.002	0.017	-0.687	-0.002	0.015	-0.657
BVTM	0.002	0.013	0.651	0.002	0.012	0.746	0.001	0.010	0.504	0.000	0.008	0.290
C12MDPSP	0.001	0.009	0.446	0.001	0.010	0.486	0.001	0.009	0.659	0.001	0.009	0.697
C24MDPSP	-0.001	0.010	-0.375	-0.001	0.011	-0.404	0.000	0.010	0.056	0.000	0.009	-0.005
VT	0.001	0.012	0.347	0.001	0.012	0.339	-0.001	0.010	-0.632	-0.001	0.011	-0.597
MOM3	0.001	0.020	0.326	0.001	0.020	0.308	0.000	0.014	-0.174	0.000	0.012	-0.239
DUMFINDI	0.001	0.025	0.281	0.001	0.025	0.296	0.001	0.020	0.207	0.001	0.011	0.306
MOM12	0.000	0.023	0.014	-0.001	0.022	-0.140	0.003	0.016	0.968	0.003	0.014	1.110
MOM6	0.000	0.023	0.006	0.000	0.023	-0.063	0.000	0.017	0.043	0.001	0.015	0.321

6.2.1 Consistency of Sign of Significant Univariate Factor Payoffs

The clustered, stacked bar chart in Figure 6.1 shows the lack of consistency in payoff signs between the in- and out-of-sample periods for the significant factors identified in Section 6.2. All factors with a positive mean payoff, except cash flow-to-price and earnings yield show a marked decrease in the percentage of positive payoffs between the respective periods. Earnings yield remains relatively unchanged between periods while cash flow-to-price displays a marginal increase in the percentage of positive months from the one sample period to the other.

The two significant factors with negative mean factor payoffs over the full- and in-sample periods, analyst downgrades and log of market value, both demonstrate a decrease in the percentage of negative payoffs from the in-sample to the out-of-sample period. The change is particularly large for log of market value which changes from having 77% of its in-sample months show negative payoffs to just 43% in the out-of-sample period.

We next consider the results of Wilcoxon signed-rank tests and van der Waerden tests of the factor payoffs for the significant factors identified in the in-sample period. A summary of the results is provided in Table 6.4. The results indicate that the null hypothesis that the median of the factor payoffs was zero (i.e. that they were not consistently positive or negative) for both tests could be rejected for all significant factors. These factors can therefore be said to have exhibited factor payoffs that are consistently positive or negative over that period and would be good candidate factors for inclusion in the multi-factor regressions according to the criteria of Haugen and Baker (1996). In the out-of-sample period the null hypothesis can be rejected for both tests for cash flow-to-price alone.

As we plan to consider all candidate factors in the multivariate analysis we next examine the consistency of those factors found to be insignificant during the univariate testing but which demonstrate consistent payoffs as per the Wilcoxon signed-rank and van der Waerden tests. Our findings are presented in Table 6.5. From the table it is evident that the all of the insignificant factors have inconsistent factor payoffs.

There therefore seems to be a clear relationship between factor significance and consistency.

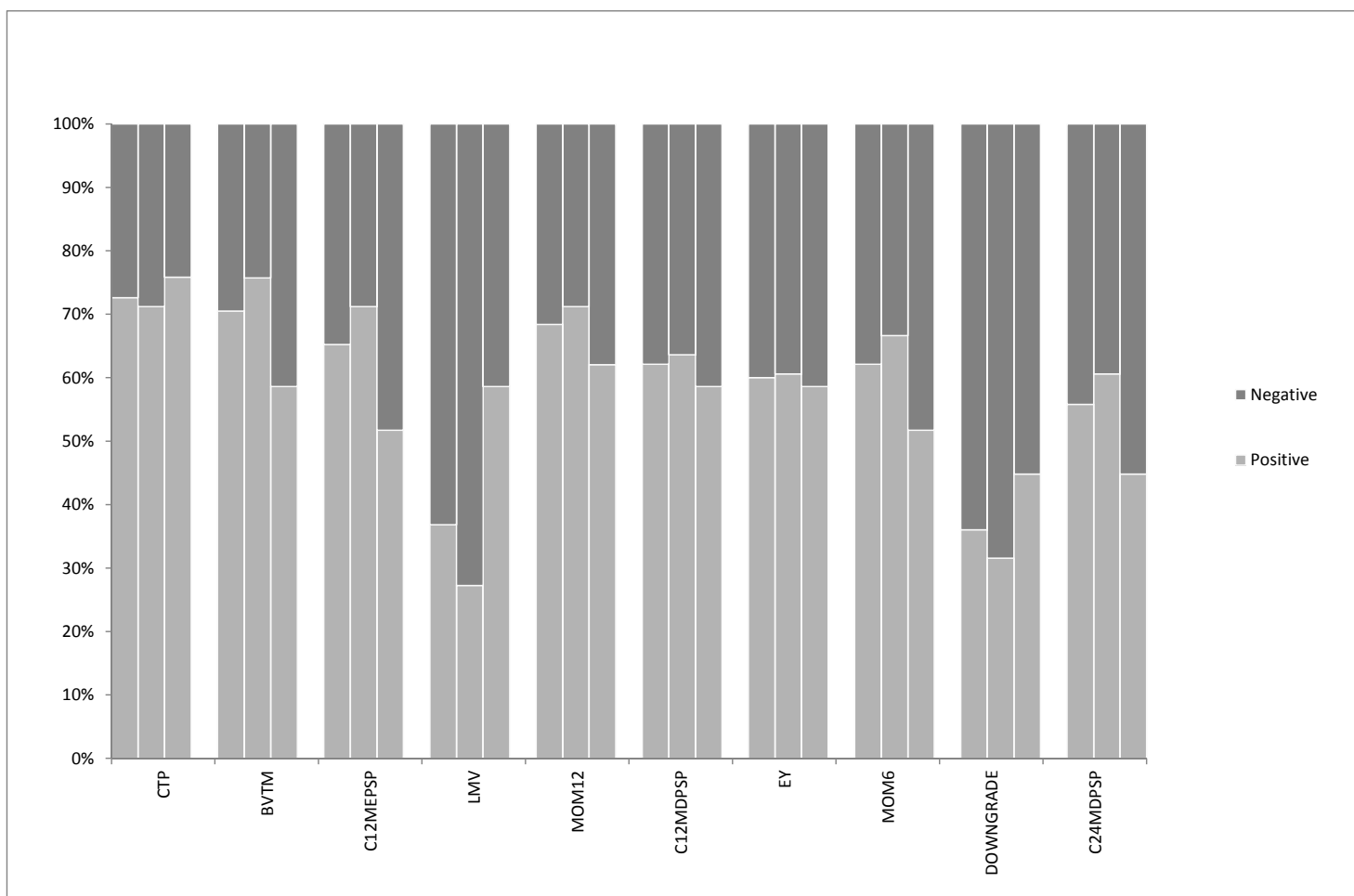


Figure 6.1 Percentage positive and negative months for the significant factors in each sample period

We conduct univariate regressions of our style attributes against realised returns over the period February 2002 to December 2009. The percentage of positive and negative months for the payoffs of each significant factor over the sample period is presented in a clustered, stacked bar chart below. Three stacked bars are displayed for each factor. The first bar represents the full sample period as above, the second represents the in-sample period (February 2002 to July 2007), and the third represents the out-of-sample period (August 2007 to December 2009).

Table 6.4 Results of Wilcoxon Signed-Rank and van der Waerden tests for the Significant Factors

We calculate the Wilcoxon Signed-Rank and van der Waerden test statistics for the significant factors identified during the univariate regression procedure over the in-sample and out-of-sample periods. These statistics are evaluated at a 5% significance level and significant statistics are indicated in bold. Where the test statistic is significant we reject the null hypothesis that the median value of the factor payoff is zero and accept that the factor's payoffs are more consistently positive or negative.

Factors	In-Sample Period				Out-of-Sample Period			
	Positive %	Negative %	Wilcoxon	Vd Waerden	Positive %	Negative %	Wilcoxon	Vd Waerden
CTP	71%	29%	3.78	3.78	76%	24%	2.70	2.58
BVTM	76%	24%	3.83	3.78	59%	41%	0.40	0.46
C12MEPSP	71%	29%	3.94	3.83	52%	48%	1.03	-1.20
LMV	27%	73%	4.15	-3.85	59%	41%	1.09	1.20
MOM12	71%	29%	3.57	3.30	62%	38%	0.27	0.06
C12MDPSP	64%	36%	2.35	2.35	59%	41%	0.54	0.56
EY	61%	39%	2.09	2.01	59%	41%	0.71	0.79
MOM6	67%	33%	3.30	3.17	52%	48%	0.29	0.10
DOWNGRADE	32%	68%	2.51	-2.47	45%	55%	0.56	0.88
C24MDPSP	61%	39%	2.05	2.02	45%	55%	0.60	-0.40

Table 6.5 Results of Wilcoxon Signed-Rank and van der Waerden tests for the Insignificant Factors

We calculate the Wilcoxon Signed-Rank and van der Waerden test statistics for the insignificant factors identified during the univariate regression procedure for the in-sample period. These statistics are evaluated at a 5% significance level and significant statistics are indicated in bold. Where the test statistic is significant we reject the null hypothesis that the median value of the factor payoff is zero and accept that the factor's payoffs are more consistently positive or negative.

Factors	In-Sample Period				Out-of-Sample Period			
	Positive %	Negative %	Wilcoxon	Vd Waerden	Positive %	Negative %	Wilcoxon	Vd Waerden
BUY	52%	48%	0.01	-0.05	62%	38%	1.33	1.21
C24MEPSP	68%	32%	1.90	1.54	59%	41%	0.91	0.90
DUMFINDI	52%	48%	0.74	0.63	48%	52%	0.10	0.26
DY	58%	42%	0.09	-0.02	59%	41%	0.87	0.95
MOM1	45%	55%	1.58	-1.52	38%	62%	1.45	-1.43
MOM3	55%	45%	0.54	0.63	55%	45%	0.22	0.28
POUT	58%	42%	0.85	0.47	55%	45%	1.22	1.44
ROE	55%	45%	0.66	0.58	52%	48%	0.79	1.00
STDEV12	53%	47%	1.61	1.61	48%	52%	0.30	-0.44
VT	53%	47%	0.19	0.28	52%	48%	0.38	0.35

Next we examine the geometric cumulative factor payoffs to the significant factors to obtain a clearer view of the time-varying nature of the factor payoffs. All factors are assigned a base value of 1 and this base is grown on a geometric basis using the factor payoffs over time.

Of the significant value factors, book value-to-market value demonstrates a strong positive trend over the full in-sample period with a final cumulative factor payoff value of 1.44. The factor flattens off significantly over the out-of-sample period, though, before peaking towards the end of 2009, ending the period with a cumulative value of 1.50. The cumulative payoff to cash flow-to-price is relatively flat for the first half of the in-sample period before displaying a positive trend over the latter half. It has a final cumulative value of 1.29 over the period. Contrary to the other significant factors in the sample, however, the factor's payoffs strengthen markedly in the out-of-sample period and the factor ends the period with a cumulative factor payoff of 1.57. This supports the finding that CTP continued to be a consistent performance over the market crisis period. Earnings yield has a small positive trend in the first half of the in-sample period but this flattens off for the remainder, and the factor ends with a cumulative factor payoff of 1.19. The subsequent factor payoffs for earnings yield are largely flat with a slightly negative trend and the factor ends with a cumulative value of 1.25. All three factors exhibit greater volatility (as measured by the standard deviation of the factor payoffs) in the out-of-sample period which corresponds with the lack of consistency of the factor payoffs determined in the rank-tests. It should be noted, however, that all three factors demonstrate positive trends beginning February 2009.

12-month earnings growth-to-price has a positive trend over the full in-sample period, cumulating to a value of 1.37. The factor maintains this trend through 2007 but then subsequently sees a shift in factor payoffs to largely negative values resulting in a final cumulative factor payoff of 1.28. 12-month dividend growth-to-price and 24-month dividend growth-to-price follow the same, albeit slightly less pronounced, positive trend initially but this is reversed in March 2005 after which the trend is negative until the end of the in-sample period. Their final cumulative factor payoffs are 1.18 and 1.10, respectively.

12-month momentum and 6-month momentum both demonstrate positive trends over the full period. Both factors are more volatile than average though with factor payoff standard deviations of 0.016 and 0.0145 respectively.

Analyst downgrades has a slight negative trend but is relatively flat over the full period with its cumulative value of 0.94 showing little deviation from the base value of 1. Lastly, log of market value has a consistently negative, albeit volatile, factor payoff throughout the period.

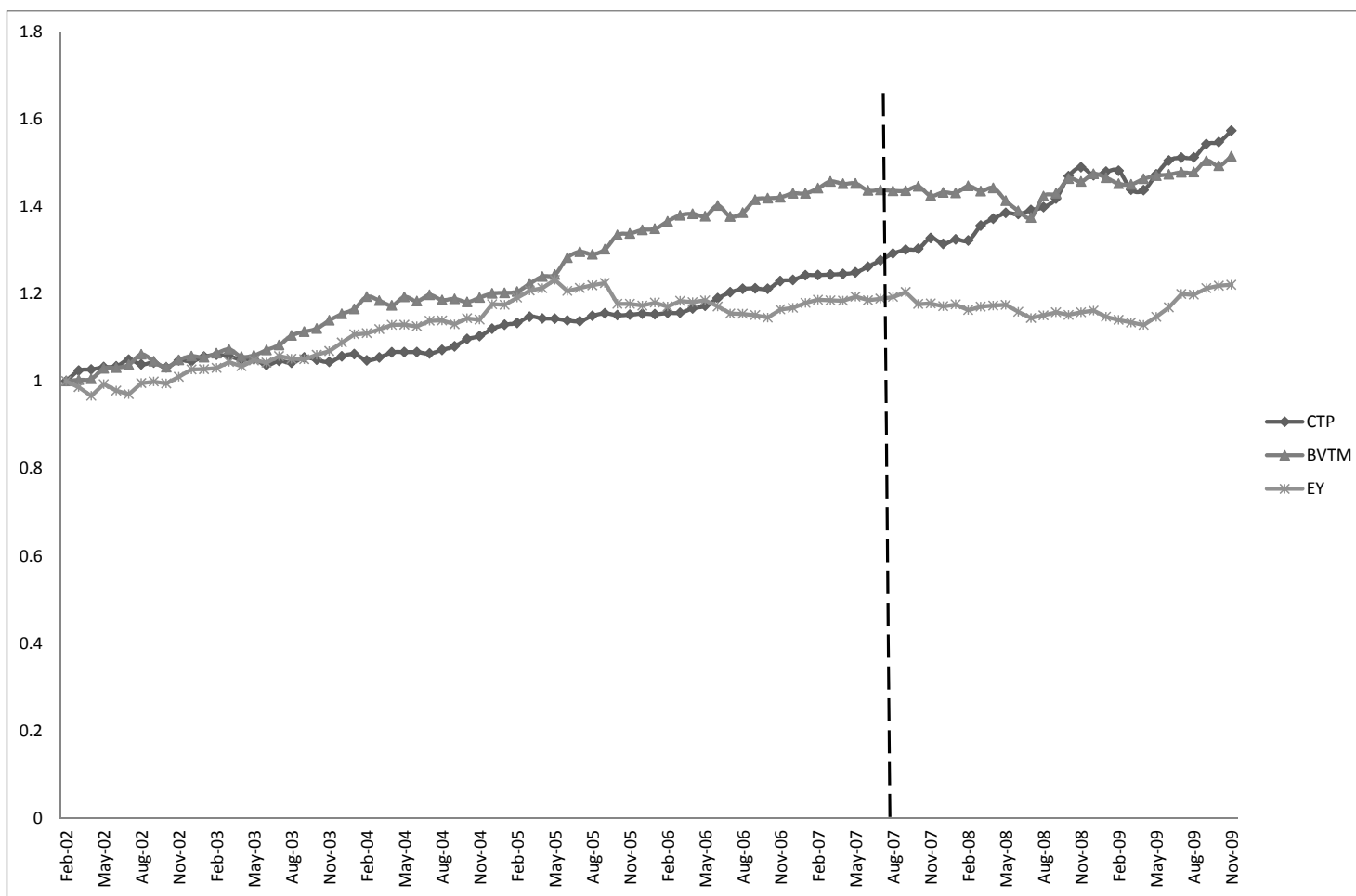


Figure 6.2 Cumulative Significant Value Factor Payoffs

We conduct univariate regressions of our style attributes against realised returns. Cumulative geometric payoffs to the significant factors in the value category are computed and displayed below. All cumulative payoffs are based to a value of 1 and calculated over the full-sample period, February 2009 to December 2009. The transition between the in- and out-of-sample periods is illustrated by a vertical dashed line.

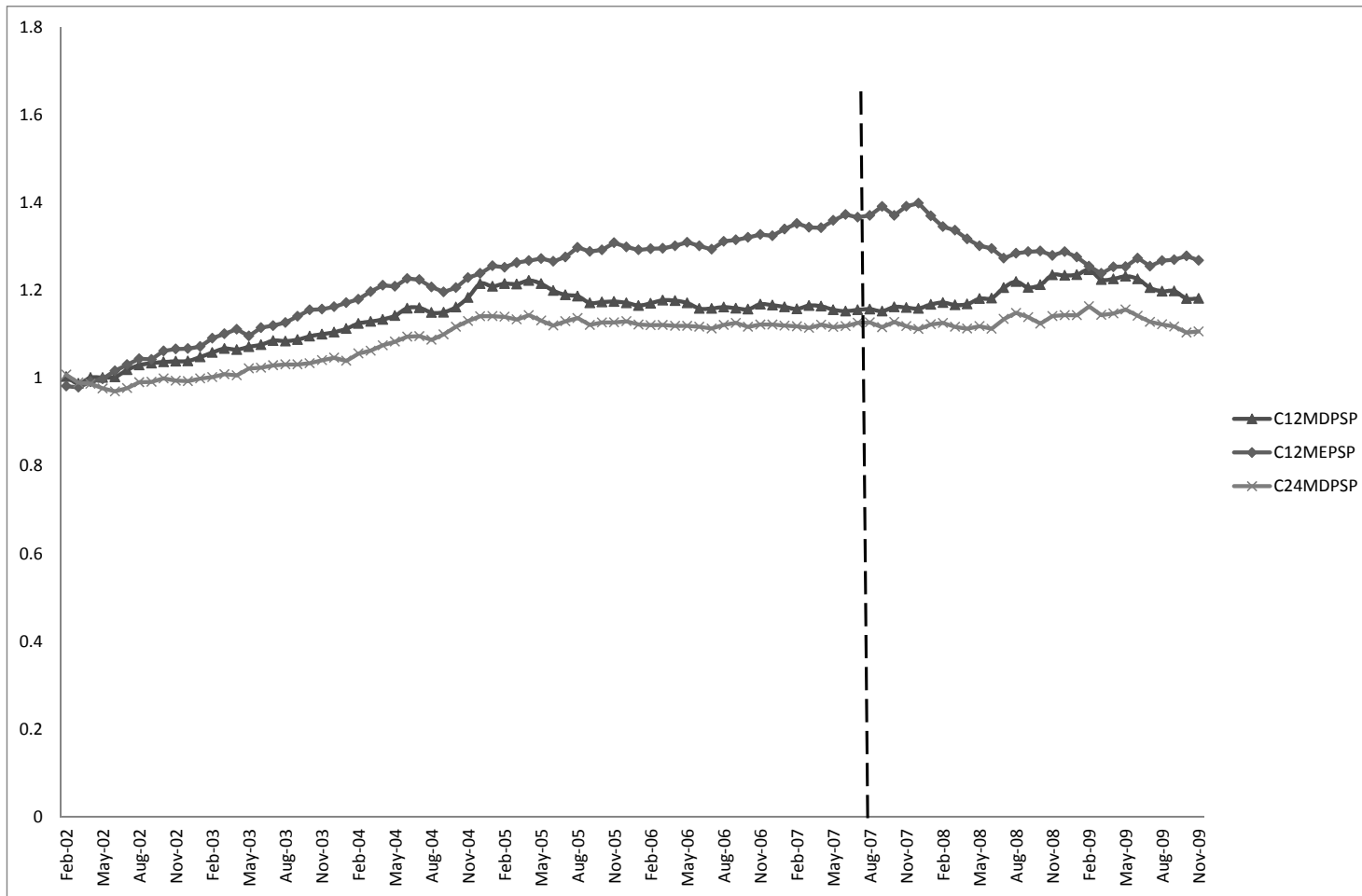


Figure 6.3 Cumulative Significant Growth Factor Payoffs

We conduct univariate regressions of our style attributes against realised returns. Cumulative geometric payoffs to the significant factors in the growth category are computed and displayed below. All cumulative payoffs are based to a value of 1 and calculated over the full-sample period, February 2009 to December 2009. The transition between the in- and out-of-sample periods is illustrated by a vertical dashed line.

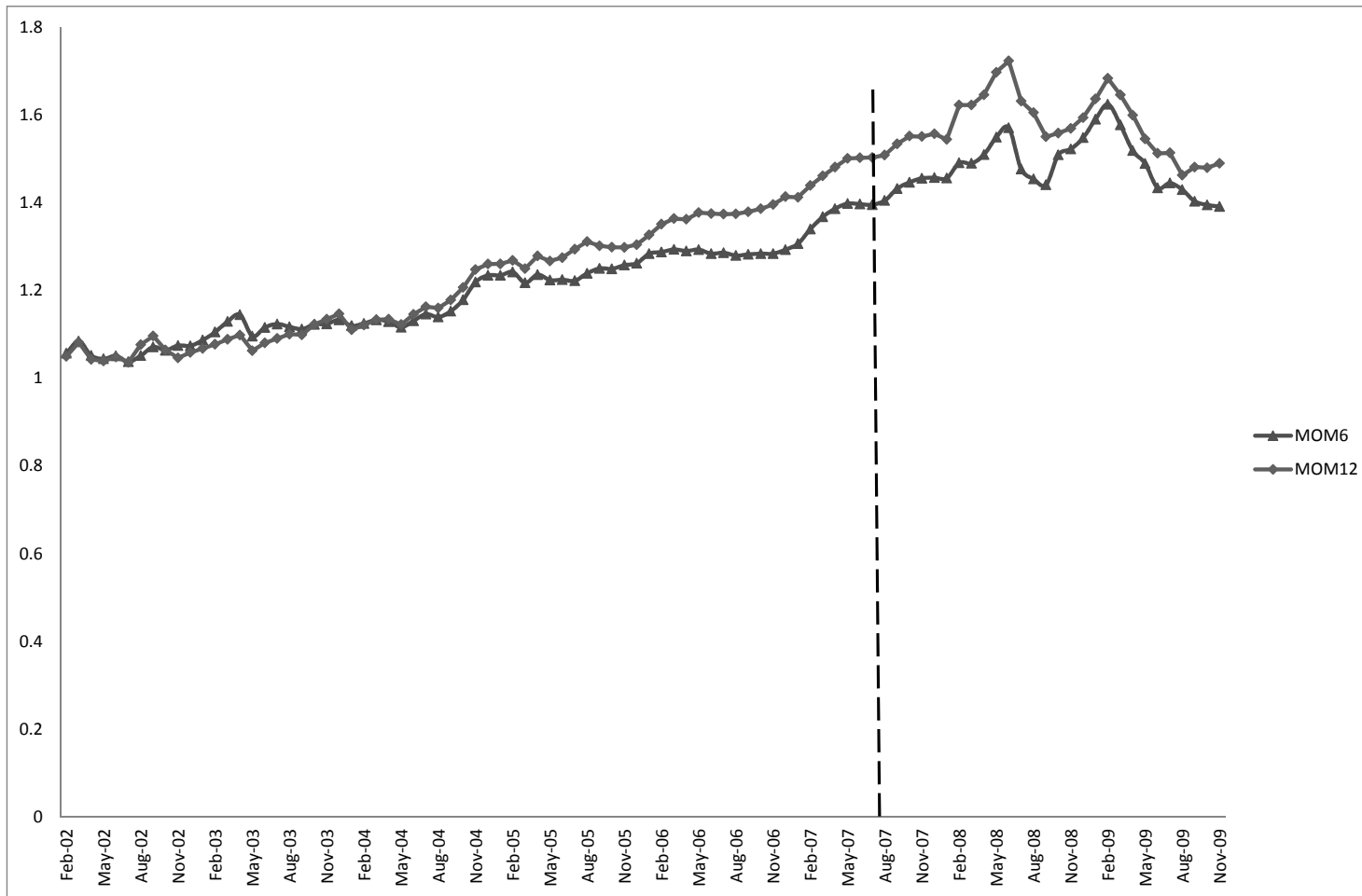


Figure 6.4 Cumulative Significant Momentum Factor Payoffs

We conduct univariate regressions of our style attributes against realised returns. Cumulative geometric payoffs to the significant factors in the momentum category are computed and displayed below. All cumulative payoffs are based to a value of 1 and calculated over the full-sample period, February 2009 to December 2009. The transition between the in- and out-of-sample periods is illustrated by a vertical dashed line.

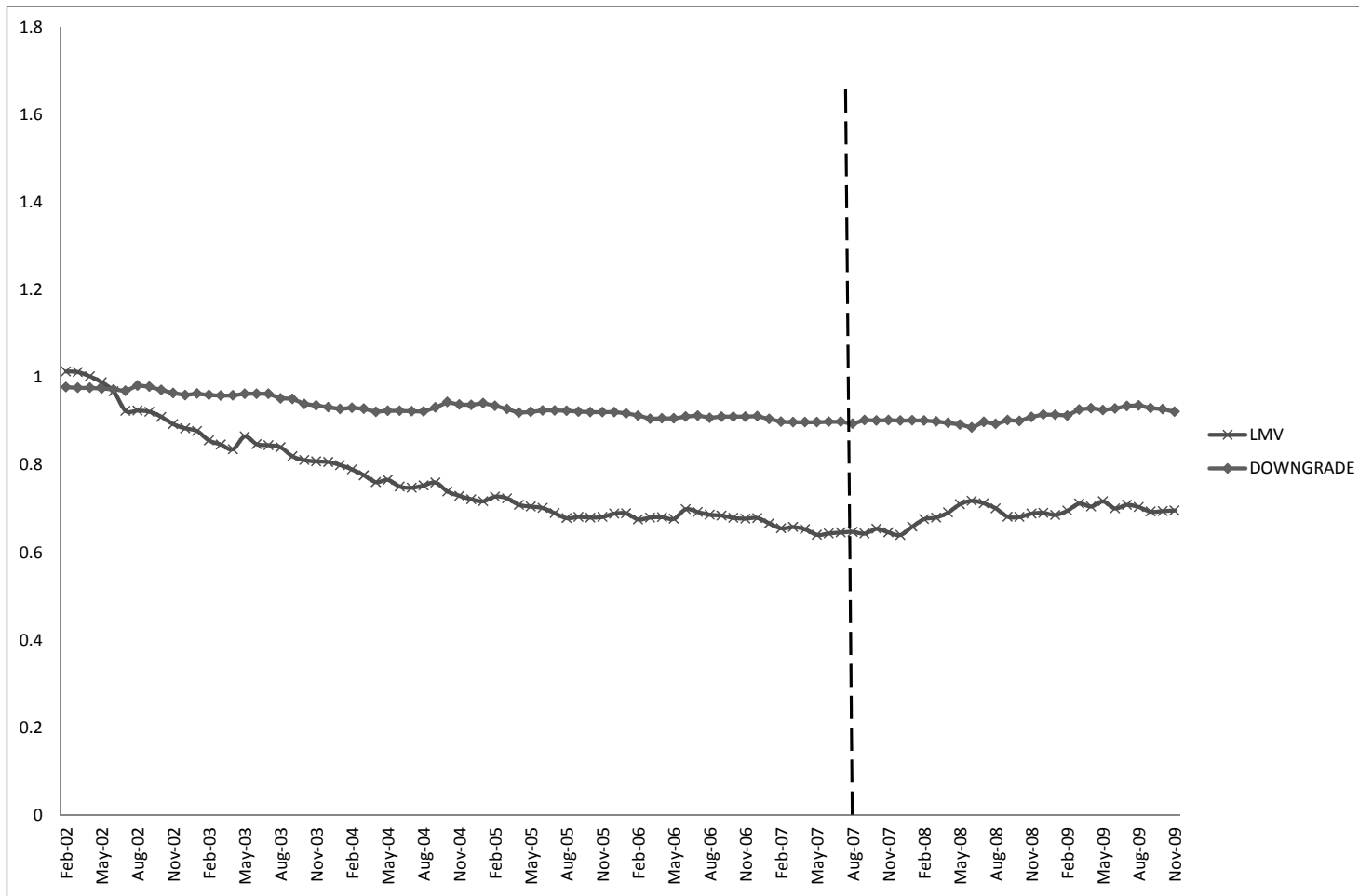


Figure 6.5 Remaining Cumulative Significant Factor Payoffs

We conduct univariate regressions of our style attributes against realised returns. Cumulative geometric payoffs to the significant factors in the analyst recommendation category are computed and displayed below. All cumulative payoffs are based to a value of 1 and calculated over the full-sample period, February 2009 to December 2009. The transition between the in- and out-of-sample periods is illustrated by a vertical dashed line.

It is clear from the results that the sign of the factor payoffs is somewhat variable even during the relatively calm bull run of the in-sample period but that the significant factors identified during this period are relatively consistent. In contrast, factor payoffs are extremely volatile during the market crash of the out-of-sample period indicating that factor models are likely to become less robust where such extreme changes in market regimes occur.

6.3 Conclusions

We employ univariate regression-based analysis to investigate the predictability of returns using style anomalies. The univariate analysis finds eight significant factors over the in-sample period. Book value-to-market value, earnings yield, log of market value, 6-month momentum, 12-month momentum, analyst downgrades and cash flow-to-price are significant anomalies which were all previously identified in the literature. Prior studies have examined the period 1990 to 2000 but if we consider the findings of Fraser and Page (2000) and van Rensburg (2001) many of these factors are found to be significant as far back as 1973, albeit for limited samples of Industrials shares only. The results indicate that the major significant anomalies identified in this and prior studies on the South African market have been consistent over at least the period 1990 to 2007.

12- and 24-month dividend growth-to-price and 12-month earnings growth-to-price are new factors identified over the period February 2002 to July 2007. These factors fall within the classification of growth anomalies and were found to be insignificant in prior studies. In addition, even though 6-month momentum and 12-month momentum were found to be significant in prior studies, this was only for samples of Industrials shares over the period 1973 to 1999 and the same factors were not found to be significant by van Rensburg and Robertson (2003a) for a more representative sample over the period 1990 to 2000.

This indicates that even though there appears to be consistency in the significant factors identified in this and prior studies, factor stability is not necessarily consistent over time. This may be due to the anomalies being sample-specific or due to problems in the data sets as the majority of these studies have been conducted on incomplete data sets and have employed listwise deletion in order to adjust for this. The analysis in this study was conducted using both the listwise deletion method as well as a multiple imputation methodology to account for the missingness in the data. The results using the multiply imputed data sets were found to be largely unchanged from those using the listwise deletion method. However, it must be stressed that the missingness in our sample was limited, and that the issue of missingness could potentially have a significant impact on any analysis of this kind.

The primary focus of this study, however, is to investigate the predictability of returns over periods of market instability. We find that for our sample period, only cash flow-to-price remains significant after the transition to the out-of-sample period, which corresponds with the subprime mortgage crisis. This indicates the instability of fundamental factors to significant changes in the market. The findings suggest that firms with greater (lower) cash-flows relative to their price outperform (underperform) during periods of market instability

Haugen and Baker (1996) suggest that factors must have consistent payoffs in order to be exploitable. Wilcoxon and van der Waerden sign rank tests indicate that all of the factors found to be significant over both the in-sample and out-of-sample periods exhibit consistent payoffs during their respective periods of significance. This seems to indicate that significant factors tend to be those with consistent payoffs.

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7. Linear Factor Model Construction: Multivariate Testing

Jensen (1978) stresses that if anomalous return behaviour cannot be profited from, it is not economically significant. Following our identification and examination of style anomalies in Chapter 6 we attempt to construct multi-factor linear predictive models of return in order to establish whether it would indeed have been possible to profit on a consistent basis from these anomalies over both the in- and out-of sample periods of the analysis.

Connor and Korajczyk (2010) provide an overview of factor modelling techniques for asset returns suggesting that the theory encompasses three primary areas – macroeconomic models, characteristic-based models and statistical models. The first two forms of models employ multi-factor regression approaches to determine the relationship between the chosen factor set and asset returns in a particular market. The two approaches differ primarily in the choice of factors, with macroeconomic models exploring the relationship between asset returns and specific macroeconomic data such as bond yields and industrial production while characteristic-based models use firm-specific characteristics such as accounting ratios as drivers for asset returns.

Statistical models in contrast employ factor and cluster analysis techniques to attempt to identify common sources of variability within the data sample based on the covariances of asset returns. These models therefore are not limited to interactions with a finite factor sample and provide a more complete description of the underlying drivers of asset returns. Unfortunately, as the nature of these relationships is not specified a priori, these underlying drivers are not always clear (see Page (1993) and van Rensburg and Slaney (1997) for a discussion of these difficulties with an emphasis on the South African market).

Consistent with our univariate analysis in Chapter 6, we employ a multiple regression approach in order to develop characteristic-based factor models for predictive purposes. The multiple OLS regression analysis simultaneously estimates the monthly cross-sectional regression coefficients of a number of selected firm characteristics against realised returns, similar to models applied by practising analysts (Haugen and Baker, 1996).

7.1 Methodology

We construct multivariate factor models similar to those of Haugen and Baker (1996). In each month a cross-sectional OLS multi-factor regression is conducted with the one-month forward return as the dependent variable and the style attributes under consideration as independent variables.

$$R_{i,t+1} = \alpha_{t+1} + \sum \beta_{k,t+1} F_{i,k,t} + \varepsilon_{i,t+1} \quad (7.1)$$

where $R_{i,t+1}$ is the return on stock i in month $t+1$, $\beta_{k,t+1}$ is the regression coefficient of factor k in month $t+1$ and $F_{i,k,t}$ is the exposure to factor k for stock i at the end of month t .

These regressions generate slope coefficients for each independent variable (factor). As per Haugen and Baker (1996) a twelve-month trailing mean of these slopes for each factor is used to estimate the following month's payoff to that factor. We can thus employ this estimated payoff to forecast returns one month ahead as follows

$$E(R_{i,t+1}) = \sum E(\beta_{k,t+1}) F_{i,k,t} \quad (7.2)$$

where $E(R_{i,t+1})$ is the expected return on stock i in month $t+1$, $E(\beta_{k,t+1})$ is the expected payoff of factor k in month $t+1$ as represented by the twelve-month trailing mean of the slopes for that factor and $F_{i,k,t}$ is the exposure to factor k for stock i at the end of month t .

Additionally, in order to assess the impact of the stability of the factor payoffs over time on the predicted returns, we amend Haugen and Baker's (1996) approach to test three-month and six-month trailing means in order to estimate the expected payoffs.

Aside from varying the periodicity of the trailing mean in estimating our expected factor payoffs, our methodology differs from that of Haugen and Baker (1996) in a few material respects. Firstly, we do not simultaneously test all factors under consideration. Michaud (1999) argues that although the inclusion of a greater number of explanatory variables is likely to increase in-sample predictive power, it is also likely to result in a decrease in out-of-sample predictive power. Instead, we employ a stepwise procedure to optimise the style attributes included in the model. All twenty variables are considered for inclusion into the model and tested individually against a pre-specified performance metric. The factor which yields the greatest average value for the metric over the sample is taken up as the first factor in the model. The process is repeated with all unselected attributes re-tested in conjunction with the first factor. The factor that yields the greatest increase in the metric on the second pass is then taken up as the second factor in the model. This procedure is repeated until further passes yield no further improvements in the stated metric.

It must be stressed that the stepwise procedure does not necessarily yield the optimal model for any specified criterion as it does not consider all possible combinations of factors. It does provide at least an approximation of the optimal model for a given criterion, however.

Secondly, Haugen and Baker (1996) replaced missing factor values with the mean for the factor in each month. This approach has been criticized as it may lower the variance of the factor series and potentially distort the relationship between variables where there is significant missingness in the data set. As the results from Chapter 6 indicated that there was no difference in findings between the univariate tests using the unadjusted data set with listwise deletion (i.e. a complete case basis) and the multiply imputed data sets, we employ the complete case data set for the analysis.

7.1.1 Selection of Evaluation Metrics

For purposes of the analysis we have employed a range of metrics and generated a single optimized model using each for purposes of comparison. The metrics tested are drawn from Frankish (2004) and include the information coefficient (IC), Qian and Hua's (2004) information ratio, the Slope T-statistic and the Quintile Spread T-statistic. A definition for each metric is provided in Table 7.1.

Table 7.1 Summary of the metrics used to evaluate factors in the step-wise procedure

We employ a stepwise procedure to optimise the style attributes included in the factor models to be constructed. The stepwise procedure is conditioned on a range of performance metrics as defined below.

Performance Metric	Description
Information Coefficient (IC)	The correlation between forecasted and realised returns in each month averaged over the sample.
Information Ratio (Qian and Hua, 2004)	$IR \approx \frac{\overline{IC}_t}{\sigma(IC_t)}$
Slope T-Statistic	The t-statistic of the slope coefficients obtained from the regression between the forecasted and realised returns in each month of the sample, defined as: $t = \frac{\beta_{R,\alpha-1}\sqrt{N}}{\sigma_{\beta,\alpha-1}}$
Quintile Spread T-Statistic	The t-statistic of the difference in mean between the returns of the 1 st and 5 th quintiles of shares ranked by forecast return in each month.

The information coefficient, the correlation between forecast returns and realised returns, has traditionally been used as a measure of the forecast ability of asset managers. Grinold and Kahn (1995) suggest that an IC of 0.1 is an indication of the potential for profitable predictability.

Grinold's (1989) fundamental law of active management relates manager skill, as measure by the information coefficient, to the breadth of manager forecasts to generate an information ratio, IR,

$$IR \sim IC\sqrt{N} \quad (7.3)$$

where IC is the information coefficient and N is the number of forecasts made.

This suggests that performance is enhanced where forecasts can be applied consistently over a broad range rather than for single, independent bets. Grinold and Kahn (1995) argue that the highest value added through active management is proportional to the squared information ratio. We do not consider the Grinold criteria for our model construction given that all factors in the data set have complete data and hence \sqrt{N} is consistent between factors.

Qian and Hua (2004) argue that there is an additional source of active risk for any investment strategy that originates from the variability of the strategy's information coefficient over time. As such they suggest that the ratio of the average information coefficient to the standard deviation of the information coefficient provides a more consistent estimate of the information ratio.

$$IR = \frac{\overline{IC}_t}{\sigma(IC_t)} \quad (7.4)$$

The Slope T-statistic attempts to maximize the t-statistic of the slope of the regression of the forecast and actual returns in each month. It can be demonstrated that the slope coefficient is a scaled function of Qian and Hua's (2004) information ratio, where

$$t = \frac{\overline{IC}_t \sigma_{R_t}}{\sigma_{IC_t R_t}} \quad (7.5)$$

Frankish (2004) suggests that the slope t-statistic may better account for extreme observations as these will have the greatest influence on the regression slope.

Finally, the Quintile Spread statistic is a fractile performance measure which maximizes the t-statistic of the mean difference in returns between the 1st and 5th quintiles in each month. The criterion is indicative of the models ability to distinguish on a consistent basis between forecast out- and underperformers.

7.1.2 Multicollinearity

Multicollinearity occurs where two or more predictors in a multiple regression are highly correlated. While multicollinearity does not impact on the ability of a multi-factor model to provide forecasts, it can result in regression coefficients that are unstable, as indicated by the standard error of the coefficient. Makridakis et al. (1998) emphasise that multicollinearity is not a concern unless the individual regression coefficients are of interest or it is necessary to isolate the contributions of individual predictors on the dependent variable, independent of the influence of the other predictors in the regression.

A common misconception is that multicollinearity can be identified by simply examining the correlations between predictors. An absence of high correlations between predictors does not necessarily indicate a lack of multicollinearity (Makridakis et al, 1998).

In the absence of high pairwise correlations between any of the predictors the determinant of the correlation matrix can be examined for evidence of multicollinearity. Where the determinant of the correlation matrix of the predictors is close to zero, some or all of the predictors are highly correlated (Mansfield et al, 1982). Alternatively, we can calculate the tolerance associated with each predictor and discard those variables with the smallest tolerance. The tolerance of a predictor is given by 1 minus the squared multiple correlation between that predictor and the remaining predictors.

7.1.3 Performance Evaluation

Model performance is evaluated by ranking forecasts in each month in descending order and then forming equally-weighted quintiles of these ranked stocks. Quintile 1 will therefore represent the top twenty percent of the sample in each month by forecast performance while Quintile 5 will represent the bottom twenty percent by forecast performance. The Sharpe ratio (Sharpe, 1966) is typically employed as a measure for ranking portfolio performance on the basis of the relationship between excess return and total portfolio risk, and is closely related to the Capital Market Line (CML) of the CAPM model. The measure is defined as

$$S_i = \frac{\bar{R}_i - \bar{r}_f}{\sigma_i} \quad (7.6)$$

where \bar{R}_i is the average rate of return on portfolio i during the specified time period, \bar{r}_f is the average rate of return on risk-free assets during the period and σ_i is the standard deviation of the returns for portfolio i during the period. Higher Sharpe ratios are indicative of greater rewards relative to risk and portfolios are therefore ranked in decreasing order of Sharpe ratio for comparison.

The Sharpe ratio may, however, lead to spurious rankings between portfolios where excess returns are negative. Consider an example of two portfolios, A and B, where A has an average excess return of -3% over some period and B has an average excess return of -2.5% over the same period. The standard deviation of A's returns is 0.15 while that of B's returns is 0.12. Calculating the Sharpe ratio for each portfolio using equation 6.6 we find a Sharpe ratio of -0.2 for A while that for B is -0.208. Portfolio A therefore has the highest (or least negative) Sharpe ratio and would therefore seem to have performed better over the period than B. Examining the information, however, it is clear that A had both lower returns over the period and a higher associated standard deviation of returns indicating underperformance relative to B.

This problem is of particular interest to this study as the out-of-sample period corresponds with a market crisis and excess returns for some or all of the portfolios formed over this period may be negative as a consequence. Israelsen (2005) suggests a modified Sharpe ratio to accommodate the impact of negative excess returns on the Sharpe ratio whereby an exponent is added to the denominator of the equation.

$$S_i = \frac{\bar{R}_i - \bar{r}_f}{\sigma_i^{|\bar{R}_i - \bar{r}_f|}} \quad (7.7)$$

Where the excess returns on the portfolio are positive, this modified Sharpe ratio provides the same result as the original equation. Where the excess return is negative, however, the consequent result is modified such that portfolios may be correctly ranked in descending order of modified Sharpe ratio regardless of the sign of the excess returns. We employ Israelsen's (2005) modified Sharpe ratio in our analysis.

The performance of the quintiles formed in the analysis will be compared between our multivariate models over the in-sample period and out-of-sample period in order to measure the ability of each factor model both to accurately forecast returns and also to differentiate between over- and under-performing stocks.

We use the equal weighting methodology for the formation of our portfolios as the method is less biased towards the excessive industry concentration which characterizes the JSE (Kruger and van Rensburg, 2008). An equally-weighted approach does, however, give rise to concerns regarding liquidity and market

capacity constraints. As small cap stocks are weighted on the same basis as large-cap stocks it is possible for an equal weighting methodology to provide a record of performance that is not practically achievable on the basis of these constraints. We therefore restate all findings using a market capitalization-weighted approach for purposes of comparison.

In addition we employ the success rate performance measures suggested by Achour et al. (1999). The success rate is measured for both the top and bottom quintiles and measures the percentage of stocks in the top quintile that outperform the average market return in each month and the percentage of stocks in the bottom quintile that underperform the average market return in each month. This figure is then used to calculate the average success rate over the sample, the standard deviation of the success rate, the success consistency ratio (the percentage of months for which the success rate was greater than 50%) and the maximum and minimum success rates observed over the sample for both the top and bottom quintiles.

7.1.4 Portfolio Rebalancing and Transaction Costs

We measure the impact of portfolio rebalancing and transaction costs on the models constructed using a measure of portfolio turnover. The measure employed captures the impact of rebalancing as a result of both selling and purchasing shares at the beginning of each month.

As per Yu (2008), we define the measure of portfolio rebalancing PR as

$$PR = \sum_{i=1}^{K_t} \Delta w_{i,t} \quad (7.8)$$

where K_t represents the number of shares in the sample in month t and $w_{i,t}$ is the rebalancing weight required for share i at the beginning of month t . The rebalancing weight required for each share is the difference between the required weight in share i at the beginning of month t , as determined by the model, and the weight in share i at the end of the previous month after accounting for the impact on the share's weight in the portfolio of the return on the share over that month.

$$\Delta w_{i,t} = w_{i,t,req} - w_{i,t-1} \times \frac{(1 + R_{i,t-1})}{(1 + R_{q,t-1})} \quad (7.9)$$

where $w_{i,t,req}$ is the required portfolio weight of share i at the beginning of month t , $R_{i,t-1}$ is the return on share i for month $t - 1$ and $R_{q,t-1}$ is the return on portfolio q for month $t - 1$.

The maximum value of the portfolio turnover measure as defined above is 200%, assuming that all shares in the portfolio are sold and replaced in any given month. In order to assess the impact of transaction costs on portfolio performance we multiply the average portfolio turnover for any given portfolio by an appropriate measure of transaction costs associated with individual trades (not a round-trip cost as our measure of portfolio turnover measures buy and sell transactions individually). Yu (2008) suggests that 20 basis points (bp) is a conservative measure of transaction costs in South Africa. As an indication of the impact of transaction costs on the performance of our models, however, we provide for transaction costs of 20bp, 50bp and 100bp in the analysis.

7.2 Results

Prior to the construction of our multi-factor models we examine the pairwise correlation matrix of the factors for evidence of multicollinearity. The correlation matrix is provided in Appendix C.1. It is evident from the correlation matrix that the only pairwise correlations above a value of 0.7 exist between MOM6 and MOM12 and MOM6 and MOM3. Where we find that combinations of these factors enter into our stepwise models we undertake to retain only the first (most significant) of the pairwise factors in the model.

7.2.1 Factor Model Construction

We employ the stepwise procedure detailed in Section 7.1 in order to construct a series of multivariate linear models for comparison. Our initial testing compares the effect of varying the periodicity of the trailing mean estimation of expected factor payoffs on the model scores as detailed in Table 7.2.

Table 7.2 The Impact of Varying the Factor Payoff Estimation Window on Model Score

Multi-factor linear predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Expected factor payoffs in each month were estimated using a trailing mean of factor payoffs from prior months over 3, 6 and 12 months. A comparison of the model scores for each criterion for the differing estimation periods is presented below.

MODEL SCORE				
TRAILING PERIOD	IC	IR (Q & H)	SLOPE T	QUINTILE SPREAD
3 Months	0.126	0.909	5.658	7.056
6 Months	0.138	1.218	8.560	6.724
12 Months	0.149	1.364	9.012	7.258

We note that the maximized model scores for all selection criteria, excepting the Quintile Spread measure, increase as the trailing estimation period increases. This suggests that the factor payoffs are more consistent over longer periods, resulting in more robust estimates of expected factor payoffs over the twelve-month trailing estimation window. The quintile spread measure is dependent on the ranking of the forecast returns as it is a measure of spread between the top and bottom quintile of forecast returns in

each month. The optimization procedure for this criterion is therefore susceptible to the influence of incorrectly forecast shares which have extreme returns.

Table 7.3 Factor Constituents for Models when Varying the Factor Payoff Estimation Window

Multi-factor linear predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Expected factor payoffs in each month were estimated using a trailing mean of factor payoffs from prior months over 3, 6 and 12 months. A comparison of the factor constituents for each criterion for the differing estimation periods is presented below.

IC MODEL

12M	SCORE	6M	SCORE	3M	SCORE
MOM12	0.085	CTP	0.067	CTP	0.067
MOM1	0.126	MOM12	0.093	C12MEPSP	0.085
CTP	0.139	MOM1	0.125	MOM1	0.095
BVTM	0.149	BVTM	0.138	MOM6	0.120
				C12MDPSP	0.124
				MOM12	0.125
				BVTM	0.126

IR (Q & H) MODEL

12M	SCORE	6M	SCORE	3M	SCORE
CTP	0.795	CTP	0.648	CTP	0.588
C12MEPSP	1.088	C12MEPSP	0.953	C12MEPSP	0.909
MOM1	1.103	MOM1	0.957		
MOM12	1.232	MOM12	1.107		
BVTM	1.302	BVTM	1.218		
VT	1.349				
STDEV12	1.364				

SLOPE T

12M	SCORE	6M	SCORE	3M	SCORE
CTP	4.822	CTP	4.572	CTP	4.773
C12MEPSP	6.232	C12MEPSP	5.912	C12MEPSP	5.658
MOM1	7.914	MOM1	5.976		
MOM12	8.303	MOM12	6.644		
BVTM	8.602	BVTM	7.391		
VT	8.909	VT	8.368		
STDEV12	9.012	DOWNGRADE	8.447		
		POUT	8.560		

QUINTILE SPREAD

12M	SCORE	6M	SCORE	3M	SCORE
CTP	5.861	CTP	5.187	CTP	4.546
BVTM	6.359	BVTM	6.446	C12MDPSP	6.067
C24MDPSP	6.909	DOWNGRADE	7.742	C12MEPSP	6.663
POUT	7.258			C24MDPSP	7.056

Table 7.3 provides a comparison of the factor constituents for each of the models constructed for the varied estimation windows detailed above. We find strong similarities in the factor constituents of models over the 6 and 12-month estimation periods, while the number of factors over the 3-month estimation period is either substantially greater or smaller than that for the longer estimation periods. This seems to support the view that factor payoffs are more consistent over longer periods.

On the basis of these initial results, the discussion to follow is confined to the models constructed using the trailing twelve-month estimation period. The factor composition of each of the models is presented in Table 7.4, with the factors listed in the order in which they entered each model. MOM6 was not found to be significant in any of the models and no adjustment was therefore required to correct for the potential multicollinearity identified above. The IR (Qian) and Slope T criteria yielded the same overall models even though the order in which the individual factors entered each model differed. This is consistent with the discussion in Section 7.1.1 which demonstrated the similarities between the two measures.

Table 7.4 Factor Model Constituent Factors

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Significant factors which constitute each model are listed in the order in which they entered each model.

FACTOR #	IC	SCORE	IR (Q & H)	SCORE	SLOPE T	SCORE	QUINTILE SPREAD	SCORE
FACTOR 1	MOM1	0.085	CTP	0.795	CTP	4.822	CTP	5.861
FACTOR 2	MOM12	0.126	C12MEPSP	1.088	C12MEPSP	6.232	BVTM	6.359
FACTOR 3	CTP	0.139	MOM1	1.103	MOM1	7.914	C24MDPSP	6.909
FACTOR 4	BVTM	0.149	MOM12	1.232	MOM12	8.303	POUT	7.258
FACTOR 5			BVTM	1.302	BVTM	8.602		
FACTOR 6			VT	1.349	VT	8.909		
FACTOR 7			STDEV12	1.364	STDEV12	9.012		

Table 7.5 lists the frequency with which the constituent factors appear in the five models constructed.

Table 7.5 Frequency of Constituent Multivariate Factors

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. The frequencies with which the factors identified appear in the four multivariate models are displayed below.

SIGNIFICANT UNIVARIATE FACTORS	# MODELS	OTHER UNIVARIATE FACTORS	# MODELS
CTP	4	MOM1	3
BVTM	4	POUT	1
MOM12	3	STDEV12	2
C12MEPSP	2	VT	2
C24MDPSP	1		

Of the nine variables identified as significant in the univariate tests, five of the variables appeared in one or more of the models constructed while LMV, C12MDPSP, MOM6, and DOWNGRADE failed to appear in any of the models constructed. The effect of these factors therefore seems to have been subsumed in the multivariate analysis. There is significant consistency in the first few factors in the models and most of the explanatory power for each model derives from the first three to four factors that enter the model. Additional factors subsequently provide only marginal improvements to each metric. It is found that BVTM and CTP are common to all four models while MOM1 and MOM12 are common to the IC, Qian and Slope T models. As the Qian and Slope T models are the same we present a single set of results for these models going forward.

7.2.2 Factor Model Performance

The results of an evaluation of the relative performance of the models in both the in- and out-of-sample periods is presented in Table 7.6. It should be noted that only 54 months of forecasts in the in-sample period is available, given the need for a 12-month rolling average period over which to estimate the forecast returns in each month. Model forecasts are therefore available from February 2003.

As discussed in Section 7.1.3, model performance is evaluated by ranking forecasts in each month in descending order and then forming equally-weighted quintiles of these ranked stocks. A long-only strategy is simulated by assuming investment in quintile 1 only. All four factor models significantly outperform the ALSI over the in-sample period with an average monthly geometric return for the models of 4.75% while that of the ALSI over the same period is 2.20%. The Quintile spread factor model demonstrates less volatility than the ALSI while the volatility for the IC and IR Qian/Slope T models is only marginally higher. We calculate modified Sharpe ratios for each quintile using the Short-term Fixed Interest (STEFI) index as the risk-free rate.

Over the in-sample period, quintile 1 for the IC model has the highest monthly geometric return and outperforms the other models on the basis of the modified Sharpe ratio. The Quintile Spread model has the lowest standard deviation of returns but also the lowest geometric average return and hence has the lowest Sharpe ratio over the period.

Examining the out-of-sample period, the geometric return for all models drops substantially with the average monthly geometric returns for the models of 0.56% below the STEFI's average monthly geometric return of 0.87%. The returns for the IC and Quintile Spread models are, however, still in excess of the monthly geometric ALSI return of 0.14% over the period. In addition, the average volatility of returns of the models increases from 0.046 over the in-sample period to 0.062 over the out-of-sample period. The result is that all of the models considered have modified Sharpe ratios below zero. The

Quintile Spread model demonstrates the highest geometric average return and the lowest standard deviation of returns over the period and therefore delivers the highest modified Sharpe ratio while the IC model is ranked second.

We also note that the multifactor models demonstrate a significantly higher number of positive return months relative to the ALSI over the in-sample period but this trend reverses during the out-of-sample period where the ALSI has a higher number of positive return months than the IC and IR/Slope T models. The fact that the ALSI underperforms relative to the multi-factor models over the in-sample period would seem to be due to the smaller average monthly returns of the ALSI over the period.

Table 7.6 Comparison of In-Sample and Out-of-Sample Performance of Multi-Factor Models

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. The monthly geometric return, standard deviation and Sharpe Ratio measures for the top quintile for each model over the in-sample and out-of-sample periods are presented below.

<u>In-Sample</u>				
<u>Top Quintile</u>	<u>IC MODEL</u>	<u>IR (Q & H)/Slope T</u>	<u>Quintile Spread</u>	<u>ALSI</u>
Monthly Geometric Return	4.89%	4.76%	4.25%	2.45%
Standard Deviation	0.046	0.047	0.043	0.045
Modified Sharpe Ratio	0.912	0.866	0.828	0.391
# Positive Months	46 (85.19%)	46 (85.19%)	47 (87.04%)	37 (68.52%)
# Negative Months	8 (14.81%)	8 (14.81%)	7 (12.96%)	17 (31.48%)
<u>Out-of-Sample</u>				
<u>Top Quintile</u>	<u>IC MODEL</u>	<u>IR (Q & H)/Slope T</u>	<u>Quintile Spread</u>	<u>ALSI</u>
Monthly Geometric Return	0.62%	0.31%	0.76%	0.14%
Standard Deviation	0.073	0.064	0.059	0.066
Modified Sharpe Ratio	-0.00018	-0.00036	-0.00007	-0.00049
# Positive Months	16 (55.17%)	17 (58.62%)	18 (62.07%)	18 (62.07%)
# Negative Months	13 (44.83%)	12 (41.38%)	11 (37.93%)	11 (37.93%)

We plot a graph, Figure 7.1, to illustrate the cumulative geometric returns for the in-sample returns of the top quintile in each model relative to the ALSI. All models significantly outperform the ALSI over the in-sample, but the result is expected given that these models were specifically optimized during the period in question.

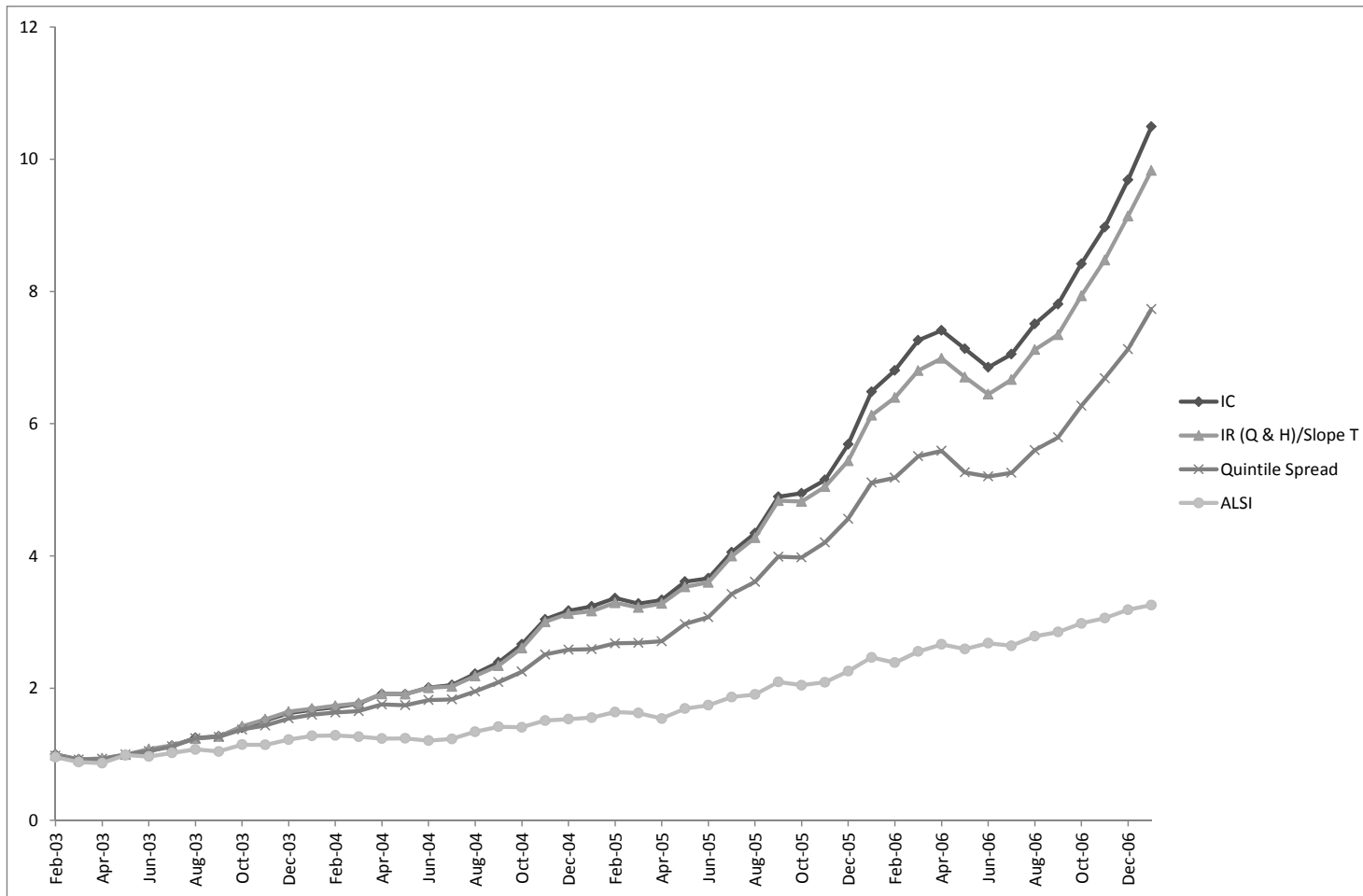


Figure 7.1 Cumulative Geometric Returns for Quintile 1 (In-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for the top quintile in each model is presented below. Cumulative geometric returns on the ALSI over the period are provided for comparative purposes.

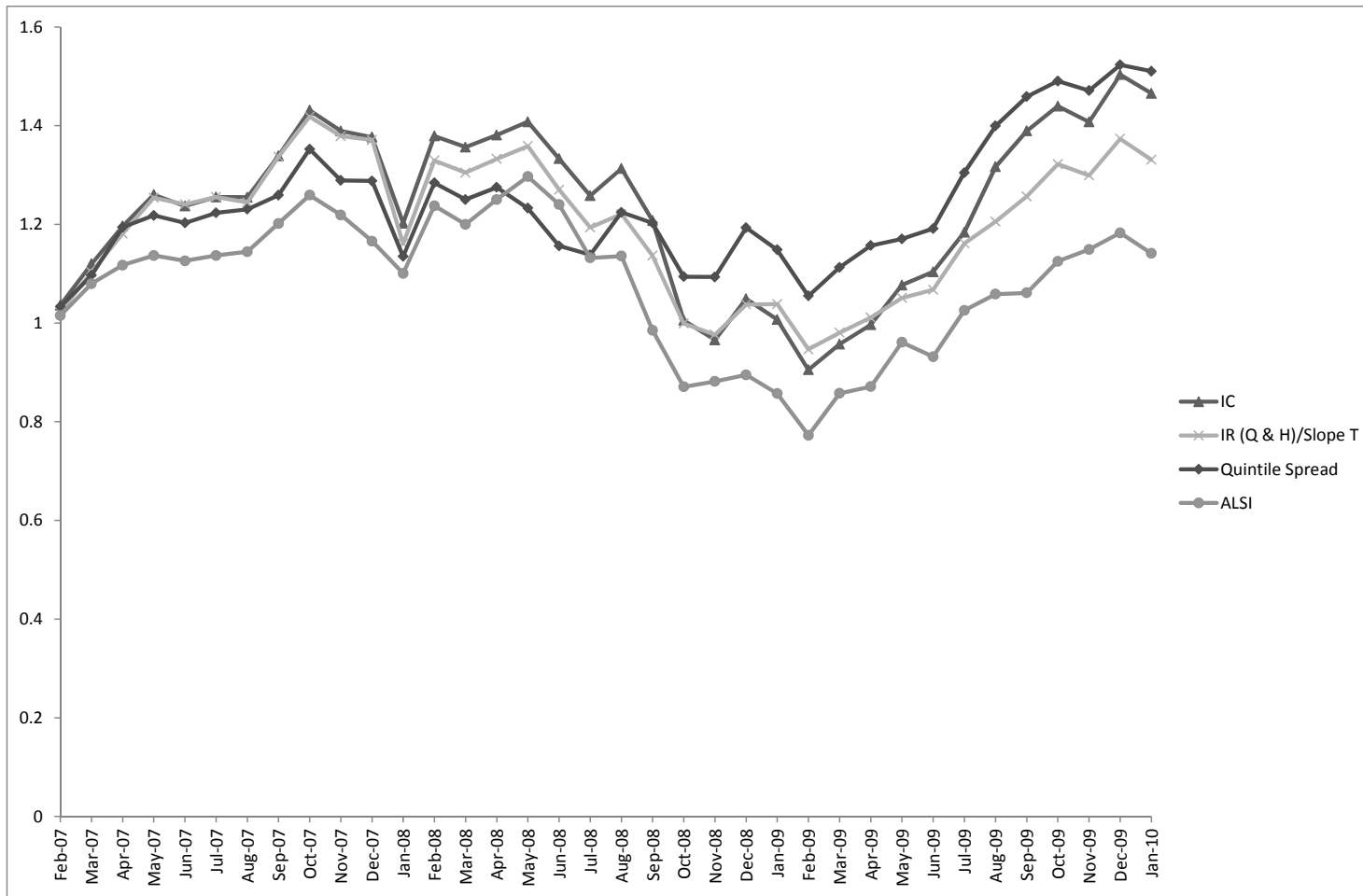


Figure 7.2 Cumulative Geometric Returns for Quintile 1 (Out-of-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for the top quintile in each model is presented below. Cumulative geometric returns on the ALSI over the period are provided for comparative purposes.

Figure 7.2 plots the cumulative geometric return of the top quintile for each of the factor models and the ALSI over the out-of-sample period. The increased volatility between the in- and out-of-sample periods is clearly evident when comparing the two graphs above. Interestingly, the Quintile spread underperforms relative to other models (excepting the Qian/Slope T model) during the early part of the out-of-sample period, even underperforming relative to the ALSI, but once the market recovery begins in February 2009 it generates substantial outperformance relative to other models and exceeds their cumulative performance over the period by January 2010.

An examination of the performance spread between the top and bottom quintiles for both the in- and out-of-sample periods is presented in Table 7.7.

Table 7.7 Comparison of Return Spread between Quintiles 1 and 5 for Multi-Factor Models

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. The monthly geometric return, standard deviation and Sharpe Ratio for the spread between the top and bottom quintile for each model is presented below.

<u>In-Sample</u>			
<u>Top - Bottom Quintile</u>	<u>IC</u>	<u>IR (Q & H)/Slope T</u>	<u>Quintile Spread</u>
Monthly Geometric Return	3.04%	2.62%	2.17%
Standard Deviation	0.029	0.030	0.015
Modified Sharpe Ratio	0.821	0.643	0.989
# Positive Months	48 (88.89%)	46 (85.19%)	52 (96.30%)
# Negative Months	6 (11.11%)	8 (14.81%)	2 (3.70%)
 <u>Out-of-Sample</u>			
<u>Top - Bottom Quintile</u>	<u>IC</u>	<u>IR (Q & H)/Slope T</u>	<u>Quintile Spread</u>
Monthly Geometric Return	1.40%	1.51%	0.95%
Standard Deviation	0.040	0.044	0.042
Modified Sharpe Ratio	0.131	0.143	0.019
# Positive Months	19 (65.52%)	20 (68.97%)	18 (62.07%)
# Negative Months	10 (34.48%)	9 (31.03%)	11 (37.93%)

We note that the excess geometric return of the top quintile over the bottom is substantial for all the models over the in-sample period. It is also evident that the use of a long-short strategy allocated to quintile 1 (long) and quintile 5 (short) would have underperformed a long-only strategy invested in quintile 1 during the in-sample period for all models, excepting the Quintile spread model. Even though the Quintile spread model demonstrates a lower geometric return over the period than the other models, it has a significantly lower standard deviation of returns and it therefore demonstrates the greatest modified Sharpe ratio over the in-sample period. This is consistent with the manner in which the criterion maximizes the t-statistic of the difference in means between the top and bottom quintiles. Over the in-

sample period, the Quintile spread model following such a naïve long-short approach would have outperformed a long-only approach using this model criterion.

The spread in return performance for the top and bottom quintiles is significant in the out-of-sample period due to the bottom quintile yielding negative returns for most of the period for all models considered. A long-short strategy over this period, as described above, would have resulted in higher returns and Sharpe ratios than those possible under a long strategy alone. We note that over the out-of-sample period the spread in quintile returns is highest for the IC model and lowest for the Quintile spread model. This indicates that the model criterion for the Quintile spread either over fit the data of the in-sample period or the factor composition of the model was significantly impacted by the market instability of the out-of-sample period.

Figures 7.3 and 7.4 illustrate the spread in returns between the top and bottom quintiles over both the in- and out-of-sample periods. Note the strong initial gains for all five models followed by substantial volatility of the spread in returns between June 2008 and January 2010. Of the four models, the Quintile Spread appears to be the most consistent during the volatile out-of-sample period which is consistent with the manner in which the criterion is optimized. It is evident from the graphs, however, that the Quintile Spread model underperforms the other models until July 2008 at which point it generates substantial returns until December 2008, subsequent to which the trend reverses. In addition, while our focus is on the relative performance between the top and bottom quintiles, we provide graphs of the overall quintile performance for all five models in Appendix C.2. The graphs illustrate a clear separation in the performance of the quintiles for each model over the in-sample but this separation is less obvious during the out-of-sample periods.

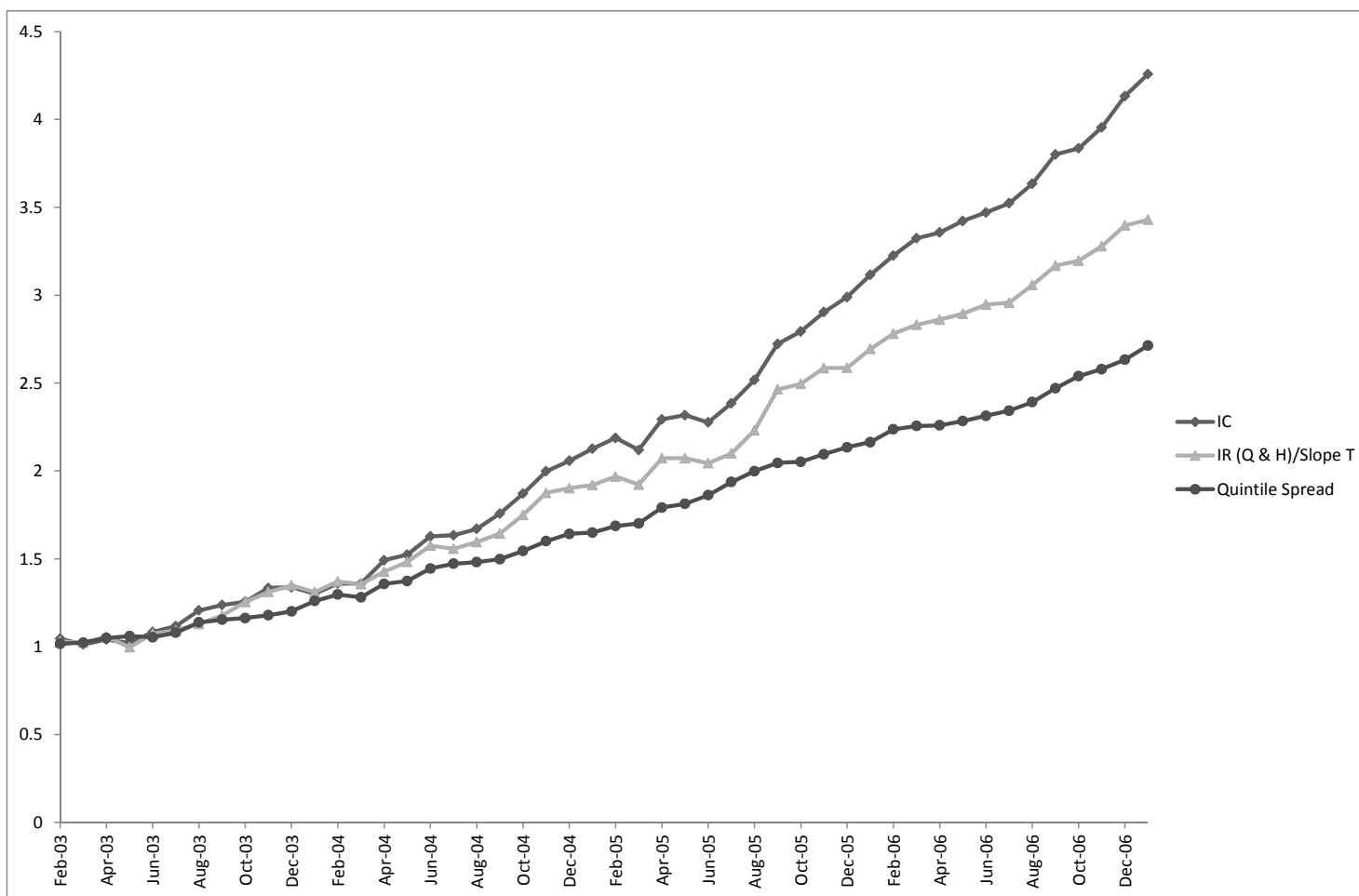


Figure 7.3 Cumulative Geometric Returns for Spread between Quintiles 1 and 5 (In-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. The cumulative geometric return for the difference in returns between the top and bottom quintile in each model is presented below.

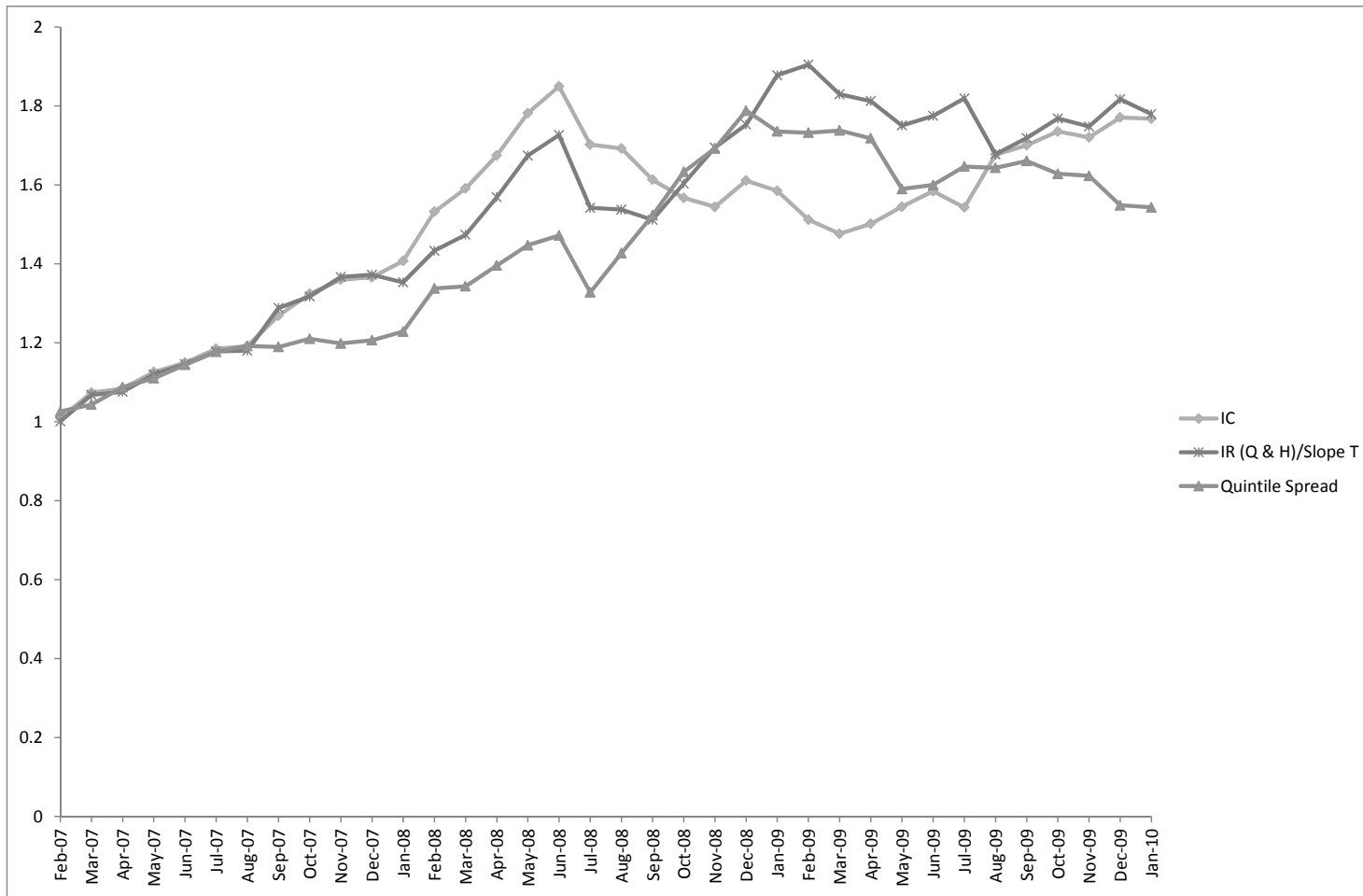


Figure 7.4 Cumulative Geometric Returns for Spread between Quintiles 1 and 5 (Out-of-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. The cumulative geometric return for the difference in returns between the top and bottom quintile in each model is presented below.

7.2.3 Equal Weightings versus Market Cap Weightings

In order to examine the impact of our weighting methodology on our performance results, we re-weight our quintiles on the basis of market capitalization rather than equal weights and restate our results in Table 7.8. The impact of the change in the weighting methodology varies across the models. The geometric returns and modified Sharpe ratios are higher over the in-sample period for the IC and Quintile Spread models but lower for the Qian/Slope T model. Over the out-of-sample period, however, it is clear from the results that performance for all the models considered, excepting the Quintile Spread model, decreases substantially when the market-weighting methodology is applied. This is consistent with findings regarding the size effect. Smaller capitalization shares are found to exhibit higher rates of return and the market capitalization method restricts the weighting of these smaller shares in the sample. This is particularly severe on the JSE due to the excessive levels of concentration that characterize the South African market.

Table 7.8 Comparison of Performance of Multi-Factor Models using Market-Weighted Quintiles

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into market capitalization-weighted quintiles. The monthly geometric return, standard deviation and Sharpe Ratio for the top quintile for each model over the in-sample and out-of-sample periods is presented below.

	<u>In-Sample</u>		
<u>Long-Only</u>	IC	IR (Q & H)/Slope T	Decile Spread
Monthly Geometric Return	5.14%	4.59%	4.55%
Standard Deviation	0.046	0.048	0.041
Modified Sharpe Ratio	0.978	0.814	0.938
# Positive Months	47 (87.04%)	45 (83.33%)	48 (88.89%)
# Negative Months	7 (12.96%)	9 (16.67%)	6 (11.11%)
	<u>Out-of-Sample</u>		
<u>Long-Only</u>	IC	IR (Q & H)/Slope T	Decile Spread
Monthly Geometric Return	0.19%	0.1%	1.27%
Standard Deviation	0.082	0.073	0.061
Modified Sharpe Ratio	-0.00056	-0.00056	0.06061
# Positive Months	16 (55.17%)	18 (62.1%)	18 (62.1%)
# Negative Months	13 (44.83%)	11 (37.9%)	11 (37.9%)

7.2.4 Success Rate Comparisons

While the prior analysis provides a comparison of the overall performance on the basis of returns for the top quintiles in each model it fails to provide any indication of the ability of the models to distinguish between winners and losers. We therefore employ the success rate measure of Achour et al. (1999) as detailed in Section 7.1.3. This success rate measure is calculated using the percentage of stocks in the top quintile that outperformed the ALSI in each month and the percentage of stocks in the bottom quintile that underperformed the ALSI in each month. The results are presented in Table 7.9.

Table 7.9 Success Rate Comparisons

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. The relative ability of each model to predict winners and losers accurately is measured by the success rate. The average success rate for quintile 1 (5) is calculated as the average percentage of forecasts in that quintile that exceed (fall short of) the return on the ALSI in each month. The success consistency ratio measures the number of months for which the success rate exceeds 50%.

<u>In-Sample</u>			
	IC MODEL	IR (Q & H)/Slope T	Quintile Spread
Average Success Rate (Top Quintile)	59.8%	58.5%	56.1%
Average Success Rate (Bottom Quintile)	55.2%	54.2%	51.7%
Standard Deviation (Top Quintile)	17.8%	18.3%	19.4%
Standard Deviation (Bottom Quintile)	16.1%	16.7%	15.1%
Success Consistency Ratio (Top Quintile)	66.7%	63.0%	61.1%
Success Consistency Ratio (Bottom Quintile)	57.4%	57.4%	51.9%
Max Success Rate (Top Quintile)	95.8%	96.6%	88.9%
Min Success Rate (Top Quintile)	22.2%	16.7%	13.3%
Max Success Rate (Bottom Quintile)	92.0%	88.0%	82.1%
Min Success Rate (Bottom Quintile)	16.0%	16.0%	13.3%
<u>Out-of-Sample</u>			
	IC MODEL	IR (Q & H)/Slope T	Quintile Spread
Average Success Rate (Top Quintile)	50.9%	51.3%	49.4%
Average Success Rate (Bottom Quintile)	54.3%	56.9%	53.4%
Standard Deviation (Top Quintile)	17.1%	20.3%	22.5%
Standard Deviation (Bottom Quintile)	20.2%	19.2%	20.8%
Success Consistency Ratio (Top Quintile)	48.3%	51.7%	48.3%
Success Consistency Ratio (Bottom Quintile)	55.2%	65.5%	51.7%
Max Success Rate (Top Quintile)	77.8%	92.6%	88.9%
Min Success Rate (Top Quintile)	15.4%	7.7%	11.1%
Max Success Rate (Bottom Quintile)	88.9%	88.9%	92.0%
Min Success Rate (Bottom Quintile)	17.9%	10.7%	3.6%

For the in-sample period the IC model has the highest average success rate for both the top and bottom quintiles. Correspondingly, the success rate for the top quintile of the IC model is also the least volatile overall while that of the Quintile Spread model is the least volatile for the bottom quintile. The success consistency ratio measures the number of months for which a particular quintile's success ratio exceeded 50% relative to the total number of months over the period examined. We find that the IC model has the highest consistency ratio for both the top and bottom quintiles while the Qian/Slope T model has the second-highest consistency ratio for both quintiles. The Quintile Spread model has the lowest success rate for both quintiles and also the lowest consistency ratio for both quintiles. All models considered appear to be better able to predict outperformers than underperformers over the in-sample period as the success rate for the top quintile substantially exceeds that for the bottom quintile across all five models.

In the out-of-sample period, the success rate for the top quintile decreases for all models to a rate closer to 50% while the success rate of the bottom quintile increases for all models. The Qian/Slope T model has the highest average success rate for both the top and bottom quintiles. The IC model also has the lowest volatility for the top quintile while the Qian/Slope T model has the lowest volatility for the bottom quintile. The success consistency ratio for the top quintile for all the models decreased significantly from that found during the in-sample period and is closer to 50%. This indicates the decreased ability of the models to consistently forecast winners over the out-of-sample period. The success consistency ratio for the bottom quintile has increased, however, in line with the increase in average success rate for that quintile across all the models.

We compare the success consistency ratio across the models in Tables 7.10 through 7.13 and find that there is significant overlap in the months in which their success ratios exceeded 50%. Over the in-sample period there are 9 shared months out of 48 for which the top quintile has success ratios below 50% and also 9 shared months out of 35 for which this occurs over the out-of-sample period. These periods represent months in which the top quintile of none of the models was able to outperform the ALSI. For the bottom quintile, there are 12 shared months of success ratios below 50% over the in-sample period and 5 months over the out-of-sample period. This corresponds with the decrease in average success ratio for the top quintile predictions and the corresponding increase in average success ratio for bottom quintile predictions as we move from the in- to the out-of-sample period.

We note that while the success ratio analysis shows that the models have a greater than 50% chance of predicting winners and losers in any month during both the in- and out-of-sample period, the consistency tables demonstrate an episodic pattern of predictive success/failure not unlike our findings for the tests of serial dependence in Chapter 4, particularly over the out-of-sample period.

Table 7.10 Comparison of Success Rate Consistency (Top Quintile, In-Sample)

The relative ability of each of the multi-factor models to predict winners and losers accurately is measured by the success rate. The average success rate for quintile 1 is calculated as the percentage of forecasts in that quintile that exceed the return on the ALSI in each month. Months in which the success rate exceeds 50% are shaded in grey to facilitate the comparison of consistency of success rate across the models.

	IC	QIAN	Slope T	Quintile Spread
	% Success Rate (T)	% Success Rate (T)	% Success Rate (T)	% Success Rate (T)
Feb-03	68%	61%	68%	61%
Mar-03	57%	57%	68%	68%
Apr-03	59%	72%	52%	59%
May-03	23%	17%	20%	13%
Jun-03	83%	97%	83%	86%
Jul-03	64%	46%	61%	57%
Aug-03	57%	54%	68%	64%
Sep-03	74%	78%	78%	74%
Oct-03	48%	56%	60%	36%
Nov-03	96%	83%	79%	71%
Dec-03	50%	50%	54%	50%
Jan-04	45%	45%	50%	50%
Feb-04	54%	54%	58%	42%
Mar-04	70%	61%	74%	70%
Apr-04	83%	79%	83%	88%
May-04	41%	44%	37%	33%
Jun-04	89%	89%	96%	85%
Jul-04	48%	44%	59%	22%
Aug-04	44%	44%	48%	30%
Sep-04	59%	59%	70%	52%
Oct-04	93%	93%	93%	81%
Nov-04	77%	77%	85%	69%
Dec-04	58%	58%	58%	62%
Jan-05	46%	42%	50%	31%
Feb-05	38%	38%	31%	31%
Mar-05	33%	33%	26%	52%
Apr-05	85%	85%	85%	74%
May-05	46%	42%	27%	50%
Jun-05	42%	46%	35%	58%
Jul-05	65%	69%	69%	62%
Aug-05	77%	77%	85%	73%
Sep-05	58%	54%	54%	50%
Oct-05	69%	62%	54%	50%
Nov-05	58%	62%	62%	73%
Dec-05	65%	46%	54%	50%
Jan-06	81%	69%	69%	69%
Feb-06	77%	77%	73%	73%
Mar-06	46%	46%	42%	42%
Apr-06	30%	33%	22%	19%
May-06	33%	30%	41%	26%
Jun-06	22%	22%	22%	30%
Jul-06	81%	81%	81%	74%
Aug-06	44%	44%	44%	41%
Sep-06	59%	52%	52%	52%
Oct-06	64%	64%	68%	79%
Nov-06	81%	85%	81%	89%
Dec-06	63%	63%	67%	63%
Jan-07	79%	79%	83%	83%
Feb-07	55%	45%	55%	55%
Mar-07	39%	39%	36%	36%
Apr-07	75%	75%	75%	79%
May-07	68%	71%	68%	32%
Jun-07	52%	52%	59%	52%
Jul-07	56%	52%	56%	63%

Table 7.11 Comparison of Success Rate Consistency (Top Quintile, Out-of-Sample)

The relative ability of each of the multi-factor models to predict winners and losers accurately is measured by the success rate. The average success rate for quintile 1 is calculated as the percentage of forecasts in that quintile that exceed the return on the ALSI in each month. Months in which the success rate exceeds 50% are shaded in grey to facilitate the comparison of consistency of success rate across the models.

	IC	QIAN	Slope T	Slope T
	% Success Rate (T)	% Success Rate (T)	% Success Rate (T)	% Success Rate (T)
Aug-07	38%	35%	35%	38%
Sep-07	52%	56%	48%	26%
Oct-07	65%	62%	58%	65%
Nov-07	46%	50%	54%	31%
Dec-07	73%	85%	69%	81%
Jan-08	15%	8%	12%	19%
Feb-08	62%	58%	58%	42%
Mar-08	50%	46%	50%	50%
Apr-08	38%	42%	46%	27%
May-08	38%	38%	50%	23%
Jun-08	54%	46%	42%	42%
Jul-08	58%	54%	50%	65%
Aug-08	63%	52%	48%	63%
Sep-08	67%	70%	67%	89%
Oct-08	50%	68%	50%	75%
Nov-08	33%	37%	33%	56%
Dec-08	78%	81%	74%	81%
Jan-09	37%	67%	37%	59%
Feb-09	44%	48%	59%	59%
Mar-09	30%	19%	30%	22%
Apr-09	63%	67%	59%	59%
May-09	30%	15%	48%	11%
Jun-09	74%	93%	78%	78%
Jul-09	37%	41%	30%	41%
Aug-09	74%	56%	85%	70%
Sep-09	70%	70%	48%	70%
Oct-09	37%	37%	30%	15%
Nov-09	27%	27%	31%	27%
Dec-09	73%	62%	62%	46%

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Table 7.12 Comparison of Success Rate Consistency (Bottom Quintile, In- Sample)

The relative ability of each of the multi-factor models to predict winners and losers accurately is measured by the success rate. The average success rate for quintile 1 is calculated as the percentage of forecasts in that quintile that exceed the return on the ALSI in each month. Months in which the success rate exceeds 50% are shaded in grey to facilitate the comparison of consistency of success rate across the models.

	IC	QIAN	Quintile Spread
	% Success Rate (B)	% Success Rate (B)	% Success Rate (B)
Feb-03	46%	43%	46%
Mar-03	39%	43%	43%
Apr-03	55%	42%	48%
May-03	82%	75%	82%
Jun-03	27%	30%	13%
Jul-03	59%	70%	56%
Aug-03	74%	63%	52%
Sep-03	44%	52%	33%
Oct-03	64%	64%	56%
Nov-03	46%	46%	42%
Dec-03	55%	68%	64%
Jan-04	45%	50%	73%
Feb-04	74%	70%	52%
Mar-04	28%	32%	28%
Apr-04	32%	16%	40%
May-04	66%	72%	55%
Jun-04	46%	46%	42%
Jul-04	64%	64%	68%
Aug-04	59%	63%	59%
Sep-04	72%	68%	60%
Oct-04	16%	16%	24%
Nov-04	43%	43%	39%
Dec-04	46%	42%	62%
Jan-05	67%	56%	63%
Feb-05	71%	71%	64%
Mar-05	50%	50%	43%
Apr-05	48%	48%	31%
May-05	67%	67%	59%
Jun-05	48%	48%	48%
Jul-05	48%	44%	44%
Aug-05	52%	56%	48%
Sep-05	75%	83%	67%
Oct-05	46%	46%	50%
Nov-05	58%	63%	42%
Dec-05	54%	54%	58%
Jan-06	54%	63%	46%
Feb-06	33%	33%	38%
Mar-06	69%	65%	62%
Apr-06	84%	80%	76%
May-06	64%	56%	72%
Jun-06	92%	88%	80%
Jul-06	31%	23%	38%
Aug-06	62%	62%	58%
Sep-06	63%	63%	56%
Oct-06	46%	42%	42%
Nov-06	39%	36%	32%
Dec-06	52%	52%	34%
Jan-07	44%	33%	30%
Feb-07	56%	56%	52%
Mar-07	83%	86%	72%
Apr-07	34%	31%	45%
May-07	63%	63%	56%
Jun-07	69%	66%	76%
Jul-07	75%	64%	71%

Table 7.13 Comparison of Success Rate Consistency (Bottom Quintile, Out-of- Sample)

The relative ability of each of the multi-factor models to predict winners and losers accurately is measured by the success rate. The average success rate for quintile 1 is calculated as the percentage of forecasts in that quintile that exceed the return on the ALSI in each month. Months in which the success rate exceeds 50% are shaded in grey to facilitate the comparison of consistency of success rate across the models.

	% Success Rate (B)	% Success Rate (B)	% Success Rate (B)	% Success Rate (B)
Aug-07	61%	64%	61%	61%
Sep-07	68%	76%	56%	56%
Oct-07	65%	58%	54%	54%
Nov-07	69%	69%	50%	50%
Dec-07	31%	31%	31%	31%
Jan-08	89%	89%	89%	89%
Feb-08	67%	58%	79%	79%
Mar-08	71%	63%	50%	50%
Apr-08	79%	83%	83%	83%
May-08	76%	76%	92%	92%
Jun-08	75%	79%	71%	71%
Jul-08	18%	11%	4%	4%
Aug-08	28%	36%	44%	44%
Sep-08	18%	25%	36%	36%
Oct-08	46%	62%	58%	58%
Nov-08	45%	66%	59%	59%
Dec-08	48%	56%	44%	44%
Jan-09	50%	77%	50%	50%
Feb-09	33%	44%	41%	41%
Mar-09	65%	65%	85%	85%
Apr-09	40%	32%	36%	36%
May-09	69%	46%	58%	58%
Jun-09	22%	41%	19%	19%
Jul-09	62%	73%	65%	65%
Aug-09	50%	31%	46%	46%
Sep-09	31%	48%	34%	34%
Oct-09	84%	68%	64%	64%
Nov-09	62%	62%	58%	58%
Dec-09	52%	64%	32%	32%

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7.2.5 Characteristics of Out- and Underperforming Shares

Like Haugen and Baker (1996), we examine the average characteristics of firms across the quintiles in each of the models in order to identify common characteristics between firms in the top, intermediate and bottom quintiles. The results of the analysis for the in-sample period are presented in Table 7.14.

It is evident from the table that the size of firms (as measured by the log of market value, LMV) increases as we move from the top to the bottom quintile for all models. This is consistent with the size effect, identified in the univariate analysis and evidenced in the difference in performance of the models on the basis of the equal and market-weighted methodologies. As the smaller shares also tend to demonstrate greater risk we find that the average prior twelve-month volatility of returns (STDEV12) for shares tends to decrease as we move from the top to the bottom quintile. This distinction is less obvious between the intermediate and bottom quintiles but is clear between the top and the remaining quintiles. This finding is contrary to that of Haugen and Baker (1996) on the U.S. market, who found that their top decile comprised shares which had higher average market values and a lower average volatility of returns. Our top quintiles, in contrast, exhibit smaller capitalizations and higher risk and lower liquidity.

Consistent with our prior results, we find that the average book-value-to-market-value, cashflow-to-price, earnings yield, twelve-month momentum, 12-month dividend growth-to-price and 12-month earnings growth-to-price show a clear decreasing trend as we move from the top to the bottom quintiles for all models considered. This indicates that outperformance is forecast for shares with greater cash value relative to price and that demonstrate greater earnings, profitability and growth. These findings related to cashflow-to-price, earnings yield and dividend yield are consistent with those of Haugen and Baker (1996). We found no significant difference in the mean value of shares between quintiles for the other fundamental factors in the sample.

Results for the out-of-sample period are presented in Table 7.15. Over this period the top quintile for all models continue to exhibit larger twelve-month momentum and cash flow-to-price values. This suggests that during the period of instability, shares which have demonstrated greater cash flows as well as long-term positive returns momentum are expected to outperform. The results for the remaining fundamental factors vary across the different models and are largely inconsistent. This is consistent with the findings that the factors found to be significant during the in-sample period do not persist into the out-of-sample period.

Table 7.14 Average Characteristics of Firms Within Each Quintile (In-Sample)

Multi-factor predictive models of returns are created using a forward stepwise regression procedure for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. The average monthly characteristics of shares in each quintile are calculated and presented below.

		IC	IR (Q & H)/Slope T	Quintile Spread
MOM12	Quintile 1	0.679	0.668	0.455
	Quintile 2	0.465	0.476	0.443
	Quintile 3	0.385	0.382	0.426
	Quintile 4	0.349	0.341	0.430
	Quintile 5	0.228	0.240	0.359
STDEV12	Quintile 1	0.094	0.097	0.087
	Quintile 2	0.077	0.078	0.081
	Quintile 3	0.079	0.076	0.086
	Quintile 4	0.080	0.076	0.079
	Quintile 5	0.085	0.088	0.085
EY	Quintile 1	0.111	0.119	0.124
	Quintile 2	0.110	0.102	0.103
	Quintile 3	0.099	0.103	0.095
	Quintile 4	0.088	0.090	0.095
	Quintile 5	0.082	0.082	0.077
LMV	Quintile 1	7.551	7.529	7.570
	Quintile 2	8.055	8.115	7.924
	Quintile 3	8.204	8.216	8.130
	Quintile 4	8.379	8.332	8.307
	Quintile 5	8.309	8.302	8.556
C12MEPSP	Quintile 1	0.222	0.278	0.214
	Quintile 2	0.149	0.150	0.151
	Quintile 3	0.126	0.114	0.121
	Quintile 4	0.110	0.095	0.111
	Quintile 5	0.116	0.087	0.130
C12MDPSP	Quintile 1	0.007	0.005	0.010
	Quintile 2	0.005	0.005	0.007
	Quintile 3	0.003	0.003	0.005
	Quintile 4	0.002	0.003	0.003
	Quintile 5	-0.001	-0.001	-0.005
BVTM	Quintile 1	0.806	0.788	0.872
	Quintile 2	0.635	0.619	0.661
	Quintile 3	0.560	0.577	0.555
	Quintile 4	0.502	0.506	0.464
	Quintile 5	0.461	0.475	0.395
CTP	Quintile 1	0.227	0.206	0.240
	Quintile 2	0.149	0.154	0.161
	Quintile 3	0.138	0.135	0.129
	Quintile 4	0.114	0.118	0.114
	Quintile 5	0.089	0.100	0.068

Table 7.15 Average Characteristics of Firms Within Each Quintile (Out-of-Sample)

Multi-factor predictive models of returns are created using a forward stepwise regression procedure for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. The average monthly characteristics of shares in each quintile are calculated and presented below.

		IC	IR (Q & H)/Slope T	Quintile Spread
MOM12	Quintile 1	0.146	0.266	0.040
	Quintile 2	0.025	0.099	0.051
	Quintile 3	0.019	0.016	0.081
	Quintile 4	0.029	-0.050	0.022
	Quintile 5	-0.049	-0.171	-0.022
STDEV12	Quintile 1	0.115	0.090	0.097
	Quintile 2	0.101	0.088	0.098
	Quintile 3	0.091	0.090	0.098
	Quintile 4	0.086	0.100	0.101
	Quintile 5	0.096	0.121	0.099
EY	Quintile 1	0.126	0.097	0.112
	Quintile 2	0.108	0.101	0.113
	Quintile 3	0.098	0.103	0.107
	Quintile 4	0.104	0.113	0.109
	Quintile 5	0.113	0.122	0.097
LMV	Quintile 1	8.874	9.039	8.814
	Quintile 2	8.724	8.748	8.578
	Quintile 3	8.658	8.549	8.574
	Quintile 4	8.670	8.626	8.763
	Quintile 5	8.614	8.519	8.743
C12MEPSP	Quintile 1	0.122	0.071	0.099
	Quintile 2	0.086	0.074	0.101
	Quintile 3	0.092	0.089	0.098
	Quintile 4	0.095	0.093	0.100
	Quintile 5	0.095	0.152	0.097
C12MDPSP	Quintile 1	0.003	0.004	0.004
	Quintile 2	0.003	0.004	0.003
	Quintile 3	0.003	0.003	0.002
	Quintile 4	0.003	0.003	0.003
	Quintile 5	0.000	-0.002	0.000
BVTM	Quintile 1	0.689	0.608	0.700
	Quintile 2	0.612	0.632	0.633
	Quintile 3	0.609	0.629	0.604
	Quintile 4	0.618	0.635	0.617
	Quintile 5	0.674	0.745	0.664
CTP	Quintile 1	0.250	0.189	0.208
	Quintile 2	0.165	0.140	0.168
	Quintile 3	0.125	0.134	0.139
	Quintile 4	0.110	0.137	0.124
	Quintile 5	0.083	0.124	0.089

7.2.6 Portfolio Turnover and Transaction Costs

Although the findings in Section 7.2.2 indicate that all four models outperformed the ALSI both in- and out-of-sample, the analysis fails to assess the impact of transaction costs on model performance using the portfolio turnover measure calculated in Section 7.1.4. The prior analysis assumed that quintiles were rebalanced on a monthly basis. This is consistent with Haugen and Baker (1996) who found that the payoffs of many of the factors they considered were mean-reverting in the short term and thus their predictive power was greater over a monthly horizon than over a one year horizon. The transaction costs arising from such a short rebalancing period may potentially erode the excess returns our models were found to earn over the ALSI. We extend the earlier analysis to consider rebalancing periods of three and six months in order to contrast the impact of rebalancing and transaction costs over longer periods with the existing one-month rebalancing results. In order to facilitate comparison, all figures are annualized, unless stated otherwise.

Table 7.16 presents the results for the in-sample period. The average annual portfolio turnover for the IC and Qian/Slope T models over the one-month rebalancing window is substantial, indicating that the top quintile for these models is turned over completely (200% as per the measure) every two months on average. The portfolio turnover for the Quintile Spread model in contrast is half that of the other models indicating that the shares forecast to out- and underperform are more consistent, necessitating less frequent rebalancing of the quintile constituents. The ranked performance of the models is little changed with the introduction of transaction costs with the IC model still ranked ahead of the Qian/Slope T model which itself is followed by the Quintile Spread model. Overall, we note that the models continue to outperform the ALSI on a cost-adjusted basis for all levels of transaction costs considered.

We note two trends from the analysis as the rebalancing period is increased. Firstly, the performance of each of the models decreases as the length of the rebalancing period increases. This is consistent with Haugen and Baker's (1996) suggestion that predictive power is greater over shorter periods. Secondly, portfolio turnover decreases as expected when the rebalancing period increases. There is therefore a trade-off between reduced transaction costs and gross model performance as we increase the rebalancing period for all the models considered. The loss in model performance outweighs the decrease in portfolio turnover and resultant transaction costs, however, as the modified Sharpe ratio decreases for all the models as the rebalancing period increases. We also note that the standard deviation associated with the returns of the models increases as the rebalancing period increases. The combined impact of these effects is that the factor models outperform the ALSI on a modified Sharpe ratio basis across all levels of transaction costs considered for the one-month and three-month rebalancing period but underperform the ALSI over the six-month rebalancing period even before transaction costs are considered. This is a result of the higher volatility in the returns for longer holding periods and implies that shorter rebalancing

periods are preferred and that the models outperform the ALSI regardless of the onerous rebalancing requirements over these shorter periods. However, it must be noted that these results reflect the in-sample period over which the models were optimized.

We next consider the out-of-sample period, results for which are presented in Table 7.17. We observe the same expected decreasing trend in portfolio turnover as the rebalancing period increases, as well as a clear decrease in quintile performance as the rebalancing period increases for all the models. The IC model continues to outperform the ALSI on a one-month rebalancing basis where transaction costs do not exceed 50 bp. The Qian/Slope T model underperforms the ALSI on the basis of the modified Sharpe ratio for all levels of transaction costs while the Quintile Spread model outperforms the ALSI for all levels of transaction cost considered.

Over the three-month and twelve-month rebalancing periods, the IC and Qian/Slope T models underperform the ALSI even before transaction costs are accounted for due to the decrease in gross quintile returns over the longer periods. The Quintile Spread model, in contrast, continues to outperform the ALSI for all levels of cost considered over the three-month period but underperforms after costs over the six-month rebalancing period.

The results agree for the most part with those of the in-sample period. The lower returns over the out-of-sample period, however, exacerbate the impact of transaction costs on performance. We note, however, that even over the out-of-sample period, the factor models outperformed the ALSI net of costs at the 20bp transaction cost level over a one-month rebalancing period. Although the results indicate that there is a reduction in transaction costs over longer rebalancing periods, these savings are more than offset by the diminished performance of the models as a consequence of ignoring monthly signals between rebalancing dates.

Table 7.16 The Impact of Portfolio Turnover and Transaction Costs on Model Performance (In-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. Models are rebalanced on a one-month, three-month and twelve-month basis and performance is compared on the basis of the annualized monthly geometric return, standard deviation and modified Sharpe Ratio measures for the top quintile for each model. A measure of portfolio turnover for each model is also provided and the impact of transaction costs on model performance is presented in the form of a cost-adjusted geometric return and associated modified Sharpe ratio. Provision is made for transaction costs of 20bp, 50bp and 100bp.

Monthly Rebalancing

<u>Top Quintile Performance (Annualized Returns)</u>	IC	IR (Q & H)/Slope T	Quintile Spread	ALSI
Geometric Return	79.97%	77.07%	66.77%	33.77%
Standard Deviation	0.164	0.161	0.149	0.156
Modified Sharpe Ratio	4.364	4.248	3.907	1.613
Average Annual Turnover	1221.7%	1177.1%	559.9%	
Cost-adjusted Geometric Return (20bp)	77.5%	74.7%	65.6%	
Modified Sharpe Ratio	4.214	4.102	3.832	
Cost-adjusted Geometric Return (50bp)	73.9%	71.2%	64.0%	
Modified Sharpe Ratio	3.990	3.883	3.719	
Cost-adjusted Geometric Return (100bp)	67.8%	65.3%	61.2%	
Modified Sharpe Ratio	3.617	3.518	3.531	

Quarterly Rebalancing

<u>Top Quintile Performance (Annualized Returns)</u>	IC	IR (Q & H)/Slope T	Quintile Spread	ALSI
Geometric Return	74.20%	73.26%	64.19%	33.77%
Standard Deviation	0.224	0.214	0.220	0.156
Modified Sharpe Ratio	2.929	3.019	2.530	1.613
Average Annual Turnover	319.4%	316.0%	235.3%	
Cost-adjusted Geometric Return (20bp)	73.6%	72.6%	63.7%	
Modified Sharpe Ratio	2.900	2.989	2.508	
Cost-adjusted Geometric Return (50bp)	72.6%	71.7%	63.0%	
Modified Sharpe Ratio	2.857	2.945	2.476	
Cost-adjusted Geometric Return (100bp)	71.0%	70.1%	61.8%	
Modified Sharpe Ratio	2.786	2.871	2.423	

Semi-Annual Rebalancing

<u>Top Quintile Performance (Annualized Returns)</u>	IC	IR (Q & H)/Slope T	Quintile Spread	ALSI
Geometric Return	62.63%	58.85%	54.81%	33.77%
Standard Deviation	0.366	0.331	0.292	0.156
Modified Sharpe Ratio	1.477	1.520	1.585	1.613
Average Annual Turnover	110.9%	112.5%	110.3%	
Cost-adjusted Geometric Return (20bp)	62.4%	58.6%	54.6%	
Modified Sharpe Ratio	1.471	1.513	1.577	
Cost-adjusted Geometric Return (50bp)	62.1%	58.3%	54.3%	
Modified Sharpe Ratio	1.462	1.503	1.566	
Cost-adjusted Geometric Return (100bp)	61.5%	57.7%	53.7%	
Modified Sharpe Ratio	1.446	1.486	1.547	

Table 7.17 The Impact of Portfolio Turnover and Transaction Costs on Model Performance (Out-of-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement for each of the performance metrics presented below. Forecasts are sorted in descending order by forecast return and then grouped into equally-weighted quintiles. Models are rebalanced on a one-month, three-month and twelve-month basis and performance is compared on the basis of the annualized monthly geometric return, standard deviation and modified Sharpe Ratio measures for the top quintile for each model. A measure of portfolio turnover for each model is also provided and the impact of transaction costs on model performance is presented in the form of a cost-adjusted geometric return and associated modified Sharpe ratio. Provision is made for transaction costs of 20bp, 50bp and 100bp.

Monthly Rebalancing

<u>Top Quintile Performance (Annualized Returns)</u>	IC MODEL	IR (Q & H)/Slope T	Quintile Spread	ALSI
Geometric Return	7.75%	3.80%	9.50%	1.65%
Standard Deviation	0.253	0.252	0.201	0.229
Modified Sharpe Ratio	-0.007	-0.017	-0.002	-0.020
Average Monthly Turnover	872.7%	836.4%	525.1%	
Cost-adjusted Geometric Return (20bp)	6.0%	2.1%	8.5%	
Modified Sharpe Ratio	-0.011	-0.021	-0.004	
Cost-adjusted Geometric Return (50bp)	3.4%	-0.4%	6.9%	
Modified Sharpe Ratio	-0.018	-0.027	-0.007	
Cost-adjusted Geometric Return (100bp)	-1.0%	-4.6%	4.3%	
Modified Sharpe Ratio	-0.029	-0.038	-0.012	

Quarterly Rebalancing

<u>Top Quintile Performance (Annualized Returns)</u>	IC MODEL	IR (Q & H)/Slope T	Quintile Spread	ALSI
Geometric Return	-1.20%	-3.50%	7.28%	1.65%
Standard Deviation	0.319	0.274	0.223	0.229
Modified Sharpe Ratio	-0.037	-0.038	-0.007	-0.020
Average Annual Turnover	255.6%	293.8%	237.5%	
Cost-adjusted Geometric Return (20bp)	-1.7%	-4.1%	6.8%	
Modified Sharpe Ratio	-0.038	-0.040	-0.008	
Cost-adjusted Geometric Return (50bp)	-2.5%	-5.0%	6.1%	
Modified Sharpe Ratio	-0.041	-0.042	-0.009	
Cost-adjusted Geometric Return (100bp)	-3.8%	-6.4%	4.9%	
Modified Sharpe Ratio	-0.045	-0.046	-0.012	

Semi-Annual Rebalancing

<u>Top Quintile Performance (Annualized Returns)</u>	IC MODEL	IR (Q & H)/Slope T	Quintile Spread	ALSI
Geometric Return	-6.77%	-3.16%	5.02%	1.65%
Standard Deviation	0.294	0.234	0.196	0.229
Modified Sharpe Ratio	-0.050	-0.032	-0.010	-0.020
Average Annual Turnover	244.7%	239.1%	211.1%	
Cost-adjusted Geometric Return (20bp)	-7.3%	-3.6%	4.6%	
Modified Sharpe Ratio	-0.052	-0.033	-0.011	
Cost-adjusted Geometric Return (50bp)	-8.0%	-4.4%	4.0%	
Modified Sharpe Ratio	-0.054	-0.034	-0.012	
Cost-adjusted Geometric Return (100bp)	-9.2%	-5.6%	2.9%	
Modified Sharpe Ratio	-0.057	-0.037	-0.015	

7.3 Conclusion

We create multi-factor predictive models of share returns using a stepwise procedure that maximizes a range of criterion used to evaluate manager performance. We then examine the performance of our model predictions over time as well as their ability to predict winners and losers.

The models outperform relative to the ALSI during the optimized in-sample period but this performance decreases significantly during the subsequent instability of the out-of-sample period. All models still outperform the ALSI over the market crisis period, however, indicating that the decrease in performance is due in large part to the regime shift that occurred in 2007. The IC model yields the highest Sharpe ratio over the in-sample period but is marginally outperformed by the Quintile Spread model over the out-of-sample period. We find that all models yield a substantial difference in returns between the top and bottom quintiles indicating an ability to distinguish between winners and losers. Further investigation demonstrates that the models are able to predict winners better than losers during the in-sample period. This is reversed during the out-of-sample period where the predictive accuracy of winners decreases while those for losers increases for all models. This is consistent with the positive return trend of the in-sample period and the negative return trend of the out-of-sample period.

We test market-capitalization share weightings relative to equal share weightings and find that portfolio performance using the market-weighted approach yields improved results for the IC and Quintile Spread models over the in-sample period, but worse results for the Qian and Slope T models. Performance for all models except the Quintile spread is substantially worse when using market capitalization weightings over the out-of-sample period. It is thus not clear which weighting methodology is superior and the choice may be dependent on the relative liquidity of shares in the sample given that there is a trade-off in portfolio construction between liquidity and firm size (see Kruger and van Rensburg, 2008).

An analysis of the characteristics of shares across the quintiles for each of the models yields interesting results. We find that stocks with forecast outperformance over the in-sample period tend to be smaller, riskier stocks which have greater cash flow relative to price and demonstrate greater earnings and profitability. Over the out-of-sample period, outperformance is forecast for stocks with greater cash flow relative to price and a history of strong 12-month price momentum. This suggests that the in-sample period is dominated by momentum investing and an appetite for risk while the out-of-sample period is characterized by risk aversion and value investing.

We examine the impact of portfolio rebalancing and transaction costs on the models constructed and find that, although portfolio turnover and the associated transaction costs are reduced by rebalancing the top quintile of each model over longer periods, gross model performance decreases substantially over longer

rebalancing periods. This agrees with the findings of Haugen and Baker (1996) who suggest that the exposures of many factors are mean-reverting and therefore have greater predictive power over shorter periods.

We find that all models continue to outperform the ALSI net of transaction costs regardless of the length of the rebalancing period for the in-sample data. This outperformance is also found for all models for the out-of-sample data but only for a one-month rebalancing period where transaction costs do not exceed 20bp. For longer periods and/or higher transaction costs the IC and Qian/Slope T models underperform the ALSI but the Quintile Spread model continues to outperform for longer rebalancing periods indicating its greater stability over the out-of-sample period.

It should be noted that our goal is not to construct the optimal linear predictive model for returns on the JSE – this is impossible to identify unless all possible variables which have a relationship with returns and all possible models and combinations of models are tested. Instead, our objective is to investigate the consistency of linearly predicted returns over time, with particular emphasis on the impact of the market instability starting 2007. Our analysis is therefore a comparison of specific models and it cannot necessarily be inferred that the results hold for linear models in general.

8. Conclusions

There is extensive literature that has uncovered evidence of the predictability of returns on the Johannesburg Stock Exchange (JSE) but these studies have focused on periods of relative market stability. Although it is generally accepted that evidence of predictability in return is not robust to changes in market regimes, there has been no examination of the extent to which return predictability persists during periods of market crisis. In addition, there is a growing interest internationally in the presence of nonlinearities in the data generation process underlying share returns and it has been suggested that emerging markets in particular may be more prone to nonlinear rather than linear behaviour. Little research has been conducted for the JSE on this promising area.

We add to the existing body of knowledge by investigating both the presence of linear and nonlinear serial dependencies in share returns on the JSE and the predictability of share returns both prior to and over the recent market crisis.

We apply the battery of tests of Ashley and Patterson (2000) to a sample of daily share and index returns which have been adjusted for thin trading. Our results indicate that there is evidence of both linear and nonlinear predictability in daily share and index returns on the JSE over both the stable and market crisis periods of the data sample. In order to assess whether these dependencies are consistent over the sample periods or due to the influence of shorter window periods of dependence over the period we apply the episodic tests of Hinich and Patterson (2001) to the data. The findings suggest that both linear and nonlinear serial dependence in returns on the JSE are episodic in nature and that the return generation process is characterized for the most part by extended periods of pure white noise. The results differ between shares and indices on the basis of market capitalization, suggesting that serial dependence may be related to size (similar to what was found for the US market by Skaradzinski, 2003), but are inconclusive and may be a consequence of the proportion of zero returns which remain in the smaller capitalization samples even after adjusting for thin trading. This is the first time that these methodologies have been applied to the South African market and the findings are consistent with those for international markets¹⁵.

We examine whether these results are robust to the construction of actual linear and nonlinear predictive models and compare AR(1) and ARMA(1,1) models to nonlinear GARCH-in-mean and Markov Switching models. Although there have been studies on the structure of volatility on the JSE using ARCH and GARCH models (see Mangani, 2008), findings on the impact of GARCH-in-mean effects on return predictability has not as yet been documented for the JSE. In addition, the only application of Markov Switching models on the JSE has been in a study to identify business cycles (see Moolman, 2004). The

¹⁵ see Hinich and Patterson, 2001, Skaradzinski, 2003, Lim and Hinich, 2005, and Bonilla, Romero-Meza and Hinich, 2006

data sample is divided into three periods – a shortened in-sample period over which the models are constructed, a stable market period and a market crisis period. Models are estimated recursively starting with the in-sample period and then expanded as the testing period is increased incrementally. The results indicate that there is a definite gain to be made in forecasting power using nonlinear models over the stable sample period. We find, however, that the simpler linear models outperform the nonlinear alternatives over the market crisis period, indicating the difficulties the nonlinear models have to adjusting to the significant structural change in the market over that period. Interestingly, we note that the GARCH family of models outperform the Markov Switching models during the stable period but this is reversed during the market crisis.

Next, we extend our investigation to the semi-strong case and examine the anomalous relationships between firm characteristics and returns identified in prior literature. Although there has been substantial literature on this area of return predictability in recent years, all prior studies have been considered over relatively stable market periods and none have examined sub-periods in their data samples in order to assess whether the observed relationships are stable over time. Our findings provide support for all anomalies identified in prior literature for the JSE over our in-sample period. In addition, we find evidence of predictability using factors representing long-term growth in dividends and earnings. Over the market crisis period (out-of-sample) we find that only the cash flow-to-price ratio continues to be a good predictor of returns. This is consistent with the expected risk aversion that investors demonstrate during periods of market upheaval. We note that much of the literature has employed incomplete data sets and conducted their analysis on a complete case basis as a consequence. Given the potential impact of this approach on the findings of regression analysis we adjust our data set, which itself is subject to missing data, for missingness using a multiple imputation methodology. We find that the results for the multiple imputation set do not differ materially from those of the unadjusted data. It should be noted that these findings do not necessarily indicate that missingness has no impact on such studies as the missingness in our data was not severe and the form of missingness (i.e. whether the data is missing completely at random, missing at random or not missing at random) has a significant impact on the effect of the missing data on the analysis.

Lastly, it has been suggested by Jensen (1987) that anomalies which question the efficiency of markets are of little benefit if they have are not economically exploitable. We therefore construct a range of multivariate characteristic-based factor models in order to forecast share returns over both the stable and market crisis periods. We form portfolios of forecast outperforming shares on the basis of these model predictions and compare their performance over time to that of the All-Share Index (ALSI). Results indicate that the portfolios formed on this basis outperform the ALSI over both the stable and market crisis periods. We adjust the returns for the portfolios to account for transaction costs at varying levels and find that this outperformance persists over the in-sample period for all transaction costs considered

but the ALSI outperforms the factor model portfolios over the market crisis period for transaction costs (one-way) in excess of 20bps.

Overall, the results provide significant evidence of return predictability, both linear and nonlinear, on the JSE but suggest that there is work to be done in identifying the most appropriate model for the data generation process underlying share returns on the JSE.

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Appendix A

A1. Descriptive Statistics

Descriptive statistics are provided for the style attributes both before and after the application of a trimming procedure to trim values at the 1st and 99th percentiles.

Unadjusted Data

	MOM1	MOM3	MOM6	MOM12	STDEV12	EY	DY	LMV	VT	C12MEPSP
Mean	0.021	0.067	0.142	0.291	0.103	0.133	0.044	8.189	0.049	0.245
Median	0.017	0.060	0.120	0.245	0.084	0.088	0.035	8.087	0.029	0.064
Maximum	2.273	5.071	6.755	16.000	7.619	95.238	2.000	13.493	6.839	66.667
Minimum	-0.958	-0.954	-0.953	-0.943	0.021	-17.097	0.000	1.206	0.000	0.000
Std. Dev.	0.111	0.210	0.340	0.604	0.214	1.214	0.073	1.758	0.140	1.343
Skewness	1.680	2.362	2.999	4.792	31.963	51.311	12.803	0.085	28.156	23.569
Kurtosis	28.841	38.380	38.422	68.867	1113.028	3493.313	236.135	3.263	1089.408	799.382

Trimmed Data

	MOM1	MOM3	MOM6	MOM12	STDEV12	EY	DY	LMV	VT	C12MEPSP
Mean	0.020	0.065	0.137	0.280	0.097	0.106	0.041	8.197	0.044	0.145
Median	0.017	0.060	0.120	0.245	0.084	0.088	0.035	8.087	0.029	0.064
Maximum	0.572	1.505	2.609	4.222	0.793	3.856	1.667	13.167	0.719	1.000
Minimum	-0.494	-0.729	-0.816	-0.892	0.027	-0.870	0.000	2.459	0.000	0.000
Std. Dev.	0.098	0.188	0.300	0.502	0.058	0.149	0.041	1.710	0.058	0.213
Skewness	0.150	0.522	0.885	1.259	4.833	8.976	7.083	0.177	4.649	2.553
Kurtosis	4.835	5.872	6.782	7.445	44.389	161.307	179.072	2.860	36.651	9.511

In-Sample

Observations	11671	11614	11524	11349	11339	11485	11671	11671	11651	10343
Missingness	0.0%	0.5%	1.3%	2.8%	2.9%	1.6%	0.0%	0.0%	0.2%	12.8%

Out-of-Sample

Observations	5014	4992	4957	4877	4877	4873	5014	5014	5004	4493
Missingness	0.0%	0.4%	1.1%	2.8%	2.8%	2.9%	0.0%	0.0%	0.2%	11.6%

A1. Descriptive Statistics

Descriptive statistics are provided for the style attributes both before and after the application of a trimming procedure to trim values at the 1st and 99th percentiles.

Unadjusted Data

	C24MEPSP	BVTM	POUT	C12MDPSP	C24MDPSP	CTP	ROE	DUMFINDI	BUY	DOWNGRADE
Mean	0.014	0.740	0.267	0.003	0.006	0.014	0.240	0.817	0.127	0.035
Median	0.020	0.553	0.248	0.000	0.001	0.121	0.216	1.000	0.000	0.000
Maximum	13.049	33.333	1.000	2.000	2.000	100.000	16.501	1.000	1.000	1.000
Minimum	-4.692	-9.091	-0.058	-1.310	-1.804	-100.000	-4.057	0.000	0.000	0.000
Std. Dev.	0.290	1.215	0.265	0.059	0.062	4.236	0.578	0.387	0.333	0.183
Skewness	23.686	12.398	0.666	5.696	5.567	-18.562	18.141	-1.635	2.243	5.074
Kurtosis	1014.645	222.030	2.554	306.079	261.405	504.085	498.119	3.675	6.031	26.744

Trimmed Data

	C24MEPSP	BVTM	POUT	C12MDPSP	C24MDPSP	CTP	ROE	DUMFINDI	BUY	DOWNGRADE
Mean	0.011	0.688	0.266	0.002	0.005	0.139	0.225	0.817	0.127	0.035
Median	0.020	0.552	0.248	0.000	0.001	0.121	0.216	1.000	0.000	0.000
Maximum	1.069	20.000	0.998	1.667	1.667	2.656	1.690	1.000	1.000	1.000
Minimum	-1.036	-0.009	-0.004	-0.531	-0.489	-5.619	-1.457	0.000	0.000	0.000
Std. Dev.	0.107	0.603	0.263	0.030	0.035	0.268	0.239	0.387	0.333	0.183
Skewness	-1.727	5.888	0.644	4.343	3.598	-4.820	0.136	-1.635	2.243	5.074
Kurtosis	26.521	99.216	2.484	487.872	284.187	124.657	11.605	3.675	6.031	26.744

In-Sample

Observations	10404	11122	11409	11671	11671	11122	10881	11671	11671	11671
Missingness	12.2%	4.9%	2.3%	0.0%	0.0%	4.9%	7.3%	0.0%	0.0%	0.0%

Out-of-Sample

Observations	4627	4744	4804	5014	5014	4710	4682	5014	5014	5014
Missingness	8.4%	5.7%	4.4%	0.0%	0.0%	6.5%	7.1%	0.0%	0.0%	0.0%

A2. Comparison of Unadjusted and Multiply Imputed Data Sets

Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.

MOM3							
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted	0.0652	0.0600	1.5050	-0.7290	0.1884	0.5218	5.8721
M1	0.0652	0.0600	1.5050	-0.7290	0.1885	0.5201	5.8533
M2	0.0653	0.0600	1.5050	-0.7290	0.1884	0.5189	5.8518
M3	0.0652	0.0600	1.5050	-0.7290	0.1884	0.5206	5.8633
M4	0.0655	0.0600	1.5050	-0.7290	0.1884	0.5209	5.8597
M5	0.0653	0.0600	1.5050	-0.7290	0.1884	0.5187	5.8635

MOM6							
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted	0.1371	0.1200	2.6090	-0.8160	0.3003	0.8846	6.7821
M1	0.1377	0.1200	2.6090	-0.8160	0.3008	0.8724	6.6979
M2	0.1382	0.1200	2.6090	-0.8160	0.3008	0.8758	6.7044
M3	0.1376	0.1200	2.6090	-0.8160	0.3007	0.8794	6.7277
M4	0.1378	0.1200	2.6090	-0.8160	0.3009	0.8759	6.7015
M5	0.1378	0.1200	2.6090	-0.8160	0.3006	0.8739	6.7154

MOM12							
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted	0.2795	0.2450	4.2220	-0.8920	0.5017	1.2588	7.4448
M1	0.2820	0.2470	4.2220	-1.2138	0.5023	1.2321	7.2929
M2	0.2821	0.2470	4.2220	-1.2963	0.5034	1.2257	7.2477
M3	0.2817	0.2470	4.2220	-0.9991	0.5034	1.2220	7.2376
M4	0.2823	0.2480	4.2220	-0.8920	0.5028	1.2291	7.2778
M5	0.2824	0.2480	4.2220	-1.0500	0.5028	1.2251	7.2592

STDEV12							
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted	0.0967	0.0840	0.7930	0.0270	0.0579	4.8333	44.3889
M1	0.0968	0.0850	0.7930	-0.0647	0.0578	4.7104	43.3274
M2	0.0969	0.0850	0.7930	-0.0573	0.0578	4.7265	43.4660
M3	0.0968	0.0850	0.7930	-0.0449	0.0578	4.7227	43.4600
M4	0.0969	0.0850	0.7930	-0.0634	0.0578	4.7186	43.4241
M5	0.0968	0.0850	0.7930	-0.0666	0.0578	4.7232	43.4820

A2. Comparison of Unadjusted and Multiply Imputed Data Sets

Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.

		EY						
		Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted		0.1055	0.0880	3.8560	-0.8700	0.1485	8.9765	161.3072
M1		0.1045	0.0880	3.8560	-0.8700	0.1492	8.6810	155.6456
M2		0.1049	0.0880	3.8560	-0.8700	0.1487	8.7808	157.6104
M3		0.1045	0.0880	3.8560	-0.8700	0.1487	8.7600	157.5028
M4		0.1045	0.0880	3.8560	-0.8700	0.1490	8.7165	156.3579
M5		0.1046	0.0880	3.8560	-0.8700	0.1488	8.7657	157.3681

		VT						
		Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted		0.0445	0.0290	0.7190	0.0000	0.0578	4.6493	36.6508
M1		0.0444	0.0290	0.7190	-0.1034	0.0578	4.6302	36.5551
M2		0.0444	0.0290	0.7190	-0.1203	0.0579	4.6158	36.3358
M3		0.0444	0.0290	0.7190	-0.1182	0.0579	4.6152	36.3139
M4		0.0445	0.0290	0.7190	-0.1279	0.0581	4.6074	36.1434
M5		0.0445	0.0290	0.7190	-0.1053	0.0581	4.6038	36.0576

		C12MEPSP						
		Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted		0.1449	0.0640	1.0000	0.0000	0.2129	2.5528	9.5110
M1		0.1524	0.0707	1.2124	-0.5217	0.2158	2.2328	8.3460
M2		0.1509	0.0700	1.1115	-0.5786	0.2143	2.2503	8.4271
M3		0.1516	0.0700	1.1413	-0.6239	0.2154	2.2481	8.4106
M4		0.1523	0.0710	1.2057	-0.4897	0.2156	2.2524	8.3490
M5		0.1510	0.0700	1.1198	-0.7182	0.2162	2.2088	8.3138

		C24MEPSP						
		Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted		0.0107	0.0200	1.0690	-1.0360	0.1066	-1.7268	26.5213
M1		0.0105	0.0190	1.0690	-1.0360	0.1075	-1.5107	23.5736
M2		0.0104	0.0190	1.0690	-1.0360	0.1073	-1.4979	23.7315
M3		0.0105	0.0190	1.0690	-1.0360	0.1069	-1.5263	24.0762
M4		0.0102	0.0190	1.0690	-1.0360	0.1069	-1.5211	24.1457
M5		0.0104	0.0190	1.0690	-1.0360	0.1069	-1.5381	24.1329

		BVTM						
		Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted		0.6884	0.5520	20.0000	-0.0090	0.6033	5.8881	99.2159
M1		0.6745	0.5460	20.0000	-1.3100	0.6061	5.5169	93.1047
M2		0.6734	0.5460	20.0000	-11.0300	0.6128	4.9215	97.0821
M3		0.6780	0.5490	39.9798	-1.6540	0.6818	15.3507	720.1441
M4		0.6741	0.5460	20.0000	-6.7611	0.6093	5.3204	92.4857
M5		0.6762	0.5490	20.0000	-5.6979	0.6088	5.3640	92.1822

A2. Comparison of Unadjusted and Multiply Imputed Data Sets

Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.

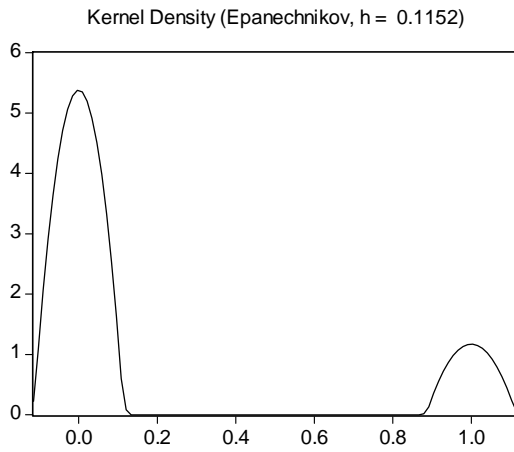
	POUT						
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted	0.2662	0.2480	0.9980	-0.0040	0.2634	0.6435	2.4839
M1	0.2756	0.2660	1.5021	-0.7120	0.2635	0.5780	2.4782
M2	0.2757	0.2640	1.2382	-1.1399	0.2634	0.5860	2.5180
M3	0.2755	0.2650	1.5263	-0.5759	0.2633	0.5892	2.4790
M4	0.2747	0.2640	1.2002	-0.8707	0.2635	0.5886	2.4914
M5	0.2753	0.2640	1.2381	-0.4650	0.2639	0.5877	2.4769

	CTP						
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted	0.1388	0.1210	2.6560	-5.6190	0.2681	-4.8199	124.6573
M1	0.1404	0.1220	2.6560	-5.6190	0.2702	-4.6737	117.3331
M2	0.1397	0.1210	2.6560	-5.6190	0.2668	-4.6553	120.4716
M3	0.1391	0.1210	2.6560	-10.6246	0.2810	-7.3526	227.0160
M4	0.1389	0.1210	2.6560	-5.6190	0.2683	-4.5972	117.7422
M5	0.1390	0.1210	2.6560	-5.6190	0.2676	-4.6260	119.0556

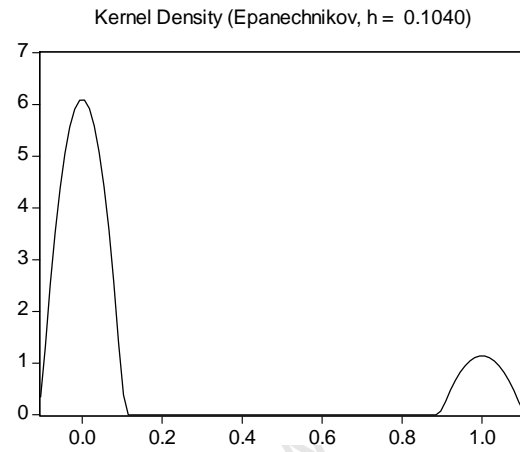
	ROE						
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Unadjusted	0.2247	0.2160	1.6900	-1.4570	0.2392	0.1364	11.6047
M1	0.2223	0.2160	1.6900	-1.5981	0.2408	0.0305	11.0678
M2	0.2225	0.2160	1.6900	-2.1117	0.2392	0.0127	11.5026
M3	0.2225	0.2160	1.6900	-1.4570	0.2394	0.0782	10.9468
M4	0.2230	0.2160	1.6900	-1.4570	0.2401	0.0958	10.9047
M5	0.2225	0.2160	1.6900	-1.4941	0.2413	0.0574	10.8822

A3. Comparison of the Relative Densities of the Actual and Mean Imputed Data

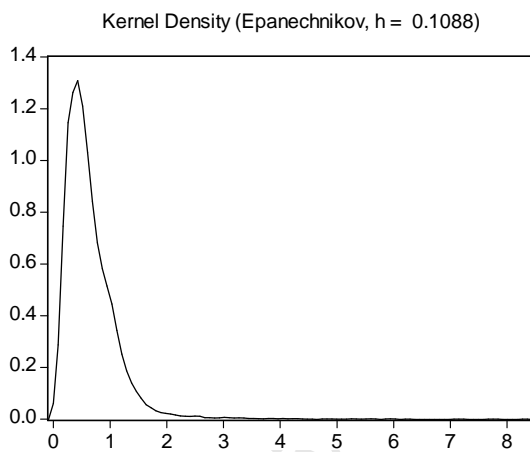
Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.



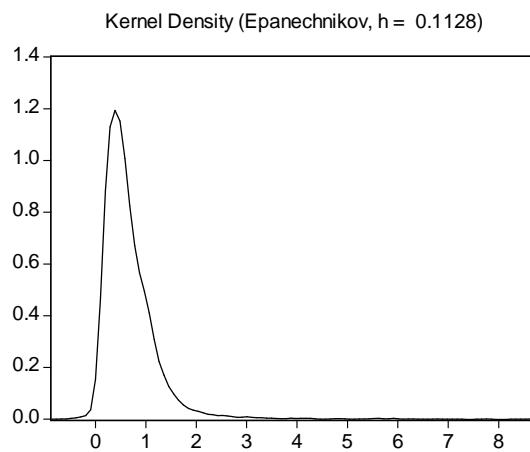
BUY – Unadjusted



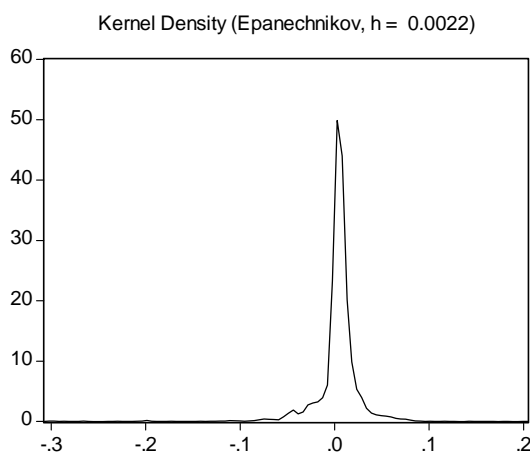
BUY – Multiply Imputed



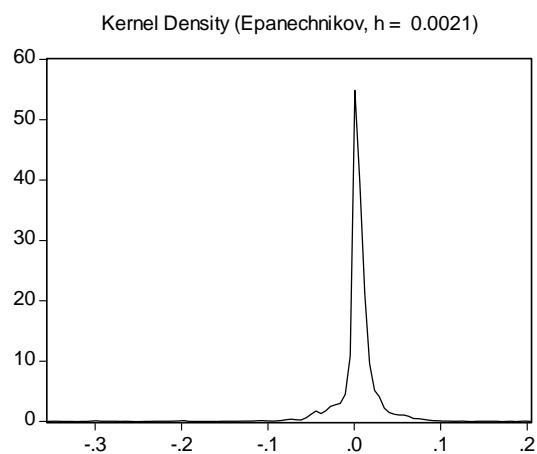
BVTM – Unadjusted



BVTM – Multiply Imputed



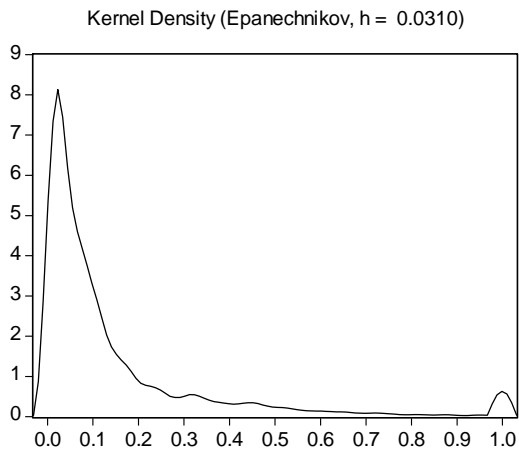
C12MDPSP – Unadjusted



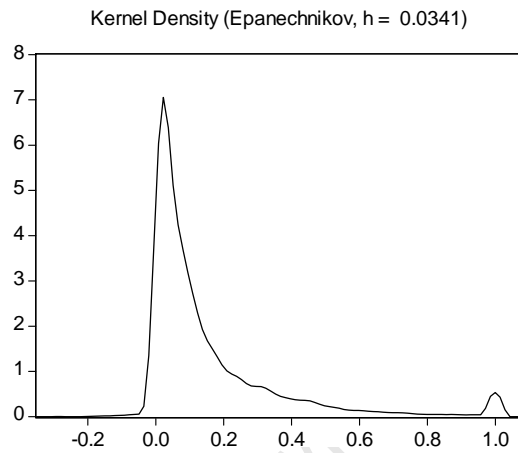
C12MDPSP – Multiply Imputed

A3. Comparison of the Relative Densities of the Actual and Mean Imputed Data

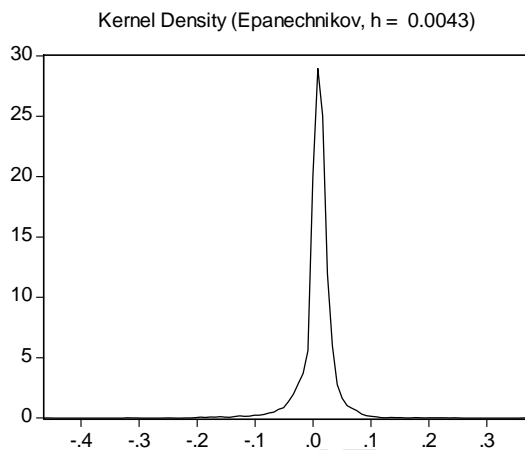
Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.



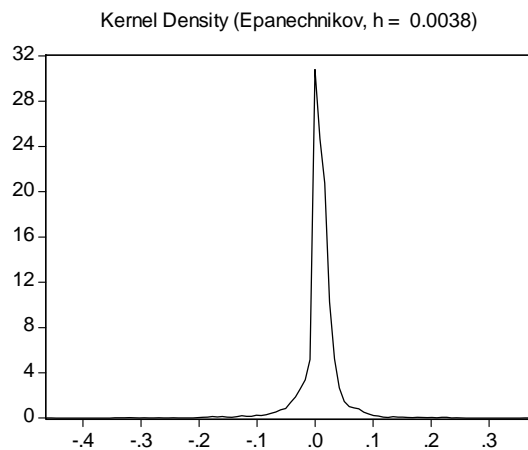
C12MEPSP – Unadjusted



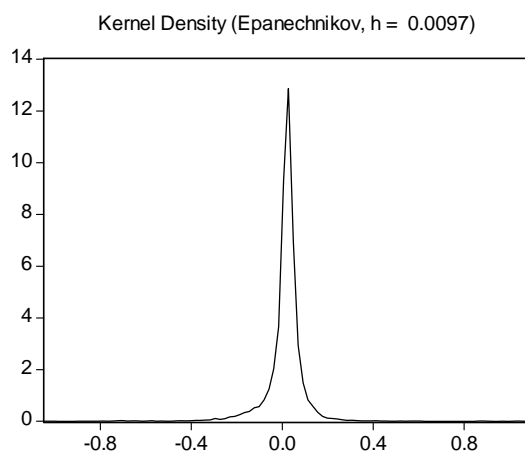
C12MEPSP – Multiply Imputed



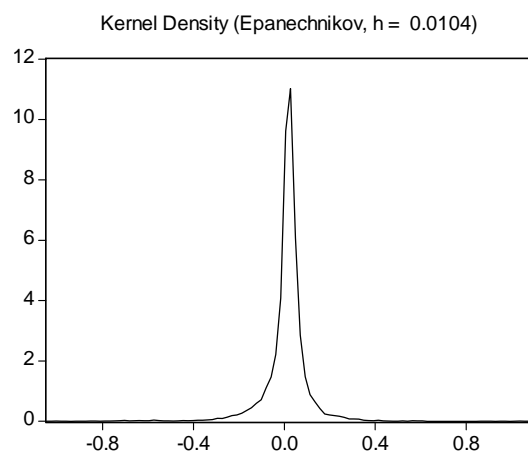
C24MDPSP – Unadjusted



C24MDPSP – Multiply Imputed



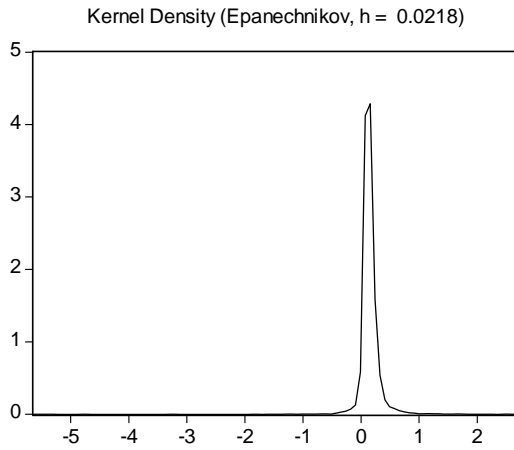
C24MEPSP – Unadjusted



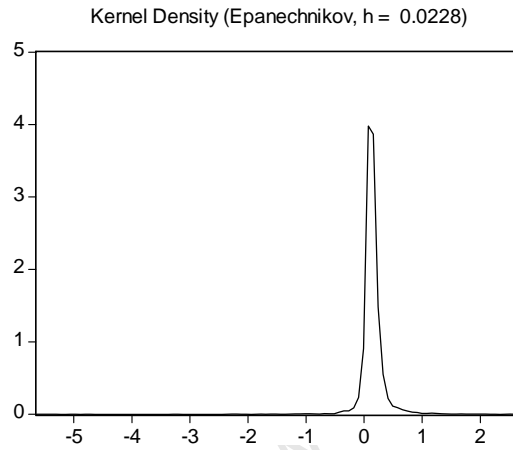
C24MEPSP – Multiply Imputed

A3. Comparison of the Relative Densities of the Actual and Mean Imputed Data

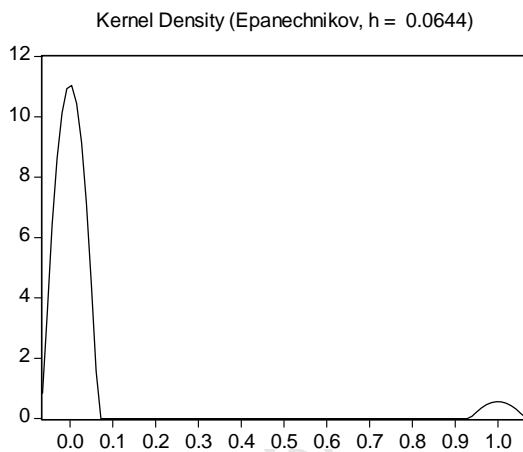
Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.



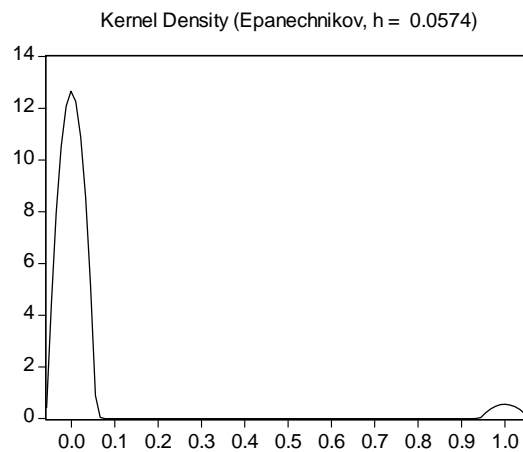
CTP – Unadjusted



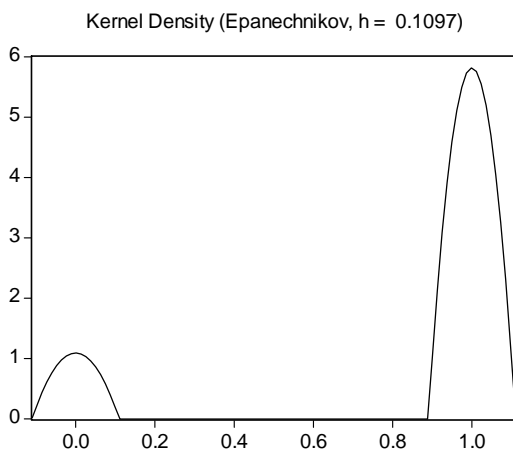
CTP – Multiply Imputed



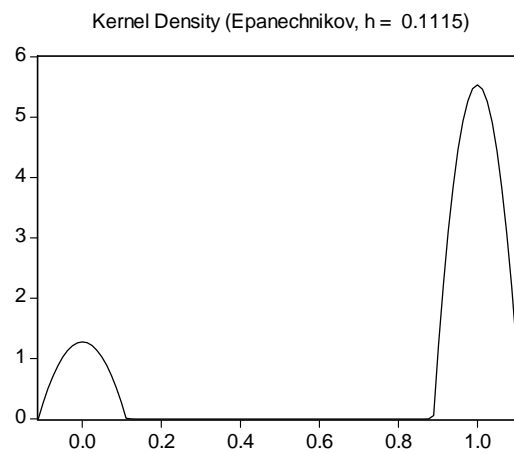
DOWNGRADE – Unadjusted



DOWNGRADE – Multiply Imputed



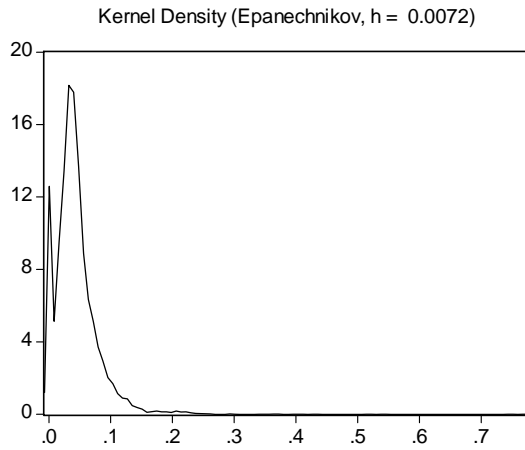
DUMFINDI – Unadjusted



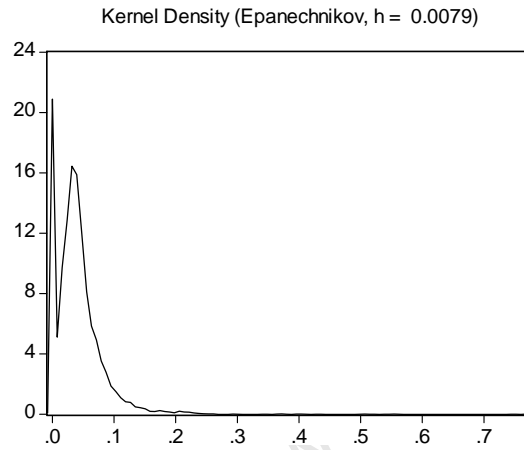
DUMFINDI – Multiply Imputed

A3. Comparison of the Relative Densities of the Actual and Mean Imputed Data

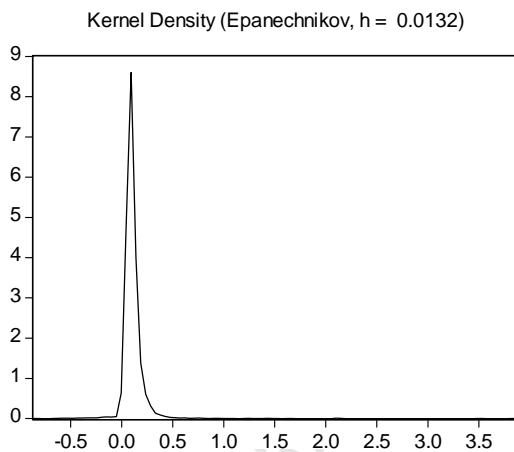
Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.



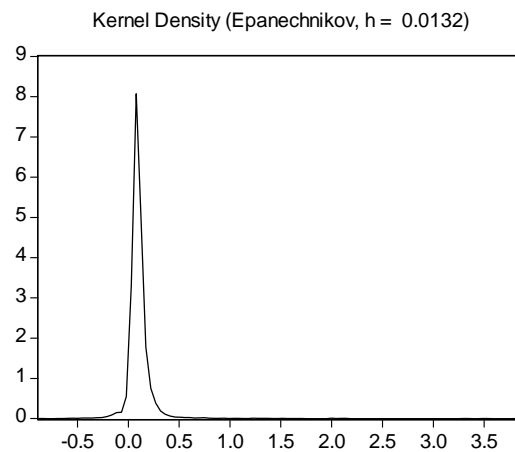
DY – Unadjusted



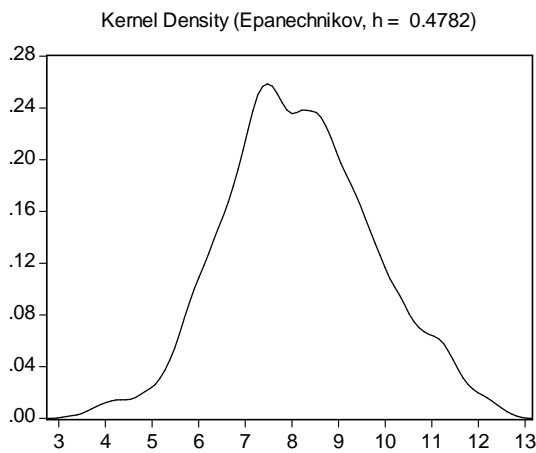
DY – Multiply Imputed



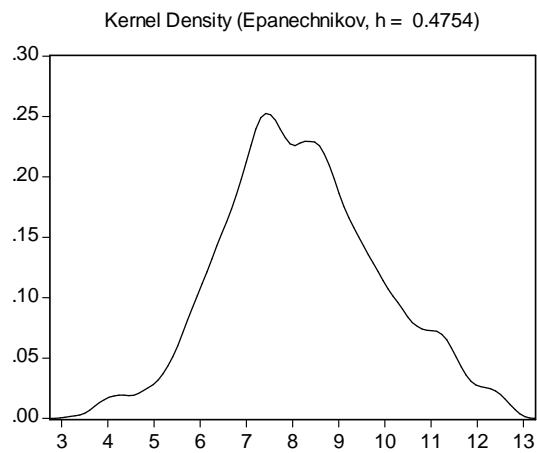
EY – Unadjusted



EY – Multiply Imputed



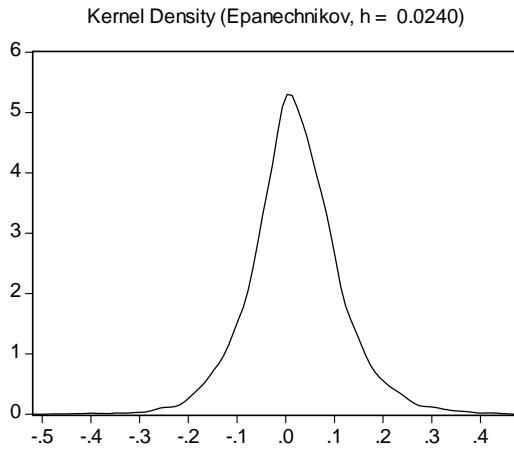
LMV – Unadjusted



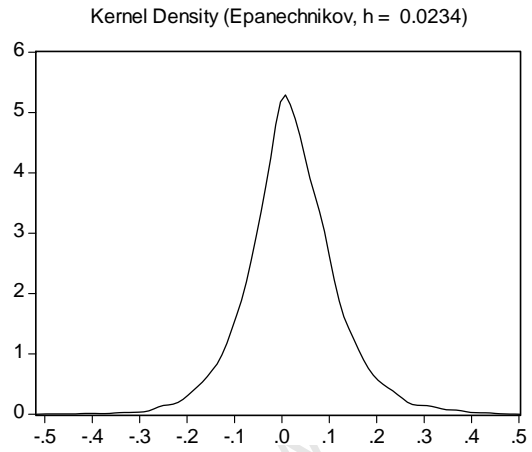
LMV – Multiply Imputed

A3. Comparison of the Relative Densities of the Actual and Mean Imputed Data

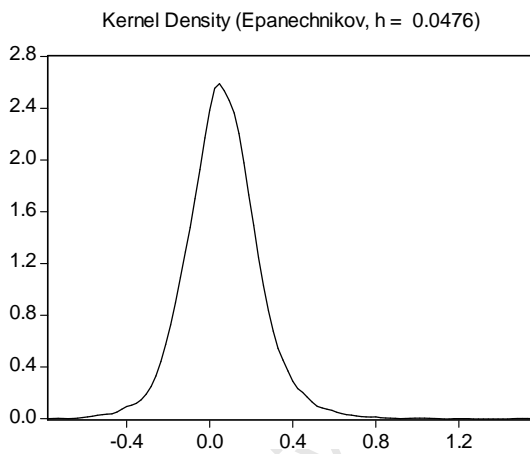
Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.



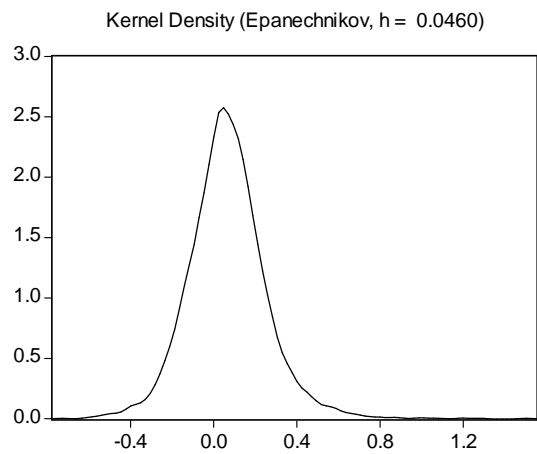
MOM1 – Unadjusted



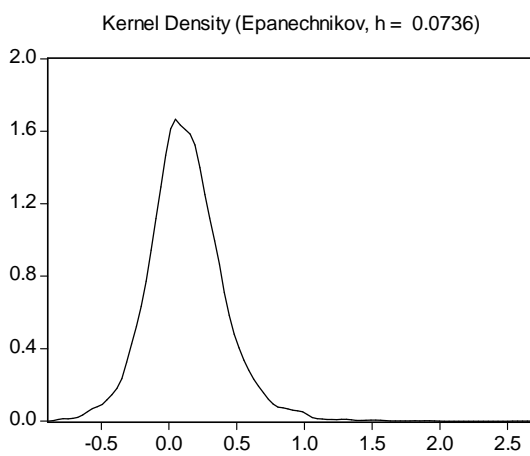
MOM1 – Multiply Imputed



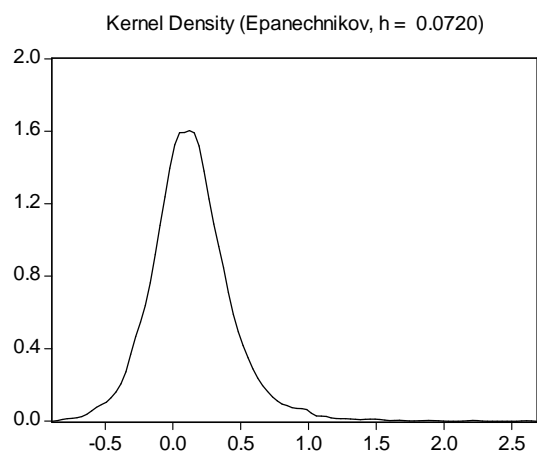
MOM3 – Unadjusted



MOM3 – Multiply Imputed



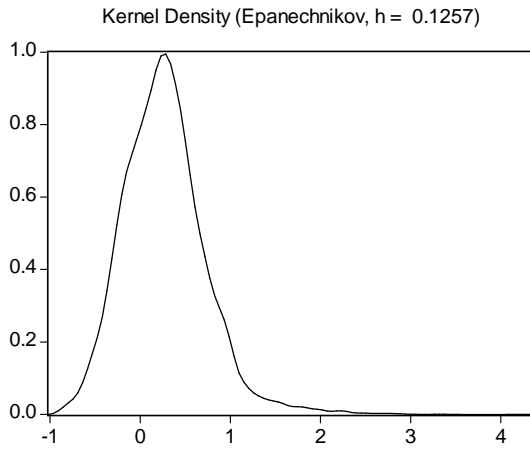
MOM6 – Unadjusted



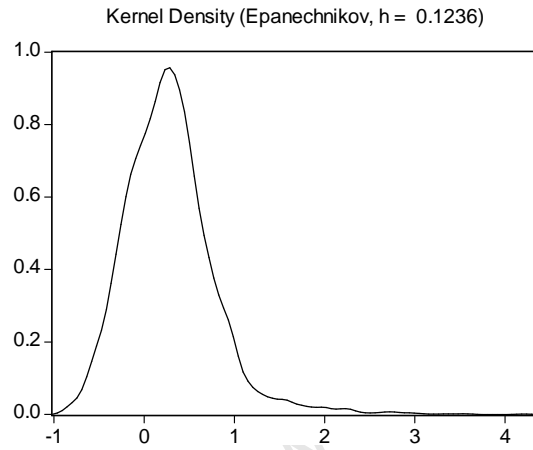
MOM6 – Multiply Imputed

A3. Comparison of the Relative Densities of the Actual and Mean Imputed Data

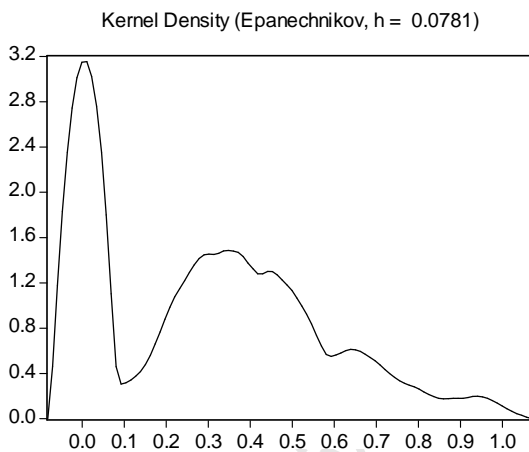
Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.



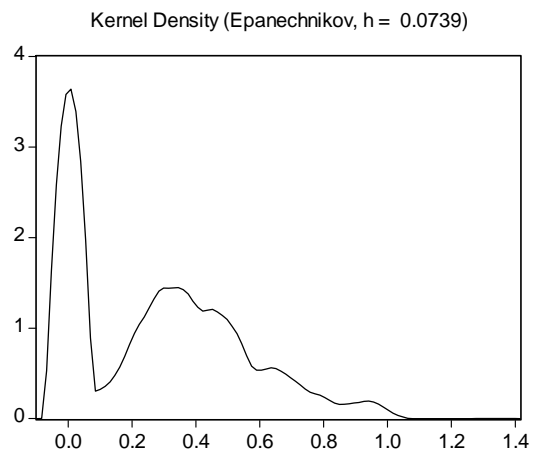
MOM12 – Unadjusted



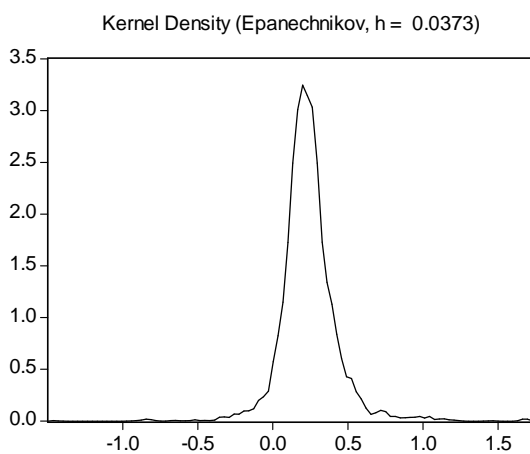
MOM12 – Multiply Imputed



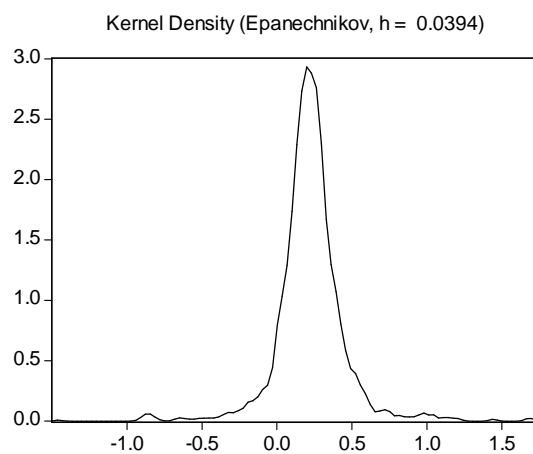
POUT – Unadjusted



POUT – Multiply Imputed



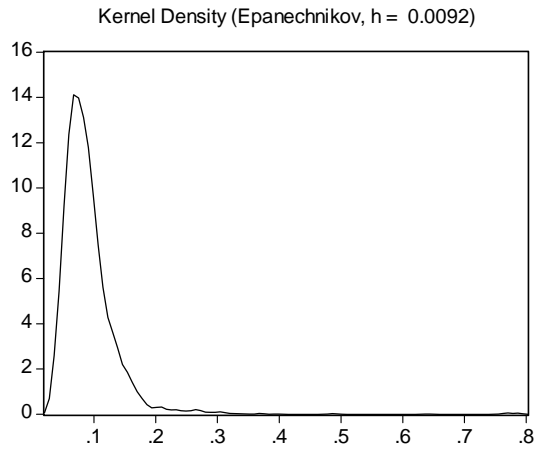
ROE – Unadjusted



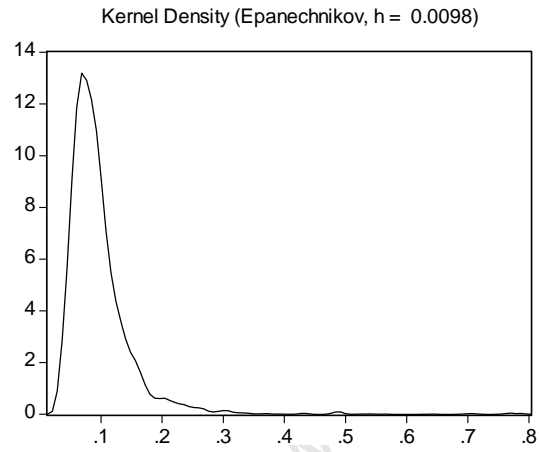
ROE – Multiply Imputed

A3. Comparison of the Relative Densities of the Actual and Mean Imputed Data

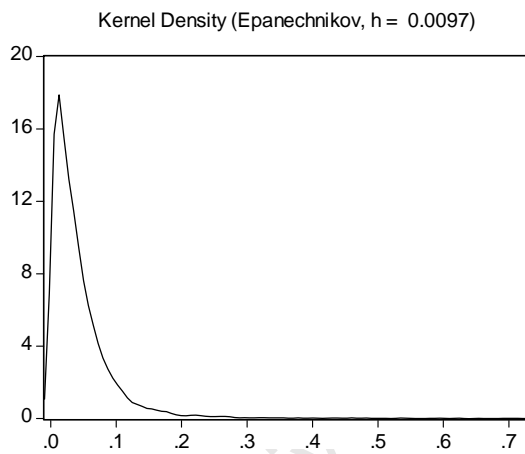
Descriptive statistics are provided for the style attributes for both the unadjusted data and five multiply imputed data sets. The multiply imputed data sets are labelled by number.



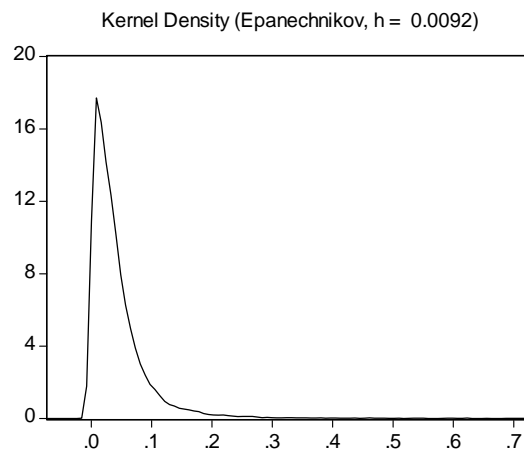
STDEV12 – Unadjusted



STDEV12 – Multiply Imputed



VT – Unadjusted



VT – Multiply Imputed

Appendix B

B1. Significant Autocorrelation Coefficients for the Large Cap Sample over the In-Sample Period

Autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ACL	0.099	0.005	0.001	0.079	-0.003	0.018	0.027	0.004	-0.019	0.012
AGL	0.039	0.009	-0.009	-0.040	-0.067	-0.013	-0.031	0.016	0.018	0.020
AMS	0.125	-0.024	-0.062	0.012	0.001	-0.019	-0.030	0.008	0.004	-0.011
ANG	0.000	-0.006	-0.051	-0.007	-0.015	-0.019	-0.008	-0.013	0.043	-0.006
ARI	0.035	0.052	0.008	-0.019	0.000	-0.041	0.014	-0.038	0.017	-0.058
ASA	0.097	-0.027	-0.074	0.004	-0.026	-0.060	-0.027	-0.038	-0.031	0.005
BAW	0.070	-0.046	-0.052	0.000	-0.069	-0.005	-0.054	0.001	0.016	0.006
BIL	-0.002	0.011	-0.071	-0.064	-0.098	-0.011	0.001	0.041	-0.010	0.028
BVT	-0.029	-0.047	-0.055	-0.033	-0.045	-0.058	-0.019	0.027	0.005	0.022
CFR	0.021	-0.020	-0.057	-0.027	-0.054	0.007	-0.001	0.041	0.010	0.006
FSR	0.022	-0.045	-0.051	0.009	-0.050	-0.038	-0.035	0.003	-0.022	-0.026
GFI	0.007	-0.015	-0.011	0.021	-0.027	-0.018	0.011	-0.047	-0.002	-0.016
HAR	0.043	-0.024	-0.029	0.023	-0.022	-0.047	0.048	0.020	-0.008	-0.008
IMP	0.098	-0.036	-0.064	0.008	-0.023	-0.054	-0.055	0.006	0.021	-0.010
INL	0.048	0.001	-0.059	-0.015	-0.065	-0.012	0.005	-0.019	-0.038	-0.009
IPL	0.045	-0.066	-0.022	-0.018	-0.038	-0.020	-0.019	0.022	-0.014	0.030
LBH	0.067	-0.009	-0.038	-0.050	0.018	-0.008	0.001	-0.032	-0.037	-0.006
LON	-0.029	0.016	0.003	-0.014	0.006	-0.033	-0.049	0.012	0.046	-0.021
MTN	0.121	0.003	-0.058	-0.044	-0.063	-0.045	-0.010	0.042	-0.063	0.002
MUR	0.091	-0.005	-0.038	-0.018	-0.020	-0.050	-0.005	-0.002	0.025	-0.009
NED	0.113	-0.040	-0.041	-0.063	-0.053	-0.025	-0.018	-0.004	0.017	-0.010
NPN	0.144	0.023	-0.015	0.097	-0.004	-0.024	-0.036	0.013	0.015	0.021
NTC	-0.046	-0.060	0.005	-0.018	-0.074	0.035	-0.006	-0.029	-0.027	0.050
OML	-0.007	-0.031	-0.047	-0.027	0.020	-0.056	0.023	-0.072	-0.072	0.003
PPC	-0.003	0.001	-0.027	0.016	-0.026	-0.007	-0.009	0.024	-0.074	-0.001
RMH	-0.053	0.004	-0.038	-0.017	-0.063	-0.035	-0.037	0.016	-0.023	-0.026
SAB	0.045	-0.040	-0.046	-0.055	-0.026	-0.001	0.058	0.005	-0.069	0.012
SAP	0.096	-0.001	-0.057	-0.051	-0.056	0.012	-0.003	-0.016	-0.026	-0.006
SBK	0.051	-0.088	-0.074	-0.033	-0.032	-0.053	-0.002	0.002	-0.009	-0.033
SHF	-0.014	-0.052	-0.065	-0.027	-0.032	-0.065	0.039	-0.017	-0.014	0.065
SLM	0.040	-0.048	-0.055	-0.033	-0.038	0.015	-0.017	-0.017	-0.013	-0.043
SOL	0.063	-0.016	-0.076	-0.035	-0.030	-0.043	0.035	-0.008	-0.024	0.001
TBS	-0.031	-0.029	-0.050	0.021	-0.054	-0.008	-0.014	0.015	0.019	-0.010
WHL	-0.051	-0.019	-0.024	-0.004	0.005	-0.021	-0.006	-0.029	-0.027	0.011

B1. Significant Autocorrelation Coefficients for the Mid Cap Sample over the In-Sample Period

Autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ABL	0.100	0.014	0.009	-0.010	-0.017	0.004	0.026	0.033	-0.036	-0.046
AEG	0.056	-0.017	0.007	0.027	-0.038	-0.061	-0.013	-0.005	0.029	-0.036
AFE	0.091	0.001	-0.001	-0.008	-0.021	-0.067	-0.030	-0.055	-0.071	-0.058
AFX	0.058	0.002	-0.026	0.016	0.007	-0.016	-0.036	0.025	-0.032	0.033
ALT	0.102	0.020	0.000	0.024	-0.020	-0.029	-0.004	-0.002	0.007	0.027
APN	0.019	-0.014	-0.025	0.003	-0.005	-0.010	0.003	0.005	0.003	-0.021
ATN	-0.016	-0.022	-0.008	-0.080	0.004	-0.024	-0.001	0.043	-0.043	0.033
AVI	-0.012	-0.006	-0.021	0.024	-0.063	-0.001	-0.013	-0.003	0.014	-0.008
CAT	-0.144	0.046	-0.024	-0.018	0.008	0.011	-0.044	0.039	-0.024	0.015
CLS	0.119	-0.083	-0.015	-0.065	-0.006	0.014	-0.047	-0.051	-0.009	-0.002
DDT	0.129	0.044	-0.006	-0.015	0.005	-0.022	0.005	0.032	0.009	0.024
DSY	0.067	0.024	-0.007	-0.030	-0.015	-0.039	-0.008	-0.019	-0.039	-0.004
DTC	0.124	0.037	0.024	0.068	0.007	0.008	-0.009	0.065	-0.004	0.025
FOS	0.070	-0.013	0.022	0.009	-0.017	-0.036	0.010	0.033	0.024	-0.007
FPT	-0.081	-0.021	0.003	-0.061	-0.051	-0.014	-0.002	-0.032	0.014	0.001
GDF	-0.112	-0.067	-0.018	-0.031	0.039	0.011	-0.031	-0.079	0.049	-0.042
GND	-0.037	0.036	-0.057	-0.046	-0.001	-0.026	-0.058	-0.024	0.072	-0.040
GRF	0.055	-0.006	-0.013	-0.011	-0.068	-0.060	0.004	-0.017	-0.013	0.030
GRT	-0.013	0.013	0.038	-0.074	0.013	-0.012	0.005	-0.063	0.019	-0.037
HVL	0.171	0.034	0.024	-0.045	-0.041	-0.020	-0.034	0.021	0.032	0.002
HYP	-0.003	0.035	-0.014	-0.017	-0.074	-0.011	0.014	-0.002	-0.005	0.013
ILV	0.102	0.022	-0.005	-0.036	-0.015	-0.026	-0.011	0.018	0.012	-0.011
JDG	0.126	0.013	-0.016	-0.014	-0.011	-0.019	0.007	0.013	0.001	-0.042
MDC	0.038	0.007	-0.035	0.019	-0.038	-0.011	-0.039	-0.037	-0.055	-0.011
MET	-0.023	-0.038	0.010	-0.002	-0.013	-0.071	0.032	0.003	-0.014	0.052
MPC	0.079	-0.003	-0.020	0.011	0.013	0.038	-0.015	0.010	0.033	-0.030
MTX	-0.097	0.011	-0.040	0.011	-0.007	0.017	-0.052	0.002	-0.020	-0.021
MVG	0.033	-0.023	0.039	0.034	-0.040	0.000	-0.003	0.002	-0.003	0.022
NHM	0.140	-0.011	-0.028	-0.013	-0.018	-0.058	0.039	-0.035	0.012	-0.038
NPK	-0.016	-0.046	-0.061	-0.052	0.005	-0.033	0.019	0.008	-0.034	0.057
PAP	-0.052	0.054	-0.001	0.017	-0.017	0.012	0.037	-0.046	0.040	0.011
PGR	-0.026	-0.029	0.037	0.011	-0.033	0.004	-0.008	0.000	0.072	0.000
PIK	-0.020	-0.065	-0.027	-0.026	-0.003	-0.043	0.006	-0.032	-0.022	-0.004
RBW	-0.059	-0.020	-0.026	-0.002	-0.059	0.004	0.012	-0.024	0.016	-0.034
RLO	0.036	-0.028	-0.004	-0.023	-0.022	-0.045	-0.038	-0.003	0.014	0.019
SAC	-0.135	-0.036	-0.049	-0.019	-0.035	-0.009	-0.022	-0.035	0.026	-0.01
SHP	0.002	-0.066	-0.061	0.023	0.048	0.018	-0.045	-0.032	-0.008	-0.013
SNT	0.013	-0.005	0.042	-0.02	-0.019	0.005	-0.015	0.001	-0.023	-0.007
SPG	-0.001	0.034	-0.047	-0.004	-0.001	-0.033	0.001	-0.039	0.046	0.019
SUI	0.131	0.029	0.01	-0.016	0.004	-0.016	0.013	-0.008	-0.04	-0.026
TON	0.013	0.002	-0.014	0	-0.019	0.01	-0.007	0.025	-0.029	0.026
TRE	-0.005	0.06	0.052	0.036	-0.018	-0.045	-0.012	-0.04	-0.016	0.037
TRU	0.042	-0.025	-0.016	-0.005	-0.016	-0.026	-0.048	-0.015	0.019	-0.004
WBO	0.028	0.037	-0.016	0.001	0.023	-0.063	0.036	-0.022	0.034	-0.02

B1. Significant Autocorrelation Coefficients for the Small Cap Sample over the In-Sample Period

Autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ADH	-0.129	-0.074	-0.026	-0.026	-0.014	-0.054	0.036	0.019	-0.051	0.009
ADR	0.010	0.051	-0.012	0.023	0.048	-0.017	0.035	0.017	-0.005	0.081
AFR	0.008	-0.035	-0.050	0.014	0.042	0.022	-0.001	-0.015	0.012	-0.022
ART	-0.146	0.029	-0.036	-0.053	0.010	-0.027	0.052	-0.053	0.003	0.008
BAT	0.113	0.061	0.040	-0.006	0.023	-0.002	0.003	-0.017	0.024	0.013
BCX	-0.017	0.006	-0.059	0.006	-0.051	-0.001	-0.031	0.043	0.044	-0.011
BEL	-0.069	0.018	0.015	0.009	0.008	-0.051	0.010	-0.001	-0.040	-0.036
CDZ	-0.056	0.002	-0.002	0.009	-0.039	0.022	0.032	-0.033	-0.004	-0.021
CLH	-0.020	0.022	-0.013	-0.036	0.017	-0.025	-0.025	0.007	-0.031	0.021
CRM	-0.143	0.022	0.036	-0.070	0.019	-0.006	0.028	0.066	-0.028	0.007
CSB	0.019	0.030	0.003	0.027	-0.029	0.066	-0.040	0.048	0.000	0.026
DAW	-0.194	-0.012	-0.062	0.009	-0.019	0.022	0.013	0.018	-0.020	0.090
DRD	-0.002	-0.041	0.000	0.017	-0.027	-0.039	0.012	0.020	0.026	-0.029
FBR	-0.209	-0.004	-0.048	0.025	-0.015	-0.037	0.005	0.037	-0.033	-0.008
HDC	0.059	0.004	-0.011	0.004	-0.025	-0.021	-0.001	-0.045	0.005	0.000
ILA	-0.101	-0.002	0.006	-0.062	-0.004	0.048	-0.043	-0.001	0.015	-0.007
IVT	-0.116	-0.058	-0.031	0.007	0.030	0.010	-0.039	0.014	0.002	-0.004
KAP	-0.112	-0.083	0.005	-0.013	-0.014	0.024	-0.025	-0.024	0.011	-0.005
MRF	-0.020	-0.003	-0.005	-0.006	-0.027	-0.022	-0.013	-0.010	0.000	-0.049
MTA	-0.049	0.030	0.042	-0.062	0.021	-0.022	0.010	0.004	-0.038	0.024
OCE	-0.106	0.050	-0.023	0.017	-0.070	-0.016	-0.001	-0.053	0.005	0.030
OCT	-0.124	0.001	-0.023	-0.034	-0.047	0.010	0.069	-0.006	-0.043	-0.033
OMN	0.075	0.101	0.024	-0.010	-0.034	-0.001	-0.001	0.020	0.007	0.032
PAM	0.016	0.062	0.013	-0.003	0.023	0.022	-0.032	-0.022	-0.016	-0.029
PMM	-0.208	-0.063	0.053	-0.073	0.051	-0.001	-0.017	0.022	-0.019	-0.015
SYC	-0.015	0.048	0.022	-0.001	0.000	0.021	-0.030	-0.004	-0.006	0.012

B1. Significant Partial Autocorrelation Coefficients for the Large Cap Sample over the In-Sample Period

Partial autocorrelation coefficients (PAC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ACL	0.099	-0.005	0.001	0.079	-0.019	0.021	0.024	-0.008	-0.017	0.013
AGL	0.039	0.007	-0.010	-0.039	-0.064	-0.008	-0.030	0.016	0.012	0.014
AMS	0.125	-0.040	-0.055	0.027	-0.008	-0.021	-0.023	0.013	-0.002	-0.014
ANG	0.000	-0.006	-0.051	-0.007	-0.016	-0.022	-0.009	-0.016	0.041	-0.008
ARI	0.035	0.051	0.005	-0.022	0.000	-0.040	0.017	-0.036	0.019	-0.058
ASA	0.097	-0.037	-0.069	0.017	-0.033	-0.060	-0.016	-0.042	-0.033	0.006
BAW	0.070	-0.051	-0.045	0.004	-0.075	0.003	-0.061	0.002	0.010	-0.007
BIL	-0.002	0.011	-0.070	-0.065	-0.098	-0.016	-0.007	0.024	-0.024	0.015
BVT	-0.029	-0.048	-0.058	-0.039	-0.053	-0.069	-0.034	0.011	-0.008	0.014
CFR	0.021	-0.020	-0.056	-0.025	-0.055	0.004	-0.007	0.035	0.006	0.004
FSR	0.022	-0.046	-0.049	0.010	-0.055	-0.038	-0.038	-0.004	-0.028	-0.032
GFI	0.007	-0.015	-0.011	0.021	-0.028	-0.017	0.011	-0.049	0.000	-0.017
HAR	0.043	-0.026	-0.027	0.025	-0.026	-0.044	0.052	0.011	-0.009	-0.002
IMP	0.098	-0.046	-0.057	0.019	-0.031	-0.053	-0.046	0.009	0.010	-0.019
INL	0.048	-0.001	-0.059	-0.010	-0.064	-0.009	0.005	-0.028	-0.038	-0.009
IPL	0.045	-0.068	-0.016	-0.021	-0.039	-0.019	-0.023	0.020	-0.021	0.031
LBH	0.067	-0.014	-0.036	-0.046	0.023	-0.013	0.000	-0.034	-0.032	-0.004
LON	-0.029	0.015	0.004	-0.014	0.005	-0.032	-0.051	0.010	0.049	-0.020
MTN	0.121	-0.012	-0.058	-0.030	-0.055	-0.035	-0.005	0.037	-0.082	0.013
MUR	0.091	-0.014	-0.036	-0.011	-0.018	-0.049	0.002	-0.004	0.021	-0.015
NED	0.113	-0.053	-0.031	-0.058	-0.043	-0.021	-0.022	-0.009	0.010	-0.020
NPN	0.144	0.002	-0.019	0.104	-0.033	-0.023	-0.024	0.011	0.015	0.020
NTC	-0.046	-0.062	-0.001	-0.022	-0.076	0.026	-0.012	-0.027	-0.034	0.040
OML	-0.007	-0.031	-0.047	-0.028	0.016	-0.060	0.021	-0.076	-0.077	-0.005
PPC	-0.003	0.001	-0.027	0.016	-0.026	-0.008	-0.008	0.023	-0.074	-0.002
RMH	-0.053	0.001	-0.038	-0.021	-0.066	-0.044	-0.043	0.006	-0.028	-0.039
SAB	0.045	-0.042	-0.042	-0.053	-0.025	-0.006	0.052	-0.005	-0.068	0.022
SAP	0.096	-0.010	-0.056	-0.041	-0.048	0.019	-0.011	-0.023	-0.025	-0.004
SBK	0.051	-0.091	-0.065	-0.034	-0.041	-0.061	-0.008	-0.015	-0.020	-0.040
SHF	-0.014	-0.052	-0.067	-0.032	-0.041	-0.075	0.028	-0.030	-0.024	0.062
SLM	0.040	-0.049	-0.051	-0.031	-0.041	0.012	-0.025	-0.019	-0.015	-0.048
SOL	0.063	-0.020	-0.074	-0.026	-0.029	-0.047	0.035	-0.019	-0.031	0.006
TBS	-0.031	-0.030	-0.052	0.017	-0.056	-0.013	-0.016	0.007	0.019	-0.012
WHL	-0.051	-0.021	-0.026	-0.007	0.004	-0.021	-0.009	-0.031	-0.031	0.006

B1. Significant Partial Autocorrelation Coefficients for the Mid Cap Sample over the In-Sample Period

Partial autocorrelation coefficients (PAC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ABL	0.100	0.004	0.007	-0.012	-0.015	0.007	0.026	0.028	-0.044	-0.039
AEG	0.056	-0.020	0.009	0.026	-0.041	-0.056	-0.009	-0.006	0.032	-0.038
AFE	0.091	-0.007	0.000	-0.008	-0.020	-0.064	-0.018	-0.052	-0.063	-0.049
AFX	0.058	-0.002	-0.026	0.019	0.005	-0.018	-0.034	0.029	-0.036	0.036
ALT	0.102	0.010	-0.003	0.024	-0.025	-0.026	0.002	-0.001	0.008	0.027
APN	0.019	-0.015	-0.025	0.004	-0.006	-0.010	0.003	0.005	0.003	-0.021
ATN	-0.016	-0.022	-0.009	-0.080	0.001	-0.028	-0.003	0.035	-0.042	0.030
AVI	-0.012	-0.006	-0.021	0.024	-0.062	-0.003	-0.013	-0.007	0.017	-0.013
CAT	-0.144	0.026	-0.014	-0.026	0.003	0.014	-0.043	0.027	-0.011	0.006
CLS	0.119	-0.098	0.007	-0.074	0.012	0.001	-0.050	-0.043	-0.006	-0.008
DDT	0.129	0.028	-0.015	-0.014	0.010	-0.023	0.010	0.032	0.001	0.021
DSY	0.067	0.019	-0.010	-0.030	-0.011	-0.036	-0.004	-0.017	-0.038	0.000
DTC	0.124	0.022	0.017	0.063	-0.010	0.005	-0.013	0.064	-0.020	0.025
FOS	0.070	-0.018	0.024	0.005	-0.017	-0.034	0.014	0.031	0.022	-0.009
FPT	-0.081	-0.027	-0.001	-0.062	-0.062	-0.027	-0.008	-0.039	0.001	-0.006
GDF	-0.112	-0.080	-0.035	-0.044	0.027	0.013	-0.026	-0.085	0.028	-0.048
GND	-0.037	0.035	-0.054	-0.052	0.000	-0.026	-0.066	-0.030	0.071	-0.044
GRF	0.055	-0.009	-0.012	-0.010	-0.067	-0.053	0.009	-0.020	-0.014	0.026
GRT	-0.013	0.013	0.038	-0.074	0.010	-0.011	0.010	-0.069	0.021	-0.038
HVL	0.171	0.005	0.018	-0.054	-0.026	-0.008	-0.027	0.032	0.022	-0.009
HYP	-0.003	0.035	-0.014	-0.019	-0.073	-0.010	0.019	-0.004	-0.009	0.008
ILV	0.102	0.012	-0.009	-0.036	-0.007	-0.023	-0.006	0.019	0.007	-0.016
JDG	0.126	-0.003	-0.018	-0.010	-0.008	-0.017	0.011	0.010	-0.002	-0.043
MDC	0.038	0.006	-0.035	0.022	-0.039	-0.009	-0.036	-0.038	-0.051	-0.010
MET	-0.023	-0.039	0.008	-0.003	-0.013	-0.072	0.028	-0.001	-0.010	0.051
MPC	0.079	-0.009	-0.020	0.014	0.011	0.036	-0.021	0.014	0.032	-0.037
MTX	-0.097	0.002	-0.039	0.003	-0.005	0.015	-0.049	-0.008	-0.019	-0.029
MVG	0.033	-0.024	0.041	0.031	-0.040	0.003	-0.008	0.004	-0.001	0.021
NHM	0.140	-0.031	-0.022	-0.006	-0.017	-0.055	0.055	-0.053	0.024	-0.045
NPK	-0.016	-0.046	-0.062	-0.057	-0.003	-0.043	0.011	0.002	-0.037	0.054
PAP	-0.052	0.052	0.005	0.015	-0.016	0.008	0.040	-0.044	0.032	0.018
PGR	-0.026	-0.030	0.036	0.013	-0.030	0.001	-0.011	0.002	0.073	0.004
PIK	-0.020	-0.065	-0.030	-0.032	-0.008	-0.049	0.001	-0.040	-0.027	-0.013
RBW	-0.059	-0.024	-0.029	-0.006	-0.061	-0.005	0.009	-0.026	0.012	-0.037
RLO	0.036	-0.029	-0.002	-0.023	-0.02	-0.045	-0.037	-0.003	0.011	0.016
SAC	-0.135	-0.055	-0.062	-0.038	-0.05	-0.028	-0.036	-0.053	0.005	-0.018
SHP	0.002	-0.066	-0.061	0.019	0.041	0.018	-0.037	-0.026	-0.013	-0.024
SNT	0.013	-0.005	0.042	-0.021	-0.018	0.003	-0.013	0.002	-0.024	-0.005
SPG	-0.001	0.034	-0.047	-0.006	0.002	-0.035	0	-0.037	0.043	0.021
SUI	0.131	0.012	0.004	-0.019	0.008	-0.017	0.018	-0.012	-0.039	-0.017
TON	0.013	0.002	-0.014	0	-0.019	0.01	-0.008	0.024	-0.03	0.026
TRE	-0.005	0.06	0.053	0.033	-0.024	-0.053	-0.014	-0.034	-0.009	0.047
TRU	0.042	-0.026	-0.014	-0.004	-0.017	-0.025	-0.047	-0.013	0.017	-0.009
WBO	0.028	0.037	-0.018	0	0.024	-0.065	0.038	-0.018	0.031	-0.02

B1. Significant Partial Autocorrelation Coefficients for the Small Cap Sample over the In-Sample Period

Partial autocorrelation coefficients (PAC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ADH	-0.129	-0.092	-0.049	-0.044	-0.031	-0.070	0.011	0.012	-0.051	-0.006
ADR	0.010	0.050	-0.013	0.021	0.049	-0.020	0.032	0.019	-0.012	0.079
AFR	0.008	-0.035	-0.049	0.013	0.039	0.020	0.003	-0.010	0.013	-0.026
ART	-0.146	0.008	-0.032	-0.064	-0.006	-0.027	0.041	-0.044	-0.015	0.009
BAT	0.113	0.049	0.028	-0.016	0.022	-0.007	0.002	-0.019	0.029	0.008
BCX	-0.017	0.006	-0.059	0.004	-0.050	-0.006	-0.030	0.036	0.046	-0.016
BEL	-0.069	0.013	0.017	0.011	0.009	-0.051	0.002	0.001	-0.039	-0.041
CDZ	-0.056	-0.001	-0.002	0.009	-0.038	0.018	0.034	-0.030	-0.007	-0.024
CLH	-0.020	0.022	-0.012	-0.037	0.016	-0.023	-0.028	0.006	-0.030	0.017
CRM	-0.143	0.002	0.040	-0.061	-0.001	-0.003	0.032	0.072	-0.009	-0.003
CSB	0.019	0.029	0.002	0.026	-0.030	0.066	-0.042	0.046	0.001	0.020
DAW	-0.194	-0.051	-0.078	-0.021	-0.028	0.008	0.018	0.024	-0.008	0.093
DRD	-0.002	-0.041	0.000	0.016	-0.027	-0.038	0.010	0.017	0.028	-0.027
FBR	-0.209	-0.050	-0.062	0.001	-0.013	-0.046	-0.014	0.033	-0.023	-0.018
HDC	0.059	0.000	-0.011	0.006	-0.026	-0.018	0.001	-0.046	0.010	-0.001
ILA	-0.101	-0.012	0.005	-0.062	-0.017	0.045	-0.034	-0.012	0.013	0.001
IVT	-0.116	-0.073	-0.047	-0.007	0.026	0.016	-0.032	0.009	0.001	-0.005
KAP	-0.112	-0.097	-0.016	-0.023	-0.020	0.017	-0.023	-0.028	0.000	-0.009
MRF	-0.020	-0.003	-0.005	-0.006	-0.027	-0.023	-0.014	-0.011	-0.001	-0.050
MTA	-0.049	0.027	0.044	-0.059	0.013	-0.019	0.012	0.002	-0.035	0.017
OCE	-0.106	0.039	-0.013	0.011	-0.066	-0.032	0.000	-0.055	-0.005	0.031
OCT	-0.124	-0.014	-0.025	-0.041	-0.058	-0.005	0.068	0.007	-0.047	-0.045
OMN	0.075	0.096	0.010	-0.022	-0.036	0.006	0.006	0.021	0.002	0.027
PAM	0.016	0.062	0.011	-0.008	0.022	0.022	-0.035	-0.025	-0.012	-0.025
PMM	-0.208	-0.111	0.016	-0.068	0.028	0.004	-0.006	0.012	-0.010	-0.021
SYC	-0.015	0.048	0.023	-0.003	-0.002	0.021	-0.029	-0.007	-0.004	0.014

B2. Significant Autocorrelation Coefficients for the Large Cap Sample over the Out-of-Sample Period

Autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ACL	0.033	-0.023	-0.028	-0.010	-0.052	-0.031	0.076	0.025	0.022	-0.150
AGL	0.031	0.010	-0.177	-0.021	-0.066	-0.021	0.066	0.022	-0.009	-0.069
AMS	0.094	-0.074	-0.046	-0.008	-0.068	0.014	0.000	0.011	0.031	-0.012
ANG	0.030	-0.060	0.008	0.040	-0.085	-0.047	0.026	-0.132	-0.012	-0.028
ARI	0.081	-0.103	-0.115	0.073	0.102	0.005	-0.017	-0.074	0.031	-0.016
ASA	0.058	-0.119	-0.094	-0.086	-0.015	-0.041	-0.071	0.091	0.101	-0.101
BAW	0.061	-0.014	-0.081	-0.057	0.006	-0.012	-0.022	-0.001	0.017	0.002
BIL	0.009	-0.007	-0.113	-0.036	-0.035	-0.044	0.041	-0.052	0.001	-0.022
BVT	-0.110	-0.063	-0.030	-0.028	0.079	-0.060	0.003	0.008	0.030	0.017
CFR	-0.066	0.039	-0.027	-0.044	0.015	0.036	0.003	-0.042	-0.027	0.006
FSR	0.044	-0.107	-0.090	-0.110	0.014	0.029	-0.009	0.094	-0.023	-0.042
GFI	0.035	-0.085	-0.023	0.017	-0.028	-0.047	-0.053	-0.014	-0.017	-0.046
HAR	0.103	0.005	0.057	-0.026	-0.002	0.003	-0.051	0.011	-0.033	0.003
IMP	0.092	-0.103	-0.121	-0.096	-0.066	0.031	0.090	0.062	0.010	-0.080
INL	0.012	-0.099	-0.066	-0.071	0.007	0.047	-0.029	0.120	0.042	-0.111
IPL	0.014	-0.033	0.037	-0.032	-0.031	0.013	0.050	0.009	0.016	-0.033
LBH	-0.164	-0.020	-0.059	-0.008	-0.022	-0.050	0.029	0.044	0.032	0.002
LON	0.035	0.038	-0.054	0.030	0.005	-0.039	0.029	0.007	0.022	-0.073
MTN	-0.005	-0.162	-0.017	-0.064	-0.007	-0.028	-0.040	0.054	0.019	0.020
MUR	0.065	-0.069	0.006	-0.036	0.000	-0.008	-0.031	0.016	0.022	0.078
NED	0.004	-0.097	-0.051	-0.032	-0.020	-0.021	-0.057	0.066	0.056	-0.031
NPN	-0.022	-0.125	-0.027	-0.030	-0.047	0.030	0.018	0.081	-0.013	-0.073
NTC	0.006	-0.033	-0.023	-0.011	-0.040	-0.009	-0.017	-0.042	0.018	0.016
OML	-0.053	-0.045	-0.071	-0.037	-0.030	0.094	0.024	0.063	0.020	-0.113
PPC	-0.054	-0.147	-0.043	-0.027	0.137	-0.021	-0.073	0.035	0.076	-0.013
RMH	-0.067	-0.073	-0.085	-0.065	0.022	-0.019	0.000	0.075	-0.012	-0.042
SAB	-0.031	-0.055	-0.007	-0.036	-0.088	0.006	0.041	0.005	-0.037	0.020
SAP	0.100	-0.019	-0.022	-0.034	-0.087	-0.031	0.032	0.011	-0.024	0.079
SBK	-0.003	-0.122	-0.024	-0.076	-0.001	-0.066	0.001	0.107	0.009	-0.081
SHF	0.055	-0.133	-0.083	-0.093	0.004	0.056	0.066	0.063	0.015	-0.070
SLM	-0.093	-0.157	-0.016	0.048	0.055	-0.034	-0.037	0.044	0.033	-0.056
SOL	-0.003	-0.005	-0.060	-0.062	-0.069	-0.086	0.055	-0.023	-0.007	0.006
TBS	0.016	-0.018	-0.066	-0.065	0.000	-0.072	0.007	0.049	0.023	-0.024
WHL	-0.041	-0.012	-0.005	0.035	-0.022	-0.001	0.020	0.040	0.064	-0.060

B2. Significant Autocorrelation Coefficients for the Mid Cap Sample over the Out-of-Sample Period

Autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ABL	-0.026	-0.097	-0.041	0.042	-0.048	-0.095	-0.034	0.074	0.004	-0.023
AEG	0.113	-0.069	-0.114	-0.080	0.019	-0.038	-0.010	0.056	0.076	0.034
AFE	-0.060	0.035	-0.034	-0.009	0.011	-0.055	0.002	0.036	0.102	-0.035
AFX	-0.034	-0.027	0.052	0.027	0.025	-0.006	-0.003	-0.028	-0.009	-0.073
ALT	-0.062	0.026	0.026	-0.009	-0.003	-0.003	-0.014	-0.062	-0.030	0.034
APN	0.082	-0.126	-0.060	-0.107	-0.016	-0.008	0.045	0.029	0.011	0.011
ATN	-0.057	-0.001	-0.096	-0.076	0.040	0.022	0.034	-0.010	-0.056	0.002
AVI	0.082	-0.060	-0.096	-0.047	-0.070	-0.013	0.016	0.038	-0.039	0.030
CAT	-0.281	-0.037	0.026	-0.017	-0.002	-0.033	0.039	0.058	-0.069	0.103
CLS	-0.084	-0.029	-0.040	-0.020	-0.028	-0.011	-0.013	-0.004	0.090	-0.077
DDT	0.015	-0.060	0.019	-0.037	-0.029	-0.073	0.095	0.002	0.000	-0.037
DSY	0.027	-0.034	-0.021	-0.059	-0.002	-0.037	-0.024	0.004	0.018	-0.057
DTC	0.108	0.044	0.051	0.005	0.037	0.035	0.032	-0.027	-0.021	-0.058
FOS	0.051	0.017	-0.027	0.005	0.053	-0.014	-0.094	0.018	0.033	0.037
FPT	0.037	-0.056	-0.029	0.029	-0.015	0.035	0.036	0.025	-0.022	-0.020
GDF	-0.041	-0.025	-0.046	0.046	-0.016	0.010	0.034	-0.044	0.023	0.012
GND	0.070	-0.005	-0.119	-0.024	0.058	0.006	0.039	-0.002	0.023	-0.057
GRF	0.104	-0.019	-0.027	0.006	0.061	-0.022	0.018	0.027	0.068	0.003
GRT	0.026	0.013	-0.020	-0.013	-0.022	-0.101	0.060	0.017	0.023	-0.044
HVL	-0.018	0.024	0.055	0.006	0.073	-0.066	0.050	0.040	0.067	0.031
HYP	-0.023	-0.013	0.039	0.039	-0.009	-0.048	0.079	-0.045	0.012	-0.020
ILV	-0.009	-0.005	-0.024	0.008	0.020	-0.062	0.053	-0.006	-0.015	-0.001
JDG	0.075	-0.086	-0.087	0.031	0.075	0.006	-0.004	-0.026	0.083	0.049
MDC	-0.109	-0.010	-0.063	-0.001	0.031	0.009	0.001	-0.036	0.018	0.016
MET	-0.143	-0.026	0.008	-0.043	-0.028	0.010	0.056	0.039	0.032	-0.056
MPC	0.075	-0.067	-0.049	-0.019	-0.039	-0.022	-0.001	-0.026	0.023	0.087
MTX	0.102	0.094	0.039	-0.025	0.024	-0.017	0.031	-0.039	-0.010	-0.045
MVG	0.052	0.060	0.018	0.036	0.012	-0.023	0.023	-0.023	0.030	0.014
NHM	0.102	-0.017	-0.104	-0.053	0.009	0.015	0.105	-0.024	-0.027	-0.072
NPK	-0.137	-0.063	-0.059	-0.079	0.014	0.047	-0.039	0.037	0.096	-0.034
PAP	0.126	0.009	-0.031	-0.021	0.021	-0.030	0.035	0.066	0.030	-0.023
PGR	0.020	-0.005	-0.025	-0.003	0.038	-0.015	0.062	0.065	0.020	-0.003
PIK	-0.115	-0.114	0.034	-0.025	0.007	-0.017	-0.039	-0.024	0.080	-0.027
RBW	-0.088	0.057	-0.068	0.045	0.010	0.001	0.023	0.013	0.004	0.023
RLO	-0.021	-0.021	-0.067	-0.125	-0.02	0.063	-0.018	0.041	0.059	-0.023
SAC	-0.035	-0.033	-0.007	-0.005	0.004	-0.059	0.07	0.056	-0.029	-0.071
SHP	-0.076	-0.087	-0.028	-0.059	0.077	-0.047	0.015	0.062	-0.064	0.029
SNT	-0.127	-0.036	0.05	0.004	0.012	-0.023	-0.017	0.031	0.01	-0.036
SPG	-0.008	0.033	-0.011	0.019	-0.008	-0.016	-0.002	0.112	0.057	0.008
SUI	0.124	-0.024	-0.061	-0.068	0.006	-0.01	0.067	0.032	0.018	-0.045
TON	-0.04	0.06	0.028	0.031	0.018	-0.013	0.019	0.033	-0.018	0.059
TRE	-0.155	-0.101	-0.023	0.055	0.046	-0.061	0.008	0.021	-0.084	0.075
TRU	-0.037	-0.021	-0.072	0.011	0.016	-0.1	-0.01	-0.002	0.062	-0.091
WBO	0.044	-0.054	0.037	0.001	0.034	0.022	-0.022	0.006	0.024	0.032

B2. Significant Autocorrelation Coefficients for the Small Cap Sample over the Out-of-Sample Period

Autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ADH	-0.130	0.053	-0.074	0.055	-0.063	0.034	0.056	-0.024	0.014	0.067
ADR	-0.163	0.008	0.001	0.010	0.005	0.046	0.089	-0.004	-0.060	0.029
AFR	-0.072	0.048	-0.020	-0.008	-0.015	-0.021	-0.046	-0.047	-0.078	-0.083
ART	0.070	0.074	0.046	-0.009	-0.021	0.004	-0.023	-0.092	0.052	-0.018
BAT	0.027	0.005	0.096	-0.032	0.068	0.041	-0.042	0.037	0.018	-0.033
BCX	-0.095	-0.073	0.060	-0.063	0.002	0.026	-0.058	-0.011	0.048	-0.034
BEL	-0.212	0.017	0.003	-0.014	-0.039	-0.035	-0.014	-0.015	0.011	0.001
CDZ	-0.043	0.002	-0.115	-0.019	-0.035	0.052	0.022	0.021	-0.030	-0.023
CLH	0.058	-0.077	-0.038	-0.022	-0.023	-0.015	0.028	-0.028	-0.003	0.007
CRM	0.134	0.049	-0.009	-0.074	0.043	0.022	0.015	0.006	-0.016	0.007
CSB	0.019	0.011	0.009	-0.070	-0.047	-0.025	0.016	-0.046	0.017	0.002
DAW	-0.017	-0.019	0.020	0.005	-0.009	-0.012	0.021	-0.038	-0.018	-0.032
DRD	0.100	0.070	0.009	-0.037	-0.075	-0.057	-0.035	0.000	0.029	0.042
FBR	-0.046	0.000	-0.008	-0.018	0.086	-0.070	-0.031	0.042	-0.034	0.041
HDC	-0.048	-0.017	-0.011	0.039	-0.020	-0.018	0.079	0.046	-0.015	0.016
ILA	-0.029	0.062	0.041	-0.028	0.014	-0.017	0.008	-0.033	-0.022	0.015
IVT	-0.056	-0.029	0.036	0.004	-0.010	0.010	-0.044	0.004	-0.043	-0.023
KAP	-0.081	-0.025	-0.012	-0.003	-0.020	-0.023	-0.003	0.013	-0.017	0.071
KGM	-0.071	-0.151	-0.151	0.097	0.029	-0.024	0.022	0.050	-0.001	-0.061
MRF	0.072	-0.033	-0.021	0.051	0.047	0.023	0.073	0.046	-0.017	-0.089
MTA	-0.359	-0.091	0.037	0.126	-0.023	-0.121	0.085	0.048	-0.040	-0.011
OCE	0.046	-0.033	0.045	0.025	0.028	-0.030	0.034	-0.011	-0.019	0.019
OCT	-0.142	0.123	0.027	-0.004	-0.055	0.039	-0.077	-0.024	0.016	0.066
OMN	0.020	0.020	-0.029	-0.062	-0.040	0.041	0.026	0.037	0.065	-0.030
PAM	0.011	0.110	0.028	-0.005	0.051	-0.033	-0.003	0.009	-0.026	0.058
PMM	-0.142	-0.082	0.111	-0.037	-0.098	0.097	0.005	-0.119	0.083	0.003
SIM	0.065	-0.008	0.010	-0.061	0.077	-0.067	-0.069	-0.114	0.044	-0.007
SYC	-0.176	-0.021	-0.032	-0.008	0.089	0.008	0.015	-0.031	-0.044	0.101

B2. Significant Partial Autocorrelation Coefficients for the Large Cap Sample over the Out-of-Sample Period

Partial autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ACL	0.033	-0.024	-0.026	-0.009	-0.053	-0.029	0.075	0.016	0.022	-0.152
AGL	0.031	0.009	-0.178	-0.010	-0.062	-0.050	0.066	-0.005	-0.026	-0.051
AMS	0.094	-0.084	-0.031	-0.007	-0.073	0.026	-0.016	0.010	0.030	-0.023
ANG	0.030	-0.061	0.011	0.036	-0.087	-0.037	0.018	-0.141	0.007	-0.049
ARI	0.081	-0.110	-0.099	0.082	0.070	-0.007	0.016	-0.064	0.030	-0.041
ASA	0.058	-0.123	-0.080	-0.092	-0.028	-0.071	-0.091	0.074	0.062	-0.117
BAW	0.061	-0.018	-0.079	-0.048	0.010	-0.021	-0.028	0.000	0.015	-0.005
BIL	0.009	-0.007	-0.113	-0.035	-0.037	-0.057	0.033	-0.064	-0.013	-0.020
BVT	-0.110	-0.076	-0.046	-0.043	0.067	-0.050	-0.001	0.005	0.033	0.018
CFR	-0.066	0.035	-0.022	-0.049	0.011	0.040	0.004	-0.046	-0.030	0.010
FSR	0.044	-0.109	-0.082	-0.116	0.004	-0.004	-0.027	0.090	-0.030	-0.020
GFI	0.035	-0.086	-0.017	0.011	-0.032	-0.043	-0.055	-0.019	-0.027	-0.050
HAR	0.103	-0.006	0.057	-0.039	0.005	-0.001	-0.048	0.020	-0.037	0.017
IMP	0.092	-0.112	-0.103	-0.089	-0.076	0.012	0.053	0.034	0.011	-0.060
INL	0.012	-0.099	-0.064	-0.081	-0.005	0.028	-0.039	0.125	0.040	-0.087
IPL	0.014	-0.033	0.038	-0.034	-0.028	0.010	0.050	0.009	0.016	-0.037
LBH	-0.164	-0.048	-0.073	-0.032	-0.036	-0.069	0.003	0.042	0.041	0.019
LON	0.035	0.037	-0.056	0.033	0.007	-0.045	0.036	0.008	0.014	-0.070
MTN	-0.005	-0.162	-0.019	-0.093	-0.015	-0.056	-0.050	0.033	0.001	0.029
MUR	0.065	-0.073	0.015	-0.043	0.007	-0.015	-0.029	0.017	0.016	0.079
NED	0.004	-0.097	-0.050	-0.042	-0.030	-0.032	-0.067	0.057	0.040	-0.028
NPN	-0.022	-0.126	-0.033	-0.048	-0.058	0.016	0.004	0.085	-0.007	-0.053
NTC	0.006	-0.034	-0.023	-0.012	-0.041	-0.010	-0.020	-0.044	0.015	0.011
OML	-0.053	-0.048	-0.077	-0.049	-0.043	0.081	0.025	0.070	0.042	-0.094
PPC	-0.054	-0.150	-0.061	-0.057	0.119	-0.019	-0.042	0.033	0.075	-0.016
RMH	-0.067	-0.078	-0.097	-0.086	-0.006	-0.040	-0.018	0.066	-0.005	-0.037
SAB	-0.031	-0.056	-0.011	-0.040	-0.092	-0.005	0.031	0.004	-0.040	0.011
SAP	0.100	-0.029	-0.017	-0.031	-0.083	-0.016	0.032	0.000	-0.030	0.079
SBK	-0.003	-0.122	-0.025	-0.092	-0.009	-0.090	-0.007	0.081	0.005	-0.072
SHF	0.055	-0.136	-0.069	-0.105	-0.007	0.025	0.049	0.062	0.032	-0.041
SLM	-0.093	-0.168	-0.051	0.015	0.054	-0.012	-0.024	0.033	0.028	-0.042
SOL	-0.003	-0.005	-0.060	-0.063	-0.071	-0.093	0.045	-0.037	-0.027	-0.005
TBS	0.016	-0.018	-0.066	-0.063	0.000	-0.080	0.000	0.043	0.013	-0.033

B2. Significant Partial Autocorrelation Coefficients for the Mid Cap Sample over the Out-of-Sample Period

Partial autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ABL	-0.026	-0.098	-0.047	0.030	-0.055	-0.095	-0.048	0.048	-0.005	-0.013
AEG	0.113	-0.083	-0.098	-0.062	0.021	-0.065	-0.011	0.052	0.057	0.017
AFE	-0.060	0.031	-0.031	-0.014	0.012	-0.055	-0.006	0.040	0.104	-0.027
AFX	-0.034	-0.028	0.050	0.030	0.030	-0.005	-0.005	-0.033	-0.013	-0.076
ALT	-0.062	0.022	0.029	-0.006	-0.005	-0.004	-0.014	-0.064	-0.037	0.034
APN	0.082	-0.133	-0.039	-0.118	-0.010	-0.040	0.036	0.002	0.016	0.012
ATN	-0.057	-0.004	-0.096	-0.088	0.029	0.016	0.021	-0.007	-0.048	0.002
AVI	0.082	-0.067	-0.087	-0.036	-0.075	-0.016	0.002	0.020	-0.052	0.036
CAT	-0.281	-0.126	-0.023	-0.022	-0.013	-0.044	0.017	0.077	-0.025	0.093
CLS	-0.084	-0.037	-0.046	-0.029	-0.035	-0.020	-0.020	-0.012	0.085	-0.066
DDT	0.015	-0.060	0.020	-0.042	-0.025	-0.078	0.097	-0.012	0.014	-0.050
DSY	0.027	-0.035	-0.019	-0.059	-0.001	-0.041	-0.024	-0.001	0.014	-0.063
DTC	0.108	0.033	0.043	-0.006	0.034	0.027	0.024	-0.038	-0.019	-0.057
FOS	0.051	0.014	-0.029	0.007	0.053	-0.021	-0.094	0.031	0.034	0.024
FPT	0.037	-0.057	-0.025	0.028	-0.021	0.039	0.033	0.025	-0.017	-0.016
GDF	-0.041	-0.027	-0.048	0.042	-0.015	0.009	0.038	-0.045	0.023	0.014
GND	0.070	-0.010	-0.118	-0.007	0.061	-0.017	0.037	0.007	0.023	-0.057
GRF	0.104	-0.030	-0.022	0.011	0.059	-0.035	0.028	0.025	0.063	-0.012
GRT	0.026	0.012	-0.021	-0.012	-0.021	-0.100	0.065	0.015	0.016	-0.046
HVL	-0.018	0.024	0.056	0.007	0.071	-0.067	0.044	0.037	0.074	0.023
HYP	-0.023	-0.014	0.039	0.041	-0.006	-0.049	0.074	-0.044	0.017	-0.023
ILV	-0.009	-0.005	-0.024	0.008	0.020	-0.062	0.053	-0.005	-0.018	0.002
JDG	0.075	-0.092	-0.074	0.036	0.058	-0.005	0.012	-0.018	0.085	0.031
MDC	-0.109	-0.022	-0.068	-0.016	0.027	0.011	0.004	-0.032	0.012	0.018
MET	-0.143	-0.048	-0.003	-0.045	-0.042	-0.003	0.056	0.056	0.049	-0.042
MPC	0.075	-0.073	-0.038	-0.017	-0.042	-0.020	-0.005	-0.033	0.024	0.078
MTX	0.102	0.084	0.022	-0.039	0.025	-0.016	0.032	-0.045	-0.005	-0.041
MVG	0.052	0.057	0.012	0.032	0.007	-0.028	0.023	-0.024	0.030	0.015
NHM	0.102	-0.027	-0.100	-0.033	0.014	0.001	0.098	-0.044	-0.015	-0.050
NPK	-0.137	-0.083	-0.082	-0.108	-0.027	0.026	-0.044	0.023	0.109	0.006
PAP	0.126	-0.007	-0.032	-0.013	0.026	-0.037	0.043	0.059	0.014	-0.030
PGR	0.020	-0.005	-0.025	-0.002	0.037	-0.018	0.063	0.065	0.017	-0.002
PIK	-0.115	-0.129	0.004	-0.036	0.004	-0.023	-0.043	-0.042	0.065	-0.017
RBW	-0.088	0.050	-0.059	0.032	0.023	-0.004	0.027	0.018	0.003	0.026
RLO	-0.021	-0.022	-0.068	-0.129	-0.031	0.051	-0.034	0.023	0.064	-0.008
SAC	-0.035	-0.034	-0.009	-0.007	0.003	-0.059	0.066	0.057	-0.022	-0.069
SHP	-0.076	-0.093	-0.043	-0.074	0.06	-0.05	0.016	0.058	-0.047	0.023
SNT	-0.127	-0.053	0.04	0.014	0.019	-0.021	-0.023	0.023	0.018	-0.029
SPG	-0.008	0.033	-0.011	0.018	-0.007	-0.017	-0.002	0.113	0.059	0.003
SUI	0.124	-0.04	-0.054	-0.055	0.019	-0.02	0.066	0.013	0.017	-0.044
TON	-0.04	0.059	0.033	0.03	0.017	-0.016	0.014	0.034	-0.018	0.053
TRE	-0.155	-0.128	-0.063	0.028	0.054	-0.036	0.006	0.014	-0.089	0.054
TRU	-0.037	-0.022	-0.074	0.005	0.013	-0.104	-0.016	-0.006	0.046	-0.09

B2. Significant Partial Autocorrelation Coefficients for the Small Cap Sample over the Out-of-Sample Period

Partial autocorrelation coefficients (AC) for the daily share sample are presented below. The share return series adjusted using the approach of Mlambo et al. (2003) is employed for the analysis as it is the only sample which has been corrected for spurious autocorrelation arising from thin trading. Results are presented up to lag 10 and coefficients which are significant at the 5% level are highlighted in bold.

	1	2	3	4	5	6	7	8	9	10
ADH	-0.130	0.036	-0.064	0.037	-0.047	0.013	0.073	-0.020	0.012	0.078
ADR	-0.163	-0.019	-0.001	0.010	0.008	0.050	0.107	0.030	-0.058	0.008
AFR	-0.072	0.043	-0.014	-0.012	-0.015	-0.022	-0.049	-0.053	-0.083	-0.094
ART	0.070	0.069	0.037	-0.019	-0.025	0.007	-0.019	-0.089	0.067	-0.012
BAT	0.027	0.004	0.095	-0.037	0.070	0.028	-0.038	0.026	0.015	-0.029
BCX	-0.095	-0.083	0.046	-0.060	-0.001	0.014	-0.049	-0.022	0.036	-0.022
BEL	-0.212	-0.029	0.000	-0.014	-0.047	-0.056	-0.035	-0.027	0.000	-0.001
CDZ	-0.043	0.000	-0.115	-0.029	-0.038	0.036	0.021	0.015	-0.020	-0.020
CLH	0.058	-0.080	-0.029	-0.024	-0.025	-0.017	0.025	-0.036	0.003	0.003
CRM	0.134	0.031	-0.020	-0.073	0.065	0.015	0.004	-0.003	-0.009	0.011
CSB	0.019	0.011	0.008	-0.071	-0.045	-0.022	0.020	-0.050	0.012	-0.003
DAW	-0.017	-0.019	0.019	0.006	-0.008	-0.013	0.020	-0.037	-0.017	-0.035
DRD	0.100	0.060	-0.003	-0.042	-0.069	-0.040	-0.016	0.011	0.027	0.029
FBR	-0.046	-0.002	-0.008	-0.019	0.085	-0.063	-0.037	0.041	-0.030	0.029
HDC	-0.048	-0.020	-0.013	0.037	-0.017	-0.019	0.078	0.052	-0.006	0.019
ILA	-0.029	0.061	0.045	-0.029	0.007	-0.015	0.008	-0.032	-0.023	0.016
IVT	-0.056	-0.032	0.032	0.007	-0.007	0.008	-0.044	0.000	-0.046	-0.026
KAP	-0.081	-0.031	-0.017	-0.006	-0.021	-0.027	-0.008	0.010	-0.017	0.069
KGM	-0.071	-0.157	-0.180	0.044	-0.009	-0.028	0.047	0.052	0.012	-0.032
MRF	0.072	-0.038	-0.016	0.052	0.039	0.020	0.076	0.037	-0.022	-0.086
MTA	-0.359	-0.253	-0.116	0.092	0.092	-0.064	0.008	0.044	0.016	0.022
OCE	0.046	-0.035	0.049	0.019	0.029	-0.034	0.038	-0.020	-0.013	0.017
OCT	-0.142	0.105	0.059	-0.006	-0.069	0.024	-0.055	-0.045	0.019	0.087
OMN	0.020	0.019	-0.030	-0.061	-0.037	0.044	0.023	0.029	0.061	-0.029
PAM	0.011	0.110	0.026	-0.018	0.046	-0.032	-0.012	0.014	-0.022	0.054
PMM	-0.142	-0.104	0.086	-0.017	-0.092	0.058	0.018	-0.092	0.039	-0.001
SIM	0.065	-0.013	0.011	-0.063	0.086	-0.081	-0.055	-0.116	0.074	-0.035
SYC	-0.176	-0.053	-0.047	-0.024	0.084	0.039	0.031	-0.015	-0.050	0.079

B3. In-Sample Results of Nonlinearity Tests for Large-Cap Sample (Adjustment of Atchison et al, 1987)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE $p = 5$	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ACL	Top 40	0.000	0.000	0.000	0.000	0.000
AGL	Top 40	0.000	0.000	0.000	0.221	0.000
AMS	Top 40	0.000	0.000	0.000	0.004	0.000
ANG	Top 40	0.000	0.000	0.000	0.135	0.000
ARI	Top 40	0.000	0.000	0.000	0.258	0.000
ASA	Top 40	0.000	0.000	0.000	0.077	0.000
BAW	Top 40	0.000	0.000	0.000	0.000	0.000
BIL	Top 40	0.000	0.000	0.000	0.879	0.000
BVT	Top 40	0.000	0.000	0.000	0.000	0.000
CFR	Top 40	0.000	0.000	0.000	0.005	0.000
FSR	Top 40	0.000	0.000	0.000	0.131	0.000
GFI	Top 40	0.000	0.000	0.000	0.000	0.000
HAR	Top 40	0.000	0.000	0.000	0.000	0.000
IMP	Top 40	0.000	0.000	0.000	0.000	0.000
INL	Top 40	0.000	0.000	0.000	0.000	0.000
IPL	Top 40	0.000	0.000	0.000	0.033	0.000
LBH	Top 40	0.000	0.000	0.000	0.014	0.000
LON	Top 40	0.133	0.101	0.000	0.104	0.012
MTN	Top 40	0.000	0.000	0.000	0.000	0.000
MUR	Top 40	0.000	0.000	0.000	0.010	0.000
NED	Top 40	0.000	0.000	0.000	0.163	0.000
NPN	Top 40	0.000	0.000	0.000	0.000	0.000
NTC	Top 40	0.000	0.000	0.000	0.050	0.000
OML	Top 40	0.000	0.000	0.000	0.043	0.000
PPC	Top 40	0.000	0.000	0.000	0.047	0.000
RMH	Top 40	0.000	0.000	0.000	0.107	0.000
SAB	Top 40	0.000	0.000	0.000	0.000	0.000
SAP	Top 40	0.000	0.000	0.000	0.115	0.000
SBK	Top 40	0.000	0.000	0.000	0.085	0.000
SHF	Top 40	0.001	0.002	0.000	0.000	0.001
SLM	Top 40	0.000	0.000	0.000	0.001	0.000
SOL	Top 40	0.000	0.000	0.001	0.034	0.000
TBS	Top 40	0.000	0.000	0.000	0.537	0.000
WHL	Top 40	0.000	0.000	0.000	0.178	0.000

B3. In-Sample Results of Nonlinearity Tests for Mid-Cap Sample (Adjustment of Atchison et al, 1987)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ABL	Mid Cap	0.000	0.000	0.000	0.002	0.000
AEG	Mid Cap	0.000	0.000	0.000	0.337	0.487
AFE	Mid Cap	0.000	0.000	0.000	0.025	0.000
AFX	Mid Cap	0.000	0.000	0.000	0.130	0.001
ALT	Mid Cap	0.001	0.000	0.000	0.339	0.679
APN	Mid Cap	0.000	0.000	0.000	0.000	0.000
ATN	Mid Cap	0.000	0.000	0.000	0.000	0.000
AVI	Mid Cap	0.000	0.000	0.000	0.035	0.000
CAT	Mid Cap	0.000	0.000	0.000	0.000	0.000
CLS	Mid Cap	0.000	0.000	0.000	0.031	0.000
DDT	Mid Cap	0.000	0.000	0.000	0.195	0.000
DSY	Mid Cap	0.000	0.000	0.000	0.000	0.000
DTC	Mid Cap	0.000	0.000	0.000	0.000	0.000
FOS	Mid Cap	0.000	0.002	0.000	0.010	0.000
FPT	Mid Cap	0.000	0.000	0.000	0.555	0.000
GDF	Mid Cap	0.000	0.000	0.000	0.000	0.000
GND	Mid Cap	0.000	0.000	0.000	0.098	0.000
GRF	Mid Cap	0.000	0.000	0.000	0.000	0.000
GRT	Mid Cap	0.000	0.000	0.000	0.015	0.000
HVL	Mid Cap	0.000	0.000	0.000	0.003	0.000
HYP	Mid Cap	0.000	0.000	0.000	0.000	0.000
ILV	Mid Cap	0.000	0.000	0.000	0.172	0.000
JDG	Mid Cap	0.000	0.000	0.000	0.006	0.000
MDC	Mid Cap	0.001	0.000	0.000	0.443	0.000
MET	Mid Cap	0.000	0.002	0.000	0.004	0.000
MPC	Mid Cap	0.060	0.071	0.000	0.011	0.002
MTX	Mid Cap	0.012	0.002	0.000	0.004	0.000
MVG	Mid Cap	0.000	0.000	0.000	0.254	0.000
NHM	Mid Cap	0.000	0.000	0.000	0.436	0.000
NPK	Mid Cap	0.000	0.000	0.000	0.093	0.000
PAP	Mid Cap	0.362	0.103	0.000	0.108	0.150
PGR	Mid Cap	0.000	0.000	0.000	0.045	0.000
PIK	Mid Cap	0.000	0.000	0.000	0.583	0.000
RBW	Mid Cap	0.024	0.028	0.000	0.253	0.001
RLO	Mid Cap	0.001	0.001	0.000	0.018	0.000
SAC	Mid Cap	0.000	0.000	0.000	0.009	0.000
SHP	Mid Cap	0.000	0.000	0.000	0.038	0.000
SNT	Mid Cap	0.002	0.003	0.000	0.001	0.049
SPG	Mid Cap	0.000	0.000	0.000	0.005	0.000
SUI	Mid Cap	0.007	0.000	0.000	0.316	0.000
TON	Mid Cap	0.000	0.000	0.000	0.001	0.000
TRE	Mid Cap	0.000	0.000	0.000	0.000	0.000
TRU	Mid Cap	0.000	0.000	0.000	0.001	0.000
WBO	Mid Cap	0.000	0.000	0.000	0.000	0.000

B3. In-Sample Results of Nonlinearity Tests Small-Cap Sample (Adjustment of Atchison et al, 1987)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ADH	Small Cap	0.000	0.000	0.000	0.002	0.000
ADR	Small Cap	0.748	0.075	0.000	0.082	0.000
AFR	Small Cap	1.000	0.513	0.000	0.023	0.000
ART	Small Cap	0.000	0.000	0.000	0.032	0.000
BAT	Small Cap	0.001	0.000	0.000	0.153	0.000
BCX	Small Cap	0.000	0.560	0.000	0.025	0.000
BEL	Small Cap	0.000	0.000	0.000	0.001	0.000
CDZ	Small Cap	0.023	0.000	0.000	0.013	0.000
CLH	Small Cap	0.000	0.000	0.000	0.000	0.000
CRM	Small Cap	0.000	0.000	0.000	0.597	0.000
CSB	Small Cap	0.000	0.000	0.000	0.001	0.000
DAW	Small Cap	0.000	0.000	0.000	0.001	0.000
DRD	Small Cap	0.000	0.000	0.000	0.000	0.000
FBR	Small Cap	0.000	0.000	0.000	0.001	0.000
HDC	Small Cap	0.003	0.000	0.000	0.000	0.080
ILA	Small Cap	0.000	0.000	0.000	0.023	0.000
IVT	Small Cap	0.000	0.000	0.000	0.000	0.000
KAP	Small Cap	0.069	0.035	0.000	0.005	0.002
KGM	Small Cap	0.384	0.077	0.000	0.024	0.054
MRF	Small Cap	0.000	0.000	0.000	0.000	0.000
MTA	Small Cap	0.103	0.047	0.000	0.154	0.002
OCE	Small Cap	0.004	0.000	0.000	0.001	0.000
OCT	Small Cap	0.000	0.000	0.000	0.021	0.000
OMN	Small Cap	0.606	1.000	0.000	0.757	0.003
PAM	Small Cap	0.000	0.002	0.000	0.046	0.000
PMM	Small Cap	0.020	0.012	0.015	0.003	0.004
SIM	Small Cap	0.007	0.000	0.000	0.000	0.005
SYC	Small Cap	0.000	0.000	0.000	0.013	0.000

B3. In-Sample Results of Nonlinearity Tests Large-Cap Sample (Adjustment of Mlambo et al, 2003)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ACL	Top 40	0.018	0.004	0.000	0.004	0.004
AGL	Top 40	0.000	0.000	0.000	0.221	0.000
AMS	Top 40	0.000	0.000	0.000	0.003	0.000
ANG	Top 40	0.000	0.000	0.000	0.146	0.000
ARI	Top 40	0.001	0.001	0.000	0.168	0.000
ASA	Top 40	0.005	0.004	0.000	0.091	0.000
BAW	Top 40	0.000	0.000	0.000	0.000	0.000
BIL	Top 40	0.000	0.000	0.000	0.088	0.000
BVT	Top 40	0.000	0.000	0.000	0.000	0.000
CFR	Top 40	0.000	0.000	0.000	0.007	0.000
FSR	Top 40	0.000	0.000	0.000	0.133	0.000
GFI	Top 40	0.000	0.000	0.000	0.000	0.000
HAR	Top 40	0.000	0.000	0.000	0.000	0.000
IMP	Top 40	0.000	0.000	0.000	0.001	0.000
INL	Top 40	0.000	0.000	0.000	0.000	0.000
IPL	Top 40	0.000	0.000	0.000	0.036	0.000
LBH	Top 40	0.001	0.000	0.000	0.000	0.000
LON	Top 40	0.135	0.052	0.000	0.175	0.018
MTN	Top 40	0.000	0.000	0.000	0.000	0.000
MUR	Top 40	0.000	0.000	0.000	0.014	0.000
NED	Top 40	0.000	0.000	0.000	0.160	0.000
NPN	Top 40	0.000	0.000	0.000	0.000	0.000
NTC	Top 40	0.000	0.000	0.000	0.064	0.000
OML	Top 40	0.000	0.000	0.000	0.051	0.000
PPC	Top 40	0.000	0.000	0.000	0.025	0.000
RMH	Top 40	0.000	0.000	0.000	0.107	0.000
SAB	Top 40	0.000	0.000	0.000	0.000	0.000
SAP	Top 40	0.000	0.000	0.000	0.115	0.000
SBK	Top 40	0.000	0.000	0.000	0.085	0.000
SHF	Top 40	0.010	0.004	0.000	0.001	0.000
SLM	Top 40	0.000	0.000	0.000	0.003	0.000
SOL	Top 40	0.000	0.000	0.001	0.034	0.000
TBS	Top 40	0.000	0.000	0.000	0.524	0.000
WHL	Top 40	0.000	0.000	0.000	0.154	0.000

B3. In-Sample Results of Nonlinearity Tests Mid-Cap Sample (Adjustment of Mlambo et al, 2003)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE $p = 5$	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ABL	Mid Cap	0.000	0.000	0.000	0.002	0.000
AEG	Mid Cap	0.000	0.000	0.000	0.334	0.371
AFE	Mid Cap	0.001	0.000	0.000	0.023	0.000
AFX	Mid Cap	0.000	0.000	0.000	0.014	0.003
ALT	Mid Cap	0.005	0.002	0.000	0.528	0.350
APN	Mid Cap	0.000	0.000	0.000	0.000	0.000
ATN	Mid Cap	0.000	0.001	0.000	0.020	0.000
AVI	Mid Cap	0.000	0.000	0.000	0.033	0.001
CAT	Mid Cap	0.007	0.000	0.000	0.001	0.021
CLS	Mid Cap	0.000	0.000	0.000	0.036	0.000
DDT	Mid Cap	0.000	0.000	0.000	0.199	0.000
DSY	Mid Cap	0.000	0.000	0.000	0.000	0.000
DTC	Mid Cap	0.000	0.000	0.000	0.000	0.000
FOS	Mid Cap	0.018	0.015	0.000	0.008	0.001
FPT	Mid Cap	0.000	0.000	0.000	0.497	0.000
GDF	Mid Cap	0.000	0.000	0.000	0.000	0.000
GND	Mid Cap	0.003	0.000	0.000	0.153	0.001
GRF	Mid Cap	0.037	0.010	0.000	0.000	0.002
GRT	Mid Cap	0.000	0.008	0.000	0.087	0.000
HVL	Mid Cap	0.000	0.001	0.000	0.027	0.000
HYP	Mid Cap	0.000	0.000	0.000	0.004	0.002
ILV	Mid Cap	0.000	0.000	0.000	0.180	0.000
JDG	Mid Cap	0.001	0.000	0.000	0.031	0.000
MDC	Mid Cap	0.000	0.002	0.000	0.226	0.000
MET	Mid Cap	0.001	0.002	0.000	0.006	0.001
MPC	Mid Cap	0.038	0.065	0.000	0.008	0.003
MTX	Mid Cap	0.018	0.004	0.000	0.002	0.001
MVG	Mid Cap	0.000	0.000	0.000	0.260	0.000
NHM	Mid Cap	0.000	0.000	0.000	0.435	0.000
NPK	Mid Cap	0.000	0.000	0.000	0.090	0.000
PAP	Mid Cap	0.152	0.083	0.000	0.207	0.174
PGR	Mid Cap	0.000	0.000	0.000	0.014	0.000
PIK	Mid Cap	0.000	0.000	0.000	0.583	0.000
RBW	Mid Cap	0.019	0.041	0.000	0.277	0.010
RLO	Mid Cap	0.000	0.000	0.000	0.020	0.000
SAC	Mid Cap	0.000	0.000	0.000	0.001	0.000
SHP	Mid Cap	0.000	0.000	0.000	0.038	0.000
SNT	Mid Cap	0.000	0.001	0.000	0.002	0.020
SPG	Mid Cap	0.000	0.000	0.000	0.012	0.000
SUI	Mid Cap	0.006	0.001	0.000	0.408	0.002
TON	Mid Cap	0.000	0.000	0.000	0.000	0.000
TRE	Mid Cap	0.000	0.000	0.000	0.002	0.001
TRU	Mid Cap	0.000	0.000	0.000	0.000	0.000
WBO	Mid Cap	0.000	0.000	0.000	0.000	0.000

B3. In-Sample Results of Nonlinearity Tests Small-Cap Sample (Adjustment of Mlambo et al, 2003)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ADH	Small Cap	0.000	0.000	0.000	0.003	0.000
ADR	Small Cap	0.192	0.034	0.000	0.143	0.009
AFR	Small Cap	0.396	0.181	0.000	0.032	0.008
ART	Small Cap	0.000	0.000	0.000	0.007	0.000
BAT	Small Cap	0.035	0.005	0.000	0.020	0.012
BCX	Small Cap	0.006	0.001	0.000	0.003	0.001
BEL	Small Cap	0.000	0.000	0.000	0.000	0.000
CDZ	Small Cap	0.000	0.000	0.000	0.005	0.000
CLH	Small Cap	0.000	0.000	0.000	0.000	0.000
CRM	Small Cap	0.017	0.005	0.000	0.068	0.003
CSB	Small Cap	0.000	0.003	0.000	0.000	0.000
DAW	Small Cap	0.000	0.000	0.000	0.002	0.000
DRD	Small Cap	0.000	0.000	0.000	0.001	0.000
FBR	Small Cap	0.038	0.030	0.000	0.013	0.000
HDC	Small Cap	0.152	0.009	0.000	0.000	0.097
ILA	Small Cap	0.001	0.002	0.000	0.016	0.000
IVT	Small Cap	0.001	0.000	0.000	0.031	0.000
KAP	Small Cap	0.000	0.001	0.000	0.034	0.000
KGM	Small Cap	0.019	0.039	0.000	0.002	0.003
MRF	Small Cap	0.000	0.000	0.000	0.000	0.000
MTA	Small Cap	0.304	0.057	0.000	0.436	0.045
OCE	Small Cap	0.007	0.000	0.000	0.757	0.044
OCT	Small Cap	0.001	0.002	0.000	0.346	0.005
OMN	Small Cap	0.032	0.036	0.000	0.610	0.001
PAM	Small Cap	0.005	0.013	0.000	0.212	0.004
PMM	Small Cap	0.022	0.013	0.000	0.002	0.005
SIM	Small Cap	0.463	0.310	0.000	0.020	0.039
SYC	Small Cap	0.000	0.000	0.000	0.411	0.000

B3. Out-of-Sample Results of Nonlinearity Tests Large-Cap Sample (Adjustment of Atchison et al, 1987)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ACL	Top 40	0.000	0.000	0.000	0.001	0.000
AGL	Top 40	0.000	0.000	0.000	0.197	0.000
AMS	Top 40	0.000	0.000	0.000	0.000	0.000
ANG	Top 40	0.000	0.000	0.000	0.000	0.000
ARI	Top 40	0.000	0.000	0.000	0.053	0.000
ASA	Top 40	0.000	0.000	0.000	0.067	0.000
BAW	Top 40	0.000	0.000	0.000	0.000	0.000
BIL	Top 40	0.000	0.000	0.000	0.008	0.000
BVT	Top 40	0.000	0.001	0.000	0.020	0.000
CFR	Top 40	0.398	0.293	0.000	0.012	0.014
FSR	Top 40	0.000	0.000	0.000	0.086	0.001
GFI	Top 40	0.000	0.000	0.000	0.001	0.000
HAR	Top 40	0.000	0.008	0.000	0.001	0.000
IMP	Top 40	0.000	0.008	0.000	0.017	0.000
INL	Top 40	0.000	0.008	0.000	0.001	0.000
IPL	Top 40	0.000	0.008	0.000	0.074	0.000
LBH	Top 40	0.130	0.030	0.000	0.011	0.056
LON	Top 40	0.533	0.564	0.000	0.769	0.038
MTN	Top 40	0.000	0.000	0.000	0.001	0.000
MUR	Top 40	0.000	0.000	0.000	0.088	0.000
NED	Top 40	0.000	0.001	0.000	0.328	0.000
NPN	Top 40	0.000	0.001	0.000	0.103	0.006
NTC	Top 40	0.000	0.000	0.000	0.024	0.000
OML	Top 40	0.000	0.000	0.000	0.001	0.000
PPC	Top 40	0.000	0.000	0.000	0.001	0.000
RMH	Top 40	0.000	0.000	0.000	0.063	0.000
SAB	Top 40	0.000	0.000	0.000	0.001	0.000
SAP	Top 40	0.000	0.000	0.000	0.001	0.000
SBK	Top 40	0.000	0.000	0.000	0.001	0.000
SHF	Top 40	0.000	0.000	0.000	0.001	0.000
SLM	Top 40	0.000	0.002	0.000	0.449	0.000
SOL	Top 40	0.000	0.002	0.000	0.004	0.000
TBS	Top 40	0.000	0.000	0.000	0.001	0.000
WHL	Top 40	0.000	0.000	0.000	0.002	0.000

B3. Out-of-Sample Results of Nonlinearity Tests Mid-Cap Sample (Adjustment of Atchison et al, 1987)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ABL	Mid Cap	0.000	0.000	0.000	0.008	0.000
AEG	Mid Cap	0.000	0.000	0.000	0.297	0.000
AFE	Mid Cap	0.210	0.225	0.000	0.216	0.057
AFX	Mid Cap	0.008	0.000	0.000	0.154	0.056
ALT	Mid Cap	0.009	0.006	0.003	0.003	0.000
APN	Mid Cap	0.000	0.000	0.000	0.288	0.000
ATN	Mid Cap	0.000	0.018	0.000	0.000	0.000
AVI	Mid Cap	0.460	0.267	0.000	0.211	0.009
CAT	Mid Cap	0.013	0.076	0.000	0.000	0.032
CLS	Mid Cap	0.001	0.000	0.000	0.010	0.004
DDT	Mid Cap	0.000	0.000	0.000	0.056	0.000
DSY	Mid Cap	0.000	0.001	0.000	0.413	0.002
DTC	Mid Cap	0.000	0.000	0.001	0.000	0.000
FOS	Mid Cap	0.000	0.000	0.000	0.004	0.000
FPT	Mid Cap	0.000	0.000	0.000	0.004	0.000
GDF	Mid Cap	0.072	0.117	0.000	0.382	0.007
GND	Mid Cap	0.000	0.000	0.000	0.004	0.000
GRF	Mid Cap	0.000	0.000	0.000	0.046	0.020
GRT	Mid Cap	0.000	0.000	0.000	0.004	0.000
HVL	Mid Cap	0.000	0.011	0.000	0.032	0.000
HYP	Mid Cap	0.000	0.000	0.000	0.359	0.000
ILV	Mid Cap	0.000	0.007	0.002	0.583	0.018
JDG	Mid Cap	0.000	0.000	0.000	0.094	0.000
MDC	Mid Cap	0.142	0.010	0.000	0.529	0.002
MET	Mid Cap	0.000	0.001	0.000	0.066	0.007
MPC	Mid Cap	0.000	0.000	0.000	0.010	0.000
MTX	Mid Cap	0.007	0.020	0.000	0.055	0.000
MVG	Mid Cap	0.001	0.003	0.000	0.000	0.001
NHM	Mid Cap	0.005	0.019	0.000	0.004	0.000
NPK	Mid Cap	0.000	0.000	0.000	0.000	0.000
PAP	Mid Cap	0.724	0.223	0.000	0.170	0.014
PGR	Mid Cap	0.008	0.005	0.000	0.002	0.000
PIK	Mid Cap	0.000	0.009	0.000	0.364	0.000
RBW	Mid Cap	0.992	0.854	0.000	0.069	0.046
RLO	Mid Cap	0.000	0.000	0.000	0.010	0.000
SAC	Mid Cap	0.009	0.003	0.000	0.000	0.003
SHP	Mid Cap	0.000	0.007	0.000	0.358	0.000
SNT	Mid Cap	0.011	0.014	0.000	0.039	0.071
SPG	Mid Cap	0.000	0.000	0.000	0.000	0.000
SUI	Mid Cap	0.002	0.000	0.000	0.244	0.000
TON	Mid Cap	0.566	0.118	0.007	0.192	0.371
TRE	Mid Cap	0.041	0.015	0.000	0.008	0.001
TRU	Mid Cap	0.000	0.000	0.000	0.001	0.000
WBO	Mid Cap	0.000	0.075	0.001	0.557	0.000

B3. Out-of-Sample Results of Nonlinearity Tests Small-Cap Sample (Adjustment of Atchison et al, 1987)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ADH	Small Cap	0.055	0.015	0.000	0.019	0.071
ADR	Small Cap	0.063	0.008	0.002	0.007	0.061
AFR	Small Cap	0.001	0.000	0.000	0.165	0.001
ART	Small Cap	0.000	0.000	0.000	0.054	0.001
BAT	Small Cap	0.020	0.002	0.000	0.136	0.002
BCX	Small Cap	0.000	0.000	0.000	0.001	0.000
BEL	Small Cap	0.085	0.016	0.000	0.000	0.000
CDZ	Small Cap	0.007	0.005	0.000	0.007	0.001
CLH	Small Cap	0.000	0.000	0.000	0.115	0.011
CRM	Small Cap	0.505	0.117	0.016	0.039	0.063
CSB	Small Cap	0.017	0.010	0.002	0.004	0.023
DAW	Small Cap	0.203	0.012	0.004	0.828	0.657
DRD	Small Cap	0.000	0.001	0.006	0.335	0.000
FBR	Small Cap	0.004	0.014	0.000	0.135	0.000
HDC	Small Cap	0.000	0.009	0.002	0.038	0.000
ILA	Small Cap	0.950	0.423	0.000	0.622	0.149
IVT	Small Cap	0.461	0.136	0.000	0.183	0.432
KAP	Small Cap	0.015	0.000	0.000	0.003	0.003
KGM	Small Cap	0.431	0.273	0.000	0.016	0.007
MRF	Small Cap	0.000	0.000	0.000	0.000	0.000
MTA	Small Cap	0.000	0.000	0.000	0.000	0.002
OCE	Small Cap	0.703	0.110	0.004	0.012	0.045
OCT	Small Cap	0.000	0.000	0.000	0.001	0.000
OMN	Small Cap	0.000	0.000	0.000	0.003	0.000
PAM	Small Cap	0.000	0.000	0.000	0.000	0.000
PMM	Small Cap	0.006	0.005	0.179	0.027	0.000
SIM	Small Cap	0.000	0.000	0.000	0.002	0.000
SYC	Small Cap	0.000	0.003	0.000	0.003	0.006

B3. Out-of-Sample Results of Nonlinearity Tests Large-Cap Sample (Adjustment of Mlambo et al, 2003)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ACL	Top 40	0.000	0.000	0.000	0.001	0.000
AGL	Top 40	0.000	0.000	0.000	0.197	0.000
AMS	Top 40	0.000	0.000	0.000	0.000	0.000
ANG	Top 40	0.000	0.000	0.000	0.000	0.000
ARI	Top 40	0.000	0.000	0.000	0.038	0.000
ASA	Top 40	0.000	0.000	0.000	0.067	0.000
BAW	Top 40	0.000	0.000	0.000	0.000	0.000
BIL	Top 40	0.000	0.000	0.000	0.008	0.000
BVT	Top 40	0.000	0.001	0.000	0.020	0.000
CFR	Top 40	0.398	0.293	0.000	0.012	0.014
FSR	Top 40	0.000	0.000	0.000	0.086	0.001
GFI	Top 40	0.000	0.000	0.000	0.001	0.000
HAR	Top 40	0.000	0.008	0.000	0.001	0.000
IMP	Top 40	0.000	0.008	0.000	0.017	0.000
INL	Top 40	0.000	0.008	0.000	0.001	0.000
IPL	Top 40	0.000	0.008	0.000	0.074	0.000
LBH	Top 40	0.130	0.023	0.000	0.013	0.054
LON	Top 40	0.533	0.564	0.000	0.769	0.038
MTN	Top 40	0.000	0.000	0.000	0.001	0.000
MUR	Top 40	0.000	0.000	0.000	0.088	0.000
NED	Top 40	0.000	0.001	0.000	0.328	0.000
NPN	Top 40	0.000	0.001	0.000	0.103	0.006
NTC	Top 40	0.000	0.000	0.000	0.024	0.000
OML	Top 40	0.000	0.000	0.000	0.001	0.000
PPC	Top 40	0.000	0.000	0.000	0.001	0.000
RMH	Top 40	0.000	0.000	0.000	0.063	0.000
SAB	Top 40	0.000	0.000	0.000	0.001	0.000
SAP	Top 40	0.000	0.000	0.000	0.001	0.000
SBK	Top 40	0.000	0.000	0.000	0.001	0.000
SHF	Top 40	0.000	0.000	0.000	0.001	0.000
SLM	Top 40	0.000	0.002	0.000	0.449	0.000
SOL	Top 40	0.000	0.002	0.000	0.004	0.000
TBS	Top 40	0.000	0.000	0.000	0.001	0.000
WHL	Top 40	0.000	0.000	0.000	0.002	0.000

B3. Out-of-Sample Results of Nonlinearity Tests Mid-Cap Sample (Adjustment of Mlambo et al, 2003)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ABL	Mid Cap	0.000	0.000	0.000	0.008	0.000
AEG	Mid Cap	0.000	0.000	0.000	0.297	0.000
AFE	Mid Cap	0.210	0.225	0.000	0.216	0.057
AFX	Mid Cap	0.008	0.000	0.000	0.154	0.056
ALT	Mid Cap	0.009	0.006	0.003	0.003	0.000
APN	Mid Cap	0.000	0.000	0.000	0.288	0.000
ATN	Mid Cap	0.000	0.018	0.000	0.000	0.000
AVI	Mid Cap	0.460	0.267	0.000	0.211	0.009
CAT	Mid Cap	0.008	0.061	0.000	0.001	0.028
CLS	Mid Cap	0.001	0.000	0.000	0.010	0.004
DDT	Mid Cap	0.000	0.000	0.000	0.056	0.000
DSY	Mid Cap	0.000	0.001	0.000	0.413	0.002
DTC	Mid Cap	0.000	0.000	0.001	0.000	0.000
FOS	Mid Cap	0.000	0.000	0.000	0.004	0.000
FPT	Mid Cap	0.002	0.000	0.000	0.003	0.000
GDF	Mid Cap	0.072	0.117	0.000	0.382	0.007
GND	Mid Cap	0.000	0.000	0.000	0.004	0.000
GRF	Mid Cap	0.000	0.000	0.000	0.046	0.020
GRT	Mid Cap	0.000	0.000	0.000	0.004	0.000
HVL	Mid Cap	0.000	0.011	0.000	0.032	0.000
HYP	Mid Cap	0.000	0.000	0.000	0.359	0.000
ILV	Mid Cap	0.001	0.004	0.050	0.587	0.013
JDG	Mid Cap	0.000	0.000	0.000	0.094	0.000
MDC	Mid Cap	0.142	0.010	0.000	0.529	0.002
MET	Mid Cap	0.000	0.001	0.000	0.066	0.007
MPC	Mid Cap	0.000	0.000	0.000	0.010	0.000
MTX	Mid Cap	0.007	0.020	0.000	0.055	0.000
MVG	Mid Cap	0.001	0.003	0.000	0.000	0.001
NHM	Mid Cap	0.004	0.019	0.000	0.003	0.000
NPK	Mid Cap	0.000	0.000	0.000	0.000	0.000
PAP	Mid Cap	0.717	0.221	0.000	0.305	0.021
PGR	Mid Cap	0.008	0.005	0.000	0.002	0.000
PIK	Mid Cap	0.000	0.009	0.000	0.364	0.000
RBW	Mid Cap	0.992	0.854	0.000	0.069	0.046
RLO	Mid Cap	0.000	0.000	0.000	0.010	0.000
SAC	Mid Cap	0.009	0.003	0.000	0.000	0.003
SHP	Mid Cap	0.000	0.007	0.000	0.358	0.000
SNT	Mid Cap	0.011	0.014	0.000	0.039	0.071
SPG	Mid Cap	0.000	0.000	0.000	0.000	0.000
SUI	Mid Cap	0.002	0.000	0.000	0.244	0.000
TON	Mid Cap	0.566	0.118	0.007	0.192	0.371
TRE	Mid Cap	0.047	0.012	0.000	0.006	0.001
TRU	Mid Cap	0.000	0.000	0.000	0.000	0.000
WBO	Mid Cap	0.001	0.080	0.002	0.531	0.000

B3. Out-of-Sample Results of Nonlinearity Tests Small-Cap Sample (Adjustment of Mlambo et al, 2003)

All tests are evaluated at a 5% significance level, obtained via 1000 bootstrap replications. Significant results are highlighted in bold. For the sake of brevity, the BDS test is reported only for $m = 2$ and $\epsilon = 2$ but these results are available on request.

TICKER	INDEX	MCLEOD-LI L = 24	ENGLE p = 5	BDS m = 2	TSAY k = 5	BICOVARIANCE l = 8
ADH	Small Cap	0.055	0.015	0.000	0.019	0.071
ADR	Small Cap	0.063	0.008	0.002	0.007	0.061
AFR	Small Cap	0.001	0.001	0.003	0.130	0.003
ART	Small Cap	0.000	0.000	0.000	0.058	0.000
BAT	Small Cap	0.016	0.001	0.016	0.001	0.000
BCX	Small Cap	0.000	0.000	0.000	0.001	0.000
BEL	Small Cap	0.080	0.022	0.001	0.000	0.003
CDZ	Small Cap	0.004	0.005	0.000	0.004	0.000
CLH	Small Cap	0.000	0.000	0.000	0.098	0.014
CRM	Small Cap	0.505	0.117	0.016	0.039	0.063
CSB	Small Cap	0.024	0.013	0.001	0.004	0.024
DAW	Small Cap	0.203	0.012	0.004	0.828	0.657
DRD	Small Cap	0.000	0.001	0.006	0.335	0.000
FBR	Small Cap	0.011	0.017	0.000	0.143	0.001
HDC	Small Cap	0.000	0.009	0.002	0.038	0.000
ILA	Small Cap	0.941	0.444	0.000	0.518	0.121
IVT	Small Cap	0.461	0.136	0.000	0.183	0.432
KAP	Small Cap	0.018	0.004	0.001	0.005	0.008
KGM	Small Cap	0.443	0.295	0.002	0.020	0.006
MRF	Small Cap	0.000	0.000	0.000	0.000	0.000
MTA	Small Cap	0.000	0.000	0.000	0.000	0.000
OCE	Small Cap	0.760	0.110	0.017	0.004	0.050
OCT	Small Cap	0.000	0.000	0.000	0.000	0.000
OMN	Small Cap	0.000	0.000	0.000	0.003	0.000
PAM	Small Cap	0.000	0.000	0.000	0.000	0.000
PMM	Small Cap	0.006	0.005	0.179	0.027	0.000
SIM	Small Cap	0.000	0.000	0.000	0.002	0.000
SYC	Small Cap	0.000	0.003	0.000	0.003	0.006

B4. In-Sample Results of Episodic Nonlinearity Tests Large-Cap Sample (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ACL	Top 40	8	1	14.8%	1.9%	0.416	0.326
AGL	Top 40	2	4	3.7%	7.4%	0.409	0.426
AMS	Top 40	4	7	7.4%	13.0%	0.442	0.400
ANG	Top 40	7	5	13.0%	9.3%	0.440	0.400
ARI	Top 40	3	5	5.6%	9.3%	0.413	0.390
ASA	Top 40	3	5	5.6%	9.3%	0.325	0.424
BAW	Top 40	5	4	9.3%	7.4%	0.430	0.351
BIL	Top 40	4	5	7.4%	9.3%	0.393	0.366
BVT	Top 40	5	4	9.3%	7.4%	0.387	0.408
CFR	Top 40	7	4	13.0%	7.4%	0.380	0.405
FSR	Top 40	7	4	13.0%	7.4%	0.420	0.393
GFI	Top 40	3	2	5.6%	3.7%	0.403	0.358
HAR	Top 40	4	7	7.4%	13.0%	0.399	0.420
IMP	Top 40	4	5	7.4%	9.3%	0.381	0.433
INL	Top 40	5	6	9.3%	11.1%	0.389	0.395
IPL	Top 40	8	10	14.8%	18.5%	0.455	0.427
LBH	Top 40	4	2	7.4%	3.7%	0.335	0.346
LON	Top 40	6	6	11.1%	11.1%	0.393	0.328
MTN	Top 40	5	3	9.3%	5.6%	0.374	0.403
MUR	Top 40	7	5	13.0%	9.3%	0.379	0.441
NED	Top 40	3	1	5.6%	1.9%	0.435	0.335
NPN	Top 40	7	5	13.0%	9.3%	0.433	0.415
NTC	Top 40	5	3	9.3%	5.6%	0.409	0.415
OML	Top 40	4	5	7.4%	9.3%	0.323	0.383
PPC	Top 40	2	9	3.7%	16.7%	0.392	0.423
RMH	Top 40	7	6	13.0%	11.1%	0.386	0.370
SAB	Top 40	3	3	5.6%	5.6%	0.382	0.373
SAP	Top 40	5	7	9.3%	13.0%	0.385	0.428
SBK	Top 40	7	2	13.0%	3.7%	0.423	0.346
SHF	Top 40	8	7	14.8%	13.0%	0.430	0.393
SLM	Top 40	7	5	13.0%	9.3%	0.451	0.460
SOL	Top 40	2	4	3.7%	7.4%	0.338	0.346
TBS	Top 40	7	3	13.0%	5.6%	0.330	0.318
WHL	Top 40	2	4	3.7%	7.4%	0.395	0.392

B4. In-Sample Results of Episodic Nonlinearity Tests Mid-Cap Sample (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	9	9	16.7%	16.7%	0.436	0.407
AEG	Mid Cap	4	5	7.4%	9.3%	0.369	0.385
AFE	Mid Cap	6	3	11.1%	5.6%	0.368	0.285
AFX	Mid Cap	3	5	5.6%	9.3%	0.372	0.433
ALT	Mid Cap	5	6	9.3%	11.1%	0.380	0.393
APN	Mid Cap	9	4	16.7%	7.4%	0.415	0.341
ATN	Mid Cap	9	5	16.7%	9.3%	0.409	0.370
AVI	Mid Cap	3	10	5.6%	18.5%	0.426	0.439
CAT	Mid Cap	9	6	16.7%	11.1%	0.405	0.365
CLS	Mid Cap	4	4	7.4%	7.4%	0.440	0.387
DDT	Mid Cap	3	8	5.6%	14.8%	0.381	0.365
DSY	Mid Cap	4	2	7.4%	3.7%	0.343	0.329
DTC	Mid Cap	12	8	11.1%	9.3%	0.483	0.419
FOS	Mid Cap	5	5	9.3%	9.3%	0.370	0.400
FPT	Mid Cap	9	6	16.7%	11.1%	0.494	0.428
GDF	Mid Cap	6	7	11.1%	13.0%	0.357	0.396
GND	Mid Cap	6	5	11.1%	9.3%	0.338	0.331
GRF	Mid Cap	12	9	22.2%	16.7%	0.471	0.399
GRT	Mid Cap	8	10	14.8%	18.5%	0.427	0.409
HVL	Mid Cap	8	4	14.8%	7.4%	0.329	0.330
HYP	Mid Cap	14	14	25.9%	25.9%	0.461	0.470
ILV	Mid Cap	2	5	3.7%	9.3%	0.372	0.442
JDG	Mid Cap	8	5	14.8%	9.3%	0.331	0.291
MDC	Mid Cap	4	8	7.4%	14.8%	0.424	0.428
MET	Mid Cap	6	7	11.1%	13.0%	0.393	0.383
MPC	Mid Cap	2	6	3.7%	11.1%	0.340	0.447
MTX	Mid Cap	8	4	14.8%	7.4%	0.437	0.337
MVG	Mid Cap	4	4	7.4%	7.4%	0.355	0.335
NHM	Mid Cap	2	8	3.7%	14.8%	0.319	0.397
NPK	Mid Cap	5	6	9.3%	11.1%	0.440	0.404
PAP	Mid Cap	9	3	16.7%	5.6%	0.378	0.308
PGR	Mid Cap	7	2	13.0%	3.7%	0.418	0.401
PIK	Mid Cap	7	4	13.0%	7.4%	0.427	0.395
RBW	Mid Cap	6	9	11.1%	16.7%	0.384	0.455
RLO	Mid Cap	5	5	9.3%	9.3%	0.389	0.385
SAC	Mid Cap	2	5	3.7%	9.3%	0.373	0.317
SHF	Mid Cap	3	4	5.6%	7.4%	0.367	0.349
SNT	Mid Cap	6	7	11.1%	13.0%	0.375	0.355
SPG	Mid Cap	11	7	20.4%	13.0%	0.445	0.481
SUI	Mid Cap	6	6	11.1%	11.1%	0.358	0.376
TON	Mid Cap	6	9	11.1%	16.7%	0.387	0.453
TRE	Mid Cap	9	13	16.7%	24.1%	0.404	0.450
TRU	Mid Cap	2	5	3.7%	9.3%	0.388	0.390
WBO	Mid Cap	13	10	24.1%	18.5%	0.519	0.469

B4. In-Sample Results of Episodic Nonlinearity Tests Small-Cap Sample (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ADH	Small Cap	6	7	11.1%	13.0%	0.369	0.363
ADR	Small Cap	10	11	18.5%	20.4%	0.453	0.390
AFR	Small Cap	5	6	9.3%	11.1%	0.391	0.349
ART	Small Cap	6	4	11.1%	7.4%	0.384	0.366
BAT	Small Cap	4	6	7.4%	11.1%	0.270	0.373
BCX	Small Cap	2	7	3.7%	13.0%	0.350	0.387
BEL	Small Cap	8	8	14.8%	14.8%	0.397	0.358
CDZ	Small Cap	6	9	11.1%	16.7%	0.365	0.370
CLH	Small Cap	11	5	20.4%	9.3%	0.426	0.340
CRM	Small Cap	7	6	13.0%	11.1%	0.443	0.375
CSB	Small Cap	8	6	14.8%	11.1%	0.448	0.397
DAW	Small Cap	7	11	13.0%	20.4%	0.391	0.420
DRD	Small Cap	6	7	11.1%	13.0%	0.390	0.385
FBR	Small Cap	7	10	13.0%	18.5%	0.409	0.400
HDC	Small Cap	5	7	9.3%	13.0%	0.408	0.353
ILA	Small Cap	5	5	9.3%	9.3%	0.359	0.392
IVT	Small Cap	12	14	22.2%	25.9%	0.420	0.431
KAP	Small Cap	6	9	11.1%	16.7%	0.319	0.329
KGM	Small Cap	14	7	25.9%	13.0%	0.493	0.368
MRF	Small Cap	8	4	14.8%	7.4%	0.361	0.274
MTA	Small Cap	9	6	16.7%	11.1%	0.330	0.301
OCE	Small Cap	8	9	14.8%	16.7%	0.437	0.396
OCT	Small Cap	11	8	20.4%	14.8%	0.471	0.352
OMN	Small Cap	10	8	18.5%	14.8%	0.423	0.399
PAM	Small Cap	8	9	14.8%	16.7%	0.440	0.411
PMM	Small Cap	12	10	22.2%	18.5%	0.394	0.352
SIM	Small Cap	11	16	20.4%	29.6%	0.423	0.432
SYC	Small Cap	9	4	16.7%	7.4%	0.442	0.309

B4. In-Sample Results of Episodic Nonlinearity Tests Large-Cap Sample (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ACL	Top 40	5	6	1	9.3%	11.1%	1.9%	0.512	0.399	0.366
AGL	Top 40	3	4	4	5.6%	7.4%	7.4%	0.499	0.411	0.420
AMS	Top 40	7	4	6	13.0%	7.4%	11.1%	0.513	0.443	0.408
ANG	Top 40	1	6	4	1.9%	11.1%	7.4%	0.446	0.440	0.400
ARI	Top 40	8	7	5	14.8%	13.0%	9.3%	0.504	0.379	0.408
ASA	Top 40	5	2	6	9.3%	3.7%	11.1%	0.479	0.346	0.440
BAW	Top 40	4	9	4	7.4%	16.7%	7.4%	0.492	0.454	0.358
BIL	Top 40	4	4	5	7.4%	7.4%	9.3%	0.534	0.393	0.366
BVT	Top 40	8	8	9	14.8%	14.8%	16.7%	0.536	0.392	0.415
CFR	Top 40	5	7	4	9.3%	13.0%	7.4%	0.451	0.377	0.405
FSR	Top 40	2	6	6	3.7%	11.1%	11.1%	0.463	0.416	0.394
GFI	Top 40	6	2	1	11.1%	3.7%	1.9%	0.486	0.380	0.344
HAR	Top 40	2	4	10	3.7%	7.4%	18.5%	0.453	0.403	0.455
IMP	Top 40	2	4	4	3.7%	7.4%	7.4%	0.508	0.381	0.433
INL	Top 40	7	5	7	13.0%	9.3%	13.0%	0.568	0.416	0.410
IPL	Top 40	1	7	6	1.9%	13.0%	11.1%	0.510	0.455	0.427
LBH	Top 40	2	5	3	3.7%	9.3%	5.6%	0.463	0.359	0.369
LON	Top 40	1	2	0	2.2%	4.3%	0.0%	0.455	0.286	0.310
MTN	Top 40	6	5	4	11.1%	9.3%	7.4%	0.519	0.360	0.384
MUR	Top 40	3	5	6	5.6%	9.3%	11.1%	0.457	0.407	0.459
NED	Top 40	5	3	1	9.3%	5.6%	1.9%	0.469	0.435	0.335
NPN	Top 40	7	8	4	13.0%	14.8%	7.4%	0.516	0.433	0.415
NTC	Top 40	4	5	3	7.4%	9.3%	5.6%	0.435	0.409	0.415
OML	Top 40	5	4	2	9.3%	7.4%	3.7%	0.446	0.323	0.383
PPC	Top 40	3	4	7	5.9%	7.8%	13.7%	0.524	0.398	0.426
RMH	Top 40	5	8	6	9.3%	14.8%	11.1%	0.478	0.377	0.367
SAB	Top 40	2	3	3	3.7%	5.6%	5.6%	0.530	0.382	0.373
SAP	Top 40	8	3	6	14.8%	5.6%	11.1%	0.532	0.385	0.428
SBK	Top 40	4	6	2	7.4%	11.1%	3.7%	0.479	0.423	0.348
SHF	Top 40	3	7	9	5.6%	13.0%	16.7%	0.479	0.450	0.376
SLM	Top 40	4	7	7	7.4%	13.0%	13.0%	0.464	0.451	0.460
SOL	Top 40	5	2	2	9.3%	3.7%	3.7%	0.474	0.338	0.346
TBS	Top 40	5	5	3	9.3%	9.3%	5.6%	0.453	0.330	0.318
WHL	Top 40	4	2	4	7.4%	3.7%	7.4%	0.514	0.400	0.382

B4. In-Sample Results of Episodic Nonlinearity Tests Mid-Cap Sample (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	6	10	8	11.1%	18.5%	14.8%	0.461	0.442	0.413
AEG	Mid Cap	2	1	4	3.7%	1.9%	7.4%	0.470	0.361	0.384
AFE	Mid Cap	6	5	3	11.1%	9.3%	5.6%	0.487	0.390	0.302
AFX	Mid Cap	5	3	4	9.3%	5.6%	7.4%	0.504	0.312	0.355
ALT	Mid Cap	3	6	4	5.7%	11.3%	7.5%	0.465	0.386	0.350
APN	Mid Cap	4	5	5	7.4%	9.3%	9.3%	0.536	0.366	0.375
ATN	Mid Cap	1	3	1	2.1%	6.4%	2.1%	0.446	0.331	0.222
AVI	Mid Cap	3	7	12	5.6%	13.0%	22.2%	0.439	0.449	0.438
CAT	Mid Cap	7	5	3	15.9%	11.4%	6.8%	0.540	0.330	0.322
CLS	Mid Cap	3	6	6	5.6%	11.1%	11.1%	0.562	0.458	0.430
DDT	Mid Cap	3	3	7	5.6%	5.6%	13.0%	0.471	0.381	0.365
DSY	Mid Cap	6	6	6	11.3%	11.3%	11.3%	0.469	0.350	0.359
DTC	Mid Cap	3	9	6	5.6%	16.7%	11.1%	0.485	0.458	0.414
FOS	Mid Cap	4	4	5	7.4%	7.4%	9.3%	0.492	0.434	0.411
FPT	Mid Cap	7	7	4	13.0%	13.0%	7.4%	0.532	0.432	0.382
GDF	Mid Cap	3	3	5	6.1%	6.1%	10.2%	0.476	0.365	0.345
GND	Mid Cap	5	6	1	12.8%	15.4%	2.6%	0.473	0.380	0.286
GRF	Mid Cap	4	9	5	7.8%	17.6%	9.8%	0.515	0.411	0.400
GRT	Mid Cap	4	6	4	9.1%	13.6%	9.1%	0.532	0.435	0.379
HVL	Mid Cap	8	6	9	16.0%	12.0%	18.0%	0.531	0.426	0.411
HYP	Mid Cap	5	5	5	11.9%	11.9%	11.9%	0.481	0.299	0.312
ILV	Mid Cap	2	2	5	3.7%	3.7%	9.3%	0.541	0.372	0.442
JDG	Mid Cap	7	8	5	13.0%	14.8%	9.3%	0.488	0.444	0.397
MDC	Mid Cap	3	5	3	5.7%	9.4%	5.7%	0.510	0.413	0.367
MET	Mid Cap	7	1	8	13.0%	1.9%	14.8%	0.574	0.385	0.395
MPC	Mid Cap	1	7	7	1.9%	13.2%	13.2%	0.466	0.393	0.386
MTX	Mid Cap	9	8	6	18.0%	16.0%	12.0%	0.501	0.381	0.362
MVG	Mid Cap	6	3	5	11.3%	5.7%	9.4%	0.446	0.322	0.350
NHM	Mid Cap	5	2	3	9.3%	3.7%	5.6%	0.520	0.335	0.348
NPK	Mid Cap	6	5	6	11.1%	9.3%	11.1%	0.619	0.440	0.404
PAP	Mid Cap	3	7	5	6.0%	14.0%	10.0%	0.503	0.308	0.349
PGR	Mid Cap	4	5	2	7.7%	9.6%	3.8%	0.484	0.450	0.382
PIK	Mid Cap	6	7	4	11.1%	13.0%	7.4%	0.525	0.427	0.395
RBW	Mid Cap	5	5	9	9.3%	9.3%	16.7%	0.454	0.407	0.473
RLO	Mid Cap	6	7	5	11.1%	13.0%	9.3%	0.503	0.361	0.407
SAC	Mid Cap	6	1	2	11.3%	1.9%	3.8%	0.527	0.303	0.277
SHX	Mid Cap	6	3	4	11.1%	5.6%	7.4%	0.526	0.367	0.349
SNT	Mid Cap	4	6	7	7.5%	11.3%	13.2%	0.443	0.384	0.354
SPG	Mid Cap	6	12	10	11.1%	22.2%	18.5%	0.476	0.478	0.466
SUI	Mid Cap	8	5	4	15.4%	9.6%	7.7%	0.441	0.375	0.350
TON	Mid Cap	3	4	7	5.6%	7.4%	13.0%	0.515	0.388	0.433
TRE	Mid Cap	11	10	9	23.4%	21.3%	19.1%	0.596	0.404	0.412
TRU	Mid Cap	2	4	8	3.7%	7.4%	14.8%	0.462	0.428	0.402
WBO	Mid Cap	1	7	5	2.0%	14.0%	10.0%	0.455	0.420	0.348

B4. In-Sample Results of Episodic Nonlinearity Tests Small-Cap Sample (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ADH	Small Cap	5	4	8	10.0%	8.0%	16.0%	0.532	0.360	0.365
ADR	Small Cap	5	9	5	10.2%	18.4%	10.2%	0.548	0.392	0.359
AFR	Small Cap	4	4	3	7.4%	7.4%	5.6%	0.553	0.396	0.372
ART	Small Cap	8	5	7	16.3%	10.2%	14.3%	0.556	0.307	0.348
BAT	Small Cap	1	3	5	1.9%	5.8%	9.6%	0.465	0.374	0.421
BCX	Small Cap	3	4	4	5.6%	7.4%	7.4%	0.436	0.349	0.376
BEL	Small Cap	1	4	5	2.2%	8.7%	10.9%	0.499	0.305	0.342
CDZ	Small Cap	4	2	2	7.8%	3.9%	3.9%	0.543	0.345	0.314
CLH	Small Cap	5	9	5	10.9%	19.6%	10.9%	0.492	0.372	0.331
CRM	Small Cap	6	1	5	14.0%	2.3%	11.6%	0.512	0.386	0.302
CSB	Small Cap	4	7	7	8.7%	15.2%	15.2%	0.458	0.346	0.346
DAW	Small Cap	10	5	6	20.8%	10.4%	12.5%	0.554	0.381	0.389
DRD	Small Cap	2	7	6	3.7%	13.0%	11.1%	0.453	0.385	0.396
FBR	Small Cap	8	9	10	18.2%	20.5%	22.7%	0.541	0.473	0.389
HDC	Small Cap	1	3	3	2.6%	7.7%	7.7%	0.454	0.369	0.336
ILA	Small Cap	5	3	4	10.6%	6.4%	8.5%	0.540	0.334	0.356
IVT	Small Cap	6	5	8	16.2%	13.5%	21.6%	0.557	0.380	0.393
KAP	Small Cap	3	2	1	7.9%	5.3%	2.6%	0.471	0.264	0.219
KGM	Small Cap	6	4	2	18.8%	12.5%	6.3%	0.621	0.395	0.377
MRF	Small Cap	4	3	4	7.4%	5.6%	7.4%	0.540	0.422	0.361
MTA	Small Cap	5	4	4	16.7%	13.3%	13.3%	0.479	0.340	0.303
OCE	Small Cap	3	6	7	6.7%	13.3%	15.6%	0.457	0.327	0.336
OCT	Small Cap	6	5	1	15.4%	12.8%	2.6%	0.558	0.393	0.268
OMN	Small Cap	5	3	4	11.1%	6.7%	8.9%	0.514	0.350	0.353
PAM	Small Cap	2	6	7	4.3%	13.0%	15.2%	0.497	0.424	0.375
PMM	Small Cap	3	3	1	8.8%	8.8%	2.9%	0.515	0.280	0.221
SIM	Small Cap	5	4	4	18.5%	14.8%	14.8%	0.530	0.352	0.328
SYC	Small Cap	7	6	4	13.7%	11.8%	7.8%	0.551	0.390	0.307

B4. Out-of-Sample Results of Episodic Nonlinearity Tests Large-Cap Sample (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ACL	Top 40	0	0	0.0%	0.0%	0.451	0.379
AGL	Top 40	1	0	5.9%	0.0%	0.378	0.349
AMS	Top 40	0	0	0.0%	0.0%	0.371	0.383
ANG	Top 40	1	1	5.9%	5.9%	0.419	0.467
ARI	Top 40	2	2	11.8%	11.8%	0.391	0.373
ASA	Top 40	1	1	5.9%	5.9%	0.493	0.292
BAW	Top 40	1	1	5.9%	5.9%	0.378	0.428
BIL	Top 40	1	0	5.9%	0.0%	0.329	0.360
BVT	Top 40	2	2	11.8%	11.8%	0.494	0.405
CFR	Top 40	0	0	0.0%	0.0%	0.262	0.364
FSR	Top 40	0	0	0.0%	0.0%	0.476	0.344
GFI	Top 40	2	2	11.8%	11.8%	0.378	0.531
HAR	Top 40	2	2	11.8%	11.8%	0.339	0.305
IMP	Top 40	2	2	11.8%	11.8%	0.359	0.462
INL	Top 40	2	2	11.8%	11.8%	0.442	0.385
IPL	Top 40	4	3	23.5%	17.6%	0.521	0.477
LBH	Top 40	1	1	5.9%	5.9%	0.477	0.267
LON	Top 40	2	2	11.8%	11.8%	0.333	0.361
MTN	Top 40	2	1	11.8%	5.9%	0.441	0.306
MUR	Top 40	1	1	5.9%	5.9%	0.504	0.389
NED	Top 40	2	2	11.8%	11.8%	0.502	0.536
NPN	Top 40	1	1	5.9%	5.9%	0.476	0.314
NTC	Top 40	4	4	23.5%	23.5%	0.493	0.488
OML	Top 40	3	2	17.6%	11.8%	0.489	0.397
PPC	Top 40	1	1	5.9%	5.9%	0.338	0.437
RMH	Top 40	1	1	5.9%	5.9%	0.449	0.358
SAB	Top 40	0	0	0.0%	0.0%	0.453	0.309
SAP	Top 40	1	1	5.9%	5.9%	0.320	0.381
SBK	Top 40	1	1	5.9%	5.9%	0.435	0.408
SHF	Top 40	2	2	11.8%	11.8%	0.489	0.478
SLM	Top 40	3	2	17.6%	11.8%	0.458	0.500
SOL	Top 40	4	4	23.5%	23.5%	0.401	0.551
TBS	Top 40	1	1	5.9%	5.9%	0.434	0.629
WHL	Top 40	0	0	0.0%	0.0%	0.384	0.277

B4. Out-of-Sample Results of Episodic Nonlinearity Tests Mid-Cap Sample (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	1	1	5.9%	5.9%	0.359	0.410
AEG	Mid Cap	1	1	5.9%	5.9%	0.352	0.317
AFE	Mid Cap	2	1	11.8%	5.9%	0.319	0.247
AFX	Mid Cap	3	3	17.6%	17.6%	0.401	0.521
ALT	Mid Cap	1	1	5.9%	5.9%	0.271	0.233
APN	Mid Cap	3	3	17.6%	17.6%	0.468	0.453
ATN	Mid Cap	2	2	11.8%	11.8%	0.334	0.389
AVI	Mid Cap	2	1	11.8%	5.9%	0.155	0.251
CAT	Mid Cap	1	1	5.9%	5.9%	0.354	0.303
CLS	Mid Cap	3	3	17.6%	17.6%	0.502	0.377
DDT	Mid Cap	2	2	11.8%	11.8%	0.412	0.406
DSY	Mid Cap	2	2	11.8%	11.8%	0.409	0.343
DTC	Mid Cap	1	0	5.9%	0.0%	0.354	0.337
FOS	Mid Cap	1	1	5.9%	5.9%	0.423	0.250
FPT	Mid Cap	4	4	23.5%	23.5%	0.310	0.381
GDF	Mid Cap	2	2	11.8%	11.8%	0.365	0.352
GND	Mid Cap	0	0	0.0%	0.0%	0.534	0.299
GRF	Mid Cap	0	0	0.0%	0.0%	0.420	0.380
GRT	Mid Cap	0	0	0.0%	0.0%	0.475	0.342
HVL	Mid Cap	2	2	11.8%	11.8%	0.487	0.432
HYP	Mid Cap	3	3	17.6%	17.6%	0.460	0.454
ILV	Mid Cap	1	1	5.9%	5.9%	0.353	0.360
JDG	Mid Cap	3	3	17.6%	17.6%	0.494	0.466
MDC	Mid Cap	1	1	5.9%	5.9%	0.337	0.386
MET	Mid Cap	1	1	5.9%	5.9%	0.394	0.447
MPC	Mid Cap	1	1	5.9%	5.9%	0.329	0.355
MTX	Mid Cap	1	1	5.9%	5.9%	0.516	0.474
MVG	Mid Cap	1	1	5.9%	5.9%	0.337	0.347
NHM	Mid Cap	1	1	5.9%	5.9%	0.338	0.329
NPK	Mid Cap	3	3	17.6%	17.6%	0.301	0.401
PAP	Mid Cap	2	2	11.8%	11.8%	0.448	0.365
PGR	Mid Cap	4	4	23.5%	23.5%	0.529	0.396
PIK	Mid Cap	1	1	5.9%	5.9%	0.466	0.409
RBW	Mid Cap	5	5	29.4%	29.4%	0.310	0.411
RLO	Mid Cap	2	2	11.8%	11.8%	0.531	0.472
SAC	Mid Cap	3	3	17.6%	17.6%	0.514	0.392
SHP	Mid Cap	2	2	11.8%	11.8%	0.401	0.471
SNT	Mid Cap	3	2	17.6%	11.8%	0.398	0.363
SPG	Mid Cap	2	2	11.8%	11.8%	0.326	0.357
SUI	Mid Cap	2	2	11.8%	11.8%	0.598	0.408
TON	Mid Cap	0	0	0.0%	0.0%	0.359	0.319
TRE	Mid Cap	3	2	17.6%	11.8%	0.366	0.361
TRU	Mid Cap	2	2	11.8%	11.8%	0.376	0.412
WBO	Mid Cap	0	0	0.0%	0.0%	0.296	0.311

B4. Out-of-Sample Results of Episodic Nonlinearity Tests Small-Cap Sample (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ADH	Small Cap	3	3	17.6%	17.6%	0.434	0.442
ADR	Small Cap	3	3	17.6%	17.6%	0.314	0.377
AFR	Small Cap	0	0	0.0%	0.0%	0.379	0.248
ART	Small Cap	2	2	11.8%	11.8%	0.484	0.350
BAT	Small Cap	4	4	23.5%	23.5%	0.610	0.424
BCX	Small Cap	2	2	11.8%	11.8%	0.501	0.396
BEL	Small Cap	0	0	0.0%	0.0%	0.252	0.336
CDZ	Small Cap	2	2	11.8%	11.8%	0.549	0.380
CLH	Small Cap	1	1	5.9%	5.9%	0.460	0.259
CRM	Small Cap	3	3	17.6%	17.6%	0.197	0.296
CSB	Small Cap	1	1	5.9%	5.9%	0.481	0.296
DAW	Small Cap	0	0	0.0%	0.0%	0.361	0.292
DRD	Small Cap	1	1	5.9%	5.9%	0.255	0.329
FBR	Small Cap	6	5	35.3%	29.4%	0.287	0.434
HDC	Small Cap	1	1	5.9%	5.9%	0.258	0.198
ILA	Small Cap	3	2	17.6%	11.8%	0.376	0.381
IVT	Small Cap	3	3	17.6%	17.6%	0.427	0.357
KAP	Small Cap	1	1	5.9%	5.9%	0.195	0.266
KGM	Small Cap	1	1	5.9%	5.9%	0.208	0.248
MRF	Small Cap	2	2	11.8%	11.8%	0.290	0.366
MTA	Small Cap	7	6	41.2%	35.3%	0.415	0.460
OCE	Small Cap	4	3	23.5%	17.6%	0.412	0.354
OCT	Small Cap	3	3	17.6%	17.6%	0.370	0.316
OMN	Small Cap	1	1	5.9%	5.9%	0.445	0.361
PAM	Small Cap	0	0	0.0%	0.0%	0.397	0.385
PMM	Small Cap	0	0	0.0%	0.0%	0.187	0.220
SIM	Small Cap	1	1	5.9%	5.9%	0.657	0.323
SYC	Small Cap	5	5	29.4%	29.4%	0.417	0.416

B4. Out-of-Sample Results of Episodic Nonlinearity Tests Large-Cap Sample (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ACL	Top 40	1	1	1	5.9%	5.9%	5.9%	0.360	0.431	0.388
AGL	Top 40	1	1	0	5.9%	5.9%	0.0%	0.557	0.404	0.338
AMS	Top 40	2	1	1	11.8%	5.9%	5.9%	0.558	0.324	0.395
ANG	Top 40	0	4	1	0.0%	23.5%	5.9%	0.386	0.419	0.467
ARI	Top 40	2	1	1	11.8%	5.9%	5.9%	0.479	0.369	0.380
ASA	Top 40	1	1	1	5.9%	5.9%	5.9%	0.531	0.493	0.292
BAW	Top 40	3	2	1	17.6%	11.8%	5.9%	0.536	0.378	0.428
BIL	Top 40	0	1	0	0.0%	5.9%	0.0%	0.342	0.329	0.360
BVT	Top 40	0	3	2	0.0%	17.6%	11.8%	0.469	0.494	0.405
CFR	Top 40	2	0	0	11.8%	0.0%	0.0%	0.478	0.262	0.364
FSR	Top 40	2	3	0	11.8%	17.6%	0.0%	0.476	0.476	0.344
GFI	Top 40	0	1	2	0.0%	5.9%	11.8%	0.423	0.378	0.531
HAR	Top 40	1	3	2	5.9%	17.6%	11.8%	0.347	0.339	0.305
IMP	Top 40	0	2	2	0.0%	11.8%	11.8%	0.501	0.359	0.462
INL	Top 40	1	3	2	5.9%	17.6%	11.8%	0.542	0.442	0.385
IPL	Top 40	0	4	3	0.0%	23.5%	17.6%	0.422	0.521	0.477
LBH	Top 40	4	4	1	23.5%	23.5%	5.9%	0.501	0.466	0.276
LON	Top 40	2	1	2	11.8%	5.9%	11.8%	0.414	0.333	0.361
MTN	Top 40	1	1	1	5.9%	5.9%	5.9%	0.554	0.441	0.306
MUR	Top 40	1	3	1	5.9%	17.6%	5.9%	0.519	0.504	0.389
NED	Top 40	1	3	2	5.9%	17.6%	11.8%	0.524	0.502	0.536
NPN	Top 40	3	1	1	17.6%	5.9%	5.9%	0.518	0.476	0.314
NTC	Top 40	0	1	4	0.0%	5.9%	23.5%	0.439	0.493	0.488
OML	Top 40	1	2	2	5.9%	11.8%	11.8%	0.508	0.489	0.397
PPC	Top 40	1	0	0	5.9%	0.0%	0.0%	0.590	0.299	0.403
RMH	Top 40	2	1	1	11.8%	5.9%	5.9%	0.485	0.449	0.358
SAB	Top 40	2	1	0	11.8%	5.9%	0.0%	0.591	0.453	0.309
SAP	Top 40	0	2	1	0.0%	11.8%	5.9%	0.438	0.320	0.381
SBK	Top 40	0	0	1	0.0%	0.0%	5.9%	0.512	0.435	0.408
SHF	Top 40	4	4	2	23.5%	23.5%	11.8%	0.609	0.489	0.478
SLM	Top 40	3	1	2	17.6%	5.9%	11.8%	0.563	0.458	0.500
SOL	Top 40	1	4	4	5.9%	23.5%	23.5%	0.434	0.401	0.551
TBS	Top 40	1	1	1	5.9%	5.9%	5.9%	0.529	0.434	0.629
WHL	Top 40	0	1	0	0.0%	5.9%	0.0%	0.431	0.384	0.277

B4. Out-of-Sample Results of Episodic Nonlinearity Tests Mid-Cap Sample (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.452	0.359	0.410
AEG	Mid Cap	0	1	1	0.0%	5.9%	5.9%	0.612	0.352	0.317
AFE	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.412	0.319	0.247
AFX	Mid Cap	0	1	3	0.0%	5.9%	17.6%	0.427	0.349	0.404
ALT	Mid Cap	2	0	1	11.8%	0.0%	5.9%	0.436	0.271	0.233
APN	Mid Cap	3	2	3	17.6%	11.8%	17.6%	0.551	0.468	0.453
ATN	Mid Cap	0	1	2	0.0%	5.9%	11.8%	0.557	0.334	0.389
AVI	Mid Cap	1	0	1	5.9%	0.0%	5.9%	0.552	0.155	0.251
CAT	Mid Cap	4	2	1	23.5%	11.8%	5.9%	0.693	0.382	0.347
CLS	Mid Cap	0	1	3	0.0%	5.9%	17.6%	0.436	0.502	0.377
DDT	Mid Cap	1	2	2	5.9%	11.8%	11.8%	0.490	0.412	0.406
DSY	Mid Cap	1	3	2	5.9%	17.6%	11.8%	0.430	0.409	0.343
DTC	Mid Cap	1	0	0	5.9%	0.0%	0.0%	0.388	0.354	0.337
FOS	Mid Cap	0	3	1	0.0%	17.6%	5.9%	0.429	0.423	0.250
FPT	Mid Cap	0	3	2	0.0%	17.6%	11.8%	0.456	0.243	0.286
GDF	Mid Cap	5	2	2	29.4%	11.8%	11.8%	0.603	0.365	0.352
GND	Mid Cap	1	3	0	5.9%	17.6%	0.0%	0.484	0.534	0.299
GRF	Mid Cap	1	0	0	5.9%	0.0%	0.0%	0.589	0.449	0.529
GRT	Mid Cap	1	4	0	5.9%	23.5%	0.0%	0.422	0.475	0.342
HVL	Mid Cap	1	5	2	5.9%	29.4%	11.8%	0.512	0.487	0.432
HYP	Mid Cap	2	3	3	11.8%	17.6%	17.6%	0.536	0.460	0.454
ILV	Mid Cap	1	1	1	5.9%	5.9%	5.9%	0.475	0.353	0.360
JDG	Mid Cap	1	1	3	5.9%	5.9%	17.6%	0.462	0.494	0.466
MDC	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.445	0.337	0.386
MET	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.555	0.394	0.447
MPC	Mid Cap	1	0	1	5.9%	0.0%	5.9%	0.497	0.329	0.355
MTX	Mid Cap	2	6	1	11.8%	35.3%	5.9%	0.502	0.516	0.474
MVG	Mid Cap	1	1	1	5.9%	5.9%	5.9%	0.467	0.337	0.347
NHM	Mid Cap	2	0	3	11.8%	0.0%	17.6%	0.499	0.386	0.485
NPK	Mid Cap	1	1	3	5.9%	5.9%	17.6%	0.524	0.301	0.401
PAP	Mid Cap	1	3	2	5.9%	17.6%	11.8%	0.582	0.496	0.339
PGR	Mid Cap	0	6	4	0.0%	35.3%	23.5%	0.437	0.529	0.396
PIK	Mid Cap	2	2	1	11.8%	11.8%	5.9%	0.541	0.466	0.409
RBW	Mid Cap	0	1	5	0.0%	5.9%	29.4%	0.583	0.310	0.411
RLO	Mid Cap	4	2	2	23.5%	11.8%	11.8%	0.527	0.531	0.472
SAC	Mid Cap	3	4	3	17.6%	23.5%	17.6%	0.515	0.514	0.392
SHF	Mid Cap	2	0	2	11.8%	0.0%	11.8%	0.621	0.401	0.471
SNT	Mid Cap	2	2	2	11.8%	11.8%	11.8%	0.604	0.398	0.363
SPG	Mid Cap	4	2	3	23.5%	11.8%	17.6%	0.642	0.331	0.373
SUI	Mid Cap	0	6	2	0.0%	35.3%	11.8%	0.448	0.598	0.408
TON	Mid Cap	1	1	0	5.9%	5.9%	0.0%	0.556	0.359	0.319
TRE	Mid Cap	3	3	2	17.6%	17.6%	11.8%	0.516	0.393	0.251
TRU	Mid Cap	1	2	2	5.9%	11.8%	11.8%	0.567	0.373	0.422
WBO	Mid Cap	0	0	0	0.0%	0.0%	0.0%	0.473	0.296	0.311

B4. Out-of-Sample Results of Episodic Nonlinearity Tests Small-Cap Sample (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ADH	Small Cap	5	1	3	29.4%	5.9%	17.6%	0.607	0.434	0.442
ADR	Small Cap	0	1	3	0.0%	5.9%	17.6%	0.505	0.314	0.377
AFR	Small Cap	3	1	0	17.6%	5.9%	0.0%	0.514	0.396	0.254
ART	Small Cap	1	3	2	5.9%	17.6%	11.8%	0.516	0.464	0.368
BAT	Small Cap	1	1	3	5.9%	5.9%	17.6%	0.436	0.453	0.382
BCX	Small Cap	4	3	2	23.5%	17.6%	11.8%	0.580	0.501	0.396
BEL	Small Cap	2	2	1	11.8%	11.8%	5.9%	0.531	0.407	0.370
CDZ	Small Cap	3	2	1	17.6%	11.8%	5.9%	0.559	0.375	0.368
CLH	Small Cap	2	2	0	11.8%	11.8%	0.0%	0.536	0.472	0.248
CRM	Small Cap	2	1	3	11.8%	5.9%	17.6%	0.477	0.197	0.296
CSB	Small Cap	1	3	1	5.9%	17.6%	5.9%	0.435	0.448	0.280
DAW	Small Cap	2	2	0	11.8%	11.8%	0.0%	0.431	0.361	0.292
DRD	Small Cap	0	1	1	0.0%	5.9%	5.9%	0.375	0.255	0.329
FBR	Small Cap	0	1	4	0.0%	5.9%	23.5%	0.422	0.358	0.387
HDC	Small Cap	1	1	1	5.9%	5.9%	5.9%	0.475	0.258	0.198
ILA	Small Cap	1	2	4	5.9%	11.8%	23.5%	0.506	0.389	0.448
IVT	Small Cap	1	0	3	5.9%	0.0%	17.6%	0.425	0.427	0.357
KAP	Small Cap	2	0	1	11.8%	0.0%	5.9%	0.516	0.170	0.261
KGM	Small Cap	3	4	0	17.6%	23.5%	0.0%	0.492	0.286	0.216
MRF	Small Cap	2	2	1	11.8%	11.8%	5.9%	0.481	0.284	0.353
MTA	Small Cap	4	4	5	23.5%	23.5%	29.4%	0.573	0.414	0.461
OCE	Small Cap	3	2	3	17.6%	11.8%	17.6%	0.422	0.403	0.307
OCT	Small Cap	3	5	5	17.6%	29.4%	29.4%	0.552	0.432	0.431
OMN	Small Cap	2	2	1	11.8%	11.8%	5.9%	0.508	0.445	0.361
PAM	Small Cap	1	2	0	5.9%	11.8%	0.0%	0.474	0.397	0.385
PMM	Small Cap	4	1	0	23.5%	5.9%	0.0%	0.493	0.187	0.220
SIM	Small Cap	0	6	1	0.0%	35.3%	5.9%	0.527	0.657	0.323
SYC	Small Cap	2	2	5	11.8%	11.8%	29.4%	0.514	0.417	0.416

B5. In-Sample Results of Episodic Nonlinearity Tests for Large-Cap Sample after 1% Trimming (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ACL	Top 40	8	2	14.8%	3.7%	0.429	0.346
AGL	Top 40	3	3	5.6%	5.6%	0.417	0.417
AMS	Top 40	4	4	7.4%	7.4%	0.446	0.402
ANG	Top 40	7	4	13.0%	7.4%	0.438	0.383
ARI	Top 40	5	3	9.3%	5.6%	0.426	0.400
ASA	Top 40	3	5	5.6%	9.3%	0.354	0.450
BAW	Top 40	5	4	9.3%	7.4%	0.437	0.369
BIL	Top 40	4	5	7.4%	9.3%	0.387	0.424
BVT	Top 40	5	3	9.3%	5.6%	0.402	0.416
CFR	Top 40	6	4	11.1%	7.4%	0.397	0.414
FSR	Top 40	7	4	13.0%	7.4%	0.433	0.410
GFI	Top 40	2	2	3.7%	3.7%	0.399	0.376
HAR	Top 40	3	6	5.6%	11.1%	0.402	0.432
IMP	Top 40	4	4	7.4%	7.4%	0.383	0.451
INL	Top 40	5	5	9.3%	9.3%	0.412	0.402
IPL	Top 40	6	6	11.1%	11.1%	0.458	0.444
LBH	Top 40	3	3	5.6%	5.6%	0.363	0.346
LON	Top 40	5	6	9.3%	11.1%	0.428	0.353
MTN	Top 40	4	5	7.4%	9.3%	0.386	0.408
MUR	Top 40	6	6	11.1%	11.1%	0.410	0.452
NED	Top 40	4	2	7.4%	3.7%	0.428	0.336
NPN	Top 40	7	7	13.0%	13.0%	0.440	0.422
NTC	Top 40	5	3	9.3%	5.6%	0.418	0.422
OML	Top 40	5	4	9.3%	7.4%	0.334	0.388
PPC	Top 40	2	8	3.7%	14.8%	0.406	0.437
RMH	Top 40	8	6	14.8%	11.1%	0.396	0.384
SAB	Top 40	3	2	5.6%	3.7%	0.374	0.389
SAP	Top 40	3	7	5.6%	13.0%	0.416	0.439
SBK	Top 40	6	2	11.1%	3.7%	0.426	0.360
SHF	Top 40	8	5	14.8%	9.3%	0.431	0.385
SLM	Top 40	6	6	11.1%	11.1%	0.454	0.455
SOL	Top 40	1	4	1.9%	7.4%	0.365	0.366
TBS	Top 40	9	4	16.7%	7.4%	0.365	0.343
WHL	Top 40	2	4	3.7%	7.4%	0.401	0.395

B5. In-Sample Results of Episodic Nonlinearity Tests for Mid-Cap Sample after 1% Trimming (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	9	9	16.7%	16.7%	0.450	0.424
AEG	Mid Cap	5	5	9.3%	9.3%	0.375	0.393
AFE	Mid Cap	7	5	13.0%	9.3%	0.416	0.314
AFX	Mid Cap	4	6	7.4%	11.1%	0.396	0.452
ALT	Mid Cap	3	6	5.6%	11.1%	0.402	0.400
APN	Mid Cap	5	4	9.3%	7.4%	0.422	0.355
ATN	Mid Cap	8	8	14.8%	14.8%	0.423	0.402
AVI	Mid Cap	6	10	11.1%	18.5%	0.437	0.452
CAT	Mid Cap	10	7	18.5%	13.0%	0.442	0.376
CLS	Mid Cap	4	4	7.4%	7.4%	0.447	0.391
DDT	Mid Cap	3	7	5.6%	13.0%	0.412	0.379
DSY	Mid Cap	3	2	5.6%	3.7%	0.359	0.340
DTC	Mid Cap	10	5	18.5%	9.3%	0.484	0.423
FOS	Mid Cap	5	5	9.3%	9.3%	0.385	0.416
FPT	Mid Cap	11	7	20.4%	13.0%	0.489	0.450
GDF	Mid Cap	6	7	11.1%	13.0%	0.375	0.427
GND	Mid Cap	4	5	7.4%	9.3%	0.393	0.359
GRF	Mid Cap	10	10	18.5%	18.5%	0.475	0.431
GRT	Mid Cap	8	11	14.8%	20.4%	0.462	0.441
HVL	Mid Cap	9	9	16.7%	16.7%	0.384	0.381
HYP	Mid Cap	12	16	22.2%	29.6%	0.466	0.476
ILV	Mid Cap	2	6	3.7%	11.1%	0.389	0.467
JDG	Mid Cap	6	4	11.1%	7.4%	0.357	0.308
MDC	Mid Cap	5	8	9.3%	14.8%	0.461	0.446
MET	Mid Cap	5	6	9.3%	11.1%	0.401	0.395
MPC	Mid Cap	2	9	3.7%	16.7%	0.355	0.489
MTX	Mid Cap	10	3	18.5%	5.6%	0.451	0.359
MVG	Mid Cap	4	5	7.4%	9.3%	0.383	0.363
NHM	Mid Cap	3	7	5.6%	13.0%	0.333	0.406
NPK	Mid Cap	5	6	9.3%	11.1%	0.444	0.409
PAP	Mid Cap	9	2	16.7%	3.7%	0.424	0.330
PGR	Mid Cap	7	3	13.0%	5.6%	0.417	0.417
PIK	Mid Cap	6	4	11.1%	7.4%	0.439	0.409
RBW	Mid Cap	7	6	13.0%	11.1%	0.424	0.465
RLO	Mid Cap	5	5	9.3%	9.3%	0.405	0.407
SAC	Mid Cap	1	3	1.9%	5.6%	0.405	0.314
SHP	Mid Cap	5	5	9.3%	9.3%	0.403	0.380
SNT	Mid Cap	8	10	14.8%	18.5%	0.407	0.365
SPG	Mid Cap	10	9	18.5%	16.7%	0.467	0.491
SUI	Mid Cap	8	8	14.8%	14.8%	0.374	0.401
TON	Mid Cap	5	8	9.3%	14.8%	0.373	0.457
TRE	Mid Cap	10	14	18.5%	25.9%	0.453	0.408
TRU	Mid Cap	2	5	3.7%	9.3%	0.411	0.389
WBO	Mid Cap	10	10	18.5%	18.5%	0.511	0.470

B5. In-Sample Results of Episodic Nonlinearity Tests for Small-Cap Sample after 1% Trimming (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ADH	Small Cap	7	8	13.0%	14.8%	0.393	0.363
ADR	Small Cap	10	9	18.5%	16.7%	0.460	0.391
AFR	Small Cap	5	6	9.3%	11.1%	0.428	0.350
ART	Small Cap	8	6	14.8%	11.1%	0.423	0.375
BAT	Small Cap	5	6	9.3%	11.1%	0.265	0.391
BCX	Small Cap	2	10	3.7%	18.5%	0.378	0.413
BEL	Small Cap	8	6	14.8%	11.1%	0.441	0.385
CDZ	Small Cap	7	5	13.0%	9.3%	0.376	0.385
CLH	Small Cap	10	6	18.5%	11.1%	0.447	0.353
CRM	Small Cap	8	7	14.8%	13.0%	0.472	0.361
CSB	Small Cap	10	6	18.5%	11.1%	0.455	0.396
DAW	Small Cap	6	9	11.1%	16.7%	0.418	0.403
DRD	Small Cap	4	6	7.4%	11.1%	0.412	0.409
FBR	Small Cap	8	11	14.8%	20.4%	0.437	0.452
HDC	Small Cap	5	9	9.3%	16.7%	0.415	0.380
ILA	Small Cap	6	6	11.1%	11.1%	0.365	0.407
IVT	Small Cap	10	11	18.5%	20.4%	0.430	0.435
KAP	Small Cap	8	12	14.8%	22.2%	0.376	0.355
KGM	Small Cap	18	9	33.3%	16.7%	0.417	0.505
MRF	Small Cap	7	3	13.0%	5.6%	0.352	0.280
MTA	Small Cap	9	10	16.7%	18.5%	0.356	0.343
OCE	Small Cap	7	9	13.0%	16.7%	0.394	0.442
OCT	Small Cap	7	8	13.0%	14.8%	0.382	0.514
OMN	Small Cap	9	8	16.7%	14.8%	0.440	0.415
PAM	Small Cap	10	10	18.5%	18.5%	0.459	0.453
PMM	Small Cap	13	13	24.1%	24.1%	0.411	0.457
SIM	Small Cap	11	11	20.4%	20.4%	0.480	0.475
SYC	Small Cap	8	5	14.8%	9.3%	0.327	0.476

B5. In-Sample Results of Episodic Nonlinearity Tests for Large-Cap Sample after 1% Trimming (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ACL	Top 40	6	7	2	11.1%	13.0%	3.7%	0.514	0.422	0.391
AGL	Top 40	3	3	3	5.6%	5.6%	5.6%	0.492	0.419	0.411
AMS	Top 40	7	4	5	13.0%	7.4%	9.3%	0.516	0.447	0.410
ANG	Top 40	1	6	4	1.9%	11.1%	7.4%	0.450	0.437	0.382
ARI	Top 40	7	9	3	13.0%	16.7%	5.6%	0.518	0.414	0.423
ASA	Top 40	5	3	5	9.3%	5.6%	9.3%	0.493	0.356	0.453
BAW	Top 40	3	9	3	5.6%	16.7%	5.6%	0.499	0.453	0.354
BIL	Top 40	2	4	3	3.7%	7.4%	5.6%	0.536	0.416	0.395
BVT	Top 40	8	7	8	14.8%	13.0%	14.8%	0.550	0.408	0.424
CFR	Top 40	6	6	4	11.1%	11.1%	7.4%	0.455	0.392	0.413
FSR	Top 40	2	7	4	3.7%	13.0%	7.4%	0.470	0.428	0.411
GFI	Top 40	4	1	2	7.4%	1.9%	3.7%	0.480	0.381	0.380
HAR	Top 40	2	3	10	3.7%	5.6%	18.5%	0.469	0.400	0.459
IMP	Top 40	2	4	4	3.7%	7.4%	7.4%	0.516	0.383	0.451
INL	Top 40	7	6	8	13.0%	11.1%	14.8%	0.566	0.395	0.433
IPL	Top 40	1	5	6	1.9%	9.3%	11.1%	0.516	0.458	0.445
LBH	Top 40	2	4	4	3.7%	7.4%	7.4%	0.458	0.371	0.391
LON	Top 40	1	3	1	2.2%	6.5%	2.2%	0.463	0.316	0.330
MTN	Top 40	4	4	3	7.4%	7.4%	5.6%	0.531	0.365	0.393
MUR	Top 40	3	5	7	5.6%	9.3%	13.0%	0.468	0.433	0.470
NED	Top 40	5	4	1	9.3%	7.4%	1.9%	0.478	0.428	0.335
NPN	Top 40	7	6	5	13.0%	11.1%	9.3%	0.514	0.440	0.422
NTC	Top 40	1	5	3	1.9%	9.3%	5.6%	0.430	0.418	0.421
OML	Top 40	3	4	2	5.6%	7.4%	3.7%	0.444	0.334	0.388
PPC	Top 40	4	4	7	7.8%	7.8%	13.7%	0.530	0.426	0.439
RMH	Top 40	6	8	6	11.1%	14.8%	11.1%	0.479	0.386	0.380
SAB	Top 40	3	3	3	5.6%	5.6%	5.6%	0.532	0.375	0.389
SAP	Top 40	7	3	5	13.0%	5.6%	9.3%	0.532	0.414	0.439
SBK	Top 40	3	5	2	5.6%	9.3%	3.7%	0.483	0.426	0.364
SHF	Top 40	3	8	4	5.6%	14.8%	7.4%	0.492	0.445	0.361
SLM	Top 40	3	5	7	5.6%	9.3%	13.0%	0.457	0.454	0.453
SOL	Top 40	5	2	2	9.3%	3.7%	3.7%	0.471	0.365	0.365
TBS	Top 40	6	6	4	11.1%	11.1%	7.4%	0.465	0.364	0.341
WHL	Top 40	4	1	4	7.4%	1.9%	7.4%	0.505	0.406	0.387

B5. In-Sample Results of Episodic Nonlinearity Tests for Mid-Cap Sample after 1% Trimming (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	6	9	7	11.1%	16.7%	13.0%	0.463	0.454	0.426
AEG	Mid Cap	2	3	4	3.7%	5.6%	7.4%	0.480	0.367	0.392
AFE	Mid Cap	5	7	4	9.3%	13.0%	7.4%	0.488	0.434	0.332
AFX	Mid Cap	4	2	2	7.4%	3.7%	3.7%	0.502	0.325	0.365
ALT	Mid Cap	3	7	4	5.7%	13.2%	7.5%	0.479	0.415	0.366
APN	Mid Cap	4	4	5	7.4%	7.4%	9.3%	0.548	0.378	0.402
ATN	Mid Cap	1	2	1	2.1%	4.3%	2.1%	0.446	0.363	0.228
AVI	Mid Cap	3	6	11	5.6%	11.1%	20.4%	0.435	0.460	0.451
CAT	Mid Cap	7	6	6	15.9%	13.6%	13.6%	0.552	0.385	0.322
CLS	Mid Cap	3	5	6	5.6%	9.3%	11.1%	0.566	0.467	0.438
DDT	Mid Cap	3	1	7	5.6%	1.9%	13.0%	0.479	0.410	0.378
DSY	Mid Cap	4	5	7	7.5%	9.4%	13.2%	0.471	0.375	0.391
DTC	Mid Cap	2	7	4	3.7%	13.0%	7.4%	0.476	0.458	0.418
FOS	Mid Cap	4	4	4	7.4%	7.4%	7.4%	0.493	0.448	0.420
FPT	Mid Cap	7	7	1	13.0%	13.0%	1.9%	0.532	0.444	0.383
GDF	Mid Cap	4	3	8	8.2%	6.1%	16.3%	0.485	0.371	0.376
GND	Mid Cap	5	7	2	12.8%	17.9%	5.1%	0.491	0.422	0.329
GRF	Mid Cap	4	10	4	7.8%	19.6%	7.8%	0.516	0.435	0.430
GRT	Mid Cap	4	3	4	9.1%	6.8%	9.1%	0.538	0.450	0.399
HVL	Mid Cap	8	7	12	16.0%	14.0%	24.0%	0.542	0.461	0.446
HYP	Mid Cap	7	5	4	16.7%	11.9%	9.5%	0.467	0.305	0.317
ILV	Mid Cap	2	2	4	3.7%	3.7%	7.4%	0.539	0.389	0.467
JDG	Mid Cap	6	8	5	11.1%	14.8%	9.3%	0.507	0.466	0.408
MDC	Mid Cap	3	4	1	5.7%	7.5%	1.9%	0.508	0.441	0.378
MET	Mid Cap	7	2	8	13.0%	3.7%	14.8%	0.567	0.407	0.409
MPC	Mid Cap	1	7	8	1.9%	13.2%	15.1%	0.491	0.408	0.431
MTX	Mid Cap	8	7	5	16.0%	14.0%	10.0%	0.504	0.414	0.386
MVG	Mid Cap	6	3	5	11.3%	5.7%	9.4%	0.445	0.336	0.375
NHM	Mid Cap	6	3	4	11.1%	5.6%	7.4%	0.520	0.351	0.355
NPK	Mid Cap	6	6	6	11.1%	11.1%	11.1%	0.626	0.444	0.409
PAP	Mid Cap	3	3	2	6.0%	6.0%	4.0%	0.510	0.338	0.373
PGR	Mid Cap	4	4	2	7.7%	7.7%	3.8%	0.483	0.454	0.399
PIK	Mid Cap	6	6	4	11.1%	11.1%	7.4%	0.524	0.439	0.408
RBW	Mid Cap	5	6	7	9.3%	11.1%	13.0%	0.466	0.450	0.489
RLO	Mid Cap	6	7	4	11.1%	13.0%	7.4%	0.520	0.382	0.431
SAC	Mid Cap	6	1	4	11.3%	1.9%	7.5%	0.525	0.329	0.291
SHF	Mid Cap	7	4	6	13.0%	7.4%	11.1%	0.534	0.401	0.379
SNT	Mid Cap	1	7	9	1.9%	13.2%	17.0%	0.443	0.440	0.374
SPG	Mid Cap	6	11	8	11.1%	20.4%	14.8%	0.485	0.479	0.466
SUI	Mid Cap	8	6	5	15.4%	11.5%	9.6%	0.458	0.392	0.385
TON	Mid Cap	2	4	7	3.7%	7.4%	13.0%	0.526	0.385	0.439
TRE	Mid Cap	12	8	10	25.5%	17.0%	21.3%	0.592	0.425	0.425
TRU	Mid Cap	2	4	6	3.7%	7.4%	11.1%	0.459	0.447	0.399
WBO	Mid Cap	0	8	8	0.0%	16.0%	16.0%	0.456	0.440	0.410

B5. In-Sample Results of Episodic Nonlinearity Tests for Small-Cap Sample after 1% Trimming (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C%	H%	T4%	AvgCStat	AvgHStat	AvgT4Stat
ADH	Small Cap	5	4	8	10.0%	8.0%	16.0%	0.532	0.360	0.363
ADR	Small Cap	5	8	5	10.2%	16.3%	10.2%	0.551	0.398	0.371
AFR	Small Cap	3	3	3	5.6%	5.6%	5.6%	0.564	0.421	0.387
ART	Small Cap	8	4	7	16.3%	8.2%	14.3%	0.563	0.355	0.369
BAT	Small Cap	3	3	5	5.8%	5.8%	9.6%	0.477	0.378	0.436
BCX	Small Cap	3	4	5	5.6%	7.4%	9.3%	0.451	0.382	0.390
BEL	Small Cap	1	3	5	2.2%	6.5%	10.9%	0.500	0.321	0.361
CDZ	Small Cap	3	2	1	5.9%	3.9%	2.0%	0.552	0.355	0.314
CLH	Small Cap	4	9	4	8.7%	19.6%	8.7%	0.488	0.394	0.352
CRM	Small Cap	6	5	5	14.0%	11.6%	11.6%	0.523	0.350	0.293
CSB	Small Cap	3	5	6	6.5%	10.9%	13.0%	0.470	0.356	0.402
DAW	Small Cap	9	5	6	18.8%	10.4%	12.5%	0.550	0.368	0.399
DRD	Small Cap	3	8	4	5.6%	14.8%	7.4%	0.465	0.407	0.420
FBR	Small Cap	7	9	7	15.9%	20.5%	15.9%	0.557	0.476	0.405
HDC	Small Cap	1	4	5	2.6%	10.3%	12.8%	0.475	0.382	0.377
ILA	Small Cap	6	5	2	12.8%	10.6%	4.3%	0.550	0.359	0.356
IVT	Small Cap	6	6	6	16.2%	16.2%	16.2%	0.563	0.419	0.393
KAP	Small Cap	3	2	2	7.9%	5.3%	5.3%	0.504	0.281	0.250
KGM	Small Cap	4	2	1	12.5%	6.3%	3.1%	0.594	0.379	0.393
MRF	Small Cap	4	3	3	7.4%	5.6%	5.6%	0.543	0.430	0.366
MTA	Small Cap	4	5	4	13.3%	16.7%	13.3%	0.525	0.346	0.342
OCE	Small Cap	3	7	7	6.7%	15.6%	15.6%	0.456	0.337	0.347
OCT	Small Cap	6	4	1	15.4%	10.3%	2.6%	0.569	0.424	0.282
OMN	Small Cap	3	3	4	6.7%	6.7%	8.9%	0.516	0.366	0.372
PAM	Small Cap	3	7	6	6.5%	15.2%	13.0%	0.496	0.459	0.395
PMM	Small Cap	4	4	3	11.8%	11.8%	8.8%	0.560	0.349	0.309
SIM	Small Cap	5	5	4	18.5%	18.5%	14.8%	0.592	0.390	0.360
SYC	Small Cap	7	5	5	13.7%	9.8%	9.8%	0.549	0.415	0.327

B5. Out-of-Sample Results of Episodic Nonlinearity Tests for Large-Cap Sample after 1% Trimming (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ACL	Top 40	1	0	5.9%	0.0%	0.450	0.403
AGL	Top 40	2	1	11.8%	5.9%	0.397	0.377
AMS	Top 40	0	1	0.0%	5.9%	0.387	0.410
ANG	Top 40	4	0	23.5%	0.0%	0.432	0.491
ARI	Top 40	3	2	17.6%	11.8%	0.397	0.364
ASA	Top 40	1	1	5.9%	5.9%	0.500	0.323
BAW	Top 40	2	1	11.8%	5.9%	0.383	0.436
BIL	Top 40	1	0	5.9%	0.0%	0.338	0.374
BVT	Top 40	3	2	17.6%	11.8%	0.496	0.429
CFR	Top 40	0	0	0.0%	0.0%	0.262	0.402
FSR	Top 40	2	0	11.8%	0.0%	0.497	0.343
GFI	Top 40	0	1	0.0%	5.9%	0.383	0.543
HAR	Top 40	2	2	11.8%	11.8%	0.333	0.309
IMP	Top 40	2	0	11.8%	0.0%	0.358	0.465
INL	Top 40	3	2	17.6%	11.8%	0.429	0.422
IPL	Top 40	6	2	35.3%	11.8%	0.539	0.486
LBH	Top 40	3	1	17.6%	5.9%	0.459	0.273
LON	Top 40	1	2	5.9%	11.8%	0.381	0.415
MTN	Top 40	0	0	0.0%	0.0%	0.407	0.281
MUR	Top 40	3	1	17.6%	5.9%	0.505	0.390
NED	Top 40	4	0	23.5%	0.0%	0.416	0.494
NPN	Top 40	1	1	5.9%	5.9%	0.497	0.334
NTC	Top 40	0	4	0.0%	23.5%	0.500	0.496
OML	Top 40	1	1	5.9%	5.9%	0.495	0.438
PPC	Top 40	1	1	5.9%	5.9%	0.362	0.449
RMH	Top 40	1	1	5.9%	5.9%	0.457	0.371
SAB	Top 40	1	0	5.9%	0.0%	0.458	0.307
SAP	Top 40	2	1	11.8%	5.9%	0.324	0.405
SBK	Top 40	1	1	5.9%	5.9%	0.441	0.411
SHF	Top 40	4	2	23.5%	11.8%	0.496	0.477
SLM	Top 40	1	2	5.9%	11.8%	0.502	0.504
SOL	Top 40	4	4	23.5%	23.5%	0.402	0.555
TBS	Top 40	1	1	5.9%	5.9%	0.433	0.624
WHL	Top 40	1	0	5.9%	0.0%	0.391	0.292

B5. Out-of-Sample Results of Episodic Nonlinearity Tests for Mid-Cap Sample after 1% Trimming (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	2	1	11.8%	5.9%	0.372	0.421
AEG	Mid Cap	0	1	0.0%	5.9%	0.358	0.325
AFE	Mid Cap	2	1	11.8%	5.9%	0.349	0.259
AFX	Mid Cap	2	3	11.8%	17.6%	0.405	0.508
ALT	Mid Cap	0	0	0.0%	0.0%	0.242	0.302
APN	Mid Cap	1	2	5.9%	11.8%	0.487	0.454
ATN	Mid Cap	1	3	5.9%	17.6%	0.341	0.413
AVI	Mid Cap	0	1	0.0%	5.9%	0.195	0.240
CAT	Mid Cap	3	1	17.6%	5.9%	0.357	0.306
CLS	Mid Cap	1	3	5.9%	17.6%	0.501	0.387
DDT	Mid Cap	3	2	17.6%	11.8%	0.450	0.432
DSY	Mid Cap	3	2	17.6%	11.8%	0.414	0.342
DTC	Mid Cap	0	0	0.0%	0.0%	0.368	0.341
FOS	Mid Cap	3	2	17.6%	11.8%	0.431	0.291
FPT	Mid Cap	1	4	5.9%	23.5%	0.320	0.390
GDF	Mid Cap	2	2	11.8%	11.8%	0.421	0.396
GND	Mid Cap	2	0	11.8%	0.0%	0.523	0.317
GRF	Mid Cap	0	0	0.0%	0.0%	0.409	0.395
GRT	Mid Cap	3	0	17.6%	0.0%	0.472	0.346
HVL	Mid Cap	3	2	17.6%	11.8%	0.472	0.451
HYP	Mid Cap	4	1	23.5%	5.9%	0.508	0.473
ILV	Mid Cap	1	1	5.9%	5.9%	0.373	0.364
JDG	Mid Cap	2	3	11.8%	17.6%	0.492	0.456
MDC	Mid Cap	2	1	11.8%	5.9%	0.351	0.401
MET	Mid Cap	2	1	11.8%	5.9%	0.427	0.427
MPC	Mid Cap	1	1	5.9%	5.9%	0.343	0.364
MTX	Mid Cap	6	2	35.3%	11.8%	0.504	0.515
MVG	Mid Cap	1	1	5.9%	5.9%	0.339	0.345
NHM	Mid Cap	0	2	0.0%	11.8%	0.402	0.396
NPK	Mid Cap	1	3	5.9%	17.6%	0.309	0.406
PAP	Mid Cap	1	2	5.9%	11.8%	0.468	0.378
PGR	Mid Cap	5	4	29.4%	23.5%	0.539	0.411
PIK	Mid Cap	2	1	11.8%	5.9%	0.478	0.407
RBW	Mid Cap	1	4	5.9%	23.5%	0.304	0.413
RLO	Mid Cap	2	2	11.8%	11.8%	0.532	0.486
SAC	Mid Cap	3	3	17.6%	17.6%	0.530	0.416
SHP	Mid Cap	0	2	0.0%	11.8%	0.404	0.499
SNT	Mid Cap	3	3	17.6%	17.6%	0.402	0.405
SPG	Mid Cap	1	1	5.9%	5.9%	0.378	0.381
SUI	Mid Cap	7	3	41.2%	17.6%	0.611	0.405
TON	Mid Cap	1	0	5.9%	0.0%	0.417	0.367
TRE	Mid Cap	3	1	17.6%	5.9%	0.481	0.415
TRU	Mid Cap	1	2	5.9%	11.8%	0.374	0.409
WBO	Mid Cap	0	0	0.0%	0.0%	0.325	0.325

B5. Out-of-Sample Results of Episodic Nonlinearity Tests for Small-Cap Sample after 1% Trimming (Adjustment of Atchison et al, 1987)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	H	T4	H %	T4 %	Avg H Stat	Avg T4 Stat
ADH	Small Cap	2	1	11.8%	5.9%	0.402	0.365
ADR	Small Cap	2	1	11.8%	5.9%	0.245	0.336
AFR	Small Cap	1	0	5.9%	0.0%	0.371	0.236
ART	Small Cap	4	2	23.5%	11.8%	0.468	0.352
BAT	Small Cap	5	5	29.4%	29.4%	0.610	0.419
BCX	Small Cap	3	2	17.6%	11.8%	0.480	0.385
BEL	Small Cap	3	3	17.6%	17.6%	0.291	0.422
CDZ	Small Cap	3	2	17.6%	11.8%	0.532	0.380
CLH	Small Cap	2	0	11.8%	0.0%	0.467	0.246
CRM	Small Cap	0	2	0.0%	11.8%	0.110	0.231
CSB	Small Cap	3	0	17.6%	0.0%	0.519	0.303
DAW	Small Cap	2	0	11.8%	0.0%	0.377	0.303
DRD	Small Cap	1	1	5.9%	5.9%	0.266	0.344
FBR	Small Cap	0	6	0.0%	35.3%	0.316	0.437
HDC	Small Cap	1	1	5.9%	5.9%	0.274	0.216
ILA	Small Cap	1	3	5.9%	17.6%	0.389	0.422
IVT	Small Cap	0	4	0.0%	23.5%	0.421	0.351
KAP	Small Cap	1	2	5.9%	11.8%	0.243	0.301
KGM	Small Cap	2	1	11.8%	5.9%	0.288	0.335
MRF	Small Cap	0	2	0.0%	11.8%	0.311	0.395
MTA	Small Cap	3	4	17.6%	23.5%	0.395	0.430
OCE	Small Cap	2	4	11.8%	23.5%	0.442	0.397
OCT	Small Cap	3	3	17.6%	17.6%	0.368	0.304
OMN	Small Cap	3	1	17.6%	5.9%	0.439	0.363
PAM	Small Cap	2	1	11.8%	5.9%	0.426	0.422
PMM	Small Cap	1	1	5.9%	5.9%	0.248	0.253
SIM	Small Cap	6	1	35.3%	5.9%	0.650	0.325
SYC	Small Cap	2	4	11.8%	23.5%	0.406	0.428

B5. Out-of-Sample Results of Episodic Nonlinearity Tests for Large-Cap Sample after 1% Trimming (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ACL	Top 40	1	1	1	5.9%	5.9%	5.9%	0.361	0.437	0.406
AGL	Top 40	1	2	0	5.9%	11.8%	0.0%	0.561	0.416	0.368
AMS	Top 40	3	1	1	17.6%	5.9%	5.9%	0.571	0.333	0.418
ANG	Top 40	0	4	0	0.0%	23.5%	0.0%	0.388	0.432	0.491
ARI	Top 40	2	0	1	11.8%	0.0%	5.9%	0.485	0.320	0.317
ASA	Top 40	1	1	1	5.9%	5.9%	5.9%	0.523	0.500	0.323
BAW	Top 40	3	2	1	17.6%	11.8%	5.9%	0.533	0.383	0.436
BIL	Top 40	0	1	0	0.0%	5.9%	0.0%	0.319	0.338	0.374
BVT	Top 40	0	3	2	0.0%	17.6%	11.8%	0.464	0.496	0.429
CFR	Top 40	2	0	0	11.8%	0.0%	0.0%	0.491	0.262	0.402
FSR	Top 40	2	2	0	11.8%	11.8%	0.0%	0.472	0.497	0.343
GFI	Top 40	0	1	0	0.0%	5.9%	0.0%	0.417	0.383	0.543
HAR	Top 40	1	2	2	5.9%	11.8%	11.8%	0.327	0.333	0.309
IMP	Top 40	0	2	0	0.0%	11.8%	0.0%	0.501	0.358	0.465
INL	Top 40	2	3	2	11.8%	17.6%	11.8%	0.521	0.429	0.422
IPL	Top 40	0	6	2	0.0%	35.3%	11.8%	0.431	0.539	0.486
LBH	Top 40	2	3	1	11.8%	17.6%	5.9%	0.496	0.461	0.266
LON	Top 40	1	1	2	5.9%	5.9%	11.8%	0.474	0.381	0.415
MTN	Top 40	1	0	0	5.9%	0.0%	0.0%	0.569	0.407	0.281
MUR	Top 40	1	3	1	5.9%	17.6%	5.9%	0.524	0.505	0.390
NED	Top 40	3	4	0	17.6%	23.5%	0.0%	0.540	0.416	0.494
NPN	Top 40	4	1	1	23.5%	5.9%	5.9%	0.520	0.497	0.334
NTC	Top 40	0	0	4	0.0%	0.0%	23.5%	0.438	0.500	0.496
OML	Top 40	1	1	1	5.9%	5.9%	5.9%	0.500	0.495	0.438
PPC	Top 40	1	0	0	5.9%	0.0%	0.0%	0.585	0.313	0.407
RMH	Top 40	3	1	1	17.6%	5.9%	5.9%	0.484	0.457	0.371
SAB	Top 40	2	1	0	11.8%	5.9%	0.0%	0.592	0.458	0.307
SAP	Top 40	0	2	1	0.0%	11.8%	5.9%	0.443	0.324	0.405
SBK	Top 40	0	1	1	0.0%	5.9%	5.9%	0.507	0.441	0.411
SHF	Top 40	4	4	2	23.5%	23.5%	11.8%	0.615	0.496	0.477
SLM	Top 40	3	1	2	17.6%	5.9%	11.8%	0.570	0.502	0.504
SOL	Top 40	1	4	4	5.9%	23.5%	23.5%	0.421	0.402	0.555
TBS	Top 40	1	1	1	5.9%	5.9%	5.9%	0.524	0.433	0.624
WHL	Top 40	0	1	0	0.0%	5.9%	0.0%	0.424	0.391	0.292

B5. Out-of-Sample Results of Episodic Nonlinearity Tests for Mid-Cap Sample after 1% Trimming (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ABL	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.460	0.372	0.421
AEG	Mid Cap	0	0	1	0.0%	0.0%	5.9%	0.606	0.358	0.325
AFE	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.414	0.349	0.259
AFX	Mid Cap	1	2	3	5.9%	11.8%	17.6%	0.446	0.401	0.521
ALT	Mid Cap	2	0	0	11.8%	0.0%	0.0%	0.430	0.242	0.302
APN	Mid Cap	3	1	2	17.6%	5.9%	11.8%	0.556	0.487	0.454
ATN	Mid Cap	0	1	3	0.0%	5.9%	17.6%	0.552	0.341	0.413
AVI	Mid Cap	2	0	1	11.8%	0.0%	5.9%	0.556	0.195	0.240
CAT	Mid Cap	4	2	1	23.5%	11.8%	5.9%	0.696	0.367	0.353
CLS	Mid Cap	0	1	3	0.0%	5.9%	17.6%	0.440	0.501	0.387
DDT	Mid Cap	1	3	2	5.9%	17.6%	11.8%	0.507	0.450	0.432
DSY	Mid Cap	1	3	2	5.9%	17.6%	11.8%	0.425	0.414	0.342
DTC	Mid Cap	1	0	0	5.9%	0.0%	0.0%	0.390	0.368	0.341
FOS	Mid Cap	0	3	2	0.0%	17.6%	11.8%	0.407	0.431	0.291
FPT	Mid Cap	0	3	2	0.0%	17.6%	11.8%	0.461	0.254	0.293
GDF	Mid Cap	4	2	2	23.5%	11.8%	11.8%	0.606	0.421	0.396
GND	Mid Cap	1	2	0	5.9%	11.8%	0.0%	0.486	0.523	0.317
GRF	Mid Cap	2	0	0	11.8%	0.0%	0.0%	0.595	0.409	0.395
GRT	Mid Cap	1	3	0	5.9%	17.6%	0.0%	0.409	0.472	0.346
HVL	Mid Cap	0	3	2	0.0%	17.6%	11.8%	0.502	0.472	0.451
HYP	Mid Cap	1	4	1	5.9%	23.5%	5.9%	0.528	0.508	0.473
ILV	Mid Cap	1	1	1	5.9%	5.9%	5.9%	0.470	0.373	0.364
JDG	Mid Cap	1	2	3	5.9%	11.8%	17.6%	0.452	0.492	0.456
MDC	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.456	0.351	0.401
MET	Mid Cap	1	2	1	5.9%	11.8%	5.9%	0.535	0.427	0.427
MPC	Mid Cap	1	1	1	5.9%	5.9%	5.9%	0.501	0.343	0.364
MTX	Mid Cap	2	6	2	11.8%	35.3%	11.8%	0.495	0.504	0.515
MVG	Mid Cap	1	1	1	5.9%	5.9%	5.9%	0.482	0.339	0.345
NHM	Mid Cap	3	0	2	17.6%	0.0%	11.8%	0.527	0.454	0.533
NPK	Mid Cap	1	1	3	5.9%	5.9%	17.6%	0.530	0.309	0.406
PAP	Mid Cap	1	2	2	5.9%	11.8%	11.8%	0.583	0.518	0.351
PGR	Mid Cap	0	5	4	0.0%	29.4%	23.5%	0.414	0.539	0.411
PIK	Mid Cap	2	2	1	11.8%	11.8%	5.9%	0.560	0.478	0.407
RBW	Mid Cap	0	1	4	0.0%	5.9%	23.5%	0.568	0.304	0.413
RLO	Mid Cap	4	2	2	23.5%	11.8%	11.8%	0.527	0.532	0.486
SAC	Mid Cap	3	3	3	17.6%	17.6%	17.6%	0.518	0.530	0.416
SHP	Mid Cap	3	0	2	17.6%	0.0%	11.8%	0.617	0.404	0.499
SNT	Mid Cap	4	3	3	23.5%	17.6%	17.6%	0.608	0.402	0.405
SPG	Mid Cap	4	2	3	23.5%	11.8%	17.6%	0.673	0.331	0.373
SUI	Mid Cap	0	7	3	0.0%	41.2%	17.6%	0.433	0.611	0.405
TON	Mid Cap	1	1	0	5.9%	5.9%	0.0%	0.594	0.417	0.367
TRE	Mid Cap	2	2	2	11.8%	11.8%	11.8%	0.517	0.444	0.310
TRU	Mid Cap	1	1	2	5.9%	5.9%	11.8%	0.565	0.373	0.421
WBO	Mid Cap	0	0	0	0.0%	0.0%	0.0%	0.471	0.325	0.325

B5. Out-of-Sample Results of Episodic Nonlinearity Tests for Small-Cap Sample after 1% Trimming (Adjustment of Mlambo et al, 2003)

Figures quoted for the number of significant windows for each share represent the average number of windows with a significant test statistic at the 5% level, obtained via 1000 bootstrap replications. The C statistic is a measure of linear serial dependence in the data while the H and T4 statistics are measures of third- and fourth-order nonlinear serial dependence, respectively.

TICKER	INDEX	C	H	T4	C %	H %	T4 %	Avg C Stat	Avg H Stat	Avg T4 Stat
ADH	Small Cap	5	2	1	29.4%	11.8%	5.9%	0.615	0.402	0.365
ADR	Small Cap	0	2	1	0.0%	11.8%	5.9%	0.527	0.245	0.336
AFR	Small Cap	2	0	0	11.8%	0.0%	0.0%	0.494	0.412	0.259
ART	Small Cap	1	2	2	5.9%	11.8%	11.8%	0.516	0.480	0.368
BAT	Small Cap	1	1	4	5.9%	5.9%	23.5%	0.447	0.459	0.404
BCX	Small Cap	4	3	2	23.5%	17.6%	11.8%	0.587	0.480	0.385
BEL	Small Cap	2	2	1	11.8%	11.8%	5.9%	0.521	0.412	0.381
CDZ	Small Cap	3	2	1	17.6%	11.8%	5.9%	0.568	0.379	0.373
CLH	Small Cap	2	2	0	11.8%	11.8%	0.0%	0.539	0.461	0.264
CRM	Small Cap	2	0	2	11.8%	0.0%	11.8%	0.471	0.110	0.231
CSB	Small Cap	1	2	2	5.9%	11.8%	11.8%	0.456	0.471	0.306
DAW	Small Cap	2	2	0	11.8%	11.8%	0.0%	0.441	0.377	0.303
DRD	Small Cap	0	1	1	0.0%	5.9%	5.9%	0.375	0.266	0.344
FBR	Small Cap	0	1	4	0.0%	5.9%	23.5%	0.424	0.365	0.416
HDC	Small Cap	1	1	1	5.9%	5.9%	5.9%	0.500	0.274	0.216
ILA	Small Cap	1	2	4	5.9%	11.8%	23.5%	0.531	0.396	0.472
IVT	Small Cap	1	0	4	5.9%	0.0%	23.5%	0.420	0.421	0.351
KAP	Small Cap	1	0	1	5.9%	0.0%	5.9%	0.531	0.187	0.277
KGM	Small Cap	2	5	1	11.8%	29.4%	5.9%	0.531	0.418	0.323
MRF	Small Cap	2	1	0	11.8%	5.9%	0.0%	0.484	0.300	0.374
MTA	Small Cap	4	3	3	23.5%	17.6%	17.6%	0.363	0.394	0.431
OCE	Small Cap	2	2	4	11.8%	11.8%	23.5%	0.455	0.436	0.351
OCT	Small Cap	3	5	5	17.6%	29.4%	29.4%	0.551	0.432	0.410
OMN	Small Cap	2	3	1	11.8%	17.6%	5.9%	0.495	0.439	0.363
PAM	Small Cap	1	2	1	5.9%	11.8%	5.9%	0.476	0.426	0.422
PMM	Small Cap	4	1	1	23.5%	5.9%	5.9%	0.504	0.248	0.253
SIM	Small Cap	0	6	1	0.0%	35.3%	5.9%	0.506	0.650	0.325
SYC	Small Cap	1	2	4	5.9%	11.8%	23.5%	0.519	0.406	0.428

Appendix C

C1. Correlation Table

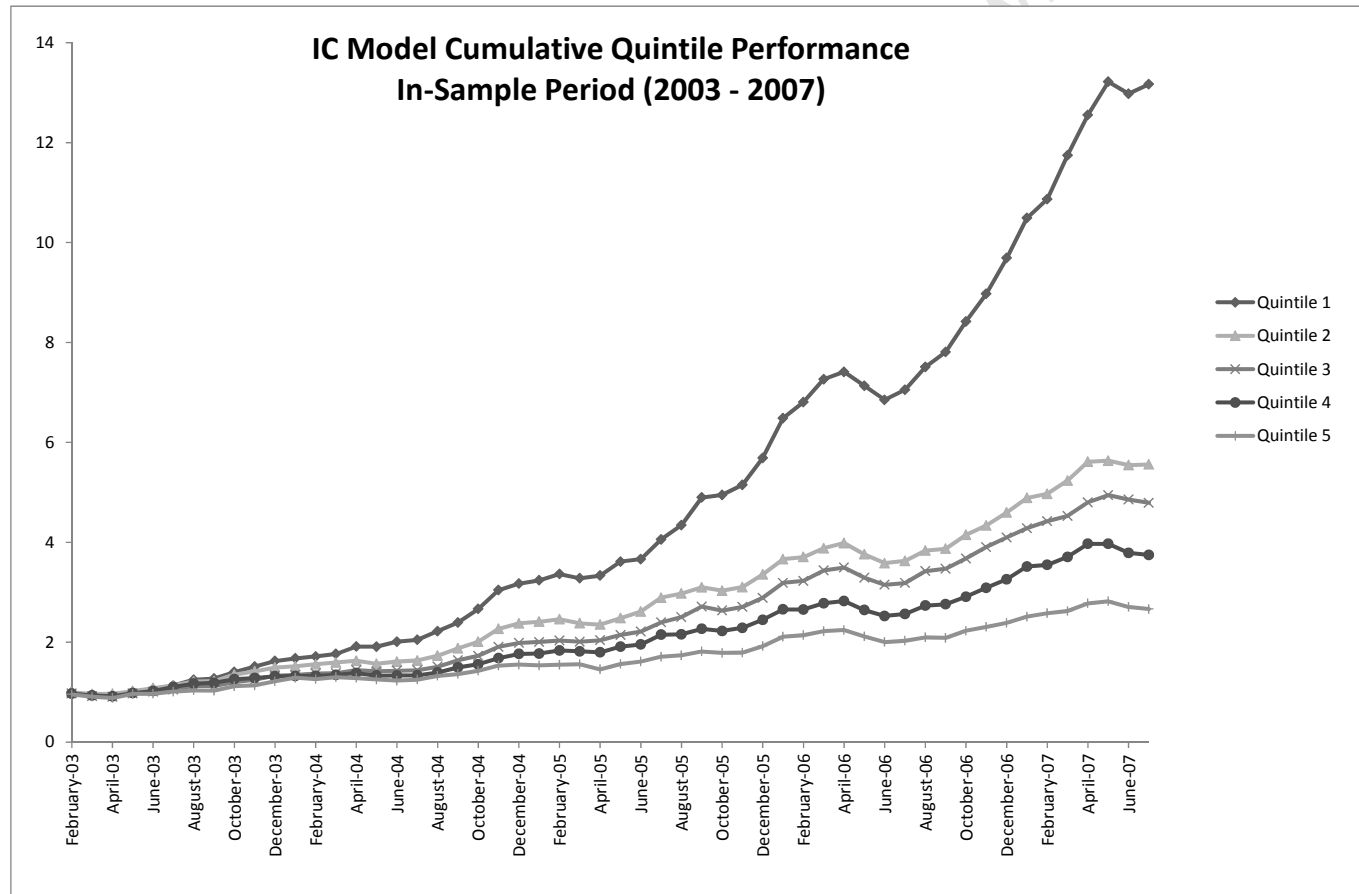
The below table summarizes the pairwise correlations for the factors examined in the study. Correlations in excess of 0.7 are shaded and highlighted in bold.

	MOM1	MOM3	MOM6	MOM12	STDEV12	EY	DY	LMV	VT	C12MEPSP	C24MEPSP	BVTM	POUT	C12MDPSP	C24MDPSP	CTP	ROE	DUMFINDI	BUY	DOWNG	
MOM1	1.000																				
MOM3	0.581	1.000																			
MOM6	0.421	0.726	1.000																		
MOM12	0.295	0.510	0.719	1.000																	
STDEV12	0.006	0.026	0.007	-0.074	1.000																
EY	-0.069	-0.105	-0.133	-0.130	0.108	1.000															
DY	-0.082	-0.137	-0.178	-0.193	-0.062	0.187	1.000														
LMV	0.002	0.003	0.011	0.006	-0.166	-0.164	-0.154	1.000													
VT	0.014	0.008	0.025	0.017	0.019	-0.027	-0.043	0.337	1.000												
C12MEPSP	-0.009	-0.010	-0.004	0.031	0.177	0.278	0.066	-0.536	-0.174	1.000											
C24MEPSP	0.010	0.024	0.036	0.104	-0.001	0.445	-0.032	0.049	-0.058	0.160	1.000										
BVTM	-0.112	-0.174	-0.233	-0.279	0.208	0.191	0.214	-0.441	-0.108	0.324	-0.136	1.000									
POUT	0.014	0.014	0.018	0.029	-0.154	-0.062	0.236	0.200	0.057	-0.199	-0.025	-0.218	1.000								
C12MDPSP	0.019	0.026	0.055	0.157	-0.103	0.037	0.217	0.022	-0.101	0.055	0.289	-0.086	0.055	1.000							
C24MDPSP	-0.003	-0.006	0.008	0.090	-0.048	0.051	0.310	0.043	-0.072	-0.008	0.372	-0.108	0.062	0.604	1.000						
CTP	-0.019	0.002	-0.012	-0.010	0.024	0.065	0.025	-0.037	-0.003	0.087	0.088	0.114	-0.033	0.028	0.026	1.000					
ROE	-0.002	0.010	0.043	0.134	-0.066	0.129	0.051	0.187	0.053	-0.081	0.289	-0.301	0.176	0.147	0.214	-0.024	1.000				
DUMFINDI	0.020	0.011	-0.003	-0.008	-0.177	0.049	0.119	-0.194	-0.048	0.074	0.033	0.012	0.086	0.048	0.055	-0.004	0.043	1.000			
BUY	-0.015	-0.035	-0.039	-0.023	-0.051	-0.024	-0.054	0.249	0.137	-0.158	0.073	-0.138	0.036	0.037	0.055	0.005	0.077	0.055	1.000		
DOWNG	-0.016	-0.012	-0.028	-0.071	0.034	-0.001	0.005	0.158	0.054	-0.093	0.021	-0.043	0.060	-0.019	-0.001	0.009	0.037	0.018	-0.105	1.000	

Appendix D

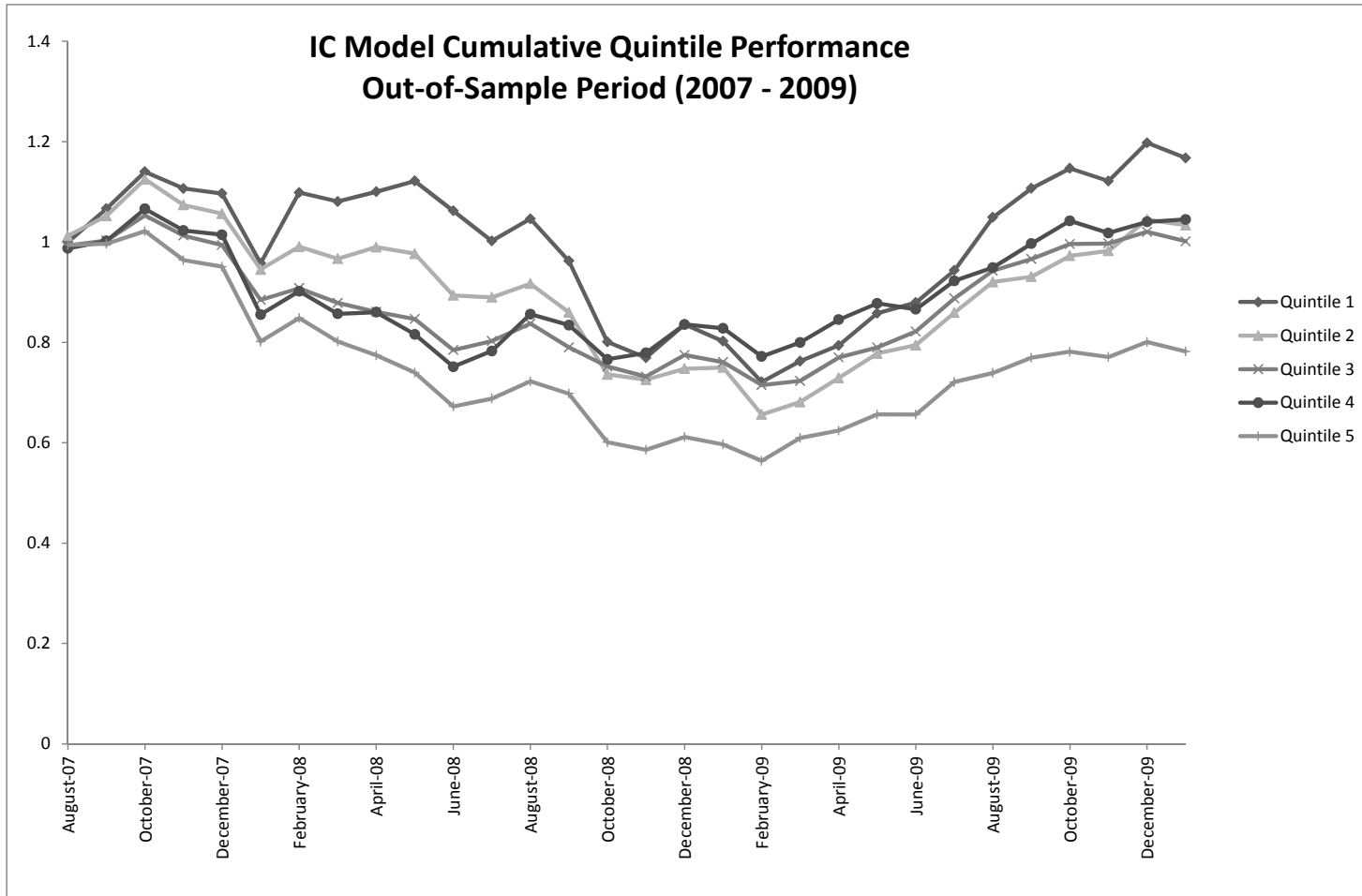
D1. Cumulative Geometric Quintile Returns for IC Model (In-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement by optimizing the information coefficient (IC) criterion. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for each quintile in the model over the in-sample period is presented below.



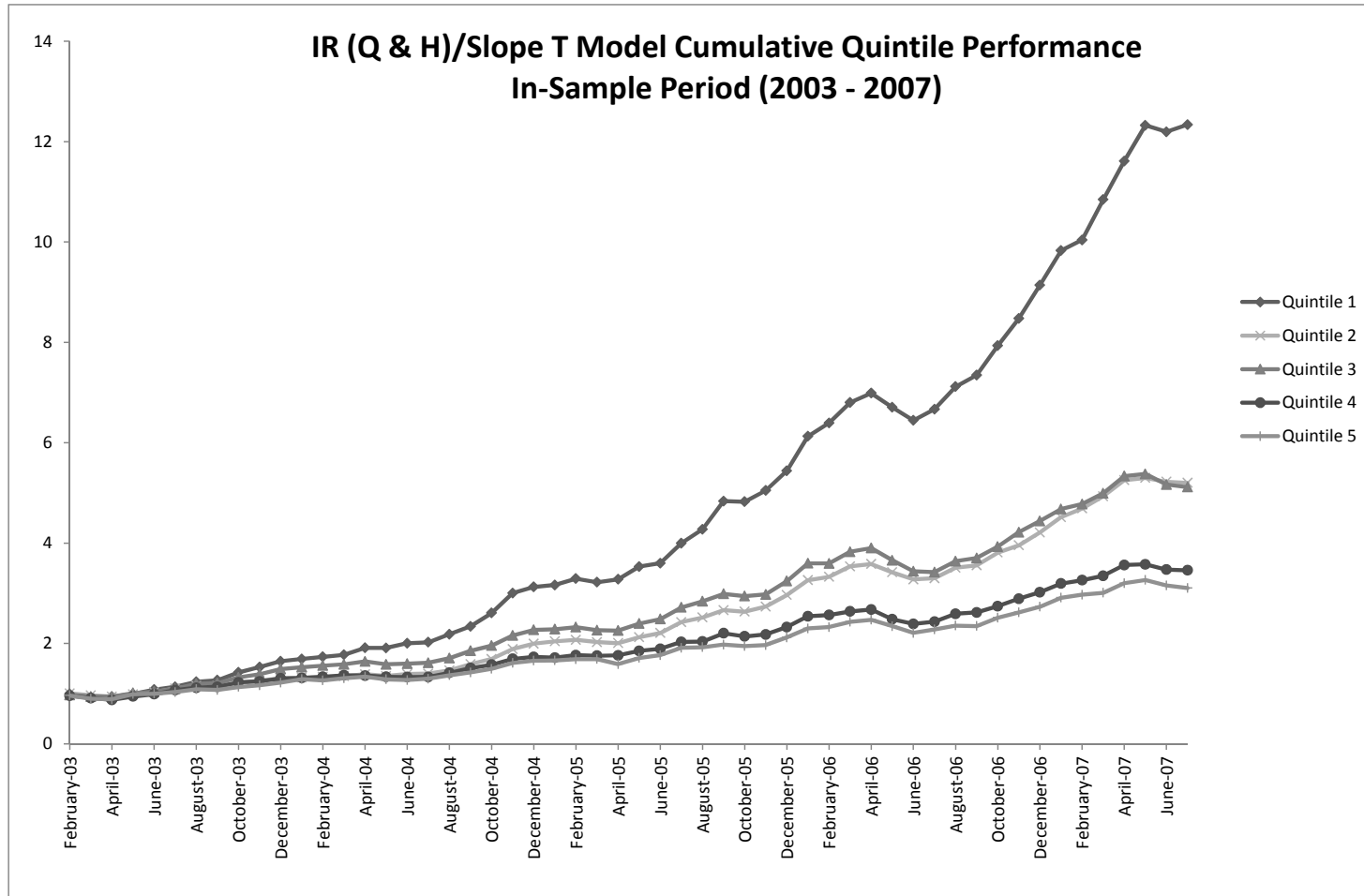
D2. Cumulative Geometric Quintile Returns for IC Model (Out-of-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement by optimizing the information coefficient (IC) criterion. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for each quintile in the model over the out-of-sample period is presented below.



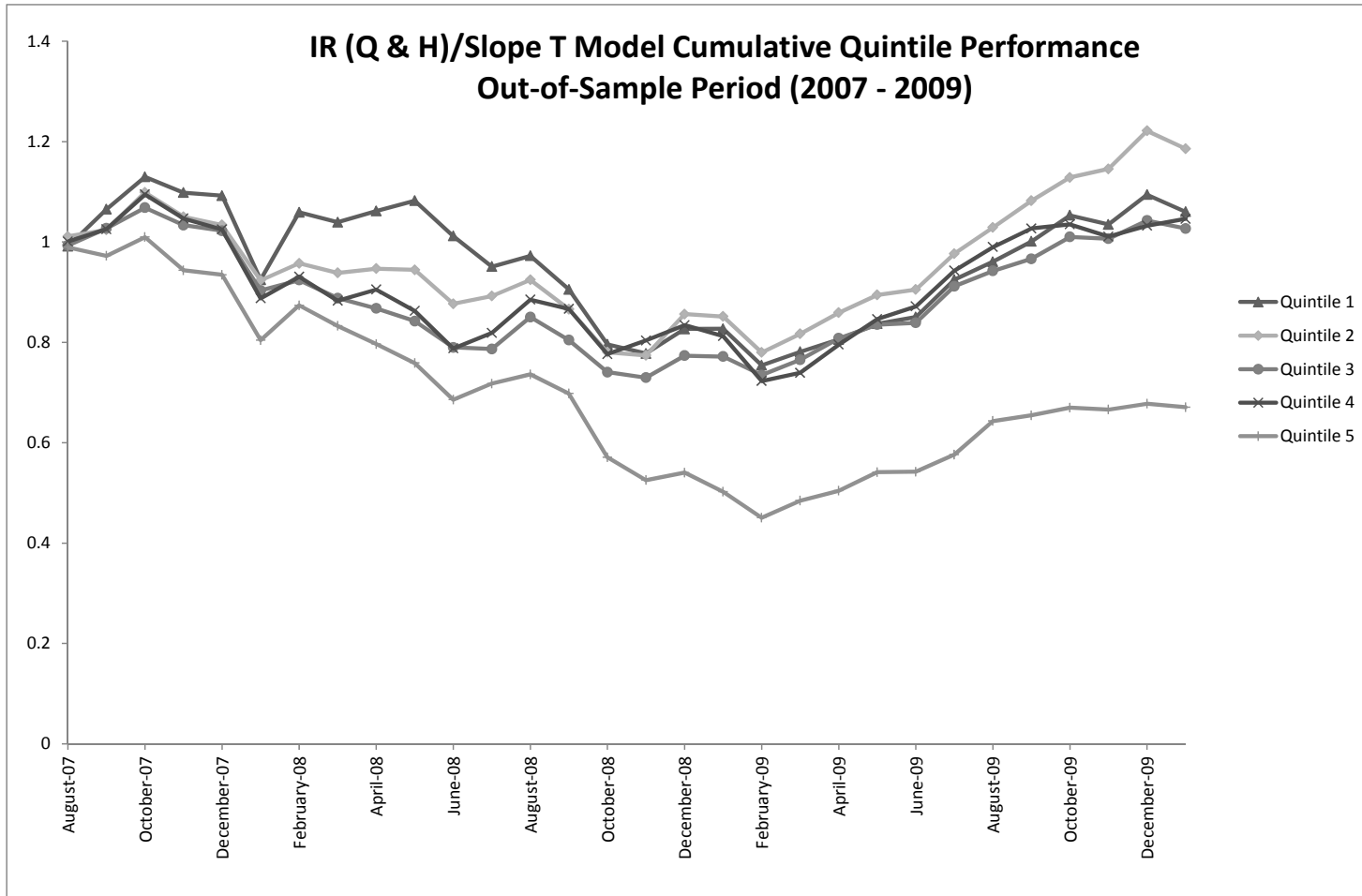
D3. Cumulative Geometric Quintile Returns for Qian Model (In-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement by optimizing the Qian IR criterion. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for each quintile in the model over the in-sample period is presented below.



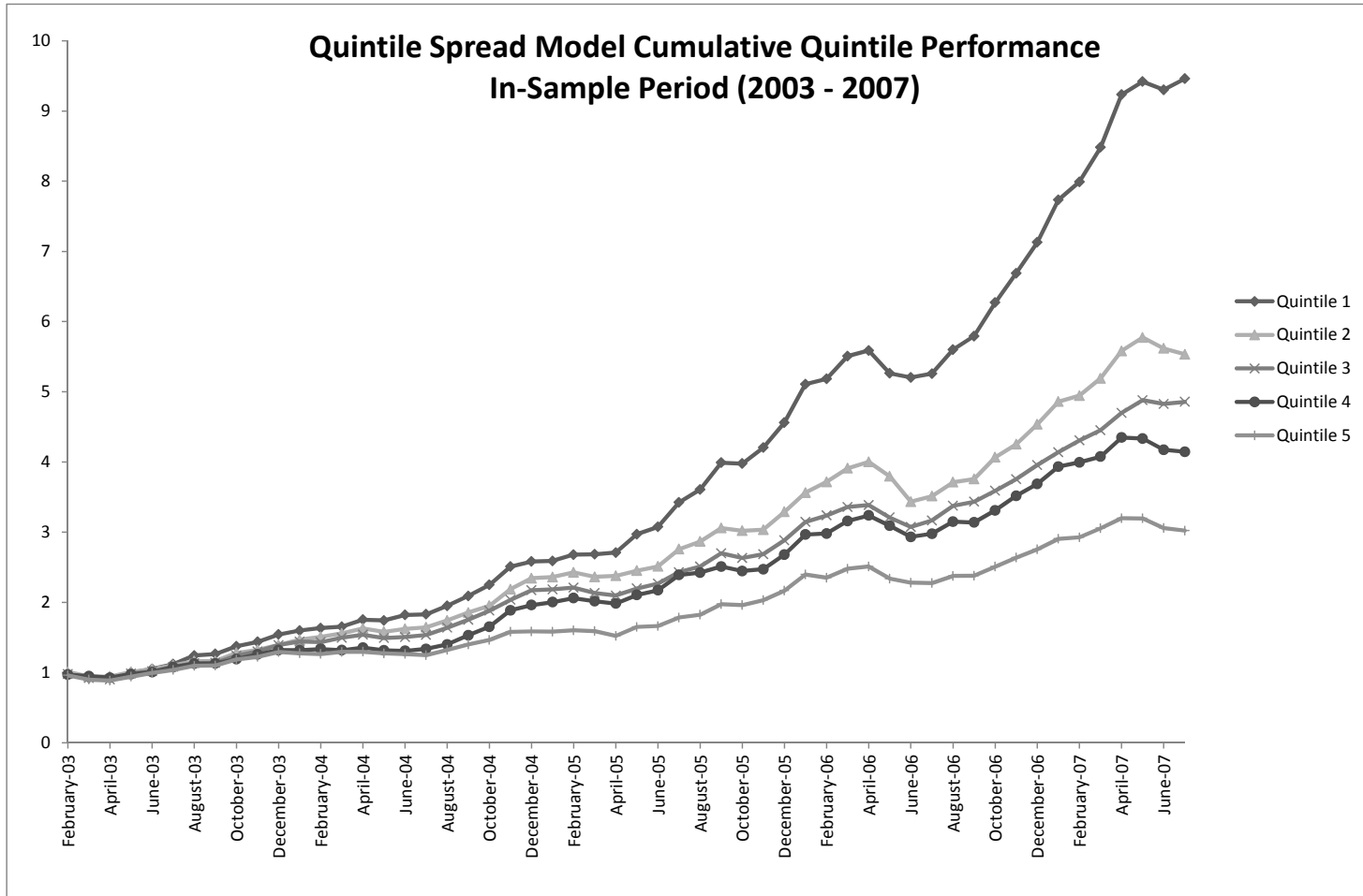
D4. Cumulative Geometric Quintile Returns for Qian Model (Out-of-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement by optimizing the Qian IR criterion. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for each quintile in the model over the out-of-sample period is presented below.



D5. Cumulative Geometric Quintile Returns for Quintile Spread Model (In-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement by optimizing the Quintile Spread criterion. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for each quintile in the model over the in-sample period is presented below.



D5. Cumulative Geometric Quintile Returns for Quintile Spread Model (Out-of-Sample)

Multi-factor predictive models of returns are created using a forward stepwise procedure with replacement by optimizing the Quintile Spread criterion. Forecasts are sorted in descending order by forecast return and then grouped into quintiles. The cumulative geometric return for each quintile in the model over the out-of-sample period is presented below.

