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**UNIVERSITY OF CAPE TOWN**  
**Faculty of Engineering and the Built Environment**  
**Department of Civil Engineering**



**Predicting and Testing the Tensile Relaxation of concrete**

*Thesis submitted in partial fulfillment of the requirements for the degree of*

**MASTER OF SCIENCE IN CIVIL ENGINEERING**

*by*

**Nicholas Kizito Magera**

**Supervisor: A/Prof. Hans-Dieter Beushausen**

**Co-supervisor Prof. Pilate Moyo**

**February 2013**

## **Plagiarism declaration**

I know the meaning of plagiarism and I declare that all of the work in this document, save for that which is properly acknowledged, is my own. I also affirm that this work has not been submitted in this, or any other university for examination, or for any other purposes.

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## **Dedication**

*Dedicated to my family and to all my friends*

University of Cape Town

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## Abstract

Tensile relaxation is an important property in structural concrete members such as concrete overlays and patch repairs under sustained imposed restraint. Tensile relaxation helps in reducing tensile stresses in concrete which leads to mitigation of cracking in restrained concrete members. Normally, it is assumed that tensile relaxation is influenced in a similar way by the same parameters that influence creep despite potential differences between the two. Moreover, relaxation is predicted from functions that link it to creep. Whereas it is easy to find published literature on creep of concrete, little can be found on the relaxation behaviour of concrete.

More research is therefore needed to understand the factors that influence relaxation and its prediction. This study aimed at comparing the influence of selected factors on creep and relaxation to establish a correlation if any between the two. The parameters investigated include: w/c ratio, age of loading, initial stress-strength ratio and aggregate content. Tensile relaxation tests are difficult to perform and hence prediction models are often relied upon to predict relaxation. Two simplified approximate methods are suggested in the fib Model Code to obtain the relaxation function from the creep function. The Age Adjusted Effective Modulus method (AAEM) given in the MC2010 and the Approximate Relaxation Function (ARF) in MC90-93 are mentioned. Values from the two were compared with experimental relaxation results to verify their accuracy in predicting the relaxation potential of selected concretes.

Results show that tensile relaxation of concrete, similar to creep is affected by parameters such as w/c ratio, age of loading and aggregate content. It is observed that the magnitude of tensile relaxation, with other factors held constant, was found to reduce with a reduction in w/c ratio, increase in age and increase in aggregate content. The initial stress-strength ratio does not seem to have a significant influence on relaxation. However, an increase in the initial stress leads to a small and un- proportional increase in relaxation.

The use of models linking the creep function and relaxation function to predict low-age relaxation is probably valid. Although both the AAEM and ARF methods yield fairly good predictions of relaxation, the simplicity of the AAEM makes it a better option.

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## List of relevant symbols and abbreviations

$t$	Time, in days
$t_o$	Age at loading, in days
$t_c$	Age when drying begins, end of moist curing, in days
$\sigma(t)$	Stress at time t
$\varepsilon_c(t)$	Total strain at time t
$\varepsilon_{cn}(t)$	Stress independent strain total strain at time t
$\varepsilon_{c\sigma}(t) = \varepsilon_c(t) - \varepsilon_{cn}(t)$	Stress dependent strain
$E_c(t_o)$	Elastic modulus of concrete at time $t_o$
$E_{c28}, E_{ci}$	Elastic modulus of concrete at time $t = 28$ days
$J(t, t_o)$	Creep or compliance function
$R(t, t_o)$	Relaxation function
$\xi(t, t_o, t_1)$	Redistribution function
$\chi(t, t_o)$	Aging coefficient
$RH\%$	Relative humidity
$h$	Relative humidity (range 0-1) in GL2000 and B3 models
$2A_c / u$	Notional size of the concrete member in mm, in CEB MC90 model
$V / S$	Volume to surface ratio of the concrete member, in GL2000 and B3 models; $(2V/S) =$ effective thickness
$f_{ck}$	Characteristic strength of concrete
$\phi$	Creep Coefficient
$\psi$	Relaxation coefficient
$Re$	Residuals
$V$	Coefficient of variation
$V_m$	Coefficient of variation for model prediction
AAEM	Age Adjusted Effective Modulus Method
ARF	Approximate Relaxation Function (Not a universal abbreviation)

# 1 INTRODUCTION

---

## 1.1 Background and problem statement

Concrete structures experience numerous stresses from the time they are built. These stresses emanate from various sources and can either be compressive, tensile, shear or torsional. A concrete structure should be able to resist these stresses. Some of these stresses are externally induced by applied loads or actions. However, the stresses can also be self-induced mainly by volumetric changes in the material and thermal strains (Hilsdorf and Mueller, 1999; Østergaard *et al.*, 2001). At the forefront of these volumetric changes is shrinkage. Shrinkage is the reduction in the volume of concrete with time mostly due to loss of moisture (Gilbert and Ranzi, 2010). When shrinkage is restrained, tensile stresses are induced in the concrete (Neville, 2002; Banthia and Gupta, 2008; Hossain and Weiss, 2004). Failure of the concrete to withstand these stresses leads to effects such as cracking (Bissonnette and Pigeon, 1995).

When tensile stresses are continuously applied to concrete, it exhibits tensile creep. Creep is the deformation of a structural concrete member under sustained stress. If strain in the concrete member is restrained, there is a reduction in the self-induced stress resulting from what is known as stress relaxation. Stress relaxation is defined as the reduction in stress when the imposed strain is sustained. If the restrained stresses are tensional, the result is what is known as tensile relaxation. When stresses are relaxed, the ultimate tensile strength of the concrete may not be exceeded and thus the onset of cracking may be prolonged (Neville, 2002; Bentur and Kovler, 2003). If the amount of induced tensile stress lost through relaxation is estimated, the onset of cracking can be modelled. A good understanding of the various factors which affect tensile relaxation is therefore critical in ensuring durable and economical design of restrained structural elements. In the same way, mix design can be optimised to ensure the benefits of relaxation are maximised.

In some literature creep and relaxation are said to be influenced in the same way by the same parameters (Hilsdorf and Mueller, 1999). Although creep and relaxation of concrete are related and may be influenced by similar factors, they may in fact be different. Generally, the factors that influence the behaviour of concrete in creep are fairly well understood compared to those that affect relaxation. The influence of certain factors on relaxation of concrete is still unclear. For example, different authors have different opinions on the effects of initial stress-strength ratios on relaxation. In addition, researchers such as Marimoto and Kayonagi (1994) found that relaxation is proportional to the initial stress-strength ratio (in the range under 80%) while Gutsch and Rostasy (1994) found no proportionality.

The study aimed at investigating if selected parameters would affect relaxation and creep of the same concrete in the same way. The parameters investigated include: w/c ratio, age of loading, initial stress-strength ratio and volume of aggregates. The first two parameters were investigated on both creep and relaxation samples. Experiments investigating the influence of initial stress-strength ratio were carried out on relaxation samples. Past literature was relied upon to make comparisons with creep behaviour of concrete at varying stress levels. In the same way, the influence of aggregate content on relaxation was investigated by comparisons with recent and past studies such as Masuku (2009) and Chilwesa (2012).

Relaxation tests are rarely carried out. This is partly because of the difficulty in carrying out such tests. Very often, prediction models are relied upon to estimate the relaxation potential of concrete. Moreover, most attempts at predicting the relaxation behaviour of concrete use creep functions (Pane and Hansen, 2001). Atrushi (2003) states that most theoretical studies on self-induced stresses use creep properties for modelling.

Creep and relaxation can be related and solved by equations linking their respective functions i.e. the compliance function  $J(t, t_0)$  and the relaxation function  $R(t, t_0)$ . The compliance function represents the strain response at a certain time  $t$  due to a constant unit stress applied at age  $t_0$ . The relaxation function represents the stress response at time  $t$  due to constant unit sustained strain applied at age  $t_0$ . The creep and relaxation functions are linked in the form of equations such as the Volterra integral equations (Sassone and Chiorino, 2005). These equations are, in their basic form, derived from application of methods developed in the theory of linear viscoelasticity. A viscoelastic material is both viscous and elastic (Whitman, 1971). Concrete is said to be a viscoelastic material. Another assumption made in the derivation of the named equations is the so called principle of superposition. The principle states that: a strain produced by an increment of stress applied at any time  $t$  is not affected by any stress applied either before or later (Gilbert and Ranzi, 2010).

Numerical procedures and computer programmes have been developed to solve the integral equations to obtain the relaxation or creep function (for example, Bazant, 1972a). However, the numerical procedures suggested are not favoured for use in practice (Bazant and Kim, 1979b). Simplified approximate equations are preferred for quick calculations (Bazant and Kim, 1979b; Bazant *et al.*, 2013). The first simplified model used was the effective modulus method. However, the accuracy of the method is poor because ageing of the material is not considered. To reduce on the error in the effective modulus method, approximate methods were developed. Two of these simplified approximate methods were used in this study. The first simplified tool used to predict the relaxation function is the Age Adjusted Effective Modulus (AAEM) method given in MC90-93 and MC2010. The AAEM method can also be

expressed in terms of the relaxation and creep coefficient as presented by Bazant (1972b) and given in Mueller and Haist (2009). The second model is the Approximate Relaxation Function (ARF) suggested by Bazant and Kim (1979b) and adopted in MC90-93. A more recent version of the ARF has been presented in Bazant *et al.* (2013). These simplified models link the relaxation and creep compliance functions. Either one of the functions can be obtained if the other is known.

Normally, the creep compliance function is easily verified by experimental work. However, the equivalent relaxation function is almost exclusively determined by solving equations relating the two above mentioned functions. Chiorino (2005) reported that there is a lack of significant experimental results to check the derived relaxation functions. Atrushi (2003) states that it is more appropriate to employ relaxation functions derived from relaxation experiments as opposed to using creep functions to derive the relaxation function. In this study, the models mentioned were tested by comparisons with results from relaxation experiments.

## 1.2 Research significance

In earlier concrete design codes (and in practice), emphasis was placed on the safety and serviceability requirements of structures. Today, new design codes such as the Model Code 2010 for concrete structures are paying attention to durability and sustainability requirements of structures. Creep, relaxation and shrinkage are some of the most important factors affecting the long term performance of old and new concrete structures. In particular, the three factors affect the durability of concrete structures in addition to the safety and serviceability requirements. Whereas creep is responsible for increased deflections in concrete structures, relaxation plays a dominant role in the proper functioning of bonded overlays and patch repairs commonly used in repairs. Underestimation or overestimation of the two (creep and relaxation) can have both economic and structural consequences.

The relaxation behaviour of concrete, despite its importance, is an area where not much research has been dedicated. Importantly, relaxation data available is often not backed up by sufficient experimental work (Atrushi, 2003; Chiorino, 2005). The experiments carried out in this study are expected to be a valuable addition to the database and will help in understanding the relaxation behaviour of concrete.

### 1.3 Aim of the research

The main aims of the study are:

- i. To investigate how key intrinsic and extrinsic factors which affect concrete creep, will affect the tensile relaxation behaviour of similar concrete. This was done to establish the general validity of the correlation between creep and relaxation.
- ii. To compare relaxation values from prediction models to experimental work with the underlying aim of investigating if the models can be used to predict early-age relaxation.

### 1.4 Research objectives

In order to achieve the aims mentioned above, the following research objectives were suggested. This research aims to

- i. Investigate and compare the effects of age at loading, w/c ratio on relaxation and creep.
- ii. Investigate the influence of aggregate content on relaxation and compare it with that of creep.
- iii. Investigate the influence of stress-strength ratio on relaxation of concrete and make comparisons with its influence on creep.
- iv. To verify the accuracy of the Age adjusted effective modulus (AAEM) method given in MC90-93 and MC2010 in estimating the relaxation potential of concrete.
- v. To verify the accuracy of the Approximate relaxation function (ARF) given in MC90-93 in estimating the relaxation of concrete.

### 1.5 Scope of the research

- i. This research focuses on normal strength concretes commonly used in the construction industry. Special concretes such as high strength concretes and fibre reinforced concretes were not investigated.
- ii. CEM-1 52.5 was used as binder. The influence of extenders was not investigated because of time constraints.
- iii. Water-cement ratios varied from 0.45 to 0.65. The values of w/c ratio were used because most concretes made in industry lie in that range (Alexander and Beushausen, 2009).
- iv. Other parameters investigated include: age of loading, aggregate content and initial stress-strength ratio.

- v. Temperature, relative humidity and size effects are important input parameters in most creep prediction models. However, they were not investigated because of inadequacies in varying them in the laboratory.
- vi. For relaxation prediction, the approximate models used are those recommended in MC90-93 and MC2010. These are: the Age adjusted effective modulus (AAEM) and the Approximate Relaxation Function (ARF).

## 1.6 Limitations of the study

- i. The main limitation of the study was shortage of equipment to carry out sufficient creep and tensile relaxation tests.
- ii. Secondly, because creep and relaxation experiments are time dependant, investigating all the parameters that influence creep and relaxation was not feasible. To study the influence of aggregate content, creep and relaxation experimental values were compared with past literature.
- iii. Regarding initial stress-strength ratio, experiments were carried out only for relaxation. For the influence of initial stress-strength ratio on creep, past literature was relied upon.
- iv. Another limitation acknowledged is the fact that while tensile relaxation is a result of tensile creep, creep tests were carried out in compression. The assumption made is that the two may be similar at low stress levels.
- v. When verifying the Approximate Relaxation Function (ARF), compliance functions that could not be obtained experimentally (because of varying time of load application) were determined using the MC90-99. With that procedure, an inconsistency is acknowledged.
- vi. Creep tests were carried out at stress levels below 30% while the longest running relaxation tests were carried out at 40% and 80% initial stress-strength ratio. The possible influence of that discrepancy is mentioned in Chapter five.

## 1.7 Outline of thesis

This thesis document is organised in six chapters. Chapter one presents an overall introduction to the study and justifies the research. In Chapter two, a detailed synthesis of the relevant literature is presented. Chapter three discusses the methodology followed to achieve the aims of the research. Chapter four discusses the results of the experimental work. In Chapter five, selected approximate relaxation prediction models are evaluated by comparisons with experimental values. Chapter six is used to present conclusions on the study. Recommendations for future work have also been suggested in this chapter.

## 2 LITERATURE REVIEW

---

### 2.1 Introduction

Chapter one introduced this study including its background, benefits and scope. Creep and relaxation of concrete are central in this research. In this Chapter, literature on the relaxation and creep behaviour of concrete is discussed. The micro mechanisms, beneficial and adverse effects of both creep and relaxation on concrete are mentioned. In addition, the factors influencing the magnitude of both are also presented. Also presented are methods of predicting and testing creep and relaxation. Particular attention is given to the relaxation function and its relation to the creep function. Effort is also spent on to the influence of selected parameters on creep and relaxation. Lastly, a summary is presented on the creep and relaxation behaviour of concrete.

### 2.2 Creep of concrete

#### 2.2.1 Introduction

Creep is defined as the continuous deformation of a body under constant or controlled load (Figure 2.1). Concrete deformations can be classified as time-independent and time dependent (Atrushi, 2003). The time-independent deformations happen as soon as the load is imposed on the structure. The deformations that take place after the immediate are termed time-dependent and chief among them is creep.

When a concrete is loaded, the total deformation comprises of an elastic, creep and shrinkage component (Townsend, 2003). Therefore, to determine the creep component of the deformation, the elastic strain and shrinkage must be subtracted off the total deformation. Some creep strain is recoverable upon removal of load while some is permanent. This means that when the applied load is removed, the strain reduces by an amount equal to the elastic strain at that age, which is generally lower than the elastic strain on loading (Neville, 2002).

In Figure 2.1, the gap between zero time and initial recorded time  $t_o$  shows that the stress is applied after some time, in this case  $t_o$ . At time  $t_o$ , there is an immediate increase in strain called the initial elastic strain due to instantaneous loads. Beyond the time  $t_o$ , the strain in the concrete is mainly caused by creep. The figure also shows after long durations of loading, creep strains may exceed the initial elastic strain. As shown in Figure 2.1, creep develops rapidly after initial loading until it reaches its limiting value as the time of loading approaches infinity. Approximately half of the final creep develops in the first 2-3 months, while about 90% develops after 2-3 years (Gilbert and Ranzi, 2010).

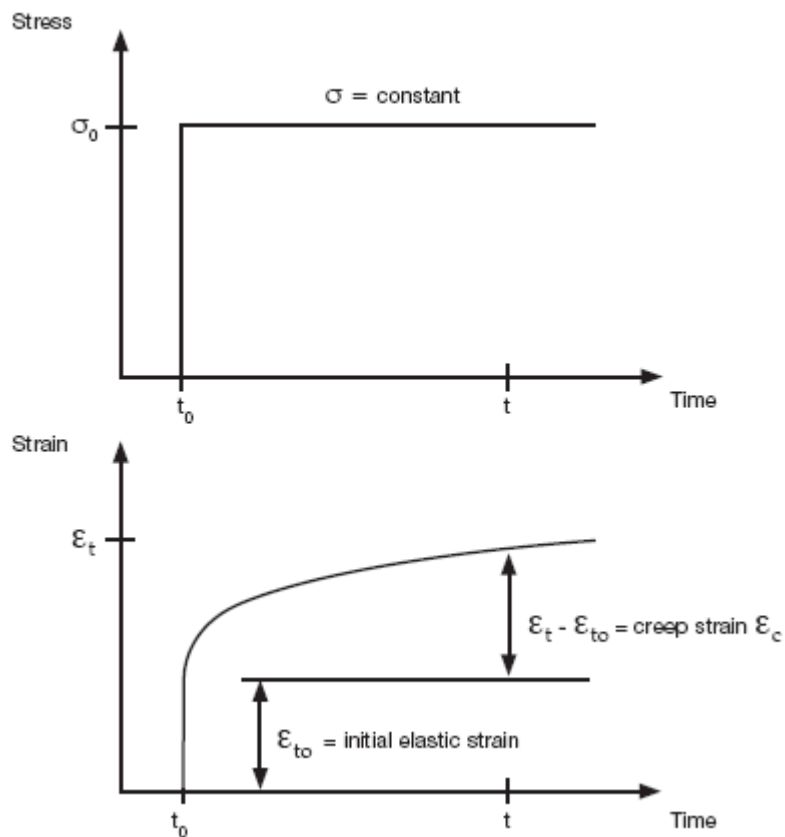


Figure 2.1: Characteristics of creep: time-dependent increase in strain under constant stress (Alexander and Beushausen, 2009).

### 2.2.2 Micro-mechanisms of creep

The creep of concrete is thought to be caused by several different complex mechanisms far from being fully understood (Gilbert and Ranzi, 2010). Gilbert and Ranzi (2010) explain that the hardened cement paste consists of a solid cement gel containing numerous capillary pores. Many researchers accept that creep of concrete is largely caused by the creep of the hydrated cement paste. This is because of the assumption that normal aggregates show little or no creep deformation. The role of aggregates according to Neville (2002) is that of restraint.

More recently Mucambe (2010) grouped the mechanisms of creep into: moisture diffusion, micro-cracking, delayed elastic strain and structural adjustment. They are discussed in the next sections.

## Structural adjustment

Structural adjustment can occur in the following ways i.e. viscous flow and local bond breakage (Mucambe, 2010). Viscous flow involves the sliding of colloidal sheets (of calcium silicate hydrates) in the cement gel between the layers of absorbed water (Gilbert and Ranzi, 2010). Feldman and Sereda (1968) similarly explained that creep is caused by movement of water i.e. interlayer water between gel layers. This movement causes gel layers to slide over each other leading to micro structural changes. Eventually, deformation occurs because of the sliding of the layers in the cement paste. Figure 2.2 shows a schematic of the Feldman and Sereda model

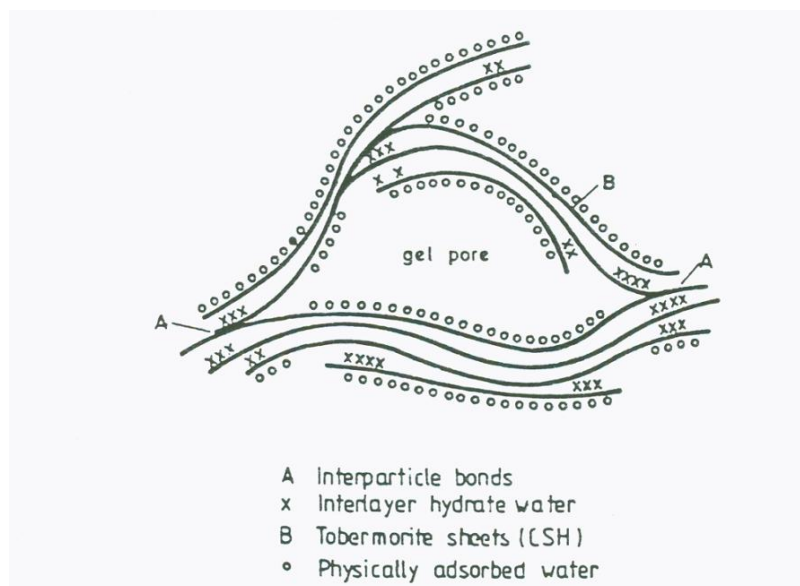


Figure 2.2: Schematic of cement paste microstructure (Feldman and Sereda, 1968).

When local bond breakage occurs, particles are moved and reconnected, this leads to increase in strains.

## Moisture diffusion

In this case, water is said to move from zones of high pressure where the stresses are applied to areas of low pressure. The water moves from smaller to larger pores Mucambe (2010). Movement of moisture occurs in the following ways:

- i. The movement of the capillary water causes deformation of the concrete
- ii. The movement of adsorbed water subject to attractive surface forces
- iii. The movement of interlayer water present in narrow spaces and held by the attractive forces of the solid surfaces.

The first two processes are reversible while the last example may not be reversible (Mucambe, 2010)

### **Micro-cracking**

Micro-cracking is caused by local fracture within the cement paste involving the breakdown of the physical bond. Most of the micro-cracking is common or begins in the interfacial transitional zone (ITZ) between the paste and aggregate. At stresses above 40% of the ultimate stress, progressive micro cracking causes strain deformations which are much larger than the creep strains (Alexander and Beushausen, 2009).

### **Delayed elastic strain**

As mentioned in Section 2.2.1, most of the creep deformation is experienced by the paste. However in some circumstances, the aggregates can deform as well. Delayed elastic strain can be explained as the deformation caused when elastic aggregates act on the viscous paste after load removal (Gilbert and Ranzi, 2010). This is referred to as the recoverable component of creep. Another way of explaining delayed elasticity is by considering the cement paste and aggregate as perfectly bonded together. As the concrete section partly fails under increased load, loads are transferred from the cement paste to the aggregates. Since they have some elasticity, aggregate will deform, and this contributes to creep. This process is reversible.

### **2.2.3 Components of creep**

As mentioned in Section 2.2.1, creep is part of the total strain on a concrete subjected to load. The components of creep can generally be separated into two: the basic creep and the drying creep (Figure 2.3). These two components are based on the influence of relative humidity (Atrushi, 2003). The basic creep is creep occurring at constant moisture content or when there is no moisture exchange with the environment. Drying creep is caused by additional drying of the concrete.

However, Gilbert and Ranzi (2010) state that the basic creep and drying creep components are sub sets of two creep strain portions: the recoverable and irrecoverable (permanent) portions. The irrecoverable creep portion is larger than the recoverable portion as shown in Figure 2.4.

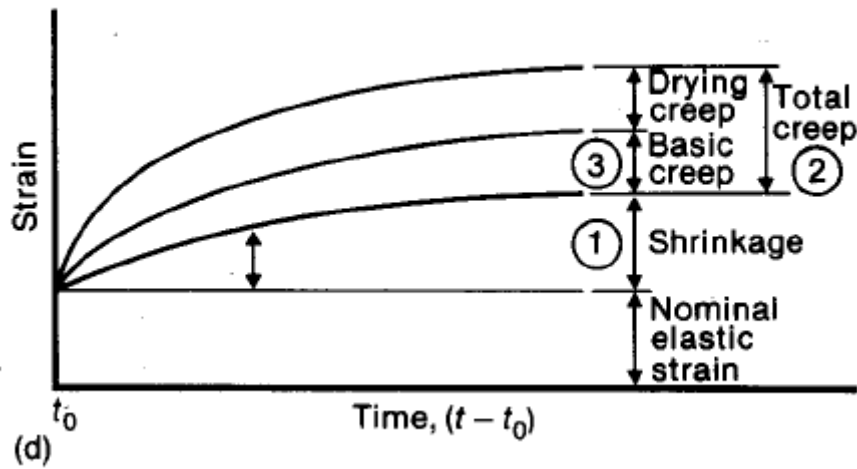


Figure 2.3: Time dependent deformation in concrete subjected to a sustained initial load at time  $t_0$  (Atrushi, 2003 cited in Neville, 1995)

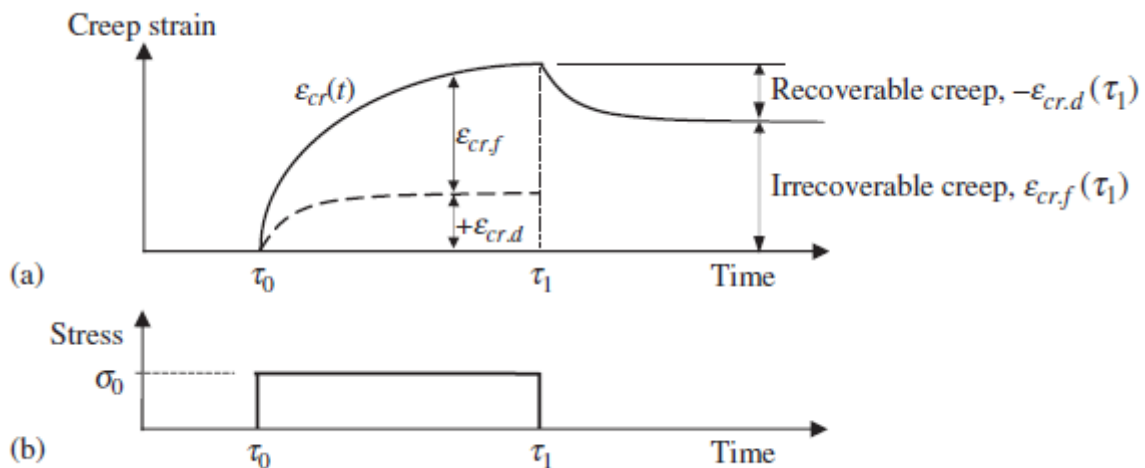


Figure 2.4: Recoverable and irrecoverable creep components (Gilbert and Ranzi, 2010)

The recoverable part of creep can also be referred to as the *delayed elastic strain*,  $\epsilon_{cr,d}(t)$  while the irrecoverable creep portion is also referred to as *flow*. As discussed in the mechanisms of creep, the delayed elastic strain is said to be caused by elastic aggregates acting on the cement paste after unloading (Gilbert and Ranzi, 2010).

The flow component of creep is then further grouped into two components: the *rapid initial flow*,  $\epsilon_{cr,fi}(t)$  and the 'remaining flow'. The rapid initial flow occurs in the first 24 hours after loading while the remainder of the flow occurs thereafter. While the rapid initial flow depends on the age at which the specimen is loaded, the remaining flow depends on the

relative humidity. According to Gilbert and Ranzi (2010), it is this ‘remaining flow’ component of the flow that is further divided into the basic flow,  $\varepsilon_{cr fb}(t)$  and drying flow  $\varepsilon_{cr fd}(t)$  or basic creep and drying creep as they are referred to in most literature.

### 2.2.4 Creep expressions

The most common creep expressions are specific creep  $C_c$  and creep factor or creep coefficient ( $\phi$ ). The specific creep is defined as the creep strain  $\varepsilon_c$  per unit stress ( $\sigma$ ).

$$C_c = \frac{\varepsilon_c}{\sigma} \quad (1)$$

For low stress levels of up to 40% of ultimate strength a linear relationship exists between applied stress and creep of concrete (Alexander and Beushausen, 2009) Specific creep provides a basis for comparing the creep behaviour of concretes at different stress levels. Otherwise concrete creep is compared on the basis of equal stress/strength ratios in which case the factors which affect the strength of concrete at the time of loading are taken into account. The creep factor ( $\phi$ ) or creep coefficient is defined as the ratio of creep strain ( $\varepsilon_c$ ) to the initial elastic strain ( $\varepsilon_o$ )

$$\phi = \frac{\varepsilon_c}{\varepsilon_o} \quad (2)$$

From Equation 1 and 2 a relation can be obtained between creep coefficient and specific creep

$$\phi(t, t_o) = C_c(t, t_o) E_c(t) \quad (3)$$

Another term used to define and quantify creep is the *creep compliance function*  $J(t, t_o)$ . It is defined as the sum of the instantaneous and creep strains at time  $t$  produced by a sustained unit stress applied at time  $t_o$ . It can be shown in the equation below

$$J(t, t_o) = \frac{1}{E_c(t)} + C(t, t_o) = \frac{1}{E_c(t)} [1 + \phi(t, t_o)] \quad (4)$$

Since the creep function defines strains caused by a unit stress, the total strain caused by a constant sustained stress  $\sigma_c(t)$  first applied at age  $t_o$  is given by

$$\varepsilon_c(t) + \varepsilon_{cr}(t, t_o) = J(t, t_o) \sigma_c(t) = \frac{\sigma_c(t)}{E_c(t)} [1 + \phi(t, t_o)] = \frac{\sigma_c(t_o)}{E_e(t, t_o)} \quad (5)$$

Where  $E_e(t, t_o)$  is known as the *effective modulus* and can be given by:

$$E_c(t, t_0) = \frac{E_c(t_0)}{[1 + \phi(t, t_0)]} \quad (6)$$

### 2.2.5 Structural effects of creep

The effects of creep on a structure can be either beneficial or detrimental (Addis and Owens, 2005). Creep transmits some degree of ductility to concrete members. This ductility is enhanced by the interface between hardened cement paste and aggregate particles. Concrete structures would be too brittle for use without this ductility. Creep reduces the internal actions caused by imposed deformations in both reinforced and plain concrete structures (Gilbert and Ranzi, 2010). In that way creep is said to be a necessary phenomenon for concrete structures. These unintentionally imposed deformations include support settlement, shrinkage and thermal gradients.

In reinforced concrete structures such as columns, creep causes a redistribution of stresses between the concrete and the steel. When axially loaded, the steel and concrete strains are identical because of compatibility and the bond between them. However, as the concrete creeps, it compresses the steel hence increasing the steel's compressive stress. Due to equilibrium the compressive stresses in the concrete are decreased to balance the increase in the compressive stress in the steel. As the compressive stresses in the steel increase, loads will gradually be transferred to the steel in increasing proportion (Gilbert and Ranzi, 2010). This is illustrated in Figure 2.

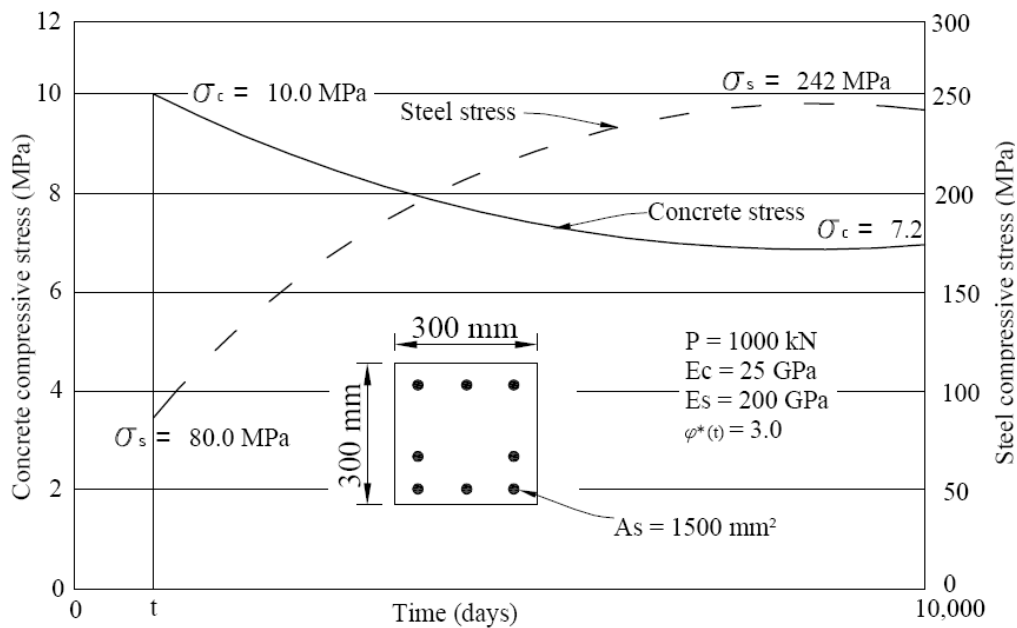


Figure 2.5: Variations of stresses with time due to creep on a symmetrically axially loaded column section (Gilbert and Ranzi, 2010).

The most obvious detrimental effect of creep is that it increases deformations in structures leading to: increased deflections of members, loss of prestress forces and buckling of long structural members such as columns (Addis and Owens, 2005). The noticeable effects of creep (and shrinkage) include the damage caused to secondary building elements which are usually not loadbearing. For example, claddings attached to creeping columns may suffer spalling or bowing, while partition walls of brick or other materials may crack as the supporting concrete members such as beams or slabs continues to deflect due to creep (Alexander and Beushausen, 2009). To avoid costly repairs, engineers should design for creep deformations.

### **2.2.6 Factors affecting creep**

Creep is affected by a number of factors which can be grouped into: intrinsic and extrinsic factors (Addis and Owens, 2005). According to Atrushi (2003) creep behaviour of concrete is strongly affected by microstructural changes of the hardened cement paste matrix. For well-cured concrete specimens, the main variables influencing the physical properties (strength, modulus, creep, shrinkage, etc.) are the porosity and the pore size distribution in concrete (Sellevoid, 1969 cited in Atrushi, 2003). This reason explains the low creep in HSC (High strength concrete) as compared to NSC (Normal strength concrete). The microstructure of HSC has a lower porosity and finer pore structure meaning it has a lower capillary porosity. HSC in addition has a more uniform hardened cement paste matrix and a different structure of the aggregate paste interface (Müller and Rübner, 1995; Atrushi, 2003). All the factors mentioned lead to a structure with a high stiffness.

#### **2.2.6.1 Intrinsic factors**

Intrinsic factors are those relating to the properties of the concrete itself. They include water-cement ratio, moisture content, cement type, cement extenders, admixtures, the content and properties of aggregates. They are discussed in more detail in the following sections.

##### ***Water-cement ratio***

The water-cement ratio is the ratio of water in a concrete to the amount of cement. The water cement ratio affects the stiffness of the cement paste. A low water cement ratio increases the stiffness of the concrete hence increasing its ability to resist deformation (Neville, 2002). Movement of inter-layer water will also be reduced when the porosity of the cement paste is reduced subsequently reducing creep. Figure 2.6 illustrates the influence of w/c ratio on the magnitude of creep. Atrushi (2003) citing Lorman (1940) reports a relation between the w/c ratio and creep. Creep is said to be proportional to the square of the w/c ratio if other factors remain constant. In past studies, Ostergaard *et al.* (2001) carried out creep tests on concretes

with w/c ratios between 0.32 and 0.5. Results confirmed that creep decreased with decreasing w/c ratio.

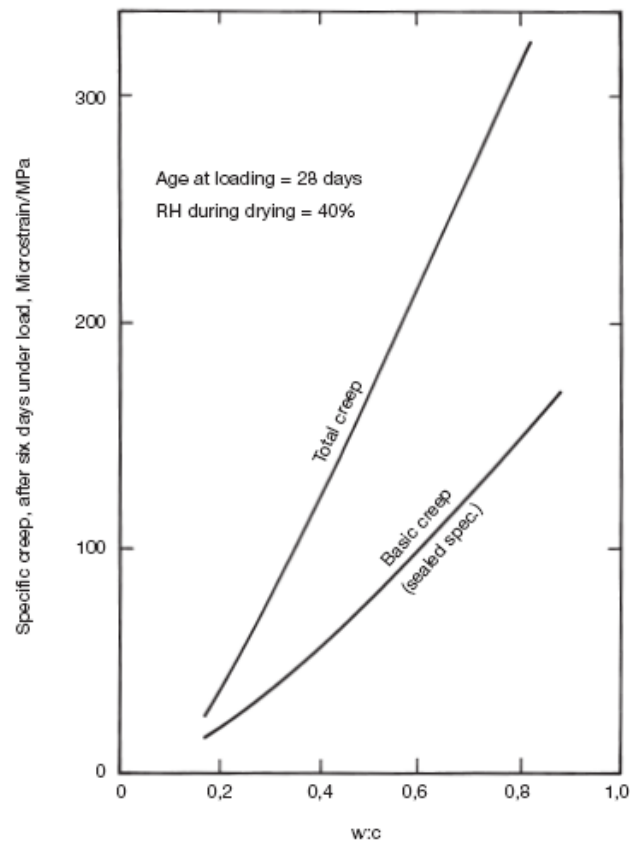


Figure 2.6: The effect of water-cement ratio on creep of the cement paste (Ruetz, 1965 cited in Alexander and Beushausen, 2009)

### ***Moisture content***

Because of the fact that it originates from the cement paste, the basic observation is that creep will decrease with decreasing moisture content (Hilsdorf and Mueller, 1999; Atrushi, 2003). This is attributed to the fact that the creep strains are caused by movement of water in the cement paste. When the rate at which water is lost before loading is high such as in areas with low relative humidity, the creep strains are reduced (Alexander and Beushausen, 2009).

### ***Cement type and cement extenders***

The cement type affects the creep of concrete. Cements with a higher tricalcium aluminate ( $C_3A$ ) content or lower tricalcium silicate ( $C_3S$ ) content experience higher creep. This is due to the fact that  $C_3A$  has a high water demand and increases temperature development during hydration which increases loss of water and subsequently increasing creep (Alexander and Beushausen, 2009).

Extenders such as condensed silica fume (CSF), which make the cement paste denser reduce the creep of the concrete. The effect of CSF (and other extenders) at early ages may however be different and has yielded contradictory results from past research (Alexander and Beushausen, 2009; Atrushi, 2003).

Atrushi (2003) found that an increase in SF content increased the tensile creep strains measured. SF replacement levels varied from 0 to 15%. This observation was made for unsealed specimen tested at the age of 3 days. For sealed specimen, there was no influence of SF reported for replacement ranges between 0 to 10%. Results for unsealed tests are presented in Figure 2.7.

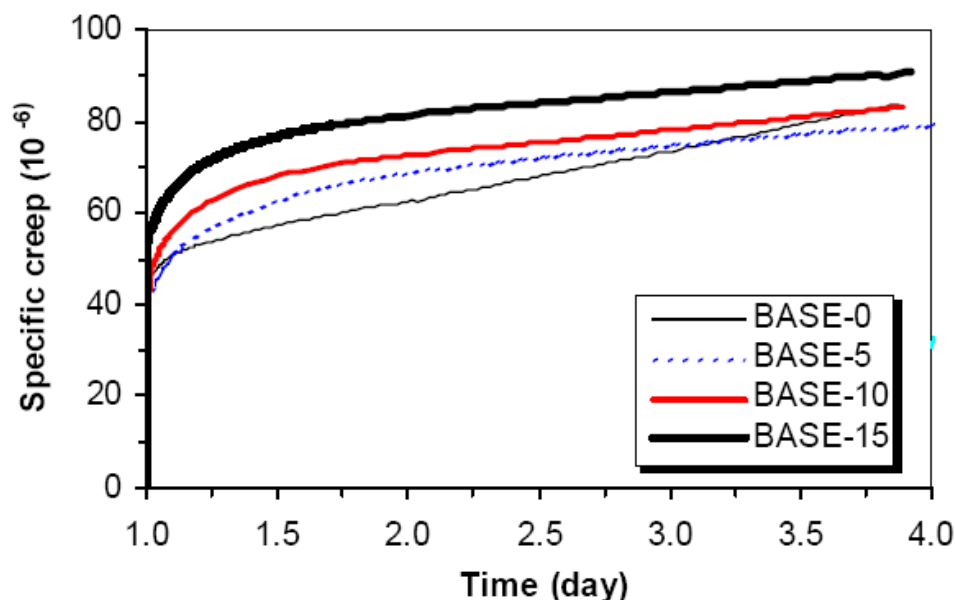


Figure 2.7: Influence of silica fume on tensile creep of concrete (Atrushi, 2003)

Results for testing up to the age of four days are presented. At low ages, the magnitude of creep is highest and hence comparisons can be made between different concretes. Similar observations were reported by Kovler *et al.* (1999) and Bissonnette and Pigeon (1995). The two studies noted that at early ages, SF increases the tensile creep of concrete with a similar w/c ratio. No explanation was forwarded for this behaviour.

Pane and Hansen (2001) presented their findings on the influence of extenders on creep in terms of the creep compliance function. This function represents the total elastic and creep strains produced by a sustained unit stress. In this study, FA and GGBS were found to reduce the creep compliance function while SF increases the compliance function, Figure 2.8. The reasons for this trend with regards to SF are discussed above.

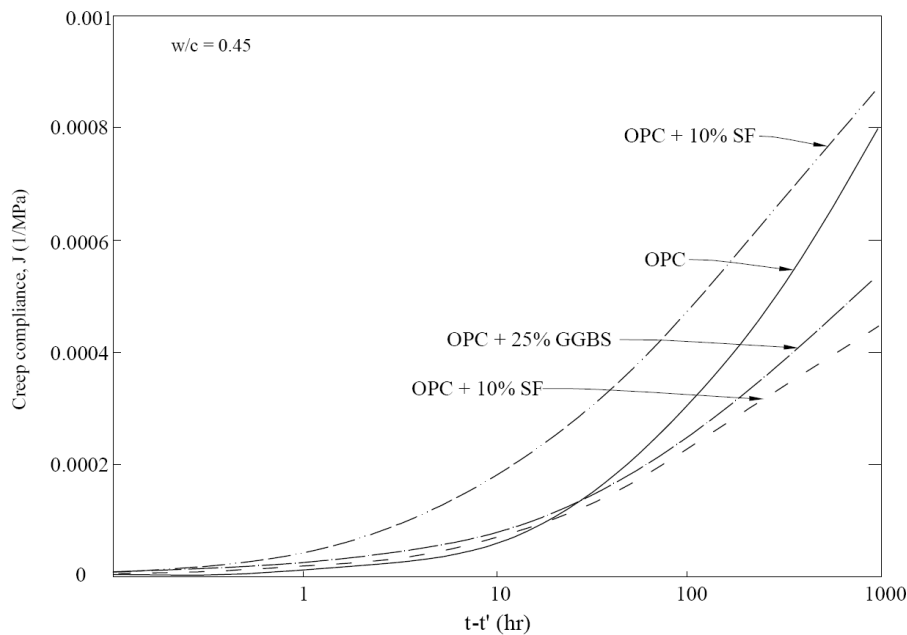


Figure 2.8: Influence of extenders on creep (Pane and Hansen, 2001)

Neville (2002) summaries the influence of cement types on creep as follows:

- i. The type of cement affects the creep of concrete as long as it affects the strength of the concrete
- ii. At the time of loading, creep is influenced by the rate of strength development of the concrete and its constituents
- iii. In cases where they affect the hydration process, the creep of concrete is affected. For example, FA and GGBS increase long term hydration hence increase long term strength gain and subsequently long term rate of creep is reduced. CSF on the other hand reduces drying creep because it takes up water from the cement gel during hydration.

### ***Admixtures***

Neville (2002) states that water-reducing admixtures and super plasticisers have been found to increase basic creep but that no reliable pattern has been established. Alexander and Beushausen (2009) state that the effect of admixtures on creep appears to be highly variable and depends on the following factors: on the specific cement and admixture chemistry, age at loading, relative humidity of the environment and time under load. The reducing of the water content or paste content does not necessarily reduce creep according to some studies done in South Africa. In summary, creep may be differently by different admixtures depending on the conditions and hence laboratory tests should be carried out to test the effect of a particular admixture used.

### Aggregate properties and content

Aggregates affect creep of concrete in two ways: depending on the content and aggregate type. Depending on their content, aggregates reduce creep when they are increased in a concrete mix. Aggregates may absorb the water in the concrete hence reducing both the w/c ratio and water content thereby reducing creep. In addition, aggregates provide restraint to movement of paste hence reducing creep. Figure 2.9 shows the effect of aggregate content on creep of concrete. The higher the aggregate content, the lower the cement paste and hence the lower the creep strains.

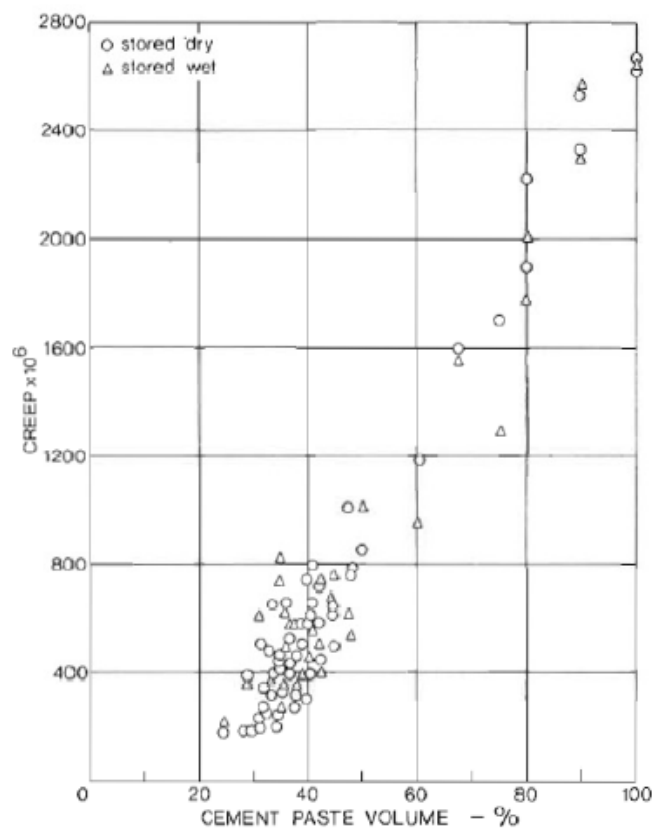


Figure 2.9: The effect of cement paste on creep (Wagner, 1958, cited in Mucambe, 2010)

With regards to aggregate type, the elastic modulus of the aggregate is the most important factor affecting the creep of concrete. Aggregates with a higher elastic modulus are stiffer and hence provide a greater restraint to paste movement. This resistance to paste movement reduces the creep of concrete. The effects of aggregate stiffness are illustrated in Figure 2.10. According to the figure, beyond an aggregate effective modulus of 70 GPa, creep of concrete is not affected.

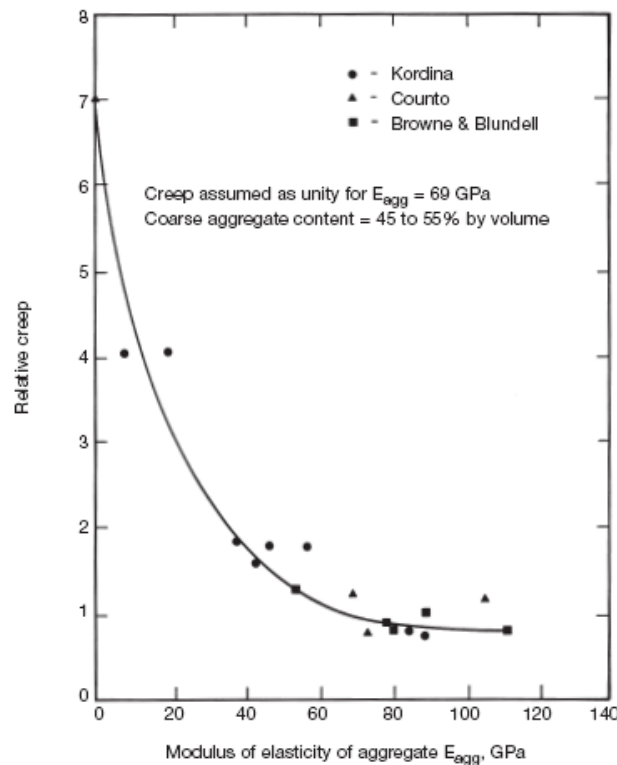


Figure 2.10: The effect of aggregate stiffness on creep potential of concrete (Alexander and Beushausen, 2009)

### 2.2.6.2 Extrinsic factors

Extrinsic factors relate to the external conditions such as the environment and loading conditions. They include: member geometry and size, drying conditions, stress-strength ratio, curing and age of loading, time under load and the state of stress.

#### *Member geometry and size*

If the drying conditions are constant, a bigger concrete specimen will creep less than a similar smaller specimen. This is attributed to the fact that drying is much slower in a bigger specimen. A thin specimen will lose water easily to the atmosphere because the water has less distance to travel (Hilsdorf and Mueller, 1999). When specimens are in sealed conditions the size effects do not matter (Atrushi, 2003). Generally, the shape and size parameters can be expressed in terms of the volume to surface ratio of the concrete member.

The same parameters are also expressed as the effective thickness which is the ratio of twice the cross-sectional area divided by the exposed perimeter  $h = 2A/u$ . Figure 2.11 shows the influence of relative humidity and effective thickness on the creep coefficient.

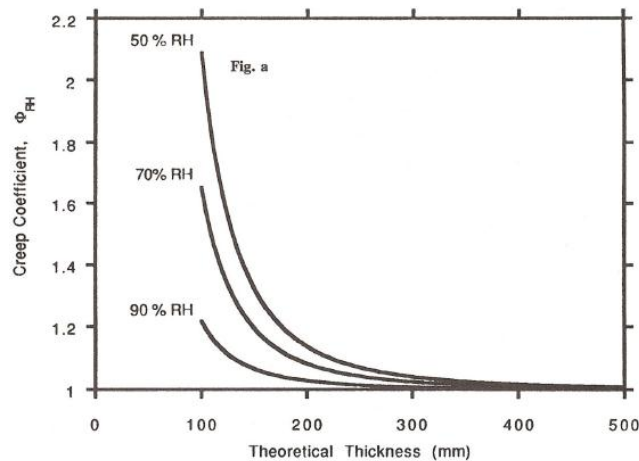
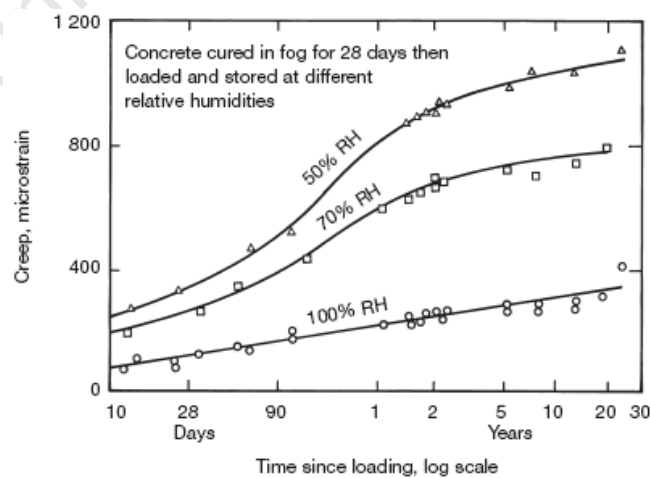


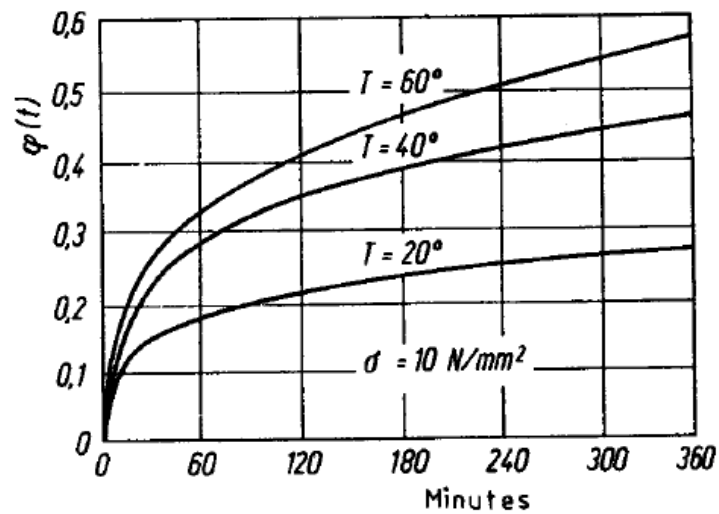
Figure 2.11: The influence of specimen size on creep coefficient (CEB FIP 1990, cited in Mucambe 2010)

### **Drying conditions**

When the relative humidity is high, water is lost to the atmosphere at a lower rate from the concrete specimen and hence reducing the creep of the specimen. The dependence of creep on relative humidity and size reduces with increase in strength (Gilbert and Ranzi, 2010). This can be explained from the view point that strength increases with time. With time, the whole concrete section has undergone sufficient drying with most of the water available for hydration used up (Neville, 2002). The temperature has an opposite effect to that of relative humidity with an increase in temperature increasing the creep of concrete because it increases the deformability of the paste and accelerates drying. Figures 2.12 and 2.13 illustrate the effects of drying conditions on the creep of concrete.



Figures 2.12: The effects of drying conditions on the creep of concrete (Troxell *et al.*, 1958 cited in Alexander and Beushausen, 2009)



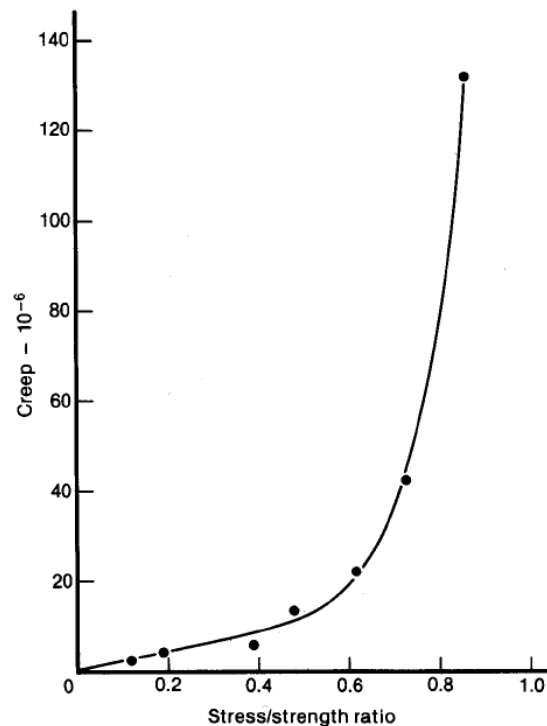
Figures 2.13: Development of creep coefficient for various temperatures for concrete subjected to constant stress (Shkoukani and Walraven, 1993 cited in Atrushi, 2003)

For saturated concrete, curing at the test temperature reduces creep in comparison with heating and loading at the same time. This is because of improved hydration (Alexander and Beushausen, 2009). An increase in temperature will initially increase creep. Eventually, the creep is reduced because an increase in the rate of hydration of the concrete increases its stiffness (Neville, 2002). The initial increase in creep is probably due to loss of water by way of evaporation. Reduction in concrete strength at high temperatures is another reason suggested for the initial increase in creep (Neville, 2002).

### ***Stress-strength ratio***

For low stress-strength ratios, the creep is said to be linear to the initial stress-strength ratio. The assumption of linearity is widely acceptable for concrete in compression. The upper and lower limits of this proportionality vary depending on researcher. Some researcher's put these limits at 0.4 – 0.6 (Mucambe, 2010). Atrushi (2003) and Neville (2002) put this ratio at between 30% and 75% of the concrete strength in compression. For mortars, Neville (2002) puts the upper limit to between 80% and 85%. Above the proportionality limits, creep increases with increase in stress levels at increasing rate due to progression of micro cracks as shown in Figure 2.14.

According to Neville (2002), the development of cracks leads to reduction of effective area resisting the applied load hence increasing local stresses. Subsequently, the local stress is larger than the nominal stress which makes the strain increase at a faster rate than the applied stress making the curve to bend towards the strain axis.



Figures 2.14: Relation between compressive creep to stress-strength ratio for a concrete loaded at 5 days (Neville *et al.*, 1983, cited in Atrushi, 2003).

### ***Curing and age of loading***

Generally, longer curing periods will result in lower creep because the cement is allowed to hydrate without moisture loss leading to a stronger gel. Ultimately the paste is less porous hence reducing on the movement of moisture. Also, the duration of curing affects the creep because, the longer the curing the stronger the ITZ becomes hence leading to high strengths (Mucambe 2010).

The curing method may also affect the creep of concrete. Steam curing for example may lead to a reduction in creep because of alteration of the structure of the gel leading to a more crystalline structure (Mucambe, 2010). The age of loading influences creep indirectly because it is directly linked to the concrete's strength (Neville, 2002). The older the concrete, the less it will creep.

### ***Time under load***

Concrete continues to creep after a long time. Various authors have presented different values with respect to this. According to Alexander and Beushausen (2009), depending on the size of the member, 50% of the 20-year creep will occur after about two to six months under load, while 80% will occur after about one to two years. However, concrete is known to creep for

30 years or more. In other studies, Gilbert and Ranzi (2010) state that half of the final creep develops in the first 2-3 months while about 90% develops after 2-3 years.

### ***State of stress***

Concrete suffers deformation not just by compressive forces, but also by other stress states such as tension, torsion, flexural, dynamic and stress due to Poisson's ratio. Each of the above states of stress can cause creep to the concrete. The tensile creep is discussed in detail in the next section. The torsional creep is said to be of the same magnitude as the compressive creep; the flexural creep is governed by cracking in the tension zone; dynamic creep is influenced by the fatigue behaviour of the concrete. Poisson's ratio creep appears to occur for uniaxial compressive loading, and is approximately the same for short-term loading provided corrections for lateral shrinkage are made (Alexander and Beushausen, 2009).

### **2.2.7 Tensile creep**

Little work has been devoted to the tensile creep behaviour of concrete. The question of whether tensile creep is equal to compressive creep is not fully answered. Carlswärd (2006) citing work by Westman (1999) and Grzybowski (1989) stated that the two are equal at moderate stress levels. However some experimental evidence from the work also showed that tensile creep is higher at lower stress/strength ratios while the difference between the two reduced as stress/strength ratios approached 50%. This observation was linked to micro cracking that is significant for loading in tension even at low stress/strength ratios.

More recently Atrushi (2003) found that compressive creep is higher than tensile creep in the first 24 hours after loading while the tensile creep becomes larger than the compressive creep in the long term. This is due to the fact that the compressive creep rate decreases while that of tensile creep remains constant. Gilbert and Ranzi (2010) stated that to produce equivalent coefficients describing tensile creep, some researchers multiply the creep coefficients measured for compressive stresses by factors in the range of 1 to 3. This implies that the tensile creep is 1 to 3 times the compressive creep.

#### **2.2.7.1 Influencing factors**

Although little research has been devoted to it, it is generally agreed that the creep mechanisms of concrete in tension are potentially different from those of concrete in compression. The age of loading and the drying conditions affect the tensile creep of concrete in the same way they do the compressive creep. Early loading and drying increase the tensile creep of concrete (Gilbert and Ranzi, 2010).

Research by Bissonnette and Pigeon (1995) focused on understanding and characterising the tensile creep of concrete. The influence of different parameters such as w/c ratio, cement type, age of loading and use of fibre reinforcement were studied. Results indicated that tensile creep is significantly influenced by the water-cement ratio, the age of the concrete at loading and the fibre reinforcement. Results from this research showed that tensile creep increases with the w/c ratio and decreases with the age of loading. Silica fume was found to have little influence on tensile creep and the reasons for that were not understood.

Atrushi (2003) studied past works done on tensile creep by a host of researchers including: Westman (1995); Gutsch and Rostásy (1995); Umehara *et al.* (1994); Hauggaard-Nielsen (1997a); Kovler *et al.* (1999); Pane and Hansen (2002); Lange and Altoubat (2002); Østergaard *et al.* (2001) and Hagihara *et al.* (2002). The response of creep to loading age, concrete viscoelasticity, creep rate, water binder ratio and temperature were all found to be remarkably similar to creep in compression. The stress-strength ratio was found to be linear for ratios up to 60% of the concrete strength. Pigeon and Bissonnette (1999) reported the value of proportionality to be up to 50% of tensile strength. These values fall within the ranges of values reported for compressive creep. A maximum value of 75% has been reported as discussed in section 2.2.6.

Generally, the effect of extenders such as FA and silica fume has yielded contradicting results as is the case with compressive creep (Bissonnette and Pigeon, 1995; Atrushi, 2003).

### **2.2.8 Prediction models**

A prediction model aims at predicting the creep, relaxation or shrinkage of a structural element, while incorporating various parameters to represent physical mechanisms (Mucambe, 2010). Most prediction models are contained in common design codes while a few are proposed in the literature. According to Carlswärd (2006) all the existing creep prediction models were developed on data obtained from compressive creep tests. There are a number of prediction models available for creep prediction. Some of the most commonly used models are summarised in the next sections. The models are also chosen because they are recent and are based on extensive research. Most of them have also been found to be accurate in predicting creep values of concrete (Goel *et al.*, 2007; Fanourakis, 2011; Ahmad and Roy, 2012; Gardner and Lockman, 2001 and Fanourakis and Ballim, 2003). The American Concrete Institute (ACI 209, 2008) guide on the modelling of creep and shrinkage also contains some of the models considered for example, the GL 2000, MC90-99, B3 and ACI209R-92.

### 2.2.8.1 Gardner and Lockman (GL 2000 Model)

Mucambe (2010) reported that the GL 2000 model was developed by Gardner and Lockman in 2001. The model is a modification of an earlier model, the GZ model, proposed by Gardner and Zau in 1993. The most recent version of this model was presented in 2004. It included minor modifications to some coefficients and to the strength development with time equation ( $f_{cmt}$ ). This model is advantageous because the input parameters are those available to the engineer at the time of design. The input parameters for this model are: modulus of elasticity at 28 days, MPa; mean compressive strength at the age of 28 days (cylinder strength), MPa. The cements types are according to ASTM C150 standard. A correction factor is provided to account for loading before drying. Account is also made for the different cement types. In this model the creep is defined in terms of the compliance function, Equation 7. The expressions are extracted from Sassone and Chiorino (2005) and ACI 209 (2008). A number of past studies have found this model to be reliable in the determination of creep values for various concretes (Fanourakis, 2011; Ahmad and Roy, 2012; Gardner and Lockman, 2001 and Goel *et al.*, 2007).

$$J(t, t_0) = \frac{I}{E_{cmt_0}} + \frac{\phi}{E_{cm28}} \quad (7)$$

$$E_{cmt} = 3500 + 4300\sqrt{f_{cmt}}$$

$$f_{cmt} = f_{cm28} \frac{t^{3/4}}{a + bt^{3/4}} \text{ for GL 20001;}$$

$$f_{cmt} = \beta_e^2 f_{cm28} \text{ for GL 2004; } \beta_e^2 = \exp\left[\frac{s}{2}\left(1 - \sqrt{\frac{28}{t}}\right)\right]$$

$s$  = strength development parameter depending on cement type

$s = 0.335, 0.4, 0.13$  for type I, II and III cements respectively

$$f_{cm28} = 1.1f_{ck28} + 5.0$$

$$\phi = \Phi(t_c) \left( 2 \frac{(t-t_0)^{0.3}}{(t-t_0)^{0.3} + 14} + \left(\frac{7}{t_0}\right)^{0.5} \left(\frac{t-t_0}{t-t_0+7}\right)^{0.5} + 2.5(1-1.086h^2) \left(\frac{t-t_0}{t-t_0+0.12(V/S)^2}\right)^{0.5} \right)$$

If  $t_0 = t_c$

$$\Phi(t_c) = 1$$

If  $t_0 > t_c$

$$\Phi(t_c) = \left[ 1 - \left( \frac{t_0 - t_c}{t_0 - t_c + 0.12(V/S)^2} \right)^{0.5} \right]^{0.5}$$

The input data necessary to perform calculation are:

$f_{cm28}$  : concrete mean compressive strength at 28 days [MPa]

$f_{ck28}$  : Specified/ characteristic strength at 28 days [MPa]

$a, b$  : coefficients related to the type of cement

$h$  : relative humidity express as a decimal

$V/S$  : volume to surface ratio [mm]

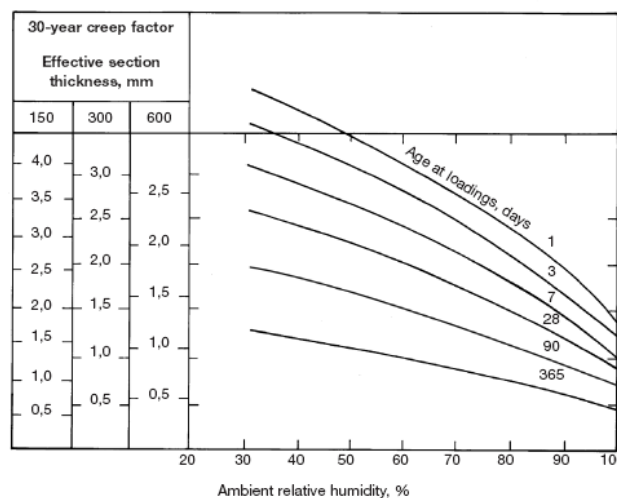
$t$  : age of concrete [days]

$t_o$  : age of concrete loading [days]

$t_c$  : age when drying begins, end of moist curing [days]. The value of  $t_c$ , representing the age when the moist curing of concrete ends and the drying starts, must be less than  $t_o$ .

### 2.2.8.2 SANS 10100 (2000) and BS 8110 Part 2– (1985)

This code (SANS 10100:2000) is a newer version of the South African Bureau of Standards, SABS 0100: 1992). The SANS code is taken up from BS 8110. The creep strain is calculated from the basic creep formula given in Equation 2. The creep coefficient is obtained from Figure 2.15.



Figures 2.15: SANS 10100:2000 creep prediction model.

Input parameters include age of loading, relative humidity and effective section thickness. The effective section thickness can be calculated from the formula  $2A/u$  where  $A$  is the cross sectional area while  $u$  is the exposed perimeter.

### 2.2.8.3 BS EN 1992:2004

Such as the BS and SANS methods, the BS EN 1992: 2004 prediction model depends on estimating a value of the creep coefficient. This coefficient can then be used in Equation 2 to calculate the creep strain. This coefficient can be calculated from equations but the code provides a simplified graphical method for determination of the creep coefficient.

The formulation of creep equations is adopted from the Model Code 1990. The creep coefficient is based on a computerised data bank with a stated coefficient of variation in the order of 20% (Alexander and Beushausen, 2009). The coefficient can be calculated for both outside conditions (in the UK) (RH = 80%) and inside conditions (RH = 50%). The graphs for calculating the creep coefficient are shown in Figure 2.16.

The figure uses the following symbols:

$\phi(\infty, t_0)$ : final creep coefficient

$t_0$ : age of the concrete at first loading (days)

$h_0$ : notional size =  $2A_c/u$ , with  $A_c$  = cross-sectional area and  $u$  = perimeter exposed to drying

S: applies to slow hardening cements

N: applies to normal cements

R: applies to rapid hardening cements

C20/25: strength classes according to EN 206-1:  $C f_{ck}/f_{ck,cube}$ ,

$f_{ck}$ : characteristic cylinder strength,

$f_{ck,cube}$ : characteristic cube strength

When using the graph, identify the age of loading  $t_0$  and draw a straight line to meet the curve which describes the cement type used (1). At the point of intersection of the two, draw a diagonal line to the right hand corner of the left graph (2). On the right graph, draw a straight line from the calculated value of the effective section thickness  $h_0$  to meet the curve that describes the concrete strength (3). From the point of intersection of the two, draw a straight line meeting the earlier drawn diagonal (4). From where line (4) meets the diagonal, draw a line to the bottom of the left graph to read off the creep coefficient (5). In summary, the input parameters for this model include age of loading, relative humidity; cement type, and effective section thickness and strength.

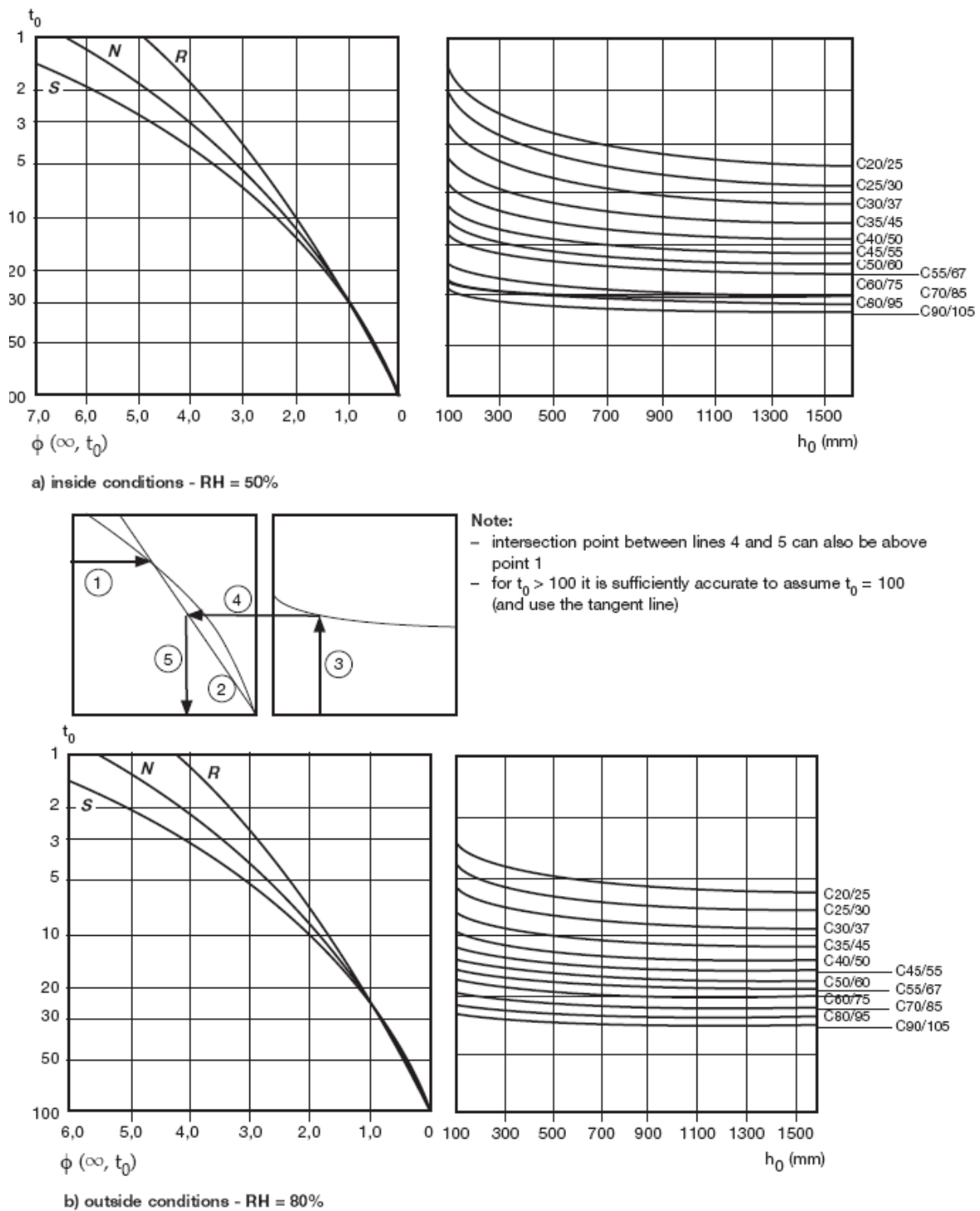


Figure 2.16: Determination of creep coefficient using BS EN: 1992:2004

### 2.2.8.4 American Concrete Institute 209R-92 (ACI 209R-92)

The ACI209R-92 model was initially developed for the precast prestressed industry by Branson and Christianson (1971), with minor modifications introduced in ACI 209R-82. In

the ACI method, the creep coefficient is given by the product of several partial coefficients (correction factors) which cover the major factors likely to influence creep. These factors include: loading age, relative humidity, slump, fine aggregate percentage, air content and volume to surface ratio factor (Mucambe 2010). Normal-density and low density concretes are relevant in this method, and ASTM type I and III cements (ordinary and rapid-hardening Portland cements) are used (Alexander and Beushausen, 2009). This model makes provision for moist and steam cured concretes (Mucambe 2010).

The creep coefficient is calculated from the Equation 8 obtained from Omar, Makhtar, Lai, Omar R and Kwong (2008).

$$\phi = \frac{(t - t_0)^{0.6}}{10 + (t - t_0)^{0.6}} v_u \cdot F_{CT} \cdot F_{Ch} \cdot F_{Ct} \cdot F_{Cs} \cdot F_{Cc} \cdot F_{Cf} \cdot F_{Ca} \quad (8)$$

where;

$t_0$  = age at loading (days)

$t$  = current age (days)

$v_u$  = ultimate creep coefficient defined as ratio of creep strain to initial strain, with recommended average value at 2.35

$F_{CT}$  = ambient relative humidity correction factor

$F_{Ch}$  = minimum thickness of member correction factor

$F_{Ct}$  = age at loading correction factor

$F_{Cs}$  = slump correction factor

$F_{Cc}$  = cement content correction factor

$F_{Cf}$  = percent fines correction factor

$F_{Ca}$  = percentage of air content correction factor

#### 2.2.8.5 CEB-FIP MC90-99

This model is an improvement of the earlier MC90 model. Important input parameters include: Age of concrete at drying and loading; mean cylinder strength; cement type and relative humidity. Relative humidity should range between 40- 100%. The development of creep with time is taken into account. The effect of different temperature ranges and stresses is provided for in this code. High strength concretes are also considered. With the exception of new coefficients that depend on the mean compressive strength, the creep prediction

procedure is similar to that in the older MC90. Creep is expressed in terms of the compliance function (Equation 9). One advantage of this model is that it does not require any information on duration and method of curing.

$$J(t, t_0) = \left[ \frac{1}{E_c(t_0)} + \frac{\phi(t, t_0)}{E_{ci}} \right] \quad (9)$$

$E_c(t_0)$  = modulus of elasticity at the time of loading

$\phi(t, t_0)$  = creep coefficient determined from equation (10)

$E_{ci}$  = modulus of elasticity at the end of loading

$$\phi(t, t_0) = \phi_0 \beta_c(t - t_0) \quad (10)$$

$\phi(t, t_0)$  = creep coefficient

$\phi_0$  = notional creep coefficient (equation 11)

$\beta_c$  = coefficient to describe the development of creep with time after loading equation (15)

$t$  = age of concrete (days) at the moment considered

$t_0$  = age of concrete at loading (days)

$$\phi_0 = \phi_{RH} \beta(f_{cm}) \beta(t_0) \quad (11)$$

$$\phi_{RH}(h) = \left[ 1 + \frac{1 - \left( \frac{h}{h_0} \right)}{\sqrt[3]{0.1 \left( \frac{V/S}{(V/S)_o} \right)}} \alpha_1 \right] \alpha_2 \quad (12)$$

$$\beta(f_{cm}) = \frac{5.3}{\left( \frac{f_{cm}}{f_{cmo}} \right)^{0.5}} \quad (13)$$

$$\beta(t_0) = \frac{1}{0.1 + \left( \frac{t_0}{t_1} \right)^{0.2}} \quad (14)$$

The development of creep with time is represented by

$$\beta_c(t-t_0) = \left[ \frac{(t-t_0)/t_1}{\beta_H + (t-t_0)/t_1} \right]^{0.3} \quad (15)$$

$$\beta_H = 150 \left[ 1 + \left( 1.2 \frac{h}{h_0} \right)^{18} \right] \frac{(V/S)}{(V/S)_o} + 250\alpha_3 \leq 1500\alpha_3 \quad (16)$$

$$\alpha_1 = \left[ \frac{3.5f_{cmo}}{f_{cm28}} \right]^{0.7} \quad \alpha_2 = \left[ \frac{3.5f_{cmo}}{f_{cm28}} \right]^{0.2} \quad \alpha_3 = \left[ \frac{3.5f_{cmo}}{f_{cm28}} \right]^{0.5}$$

$h$  = is the relative humidity in decimals;  $h_0 = 1$  mm

$V/S$  is the volume-surface ration (mm);  $(V/S)_o = 50$  mm

$f_{cm28}$  = mean compressive strength of concrete at 28 days;  $f_{cmo} = 10$  MP;  $t_1 = 1$  day

It should be noted that  $h$  is used to represent the humidity in MC 90-99, and not the nominal size as in MC 90.

#### 2.2.8.6 RILEM Model B3 (Bazant and Baweja, 1995)

The RILEM Model B3 incorporates both extrinsic and intrinsic variables that affect creep, such as relative humidity, temperature, concrete age at first loading, section size, w: c ratio, cement type, cement content and aggregate-cement ratio. This model is complex, requires detailed information on the concrete to be used, and is generally not suitable at the initial structural design stage. For creep prediction, the model separates basic and drying creep components. The creep is expressed in terms of the compliance function. The model is derived from work of RILEM Technical Committee 107-GCS (Alexander and Beushausen, 2009). Fanourakis and Ballim (2003) found this model to be the most accurate among a number of models including SABS 0100, BS 8110, ACI 209, AS 3600, CEB-FIB (1970:1990). The expressions defining the creep compliance function was extracted from Sassone and Chiorino (2005) and is given by Equation 17.

$$J(t, t_0) = q_1 + C_0(t, t_0) + C_d(t, t_0, t_c) \quad (17)$$

$$q_1 = 0.6 / E_{cm28}$$

$$E_{cm28} = 4734 \sqrt{f_{cm28}} \quad (18)$$

$$C_0(t, t_0) = q_2 Q(t, t_0) + q_3 \ln[1 + (t - t_0)^n] + q_4 \ln(t/t_0)$$

$$q_2 = 185.4 \times 10^{-6} c^{0.5} f_{cm28}^{-0.9}$$

$$Q(t, t_0) = Q_f(t_0) \left[ 1 + \left( \frac{Q_f(t_0)}{Z(t, t_0)} \right)^{r(t_0)} \right]^{-1/r(t_0)}$$

$$Q_{f(t_0)} = [0.086(t_0)^{2/9} + 1.21(t_0)^{4/9}]^{-1}$$

$$Z(t, t_0) = (t_0)^{-m} \ln[1 + (t - t_0)^n]$$

$$r(t_0) = 1.7(t_0)^{0.12} + 8$$

$$q_3 = 0.29q_2(w/c)^4$$

$$q_4 = 20.3 \times 10^{-6} (a/c)^{-0.7}$$

$$C_d(t, t_0, t_c) = q_5 [\exp\{-8H(t)\} - \exp\{-8H(t_0)\}]^{1/2}$$

$$q_5 = 0.757 f_{cm28}^{-1} |\varepsilon_{sh\infty} \times 10^6|^{-0.6}$$

$$\varepsilon_{sh\infty} \approx -\alpha_1 \alpha_2 [0.019w^{2.1} f_{cm28}^{-0.28} + 270] \times 10^{-6}$$

$$H(t) = 1 - (1 - h)S(t)$$

$$H(t_0) = 1 - (1 - h)S(t_0)$$

$$S(t) = \tanh \left[ \left( \frac{t - t_c}{\tau_{sh}} \right)^{1/2} \right]$$

$$S(t_0) = \tanh \left[ \left( \frac{t_0 - t_c}{\tau_{sh}} \right)^{1/2} \right]$$

$$\tau_{sh} = 0.085 t_c^{-0.08} f_{cm28}^{-0.25} [2k_s (V/S)]^2$$

The input data necessary to perform calculation are:

$f_{cm28}$  concrete mean compressive strength at 28 days [MPa]

$a$  aggregate content in concrete [ $kg/m^3$ ]

$c$  cement content in concrete [ $kg/m^3$ ]

$w$  water content in concrete [ $kg/m^3$ ]

$a_1$	coefficient related to the type of cement
$a_2$	coefficient related to the curing method
$h$	relative humidity express as a decimal
$k_s$	coefficient related to the cross-section shape
$V/S$	volume to surface ratio [mm]
$t$	age of concrete [days]
$t_0$	age of concrete loading [days]
$t_c$	age when drying begins, end of moist curing [days]

The value of  $E_{cm28}$  is computed from Equation 18 which is considered as part of the model formulation. The exponent  $m$  and  $n$  are empirical quantities assumed to be equal to 0.5 and 0.1. The value of  $t_c$ , representing the age when the moist curing of concrete ends and start the drying, must be less than  $t_0$ .

### 2.2.8.7 Summary on creep models

The models discussed above cannot be relied upon to give accurate results and should therefore be used carefully. This is because the conditions and materials used in different regions vary, meaning that the deformation characteristic of concrete can change. The compressive strength (cylinder strength), volume to surface ratio and relative humidity are found to be the most common input parameters for most of the creep prediction models. In this research, creep values were experimentally determined and compared with selected models. The creep coefficient was compared with values predicted by the MC90-99 and GL 2001(4). The creep compliance was compared with values predicted by the MC90-99, GL 2001(4) and B3. The MC90-99 and GL 2001(4) models were chosen because past literature has found them to be accurate at predicting creep values of concrete (Fanourakis, 2011; Ahmad and Roy, 2012; Gardner and Lockman, 2001 and Goel *et al.*, 2007). Because of its slightly different method of obtaining the compliance function, the B3 model was added for comparison purposes. In the method, the material composition influences the predicted creep compliance function which is not the case with the other two models.

Fanourakis and Ballim (2003) investigated the accuracy of creep predictions yielded by eight commonly used international “code type” prediction models. The models studied do not all consider the same material parameters. Results from predicted strains varied when compared

with those from measured samples. The models assessed were the: SABS 0100 (1992), BS 8110 (1985), ACI 209 (1992), AS 3600 (1988), CEB-FIP (1970, 1978 and 1990) and the RILEM Model B3 (1995). Their research found the RILEM Model B3 (1995) and CEB-FIP (1978) to be the most and least accurate models respectively.

More recently, Fanourakis (2011) assessed the accuracy of six international code type models in predicting creep strains of a wide range of South African concretes. Tests were done under laboratory control conditions. The models considered were those contained in AS 3600 (2001), AS 3600 (2009), Euro code EC 2 (2004), GL (2000), GL (2004) and GZ (1993). The GL (2000) model yielded the most accurate predictions while the least accurate method was the AS 3600 (2009).

### **2.2.9 Test methods for creep of concrete**

Gilbert (2005) states that the most accurate way of determining the creep coefficient is through tests on similar local concrete. However, testing is time consuming and is often not practical for a structural designer. In that case, models such as those discussed in section 2.2.8 can be used. For research purposes and on important jobs, determining the creep coefficient through laboratory tests is unavoidable. Some of the methods recommended are discussed in the following

The most common international known standards for creep testing are ASTM C 512-02 and RILEM CPC-12, 1983 (Alexander and Beushausen, 2009). For the former, creep testing involves applying the stress over a short time period usually two to five minutes and then measuring long-term total strain. Companion unloaded specimens are used for measuring shrinkage/swelling strains. The RILEM CPC-12, 1983 (Measurement of creep in compression) is a European document on creep testing which allows for various environments of storage.

For tensile creep, Bissonnette and Pigeon (1995), Atrushi (2003) used systems which consist of a vertical steel creep frame and a horizontal loading frame. In Figure 2.17, the experimental set up used by Bissonnette and pigeon (1995) is presented. Loading is applied by lever arm system and is transferred to the samples through steel plates at the end of the samples. At both ends of the samples, the load is transmitted through hinges to prevent development of moments. A strain measurement device is provided to measure changes in strain.

Pane and Hansen (2001) determined the creep strains in terms of compliance function using a set up consisting of a level type loading apparatus with a pneumatic pump that allows control

of the loading rate. In order to ensure that loads remain axial, a guiding apparatus was installed to the system.

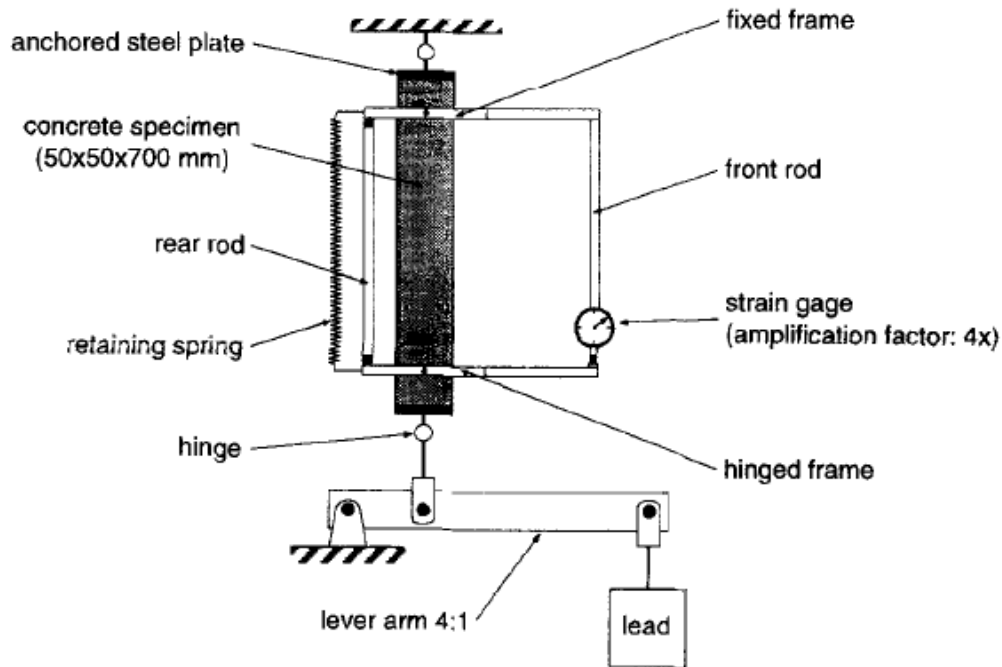


Figure 2.17: Test apparatus and strain measuring device (Bissonnette and Pigeon, 1995)

Two specimens are tested at stress levels of approximately 30 % of split tensile strength. Changes in strain are monitored by a Linear Variable Differential Transformer (LVDT).

Locally, numerous types of creep apparatus have been designed usually based on mechanical or hydraulic principles. An example of such apparatus was used in this research (see chapter 3). Also a system based on the same principal and equipped with oil pressure accumulators was used by Atrushi (2003) for creep in compression (Figure, 2.18).



Figure 2.18: Compressive creep rigs used by Atrushi (2003)

## 2.3 Relaxation of concrete

Relaxation can be defined as the reduction of stress under constant strain, Figure 2.19. In the figure, the initial stress  $\sigma_{t_0}$  is applied at a time  $t_0$ . When the strain is kept constant, there is a reduction in the initial stress  $\sigma_{t_0}$  to  $\sigma_t$ . In most literature, the term creep is used to refer to both creep deformation and relaxation of stress (Atrushi, 2003).

### 2.3.1 Importance of studying relaxation

It is important to study the tensile relaxation behaviour of concrete because of its role in reducing, prolonging or eliminating the manifestation of cracks in concrete. Bentur and Kovler (2003) state that early age cracking in concrete members is governed by the extent of shrinkage, but whether cracking will occur or not is dependent on the restraint and the stress relaxation. Restraint can be of two kinds i.e. external or internal.

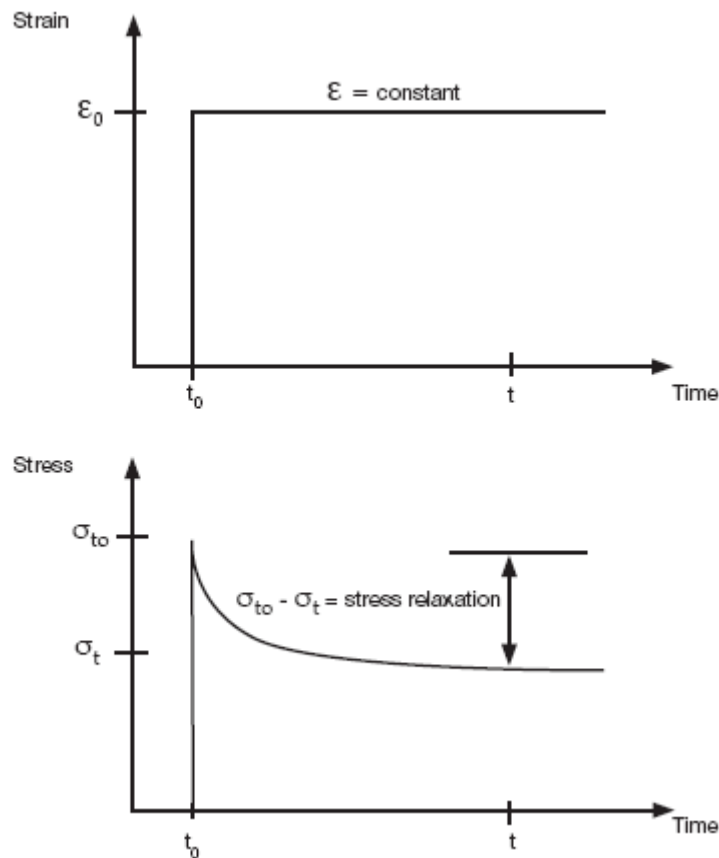


Figure 2.19: Characteristics of relaxation: time-dependent decrease in stress under constant imposed strain (Alexander and Beushausen, 2009)

External restraint occurs when the free movement of the concrete is prevented (Alfredsson and Spals, 2008). Members which are prone to external restraint include bonded overlays, repair systems and hybrid construction systems (Alexander and Beushausen, 2009). The bonded overlay technique is used in the repair of or strengthening deteriorated bridge and parking decks, damaged industrial floors or as finishing layers on prefabricated elements (Carlsw ard, 2006).

Internal restraint can occur because of the interaction between the concrete and the reinforcement. It can also be caused by varying temperatures over a concrete section or by moisture gradients (Alfredsson and Spals, 2008). A totally unrestrained member is not easy to find, and therefore internal and external restraints nearly always coexist in concrete (Atrushi, 2003). When concrete is restrained, “*self-induced stresses*” are induced in the member. Self-induced stresses are caused by anything but external loading (Atrushi, 2003). Cracking occurs when the stresses induced in the concrete exceed its tensile strength. Relaxation is important because it reduces these stresses. Figure 2.20 shows how tensile relaxation prolongs cracking in a concrete member subject to restrained shrinkage.

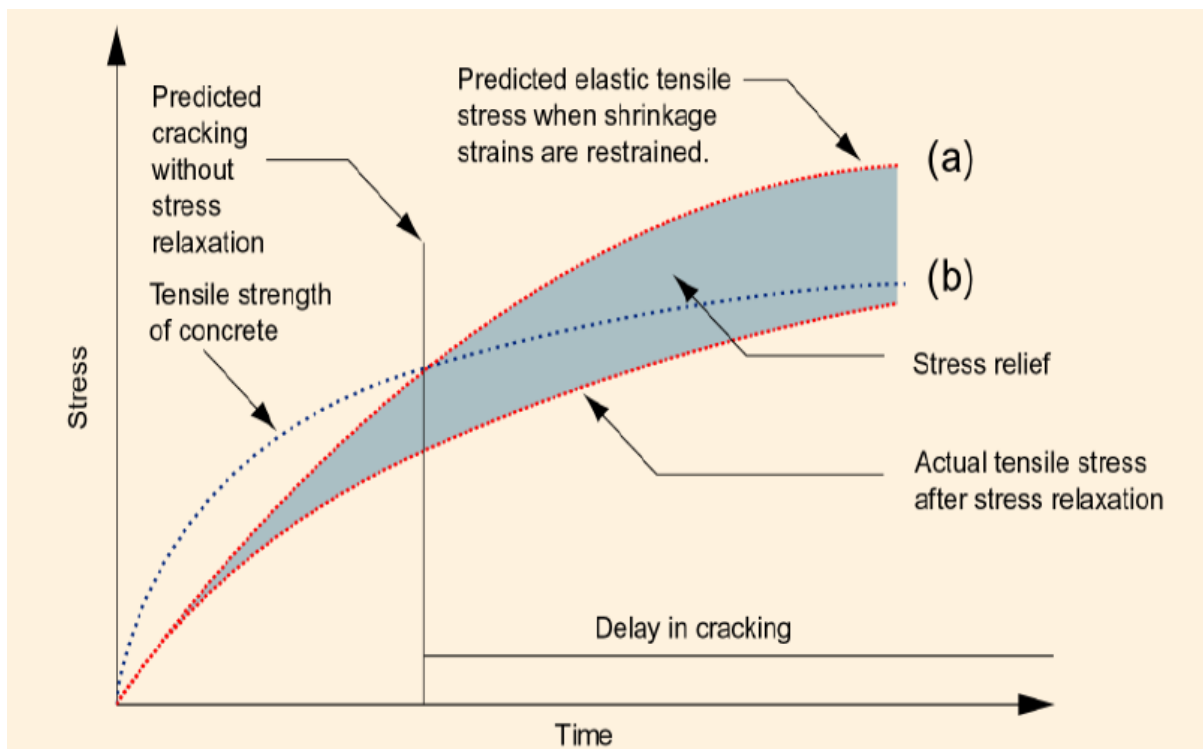


Figure 2.20: How tensile relaxation prolongs cracking in concrete (Mont, 2011)

Without relaxation, the tensile capacity of the concrete would be exceeded hence leading to cracking. With tensile stress relaxation, the cracking is delayed or prevented. The amount of stress that can be reduced by relaxation varies depending on age and material composition. Beushausen and Alexander (2006) found that relaxation releases approximately 40-50% of tensile overlay stress. Pigeon *et al.* (2001) measured 67% relaxation in fully restrained specimens. Morimoto and Koyanagi (1995) reported values of 20% relaxation after 96 hours. The magnitude of relaxation is discussed in subsequent sections.

A greater emphasis is nowadays placed on performance based durability design of concrete structures as opposed to the prescriptive design (Alexander and Beushausen, 2009). In such cases, the quality of the concrete and its ability to remain crack free is an important consideration. New emerging high strength concretes with low w/c ratio are prone to cracking because of autogenous shrinkage. Cracking impairs the durability of the structure by exposing it to harmful substances (Beushausen *et al.*, 2012). Cracking results in spalling and de-bonding of overlays. The stiffness of the overlay is reduced hence leading to increased deformations. Moreover, the function of water retaining structures is destroyed by cracks. It is therefore important that the factors that influence the relaxation of concrete are understood. This can help material designers take advantage of relaxation of concrete.

### 2.3.2 Mechanisms of relaxation

The mechanisms of relaxation are not well understood. Very little work has been done by researchers to explain the micro-mechanisms of relaxation. Most researchers tend to agree that the mechanisms of relaxation are similar to those of creep.

According to Gilbert and Ranzi (2010), creep and relaxation are similar problems but with different boundary conditions. To explain how stresses are relaxed in concrete, they use the argument that the total strain in a concrete member is composed of the elastic strain, creep strain and shrinkage strain (see Equation 1). During relaxation, the total change in strain is equal to zero because the strain is held constant i.e.  $\Delta\varepsilon_e + \Delta\varepsilon_{cr} + \Delta\varepsilon_{sh} = 0$ . When the creep and shrinkage strain increase, the elastic strain has to decrease to keep the total change in strain equal to zero. This reduction in elastic strain is what is said to lead to reduction in initial applied stress. ( $\sigma = \varepsilon E$ )

Gutsch and Rostasy (1994) investigated the creep and relaxation behaviour of concrete under high tensile stresses. At initial stresses-strength ratios of 30% and above, micro cracking within the paste was found to cause loss of stiffness and subsequent relaxation.

Hossain and Weiss (2004) found that concrete specimens subjected to a higher degree of restraint in the ring test developed higher micro cracking and a subsequently higher relaxation. This supports the argument that the micro-cracking is an important factor in the relaxation of stresses.

Pigeon and Bissonnette (1999) also concluded that viscous shear and micro cracking theory can be advanced to explain relaxation. Viscous shear (flow) involves the sliding of colloidal sheets in the cement gel between the layers of absorbed water as explained in Section 2.2.2.4. They attributed this to the loss of rigidity their samples experienced. Although viscous shear was found to be the predominant mechanism, micro cracking could not be entirely ignored.

In summary, most researchers agree that relaxation, similar to creep is a result of the viscoelastic nature of the concrete. Micro cracking is another mechanism advanced to explain the same phenomenon. The mechanisms responsible for creep discussed in section 2.2.2 are believed to explain the relaxation of concrete materials.

### 2.3.3 Test methods (tensile relaxation)

Tensile relaxation tests involve applying a stress to a specimen and maintaining the strain. Not many researchers have attempted to carry out relaxation tests. This is because relaxation experiments are difficult to perform. Some of the researchers that have done work on

relaxation tests include Morimoto and Koyanagi (1994), Gutsch and Rostasy (1994), Beushausen and Alexander (2006), Masuku (2009) and Chilwesa (2012).

Morimoto and Koyanagi (1994) used a universal testing machine (UTM) capable of controlling the strain of the specimen to calculate compressive stress relaxation. The amount of stress relaxation of the specimen was then detected from the output of the potentiometer used to indicate the loads. In tension, the tensile tests were performed using a high-rigidity loading frame. The amount of stress relaxation of the specimens was then determined through wire strain gages. Chilwesa (2012) and Masuku (2009) used a Zwick Roell (Z20) Universal Testing Machine (Figure 2.21) to carry out tensile relaxation tests on concrete specimens.



Figure 2.21: The Zwick Roell Universal Testing Machine (Masuku, 2009).

The concrete specimen being tested was held in position by aluminium gripping jaws that ensure that applied loading remains axial. The tensile stress relaxation was measured by subjecting specimens to sustained constant strain and observing the reduction in applied stress. The relaxation can be monitored from the computer using the software “testXpert”. The computer is connected to the UTM and the two have to be initialised before an experiment can run.

### 2.3.4 Factors affecting relaxation

Some of the most common factors affecting the relaxation of concrete are discussed in the following section.

### 2.3.4.1 Age of loading

Similar to creep, relaxation is affected by the age at which the concrete is loaded. Most researchers agree that relaxation is higher in younger concretes and reduces with age. Figure 2.22 shows that if the time increment is kept constant, the older concrete will have a smaller final magnitude of relaxation than the younger concrete if they are subjected to equal initial strain.

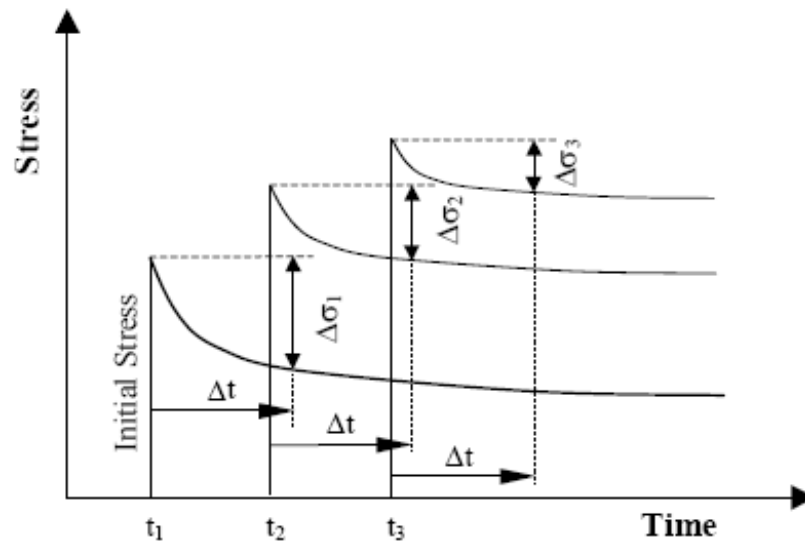


Figure 2.22: Effect of loading age on relaxation in concrete subjected to equal initial strain (Atrushi, 2003)

The rate at which the stress relaxes will also be higher in the young concrete than in the old concrete (Atrushi 2003).

Atrushi (2003) reported on work by Serda and Kristeck (1988) which showed that the relaxation process developed more rapidly at the beginning and approached its final value asymptotically. Beushausen et al. (2012) tested different concrete mortar specimens for 72 h relaxation values.

Samples were water cured for 2, 7 and 28 days. Samples were subjected to an initial stress of 80% of their tensile strength. Results showed that the older the concrete, the lower the relaxation. Their research also found that the difference between relaxation values for the 2 day and 7 day specimen was higher than that between the 7-day and 28-day specimen. This confirms what Atrushi (2003) found - that the rate of relaxation is higher in the younger concrete. Figure 2.23 illustrates their findings for all the mixes investigated in the research.

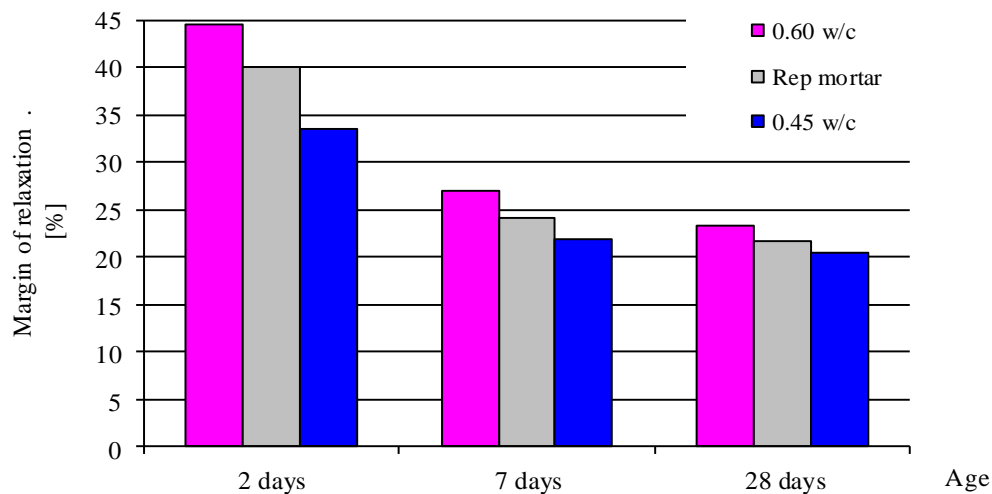


Figure 2.23: Stress relaxation at ages of 2, 7 and 28 days (Beushausen *et al.*, 2012).

Marimoto and Kayonagi (1994) carried out compressive and tensile relaxation tests on different concretes. The two concrete mixes tested had w/c ratios of 0.5 and 0.59 respectively. They loaded their specimens at ages of 1, 3, 7, 14 and 21. Using a universal testing machine capable of controlling strain, the following observations were made with respect to the influence of age on relaxation. The ultimate amount of relaxation and the half relaxation times decrease as the age of loading increases. The half relaxation time is the time that elapses before the relaxation reaches half the ultimate value. These values can give an indication of the rate at which the relaxation happens. The results from their studies are shown in the Table 2.1

Table 2.1: 1/2 relaxation time (r) and ultimate amount (Q) Morimoto and Koyanagi (1994)

Age (days)	r (hours)	Q (%)
1	25-30	95-100
3	10-14	60-65
7	5-15	40-50
14	7-8	30-40
21	7-15	35-45

Gutsch and Rostasy (1994) carried out tests on the relaxation and creep behaviour of early age concrete. Three concrete mixes of w/c ratio 0.65, 0.47 and 0.42 were used in the experimental work. Mixes were made of rapid hardening cement (PZ 35 F), slow hardening blend (HOZ 35 L) 65% GGBS to 35% OPC and OPC (PC 45). In all mixes, quartz aggregate

of maximum size 16 mm was used. w/c ratios results of the axial relaxation tests showed that relaxation increases with a decrease of the equivalent age ( $t_e$ ). Results are shown in Figure 2.24

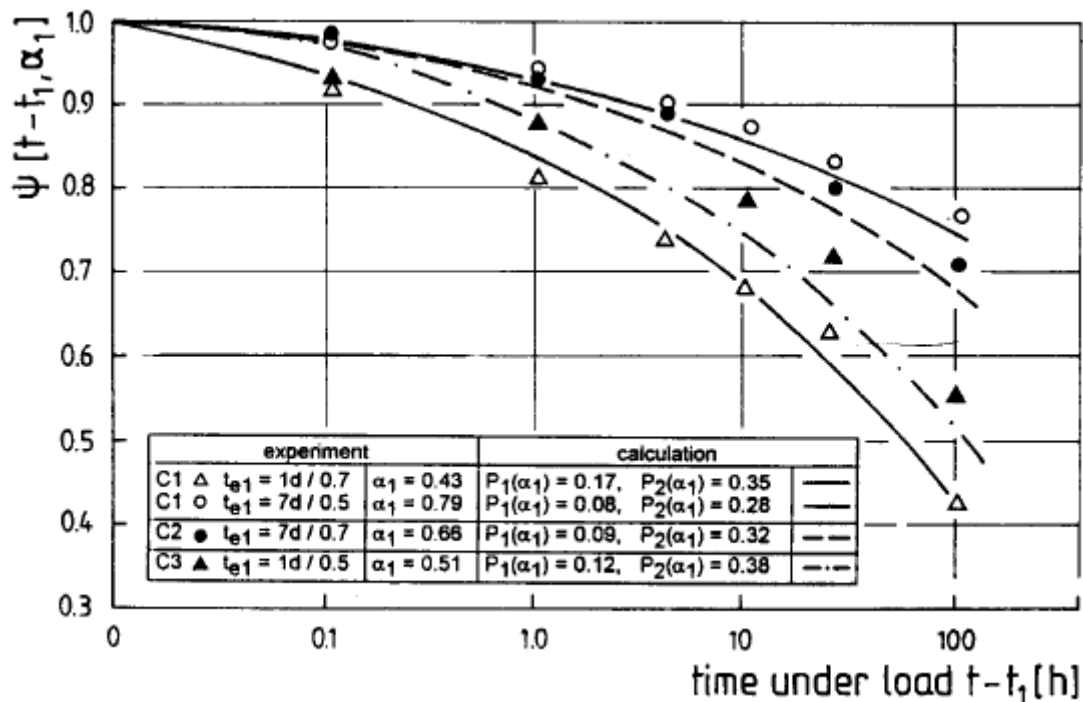


Figure 2.24: Relaxation function depends on age at first loading. Test results are shown in dots while model results are shown by lines (Gutsch and Rostasy, 1994).

#### 2.3.4.2 Time under load (Development of relaxation)

It is generally agreed in literature that relaxation proceeds at a higher rate than creep in the first hours and days (Atrushi, 2003). Beyond 3 days, relaxation proceeds at a lower rate when compared with creep. The rate of development of relaxation can be monitored by calculating the slope of the relaxation curve (Masuku, 2009). After 1-2 years, only 80% of the creep has occurred (Alexander and Beushausen, 2009). Creep is said to occur up to ages of between 30 – 50 years. Relaxation on the other hand reaches its maximum values after approximately 100 hours (Marimoto and Kayonagi, 1994). Gutsch and Rostasy (1994) and Marimoto and Kayonagi (1994) reported that ultimate relaxation values were reached after approximately 2-3 days i.e. 48 hours.

The latter also reported that 25% of ultimate relaxation occurs in the first hour. In work by Beushausen *et al.* (2012), it was found that the relaxation of a 7 day old mortar specimen increased by only 5 % for the period between 72 hours and 400 hours. Atrushi (2003) reported a similar trend in work carried out on three mixes with a similar w/c ratio of 0.4 and

5 % SF replacement. The three mixes were made under different conditions leading to different autogenous shrinkage in each of them. In figure 2.25, the initial increase in stresses before 24 hours is said to be probably caused by autogenous shrinkage.

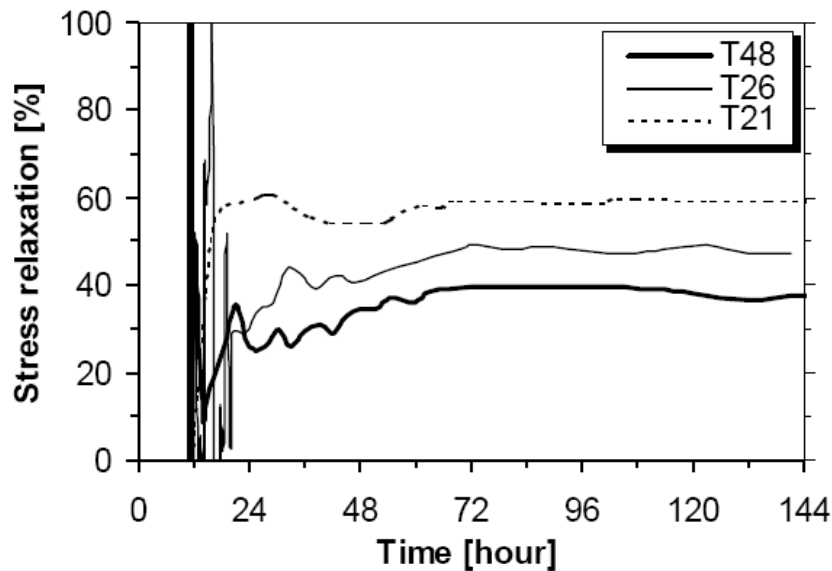


Figure 2.25: Relaxation and residual stresses in concrete BASE-5 (Atrushi, 2003).

Although the different mixing conditions led to a different magnitude of relaxation, all 3 mixes show that beyond 72 hours, the relaxation remains relatively constant (Figure 2.25). Going by the research findings discussed, the 72 hour mark was chosen to represent the ultimate value for relaxation tests in this research.

#### 2.3.4.3 Water-cement ratio

Past research on the relaxation behaviour of concrete indicates that w/c ratio affects its magnitude. A lower w/c ratio reduces the ultimate amount of relaxation. Beushausen *et al.* (2012) attributed this to the denser microstructure of low w/c ratio concretes. The fact that low w/c ratio concretes have a high strength and elastic modulus makes them resist the mechanisms which promote relaxation (Masuku, 2009). The influence of the w/c ratio was found to decrease with age. In Figure 2.26 below, it can be seen that specimens with a w/c ratio of 0.6 had higher relaxation values than specimens with a w/c ratio of 0.45 for all the days of testing.

Bissonnette and Pigeon (1995) carried out a study on the viscoelastic behaviour of repair concretes in tension. Two w/c ratio concretes were investigated i.e. 0.35 and 0.55.

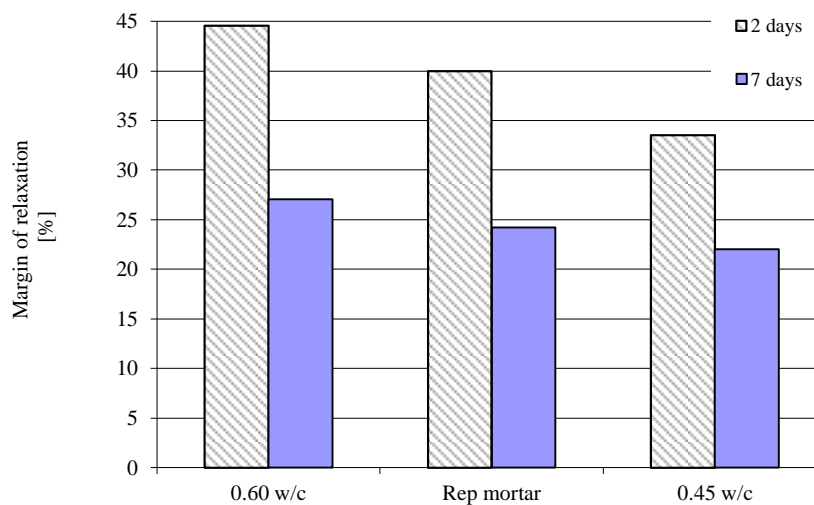


Figure 2.26: 72 hour relaxation of repair mortar, 0.60 and 0.45 w/c samples (Masuku, 2009).

The tensile creep was found to increase with w/c ratio and was found to play an important role in relaxing stresses due to restrained shrinkage.

Östergaard *et al.* (2001) found similar results to those of Bissonnette and Pigeon (1995). The basic tensile creep of concrete with w/c ratios between 0.32 and 0.50 was found to decrease with decreasing w/c ratio.

More recently Chilwesa (2012) reported a somewhat similar trend to that reported by Masuku (2009). Relaxation tests were carried out on dog-bone mortar specimens of 0.45 and 0.65 w/c ratios. In addition, tests were carried out on similar repair mortar specimens whose mix composition is not reported. After two days of curing, the 0.6 w/c ratio samples relaxed by 46% as opposed to 40% relaxation by 0.45 w/c ratio samples. As mentioned earlier, the influence of w/c ratio reduces with age. After 7 days of curing, the 0.6 w/c samples relaxed 38% of initial stress while the 0.45 samples relaxed by 35%.

#### 2.3.4.4 Cement extenders

The effect of cement extenders on the creep and relaxation of concrete is a subject of continuous debate. Some researchers agree that extenders such as FA and SF which increase the strength of the concrete will additionally reduce its relaxation potential in the same way they do to creep of concrete. Pane and Hansen (2001) investigated the effects of cement extenders such as fly ash, ground granulated blast furnace slag and silica fume on early age stress development of concrete. Replacement amounts of extenders by weight of total binder used in the blended cements were fly ash (25%), ground granulated blast furnace slag (25%), and silica fume (10%). The addition of the above mentioned extenders was found to reduce stress relaxation of concrete. However, the conclusions made by Pane and Hansen (2001) are

largely based on the assumption that concrete creep and relaxation are affected in the same way by the extenders.

Atrushi (2003) carried out research on the influence of SF on relaxation and tensile creep. Similar to the case of tensile creep described in Section 2.2.6.1, SF was found to increase relaxation when its content is increased. Addition of SF was also found to increase the rate of relaxation until maximum values are reached at 72 hours. After 72 hours SF mixes and the control mixes had similar rates of relaxation. Mixes with replacement levels between 0 to 15% were tested and the results are shown in Figure 2.27

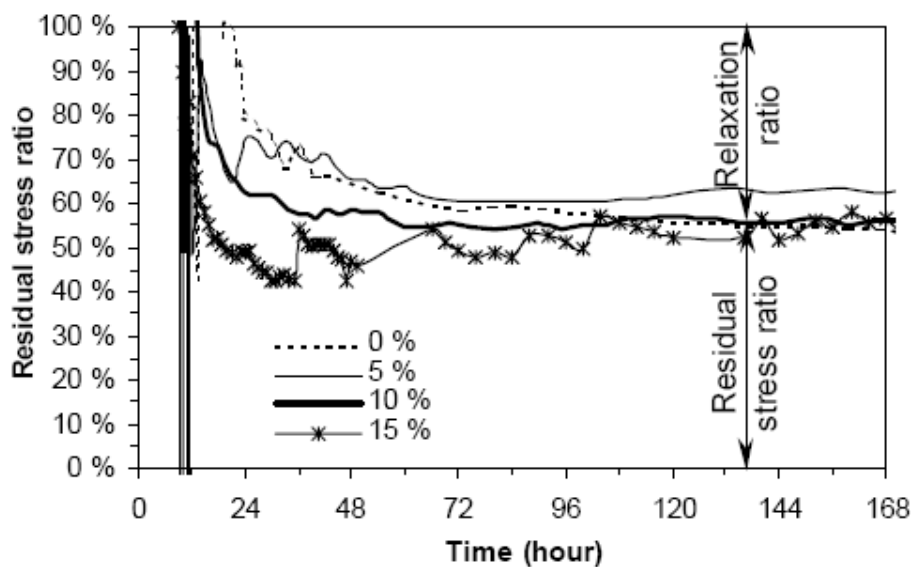


Figure 2.27: Influence of SF on relaxation on a 0.4 w/c mix (Atrushi, 2003)

#### 2.3.4.5 Initial stress-strength ratio

Similar to creep, the relaxation of concrete is assumed to be proportional to the initial stress-strength ratio at low stress levels. Marimoto and Kayonagi (1994) carried out on both the compressive and tensile relaxation behaviour of concrete. The details of the materials used are discussed in section 2.3.4.1.

Both compressive and tensile relaxation was found to be proportional to the initial stress in the range of initial stress levels up to 80%. Gutsch and Rostasy (1994) however found that for initial stress-strength ratios up to 90%, the relaxation of the concrete is not influenced.

Samples were loaded with stresses ranging from 30% to 90% of their failure strength. More details of the mix compositions can be found in Section 2.3.4.1. Tanabe and Ishikawa (1993) carried out relaxation tests in both compression and tension. In tension, samples were loaded at three stress levels: below 30%, between 30% -50% and over 60% of ultimate strength. In

compression, samples were loaded at 30%, 50% and 80% stress levels. Results showed that an increase in initial stress level increased relaxation only slightly in both tension and compression. For example, samples loaded at one day at initial stress ratios of 0.3 and 0.8 experienced similar relaxation values of 60%. This observation was observed for samples loaded in compression.

Atrushi (2003) citing earlier work by Serda and Kristeck (1988) found a non-linear relation between initial stress and relaxation. Figure 2.28 shows the effects of the magnitude of the initial stress.

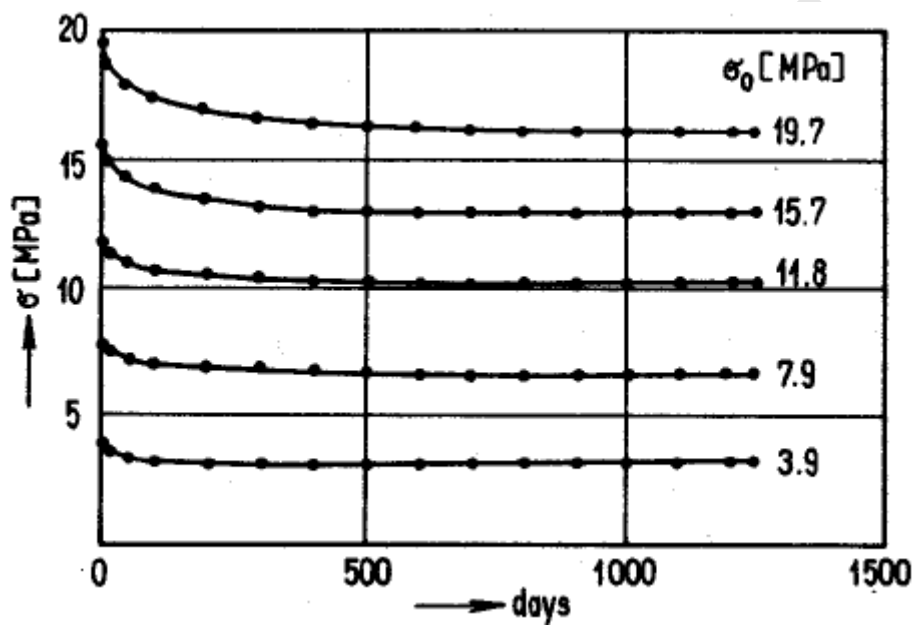


Figure 2.28: The effects of stress magnitude on the relaxation process (Serda and Kristek, 1988 cited in Atrushi 2003)

The relative increase in relaxation was found to be higher than the relative increase of stresses above 11.8 MPa, meaning that the initial stress to relaxation relationship is not linear. Although the initial stress is increased by 4 MPa, the magnitude of relaxation does not change proportionally.

The degree of restraint is another way of studying the effects of stress levels on relaxation. Hossain and Weiss (2004) carried out experiments on the ring test. In these experiments, concrete is cast around a core (usually made of steel) which provides restraint to the shrinking concrete (Bentur and Kovler, 2003). These tests were used to provide quantitative information about stress development, which could be used to assess the potential for

cracking in a concrete member. The steel cores used in this study were of 3.1, 9.5 and 19 mm thickness. It was found that the thicker steel walls exhibited a higher degree of restraint hence subjecting the concrete to a higher load or a higher stress to strength ratio. Consequently, the stress relaxation effect is more pronounced in such concretes. The relaxation was evaluated by measuring the reduction in residual stress over time, Figure 2.29. The influence of initial stress-strength ratio was investigated in this research.

#### 2.3.4.6 Temperature effects

Research on the effects of temperature on relaxation has yielded somewhat contrasting results. Gutsch and Rostasy (1994) found that relaxation of early age concrete was accelerated at temperatures beyond 20 °C under loading. On the contrary, Marimoto and Kayonagi (1994) found the influence of temperature on relaxation to be marginal in the ranges up to 60 °C. Shkoukani and Walraven (1993) investigated the effect of temperature on old concretes. One particular example is a 5 year old concrete in which 20% relaxation was reported after just 3 hours. The concrete was subjected to temperature between 40-60 °C. These findings seem to suggest that if subject to initial imposed thermal deformation, concrete relaxes more than it would if subjected to say an initial external tensile stress.

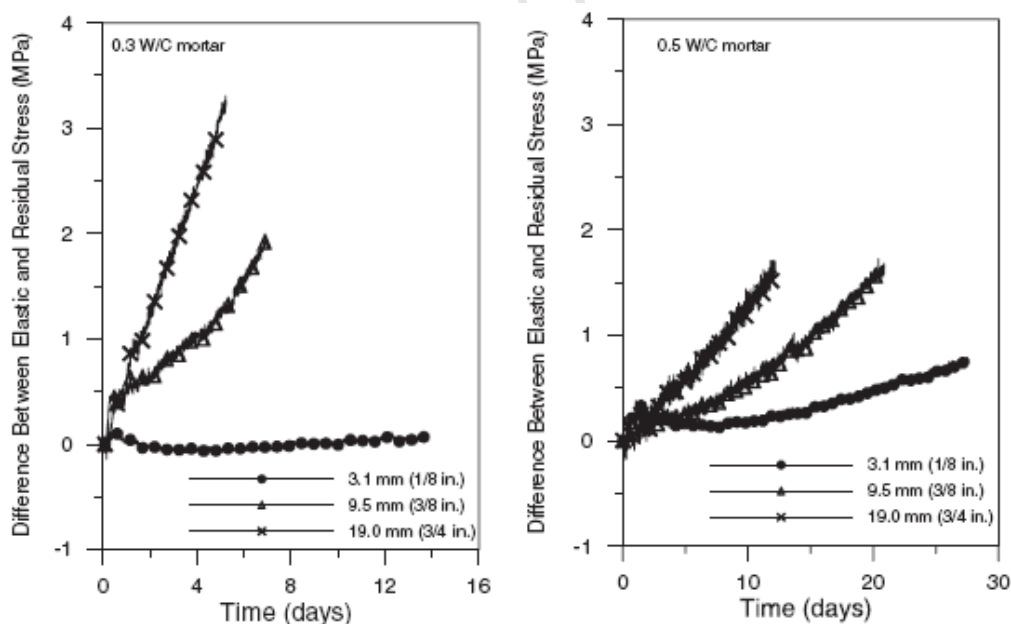


Figure 2.29: Reduction in stress due to relaxation (Hossain and Weiss, 2004)

Atrushi (2003) carried out a comprehensive study on the effect of temperature on relaxation of a particular mix of w/c 0.4 and containing 5% silica fume. Samples tested at 20 °C were used as a control. Another set of samples were tested at 5 °C, 13 °C and 45 °C. Samples tested at temperatures below 5 °C and 13 °C relaxed more than those tested at 20 °C while samples

tested at 45 °C had a smaller relaxation. The results of the study are summarised in Figure 2.30.

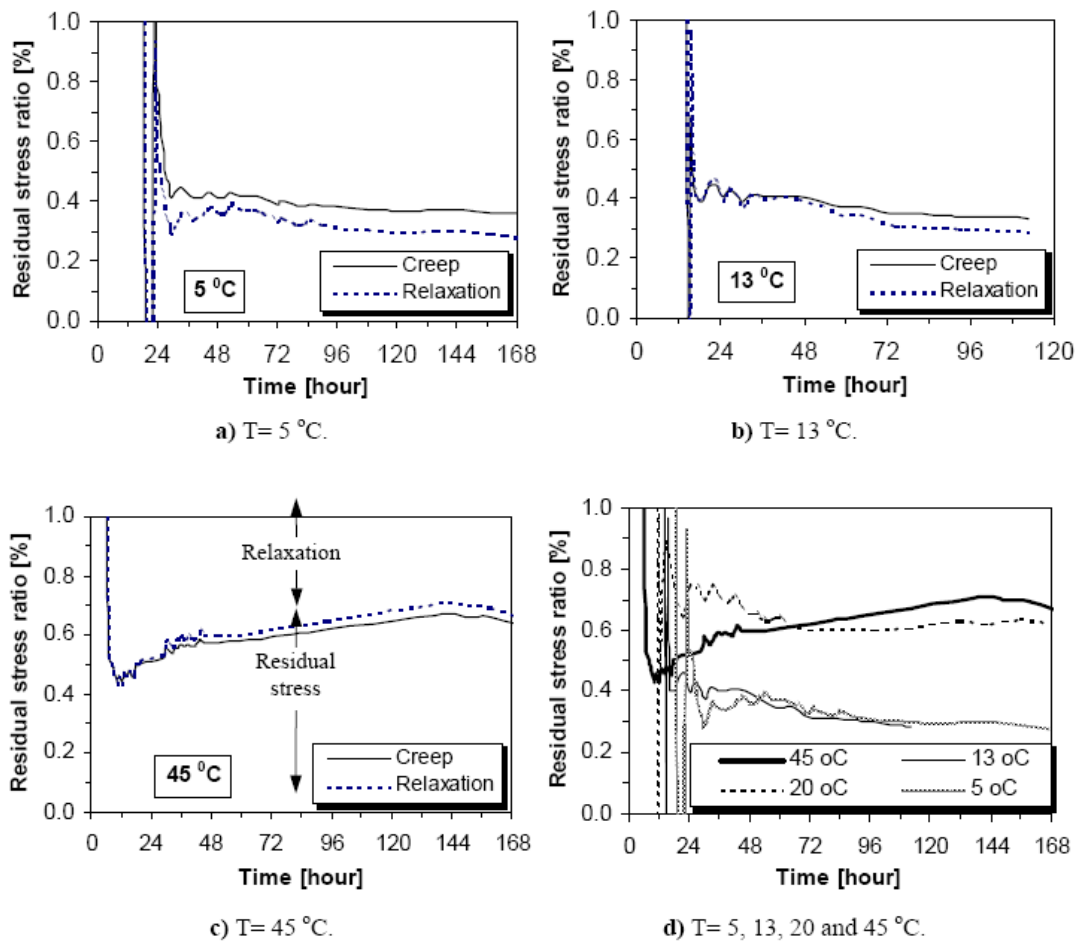


Figure 2.30: Influence of temperature on relaxation (Atrushi 2003)

The study initially suggested that relaxation is inversely proportional to the temperature. However the results were found not to be systematic. The researchers hence did not come with a conclusion on the effect of temperature on relaxation.

#### 2.3.4.7 Aggregates

Aggregates seem to reduce relaxation of concrete. The same reasons forwarded for this behaviour for creep can possibly be used to explain their influence on relaxation. The absorption of water by the aggregates which reduces the w/c ratio is one of the reasons. The other reason is that aggregates provide restraint to the paste hence reducing its ability to deform. Past work seems to show that aggregates reduce the relaxation of concrete. Morimoto and Koyanagi (1994) measured 15% relaxation for samples which had 70% by volume of aggregates. Coarse aggregates were 39% total of the aggregates. Masuku (2009)

and Chilwesa (2012) measured 34% and 40% relaxation values respectively. Their mixes had no coarse aggregates while the volume of fine aggregate was 56% for both studies.

#### 2.3.4.8 State of stress

Relaxation is said to be influenced by the state of stress. According to Tanabe and Ishikawa (1993) whose work is described in Section 2.3.5.5, compressive relaxation is higher than tensile relaxation. For a sample loaded at 1 day, compressive relaxation was found to be 67% compared to 25% tensile relaxation. At the age of 3 days, 60% compressive relaxation was measured compared to 20% in tension.

Morimoto and Koyanagi (1994) reported similar findings. Tensile relaxation was found to be smaller in magnitude when compared to compressive relaxation. Moreover, ultimate values were quickly reached in tensile relaxation than in compressive relaxation. Figure 2.31 illustrates these findings.

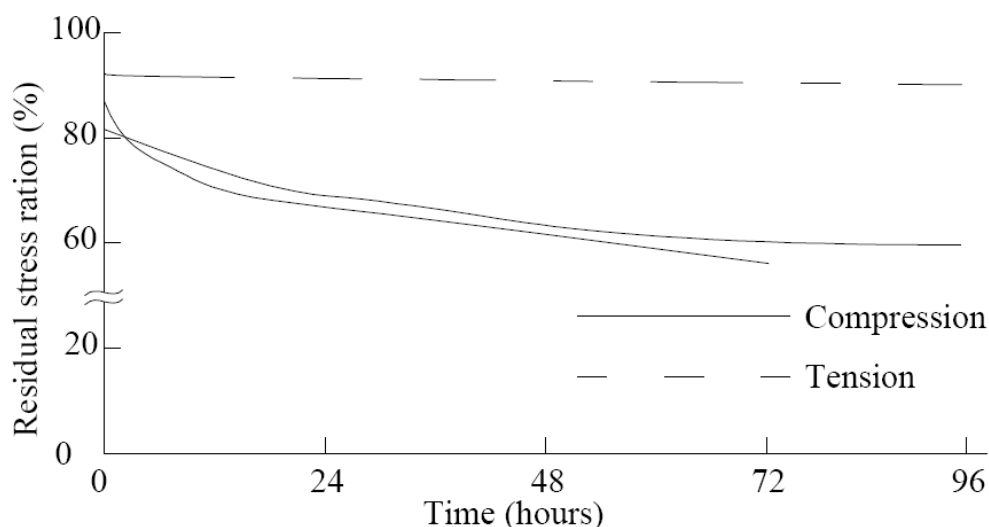


Figure 2.31: Influence of state of stress on relaxation (Morimoto and Koyanagi, 1994)

## 2.4 Relaxation function and its relation to the creep function

### 2.4.1 Theory of linear viscoelasticity, rheological models and integral equations

The creep function  $J$  and the relaxation function  $R$  are important parameters in the definition of time dependent properties of concrete. The creep analysis of structures is performed on the basis of the theory of linear viscoelasticity of materials (Sassone and Chiorino, 2005). All past attempts to link creep functions and relaxation functions have been done by using methods developed in the theory of linear viscoelasticity (Wittmann, 1971). In simple terms, a viscoelastic material exhibits both viscous and elastic behaviour. In more complex analysis,

plasticity can be modelled. The material is considered viscous because it obeys Newton's law of viscosity ( $\tau = \mu \dot{\epsilon}$ ). It is considered elastic because it obeys Hooke's law of elasticity ( $\sigma = E \epsilon$ ). Concrete complies with the theory of linear viscoelasticity (Chiorino, 2005).

One of the assumptions made in this theory is that creep strains under constant stress are linearly related to the stress level. This assumption is made with the knowledge of the non-linear behaviour of creep but it is justified by the fact that only service stresses are applied to the concrete (Atrushi, 2003). Service stresses are generally taken to be less than 40% of concrete compressive strength.

The other assumption or principal applied to the creep and relaxation of concrete is the principal of superposition. This again is applied with the assumption of linearity in mind. The principal states that: the strains produced in concrete at any time  $t$  by either a tensile or compressive stress increment applied at time  $t_0$  are independent of the effect of any stress applied either earlier or later than time  $t_0$ .

As stated above, the theory of linear viscoelasticity is used to link creep and relaxation of concrete. The two are linked by way of mathematical equations. The majority of equations of the theory of linear viscoelasticity are derived by means of rheological models (Wittmann, 1971). These models are used to simulate the real behaviour of concrete (Atrushi, 2003). In these models, the elastic element is represented by a spring while the viscous element is represented by a dashpot. The spring and dashpot are then arranged in various ways to simulate the material behaviour. One of the simplest rheological models is the 'Maxwell model'. In this model, the spring and the dashpot are arranged in series, Figure 2.32 (b)

To calculate the total strains for two elements in series, the strain due to the spring is added to the strain due to the dashpot. Another basic model is the Kelvin model in which the elements (spring and dashpot) are arranged in parallel. The two basic models are used to model more complex models. Complex models give better approximation of the material behaviour. Examples of complex models include the Burger model, kelvin chain model and the Maxwell chain model. All models can be seen in Figure 2.32. The different combinations of models are used to derive integral and differential equations linking stresses and strains in the elements (or the material in reality). It is from such equations that formulations solving concrete deformation problems (such as creep) are built.

The most commonly used integral equations to relate the creep and relaxation function are known as the Volterra linear integral equations. As mentioned before, these equations are derived with two assumptions in mind: the linearity of the material and the fact that the principal of superposition is applied.

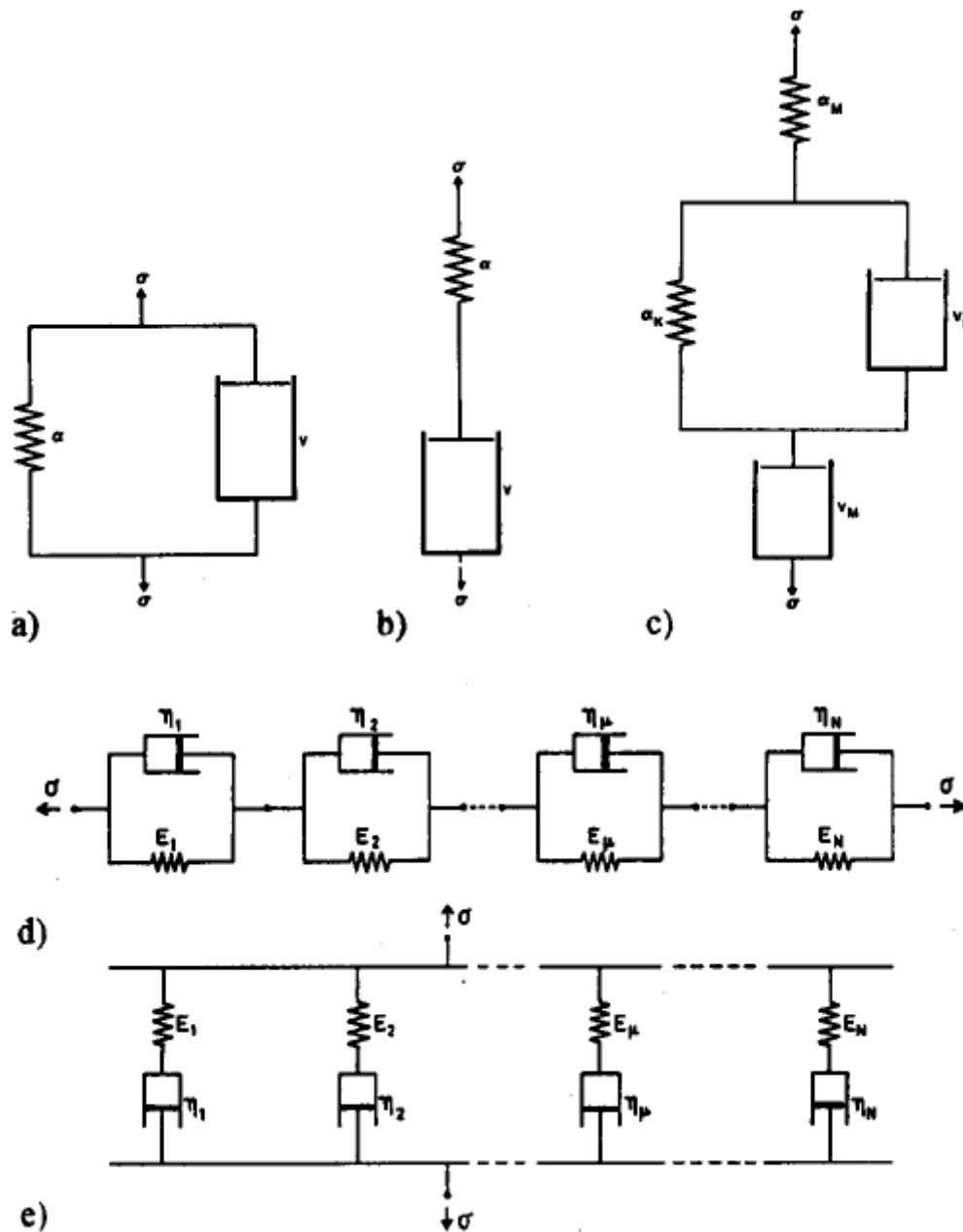


Figure 2.32: Rheological models: a) Kelvin-Voigt model, b) Maxwell model, c) Burger model, d) Kelvin Chain Model and Maxwell Chain Model (after Emborg, 1998)

When the strain history due to all small increments before time  $t$  are summed up, the following integral equation for uniaxial stress is written (Sassone and Chiorino, 2005; Chiorino, 2005).

$$\varepsilon_{\sigma}(t) = \sigma(t_0)J(t, t_0) + \int_{t_0}^t J(t, t')d\sigma(t') \quad (19)$$

where:

$\varepsilon_{\sigma}(t) = \varepsilon(t) - \varepsilon_n(t)$  = stress-dependent strain,

$\varepsilon(t)$  = total strain at time  $t$ ,

$\varepsilon_n(t)$  = stress-independent strain (e.g. shrinkage and /or temperature effect)

$\sigma(t)$  = stress at time  $t$ .

$J(t, t')$  = represents the stress dependent strain per unit stress, i.e. the response at time  $t$  due to a sustained constant unit imposed stress applied at time  $t'$

Likewise, the stress response to a variable strain history with initial value  $\varepsilon_{\sigma}(t_0)$  may be written as:

$$\sigma(t) = \varepsilon_{\sigma}(t_0)R(t, t_0) + \int_{t_0}^t R(t, t')d\varepsilon_{\sigma}(t') \quad (20)$$

where:

$R(t, t')$  = represents the stress response at time  $t$  to a sustained constant unit imposed strain applied at time  $t'$

The creep (compliance) and the relaxation function can be reciprocally related by the integral equation.

$$I = R(t_0, t_0)J(t, t_0) + \int_{t_0}^t J(t, t')dR(t', t_0) \quad (21)$$

To solve the integral equations above numerical integration is necessary (Sassone and Chiorino, 2005). Various numerical procedures for solving the integral equations have been suggested by Bazant (1972a). Integral equations can be solved by hand methods such as the trapezoidal rule. Computers can also be used to quicken the process. Numerical solution of the Volterra integral equations to obtain the relaxation function is a complex and time consuming process (Bazant and Kim, 1979; Lacidogna and Tarantino, 1996).

This problem lead to the necessity of developing simplified expressions to link the creep and relaxation functions. The simplified methods relating creep and relaxation functions are based on the assumption that the loads and stresses in a structure have a single-step history. This

means that they remain constant from when the structure is first loaded. This is not true in real structures. Integral equations and the principal of superposition described above would be ideal to solve real problems. In the next sections, simplified methods relating creep and relaxation are discussed.

#### 2.4.2 Effective modulus method

The oldest simplified method of solving the creep or relaxation function is the effective modulus method represented by Equation 22. According to Goel *et al.* (2007), the method was first introduced by McMillan in 1916. In this method, the relaxation function can be inversely related to the creep function. When the inverse of the creep function  $J$  (i.e. effective modulus) is equated to the relaxation function, the expression is termed the *classical approximate formula* (Bazant and Kim, 1979). This is the simplest way of relating the two functions (Carlswärd, 2006). In this method, the influence of ageing is not considered. Only the load duration ( $t-t_o$ ) and the time of load application ( $t_o$ ) affect creep (Bazant and Kim, 1979). This is not true for concrete as the material ages with time. This implies that in relaxation,  $R(t, t_o) < \frac{I}{J(t, t_o)}$  because the stress reduces at constant strain

$$E_{c, ef}(t, t_o) = \frac{I}{J(t, t_o)} = \frac{E_c(t_o)}{1 + \phi(t, t_o)} = R(t, t_o) \quad (22)$$

#### 2.4.3 Age-adjusted effective modulus method (MC90-93:2010)

The accuracy of the effective modulus method was known to be very poor because the ageing of the material was not considered. Trost (1967) cited in Bazant and Kim (1979b) proposed an algebraic expression which reduced the error in the first method. A further refinement called the age-adjusted effective modulus method (AAEM) was proposed and mathematically proven by Bazant (1972b). In this method, the effective modulus is corrected by a coefficient called the aging coefficient. This coefficient can be calculated and tabulated for a given creep function. Carlswärd (2006) citing Westman (1999) explains the effect of the ageing coefficient as a result of continuous hydration of the cement gel when concrete is loaded at early ages. At early ages, the available cement gel carries all the stress, but as hydration continues, new gel is formed which relieves the stress on the original cement gel.

Therefore, by incorporating the aging coefficient ( $\chi$ ), the solutions are in some cases similar to those got from solving integral equations (Chiorino, 2005). The following algebraic expression is presented in MC90-93 and MC2010 (Equation 23)

$$R(t, t_0) = E_c(t_0) \left[ 1 - \frac{\phi(t, t_0)}{(E_c / E_c(t_0) + \chi \phi(t, t_0))} \right] \quad (23)$$

where:

$R(t, t_0)$ : is the relaxation function, which represents the axial stress produced at a time  $t$  by a constant unit strain applied at time  $t_0$

$E_c(t_0)$ : is the concrete modulus of elasticity at time  $t_0$

$E_c$ : is the concrete modulus of elasticity at 28 days

$\chi$ : is the concrete ageing coefficient set at 0.8, but can be determined more accurately as given in CEB MC90-93 and presented in Equation 24.

$$\chi = \frac{t_0^{0.5}}{1 + t_0^{0.5}} \quad (24)$$

$\phi$ : creep coefficient that can be determined experimentally or estimated using prediction models

The AAEM can be expressed in terms of the relaxation coefficient ( $\psi$ ) as shown by Bazant (1972b). In Equation 23, when the relaxation function  $R(t, t_0)$  is replaced with the ratio of stress  $\sigma(t)$  divided by the initial strain  $\varepsilon(t_0)$ , the result is equation 25.

If  $R = \frac{\sigma(t)}{\varepsilon(t_0)}$ ;  $\varepsilon(t_0) = \frac{\sigma(t_0)}{E(t_0)}$ ;  $\Delta\sigma(t, t_0) = \sigma(t_0) - \sigma(t)$ ;  $\psi = \frac{\Delta\sigma(t, t_0)}{\sigma(t_0)}$ , and assuming constant strain,

Equation 23 becomes

$$\psi(t, t_0) = \frac{\phi(t, t_0)}{E_c / E_c(t_0) + \chi \cdot \phi(t, t_0)} \quad (25)$$

This expression is also given in the fib structural concrete text book on behaviour, design and performance (Mueller and Haist, 2009).

#### 2.4.3.1 Advantages and disadvantages of AAEM

The biggest advantage of the AAEM is its simplicity. All input parameters are obtainable from the laboratory or from prediction models. Calculations are quicker and less cumbersome. This is because they do not involve numerous steps of integration similar to the case with integral formulas relating the creep and relaxation function. Moreover, the AAEM

be used with some degree of accuracy at the initial design stage especially for simple problems. For more complex problems, the AAEM can be used by summing a series of single step stress histories (Sassone and Chiorino, 2012).

The disadvantage of the AAEM is that it cannot be used in complex situations such as those dealing with hygrothermal effects and cracking. Calculation of the ageing coefficient is time consuming in cases where it cannot be avoided (Sassone and Chiorino, 2012).

#### 2.4.4 Approximate relaxation function (ARF) (MC90-93)

As is the case with the AAEM, the principal of this method is to reduce the error in the effective modulus method. This expression was first developed by Bazant and Kim (1979b). It was developed because of the disadvantage of the AAEM. In the AAEM, different and long tables of ageing coefficients would have to be developed to cater for different humidity conditions and size of cross-sections. Therefore an expression that does not require tabulation of any coefficient and one which worked well for most conceivable time shapes of the concrete creep function was proposed (Bazant and Kim, 1979b). Again, the validity of the principal of superposition for concrete is assumed. In this expression, the relaxation function is given in terms of the creep compliance function, Equation 25.

$$R(t, t_0) = \frac{1 - 0.008}{J(t, t_0)} - \frac{0.115}{J(t, t - 1)} \left[ \frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \right] \quad (25)$$

$R(t, t_0)$  = relaxation function at time,  $t$  due to constant imposed unit strain at time  $t_0$

$t$  = age after loading at which any given parameter is calculated;

$t_0$  = age of member during loading.

$J(t, t_0)$  = creep compliance function representing the total stress dependant strain per unit stress. The creep compliance function  $J(t, t_0)$  can be determined using models such as the MC90-99, B3 Model (Bazant and Baweja, 2000) or the GL2000 (Gardner and Lockman, 2001).

$$\Delta = \frac{t - t_0}{2}$$

Equation 25 can be re-written in the form;

$$\frac{I - 0.008}{J(t, t_0)} - \frac{c_1 \alpha_0}{J(t, t-1)} \text{ where } c_1 = 0.115 \text{ and } \alpha_0 = \left[ \frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \right] \quad (25a)$$

The ratio  $\alpha_0$  is used to introduce the effect of reducing creep with ageing. The numerator and denominator in that expression have the same load duration. If ageing is not considered,  $\alpha_0$  vanishes. Because  $\alpha_0$  is dimensionless,  $J(t, t-1)$  is introduced to make the right hand side of the equation dimensionally correct. Constant  $c_1$  is introduced to allow an even better approximation of the relaxation function.

Recently a new version of the method has been proposed by Bazant *et al.* (2013). The new formula is proposed because the old formula yielded poor results for the long-time relaxation function obtained from the creep function. The constants in both the old and new expressions of the approximate relaxation function were obtained from curves drawn using the accurate numerical integration. Both methods are semi-empirical.

$$R(t, t_0) = \frac{I}{J(t, t_0)} \left( 1 + \frac{c_1 \alpha(t, t_0) J(t, t_0)}{q J(t, t-1)} \right)^{-q} \quad (26)$$

$$\text{where } c_1 = 0.0119 \ln t_0 + 0.08; \quad q = 10 \quad \alpha(t, t_0) = \frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \quad \Delta = \frac{t - t_0}{2}$$

Both the old and new expressions of the relaxation function were tested in this study.

#### 2.4.5 Design aids

The numerical procedures of solving integral equations linking the relaxation and creep functions can be made simpler by using calculation (design) aids. One such design aid 'creepanalysis' is recommended by MC2010 and is available for download at the website [polito.it/creepanalysis](http://polito.it/creepanalysis) (Sassone and Chiorino, 2005).

#### 2.4.6 Summary on relaxation prediction

Different researchers have over the years developed problem specific expressions relating creep and relaxation. These include: Marimoto and Kayonagi (1994) who developed a model based on experimental data gathered by preparing and testing 2 concrete samples. In their research, equations that estimate relaxation at early ages were proposed. The research also investigated the effects of age of concrete, initial stress level and testing temperature on stress relaxation.

Gutsch and Rostásy (1994) also developed a model based on linear creep theory. The model was suggested for the prediction of creep and relaxation of early age concrete. This model was based on experimental work. Tests were carried out under tensile stresses. Tests carried out include: axial tensile tests, creep and relaxation tests. Ghali and Favre (1994) presented a mathematical expression for relaxation based on instantaneous stress and creep functions. As is the case with most of the models, Ghali and Favre (1994) derived a relaxation model from the basis of creep. The derivation was not specific to a particular magnitude of stress, sample size and loading conditions.

Details concerning these works can be seen in the respective literature. In this research AAEM method and ARF were tested and compared with laboratory experimental work. These expressions are comprehensive and data necessary to perform calculations is obtainable through experiments. The equations are found in the fib Model Code which is the basis of new and future concrete codes.

## **2.5 Summary of literature review**

Available information and data on relaxation is scanty and is authored by a handful of researchers. Some aspects of relaxation are agreed upon by most researchers for example the fact that similar to creep, relaxation is predominantly caused by movement of water in the cement gel. Micro cracking is also suggested to explain this phenomenon especially above service stresses. It is also generally accepted that similar to creep, relaxation is affected by age of loading, water-cement ratio and initial stress-strength ratio. Another important aspect recognised by all researchers is the importance of relaxation in the mitigation and prolonging of crack development. Relaxation function is in most cases derived from the creep function. The data base on relaxation experimental work is insufficient to be confidently used by researchers. More experimental work needs to be done.

Unlike relaxation, the mechanisms and factors that influence creep are fairly well understood. Data on the creep behaviour of concrete is also available and accessible. As mentioned above, the movement of water in the cement gel and micro-cracking are the most common mechanisms forwarded to explain how creep occurs in concrete. The factors that affect creep are also very well-known and include: water binder ratios, moisture content, cement type, aggregate properties and content, member geometry and size, drying conditions, stress-strength ratios, curing and age of loading. It is important to study and design for creep in structures because it influences the serviceability and durability of structures.

A number of gaps still exist in the studies on relaxation and its relation to creep. From all the studies on relaxation, most researchers agree that more raw data is required on the relaxation

behaviour of concrete. For example Masuku (2009) suggests that research on the effect of mix components such as aggregates is needed. Because some of the present work on relaxation has been carried out on mortar specimens, there is need to investigate typical concrete mixes commonly used in bonded overlays. Furthermore, contradictions still exist on how some important parameters affect relaxation. For example Marimoto and Kayonagi (1994) found that compressive and tensile relaxation is proportional to initial stress-strength ratio in the range below 80%. Gutsch and Rostasy (1994) and Atrushi (2003) [citing work by Serda and Kristeck (1988)] however found no proportionality.

Most importantly, it is known that available simplified relaxation prediction models are based on creep functions. Laboratory experimental work to determine the relaxation function needs to be carried out in order for available expressions to be tested (Chiorino, 2005; Atrushi 2003; Wittmann, 1971). Simplified algebraic expressions such as the AAEM and ARF are verified in chapter five. Compared to numerical integration and computers, these models enable quick calculations (Bazant *et al.*, 2013; Bazant and Kim, 1979; Carlswärd, 2006).

## 3 METHODOLOGY

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### 3.1 Introduction

Experimental work was carried out to investigate whether creep and relaxation are influenced in the same way by the same factors. In addition, experimental values were used to check if relaxation models give good results for low-age relaxation prediction. Tests were carried out on the same concretes. In some cases, laboratory results were compared with past studies in order to understand the influence of selected parameters. Test variables in this study included: w/c ratio, age of loading, initial stress-strength ratio and influence of aggregate content. W/c ratio and age of loading were varied for both relaxation and creep tests. The influence of initial stress-strength ratio was investigated on relaxation and compared with available literature for creep. Likewise, the influence of aggregates on relaxation was investigated by making comparisons with past studies.

To achieve the first aim of this study, the following tests were carried out: compressive strength, tensile strength, elastic modulus, creep, relaxation and drying shrinkage tests. Details of test procedures and experimental designs are discussed in the relevant sub-sections of Section 3.4. Specific reasons for carrying out the named tests are given in the relevant sections.

One of the challenges encountered in this study was the shortage of equipment used for long term tests such as creep and relaxation tests. This reduced the possibility of additional replicate testing for the said tests. With replicate testing, certain trends in the experimental work would be confirmed. Figure 3.1 illustrates the experimental approach which was followed in this research.

### 3.2 Test Variables

#### 3.2.1 Concrete mix design and material selection

3 mixes were used in this study. W/c ratios of 0.45, 0.55 and 0.65 were chosen based on past research. The three w/c ratios reflect some of the most commonly used w/c ratios in industry. Throughout this research, CEM 1-52.5 (OPC) was used as binder. This is the cement grade currently manufactured by South African cement producers. No extenders were used in this research.

Klipheuwel sand was used for all mixes because of its advantages. It is well graded and has a low water demand hence good workability (Grieve, 2009). Zwarteboosch stone, 9 mm in size was used as aggregate for all the three mixes. All mixes had the same volume of coarse aggregates. 9 mm aggregates were used because of the size of the relaxation samples. With

an effective area of 40 x 40 mm, 9 mm aggregates can fit in the moulds without undermining the strength of the concrete.

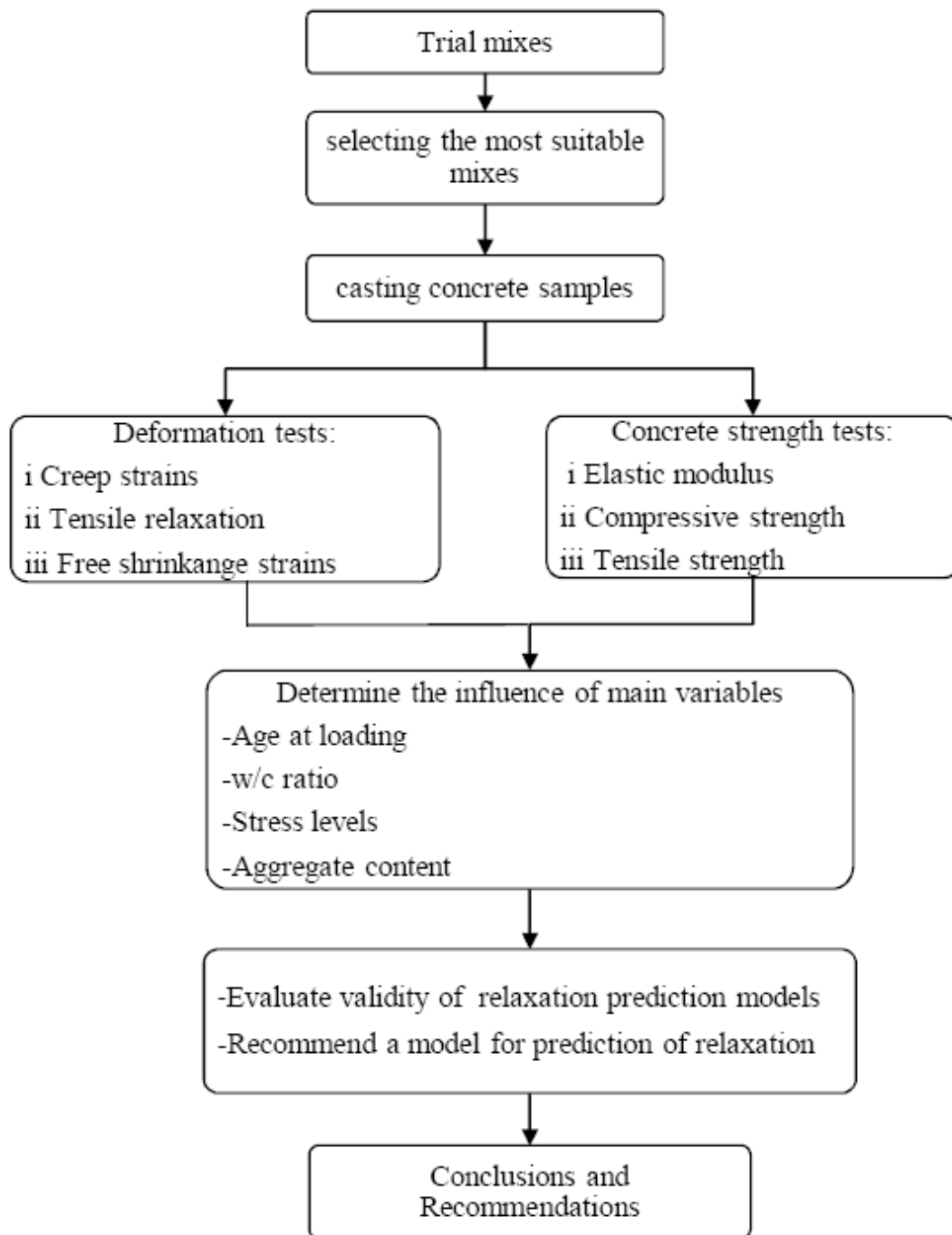


Figure: 3.1: Schematic structure of experimental research.

Some work has been done on the relaxation behaviour of mortars by Beushausen *et al.* (2012) and Chilwesa (2012). In one of their recommendations, Beushausen *et al.* (2012) called for work to be done on the relaxation behaviour of concrete containing aggregates. Relaxation results from this research were compared with related work in past literature in order to

identify the influence of aggregates. Superplasticiser was used to improve the workability of the 0.45 mix. Table 3.1 shows the mix proportions of the different constituents.

Table 3.1: Mix composition, proportions and selected properties of concrete mixes

<i>Constituent</i>	<i>Units</i>	<i>Concrete types</i>		
<b>w/c ratio</b>		<b>0.45</b>	<b>0.55</b>	<b>0.65</b>
<b>CEM 1 52.5N</b>	(kg/m <sup>3</sup> )	400	336	292
<b>Water content</b>	(kg/m <sup>3</sup> )	180	185	190
<b>Superplasticiser (Visco Crete 10)</b>	(l/m <sup>3</sup> )	0.63	-	-
<b>Klipheuwel sand</b>	(kg/m <sup>3</sup> )	1063	1104	1128
<b>9 mm Zwartebosch stone</b>	(kg/m <sup>3</sup> )	801	801	801
<b>Paste content</b>	(l/m <sup>3</sup> )	307	292	284
<b>Slump</b>	(mm)	130 ± 10	85 ± 10	95 ± 10
<b>28-Day cube compressive strength</b>	(MPa)	59	45	34

### 3.2.2 Age of loading

Two ages of loading were considered i.e. 3 and 10 days. These ages are calculated from the day of casting. Samples were water cured for the durations mentioned prior to testing or loading. The low ages of testing were chosen primarily because relaxation is important in early ages of loading when the concrete is susceptible to cracking (Banthia and Gupta, 2009; Pigeon *et al.*, 2005; Bentur and Kovler, 2003; Atrushi, 2003). Additionally, similar to creep, the magnitude of relaxation is higher in young concretes (Beushausen *et al.*, 2012; Masuku, 2009; Morimoto and Kayonagi, 1994; Atrushi, 2003). The ages of testing and loading were kept similar for both creep samples and relaxation samples to enable a comparison between the two. For comparison purposes and because of their importance in some prediction models, some concrete properties were tested at the age of 28 days. The particular tests carried out are discussed in Section 3.4.

### 3.2.3 Stress levels

Owing to the limitation in the number of available creep rigs, all samples were loaded at the same stress level. The pressure to be loaded on the cylinders was determined by the strength of the weakest samples. However, samples were of different compressive strengths, which meant that different stress-strength ratios were applied to each of them. Stress-strength ratios ranged from 10% to 30% depending on concrete age and strength. These stress-strength ratios are summarised in Table 3.2.

Table 3.2: Applied stresses on creep samples at the time of initial loading.

	$w/c = 0.45$		$w/c = 0.55$		$w/c = 0.65$	
Age	Applied stress (MPa)	Stress-strength ratio (%)	Applied stress (MPa)	Stress-strength ratio (%)	Applied stress (MPa)	Stress-strength ratio (%)
<b>3 days</b>	6	15	6	21	6	30
<b>10 days</b>	6	12	6	15	6	20

At the stress levels shown in Table 3.2, the stress is assumed to be proportional to the strain in concrete. A non-linear relationship exists when stress-strength ratios above 40 % are applied to the concrete. Thereafter which micro cracks start to develop in the cement paste. The actual ratio at which non-linearity is assumed varies depending on the concrete strength (Neville 2002). For that reason, stress-strength ratios were kept at a maximum of 30% for creep of concrete in compression.

In reality, the stress-strength ratios in Table 3.2 cannot be maintained with time as the concrete continues to creep (Atrushi, 2003). This is because of two main reasons: first, as the concrete creeps or deforms, the stress applied is reduced (if it is not controlled). Secondly, the concrete strength increases with age hence reducing the stress-strength ratio. To maintain the same stress-strength ratio, the stress applied on the concrete specimens would have to be increased in proportion to the strength increment.

In tension, the concrete starts to crack when its tensile capacity is exceeded (Pigeon and Bissonnette, 1999). For that reason, the relaxation potential of a concrete is of most importance when stress-strength ratios of about 80% are applied to the concrete. In order to study the effect of stress levels on relaxation, two stress levels were initially investigated i.e. 80% and 40%. 72-hour relaxation tests were also carried out at the two stress levels in order to investigate the effect of stress level in the prediction of relaxation. For a more detailed investigation of the effect of initial stress-strength ratios, two additional stress-strength ratios i.e. 20% and 60% were investigated.

To estimate stress levels to apply on relaxation samples, their tensile strength was first determined. Two samples were used to determine the tensile strength of the concrete for each mix, day and round of testing. From it, the average stress was determined. Table 3.3 shows how stresses and forces were calculated for relaxation experiments.

Table 3.3: Stress calculation for relaxation experiments

Ultimate failure load (N)	Area (mm <sup>2</sup> )	Stress (MPa)	Average Stress (MPa)	Stress applied to relaxation sample (MPa)			
				20 %	40 %	60 %	80 %
5492	1200	4.6	4.44	0.9	1.8	2.7	3.5
5154	1200	4.3					

### 3.3 Experimental procedures

In this section, the procedures used to carry out the various tests are discussed. In cases where code procedures are followed, mention is made of the specific code used. Testing was carried out in order to obtain material properties that can be used to describe the concretes. Tests carried out include compressive strength, tensile strength tests, creep tests, relaxation tests, elastic modulus and free shrinkage tests.

#### 3.3.1 Compressive strength tests

The compressive strength is the property generally used to identify the quality of concrete (Omar *et al.*, 2008). Moreover, it is an input parameter for some prediction models, for example those used to predict creep, shrinkage and elastic modulus. For this research, the compressive strength was also important because it was used to estimate the load to apply to samples when carrying out creep and elastic modulus tests.

For each mix and day of testing, 3 no 100 mm concrete cubes were cast and wet cured 3, 10 and 28 days before testing. Cubes were tested at the ages of 3, 10 and 28 days after casting. The Amsler compression testing machine was used for compressive strength tests. The machine has a capacity of 3000 kN.

Compressive strength testing was carried out following procedures similar to those described in the South African Code - SANS 5863: 2006. Prior to loading, the weight of the cubes was determined using an electronic balance. Samples were loaded until failure. The load was applied at a rate of 2 kN per second up to failure.

The compressive strength was calculated as the load at failure divided by the cross sectional area of the cube. The failure load was read off the compression testing machine while the average cross sectional area of the cube is determined prior to loading. The compressive strength of the mix was determined as the arithmetic mean of the three cubes tested.

### 3.3.2 Elastic modulus

The elastic modulus of the concretes was determined because it is an input parameter in relaxation and creep prediction models. See Sections 2.2.8 and 2.4.4.1 for details on such models. Prior to the determination of the elastic modulus, the compressive strength of the samples was determined

Determining the elastic modulus involved the loading concrete specimens incrementally while measuring the changes in strain. Static tests were carried out as opposed to dynamic tests. In the former method, loads are applied over a period of minutes while in the latter loads are applied over microseconds (Alexander and Beushausen, 2009). The procedures followed are similar to those prescribed in BS 1881-121: 1983. 3 samples were cast for each mix and age of testing. Concrete cylinders whose dimensions were 100 mm diameter by 200 mm length were used to determine the static elastic modulus. Targets were placed on the specimens and changes in strains measured off by a measuring instrument- the Demec strain gauge such as that shown in Figure 3.2.



Figure 3.2: Mechanical strain gauge and reference bar

Loads were applied using the compressive testing machine the Amsler which has a capacity of 3000 kN. The samples were loaded for four cycles of loading and unloaded at a rate of 2 kN per second. Changes in strains were measured at each cycle. A stress versus strain graph was then plotted and a 'secant modulus' calculated as the elastic modulus. The 33 % value of ultimate compressive strength was used as the upper limit of proportionality when calculating the secant modulus.

### 3.3.3 Creep tests

Creep tests were carried out to determine the time dependent deformation of the 3 mixes investigated in this research. Moreover, expressions such as the creep coefficient and creep compliance function are input parameters in relaxation prediction models. Only the influence of age of loading and w/c ratio was investigated for creep tests. Past studies on creep were used to compare influence of aggregate and initial stress-strength ratio of creep and relaxation.

When placing concrete in creep moulds care was taken to ensure that it was well tamped in three layers. This was done to avoid honeycombing as this would undermine the ability of the concrete samples to withstand load. One cylinder (105 mm diameter x 300 mm length) was cast for each mix and age of loading. This was because of insufficient creep rigs to accommodate more cylindrical samples. The test specimen, in their moulds, were covered with a plastic sheet and stored overnight in a place free from vibration and in a room that has a temperature of  $23 \pm 2^\circ\text{C}$  and relative humidity of  $50 \pm 5\%$ . The specimens were then demoulded one day after casting and wiped with a damp cloth. Targets were glued onto the specimen using a paste made of epoxy resin. The targets were placed at a gauge length of 100 mm between any two adjacent targets. Three targets in a line are placed on three equidistant positions along the face of the cylindrical specimens. This provided 6 measuring points for each sample.

Creep tests were carried out following procedures similar to those prescribed by the American Society of Testing Materials (ASTM C512M-10) and RILEM CPC-12, 1983 measurement of creep in compression. This method involves initially applying stress on concrete specimens over a short time period as will be described later in subsequent sections. Stress-strength ratios have been shown and described in Table 3.2. Strain readings were immediately taken before and after loading. To ensure the correct functioning of the creep rigs, the procedure of loading and unloading while readings are taken was repeated and readings compared. Initially readings were taken daily for one week. Thereafter, readings were taken thrice a week and eventually readings were taken once a week. Readings were taken for just over 90 days.

The creep rig is equipped with a hydraulic jack and a spherical bearing plate to ensure uniform distribution of imposed load over the specimen. Hydraulic pressure in the jacks is individually measured and controlled by a central hydraulic system known as the accumulator. The load is applied by a hand pump at a rate of 3 kN per second.



Figure 3.3: Creep rig set-up

Concrete samples were checked for level after being placed in the creep rig. This was to ensure that stresses are distributed evenly over the cross-section area of the sample. Strains were measured using a Demec strain gauge (Figure 3.2). The measuring equipment and reference bar were kept in the creep test room. Creep test samples together with the test rigs were kept in the same room. The creep strain was calculated as the average of the 6 readings on each sample. The stress applied on the specimens drops as the samples creep. This is because of the reduction in strain in the sample. The stress was re-adjusted back whenever it was noted to drop below the set value.

### 3.3.4 Tensile strength tests

The tensile strength of the concrete mixes was determined in order to estimate stress levels to apply on relaxation samples. In relaxation tests, a fraction of the concrete's ultimate failure load (tensile) is applied to the sample. The sample is then held at constant strain. Immediately, the stress/ force decay is monitored.

For tensile strength tests, the same dog-bone used in the relaxation test were used. The geometry of the samples is described in Section 3.4.1.1. The specimens were subjected to uniaxial tensile force using the Zwick Roell Z020 Universal Testing Machine (UTM) until failure. Tests were carried out at the ages of 3, 10 and 28 days. Two specimens were tested for each mix and for each day of testing. Tensile tests were repeated whenever relaxation tests were performed. This was done because of slight variations in strength that similar concretes cast on different days might have. Curves of stress vs. strain were obtained from

the UTM via a connected computer. The tensile stress was calculated as the failure load divided by the cross-sectional area at the point of failure.

### 3.3.4.1 Specimen geometry for tensile and relaxation tests

Dog-bone specimens with dove tail ends (Figure 3.4) were used for direct tensile strength and relaxation tests. The overall dog bone specimen length was 270 mm. The widest section was 90 mm at the gripping area and was reduced gradually to 40 mm over a 50 mm length. To connect the wide and uniform profile of the specimen, it is curved at the connection of the two profiles in order to make the change less pronounced. Figure 3.4 below shows dimensions of the test sample.

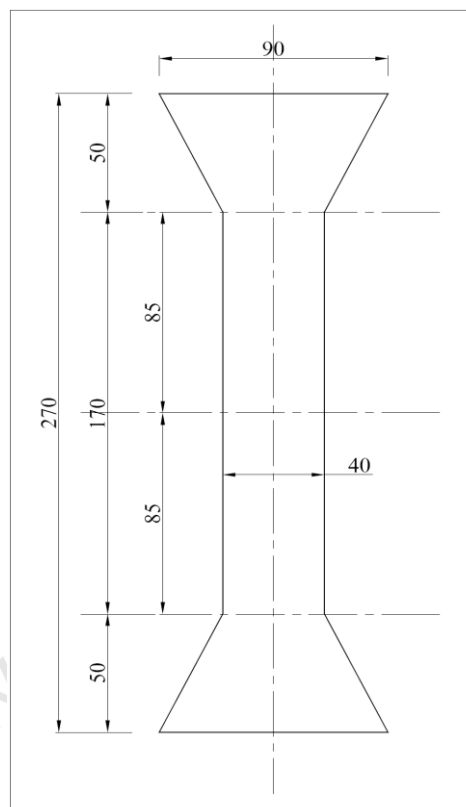


Figure 3.4: Basic geometry of the test specimen (All dimensions in millimetres)

During a direct tensile test, the portion of the test specimen with a uniform profile (40 mm thick and approximately 150 mm long) is expected to experience the highest tensile stresses. The specimen is therefore expected to break within that area.

To solve the problem of local failures (Figure 3.5), a notch was incorporated in the specimen in order to induce failure in the uniform section of the specimen.



Figure 3.5: Local specimen failure in non-prismatic region

A notch of dimensions 1 mm thick by 5 mm wide by 40 mm long was situated centrally on both sides of the specimen as shown in Figure 3.6. That size of notch was arrived at after a trial run of tests on samples with a notch of 2.5 mm width did not produce the desired effect consistently.

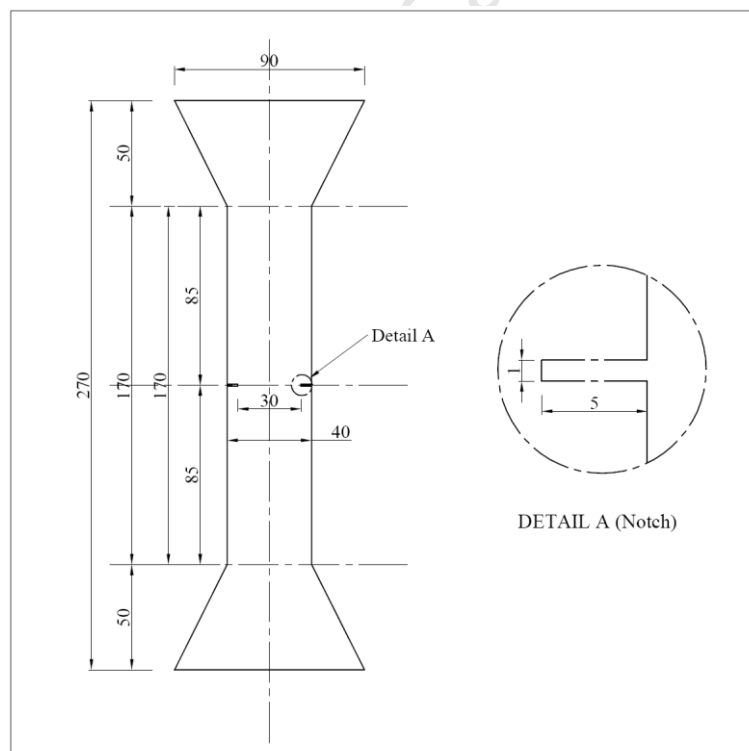


Figure 3.6: Modified dog-bone specimen showing notch (All dimensions in millimetres).

A trial run was carried out on two specimens to check the effect of the notch on failure loads. One sample had a notch and one had no notch. The two were found to give similar value of

tensile stress as long as the appropriate failure cross sectional area was considered in stress calculations. In fact it was observed during the course of the research that values of tensile stress calculated with notched samples were consistent while those calculated with un-notched samples were inconsistent and underestimated strength. The notched dog bone specimen was used primarily to determine the tensile strength of the concrete samples. These stresses were then used to estimate stress-strength ratios for relaxation experiments. Figure 3.7 shows samples that were tested for tensile strength using the improved notch.



Figure 3.7: Induced failure point in the prismatic section of modified dog-bone specimen

#### 3.3.4.2 Zwick Roell Universal Testing Machine (UTM)

The Zwick Roell UTM was used for tensile and relaxation testing in this research. Two machines were available for use during the testing period. The two Zwick Roell machines have capacities of 20 kN and 100 kN respectively. Trial tensile and relaxation tests were carried out on both machines to compare results from similar concretes. Results from both machines were found to be similar. This meant that depending on availability, any one of the machines would be used in this study. Most importantly, this meant that the machines would confidently be used in comparative studies like in the investigation of the influence of different stress-strength ratios on the relaxation behaviour of concrete. Figure 3.8 shows the 20 kN Zwick Roell UTM used in this research.

The Zwick Roell UTM is a multifunctional machine that can be used for a number of tests for example elastic modulus, compression, tensile and relaxation tests. For a particular test, the UTM is fitted with the appropriate loading frame.

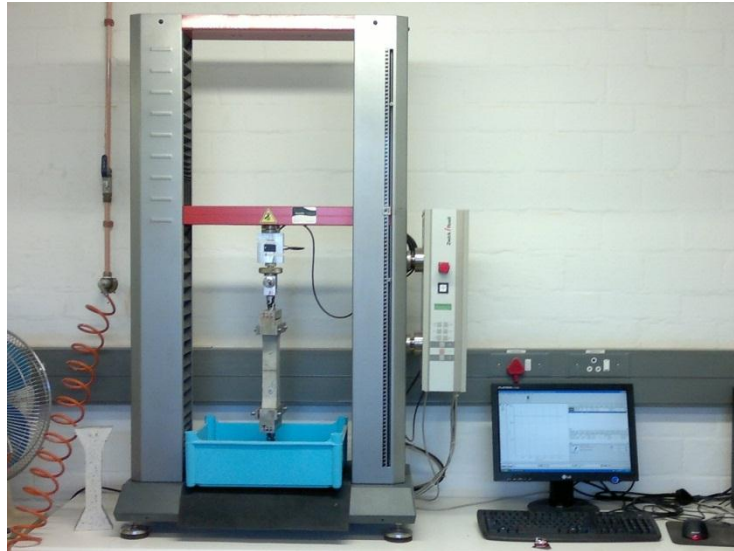


Figure 3.8: Zwick Roell Universal Testing Machine (Z20)

Tensile and relaxation experiments were carried out using the same loading frame fabricated in such a way that it was compatible with standard fixtures of the UTM (Masuku 2009). Of most importance for the tensile and relaxation loading frame are the aluminium gripping jaws designed to hold the specimen in place during testing. The gripping jaws were made of aluminium alloy and stiffened with a metal plate also made of aluminium alloy. Bearings are fitted on the jaws to ensure axial loading and rotational freedom to prevent build-up of moment. Figure 3.9 shows the aluminium gripping jaws used to hold specimen in place.



Figure 3.9: Aluminium gripping jaws for tensile strength (and relaxation) tests.

The dimensions of the gripping jaws were 124 mm x 50 mm x 50 mm. The overall thickness of the plates was 20 mm. Some of the input parameters when using the Zwick Roell for

tensile strength tests include the loading rate and specimen dimensions. The load was applied by the upper crosshead which travels at a speed of 0.2 mm per minute until failure.

### **3.3.5 Tensile relaxation tests**

#### **3.3.5.1 Introduction**

One of the main aims of this research is to test some of the available relaxation prediction models by experimental work. Thus, relaxation experiments were carried out to obtain relaxation data on the different concrete mixes. From relaxation experiments, the magnitude of relaxation with time and the relaxation coefficient were obtained. To simulate the relaxation behaviour of the material, the Zwick UTM is used to apply a tensile force on a sample and maintain it at constant strain.

#### **3.3.5.2 Sample preparation**

Relaxation specimens were cleaned after the end of their curing period in readiness for testing. All tensile relaxation specimens were tested under sealed conditions. This was done in order to avoid additional stress due to shrinkage. This meant that only the relaxation of the applied stress was monitored. Sealing of the tensile relaxation samples was done by applying a thin layer/coating of hot melted paraffin wax on all the exposed surfaces.

The sealing of the specimen does not however eliminate all the shrinkage in the concrete. Any autogenous shrinkage in the mixes studied in this research was considered negligible as the lowest w/c ratio was 0.45. According to Alexander and Beushausen (2009) autogenous shrinkage is not significant in normal strength concretes having a w/c greater than 0.40. Thermal strains could also lead to thermal stresses. To avoid additional self-induced stresses, experiments were carried in a room whose temperature is maintained at  $23 \pm 2^\circ\text{C}$  and a relative humidity of at least  $50 \pm 5\%$

Before placing tensile relaxation samples in the testing frame, the tensile strength of the specimen was determined. This was done to estimate the stress to be applied on the relaxation samples. The 72-hour relaxation test samples were subjected to a stress equal to 80% and 40% of their ultimate tensile strength. See Section 3.3.3 for calculation of stresses to be applied on relaxation samples. Test specimens were then placed into gripping jaws and held in position within the testing frame. The calculated initial direct tensile stress was then applied by the UTM and strain kept constant. The working of the UTM is discussed in the Section 3.4.4.3. The computer is the main component for data output attached to the UTM.

#### **3.3.5.3 Results output**

The UTM software -'testXpert' enabled the computer to record the increase in strain as well as the final strain reached by the loaded specimen. The software was also used to monitor the

tensile stress decay within the specimen under test with the time. The recorded data was printed out and analysed. Stress relaxation analysis involved relating initial stresses applied to the sample to observed stress at any time along the curve. The gradient of the curve was used to monitor the rate of stress relaxation. This is because relaxation proceeds at a decreasing rate after 3 days. Examples of studies in support of that argument are presented in Section 2.3.4.2

### 3.3.6 Drying shrinkage

Shrinkage which involves withdrawal of water from concrete stored in conditions of normal relative humidity is known as drying shrinkage (Neville, 2002). The drying shrinkage test was performed because shrinkage data is required in the analysis of the creep deformation characteristics of concrete.

Test specimens were stored in a drying room with a clearance of at least 25 mm on all sides. In this experiment, concrete specimens are stored in a controlled environment of at least  $50 \pm 5\%$  relative humidity and  $23 \pm 2^\circ\text{C}$ . The shrinkage deformation was measured over a period of over 90 days. The test is carried out following procedures similar to those prescribed by the American Society of Testing Materials (ASTM C 157/C 157N-92) and the Australian Standard (AS 1012.13-1992). 2 No 105 mm diameter by 300 mm height cylindrical samples were used for shrinkage tests for each mix and age of testing. Shrinkage tests were carried out at the ages of 3, 10 and 28 days. The samples were water cured for 3, 10 and 28 days respectively.

A pair of strain targets (gauge points) were attached to each of two opposite positions along the face of the cylindrical specimens to provide a gauge length of 100 mm. Strain readings were taken using the demec strain gauge. Before readings are taken, the demec gauge was calibrated with a reference bar. When readings were first taken, the samples were marked so that one end always remained oriented in the same direction in relation to the measuring equipment. The shrinkage for each mix and age of testing was calculated as the average of the six shrinkage strain readings measured on the two cylinders and. Readings were taken daily for the first week, every other day for the second week, thrice for the third and fourth week, there after readings were taken once a week.

## 3.4 Conclusion

In the sections above, information on the research methods as well as experimental procedures followed have been presented. A summary of test types and numbers is provided in Table 3.4. Chapter four is used to present the results and discussions for the different experiments carried out.

Table 3.4: Summary of types and number of tests carried out in study

<i>Type of test</i>	<i>Reasons for carrying out test</i>	<i>Number of tests/specimen</i>
<b>Compressive strength</b>	<ul style="list-style-type: none"> <li>- Characterise the material</li> <li>- Input parameter in creep models</li> <li>- Estimate loads on creep and elastic modulus samples</li> </ul>	27
<b>Elastic modulus</b>	<ul style="list-style-type: none"> <li>- Input parameter in creep and relaxation models</li> <li>- Check functioning of creep rig by comparison of values</li> </ul>	27
<b>Tensile strength</b>	<ul style="list-style-type: none"> <li>- To determine stress level to apply on relaxation samples</li> <li>- Characterise material</li> </ul>	48
<b>Creep tests</b>	<ul style="list-style-type: none"> <li>- To determine creep potential of concrete</li> <li>- Creep values used as input parameters in relaxation models</li> <li>- To investigate influence of parameters</li> </ul>	6
<b>Tensile relaxation</b>	<ul style="list-style-type: none"> <li>- To determine the relaxation potential of the concrete</li> <li>- To investigate influence of parameters</li> </ul>	47
<b>Shrinkage</b>	<ul style="list-style-type: none"> <li>- To determine shrinkage potential of concrete</li> <li>- Used in analysis of creep data</li> </ul>	27

## 4 EXPERIMENTAL RESULTS AND DISCUSSIONS

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### 4.1 Introduction

In this Chapter, results from laboratory tests are presented and discussed. The discussed test results include compressive strength, tensile strength, elastic modulus, creep, relaxation and shrinkage. The influences of the different test variables such as w/c ratios, age of loading, aggregates and initial stress-strength ratios on creep and relaxation will be discussed.

Creep, shrinkage, and elastic modulus experimental results are compared with past research and with selected prediction models. It should be noted that this was to ensure laboratory results were within acceptable limits. For that reason, statistical analysis to rank the selected models was not carried out. For relaxation, a detailed comparison of laboratory results and values from prediction models is done in Chapter 5. This was one of the main aims of this study.

#### 4.1.1 Programing laboratory results and models

Laboratory results are presented in form of graphs and tables. These graphs and tables were developed using Microsoft Excel. In addition, the equations that make up the models used to verify laboratory results had to be modelled. A few options were available to model these equations including MATLAB and Microsoft Excel. Eventually Microsoft Excel was used because of its simplicity. Moreover, some of the prediction models were already available in MS Excel making it easier to continue with the same package.

Before being used, all the models were checked and validated by using hand worked examples to rule out errors due to poor programming. The American Concrete Institute (ACI 209, 2008) guide includes specific model solutions which can be used to validate Excel programs. The models included in the ACI report include: ACI 209R-92; Bazant-Baweja B3; CEB MC90-99 and GL2000. From those solutions, equations which predict the 28-day elastic modulus, shrinkage strains and creep strains (creep coefficient and compliance functions) for the respective models were programmed. Once the Excel programme can replicate the results in the guide, it can be used with confidence. After verifying the accuracy of the programmed Excel model, it was then used to predict the deformation properties for other mixes. This was done by varying the necessary input parameters.

## 4.2 Compressive strength

Compressive strength tests were performed on 100 mm cubes. The compressive strength tests were performed at the ages of 3, 10 and 28 days. The compressive strength tests were carried out in order to characterise the material. In addition, these tests were carried out in order to chose stresses in elastic modulus and creep experiments. During these experiments, stresses applied to samples are usually a fraction of the 28-day compressive strength. The development of the compressive strength is shown in Figure 4.1.

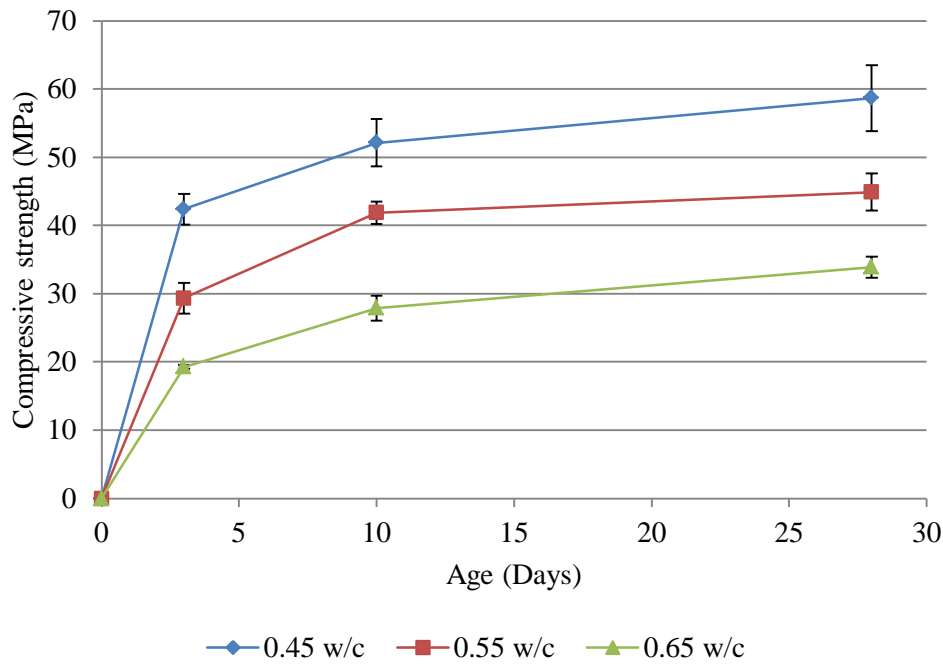


Figure 4.1: Influence of w/c ratio and age on compressive strength.

As expected, the compressive strength of concrete increases with decreasing w/c ratio and increasing age. This can be attributed to a stronger paste of the lower w/c ratio mix. Higher w/c ratio pastes are more porous than low w/c pastes because they contain more water and air as the concrete dries out.

## 4.3 Elastic modulus

As mentioned in Section 3.3.2, the secant elastic modulus of the three mixes was measured. Loads applied to the concrete were kept below 1/3 of its compressive strength at the age of testing. The magnitude of elastic modulus for the three mixes is presented in Figure 4.2. The elastic modulus increases from age 3 days to 10 days. Although they are within acceptable ranges, the 28 day elastic modulus results do not follow the trend expected because the measured values slightly reduce from the age of 10 to 28 days. This is probably because of experimental errors.

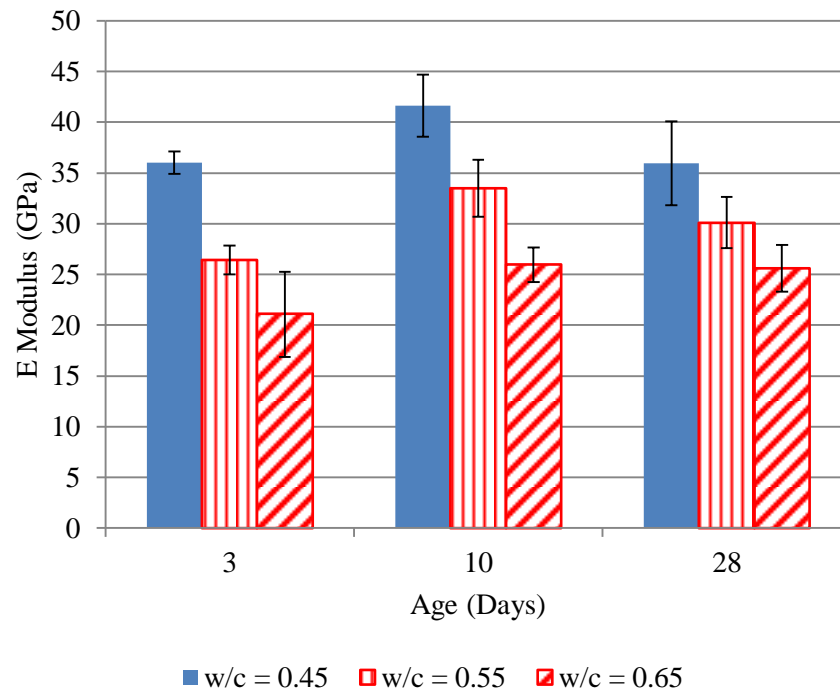


Figure 4.2: Influence of w/c ratio and age on elastic modulus.

Measured values would be expected to remain constant or to increase. The 0.45 w/c ratio mix had an average standard deviation of 2.7 MPa. The 0.55 w/c and 0.65 w/c ratio mixes had standard deviations of 2.3 MPa and 2.7MPa respectively.

The elastic modulus of the mixes is plotted against the compressive strength at all ages (Figure 4.3). The Figure shows that both compressive strength and elastic modulus increase with decreasing w/c ratio. The three points on each line represent the three mixes in order of decreasing w/c ratio i.e. 0.65, 0.55 and 0.45. Results by Pane and Hansen (2001) showed similar trends with both the elastic modulus and compressive strength increasing with age and decreasing w/c ratio. Details of elastic modulus results are presented in Appendix B.

Experimental results obtained for elastic modulus are similar to those suggested in Table 1 of SANS 10100 reproduced here as Table 4.1. The values in Table 4.1 are for elastic modulus determined under service stress. Service stresses are considered to be 33% of 28-day compressive strength (Neville, 2002). Comparisons were also made with selected prediction models as shown in Table 4.2. The elastic modulus predicted by the two models depends on the concrete's mean compressive strength.

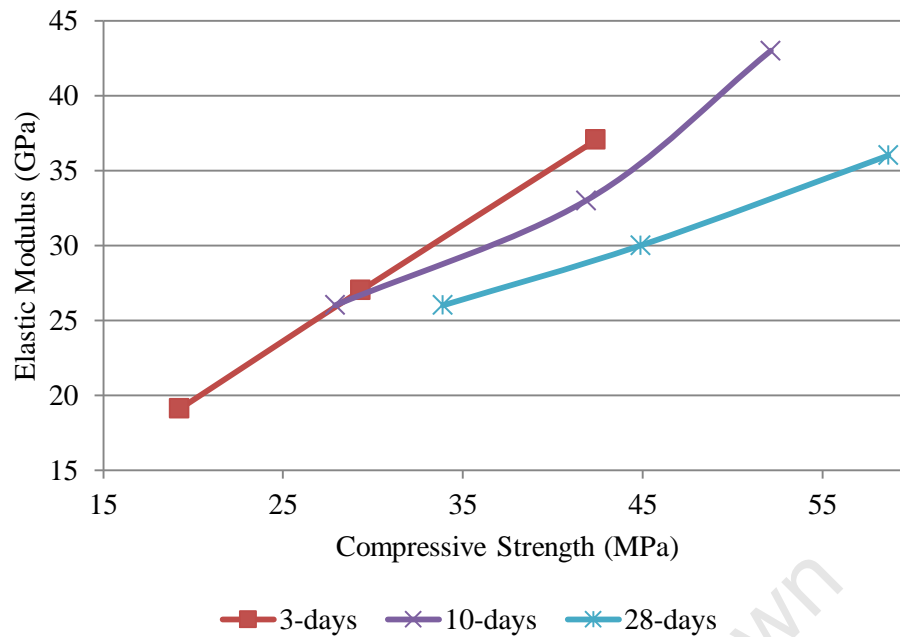


Figure 4.3: Relationship between elastic modulus and compressive strength

Table 4.1: Values for modulus of elasticity of concrete,  $E_c$  (SANS 10100)

1	2
<b>Cube strength of concrete at the appropriate age or stage under consideration MPa</b>	<b>Modulus of elasticity Of concrete, <math>E_c</math></b>
20	25
25	26
30	28
40	31
50	34

It is agreed that the main factors affecting elastic modulus of concrete are: the stiffness of the paste, the volume and stiffness of aggregates, the nature of the interfacial transitional zone (ITZ) between aggregates and the paste (Omar *et al.* 2008; Alexander and Beushausen, 2009). In this research, the coarse aggregates were kept constant.

Table 4.2: Comparisons of experimental elastic modulus results with selected models

Comparisons of 28-day Elastic modulus				
w/c		0.45 w/c	0.55 w/c	0.65 w/c
<b>Measured</b>		35.9	30.1	25.6
<b>MC99</b>		38.9	35.5	32.0
<b>GL 2000</b>		36.5	32.4	28.2
<b>Residuals, R<sub>e</sub></b>	<b>MC99</b>	3.00	5.40	6.40
	<b>GL 2000</b>	-2.40	-3.10	-3.80

This means that the stiffness of the paste is responsible for the differences in elastic modulus of the mixes. The stiffer the paste (concrete) the harder it is for it to be deformed. The stiffness of the paste also increases with decreasing w/c ratio and increasing age. The elastic modulus in compression was obtained in this research. Past research by Mehta and Monteiro (2006a); Atrushi (2003) has shown that the compressive and tensile elastic moduli are similar. Therefore, values of compressive elastic modulus obtained were used throughout the research.

#### 4.4 Tensile strength results

The tensile strength of concrete is one of the most important parameters in the concrete's ability to remain crack free (Atrushi 2003). When tensile stresses exceed the tensile strength of concrete, cracks develop. In this research, direct axial tensile tests were carried out on dog bone specimen whose geometry is described in Chapter Three. Tensile strength results are presented in Figure 4.4. The 3 day samples of all mixes have an average standard deviation of 0.41 while 10 day samples have a standard deviation of 0.37.

Similar to the compressive strength of concrete, the tensile strength is influenced by the w/c ratio and age of the concrete. The tensile strength increases with decreasing w/c ratio and increasing age of the concrete. The same reasons advanced for the increase of compressive strength of concrete apply for its tensile strength development (Mueller and Haist, 2009). The hydration process and the rate at which it occurs is the main factor that governs the concrete's increase in strength. Measured tensile strength test results are similar to those reported in literature (Pane and Hansen, 2001; Bissonnette and Pigeon, 1995; Masuku, 2009; Chilwesa, 2012). Details of tensile strength results are presented in Appendix C.

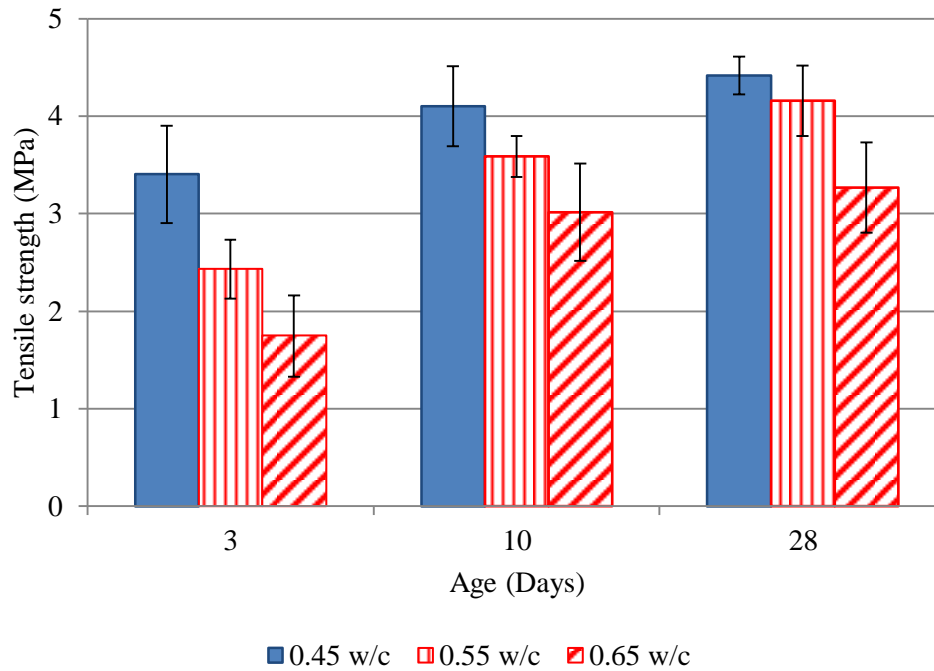


Figure 4.4: Influence of w/c ratio and age on tensile strength.

## 4.5 Creep results

In this section, creep test results are presented. Creep tests were performed on 3 day and 10 day concrete samples for the 3 mixes investigated in this research. The tests were carried out for a period of just over 90 days. In the calculation of expressions relating to creep, the initial elastic deformation and drying shrinkage are subtracted from the total strain recorded. This leaves the total creep of the concrete specimens. The total creep comprises of the drying creep and the basic creep. The basic creep is the creep undergone by a specimen when moisture loss is prevented. The drying creep is that undergone by a sample when drying (or moisture loss) is not prevented. In this research, the two will not be separated because the basic creep component of the total creep was not obtained. In Figure 4.5, the 3 day and 10 day total strain, shrinkage and total creep results are presented. Details on the development of the total strain, total creep, shrinkage, specific creep and creep coefficient are presented in Appendix D.

Samples were subjected to the same pressure (stress) but different stress-strength ratios (Section 3.2.3). Therefore, in order to compare the creep potential of the different concretes, the results were expressed in terms of the specific creep, creep coefficient and compliance function. Creep strains expressed in terms of the compliance function are presented in Section 4.5.1.

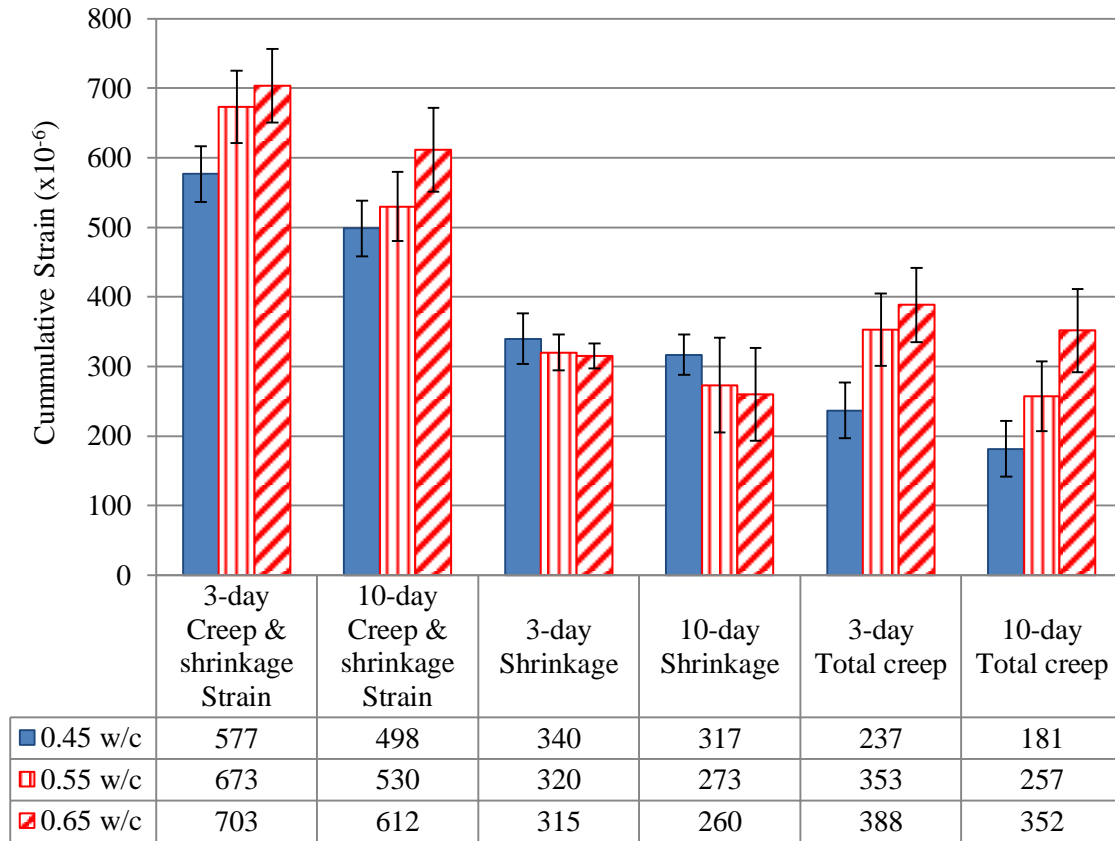


Figure 4.5: 90-day Strain, drying shrinkage and total creep results

In that section, laboratory results are compared with values from selected prediction models. The specific creep is the creep strain per unit stress. Figure 4.6 shows the specific creep of the 3 mixes when load is applied at 3 and 10 days respectively.

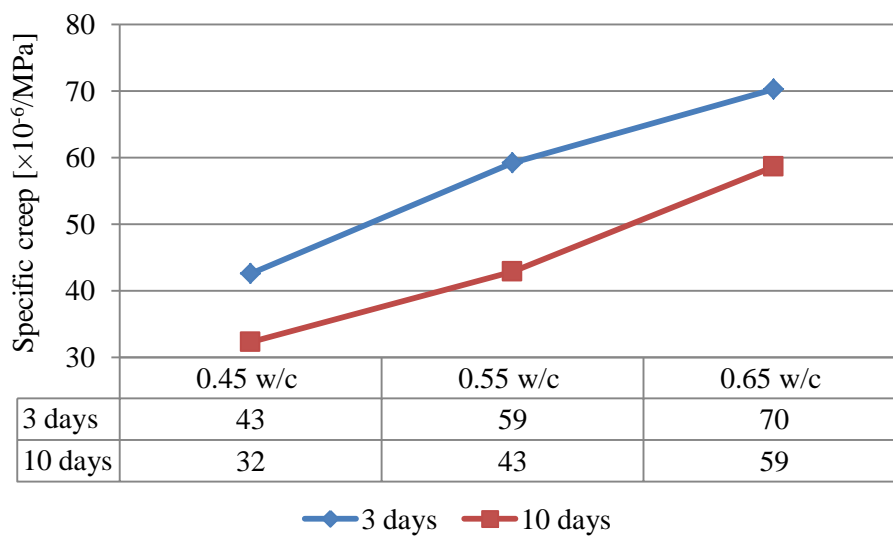


Figure 4.6: Influence of w/c and age of loading on 90-day specific creep

Another way to compare the creep potential of the concretes is by expressing them in terms of the creep coefficient (Figure 4.7).

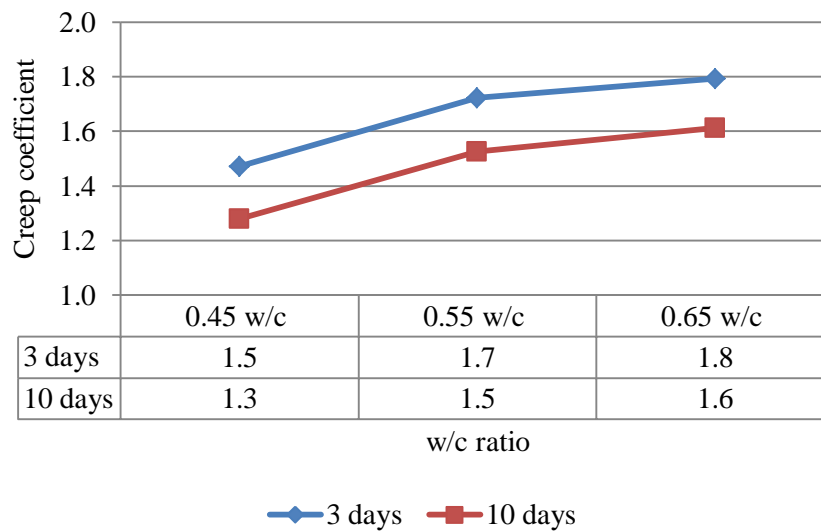


Figure 4.7: Influence of w/c and age of loading on 90-day creep coefficient

The creep coefficient is the ratio of the creep strain to the initial elastic strain. The creep coefficient enables the elasticity of the material to be captured when comparing their creep potentials (Omar *et al.*, 2008). The values of basic creep and creep coefficient presented were obtained after a period of 90 days. Ultimate values of total strain, total creep, and shrinkage strain are higher than those presented in this section. Figures showing detailed creep results can be seen in Appendix D. In particular, figures that show the development of creep with age are presented in the same Appendix.

The influence of w/c ratio on creep strains is apparent. The total creep and consequently the specific creep and creep coefficient increase with higher w/c ratio. The smaller creep strains in the low w/c ratio mixes can be attributed to a stronger, stiffer and less permeable paste (Alexander and Beushausen, 2009). Omar *et al.* (2008) citing work by Smadi *et al.* (1987) states that the high strength of low w/c mixes is responsible for the low creep values in those mixes. Low w/c concretes have fewer pores and a rigid solid matrix hence leading to decreased deformation.

The total strain, total creep and subsequently specific creep and creep coefficient reduce with age at loading. The same can be said for the creep coefficient. Concrete becomes stiffer and less porous with age. This is because more hydration of the cement paste has taken place in older concrete. The hydration process consumes water in the mix making concrete become a solid matrix subsequently reducing its deformation capacity. The observations reported on the influence of w/c ratio and age on creep of concrete are in accordance with those reported by

Bissonnette and Pigeon (1995); Østegaard *et al.* (2001); Atrushi (2003) and Pane and Hansen (2001)

#### 4.5.1 Comparison of experimental creep results to selected models

Laboratory creep results were compared with values from selected creep prediction models. The models considered include the CEB MC90-99, GL2004 and B3 Model. The GL 2004 is the most recent version of the GL 2000 Model. The models are chosen based on the fact that past research has shown that these models are the closest to predicting deformation properties of concrete. Locally, Fanourakis (2011) found the GL2000 Model as the most accurate creep prediction model for local concretes. Fanourakis and Ballim (2003) found the B3 model as the most accurate model after considering a range of concretes. Goel *et al.* (2007) found creep predictions by the GL 2000 model the closest to experimental results. The GL 2000 and MC90-99 are easy to use with quickly obtainable input parameters such as compressive strength and relative humidity.

As was the case with laboratory results, the creep potential of the different mixes is presented in terms of creep expressions such as the creep coefficient. The creep compliance function,  $J$  is introduced to compare the creep potential of different concretes. According to Carlswård (2006), the compliance function includes both the instantaneous and long term characteristics of a concrete. This makes it preferable as discussed in Section 4.5.4.1.

##### 4.5.1.1 Comparison of 90-day creep coefficient

The 90 day creep coefficient for the three mixes is compared with values from selected models. The models considered are the MC 90-99 and the GL 2004. Figure 4.8 is presented for the mixes loaded at 3 days.

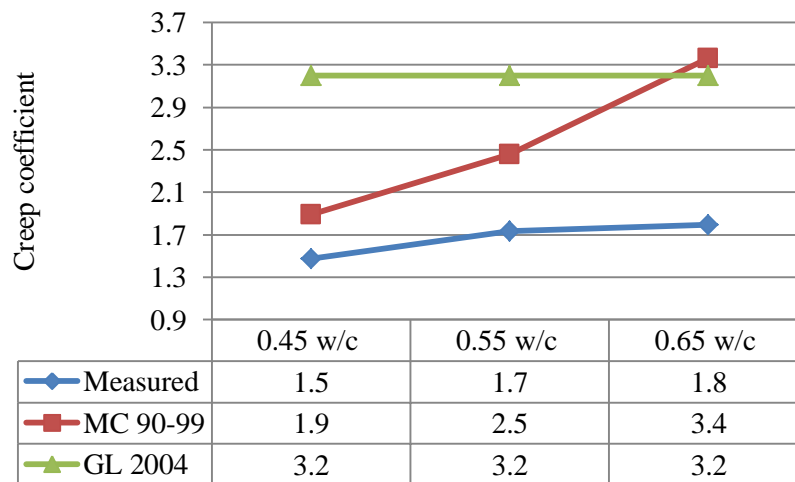


Figure 4.8: Graphical comparison of 90-day creep coefficient for 3 day loaded samples

Figure 4.9 presents a comparison of creep coefficient for similar mixes loaded at 10 days.

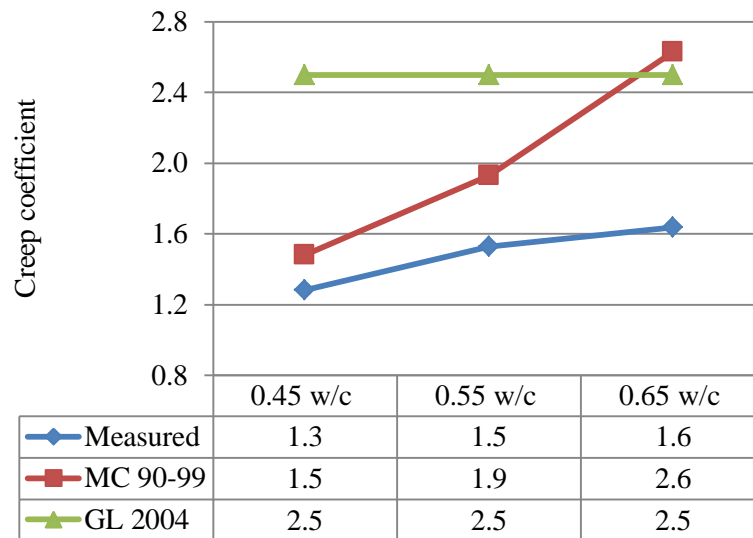


Figure 4.9: Graphical comparison of 90-day creep coefficient for 10 day loaded samples

Both models overestimate the creep coefficient. As seen from Figures 4.8 and 4.9, the GL 2004 model gives a constant value of creep coefficient. This is because the creep coefficient at the desired age depends only on the age of loading, volume/surface ratio and relative humidity (See model equations presented in Section 2.8.1). This is in contrast to the MC 90-99 model in which the creep coefficient depends on the strength among other parameters. The dependence of the creep coefficient on strength explains why the creep coefficient increases with decreasing w/c ratio for the MC 90-99 model.

The MC90-99 predicts values of creep coefficient which are closer to measured one as compared to the GL2004. This is in contrast to what most researchers have found. The earlier version of the model, the GL 2000 has been found by many to be the most accurate model in predicting creep strains of concrete. The reason for this deviation could probably be attributed to differences in material composition. Further focused research is required to verify this.

However, as is stated in the ACI guide (ACI 209, 2008) these models are developed for 'office use'. The values predicted are supposed to be used by engineers in estimating loads at the preliminary design stage. Moreover, different models have been found to be more accurate for different situations (Fanourakis, 2011; Ahmad and Roy, 2012; Gardner and Lockman, 2001 and Goel *et al.*, 2007). The creep compliance function presented in the next section is nowadays preferred to compare creep strains of different concretes.

#### 4.5.1.2 Comparison of 90 day creep compliance function

A comparison of creep values of concrete by the compliance function is preferred to the creep coefficient (ACI 209, 2008). The compliance function takes into account both elastic and creep deformation properties of the concrete. The creep coefficient depends on the creep strain at an instant and the initial elastic modulus. The initial elastic modulus is affected by the rate of loading which is not important in the long term deformation of the material.

In addition to the GL 2004 and the MC 90-99, the B3 model is used to compare compliance values. In Figure 4.10, 90 day compliance values of mixes loaded at 3 days are presented.

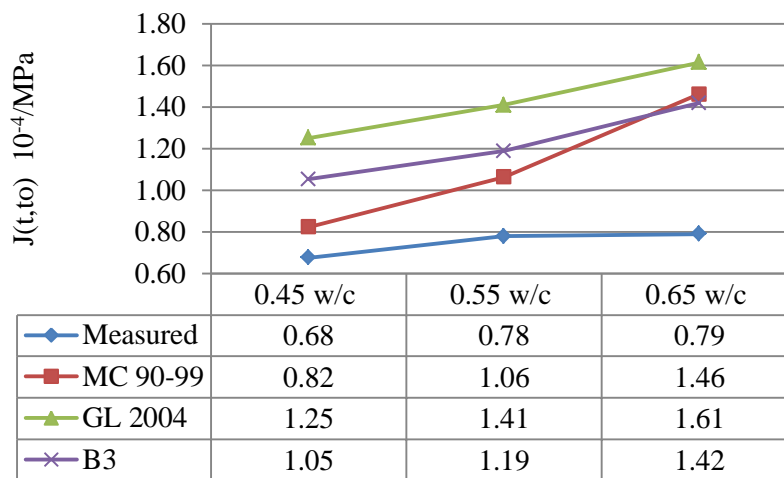


Figure 4.10: Comparison of 90-day creep compliance between experimental results and 3 day loaded samples

Figure 4.11 shows values of 90 day compliance function for 10 day loaded samples.

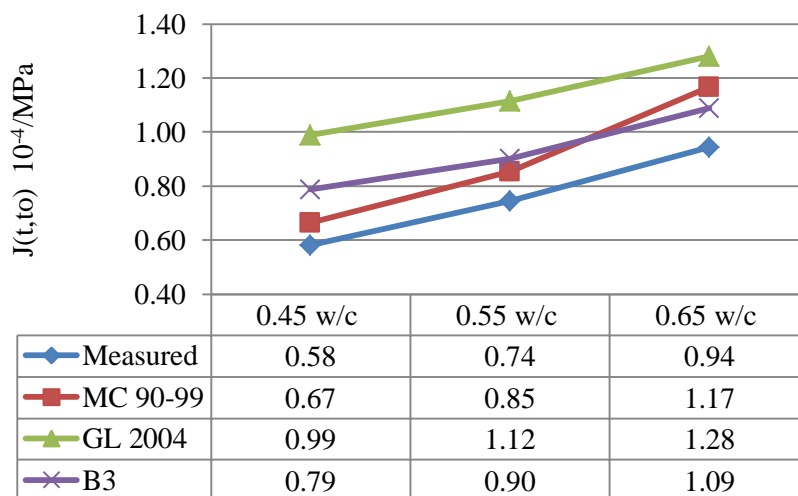


Figure 4.11: Comparison of 90-day creep compliance between experimental results and 10 day loaded samples

Values of compliance functions obtained from the models seem to be closer to experimental results at lower w/c ratios for 3 day tested samples. Figures that show the development of the creep compliance function for the measured and prediction models can be seen in Appendix D. An example of such is presented in Figure 4.12 for the 0.45 w/c ratio mix loaded at 10 days. Unlike the creep coefficient, compliance results show a relatively similar trend for both measured and predicted values. The creep compliance increases with increase in w/c ratio and reduces with age in all cases considered. All selected models over predict the compliance function.

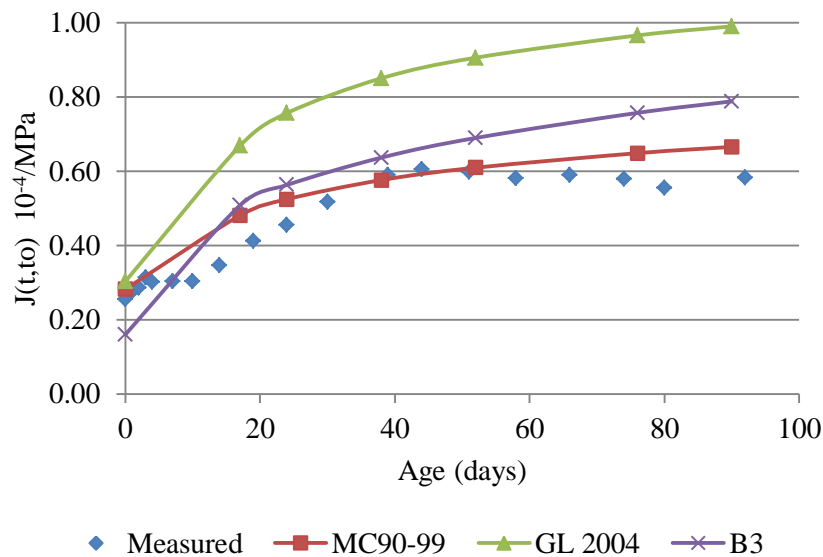


Figure 4.12: Development of creep compliance for 0.45 w/c mix loaded on the 10<sup>th</sup> day.

The MC 90-99 is again found to predict compliance values closer to experimental results. The Model B3 follows with the GL 2004 providing the worst predictions.

Measured creep compliance values reduced by 20% from 3 days to 10 days. Interestingly, values from the most accurate prediction model, the MC90-99 reduced by the same margin for the same ages. Supporting example calculations for model predictions are presented in Appendix G.

#### 4.5.2 Shrinkage

In this research, the results obtained appear to indicate that the higher the paste content, the higher the shrinkage. However the difference in magnitude is relatively small. The same can be said of the influence of onset of drying on shrinkage values. The relatively equal shrinkage strains can be attributed to the small difference in paste content of the three mixes used in this research. In Figure 4.13, 90 day laboratory measured shrinkage results are presented for samples tested from the age of 3 days.

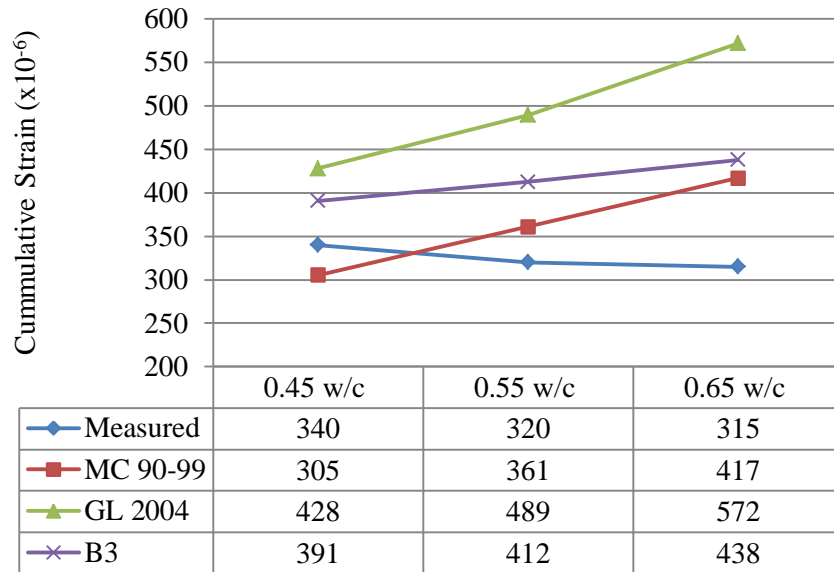


Figure 4.13: Comparison of 90-day shrinkage strain between experimental results and 3 day loaded samples

In the same figure, comparisons are made with shrinkage values from prediction models. Equations used to formulate the selected shrinkage models are given in Appendix F while example calculations are given in Appendix G. Figure 4.14 is used to present 90 shrinkage results for samples tested from 10 days.

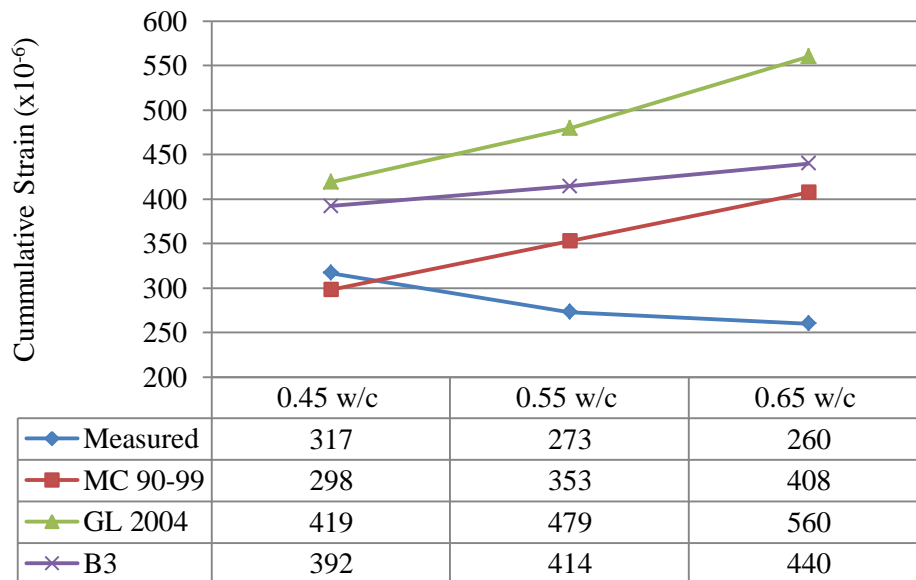


Figure 4.14: Comparison of 90-day shrinkage strain between experimental results and 10 day loaded samples

As can be seen in Figure 4.13 and 4.14, prediction models are sensitive to w/c ratio and age at onset of drying. Shrinkage reduces with age and increases with decreasing w/c ratio. However, as seen in this study, model formulations can easily lead to wrong predictions of shrinkage. This is because as observed in the figures, other factors such as the paste content play a big role in a concrete's shrinkage potential. This is probably the reason why experimental shrinkage values reduce with w/c ratio. The shrinkage of the 3 mixes appears to reduce with age at the onset of drying although differences are relatively small. This is because at higher ages, water content is reduced and hence the concrete loses less water to its surroundings. Most importantly, the reduced pore spaces make the concrete stiffer which reduces any kind of deformation (Omar *et al.*, 2008).

Generally, shrinkage of concrete depends on the w/c ratio, the water content, cement content and the paste content. The influence of these factors is difficult to generalise. This is especially true in practical terms where constituents are usually varied. For example, an increase in water content normally increases the shrinkage strain. However, low w/c ratio mixes with low water content and higher paste content have been found to shrink more than higher w/c ratio mixes. Although the water content increases with higher w/c ratio, the influence of the paste (and cement) content probably outweighs that of the water content as explained below.

The higher shrinkage in low w/c ratio mixes (loaded at 3 days) can probably partly be linked to autogenous shrinkage in these mixes. Autogenous shrinkage is defined as the shrinkage undergone by concrete during the early days when drying is prevented. Endogenous shrinkage is a term used to refer to the sum of autogenous and thermal shrinkage. Endogenous shrinkage increases with decreasing w/c ratio and increasing cement content and this can possibly explain the high shrinkage in the lower w/c ratio mixes (Gilbert and Ranzi, 2010). Bissonnette and Pigeon (1995) observed a similar trend in their study. Plain concrete (OPC) was used. Drying shrinkage was found to be higher in mixes with w/c ratio of 0.35 as compared to 0.55 w/c mixes. The paste content was kept constant. This is explained from the view point of Neville (2002) who states that the more cement available in a concrete mix, the larger the volume of hydrated cement paste which is liable to shrinkage hence the higher the shrinkage.

It can be concluded that while the total strain and total creep depend on the stiffness of the material (i.e. w/c ratio), the shrinkage strain is influenced by the paste content. Additional results and figures on creep and shrinkage can be found in Appendix D.

## 4.6 Relaxation results

### 4.6.1 Introduction

Similar to creep tests, relaxation tests were carried out starting at the ages of 3 and 10 days. In these experiments, concrete dog-bone specimens were subjected to a direct tensile force which is a fraction of their ultimate tensile force. The reduction in force was then plotted against time. To prevent shrinkage, samples were sealed with a thin layer of wax on all faces.

The initial applied stress is denoted  $\sigma_0$  while the stress after a certain time period is denoted  $\sigma_t$ . The relative relaxation is the ratio of stress after a certain time, to the initial applied stress expressed as a percentage. This relationship is used to compare the relaxation behaviour of different mixes. This comparison can be based on w/c ratio, stress-strength ratio and age of loading. 72-hour relaxation tests were carried out at two initial stress levels i.e. at 40% and 80% of ultimate tensile force. This was done to investigate the effect of stress level in the prediction of relaxation. More on the same is discussed in Chapter 5. Laboratory relaxation results are compared with values from prediction models in the same chapter. An example of a typical relaxation curve (for a 0.65 w/c ratio concrete at 80% initial stress-strength ratio) is shown in Figure 4.15.

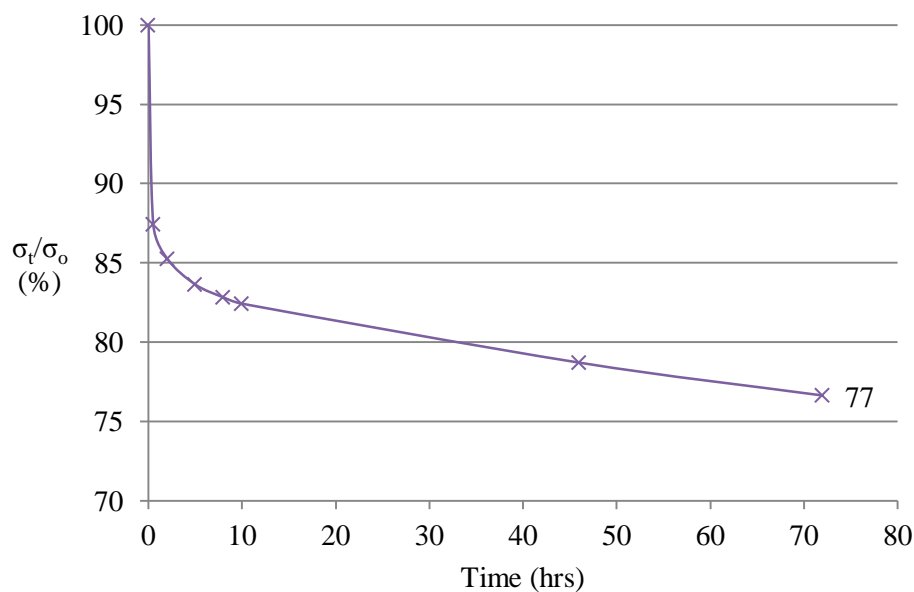


Figure 4.15: Typical relative relaxation vs. time curve

### 4.6.2 Time development of relaxation

In this research it was assumed that relaxation reached ultimate values after 72 hours. Justification for this assumption can be found in literature review, Section 2.3.4.2. In total over 40 relaxation tests were carried out in this research. It was observed that relaxation

proceeds at a high rate in the first hours with up to 67% of the relaxation occurring in the first 2 hours. On average, 74% of the relaxation occurs in the first 5 hours (Figure 4.16).

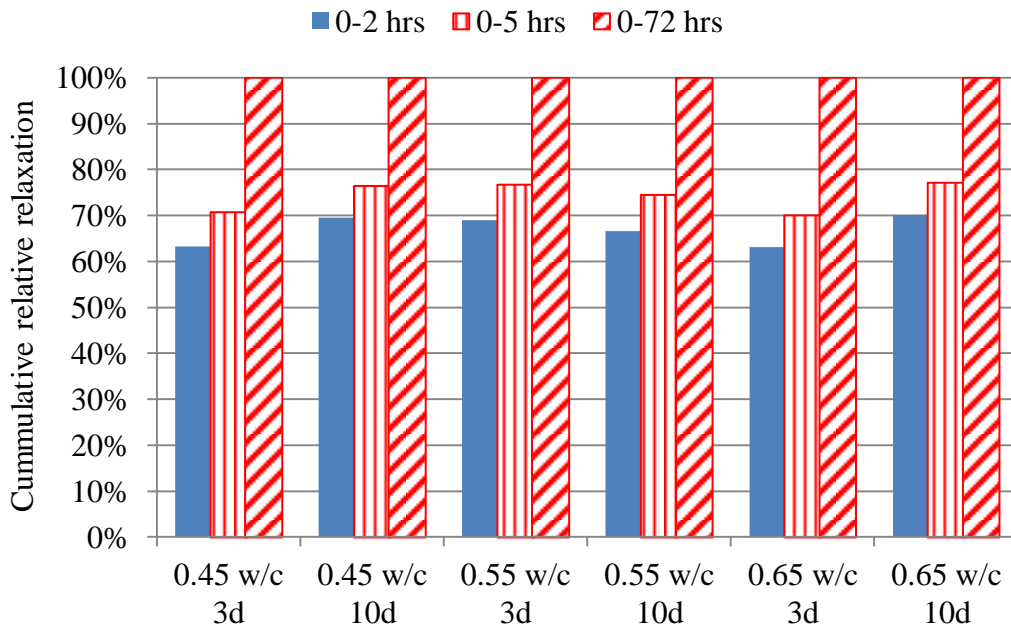
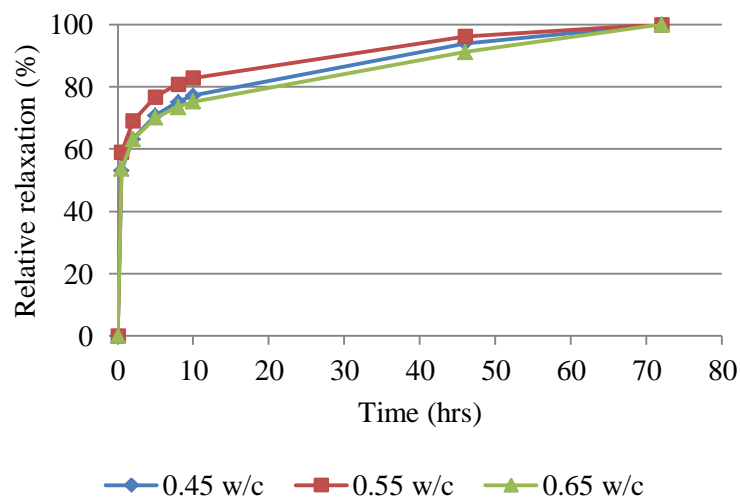


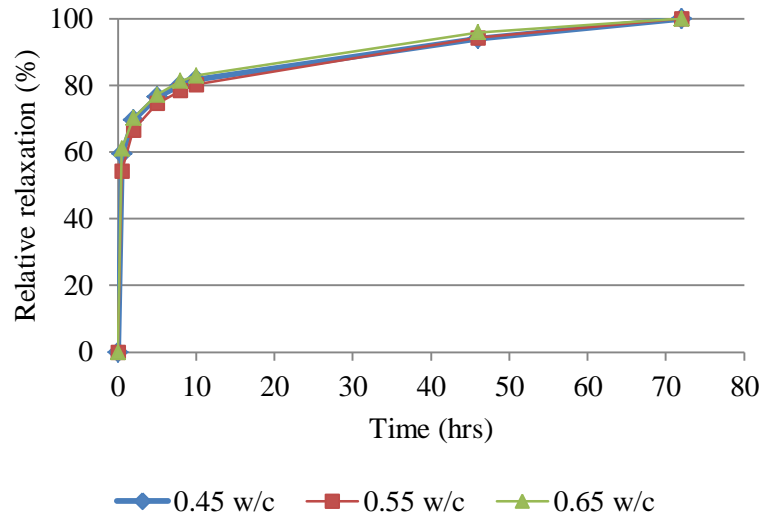
Figure 4.16: Influence of time on cumulative relative relaxation

#### 4.6.2.1 Rate of development of relaxation

It appears that the rate of development of relaxation is not greatly influenced by either one of w/c ratio, age of loading and initial stress levels. At the age of 3 days, the 0.55 w/c ratio samples exhibit a more rapid development of relaxation followed by the 0.45 and 0.65 w/c samples in that order, Figure 4.17 (a). At 10 days, the order is reversed with the 0.65 w/c showing a more rapid development. The rates at this age are almost similar, Figure 4.20 (b).



(a) at 3 days



(b) at 10 days

Figure 4.17: Rate of development of relaxation for different w/c ratios

The rate of development of relaxation is also not influenced by the age of loading, w/c ratio and initial stress-strength ratio. Again, the results are inconsistent and hence no clear trend can be drawn from them. The observations made on the rates of relaxation are similar to those made by Beushausen *et al.* (2012). See more figures of these results in Appendix E.

#### 4.6.3 Influence of w/c ratio

To investigate the influence of w/c ratio, the age testing of the different mixes is kept constant. Relaxation, similar to creep appears to decrease with decreasing w/c ratio. This can be attributed to a stiffer, less porous paste which ensures reduction in movement of water through pore spaces (Masuku, 2009; Wittmann, 1981).

There seems to be no significant influence of w/c ratio on the relaxation potential of the 0.45 w/c versus the 0.55 mix. For samples tested at the age of 3 days, average relaxation values were found to be similar for the two mixes (Figure 4.18). For the age of 3 days, relaxation in the 0.45 and 0.55 w/c ratio mixes is 17% while that in the 0.65 w/c mix is 23%. Figure 4.19 shows the relative relaxation values at the age of 10 days.

At the age of 10 days, relaxation again appears to depend on the w/c ratio to a certain extent. The influence of w/c ratio on relaxation reduces with age. The difference between maximum and minimum relaxation is reduced from 6% at 3 days to 4% at 10 days. Similar figures for samples loaded at 40% initial stress-strength ratio are presented in Appendix E.

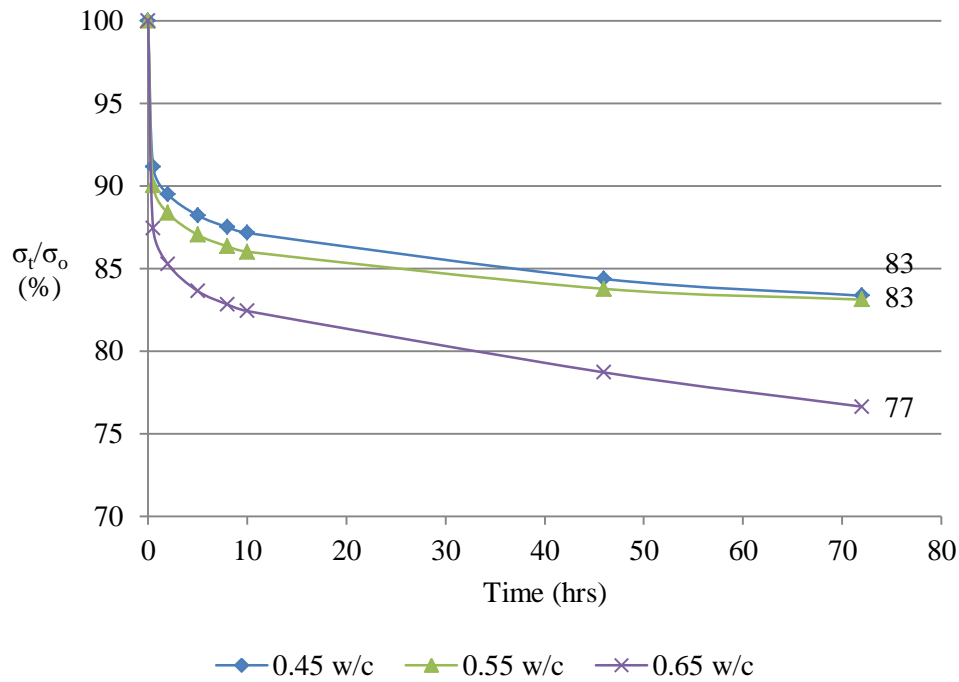


Figure 4.18: 3 day tensile relaxation at 80 % initial stress-strength ratio

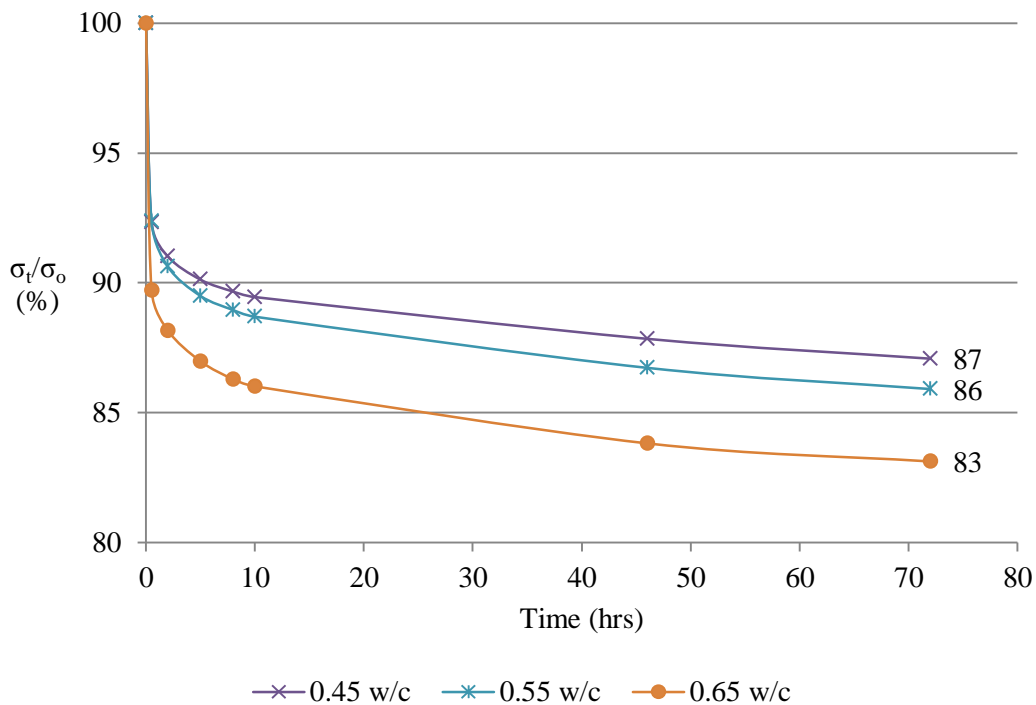


Figure 4.19: 10 day tensile relaxation at 80 % initial stress-strength ratio

#### 4.6.4 Influence of age

Two ages of loading were considered i.e. 3 and 10 days. To study the influence of age, comparisons are made with the same w/c ratio specimens. The relaxation of concrete reduces

with an increase in age. This is similar to what happens with creep. Figure 4.20 summarizes the effect of age and w/c ratio. The samples in the figure were subjected to an initial stress-strength ratio of 80% of their tensile strength. The reduction in relative relaxation from 3 to 10 days is 24% for 0.45 w/c ratio mixes, 18% for 0.55 w/c ratio and 26% for the 0.65 w/c mixes. Error bars show the standard deviation.

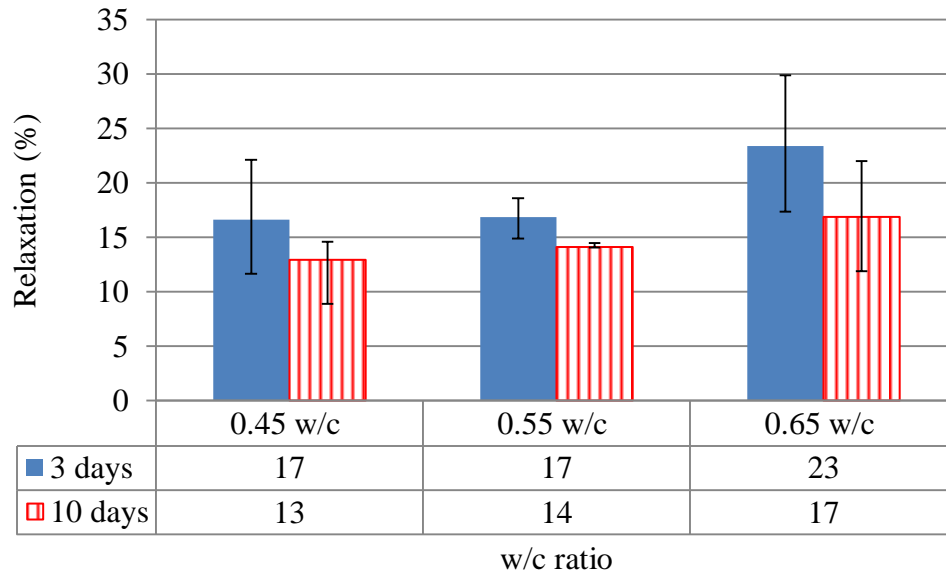


Figure 4.20: Influence of age and w/c ratio on relaxation at 80% initial stress-strength ratio

Figure 4.21 shows a summary of the 72-hour relaxation results for samples loaded at 40% initial stress-strength ratio.

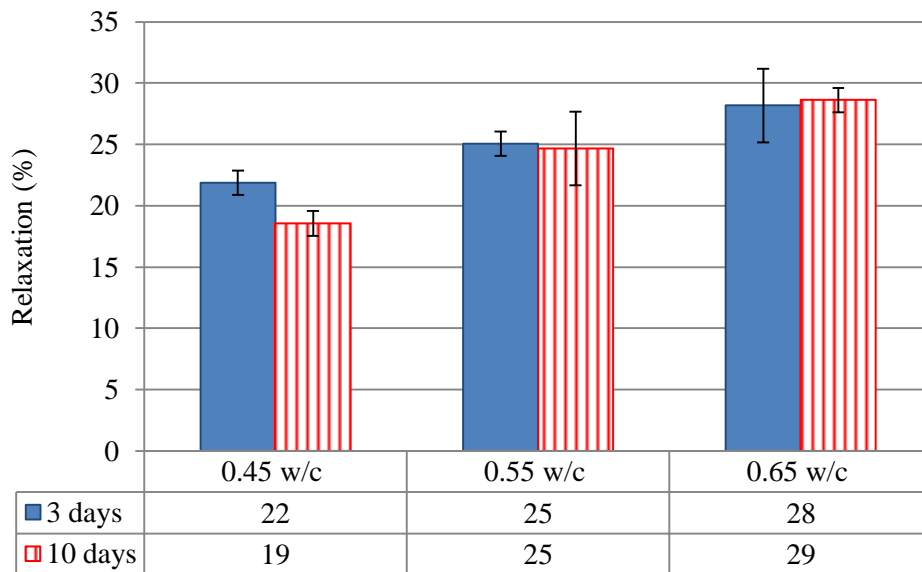


Figure 4.21: Influence of age and w/c ratio on relaxation at 40% initial stress-strength ratio

For samples loaded at 40% initial stress-strength ratio, w/c ratio seems to have an influence on relaxation. Relaxation increases with increasing w/c ratio. The influence of age of loading is scarcely observed.

The reduction in relaxation at later ages as observed on 80% samples can be attributed to a more stiffened concrete. Relaxation is high when the rate of hydration is high (Neville, 1981). The observations reported on the influence of w/c ratio and age on relaxation of concrete are in accordance with those reported by other researchers (Masuku, 2009; Chilwesa, 2012; Marimoto and Kayonagi, 1994; Gutsch and Rostasy, 1994 and Atrushi, 2003).

#### 4.6.5 Influence of aggregate (and paste) content

If all other factors are kept constant, the volume of aggregates in the concrete probably reduces its relaxation. For example, Masuku (2009) carried out work on the relaxation of a 0.45 w/c ratio mix. No coarse aggregates were used in the study. The mix used had 56% fine aggregate by volume. At the age of 2 days, relaxation was found to be 34 %. More recently, Chilwesa (2012) carried out work on mortar samples of w/c ratio of 0.45. Their mix was similar to that used by Masuku (2009). Relaxation was found to reduce stresses by 40% after two days. In this research, the 0.45 w/c ratio mix had 69% total aggregate by volume of which 29% constituted coarse aggregate. Relaxation was found to be 17%. Table 4.3 summarises the influence of aggregates on relaxation by comparisons of different studies.

Table 4.3: Influence of aggregate (or paste content) on relaxation

Study	w/c ratio	Age of testing (Days)	Total volume of aggregates (%)	Volume of coarse aggregates (%)	Relaxation (%)
<b>Chilwesa (2012)</b>	0.45	2	56	0	40
<b>Masuku (2009)</b>	0.45	2	56	0	34
<b>Marimoto and Kayonagi (1994)</b>	0.5	3	70	39	15
<b>Tanabe and Ishikawa (1993)</b>	0.5	3	70	38	14
<b>This study</b>	0.45	3	69	29	17

The studies discussed cannot give a quantitative relation between relaxation and volume of aggregate because of slight differences in mix design and age of testing. However, these results can give an indication of the influence of aggregates on relaxation. It is apparent that

aggregates reduce the relaxation potential of concrete. One reason for this phenomenon is probably the restraint to paste movement the aggregates provide in concrete. Another explanation is that the presence of aggregates reduces the paste content; which is the main source of the concrete's ability to deform.

The volume of coarse aggregates was kept constant in all mixes used in this study. The 0.45 w/c ratio mix had higher paste content than the rest of the mixes. However, results show that this mix exhibits lower relaxation than the 0.55 and 0.65 w/c ratio mixes. This implies that similar to creep, the stiffness of the paste is more influential in the concrete's relaxation potential than the paste content. However, more testing would be required to confirm this.

The addition of coarse aggregates appears to increase the rate of relaxation. Masuku (2009) reported that on average 57% of 72-hour relaxation had occurred after 2 hours for 2 day loaded samples. In addition, 78% relaxation occurred after the first 12 hours. No coarse aggregates were used in the study (Table 4.2). In this study, 67% relaxation on average occurred after 2 hours while 74% of ultimate relaxation occurred after only the first 5 hours. Samples were loaded at 3 days. In both studies, samples were loaded at 80% initial stress-strength ratio. These observations seem to suggest that aggregates lead to a higher rate of relaxation. The reason for the possible increase in rate of relaxation in mixes containing aggregates is probably attributed to the ease in the formation of micro-cracks in these mixes. With aggregates, concrete is weaker at the interfacial transitional zone (ITZ) and hence cracks propagate easily, that way the rate of relaxation increases. The influence of aggregates discussed here cannot be conclusive because of insufficient experimental work. A more systematic and comprehensive study is therefore necessary to quantify the influence of aggregates on relaxation.

#### **4.6.6 Influence of initial stress-strength ratio**

To investigate the influence of initial stress-strength ratios on relaxation four initial stress-strength ratios were applied on the concrete samples. The four stress levels were 20%, 40%, 60% and 80%. These values represent a fraction of the ultimate failure load of the specimen. Therefore, before the effect of stress levels was investigated, the tensile strength of the concrete was determined.

5-hour relaxation was taken as the basis of comparison due to time constraints. As mentioned in Section 4.6.2, approximately 75% of the relaxation commonly occurs after 5 hours. Therefore, although the final magnitude of relaxation is underestimated, a fair comparison of the relaxation behaviour of similar mixes subject to a different initial stress is possible. Tanabe and Ishikawa (1993) also used the 5-hour relaxation as ultimate when investigating the effects of stress levels on different concretes.

There seems to be no clear influence of the initial stress applied on relaxation as illustrated in Figures 4.22 to 4.24. The figure summarises results on the influence of initial stress-strength ratio on selected mixes. More results can be seen in Appendix E.

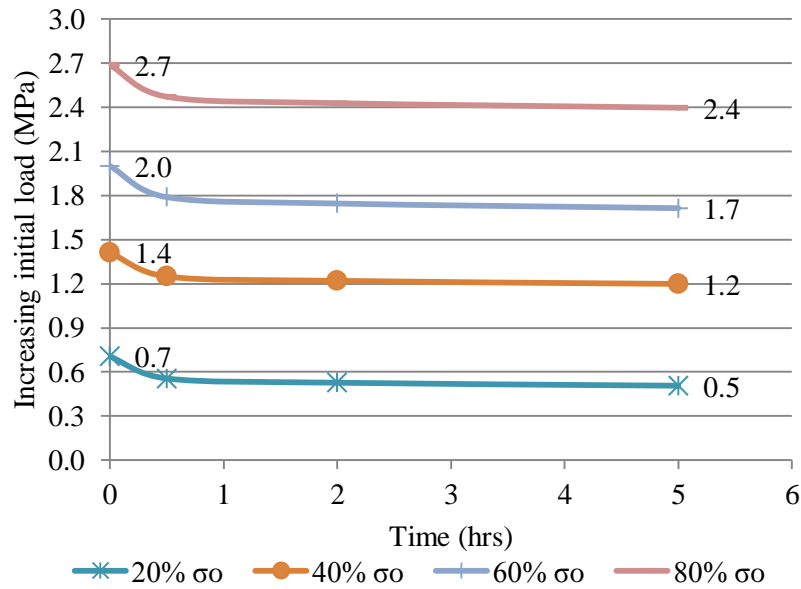


Figure 4.22: Influence of initial stress-strength ratio on 5-hour relaxation of 0.55 w/c mix tested at 10 days

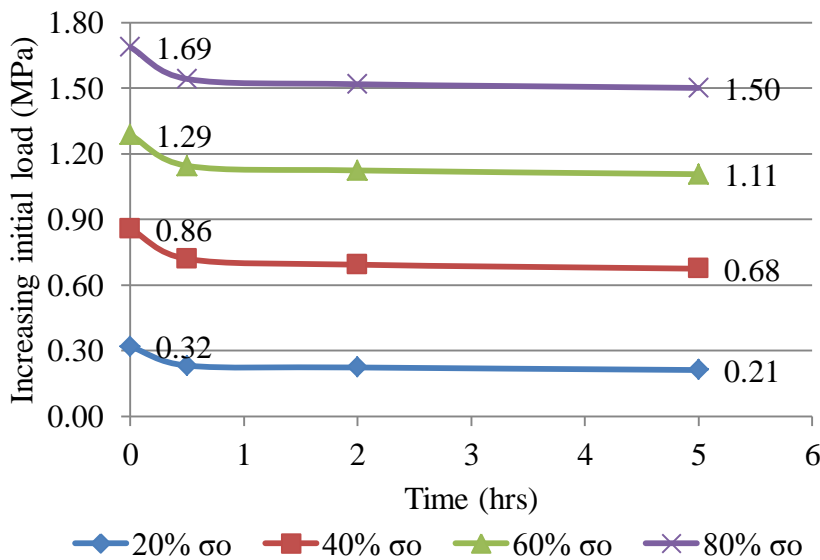


Figure 4.23: Influence of initial stress-strength ratio on 5-hour relaxation of 0.65 w/c ratio mix tested at 3 days

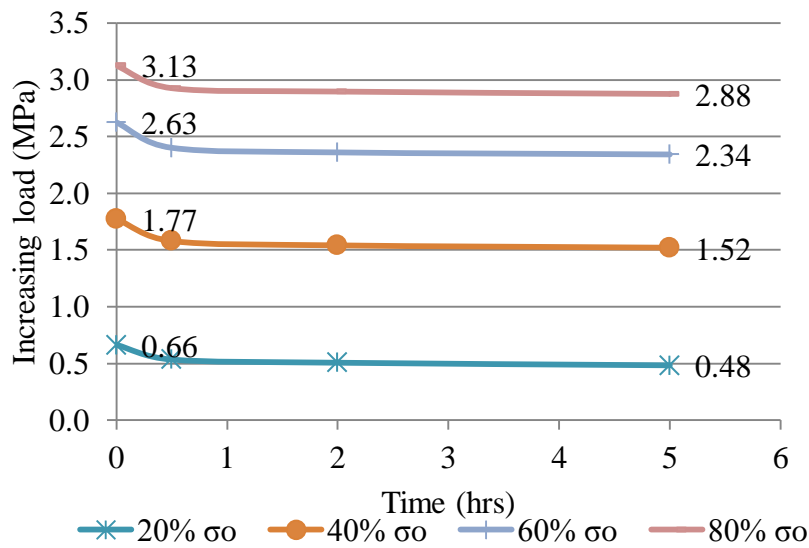


Figure 4.24: Influence of initial stress-strength ratio on 5-hour relaxation of 0.45 w/c ratio mix tested at 10 days.

Initial and final stresses after relaxation are shown on the figures. At 20% initial stress-strength ratio, a relaxation of 0.2 MPa was measured while the values at 40%, 60% and 80% were 0.2 MPa, 0.3 MPa and 0.3 MPa respectively for the 0.55 w/c ratio mix tested at the age of 10 days. From the results obtained, it can be seen that an increase in initial load did not lead to a proportionate increase in relaxation. A similar trend was also observed in all other tests. In this study, it was found that on average an increase of initial stress-strength ratio of 300% led to an increase of relaxation of 58% (See figure E12 in Appendix). A more extensive study needs to be carried out to verify these findings although all results are presented for completeness. Longer times of testing and varying stress levels should be considered in the experimental programme.

The findings reported here are somewhat similar to what Gustch and Rostasy (1994) observed in their research. Initial stress-strength ratio was found to have no significant effect on relaxation of concrete. The results are also similar to those of Tanabe and Ishikawa (1993) who found that relaxation increases only slightly with increase in initial stress level. However, the results are different from findings by Morimoto and Koyanagi (1994) who stated that relaxation is proportional to initial stress-strength ratio in the ranges up to 80%. For additional results and figures on relaxation, see Appendix E.

## 4.7 Summary

The following observations can be made from the results and discussions in the previous section.

### 4.7.1 Deformation properties

The total strain, total creep and associated creep coefficient, specific creep and creep compliance function all increase with increasing w/c ratio and reduce with increasing age of loading. A less porous and stiffer paste is used to explain the reduced deformation of low w/c ratio and older concrete mixes.

- Creep and shrinkage prediction models used in this study are generally sensitive to the w/c ratio and age of loading. However, the GL2000 predicts a constant value of creep coefficient for different w/c ratio mixes. This is because the formulation for creep coefficient in this model is based on the age of loading, relative humidity and volume-surface ratio and not strength or w/c.
- The compliance function should be used in comparing creep potential of different concretes. This is because both the elastic and creep strain components are incorporated in the analysis. In this study, compliance function values obtained by the MC90-99 are closest to experimental work. However, all models over-predicted the deformation properties of the concretes.
- The parameters that influence shrinkage may not be easily identified without testing. Whether a concrete will shrink more than another depends on the paste and cement content and not w/c ratio or strength. In this study, shrinkage generally reduces with age. This is probably because the reduction in pore spaces makes the concrete matrix stiffer hence improving its ability to resist deformation. Although shrinkage increases with decreasing w/c ratio, the overall strain of concrete is not affected. Strain decreases with decreasing w/c ratio, owing to lower creep at lower w/c.

### 4.7.2 Relaxation

With regard to the relaxation potential of the concrete mixes investigated, the following observations were made.

- The magnitude of relaxation is influenced by the w/c ratio and age of loading. Relaxation generally reduces with decreasing w/c ratio and increasing age at loading.
- Aggregates reduce the magnitude of relaxation in a similar way as they do for creep. The paste is the main source of deformation in concrete; therefore the restraint that aggregates provide to paste movement can probably be used to explain this phenomenon. Aggregates reduce the paste content of concrete. This can also be used to explain why relaxation reduces when aggregate content is increased.

- The initial stress-strength ratio was found to not be proportional to relaxation. When initial stress-strength ratio was increased, relaxation initially increased by a relatively small amount but thereafter, not much change was observed. Generally relaxation was found to increase at a rate 5 times less than the rate at which the initial stress-strength ratio is increased. Further testing is needed to verify this.
- In this research, the magnitude of relaxation was found to be in the range of 13%-23% for initial stress levels of 80%. When initial stress level is reduced to 40%, 72-hour relaxation values range from 19%- 29%. These values are the same order of magnitude to those reported in literature considering the mix constituents
- As expected, the rate of relaxation is higher than that of creep in the first hours. Moreover, the rate of relaxation is not influenced by the w/c ratio, age and initial stress-strength ratio. The presence of coarse aggregates probably increases the rate of relaxation.

#### 4.7.3 Summary of the influence of selected parameters on creep and relaxation

From the observations made for both deformations and relaxation properties of the concrete tested in this research, it can be seen that relaxation and creep are largely influenced in a similar way by the same parameters. Apart from the influence of initial stress-strength ratio, the other parameters investigated, seem to influence both creep and relaxation in the same way. Table 4.4 summarises the influence of the different parameters on creep and relaxation.

Table 4.4: observed effect of various parameters on creep and relaxation.

Parameter	Creep factor	Relaxation (%)
Increase in w/c ratio	increase	increase
Increase in age at loading	decrease	decrease
Increase in aggregate content	decrease	decrease
Increase in initial stress-strength ratio	Constant (under service loads)	decrease**:

\*\* : needs verification in further research

## 5 VALIDATION OF RELAXATION MODELS

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### 5.1 Introduction

In Chapter 4, the first aim of this study was achieved by investigating how selected parameters that affect creep would influence relaxation in similar concrete. The parameters considered were: w/c ratio, age of loading, volume of aggregate and initial stress-strength ratio. Chapter 5 is dedicated to achieving the second aim of this research; which is to compare experimental relaxation results with selected approximate relaxation prediction models.

The relaxation function can be determined from equations linking it to the creep compliance function. This is made possible using numerical integration or computer programs. However, for quick but fairly accurate results, simplified algebraic equations are preferred (Bazant and Kim, 1979a; Carlswärd, 2006; Lacidogna and Tarantino, 1996). The approximate models used are derived mathematically from numerical integration and optimisation of equations linking the creep function  $J(t, t_0)$  and the relaxation function  $R(t, t_0)$  (Bazant *et al.*, 2013). While the creep function is easily verified by experimental data, the equivalent relaxation function is almost exclusively determined mathematically (Chiorino, 2005; Atrushi, 2003; Wittmann, 1971). It is therefore important that experimental relaxation tests are carried out to check the validity of prediction models.

Two approximate methods were used to verify experimental relaxation data i.e. the Age Adjusted Effective Modulus Method (AAEM) given by equations 5.1 and 5.1a, and the Approximate relaxation function (ARF), given by equations 5.2 and 5.2a. Both approximate methods considered were developed based on the same principle, which is to reduce the error in the effective modulus method. Since ageing of the material was not considered, the first effective modulus method produced inaccurate deformation and stress predictions. The simplified models considered are recommended in the old and recent Model Code. The AAEM method is recommended by the MC90-93 and its recent version the MC2010. Expressed differently (Equation 5.1a), the same method is recommended by the fib textbook on structural concrete behaviour, design and performance (Mueller and Haist, 2009). The ARF is recommended in MC90-93. A more recent version of the same model has been developed. It is said to be more accurate in estimating the long-time relaxation of concrete loaded at younger ages (Bazant *et al.*, 2013). Both the old and new approximate relaxation functions were used in verifying experimental results.

In Chapter 2, the approximate relaxation prediction models/methods were comprehensively discussed. The two methods considered are presented again for convenience.

- The age adjusted effective modulus method (AAEM) given in MC90-93 and MC2010

$$R(t, t_0) = E_c(t_0) \left[ 1 - \frac{\phi(t, t_0)}{E_c / E_c(t_0) + \chi \phi(t, t_0)} \right] \quad (5.1)$$

Equation 5.1 can be expressed in terms of the relaxation coefficient and creep coefficient as shown in Equation 5.1a given in Mueller and Haist (2009).

$$\psi(t, t_0) = \frac{\phi(t, t_0)}{E_c / E_c(t_0) + \chi \cdot \phi(t, t_0)} \quad (5.1a)$$

- The approximate relaxation function (ARF) given in MC90-93

$$R(t, t_0) = \frac{1 - 0.008}{J(t, t_0)} - \frac{0.115}{J(t, t-1)} \left[ \frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \right] \quad (5.2)$$

A more recent improved version of equation 5.2, the improved approximate relaxation function (ARF2) is given by equation 5.2a (Bazant *et al.*, 2013)

$$R(t, t_0) = \frac{1}{J(t, t_0)} \left( 1 + \frac{c_1 \alpha(t, t_0) J(t, t_0)}{q J(t, t-1)} \right)^{-q} \quad (5.2a)$$

$$\text{where } c_1 = 0.0119 \ln t_0 + 0.08; \quad q = 10 \quad \alpha(t, t_0) = \frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \quad \Delta = \frac{t - t_0}{2}$$

Graphical comparisons of results and formulas used were done using Microsoft Excel. Microsoft Excel was chosen over other programmes such as MATLAB because of its simplicity. Hand calculations were used to validate the programed models.

## 5.2 Evaluating model validity

Relaxation tests were carried out for 72 hours. Thus the models that were used in this study were not experimentally tested for long term prediction of relaxation. For simplicity, and to enable a comparison, all results are expressed in terms of relative relaxation. The relative relaxation is the ratio of stress at a time to the stress at the start of loading.

72-hour relaxation tests were carried out at two initial stress-strength ratios i.e. 40% and 80% of their ultimate tensile strength. In addition to investigating the influence of initial stress-strength ratio, these two values enabled the investigation of varying stress levels on the prediction of relaxation. The prediction models were checked for their ability to predict the relaxation potential of the material after 3 days of loading. This is because relaxation tests were carried for a maximum 3 days. Moreover, creep values at low ages, from which

relaxation is predicted were determined at 24 hour intervals for the first week. In essence, measured creep values were used in the models that link creep and relaxation.

For the AAEM method, the second expression given in equation 5.1a was used. This is because the parameters necessary for comparison were all determined experimentally in the laboratory. The relaxation coefficient ( $\psi$ ) was obtained by dividing the decrease of stress at a certain time by the initial stress. On the right side of the equation, the laboratory measured creep coefficient and elastic modulus were the parameters used in the model. The ageing coefficient ( $\chi$ ) was calculated from the relevant formula. A single value of 0.8 is recommended by in the MC90-93. Particular attention was paid to the effect of using the formula as opposed to the recommended value.

In the ARF method (Equation 5.2), the compliance function  $J(t, t_0)$  was obtained from experimental creep data using the effective modulus method. However, two of the compliance functions i.e.  $J(t, t-1)$  and  $J(t, t_0 + \Delta)$  have a varying time of load application. Thus, their determination from pure experimental work was not feasible. The creep prediction model suggested in MC90-99 was hence used to obtain these compliance functions. The model was used because the values it predicted were the closest to experimental results. In support of that, it should be remembered that the function of the additional compliance functions in the ARF method is that of reducing the error in the effective modulus method as explained in the Section 2.4.4. However, the limitation is nevertheless mentioned.

### 5.2.1 Graphical comparison of model predictions

In this section, experimental relaxation results are graphically compared with values from the different approximate relaxation models. For the Approximate relaxation function, predictions from both the old (ARF) and new expressions (ARF2) are initially included. For clarity and comparisons, experimental results are first presented in Table 5.1 and Table 5.2 for the 3 and 10 day loaded samples respectively. As can be seen in Table 5.1 and 5.2, both ARF and ARF2 methods yield relatively similar predictions for low age relaxation. Therefore, predictions by the new ARF2 are not included in subsequent figures.

In Figures 5.1 to 5.3, comparison of experimental values and model predictions for 3 day loaded samples is presented. As was explained in Chapter four, an increase in stress level leads to small increase in magnitude of relaxation; therefore the residual relaxation ratio will reduce with increase in stress level. This explains why the relaxation curves for experiments carried out at 40% initial stress-strength ratio show a higher percentage relaxation.

Table 5.1: Comparison between measured and predicted values of residual stress ratios (loaded at 3 days)

Age of loading (days)	Mix	Age (hrs)	Measured values of residual stress ratio (%)		Predicted values of residual stress ratio (%)		
			( $0.8\sigma$ )	( $0.4\sigma$ )	AAEM	ARF <sub>1</sub>	ARF <sub>2</sub>
3	0.45	0	100	100	100	100	100
		24	86	81	93	93	92
		48	84	79	84	84	83
		72	83	78	75	76	74
	0.55	0	100	100	100	100	100
		24	85	78	84	82	81
		48	84	76	82	80	79
		72	83	75	75	74	72
	0.65	0	100	100	100	100	100
		24	82	75	79	77	76
		48	79	73	77	75	74
		72	77	72	71	70	68

Table 5.2: Comparison between measured and predicted values of residual stress ratios (loaded at 10 days)

Age of loading (days)	Mix	Age (hrs)	Measured values of residual stress ratio (%)		Predicted values of residual stress ratio (%)		
			( $0.8\sigma$ )	( $0.4\sigma$ )	AAEM	ARF <sub>1</sub>	ARF <sub>2</sub>
10	0.45	0	100	100	100	100	100
		24	89	84	92	93	93
		48	88	83	88	89	88
		72	87	81	81	81	80
	0.55	0	100	100	100	100	100
		24	88	78	96	96	96
		48	87	77	91	88	87
		72	86	75	82	84	83
	0.65	0	100	100	100	100	100
		24	85	76	92	92	91
		48	84	73	73	75	73
		72	83	71	71	73	71

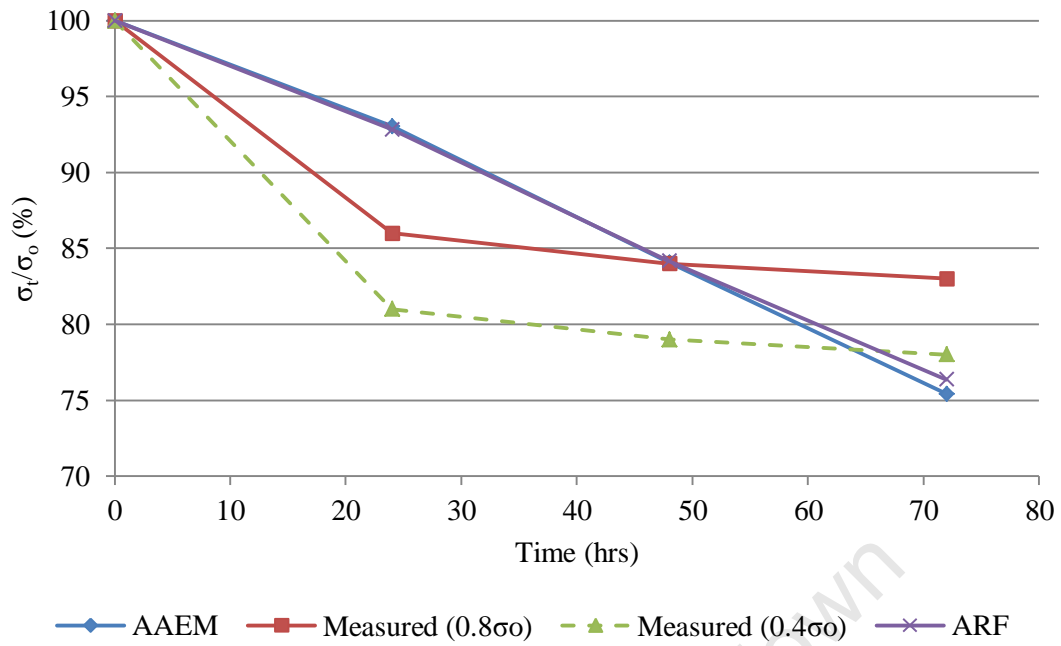


Figure 5.1: Graphical comparison of relaxation results, 0.45 w/c loaded at 3 days

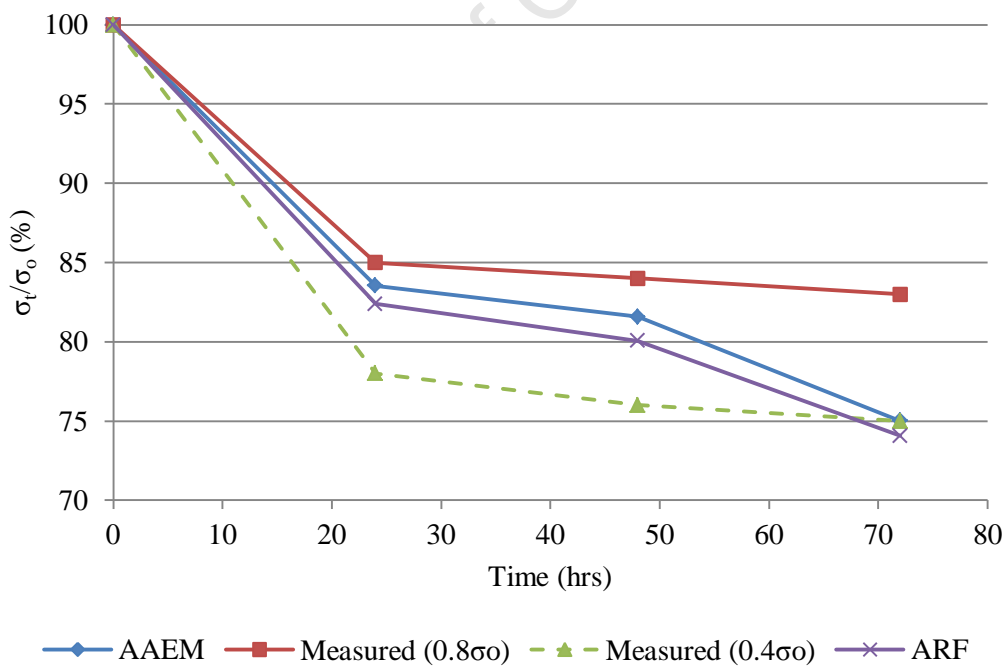


Figure 5.2: Graphical comparison of relaxation results, 0.55 w/c loaded at 3 days

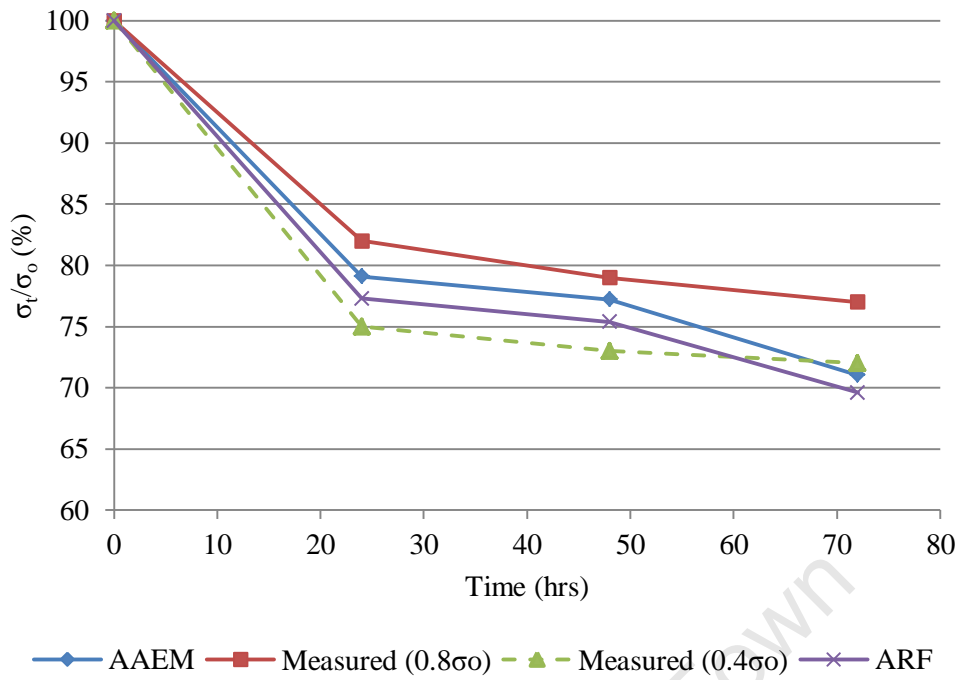


Figure 5.3: Graphical comparison of relaxation results, 0.65 w/c loaded at 3 days

Figures 5.4 to 5.6 are used to present comparisons of relaxation results of 10 day loaded samples.

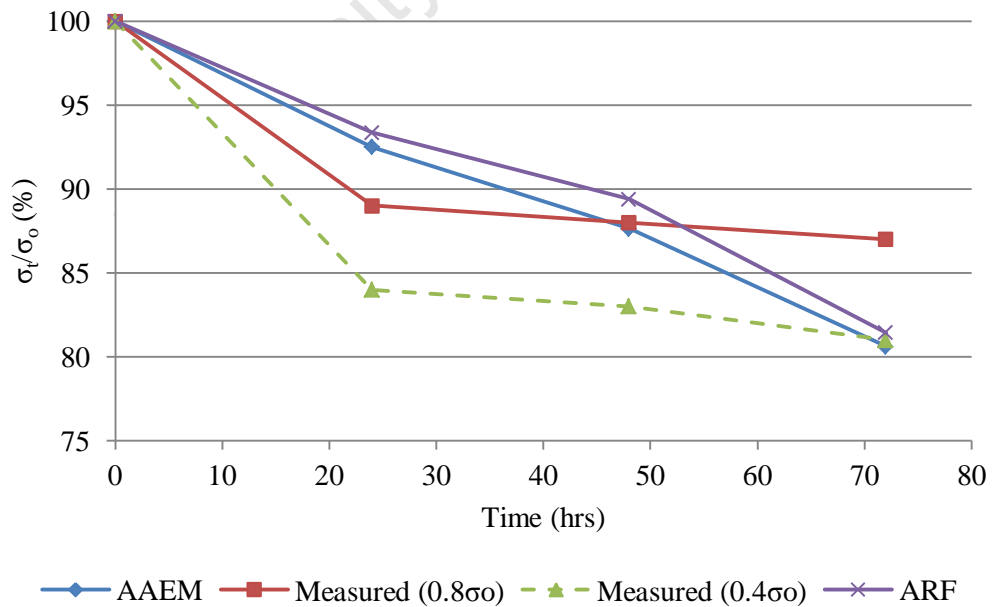


Figure 5.4: Graphical comparison of relaxation results, 0.45 w/c loaded at 10 days

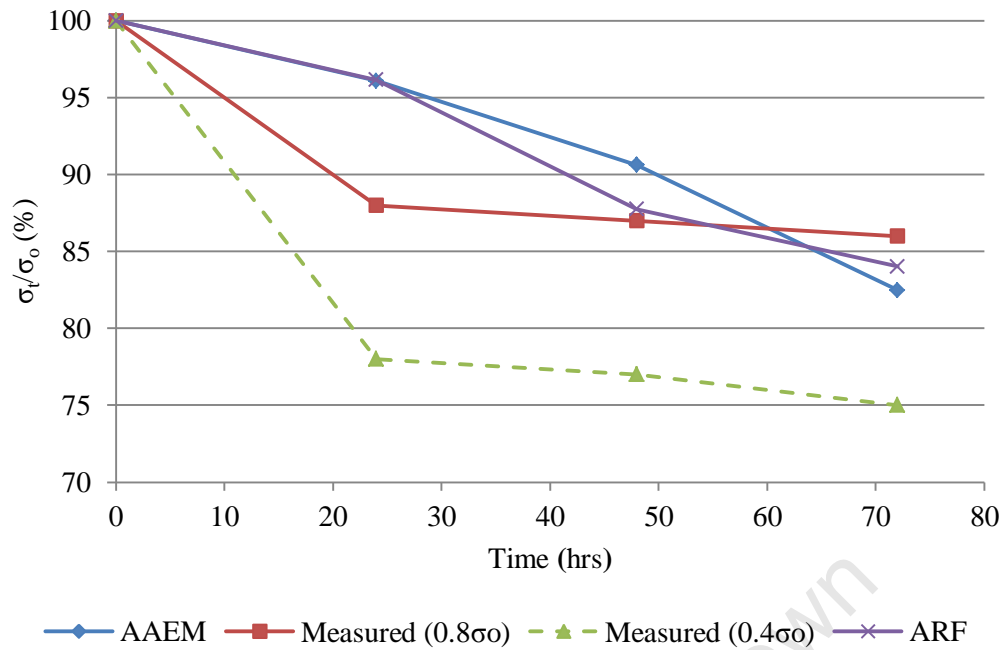


Figure 5.5: Graphical comparison of relaxation results, 0.55 w/c loaded at 10 days

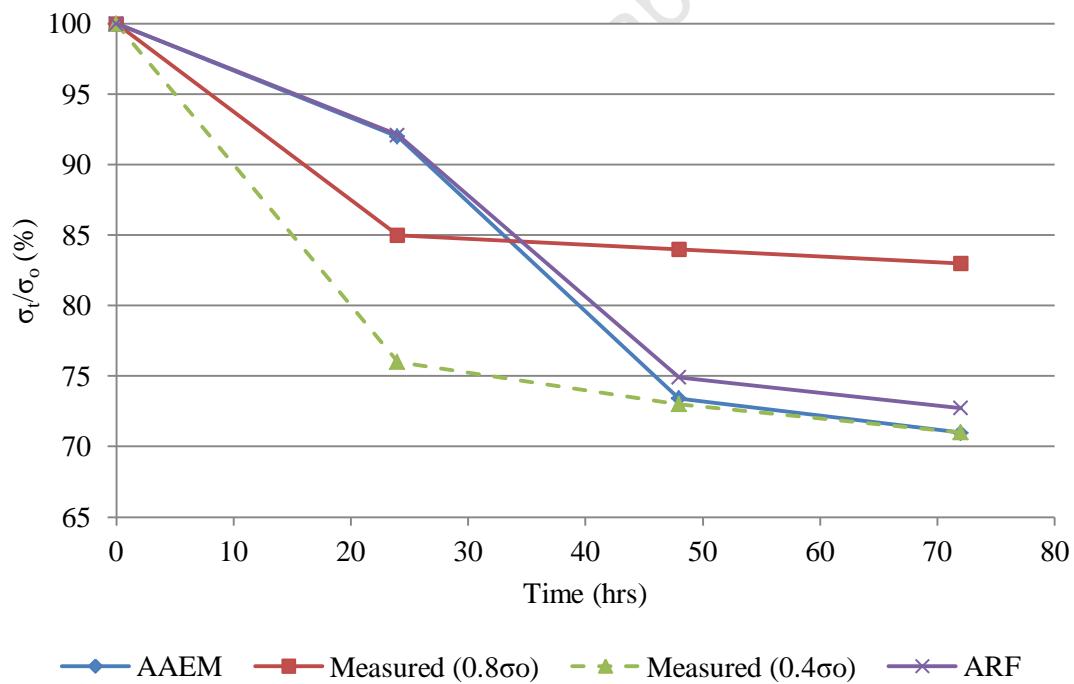


Figure 5.5: Graphical comparison of relaxation results, 0.65 w/c loaded at 10 days

Additional supporting tables for relaxation model values are presented in Appendix H.

Prediction models, similar to experimental tests were found to be sensitive to changes in the parameters considered. Both the AAEM and the ARF are sensitive to w/c ratio, age of loading and initial stress-strength ratio. The following observations were made:

Generally, there is an increase in relaxation with reduced w/c ratio in model predictions. The same observations were made with experimental results as shown in Table 5.1. The models predict a reduction in relaxation with increase in age. Again this is similar to what is obtained using experimental results. Using the AAEM, there is an average reduction of 4% relaxation between 3 day and 10 day loaded samples. A reduction of 6% is predicted for the ARF for the same ages. The algebraic relaxation prediction models considered seem to be stress sensitive. Better 72-hour predictions were generally achieved for relaxation tests carried out at 40% initial stress-strength ratio than those carried out at 80% initial stress-strength ratio. This is probably because of the fact that service loads (i.e. loads not exceeding 40% of compressive strength) are applied on creep samples in experiments and in model formulations.

When using the AAEM, the ageing coefficient was calculated using the appropriate formula. Values were 0.63 and 0.76 for the 3 and 10 day mixes respectively. The effect of using the calculated ageing coefficient was scarcely observed for the ages considered. This means that the code recommended (MC90-93) value of 0.8 can be used in cases where quick results are required. Past studies for example Carlswärd (2006) and Sassone and Chiorino (2005) have made similar observations.

Both the AAEM and ARF provide good predictions of the relaxation potential of concrete after 72 hours. For the ARF method, various compliance functions have to be obtained. This makes the process cumbersome, long and hence less attractive than the AAEM. Overall, the AAEM is found to be accurate and at the same time easy to use. Creep coefficient, elastic modulus and the ageing coefficient are some of its input parameters. All the three can be obtained either through experiments or by using prediction models and formulas.

### 5.3 Summary

Generally, it can be concluded that the use of creep values to estimate low-age relaxation potential of concrete is probably valid. If the necessary input parameters can be obtained, the AAEM method is a reasonable start in the event that a quick estimate of the low-age relaxation potential of concrete is required. This can lead to saving of time in cases when experiments cannot be carried out. The AAEM has also been found adequate by past studies including Carlswärd (2006), Tran *et al.* (2008) and Larsson (2003) cited in Carlswärd (2006).

In addition, relaxation models are found to be more accurate than creep models. This is probably because of the fact that experimental creep values are used in the relaxation models. Creep models have strength and elastic modulus as important input parameters. However the paste content plays an important role in the creep potential of the material. This probably explains why creep predictions are not as accurate as relaxation predictions.

## 6 CONCLUSIONS AND RECOMMENDATIONS

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The deformation and durability of concrete structures is an important consideration for the designer. Two of the important parameters affecting the durability and deformation of concrete are creep and tensile relaxation. Creep is the deformation of a structure under constant load while tensile relaxation is the reduction in stress when strain is kept constant. Creep has numerous effects; the most common being increase in deformation of structures which, if not checked can lead failure. Relaxation is important because it leads to reduction in tensile stresses induced in restrained concrete members. This reduction in stresses leads to prolonging and/or eliminating of cracking; the effects of which are undesirable (Chilwesa, 2012; Masuku, 2009; Carlward, 2006). In some literature, creep and relaxation are said to be similar phenomenon having different boundary conditions (Gilbert and Ranzi, 2010). Hilsdorf and Mueller (1999) state that in fact, creep and relaxation are influenced by the same factors.

This study aimed at investigating whether selected parameters that affect creep, would have the same effect on relaxation of concrete. The parameters investigated include: age of loading, w/c ratio, initial stress-strength ratio and aggregate content. Because relaxation tests are rarely carried out, predictions models are often used to calculate the relaxation function from the creep function (Chiorino, 2005; Atrushi, 2003; Wittmann, 1971). In this study, the accuracy of two algebraic relaxation prediction models was investigated. The selected models were: The Age Adjusted Effective Modulus (AAEM) and the Approximate Relaxation Function (ARF).

The first aim of the study was achieved by carrying out relaxation and creep experiments on similar concrete and by comparing experimental work with past studies. The second objective of the study was achieved by comparing experimental relaxation values with those from prediction models.

The following conclusions have been drawn based on the research aims and objectives of the study.

### **Influence of w/c ratio**

With regards to the influence of w/c ratio, a similarity between creep and relaxation is reported. It appears both relaxation and creep generally increase with increasing w/c ratio. Creep and relaxation are influenced by the stiffness of the paste in addition to its content. Reduced relaxation and creep in low w/c ratio mixes can be attributed to a stronger, stiffer

and less permeable paste (Alexander and Beushausen, 2009). Omar *et al.* (2008) citing work by Smadi *et al.* (1987) attributes this to the fewer pores and a rigid solid matrix of low w/c ratio concretes. This observation has been reported in previous studies on relaxation (Masuku, 2009; Chilwesa, 2012; Marimoto and Kayonagi, 1994; Gutsch and Rostasy, 1994 and Atrushi, 2003).

### **Influence of age**

Creep and relaxation are probably influenced by age in the same way. Both reduce at higher ages of first loading. In experimental work, an average reduction of 3% relaxation was measured for samples tested at 3 days when compared to those tested at 10 days. Using prediction models, an average reduction of 5% was predicted for same ages of testing. This reduction in creep and relaxation can be attributed to an increasingly stiffened concrete which reduces the ability of the material to deform. For creep of concrete, the observation is in accordance with that reported by Bissonnette and Pigeon (1995), Østegaard *et al.* (2001), Atrushi (2003) and Pane and Hansen (2001). In the case of relaxation, these observations were made by past studies for example Masuku (2009), Chilwesa (2012) and Marimoto and Kayonagi (1994).

### **Influence of aggregate content**

Both creep and relaxation are reduced when the volume of aggregates is increased. This statement is true if other factors are kept constant. When past studies were compared, it was found that on average increasing volume of coarse aggregate content from 0 to 35%, lead to a reduction in relaxation of 20%. Aggregates reduce the creep of concrete because they reduce the paste content by the 'dilution effect'. The paste is the main source of deformation in concrete; therefore, the restraint aggregates provide to its movement can additionally be used to explain this phenomenon.

### **Influence of initial stress-strength ratio**

The initial stress-strength ratio was found not to have a clear influence on relaxation. When initial stress-strength ratio is increased, relaxation generally increases by a relatively small amount. Relaxation was found to increase at a rate approximately 5 times less than the rate of increase of initial stress-strength ratio. The reason for this is not understood. Creep is said to be proportional to the applied stress below service loads. Beyond service loads, creep strain increases at an increasing rate and is thus said to be non-linear. The non-linearity of creep (and possibly relaxation) at increased stress levels is probably caused by the formation of micro cracks at the Interfacial transitional zone (ITZ).

### **Prediction of relaxation using algebraic relaxation models**

The prediction models, similar to experimental tests are sensitive to changes in the selected parameters. Both the AAEM and the ARF are sensitive to w/c ratio, age of loading and initial stress-strength ratio. Relaxation reduces with reduced w/c ratio and increased age of first loading. Stress levels are found to have an effect on the prediction of relaxation. Better predictions are achieved at initial stress-strength ratio of 40%. The reason for this is probably because creep models and experiments, from which relaxation functions are derived, are carried out at stress levels which don't exceed 40%. The value of 0.8 recommended for the ageing coefficient by MC90-93 is adequate. Calculation of the coefficient, especially for ages of concrete beyond 10 days did not yield substantial differences in predictions.

Generally, low-age relaxation can be predicted using the AAEM and ARF methods. It is however important that the creep and elastic modulus values are accurately determined preferably by experimental work. The AAEM method is preferred for the prediction of relaxation because of its combined effect of simplicity and accuracy. This can lead to saving of time in cases where experiments cannot be carried out.

### **Miscellaneous**

Besides the main conclusions of the study, it was observed in this research that:

Creep and shrinkage prediction models are mainly sensitive to strength, size effects and relative humidity. These models should be used carefully especially in the prediction of shrinkage as the paste content seems to be the underlying factor in a concrete's ability to shrink. Creep compliance function should be used to describe the concrete's creep deformation potential because it includes both short term and long term components.

In this study, relaxation is found to reduce stresses by 13- 23% for initial stress levels of 80% and by 19- 29% for 40% initial stress-strength ratio. The rate of relaxation is not influenced by either one of w/c ratio, age and initial stress-strength ratio but is probably influenced by the aggregate content. In addition, it was observed that the rate of relaxation is higher than that of creep in the first hours.

### **Recommendations for further research**

In this study, the amount of experimental work carried out was in some instances not sufficient to draw definitive conclusions. It is therefore clear that there is a need for further research into this topic and related topics. A number of future research options are thus recommended.

- The most obvious future research that can be recommended from this study is the investigation of the possibility of available models to predict long term relaxation. The only difference in the study would be that relaxation is measured for longer durations.
- The influence of thermal effects on relaxation should be investigated. Thermal effect cause additional stresses and hence it is important that they are considered.
- With regards to the effects of aggregate content, more work needs to be done in order to quantify their influence. The influence of aggregate properties such as stiffness on relaxation is still open to research.
- Generally, mix design optimisation to maximise the benefits of relaxation is a topic that is far from being fully understood and is therefore worth studying
- Influence of initial stress-strength ratio on relaxation needs further study in order to be quantified. For example, is relaxation proportional to initial stress-strength ratio at very low stress levels? More stress levels starting at 10% can be investigated to answer that question. Longer durations of testing should also be considered. In addition, the influence of initial stress-strength ratio should be investigated on mortars in comparison with concrete.
- Relaxation is predicted from creep values. Creep prediction models are predominantly based on strength, elastic modulus and relative humidity. A model or guide that enables prediction of relaxation based on the same parameters (i.e., strength, elastic modulus, and relative humidity) is therefore a possibility.
- Tensile relaxation was obtained from creep values and yet these values were compressional. More research is needed with respect to making comparisons between compressive and tensile creep with respect to relaxation prediction.

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**APPENDICES**

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## Appendix A: Compressive strength results

Table A1: 3 day compressive strength results

3 day compressive strength results 07/06/12					
Mix 1 (w/c = 0.45)					
Specimen mark	Weights	Compressive Load at failure	Compressive strength (MPa)	Mean Compressive strength (MPa)	Standard deviation
	(kg)	kN			
Mix 1,3a	2.337	412	41.2	42.4	1.04
Mix 1,3b	2.3696	430	43		
Mix 1,3c	2.3525	430	43		
Mix 2 (w/c = 0.55)					
Specimen mark	Weights	Compressive Load at failure	Compressive strength (MPa)	Mean Compressive strength (MPa)	Standard deviation
	(kg)	kN			
Mix 2,3a	2.326	305	30.5	29.3	1.04
Mix 2,3b	2.3023	285	28.5		
Mix 2,3c	2.2923	290	29		
Mix 3 (w/c = 0.65)					
Specimen mark	Weights	Compressive Load at failure	Compressive strength (MPa)	Mean Compressive strength (MPa)	Standard deviation
	(kg)	kN			
Mix 3,3a	2.285	192	19.2	19.3	0.12
Mix 3,3b	2.332	194	19.4		
Mix 3,3c	2.302	192	19.2		

Table A2: 10-day compressive strength results

<b>10 day compressive strength results 14/06/12</b>					
<b>Mix 1 (w/c = 0.45)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Compressive Load at failure</b>	<b>Compressive strength (MPa)</b>	<b>Mean Compressive strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 1,10a	2.345	510	51	52.1	1.47
Mix 1,10b	2.374	538	53.8		
Mix 1,10c	2.3654	516	51.6		
<b>Mix 2 (w/c = 0.55)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Compressive Load at failure</b>	<b>Compressive strength (MPa)</b>	<b>Mean Compressive strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 2,10a	2.364	418	41.8	41.9	0.70
Mix 2,10b	2.335	426	42.6		
Mix 2,10c	2.311	412	41.2		
<b>Mix 3 (w/c = 0.65)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Compressive Load at failure</b>	<b>Compressive strength (MPa)</b>	<b>Mean Compressive strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 3,10a	2.295	274	27.4	27.9	0.78
Mix 3,10b	2.343	288	28.8		
Mix 3,10c	2.321	275	27.5		

Table A3: 28-day compressive strength results

<b>28 day compressive strength results 02/07/12</b>					
<b>Mix 1 (w/c = 0.45)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Compressive Load at failure</b>	<b>Compressive strength (MPa)</b>	<b>Mean Compressive strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 1,28a	2.364	572	57.2	58.7	2.04
Mix 1,28b	2.368	610	61		
Mix 1,28c	2.378	578	57.8		
<b>Mix 2 (w/c = 0.55)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Compressive Load at failure</b>	<b>Compressive strength (MPa)</b>	<b>Mean Compressive strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 2,28a	2.357	436	43.6	44.9	1.15
Mix 2,28b	2.336	453	45.3		
Mix 2,28c	2.322	458	45.8		
<b>Mix 3 (w/c = 0.65)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Compressive Load at failure</b>	<b>Compressive strength (MPa)</b>	<b>Mean Compressive strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 3,28a	2.323	338	33.8	33.9	0.66
Mix 3,28b	2.307	346	34.6		
Mix 3,28c	2.278	333	33.3		

## Appendix B: Elastic modulus results

Table B1: Elastic modulus results for 0.45 w/c mix

Summary of Elastic modulus results for 0.45 w/c mix				
Age (Days)	No.	Elastic Modulus (GPa)	Mean Elastic Modulus (GPa)	Standard deviation
3	1	36.0	36.0	1.1
	2	37.1		
	3	35.0		
10	1	45.0	41.6	3.07
	2	40.9		
	3	39.0		
28	1	33	35.9	4.13
	2	38.84		
	3			

Table B2: Elastic modulus results for 0.55 w/c mix

Summary of Elastic modulus results for 0.55 w/c mix				
Age (Days)	No.	Elastic Modulus (GPa)	Mean Elastic Modulus (GPa)	Standard deviation
3	1	28.0	26.4	1.4
	2	25.2		
	3	26.0		
10	1	36.0	33.5	2.84
	2	30.4		
	3	34.0		
28	1	28.29	30.1	2.54
	2	31.88		
	3			

Table B3: Elastic modulus results for 0.65 w/c mix

<b>Summary of Elastic modulus results for 0.65 w/c mix</b>				
<b>Age (Days)</b>	<b>No.</b>	<b>Elastic Modulus (GPa)</b>	<b>Mean Elastic Modulus (GPa)</b>	<b>Standard deviation</b>
3	1	16.7	21.1	4.2
	2	21.6		
	3	25.0		
10	1	24.0	26.0	1.71
	2	26.9		
	3	27.0		
28	1	24	25.6	2.28
	2	27.22		
	3			

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## Appendix C: Tensile strength results

Table C1: 3-day Tensile strength results

3 day Tensile strength results					
Mix 1 (w/c = 0.45)					
Specimen mark	Weights	Tensile Load at failure	Tensile strength (MPa)	Mean Tensile strength (MPa)	Standard deviation
	(kg)	kN			
Mix 1,3a	1.38	4.70	2.94	3.40	0.50
Mix 1,3b	1.39	4.38	2.74		
Mix 1,3c	1.38	4.72	3.37		
Mix 1,3e	1.37	4.35	3.63		
Mix 1,3f	1.37	4.39	3.66		
Mix 1,3g		4.90	4.08		
Mix 2 (w/c = 0.55)					
Specimen mark	Weights	Tensile Load at failure	Tensile strength (MPa)	Mean Tensile strength (MPa)	Standard deviation
	(kg)	kN			
Mix 2,3b	1.37	4.37	2.73	2.43	0.30
Mix 2,3c	1.38	3.47	2.17		
Mix 2,3d	1.38	3.57	2.23		
Mix 2,3g	1.37	3.34	2.08		
Mix 2,3h	1.35	3.33	2.77		
Mix 2,3i	1.35	3.11	2.60		
Mix 3 (w/c = 0.65)					
Specimen mark	Weights	Tensile Load at failure	Tensile strength (MPa)	Mean Tensile strength (MPa)	Standard deviation
	(kg)	kN			
Mix 3,3b	1.34	2.27	1.42	1.75	0.42
Mix 3,3c	1.37	3.17	2.27		
Mix 3,3d	1.30	3.72	2.33		
Mix 3,3e	1.34	3.14	1.31		
Mix 3,3f	1.35	3.35	1.40		
Mix 3,3g	1.35	2.19	1.83		
Mix 3,3h	1.37	2.71	1.69		

Table C2: 10-day Tensile strength results

<b>10 day Tensile strength results</b>					
<b>Mix 1 (w/c = 0.45)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Tensile Load at failure</b>	<b>Tensile strength (MPa)</b>	<b>Mean Tensile strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 1,10a	1.39	5.76	3.60	4.10	0.41
Mix 1,10b	1.37	5.28	3.74		
Mix 1,10d	1.37	6.00	4.29		
Mix 1,10f	1.40	5.49	4.58		
Mix 1,10g	1.40	5.15	4.30		
<b>Mix 2 (w/c = 0.55)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Tensile Load at failure</b>	<b>Tensile strength (MPa)</b>	<b>Mean Tensile strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 2,10a		5.35	3.82	3.59	0.21
Mix 2,10b		5.23	3.27		
Mix 2,10c	1.40	5.43	3.88		
Mix 2,10e	1.35	4.30	3.59		
Mix 2,10f	1.33	4.19	3.49		
Mix 2,10g		4.29	3.57		
Mix 2,10h		4.19	3.49		
<b>Mix 3 (w/c = 0.65)</b>					
<b>Specimen mark</b>	<b>Weights</b>	<b>Tensile Load at failure</b>	<b>Tensile strength (MPa)</b>	<b>Mean Tensile strength (MPa)</b>	<b>Standard deviation</b>
	<b>(kg)</b>	<b>kN</b>			
Mix 3,10a		4.29	2.38	3.01	0.50
Mix 3,10b		4.77	3.38		
Mix 3,10c		3.91	2.44		
Mix 3,10d		4.50	3.22		

Table C3: 28-day Tensile strength results

28 day Tensile strength results Date.....					
Mix 1 (w/c = 0.45)					
Specimen mark	Weights	Tensile Load at failure	Tensile strength (MPa)	Mean Tensile strength (MPa)	Standard deviation
	(kg)	kN			
Mix 1,28a	1.393	7.06	4.42	4.42	0.19
Mix 1,28b		7.50	4.69		
Mix 2 (w/c = 0.55)					
Specimen mark	Weights	Tensile Load at failure	Tensile strength (MPa)	Mean Tensile strength (MPa)	Standard deviation
	(kg)	kN			
Mix 2,28a	1.412	6.24	3.90	4.16	0.36
Mix 2,28b		5.30	4.41		
Mix 3 (w/c = 0.65)					
Specimen mark	Weights	Tensile Load at failure	Tensile strength (MPa)	Mean Tensile strength (MPa)	Standard deviation
	(kg)	kN			
Mix 3,28a	1.349	3.41	2.84	3.27	0.46
Mix 3,28b		4.51	3.76		
Mix 3,28a		5.13	3.20		

Table C4: Summary of concrete mechanical properties

Mix (w/c)	Compressive Strength (MPa)			Mean Tensile strength (MPa)			Mean Elastic Modulus (GPa)		
	3d	10d	28d	3d	10d	28d	3d	10d	28d
<b>0.45</b>	42.4	52.1	59.0	3.4	4.1	4.4	36.0	41.6	35.9
<b>0.55</b>	29.3	42.9	45.0	2.4	3.6	4.2	26.4	33.5	30.1
<b>0.65</b>	19.3	27.9	33.0	1.7	3.0	3.3	21.1	26.0	25.6

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### Appendix D: Creep and Shrinkage results

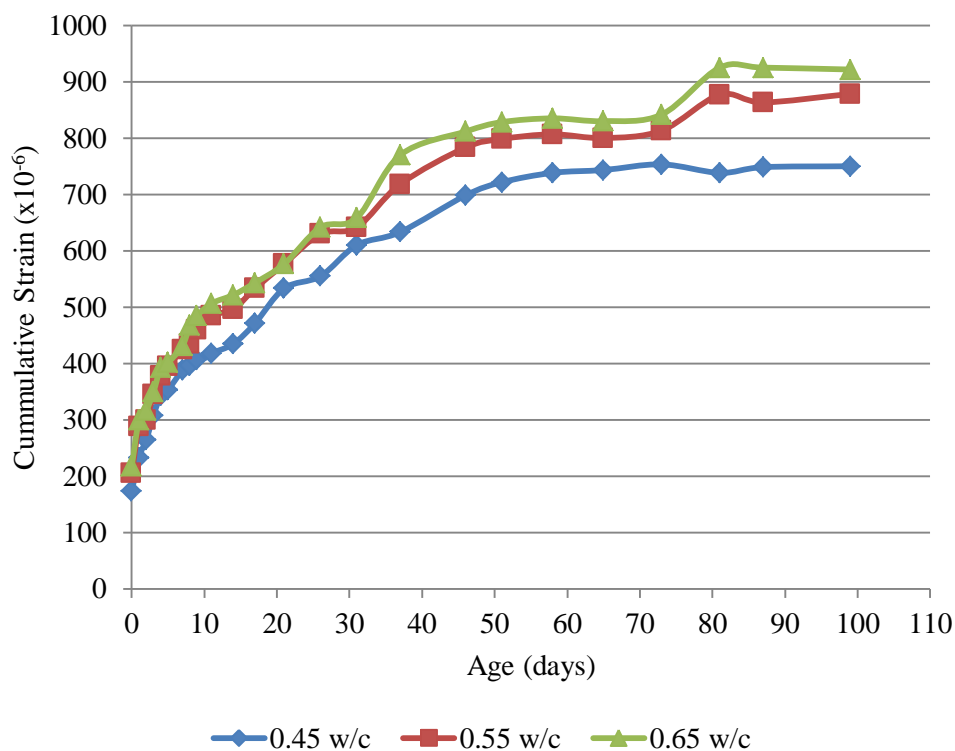


Figure D1: Development of total strain for all mixes loaded at 3 days

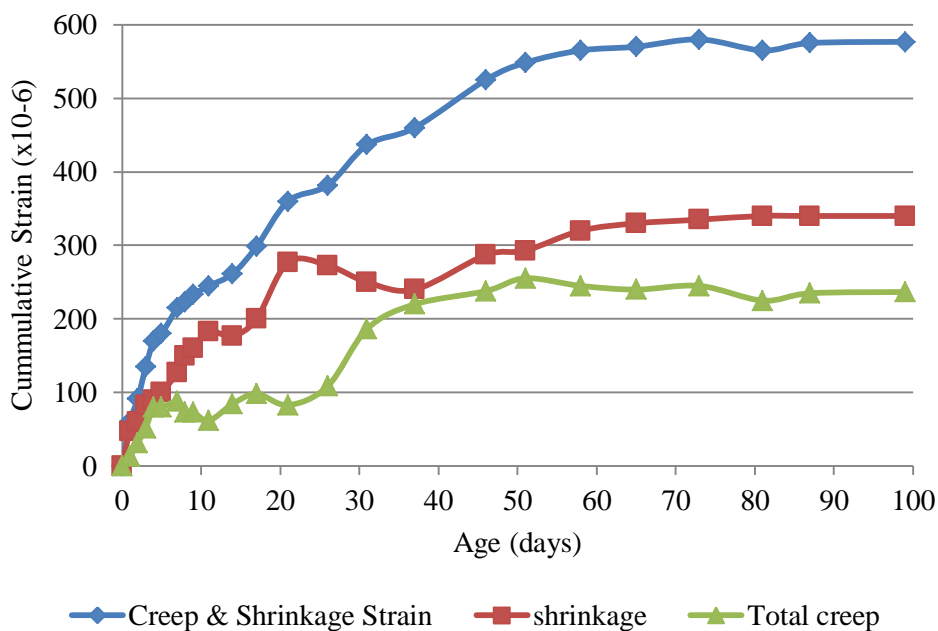


Figure D2: Development of strain, shrinkage and total creep for 0.45 w/c mix (3-day loaded)

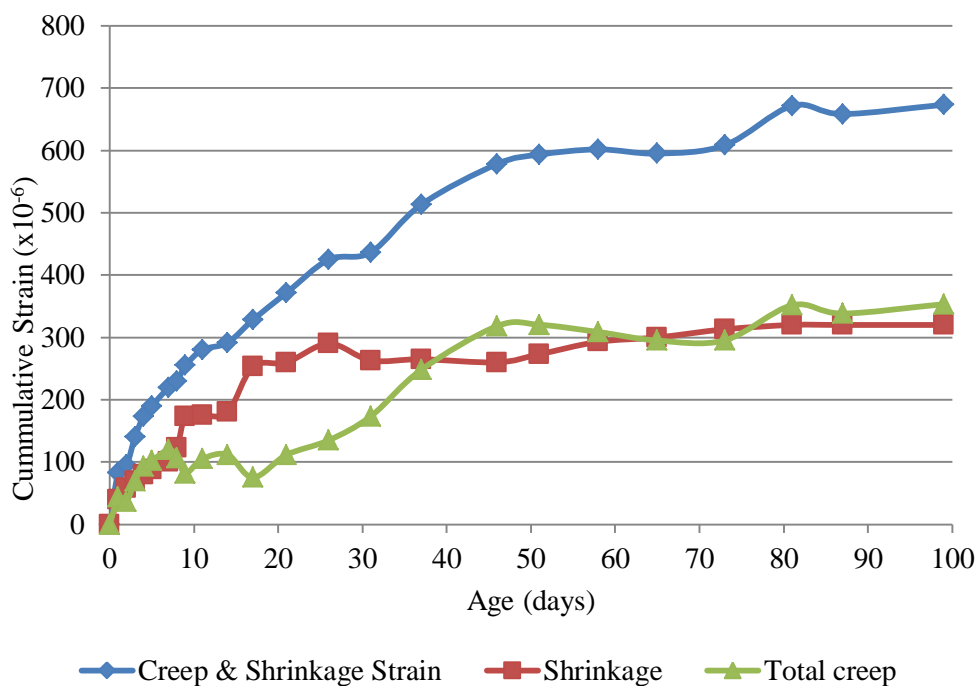


Figure D3: Development of strain, shrinkage and total creep for 0.55 w/c mix (3-day loaded)

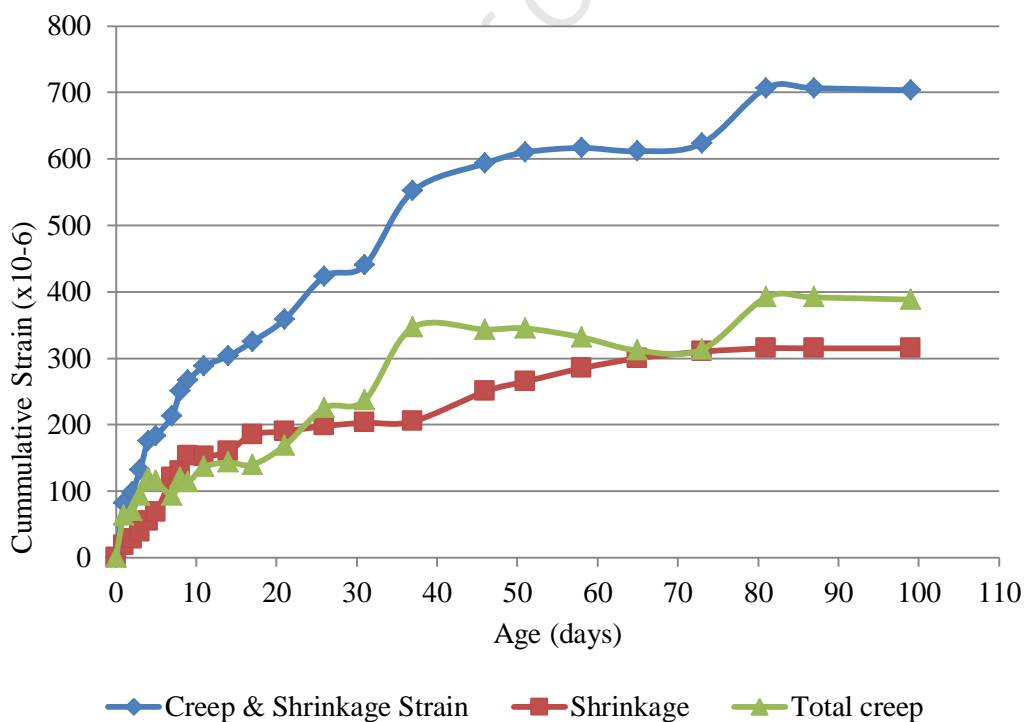


Figure D4: Development of strain, shrinkage and total creep for 0.65 w/c mix (3-day loaded)

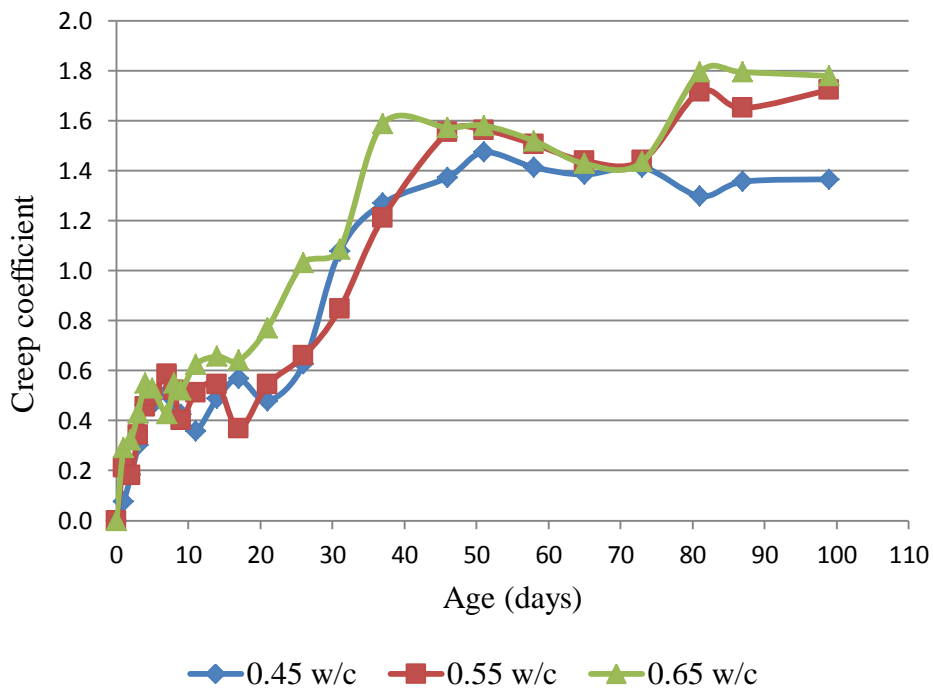


Figure D5: Development of creep coefficient for 3-day loaded creep samples

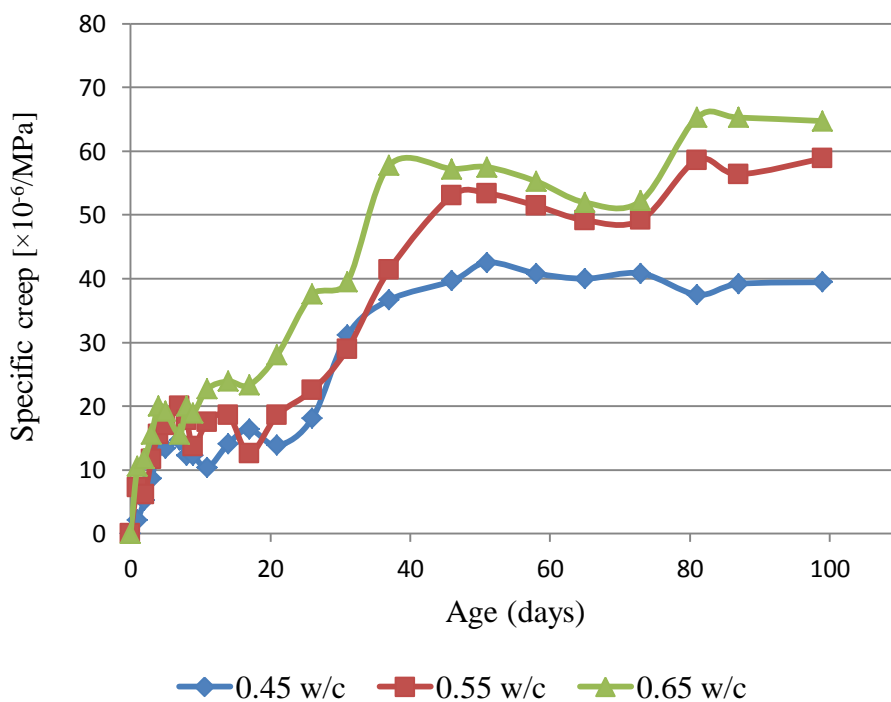


Figure D6: Development of specific creep for 3-day loaded creep samples

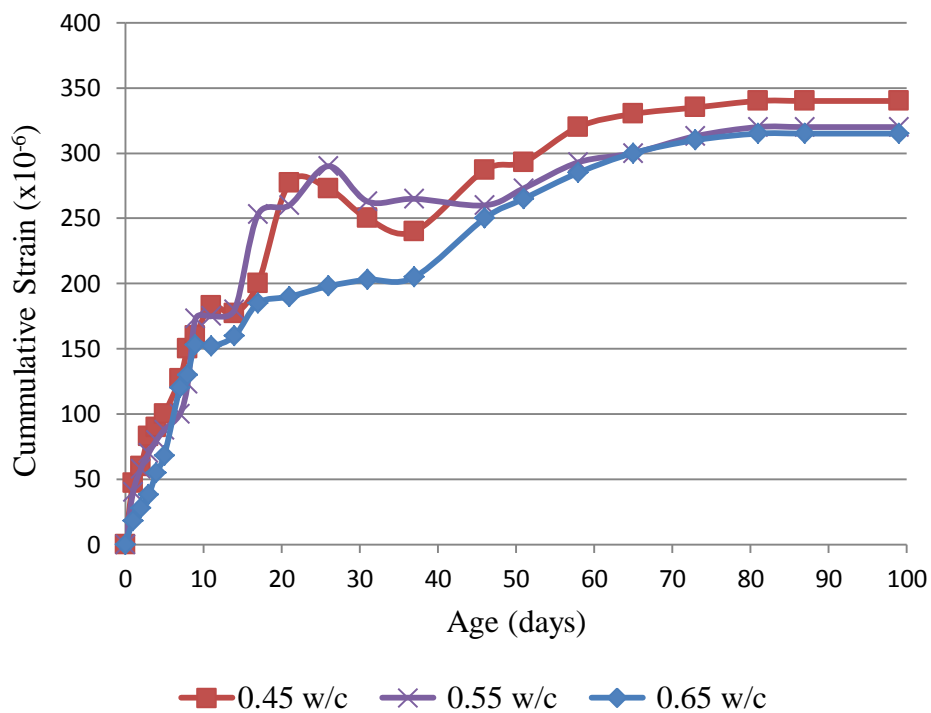


Figure D7: Development of shrinkage for 3-day loaded samples

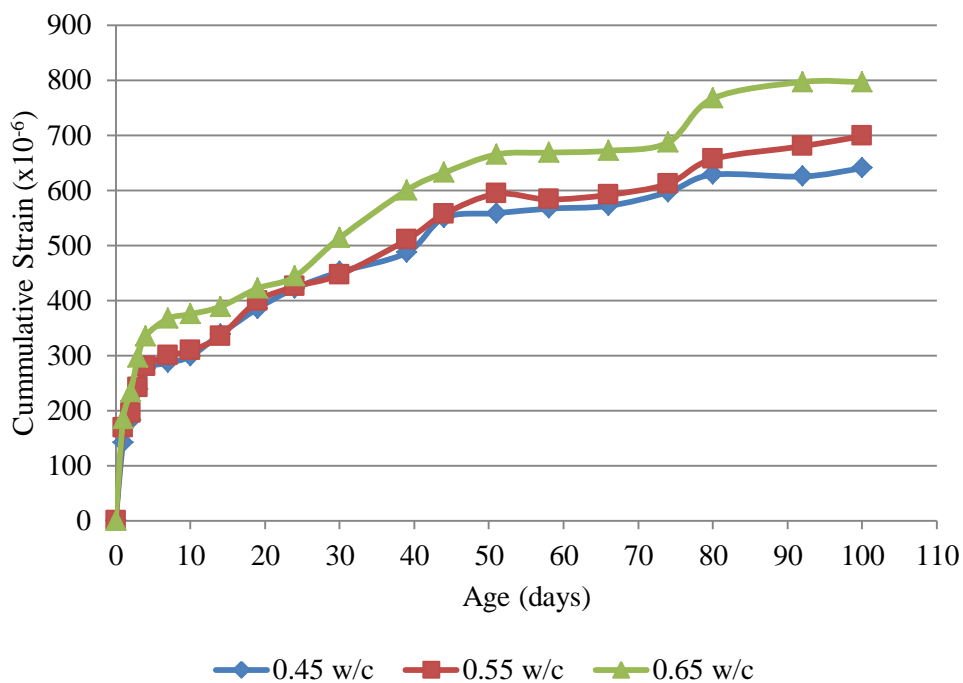


Figure D8: Development of total strain for all mixes loaded at 10 days

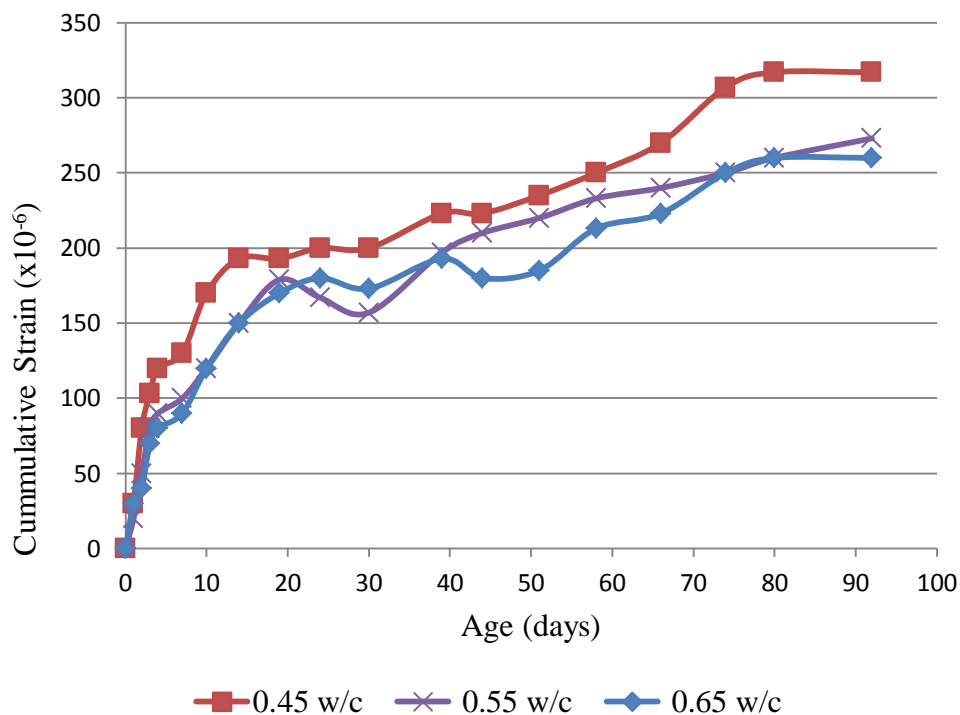


Figure D9: Development of shrinkage for 10-day loaded samples

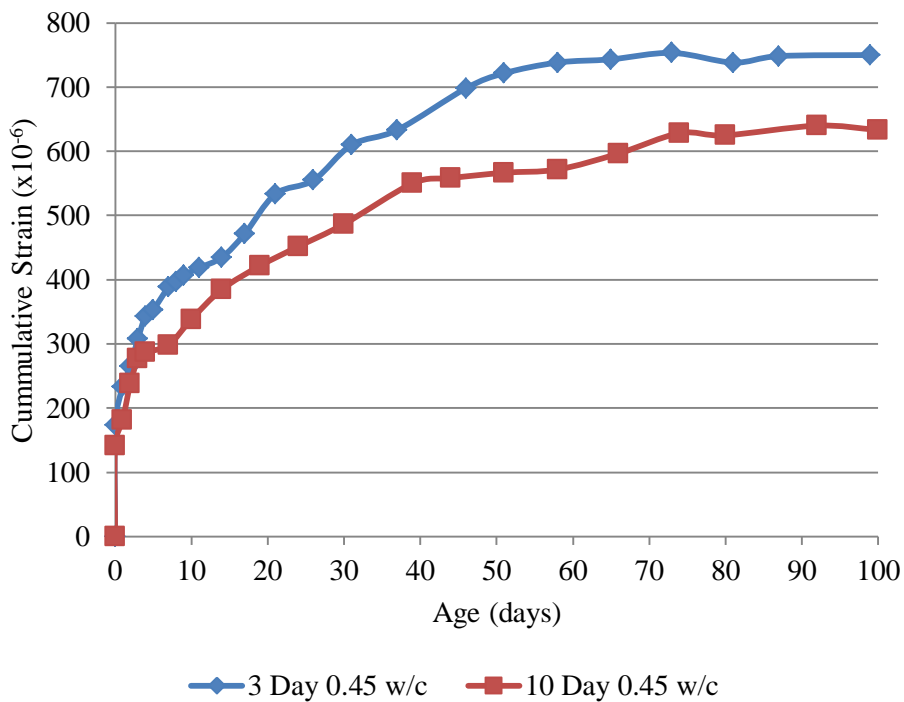


Figure D10: Development of total strain for mix with w/c 0.45 at different ages

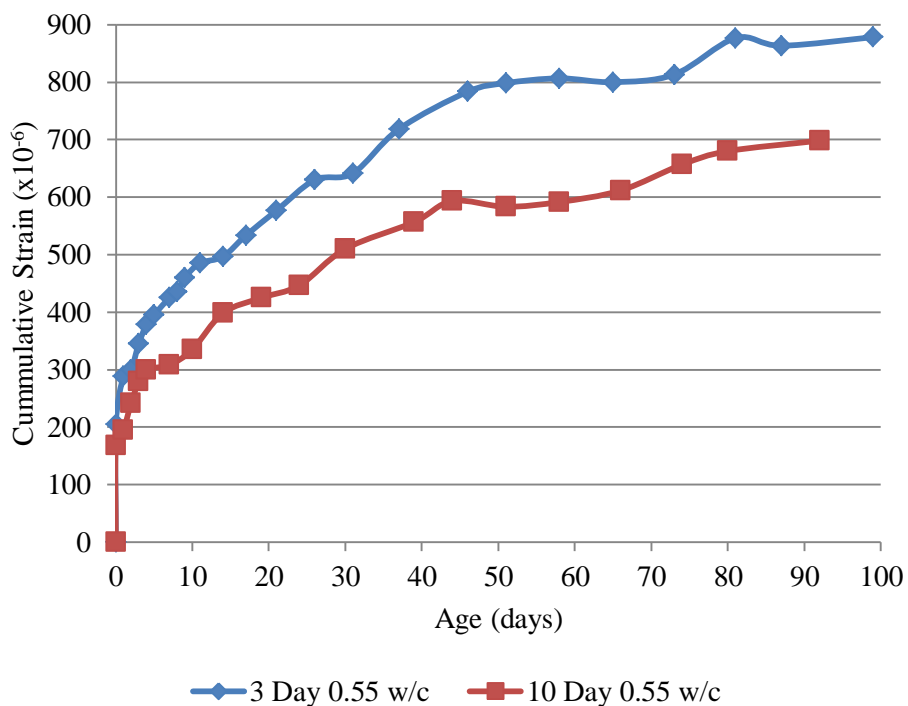


Figure D11: Development of total strain for mix with w/c 0.55 at different ages

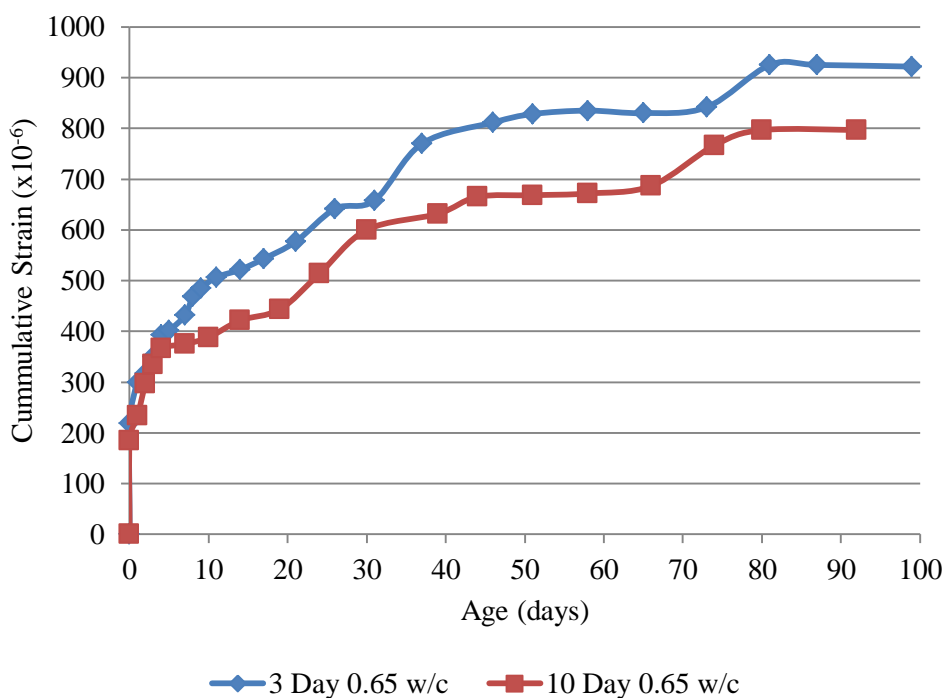


Figure D12: Development of total strain for mix with w/c 0.65 at different ages

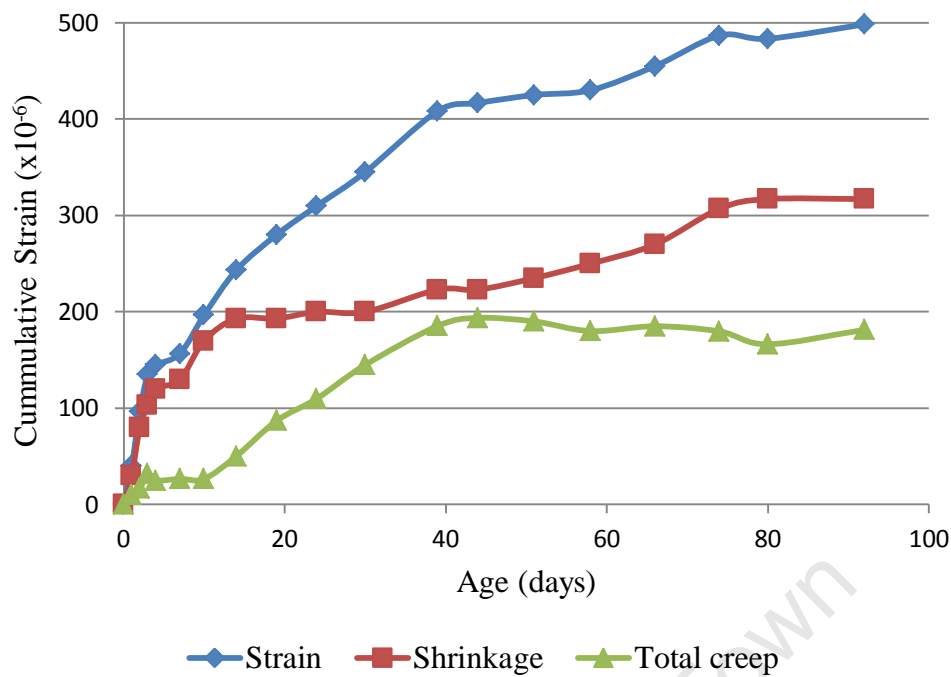


Figure D13: Development of strain, shrinkage and total creep for 0.45 w/c mix (10-day loaded)

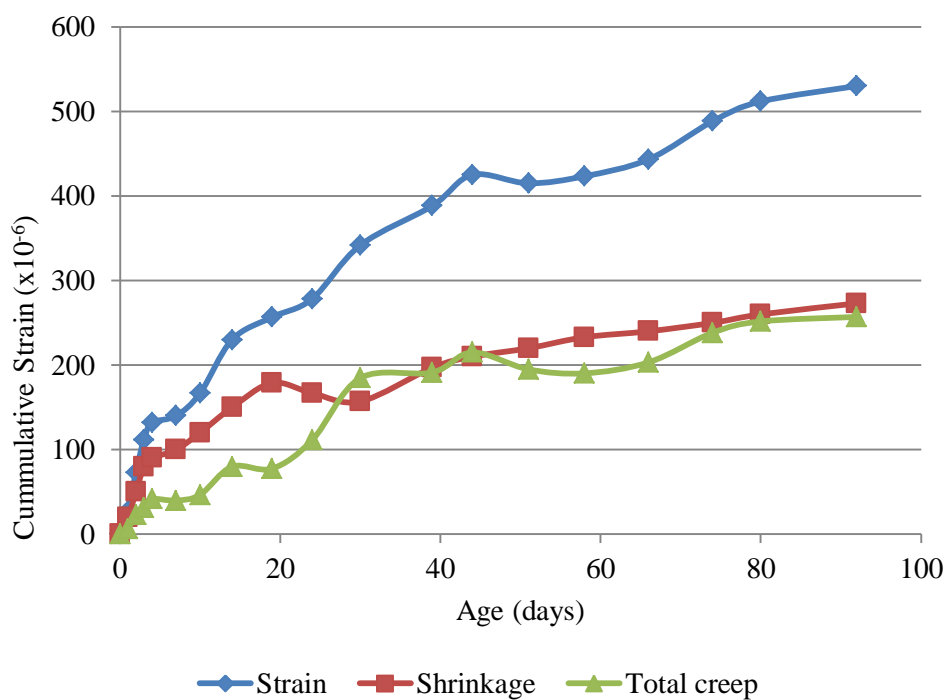


Figure D14: Development of strain, shrinkage and total creep for 0.55 w/c mix (10-day loaded)

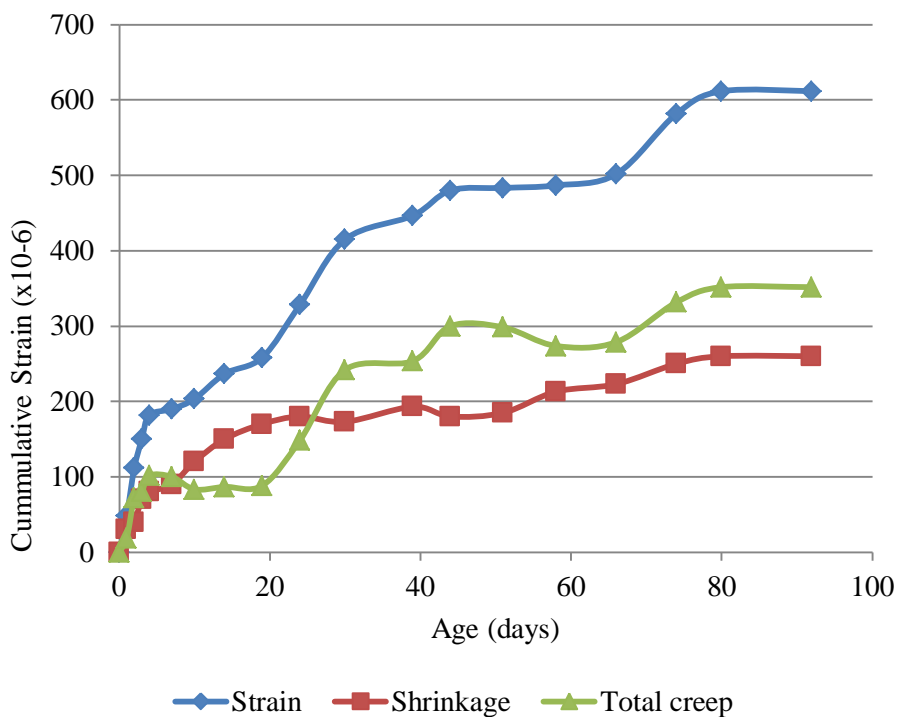


Figure D15: Development of strain, shrinkage and total creep for 0.65 w/c mix (10-day loaded)

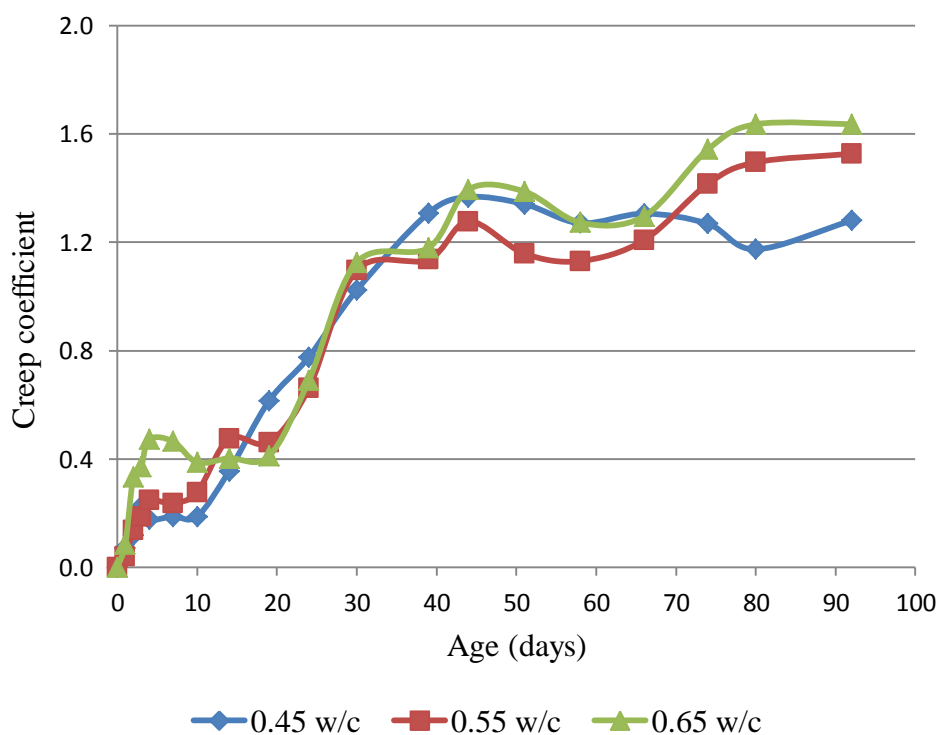


Figure D16: Development of creep coefficient for 10-day loaded creep samples

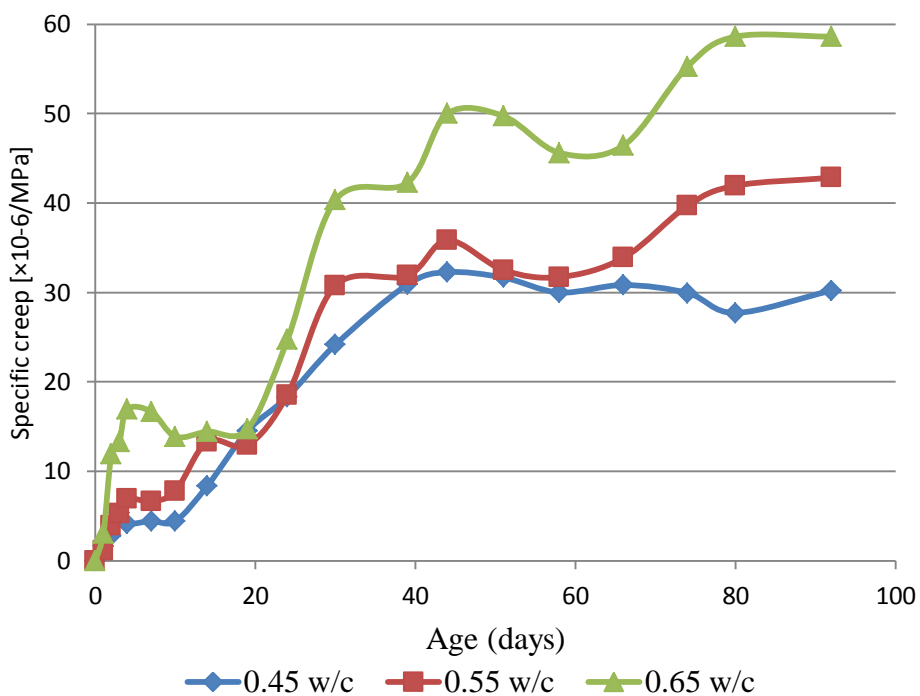


Figure D17: Development of specific creep for 10-day loaded creep samples

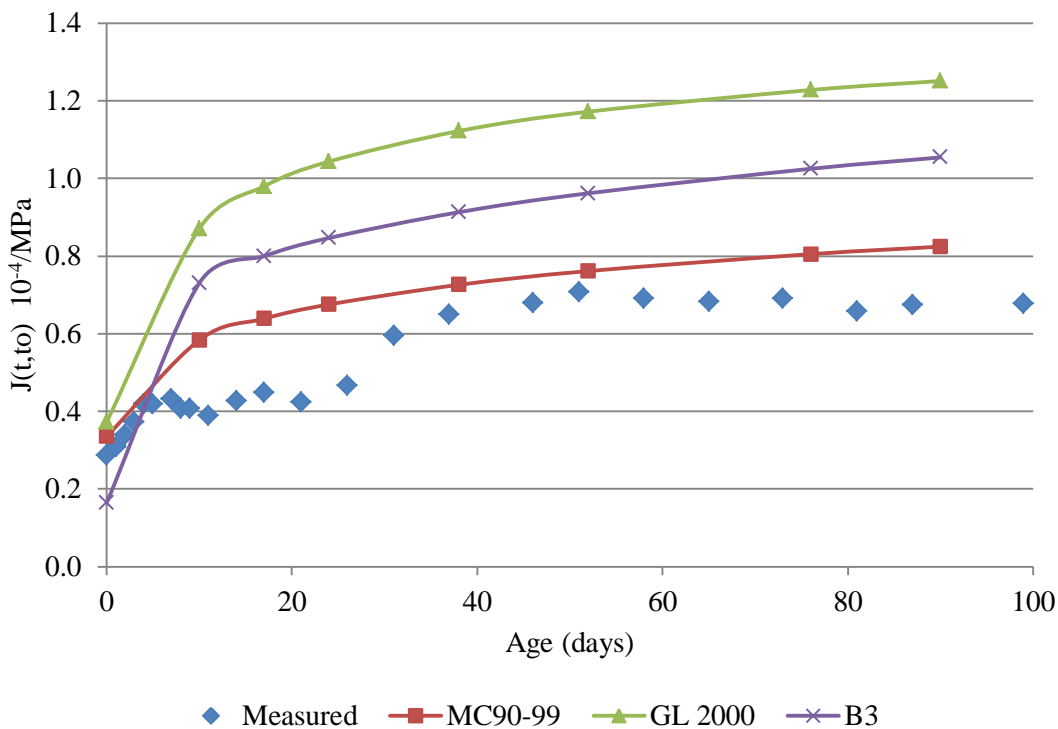


Figure D18: Development of creep compliance for 3-day loaded creep samples with 0.45 w/c

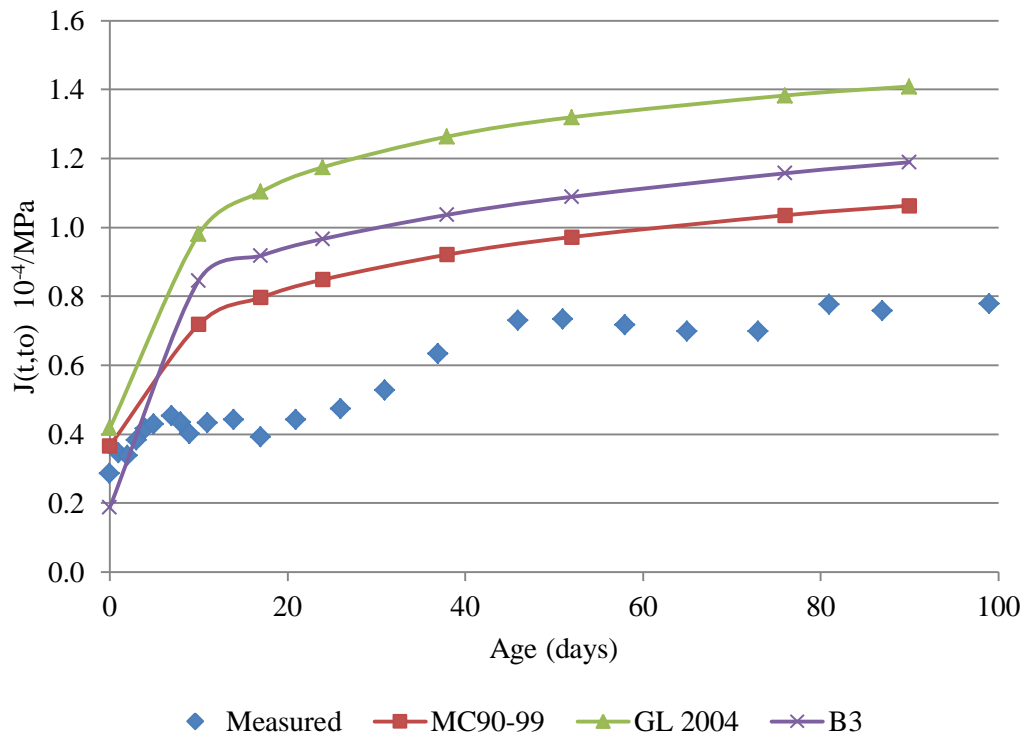


Figure D19: Development of creep compliance for 3-day loaded creep samples with 0.55 w/c

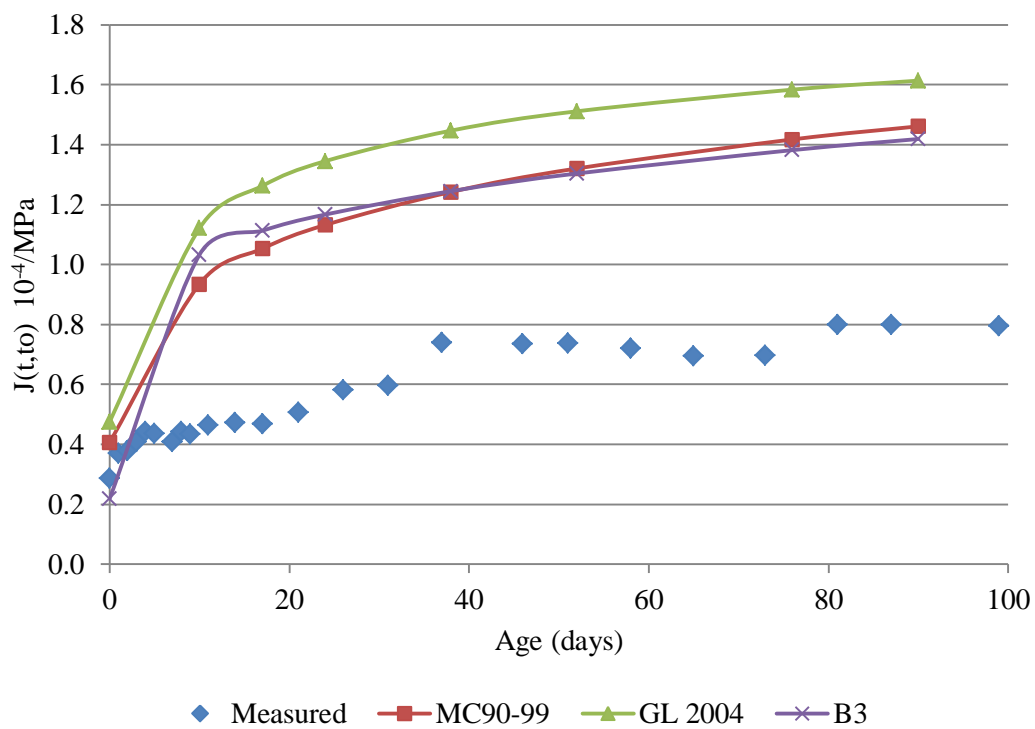


Figure D20: Development of creep compliance for 3-day loaded creep samples with 0.65 w/c

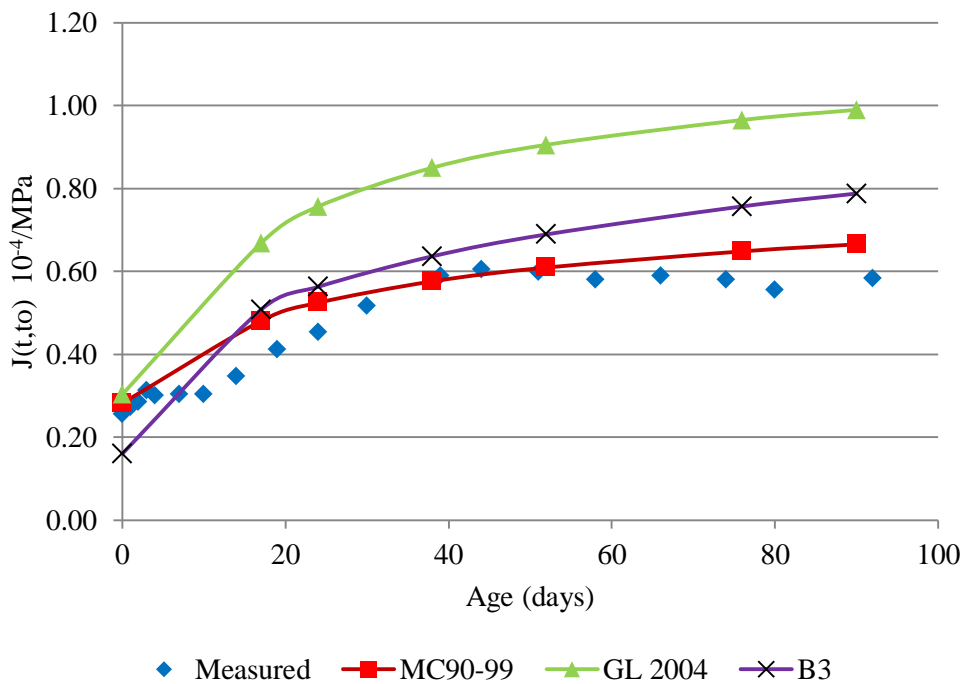


Figure D21: Development of creep compliance for 10-day loaded creep samples with 0.45  $w/c$

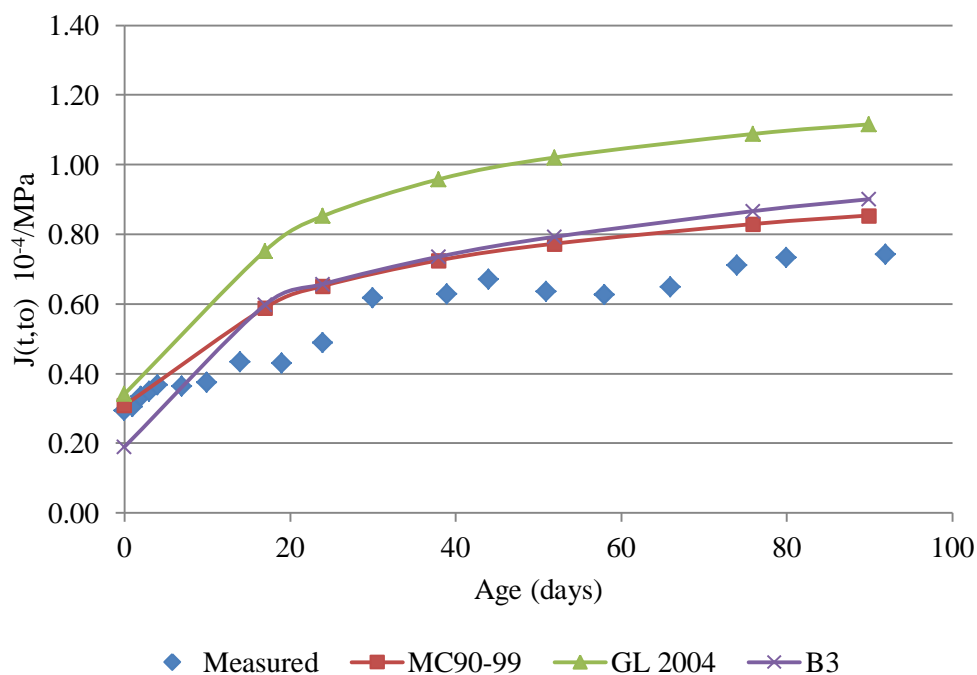


Figure D22: Development of creep compliance for 10-day loaded creep samples with 0.55  $w/c$

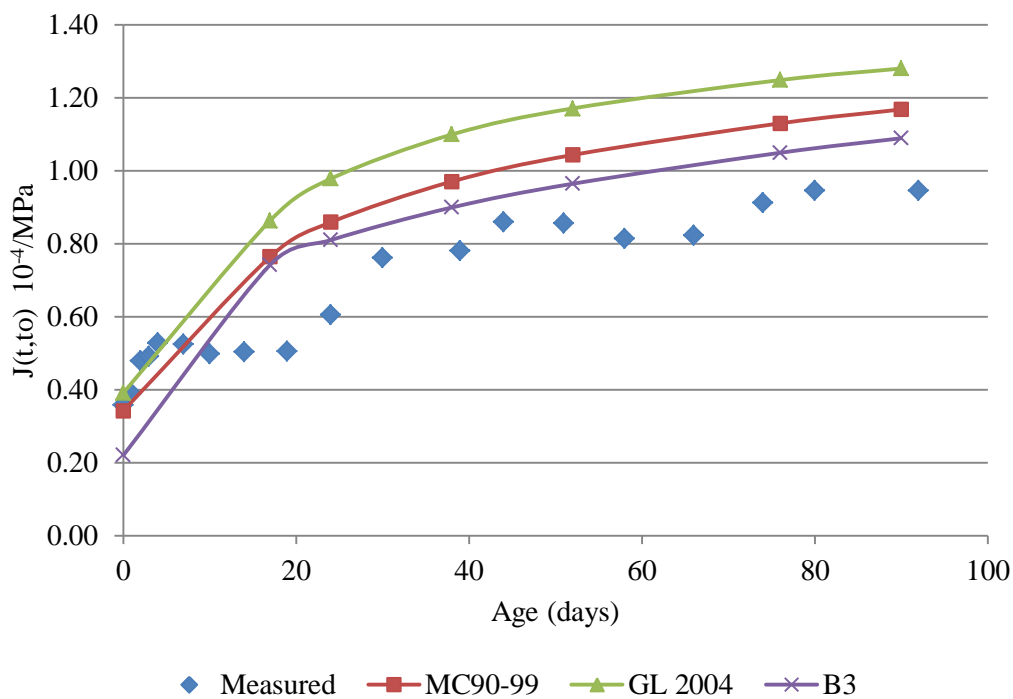


Figure D23: Development of creep compliance for 10-day loaded creep samples with 0.65 w/c

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### Appendix E: Relaxation results

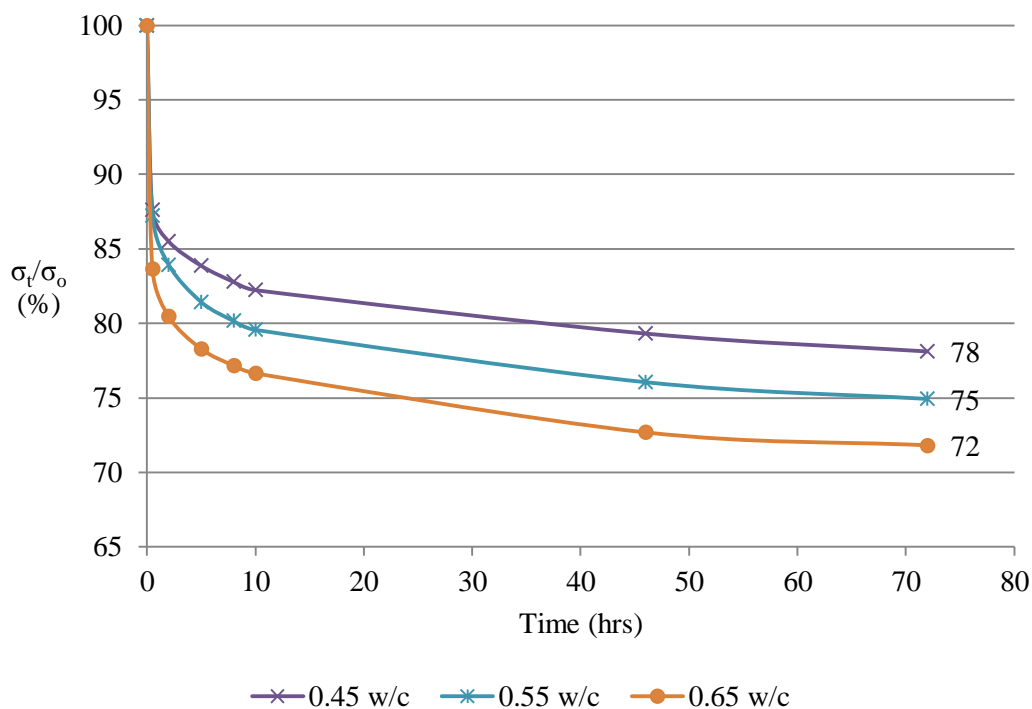


Figure E1: 3 Day Tensile relaxation at 40 % initial stress-strength ratio

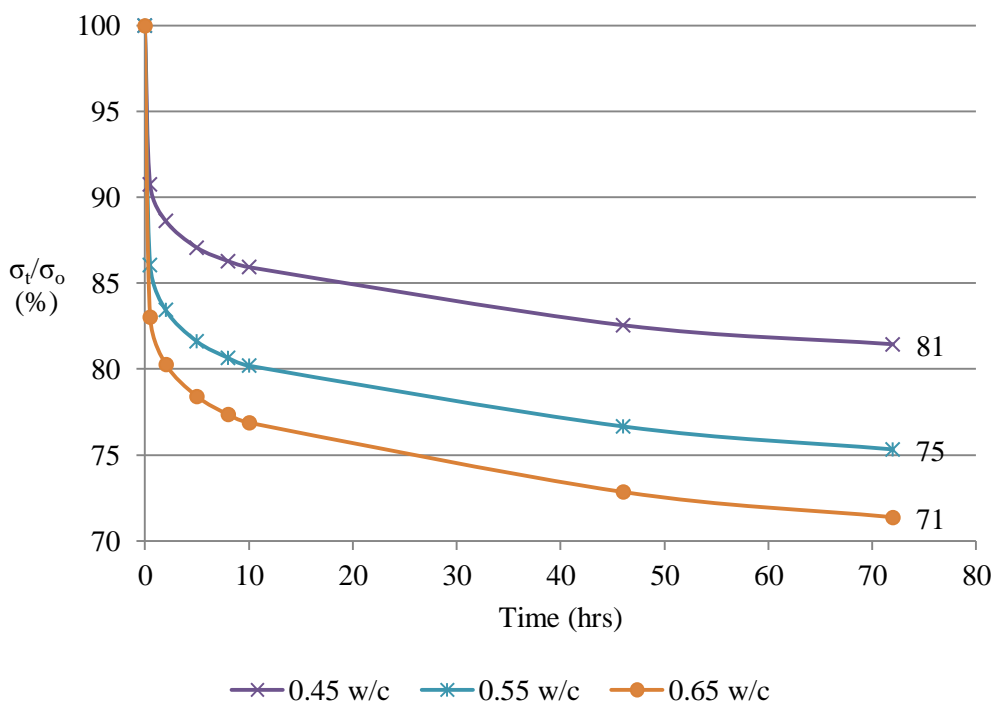


Figure E2: 10 Day Tensile relaxation at 40 % initial stress-strength ratio

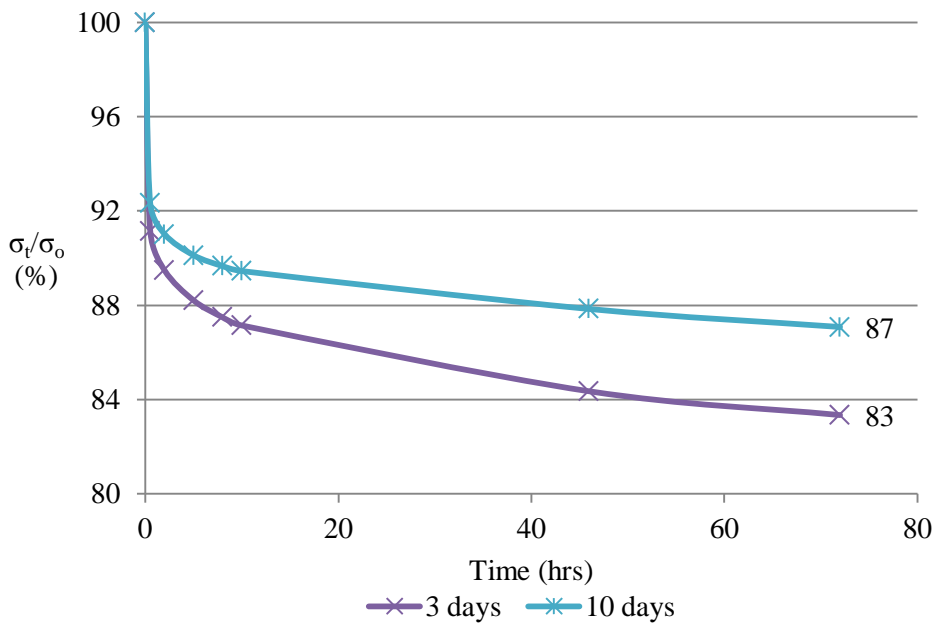


Figure E3: Influence of age of loading on development of relaxation of 0.45 w/c ratio mix

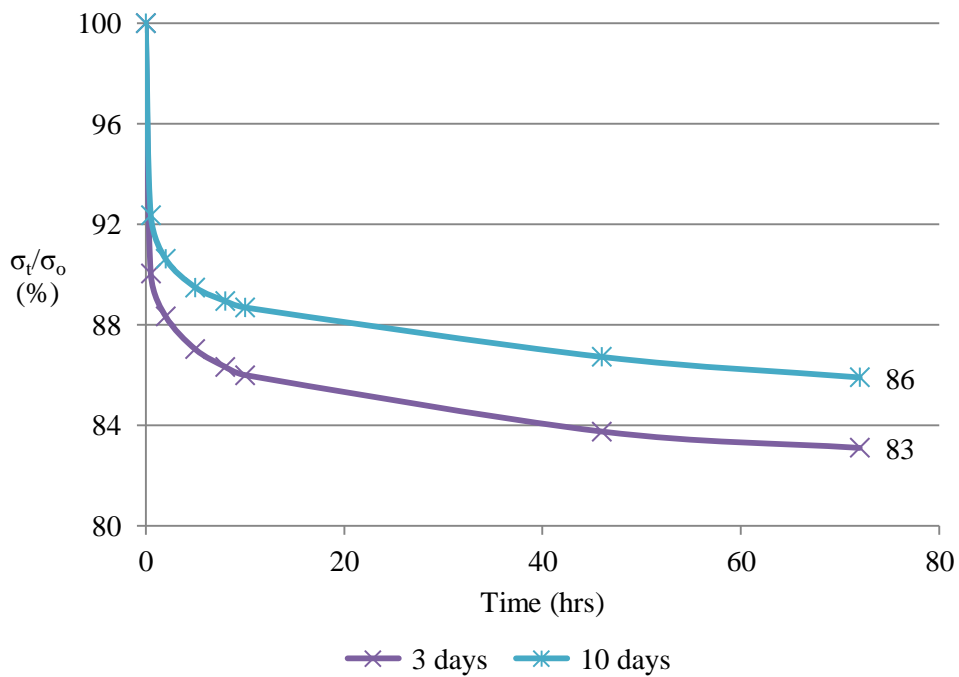


Figure E4: Influence of age of loading on development of relaxation of 0.55 w/c ratio mix

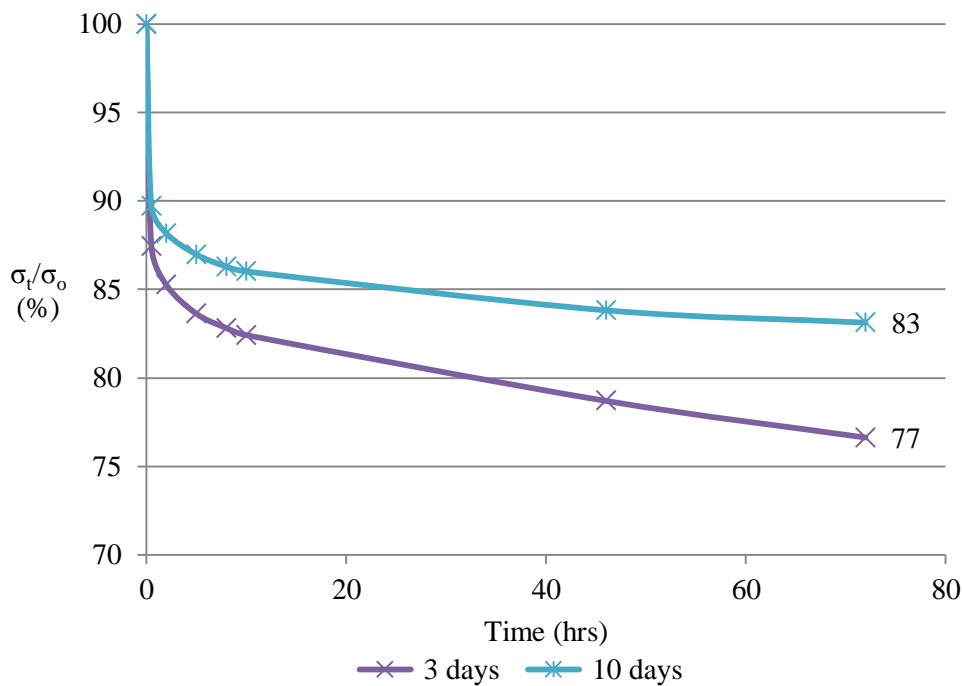


Figure E5: Influence of age of loading on development of relaxation of 0.65 w/c ratio mix

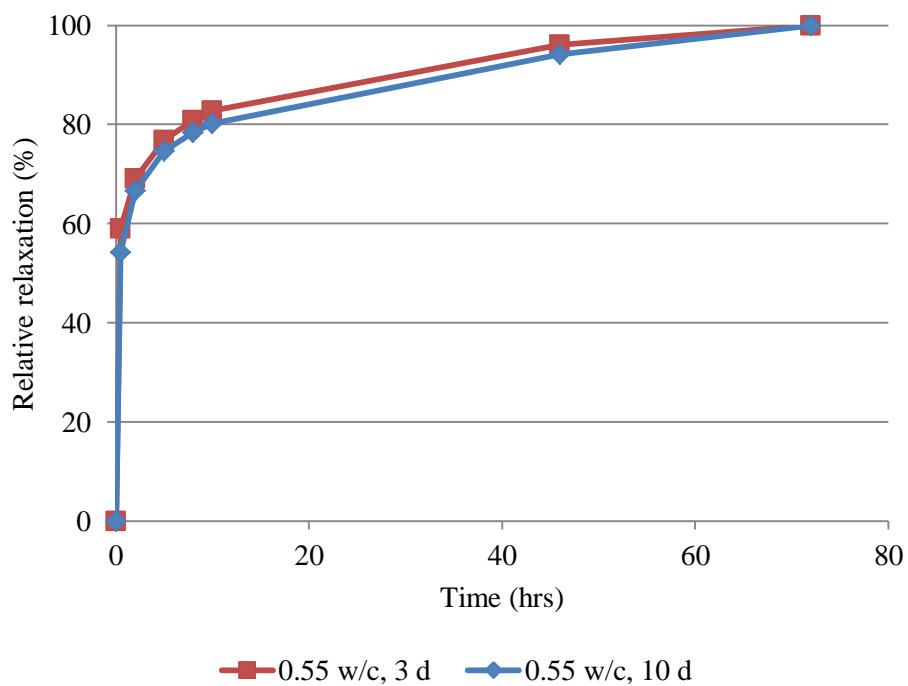


Figure E6: Influence of age on rate of development of relaxation of 0.55 w/c ratio mix

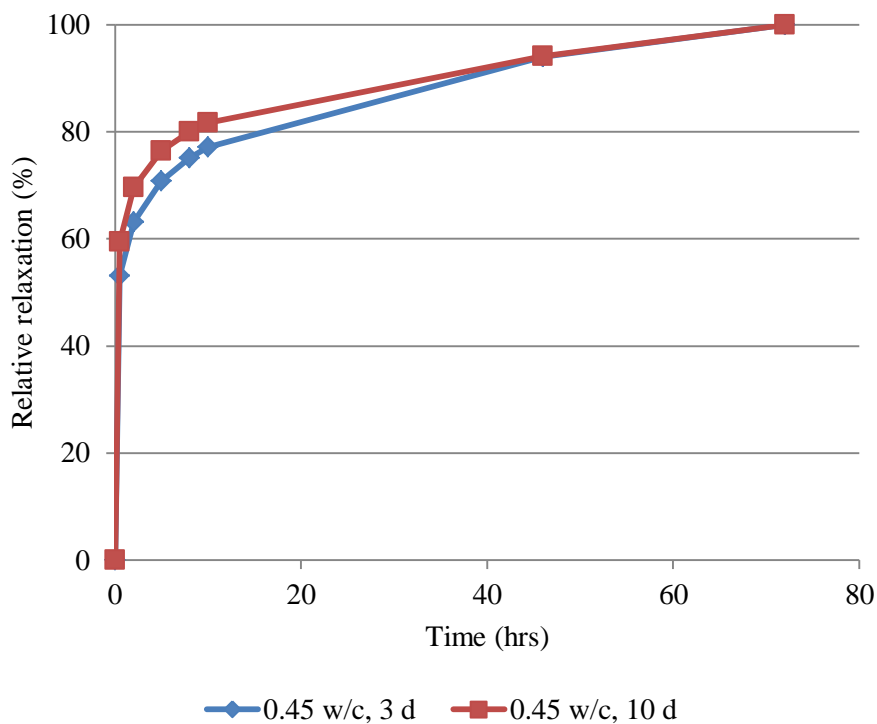


Figure E7: Influence of age on rate of development of relaxation of 0.45 w/c ratio mix

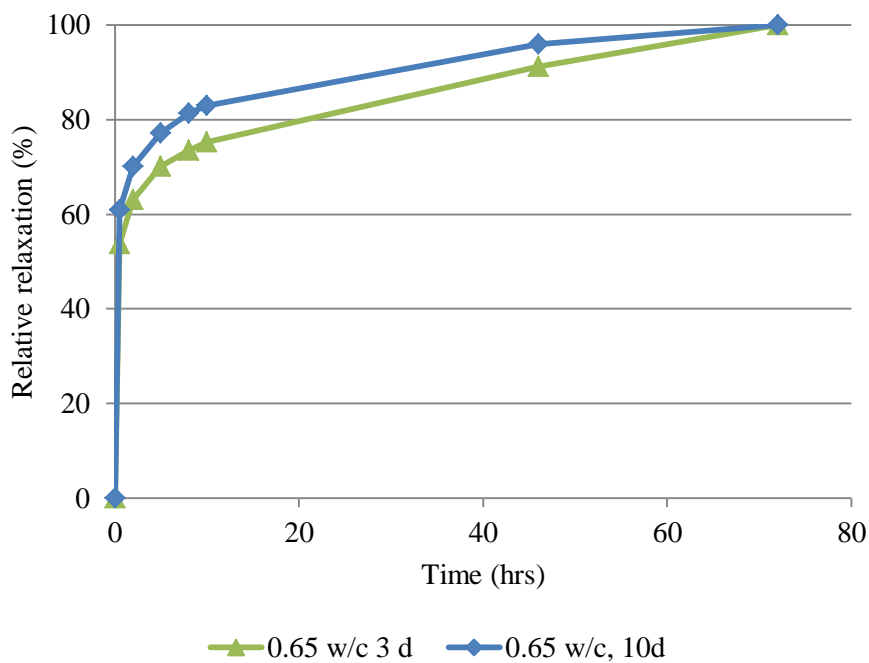


Figure E8: Influence of age on rate of development of relaxation of 0.65 w/c ratio mix

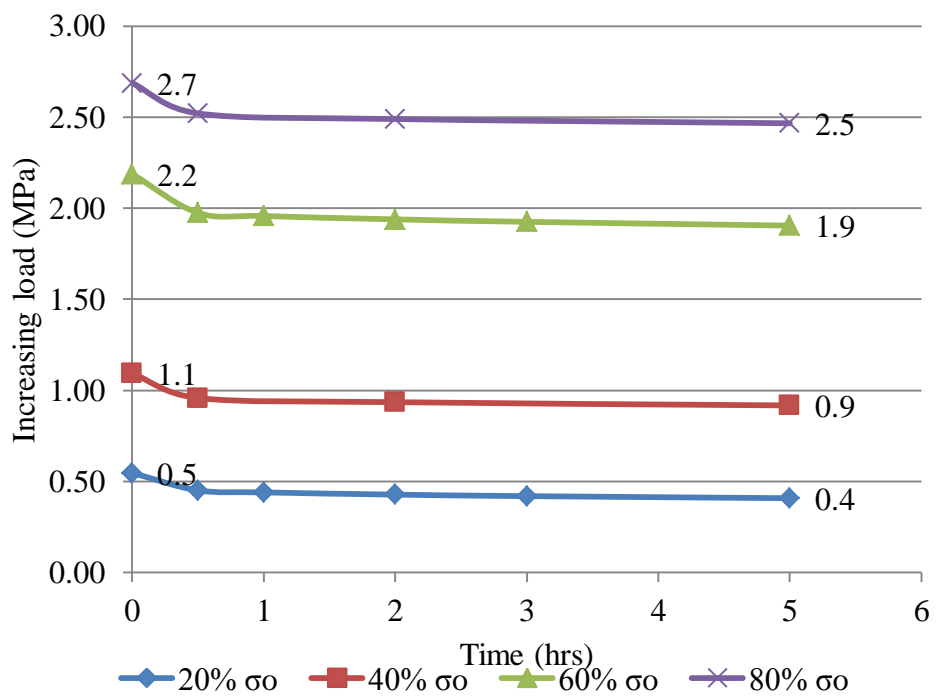


Figure E9: Influence of initial stress-strength ratio on 3-day relaxation of 0.45 w/c ratio mix

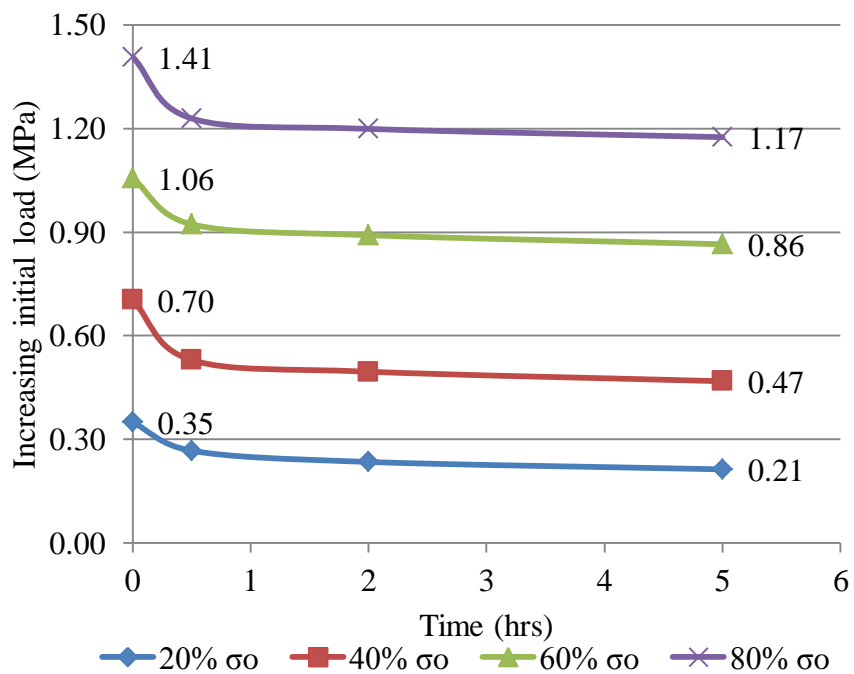


Figure E10: Influence of initial stress-strength ratio on 3-day relaxation of 0.65 w/c ratio mix

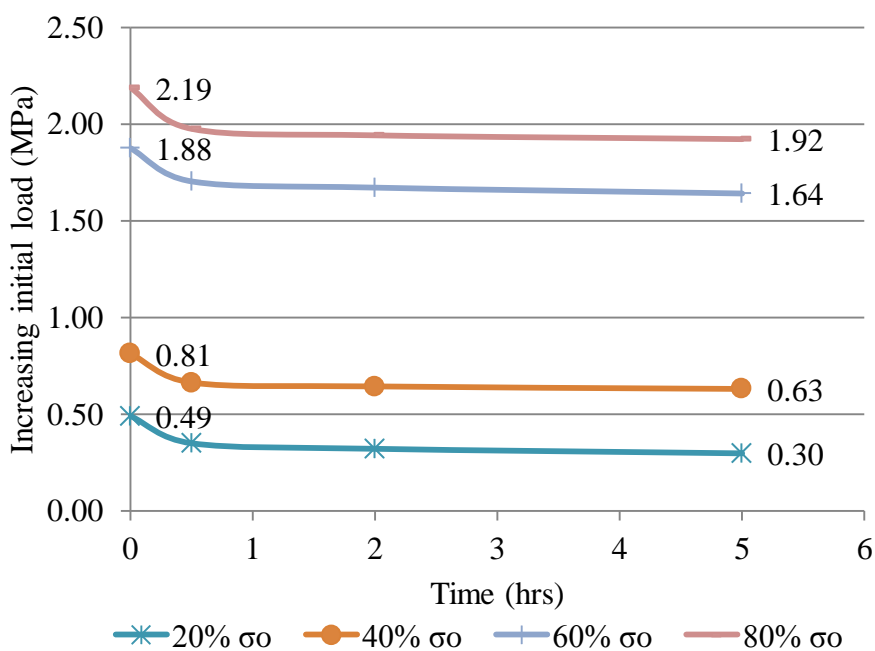


Figure E11: Influence of initial stress-strength ratio on 10-day relaxation of 0.65 w/c ratio mix

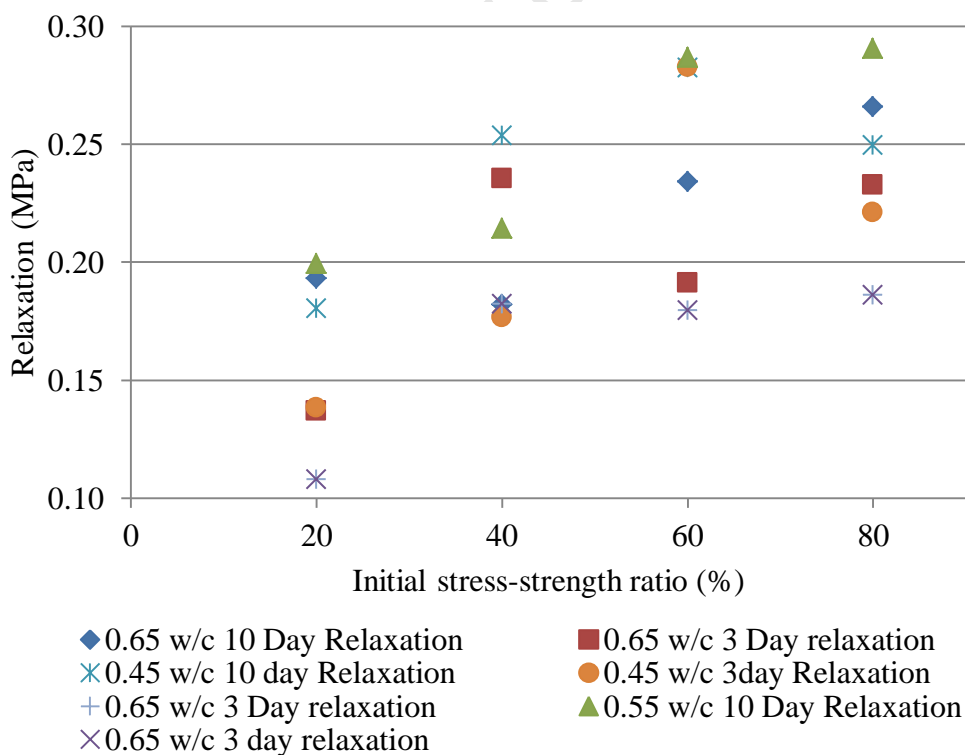


Figure E12: Influence of initial stress-strength ratio on 5-hour relaxation

**Scheduling of relaxation tests**

Tensile and relaxation samples were cast on Tuesdays and Fridays of every week. Both 3 day and 10 day specimen were cast on each day of casting. To avoid having to cast small mixes, only one w/c ratio mix was cast on each round of casting. This was done in order to have specimen from the same batch tested for the different experimental variables. Samples cast on Tuesday were tested for three day tensile strength on Friday. Thereafter, similar specimens were tested for relaxation after ascertaining their tensile strength.

On Monday different mix of samples, cast on Friday were tested for both 3 day tensile strength and relaxation. Relaxation experiments for these samples went on up to Thursday. On Friday of the second week, samples cast on Tuesday of the first week were tested for 10 day tensile strength and relaxation. While samples cast on Friday of the first week were tested for 10 day tensile strength and relaxation on Monday of the third week.

Specimens were cast on Tuesday and Friday of every other week, meaning there was no casting in the second week of each cycle. The scheduling of tests is shown schematically in Table E1. On average, 3 samples were testes for each mix and age of loading. In total over 100 samples were cast for both tensile and relaxation tests.

Table E1: Scheduling of relaxation tests

Task Name	Duration	Start	Finish	MON	TUE	WED	THU	FRI	SAT	SUN
CA 3DR & 10DR (1)	1 day	Tue	Tue							
CA 3DTT & 10TT (1)	1 day	Tue	Tue							
T 3DTT (1)	1 day	Fri	Fri							
T 3DR (1)	3 days	Fri	Mon							
CA 3DR & CA 10DR (2)	1 day	Fri	Fri							
CA 3DTT & CA 10DTT (2)	1 day	Fri	Fri							
<b>Week 2</b>										
T 3DTT (2)	1 day	Mon	Mon							
T 3DR (2)	3 days	Mon	Thu							
T 10DTT (1)	1 day	Fri	Fri							
T 10DR (1)	3 days	Fri	Mon							
<b>Week 3</b>										
T 10DTT (2)	1 day	Mon	Mon							
T 10DR (2)	3 days	Mon	Thu							

**Legend:**

CA: Casting

T: Testing

3DR: 3 days relaxation; 10DR: 10 day's relaxation

3DTT: 3 days tensile tests; 10DTT: 10 days tensile tests

(1): indicates the mix number i.e. mix 1

Therefore, CA 10DR and CA 10DTT (2) means casting of 10 day relaxation samples and casting of 10 day tensile test samples all of mix 2. At the end of each cycle (i.e. after all the 3 mixes have been tested), the tests were repeated in order to obtain average values

## Appendix F: Shrinkage models

### 1. CEB MC90-99

Mean shrinkage strain the cross section  
(Drying shrinkage + Autogenous shrinkage)

$$\varepsilon_{sh}(t - t_0) = -[\varepsilon_{sd}(t - t_0) + \varepsilon_{sa}(t)]$$

Drying shrinkage

$$\varepsilon_{sd}(t - t_0) = \varepsilon_{sd,0}S(t - t_0)$$

Autogenous shrinkage

$$\varepsilon_{sa}(t) = \varepsilon_{sa,0}S^*(t)$$

Basic drying shrinkage strain

$$\varepsilon_{sd,0} = (220 + (110 \cdot 4)) \exp\left(-0.12 \frac{f_c}{10MPa}\right) \cdot 10^{-6} k_h$$

Notional autogenous shrinkage

$$\varepsilon_{sa,0} = 700 \left[ \frac{f_c / 10MPa}{6 + f_c / 10MPa} \right]^{2.5} \cdot 10^{-6}$$

Time dependence

$$S(t - t_0) = \frac{(t - t_0)}{\sqrt{350 \left[ \frac{(V/S)}{50mm} \right]^2 + (t - t_0)}}$$

Humidity dependence

$$k_h = 1.55 [1 - h^3] \quad \text{for } 0.4 \leq h \leq 0.99 \cdot \left( \frac{3.5 \cdot 10MPa}{f_c} \right)^{0.1}$$

$$= 0.25 \quad \text{for } h \geq 0.99 \cdot \left( \frac{3.5 \cdot 10MPa}{f_c} \right)^{0.1}$$

Time dependence

$$S^*(t) = 1 - \exp[-0.2\sqrt{t}]$$

## 2. RILEM B3

Mean shrinkage strain in the cross section  $\varepsilon_{sh}(t-t_0) = -[\varepsilon_{sh\infty} k_h S(t-t_0)]$

Ultimate shrinkage  $\varepsilon_{sh\infty} = \varepsilon_{s\infty} \frac{E(607)}{E(t_0 + \tau_{sh})}$

Humidity dependence  $k_h = \left\{ \begin{array}{ll} 1-h^3 & h \leq 0.98 \\ -0.2 & h = 1 \\ \text{linear interpolation} & 0.98 \leq h \leq 1 \end{array} \right\}$

Time dependence  $S(t-t_0) = \tanh \sqrt{\frac{t-t_0}{\tau_{sh}}}$

Shrinkage  $\varepsilon_{sh\infty} = \alpha_1 \alpha_2 [0.019 w^{2.1} f_c^{-0.28} + 270]$

Elastic modulus at age t  $E(t) = E(28) \sqrt{\frac{t}{4 + 0.85t}}$

$\alpha_1 = 1.0$  for CEM I  
 $0.85$  for CEM II  
 $1.1$  for CEM III

$\alpha_2 = 0.75$  for steam curing  
 $1.2$  for sealed curing in air

$1.1$  for curing in water or at 100% relative humidity

Size dependence  $\tau_{sh} = k_t (k_s D)^2$

$k_t = 0.085 t_0^{-0.08} f_c^{-0.25}$

Effective cross-sectional thickness  $D = 2 \frac{V}{S}$

$k_s = 1.0$  infinite slab

Cross sectional shape factor

$1.15$  infinite cylinder

$1.25$  infinite square prism

$1.30$  sphere

### 3. GL 2001(4)

Mean shrinkage strain in the cross section	$\varepsilon_{sh}(t-t_0) = -\varepsilon_{sh\infty} k_h S(t-t_0)$
Ultimate shrinkage	$\varepsilon_{sh\infty} = 900\alpha_5 \sqrt{\frac{30MPa}{f_c}} \cdot 10^{-6}$
Humidity dependence	$k_h = 1 - 1.18h^4$
Time dependence	$S(t-t_0) = \sqrt{\frac{(t-t_0)}{0.12(V/S)^2 + (t-t_0)}}$
Cement type factor	$\alpha^3 = 1$ for CEM I 0.75 for CEM II 1.15 for CEM III
28 day standard cylinder compressive strength (MPa)	$f_c$
Age of concrete (days)	$t$
Age at first drying of concrete (days)	$t_0$
Specimen surface area (mm <sup>2</sup> )	$S$
Specimen volume (mm <sup>3</sup> )	$V$
Relative humidity	$h$

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## Appendix G: Creep and shrinkage models: example calculations

### 1. CEB MC90-99

CEB MC90-99 Model					
<b>Problem Data</b>					
<i>Concrete Data:</i>		SI UNITS	US CUSTOMARY UNITS		
Specified 28 days strength	$f'_c =$	51.0	MPa	7397	psi
<i>Ambient Conditions:</i>					
Relative humidity:	$h =$	0.5		0.5	
Temperature	$T =$	22	°C	71.6	°F
<i>Specimen:</i>					
Volume-to surface ratio	$V/S =$	26	mm	1.0	in
Shape					
<i>Initial Curing:</i>					
Curing time	$t_c =$	3	days	3	days
Curing condition			Moist cured		Moist cured
<i>Concrete at loading:</i>					
Age at loading	$t_o =$	3	days	3	days
Applied stress range	$k_s =$	33	%	33	%
<b>SOLUTION</b>					
Estimated Concrete Properties					
Mean 28 days strength	$f_{cm28} =$	<b>59.0</b>	MPa	<b>8557</b>	psi
Strength constant	$f_{cmo} =$	<b>10.0</b>	MPa	<b>1450</b>	psi

<b>Mean 28 days elastic modulus</b>	Ecm28 =	<b>38850</b>	MPa	<b>5635111</b>	psi
Estimated Concrete Mixture					
Cement type		N		N	
Maximum aggregate size		10	mm	3/4	in.
Cement content	c =	400	kg/m3	674	lb/yd3
Water content	w =	180	kg/m3	303	lb/yd3
Water-cement ratio	w/c =	0.450	(4-1)	0.450	(4-1)
Aggregate-cement ratio	a/c =	4.66		4.04	
Fine aggregate percentage	ψ =	57	%	57	%
Air content	α =	5.0	%	5	%
Slump	s =	130	mm	5.12	in
<b>Unit weight of concrete</b>	gc =	2445	kg/m3	4121	lb/yd3
1) CEB MC90 Shrinkage Strain					
		SI UNITS			
Cement type factor					
	bsc =	5	Table A.10		
Concrete strength factor	$\epsilon_s(f_{cm28}) = [160 + 10 \cdot b_{sc} \cdot (f_c^0 - f_{cm28}/f_{cm0})] \cdot 10^{-6}$				
	$\epsilon_s(f_{cm28}) =$	3.150E-04	(A-56)		
Ambient relative humidity factor	$b_{RH}(h) = -1.55 \cdot [1 - (h/h_0)^3]$ for $0.4 \leq h < 0.99$				
	$b_{RH}(h) = 0.25$ for $h \geq 0.99$				
	h <sub>0</sub> =	1			
	b <sub>RH</sub> (h) =	-1.356	(A-57)		
Notional shrinkage coefficient	$\epsilon_{cs0} = \epsilon_s(f_{cm28}) \cdot b_{RH}(h)$	(A-55)			
	$\epsilon_{cs0} =$	-4.272E-04	(A-55)		
Shrinkage time-function	$\epsilon_{sh}(t, t_c) = \epsilon_{cs0} \cdot b_s(t - t_c)$	(A-54)			

	$bs(t-tc) = \left\{ \frac{(t-tc)}{t1} \right\} / \left\{ 350 \cdot \left[ \frac{(V/S)}{(V/S)_0} \right]^2 + (t-tc) \right\}^{0.5}$	(A-58)		
	t1 =	1	day	
	(V/S) <sub>0</sub> =	50	mm	
	t (days)	bs(t-tc)	εsh(t,tc)	
	3	0.000	0.000E+00	
	10	0.262	-1.121E-04	
	17	0.359	-1.534E-04	
	24	0.426	-1.821E-04	
	38	0.520	-2.220E-04	
	52	0.584	-2.495E-04	
	76	0.660	-2.819E-04	
	90	0.692	-2.957E-04	
	100	0.711	-3.039E-04	

<b>2) CEB MC90-99 Shrinkage Strain</b>				
<b>2.1 Autogenous Shrinkage</b>				
Cement type factor	aas =	700	Tabla A.11	
Notional autogenous shrinkage	$\epsilon_{caso}(fcm28) = -aas \cdot \left[ \frac{(fcm28/fcmo)}{6 + (fcm28/fcmo)} \right]^{2.5} \cdot 10^{-6}$		(A-63)	
	$\epsilon_{caso}(fcm28) =$	-1.212E-04	(A-63)	
Autogenous shrinkage time-function	$\epsilon_{cas}(t) = \epsilon_{caso}(fcm28) \cdot bas(t)$		(A-62)	
	$bas(t) = 1 - \exp[-0.2 \cdot (t/ti)^{0.5}]$		(A-64)	
	t1 =	1	day	

	t (days)	bas(t)	εcas(t)		
	0	0.000	0.000E+00		
	3	0.293	-3.547E-05		
	10	0.469	-5.679E-05		
	17	0.562	-6.804E-05		
	24	0.625	-7.568E-05		
	38	0.709	-8.585E-05		
	52	0.764	-9.252E-05		
	76	0.825	-9.997E-05		
	90	0.850	-1.030E-04		
	100	0.865	-1.048E-04		
<b>2.2 Drying Shrinkage</b>					
<b>Cement type factors</b>					
	ads1 =	4	Table A.11		
	ads2 =	0.12	Table A.11		
Notional drying shrinkage coefficient	ecdso(fcm28) = [(220+110·ads1)·exp(- ads2·fcm28/fcmo)]·10 <sup>-6</sup>	(A-66)			
	ecdso(fcm28) =	3.251E-04	(A-66)		
<b>Ambient relative humidity factor</b>					
	ho =	1			
	bs1 = [3.5·fcmo/fcm28]0.1 £ 1.0				
	bs1 =	0.949	(A-69)		
	bRH(h) = -1.55·[1- (h/ho) <sup>3</sup> ] for 0.4 £ h < 0.99·bs1				
	bRH(h) = 0.25 for h ≥ 0.99·bs1				

	$bRH(h) =$	-1.356	(A-67)		
Drying shrinkage time-function	$\epsilon_{cds}(t,tc) = \epsilon_{cdso}(f_{cm28}) \cdot bRH(h) \cdot bds(t-tc)$	(A-65)			
	$bds(t-tc) = [ \{ (t-tc)/t_1 \} / \{ 350 \cdot [ (V/S)/(V/S)_0 ]^2 + (t-tc)/t_i \} ]^{0,5}$	(A-68)			
	$t_1 =$	1	day		
	$(V/S)_0 =$	50	mm		
	$t$ (days)	$bds(t-tc)$	$\epsilon_{cds}(t,tc)$		
	3	0.000	0.000E+00	0	
	10	0.262	-1.157E-04	116	
	17	0.359	-1.583E-04	158	
	24	0.426	-1.879E-04	188	
	38	0.520	-2.291E-04	229	
	52	0.584	-2.576E-04	258	
	76	0.660	-2.910E-04	291	
	90	0.692	-3.052E-04	305	
		#NUM!	#NUM!	#NUM!	
2.3 Total Shrinkage					
	$\epsilon_{sh}(t,tc) = \epsilon_{cas}(t) + \epsilon_{cds}(t,tc)$			(A-61)	
	$t$ (days)	$\epsilon_{cas}(t)$	$\epsilon_{cds}(t,tc)$	$\epsilon_{sh}(t,tc)$	
	0	0.000E+00		0.000E+00	
	3	-3.547E-05	0.000E+00	-3.547E-05	
	10	-5.679E-05	-1.157E-04	-1.725E-04	
	17	-6.804E-05	-1.583E-04	-2.263E-04	

	24	-7.568E-05	-1.879E-04	-2.636E-04	
	38	-8.585E-05	-2.291E-04	-3.150E-04	
	52	-9.252E-05	-2.576E-04	-3.501E-04	
	76	-9.997E-05	-2.910E-04	-3.910E-04	
	90	-1.030E-04	-3.052E-04	-4.082E-04	
	100	-1.048E-04	#NUM!	#NUM!	

<b>3) Compliance</b>					
	<b>SI UNITS</b>				
<b>3.1 Elastic compliance</b>					
Cement type	N				
	s =	0.25	Tabla A.12		
Mean strength at age to	$be = \exp[s/2 \cdot \{1 - (28/to)^{0.5}\}]$				
	be =	0.773	(A-97)		
	$fc_{mto} = be \cdot f_{cm28}$				
	$fc_{mto} =$	35.3	MPa	(A-96)	
Mean elastic modulus at age to	$E_{cmto} = E_{cm28} \cdot \exp[s/2 \cdot \{1 - (28/to)^{0.5}\}]$				
	$E_{cmto} =$	30049	MPa	(A-71)	
Elastic compliance	$J(t_0, t_0) = 1/E_{cmto}$			(A-70)	
	$J(t_0, t_0) =$	3.328E-05	1/MPa	(A-70)	
Effect of temperature on modulus of elasticity	$E_{cm28}(T) = E_{cm28} \cdot (1.06 - 0.003 \cdot T/T_0)$			(A-85)	
	$E_{cm28}(T) =$	38617	MPa	(A-85)	
	$E_{cmto}(T) = E_{cmto} \cdot (1.06 - 0.003 \cdot T/T_0)$			(A-85)	

	$E_{cmto}(T) =$	29868	MPa	(A-85)	
Elastic compliance temperature adjusted	$J(t_0, t_0) = 1/E_{cmto}$				
	$J(t_0, t_0) =$	3.348E-05	1/MPa	(A-70)	
<b>3.2 Creep Coefficient</b>					
Compressive strength factors	$a_1 = [3.5 \cdot f_{cmo}/f_{cm28}]^{0.7}$				
	$a_2 = [3.5 \cdot f_{cmo}/f_{cm28}]^{0.2}$				
	$a_1 =$	0.694	(A-79)		
	$a_2 =$	0.901	(A-79)		
Ambient relative humidity and volume-surface ratio factor	$fRH(h) = [1 + \{(1-h/h_0) \cdot a_1 / (0.1 \cdot (V/S)/(V/S)_0)\}] \cdot a_2$			(A-76)	
	$h_0 =$	1			
	$(V/S)_0 =$	50	mm		
	$fRH(h) =$	1.738	(A-76)		
Concrete strength factor	$b(f_{cm28}) = 5.3 / (f_{cm28}/f_{cmo})^{0.5}$			(A-77)	
	$b(f_{cm28}) =$	2.182	(A-77)		
Temperature adjusted age of loading	$t_{0,T} = SDt_i \times \exp[13.65 - 4000 / \{273 + (T(Dt_i)/T_0)\}]$			(A-87)	
	$T_0 =$	1	°C		
	$t_{0,T} =$	3.3	days	(A-87)	
	$t_0 = t_{0,T} \cdot [9 / \{2 - (t_0/T_1, T)^{1.2}\} + 1]^a \geq 0.5$ days			(A-81)	
	$a =$	0			
	$t_1, T =$	1	day		
	$t_0 =$	3.3	days	(A-81)	
Adjusted age of loading factor	$b(t_0) = 1 / [0.1 + (t_0/t_1)^{0.2}]$			(A-78)	
	$b(t_0) =$	0.731	(A-78)		
Notional creep coefficient	$f_0 = fRH(h) \cdot b(f_{cm28}) \cdot b(t_0)$		(A-75)		
	$f_0 =$	2.771	(A-75)		

Effect of temperature conditions	$f_T = \exp[0.015 \cdot (T/T_o - 20)]$	(A-91)		
	$f_T =$	1.030	(A-91)	
	$f_{RH,T} = f_T + [f_{RH}(h)-1] \cdot f_T^{1.2}$		(A-90)	
	$f_{RH,T} =$	1.796	(A-90)	
	$f_o = f_{RH,T} \cdot b(f_{cm28}) \cdot b(t_o)$		(A-75)	
	$f_o =$	2.863	(A-75)	
Effect of high stresses	$f_{o,k} = f_o \cdot \exp[1.5 \cdot (k_s - 0.4)]$	(A-93)		
	$f_{o,k} =$	1.000	(A-93)	
Notional creep coefficient temperature and stress adjusted	$f_o = f_{ck}$			
	$f_o =$	1.000		
Creep coefficient time-function	$a_3 = [3.5 \cdot f_{cmo}/f_{cm28}]^{0.5}$	(A-84)		
	$a_3 =$	0.770	(A-84)	
	$b_H = 150 \cdot [1 + (1.2 \cdot h/h_o)^{18}] \cdot (V/S)/(V/S)_o + 250 \cdot a_3 \leq 1500 \cdot a_3$			
	$b_H =$	270.560	(A-83)	

Effect of temperature conditions	$b_T = \exp[1500/(273+T/T_o)-5.12]$	(A-89)		
	$b_T =$	0.965	(A-89)	
	$b_{H,T} = b_H \cdot b_T$		(A-88)	
	$b_{H,T} =$	261.188	(A-88)	
	$\Delta f_{T,trans} = 0.0004 \cdot (T/T_o - 20)^2$		(A-92)	
	$\Delta f_{T,trans} =$	0.002	(A-92)	

Creep coefficient time function	$bc_{(t-t_0)} = [(t-t_0)/t_1 / \{bH + (t-t_0)/t_1\}]^{0.3}$	(A-82)		
	$t_{(days)}$	$bc(t-t_0)$	$bc(t-t_0)$ Temp. Adj.	
	3	0.000	0.000	
	10	0.332	0.335	
	17	0.405	0.409	
	24	0.454	0.459	
	38	0.522	0.527	
	52	0.570	0.575	
	76	0.628	0.634	
	90	0.654	0.660	
		#NUM!	#NUM!	
Creep Coefficient	$f_{28(t, t_0)} = f_0 \cdot bc(t-t_0)$		(A-74)	
	$t_{(days)}$	$f_{28(t, t_0)}$		
	3	0.000		
	10	0.919		
	17	1.123		
	24	1.259		
	38	1.447		
	52	1.579		
	76	1.741		
	90	1.814		
	0	#NUM!		
Creep Coefficient Temperature and Stress Adjusted	$f_{28(t, t_0, T)} = f_0 \cdot bc(t-t_0) + \Delta f_{T, trans}$		(A-86)	
	$t_{(days)}$	$f_{28(t, t_0, T)}$		
	3	0.002		
	10	0.961		
	17	1.173		
	24	1.315		
	38	1.510		
	52	1.647		

	76	1.815			
	90	1.890			
	0	#NUM!			
3.3 Compliance	$J_{(t,to)} = 1/E_{cm}t_0 + f_{28}(t,to)/E_{cm}28$		(A-70)		
	$t_{(days)}$	J(t <sub>0</sub> ,t <sub>0</sub> )	$f_{28}(t,to)/E_{cm}28$	J(t <sub>0</sub> ,t <sub>0</sub> ) (1/MPa)	
	3	3.328E-05	0.000E+00	3.328E-05	0.33
	10	3.328E-05	2.365E-05	5.693E-05	0.57
	17	3.328E-05	2.890E-05	6.218E-05	0.62
	24	3.328E-05	3.240E-05	6.568E-05	0.66
	38	3.328E-05	3.724E-05	7.052E-05	0.71
	52	3.328E-05	4.064E-05	7.392E-05	0.74
	76	3.328E-05	4.482E-05	7.810E-05	0.78
	90	3.328E-05	4.668E-05	7.996E-05	0.80
	0				
Compliance Temperature and Stress Adjusted	$J_{(t,to)} = 1/E_{cm}t_0(T) + f_{28}(t,to)/E_{cm}28(T)$		(A-70)		
	$t_{(days)}$	J(t <sub>0</sub> ,t <sub>0</sub> )	$f_{28}(t,to)/E_{cm}28$	J(t <sub>0</sub> ,t <sub>0</sub> ) (1/MPa)	
	3	3.348E-05	4.143E-08	3.352E-05	0.34
	10	3.348E-05	2.487E-05	5.835E-05	0.58
	17	3.348E-05	3.038E-05	6.386E-05	0.64
	24	3.348E-05	3.405E-05	6.753E-05	0.68
	38	3.348E-05	3.911E-05	7.259E-05	0.73
	52	3.348E-05	4.266E-05	7.614E-05	0.76
	76	3.348E-05	4.701E-05	8.049E-05	0.80
	90	3.348E-05	4.895E-05	8.243E-05	0.82

## 2. RILEM B3

<b>Bazant-Baweja B3 Model</b>					
<b>Problem Data</b>					
<b>Concrete Data:</b>		<b>SI UNITS</b>		<b>US CUSTOMARY UNITS</b>	
Specified 28 days strength	$f'_c =$	51.5	MPa	7469	psi
<b>Ambient Conditions:</b>					
Relative humidity:	$h =$	0.5		0.5	
Temperature	$T =$	23	°C	73.4	°F
<b>Specimen:</b>					
Volume-to surface ratio	$V/S =$	26	mm	1.0	in
Shape		infinite cylinder		infinite cylinder	
<b>Initial Curing:</b>					
Curing time	$t_c =$	3	days	3	days
Curing condition		Moist cured		Moist cured	
<b>Concrete at loading:</b>					
Age at loading	$t_o =$	3	days	3	days
Applied stress range	$ks =$	33	%	33	%
<b>SOLUTION</b>					
Estimated Concrete Properties					
Mean 28 days strength	$f_{cm28} =$	<b>59.8</b>	MPa	<b>8673</b>	psi
Mean 28 days elastic modulus	$E_{cm28} =$	<b>36608</b>	MPa	<b>5308428</b>	psi

Estimated Concrete Mixture					
Cement type		<b>I</b>		<b>I</b>	
Maximum aggregate size		<b>10</b>	mm	<b>3/4</b>	in.
Cement content	c =	<b>400</b>	kg/m <sup>3</sup>	<b>674</b>	lb/yd <sup>3</sup>
Water content	w =	<b>180</b>	kg/m <sup>3</sup>	<b>303</b>	lb/yd <sup>3</sup>
Water-cement ratio	w/c =	<b>0.450</b>	(4-1)	<b>0.450</b>	(4-1)
Aggregate-cement ratio	a/c =	<b>4.66</b>		<b>4.04</b>	
Fine aggregate percentage	ψ =	<b>57</b>	%	<b>57</b>	%
Air content	α =	<b>2</b>	%	<b>2</b>	%
Slump	s =	<b>130</b>	mm	<b>5.12</b>	in
Unit weight of concrete	gc =	<b>2445</b>	kg/m <sup>3</sup>	<b>4121</b>	lb/yd <sup>3</sup>

<b>1) Shrinkage Strain</b>					
	<b>SI UNITS</b>				
Ambient relative humidity factor	kh = -0.2 if h = 1				
	kh = 12.74-12.94·h if 0.98 < h < 1				
	kh = 1-h <sup>3</sup> if h ≤ 0.98				
	kh =	0.875	Table A.6		
Cement type factor	a1 =	1.000	Table A.7		
Curing condition factor	a2 =	1.000	Table A.8		
Nom. ultimate shrinkage	ε <sub>sγ</sub> = -a1·a2·[0.019·w <sub>2.1</sub> ·f <sub>cm28-0.28+270</sub> ]·10 <sup>-6</sup>				
	ε <sub>sγ</sub> =	-5.991E-04	(A-33)		
Member shape factor					

	ks =	1.150	Table A.9		
Shrinkage half-time	$tsh = 0.085 \cdot tc - 0.08 \cdot f_{cm} \cdot 28 - 0.25 \cdot [2 \cdot ks \cdot (V/S)]^2$				
	tsh =	100.110	(A-36)		
Time dependence factor	$E_{cm607}/E_{cm(tc+tsh)} = 1.0805 / [(tc+tsh)/(4+0.85 \cdot (tc+tsh))]^{0.5}$				
	$E_{cm607}/E_{cm(tc+tsh)}$	1.019	(A-32 & A-34)		
Ultimate shrinkage strain	$\epsilon_{sh\infty} = -\epsilon_{s\infty} \cdot E_{cm607}/E_{cm(tc+tsh)}$				
	$\epsilon_{sh\infty}$	-6.103E-04	(A-32)		
Shrinkage time-function	$\epsilon_{sh}(t,tc) = -\epsilon_{sh\infty} \cdot kh \cdot \tanh[(t - tc)/tsh]^{0.5}$				
	$S(t-tc) = \tanh[(t - tc)/tsh]^{0.5}$			(A-35)	
	t (days)	S(t-tc)	$\epsilon_{sh}(t,tc)$		
	3	0.000	0.000E+00	0	
	10	0.258	-1.380E-04	138	
	17	0.357	-1.909E-04	191	
	24	0.428	-2.288E-04	229	
	38	0.531	-2.835E-04	283	
	52	0.604	-3.226E-04	323	
	76	0.693	-3.701E-04	370	
	90	0.732	-3.907E-04	391	

<b>2) Compliance</b>	$J(t,t_0) = q_1 + C_0(t,t_0) + C_d(t,t_0,t_c)$			(A-37)	
	<b>SI UNITS</b>				
<b>2.1 Instantaneous compliance</b>	$q_1 = 1/E_0 = 0,6/E_{cm28}$				
	$q_1 =$	1.639E-05	1/MPa	(A-38)	
<b>2.2 Compliance function for basic creep</b>	$C_0(t,t_0) = q_2 \cdot Q(t,t_0) + q_3 \cdot \ln[1 + (t-t_0)^n] + q_4 \cdot \ln(t/t_0)$			(A-40)	
Ageing viscoelastic term	$q_2 = 185.4 \times 10^{-6} \times c_0.5 \times f_{cm28}^{-0.9}$				
	$q_2 =$	9.335E-05	1/MPa	(A-41)	
	$Q_f(t_0) = [0.086 \cdot (t_0)^{2/9} + 1.21 \cdot (t_0)^{4/9}]^{-1}$				
	$Q_f(t_0) =$	4.804E-01		(A-43)	
	$m =$	0.5			
	$r(t_0) = 1.7 \cdot (t_0)^{0.12} + 8$				
	$r(t_0) =$	9.940		(A-45)	
	$Z(t,t_0) = (t_0)^{-m} \cdot \ln[1 + (t-t_0)^n]$			(A-44)	
	$Q(t,t_0) = Q_f(t_0) [1 + \{Q_f(t_0)/Z(t,t_0)\}^{r(t_0)}]^{-1/r(t_0)}$			(A-42)	
	Ageing viscoelastic term				
	t (days)	Z(t,t <sub>0</sub> )	Q(t,t <sub>0</sub> )	$q_2 \cdot Q(t,t_0)$ (1/MPa)	
	3	0.000	0.000	0.000E+00	
	4	0.400	0.394	3.680E-05	
	17	0.481	0.449	4.187E-05	
	24	0.495	0.454	4.240E-05	
	38	0.512	0.460	4.296E-05	
	52	0.523	0.464	4.327E-05	
	76	0.537	0.467	4.358E-05	
	90	0.543	0.468	4.370E-05	

Nonageing viscoelastic term	$q3 = 0.29 \times (w/c)^4 \times q2$				
	q3 =	1.110E-06	1/MPa	(A-46)	
	n =	0.1			
	Nonageing viscoelastic term				
	t (days)	$\ln[1 + (t-t_0)^n]$	$q3 \cdot \ln[1 + (t-t_0)^n]$ 1/MPa		
	3	0.000	0.000E+00		
	10	0.795	8.827E-07		
	17	0.834	9.256E-07		
	24	0.857	9.512E-07		
	38	0.887	9.842E-07		
	52	0.907	1.006E-06		
	76	0.931	1.033E-06		
	90	0.941	1.045E-06		

Ageing flow term	$q4 = 20.3 \times 10^{-6} \times (a/c)^{-0.7}$				
	q4 =	6.910E-06	1/MPa	(A-47)	
	Ageing flow term				
	t (days)	$\ln(t/t_0)$	$q4 \cdot \ln(t/t_0)$ 1/MPa		
	3	0.000	0.000E+00		
	10	1.204	8.319E-06		
	17	1.735	1.199E-05		
	24	2.079	1.437E-05		
	38	2.539	1.754E-05		

	52	2.853	1.971E-05		
	76	3.232	2.233E-05		
	90	3.401	2.350E-05		
Compliance function for basic creep	$Co(t,t_0) = q_2 \cdot Q(t,t_0) + q_3 \cdot \ln[1 + (t-t_0)^n] + q_4 \cdot \ln(t/t_0)$			(A-40)	
	t (days)	$q_2 \cdot Q(t,t_0)$	$q_3 \cdot \ln[1 + (t-t_0)^n]$	$q_4 \cdot \ln(t/t_0)$	$Co(t,t_0)$ (1/MPa)
	3	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	10	4.078E-05	8.827E-07	8.319E-06	4.999E-05
	17	4.187E-05	9.256E-07	1.199E-05	5.478E-05
	24	4.240E-05	9.512E-07	1.437E-05	5.772E-05
	38	4.296E-05	9.842E-07	1.754E-05	6.149E-05
	52	4.327E-05	1.006E-06	1.971E-05	6.399E-05
	76	4.358E-05	1.033E-06	2.233E-05	6.695E-05
	90	4.370E-05	1.045E-06	2.350E-05	6.825E-05
<b>2.3 Compliance function for drying creep</b>	$Cd(t,t_0,t_c) = q_5 \cdot [\exp\{-8 \cdot H(t)\} - \exp\{-8 \cdot H(t_0)\}]^{0.5}$			(A-48)	
	$q_5 = 0.757 \cdot f_{cm28-1} \cdot \exp\{-106 \cdot \frac{t_0}{t_0+6}\}$				
	$q_5 =$	2.698E-04	1/MPa	(A-49)	
	$S(t_0-t_c) = \tanh[(t_0 - t_c)/t_{sh}]^{0.5}$				
	$S(t_0-t_c) =$	0.000E+00		(A-53)	
	$H(t_0) = 1 - (1-h) \cdot S(t_0-t_c)$				
	$H(t_0) =$	1.000E+00		(A-51)	
	$S(t-t_c) = \tanh[(t - t_c)/t_{sh}]^{0.5}$				
	$H(t) = 1 - (1-h) \cdot S(t-t_c)$				
	t (days)	$S(t-t_c)$	$H(t)$	$[\exp\{-8 \cdot H(t)\} - \exp\{-8 \cdot H(t_0)\}]^{0.5}$	$Cd(t,t_0,t_c)$ 1/MPa
	3	0.0000	1.0000	0.0000E+00	0.000E+00
	10	0.2584	0.8708	2.4652E-02	6.652E-06

	17	0.3575	0.8213	3.2651E-02	8.810E-06
	24	0.4285	0.7858	3.9069E-02	1.054E-05
	38	0.5308	0.7346	4.9684E-02	1.341E-05
	52	0.6041	0.6979	5.8514E-02	1.5788E-05
	76	0.6931	0.6534	7.0932E-02	1.9139E-05
	90	0.7316	0.6342	7.6975E-02	2.0769E-05
<b>2.4 Compliance</b>	$J(t,t_0) = q_1 + C_0(t,t_0) + C_d(t,t_0,t_c)$			(A-37)	
	t (days)	q1	C <sub>0</sub> (t,t <sub>0</sub> )	C <sub>d</sub> (t,t <sub>0</sub> ,t <sub>c</sub> )	J(t,t <sub>0</sub> ) (1/MPa)
	3	1.639E-05	0.000E+00	0.000E+00	1.639E-05
	10	1.639E-05	4.999E-05	6.652E-06	7.303E-05
	17	1.639E-05	5.478E-05	8.810E-06	7.998E-05
	24	1.639E-05	5.772E-05	1.054E-05	8.465E-05
	38	1.639E-05	6.149E-05	1.341E-05	9.129E-05
	52	1.639E-05	6.399E-05	1.579E-05	9.617E-05
	76	1.639E-05	6.695E-05	1.914E-05	1.025E-04
	90	1.639E-05	6.825E-05	2.077E-05	1.054E-04

## 3. GL 2001(4)

<b>GL2000 Model</b>					
<b><u>Problem Data</u></b>					
<b>Concrete Data:</b>					
Specified 28 days strength	$f'_c =$	49.0	MPa	7107	psi
<b>Ambient Conditions:</b>					
Relative humidity:	$h =$	0.5		0.5	
Temperature	$T =$	23	°C	73.4	°F
<b>Specimen:</b>					
Volume-to surface ratio	$V/S =$	26	mm	1.0	in
<b>Initial Curing:</b>					
Curing time	$t_c =$	3	days	3	days
Curing condition		Moist cured		Moist cured	
<b>Concrete at loading:</b>					
Age at loading	$t_o =$	3	days	3	days
Applied stress range	$k_s =$	33	%	33	%
<b>SOLUTION</b>					
Estimated Concrete Properties					
Mean 28 days strength	$f_{cm28} =$	<b>58.9</b>	MPa	<b>8518</b>	psi
Mean 28 days elastic modulus	$E_{cm28} =$	<b>36501</b>	MPa	<b>5299105</b>	psi
Estimated Concrete Mixture					
Cement type		I		I	
Maximum aggregate size		10	mm	3/4	in.
Cement content	$c =$	400	kg/m <sup>3</sup>	674	lb/yd <sup>3</sup>
Water content	$w =$	180	kg/m <sup>3</sup>	303	lb/yd <sup>3</sup>

Water-cement ratio	w/c =	0.450	(4-1)	0.450	(4-1)
Aggregate-cement ratio	a/c =	4.66		4.04	
Fine aggregate percentage	$\psi$ =	57	%	57	%
Air content	$\alpha$ =	2.0	%	2	%
Slump	s =	130	mm	5.12	in
Unit weight of concrete	gc =	2445	kg/m <sup>3</sup>	4121	lb/yd <sup>3</sup>

<b>1) Shrinkage Strain</b>					
<b>SI UNITS</b>					
Cement type factor					
	k =		1.000	Tabla A. 14	
Ultimate shrinkage strain	$\epsilon_{shu} =$ $900 \cdot k \cdot [30/f_{cm28}]^{0.5} \cdot 10^{-6}$				
	$\epsilon_{shu} =$		6.423E-04	(A-99)	
Ambient relative humidity factor	b(h) = (1-1.18·h <sup>4</sup> )				
	b(h) =		0.926	(A-100)	
Shrinkage time-function	b(t-tc)=[(t-tc)/(t-tc+0.12(V/S) <sup>2</sup> )] <sup>0.5</sup>			(A-101)	
	$\epsilon_{sh}(t,tc) = \epsilon_{shu} \cdot b(h) \cdot b(t-tc)$			(A-98)	
	t (days)	b(t-tc)	$\epsilon_{sh}(t,tc)$		
	3	0.000	0.000E+00	0	
	10	0.282	1.677E-04	168	
	17	0.384	2.282E-04	228	
	24	0.453	2.698E-04	270	
	38	0.549	3.266E-04	327	
	52	0.614	3.651E-04	365	
	76	0.688	4.095E-04	409	
	90	0.719	4.280E-04	428	

	100	0.738	4.390E-04	439		
2) Compliance						
	SI UNITS					
2.1 Elastic compliance						
Cement type	I					
	s =		0.335	Tabla A.14		
Mean strength at age to	$be = \exp[\frac{s}{2} \{ 1 - (28/t_0)^{0.5} \}]$	(A-97)				
	$fc_{m,t_0} = be^2 * fc_{m,28}$		(A-96)			
	be =	0.709	(A-97)			
	$fc_{m,t_0} =$		29.6	MPa	(A-96)	
Mean elastic modulus at age to	$E_{c,m,t_0} (MPa) = 3500 + 4300 * (fc_{m,t_0})^{0.5}$					
	$E_{c,m,t_0} =$	26890	MPa	(A-95)		
Elastic compliance	$J(t_0, t_0) = 1/E_{c,m,t_0}$					
	$J(t_0, t_0) =$	3.719E-05	1/MPa	(A-102)		
2.2 Creep Coefficient	$f_{28}(t, t_0)$					
	Effect of drying before loading factor					
	$F(tc) =$	1.000	(A-104) & (A-105)			
	Basic creep					
	1st term	$2 * [(t-t_0)^{0.3} / \{(t-t_0)^{0.3} + 14\}]$				
	2nd term	$[7/t_0] * 0.5 * [(t-t_0) / \{(t-$				

	t (days)	1st term	2nd term	Basic creep
	3	0.000	0.000	0.000
	10	0.227	1.080	1.307
	17	0.272	1.247	1.520
	24	0.302	1.323	1.625
	38	0.344	1.394	1.738
	52	0.373	1.429	1.802
	76	0.411	1.459	1.870
	90	0.429	1.470	1.898

<b>Drying creep</b>				
Ambient relative humidity factor =	$2.5*(1 - 1.086 \cdot h^2)$			
		1.821		
Time function =	$f(t,t_0) = [(t-t_0)/\{(t-t_0)+0.12*(V/S)^2\}]^{0.5}$			
	t (days)	f(t,t <sub>0</sub> )	Drying creep 3rd term	
	3	0.000	0.000	
	10	0.282	0.513	
	17	0.384	0.699	
	24	0.453	0.826	
	38	0.549	1.000	
	52	0.614	1.118	
	76	0.688	1.253	
	90	0.719	1.310	
Creep coefficient	$f_{28}(t,t_0) = F(tc) \cdot [\text{Basic} + \text{Drying creep}]$		(A-103)	
	t (days)	Basic + Drying creep	$f_{28}(t,t_0)$	
	3	0.000	0.000	

	10	1.820	1.820		
	17	2.218	2.218		
	24	2.451	2.451		
	38	2.738	2.738		
	52	2.920	2.920		
	76	3.124	3.124		
	90	3.208	3.208		
2.3 Compliance					
	$J(t,t_0) = \frac{1}{E_{cm}t_0} + \frac{f_{28}(t,t_0)}{E_{cm}28}$		(A-102)		
	t (days)	J(t <sub>0</sub> ,t <sub>0</sub> )	f <sub>28</sub> (t,t <sub>0</sub> )/E <sub>cm</sub> 28	J(t,t <sub>0</sub> ) (1/MPa)	
	3	3.719E-05	0.000E+00	3.719E-05	0.37
	10	3.719E-05	4.987E-05	8.706E-05	0.87
	17	3.719E-05	6.077E-05	9.796E-05	0.98
	24	3.719E-05	6.715E-05	1.043E-04	1.04
	38	3.719E-05	7.501E-05	1.122E-04	1.12
	52	3.719E-05	8.000E-05	1.172E-04	1.17
	76	3.719E-05	8.558E-05	1.228E-04	1.23
	90	3.719E-05	8.790E-05	1.251E-04	1.25

**Appendix H: Evaluating relaxation model validity: AAEM and ARF results and tables**

Table H1: Approximate Relaxation Function (ARF) results

<b>APPROXIMATE RELAXATION FUNCTION (ARF)</b>															
Values that involve delta in relaxation function formula were derived from the MC9099 Model															
		<b>0.45 w/c 3 day loading</b>								<b>0.45 w/c 10 day loading</b>					
R (t,t0)	t	t0	J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	σt/σo	R (t,t0)	t	t0	J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	σt/σo
							(%)								(%)
<b>34672</b>	0	3	2.86E-05	5.16E-05	3.35E-05	3.35E-05	<b>100</b>	<b>38817</b>	0	10	2.56E-05	3.99E-05	2.82E-05	2.82E-05	<b>100</b>
<b>32183</b>	1		3.08E-05	4.75E-05	4.49E-05	4.36E-05	<b>93</b>	<b>36240</b>	1		2.74E-05	3.93E-05	3.73E-05	3.70E-05	<b>93</b>
<b>29180</b>	2		3.38E-05	4.51E-05	4.75E-05	4.51E-05	<b>84</b>	<b>34696</b>	2		2.86E-05	3.89E-05	3.93E-05	3.89E-05	<b>89</b>
<b>26474</b>	3		3.72E-05	4.34E-05	4.93E-05	4.59E-05	<b>76</b>	<b>31611</b>	3		3.13E-05	3.85E-05	4.08E-05	4.01E-05	<b>81</b>
		<b>0.55 w/c 3 day loading</b>								<b>0.55 w/c 10 day loading</b>					
R (t,t0)	t		J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	σt/σo	R (t,t0)	t	10	J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	σt/σo
							(%)								(%)
<b>34672</b>	0		2.86E-05	6.12E-05	3.67E-05	3.67E-05	<b>100</b>	<b>33691</b>	0		2.94E-05	4.73E-05	3.09E-05	3.09E-05	<b>100</b>
<b>28563</b>	1		3.47E-05	5.64E-05	5.27E-05	5.12E-05	<b>82</b>	<b>32392</b>	1		3.06E-05	4.66E-05	4.37E-05	4.34E-05	<b>96</b>
<b>27759</b>	2		3.56E-05	5.36E-05	5.64E-05	5.36E-05	<b>80</b>	<b>29558</b>	2		3.35E-05	4.61E-05	4.66E-05	4.61E-05	<b>88</b>
<b>25680</b>	3		3.84E-05	5.16E-05	5.90E-05	5.49E-05	<b>74</b>	<b>28309</b>	3		3.50E-05	4.55E-05	4.87E-05	4.78E-05	<b>84</b>
		<b>0.65 w/c 3 day loading</b>								<b>0.65 w/c 10 day loading</b>					
R (t,t0)	t		J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	σt/σo	R (t,t0)	t	10	J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	σt/σo
							(%)								(%)
<b>34672</b>	0		2.86E-05	7.63E-05	4.07E-05	4.07E-05	<b>100</b>	<b>27684</b>	0		3.58E-05	5.88E-05	3.42E-05	3.42E-05	<b>100</b>
<b>26797</b>	1		3.70E-05	7.03E-05	6.48E-05	6.29E-05	<b>77</b>	<b>25496</b>	1		3.89E-05	5.79E-05	5.35E-05	5.31E-05	<b>92</b>
<b>26132</b>	2		3.78E-05	6.67E-05	7.03E-05	6.67E-05	<b>75</b>	<b>20736</b>	2		4.78E-05	5.71E-05	5.79E-05	5.71E-05	<b>75</b>
<b>24129</b>	3		4.09E-05	6.43E-05	7.41E-05	6.90E-05	<b>70</b>	<b>20136</b>	3		4.92E-05	5.65E-05	6.09E-05	5.98E-05	<b>73</b>

Table H2: Approximate Relaxation Function (ARF<sub>2</sub>) results

APPROXIMATE RELAXATION FUNCTION (ARF <sub>2</sub> )															
Values that involve delta in relaxation function formula were derived from the MC9099 Model															
c <sub>1</sub>	0.093	0.45 w/c 3 day loading			q	10.00	σ <sub>t</sub> /σ <sub>o</sub>	c <sub>1</sub>	0.107	0.45 w/c 10 day loading			q	10.00	σ <sub>t</sub> /σ <sub>o</sub>
R (t,t0)	t	t <sub>0</sub>	J(t,t)	J(t,t-1)	J(t-Δ,t <sub>0</sub> )	J(t,t+Δ)	(%)	R (t,t0)	t	t <sub>0</sub>	J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t0+Δ)	(%)
33196	0.00	3.00	2.86E-05	5.16E-05	3.35E-05	3.35E-05	100	36537	0.00	10.00	2.56E-05	3.99E-05	2.82E-05	2.82E-05	100
30563	1.00		3.08E-05	4.75E-05	4.49E-05	4.36E-05	92	33914	1.00		2.74E-05	3.93E-05	3.73E-05	3.70E-05	93
27463	2.00		3.38E-05	4.51E-05	4.75E-05	4.51E-05	83	32334	2.00		2.86E-05	3.89E-05	3.93E-05	3.89E-05	88
24689	3.00		3.72E-05	4.34E-05	4.93E-05	4.59E-05	74	29212	3.00		3.13E-05	3.85E-05	4.08E-05	4.01E-05	80
		0.55 w/c 3 day loading					σ <sub>t</sub> /σ <sub>o</sub>			0.55 w/c 10 day loading					σ <sub>t</sub> /σ <sub>o</sub>
R (t,t0)	t		J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	(%)	R (t,t0)	t	10.00	J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	(%)
33467	0.00		2.86E-05	6.12E-05	3.67E-05	3.67E-05	100	31773	0.00		2.94E-05	4.73E-05	3.09E-05	3.09E-05	100
27209	1.00		3.47E-05	5.64E-05	5.27E-05	5.12E-05	81	30439	1.00		3.06E-05	4.66E-05	4.37E-05	4.34E-05	96
26332	2.00		3.56E-05	5.36E-05	5.64E-05	5.36E-05	79	27566	2.00		3.35E-05	4.61E-05	4.66E-05	4.61E-05	87
24194	3.00		3.84E-05	5.16E-05	5.90E-05	5.49E-05	72	26290	3.00		3.50E-05	4.55E-05	4.87E-05	4.78E-05	83
		0.65 w/c 3 day loading								0.65 w/c 10 day loading					σ <sub>t</sub> /σ <sub>o</sub>
R (t,t0)	t		J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	(%)	R (t,t0)	t	10.00	J(t,t0)	J(t,t-1)	J(t-Δ,t0)	J(t,t+Δ)	(%)
33754	0.00		2.86E-05	7.63E-05	4.07E-05	4.07E-05	100	26143	0.00		3.58E-05	5.88E-05	3.42E-05	3.42E-05	100
25736	1.00		3.70E-05	7.03E-05	6.48E-05	6.29E-05	76	23920	1.00		3.89E-05	5.79E-05	5.35E-05	5.31E-05	91
25011	2.00		3.78E-05	6.67E-05	7.03E-05	6.67E-05	74	19118	2.00		4.78E-05	5.71E-05	5.79E-05	5.71E-05	73
22955	3.00		4.09E-05	6.43E-05	7.41E-05	6.90E-05	68	18498	3.00		4.92E-05	5.65E-05	6.09E-05	5.98E-05	71

Table H3: Summary of relaxation results from models and experiments

$\psi(t, t_0) = \frac{\phi(t, t_0) \chi = \frac{t_0^{0.5}}{t + t_0^{0.5}}}{E_c / E_c(t_0) + \chi \cdot \phi(t, t_0)}$										
					AAEM	Age adjusted effective modulus method				
					ARF	Approximate relaxation function (Bazant-Kim, 1979 )				
Age of loading (days)					ARF <sub>2</sub>	New Approximate relaxation function (Bazant-Mija, 2013)				
t <sub>0</sub>	χ	t	φ(t, t <sub>0</sub> )	ψ(t, t <sub>0</sub> )						
<b>0.45 w/c, 3days</b>					<b>Predicted</b>	<b>Average measured</b>		<b>Predicted</b>		
<b>3</b>	<b>0.63</b>				<b>AAEM</b>	<b>Measured (0.8σ)</b>	<b>Measured (0.4σ)</b>	<b>ARF<sub>BK</sub></b>	<b>ARF<sub>2</sub></b>	
		0	0.00	0.00	100	100	100	100	100	
E <sub>C</sub> /E <sub>t0</sub>	1.03	24	0.08	0.07	93	86	81	93	92	
		48	0.18	0.16	84	84	79	84	83	
		72	0.30	0.25	75	83	78	76	74	
<b>0.55 w/c, 3days</b>										
			φ(t, t <sub>0</sub> )	ψ(t, t <sub>0</sub> )	AAEM	Measured (0.8σ)	Measured (0.4σ)	ARF	ARF <sub>2</sub>	
E <sub>C</sub> /E <sub>t0</sub>	1.15	0	0.00	0.00	100	100	100	100	100	
		24	0.21	0.16	84	85	78	82	81	
		48	0.24	0.18	82	84	76	80	79	
		72	0.34	0.25	75	83	75	74	72	
<b>0.65 w/c, 3days</b>										
E <sub>C</sub> /E <sub>t0</sub>	1.21		φ(t, t <sub>0</sub> )	ψ(t, t <sub>0</sub> )	AAEM	Measured (0.8σ)	Measured (0.4σ)	ARF	ARF <sub>2</sub>	
		0	0.00	0.00	100	100	100	100	100	
		24	0.29	0.21	79	82	75	77	76	
		48	0.32	0.23	77	79	73	75	74	
		72	0.43	0.29	71	77	72	70	68	

Table H4: Summary of relaxation results from models and experiments

		0.45 w/c, 10days			Predicted	Average measured				
10	0.76		$\phi(t, t_o)$	$\psi(t, t_o)$	AAEM	Measured (0.8 $\sigma$ )	Measured (0.4 $\sigma$ )	ARF	ARF <sub>2</sub>	
		0	0.00	0.00	100	100	100	100	100	
E <sub>C</sub> /E <sub>t0</sub>	0.88	24	0.07	0.08	92	89	84	93	93	
		48	0.12	0.12	88	88	83	89	88	
		72	0.20	0.19	81	87	81	81	80	
		0.55 w/c, 10days								
			$\phi(t, t_o)$	$\psi(t, t_o)$	AAEM	Measured (0.8 $\sigma$ )	Measured (0.4 $\sigma$ )	ARF	ARF <sub>2</sub>	
E <sub>C</sub> /E <sub>t0</sub>	0.99	0	0.00	0.00	100	100	100	100	100	
		24	0.04	0.04	96	88	78	96	96	
		48	0.10	0.09	91	87	77	88	87	
		72	0.20	0.18	82	86	75	84	83	
		0.65 w/c, 10days								
			$\phi(t, t_o)$	$\psi(t, t_o)$	AAEM	Measured (0.8 $\sigma$ )	Measured (0.4 $\sigma$ )	ARF	ARF <sub>2</sub>	
E <sub>C</sub> /E <sub>t0</sub>	1.00	0	0.00	0.00	100	100	100	100	100	
		24	0.09	0.08	92	85	76	92	91	
		48	0.33	0.27	73	84	73	75	73	
		72	0.37	0.29	71	83	71	73	71	