

PRODUCTION SCHEDULING
IN A FOUNDRY MACHINE SHOP

by

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in partial fulfilment of the requirements
for the degree of Master of Science in Engineering

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ABSTRACT

The scheduling of production in job shops is generally accomplished in four stages; aggregate planning, machine loading, sequencing and detailed scheduling. In industrial job shops, the number of jobs and machines makes detailed scheduling a particularly complicated and unwieldy task. When faced with this situation, a typical response of managements is to simply ignore the problem and apply some remedial action by adapting existing company operation procedures.

The first objective of this dissertation is to indicate the dangers and inefficiencies which result when the problem of detailed scheduling is ignored. This is done in terms of a case study analysis in which the problems which currently exist in the machine shop at Atlantis Aluminium, a jobbing foundry, are illustrated.

The second objective is to develop a systematic approach for the solution of detailed scheduling in job shops. Major steps in this approach are:

- i) a classification of shop scheduling problems
- ii) a survey of relevant scheduling literature in order to determine existing detailed scheduling techniques

iii) the design of the scheduling system

This approach is illustrated by applying it to the machine shop at Atlantis Aluminium.

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GLOSSARY

Active schedules	The set of all schedules in which no global left-shift can be made
Deterministic problems	Here, all problem elements such as due dates, processing times, etc. are assumed to be known and fixed
Due date	The time at which the job should be completed and ready for delivery. The due date is usually negotiated by job shop manufacturing organizations and customers, forming an important element of the formal work contract
Dynamic problems	In these problems, jobs arrive randomly at times that are given by a probability distribution
Expedition	Giving a job top priority status
Flow shop case	All jobs have the same routing through the shop
General job shop case	Individual jobs have unique routings through the shop
Global left-shift	An adjustment made to a schedule in which an operation is begun earlier without delaying the start of another operation
Job routing	The path followed by each job through the machine shop. Each job visits only those machines required and in the order specified
Local left-shift	An adjustment made to a schedule in order to reduce superfluous idle time
Measure of performance	The yardstick used for the evaluation of alternative feasible schedules

GLOSSARY (continued)

Non-delay schedules	The set of schedules obtained by specifying that no machine is kept idle at a time when it could begin processing of some operation
Partial schedule	An incomplete schedule which consists of those operations that have had start times assigned
Permutation flow shop case	All jobs have the same routing through the shop with all machines having the same processing order as well
Priority rule	A method of selection used for choosing amongst competing operations
Processing time	Manufacturing time required for a particular operation and machine. Included is any set-up time and transportation time
Schedule	A detailed description of start and end times for all operations, which is displayed graphically for each machine
Schedulable operation	An operation which has had its predecessor operation scheduled
Semi-active schedule	The set of all schedules in which no local left shift can be made
Static scheduling problems	In these problems, jobs either arrive simultaneously or have known ready times
Stochastic problems	Here, uncertainties are found in the problem elements

LIST OF SYMBOLS

a_i	Flow allowance for Job i
b	Resource constraints
c	Value of the performance measure for schedule y
d_i	Due date of Job i
g_i	Number of operations for Job i
k	Machine number
lb	Lower bound
n	Number of jobs in a static problem
n_T	The number of tardy jobs
O_{ijk}	The j th operation of Job i which requires machine k
O_j	The operation which is chosen from S_t
P_{ijk}	Processing time of the j th operation of Job i which requires machine k
p_i	The sum of the processing times for Job i
r_i	Ready time or release time of Job i
x_l	Objective function variables
y	A particular schedule
y^*	The trial schedule
A	Job arrival process (either static or dynamic)
B	Number of machines (m) in the shop
C	Flow pattern or discipline within the shop
C_i	completion time of Job i
\bar{C}	Average completion time
C_{max}	Maximum completion time
D	Criterion by which schedules are to be evaluated

LIST OF SYMBOLS (continued)

E_i	Earliness of Job i
F	The flow shop
F_i	Flow time of Job i
F_{\max}	Maximum flow time
\bar{F}	Average flow time
G	The general job shop
I_k	Idle time on machine M_k
\bar{I}	Average idleness
I_{\max}	Maximum idleness
J_i	Job i
L_i	Lateness of Job i
\bar{L}	Average lateness
L_{\max}	Maximum lateness
M_k	The k th machine in the shop
M^*	The machine on which ϕ or σ occurs
$N_w(t)$	Number of jobs waiting between machines
$N_p(t)$	Number of jobs actually being processed at time t
$N_c(t)$	Number of jobs completed at time t
$N_u(t)$	Number of jobs still to be completed at time t
\bar{N}_w	Average number of jobs waiting for machines
\bar{N}_u	Average number of unfinished jobs
\bar{N}_c	Average number of jobs completed
\bar{N}_p	Average number of jobs being processed
O	The set of all operations
P	The permutation flow shop

LIST OF SYMBOLS (continued)

P_t	The partial schedule
R	Regular measure of performance
S	Sequence (for the type of jobs used in Johnsons algorithm)
S_t	The set of schedulable operations at stage t
T_i	Tardiness of Job i
\bar{T}	Average tardiness
T_{\max}	Maximum tardiness
W_{ij}	Waiting time which precedes the j th operation of Job i
W_i	Total waiting time of Job i
Y	The subset of schedules corresponding to a particular node
Z_1	The class of fully explored nodes
Z_2	The class of partially explored nodes
Z_3	The class of unexplored nodes
ϕ	Earliest completion time
ϕ^*	The minimum of the earliest completion time
σ	The earliest start time
σ^*	The minimum of the earliest start time

CHAPTER 1

INTRODUCTION

In manufacturing organizations, the scheduling of production is a unifying problem which relates such diverse elements of the organization as sales, cost control, purchasing, capital budgeting and many others. In general terms, it is defined as the allocation of resources over time, to provide goods and services when demanded. (Lockyer, Reference 13) Consequently, it is primarily a planning function and is closely associated with the production control system. Where these two systems have been integrated and co-ordination optimised, profit benefits result due to improved management performance and a reduction of operating costs. Advantages over market competitors would also be gained through improved customer relations, due mainly to reliable due date performance and shortened delivery times.

The application of a production scheduling system, together with its related control system, is strongly dependent upon the organisation and functions of the overall production system. Two extreme organisational forms have been identified, viz. the intermittent and the continuous flow process operation.

Continuous processes are characterised by the large or indefinite number of units of a single product that are produced; and consequently, the simplest of scheduling and control techniques are applicable. Examples of this type of process are the petrochemical and synthetic fibre industries. Intermittent systems, on the other hand, either produce a variety of products one at a time (i.e. custom made) or finite numbers of different products in batches. Here, individual products may have different machine routings, input materials, completion schedules and due dates. Thus, sophisticated and complex production planning

and control techniques are required to bring together in proper sequence and the right time and place, the results of these interrelated activities.

In a manufacturing context, intermittent systems are traditionally referred to as job shops, with the load on the facility increasing as work orders arrive. Some work centres may be idle while others are severely loaded, leading to the build up of work-in-progress queues at some centres. The sequence in which these waiting jobs are processed through the machines in the facility is of particular importance, as it determines the efficiency and effectiveness of the intermittent system. Specifically, sequencing determines the amount of job lateness, costs incurred for set up and change over, delivery lead times, inventory costs and the degree of congestion in the facility.

The scheduling of production in job shops involves everything from the receipt of specific demand information, through the setting up of starting and stopping times for individual operations, issuing of shop orders, the routing of materials and work flow, to the actual authorization and completion of work activities. This scheduling process, as discussed in Monks (Reference 16), takes place in a number of stages and is given below:

- 1) Aggregate Planning - in this, the first stage, the organisation's overall level of output and the resource inputs required for achieving it are determined for each of several future time periods (a time period is typically a month). A general picture is thus built up of the workload assigned to the facility in relation to the productive capacity available. At this level, capacity refers to the labour and machine time available (in hours)

- 2) Machine Loading - this stage is more specific and detailed than aggregate planning, and differentiates between the various work centres within the overall facility. The planned units (specific jobs) to be produced during a period are allocated among the work centres, thus establishing the load that each work centre must carry

- 3) Sequencing - this stage establishes the priorities of jobs in the queues (waiting lines) at each of the work centres, thus specifying the order in which these waiting jobs will be processed

- 4) Detailed Scheduling - here, calendar times are specified for when job orders, employees, materials (input) and job completion (output) should occur at each work centre. This stage supplements the preceding loading and sequencing stages, as detailed dates and times cannot be specified until the processing sequence of waiting jobs has been determined. Using estimates of processing durations and due dates for all jobs, schedulers can establish their beginning and end dates, thus developing a detailed schedule

Only in the simplest of cases, such as the processing of jobs on a single machine, is the solution of the sequencing problem and consequent derivation of detailed schedules a straightforward exercise. In other manufacturing situations, such a solution is not possible as the preferred sequence of operations for one job is a function of the sequence of operations chosen for another, or perhaps, many other jobs. In this case, in order to determine the preferred sequence for one job, it is necessary to determine the preferred sequence for all jobs simultaneously. As a result, the sequencing problem becomes one of considerable size and complexity when considering industrial job shops, operating in the real-world environment.

When faced with the complex problem of scheduling production in job shops, a fairly typical response of industrial schedulers has been to simply ignore the problem of detailed scheduling. To these schedulers, no scheduling problem exists as the organisation which surrounds them has reacted and adapted itself to protect them from the need to perform detailed scheduling. Typically, protection is afforded by the production department interacting with the sales department, laying down guidelines based upon perceived shop load, for the acceptance of orders and setting of due dates.

The first objective of this thesis is to indicate the dangers and inefficiencies which result when the problem of full and detailed scheduling in job shops is ignored, and where the scheduling function is protected. This is done in terms of a case study analysis in which the problems which currently exist at Atlantis Aluminium, a jobbing foundry, are illustrated. The second and primary objective of the thesis is to review and evaluate the existing, detailed production scheduling techniques, with a view to integration into the overall production scheduling system at Atlantis Aluminium. The study was initiated by the machine shop superintendent and concentrates specifically on this area of activity.

CHAPTER 2

ATLANTIS ALUMINIUM - BACKGROUND DETAILS2.1 GENERAL INFORMATION

Atlantis Aluminium is a manufacturer of aluminium components primarily for Atlantis Diesel Engines (A.D.E.), a local supplier of diesel engines to the South African market. These aluminium components are finished products which have been machined to the high degree of accuracy demanded by A.D.E. This means that once these components have been approved by A.D.E.'s quality control department, they are fitted directly onto the diesel engines. Typical products produced by Atlantis Aluminium for A.D.E. include cylinder head covers, oil sumps and a range of aluminium piping. A full list of components produced by A.D.E. is shown in Appendix A. The associated part numbers designated by A.D.E. and used by Atlantis Aluminium during the production process are also indicated.

The plant is situated in the industrial township of Atlantis, located some 45 km north of Cape Town. It has a planned capacity of processing 10 000 tons of aluminium by 1986. However, it is doubtful whether this target will be achieved in the near future, as company expansion has been retarded by the downturn in the South African economy.

Atlantis Aluminium commenced operations as an independent company, and was subsequently taken over by A.D.E. Due to group rationalization, it today receives production engineering assistance (in the form of work studies, production problem solving and production planning) and accounting services from A.D.E. No specific production scheduling assistance is provided by A.D.E. however.

Although Atlantis Aluminium produces components mainly for A.D.E., it does take on work from outside companies. At the moment, however, the volume of outside work taken as a percentage of the total work accepted, is very small (maximum of 5%).

2.2 DESCRIPTION OF THE PLANT AND PROCESSES INVOLVED

Atlantis Aluminium is made up of three departments. Two of these, the foundry and the machine shop, are direct production departments while the third, the quality control department, is a production related department.

2.2.1 The Foundry

The foundry is responsible for the production of aluminium castings. Two casting processes are used in the foundry, permanent mould casting and sand casting. In permanent mould casting, molten aluminium is poured under gravity into a metal mould, the cavity of which has been coated with special refractory washes. As soon as the aluminium has solidified, the mould is opened and the casting withdrawn. Mould movement, core retraction and casting ejection are accomplished with a manually operated lever and pinion arrangement.

Sand casting practices for aluminium are not greatly different from that of other metals. (Van Horn, Reference 23). At Atlantis Aluminium, the only difference is that a synthetic sand which has been specially prepared is used. This sand is fed to the moulding box where it is packed by hand and left to harden.

The aluminium charge, made up of aluminium ingots and scrap, is melted in small electric induction furnaces. When needed, the molten metal is emptied into hand ladles for pouring into the moulds.

A small pattern shop and a fettling shop are also under the control of the foundry. The pattern shop is used mainly for the repair of, or modification to existing patterns, as the majority of patterns used at Atlantis Aluminium are imported. In the fettling shop, castings are dressed by trimming off runners and risers and grinding off minor irregularities. Common tools used in the fettling shop are high speed wood-working bandsaws, chipping hammers, power driven portable grinders and sanding discs.

2.2.2 The Machine Shop

The cast articles produced in the foundry are machined in the machine shop to the specified dimensions and tolerances indicated on the component drawings. The shop consists of ten individual work stations, viz. 3 milling machines, 1 combination milling and drilling machine, 3 drilling machines, 2 lathes and a bench area where techniques of helicoiling and deburring of components are applied.

Each machine in a particular class (e.g. milling) is unique, differing in capacity, processing rate and technical capability from the other machines in that class. Although machines of a particular class are generally grouped into a single department, the small size of the shop and number of machines, results in each machine being regarded as an individual work centre. The machines are also relatively new, having been only recently acquired. In particular, the milling machines and the combination milling/drilling machine are respectively numerical-control (NC) and computer numerical control (CNC) machines.

A storage area is located at the entrance to the machine shop, where castings are stored after arriving from the foundry. Storage areas are also located at each machine for holding those castings being processed or waiting to be processed on that machine.

The cast articles are machined in batches with the quantity in each batch made up solely by the number of articles that have been cast. This number is always greater than that required, in order to compensate for any articles which have to be scrapped because of incorrect machining or moulding defects. Each component has such an allowance which is based upon the results of a monthly scrap report. Once castings have accumulated to this predetermined amount, it is known as a job.

Each job follows a predetermined routing through the machine shop visiting only those machines required and in the order specified. The choice of machine for each stage of the production process (or operation) is based upon the technical and machining requirements and is also termed the technological constraints. Job routings are assessed individually by the machine shop superintendent and his shop foremen. Consequently, all jobs are independent of each other and where similarities do occur, it is purely accidental.

2.2.3 The Quality Control Department

The quality control department ensures that the planned standards of quality are maintained through each stage of production. In the foundry, initial samples of a production run are examined and approved before full-scale production can begin. At the end of the run, castings are selected from the batch on a statistical basis, and inspected before machining is allowed.

In the machine shop, patrol inspectors carry out frequent and infrequent inspections of machined components. When errors are detected in a component the entire batch is quarantined and examined. The quality control department also authorises the start of a production run by ensuring that the machines have been set up in accordance with the specifications.

The quality control department also has a sophisticated impregnation plant under its control. This plant is used for the salvage of castings where surface imperfections and leakage have resulted from poor foundry techniques.

2.3 MANPOWER RESOURCES

2.3.1 The Management Team

The management structure for Atlantis Aluminium is shown in Appendix B. Heading the company is John Proffitt, the general manager. Reporting to him are the three heads of department which constitute Atlantis Aluminium, viz. (1) Allan Wood, the foundry superintendent, (2) John Millar, the machine shop superintendent and (3) Johan van der Merwe, the quality control superintendent. Each of these superintendents has foremen in charge of the sections within their departments.

As mentioned earlier, the parent company A.D.E., provides additional services. Coordinating these are A.D.E.'s Barry Le Hair for production engineering, and C. Robertson for accounting.

Atlantis Aluminium is also considering the appointment of a production planner to plan and coordinate all aspects of manufacturing in the plant.

2.3.2 The Machine Shop Workforce

The breakdown of workers in the machine shop is as follows:

- 3 machine setters
- 11 operator machinists
- 2 deburrers (final component preparation)
- 1 storeman
- 1 material handler

Based on these numbers, it would appear that the machine shop is quite adequately staffed.

2.4 WORK ACCEPTANCE PROCEDURES

Two distinct forms of work acceptance procedures are used. First is that for the acceptance of work from A.D.E., and secondly, for work accepted from outside customers.

2.4.1 The Work Acceptance Procedure for A.D.E. Components

The main steps in the procedure are shown on a flowchart (Fig. 2.1) The initial step is taken by A.D.E., who, based upon the release statement generated by their Manufacturing Resources Planning (M.R.P.) system, determine the type and quantity of aluminium components required. The list of these requirements is then sent to Atlantis Aluminium, who decide whether the manufacture of new components is within the technical capabilities of the company (either manpower or equipment). If a component is accepted for manufacture, it is incorporated into the A.D.E. forecast of components for Atlantis Aluminium. An example of this forecast is given in Appendix C. The forecast, which contains such job information as quantity and due dates, is spread over a period of five months, of which the first three are firm. This means that if any component data are listed on the first three months of the forecast, the requirements of that component are fixed and cannot be altered. For example (with reference to Appendix C), the 100 articles of component 0204 required by March 28 is fixed. However, the 10 articles of component 0107 by April 30 may be changed.

At this point, the forecast is passed to each of the departments, who determine what equipment will be needed for the manufacture of a component. For example, the foundry might require new patterns, the machine shop new jigs and fixtures and the quality control department new measuring

jigs and templates. The departments have to ensure that this equipment is either on hand, or has been ordered, and will be ready at the commencement of production.

Once the departmental needs are satisfied, the jobs are ready to be entered into a monthly plant loading statement. Appendix A displays the actual plant loading statement for the month of February, containing those jobs with due dates in February. The purpose of this statement is to determine the monthly percentage utilization of each of the 10 work centres in the machine shop, and is based upon component machining times. These machining times are then adjusted by an overall efficiency of 0,55 accounting for inefficiencies such as labour variances, machine breakdowns etc. The use of this adjusting efficiency is an attempt by management to model actual shop performance, which in most cases is substantially different from predicted performance.

When the plant loading statement has been drawn up and approved by management, details concerning the jobs in the plant loading statement are sent to each of the foremen in charge of the foundry, machine shop and quality control departments. These foremen then assume responsibility for the processing of the job through the plant.

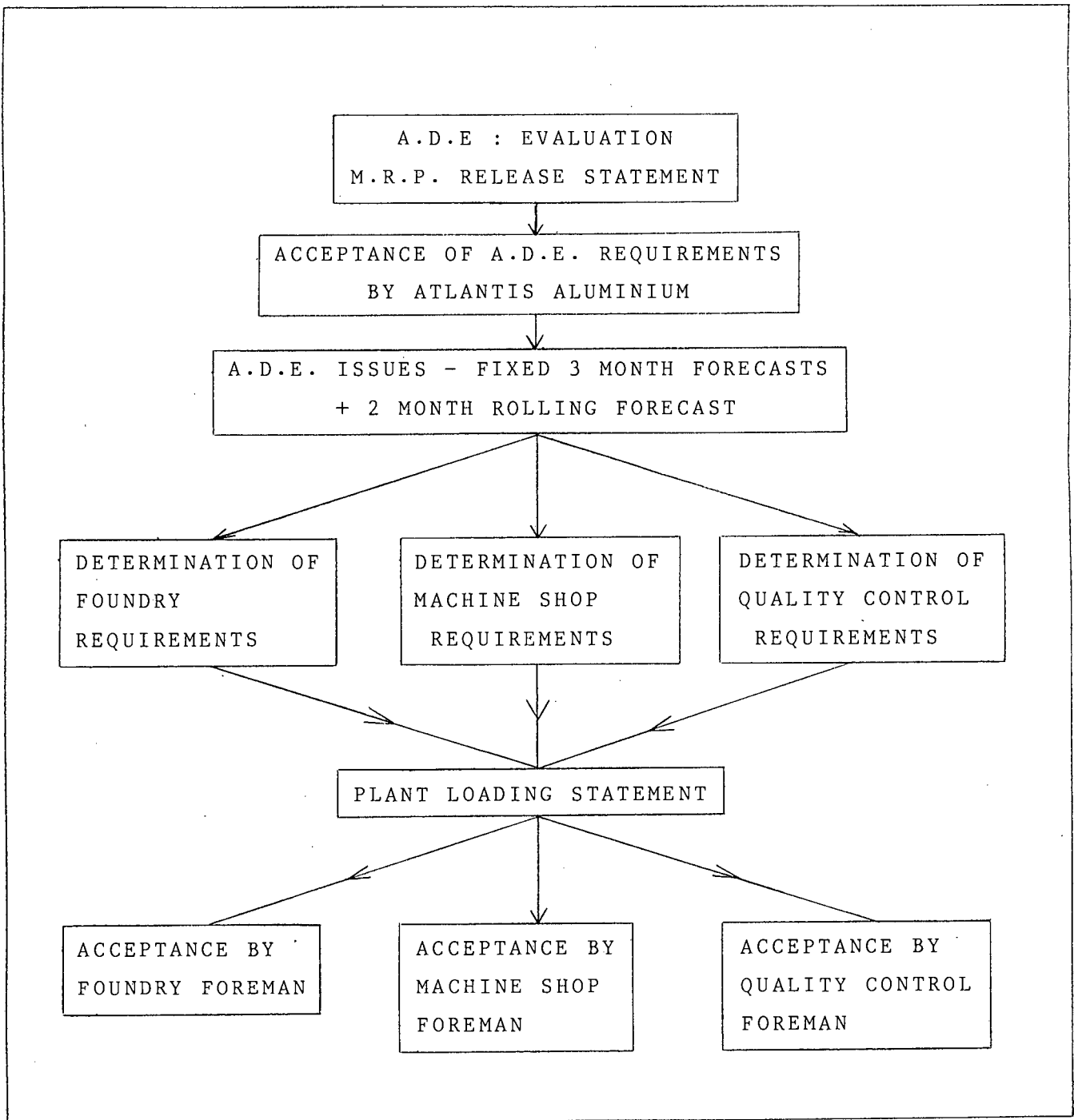


FIGURE 2.1 : ATLANTIS ALUMINIUM -WORK ACCEPTANCE PROCEDURE
FOR A.D.E. COMPONENTS

2.4.2 The Work Acceptance Procedure for Outside Customers

The procedure followed for the acceptance of work from outside customers is shown in Figure 2.2. On receipt of an enquiry from a prospective customer, the details of the required component are entered into a product enquiry file. The original enquiry together with component drawings are then sent to the general manager, who, together with his department heads, either accepts or rejects the proposed order.

If the order is accepted, the enquiry file is circulated amongst the department heads, who decide on their requirements for the manufacture of that component. This information is then sent to the accounting department where the resource requirements (manpower, raw material, machine time, etc.) needed for the manufacture of the component are costed and a final quotation is made to the customer.

On acceptance of the quotation by the customer, the order is assigned a job code number. The procedure at this point then follows that of the A.D.E. component procedure, with the job being incorporated into the plant loading statement, followed by job detail notification of the department foremen.

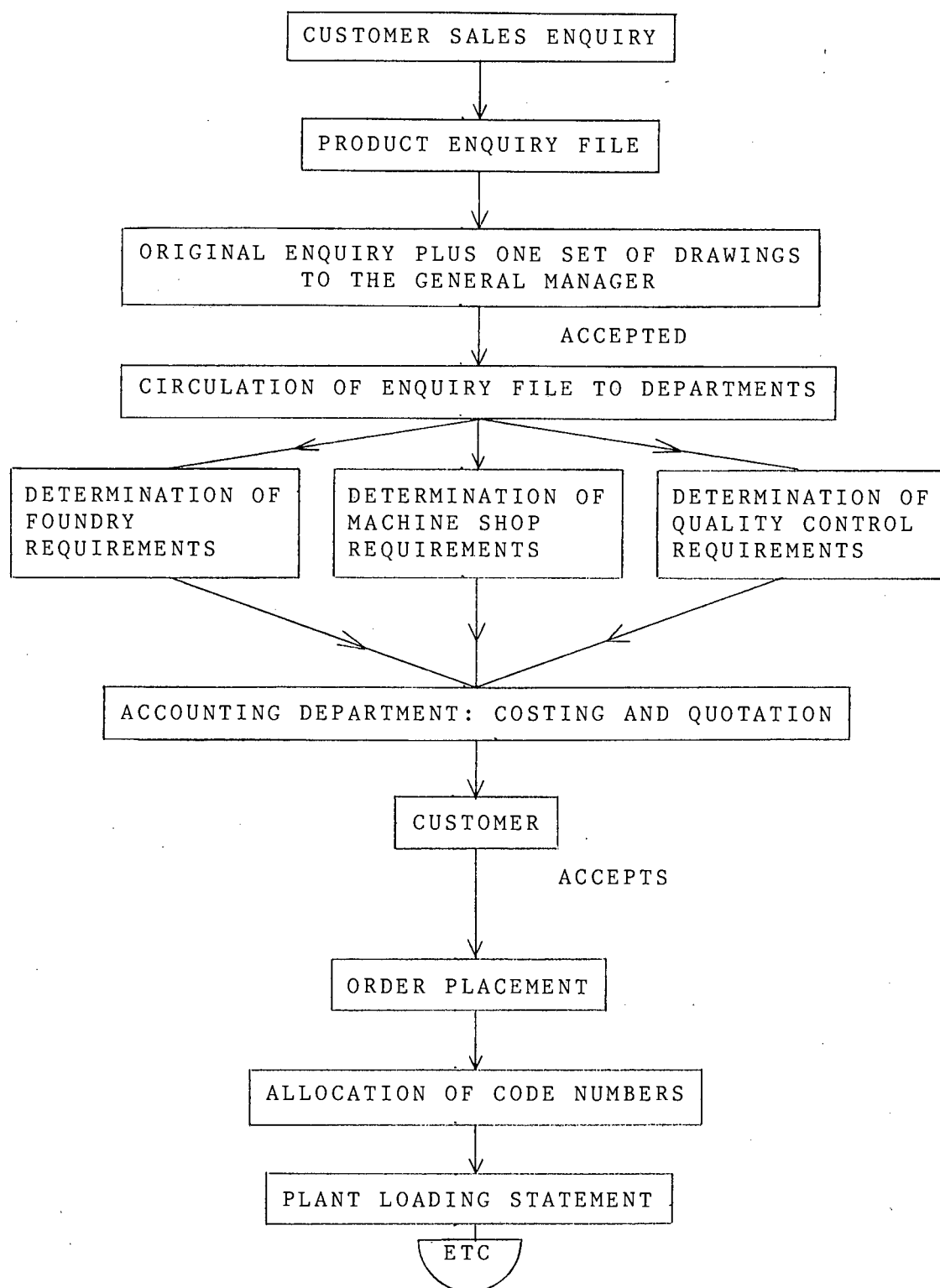


FIGURE 2.2 : ATLANTIS ALUMINIUM - WORK ACCEPTANCE PROCEDURE
FOR OUTSIDE CUSTOMERS

CHAPTER 3

EVALUATION OF THE EXISTING PRODUCTION SCHEDULING SYSTEM USED IN THE MACHINE SHOP

3.1 INTRODUCTION

The evaluation of the production scheduling system currently in operation in the machine shop, formed the initial part of the study, and was carried out locally at Atlantis Aluminium. Relevant information concerning system operation and performance, was gathered from personal investigation, observation of shop floor conditions and in discussion with management and shop floor personnel.

This chapter presents the results of this evaluation by briefly describing and discussing the existing scheduling system as well as the problems and inefficiencies that have resulted.

3.2 DESCRIPTION OF MACHINE SHOP SCHEDULING SYSTEM

At present, no formal production scheduling system, which systematically schedules jobs over the production planning period, is in use. The allocation of work to machines is done on an arbitrary basis by the machine shop foreman. These allocation decisions are made whenever a machine becomes available and there are jobs waiting to be processed on that machine. The scheduling conflicts which arise when more than one job is available for processing are usually settled unwittingly, with the FIFO principle (First in - First out), although the final decision rests with the foreman.

The present system of work allocation is in some ways similar to the Loading Systems in common use in industry for job allocation. The distinguishing characteristic of the Loading System is the fact that no detailed schedules are provided by management nor is there central control. Rather, decentralized control is emphasized with the shop foreman being left to make scheduling decisions when required, at individual work centres. However, total control is not decentralized as management specifies the rules for the resolution of conflicts on machines. These rules are expected to be followed unless there is some overriding reason for alternative action.

The latter point serves to indicate the significant difference between Loading Systems and the Atlantis Aluminium machine shop system. In the latter, control is completely decentralized with the shop foreman making all decisions and effectively controlling production. The machine shop superintendent simply has to accept the production schedules followed by the foreman, as well as the consequences of those schedules. Furthermore, where there are two or more foremen, as is the case in this machine shop, scheduling decisions are made independently of one another, as they are based upon the information available to each foreman and his level of skill. This situation results in an additional inefficiency being added to an already poor system.

3.3 EFFECTS OF THE PRESENT SYSTEM

The production schedules used in the machine shop are based upon the decisions taken by the machine shop foreman. These decisions do not take into account the status or importance of a job and have resulted in the emergence and continued existence of the following fundamental problems:

- 1) Excessive congestion on the shop floor. High volumes of work-in-progress has led to jobs building up in the limited storage space at each machine
- 2) Unnecessarily long lead times. Although these are a direct result of a poor scheduling system, the situation is further aggravated by the arbitrary manner in which job start dates are set
- 3) Excessive and regular overtime. The lack of a good scheduling system has given an incorrect indication of monthly production requirements, causing management to authorise the institution of expensive overtime. Currently, a permanent night-shift is worked in the machine shop with the option of further overtime at weekends. In other words, unnecessary and costly management decisions have been made in order to operate the present system
- 4) Job due dates are frequently not met. This situation has arisen although management realises the importance of meeting due dates. Where there is a possibility that jobs will be late, typical management actions include, firstly expediting and then the authorisation of overtime. However, despite these actions jobs are regularly late, leading to a loss of confidence on the part of customers in the ability of Atlantis Aluminium to produce to a due date. For example, A.D.E. have inserted an additional time safety margin to compensate for the variability in supply times of Atlantis Aluminium
- 5) An uneven monthly production load. The actual production schedule is made up of periods of normal pace activity and periods of intense activity. The latter periods occur mainly due to the expedition of jobs (as discussed earlier) and are concentrated around job due dates. As the majority of job due dates occur

at the end of a month, shop loads are particularly severe during this period

A further complication of the concentration of activity is that the capital intensive milling machines used mainly for first operations are under-utilized during this period, with all activity focussed on the later finishing operations

In addition, the following production-related problems have also been noted:

- 6) As the arbitrary method of work allocation in use at the moment is a manual method, it is time-consuming and expensive. This is particularly emphasised at Atlantis Aluminium where each operation of each job is scheduled by the foreman. Consequently, the foreman has to spend a significant proportion of his time making these decisions and spending less time supervising the production activity on the shop floor. Furthermore, an increase in production volumes due to company expansion results in these scheduling decisions becoming more complicated and difficult to take, thus exacerbating the problem

- 7) The operation of the present inefficient scheduling system has resulted in a lack of co-ordination between the three departments which constitute Atlantis Aluminium (viz. the foundry, the machine shop and quality control departments). As discussed and illustrated in Chapter 2, for each component there is a close interaction between these departments, from the planning stage through manufacturing and, finally, dispatch. The inefficiency of the scheduling system used in the machine shop therefore also tends to disrupt the activities of the other departments

- 8) The problems mentioned above not only affect the utilization of machinery, but more importantly, the people that are associated with the production activity. Firstly, total decentralization of control results in the machine shop foreman effectively controlling production. The situation which currently exists, where job due dates are frequently missed, makes the positions of both the machine shop foreman and the man ultimately responsible for timely production, the machine shop superintendent, particularly stressful. The same applies to their working relationship. Secondly, as stated earlier, there is a lack of co-ordination between departments. This situation has led to considerable friction and differences in opinion between heads of departments. Finally, the arbitrary method of work allocation, by its very nature, frequently results in jobs being late. This lack of commitment to the meeting of job due dates has on the whole, a negative influence upon the productivity and effectiveness of the personnel at Atlantis Aluminium

A well-designed scheduling system on the other hand, will get the same work done on time while making optimal use of the required resources.

CHAPTER 4

CLASSIFICATION OF MACHINE SCHEDULING PROBLEMS4.1 INTRODUCTION

The first part of this study, discussed in the previous chapter, indicated the need for an effective scheduling system at Atlantis Aluminium. The design of this scheduling system forms the second major part of the study, and is split up into a number of stages. The first stage of the design process involves the classification in scheduling terms of the machine shop problem. This classification process is spread over the next three chapters and is particularly useful when conducting a survey of scheduling literature, as research results have been arranged along classification category lines. Thus, once a machine shop has been classified in scheduling terms, it becomes a simple matter to refer to the literature developed specifically for that category of problem.

This chapter looks at the characteristics which classify and distinguish machine scheduling problems. These are then translated into a standard four parameter notation. Based on this notation the machine shop at Atlantis Aluminium is partially classified with the remaining parameter being the subject of more in-depth analysis and discussion in Chapter 6.

4.2 CLASSIFICATION PARAMETERS

In general, any scheduling problem can be described by four types of information as stated in Conway et al. (Reference 8):

- 1) The jobs and operations to be processed
- 2) The number and types of machines that make up the shop
- 3) Work flow disciplines that restrict the manner in which assignments of jobs to machines can be made
- 4) The criteria by which a production schedule is to be evaluated

Based on the above, individual problems can be distinguished from one another by the number of jobs to be processed, the manner in which jobs arrive at the shop and the order in which different machines appear in the operations sequence of single jobs.

An important distinction given by the nature of job arrivals, is of that between static and dynamic problems. In a static problem, a fixed number of jobs either arrives simultaneously or have known ready times (potential start times), in a shop that is immediately available for work. No further jobs will arrive so attention can be focussed on scheduling this completely known and available set of jobs. In a dynamic problem, jobs arrive randomly at times that are only predictable in a statistical sense, and continue indefinitely into the future.

A further distinction is of that between deterministic and stochastic problems. In the deterministic case, all elements of the problem such as due dates, processing times, etc. are known and fixed. Stochastic problems on the other hand, contain uncertainties in these problem elements.

The machine order of different jobs forms the basis for the classical descriptions of industrial machine shops. First of these is the flow shop in which all jobs follow the same route through the machine shop. A derivative of the conventional flow shop is the permutation flow shop where not only is the machine order the same for all jobs, but the

processing order for jobs on each machine is the same as well. Differing greatly from the flow shop is the general job shop. In this shop, work flows are general or arbitrary, with each job visiting only those machines required and in the order specified.

The above discussion can now be translated into a four parameter notation, which is then used to identify and classify individual scheduling problems, viz.:

A/B/C/D

here A - denotes the job arrival process. For dynamic problems, A will identify the probability distribution of the times between job arrivals. For static problems, it will specify the number of jobs (n) assumed to arrived simultaneously or with known ready times. Thus, when n is given as the first term, it denotes an arbitrary, but finite, number of jobs in a static problem

B - denotes the number of machines (m) in the shop

C - describes the flow pattern or discipline within the shop. Principal symbols are:

F for the flow shop case

P for the permutation flow shop case

G for the general job shop case where there are no restrictions on the form of the technological constraints

D - denotes the criterion by which the schedule is to be evaluated. The alternative criteria and associated notation are described and discussed in some depth in Chapter 6

4.3 PARTIAL CLASSIFICATION OF THE MACHINE SHOP AT ATLANTIS ALUMINIUM

4.3.1 The Job Arrival Process - A

As stated in Chapter 2, by far the majority of work undertaken at Atlantis Aluminium is for A.D.E. These jobs have fixed due dates, ready times and processing times, as discussed in Section 2.4.1. Consequently, the conditions in the machine shop are classified as a static problem. It should be noted that the small component of jobs for outside customers forms a dynamic element of the problem. The eventual solution for Atlantis Aluminium will have to cater for this situation as well.

The machine shop is also taken to be deterministic because with the use of modern NC and CNC machines, a greater degree of certainty in processing times is attained.

Thus the parameter A for the static and deterministic machine shop problem is taken to be:

$$n = 15$$

where 15 is the average monthly quantity of jobs undertaken and completed.

4.3.2 Number of Machines - B

As stated in Section 2.2.2, there are 10 individual work centres. These work centres are taken as being machines in a scheduling sense.

4.3.3 Work Flow Discipline - C

Based on the discussion in Section 2.2.2 of actual work flow patterns and the manner in which they are determined, the machine shop is classified as a general job shop (G).

In summary then, the machine shop problem is taken to be static and deterministic with the following partial classification parameters:

15/10/G/D

The choice of the performance measure D is still to be discussed and specified.

CHAPTER 5

DETAILED DESCRIPTION OF THE GENERAL JOB SHOP PROBLEM5.1 INTRODUCTION

In the previous chapter the machine shop at Atlantis Aluminium was partially classified as a static and deterministic general job shop.

The purpose of this chapter is to state and detail the problem of scheduling in this general job shop environment. This is done firstly by describing the machine shop in scheduling terms, and secondly, by defining relevant problem element notation which will be used in this thesis. Based on this shop description and element definition, the machine shop scheduling problem is then discussed.

5.2 SCHEDULING DESCRIPTION OF THE MACHINE SHOP

In the static, deterministic general job shop, n jobs ($J_1 ; J_2 ; \dots ; J_n$) have to be scheduled through a fixed number of machines, m . Each job consists of a number of operations which have to be carried out in a specified order. An operation is also associated with a particular machine and consequently each job has a unique routing (or the technological constraints) from machine to machine, through the shop. This is as opposed to the flow shop case where all jobs have the same routing through the shop.

Each machine in the machine shop can now be viewed as having input and output workflows as shown in Figure. 5.1 which follows:

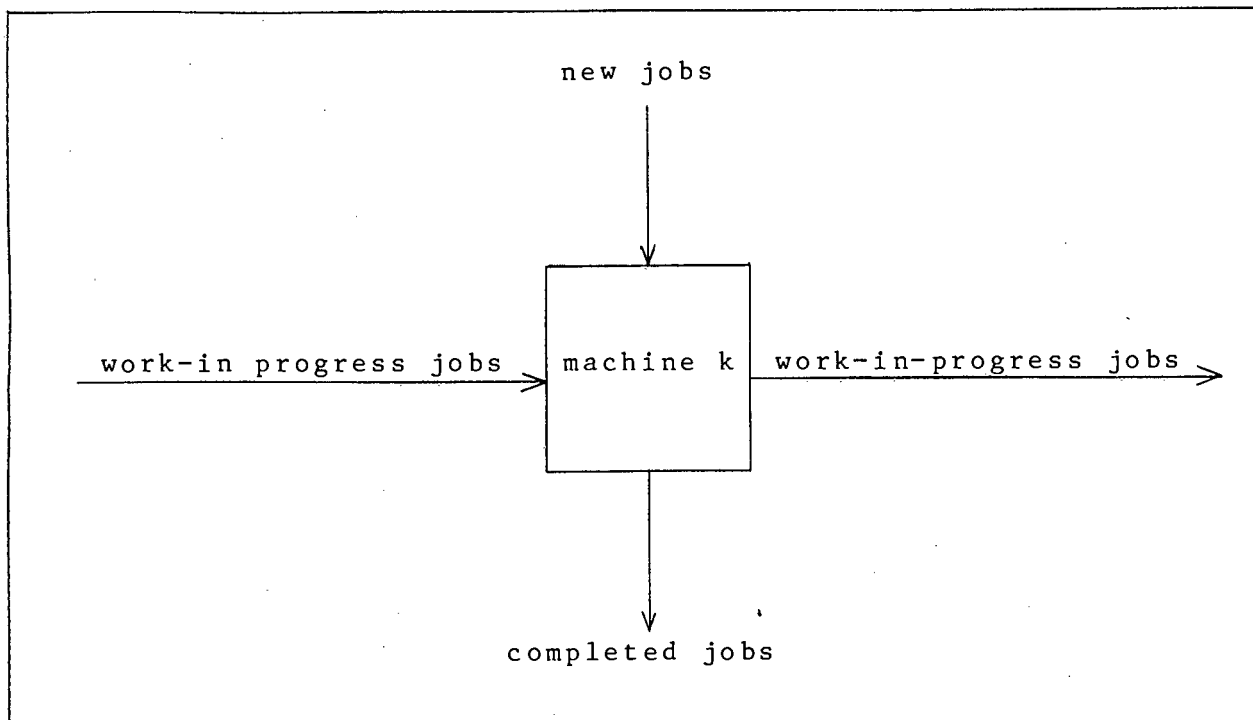


FIGURE 5.1 : WORKFLOW AT A MACHINE IN THE SHOP

Strictly speaking, there is no initial machine that performs only the first operation or a terminal machine which performs only the last operation of a job. At Atlantis Aluminium, milling tends to be the first operation but it is usually spread over three machines.

5.3 MACHINE SHOP NOTATION

In this thesis, the operation is denoted by the (i,j,k) triplet which indicates that the j th operation of job i requires machine k . It can also be stated as the job-operation-machine triplet (JOM).

The processing time for each operation (o_{ijk}) is denoted by p_{ijk} . By convention, included in the processing time (p_{ijk}) is any time required to adjust or set-up the machine for that operation. Also included is any time required to transport the job from the machine which performed the previous operation to the machine k on which operation

(o_{ijk}) is to be processed. The latter is assumed to be negligible in the case of Atlantis Aluminium as the 10 work centres are in close proximity to each other.

5.4 DESCRIPTION OF THE MACHINE SHOP SCHEDULING PROBLEM

Using the concepts discussed above, the machine shop problem can be shown in its simplest and most useful form, by displaying it graphically. This is done with gantt charts on which blocks denoting operations are arranged. Each job i is given a set of blocks, one for each of its operations. Associated with each operation-block is its identifying job-operation-machine triplet (i,j,k) . The length of each block is proportional to the processing time required to perform that operation.

The use of the gantt charts is illustrated by the following problem (taken from Baker, Reference 3). The data for this problem is shown in Tables 5.1 and 5.2 below with each job having 3 operations to be processed through 3 machines.

TABLE 5.1 : Processing Times

		Operation		
		1	2	3
	1	4	3	2
Job	2	1	4	4
	3	3	2	3
	4	3	3	1

TABLE 5.2 : Routing

		Operation		
		1	2	3
	1	1	2	3
Job	2	2	1	3
	3	3	2	1
	4	2	3	1

Table 5.1 gives the operation processing times for each job while Table 5.2 gives the machines required and their order for each job.

In the first step, the operation-blocks are arranged in rows according to their parent jobs and in numbered order, as shown in Fig. 5.2 which follows:

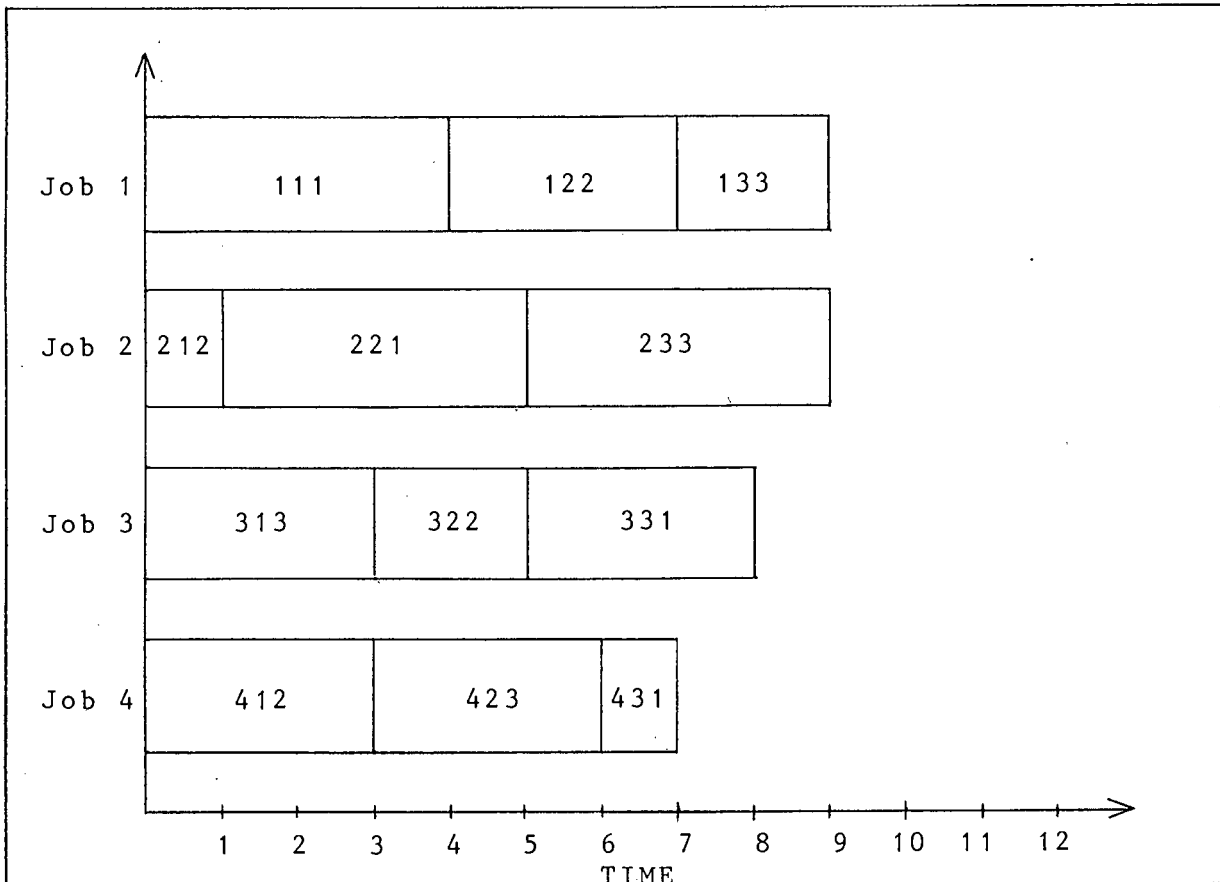


FIGURE 5.2 : JOB-BY-JOB DESCRIPTION OF WORK

Arrangement by job implies that the first identifier i of each operation in a row is the same. The second identifier j forms an increasing sequence indicating the linear operation sequence.

The next step is to arrange these twelve operation-blocks into rows by machine, as compactly as possible. Figure. 5.3 below shows this arrangement when compacted in an arbitrary fashion.

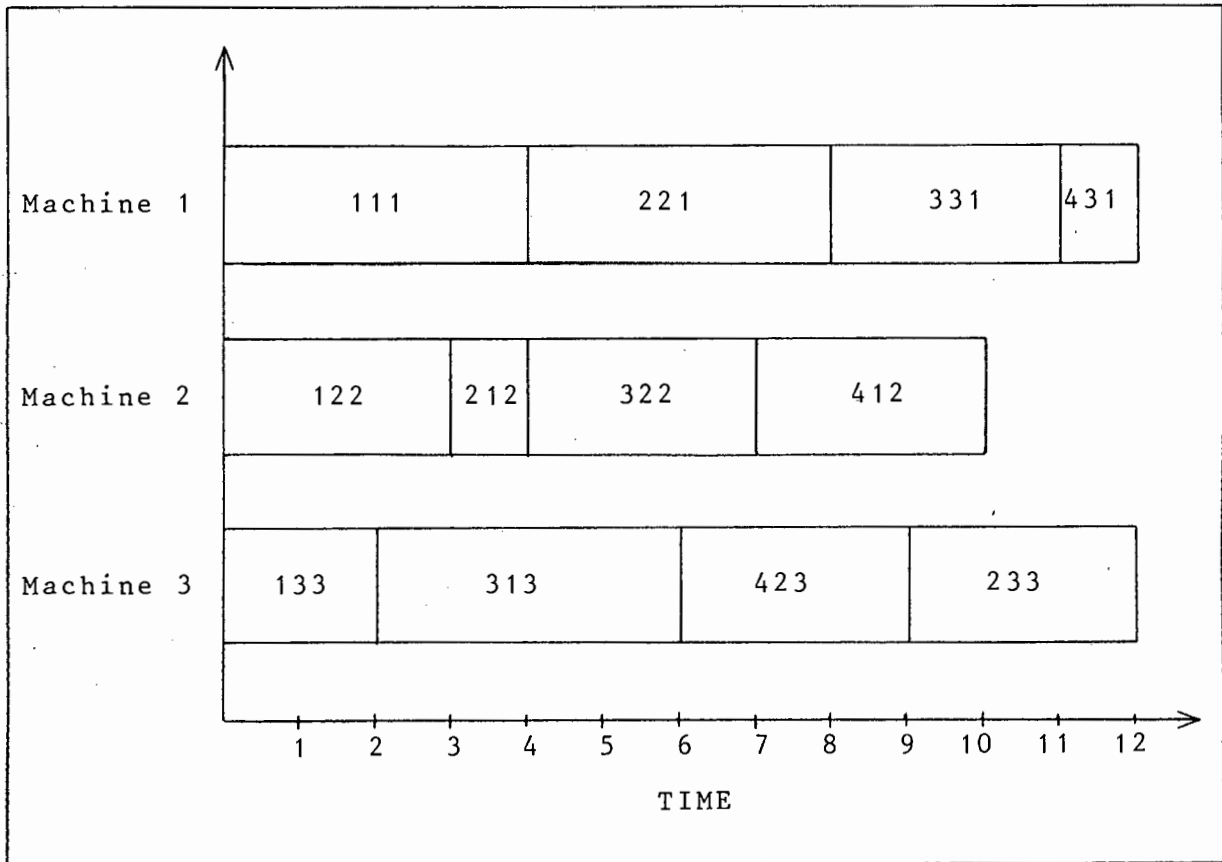


FIGURE 5.3 : MACHINE-BY-MACHINE DESCRIPTION OF WORK

This arrangement gives an indication of the work load for each machine, but does not represent a valid schedule since the operations cannot be done in the order indicated and at the times implied.

The key to the problem is to construct an arrangement by machine such that if the operations were projected back onto an arrangement by job, the operations would be in the original order and would be a valid or feasible solution, firstly, if the schedule is a feasible resolution of the resource constraints, i.e. when no two operations ever occupy the same machine simultaneously. Secondly, a valid schedule exists if it is a feasible resolution of the logical constraints, i.e. when all operations of each job can be placed on one time axis in the required order and

without overlap. An example of a feasible schedule is shown in Figure 5.4 below:

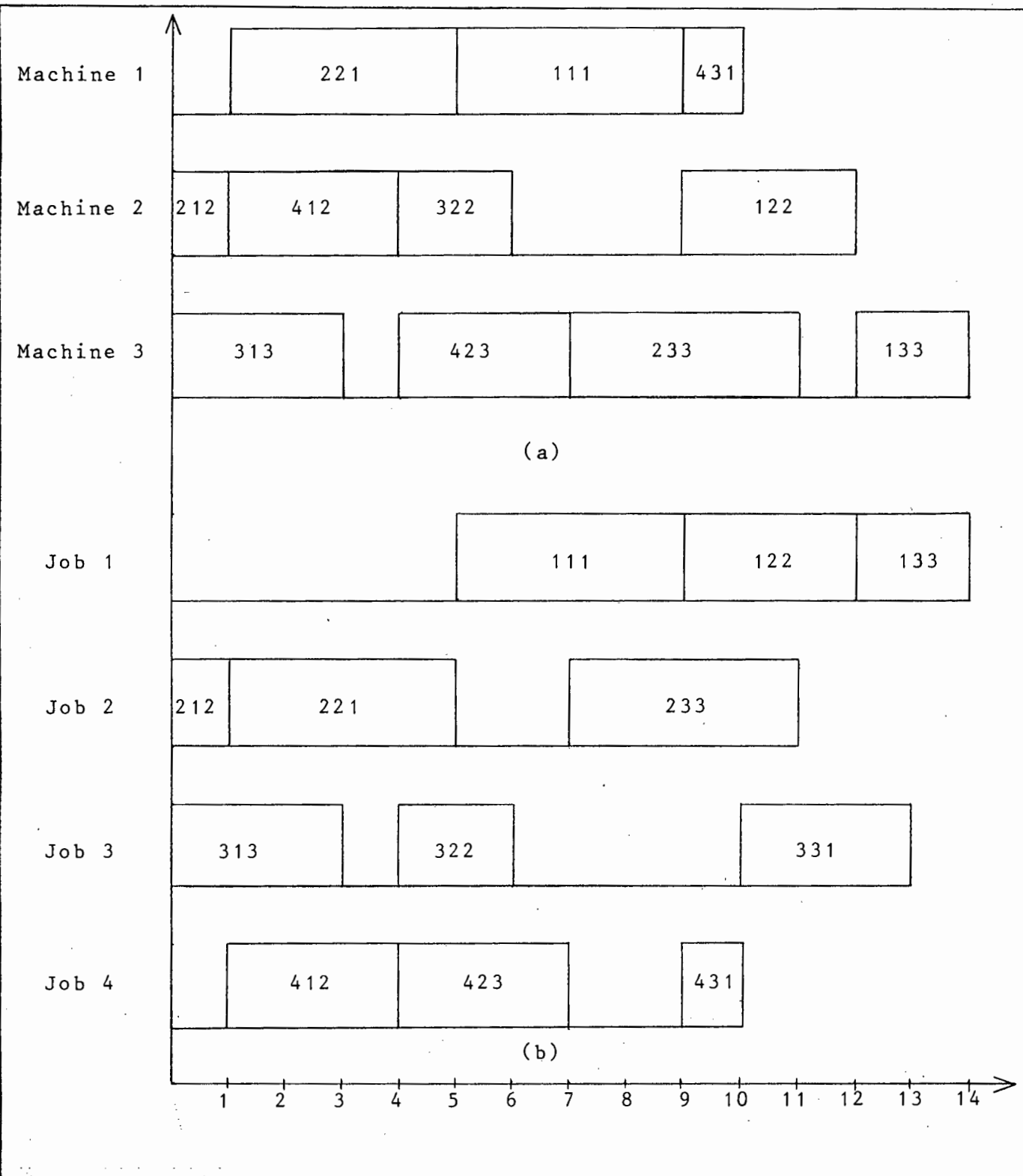


FIGURE 5.4 : EXAMPLE OF A FEASIBLE SCHEDULE
 : (a) Machine Gantt Chart : (b) Job-by-Job Chart

To complete the problem description, a measure of performance must be specified. This measure is used as a yardstick for the evaluation of alternative schedules. The machine shop problem now becomes one of constructing a schedule that is feasible and which optimises the measure of performance.

CHAPTER 6

MEASURES FOR SCHEDULE EVALUATION

6.1 INTRODUCTION

In this chapter, the measures of performance needed for the evaluation of the feasible schedules associated with the general job shop problem (as discussed in Chapter 5), are derived. These derivations are formulated around the objectives of the scheduling system, together with the variables which define the job shop scheduling problem. Both formulation elements are stated and described, with the resultant performance measures being placed into categories according to function.

Based on these derivations, a measure of performance can be chosen for Atlantis Aluminium. This choice is the final parameter required for the complete classification of the machine shop.

6.2 SCHEDULING OBJECTIVES

For a particular general job shop, it is not a simple matter to state the objectives required in scheduling, as they are numerous, complex and often conflicting. Mellor (Reference 41) lists 27 distinct scheduling goals and it has subsequently been shown that even with the simplest of these objectives, the mathematics of the general job shop problem becomes extremely difficult. Consequently this chapter is based upon those scheduling objectives that have been used in practical job shops and proved successful.

These objectives are:

- 1) Keeping promised delivery dates. If delivery dates are not met, financial penalties are usually incurred. In addition, constantly missing due dates or excessive lateness, will lead to a loss of goodwill, damaging the company image irreparably
- 2) Minimising the overall length of the scheduling period. In this case, once all jobs are completed, the machines which were tied up in that scheduling period may be released
- 3) Minimising the time for which machines are idle, as idle machines mean idle capital investment
- 4) Minimising inventory costs. (This includes the cost of storing raw material, work-in-progress jobs and finished jobs)
- 5) The maintenance of a uniform rate of production activity throughout the scheduling period, stabilising demands for labour and machine capacity

6.3 SCHEDULING VARIABLES

In the scheduling problem, it is important to clearly distinguish between variables which are assumed to be given by some external agency, and those variables that describe the solution produced by the scheduling process. Notation used in this chapter will be lower case letters for the given variables and capital letters for those determined by scheduling.

6.3.1 Variables that Define a Scheduling Problem

The problem description starts with a shop and a set of jobs:

- i) n - Jobs are identified by intergers 1, 2, ...; n
- ii) m - Machines are identified by intergers 1, 2, ...; m

The characteristics of job J_i are denoted by:

- iii) r_i - the ready or release time, is the time at which the job is released to the shop by some external job generating process. It is significant as the earliest time that processing of the first operation of the job can begin
- iv) d_i - the due date or the promised delivery date. It is the time by which the job ideally should be completed
- v) a_i - the allowance for J_i . This is the period allowed for processing between the ready time and the due date

$$\text{i.e. } a_i = d_i - r_i$$

- vi) p_{ij} - the processing time for the j th operation of job i (with k being omitted for simplicity). In the case of an industrial job shop, a job will consist of a batch of identical pieces each of which is to be processed in the same manner. The batch size is the number of pieces in the batch and the cycle time is taken on one piece. The symbol p_{ij} will thus be used to denote the total time needed to perform the operation, i.e. the cycle time multiplied by the batch size
- vii) g_i - the number of operations for job i , i.e. operation from 1, 2, 3, ..., g_i for J_i
- viii) p_i - sum of processing times for job J_i , i.e.
- $$p_i = \sum_{j=1}^{g_i} p_{ij}$$

6.3.2 Variables that Describe the Solution to a Scheduling Problem

The sequencing of jobs before machines simply determines when each operation of each job should be done. It is equivalent to determining how long each operation of each job should wait before processing begins and is denoted by:

- i) W_{ij} - the waiting time preceding the j th operation of job i . It is the time that job J_i must wait after completion of processing of the $(j-1)$ operation till processing of the j th operation begins
- ii) W_i the total waiting time. It is the sum of waiting times for all the operations for job i

$$\text{i.e. } W_i = \sum_{j=1}^{g_i} W_{ij}$$

The resultant schedule which is generated by the scheduling process is completely specified by giving a set of W_{ij} . Several important variables may be derived from the W_{ij} :

- 1) the time at which jobs leave the shop
- 2) the length of time that particular jobs spend in the shop
- 3) the difference between the times when jobs leave the shop and when they were supposed to leave

These variables are defined as:

- iii) C_i = the completion time of job J_i . The time at which processing of the last operation is completed

$$\begin{aligned} \text{i.e. } C_i &= r_i + W_{i,1} + p_{i,1} + W_{i,2} + p_{i,2} + \dots + W_{i,g_i} + p_{i,g_i} \\ &= r_i + \sum_{j=1}^{g_i} p_{ij} + \sum_{j=1}^{g_i} W_{ij} \\ &= r_i + p_i + W_i \end{aligned}$$

iv) F_i = The flow time of job J_i . The total time that job J_i spends in the shop

$$\begin{aligned} \text{i.e. } F_i &= W_{i,1} + p_{i,1} + W_{i,2} + p_{i,2} + \dots + W_{i,g_i} + p_{i,j_i} \\ &= \sum_{j=1}^{g_i} W_{ij} + \sum_{j=1}^{g_i} p_{ij} \\ &= W_i + p_i \end{aligned}$$

and from (iii) above

$$= C_i - r_i$$

v) L_i - The lateness of job J_i

$$\begin{aligned} L_i &= C_i - d_i \\ &= F_i - a_i \end{aligned}$$

Note that when a job is early, i.e. when it completes before its due date, L_i is negative (denoting the earliness of the job). In order to deal with variables which handle only non-negative values, L_i is defined as:

$$\text{vi) } E_i = \max(-L_i, 0)$$

Similarly, when a job completes after its due date (positive lateness) it is said to be tardy. Job tardiness is defined as:

$$\text{vii) } T_i = \max(L_i, 0)$$

viii) idle time on machine M_k is defined as

$$I_k = C_{\max} - \sum_{i=1}^n p_{ik}$$

where C_{\max} is the time when all processing ceases and

$\sum_{i=1}^n p_{ik}$ is the total processing time on machine

M_k

Variables which indicate the number of jobs in various states of completion at any given time are now introduced.

- ix) $N_w(t)$ - the number of jobs waiting between machines or not ready for processing at time t
- x) $N_p(t)$ - the number of jobs actually being processed at time t
- xi) $N_c(t)$ - the number of jobs completed by time t
- xii) $N_u(t)$ - the number of jobs still to be completed by time t

It follows from these definitions that:

- 1) $N_w(t) + N_p(t) + N_c(t) = n_T$
- 2) $N_w(t) + N_p(t) = N_u(t)$ for all t
- 3) $N_u(0) = n$
- 4) $N_u(C_{\max}) = 0$

6.4 MEASURES OF PERFORMANCE

Based on the above definitions of scheduling variables, measures of performance which encompass the general objectives as stated in Section 6.2 earlier, can now be identified. These measures or criteria have been placed into three categories as detailed below:

6.4.1 Performance Criteria Based Upon Completion Times

The important criteria in this category are:

- 1) F_{\max} - minimising the maximum flow time implying that a schedule cost is directly related to its longest job

- 2) C_{\max} - minimising the maximum completion time is used in the case where the cost of the schedule depends on how long the shop is devoted to the entire set of jobs
- 3) \bar{F} - minimising the average flow time is used where the schedule's cost is directly related to the average time it takes to process a single job
- 4) \bar{C} - minimising the average completion time is used where the schedule's cost is directly related to the average time it takes to complete each job

6.4.2 Performance Criteria Based Upon Due Dates

As the cost of a schedule is related to tardiness (positive lateness), obvious measures of performance are:

- 1) \bar{L} , the mean lateness
- 2) L_{\max} , the maximum lateness
- 3) \bar{T} , the mean tardiness
- 4) T_{\max} , the maximum tardiness

Minimising either \bar{L} , or L_{\max} is appropriate when there is a positive reward for completing a job early, and that reward is larger the earlier the job is completed.

Minimising \bar{T} or T_{\max} is appropriate when early jobs bring no reward, there are only the penalties incurred for late jobs.

6.4.3 Performance Criteria Upon the Inventory and Utilization Costs

Important criteria to minimise in order to optimise inventory carrying costs are:

\bar{N}_w - the mean number of jobs waiting for machines

\bar{N}_u - the mean number of unfinished jobs

Both these criteria are roughly related to the in-process inventory costs. Measures which minimise the inventory cost of finished goods can also be considered, i.e.:

\bar{N}_c - the mean number of jobs completed

Turning now to the efficient utilization of machines, measures to be considered are:

Maximise \bar{N}_p - the mean number of jobs actually being processed at any time

or minimise \bar{I} or I_{\max} - the mean or maximum machine idle time.

6.5 REGULAR MEASURES OF PERFORMANCE

An important classification of the measures of performance described in the previous section is into those that are regular and those that are not. A regular measure of performance R is one that is non-decreasing in the completion times. Thus R is a function of $C_1, C_2, C_3, \dots, C_n$ such that

$$C_1 \leq C'_1; C_2 \leq C'_2; \dots; C_n \leq C'_n$$

if these are taken together it is implied that $R(C_1; C_2; \dots, C_n) \leq R(\bar{C}_1; \bar{C}_2; \dots; \bar{C}_n)$

French (Reference 9) shows that \bar{C} , C_{\max} , \bar{F} , F_{\max} , \bar{L} , L_{\max} , \bar{T} , T_{\max} and n_T (number of tardy jobs) are all regular measures of performance.

6.6 THE MEASURE OF PERFORMANCE FOR ATLANTIS ALUMINIUM

The selection of the measure of performance to be used as part of the scheduling system for the machine shop was made by the author in conjunction with the management of Atlantis Aluminium. Management was made aware of the scheduling objectives and associated performance measures which have been stated and discussed in this chapter.

Based on this information, the considered and unanimous decision of the management team was that as the production at Atlantis Aluminium is virtually totally dedicated to A.D.E. components, the meeting of these externally set due dates should be given top priority. Consequently, the measure of performance chosen for Atlantis Aluminium is the meeting of due dates (more specifically, minimising \bar{T} or T_{\max}) leading to the full classification of the machine shop as follows:

15/10/G/meeting job due dates (min \bar{T} or T_{\max})

CHAPTER 7

SCHEDULE CHARACTERISTICS

7.1 INTRODUCTION

This chapter focusses on the feasible schedules which characterise the general job shop problems as discussed in Chapter 5. Schedules can be categorised into three groups which are particularly important, as they form the foundation of many of the methods used in solving the general job shop problem. For this reason, these categories of schedules are stated and discussed in some detail, together with the relationships between them.

7.2 CATEGORIES OF SCHEDULES

7.2.1 Semi-active Schedules

For every job shop problem an infinite number of schedules exists as idle time can be inserted between two operation-blocks on any machine. However, the insertion of unnecessary idle time is contrary to the objective of minimising the scheduling cost where operations are processed as quickly (i.e. compactly on the gantt chart) as possible. This unnecessary idle time is said to exist in a schedule if an operation can be started earlier without altering the operation sequence on any machine. Adjusting the start time in this way is equivalent to moving an operation-block to the left on the gantt chart, and is known as a local left-shift. It can be seen that for a fixed operation sequence on a machine, there is only one schedule in which no local left-shift can be made. The set of all schedules in which no local left-shift can be made, is called the set of semi-active schedules and is equivalent to the set of all schedules that contain none of the

unnecessary idle time described above. This set dominates the set of all schedules, which means that it is sufficient to consider only semi-active schedules to optimise any regular measure of performance.

The number of semi-active schedules in the set, although large, is at least finite. In the case of the classical job shop in which each job has exactly one operation on each machine, each machine must process n operations. The number of possible sequences for each machine is therefore $n!$, and if the sequences on each machine were entirely independent, there would be $(n!)^m$ semi-active schedules. If the classical job shop problem were to be made more realistic through the introduction of technological constraints, the effect would be to make some of the combinations unfeasible, thus reducing the number of feasible schedules.

7.2.2 Active Schedules

In semi-active scheduling the start of an operation is constrained by either the processing of a different job on the same machine, or by the processing of the directly preceding operation on a different machine. In the case of the former, where the completion time of an earlier operation on the same machine is constraining, it may still be possible to find a means of improvement.

Suppose that for the example used in Section 5.4 the job sequence 4-3-2-1 is used at each machine. The associated semi-active schedule is displayed in Figure 7.1

While no local left-shifts are possible in the schedule, an improved schedule can be devised. For instance, it is possible to start operation (1,1,1) earlier than at the time indicated without delaying any other operation. Indeed, operation (1,1,1) can be started at time zero, with all three operations of job 1 having the potential to start earlier, without delaying any of the other operations. On the gantt chart, such an alteration would correspond to shifting the first operation of job 1 to the left and beyond other operations already scheduled on machine 1. This type of adjustment in which an operation is begun earlier without delaying the start of any other operation, is called a global left-shift. The set of all schedules in which no global left-shift can be made is called the set of active schedules, and is clearly a subset of the set of semi-active schedules.

Just as the set of semi-active schedules dominates the set of all schedules, so the set of active schedules dominates the set of semi-active schedules. In other words, in optimising any regular measure of performance, it is sufficient to consider only the set of active schedules. This is due to the fact that the number of active schedules is a function of both the routing and the processing times for a given problem, whereas the number of semi-active schedules is a function solely of the routing. Thus while one semi-active schedule corresponds to each feasible combination of machine routing, many semi-active schedules can be compacted into the same active schedule through a series of left-shifts.

An alternative way of looking at the role of semi-active and active schedules is with the use of a Venn diagram. In Figure 7.2 which follows, the contents of the rectangle represent the infinite set of all schedules.

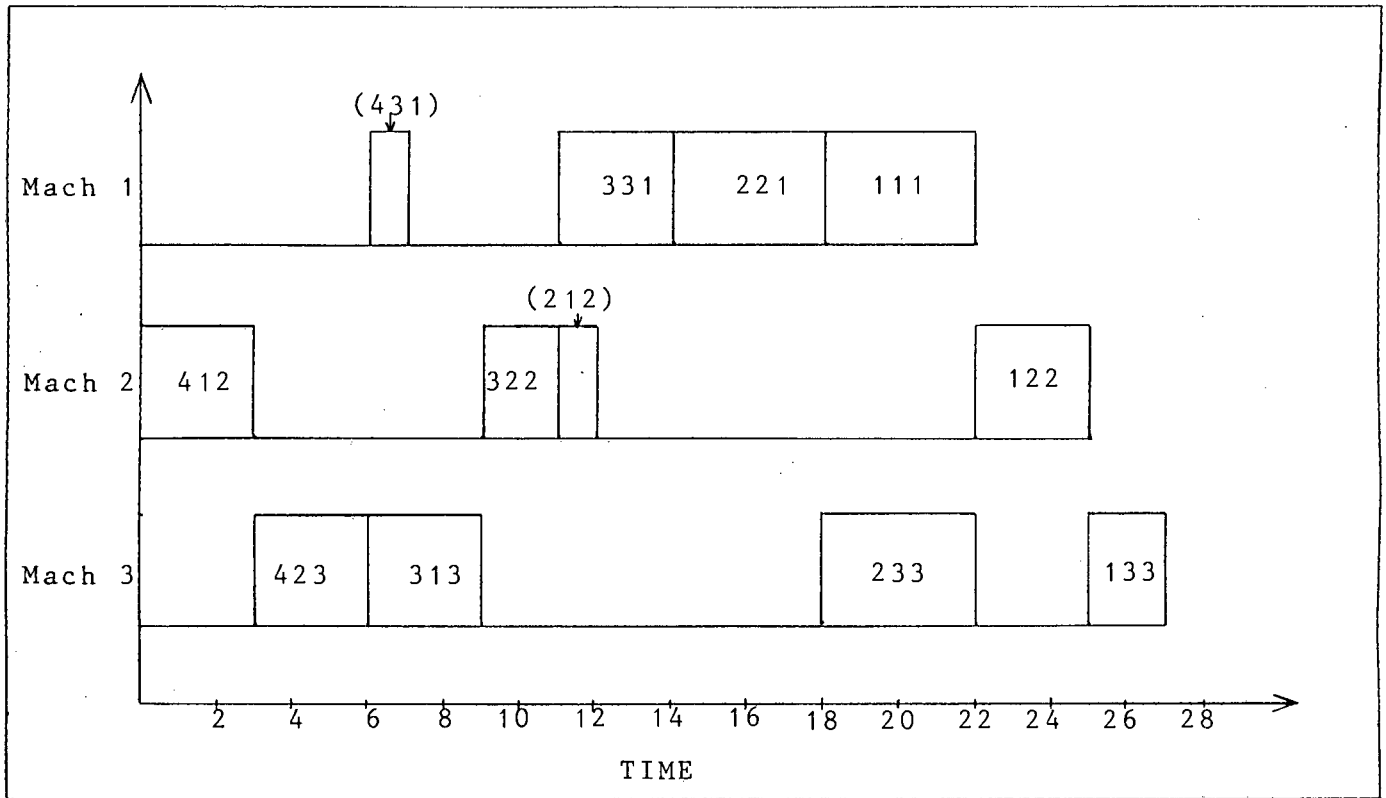


FIGURE 7.1 : THE SEMI-ACTIVE SCHEDULE FOR EXAMPLE OF SECTION 5.4

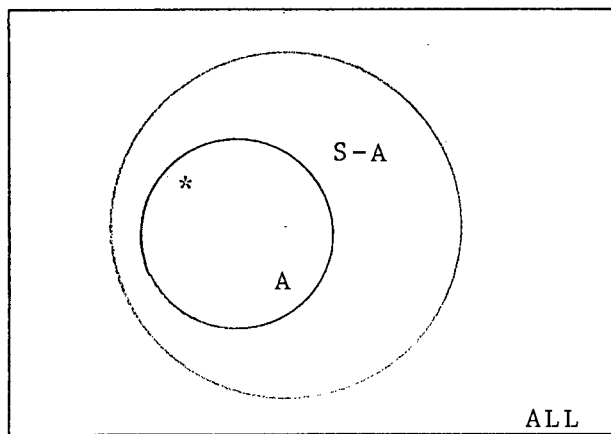


FIGURE 7.2 : A VENN DIAGRAM OF SEMI-ACTIVE AND ACTIVE SCHEDULES

The interior of the region labelled (S-A) represents the finite set of semi-active schedules. Contained in the semi-active region is the set of active schedules (A) with the asterisk indicated in this set representing some optimal schedules. There is at least one such optimum which is always found in this subset.

7.2.3 Non-delay Schedules

In large job problems (approximately > 5 machines), the set of active schedules reaches unmanageable proportions, and it becomes convenient to focus on a smaller subset, called the non-delay schedules. Non-delay schedules are obtained by specifying that no machine is kept idle at a time when it could begin processing some operation. For example, consider the schedule given in Figure 7.3 below:

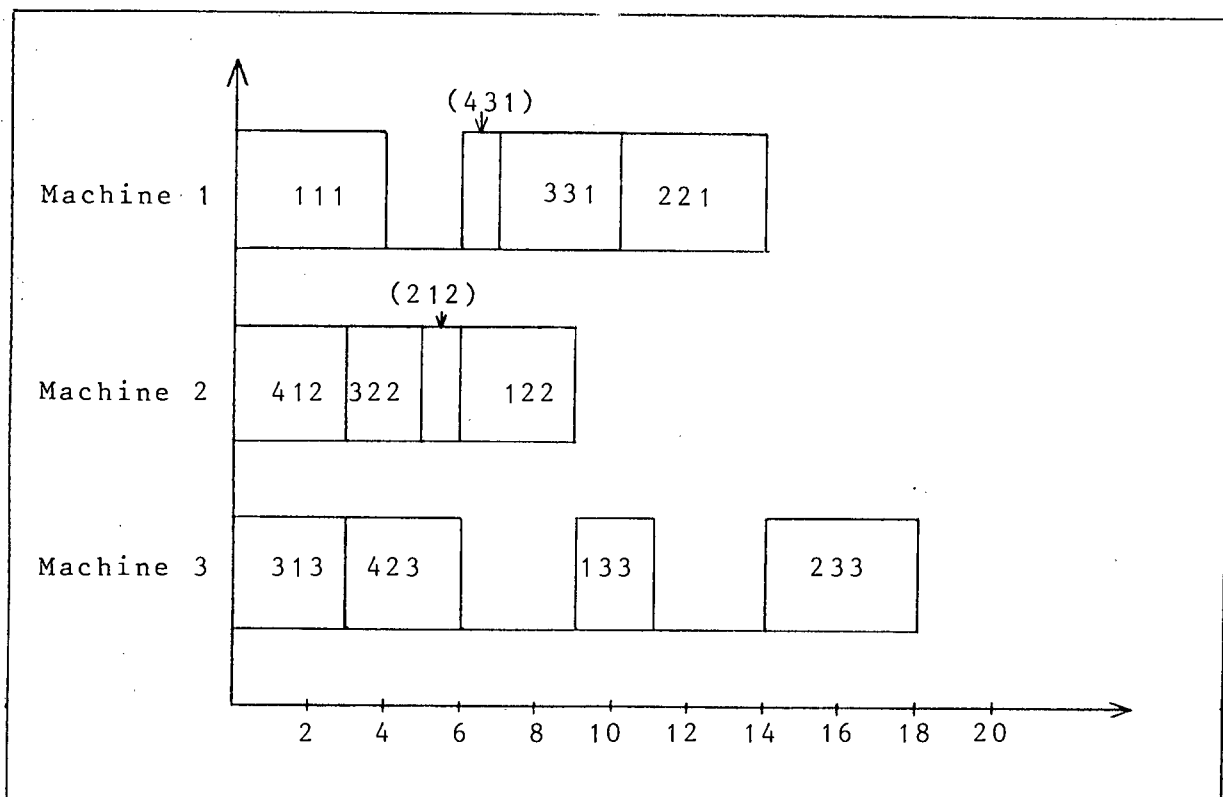
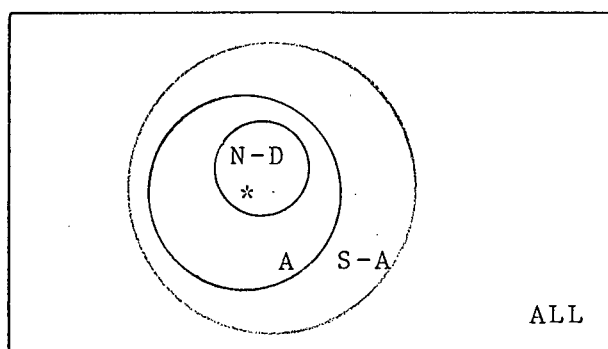


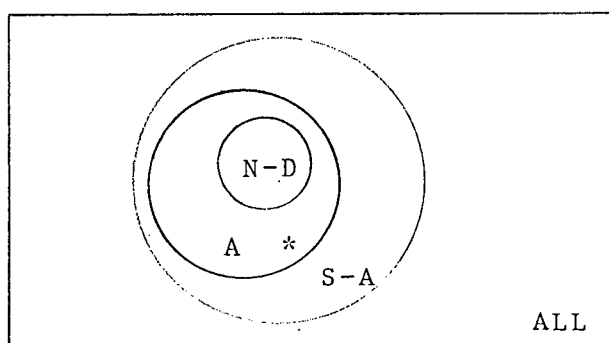
FIGURE 7.3 : EXAMPLE OF SCHEDULE FOR APPLYING THE NON-DELAY PRINCIPLE

Note that machine 1 remains idle at time 5 when it could start processing of operation (3,3,1). Consequently, the schedule shown is not a non-delay schedule.

All non-delay schedules are also active schedules since no left-shifting would be possible. On the other hand, many active schedules may not be non-delay schedules which results in there being significantly fewer non-delay schedules than active schedules. The dilemma here is that although there are fewer non-delay schedules than active schedules, the set of non-delay schedules is not guaranteed to contain an optimal solution. This situation is illustrated using Venn diagrams in Figure 7.4 below:



(a)



(b)

FIGURE 7.4 : VENN DIAGRAMS ILLUSTRATING THE RELATIONSHIP OF NON-DELAY SCHEDULES TO ACTIVE AND SEMI-ACTIVE SCHEDULES

Figure 7.4(a) depicts the situations where at least one optimal solution is a non-delay schedule, while in Figure 7.4(b), no optimal solution is a non-delay schedule.

In summary then, the smallest set of dominant schedules (containing an optimal solution) is that of active schedules. The set of non-delay schedules, although smaller in number, is not guaranteed to contain an optimum. However, as will be discussed in a following section, the best sub-optimal non-delay schedules can usually provide a very good solution which is quite close to the optimum.

CHAPTER 8

METHODS OF SOLUTION8.1 INTRODUCTION

In the final stage of the scheduling system design process for the machine shop at Atlantis Aluminium, the job shop was classified as a 15 job/10 machine general job shop with the associated measure of performance being the meeting of due dates. The second stage of the design process involved carrying out a survey of the relevant scheduling literature based upon the classification of the machine shop. The results of this survey are detailed and discussed in this chapter.

8.2 CONSTRUCTIVE METHODS8.2.1 Constructive Algorithms

A constructive algorithm is a method for building up an optimal schedule from the data of a problem. This is done by following a simple set of rules which exactly determines the processing order. To illustrate this concept, consider the following two-machine job shop problems:

$$n/2/G/F_{\max}$$

This problem, which is restricted in that each job has at most two operations, can be solved using the constructive algorithm developed by Johnson (Reference 38) and given in Appendix D.

Indeed, the research indicates that the only constructive algorithm which is applicable in all cases and without restriction is the two machine problem algorithm described earlier. Several researchers, notably Szwarc (Reference 45) and Panwalker and Khan (Reference 44) have reported on further work with constructive algorithms. Their results,

although applicable mainly to flow shops, further serve to indicate the poorness of this approach for the application to general job shop problems.

8.2.2 Graphical Construction Methods

A graphical procedure for the solution of the general job shop problem was first proposed by Akers (Reference 25). The procedure was developed for the two-job shop problem ($2/M/G/F_{\max}$), and quite favourable results have been achieved under these conditions. However, due to the graphical nature of the method, it is confined to applications of two dimensions, with generalisations for three and more jobs becoming increasingly unwieldy. Consequently, no detailed description of the graphical method is provided here.

8.3 ENUMERATION METHODS

As discussed in Section 8.2, constructive methods which find optimal solutions directly by building up schedules from the data of a problem, are very limited in their application to the general job shop problem.

The next method to be considered for finding optimal solutions is that of enumeration. Enumeration methods simply list or enumerate all possible schedules that have been generated, and then eliminate those that are non-optimal from the list. The schedules that remain are then optimal.

The two enumeration methods investigated were:

- 1) Complete Enumeration
- 2) Branch and Bound

and are discussed below.

8.3.1 Complete Enumeration

Complete enumeration is the most obvious and extreme case of the enumeration methods. This method enumerates each of the possible schedules for any n job/ m machine job shop problem. As discussed in Chapter 7, these possible schedules are known as the set of the semi-active schedules and tend to be very big. The upper limit on the number of schedules in the case of the classical general job shop is $(n!)^m$, leading to the situation where there are enormous quantities of schedules to be evaluated even for small problems.

For example, consider the 5 job/5 machine job shop case. Here the number of possible schedules is:

$$(5!)^5 = 2,488 \times 10^{10}.$$

In the case of Atlantis Aluminium (15 job/10 machine case), the number of possible schedules is:

$$(15!)^{10} = 1,4623 \times 10^{113}$$

If it is assumed that these schedules are to be evaluated using a computer capable of processing 100 000 schedules per second (as in the study of Rinnooy Kan (Reference 20), the time required to evaluate the $n = m = 5$ problem above would be 3 days. In the case of the Atlantis Aluminium problem, the time required to perform a complete enumeration procedure would be $4,637 \times 10^{113}$ years.

Although these figures are based on the classical job shop problem, a reduction in the number of schedules which would be accomplished with the introduction of technological constraints, would still result in an astronomical figure. Indeed, Conway et al. (Reference 8) stated that: "... it is hardly worth very much effort to find such an upper bound or estimate since the only practical use would be to demonstrate the enormous size of the problem and to discourage attempts at enumeration."

Even for problems of smaller size (e.g. 4/4), it would be ludicrous to devote expensive computer time for the solution of the general job shop problem by complete enumeration. Furthermore, if this were indeed contemplated, it is doubtful whether the necessary computer capacity would be available.

In conclusion then, only job shop problems of a very small size (3/3) would be worth solving by the complete enumeration technique, ruling out the solution of the larger and more realistic problems.

8.3.2 The Branch and Bound Method

The second enumeration technique which could theoretically be utilised to solve the general job shop problem is the

Branch and Bound method. However, unlike complete enumeration, it does not evaluate every possible schedule. Rather the schedules are evaluated in an intelligent manner exploring and determining, during the process of solution, which possibilities would be more likely to produce an optimal schedule, while disregarding those which indicate unlikely possibilities. For this reason, the Branch and Bound method is an implicit enumeration technique. Implicit in its logic is the checking of every possible schedule, but unlike complete or explicit enumeration, it does not consider every possibility explicitly.

The purpose of this section is to present and discuss the salient features and operational characteristics of the Branch and Bound method. Detailed explanations of the workings of the method and applicable algorithms can be found in Bellman et al. (Reference 5).

8.3.2.1 Features of the Branch and Bound Method

Use of the Branch and Bound method is based upon four important features which are stated and then discussed below:

- 1) the elimination or enumeration tree
- 2) the bounding function
- 3) the trial schedule
- 4) the search strategy

1) The Elimination Tree

Every scheduling problem can be visualised as having its own unique elimination or enumeration tree. Each branch of the enumeration tree corresponds to a particular feasible schedule. To illustrate this concept, consider the simple case of the 4/1/C-D. problem (i.e. 4 jobs to be scheduled on one machine). A schedule for this problem corresponds to an

assignment of a different job to each of the four positions in the processing sequence. Thus, the number of schedules is $n! = 24$. The 24 possible combinations can be shown graphically as the hierarchal branching structure in Figure 8.1.

The construction of a schedule is started with no job sequenced, indicated by the point or node (----). An (-) in the processing order denotes that no job has yet been assigned to that position. The next stage is to assign a job to the first position in the sequence. As there are four jobs there are four possibilities (1----, 2----, 3----, 4----) i.e. each branch which descends from a node represents the selection of one of the competing operations.

A second job is now assigned. As one job has already been assigned, there are three possibilities for each branch. For example, consider the first branch (1----). It has already been assigned, thus any of the three remaining jobs 2, 3 or 4, can be assigned to the second position in the processing sequence. This assignment procedure is carried out until each schedule has been completed. The resultant branching structure which is obtained is called a tree and as the enumeration methods investigate or enumerate some or all of the branches, it is called an enumeration tree. Further, as the tree is used to eliminate non-optimal schedules, it is also called an elimination tree. It can also be seen from the enumeration tree, that each node corresponds to a subset of schedules. For example, the node (1----) represents the subset of schedules which have job 1 first in the processing sequence.

The construction of the enumeration tree is the first step of the Branch and Bound method and forms the basis for all further manipulation.

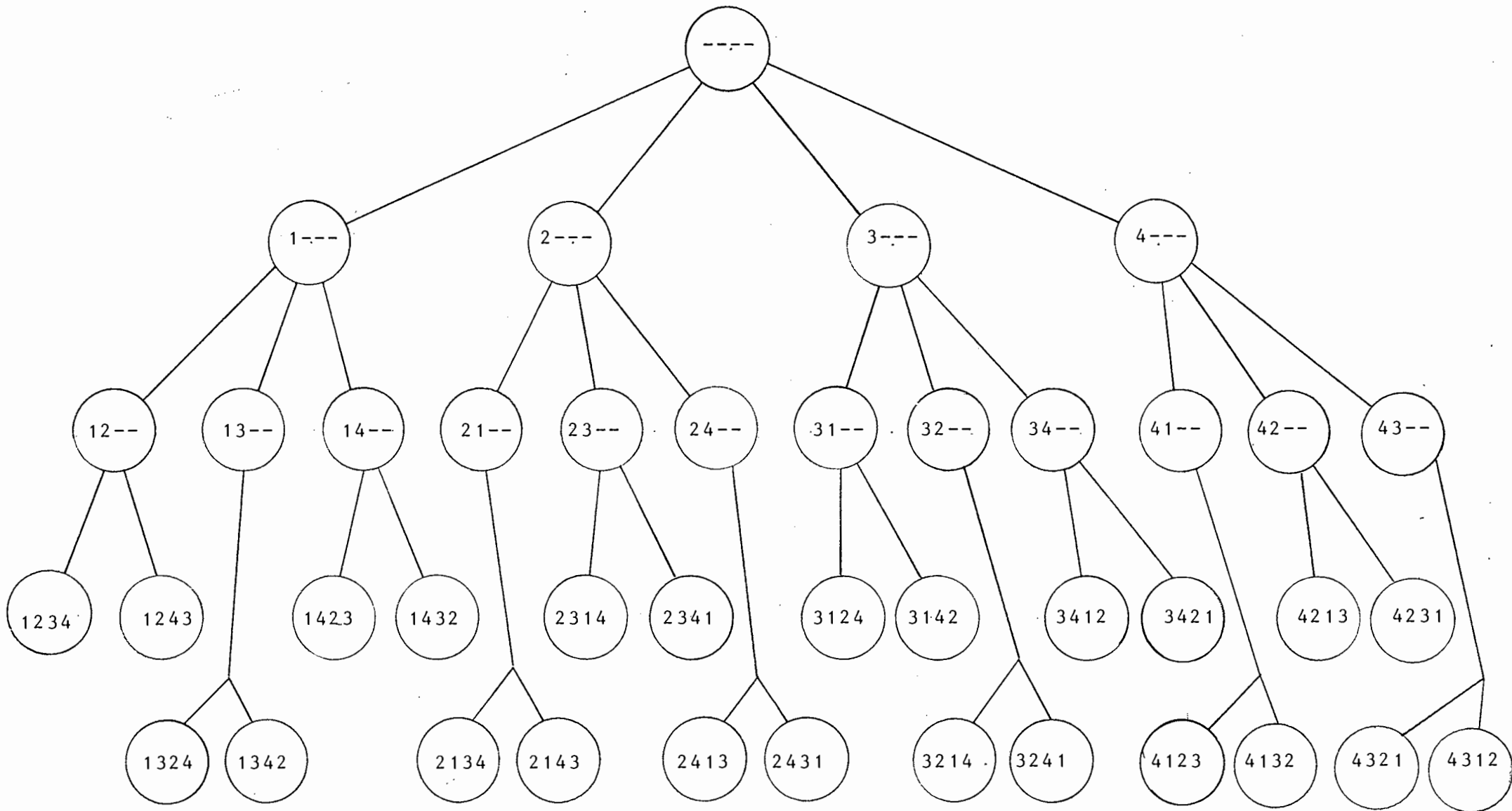


FIGURE 8.1 : THE ELIMINATION TREE FOR THE 4/1/C/D PROBLEM

8.3.2.2. The Bounding Function

The second requirement for the Branch and Bound method is a bounding function. The bounding function serves as the mechanism for the selection of those promising branches which will be evaluated further.

Brooks and White (Reference 24) have developed lower bounds known as job-based and machine-based bounds, for the maximum flow time measure of performance. The selection of a particular branch is based on the minimum value of a lower bound, determined in terms of maximum flow time, for each possible branch. The Brooks and White approach is given here in order to illustrate typical lower bound calculations and procedures.

Job-based bounds: where it is assumed that there is no resource conflict when the remaining processing for all jobs is scheduled as compactly as possible

procedure - For each job, find the earliest time at which it could possibly start its next scheduled operation. Add to this the sum of the processing times of all the unscheduled operations of the job. Take as a lower bound the maximum of these quantities. Take the minimum of these lower bounds as the job-based bounds

Machine-based bounds : here it is assumed that there is no logical or precedence conflict when the remaining processing for all machines is scheduled as compactly as possible

procedure - For each machine, find the minimum time at which an unscheduled operation could be started. Add to this the sum of the processing which requires that machine. Take as a lower bound the maximum of these

quantities. Take the minimum of these lower bounds as the machine-based bounds .

The final step is to take the maximum of the job-based bounds and the machine-based bounds as the final lower bound. An example of the procedure and its associated calculations is shown in Appendix E.

In general, the bounding function can be stated mathematically by the following. If Y is the subset corresponding to a particular node, a lower bound (lb) can be calculated on the performance measure for all schedules in Y . If $c(y)$ is the value of the performance measure for a schedule y , then it is required that:

$$lb(Y) \leq c(y) \text{ for all } y \text{ in } Y$$

8.3.2.3 The Trial Schedule (y^*)

This schedule may not be set initially, but at some point in the procedure it becomes set to the schedule which has yielded the best value of the performance measure found so far.

8.3.2.4 Search Strategy

The search strategy is simply the method which is used in searching the tree. The two methods that are commonly used are: 1) the depth-first search strategy and, 2) the frontier search strategy.

No detailed descriptions of these strategies are given here. Their effects, however, are given in the following section.

8.3.3 The Branch and Bound Procedure

The Branch and Bound procedure can best be explained by noting that, at any stage, the nodes of the elimination tree can be classified into three classes:

Z_1 - the class of nodes which have either been eliminated or fully explored. A fully explored node is either a terminal node for which the performance measure has been evaluated exactly, or a node in the main body of the tree for which lower bounds have been evaluated at all nodes immediately beyond it

Z_2 - the class of partially explored nodes. The lower bound has been evaluated for any node in this class, but the node itself has neither been eliminated nor fully explored

Z_3 - the class of unexplored nodes. These nodes have neither been eliminated implicitly by the elimination of a node that precedes them in the tree, nor been examined so far by the search procedure

At the beginning of a Branch and Bound procedure, all the nodes are in class Z_3 . As the procedure progresses, nodes are moved either directly to class Z_1 or via the intermediate class Z_2 . The procedure ends with all nodes in Z_1 with the trial schedule y^* taken as being optimal.

The movement of nodes from Z_3 to Z_1 is illustrated by considering a stage in the Branch and Bound solution. The search procedure selects a pair of nodes Y and Y_i , such that Y_i lies directly beyond Y in the tree. Furthermore, Y lies in Z_2 and Y_i in Z_3 . In other words, the tree branches from node Y to nodes $Y_1; Y_2; Y_v$ with Y_i being one of those subsequent nodes which has yet to be explored. When Y and Y_i have been selected, there are two possibilities:

- 1) Y_i contains more than one schedule. Here the lower bound $lb(Y_i)$ is calculated and if $lb(Y_i) \geq c(y^*)$, the best value of the performance measure to date, then Y_i and the nodes that lie beyond it are eliminated. These nodes thus move directly from Z_3 to Z_1 . If

$lb(Y_i) \leq c(y^*)$, Y_i cannot be eliminated and is moved from Z_3 to Z_2 .

- 2) Y_i contains exactly one schedule i.e. $Y_i = (y)$. Here $c(Y)$ is calculated and if $c(y) \geq c(y^*)$, Y_i is eliminated and moved from Z_3 to Z_1 . If $c(y) \leq c(y^*)$ then y becomes the new trial schedule. In this case, all the nodes in Z_2 must be examined to see if they are eliminated by the value of the performance measure for this new trial schedule.

8.3.4 Practical Applicability of the Branch and Bound Method

As stated in Section 8.2, the solution of practical job shop problems by complete enumeration is computationally unfeasible for two reasons. Firstly, the technique requires many quantities to be remembered during the procedure, and secondly, it takes prohibitively long to solve medium to large problems. These reasons now serve as the evaluation criteria for the Branch and Bound method.

French (Reference 9) in his research, shows that in the case of the depth-first search strategy, the maximum storage requirements are fixed and are only a small percentage of that which would be required for complete enumeration. In the case of the frontier search strategy, the storage requirements are completely unpredictable as theoretically the size of Z_2 may become extremely large during the solution of some problems. However, practical experience gained with these problems, as detailed in French (Reference 8), has shown that this does not seem to happen, and that frontier searches may be used with only a small risk that they will exhaust available capacity.

The number of operations required and hence the time required to solve a problem by the Branch and Bound procedure is, however, unpredictable whatever search strategy is used. It might happen that the procedure has to explore fully virtually every node, in which case it would

take as long as complete enumeration. Indeed, it might take even longer, because the Branch and Bound procedure involves more computation per node than complete enumeration.

Nevertheless, in general, Branch and Bound does perform a great deal better than complete enumeration. It should not be assumed from this, however, that it can solve any problem in practice, as although theoretically it always finds an optimal solution, it may take prohibitively long to do so. For instance, the 10/10/G/D problem proposed by Muth et al. (Reference 34) still has to be solved optimally.

In conclusion then, the Branch and Bound procedure is ruled out as a method of solution for the general job shop problem because of its unpredictable nature and doubt as to its applicability to the large job shop problems such as that at Atlantis Aluminium.

8.4 INTEGER PROGRAMMING

A sizeable body of literature exists which suggests the use of mathematical programming techniques for solving the general job shop problem. In general, mathematical programming techniques model real world problems in which the objective is to optimise a value criterion. Optimisation of the value function may take the form of maximisation in the case of profit, benefit, revenue and efficiency or minimisation in the case of costs, time and effort. The optimisation is restricted as the problem is subject to resource constraints (manpower, raw material and machines).

These problems can be stated mathematically as:

$$\text{Minimise (or maximise) } f(x_1; x_2; \dots; x_n)$$

with respect to $(x_1; x_2; \dots; x_n)$, subject to the constraints

$$g_1(x_1; x_2; \dots; x_n) \leq b_1$$

$$g_2(x_1; x_2; \dots; x_n) \leq b_2$$

$$g_k(x_1; x_2; \dots; x_n) \leq b_k$$

The concepts stated above can be applied to the production scheduling problem in which the objective is to optimise the measure of performance subject to the technological constraints on the allowable processing order.

The mathematical programming technique which is most commonly used for modelling the general job shop problem, is integer programming, more specifically mixed-integer programming. This type of programming is particularly suited to this application as some of the variables (e.g. machines) have to be integers (whole numbers without

fractional or decimal parts). Some of the variables can also have fractional allocations (e.g. time). Furthermore, the functions f and $(g_1; g_2; \dots; g_k)$ are linear.

The standard problem takes the form:

$$\text{Minimise } C_1 x_1 + C_2 x_2 + C_3 x_3 + \dots + C_n x_n$$

subject to constraints

$$\begin{aligned} g_{11} x_1 + g_{12} x_2 + g_{13} x_3 + \dots + g_{1n} x_n &\leq b_1 \\ g_{21} x_1 + g_{22} x_2 + g_{23} x_3 + \dots + g_{2n} x_n &\leq b_2 \end{aligned}$$

$$g_{k1} x_1 + g_{k2} x_2 + g_{k3} x_3 + \dots + g_{kn} x_n \leq b_k$$

with some of the $(x_1; x_2; x_3; \dots; x_n)$ variables restricted to being integral values.

Several researchers have developed the idea of using mixed-integer programming for modelling the general job shop. A number of models have been developed, notably those of Bowman (Reference 28), Wagner (Reference 47) and Manne (Reference 40). Appendix F describes the model developed by Manne.

After formulation of the problem, as described above, methods have to be employed in the solution thereof. These methods are reviewed in Garfinkel and Nemhauser (Reference 11) and can be broadly classified into either implicit enumeration (in particular the Branch and Bound method) and cutting plane techniques (Gomory method). Both these techniques require much computation and, furthermore, both are based upon the properties of integer programmes in general, paying no regard to the particular properties of the problem being solved. As a result, these techniques tend to take longer to find a solution than implicit enumeration algorithms, designed specifically for a particular class of problem. For example, consider two

methods of solving the scheduling problem. Firstly, it may be translated into an integer programme and solved by Branch and Bound with bounds based upon general integer programming theory. Secondly, the problem could be tackled directly by Branch and Bound, with bounds based upon knowledge of the physical properties of schedules. The lower bounds found in the first case, are usually poorer than those found in the second case, with the Branch and Bound search being correspondingly larger. Consequently, it would be better to approach scheduling problems directly rather than indirectly via integer programming.

A second disadvantage associated with the use of the integer programming approach for modelling the general job shop is the number of variables and constraint equations required for realistic job shops. For the classical job shop problem, Appendix F gives:

the number of variables as : $mn + m \frac{n(n-1)}{2}$

and constraint equations : $mn(n-1) + (m-1)n$

Thus, in the case of 10 job/4 machine problems, the number of variables would be:

$$4 \times 10 + 4 \frac{10(10-1)}{2} = 220 \text{ variables}$$

and constraint equation : $4 \times 10(10-1) + (4-1)10 = 390$
constraint equations

In the practical case of Atlantis Aluminium (15 job/10 machine problem):

the number of variables = 1200

the number of constraint equations = 2235

As stated in Appendix F, the constraint and variable equations are based on the classical job shop. By modelling

more realistic problems, the number of variables and constraint equations will increase still further, leading to a bigger formulation problem.

In summary then, the use of mixed-integer programming for solving the general job shop problem is deemed to be inappropriate, firstly because of the indirect and thus slower approach, and secondly, because of the enormous size of the formulated integer programme.

8.5 HEURISTIC METHODS

The procedure followed in the search for a solution to the machine shop scheduling problem was to analyse firstly constructive algorithms, and secondly, implicit enumeration techniques. This analysis showed that constructive algorithms were only applicable for the simplest of problems, while the solution of large practical problems by implicit enumeration techniques was found to be unpredictable and computationally unfeasible.

Furthermore, the mathematical theory of NP-Completeness (which is not discussed) predicts that no constructive algorithms will ever be developed for the vast majority of scheduling problems. Further, large scheduling problems (the general job shop with more than two machines) are for all practical purposes insoluble (or NP-hard). These mathematical developments are discussed in some detail by Garey and Johnson (Reference 10).

This pessimistic outlook given by the theory, leads to the conclusion that practical job shop problems cannot be solved optimally, and a suboptimal solution will have to be accepted. The objective now is to consider techniques that would find a schedule which, if not optimal, may at least be expected to perform better than average. These techniques, called heuristic or approximation algorithms, are based upon the philosophy of schedule generation, and will be discussed in this section.

8.5.1 Schedule Generation

The techniques which are embodied in this philosophy span the spectrum between complete enumeration and heuristic solution, i.e. algorithms which produce all, some, or just one schedule of a particular class. If that class is sure to contain an optimal solution and all schedules in that class are generated, then the optimal solution is obtained

by complete enumeration. On the other hand, if either the class is not guaranteed to contain an optimal schedule, or not all the schedules in that class are generated, then only an approximate solution can be found.

All schedule generation techniques have a basic similarity

in that each operates on the set of $O = \left(\sum_{i=1}^n o_{ijk} \right)$ operations, selecting one at a time and assigning a ready time to each. The order in which the operations are selected and the manner in which the ready time is determined characterise a schedule generation technique. An important distinction is of that between single pass and adjusting techniques. In the former, the start time of an operation is permanently fixed the first time it is assigned, thus generating a full schedule with a single pass through the list of operations. In an adjusting technique, a start time is tentatively assigned and subject to repeated modification until the entire schedule has been completed. However, as discussed in Conway et al. (Reference 8), serious difficulties have been encountered with the application of adjusting techniques to practical scheduling problems, leading to almost exclusive use of single pass techniques in practice. All further discussion will thus be based on single pass techniques.

As discussed in Chapter 7, the two schedule types which are preferable for manipulation are active and non-delay schedules. The larger of the two, active schedules, will always contain an optimal solution whereas the smaller non-delay class does not guarantee that the best solution found is optimal (although the best solutions are always very close to being optimal). Methods used for the generation of active and non-delay schedules have been developed by Giffler and Thomson (Reference 33) and are discussed below.

8.5.1.1 Active Schedule Generation

Appendix G gives the detailed description of the active generation algorithm developed by Giffler and Thompson. Essentially, the method looks at the set of schedulable operations, determining for each constituent the earliest completion time (ϕ). The minimum of these completion times is then found and denoted by ϕ^* , with the machine on which it occurs denoted by M^* . Step 3 of the method examines all schedulable operations that need M^* and can start before ϕ^* . There is at least one such operation, namely the one that completes at ϕ^* .

Of these one is selected and scheduled as soon as possible. Further, if any of the other operations schedulable on M^* are started before the selected operation O_{ijk} , they must complete at some time ϕ^* thus delaying the start of O_{ijk} . In this way the algorithm is steadily building up an active schedule.

8.5.1.2 Non-delay Schedule Generation

The algorithm designed to generate non-delay schedules is described in Appendix G and is a modification of the active schedule generation technique. In this technique, the start times (σ) for the schedulable operations are examined and the minimum (σ^*) is found. The machine (M^*) on which the σ^* operation occurs is noted. Step 3 examines all the schedulable operations that need M^* and start at σ^* , and selects one of these operations which is then scheduled. In this way, a non-delay schedule is built up.

However, both the active and non-delay schedule generation algorithms are not fully defined as a choice amongst acceptable competing operations has to be made in step 3. On the one extreme, by making all possible choices at step 3, the complete set of active and non-delay schedules is generated. As stated earlier, however, the solution of job shop problems through complete enumeration is only

applicable in small problems. In the case of practical industrial problems, solution by complete enumeration has been found to be computationally unfeasible.

The algorithms used for the generation of active and non-delay schedules are based upon a branching structure, similar to the elimination tree (as discussed in Section 8.3.2.1). Here the nodes in the tree correspond to partial schedules, and each time a new operation is added to a partial schedule the algorithm proceeds from one level to the next. If the tree is constructed completely, then all active and non-delay schedules are enumerated under their respective algorithms. Given this tree-structured mechanism, the next obvious progression from complete enumeration would be to consider implicit enumeration, where the algorithms can be used as a basis for a Branch and Bound solution. As in the Branch and Bound method discussed in Section 8.3.2, potentially all possible sequences of choices have to be enumerated, with the choice in this case concerning the operation to schedule in step 3 of the generation algorithm. However, similar results to that of Section 8.3.2 have been achieved with Lageweg et al. (Reference 39) showing that even with the best bounding schemes available it becomes computationally unfeasible to apply these methods to practical problems.

Thus the problem of choosing an operation at step 3 cannot be avoided by simply making all possible choices. The objective now, is to be more selective. The methods of selection which have been analysed are:

- 1) Priority Dispatching Rules
- 2) Sampling Procedures
- 3) Probabilistic Dispatching Procedures

8.5.2 Priority Dispatching Rules

This procedure resolves the conflict on machine M^* (step 3) by calculating priorities for each of the competing jobs. The job with the highest priority is then selected and is processed next. In this way, each time a conflict arises, one job is selected, thus building up a single unique schedule.

Job priorities are calculated by using priority rules. Iskander and Panwalker (Reference 36) present a survey of over 100 scheduling rules. Examples of these (which are also the most studied) are:

- SPT - (Shortest Processing Time) - Select the operation with the shortest processing time
- FCFS - (First Come First Served) Select the operation that has entered the set of schedulable operations (S_t) the earliest
- MWKR - (Most Work Remaining) Select the operation associated with the job having the greatest total processing time remaining
- LWKR - (Least Work Remaining) Select the operation associated with the job having the least total processing time remaining
- MOPNR - (Most Operations Remaining) Select the operation associated with the job having the greatest number of operations still to be processed
- Random - (Random) Select any operation at random

It should be noted that the application of one priority rule may not resolve the conflict in all cases. Consider the case where two jobs which are identical are to be processed (same components, due dates, availability, batch size, etc.)

Here the SPT rule would not be able to choose between the two. The solution to this problem is to specify a hierarchy of priority rules. For example, first use:

- 1) SPT, then
- 2) MWKR, then
- 3) Random

A study of the various factors involved in the priority dispatching technique was undertaken by Jeremiah et al. (Reference 37). The authors generated both active and non-delay schedules under each of the priority rules listed earlier, for some 84 different problems. These experiments demonstrated that schedule generation based on priority dispatching rules is a practical method of obtaining suboptimal solutions to the general job shop problem. In addition, the results indicate that the set of non-delay schedules is a better basis for schedule generation than the set of active schedules.

8.5.3 Random Sampling

As stated earlier, two computational extremes were identified in heuristic techniques. Firstly, complete enumeration and, secondly, priority dispatching rules where a single schedule is constructed.

Random sampling and probabilistic dispatching techniques lie somewhere between these two extremes. The justification for the development and use of these techniques is based on the observation that even when the enumeration approach is impractical for large job shop problems, the construction of only a single schedule by priority dispatching may still be a very brief computational task. Consequently, it might be worthwhile to repeat the technique by which the single schedule is obtained but with some simple variations.

In the Random Sampling technique, once the conflicting operations have been identified, the technique chooses

amongst these by using a random mechanism that assigns to each operation an equal probability of being chosen. Thus, if there are k conflicting operations at some stage, then any particular operation will be selected in step 3 with a probability of $1/k$. This procedure is continued until the entire schedule has been constructed, virtually as rapidly as the priority dispatching algorithm. In addition, when the sampling algorithm is repeated several times, a collection of different schedules will usually be generated as long as the random mechanism resolves conflicts differently in each repetition. The solution chosen will be the best schedule contained in that collection or sample.

A detailed description of the theory and steps in the technique, together with that of Probabilistic Dispatching which follows, is given in Conway et al. (Reference 8).

8.5.4 Probabilistic Dispatching Procedures

It seems intuitively incorrect to choose amongst competing operations that have equal probabilities as in the case of Random Sampling techniques. The Probabilistic Dispatching procedures improves upon this situation by biasing the probabilities used in the random selection to favour those operations that seem the most sensible choice. The operations are first ranked by the given priority rule and then the probabilities are assigned in order; with the first operation being assigned the largest probability, the next operation the second largest probability and so on.

In general, the results of several researchers, notably that of Jeremiah et al. (Reference 37), indicate that where it is feasible to solve the general job shop problem by selecting the best from a sample of schedules, probabilistic dispatching is more effective than random sampling and priority dispatching. This conclusion has been based upon research work that has examined mean flow time (\bar{F}) and maximum flow time (F_{\max}) with the results pointing to the use of suitable priority rules. However, in the case of the

mean tardiness measure of performance (\bar{T}), the choice between Probabilistic Dispatching and Random Sampling may not be as clear cut without some idea of which priority rule is suitable. Furthermore, Probabilistic Dispatching like Random Sampling suffers from the lack of quantitative knowledge about the best schedule in the sample and related implications for selecting a sample size. Consequently, it is really difficult to know whether the improvement of Probabilistic Dispatching over Priority Dispatching is worth the added computational effort. Indeed, Conway et al. (Reference 8) conclude that probabilistic dispatching will provide diminishing returns for larger problems.

CHAPTER 9

DISCUSSION OF THE PROPOSED SOLUTION FOR ATLANTIS ALUMINIUM9.1 INTRODUCTION

The results of the research detailed in the previous chapter indicate that the most suitable method for application to the machine shop at Atlantis Aluminium is the priority dispatching schedule generation technique. The smallest set of schedules, the non-delay schedules, is chosen in preference to the set of active schedules as the basis of the schedule generation technique.

In addition, the priority dispatching schedule generation technique has the following operational advantages which make it particularly suited for application in the machine shop at Atlantis Aluminium:

- 1) Capable of handling unexpected job arrivals, machine breakdowns or other factors that could affect shop status over time
- 2) Allows a job to be worked on a machine more than once
- 3) Allows any number of operations per job
- 4) Flexibility with regard to varying the number of jobs and machines due to company expansion
- 5) Simple to use and operate
- 6) Allows evaluation of the derived schedule. If needed, alterations could be made to optimise the schedule still further
- 7) The results of the technique are rapidly acquired

8) Widely used in practice

The major disadvantage of this technique is that the solution which is generated will not be optimal. However, the technique has been found to work quite well in practice, often producing results quite near to the optimum when the number of jobs and machines becomes large (>5) and tends to infinity.

The fundamental design principle of the scheduling system which incorporates the technique mentioned above, is the accurate modelling of actual shop performance. To this end, the proposed solution gives due consideration to the interactive problems involving resources (men, machines and work material) which have a detrimental effect on the underlying principle of operation.

The procedure followed in this chapter is firstly to select the hierarchy of priority rules to be used with the priority dispatching schedule generation technique. Secondly, the full job scheduling procedure, in which individual operation start and end times are derived for each machine, is detailed. Finally, the two scheduling related subsystems of the overall shop management system, the shop scheduling subsystem and the production and schedule control subsystem are introduced. The outputs of the combined subsystems viz. the shop schedule, job progress report, deviation report and the report of the operations completed, are discussed in some detail.

9.2 THE SELECTION OF THE HIERARCHY OF PRIORITY RULES

The selection of priority rules for use with the priority dispatching schedule generation technique is complicated by the dynamic characteristic of the machine shop, i.e. where jobs arrive at the shop randomly over time. Thus, as discussed earlier, the machine shop behaves as a network of queues. The effects of dispatching procedures in queueing

networks however, are very difficult to describe by means of analytical techniques. To this end, several researchers have resorted to the use of computer simulation models. These models have enabled different dispatching rules to be compared and evaluated in order to determine those rules which optimise good performance.

The results of this research are summarised in Baker (Reference 3) and can be applied to the Atlantis Aluminium machine shop problem for the following reasons. Firstly, it has been found that the scale of any problem (i.e. the number of machines) does not exert a crucial influence upon the relative performance of scheduling rules. Secondly, the assumptions upon which the simulation models are based, are very similar to the actual conditions at Atlantis Aluminium. Where modifications to the standard set of assumptions have been made, it was done specifically to permit generalisation of the research results.

The choice of priority rules for Atlantis Aluminium is based upon the selected measure of performance (i.e. meeting due dates). Conway (Reference 8) performed simulation studies of the general job shop under these conditions. The most significant result of this study was that the two rules, dynamic slack per operation (S/OPN) and shortest processing time (SPT), continually performed substantially better than any of the other rules. However, preference for one or the other is a function of such factors as the manner in which due dates are set, the tightness of these due dates and the prevailing shop load.

As described in Chapter 2, job due dates at Atlantis Aluminium are set externally by the Atlantis Diesel Engine M.R.P. release statement. With the appropriate selection of a method for the calculation of job lead times, the job due date ceases to be a major influencing factor. The choice of priority rule is now merely a function of the prevailing shop load which affects the performance of these rules as

overall machine utilization increases. The performance of the dynamic slack per operation rule is quite sensitive to this increase with numerical results indicating that when shop loads are heavy ($> 88\%$ in Conway's study) SPT is preferable. For shop loads less than the initial heavy load, S/OPN is preferable. Thus there appears to be a crossover point, beyond which SPT achieves better performance than S/OPN. This crossover point is a unique characteristic of a particular job shop, and will have to be determined.

Although no historical data relating to shop load at Atlantis Aluminium is available, it would seem by observation and discussion with management to be low to medium with, on average, fifteen jobs processed monthly (roughly six operations per job) through ten machines. Consequently, the dynamic slack per operation rule is chosen as first preference in the hierarchy of priority rules, followed by SPT. If a conflict still exists after the second priority rule has been applied, the conflicting jobs are identical in both job description (component and batch size) and job status (ready times, due dates and operations completed). In this case, the choice is made randomly using the RANDOM rule.

Due to the time and effort that would be involved, the determination of the precise crossover point at Atlantis Aluminium by computer simulation methods is deemed to be an unnecessary exercise. The existence of this point has been noted and its effect at high shop load can easily be checked by modifying the hierarchy of priority rules to see whether a better schedule can be obtained.

In summary then, the hierarchy of priority rules to be used with the priority dispatching schedule generation technique when applied to Atlantis Aluminium is:

- 1) Dynamic Slack per Operation (S/OPN)

2) Shortest Processing Time (SPT)

3) Random (RANDOM)

At high shop load (approximately 90%), the hierarchy of priority rules changes to (2), (1) then (3).

9.3 THE JOB SCHEDULING PROCEDURE

The method of job scheduling using the priority dispatching schedule generation technique is illustrated by applying the technique to the conditions of the machine shop at Atlantis Aluminium. Figure 9.1 briefly displays the method in diagrammatic form, while detailed explanations are given below:

Step (1): This first step entails the coding or numbering of machines in the machine shop as shown in Appendix I. Machines, in this sense, are not restricted to conventional forms of machinery, e.g. lathes, milling machines, etc., but include other work centres such as deburring and inspection, quality control, stores, etc.

Step (2): The available standard production time during the planning period is now determined. In the case of Atlantis Aluminium, this planning period is taken to be three months, corresponding to the fixed period as discussed in Chapter 2. Daily standard production hours for use in the planning period have been determined, and are shown in Appendix J.

Unavailable production time such as tea breaks, lunch breaks, weekends and public holidays, as well as any other which occurs during the planning period are noted. In this way, an overall picture of the available production time is developed.

Step (3): The production requirements for the planning period are now determined. The list of components required to be produced during this period is made up of the fixed Atlantis Diesel Engine MRP release statement, external work and uncompleted work from the previous planning period. Each new batch of

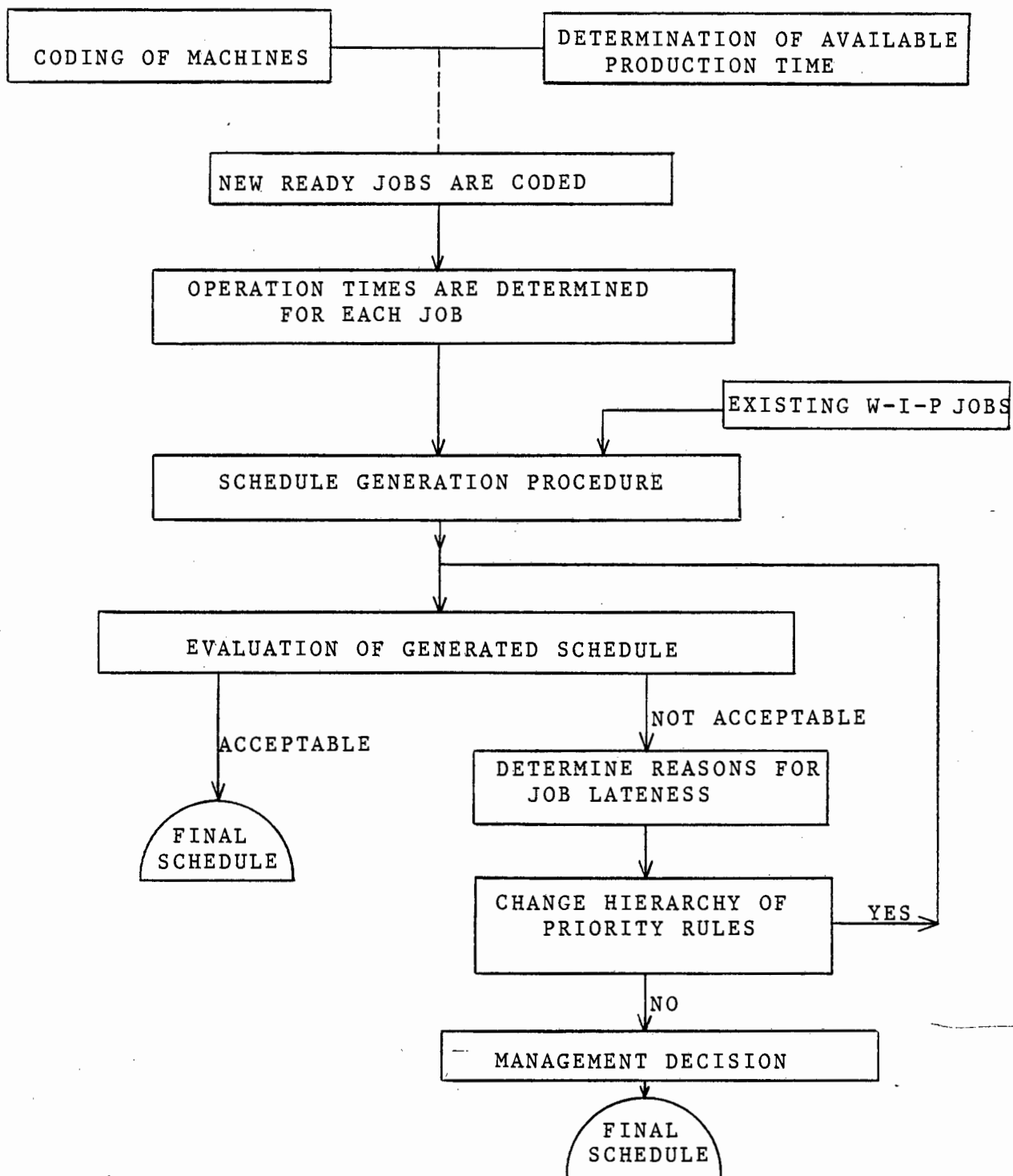


FIGURE 9.1 :DIAGRAMMATIC REPRESENTATION OF THE JOB SCHEDULING PROCEDURE

components is allocated a job number which is retained through its time in the machine shop and is used in the scheduling procedure. This is illustrated in Appendix K and is based on the actual production requirements for February (start of production year).

The information contained in steps (1), (2) and (3) is not only required by the job scheduling procedure. It is also an important input into the shop loading statement.

Step (4): Operation processing times are now obtained for each job which is to be processed during the planning period. These times have been determined from time and motion studies and include the machine set-up time. The standard production times of the jobs to be processed in the planning period are given in Appendix L (using production requirements for February) and are adjusted by an efficiency value which is set by management. In the example mentioned, an efficiency value of 70% is used. This value is based upon the experience of the machine shop superintendent and is taken as representative of the machine shop efficiency at the present moment.

Step (5): Using the information gathered in steps 1 to 4 above, the full production schedule for the planning period is generated using the technique as detailed in Appendix H. Appendix M shows the applications of this technique to the example used earlier, producing the first two stages (2 scheduled operations).

Step (6): The production schedule generated in step 5 above is now evaluated. If all jobs complete before their due dates the schedule is accepted. If any job completes later than its due date, the reasons

for this situation are determined and corrective measures taken. This is done firstly by generating a new schedule using the revised hierarchy of priority rules viz.:

- 1) SPT
- 2) S/OPN
- 3) RANDOM

If the new schedule is still unacceptable, a decision will have to be made by management whether to give the late job top priority (i.e. expedite, achieved by giving job an earlier due date), subcontract or to negotiate with the customer. The schedule which is generated after this management decision is taken to be final and sent to the machine shop.

This final production schedule, which is to be produced weekly, will show for each machine the start time and end time of each operation which requires that machine. This information is then displayed on Gantt charts for use by management.

The combination of job scheduling, the role of management and any other additional input described above form the shop scheduling subsystem.

9.4 THE PRODUCTION AND SCHEDULE CONTROL SUBSYSTEM

The main function of this subsystem is to constantly evaluate and report on the actual performance of the shop with the scheduled performance. Figure 9.2 below shows the integrated shop scheduling/production and schedule control subsystem.

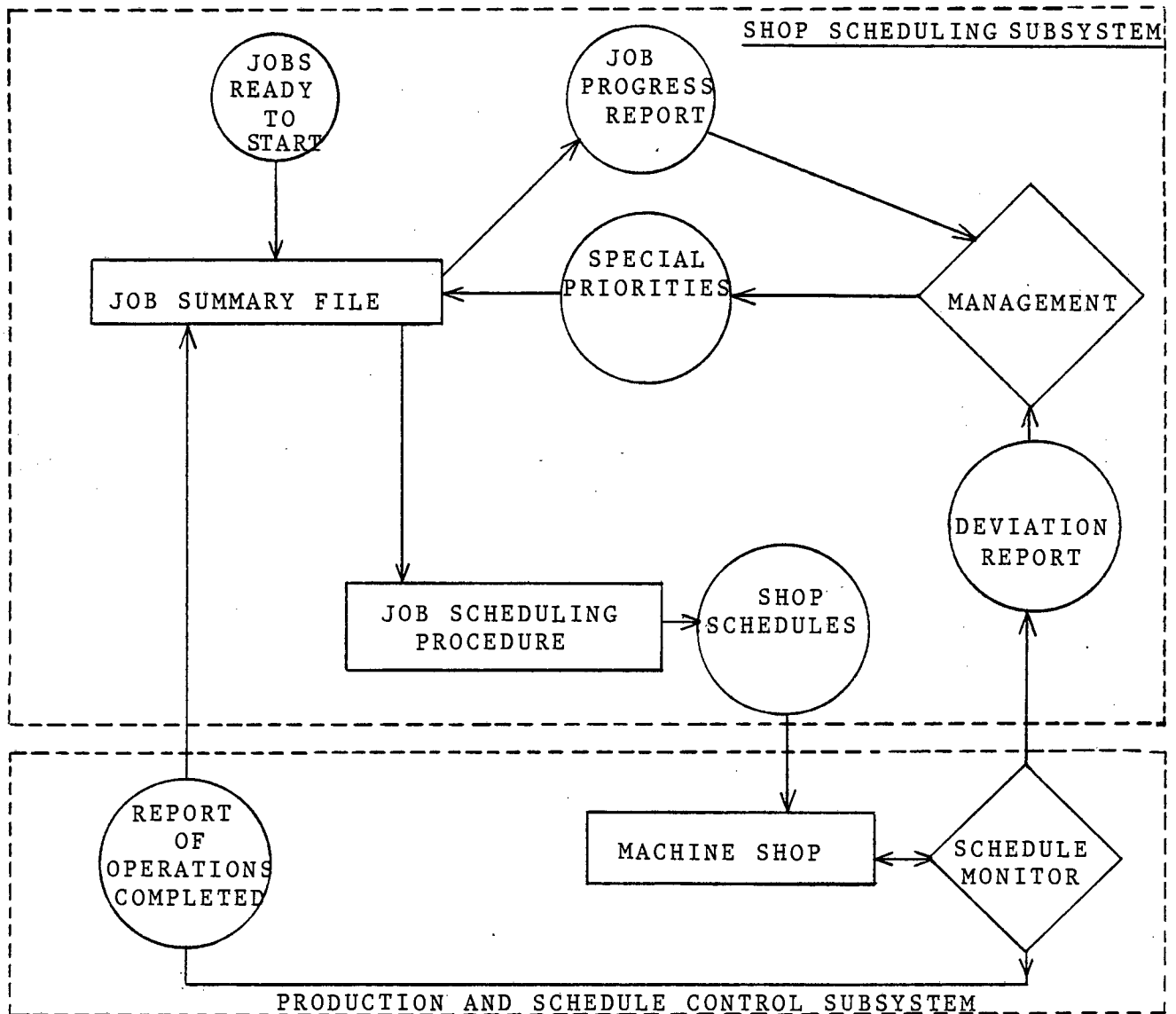


FIGURE 9.2 : INTEGRATED SHOP SCHEDULING/PRODUCTION AND SCHEDULE CONTROL SUBSYSTEMS

The evaluation of shop performance with respect to scheduled performance is carried out by the schedule monitor activity. This is done by comparing reports of actual job progress obtained from the shop. The information needed for the comparison can be obtained from the job card system currently employed.

Based on the results of the evaluation of shop performance, the schedule monitor produces two reports:

- 1) Report of Operations Completed - The contents of this report are entered into the job summary file, updating the status of each job. These reports are to be produced daily and if a new schedule is to be generated, the number of components still to be processed for unfinished operations is noted and included in the report.

- 2) The Deviation Report - Where substantial deviations occur between the shop performance and schedule performance, e.g. machine breakdowns, an immediate deviation report is generated. This report is sent to management who have to decide upon the best course of action to be followed in that situation. Based upon this best course of action, a new schedule is generated and implemented. The limits for what constitutes a substantial deviation will have to be prescribed by management.

A third report produced by the combined subsystems is a job progress report, based upon the contents of the job summary file. It indicates to management the progress or status of each job, emphasising those that have fallen behind schedule and may not meet promised due dates. Here again, a management decision is required similar to that under schedule evaluation (i.e. expedite, subcontract or negotiate).

CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS10.1 CONCLUSIONS

Based on the investigative and research studies performed and detailed in the preceding text, the following conclusions were drawn with regard to production scheduling at Atlantis Aluminium.

1) The informal production scheduling system in operation at the moment, in which control is totally decentralized and operations are scheduled in an arbitrary manner, has led to the inefficiencies which characterise the manufacturing situation at Atlantis Aluminium:

(i) high volumes of work-in-progress (resulting in congestion on the shop floor)

(ii) unnecessarily long lead times

iii) excessive and regular overtime

iv) consistently missing promised delivery dates (due dates)

v) uneven monthly production loads

In general, the above characteristics exist in job shops where no detailed scheduling is undertaken and they serve to indicate the need for an effective, well-designed production scheduling system.

2) A direct spin-off which results from the use of an inefficient informal production scheduling system, and one which is often ignored, is the adverse effect that

it has on the personnel and human relations in organisations. At Atlantis Aluminium, the situation is highlighted by the positions of:

- i) the machine shop foreman, who through decentralization of control, effectively controls production and is thus placed in a position that he is not qualified to handle
- ii) the machine shop superintendent, the man ultimately responsible for the effects of the ineffective and inefficient production scheduling system
- iii) the three heads of department which constitute Atlantis Aluminium, who due to the lack of co-ordination which results when detailed scheduling is ignored, are constantly at loggerheads
- iv) the workforce at Atlantis Aluminium, who consistently miss promised delivery dates and have become de-motivated

The effect of the production scheduling system on the personnel is thus an important consideration, and will have to be accounted for in any proposed system.

- 3) The second major part of the study involved the design of the production scheduling system for Atlantis Aluminium. In the first stage of this design process, the machine shop was classified in scheduling terminology as a static and deterministic general job shop. The associated measure of performance to be used for the evaluation of schedules, is the meeting of due dates. This classification is described more specifically in the four parameter notation as:

15/10/G/meeting due dates

with on average fifteen jobs being processed monthly through ten machines.

Although the problem has been classified as static, it does contain a dynamic element contributed by the acceptance of outside work which will have to be accounted for in the first solution.

4) In the second stage of the design process (Chapter 8), a literature survey was carried out into existing scheduling techniques. The results of this survey indicated that optimal solutions are unattainable for practical job shop problems and that a sub-optimal solution will have to be accepted. The research also indicates that the most appropriate production scheduling technique which meets the requirements, constraints and criteria of Atlantis Aluminium, is the Priority Dispatching Schedule Generation technique. Research results also suggest the use of non-delay schedules as the basis of the schedule generation technique, with scheduling conflicts being settled with the use of a hierarchy of priority rules.

5) The most effective hierarchy of priority rules for use with the Priority Dispatching Schedule Generation technique is:

(i) Dynamic Slack per Operation (S/OPN)

(ii) Shortest Processing Time (SPT)

(iii) Random (RANDOM)

At high shop loads (approximately 90%) the hierarchy of priority rules changes to (ii), (i) and (iii).

10.2 RECOMMENDATIONS FOR FUTURE WORK

On the basis of the results obtained from investigations and research performed and detailed in this thesis, the following recommendations concerning or relating to production scheduling were made to the management of Atlantis Aluminium:

- 1) The shop scheduling sub-system (discussed in section 9.3) which incorporates the Priority Dispatching Schedule Generation technique described earlier, together with its associated production and schedule control sub-system (discussed in section 9.4) should be introduced as the basis of the production scheduling system
- 2) The production scheduling system has to be computer-based in order for it to be utilized effectively. Programming of the system should preferably be a joint project between the author, the Atlantis Aluminium production planner and an A.D.E. systems analyst
- 3) Although the scheduling system has been designed specifically for application in the machine shop, it should be expanded to cover other production and production-related activities viz. the foundry, quality control including the impregnation plant, dispatch, stores, etc.
- 4) On introduction of the production scheduling system, staff, from managerial level through to the foreman on the shop floor, should be made familiar with the aims, objectives and workings of the system. In this way, staff co-operation would be obtained together with the understanding of the role of the individual in the scheduling system

- 5) Record-keeping systems for the important production parameters should be devised and introduced immediately. The introduction of these systems would have the following important effects:
- i) a more accurate and realistic plant loading statement which would be based on the individual machine characteristics and efficiency ratings. This is as opposed to the current use of an overall efficiency value of 0,55 to account for all unfavourable variances from the norm
 - ii) the compilation of a machine analysis chart, which would allow a more in-depth analysis to be carried out into the utilization, efficiency and additional requirements of machinery
 - iii) the determination of accurate lead times. Based on the data obtained from the record system, further research can be carried out into the methods which can be applied in the determination of these lead times. Ragatz and Mabert (Reference 43) present some of the latest models for lead time determination in manufacturing organizations, where the objective of the scheduling system is the meeting of job due dates
- 6) A further study should be carried out into economic production quantities, particularly for A.D.E. components, in order to determine the most cost-effective batch size for each component manufactured at Atlantis Aluminium.
- 7) As standard operation processing times are an important input into the production scheduling system, these times should be continuously monitored and updated when necessary. In this case, closer co-operation is envisaged between the A.D.E. Production Engineering

department and the Atlantis Aluminium production planner.

REFERENCES

BOOKS

1. ADAM, E.E. and EBERT, R.J. (1982). Production and Operations Management. Prentice-Hall, Englewood Cliffs, U.S.A.
2. ALUMINIUM COMPANY OF CANADA (1961). Handbook of Aluminium. Montreal, Canada.
3. BAKER, K.R. (1974). Introduction to Sequencing and Scheduling. John Wiley, New York.
4. BEDWORTH, D.D. and BAILEY, J.E. (1982). Integrated Production Control Systems. John Wiley, New York.
5. BELLMAN, R., ESOGBUE, A.O. and NABESHIMA, I. (1982). Mathematical Aspects of Scheduling and Applications. Pergamon Press, Oxford, Great Britain.
6. BUFFA, E.S. and TAUBERT, W.H. (1972). Production - Inventory Systems. Planning and Control. Richard Irwin Incorporated, Illinois.
7. CARSON, G.B. BOLZ, H.A. and YOUNG, H.H. (1972). Production Handbook. Third Edition. John Wiley, New York.
8. CONWAY, R.W., MAXWELL, W.L. and MILLER, L.W. (1967). Theory of Scheduling. Addison-Wesley, Reading, Mass.
9. FRENCH, S. (1982). Sequencing and Scheduling. An Introduction to the Mathematics of the Job Shop. Ellis Horwood Limited, Chichester, England.
10. GAREY, M.R., and JOHNSON, D.S. (1978). Computers and Intractability. A Guide to the Theory of NP-Completeness. Freeman, San Francisco.
11. GARFINKEL, R.S., and NEMHAUSER, G.L. (1972). Integer Programming. John Wiley, New York.
12. HOUSE, W.C. (1972). Operations Research. Auerbach, New York.
13. LOCKYER, K. (1985). Production Management. Fourth Edition. Pitman Publishing Limited, London.

14. MAGEE, J.F. (1958). Production Planning and Inventory Control. McGraw-Hill, New York.
15. MAYNARD, H.B. (1971). Industrial Engineering Handbook. Third Edition. McGraw-Hill, New York.
16. MONKS, J.G. (1977). Operations Management. McGraw-Hill, New York.
17. MUTH, J.F. and THOMPSON, G.L. (1963). Industrial Scheduling. Prentice-Hall, Englewood Cliffs, U.S.A.
18. NICHOLSON, T.A.J. (1978). Managing Manufacturing Operation. Macmillan Press Limited, London.
19. NICHOLSON, T.A.J. (1971). Optimisation in Industry. Volume I. Longman Group Limited, London.
20. RINNOOY, KAN A.H.G. (1976). Machine Scheduling Problems. Martinus Nijhoff, The Hague.
21. SCHMENNER, R.W. (1981). Production/Operations Management. Science Research Associates, Chicago, U.S.A.
22. SZENDROVITS, A.Z. (1973). An Introduction to Production Management. Faculty of Business, McMaster University, Ontario, Canada.
23. VAN HORN, K.R. (1968). Aluminium. Volume III. American Society of Metals, Ohio.
24. WOOLSEY, R.E.D., HUNTINGTON, S.S. (1975). Operations Research for Immediate Application : A Quick and Dirty Manual. Harper and Row, New York.

JOURNALS

25. AKERS, S.B. (1956). A Graphical Approach to Production Scheduling. Operations Research, Volume 24, pp.244-245.
26. BAKER, K.R. (1984). Sequencing Rules and Due Date Assignments in a Job Shop. Management Science, Volume 30, Part 9, pp.1093-1104.
27. BAKER, K.R. and KLANET, J.J. (1984). Improved Decision Rules in a Combined System for Minimizing Job Tardiness. Int. J. Prod. Res., Volume 22, Part 6, pp.917-921.

28. BOWMAN, E.H. (1959). The Schedule-Sequence Problem. Operations Research, Volume 7, pp.621-624.
29. BROOKS, G.H. and WHITE, C.R. (1965). An Algorithm for Finding Optimal or Near Optimal Solutions to the Production Scheduling Problem. Journal of Industrial Engineering, Volume 16, pp.34-40.
30. BUCK, J.R., HAIDER, S.W. and MOODIE, C.L. (1981). An Investigation of the Advantages of Using a Man-Computer Interacting Scheduling Methodology for Job Shops. Int. J. Prod. Res., Volume 19, Part 4, pp.381-392.
31. CAMPBELL, H.G., DUDEK, R.A. and SMITH, M.L. (1970). A Heuristic Algorithm for the n-job, m-machine Sequencing Problem. Management Science, Volume 16, pp.630-637.
32. DAY, J.E. and HOTTENSTEIN, M.P. (1970). Review of Sequencing Research. Naval Research Logistics Quarterly.
33. GIFFLER, B. and THOMPSON, G.L. (1960). Algorithms for Solving Production Scheduling Problems. Operations Research, Volume 8, pp.487-503.
34. GRAHAM, R.L., LAWLER, E.L., LENSTRA, J.K. and RINNOY KAN, A.H.G. (1979). Optimisation and Approximation in Deterministic Sequencing and Scheduling : A survey. Ann. Discrete. Math., Volume 5, pp.287-326.
35. HOLLOWAY, C.A. and NELSON, R.T. (1974). Job Shop Scheduling with Due Dates and Overtime Capability. Management Science, Volume 21, pp.68-78.
36. ISKANDER, W. and PANWALKER, S.S. (1977). A Survey of Scheduling Rules. Operations Research, Volume 25, Part 1, pp.45-61.
37. JEREMIAH, B., LALCHANDANI, A., and SCHRAGE, L. (1964). Heuristic Rules Towards Optimal Scheduling. Research Report, Department of Industrial Engineering, Cornell Univeristy.
38. JOHNSON, S.M. (1954). Optimal Two and Three Stage Production Schedules with Set-up Times Included. Naval Research Logistics Quarterly, Volume 1, pp.61-68.
39. LAGEWEG, B.J. LENSTRA, J.K. and RINNOOY KAN, A.H.G. (1977). Job Shop Scheduling by Implicit

- Enumeration. Journal of Management Science, Volume 24, Part 4, pp.441-450.
40. MANNE, A.S. (1960). On the Job Shop Scheduling Problem. Operations Research, Volume 8, pp.219-223.
41. MELLOR, P. (1966). A Review of Job Shop Scheduling. Operational Research Quarterly, Volume 17, pp. 161-171.
42. MIYAZAKI, S. (1981). Combined Scheduling System for Reducing Job Tardiness in a Job Shop. Int. J. Prod. Res., Volume 19, Part 2, pp.201-211.
43. RAGATZ, G.L. and MABERT, V.A. (1984). A Framework for the Study of Due Date Management in Job Shops. International Journal of Production Research, Volume 22, Part, 4, pp.685-695.
44. PANWALKER, S.S. and KHAN, A.W. (1976). An Ordered Flow Shop Sequencing Problem with Mean Completion Time Criterion. International Journal of Production Research, Volume 14, pp.631-635.
45. SZWARC, W. (1977). Optimal Two Machine Orderings in the 3 x n Flow Shop Problem. Operations Research, Volume 25, pp. 70-77.
46. TRUSSCOTT, W.G. (1985). Scheduling Production Activities in Multi-Stage Batch Manufacturing Systems. International Journal of Production Research, Volume 23, Part 2, pp.315-328.
47. WAGNER, H.M. (1959). An Integer Programming Model for Machine Scheduling. Naval Research Logistics Quarterly, Volume 6, pp.131-140.

APPENDIX A

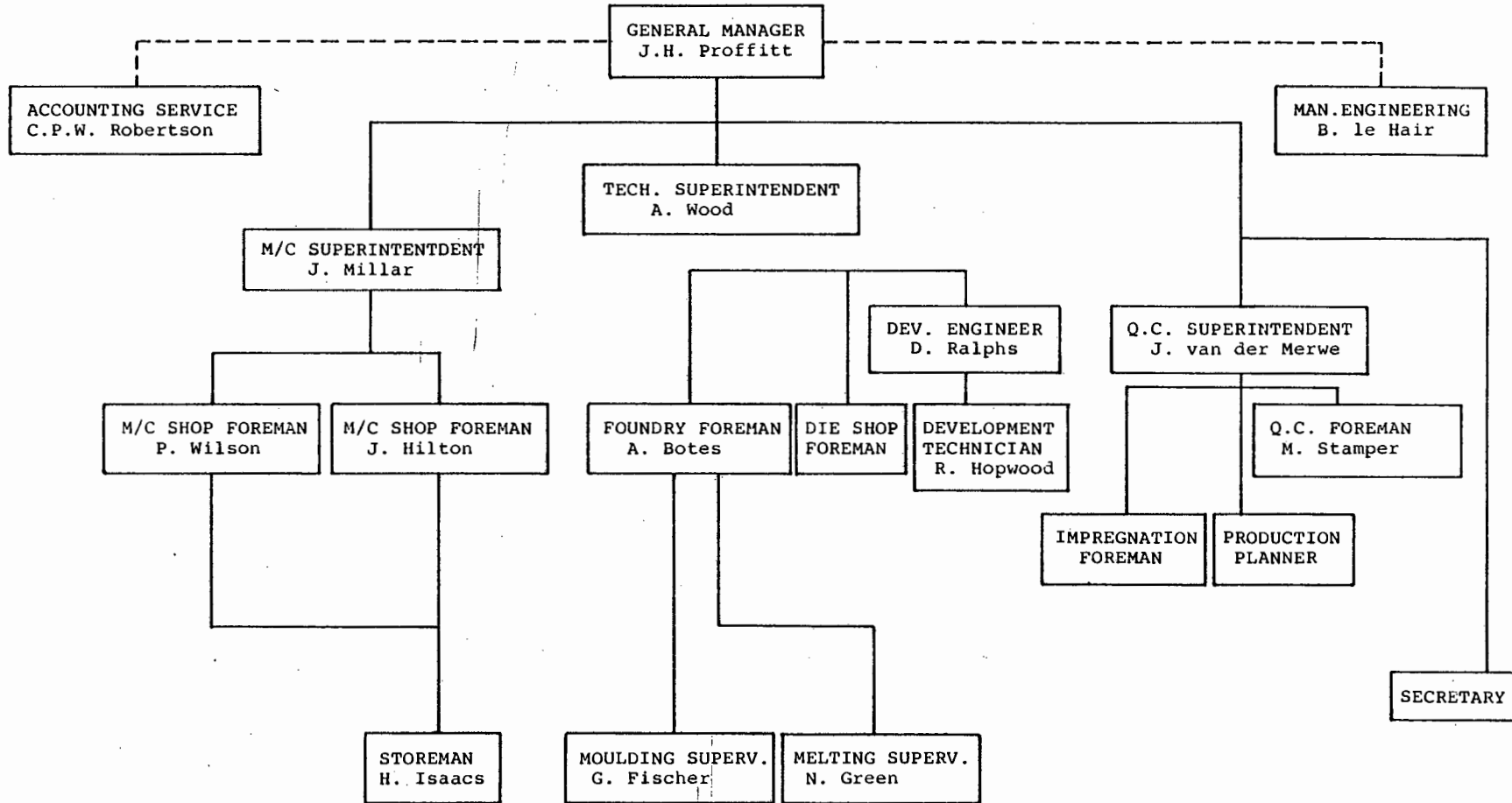
PLANT LOADING STATEMENT GIVING COMPONENTS
AND ASSOCIATED PART NUMBERS

APPENDIX B

ATLANTIS ALUMINIUM MANAGEMENT STRUCTURE

APPENDIX B

ATLANTIS ALUMINIUM MANAGEMENT STRUCTURE



APPENDIX C

A.D.E. FORECAST OF COMPONENTS FOR ATLANTIS ALUMINIUM

APPENDIX C

A.D.E. FORECAST OF COMPONENTS FOR ATLANTIS ALUMINIUM

14 January 85 ATLANTIS ALUMINIUM (PTY) LTD

ISSUE NO.

COMPN. DESCRIPTION NO.	JANUARY		FEBRUARY		MARCH		APRIL		
	QTY	DUE DATE FOUNDRY M/SHOP	QTY	DUE DATE FOUNDRY M/SHOP	QTY	DUE DATE FOUNDRY M/SHOP	QTY	DUE DATE FOUNDRY M/SHOP	
0107	CYL. HEAD COVER	0	0	0	0	10	30-04		
0203	AIR INTAKE	0	50	28-02	0	0	0		
0204	ASSY. INT. MANIFOLD	0	0	100	28-03	0	0		
0206	ASSY. INT. MANIFOLD	0	0	100	21-03	0	0		
0208	ASSY. INT. MANIFOLD	0	0	100	21-03	0	0		
0210	AIR INTAKE	0	50	28-02	0	0	0		
0211	AIR INTAKE	0	0	2	21-03	0	0		
0302	OIL SUMP	100	13-01	50	15-02	50	21-03		
0305	OIL SUMP	0	77	01-02	0	0	0		
0501	C. WATER OUTLET	60	24-01	100	07-02	100	14-03		
0502	C. WATER OUTLET	74	24-01	100	07-02	100	21-03		
0506	C. WATER OUTLET	0	50	08-02	0	0	0		
0702	COVER T. CASE	0	363	10-02	0	0	0		
0703	COVER T. CASE	67	03-01	200	17-02	500	23-03		
0704	COVER T. CASE	100	14-01	100	27-02	0	0		
1001	COVER	0	0	10	31.03	0	0		
1002	TIMING CASE	479	24-01	300	07-02	300	19-03		
		0	200	21-02	0	0	0		
1202	SPACER	187	14-01	0	0	0	0		
1501	FAN SPACER	114	24-01	0	100	14-03	100	30-04	
1503	FAN SPACER	0	0	0	100	23-03	100	10-04	
1508	FAN SPACER ASSY.	100	15.01	0	100	11-03	0	0	
1509	FAN SPACER	0	0	0	10	03-03	0	0	
1510	FAN SPACER	0	0	0	10	03-03	0	0	
1511	FAN SPACER	0	0	0	10	03-03	0	0	
1602	ELBOW TURBO	0	67	28-02	0	0	0		
1603	ELBOW PIPE	0	0	0	0	10	01-04		
1801	COVER	0	0	0	10	31-03	0	0	
1901	REAR OIL SEAL HSG.	200	24-01	300	03-02	600	11-03	800	10-04
		373	14-01	400	07-02	0	0	0	
2401	FLANGE ASSY.	10	01-01	0	0	0	0	0	
2601	TURBO PIPE	0	0	0	10	03-03	0	0	
		1864		2407		2302		1570	

APPENDIX D

JOHNSON'S ALGORITHM FOR THE $n/2/G/F_{\max}$ PROBLEM

APPENDIX D

JOHNSON ALGORITHM FOR THE $n/2/G/F_{\max}$ PROBLEM

The starting point of Johnson's algorithm is the set of n jobs ($J_1; J_2; J_3; \dots; J_n$). This set can be partitioned into four types of jobs:

- 1) Type A - those jobs to be processed on the first machine (M_1), only
- 2) Type B - those jobs to be processed on the second machine (M_2) only
- 3) Type C - those jobs to be processed on both machines in the order M_1 then M_2
- 4) Type D - those jobs to be processed on both machines in the order M_2 then M_1

The procedure for the construction of the optimal schedule is as follows:

- i) schedule the jobs of type A in any order to give the sequence S_A
- ii) schedule the jobs of type B in any order to give the sequence S_B

- iii) schedule the jobs of type C according to Johnson's Algorithm to give the sequence S_C
- iv) schedule the jobs of type D according to Johnson's Algorithm to give the sequence S_D

The optimal schedule is then:

<u>Machine</u>	<u>Processing Order</u>
M_1	(S_C, S_A, S_D)
M_2	(S_D, S_B, S_C)

Johnson's algorithm for use in steps 3 and 4 of the procedure is as follows:

- 1) find the smallest processing time for any job on any machine
- 2) for the job found, if the smallest processing time is on the first machine (M_1), schedule this job first on this machine

If the smallest processing time is on the second machine (M_2), schedule this job last on M_2

- 3) cross off this job and go to step 1

For example, consider the $9/2/G/F_{\max}$ problem which follows:

PROCESSING ORDER AND TIMES

<u>Job</u>	<u>First Processing Machine</u>		<u>Second Processing Machine</u>	
1	M_1	8	M_2	2
2	M_1	7	M_2	5
3	M_1	9	M_2	8
4	M_1	4	M_2	7
5	M_2	6	M_1	4
6	M_2	5	M_1	3
7	M_1	9		
8	M_2	1		
9	M_2	5		

Type A jobs - J_7 i.e. $S_A = (J_7)$

Type B jobs - J_8, J_9 i.e. $S_B = (J_8; J_9)$

Type C jobs - J_1, J_2, J_3, J_4

Using Johnson's algorithm:

M_1 M_2
 (J_4, J_3) (J_2, J_1)

i.e. $S_C = (J_4; J_3; J_2; J_1)$

Type D jobs - J_5, J_6

Using Johnson's algorithm: $S_D = (J_5, J_6)$

The optimal which is derived is then:

$M_1 - (J_4; J_3; J_2; J_1; J_7; J_5; J_6)$

$M_2 - (J_5; J_6; J_8; J_9; J_4; J_3; J_2; J_1)$

APPENDIX E

AN EXAMPLE OF THE BROOKS AND WHITE APPROACH

APPENDIX E

AN EXAMPLE OF THE BROOKS AND WHITE PROCEDURE

	1		2		3		4	
	machine	p	machine	p	machine	p	machine	p
1	1	6	4	8	3	9	2	4
Job 2	1	1	2	3	3	9	4	6
3	1	5	3	5	2	3	3	6

Since all the jobs have their first operation on machine 1, there is a conflict on machine 1 at time zero. The three branches out of the initial node of the enumeration tree correspond to the selection of job 1, 2, 3 at time zero. These branches are denoted by B_1 , B_2 and B_3 .

Using the job-based bounding procedure. If job 1 is chosen, then each job could theoretically start at time 6 and the remaining processing times would be 21, 19 and 19, so that the lower bound for B_1 is 27. For B_2 it is 28 ($1 + \max(27, 18, 19)$) and for B_3 it is 32 ($5 + \max(27, 19, 14)$). Therefore branch B_1 will be followed.

Using the machine-based bounding procedure. If job 1 is chosen then the earliest possible times that each machine could start on the remaining operations are 6, 7, 10, 6 (for

machines 1 through 4). The sum of the processing times of the unscheduled operations on each of the four machines are 6, 10, 29 and 14. The lower bound associated with B_1 is then $\max (6 + 6, 7 + 10, 10 + 29, 6 + 14) = 39$. For B_2 the lower bound is 33 and for B_3 it is 34 so that B_2 would be followed.

APPENDIX F

MANNE'S MODEL FOR MODELLING THE GENERAL JOB SHOP
PROBLEM USING THE INTEGER PROGRAMMING TECHNIQUE

APPENDIX F

MANNE'S MODEL FOR MODELLING THE GENERAL JOB-SHOP PROBLEM

USING THE INTEGER PROGRAMME TECHNIQUE

The formulation is based on the assumption that each job requires processing on each machine once and once only (classical job shop problem). This restriction can be removed if required, resulting in increased problem complexity.

MODEL FORMULATION

Set p_{ik} = the processing time of job i on machine k
 Z_{ijk} = 1 if the j th operation of job i requires machine k
= 0 otherwise
and T_{ik} = the starting time of job i on machine k

From the requirement that only one job can be processed on a machine at any time, then for two jobs (I and J), either job J precedes job I or job I precedes job J

$$\text{i.e. } T_{Ik} - T_{Jk} \leq P_{Jk} \text{ or } T_{Jk} - T_{Ik} \leq P_{Ik} \dots (1)$$

These either-or restrictions cannot be handled by ordinary linear programming and require the introduction of integer variables (Y).

let $Y_{IJk} = 1$ if job I precedes job J on machine k
 $= 0$ otherwise

The two either-or constraints can now be written as two independent constraints, both of which must hold.

$$(M + p_{Jk}) Y_{IJk} + (T_{Ik} - T_{Jk}) \leq p_{Jk} \quad (2)$$

$$(M + p_{Ik})(1 - Y_{IJk}) + (T_{Jk} - T_{Ik}) \leq p_{Ik} \quad (3)$$

Where M is a constant which is chosen sufficiently large so that only one of the constraints (2) and (3) is binding for $Y_{IJk} = 0$ or 1. The value of M may be set equal to $\sum_i \sum_k p_{ik}$.

The operation precedence constraints are handled by noting that $\sum_k Z_{ijk} T_{ik}$ is the starting time of the jth operation of job i.

Thus, for all but the last operation of a job:

$$\sum_k Z_{ijk} (T_{ik} + p_{ik}) \leq \sum_k Z_{i(j+1)k} T_{ik} \quad (4)$$

For the n job m machine problem the number of variables and constraint equations can be calculated.

In the case of the variables:

	<u>Number</u>
$T_{ik} \leq 0$	mn
$Y_{IJK} = 0 \text{ or } 1$	$m \frac{n(n-1)}{2}$

constraint equations:

$$(2) \quad m \frac{n(n-1)}{2}$$

$$(2) \quad m \frac{n(n-1)}{2}$$

$$(4) \quad (m-1)n$$

Turning to the objective function i.e. the measure of performance. In the case of A.A. the objective is to minimise the mean tardiness \bar{T}

i.e. $T_i - E_i = F_i - d_i$, $i = 1, 2, \dots, n$, are firstly added to the constraints

The objective is then to minimise $\sum T_i$

APPENDIX G

ACTIVE SCHEDULE GENERATION TECHNIQUE
(GIFFLER AND THOMPSON)

APPENDIX G

ACTIVE SCHEDULE GENERATION TECHNIQUE (GIFFLER AND THOMPSON)

Set P_t - the partial schedule of $(t-1)$ scheduled operations

S_t - the set of operations schedulable at stage t i.e. all the operations that must precede those in S_t are in P_t

σ_k - the earliest time that operation O_k in S_t could be started

ϕ_k - the earliest time that operation O_{ijk} in S_t could be finished i.e. $\phi_k = \sigma_k + p_k$

STEP 1 : Set $t = 1$ with P_1 being null. S_1 will be the set of all operations with no predecessors i.e. those that are first their job

STEP 2 : Find $\phi^* = \min_{O_k \text{ in } S_k} (\phi_k)$ and the machine M^* on which ϕ^* occurs. If there is a choice for M^* choose arbitrarily

STEP 3 : Choose an operation O_j in S_t such that:
 1) it requires M^* , and
 2) $\sigma_j < \phi^*$

STEP 4 : Move to next stage by:

- 1) adding O_j to P_t , so creating P_{t+1}
- 2) deleting O_j from S_t and creating S_{t+1} by adding to S_t the operation that directly follows O_j in its first job
- 3) incrementing t by 1

STEP 5 : If there any operations left unscheduled go to Step 2. Otherwise, stop

APPENDIX H

NON-DELAY SCHEDULE GENERATION TECHNIQUE

APPENDIX H

NON-DELAY SCHEDULE GENERATION TECHNIQUE

- STEP 1 : Let $t = 1$, P_1 being null. S_1 will be the set of all operations with no predecessors i.e. those that are first in their job
- STEP 2 : Find $\sigma^* = \min O_k$ in $S_k^{(\sigma_k)}$ and the machine M^* on which σ^* occurs. If there is a choice for M^* , choose arbitrarily
- STEP 3 : Choose an operation O_j in S_t such that:
- 1) it requires M^*
 - 2) $\sigma_j = \sigma^*$
- STEP 4 : Move to the next stage by:
- 1) adding O_j to P_t so creating P_{t+1}
 - 2) deleting O_j from S_t and creating S_{t+1} by adding to S_t the operation that directly follows O_j in its job (unless O_j completes its job)
 - 3) incrementing t by 1
- STEP 5 : If there are any operations left unscheduled, go to Step 2. Otherwise, stop

APPENDIX I

MACHINE NUMBERS

APPENDIX I

MACHINE NUMBERS

<u>MACHINE DESCRIPTION</u>	<u>MACHINE NUMBER</u>
Kearney and Trecker Milling Machine	1
Heckert Milling Machine	2
Legun Milling Machine	3
Fanuc C.N.C. Milling/Drilling Machine	4
Otto Muller Drilling Machine	5
Herbert Radial Drilling Machine	6
Voest lathe 250	7
Voest lathe 210	8
Pedestal Drilling Machine	9
Bench	10

APPENDIX J

PRODUCTION TIMES

APPENDIX J
PRODUCTION TIMES

<u>MONDAY TO THURSDAY:</u>	<u>DAY SHIFT</u>
7.00 - 10.00	
10.00 - 10.10	Tea break
10.10 - 12.30	
12.30 - 13.00	Lunch break
13.00 - 15.00	
15.00 - 15.10	Tea break

Therefore Total Day Shift Production Time = 9 hours 10 minutes

	<u>NIGHT SHIFT</u>
19.00 - 22.00	
22.00 - 22.10	Tea Break
22.10 - 0.30	
0.30 - 1.00	Lunch Break
1.00 - 3.00	
3.00 - 3.10	Tea Break
3.10 - 5.00	
5.00 - 5.10	Tea Break
5.10 - 7.00	

Therefore Total Night Shift Production Time = 10 hours 55 minutes

APPENDIX K

FEBRUARY PRODUCTION REQUIREMENTS

APPENDIX K

FEBRUARY PRODUCTION REQUIREMENTS

<u>JOB NO.</u>	<u>PART NO.</u>	<u>QUANTITY</u>	<u>START DATE</u>	<u>DUE DATE</u>
1	0203	50	29/1	26/2
2	0210	50	29/1	26/2
3	0302	50	29/1	13/2
4	0501	100	21/1	5/2
5	0502	100	21/1	5/2
6	0506	50	21/1	6/2
7	0702	363	29/1	7/2
8	0703	267	5/2	14/2
9	0704	200	7/2	25/2
10	1002	300	30/1	5/2
11	1002	200	5/2	19/2
12	1602	67	3/2	26/2
13	1901	100	29/1	31/1

APPENDIX L

OPERATION PROCESSING TIMES

APPENDIX L

OPERATION PROCESSING TIMES (HRS : ADJUSTED FOR AN
EFFICIENCY OF 70%)

OPERATION	10	20	30	40	50	60	70	80	90	100
JOB NO:										
1 OPERATION TIME MACHINE NO:	5,5 2	10,2 6	4,5 6	3,6 10						
2	5,3 2	5,3 6	6,5 6	2,5 10						
3	7,5 2	4,3 3	31,6 5	4,6 2	9,3 6	7,7 10	23,9 10	5,5 5		
4	7,9 3	14,6 4	11,0 5	7,0 10						
5	7,9 3	14,6 4	11,0 5	7,0 10						
6	4,3 3	8,4 4	6,3 5	3,9 10						
7	78,6 4	34,6 9	9,3 3	18 8	17,9 10					
8	20,6 1	49,9 4	19,1 10							
9	32 2	62,9 5	13,3 8	19,51 4	16,3 10	12,9				
10	24,3 1	32,9 2	13,2 5	42,2 5	26 2	54 6	30,7 4	27,2 4	71,5 10	
11	16,8 1	22,5 2	87,6 5	28,5 5	17,6 2	36,3 6	21,2 4	18,3 4	47,8 10	
12	6,8 3	8,9 5	7,3 3	8,6 3	3,3 10					
13	29,6 2	73,6 4	21,5 10							
14	39,2 2	9,9 4	28,6 10							

APPENDIX M

EXAMPLE OF THE SCHEDULE GENERATION PROCEDURE

APPENDIX M

EXAMPLE OF THE SCHEDULE GENERATION PROCEDURE

The data for this example is taken from the production requirements for February (Appendix K). For the purposes of illustration, it is assumed that:

- 1) all machines are available at the start of day shift
i.e. 7H 00
- 2) all jobs are unworked i.e. the set of schedulable operations consists of the first operation of each job
- 3) the priority rules used are SPT and Random
- 4) the present date is 21/1/86

Following the steps of the algorithm (Appendix H), each scheduling stage is developed and formatted as indicated in Table M.1 below. In the initial stage, machine ready time and the list of schedulable operations ((i, j, k) notation simplified to (job, machine)) are inserted. Potential start times (σ) corresponding to each of the schedulable operations are entered and the minimum of these (σ^*) is noted i.e. jobs 4, 5 and 6. The machine (M^*) on which (σ^*) occurs is machine 3. The choice of the scheduled operation

TABLE M.1 : EXAMPLE OF A SCHEDULING STAGE

Stage t	MACHINE										O_k in S_t	σ_t DATE/TIME	σ^*	O_j SCHEDULED OPERATIONS	
	1	2	3	4	5	6	7	8	9	10					
0	7:00	7:00	7:00	7:00	7:00	7:00	7:00	7:00	7:00	7:00	7:00	(1;2) (2;2) (3;2) (4;3) (5;3) (6;3) (7;4) (8;1) (9;2) (10;1) (11;1) (12;3) (13;2) (14;2)	29/1 29/1 29/1 21/1 (7:00) 21/1 (7:00) 21/1 (7:00) 29/1 5/2 7/2 30/1 5/2 3/2 29/1 7/2	7:00 7:00 7:00	(6;3)
	+ 4.3hrs														
1	7:00	7:00	11:28	7:00	7:00	7:00	7:00	7:00	7:00	7:00	7:00	(1;2) (2;2) (3;2) (4;3) (5;3) (6;4) (7;4) (8;1) (9;2) (10;1) (11;1) (12;3) (13;2) (14;2)	29/1 29/1 29/1 21/1 (11:28) 21/1 (11:28) 21/1 (11:28) 29/1 5/2 7/2 30/1 5/2 3/2 29/1 7/2	11:28 11:28 11:28	(6;4)

(O_j) lies between jobs 4, 5 and 6 and is made using the SPT rule. Thus job 6 (processing time 4,3 hours) is chosen.

The machine ready times in the next stage are updated with machine 3 ready time now 11 hours 28 minutes (taking into account the 10 minute tea break). The list of schedulable operations is updated by adding the next operation required by job 6 i.e. (6,4). Potential start times for this revised list are now determined. Note that now M^* occurs on more than one machine. The choice for M^* is made arbitrarily (e.g. machine 4) leading to (6,4) being scheduled. This procedure is continued until all operations have been scheduled providing a complete schedule.