Fuzzy Modelling of the Johannesburg Security Exchange Overall Index

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DECLARATION

I declare that this is own, unaided work. It is being submitted for the degree of Doctor of Philosophy in the University of Cape Town. It has not been submitted before for any degree or examination in any other university.

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ABBREVIATIONS

The following abbreviations and symbols are used in this thesis:

H:	Hurst exponent
P:	Probability
N:	Necessity measure
Π:	Possibility measure
γ:	Possibility - probability consistency
R/S:	Rescaled range
AR:	Autoregressive model
NY:	NYSE
NQ:	Nasdaq
DJ:	Dow Jones
DX:	DAX
NK:	Nikkei500
IB:	IBOVESPA
JS:	JSE
JG:	JSE Gold
MS:	Money Supply
MP:	Manufacturing Production Index
CONS:	Consumer Price Index
ANG:	Anglo Gold share price
ANGF:	Anglo Gold share price due to fundamentals
PROD:	Producer Index
ARCH:	Autoregressive conditional heteroscedasticity
EWMA:	Exponentially weighted moving average
GARCH:	Generalised autoregressive conditional heteroscedasticity

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ABSTRACT

This thesis focuses on the empirical analysis of the fuzzy feature of the Johannesburg Stock Exchange Overall Index using fuzzy logic techniques. The data for the periods 1985 - 2001 is used in the analysis. Description of the fuzzy feature is crucial to the proper understanding of the movement of the JSE Overall Index and the South African economy. The fuzzy feature of the Johannesburg Security Exchange Overall Index if understood would impact on the financial and economical decisions.

A preliminary Fractal analysis is carried out before the fuzzy analysis to investigate the nature of the Johannesburg Security Exchange Overall Index. The Johannesburg Security Exchange Overall Index experiences the Hurst phenomena of long memory for periods of 100 days (approximately three months). Outside the long memory periods, the Johannesburg Security Exchange Overall Index is found to exhibit antipersistent or short-range dependency characteristics.

The fuzzy feature of the Johannesburg Security Exchange Overall Index is described by many aspects of fuzzy logic. The analysis of the fuzzy feature is carried out according to time periods of approximately 4 years each of the Johannesburg Security Exchange Overall Index. The index in each time period is partitioned in three fuzzy states: "low", "middle" and "high". The fuzzy states are important in assessing the fuzzy nature of the Johannesburg Security Exchange Overall Index. The partitioning reveals that the fuzzy states of the Johannesburg Security Exchange Overall Index do not possess sharp boundaries. The sizes of the fuzzy states are found to change with time. This reflects changes in the forces behind the dynamics of the index.

Information and knowledge about the movement of the Johannesburg Security Exchange Overall Index are presented by computing the possibility distributions of the fuzzy states. The possibility distributions of the fuzzy states play a key role in describing the fuzzy feature of the Johannesburg Security Exchange Overall Index. The possibility distributions reveal the meaning of the index values through the possibility grades. The possibility grades expose the compatibility of the index values with the states. The meaning and compatibility of the index values are found to change with the time periods. Fuzziness can be used to estimate how obscure the movement of the index is in the fuzzy states. This has been documented by approximating the area under the possibility distributions. The fuzziness in the movement of the Johannesburg Security Exchange Overall Index is found to vary between the states and the time. The subjective belief of the stake holders about the movement of the Johannesburg Security Exchange Overall Index contributes to its fuzzy feature. The coherence between the subjective

belief about the Johannesburg Security Exchange Overall Index events and the frequency of the occurrence of the events has been documented by the possibility - probability consistency. The possibility - probability consistency, is computed from the possibility distributions of the fuzzy states. The coherence between the subjective belief and the frequency is found to be high in the "middle states and low in the "low" and "high" states. This result shows that investors find it easier to deal with the movement of the Johannesburg Security Exchange Overall Index when it is in the "middle" state than when it is in the "low" and "high" states.

The stability of the Johannesburg Security Exchange Overall Index is crucial and reflects stability of the economy, as well as that of the listed companies on the market. The necessity measure from possibility theory is identified as a descriptor of the Johannesburg Security Exchange Overall Index stability. The necessity measure is found to show that the Johannesburg Security Exchange Overall Index has been relatively stable for the periods 1985 - 2001.

The transitions of the Johannesburg Security Exchange Overall Index between its states is characterised by vagueness and not by sharp jumps. The dynamics of the Johannesburg Security Exchange Overall Index between its fuzzy states has been described by fuzzy Markov chain. The fuzzy Markov chain shows that the Johannesburg Security Exchange Overall Index has a tendency to move to the "middle" state. This feature changes with time. The prediction of the Johannesburg Security Exchange Overall Index using the fuzzy Markov chain model is carried out. The method gives another way of predicting the index.

Financial markets are often affected by the mood of investors. The mood of investors is normally qualitative. A mood index has been derived to quantify the mood of investors. The index has been applied to data of different market indices. The nature of the mood of investors for the different stock market indices has been explored by way of Fractal analysis. The Hurst phenomena of long memory is found to characterise the mood of investors. The fractal dimensions have shown that the mood of investors is formed differently for the different market indices. The impact of the mood of investors on the future of the market has been documented. The mood of investors is found to have high impact on the future of the markets. The possibility distributions of the mood indices have been computed in order to document information and knowledge about the mood of investors. The mood of investors for most market indices is discovered to have a high possibility of being stable. The stability varies between the market indices. The meaning of the mood index values has been achieved. The values are found to have different meanings according to the mood indices. Fuzziness of the mood of investors that could be used to estimate how obscure the mood of investors could be, was documented. The vagueness of the mood of investors is found to vary between the market indices.

Both fundamentals and sentiment contribute to what constitutes a share price. The Johannesburg Anglo Gold share price is decomposed into fundamentals and sentiment. Such decomposition is vital for investors to catch up with the market characteristic to better their portfolio management both in the short term and long term. The fundamental component of the share price was found to have an improved linear relationship with the South African economic indicators.

Chapter 1

Introduction

1.1 Problem Statement

Financial markets are considered very important to the economy of a country. They are places where financial instruments and securities are traded. These markets among other things, reflect the government's economic policy and the political situation of a country. The markets are places through which capital is distributed into various sectors of the economy. Some of the major financial markets are: the Stock markets, Currency markets, Commodity markets and Futures markets. The stock market provides an environment where corporate firms and governments can raise funds for investment. The stock market is also a place for investors to invest in shares of their choice. Thus market movement is of major concern to economists, policy makers, investors and security analysts.

Movement of the stock market is summarised by a market index. The index is constructed from a collection of the listed stocks or all the listed shares. There are many factors responsible for market movement. One of the factors is changes in share prices (Brealey 1983). Others include company performance and expected profits, general economic conditions, and mood of investors (Fischer and Jordan 1975, Scott 1999). It is impossible to describe and predict each and every change in a share price accurately, and this has led to the study of a wide range of possible links to changes in share price (Shiller 1989). A major concern regarding market movement is to determine the distribution of the stock index. The distribution is important in determining risk of investment and also gives indirect information on the underlying factors affecting the economy (Fama 1976).

From the literature, the uncertainty in financial markets is a fundamental issue. Economic fundamentals, investors' psychology and other factors contribute to what constitutes changes in the stock market indices. Knowledge in the markets is generated from human thinking. Natural language is used to describe the market movement. Human thinking and natural language are intrinsically fuzzy (Zadeh 1976, 1978a, 1978b). Accordingly, much of the information and knowledge about the movement of the market may be considered to be imprecise. This implies that the information provided by the indices should be fuzzy in nature though the indices are expressed by accurate numbers. Vagueness of information is inherently part of the market indices.

However, most of the research done so far in the financial markets, has mainly addressed the uncertainty only from one of its intrinsic features - randomness. This has been done with the aid of mathematical and statistical models (Fischer and Jordan 1975, Fama 1976, Peters 1994) and the models and techniques surveyed in Sundaresan (2000). These methods are not capable of dealing with the vagueness of information (Zadeh 1983). The fuzziness feature of the financial market has been for a long time unnoticed, ignored and unaddressed. Studies that have considered vagueness in the analysis of market movement, however, exist in the literature. They include the analysis of the German DAX index, the Brazilian IBOVESPA index, portfolio selection using possibility analysis and other subjects related to vagueness in financial markets (Ribeiro et al. ed. 1999, Tanaka and Guo 1999). Some of them are highly theoretical and mathematical.

This study will focus on the empirical description of the movement of the Johannesburg Security Exchange (JSE) Overall Index using fuzzy techniques. The main aim of the study is to investigate and reveal the fuzzy feature hidden behind the crystal clear numbers of the JSE Overall Index. The index will be partitioned into its fuzzy states in order to explore the nature of its movement. Information and knowledge about the movement of the index will be computed and presented. This will achieve the meaning of the index relative to the fuzzy states. Coherence between subjective belief about the events of the index and the frequency of their occurrence will be documented. The movement of a market index may be perceived as obscure. To measure this, fuzziness of the movement of the index within the states will be documented. certain aspects in the movement of the index can be associated with stability or instability of the national economy. The necessity measures of the JSE Overall Index will be computed in order to explore the nature of the South African economy. The movement of the index between the fuzzy states is not sharp. This movement will be described by the fuzzy Markov chain model. Prediction of the index will be explored using the fuzzy Markov chain model. By doing this, insight about the movement of the JSE Overall Index will be gained.

Human behaviour has been considered an important aspect of financial markets (De Bondt and Thaler 1995). Herd behaviour such as information cascades, in which people acquire information by observing the behaviour of others, and fads, in which people behave according to fashion have been identified. Their effects to financial markets have been studied (Shiller 1990, 1995). The herd behavior of investors could be linked to the mood of investors. The mood of investors plays a critical role in describing the state of the market at a particular time. One use of the mood of investors is to gauge the direction and performance of markets (Upgrave 1995). In the literature, the mood of investors has been described qualitatively (Sheeline 1992, Lee 1999). Little attempt has been made to quantify the mood of investors. It is one of the purposes of this study to fill up this gap. The study will create a mood index to quantify the mood of investors in the market. The mood index will be applied to various market indices. The characteristics of the mood of investors in different markets will be investigated and compared. The impact of present mood of investors to the future of the market will be computed. Knowledge and information about mood of investors will be documented. Fuzziness of the mood of investors in different markets will be analysed. The relationship between the subjective belief about the occurrence of mood of investors and the frequency of the mood of investors will be documented. Quantifying the mood of investors will give insight into the structure underlying the distribution of the mood.

Economic fundamentals and investor sentiment are important to market movement. Evidence exists to show that fundamentals and sentiment constitute the price of a share or indeed changes in the market index (Shiller 1989). However, there is no clear distinction between movement due to fundamentals or movement due to sentiment of share price or market index. Thus decomposing share price or market index movement into fundamentals and sentiment is not easy to deal with. Some of the models that have been used to decompose share price into fundamentals and sentiment include the K-Z model (Peters 1991). This study will explore the decomposition of share price into fundamentals and sentiment using the mood of investors computed from share price data. A successful decomposition of share price will reveal the distribution of share price movement due to fundamentals and distribution of share price due to sentiment separately. This will show the extent to which fundamentals and sentiment contribute to the overall distribution of share price movement.

1.2 Motivation

This study is motivated by the desire to address the unnoticed fuzzy feature in the movement of the JSE Overall Index. Uncertainty is intrinsically part of market movement (Peters 1999). The random aspect of uncertainty in the financial markets has been covered widely in the literature. However, the fuzzy component of uncertainty in the financial markets to a large extent has only received attention at a philosophical level. Considerations of fuzziness in financial markets in a practical sense has been limited to a few cases (Ribeiro et al. ed. 1999). Literally no previous attempt has been made to describe and expose the fuzzy character of the JSE Overall Index. It is the intention of this study to fill up this gap using fuzzy techniques. This will create an opportunity to view the JSE Overall Index in a different way.

The study is also motivated by the need to quantify the mood of investors and to use the mood to decompose the share price movement into fundamentals and sentiment. Not much effort has been made to quantify the mood of investors. This study will explore the possibility to quantify the mood of investors by deriving a mood index.

1.3 Objectives of the thesis

The intention of this study is to:

1. Describe the fundamental fuzzy feature in the movement of the Johannesburg Security Exchange Overall Index using the fuzzy logic techniques; Reveal the subjective belief about the movement of the index and give insight into its meaning; Explore the predictability of the JSE Overall Index from the fuzzy Markov chain model.

2. Create a mood index to quantify and describe the mood of investors. Apply the mood index to various market indices and investigate the characteristics and meaning of mood of investors in these markets.

3. Propose empirically a decomposition scheme of a share price into fundamentals and sentiment.

The first objective will be achieved by partitioning the JSE overall index into its fuzzy states: "low", "middle", "high", through the use of properties for membership functions of fuzzy sets. Possibility distributions which represent knowledge and information about the movement of the index will be computed for each fuzzy state. The related necessity measures to determine the deterministic characteristics of the index, and possibility - probability consistency to measure coherence between randomness and vagueness of the index will be computed for each fuzzy state. The movement of the index between the fuzzy states will be presented by a fuzzy Markov chain transition matrix which will also be used in an attempt to predict the index.

The second objective will be accomplished by defining three types of moods of investors. The first will be mood of investors relative to record maximum and record minimum, then mood relative to daily maximum and daily minimum and the final mood of investors will be defined in terms of the cumulative maximum and the cumulative minimum. The three types of mood will be combined to create a mood index. Possibility distributions will be computed to give meaning to the mood index.

The third objectives will be achieved by the assumption that a stable mood of a share price is achieved when share price movement depends on fundamentals. The mood is classified into three states namely, "low", "middle" and "high". The states are fuzzy in nature and each is represented by a possibility distribution. The value with the highest possibility grade in the "middle" state is used to decompose the share price into its fundamentals and sentiment. Values with lesser possibility grades in the "middle" state may also be used in the decomposition depending on the circumstances.

In order to achieve the objectives, the study is divided into a number of chapters. These are discussed below.

1.4 Outline of the thesis

The purpose of this section is to provide the outline of the study. The above mentioned objectives will be addressed in this study which consists of seven chapters. In the first chapter, the introduction and motivation of the study have been given.

Chapter 2 provides literature review. Stock markets and market indices used to measure market movement are described. The characteristics of the indices are mentioned. Factors responsible for changes in share prices are discussed. Some commonly models used in the analysis of market indices are cited.

Chapter 3 is concerned with the Statistical techniques. One of the important techniques to investigate the characteristics of a time series is the fractal dimension. This technique is described briefly. The fuzzy techniques to describe vagueness of data (time series) are: membership functions, possibility distributions, and fuzzy Markov chains. These techniques are described. Necessity measures to assess the deterministic aspects of data is discussed. Classical Markov chain to compute transition probabilities of a process are given.

Chapter 4 describes data that will be used to carry out the study. The financial data are briefly discussed. Preliminary results are presented.

Chapter 5 documents the use of fuzzy set theory techniques to describe movement of the JSE overall index. The index is partitioned into fuzzy states: Low, Middle and High. It is revealed that the states are not sharply defined. The possibility distributions of the states representing information and knowledge about the movement of the JSE Overall Index are evaluated. Possibility - probability consistency to asses coherence between subjective belief about events and the frequency of occurrence is computed. Possibility measures, ignorance or vaguness, and necessity measures are also computed. Fuzzy Markov chains transition matrix is computed in order to describe movement of the index between the fuzzy states. Chapter 5 also attempts to predict the index. The fuzzy Markov chain model is used for prediction. To improve the results, the AR(2) model and the fuzzy Markov chain models of its residuals are used.

Chapter 6 defines mood of investors in terms of changes in the market indices or share price. Quantifying mood of investors is important to gauge market performance and has not been done previously. Chapter 6 quantifies three types of investors mood. A composite mood index is created from the three types. Characteristics of the mood of investors need to be investigated. Fractal dimensions of the mood indices are computed and reveal that mood of investors in different markets are persistent not independent processes. Subjective belief and meaning of the mood of investors are summarised by possibility distributions. The chapter reveals that subjective belief about mood and frequency of mood are reasonably coherent. The impact of present mood of investors on the future is computed.

Chapter 7 presents conclusions and discusses the main findings of the study.

In the next chapter literature review describing aspects of service quality and stock markets is presented. This gives a brief review of the techniques used in the analysis of service quality and stock markets.

Chapter 2

Literature Review

The objective of this chapter is to provide a literature review related to stock markets movement. The literature review in this chapter will serve as a background to this thesis. Factors affecting share price changes will be discussed. Financial markets are characterised by uncertainty (Peters 1999). There are two recognised types of uncertainty: randomness and vagueness (Kruse and Meyer 1987). Method dealing with both randomness and vagueness in market movement analysis will be reviewed. Methods used to analyse the JSE overall index will also be reviewed.

2.0.1 Stock Market indices

Most economies of the world consider stock markets very important in that they act as indicators of economic performance. As such, stock market movement provide information about the underlying economic policies of a country (Fama 1976). A summary of market movement is measured by a market index. A stock index is derived from shares. To create a stock index, the following are considered: selection of stocks to be included in the construction, determining the relative importance of each included stock and averaging included stocks (Lorie and Hamilton 1973, Fama 1976). Another important index is the investment performance index. The investment performance index is created from rates of return and it is used to measure total return. Stock indices and investment performance indices have been shown to be positively correlated (Lorie and Hamilton 1973). This indicates that variance in one type of index may be explained by variance in the other.

In South Africa, the Johannesburg Security Exchange (JSE) Overall Index has been used to measure the Johannesburg Security Exchange market movement. The main function of the JSE is raising primary capital by rechannelling cash resources into viable sectors of the South African economy while enhancing job opportunities and wealth creation (Berg 1998). The JSE overall index is the composite index of all the listed companies on the JSE. Its main purpose is to monitor the overall market performance. Some internationally known stock indices are the New York Stock Exchange index (NYSE), the Dow Jones, the Nasdaq, the German DAX, the Nikkei, the Brazilian IBOVESPA etc.

Stock market movement has been widely studied (Lorie and Hamilton 1973, Geisst 1982, Pratten 1993). A wide range of factors have been known to influence market movement. The factors include: changes in share prices and dividend payments, economical environment etc. (Fischer and Jordan 1975). Market movement is an important aspect of investment and affects changes in money supply, industrial production, corporate profits (Lorie and Hamilton 1973). The index makes it possible to compare the price of an individual stock to movement in the market (benchmarking). This is important in portfolio analysis and allocation of funds rationally among stocks (Lorie and Hamilton 1973).

Distributions of market indices are important and will be considered in the next section.

2.0.2 Distributions of market indices

The distribution of an index is a major determinant of risk of investment and gives indirect information about the underlying factors affecting the economy (Fama 1976). Many studies have stated and discussed that market movement is based on the efficient market hypothesis which implies that price changes are random (Brealey 1983). This has led to the assumption and use of the normal distribution in the analysis of market indices returns. The use of the normal distribution implies that knowledge about the mean and the variance of the distribution is adequate to describe the aggregate movement of the index. The variance measures the risk (volatility) of the index (Fama 1976).

However, empirical evidence has shown that returns on most stock indices do not follow the normal distribution. For example Fama (1965) showed that the daily returns had more observations in the left hand tail than in the right-hand tail and that the tails were fatter and the peak around the mean was higher than that expected of the normal distribution. A study of volatility on the daily S&P 500 and the Dow Jones for the period 1928 to 1990 showed that the indices were negatively skewed, with high peaks around the mean compared to the normal distribution (Turner and Weigel 1990). The condition of obtaining fat tails with a high peak around the mean that expected of the normal distribution is called "leptokurtosis". Other studies that have confirmed this condition for market indices include the quarterly S&P 500 returns, from 1946 through 1988 (Friedman and Laibson 1989), the study of financial futures prices (Sterge 1989).

These results show that the normal distribution may not be appropriate to efficiently deal with market movement. The normal distribution has no ability to deal adequately with events which are high standard deviations away from the mean. The normal distribution also assumes stable variance which differs from market movement in reality.

Fractal analysis has shown that market indices posses characteristics of long memory (Peters 1991, 1994). Literature review of chaos and fractal theory models applied to stock markets analysis will be given in the later sections.

2.0.3 Factors affecting market indices

This section will present some of the factors that are understood to affect changes in the stock indices. Since a stock index is derived from shares, its movement is affected by changes in share price. Some of the key factors affecting the movement of share's price have been identified as: company actual performance versus its expected performance, the general economic climate, and the mood of investors.

Company actual performance

Company news is very important to the share price on the stock market. Information about profit predictions are important in that if a gloomy future for profits is forecast, the share price is expected to go down. If predictions show an undervalued company, the share price will rise (Scott 1999). Share price is also affected by the rate at which certain expected earnings are discounted to determine the present value (Lorie and Hamilton 1973, Fischer and Jordan 1975, Brealey and Myers 1984). News about expansion of a company, mergers or take overs can make the share price to rise (Dodd 1980, Brealey and Myers 1984). In a merger investors believe efficiency and profits will increase and this pushes up the share price. New product launches, changes in members of the board etc. also affect the share price. Public information such as dividend announcements, money supply, stock splits and secondary distributions i.e. sales of large quantities of stock that cannot easily be disposed of in the usual manner, have a direct bearing on price changes (Fama, Fisher et al. 1969, Homa and Jaffee 1971, Pettit 1972, Humburger and Kochin 1972, Scholes 1972, Cooper 1974, Rogalski 1977, Charest 1978a, 1978b, Brealey 1983). Insider trading (Lorie and Niederhoffer 1968, Chakravrty and McConnell 1999), estimates of earnings per share (Foster 1973, Patell 1976, Brown 1978) and location of trade (Froot and Dabora 1999) as well as corporate events (Biger and Page 2000) influence share price changes.

General economic climate

The subject of share price relationship with general economic conditions is wide and has been studied extensively. Many aspects of the economy perceived to affect the share price have been identified and studied. Some have been outlined as inflation, unemployment, interest rates, changes in money supply, income and gross national product (Lorie and Hamilton 1973, Rogalski 1977, Brealey 1983).

Stock prices have been known to perform rather less well when inflation is low than when it is high. A possible reason is that inflation is associated with low level of industrial activity which may induce a decrease in company cash flow. At the same time, during inflation periods investors ask for high returns (Moosa 1980, Brealey 1983). Interest rates affect the share price in that if they are high, they make savings accounts more attractive and encourage share holders to sell off their share and deposit their money in low-risk fixed income products. The high rates are a sign of slow growth which affects the companies' profits negatively and adversely affect the share price. When interest rates are high, companies experience high borrowing costs, which in turn affects the business operations and share prices are affected (Scott 1999). Changes in money supply have been known to pave way for changes in general economic conditions. Investors react to money supply information. Thus changes in the growth rate of the money supply have significant effect on business conditions and hence the stock price (Sprinkel 1971, Brealey 1983). Evidence shows that changes in the growth of money supply explains some variation in stock returns (Pesando 1974).

Mood of investors

The mood of investors plays a crucial role in the market movement (Barrett et al. 1988, Lee 1999, Updegrave 2000, Zweig 2001). Mood is regulated by various factors. For example depending on company news and many other things, the mood of investors may be low or optimistic. The general mood of investors is sometimes influenced by information from news papers.

Past work on mood of investors has been documented using qualitative and subjective techniques. The mood of investors will be discussed in detail in chapter 7.

Other factors

Several factors responsible for changes in market indices and share price have been document and studied in the literature as season of year (Keim 1983) and certain months of year (Thaler 1987a, 1987b, Mills and Couts 1995, Mills et al. 2000), the day of the week referred to as the day of the week effect (Harris 1986, Rogaski 1984, Lakonishok and Levi, 1982, Sias and Starks 1995, Wang and Erickson 1997), Friday - Monday effect called the weekend effect (Keim and Stambaugh 1984, Brusa, Liu and Schulman 2000, Connoly 1989). Other studies have shown that political situation influence market movement (David 1992, Ford 1996, Ashurst 1996).

More recently, terrorist attacks such as those targeted at the World Trade Centre in New York and the Pentagon in Washington are also expected to affect financial markets adversely (cnn.com 2001).

Uncertainty in the changes of share price is known as volatility or risk and is commonly measured by standard deviation (Fama 1976). Some of the models frequently used to measure and forecast volatility will be named in the following section.

2.0.4 Models for stock market movement

This section will be concerned with the literature review of models used in describing stock market movement. The autoregressive models, stochastic models, chaos models and fuzzy models will be reviewed. Some of these models have successfully been applied to predict market movement. The estimated market movement variability (i.e. volatility) is associated with many aspects of the stock market such as investment decisions, changes in money supply, corporate returns, industrial production, pricing of financial securities etc. Volatility of stock returns may change over time (Schwert 1989).

Autoregressive models

The use of autoregressive models to describe stock market movement has been documented in many studies. These studies generally measure and forecast market volatility. For example the stationary autoregressive, AR(1) model was used to describe volatility of monthly returns on the Standard & Poor's (S&P) composite index (Poterba and Summers 1986). A linear AR(12) was used to approximate monthly volatility (Schwert 1990, Schwert and Seguin 1990). The univariate autoregressive-integrated-moving average (ARIMA) models were used to investigate the relationship between expected stock returns and volatility of the NYSE and the S&P indexes (French et al. 1987). The models showed a positive relationship between expected risk premium on stocks and the predictable level of volatility.

It has been observed in the literature that some features, such as nonlinear patterns of volatility cannot be described by linear time series models. Some most frequently nonlinear models used in the analysis of stock markets volatility are the autoregressive conditional heteroscedasticity (ARCH) (Engle 1982) and the generalised autoregressive conditional heteroscedasticity (GARCH) (Bollerslev 1986). The ARCH/GARCH models have been applied widely. For example the GARCH methods were used to show that Taiwan's stock market liberalisation induced some changes in the returns distribution of the Taiwan weighted index (Kwan and Reyes 1997). The mean reverting patterns of monthly return indexes of the NYSE, AMEX and NASDAQ were investigated using the GARCH models, in which the analysis showed that negative returns reverse to positive returns quicker than positive reverted to negative ones, and negative returns reduced risk premiums from predictable high volatility (Nam, Pyun and Avard 2001). A special case of the ARCH/GARCH models is the exponentially weighted moving average (EWMA) models. The EWMA has been used to model volatility. For example in a study comparing volatility forecasting techniques on Australian value weighted indices, the EWMA performed better that the ARCH/GARCH models (Walsh and Tsou, 1998).

The ARCH and the GARCH models have a unique characteristic that recognises that volatility is not constant. The models attempt to keep track of the variations in the volatility through time. In general terms estimate of volatility in the ARCH model is based on a long-run variance and more recent observations. The GARCH is generally based on the most recent observations and the most recent estimate of variance rate. The application of these models has been widening and they are surveyed by Bollerslev, Chou and Kroner (1992).

Apart from the ARCH models other non linear models such as the threshold autoregressive models (TAR) are used to produce estimates of volatility. The TAR models were used to estimate volatility of the stock returns of the monthly value-weighted, equal-weighted and S&P composite portfolio excess returns for the NYSE and AMEX, in which the results showed that the threshold autoregressive models were preferable to the ARCH models in the modelling of monthly excess returns (Cao and Tsay 1993). These models take account of asymmetric news, they also measure and test the impact of news on volatility. Threshold GARCH models that take account of asymmetries in volatility have been discussed and analysed (Zakoïan 1991). The threshold GARCH models have been applied to describe asymmetries in the volatility of the daily series of the French CAC index (Rabemananjara and Zakoïan 1993). The results showed evidence of asymmetries in the volatility of the daily series of the French CAC index. Models to handle asymmetric news in the estimation of volatility have been discussed in Nelson (1990).

More recently studies to investigate memory in volatility have been carried out. Beran and Ocker (2001) have developed SEMIFAR model to investigate long memory in volatility. Data from nominal closing indices of 19 stock markets between 1992 and 1995 have been used. The results show that long memory in volatility of stock market indices exists. This suggest that realistic models of volatility should take account of long memory. Volatility persistence was also earlier shown in the study to explore stochastic properties of the quarterly earnings per share series for industrials, railroads and utilities since 1935, in which volatility persistence appeared to be high in all series (Laopodis 1999).

Stochastic models

Stochastic volatility has been studied. Models where volatility follows a stochastic process have been developed in Hull and White (1987). Most stochastic volatility models are used in option pricing. However some of these models have been used to model volatility of stock market indices.

A wide range of other Statistical and probabilistic models have been derived for the analysis of stock markets and financial markets in general. Some of the methods and models are surveyed in (Sundaresan 2000). The methods reviewed include those modelling options and other derivatives valuation in which work by Black and Scholes and Merton have played a central role. Others are methods in the analysis of term structure of interest rates, asset pricing, dynamic consumption and portfolio choice, default risk and credit spreads, capital market frictions, estimation of continuous time-models and international markets and exchange rate dynamics. The most significant developments in each one of these fields are mentioned. Most Statistical and probabilistic models have mainly focussed on the random aspect of uncertainty of stock markets.

Chaos and fractal models

The last few years have seen great interest developed in the study of financial markets using chaos and fractal theory. The origins of the theory are not known exactly, but the work of Lorenz (1963) during his investigations involving weather predictions popularised the theory. One of the motivation for the use of chaos is that it offers a possibility of describing randomness as a result of a known deterministic process. There is also no assumption of the distribution of the process. One of the models used to present chaotic behaviour of a process is the logistic function. May (1985) and Rogers et al. (1986) have studied the *logistic model* depicting different forms of chaotic behaviour, depending on its parameters and initial conditions. One of the approaches to chaos is through the study of fractals. Fractals are sets that exhibit properties of self-similarity (Mandelbrot 1982). The simplest geometric property of a fractal is measured by its fractal dimension. The fractal dimension of a time series is to reveal its characteristics in terms of how it occupies its space. Chatterjee and Yilmaz (1992) and Berliner (1992) have discussed some of the methods to compute fractal dimensions. Mandelbrot (1982) showed that the fractal dimension of a time series can be computed as the inverse of its Hurst exponent. Computation of the Hurst exponent has been discussed and outlined in Peters (1991, 1994).

Fractal dimensions have been used in the analysis of financial time series to determine the characteristics of stock index. Methods of fractal analysis were applied to the daily New Zealand Stock Exchange index (NZSE40) time series. The fractal dimension analysis revealed that the time series had a memory effect of up to 15 days. It also showed that the probability density for the index was far from having a Gaussian behaviour (Kozma and Kasabov 1999). Fractal analysis has also been applied to many stock market indices such as the S&P 500, the Japan, U.K. and German markets each represented by Morgan Stanley Capital International (MSCI) index, individual stocks such as IBM, Mobil Oil, Coca Cola, Niagra Mohawk. The rescaled range (R/S) analysis was used to estimate the Hurst coefficient (H) for each series. The fractal dimensions were computed by taking the inverse of H. In all the computations H was estimated to be $0.5 < H \leq 1$, implying the series were persistent, i.e. had memory. These values of H also suggest that the series were not normally distributed (Peters 1991, 1994). Rescaled range analysis (R/S) was also used to examine the fractal structure of real estate and stock market returns, in which it was shown that the stock market have tendencies consistent with a random walk (Ambrose et al. 1992). In other applications of chaos theory to stock markets, key features of deterministic chaotic systems are discussed in terms of the movement of financial prices (Hsieh 1991).

Methods of chaos analysis do not make any assumption such as Gaussian behaviour for time series. An added advantage of chaos theory is that it has potential to described and explain fluctuations for example in the economy and financial markets that seem to be random using simple deterministic models. Interest in the use of chaos theory in the analysis of stock markets has been on the increase. This increase could be attributed to the realisation that movements in the financial markets are not completely in line with the efficient-market hypothesis and thus cannot be handled by the normal distribution (Peters 1994).

So far the methods and models reviewed deal only with uncertainty due to randomness. Studies that take account of vagueness in financial markets have been done. These will be reviewed in the next section.

Fuzzy models

This section will be concerned with the literature review related to application of fuzzy set theory to stock markets. For a long time analysis of uncertainty was based on probability theory until fuzzy set theory was discovered (Zadeh 1965). Probability theory did not allow the modeling of partial knowledge, mainly from human thinking. The main objective of fuzzy set theory is to build a quantitative frame work that captures the vagueness of human knowledge as it is expressed via the natural language (Dubois and Prade 1991). Among the many fields of study in which fuzzy set theory techniques have been applied include regression analysis (Kacprzyk and Fedrizzi eds, 1992), time series analysis (Song and Chisson 1993, Hwang et al. 1998, Chen and Hwang 2000). In industry, fuzzy set theory has been applied widely (Yen et al. eds. 1995).

The last decade has seen fuzzy techniques being applied in the financial markets, though the techniques have been very technical. The motivation to apply the fuzzy logic techniques has been the realisation that vagueness is an intrinsic part of the stock market movement. Sources of vagueness in stock markets include ambiguity of stock value as stock prices are relatively vulnerable to pure social movements because of no acceptable theory by which to understand the worth of stocks and no clearly predictable consequences of changing one's investments (Shiller 1997), the use of verbal expressions in the analysis of economic and financial problems as well as vagueness in expected returns (Mereš and Mesiar 1999, Mereš 1999). Ambiguous information from political issues, varying degrees of freshness in economic data, interpretation of the meaning of numerical values in the context of the markets, economic knowledge, subjective evaluation of important economic indices by individuals or dealers, etc. are also known to be responsible for vagueness in the markets (Tano 1999). Uncertainty in the enormous number of assets on the market to choose from, vagueness and ambiguity of human understanding have also been documented to cause fuzziness in the financial markets (Tanaka and Guo 1999a, 1999b)

A number of applications of the fuzzy techniques to finance have been documented. For example fuzzy neural systems have been used in the analysis of stock selection (Wong et al. 1992). Karsak (1998) employed fuzzy techniques to measure liquidity risk in capital budgeting. The results showed that fuzzy techniques overcome the shortcomings of the classical techniques. Chyi (1997) used fuzzy techniques to examine the nonlinear dynamics of the stock returns on the Taiwan Stock Exchange (TSE). Data of the daily stock returns of five companies most actively traded on the TSE were used. Classical techniques were also applied to the data. Results showed that the fuzzy model could be a more useful tool in forecasting stock returns. Fuzzy techniques have also been used in technical analysis to identify several candlestick patterns (Ruggiero 1995). Membership functions are defined with respect to the data for each variable involved. The membership functions are programmed into a function which is then used to detect patterns. The win/loss ratio of more than 2:1 is achieved.

Fuzzy set theory techniques combined with neural network methods have been used on the German Stock index (DAX). A system to predict daily return on the DAX using a semantic learning algorithm in making an effective and efficient use of expert knowledge and historic data is developed (Siekmann et al. 1999). Fuzzy-neuro techniques have also been applied in the analysis of the Brazilian Capital Markets. A model that includes empirical knowledge for the prediction of the Brazilian IBOVESPA nominal spot index is developed (Machado et al. 1999). In other applications, fuzzy regression models have been used to predict exchange rates for composite currencies. The models are concerned with issues related to the prediction of composite currencies characterised by an inherent volatility, and uncertain and vague knowledge (Yen and Ghoshray 1999). In market equilibrium conditions analysis, fuzzy sets have been applied, where the concept of fuzzy financial law useful in dealing with some financial phenomena in cases of uncertainty is introduced (Greco 1999). In general finance pattern recognition fuzzy techniques have been used in the analysis of risk and claim classification. The technique focuses on models of uncertainty different from models by probability. The models are applied to the Massachusetts private-passenger automobile insurance data. The results suggest that fuzzy techniques are a valuable tool in evaluating data provided by claims adjusters (Derrig and Ostaszewski 1995).

Fuzzy logic is perceived to offer advantages over traditional analysis methods. Fuzzy logic intentionally makes vague the sharp boundaries in logic and allows the recognition of outlying factors (Zadeh 1965, Stein 1991). Other benefits include increased flexibility, greater tolerance of imprecise data and capacity to model nonlinear information of arbitrary complexity, (Malhotra, R. and Malhotra, D. K. 1999).

Vagueness in stock markets show that markets characterized by partial knowledge and information. Systems with partial knowledge and information are effectively modeled by possibility theory (Tanaka and Guo 1999b). From the literature review fuzzy techniques have not been applied to analyse vagueness of the JSE Overall Index. This thesis is concerned with exposing the fundamental feature of fuzziness of the JSE Overall Index.

2.1 Summary

This section will present a summary of the literature review on stock market. The literature review has shown that market movement is measured by market indices. Changes in a market index are affected by factors responsible for changes in share price. The factors have been summarised as company performance, general economic pattern and mood of investors (Lorie and Hamilton 1973, Brealey 1983, Brealey and Myers 1984, Scott 1999).

Stock markets are characterised by uncertainty (Peters 1999). Uncertainty of a market index is measured by volatility. Models used to measure and predict volatility include the ARCH, GARCH models (Engle 1982, Bollerslev 1986, Bollerslev, Chou and Kroner 1992). A major advantage of these models is that they attempt to keep track of the variations in the volatility through time. The threshold models such as the TAR have been used to measure volatility (Cao and Tsay 1993). Models that take account of asymmetric news in the analysis of volatility have been used (Nelson 1990). Stochastic models to measure and predict volatility in financial markets have been used (Hull and White 1987). Models to analyse financial markets in general have been surveyed (Surandesen 2000).

The distribution of a market index is important. It is a major determinant of risk of investment and gives indirect information about the underlying factors affecting the economy (Fama 1976). Many studies have shown that most market indices exhibit a high peak at the mean and fat tails (Fama 1965, Friedman and Laibson 1989, Turner and Weigel 1990). The rescaled range analysis to compute the Hurst exponent and fractal dimension has been on most stock indices (Peters 1991, 1994). The results have shown that most stock indices are not independent processes and thus should not be modelled by the normal distribution. The indices are persistence and thus posses characteristics of long memoriness. An advantages of chaos analysis is that no assumption about the distribution of the time series to be analysed is made. Chaos analysis of the JSE overall index to determine its characteristics has not been performed. This study will compute the Hurst exponent and fractal dimention of the JSE overall index to determine the nature of the index.

Vagueness plays a critical role in the movement of stock markets. Fuzzy set theory is used to deal with Vagueness (Zadeh 1965). Sources of vagueness in market indices have been identified (Mereš and Mesiar 1999, Tano 1999, Tanaka and Guo 1999a, 1999b). Fuzzy techniques and euro-fuzzy systems have been used to analyse financial markets and predict some stock market indices (Ribeiro et al. eds. 1999). Vagueness also exits in portfolio selection and possibility theory techniques have been used to deal with the problem (Tanaka and Guo 1999a).

Vagueness of the JSE Overall Index has not been studied in the literature. This study will apply fuzzy techniques to deal with vagueness in the movement of the JSE Overall Index.

Chapter 3

Statistical and Fuzzy Techniques

3.1 Introduction

The purpose of this chapter is to describe briefly the techniques that will be used in the analysis of the JSE Overall Index and other international indices data. Data analysis is crucial to gain insight into the movement of stock market indices. Stock market indices are characterised by uncertainty (Peters 1999), for example randomness and vagueness. Randomness is related to the frequency of occurrence of events captured by repeated experiments whose outcomes are corded, and is dealt with by probability models. Vagueness, on the other hand, is related to the imprecision of natural language and partial belief due to incomplete information and is modelled by fuzzy logic (Zadeh 1978a). Data to which probability and fuzzy logic models are applied are observed over time and sometimes presented as a time series. Rescaled range analysis will be used to compute the fractal dimension which is used to determine the nature of a time series. This study will focus on the analysis of vagueness in the movement of the JSE Overall Index using fuzzy logic techniques. Fuzzy membership functions, possibility distributions, possibility measures, possibility - probability coherence, fuzzy Markov chains, and necessity measures will be used in this study. A brief outline of these techniques will be given in the following sections.

3.2 Fractal dimensions

Fractal dimension to determine the characteristics of a time series have been studied and applied widely (Mandelbrot 1997, Kozma and Kasabov 1999, Higuchi 1988, Tsonis and Elsner 1992, Peters 1991, 1994). Some of the features revealed by the fractal dimensions of a time series include stationarity or non-stationarity, periodic components, randomness or biasedness. One of the methods to compute the fractal dimension of a time series is by using the Hurst exponent (H). The Hurst exponent is computed using the method of rescaled range analysis (R/S).

Steps to apply the R/S analysis in computing the Hurst exponent have been outlined as follows (Peters 1991, 1994): The rescaled range (R/S) analysis was first employed by Hurst in the 1940s and later by Mandelbrot in the 1960s and 1970s to study biased random walks. Hurst worked on the Nile river dam project to design a policy of managing the dam such that the reservoir never overflowed or emptied. In the process of this work, he developed a new statistic now called the Hurst exponent (H). The Hurst analysis is based on the principle that time is an iterative process, and the impact of the present on the future can be expressed as a correlation

$$C = 2^{(2H-1)} - 1$$

where C = correlation measure, H = Hurst exponent. There are three distinct classification for the Hurst exponent (H):

$$\begin{aligned} H &= 0.50 \\ 0 &\leq H < 0.50 \\ .50 &< H < 1.0 \end{aligned}$$

H equal to 0.50 denotes a random series. $0 \leq H < 0.50$ implies that the system is antipersistent. When $.50 < H < 1.0$, we have a persistent, or trend-reinforcing series.

The procedure to compute the Hurst exponent is given in Peters (1991, 1994). The method of testing the significance of the computed Hurst exponent is explained in Peters (1994). The method involves computing the expected R/S_n values i.e. $E(R/S_n)$. The $E(R/S_n)$ is computed as a comparison against the null hypothesis that the system is an independent process. The expected value of the Hurst exponent $E(H)$ is computed from the $E(R/S_n)$ values.

The R/S values are random variables and normally distributed, thus the values of H are expected to also be normally distributed and with variance

$$Var(H) = \frac{1}{T}$$

where T is the number of observations in the sample. This is expected to be the variance around the $E(H)$, as calculated from the $E(R/S_n)$.

The V-statistic which is a ratio of R/S_n to $\sqrt{n}$ is used to determine the periods when the process will exhibit antipersistence ($H < 0.5$), randomness ($H = 0.5$) or persistence ($H > 0.5$).

In this study, the Hurst exponent phenomena for the JSE overall index, and the mood indices of financial markets will be investigated. The Gauss Light programme codes given in Peters (1994) will be used to compute the Hurst exponent, and $E(R/S_n)$.

3.3 Fuzzy Sets

Fuzzy logic was initiated by (Zadeh 1965). Since then it has been applied widely to solve many real life problems in various fields such as the medical diagnosis of diseases (Adlassnig, 1982, Sanchez, 1986), manufacturing and food processing (AbdelRahman 1995), intelligent sensors systems for space operations (Lea and Jani 1995), intelligent manufacturing systems (Jamshidi 1995), improving features of imaging processing equipment (Tagaki 1995, Kerre and Nachtegael, eds. 2000). In cluster analysis and pattern recognition, fuzzy logic has been used to successfully classify features extracted from real data (Newton et al. 1992, Wolf et al. 1996, Frank et al. 1998). In regression analysis, a wide range of fuzzy regression models have been developed and used for forecasting under uncertainty (Heshmatty and Kandel 1985, Celminš 1987, Chang and Lee 1996, Diamond and Körner 1997). A summary of development and examples of application of fuzzy regression models can be found in Kacprzyk and Fedrizzi 1992). In time series analysis, fuzzy time series models and forecasting procedures have been suggested (Song and Chisson 1993). Fuzzy time methods are used to predict university enrollments (Chen 1996) and to predict temperature (Chen and Hwang 2000). More recently fuzzy logic techniques have been applied to stock markets, finance and economics (Ribeiro et al. eds. 1999). The areas of fuzzy applications will continue to widen.

The motivation of fuzzy logic is mainly to provide formal, powerful and quantitative frame work to cope with the vagueness of human knowledge as it is expressed by means of natural language (Dubois and Prade 1991). One of the basic tools of fuzzy logic and approximate reasoning is the notion of linguistic variables. The motivation of the use of linguistic variables is that they are less specific than numerical ones and are thus more in line with human thinking and perception of things (Zadeh 1973, Zimmermann 1991).

An important aspect of fuzzy analysis is the determination or derivation of membership functions. Some of the types of membership functions used in the literature are mentioned in the next section. The membership functions are the most important aspects of fuzzy analysis.

3.3.1 Membership functions

The membership functions are a cornerstone for any meaningful fuzzy analysis. Thus determination of a membership function is crucial in practical application of fuzzy set theory. A meaningful membership function aught to take account of the context of data or situation being modelled.

3.3.2 Definition of membership function

The purpose of this section is to give the give the definition of a membership function. Mathematically a membership function is an extension of the indicator function.

Define the indicator function

$$I_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

of a crisp set A assigns a value of either 1 or 0 to each individual in the universal set, thereby discriminating between members and nonmembers of the crisp set under consideration. This function can be generalised such that the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set in question. Such a function is called a membership function and the set defined by it a fuzzy set.

Let X denote a universal set. Then, the membership function μ_A by which a fuzzy set A is usually defined has the form

$$\mu_A : X \to [0,1],$$

where $[0, 1]$ denotes the interval of real numbers from 0 to 1 inclusive (Zadeh 1968, Dubois and Prade 1980)

3.3.3 Approaches to membership function derivation

Several different approaches to membership derivation have been widely addressed and surveyed (Dubois and Prade 1980).

It is desirable to derive membership functions from context. One of the most natural way to derive a membership function has been the use of expert opinion. Experts are polled on an opinion say x "is small". If M out of N answers say yes to x "is small" then $\frac{M}{N}$ is the degree of belief x "is small". $\frac{M}{N}$ is interpreted as frequency or subjective probability that x "is small" (Klir and Folger 1988, Ramer and Kreinovich 1994).

In other instances membership functions are derived from statistical data when elements of the fuzzy set have defining features with a known probability densities (Nowakowska 1977, Civanlar and Trussel 1986).

The S function is used as membership function for fuzzy sets (Zadeh 1978a, 1978b, Zimmermann 1991). For example, if U is universe of discourse with a generic element of U denoted by u, the S function is defined by

$$S(u;\alpha,\beta,\gamma) = \begin{cases} 0 & \text{for } u \leq \alpha \\ 2\left(\frac{u-\alpha}{\gamma-\alpha}\right)^2 & \text{for } \alpha \leq u \leq \beta \\ 1-2\left(\frac{u-\gamma}{\gamma-\alpha}\right)^2 & \text{for } \beta \leq u \leq \gamma \\ 1 & \text{for } u \geq \gamma \end{cases}$$

where α, β, and γ are parameters. The parameter $\beta \triangleq (\alpha+\gamma)/2$ is the cross over point, that is, $S(\beta;\alpha,\beta,\gamma) = 0.5$.

To model verbal vagueness, a membership function was developed such that the source of vagueness is represented by a generating function with an increasing continuous mapping

$$f : R \rightarrow R \quad \text{with } f(0) = 0.$$

Such a generation function representations the distribution of vagueness. A shape generator

$$\varphi : R \rightarrow [0, 1]$$

is found which generates individual fuzzy quantities with respect to the structure of the generation function (Mereš and Mesiar 1999).

In the analysis and prediction of the DAX index applying the neural fuzzy methods, each input variable created for the index has three defined fuzzy sets: *decreasing*, *stable* and *increasing*. The fuzzy set *stable* is realised by the Gaussian membership function, and the other states logistic membership functions. The Gaussian and the logistic functions are respectively defined:

$$m_j(x_i) = \exp\left(-0.5\frac{(x_i - \mu_{ij})^2}{\sigma_{ij}^2}\right) \quad \text{(Gaussian)}$$

$$m_j(x_i) = \left(1 + \exp\left(-4\frac{(x_i - \mu_{ij})}{\sigma_{ij}}\right)\right)^{-1} \qquad \text{(logistic)}$$

where μ_{ij} and σ_{ij} are parameters of the membership functions and defined by analysing the distributions of the input variables (Siekmann et al. 1999).

In the modelling of group decisions dealing with linguistically quantified propositions, two basic types of quantifiers are identified i.e. absolute and proportional (Kacprzyk 1986) which are defined as membership functions.

A method to extract membership functions from histogram is defined in Dubois and Prade (1986). This method is justified by the fact that the grades of belief (from membership function) cannot be thought of independently of the frequency of occurrence of events. In Civanlar and Trussell (1986), a method to construct membership of fuzzy sets whose elements have features of known probability density functions is presented. The method is based on the possibility-probability consistency principle.

A fuzzy set can also be given by the $L - R$ representation (Dubois and Dubois 1980). That is, a fuzzy number M is of the form $L-R$ type if

$$\mu_M = \begin{cases} L\left(\frac{m-x}{\alpha}\right) & for \quad x \leq m, \alpha > 0 \\ R\left(\frac{x-m}{\beta}\right) & for \quad x \geq m, \beta > 0 \end{cases}$$

denoted by

$$M = (m, \alpha, \beta)_{LR}$$

where $L(\cdot)$ is for left and $R(\cdot)$ right reference. m is the mean value of M. α and β are called the left and right spreads, respectively. $L(\cdot)$ and $R(\cdot)$ satisfy the following conditions:

1. $L(x) = L(-x), \qquad R(x) = R(-x)$,
2. $L(0) = 1, \qquad R(0) = 1$,
3. $L(x)$ and $R(x)$ are strictly decreasing for $x \geq 0$.

Examples of $L(x)$ are as follows

$$\begin{aligned} L_1(x) &= \max(0, 1 - |x|^p), \\ L_2(x) &= e^{-|x|}, \\ L_3(x) &= \frac{1}{1+|x|^p}, \end{aligned}$$

where $p \geq 0$. $R(x)$ can be given as the same as $L(x)$, see also (Tanaka and Guo 1999b)

3.3.4 Notation of a fuzzy set

The standard notation of a fuzzy set is described and presented. A fuzzy set in X is denoted by $A \subseteq X$; its membership function is

$$\mu_A : X \rightarrow [0, 1].$$

Informally fuzzy sets are equated with their membership functions. When X is finite, say, $X = \{x_1, x_2, \dots, x_n\}$, a fuzzy set $A \subseteq X$ is written as

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \dots + \frac{\mu_A(x_n)}{x_n}$$

where + is meant in the set-theoretical sense (Zadeh 1973, Kacprzyk 1986).

3.3.5 Mean and Variance of Fuzzy Set

In this section, the mean and variance of a fuzzy set are defined. The definitions of mean and variance of a fuzzy set involves the probability of the set. Probability of a fuzzy set is defined in Zadeh (1968) as:

$$\begin{aligned}\Pr(A) &= \int_{R^n} \mu_A(x)dP \\ &= E(\mu_A).\end{aligned}$$

Further Zadeh (1968) has defined the mean and variance of a fuzzy set as

$$m_p(A) = \frac{1}{\Pr(A)} \int_{R^n} x\mu_A(x)dP$$

and

$$G_p^2(A) = \frac{1}{\Pr(A)} \int_{R^n} (x - m_p(A))^2 \mu_A(x)dP$$

respectively, where μ_A is the membership function of fuzzy set A and $\Pr(A)$ serves as a normalising factor. The entropy of a fuzzy subset, A, of the finite set $\{x_1, x_2, \dots, x_n\}$ with respect to a probability distribution $P = \{p_1, p_2, \dots, p_n\}$ is defined as

$$H^p(A) = -\sum_{i=1}^{n} \mu_A(x_i) p_i \log p_i$$

where μ_A is the membership function of A. $H^p(A)$ may be interpreted as the uncertainty associated with a fuzzy event (Zadeh 1968).

3.3.6 The Prediction Problems Of Fuzzy Markov Chains

In this section, the issue of fuzzy Markov chain prediction is described. A detailed outline of the fuzzy Markov chains is given in Xiang (1982).Assuming that a Markov chain has sates $E_i, i = 1, 2, ..., k$. The one step transition probabilities p_{ij} are obtained by statistical estimation. In the usual Markov chain prediction practices, predicting the future state the system will enter is based on the maximum transition probability criterion: assuming the current state is E_i, if

$$p_{ij^*} = \max_{j \in K}\{p_{ij}\}$$

where $K = \{1.2.3, ..., k\}$, the system will transit to state E_{j^*}. if

$$p_{ij^*} \gg p_{ij}, \quad j \neq j^*$$

then the confidence is high to predict that the next state will be E_{j^*}, otherwise the confidence is low. For Markov chains with fuzzy states, assume a time series $X(t) : x_1, x_2, ..., x_n$. Let $E_i, i = 1, 2, ..., k$ be a fuzzy partition of the range in which $X(t)$ takes its values.

$$\sum_{i=1}^{k} \mu_{\underline{E_i}}(x) = 1, \forall x \in X.$$

Notice that practically $k \ll n$.

It is critical to determine the initial probability $p_i^0, i = 1, 2, \ldots, k$, for each of the fuzzy states and transition probability p_{ij}. Let $\widetilde{n}_i$ denote the number of $x_1, x_2, ..., x_{n-1}$ falling into fuzzy set $\underline{E_i}, i = 1, 2, ..., k$. Define

$$\widetilde{n}_i = \sum_{l=1}^{n-1} \mu_{\underline{E_i}}(x_l), i = 1, 2, ..., k.$$

The quantities

$$\widetilde{F}_i = \frac{\widetilde{n}_i}{n-1}, i = 1, 2, ..., k.$$

are defined as the fuzzy frequency in which fuzzy state $\underline{E_i}$ happened. The initial probability of state $\underline{E_i}$ is

$$p_i^0 = \widetilde{F}_i, i = 1, 2, ..., k.$$

Define

$$\widetilde{n}_{ij} = \sum_{l=1}^{n-1} \mu_{\underline{E_i}}(x_l)\mu_{\underline{E_j}}(x_{l+1}), i = 1, 2, ..., k.$$

Then the transition probabilities μ

$$p_{ij} = \frac{\widetilde{n}_{ij}}{\widetilde{n}_i}.$$

For a given realisation x_n at time n,the membership grades with respect to each state E_i,$\mu_{E_i}(x_n), i = 1, 2 \ldots, k$, denoted as $\widetilde{\mu}$ is given by

$$\widetilde{\mu}(x_n) = (\mu_{E_1}(x_n), \mu_{E_2}(x_n), \ldots, \mu_{E_k}(x_n))$$

Then

$$\begin{aligned} \widetilde{\mu}(x_{n+1}) &= (\mu_{E_1}(x_{n+1}), \mu_{E_2}(x_{n+1}), \ldots, \mu_{E_k}(x_{n+1})) \\ &= \widetilde{\mu}(x_n) \cdot P, \end{aligned}$$

where $P = (p_{ij})_{k \times k}$.

In terms of maximum membership principle

$$\mu_d(x_{n+1}) = \max_{i \in K}\{\mu_{E_i}(x_{n+1})\},$$

the next state will transit to state E_i.

3.4 Possibility distributions

In order to represent knowledge and information about data, possibility distributions are used. Most information about service quality, stock market index, and mood of investors is generated by human thinking and summarised by natural language. Thus information and knowledge about these systems are impartial and possibilistic in nature. In this study, possibility distributions will be use to describe information and knowledge about the JSE Overall Index data and the mood indices of various stock market indices.

Possibility is interpreted in two ways: physical as a measure of material difficulty of performing an action, and epistemic as subjective judgement that does not commit its maker (Dubois and Prade 1988). Possibility is also understood as imprecise boundaries or fuzziness or lack of specificity (Yager 1986). Possibility theory was proposed by Zadeh (1978a). In possibility theory, fuzzy variables are associated with possibility distributions in much the same manner as random variables are associated with probability distributions. Possibility theory is considered part of fuzzy set theory. Possibility distributions are applied to data analysis where the problem is to deal with uncertainty due to subjective belief rather than randomness. A wide application of possibility include portfolio selection (Tanaka and Guo 1999a).

3.4.1 Definition of possibility

The fuzzy concept of possibility was introduced by Zadeh (1978a) who defined it as:

Definition 1 *A fuzzy set (Zadeh 1965) F in a given set U is characterized by a membership function $\mu_F(u)$ which associates with each point u in U a real number in the interval* $[0, 1]$, *with the value $\mu_F(u)$ representing the grade of membership of u in F.*

The concept of possibility distribution bears a close relation to the concept of fuzzy restriction (Zadeh, 1978a). Let X be a variable which takes values in a finite set U and suppose we have fuzzy information about the values of X, that is a fuzzy restriction "X is F", where F is a fuzzy subset of U. In this situation the fuzzy information is associated with a possibility distribution which coincides with the membership function of F (Moral 1986). According to Zadeh (1978a) a formal definition of a possibility distribution is as follows:

Definition 2 *Let F be a subset of a universe of discourse U which is characterized by its membership function μ_F, with grade of membership, $\mu_F(u)$, interpreted as the compatibility of u with the concept labeled F. Let X be a variable taking values from U, and let F act as a fuzzy restriction, $R(X)$, associated with X. Then the proposition "X is F", which translates into*

$$R(X) = F,$$

associates a possibility distribution, Π_X, with X which is postulated to be equal to $R(X)$, i.e.,

$$\Pi_X = R(X).$$

Correspondingly, the possibility distribution function associated with X (or the possibility distribution function of Π_X) is denoted by π_X and is defined to be numerically equal to membership function of F, i.e.,

$$\pi_X \triangleq \mu_F.$$

Thus, $\mu_F(u)$, the possibility that $X = u$, is postulated to be equal to $\mu_F(u)$. And $\mu_F(u)$ is interpreted as the degree to which the constraint represented by F is satisfied when u is assigned to X. Equivalently $1 - \mu_F(u)$ is the degree to which the constraint in question must be stretched in order to allow the assignment of u to X.

If $A(X)$ is an implied attribute of X taking values in U, then $R(X) = F$ is written

$$R(A(X)) = F.$$

In this study the procedure used to compute the possibility distributions is based on extracting membership functions from a histogram given as follows:

Let the $w_i's$ be reodered such that $p_1 \geq p_2 \geq \cdots \geq p_n$ and

$$A_j = \{w_1, w_2, \ldots, w_j\} \text{ for } j = 1, 2, \ldots, n; \; A_0 = \emptyset.$$

The possibility distribution $\Pi_A = (\pi_1, \ldots, \pi_n)$ of A is such that the $\pi_i's$ are computed by

$$\pi_i = \sum_{k=1}^{n} \min(p_i, p_k), \; i = 1, ..., n.$$

A detailed exposition of the procedure is given in Dubois and Prade (1986).

3.4.2 Possibility Measure

In order to gain some insight into the difference between probability and possibility, we need to compare the concept of a possibility measure with the concept of probability measure (Zadeh

1978a). In this section, the possibility measure of a set is described. In Zadeh (1978a) the possibility measure of a non fuzzy subset A of U is defined by

$$\pi(A) \triangleq \sup_{u \in A} \pi_X(u), \tag{3.1}$$

where $\pi_X(u)$ is the possibility function of Π_X. If A is a fuzzy subset of U then the possibility of A is

$$\pi(A) \triangleq \sup_{u \in U} \mu_A(u) \wedge \pi_X(u), \tag{3.2}$$

where Π_X is the possibility distribution associated with a variable X which takes values in U and $\wedge$ stands for minimum. If the height of a fuzzy set is defined as the supremum of its membership function then (3.2) may be expressed as

$$\pi(A) \triangleq \text{ Height } (A \cap \Pi_X). \tag{3.3}$$

In Dubois and Prade (1988), to an event $A \subseteq \Omega$ we can associate a real number $g(A)$ which measures the confidence one may have in the occurrence of the event A taking the state of knowledge into account. If A is a sure event then $g(A) = 1$, and if A is an impossible event, then $g(A) = 0$. For all $A, B \in \Omega$

$$g(A \cup B) \geq \max(g(A), g(B)). \tag{3.4}$$

The limiting measure of (3.4) denoted by Π such that

$$\forall A, B, \qquad \Pi(A \cup B) = \max(\Pi(A), \Pi(B)) \tag{3.5}$$

is called possibility measure and for finite Ω, $\Pi(A)$ is defined as in (3.3).

$\Pi(A) = 1$ means A is possible. In particular if A and $\overline{A}$ are two contradictory events then (3.5) implies

$$\max(\Pi(A), \Pi(\overline{A})) = 1 \tag{3.6}$$

interpreted as the fact that of two contradictory events, one at least is completely possible.

3.4.3 Possibility-Probability Consistency

In order to measure coherence between randomness and vagueness, possibility - probability consistency is used. The possibility-probability consistency is defined by Zadeh (1978a). If a variable X can take the values $u_1, u_2, \ldots, u_n$ with respective possibilities, $\Pi = (\pi_1, \pi_2, \ldots, \pi_n)$ and probability distribution, $P = (p_1, p_2, \ldots, p_n)$, then the degree of consistency of the possibility distribution Π with the probability distribution P is expressed by

$$\gamma = \sum_{i=1}^{n} \pi_i p_i.$$

It is also described as the coherence between information provided by the possibility distribution, Π and the probability distribution P (Moral 1986).

In order to measure the relationship between subjective belief about the occurrence of events and chance of events occurring, the possibility - probability consistency will be computed for the JSE Overall Index data, and for mood indices of different market indices..

In this study, basic principles of possibility distributions will be applied in the analysis of partial information and knowledge about the aspects of the JSE Overall Index, and the mood indices of financial markets. The study will show that by computing possibility distributions of the JSE Overall Index, the meaning of various aspects of the index movement will be achieved. The possibility distributions of the index will reveal that the meaning of the index changes with time. Subjective belief about the movement of the JSE Overall Index will be described. For the mood indices, possibility distributions will reveal the characteristics and meaning of the mood of investors in different markets.

3.5 Necessity measures

Necessity which is a complement of possibility measures describes the deterministic aspect of an event. In this section, the reviewed necessity measure concepts are extracted from Dubois and Prade (1983, 1988b). The necessity measure denoted by N, is the limiting function of the confidence measure

$$g(A \cap B) \leq \min(g(A), g(B))$$

where g is defined in section 4.1. Thus

$$\forall\ A,\ B, \qquad N(A \cap B) = \min(N(A), N(B)). \tag{3.7}$$

If the possibility distribution is known the necessity function can always be constructed by means of

$$N(A) = \inf\{1 - \pi(\omega) \mid \omega \notin A\}. \tag{3.8}$$

(Dubois and Prade 1988b) where $\pi(\omega)$ is a possibility function.

In (Dubois and Prade 1983) a general finite case of computing the necessity $N(A)$ of a set A is given as follows; Let

$$U = \{u_i \mid j = 1, 2, 3, ..., n\}$$

The u_i are ranked such that

$$p_1 \geq p_2 \geq \cdots \geq p_n$$

where

$$p_i = P(\{u_i\}), \quad \sum_{i=1}^{n} p_i = 1 \tag{3.9}$$

and P is a probability measure. A_i denotes the set $\{u_1, u_2, \ldots, u_i\}$. $A_0 = \emptyset$ by convention. The degree of necessity of event $A \subseteq U$ is then the extra amount of probability of elementary events in A over the amount of probability assigned to the most frequent elementary event outside A. That is

$$N(A) = \sum_{x_j \in A} \max\left(p_j - \max_{x_k \notin A}\ p_k, 0\right).$$

If $A = A_i$ we get

$$N(A_i) = \sum_{j=1}^{i} (p_j - p_{i+1}), \quad i = 1, 2, 3, ..., n$$

where by convention $p_{n+1} = 0$. If necessity measure of an event exits i.e. greater than zero, then pure randomness of occurrence of the events is removed. In other words there is the necessity of events to occur. $N(A)$ is also viewed as the grade of impossibility of the events not A i.e. the event $\overline{A}$.

$N(A) = 1$ means that A is sure (necessarily true). Necessity measures satisfy the relation

$$\min(N(A), N(\overline{A})) = 0 \tag{3.10}$$

which prohibits two contradictory events from both being slightest bit necessary at the same time. And

$$\forall\ A \subseteq \Omega, \qquad \Pi(A) \geq N(A) \tag{3.11}$$

which agrees with the intuition that an event becomes possible before becoming necessary. In addition,

$$\begin{aligned} N(A) &> 0 \Rightarrow \Pi(A) = 1 \\ \Pi(A) &< 1 \Rightarrow N(A) = 0. \end{aligned} \tag{3.12}$$

Measures of probability, possibility, and necessity have in common that all three can be characterized by a distribution on the elements of the reference set.

In this study necessity measure of the states of the queue size and the states of the JSE Overall Index are computed. The necessity measure will be used to explore the stability of the index. The stability of the JSE Overall Index may be linked to how stable the major contributors to the index are the necessity measures will also give an indication of how stable the South Africa economy is.

3.6 Summary

This chapter has briefly described and presented statistical and fuzzy techniques that will be used to describe data in this study. The following techniques have been discussed: fractals, fuzzy sets techniques, membership functions, possibility distribution and necessity measures. The fractal dimensions will be used to investigate the characteristics of the data. The fuzzy techniques are necessary to capture and describe vagueness of data. This will be achieved by using membership functions. Possibility distributions will be used to asses the possibility of events in the data. Possibility distributions will also reveal meaning of data. The movement of time series between their fuzzy states will be represented by the fuzzy Markov chain transition matrices. The fuzzy matrix will be used in an attempt to predict the states of time series. To investigate the deterministic aspects of data, the necessity measures will be computed.

Chapter 4

Data and preliminary analysis

The purpose of this chapter is to describe the data that will be used to achieve the stated objective. The data includes the JSE overall index and other financial indices (NSYE index, Dow Jones, Nasdaq, DAX, NIKKEI500, IBOVESPA, JSE Gold). The preliminary results of the data will be presented and involves:computations of the correlation matrix,simple statistics and the Hurst exponent . The purpose of the computations is to farmiliarise with the structure of the data.

4.1 Financial data

4.1.1 The JSE Overall Index Data

In this section,the JSE Overall Index data will be describe. The data comprises the daily returns for the period 27th March, 1985 to 1st November, 2001. The data were obtained from the University of Cape Town, Department of Statistics data base. The data comprised, the last trading values, the highest values, the lowest and the closing values. The data is plotted in Figure 4.1. The plot shows a general upward trend with local fluctuations in the movement of the JSE Overall Index. The mean and variance of the series are not constant.

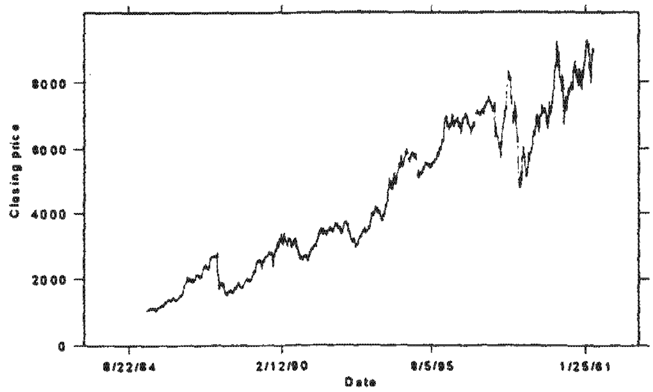

Figure 4-1: JSE Overall Index: 27th March 1985 to 1st November 2001.

The studies carried out on stock markets have suggested that stock market index data can be described and modeled by stochastic processes; that is if S denotes an index, then it can be expressed as

$$S = \{S_t, \quad t \in \mathbb{R}^+\}.$$

Models such as the geometric Brownian Motion are suggested for such a process (Wilmott 1999) to sufficiently describe the data. If a Brownian motion model is assumed the log ratio transformation of the daily closing JSE Overall Index data such that

$$u_t = \log \frac{S_t}{S_{t-1}}, \quad t = 1, 2, 3, \ldots, n$$

results in an independent increments sequence and again produces a Brownian motion

$$U = \{u_t\}, \quad t \in \mathbb{R}^+.$$

In this study $S = \{S_t, \quad t \in \mathbb{R}^+\}$ denotes daily closing value for the JSE overall index and $u_t = \log \frac{S_t}{S_{t-1}}$ its log ratio.

The plot of u_t computed from the daily closing JSE Overall Index is given in Figure 4.2.

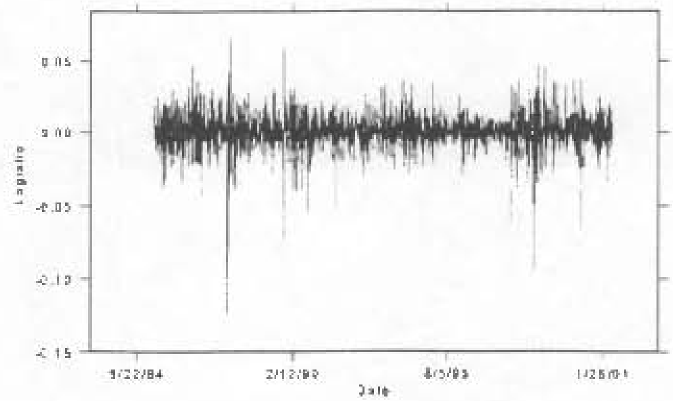

Figure 4-2: Log ratio of the JSE Overall Index: 27th March 1985 to 1st November 2001.

It is seen from the plot that most u_t values are near 0. For such values that S_t and S_{t-1} are not very different. It is also seen that some u_t values are way below 0. In such cases S_{t-1} is far much greater than S_t and are associated with market crash. The instances when u_t is way above 0 are when S_t is much greater than S_{t-1} and are associated with the rise in the index.

The histogram of u_t is given in Figure 4.3.

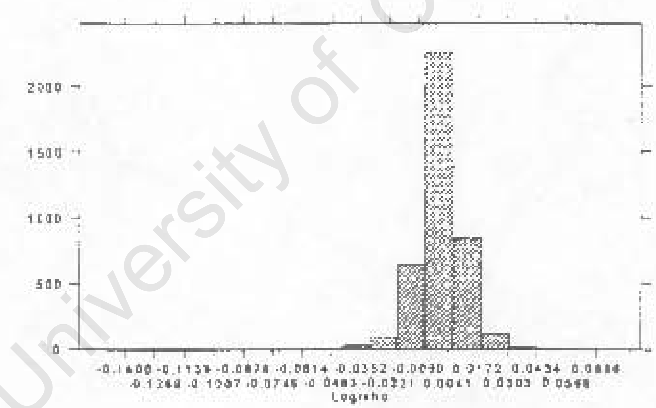

Figure 4-3: Histogram: JSE Overall Index, 27th March 1985 to 1st November 2001.

The histogram shows a high peak at $u_t = 0$. This implies there was not much change in the S_t values for most of the time the index was observed. The frequency of very small values of u_t signify the times of market crash. The frequency of big values of u_t show market rise.

The estimated density of u_t is shown in Figure 4.4.

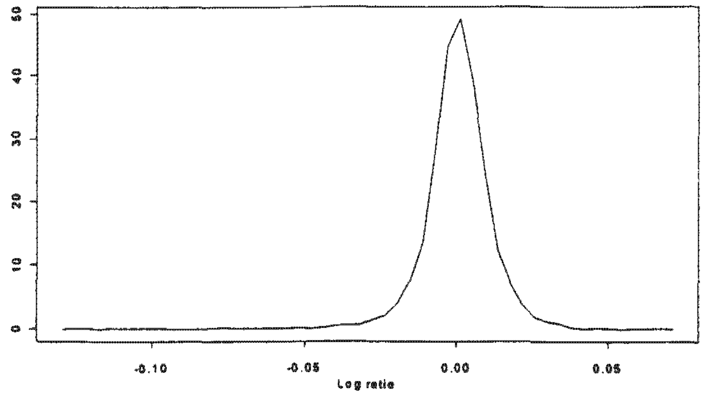

Figure 4-4: Density estimate: JSE Overall Index, 27th March 1985 to 1st November 2001.

with mean

$$\overline{u} = 0.0005365679$$

and the sample standard deviation

$$s = 0.01106875$$

The transformed data is assumed to follow a normal distribution (Wilmott 1999, Hull 2000). In the figure above, the distribution of the log index has a long but thin lower tail, a short but fat upper tail and a sharp pick around values near the mean. The reason for fat tails for stock indices is given in Peters (1991): that is, information shows up in infrequent clumps, rather than in a smooth and continuous manner. Visual examination clearly shows that the distribution of the index is not normal. The One sample Kolmogorof-Smirnov test of normality (Zar 1984) computes $ks = 0.0792$ and $p - value = 0$ for the u_t data and strongly shows that the data is not normally distributed.

In this study, u_t will be referred to as the Johannesburg Security Exchange (JSE) Overall Index.

4.1.2 The Hurst exponent for the JSE Overall Index data

In this section, the JSE Overall Index will be explored for evidence of persistence, randomness and antipersistence. This has important implications on the choice of modelling methods of the index. If the JSE Overall Index is characterised by the antipersistence or persistence, then it may be possible to observe and describe the future movement of the index.

Previous studies have documented persistence (memory) for the JSE. Bradfield and Ardington (1997) published evidence of non random pattern from the JSE for the periods 1980 - 1996. Polakow(2000) examined the historical (1925-1999) returns of the JSE and showed that they had significant statistical memory.

The previous studies used statistical techniques to test for memory in the JSE. In this study, the rescaled range (R/S) analysis will be adopted to investigate the Hurst phenomena of persistence, independence or antipersistence for JSE Overall Index. The daily data from the 27th March 1985 to 1st November 2001 is used. The data has 4147 data points, but only the last 4000 observations will be used in the analysis. The R/S is described in Peters (1991, 1994)

Peters (1994) has explained that financial time series of high frequency generally exhibit significant autoregressive tendencies, and that an autoregressive (AR) process can bias the Hurst exponent. The use of the residuals of the AR(1) model is recommended to compute the Hurst exponent of a financial time series. Thus to compute the Hurst exponent of the JSE Overall Index, the residuals of the AR(1) model of the data are used.

The results (using column two and column three of Table 4.2) of the R/S analysis of the residuals of the AR(1) model for the JSE Overall Index are summarised and shown in Table 4.1.

Constant	0.0050
Standard error of Y (estimated)	0.02506
R squared	0.9932
Hurst exponent	0.5334
Standard error of coefficient	0.0127
Significance	42.0045
Sample size used in the analysis	4000

Table 4.1: R/S analysis: JSE Overall Index.

The log log plot is shown in the Figure 4.5.

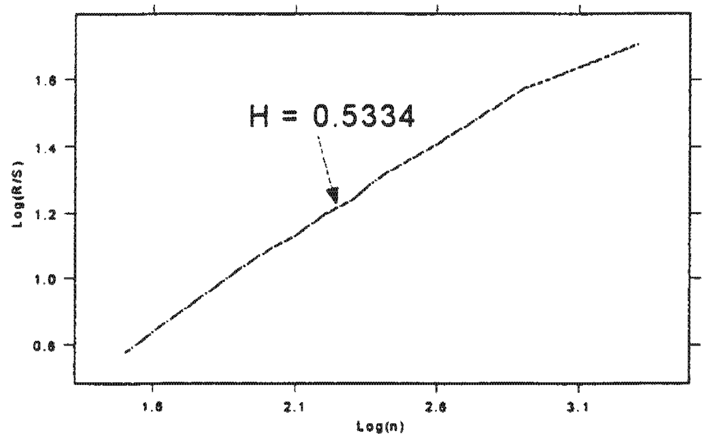

Figure 4-5: R/S analysis, JSE Oerall Index: March 1985 through November 2001.

The analysis gives the Hurst exponent $H = 0.5334$. Since $0.5 < H < 1$, this suggests that the JSE Overall Index exhibits the Hurst phenomena of persistence.

Significance test

To test the significance of this result, the expected value of the Hurst exponent $E(H)$ is computed from the $E(R/S)$. The expected value is used as a comparison against the null hypothesis that the JSE Overall Index is an independent process.

The plots of the log R/S and the $E(R/S)$ for the JSE Overall Index for $N = 4000$ is given in the Figure 4.6 below. The R/S deviates from the $E(R/S)$ line in an orderly fashion. The break in the R/S line appears to occurs at $n = 100$ ($log(100) = 2$).

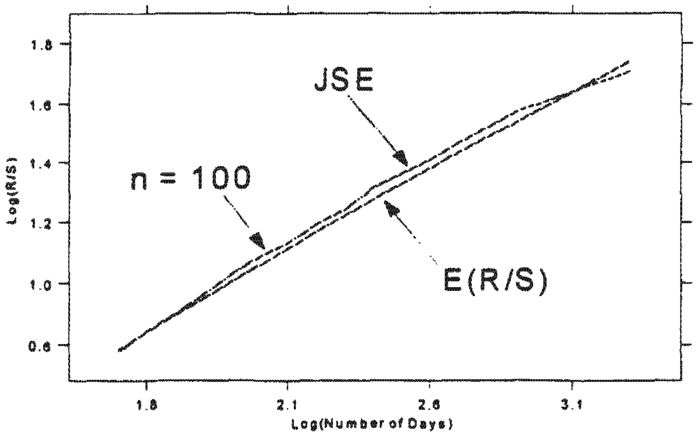

Figure 4-6: R/S analysis, JSE Overall Index, daily returns.

In order to identify precisely when the R/S line starts deviating from the E(R/S) values, the V statistic (Peters 1994) is computed. The results of the computations are shown in Table... The plot of the V-statistic is shown in the following Figure 4.7. The break in the V-statistic may be taken to be at $n = 100$ observations in the plot.

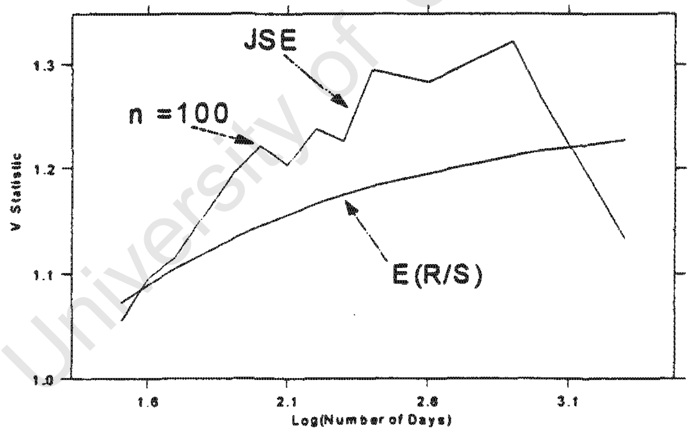

Figure 4-7: V statistic, JSE Oerall Index, daily returns.

The values that have been used in the plots of the R/S_n and the V_n-statistic are shown in Table 4.2.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(R/S)
32	1.51	0.78	0.78	1.06	1.07
40	1.60	0.84	0.84	1.10	1.09
50	1.70	0.90	0.89	1.12	1.10
80	1.90	1.03	1.01	1.20	1.13
100	2.00	1.09	1.06	1.22	1.15
125	2.10	1.13	1.11	1.20	1.16
160	2.20	1.19	1.17	1.24	1.17
200	2.30	1.24	1.22	1.23	1.18
250	2.40	1.31	1.27	1.29	1.18
400	2.60	1.41	1.38	1.28	1.20
500	2.70	1.46	1.43	1.29	1.20
800	2.90	1.57	1.54	1.32	1.21
1000	3.00	1.60	1.59	1.27	1.22
2000	3.30	1.70	1.74	1.13	1.23

Table 4.2: JSE Overall Index, daily returns.

Since the break is at $n = 100$, the H and E(H) are estimated from the R/S_n and $E(R/S_n)$ values respectively for $32 \leq n \leq 100$. The following Table 4.3 gives the regression output for $32 \leq n \leq 100$.

	JSE	E(R/S)
	$32 < n < 100$	$32 < n < 100$
Constant	-0.1675	-0.0552
Standard error of Y (estimated)	0.00234	0.00094
R squared	0.9998	0.9999
Number of observation	5	5
Degrees of freedom	3	3
Hurst Exponent	0.6277	0.5575
Standard error of coefficient	0.0057	0.0023
Significance	110.7996	244.73

Table 4.3: Regression Results.

The computed Hurst exponent is $H = 0.6277$ and the expected H is $E(H) = 0.5575$. The variance of $E(H)$ is given by $\frac{1}{T}$ where T is the number of observations used in the analysis (Peters 1994). The daily data used in the analysis had 4000 observations. Thus the variance of $E(H)$ is $\frac{1}{4000} = 0.00025$. The standard deviation of $E(H)$ is 0.0158. The Hurst exponent $H = 0.6277$ for the daily closing JSE Overall Index is thus 4.44 standard deviations above the $E(H)$. This is highly significant.

The next subperiod $125 \leq n \leq 2000$, is considered. The regression results for the sub period $125 \leq n \leq 2000$ are shown in the Table 4.4 below.

	JSE	E(R/S)
	$125 < n < 2000$	$125 < n < 2000$
Constant	0.1151	0.0202
Standard error of Y (estimated)	0.02193	0.001567
R squared	0.9894	1
Number of observation	9	9
Degrees of freedom	7	7
Hurst Exponent	0.4931	0.5215
Standard error of coefficient	0.0193	0.0014
Significance	25.5546	378.2968

Table 4.4: Regression Results.

The Hurst exponent in this period has dropped to $H = 0.4931$. The expected Hurst exponent in this period is $E(H) = 0.5215$. Therefore the Hurst exponent $H = 0.4931$ is 1.8 standard deviations below $E(H)$ in this period. This implies that $H = 0.4931$ is significant at 95% confidence level. In this analysis, two characteristics of the JSE Overall Index for this data have been identified.

The JSE Overall Index is characterised by Hurst phenomena of persistence ($H = 0.6277$) which is significantly different from an independent process. The persistence occurs only for periods of about 100 days (approximately three months). That is in these three months, if the JSE Overall Index has been up (down) in the previous period, it is likely to be up (down) in the next period. Since the series used in the analysis consists of the AR(1) residuals (Peters 1994), the process in the periods of about three months has memory at work.

After the periods of persistence, the movement of the JSE Overall index exhibits the Hurst antipersistent features with $H = 0.4931$, this may be suggesting a mean reversion phenomena in these periods. During these times, the JSE Overall Index will not sustain the upward or downward movement. The process once up, will move downwards and when it is down, will move upwards. The Mean reversion for the JSE has also been documented by other studies. Bradfield and Ardington (1997) based on annual fixed-interval returns for the periods 1980 - 1996, showed that the JSE market returns slowly revert to some average value. High (1999) studied the mean reversion of the JSE for the periods 1961 - 1997. The mean reversion phenomena has been studied and confirmed for other market indices and stock prices (Peters 1994, Fama and French 1988, 1992, Poterba and Summers 1988).

The two characteristics of persistent and mean reversion, and their fractal dimension values, 1.59 and 2.03 respectively, may be suggesting two of types modelling and prediction for the JSE Overall Index.

4.2 International indices

In this section the time series of the international indices are presented. Table 4.5 below shows the indices and their corresponding periods of observations.

Index	From	To
NYSE	9/13/99	10/25/01
NASDAQ	12/21/98	10/25/01
DOW JONES	9/16/99	10/25/01
DAX	9/28/99	10/26/01
NIKKEI500	2/26/99	10/26/01
IBOVESPA	8/11/99	10/25/01
JSE GOLD	7/13/99	10/31/01

Table 4.5: International Indices and their periods of observation.

The data were obtained from Reuters. The way these indices are derived and used has already been discussed in chapter 2.

The following time series are given and subject to analysis as same as the JSE Overall Index later.

NYSE

The plot of the NYSE is given in Figure 4.8.

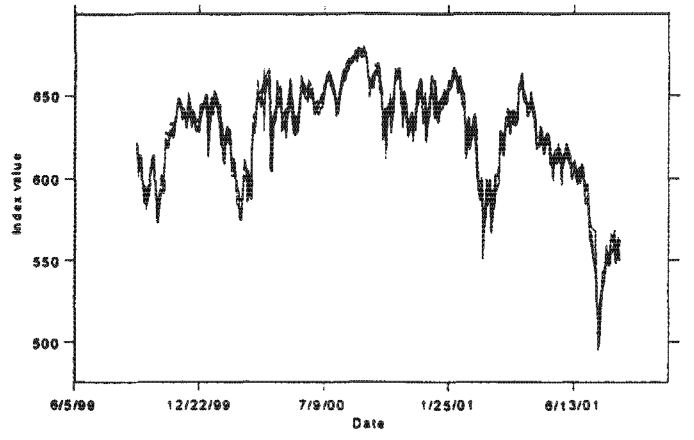

Figure 4-8: NYSE daily returns.

NASDAQ

The Nasdaq data is plotted in Figure 4.9.

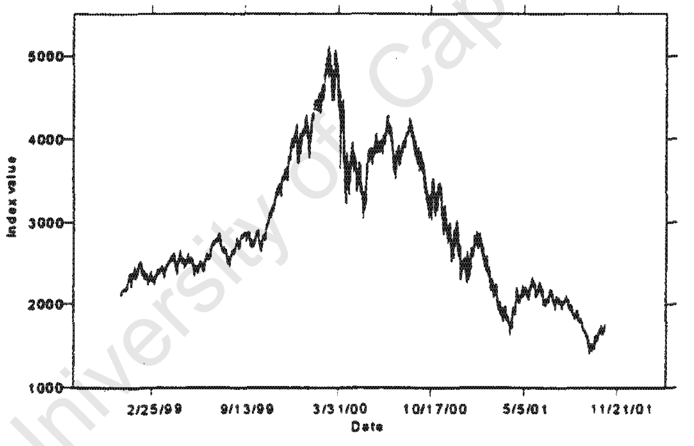

Figure 4-9: Nasdaq daily returns.

DOW JONES

The daily returns of the Dow Jones is plotted in Figure 4.10.

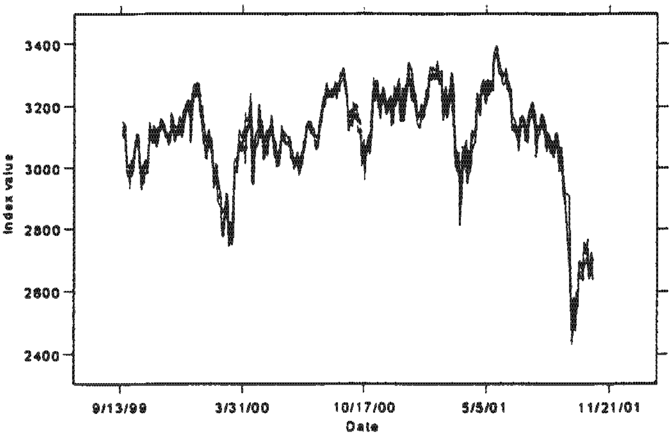

Figure 4-10: Dow Jones daily returns.

DAX

The DAX daily returns are plotted in Figure 4.11.

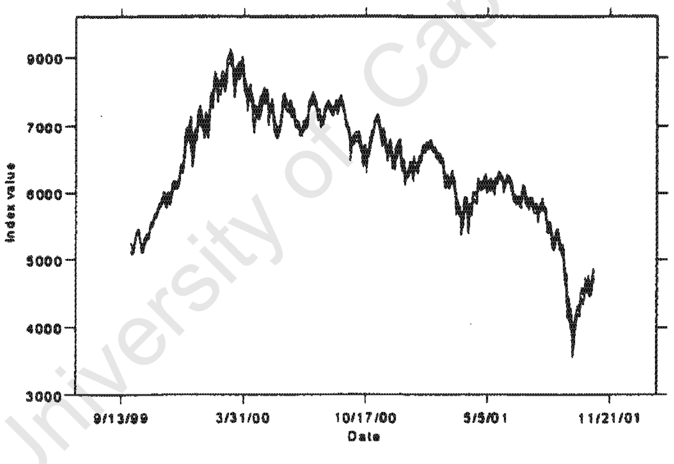

Figure 4-11: DAX daily returns.

NIKKEI500

The daily returns of the Nikkei500 are plotted in Figure 4.12.

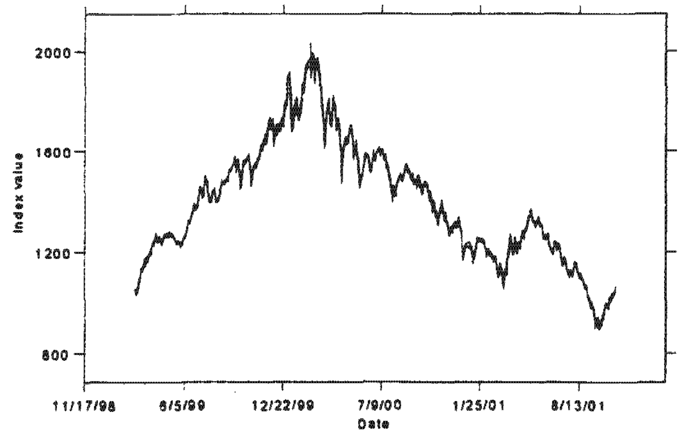

Figure 4-12: Nikkei500 daily returns.

IBOVESPA

The daily returns of the IBOVESPA are plotted in Figure 4.13.

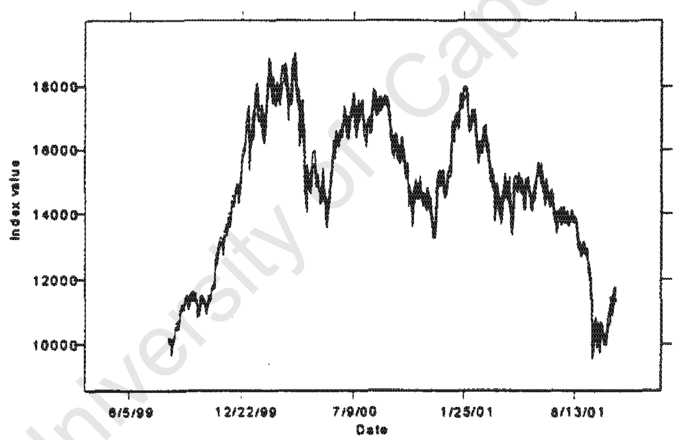

Figure 4-13: The IBOVESPA daily returns.

JSE GOLD

The JSE Gold daily returns are plotted in figure 4.14.

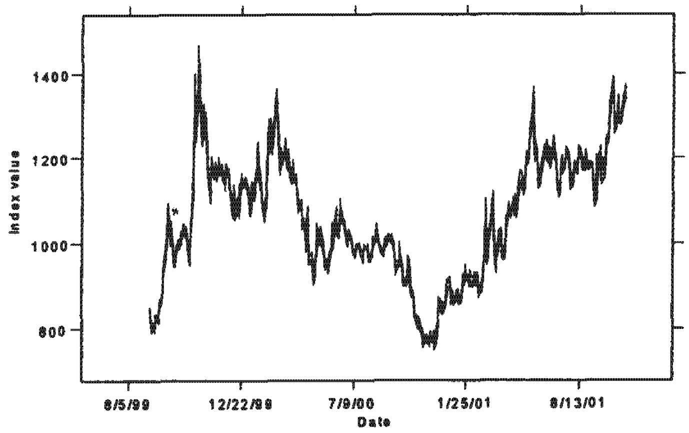

Figure 4-14: JSE Gold daily returns.

4.3 Summary

In this chapter, the financial data have been briefly discussed. The following are the preliminary findings of the log ratio of the JSE overall index:

- It is not normally distributed. It has a sharp peak in the middle and is characterised by fat and long tails.
- The JSE Overall Index has a persistence cycle of about three months with a significant Hurst exponent ($H = 0.6277$). For the rest of the period, the JSE Overall Index is antipersistent (probably mean reverting) with a significant Hurst exponent $H = 0.4931$.
- The fractal dimension 1.59 shows that the index is quite jugged

The preliminary analysis of the JSE Overall Index using classical methods did not take account of vagueness of the index. The next chapter will be concerned with the modelling of the fuzzy feature of the JSE Overall Index using fuzzy logic techniques.

Chapter 5

Application of Fuzzy Techniques to the JSE Overall Index

5.1 Introduction

The movement of the JSE Overall Index is largely an expression of many factors affecting the market such as the economic fundamentals, the investor psychology, etc. Thus the information provided by the index should be fuzzy in nature though the index itself is expressed as an accurate number. The purpose of this chapter is to apply fuzzy logic techniques to the JSE Overall Index in order to reveal certain information or characteristics behind the index.

The data of the JSE Overall Index for the periods 1985 - 2001 will be used in the investigation. In order to review the temporal characteristics of the overall index, the data is divided into time periods: 1985 to 1987, 1988 to 1991, 1992 to 1995, 1996 to 2000, 1985 to 2000 and 1985 to 2001. The division of the index into the four periods is done in order to explore the possibility that information about the index or some characteristics of the index change with time.

Financial decisions, economic monitoring and interpretation etc. are linked to the movement of the market indices. Information about the movement of the indices is normally described by linguistic variables such as "low", "middle" (or "stable") or "high". Such variables are intrinsic to the fuzzy nature of the indices. In order to capture the fuzziness in the dynamics of the JSE Overall Index, the data of the index in each time period will be partitioned into fuzzy states:

"low" "middle" and "high".

The possibility distributions of the fuzzy states will be computed in order to represent knowledge and information about the movement of the index. The possibility measures of each fuzzy state will be presented in order to assess the possibility of the index ever reaching the states. The frequency distributions of the fuzzy states will be presented to assess the probability of the index in the states. The subjective belief about the events of the index and the frequency of occurrence of the events is important. The coherence between the subjective belief and the frequency of occurrence will be measured by the possibility - probability consistency. The fuzziness in the movement of the index in each state will be explored. This will be measures by the approximate area under the possibility distributions. The movement of the index is characterised by uncertainty. however the index may not face pure uncertainty, there is necessity for the index to follow some deterministic pattern in the movement. The necessity measures will be computed to assess the deterministic aspects of the index in each time period.

The dynamics of the JSE Overall Index between the fuzzy states is not characterised by sharp jumps. The values of the index very often change by small increments. Each value of the index at an instant implies the states of the index only to a certain degree. In order to take care of fuzziness in the movement of the index between the states, the fuzzy Markov chain transition matrices will be computed. The prediction of the overall index will be explored using the fuzzy Markov chain model. To improve the prediction results the AR(2) model will be fit to the index. The residuals of the AR(2) will be described using the fuzzy techniques. The AR(2) and the fuzzy model of the residuals will be used together to predict the index.

5.2 Fuzzy analysis of the JSE Overall Index

In order to investigate the movement of the JSE Overall Index in each time period, the index is partitioned into three fuzzy states: "low", "middle", "high". To do this the method of extracting membership grades from a histogram suggested by Dubois and Prade (1986) is used. The partition of the index into the three fuzzy states is achieved by the assumption that

$$\sum_{i=1}^{n} \mu_{A_i}(x) = 1, \qquad \forall \quad x \in A_i, i = 1, 2, \ldots, n$$

where $\mu_{A_i}(x)$ is the membership grade of x in the fuzzy set A_i. The partitions of the index in each time period are given in the next section.

5.2.1 Partitioning of the JSE Overall Index into fuzzy states

The purpose of this section is to partition the JSE Overall Index into the fuzzy states: "low" "middle" and "high". The partition is done in order to capture the fuzziness in the dynamics of the index. Before the partitioning, the JSE Overall Index data were divided into the following time periods: 27/3/85 - 31/12/87, 4/1/88 - 31/12/91, 2/1/92 - 29/12/95, 2/1/96 - 26/7/00. The overall time periods: 27/3/85 - 26/7/00 and 27/3/85 - 2001 were also considered. The results of partitioning the index in each time period are given in Table5.1 and Table 5.2.

	1985 - 1987	1988 - 1991
Low	$u \leq -0.01$	$u \leq -0.01$
Mid	$-.03 \leq u \leq .03$	$-.03 \leq u \leq .03$
High	$u \geq 0.01$	$u \geq 0.01$

Table 5.1: Partitioning of JSE Overall Index into fuzzy states: 1985 - 1991

	1992 - 1995	1996 - 2000
Low	$u \leq -0.01$	$u \leq -0.01$
Mid	$-.02 \leq u \leq .02$	$-.02 \leq u \leq .02$
High	$u \geq 0.01$	$u \geq 0.01$

Table 5.2: Partitioning of JSE Overall Index into fuzzy states: 1992 - 2000

The partitioning of data for the periods 27/3/85 - 26/7/00 and 27/3/85 - 2001 are shown in Table 5.3.

	27/3/85 - 26/7/00	27/3/85 - 2001
Low	$u \leq -0.01$	$u \leq -0.01$
Mid	$-.03 \leq u \leq .03$	$-.05 \leq u \leq .05$
High	$u \geq 0.01$	$u \geq 0.01$

Table 5.3: Partitioning of JSE Overall Index: 1985 - 2000 and 1985-2001

The partitions show that the "middle" states in the time periods 2/1/92 - 29/12/95 and 2/1/96 - 26/7/00 were narrower compared to the "middle" states of the periods 27/3/85 - 31/12/87, 4/1/88 - 31/12/91 and 27/3/85 - 26/7/00. The overall time period 27/3/85 - 2001 showed a wider tolerance for the middle state compared to all the time periods. The intervals for the "low" and "high" states remained the same in all the time periods. The changing intervals of the fuzzy states in time may be linked to the changes in the general perception of the JSE due to the forces of the market. The intersection of the fuzzy states shows that certain values of the index imply more than one fuzzy state.

5.2.2 Probabilities of the fuzzy states

To compare the frequency of occurrence of the JSE Overall Index in the time periods for each fuzzy state, the probabilities were computed. The results are presented in Table 5.4

	1985 - 1987	1988 - 1991	1992 - 1995	1996 - 2000
Low	0.25	0.25	0.21	0.23
Mid	0.98	0.99	0.99	0.99
High	0.35	0.32	0.27	0.28

Table 5.4: Probability of the JSE Overall Index in each fuzzy state.

The probabilities of the index in the fuzzy states for the periods 27/3/85 - 26/7/00 and 27/3/85 - 2001 are shown in Table 5.5.

States	1985 - 2000	1985 - 2001
Low	0.23	0.24
Mid	0.99	0.97
High	0.30	0.30

Table 5.5: Probability of the JSE Overall Index in each fuzzy state: 1985 -2000 and 1958 - 2001.

The probability of middle state was the highest in all the time periods. The frequency of the "low" and "high" states dropped during the time periods: 2/1/92 - 29/12/95, 2/1/96 - 26/7/00 and 27/3/85 - 26/7/00. The overall period 27/3/85 - 2001 also shows a drop in the probability of "low" and "high" states, compared with the time periods 27/3/85 - 31/12/87 and 4/1/88 -

31/12/91.

5.2.3 Possibility Distributions of the states

To measure how compatible each observation was with the fuzzy sates, the possibility distribution of each state in each time period were computed. The possibility distribution also represent knowledge and information about movement of the index in the fuzzy states.

The possibility distributions for the "low", "middle" and "high" states for the period 27/3/85 - 31/12/87, were computed as

$$\Pi_{Low} = \frac{1}{-.12} + \frac{1}{-.08} + \frac{1}{-.07} + \frac{1}{-.06} + \frac{1}{-.04} + \frac{.9}{-.03} + \frac{.7}{-.02} + \frac{.3}{-.01}$$

$$\Pi_{Mid} = \frac{.1}{-.03} + \frac{.3}{-.02} + \frac{.7}{-.01} + \frac{1}{0} + \frac{.8}{.01} + \frac{.5}{.02} + \frac{.1}{.03}$$

$$\Pi_{High} = \frac{.2}{.01} + \frac{.5}{.02} + \frac{.9}{.03} + \frac{1}{.04} + \frac{1}{.05} + \frac{1}{.06}.$$

The plot of the possibility distributions for the states is given in Figure 5-1.

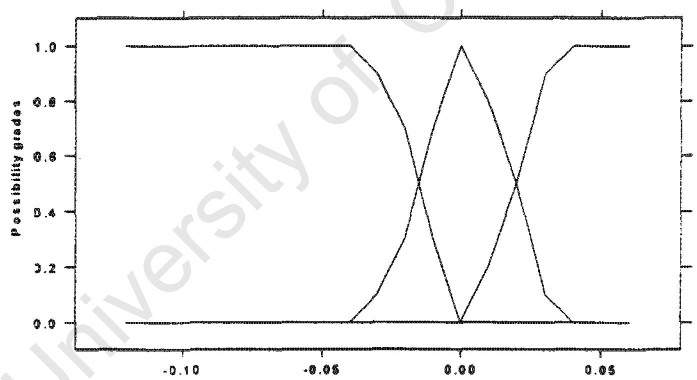

Figure 5-1:
Possibility distributions of the "low", "middle" and "high" states: JSE Overall Index 27/3/85 - 31/12/87

The possibility distributions of the "low", "middle", and "high" states for the period 4/1/88 - 31/12/91 are computed as

$$\Pi_{Low} = \frac{1}{-.11} + \frac{1}{-.05} + \frac{1}{-.04} + \frac{.9}{-.03} + \frac{.8}{-.02} + \frac{.3}{-.01}$$

$$\Pi_{Mid} = \frac{.1}{-.03} + \frac{.2}{-.02} + \frac{.7}{-.01} + \frac{1}{0} + \frac{.8}{.01} + \frac{.3}{.02} + \frac{.1}{.03}$$

$$\Pi_{High} = \frac{.2}{.01} + \frac{.7}{.02} + \frac{.9}{.03} + \frac{1}{.06}.$$

The plot of the possibility distributions is given in Figure 5-2.

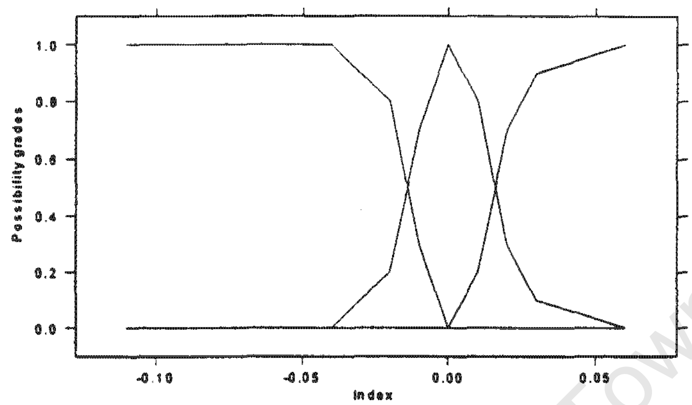

Figure 5-2:
Possibility distributions of the"low", "middle" and "high" states: JSE Overall Index 4/1/88 - 31/12/91

The possibility distributions of the "low", "middle" and "high" states for the period 2/1/92 - 29/12/95 are computed as

$$\Pi_{Low} = \frac{1}{-.03} + \frac{.9}{-.02} + \frac{.4}{-.01}$$

$$\Pi_{Middle} = \frac{.1}{-.02} + \frac{.6}{-.01} + \frac{1}{0} + \frac{.7}{.01} + \frac{.2}{.02}$$

$$\Pi_{High} = \frac{.3}{.01} + \frac{.8}{.02} + \frac{1}{.03} + \frac{1}{.04}.$$

The plots are given in Figure 5-3.

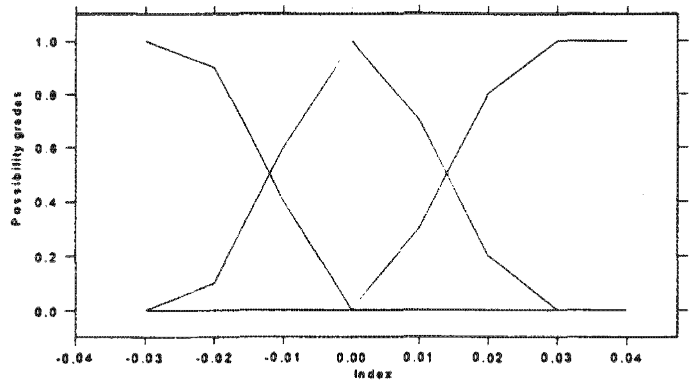

Figure 5-3:
Possibility distributions of the "low", "middle" and "high" states: JSE Overall Index 2/1/92 - 29/12/95

The possibility distributions of the "low" state, "middle" state and the "high" state for the period 2/1/96 - 26/7/00 are computed as

$$\Pi_{Low} = \frac{1}{-.11} + \frac{1}{-.10} + \frac{1}{-.08} + \frac{1}{-.05} + \frac{1}{-.04} + \frac{.9}{-.03} + \frac{.7}{-.02} + \frac{.4}{-.01}$$

$$\Pi_{Middle} = \frac{.1}{-.03} + \frac{.3}{-.02} + \frac{.6}{-.01} + \frac{1}{0} + \frac{.7}{.01} + \frac{.2}{.02} + \frac{.1}{.03}$$

$$\Pi_{High} = \frac{.3}{.01} + \frac{.8}{.02} + \frac{.9}{.03} + \frac{1}{.04} + \frac{1}{.05}.$$

The plot of the possibility distribution for the sates is given in Figure 5-4.

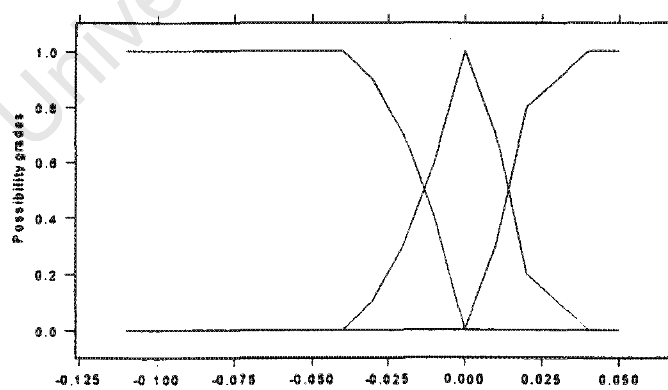

Figure 5-4:
Possibility distributions of the "low", "middle" and "high" states: JSE Overall Index 2/1/96 -

26/7/00

The possibility distributions of the "low" state, "middle" state and the "high" state for the period 27/3/85 - 26/7/00 are computed as

$$\Pi_{Low} = \frac{1}{-0.12} + \frac{1}{-0.11} + \frac{1}{-0.10} + \frac{1}{-0.08} + \frac{1}{-0.07} + \frac{1}{-0.06} + \frac{1}{-0.05} + \frac{1}{-0.04} + \frac{0.9}{-0.03} + \frac{0.8}{-0.02} + \frac{0.3}{-0.01}.$$

$$\Pi_{Middle} = \frac{0.1}{-.03} + \frac{0.2}{-.02} + \frac{0.7}{-0.01} + \frac{1}{0} + \frac{0.8}{0.01} + \frac{0.3}{0.02} + \frac{0.1}{0.03}$$

$$\Pi_{High} = \frac{0.2}{0.01} + \frac{0.7}{0.02} + \frac{0.9}{0.03} + \frac{1}{0.04} + \frac{1}{0.05} + \frac{1}{0.06} + \frac{1}{0.07}.$$

The plot of the possibility distributions is given in Figure 5-5.

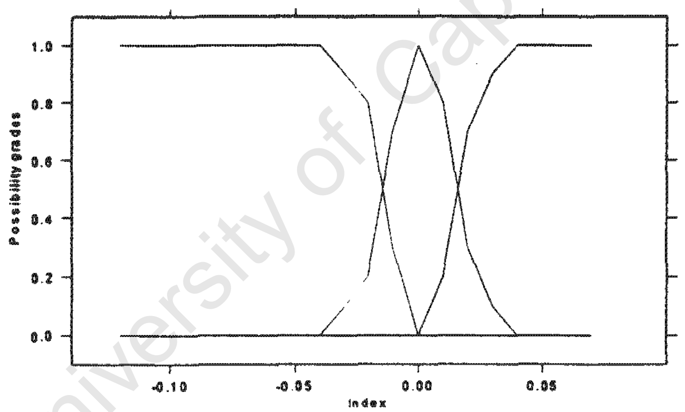

Figure 5-5:
Possibility distributions of the "low", "middle" and "high" states: JSE Overall Index 27/3/85 - 26/7/00

The possibility distributions of the states reveal that the index was fully compatible with the "low" state for values $u \leq -0.04$ in the periods 27/3/85 - 31/12/87, 4/1/88 - 31/12/91, 2/1/96 - 26/7/00 and 27/3/85 - 26/7/00. In the period 2/1/92 - 29/12/95 the index was fully compatible with the "low" state for $u \leq -0.03$. The index was fully compatible with the "middle" state in all the time periods for the value $u = 0$. It was impossible to associate

$u < -0.03$ or $u > 0.03$ with the "middle" state in the periods 27/3/85 - 31/12/87, 4/1/88 - 31/12/91, 2/1/96 - 26/7/00 and 27/3/85-26/7/00. For the period 2/1/92-29/12/95, it was impossible for $u < -0.02$ or $u > 0.02$ to be viewed as part of the "middle" state. The index $u \geq 0.04$ was fully compatible with the "high" state for the periods 27/3/85 - 31/12/87, 4/1/88 - 31/12/91, 2/1/96 - 26/7/00 and 27/3/85 - 26/7/00. In the period 2/1/92 - 29/12/95 the index $u \geq 0.03$ was fully compatible with the "high" state.

The possibility distributions show that the perception of the states of the index and that of the market differs with time. The possibility grades show that the subjective belief about the occurrence of events about the index and their meaning change with time. This may be reflecting the diversity of human thinking in the interpret factors affecting the market.

5.2.4 Fuzziness of the JSE Overall Index

Vagueness of the index in the fuzzy states is important to assess how obscure the index is in its movement. In the literature, the area under the possibility distribution can be regarded as a sort of measure of fuzziness (Tanaka and Guo 1999). The fuzziness of the states of the JSE Overall Index were measured. Table 5.6 gives the fuzziness of each state in the different time periods.

Time period	Low	Middle	High
1: 1985-1987	0.112	0.04	0.041
2: 1988-1991	0.1	0.036	0.05
3: 1992-1995	0.022	0.028	0.032
4: 1996-2000	0.102	0.034	0.042
5: 1985-2000	0.112	0.036	0.038

Table 5.6: Vagueness of the fuzzy states.

The plot of vagueness of the states is given in Figure 5.6.

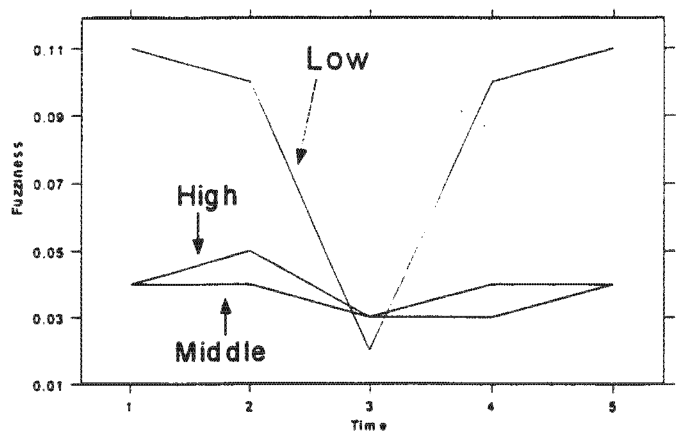

Figure 5-6: Fuzziness: States of the JSE Overall Index.

For the periods 1985 - 1991 and 1996 - 2000 vagueness (ignorance) of the "low" state was high (0.1). The period 1992 - 1995 showed low value (0.02) of vagueness for the "low" state. For the "middle" state, vagueness ranged from 0.028 to 0.04 indicating small variations. Similarly vagueness for the "high" state ranged from 0.032 to 0.05 with little changes. For the intersection between "low" and "middle" states, and "middle" and "high" states, vagueness values were constant.

In the period 1992 - 1996 generally vagueness values were low for all the states, suggesting the market was better understood during this period compared to the others.

5.2.5 Possibility Measures

In order to assess the possibility of the JSE Overall Index in the states, the possibility measures of the states were computed. Table 5.7 shows the results of the possibility measures for the states.

State	Possibility measure
Low	1
Mid	1
High	1

Table 5.7: Possibility measures of the fuzzy states of the JSE Overall Index

The results of the possibility measures show that the JSE Overall Index is bound to be in any state of the "low", "middle" or "high" states.

5.2.6 Possibility-Probability Consistence

In order to measure the coherence between subjective belief and the frequency of occurrence of events about the movement of JSE Overall Index in the states, the possibility-probability consistency were computed. The results are shown in Table 5.8.

States	1985-1987	1988-1991	1992-1995	1996-2000
Low	0.12	0.10	0.10	0.12
Mid	0.77	0.80	0.80	0.77
High	0.11	0.10	0.10	0.12

Table 5.8: Possibility-probability consistency: States of the JSE Overall Index.

The possibility - probability consistency for the overall periods are presented in Table 5.9.

States	27/3/85-31/12/87	1985-2001
Low	0.10	0.11
Mid	0.80	0.79
High	0.10	0.10

Table 5.9: Possibility-probability consistency of the JSE Overall Index: 1985 -2000 and 1985 - 2001.

The subjective belief about the occurrence of events was in conformance with the frequency of occurrence in the "middle" state. However, subjective belief was not in line with the frequency about the occurrence of events in the "low" and "high" states. This may suggest that the "low" and "high" states are difficult to deal with in the movement of the index.

5.2.7 Necessity measures

In order to investigate the deterministic characteristics of the index movement, the necessity measures in each time period were computed. The necessity measures of the "low" and the "high" states were zero. The results of the "middle" state were computed and are given in Table 5.10.

	1985-1987	1988-1991	1992-1995	1996-2000
N	0.92	0.95	0.95	0.73

Table 5.10: Necessity measures of the middle state: JSE Overall Index.

The necessity measures for the periods 27/3/85-26/7/00 and 1985-2001 are computed in Table5.11.

	1985 - 1900	1985 - 2001
N	0.96	0.99

Table 5.11: Necessity measures of the middle state: JSE Overall Index 1985-2000 and 1985-2001.

The necessity measures show that the middle part of the index over the whole period 27/3/85 - 2001 was very deterministic. However, in the period 2/1/96/00 - 26/7/00 the middle part of the index was less stable with necessity measure 0.73. In general there was high necessity of the index to be in the middle.

The necessity measures of the middle part of the index for the whole period in general show consistency in the behaviour of the market. These necessities may also reflect the economic position of South Africa and also may act as indicators of how South Africa has been viewed internationally. For instance if necessity of the middle parts was low, that would indicate high level instability in the market. Apart from other causes this would reflect the economic situation of the country and how both local and international investors view and react to such a situation.

5.2.8 Fuzzy Transition Matrices

In order to investigate the movement of the index between its fuzzy states, the fuzzy Markov chain transition matrices are computed in each time period. The fuzzy Markov matrices of the transitions between the states in each time period are computed as:

27/3/85 - 31/12/87

$$\begin{array}{cc} & \begin{matrix} Low & Mid & High \end{matrix} \\ \begin{matrix} Low \\ Mid \\ High \end{matrix} & \begin{pmatrix} 0.20 & 0.70 & 0.10 \\ 0.10 & 0.79 & 0.11 \\ 0.14 & 0.73 & 0.13 \end{pmatrix} \end{array}$$

4/1/88 - 31/12/91

$$\begin{array}{cc} & \begin{matrix} Low & Mid & High \end{matrix} \\ \begin{matrix} Low \\ Mid \\ High \end{matrix} & \begin{pmatrix} .19 & .73 & .08 \\ .09 & .81 & .10 \\ .08 & .78 & .14 \end{pmatrix} \end{array}$$

and

2/1/92 - 29/12/95

$$\begin{array}{cc} & \begin{matrix} Low & Mid & High \end{matrix} \\ \begin{matrix} Low \\ Mid \\ High \end{matrix} & \begin{pmatrix} 0.18 & 0.74 & 0.08 \\ 0.09 & 0.81 & 0.1 \\ 0.06 & 0.78 & 0.16 \end{pmatrix} \end{array}$$

2/1/96 - 26/7/00

$$\begin{array}{cc} & \begin{matrix} Low & Mid & High \end{matrix} \\ \begin{matrix} Low \\ Mid \\ High \end{matrix} & \begin{pmatrix} .27 & .65 & .08 \\ .10 & .79 & .11 \\ .07 & .73 & .20 \end{pmatrix} \end{array}$$

and for overall period

27/3/85 - 26/7/00

$$\begin{array}{cc} & \begin{matrix} Low & Mid & High \end{matrix} \\ \begin{matrix} Low \\ Mid \\ High \end{matrix} & \begin{pmatrix} 0.21 & 0.71 & 0.08 \\ 0.09 & 0.82 & 0.09 \\ 0.08 & 0.77 & 0.15 \end{pmatrix} \end{array}$$

In all the time periods the probability of the index movement form low to the middle state, and from high to the middle sate is generally high. The movement of the index: "low" to "low", "low" to "high", "high" to "low", "high" to "high", "middle" to "low" and "middle" to "high" had very small probabilities in all the periods. The probability of the index to remain in the middle state once it was entered was high and ranged from 0.79 to 0.82.

In order to check the stability of the movement of the index between the fuzzy states, in each time period with the period 3/3/85 - 26/7/00, the transition matrices were subtracted from the transition matrix of the time interval 3/3/85 - 26/7/00. The results of the subtraction are shown below.

For the period 3/3/85-26/7/00 and 27/3/85-31/12/87 we have

$$\begin{pmatrix} 0.21 & 0.71 & 0.08 \\ 0.09 & 0.82 & 0.09 \\ 0.08 & 0.77 & 0.15 \end{pmatrix} - \begin{pmatrix} 0.20 & 0.70 & 0.10 \\ 0.10 & 0.79 & 0.11 \\ 0.14 & 0.73 & 0.13 \end{pmatrix} = \begin{pmatrix} .01 & .01 & -.02 \\ -.01 & .03 & -.02 \\ -.06 & .04 & .02 \end{pmatrix}$$

For the period 3/3/85 - 26/7/00 and 4/1/88 - 31/12/91 we have

$$\begin{pmatrix} 0.21 & 0.71 & 0.08 \\ 0.09 & 0.82 & 0.09 \\ 0.08 & 0.77 & 0.15 \end{pmatrix} - \begin{pmatrix} .19 & .73 & .08 \\ .09 & .81 & .10 \\ .08 & .78 & .14 \end{pmatrix} = \begin{pmatrix} .02 & -.02 & 0 \\ 0 & .01 & -.01 \\ 0 & -.01 & .01 \end{pmatrix}$$

For the period 3/3/85 - 26/7/00 and 2/1/92 - 29/12/95

$$\begin{pmatrix} 0.21 & 0.71 & 0.08 \\ 0.09 & 0.82 & 0.09 \\ 0.08 & 0.77 & 0.15 \end{pmatrix} - \begin{pmatrix} 0.18 & 0.74 & 0.08 \\ 0.09 & 0.81 & 0.1 \\ 0.06 & 0.78 & 0.16 \end{pmatrix} = \begin{pmatrix} .03 & -.03 & 0 \\ 0 & .01 & -.01 \\ .02 & -.01 & -.01 \end{pmatrix}$$

For the period 3/3/85 - 26/7/00 and 2/1/96 - 26/7/00 we have

$$\begin{pmatrix} 0.21 & 0.71 & 0.08 \\ 0.09 & 0.82 & 0.09 \\ 0.08 & 0.77 & 0.15 \end{pmatrix} - \begin{pmatrix} .27 & .65 & .08 \\ .10 & .79 & .11 \\ .07 & .73 & .20 \end{pmatrix} = \begin{pmatrix} -.06 & .06 & 0 \\ -.01 & .03 & -.02 \\ .01 & .04 & -.05 \end{pmatrix}$$

In general major changes occurred in the movement from "high" to "low" and "high" to "middle" in the period 27/3/85-31/12/87 and in the movement from "low" to "low" and "low" to "middle" in the time periods 4/1/88 - 31/12/91, 2/1/92 - 29/12/95 and 2/1/96 - 26/7/00.The largest changes happened in the period 2/1/96 - 26/7/00. The movements of the index between most states were stable.

5.2.9 Fuzzy statistics of JSE Overall Index

In order to compare the fuzzy statistics of the middle parts of the index of each time period, the fuzzy mean, fuzzy variance have been computed and shown in Table 5.12. In all the periods both the fuzzy means and fuzzy variances were stable.

	1985-1987	1988-1991	1992-1995	1996-2000
Fuzzy Mean	0.002	0.001	0.001	0.001
Fuzzy Var	0.00007	0.00006	0.00004	0.00005

Table 5.12: Fuzzy statistics of the middle state of the JSE Overall Index.

5.3 Prediction of the JSE overall index

The purpose of this section is to explore the prediction of the JSE Overall Index on a daily basis using the 'maximum membership principle' method. To do this, membership grades and fuzzy Markov chain transition matrix will be used. The transition matrix for the period 27/3/85 - 26/7/00 will be used.

Suppose u_t (the log ratio of the index at time t) is equal to -0.02. The membership grades of -0.02 with respect to all the states "low", "middle" and "high" are $(0.8, 0.2, 0.0)$ respectively. To predict the state of the index at $t+1$, we pre multiply $(0.8, 0.2, 0.0)$ with the fuzzy Markov chain matrix as follows:

$$\begin{pmatrix} 0.8 & 0.2 & 0.0 \end{pmatrix} \begin{pmatrix} 0.21 & 0.71 & 0.08 \\ 0.09 & 0.82 & 0.09 \\ 0.08 & 0.77 & 0.15 \end{pmatrix} = \begin{pmatrix} 0.186 & 0.732 & 0.082 \end{pmatrix}.$$

This shows that the index has the highest membership grade of 0.732 in the "middle" state compared with 0.186 in the "low" state and 0.082 in the "high" state. Thus from maximum membership principle, the index will most likely be in the "middle" state at time $t+1$. The membership grade 0.732 locates the interval of prediction for the values of u_{t+1} from the membership or possibility distribution graphs (Figure5.5). For the grade 0.732, the interval is approximately $(-0.01, 0.01)$. Given

$$u_{t+1} = \alpha \in (-0.01, 0.01),$$

then

$$u_{t+1} = \log \frac{S_{t+1}}{S_t} = \alpha$$

or

$$S_{t+1} = S_t e^{\alpha}$$

which should give a daily based prediction from fuzzy Markov chain. $\alpha \in (-0.01, 0.01)$, the prediction of S_{t+1} will be done using two values of α as follows: the lower $\alpha = -0.01$ denoted α^l will predict the lower S_{t+1} denoted S^l_{t+1}, while the upper $\alpha = 0.01$ denoted by α^u will predict the upper S^u_{t+1}. The predicted S_{t+1} will be obtained as the average of S^l_{t+1} and S^u_{t+1}.

The following Table 5.13 gives a day to day prediction using the above method for part of the observed JSE Overall Index data.

S_t	u_t	α^l	S^l_{t+1}	α^u	S^u_{t+1}	S_{t+1}
7987	-0.001					
7978	0.000	-0.009	7915.4	0.01	8067.3	7999.4
7982	-0.010	-0.009	7906.5	0.01	8058.2	7982.4
7930	-0.001	-0.009	7910.5	0.01	8062.2	7986.4
7927	-0.001	-0.009	7859	0.01	8009.7	7934.4
7903	-0.001	-0.009	7856	0.01	8006.7	7931.4
7872		-0.009	7832.2	0.01	7982.4	7907.2

Table 5.13: JSE Overall Index daily prediction.

The model used the same values of α^l and α^u in all the predictions. This resulted from the difficulty to read off accurately the values of α^l and α^u from the possibility distribution graphs in Figure 5.5. The model seems to predict certain values of the JSE Overall Index well. In other cases the model over predicted the index. An effort to improve the prediction is made in the following section.

5.4 Modelling and predicting the JSE overall index using the AR(2) model and fuzzy analysis of the residuals

The purpose of this section is to improve the prediction of the JSE Overall Index from the results in the previous section. To do this an autoregressive model will be fit to the JSE Overall Index data. Detailed discussions of autoregressive processes have been in Chatfield (1989), Box and Jenkins (1976) and others. An autoregressive process of order p is denoted by $AR(p)$ and is defined by

$$u_t = \alpha_1 u_{t-1} + \cdots + \alpha_p u_{t-p} + \epsilon_t,$$

where the term ϵ_t is assumed to follow a normal distribution such that

$$\epsilon_t \sim N(0, \sigma^2).$$

The $AR(p)$ model will be assumed to take care of the deterministic part of the JSE Overall Index while its residuals will take care of the noise part in the movement of the index. The Yule Walker method of fitting the $AR(p)$ model ((Box and Jenkins 1976)) showed that an $AR(2)$ model was suitable for the JSE Overall Index data. The parameters of the $AR(2)$ model were estimated as

$$\widehat{\alpha}_1 = 0.19053915, \quad \widehat{\alpha}_2 = 0.02704578.$$

Thus

$$\widehat{u}_t = 0.19053915 u_{t-1} + 0.02704578 u_{t-2}.$$

The fit $AR(2)$ model reflects the deterministic part of the JSE Overall Index movement. The AR(2) model removes the most influential part of the series. The residuals carter for the noise component of the index and are expected to be uncorrelated.

The residuals are given by

$$\epsilon_t = u_t - \widehat{u}_t.$$

The plot of the residuals of the $AR(2)$ model is given in Figure 5.7.

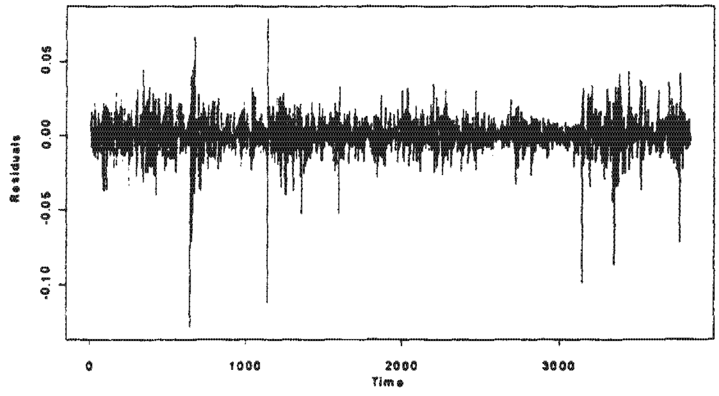

Figure 5-7: Residuals of the AR(2) model of the JSE Overall Index.

The estimated density of the residuals is shown in Figure 5.8.

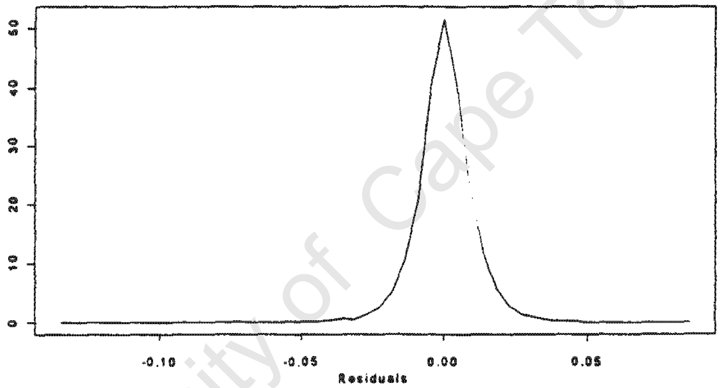

Figure 5-8:
Density estimate of the residuals: AR(2) model of the JSE Overall Index.

The mean and variance of the residuals are give as

$$mean = -5.419054e - 006$$

$$variance = 0.000117743$$

The plot of the estimated density of the residuals has long tails and a sharp peak at the mean. The one sample Kolmogorov-Smirnov Test for normality (Zar 1984) computed $ks = 0.079$ and

$p-value = 0$ and showed that the distribution of the residuals for the AR(2) model of the JSE Overall Index do not follow a normal distribution.

The residuals of the AR(20 model were investigated for randomness, persistence and antipersistence using the Hurst phenomena. The Hurst exponent of the residuals was computed as $H = 0.5521$. Since $0.5 < H < 1$, the residuals of the $AR(2)$ models of the index may have the Hurst memory phenomena. The fractal dimension of the residuals is $\frac{1}{H} = 1.81$ shows that the series of the residuals is very spiked.

The factors underlying the residual are not known. Thus the information represented by the residuals vague in nature though the residuals themselves are presented as actual numbers. Thus fuzzy techniques will be used to analyse the residuals. By extracting membership functions from the histogram, the residuals are partitioned into three fuzzy states: "low", "middle" and "high". The related possibility distribution, possibility measures, necessity measures and possibility probability consistency are computed. Movement of the residuals between the fuzzy states is measured in terms of a fuzzy Markov transition matrix. A possible prediction of the JSE Overall Index movement is explored by using the AR(2) model and the fuzzy Markov model of the residuals.

5.4.1 Fuzzy analysis of the AR(2) residuals

In this section, the possibility distributions of the states of the residuals will be computed. The residuals are assumed to be characterised by uncertainty which is partly due to vagueness. Using the method of extracting membership grades from a histogram (Dubois and Prade1986), the residuals are partitioned into three fuzzy states: "low". "middle", and "high". The possibility distributions of the states are computed as follows:

$$\begin{aligned}\Pi_{Low} &= \frac{1}{-0.13} + \frac{1}{-0.11} + \frac{1}{-0.1} + \frac{1}{-.0.09} + \frac{1}{-0.07} + \frac{1}{-0.06} + \frac{1}{-0.05} + \frac{1}{-0.04} \\ &\quad + \frac{0.9}{-0.03} + \frac{0.8}{-0.02} + \frac{0.3}{-0.01}\end{aligned}$$

$$\Pi_{Middle} = \frac{0.1}{-0.03} + \frac{0.2}{-0.02} + \frac{0.7}{-0.01} + \frac{1}{0} + \frac{0.7}{0.01} + \frac{0.3}{0.02} + \frac{0.1}{0.03}$$

$$\Pi_{High} = \frac{0.3}{0.01} + \frac{0.7}{0.02} + \frac{0.9}{0.03} + \frac{1}{0.04} + \frac{1}{0.05} + \frac{1}{0.07} + \frac{1}{0.08}.$$

Figure 5.9 gives the plot of the possibility distributions.

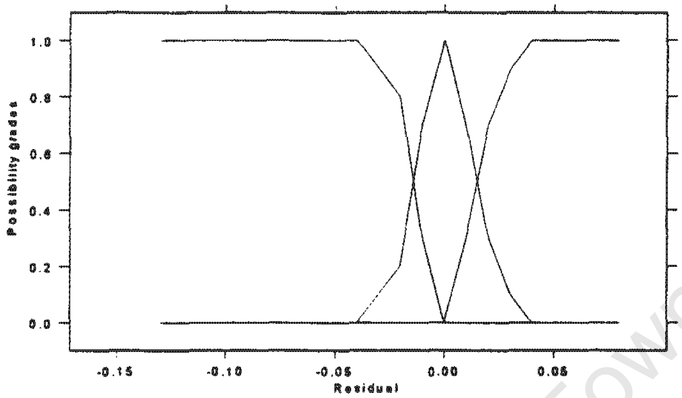

Figure 5-9:
Possibility distributions of the "low", "middle", and "high" states: AR(2) residuals of JSE Overall Index

The possibility distribution of the "low" state shows that the residuals −0.04 to−0.13 are fully compatible with the "low" state, while the residuals −0.03 to −0.01 are partially compatible with the "low" state. The possibility distribution of the "middle" state shows that residuals from −0.03 to 0.03 except for 0, are partially compatible with the "middle" state, while 0 is fully compatible with the "middle" state. The possibility distribution of the "high" state shows that the residuals from 0.01 to 0.03 are partially imply the high state while the residuals 0.04 to 0.08 are fully compatible with the "high" state.

The following Table 5.14 gives the probabilities (P), Possibility measures (Π_A), necessity measures (N) and possibility - probability consistency (γ) of each fuzzy state.

States	P	Π_A	N	γ
Low	0.25	1	0	0.11
Midle	0.99	1	0.96	0.79
High	0.26	1	0	0.10

Table 5.14: Summary computations for the AR(2) residuals.

The table shows that the "middle" state is more frequent than the other states. The residuals have a high necessity of being in the "middle" sate. That is the "middle" state is more deterministic than the "low" and "high" states. The "low" and "high" states are prone to randomness as shown by their necessity values equal to 0. The coherence $\gamma = 0.79$ between subjective belief about the occurrence of residuals and their frequency in the "middle" is quite high, showing that subjective belief and frequency of occurrence are almost in agreement. A high degree of belief about the residuals in the "low" and "high" states does not imply a high degree of frequency, nor does a low degree of frequency imply a low degree of belief. The possibility measure of each state equals 1 and implying all the states are possible.

5.4.2 Prediction of the JSE overall index by AR(2) and fuzzy Markov transition matrix of residuals

In this section, prediction of the JSE Overall Index expected to improve the prediction results of section 5.3 will be carried out using the AR(2) model and the fuzzy Markov transition matrix of the AR(2) residuals. The fuzzy Markov transition matrix between the fuzzy states of the residuals is computed as:

$$\begin{array}{c c} & \begin{array}{ccc} Low & Mid & High \end{array} \\ \begin{array}{c} Low \\ Mid \\ High \end{array} & \begin{pmatrix} 0.15 & 0.72 & 0.13 \\ 0.10 & 0.80 & 0.10 \\ 0.12 & 0.76 & 0.12 \end{pmatrix} \end{array}$$

Prediction

If the residual at time t is -0.02, its membership grades with respect to the "low", "middle", and "high" states are $(0.8, 0.2, 0)$ respectively. To predict the residual at time $t+1$, $(0.8, 0.2, 0)$ and the fuzzy Markov matrix are multiplied in the following manner.

$$\begin{pmatrix} 0.8 & 0.2 & 0.0 \end{pmatrix} \begin{pmatrix} 0.15 & 0.72 & 0.13 \\ 0.10 & 0.80 & 0.10 \\ 0.12 & 0.76 & 0.12 \end{pmatrix} = \begin{pmatrix} 0.14 & 0.736 & 0.124 \end{pmatrix}.$$

The results show that the residual at time $t+1$ is more likely to be in the state "middle" with the possibility grade 0.736. The grade 0.736 will locate the residual in the interval $(-0.01, 0.01)$.

The residuals ϵ_t of the AR(2) models are given by

$$\epsilon_t = u_t - \widehat{u}_t$$

so that

$$u_t = \widehat{u}_t + \epsilon_t, \text{ where } u_t = \log\left(\frac{S_t}{S_{t-1}}\right).$$

Thus

$$\log\left(\frac{S_{t+1}}{S_t}\right) = \widehat{u}_{t+1} + \epsilon_{t+1}$$

or

$$S_{t+1} = S_t e^{\widehat{u}_{t+1} + \epsilon_{t-1}}.$$

The quantity $\widehat{u}_{t+1}$ is predicted from the $AR(2)$ model. The residual at time $t+1$ i.e. ϵ_{t+1} is predicted from the fuzzy Markov chain model. The membership grade shown by "g" in the table below is the highest predicted grade indicates the predicted state. Two values of ϵ_{t+1}, the lower ϵ^l_{t+1} and the upper ϵ^u_{t+1} are predicted from the predicted state and obtained from Figure 5.9. The average value of ϵ^u_{t+1} and ϵ^u_{t+1} denoted by $\widehat{\epsilon}_{t+1}$ (and was 0 for all the predicted ϵ^l_{t+1} and ϵ^u_{t+1}) is then used together with $\widehat{u}_{t+1}$ from the AR(2) model to obtain S_{t+1}.

Table 5.15 shows the predictions made for a few observations.

S_t	u_t	$\widehat{u}_{t+1}$	ϵ_t	g	$\epsilon^l_{t+1}, \epsilon^u_{t+1}$	$\widehat{\epsilon}_{t+1}$	S_{t+1}
7632	.007	.000	.01				
7637	.001	.000	.00				
7656	.002	.0003	.00	.8	.-009, .009	0	7659
7640	-.002	.0005	.00	.8	-.009, .009	0	7660
7709	0.009	-.0003	.01	.8	-.009, .009	0	7638
7746	.005	.0017	.00	.78	-.01, 01	0	7722
7784	.005	.0012	0	.8	-.009, 009	0	7755
7825	.005	.0011	0	.8	-.009, 009	0	7792
7770	-.007	.0011	-.01	.8	-.009, 009	0	7834
7743	-.003	-.0012	0	.78	-.01, 01	0	7761
7775	.004	-.0009	0	.8	-.009, 009	0	7736
7801	.003	.0007	0	.8	-.009, 009	0	7780
7888	.011	.0007	.01	.8	-.009, 009	0	7806
7907	.002	.0022	0	.78	-.01, 01	0	7905
8003	.012	.0008	.01	.8	-.009, 009	0	7913
7987	-.002	.0024	0	.78	-.01, 01	0	8022
7978	-.001	-.0001	0	.8	-.009, 009	0	7986
7982	.001	-.0003	0	.8	-.009, 009	0	7975
7930	-.007	.0001	-.01	.8	-.009, 009	0	7983
7927	.00	.0012	0	.78	-.01, 01	0	7940
7903	-.003	-.0002	0	.8	-.009, 009	0	7925
7872	-.004	-.0006	0	.8	-.009, 009	0	7898

Table 5.15: Prediction of the JSE Overall Index :AR(2) model and fuzzy model of residuals

The model faced problems in reading off the predicted residuals from the possibility distributions graphs (Figure5.9). The values were too small and could not be read off accurately. As a result the predicted residual seemed to be zero in all the cases. If it were possible to read off correctly the values of predicted residuals, they would contribute effectively to the prediction of the index. The model over predicted the index values in most cases. The model showed an improvement in the prediction of the Overall Index from the previous prediction in section 5.3.

5.5 Summary

This chapter has achieved describing the JSE overall index using fuzzy techniques. The index was divided and analysed in time periods. The time periods were partitioned in three fuzzy states "Low", "Middle" and "High". The possibility distributions, probabilities, possibility - probability consistency, fuzzy statistics and necessity measures were computed for each state. To investigate the movement of the index, the fuzzy transition matrices between the fuzzy states were computed. An attempt to predict the daily closing JSE overall index using a fuzzy Markov model was made. To improve the prediction the AR(2) model was used together with the fuzzy Markov chain model of the AR(2) residuals.

The JSE overall index has been successfully partitioned into fuzzy states: Low, Middle and High using a method of extracting membership functions from a histogram. The fuzzy constraints on the values of the index that may be assigned to the fuzzy states have been computed. It is revealed that the states are not sharply defined. The constraints represent the degrees of ease with which the index may be associated with the states. Knowledge and information about movement of the index with respect to the states are presented. They reveal that meaning of the index with respect to the states differs with time. The implication of the states on the index also differ with time.

The possibility - probability consistency to measure coherence between vagueness and randomness of the index were computed. Subjective belief about the occurrence of events about the index and the frequency of the events were only highly coherent in the "middle" state. A high degree of belief did not imply a high degree of frequency, nor did a low degree of frequency imply a high degree of belief about the occurrence of the index events in the "low" and "high" states. Ignorance about the movement of the index in the "low" state was generally higher than in the "middle" and the "high" states. In other words, the "low" had high vagueness compared to the "middle" and the "high" states. Possibility measures have shown that the index is bound to be in any of the states.

Other results showed that the "middle" state of the index had the highest frequency. Thus the index was most frequently in the "middle" state than it was in the "low" and "high" states. There necessity values showed that the index had a tendency to be in the "middle" state. Whenever the index was in the "low" or "high" states, it was pulled to the "middle" state.

The fuzzy transition matrices to describe the movement of the index between the fuzzy states were computed. The matrices show that there was high probability of the index to move from "low" or "high" states to the "middle" state in all the time periods. The probability of getting out of the "middle" state once in it was small.

The movement of the index in each time period is contrasted with the movement in the period of 27/3/85 - 26/07/00. In general, major changes occurred in the movement from "high" to "low" and "high" to "middle" in the period 27/3/85-31/12/87 and in the movement from "low" to "low" and "low" to "middle" in the time periods 4/1/88 - 31/12/91, 2/1/92 - 29/12/95 and 2/1/96 - 26/7/00. The largest changes happened in the period 2/1/96 - 26/7/00. Otherwise other movements between most of the states were stable.

An attempt to predict the daily closing JSE overall index was made. The models over predicted the index. This was explained by the inability to read off accurately the predicted small values from the graphs which were supposed to contribute to the prediction.

This chapter has described the movement of the JSE overall index using fuzzy set theory. An attempt to predict the index has been made using the fuzzy Markov chain model. The AR(2) model for the index has been combined with fuzzy analysis of its residuals to improve the prediction of the index.

The mood of investors greatly affects the movement of markets. In the next chapter, the mood index to measure the mood of investors will be derived.

Chapter 6

Market mood index development and decomposition of share price

6.1 Introduction

In this chapter, a mood index to measure the mood of investors in a market will be derived. The decomposition of a share price into fundamentals and sentiment will be done. The mood of investors has wide implications in financial markets. The mood of investors may generally act as an indicator to the state of the market in terms of investors' feeling and may signal the trend the markets will take. In certain instances, the mood of investors is used as a gauge to give insight into how stocks will perform in the future (Updegrave 2000). Thus the mood is critical in making investment as well as financing decisions. In many studies, the mood of investors is described in a qualitative manner. The terms such as "low", "stable", "high", "pessimistic", "optimistic" etc. are used to describe the mood. Little effort has been made to quantify the mood of investors. It is the purpose of this chapter to fill up this gap.

The factors influencing the mood of investors in stock markets are not easy to identify. However, some of the behavioral concepts that have been found to be most useful to finance such as: overconfidence with the predictive methods and knowledge, attitude and perception of gains and losses, mental accounting, herd behaviour, regret as well as prudence in decision making etc. (Shiller 1990, 1995, De Bondt and Thaler 1996), may play a crucial role in the way market movement is perceived and interpreted. Thus, it may be reasonable to assume that these behavioral concepts are also linked to the mood of investors. The mood of investors may also be attributed to changes in share price and dividend. Thus factors which affect price and dividend changes are also linked to the mood of investors. In the literature, the course of national economy, corporate profits, interest rates, company earnings, political events (Fischer and Jordan 1975, Brealey 1983, Sirakaya et al. 1995, Ryes 1998) have been identified to affect changes in share price and dividend.

Overconfidence (or confidence) and regret experienced by investors are some of the important behavioral concepts to market movement. Overconfidence in investors is related to overestimating their reliability of knowledge, while regret is the feeling of remorse about a decision that turned out badly (De Bondt and Thaler 1996). Confidence in investors may occur when high returns are predicted, market index or share price goes up or the market movement shows a desirable trend. Some of the sources of regret could be the falling of market index or share price, low predicted returns or negative trend in the market movement.

In this study confidence and regret experienced by investors will be assumed important to the mood of investors. The mood of investors will be quantified by assuming that the investors are sensitive the market index changes, and that the record maximum or minimum, the cumulative maximum or minimum, and the daily maximum or minimum of the market index have a bearing on the way the current market index values are perceived. For example, to form an opinion about the market, some investors may compare the current changes with the record maximum or minimum of the market index. Other investors may consider the cumulative maximum or minium of the market index or the daily maximum and minium of the index to form an opinion about the market. Furthermore, when the market index is relatively low, investors may experience regret which may causes low mood in the market. When the market index is relatively high, investors may feel confident and consequently induce high mood in the market.

In this chapter, functions to measure three different types of the mood of investors in the market will be constructed. The first function will be based on the mood of investors with respect to the record maximum and record minimum in the near past and the daily closing values of the index. The second function will describe the mood of investors with respect to the daily maximum, the daily minimum and the daily closing index value. The third function will describe the mood of investors in terms of the cumulative maximum, cumulatively minimum, and the daily closing index value. The three functions describing the different types of the mood of investors will be used to construct a composite market mood index. The mood index will be used to quantify the general mood of investors in the market.

The composite market mood index will be applied to data of various international market indices. Characteristics of the mood index for the markets will be investigated. To measure subjective belief and present knowledge and information about mood of investors, the possibility distributions of the mood indices for the markets will be computed. The relationship between subjective belief and frequency of the mood will be computed for the markets.

The mood of the JSE and the JSE gold will be treated as a special case. The daily and monthly mood of investors for the JSE Anglo Gold share price will be computed. The monthly mood will be used to decompose the share price into fundamentals and sentiment components. The price due to fundamentals will be correlated with some of the economic indicators for South African.

In the following sections functions to quantify mood of investors will be constructed.

6.1.1 Mood of investors relative to previous maximum and previous minimum

In this section the mood of investors relative to the record maximum and record minimum in the near past is constructed. Let a be the record maximum of share price or index, b the record minimum share price or index in the near past and S_t^C denoted the daily closing share price or index. The quantity $a - S_t^C$ is the distance of daily closing price or index from the record maximum price or index. Intuitively, the shorter the distance, the higher the closing share price or index and more hope of good performance of the share price or index. Similarly the quantity $S_t^C - b$ is the distance of daily closing price or index from record minimum price or index. Intuitively, the shorter the distance, the lower the closing price or index and poor performance of the share price or index. In this case the share price or index is in a panic situation and investors may engage more in selling their shares to pull out from the market. The distance $a - S_t^C$ may measure the regret the investors have for the share price or index moving away from the record maximum, while $S_t^C - b$ may describe investors' confidence in share price or index improving and moving up away from the record low. The ratio

$$\frac{a - S_t^C}{S_t^C - b}$$

defines the mood of the market relative to record maximum and record minimum of share price or index. The ratio $\frac{a-S_t^C}{S_t^C-b}$ takes values from $-\infty$ to $+\infty$. The values are transformed to the interval $[0,1]$ by taking the exponential function. Thus the mood is defined as

$$\kappa_t(a, b, S_t^C) = \exp(-\frac{a - S_t^C}{S_t^C - b}).$$

When $a = S_t^C = b$, by convention the value of κ_t is recorded as 0.37.

The interpretation of the mood is given as follows:

$$\kappa_t = \begin{cases} 1 & \text{if markets is in high mood} \quad a = S_t^C \\ 0.5 & \text{if the market is calm and stable} \quad S_t^C = \frac{a+b\ln 2}{1+\ln 2} \\ 0 & \text{if market is in low mood} \quad S_t^C = b \end{cases}$$

6.1.2 Mood of investors relative to daily maximum and daily minimum

The mood relative to daily maximum and daily minimum is constructed in a similar manner like in the previous section. It is defined in terms of the daily maximum share price or index, daily minimum of share price or index and the daily closing share price or index (daily mood of investors). Let the mood be denoted by σ, then

$$\sigma_t = \exp(-\frac{S_t^M - S_t^C}{S_t^C - S_t^m}),$$

where S_t^M is the daily maximum share price or index, S_t^C the daily closing share price or index and S_t^m the daily minimum share price or index. The quantity $S_t^M - S_t^C$ is the distance of daily closing price or index from the daily maximum price or index. Intuitively, the shorter the distance, the higher the closing price or index depicting a high mood for the share. The distance $S_t^M - S_t^C$ also measures the daily regret in the trading of the share or index. The bigger the distance the higher the regret. Similarly, the quantity $S_t^C - S_t^m$ is the distance of daily closing price or index from daily minimum price or index. Intuitively, the shorter the distance, the lower the closing price or index depicting a low mood for the share or index. In the circumstances the investors may be willing to sell off their shares at any price and willing to pull out from the market. The distance $S_t^C - S_t^m$ measures the daily confidence investors have in the reading of a share or index. The wider the distance the more confidence investors have in the share price or index rising up away from the minimum price or index. If the quantities $S_t^M - S_t^C$ and $S_t^C - S_t^m$ are close to each other, the market is relatively stable. When $S_t^M = S_t^C = S_t^m$, σ_t is taken to be 0.37.

The meaning and understanding of the mood is subjective and dependent on traders beliefs. The following is a guide as how the mood can be viewed and perceived.

$$\sigma_t = \begin{cases} 1 & \text{if markets is in high mood} \quad S_t^M = S_t^C \\ 0.5 & \text{if the market is calm and stable} \quad S_t^C = \frac{S_t^M + S_t^m \ln 2}{1+\ln 2} \\ 0 & \text{if market is in low mood} \quad S_t^C = S_t^m \end{cases}$$

The daily mood of investors is useful for short term investors looking at the market in terms of daily fluctuations. The mood of investors relative to the highest and lowest price or index headed by the share or index will be presented in the next section.

6.1.3 Mood of investors in terms of cumulative maximum and cumulative minimum

The purpose of this section is to define the mood of investors relative to the cumulative maximum, cumulative minimum and the daily closing share price or market index values.

Let $S_t^{M_r}$ be the cumulative maximum of the daily maximum share price or index, and $S_t^{m_r}$ the cumulative minimum of the daily minimum share price or index. Let S_t^C denote the daily closing price or index. The quantity $S_t^{M_r} - S_t^C$ is the distance of the daily closing price or index from the highest prices or index headed by the share or index with respect to the chosen reference time point in the past. This intuitively reveals the regrets or pity according to the last maximum share price or index in the market. A short distance shows closing price or index going toward the last maximum price or index implying improvement in the share price or index. The quantity $S_t^C - S_t^{m_r}$ is the distance of the daily closing price or index from the lowest price or index reached by this share or index with respect to the same chosen in the past. $S_t^C - S_t^{m_r}$ reveals confidence in the share or index trading with respect to the last minimum price or index. A short distance depicts low confidence and market may be in a panic situation. The ratio

$$\frac{S_t^{M_r} - S_t^C}{S_t^C - S_t^{m_r}}$$

gives mood of investors with respect to cumulative maximum and cumulative minimum share prices or index. With the exponential transformation the mood of investors in terms of the cumulative maximum and cumulative minimum share prices is defined by

$$\phi_t = \exp(-\frac{S_t^{M_r} - S_t^C}{S_t^C - S_t^{m_r}})$$

where

$$\phi_t = \begin{cases} 1 & \text{if markets is in high mood} \qquad S_t^{M_r} = S_t^C \\ 0.5 & \text{if the market is calm and stable} \quad S_t^C = \frac{S_t^{M_r} + S_t^{m_r} \ln 2}{1 + \ln 2} \\ 0 & \text{if market is in low mood} \qquad S_t^C = S_t^{m_r} \end{cases}$$

The values, 1 for high mood, o.5 for stable mood and 0 for low mood, are just guidelines. The mood concept is inherently fuzzy in nature. Thus other values near 1 also imply the high mood, and the other values near 0.5 and 0 imply the stable mood and the low mood respectively.

6.2 Market mood index

The markets are mixed. They consist of investors who view the market movement in terms of daily changes in the index. Others view the markets in terms of record maximum and record minimum. While other investors may view the markets in terms cumulative maximum and minimum. This fact suggests the construction of a composite mood index which is derived from the mood of investors κ, σ, and ϕ constructed in the previous sections. Each term, κ, σ, and ϕ is given equal weight in the construction of the composite mood index. Thus the formula for the composite mood index can be written as

$$\psi = \frac{1}{3}(\kappa + \sigma + \phi).$$

The mood index ψ, is computed for the JSE Overall Index (3/27/85 - 11/1/01), JSE GOLD (7/13/99 - 10/31/01), NYSE index (9/13/99 - 10/25/01), NASDAQ (12/21/98 - 10/25/01), DOW JONES (9/16/99 - 10/25/01), DAX (9/28/99 - 10/26/01), NIKKEI500 (2/26/99 - 10/26/01), and the IBOVESPA (8/11/99 - 10/25/01) data. The computed κ, σ, and ϕ for each market index are given in the appendix at the end of the study. The plots and histograms of mood index ψ for each market index are given in the following section.

6.2.1 The mood indices plots

This section provides the plots of the mood index for various markets in the following figures.

The mood index for the JSE Overall Index is plotted in Figure 6.1. The histogram of the mood index is shown in Figure 6.2.

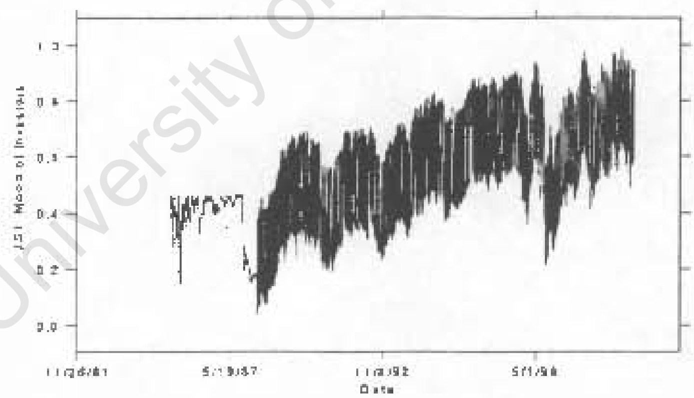

Figure 6-1: Mood index: JSE Overall Index 3/27/85 - 11/1/01

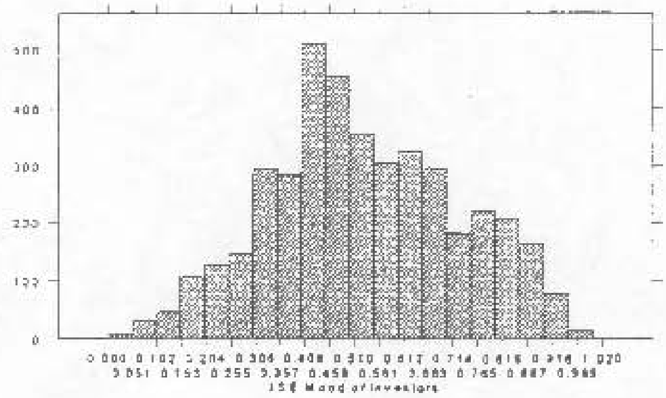

Figure 6-2: Histogram: Mood index of the JSE Overall Index 3/27/85 - 11/1/01

The mood index for the JSE Gold is shown in Figure 6.3. Figure 6.4 shows the histogram of the mood index for the JSE Gold.

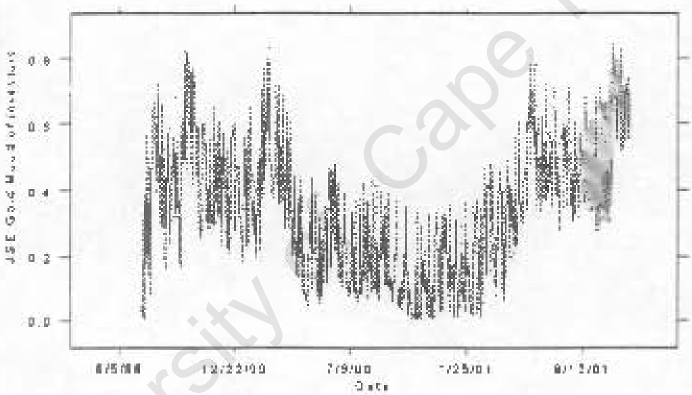

Figure 6-3: Mood index: JSE Gold index 7/13/99 - 10/31/01

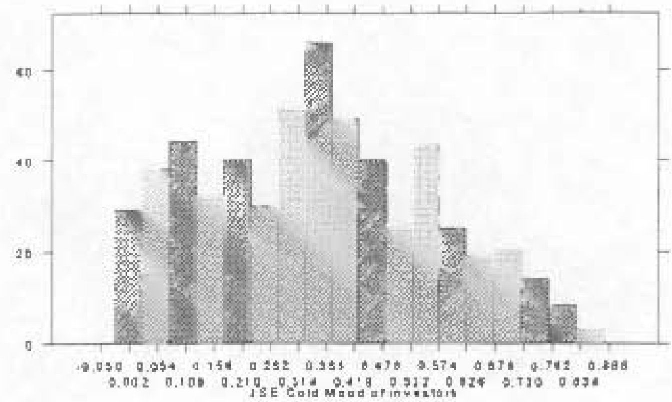

Figure 6-4: Histogram: Mood index of the JSE gold index 7/13/99 - 10/31/01

The plot in Figure 6.5 shows the mood index for the NYSE. The histogram of the mood index is given in Figure 6.6.

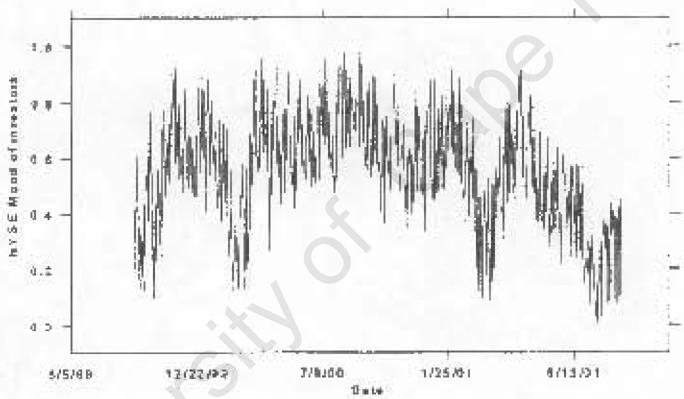

Figure 6-5: Mood index: NYSE index 9/13/99 - 10/25/01

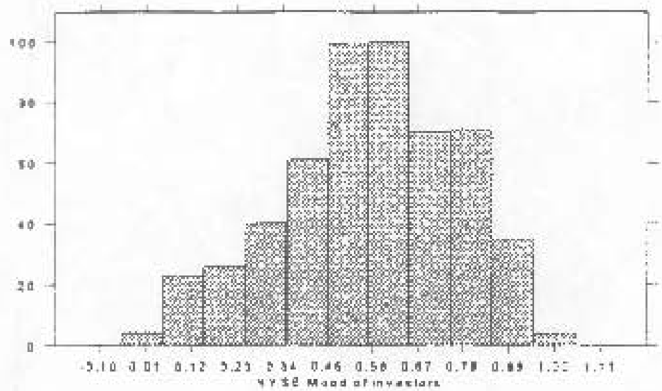

Figure 6-6: Histogram: mood index of the NYSE index 9/13/99 - 10/25/01

The plot of the mood index for the Nasdaq is shown in Figure 6.7. The histogram of the mood index for the Nasdaq is shown in Figure 6.8.

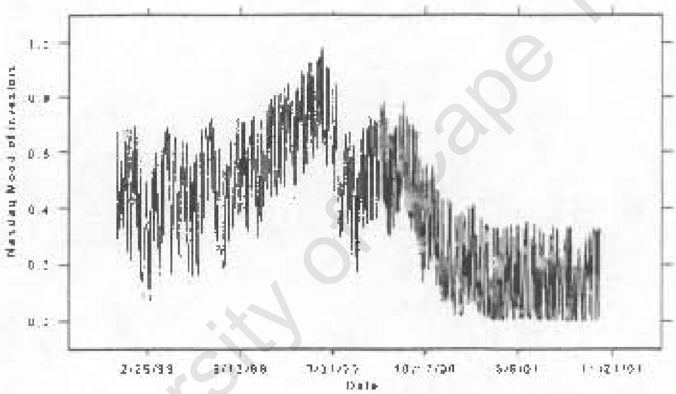

Figure 6-7: Mood index: Nasdaq index 12/21/98

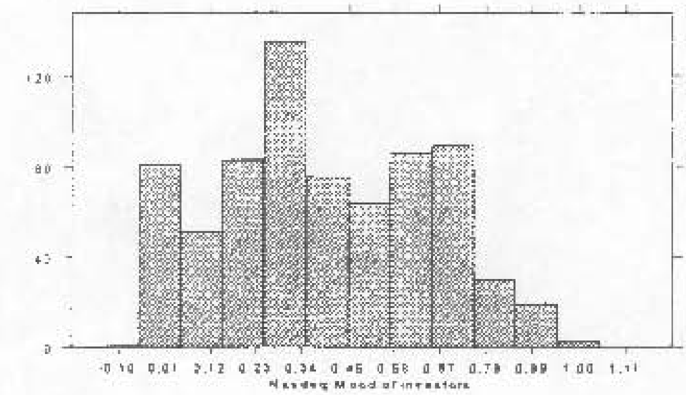

Figure 6-8: Histogram: Mood index of the Nasdaq index 12/21/98

The plot of the mood index for the Dow Jones is shown in Figure 6.9, and the histogram is given in Figure 6.10.

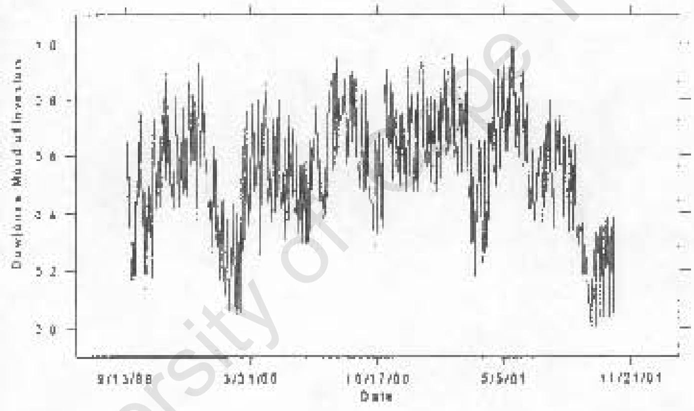

Figure 6-9: Mood index: Dow Jones index 9/16/99 - 10/25/01

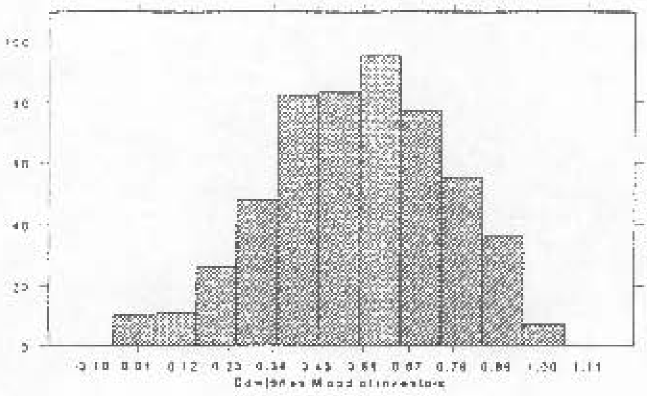

Figure 6-10: Histogram: Mood index of the Dow Jones index 9/16/99 - 10/25/01

The mood index of the DAX is plotted in Figure 6.11. The histogram of the mood index is given in Figure 6.12.

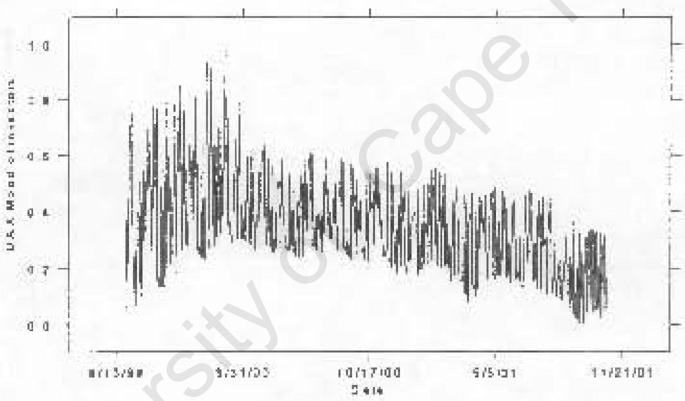

Figure 6-11: Mood index: DAX 9/28/99 - 10/26/01

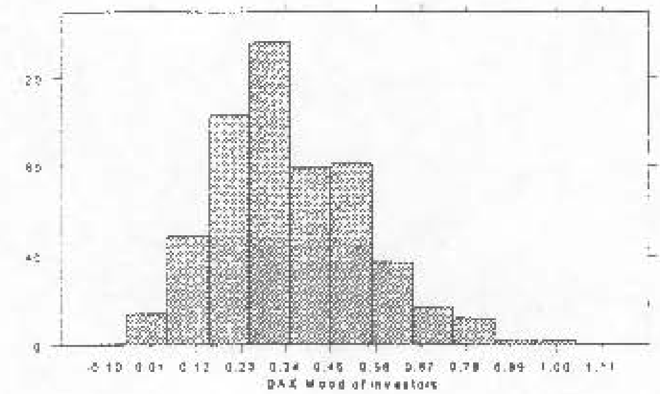

Figure 6-12: Histogram: Mood index of the DAX 9/28/99 - 10/26/01

The plot and histogram of the mood index for the Nikkei500 are given in Figure 6.13 and Figure 6.14 respectively.

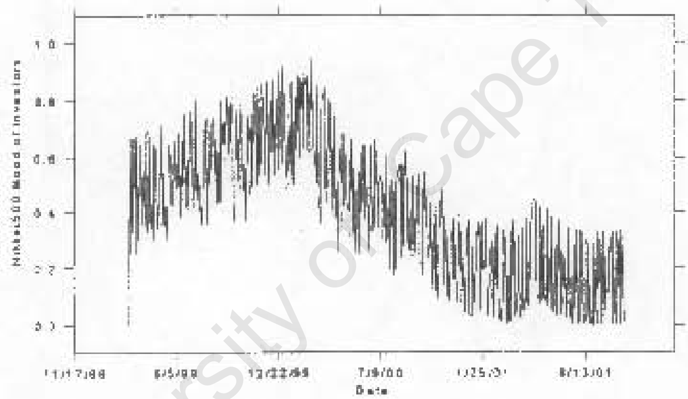

Figure 6-13: Mood index: Nikkei500 index 2/26/99 - 10/26/01

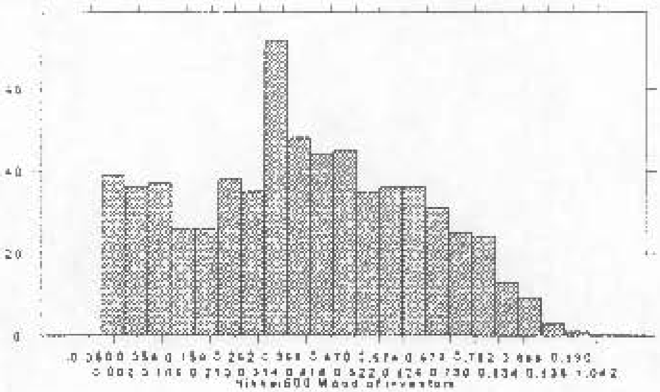

Figure 6-14: Histogram: mood index of the Nikkei500 index 2/28/99 - 10/26/01

The plot of the mood index for the IBOVESPA is shown in Figure 6.15. The histogram of the mood index for the IBOVESPA is given in Figure 6.16.

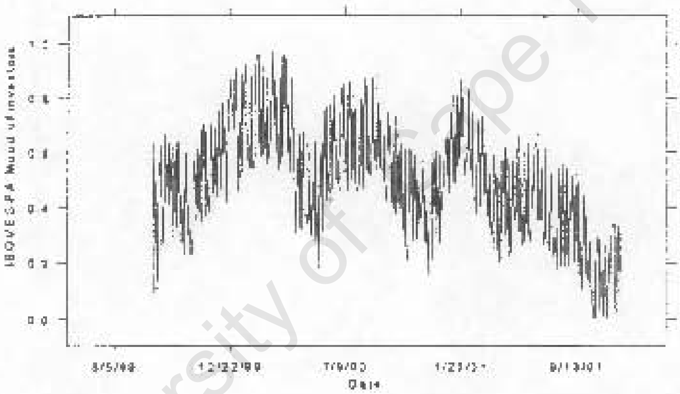

Figure 6-15: Mood index: IBOVESPA 8/11/99 - 10/25/01

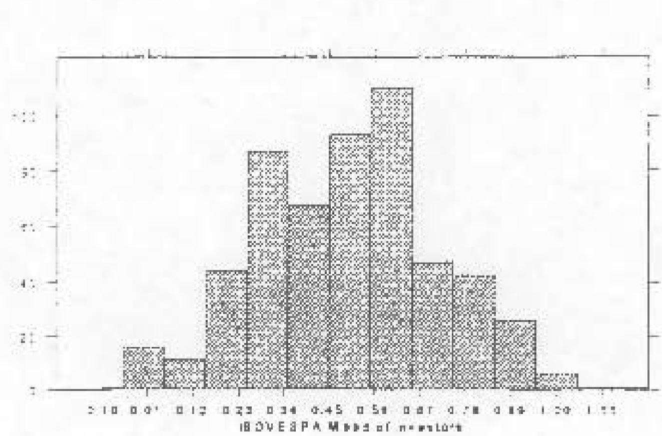

Figure 6-16: Histogram: mood index of the IBOVESPA 8/11/99 - 10/25/01

The plots of the mood indices for various markets show that, the mood of investors for the JSE had an upward trend from about 5/19/87. The mood of investors for the JSE gold index showed a general decline from around 7/13/99 to about 1/25/01 and then generally started to rise. The mood of investors for the NYSE, Dow Jones and Nasdaq had a general upward trend from about 9/16/99 and then had a general downward trend from about 3/31/00 for the NYSE and the Nasdaq, while the downward trend for the Dow Jones was from about 5/5/01. The mood of investors for the Dow Jones and the Nasdaq was very low around the 8/11/01. The mood of investors for the DAX had a general downward trend from about 3/31/00. The Nikke500 had its mood of investor generally rising from 2/26/99 to just after 12/22/99, then assumed a general downward trend. The mood of investors for the IBOVESPA was characterised by an upward trend from about 8/11/99 to about 12/22/99 and then had a general downward trend. The mood was lowest after 8/11/01.

6.2.2 Linear relationship between mood indices for various markets

Correlations between the mood indices of various markets; NYSE (NY), Nasdaq (NQ), Dow Jones (DJ), DAX (DX), Nikkei500 (NK), IBOVESPA (IB), JSE Overall Index (JS) and the JSE Gold (JG) are computed for the period 9/28/01 and 10/2/01 in order to investigate their relationships. The results are shown in the following correlation matrix given in Table 6.1.

	NY	NQ	DJ	DX	NK	IB	JS	JG
NY	1	0.139	0.375	0.232	0.104	0.341	-0.008	-0.329
NQ	0.139	1	0.057	0.385	0.664	0.469	-0.147	0.044
DJ	0.375	0.057	1	0.065	-0.167	0.145	0.023	-0.250
DX	0.232	0.385	0.065	1	0.400	0.309	-0.062	-0.062
NK	0.104	0.664	-0.167	0.400	1	0.454	-0.163	0.093
IB	0.341	0.469	0.145	0.309	0.454	1	-0.006	-0.162
JS	-0.008	-0.147	0.023	-0.062	-0.163	-0.006	1	0.055
JG	-0.329	0.044	-0.250	-0.062	0.093	-0.162	0.055	1

Table 6.1: Correlation matrix of mood indices for various market indices

Mood of investors for the DAX, NIKKEI500 and IBOVESPA all had a reasonably high correlation with the NASDAQ mood. The DAX and IBOVESPA mood also had some high correlation with the NIKKEI500 mood. The mood of the JSE was negatively correlated to the mood of most markets. This may be explained by the fact that when emerging markets like the JSE experience low mood, investors shift to the developed markets.

6.3 Persistence and randomness of mood indices for various markets

In this section, the nature of the computed mood of investors from the mood indices for various markets will be investigated. The R/S analysis will be applied to inspect whether the mood of investors are independent processes or posses the Hurst exponent memory phenomena. Peters (1994) has explained that autocorrelated data can influence the Hurst exponent. Thus the AR(1) model was fit to the mood indices for the various markets. The R/S analysis was performed on the residuals of the AR(1) models. The AR(1) model is given by

$$X_t = \alpha X_{t-1} + \xi_t.$$

The residual ξ_t at time t is expressed as

$$\xi_t = X_t - \widehat{\alpha} X_{t-1}$$

where X_t is the mood index value at t, $\widehat{\alpha}$ is the value obtained from fitting an $AR(1)$ model to the mood index, and X_{t-1} is the mood index value at $t-1$.

The summary of the AR(1) fitting for the various mood indices is shown Table 6.2.

Index	$\widehat{\alpha}$
NYSE	0.525
NASDAQ	0.578
DOW JONES	0.526
DAX	0.284
NIKKEI500	0.692
IBOVESPA	0.554
JSE	0.606
JSE GOLD	0.604

Table 6.2: AR(1) model coefficients for the mood indices of various market indices.

The Hurst exponent for each mood index will be computed from the residuals of the respective $AR(1)$ models. The computed Hurst exponent is tested for significance. The computations are carried out as follows:

The JSE Mood of investors

Figure 6.17 below shows the R/S plot for the JSE Mood Index. The $E(R/S_n)$ is also shown in the plot. 4000 observations are used in the analysis.

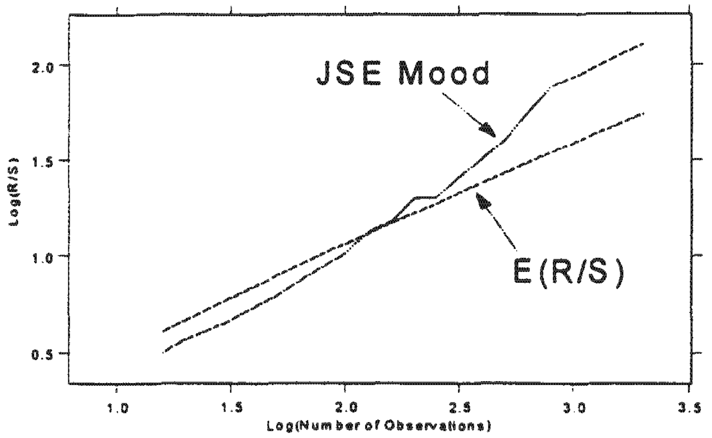

Figure 6-17: R/S analysis: JSE Mood Index

The V-statistic which is the ratio of R/S_n to $\sqrt{n}$, is computed and plotted against $Log(n)$ in Figure 6.18.

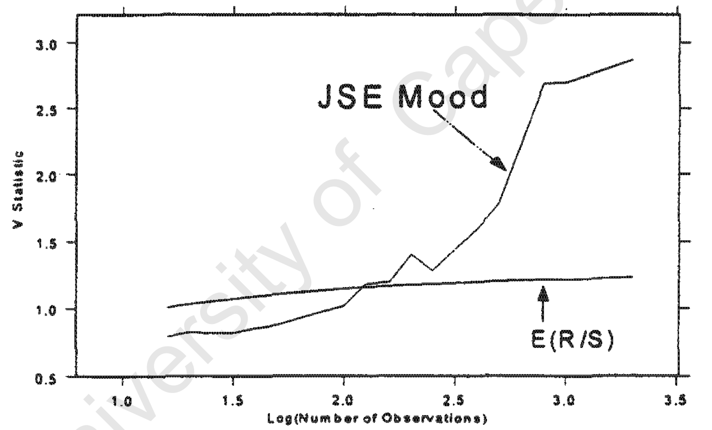

Figure 6-18: V statistic: JSE Mood Index

The V-statistic seems to increase exponentially. This suggest that the mood of investors for the JSE Overall index exhibits some Hurst phenomena of persistence.

The regression output for the R/S analysis is shown in Table 6.3.

Regression output	R/S	E(R/S)
Constant	-0.5307	-0.0294
Standard error of Y (estimated)	0.0521	0.0080
R squared	0.9900	0.9995
Number of observations	4000	4000
Degrees of freedom	15	15
Hurst exponent	0.7965	0.5402
Standard error of coefficient	0.0206	0.0032
Significance	38.626	170.35

Table 6.3: Regression results: JSE Mood Index.

From the regression, $H = 0.7965$ and $E(H) = 0.5402$. Variance of $E(H)$ is $\frac{1}{4000} = 0.00025$. Thus the standard deviation of $E(H)$ is 0.0151581. The H value for the JSE mood is therefore 16.209 standard deviations above the expected value. This is a highly significant result.

Nasdaq Mood Index

The R/S and E(R/S) plots for the Nasdaq mood are shown in Figure 6.19.

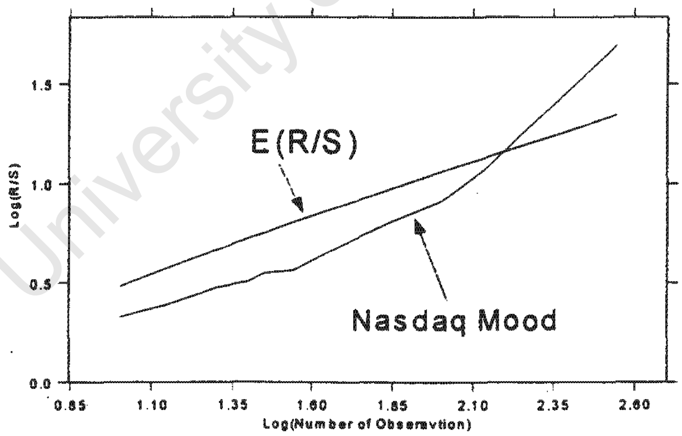

Figure 6-19: R/S analysis: Nasdaq Mood Index

The V-statistic for the Nasdaq mood is shown in Figure 6.20.

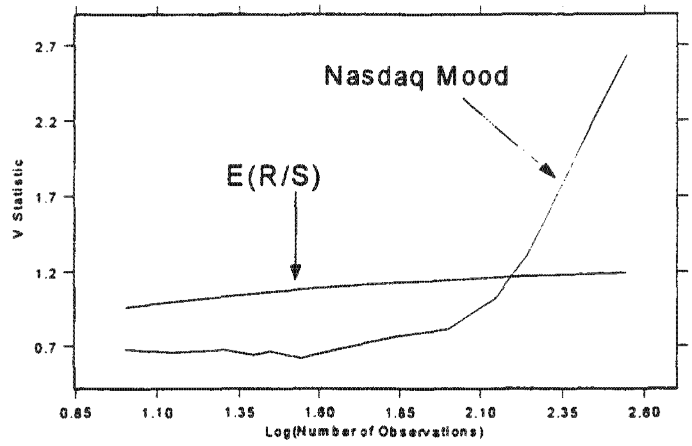

Figure 6-20: V statistic: Nasdaq Mood Index

The V-statistic is increasing, suggesting persistence in the mood of investors for the Nasdaq.

The regression output for the R/S and E(R/S) analysis for the Nasdaq Mood is given in Table 6.4.

Regression output	R/S	E(R/S)
Constant	-0.6284	-0.0671
Standard error of Y (estimated)	0.1058	0.0073
R squared	0.9373	9993
Number of observations	700	700
Degrees of freedom	10	10
Hurst exponent	0.8257	.5610
Standard error of coefficient	0.0675	0.0046
Significance	12.227	120.733

Table 6.4: Regression results: Nasdaq Mood Index

From the table, $H = 0.8257$, $E(H) = 0.5610$. The variance of $E(H)$ is $\frac{1}{700}$. The standard deviation of $E(H)$ is 0.0378. Thus H is 7.0026 standard deviations above the expected value. The result is highly significant.

Dow Jones Mood Index

The log log plots of R/S and the E(R/S) for the Dow Jones Mood Index are given in the Figure 6.21.

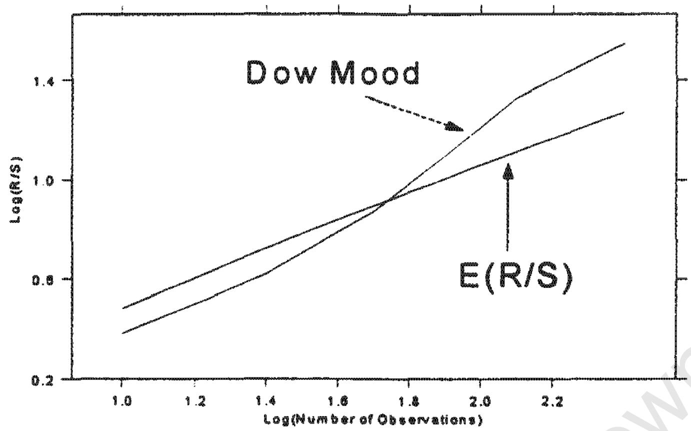

Figure 6-21: R/S analysis: Dow Jones Mood Index

The plot of the V statistic for the Dow Jones Mood Index is given in Figure 6.22. The V statistic is increasing and suggesting persistence in the mood of investors for the Dow Jones.

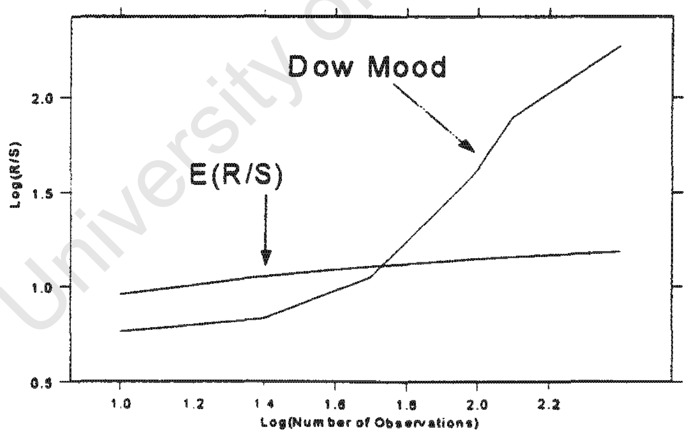

Figure 6-22: V statistic: Dow Jones Mood Index

The regression output is given in Table 6.5.

Rgression output	R/S	E(R/S)
Constant	-0.5701	-0.0739
Standard error of Y (estimated)	0.0509	0.0074
R squared	0.9889	0.9994
Number of observations	500	500
Degrees of freedom	5	5
Hurst exponent	0.8836	0.5648
Standard error of coefficient	0.0419	0.0061
Significance	21.071	92.164

Table 6.5: Regression results: Dow Jones Mood Index.

The regression output gives $H = 0.8836$, $E(H) = 0.5648$. The variance of $E(H)$ is $\frac{1}{500}$. The standard deviation of $E(H)$ is 0.0447. Thus H is 7.132 standard deviations above the expected value. The result is highly significant.

NIKKEI500 Mood Index

The R/S plots and that of the E(R/S) for the Nikkei500 Mood Index are given in Figure 6.23.

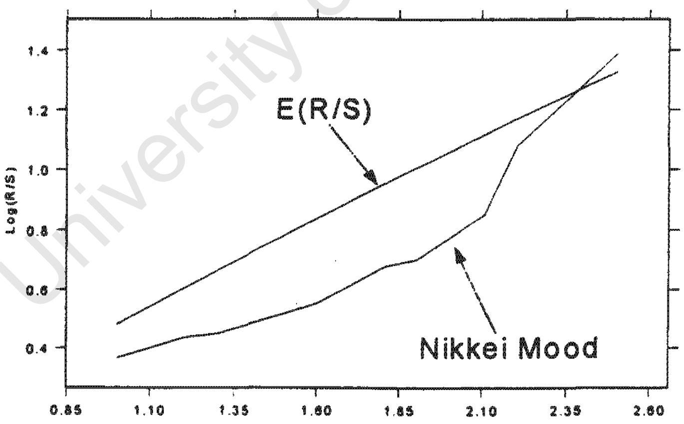

Figure 6-23: R/S analysis: Nikkei500 Mood Index

The corresponding V-statistic for the Nikkei500 mood is shown in Figure 6.24.

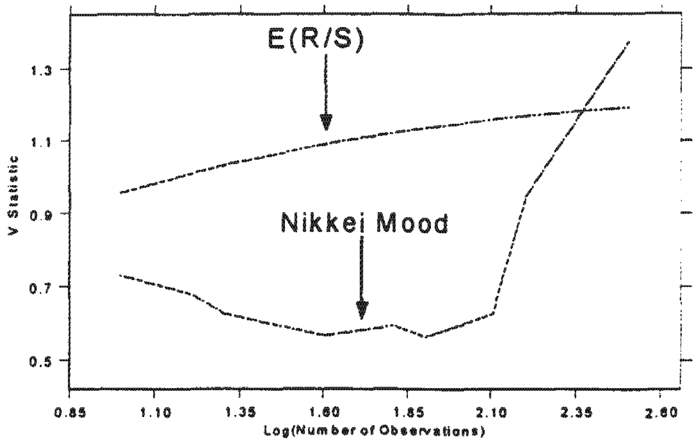

Figure 6-24: V statistic: Nikkei500 Mood Index

The regression output is given in Table 6.6.

Regression output	R/S	E(R/S)
Constant	-0.3976	-0.0695
Standard error of Y (estimated)	0.1094	0.0073
R squared	0.8986	0.9993
Number of observations	640	640
Degrees of freedom	8	8
Hurst exponent	0.6413	0.5624
Standard error of coefficient	0.0762	0.0051
Significance	8.4194	110.396

Table 6.6: Regression results: Nikkei500 Mood Index.

The regression output has $H = 0.6413$ and $E(H) = 0.5624$. The variance of $E(H)$ is $\frac{1}{640}$. The standard deviation of $E(H)$ is 0.0395. The H values is 1.997 standard deviations from the expected value. The results is significant.

NYSE Mood Index

The R/S and E(R/S) plot for the NYSE Mood Index are given in Figure 6.25.

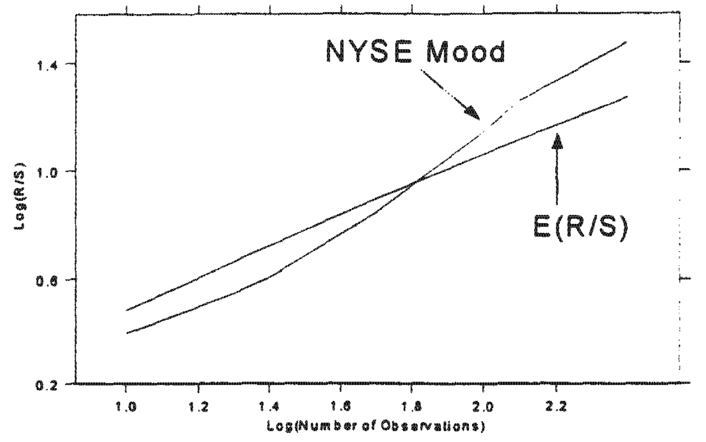

Figure 6-25: R/S analysis: NYSE Mood Index

The V-statistic for NYSE Mood Index is shown in Figure 6.26. The V-statistic is increasing and may suggest persistency in the mood of investors for the NYSE.

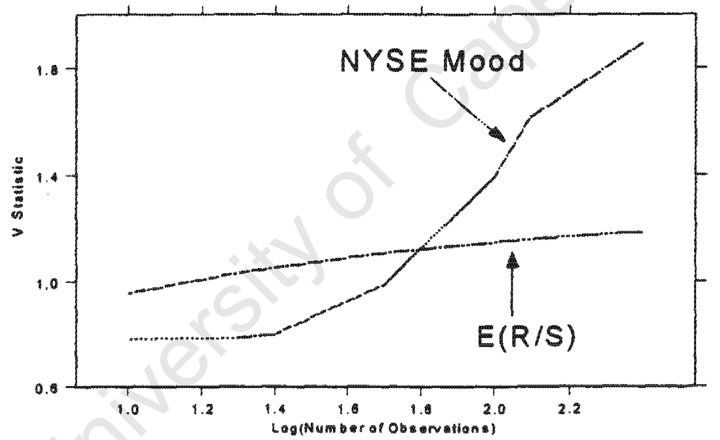

Figure 6-26: V statistic: NYSE Mood Index

The regression output for the R/S and E(R/S) of the NYSE Mood Index is shown in Table 6.7.

Regression output	R/S	E(R/S)
Constant	-0.4896	-0.0739
Standard error of Y (estimated)	0.0481	0.0074
R squared	0.9883	0.9994
Number of observations	500	500
Degrees of freedom	5	5
Hurst exponent	0.8150	0.5648
Standard error of coefficient	0.0396	0.0061
Significance	20.589	92.164

Table 6.7: Regression results: NYSE mood Index.

From the table, $H = 0.8150$ and $H = 0.5648$. The variance of $E(H)$ is $\frac{1}{500}$. The standard deviation of $E(H)$ is 0.0447. The value of H is therefore 5.6 standard deviations from the expected value. The result is highly significant.

DAX Mood Index

The R/S and E(R/S) plots for the DAX Mood Index are shown in Figure 6.27.

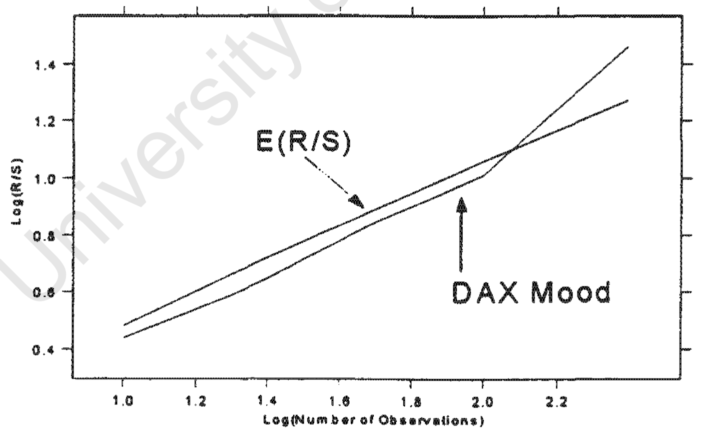

Figure 6-27: R/S analysis: DAX Mood Index

The V-statistic for the DAX Mood Index is plotted in Figure 6.28. The V-statistic suggests persistence in the mood of investors for the DAX.

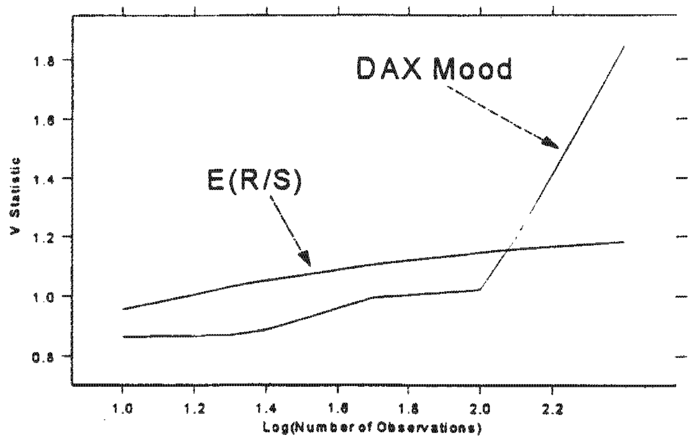

Figure 6-28: V statistic: DAX Mood Index

The regression output for R/S and E(R/S) analysis for the DAX Mood Index is given in Table 6.8.

Regression output	R/S	E(R/S)
Constant	-0.3254	-0.0739
Standard error of Y (estimated)	0.0642	0.0074
R squared	0.9727	0.9994
Number of observations	500	500
Degrees of freedom	5	5
Hurst exponent	0.7061	0.5648
Standard error of coefficient	0.0529	0.0061
Significance	13.350	92.164

Table 6.8: Regression results: DAX mood Index.

The tables shows, $H = 0.7061$ and $E(H) = 0.5648$. The variance of E(H) is $\frac{1}{500}$. The standard deviation of $E(H)$ is 0.0447. H of the DAX is 3.161 standard deviations above the expected value. The result is highly significant.

JSE Gold Mood Index

The plots of the R/S and the E(R/S) for the JSE Gold Mood Index are given in Figure 6.29.

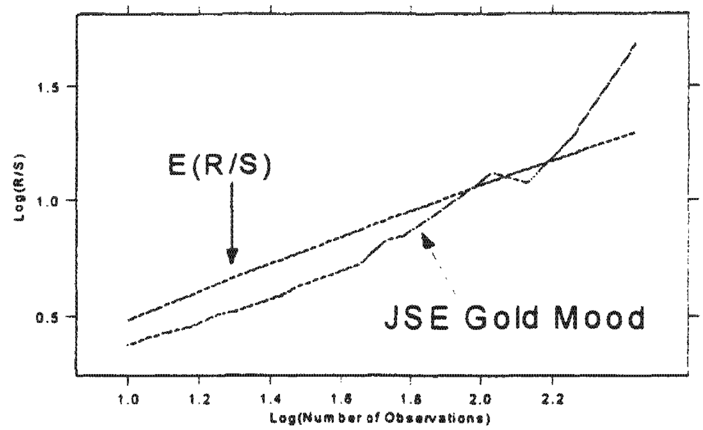

Figure 6-29: R/S analysis: JSE Gold Mood Index

The V-statistic plot for the JSE Gold Mood Index is given in Figure 6.30.

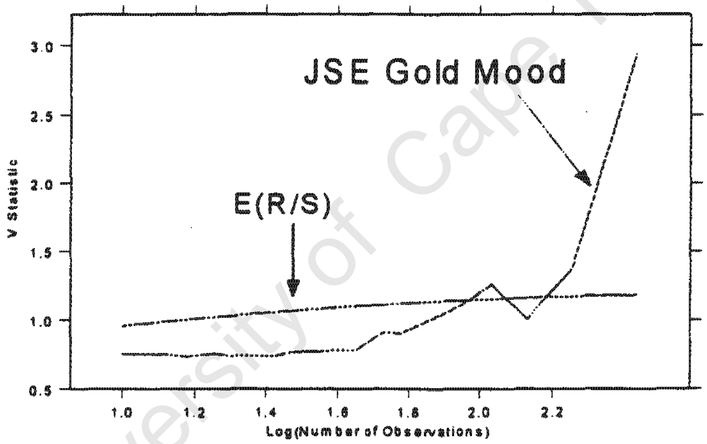

Figure 6-30: V statistic: JSE Gold Mood Index

The regression output of the R/S and E(R/S) analysis for the JSE Gold Mood Index is given in Table 6.9.

Regression output	R/S	E(R/S)
Constant	-0.5297	-0.0728
Standard error of Y (estimated)	0.0908	0.0061
R squared	0.9405	0.9994
Number of observations	540	540
Degrees of freedom	14	14
Hurst exponent	0.8063	0.5649
Standard error of coefficient	0.0542	0.0037
Significance	14.880	154.036

Table 6.9: Regression results: JSE Gold Mood Index.

The table shows $H = 0.8063$ and $E(H) = 0.5649$. The variance of $E(H)$ is $\frac{1}{540}$. The standard deviation of $E(H)$ is 0.043. Thus H of the JSE Gold Mood Index is 5.614 standard deviations above the expected value. The result is highly significant.

IBOVESPA Mood Index

The R/S and E(R/S) for the IBOVESPA Mood Index are plotted in Figure 6.31.

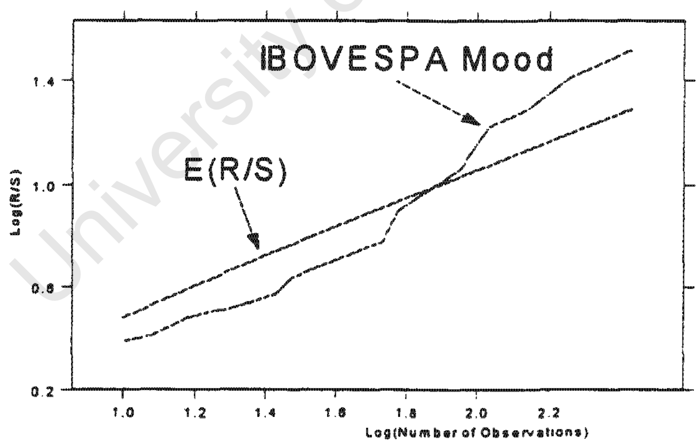

Figure 6-31: R/S analysis: IBOVESPA Mood Index

The V-statistic plot IBOVESPA Mood Index is given in Figure 6.32.

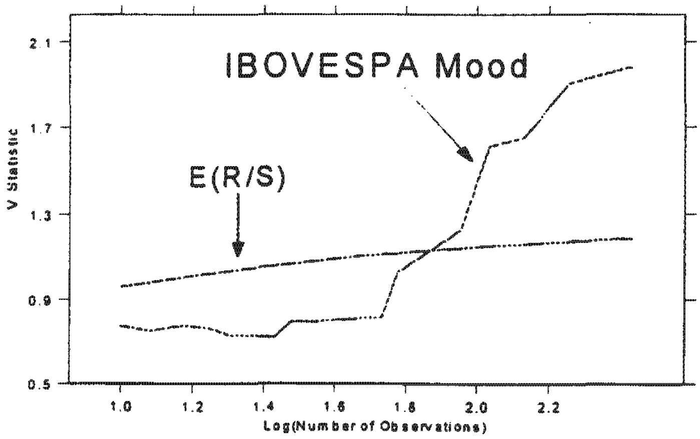

Figure 6-32: V statistic: IBOVESPA Mood Index

The regression output for the R/S and E(R/S) analysis for the IBOVESPA Mood Index is given in Table 6.10.

Regression output	R/S	E(R/S)
Constant	-0.5625	-0.0728
Standard error of Y (estimated)	0.0706	0.0061
R squared	9661	9994
Number of observations	540	540
Degrees of freedom	14	14
Hurst exponent	0.8413	0.5649
Standard error of coefficient	0.0421	0.0037
Significance	19.967	154.036

Table 6.10: Regression results: IBOVESPA Mood Index.

The table gives, $H = 0.8413$ and $E(H) = 0.5649$. The variance of E(H) is $\frac{1}{540}$. The standard deviation of $E(H)$ is 0.043. H is thus 6.4279 standard deviations above the expected value. The result is highly significant.

The results show that the mood of investors computed in this study are characterised by the Hurst phenomena of long memory. This implies that the present mood of investors is some what dependent on the past mood of investors. It also means that present mood in the market will most likely influence the mood of investors in the future. Investors remember their reaction to market events in the past and some how this has influence on their reaction to the current or future events. Thus the mood of investors do not follow a random walk. They are fractional Brownian time series and not normally distributed.

Since the AR(1) residuals were used (Peters 1994), the mood of investors are characterised by a true long memory. The long memory results of the mood of investors (except for the JSE Mood Index) have been obtained from small sets of data. Thus the results might suffer from Type I undersampling discussed in Peters (1994). However, the results might be indicative of the true nature of the mood of investors. In fact the results are consistent with the results of the JSE Mood computed from a larger data set of 4000 observations.

The impact of present mood of investors on the future can be expressed as a correlation:

$$C = 2^{(2H-1)} - 1,$$

where C is the correlation measure and H is the Hurst exponent (Peters 1991). The impact of the present mood of investors on the future of their corresponding markets are computed in Table 6.11.

Mood Index	C	Mood Index	C
NYSE	0.55	NIKKEI500	0.22
NASDAQ	0.57	IBOVESPA	0.61
DOW JONES	0.70	JSE Overall Index	0.51
DAX	0.33	J SE GOLD	0.53

Table 6.11: Impact of present mood on the future.

The impact of present mood on the future is quite big in all the markets except for the DAX at 0.46 and the JSE at 0.67.

The fractal dimension shows how a time series fills its space. The way an objects fills its space is determined by the forces involved in its formation (Peters 1991). Given the Hurst exponent H of a time series, the fractal dimension of the series is computed as $\frac{1}{H}$ (Mandelbrot 1972). The fractal dimensions of the mood indices are computed and shown in Table 6.12.

Mood Index	F/dimension	Index	F/Dimension
NYSE	1.23	NIKKEI500	1.56
NASDAQ	1.21	IBOVESPA	1.19
DOW JONES	1.13	JSE	1.26
DAX	1.42	JSE GOLD	1.24

Table 6.12: Fractal dimension for the Mood Indices.

The fractal dimensions for the mood indices of various market indices are different. This shows that the investors in different markets react differently to forces behind the markets. Thus the fractal dimensions of the mood index can be viewed as a measure of investors' reaction to the market events.

The different fractal dimensions also enables us to compare the different mood index series. From the table of the fractal dimensions, the mood of investors for the DAX and the Nikkei500 indices are more jagged than the rest of the mood indices. The least jagged mood of investors is that for the Dow Jones. Thus the mood of investors for the DAX and Nikkei500 have more noise and may be more difficult to deal with. The mood of investors for the Dow Jones has the least noise and thus may be easier to deal with. Various factors may be responsible for the different shapes of the mood of investors in each market index.

6.3.1 Possibility distributions of mood indices for various markets

In this section the possibility distributions of the mood indices will be computed and presented in order to gauge possibility and meaning of the mood events in the markets.

The NYSE Mood Index has been computed as:

$$\Pi_{NYSE} = \frac{.08}{0} + \frac{.40}{.1} + \frac{.45}{.2} + \frac{.62}{.3} + \frac{.82}{.4} + \frac{1}{.5} + \frac{1}{.6} + \frac{.89}{.7} + \frac{.89}{.8} + \frac{.57}{.9} + \frac{.08}{1}$$

and its plot is given in Figure 6.33.

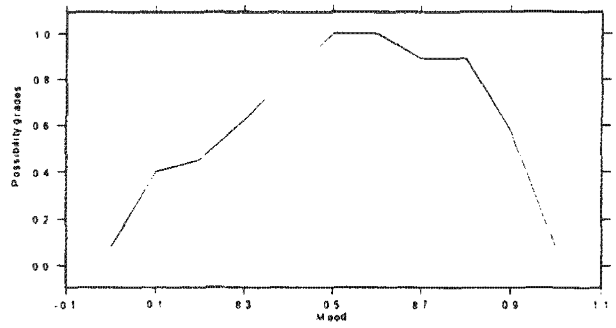

Figure 6-33: Possibility distribution: Mood Index of the NYSE 9/13/99 - 10/25/01

The possibility distribution of the Nasdaq Mood Index is computed as

$$\Pi_{NASDAQ} = \frac{.90}{0} + \frac{.64}{.1} + \frac{.91}{.2} + \frac{1}{.3} + \frac{.86}{.4} + \frac{.77}{.5} + \frac{.93}{.6} + \frac{.94}{.7} + \frac{.41}{.8} + \frac{.27}{.9} + \frac{.03}{1}$$

with the plot shown in Figure 6.34.

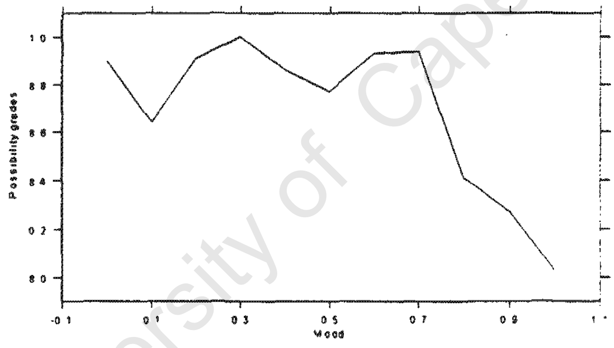

Figure 6-34: Possibility distribution: Mood Index of the Nasdaq 12/21/98 - 10/25/01

The possibility of the Dow Jones Mood Index is computed as

$$\Pi_{D.JONES} = \frac{.20}{0} + \frac{.22}{.1} + \frac{.45}{.2} + \frac{.71}{.3} + \frac{.97}{.4} + \frac{.98}{.5} + \frac{1}{.6} + \frac{.95}{.7} + \frac{.78}{.8} + \frac{.58}{.9} + \frac{.15}{1}$$

and the plot of the possibility distribution presented in Figure 6.35.

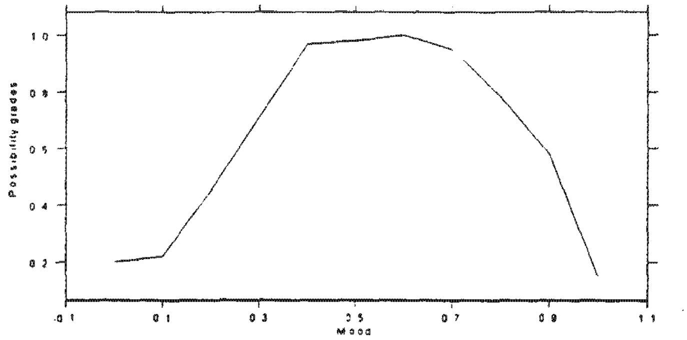

Figure 6-35: Possibility distribution: Mood Index of the Dow Jones 9/16/99 - 10/25/01

The possibility distribution of the DAX Mood Index is computed as

$$\Pi_{DAX} = \frac{.24}{0} + \frac{.62}{.1} + \frac{.94}{.2} + \frac{1}{.3} + \frac{.84}{.4} + \frac{.86}{.5} + \frac{.51}{.6} + \frac{.28}{.7} + \frac{.21}{.8} + \frac{.04}{.9} + \frac{.04}{1}.$$

The plot of the distribution is given in Figure 6.36.

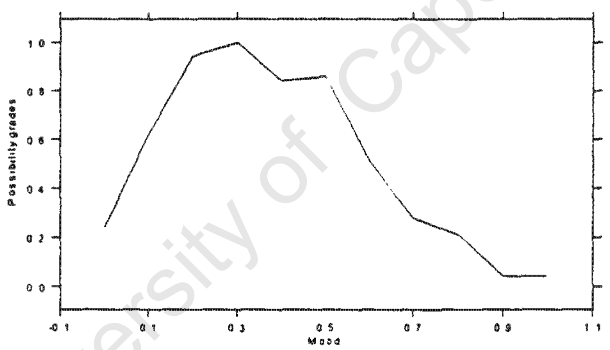

Figure 6-36: Possibility distribution: Mood Index of the DAX 9/28/99 - 10/26/01

The possibility distribution of the Nikkei500 is computed as

$$\Pi_{NIKKEI500} = \frac{.83}{0} + \frac{.83}{.1} + \frac{.78}{.2} + \frac{.99}{.3} + \frac{1}{.4} + \frac{.97}{.5} + \frac{.85}{.6} + \frac{.88}{.7} + \frac{.54}{.8} + \frac{.31}{.9} + \frac{.02}{1}.$$

The plot of the possibility distribution is given in Figure 6.37.

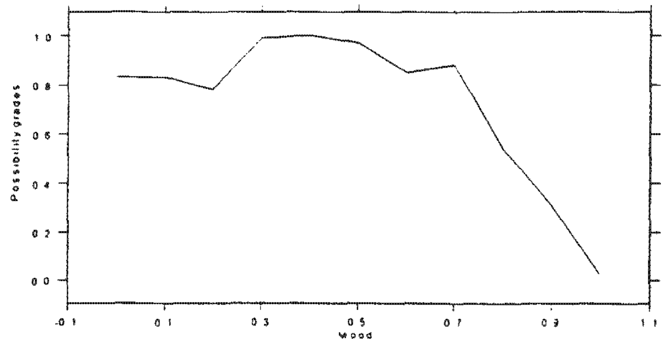

Figure 6-37: Possibility distribution: Mood Index of the Nikkei500 2/26/99 - 10/26/01

The possibility distribution of the IBOVESPA is computed as

$$\Pi_{IBOVESPA} = \frac{.29}{0} + \frac{.21}{.1} + \frac{.66}{.2} + \frac{.95}{.3} + \frac{.84}{.4} + \frac{.97}{.5} + \frac{1}{.6} + \frac{.69}{.7} + \frac{.64}{.8} + \frac{.43}{.9} + \frac{.12}{1}.$$

The plot is given in Figure 6.38

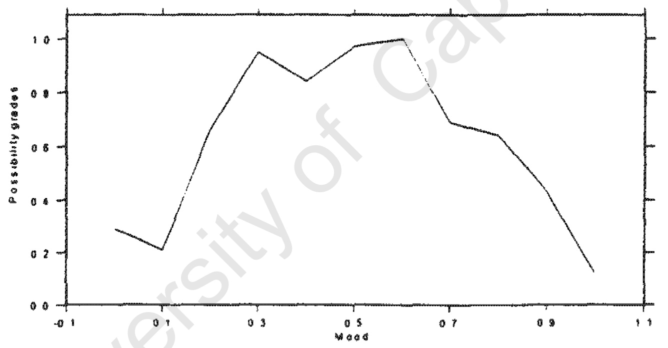

Figure 6-38: Possibility distribution: Mood Index of the IBOVESPA 8/11/99 - 10/25/01

The possibility distribution is computed as

$$\Pi_{JSE} = \frac{.02}{0} + \frac{.17}{.1} + \frac{.47}{.2} + \frac{.78}{.3} + \frac{.99}{.4} + \frac{1}{.5} + \frac{.92}{.6} + \frac{.81}{.7} + \frac{.75}{.8} + \frac{.48}{.9} + \frac{.03}{1}.$$

The plot of the distribution is given in Figure 6.39.

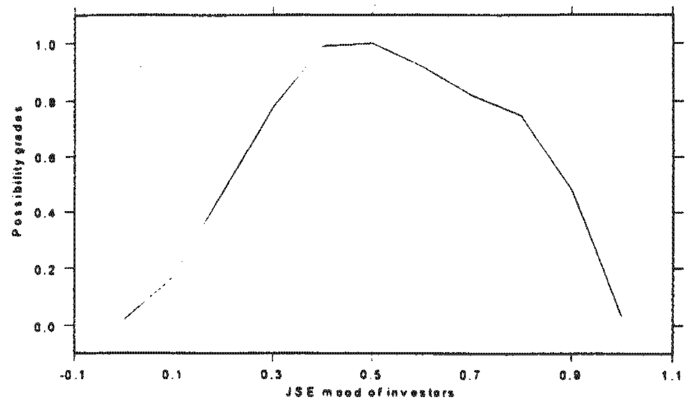

Figure 6-39: Possibility distribution: Mood Index of the JSE Overall Index 3/27/85 - 11/1/01

The possibility distribution of the JSE Gold is computed as

$$\Pi_{JSEGOLD} = \frac{.63}{0} + \frac{.95}{.1} + \frac{.92}{.2} + \frac{.99}{.3} + \frac{1}{.4} + \frac{.89}{.5} + \frac{.76}{.6} + \frac{.56}{.7} + \frac{.24}{.8} + \frac{.02}{.9} + \frac{0}{1}.$$

Its plot is given in Figure 6.40.

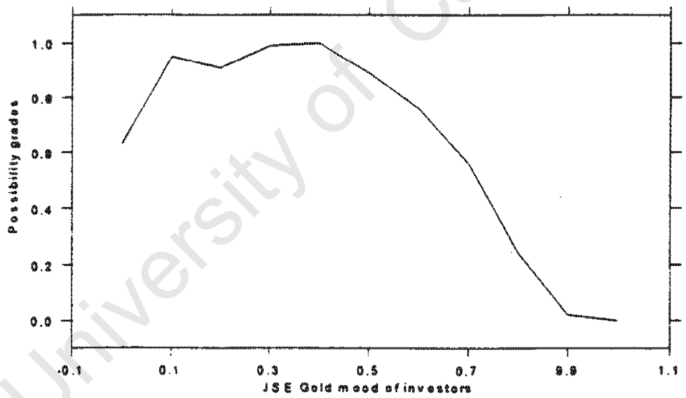

Figure 6-40: Possibility distribution: Mood Index of the JSE Gold 7/13/99 - 10/31/01

Assuming the states used to describe mood of investors are "Low", "Stable" or "High", then it is not easy to partition the mood of investors according to these states. However, if the mood is partitioned as: Low $= [0, 0.3)$, Stable $= [0.3, 0.7]$, and High $= (.7, 1]$, then all the markets considered here are characterised by stable mood of investors. That is, the possibility distributions of their mood indices all have values with possibility 1 in the "stable" state. The values from the stable states with possibility grade 1, differ between the markets. This implies that stability of mood differs from market to market. Some markets are characterised by a stable mood bordering the low mood. Others have their stable mood bordering the high mood.

The possibility grades of the mood indices differ between the markets. This shows that the subjective beliefs about the states and meaning of mood is not the same between the markets.

It can be noted that all the markets have very small possibility of the highest (i.e. 1) mood. It is in fact impossible for the JSE to attain the highest mood of investors. The Nasdaq, Nikkei500 and JSE gold have quite high possibility of the lowest (i.e. 0) mood, and the rest have very small possibility of the lowest mood.

Fuzziness of the mood of investors

In order to investigate how obscure the mood of investors can be, the fuzziness of mood of investors is measured. The measure of fuzziness is given by the area under the possibility distribution. Table 6.13 gives the estimated areas under the possibility distributions of the mood indices.

Mood Index	Fuzziness
NYSE	0.66
NASDAQ	0.76
DOW JONES	0.70
DAX	0.56
NIKKEI500	0.80
IBOVESPA	0.66
JSE	0.64
JSE GOLD	0.74

Table 6.13: Fuzziness of the Mood Indices.

possibility - probability consistency

The possibility probability consistency to measure coherence between subjective belief and frequency about the mood of investors events is computed for each market and shown in Table

6.14.

Mood Index	Possibility - probability consistency
NYSE	0.83
NASDAQ	0.85
DOW JONES	0.84
DAX	0.81
NIKKEI500	0.87
IBOVESPA	0.81
JSE	0.84
JSE GOLD	0.86

Table 6.14: Possibility - probability consistency of the Mood Indices.

The results show that in all the markets, subjective belief about mood of investors events and their chances to occur were highly consistent.

6.3.2 Mood index for the JSE Overall Index and the JSE Gold Index

The mood of the JSE Overall Index and JSE Gold Index are investigated further. The plot of the possibility distribution for the JSE Mood Index in Figure 6.39 suggests that the stable mood has high possibility followed by the high mood of investors for the JSE Overall Index. The highest mood of 1 and the lowest mood of 0 only have possibility grades of 0.03 and 0.02 respectively. Thus its almost impossible to attain the lowest and the highest mood of the JSE according to past knowledge (data). The prediction of the JSE mood of investors is stable to high mood.

The possibility distribution of the JSE Gold Mood Index and plotted in Figure 6.40 suggest that low to stable mood are more possible for the JSE Gold Index. Possibility of the lowest mood of 0 is high with possibility grade 0.63. The graph also shows that it is impossible for the JSE Gold to reach the highest mood.

6.4 Decomposition of share price of JSE Anglo Gold

The JSE overall index is derived from all the listed companies and Anglo Gold is one of them. Anglo contributes a significant portion of 23% to the JSE. In this section the daily and monthly mood of investors for JSE Anglo Gold share price are computed. The mood index of the JSE Anglo Gold share price is computed from the monthly data. The data used to compute the daily mood was for the period 5/14/98 - 11/22/01. The monthly data was for the period 3/31/90 to 8/31/01. The computations are done as follows:

6.4.1 Daily mood

To compute the daily mood of investors, the daily regret and daily confidence are computed. These are give as follows:

Daily regret

The daily regret is computed and its plot is given in Figure 6.41.

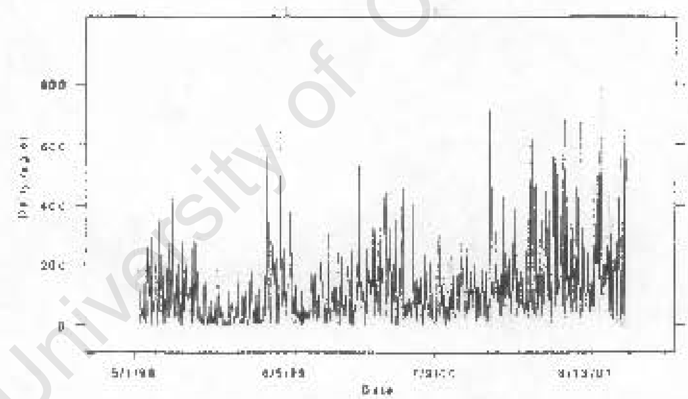

Figure 6-41: Daily regret: JSE Anglo Gold share price 5/14/98 - 11/22/01

The daily regret seemed to be very spiked. The size of the spikes increased with time.

Daily confidence

The daily confidence is plotted in Figure 6.42.

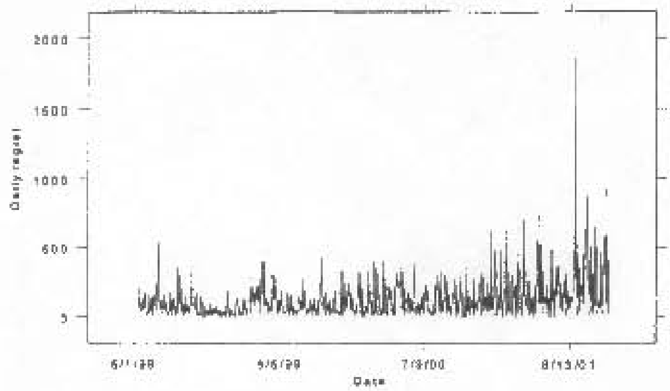

Figure 6-42: Daily confidence: JSE Anglo Gold share price 5/14/98 - 11/22/01.

The daily confidence seemed to be more stable than daily regret. Sharp spikes are seen towards the end indicating high confidence in the share price in this period.

Daily mood

The daily regret and daily confidence are combined to compute the daily mood of investors. The computed daily mood is plotted in Figure 6.43.

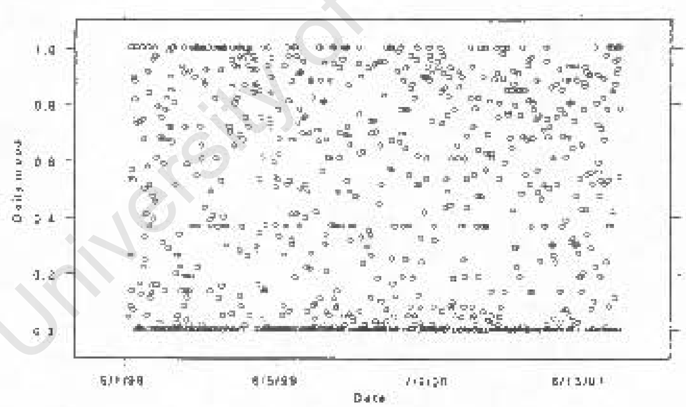

Figure 6-43: Daily mood: JSE Anglo Gold share price 5/14/98 - 11/22/01.

The distribution of the daily mood is represented by the histogram in Figure 6.44.

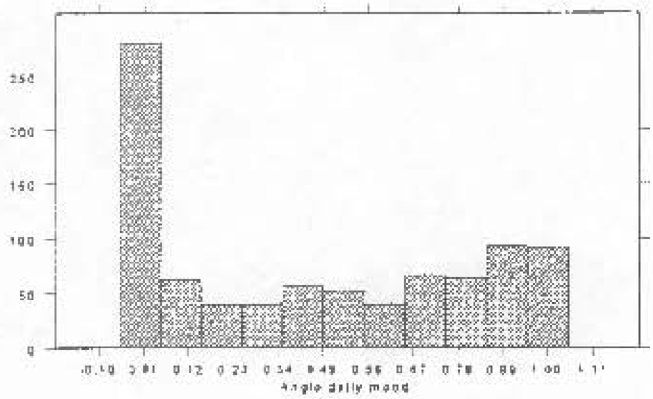

Figure 6-44:
Distribution of daily mood of investors: JSE Anglo Gold share price 5/14/98 - 11/22/01.

Characteristics of the daily mood (Anglo Gold share price) The Hurst (H) exponent is computed to investigate the nature of the daily mood of investors for the Anglo Gold share price. The rescaled range analysis was used. The computed Hurst exponent $H = 0.5578$ may imply that the daily mood for the Anglo Gold is persistent and thus exhibits the Hurst memory. The mood of the share price in the past affects the present mood and that of the future. The log-log plot of the analysis is given in Figure 6.45.

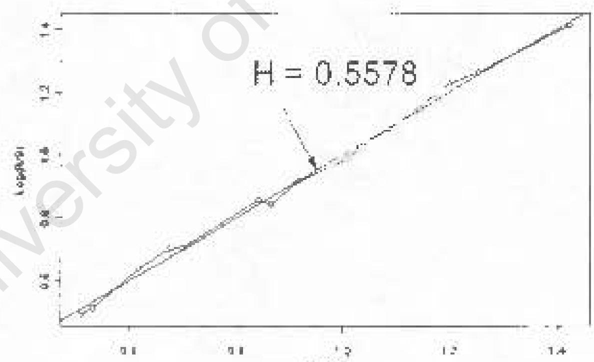

Figure 6-45:
R/S analysis, daily mood of investors: JSE Anglo Gold share price 5/14/98 - 11/22/01.

Possibility distribution

The possibility distribution of the daily mood is computed in order to represent knowledge and information about the Anglo gold share price as well to investigate subjective belief of investors about the share price. The possibility distribution is computed as follows:

$$\pi_{ANGLO} = \frac{1}{0} + \frac{.68}{.1} + \frac{.48}{.2} + \frac{.50}{.3} + \frac{.64}{.4} + \frac{.60}{.5} + \frac{.50}{.6} + \frac{.70}{.7} + \frac{.69}{.8} + \frac{.79}{.9} + \frac{.79}{1}$$

According to knowledge and information obtained from data, the lowest mood (value 1) is the most possible. The highest mood (value 1) is the next most possible daily mood of investors for the Anglo Gold share price. The low mood of investors is the most possible for Anglo Gold share price in the future.

6.4.2 Mood of JSE Anglo Gold due to monthly share price changes

This section will compute the mood index for the Anglo Gold share price. The index will be based on monthly changes of share price. The monthly data for the period 3/31/90 to 8/31/01 will be used. To compute the mood index, mood of investors relative to: record maximum and minimum (considered period); monthly maximum and minimum; and cumulative maximum and minimum are computed. The mood index is the composite of these three moods of investors. To compute each mood of investors, the regret and confidence are computed, however these will not be shown here.

The mood of investors due to monthly maximum, monthly minimum and the monthly closing price will be used to decompose the closing share price into its fundamentals and sentiment. The closing price and the decomposed (fundamentals) share price will be correlated with some of the South African economic indicators (fundamentals) namely: money supply, to investigate any relationship.

The computed mood of investors relative to record maximum, record minimum and the monthly closing values is computed and plotted in Figure 6.46.

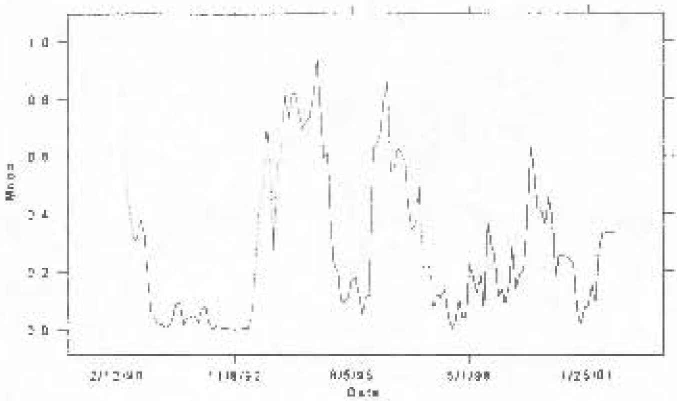

Figure 6-46:
Mood of investors relative to record maximum, record minimum and monthly closing price: JSE Anglo Gold 3/31/90 - 8/31/01.

Between 11/30/90 and 3/31/93, 1/31/95 and 12/31/95, 3/31/97 and 8/31/99 the values of this mood of investors were low. Between 4/30/93 and 12/31/94, 1/31/96 and 2/28/97, 9/30/99 and 9/30/00 the values were high.

The mood of investors due to monthly maximum, monthly minimum and the monthly closing price was computed and plotted in Figure 6.47.

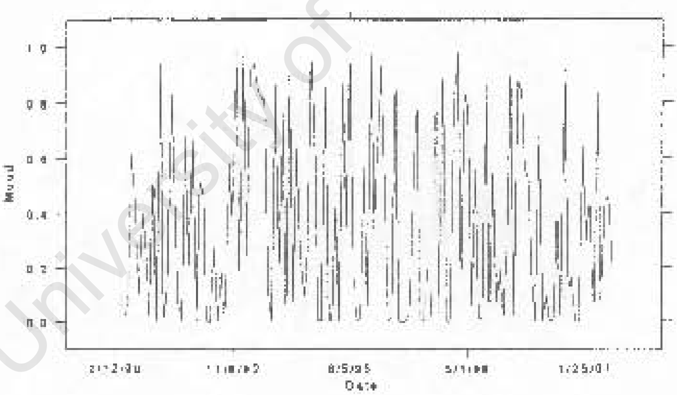

Figure 6-47:
Mood of investors due to monthly maximum, minimum and monthly closing price: JSE Anglo Gold 3/31/90 - 8/31/01.

The mood with respect to the monthly changes of the share price was highly spiked through out the data period. The monthly mood seemed stationary about the mean. The distribution of monthly mood if given by the histogram in Figure 6.48.

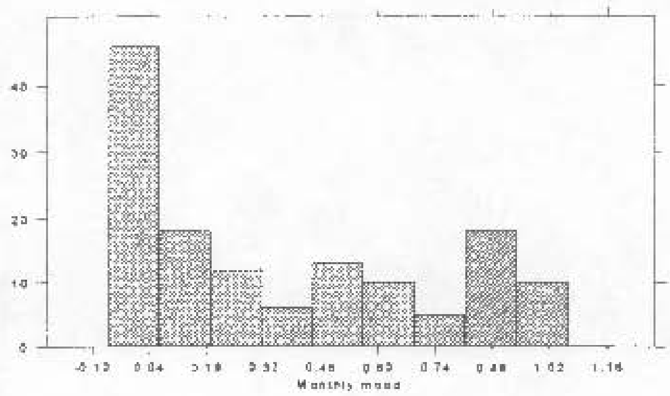

Figure 6-48:
Distribution of monthly mood of investors: JSE Anglo Gold 3/31/90 - 8/31/01.

The monthly mood of investors is characterised by low mood.

The mood relative to cumulative maximum and minimum was computed and plotted in Figure 6.49.

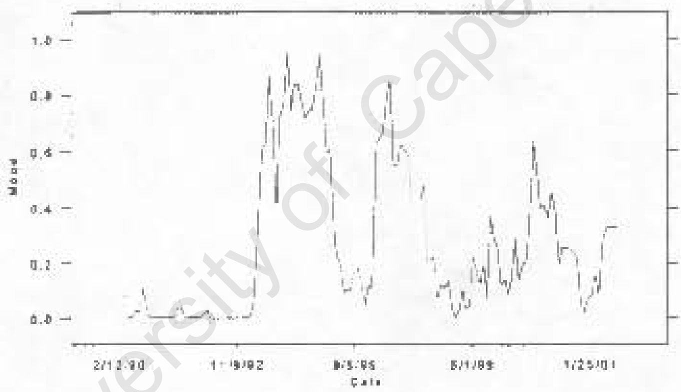

Figure 6-49:
Mood of investors relative to cumulative maximum, cumulative minimum and monthly closing price: JSE Anglo Gold 3/31/90 - 8/31/01.

This mood had between 3/31/90 and 3/31/93, 1/31/95 and 12/31/95, 3/31/97 and 8/31/99 low values. Between 4/30/93 and 12/31/94, 1/31/96 and 2/28/97, 9/30/99 and 9/30/00 the values were high.

The mood index computed as the average of the above moods of investors is computed and the plot is given in Figure 6.50.

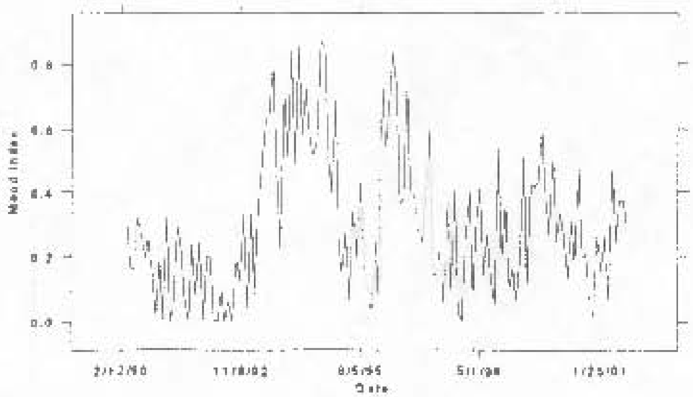

Figure 6-50: Mood index: JSE Anglo Gold monthly 3/31/90 - 8/31/01.

The movement of the mood index was characterised by an upward and downward movement. The index shows that between 4/30/93 and12/31/94, 1/31/96 and 2/28/97 the mood for the Anglo gold share price was high.

6.4.3 Decomposition of share price using the monthly mood index

Both fundamentals and market sentiments contribute to what constitutes a share price. In this section an attempt is made to decompose the JSE Anglo Gold share price into fundamentals and sentiments.

To decompose the share price into fundamentals and sentiment, it will be assumed that price changes and change in the mood of investors are related only to unanticipated events. in this case investors may not have adequate time to analyse all the necessary information to make rational decisions. decisions are influenced by sentiment. this may cause mood of investors to swing to high or low. Without the unanticipated events, price movement are due to economic fundamentals and are stable. The mood of investors is also stable in this situation.

The mood of investors due to monthly maximum, monthly minimum and the monthly closing are used in the decomposition.

The monthly mood of investors is given by

$$\xi_t = \exp\left(-\frac{S_t^M - S_t^C}{S_t^C - S_t^m}\right),$$

where S_t^M is the monthly maximum, S_t^m is the monthly minimum and S_t^C the monthly closing price. It is assume that the closing price S_t^C can be written as a sum of fundamentals and sentiments such that

$$S_t^C = \eta_t + \zeta_t$$

where η_t is the price due to fundamentals at time t and ζ_t the change in price due to sentiments at time t. The market mood is perceived to stable within a certain range of values, say (0.3, 0.7). If $\kappa_t = 0.5$ indicates stable market mood and implies $S_t^C = \eta_t$ (i.e. the share price when mood is stable is due to fundamentals), it be shown that

$$\eta_t = \frac{S_t^M + S_t^m \ln 2}{1 + \ln 2}$$

which is the contribution portion from the economic fundamentals.

The market share price due to fundamentals η_t is computed from the Debeers share price data for the period 24/03/86 to 14/05/01. The plot of η_t given in Figure 6.51.

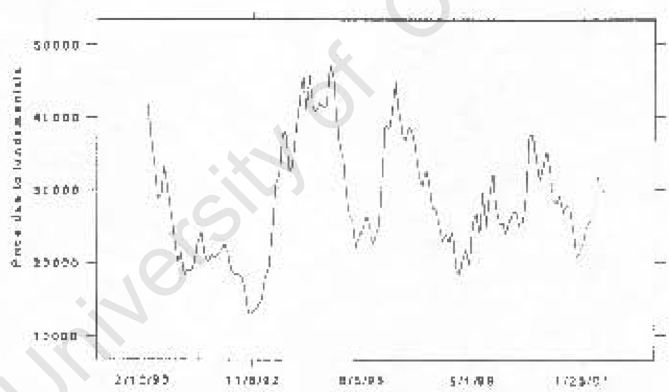

Figure 6-51: Price due to fundamentals: JSE Anglo Gold monthly price 3/31/90 - 8/31/01.

The plot indicates that change in prices due to fundamentals over the period was upward. The mean and variance of the price due to fundamentals were both changing and increasing. This could be explained by the changing economic fundamentals over time.

The change in market movement due to market sentiments ζ_t for the period 24/03/86 to

14/05/01 was computed as

$$\zeta_t = S_t^C - \eta_t$$

and plotted in Figure 6.52.

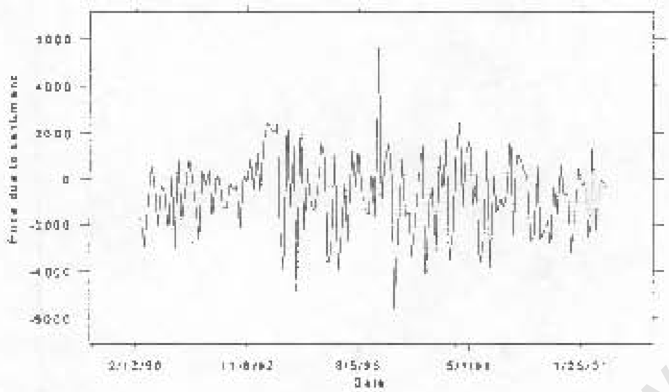

Figure 6-52: Price due to investor sentiment: JSE Anglo Gold monthly price 3/31/90 - 8/31/01.

The graph shows that price change due to sentiments was stationary about the mean with increasing variation. This implies that variations in sentiments was increasing with time. This can be explained by increased awareness by investors due to readily available market information brought about by technology advancement. Increased availability of multiple analyses and interpretation of market information induced increased variation is traders' sentiments. The tow spikes at the end of the plot suggest that at times market sentiments was quite big.

The ratio

$$\psi_t = \frac{\zeta_t}{\eta_t} \times 100$$

to compare market fundamentals and market sentiments is computed. The plot of ψ_t is given in Figure 6.53.

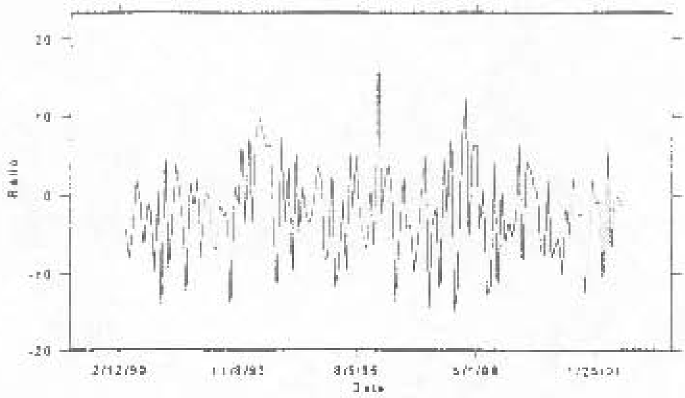

Figure 6-53:
Ratio of sentiment to fundamentals: JSE Anglo Gold monthly 3/31/90 - 8/31/01.

The ratios are small due to small values of ζ_t. However, the spikes show that sentiment was sometimes quite influential in the share price movement. The spike in the direction may be associated with the daily market crashes. Although ψ_t is small due to small values of ζ_t, it has an important effect on the share price movement in the market.

Correlation

The correlations of the log ratios of the monthly economic indicators: Producer Index (PROD), Money Supply (MS), Manufacturing Production (MP) and Consumer Price Index (CONS), with log ratio of the JSE Anglo Gold monthly closing price before decomposition (ANG) and after decomposition into fundamentals (ANGF) are computed and shown below. The correlation of the economic indicators with ANG and ANGF (Table 6.15) are very small and showing an almost none existence of linear relationship.

	ANG	*ANGF*	*PROD*	*MS*	*MP*	*CONS*
ANG	1.000	0.973	0.024	0.018	0.048	0.089
ANGF	0.973	1.000	0.047	0.037	0.090	0.101

Table 6.15: Correlation: fundamentals and sentiment of JSE Gold share price with economic indicators.

After the decomposition, the correlation between the Anglo Gold share price due to fundamentals with the economic indicators increased. Thus removing sentiment from the Anglo Gold share price improves the relationship between the share price of Anglo Gold due to fundamentals and the economic indicators.

6.5 Summary

This chapter has constructed the mood of investors relative to the record maximum and minimum, daily maximum and minimum, cumulative maximum and minimum, and daily closing values. A composite mood index has been derived from the three constructed mood of investors. This chapter has also decomposed the JSE Anglo Gold share price into fundamentals and sentiment.

The Mood Index

The mood index was applied to the JSE Overall Index, NYSE, Nasdaq, Dow Jones, DAX, Nikkei500, IBOVESPA and the JSE Gold data. The relationship between the mood indices of various markets was investigated. The Hurst phenomena of randomness, persistence and antipersistence of the mood indices were explored. The impact of the present mood of investors on the future of the markets has been computed. Information and knowledge about the mood indices of the markets were represented by possibility distributions. The relationship between subjective belief about the mood of investors events and their frequency to occur was computed for the markets. The fuzziness of the mood of investors were computed by approximating the areas under the possibility distributions of the Mood Index. Daily and monthly mood of the JSE Anglo Gold were computed. The monthly mood was used in an attempt to decompose the Anglo gold share price into fundamentals and sentiment.

Linear relationship

Correlation of mood indices between the indices showed very little linear relationship between the markets. Only a few mood indices were reasonably highly correlated. The mood of the JSE Overall Index was negatively correlated with mood of most international indices. This could be explained by the fact that a weakness in an emerging market is followed by investors shifting to stronger developed markets. Mood of the JSE Gold was also negatively correlated with some markets and had small positive correlation with the others and probably for the same reason as that given for the JSE Overall Index.

Persistence and randomness

The rescaled range analysis was used to compute the Hurst coefficient in order to investigate the characteristics of the daily mood of investors in different markets. The results showed that the mood in all the markets was persistent. That is the past mood of investors had an influence on the present or future mood i.e. memory exists in mood of investors. The impact of present mood of investors on the future was generally high for all the indices. The DAX, JSE and Nikkei500 showed lower impact of present mood on the future. The present mood of the Nasdaq had the highest impact on the future. The fractal dimensions revealed that mood of investors in different markets was not the same. Some markets experienced a more spiked mood than others. This may be linked to the different reasons behind mood of investors in each market.

Possibility distributions

The computed possibility distributions of the mood indices revealed that the markets considered here were characterised by the stable mood of investors. What is termed "stable" differed from market to market. The possibility grades of the mood values showed that, the subjective belief and the meaning of mood of investors was not the same in different markets. From the possibility distributions, it would be reasonable to predict stable mood of investors for all the markets in the near future. Stability differs between markets.

Mood of the JSE overall index was mainly stable and high as seen from the possibility distribution. It had very small possibility of the highest and lowest possible mood. The possibility distribution of the JSE Gold mood was stable and low. It had very high possibility of the lowest mood. It was impossible for the JSE Gold index to gain the highest possible mood.

The results of the possibility - probability consistency reveled that the relationship between subjective belief about mood in the markets was highly coherent with the frequency of mood events.

Decomposition

The histograms of the daily mood and the monthly mood of the JSE Anglo Gold share price almost looked similar. After the decomposition of the share price, the share price due to fundamentals showed an improved correlation with some economic fundamentals.

Chapter 7

Summary of findings and conclusions

7.1 Introduction

The purpose of this chapter, is to give the summary of findings and conclusion of the study. In this study, the fuzzy feature in the movement of the JSE Overall Index, and the mood of investors for various financial indices have been examined by means of empirical studies. The decomposition of a share price into fundamentals and sentiment has also been done empirically Fractal techniques have been used to determine the nature of the JSE Overall Index and the market mood indices for various markets. Fuzzy techniques have been used to describe fuzziness in the movement of the JSE Overall Index. They have also been used to described information and knowledge about the mood of investors in the market.

The index was divided into time periods to compare the fuzzy characteristics of the JSE Overall Index over time. In order to gain insight into the fuzzy feature of the JSE Overall Index, the index was partitioned into three fuzzy states: "low", "middle" and "high". The important aspects of the fuzziness of the index have been computed from the fuzzy states and presented. The frequency of the index in each fuzzy state has been computed to reveal the distribution of the index. Information and knowledge about the movement of the index have been presented by computing the possibility distributions of the fuzzy states. Meaning of the index values has been achieved. The measure of vagueness for the states of the JSE Overall Index has been documented by estimating the area under the possibility distributions of the fuzzy states. The possibility - probability consistency were computed in order to measured and present the relationship between subjective belief about the occurrence of the index events, and the frequency of occurrence of the events. The extent of certainty about the movement of the index has been determined by computing the necessity measures. This was done in order to investigate the deterministic aspects of the movement of the index and to reveal the stability of the JSE and the South Africa economy. In order to gain insight into the transition of the index between the fuzzy states, the fuzzy Markov transition matrix was computed. Short term prediction of the index using the fuzzy Markov chain model has been explored. An attempt to improve the predictions was carried out by fitting an AR(2) model to the JSE Overall Index, and applying the fuzzy techniques to describe the residuals of the AR(2) model.

In order to quantify the mood of investors, three types of the mood of investors were constructed by considering changes in market indices. The mood index to measure the overall mood of investors in the market was computed as a composite of the three constructed mood of investors. The mood index was applied to various financial indices. In order to examine the relationships of the mood of investors between different markets, correlations between the mood indices have been computed and presented. The nature of the daily and overall mood of investors for various financial indices have been investigated using the Hurst exponent to determine whether or not the mood of investors are independent processes. Fuzzy techniques have been utilized in order to describe and explain vagueness of the overall mood of investors of various markets. Knowledge and information about the market mood, and vagueness of the market mood of investors have been achieved by computing possibility distributions. The relationship between subjective belief about the events of the market mood of investors and their frequency of occurrence has been computed and presented.

Both economic fundamentals and sentiment contribute to what constitutes a share price. The monthly mood of investors for JSE Aglo Gold share price was computed in order to partition the share price into its fundamentals and sentiment. A correlation matrix was computed in order to investigate the linear relationship between the share price before decomposition, the share price due to fundamentals after the decomposition, and some economic indicators of the South African economy.

The main findings of the study are summarised below:

7.2 Findings

7.2.1 Characteristics of financial data (JSE overall index)

The preliminary results in chapter 4 of the daily closing JSE Overall Index data from 1985 to 2000 showed that the index has fat tails and a sharp peak near the mean. The Kolmogorov-Smirnov goodness of fit showed that the JSE Overall Index was not normally distributed.

From the rescaled range analysis (R/S), the JSE Overall Index is characterised as persistent Hurst process for periods of 100 days (approximately three months) with $H = 0.6277$. The tests for significance showed that this is different from an independent process. The R/S analysis was done on the AR(1) residuals of the JSE Overall Index. This implies that the index experiences true persistence or the Hurst phenomena long memory for periods of three months. The JSE Overall Index therefore does not exhibit persistence forever but for short periods of three months. During these periods, if the JSE Overall Index has been up (down) in the previous period, it is likely to be up (down) in the next period.

After the three moths of persistence, the JSE Overall Index exhibits characteristics of antipersistence with $H = 0.4931$, which is significantly different from the independent process. During these times of antipersistence, the index experiences short-range dependences.

7.2.2 Fuzzy analysis of the JSE overall index

The first objective of the thesis was to describe the fuzziness feature in the movement of the Johannesburg Security Exchange Overall Index using the fuzzy logic techniques. This was to enable reveal the subjective belief (vagueness) about the movement of the index and give insight into its meaning. Furthermore, prediction of the JSE Overall Index from the fuzzy Markov chains was to be explored. The objective was fulfilled as follows:

Fuzzy partition

To investigate temporal movements the index was divided into time periods. The study successfully partitioned the JSE Overall Index into its fuzzy states: "low", "middle", and "high" to facilitate investigations into the movement of the index. The partitioning of the index were carried out in each time period. The results of the partitioning revealed that the states of the index did not posses sharp boundaries. The fuzzy states were not necessarily the same size in the different time periods Tables 5.1, 5.2 and 5.3. This suggests that the movement of the index is perceived differently at different times. The forces behind the index also differ with time. The partitioning also showed that the states of the index intersect. Thus, the index at an instant implies more than one state. The index value is in a particular state to a certain degree.

Probability of the states

The frequency of the JSE Overall Index in each state was computed in order to determine the distribution of the index among the fuzzy states. The probability of the index in the middle state was higher than that of the low and the high states in all the time periods. The frequency of the index in the middle state was almost the same in all the time periods (1985 - 1987: 98%, 1988 - 1991: 99%, 1992 - 1995: 99%, 1996 - 2000: 99%, 1985 - 2000: 99%, 1985 - 2001: 97%). The frequency of the index in the "low" and "high" states had noticeable fluctuations over time. The probability of the index in the low state fluctuated between 21% and 25%, and the probability of the index in the "high" changed between 27% and 35%. This shows that the movement of the index is almost predictable to be stable over time. The index is less frequently in the "low" and "high" states. The frequency of falling and rising of the index changes with time.

Knowledge and information about the index

Knowledge and information about the movement of the index was achieved by computing the possibility distributions of the fuzzy states in each time period (section 5.2.3). The computed possibility distributions represent information conveyed by propositions about the movement of the index about the movement of the index such as: "the index is low", "the index is stable" or "the index is high". The possibility grades expressing to what extent the index values are compatible with a particular fuzzy state have been revealed and documented. Such grades reveal the subjective meaning of the index values. Thus subjective belief about the index and meaning of the index have been achieved.

The index values in a particular state had different degrees of compatibility to that state.

Thus, despite being in the same state, the index values are perceived differently and do not have the same meaning. The computations have also revealed that information and knowledge extracted from the movement of the index, show that the meaning of the index values changes with the time periods. The differences in the meaning of the index values from the computations, may affirm the diversity in the interpretation of the market by the stake holders.

The meaning of the extreme values of the index in the "low" and "high" states were achieved. The results shoved the extreme values were fully compatible (possibility grade 1) with the "low" or the "high" states. Some of the smallest extreme values in the "low" state were observed in the time periods 1985 - 1987, 1988 - 1991, and 1996 - 2000. Some of the highest extreme values were observed in time periods 1985 - 1987, and 1988 - 1991. The extreme values differed between the time periods. For example the extreme values, in the different time periods and were: time period 1985 - 1987, "low" state: {-0.12, -0.08, -0.07, -0.6, -0.04} and "high" state: {0.04, 0.05, 0.06}; time period 1988 - 1991, "low" state: {-0.11, -0.05, -0.04} and "high" state: {0.06}; time period 1992 - 1995, "low" state: {-0.03} and "high" state: {0.03, 0.04}; and in the time period 1996 - 2000, low state: {-0.11, -0.10, -0.08, -0.05, -0.04} and "high" state: {0.04, 0.05}, but had the same meaning (possibility grade 1). This suggest that what is perceived as an extreme value for the JSE Overall Index will change with time.

Fuzziness of the states in the movement of the index

In order to explore the vagueness in the movement of the JSE Overall Index, the fuzziness was measured for each state. The measure of fuzziness was computed by approximating the area under the possibility distributions of the states. The measures revealed that the "low" state had the highest fuzziness followed by the "high" state and then the "middle" state in all the time periods except the 1992 - 1995 time period as presented in Table 5.6. The values show that the movement of the JSE Overall Index index in the "low" state is obscure and may not be understood clearly compared with the other states. Thus, when the index is in the low state, the feeling of doubt and anxiety may be experienced by the investors. The movement of the index in the "middle" state is almost clearly understood. The results also reveal that in the period1992 to 1995, the movement of the index in the "low" state was understood better than it was in the "middle" and "high" states. The fuzziness in the movement of the JSE Overall

Index in each state changes with time. Therefore, the of fuzziness in the movement of the JSE Overall Index is not a constant feature. This should have an impact on the decisions which are based on the movement of the JSE Overall Index over time.

Possibility measure

The possibility measures of the states in the movement of the JSE Overall Index the were computed. The results showed that each state had the possibility measure of 1. This implied that the index had potential to be in any one of the state.

Coherence between subjective belief and frequency of occurrence of events

Documentation of the relationship between the subjective belief about the occurrence of events of the JSE Overall Index, and the frequency of occurrence was achieved by computing the possibility - probability consistency of each fuzzy state. The study has revealed that the association between the subjective belief about events and their frequency, differs according to the states of the index and with time. For the "low" state, the time periods 19985 - 1987, 1988 - 1991, 1992 - 1995, 1996 - 2000, 1985 - 2000 and 1985 2001 had the possibility - probability consistency 0.12, 0.10, 0.10, 0.12, 0.10, 0.11 respectively. For the "middle" state, the possibility - probability consistency for the time periods were 0.77, 0.80, 0.80, 0.77, 0.80, 0.79 respectively, while for the "high" state the values were 0.11, 0.10, 0.10, 0.12, 0.10, 0.10 respectively. There was high degree of relationship between the belief about events in the middle state and the frequency of their occurrence. Thus what the market stake holders think about the events of the middle state, is approximately in agreement with the frequency of their occurrence. Thus, a high degree of belief about the events of the middle state, tended to coincide with a high degree of frequency. The association between belief and frequency of events in the "low" and the "high" states was small for the JSE Overall Index. In these states, a high degree of belief about events did not comply with a high degree of frequency, nor did a low degree of frequency comply with a low degree of belief about the occurrence of the JSE Overall Index events. This suggests that the investors and other market stake holders are almost able to predict and deal with the events of the "middle" state easily, while events of the "low" and the "high" states are difficult to predict and may be a source of difficulty in the market. The study shows that the degree of ease with which the investors and others deal with the events of the "middle" state changes with time. Similarly, the difficulty with which the events of the "low" and the "high" states are dealt with, changes with time.

Certainty about the movement of the index

In order to investigate the deterministic aspects of the movement of the index, the necessity measures were computed. The necessity measures reveal that, the movement of the JSE Overall Index had some deterministic aspect, suggesting that the movement of the index is not purely random. The necessity values of the "middle" state for the time periods 1985 - 1987, 1988 - 1991, 1992 - 1995, 1996 - 2000, 1985 - 2000 and 1985 - 2001 were 0.92, 0.95, 0.95, 0.73,0.96 and 0.99 respectively. These values suggest that there was high necessity for the index to be in the middle state in all the time periods. The high values of necessity show that the movement of the JSE Overall Index in the "middle" state was to a very large extent deterministic and therefore more knowable than in the low and high states. This also implies that the JSE Overall Index from 1985 to 2001 was relatively stable. This is linked to the stability in the performance of the companies listed on the JSE and especially that of the major contributors in the same period. The stability of the JSE Overall Index also reveals relative stability in the South African economy in the same period. The necessity measures of the middle states changed with time, indicating that the movement of the index was more knowable in some periods and less knowable in others. Thus the stability of the JSE Overall Index and that of the South African economy are not constant. They fluctuate with time. The times when the necessity measures were low could be suggesting instability in the movement of the index as well as that of the economy and the listed companies on the JSE.

Movement of the index between fuzzy states

To describe movement of the index between the fuzzy states, the fuzzy Markov chain transition matrices were computed. The high probability of the index moving from the "low", "middle" and the "high" states to the "middle" state (section 5.28), reveals the tendency for the index to always move to the "middle" state and rest there. The small transition probabilities of the index from the "middle" to the "low" or "high" states shows that the index rarely left the "middle" state to the other states. The fuzzy transition probabilities of the index between the states varied with time. There were time periods 1985 - 1987 and 1996 - 2000 when the index did not for example, frequent the "middle" state as much as it did in the other time periods. In these time period the frequency of the index visiting the "low" or "high" states increased. This may suggest and confirm that the forces behind the movement of the index change with time.

The movement of the index in each time period was contrasted with the movement in the overall period of 27/3/85 - 26/07/00. The results show that no major changes were experienced

JSE Overall Index prediction

An attempt to predict the index using the fuzzy Markov chain model showed that the model over predicted the index. To improve prediction the AR(2) model for the index was combined with fuzzy Markov chains model of its residuals. The model face inaccuracies in the reading off small values of the predicted residuals from the possibility distribution graphs. The correctly read off predicted residuals would enhance significantly the prediction of the index. The model also showed over prediction.

7.3 Mood indices

The second objective of the study was to create a mood index to measure the mood of investors in the stock market. The third objective of the thesis was to decompose a share price into fundamentals and sentiment. These objective have been achieved.

The study has successfully created a composite mood index to measure the mood of investors of market indices. The mood index was based on the record maximum and minimum, cumulative maximum and minimum, the daily maximum and minimum, and the daily closing market index returns. The mood index was applied to various market indices data (NYSE, Nasdaq, Dow Jones, IBOVESPA, Nikkei500, JSE Overall Index, DAX, JSE Gold). The findings of the analysis of the mood indices are presented as follows:

Association between the mood of investors for different market indices

In order to investigate the relationships between the mood of investors for different market indices, the correlations between the mood of investors were computed and presented. The correlations between the mood of investors ranged from −0.329 to 0.664. The correlation between the Nasdaq mood of investors and the Nikkei500 mood of investors was high at 0.664. The JSE Gold and the NYSE had the least negative correlation of −0.329. The emerging market like the JSE mood of investors exhibited very small but negative correlation with the mood of investors for most international market indices (−0.008 with the NYSE, −0.147 with the Nasdaq, −0.062 with the DAX, −0.163 with the Nikkei500 and −0.006 with the IBOVESPA). In general, correlations between the mood of investors were very small. The results show that there is very little linear relationship between the mood of investors between the different market indices. The poor correlations between the mood of investors may imply that the relationship between the mood of investors for the different markets indices, is not simple. This may also suggests a nonlinear relationship between the mood of investors for the different market indices.

Memory for the mood of investors The mood of investors for the different market indices were investigated for the Hurst phenomenon of antipersistence, independence and persistence, using the R/S analysis. The mood of investors were found to be characterised by the Hurst phenomena of persistence or long memory. The Hurst exponent values for the various mood indices: IBOVESPA (0.8413), Nikkei500 (0.6413), Nasdaq (0.8257), JSE Overall Index (0.7965), Dow Jones (0.8836), NYSE (0.8150), DAX (0.7061) and JSE Gold (0.8063) were significant. The results were from the AR(1) residuals of the mood indices. Thus the mood of investors are really long memory processes. The Hurst exponent values (except for the mood of the JSE Overall Index with 4000 observations) were computed from relatively small samples and thus the results may be biased. However the results still reveal that the mood of investors in the markets are persistent or trend-reinforcing. Thus that if the mood of investors has been stable (low or high) in the last period, then the chances are that it will continue to be stable (low or high) in the next period. The result of persistence implies that timing of the mood of investors is possible.

Impact of the present mood of investors on the future

The impact of the mood of investors on the future of the markets was documented by computing the correlation coefficient C, from the Hurst exponent values. The results: IBOVESPA (0.61), Nikkei500 (0.22), Nasdaq (0.57), JSE Overall Index (0.51), Dow Jones (0.70), NYSE (0.55), DAX (0.33) and JSE Gold (0.53), suggest a reasonably high impact of present mood of investors on the future for many market indices. The small values of C, 0.22 for the Nikkei500 mood and 0.33 for the DAX mood show that for these indices, the impact of present mood of investors on the future was not strong.

Structure of the mood of investors

To understand the physical appearance of the mood of investors processes, the fractal dimensions were computed. The fractal dimensions: IBOVESPA (1.19), Nikkei500 (1.56), Nasdaq (1.21), JSE Overall Index (1.26), Dow Jones (1.13), NYSE (1.23), DAX (1.42) and JSE Gold (1.24), reveal that the mood of investors and hence the market indices are structured differently. The mood of investors for the Nikkei500 and the DAX are much different from the others. The differences in the fractal dimensions may imply that investors in the different markets react differently to forces behind the markets. The fractal dimensions results also indicate that different scales of modelling and analysis of the mood of investors in different markets would be appropriate.

Information and knowledge about the mood of investors

The representation of information and knowledge about the mood of investors was achieved by computing the possibility distributions of the mood indices. The possibility distributions reveal the meaning of the mood index values through the possibility grades. The mood of investors are found to be stable for all the market indices. The mood index values with possibility grade 1 are: IBOVESPA (0.6), Nikkei500 (0.4), Nasdaq (0.3), JSE Overall Index (0.5), Dow Jones (0.6), NYSE (0.5), DAX (0.3) and JSE Gold (0.4) and are all in the "stable" state. These results show that stability of the mood of investors differs with the market indices. The lowest mood of investors is depicted by the mood index value 0. The possibility grades of the mood index value 0, ranged from 0.02 to 0.9 between the market indices. The Nasdaq mood index value 0, had a possibility grade 0.90, the Nikkei500 mood index value 0, had a possibility grade 0.83. This shows that the Nasdaq and the Nikkei500 have a high possibilities of experiencing very low mood of investors. The highest mood value 1, had possibility grades ranging from 0 to 0.15 across the market indices. This reveals the fact that most market indices do not experience very high mood of investors. The low mood of investors is much more possible for all the market indices. The meaning of the same mood index values differed between the market indices. For example, the mood value 0.1 for the DAX had possibility grade 0.62, for the Nikkei500 the same value had a possibility grade 0.83, for the JSE it had possibility grade 0.17 etc. This suggests that, the mood of investors is unique to the market.

Subjective belief and frequency of the mood of investors

In order to document coherence between the subjective belief about the mood of investors and the occurrence of the mood of investors, the possibility - probability consistency were computed for each mood index and presented in Table 6.14. The coherence ranged between 81% and 87% for the different mood indices. The results reveal a connection between the subjective belief about the mood of investors and the occurrence of the mood of investors. The high coherence implies that, the feeling of the stake holders in the markets is very much in line with the events of the markets. And indeed what happens in the markets highly affects the feeling of the stake holders such as the investors etc.

Fuzziness of the mood of investors

The fuzziness of the mood of investors have been computed to assess the vagueness of the mood of investors in various markets. The fuzziness can in a way help to understand how obscure the mood of investors can be. The results: IBOVESPA (0.66), Nikkei500 (0.80), Nasdaq (0.76), JSE Overall Index (0.64), Dow Jones (0.70), NYSE (0.66), DAX (0.56) and JSE Gold (0.74), reveal that the degree with which the mood of investors is obscure, differs with the markets. The implication is that the markets react to different forces differently.

The analysis of the mood of investors, through the computation of the correlations, the Hurst exponents, the fractal dimensions, information and knowledge, fuzziness, and the possibility - probability consistency, shows that the mood of investors is unique to a particular market index.

Thus the analysis of the mood of investors would require taking account of the special features unique to the particular market. For example, the investors and their culture, the economic structure and strength, technology, etc.

7.3.1 Decomposition of share price

The study made an attempt to decompose the share price of the JSE Anglo Gold into fundamentals and sentiment using the monthly mood of investors. The raw data of the JSE Anglo Gold share price was correlated with the South African economic fundamentals such as the Producer Index, Money Supply, Manufacturing Production and the Consumer Price Index. The correlations were as follows: Producer Index (0.024), Money Supply (0.018), Manufacturing Production (0.048) and the Consumer Price Index (0.089). The correlation of the JSE Anglo Gold share price due to fundamentals (after removing sentiment) with the economic indicators improved to: Producer Index (0.047), Money Supply (0.037), Manufacturing Production (0.090) and the Consumer Price Index (0.0101). The results show that after removing sentiment from the share price, the association between share price due to fundamentals with the economic indicators became stronger. This may suggest that the mood of investors can be used to decompose share price into fundamentals and sentiment. It is important to note though that the association between the share price before and after removing sentiment with the economic fundamentals was not strong, which may be suggesting a nonlinear relationship.

APPENDIX

.1 Maple V Code for computing the fuzzy Markov chain transition matrix

The following procedure calculates the membership grades of the states "low", "middle", "high" states, and the fuzzy Markov chain Model of the JSE Overall Index for the period 3rd March 1985 to 26th July, 2000.

>QS := readdata('c:\\My Documents\\ALL.JSE1.txt'. 1): # Reads data to QS from the file "ALL.JSE1" placed in My Documents .

> a := array(1..1,1..3825,[QS]): # Creates an array with one row and 3825 columns (number of columns = size of QS).

> QSC := readdata('c:\\My Documents\\ALL.JSE0.txt', 1):# Reads data of zeros from file "ALL.JSE0", same size as "ALL.JSE1".

> b1 := array(1..1,1..3825,[QSC]): # Creates an array of zeros with one row and 3825 columns to be used in the computation of membership grades for the "low" state.

```
> Q1:=proc(X::array,Y::array) # "low" state membership grades
> local i,x,s;
> x:=X;
> s:=Y;
> for i from 1 to 3825 do
> if x[1,i] <= -.04 then
> s[1,i]:=1;
> elif x[1,i] = -.03 then
> s[1,i] := .9;
> elif x[1,i] = -.02 then
> s[1,i] := .8;
> elif x[1,i] = -.01 then
> s[1,i] := .3;
> else
> s[1,i] := 0;
> fi;
> od;
> RETURN(s);
> end;
```

> E1:=Q1(a,b1);# Computes and transfers membership grades for the "low" state to E1.

> b2:= array(1..1,1..3825,[QSC]): # Creates an array of zeros with one row and 3825 columns to be used in the computation of membership grades for the "middle" state.

```
> Q2:=proc(X::array,Y::array)# "middle" state membership grades
> local i,x,s;
> x:=X;
> s:=Y;
```

> for i from 1 to 3825 do

> if x[1,i] <= -0.04 then

> s[1,i]:=0;

> elif x[1,i] = -.03 then

> s[1,i] := .1;

> elif x[1,i] = -.02 then

> s[1,i] := .2;

> elif x[1,i] = -.01 then

> s[1,i] := .7;

> elif x[1,i] = 0 then

> s[1,i] := 1;

> elif x[1,i] = .01 then

> s[1,i] := .8;

> elif x[1,i] = .02 then

> s[1,i] := .3;

> elif x[1,i] = .03 then

> s[1,i] := .1;

> else

> s[1,i] := 0;

> fi;

> od;

> RETURN(s);

> end;

> E2:=Q2(a,b2); # Computes and transfers membership grades for the "middle" state to E2.

> b3:= array(1..1,1..3825,[QSC]): # Creates an array of zeros with one row and 3825 columns to be used in the computation of membership grades for the "high" state.

> Q3:=proc(X::array,Y::array) # "high" state membership grades

> local i, x, s;

> x := X;

```
> s := Y:
> for i from 1 to 3825 do
> if x[1,i] <= 0.0 then
> s[1,i] := 0;
> elif x[1,i] = .01 then
> s[1,i] := .2;
> elif x[1,i] = .02 then
> s[1,i] := .7;
> elif x[1,i] = .03 then
> s[1,i] := .9;
> else
> s[1,i] := 1;
> fi;
> od;
> RETURN(s);
> end;
> E3:=Q3(a,b3); # Computes and transfers membership grades for the "high" state to E3.
>
> V:=array([[E1],[E2],[E3]]); # Creates an array of the "low" , "middle" and "high" states.
> # Q5, computes the sum of the membership grades in each state.
> Q5 := proc(L::array)
> local j, k, X, N;
> X := L;
> N := array(1..1,1..3,[]);
> for j from 1 to 3 do
> N[1,j] := add(X[j,1][1,k], k = 1..3824);
> od;
> RETURN(N);
> end;
```

```
> N1 := Q5(V); # The sums of membership grades in each state are computed and put in N1.
> print(N1); # Prints the sums of each state.
> # Q6, Computes the fuzzy frequency and the initial probabilities of the states.
>
> Q6 := proc(P:: array)
> local j, F, Y;
> Y := P:
> F := array(1..1,1..3,[]);
> for j from 1 to 3 do
> F[1,j] := Y[1,j]/3824;
> od;
> RETURN(F);
> end;
> F1 := Q6(N1); # The initial probabilities computed and put in F1.
> print(F1); # Prints the initial probabilities.
> # MAT, Computes the fuzzy transition probability matrix.
> MAT := proc(M::array)
> local i, j, k, D, B;
> D := M:
> B := array(1..3,1..3,[]);
> for i from 1 to 3 do
> for j from 1 to 3 do
> B[i,j] := add(D[i,1][1,k]*D[j,1][1,k+1], k = 1..3824 )/N1[1,i];
> od;
> od;
> RETURN(B);
> end;
> G := MAT(V); # Computes the fuzzy transition matrix from the array V of the states and put it in G.
```

```
> print(G); # Prints the computed matrix G.
> # Q7 checks whether summation of rows of the matrix G adds to 1.
> Q7 := proc(K:: array)
> local i, j, T, S;
> T := K;
> S := array(1..3,1..1,[]);
> for i from 1 to 3 do
> S[i,1] := add(T[i,j], j = 1..3);
> od;
> RETURN(S);
> end;
> S1 := Q7(G); # The summations are computed and put in S1.
> print(S1); # Prints S1
```

.2 Graphs of functions of the mood of investors used in the construction of the mood indices for the different market indices

The following are plots for the mood of investors relative to record maximum and record minimum for the various market indices:

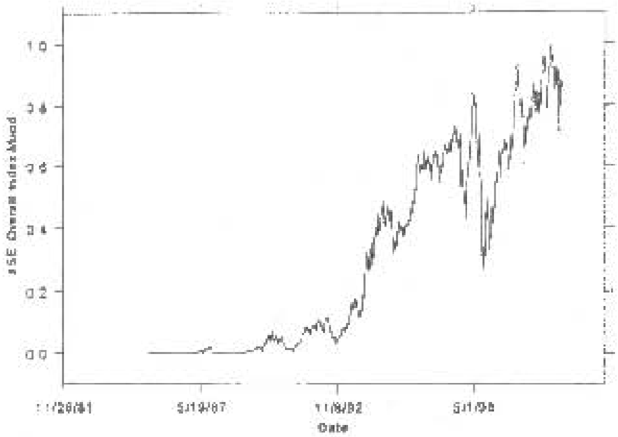

Figure -1: Mood of investors relative to record maximun and minimum: JSE Overall Index.

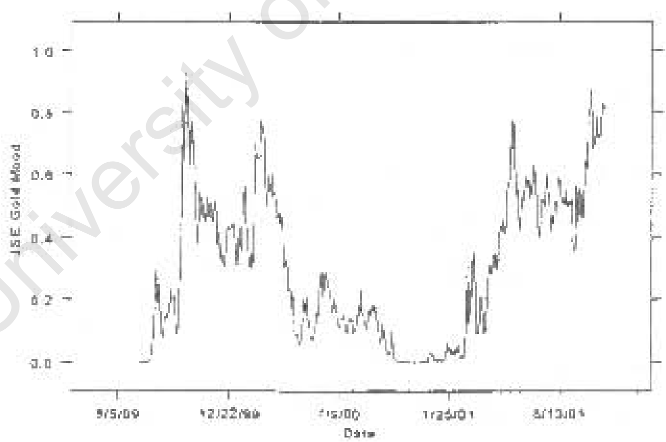

Figure -2: Mood of investors relative to record maximun and minimum: JSE Gold Index

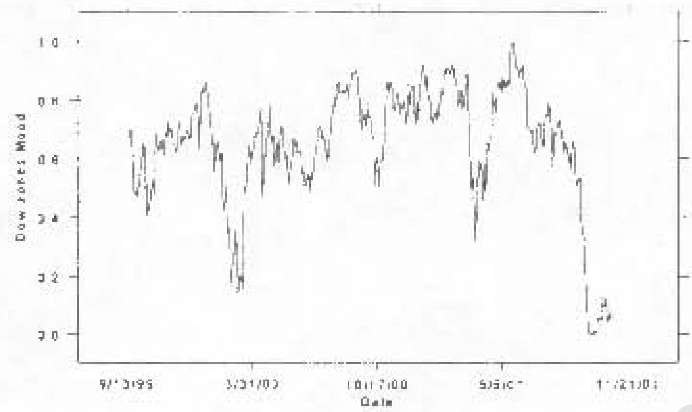

Figure -5: Mood of investors relative to record maximum and minimum: Dow Jones.

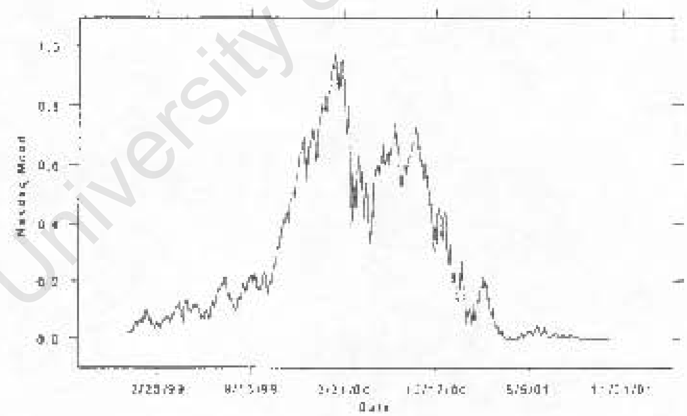

Figure -6: Mood of investors relative to record maximum and minimum: Nasdaq.

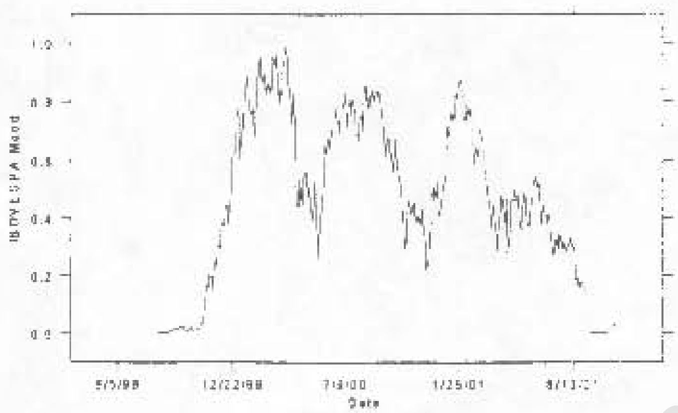

Figure -7: Mood of investors relative to record maximun and minimum: IBOVESPA.

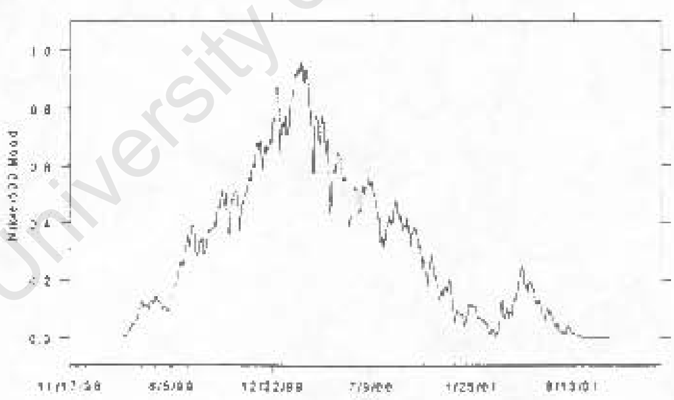

Figure -8: Mood of investors relative to record maximun and minimum: Nikkei500.

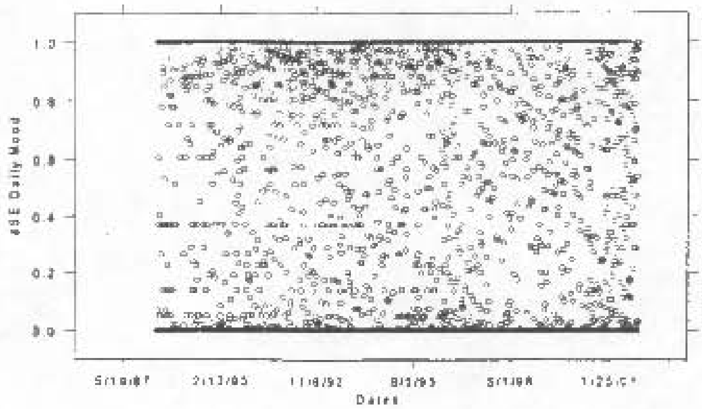

Figure -9: Plot of the daily mood of investors for the JSE Overal Index

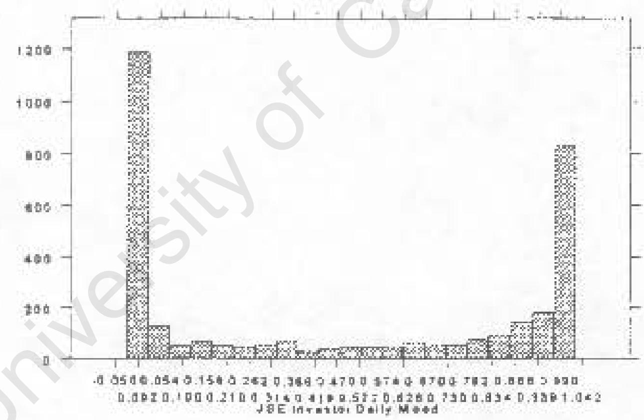

Figure -10: Histogram of the daily mood of investors for the JSE Overall Index.

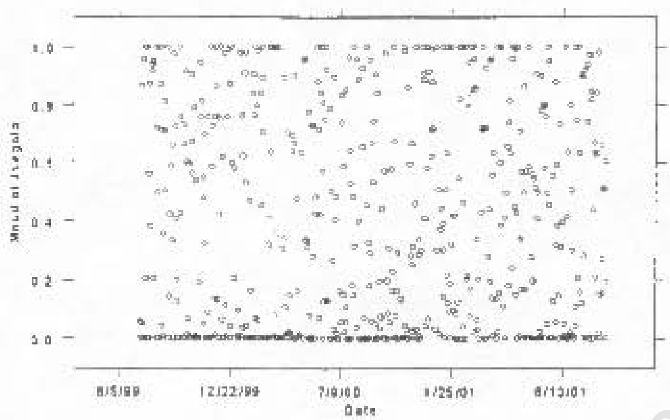

Figure -11: Plot of the daily mood of investors for the JSE Gold Index

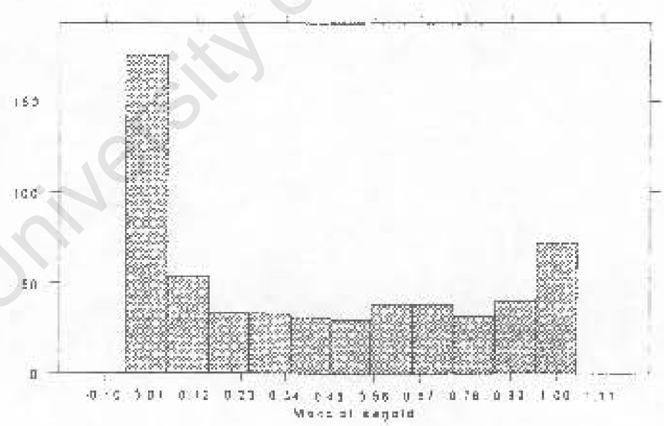

Figure -12: Histogram of the daily mood of investors for the JSE Gold Index

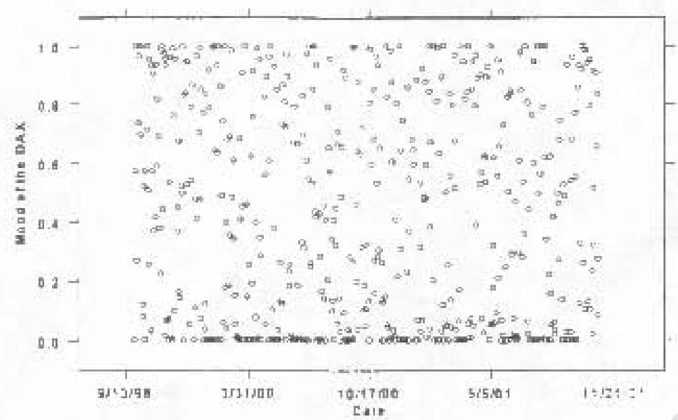

Figure 13: Plot of the daily mood of investors for the DAX.

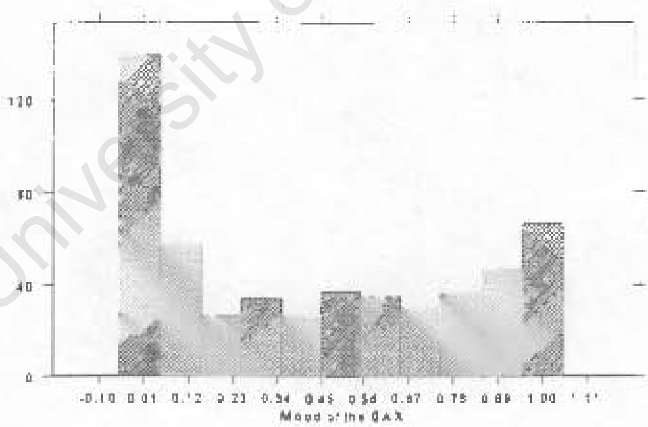

Figure 14: Histogram of the daily mood of investors for the DAX.

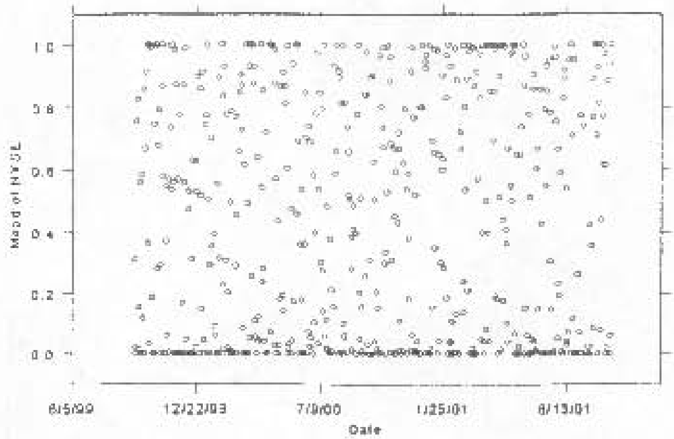

Figure -15: Plot of the daily mood of investors for the NYSE.

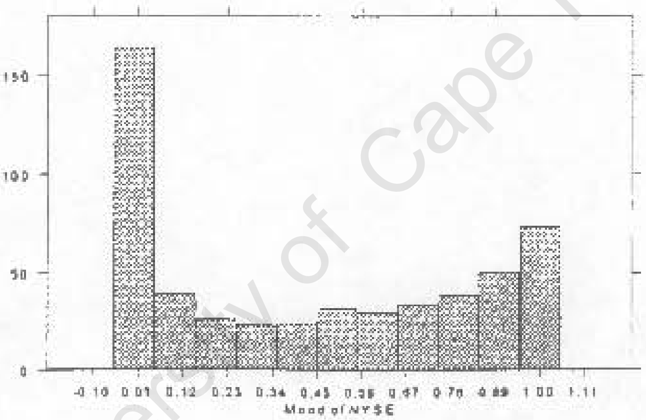

Figure -16: Histogram of the daily mood of investors for the NYSE.

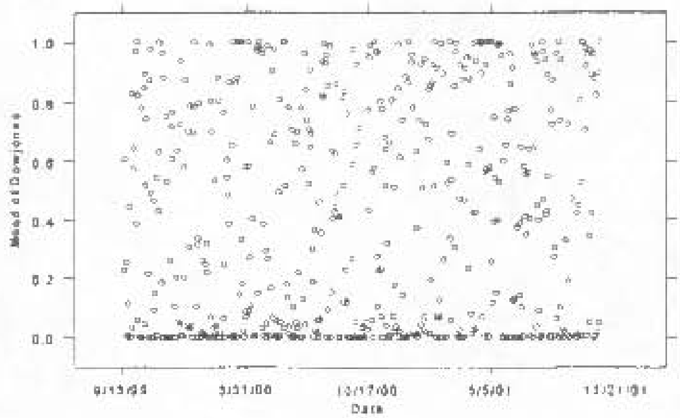

Figure 4.17: Plot of the daily mood of investors for the Dow Jones.

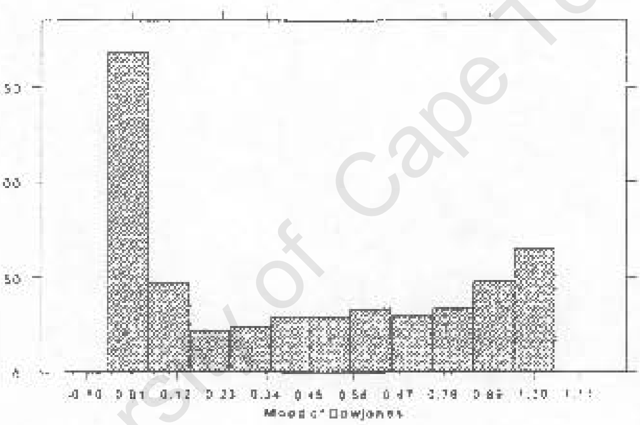

Figure 4.18: Histogram of the daily mood of investors for the Dow Jones.

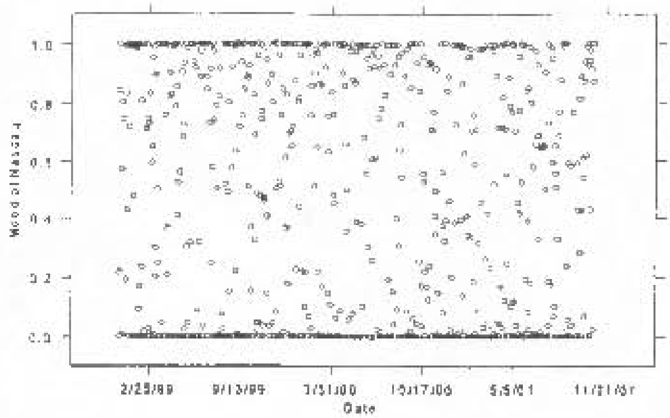

Figure -19: Plot of the daily mood of investors for the Nasdaq.

Figure -20: Histogram of the daily mood of investors for the Nasdaq.

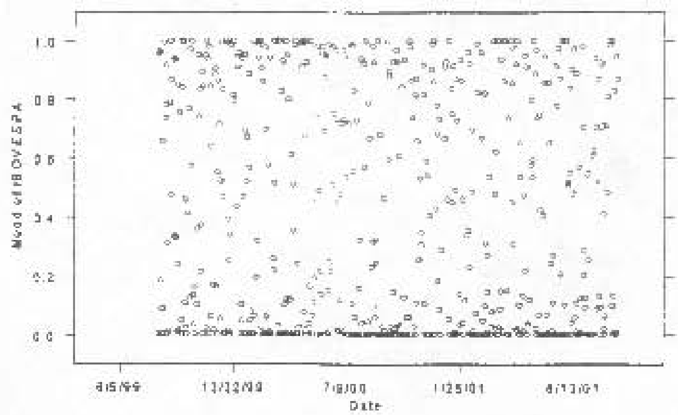

Figure -21: Plot of the daily mood of investors for the IBOVESPA.

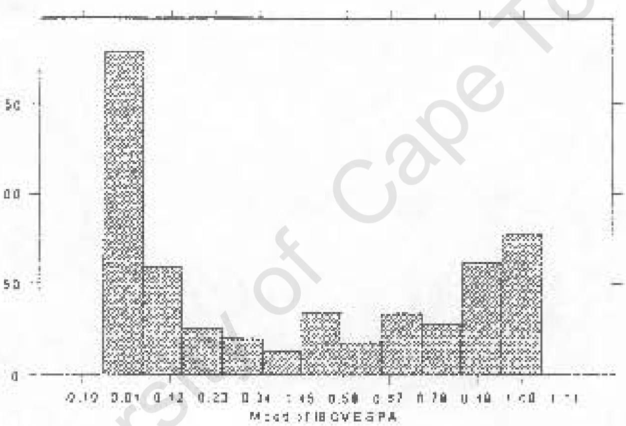

Figure -22: Histogram of the daily mood of investors for the IBOVESPA.

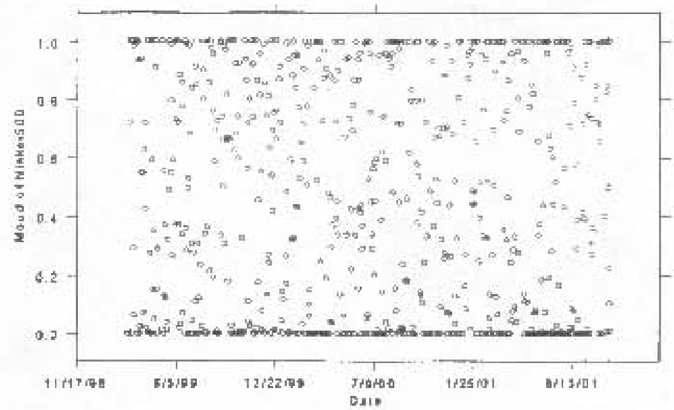

Figure -23: Plot of the daily mood of investors for the Nikkei500.

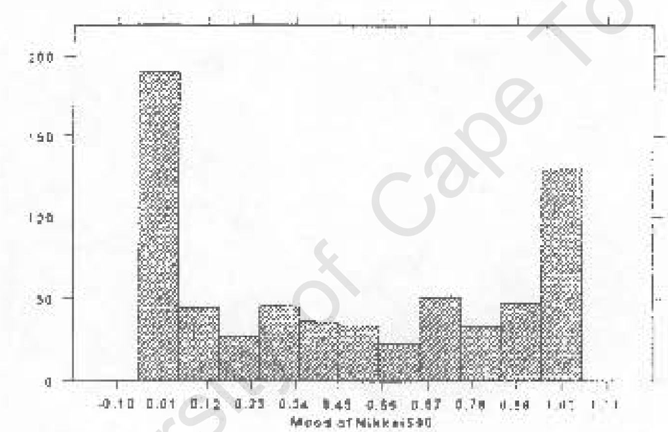

Figure -24: Histogram of the daily mood of investors for the Nikkei500.

The following are the plots of the mood of investors relative to cumulative maximum and minimum for various market indices.

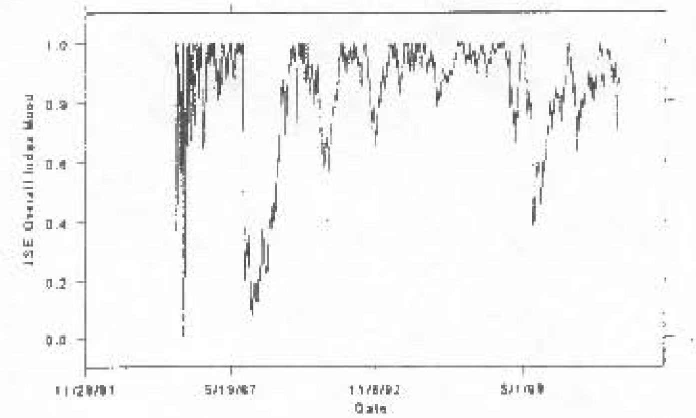

Figure -25:
The mood of investors relative to the cumulative maximum and minimum: JSE Overall Index.

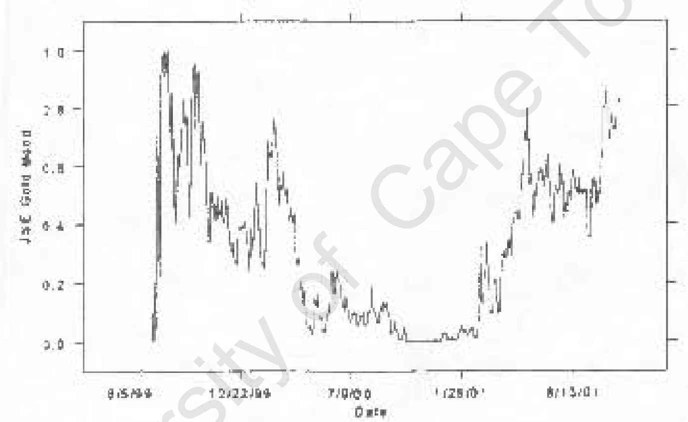

Figure -26:
The mood of investors relative to the cumulative maximum and minimum: JSE Gold Index.

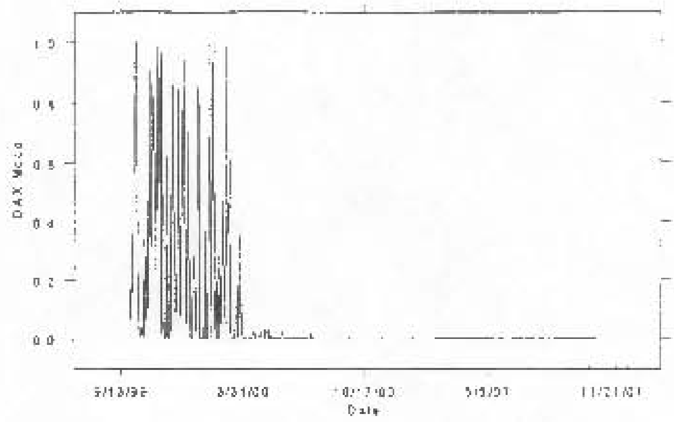

Figure -27: The mood of investors relative to the cumulative maximum and minimum: DAX.

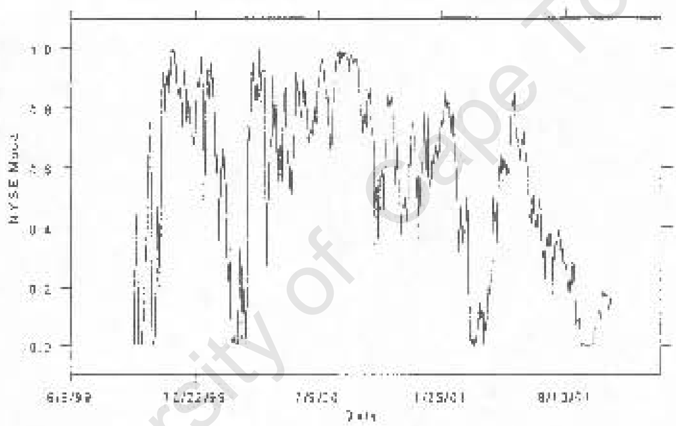

Figure -28: The mood of investors relative to the cumulative maximum and minimum: NYSE.

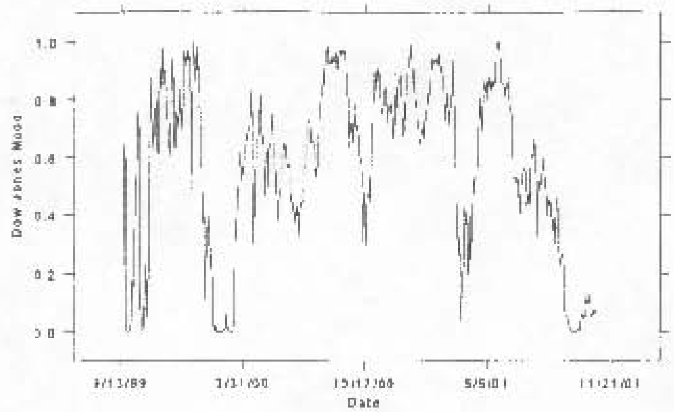

Figure -29:
The mood of investors relative to the cumulative maximum and minimum: Dow Jones.

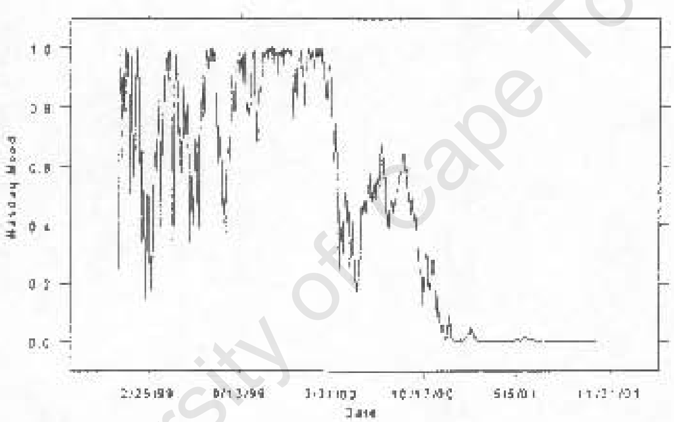

Figure -30:
The mood of investors relative to the cumulative maximum and minimum: Nasdaq.

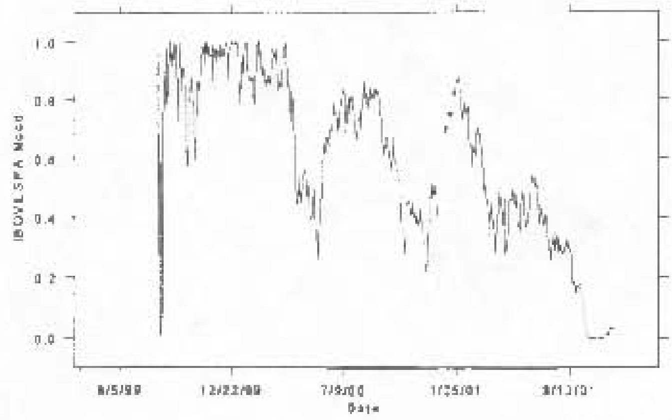

Figure -31:
The mood of investors relative to the cumulative maximum and minimum: IBOVESPA.

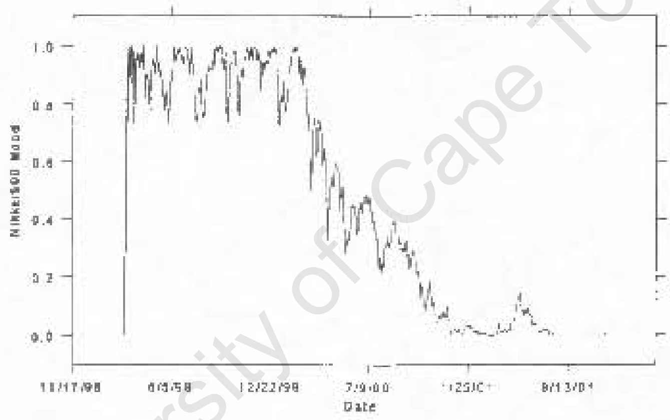

Figure -32:
The mood of investors relative to the cumulative maximum and minimum: Nikkei500.

.3 Tables of values used in the R/S analysis of the Mood Indices

The following tables give values used in the R/S analysis of the mood indices for various market indices.

The following table gives the values used in the R/S analysis of the Mood Index for the JSE Overall Index.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(R/S)
16	1.20	0.50	0.61	0.79	1.01
20	1.30	0.57	0.66	0.82	1.03
25	1.40	0.61	0.72	0.82	1.05
32	1.51	0.67	0.78	0.82	1.07
40	1.60	0.73	0.84	0.85	1.09
50	1.70	0.79	0.89	0.88	1.10
80	1.90	0.94	1.01	0.97	1.13
100	2.00	1.01	1.06	1.02	1.15
125	2.10	1.12	1.11	1.18	1.16
160	2.20	1.18	1.17	1.19	1.17
200	2.30	1.30	1.22	1.40	1.18
250	2.40	1.31	1.27	1.28	1.18
400	2.60	1.51	1.38	1.60	1.20
500	2.70	1.60	1.43	1.78	1.20
800	2.90	1.88	1.54	2.69	1.21
1000	3.00	1.93	1.59	2.70	1.22
2000	3.30	2.11	1.74	2.87	1.23

The following table gives the values used in the R/S analysis of the Mood Index for the Nasdaq.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(/R/S)
10	1.00	0.33	0.48	0.68	0.96
14	1.15	0.39	0.57	0.66	0.99
20	1.30	0.48	0.66	0.67	1.03
25	1.40	0.50	0.72	0.64	1.05
28	1.45	0.54	0.75	0.66	1.06
35	1.54	0.56	0.81	0.62	1.08
50	1.70	0.69	0.89	0.70	1.10
70	1.85	0.81	0.97	0.77	1.13
100	2.00	0.91	1.06	0.81	1.15
140	2.15	1.08	1.14	1.02	1.16
175	2.24	1.24	1.19	1.31	1.17
350	2.54	1.69	1.35	2.64	1.19

The following table gives the values used in the R/S analysis of the Mood Index for the Dow Jones.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(R/S)
10	1.00	0.38	0.48	0.76	0.96
20	1.30	0.56	0.66	0.81	1.03
25	1.40	0.62	0.72	0.83	1.05
50	1.70	0.87	0.89	1.05	1.10
100	2.00	1.21	1.06	1.61	1.15
125	2.10	1.32	1.11	1.89	1.16
250	2.40	1.56	1.27	2.28	1.18

The following table gives the values used in the R/S analysis of the Mood Index for the Nikkei500.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(R/S)
10	1.00	0.36	0.48	0.73	0.96
16	1.20	0.44	0.61	0.68	1.01
20	1.30	0.45	0.66	0.63	1.03
32	1.51	0.52	0.78	0.59	1.07
40	1.60	0.55	0.84	0.57	1.09
64	1.81	0.68	0.95	0.59	1.12
80	1.90	0.70	1.01	0.56	1.13
128	2.11	0.85	1.12	0.63	1.16
160	2.20	1.08	1.17	0.94	1.17
320	2.51	1.39	1.33	1.38	1.19

The following table gives the values used in the R/S analysis of the Mood Index for the NYSE.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(R/S)
10	1.00	0.39	0.48	0.78	0.96
20	1.30	0.55	0.66	0.79	1.03
25	1.40	0.60	0.72	0.80	1.05
50	1.70	0.84	0.89	0.99	1.10
100	2.00	1.14	1.06	1.39	1.15
125	2.10	1.26	1.11	1.62	1.16
250	2.40	1.48	1.27	1.89	1.18

The following table gives the values used in the R/S analysis of the Mood Index for the DAX.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(R/S)
10	1.00	0.44	0.48	0.87	0.96
20	1.30	0.59	0.66	0.87	1.03
25	1.40	0.65	0.72	0.89	1.05
50	1.70	0.85	0.89	1.00	1.10
100	2.00	1.01	1.06	1.02	1.15
125	2.10	1.12	1.11	1.19	1.16
250	2.40	1.47	1.27	1.85	1.18

The following table gives the values used in the R/S analysis of the Mood Index for the JSE Gold Index.

n	Log(n)	Log(R/S)	Log(E(R/S))	V Statistic	E(R/S)
10	1.00	0.38	0.48	0.75	0.96
12	1.08	0.42	0.53	0.75	0.98
15	1.18	0.45	0.59	0.73	1.00
18	1.26	0.50	0.64	0.75	1.02
20	1.30	0.52	0.66	0.73	1.03
27	1.43	0.59	0.74	0.74	1.06
30	1.48	0.62	0.77	0.77	1.07
36	1.56	0.67	0.81	0.77	1.08
45	1.65	0.72	0.87	0.78	1.10
54	1.73	0.82	0.91	0.91	1.11
60	1.78	0.84	0.94	0.90	1.12
90	1.95	1.02	1.03	1.11	1.14
108	2.03	1.12	1.08	1.26	1.15
135	2.13	1.07	1.13	1.01	1.16
180	2.26	1.27	1.20	1.38	1.17
270	2.43	1.68	1.29	2.93	1.19

The following table gives the values used in the R/S analysis of the Mood Index for the IBOVESPA.

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