

Employee Stock Option Valuation with Earnings-Based Vesting Condition

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A dissertation submitted to the Faculty of Commerce, University of Cape Town, in partial fulfilment of the requirements for the degree of Master of Philosophy.

June 10, 2018

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy to the University of Cape Town. It has not been submitted before for any degree or examination to any other university.

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Abstract

The valuation of employee stock options has become a key requirement due to the rapid growth in the use of these options as a means of employee compensation. IFRS 2 Share-based Payment stipulates that these instruments must be valued and expensed on the date the awards are issued. This dissertation aims to value an employee stock option, in a case where both the equity and vesting (performance) condition are based on a reported earnings process. The equity dependency on earnings stems from the fact that we are primarily concerned with the valuation of employee stock options that are issued by a private firm. We implement a capital structure framework provided by [Goldstein, Ju and Leland \(2001\)](#). In this framework, equity and debt are derived from an underlying EBIT process that is governed by a geometric Brownian motion. The model also accounts for taxation and bankruptcy. The research aim is addressed by incorporating the capital structure model into our employee stock option pricing framework.

Acknowledgements

Firstly, I would like to thank my creator, The Almighty Lord Krishna, for making me the person I am today and granting me the opportunity and the means to achieve all my goals. I would also like to express my immense gratitude to the following people and organisations:

- My supervisors, Thomas McWalter and Chiedza Musvosvi. Thank you for all the continuous guidance, help and support.
- The AIFMRM team, for providing me the platform to learn the extraordinary field of financial mathematics.
- Chetna Patel, for editing my dissertation.
- To my loving girlfriend, Jayati Patel. Words cannot describe how much you mean to me. You have been my biggest influence and always believed in my abilities. Thank you for all your endless love, motivation and support.
- Finally, to my family. Need I say more? Thank you for always believing in me, giving me strength and hope, and most importantly, inspiring me to achieve.

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Chapter 1

Introduction

Employee stock options (ESOs) are contingent claims on the equity of a firm. These are contracts that an entity grants to its employees. Such contracts entitle the holder to an option payoff, based on the equity value of the firm. These contracts are particularly common among executive and management remuneration packages, as a way to incentivise their performance. Generally, employees are only entitled to the option payoff if certain vesting conditions are satisfied. A vesting condition is either a service condition or a performance condition that must be met in order for an employee to exercise the employee stock option (ESO).

The valuation of employee stock options has become a key requirement due to the rapid growth in the use of these options as a means of employee compensation. Importantly, employee stock option valuation methodologies must comply with general accounting standards. From an accounting point of view, these instruments are considered to be a share-based payment, and should therefore be recognised as an expense. A guide to the accounting treatment and valuation principles of employee stock options is detailed by IFRS 2 Share-based Payment.

IFRS 2 Share-based Payment requires an entity to value and recognise share-based payment awards to employees or other parties in its financial statements. The act states that a grant date model needs to be used to value these awards. Under this model, an entity is required to estimate the fair value of a share-based payment award that is issued to an employee on the grant date. To calculate the fair value of share-based payments, IFRS 2 does not require entities to use specific option pricing techniques. However, the guide does suggest the supplementary use of Monte Carlo simulation techniques for share-based payment awards that are based on vesting conditions.

A complicating factor in the valuation of employee stock options is the fact that these instruments are non-transferable; the employee cannot sell these options. Holders of ESOs are therefore at a disadvantage to holders of traded stock options, who are entitled to sell their options at any point in time. Furthermore, since the

employee cannot trade freely in either the option or the underlying stock, this suggests that the pricing of ESOs may not follow classical replicating portfolio pricing arguments. However, the IFRS guide specifies that the option must be priced using traditional risk-neutral option pricing techniques. Additionally, to price the instrument under the risk-neutral measure, the risk-neutral dynamics of the real-world vesting conditions need to be determined. This is particularly important if vesting conditions are based on dynamic market or company performance measures.

The central aim of this study lies in the valuation of employee stock options issued by a private firm. Given that we are concerned with a non-listed firm, the option pricing framework incorporates a model which estimates the equity value of the firm from a reported earnings process. In addition to this, we are particularly concerned with a vesting condition that relates to the same earnings process. In this investigation, the EBIT (earnings before interest and taxation) process serves as our proxy for earnings. It follows that the underlying process in the option pricing framework is the firm's EBIT, which is not a traded asset. Consequently, we are faced with another complexity in that the risk-neutral drift of the EBIT process needs to be determined. Accordingly, this dissertation also presents various methods to calibrate for this unknown parameter. In this study, we use a static capital structure framework proposed by [Goldstein *et al.* \(2001\)](#), which includes our desired model of equity as a function of an underlying EBIT process. However, despite the fact that the study by [Goldstein *et al.* \(2001\)](#) concentrates on an optimal capital structure model, we focus on rather using a version of the model that remains consistent with the firm's current capital structure. The optimal capital structure condition is only considered to serve as a means of obtaining comparable results.

The structure of the dissertation is as follows. Chapter 2 discusses previous literature relating to employee stock option pricing and capital structure models. Chapter 3 then defines the option pricing framework, along with the static capital structure model proposed by [Goldstein *et al.* \(2001\)](#). Chapter 4 begins by describing the data used in the analysis. It then details the methodology that has been followed to implement the option pricing framework. Chapter 5 applies the described methodology to value the employee stock option, and presents the results along with a detailed explanation outlining the necessary calibration procedures. Finally, Chapter 6 concludes the dissertation with a brief discussion of key findings and a discussion around potential avenues for future research.

Chapter 2

Literature Review

2.1 Valuation of Employee Stock Options

[Hull and White \(2004\)](#) emphasise that the non-marketability of an employee stock option is a problematic feature from a derivatives pricing perspective. [Hull and White \(2004\)](#) value non-vanilla type employee stock options using a binomial tree, similar to that proposed by [Cox, Ross and Rubinstein \(1979\)](#). The model accounts for the non-transferability and the early exercise behaviour of employees. The early exercise opportunities are available whenever an option has vested, the stock price has reached a certain multiple of the strike price, or when the option has terminated or forfeited. In a similar study, [Aboody \(1996\)](#) uses a modified binomial option pricing model provided by [Cox *et al.* \(1979\)](#). The employee stock option is valued by modifying the model to account for the differences between ESOs and traded stock options. These differences are the non-transferability, vesting schedule and premature job termination features inherent in these contracts.

Despite the clear disparities between employee stock options and traded stock options, industry practitioners are routinely using the [Black and Scholes \(1973\)](#) formula (or a non-risk adjusted modification of this) to value these contracts ([Hall and Murphy, 2002](#)). [Hall and Murphy \(2002\)](#) argue that such an approach would fail to distinguish between the employee stock option's company cost and employee value. The fact that these options are non-transferable certainly implies that there is a divergence between employee value and company cost.

2.2 Modelling Earnings Performance Measures

In order to derive a suitable valuation of ESOs, the dynamics of the vesting conditions need to be modelled. Generally, performance-based vesting conditions are based on the operational performance of the firm and it usually relates to some form of earnings metric.

It is evident that throughout the financial literature, geometric Brownian motion (GBM) is a common assumption for modelling the behaviour of financial processes. Besides modelling stock price behaviour, GBM is also commonly used to model the behaviour of the firm's asset process and performance measures within a firm, such as EBIT and EBITDA. This is evident in the works presented by [Hackbarth, Hennesy and Leland \(2007\)](#), [Sundaresan and Wang \(2007\)](#), [Duffie and Lando \(2001\)](#) and [Goldstein *et al.* \(2001\)](#). On the contrary, [Genser \(2006\)](#) presents an arithmetic Brownian motion (ABM) approach to model EBIT. His main motivation in choosing this model is that it allows for earnings performance measures such as EBIT and EBITDA to be negative. This phenomenon is possible in reality and is not captured by GBM. Additionally, we see that [Chiang, Davidson and Okunev \(1997\)](#) present a stochastic mean-reverting model as an alternate form of modelling earnings. Furthermore, [Sarkar and Zapatero \(2003\)](#) model earnings as a mean-reverting process, and defend the use of such dynamics with sound economic reasoning and empirical support.

This study aims to derive a valuation methodology of an employee stock option for non-listed firms. It implies that the equity value may need to be derived by utilising suitable capital structure models and observable performance metrics. In particular, we attempt to model the firm's equity value based on an observable EBIT process.

2.3 A Review of Capital Structure Models

The theory of capital structure models was first pioneered by [Modigliani and Miller \(1958\)](#). Throughout the literature, it is evident that the pricing of equity and debt in these models are based on an assumption on the dynamics of the firm's value process. The firm's debt and equity are then priced as contingent claims on the value process. The most notable studies where this is evident are [Black and Scholes \(1973\)](#), [Merton \(1974\)](#), [Black and Cox \(1976\)](#), [Geske \(1977\)](#), [Leland \(1994\)](#), [Longstaff and Schwartz \(1995\)](#), [Leland and Toft \(1996\)](#) and [Goldstein *et al.* \(2001\)](#).

The modelling and pricing of debt securities introduce a notion of credit risk and default. The literature is seen to focus on two types of bankruptcy. In the first kind, bankruptcy occurs when the firm value decreases to such an extent that the firm cannot issue additional equity to keep the firm alive. Such a bankruptcy condition is known as an endogenous decision, and is outlined in the works presented by [Leland \(1994\)](#) and [Goldstein *et al.* \(2001\)](#). In the second type of bankruptcy, the default condition is exogenously specified. For example, bankruptcy is declared due to some contractual condition that is not met (the firm is bankrupt due to some

exogenously specified threshold). Notable studies that use the latter approach include [Black and Cox \(1976\)](#) and [Longstaff and Schwartz \(1995\)](#). It is apparent that more recent capital structure literature tends to focus on incorporating an endogenous bankruptcy condition. [Sarkar and Zapatero \(2003\)](#) claim this approach is intuitive in the sense that as long as equity has residual value, a firm will still be motivated and be able to issue new equity to pay off any outstanding debt obligation, as opposed to defaulting. [Leland \(1994\)](#), [Goldstein *et al.* \(2001\)](#) and [Sarkar and Zapatero \(2003\)](#) also argue that an endogenous bankruptcy condition is more suitable for firms with long-term debt obligations.

In their seminal papers, [Black and Scholes \(1973\)](#) and [Merton \(1974\)](#) established the concept of structural credit risk models. In their framework, it is assumed that firm value is governed by a geometric Brownian motion and that the firm is fully financed by one zero coupon bond with finite maturity. The debt will be honoured only if the firm value exceeds the outstanding debt on maturity. If this is not the case, the firm will declare bankruptcy. [Black and Cox \(1976\)](#) expanded on this framework by acknowledging the possibility of default before the debt expires.

[Leland \(1994\)](#) made further developments in generalising capital structure models by incorporating taxation and bankruptcy costs. In this framework, management (who acts on behalf of equity holders) maximise the value of the firm by setting an optimal bankruptcy level and leverage ratio. [Duffie and Lando \(2001\)](#) label the class of such models as second-generation models. These models vary from traditional capital structure models by allowing for an endogenous level of default, which is optimally set to maximise the wealth of shareholders. Leland's model therefore allows for a formal characterisation of optimal capital structure. The framework also incorporates the considerations of [Modigliani and Miller \(1958\)](#), who emphasised that taxes are an important factor in assessing the optimal capital structure of a firm due to the tax benefits of leverage. In Leland's model, the firm's capital structure is comprised of both equity and a single perpetual bond that pays a constant continuous coupon. [Leland \(1994\)](#) supports characterising debt with a perpetual bond in two ways. He argues that debt with a long maturity implies the principle has negligible value and can safely be ignored. He further states that long time horizons are common in both theory and practice, where even the original capital structure works by [Modigliani and Miller \(1958\)](#) assumed debt with an infinite maturity. Lastly, he argues that another benefit of this approach is that a perpetual debt permits the development of closed-form solutions for the value of debt and equity, where equity and debt are claims on the firm's value process. The model further assumes a static capital structure, where it is assumed that the nominal value of debt (once issued), remains unchanged through time. The same static

capital structure arguments are also presented by [Modigliani and Miller \(1958\)](#) and [Merton \(1974\)](#). [Leland \(1994\)](#) supports the use of a static capital structure, arguing that additional debt issuances may hurt current debt holders and debt reductions (via repurchases) may hurt current equity holders. However, he does note that a dynamic optimal capital structure may be desirable, even though it is more difficult to model.

[Goldstein *et al.* \(2001\)](#) addressed the dynamic optimal capital structure issue raised by [Leland \(1994\)](#), by developing both static and dynamic EBIT-based capital structure models. In traditional (static) capital structure models¹, the state variable is taken to be the firm value and is presumed to follow lognormal dynamics. A common feature among these models is their treatment of cash flows to the government (via taxes). Specifically, these cash flows are accounted in a way that is fundamentally different to the treatment of cash flows owed to equity and debt holders. On the other hand, in the framework presented by [Goldstein *et al.* \(2001\)](#), cash flows to the government are treated in the same manner to that of equity and debt. Therefore, in this framework, the total value of the entity is seen to be redistributed among all claimants (equity, debt, government and bankruptcy). The model also implies a significantly lower risk-neutral drift of the firm value process, as opposed to those seen in traditional frameworks. Consequently, the model predicts a higher probability of default, which in turn leads to lower optimal leverage ratios. On the basis of empirical support provided by [Toft and Prucyk \(1997\)](#), [Goldstein *et al.* \(2001\)](#) model the dynamics of EBIT as lognormal, with the implication that all claimants to EBIT (equity, debt, government and bankruptcy) are treated consistently. The framework also adopts the common assumption of the separation of investment and financing policy. This follows from the fact that the EBIT process is assumed to be invariant to changes in capital structure. The authors argue that their approach has an intuitive appeal, as the EBIT process, which is the source of firm value, runs independently of the manner in which EBIT is redistributed among the claimants. Specifically, an extra unit of currency paid out, whether in dividend payouts, taxes or interest payments, will affect the firm value in the same way ([Goldstein *et al.*, 2001](#)). Hence, on this basis, the authors claim that the choice of EBIT is indeed an appropriate state variable. [Goldstein *et al.* \(2001\)](#) utilise a capital structure consisting of both equity and a perpetual debt that pays a constant continuous coupon, where the bankruptcy threshold and coupon amount are determined endogenously by management acting on behalf of equity holders. The same capital structure and endogenous conditions are evident in [Leland \(1994\)](#).

¹Examples of such models are presented by [Brennan and Schwartz \(1978\)](#), [Leland \(1994\)](#), [Longstaff and Schwartz \(1995\)](#) and [Leland and Toft \(1996\)](#).

Overall, a clear theme in the literature that is related to the valuation of employee stock options, is the focus on a valuation that accounts for the early exercise opportunities and non-transferability of these contracts. It is evident that there is a gap in the literature dedicated to the valuation of employee stock options that incorporates an earnings-based equity model and an earnings-based vesting condition. This is our primary task. Based on the literature survey, we now apply the capital structure framework provided by [Goldstein *et al.* \(2001\)](#) to achieve this research aim.

Chapter 3

Mathematical Preliminaries

This chapter attempts to explain the valuation problem using sound economic and mathematical finance arguments.

3.1 Introduction

A central aim of this study is to utilise a suitable model that estimates the equity value of a firm from an earnings process. To solve this issue, we adopt a static capital structure model proposed by [Goldstein *et al.* \(2001\)](#), who start their analysis by taking the firm's EBIT process as the state variable. This goes against most structural models, which instead take the firm's value process as a given state variable. Regarding EBIT as a state variable has a clear advantage in that it introduces a notion of cash flows in both firm value and in the valuation of corporate securities. In this setting, the EBIT process is seen as a source of firm value, which implies that the value of the firm is a claim on EBIT. This ultimately forces a split of EBIT into various claims, easing the interpretation of derived security values from an economic point of view. In contrast to traditional capital structure models, this framework allows for bankruptcy and taxation, which adds an intuitive economic appeal.

3.2 Model of Equity

3.2.1 The EBIT and Value Process

In this framework, the EBIT process X_t , is assumed to follow a geometric Brownian motion with real-world dynamics given by

$$\frac{dX}{X} = \mu_p dt + \sigma dW^{\mathbb{P}}, \quad (3.1)$$

where μ_p and σ are constants representing the instantaneous growth rate (drift) and the volatility of the EBIT process respectively under the real world measure \mathbb{P} . $W^{\mathbb{P}}$

is a standard Brownian motion under \mathbb{P} . The evolution of the EBIT process under the risk-neutral measure \mathbb{Q} can be specified by

$$\frac{dX}{X} = \mu dt + \sigma dW^{\mathbb{Q}}, \quad (3.2)$$

where μ is the drift of the EBIT process under \mathbb{Q} . By Girsanov's Theorem, we have that $\mu = \mu_p - \theta\sigma$, where θ is the risk premium or the Girsanov kernel which affects the change of measure from \mathbb{P} to \mathbb{Q} . Note, in this framework, the EBIT process under the risk-neutral measure does not inherit a drift rate equal to the risk-free rate. This is simply because the EBIT process itself does not represent the value of a traded security.¹

We pause here and reflect on the necessity of considering the dynamics of EBIT under the risk-neutral measure. We ultimately seek to price a contingent claim (ESO) whose underlying process is the EBIT value of the firm. Such a price can be derived using the principles of risk-neutral valuation. In addition, the existence of a martingale measure \mathbb{Q} is useful in the sense that it allows for the discounting of future EBIT payments with the risk-free interest rate — a fact that is used to derive the firm value.

The total firm value V_t , is defined as the discounted value of all future EBIT payments under the risk-neutral measure,

$$V_t = \mathbb{E}_{\mathbb{Q}} \left[\int_t^{\infty} e^{-r(s-t)} X_s ds \right]. \quad (3.3)$$

Simplifying this expression yields

$$V_t = \frac{X_t}{r - \mu}, \quad (3.4)$$

provided that $r > \mu$, where r denotes a constant risk-free interest rate. In the case where $r < \mu$, the present value of cash flows will be infinite — a case we do not consider here. A detailed proof of the simplification is presented in Appendix A. As discussed by Goldstein *et al.* (2001) and Genser (2006), this definition of firm value also follows naturally if you consider the common (market practice) assumption of the separation of the firm's operating and financial decisions. That is, the value of the firm's assets is assumed to be independent of the capital structure. Since r and μ are assumed constants, an application of Itô's formula on Equation (3.4) yields that both the firm value and EBIT process share the same dynamics:

$$\frac{dV}{V} = \mu dt + \sigma dW^{\mathbb{Q}}. \quad (3.5)$$

¹The implication here is that the risk premium θ , differs from the usual risk premium $\frac{\mu_p - r}{\sigma}$, which reflects the risk premium of traded securities and is enforced by arbitrage conditions.

It follows that

$$\frac{dV + Xdt}{V} = rd_t + \sigma dW^{\mathbb{Q}}. \quad (3.6)$$

Equation (3.6) simply states that under the risk-neutral measure, the total return on the firm's value process (which is a claim on EBIT) is the risk-free rate. In this expression, $\frac{X}{V}$ is defined as a payout ratio. Equation (3.4) dictates that this ratio remains constant since r and μ are assumed to be constants. Therefore, EBIT is proportional to the firm value under geometric Brownian motion. [Goldstein *et al.* \(2001\)](#) justify a constant payout ratio using an economic argument. They argue that their framework treats cash flows to the government, equity and debt all in the same manner. In addition, since an increase in firm value leads to an increase in tax payments, this leads to a reasonable assumption that the payout ratio remains constant.

3.2.2 Tax System

This framework incorporates the effects of taxation. It is an important consideration as taxation plays a significant role in determining the actual amount owed to equity holders. In addition, the model can account for the tax deductibility (advantages) of debt. Consequently, an inclusion of a tax-regime allows for a more complete model in describing the economic environment of the firm.

The model assumes a simple tax structure that includes three different kinds of taxes. Interest payments on debt are taxed at a rate τ_i . Corporate earnings after tax are paid out as dividends, which are then taxed at τ_d . Finally, corporate profits are taxed at τ_c , with full loss offset provisions. This implies negative corporate earnings (occurs when EBIT net of coupon payment is negative) are eligible for a tax refund. That is, the firm will always earn the tax shield of debt, even when the taxable operating income is less than the outstanding debt (coupon payment). The effective tax rate for equity holders can therefore be expressed as

$$(1 - \tau_{eff}) = (1 - \tau_c)(1 - \tau_d). \quad (3.7)$$

Table 3.1 illustrates the tax structure in the case of an unlevered and levered firm. For an unlevered firm, corporate taxes are paid on the firm's EBIT, X . The residual amount is then paid out as dividends, which is taxed at the dividends tax rate. In the case of a levered firm, the company taxes are reduced as the interest payment on debt (the coupon amount, C) is tax deductible.

Tab. 3.1: Tax regime

	Corporate Taxes	Interest Payment (Coupon) Taxes	Dividend Taxes	Total Taxes
Unlevered Firm	$\tau_c X$	0	$\tau_d(1 - \tau_c)X$	$\tau_{eff} X$
Levered Firm	$\tau_c(X - C)$	$\tau_i C$	$\tau_d(1 - \tau_c)(X - C)$	$\tau_{eff}(X - C) + \tau_i C$

3.2.3 The Case of a Debtless (Unlevered) Firm: Two Claimants

In this scenario, an unlevered firm is considered. Both equity and government (or equivalently, cash flows owed to the government through taxation) will have a claim on the firm's earnings. Therefore, the current firm value, denoted by V_0 , is shared between equity and government in the following manner:

$$Eq(V_0) = (1 - \tau_{eff})V_0, \quad (3.8)$$

$$G(V_0) = \tau_{eff}V_0. \quad (3.9)$$

We now proceed to the heart of the analysis. What follows are mathematical arguments describing how a levered firm's value process is redistributed among all four claimants considered in this study. In the case of a levered firm, equity, debt, government (through taxation), and bankruptcy are all claims on the firm's value process. The value of the firm is seen as a claim on EBIT.

3.2.4 The Case of a Levered Firm: Four Claimants

In the model presented by [Goldstein et al. \(2001\)](#), it is assumed that management (acting on behalf of equity holders) chooses a static (one-time decision) debt level that maximises the wealth of current equity holders. The firm is financed by both equity and debt. The debt is characterised by a single perpetual bond with a constant continuous coupon level, denoted by C .² Debt holders will receive this payment as long as the firm remains solvent. The issuance of the perpetual debt implies that the threshold at which the firm declares bankruptcy, denoted by V_B , is time independent. Should the firm choose to default, the value of the firm will be V_B and an amount αV_B will be forfeited to bankruptcy costs.

Given the dynamics of the value process of the firm,

$$\frac{dV}{V} = \mu dt + \sigma dW^{\mathbb{Q}},$$

it follows that any claim/derivative whose value can be expressed as a function of the firm's value and time, must satisfy the following partial differential equation

²In continuous time, the actual coupon payment is $C dt$ over the infinitesimal interval dt .

(PDE):

$$\mu V F_V + \frac{1}{2} \sigma^2 V^2 F_{VV} + F_t + P = rF, \quad (3.10)$$

where $F(V, t)$ denotes the value of this claim and P is the payout flow associated with this claim. This result can easily be derived by using an application of the famous Feynman Kâc Theorem. A description of this theorem is outlined in Appendix A. Due to the issuance of perpetual debt, the value of all debt and default claims considered in this framework will be time-independent. The PDE therefore simplifies to the following ordinary differential equation (ODE):

$$0 = \mu V F_V + \frac{1}{2} \sigma^2 V^2 F_{VV} + P - rF. \quad (3.11)$$

The general solution³ to the ODE is given by

$$F(V) = A_0 + A_1 V^{-y} + A_2 V^{-x}, \quad (3.12)$$

with

$$x = \frac{1}{\sigma^2} \left[\left(\mu - \frac{\sigma^2}{2} \right) + \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right],$$

and

$$y = \frac{1}{\sigma^2} \left[\left(\mu - \frac{\sigma^2}{2} \right) - \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right].$$

The parameters A_0 , A_1 , and A_2 are constants determined by considering the boundary conditions of the securities under consideration. Note, $A_1 = 0$ for all claims of interest. This is simply due to the fact that y is negative, leading to an explosion of the first term in Equation (3.12) as V becomes large.

Equation (3.12) represents the value of a default or debt claim on the value process that is governed by Equation (3.5). This relationship is important, as it allows one to derive relationships for the claimants of concern (equity, debt, government and bankruptcy), by carefully considering various claims and using the principles of the accounting equation. It is well known that the accounting equation essentially states that all assets of a firm are either financed by debt or equity. Expressing this analytically we have,

$$A = Eq + L, \quad (3.13)$$

where A denotes the value of the firm's assets, Eq represents the shareholders' equity and L denotes the firm's liabilities. One of these claims considered is the

³An example of the proof can be found in Shimko (1992).

Arrow-Debreu bankruptcy claim, denoted by $P_B(V)$. $P_B(V)$ defines the present value of a claim that will pay one unit of currency in the event of bankruptcy (firm value reaching V_B) and zero otherwise. The value of this claim satisfies Equation (3.12) and can therefore be expressed in the form,

$$P_B(V) = A_0 + A_1 V^{-y} + A_2 V^{-x}, \quad (3.14)$$

where the constants A_0, A_1 and A_2 are determined by considering the following limiting behaviour. If the firm's value process V increases, the security's value should approach zero since it will become less likely that the firm will default. In addition, should V approach the bankruptcy threshold V_B , the security's price must approach one. Expressing this mathematically, we have,

$$\lim_{V \rightarrow \infty} P_B(V) = 0 \quad \text{and} \quad \lim_{V \rightarrow V_B} P_B(V) = 1.$$

Both these boundary conditions infer that the value of the Arrow-Debreu bankruptcy claim can be given as

$$P_B(V) = \left(\frac{V}{V_B} \right)^{-x}. \quad (3.15)$$

Additionally, a default claim denoted by $V_{def}(V)$, is defined as a claim that pays the value of the firm in the event of default (V_B), and zero otherwise. Using similar limiting arguments, it follows that

$$\lim_{V \rightarrow \infty} V_{def}(V) = 0 \quad \text{and} \quad \lim_{V \rightarrow V_B} V_{def}(V) = V_B.$$

Therefore, the firm value in the event of bankruptcy can be expressed as

$$V_{def}(V) = V_B P_B(V) = V_B \left(\frac{V}{V_B} \right)^{-x}. \quad (3.16)$$

Since the total value of the firm can be split between solvent and insolvent claims, the following relation holds:

$$V = \frac{X}{r - \mu} = V_{solv}(V) + V_{def}(V), \quad (3.17)$$

where $V_{solv}(V)$ denotes the firm value during continued operations. While the firm remains solvent ($V > V_B$), equity, debt and government share the earnings X through dividends, coupon payments and taxes respectively. Using Equation (3.17), the solvent value is described as

$$V_{solv}(V) = V - V_B P_B(V). \quad (3.18)$$

Next, the solvent value of a perpetual debt issue before taxes is considered, and is denoted by $V_{int}(V)$. This claim pays the coupon amount C as long as the firm

remains solvent and pays nothing in the case of default. During solvency, the relevant payout of this claim is the coupon amount, $P = C$. This implies that the value of coupon payments to debt holders in perpetuity can be expressed as

$$F_C = \frac{C}{r}, \quad (3.19)$$

noting that Equation (3.19) is a particular solution to Equation (3.11). The limit condition, $\lim_{V \rightarrow \infty} V_{int}(V) = \frac{C}{r}$, implies that $A_0 = \frac{C}{r}$ and $A_1 = 0$. For $V = V_B$, the value of this claim vanishes. This boundary condition determines A_2 , giving

$$V_{int}(V) = \frac{C}{r}[1 - P_B(V)]. \quad (3.20)$$

An application of Equation (3.13) immediately defines the equity value of the firm before taxes,

$$Eq_{solw}(V) = V_{solw}(V) - V_{int}(V). \quad (3.21)$$

Under the proposed tax regime, the solvent value of equity becomes

$$Eq_{solw}(V) = (1 - \tau_{eff}) [V_{solw}(V) - V_{int}(V)]. \quad (3.22)$$

Investors will value the solvent value of the perpetual debt as

$$D_{solw}(V) = (1 - \tau_i)V_{int}(V). \quad (3.23)$$

Finally, the share of the solvent firm's value owed to the government amounts to

$$G_{solw}(V) = \tau_{eff} [V_{solw}(V) - V_{int}(V)] + \tau_i V_{int}(V). \quad (3.24)$$

The first term in Equation (3.24) represents the taxation due to dividends and corporate earnings. The last term reflects the tax payments on the interest (coupon) payments. In this framework, we can easily see that the sum of all three claims adds up to the firm's solvent value. Concretely, the following relation holds:

$$V_{solw}(V) = Eq_{solw}(V) + G_{solw}(V) + D_{solw}(V). \quad (3.25)$$

The model also considers the possibility of bankruptcy for a levered firm. Due to the limited liability nature of the firm, the value of equity in the event of default is zero. That is,

$$Eq_{def}(V) = 0. \quad (3.26)$$

At this point, the remaining firm value (the value of the insolvent firm as specified by Equation (3.16)) will be divided among debt holders, the government and bankruptcy costs as follows:

$$D_{def}(V) = (1 - \alpha)(1 - \tau_{eff})V_{def}(V), \quad (3.27)$$

$$G_{def}(V) = (1 - \alpha)\tau_{eff}V_{def}(V), \quad (3.28)$$

$$BC_{def}(V) = \alpha V_{def}(V). \quad (3.29)$$

It is easy to see that these three claims sum to the value of the insolvent firm,

$$V_{def}(V) = D_{def}(V) + G_{def}(V) + BC_{def}(V). \quad (3.30)$$

To summarise, we note that the total firm value V , is the sum of the solvent part of the firm (Equation (3.18)) and the value of the firm in the case of bankruptcy (Equation (3.16)). Accordingly, the total value process can be expressed in the following manner:

$$\begin{aligned} V_t &= \mathbb{E}_{\mathbb{Q}} \left(\int_t^{\infty} e^{-r(s-t)} X_s ds \right) \\ &= V_{solv}(V_t) + V_{def}(V_t) \\ &= Eq(V_t) + D(V_t) + G(V_t) + BC(V_t) \\ &= [Eq_{solv}(V_t) + Eq_{def}(V_t)] + [D_{solv}(V_t) + D_{def}(V_t)] + [G_{solv}(V_t) + G_{def}(V_t)] \\ &\quad + BC(V_t). \end{aligned}$$

Hence, we can see that the total value of the firm is redistributed among the equity, debt, government and bankruptcy claimants. This result is similar to the “pie” model presented by [Modigliani and Miller \(1958\)](#). The model presented by [Goldstein et al. \(2001\)](#) shows that the total pie, which represents the value of all claims, is split among the four claimants in an optimal and economically intuitive manner. In this framework, the total values for equity, debt, government and bankruptcy can be expressed as follows:

$$Eq(V) = (1 - \tau_{eff}) [V_{solv}(V) - V_{int}(V)], \quad (3.31)$$

$$D(V) = (1 - \tau_i)V_{int}(V) + (1 - \alpha)(1 - \tau_{eff})V_{def}(V), \quad (3.32)$$

$$G(V) = \tau_{eff} [V_{solv}(V) - V_{int}(V)] + \tau_i V_{int}(V) + (1 - \alpha)\tau_{eff}V_{def}(V), \quad (3.33)$$

$$BC(V) = \alpha V_{def}(V). \quad (3.34)$$

Thus, from Equations (3.4) and (3.31), the closed form solution for the equity value of the firm as a function the firm’s EBIT process is given as

$$Eq(X) = \max \left((1 - \tau_{eff}) \left[\frac{X}{r - \mu} + \left(\frac{X}{(r - \mu)V_B} \right)^{-x} \left(\frac{C}{r} - V_B \right) - \frac{C}{r} \right], 0 \right), \quad (3.35)$$

where the max condition enforces limited liability.

3.2.5 The Bankruptcy Level and Optimal Coupon Amount

In the static capital structure model proposed by Goldstein *et al.* (2001), it is assumed that management sets a constant default barrier V_B to maximise shareholder wealth, subject to limited liability. Therefore, the bankruptcy level is chosen endogenously and the firm is not constrained by any covenants. Consequently, default will only occur if the firm cannot pay the instantaneous coupon payments by issuing additional equity.

Equations (3.16) – (3.18) imply that the total value of the firm is maximised by fixing V_B as low as possible. However, due to the limited liability of the firm, equity is required to be positive for all values of $V > V_B$. The lowest possible value for V_B that allows positive equity values in this range is achieved by invoking the “smooth pasting condition”⁴,

$$\left. \frac{\partial E}{\partial V} \right|_{V=V_B} = 0. \quad (3.36)$$

Solving this condition yields the default level V_B ,

$$V_B = \lambda \left(\frac{C}{r} \right), \quad (3.37)$$

where,

$$\lambda = \frac{x}{x+1}. \quad (3.38)$$

To determine the optimal capital structure, it is assumed that management chooses the coupon amount (debt level) to maximise the wealth of equity holders. Specifically, a coupon level is set to maximise the sum of debt and equity. In this scenario, the debt does not expand the assets of the firm but instead gets passed on to the equity holders. In other words, the current equity holders will earn the fair value of the debt claim sold. If V_0 is the firm value on the bond issuing date, the optimal coupon C^* is obtained by solving for,

$$\frac{\partial [D(V_0, C, V_B(C)) + Eq(V_0, C, V_B(C))]}{\partial C} = 0, \quad (3.39)$$

which yields

$$C^* = V_0 \left(\frac{r}{\lambda} \right) \left[\left(\frac{1}{1+x} \right) \left(\frac{A}{A+B} \right) \right]^{\frac{1}{x}}, \quad (3.40)$$

where

$$A = \tau_{eff} - \tau_i \quad (3.41)$$

and

$$B = \lambda \alpha (1 - \tau_{eff}). \quad (3.42)$$

⁴See Dixit (1993) for a detailed description of this condition.

Note, we assume that there are no issuance (restructuring) costs of the debt. The full derivations of the bankruptcy level (Equation (3.37)) and the optimal coupon level (Equation (3.40)) can be found in Appendix A.

To summarise, the optimal capital structure model provided by Goldstein *et al.* (2001), enforces a one-time capital structure decision of management, whose objective is to maximise current shareholder wealth. However, it should be noted that we need not be constrained to use the optimal coupon level C^* . We can however, use an arbitrary coupon level C that remains consistent with the current capital structure of the firm in question. With a perpetual debt issuance, and assuming that EBIT is governed by a geometric Brownian motion, it is possible to derive closed-form solutions for all claimants on the firm's value. In particular, Equation (3.35) expresses a relationship for the equity value of the firm as a function of the firm's EBIT. This is our desired model of equity and is directly used to evaluate the payoff of the employee stock option.

3.3 Option Pricing Model

On the basis of the mathematical arguments presented so far, the option pricing framework can now be defined. The contract only permits one exercise date, where at maturity, the employee is entitled to the option payoff if the vesting condition is satisfied. The payoff for this instrument is given as,

$$\mathbb{I}_{\{X_{T^*} \geq X_0 \beta\}} (S_T(X_T) - K)^+, \quad (3.43)$$

where

$$X_{T^*} = \sup_{s \leq T} X_s \quad (3.44)$$

is the running maximum of the firm's EBIT. T denotes the term of the contract, $S_T(X_T)$ represents the terminal share price, K is the strike of the employee stock option and X_0 denotes the value of EBIT on the grant date. Typically, in order to exercise these options, the firm's EBIT is required to at least surpass the risk-free rate of growth over the term of the option. In this case, we have $\beta = 1 + r$, where β serves as our performance-based vesting condition. The payoff is equivalent to that of a vanilla call with a barrier condition. The indicator serves as a knock-in barrier and essentially states that the contract will only vest should EBIT breach the performance barrier. The strike is structured as the product of a predefined EBIT multiple (κ) and the value of EBIT on maturity of the contract (X_T), less the total outstanding debt ($D(V_T)$). Therefore, the strike considered here is stochastic in nature. The EBIT multiple κ is generally determined by management and is specified on the grant date (valuation date) of the option.

Based on the context given so far, it is clear that we can derive the premium of an employee stock option using the principles of risk-neutral valuation. Specifically, the value of an ESO can be expressed as,

$$C_0 = \mathbb{E}_{\mathbb{Q}} \left[e^{-rT} \mathbb{I}_{\{X_{T^*} \geq X_{0\beta}\}} (S_T(X_T) - K)^+ \right], \quad (3.45)$$

where,

$$S_T(X_T) = \frac{Eq(X_T)}{\text{no. of shares}} \quad (3.46)$$

and

$$K = \max \left(\frac{\kappa X_T - D(V_T)}{\text{no. of shares}}, 0 \right). \quad (3.47)$$

Using Equations (3.4), (3.32) and (3.35), it follows that

$$Eq(X_T) = \max \left((1 - \tau_{eff}) \left[\frac{X_T}{r - \mu} + \left(\frac{X_T}{(r - \mu)V_B} \right)^{-x} \left(\frac{C}{r} - V_B \right) - \frac{C}{r} \right], 0 \right),$$

$$D(V_T) = (1 - \tau_i)V_{int}(V_T) + (1 - \alpha)(1 - \tau_{eff})V_{def}(V_T),$$

where

$$V_B = \lambda \left(\frac{C}{r} \right)$$

and

$$V_T = \frac{X_T}{r - \mu}.$$

The formulation of the strike will incentivise staff to increase the net earnings (profitability) of the firm. Specifically, employees will attempt to maximise earnings after debt repayments. Since we are using a static coupon repayment C in our model, we are imposed to use the value of the debt claim $D(V_T)$ in the strike, as this quantity is allowed to vary and further incorporates the possibility of default. Therefore, using $D(V_T)$ will essentially allow a penalty to be incurred if the value of EBIT falls to very low levels. The EBIT multiple κ can be regarded as the following performance multiple:

$$\frac{\text{Enterprise Value}}{\text{EBIT}},$$

where the enterprise value represents the market or fair value of the firm's assets. In this regard, the strike can be interpreted as a measure of the expected terminal equity value of the firm (by principle of the accounting equation).

3.3.1 Liquidity and Dilution Factors

As discussed earlier, employee stock options are not traded. This implies that traded stock options should have a greater value, since these instruments can be sold at any stage before maturity of the option. Accordingly, we could include a discount (liquidity) factor in the option payoff to account for the non-transferability of an employee stock option. Furthermore, a dilution factor should also be considered. At maturity, the firm is required to settle the value of the option. This payment is sourced from the firm, necessitating a change in the capital structure. The fact that we have imposed a perpetual debt implies that the firm cannot raise additional debt to finance the option payout. Therefore, the firm will list additional shares to raise the necessary capital. In doing so, the current shareholders' claim to equity is diluted. Accordingly, the value of the option needs to be reduced by a relevant dilution ratio. This ratio can be given as:

$$\text{Dilution ratio} = \frac{\text{no. of shares in issue}}{\text{no. of shares in issue} + \text{no. of new shares issued}}. \quad (3.48)$$

We have mentioned approaches to account for the non-transferability and dilution effect of ESOs. In the remainder of this study, we do not consider these effects. However, we stress that an adjustment for both the liquidity and dilution is necessary to calculate the fair price of an employee stock option. Moreover, our model does not account for forfeiture of the contract, due to the employee leaving the firm.

3.4 Discussion

We end this chapter by discussing two main assumptions inherent in this framework, in the context of an employee stock option valuation.

3.4.1 Perpetual Debt Issuance

We can justify the perpetual nature of debt by noting that companies tend to refinance or replace their long-term debt, in order to reduce the cost of capital, maintain lower effective tax rates and to increase/leverage equity returns. However, we note that utilising optimal perpetual debt in the context of an employee stock option valuation may be unrealistic for a particular firm, based on their current capital structure. Accordingly, in Chapter 5, we price the option under a debt level (coupon) which remains consistent with the firm's current capital structure. The option is also priced under an optimal debt level, although this is purely for comparative purposes.

We have presented a framework which incorporates a static capital structure (the coupon level remains unchanged). Employee stock options typically do not possess long-term maturities. Therefore, this may motivate the use of a static structure. However, a dynamic-based capital structure model may be more realistic, although this significantly complicates the exposition and is beyond the scope of this research.

3.4.2 A GBM-based EBIT Process

Assuming that EBIT follows a geometric Brownian motion has a distinct disadvantage that EBIT may not be negative. However, since employee stock options are generally not characterised by long-term maturities, one could argue that the occurrence of negative EBIT would be rather rare. In this regard, GBM may be an appropriate model. Furthermore, it should be noted that it is common market practice to model a wide array of financial processes with GBM, and that the great benefit of using GBM lies in its tractability. To account for the shortcomings of GBM, we note that [Sarkar and Zapatero \(2003\)](#) and [Genser \(2006\)](#) model EBIT as a mean-reverting process and an arithmetic Brownian motion process respectively. Both frameworks also model the same claimants on the firm's value process (equity, debt, government and bankruptcy).

Chapter 4

Data and Methodology

4.1 Data

The base case parameters presented by [Goldstein *et al.* \(2001\)](#) are used to provide insight into the economic characteristics that are inherent in their framework.

The data of a prominent publicly traded company are used to price an employee stock option. The data comprise of the firm's bi-annual financial statements, spanning over a period of June 2000 to June 2017. Although this is a publicly traded firm, we shall assume that it is a South African private company, and refer to it as Company A. The valuation date (grant date) for the employee stock option is 25 June 2017.

4.2 Methodology

We aim to derive the fair value (premium) of an employee stock option for the firm in question. This is achieved by directly computing Equation (3.45), with the aid of a Monte-Carlo simulation. Specifically, the valuation is derived by performing a Monte-Carlo simulation using 5,000,000 sample paths for the firm's EBIT process. The simulation checks for default using monthly EBIT observations. Furthermore, we check that the vesting condition is satisfied by verifying (path-wise) if the EBIT process outperforms the vesting condition β over the tenor of the option. Equation (3.47) highlights that the strike of the option is highly dependent on the pre-defined EBIT multiple κ . Management would ultimately decide on this multiple by considering the corresponding option premium (aka IFRS expense). Therefore, we present prices with respect to various EBIT multiples as a solution to the valuation problem. In addition to this, it was previously mentioned that κ can be regarded as a suitable performance multiple, in order to allow the strike to be interpreted as an expected equity value. Therefore, management can look at a history of these performance multiples as a guide in selecting an appropriate κ value.

In order to implement the option pricing model, the complete dynamics of the EBIT process needs to be determined. Namely, the drift and volatility of the firm's EBIT process must be derived. We resort to common market practice by estimating the volatility using historical data. Specifically, historical EBIT values that are reported in the firm's bi-annual financial statements are considered to estimate the volatility. In this option pricing framework, the underlying process is the firm's EBIT. Given that the employee stock option is not a traded instrument and the underlying EBIT process is not a traded asset, it is not possible to construct a portfolio that replicates the value of the option. This restriction implies that we may have to value the reward by incorporating the real-world risk and perform real-world pricing techniques. However, the IFRS guide clearly states that we need to price employee stock options using traditional risk-neutral option pricing techniques. Therefore, the option must be priced under the risk-neutral measure \mathbb{Q} . A unique risk-neutral measure implies that there is a unique risk-neutral drift of the EBIT process.¹ Chapter 5 demonstrates that this drift can be obtained by performing a calibration procedure using market data. Specifically, it is shown that there is only one risk-neutral drift that coincides with the equity value of the firm. Two risk-neutral drift calibration procedures are considered; an instantaneous and historical calibrated drift approach. A full description of these procedures is presented in Chapter 5.

¹This follows directly from Girsanov's Theorem.

Chapter 5

Results and Analysis

5.1 Economic Intuition of the Model

This section attempts to explain certain economic phenomena inherent in this framework. The parameters considered here are highlighted in Table 5.1 below.

Tab. 5.1: Base-case parameters

Corporate Tax Rate, τ_c	0.35	Risk-Neutral Drift of EBIT, μ	0.03
Dividends Tax Rate, τ_d	0.20	Volatility of EBIT, σ	0.25
Interest Payments Tax Rate, τ_i	0.35	Initial EBIT, X_0	100
Risk-Free Rate, r	0.045	Bankruptcy Loss Ratio, α	0.05

Figure 5.1 illustrates the relationship between the debt value of the firm and the coupon level C for various levels of EBIT volatility σ and bankruptcy costs α . The volatility of EBIT also conveys the risk of the firm, as we have the same σ present for both the firm value and EBIT processes in this framework. At the outset, we can see that the bond value first rises and then falls as the coupon level increases. This is a direct result of the endogenous default level present in this model. A higher level of C has two effects on the debt value of the firm. The direct effect is that it increases the bond value. The indirect effect is that it raises the bankruptcy threshold V_B . A higher bankruptcy trigger implies a greater probability of default, which in turn reduces the bond value. For higher coupon levels, this indirect effect will dominate and the debt value will eventually become a decreasing function of C . The peak of each curve indicates that the debt value reaches a maximum value for some coupon level. In other words, there is a debt capacity for the firm. Figure 5.1a shows that firms with lower risk will have a higher debt capacity (and vice versa). Furthermore, we note that riskier firms will have a higher debt value than less risky firms for very large coupon levels. As expected, Figure 5.1b highlights that both the debt capacity and debt value of the firm fall with an increase in bankruptcy costs.

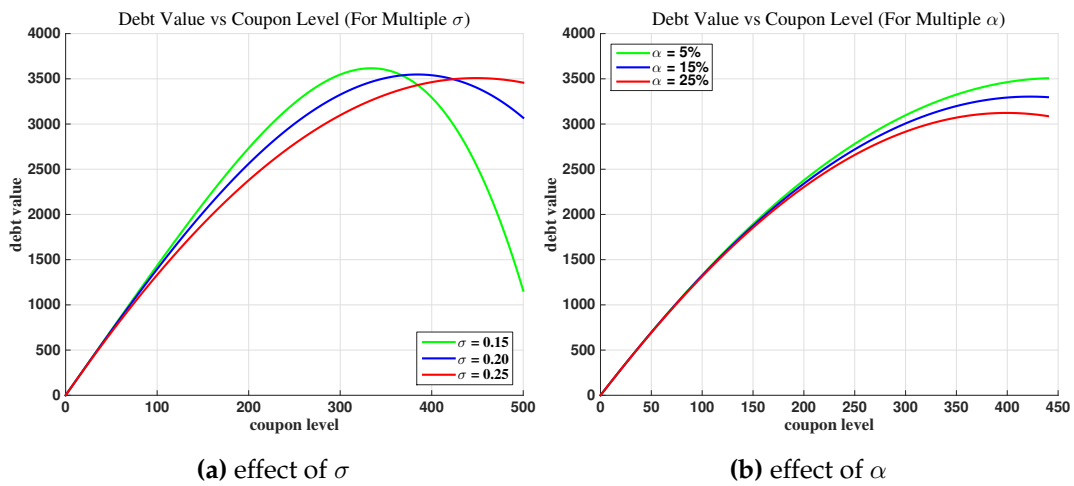


Fig. 5.1: Debt value vs coupon level for various σ and α

Figure 5.2 illustrates the relationship between the sum of debt and equity and the coupon level for various levels of volatility. The optimal coupon is that which maximises this relationship. Therefore, the optimal leverage ratio for each level of volatility is determined by the values of equity and debt at the peak of each of these curves. Table 5.2 displays the optimal capital structure for each level of firm risk, and clearly indicates a negative correlation between the optimal leverage ratio and the earnings volatility. In other words, the optimal leverage ratio of less risky firms (firms with smaller σ) is always greater than that of riskier firms (firms with higher σ). The results also show that the optimal coupon is smaller to the coupon which maximises the debt value of the firm. The model therefore suggests that it is not optimal for firms to utilise their entire debt capacity.

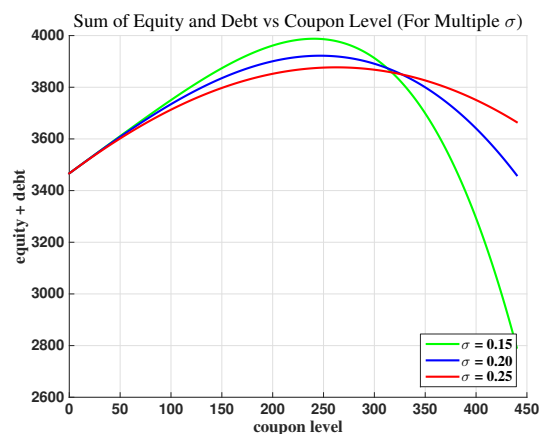


Fig. 5.2: Sum of equity and debt vs coupon level for various σ

Tab. 5.2: Effect of σ on the optimal capital structure

σ	Optimal Coupon	Equity	Debt	Optimal Leverage Ratio
0.15	240.51	838.44	3149.34	0.79
0.20	246.67	945.50	2976.57	0.76
0.25	262.44	1009.76	2867.31	0.74

Figure 5.3 indicates that the optimal coupon is a decreasing function of bankruptcy costs. As bankruptcy costs rise, the probability of default decreases¹. This, in turn, decreases the yield spread on the perpetual bond. This phenomenon is illustrated in Figure 5.4. The graph also shows that the yield spread on an optimal perpetual debt rises when there is an increase in firm risk. Figure 5.3 further highlights that the optimal coupon is a convex function of volatility. This means that firms with low or high risk will optimally promise to pay a larger coupon. On the other hand, firms with intermediate levels of risk will optimally commit to paying a smaller coupon. The effect of the bankruptcy loss ratio on the optimal capital structure is shown in Figure 5.5 and Table 5.3. These results show that the sum of debt and equity decreases for higher bankruptcy costs.

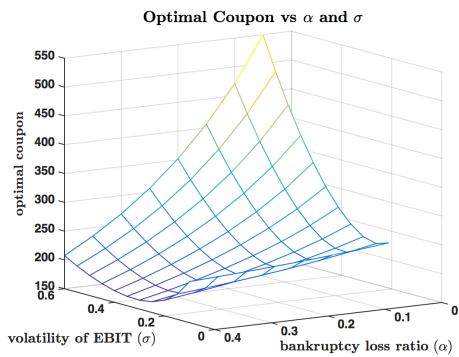


Fig. 5.3: Optimal coupon sensitivity

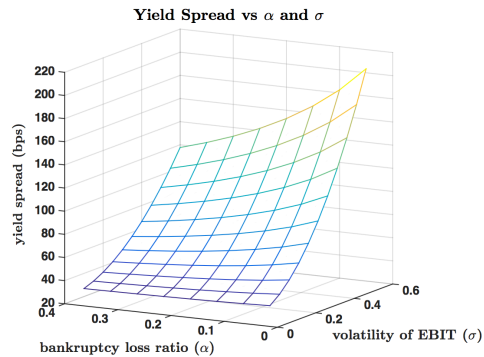


Fig. 5.4: Sensitivity of yield spread

¹A lower coupon level implies a lower default trigger V_B .

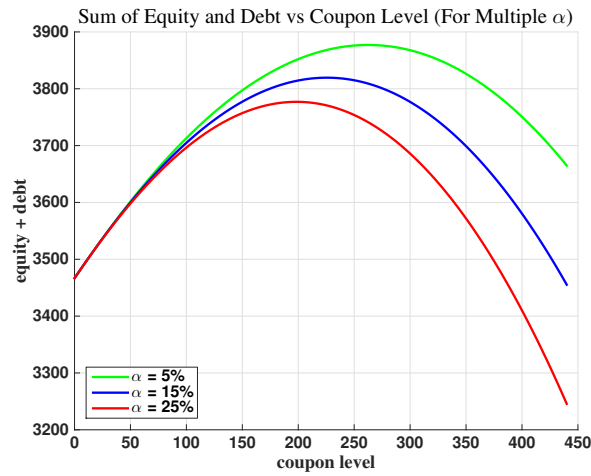


Fig. 5.5: Effect of α on the sum of debt and equity

Tab. 5.3: Effect of α on the optimal capital structure

α	Optimal Coupon	Equity	Debt	Optimal Leverage Ratio
0.05	262.44	1009.76	2867.31	0.74
0.15	225.61	1273.67	2545.80	0.67
0.25	198.45	1486.52	2290.48	0.61

The effect of taxation on the equity, debt and government claimants is shown in Figure 5.6. We can see that the equity value of the firm is a decreasing function of the corporate tax rate τ_c . This is due to the fact that a rise in the corporate tax rate increases the cash flows owed to the government, at the expense of equity.

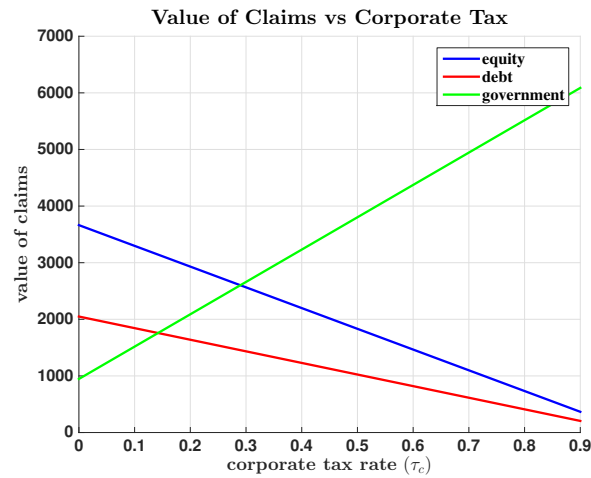


Fig. 5.6: Effect of corporate tax on claimants

The sensitivity of the equity value to changes in the initial EBIT and EBIT volatility is displayed in Figure 5.7. As expected, higher EBIT values result in greater equity values. Additionally, we can see that given an initial EBIT value, the equity value rises steeply for an increase in volatility from low levels. On the other hand, the relationship between equity and volatility flattens for an increase in volatility from high levels. This is primarily due to the fact that the value of default losses dominates the equity value for high levels of firm risk, as there is a greater chance of default for riskier firms.

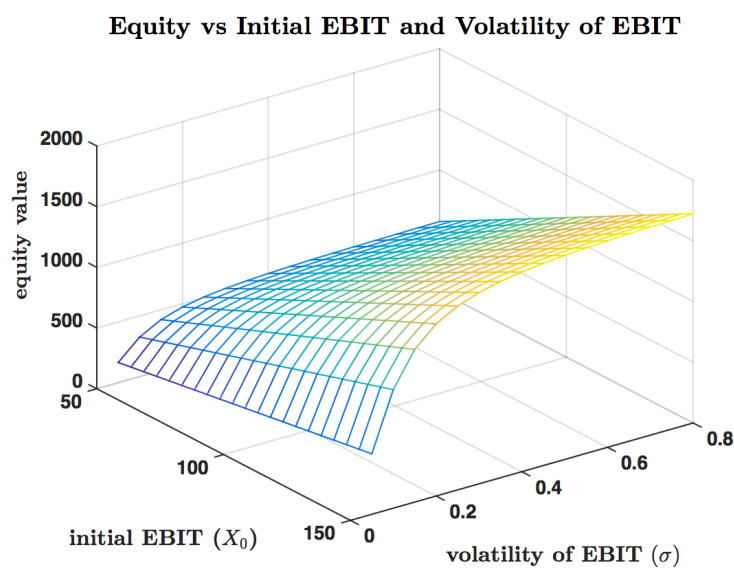


Fig. 5.7: Sensitivity of equity value

5.2 Preamble to Pricing the Employee Stock Option

Before we attempt to price the employee stock option, we need to highlight how certain values reported in the firm's financial statements relate to the parameters and variables in the framework presented by Goldstein *et al.* (2001). This interpretation is vital, as it allows one to correctly calibrate the risk-neutral drift of the EBIT process. The reported enterprise value is a representation of the fair market value of the firm's assets. Therefore, we can think of the enterprise value as a market value that is derived after the effects of taxation have been considered. To elaborate, any buyer of the firm's assets will only pay for the net asset value of the firm, i.e. they will pay for the income stream generated by the assets after tax. The fair asset value should therefore exclude any government claims. Using similar arguments, any buyer in the market will not be prepared to pay for bankruptcy costs. Therefore, the fair asset value should exclude bankruptcy claims as well. The

model proposed by Goldstein *et al.* (2001) states that the total firm value V_0 , is generated from an income stream that is redistributed among all claimants (including the government and bankruptcy). It is therefore evident that this notion of firm value does not coincide with the firm's market value of assets (reported enterprise value).

The total debt value reflected in the balance sheet is the total amount the firm must pay to settle their outstanding debt, and can be interpreted as the face value of debt. Fundamentally, this amount does not coincide with the variable $D(V_0)$ in the model. The total debt claim $D(V_0)$, represents the bondholders' valuation (or market/fair value) of the debt. Furthermore, this claim includes bankruptcy, whereas the debt value on the balance sheet does not consider bankruptcy. On the basis of these arguments, we represent the total debt value reported on the balance sheet as the model parameter V_B (i.e. bankruptcy level = face value of debt). This will enforce a natural scenario where the firm will be declared insolvent, if the total firm value falls below the value owed to debt holders. The reported market capitalisation (total equity value of the firm) can be represented by $Eq(V_0)$ in the model. This is the case, as the reported market value for equity should include the effects of taxation and bankruptcy. On the basis of these arguments, the total debt and equity values reflected in the balance sheet are governed by Equations (3.37) and (3.31) respectively. It should be noted that the model assumes a constant default trigger V_B . This implies that we are assuming that the total outstanding debt remains constant over the tenor of the option. To conclude, we stress that the smooth pasting condition (Equation (3.37)), must hold whether the coupon level C is optimal or not, to enforce limited liability. Furthermore, it should be noted that by allowing the total debt to be governed by Equation (3.37), we are forcing the coupon amount to be intricately linked to the probability of default. This is required as the coupon should always reflect the fair spread, which is related to the probability of bankruptcy. To elaborate, in a scenario where we have a higher coupon level C , the bondholders are extracting more cash from the EBIT stream. Consequently, the firm value is more likely to reach the level of bankruptcy V_B . Such a scenario explains why C is required to be related to the notion of default.

5.3 Pricing the Employee Stock Option

In this section, we price the employee stock option across two main scenarios. In the first scenario, the option is priced under an instantaneously calibrated risk-neutral drift that is derived from spot EBIT and balance sheet values of the firm. Although this method recovers the current EBIT value, we argue that this may be

a naive approach by noting that the EBIT process is sensitive to the current state of the firm and can vary drastically depending on the current economic climate and seasonal changes. To resolve this issue, we also use an option pricing framework which incorporates an average EBIT process. Specifically, in this approach, the option is priced using a historically calibrated drift that is derived from data over a range of three years prior to the valuation date in question. We choose to calibrate the risk-neutral drift over three years and assume the EBIT behaviour over this tenor is indicative of future behaviour. The three-year period is also chosen as it coincides with the option maturity length in question. Furthermore, the same risk-free rate that is used in the option pricing model is applied to estimate the historical drift. We therefore use the best estimate of the constant r that we believe rules for the last three years and the next three years from the valuation date. In an ideal world, we would need a term structure of risk-free and risk-neutral drift rates that can be utilised in a model, which incorporates the time dependency of these rates. However, the model considered in this framework relies on constant rates, and we leave the development of a time-dependent model for future research.

Within each scenario, we also price an employee stock option under two capital structure regimes. In the first regime, the option is priced in a framework which coincides with the firm's current capital structure. In the second regime, we implement an optimal capital structure condition that is presented by [Goldstein *et al.* \(2001\)](#). It is assumed that the firm will not restructure their debt (coupon level) over the tenor of the option, as we have imposed a static capital structure model in the option pricing framework. Pricing under the first regime implies that we must use a coupon level C that remains consistent with the firm's current capital structure. To justify the use of this sub-optimal² debt (coupon) level, it should be noted that in reality, management may not want or be able to adjust their current capital structure due to constrained market conditions and the elasticity of debt. For example, risky firms may find it extremely hard to increase their debt levels, as there may not be enough appetite in the market to absorb this risk. In addition, high leveraged firms may find it difficult to reduce their debt levels, as it may come at the expense of the firm selling their assets. In the optimal setting, the option is priced under the assumption that management executes an optimal capital structure strategy (finds an optimal coupon level C^*) in order to maximise the wealth of current equity holders.

Table 5.4 presents the data and parameters required to price an employee stock option for the South African private company, Company A.

²Note, for the remainder of this study, "current capital structure" and "sub-optimal capital structure" are used interchangeably.

Tab. 5.4: Firm and option pricing parameters

Valuation Date, t_0	25 June 2017
Corporate Tax Rate, τ_c	0.28
Dividends Tax Rate, τ_d	0.20
Interest Payment Tax Rate, τ_i	0.28
Risk-Free Rate, r	0.0875
Bankruptcy Loss Ratio, α	0.05
EBIT Vesting Condition, $\beta = 1 + r$	1.0875
Option Maturity (Years), T	3
Volatility of EBIT, σ	0.3054
EBIT on Valuation Date (Millions of ZAR), X_0	6438
Total Equity Value (Millions of ZAR), $Eq(V_0)$	57623.44
Total Outstanding Debt (Balance Sheet) (Millions of ZAR), V_B	13352
Number of Shares Outstanding (Millions), ns	960.548602
Current Earnings Performance Multiple, $\frac{\text{Enterprise Value}}{\text{EBIT}}$	9.89

The risk-free estimate was obtained from yield data of a ten-year South African government bond. We choose a bankruptcy loss ratio of $\alpha = 5\%$. In other words, only $1 - \alpha = 95\%$ of the available bankruptcy firm value V_B can be distributed to all bankruptcy claims.³ Lawyers therefore have a claim on 5% of the firm value in the event of default.

5.3.1 Option Pricing Approach 1: Instantaneous Calibration

In order to implement the option pricing model, we need to calibrate for the three unknown parameters. These parameters are; C (instantaneous coupon level), V_0 (initial total firm value) and μ (risk-neutral drift of EBIT). This is achieved using our known parameter values; X_0 (initial EBIT value), $Eq(V_0)$ (current market value of equity) and V_B (bankruptcy level = face value of debt). Through the aid of the mathematical relationships presented in Chapter 3, we can express the relationship between the known and unknown parameters in a system of three equations with three unknowns, and therefore obtain unique values for our parameters of concern. Concretely, the calibration procedure is conducted in the following manner:

The current equity value of the firm is governed by:

$$Eq(V_0, \mu, C) = (1 - \tau_{eff}) [V_{solv}(V_0, \mu) - V_{int}(V_0, \mu, C)], \quad (5.1)$$

³Goldstein *et al.* (2001) use this value of α and base it on empirical evidence provided by Gruber and Warner (1977).

where,

$$V_{solv}(V_0, \mu) = V_0(\mu) - V_B P_B(V_0, \mu), \quad (5.2)$$

$$V_{int}(V_0, \mu, C) = \frac{C}{r} [1 - P_B(V_0, \mu)], \quad (5.3)$$

$$P_B(V_0, \mu) = \left(\frac{V_0(\mu)}{V_B} \right)^{-x(\mu)}, \quad (5.4)$$

$$V_B(\mu, C) = \frac{x(\mu)}{x(\mu) + 1} \left(\frac{C}{r} \right), \quad (5.5)$$

$$V_0(\mu) = \frac{X_0}{r - \mu}, \quad (5.6)$$

and

$$x(\mu) = \frac{1}{\sigma^2} \left[\left(\mu - \frac{\sigma^2}{2} \right) + \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right]. \quad (5.7)$$

Substituting Equations (5.2), (5.3), (5.4) and (5.6) into Equation (5.1) we have:

$$Eq_{solv}(\mu, C) = (1 - \tau_{eff}) \left[\frac{X_0}{r - \mu} + \left(\frac{X_0}{(r - \mu)V_B} \right)^{-x(\mu)} \left(\frac{C}{r} - V_B \right) - \frac{C}{r} \right], \quad (5.8)$$

From Equation (5.5),

$$C = \frac{V_B(x(\mu) + 1)r}{x(\mu)}. \quad (5.9)$$

Substituting Equation (5.9) into Equation (5.8) yields

$$Eq(\mu) = (1 - \tau_{eff}) \left[\frac{X_0}{r - \mu} + \left(\frac{X_0}{(r - \mu)V_B} \right)^{-x(\mu)} \left(\frac{\frac{V_B(x(\mu)+1)r}{x(\mu)}}{r} - V_B \right) - \frac{\frac{V_B(x(\mu)+1)r}{x(\mu)}}{r} \right]. \quad (5.10)$$

We have now reduced the system to one equation in one unknown. Accordingly, we can solve for μ using Equation (5.10), given that we have an initial value for the equity value of the firm. Having obtained our estimate for μ , it follows that we can solve for the remaining two parameters, C and V_0 using Equations (5.9) and (5.6) respectively. From this procedure, we obtain a total firm value estimate of $V_0 = 123386.20$ (millions of ZAR), an instantaneous coupon level of $C = 2100.07$ (millions of ZAR), and an instantaneous market calibrated risk-neutral drift of $\mu = 3.5322\%$. We refer to this drift as being “market calibrated” as we are estimating μ based on the market/fair value of equity.

In Chapter 3, we explained that the optimal debt level is one that maximises the sum of debt and equity. Figure 5.8 shows this sum as a function of the coupon level C . Both the optimal and sub-optimal coupon levels are clearly depicted. The sub-optimal coupon level is determined directly from the above calibration procedure.

In this scenario, C is determined such that the default level V_B coincides with the total outstanding debt given in Table 5.4. As discussed earlier, it follows that we have allowed C to relate to the probability of default, based on the firm's current debt level. This sub-optimal coupon remains consistent with the firm's current capital structure. The optimal coupon is determined directly by Equation (3.40).

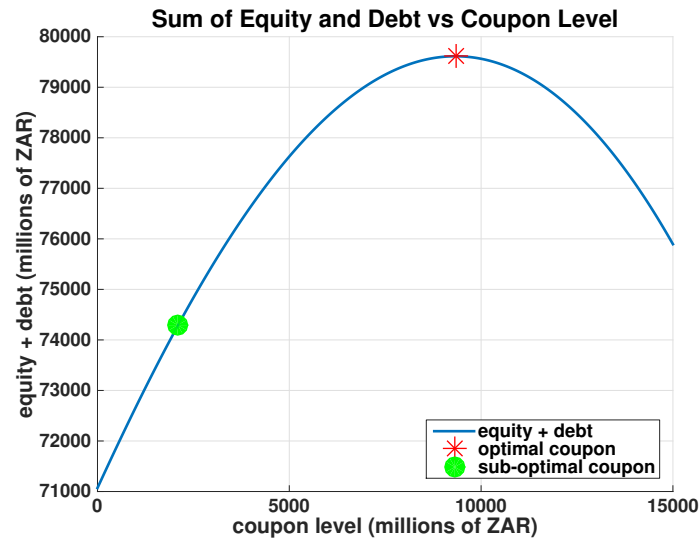


Fig. 5.8: Sum of equity and debt vs coupon level: approach 1

Table 5.5 summarises the firm's capital structure under both optimal and sub-optimal conditions.

Tab. 5.5: Capital structure summary: approach 1

	Optimal Capital Structure	Sub-Optimal (Current) Capital Structure
Coupon Level (Millions of ZAR), C	9331.25	2100.07
Debt (Millions of ZAR), $D(V_0)$	59087.95	16666.79
Equity (Millions of ZAR), $Eq(V_0)$	20525.60	57623.44
Leverage Ratio, $\frac{D(V_0)}{Eq(V_0)+D(V_0)}$	0.74218	0.2243
Corporate Bond Yield, $\frac{C}{D(V_0)}$	0.1579	0.1260
Default Level (Face Value of Debt) (Millions of ZAR), V_B	59327.04	13352.00

The results show that should the firm opt to implement an optimal capital structure strategy, it should acquire additional debt (demand a higher coupon level). However, such a drastic change may not be achievable based on current market conditions and management policy. Hence, we also present results that are consistent with the firm's current capital structure.

The value of an employee stock option under each capital structure regime is presented in Figure 5.9. The valuation is based on the data presented in Table 5.4 and 5.5, and further considers various EBIT multiples.

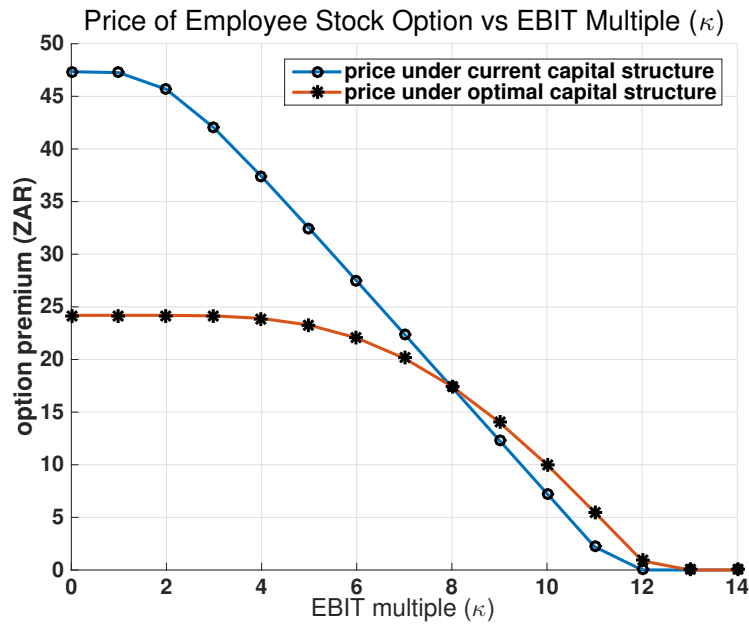


Fig. 5.9: Option premium (IFRS Expense) of employee stock option: approach 1

As expected, a higher strike (EBIT multiple κ) yields a greater probability of the option ending out the money. Equivalently, a lower specified κ results in a larger option premium. Our simulation results also show that 78.44% of the EBIT sample paths satisfied the vesting condition. Table 5.6 gives the final premium values for a single employee stock option under an optimal and sub-optimal capital structure regime.

Tab. 5.6: Value of employee stock option for Company A: approach 1

EBIT Multiple	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Option Premium Using Current Debt Level														
Upper Three-Standard Deviation Bound (ZAR)	47.40	47.30	45.71	42.04	37.42	32.50	27.49	22.44	17.39	12.33	7.27	2.22	0.01	0.00
Option Premium (IFRS Expense) (ZAR)	47.34	47.25	45.66	42.00	37.38	32.47	27.46	22.42	17.37	12.32	7.27	2.21	0.01	0.00
Lower Three-Standard Deviation Bound (ZAR)	47.29	47.19	45.61	41.95	37.34	32.43	27.43	22.40	17.35	12.31	7.26	2.21	0.01	0.00
Option Premium Using Optimal Debt Level														
Upper Three-Standard Deviation Bound (ZAR)	24.24	24.24	24.23	24.18	23.94	23.31	22.08	20.15	17.47	14.05	10.02	5.51	0.88	0.00
Option Premium (IFRS Expense) (ZAR)	24.20	24.20	24.19	24.14	23.90	23.27	22.05	20.12	17.44	14.04	10.01	5.50	0.88	0.00
Lower Three-Standard Deviation Bound (ZAR)	24.15	24.15	24.15	24.10	23.86	23.23	22.01	20.09	17.42	14.02	10.00	5.49	0.88	0.00

We have merely presented the risk-neutral price implied by the market. To elaborate, the fact that we using the spot equity value in our model immediately dictates that we are using a market implied risk-neutral drift. We now present a price surface which highlights the sensitivity of the option price, as a function of the risk-neutral drift parameter in the EBIT process. For illustrative purposes, the sensitivity analysis is conducted under the assumption the firm issues an optimal level of debt. In addition, this procedure is performed by fixing the firm value estimate

V_0 , and allowing the EBIT value X_0 , to vary to accommodate various levels of μ . Figure 5.10 highlights that the option premium is sensitive to the risk-neutral drift, where it can be seen that the option price increases for larger values of μ . The price increase stems from the fact that higher μ values imply greater terminal EBIT and equity values. This results in a larger probability of the option ending in-the-money. Furthermore, higher EBIT values imply a greater likelihood of the vesting condition being met, which in turn translates to larger option values. Figure 5.10 also illustrates that larger values for μ allow strictly positive option values to be defined over a greater range of κ values. This is a direct result of the positive correlation between the equity value of the firm and the risk-neutral drift of EBIT.

The advantage of using an instantaneously calibrated risk-neutral drift lies in the fact that we are using an estimate that is consistent with the spot values reflected in the firm's financial statements. However, the high degree of sensitivity illustrated in Figure 5.10, suggests that an instantaneously calibrated drift may be a naive approach and that a historically calibrated drift should be considered. Furthermore, a clear shortcoming in this calibration method lies in the fact that we are using a spot EBIT value, which has its disadvantages (as alluded to earlier). Thus, under this approach, we may derive significantly different premiums across different valuation dates. Accordingly, we now resort to an option pricing framework that incorporates an average EBIT process and utilises a historical calibrated risk-neutral drift parameter.

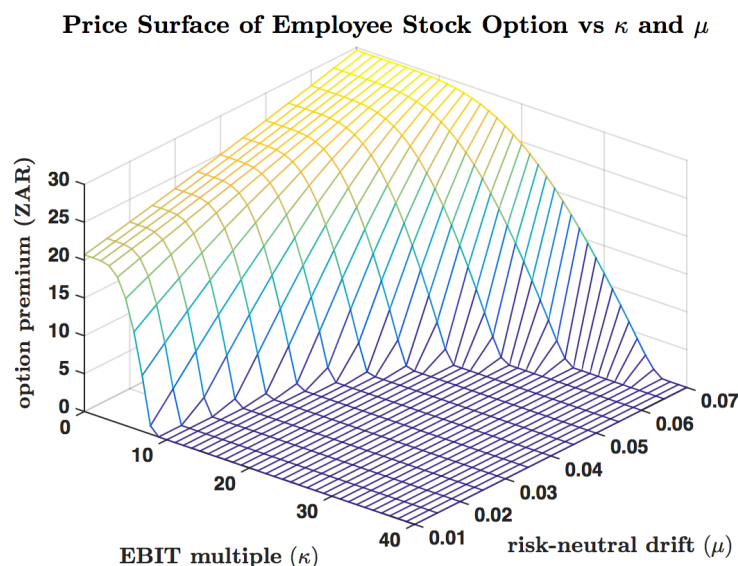


Fig. 5.10: Sensitivity of employee stock option premium to μ

5.3.2 Option Pricing Approach 2: Historical Calibration

In this approach, we first perform the same calibration procedure (outlined in Section 5.3.1) across multiple dates, where annual reported EBIT, equity and debt values over a period of three years prior to the valuation date are considered. From this procedure, we obtain instantaneously calibrated μ values across each date. These values are then averaged to derive a historical estimate of the risk-neutral drift. This is denoted by $\bar{\mu}$. Once this is obtained, our unknown parameters are now C , \bar{X}_0 and \bar{V}_0 . \bar{X}_0 and \bar{V}_0 denote average values for the EBIT and firm value. Using an average risk-neutral drift implies that we are using an average EBIT and firm value process. Furthermore, if an average risk-neutral drift is utilised, we have to use average EBIT and firm values to recover the current equity and debt level. Lastly, to solve for the three unknown parameters, we perform a similar calibration procedure, where our known parameter values are now $Eq(\bar{V}_0)$, V_B and $\bar{\mu}$. From these procedures, we obtain an average risk-neutral drift of $\bar{\mu} = 4.6353\%$, an instantaneous coupon level of $C = 2023.09$ (millions of ZAR), a total firm value estimate of $\bar{V}_0 = 122690.47$ (millions of ZAR) and an EBIT estimate of $\bar{X}_0 = 5048.35$ (millions of ZAR).

In this approach, we have essentially averaged the risk-neutral drifts across hypothetical valuation dates. Although Equation (3.4) only holds under the risk-neutral measure \mathbb{Q} , it should be noted that we are taking the reported EBIT values and estimated total firm values over the three-year period to serve as initial conditions in the GBM diffusions under \mathbb{Q} . In other words, each reported EBIT value and firm value estimate is denoted by X_0 and V_0 respectively, as we are effectively assuming each reported date over the period serves as a valuation date. We are therefore averaging the instantaneously calibrated risk-neutral drifts using market data across each “valuation date.”

Figure 5.11 and Table 5.7 summarise the optimal and current capital structure of the firm with $\bar{\mu} = 4.6353\%$.

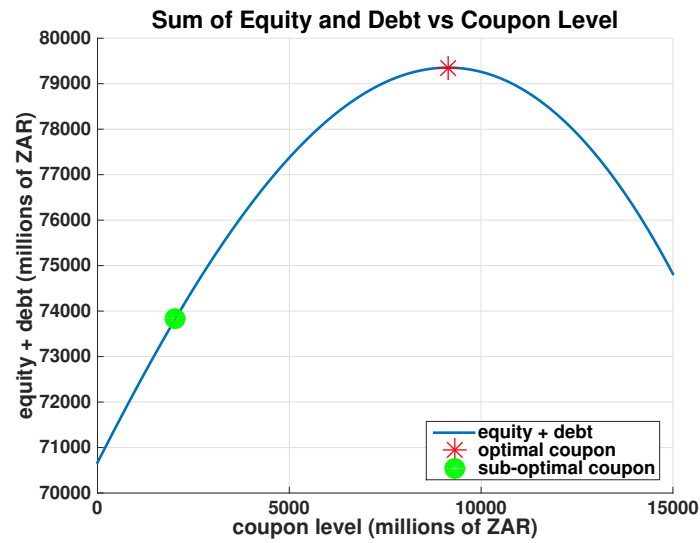


Fig. 5.11: Sum of equity and debt vs coupon level: approach 2

Tab. 5.7: Capital structure summary: approach 2

	Optimal Capital Structure	Sub-Optimal Capital Structure
Coupon Level (Millions of ZAR), C	9136.88	2023.09
Debt (Millions of ZAR), $D(\bar{V}_0)$	59204.39	16196.53
Equity (Millions of ZAR), $Eq(\bar{V}_0)$	20148.73	57623.44
Leverage Ratio, $\frac{D(\bar{V}_0)}{Eq(\bar{V}_0)+D(\bar{V}_0)}$	0.7461	0.2194
Corporate Bond Yield, $\frac{C}{D(\bar{V}_0)}$	0.1543	0.1249
Default Level (Face Value of Debt) (Millions of ZAR), V_B	60301.46	13352.00

The value of an employee stock option, for both levels of debt, is illustrated in Figure 5.12. The numerical option values for each specified EBIT multiple are presented in Table 5.8. Additionally, with a drift of $\bar{\mu} = 4.6353\%$, there is a 79.74% probability of satisfying the earnings-based vesting condition.

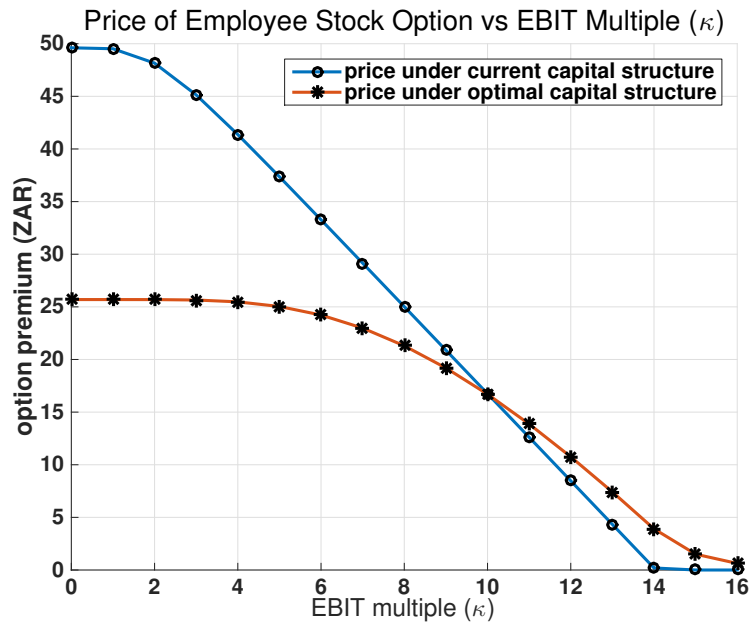


Fig. 5.12: Option premium (IFRS Expense) of employee stock option: approach 2

Tab. 5.8: Value of employee stock option for Company A: approach 2

EBIT Multiple	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Option Premium Using Current Debt Level																	
Upper Three-Standard Deviation Bound (ZAR)	49.67	49.58	48.21	45.18	41.41	37.40	33.30	29.18	25.05	20.91	16.77	12.62	8.48	4.34	0.19	0.00	0.00
Option Premium (IFRS Expense) (ZAR)	49.61	49.52	48.15	45.14	41.37	37.36	33.27	29.15	25.02	20.89	16.75	12.61	8.47	4.33	0.19	0.00	0.00
Lower Three-Standard Deviation Bound (ZAR)	49.55	49.47	48.10	45.09	41.33	37.32	33.24	29.12	24.99	20.86	16.73	12.60	8.46	4.33	0.19	0.00	0.00
Option Premium Using Optimal Debt Level																	
Upper Three-Standard Deviation Bound (ZAR)	25.74	25.74	25.73	25.69	25.52	25.08	24.26	23.02	21.33	19.22	16.72	13.89	10.76	7.42	3.91	1.51	0.63
Option Premium (IFRS Expense) (ZAR)	25.69	25.69	25.69	25.65	25.48	25.04	24.22	22.98	21.30	19.20	16.70	13.87	10.75	7.42	3.91	1.50	0.62
Lower Three-Standard Deviation Bound (ZAR)	25.65	25.65	25.65	25.61	25.43	24.99	24.19	22.95	21.27	19.17	16.68	13.85	10.74	7.41	3.91	1.50	0.62

In comparison to the results presented in the previous section, we can see that a larger μ demands a larger option premium, and further allows strictly positive option premiums to be defined over a greater range of EBIT multiples. Furthermore, the results show larger option premiums under an optimal debt level for larger values of κ . For lower κ values, the price of the option is higher under the firm’s current capital structure. This phenomenon follows mainly by considering the formulation of the strike, and noting that the optimal capital structure commands a higher coupon level than one which coincides with the firm’s current debt balance.

Figure 5.13 presents a typical distribution under the risk-neutral measure \mathbb{Q} for the terminal equity price of the firm ($S_T(X_T)$ in Equation (3.45)) using 50,000 sample EBIT paths.

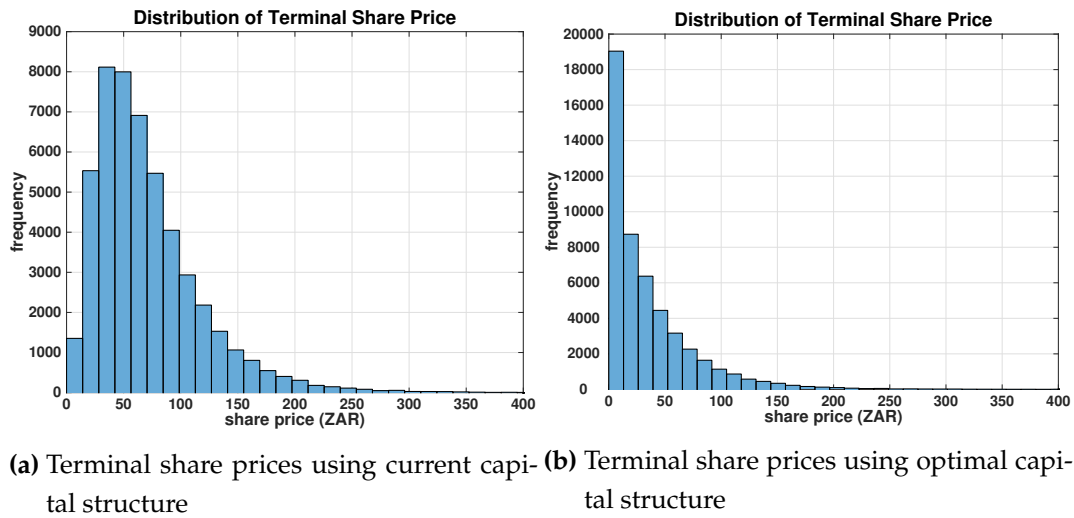


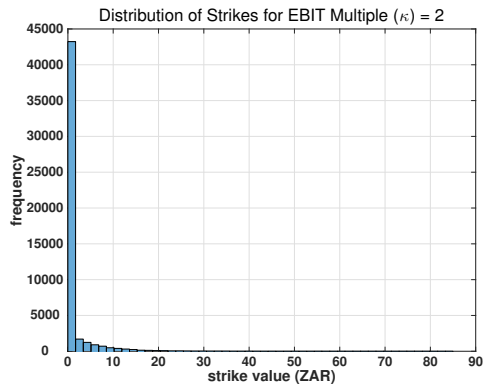
Fig. 5.13: Distribution of terminal equity price

Figure 5.13 illustrates that the average terminal equity price (in expectation) is higher under the sub-optimal debt level. This is expected as the model suggests that the optimal debt is higher than the firm's current debt level. Furthermore, the optimal coupon is derived by maximising the asset value (sum of debt and equity) and not equity alone. Figure 5.13 also highlights that the terminal equity prices will typically exhibit heavy-tailed and positively skewed distributions. Table 5.9 presents a summary of the terminal equity price statistics (under the measure \mathbb{Q}) for both capital structure conditions considered in this study.

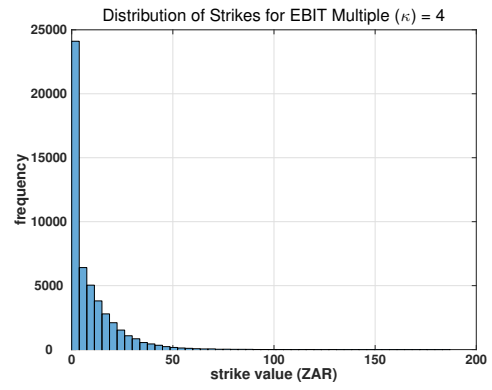
Tab. 5.9: Summary statistics for terminal share price

	Current Debt Level	Optimal Debt Level
Average Terminal Share Price	71.04	34.96
Sample Kurtosis of Terminal Share Price	10.19	14.44
Sample Skewness of Terminal Share Price	1.91	2.55
Standard Deviation of Terminal Share Price	47.88	41.56

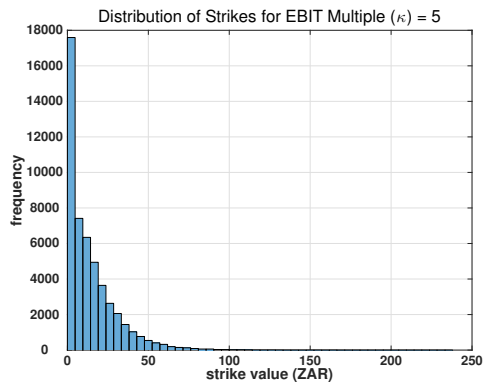
Figure 5.14 illustrates a typical distribution of the strike of the employee stock option (Equation (3.47)), for various EBIT multiples κ under the firm's current debt level. This distribution is also seen to be positively skewed. Furthermore, it can be observed that we have a larger range of strike values for higher values of κ . This is due to the fact that the minimum value allowed for the strike K is zero, and noting that the larger the EBIT multiple, the smaller the likelihood of obtaining a strike value of zero.



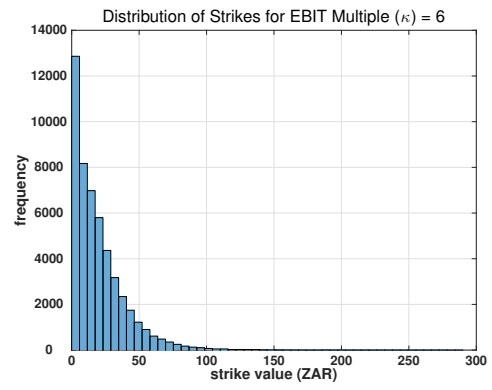
(a) Distribution of strikes for $\kappa = 2$



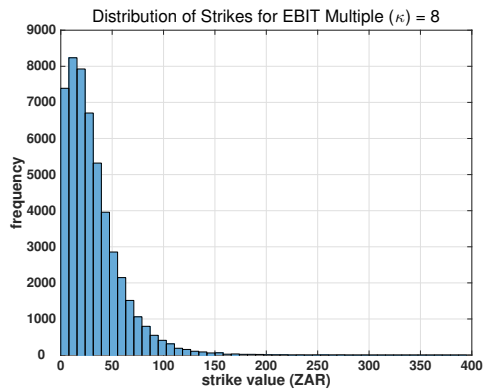
(b) Distribution of strikes for $\kappa = 4$



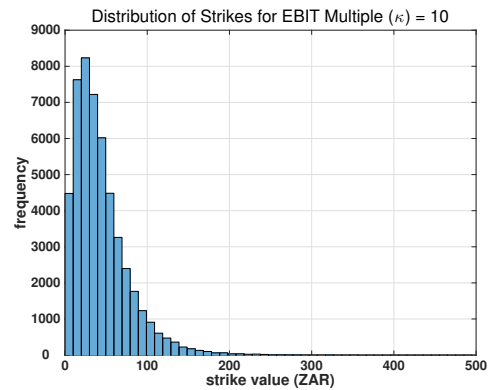
(c) Distribution of strikes for $\kappa = 5$



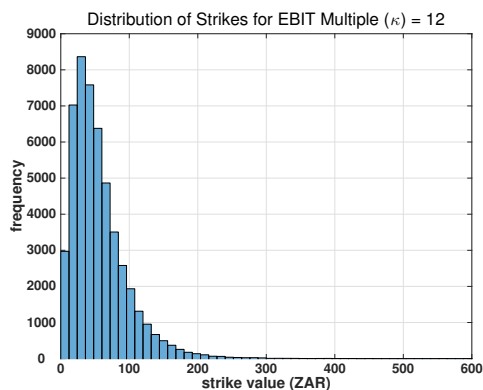
(d) Distribution of strikes for $\kappa = 6$



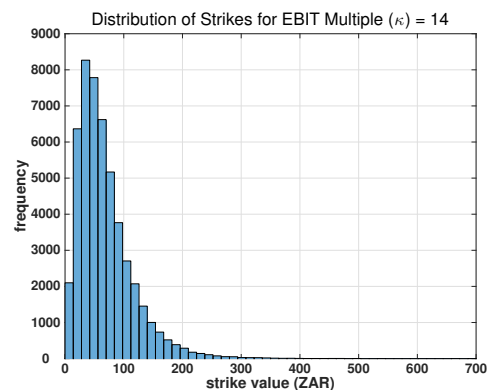
(e) Distribution of strikes for $\kappa = 8$



(f) Distribution of strikes for $\kappa = 10$



(g) Distribution of strikes for $\kappa = 12$



(h) Distribution of strikes for $\kappa = 14$

Fig. 5.14: Distribution of strike values under current capital structure

5.3.3 Sensitivity Analysis

Since the option pricing framework assumes a constant volatility and risk-free rate, it is worth exploring the effect of changes in these model input parameters on the value of the option. The sensitivity analysis is performed using the same calibration procedure to recover the current equity and debt values of the firm. The analysis also utilises an average risk-neutral drift for EBIT. Figure 5.15 illustrates an option pricing surface for various values of the volatility of EBIT, and highlights that the option premium is an increasing function of volatility. Figure 5.16 portrays the option's sensitivity to changes in the risk-free rate. The illustration shows that a change in the risk-free rate over the given range does not alter the option premium to a great degree.

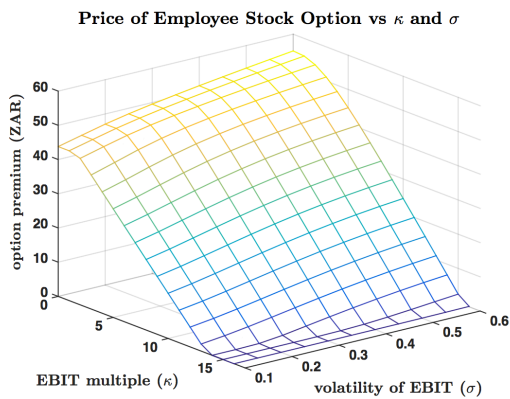


Fig. 5.15: Volatility sensitivity



Fig. 5.16: Risk-free rate sensitivity

Chapter 6

Conclusion and Recommendations for Future Research

This dissertation aimed to value an employee stock option in a framework where both the equity value and vesting condition are based on an underlying EBIT process. The research aim was addressed by incorporating a static capital structure model, provided by [Goldstein *et al.* \(2001\)](#), in the option pricing framework. The model assumes that the EBIT process is the source of firm value and is governed by a geometric Brownian motion. The model further assumes that the firm is fully financed by equity and one perpetual bond that pays a constant continuous coupon. With these assumptions, we have presented mathematical arguments that describe the derivation of closed-form solutions, for all claims on the firm's value process. The claims of concern are equity, debt, government (through taxation) and bankruptcy costs.

We considered two calibration approaches to estimate the risk-neutral drift parameter of the firm's EBIT process. In the first procedure, an instantaneous risk-neutral drift estimate was obtained, allowing the recovery of the spot EBIT value. In the second approach, we utilised an average EBIT process, which allowed us to obtain an average risk-neutral drift parameter. We argued that pricing using an average risk-neutral drift may be more reasonable. We supported this argument by noting that the EBIT process is sensitive to the current economic state of the firm, and further noting a high degree of sensitivity between the option price and the risk-neutral drift parameter. Additionally, we priced the ESO under a model which recovered the firm's current equity and debt value. The option was also priced under an optimal capital structure condition (provided by [Goldstein *et al.* \(2001\)](#)), as a means to obtain comparable results. The results showed that we can obtain significantly different option premiums if the valuation is performed under an optimal capital structure, as opposed to a valuation which remains consistent with the firm's current equity and debt level.

In this dissertation, we have presented a European-style employee stock option pricing framework, where both the equity value and vesting condition are based on an underlying earnings process. There are a number of possible extensions that can be considered. Firstly, the framework can be extended to include early exercise opportunities. In addition, a dynamic capital structure model could be incorporated in the option pricing framework. A further extension could see the development of a model which incorporates a term structure of risk-free rates and risk-neutral drifts of the earnings process. Lastly, a valuable consideration lies in the development of an option pricing framework, which assumes alternate dynamics for the firm's earnings (for example, mean-reverting, arithmetic Brownian motion, or stochastic volatility process).

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Appendix A

Relevant Derivations and Theorems

A.1 Derivation of Firm Value

In the case where for $r > \mu$, we have:

$$\begin{aligned} V_t &= \mathbb{E}_{\mathbb{Q}} \left(\int_t^{\infty} e^{-r(s-t)} X_s ds \right), \\ &= \int_t^{\infty} e^{-r(s-t)} \mathbb{E}_{\mathbb{Q}}(X_s) ds, \\ &= \int_t^{\infty} e^{-r(s-t)} X_t e^{\mu(s-t)} ds, \\ &= X_t e^{rt} e^{-\mu t} \int_t^{\infty} e^{-s(r-\mu)} ds, \\ &= X_t e^{(r-\mu)t} \left[0 + \frac{e^{-t(r-\mu)}}{r-\mu} \right], \\ &= \frac{X_t}{r-\mu}. \end{aligned}$$

A.2 Feynman - Kac Theorem in One Dimension

A generalisation of the Feynman-Kac argument shows that the solution of a PDE of the form:

$$\mu(t, V_t) F_V + \frac{1}{2} \sigma(t, V_t)^2 F_{VV} + F_t + P - rF = 0,$$

with boundary condition

$$F(T, V_T) = \Phi(T, V_T),$$

can be expressed as an expectation:

$$F(t, V) = \mathbb{E}_{\mathbb{Q}} \left[\int_t^T e^{-\int_t^u r_s ds} P du + e^{-\int_t^T r_s ds} \Phi(T, V_T) \right],$$

involving a diffusion of the form:

$$dV_s = \mu(s, V_s) ds + \sigma(s, V_s) dW_s^{\mathbb{Q}}, \quad \text{for } t \leq s \leq T,$$

where $F(t, V)$ is a differentiable function of t and V , μ is the drift of V under \mathbb{Q} , r is the risk-free rate and P is a payout flow.

A.3 Default Level

We require

$$\left. \frac{\partial E}{\partial V} \right|_{V=V_B} = 0.$$

Since

$$E_{q_{solv}}(V) = (1 - \tau_{eff}) [V_{solv}(V) - V_{int}(V)],$$

and using Equations 3.15, 3.18 and 3.20, we have that

$$\left. \frac{\partial E}{\partial V} \right|_{V=V_B} = (1 - \tau_{eff}) \left(\left. \frac{\partial V_{solv}(V)}{\partial V} \right|_{V=V_B} - \left. \frac{\partial V_{int}(V)}{\partial V} \right|_{V=V_B} \right),$$

where

$$\begin{aligned} \left. \frac{\partial V_{solv}(V)}{\partial V} \right|_{V=V_B} &= 1 - V_B(-x) \left(\frac{1}{V_B} \right) \left(\frac{V}{V_B} \right)^{-x-1}, \\ \left. \frac{\partial V_{int}(V)}{\partial V} \right|_{V=V_B} &= -\frac{C}{r}(-x) \left(\frac{1}{V_B} \right) \left(\frac{V}{V_B} \right)^{-x-1}. \end{aligned}$$

Thus,

$$\left. \frac{\partial E}{\partial V} \right|_{V=V_B} = 0,$$

\Rightarrow

$$\left. \frac{\partial V_{solv}(V)}{\partial V} \right|_{V=V_B} - \left. \frac{\partial V_{int}(V)}{\partial V} \right|_{V=V_B} = 0.$$

\Rightarrow

$$V_B = \lambda \left(\frac{C}{r} \right),$$

where

$$\lambda = \frac{x}{x+1}.$$

A.4 Optimal Coupon

We require

$$\left. \frac{\partial EV}{\partial C} \right|_{V=V_0} = 0,$$

where

$$EV = D + E.$$

Using Equations 3.31 and 3.32 we have,

$$EV = [(1 - \tau_i) - (1 - \tau_{eff})] V_{int}(V) + (1 - \alpha)(1 - \tau_{eff}) V_{def}(V) + (1 - \tau_{eff}) V_{solv}(V).$$

⇒

$$\begin{aligned} & [(1 - \tau_i) - (1 - \tau_{eff})] \frac{\partial V_{int}(V)}{\partial C} \Big|_{V=V_0} + (1 - \alpha)(1 - \tau_{eff}) \frac{\partial V_{def}(V)}{\partial C} \Big|_{V=V_0} \\ & + (1 - \tau_{eff}) \frac{\partial V_{solv}(V)}{\partial C} \Big|_{V=V_0} = 0. \end{aligned}$$

⇒

$$\begin{aligned} & [(1 - \tau_i) - (1 - \tau_{eff})] \left[\frac{1}{r} - \frac{1}{r}(1 + x) \left(\frac{\lambda C}{r} \right)^x V_0^{-x} \right] \\ & + (1 - \alpha)(1 - \tau_{eff}) \left[\frac{\lambda}{r}(x + 1) \left(\frac{\lambda C}{r} \right)^x V_0^{-x} \right] \\ & + (1 - \tau_{eff}) \left[-\frac{\lambda}{r}(x + 1) \left(\frac{\lambda C}{r} \right)^x V_0^{-x} \right] = 0. \end{aligned}$$

Hence, we have the following expression for the optimal coupon level,

$$C = V_0 \left(\frac{r}{\lambda} \right) \left[\left(\frac{1}{1 + x} \right) \left(\frac{A}{A + B} \right) \right]^{\frac{1}{x}},$$

where

$$\begin{aligned} A &= \tau_{eff} - \tau_i, \\ B &= \lambda \alpha (1 - \tau_{eff}). \end{aligned}$$