AN ANALYSIS INTO THE
HEDGING EFFECTIVENESS AND EFFICIENCY
OF THE SHARE INDEX FUTURES MARKET IN SOUTH AFRICA

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I certify that, except as noted above, this work is entirely my own and it has not been submitted as a dissertation for a degree at any other university.

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ABSTRACT

There has been much written on the ability of futures to reduce risk thereby hedging against potential market declines. However, the effect on return has been largely overlooked. This study investigates the risk *and* return effectiveness of hedging and hedging strategies using share index futures (SIF) market in South Africa.

The empirical analysis is based on actual market data applied in terms of the most prominent hedging strategies, namely the traditional, minimum-variance, beta and Howard & D'Antonio (H&D) strategies. As hedging effectiveness is dependant on market efficiency, an analysis of the pricing efficiency of the South African market is performed with reference to the cost-of-carry valuation model and arbitrage pricing techniques.

The results overwhelmingly indicate that the minimum-variance hedge strategy is the most optimal of the four strategies in terms of both risk and return. The beta hedge performed badly in terms of both risk and return (even worse than the naive traditional hedge strategy) and often led to overhedging. The beta strategy is not considered appropriate as an estimate of the minimum-variance hedge ratio in the South African situation because the futures price fluctuates significantly more than the spot index resulting in overstated hedge ratios.

The H&D hedge strategy provided results which were exceptionally poor in terms of both risk and return. Moreover, the H&D hedge ratios and measures of hedging effectiveness did not reflect what was anticipated from the theory. The failure of the strategy may be ascribed to the

impreciseness of imperfect financial markets coupled with the sensitivity of the H&D ratios and effectiveness measures to changes in market variables. This study has proposed new measures of hedging effectiveness which solve the problems encountered with the H&D measures.

Both risk and return increases with the increase in hedge length for all strategies. However, even hedges at longer duration displayed a substantial amount of residual (basis) risk. The basis risk is greater in share portfolios which are not adequately representative of the index. The level of basis risk was not sufficiently compensated for by increased return. This is considered the major reason why all hedges (bar two exceptions) were found to be less effective than the unhedged portfolio in terms of the risk-return measures of hedging effectiveness.

The basis risk is attributable to the large pricing differences observed between the actual futures price and the theoretical futures price. These pricing differences persisted owing to the inability to effectively short the index to reap arbitrage profits. Therefore, although the futures pricing was seen to be optimal in terms of pricing theory, the very large arbitrage bounds still permitted the futures price to fluctuate considerably resulting in basis risk. (The problem of actual dividends is considered to have a relatively minor effect on futures pricing in relation to the size of the arbitrage bounds). Unless the extent of the arbitrage bounds can be reduced, hedgers will be forced to bear this additional risk without any additional return in compensation for this additional risk.

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CHAPTER ONE

INTRODUCTION

1.1. INTRODUCTION

> "The growth and activity on financial futures markets has been spurred by the need for more meticulous financial management in times of economic uncertainty. Financial futures markets meet this need by providing a mechanism for price "discovery" and hedging - the two most important economic benefits of a futures market" (Stals, 1988: 7).

Markets are established to facilitate the transfer and allocation of resources between those that provide resources and those that consume resources. However, both providers and consumers of resources are affected by economic uncertainty with respect to the level and variability of prices. The dual role of price discovery and hedging through futures has enabled market users to reduce these two uncertainties.

The discovery of prices results from futures market participants trading and establishing expected future prices, thereby reducing the uncertainty regarding the level of prices in the future. Hedging using futures contracts allows users to eliminate the variability of market prices. Thus, through price discovery and hedging, futures markets have aided the efficient allocation of resources (Blank et al, 1991: 1) and have led to increased real production (Anderson & Danthine, 1981: 1192).

1.1.1. Definition of a Futures Contract

A futures contract is defined in the Stals Commission Report investigating financial futures in South Africa as follows (Rand Merchant Bank Ltd et al, 1988: 41);

> "A standardised contract which embodies an obligation to deliver (i.e. sell) or take delivery of (i.e. buy) a standard quantity of a commodity or financial instrument at a standard future date at a price agreed on by the parties to the contract."

A futures contract is a type of deferred delivery or forward contract where the price for a particular commodity or asset to be delivered at a future date is contracted for today. However, futures contracts differ from forwards in two aspects, namely they are standardised except for price and are almost always traded via structured futures exchanges.

1.1.2. Types of futures contracts

A futures contract can be established for any asset. The normal classification made for futures contracts is according to the underlying asset upon which the future is derived (hence "derivative instrument"). The major categories of futures derivatives are those based on commodities, short and long term interest bearing instruments, and financial indexes.

A futures contract can also be categorised by the particular characteristic it displays (Falkena et al, 1989: 3-6). Firstly, futures contracts can be based on specific or notional assets. Specific futures contracts are either based on physical assets (e.g. commodities) or financial assets (e.g. interest bearing scrip). However, futures may also be based on fictitious (or notional) assets such as share index futures.

Further classifications can be made between storable and perishable asset contracts, and between income and non-income producing asset contracts. Finally futures contracts may be settled in cash or through the physical delivery of the underlying asset.

Owing to the extensive nature of futures contracts it is not possible to address all futures types. This study focuses on share index futures (SIF), which are financial futures based on equity indexes. SIF are notional contracts so physical delivery is not possible; therefore, delivery (or settlement) is made in cash based on the value of the index at the expiration date (or sooner if the contract is closed out before expiration).

To understand share index futures it is important to understand the notional asset upon which the future is based. Stock market indexes fulfil two major functions, namely to present a standard against which portfolio performance can be objectively measured, and to provide a description of the market at any point in time from which an historical record of the market can be built (JSE, 1988: 3). In essence, the index is a barometer of the market as a whole.

Indexes are normally calculated on a weighted arithmetic average basis but occasionally a geometric average basis is used (for example the Value Line Index quoted on the Kansas City Board of Trade). The Johannesburg Stock Exchange calculates its equity indices on a weighted arithmetic average basis according to the formula below (JSE, 1988: 22):

$$I(t) = \frac{k \sum_{i=1}^{n} P_{it} W_{it}}{\sum_{i=1}^{n} W_{it}} \qquad (1\text{-}1)$$

where

$I(t)$: Value of index at time t.

P_{it} : Price per share of constituent i at time t.

n : Number of constituents in the index.

W_{it} : Number of eligible shares for constituent i at time t.

k : Constant that is chosen to base the index to a value at the starting date of the index.

Not all shares are selected for inclusion in the index calculation; instead, selection is made of a relatively small proportion of the total number of shares which represent the movement in the market as a whole. In general, selection is made of the largest shares (on a market capitalisation basis) per industry sector until 80% of the sector capitalisation has been selected (JSE, 1988: 4).

1.1.3. Uses of Futures Contracts

The uses of futures may broadly be divided between the reduction of risk (hedging) and the taking of risk in the desire for profit (speculating). Specifically, participation in futures markets may be motivated by several different reasons, including:

1. The reduction of risk due to price fluctuations (hedging).

2. The simultaneous purchase of spot and sale of futures instruments (or vice-versa) with the intention of yielding a profit (at low risk) from perceived discrepancies between cash and futures prices. This is inter-market arbitrage.

3. The simultaneous purchase and sale of different futures contracts to exploit perceived pricing discrepancies. This is intra-market arbitrage (or spreading).

4. The intention to profit from expected movements in the underlying asset by taking a position in the asset's derivative futures market. These are known as traders which may be very short term (day traders) or longer term (position traders).

5. The gearing up of a spot portfolio to enable participation in the market at greater leverage.

6. The intention to make a small turn on a trade by satisfying market participants orders. These participants are known as scalpers or locals.

Traditionally, only the first motive above has been included under hedging activities and the rest fit under the umbrella of speculation (Schwarz et al, 1986: 7). Even though arbitrage trading is risk-averse, we will abide by this classification owing to its acceptance in the futures industry.

Speculators fulfil a vital function enabling the futures market to achieve its basic economic role. Most important in this regard is that speculators provide the risk capital necessary to offset hedging transactions. Without speculators, there may be many times where individual hedgers would be unable to trade, i.e. in cases where the number of long hedgers and short hedgers are unequal. Speculators in the market are prepared to take up this shortfall (Blank et al, 1991: 183).

Speculators also assist in other aspects. They provide a significant share of market liquidity allowing the smooth flow of futures transactions. Speculative activities, and the related increase in volumes, improve the efficiency of futures markets through the increased marketability of contracts, reduction in transaction costs and ensuring that pricing between the various markets is kept in alignment through arbitrage (Blank et al, 1991: 183).

It is not possible in this study to address all hedging and speculation motives owing to the vastness of the topic. This study will concentrate on the economic purpose of futures, namely hedging (motive 1 above), using share index futures; although inter-market arbitrage (motive 2 above) will be dealt with in regard to the pricing of futures. Hedging using futures contracts will be discussed in more detail below.

1.2. HEDGING

The Commodity Futures Trading Commission (CFTC), the futures regulatory body in the United States of America, developed a comprehensive definition of hedging. The major points from the definition state that bona fide hedging transactions (or positions) are those transactions (or positions) which (Blank et al, 1991: 59):

- relate to a futures or forward or similar contract;
- are substitutes for transactions to be made or positions to be taken in the spot market;
- are economically appropriate to the reduction of risks of a commercial enterprise; and,
- where the risks which are reduced arise from the potential change in value of assets, liabilities or services that the hedger owns or that the hedger anticipates to own.

With futures it is possible to hedge against fluctuating asset (spot) prices because spot and futures prices tend to rise and fall in parallel movements in response to a common underlying set of supply and demand factors. The parallel co-movement follows naturally since the futures instrument is based on the underlying asset itself.

1.2.1. Types of Hedging Positions

The two principle types of hedging positions are the short hedge (where futures are sold) and the long hedge (where futures are bought). Examples of short and long hedges using share index futures are given below.

An investor wishing to insure a portfolio of shares against a possible market decline may sell (or "short") a futures contract. If the market declines, any loss on the share portfolio will be offset by a profit on the futures portfolio. Conversely, if the market rises the profit on the share portfolio will be matched by a loss on the futures portfolio. In this way, the extent of price fluctuations (both upward and downward) will be reduced.

A target company in a prospective merger may wish to enter into a short hedge if the purchase price is based on the target share's market price at some future date. The proceeds of the sale will be protected, provided that the target's shares move in line with the market.

The main use of a long hedge involves anticipatory transactions to be made (or positions to be established) in the foreseeable future. An investor may wish to purchase shares at some time in the future at which time the purchase price may be significantly greater if the market escalates rapidly (a bull market) before the date of acquisition. To hedge against such an expected market increase, the investor may fix the acquisition price now by buying futures.

Similarly, an acquiror company may wish to insure against an expected increase in the consideration payable on acquiring the shares in a prospective target company. This can be effected by means of a long futures hedge.

1.2.2. Reduction of Portfolio Risk

Risk can be defined as the variability of returns, i.e. the degree to which actual future returns may differ from expected returns. This risk is normally quantified by calculating the variance (or standard deviation) of the investment's returns.

Investors would like to increase their expected returns but at the same time wish to minimise the uncertainty of these returns. This trade-off between risk and return forms the basis of portfolio theory originally formulated by Markowitz (1952). According to Koutsoyiannis, portfolio theory is formulated on the following assumptions (Bowen, 1984: 19):

1. Capital markets are efficient. (The efficiency of markets and futures markets in particular, is discussed in chapter 5.)

2. The rational investor aims to choose the portfolio which maximises his expected utility.

3. The utility derived from a portfolio depends on its risk and return.

4. The investor is risk averse.

5. The investor can rank the various portfolios on the basis of their risk and return by means of a set of indifference curves.

Modern portfolio and capital market theory identify two sources of fluctuating returns of a single share (or security); namely, systematic and unsystematic risk (Weiner, 1981: 59). Systematic (market) risk stems from general market movements: in statistical terms it is the degree of correlation of a share's price movements with the general market index (often known as the beta of the share). On the other hand, unsystematic (nonmarket) risk occurs as a result of events unique to a particular share or industry (market sector).

Unsystematic risk can be diversified away by combining the share with others in a portfolio. This is because the unsystematic returns of individual shares tend to occur randomly and offset one another thereby reducing the risk. Evans & Archer (1968) showed that a random portfolio of only ten shares materially reduces unsystematic risk. In practice, according to Evans et al (1968: 707) and Mayshar

(1979), transaction costs impede full diversification so that there is normally a degree of unsystematic risk in every portfolio.

However, systematic (or market risk) cannot be diversified away through the construction of spot portfolios as this risk "results from changes in basic economic forces that affect the value of all securities" (Nordhauser, 1984: 56). A fully diversified portfolio will still be subject to systematic risk; no portfolio selection policy will protect the portfolio from a general market downturn.

Weiner (1981: 60) estimated that the level of systematic risk amounts to about 34% of the variability of common stocks, while Evans and Archer (1968) estimated the absolute level of systematic risk at 11.66%. Thus, the level of systematic risk in portfolios is substantial; consequently investors have searched for procedures to eliminate market risk.

The selection of negative beta shares (i.e. shares which tend to rise in a falling market) is not a practical solution owing to the very small number of securities which move against market trends. Blume (1971: 6) found that only 7 securities out of 4357 studied had negative betas.

Other means to protect portfolios against market declines includes the liquidation of the portfolio, selling shares short or buying put options (Nordhauser, 1984: 57). The selling of the portfolio is undesirable owing to the transaction costs incurred in effecting such a procedure and the potential difficulty of reconstructing the portfolio once the market decline has bottomed out. It may even be impossible to sell the shares if they are held for strategic reasons. Moreover, in South Africa, the last two solutions are not practically viable as, firstly, it is difficult to sell shares short, and secondly, there is no established market for share put options at present (although a traded options market is being developed).

Fortunately, share index futures provides a viable, flexible and cost-effective alternative to safeguarding the share portfolio against a general market decline. Nordhauser (1984: 61) concluded from empirical research that the use of SIF substantially reduces the variability of returns with only a minor reduction in the level of return.

1.2.3. Basis Risk

Hedging using SIF will substantially reduce portfolio risk. However, to eliminate all risk, the movement in the futures position must exactly offset the movement in the spot position, i.e. the share and the futures need to be perfectly correlated. In practice this is seldom the case and there is therefore a degree of residual risk remaining in the hedged portfolio. This risk is referred to as basis risk, where the basis is defined as the difference between the futures price and the spot price. Basis risk is neatly summed up by Figlewski (1984a: 657):

> "Hedging a position in stock will necessarily expose it to some measure of basis risk - risk that the change in the futures price over time will not track exactly the value of the cash position."

In share portfolio hedges, basis risk is introduced from two major sources. Firstly, nonmarket risk in share portfolios cannot be hedged through the use of futures (Fitzgerald, 1983: 105) and so this is often the cause of basis risk.

Evans & Archer (1968) showed that a portfolio will always contain a level of unsystematic risk. Secondly, basis risk may be introduced through uncorrelated fluctuations between the share and futures price. These fluctuations are able to take place because imperfect markets with transaction costs permit the futures price to move freely in a range of prices without the arbitage mechanism bringing them back in line (this will be discussed in depth in chapter 5 on pricing). Basis risk is only nil at the date of expiration where the closing futures prices is equal to the closing spot price as defined in the exchange rules (refer Appendix A for exchange rules for SIF).

1.3. STATEMENT OF THE PROBLEM

It has been shown above that share index futures provide a means to reduce systematic risk in portfolios. Of course, the hedger has the decision to fully hedge the portfolio, partially hedge the portfolio or to leave the portfolio unhedged; he/she may even wish to increase the leverage of the portfolio through the use of futures. In so doing, the risk-return relationships of the spot portfolio may be altered by hedgers using futures.

The problem facing hedgers is whether they are disadvantaged in terms of risk and return for entering into hedging positions. For example, while the risk of the spot portfolio is reduced, at the same time the portfolio return may be reduced to a level that is unacceptable to the hedger in view of the basis risk borne.

It is important that share index futures are effective in achieving the desired hedging objective from both a risk and a return perspective. This study will consequently investigate the effectiveness of hedging using share index futures in South Africa. Specifically, the effectiveness of four well established hedge strategies, which mirror the objectives applicable to most hedgers, will be investigated in a risk-return framework. Hedging effectiveness depends largely on the efficient operation of the futures market itself, and therefore, the pricing efficiency of the futures market is also assessed in this study.

The study is organised into eight chapters, of which the last chapter is the conclusion. Chapter two provides a brief discussion of futures and futures markets in general, and looks more closely at the South African market situation. Chapters 3 and 4 discuss the hedging strategies and previous empirical research relating to futures hedging while chapter 5 deals with futures pricing. Chapters 6 and 7 empirically appraise the pricing efficiency and hedging effectiveness of the South African futures market respectively.

CHAPTER TWO

OVERVIEW OF FUTURES MARKETS

2.1. DEVELOPMENT OF FUTURES

Futures markets developed naturally from forward markets although there is lack of consensus as to when this transformation took place (see Blank et al, 1991; Duffie, 1989; and Tewelles & Jones, 1987). However, it was not until 1848, when the Chicago Board of Trade (CBOT) was opened, that futures (as we know them today) started being traded. The CBOT was instituted to encourage grain storage and to limit grain price fluctuations (Blank et al, 1991: 3). The greater standardisation of futures contracts led to their success, and as a result, other exchanges followed suit and commodity futures spread worldwide.

The need for financial futures started with the breaking of the Bretton Woods Agreement in 1973, whereafter the financial calm of the 'golden years' was replaced by an increasingly stormy financial world of uncertainty with volatile inflation, interest and exchange rates. The Chicago Mercantile Exchange (CME) was the first to introduce financial futures in 1972 with certain foreign currency futures contracts. Various interest rate futures contracts followed shortly afterwards fueled by the increase in inflation and interest rate volatility in the 1970's and the oil shock of 1978. Volumes in financial futures soared as interest rate management became a major concern in the business community.

Increased stock market activity and fluctuations necessitated the introduction of equity index futures. In 1982, the first index future started trading on the Kansas City Board of Trade followed (only by a matter of months) by the CME and the newly formed New York Futures Exchange (NYFE). The S&P 500 index future traded on the CME became the fastest growing futures contract in history. Successes in share index futures (SIF) led to similar contracts being marketed around the world.

SIF suffered a temporary setback in 1987 after the October stock market "crash" when worldwide volumes dropped 30%. Index futures were accused as being "significant factors in accelerating and exacerbating the declines" (SEC, 1988: xiii). The spotlight was lifted when the true cause was traced to the failure of the financial market segments to act as one (Brady, 1988: vi).

The growth in the futures markets has been staggering. On U.S. exchanges (where the majority of the world's trading takes place) volumes in futures contracts have grown from only 13.6 million contracts per annum in 1970 to 245.9 million in 1987. Today, financial futures predominate, with interest rate futures accounting for 42.6% and SIF for 11.2% of the market in 1987 (Blank, 1991: 4). The value of futures contracts traded today, in many cases, exceed the value of the underlying spot markets themselves.

2.1.1. South African Developments

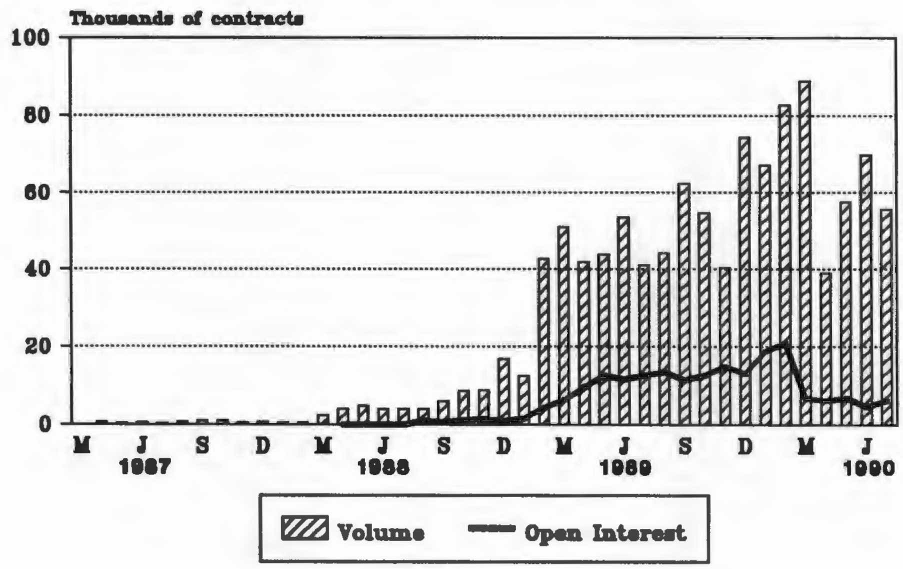
Commodity futures trading started in 1973 with South Africa's first commodity futures broking house, Holcom Futures (JSE, 1987); yet it took another thirteen years before another was established.

It was the introduction of share index futures in South Africa that gave impetus to the local futures market. Three share index futures were introduced on an over-the-counter basis in March/April 1987 by Rand Merchant Bank (RMB), and were based on the All Share, All Gold and Industrial indexes quoted on the JSE. RMB was joined by Cape Investment Bank and National Discount House and later by other market makers. Initially the futures market was not successful in attracting participants or market makers. Potential futures investors saw RMB as the only player and were reluctant to enter the market due to its lack of liquidity. Lack of education and market turnover were the biggest barriers to the development of the futures market (Harper & Williams, 1987; Arthur Anderson, 1988).

Fortunately, the fragile market of the first year has reversed itself to the benefit of all. The volume of trading of the three SIF contracts has increased substantially over the last few years as indicated by Figure 2.1. The increased market volumes can be ascribed to increased investor confidence and knowledge, and the formation of a formal futures market structure centred around the establishment of the South African Futures Exchange (SAFEX). The South African Futures Industry Association (SAFIA) helped greatly in the creation of public awareness and the dissemenation of knowledge to the public.

At the time of writing, six instruments are being traded by SAFEX, all of which are financial futures. Besides the three SIF contracts there is a Dollar Gold future, Escom 168 Gilt future and a 3 month Liquid Bankers Acceptance. Commodity futures are intended to be traded from 1993/4 (SAFIA, 1991). Full details of SAFEX requirements for share index futures contracts can be found in Appendix A.

Figure 2.1.
Share Index Futures in South Africa
Monthly Volume and Open Interest



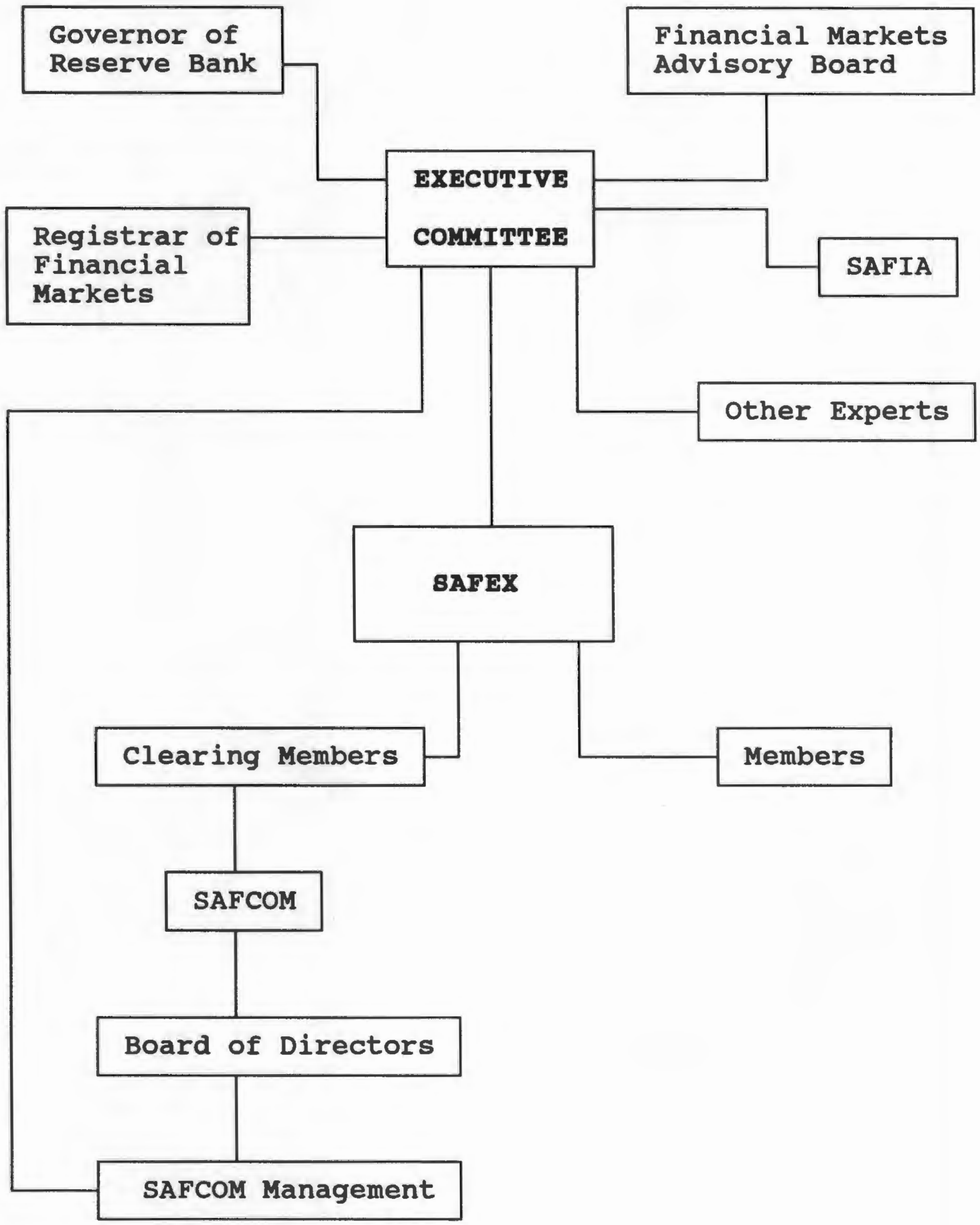
2.2. THE FUTURES MARKET STRUCTURE IN SOUTH AFRICA

The structure of the South African futures market is illustrated in Figure 2.2, the major aspects of which are discussed in turn. Unless otherwise indicated, the information is obtained from SAFIA (1991).

2.2.1. South African Futures Exchange (SAFEX)

Futures markets are either traded by open outcry on a trading floor (or pit), or by screen trading where deals are concluded telephonically. The South African Futures Exchange (SAFEX) operates on a screen trading basis via the Reuters

FIGURE 2.2.: STRUCTURE OF THE FORMAL FUTURES MARKET



network. Screen trading benefits SAFEX as it is cheap and efficient to run.

SAFEX was licensed as a financial exchange in terms of the Financial Markets Control Act on 10 August 1990 and is run by an elected Executive Committee as a non-profit making organisation. Clearing and management facilities are provided by SAFEX Clearing Company (Pty) Ltd (SAFCOM) which also guarantees the market.

SAFEX consists of three tiers of participants, namely clearing members, trading non-clearing members and market users (clients). Only members in the first two category groups are authorised to trade on the futures exchange (i.e. become market makers). Non-members must deal through authorised trading members. Further, it is a prerequisite that all trades be cleared through clearing members, who in turn guarantee all trades.

SAFEX has a code of ethics and disciplinary procedures laid down in the Rules of the Exchange (approved by the Registrar of Financial Markets) to ensure that business is done in an orderly and professional manner. The rules govern, inter alia, the registration of clients, defaults by members or clients, exchange membership, disputes and arbitration.

2.2.2. Clearing House (SAFCOM)

The clearing house is responsible for the clearing, recording, administration and fulfilment of all futures contracts. When a trade is reported to SAFCOM by the respective clearing members of both parties to the trade, the transaction is cleared by matching and reconciling the buy and the sell legs. After clearing has been completed, the clearing house becomes the party to both legs of the trade (a process called novation). In this way, the clearing house ensures that all contract provisions will be met as the original parties no longer have any rights and obligations to each other.

The clearing house financially guarantees every trade. As a result of this financial burden, market participants are required to provide a "good faith" deposit or initial margin as set by the clearing house (a variation margin may also be required by the exchange as additional security). These margins only represent a small proportion (normally less than 10%) of the total contract value. Assuming all requirements are met, margin deposits are refundable at the conclusion of the contract.

A mark-to-market process is effected daily by the clearing house which re-prices each and every contract to the current market value of each contract at the close of each business day. Profits made are credited, and losses incurred are debited, to the clients account. In terms of the mark-to-market process the participant may have to make good any losses made through topping up payments (known as margin calls).

2.2.3. Regulation

The Stals Commission Report, which investigated the development of financial futures transactions in South Africa, favoured what it called the "Minimum Intervention" approach to regulating the financial futures markets (Stals, 1988). In essence this meant that the authorities would assume only a supervisory role over the market to safeguard the public interest but not to impede natural market growth.

In view of the above recommendations, the futures market is primarily self-regulated. This in essence involves the maintainance of professional and ethical standards, and exercising the proper financial control over the market to ensure that the costs of regulation are borne by market-users rather than the taxpayer. The self-regulation is effected through the Rules of the Exchange and the control of the exchange by the Executive Committee.

To set the necessary boundaries and structures within which the market operates, certain statutory legislation has been passed, namely The Financial Markets Control Act of 1989 ("the Act") as amended by the Financial Institutions Amendment Act of 1991. The Act lays down, inter alia, certain minimum requirements for the Exchange, with specific provisions directed at the protection of market participants. The Financial Markets Advisory Board (formed in terms of the Act) and the Governor of the Reserve Bank are both official appointments to the Executive Committee and fulfil a watchdog role for the market.

2.2.4. Financial Integrity of the market

The financial integrity and soundness of the market is vital to its survival. It is well assured by a number of factors, of which the margining system is of primary importance. The receipt of margin deposits and the continual assessment of positions (requiring margin calls where necessary) limits the possibility of loss through default by market participants. Default rules of the exchange are extensive and clients, members or clearing members may have their positions closed out at short notice.

Clearing members represent large financial and banking institutions. In terms of the exchange rules the clearing mambers must possess a total net worth in excess of R20 million, and must provide a third party suretyship of R10 million to SAFCOM. The clearing members form the backbone of the exchange and, through their size and financial muscle, they impart great financial security to the market.

SAFEX requires that all trades must be cleared through a clearing member and that all margins must be paid by the clearing members on behalf of their clients. The clearing members, in turn, are responsible for collecting and controlling margins in respect of the deals passed through them. All funds received by clearing members (including margin deposits) must be maintained in trust accounts which are strictly controlled and enforced.

If a client defaults, the financial burden is taken up by the particular broker concerned; and if the broker has insufficient funds to meet the debt, this is shifted to the clearing member who must make good the loss out of its own funds. In any event, there is no recourse to the exchange itself.

2.3. ADVANTAGES OF FUTURES CONTRACTS

Forward contracts perform the same basic functions as futures contracts, but futures contracts are preferable for two main reasons. Firstly, futures contracts are standardised (i.e. where everything is fixed except the price) which enables participants to trade with the precise knowledge regarding the characteristics of the contracts in question (Falkena et al, 1989: 3).

Secondly, futures are backed by formal exchanges which guarantee the futures transactions. Formal exchanges also provide a venue for trading to take place allowing trades to be made easily and quickly. Owing to novation, the futures contract holder has total control over the particular contract and thus may terminate the contract before expiration date if so desired. The margining system, financial backing and strict regulation ensure the financial integrity and security of futures contracts and exchanges.

Futures also have other significant advantages, most important of which is that transaction costs of futures are substantially lower than other hedging alternatives (Nordhauser, 1984; Niederhoffer & Zeckhauser, 1983). For example, in South Africa (taking actual market data for 1989) it is estimated that the transaction costs for one share index futures contract was approximately 18 times cheaper than a similar value trade on the share market; this figure increases to about 77 times for 10 futures contracts (see Appendix B for details of the analysis).

Futures contracts allow participants to trade after placing a relatively small margin deposit (approximately 10% of the total contract value) compared to spot markets where normally the full consideration must be paid up front. This has allowed a greater number of participants to enter the market due to the small capital requirements, and has provided greatly improved leverage for speculators. The improved liquidity resulting from this has substantial benefits for hedgers, such as greater marketability and transferability of contracts and reduction in transaction costs.

Share index futures have advantages in addition to those discussed above. Firstly, they are very difficult to manipulate (via squeeze[1] or corner[1]) by market participants because they are based on notional assets representing the share market as a whole (Niederhoffer et al, 1980: 49; Falkena et al, 1989: 4). Secondly, SIF can assist in investment portfolio management (Figlewski & Kon, 1982: 56). They can help share selectivity by eliminating the market risk of the share portfolio: the portfolio manager can now concentrate on purchasing undervalued shares (selling overvalued shares) without worrying about the possible effects of a market decline. Additionally, the portfolio manager is able to use SIF to leverage the portfolio up or down more quickly and efficiently than with alternative methods.

2.4. CONCLUSION

Futures contracts are specialised instruments which have been designed and developed in response to large fluctuations in world commodity and financial markets. They form an integral part of the world trade and financial structure today. In South Africa, too, futures are successful despite a difficult beginning. The reason for the success of futures is attributed to their flexibility as hedging instruments, strong and unique market structure and low transaction costs.

1. Refer Glossary for definition of these terms.

CHAPTER THREE

THEORY OF FUTURES HEDGING

3.1. INTRODUCTION

The key decision to be made by a prospective hedger is the selection of the hedge ratio which will satisfy the particular hedging goal. The hedge ratio (H) expresses the value of the futures position (X_f) in the hedge as a proportion of the value of the spot position (X_s), given as;

$$H = X_f / X_s \tag{3-1}$$

Conventionally, a negative hedge ratio indicates that the position in the futures market should be opposite that of the spot market, that is, one position must be long (normally the spot market) and the corresponding position short (normally the futures market). A positive hedge ratio indicates that the futures position should move in line with the spot position (normally both are long positions).

The selection and estimation of the hedge ratio is critical as this determines, to a very large extent, the outcome of the hedged portfolio. If the hedge ratio is not estimated correctly then the desired hedging objective is unlikely to be met. It is imperative that the hedge strategy (and concomitant hedge ratio) is appropriate in meeting the hedging objective.

Allied to the selection of the appropriate hedge ratio, it is essential to adequately measure the degree of effectiveness of a prospective hedge so that the likelihood of hedging success can be assessed before the hedging decision is made. Further, the measure of hedging effectiveness is important to evaluate the performance of the hedge at any point over its life so that any necessary adjustments to the hedge ratio can be timeously made. Measuring the effectiveness of the hedge is part and parcel of the continuous decision-making and appraisal process, and is integral to the hedging strategy.

Responding to the need for effective hedges (and measures of hedging effectiveness), various hedge strategies have been formulated (Junkus & Lee, 1985) in terms of perceived hedging objectives, namely;

- The traditional hedge strategy,
- The Working basis hedge strategy, and,
- The portfolio hedge strategy, including both risk-minimising and utility-maximising strategies.

While the traditional and Working hedge strategies are still applicable to certain futures applications, the portfolio hedge strategy is more suited to share index futures and the management of equity risk. Therefore, greater focus will be given to this area. Each of the broad strategy groups is discussed in turn.

For convenience a common notation is used throughout as presented in Table 3.1. While this notation is not always consistent with the empirical articles referenced, it makes comparison between the strategy groupings easier.

Table 3.1. : Notation for Share Index Future Hedging

t	:	Time when the futures contract is taken out.
T	:	Time when the futures contract ends, either by voluntary closure or at expiration.
X_s	:	Value of the spot portfolio at t.
μ_s	:	Expected return of X_s at t for period T-t.
σ^2_s	:	Expected variance of returns of X_s at t for period T-t.
σ_s	:	Expected standard deviation of returns of X_s at t for period T-t.
X_f	:	Value of the futures portfolio at t.
μ_f	:	Expected return of X_f at t for period T-t.
σ^2_f	:	Expected variance of returns of X_f at t for period T-t.
σ_f	:	Expected standard deviation of returns of X_f at t for period T-t.
X_p	:	Total value of the combined (hedged) spot and futures portfolio at t.
μ_p	:	Expected return of X_p at t for period T-t.
σ^2_p	:	Expected variance of returns of X_p at t for period T-t.
σ_p	:	Expected standard deviation of returns of X_p at t for period T-t.
σ_{sf}	:	Expected covariance of returns between X_s and X_f for period T-t.
r	:	Correlation coefficient between the spot (X_s) and futures (X_f) prices.
σ_{si}	:	Expected covariance between the spot and index portfolios at t for period T-t.
σ^2_i	:	Expected variance of returns of the index at t for period T-t.

3.2. TRADITIONAL HEDGE STRATEGY

The traditional hedge strategy assumes that the primary goal of hedging is risk minimisation (Cootner, 1967; Blank et al, 1991: 237). In terms of this strategy, the only decision required to be made by the hedger is whether the futures market is sufficiently correlated to the spot market for the effective reduction of risk.

The traditional hedging theory advocates that the hedger holds an equal and opposite (one-to-one) position in the futures market. Given that the futures and the cash markets move in direct harmony with each other, in terms of the traditional theory, any losses incurred in the spot market will be compensated by gains in the futures market, and vice versa.

The traditional hedging strategy did not formally address the question of a measure of hedging effectiveness. However, according to Schwarz et al (1986: 186), the "traditional method of evaluating hedging potential is to focus on the variability of the basis relative to the variability of the spot price". This is given as;

$$E_f = \frac{\sigma^2{}_s - \sigma^2{}_b}{\sigma^2{}_s} \qquad (3\text{-}1)$$

which measures the proportional reduction of risk of the hedged portfolio (i.e. spot portfolio risk ($\sigma^2{}_s$) less basis risk ($\sigma^2{}_b$)) compared to the risk of the spot portfolio. In simplified terms, E_f measures the percentage decrease in the spot portfolio risk through hedging.

3.3. WORKING HEDGE STRATEGY

Working (1953) criticised the traditional approach on the grounds that it did not take into account the speculative aspect of hedging which attempts to benefit from relative price movements in the spot and futures prices. He states (ibid.: 549 & 561):

> "...the basic idea that complete effectiveness of hedging depends on parallelism of movement of spot and futures prices is false, and an improper standard by which to test the effectiveness of hedging. The effectiveness of hedging intelligently used with commodity storage, depends on *inequalities*[1] between the movements of spot and futures prices and on reasonable predictability of such inequalities. ... Any curtailment of risk may be only an incidental advantage gained, not a primary or even a very important incentive to hedging."

Working developed an arbitrage approach to hedging which argued that hedgers aimed at profit maximisation and not solely pure risk minimisation. The introduction of basis risk was seen as the major drawback of the traditional approach, but Working regarded this as a potential advantage as hedgers could benefit from the movement in the basis. According to Working, the effectiveness of a hedge was not measured by the degree to which risk was reduced, but by the level of overall profit.

Unfortunately, while the Working strategy is useful in certain arbitrage situations, it is not applicable to the majority of hedging applications. Nevertheless, the strategy did provide valuable input into the development of hedging theory.

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1. Italics in the original.

3.4. PORTFOLIO HEDGE STRATEGIES

Largely due to Working, greater focus was placed on the basis and basis risk. Analysts studying basis risk recognised that it was more relevant to evaluate the two components which together generated the basis (i.e. the spot and futures prices) rather than evaluating the basis itself. The newly introduced portfolio theory principles, formulated by Markowitz (1952) and developed by Sharpe (1964) and others, enabled hedges to be studied as a two-product portfolio model. Johnson (1959) and Stein (1961) pioneered this approach at looking at hedging.

Johnson (1959) could not adequately explain certain phenomena in the New York coffee trade market by reference to the body of hedging theory developed up to that time (i.e. the traditional and Working theories). In particular, Johnson found that hedgers were concerned not only with expected relative price movements between spot and futures (as was the basis of Working's theory) but also with the absolute expected price movements of the spot portfolio. The hedger, in terms of this, entered into speculative positions depending on expected spot price movements. For example, where expected price movements of the spot market were upward (bullish) hedgers often would reduce the level of hedging, and might even lever the portfolio up by entering into long positions in the futures market. Johnson sought to develop an appropriate model to explain the phenomena he observed.

Stein (1961) investigated the relationship between spot and futures prices in his seminal article "The Simultaneous Determination of Spot and Futures Prices". The article examined the relative supply and demand factors affecting spot and futures prices under certain circumstances. Written independently of Johnson, Stein also reformulated the hedging theory by using the two-product portfolio approach.

The model is expressed under certain simplifying assumptions which are over and above the basic assumptions of portfolio theory, namely that (Johnson, 1959: 150);

- only price changes affect expected return,
- all business risk is confined to price risk, and
- there are no money budgetary constraints, imputed interest costs or transaction costs.

Johnson and Stein defined risk to be the variability of prices (or returns) of the combined portfolio (spot plus futures) and not just the spot portfolio. The standard portfolio approach is used to isolate the optimal combined spot and futures portfolio position. This process is discussed below.

The first step is to construct the expected mean-variance two-dimensional plot for the combined futures and spot portfolio, from which the Markowitz efficiency frontier is identified. The efficiency frontier is the locus of positions of greatest return for every level of risk. The optimal combined portfolio is then defined by the position where the hedger's indifference curve tangentially touches the efficiency frontier. The indifference curve is unique for each individual and is the vehicle used to make choices under conditions of uncertainty (Copeland & Weston, 1988: 88).

The hedger's indifference curve is a line of points of equal utility based upon the hedger's particular utility function. The development of the utility function demands that individuals are rational decision makers who aim to maximise their own wealth (Copeland & Weston, 1988: 80). The utility function under situations of uncertainty determines the level of expected wealth (or return) required for each level of risk. Portfolio models assume (implicitly or explicitly) that the hedger has a quadratic utility function (Benninga et al, 1984: 155). This is a problematic assumption since quadratic utility functions have many undesirable properties (Copeland & Weston, 1988: 90; Benninga et al, 1984: 55). However, to facilitate the development of the portfolio hedging model it is assumed that hedgers have quadratic utility functions.

The portfolio model accommodates all possible hedging situations and changes in hedging positions. In terms of the model, it is possible to reduce the level of hedging (or even lever up the portfolio risk via a long hedge) if the increase in expected return of the new position sufficiently compensates for the increase in risk, with reference to the investors particular indifference curve. By doing this, the position along the efficiency frontier is merely adjusted in accordance with the investor's particular utility function. Similarly, the level of hedging could be increased if the reduced risk sufficiently compensates for the decrease in expected return.

The only restriction, however, is that a hedge can be meaningfully defined only in terms of "given a position in one market", i.e. one market must be regarded as a primary market (Johnson, 1959: 150). The concept of a primary market was also implicitly assumed under the traditional and Working theories. This restriction is well summed up by Ederington (1979: 161);

> "One difference between this and the more familiar portfolio model is that cash and futures market holdings are not viewed as substitutes. Instead, spot market holdings ... are viewed as fixed and the decision is how much of this stock to hedge."

Thus, the two-dimensional plot of the combined portfolio as explained above will be constructed by taking the fixed spot position (primary market) together with all possible positions of the futures portfolio. A hedge encompasses any position in the futures market, including those of a speculative nature. Consequently, as stated by Johnson (1953: 150);

> "there is no distinction between the hedger and the 'ordinary' speculator insofar as both are motivated by a desire to obtain a for-them optimum combination of E(R) and V(R)[2] as determined by their respective utility functions. The only essential distinction between them is that the hedger has a primary market which in this model gives rise to a merchandising profit."

This portfolio approach is able to satisfactorily explain the speculative positions sometimes taken by investors as seen by Johnson, while also accommodating Working's arbitrage hedge theory that hedgers speculate on the relative price movements of the spot and futures market.

Thus, the traditional risk-minimisation role of hedging using futures has been broadened. In terms of the portfolio model, the role of the futures market for hedging purposes is to adjust the combined portfolio's risk-return characteristics (through long or short futures positions) to correspond to the investor's expectations and his particular risk-return profile. As portfolio theory assumes that investors are risk-averse, under normal circumstances one would expect them to hold short futures positions (assuming long spot positions) more often than long futures positions.

The Johnson-Stein (JS) portfolio model is theoretically

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2. E(R) and V(R) are abbreviations for expected return and variance of returns respectively.

accurate but it has significant practical application problems. Firstly, to calculate the hedge ratio the hedger must first construct the two-dimensional plot and his/her particular indifference curves. As this procedure is extremely judgmental it provides no real benefit to the hedger other than theoretical insight.

Secondly, no easy to use measure exists which determines the effectiveness of the hedge. A measure of effectiveness is important to allow consistent management and evaluation of the hedge. The assessment of effectiveness of a particular hedge at a point in time under this strategy would involve the comparison of the hedge position with the optimal position through the complete reconstruction of the efficiency frontier and indifference curves.

3.4.1. Minimum-Variance Hedge Strategy

Working suggested that the one-to-one hedge position was not necessarily the lowest risk strategy. This prompted analysts to search for risk-minimising hedge ratios rather than to merely accept the traditional one-to-one ratio. The portfolio approach assisted in finding a more optimal hedge strategy under the assumption of risk-minimisation. It must be noted that the portfolio minimum-variance hedge strategy is only a special case of the general JS portfolio hedge strategy.

The traditional hedge assumes that the futures price moves in a one-to-one relationship with the spot price. While for some spot-futures situations this may be the case, this does not apply to share index futures and share portfolios. The reason for this can be divided into two legs.

Firstly, the share portfolio seldom moves in a one-to-one line with the index upon which the futures contract is based. The measure of co-movement between a share portfolio and the share index is commonly known as the beta (ß) of the portfolio, defined in equation 3-2. Many research articles (including Fabozzi & Francis, 1978; Uliana et al, 1987: 76) have discovered that betas differ from portfolio to portfolio and may change over a period in time.

$$\beta = \frac{\sigma_{si}}{\sigma^2_i} \qquad (3\text{-}2)$$

The second reason is that the futures price may not move in a one-to-one relationship with the index. Studies have shown that the volatility of the futures markets does sometimes differ to the volatility of the spot markets (Anderson, 1985; and Milonas, 1986).

The same two product portfolio approach was adopted by Johnson (1953) to calculate the minimum-variance hedge ratio. Given that the expected return of the combined portfolio (μ_p) and the variance of this return (σ^2_p) from time t to T are;

$$\mu_p = (X_s/X_p)\,\mu_s + (X_f/X_p)\,\mu_f; \text{ and,} \qquad (3\text{-}3)$$

$$\sigma^2_p = (X_s/X_p)^2\sigma^2_s + (X_f/X_p)^2\sigma^2_f + 2(X_s/X_p)(X_f/X_p)\sigma_{sf}; \qquad (3\text{-}4)$$

Johnson (1959: 143) used differential calculus to derive an expression for the position on the futures market (X_f^*) required to minimise price risk. This is normally stated in the form of the optimal risk-minimising hedge ratio (h^*) as;

$$h^* = -\sigma_{sf} / \sigma^2_f \qquad (3\text{-}5)$$

The minimum-variance hedge ratio is, thus, the covariance between the spot and futures price (σ_{sf}) as a ratio of the variance of the futures price (σ^2_f). This is statistically calculated as the beta coefficient of the ordinary least squares regression between the spot and futures prices (Ederington, 1979; Duffie, 1989; Schwarz et al, 1986: 180; Blank et al, 1991: 243-4; and Bell et al, 1986).

Where the volatility of the share index futures price is equal to the volatility of index value itself and the two prices are highly correlated, the risk-minimising hedge ratio can be simplified. Under these situations, the variance of the futures price (σ^2_f) will equal the variance of the index (σ^2_f); and the covariance between the spot share portfolio and the index (σ_{si}) will equal the covariance between the spot share portfolio and the futures price (σ_{sf}). Accordingly, the risk-minimising hedge ratio given in 3-5 can be reformulated as (Figlewski & Kon, 1982: 54; Figlewski, 1984a: 660);

$$h^* = \sigma_{si} / \sigma^2_i = \beta \qquad (3\text{-}6)$$

Thus, a good estimate of the risk-minimising hedge ratio under normal market conditions is simply the beta (β) of the spot share portfolio which is calculated statistically as the beta coefficient of the ordinary least squares regression between the return on the portfolio and the return on the index.

An ex ante expression for the total variance of the combined portfolio returns ($\sigma^2_p{}^*$) can be derived by substituting the risk-minimising futures position X_f^* into equation 3-4.

After simplification this is given as;

$$\sigma^2{}_p{}^* = \sigma^2{}_s (1 - r^2) \qquad (3\text{-}7)$$

From equation 3-7, given that there is no change in the primary market (i.e. $\sigma^2{}_s$ is constant for all hedge positions), the combined portfolio risk depends varies depending on the value of the squared correlation coefficient between the spot and futures markets (r^2). The correlation coefficient was also an important consideration in the traditional hedge strategy.

Risk is completely eliminated if r^2 equals one, i.e. when the correlation between the spot and futures markets is perfect, and there is no reduction of risk if r equals zero. In general, the greater the correlation coefficient between the spot and futures market, the lower the risk of the combined portfolio. It follows therefore, that the ex ante measure of effectiveness (E) for the minimum variance hedge strategy is the square of the correlation between the futures and the spot (Johnson, 1959: 144),

$$E = r^2 \qquad (3\text{-}8)$$

The value of r^2 is normally calculated though the regression of the spot and futures markets, referred to as the coefficient of determination (R^2). The R^2 is an estimate of the percentage reduction in the variability of changes in the spot position from holding the futures position (Schwarz et al, 1986: 186).

In conclusion, under the limited assumption of risk-minimisation, the Johnson minimum-variance hedge strategy has successfully established the optimal risk-minimising hedge ratio and an ex ante measure of hedging effectiveness. Unfortunately, these are not relevant in a mean and variance framework.

3.4.2. Rutledge Utility Maximising Strategy

To try to solve some of the practical problems identified with the JS portfolio strategy, a number of researchers (including Anderson and Danthine, 1981; Junkus & Lee, 1985; and Rutledge, 1972) attempted to quantify the JS portfolio optimisation model. These reformed portfolio models, which can be classed together as utility maximising strategies, do not collectively constitute a different hedging approach but are merely an extension of the JS portfolio model.

The quantification was made possible by incorporating a term for the investors utility function, but is not defined explicitly in any of the above papers. This is because it is impossible to explicitly define the utility parameter as it varies from individual to individual (Copeland & Weston, 1988: 84).

Rutledge's model is to date the most definitive utility maximising model. He included into the model convenience and capacity parameters which collectively constitute the individual's utility function. Rutledge also incorporated part of Working's arbitrage hedging strategy by adding the mean expected basis and expected change in the basis as elements in the model.

Rutledge (1972: 240-2) formulated sets of equations for the utility maximising optimal hedge ratio (H) which was simplified by Junkus & Lee (1985: 207) as given in 3-9. Junkus et al ignored the individual's risk aversion parameter from the above expression, so a factor (δ) has been added to reflect the more correct situation according to Rutledge's theory.

$$H = \delta + \frac{\sigma_{sb}(\mu_s - c) + \sigma^2_s (\mu_b - c)}{(\mu_s - c)(\sigma^2_b + \sigma_{sb}) + (\mu_b - c)(\sigma^2_s - \sigma_{sb})} \qquad (3\text{-}9)$$

where:

H	:	Utility maximising hedge ratio.
σ_{sb}	:	Covariance of spot and basis price movements.
c	:	Cost of carrying the spot commodity.
μ_b	:	Mean expected basis.
σ_b^2	:	Expected variance of basis movements.
δ	:	Factor required to adjust the hedge ratio to correspond to the individual's risk profile.

Rutledge went an additional step further and established a measure of hedging effectiveness, U(R), with ϕ being included as the individual's risk aversion factor.

$$U(R) = \mu_s - H\mu_b - \phi\ (\sigma^2_s + H^2\sigma^2_b - 2H\sigma_{sb}) \qquad (3\text{-}10)$$

While Rutledge has established satisfactory expressions for the optimal utility-maximising hedge ratio and measure of hedging effectiveness, both of these depend on the inclusion of the individual's risk aversion parameter which in turn depends on the individual's utility function. Thus, the utility-maximising strategies do not in the end solve the problems of applying the portfolio hedge strategy.

3.5. HOWARD & D'ANTONIO

None of the hedging strategies described above could provide a suitable ex ante measure of hedging effectiveness which incorporates both risk and return and is capable of being practically applied.

The traditional hedge's effectiveness measure given in equation 3-1 is an ex post measure and is only concerned with the reduction in risk. The Working strategy measure of hedging effectiveness (i.e. overall hedge profit) is also an ex post measure and thus cannot be practically applied. Furthermore, both the traditional and Working strategies are very limited in their uses as hedging strategies.

The minimum-variance portfolio strategy did develop an ex ante measure of hedging effectiveness (r^2) but this is only suitable in a risk-minimisation situation. Neither Johnson, Stein, Rutledge nor any of the other proponents of the utility maximising (portfolio) strategy could establish a suitably practical risk-return measure of hedging effectiveness.

However, Howard & D'Antonio (hereafter H&D) did establish an ex ante risk-return measure of hedging effectiveness, and also developed a closed form expression for the optimal hedge ratio.

3.5.1. Graphic Development of the H&D Model

The measure of hedging effectiveness requires initial explanation. The following analysis (H&D, 1984: 102-4), adapted for a share index futures situation, graphically conceptualises H&D's measure of hedging effectiveness.

Consider an investor who has the choice of investing in one or more of the following assets: shares, share index futures and risk-free assets. The investor would fall into one of the three graphical scenarios illustrated in Figures 3.1. to 3.3. (H&D, 1984: 103).

In terms of modern portfolio theory as developed by Sharpe (1964), all optimal portfolios lie along the line from point i (i is the percentage return of the risk-free assets) through the tangent point T and beyond. This line is called the Capital Market Line or CML (Uliana et al, 1987: 75). The H&D strategy assumes that individuals are risk-averse and the optimal portfolio position is determined solely by the expected total portfolio return (μ_p) and the standard deviation (σ_p) of this return.

In each of the figures the point S depicts the spot share portfolio only. The remainder of the efficiency frontier represents the share portfolio supplemented by varying proportions of SIF contracts. This reflects the requirement of the portfolio theory where the spot share portfolio is unchanged as the primary market.

The model also allows the investor to reduce the overall risk of the portfolio by holding proportionately more of the risk-free asset, in so doing moving the combined portfolio along the CML. The decision to hold futures is to adjust the risk-return characteristics of the share portfolio, and is independent of the decision to invest in risk-free assets.

In Figure 3.1, the optimal portfolio of shares and futures in terms of portfolio theory will be the tangent portfolio T. The optimal position constitutes a long futures position which has levered up the spot share portfolio in terms of both risk and return. In Figure 3.2, the CML passes through S so no adjustment needs to be made to the share portfolio.

Figure 3.1.
H&D Hedge Strategy
Long Futures Position

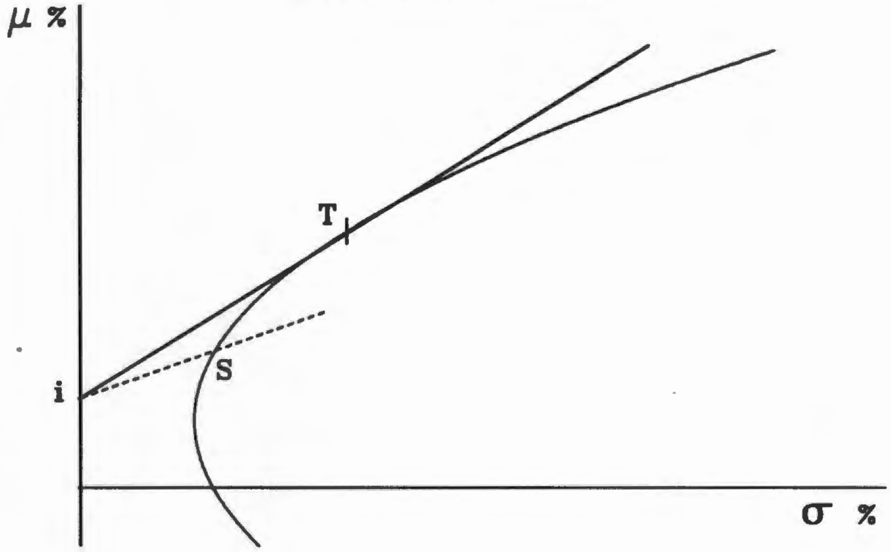


Figure 3.2.
H&D Hedge Strategy
Zero Futures Position

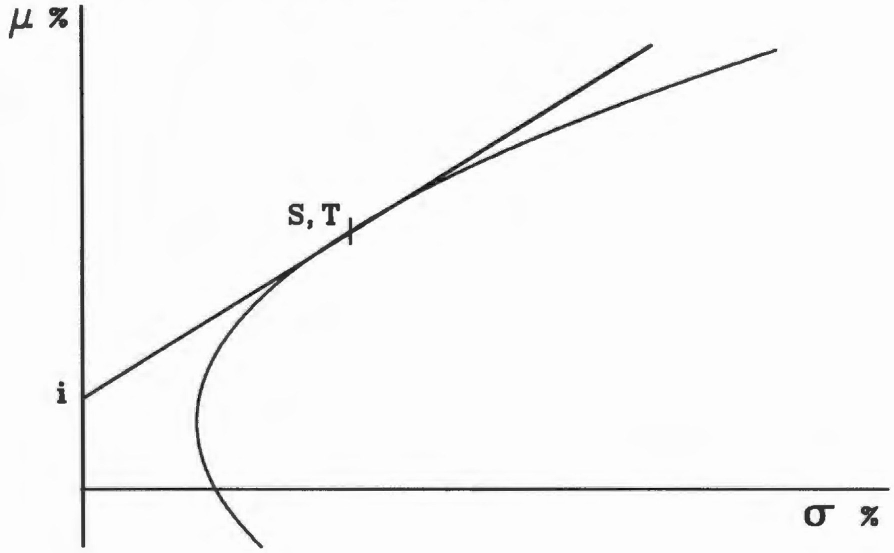


Figure 3.3.
H&D Hedge Strategy
Short Futures Position

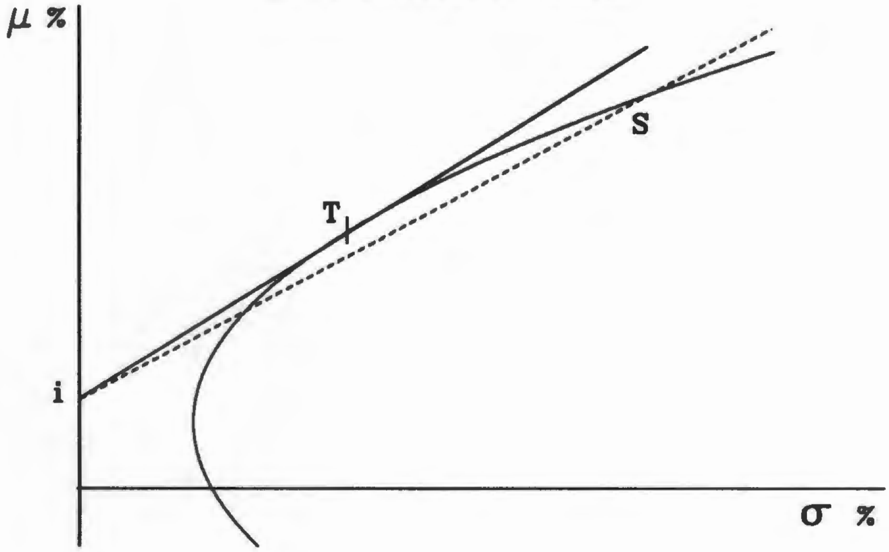


Figure 3.3. is similar to Figure 3.1. except that the optimal position consists of short futures which are required to adjust the spot portfolio downwards to the optimal position. Thus, the optimal portfolio determined by portfolio theory fundamentals may contain short futures, long futures or no futures at all.

The difference between T and i in Figures 3.1. to 3.3. represents the excess return of the optimal hedged portfolio above the risk-free rate of return. Consequently, the slope of the line i-T reflects the increase in excess return per unit of risk of the combined portfolio (denoted as θ_p). The above applies *mutatis mutandis* to the share portfolio (denoted as θ_s). These can both be expressed as follows:

$$\theta_p = (\mu_p - i) / \sigma_p \qquad (3\text{-}11)$$
$$\theta_s = (\mu_s - i) / \sigma_s \qquad (3\text{-}12)$$

where

θ_p : Excess return (percent) per unit of risk of the combined portfolio.

θ_s : Excess return (percent) per unit of risk of the spot portfolio.

μ_p : Expected return (percent) of the combined portfolio.

i : Risk-free rate of return (percent).

σ_p : Standard deviation of expected returns (percent) of the combined portfolio.

μ_s : Expected return (percent) on the spot portfolio.

σ_s : Standard deviation of expected returns (percent) of the spot portfolio.

3.5.2. Measure of Hedging Effectiveness

H&D (1984: 102-4) define hedging effectiveness as the measure of "the increase in expected portfolio returns for a portfolio containing futures to one without futures at the same level of risk". This statement follows from Markowitz portfolio theory which states that a risk-averse investor will select the greatest expected return at a given level of risk. Therefore, H&D measures the hedging performance of the combined portfolio versus the spot portfolio on a per unit of risk basis, as follows:

$$HE = \theta_p / \theta_s \qquad (3\text{-}13)$$

Essentially, the measure of hedging effectiveness (HE) compares the coefficient of variation (CV) of the hedged portfolio (numerator) with the CV of the spot portfolio (denominator). The CV is a commonly used statistic for comparing two portfolios, which is normally computed by simply dividing the total return by the total risk (Uliana et al, 1987: 57). H&D have altered this form slightly and are basing their measure on the excess return rather than the total return in terms of their graphical analysis and capital market theory above.

After substituting values for θ_p and θ_s by the standard two product expressions for mean and variance (equations 3-3 and 3-4 respectively), the measure of hedging effectiveness (HE) in 3-13 can be restated as (H&D, 1984);

$$HE = \left[\frac{1 - 2\alpha r + \alpha^2}{1 - r^2} \right]^{1/2} \quad (3\text{-}14)$$

where

$$\alpha = \frac{\mu_f / \sigma_f}{(\mu_s - i) / \sigma_s} \quad (3\text{-}15)$$

It can be seen from 3-14 that HE is a simple deterministic measure of hedging effectiveness using only two variables, namely α and the correlation coefficient of the spot and futures markets (r).

The variable α (denoted by lambda in H&D's paper) can be explained as the excess return to risk relative between the futures and spot position. In other words, α indicates the relative attractiveness of investing in futures versus the spot on a per unit of risk basis. If α is greater than one then this indicates that the futures market is more attractive on a relative basis than the spot market. Similarly, if α is less than one then the spot position would be more attractive. The numerator of α does not include the risk-free rate as zero margin requirements have been assumed (H&D, 1984).

The value of HE depends on the relative values of α and r. Accordingly, this has been illustrated graphically in Figures 3.4. to 3.6. (H&D, 1984: 109), for three different values of r; namely r=0, r=0.5, and r=0.9.

From the graphs two things are noticeable. Firstly, the degree of hedging effectiveness increases with the increase in the correlation coefficient (r). This is obvious from the denominator of 3-14 where, as r tends towards 1, the value of HE tends towards infinity. This outcome is expected in the light of Johnson's minimum-variance hedge strategy.

Figure 3.4.
H&D Hedge Strategy
Values of HE for r = 0

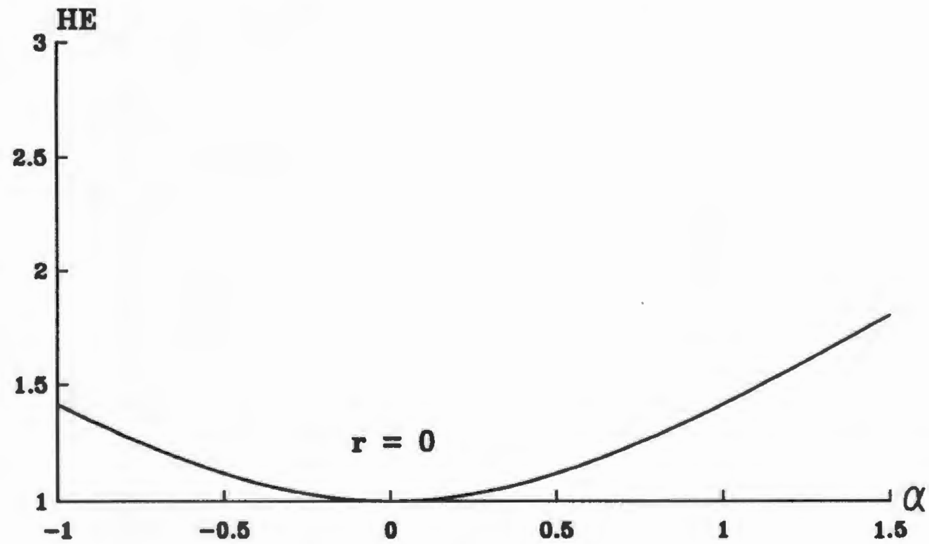


Figure 3.5.
H&D Hedge Strategy
Values of HE for r = 0.5

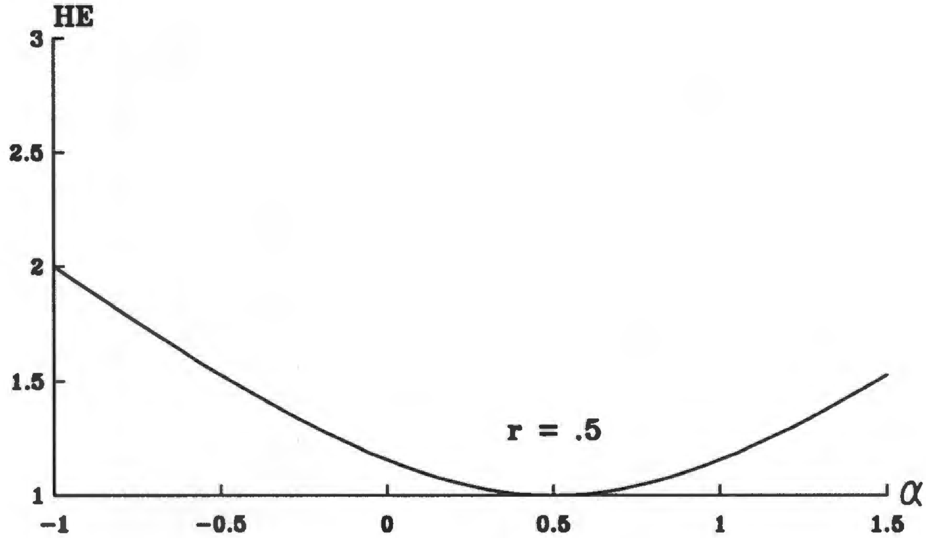
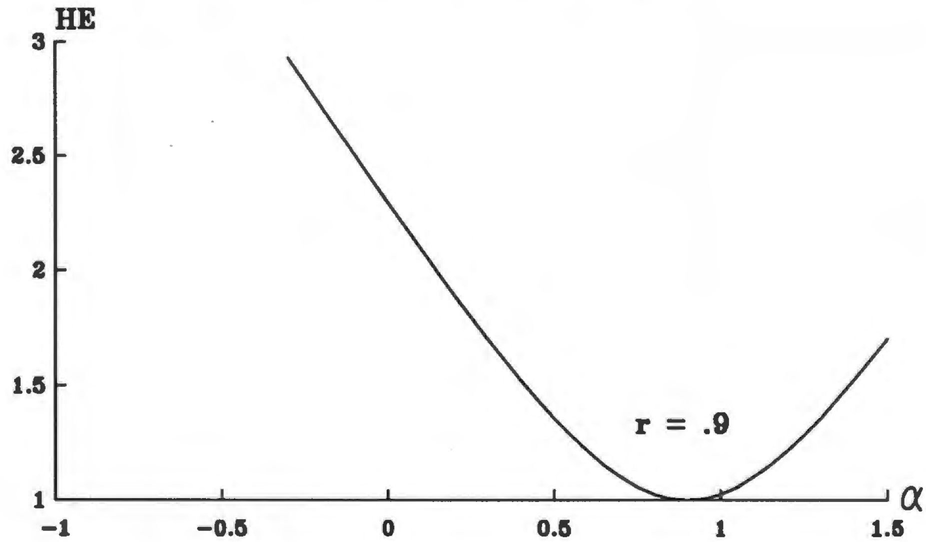


Figure 3.6.
H&D Hedge Strategy
Values of HE for r = 0.9



Secondly, the greater the difference between r and α the greater the degree of hedging effectiveness. This finding is against previous thinking as even if r is close to one there may be little benefit of holding futures if α is also close to one. Conversely, even if r is close to zero there may be a benefit of hedging if α is significantly different from zero. Therefore, in certain cases a cross hedge may be better, in terms of risk and return considerations, than the straight hedge.

Chang and Shanker (1987) noticed ambiguous results which HE presents in certain situations, but their measure (HE_1) did not solve these problems completely. Consequently, H&D (1987: 380) derived a better measure of hedging effectiveness, namely the hedging benefit per unit of risk (HBS) given as;

$$HBS = \theta_p - \theta_s \quad (3\text{-}16)$$

The HBS measure is derived using the same basis as the HE measure except that the difference between the excess return per unit of risk of the combined hedged portfolio (θ_p) and the spot portfolio (θ_s) is calculated, instead of the ratio between the two values. A positive HBS indicates that the hedged portfolio will be preferable to the spot portfolio; similarly, a negative HBS indicates that hedging would not be worthwhile.

3.5.3. H&D Hedge Ratio

The optimal hedge ratio (b^*) established by maximising θ_p (using differential calculus) with respect to the futures position is (H&D, 1987: 105);

$$b^{*} = \frac{\sigma_s \; (\alpha - r)}{\sigma_f \; (1 - \alpha r)} \qquad (3\text{-}17)$$

As with the effectiveness measures, the relationship between α and r is important in determining the optimal hedge ratio. If α is less than r then b^* will be negative, indicating that the investor should be short in futures. Similarly, if α is greater than r the investor should be long in futures. The result is different to that of the risk-minimising strategies in that the choice of holding futures does not solely depend on the correlation coefficient (r) but also on the risk-return relative.

For equation 3-17 to be satisfied, according to H&D (1984, 105-6) two conditions, known as second-order conditions (hereafter SOC), need to be satisfied. These conditions are:

$$\mu - i > 0_s \qquad (3\text{-}18)$$

and

$$1 - \alpha r > 0 \qquad (3\text{-}19)$$

With respect to the SOC, distinction must be made between ex post and ex ante situations. In an ex post situation, if the SOC do not hold then the optimal hedge ratio (b^*) will be infinite (H&D, 1987). An infinite hedge ratio indicates unlimited potential benefit from investing in futures (a zero margin requirement has been assumed so there is no theoretical limit to the level of the futures holding). In practical terms, this indicates that the optimal portfolio would be to hold only risk-free assets and futures, i.e. none of the spot portfolio should be held.

In an ex ante situation it is virtually impossible to predict the above situation and so the ex post discussion has academic merit only. From an practical ex ante perspective the second order conditions hold according to Howard & D'Antonio (1987: 378):

> "If the SOC does not hold, an infinite futures position will be taken. From a practical standpoint, this means that the agent would invest only in the risk-free and futures markets and not in the spot market. In other words, the risk-free and futures portfolios completely dominate all portfolios with *any*[3] position in the spot market. Since this is very unlikely on an *ex ante*[3] basis, we choose to ignore this possibility in our original paper and therefore assumed the SOC would hold."

Moreover, if the SOC are viewed from an ex ante perspective, it can be shown that they do hold under normal circumstances. Firstly, the spot risk premium (μ_S-i) will not be negative as risk averse investors will not hold risky assets yielding an expected return below the risk-free rate. The first condition should therefore be easily fulfilled. Secondly, in an ex ante situation, the risk-return relative α equals one while the correlation coefficient (r) is less than or equal to one, thus, the second condition is valid.

3.6. CONCLUSION

The establishment of the appropriate hedge ratio is all important in achieving a particular hedging objective. Associated with this is the need for a measure of hedging effectiveness so that timely hedging decisions can be made, both before and during a hedge.

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3. Italics in the original.

Under the objective of risk-minimisation, two strategies have been developed. It has been shown that the traditional hedge strategy is not appropriate in a share portfolio situation. Instead, the optimal risk-minimisation hedge strategy is the Johnson minimum-variance hedge based on modern portfolio theory. This strategy provides a practical formula for the hedge ratio (h^*) and an ex ante measure of hedging effectiveness (r^2). The portfolio beta (β) is theoretically a good estimate of the risk-minimising hedge ratio.

However, the minimum-variance, beta and traditional hedge strategies only suffice under the objective of risk-minimisation. The decision to hedge in the futures market is "no different from any other decision - investors hedge to obtain the best combination of risk and return" (Ederington, 1979: 169). Under the objective of maximising the combination of risk and return, the Johnson-Stein portfolio theory is the optimal strategy. Unfortunately, the JS portfolio model is based on a hedger's utility function which is unique to that particular hedger. Therefore, both the hedge ratio and the measure of hedging effectiveness cannot be readily applied in a practical situation. Howard & D'Antonio (1984) have solved this problem by deriving a suitable measure of hedging effectiveness (HBS) and establishing a closed-form expression for the optimal hedging ratio (b^*).

CHAPTER FOUR

REVIEW OF MAJOR EMPIRICAL HEDGING STUDIES

4.1. INTRODUCTION

Many empirical studies have been published in the literature which have analysed the effectiveness of futures markets in general, and the relative performance of the hedging strategies using share index futures in particular. The major studies which relate to share index futures are briefly reviewed and discussed below.

4.2. EDERINGTON (1979)

Although Ederington's seminal paper was published before the introduction of share index futures, it is a pioneering article into the hedging performance of the new financial futures markets which is relevant to share index futures.

Ederington evaluated two interest rate futures contracts in the United States of America, namely contracts based on the Government National Mortgage Association (GNMA) 8% Pass-through Certificates and the 90 day Treasury bill. The objective of Ederington's study was to evaluate these financial futures markets using the portfolio minimum-variance hedge strategy, and then to compare them with the corn and wheat commodity futures markets (the two most heavily traded commodity futures contracts at that time).

Data were extracted weekly starting from January 1976 and March 1976 for the GNMA and the Treasury bill contracts respectively; both ending in December 1977. The ex post optimal hedge ratio was calculated using ordinary least squares regression. For each contract type, arbitrarily selected hedging periods of two and four weeks were chosen. A summary of Ederington's results (two and four week results have been combined) is given in Table 4.1.

Table 4.1.: Ederington (1979)
Average hedge ratios and R^2 measures

	Average h^*	Average R^2
GMNA	0.90	0.73
Treasury bills[1]	0.71	0.68
Wheat	0.84	0.89
Corn	0.92	0.59

The minimum variance hedge ratios estimated were found to be less than one in 28 of the 32 contracts tested, 15 of which were tested to be significantly different from one. In view of his results, Ederington (1979: 166) rejected the hypothesis that the minimum-variance hedge ratio equals one, and as a consequence the traditional hedge strategy was rejected as an appropriate risk-minimising hedging model for the financial futures markets.

Of concern in an ex ante situation is the sensitivity of the hedging performance to changes in the hedge ratio.

1.The results for the Treasury bills have been extracted from Frankle (1980: 1274) owing to an error in Ederington's methodology arising from the use of average rather than actual Treasury bill rates. The results were recalculated by Frankle on the same basis but using actual Treasury bill rates.

Ederington (1979: 166) found that a 10% change in the value of the hedge ratio resulted in only a 1% reduction in the coefficient of determination (R^2). Thus, any hedge ratio estimation error is unlikely to severely affect the level of hedging effectiveness.

Both financial futures contracts proved to be fairly effective; and compared favourably to the commodity futures. Based on the R^2 statistic, 80% of the financial futures price movement could on average be explained by the index indicating that the vast majority of market risk could be eliminated using financial futures, although a fair amount of basis risk still remained.

4.3. HILL & SCHNEEWEIS (1984)

Hill & Schneeweis investigated major indexes using weekly data over the period from May 1982 to August 1982. Table 4.2. presents the results of the coefficient of determination (R^2), the minimum-variance hedge ratio (h), and the standard deviation of price returns for the unhedged position, the traditional hedge position (H=1) and the minimum-variance position.

The results show that the minimum-variance hedge would have eliminated on average 58.5% of the unhedged portfolio standard deviation, substantially better than the traditional strategy at 43.8%. The minimum-variance hedge ratios for share indexes were found to be considerably less than one.

Table 4.2.: Hill & Schneeweis (1984)
Weekly SIF hedging effectiveness

Index	R^2	h^*	Standard Deviation		
			Unhedged	H=1	h^*
S&P 500	0.82	0.69	4.09	3.09	2.13
Value Line	0.70	0.60	4.03	2.19	1.52
NYSE	0.78	0.65	2.45	0.95	0.85

4.4. FIGLEWSKI (1984a)

Figlewski examined the effectiveness of selling Standard & Poors (S&P) 500 share index futures against the underlying portfolios of five major stock indexes, namely the S&P 500, the New York Stock Exchange (NYSE) composite, the American Stock Exchange (AMEX) composite, the National Association of Securities Dealers Automated Quotation System (NASDAQ) index of over-the-counter (OTC) stocks, and the Dow Jones Industrial (DJI) index. The NYSE and the DJI indexes are closely linked to the S&P 500, but this is not the case for the AMEX and OTC indexes.

A one week hedging period was used for the period from June 1982 to September 1983. The results of the unhedged position, the minimum-variance hedge position and the beta hedge (β) position for all five portfolios (ibid.: 662) are given in Table 4.3. Mean returns (denoted by μ) and standard deviations (denoted by σ) are in percent annualised.

Table 4.3.: Figlewski (1984a)
Effectiveness of the S&P 500 index future in hedging major share index portfolios over a one week period

	Unhedged		Minimum Risk Hedge			Beta Hedge		
Portfolio	μ_s	σ_s	h^*	μ_p	σ_p	β	μ_p	σ_p
S&P 500	39.1	19.0	.85	9.7	4.6	1.00	4.7	5.6
NYSE	39.3	18.3	.82	11.3	5.0	.96	6.3	5.9
AMEX	45.3	20.9	.78	18.5	12.4	.94	13.0	12.8
OTC	46.5	17.6	.64	24.4	10.9	.77	20.0	11.2
DJI	39.4	19.4	.86	9.9	5.8	.99	5.3	6.4
Futures	34.4	21.6						

The results show that the reduction of standard deviation using the minimum-variance hedge on the S&P 500, the DJI and the NYSE is significant with an average of 72.9% of unhedged portfolio risk being reduced. However, the other indexes did not prove to be as effective in terms of risk reduction, with residual risk after hedging still amounting to approximately 60% of the unhedged portfolio. Figlewski concluded from these results that hedging effectiveness is substantially reduced through the introduction of unsystematic (specific) risk. However, the cross hedges did perform better than the straight hedges in terms of return.

When portfolio betas were used as the hedge ratios, hedging performance in terms of risk and return materially deteriorated in all cases, particularly with respect to substantially lower returns. The beta hedge ratio was observed to be more than the minimum-variance hedge ratio in every case introducing a degree of overhedging.

As basis risk was seen to largely impact hedging performance, Figlewski analysed the components of basis risk. A factor affecting basis risk is the variability of expected dividend payments (ibid. 663). However, additional empirical work into this area revealed that the variability of dividends had only a small impact on basis risk and that this was insignificant compared to the overall hedged portfolio risk faced. No other factors were found responsible for basis risk. Thus, Figlewski attributed the "noise in the price relationship between the cash and futures markets as the primary source of basis risk" (ibid.: 664). This noise could come from many sources including temporary imbalances between supply and demand in either the spot or futures market or expectation differences between spot and futures market traders.

Additional research by Figlewski in the same study established that as the hedging period increased from overnight hedges to one week hedges there was a significant reduction in basis risk. This outcome is understandable because basis risk cannot become arbitrarily large (otherwise arbitrage profits can be taken), and so the longer the hedging period, the lower the basis risk is likely to be in relation to the total risk of the hedge.

Figlewski (ibid.: 665) established that hedges taken out long before expiration (over two months) tended to exhibit a greater degree of basis risk, and lower effectiveness, than hedges taken out closer to the expiration date. This result is expected since the variations in the basis from its equilibrium position are likely to become progressively smaller towards expiration (at which point there is no basis risk).

4.5. JUNKUS & LEE (1985)

Junkus & Lee (1985) conducted an investigation into the applicability of the major hedging models to the share index futures markets. Each model was evaluated in terms of their own, and each other's, effectiveness measure. The objective was to ascertain whether any particular strategy performed satisfactorily under all decision rules from which a conclusion could be made as to the most universally effective hedging strategy.

The models investigated were the traditional, Working, minimum-variance, and Rutledge utility maximising strategies. The hedge ratios and measures of hedging effectiveness used are those discussed in chapter 3, except for the following additional conditions. A simple basis hedging rule was established for the Working strategy, while the risk-aversion parameter (δ) was excluded from the Rutledge hedge ratio. In the Rutledge measure of hedging effectiveness, the risk-aversion parameter (ϕ) was arbitrarily selected as 0.1 and 0.01 for the purposes of the study. The traditional model was used for comparison purposes; thus, no measure of effectiveness was devised for this strategy.

The study was based on data extracted from June to December 1982 on the three major index futures, viz. the New York Stock Exchange (NYSE) composite index future, the Standard & Poors (S&P) 500 index future and the Kansas City Board of Trade Value Line index future. Three different contract maturities were used, namely the three, six and nine month contracts. The hedge ratios calculated in terms of each strategy were based on the actual observed data, that is from an ex post viewpoint.

Junkus & Lee found that the optimal hedge ratios under the three strategies differed markedly from the one-to-one hedge. The minimum-variance hedge ratios were actually substantially below one, implying that the traditional hedge would result in significant overhedging. The hedge ratios were comparable to those found for other financial and commodity futures (ibid.: 220).

The research did highlight potential problems with the estimation of the minimum-variance and Rutledge hedge ratios in an ex ante situation owing to the poor correlation between the hedge ratios from one period to the next. Also, hedge ratios tended to decrease across maturity.

Each strategy proved to be optimal in terms of its own effectiveness criterion, but no clear second best strategy could be found. Exceptions to this were the poor performance of the Working basis arbitrage strategy under relatively efficient markets, and the only moderate effectiveness of the Rutledge strategy at lower risk averse levels. However, the application of Rutledge hedge ratio was not strictly in terms of the model as defined so there is doubt regarding the reliability of Junkus & Lee's conclusions in regard to the Rutledge strategy.

4.6. GRAHAM & JENNINGS (1987)

Graham & Jennings investigated the risk-return effectiveness of share portfolios themselves. An extensive sample of shares from the New York Stock Exchange was used to construct portfolios with different investment characteristics classified according to beta and dividend yield. Short futures were combined with these portfolios in one, two and four week hedges according to the traditional, beta and minimum-variance strategies. An abridged form of the results is given in Table 4.4.

Table 4.4.: Graham & Jennings (1987)
Hedge return and risk expressed as a percentage of unhedged portfolio return and risk

	Trad	Beta	Min-Var
Hedged Return	33.85%	31.78%	56.55%
Hedged Risk	78.59%	77.22%	71.06%

The minimum-variance strategy consistently outperformed the other two strategies in terms of return. While the beta hedge performed satisfactorily for low beta portfolios (i.e. where the hedge ratio was relatively low), beta hedges consistently overhedged high beta portfolios. Return retention improved with the increase in the hedge period.

In terms of risk, the results indicate that the share portfolio hedges are only about one-half as effective as those for index portfolios according to Figlewski's results above (Graham & Jennings, 1987: 9). The minimum-variance strategy performed the best on average but did not dominate the other two strategies: the relative advantage of the minimum-variance hedge over the other strategies diminished as the hedge period lengthened.

4.7. SMITH (1989)

Smith conducted research in South Africa on the hedging effectiveness of the minimum-risk, beta and H&D hedges using the All Gold, Industrial and All Share index futures contracts. The actual indexes themselves were used as the spot portfolios. The observation period from 31 December 1987 to 15 September 1989 of 90 weeks was divided into two periods, namely an initial 42 week period and a final 48 week period.

Two sets of hedge ratios were established, one from the 42 week period and one from the 48 week period. The 48 period was divided up into one, two and four weeks which represented the hedge periods to be observed. Both hedge ratios were applied to these hedge periods: the hedge ratios based on the 42 week period represented the historic case while the hedge ratios from the 48 period represented the within period case.

The two sets of hedge ratios were not altered throughout the hedging period. This is a weakness in the research methodology, admitted by Smith (1989: 31): "A more rigorous approach would have been to use a moving horizon where the parameters are estimated from the immediate past period".

In terms of the H&D model, cognisance of the second order conditions were taken by Smith. In view of the discussion in chapter 3, it is felt that this approach is correct in an ex post situation (within period case) but is not correct in an ex ante situation. In cases where the conditions were not satisfied, arbitrarily long futures positions of 1, 5 and 10 contracts were chosen by Smith to represent the infinite futures position advocated by the H&D Strategy in an ex post situation. The results revealed that the H&D model did not apply in over half the hedging cases observed because the second order conditions were not satisfied. Moreover, certain of the H&D hedges and hedge ratios behaved in a manner opposite to what was anticipated in terms of the theory.

The minimum-variance hedge ratio was, with one exception, less than one and generally less than the portfolio beta ratio. The minimum-variance hedge performed significantly better than the beta hedge in terms of risk and return. However, comparing the minimum-variance hedges to the unhedged portfolio, there was a large decrease in return through hedging despite a large incidence of residual (basis) risk still remaining in the hedges.

Direct hedges performed better than cross hedges in terms of risk reduction, and the performance of four week hedges in terms of both risk and return was better than the one week hedges.

4.8. CONCLUSION

Most studies in the literature have concentrated on the minimum-variance hedge. The minimum-variance hedge ratio was significantly less than one under all the studies, but was seen to fluctuate between hedge periods in an ex post scenario. Ederington discovered that the effectiveness of the hedge was fairly insensitive to changes in the hedge ratio of the new financial futures markets. The minimum-variance hedge was significantly more effective than any other hedge in terms of risk reduction, but still was subject to a significant amount of basis risk. Further, the minimum-variance hedge also suffered from a substantial drop in return.

The level of residual (basis) risk was greater for cross hedges than straight hedges. On average, from the studies above in respect of minimum-variance hedges, 58% of the unhedged portfolio risk remained in cross hedges (Figlewski and Graham studies) compared to 34% in straight hedges (Figlewski and Hill studies). The basis risk was ascribed to the "noise" in price movements between the spot and futures portfolios. The level of basis risk could be limited by increasing the hedging period and using the closest maturing futures contract for hedging purposes.

Surprisingly, the beta hedge was found to be very ineffective in terms of both risk and return. The beta hedge ratio was found to be greater than the minimum-variance hedge. The traditional and Working hedge strategies also did not perform very well.

Very little research has been conducted into utility-maximising strategies. Smith discovered that the Howard & D'Antonio strategy was not applicable in over half the hedges investigated. The Rutledge strategy also did not prove to be effective. No formal studies have been done on the portfolio theory using SIF, although using a number of assumptions and decision rules Cecchetti, Cumby and Figlewski (1988) did conclude that the portfolio model was optimal in the Treasury bond futures market. The practical difficulties of the portfolio theory remains the stumbling block to applying the portfolio theory, and the empirical research thereof.

CHAPTER FIVE

THEORY OF SHARE INDEX FUTURES PRICING

5.1. INTRODUCTION

A large amount of human and financial resources have been invested internationally into the analysis of futures markets, including those of share index futures. The most obvious reason for this investment is to identify potential speculative opportunities through the correct forecasting of price movements. However, how does the hedger benefit from the improved knowledge of futures pricing?

Knowledge of the pricing mechanisms amongst futures market participants, as with any financial or commercial market, helps to keep the pricing of futures contracts as theoretically optimal as possible. Futures markets which reflect a large incidence of sub-optimal pricing reveal the two characteristics discussed below.

Firstly, the correlation between the spot and futures prices in sub-optimal markets is not as high as that in optimally priced markets *ceteris parabus*. The correlation coefficient (r) plays an important role throughout hedging theory. It is an important hedging consideration under the traditional hedging strategy, determines the degree of hedging effectiveness under the Johnson minimum-variance strategy, and is incorporated into Howard and D'Antonio's effectiveness measure (HE) and hedge ratio (b).

Secondly, sub-optimally priced markets display a large amount of basis risk as a result of the poor correlation. From the hedging theory and extensive empirical research, the introduction of basis risk into a portfolio lessens the effectiveness of a hedge, particularly in respect of risk reduction (although return is sometimes seen to improve in cross hedges).

Thus, from the above, sub-optimally priced markets lessen the effectiveness of a hedge. Hedgers have control over decisions regarding the particular hedging objective and related hedge ratio. Unfortunately, they have limited or no influence over the effectiveness of a particular hedge because this is determined to a very large extent by the degree to which the market is optimally priced.

The level of optimal pricing in a market can be referred to as the degree of pricing efficiency. Efficiency is the "ability to produce the desired effect with a minimum of effort, expense or waste" (McKechnie, 1983). As the effectiveness of a futures hedge is intrinsically dependent on the pricing efficiency of the futures market, it is necessary to examine this aspect of share index futures markets.

This chapter examines the development of futures pricing. Initially, it develops the pricing equation in a perfect market. The restrictions of a perfect market are then relaxed in turn, and the impact of these conditions on the pricing model are analysed. For convenience the core terms used in this chapter are defined foremost in Table 5.1.

Table 5.1.
Notation for Share Index Futures Pricing

t	: Time when the futures contract is taken out.
T	: Time when the futures contract expires.
F(t,T)	: Futures Price at time t given the expiration of contract at time T.
I(t)	: Index value at time t.

5.2. OPTIMAL PRICING MODEL IN A PERFECT MARKET

A perfect financial securities market assumes that (Copeland & Weston, 1988: 331);

(1) "Markets are informationally efficient; i.e., information is costless, and it is received simultaneously by all individuals."

(2) "Markets are frictionless; i.e. there are no transaction costs or taxes, all assets are perfectly divisible and marketable, and there are no constraining regulations." This includes, inter alia, the assumption margin deposits are not required.

(3) "There is perfect competition in product and securities markets. ... In securities markets this means that all participants are price takers."

(4) "All individuals are rational expected utility maximisers."

While not a requirement for a perfect markets, it is initially assumed that dividends are paid out at a constant rate to facilitate the development of the optimal pricing model known as the cost-of-carry valuation model. It is also assumed that investors can borrow and lend at the risk-free rate; perfect capital markets only assume that borrowing and lending rates are equal (Copeland & Weston, 1988: 330).

5.2.1. Cost-of-Carry Valuation Model

In their simplest form, futures prices are prices set today, given current market information, for commodities (goods or securities) in the future. If a commodity is needed at a point of time in the future, one could acquire the underlying asset by purchasing the asset today and keeping it till the future point in time. Alternatively, by entering into a futures contract, one could simply purchase the asset at a future point in time based on today's prices. In a perfect market both options should be equal.

From this, it can be concluded that the difference between the futures price and the spot price of the underlying asset can be attributed to fundamental differences between holding the future and holding the asset. Fundamental pricing differences between spot and futures comprise the opportunity cost or benefit of holding a futures contract compared to holding the underlying asset; opportunity costs arise as a result of differences in income earned and expenses incurred. The net difference represents the net opportunity cost (or benefit) of carrying (holding) the asset rather than entering into the futures contract, and is thus known as the "cost-of-carry". In terms of the above, the futures price can be established through the basic model.

$$F(t,T) \quad = \quad I(t) \; + \; CC \qquad (5\text{-}1)$$

where CC : Cost-of-carry.

In a perfect market there are no income or expenditure implications of holding a share index futures contract. A SIF contract is simply a standardised agreement between two parties in which no consideration passes until the futures contract expiration date. Consequently, there are no costs of financing the up front capital investment (zero margin deposits have been assumed) and there is no return from the futures contract over the contract period.

Although one cannot physically hold the index, it can be "held" by constructing a share portfolio containing the identical constituents in exactly the correct proportions as the index is composed itself. Thus, there are income and expenditure implications of holding the index, namely the cost of financing the initial investment and the dividend income receivable over the holding period. The cost-of-carry model in 5-1 will be expanded to take into account these factors.

The finance costs of the investment in the index increases the cost of holding the index and consequently increases the cost-of-carry (CC) in equation 5-1. In simple terms, for holding the index to be equivalent to holding the futures contract, the future price must include the benefit of not paying finance costs, according to equation 5-2;

$$F(t,T) \quad = \quad I(t)\, e^{r(t,T)(T-t)} \qquad (5\text{-}2)$$

where $r(t,T)$ is the interest rate (r) on a risk-free bond that is issued at time t and will mature at time T. The risk-free interest rate has been used as it has been assumed that investors can borrow at the risk-free rate, and the term (t,T) takes into account stochastic interest rates.

However, there is a benefit of holding the index as dividend income accrues. The futures holder must, therefore, be compensated for this income foregone. Accordingly, we can extend the model in 5-2 as;

$$F(t,T) = I(t)\, e^{[r(t,T)-d](T-t)} \qquad (5\text{-}3)$$

where d is the constant dividend payout rate. Equation 5-3 is the cost-of-carry valuation model in a perfect market assuming a constant dividend payout rate and a risk-free borrowing rate. It is simply the deferred (future) value of the index less the deferred (future) value of the dividends. The proof that the cost-of-carry valuation model is indeed the optimal model can be established through arbitrage pricing.

5.2.2. Arbitrage pricing

An arbitrage is a position held where a risk-free profit can be made through pricing imperfections between two related markets, such as between the share and share index futures markets. If there is no mispricing between the two respective markets then the futures price is optimal in terms of arbitrage pricing.

The arbitrage pricing model is developed below using the illustration of Cornell & French (1983a) adapted for share index futures markets. The arbitrage model is proved by contradiction: initially, the situation where the futures price is greater than the price per the cost-of-carry valuation model will be considered; and then the reverse situation will be looked at.

Suppose that the futures price is greater than the effective cost of purchasing the index and holding it till expiration date, i.e. the following inequality holds:

$$F(t,T) > I(t)\ e^{[r(t,T)-d](T-t)} \qquad (5\text{-}4)$$

At time t, the arbitrageur can undertake the following trades:

(i) Borrow I(t) and purchase the index.

(ii) Short one share index futures contract.

No cash is required to create the above portfolio, while the debt incurs interest at the risk-free rate (assumed in section 5.2.). At expiration (time T) the futures contract is settled, the index portfolio is sold and the debt is repaid. As a result, the overall position generates the following three cash flows (negative amounts indicate cash payments):

(a) $- I(t)\ e^{r(t,T)(T-t)}$ from the debt.

(b) $I(T)\ e^{d(T-t)}$ from the share position.

(c) $F(t,T) - I(T)$ from the short futures position.

The cash flows when summed up equal the following total pay-out:

$$\text{Pay-out} = F(t,T) - I(t)\ e^{[r(t,T)-d](T-t)} \qquad (5\text{-}5)$$

Since equation 5-4 has been assumed, the pay-out must be a positive value. The arbitrageur has thus earned a profit free of risk and without making any initial investment. Given this disequilibrium, all investors would seek to enter into short future and long index arbitrage positions. The increased demand places downward market pressure on the futures price and/or upward pressure on the spot price, which in time restores the futures price back to equilibrium in terms of the cost-of-carry model.

Arbitrage opportunities also exist if the reverse situation is true, i.e. the futures price is less than the deferred cost of the share index less the deferred value of the dividends, given in Equation 5-6.

$$F(t,T) \quad < \quad I(t)\ e^{[r(t,T)-d](T-t)} \qquad (5\text{-}6)$$

In this case, the arbitrageur can make the following trades at time t:

(i) Short the share index[1] and invest the proceeds at the risk-free rate.

(ii) Enter into a long futures contract.

Once again the initial investment is nil. At time T the risk-free investment is realised, the long futures position expires and the short index position is settled. The cash flows yielded as a result are given as;

(a) $I(t)\ e^{r(t,T)(T-t)}$ from the risk-free investment.

(b) $-\ I(T)\ e^{d(T-t)}$ from the short index position.

(c) $I(T) - F(t,T)$ from the long futures position.

1. The index can be sold short by selling the index constituents themselves short.

The settlement amount in respect of the short index position will include the value of the dividends for the period, hence the dividend term above. The total pay-out will amount to:

$$\text{Pay-out} = I(t)\, e^{[r(t,T)-d](T-t)} - F(t,T) \qquad (5\text{-}7)$$

The net pay-out will again will be positive according to our assumption in 5-6 above. Similarly, given the long futures arbitrage scenario, demand for long (buy) futures positions and short spot positions will put upward pressure on the futures price and/or downward pressure on the spot price until the inequality 5-6 no longer applies. By so doing, the futures price moves back into equilibrium in terms of the cost-of-carry valuation model.

This analysis proves through arbitrage pricing that the cost-of-carry valuation model given in equation 5-3 is the optimal pricing model. No arbitrage opportunities exist if futures are priced in terms of this model. Moreover, the arbitrage mechanism itself keeps futures pricing at its optimal position by enabling market participants to exploit arbitrage profits thereby exerting pressure on prices to move back to their correct positions.

5.3. PRICING IN IMPERFECT MARKETS

In this section we will now address pricing in imperfect markets by analysing the assumptions of a perfect market given earlier with particular reference to the financial markets in South Africa. At this stage it is still assumed that dividends are paid out at a constant rate: this assumption will be investigated in depth in section 5.4.

5.3.1. Market Information and Efficiency

The most important factor affecting the price of any futures contract is the value of the underlying spot asset. In perfect markets there is a one-to-one relationship between the value of the spot and the value of the future. However, if the spot or the futures market is not information efficient, then information disturbances between the two markets may lead to differences between the spot and the futures price. For this reason, it is important to assess the efficiency of the share index and the share index futures markets.

The efficient market hypothesis (EMH), as pioneered by Fama (1970), is based essentially on the premise that all available information regarding a particular share is impounded immediately in the security price. The current price fully reflects all available market information and, therefore, there is no additional information on which to predict future prices. Security prices which obey this hypothesis are said to be in market equilibrium. A market may still be in equilibrium in terms of the EMH even if the other conditions of a perfect market do not hold (e.g. the presence of transaction costs).

Fama (1970: 383) identified three basic types of market efficiency, namely, the weak, semi-strong and strong forms. The weak form is where the 'information set' available includes historical prices only, the semi-strong form includes all publicly available information (such as earnings announcements and press reports) while the strong form goes one step further and includes inside information not available to the public at large.

The tests of the EMH are weak and cannot conclusively prove the markets efficiency. The EMH is "inherently untestable" (Hatjoullis & Stark, 1981) because there is no testable null hypothesis for inefficiency (Brenner, 1977; Summers, 1986). However, the large amount of empirical studies performed on the EMH does give us fairly good corroborative evidence of the validity of the EMH.

Tests of efficiency in the weak form involve mainly the investigation of the random nature of security prices (known as the random-walk hypothesis) and studies of filter or trading rules. Voluminous tests have been performed which support the random-walk hypothesis and trading rules have been shown to not produce consistently superior returns (Firth, 1977; Affleck-Graves & Money, 1975). This gives support of the EMH in the weak form.

At the semi-strong level tests have been performed using specific economic events such as share splits, capitalisation issues and earnings announcements. No test as yet has disproved the EMH at the semi-strong level (Levy & Sarnat, 1984). Fama (1970: 383) concluded from his analysis that "the efficient market stands up well in the semi-strong form". Tests of the strong form have, not surprisingly, shown that the market is inefficient with respect to non-public information (Levy et al, 1984; Firth, 1977). Individuals having access to insider information have been able to reap superior returns.

A number of EMH research studies been performed on the Johannesburg Stock Exchange (JSE). Affleck-Graves & Money (1975) concluded that at least seventy to eighty percent of shares quoted on the JSE are efficient in the weak-form, so "any form of technical analysis based on past prices is worthless".

Gilbertson & Roux (1977) later found that the JSE share prices were consistent with the EMH in the weak and semi-strong forms. This research was criticised by Strebel (1977) because the tests were inappropriate for shares with low trading volumes. The JSE according to Strebel is efficient in the weak and semi-strong forms for shares with annual trading volumes in excess of a quarter of a million shares. In terms of this approximately half the shares listed on the JSE are efficient in the semi-strong form (Strebel, 1977).

It is reasonable to assume from the above empirical evidence that the major share indexes in South Africa (namely the All Share, All Gold and Industrial indexes) behave in a manner consistent with the EMH on at least the semi-strong level. Therefore, as the share indexes fully reflect all publicly available information, it is reasonable to conclude that there is no additional public information available which will impact a derivative of the share index such as a share index futures contract. Even in imperfect markets, therefore, no adjustment to the futures price is necessary as markets are information efficient.

5.3.2. Transaction Costs

The optimal pricing model has been established through the arbitrage model framework assuming a market free of restrictions of any kind. However, in an imperfect market situation there are many factors which may impede both share and futures transactions. These include physical costs incurred in entering transactions, such as brokerage, futures bid-ask spread, commissions and taxes; and other intangible factors which impede transactions and increase the cost of trading, for example imperfect divisibility of futures contracts and marketability constraints. All these costs of imperfect markets are collectively referred to as transaction costs.

With the introduction of transaction costs, the arbitrageur will now only enter into an arbitrage position if the net overall arbitrage proceeds (i.e. after transaction costs) are positive. Therefore, the short futures hedge will now only be profitable if the inequality 5-8 holds, and similarly, inequality 5-9 will be required for the long hedge to be profitable. The proof of these are straightforward following the approach in section 5.2.2.

$$F(t,T) > I(t)\, e^{[r(t,T)-d](T-t)} + T_f + T_l \qquad (5\text{-}8)$$

$$F(t,T) < I(t)\, e^{[r(t,T)-d](T-t)} - T_f - T_s \qquad (5\text{-}9)$$

where

T_f : Transaction costs relating to the futures arbitrage leg.

T_s : Transaction costs relating to the share index arbitrage leg in a short index and long futures arbitrage position.

T_l : Transaction costs relating to the share index arbitrage leg in a long index and short futures arbitrage position.

The transaction costs relating to the spot share index position are distinguished between those relating to the short spot arbitrage position (T_s) and those costs relating to the long spot arbitrage position (T_l). This is because transaction costs relating to shorting the index often differs from that relating to purchasing the index. Futures transaction costs do not ordinarily differ between short and long positions, thus no distinction is made above.

Equation 5-8 and 5-9 can be combined to give a range of prices in which no arbitrage profits can be made, i.e. when neither of the inequalities 5-8 or 5-9 are satisfied. This range is given in expression 5-10 as;

$$I(t)\ e^{[r(t,T)-d](T-t)} + T_f + T_l \geq F(t,T) \geq I(t)\ e^{[r(t,T)-d](T-t)} - T_f - T_s \qquad (5\text{-}10)$$

Therefore, an arbitrageur cannot reap a risk-free profit unless the extent of the futures price anomaly from the optimal position exceeds the total transaction costs to be incurred, both on the shares and futures. This is well summed up by Gould (1988: 48):

> "When transaction costs are considered, it can be seen that the relationship between a futures contract and its underlying stock index is defined, not by a 'fair price', but by a 'fair range' of prices that lie in a window about the value of the underlying index. Opportunities to arbitrage profitability between the cash and futures markets (by either buying futures or selling them short) will occur only when futures price lie outside this fair range, or window. In other words, any prices within this window are fair."

The window has been defined by expression 5-10. Prices within the two arbitrage bounds, designated by the left and right hand sides of 5-10, are considered to be optimally (or fairly) priced because arbitrage opportunities do not exist for prices within this range. Therefore, there is a range of optimal futures prices in imperfect markets and not a single optimal price as was the case in perfect markets.

5.3.3. Borrowing costs

In addition to the above costs, an arbitrageur may not necessarily be able to borrow at the risk-free rate. The term representing the financing costs ($e^{r(t,T)(T-t)}$) is likely to be greater in imperfect markets where borrowing and lending rates are above the risk-free rate and different from each other. As a consequence, the fair range expressed per 5-10 will increase by the following amount (adapted from Gould, 1988: 48):

$$I(t)\ (e^{rb(t,T)(T-t)} - e^{r(t,T)(T-t)}) \qquad (5\text{-}11)$$

where $e^{rb(t,T)(T-t)}$ is the effective continuously compounded annualised interest at the individual's borrowing rate for the period from entering into the futures contract (t) to the date of expiration of futures contract (T). Therefore, the futures price will need to be at an even greater premium or discount to the optimal cost-of-carry valuation before risk-free arbitrage opportunities exist. This is because the arbitrageur will have to recoup the extra borrowing costs before any arbitrage profit can be made.

If the arbitrageur has capital funds, however, the opportunity cost of using those funds to enter into an arbitrage trade will be the risk-free rate of return and so in this case there will be no adjustment to the fair range. The opportunity cost of capital funds is normally considered to be the individual's investing rate (cost of capital) but the investing rate assumes a level of risk associated with the risky investment. However, arbitrage transactions are risk-free and so the opportunity cost of using the funds for a similar investment is the risk-free rate of return.

In conclusion, "market aberrations must be more significant in order for those without capital to participate in opportunities to earn risk-free returns above the risk-free rate" (Gould, 1988: 50).

5.3.4. Other Issues

The third assumption of perfect markets states that all participants in securities markets are price takers. As the index represents the total share market (or significant portion of the market), manipulation of the SIF price through squeeze[2] or corner[2] is virtually impossible (Falkena et al, 1989: 4). In view of this, coupled with the large volumes and competition in futures markets, it is reasonable to conclude that all participants of the share index futures markets are price takers.

The assumption of utility maximisation is axiomatic in that it is extremely difficult to prove (or disprove) but is the foundation of modern portfolio theory[3] and hedging theory. It must, therefore, be assumed that this assumption also holds in imperfect markets.

The arbitrage pricing model assumes that the only investment needed to be made in an arbitrage situation is the investment in the underlying index. However, additional investment is required in the form of margin payments in

2. See Glossary for terminology used.
3. Refer section 1.2.2.

respect of futures contracts. The finance capital costs associated with making margin deposits increase the arbitrage bounds. Where a money market rate of interest is paid on margins, however, (for example on the South African Futures Exchange) the effect on the arbitrage bounds will be reduced (or even eliminated) by the compensating return received.

5.4. DIVIDENDS

In formulating the optimal cost-of-carry valuation model and arbitrage bounds it was assumed that dividends are paid out constantly over the year. However, this is not true as companies tend to be pay dividends in discrete amounts a few times a year, normally twice in the form of an interim and a final dividend. This section will evaluate the effect of lumpy dividend payments by firstly reviewing the major local and international literature, and secondly assessing the payment of dividends in the South African situation. Finally, the impact of lumpy dividend payments on the cost-of-carry valuation model will then be discussed.

5.4.1. Review of Literature

Gastineau and Madansky (1983) studied the timing consideration of dividend payments of the shares constituting the Standard & Poors (S&P) 500 share index for January, February and March 1982. The paper illustrated the disparity of dividend payments between months; for example, about seven times more dividends are paid in the first half of February compared to the second half of January.

Gastineau et al (1983: 69-76) noticed a pattern of these dividend payments and constructed "Evaluation Tables" to calculate the futures price taking into account the appropriate pattern of dividend payments. Modest (1984) also examined the effect that lumpy dividends have on futures markets, and supported the views of Gastineau & Madansky.

Peters (1985) studied the lumpiness of dividend payments on the S&P 500 index over 1982 and 1983. His paper illustrated the considerable impact that lumpy dividends had on pricing. Peters discovered that the pricing of the S&P 500 index future was becoming increasingly more efficient owing to the better estimation by market participants of the future lumpy dividend stream.

Turning to the South African situation, Lambrechts (1988) recognised the seasonality of dividends in South Africa since 96% of companies have their year-end in either December, February, March, June or September. He commented that "a fluctuating dividend payout pattern may therefore significantly impact on theoretical (futures) prices". King (1989) also illustrated the seasonality of dividends on the South African markets, with June, August and December being the most common months in which the last day to register (LDR) fell in respect of shares comprising the All Share and All Gold index.

5.4.2. Evaluation of the South African Situation

To assess the extent of lumpy dividend payments in South Africa, it is first necessary to obtain the actual dividend payments made by the index constituents. This was done for the All Share index (ALSI), All Gold index (ALGI) and Industrial index (INDI) for the period from April 1987 to

December 1989. The data were extracted from JSE Monthly Bulletins.

Given the importance of the timing of dividend payments, the selection of the date of the dividend payment is vital. Only shareholders registered on the last day to register (LDR) are entitled to the dividend declared and so this is regarded as the effective payment date. It must be noted that the date of the actual dividend payment (payment date) to all shareholders registered on the LDR date is, strictly speaking, the correct date. However, this simplification results in only a minor difference, that being the loss of a few weeks reinvestment earnings. The LDR date was also used by other studies including Peters (1985) and Gastineau & Madansky (1983).

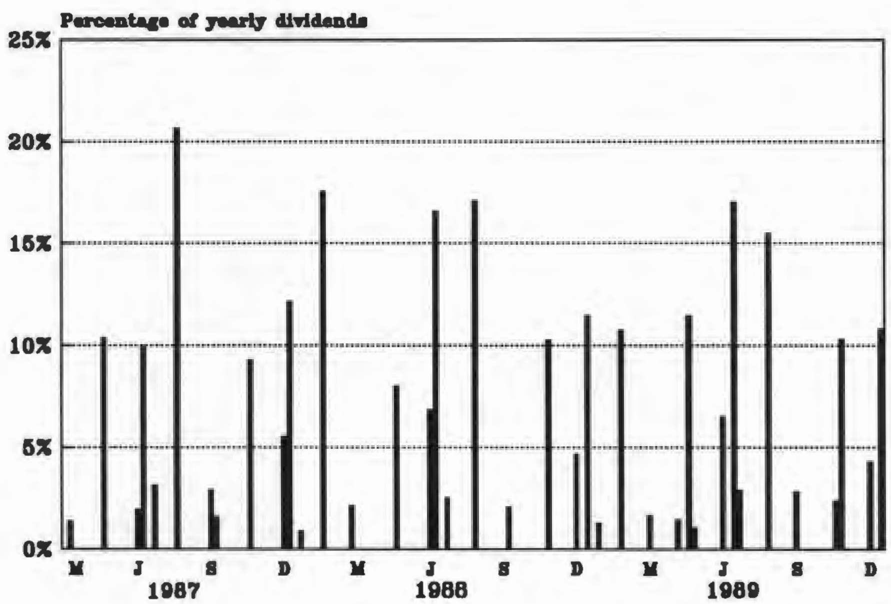
Dividend payments were accumulated per week and not daily; as nearly all company's LDR dates fall on a Friday the collection of data per day would have been unnecessary. The results for each index are discussed in turn below.

5.4.2.1. All Gold Index (ALGI)

From Figure 5.1. of ALGI dividend payments, a sharp contrast can be seen between relatively few weeks of very high dividend payments and a large number of weeks where no dividends are paid at all.

The large dividend payments on the index are made at the same times of the year each year because they are made by a few very large index constituents which pay their dividends consistently at the same time in each year. Conversely, the periods where no payments are found is due to there being no index constituents which pay their dividends at these times.

Figure 5.1.
Dividends Paid on All Gold Index



The weeks of the year in which the majority of dividend payments were paid are listed in Table 5.2. together with the index constituents contributing to these large payments. The timing of dividends paid by these constituents over the observation period were so consistent that they could be identified to a particular week of the year (and not just to the particular month).

In the six weeks mentioned in Table 5.2, an average of R1510 million (76% of the total dividends) were paid by the above constituents in 1988 and 1989. This not only indicates the degree to which lumpy dividend payments are made on the ALGI, but moreover, it displays the extent to which dividends are paid by companies in the same week of the year each year. There is a definite pattern in the timing of dividend payments which suggests that future dividend payment timings can be predicted fairly accurately.

Table 5.2. All Gold Index
Highest Dividend Paying Periods and Responsible Constituents

Period	Constituents
Second week in February	Southvaal, Western Deep, & Vaal Reefs
First Week in May	ERGO, Freegold
Last week in June	Driefontein, Kloof
Second week in August	Southvaal, Western Deep, & Vaal Reefs
Second Week in November	ERGO, Freegold
Last week in December	Driefontein, Kloof

Table 5.3.: All Gold Index
Market Capitalisation and Dividend Payments

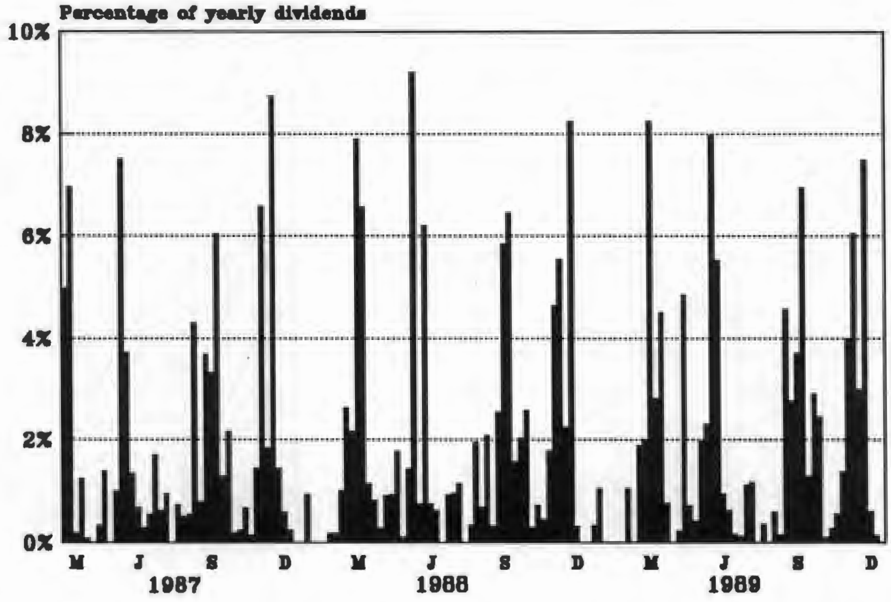
	Average Market Cap (Rm)	% of Total Mkt Cap	Average Dividends p.a.(Rm)	% of Total Dividends Paid p.a.
Driefontein	15606	20.0	393	19.8
Vaal Reefs	11391	14.6	253	12.8
Kloof	8359	10.7	161	8.1
Freegold	7566	9.7	343	17.3
	42922	55.0	1150	58.0

The dominance of a few very large companies within the index constituents is illustrated in Table 5.3. It shows that only four companies constitute 55% of the index capitalisation and pay 58% of the dividends.

5.4.2.2. Industrial Index (INDI)

As with the ALGI, Figure 5.2. illustrates the contrast on the INDI between relatively few weeks of extremely high dividend payments and the greater proportion of the year where relatively small amounts of dividends are paid. The variability is not as marked with the Industrial index compared to the ALGI owing to the greater number of index constituents and reduced dominance by large corporations on the INDI.

Figure 5.2.
Dividends Paid on Industrial Index



However, a number of weeks can still be identified in which a significant portion of the dividends are paid, as given in Table 5.4. In the seven weeks identified in Table 5.4. an average of R938 million was paid in 1988 and 1989 by only the five constituents mentioned. This represents 34% of the total dividends paid on the INDI. The dominance of these companies on the index is further illustrated in Table 5.5.

Table 5.4.: Industrial Index
Highest Dividend Paying Periods and Responsible Constituents

Period	Constituents
Last two weeks in March	AMIC, Remgro, Sasol
First two weeks in June	Barlows, SAB
Third and Fourth week in September	AMIC, Sasol, Remgro
Second week in December	Barlows

Table 5.5. Industrial Index
Market Capitalisation and Dividend Payments

	Average Market Cap (Rm)	% of Total Mkt Cap	Average Dividends p.a. (Rm)	% of Total Dividends Paid p.a.
Remgro	11680	8.5	101	3.7
Barlows	11090	8.1	268	9.8
SAB	11052	8.1	213	7.8
SASOL	10648	7.8	265	9.7
AMIC	9459	6.9	151	5.5
	53929	39.4	998	36.5

5.4.2.3. All Share Index (ALSI)

There are a far greater number of shares making up the ALSI than the ALGI and INDI, resulting in the smoother pay-out of dividends on the ALSI as illustrated in Figure 5.3.

Figure 5.3.
Dividends Paid on All Share Index

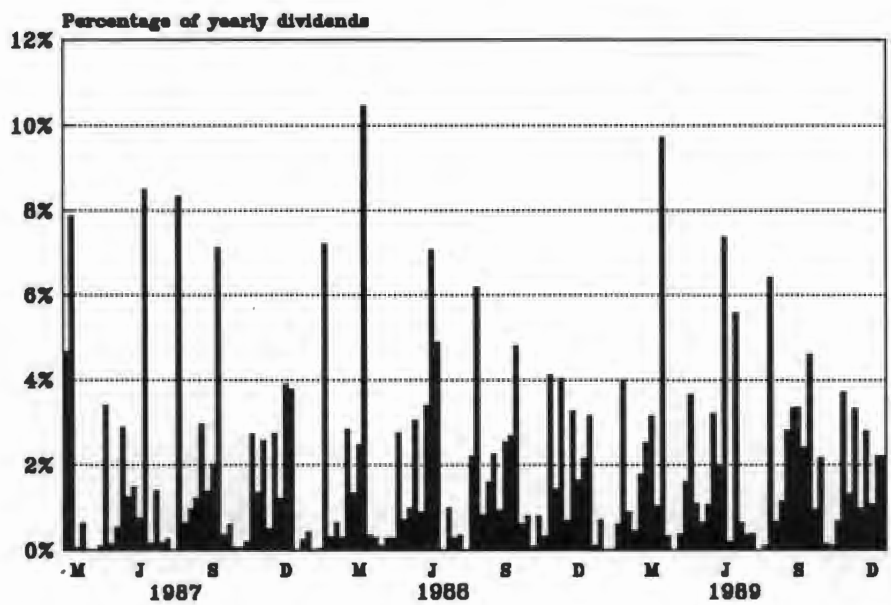


Table 5.6.: All Share Index
Highest Dividend Paying Periods and Responsible Constituents

Period	Constituents
Second week in February	Southvaal, Vaal Reefs, Western Deep, Rusplats, Implats
Last week in March	De Beers, Amgold
Last two weeks in June	AAC, Barlows, Driefontein, Harties
Second week in August	Vaal Reefs, Southvaal, Western Deep, Rusplats, Implats
Last week in September	De Beers, Amgold
Second and third week in December	AAC, Driefontein, Harties, Barlows

However, Figure 5.3. shows that there are still periods where very small amounts of dividends are paid and others where extremely large payments are made. The major constituents responsible for these payments and the weeks of the year in which they are consistently paid are given in Table 5.6. An average of R3221 million was paid in the eight weeks identified in Table 5.6. by the 11 constituents during 1988 and 1989. Thus, only 11 shares (out of a total of 164 constituents) were responsible for paying 31% of the dividends over only eight weeks of the year.

Table 5.7. All Share Index
Market Capitalisation and Dividend Payments

	Average Market Cap (Rm)	% of Total Mkt Cap	Average Dividends p.a. (Rm)	% of Total Dividends Paid p.a.
De Beers	34201	8.9	472	4.5
Anglo Am	33498	8.7	591	5.7
Driefontein	15606	4.0	393	3.7
Gencor	13488	3.5	280	2.7
Rusplats	12688	3.3	326	3.1
Amgold	12164	3.2	299	2.9
Remgro	11680	3.0	101	1.0
Vaal Reefs	11391	3.0	253	2.4
Barlows	11089	2.9	268	2.6
SAB	11052	2.9	213	2.0
GFSA	11332	2.9	160	1.5
SASOL	10648	2.8	265	2.5
Freegold	7566	2.0	343	3.3
	196403	51.1	3964	37.9

This variability of dividend payments is once again a result of a few companies' considerable influence on the total market capitalisation and dividends paid on the index. This influence is indicated in Table 5.7. where only 13 companies constitute over 50% of the market capitalisation of the All Share Index.

5.4.3. Impact of Dividends on the Pricing Model

Thus, from the above analysis, the assumption of constant dividend payout cannot be accepted in the South African situation. This is in line with research both locally and abroad. Accordingly the cost-of-carry valuation model is altered to take into account seasonal dividends, as follows:

$$F(t,T) = I(t)e^{r(t,T)(T-t)} - \int_t^T D(w)e^{R(t,w,T)(T-w)} \qquad (5\text{-}12)$$

where D(w) is the instantaneous dividend at time w and R(t,w,T) is the forward rate at time t for a loan that will be made at time w and that will mature at time T. This will also change the arbitrage bound to;

$$I(t)\ e^{r(t,T)(T-t)} - \int_t^T D(w)e^{R(t,w,T)(T-w)} + T_f + T_l$$

$$\geq \quad F(t,T) \quad \geq \qquad (5\text{-}13)$$

$$I(t)\ e^{r(t,T)(T-t)} - \int_t^T D(w)e^{R(t,w,T)(T-w)} - T_f - T_s$$

5.4.4. Estimation of Dividend Payments

The revised cost-of-carry valuation model (5-12 above) now reflects the sum of the deferred value of expected dividends receivable on the index. Unfortunately, while the computation of this term is straightforward in principle, in practice it is a lengthy process involving the estimation of the timing and amount of dividends declared for each index constituent. For example, to calculate the dividends receivable on the All Share Index, estimates will need to be made for all the 164 constituents (JSE, 1988: 8-14). This would be impracticable and very expensive for a hedger to follow every time a futures hedge (or for that matter any futures position) is considered.

Owing to these problems faced with estimating the expected dividends receivable over the hedge, the quoted dividend yield on the index if often resorted to by hedgers as a surrogate measure for the expected dividends to be received. The dividend yield on the index is defined as the ratio between the most recently declared twelve months dividends for the shares constituting the index and the total market capitalisation of those constituents at the date of calculation (JSE, 1988: 19). The dividend yield (DY(t)) can be represented by (JSE, 1988: 23);

$$DY(t) = \frac{\sum_{i=1}^{n} D_{it} W_{it}}{\sum_{i=1}^{n} P_{it} W_{it}} \qquad (5\text{-}14)$$

where

D_{it} : Total dividends per share of constituent i declared for the twelve months ending at time t.

W_{it} : Total eligible shares of constituent i at time t.

P_{it} : Market price per share of constituent i at time t.

n : Number of constituents in the index.

The dividend yield above is essentially a twelve month moving average of dividend return which has the effect of smoothing the dividend return on the index. Moreover, the dividend yield is based on past dividends and not expected dividends; consequently, the dividend yield will have the effect of understating the dividend return on the index assuming a general trend of increasing dividend payments. Thus the dividend yield is inaccurate in terms of both the timing and level of dividend payments.

It is necessary to assess the margin of error associated with using the dividend yield as a surrogate for actual dividends. For hedges of long duration (i.e. approaching a year in length) the difference between the dividend yield and the actual dividend return is likely to be small. However, hedge positions are often taken out for short periods (three months or less), in which case there may be a significant difference between the quoted index dividend yield and the effective actual dividend return.

Some research has been done on this in South Africa. Lambrechts (1988) recognised the problem with the use of dividend yields, but unfortunately he did not take into account the seasonality of futures prices into his empirical analysis.

King (1989) found that the difference due to dividends had a small impact on the futures price with a maximum deviation of 1.3% of the futures price. However, while the difference between the two is small on a percentage basis, the pricing efficiency criteria is dictated by the arbitrage model. A small difference compared to the total futures price may still mean that the futures are not optimally priced in terms of the arbitrage model. Unfortunately, King did not investigate the dividend error in absolute terms and relate this to arbitrage pricing.

In chapter 6, an assessment will be made regarding the applicability of using the dividend yield as an estimate for actual dividends. This will be done by establishing the absolute pricing error and relating this to the general arbitrage argument.

5.5. CONCLUSION

In this chapter the cost-of-carry valuation model has been developed based on arbitrage pricing theory. The cost-of-carry model given in expression 5-12 defines the fair range within which the futures price is optimally priced. Any price outside this range is mispriced and is subject to arbitrage opportunities. This fair range will differ from individual to individual depending on the particular market barriers and costs encountered.

The lumpiness of dividend payments was evaluated. The large degree of lumpy dividends paid on the major indexes in South Africa was illustrated which is similar to that found on overseas markets. The lumpy dividends may have a significant impact on the optimal futures price in terms of the cost of carry model. For this reason, the dividend yield should not be accepted, without further analysis, as a surrogate for expected future dividends on the index.

CHAPTER SIX

EMPIRICAL RESEARCH: FUTURES PRICING

6.1. INTRODUCTION

The pricing efficiency of the share index futures market is evaluated with respect to the cost-of-carry valuation model and arbitrage principles. This is achieved in two ways, namely through regression techniques and the construction of arbitrage bounds around the theoretical futures price.

Initially, the regression procedure is used to evaluate whether the actual futures price is significantly different from the theoretical futures price, thereby identifying periods with significant pricing differences. Thereafter, pricing optimality is assessed by reconstructing arbitrage bounds from which the incidence of arbitrage opportunities can be established. However, before investigating futures pricing it is necessary to ascertain whether the dividend yield can be used as an effective surrogate for actual dividends in the calculation of the theoretical futures price.

6.2. SELECTION AND EXTRACTION OF DATA

While each analysis will be performed in turn, they all are based on the same data set. All three share index futures contracts in South Africa are looked at, namely futures derivatives based on the All Share Index (ALSI), All Gold Index (ALGI) and Industrial Index (INDI). The period investigated extends from the inception of SIF in March 1987 to December 1989.

The trading volumes of the three (near), six (mid) and nine months (far) contracts of the three different index futures from August 1988 to July 1990 are illustrated in Figures 6.1. to 6.3. (volumes per index contract type are not available for the period prior to August 1988). The graphs illustrate the relatively low trading of nine month futures contracts compared to that of three and six month contracts. Other than the relatively high level of trading of nine month ALSI futures contracts during the period from February 1989 to September 1989, monthly volumes on the nine month futures derivative have been very low, with a significant number of months having no trading volumes at all.

On the other hand, the three and six month contracts have consistently maintained reasonably high levels of trading volumes despite the fluctuating volumes from month to month. In view of this, empirical research has been restricted to three month and six month futures contracts as the nine month futures contract is considered to be too illiquid for any meaningful conclusion to be drawn regarding futures pricing.

Friday weekly close futures prices and spot index values are used throughout the study. They were obtained from Rand Merchant Bank (RMB) Limited and verified by reference to the financial press and magazines. The quoted dividend yields on the index were obtained from the financial press and publications. Extraction of actual dividends has already been discussed in section 5.4.

Figure 6.1.
All Gold Index Futures
Monthly Volumes in Contracts

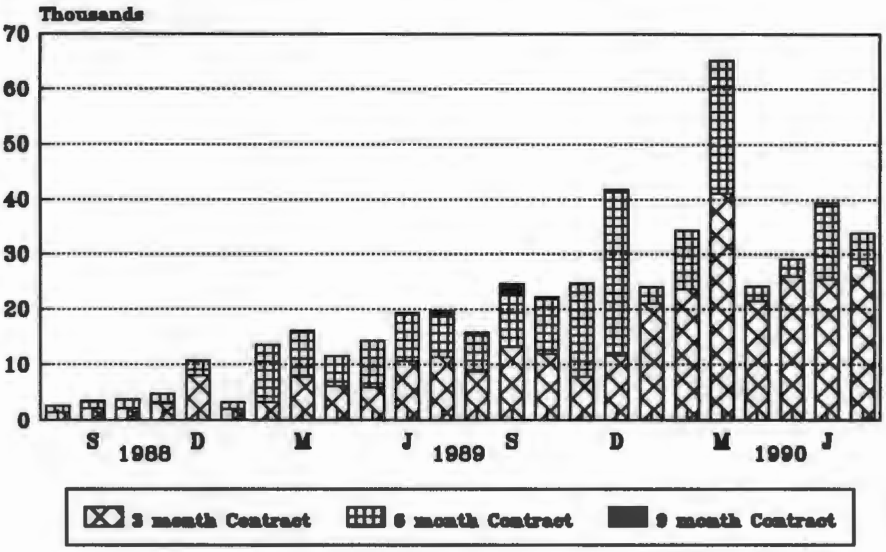


Figure 6.2.
Industrial Index Futures
Monthly Volumes in Contracts

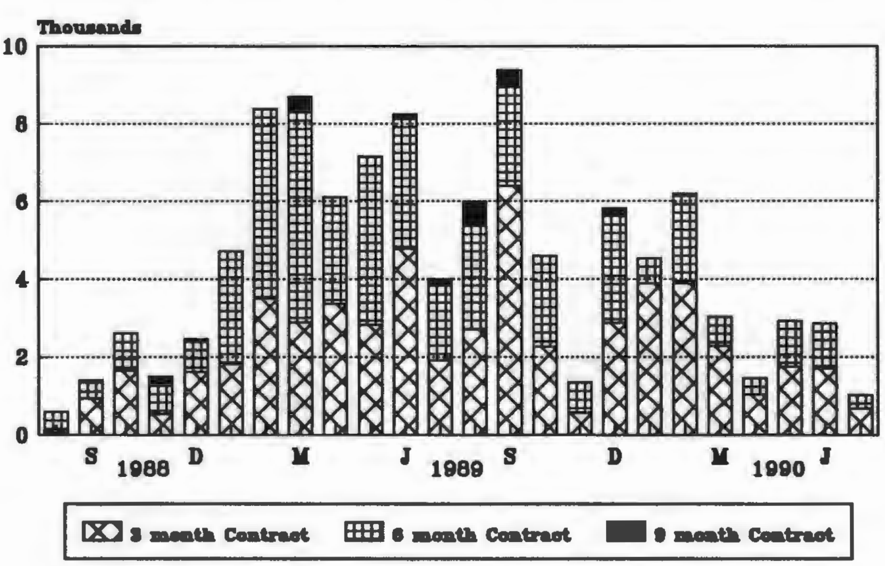
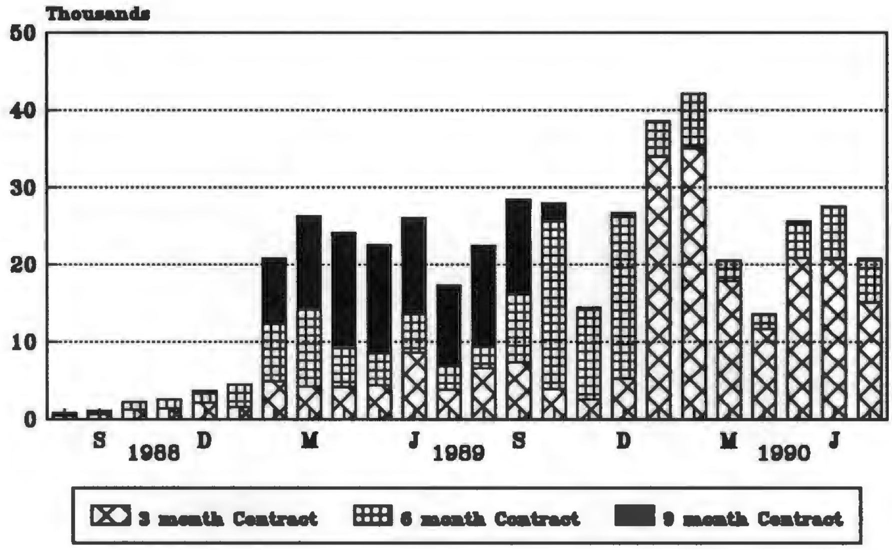


Figure 6.3.
All Share Index Futures
Monthly Volumes in Contracts



The 91-day treasury bill rate, obtained from the South African Reserve Bank Quarterly Bulletins, are Friday rates and thus the timing of all data is uniform. The treasury bill rate is used as a measure of the risk-free interest rate which is consistent with the literature (Cornell & French, 1983a; Fama & French, 1987; Modest, 1984; Gastineau & Madansky, 1983).

6.3. DIVIDEND YIELD VERSUS ACTUAL DIVIDENDS

The likely pricing error needs to be established to assess the impact of using the dividend yield instead of using actual dividends. This is done for two reasons, namely to assess whether the futures market can be empirically analysed using the dividend yield instead of actual dividends, and secondly to estimate the size and impact of the estimation error to provide information to hedgers and other market participants.

6.3.1. Methodology

Two theoretical futures prices are calculated, one based on the dividend yield at the time of observation and the other based on actual dividends receivable to expiration of the contract. The difference given between the two values will provide the absolute pricing error of using the dividend yield.

$$\text{Difference} = FP(dy) - FP(actdiv) \qquad (6\text{-}1)$$

where

Difference	:	Difference in the theoretical futures price at time t between the two dividend bases.
FP(dy)	:	Theoretical futures price per the cost-of-carry valuation model using the dividend yield at time t.
FP(actdiv)	:	Theoretical futures price per the cost-of-carry valuation model using actual dividends from time t to expiration of the futures contract (T).

The theoretical futures price based on the dividend yield is calculated in the following manner:

$$F(t,T) = I(t) + I(t)\ (TB*d/365 - DY*d/365) \qquad (6\text{-}2)$$

where

- TB : Treasury bill rate at time t.
- d : Days to expiration of the futures contract.
- DY : Dividend yield on the index at time t.

On the other hand, the theoretical futures price based on actual dividends is calculated in the following way:

$$F(t,T) = I(t) + I(t)\left(TB*d/365 - \sum_{j=1}^{k} D_j/W_j\right) \qquad (6\text{-}3)$$

where

- D_j : Total dividends paid by index constituents in week j.
- W_j : Total capitalisation of the index in week j.
- k : Number of weeks to expiration of the contract.

The risk-free rate (TB) used for both prices was taken at the date of the observation. Thus, the finance cost term calculated on a straight line basis is the same for both; only the dividend terms differ. The dividends foregone on the dividend yield basis is calculated on a simple straight line basis using the quoted dividend yield at the date of the observation. However, the actual dividend term calculated per equation 6-3 represents the sum of the weekly dividend returns of the index from the date of the observation to the date of expiration of the futures contract. The reasonableness of the weekly actual dividend returns was checked by summing each 52 consecutive weekly returns and comparing this to the quoted dividend yield.

The calculation above ignores the time value of money with respect to both the finance costs, dividend yield and actual dividends. However, despite representing a limitation to the analysis, it is firstly unlikely that these differences will have a significant effect on the theoretical futures price, and secondly, ignoring the time value of money aspect has been applied consistently to both futures prices.

6.3.2. Results

Results are provided graphically for each contract type in terms of equation 6-1. The difference shown in the graphs is positive if the actual dividends payable by the index constituents is greater than the dividend yield on the index. Conversely, if the dividend yield is greater than the actual dividends then the difference is negative.

Two graphs per contract type are given to enable all contract observations to be shown. One graph illustrates the situation where the futures contract has three months or less to expiration (near contracts), and the other illustrates contracts with three to six months to expiration (mid contracts). Each share index future is investigated in turn.

6.3.2.1. All Gold Index future

The difference between the two bases amounts to up to 12 index points in respect of the three month contracts (Figure 6.4.), and up to 17 points in respect of the six month contracts (Figure 6.5.). The six month contracts are expected to have the greatest difference because, being further away from expiration, these contracts have a greater premium (or cost-of-carry) and consequently a greater chance

Figure 6.4. All Gold Near Contract
Difference in futures price between actual dividends and dividend yield

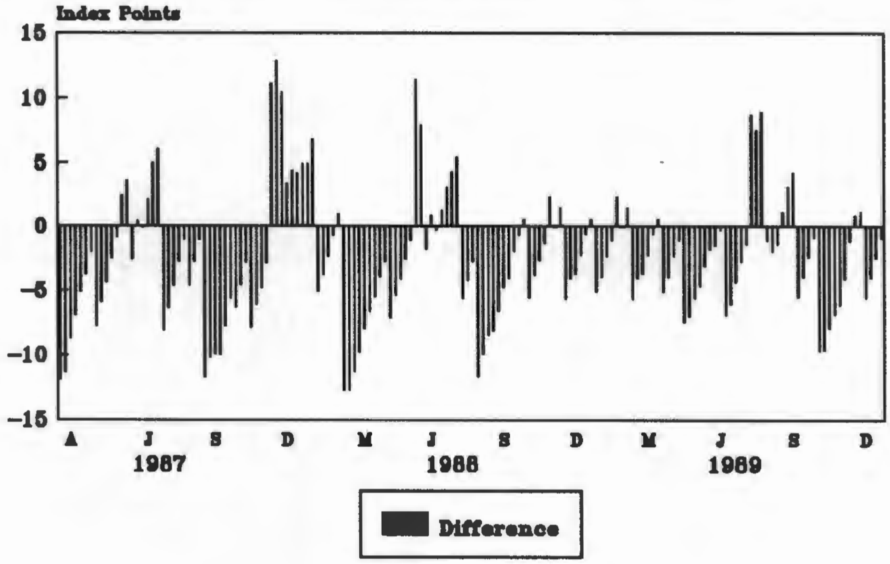
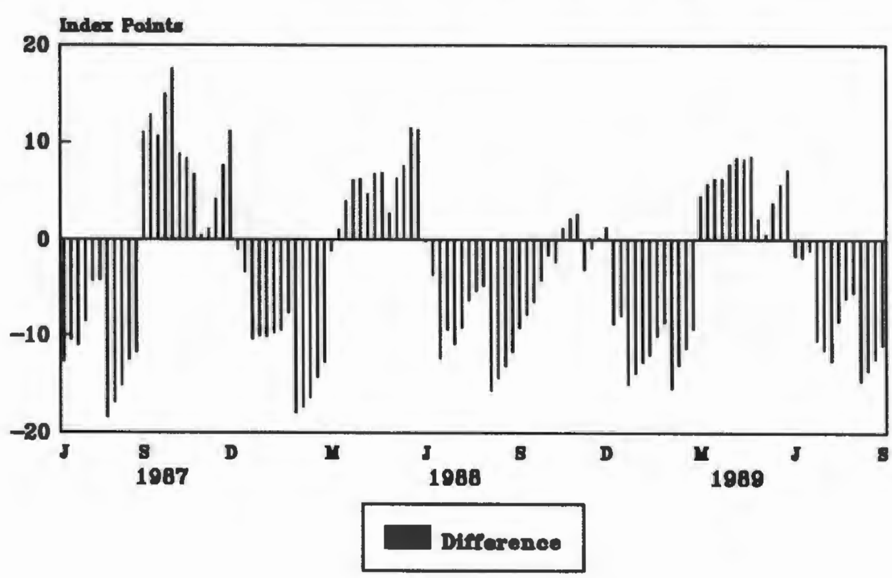


Figure 6.5. All Gold Mid Contract
Difference in futures price between actual dividends and dividend yield



of the absolute variability of this premium. However, relative to their respective premiums, the percentage differences are comparable between three and six month contracts.

The graphs reflect situations where there is a sudden increase in the amount of the difference. This is a direct result of the lumpy nature of actual dividend payments. The amount of the difference would, therefore, react sharply to any large dividends payable.

This reaction to large dividend payments can be identified with reference to the few high dividend paying weeks detailed in Chapter 5. Therefore, positive differences in the three month contracts can be seen in February, May, June, August, November and December when large dividend payments are made. (Only small positive differences can be seen in December 1988 because of an extremely low interim dividend from Driefontien, and also in December 1989 since the last observation was 10 December 1989). Similar movements can be identified in the six month contracts after taking into account that these are out of phase by three months (i.e. observation in September relates to the contract expiring in December).

6.3.2.2. Industrial Index future

As illustrated by Figures 6.6. and 6.7, the difference between the two bases in the Industrial index approaches 17 points in respect of the three month contracts and up to 19 points for the six month contracts.

Figure 6.6. Industrial Near Contract
Difference in futures price between actual dividends and dividend yield

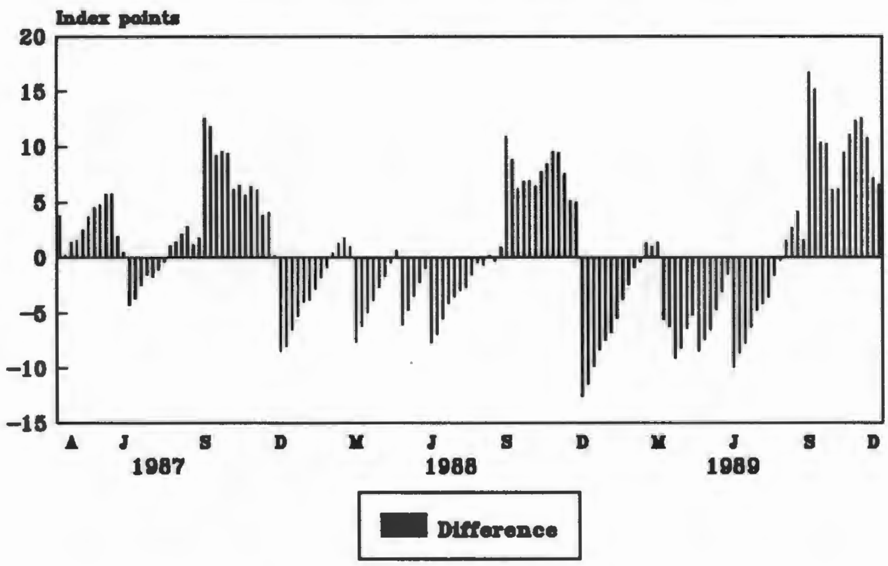
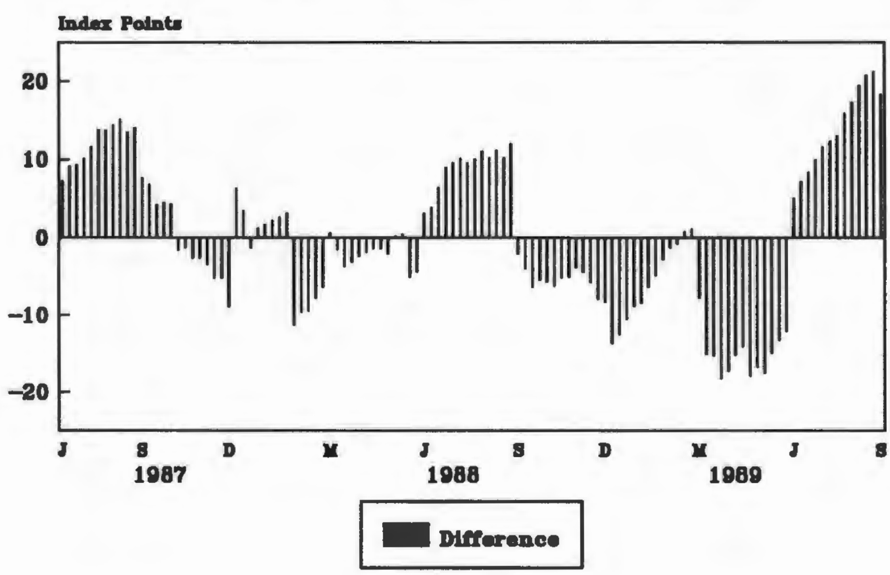


Figure 6.7. Industrial Mid Contract
Difference in futures price between actual dividends and dividend yield



Like the ALGI, the differences show a definite pattern which can be explained by the lumpy nature of dividend payments. The positive differences are as a result of large dividends paid by index constituents in the months of March, September and December.

6.3.2.3. All Share Index future

There is a substantial drop in the differences found in regard to the All Share index future compared to the ALGI and INDI. This is illustrated in Figures 6.8. and 6.9.

Figure 6.8. All Share Near Contract
Difference in futures price between
dividend yield and actual dividends

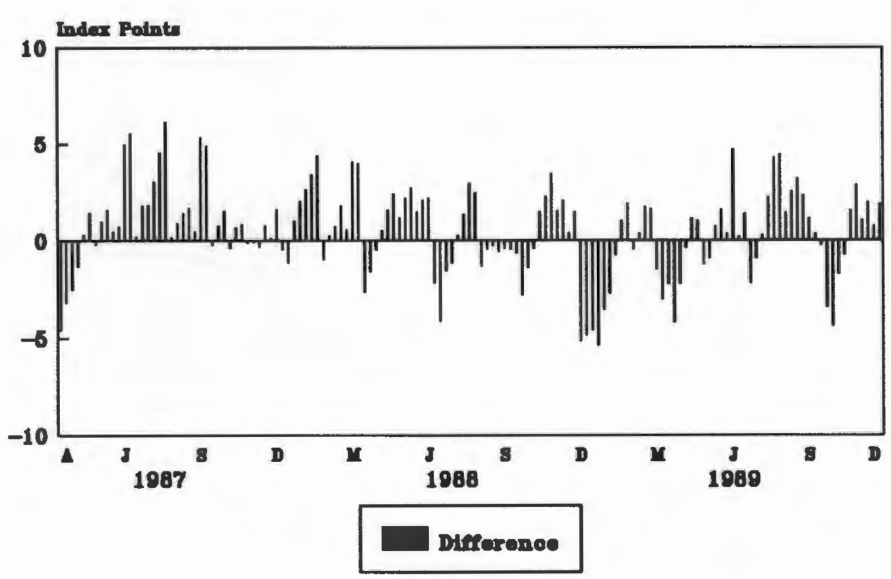
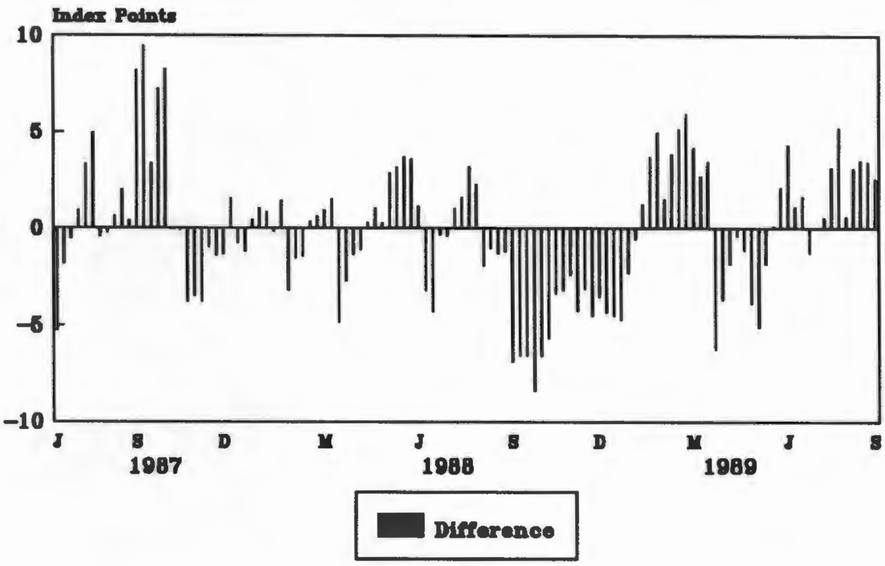


Figure 6.9. All Share Mid Contract
Difference in futures price between dividend yield and actual dividends



Differences of to up to 6 index points were recorded for the three month contract and up to 10 index points in respect of the 6 month contract. The majority of the differences computed were well below five index points. The lower differences seen in the All Share index future is attributed to the more even distribution of dividend payments on the ALSI.

6.3.3. Conclusion

The decision to use the dividend yield to calculate the futures price in terms of the cost-of-carry valuation model is a trade off between the saving of the time and cost involved in estimating the actual dividends on the one hand and bearing the cost of any estimation error on the other.

This trade off decision will depend on the particular circumstances and objectives of the futures market participant.

The magnitude of the estimation error has been calculated in this study. The estimation error of the ALGI futures contract and the INDI futures contract were found to be greater than the ALSI futures contract. This is to be expected in view of the much smoother distribution of dividend payments on the ALSI.

With respect to the ALGI and INDI futures contracts the difference of up to nearly 20 points is large in relation to the average cost-of-carry calculated over the observed period. The average cost-of-carry based on the dividend yield equalled 28.00 points for ALGI futures and 47.99 points for the INDI future over the observation period. However, as far as the ALSI is concerned, the error is likely to amount to less than 10% in most cases of the average cost-of-carry on the ALSI which averaged 47.53 points.

The materiality of the estimation error depends on the relative extent of the arbitrage bounds. In other words, the greater the size of the fair range within which the actual futures price may move freely, the lower the need is likely to be for an accurate calculation of the theoretical futures price; and vice versa. This consideration is evaluated later in this chapter after the construction of the arbitrage bounds. However, at this stage, in view of the size of the differences in relation to the cost-of-carry, the dividend yield will not be used as a surrogate for expected dividends in the assessment of pricing efficiency.

Thus, pricing in the futures market is investigated with reference to the theoretical futures price calculated by using actual dividends. This is done by firstly regressing the theoretical futures price with the actual futures price, and secondly, by identifying whether arbitrage opportunities exist through the construction of arbitrage bounds.

6.4. REGRESSION

Regression in this study is used to satisfy two objectives. The first objective is to ascertain the degree to which the futures markets are optimally priced by assessing the extent to which the actual futures price matches the theoretical futures price. The second objective is aimed to establish the correlation between the futures price and the spot price, i.e. the extent to which the futures markets are effective in reducing risk. The second objective relates more particularly to hedging effectiveness but has been included here for convenience.

6.4.1. Methodology

The first objective may be achieved by regressing the actual futures price with the theoretical futures price. However, owing to potential statistical problems with using actual price levels (Brown, 1985), the regression is performed using price changes instead, as;

$$[TFP(t,T)-TFP(t-1,T)] = c + \beta\,[MTM(t,T)-MTM(t-1,T)] + \epsilon \qquad (6\text{-}4)$$

where

TFP(t,T)	:	Theoretical futures price at time t.
TFP(t-1,T)	:	Theoretical futures price at time t-1.
MTM(t,T)	:	Actual (mark-to-market) futures price at time t.
MTM(t-1,T)	:	Actual (mark-to-market) futures price at time t-1.
c	:	Regression constant.
ß	:	Coefficient of the dependent variable.
ϵ	:	Residual or error term.

If the slope of the regression line (ß) is significantly different from one, then this signifies that there is not a one-to-one (equal) relationship between the actual futures price and the theoretical futures price. If the constant term (c) is significantly different from zero then this indicates an unaccounted for difference between the two values (i.e. a sub-optimal situation). Thus, for the actual futures price to be equal to the theoretical futures price the regression coefficient of the dependent variable (ß) should be equal to one and the constant term (c) should be zero. A hypothesis test can be formally stated as follows:

Null (H_0): The actual futures price (MTM) equals the theoretical futures price (TFP).

Alternative (H_a): The actual futures price is significantly different from the theoretical futures price.

The null hypothesis is rejected if either the beta coefficient is significantly different from one, or the constant term is significantly different from zero. The test is based on the relevant t-statistics of ß and c at a 95% level of significance.

As far as the second objective is concerned, regression techniques have been used in the majority of published empirical studies to establish the degree of correlation between the actual futures price and spot price (Ederington, 1979; Hill & Schneeweis, 1984; Figlewski, 1984a; Junkus & Lee, 1985). However, regressing the futures price changes directly with spot price changes may provide misleading results in a share index futures situation. This is because the decline in the futures premium towards expiration in terms of pricing theory will impact the return on the futures price, i.e. regression against the spot index will not show perfect correlation even if the futures price is acting exactly in terms of the cost-of-carry model. To eliminate this problem, actual future price changes will not be regressed against the index, but instead against the theoretical futures price.

Therefore, the same regression expression per 6-4 will satisfy both objectives. Ordinary least squares regression is accordingly performed over the observation period from March 1987 to December 1989. The results are analysed below for each contract type. Each table of regression results contains the beta coefficient (ß), constant term (c), coefficient of variation (R^2) and Durbin-Watson test statistic (DW). The t-statistics for ß and c are given in brackets below the respective terms. An asterisk (*) indicates whether the ß term is significantly different from one, c is significantly different from zero or that the DW statistic indicates significant autocorrelation (all these at the 95% level of significance).

6.4.2. All Gold

Table 6.1. indicates a fairly good overall relationship between the actual futures price and the theoretical futures price, with the average beta coefficient of 0.9244.

Table 6.1.
All Gold Index Future
Regression of Actual vs Theoretical Futures Price

Contract	Obs	Beta (t-stat)	Constant (t-stat)	R^2	DW
June 1987	10	*0.7219 (-2.4309)	4.5651 (0.3940)	0.8327	2.456
Sep 1987	12	0.7159 (-1.3765)	-3.7178 (-0.2810)	0.5461	1.495
Dec 1987	23	1.0439 (0.2892)	0.8866 (0.0474)	0.6926	2.522
Mar 1988	25	1.0991 (0.7865)	-1.3028 (-0.0805)	0.7679	*2.579
June 1988	25	1.1019 (1.6226)	-2.3694 (-0.4765)	0.9304	*2.669
Sep 1988	25	1.0264 (0.2664)	0.6133 (0.1163)	0.8235	*2.865
Dec 1988	25	0.8879 (-0.9257)	0.3206 (0.0565)	0.7002	2.503
Mar 1989	25	*0.8171 (-2.7924)	-5.2150 (-1.4288)	0.8714	*2.591
June 1989	24	0.9334 (-0.7947)	-0.7148 (-0.1322)	0.8493	2.254
Sep 1989	25	0.8610 (-1.4978)	3.4205 (0.5824)	0.7893	*2.728
Dec 1989	25	0.9596 (-0.5519)	-1.9034 (-0.2803)	0.8821	2.277

In two instances (June 1987 and March 1989) the beta coefficient is found to be significantly different from one and on no instances is the constant term significantly different from zero. A beta significantly different from one could be expected from the June 1987 contract in view of pricing inefficiencies which are present in an infant market. However, the inefficiency in the March 1989 contract is more difficult to understand as at this stage the market was two years old and much better developed.

The average R^2 for all contracts equalled 0.7896. The low R^2 is largely attributable to the first year of operation (particularly September 1987 and December 1987 contracts) when markets were undeveloped. This position improved substantially from June 1988 with the average R^2 equalling a more respectable 0.8352. However, even this level indicates a fair amount of unexplained variance between the actual and theoretical futures price.

If market participants are functionally fixated on the dividend yield valuation, then it is possible that a portion of the unexplained variance is due to the different basis of calculating the cost-of-carry futures price. However, this is unlikely since market participants quickly adapt for such factors, as was identified by Peters (1985). The unexplained variance is likely to be due to the noise in the fluctuations between the futures price and the spot price (Figlewski, 1984a). It is this unexplained variance which is responsible for basis risk introduced into hedged portfolios.

The significant Durbin-Watson statistic in five out of the eleven contracts indicates the possible problem of serial correlation.

6.4.3. Industrial

As the results in Table 6.2. indicate, the constant term (c) is found to be not significantly different from zero for all contracts. However, the beta coefficient proved to be significantly different from one for the June 1989 and September 1989 contracts. The beta coefficients for June and September 1987 were also very low, but were not significantly different from one owing to the very large standard errors of the beta coefficients.

No readily apparent reason could be found to explain why the betas in the June and September 1989 contracts were significantly different from one. The R^2 statistic which indicates the goodness of fit of the regression line was higher than average. The investigation of the regression residual distribution did not add any further insight to this problem. Even when outliers were omitted from the sample there is only a slight improvement in the beta coefficient towards one.

The R^2 for the Industrial futures averaged 0.7145, which improves to 0.7866 if the first two contracts are excluded. This average is below that of the All Gold index and again indicates the fairly large variability in the basis. What is noticeable from the R^2 statistics is that they have steadily improved over time to a very respectable R^2 of 0.9055 for December 1989. The DW statistic indicated negative serial correlation for four out of the eleven futures contracts.

Table 6.2.
Industrial Index Future
Regression of Actual vs Theoretical Futures Price

Contract	Obs	ßeta (t-stat)	Constant (t-stat)	R^2	DW
June 1987	10	0.7680 (-0.7407)	2.0328 (0.1726)	0.4291	1.462
Sep 1987	12	0.7811 (-0.6513)	-1.0666 (-0.0854)	0.3507	*2.711
Dec 1987	23	0.8937 (-0.9551)	4.6358 (0.3722)	0.7543	*2.638
Mar 1988	25	0.8879 (-1.0399)	4.5497 (0.3786)	0.7469	2.529
June 1988	25	0.8397 (-1.4856)	0.6874 (0.1715)	0.7249	1.719
Sep 1988	25	0.9411 (-0.5013)	-0.5984 (-0.1647)	0.7361	*2.663
Dec 1988	25	1.0466 (0.3749)	-1.1006 (-0.2708)	0.7550	1.834
Mar 1989	25	1.0207 (0.1832)	-3.6262 (-0.7573)	0.7800	2.399
June 1989	24	*1.5241 (3.9495)	8.0504 (0.9303)	0.8570	*2.775
Sep 1989	25	*1.4902 (3.3644)	5.0747 (0.6416)	0.8197	1.438
Dec 1989	25	1.0387 (0.5529)	-9.6055 (-1.3700)	0.9055	2.402

6.4.4. All Share

The results in Table 6.3. illustrate that the beta coefficients were significantly different from one for the

June 1989 and September 1989 contracts, as was also the case for the Industrial futures contract tested. The betas in the June and September 1987 contracts were very low but they could not be statistically proven different from one owing to the large standard error of the coefficients.

Table 6.3.
All Share Index Futures
Regression of Actual vs Theoretical Futures Price

Contract	Obs	ß eta (t-stat)	Constant (t-stat)	R^2	DW
June 1987	10	0.7774 (-0.9064)	3.9664 (0.2386)	0.5561	2.047
Sep 1987	12	0.6270 (-1.1906)	-2.7320 (-0.1915)	0.2860	2.374
Dec 1987	23	0.9374 (-0.6336)	3.7829 (0.2744)	0.8109	*2.747
Mar 1988	25	0.9227 (-0.9336)	5.2643 (0.4529)	0.8438	*2.643
June 1988	25	1.0934 (0.6853)	-1.6204 (-0.2140)	0.7367	*2.725
Sep 1988	25	1.0350 (0.3321)	0.1734 (0.0382)	0.8075	2.284
Dec 1988	25	0.9561 (-0.4205)	-1.6789 (-0.3531)	0.7848	2.328
Mar 1989	25	0.9797 (-0.1943)	-3.9804 (-0.6815)	0.7927	2.503
June 1989	24	*1.3694 (2.8814)	6.3352 (0.7106)	0.8384	*3.097
Sep 1989	25	*1.3430 (2.3160)	3.9383 (0.4597)	0.7814	2.223
Dec 1989	25	0.9801 (-0.2839)	-9.2824 (-1.3230)	0.8949	2.122

Again no explanation for the mispricing of the June and September 1989 contracts could be found through investigation of the residual pattern or by removing outliers. Moreover, the R^2 values of 0.8384 and 0.7814 for the June and September 1989 contracts respectively is reasonable in relation to the R^2 of the other contracts.

The average R^2 of the contracts is found to be 0.7394 but this improves substantially to 0.8101 if the June 1987 and September 1987 contracts are left out. The very low R^2 for these contracts can be explained by the pricing inefficiencies arising from an underdeveloped market. The DW statistic is found to be significant on four occasions.

6.4.5. Conclusion of Regression

The results indicate that the pricing of the futures contracts observed is sub-optimal in 6 out of the 33 tested over all three index futures; two each for all three contract types. Five of these contracts were in 1989, over two years after the introduction of SIF in South Africa. This is unlikely to be a result of inefficiencies associated with a developing market; this issue is addressed in the analysis of arbitrage opportunities.

Thirteen out of the 33 contracts exhibited statistically significant negative serial correlation. While the low sample sizes are likely to a contributory factor, most of the significant Durbin-Watson statistics indicated negative serial correlation. Thus, the non-random nature of the serial correlation may indicate a possible bias in the regression caused by correlation between regression residuals. This bias is discussed with respect to the investigation into arbitrage pricing.

The average R^2 values of 0.79 for All Gold, 0.71 for Industrial and 0.74 for All Share index futures contracts compare favourably with the R^2 values calculated by Ederington (1979), Hill & Schneeweis (1984) and others. Despite this, however, there is still a substantial amount of unexplained variance between the index and futures price owing to the introduction of basis risk.

6.5. ARBITRAGE BOUNDS

The regression above has indicated periods where the actual futures price is significantly different from the theoretical futures price. However, this does not necessarily mean that mispricing has occurred; i.e. the difference between the two values may be large but the futures price will still be optimally priced in terms of arbitrage pricing if the actual futures price remains within the fair range. The ultimate test of pricing efficiency is the extent to which arbitrage opportunities are available.

6.5.1. Methodology

This investigation of arbitrage opportunities is done in accordance with the general pricing expression 5-13, which has been reproduced in a slightly different form in expression 6-5:

$$T_f + T_s \geq$$

$$I(t)\, e^{r(t,T)(T-t)} - \int_t^T D(w)\, e^{R(t,w,T)(T-w)} - F(t,T) \qquad (6\text{-}5)$$

$$\geq -T_f - T_l$$

The middle section of expression 6-5 represents the difference between the theoretical futures price in terms of the cost-of-carry model and the actual futures price (F(t,T)). This will be referred to as the pricing difference.

The theoretical futures price is calculated in the same manner as that used for regression based on actual dividends. The left and right hand sides of the expression above will be referred to as the upper and lower arbitrage bounds respectively. These bounds are established with reference to the South African situation to assess the pricing efficiency of the South African futures market.

At this stage it is assumed that there are no market restrictions to shorting the index, i.e. the transaction cost of the short share index position (T_s) equals the transaction cost of the long share index position. Later this assumption is relaxed when it is shown that this element has a dominant impact on futures pricing in South Africa.

Various factors affect the magnitude of the arbitrage bounds and the ability to effect arbitrage trades. These include the availability and cost of capital, market transaction costs, and the ease at which participants can enter and leave the markets in terms of exchange requirements and regulations. These factors differ from individual to individual and from situation to situation; thus the arbitrage bounds fluctuate from person to person and from time to time.

The availability of arbitrage opportunities are viewed from the position of participants who incur the lowest transaction costs in effecting arbitrage transactions. In practice, it is actually these participants who are the first to reap any available arbitrage profits. Consequently, the minimum possible arbitrage bounds are constructed.

The transaction costs are converted to an index points basis to facilitate the analysis of futures pricing differences. According to the rules of the South African Futures Exchange (SAFEX) one index point is equivalent to R10 and so conversion to index points is made on this basis.

6.5.1.1. Costs on the Futures Arbitrage leg

Commission payable to market makers is charged mainly through the difference between the bid and ask price. To calculate the minimum arbitrage bounds the lowest bid-ask spread has been used; namely the difference between the highest bid and lowest offer quoted at the close of the particular business day. Futures prices are freely an readily available to market participants (through the Reuters network); therefore, potential information costs faced by market participants are limited.

The full bid-ask spread has been taken into account here as most arbitrage positions are only held for a short period of time before being closed out by an equal and opposite transaction. This is because arbitrage positions are only maintained until the mispricing is corrected after which the position is quickly closed out to take the available profits. Very seldom is the arbitrage position maintained till expiration date (Duffie, 1989: 42).

Booking or clearing house fees as laid down by SAFEX are payable on each and every futures transaction. Fees on share index futures over the observation period are given in Table 6.4. (source: Rand Merchant Bank). Both the transaction and contract fees are taken into account in calculating the arbitrage bounds.

Table 6.4.
Share Index Futures Booking Fees

Period		**Per**	**Per**
From	**To**	**Transaction**	**Contract**
March 1987	31 May 1988	Nil	Nil
1 June 1988	16 July 1989	R10	R2
17 July 1989	December 1989	R15	R3

The bid-ask spread and the booking fees are the minimum transaction costs required to enter the futures market. Additional costs may be incurred over and above these costs if trading is done through a futures broker, but these are not incurred if a clearing member trades directly with another clearing member. On the grounds that minimum arbitrage bounds are constructed, and that in all likelihood it is the clearing members themselves who will exploit arbitrage opportunities, additional brokerage costs are ignored.

The only other cash requirement when entering a futures contract is the payment of margin deposits at the time the contract is initiated. During the observation period the initial margin payable on SIF contracts amounted to R2000 per contract with no variation or maintenance margin (Falkena et al, 1989). SAFEX pays a market related money market rate of interest on these margins. As market participants are compensated in this manner for margin deposits made, it is the opinion of the author that margin deposits can be ignored for the purposes of calculating the arbitrage bounds. It must be stressed, however, that there may be a opportunity cost of making margin deposits for some participants where the margin interest rate payable does not adequately compensate for lost earnings on the margin capital; but it is difficult to estimate the effect of this.

Over the period of the futures contract, through the marking-to-market process, margin calls may be required when the margin deposits fall below certain limits. This situation is different to other investment types (such as shares) where losses are on paper only. This requirement to make good losses incurred on futures contracts adds additional risk to the contract position and may necessitate the participant to maintain capital reserves for this purpose (Blank, 1990).

Blank (1990: 170) points out that the level of margin reserves kept is a trade-off between the desire to keep the risk of not being able to meet margin calls to a minimum on the one hand, and the desire to keep the margin reserves to a minimum on the other hand. The maintenance of capital reserves has a cost associated with it (cost of capital) which may widen the arbitrage bounds faced by these individuals. The cost depends on the participant's position, inter alia, the availability of capital funds and the relative liquidity of the spot position to meet margin calls. However, while the maintenance of reserves to meet margin calls may be costly for futures participants, this is not likely to impede all market participants in making arbitrage trades. For this reason costs associated with margin call reserves are ignored in calculating the arbitrage bounds.

As far as the market and restricting regulations are concerned, the South African futures market is relatively restriction free compared to world markets. For example, the futures markets in the USA are totally controlled and regulated by a government body, namely The Commodity Futures Trading Commission (Tewelles & Jones, 1987). In South Africa, the policy of "Minimum Intervention" and control of the market through self-regulation has resulted in very few restrictions faced by market participants.

6.5.1.2. Costs on the Spot Arbitrage leg

The other leg of the arbitrage trade involves transaction costs on the spot index position. These costs include brokerage and other costs associated with the purchase and sale of the market portfolio.

Brokerage fees charged in South Africa are regulated and defined in terms of section 6.20.2. of the Rules and Directives of the Johannesburg Stock Exchange (1984). The fees are levied in the form of a basic charge plus brokerage commission charged on a sliding scale. For the period under investigation the basic charge equalled one cent a share to a maximum of R25, and the brokerage commission varied from 0.2% (on considerations greater than R1.5 million) to 1.2% (on consideration less than R5000) for both buy and sell legs (JSE, 1989).

It is reasonable to assume that arbitrageurs (or at least some of them) trade in large volumes of shares at a time. On this basis the R25 basic charge is negligible and may be ignored. For example, on a consideration of R1.5 million the basic charge reflects a cost of only 0.0017%. Also, it may be assumed that the marginal cost to an arbitrageur is the minimum brokerage rate of 0.2%.

Arbitrageurs can only trade in volumes if the futures market is capable of handling the large volumes. A consideration of R1.5 million is equivalent to approximately 60 futures contracts (i.e. assuming an index value of 2500). As illustrated by Figure 2.1. in chapter 2, the volumes in 1987 were below 1000 contracts per month. Thus, over this period there were few, if any, trades in excess of 60 contracts; therefore, any arbitrage trades over this period are likely to have incurred transaction costs above the minimum on both the share and futures leg. However, since March 1988 (and since February 1989 in particular) contract volumes were sufficiently high for arbitrage transactions of 60 contracts to be accommodated.

As far as taxes are concerned, over the observation period 1.5% marketable securities tax (MST) is due on the buying leg only. Other costs which impede transactions in an imperfect market such as indivisibility of futures contracts and restraining regulations are ignored as these costs are likely to be immaterial.

Arbitrage transactions assume that the arbitrageur can easily buy and sell the spot asset. However, as share index futures are notional assets this is more difficult to accomplish in a SIF arbitrage situation. The index is bought and sold by buying and selling a portfolio containing a representative proportion of index constituents. Unfortunately, in practice the portfolio inevitably differs from the index to a certain degree, which exposes the portfolio to an amount of basis risk (Figlewski, 1984a). Thus, in practice arbitrage transactions involve a certain amount of risk. This additional risk should be compensated for by increased expected return; consequently the arbitrage bounds extend by the amount of additional return required. This consideration has not been taken into account into the construction of the arbitrage bounds because it is very difficult to quantify, and depends on the level of basis risk faced and the risk profile of the particular arbitrageur.

From the above, to effect an arbitrage trade, the minimum cost on both spot transaction legs (i.e. buy and sell) for the period under observation amounted to 1.9% of the transaction consideration over the observation period (namely brokerage of 0.2% on both legs and 1,5% MST on the buying leg). This can easily be converted to a points basis by multiplying the index value by 1.9% as in effect the arbitrageur is 'buying' (or 'selling') the index.

6.5.1.3. Construction of Arbitrage Bounds

To illustrate the costs on each transaction leg two arbitrage bounds are calculated. The first bound, which has been called Bound A, constitutes the costs on futures transactions only, namely the bid-ask spread and the booking fees. The second bound, Bound B, includes transaction costs on both the share and futures transaction legs.

The results are shown graphically and in tabular form for each index future type. The tables provide the number, and size, of arbitrage opportunities, while the graphs illustrate the difference between the theoretical futures price and the actual futures price according to 6-5 together with the constructed arbitrage bounds. Two graphs per index future type are provided, one showing the near (3 month) futures contract and the other giving the mid (6 month) futures contract (i.e. contracts with between three and six months to expiration).

6.5.2. All Gold

Figures 6.10. and 6.11. indicate that the period from April 1987 to December 1987 was characterised by large differences between the actual and theoretical futures price; on most occasions the actual futures price was greater than that of the theoretical futures price. There were a number of instances where the mispricing could have resulted in arbitrage profits (as shown by the Table 6.5.) and the level of mispricing reached up to 91 index points. Moreover, both at the end of July and the end of September 1987 three consecutive observations were seen to be mispriced beyond the arbitrage bounds. These results confirm the low betas and R^2 for the June 1987 and September 1987 contracts in the regression analysis.

Table 6.5. and Figures 6.10. and 6.11. exhibit optimal pricing for the period from December 1987 to the close out of the September 1988 contract where differences between the theoretical and actual futures prices were very low. However, from the December 1988 contract the incidence of mispricing increased, especially in October and November 1988 confirmed by the statistical significance of the beta coefficient in the March 1989 regression.

Figure 6.10. All Gold Index Futures
Pricing Differences and Arbitrage Bounds
Near Contracts

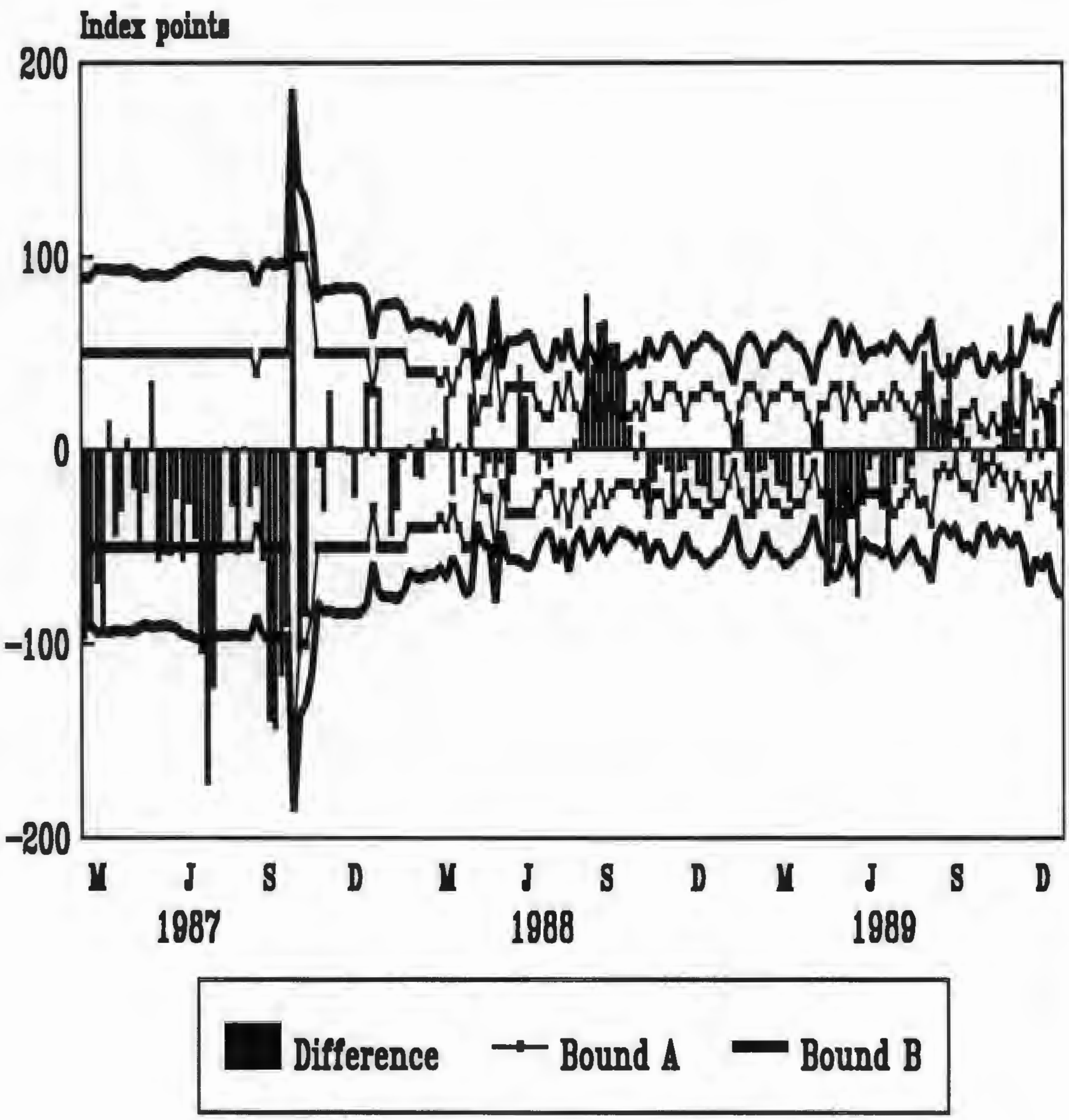


Figure 6.11. All Gold Index Futures
Pricing Differences and Arbitrage Bounds
Mid Contracts

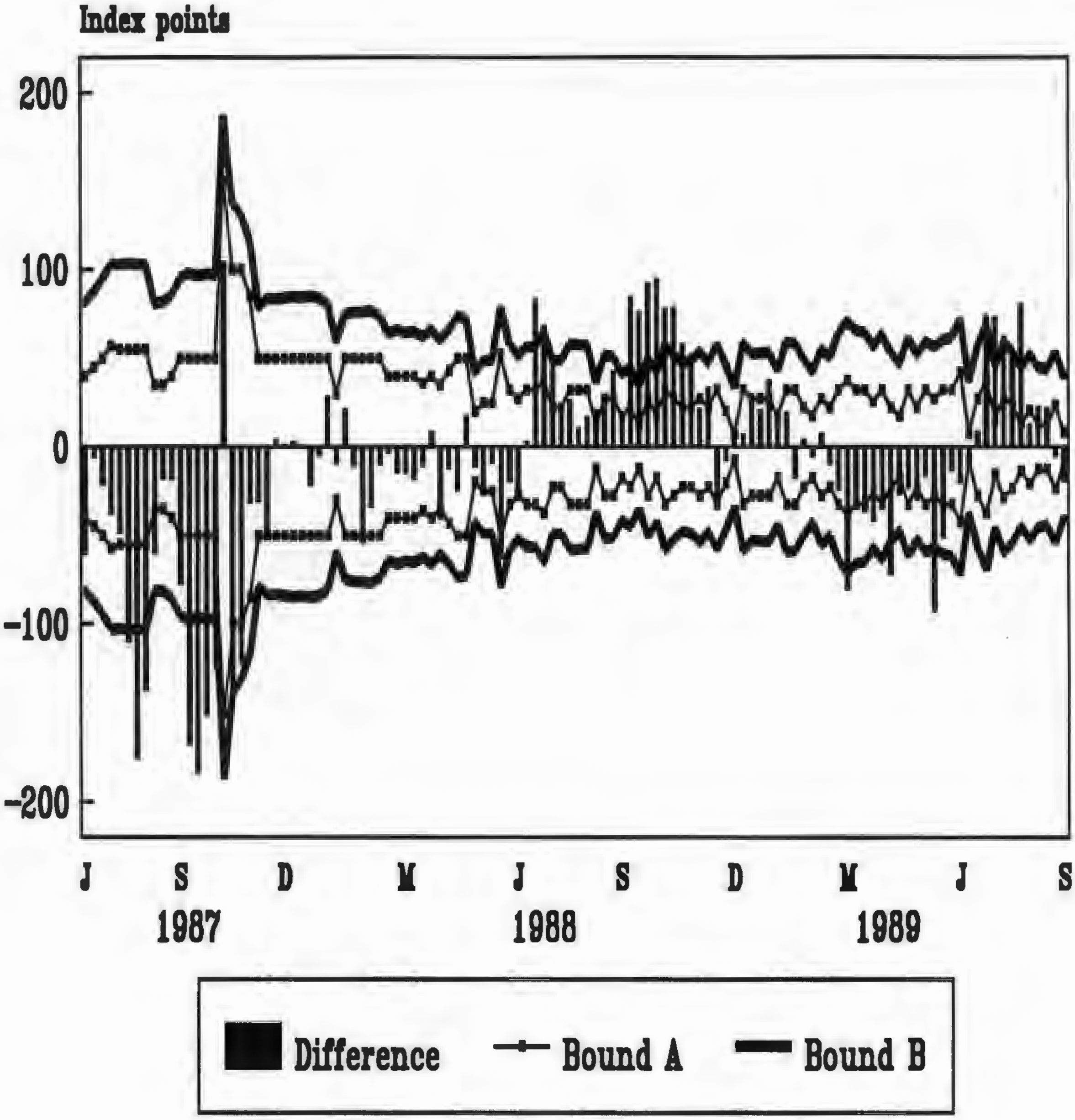


Table 6.5.

All Gold Futures Arbitrage Differences

Contract	# obs	# Arbitrage opportunities	Average mispricing Index pts	Largest mispricing Index pts
Jun 1987	12	2	7.18	14.15
Sep 1987	13	3	38.45	121.72
Dec 1987	24	6	40.21	120.04
Mar 1988	26	5	50.40	133.42
Jun 1988	26	0	-	-
Sep 1988	26	0	-	-
Dec 1988	26	7	15.35	25.62
Mar 1989	26	8	28.62	51.29
Jun 1989	25	4	14.70	36.53
Sep 1989	26	4	18.89	38.09
Dec 1989	26	6	14.15	34.99

During the second half of 1989 the size of pricing differences increased, and the number of arbitrage opportunities rose. However, the incidences of mispricing observed during this period were relatively low and isolated, indicating that the arbitrage mechanism had brought the futures price quickly back in line when the arbitrage bound had been transgressed.

6.5.3. Industrial

An extraordinary observation on 6 November 1987 shortly after the 1987 stock market crash, and a solitary observation on 25 March 1988 (where the mispricing only is recorded as 8.15 points) were the only arbitrage opportunities observed for the period from March 1987 to June 1988. In fact, the pricing differences over this period on most occasions were also within Bound A as illustrated by Figures 6.12. and 6.13. These results are impressive considering that this is the period where the market was in its infancy, contract volumes were low and knowledge of market participants was likely to be below par.

On the other hand, during the period from June 1988 to December 1989 a total of 71 arbitrage opportunities (refer Table 6.6.) were available representing nearly half of the total of 155 weekly observations on both near and mid contracts. This was also identified in the regression where the actual futures were found to be significantly different from the theoretical futures price for the September and June 1989 contracts.

Table 6.6.
Industrial Futures Arbitrage Differences

Contract	# obs	# Arbitrage opportunities	Average mispricing	Largest mispricing
Jun 1987	12	0	-	-
Sep 1987	13	0	-	-
Dec 1987	24	1	27.74	27.74
Mar 1988	26	1	41.66	41.66
Jun 1988	26	1	8.15	8.15
Sep 1988	26	7	5.86	17.11
Dec 1988	26	14	15.95	46.15
Mar 1989	26	12	60.75	90.29
Jun 1989	25	7	38.77	80.55
Sep 1989	26	17	58.75	163.13
Dec 1989	26	14	95.31	166.13

Figure 6.12. Industrial Index Futures
Pricing Differences and Arbitrage Bounds
Near Contracts

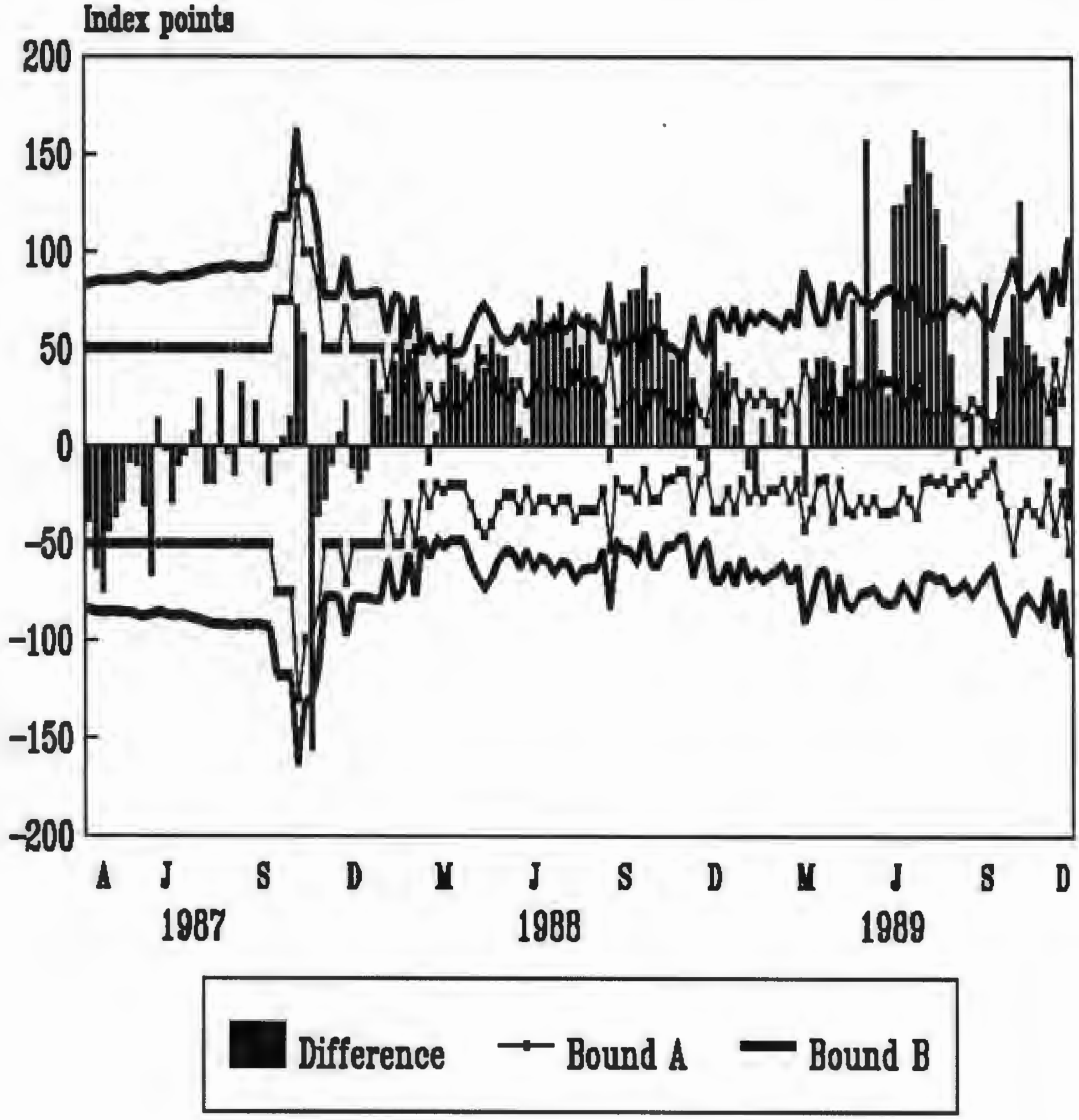
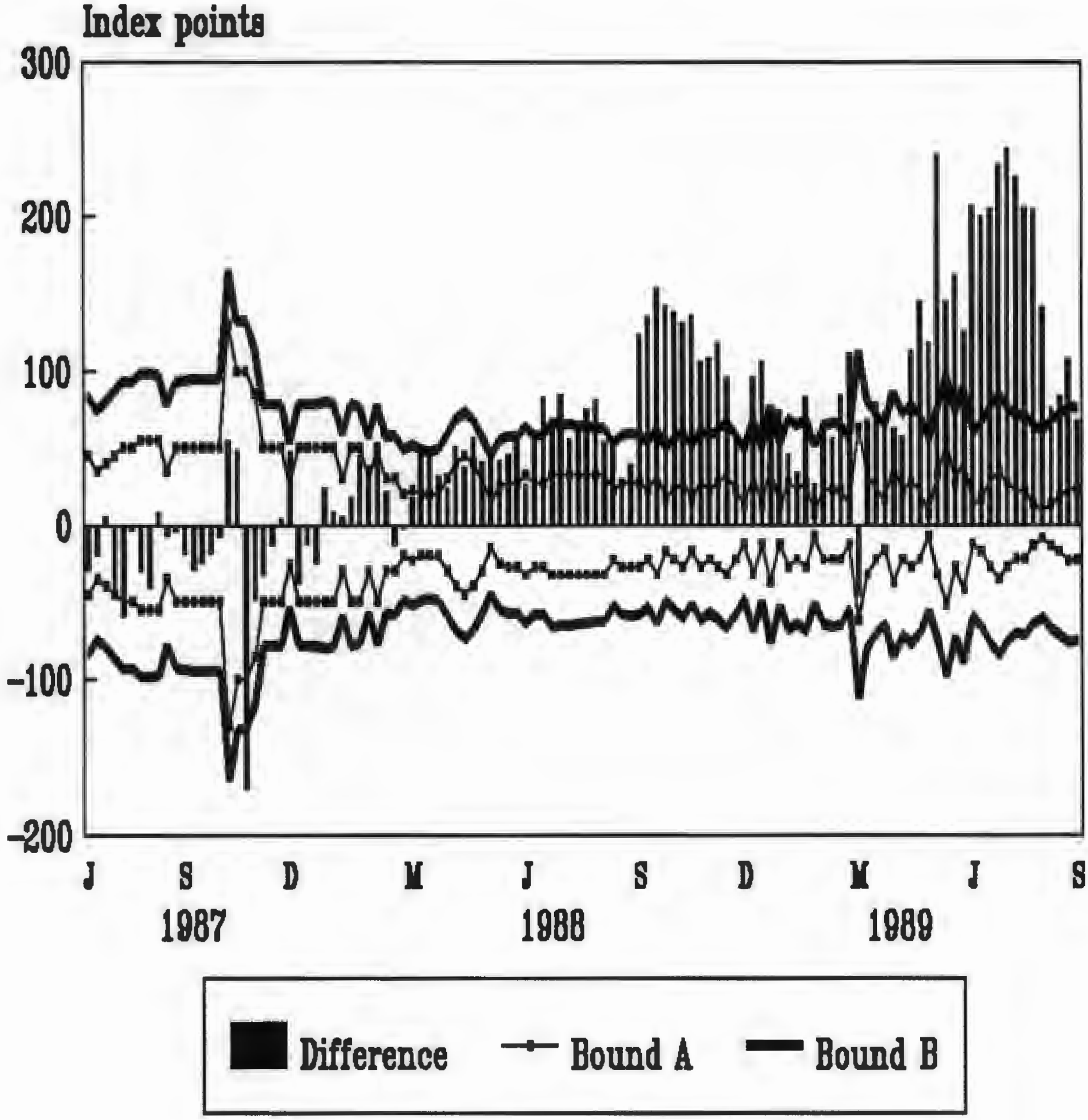
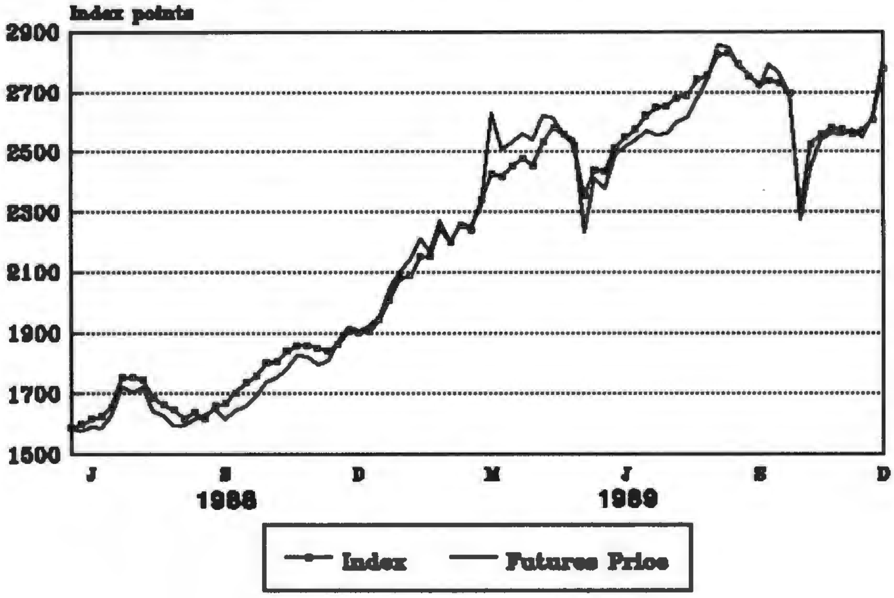


Figure 6.13. Industrial Index Futures
Pricing Differences and Arbitrage Bounds
Mid Contracts



All of the arbitrage opportunities resulted from the actual mark-to-market futures price being significantly less than (up to 166 index points) the theoretical futures price. Over this period the pricing differences were positive for 145 out of the 155 weekly observations. The mispricing bias is likely to have resulted in the negative serial correlation. Figure 6.14. illustrates that the futures price traded at a discount to even the index on many occasions.

Figure 6.14.
Industrial Index Value and Futures Price



Niederhoffer & Zeckhauser (1983) postulated that the futures price can trade at a discount to the index value due to downward pressure on futures prices because the futures market provides the only convenient instrument for selling the index short. While this may be true over short periods of time, this reason does not explain this situation where there is continual mispricing.

Thus, the overwhelming extent and bias of pricing differences over the second half of the observation period is very unlikely to be as a result of sub-optimal pricing. The considerations ignored in calculating the arbitrage bounds (including additional brokerage, margin deposits and reserves, arbitrage risk and indivisibility of contracts) could not satisfactorily account for the very high level of mispricing. It is shown later that these pricing differences do not present arbitrage opportunities, but persist due to difficulties in shorting the index to effect long futures arbitrage trades.

6.5.4. All Share

The arbitrage pricing results of the All Share index future, given in Figures 6.15. and 6.16, are remarkably similar to the Industrial index future in many ways.

Firstly, the period from March 1987 to June 1988 presented a very small number of arbitrage opportunities and the actual futures price matched the theoretical futures price very closely (most of the price differences actually remained within Bound A). However, the period subsequent to this exhibited 66 arbitrage opportunities out of 155 observations (refer Table 6.7.). Of particular note, the actual futures price traded beyond the fair range denoted by Bound B for 19 consecutive weeks on the mid (6 month) contract.

Figure 6.15. All Share Index Futures
Pricing Differences and Arbitrage Bounds
Near Contracts

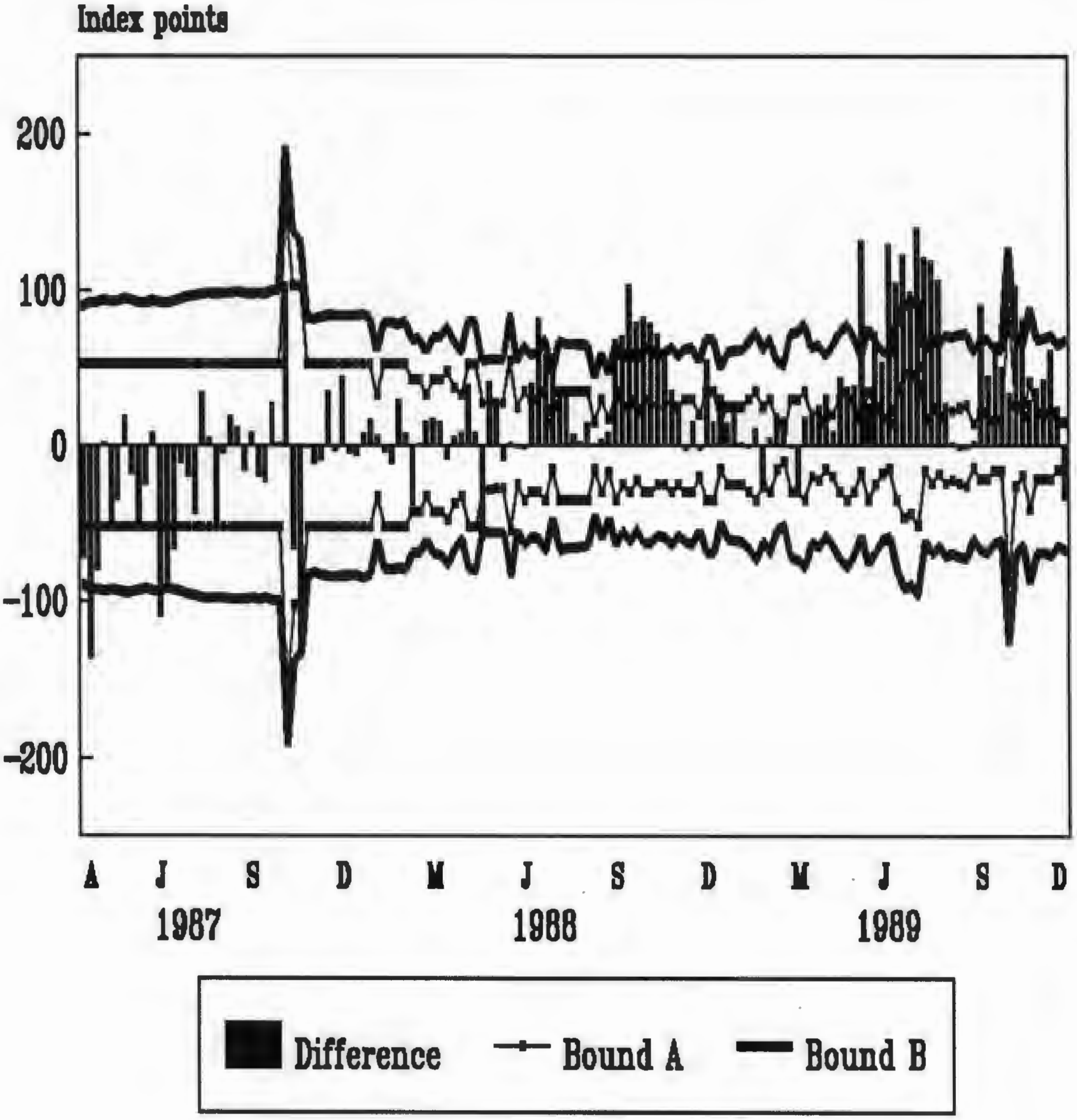


Figure 6.16. All Share Index Futures
Pricing Differences and Arbitrage Bounds
Mid Contracts

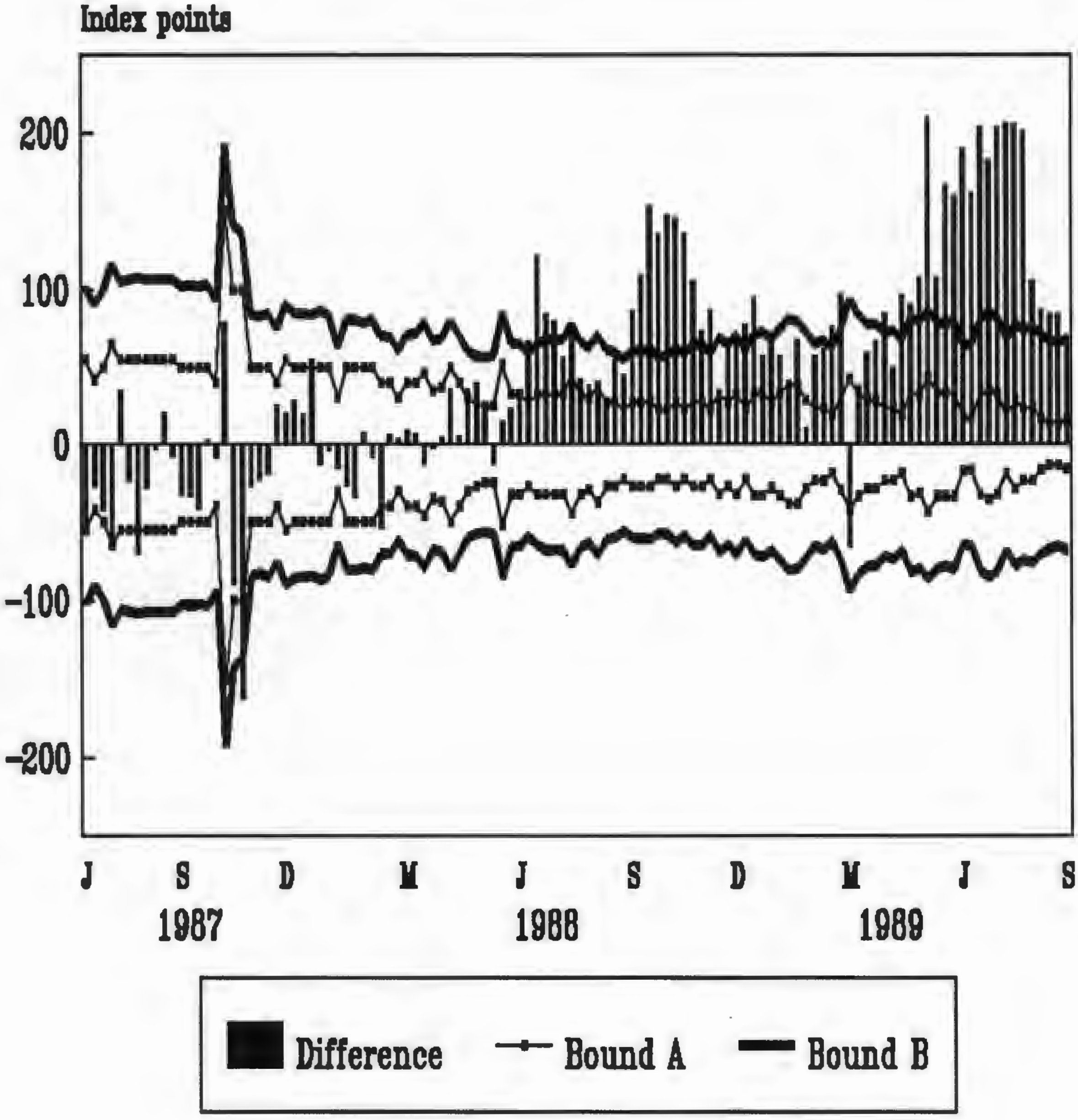


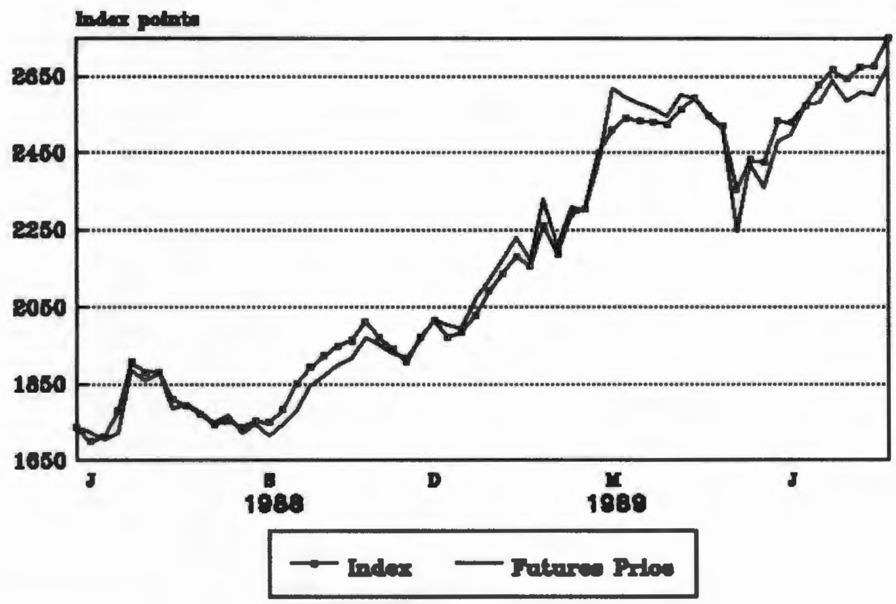
Table 6.7.
All Share Arbitrage Differences

Contract	# obs	# Arbitrage opportunities	Average mispricing	Largest mispricing
Jun 1987	11	1	42.10	42.10
Sept 1987	13	1	15.01	15.01
Dec 1987	24	1	2.94	2.94
Mar 1988	26	1	29.17	29.17
Jun 1988	26	1	5.41	5.41
Sep 1988	26	1	20.33	20.33
Dec 1988	26	12	18.14	55.18
Mar 1989	26	11	37.50	90.22
Jun 1989	25	9	26.17	78.55
Sep 1989	26	16	43.29	121.09
Dec 1989	26	17	64.42	136.29

Secondly, the regression procedure identified sub-optimal pricing in the June and September 1989 futures contracts for both the Industrial and Share Index futures. Thirdly, except for two observations in March 1989 and one in December 1989, all pricing differences subsequent to June 1988 were due to the actual futures price being less than the theoretical futures price. This situation was also identified by King (1989). A large proportion of the mispricings originated from the futures price once again trading below the index value as indicated by Figure 6.17.

As with the Industrial futures mispricing, the mispricing in the latter portion of the observation period is too extensive to represent a period of available opportunities. Once again the cause is the inability to effectively short the index. This will be discussed in the next section.

Figure 6.17.
All Share Index Value and Futures Price



6.5.5. Long Futures Arbitrage in South Africa

Over the period from approximately June 1988 to December 1989 arbitrage opportunities were available on the Industrial and All Share index futures as a result of the theoretical futures price being greater than the actual futures price. To reap these arbitrage profits a long hedge would need to be entered into which involves the selling (shorting) of the index and the buying of the futures contract. The buying of the futures contract poses no problems and would follow the normal manner involving similar transaction costs to selling a futures contract. However, it is not as easy to sell the index short.

To sell then index short do this the arbitrageur would need to sell the shares comprising the index short. A short sale of a share is known as a "bear sale" in terms of the Stock Exchange Control Act, No. 1 of 1985 ("the Act") and is defined as a sale "of listed securities which the seller is not the owner at the time the sale is entered into, and of which he is not at the time entitled to become the owner by virtue of an inheritance or in terms of a transaction entered into before the sale is effected" (South Africa, 1985: S1). Bear sales are strictly regulated in the Act which includes, inter alia, the payment of a minimum cover deposit (in respect of bear sales contracted for) equivalent to the most recent market value of the applicable securities (South Africa, 1985: S24(3)).

Long futures and short spot arbitrage (outlined in section 5.2.2.) is based on the assumption that short proceeds are received immediately and invested at the risk-free interest rate. Unfortunately this is not possible in South Africa where the sales proceeds are received subsequently at the settlement date. Moreover, an initial investment more or less equivalent to the sales proceeds needs to be deposited up front, in terms of the regulations, to make good any short sale. The net effect is that it is not possible to receive short sale proceeds initially when the bear sales are made.

Modest (1984) investigated the effect of short sale proceeds on arbitrage opportunities on the Standard & Poors (S&P) 500 stock index futures market. He noticed that where investors did not have use of short sale proceeds futures prices almost always lay within the theoretical arbitrage bounds. It is possible for futures prices to even trade at a discount to the spot value of the index without giving rise to arbitrage opportunities (ibid: 54). He concluded: "the historical experience reveals that arbitrage opportunities are consistently available only for traders who have full use of proceeds" (ibid: 56). As a result of the inability to use the short sale proceeds, the upper arbitrage bound can be reformulated as;

$$T_f + T_l + e^{r(t,T)(T-t)} \geq$$

$$I(t)\ e^{r(t,T)(T-t)} - \int_t^T D(w)e^{R(t,w,T)(T-w)} - F(t,T) \qquad (6\text{-}6)$$

$$\geq\ - T_f - T_l$$

where $e^{r(t,T)(T-t)}$ represents the interest foregone. The costs of the long spot position (T_l) is included in the upper arbitrage bound as the transaction costs incurred (short sale restrictions aside) are the same as those for the long futures position.

The revised upper arbitrage bound (Bound C) given by the left hand side of 6-6, has been calculated for the period from June 1988 to December 1988. The interest foregone term is computed by multiplying the index value by the risk-free treasury bill rate for the period to expiration of the futures contract. The results are provided for both the All Share and Industrial Index futures in Figures 6.18. to 6.21.

The effect of adjusting for interest foregone on bear sales is striking. Analysis of Figures 6.18. and 6.19. of the All Share Index futures near and mid contracts respectively reveals that all observations (except for one solitary observation on 19 May 1989) which were seen to provide arbitrage opportunities under the Bound B scenario were in fact not arbitrage opportunities. This situation is also the same with the Industrial Index near and mid futures contracts (Figures 6.20. and 6.21. respectively); except for the extraordinary observation once again on 19 May 1989, and

Figure 6.18. All Share Index Futures
Revised Upper Arbitrage Bound C
Near Contracts

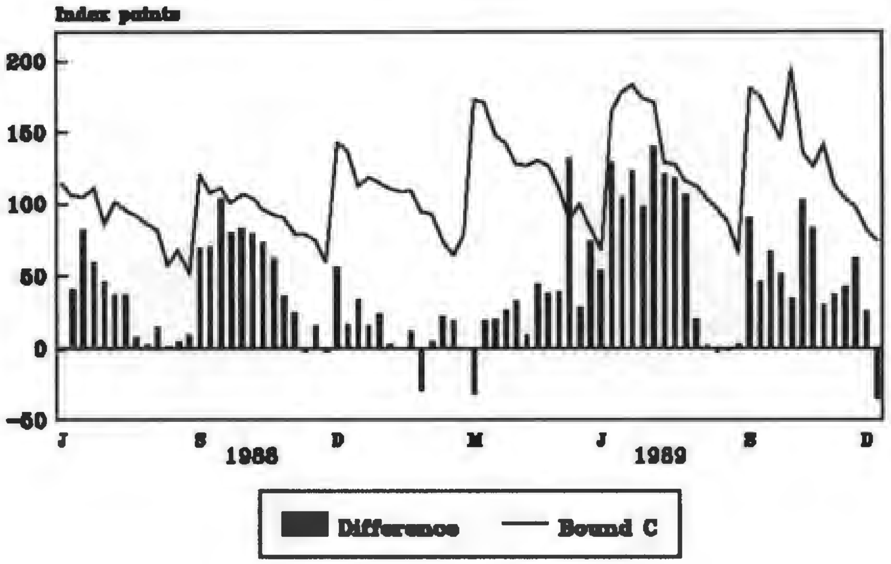


Figure 6.19. All Share Index Futures
Revised Upper Arbitrage Bound C
Mid Contracts

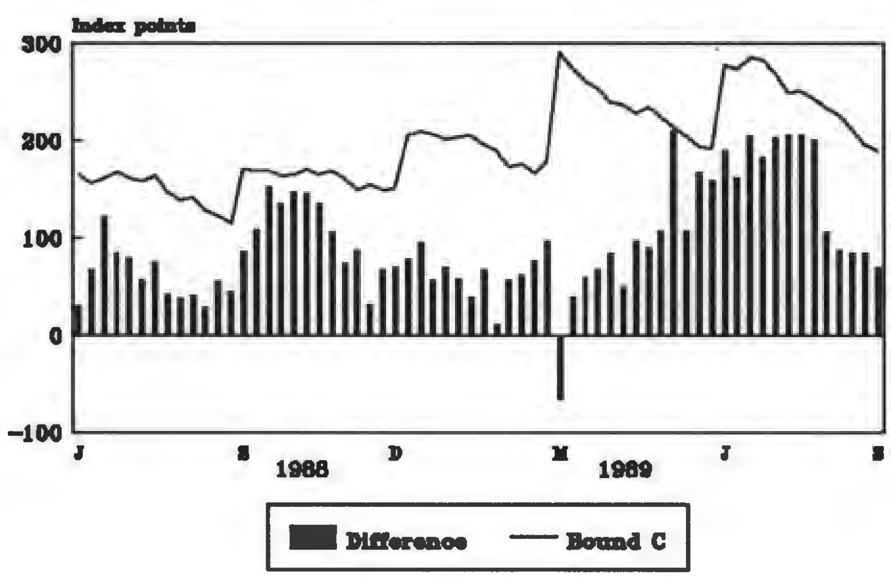


Figure 6.20. Industrial Index Futures
Revised Upper Arbitrage Bound C
Near Contracts

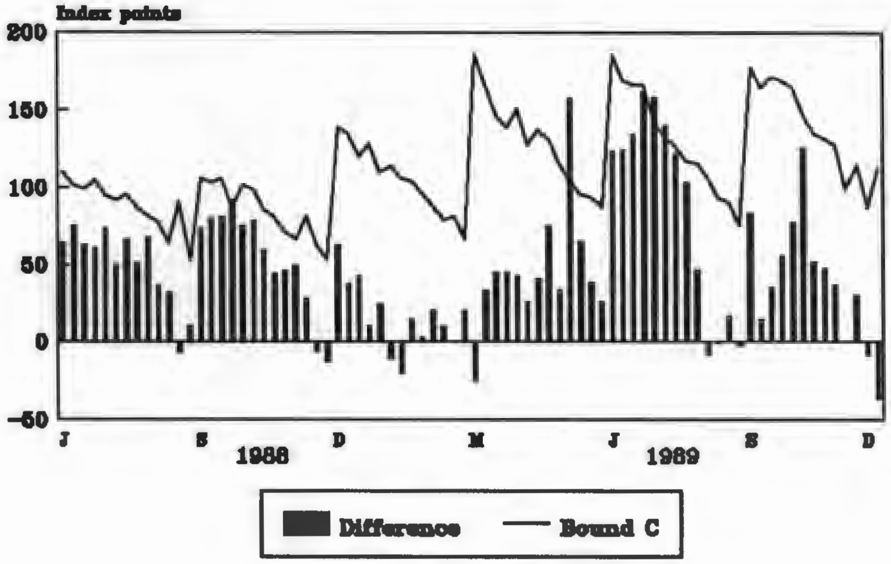
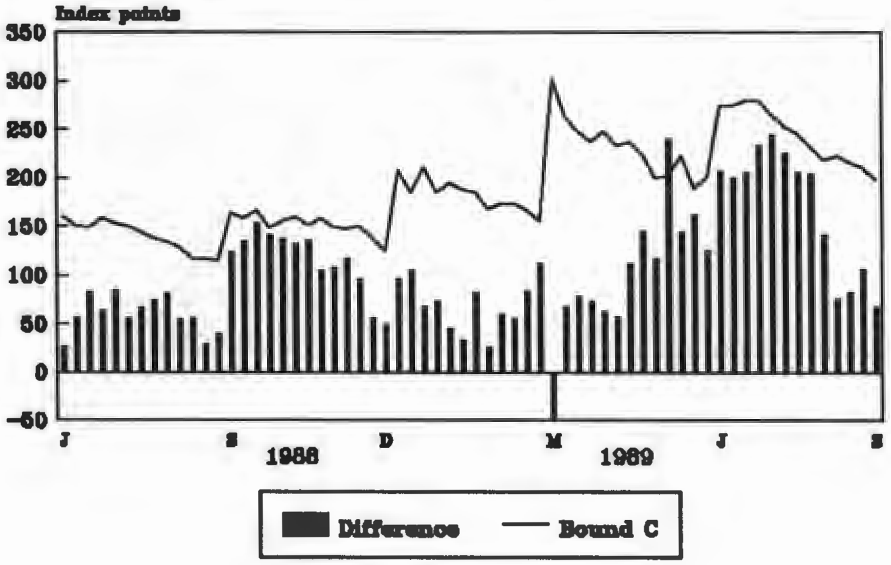


Figure 6.21. Industrial Index Futures
Revised Upper Arbitrage Bound C
Mid Contracts



two observations in July 1989 whose pricing differences only marginally extend beyond Bound C. These results are very similar to that found on the S&P 500 futures market (Modest, 1984).

What is noticeable from the graphs is the way in which the trend in the pricing differences follows Bound C. This is expected to a certain degree because the actual futures price will naturally draw closer to the theoretical futures price as the date of expiration approaches; at the same time the interest foregone also decreases towards expiration. Despite this mutual trend, the closeness to which the trend is followed at times suggests that the above method of calculating the arbitrage bound correctly reflects the arbitrage bounds facing market participants.

6.5.6. Conclusion: Arbitrage Pricing

Once the costs of short selling has been taken into account for the All Share and Industrial index futures (and accepting that this also applies to the All Gold index future) the number of arbitrage opportunities are relatively few.

In total, only 32 arbitrage opportunities were available on both contracts over the observation period; namely, 20 in respect of the All Gold index, and 6 each in respect of the Industrial and All Share indexes. This total represents only 4.08% of the grand total of 785 weekly observations (i.e. for both near and mid contracts on all three contract types). Moreover, only four of the observations were recorded since June 1988 indicating an improvement in arbitrage pricing over the observation period.

These results contradict the results of Lambrechts (1988) who concluded that there were "rather obvious arbitrage opportunities" in the South African share index futures market. His conclusion is based on results for the period from May to September 1988 which showed that the theoretical futures prices were higher than the actual futures prices in 70% of the cases (his results favourably compares to those found in this study). However, Lambrechts ignored the obstacle of shorting the index faced by arbitrageurs and the related extension to the arbitrage bound.

6.6. CONCLUSION OF CHAPTER

The analysis of futures pricing through regression techniques and the analysis of arbitrage bounds has established that the share index futures markets were optimally priced over the period investigated. Even though the extent of pricing differences were large over the second half of the observation period, very few arbitrage opportunities were available. This is due to the very large arbitrage bounds faced by arbitrageurs mainly as a result of restrictions imposed to shorting the index.

While the lack of arbitrage opportunities represents a positive aspect as far as pricing efficiency is concerned, the large extent of the arbitrage bounds has introduced substantial variability between the spot and futures price. The upper arbitrage bound approaches approximately 300 points and the lower arbitrage bound reaches above 100 index points; therefore, the futures price can freely move within an arbitrage bound of up to 400 index points. This large range explains the basis risk introduced.

This variability between the spot and futures price is noticeable in the regression where the coefficients of variation (R^2) were very low considering the fact that the futures instrument is directly based on the underlying asset. The average R^2 values for the whole observation period indicates unexplained variability (i.e. basis risk) of between 21% and 29% for the three contracts.

Unfortunately hedgers are forced to bear this basis risk unless the extent of the arbitrage bounds are reduced. This can be done by either reducing the cash costs of making arbitrage transactions (i.e. bid-ask spread, booking fees and share transaction costs) or relaxing the restrictions regarding short sales. While the share transactions costs were reduced in 1990 by the phasing out of the marketable securities tax, the reduction in the other arbitrage costs are likely to remain for the foreseeable future.

In the light of the above results, the impact of using the dividend yield as a surrogate of the actual dividends needs to be reassessed. The effect of using the quoted dividend yield is assessed to result in an estimation error of the futures price of up to 20 index points. The maximum error is relatively insignificant if viewed in relation to the fair range of up to 400 index points. Therefore, the dividend yield is likely be satisfactory for most hedging motives. However, at the margin this difference may be material to arbitrage and short term speculative traders. This study has quantified the likely magnitude of the estimation error allowing users to make their own decisions depending on the sensitivity of the particular objective.

CHAPTER SEVEN

EMPIRICAL RESEARCH: FUTURES HEDGING

7.1. INTRODUCTION

The vital consideration facing a prospective hedger is the risk-return performance of the futures hedge. In this chapter we will look at the effectiveness of hedging in South Africa with the following main objectives:

i) To compare the performance of the major hedging models in terms of hedging risk and return. The major hedging models are the traditional, minimum-variance, beta and the Howard & D'Antonio (H&D) hedge strategies.

ii) To evaluate the hedging effectiveness of the above models by using the H&D effectiveness measures, namely hedging benefit per unit of risk (HBS) and measure of hedging effectiveness (HE).

iii) To assess the hedging effectiveness of the South African market in terms of both risk and return.

7.2. METHODOLOGY

This study focuses on hedging from an ex ante viewpoint which differs from other studies (such as Ederington, 1979; Junkus & Lee, 1985) where an ex post approach was maintained. While these studies did establish the optimal ex post hedge ratios and hedge results, they did not adequately assist users to evaluate probable future hedging performances. The ex ante focus enables informative conclusions to be drawn regarding the potential hedging results obtainable from the application of a particular hedging strategy. Consequently, this study recognises that, at any point in time, the hedger has only the knowledge of historical events. In this regard, it is assumed that the futures market is efficient in the semi-strong form; i.e. there is no information available (other than that which has already been impounded on historical prices) with which the hedger can predict future prices. This assumption is based on the analysis and discussion in section 5.3.1.

The empirical analysis is based on hedging results obtained from selected spot portfolios combined with a representative futures portfolio. The size of the futures portfolio is determined by the hedge ratio calculated in terms of each particular hedge strategy.

The futures portfolios are limited to contracts based on the respective index which best resembles the particular spot portfolio. Hedging with futures contracts in a cross hedge situation (e.g. using All Gold index futures contracts to hedge Industrial spot portfolios) is out of the scope of this study. Hedging using the most appropriate share index future is more often and widely used than cross hedging using other index futures. Thus, this study will assist in the majority of hedging applications.

The observation period stretched from 15 December 1987 to 15 December 1989; which consists of eight futures contract expiration dates. The starting date is chosen to allow time for the futures market to develop (Ederington, 1979) and to establish a series of historical futures data from which the hedge ratios could be estimated. Delaying the starting point has the additional advantage of eliminating the October 1987 Stock Market Crash, thereby limiting the possibility of this extraordinary event leading to anomalous results.

Friday weekly data is used except for a few observations where Friday data was not available; in these cases the nearest available data is used. Actual share data was obtained from the Beltel information system while the source for all other data is the same as that used in chapter 6 on futures pricing.

Hedging periods of one, two, four, eight and twelve weeks are held, which compares favourably to other studies. For example, Ederington (1979) based his analysis on two and four week hedging periods and Smith (1989) used one, two and four week hedging periods. The hedging periods start at weekly intervals and thus overlap one another (except for the one-week hedging period). In all, 120 hedge portfolios have been selected (i.e. 6 spot portfolios, 4 hedge strategy types and 5 hedge lengths).

As the maximum hedging period selected is three months (12 weeks), the three month futures price is sufficient for hedging purposes. In situations when the hedging period crosses a futures expiration date, it is assumed that the futures contract is rolled over into the following three month futures contract.

7.2.1. Futures Portfolios

The formulae for the respective hedge strategies analysed are given in Table 7.1. (definition of terms can be found at the equation references provided):

Table 7.1.
Hedging Strategies and Ratios

Strategy	Symbol	Eqn	Hedge Ratio
Traditional			1
Minimum Variance	h	3-5	$\sigma_{sf} / \sigma^2{}_f$
Beta Hedge	ß	3-6	$\sigma_{si} / \sigma^2{}_i$
H&D	b	3-17	$\sigma_s(\alpha - r) / \sigma_f(1 - \alpha r)$
			$\alpha = [\mu_f/\sigma_f]/[(\mu_s - i)/\sigma_s]$

To calculate the hedge ratio at any point in time over the observation period, firstly the appropriate formula is selected for each hedge strategy investigated. Other than the traditional hedge strategy (where the hedge ratio is one), the terms of the hedge ratio per Table 7.1. above is computed with reference to the 52 weeks of historical data immediately preceding the observation. It must be noted that for the first six months of the observation period a full year's futures historical data was not yet available so the lesser period is used. This basis provides a sufficiently long historical series upon which a reliable hedge ratio estimate can be made, while at the same time, still being short enough to accommodate changes in the hedge ratios in response to market changes.

The minimum hedge ratio (h) at any point in time is computed by regressing the spot portfolio one-week returns for the previous 52 observations against the previous 52 futures portfolio one-week returns. Similarly, the beta hedge ratio is calculated by regressing the preceding 52 spot portfolio one-week returns against the index one-week returns for the previous 52 weeks. The regression technique is the normal procedure used to calculate these hedge ratios (Ederington, 1979; Duffie, 1989: 219; Schwarz et al, 1986: 180; and Blank et al, 1991: 243-4).

Regression cannot be used to calculate the H&D hedge ratio, therefore the variables according to Table 7.1. must be estimated in turn. In this regard, the futures return (μ_f) at time t is taken to be the average one-week return for the immediately preceding 52 observations. However, the excess return on the spot portfolio (μ_s-i) must, in terms of Howard and D'Antonio (1984), also include the dividends receivable on the spot portfolio and the risk-free rate of return. Thus, the value of the excess return on the spot portfolio (μ_s-i) at time t is calculated as;

$$\mu_s - i \quad = \quad R_s(t) \; + \; DY(t) \; - \; TB(t) \qquad (7\text{-}1)$$

where

$R_s(t)$: Average spot return (percent) calculated at time t based on the immediately preceding 52 spot returns.

DY(t) : Dividend yield (percent) at time t.

TB(t) : Treasury bill rate at time t.

The quoted dividend yield is used as an estimate of the dividends receivable on the spot portfolio, and the treasury bill rate is taken as a surrogate of the risk-free rate. The difference between the actual dividend yield and quoted dividend yield, as identified in chapter 6, is not relevant here as a full year's historical data has been used to calculate all necessary variables (the dividend yield is the yield of the previous year's actual dividends).

The standard deviations of the future (σ_f) and spot (σ_s) portfolios used in calculating the H&D hedge ratio is estimated by taking the standard deviation of the immediately preceding 52 one-week returns of the futures and spot markets respectively. The correlation coefficient between the spot and futures portfolios (r) is obtained from the square root of the coefficient of determination (R^2) computed from the regression between the previous 52 spot and futures one-week returns.

The second-order conditions (SOC) are ignored in calculating the H&D hedge ratios as in an ex ante situation they are assumed to be satisfied[1]. This study differs from Smith (1989) who assumed an arbitrary long futures position where the second-order conditions did not hold.

7.2.2. Spot Portfolios

Six spot portfolios have been selected, two for each share index futures contract. The first portfolio, to be known as the index portfolio, is established by assuming that the value of the index portfolio at any point in time is the index itself. Consequently, the period return on the index portfolio is simply the percentage period movement of the index. The index portfolio is selected because the movement in the index should match the futures price more closely than any other portfolio. In this manner, the lowest basis risk situation is established.

The index portfolio is idealistic in that it is virtually impossible in practice to establish and maintain such a diversified portfolio. Thus, share portfolios for each index are established to illustrate the more realistic hedging performance that could be expected.

The share portfolios were formed by selecting a few of the

1. Refer section 3.5.3.

largest constituents in the respective index by reference to the market capitalisation of index constituents as at 2 January 1990. Selection is made in descending order of market capitalisation until a reasonable percentage (approximately 50%) of the total index capitalisation is obtained. The share portfolios selected and their percentage of the total market capitalisation of the index are given in Tables 7.2. to 7.4. Richemont and Minorco are large capitalised shares in the Industrial and All Share index respectively, but were not chosen as they were not part of the index for the full observation period.

Table 7.2.

All Share share portfolio

Share	% of AS Index
Anglo American Corp	9.0%
De Beers	8.3%
Gencor	4.4%
Driefontein	3.7%
Rustenburg Platinum	3.7%
Gold Fields of SA	3.4%
Barlow Rand	2.9%
SA Breweries	2.8%
Anglo American Gold	2.7%
Vaal Reefs	2.6%
SASOL	2.6%
Rembrandt Group	2.5%
Freegold	2.0%
	53.6%

Table 7.3.

Industrial share portfolio

Share	% of IN Index
Barlow Rand	8.3%
SA Breweries	8.0%
SASOL	7.4%
Rembrandt Group	7.2%
AMIC	5.4%
SAPPI	3.4%
AECI	2.6%
	42.3%

Table 7.4.

All Gold share portfolio

Share	% of AG Index
Driefontein	19.3%
Vaal Reefs	13.8%
Freegold	10.7%
Kloof	9.8%
	53.6%

7.2.3. Creation of the Hedge

Each hedge is created by calculating the hedge ratio at the starting point of the hedging period and keeping the hedge ratio constant throughout the hedging period. Dynamic hedging (i.e. the altering of the hedge ratio during the hedge) is not considered owing to the relatively short hedging period and the difficulty of establishing an objective dynamic hedging strategy. The hedged portfolio period return is calculated for each separate hedge period according to the following manner:

$$R_p(T-t) = \frac{[X_s(T) - X_s(t)] - H(t)\,[X_f(T) - X_f(t)]}{X_s(t)} \qquad (7\text{-}2)$$

where

t	:	Time when the hedge period starts.
T	:	Time when the hedge period ends.
$R_p(T-t)$	:	Return of the hedged portfolio over the hedging period starting at time t and ending at time T.
$X_s(t)$	:	Value of the spot portfolio at time t.
$X_s(T)$	:	Value of the spot portfolio at time T.
$X_f(t)$	:	Futures price at time t.
$X_f(T)$	:	Futures price at time T.
H(t)	:	Hedge ratio calculated at the beginning of the hedging period (t).

The denominator in 7-2 above does not include the futures position as only margin deposits are made which earn a money market rate of return in compensation[2]. Transaction costs and other expenses (including any financial costs associated with making margin deposits and margin calls) have been ignored in calculating the hedge return. This is because these costs vary from participant to participant making it difficult to objectively include these costs into the hedging analysis. Nevertheless, transaction costs are likely to have an impact on the performance of the hedge and thus must be considered before any hedge is entered into.

The standard deviation of each hedged portfolio at time t is then calculated based on the previous 52 hedge period hedged returns as computed per 7-2 above. Howard & D'Antonio's HE and HBS measures of hedging effectiveness have also been

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2. Refer sections 5.3.4. and 6.5.1.1.

computed for each individual hedge using the hedge period return and standard deviation as calculated above, together with the treasury bill rate taken at the start of the hedge.

Results have been illustrated graphically throughout the text; the actual data produced from the analysis are given in Tables C.1. to C.6. in Appendix C. Flowchart 7.1. summarises how the results are obtained for each hedge strategy.

To facilitate better comparisons between different portfolio types, the hedged portfolio return and risk are expressed graphically as a percentage of the unhedged (spot) portfolio return and risk respectively. In this manner, differences between portfolio types and hedge lengths are eliminated allowing comparisons to be made. Each portfolio has been abbreviated for purposes of graphical presentation according to Table 7.5.

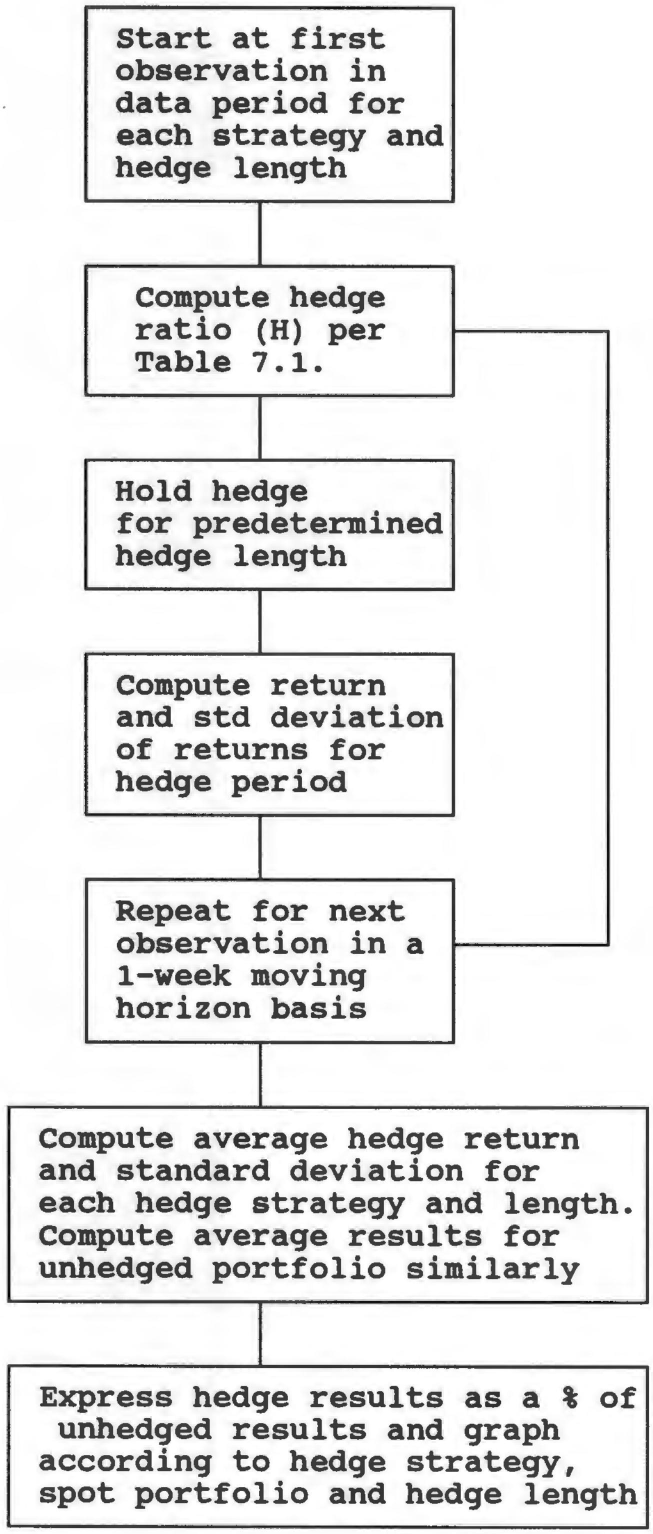
Table 7.5.
Portfolio Abbreviations

AGI	All Gold index portfolio
AGP	All Gold share portfolio
INI	Industrial index portfolio
INP	Industrial share portfolio
ASI	All Share index portfolio
ASP	All Share share portfolio

The discussion and evaluation of the results has been divided into four major sections. Firstly the hedge ratios for all strategies are discussed, followed by results of the Howard & D'Antonio strategy and then those on the risk-minimising strategies. Finally hedging effectiveness of all strategies are investigated.

FLOWCHART 7.1.

DIAGRAM OF HOW AVERAGE HEDGE RETURN AND AVERAGE HEDGE STANDARD DEVIATION RESULTS ARE OBTAINED



7.3. HEDGE RATIOS

The hedge ratios for the share and index portfolios are illustrated graphically. For ease of presentation, short futures positions are shown as positive hedge ratios while long futures positions are shown by negative hedge ratios. The hedge ratios are discussed according to contract type.

7.3.1. All Gold Hedge Ratios

Looking first to the share portfolio in Figure 7.1, the minimum-variance hedge ratio is substantially lower than the beta hedge ratio. The difference between the two ratios range from a minimum of 0.077 to a maximum of 0.385 with an average difference of 0.2023 (equal to 24.59% of the average minimum-variance hedge ratio). The average minimum-variance hedge ratio of 0.8067 is significantly lower than the traditional one-to-one hedge ratio. These results are consistent with the findings of the empirical studies reviewed (refer chapter 4).

The minimum-variance hedge ratio of the index portfolio (refer Figure 7.2.) follows the pattern of the share portfolio, with the average hedge ratio only marginally higher at 0.8264. The share portfolio and the index returns are highly correlated, evident by the very high average R^2 statistic between the share portfolio and the index of 0.9604. Also, the standard deviations of the share and index one-week returns are very similar, namely at 4.4467% and 4.3223% respectively. The combination of these two components has resulted in only a small difference in the minimum-variance hedge ratio between the share and index portfolios.

Figure 7.1.
All Gold Share Portfolio
Minimum-Variance & Beta Hedge Ratios

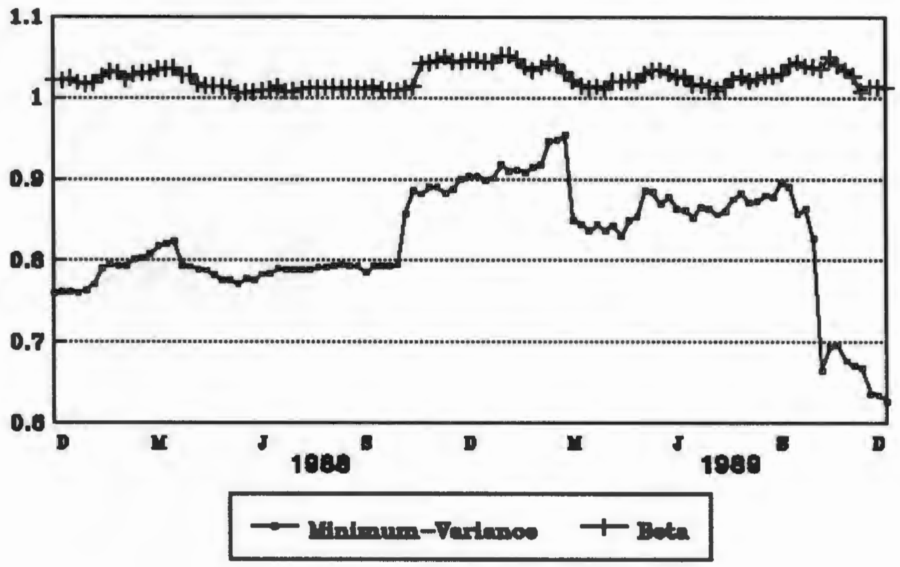
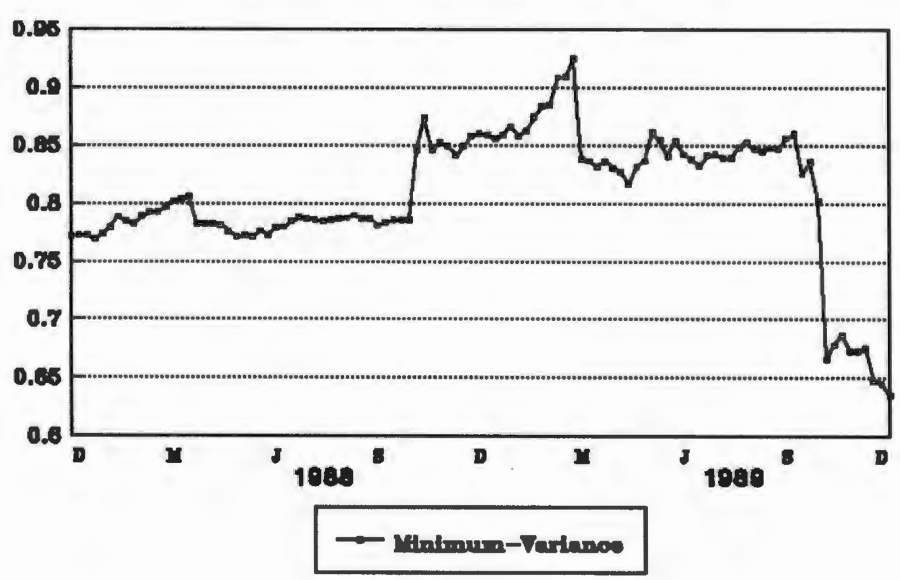


Figure 7.2.
All Gold Index Portfolio
Minimum-Variance Hedge ratio



The reason that the minimum-variance hedge ratio is lower than the beta hedge ratio is due to two factors. Firstly, the variance of the futures prices is greater than that of the index and so the denominator of the minimum-variance hedge ratio (σ^2_f) is greater than the denominator of the beta hedge ratio (σ^2_i). The average standard deviation of one-week futures returns (σ_f) is 5.1190% compared to the average standard deviation one-week All Gold index returns (σ_i) of only 4.5358%.

Secondly, the covariance between the spot portfolio and the index (σ_{si}) is greater than the covariance between the spot portfolio and the futures portfolio (σ_{sf}). This is apparent from the coefficient of variation (R^2) of the respective regressions. The average R^2 for the beta hedge ratios are 1.0000 and 0.9604 compared to only 0.7643 and 0.7213 for the minimum-variance hedge ratio.

In both the index and share portfolios, the minimum-variance hedge depicts a large drop in the hedge ratio at the end of October 1989. This drop is generated by a large increase in the standard deviation of the futures portfolio: for the week from 20 October 1989 to 27 October 1989 the standard deviation of the All Gold index future one-week returns increased by 13.96% from 3.1701% to 3.6125%.

The large increase in the futures price standard deviation was sparked by a decline of 13.015% in the futures price between two adjacent observations, namely the 6 October 1989 and 16 October 1989. The index also dropped during this period but not to nearly the same extent; the balance of the decrease during this 10 day period resulted because the futures price moved from a small premium to the index situation to a large discount to the index position. This large drop is apparent from the pricing differences for the All Gold three month index futures contracts in Figure 6.10.

The Howard & D'Antonio hedge ratios reveal a different picture to that of the other hedge strategies. The graphs illustrate that the hedge ratio is substantially greater than one: the average hedge ratios are 3.6879 and 2.7313 for the index and share portfolios respectively. For graphical purposes a number of observations have been omitted from the H&D hedge ratio. These are hedge ratios of 165.471 (19/8/1988) and 103.845 (14/10/1988) for the index portfolio, and 34.251 (15/12/1989) and 20.991 (6/1/1989) for the share portfolio.

The H&D hedge ratio depends on the interaction between the risk-return relative (α) and the correlation coefficient (r) between the spot and futures portfolio. Despite the large difference differences in the average value of α between the index and share portfolios, as indicated in Table 7.6. below, the relationship between the variables of α and r under both portfolios is such that the average hedge ratios are fairly similar in size.

Table 7.6.
All Gold Risk-Return Relative and Correlation Coefficient

	Ave α	**Ave r**	**Ave b**
All Gold Index Portfolio	8.9627	0.8732	3.6879
All Gold Share Portfolio	2.7164	0.8511	2.7313

It is surprising that the hedges calculated are mostly short hedges considering that the risk-return relatives are all well above one, i.e. the return on the futures market exceeds the excess return on the spot portfolio. However, the large values of α had the opposite effect: in most cases the factor ($1-\alpha r$) in the denominator was negative while the numerator ($\alpha-r$) was positive.

Figure 7.3.
All Gold Share Portfolio
H&D Hedge Ratio

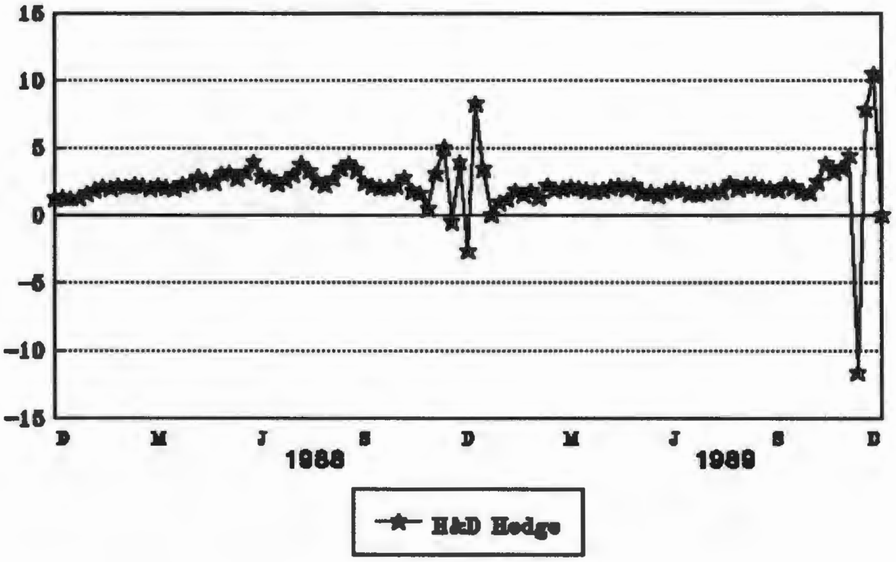
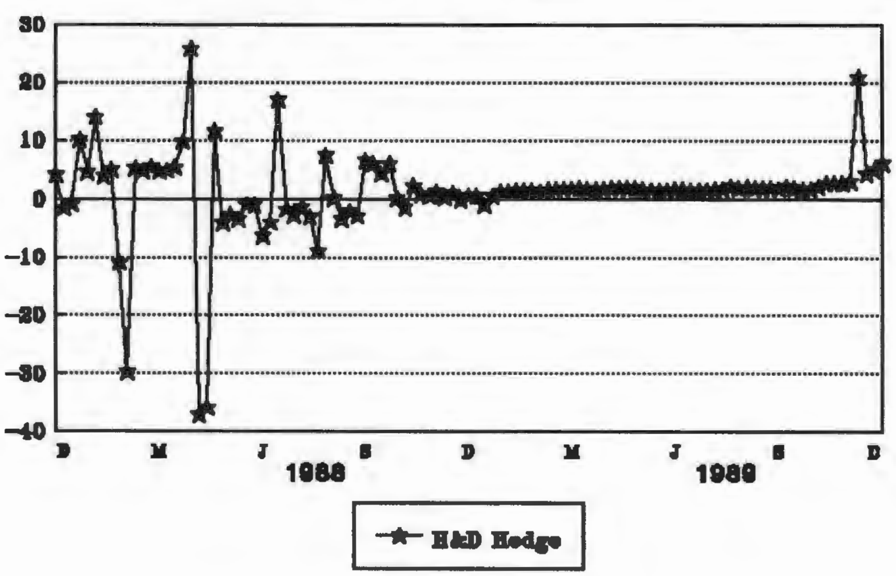


Figure 7.4.
All Gold Index Portfolio
H&D Hedge Ratio



7.3.2. Industrial Hedge Ratios

As illustrated in Figure 7.5, the average beta hedge ratio of the share portfolio observed was 1.2282, increasing over the observation period from 1.0962 in December 1987 to 1.3825 in December 1989. The rise in the beta from November 1988 to December 1989 is steeper than the initial period. This is attributed to the greater one-week standard deviation of returns for the share portfolio (σ_s) for 1989 of 3.2244% compared to 2.9534% for 1988. (The covariance between the share portfolio and the index remained fairly constant: the R^2 values are 0.9341 and 0.9166 for 1988 and 1989 respectively).

The beta hedge ratio did not fall below the one-to-one traditional hedge ratio at any stage. In contrast, the minimum-variance hedge ratio was found to be below the one-to-one hedge ratio throughout the observation period, averaging 0.8356 for the share portfolio and 0.6993 for the index portfolio. The lower variability of returns of the index portfolio (average standard deviation (σ_i) of 2.9534%) compared to the share portfolio (average standard deviation of 2.1686%) is the major reason for the lower minimum-variance hedge ratio of the index portfolio.

The general downward trend in the minimum-variance hedge ratio in 1989, especially with respect to the index portfolio, is a result of the greater standard deviation of one-week futures returns in 1989 of 3.9778% compared to only 2.2747% for 1988. This is out of proportion to the increase in the standard deviation of the index which only increased marginally from 2.0697% in 1988 to 2.2299% in 1989. The greater standard deviation of futures returns can be attributed to the large size of the pricing differences observed from September 1988 to December 1989.

Figure 7.5.
Industrial Share Portfolio
Minimum-Variance and Beta Hedge Ratios

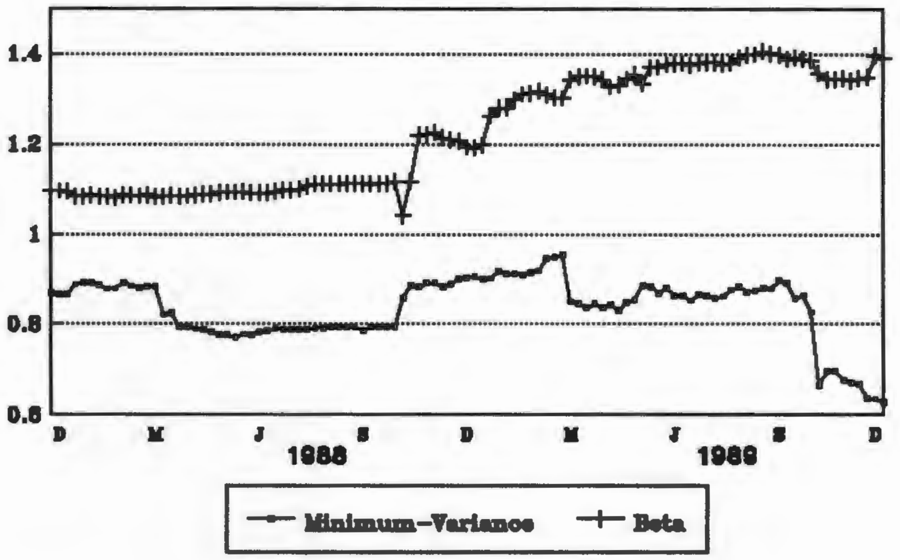
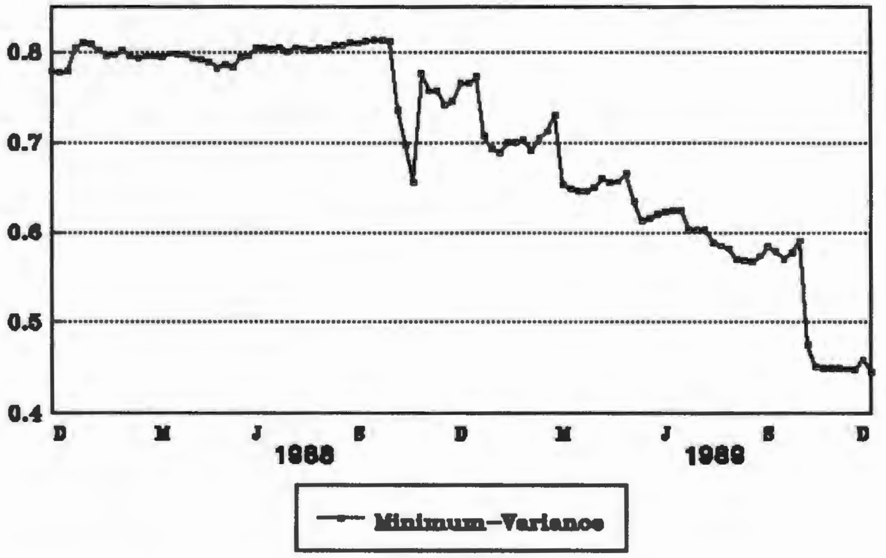


Figure 7.6.
Industrial Index Portfolio
Minimum-Variance Hedge Ratio



As with the All Gold portfolios, the reason that h is less than ß is due to the greater standard deviation of the futures one-week returns (3.6203%) compared to the spot portfolio one-week returns (2.1686%). This was compounded by the lower covariance between the spot portfolio and the futures portfolio (R^2 equals 0.6917 and 0.1538) compared to the covariance of the spot portfolio and the index (R^2 equals 1.0000 and 0.9253).

The H&D hedge ratios are substantially greater than one with the average hedge ratios being 1.5506 (short position) for the index portfolio and -1.8394 (long position) for the share portfolio. A number of observations have been omitted to facilitate graphing, namely 45.407 (8/12/1989), 37.065 (26/8/1988) and -18.063 (15/1/1988) in the index portfolio H&D ratio graph; and 116.56 (1/7/1988) in the share portfolio H&D ratio graph.

Table 7.7.
Industrial Risk-Return Relative and Correlation Coefficient

	Ave α	Ave r	Ave b
Industrial Index Portfolio	1.1058	0.8305	1.5506
Industrial Share Portfolio	1.0049	0.3521	-1.8394

The very low correlation coefficient (r) between the Industrial share portfolio and the futures portfolio is largely responsible for the net long futures position of the Industrial share portfolio. The low r factor combined with the relatively low risk-return relative ensures that both the numerator and the denominator of the H&D hedge ratio are positive.

Figure 7.7.
Industrial Share Portfolio
H&D Hedge Ratio

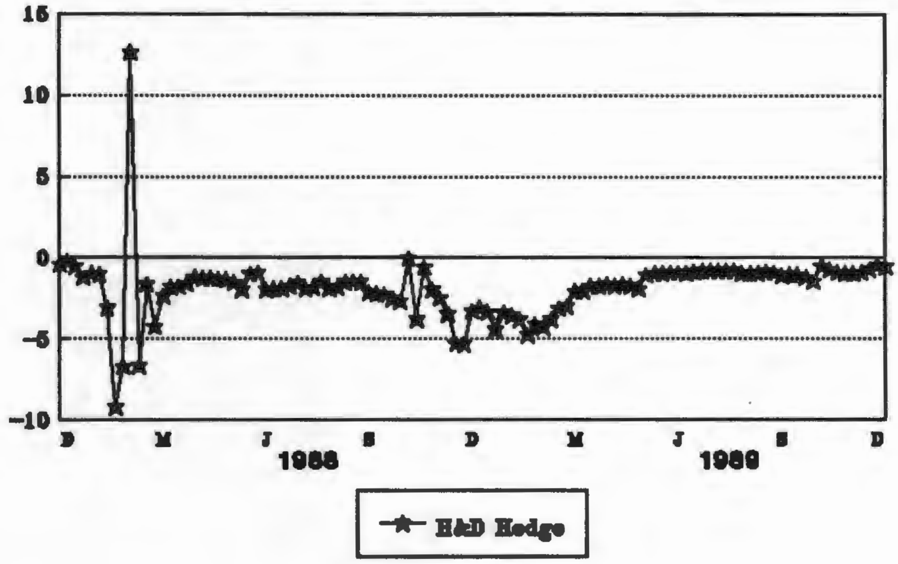
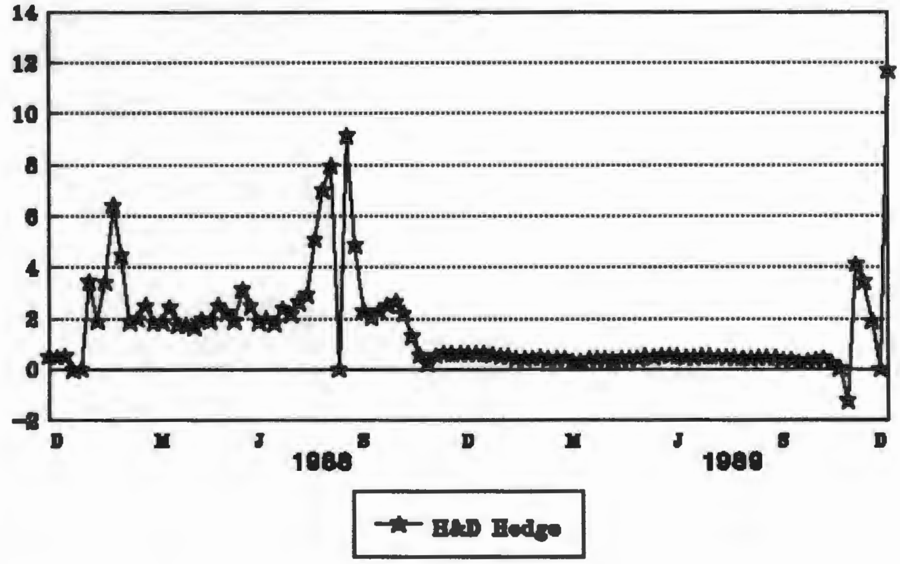


Figure 7.8
Industrial Index Portfolio
H&D Hedge Ratio



7.3.3. All Share Hedge Ratios

The beta hedge ratio of the All Share share portfolio was always above the traditional one-to-one hedge ratio. However, as with the All Gold and Industrial portfolios, the minimum-variance hedge ratio is substantially below this with an average of 0.8227. The minimum-variance hedge ratio of the index portfolio is lower than that of the share portfolio, with the average hedge ratio of 0.7485. The reasons for both of these situations are the same as those for the All Gold hedge portfolios.

Figure 7.9.
All Share Share Portfolio
Minimum-Variance and Beta Hedge Ratios

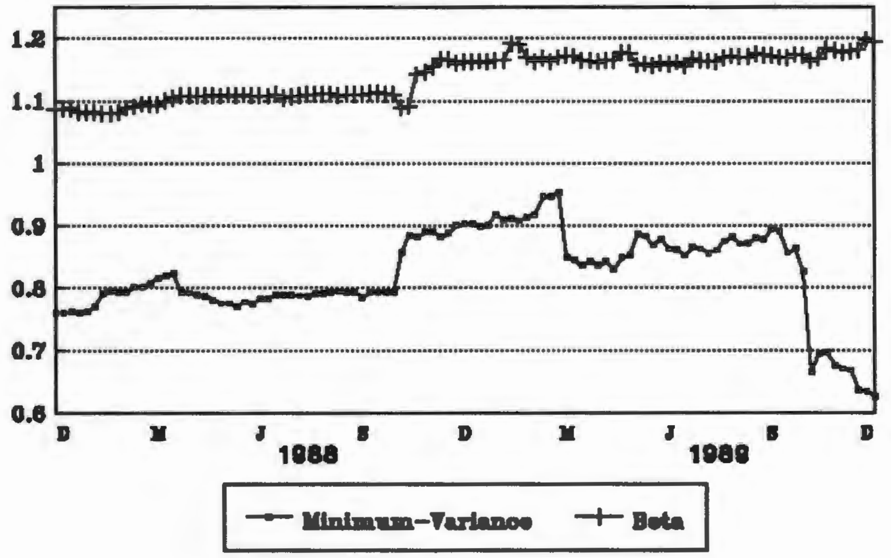
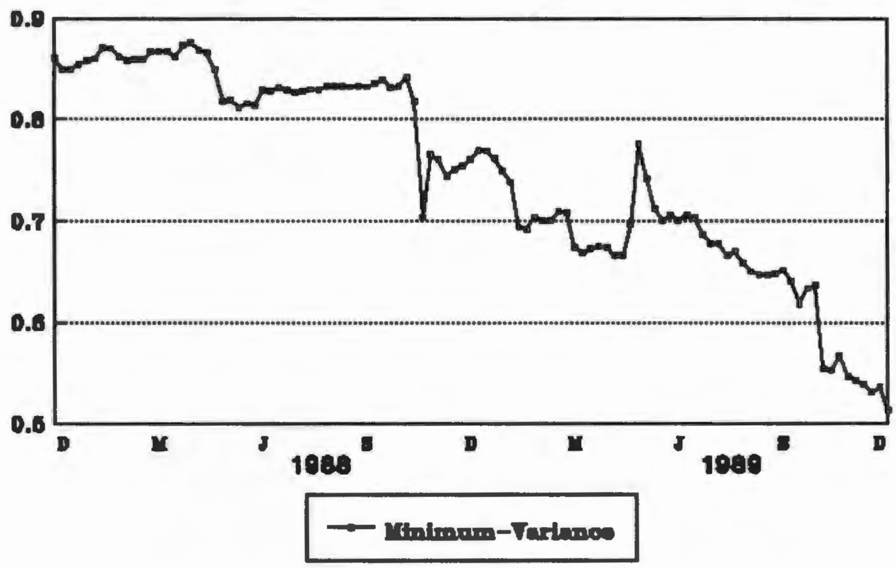


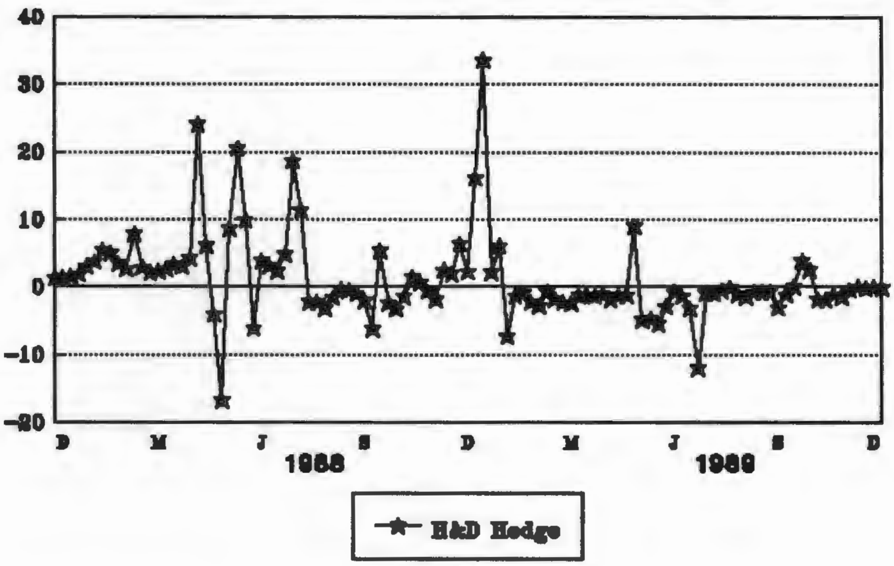
Figure 7.10.
All Share Index Portfolio
Minimum-Variance Hedge Ratio



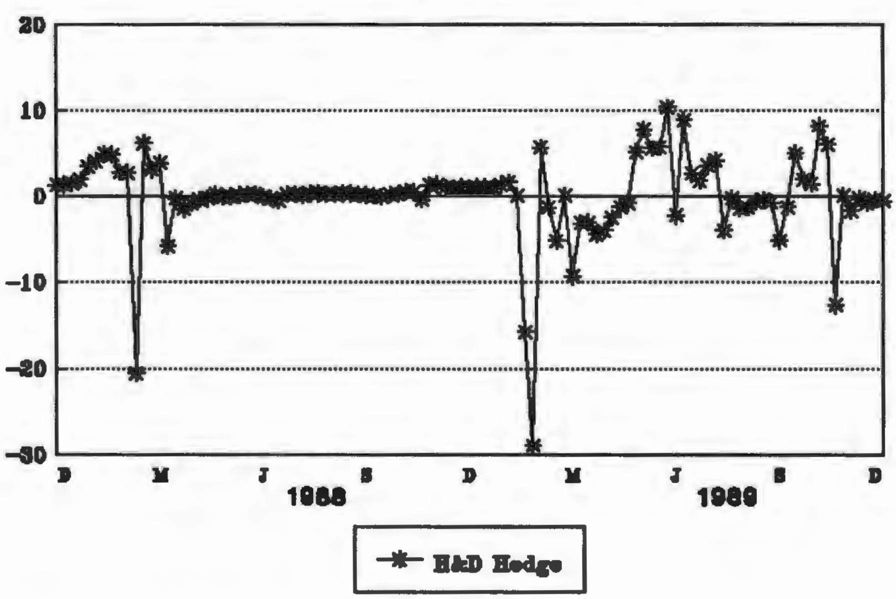
The Howard and D'Antonio hedge ratios generally represent short futures positions with the average ratios for the index and share portfolios being 3.6879 and 2.6975 respectively. Figures 7.11. and 7.12. illustrate the large variability of the H&D hedge ratio, particularly with respect to the share portfolio.

Moreover, the graphs do not take into account a number of abnormally large hedge ratios which are omitted to facilitate graphical presentation. These are 165.787 (27/1/1989), 57.902 (10/3/1989) and 72.119 (10/11/1989) in respect of the index portfolio; and 164.326 (29/9/1989) in the share portfolio.

Figure 7.11.
All Share Share Portfolio
H&D Hedge Ratio



All Share Index Portfolio
Hedge ratios



The short position calculated for the share portfolio is mainly attributable to the large value of the futures-spot excess return to risk relative as indicated in Table 7.8.

Table 7.8.
All Share Risk-Return Relative and Correlation Coefficient

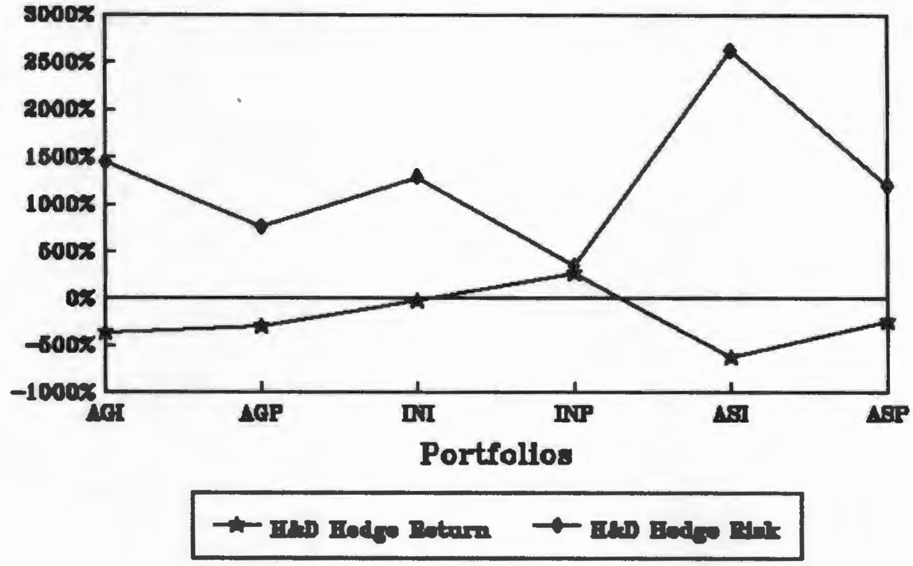
	Ave α	Ave r	Ave b
All Share Index Portfolio	1.0793	0.8489	3.6879
All Share Share Portfolio	1.6293	0.8611	2.6975

However, in the index portfolio on average the risk-return relative of the futures position is only marginally greater than the excess return to risk relative of the spot position. These averages suggest a long futures position. The net short position in this case resulted because of circumstances, i.e. either the (α-r) in the numerator was negative and the value (1-αr) in the denominator was positive, or vice versa. Very seldom are both positive (or both negative).

7.4 HOWARD AND D'ANTONIO HEDGING

The Howard and D'Antonio strategy is discussed first as its results are vastly different from the other three strategies in view of the abnormally large H&D hedge ratios. The risk and return results of the H&D strategy classified by portfolio type are illustrated in Figure 7.13.

Figure 7.13. H&D Hedge
Average risk and return given as % of unhedged portfolio ave risk and return

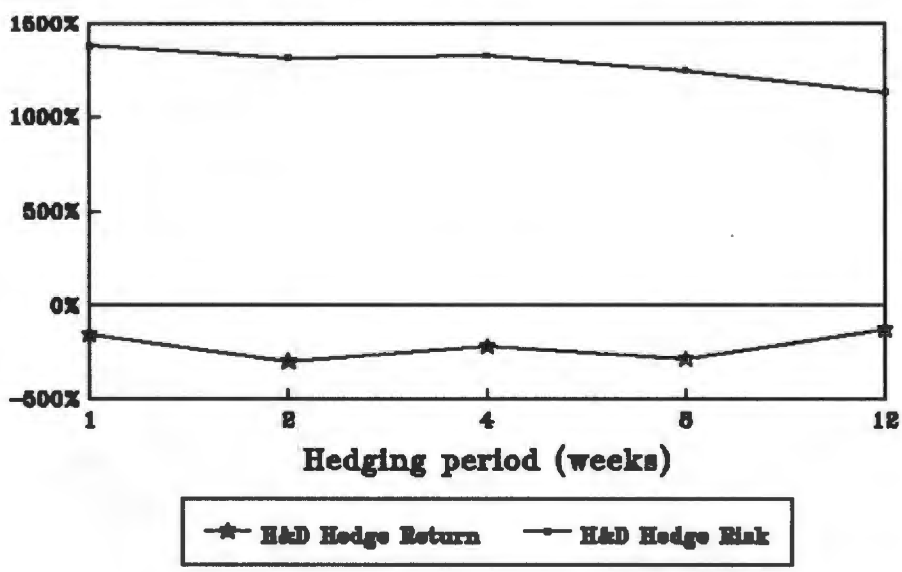


On a portfolio basis, the standard deviation of returns averaged 12.81 times the standard deviation of the unhedged portfolio. Even the Industrial share portfolio, which recorded the lowest standard deviation of the H&D portfolio hedges, was far greater (3.85 times) than the standard deviation of the unhedged portfolio. On average, the share portfolios have performed better in terms of risk than the index portfolios: the standard deviation of share portfolio returns is less than half that of the index portfolios.

The large risk associated with the H&D hedge, in terms of portfolio theory, should be compensated for by greater returns. Contrary to this premise, the average returns are negative except for the Industrial share portfolio. Moreover, these negative returns average 2.16 times greater in magnitude than the unhedged portfolio return; and reach a maximum of 6.25 times greater for the All Share index portfolio.

No significant trends can be seen when looking at the H&D hedge classified by hedge lengths in Figure 7.14, except for a marginal decrease in risk as the hedging period increases. Despite the decrease in risk, the variability of returns of the 12 week hedge was still very high at 11.31 times the standard deviation of the unhedged spot portfolio.

Figure 7.14. H&D Hedge
Period ave risk and return given as % of unhedged portfolio ave risk and return

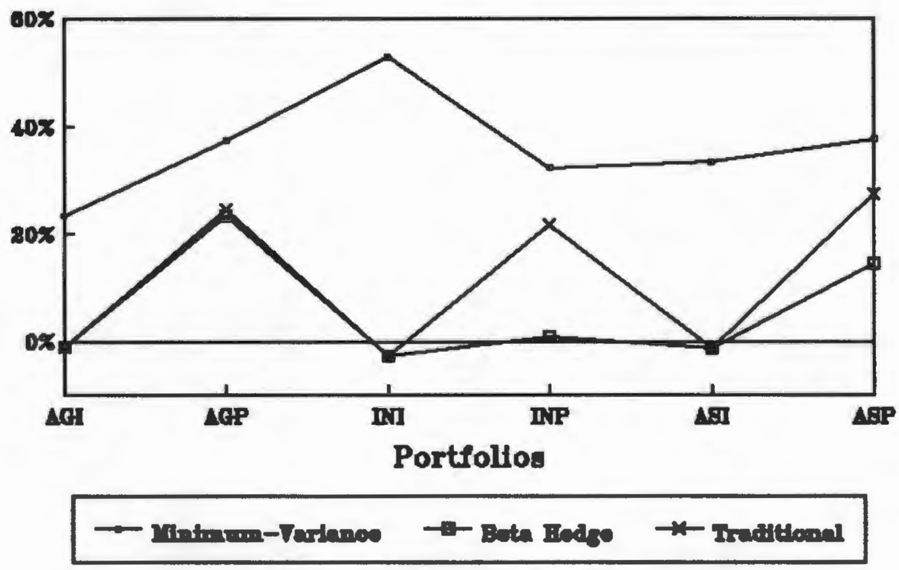


7.5. MINIMUM-VARIANCE, BETA AND TRADITIONAL HEDGES

7.5.1. Effect on Return

A decrease in hedging return would be expected in a rising market where short positions are taken. In this regard, over the two year observation period, the All Gold, Industrial and All Share indexes have increased by 26.86%, 97.12% and 70.19% respectively. It is not surprising, therefore, that the average hedging returns given in Figure 7.15. are substantially below that of the unhedged portfolio.

Figure 7.15. Hedged Portfolio Ave Return
Expressed as a percentage of unhedged portfolio average return



The return of the minimum-variance strategy was higher for all portfolios than that of the beta and traditional strategies, in many cases by a substantial margin. The average minimum-variance return equalled 36.26% of the unhedged portfolio compared to only 6.13% and 11.96% for the beta and traditional strategies respectively.

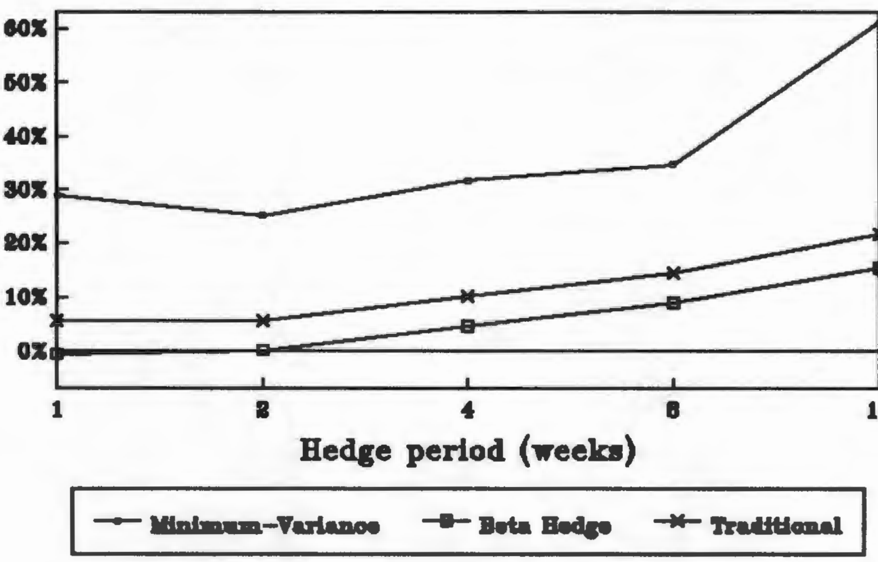
The beta hedge performed worse than both the minimum-variance and traditional hedge. Small negative returns for the Industrial and All Gold index portfolios are observed for the beta (and traditional) hedges indicating that the hedge ratios resulted in a certain amount of overhedging.

The share portfolios on average performed better than the index portfolios for all cases, except for the minimum-variance Industrial hedge. In all, the average share portfolio return was over twice as high as the index portfolio return, with 25.22% and 11.69% of the unhedged portfolio return respectively. Therefore, the slight cross-hedges (i.e. the share portfolios) have performed better in terms of return than the straight hedges (i.e. the index portfolios). A number of studies have also identified situations where cross hedges have performed better in terms of return than straight hedges, including Anderson & Danthine (1981); Howard & D'Antonio (1984) and Figlewski (1984a). However, an extensive analysis of hedging using many different portfolios will need to be undertaken before any definite conclusion can be drawn in this regard.

Comparing futures markets, hedges using All Share futures (return of 19.37% of the unhedged return) proved to be marginally better in terms of return than the All Gold and Industrial hedges (17.89% and 17.09% respectively). However, the differences are too insignificant for any unequivocal conclusion to be made.

The minimum-variance, beta and traditional hedges all display an upward trend in return corresponding with the increase in the hedging period as illustrated in Figure 7.16. However, as the beta and traditional hedge length shorten the return generated was very poor dropping below 6% of the unhedged return. This drop was not as steep for the minimum-variance hedge: less than 10% separate the returns for hedging periods up to and including 8 weeks in length.

Figure 7.16. Hedge Period Port Return
Expressed as a percentage of unhedged portfolio average return

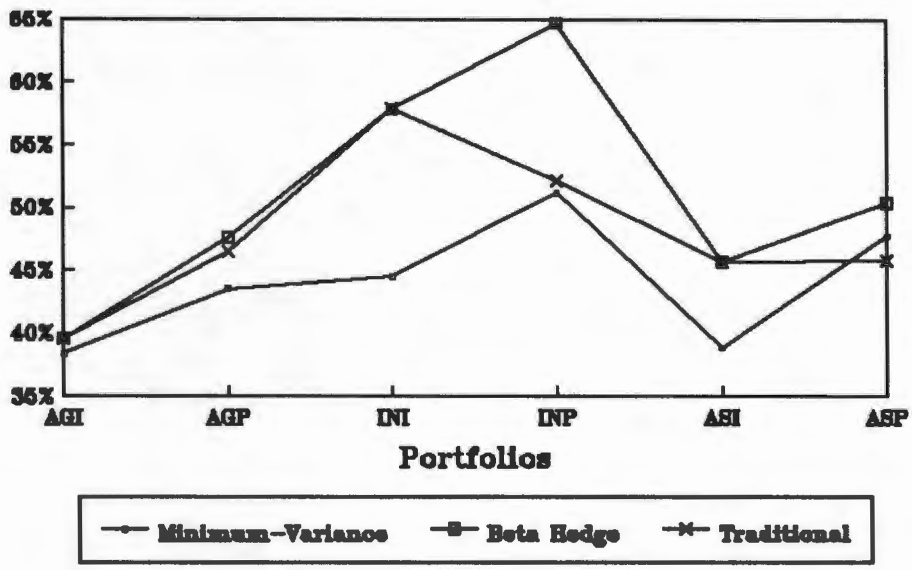


7.5.2. Effect on Risk

The minimum-variance hedge has performed best overall in terms of limiting the variability of returns (refer Figure 7.17.). Average standard deviations are 44.01%, 50.96% and 47.89% of the unhedged portfolio standard deviation for the minimum-variance, beta and traditional hedges respectively. The minimum-variance hedge has also the lowest variability of returns for all portfolio types, except for the All Share share portfolio where the traditional hedge standard deviation was slightly lower than the minimum-variance strategy.

The beta hedge strategy ended worst off of the three strategies particularly with respect to the Industrial share portfolio where 64.72% of the original spot portfolio remained after hedging. The poor performance can be attributed to overhedging which has increased the risk in the hedged portfolio.

Figure 7.17. Hedged Ave Standard Devn
Expressed as a percentage of average unhedged portfolio standard deviation



The index portfolios, as expected, proved to be substantially better than the share portfolios in terms of risk minimisation. The difference between the index portfolio risk and the share portfolio risk represents additional basis risk introduced into the share portfolios. The average amount of additional basis risk involved is given in Table 7.9.

The additional basis risk introduced depends largely on the share portfolios selected. Since the portfolios in this study selected index constituents (and in their correct relative proportions) contributing a large percentage of the total index value, it is probable that the results above indicate the low limit rather than the average. For example, based on an extensive sample of share portfolios, Graham & Jennings concluded that the risk of share portfolios were twice that of the risk of index portfolios.

Table 7.9.

Basis risk in Share Portfolios due to Cross hedging

Share Portfolio	**Additional Risk**[2]	**% Increase**[3]
All Gold	6.65%	16.97%
Industrial	2.61%	4.90%
All Share	4.54%	10.47%

The results for the Industrial share portfolio in Table 7.9. are biased by the large drop in risk in respect of the traditional hedge. If the traditional hedge is omitted then the additional basis risk would be 6.79% of the unhedged portfolio standard deviation, representing an increase of 13.27%.

In the light of the above, it is estimated that share portfolio hedges are subject to at least an additional 10% of the hedged portfolio risk because the portfolio represents a cross hedge. This risk is over and above the basis risk arising from the uncorrelated fluctuations between the futures price and the index. The amount of additional basis risk will depend on the degree to which the portfolio matches the index through diversification.

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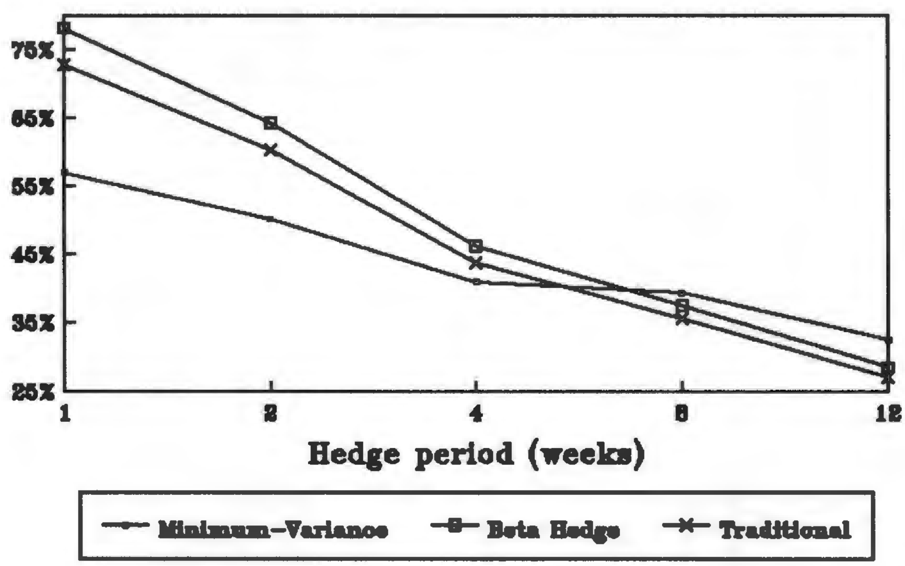
2. Based on the average of the minimum-variance, beta and traditional hedge strategies.
3. Increase based on the average standard deviation of the hedged index portfolio of the three risk-minimising strategies.

Looking at the differences in standard deviation between hedging periods per Figure 7.18, there was a substantial decrease in risk as the hedging period increases. The average standard deviation for the beta, traditional and minimum-variance hedges drops steadily from a very high level of 68.30% of the unhedged spot portfolio for one week hedges to 29.38% for 12 week hedges. This result is expected as the relative affect of basis risk should diminish as the hedge length increases (refer Figlewski, 1984a).

Figure 7.18. Hedged Period Ave Std Devn
Expressed as a percentage of unhedged portfolio standard deviation



The minimum-variance hedge ratio performs far better than the traditional or beta hedge strategies for four week hedges and less. The shorter the hedge the better the minimum-variance hedge ratio does compared to the other two hedge approaches. In this regard, the difference between the minimum-variance and beta one-week hedges was 21.12 percentage points of the unhedged portfolio, representing

37.02% of the minimum-variance one-week hedge standard deviation. However, the trend reverses itself for the 8 and 12 week hedges where both the beta and traditional hedges perform better than the minimum-variance hedge although the difference in this case is not as significant. The decrease in the relative advantage of the minimum-variance hedge over the other hedges with the increase in hedge length is consistent with the findings of Graham & Jennings (1987).

7.5.3. Effect of Pricing Inefficiencies on Hedging Risk

The level of risk remaining after hedging was still fairly high at an average of 44.01% of the unhedged portfolio for the minimum-variance hedge. The lowest recorded risk for any hedge in the sample was 21.67% in the 12-week beta ASI portfolio hedge. While it is likely that longer hedges may have even lower hedge portfolio risk, there will always be an element of residual risk in any hedge.

This residual risk is present due to two factors. Firstly, risk is present owing to errors in estimating the risk-minimising hedge ratio. These errors arise because the regression coefficients estimated from the historical data may not correctly reflect the true hedge value. Problems with the estimation of hedge ratios are highlighted by Ederington (1979) and Junkus & Lee (1985). However, it is unlikely that these estimation errors will impact the hedge risk to the extent shown by the empirical results. The second, and more dominant, factor is basis risk resulting because the fluctuations in the futures price which are not perfectly correlated with the fluctuations in the spot price.

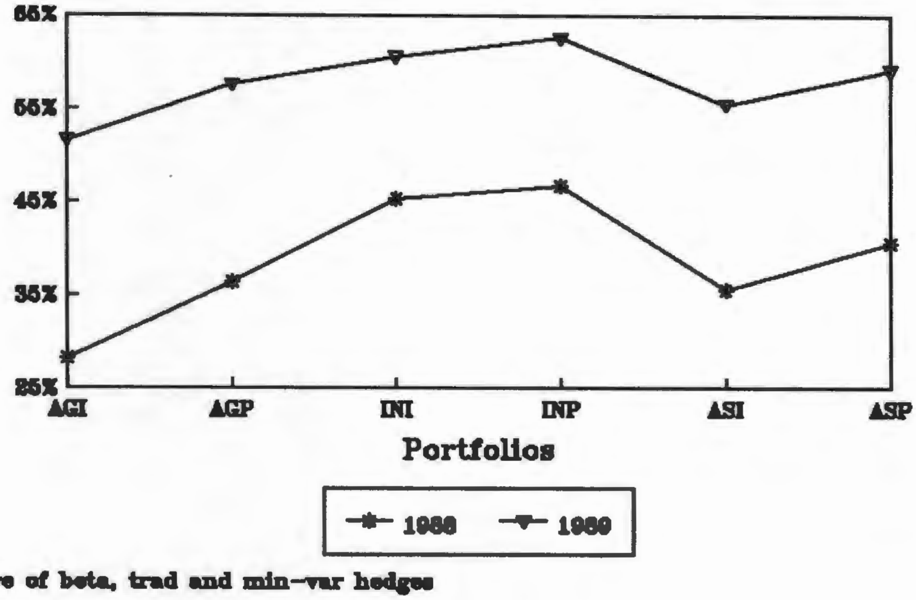
The previous chapter identified periods where large differences occurred between the actual and theoretical futures price. The reason for the large differences are attributed to the presence of wide arbitrage bounds around the theoretical futures price. The actual futures price could move freely within these bounds, thereby introducing basis risk into hedged portfolios. It follows that the greater the level of pricing differences, the greater the level of basis risk is likely to be present.

The high level of pricing differences was most prevalent in the latter part of the observation period. This is especially the case in respect of the All Share and Industrial index futures contracts, although the same trend was also noticed in the All Gold index futures contract. In order to investigate this effect, standard deviations of hedges in the year ending 15 December 1989 are compared to hedge standard deviations for the year ending 15 December 1988. The standard deviations (expressed as a percentage of unhedged portfolio standard deviations) have been averaged per hedge strategy type and presented in Figure 7.19.

It is apparent from the graph that the proportionate amount of risk remaining in the portfolio after hedging was greater for 1989 than 1988. The increase in hedge risk from 1988 and 1989 was 19.1%, i.e. from 38.8% (1988) to 57.9% (1989). This represents a 49.23% increase in risk over the 1988 level.

The increase from 1988 to 1989 in the standard deviation of the All Gold, All Share and Industrial portfolios amount to an average of 22.3%, 19.3% and 15.6% of the unhedged portfolio standard deviation respectively. The large increase in risk in the 1989 period over the 1988 period must be ascribed to the greater incidence of pricing differences in 1989. This can be seen by comparing the extent of pricing differences between 1988 and 1989 as given in the near contract arbitrage pricing graphs in chapter 6.

Figure 7.19. Ave 1988 and 1989 Std Devn
Expressed as a % of ave 1988 and 1989 unhedged portfolio standard deviation



7.6. HEDGING EFFECTIVENESS

Howard & D'Antonio (1984, 1987) established two measures of hedging effectiveness (HBS and HE) based on the risk to excess return relatives between the hedged portfolio relative to the spot portfolio. These have already been discussed in chapter three but are reproduced below for ease of reference (all terms have been previously defined).

$$HE = \theta_p / \theta_s \quad (7\text{-}2)$$
$$HBS = \theta_p - \theta_s \quad (7\text{-}3)$$

where

$$\theta_p = (\mu_p - i) / \sigma_p \quad (7\text{-}4)$$
$$\theta_s = (\mu_s - i) / \sigma_s \quad (7\text{-}5)$$

Although these measures of hedging effectiveness were developed by H&D (1984) on an ex ante basis, they have been calculated for the hedged portfolios on an ex post basis in order to evaluate the particular hedging strategies. The values of HE and HBS were calculated for each individual hedge. The average results are provided graphically in Figures 7.22. and 7.23. (Appendixes C.1. to C.6. contain the actual data).

Unfortunately, the measures do not correctly measure the relative hedging performance of the hedge strategies if they are compared to the risk and return of the different strategies. The distortion is very noticeable for the H&D hedge, which has been ranked first in four out of six portfolios according to the HBS measure (three out of six per the HE measure). However, the results show that the H&D strategy recorded the lowest returns for five portfolios and the highest standard deviation for all six portfolios.

Figure 7.20.
Hedging Benefit per Unit of Risk (HBS)

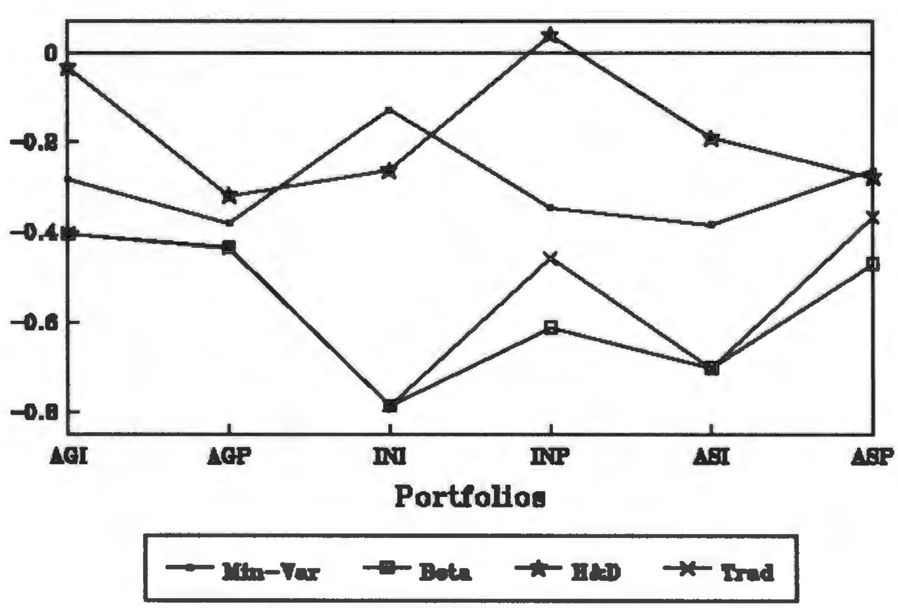
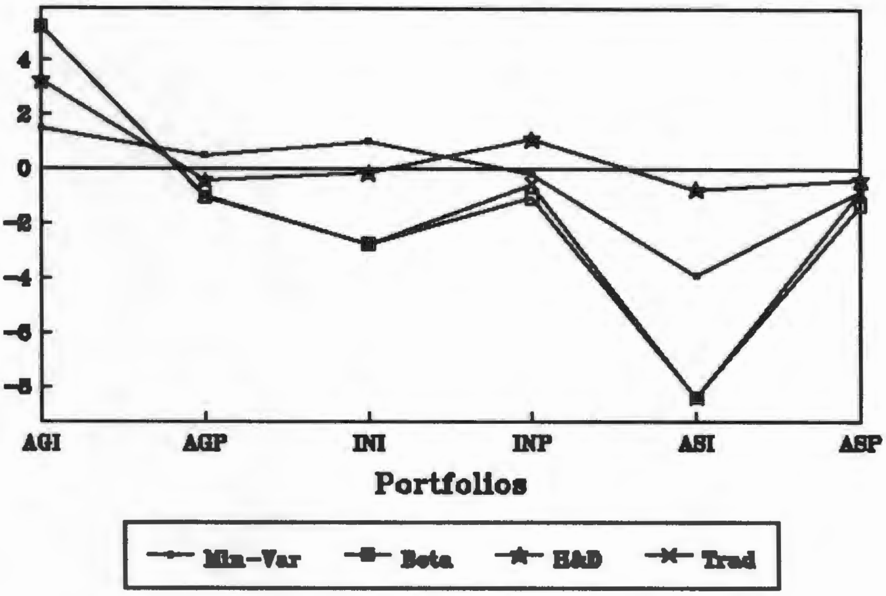


Figure 7.21.
Measure of Hedging Effectiveness (HE)



The reason for this anomaly arises from the combination of the very high standard deviation of returns of the H&D hedge and the negative excess returns for most of the hedged portfolios. The excess return of the hedged portfolios are positive for only 9 out of the 120 portfolio hedges evaluated. On the other hand, the excess return on the unhedged spot portfolios are positive for 27 out of the 30 different portfolios (i.e. 6 spot portfolios, 5 hedge lengths per spot portfolio).

As Θ_s remains constant when comparing hedge strategy types, the relative HBS and HE measures depend on outcome of Θ_h. Owing to the fact that the excess returns of the hedged portfolios are predominantly negative, the greater the standard deviation of the hedged returns, the greater (less negative) the value of Θ_h will be. It was for this reason that the H&D strategy has been erroneously ranked first according to the HBS and HE measures.

The measures of hedging effectiveness are designed to evaluate the effectiveness of the hedge with reference to the unhedged portfolio. All hedges (except the H&D Industrial share portfolio hedge) have recorded HBS values which are less than zero (HE less than one), and thus are all less effective than the unhedged portfolio. This conclusion is reasonable considering the relatively poor returns of the hedge portfolios (except for the H&D Industrial share portfolio hedge). Thus, in this narrow sense, this objective of the hedging measures have been met. Unfortunately, the measures are inappropriate when comparing hedging strategies.

Further anomalous situations are identified in the HE measure where the AGI values are exceptionally high and the ASI are exceptionally low. This has been brought about because the excess return to risk ratio of the unhedged portfolios are very close to zero resulting in large HE absolute values. The unhedged portfolio one-week excess return to risk relative was only 0.0090 and 0.0404 for AGI and ASI portfolios respectively. This problem was also recognised by Chang & Shanker (1987).

7.6.1. New Measures of Hedging Effectiveness

To eliminate the problem with the effectiveness measures identified above, new measures for HE and HBS are suggested below which will enable more accurate comparisons between hedge strategies to be made.

$$HE^{*} = (\mu_p/\sigma_p) \;/\; (\mu_s/\sigma_s) \qquad (7\text{-}6)$$

$$HBS^{*} = (\mu_p/\sigma_p) - (\mu_s/\sigma_s) \qquad (7\text{-}7)$$

Both the new HE^* and HBS^* measures have the same form as the H&D measures except they are based on total portfolio returns and not excess portfolio returns. The new expressions are now based directly on the coefficient of variation used to evaluate portfolios (Uliana et al, 1987).

While the new formulas do not measure the slope of the lines as envisaged by Howard & D'Antonio (1984), they adequately measure the relative risk-return performances between a particular hedge and the unhedged portfolio, and between hedge strategies themselves. The cutoff points in respect of the unhedged portfolio of 0 and 1 for HBS^* and HE^* respectively are still applicable. In addition, the new measure of HE has the advantage of solving the problem arising from near-zero denominators in the HE measure outlined above.

The values of HE^* and HBS^* for the hedge portfolios have been calculated in the same manner as the old measures, and are presented graphically in Figures 7.22. and 7.23. The results are provided in full in Tables D.1. to D.6. in Appendix D.

Under both the HE^* and the HBS^* criteria the H&D hedge was ranked last in four of the six portfolios. In the Industrial index portfolio the H&D hedge was only marginally ahead of the beta and traditional hedge portfolios. The reformulated ranking is now in line with what would be expected given the risk and return results obtained for the H&D hedge.

Figure 7.22.
Hedging Benefit per Unit of Risk (HBS*)

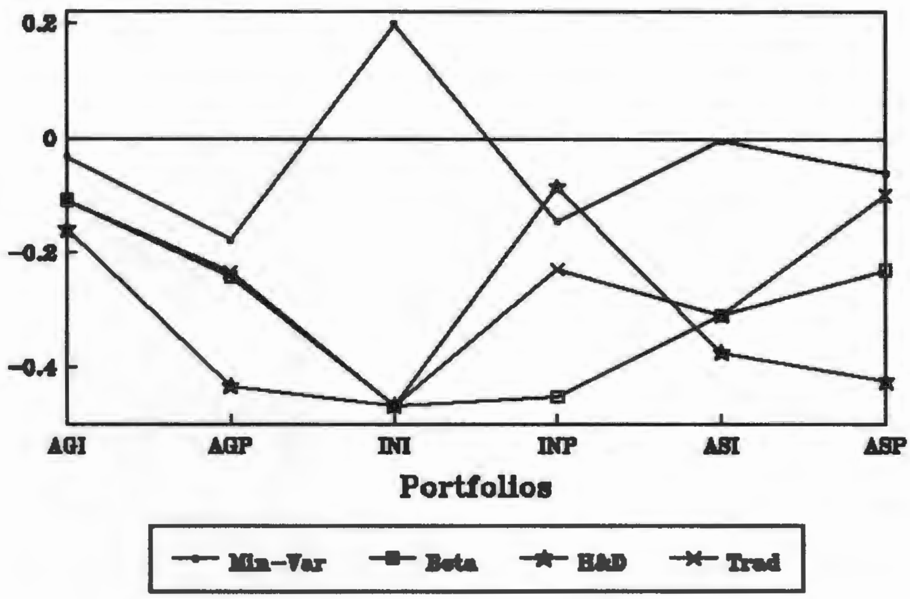
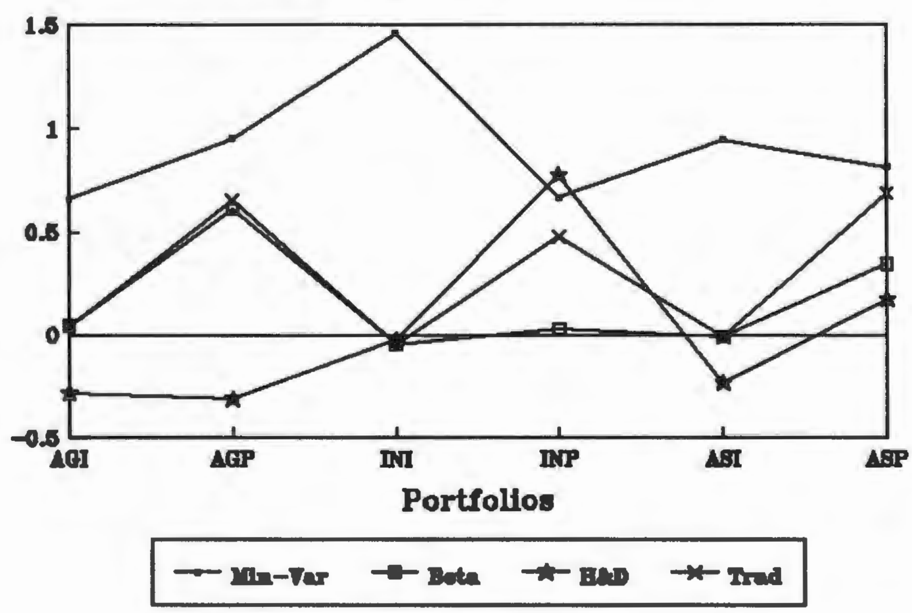


Figure 7.23.
Measure of Hedging Effectiveness (HE*)



The H&D hedge is ranked first for the Industrial share portfolio since the return for this portfolio was high at 2.63 times the unhedged portfolio return and the risk relatively low (compared to the other H&D portfolios) at 3.48 times the unhedged portfolio standard deviation.

As would be expected from the risk and return analysis, the minimum-variance hedge performed the best overall of all the hedging strategies. The minimum-variance strategy HE^* and HBS^* measures are significantly above those of all the other strategies with the exception of the Industrial share portfolio mentioned above.

The average HBS^* and HE^* measures for the minimum-variance strategy are calculated as -0.0359 and 0.9177 respectively, which is not far from the respective cutoff points of 0 and 1. Thus, the hedge performance of the minimum-variance hedge is only marginally less effective on average than the unhedged position.

The minimum-variance Industrial index portfolio hedge was the only strategy which was found to be more effective than the unhedged portfolio (under both HBS^* and HE^* statistics). The return of the Industrial index portfolio dropped 47.08% after hedging but the percentage drop in the standard deviation of returns of the unhedged portfolio was greater than this at 55.54%. On a straight line risk-return benefit, this represents a net improvement on the unhedged portfolio for the strategy as indicated by the effectiveness measure.

The beta and traditional hedge HE^* values averaged 0.163 and 0.302 respectively, with HBS^* values being -0.302 and -0.241 for the two strategies respectively. These values are substantially less than the averages recorded for the minimum-variance hedge. The traditional hedge recorded better HE^* and HBS^* scores than the beta hedge especially with respect to the Industrial and All Share share portfolios.

No definite trend can be identified between the effectiveness of the share portfolios compared to the index portfolios. The index portfolios performed better in terms of the HBS^* measure on average (average of the minimum-variance, beta, and traditional hedges), i.e. -0.179 for the index portfolio and -0.207 for the share portfolio. However, in terms of the HE^* measure, the share portfolio (average HE^* of 0.583) was more effective than the index portfolio (average HE^* of 0.341).

The share portfolios recorded higher average returns than the index portfolio at the expense of higher risk. The share portfolios have achieved average hedging returns of 25.22% of the unhedged portfolio compared to only 11.69% for the index portfolios. At the same time, the share portfolio was subjected to 4.6% more risk at 49.92% of the unhedged portfolio. On the basis of straight-line risk-return benefit, the share portfolios have performed marginally better over the observation period.

Comparing futures contract types (based on effectiveness measures of the three hedge strategies) the All Gold futures hedge was most effective on average, marginally ahead of the All Share futures hedge and the Industrial futures hedge. If the ranking is based solely on the minimum-variance hedge, however, the order would be completely reversed with the Industrial futures first and the All Gold futures last. Owing to this conflict, and the small differences in the effectiveness measures, no definite conclusion can be established as to which futures contract was most effective for hedging.

Figure 7.24.
Hedged Period HBS*

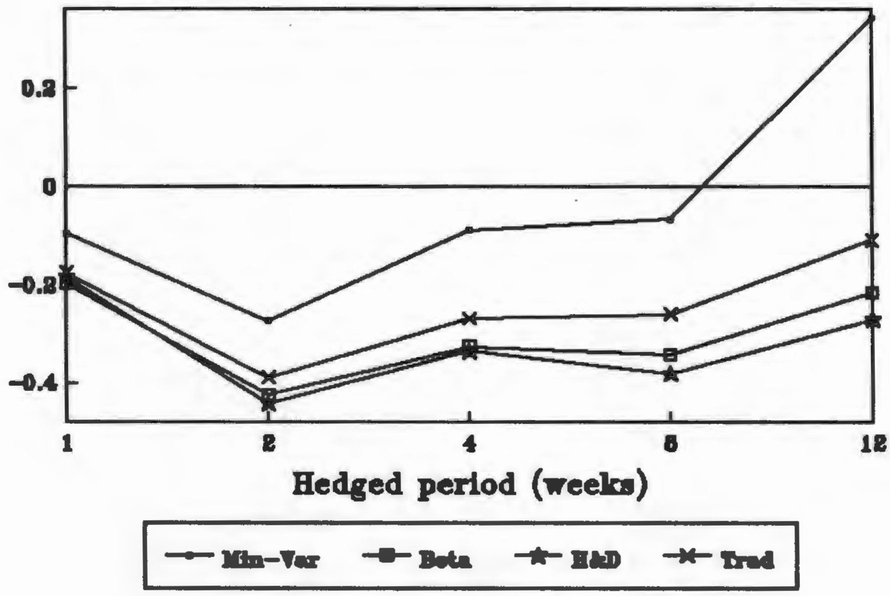
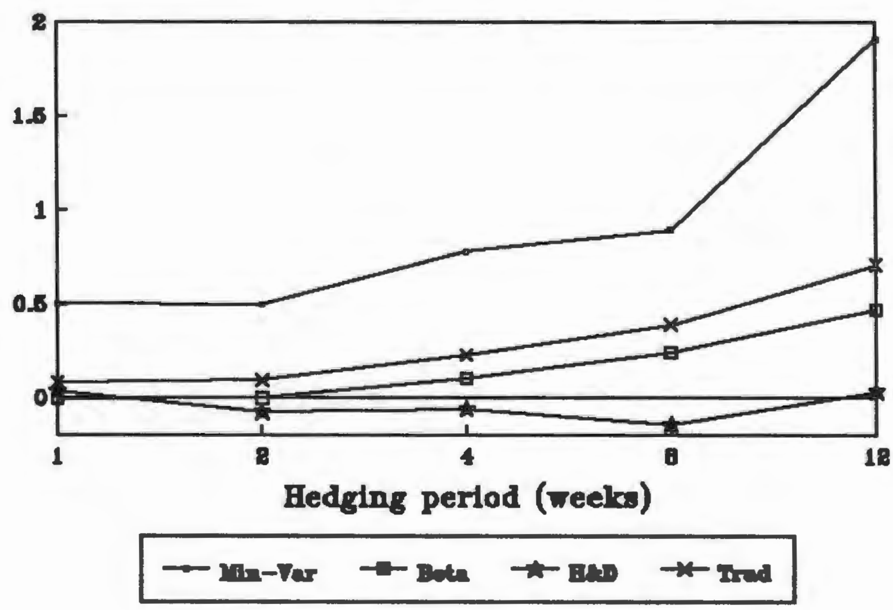


Figure 7.25.
Hedged Period HE*



The benefit of increasing return coupled with decreasing risk as the hedging period increases has been clearly reflected in Figures 7.24. and 7.25. The increasing hedging effectiveness was more prominent in the minimum-variance hedge than the other hedges. The 12-week hedge was greater than the respective HBS^* and HE^* cutoff points indicating that this hedge was more risk-return effective than the unhedged portfolio. The drop in return for 12-week hedges from the unhedged portfolio return was only 39.03% compared to the drop of 67.49% in the unhedged portfolio standard deviation.

7.6. CONCLUSION

The striking conclusion from the research is the lack of success of the Howard & D'Antonio strategy in a practical situation; the root cause of the failure is the problems encountered in calculating the hedge ratio. The hedge ratio is found to be very sensitive to the changes in the factors $(\alpha-r)$ and $(1-\alpha r)$, both dependent on the futures-spot risk to excess return relative (α) and the correlation between the futures and spot portfolios (r).

Under the impreciseness of an actual market situation, given the sensitivity of the hedge ratio, widely fluctuating hedge ratios are computed for the H&D strategy. Moreover, most of the hedge ratios calculated are short positions which, in many cases, did not correctly reflect the relative attractiveness of a long futures position indicated by high values of α. As a consequence of the widely fluctuating hedge ratio and large short futures positions, the hedged returns are very low while the variability of returns are very high.

The H&D measures of hedging effectiveness, namely HBS and HE, are found to be unsuitable as the rankings between various hedge strategies conflicted with the risk and return results obtained. The reason for this is attributed to the narrow objective of the measure and erroneous results provided in situations where the excess return of hedged portfolios are negative. Two new measures of hedging effectiveness, HBS^* and HE^*, have been designed which have solved these problems.

It terms of the new hedging effectiveness measures HBS^* and HE^*, the only portfolio which was more effective on average than the unhedged portfolio was the minimum-variance Industrial index portfolio. Overall, the minimum-variance strategy was only marginally less effective in terms of risk and return compared to the unhedged portfolio.

The minimum-variance hedge strategy performed significantly better in terms of both risk and return compared to the beta and traditional strategies. However, while the minimum-hedge ratio performed considerably better in terms of risk for hedge periods of four weeks or less, the minimum-variance strategy was marginally riskier than the other two strategies for hedge periods of longer duration.

The minimum-variance hedge ratio was substantially less than the beta and one-to-one ratios. The low hedge ratios are due to the high variability of the futures price relative to the spot price, and the lower correlation between the spot and futures prices compared to the correlation between the spot and index. The lower hedge ratio is also beneficial to the hedger in view of reduced transaction costs.

The beta hedge strategy performed particularly badly both in terms of risk and return, being less effective than even the traditional hedge strategy. The beta hedge did perform satisfactorily in terms of risk for hedges of 8 to 12 weeks but this was unfortunately marred by low returns. The poor performance was aggravated by overhedging which resulted in very poor returns and increased variability of returns.

The beta hedge strategy is based on the assumption that the futures price moves in harmony with the index (both in size and in direction), in which case the beta hedge ratio is a good approximate for the minimum-variance ratio. However, the assumption does not hold in the South African market where the fluctuations in the futures price was seen to be much higher than the index. To illustrate the difference in variability, the one-week standard deviation of returns of the index and futures price are given in Table 7.10. In terms of the above analysis, minimum-variance hedge is much more appropriate than the beta hedge strategy in the South African situation.

Table 7.10.
Standard Deviation of One-Week Returns

	Index	**Futures**
All Gold	4.3223%	5.1190%
Industrial	2.2299%	3.2464%
All Share	2.4611%	3.5659%

Hedging effectiveness increased greatly as the hedge length increased particularly with respect to the 12-week hedge. The minimum-variance 12-week hedge strategy was actually found to be more effective than the unhedged portfolio. This trend is to be expected since the proportion of basis risk in a portfolio must decrease as the hedge length increases.

No significant differences in the hedging performance between the different index futures contracts are identified. However, there are differences between the index portfolios and the share portfolios. The share portfolios are subject to greater basis risk than the index portfolio owing to the slight cross hedge. The amount of additional basis risk was estimated to be at least 10% on average of the unhedged portfolio, but this figure largely depends on how well the share portfolio is correlated with the index. The share portfolios performed slightly better, on the other hand, in terms of return and overall effectiveness; but without extensive analysis of other portfolios no definite conclusion can be drawn in this regard.

The effectiveness of the South African futures market depends to a large extent on the optimal pricing structure of the market. However, large pricing differences between the theoretical and actual futures price, identified in the previous chapter, impacted heavily on the risk profile of the hedge. This could be identified by comparing the residual hedge portfolio risk for 1988 and 1989.

Additional risk due to pricing differences are introduced into hedges from two main sources. Firstly, the large pricing differences tend to increase the variability of the futures price which in turn impacts the hedge ratio, as could be seen by the decrease in the minimum-variance hedge ratio in 1989. Consequently, the estimation of the hedge ratio becomes more susceptible to error. Secondly, and more importantly, the greater the variability of the futures price (unexplained by the variability in the index) the greater the basis risk will be. Unless the arbitrage bounds narrow through the reduction in transaction costs and/or the relaxation of restrictions over short sales, hedgers will have to bear this basis risk.

CHAPTER EIGHT

CONCLUSION

8.1. INTRODUCTION

Futures are unique financial instruments designed and introduced to enable the fixing of prices (price discovery) and the curtailing of price risk (hedging). However, for maximum benefit to be achieved by the hedger it is important that all hedges entered into are efficient and effective in terms of meeting the particular hedging objective. Effectiveness entails the degree to which the hedging objective is achieved and efficiency embodies pricing optimality so that the market is efficient in achieving the particular hedging objective. Thus, this study looks at both aspects with reference to the South African futures market.

8.2. PRICING EFFICIENCY

The South African SIF market is found to be efficiently priced in terms of arbitrage pricing theory, i.e. very few arbitrage opportunities are seen to be available. Although the regression analysis did identify a number of contracts where the actual futures price is significantly different from the theoretical futures price, the futures prices are still found to be within the fair range. This is mainly a result of restrictions imposed by the JSE effectively preventing arbitrageurs from obtaining the use of proceeds on short selling. As a consequence of this, the upper arbitrage bound is increased considerably by the loss of income on the short proceeds, thereby eliminating any chance of reaping arbitrage profits.

The error of using the dividend yield as a surrogate for actual dividends was estimated to not exceed 17, 19 and 10 index points for the All Gold, Industrial and All Share index futures respectively. This difference may be significant for certain futures applications such as arbitrage and speculation strategies. However, the fair range within which the futures price may move freely is so large that the greater accuracy obtained in calculating the theoretical futures price using actual dividends will not be greatly beneficial to most futures participants. This is particularly applicable to hedgers who may only wish to assess the efficiency of the futures market before entering into hedging positions.

8.3. HEDGING EFFECTIVENESS

The Howard & D'Antonio hedge strategy does not stand up to practical application in a number of respects. Firstly, the impreciseness of an imperfect market combined with the sensitivity of the hedge ratio variables resulted in very large, fluctuating hedge ratios. Many H&D hedge ratios did not behave in the manner anticipated in terms of the theory; this conclusion is consistent with the findings of Smith (1989). Owing to the abnormal hedge ratios, the strategy performed exceptionally poorly in terms of both risk and return. Secondly, the H&D measure of hedging effectiveness provided erroneous results in the practical situation. This study makes slight adjustments to the measures which satisfactorily eliminates the problems encountered.

The minimum-variance hedge strategy proved to be the most optimal in terms of both risk and return. The minimum-variance hedge ratio was also the lowest of the strategies, which of course will entail benefits to hedgers in terms of reduced transaction costs. In contrast, the traditional and beta hedge ratios performed particularly badly with the beta hedge ratio resulting in overhedging in many instances. The portfolio betas are not a good estimate for the minimum-variance hedge ratio in South Africa because the futures price fluctuates more widely than the index. Consequently, the minimum-variance hedge is recommended above the beta hedge strategy.

As expected from the theory, the index portfolios were more effective in terms of risk reduction than the spot portfolios. This additional basis risk owing to the cross hedging was estimated to amount to an additional 10% of the hedged index portfolio risk. However, this figure is likely to be far higher for portfolios who are less representative of the index. The findings indicate that the level of additional basis risk due to cross-hedging can be limited by ensuring that the portfolios match the index as far as possible.

Despite the better performance of the index portfolios in terms of risk, in absolute terms the basis risk after hedging in the index portfolios is still significant as indicated by the hedging results and the regression procedure. The level of basis risk can be minimised by increasing the hedge length; which also has the benefit of improving the hedge return.

Except for the minimum-variance Industrial index portfolio, all hedge strategies are found to be less effective than the unhedged portfolio in terms of risk and return. This result is ascribed to the large amount of basis risk introduced into portfolios which is out of proportion to the return obtained. The large arbitrage bounds (and large pricing differences as a consequence) are largely responsible for the significant basis risk. Unless the arbitrage bounds are reduced in some way (e.g. by reducing transaction costs or relaxing restrictions on bear sales), hedgers will have to incur this additional basis risk without being compensated for by additional return.

8.4. SCOPE FOR FURTHER RESEARCH

There is a large amount of scope to research the other futures markets in South Africa in terms of the same objectives as this study. With reference to share index futures, additional work can be performed into futures pricing in terms of the applicability of the Efficient Market Hypothesis and day-to-day pricing effects of futures markets. As far as hedging is concerned, there is scope for additional work to be performed on cross hedging using different index future contracts, and dynamic hedging techniques. It would be very useful if additional work is performed on the applicability and reliability of the new measures of hedging effectiveness proposed in this study.

APPENDIX A

SAFEX SHARE INDEX FUTURES CONTRACT TERMS[1]

Underlying asset	All Share, All Gold and Industrial Indexes
Contract size	R10 times index value
Minimum price movement	1 Point, i.e. R10 per contract
Issued in the name of	Investor
Quotation	In index notation on Reuters screen
Expiration date	At close on 15th or next business day of March, June, September and December.
Clearing fees	R3 per contract and R15 per transaction (subject to change)
Initial Margin	R2000 per contract (subject to change)
Mark-to-Market	At mid-market price at close of each business day
Market price	On an index value basis
Settlement form	Cash
Settlement period	Until close on subsequent business day
Settlement price	Closing index value on Expiration date.

1. Source: Falkena, Kok & Raine (1989) (as amended)

APPENDIX B

ILLUSTRATION OF TRANSACTION COSTS: SHARES VERSUS FUTURES[1]

ROUND TRIP TRANSACTION OF 1 AND 10 FUTURES CONTRACTS

	ALSI		ALGI		INDI	
Futures						
Index Level	2750	2750	1850	1850	2500	2500
No. contracts	1	10	1	10	1	10
Value	27500	275000	18500	185000	25000	250000
Book Fees In	18	45	18	45	18	45
Sub-Total	27518	275045	18518	185045	25018	250045
Book Fees Out	18	45	18	45	18	45
Total	27536	275090	18536	185090	25036	250090
Tot T-costs	36	90	36	90	36	90
% T-costs	0.131%	0.033%	0.195%	0.049%	0.144%	0.036%
Shares						
Price	27.5	27.5	18.5	18.5	25	25
No. Shares	1000	10000	1000	10000	1000	10000
Value	27500	275000	18500	185000	25000	250000
Basic	10	25	10	25	10	25
MST	413	4125	278	2775	375	3750
Brokerage	234	1890	166	1330	215	1753
Sub-Total	28156	281040	18954	189130	25600	255528
Basic	10	25	10	25	10	25
Brokerage	239	1923	170	1357	220	1783
Total	28405	282988	19133	190512	25830	257335
Tot T-costs	905	7988	633	5512	830	7335
% T-costs	3.291%	2.905%	3.424%	2.979%	3.320%	2.934%
TIMES CHEAPER	25	88	18	61	23	82

1. Source: Rand Merchant Bank (adapted)

APPENDIX C: RESULTS OF EMPIRICAL HEDGING

TABLE C.1.

Spot Portfolio — All Gold Index
Futures Portfolio — All Gold Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Futures
Hdg Ratio			1.0000	0.8067	3.6879	
R-Squared			1.0000	0.7643		
Average	1	0.3077	-0.0270	0.0551	1.1587	0.3356
Period	2	0.6111	-0.0520	0.1098	-4.8920	6.6340
Return	4	1.3688	-0.0150	0.3298	-8.6390	1.3843
(%)	8	2.0753	0.0577	0.5674	-17.9450	2.0476
	12	1.1243	0.1219	0.3387	0.9046	1.0024
Return as	1		-0.0877	0.1791	3.7657	
Ratio of	2		-0.0851	0.1797	-8.0052	
Unhedged	4		-0.0110	0.2409	-6.3114	
Return	8		0.0278	0.2734	-8.6469	
	12		0.1084	0.3013	0.8046	
	1	4.5358	2.7204	2.3075	65.2361	5.1190
Standard	2	6.4364	3.4154	2.8679	118.5372	6.9733
Deviation	4	9.6089	3.2856	3.1756	222.7401	9.8874
(σ)	8	13.4572	3.6327	4.4860	109.0494	12.4256
	12	14.4901	3.4478	4.3612	124.5334	12.3325
σ as			0.5998	0.5087	14.3825	
Ratio of	2		0.5306	0.4456	18.4167	
Unhedged	4		0.3419	0.3305	23.1806	
Std Dev	8		0.2699	0.3334	8.1034	
	12		0.2379	0.3010	8.5944	
Excess	1	0.0073	-0.1108	-0.0951	0.0136	
Return	2	0.0096	-0.1760	-0.1532	-0.0459	
per unit	4	0.0282	-0.3388	-0.2419	-0.0437	
of Risk	8	-0.0090	-0.5887	-0.3631	-0.1847	
(Θ)	12	-0.1498	-0.9201	-0.6777	-0.0192	
	1		-0.1182	-0.1024	0.0062	
H&D	2		-0.1856	-0.1628	-0.0555	
HBS	4		-0.3670	-0.2701	-0.0719	
	8		-0.5797	-0.3541	-0.1757	
	12		-0.7704	-0.5279	0.1306	
	1		-15.1527	-12.9999	1.8529	
H&D	2		-18.2525	-15.8855	-4.7609	
HE	4		-12.0249	-8.5874	-1.5517	
	8		65.5378	40.4221	20.5626	
	12		6.1441	4.5254	0.1281	

TABLE C.2.
Spot Portfolio — All Gold Portfolio
Futures Portfolio — All Gold Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Trad	Futures
Hdg Ratio			1.0250	0.8227	2.7313	1.0000	
R-Squared			0.9604	0.7213			
Average	1	0.3895	0.0399	0.1247	-0.0264	0.0539	0.3356
Period	2	7.6040	0.0774	0.2426	-2.5020	0.0969	6.6340
Return	4	1.6910	0.2879	0.6426	-4.5380	0.3067	1.3843
(%)	8	2.9134	0.8698	1.3864	-18.7860	0.8958	2.0476
	12	2.4972	1.4853	1.6617	-13.2810	1.4948	1.0024
Return as	1		0.1024	0.3202	-0.0677	0.1384	
Ratio of	2		0.0102	0.0319	-0.3290	0.0127	
Unhedged	4		0.1703	0.3800	-2.6836	0.1814	
Return	8		0.2986	0.4759	-6.4481	0.3075	
	12		0.5948	0.6654	-5.3184	0.5986	
	1	4.7640	3.0026	2.5797	22.5691	2.9421	5.1190
Standard	2	6.6923	3.9787	3.3698	26.3704	3.8797	6.9733
Deviation	4	9.6723	4.2511	3.8415	63.7133	4.1484	9.8874
(σ)	8	13.4688	4.8243	5.2308	175.8644	4.7407	12.4256
	12	13.5995	4.8607	4.6830	129.6121	4.6708	12.3325
σ as	1		0.6303	0.5415	4.7374	0.6176	
Ratio of	2		0.5945	0.5035	3.9404	0.5797	
Unhedged	4		0.4395	0.3972	6.5872	0.4289	
Std Dev	8		0.3582	0.3884	13.0572	0.3520	
	12		0.3574	0.3444	9.5307	0.3435	
Excess	1	0.0241	-0.0781	-0.0581	-0.0133	-0.0750	
Return	2	1.0542	-0.1185	-0.0909	-0.1157	-0.1165	
per unit	4	0.0613	-0.1906	-0.1186	-0.0885	-0.1908	
of Risk	8	0.0533	-0.2749	-0.1548	-0.1193	-0.2743	
(θ)	12	-0.0586	-0.3722	-0.3486	-0.1279	-0.3853	
	1		-0.1023	-0.0822	-0.0375	-0.0991	
H&D	2		-1.1727	-1.1451	-1.1699	-1.1707	
HBS	4		-0.2519	-0.1799	-0.1498	-0.2521	
	8		-0.3282	-0.2081	-0.1726	-0.3276	
	12		-0.3136	-0.2900	-0.0693	-0.3266	
	1		-3.2376	-2.4064	-0.5524	-3.1070	
H&D	2		-0.1124	-0.0863	-0.1098	-0.1106	
HE	4		-3.1090	-1.9343	-1.4431	-3.1121	
	8		-5.1630	-2.9072	-2.2405	-5.1511	
	12		6.3498	5.9481	2.1820	6.5733	

TABLE C.3.

Spot Portfolio — Industrial Index
Futures Portfolio — Industrial Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Futures
Hdg Ratio			1.0000	0.6993	1.5506	
R-Squared			1.0000	0.6917		
Average Period Return (%)	1	0.6529	-0.0450	0.1725	2.0722	0.6985
	2	1.2718	-0.0330	0.3578	-0.5630	1.3058
	4	2.4531	-0.0520	0.6640	-1.0830	2.5060
	8	4.3264	-0.0440	1.2036	-4.6390	4.3713
	12	5.1553	-0.0650	8.0000	-12.8910	5.2210
Return as Ratio of Unhedged Return	1		-0.0689	0.2642	3.1738	
	2		-0.0259	0.2813	-0.4427	
	4		-0.0212	0.2707	-0.4415	
	8		-0.0102	0.2782	-1.0723	
	12		-0.0126	1.5518	-2.5005	
Standard Deviation (σ)	1	2.1686	2.0647	1.2574	32.2504	3.6203
	2	3.2222	2.4183	1.6451	35.0567	4.5508
	4	4.6324	2.5467	1.9344	34.0495	6.4214
	8	7.3091	2.9328	2.9422	104.3187	8.8832
	12	11.9651	2.8539	3.7411	205.1743	10.4908
σ as Ratio of Unhedged Std Dev	1		0.9521	0.5798	14.8715	
	2		0.7505	0.5106	10.8797	
	4		0.5498	0.4176	7.3503	
	8		0.4013	0.4025	14.2724	
	12		0.2385	0.3127	17.1477	
Excess Return per unit of Risk (θ)	1	0.1745	-0.1548	-0.0811	0.0557	
	2	0.2243	-0.2407	-0.1163	-0.0317	
	4	0.2925	-0.4516	-0.2244	-0.0641	
	8	0.2914	-0.7638	-0.3374	-0.0655	
	12	0.1555	-1.1771	1.2578	-0.0789	
H&D HBS	1		-0.3292	-0.2556	-0.1187	
	2		-0.4650	-0.3406	-0.2560	
	4		-0.7441	-0.5169	-0.3566	
	8		-1.0553	-0.6288	-0.3570	
	12		-1.3326	1.1023	-0.2344	
H&D HE	1		-0.8869	-0.4650	0.3195	
	2		-1.0730	-0.5183	-0.1414	
	4		-1.5439	-0.7672	-0.2190	
	8		-2.6208	-1.1575	-0.2248	
	12		-7.5677	8.0870	-0.5072	

TABLE C.4.

Spot Portfolio — Industrial Portfolio
Futures Portfolio — Industrial Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Trad	Futures
Hdg Ratio			1.2282	0.8356	-1.8394	1.0000	
R-Squared			0.9253	0.1538			
Average Period Returns (%)	1	0.8416	-0.0290	0.2509	1.6602	0.1431	0.6985
	2	1.6468	0.0218	0.5322	3.6004	0.3409	1.3058
	4	3.1461	0.0283	0.9869	8.9682	0.6401	2.5060
	8	5.7050	0.1584	1.8722	17.3402	1.3336	4.3713
	12	7.1706	0.2425	2.5835	22.2225	1.9495	5.2210
Return as Ratio of Unhedged Return	1		-0.0345	0.2981	1.9727	0.1700	
	2		0.0132	0.3232	2.1863	0.2070	
	4		0.0090	0.3137	2.8506	0.2035	
	8		0.0278	0.3282	3.0395	0.2338	
	12		0.0338	0.3603	3.0991	0.2719	
Standard Deviation (σ)	1	2.9534	2.8565	1.8825	10.6883	2.2073	3.6203
	2	4.3005	3.3413	2.4809	16.7265	2.6616	4.5508
	4	6.0792	3.6659	3.0808	23.6218	3.0146	6.4214
	8	8.9400	4.6529	4.2032	27.9847	3.8590	8.8832
	12	13.7117	5.0494	5.0259	39.7468	4.2666	10.4908
σ as Ratio of Unhedged Std Dev	1		0.9672	0.6374	3.6190	0.7474	
	2		0.7770	0.5769	3.8894	0.6189	
	4		0.6030	0.5068	3.8857	0.4959	
	8		0.5205	0.4702	3.1303	0.4317	
	12		0.3683	0.3665	2.8988	0.3112	
Excess Return per unit of Risk (θ)	1	0.1920	-0.1063	-0.0125	0.1296	-0.0595	
	2	0.2553	-0.1578	-0.0068	0.1824	-0.0782	
	4	0.3369	-0.2918	-0.0361	0.3332	-0.1519	
	8	0.3925	-0.4380	-0.0771	0.5412	-0.2235	
	12	0.2827	-0.6044	-0.1414	0.4762	-0.3152	
H&D HBS	1		-0.2983	-0.2046	-0.0624	-0.2515	
	2		-0.4131	-0.2621	-0.0728	-0.3335	
	4		-0.6287	-0.3730	-0.0037	-0.4888	
	8		-0.8304	-0.4696	0.1487	-0.6160	
	12		-0.8871	-0.4241	0.1935	-0.5979	
H&D HE	1		-0.5534	-0.0654	0.6752	-0.3101	
	2		-0.6182	-0.0266	0.7147	-0.3064	
	4		-0.8662	-0.1071	0.9890	-0.4510	
	8		-1.1159	-0.1964	1.3788	-0.5695	
	12		-2.1379	-0.5002	1.6845	-1.1149	

TABLE C.5.

Spot Portfolio — All Share Index
Futures Portfolio — All Share Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Futures
Hdg Ratio			1.0000	0.7485	3.6879	
R-Squared			1.0000	0.7420		
Average	1	0.5377	-0.0310	0.1631	-6.0250	0.5690
Period	2	1.0521	-0.0330	0.3297	-9.9260	1.0851
Return	4	2.1003	-0.0140	0.6659	-10.0040	2.1151
(%)	8	3.5989	0.0683	1.2420	-11.2350	3.5306
	12	3.7835	0.0282	1.4917	-10.2310	3.7553
Return as			-0.0577	0.3033	-11.2051	
Ratio of	2		-0.0314	0.3234	-9.4345	
Unhedged	4		-0.0067	0.3170	-4.7631	
Return	8		0.0190	0.3451	-3.1218	
	12		0.0075	0.3943	-2.7041	
	1	2.6873	2.0582	1.4898	66.7275	3.5659
Standard	2	3.9268	2.3280	1.7720	121.3708	4.8737
Deviation	4	5.9487	2.3402	2.0928	175.9563	6.9922
(σ)	8	8.6308	2.7075	2.8494	226.1459	9.4016
	12	12.1039	2.6227	3.0862	240.4662	10.3519
σ as	1		0.7659	0.5544	24.8307	
Ratio of	2		0.5928	0.4513	30.9083	
Unhedged	4		0.3934	0.3518	29.5790	
Std Dev	8		0.3137	0.3301	26.2022	
	12		0.2167	0.2550	19.8668	
Excess	1	0.0979	-0.1484	-0.0748	-0.0944	
Return	2	0.1281	-0.2500	-0.1238	-0.0863	
per unit	4	0.1685	-0.4752	-0.2065	-0.0631	
of Risk	8	0.1625	-0.7859	-0.3349	-0.0594	
(θ)	12	0.0404	-1.2453	-0.5841	-0.0562	
	1		-0.2464	-0.1727	-0.1923	
H&D	2		-0.3781	-0.2519	-0.2144	
HBS	4		-0.6437	-0.3750	-0.2316	
	8		-0.9484	-0.4974	-0.2219	
	12		-1.2857	-0.6245	-0.0967	
	1		-1.5157	-0.7637	-0.9640	
H&D	2		-1.9516	-0.9662	-0.6737	
HE	4		-2.8206	-1.2258	-0.3745	
	8		-4.8357	-2.0604	-0.3654	
	12		-30.8093	-14.4502	-1.3915	

TABLE C.6.

Spot Portfolio — All Share Portfolio
Futures Portfolio — All Share Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Trad	Futures
Hdg Ratio			1.1385	0.8227	2.6975	1.0000	
R-Squared			0.9703	0.7266			
Average	1	0.7496	0.0815	0.2739	-5.0747	0.1805	0.5690
Period	2	1.4593	0.1873	0.5506	-2.5452	0.3742	1.0851
Return	4	2.8591	0.3874	1.0801	-4.8540	0.7439	2.1151
(%)	8	4.9906	0.8332	1.9113	-3.6546	1.4600	3.5306
	12	5.6082	1.0667	2.1600	-6.8079	1.8529	3.7553
Return as	1		0.1087	0.3654	-6.7699	0.2408	
Ratio of	2		0.1283	0.3773	-1.7441	0.2564	
Unhedged	4		0.1355	0.3778	-1.6977	0.2602	
Return	8		0.1670	0.3830	-0.7323	0.2925	
	12		0.1902	0.3852	-1.2139	0.3304	
	1	3.2133	2.4894	1.9308	65.4614	2.1934	3.5659
Standard	2	4.6842	2.8820	2.4456	51.4828	2.5686	4.8737
Deviation	4	7.1139	3.1471	3.2316	64.7376	2.9109	6.9922
(σ)	8	10.1739	3.9713	4.4337	102.1348	3.7645	9.4016
	12	13.8299	4.0773	5.1289	136.1745	3.8310	10.3519
σ as	1		0.7747	0.6009	20.3720	0.6826	
Ratio of	2		0.6153	0.5221	10.9907	0.5484	
Unhedged	4		0.4424	0.4543	9.1002	0.4092	
Std Dev	8		0.3903	0.4358	10.0389	0.3700	
	12		0.2948	0.3709	9.8464	0.2770	
Excess	1	0.1478	-0.0775	-0.0003	-0.0817	-0.0429	
Return	2	0.1943	-0.1255	0.0006	-0.0601	-0.0681	
per unit	4	0.2475	-0.2258	-0.0056	-0.0919	-0.1217	
of Risk	8	0.2747	-0.3432	-0.0643	-0.0573	-0.1956	
(Θ)	12	0.1673	-0.5463	-0.2212	-0.0742	-0.3762	
	1		-0.2254	-0.1482	-0.2296	-0.1907	
H&D	2		-0.3198	-0.1937	-0.2544	-0.2624	
HBS	4		-0.4734	-0.2531	-0.3395	-0.3692	
	8		-0.6179	-0.3389	-0.3320	-0.4702	
	12		-0.7136	-0.3885	-0.2415	-0.5436	
	1		-0.5244	-0.0022	-0.5527	-0.2899	
H&D	2		-0.6459	0.0033	-0.3093	-0.3503	
HE	4		-0.9122	-0.0225	-0.3714	-0.4915	
	8		-1.2495	-0.2339	-0.2086	-0.7120	
	12		-3.2653	-1.3218	-0.4434	-2.2487	

APPENDIX D: REVISED HEDGING EFFECTIVENESS MEASURES

TABLE D.1.

Spot Portfolio — All Gold Index
Futures Portfolio — All Gold Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D
Coeff of Variation (CV)	1	0.0678	-0.0099	0.0239	0.0178
	2	0.0949	-0.0152	0.0383	-0.0413
	4	0.1425	-0.0046	0.1039	-0.0388
	8	0.1542	0.0159	0.1265	-0.1646
	12	0.0776	0.0354	0.0777	0.0073
HE*	1	1.0000	-0.1463	0.3520	0.2618
	2	1.0000	-0.1604	0.4032	-0.4347
	4	1.0000	-0.0320	0.7291	-0.2723
	8	1.0000	0.1030	0.8202	-1.0671
	12	1.0000	0.4557	1.0009	0.0936
HBS*	1	0.0000	-0.0778	-0.0440	-0.0501
	2	0.0000	-0.1102	-0.0567	-0.1362
	4	0.0000	-0.1470	-0.0386	-0.1812
	8	0.0000	-0.1383	-0.0277	-0.3188
	12	0.0000	-0.0422	0.0001	-0.0703

TABLE D.2.

Spot Portfolio — All Gold Portfolio
Futures Portfolio — All Gold Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Trad
Coeff of Variation (CV)	1	0.0818	0.0133	0.0483	-0.0012	0.0183
	2	1.1362	0.0195	0.0720	-0.0949	0.0250
	4	0.1748	0.0677	0.1673	-0.0712	0.0739
	8	0.2163	0.1803	0.2650	-0.1068	0.1890
	12	0.1836	0.3056	0.3548	-0.1025	0.3200
HBS*	1		-0.0685	-0.0334	-0.0829	-0.0634
	2		-1.1168	-1.0642	-1.2311	-1.1113
	4		-0.1071	-0.0076	-0.2461	-0.1009
	8		-0.0360	0.0487	-0.3231	-0.0273
	12		0.1219	0.1712	-0.2861	0.1364
HE*	1		0.1625	0.5912	-0.0143	0.2241
	2		0.0171	0.0634	-0.0835	0.0220
	4		0.3874	0.9568	-0.4074	0.4229
	8		0.8335	1.2253	-0.4938	0.8736
	12		1.6641	1.9324	-0.5580	1.7429

TABLE D.3.

Spot Portfolio — Industrial Index
Futures Portfolio — Industrial Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D
Coeff of Variation	1	0.3011	-0.0218	0.1372	0.0643
	2	0.3947	-0.0136	0.2175	-0.0161
	4	0.5296	-0.0204	0.3433	-0.0318
	8	0.5919	-0.0150	0.4091	-0.0445
	12	0.4309	-0.0228	2.1384	-0.0628
HBS*	1		-0.3229	-0.1639	-0.2368
	2		-0.4083	-0.1772	-0.4108
	4		-0.5500	-0.1863	-0.5614
	8		-0.6069	-0.1828	-0.6364
	12		-0.4536	1.7075	-0.4937
HE*	1		-0.0724	0.4557	0.2134
	2		-0.0346	0.5510	-0.0407
	4		-0.0386	0.6482	-0.0601
	8		-0.0253	0.6911	-0.0751
	12		-0.0529	4.9631	-0.1458

TABLE D.4.

Spot Portfolio — Industrial Portfolio
Futures Portfolio — Industrial Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Trad
Coeff of Variation (CV)	1	0.2850	-0.0102	0.1333	0.1553	0.0648
	2	0.3829	0.0065	0.2145	0.2153	0.1281
	4	0.5175	0.0077	0.3203	0.3797	0.2123
	8	0.6381	0.0340	0.4454	0.6196	0.3456
	12	0.5230	0.0480	0.5140	0.5591	0.4569
HBS*	1		-0.2951	-0.1517	-0.1296	-0.2201
	2		-0.3764	-0.1684	-0.1677	-0.2549
	4		-0.5098	-0.1972	-0.1379	-0.3052
	8		-0.6041	-0.1927	-0.0185	-0.2926
	12		-0.4749	-0.0089	0.0361	-0.0660
HE*	1		-0.0356	0.4677	0.5451	0.2275
	2		0.0170	0.5602	0.5621	0.3345
	4		0.0149	0.6190	0.7336	0.4103
	8		0.0533	0.6980	0.9710	0.5415
	12		0.0918	0.9829	1.0691	0.8737

TABLE D.5.
Spot Portfolio — All Share Index
Futures Portfolio — All Share Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D
	1	0.2001	-0.0151	0.1095	-0.0903
	2	0.2679	-0.0142	0.1861	-0.0818
CV	4	0.3531	-0.0060	0.3182	-0.0569
	8	0.4170		8	0.4170
	12	0.3126	0.0108	0.4833	-0.0425
	1		-0.2152	-0.0906	-0.2904
	2		-0.2821	-0.0819	-0.3497
HBS*	4		-0.3591	-0.0349	-0.4099
	8		-0.3918	0.0189	-0.4667
	12		-0.3018	0.1708	-0.3551
	1		-0.0753	0.5471	-0.4513
	2		-0.0529	0.6944	-0.3052
HE*	4		-0.0169	0.9012	-0.1610
	8		0.0605	1.0453	-0.1191
	12		0.0344	1.5463	-0.1361

TABLE D.6.
Spot Portfolio — All Share Portfolio
Futures Portfolio — All Share Index Futures

	Wks	Unhedged	Beta	Min-Var	H&D	Trad
	1	0.2333	0.0327	0.1419	-0.0775	0.0823
Coeff of	2	0.3115	0.0650	0.2251	-0.0494	0.1457
Variation	4	0.4019	0.1231	0.3342	-0.0750	0.2556
(CV)	8	0.4905	0.2098	0.4311	-0.0358	0.3878
	12	0.4055	0.2616	0.4211	-0.0500	0.4837
	1		-0.2005	-0.0914	-0.3108	-0.1510
	2		-0.2465	-0.0864	-0.3610	-0.1659
HBS*	4		-0.2788	-0.0677	-0.4769	-0.1463
	8		-0.2807	-0.0594	-0.5263	-0.1027
	12		-0.1439	0.0156	-0.4555	0.0781
	1		0.1403	0.6081	-0.3323	0.3528
	2		0.2086	0.7227	-0.1587	0.4676
HE*	4		0.3063	0.8316	-0.1866	0.6359
	8		0.4277	0.8788	-0.0729	0.7906
	12		0.6452	1.0385	-0.1233	1.1927

GLOSSARY OF TERMS[1]

Arbitrage: Simultaneous purchase and sale of financial instruments yielding a riskless profit owing to temporary price discrepancies between markets.

Arbitrageurs: Traders who profit through arbitrage.

Basis: The difference between the spot and futures market prices.

Basis Risk: The variability in the basis. This term is normally refers to the residual risk present in hedge portfolios owing to imperfect correlation between the spot and futures prices.

Bid Price: The futures price at which the market maker is willing to buy, i.e. the futures price at which participants must contract at if they wish to sell.

Clearing: Process of matching, registering and guaranteeing futures transactions.

Close out (offset): A transaction entered into which nets off a previous position to zero; or the final settlement of the contract at its conclusion.

Convergence: The process where the futures price and commodity price approach each other towards the completion of the contract. At the end of the contract they are equal.

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1. Glossary terms have been restricted to futures markets only. Source of terms is Schwarz, Hill and Schneeweis (1986) and Falkena, Kok and Raine (1989).

Corner: Control by a market participant or group of participants of the entire quantity of the asset underlying the futures contract.

Cost-of-carry: The net cost of holding (carrying) the spot versus holding the futures contract.

Cross hedge: A hedge position where the asset underlying the futures contract is similar, but not identical, to the spot position.

Delivery: The act of delivering or taking delivery of the underlying instruments. Share index futures are effectively delivered through cash settlement.

Derivative instruments: Futures or options taken (or derived) from other financial instruments.

Expiration date: Date on which the futures contract matures.

Financial futures: A futures contract written on a financial instrument (rather than on a physical commodity).

Futures contract: A standardized contract that embodies an obligation to deliver (i.e. sell) or take delivery of (i.e. buy) a standard quantity of a commodity or financial instrument at a standard future date at a price specified by the parties to the contract.

Forward contract: A customised futures contract.

Hedge: A position taken in the futures market aimed to minimize the risk of a change in value of an underlying asset or liability.

Hedge ratio: The proportion of futures contracts needed to hedge an open spot position.

Hedgers: Investors and others who wish to minimize their risk exposure by entering into futures contracts (as opposed to speculators).

Initial Margin: The deposit payable as security by the participant on entering into a futures contract.

Liquidity: Depth of potential and existing trading volumes in the market.

Maintenance Margin: Minimum net amount that must be held by the exchange or market maker throughout the life of the futures contract.

Margins: Good faith deposits made to provide collateral for the futures contract.

Margin call: The requirement by the exchange to top up the margin deposit if the mark-to-market process reduces the margin below the minimum required limit.

Market Maker: A market participant (usually a member of an exchange) who holds himself out to be prepared to make a price in the relevant futures market instrument as principal.

Marking-to-market: The process of recalculating and marking the futures positions to the price at the end of each business day. Margin accounts are debited or credited to reflect changes in the current market prices on the positions held.

Offer Price: The futures price at which a market maker is willing to sell.

Open contracts: Futures contracts which are still in force, i.e. they have neither expired nor been closed out.

Open interest: The total number of all open contracts at a particular point in time.

Over-the-counter market: A market where futures are traded one-to-one between the participants and the market maker, but not on an exchange.

Short: A sale of a futures or spot contract. Shorting the spot normally indicates the selling of the spot asset before ownership of the spot asset is actually obtained.

Speculators: Traders who attempt to make profits by dealing in the futures markets.

Spot asset: The asset upon which the futures contract is based.

Spot (cash) market: The market for all the underlying instruments on which futures are based.

Squeeze: Control by a market participant or group of participants of a sufficient quantity of an asset underlying a futures contract to exert significant pressure on prices.

Straight hedge: A hedge position where the asset underlying the futures contract is the same as the spot portfolio.

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