OPTIMAL GOVERNMENT POLICY:

A Refutation of Claims that Optimal Policy may be Time Inconsistent.

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INTRODUCTION

The original purpose of this thesis was to derive and study optimal central bank interest rate setting policies in a number of different stochastic settings. The formal derivation of these optimal policies was to be guided by optimal control techniques (see for example Aström (1970), Intrilligator (1971), Chow (1975), Kamien and Schwartz (1981)). A review of the economics literature revealed, however, that the use of these techniques in this context was not universally accepted by the profession. Arguments against their use were forwarded by economists associated with the Rational Expectations school. These arguments focused on the forward looking nature of economic agents, and their awareness of the incentives facing policy makers. Expectations formed by agents on the basis of this knowledge presented, it was argued, a case against the use of control methods (see for example Sheffrin (1983)). Control techniques were, purportedly, unable to incorporate expectations into calculations of optimal policy, unless expectations were treated as parametric. As a consequence of this, it was claimed that the use of techniques other than control methods would, in general, give rise to relatively superior outcomes.

These alternative techniques were argued to give rise to 'time inconsistent'[1] policies. Furthermore, 'time inconsistent' policies were identified with 'rules', whilst 'consistent' policies were identified with 'discretion'. This association contributed to the well known debate in the policy literature regarding the superiority of 'rules' over 'discretion'.

This debate, in which Simons (1936) and Friedman (1959) were early proponents of 'rules', was given new impetus by the claim of Kydland and Prescott (1977) that optimal control techniques were 'discretionary' or consistent and hence sub-optimal, and that 'rules' were therefore superior.

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[1]As opposed to the 'consistent' nature of control theory solutions which are related to Dynamic Programming methods (see Intriligator (1971), Bellman (1957)).

A review of the literature reveals that 'rules' have generally been interpreted as a commitment, **ex ante**, to a particular policy, whilst 'discretion' has generally been interpreted as sequential decision making, where new decisions are made at each point in time. The question we need to ask in the light of these definitions is, "Why do 'rules' and 'discretion' give rise to different outcomes ?"

Simons' explanation of the different outcomes resulting from utilising 'discretionary' rather than 'rules' policies was based on the idea that sequential decision making inevitably leads to "endless concessions to organized minorities" (Quoted in Sheffrin (1983), p98). Essentially, his argument rested on the premise that organized minorities continually change the nature of the environment in which policies are to be made. As the environment changes, so do the objectives of the policy maker and, consequently, the policies themselves. The instability that results from this undermines the "'constitutional structure' under which free enterprise and representative government can function" (Ibid, p98). 'Rules' and 'discretion' differ because in the latter, objectives change, whilst in the former they remain constant.

Friedman saw the problem as one of timing. 'Discretionary' policy, because it is sequential in nature, enables the policy maker to respond to downturns and upturns in the cycles of the economy. If these responses are mistimed, which is likely given that the dynamics of the economic system are poorly understood, sequential or 'discretionary' policy making can contribute to economic instability by accentuating these cycles (Phillips (1957) discusses in detail the effects of mistimed control in a dynamic system). 'Rules' do not respond to these changes, and hence give rise to different, supposedly superior, outcomes.

The 'time inconsistency' literature has interpreted the disparity between 'rules' and 'discretion' as being a consequence of agents' expectations. Expectations, it was argued, present the government or some optimising authority with two 'different' optimisation procedures for the <u>same</u> economy. Using the 'rules' optimisation procedure gives rise to better ('inconsistent') outcomes than using the 'discretionary' ('consistent') procedure.

The arguments forwarded by Simons and Friedman have received sufficient coverage in the literature and are not considered here. Instead we proceed with an examination of the question of 'time inconsistency'.

Specifically, we are interested in the question of whether or not the supposed superiority of 'time inconsistent' policies vis-a-vis 'consistent' ones is correct. Given that the notion of 'inconsistency' forms the basis for the recent arguments in favour of 'rules' and against the use of control techniques, this question is of obvious importance.

Briefly, the structure of the thesis is as follows. The first chapter concentrates on specifying the general nature of the problems tackled in the 'time inconsistency' literature, and forms the framework for the analysis which follows. Seminal papers on the issue of the 'inconsistency' of optimal plans and the inferiority of 'consistent' solutions are then examined in chapters two and three. A synthesis of the literature is reviewed in chapter four. Chapter five stands by itself as an illustration of the logical difficulties underlying claims that 'optimal' policies are in general 'time inconsistent'. The conclusion is in chapter six.

CHAPTER ONE

1) Introduction

This chapter presents the general structure of the problems tackled in the literature on time inconsistency and reviewed in this thesis[2]. The terminology to be used throughout the paper is defined in this section. Our primary focus will be on defining time inconsistency.

We proceed firstly by specifying the common form of the optimisation problem facing an authority. Secondly, we define time inconsistent behaviour. Three cases are then presented, in two of them agents are 'rational'[3] whilst in the other, agents are 'non-rational'. The possibility of time inconsistent behaviour is considered in each. Finally, the issue of optimality and time inconsistency is discussed.

1.1) The Optimisation Problem.

The objective of the authority is to choose a policy sequence, known as an optimal policy, which minimises or maximises some loss or objective function. More formally, the authority wishes to solve the problem

$$\min_{\{\mathbf{u}(t),\ \mathbf{u}^a(t)\}} \text{ or } \max. \qquad L = L(\ \mathbf{x}(t),\ \mathbf{u}\ (t),\ \mathbf{x}^e\ (t,\ \mathbf{u}^a\ (t),\ S)) \qquad (1)$$

where $\mathbf{u}$ (t) is a vector of control or instrumental variables whose value is chosen by the authority, $\mathbf{x}$ (t) is a vector of state variables describing the current state of the economy, and $\mathbf{x}^e$ (t, $\mathbf{u}^a$, S) are expectational values of $\mathbf{x}$ (t). The arguments of $\mathbf{x}^e$ are time, t, a vector of announcements, $\mathbf{u}^a$ (t), and the structure of the optimisation problem, S.

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[2]Except for the problem in chapter 5 which assumes a different form.

[3]'Rational' is defined here in the sense of agents not making systematic forecasting errors of future variables in the economy (i.e. no serially correlated error terms).

Announcements are made by the same authority that sets the values of the instrumental variables, and relate to what the values of the instrumental variables <u>might</u> be in the future. The structure, S, of the optimisation problem is the specific functional form of the problem, and might include constraints on the values of the control and state variables as well as a description of how expectations evolve through time.

The problems to be treated later are all <u>deterministic</u>[4]. This is of importance for our purposes because the optimal trajectories of the instumental variables and the announcements can be set and known with certainty at the beginning of the planning interval. Consequently, expectations and the trajectory of the state variables are known at the beginning of the planning interval too. This reflects the well known equivalence of open and closed loop controls in a deterministic system[5] (Intriligator (1971)).

With the general structure of the problem defined, we can now turn to defining 'time inconsistency'.

1.2) Time Inconsistent Behaviour Defined.

Consider the temporal ordering $t_k > t_j > t_i$, where it is understood that later events (for instance, at t_k) cannot influence earlier events (for instance, at t_j). 'Time inconsistency', as uniformly defined in the literature, refers to the situation in which an 'optimal' plan formed at t_i for execution at time t_k will differ from a plan formed at t_j for execution at the same time, t_k . More formally, we can write

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[4]A deterministic system is one where no new information relevant to the optimisation process becomes avaiable during the planning interval.

[5]Two types of control are possible, known as open and closed loop control. With open loop control, the optimal control and announcement trajectory is determined as a function of <u>time</u>. These trajectories are completely specified at the initial time, t_0 . With closed loop control, the optimal control trajectory is determined as a function of the current <u>state variables and time</u>, and is not fully specified at time t_0.

$$(u^* (t_k, t_i) \mid I_0) \neq (u^*(t_k, t_j) \mid I_0), \quad t_i < t_j < t_k \quad (2)$$

where $u^*(t_j,t_i)$ is an 'optimal' plan for the instrument, u, at time t_j formed at time t_i (a similar 'inconsistent' optimal plan for announcements could be defined).

More generally, for each planning point between t_i and t_j where $(j < k)$, there is a different 'optimal' plan for policy to be executed at time t_k. It is important to note that the information set, I_0, does not change.

Clearly, 'time inconsistency' is in violation of Bellman's (1957) 'Principle of Optimality' which defines an optimal policy to be one that "has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." (p 83).

This notion of optimality forwarded by Bellman implies that <u>if</u> a policy is optimal at some time point, and we move forward in time and reoptimise, the original policy <u>will</u> <u>not</u> <u>change</u>. If the original 'optimal' plan does change as we reoptimise sequentially, it could not have been optimal in the first instance. Certainly, in a deterministic problem, where the authorities know <u>all</u> relevant information for <u>all</u> future periods and account for this properly at the start of the planning interval, optimal plans cannot change (we consider an explicit example in chapter five, and argue that this is indeed the case).

With optimality now properly defined, we observe that it implies one of two things about the 'time inconsistency' literature. Either this literature has concerned itself with models that are different to the one described in section 1.1, or else there are logical errors in the way in which optimal policies have been identified and understood. It is our contention, as becomes clear later on, that the literature <u>has</u> utilised models of the type described in section 1.1. The identification of 'inconsistent' behaviour is, consequently, the result of logical errors. The primary source of error common to most of the literature reviewed is identified by considering the connection between agents beliefs or expectations and the authorities' announcements[6].

1.3) Announcements and Agents' Beliefs.

Let us proceed by considering why the authorities would wish to make announcements. Clearly, announcements are used by authorities in an attempt to influence agents' behaviour.

There are three classes of agents' responses to government announcements. Firstly, agents may believe the government's announcements regardless of the structure of the problem (a so-called 'non-rational' case). Secondly, agents may realise that the incentives facing government will result in announced plans not being carried through. Thirdly, agents may believe the governments' announcements because they realise that government has the incentive to carry through with announced plans (in both cases two and three, agents are 'rational'). We can now discuss which plans, if any, are time inconsistent in each of these classes of response.

1.3.1) Class One.

The first class of responses outlined above comes closest to most of the examples which have been used to demonstrate time inconsistency. Let us assume that the authorities can benefit from inducing agents to expect levels of the instruments in the economy that differ from those that will actually prevail. By announcing certain future levels of the instrument (which agents believe) and setting actual levels of the instrument differently, the government obtains superior values of their loss function. This superiority is, of course, relative to the value of the loss function that would obtain if they followed through with their announcements. In a deterministic setting, government knows what its optimal actual path of the control will be in the future for all possible announced and believed levels of the control, and that in general,

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[6]Other sources are identified seperately in each chapter.

the announced and actual paths will differ[7].

According to our earlier specification of the problem, and in the light of the discussion immediately above, neither the actual nor the announced plan will change on sequential optimisation, and 'time inconsistency ' is not exhibited (**ex ante**, both paths are known with certainty). The presence of 'time inconsistency' has nevertheless been claimed in examples of this class. This claim is false, however, on the grounds that what is claimed as 'time inconsistency' is merely the difference between the announced and actual values of the instrument. We shall use the label pseudo-time-inconsistency to identify this situation[8].

This class of responses suggests that agents could be systematically fooled if there was continued interaction between them and the authorities over a number of periods (their prediction of the actual level of the instrument is equal to the authorities' announcement which, in general, does not equal the actual value of the instrument in the future). If, however, agents know the model of the economy, and hence the authorities' optimisation problem, as would be expected if they were endowed with rational expectations, they would anticipate any incentive on the part of the authorities to deviate from announced plans, **ex ante**. This is our second class of responses.

1.3.2) Class Two.

If it is in agents' interests[9] to attempt to anticipate cheating by the authorities, they would realise that the actual and announced paths will differ, and would ignore any announcements.

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[7]This point is made explicit in a later chapter on Fischer (1980) and Blanchard and Fischer (1990) (see section 4.4 of chapter 4).

[8]The really interesting problem would be if **ex ante** planned did not equal **ex post** actual policy, but this is not the case. The difference between announced and actual values is nothing more than cheating (Modigliani(1977)).

[9]i.e. agents derive disutilty from being cheated when announced levels of the instrument do not equal actual levels in the future.

Although the authorities could continue to make announcements, they serve no purpose, and both agents and government know the actual path of the instrument in all future periods, and this path would not change through time.

1.3.3) Class Three.

The final case, where agents believe announcements because of the incentives facing the authorities, does not, as can be easily surmised, exhibit time inconsistency of any type (i.e. announced = actual = expected).

In the chapters which follow, we will utilise the basic framework developed in sections 1 to 1.2.3 to argue that papers that have emerged from the literature in recent years purporting to demonstrate time inconsistency are usually demonstrating pseudo-time- inconsistency. We conclude this chapter with a discussion of the optimality of the results obtained in the three cases presented.

1.4) Optimality and Time Inconsistency.

Let us consider what constitutes an optimal policy when agents always believe announcements (our first class of problems). Following our discussion above, there are two sets of 'optimal' paths, which will, in general, differ. One set is the actual path of the instrument that will prevail in future periods and the other set defines the announced path of the instruments. These plans are properly understood to be optimal in the context of the unique constraints of this problem, and any deviations from these plans would give rise to inferior values of the loss function. It is optimal, in this economy, to cheat and pursue a pseudo-time-inconsistent policy.

If, on the other hand, agents do not believe that the authorities will follow through with its announcements (i.e. as in class two of responses above, the constraint that agents believe any announcement by the authorities has been removed), they will expect actual policy to deviate from announced policy, and will ignore announcements when forming their expectations.

The issue of inconsistency and optimality arises in this class of responses because, as Blanchard and Fischer (1990, p596) argue, the value of the loss function will, in general, be inferior in class two responses compared to class one responses. The crucial observation to make, however, is that there is no change that the authorities can make to their policy, given these constraints[10], that would give rise to a superior value of the loss function. This is correctly understood to be an optimal policy.

If the constraints of the problem change so as to induce the policy maker to carry through its announcements, 'rational' agents will correctly believe these announcements, and act accordingly. These new constraints induce the authority optimally to honour its announcements. If government deviated in any way from the announced path, an inferior value of their objective function would be obtained. The announced and actual plans, which coincide, are understood to be optimal[11].

We will attempt throughout this paper to come to a proper understanding of what constitutes an optimal plan in a variety of models of the economy, and will argue that often plans identified as sub-optimal are correctly interpreted as being optimal.

We are now in a position to evaluate some of the more important papers which have emerged from the time inconsistency literature in recent years. Chapter one forms the basis of our analysis of these works, although unique problems in each paper will also be highlighted.

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[10] Agents would only believe announcements that they know government has the incentive to follow through with.

[11] An example of this sort of constraint would be where the authority agrees to pay out some penalty sum that is greater than the potential benefit of deviating from the announced path (e.g Hughes Hallett (1986)).

CHAPTER TWO

Kydland and Prescott (1977):The Inconsistency of Optimal Plans.

2) Introduction.

We start our examination of the 'time inconsistency' literature with what is possibly the most widely cited paper on this topic, Kydland and Prescott (1977). Kydland and Prescott argue in this paper in favour of rules over discretion, based on the alleged superiority of 'time inconsistent' solutions over 'time consistent' ones.

For the purposes of this argument, Kydland and Prescott (hereafter K&P) distinguish between what they term a time consistent policy and an optimal policy. Time consistent policy is associated both with Control Theory and what is termed discretion, whilst optimal policy, which they argue is time inconsistent, is associated with a rule[12]. On the basis of these associations, K&P argue that in general, time consistent policy will not be optimal. Following a rule would lead to a higher value of the social objective function. This is demonstrated by way of formal and verbal examples.

This chapter is comprised of three parts. Firstly, the formal model used by K&P to illustrate the difference between time consistency and optimality is critically examined. Secondly, a new interpretation of one of their verbal examples is given. Finally, we conclude that in each case K&P mistakenly identify two distinct models of the economy as being identical, and that, consequently, time inconsistency is not demonstrated.

2.1) Kydland and Prescott's 2-period model.

The government's objective is to maximise the social welfare function

(1) $S(x_1 , x_2 , \pi_1 , \pi_2)$

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[12]Announcements are implicit in K&P's 'inconsistent' policy, as we argue below.

where the arguments of this function are values at points 1 and 2 in time of the instrumental variable π and the state variable x. The state variable is chosen by economic agents according to

(2) $$x_1 = X_1(\pi_1, \pi_2),$$

(3) $$x_2 = X_2(x_1, \pi_1, \pi_2).$$

Under the optimal solution, π_1 and π_2 are chosen simultaneously so as to maximise S under the constraints of both (2) and (3), yielding the two first order conditions

(4) $$\frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial \pi_i} + \frac{\partial S}{\partial \pi_i} + \frac{\partial X_1}{\partial \pi_i}\left(\frac{\partial S}{\partial x_1} + \frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial x_1}\right) = 0, \quad i = 1,2.$$

This is contrasted with the consistent solution for period 2 obtained by maximising S subject to (3) only, but taking the past values x_1 and π_1 as given parameters. Under this procedure, the optimal value of $\pi_2 = \pi_2(x_1, \pi_1)$ must satisfy[13]

(5) $$\frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial \pi_2} + \frac{\partial S}{\partial \pi_2} = 0.$$

This result differs from eqn. (4) where i = 2, unless $\partial X_1/\partial \pi_2 = 0$ (or unless, trivially, the bracket in eqn. (4) is zero), which implies that the period 2 policy differs under the two procedures.

Let us first consider the causal relationships between the variables in this example. The symbols x_1, x_2, π_1, π_2 represent unique values of the variables x and π at moments of time (or 'periods') labelled 1 and 2, where time 1 is understood to precede time 2. With this temporal ordering, whilst an expectation of π_2 (formed at time 1 or prior to time 1) may influence x_1, the value π_2 itself cannot influence x_1 as implied by eqn. (2) unless π_2 is predetermined and known with certainty by agents when they choose x_1. Unless, at time 1, the value π_2 is predetermined[14] and known to

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[13]The consistent solution procedure could also be applied in period one, taking eqn. (5) as a constraint in addition to (2) and (3) when deriving π_1.

[14]As we argue in the section on the verbal examples below, pre-commitment relates to announced and believed values for the instrument in period two.

agents with certainty, eqn. (2) is in violation of causality.

Let us accept, then, that either eqn. (2) is non-causal[15] or π_2 is predetermined, and now consider the logical validity of the above demonstration. In the optimal solution, eqn. (2) is one of the constraints: π_2 plays a part in agents' choice of x_1 , i.e. x_1 does depend on π_2 . In the consistent (second period) solution, the constraint of eqn. (2) is replaced by the constraint that x_1 (and π_1) is a fixed parameter: x_1 does not depend on π_2 . Since these solutions are derived under different assumed constraints, it is not surprising that the first order conditions are different. But this difference does not arise, as Kydland and Prescott imply that it does, because of different decision procedures by the government (optimal/rule/inconsistent as opposed to consistent/discretion); it arises because the assumed structure of the economy is different.

For instance, whereas the consistent or sequential optimisation problem explicitly depends on time, with an associated set of constraints (where period one is assumed to precede period two, and cannot be affected by actual events in period two), the time inconsistent 'optimal' policy does not incorporate time in this sense, and has a different set of associated constraints. It is an illegitimate exercise to assume that x_1 does depend on π_2 in one instance, and then to compare it to a situation in which x_1 does not depend on π_2 [16] as the basis for a claim that Bellman's principle has been violated.

We conclude, therefore, that the apparent difference between the optimal and consistent policies in this deterministic model is a result of an illegitimate comparison. Depending on the models'

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[15]The non-causal nature of the optimal plan has been mentioned by Buiter (1981) and Chow (1981).

[16]The replacement of eqn (2) with $x_1 = X_1(\pi_1, E_1(\pi_2))$, where $E_1(\pi_2)$ is a rational expectation of π_2 given information available at time 1, would clearly restore causality. In this demonstration, the only variable which is observable at time 1 is, however, π_1 , thus eqn. (2) becomes $x_1 = X_1'(\pi_1)$; then $\partial X_1/\partial \pi_2 = 0$ and solution (4) coincides with solution (5).

assumptions, the derived first order conditions are the correct ones. If x_1 <u>does</u> <u>not</u> depend on π_2 , then solutions (4) and (5) are the same. If x_1 <u>does</u> depend on π_2 through eqn. (2), then solution (4) is correct and there is no scope for denying eqn. (2) to find a second, different solution.

The two policies identified by K&P are optimal, albeit for different models of the economy.

2.1.2) Expectations.

K&P attempt to justify their conclusions about 'time inconsistency' on the basis of expectations. A brief look at K&P's model again reveals, however, that this exercise is not justified.

This follows directly from the observation that <u>nowhere</u> in the model do expectations feature. Although K&P's <u>verbal</u> explanation of the dynamics of this particular model contain an expectational component, the formal model itself does not justify this procedure.

Only if an expectations formation schema such as Chow's (1981) is used, and expectations are formally added to the system of equations describing the economy, can the verbal description of the economy incorporate expectations. This process also eliminates the problem of non-causality. The addition of an expectational constraint restores real time dependence by allowing for <u>expectations</u> of future events to influence current events, and unlike in the K&P model, the effects of actual future policy on present decisions can be ignored.

The role of announcements has not been made explicit in this discussion, primarily because K&P do not utilise announcements in their exposition. The linkages between their paper and our previous chapter are more easily seen in their verbal examples.

2.2) An example.

K&P provide two verbal examples, intended to illustrate the difference between optimal and consistent policy. Consider the 'flood plain' example.

In the flood plain example, agents choices are whether to build houses in a flood plain (H) or not ($\overline{H}$), and the government's choice is whether to build protective dams (D) or not ($\overline{D}$). The government's known social welfare function is the ranking

$$\overline{HD} > HD > H\overline{D},$$

and agents' preferences are

$$HD > \overline{HD} > H\overline{D}.$$

There are two time periods and decisions in each period are denoted by a time subscript (e.g. D_1 would be a dam in period one).

It is K&P's claim that given this preference ordering, optimal and consistent policies (to use their distinction) will differ: in the 'consistent' case houses and dams will be built; in the 'optimal' case, no houses and dams are built.

Furthermore, K&P claim that consistent and optimal outcomes are possible in an identical economy i.e. given the <u>same</u> economy but differing solution procedures, one outcome (following a consistent policy) is inferior to the other (following an optimal or time inconsistent policy) in terms of government's preference ordering.

As in their formal example, these claims are shown to be wrong because the economy postulated in each case is different.

<u>2.2.1) Economy One.</u>

In this economy, the government is a leader in the policy game, and sets its policies **ex ante**, announcing what it will do in the next period. Agents on the other hand are followers in the sense that they optimise subject to the constraints of what government does in the present and announces it will do in future. The problem for government is thus to optimise the social welfare function subject to the constraint :{ $\overline{D}_1 + \overline{D}_2 \Rightarrow \overline{H}$ and $D_1 \times D_2 \Rightarrow H$ }.

The precommitment in this case results in government not building dams in either period. Agents, who maximise subject to this constraint (believing that no dams will be built in either period ie. $\bar{D}$), do not build houses. This outcome is first best for government and second best for agents, and is a direct result of the specification of the problems' constraints.

This corresponds with case three identified in section 1.2.3. There is an implicit constraint that induces government to follow through with its announcements, and they cannot build dams. Agents, who are aware of this, do not build houses.

2.2.2) Economy Two.

In the consistent, sequential solution problem, the rules of the game change. Here agents, knowing government's ranking of outcomes, build houses in the certain knowledge that it will cause dams to be built in period two, regardless of what government announces at the start of the period (i.e. as in section 1.2.2. agents do not believe announcements). Government knows this, and to avoid its worst outcome ($H\bar{D}$), builds dams. The constraint facing government in this world is just (H).

Government would like to 'cheat' in this scenario by announcing that they will not build dams, but agents, knowing governments' preference ordering and the constraints they face, realise that when faced with houses in period two, government <u>will build dams.</u>

Superficially, it may appear as if K&P's point regarding the inferiority of 'consistent' policy procedures has been made. This is, however, not true, and the different outcomes are optimal given that unique constraints are in operation in case (which, it will be recalled, is the basis for the distinctions made between cases 1, 2 and 3 in chapter one).

2.3) Optimality and K&P's Examples

The impression which K&P create is that they are deriving two <u>different</u> 'optimal' policies for the same economy. We have shown that, on the contrary, the policies derived are a function of <u>different</u> constraints, and are optimal given these different constraints.

Whilst comparing the performance of different economies may be a valid exercise (see for example Blanchard and Fischer (1990), Chap. 11), this is not what K&P purport to show in their verbal examples.

We can conclude that there is no time inconsistency in any of their examples, and that each outcome is optimal. Where announced plans are not carried through (i.e. a pseudo-time-inconsistent outcome), the separate paths of the announcement and actual controls are always known, **ex ante**, by agents and government.

2.4) Conclusion

From the preceeding discussion it is clear that the influence of this paper has been perpetuated by a misunderstanding of what K&P really demonstrate. All that is in fact demonstrated is that optimal policies in different economies differ, but these optimal plans are not time inconsistent. Nothing about the applicability of control theory in dynamic settings nor the superiority of rules can be deduced from their work.

CHAPTER THREE

Barro and Gordon's (1983) Reputational Equilibrium

3) Introduction

Another widely cited paper to emerge from the Time Inconsistency literature in recent years, spawned in part by Kydland and Prescott's paper, is Barro and Gordon's (hereafter B&G) work on reputational equilibria. In this paper, B&G argue that only announcements which the authorities have no incentive to deviate from will be believed by 'rational' agents (as in case three in chapter one). Other announcements are ignored. 'Time inconsistency' purportedly is the basis of the derivation of the set of announcements that are believed and carried out.

We proceed by setting out their model in which a 'discretionary', 'rules' and 'reputational' scenario is presented. Time inconsistency and pseudo-time-inconsistency in the model is then discussed. Thereafter, the operation of expectations in generating their results is examined, and finally, certain of their assumptions are questioned.

Our conclusion is that it is not 'time inconsistency' that underpins their derived results, but pseudo-time-inconsistency. Furthermore, we conclude that B&G's use of expectations is contradictory and that their results are arbitrary, given that they are derived from a set of arbitrary assumptions.

3.1) Barro and Gordon's Model

B&G assume that the monetary authorities wish to minimise a social objective function [17] (which is common to both the government and private agents)

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[17] B&G treat certain of the terms in equation one as stochastic (and later on, the discount rate too). We will, however, ignore the stochastic nature of these terms because B&G's analysis is unaffected by assuming agents know all parameter values with certainty.

$$Z_t = \left(\frac{a}{2}\right)(\pi_t)^2 - b\,(\pi_t - \pi_t^e), \qquad a, b > 0 \qquad (1)$$

where π_t is inflation, $(\pi_t - \pi_t^e)$ reflects a Phillips' type inflation/employment tradeoff (π_t^e is expected inflation), and the t subscript represents time. A further assumption is that the monetary authorities are able to set any level of inflation π_t they desire i.e. inflation is both the instrument and the state variable.

B&G examine three policy options facing the authorities. One is to conduct policy in a 'discretionary' fashion, another to follow a 'rule', and the other to establish a 'reputational' equilibrium.

3.1.1) 'Discretionary' Policy.

Discretionary policy is defined as the authorities optimising under the assumption that expectations are fixed in each period (and are hence parametric), and that all future costs of inflation are fixed and unaffected by what is done in the present[18]. The policy maker, in minimising (1) under these conditions would set inflation at

$$\hat{\pi}_t = \frac{b}{a}\,. \qquad (2)$$

B&G argue, however, that if agents have rational expectations and understand the authorities' optimisation problem, they realise that this will be the inflation rate and set their expectations accordingly (actual and expected inflation coincide). The value of the loss function in this scenario is

$$\hat{Z}_t = \frac{1}{2}\,\frac{b^2}{a} \qquad (3)$$

and inflation incurs positive costs (a (^) reflects a discretionary outcome). This scenario corresponds to case two in chapter one where, even if the authorities were to announce a

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[18]One could interpret this as reflecting optimisation with an infinite discount rate, and characterise the problem in this way.

level of inflation other than b/a, agents would not believe them, and would expect b/a inflation to prevail.

3.1.2) 'Rules'.

In the 'rules' scenario, the policy maker credibly announces and binds himself to all future policy, which then sets expectations to the same level as the announced and executed policy. With the second term in the loss function now equal to zero, the optimal value for inflation is then

$$\pi_t^* = 0 \quad , \tag{4}$$

and the value of the loss function is

$$Z_t^* = 0 \quad . \tag{5}$$

(values associated with a 'rule' or announcement appear with a (*) superscript). Given that agents are aware that the announcement is binding, this outcome corresponds with case three in chapter one.

The authorities cannot, however, bind themselves to an announcement of zero inflation in B&G's exposition. Although the 'rule' appears to be superior to the discretionary outcome because of the lower value of the loss function, it is not, according to B&G, a feasible[19] equilibrium. This is because the authorities have a temptation to 'cheat' and gain an even lower value of the loss function, and it is assumed that rational agents are aware of this.

A comparison of the gains and costs of this exercise highlights this temptation, and shows that an inflationary bias exists in the economy.

Let us calculate the net gains from cheating (cheating is denoted by a (~)) in period t, assuming $\pi_t^e = 0$. In period t, the

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[19]A feasible equilibrium would, according to B&G, be where actual and expected inflation are equal to each other, and where they are in turn equal to the 'rule' announced by the authorities.

authority gains

$$\tilde{Z}_t = -\frac{1}{2}\frac{b^2}{a} \tag{7}$$

but in period t + 1, when expectations have, by assumption, become $\pi^e_{t+1} = b/a$, the authority is 'punished' by

$$\hat{Z}_{t+1} = q\ \frac{1}{2}\ \frac{b^2}{a} \qquad (0 \le q \le 1) \tag{8}$$

where q is a discount factor. For any discount factor less than one, present gains always outweigh future costs, and the authorities are tempted to inflate.

In the absence of credible binding, rational agents would never expect zero inflation, even if it were announced as such, because of this temptation (recall case two in chapter one where agents did not believe announcements). The incentives facing the authorities militate against a feasible outcome with a 'rule' of zero inflation.

<u>3.1.3) A 'Reputational' Equilibrium</u>.

Given that zero inflation is not a feasible, rational expectations equilibrium, B&G undertake the search for a 'feasible' and optimal 'rule' by utilising the following expectations formation process (which they subsequently claim is 'rational')

$$\pi^e_t = \pi^*_t \quad \text{if} \quad \pi_{t-1} = \pi^e_{t-1}\text{, and} \tag{6)(i}$$

$$\pi^e_t = \hat{\pi}_t = \frac{b}{a} \quad \text{if} \quad \pi_{t-1} \neq \pi^e_{t-1}. \tag{6)(ii}$$

Equation 6(i) indicates that if expectations are fulfilled in the previous period, expectations for the present period are that the announcement or 'rule' will prevail. If on the other hand, expectations are not fulfilled (equation 6(ii)), agents expect that the 'discretionary' level of inflation will prevail in the present period. It is important for our future commentary on this work to note that these equations specify a <u>one period</u> 'punishment' interval, where this 'punishment' is defined as agents setting expectations at the discretionary level. After one period of 'punishment', equation (6) implies that agents revert to expecting the announcement.

B&G find that, given (6), there is a range of 'rules' or announcements, π^* which are stable or enforceable: any deviation from these give rise to inferior values of the loss function. This result is derived firstly by comparing the value of the loss function obtained by announcing and executing any level of inflation π^* in the first period, with the value of the loss function from announcing π^* but setting the discretionary level of inflation, $\hat{\pi} = b/a$ i.e. cheating. This nett, first period gain from cheating can be written as

$$Y_t = \frac{a}{2}(\pi^*)^2 - [\frac{a}{2}(\frac{b}{a})^2 - b(\frac{b}{a} - \pi^*)] . \tag{9}$$

Secondly, these gains are compared with the discounted costs in the following period. The costs in the second period are calculated by comparing the value of the loss function (in period two) when the announcement had not been honoured in the first period, with the value of the loss function when it had. This nett, period two loss is

$$Y_{t+1} = q\,(\frac{a}{2})\,[\,(\frac{b}{a})^2 - (\pi^*)^2\,]. \tag{10}$$

To calculate the range of announcements from which it would not pay the policy maker to deviate, we need to compare the benefits and costs reflected in these two equations.

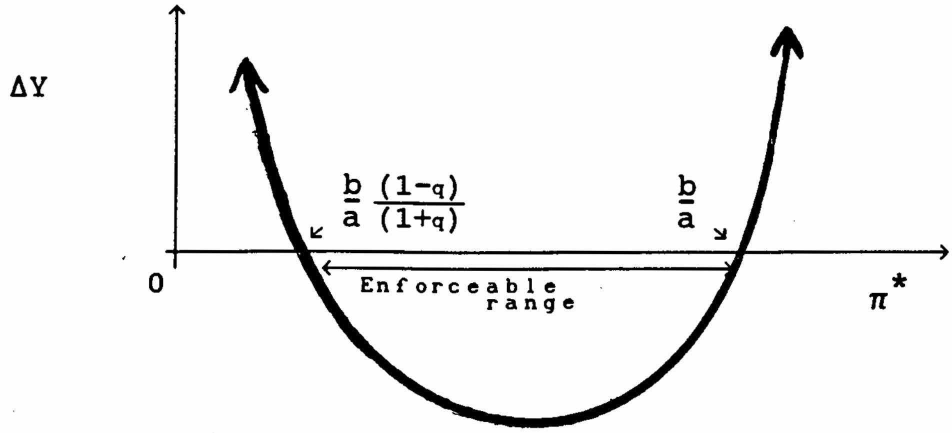
Defining $\Delta Y = Y_1 - Y_2$, we require that $\Delta Y \leq 0$ for there to be no gain from deviating from the announcement.

It is found that $\Delta Y \leq 0$ if π^* lies in the 'enforceable' range $BER \leq \pi^* \leq b/a$, where BER is the 'best enforceable rule':

$$BER = \pi_t = \pi^* = \pi_t^e = \left[\frac{1-q}{1+q}\right]\frac{b}{a} \quad (\,0 \leq BER \leq b/a\,). \tag{11}$$

This can be shown diagramatically as follows (figure one)

Figure one



With inflation announced and set at the BER, the outcome is both feasible and optimal. Agents believe the announcement because of the incentives facing the authorities to follow through with this announcement (recall case three in chapter one).

Having briefly discussed B&G's model, we now address the first of our stated objectives for this chapter.

3.2) 'Reputational' Equilibrium and Time Inconsistency.

It follows from the derivation of the 'enforceable' range that if the government announces any level of inflation in this range, no time inconsistency or pseudo-time-inconsistency results. This is seen by recognizing the absence of any incentive for the authorities to renege on this announcement or to change the actual level of inflation planned for **ex ante**. Changing the announced or planned path leads to an inferior value of the loss function relative to the value of the loss function if the authorities fulfilled their announcements and agents' expectations.

A result of primary importance for our purposes is B&G's implication that pseudo-time-inconsistency is not possible if agents are rational, regardless of what announcement is made. Recall that the enforceable range is calculated on the basis of, among other things, the authorities' incentive to indulge in pseudo-time-inconsistent behavior by making announcements not in this range. These incentives are, however, recognized by rational agents. Any incentive to renege on announcements is always fully anticipated, and there is a net loss associated with this action (compared to announcing and following through with the 'best enforceable' rule. Hughes Hallett (1986) also makes this observation).

The implication of this is simply that although the potential for pseudo-time-inconsistent behaviour is essential to the derivation of the 'enforceable' range, it is never an exploitable policy option because non-rational expectations outcomes are ruled invalid. More importantly, if the authorities opt for a stationary outcome and set announced and actual inflation equal to the BER, no other path of announcements and actual inflation can give rise to superior values of the loss function, and the authorities would not wish to indulge in pseudo-time-inconsistent behaviour.

We can conclude that neither time inconsistent or pseudo-time-inconsistency is exhibited by B&G's optimal policy As in case three in chapter 1, announcements, if they are in the 'enforceable' range, are believed because of the incentives for the authorities to honour them.

3.3) Expectations

B&G claim that the expectations formation process represented by equation (6) above is rational. This rationality is justified by showing that when the authorities' 'rule' is in the enforceable range, a feasible stationary outcome obtains in which expectations are always fulfilled. Two questions emerge about their methodology. Firstly, is equation (6) in itself a sufficient condition for rationality? Secondly, given B&G's loss function, is the expectational behaviour that they ascribe to agents really what we would expect from 'rational' agents?

According to B&G, equation (1) subject to equation (6) is sufficient for the purpose of deriving their enforceable result. To evaluate this claim, assume an initial stationary level of inflation that is less than the 'best enforceable rule'. Specifically, allow this rate to equal zero (actual and expected inflation are zero)[20]. Following the first line of equation (6),

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[20]The argument that follows applies equally well to any starting

6(i), expected inflation will be equal to the value of inflation associated with a 'rule' (which according to B&G "might" prescribe zero inflation (p108), and here is assumed to do so[21]) because past expectations have been 'confirmed':

$$\pi = \pi^e = \pi^* = 0 \ . \tag{12}$$

What happens if the authorities 'suddenly' inflate?

In the first period, inflation would be set equal to the 'discretionary' level[22] (assuming that the discount rate is not zero), and the value of the loss function in this period would simply be the same as in the case of 'fooling' described by equation (7) above. The authorities are, however, 'punished' in the second period by inflation with no employment gain (following equation (6)), although it remains optimal for them to keep inflation in that period equal to the discretionary rate, b/a, and the loss is given by equation (8).

Since expected and actual inflation are equal in period two, period three expected inflation returns to $\pi^e = 0$ according to (6(ii)). Period three thus replicates period one, and this oscillatory process, illustrated below (figure two), may continue in perpetuity, providing lower values of the loss function than $\pi = \pi^e = \pi^* = 0$.

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level of inflation. The boundary condition (i.e. the starting level of inflation) is not crucial to the arguments that follow.

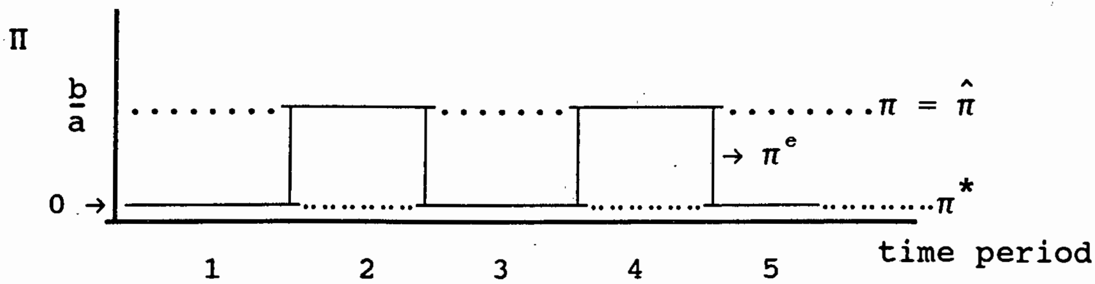
[21]There is no **a priori** reason to eliminate zero inflation as an option that the authorities can choose.

[22]The problem here is to compare the value of the loss function if announced and actual levels of inflation are equal to zero, giving a value of the loss function of zero, against the option of setting inflation at some level other than zero. This particular problem

$$\max L = \frac{a}{2}(\pi)^2 - b(\pi - 0) + q[\frac{a}{2}(\pi)^2 - b(\pi - \frac{b}{a})]$$

gives the optimal value of $\pi = \frac{b}{a}$. The value of the loss function when $\pi = \frac{b}{a}$ is easily shown to be negative, and hence preferred to the option of following through with the announcement (i.e $\pi^* = \pi = 0$).

Figure Two.



.... = actual inflation = $\pi = \frac{b}{a}$.

= expected inflation = π^e.

.... = announced inflation = $\pi^* = 0$.

Equations (1) and (6) are clearly not sufficient to generate <u>only</u> values of announced inflation in the enforceable range.

More importantly, we observe that if the whole range of announced and actual levels of inflation are available to the authorities, oscillatory outcomes, with the announcement in the range ($0 \le \pi^* \le$ BER) and actual inflation, π, equal to b/a, are strictly preferred to stationary solutions with $\pi^* = \pi =$ BER (see the appendix to this chapter).

How then do B&G avoid outcomes such as the oscillatory one identified above? Oscillatory outcomes are eliminated by B&G through an additional assumption that expectations must be stationary at a level of inflation in the enforceable range[23]. This assumption is applied to the system **ex post**, restricting the possible range of <u>announcements</u> or <u>rules</u> that face the authorities. B&G's solution and equation (6) are, consequently, only rendered rational **ex post**.

According to B&G, the authorities' announced level of inflation must fall within the enforceable range of inflation for

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[23]If expectations are stationary, and at a level in the enforceable range, a feasible outcome would obtain. Stationarity of expectations is then <u>sufficient</u> for feasible outcomes to obtain.

agents to believe it. If this is not the case, agents simply disregard equation (6) and form their expectations in some other unspecified way (B&G, p110). It follows immediately from this that we need to know the range of enforceable levels of announced inflation before applying equation (6).

This, however, contradicts B&G's claim that the expectational schema described by equation (6), together with equation (1) is sufficient to generate their result. We only know if equation (6) is going to be operational after the enforceable range of inflation has been derived. Clearly, equation (6) (and (1)) cannot be both generating the result and be contingent on the result at the same time. Potential ranges of announcements have to be restricted **ex post** by the introduction of the stationarity assumption if 'unwanted' outcomes such as the oscillatory one are to be avoided.

To complete this discussion of B&G's expectational schema, we need to consider the possibility of imposing the stationarity assumption **a priori**. We argue below, however, that this is not justifiable on the basis of the optimisation problem facing the government and agents. Specifically, we argue that the imposition of this assumption, **a priori**, would unreasonably and arbitrarily restrict the behaviour of optimising agents.

Recalling that the loss function (equation one) is assumed common to both the authorities and agents, let us construct a pay-off matrix for agents and government.

Authorities' Strategies

Actual Inflation, π = 0, b/a or BER.

Agents' Strategies π^e = 0, b/a or BER.		0	b/a	BER
	0	0	negative	negative
	$\frac{b}{a}$	positive	positive	positive
	BER	positive	positive	positive

BER is the best enforceable rule. Pay-offs are values of the loss function.

Given this payoff matrix, and without arbitrarily restricting the choices of agents and government, both would opt for a strategy that gives rise to a **negative** payoff, and these pay-offs are preferred to <u>all</u> <u>other</u> possible outcomes. Both the outcomes resulting in a negative value of the loss function are the result of what B&G call 'fooling', which they argue, would not be a 'rational' or feasible outcome. 'Fooling', however, would be desired by rational agents, as can be seen from the pay-off matrix. Agents fully understand what level of inflation the government is going to set, but find it optimal, in the context of their optimisation process, to 'set' their expectations of inflation at zero.

Just why agents would not allow themselves to be 'fooled' is not clear, but this behaviour is arbitrarily excluded by B&G's stationarity assumption. Behaviour that agents would clearly prefer to engage in, is assumed away.

Having established how it is that expectations are assumed to operate in B&G's model, we can now turn to other criticisms of the model. Specifically, we examine the implications of the length of the punishment interval postulated by B&G and the form of the loss function for the generality of their results.

<u>3.4)</u> <u>B&G's</u> <u>Punishment</u> <u>Interval</u>.

It will be recalled that B&G assume a punishment interval of one period. Let us change this punishment interval to an equally arbitrary length of two periods. What would rational agents' expectations of inflation be in this scenario?

They know that if they set expectations of inflation at zero, the authorities have a temptation to cheat, the benefit of which is given in equation (7). They are equally aware, however, that they can 'punish' the authorities for two periods after that by setting their expectations of inflation at the 'discretionary' level of (b/a).

There is now not only the punishment associated with the next period, given by equation (8), but the additional punishment of two periods hence given by

$$Z_{t+2} = q^2 \frac{1}{2} \frac{b^2}{a} . \qquad (13)$$

Here it is no longer the case that except for zero discount rates the authorities always inflate. In fact we observe a range of values for the discount factor[24] over which it is optimal for the authorities to set inflation at zero for all periods, and a stationary result of zero inflation does in fact obtain. As the punishment length increases, it is clear that the zero inflation option becomes more likely. B&G's conclusions about inflationary bias are dependent on their arbitrary assumption about the length of the punishment interval.

3.5) The Loss Function.

The arbitrary form of B&G's loss function also impinges directly on the generality of their result[25].

Firstly, changes to the form of the loss function change the 'best enforceable rule', and hence the stationary value of inflation. Secondly, Blanchard and Fischer ((1990), p603) argue

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[24] In this case, values of the discount factor between 0.618 and 1.

[25] B&G do acknowledge this problem, but appear to ignore it when it comes to proclaiming the generality of their results.

that particular forms of the loss function, with all other aspects of the problem left unchanged, can actually generate stationary levels of zero inflation.

3.6) Conclusion

If one accepts B&G's exposition, with all of its problematic assumptions and methodology, it is clear that the BER is indeed the optimal announcement of and actual level of inflation. There is no announcement and actual level of inflation that would give rise to a superior value of the loss function over time. Neither time inconsistency nor pseudo-time-inconsistency is exhibited by this optimal policy. In fact, their result depends on the avoidance of pseudo-time-inconsistent behaviour. This corresponds to case three in chapter one.

If their method is not accepted, we have forwarded one other possible optimal plan which is oscillatory, and <u>does</u> exhibit pseudo-time-inconsistency. It does not, however, exhibit time inconsistency. Our optimal plan corresponds to case one in chapter one.

As regards their assumptions, we have shown that there is in fact a logical contradiction in the way expectations are assumed to influence the 'feasible' outcome, as well as showing that behaviour that optimising agents would choose to engage in is arbitrarily assumed away.

We conclude that general statements about monetary policy and inflation cannot be made on the basis of these results. This contradicts B&G's claim to generality[26].

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[26]For instance, they state in their abstract, "We extend the positive theory of monetary policy...(and) generate predictions about the behaviour of monetary growth and inflation...".

Appendix .

In this appendix, we make the general case for the superiority of oscillatory over stationary solutions[27]. We show that superior values of the loss function are obtained when the authorities make announcements in the non-enforceable region and then opt for the discretionary level of inflation, than when they make announcements in the enforceable region that are adhered[28] to. We proceed by comparing the value of the loss function in each instance.

The Value of the Loss Function.

a) We examine firstly the stationary case where announced and actual inflation are equal for all periods, and the announcement is the best enforceable rule[29]:

The loss over two periods is, in this instance,

$$L_1 = \frac{a}{2}\left[\frac{b}{a}\frac{(1-q)}{(1+q)}\right]^2 + q\,\frac{a}{2}\left[\frac{b}{a}\frac{(1-q)}{(1+q)}\right]^2 \qquad \begin{matrix} a,b > 0 \\ 0 \le q \le 1 \end{matrix}$$

$$= \frac{b^2}{2a}\left(\frac{(1-q)^2}{1+q}\right) \qquad \text{(A1)}$$

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[27]We have already demonstrated one special case of this, namely announcements of zero inflation.

[28]The enforceable range is defined as that range from which the authorities have no incentive to deviate.

[29]We need only concern ourselves with the so-called Best Enforceable Rule (BER) because, as Barro and Gordon show, this is the optimal stationary outcome in the enforceable range.

b) In the second case, announced inflation, π^*, is not in the enforceable range.

Let $0 \leq \pi^* < \frac{b}{a}\left(\frac{1-q}{1+q}\right)$.

The loss over two periods is, given $\pi = b/a$, $\pi_1^e = 0$, $\pi_2^e = b/a$

$$L_2 = \frac{a}{2}\left(\frac{b}{a}\right)^2 - b\left(\frac{b}{a} - \pi^*\right) + q\,\frac{a}{2}\left(\frac{b}{a}\right)^2$$

$$= \frac{b^2}{2a}(q-1) + b\pi^*. \qquad \text{(A2)}$$

We now compare the loss in (2) with the loss in (1).

Taking the difference:

$$L_2 - L_1 = \frac{b^2}{2a}(q-1) + b\pi^* - \frac{b^2}{2a}\left(\frac{(1-q)^2}{(1+q)}\right)$$

$$= \frac{-b^2}{a}\left(\frac{(1-q)}{(1+q)}\right) + b\pi^* \qquad \text{(A3)}$$

which is clearly negative provided $\pi^* <$ BER where BER $= \frac{b}{a}\left(\frac{1-q}{1+q}\right)$.

The oscillatory outcome is superior for any announcement in the range $0 \leq \pi^* \leq$ BER. QED.

Blanchard and Fischer (1990,(a))[30].

4) Introduction.

The next paper discussed, Blanchard and Fischer (1990,(a)) (hereafter B&F(a)), is important in that it attempts a synthesis of the time inconsistency literature from Kydland and Prescott's (1977) paper up until the mid-1980's[31]. Central to this attempted synthsis is a comparison of the values of a uniform loss function obtained under different constraints, as well as a comparison of the values of the loss function obtained when 'different' solution procedures are used to 'optimise' in what is purportedly the same economy.

Our main interest in B&F's work is to see if they avoid the types of errors committed in previous time inconsistency work, in particular those papers that we have reviewed above.

We proceed firstly by setting out B&F(a)'s three models of the economy. Secondly, we examine these models to establish the basis on which the authors claim that they demonstrate time inconsistency. The third section considers B&F(a)'s use of expectations and, finally, the usefulness of comparing the values of loss functions in different economic models is questioned.

Our first conclusion is that B&F(a) perpetuate Kydland and Prescott's error of interpreting two distinct models of the economy as being identical when in fact they are different. Consequently, their claim to have demonstrated time inconsistency (i.e. a violation of the Principle of Optimality) is false. Secondly, we argue that their use of expectations allows for the systematic fooling of agents i.e. the authorities could persistently gain from engaging in pseudo-time-inconsistent

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[30] Blanchard and Fischer (1990, (a)) refers to chapter 11 of the text.

[31] See for example Backus and Driffill (1985), Canzeroni (1985), Rogoff (1985), Cuikerman and Meltzer (1986) and Currie and Levine (1986).

behaviour. Finally, we argue that superior values of a loss function obtained under one set of constraints does not constitute a sufficient condition for arguing in favour of that set of constraints over another set of constraints.

4.1) The Models[32].

Fischer identifies three distinct models of the economy, the so-called 'command' economy, the 'optimal' tax problem economy and the 'consistent' solution economy. It is assumed that there are two periods and the same objective function in all three economies.

4.1.1) The 'Command' Economy.

In this economy, the government maximises the utility of the representative agent, given by

$$U(c_1,c_2,g_2) = \ln c_1 + \delta(\ln c_2 + \alpha\ln(\bar{n} - n_2) + \beta\ln g_2) \quad (1)$$

where c_i is the rate of consumption in period i, n_2 is the amount of work in period two, g_2 is the level of government spending in period two, and $\bar{n}$ is a constant, subject to the following technological constraints

$$c_1 + k_2 \le R_1 k_1 , \quad (2)$$

$$c_2 + g_2 \le an_2 + R_1 k_2 , \quad (3)$$

where k_i is given capital in period i, and a and R_1 are constants.

This economy is a 'command' economy in the sense that government chooses agents' consumption and work effort as well as their own spending. Their instruments are, consequently, c_1, c_2, n_2 and g_2 ($n_1 = 0$). Maximising (1) subject to (2) and (3), government would choose

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[32] B&F(a) draw entirely on Fischer (1980) with regard to their models, and we consequently discuss Fischer's demonstration of time inconsistency.

$$c_1 = [1 + \delta(1 + \alpha + \beta)]^{-1}[a\bar{n}/R_1 + R_1k_1], \tag{4}$$

$$c_2 = \delta R_1 c_1, \tag{5}$$

$$\bar{n} - n_2 = \alpha c_2/a , \tag{6}$$

$$g_2 = \beta c_2 . \tag{7}$$

There is no time inconsistency[33] problem with these optimal values as can be shown by reoptimising in period two subject to the value of consumption in period one. Maximising equation (1) subject to equations (2) and (3), where c_1 is now treated as given (by equation (4)), leaves the remaining first order conditions for a maximium unchanged. Consumption and work decisions would not change.

Sequential optimisation does not alter the optimal path of the governments' control variables, and Bellman's Principle of Optimality is not violated.

4.1.2) The 'Optimal' Tax Problem.

In the second economy considered by Fischer, the government has to use taxes rather than command to bring about desired results, and they (government) do not choose agents' consumption and work decisions directly. There are, consequently, two optimisation problems to be dealt with here, the consumers' problem and the governments'problem.

Consumers optimise (1) subject to constraints reflecting future tax rates. These constraints are

$$c_1 + k_2 = R_1k_1 , \tag{8}$$

$$c_2 = R_2k_2 + a(1-\tau_2)n_2, \tag{9}$$

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[33]There is obviously no pseudo-time-inconsistency either. No announcements are necessary in this economy as agents are instructed by the authorities as to what they must do, and presumably comply with these instructions. This section, although it contains no time inconsistency of either type, is included for completeness.

where R_2 is the after tax rate of return on capital and τ_2 is the tax rate on labour income in period two.

The consumer forms expectations[34] of R_2 , τ_2 and g_2 , R_2^e , τ_2^e, g_2^e , and the first order conditions for a maximum are

$$c_1 = [1 + \delta(1 + \alpha)]^{-1}[a\bar{n}(1 - \tau_2^e)/R_2^e) + R_1k_1], \quad (10)$$

$$c_2 = \delta R_2^e c_1, \quad (11)$$

$$\bar{n} - n_2 = \alpha c_2/a(1 - \tau_2^e) \quad (12)$$

The government in turn maximises (1) subject to equations (10) to (12), as well as their budget constraint

$$g_2 = (R_1 - R_2)k_2 + \tau_2 a n_2. \quad (13)$$

The government needs to know, however, what private sector beliefs are before they themselves can maximise. <u>Assuming</u> that expected and actual values coincide[35], a set of first order conditions can be calculated which, in general, give values for τ_2 and R_2 that are non-zero[36]. These are solutions to the following pair of simultaneous equations.

$$a\bar{n}(1 - R_1/R_2)(1 - \tau_2)/\alpha\delta R_2) + \tau_2 R_1 k_1/(1 - \tau_2) = 0 \quad (14)$$

$$[R^2k_1\delta(1 + \alpha) + a\bar{n}(1 + \delta)] - \delta R k_1 R_2[1+\alpha/(1 - \tau_2) + \beta]$$
$$= a\bar{n}(1 - \tau_2)[R_1/R_2 + (1 + \beta)\delta] \quad (15)$$

The optimal value of government expenditure in period two is

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[34]It is not clear where the expectations suddenly come from in Fischer's exposition, what values they take on, or how they are derived. We deal with this important issue in some detail in the section on expectations.

[35]Fischer claims that they must coincide for a rational expectations equilibrium to obtain.

[36]This result is an open loop control for a non-causal model. The implications of non-causality are discussed in section 4.2.

given by

$$g_2 = \beta c_2 \quad . \tag{16}$$

4.1.3) The 'Time Consistent' Economy.

It is Fischer's claim that the results calculated in (14) and (15) above are time inconsistent: if the government re-optimised in period two, taking all past values of variables as given, the optimal values for the choice variables in period two would differ from those calculated in period one - an apparent violation of the Principle of Optimality. With c_1 given, the consumer's objective is

$$\text{MAX } U_2(\ ) = \ln c_2 + \alpha \ln(\bar{n} - n_2) + \beta \ln g_2, \tag{17}$$

$$\text{s.t. } c_2 = R_2 k_2 + (1 - \tau_2) a n_2 \ , \tag{18}$$

taking R_2 , τ_2 and g_2 as given i.e with expectations R_2^e , τ_2^e , g_2^e equal to these known values. The first order conditions are

$$c_2 = (1/(1 + \alpha))[a\bar{n}(1 - \tau_2) + R_2 k_2], \tag{19}$$

$$\bar{n} - n_2 = \alpha c_2 / a(1 - \tau_2). \tag{20}$$

The government, maximising the same utility function subject to equations (13), (18) and (19) sets

$$\tau_2 = 0 \tag{21}$$

$$R_2 = [1 + \alpha + \beta]^{-1}[R_1(1 + \alpha) - \beta a\bar{n}/k_2]. \tag{22}$$

These optimal values differ from the optimal values calculated in period one (compare with equations (14) and (15)), and, it is claimed, time inconsistency has been demonstrated. This result is then contrasted with the 'consistent' solution to the problem.

The 'consistent' solution (equations (21) and (22)) is derived by Fischer using Dynamic Programming methods (see the chapter on Kydland and Prescott where the association is made between consistent solutions and Dynamic Programming methods). This entails optimising backwards in time from the final period.

In period two then, agents and government, treating the past as a bygone, optimise. This is the problem solved in equations (17) to (22) above, and gives values for second period variables on the basis of which both agents and government can optimise in period one (τ_2, R_2, g_2 are known in period one).

The 'optimal' values of the instruments obtained when using the 'consistent' method of solution differ, in general, to those obtained when using the 'inconsistent' method. Fischer then argues that the optimised value of the objective function in the consistent case[37] is also, in general, lower than that in the 'inconsistent' case.

The similarity between this demonstration of 'consistent' and 'inconsistent' solutions and Kydland and Prescott's demonstration of the same phenomenon in chapter 2 is not coincidental. Fischer's demonstration, although more sophisticated than Kydland and Prescotts', is in principle the same. As a consequence, his attempt to demonstrate 'time inconsistency' perpetuates the same logical errors identified in our section on K&P.

4.2) Inconsistency in Fischer's 'Optimal' Tax Example.

It will be recalled that in chapter 2, we argued that Kydland and Prescott's comparison of the optimal and consistent policy was invalid because different economies were being compared. Similar critisism can be levelled at Fischer and B&F. Consider the 'optimal' and 'consistent' economies again.

In the 'optimal' tax example, the government optimises subject to the constraint that agents' consumption in period one is a function of actual policy tomorrow. This can be read directly from equation (10) where agents' optimal consumptive decision in period one is a function of R_2 and τ_2. In a non-causal model of this sort, the only way to solve the problem is **ex ante** and there is no scope for solving it sequentially and retaining the non-causal dependence reflected in equation (10). In fact, the

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[37]The consistent result does not of course suffer from the problem of time inconsistency.

notion of real time sequential optimisation in a non-causal model lacks meaning. In the consistent example the problem is no longer the same.

In this problem, government optimises subject to the constraint that consumption in period one is a fixed parameter and is no longer a function of R_2 and τ_2 . This follows immediately from the observation that in period two, when R_2 and τ_2 are chosen, c_1 is a fixed number that cannot be influenced by the values of R_2 and τ_2 (which it is in the 'optimal' economy).

The economies are different in each case, and it is for this reason only that the optimal policies differ. Neither the 'optimal' tax problem nor the 'consistent' problem suffer from time inconsistency, as we discovered in the chapter on Kydland and Prescott.

4.3) Expectations in the 'Optimal' Tax Problem.

Unlike K&P, Fischer attempts to treat expectations explicitly in his solution of the 'optimal' tax problem. In this model, consumers maximise (1) subject to (8) and (9). Since their optimal behaviour is assumed to depend on expected government action in the future, their first order conditions ((10) - (11)) are functions of expected government controls, τ_2^e , g_2^e , in period two.

The government, in its turn, maximises subject to expected consumer behaviour; hence to make the problem soluble it must be the case that the government knows how consumers expectations of its own behaviour are formed.

To circumvent the problem of just what those expectations might be (and how they are formed), Fischer invokes the equilibrium rational expectations condition that actual values of the control variables equal the expected values of the control variables (recall Barro and Gordon's **ex post** restrictions on outcomes). Once this is assumed, the optimal values of R_2 and τ_2 can be calculated by government (equations (14-15)). This implies that the government is solving not only for their own optimal values of the controls, but for agents' expectations of these controls too (the values of the controls and expectations are arrived at simultaneously). The model is thus non-causal in the same sense as K&P's model: agents' current actions depend on actual future government policy.

A further implication of the observation that government determines agents' expectations is that agents would be 'fooled' in period two (and could be systematically fooled if the model was extended to more than two periods). Government can announce[38] any future level of the control with the full knowledge that this is what agents will expect (this corresponds to case one in chapter one). With this power to preset expectations, the governments' optimal behaviour in period two is to renege on its announcement, thus exhibiting pseudo-time-inconsistency.

This 'switch' in policy is identified incorrectly by Fischer as 'time inconsistency' of the original optimal policy. The reason that he is incorrect follows from his failure to distinguish adequately between announced and actual levels of the controls (recall case one in chapter one). Government knows in period one what the <u>actual</u> level of the control variables will be in period two, and in this sense there is no time inconsistency problem.

To recognise this, we need only point out that the government could conceivably conduct the <u>same formal exercise</u> as Fischer did to show 'time inconsistency', and in the process, 'discover' the gains that are possible from pseudo-time-inconsistent behaviour. The implication of Fischer's use of 'inconsistency' in this context implies government sets up its optimal plan in period one and is suprised to discover in period two that this plan is no longer optimal. This is clearly not the case. They are fully aware in period one what the path of <u>actual</u> levels of the control will be in period two, and they are similarly aware that their announced levels of the controls will in fact not be adhered to. Government optimally <u>cheats</u>.

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[38]The issue of announcements is taken up immediately below.

4.4 Comparing Values of the Loss Function.

Fischer's and B&F(a)'s work on the question of dynamic inconsistency, although incorrect as we have demonstrated, raises the important question of the comparison of the values of loss functions derived in <u>different</u> economies i.e. the value of the same loss function under different constraints.

The methodology of comparing the values of loss functions in different economies is utilised by B&F(a) to make substantive claims as to the superiority of one system over another. This is reflected in their ranking of the 'command optimum', the 'fooling or inconsistent' result, the 'precommitted' outcome[39] and the 'consistent' outcome (p596). The rankings are as follows

$$U^* > U_f > U_p > U_c$$

where the (*) superscript denotes the command optimum, and subscript f, p and c stand for fooling (i.e pseudo-time-inconsistent), precommitted (**ex ante** solution of the optimal tax problem) and consistent respectively.

Is it, in the context of B&F's demonstration[40], justifiable to argue in favour of a command economy? This is undeniably an important question given the proclivity in the literature for utilising such simple models to make broad policy recommendations (such as the 'rules' over 'discretion' debate about policy making).

We argue that different economies cannot be compared unless the costs of restructuring the economy in a particular way are also considered. For instance, we have alluded earlier to the implicit constraints on government in the 'precommitted' example. What form do these constraints take, and what are the costs of imposing them? Following Demsetz (1969), we argue that not only

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[39] Strictly speaking, Fischer and B&F only acknowledge that the command and 'optimal' economies differ in terms of their constraints.

[40] Assuming different economies in each case.

must the economy be considered after we have imposed a particular structure on it, but also that the costs of reaching that new institutional arrangement need to be included in a valid comparison.

4.5) Conclusion

In conclusion, we do indeed observe that Fischer and B&F perpetuate the same error as Kydland and Prescott (1977), namely the mistaken belief that different optimising procedures in the same economy are being compared when in fact different economies (i.e. models with different constraints) are being compared. We have also demonstrated the problems with the expectational schema of Fischer's, and shown how it implies the fooling of agents.

Blanchard and Fischer (1990, (b))[41].

5) Introduction.

Our previous chapters have examined the issue of inconsistency from a number of differing perspectives, but a common theme has been in evidence throughout: expectations are believed to give rise to the problem of 'time inconsistency'. In this chapter we examine the issue of time inconsistency in a slightly 'purer' setting where expectations do not feature.

We proceed by setting out another of B&F's examples in which they purport to demonstrate time inconsistent policy. We then argue that their procedure is logically flawed, and that what they claim is originally an optimal policy cannot in fact be optimal.

5.1) The Model.

B&F(b) present an infinite horizon problem in which utility is maximised over the interval (s to ∞). The objective function is

$$U_s = \int_s^{\infty} u(c_t)\, D[t, t-s, x(t)]\, dt \,. \qquad (1)$$

Utility (U) is a weighted integral of utility flows at different times, where the weighting function or discount function, D is a function of calendar time, t, time 'distance', t-s, and possibly other variables, x(t) (which B&F(b) ignore in their analysis). The absence of any stochastic terms reflects full certainty.

Using this formulation of the problem, B&F compare marginal rates of substitution between consumption at different future times when viewed from different planning dates . Assuming that

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[41] Blanchard and Fischer (1990, (b)) refers to chapter 2 of the text, especially pp 70-72.

$$t_2 > t_1 > \tau_2 > \tau_1 >, s$$

where the t's are the dates planned for and the τ's are the planning dates, B&F claim that different rates of substitution will obtain between consumption at t_1 and t_2, depending on the planning date τ_1 or τ_2 that happens to be chosen. They illustrate this as follows. At planning time τ_1 we have rates of substitution[42]

$$\frac{u'(c(t_1))\ D(t_1,\ t_1 - \tau_1)}{u'(c(t_2))\ D(t_2,\ t_2 - \tau_1)} \qquad (2)$$

and at planning time τ_2,

$$\frac{u'(c(t_1))\ D(t_1,\ t_1 - \tau_2)}{u'(c(t_2))\ D(t_2,\ t_2 - \tau_2)} \qquad (3)$$

B&F assert that these two rates of substitution are not equal, because in general,

$$D(t,\ t - \tau_1) \neq D(t,\ t - \tau_2) \quad \forall\ \tau_1 \neq \tau_2 .$$

Optimising at time τ_1 by setting the rate of substitution equal to some known future price ratios would not be the same as the optimal rate of substitution as of time τ_2, and we have a 'time inconsistent' result.

This implies that as we move forward in calendar time, the rate at which we discount the future changes, giving rise to different optimal paths (recall our definition of time inconsistency in chapter one). In this continuous time setting an

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[42] By setting up a present value Hamiltonian function (see B&F(b), p39 where a technological constraint on equation (1) is considered), we can derive the Keynes-Ramsey rule that the marginal rates of sustitution (MRS) in consumption between times t and $t + \Delta t$ (where $\Delta t > 0$) equals the marginal rates of transformation between the same points in time. For the purposes of B&F(b)'s demonstration of inconsistency, however, only the MRS between consumption needs to be considered.

infinite number of optimal paths for the state variable result as we move forward in time i.e. as the planning point changes.

This result is suprising, for, although no new information has become available, the optimal path keeps on changing[43]. It appears that time inconsistency has indeed been demonstrated, and that there is a violation of the Principle of Optimality (Bellman (1957)). Our criticism of this contention is really a re-statement of Bellman's principle.

Let us start by assuming that at time (s), a planner has identified the optimal path through until (t). Let us also assume that at time (s), this planner decides to experiment by dividing the entire planning interval (s → t) into two arbitrary intervals. Let the dividing point in time be (τ).

Using B&F(b)'s construction of the optimisation problem, the planner should find when recalculating the optimal path at time (τ) that it (the optimal path) calculated at (t) is no longer optimal. The important point to note is that in a certain world, all this can be done at time (s) because there are no information suprises as we move through time.

What can the planner ascertain from this disparity? If we assume that the sub-path (τ → t, calculated at τ) is indeed optimal, then at time (s) it must be clear to the planner that the first calculation and path (the entire interval s → t) was in fact not optimal. The planner could recalculate at time (s) to achieve a superior outcome. Certainly, if the planner knows <u>now</u> how things are to change in the future, these changes must be incorporated into the optimal plan initially to render it optimal in a meaningful way, not at some later stage when the change occurs and recalculation is deemed necessary (when the planner "suddenly" finds that the original 'optimal' plan no longer holds[44]).

In a world of certainty B&F's demonstration must be false,

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[43]Note that unlike any of the previous work reviewed, this result does not rely on non-causal models nor on any assumed expectational schema.

[44]Recall our discussion in chapter one on this very issue.

and the form of the discount function is irrelevant as long as we are certain at the beginning of the period how it operates over the entire interval. Inconsistency would only be possible with myopic planners.

One more demonstration that optimal policy is time inconsistent has been shown to be incorrect.

6) Conclusion.

The purpose of this thesis has not been to argue that control theory is applicable in economic settings, or that 'rules' are unimportant to the workings of the economic system. The primary purpose has been to demonstrate that the notion of 'time inconsistency' identified in the literature is logically flawed, and hence cannot form the basis for arguments in favour of 'rules' and against control theory.

The rejection of the idea that optimal policies may be 'time inconsistent' has centred around the logical impossibility of time inconsistent 'optimal' policies. It will be recalled that the definition of an optimal policy forwarded by Bellman (see chapter 1) explicitly excludes the possibility that the optimal plan may change upon reoptimisation through time (see also chapters 4 and 5). The literature, however, has claimed that reoptimisation does indeed give rise to different paths of the instrumental variables.

The logical impossibility of the optimal plan being 'time inconsistent' suggests that the literature purporting to demonstrate this phenomenon must be erroneous. Three principal sources of error were identified.

a) Firstly, the comparison of causal and non-causal models to demonstrate time inconsistency was shown to be invalid. Simply, different models of the economy, reflected in their different constraints, were being compared (see chapters 2 and 4).

b) Secondly, it was argued that appearances of time inconsistency were in fact simply pseudo-time-inconsistency i.e. a divergence of announced and actual values of the instrumental variable/s (see for example chapter 3). Neither the announced nor the actual planned paths changed upon reoptimisation.

c) Finally, it was argued that the use of expectations to generate 'inconsistent' outcomes was methodologically unsound. More specifically, it was argued that the methodology utilised in the literature to incorporate expectations into the optimisation exercise is contradictory. Consequently, the demonstration of 'inconsistency' based on this methodology is spurious (see for example chapter 4)

Notwithstanding these errors, the literature has made an important contribution to the theory of policy making. This contribution has taken the form of emphasising the role of credibility in policy formulation. A lack of credibility on the part of the policy maker may induce agents to behave in a way that gives rise to inferior values of the social objective function. This is well illustrated in Kydland and Prescott's flood plain example and Barro and Gordon's inflation example. The intuitive appeal of these results, although formally wrong, probably provides the reason why the time inconsistency literature has achieved such prominence.

The upshot of these observations is the somewhat counter-intuitive result that it may be optimal to have more rather than fewer constraints on government, constraints that impart credibility to their announcements. The incentive to cheat is, we argue, the 'true' nature of the inconsistency problem with which the literature has been grappling. Once it is understood that the notion of 'time inconsistency' cannot inform decisions about the nature of the constraints that obviate the incentive to cheat, a more focussed research program might emerge.

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