University of Cape Town
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Mirror, Mirror, on the wall, will the market rise or will it fall?
A study into the effectiveness of Japanese Candlestick Charting on the Johannesburg Stock Exchange from 2010 to 2019

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Submitted in partial fulfilment of the requirements for the degree of Master in Commerce in Finance in the field of Corporate Finance and Valuations

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Abstract

Using the methodology developed by Caginalp and Laurent (1998), this study tests the predictive ability of eight three-day reversal candlestick patterns on a sample of sixty listed shares on the Johannesburg Stock Exchange (JSE) from 1 January 2010 to 31 December 2019. The study further investigates the predictability of the patterns on three sub-sectors of the JSE; this included the resource, financial and industrial sectors. As the final step, the study tests the use of the candlestick pattern as a trading strategy before and after costs. To the authors knowledge, it is the first study of candlestick patterns on South Africa's JSE. However, similar studies have been conducted in other emerging markets such as Thailand, Taiwan and Brazil. The study finds that only one of the candlestick patterns, the Three Outside Down pattern, has predictive ability on the JSE. However, when looking at the JSE sub-sectors, the results are more divergent. Candlestick patterns have no predictive ability on the resources and financial sector but on the industrial sector three patterns (Three Inside Up, Three Black Crows and Morning Star) were significant at the ten percent level of significance; and the Three Outside Up and Three Outside Down were at the five percent level of significance. The study finds that after fees the use of candlestick patterns is unable to outperform passive benchmarks. The evidence suggests some violations of the Random Walk Theory, but the study finds that the JSE and its sub-indices are weak-form efficient in terms of the Efficient Market Hypothesis. The Thai and Brazilian markets were also found to be weak-form efficient.

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1. Introduction

One of the most influential concepts in finance is the Efficient Market Hypothesis (the EMH). The EMH is the idea that market prices incorporate all new information on an asset rationally and instantaneously. In an informationally efficient market, price changes must be unforecastable; which is suggested by the random walk theory (the RWT). The more efficient the market, the more random the sequence of price changes. The most efficient market is one in which price changes are completely random and unpredictable (Lo, 2004).

However, an increasing body of evidence contradicts the predictions of the EMH. For example, when United States equity market constituents from January 1926 to December 1987 were ranked according to their previous twelve-month performance, the top decile of constituents significantly outperformed the bottom decile over the next twelve months (Jegadeesh, 1990). This serial correlation has more recently been termed momentum. Asness et al. (2014), in a study covering 40 equity markets over the period 1801 to 2012, found momentum in various markets and across different time periods. Studies such as those of Jegadeesh (1990) and Asness et al. (2014) were conducted using monthly return data. However, studies on higher frequency (daily and weekly) data have been mixed (Smith, 2008). For example, in South Africa, Smith, Jefferis and Ryoo (2002) conducted a random-walk test on eight of the largest African equity markets using weekly data from January 1990 to August 1998. The authors found that the South African market followed a random-walk. In contrast, a test of weekly data on the -Johannesburg Stock Exchange (JSE) and its sub-indices from January 1993 to March 2001, found a divergence in random-walk findings. Of the six indices tested, the All Share Top 40, Industrial 25 and Gold indices all followed a random walk. On the other hand, the Mid Cap, Small Cap and broader Industrial Index failed the random walk test (Jefferis and Smith, 2004). These studies highlight the potential of the use of technical analysis in the longer-term (for periods greater than one year), while raising questions about its use over short-term time horizons (daily or weekly).

Technical analysis is the use of past price and trading volume trends to infer the direction of future prices. Technical analysts argue that the technique allows them to profit from changes in the market psychology (Neely, Wheller and Dittmar, 1997). This is observed as changes of supply and demand, and ultimately through price action (Brock, Lakonishok and LeBron, 1992). In a world where the weak-form EMH holds, technical analysis would be useless. Empirical studies have been mixed. For example, studies by Brock, Lakonishok and LeBron (1992) and Lo (2004) find that technical trading rules add value, whilst early studies, such as that of Levy (1971), find that these rules have very little predictive value.

These inconclusive results have not stopped technical analysis from being widely used by practitioners, however. Thus, for example, Schwager (1989), in interviews with top traders, documents a common theme - technical analysis played an integral role in their speculative or investment process. All the traders interviewed discussed the importance of identifying the trend, and many used technical analyses to gauge the mood of the market. Fundamental analysts, that is, investors that use publicly available information in the decision-making process, often question the value of technical analysis. However, “fighting the tape” or “catching a falling knife” – trying to buy near the market bottom – are problems often also faced by the fundamental investor. Technical analysis, for the fundamental analyst, may play a role in determining entry into a position, and the subsequent exit.

A technical analysis tool that features relatively seldom in the literature is Candlestick Charting (Candlesticks). Candlesticks were first developed by Muneshisa Homma in Japan in the 18th century and in 1991, the technique was introduced to the western world by Nison (Lu, 2014; Prado, 2013; Lu, Shiu and Liu, 2012). The earliest form of candlestick charts was used in the Dojima Rice Exchange in Osaka, Japan. Although the market started as a central point for the physical exchange of rice in the late 1600s, after 1710, the exchange began to issue and receive rice warehouse receipts called rice coupons. These rice receipts were the first futures contracts. The Homma family had a prominent rice farming estate and it through this estate that Muneshisa Homma had access to rice market information. This included records of weather and pricing information from the inception of the exchange that allowed Homma to learn about the psychology of investors. Homma’s books and trading principles evolved into the candlestick methodology used today (Nison, 2001).

Candlesticks read changes in the makeup of value perceived by investors in financial markets (Morris, 2006). What differentiates the technique from other technical analysis tools, such as momentum and moving averages, is that it is more than a trend following strategy; that is, it also attempts to predict the potential reversal of the trend and attempts to visually depict the psychological component of the market.

Candlesticks use the open, high, low and closing price of a financial instrument as a source of information. The rectangle section of the candlestick (refer to Figure 1) is known as the real body (or “body”, in the remainder of this document); this represents the price range between a specific trading session’s open and close. A filled body means that the closing price of the session was lower than the opening price, whilst an empty body indicates that the close was higher than the open. The vertical lines above and below the body are known as the shadows. The upper shadow (above the real body) shows the highest price achieved during the session. The lower shadow (below the real body) shows the lowest price reached during the session. Candlesticks can be drawn using any time frame. However, it is recommended that it used over the short-term (one to seven days) (Morris, 2006). A candlestick pattern can consist of one candlestick, or as many as three. The technique can be used in equity, futures, options and foreign exchange markets.

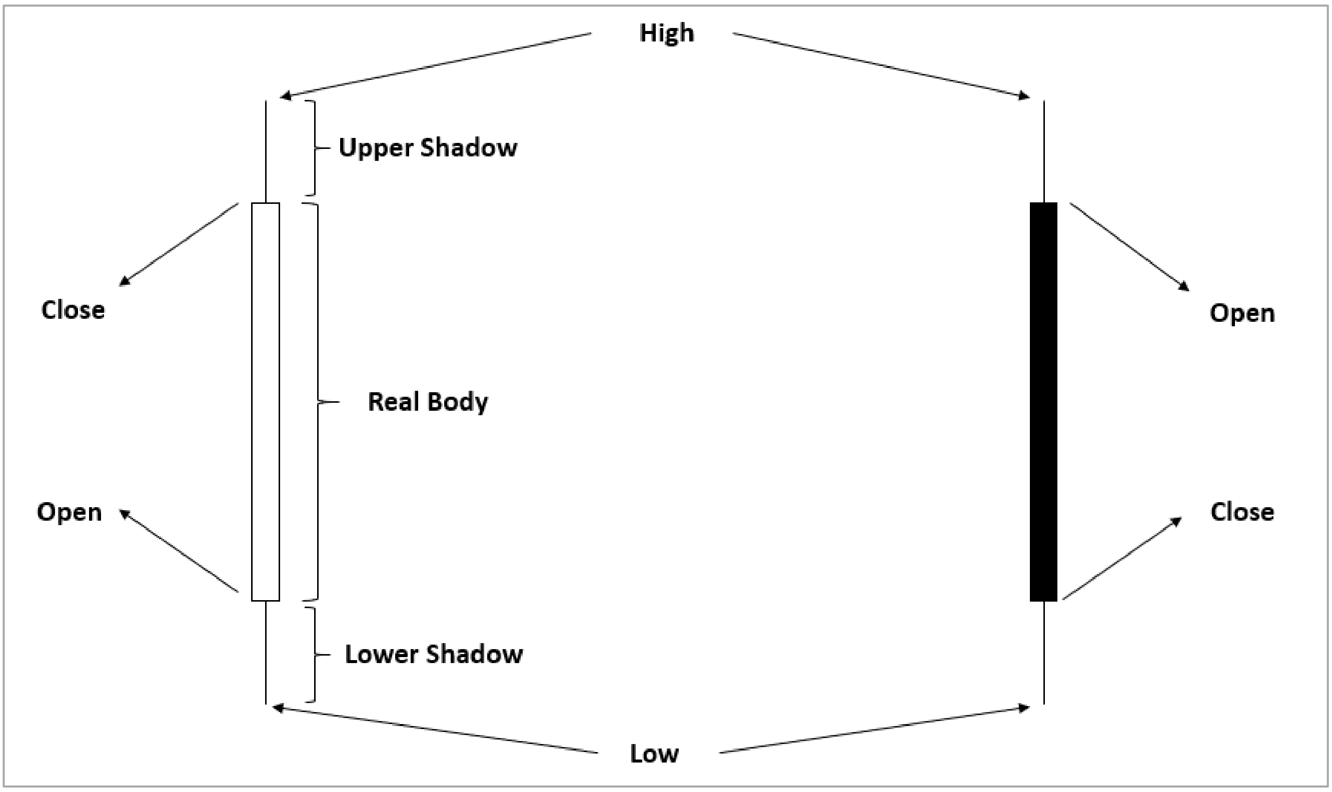

Figure 1: Features of candlesticks
Source: Nison (2001)

Candlestick patterns fall into two general categories: the first group indicates a reversal pattern, and the second indicates a trend continuation. The differentiator of candlesticks as a technical analysis tool is that it attempts to indicate turning points (reversals) of the market. Thus, most candlestick signals are reversal patterns. The term reversal is often thought of as an old trend ending abruptly and moving in the opposite direction – this is rare. A trend reversal signal implies that the prior trend may change; it is therefore a signal of caution, rather than a precise indicator. The reversal signal often requires a confirmation, soon after the signal, that the prior trend has reversed. An important principle is to initiate a new position only if the signal is in the direction of the major trend. For example, in a bull (positively trending) market, a bearish (negative) signal would not warrant a short sale. However, it may signal a sale of long positions (Nison, 2001).

A common problem with candlesticks (as with many technical analysis patterns) is that there is often subjectivity in deciding whether a certain candle formation meets the guidelines for a particular formation (Nison, 2001). In order to measure the success of a pattern, the pattern must be identified, but it is dependent on the trend. Lastly, there is a prediction interval to measure the success or failure of the candlestick signal (Morris, 2006). This time period varies for each user.

1.1 Purpose and research question

The purpose of this study is to empirically test Japanese Candlesticks Charting, a technical analysis tool that features less in the literature, within the context of the South African listed equity market. The study investigates the power of eight reversal patterns as defined by Caginalp and Laurent (1998). This includes four bullish reversal patterns and four bearish patterns. The bullish patterns consist of the Three White Soldiers (TWS), Three Inside Up (TIU), Three Outside Up (TOU) and Morning Star (MS) patterns. The bearish patterns consist of the Three Black Crows (TBC), Three Inside Down (TID), Three Outside Down (TOD) and Evening Star (ES) patterns.

The study is also a test of market efficiency, which is of particular importance for the finance practitioner. For the asset allocator, in an efficient market, the incentive to allocate to active managers is diminished. In aggregate, equity fund returns must equal those of the market and will ultimately be short of the market returns by the amount of costs (Bogle, 2005). Greater market efficiency makes the task of choosing an investment manager increasingly difficult because as market efficiency increases, the only way an investor generates superior returns will be through luck or taking additional risk (Fama, 1965; Malkiel, 2003). In an efficient market, the role of passive investments increases.

The research questions are therefore:

1. Do candlestick patterns have predictive ability on the JSE and its sub-indices?
2. Is the JSE weak-form efficient?

The hypothesis of the study is that at least some of the eight candlestick patterns being tested will have predictive ability.

1.2 Novelty and Contribution

Up to this point the candlestick technique has not been tested on the South African equity market. The South African equity market is of interest because, from an economic perspective, South Africa is an emerging economy, but the country's financial markets share more characteristics with developed markets (World Bank, 2020). In addition, in a study of fifty equity markets, including both emerging and developing markets, South Africa was found to be the sixth most inefficient, indicating its suitability for a study that hypothesises weak form inefficiency (Lim and Brook, 2010). For the practitioner, the question of market efficiency will always be an important one, as where there are inefficiencies there are opportunities (Lim and Brook, 2010). In South Africa, from 1988 to 2005, South African equity funds underperformed the market by over five percent, raising questions of fees and market efficiency (Malefo, Hsieh and Hodnett, 2016).

This study fills the current gap in the literature with regards to the use of candlestick share price patterns as investment tools, which have not been tested on the South African equity market. The study is not only a test of the random walk theory on the South African equity market, but goes a step further in determining whether the weak-form Efficient Market Hypothesis holds. Further, the study covers the period from 1 January 2010 to 31 December 2019, and is thus a good representation of the current equity market.

1.3 Dissertation structure

The remainder of the study proceeds as follows. Chapter 2 provides an overview of the literature on the RWT, the EMH, Behavioral Finance and previous Candlestick studies. In Chapter 3, the data and methodology used in this study is discussed. This is followed by Chapter 4, where the results and findings of the study are presented. Chapter 5 concludes and presents recommendations for future studies.

2. Literature Review

Tests of traditional finance theory have found that it does not always accurately reflect the reality of financial markets. Many studies have found evidence inconsistent with the RWT and the EMH[1]. However, data frequency appears to be a factor in determining each study's findings; high frequency daily data tend to suggest randomness, while lower frequency data does not (Smith, 2008). Differences between emerging and developed markets are also evident (Lim and Brook, 2010). More recently, behavioural finance research – that is the study of individuals' biases and their limit to rationality - has grown in prominence as a bridge between theory and observed behaviour. However, in the face of contradictory evidence, behavioural finance does not necessarily provide predictive power.

This study serves, by implication, as a test of the EMH with respect to the JSE. In this chapter, academic literature relevant to both the RWT and the EMH is reviewed, with specific reference to empirical studies that highlight the shortcoming of both theories. Further, empirical studies conducted in South Africa is presented, followed by behavioural literature that highlights several reasons for the mismatch between traditional finance theory and empirical evidence. Thereafter, given that this is the basis of this study, the empirical literature on Japanese Candlestick charting and its predictive power are reviewed as justification for its use as a test of the RWT, and ultimately the weak-form EMH, on the Johannesburg Stock Exchange, using high frequency data.

2.1 The Random Walk Theory (RWT) and the Efficient Market Hypothesis (EMH)

The RWT holds that a series of stock price movements are completely independent of each other. The implication of this theory is that the future path of prices is random (Fama, 1965), and can therefore not be predicted. The EMH, which evolved from RWT, proposes that security markets are extremely efficient in reflecting new information about stocks (Malkiel, 2003). The connection between the RWT and EMH arises from research conducted by Paul Samuelson, which concluded in an informationally efficient market price change must be unforecastable (random) (Lo, 2004).

Market efficiency holds when there is a large number of rational profit maximisers actively competing with each other in an attempt to predict future market values of securities, and where important current

[1] See for example, Leroy (1973), Lucas (1978) and Lo and MacKinlay (2002). The authors highlight inconsistencies between the RWT and the EMH

information is almost freely available to all market participants. In addition, it requires there to be no trading costs (Fama, 1965).

Fama (1965) separated the EMH into three categories, with each successive level being increasingly stringent in its assumptions. The first, the weak-form, states that information in past prices is reflected in the current price of a security. The second, the semi-strong form, states that all public information is reflected in the current price of a security. Lastly, the strong form states that all information – public and private – is reflected in the price of a security.

If markets are weak-form efficient, technical analysis – the study of past prices for an informational advantage with regards to future prices – would not enable an investor to generate returns greater than that of the market. A consequence of the semi-strong efficiency is that fundamental analysis – the study of public information – would not be to the investor's advantage. Lastly, if strong form efficiency holds in a market, it is impossible for an investor to generate superior returns without luck or taking more risk (Fama, 1965; Malkiel, 2003).

2.1.1 Empirical evidence on the RWT and EMH

Early evidence supported both the RWT and the EMH. The most prominent methodology in the early literature used tests of serial correlation. For example, Kendall (1953) found evidence of the random walk for both British industrial companies and on spot prices of cotton (in New York) and wheat (in Chicago). Granger and Morgenstern (1963), using spectral analysis of weekly New York Stock Exchange returns, concluded that the series obeyed the simple RWT. However, with such strict assumptions, it is acknowledged that it is unlikely that the RWT and the EMH provide an exact description of stock market behaviour (Fama, 1991).

Not long after the RWT was proposed, empirical studies found empirical evidence inconsistent with the theory (Jensen, 1978). For example, Niederhoffer (1965) found that in a sample of 141 successive ticks at $\frac{8}{8}$ and $\frac{7}{8}$, an advance followed 94 times. However, in a sample of 135, where the successive ticks occurred at $\frac{6}{8}$ and $\frac{7}{8}$, a price advance only occurred 37 times. According to the RWT, the proportions of advances and declines should be equal. The sample studied included 700 randomly chosen stocks on the New York Stock Exchange on a single day. The author concludes that these findings are due to the mechanism of the auction market - stock prices show regularities and structures in their movement. In a similar study done on six of the first seven stocks on the Dow Jones Industrial

Index for twenty-two trading days of October 1964, Niederhoffer and Osbourne (1966) found that after two prices changes in the same direction, the odds of a continuation in that direction are almost twice as great as two changes in the opposite direction.

More recently, momentum – the relationship between past and future stock prices – features prominently in the literature. This differs from the earlier empirical work in that it places greater emphasis on longer-term relationships. For example, there is strong evidence of twelve-month serial correlation. Jegadeesh (1990) formed ten portfolios based on the rank of the stocks in terms of their trailing one-month and twelve-month performance (that is, the top decile shares from a return perspective formed one portfolio and the second decile formed the next, until ten portfolios were formed). The data used in the study included United States equity market constituents from January 1926 to December 1987. These portfolios were then rebalanced after 12 months based on the previous steps. The top quartile performance based on twelve-month lag and one-month lag outperformed the lowest portfolio by 1.99 percent and 0.93 percent, respectively. The study was conducted on monthly share data from January 1926 to December 1987 on the United States equity market. Asness et al (2014), in a study covering 40 equity markets over the period 1801 to 2012, found momentum in various markets and across different time periods. Lo (2004) highlights several studies that find randomness is neither a necessary condition for rationally determined share prices. Unforecastable prices do not imply an efficient market, and forecastable prices do not imply the opposite. For example, LeRoy (1973), in a multiperiod theoretical model, finds that a risk-averse investor, when faced with a risk-free and risky asset that has an unforecastable return, the investor is willing to pay not to bear the risk of risky asset. This would lead to the value of the asset declining in successive periods, even if the prices reflect all available information.

In contrast to the RWT, the test of the EMH involves added complexity due to the joint-hypothesis involved. The joint-hypothesis problem arises because the asset pricing model – which determines whether the asset is fairly, over or undervalued – may be the reason for the observed anomaly - that is, the asset pricing model may be misspecified (Roll, 1977). As a result, when testing the EMH, both the efficiency of the market and the accuracy of the asset pricing model used are jointly tested. Thus, any anomalies found may be due to either an inefficient market, or the asset pricing model used being inaccurate (Konte, 2010). For example, Brenner (1977) highlights that misspecification in the form of an omitted variable yields biases that depend on the correlation between the omitted variable and the existing variables. In the case where there is covariance between the omitted variable and the existing model, bias in the residuals used to test the EMH may result.

In a study of the impact of the underlying market model on tests of market efficiency, Brenner (1979), using a US data set covering the period January 1936 to June 1967 that includes 830 companies and 1188 stock splits, finds that the use of different market models leads to different conclusions. To test the difference between models, an event (stock split) is used as the source of new information, and the difference of the residuals from the application of the event in each model is tested. The asset pricing models used included the single-factor Capital Asset Pricing Model and four variations of a two-factor model.

Ball (1978) highlights the complexity of the joint-market hypothesis in a literature survey. The author finds consistent excess returns after public announcements of firms' earnings. In several of the studies featured, excess return is defined by the equilibrium returns implied by a two-parameter model. The model includes the market portfolio and either an earnings or dividend related measure. The magnitude of the excess return was found to be reliant on the asset pricing model used. The author also highlights the impact of using earnings as an input variable; it is likely to correlate with an omitted variable. There may have been other misspecification effects. Notwithstanding this limitation, other studies have found statistically significant abnormal returns post-earnings announcements (Watts, 1978). In a separate event study focused on stock splits, Fama et al. (1969) finds that the cumulative average return drifts upwards after the stock split announcement. This is due to investors believing that the split is a signal for the future prospects of the business. However, after the actual split, the cumulative average residual (excess return) falls dramatically in the few months after the split, when the anticipated dividend increase does not materialise. Importantly, when all the splits are examined together, there is no net movement up or down in the cumulative average residual. This result suggests the market provides an unbiased forecast of the implications of a split for future dividends, and these forecasts are fully reflected in the prices of the security by the end of the split month. The findings support market efficiency, as prices reflect information once it becomes known.

A different approach to test for market efficiency is employed by Lehmann (1990), who examines the profits on a feasible ex-ante costless portfolio that should not earn riskless profits in an efficient market, but could earn such profits if stock price overreaction affects many equity returns. The study included the daily returns of securities listed on the New York and American Stock Exchanges between 1962 and 1988. This avoided the problems associated with specifying a model for variations in expected returns. The author found that portfolios of securities that had positive returns in one week typically had negative returns in the next week, while negative returns in one week typically had positive returns in the next week. In addition, the costless portfolio, the difference between the winners and losers' portfolios, had profits in forty-nine six-month periods covered by the data. These findings are clearly in contradiction of the weak-form EMH.

2.1.2 Empirical Evidence of the RWT and the EMH in South Africa

Empirical evidence in South Africa is also inconclusive with regard to the RWT. For example, Smith (2008) conducted a study on eleven African markets, including South Africa, using weekly and monthly observations. The study found that the random walk hypothesis was rejected by the empirical evidence in all eleven markets. In contrast, a test of weekly data on the Johannesburg Stock Exchange and its sub-indices from January 1993 to March 2001, found a divergence in random-walk findings. Of the six indices tested, the All Share Top 40, Industrial 25 and Gold indices all followed a random walk. On the other hand, the Mid Cap, Small Cap and broader Industrial Index failed the random walk test (Jefferis and Smith, 2004). These two studies highlight the impact of using differing data frequencies, with monthly series appearing to tend not to follow the random walk (Smith, 2008).

Researchers have also tried to answer whether the South African market *consistently* follows the random walk. Lim and Brook (2010) conduct a study using daily data from 1995 to 2005 and a bicorrelation test to measure how often several stock markets depart from random walk. The authors found that the JSE failed the random walk test 58% of the time. An additional finding from this study was that the average deviation from random walk was greater for emerging markets than for developed markets. Of the 50 markets in the study, sixteen out of the twenty with the greatest deviation from random walk were emerging markets; South Africa was sixth.

In addition, studies have also been conducted to test the market efficiency of the JSE beyond the RWT. Kruger and Toerien (2013) conduct a study on the impact of addition and deletions from the Johannesburg Stock Exchange (JSE) Top 40 Index. The authors found evidence of anticipatory price movement. The results imply the information inefficiencies could be used to generate profits. Using the Augmented Dickey-Fuller and Philips-Peron and monthly JSE return data from October 1998 to April 2014, Grater and Struweg (2015) found the JSE All Share Index is not weak-form efficient.

In summary, empirical studies of the RWT and the EMH have been mixed, both globally and on the JSE. Early studies found the RWT to hold in practice, but as data has become more abundant and methodologies evolved, more contradictory evidence have been found. Studies that focus on less frequent data, for example, weekly or monthly data, are more likely to find violations of the RWT. However, empirical studies using daily data tend to find that the RWT holds. This highlights an opportunity for more high frequency random walk studies.

Tests of market efficiency are complex due to the joint-hypothesis problem. Results are dependent on the market model that is used. This important factor has meant that there have been mixed results with regard to EMH, both globally and in South Africa. The market model is an important basis for most efficiency tests, but the true market model is unknowable. Importantly, the literature suggests that South Africa may not be weak-form efficient. Of fifty equity markets studied, South Africa was found to be the sixth most inefficient (Lim and Brook, 2010). The evidence also suggests that studies across the market, using market constituents across a broad set of industries and market capitalisations, may provide an interesting set of results (Jefferis and Smith, 2004).

2.2 Behavioural Finance

The behavioural finance literature attempts to reconcile the mismatch between the theory of the EMH and the observed behaviour of market participants and the market itself. Classical finance rests on principles that are largely drawn from economics (Ibbotson et al., 2018). These include:

1. Rationality; investors are rational utility maximisers who care about cash flows, expected return and risk. In addition, upon receiving new information, agents update their beliefs correctly (Constantinides, Harris and Stulz, 2003).
2. Risk-free arbitrage; the law of one price underlies this principle. In classical finance, in the absence of frictions, two securities with the same risk characteristics and pay-out must have the same price
3. Equilibrium; that is, supply equals demand. The prices of securities are such that every available security is held by some investor in the quantity that the investor wants to hold at the prevailing prices. Provided the equilibrium is frictionless, there will be no transaction costs and no risk-free arbitrage opportunities.
4. Efficient Market; security prices reflect all available information. In an efficient market, all securities' prices equate to their intrinsic value.

2.2.1 Rationality

Behavioural finance highlights that individuals suffer from various biases that puts limits on rationality (Ibbotson et al., 2018). Adam Smith, in his book *The Theory of Moral Sentiments*, argued that behaviour is determined by emotions and the impartial spectator. Smith noted the impact that loss aversion, self-control, altruism, fairness and overconfidence has on an individual's decision-making (Ashraf, Camerer and Loewenstein, 2005). Herbert Simon also challenged the economic rational man, arguing that humans do not have the cognitive ability to recognise all possible alternatives and to calculate optimum solutions. As a result, he proposed the idea of "bounded rationality" and a "satisficing man"; a man who makes satisfactory, rather than optimal, choices (Huppatz, 2015).

More recently, the behavioural finance literature primarily leans on experimental evidence by cognitive psychologists with regards to biases that may cause irrational behaviour by market participants (Constantinides, Harris and Stulz, 2003). For example, De Bondt and Thaler (1985) find that individuals tend to overweight recent information and underweight the base rate. Kahneman and Tversky (1974) highlight the bias of anchoring ; that is, when forming estimates, people start with some initial value and adjust from it. The authors also find when judging the probability of an event people often search their memories for relevant information to assign probabilities (Kahneman and Tversky, 1974). These cognitive errors are shared by experts and laymen (Kahneman and Tversky, 1982).

2.2.2 Risk-free arbitrage

In contrast to theoretical arbitrage, real world arbitrage entails cost and risks (Constantinides, Harris and Stulz, 2003). Other than fees, there may be legal constraints. For some, short-selling is not allowed. An additional limitation to the risk-free arbitrage condition is that when a mispriced security has no close substitute, the arbitrageur is exposed to fundamental risk (Constantinides, Harris and Stulz, 2003). Even if there are perfect substitutes, arbitrage can still be limited because:

1. Arbitrageurs are risk averse and have short horizons, and
2. The noise trader risk is systematic, or the arbitrage requires specialised skills, or there are costs to learning about such opportunities

The limits of the risk-free arbitrage assumption are shown in several studies. For example, De Long et al. (1990) highlights that traders may prefer trading in the same direction as the noise traders, thereby exacerbating the mispricing. Most studies that test the limits to risk-free arbitrage found that the inclusion of shares into the S&P 500 results in statistically significant returns without any informational change (Shleifer, 1986; Harris and Gurel, 1986; Wurgler and Zhuravskaya, 2002). These studies therefore concluded that the condition of a horizontal demand curve is violated. This violation means that investors are unable to buy and sell any amount of equity without significantly affecting the price.

2.2.3 Equilibrium

The existence of a frictionless equilibrium is challenged by the literature. Early research was focused on theoretical considerations. Using the second-hand car market as an example, Akerlof (1970) explains by using game theory why the value of “lemons” (secondhand cars), falls dramatically after purchase. The author highlights the role of information asymmetry in the lemon market; that is the seller (owner) of the car knows more than the buyer, and as a result the pricing mechanism is distorted. Rothschild and Stiglitz (1976) reach a similar conclusion using the insurance market as a framework. The authors concluded that the single price mechanism of conventional competitive analysis is no longer viable. The impact of imperfect and asymmetric information on pricing is a common theme in the literature. The result is prices that do not correctly reflect the value of the asset, or the underlying risk.

More recently, authors have empirically tested the assumption that supply equals demand, and the implications when there is an imbalance. For example, Kempf and Korn (1997) conducted such a study using intraday data on German Stock Index futures between 17 September 1993 and 15 September 1994. The authors analysed the permanent price impact caused by order flows and found that an unexpected net supply (demand) of 100 units will on average cause a price decrease (increase) of 0.07 percent. A similar study, drawing from physics models and using General Electric’s 1997 daily trade data, found a positive linear relationship between order imbalances and stock price (Rosenow, 2002). Several other studies applying various physics models reached similar conclusions[2].

The limitation of the assumptions underlying classic finance, particularly the RWT and the EMH, are well known. The behavioural finance literature highlights three factors that often lead to violations of either the RWT or the EMH. These factors include limits to rationality, risk-free arbitrage and equilibrium. However, even in the face of contradictory evidence, many economists continue to ignore behavioural finance. This is because the evidence usually has little predictive power for market behaviour (Heaton, 2019). Regardless, the behavioural finance literature provides reasons for non-randomness in financial markets. Importantly, the studies are conducted in several markets and imply that human and market behaviour may be repetitive and predictable. The literature also provides evidence of market frictions, which may prevent the market from reaching equilibrium. This provides the theoretical basis for technical analysis in general, and candlestick charting in particular.

[2] See Lillo, Farmer and Mantegna (2003), Daniels et al. (2003) and Plerou, Gopikrishnan and Stanley (2003)

2.3 Technical Analysis

Technical analysis uses past security price and volume data to predict future price movements (Han, Yang and Zhou, 2013). Early evidence from technical analysis covered a broad range of strategies and the findings of the studies were largely inconclusive. However, more recently findings have been more positive with regards to the practical value of technical analysis techniques.

2.3.1 Technical Analysis Trading Based Rules

Fama and Blume (1966) conducted one of the earliest technical analysis studies. The authors test Alexander's filter rule on 30 individual shares on the Dow Jones Industrial Index from January 1956 to April 1958. The Alexander filter is a mechanical trading rule which buys and holds a security if the closing price moves x percent and sells – while subsequently going short - when the price decreases x percent. Three filter rules, which included 0.5%, 1.0% and 1.5%, generated higher returns than a simple buy and hold strategy. However, after trading costs all strategies underperformed a buy and hold strategy. Levy (1971) conducted one of the early studies of thirty-two five-point chart patterns, including four variations of the head-and-shoulders pattern. The dataset included 548 securities listed on the New York Stock Exchange between 3 July 1964 and 4 July 1969, and the measurement interval after the pattern varied from one to twenty-six weeks. The author found that none of the patterns tested could be used to outperform the market.

Osler and Chang (1995) highlight three potential flaws in the initial study by Levy (1971). Firstly, the study did not impose a symmetry requirement into the head-and-shoulders pattern. Secondly, the study did not require the price to cross the neckline before entering the position and lastly, it did not incorporate the exit rules central to technical analysis; the initial study calculated profits on fixed and arbitrary intervals of one, four, fourteen and twenty-six weeks. In a similar study focused on the head-and-shoulders pattern and its predicative ability. Thus, Osler and Chang (1995) found that the head-and-shoulder trading rule generated profits that were both statistically and economically significant.

In a review of the empirical literature, Park and Irwin (2007) found that even early technical analysis studies conducted on foreign exchange and futures markets generated substantial profits. For example, in a study of the Alexander filter on July corn and soybean contracts from 1957 to 1968, it was found that the futures did not follow a random-walk and using a technical trading rule, outperformed a simple buy-and-hold strategy (Stevenson and Bear, 1970). The authors, however, did not test the statistical significance of the returns. In a similar study on live cattle futures, from April 1965 to February 1970, Leuthold (1972) found that the Alexander filter rule was profitable when applied at the two, three, four and ten percent level. Neely, Weller and Dittmar (1997) made a similar finding on currency markets. In a study of technical trading-based rules for six developed market cross currency pairs from 1981 to 1995, the authors find strong evidence of economically significant out-of-sample excess returns.

2.3.2 Moving Averages

Brock, Lakonishok and LeBron (1992) test the moving average (MA) utilising the Dow Jones Index from 1897 to 1986. The authors included the bootstrapping methodology for statistical robustness. The null models used in the study include the random walk with drift, AR (1), GARCH-M and Exponential GARCH. In the study, buy signals consistently generate higher returns than sell signals; over a ten-day period - this return averaged 0.8 percent, whereas for the null models it averaged 0.17 percent. Okunev and White (2001) also found that moving averages generated positive returns that are statistically different from zero. The authors conduct a study on three-month government interest rates and exchange rate for several countries from January 1975 to June 2000 [3]. The currencies were ranked from most to least attractive using moving average rules that ranged from one to twelve months. Each currency received an equal allocation at the beginning of the period and after each month, the positions were reweighted to increase the allocation of higher-ranking currencies. Although early studies typically highlight the returns of the various strategies, they do not provide possible reasons for their occurrence. One possible reason for the statistically significant results when using MA strategies is investor herding (Ni, Liao and Huang, 2015)

Han, Yang and Zhou (2013) found that the MA outperformance had a positive relationship with asset volatility. The authors use one set of ten portfolios constructed using New York Stock Exchange data from 1 July 1963 to 31 December 2009; the portfolios were constructed based on annual standard deviation, estimated using daily returns in the preceding twelve months. The second set of ten portfolios were constructed using firm size (market capitalisation). Both sets of portfolios were rebalanced at the end of each year. For a given portfolio the moving average timing strategy is to buy or continue to hold the portfolio today when yesterday's price is above its ten-day moving average and to otherwise invest in the risk-free asset (30-day Treasury Bill). This is measured versus a buy-and-hold strategy. A secondary finding in the study was MA outperformance had an inverse relationship with market capitalisation. The authors suggest that this finding is also due to volatility. That is, small and medium capitalisation stocks generally having a higher level of volatility than large

[3] The countries included in the study were Australia, Canada, France, Germany, Japan, Switzerland, the United Kingdom and the United States.

capitalisation stocks. Technical analysis and trading based rules were also found to work better in emerging markets, in comparison to developed markets (Hsu, Hsu and Kuan, 2010). This finding may also have been due to differences in volatility.

The literature indicates that findings of early studies generally found technical analysis to be ineffective in predicting future share prices. However, as data and analysis techniques have improved, the frequency of more positive results have increased. The technical analysis literature also highlights that this technique has uses in more than one type of financial market; thus, there have been positive findings in both currency and equity markets. Lastly, studies suggest technical analysis works optimally on more volatile instruments (Han, Yang and Zhou, 2013; Lu, Chen and Hsu, 2015).

2.4 Candlestick Charting

This section discusses the empirical literature on Candlestick Charting. Conclusions have been mixed and studies have used different methodologies to test the predicative ability of candlestick patterns.

2.4.1 Developed Markets

In the US, Caginalp and Laurent (1998), using binomial distribution to test three-day candlesticks of the constituents of the S&P 500 and a world equity closed-end fund between 1992 and 1996, found eight patterns to be significant at the five percent significance level in terms of predicting price direction over a three-day period after the signal. These patterns also generated alpha. For the S&P 500, bullish reversal patterns generated on average 0.9 percent, while bearish signals generated 0.27 percent – both before costs. The patterns tested were Three White Soldiers, Three Inside Up, Three Outside Up and Morning Star, and their inverse patterns. Caginalp and Laurent (1998) define a downtrend as a decreasing 3-day moving average over 6 days. In contrast, Horton's (2007) study found little value in the use of candlesticks. The analysis covered 349 randomly selected stocks listed in the United States, and made use of non-parametric tests, namely the Kolmogorov-Smirnov (KS) test, the Cramer-von-Mises (CVM) test, and the Birnbaum-Hall (BH) test. The author concludes that the probability of choosing a correct result using candlesticks is not statistically different from choosing at random. Marshall, Young and Rose (2006), using a bootstrap methodology and the constituents of the Dow Jones Industrial Index between 1992 and 2002, found no evidence that candlesticks have predictive value. The key differentiator of this study was the use of a 10-day exponential moving average to determine the trend, and the 10-day predictive interval to determine the success or failure of the pattern.

Each of the above studies used different methodologies. These include differing trend methodologies and predictive intervals. Lu, Chen and Hsu (2015), applied the three methodologies discussed above to determine the factors contributing to candlestick profitability, and concluded that the holding strategy is the key to the effectiveness of candlestick charting. The three-day holding period after the signal was found to be most profitable, with returns ranging from zero to three percent, depending on the pattern. The fact that transaction costs of 0.5% are incorporated into the study provides more validity to the findings.

Technical analysis, in particular moving average rules, are more profitable on more volatile assets. Thus, Lu, Chen and Hsu (2015) found the use of candlesticks to be more profitable when applied to the constituent stocks of the NASDAQ 100, which includes technology and internet stocks that are more volatile than the Dow Jones Industrial Index (DJIA). The authors found, for example, that the Three White Soldiers (TWS) generated a mean daily return of 1.45 percent on the NASDAQ 100, while it generated a mean daily return of 0.23 percent on the DJIA. The pattern was only found to be significant on the NASDAQ 100 constituents. Pastor and Veronesi (2006), found that the reason the reason the NASDAQ was more volatile was due to uncertainty around their growth prospects. Tests on more volatile assets have been limited, but this evidence suggests that candlestick patterns are capable of generating more profits on a more volatile asset.

Erb, Harvey and Viskanta (1995) conducted a study of forty countries' credit ratings and market volatility between March 1980 and December 1993. The authors found that countries with lower credit rating (higher risk of default) were more volatile; that is there was negative correlation between credit risk and stock market volatility. In essence, emerging markets are more volatile than their developed market counterparts. It is against this backdrop that the candlestick research in emerging market follows.

2.4.2 Emerging Markets

The candlestick research in emerging markets is limited. Two studies, conducted on two different markets, reached different conclusions about the effectiveness of candlestick patterns.

A two-day candlestick study by Lu, Shiu, and Liu (2012) on the constituents of the Taiwan 50 Index's for the period January 2002 to December 2009, found that candlesticks have predicative power in the Taiwanese market; in particular the three bullish patterns, which were found to have an accuracy of

61 percent to 91 percent. In contrast to previous studies, this study used a holding period strategy of only exiting a position once a contradictory signal occurs. For example, after a bullish signal, a long position is entered, and is only sold after a bearish signal. The average return for the bullish patterns ranged from four percent to thirteen percent. The findings for the three bearish patterns were less promising. This highlights that not all patterns will necessarily have the same magnitude of predicative value. In contrast, Prado et al. (2013) found no evidence that candlesticks have predictive value in the Brazilian market.

Technical analysis uses past prices to predict future prices, but research that has focused on developed markets have met with mixed results. A possible reason for this appears to be the widely differing methodologies and assumptions used in these studies. The predictive interval or holding period appears to be of particular importance in determining the accuracy and profitability of candlesticks. A promising result in the technical analysis literature is that the technique is more profitable for assets with more volatility. Thus, positive results were found for the more volatile NASDAQ 100 and in Taiwan, an emerging market. South Africa is an emerging market, but whether it will yield similar result to Taiwan has not been determined.

2.5 Conclusion

Empirical evidence suggests that the RWT does not accurately depict financial markets. The evidence in the literature, both in South Africa and globally, suggests that the RWT does not fully hold, at least not in all markets and at all times. In addition, developed markets and emerging markets operate across a spectrum of randomness; evidence suggests emerging markets are more serially correlated (Smith, 2008; Lim and Brook, 2010). The frequency of the data used appears to be a factor in the conclusion of the studies. For example, weekly and monthly data tend to display serial correlation, while daily data tend to follow the random walk.

The RWT forms the foundation of the EMH. However, many studies caution that the violation of the RWT does not imply violation of the EMH. The EMH proposes that share prices instantaneously reflect all available information correctly. The first of the three forms of the EMH (the weak form) states that an investor cannot use past price information to outperform the market. That is, there is no informational advantage in examining past price movements.

The mismatch of theory and empirical evidence has led many academics to question the EMH. The behavioural literature has grown in importance as the bridge between what the theories postulate, and real-world experience. The experimental research in behavioural finance finds that individuals, who ultimately form the market, have cognitive limitations, and often incorrectly process information. The behavioural literature has, however, been largely ignored by practitioners, as it has limited use in predicting the direction of financial markets. The technical analysis literature highlights that past prices may have predictive ability.

The candlestick technical share price pattern is a tool used by some investors to attempt to gauge the emotions underpinning markets. Based on the underlying behavioural finance theory and due to potential market inefficiency, the technique is promising. However, empirical evidence found in the literature has been mixed with regards to the predictive power of the candlestick technique. To date, empirical tests of the effectiveness of this pattern has mainly been limited to developed markets (*e.g.* the United States, Europe and Japan) and a few emerging markets (*e.g.* South-East Asia and Brazil). The mixed results appear to be due to differing experimental designs. Thus, Lu, Chen and Hsu (2015) highlight that the post signal holding period is of particular importance in affecting the test outcome. To date, no study of the candlestick pattern has been conducted on the JSE. As the technique can also be applied to daily data, it further allows a new way in which to test the RWT. In addition, with the incorporation of transaction costs, this test also addresses the question of whether or not the JSE displays weak-form market efficiency.

3. Data and Methodology

The study was conducted in Microsoft Excel (Excel). The primary reason for this was its ease of use and affordability. The data source (Bloomberg) is easily integrated into the spreadsheet through an Excel Add-in. In addition, Excel allows the inclusion of formulae, which are crucial for the study.

In order to test the hypothesis, the daily low, high, open and close price is required for each security in the study. Caginalp and Lauren (1998) provide the testing methodology. Although the methodology used by Caginalp and Lauren (1998) is non-parametric, provided the sample size is large enough and the condition of independent and identically distributed (i.i.d) is largely met, the normal distribution approximates the binomial distribution.

This following section gives more details on the dataset used in the study, as well as limitations or risks that may exist. Thereafter, the methodology, which includes the trend analysis and candlestick pattern criteria, are discussed.

3.1 Data

In order to test the hypothesis, the daily low, high, opening and closing price is required for each security in the study. Using the Bloomberg database, the top 100 shares listed on the Johannesburg Stock Exchange (JSE) (as measured by market capitalisation) on 31 December 2009 were considered. These shares had to still be in the top 100 as at 31 December 2019. The primary reason for choosing the top 100 shares, was to ensure the study included liquid shares. The ten-year period was relevant for the study as it includes more than one business cycle. Evidence from developed market business cycles suggests they last 3 to 6 years (Cotis, Coppel and Ortega, 2005). The final sample included sixty shares, across a variety of industries (see Appendix A for the list of shares used in the study). The primary reason for requiring the shares to be in the sample for the ten-year period was to simplify the use of the dataset.

As the data only considers companies that remained listed on the JSE by the end of the sample period. It may therefore at first glance appear that the study potentially suffers from survivorship bias; *i.e.* the sample does not include companies that did not survive or remain listed (Horton, 2007). A result of the bias in most studies is that the average return of a sample will tend to be overstated, leading to misleading findings (Rohleder, Scholz and Wilkens, 2011). However, survivorship bias is not likely to affect the results of candlestick studies (Marshall, Young and Rose, 2006), as the performance of the patterns that are assessed in these studies are influenced by day-to-day volatility, and not the longer-term characteristics of the underlying stocks. The data also considers a diverse group of shares, across various sectors, and with a diverse set of products or services. Table 1 describes the data sample in more detail.

Table 1: Descriptive statistics of sample

	No of securities	No of daily observations	Mean Return (Daily)	Standard Deviation (Daily)	Skewness	Kurtosis	Positive Days (%)
Sample	60	149 880	0.04%	1.91%	0.39	16.71	49.48%
Sectors							
Resources	14	37 470	0.04%	2.49%	0.47	5.55	49.47%
Financials	19	47 462	0.04%	1.58%	-0.06	4.10	48.86%
Industrials	27	67 446	0.05%	1.79%	0.45	35.20	49.92%

The data set includes 60 company shares that represent a good balance between resource, financial and industrial companies. Thus, 27 of the 60 securities are in the industrial sector, while 19 and 14 are in the financials and resources sectors, respectively. The total dataset is approximated by a normal distribution. However, the kurtosis of 16.71 highlights that the dataset has a greater concentration of observations about the mean and is therefore leptokurtic. The implication is that the dataset has larger tails; positive and negative extreme events are underestimated by the normal distribution. This should not have an impact on the predicative ability of the candlesticks, as the first step of the candlestick study is concerned with a binary outcome. Although the second part of the study is concerned with the returns generated using candlesticks, the null sample is equally impacted and therefore the leptokurtic data should influence both datasets. Therefore, it should not impact the study.

The other observation is the different standard deviation (volatility) of each sector. The resources sector has the largest daily volatility of 2.49%, followed by the industrials sector that has a volatility of 1.79%. Lu, Chen and Hsu (2015) highlight that technical trading rules are more profitable on volatile assets. The difference in volatility between sectors provides an additional component to the study. The three sectors have average daily returns of either approximately 0.04% and 0.05%. All the data and sub-samples are negatively skewed, except for the resources sector. In addition, the financial sector is the closest to the normal distribution. Interestingly, the positive days in the sample are 50 percent of the daily observations. This highlights that the data is not biased from a business or market cycle perspective.

Table 2: Market capitalisation statistics of each sector

Sector		Market capitalization (in Rand Millions)	
		Start of Period	End of Period
Resources	Low	17 313	23 933
	Average	201 975	263 234
	High	1 484 206	1 821 173
Financials	Low	5 132	7 744
	Average	39 287	93 779
	High	158 942	352 276
Industrials	Low	4 298	6 870
	Average	34 864	103 486
	High	216 997	1 020 842

Table 2 highlights the lowest, average and largest market capitalisation in each sector over the sample period. In general, the resources sector on average was larger than the other two sectors. Both its lowest and highest market capitalisation were also greater than financial and industrial sector at the beginning and the end of the period. The market capitalisation statistics of the financial and industrial sector are similar at the beginning of the period; the average of the financial sector was R39.287 billion, while for the financial sector was R34.864 billion. The largest market capitalisation at the beginning of the sample period was greater in the industrial sector in comparison to the financial sector. In the industrial sector, at the end of the period the largest market capitalisation grew to three time the financial sector. The overall change of the sectors is not relevant for the study, as the methodology focusses on short-term price movements. Table 2 does not explain the difference in the sector kurtosis.

3.2 Methodology

The objective of the methodology used in this paper is to test whether candlestick charting has any predicative value and whether or not the JSE is weak-form efficient. The first null hypothesis in the test is:

$$H_0: Candlestick\ patterns\ have\ no\ predicative\ ability\ on\ the\ JSE\ at\ the\ 5\%\ level\ of\ significance$$

$$H_1: Candlestick\ patterns\ have\ predicative\ ability\ on\ the\ JSE\ at\ the\ 5\%\ level\ of\ significance$$

The second null hypothesis is:

$$H_0: The\ JSE\ is\ weak - form\ efficient$$

$$H_1: The\ JSE\ is\ not\ weak - form\ efficient$$

To measure the success of candlestick patterns the following four steps must be taken: firstly, the trend must be identified. In this study the same trend identification definition is followed as that used by Caginalp and Laurent (1998). A downtrend is defined as a decreasing three-day moving average over a rolling six-days; an uptrend is the opposite definition. Other trend definitions do exist, and are provided later in this section. Secondly, the pattern must be identified. A key issue in the pattern identification process is the subjectivity of patterns (Morris, 2006). Caginalp and Laurent provide the definitions of the candlestick patterns used in the study. This methodology is consistent across studies and is highlighted in the following section.

The penultimate step in testing the predicative ability of candlesticks is to determine a basis of measurement (Morris, 2006). In this study, a three-day (holding) period after the signal is used. Lu, Chen and Hsu (2015) discuss the impact of various holding periods on the success. These authors indicate that the three-day period used by Caginalp and Laurent (1998) is the best methodology, from a predictability and return perspective. This test provides binary data (success or failure), which is compared to a null hypothesis. Finally, a statistical test was conducted on the binary data. This study uses the normal approximation to the binomial distribution to provide robustness to the study - this is done at the five percent significance level. The alternative statistical methodologies will be briefly discussed.

To provide legitimacy to the study and test the weak-form market efficiency, the candlestick patterns were tested against a market return that does not include the use of candlesticks. This was done after costs. The average return of each candlestick pattern was statistically tested using the normal distribution.

3.2.1 Trend identification

Crucial to the identification of a candlestick pattern is the definition of the uptrend or downtrend, which is the first step after the data aggregation. A three-day pattern without the correct trend is irrelevant (Caginalp and Laurent, 1998). In the study, a downtrend was identified by a three-day moving average, that is, decreasing each of the past six days (see definition 3.2).

$$MA_t = \frac{1}{3} * (P_{(t-2)} + P_{(t-1)} + P_{(t)}), \text{ where } P \text{ is the closing price} \quad (3.1)$$

$$MA_{(t-6)} > MA_{(t-5)} > \cdots > MA_{(t)} \quad (3.2)$$

The uptrend is defined opposite to the downtrend as per Equation 3.2. All downtrend and uptrends in the pattern follow the above definition. This was done for each of the sixty shares in the study. The result was two spreadsheets; one that only considers downtrends and the other uptrends.

The use of a three-day moving average is somewhat arbitrary. In a comprehensive study of the impact of three different trends on the effectiveness of candle patterns Lu, Chen and Hsu (2015), used a three-day moving average as per the study of Caginalp and Laurent (1998), a ten-day exponential moving average and, lastly, the trend identification pattern used in Levy (1971). The latter approach was based on average daily share price changes over 131 days, and the uptrend was defined as:

$$\frac{P_t^c - P_{t-6}^c}{P_{t-6}^c} > 6\theta \quad (3.3)$$

Where θ is the average daily price change over the 131-day period. Importantly, the definition of the trend did not impact results. In the present study, a six-day period was used in accordance with that used by Caginalp and Laurent (1998).

3.2.2 Pattern identification

After the trend identification in the previous step, the patterns to be used in the study can be identified. This study focusses on eight (non-overlapping) reversal patterns and their ability to forecast the change in the direction of the trend (see Appendix B for the eight patterns tested in the study and their definitions). Subjectivity is a hurdle in identifying candlestick formations (Nison, 2001). Caginalp and Laurent (1998), simplify the interpretation of the traditional definitions in order to maintain the non-parametric testing. The expectation is that omission of certain conditions may reduce, but not eliminate, positive results.

For each of the unique patterns used in this study, the pattern identification system only considers the body of the candles and not the shadows. The body of the candle only measures the open and close, while the shadows consider the high and the lows of the day. The size of the body is important and provides information about the momentum of market, while the shadows are less important, particularly in the case of three-day patterns (Nison, 2001).

Several studies follow the same pattern identification methodology; each studied a different set of patterns (see Horton, 2007; Lu, Shiu and Liu, 2012; Lu, 2014; Lu, Chen and Hsu, 2015). Lu (2014) uses a slightly different methodology for pattern identification, namely a 1x4 vector, to categorise candlestick patterns. Importantly, objective measurement of the patterns is maintained. Once the patterns have been identified, the next step requires the selection of a holding period after the signal in order to determine the success or failure of the approach as an investment tool; as well as a method to measure whether the returns achieved by the use of the candlestick method is statistically significant from a passive strategy.

3.2.3. Measurement interval (holding period)

In the literature there is great diversity with regard to the measurement interval and the testing of the predicative power of the pattern once it has occurred. As the test is concerned with two outcomes (success or failure), several studies use non-parametric tests. A non-parametric test is one where the underlying distribution of the population is unknown. Caginalp and Laurent (1998) and Tharanvanij, Siraprapasiri and Rajchamaha (2017) use the binomial test to measure the predicative ability of the candlestick charts, whilst Horton (2009) uses three non-parametric tests, namely the Kalmogorov-Smirnov (K-S) test, the Cramer-von Mises (CvM) test and Birnbaum-Hall test.

Both the K-S and CvM tests are goodness-of-fit test that compares a hypothetical or fitted cumulative distribution function (cdf) with an empirical cdf in order to assess fit (Evans, Drew and Leemis, 2008). These tests are applied to independent samples to determine whether they are likely drawn from the same underlying population distribution (Horton, 2009). In this case, this was a comparison of a sample using candlesticks, while the alternative sample would be random. The underlying assumptions of the test is that the samples are random, mutually independent and that the measurement scale is ordinal.

Similarly, to Caginalp and Laurent (1996), this study utilised the normal approximation to the binomial distribution. This methodology was preferred because of its familiarity. The probability of success of the pattern (p) is tested against a null probability (p_0). The difference between the means np (the actual number) and np_0 (the expected number of success), is then divided by the standard deviation to get the z-statistic for the hypothesis test (see Sections 3.8 and 3.9).

An event was defined over a three days period (measurement interval or holding period) after the candlestick signal. Calculating the null hypothesis or benchmark was the first step in determining the success or failure in the measurement interval. The rationale behind the null hypothesis was to establish the three-day return regardless of whether or not the three-day candlestick was used. Note that in the section that follows, only the methodology for a downtrend and bullish signal is discussed and illustrated, with that applicable to an uptrend and bearish signal being the exact opposite.

The event for a downtrend is calculated as follow

$$n = Event_{with\ or\ without\ 3-day\ signal} = Price_{t+3} < Price_{average}(t+4, t+5, t+6) \qquad (3.4)$$

The sum of all these events; that is, where the average price three days after the waiting period is greater than $Price_{t+3}$, is divided by the total number of downtrends in the sample. This then provides the probability of a downtrend turning positive irrespective of the three-day candlestick signal (see Section 3.5)

$$p_0 = \frac{\sum Event_{with\ or\ without\ signal}}{total\ downtrends\ in\ the\ sample} \qquad (3.5)$$

An event is a success, in the case of bullish candle stick signal, when:

$$Event_{after\ signal} = Open_{t+4} < Price_{average}(t+4, t+5, t+6) \qquad (3.6)$$

$$p = \frac{\sum Event_{after\ signal}}{n} \qquad (3.7)$$

Where, $P_{average}(t+4, t+5, t+6)$ is the average of the closing prices on those days, and where n is the total number of events in the study. This is the same definition used by Caginalp and Laurent (1998). Lu, Chen and Hsu (2015) tested the impact of the measurement interval and exit methodology by performing comparative testing of the Caginalp Laurent (CL) strategy (as above), and the methodology used by Marshall, Young and Rose (2006), which exits the investment entirely at the end of the measurement interval. The above authors find that the exit methodology is important in determining the success of the candlestick signal, and that the averaging out CL methodology is superior to the full end of measurement interval exit from a predictability and return perspective.

$$Z = \frac{n(p - p_0)}{\sigma} \tag{3.8}$$

Based on the central limit theorem (CLM), which establishes that when random variables are added, their normalised sum tends towards a normal distribution irrespective of the original variables' distribution, the Z-statistic (Equation 3.8) has a normal distribution. This feature allows the normal distribution to be used to statistically test the predicative ability of candlestick patterns. Caginalp and Laurent (1998), highlight that although the sample of moving averages defined in Equation 3.1 are not completely independent, the moving averages contain few points in comparison to the sample, such that the CLM applies.

$$\sigma = \sqrt{np_o(1 - p_0)} \tag{3.9}$$

For each of the eight patterns in the study, a z-score was therefore calculated. The final step of the statistical test is to determine a rejection region for the normal distribution. Several studies use the one, five and ten percent levels of significance as the level at which the null hypothesis is rejected (see Marshall, Young and Cahan, 2007; Horton, 2009, Lu, Chen and Hsu, 2015; and Tharanvanij, Siraprapasiri and Rajchamaha, 2017). For all of these studies the test was one-sided. The use of the one, five and ten percent level of significance is conventional, but there is no scientific basis for it (Kim and In Choi, 2019). Kim and In Choi (2019), highlight concerns with statistical decisions being made at conventional levels. These include p-hacking and data mining. Both practices involve analysing data that provides pleasing results. However, due to the size of the sample and its diversification from a sectoral perspective, p-hacking and data mining are not a major concern. There is therefore no need to deviate from convention.

For this study, a one-side test is used. The null hypothesis was rejected if the z-score was greater than 1.645 for the patterns. In a one-sided test, this corresponds to a 95% confidence level (i.e. a 5% significance level). For the sake of simplicity, the p-values will be presented for each of the patterns in the study. The p-value is the probability of obtaining results at least as extreme as the observed results – thus, when this is lower than five percent this resulted in the rejection of the null hypothesis.

Figure 2 highlights the methodology from trend identification to the holding period. The trend is identified over six three-day moving averages. Thereafter, if a downtrend or uptrend has been identified, the respective candlestick pattern occurs over the next three-days (t+1, t+2, t+3). If a sell signal is indicated, the entire short-sale occurs on the open of t+4. Subsequently, a third is then covered on the close of t+4, t+5 and t+6. While for a bullish pattern, the purchase occurs on the open of t+4 and the subsequent sale occurs on t+4, t+5 and t+6. In the study, a successful prediction of a bearish candlestick pattern is measured when there is a negative return during the holding period, as a decrease in prices is desired. However, in order to take advantage of the signal the investor would execute a short-sale. Thus, in the tables presented in the results section, a positive return is presented as the desired outcome.

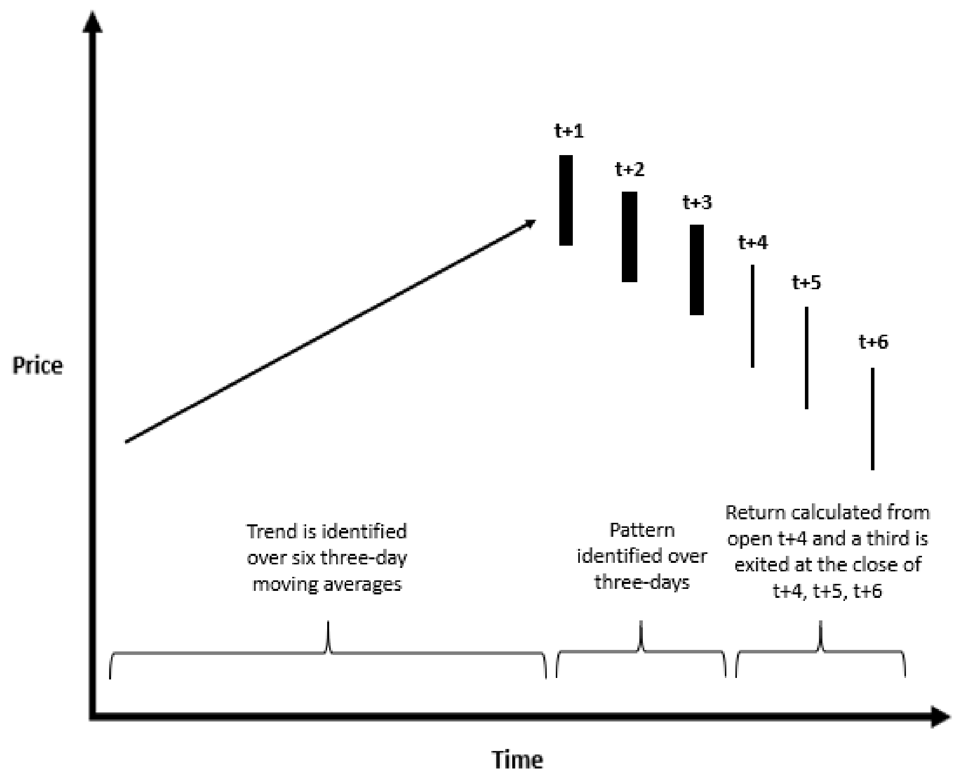

Figure 2: Timeline of candlestick efficacy test from start to finish

Source: Author

3.2.4. Transaction costs

An often-highlighted shortfall of many technical based trading rule studies is that that these non-effects can be small relative to the transaction costs needed to exploit them (Malkiel, 2003). Park and

Irwin (2007), in a survey, similarly highlight that no studies have reliably been able to outperform after transaction costs. Lu, Chen and Hsu (2015), assume a total transaction cost of 0.5%. This is based on the bid-ask spread of 0.1 to 0.3 percent used by Caginalp and Laurent (1998). A review of the trading fees on Easy Equities, a prominent trading platform in South Africa, highlighted the following fees:

Table 3: EasyEquities' cost per transaction

Description	
Broker commission	0.25%
Securities Transfer Tax and Administration	0.25% of value traded
Settlement and Administration	0.075% of value traded
Investor Protection Levy and Administration	0.0002% of value traded
Value added tax	15% on brokerage & settlement

Source: EasyEquities (2020)

The one-way fee based on this information is therefore around 0.5% to 0.6%. For this study the after-transaction cost return was therefore calculated as:

$$after - fee\ return = \frac{P_{average}(t+4,t+5,t+6)}{Open_{t+3}} - 1 - 1{,}0\% \tag{3.9}$$

For short trades based on bearish patterns, the one percent transaction cost is also deducted to the pre-transaction cost return. For each of the patterns in the study the respective after-fee returns were used, and then divided by the total number of observations (n) to determine the average return obtained by using the relevant candlestick pattern (see Section 3.10) as trading rule.

$$\bar{x} = \frac{\sum after\ fee\ returns}{n} \tag{3.10}$$

In order to determine whether the return generated by using candlesticks is significant, a benchmark return is required. The benchmark return used were the average daily return and standard deviation as in Table 1 which describes the descriptive statistics of the data. For example, the benchmark daily return and volatility for the sample was 0.04% and 1.91%, respectively. The intuition behind this is the benchmark is used as a passive benchmark.

The penultimate step was the calculation of the normal distribution, in order to determine the statistical significance of the candlestick returns. For each candlestick pattern the z-score was calculated as:

$$Z = \frac{\bar{x} - \mu}{\sigma} \quad (3.11)$$

As previously indicated, the level of significance for the study was five percent. Therefore, if the z-score calculated using Equation 3.13 is greater than 1.645 (or the p-value is less than five percent), the findings are considered to be statistically significant.

In the section that follows the findings of the study will be presented. This includes the findings of the effectiveness of the patterns in the subsectors.

4. Results and Findings

4.1 Sample findings

Table 4 provides information of the sample in the study. In the period under consideration, there were 19 536 uptrend reversals, while there were 16 642 downtrend reversals. Of the 19 536 downtrend reversals, 9 609 (49%) had a negative three-day return after a three-day waiting period – this served as the null hypothesis for the bearish patterns in the sample (p) . Of the 16 642-downtrend reversals, 8 415 (51%), had positive returns over the three-day holding period. In their study of the S&P 500 between 1992 and 1996, Caginalp and Lauren (1998) found the bull and bear pattern null probability to be 45% and 53% respectively.

Table 4: Candlestick results of the entire sample

Sector	Observations					σ			Daily return	
Sample	149 880					1.91%			0.04%	
Pattern	**Observations**	**Predicted Success**	**Actual Success**	p	$n(p - p_0)$	σ	**Z-score**	**P-value**	**Daily return**	**Daily return ****
Uptrend reversal	19 536		9 609	49%		2.3%				
Downtrend reversal	16 642		8 415	51%		2.6%				
Three White Soldiers (TWS)	1 991	1007	984	49%	-22.75	22.31	-1.02	0.85	0.02%	-0.98%
Three Black Crows (TBC)*	2 391	1 176	1 208	51%	31.96	24.45	1.31	0.10	0.00%	-1.00%
Three Inside Up (TIU)	1 940	981	997	51%	16.04	22.02	0.73	0.23	0.17%	-0.83%
Three Inside Down (TIU) *	2 088	1 027	1 052	50%	24.99	22.84	1.09	0.14	0.02%	-0.98%
Three Outside Up (TOU)	1 294	654	665	51%	10.69	17.98	0.59	0.28	0.19%	-0.81%
Three Outside Down (TOD) *	1 803	887	928	51%	41.17	21.23	1.94	0.03	0.10%	-0.90%
Morning Star (MS)	943	477	482	51%	5.17	15.35	0.34	0.37	0.12%	-0.88%

* Indicates bearish pattern
**Indicates after-fee return

Of the eight patterns in the study, the ES pattern did not feature in the sample data. The TOD pattern was the only pattern in the sample to indicate predictive ability at the five percent level of significance. Importantly as well, the return of the TOD pattern is 0.10%. This return was, however, statistically non-significant. The TBC pattern was significant at the ten percent level of significance. Generally, bearish patterns indicated more promise from a predictive perspective.

The results are less compelling than those of Caginalp and Laurent (1998). These authors found the predicative ability of the four bullish patterns to be 71 percent, while for bearish patterns the success rate was 67 percent. Similarly, Lu, Chen and Hsu (2015), in a study of thirty constituents of the DJIA index between January 1992 and 2012, found the success of bullish patterns ranged from 54 to 70 percent, whilst the success of bearish patterns ranged from 80 to 92 percent. Lu (2013) found a similar range. The difference between the United States and the South African sample in our study may be due to differences between emerging and developing markets. However, Lu, Shiu, and Liu (2012), found strong predicative ability for candlestick patterns for both bullish and bearish patterns in the Taiwanese equity market; an emerging market (Lim and Brooks, 2010).

The results show that aside from the TOD candlestick pattern, the patterns tested have no predicative ability, except for the TBC pattern, which provides inconclusive results. The predictive ability of the TOD highlights that the JSE may not completely follow the RWT. However, the patterns fail to provide evidence against the weak-form EMH. None of the returns were significant and may therefore have been due to coincidence or luck. The reason for the differences between the results of this study and some of those mentioned above may have less to do with differences between emerging and developed markets, and more to do with the underlying constituents. Jefferis and Smith (2004), in conducting a random-walk study on six sub-indices of the JSE, found differences between the behaviour of the indices. For example, the Industrial and Gold indices followed the random walk, while the Small cap, Mid cap and broader Industrial index did not. Therefore, based on these different index behaviours, the next sections present the results of the patterns for the different index sectors.

4.2 Financials

Table 5 highlights the difference in the predictive ability of candlesticks for the financials sector. The sample for this sector contained 19 shares. In financial sector sample there were 6 078 uptrend reversals. Of the 6 078 uptrend reversals, 2 963 (49%) had a negative three-day return after a three-day waiting period – this served as the null hypothesis for the bearish patterns in the sample (p). Of the 4 791 -downtrend reversals, 2 480 (52%), had positive returns over the three-day holding period. In the financial sector no pattern had any predicative ability at the five or the ten percent level of significance. The daily returns for bullish patterns had positive returns and bearish patterns had negative returns. These too, were not significant at the five percent level of significance. Once the one percent commission fee was introduced, evidence against the use of candlestick patterns as investment tools increased.

Table 5: Candlestick results in the financial sector

Sector	Observations					σ			Daily return	
Financials	47 462					1.58%			0.04%	
Pattern	**Observations**	**Predicted Success**	**Actual Success**	p	$n(p - p_0)$	σ	**Z-score**	**P-value**	**Daily return**	**Daily return ****
Uptrend reversal	6 078		2 963	49%						
Downtrend reversal	4 791		2 480	52%						
Three White Soldiers (TWS)	572	296	288	50%	-8.09	11.95	-0.68	0.75	0.12%	-0.88%
Three Black Crows (TBC)*	669	326	339	51%	12.87	12.92	1.00	0.16	0.02%	-0.98%
Three Inside Up (TIU)	548	284	281	51%	-2.67	11.70	-0.23	0.59	0.09%	-0.91%
Three Inside Down (TIU) *	610	297	300	49%	2.63	12.34	0.21	0.42	0.06%	-0.94%
Three Outside Up (TOU)	377	195	192	51%	-3.15	9.70	-0.32	0.63	0.07%	-0.93%
Three Outside Down (TOD) *	515	251	261	51%	9.94%	11.35%	0.88	0.19	0.11%	-0.89%
Morning Star (MS)	283	146	139	49%	-7.49	8.41	-0.89	0.81	0.17%	-0.83%

*Indicates bearish patterns
**Indicates after-fee return

Studies of candlestick patterns on the financial sector are limited, and thus the results of this study cannot be compared to results from other markets. Damodaran (2010) highlights the different characteristics of the firms in the financial sector. The sector consists of banks that make money on the spread between the interest it borrows from and those it lends to, insurance companies that make money from the premiums it receives and from the investment portfolios they manage to service future obligations, investment banks that make their money from providing corporate banking services (including advisory or sales fees), and lastly, investment firms provide investment advice or manage portfolios on the behalf of clients. A feature of these business models is that they are vulnerable to exogenous factors. For example, the change in interest rates or market performance may be unpredictable and the information is available publicly. As a result, candlestick chart methods may not have predictive power for share prices in this sector. This study suggest that the financial sector follows the random walk and is weak-form efficient.

4.3 Industrials

Table 6 highlights the difference in the predictive ability of candlesticks in the industrial sector. Of the sectors tested, the industrial sector showed the most promising results for the use of the candlestick approach as an investment tool. The sector was the largest in the data set, with 27 shares. The sector had 8 914 uptrends and 7 406 downtrends. Fifty percent of the uptrends reversed, while fifty-one percent of the downtrends reversed over the three-day holding period. Two of the seven patterns were found to have predicative ability at the five percent level of significance. The statistically significant patterns at the five percent level include the TOU and TOD patterns, whilst the MS pattern was found to be inconclusive at the five percent level of significance. Two additional patterns, namely the TBC and TIU, were predictive at ten percent level of significance. Thus, the above evidence suggests that the industrial sector may not follow the random walk, although it is difficult to speculate as to why this could be the case.

Jefferis and Smith (2004) also found the industrial sector of the JSE to be weak-form inefficient, and that smaller capitalisation shares were generally less efficient than larger capitalisation shares. One reason for candlesticks being more predicative on the industrial sector may be due to the sector, on average, having a smaller capitalisation shares.

Although the candlestick patterns were found to be predictive in the industrial sector and the gross of fee returns were directionally correct, the gross of fee returns versus the passive benchmark was found to be statistically insignificant. Including the trading costs exacerbated this finding further, with the p-value for all the returns in the table increase to 100 percent.

There is evidence that the industrial sector does not follow a random walk. The exact reason for this is not known. There is, however, no evidence that the industrial sector is weak-form inefficient. Therefore, the use of candlestick patterns does not outperform the benchmark after-fees.

Table 6: Candlestick results in the industrial sector

Sector	Observations					σ			Daily return	
Industrials	67 446					1.91%			0.05%	
Pattern	**Observations**	**Predicted Success**	**Actual Success**	p	$n(p-p_0)$	σ	**Z-score**	**P-value**	**Daily return**	**Daily return ****
Uptrend reversal	8 914		4 433	50%						
Downtrend reversal	7 406		3 751	51%						
Three White Soldiers (TWS)	930	471	473	51%	1.97	15.25	0.13	0.45	-0.03%	-0.97%
Three Black Crows (TBC)*	1 127	560	584	52%	23.53	16.79	1.40	0.08	0.09%	-0.91%
Three Inside Up (TIU)	845	428	441	42%	13.02	14.53	0.90	0.19	0.17%	-0.83%
Three Inside Down (TIU) *	928	462	483	52%	21.50	15.23	1.41	0.08	0.14%	-0.86%
Three Outside Up (TOU)	593	300	322	54%	21.66	12.17	1.78	0.04	0.28%	-0.72%
Three Outside Down (TOD) *	820	408	437	53%	29.21	14.32	2.04	0.02	0.22%	-0.78%
Morning Star (MS)	382	193	210	55%	16.52	9.77	0.34	0.05	0.24%	-0.76%

* Indicates bearish pattern
**Indicates after-fee return

4.4 Resources

Table 7 highlights the difference in the predictive ability of candlesticks in the resources sector. The sector was the smallest in the data set, with 14 shares. Surprisingly, candlesticks performed poorly in the resources sector. Han, Yang and Zhou, 2013 and Lu, Chen and Hsu (2015), found that the candlesticks performed better in a more volatile environment. Table 6 highlights that the resources sector was the most volatile sector of the three JSE sector samples investigated. The resources sector had 4 544 uptrends and 4 445 downtrends. For both uptrends and downtrends, forty-nine percent successfully reversed over the three-day holding period. However, even though it was the most volatile of the three sectors (see Table 1), the predicative ability of the candlestick patterns was found to be statistically insignificant at both the five and ten percent level of significance.

Similarly, the return of the candlesticks is insignificant. In addition, the returns in many cases are loss making. For example, the TBC and TID, both bearish patterns, generate an average return of 0.22% in both cases. The gross of fee returns versus the respective passive benchmark was found to be statistically non-significant. The inclusion of trading costs exacerbated this problem further, with the p-value for all the returns in the table increasing to 100 percent.

Table 7: Candlestick results in the resources sector

Sector	Observations					σ			Daily return	
Resources	37 470					2.49%			0.04%	
Pattern	**Observations**	**Predicted Success**	**Actual Success**	**p**	**$n(p - p_0)$**	**σ**	**Z-score**	**P-value**	**Daily return**	**Daily return ****
Uptrend reversal	4 544		2 213	49%						
Downtrend reversal	4 445		2 184	49%						
Three White Soldiers (TWS)	489	240	223	46%	-17.26	11.06	-1.56	0.94	-0.10%	-1.10%
Three Black Crows (TBC)*	595	290	285	48%	-4.77	12.19	-0.39	0.65	-0.22%	-1.22%
Three Inside Up (TIU)	547	269	275	50%	6.24	11.69	0.53	0.30	0.24%	-0.76%
Three Inside Down (TIU) *	550	268	269	49%	1.14	11.72	0.10	0.46	-0.22%	-1.22%
Three Outside Up (TOU)	324	159	151	47%	-8.19	9.00	-0.91	0.82	0.18%	-0.82%
Three Outside Down (TOD) *	468	228	230	49%	2.08	10.81	0.19	0.42	-0.14%	-1.14%
Morning Star (MS)	278	137	133	48%	-3.59	8.34	-0.43	0.67	-0.10%	-1.10%

*Indicates bearish pattern
**Indicates after-fee returns

The evidence in the resources sector supports the RWT and weak-form EMH. This is evidenced through lack of predictive ability of the candlestick patterns in the resources sector and its inability to generate statistically significant returns. The reason for the finding may be that the resources sector is historically considered the core of the South African equity market. Many of the companies in the resources sector tend to be exposed to one commodity and are uncomplicated to understand. These companies also tend to be driven by a small number of variables, such as international commodity price and the United States Dollar to South African Rand (USD-ZAR) exchange rate. Both of these variables are public and available in real time (Jefferis and Smith, 2004).

5. Conclusion

The objective of this study was to investigate the efficacy of the 18th century Japanese Candlestick technique. The study was two-fold. Firstly, the predicative ability of the candlesticks was tested. Secondly, the use of the candlesticks as a trading strategy was tested. The first aspect of the study tested the RWT, while the second tested the weak-form EMH.

Although the technique is centuries old, there is little empirical evidence to support it, particularly in emerging markets. In both emerging and developed markets, where studies have been conducted there have been mixed results. For example, in developed markets, Caginalp and Laurent (1998) and Lu, Chen & Hsu (2015) found strong evidence that candlestick patterns can be profitably used as investment strategy; the patterns generated statistically significant returns after costs. In contrast, Marshall, Young and Rose (2006) and Horton's (2007) found no evidence in support of the predictive power of candlestick patterns. In emerging markets, there have also been diverging findings. For example, Lu, Shiu, and Liu (2012) find evidence in support of the predictive power of candlesticks in the Taiwanese market, whilst Prado et al. (2013) found no evidence that candlesticks have predictive value in the Brazilian market.

Empirical studies of candlestick charts in emerging markets have been limited. The studies that have been conducted have primarily been done in the Asian market. South Africa does not feature in the literature. The South African equity market provides an interesting market to study as Lim and Brook (2010) found that the JSE failed the random walk test 58% of the time between 1995 and 2005. In that study, of the 50 markets investigated, South Africa ranked sixth in terms of the greatest deviation from the random walk.

This study aimed to address this gap in the literature. Using the same methodology as Caginalp and Laurent (1998), eight three-day patterns were tested, using the normal approximation to the binomial distribution. This study used daily prices of JSE listed shares from January 2010 to December 2019 as the basis of the study. This consisted of sixty shares; fourteen were in the resources sector, nineteen in the financial sector and twenty-seven in the resource sector. All the data was sourced from Bloomberg.

In the 60-share sample, the TOD pattern was found to be predicative at the five percent level of significance. The TBC was inconclusive. All the patterns in the sample had the correct return directionally. However, none of the gross of fee returns were found to be statistically significant. The addition of transaction costs further exacerbated this finding.

Interestingly, the predicative ability of candlesticks differed across the three sectors. There was no evidence that the candlesticks patterns used in the study have any predicative ability on the financial and resource sector. On the other hand, in the industrial sector, the TOU and TOD patterns were found to be predictive at the five percent level of significance. The TBC and TIU patterns were found to be significant at the ten percent level of significance. In the case of industrial sector, this provides evidence against the RWT. Although the patterns were found to be predictive in the industrial sector, none of the gross of fee returns generated by the candlesticks, across the three sectors, were found to be statistically significant. After costs this performance was, as expected, worse.

The reason for the difference in predictive performance goes beyond the scope of the study, but some very speculative possibilities can be offered. Thus, resource companies tend to be driven by a few variables, such as the commodity price and the USD-ZAR exchange rate (Jefferis and Smith, 2004). The evidence in favour of the random walk in the resources sector highlights that these variables are public. Similarly, the financial sector is driven by market performance and interest rates (Damodaran, 2010), both of which are public information. The above may not apply to the industrial sector.

Although the South African equity market does not completely follow the RWT, the results of the study suggest the South African equity market is weak-form efficient; that is, past prices alone cannot be used to generate statistically significant positive returns. The findings of the study do, however, indicate that some candlestick patterns are able to predict detect turning points, which may be value adding for market practitioners.

This study could in future be extended by explicitly testing whether candlestick patterns have better predictive ability on more volatile assets. Lu, Chen and Hsu (2015) found that candlesticks have superior predicative ability on more volatile assets. This study however, did not find that candlestick patterns study have predictive ability on more volatile assets. The study could be extended to include the Alt-X which may be less informationally efficient and more volatile than the JSE.

A further suggestion is to extend the research into other markets in Africa. The study of candlestick patterns on emerging markets remains limited. South Africa’s equity market is the largest in Africa and this study indicates that it is weak-form efficient. Jefferis amd Smith (2008) however, find differences in efficiency in several African equity markets. This raises questions of the effectiveness of candlesticks on less efficient markets.

This study could in future be extended to include other candlestick patterns. This study includes eight three-day patterns, however Morris (2006) highlights more than twenty three-day patterns and several more one, two, four-or more-day patterns. Nison (2001) similarly highlights several continuation patterns.

Lastly, this study focusses on the South African equity market. Studies by Neely, Weller and Dittmar (1997) and Okunev and White (2001) found that technical analysis generated statistically significant returns on developed market currencies. Earlier studies by Stevenson and Bear (1970) and Leuthold (1972) on the futures market also found strong evidence in support of technical trading rules. This study could be extended to include testing candlestick patterns on emerging market currencies or futures markets.

References

Akerlof, G., 1970. The Market for "Lemons": Quality Uncertainty and the Market Mechanism. *The Quarterly Journal of Economics*, 84(3), pp.488-500.

Ashraf, N., Camerer, C. and Loewenstein, G., 2005. Adam Smith, Behavioral Economist. *Journal of Economic Perspectives*, 19(3), pp.131-145.

Asness, C., Frazzini, A., Israel, R. and Moskowitz, T., 2014. Fact, Fiction and Momentum Investing. *The Journal of Portfolio Management*, (Special 40th Anniversary Issue).

Ball, R., 1978. Anomalies in relationships between securities' yields and yield-surrogates. *Journal of Financial Economics*, 6(2-3), pp.103-126.

Bogle, J., 2005. The Mutual Fund Industry 60 Years Later: For Better or Worse?. *Financial Analysts Journal*, 61(1), pp.15-24.

Brenner, M., 1977. The Effect of Model Misspecification on Tests of the Efficient Market Hypothesis. *The Journal of Finance*, 32(1), pp.57-66.

Brenner, M., 1979. The Sensitivity of the Efficient Market Hypothesis to Alternative Specifications of the Market Model. *The Journal of Finance*, 34(4), pp.915-929.

Brock, W., Lakonishok, J. and LeBaron, B., 1992. Simple Technical Trading Rules and the Stochastic Properties of Stock Returns. *The Journal of Finance*, 47(5), pp.1731-1764.

Caginalp, G. and Laurent, H., 1998. The predictive power of price patterns. *Applied Mathematical Finance*, 5(3-4), pp.181-205.

Constantinides, G., Harris, M. and Stulz, R., 2003. *Handbook of The Economics Of Finance*. 1st ed. Amsterdam: Elsevier/North-Holland, pp.1053-1128.

Cotis, J., Coppel, J. and Ortega, E., 2005. *Business Cycle Dynamics In OECD Countries: Evidence, Causes And Policy Implications*. Sydney: Reserve Bank of Australia.

Damodaran, A., 2010. *The Dark Side of Valuation: Valuing Young, Distressed, And Complex Businesses*. 2nd ed. New Jersey: Pearson Education, pp.312-476.

De Bondt, W. and Thaler, R., 1985. Does the Stock Market Overreact?. *The Journal of Finance*, 40(3), pp.793-805.

De Long, J., Shleifer, A., Summers, L. and Waldmann, R., 1990. Noise Trader Risk in Financial Markets. *Journal of Political Economy*, 98(4), pp.703-738.

Easy Equities, 2019. *Cost Profile: Easy Equities*. [online] p.2. Available at: <https://resources.easyequities.co.za/EasyEquities_CostProfile.pdf?_ga=2.106498309.746616299.1611776315-1135730615.1611776315> [Accessed 4 December 2020].

Erb, C., Harvey, C. and Viskanta, T., 1995. Country Risk and Global Equity Selection. *The Journal of Portfolio Management*, 21(2), pp.74-83.

Evans, D., Drew, J. and Leemis, L., 2008. The Distribution of the Kolmogorov–Smirnov, Cramer–von Mises, and Anderson–Darling Test Statistics for Exponential Populations with Estimated Parameters. *Communications in Statistics - Simulation and Computation*, 37(7), pp.1396-1421.

Fama, E., 1965. Random Walks in Stock Market Prices. *Financial Analysts Journal*, 21(5), pp.55-59.

Fama, E., 1991. Efficient Capital Markets: II. *The Journal of Finance*, 46(5), pp.1575-1617.

Fama, E. and Blume, M., 1966. Filter Rules and Stock-Market Trading. *The Journal of Business*, 39(S1), pp.226-241.

Fama, E., Fisher, L., Jensen, M. and Roll, R., 1969. The Adjustment of Stock Prices to New Information. *International Economic Review*, 46(5), pp.1575-1617.

Granger, C. and Morgenstern, O., 1963. Spectral Nalaysis of New York Stock Market Prices. *Kyklos*, 16(1), pp.1-27.

Grater, E. and Struweg, J., 2015. Testing weak form efficiency in the South African market. *Journal of Economic and Financial Sciences*, 8(2), pp.621-632.

Han, Y., Yang, K. and Zhou, G., 2013. A New Anomaly: The Cross-Sectional Profitability of Technical Analysis. *Journal of Financial and Quantitative Analysis*, 48(5), pp.1433-1461.

Harris, L. and Gurel, E., 1986. Price and Volume Effects Associated with Changes in the S&P 500 List: New Evidence for the Existence of Price Pressures. *The Journal of Finance*, 41(4), p.815.

Heaton, J., 2019. Managerial optimism: New observations on the unifying theory. *European Financial Management*, 25(5), pp.1150-1167.

Horton, M., 2009. Stars, crows, and doji: The use of candlesticks in stock selection. *The Quarterly Review of Economics and Finance*, 49(2), pp.283-294.

Hsu, P., Hsu, Y. and Kuan, C., 2010. Testing the predictive ability of technical analysis using a new stepwise test without data snooping bias. *Journal of Empirical Finance*, 17(3), pp.471-484.

Huppatz, D., 2015. Revisiting Herbert Simon's "Science of Design". *Design Issues*, 31(2), pp.29-40.

Ibbotson, R., Idzorek, T., Kaplan, P. and Xiong, J., 2018. *Popularity: A Bridge Between Classical And Behavioral Science*.

Jefferis, K. and Smith, G., 2004. Capitalisation and Weak-form Efficiency in the JSE Securities Exchange. *South African Journal of Economics*, 72(4), pp.684-707.

Jegadeesh, N., 1990. Evidence of Predictable Behavior of Security Returns. *The Journal of Finance*, 45(3), pp.881-898.

Jensen, M., 1978. Some anomalous evidence regarding market efficiency. *Journal of Financial Economics*, 6(2-3), pp.95-101.

Kahneman, D. and Tversky, A., 1982. *Intuitive Prediction: Biases And Corrective Procedures*. Cambridge: Cambridge University Press, pp.414-421.

Kendall, M., 1953. The Analysis of Economic Time-Series-Part I: Prices. *Journal of the Royal Statistical Society. Series A (General)*, 116(1), pp.11-34.

Kenneth Malefo, B., Hsieh, H. and Hodnett, K., 2016. Performance evaluation of actively managed mutual funds. *Investment Management and Financial Innovations*, 13(4), pp.188-195.

Kim, J. and Choi, I., 2019. Choosing the Level of Significance: A Decision-theoretic Approach. *Abacus*, pp.1-16.

Konte, M., 2010. Behavioral Finance and Efficient Markets: The Joint Hypothesis is it Really the Problem?. *The IUP Journal of Behavioral Finance*, 7(1), pp.19-29.

Korn, O. and Kempf, A., 1997. Market depth and order size: an analysis of permanent price effects of DAX futures' trades. *SSRN Electronic Journal*,.

Kruger, R. and Toerien, F., 2013. The Impact Of Index Migrations On Share Prices: Evidence From The Johannesburg Stock Exchange. *Journal of Applied Business Research (JABR)*, 29(6), pp.1861-1872.

Lehmann, B., 1990. Fads, Martingales, and Market Efficiency. *The Quarterly Journal of Economics*, 105(1), pp.1-28.

LeRoy, S., 1973. Risk Aversion and the Martingale Property of Stock Prices. *International Economic Review*, 14(2), pp.436-446.

Leuthold, R., 1972. Random Walk and Price Trends: The Live Cattle Futures Market. *The Journal of Finance*, 27(4), pp.879-889.

Levy, R., 1971. The Predictive Significance of Five-Point Chart Patterns. *The Journal of Business*, 44(3), pp.316-323.

Lim, K. and Brooks, R., 2010 Why Do Emerging Markets Experience More Persistent Price Deviations from a Random Walk Over Time? A Country-Level Analysis. *Macroeconomic Dynamics*, 14(S1), pp.3-41.

Lo, A., 2004. The Adaptive Markets Hypothesis. *The Journal of Portfolio Management*, 30(5), pp.15-29.

Lo, A. and MacKinlay, A., 2002. *A Non-Random Walk Down Wall Street*. Princeton: Princeton Univ. Press, p.6.

Lu, T., 2014. The profitability of candlestick charting in the Taiwan stock market. *Pacific-Basin Finance Journal*, 26, pp.65-78.

Lu, T., Chen, Y. and Hsu, Y., 2015. Trend definition or holding strategy: What determines the profitability of candlestick charting?. *Journal of Banking & Finance*, 61, pp.172-183.

Lu, T., Shiu, Y. and Liu, T., 2012. Profitable candlestick trading strategies—The evidence from a new perspective. *Review of Financial Economics*, 21(2), pp.63-68.

Malkiel, B., 2003. The Efficient Market Hypothesis and Its Critics. *Journal of Economic Perspectives*, 17(1), pp.59-82.

Marshall, B., Young, M. and Rose, L., 2006. Candlestick technical trading strategies: Can they create value for investors?. *Journal of Banking & Finance*, 30(8), pp.2303-2323.

Morris, G., 2006. *Candlestick Charting Explained*. 3rd ed. New York, USA: McGraw-Hill.

Neely, C., Weller, P. and Dittmar, R., 1997. Is Technical Analysis in the Foreign Exchange Market Profitable? A Genetic Programming Approach. *The Journal of Financial and Quantitative Analysis*, 32(4), pp.405-426.

Ni, Y., Liao, Y. and Huang, P., 2015. MA trading rules, herding behaviors, and stock market overreaction. *International Review of Economics & Finance*, 39, pp.253-265.

Niederhoffer, V., 1965. Clustering of Stock Prices. *Operations Research*, 13(2), pp.258-265.

Niederhoffer, V. and Osborne, M., 1966. Market Making and Reversal on the Stock Exchange. *Journal of the American Statistical Association*, 61(316), pp.897-916.

Nison, S., 2001. *Japanese Candlestick Charting Techniques*. 2nd ed. New York: New York Institute of finance.

Okunev, J. and White, D., 2003. Do Momentum-Based Strategies Still Work in Foreign Currency Markets?. *The Journal of Financial and Quantitative Analysis*, 38(2), pp.425-447.

Osler, C. and Chang, P., 1995. Head and Shoulders: Not Just a Flaky Pattern. *SSRN Electronic Journal*, [online] Available at: <https://papers.ssrn.com/sol3/papers.cfm?abstract_id=993938>.

Park, C. and Irwin, S., 2007. What do we know about the profitability of technical analysis? *Journal of Economic Surveys*, 21(4), pp.786-826.

Pastor, L. and Veronesi, P., 2006. Was there a Nasdaq bubble in the late 1990s?. *Journal of Financial Economics*, 81(1), pp.61-100.

Prado, H., Ferneda, E., Morais, L., Luiz, A. and Matsura, E., 2013. On the Effectiveness of Candlestick Chart Analysis for the Brazilian Stock Market. *Procedia Computer Science*, 22, pp.1136-1145.

Rohleder, M., Scholz, H. and Wilkens, M., 2011. Survivorship Bias and Mutual Fund Performance: Relevance, Significance, and Methodical Differences*. *Review of Finance*, 15(2), pp.441-474.

Roll, R., 1977. A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory. *Journal of Financial Economics*, 4(2), pp.129-176.

Rosenow, B., 2002. Fluctuations and Market Friction in Financial Trading. *International Journal of Modern Physics C*, 13(03), pp.419-425.

Rothschild, M. and Stiglitz, J., 1976. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *The Quarterly Journal of Economics*, 90(4), pp.629-649.

Schwager, J., 1989. *Market Wizards*. New York, NY: New York Institute of Finance.

Shleifer, A., 1986. Do Demand Curves for Stocks Slope Down?. *The Journal of Finance*, 41(3), pp.579-590.

Smith, G., 2008. Liquidity and the Informational Efficiency of African Stock Markets. *South African Journal of Economics*, 76(2), pp.161-175.

Smith, G., Jefferis, K. and Ryoo, H., 2002. African stock markets: multiple variance ratio tests of random walks. *Applied Financial Economics*, 12(7), pp.475-484.

Stevenson, R. and Bear, R., 1970. Commodity Futures: Trends or Random Walks?. *The Journal of Finance*, 25(1), pp.65-81.

Tharavanij, P., Siraprapasiri, V. and Rajchamaha, K., 2017. Profitability of Candlestick Charting Patterns in the Stock Exchange of Thailand. *SAGE Open*, 7(4), pp.1-18.

Tversky, A. and Kahneman, D., 1974. Judgment under Uncertainty: Heuristics and Biases. *Science*, 185(4157), pp.1124-1131.

Watts, R., 1978. Systematic ‘abnormal’ returns after quarterly earnings announcements. *Journal of Financial Economics*, 6(2-3), pp.127-150.

World Bank, 2020. *Doing Business 2020: Economy Profile South Africa*. Doing Business. [online] World Bank Group, p.39.

Wurgler, J. and Zhuravskaya, E., 2002. Does Arbitrage Flatten Demand Curves for Stocks?. *The Journal of Business*, 75(4), pp.583-608.

Appendices

Appendix A: Sector breakdown

The table below indicates the names, tickers and sectors of the shares included in the study. In total there are sixty shares, with fourteen in the resources sector, nineteen in the financials sector and twenty-seven in the industrial sector.

Name	Ticker	Sector
ABSA Group	ABG	Financial
AECI	AFE	Industrial
Anglo American	AGL	Resources
AngloAmerican Platinum	AMS	Resources
AngloGold Ashanti	ANG	Resources
Aspen	APN	Industrial
African Rainbow Minerals	ARI	Resources
Astral Food	ARL	Industrial
AVI	AVI	Industrial
Barloworld	BAW	Industrial
BHP Group PLC	BHP	Resources
Bidvest Group	BVT	Industrial
Compagnie Financiere Richemont	CFR	Industrial
Clicks	CLS	Industrial
Discovery	DSY	Financial
Datatec	DTC	Industrial
Exxaro	EXX	Resources
FirstRand	FSR	Financial
Gold Fields	GFI	Resources
Growthpoint Properties	GRT	Financial
Harmony Gold Mining Company	HAR	Resources
Hyprop Investments	HYP	Financial
Impala Platinum Holdings	IMP	Resources
Investec	INL	Financial
Investec plc	INP	Financial
Imperial Logistics	IPL	Industrial
JSE	JSE	Financial
Kumba Iron Ore	KIO	Resources
Liberty Holdings	LBH	Financial
Mediclinic International PLC	MEI	Industrial
Mondi Plc	MNP	Resources
MR Price Group	MRP	Industrial
Momentum Metropolitan Holdings	MTM	Financial
MTN	MTN	Industrial
Nedbank Group	NED	Financial
Northam Platinum	NHM	Resources

Naspers	NPN	Industrial
Netcare	NTC	Industrial
Pioneer Food Group	PFG	Industrial
Pick ‘n Pay Stores	PIK	Industrial
Redefine Properties	RDF	Financial
Remgro	REM	Financial
Resilient Reit	RES	Financial
Reunert	RLO	Industrial
RMN Holdings	RMH	Financial
SA Corporate Real Estate Fund Managers	SAC	Financial
Sappi	SAP	Resources
Standard Bank Group	SBK	Financial
Shoprite Holdings	SHP	Industrial
Sanlam	SLM	Financial
Santam	SNT	Financial
Sasol	SOL	Resources
Spar Group	SPP	Industrial
Tiger Brands	TBS	Industrial
The Foschini Group	TFG	Industrial
Telkom	TKG	Industrial
Truworths Internationals	TRU	Industrial
Vodacom	VOD	Industrial
Wilson Bayly	WBO	Industrial
Woolworths Holdings	WHL	Industrial

Appendix B: Pattern definitions

Each of patterns in the study are three-day patterns with t+1 being the first day, and t+3 being the last day of the pattern. The return of the pattern is measured from the open of t+4 to the close on t+6. See Figure 3 for the three-day candlestick pattern.

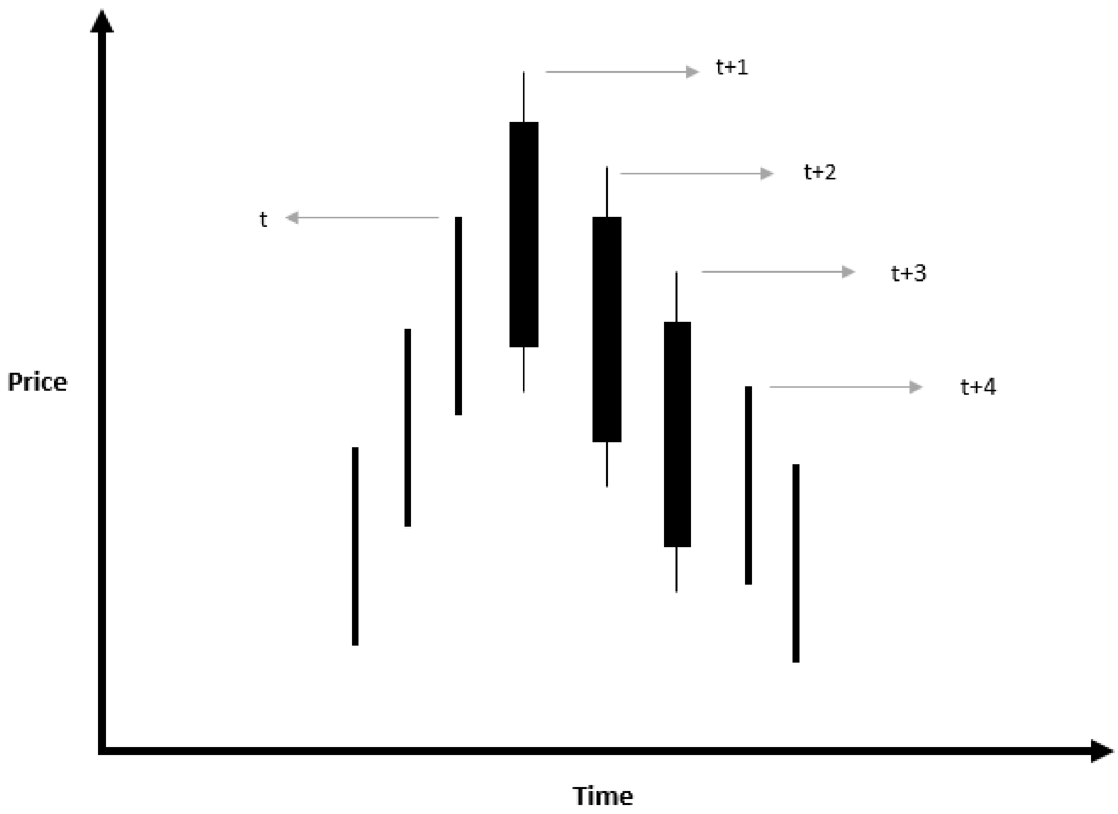

Figure 3: Three-day candlestick patterns

Three White Soldiers

The Three White Soldier (TWS) pattern is comprised of three white candles which close at progressively higher prices, and begin after a downtrend. To maintain mathematical precision and non-parametric criteria, the condition on the length of the candle body is eliminated; in practice, the length of the candle on t+2 and t+3 is of importance (see Figure 4).

Mathematically, the TWS is defined as:

1. The first day (t+1), occurs after a downtrend as defined in 3.1.
2. Three consecutive candles occur; each closing at a higher price

$$c_i - o_i > 0 \; for \; i = t+1, t+2, t+3$$

$$c_{t+3} > c_{t+2} > c_{t+1}$$

3. Each day opens within the previous day's range

$$c_{t+1} > o_{t+2} > o_{t+1}$$

$$c_{t+2} > o_{t+3} > o_{t+2}$$

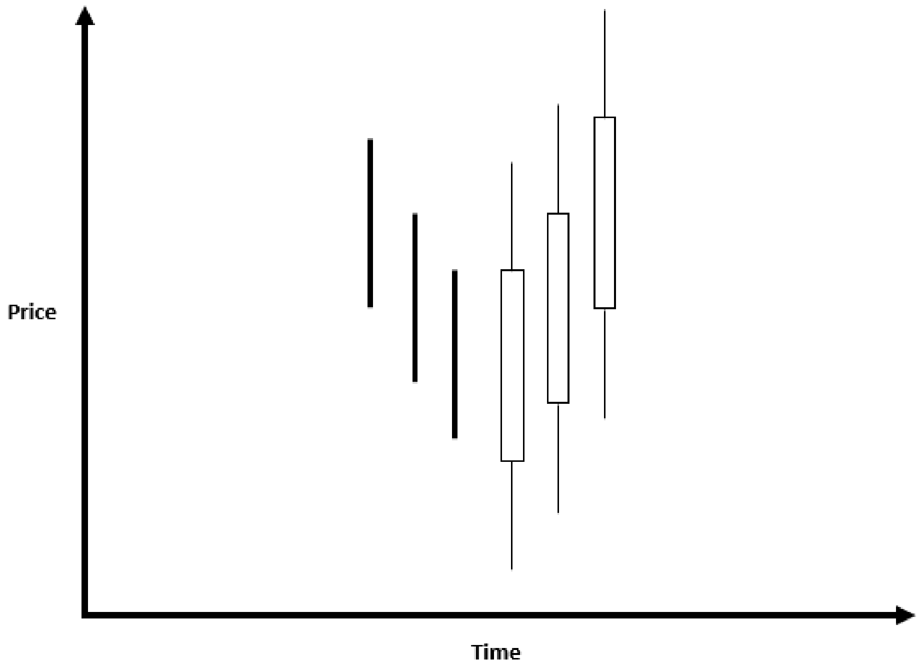

Figure 4: Three white soldiers

Three Black Crows

The Three Black Crows (TBC) is the bearish counterpart of the TWS. Ideally, the three candles should close at, or near, their lows (Nison, 2001). In addition, each day opens within the body of the previous candle (see Figure 5).

TBC is defined as:

1. The first day (t+1), occurs after an uptrend as defined in 3.1.
2. Three consecutive candles occur; each closing at a lower price

$$o_i - c_i > 0 \; for \; i = t+1, t+2, t+3$$

$$o_{t+1} > o_{t+2} > o_{t+3}$$

$$c_{t+1} > c_{t+2} > c_{t+3}$$

3. Each day opens within the previous day's range

$$o_{t+1} > o_{t+2} > c_{t+1}$$

$$o_{t+2} > o_{t+3} > c_{t+2}$$

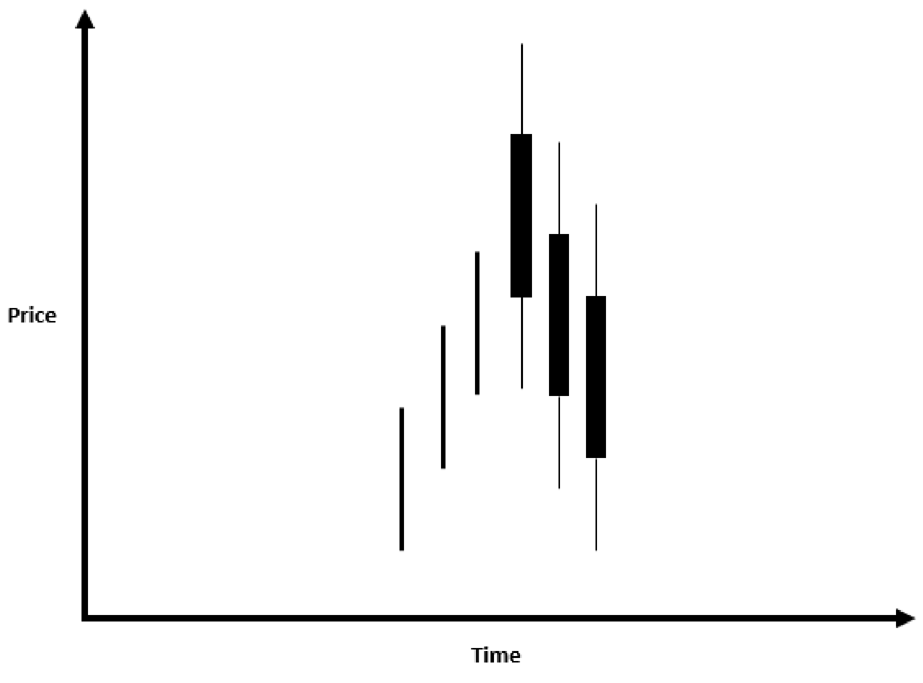

Figure 5: Three black crows

Three Inside Up

The Three Inside Up (TIU) is a three-day pattern that does not appear in the Japanese literature. The pattern was developed by Morris (2006). The Harami pattern – which features in the Japanese literature - occurs on the first two days. The third day serves as a confirmation. The first day of the pattern is a black candle followed by a smaller candle on the second day, that is either black or white, but must open and close within the first body. The third day is a white candle that closes at a new high (see Figure 6).

TIU is defined as:

1. The first day (t+1), occurs after a downtrend as defined in 3.1.
2. The first day of the pattern (t+1) should be a black day

$$o_{t+1} - c_{t+1} > 0$$

3. The second day (t+2) must be contained within the body of the first day of the pattern

$$o_{t+1} \geq o_{t+2} > c_{t+1}$$

$$o_{t+2} > c_{t+2} \geq c_{t+1}$$

Only one of the two inequalities can hold. Either the opening prices or the closing price of t+1 and t+2 may be equal.

4. Day t+3 has a higher closing price than opening price, and closes above the opening price of day t+1

$$c_{t+3} > o_{t+3}$$

$$c_{t+3} > o_{t+1}$$

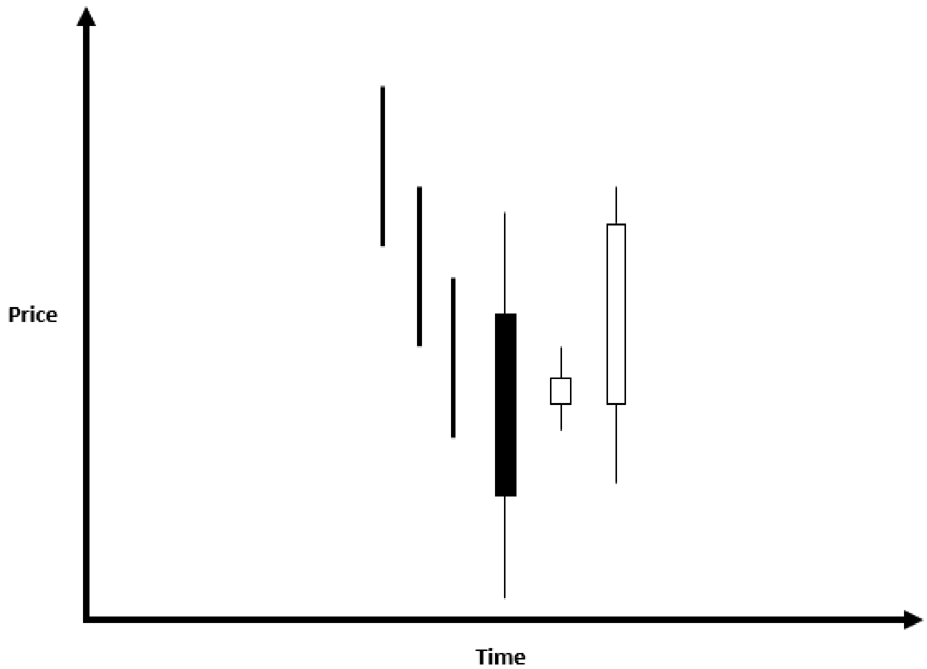

Figure 6: Three inside up

Three Inside Down

The Three Inside Down (TID) pattern is the bearish counterpart of the TIU. It similarly does not feature in the Japanese literature. The first day of the pattern (t+1) is a white candle followed by smaller candle on the second day (t+2), that can be either black or white, but the opening and closing prices are within the first body. The third day (t+3) is a black candle that closes at a new low (See Figure 7).

TID is defined as:

1. The first day (t+1), occurs after an uptrend as defined in 3.1.
2. The first day of the pattern (t+1) should be a white candle

$$c_{t+1} - o_{t+1} > 0$$

3. The second day (t+2) must be contained within the body of the first day of the pattern

$$c_{t+1} > o_{t+2} \geq o_{t+1}$$

$$c_{t+1} \geq c_{t+2} > o_{t+1}$$

Only one of the two inequalities can hold. Either the opening prices or the closing price of t+1 and t+2 may be equal.

4. Day t+3 has a lower close than open and closes below the open of day t+1

$$o_{t+3} > c_{t+3}$$

$$c_{t+3} > o_{t+1}$$

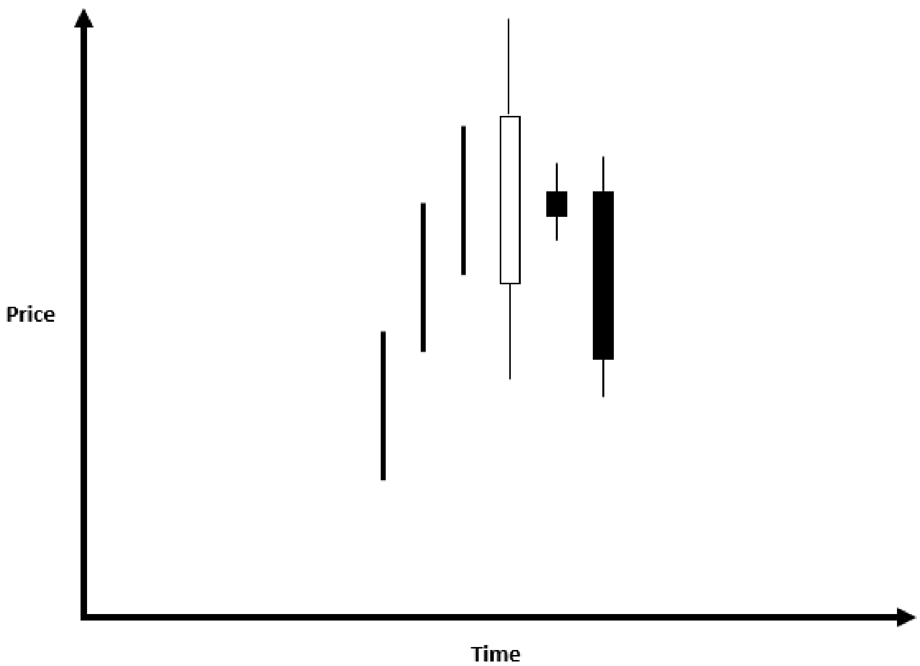

Figure 7: Three inside down

Three Outside Up

The Three Outside Up (TOU) is a pattern that does not feature in Japanese literature (Morris, 2006), and was developed as confirmation pattern of the engulfing pattern. The engulfing pattern is a major reversal signal that consists of two candles of the opposite colour (Nison, 2001). In the case of the TOU, after a downtrend, the first candle is a black candle. This is followed by a large second candle that is larger than the first candle. In addition, the second candle is white and the whole of the first body lies within the second candle's body. The third candle is a white candle that closes at a new high (see Figure 8).

TOU is defined as:

1. The first day (t+1), occurs after a downtrend as defined in 3.1.
2. The first day of the pattern (t+1) should be a black candle

$$o_{t+1} - c_{t+1} > 0$$

3. The second day (t+2) must engulf the body of the first day of the pattern

$$c_{t+2} \geq o_{t+1} \geq c_{t+1} \geq o_{t+2}$$

$$|c_{t+2} - o_{t+2}| > |c_{t+1} - o_{t+1}|$$

4. Day t+3 has a higher closing price than opening price, and closes above the opening price of day t+2

$$c_{t+3} > o_{t+3}$$

$$c_{t+3} > c_{t+2}$$

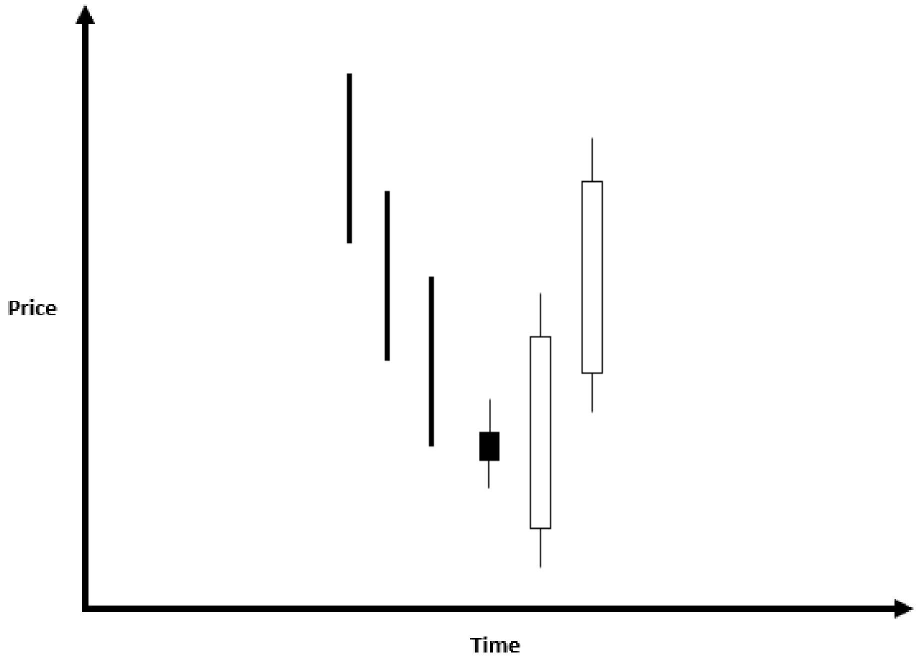

Figure 8: Three outside up

Three Outside Down

The Three Outside Down (TOD) pattern also does not feature in Japanese literature (Morris, 2006), and is the bearish counterpart of the TOU (see Figure 9).

TOD is defined as:

1. The first day (t+1), occurs after an uptrend as defined in 3.1.
2. The first day of the pattern (t+1) should be a white candle

$$c_{t+1} - o_{t+1} > 0$$

3. The second day (t+2), a black candle, must engulf the body of the first day of the pattern

$$o_{t+2} \geq c_{t+1} > o_{t+1} \geq c_{t+2}$$

$$|c_{t+2} - o_{t+2}| > |c_{t+1} - o_{t+1}|$$

4. Day t+3 has a higher closing than opening price, and closes above the opening price of day t+2

$$o_{t+3} > c_{t+3}$$

$$c_{t+3} < c_{t+2}$$

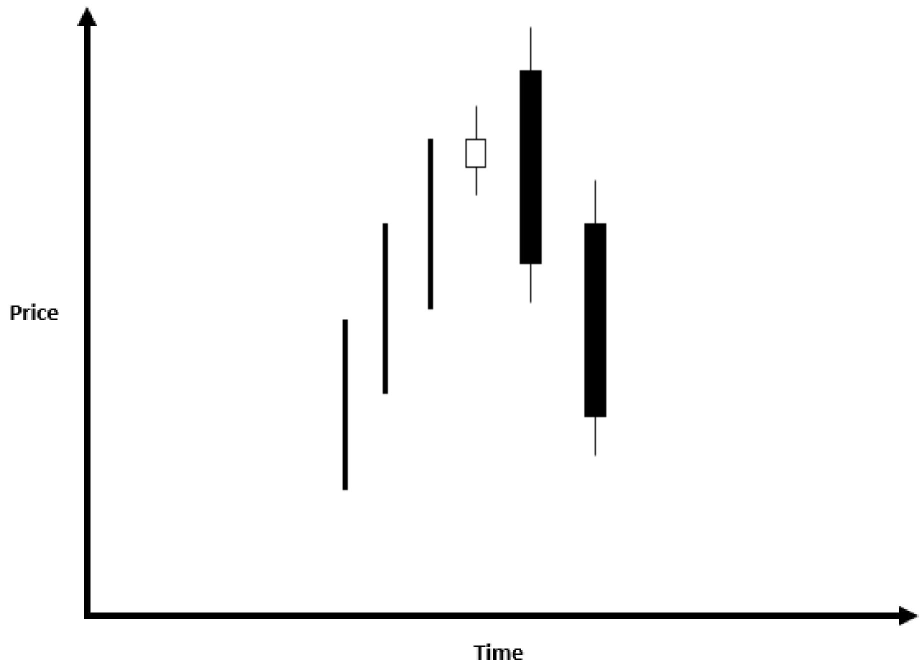

Figure 9: Three outside down

Morning Star

The Morning Star (MS) is a bullish reversal pattern. It consists of a long body followed by a small body that gaps lower; the second candle be either black or white. The smaller second body describes some market indecision. The third day is a white candle that closes past the midpoint of the first day (see Figure 10).

MS is defined as:

1. The first day (t+1), occurs after a downtrend as defined in 3.1.
2. The first day of the pattern (t+1) should be a black candle

$$o_{t+1} - c_{t+1} > 0$$

3. The second day (t+2), must be gapped from the first candle.it can be either colour

$$|o_{t+2} - c_{t+2}| > 0$$

$$c_{t+1} > c_{t+2} \ and \ c_{t+1} > o_{t+2}$$

4. Day t+3 is a white candle, and ends higher than the midpoint of the first day (t+1)

$$c_{t+3} > o_{t+3}$$

$$c_{t+3} > \frac{o_{t+1} - c_{t+1}}{2}$$

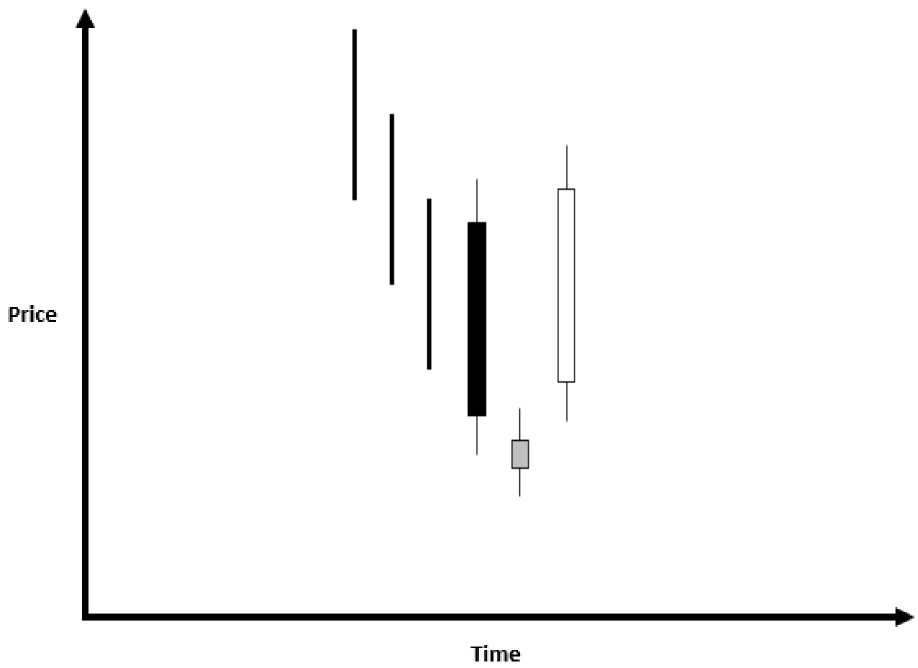

Figure 10: Morning star

Evening Star

The Evening Star (ES) is the mirror image of the MS, and is a bearish reversal pattern (see Figure 11).

ES is defined as:

1. The first day (t+1), occurs after an uptrend as defined in 3.1.
2. The first day of the pattern (t+1) should be a white candle

$$c_{t+1} - o_{t+1} > 0$$

3. The second day (t+2), must be gapped from the first candle.it can be either colour.

$$|o_{t+2} - c_{t+2}| > 0$$

$$c_{t+2} > c_{t+1} \; and \; o_{t+2} > c_{t+1}$$

4. Day t+3 is a black candle, and ends lower than the midpoint of the first day (t+1)

$$o_{t+3} > c_{t+3}$$

$$c_{t+3} > \frac{c_{t+1} - o_{t+1}}{2}$$

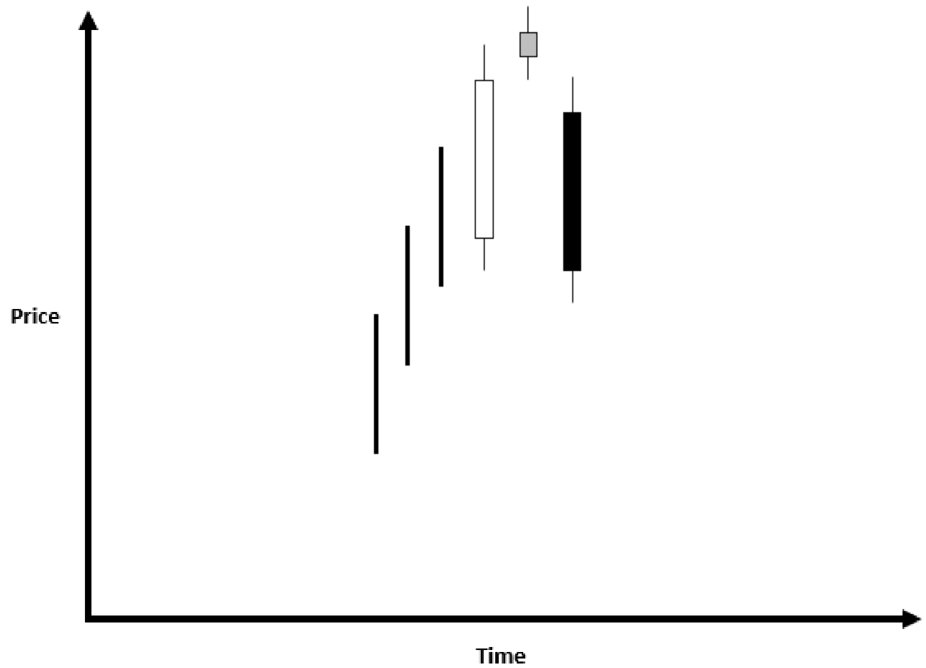

Figure 11: Evening star