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**UNIVERSITY OF CAPE TOWN**

**FACULTY OF HUMANITIES**

**DEPARTMENT OF EDUCATION**

**An Investigation of Mathematical Misconceptions through an Analysis of  
Grade 7 Learners' Responses to Test Items on Decimals, Percentages  
and Measurement**

**A minor dissertation submitted to the University of Cape Town in partial  
fulfilment of the requirements for the degree of Master of Education  
(Mathematics Education)**

**By**

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**October 2001**

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## *DECLARATION*

I declare that this dissertation is my own unaided work. It is being submitted for the degree of Master of Education of the University of Cape Town. It has not been submitted before for any degree of examination in any other university

GEORGE TAWODZERA

SIGNATURE signature removed

OCTOBER 2001

University of Cape Town

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## ***ABSTRACT***

This research dissertation emerges as a component of a broader research study, which sought to determine the impact of a mathematics textbook, *Maths for all (Mfa)* on teaching and learning, in general, and on learners' performance, in particular. The impact evaluation study focused on Grade 7 learners, from a sample of formerly DET primary classrooms in townships near Cape Town. It focused particularly on the teaching of decimals, percentages and measurement which 14 teachers in these schools agreed to teach in the second term of 2000. The 538 learners, from 10 experimental classrooms (with access to *Mfa*) and 4 control classrooms (with no access to *Mfa*), were given a pre-test at the beginning of the second term, and the same test as a post-test towards the end of the same term of the year 2000.

The present study aims to analyse possible patterns of error in learners' responses to the test and investigate whether these patterns suggest underlying misconceptions held by the learners on decimals, percentages and measurement. As a secondary aspect, the study also set out to evaluate the test instrument as a measure of achievement and of potential misconceptions.

Through an analysis of the learners' incorrect responses to clusters of similar, topic-related post-test items, several possible underlying misconceptions were uncovered. A substantial number of the identified possible underlying misconceptions were linked to a lack of, or limited conceptual understanding of, the symbolic representations of fractions, decimals and percentages. The research findings revealed that some learners failed to construct appropriate meanings of fraction, decimal and percentage symbols. Several possible reasons are suggested for this. Firstly, some learners had not been taught the topics of decimals, percentages and measurement and did not have sufficient informal mathematical pre-knowledge to assist them in answering some of test questions. Secondly, some learners seem to have developed incorrect strategies, which at times resulted by chance in correct answers, but in most cases led them to incorrect solutions. A comparison between pre- and post-test multiple-choice results revealed that there were no significant differences in learners' answer choice preferences.

## Abbreviations

CRESST	The National Center for Research on Evaluation, Standards, and Student Testing
DET	Department of Education and Training
DIFF	Difference
FDE	Further Diploma in Education
FREQ	Frequency
INSET	In-service education and training
MALATI	Mathematics Learning and Teaching Initiative
Mfa	Maths for all
MCQ	Multiple-choice questions
MEP	Mathematics Education Project
TIMSS	Third International Mathematics and Science Study
SDMT	Stanford Diagnostic Mathematics Test

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## Chapter 1

### INTRODUCTION

#### 1.1 The statement of the problem

Among the salient issues mathematics education attempts to address is the improvement of the quality of mathematics teaching and learning, and subsequently learners' performance or achievement in mathematics. However, there is an apparent plethora of factors that militate against the amelioration of classroom practice. Although on the face of it, misconceptions seem to be a direct consequence of the teaching-learning process, they are a major obstacle to successful learning. None the less, some misconceptions are not only obdurate, but also pervasive in diverse contexts. Hence, in order to improve the quality of classroom practice, in general, and learners' mathematics achievement in particular, there is need to critically address the issue of misconceptions.

Drawing from such a backdrop, there is a related topical issue, which is closely linked to the investigation of learners' mathematical misconceptions. This seemingly subtle but salient issue pertains to the type and nature of assessment and the related research methods that can adequately uncover learners' misconceptions on decimals, percentages and measurement. What are the most effective research methods (in particular, assessment instruments) which can adequately expose and measure mathematical performance and potential misconceptions?

When learners' performance results are apparently poor at whatever level of any education system various questions are posed by different stakeholders, who include parents, teachers, government and educationists. Particularly interesting are the seemingly inevitable questions such as, "What went wrong? Is it the teacher or the learner? Are the standards of the education system falling?" In most cases the concerned stakeholders do not bother to probe the appropriateness or the suitability of the assessment instruments used. In the light of this, Thompson and Senk (2001) argue that sceptics and critics of the current reform efforts are not only concerned

about students' performance, but also about how the performance is measured. This evokes the issue of the suitability of the assessment instrument.

## 1.2 Context of the present study

This present research study emerged as a component of a larger study, which sought to determine the impact of a Grade 7 mathematics textbook *Maths for all (Mfa)* on teaching and learning, and ultimately on learners' mathematical performance or achievement. The textbook series *Mfa* for primary schools was developed by MEP (Mathematics Education Project) at the University of Cape Town. *Mfa* was designed and developed to support the visions of the outcomes-based education reforms currently being implemented in South Africa under the banner *Curriculum 2005*. Curriculum 2005 was launched in 1997. The textbook series also privileged a view of teaching and learning that had been developed through both in-service and pre-service projects, which were conducted with primary school teachers over a period of about 10 years.

The larger impact evaluation study adopted a quasi-experimental research design in which the same test was given to the research subjects at the beginning of the second term of 2000 as a pre-test, and as a post-test at the end of the same term. The research study aimed at measuring the impact of the textbook on learner achievement through a pre- and post-test. While the focus of the impact evaluation study was the influence of *Mfa* on mathematics teaching and learning and subsequently on the learners' performance, the study also explored learning obstacles that militate against the development of a conceptual understanding of mathematics.

As a part of the larger research project, this study attempts to establish the extent to which the post-test could expose error patterns that could be used to detect underlying misconceptions on decimals, percentages and measurement of Grade 7 learners<sup>1</sup>. This study attempts to explore and uncover learners' possible underlying misconceptions

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<sup>1</sup> All learners attending the ex-DET schools came from impoverished backgrounds, that is, they came from poor and predominantly black townships around Cape Town (Ensor et al 2001).

through analysis of learners' responses to clusters of topic-related test items and cluster analysis.

Data analysis primarily focused on the analysis of learners' responses to post-test items, particularly their incorrect responses to similar, topic-related items. The use of the post-test results as the main data source was intended to identify learners' possible underlying misconceptions after the teachers had had the opportunity to teach decimals, percentages and measurement. Nevertheless, not all teachers had *Mfa*, only 10 of the 14 teachers did. Although the main research project differentiated between experimental and control groups of learners this study does not differentiate the two groups. In addition to a study of misconceptions, the study also aimed at evaluating the post-test as a measure of misconceptions, and of learners' achievement. The present study seeks to further contribute to the larger impact evaluation study by comparing the pre- and post-test results in terms of learners' responses to multiple-choice items. It aims at establishing whether there were significant differences between the pre- and post-test results with respect to learners' responses to multiple-choice items. This was undertaken to see if there were shifts in performance between pre- and post-test that could shed light on our understanding of misconceptions.

### **1.3 Significance of the research study**

This study has significance at two levels; one which is local and 'specific', and the other which is much broader. This particular study, as has been stated, forms part of a broader research project that investigated the impact of a textbook scheme on Grade 7 mathematics teaching and learning in 10 ex-DET primary schools. (These 10 classrooms were contrasted with 4 'control' classrooms, which were not given *Mfa* until the end of the research project). My task, in the context of this study, was to analyse post-test results in order to establish a) patterns of error in learners' responses to test items b) whether these signalled underlying misconceptions and c) what criticisms might be made of the test as a measure of achievement and of misconceptions. This is the local, specific significance of my study.

In addition to addressing the specific requirements of the research study, I wished to engage more broadly with learners' misconceptions in decimals, percentages and measurement (and the other side of this – conceptions and theories of learning). The present study, then, is broadly positioned in a study of misconceptions in decimals, percentages and measurement, while at the same time addressing the particular requirements of the research project for an analysis of test results, error patterns and possible underlying misconceptions, and an evaluation of the test instrument.

Being part of a research project has great advantages for a beginning researcher. Since the broad research project is conceptualised and operationalised by a team rather than a single researcher, a beginning researcher benefits from the skills and knowledge of experienced participants. However, the limitations for me were that I was not in Cape Town when the research was designed, the instruments created and the data collected. I entered the project after all these things were completed, and my task was to analyse the test results. This analysis makes an authentic contribution to creating new knowledge and understanding. I have done this in order to address the needs of the project, but have gone beyond this to extend my own understanding of many of the issues involved in conceiving, misconceiving and measuring both of these phenomena. Analysis of the test results allowed me to develop hunches, theses and possibilities, but these needed to be tested out in clinical interviews to make confident claims. What will become clear in the discussion that follows is that attempting to uncover misconceptions on the basis of a largely multiple-choice test designed to measure achievement is problematic.

#### **1.4 Relevance of the study to mathematics teaching and learning**

According to Sierpinska and Kilpatrick (1999) “No one disputes how important it is, in today's world, to prepare students to understand mathematics as well as to use and communicate mathematics in their future lives” (p. ix). Similarly, considering the ever-increasing importance that is attached to science and mathematics in the school curriculum today, there is the need to critically consider different factors that influence students' performance in these subjects. However, the central question that needs to be addressed is, how do we improve the learners' mathematical performance

or achievement? In an attempt to answer such a question, on the one hand, there is need to focus on different factors that facilitate improvement on instructional materials, improvement on teaching and learning classroom practices (pedagogy) and, last but not least, assessment procedures and design. On the other hand, the issue of eliminating or reducing the incidents of mathematical misconceptions in mathematics classrooms needs to be addressed. Misconceptions seem to be the major obstacle to successful learning of new concepts, and are prevalent in diverse classroom contexts.

Drawing from constructivist tenets, Putnam and Borko (1997) assert that

Indeed, there is considerable evidence that students often develop or cling to understandings that are markedly different from what curriculum developers and teachers intend. This has led to a good deal of research on the various misconceptions that students hold, particularly in science and mathematics...(p. 1228).

This raises further concern on the issues that impinge upon student learning. In this regard, Swedosh and Clark (1998) argue that

Since subsequent learning of mathematics relies to such an extent on the mastery of pre-requisite knowledge, there has been a high level of interest amongst mathematics educators (teachers, lecturers, and tutors) regarding mathematical misconceptions, their frequencies of occurrence, and whether a method exists which effectively reduces these frequencies (p. 2).

In this context, misconceptions are regarded as a major obstacle to successful learning of new concepts. If students' prior knowledge is contaminated with their erroneous constructions or conceptions, then their subsequent learning of new concepts is inevitably distorted. In this study, assessment is concurrently considered because it is through assessment that we can possibly isolate students' learning difficulties and misconceptions. The issues of assessment and misconceptions are closely related.

Focusing on the issue of misconceptions, there is undoubtedly much research evidence to confirm that conceptual obstacles abound for such mathematical concepts as decimals, fractions, percentages and measurement (length, area and perimeter). Saxe, Gearhart and Nasir (2001) purport that "... the domain of fractions is deeply related to the other forms of important rational number concepts, including... measures, percents, and decimals, and therefore is a critical curriculum target for the upper elementary grades..." (p. 55). On the other hand, mathematics has become an

indispensable subject in today's world in terms of its ubiquitous utility. Hence considering the issue of mathematical misconceptions and the current importance that has been attached to mathematics in the school curriculum, research on misconceptions becomes paramount.

The issue of exploring students' misconceptions in mathematics, in general, and on decimals, percentages and measurement in particular, is based on the premise that for analysis and identification of misconceptions, a background understanding of how concept development takes place in learners is required. In the light of the foregoing, this research study draws from theories on concept development from the perspective of cognitive psychology (see for example Skemp, 1979, Sfard, 1991, Tall, 1995, 1996). Attempts are then made to analyse learners' responses to post-test items, particularly their incorrect responses to similar, topic-related items as a basis for making inferences about their possible underlying misconceptions. The learning theories serve as lens for analysing learners' incorrect responses to the post-test items.

Alan Schoenfeld (1983) in his attempt to address the issue of how learning takes place in mathematics classrooms, argues that:

All too often we focus on a narrow collection of well-defined tasks and train students to execute these tasks in a routine, if not an algorithmic fashion. Then we test students on tasks that are very close to the ones they have been taught. If they succeed on these problems, we and they congratulate each other.... To allow them and ourselves to believe that they "understand" mathematics is deceptive and fraudulent (p. 29).

According to Schoenfeld (ibid.) a procedural or an algorithmic approach to teaching and learning does not nurture conceptual understanding of mathematics concepts. Further, if students are tested on certain tasks which require the application of the acquired algorithmic skills, particular misconceptions can be masked and hence are not identified. A study of students' misconceptions provides enormous insight into students' mathematical understanding (whether relational or instrumental) and conceptual knowledge, and may provide a basis for the diagnosis of students' learning difficulties. Along these lines, an awareness of the nature of students' misconceptions on decimals, percentages and measurement may provide guidelines for teachers in the selection of appropriate teaching-learning materials. It may also encourage them to reflect upon the relationships among decimals, fractions, percentages and

measurement in terms of equivalence and sequencing of topics and topic strands. Swedosh and Clark (1998) assert that "... by considering the mathematical misconceptions which are exhibited by students, teaching techniques can be developed which aim at diagnosing and eliminating those misconceptions" (p. 1). Learning theories may also induce the teachers to constantly reflect on their classroom teaching practices, and ultimately embody reflection as part of everyday teaching strategies.

#### **1.4.1 Statement of the research question**

This research study will therefore focus on and attempt to address the following questions:

##### **Main question:**

What possible misconceptions do Grade 7 learners from a sample of 14 formerly DET primary classrooms in predominantly black townships near Cape Town display in a test on decimals, percentages and measurement?

##### ***Sub-questions:***

1. What common errors do learners make in their responses to the test items on decimals, percentages and measurement?
2. Do patterns emerge in learners' responses, which suggest underlying misconceptions of decimals, percentages and measurement?
3. How useful is the test used for measuring the learners' underlying misconceptions?

In an attempt to address question 1, an analysis of the frequency distribution of learners' responses to all test items was conducted. For question 2, an analysis of frequency distribution of learners' responses to clusters of topic-related test items, and cluster analysis were conducted. The discussion of question 3 was based on the results for question 1 and 2, and a review of relevant literature.

## **1.5 An outline of the research study: topic summary**

Chapter 2 explores different theories on concept development from a cognitive psychologist's perspective and attempts to deliberate on how mathematics learning takes place at primary school level. This provides a lens for analysing and understanding misconceptions.

Chapter 3 gives an overview of literature related to errors and misconceptions on fractions, decimal fractions, percentages and measurement. It also attempts to address issues on test design.

Chapter 4 focuses on the research design, which includes the research method used and the data analysis procedures adopted for analysing the learners' responses for each test item, particularly their incorrect responses to similar or topic-related items.

Chapter 5 involves the data analysis, presentation and research findings.

Chapter 6 focuses on the discussion of the research findings, implications and limitations.

Chapter 7 highlights the research conclusions and recommendations.

## Chapter 2

### LEARNING THEORIES AS A LENS

#### 2.1 Concept development: a cognitive psychologist's perspective

This chapter focuses on various theories of concept development from the perspective of cognitive psychology in order to provide a basis for discussing mathematical misconceptions in the following chapter. Although this present study seeks to investigate learners' misconceptions, the issue of concept development needs to be addressed. How mathematics concepts are developed and how mathematical misconceptions arise are intricately interwoven, and hence inform each other. Recent research evidence has shown that the quest to improve classroom teaching and learning of mathematics cannot possibly be realised without evoking parallel research on the cognitive development of concepts. This being the case, several research studies on the mental processes by which mathematics is conceived and learnt have emerged in the mathematics education arena (see for example Skemp, 1971; Tall, 1995; Bowie, 1998).

It is in my view that a background understanding of the cognitive growth of mathematics concepts will inform the study of misconceptions, particularly on such topics like decimals, percentages and measurement, which are fundamental in elementary school mathematics.

#### 2.2 Learning theories

Current learning theories seem to differ in many respects, including approach, focus and detail, but they nevertheless concur on particular notions. In the light of this, Putnam and Borko (1997), drawing from Resnick (1991), suggest that

Virtually all current cognitive theories of learning hold some form of a *constructivist* assumption – that knowledge is a form of interpretation based on learners' existing conceptions, and that learning is the modification of those conceptions... (p. 1227).

Therefore, in this chapter I have attempted to bring together a number of recent theories on cognitive constructions of mathematical concepts in order to develop a learning framework based on the common themes. Focusing on learning from a constructivist's lens, "... it has been increasingly accepted that students construct their own knowledge as they engage in the processes of interpreting and making sense of their classroom experience" (Nuthall, 1997: 684). However, from a cognitive psychologist's perspective, learning is regarded as the conceptual restructuring that results from the cognitive processes (Nuthall, 1997).

### 2.3 Process-object duality theory: *reification, encapsulation and procept*

According to Ari & Tall (1996) "... the growth of human knowledge starts with the actions (first on the environment), some of which become repeatable processes and later conceived as objects in their own right to be manipulated on higher level by further mental processes" (p. 293). In Ari & Tall's (1996) assertion, following Bowie (1998), I wish to underline the notion of "process-object" duality, which seems to characterise the theory of the cognitive growth of mathematical concepts.

Piaget (1972) asserts that

... mathematics entities move from one level to another; an operation on such "entities" becomes in its turn an object of the theory, and this process is repeated until we reach structures that are alternately structuring or being structured by "stronger" structures (Piaget, cited in Tall, 1995: 62).

Tall (1995) concedes that it is Piaget's theory on conceptual cognitive development that influenced a number of other authors who later foregrounded the notion of "process-object" duality nature of mathematical concepts. Among such authors he cites Davis, 1975; Skemp, 1979; Greeno, 1983; Dubinsky, 1991, Sfard, 1991; Gray & Tall, 1994 (Tall, 1995: 62). In the light of this, Skemp (ibid.) proposed the 'varifocal theory' in which there is a back-and-forth hierarchical transition between 'schema' and 'concept' (Tall, 1995). Skemp (ibid.) suggests that a 'schema' seen as a whole is regarded as a 'concept' and a 'concept' seen in detail is a 'schema'. Along these lines, Dubinsky (1991) and Sfard (1991) recently came up with the notions of *encapsulation*

and *reification* respectively. Gray and Tall (1994) subsequently developed their notion of *procept*.

Bowie (1998) defines ‘encapsulation’ “... as the leap to seeing as an object what was previously been conceived of as a process” (p. 11). Similarly, according to Sfard (1991), processes are reified into new objects and she regards ‘reification’ as the ability to detach an idea from the processes that produced it and see it as an object (Bowie, 1998). Still in the same context, Gray and Tall (1996) propose that central to their notion of procept is the view that the symbol is the pivot between process and concept. It appears from the various authors’ perspectives that the notion of the ‘process-object’ duality is central to the cognitive constructions of mathematics concepts.

This notion of “process-object” duality for concept development was elaborated in an attempt to understand the cognitive growth of mathematical concepts at an advanced stage (Tall, 1995) and focused on such topics as functions and calculus. A detailed study and discussion of these notions and theories is provided in Bowie (1998).

## 2.4 The theory of abstraction

In the discussion that follows I am going to foreground the ideas of Skemp (1971), in particular, and Tall (1995). Skemp’s (1971) theory on concept formation will form the basis of the proposed learning framework.

According to Skemp (*ibid.*), the process that is fundamental to concept formation is ‘abstraction’. He views concept formation as a process, which involves other fundamental processes including ‘abstracting’ and ‘classifying’. Skemp (*ibid.*) defines ‘abstracting’ as “... an activity by which we become aware of similarities (in everyday sense, not the mathematical sense) among our experiences” (p. 22). ‘Classifying’ refers to the process of “... collecting together our experiences on the basis of these similarities” (Skemp, 1971: 22). It follows from his argument that abstracting precedes classifying.

Skemp (ibid.) further argues that "... concepts are formed through abstracting an activity by which we become aware of similarities and an 'abstraction' as an end point of the activity is the 'concept'" (p. 22). Mitchelmore and White (2000) also argue that concepts are formed when experiences are connected to one another on the basis of their similarities. On the other hand, Tall (1995) suggests that "elementary mathematics begins with *perceptions of* and *actions on* objects in the external world" (p. 61). Skemp (ibid.) identified two categories of concepts, the *primary* concepts, which are derived from our sensory and motor experiences of the outside world and, *secondary* concepts, which are abstracted from other concepts. Here Skemp (ibid.) underscores the issue of hierarchical formation of concepts, which is 'linear'.

Skemp (ibid.) further determines that abstraction refers to the degree of detachment from the real world, which is, however, made possible by language. Put in a different way, the concepts formed (through further abstraction of other concepts) get detached from the experiences from which they were derived, and they get attached instead to language in the form of symbols. Therefore, formation of secondary concepts assumes the primary concepts as objects. It is at this point that Skemp (ibid.) further purports that "... only by being detachable from the sensory experiences from which they originated can concepts be collected together as examples from which new concepts, of greater abstraction, can be formed" (p. 28).

Tall (ibid.) concurs with the above assertion on the cognitive development of concepts, namely that this "... requires significant constructions and reconstructions of recurring knowledge cycles of activities in which a *process* such as counting becomes a concept such as of number" (p. 293). Other examples given include the process of addition becoming the concept of sum and the process of equal sharing becoming the concept of fraction. Seemingly, Skemp's (1971) argument forms the basis for Sfard (1991)'s notion of reification, which involves essentially a qualitative shift in conceptual understanding. This shift is said to occur when a student is able to detach the mathematical idea from the process that produced it and see it as an object (Bowie, 1998). However, Mitchelmore and White (2000) suggest that "Sfard (1991) uses the term *reification* for the final stage of the abstraction process where the learner constructs a new mental object to embody a perceived similarity" (p. 433). In this context, reification is regarded as a component of abstraction.

Piaget (ibid.) came up with another kind of abstraction involved in the formation of elementary logico-mathematical concepts, which he referred to as *reflective* abstraction. He made a distinction based on its active nature, which he emphasised and described as following: “The [mathematical] abstraction is drawn not from the object that is acted upon, but from the action itself” (Piaget, cited in Mitchelmore & White, 2000: 433). On the other hand, “Reflective abstraction, involves the construction of relationships between/among objects...” (Kamii (1985), cited in Confrey 1990: 15). Furthermore, within the process of abstraction, there is a component of generalisation whereby a particular concept undergoes some modifications, thus rendering it more general in a wider set of situations (Mitchelmore & White, 2000).

The issue that seems central to Skemp’s (1971) theory of concept formation is that conceptual development is hierarchical in nature, and abstraction nurtures fertile ground for the interconnectedness of mathematical concepts. At the lowest level of abstraction, connections between concepts are based on similarities. Nevertheless, through further abstraction, similar concepts are made distinct or get separated through different symbolic representations. However, it is at the stage where previously embodied concepts are separated to stand as distinct concepts which is crucial. Seemingly, that is the stage, where most learners encounter problems. For instance, some learners tend to overgeneralise their whole number schemas to decimals and fractions.

In mathematics, Skemp (1971) asserts that “... the direction of learning is for the most part in the direction of still greater abstraction” (p. 26). In other words, mathematical concepts have to be developed in a particular ‘linear’ order. Certain concepts may be taken as primary concepts for other concepts and, as such, their formation should precede the formation of secondary concepts. Succinctly expressing the same notion, Skemp (ibid.) asserts that “... to understand algebra without ever having really understood arithmetic is an impossibility” (p. 35).

## 2.5 Forms of understanding: *instrumental and relational*

In an attempt to expand the frame of the classroom learning process, I focus also on Skemp's (1979) forms of understanding. Emerging from a linguistic domain, Skemp (ibid.) foregrounds the notion of words that have the capacity of holding two meanings in the same context. As an example, he distinguishes between two forms of understanding: *relational* understanding, which he describes as "... knowing what to do and why..." and *instrumental* understanding which he refers to as knowing the rule and being able to use it, that is, a procedural or mechanical kind of understanding. Similarly, on the issue of instrumental understanding, Rees and Barr (1984) concur:

Students can fool us by their apparent ability to solve many tasks correctly. Yet if or when the understanding is probed we realize that they have not the foggiest understanding of why they are doing what they are doing (p. 1).

Hence, memorised procedures tend to mask learners' misconceptions. Seemingly, these are the two forms of understanding that learners manifest in mathematics classrooms.

Skemp's (1979) notion of categorising mathematical understanding seems to be echoed by Hiebert and Lefevre's (1986) notion of conceptual and procedural understanding (see Bowie, 1998). They characterise conceptual knowledge as rich in relationships, contrary to procedural knowledge which is mainly constituted by symbolic representations and algorithms for completing mathematical tasks (Bowie, 1998). This appears to concur with Sfard's (1991) notion of 'operational' thinking, which Bowie asserts would lead to procedural knowledge and 'structural' thinking which would lead to conceptual understanding. However, the issue of categorisation of mathematical understanding is not only hierarchical in nature, but must also be conceived as a duality not a dichotomy. In line with this, Putnam et al. (1990) assert that "Indeed, in at least some cases, procedural knowledge must form the basis for conceptual understanding" (p. 84). The two forms of understanding are complementary, though according to Sfard (ibid.) operational conceptions should precede structural ones. Similarly, instrumental understanding is inferior to relational understanding, but their co-existence and inter-dependence is paramount.

## 2.6 Visualisation

Powell (1983) suggests that “Misconceptions are misunderstandings which are based upon incorrect meanings” (p. 1). He further argues that since mental imagery precedes meaning construction, the study of misconceptions is therefore closely related to visualisation. This rather vexed issue of visualisation in mathematics learning has attracted many researchers internationally. This has been accelerated by the advent of computer technology, which allows multiple modes of visual displays or representations (see for example Tall & Thomas, 1991; Presmeg, 1992).

The term visualisation carries different meanings for different people in different contexts. According to Solano and Presmeg (1995) “... the difficulty in defining visualisation stems from the issue of whether visual representations are internal or external...”, derived from the dichotomous approach in which internal refers to the mind and external refers to what is on paper or a computer screen (p. 67). However, the issue at stake is that visualisation should not be approached from a dichotomous perspective, whether it is internal or external. Instead the focus should be on the relationship between what learners see and their mental images, and how this affects the cognitive processes of meaning constructions. Along this line, they further suggest that visualisation can be “... defined as the relationship among images” (Solano & Presmeg, 1995: 67). Nevertheless, in their discussion, they make no explicit distinction between imagery and visualisation.

Lowrie and Pegg (2000) suggest that visualisation in mathematics “... contains four main elements, mental images, external representations, processes and abilities of visualisation” (p. 233). In line with the foregoing, Everett and Mulligan (2000) assert that “... visualisation is essential for both spatial concept development and spatial problem solving” (p. 243). However, for the purpose of this study, and in an attempt to understand the issue of concept development and how learning takes place, visualisation is viewed as an essential mathematical process which enhances meaning construction and, as such, is linked to the cognitive conceptual growth in mathematics. Even though some research studies foreground the visual strategies used by students, based on different kinds of visual imagery, many authors emphasise the

importance of visualisation and visual reasoning for learning mathematics, particularly for meaning construction. In line with this, Presmeg (1995) argues that though some research has demonstrated that students are reluctant to visualise, "... many students use their visualisations as a tool for meaning-making in mathematics" (p. 59).

In line with the cognitive theories on concept development, Lowrie and Kay (2001) assert that "The cognitive processes that construct meaning are usually categorised as verbal, visual or some combination of both..." (p. 248). In consonant with the foregoing, Presmeg (1986) suggests that "... visualisation is emerging as an avoidable integral component of learning mathematics..." (p. 42). Apparently the issue at hand is the relationship between what students see and their mental imagery, which most researchers have focused on in an attempt to understand students' processes of mathematical reasoning and thinking. (see for example Kruteskii, 1976; Presmeg, 1986, 1986a). Recent research on problem solving abilities of students has also emphasised the role that imagery plays in information processing (see for example Lowrie and Kay, 2001).

In an attempt to explicate how imagery influences the process of cognitive constructions of mathematics concepts, Lowrie and Kay argue that "... diagrams or pictures (either on paper or in one's mind) seemed to provide additional information that helped link conceptual understandings" (p. 253). Therefore, I suggest that though mental imagery is idiosyncratic and not accessible to the public, the relationship between what learners see and their mental images can enhance or impinge on the cognitive processes of concept development. If properly developed, visual imagery can make substantial contributions towards learners' development of appropriate cognitive constructions of meanings for the various modes of symbolic representations of decimal, percentage and measurement concepts.

## **2.7 Implications for the study**

According to Bowie (1998),

The relationship between learning theories based on the process-object duality in mathematics and the categorisation of mathematical understanding into conceptual and procedural (or relational and instrumental) is, I believe, a crucial component of being able to build an understanding of what can go wrong in student learning (p. 16).

The question, "what can go wrong in student learning", I believe forms the backdrop to the analysis of learners' misconceptions. However, the theories on the cognitive growth of concepts and the different forms of understanding will essentially form the basis of the framework for analysing and identifying learners' misconceptions.

Drawing from the notion that concept development is hierarchical in nature, it is therefore suggestive that misconceptions in mathematics are to a greater extent attributed to a limited understanding of background concepts. In line with this, Putnam and Borko (1997) purport that "All experience is filtered through the individual's existing cognitive structures; therefore, an individual's current understandings must provide the starting point for developing richer, more complex understandings" (p. 1228). As such, prior learning experiences play a considerable role in learning new material or concepts since certain concepts should form the foundation of other concepts, and if there are gaps somewhere then the concept development process collapses. It therefore follows that in the teaching-learning process due attention should be paid in the ordering or sequencing of topics or topic strands, and special emphasis made on interconnectedness of mathematics concepts.

More importantly, at primary school level, instrumental or procedural understanding tends to be predominant. Greater emphasis is put on getting the right answer than upon relational understanding. This is due in part to the pressure exerted by the national assessment systems. However, the issue at hand for this particular research is that at times it becomes difficult to tell whether a learner has grasped particular concepts instrumentally or relationally by using, say, a multiple-choice assessment instrument. Putting it differently, for particular questions in multiple-choice, it is difficult to distinguish the forms of understanding and, as such, identification of learners' misconceptions becomes problematic.

Though the effect of visualisation on concept development has not been made so explicit in various research studies, there are some inherent matching relationships,

between mental imagery (in the process of concept development) and learners' preferences for solving mathematics problems. This aspect of matching learners' cognitive concept development processes and their preferences in tackling mathematics problems can provide some insight into the nature of the errors learners make in response to the post-test items. Along this line, Lowrie and Kay purport that "Learners develop preferences for tackling mathematics problems using either visual or nonvisual (more verbal) approaches..." (p. 248).

Although, the learning framework has been largely informed by cognitive theories based on *constructivism*, learning can also be viewed from various perspectives, which include a socio-cultural perspective (see for example, Vygotsky, 1978; Rogoff, 1990; Lave, 1991, cited in Nuthall, 1997:758) and, not less, from a linguistic perspective (Nuthall, 1997). Besides the social nature and linguistic nature of cognition, there is the *situated* nature of cognition and the *distributed* nature of cognition as well (Putnam & Borko, 1997). Though the alternative perspectives might be conflicting on various aspects, in order to develop a more holistic understanding of how learning takes place, it is essential that we embrace the different perspectives. Along this line, Nuthall (1997) suggests that,

... if we incorporate the sociocultural and linguistic perspectives into a cognitive constructivist model of the development of mental processes, then it is possible to see how the language and social processes of the classroom construct the ways in which students acquire and retain knowledge (p. 758).

It can be emphasised that learning is influenced by a plethora of variables, some of which are visible and others that are difficult or even impossible to identify, let alone control. Therefore, it is also necessary to highlight that the alternative perspectives (though not elaborated in this section) are not mutually exclusive and, as such, should inform each other. Furthermore, it is through an adoption of such a holistic perspective that "... we will begin to see the concept of individual cognition as nothing more significant than an embedded fragment of larger and more inclusive cultural processes" (Nuthall, 1997: 711). Cognition is a multifaceted phenomenon, which involves interplay among persons, cognitive tasks, resources and tools like computers, cultural artefacts, and is embedded in particular classroom environments or settings (Putnam & Borko, 1997).

In the light of the foregoing, it can be said that the classroom ecology has a substantial impact on learning, and hence upon the formation of misconceptions. However, if mathematics teachers realise how misconceptions can arise, particularly in the classroom situation, they can deliberately attempt to improve their instructional practices. Teaching strategies that are based on the current theories of concept development and the different forms of understanding of mathematical concepts can be developed for use in different classroom contexts. Furthermore, mathematics teachers need to be equipped with the necessary assessment skills that can enable them to identify the learners' misconceptions. In summary, this chapter provides a basis for analysing and understanding misconceptions through the lens of different learning theories. This leads to the next chapter which focuses on how students' misconceptions develop or how they can arise in a classroom context and other sources of mathematical misconceptions, particularly in decimals, percentages and measurement.

## Chapter 3

### MATHEMATICAL MISCONCEPTIONS

#### 3.1 Misconceptions in mathematics in general, and their possible sources

The purpose of this chapter is to consider the prevalence of students' misconceptions in mathematics, in general, and in decimals, percentages and measurement, in particular. Also in the ensuing discussion, a deliberate attempt is made to invoke the possible sources or origins of misconceptions. A substantial amount of literature supports the view that students' learning is characterised by a wide spectrum of misconceptions in almost every field of knowledge<sup>2</sup>.

A review of the literature suggests that discussions on misconceptions tend to focus on two major issues:

- 1) the identification of students' misconceptions and attempts to uncover the origins of the misconceptions, and
- 2) the pervasiveness of misconceptions in diverse contexts (see for example Helme & Stacey, 2000).

According to Bowie (1998) "The study of students' misconceptions in mathematics has attracted a large number of researchers..." (p. 40). The present research study bears testimony to her assertion, focusing in particular on the identification of learners' misconceptions on decimals, percentages and measurement as well as attempting to trace the origins of the identified misconceptions. In addition, the study also attempts to address the issue of the suitability of the research instrument used to uncover the misconceptions.

A brief background on how misconceptions evolve, their persistence, and how existing misconceptions influence the learning of new concepts, is necessary in a

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<sup>2</sup> See for example the abstracts of papers presented at the Proceedings of the Misconceptions in Science and Mathematics: Cornell University, Ithaca, NY, USA, June 20-22, 1983.

study of misconceptions. Mack (1995) purports that learners' *intuitive, situated or informal knowledge*, which can be characterised generally as applied, real-life circumstantial knowledge constructed by the individual learner, can either be correct or incorrect in relation to other concepts. If this knowledge is incorrect then it forms the learners' misconceptions (Mack, 1995). According to Pines (1985):

Certain conceptual relations that are acquired may be inappropriate within a certain context. We term such relations as "misconceptions". A misconception does not exist independently, but is contingent upon a certain existing conceptual framework (Pines, cited in Confrey, 1990: 22).

Misconceptions can be viewed as incorrectly conceived conceptions (incorrect preconceptions) derived from a learner's intuition or naïve ideas with respect to a particular conceptual framework. Mathematical misconceptions cannot be stated without making a reference to specific concepts or a particular conceptual framework. For instance, in the present study, the focus is on misconceptions on decimals, percentages and measurement. Misconceptions can be derived from inappropriate conceptual relations based on a particular existing conceptual framework. However, such conceptions can always be modified, rejected or extended (Confrey, 1990).

Cooney (1990) suggests that misconceptions in mathematics can be attributed to how learners' perceive mathematics. Their perceptions are, to a great extent, influenced or shaped by their teachers. He argues that misconceptions, as dysfunctional or incorrectly conceived conceptions of mathematics, derive largely from a simplistic view of mathematics as a static or fixed body of knowledge, which can be learnt through memorisation of skills, facts, algorithms and repeated practice. In resonance with Cooney's propositions, Kinner (1983) purports that misconceptions are due to factors *extrinsic* to the learner, and are *engendered* as a consequence of limitations and biases in experiences and materials like textbooks to which students are exposed.

In the light of the foregoing, Newstead and Murray (1998) assert that "In South African primary schools fractions are usually introduced by presenting halves and quarters using pre-partitioned geometric shapes or other manipulatives" (p. 296).

This suggests that an impoverished introduction, with limited exposure to various fractions and their representations, to some extent accounts for learners' misconceptions on fractions. This phenomenon is echoed by Steinle and Stacey (1998) who argue that "An indication of the variability of students' thinking patterns by school is presented which provides preliminary evidence that sometimes students' misconceptions about decimals are a direct result of instruction" (p. 1).

Some arguments on the origins of misconceptions seem to suggest that the development of misconceptions can be explained in terms of what happens in the classroom context. Perry and Howard (2000) assert that "It is recognised that a student's prime, but by no means only, source of mathematical experiences is the classroom ... and what occurs in the mathematics classroom influences student beliefs..." (p. 331). Learners' misconceptions can therefore be attributed to how mathematics is presented and the beliefs that the teachers hold on the teaching and learning of mathematics. In line with this, Perry and Howard suggest that "All teachers hold beliefs towards this teaching and learning" (p. 331)

The top-down influence of teachers' beliefs upon their students' beliefs to some extent shapes the learners' misconceptions. Put in a different way, learners' misconceptions can be developed *in situ*, that is, in the classroom situation. Along these lines, Vygotsky (1978) emphasised that learning takes place most effectively under the guidance of an expert, in this case the teacher. This is outlined in his concept of a *zone of proximal development*. "This 'zone' is defined as the distance or space between what a child can do alone and what a child can do with adult or expert assistance" (Vygotsky, cited in Nuthall, 1997: 707). Vygotsky (*ibid.*) emphasised that the learning-teaching interactive discourse should be constituted between participants of unequal expertise or knowledge. This seems to support that learners' misconceptions to some extent stem from their teachers.

The underlying causes of misconceptions can be attributed to serious mismatches between the goals and views of teachers and students (Ellerton and Clarkson, 1996). This is also echoed by Skemp (1979) when he suggests that mismatches in the mathematics classroom may occur when the learners' goal is to understand instrumentally whilst they are being taught by a teacher who wants them to

understand relationally (or vice versa). As Skemp (*ibid.*) argues; “The other mismatch, in which pupils are trying to understand relationally but the teaching makes this impossible, can be a more damaging one” (p. 19). Similarly, Lowrie and Kay (2001) assert that “... learners develop preferences for tackling mathematics problems using either visual or nonvisual (more verbal) approaches...” (p. 248). Again, the possible mismatches between the learner and the teacher approaches in this regard might give rise to the formation of misconceptions.

Hiebert and Carpenter (1992) assert that “The analysis of students’ errors in arithmetic and algebra suggests that errors frequently are caused by extending learned procedures to new problem situations incorrectly” (p. 88). This, to some extent, explains why learners have difficulties in handling decimals and fractions. It demonstrates their tendency to extend whole number rules to fractions and decimals. Along these lines, Hiebert & Carpenter (*ibid.*) argue that “...errors are a natural consequence of attempting to integrate new procedures with prior knowledge” (p. 88). Consequently, learners’ prior knowledge can be a possible source of misconceptions. Confrey (*ibid.*) concurs “... preconceptions will act as a filter for new concepts” (p. 36). Putting it differently, Steinle and Stacey (*ibid.*) suggest that “In other cases, misconceptions arise when ideas interfere with each other” (p. 1).

From a constructivist’s perspective, it can also be argued that in learning new and key concepts, students are actively transforming what the teacher says in an attempt to make sense of the new concepts, and these transformations may lead to serious misconceptions (Confrey, 1990). However, the origins of misconceptions are not necessarily limited to the classroom ecology or contexts, particularly the teacher, because some origins of misconceptions are culture-based (Strike, 1983). We need to take cognisance and acknowledge that even the classrooms themselves are embedded in particular socio-cultural contexts. Therefore the root causes of misconceptions in mathematics are not only numerous and varied, but are also contentious and debatable. Along these lines Boero (2000) suggests that:

In mathematics education we cannot isolate “learning” from “teaching”, nor “learning mathematics” from socially “situated” intellectual and cultural development, including linguistic competencies, metacognitive aspects, rational attitudes, etc (p. 76).

Misconceptions can thus be a result of a single factor or a combination of two or more factors. Notwithstanding this, Garfield and Ahlgren (1988) assert that “Some misconceptions are quite widespread and can persist in spite of relevant instruction” (p. 44).

### 3.2 Misconceptions in relation to decimals

According to Helme and Stacey (2000) “It is well documented that many students throughout schooling, and indeed many adults, have difficulties with decimals” (p. 299). This is also supported by Steinle and Stacey (1998) in their assertion that:

It is now well documented that many students throughout schooling and indeed many adults have difficulty in understanding the notation used for decimal fractions. The recent Third International Mathematics and Science Study showed that internationally about a half of 13 year old students could select the smallest decimal number from a multiple choice list of five decimals.... (p. 1)

It is argued that students’ poor understanding of decimals, demonstrated by the errors they make in ordering decimals and in predicting the results of operations involving decimals, derives from the students’ previously developed whole number schemas (Irwin, 1995). In fact, Irwin (*ibid.*) asserts that

These errors indicate that students often deal with decimal fractions as though they were whole numbers, or that they deal solely with symbolic features of the notation rather than relating the decimal fractions to appropriate quantity (p. 50).

It appears that many students lack the number sense for decimals. For the purposes of this research the terms: decimals, decimal numbers or decimal fractions are used interchangeably.

In his investigation of beliefs related to the misconception “A decimal number is a pair of whole numbers”, Brekke (1996) purports that:

We regard the assumption that a decimal number is composed of two whole numbers separated by a point to be the most important underlying misconception linked to the conceptual knowledge of decimal numbers (p. 137).

He further argues that difficulties of operating with decimal numbers stem from the fact that they mean different things to different students. First, they can be interpreted concretely in various ways, as a result of division, a part of a unit, a comparison or a point on a line. Secondly, the interpretation of a decimal point as a separator of two whole numbers, seems to be prevalent in diverse mathematics classrooms (Brekke, 1996).

Baturo and Cooper (1995) in their exploration of learners' strategies for comparing decimal numbers with the same whole number part, concur with Brekke on the view of a decimal number as being composed of two whole numbers separated by a point. However, they suggest that "Comparing decimal numbers which have the same whole number part requires both an understanding of place-value system and fraction concept" (Baturo & Cooper, 1995: 73). This is echoed by Irwin (2000) in her assertion that "There are several aspects of decimal fractions which students find difficult... because they do not follow the rules, or schema, that the students have developed for whole numbers..." (p. 339).

Drawing from various authors (see for example Sackur-Grisvard & Leonard, 1985; Resnick et al., 1989), Baturo and Cooper (ibid.) highlighted three categories of strategies that children incorrectly use in determining the bigger of two decimal numbers with the same whole number part. In fact, they elaborated on Resnick et al.'s (1989) categories of the incorrect strategies as follows: *whole number*, *zero* and *fraction* rule.

According to Baturo and Cooper (ibid.), the *whole number* rule points to the fact that the decimal number with more decimal places is larger, that is the "longer" the larger. For instance, 4.156 would be considered larger than 4.3 because it has three decimal places while 4.3 has one. It is asserted that this kind of error originates from treating the decimal parts as whole numbers where 156 is far greater than 3. The underlying cause, however, seems to be "... an overgeneralisation of an impoverished method for comparing whole numbers, namely 'the number with most digits is the largest'" (Baturo & Cooper, 1995: 73). Along this line, Irwin (2000) argues that "If students try to generalise from their understanding of whole numbers, they are likely to be incorrect" (p. 339).

Irwin (ibid.) further brings another dimension of students' errors on decimals. She asserts that some students "... may believe that decimal fractions work in a manner that is a mirror image of whole numbers, so that 0.0023 is larger than 0.23" (p. 340). Irwin (ibid.) further argues that "All of these misconceptions are common. They all reflect an imperfect understanding of what a decimal fraction is and how it is shown in writing" (p. 340).

The *zero* rule dictates that the decimal number with one or two zeros to the immediate right of the decimal point is the smaller. In this case, 4.09 is correctly considered to be smaller than 4.1. The zero rule therefore always produces a correct result but for an inappropriate reason, which clearly demonstrates the possibility of some students developing procedural or instrumental understanding at the expense of relational understanding. This makes it difficult for an assessment instrument like a multiple-choice test to detect the underlying conceptual understanding of the learner.

The *fraction* rule, as the name implies, derives from "... an overgeneralisation of the principle for comparing common fractions, namely 'the larger the denominator, the smaller the fraction'" (Baturu & Cooper, 1995: 73). In this case the decimal number with fewer decimal places is the larger, that is, the 'shorter' the larger. For example 4.2 is considered to be larger than 4.865 because 4.2 has one decimal place and 4.865 has three. This again demonstrates an impoverishment of the learners' understanding of the decimal number concept and its symbolic representation.

Baturu and Cooper (ibid.), nevertheless, developed a fourth category, which they referred to as the *expert* rule. The *expert* rule produces correct comparisons in all situations and assumes a tacit understanding that experts compare digits in like places from left to right. Students who successfully use the expert rule are referred to as *Task experts* (Steinle & Stacey, 1998). However, as in the case of the zero rule, a multiple-choice form of assessment cannot expose the expert rule and its related underlying misconceptions. This is supported by Steinle and Stacey (1998) who assert that "A written test cannot distinguish between these various forms of thinking, but it may be straightforward in an interview situation" (p. 2).

Similarly, drawing from Resnick et al's (1989) erroneous strategies for comparing decimals, Steinle and Stacey (ibid.) elaborated the three categories, *longer-is-larger*, *shorter-is-larger* misconceptions and the *apparent-expert* rule into almost ten smaller sub-categories, which demonstrated the various ways of thinking students engage on when comparing decimals. According to Steinle and Stacey (ibid.) students in the *longer-is-larger* "...misconception category generally believe that a longer decimal is larger than a shorter decimal. It includes both "whole number rule" and "zero rule" as described by Resnick et al (1989)" (p. 2). Students in the *shorter-is-larger* misconception category compare the sizes of decimal numbers on the basis of the place-value column names. For instance, drawing from the notion that one tenth is larger than one hundredth, the students tend to erroneously generalise that tenths, irrespective of size are greater than hundredths. These incorrect strategies are based on the consideration of the size of the parts of a decimal number in isolation (Steinle and Stacey, 1998).

To conclude Irwin (2000) purports that "A wealth of students' misconceptions can be traced to their failure to understand these basic qualities of decimals" (p. 340). Along these lines, Hiebert and Wearne (1992) assert that students "... fail to connect decimal symbols with meaningful referents" (p. 208). Therefore, it follows that emphasis should be put on the meaning of symbolic notation and the relationship of decimal fractions and the language used to describe them.

### 3.3 Misconceptions on fractions

According to Saxe, Gearhart and Nasir (2001) "Many upper elementary children do not understand what fraction symbols represent..." (p. 56). It appears that the major difficulty that students face on fractions is related to the interpretation and meaning constructions of the symbolic representations of fraction concepts. In the light of this, Mack (1995) purports that

... as students attempt to construct meaning for symbolic representations that are new to them, they may draw on prior knowledge of other mathematical symbol systems, consequently overgeneralizing and constructing inappropriate meanings for the new representations... (p. 423).

Students' failure to construct appropriate meanings for fraction symbols can force them to rely on their prior knowledge of whole numbers. Along these lines, Mack (ibid.) asserts that "Students bring to the study of fractions a wealth of knowledge related to whole numbers" (p. 423). Similarly, D'Ambrosio and Mewborn (1994) identify schemes based on students' prior knowledge that would limit further understandings which are derived from "...the language of instruction; the visual representations used, including the types of examples and non-examples used; and the textbook sequencing of topics" (p. 153). They referred to these components as sources of students' limiting constructions.

The problem appears not to be based on the tendency of the students to over-generalise, but lack of appropriate meaning constructions for fraction symbols and concepts, which rather forces them to draw from their prior knowledge of whole numbers and therefore develop misconceptions. According to Mack (1995) "... a number of studies have documented that many students' misconceptions related to symbolic representations for fractions are tied to their knowledge of whole numbers..." (p. 423).

Mack's (1995) research findings show that:

- students confounded the meanings of symbolic representations for whole numbers and fractions, and
- students integrated symbolic representations for whole numbers and fractions (p. 424).

This apparent confusion of students in interpreting symbolic representations for fractions has further consequences when it comes to carrying out operations like addition, subtraction, multiplication and division with fractions. To support this, Mack states that "...a substantial body of literature has suggested that many students perform operations on symbolic representations with little understanding of the meaning underlying the representations..." (p. 422).

Drawing from a South African-based research study, Newstead and Murray (1998) assert that "There are many factors that contribute towards elementary school

students' poor understanding of common fractions" (p. 295). Among the various factors, they focus on the following:

- The *way and sequence* in which the content is initially presented to the students, in particular exposure to a limited variety of fractions (only halves and quarters), and the use of pre-partitioned manipulatives;
- A *classroom environment* in which, through lack of opportunity, *incorrect intuitions and informal (everyday) conceptions of fractions are not monitored or resolved*; and
- The inappropriate application of *whole-number schemes*, based on the interpretation of the digits of a fraction at face value or seeing the numerator and denominator as separate whole numbers. This can be seen as a special case of the previous problem, but is commonly reported in the literature and will thus be considered separately (Newstead & Murray, 1998: 296, emphasis in the original.).

Drawing from their research findings, Newstead and Murray (ibid.) point out that "One of the most common general errors identified in this study was the students' inability to see a fraction as a quantity - a quotient relation between two numbers - rather than two separate whole numbers" (p. 300). They further argue that "...this is an example of a misconception or a limiting construction based on students' own intuitions and previous experience" (Newstead & Murray, 1998: 300).

Newstead and Murray's (1998) research findings also suggest that the misconception of comparing the size of fractions with the same numerator by considering the size of the denominator was prevalent with Grade 6 students rather than Grade 4 students. It seemed that the misconception derived from the tendency of the students to consider the denominator and the numerator of a fraction as two unrelated whole numbers. In this context, it can be argued that this failure to develop a relational understanding of fraction symbols can result in learners memorising procedures and hence developing mechanical skills of solving some of the problems on fractions at 'face value'.

This, however, tends to mask their conceptual understanding of fraction concepts and subsequently nurtures a separation between the students' procedural and conceptual knowledge. For instance, it is believed that for division with fractions, some students simply memorise the rule "invert the divisor and then multiply simultaneously". By using the rule, the students get the correct answers but without understanding why the rule works that way. It is also suspected that even most of the mathematics teachers themselves do not know how the rule and where the rule was derived from, and more importantly why it works that way. In line with this, Webb et al (cited in Taylor &

Vinjevold, 1999) in their study of an accredited INSET (in-service education) programme for under-qualified science and mathematics teachers in South African primary schools grades 5, 6 and 7 assert that "... in questions on fractions, teachers and pupils both had difficulty with fractions other than halves and quarters, and with decimals" (p. 141).

On percentages, Rees and Barr (1984) suggest that misconceptions "... probably follow quite naturally from those associated with common fractions" (p. 81). They, however, purport that the misconceptions on percentages might be a culmination of a lack of a concrete grasp of the meaning of percentage.

### **3.4 Misconceptions on measurement**

Misconceptions related to measurement concepts, particularly length concepts, are attributed to learners' impoverished understanding of the decimals. As such, the development of length concepts should assume the decimal number concepts as the primary concepts (Skemp, 1979). However, according to Outhred and McPhail (2000) students begin to compare quantities on the basis of perceptual judgements to develop a sense of what length, area or volume is. Then, the perceptual judgements are later, gradually modified by cognitive processes and thus the students build a firm understanding of measurement units. They further argue that although "Measurement concepts are introduced in kindergarten, then developed and extended throughout the primary years...students will not learn to estimate and measure successfully if they have inadequate understandings of measurement attributes and processes" (p. 487).

There seems to be a general consensus among researchers (see for example Hart, 1981; Rowland & Wilson, 1993; Outhred & McPhail, 2000) that the sources of common misconceptions on measurement arise from:

- 1) conservation of the quantity being measured;
- 2) the meaning of the measurement unit and unit iteration (arbitrary units and standard units) as well as correct use of the measurement tools;
- 3) underdeveloped perceptions and representations;

- 4) lack of understanding about the properties being measured and transitivity;
- 5) over-emphasis on numerical cues.

Along these lines, Outhred and McPhail (ibid.) assert that “Commonly reported errors in these studies involve inadequate understanding of:

- the attribute being measured;
- what is counted when informal units are used;
- the use of measurement instruments” (p. 488).

The issue of the inappropriate use of the measurement instruments like a straight edge ruler, can be derived from lack of understanding of the graduation, that is, failure to read the scale correctly and failure to link the scale to the decimal number concepts. Considering misconceptions on measurement of length, Hart (1981) points out that the most prevalent error which students make when measuring length using rulers is the tendency to take readings at the end points instead of counting ‘spaces’, especially in cases where the starting point is not at the zero line of the ruler.

Although there is not much literature on learners’ misconceptions on area and perimeter of plane shapes, Rowland and Wilson (1993) purport that when comparing areas of plane shapes like rectangles, learners tend to focus on one dimension, for instance, the length and make conclusions based on the assumption that the longer the length, the bigger the area.

In a sense, distilling from issues and themes about learners’ common errors and misconceptions on decimals, percentages and measurement, a substantial amount of research has suggested that many learners have problems in constructing appropriate meanings for the symbolic representations of related concepts. This subsequently impinges upon their development of conceptual knowledge in these related topics. This being the case, the notion of equivalence becomes elusive to many learners. In line with this, Hiebert and Wearne (1985) purport that there is an apparent divorce between the students’ procedural knowledge and conceptual knowledge. This separation between procedural and conceptual knowledge clearly demonstrates the absence of connections or relationships between related concepts or the same concept

but represented by different symbols (equivalent concepts) in the learners' cognitive concept growth. Consequently, procedural knowledge tends to overshadow the conceptual knowledge and can eventually mask the learners' misconceptions. It should be borne in mind that although procedural and conceptual knowledge are somehow hierarchical they are not mutually exclusive or dichotomous and, as such, should inform each other.

A substantial amount of research has established that a wide variety of such misconceptions in decimals, percentages and measurement interfere with new learning and for many learners tend to persist through years of schooling (see for example Stacey, MacIntosh et al, 2000; Helme & Stacey, 2000). As such, it is important that the assessment instruments designed to uncover learners' misconceptions should be adequately informed so as to ensure validity and reliability in relation to the purpose of the instrument. An analysis of learners' possible underlying misconceptions will be based on their responses to post-test items. Therefore, a background knowledge or view of the types and nature of errors that will be expected is necessary.

### **3.5 Categorisation of learners' common errors**

Since data analysis for this study is based on learners' incorrect responses to post-test items, errors can be viewed as the primary source of information concerning learners' possible misconceptions. In line with this, Mundy (1985) suggests that a

Study of student errors and misconceptions, which holds a view of errors as results of systematic, rational application of student constructions, is promising as a means of gaining insight to student understanding and as a starting point for diagnosis of student difficulties (p. 170).

There are several research studies that have focused on the analysis of students' errors on their responses to certain test instruments, and subsequently, made bold and candid inferences about related misconceptions (see for example Baturo & Cooper, 1985; Ellerton & Clements, 1985; Brekke, 1996; Newstead & Murray, 1998; Steinle & Stacey, 1998; Swedosh & Clark, 1998).

In consonant with the foregoing, Confrey (1990) argues that:

Errors are like misconceptions in that they result from non-random applications of rules based on certain beliefs. They are unlike misconceptions (...) in that they typically are not well-connected into a theoretical position, or the theoretical connections lack articulation by the researcher (p. 33).

On the other hand, Confrey argues that "... many researchers in the information-processing tradition use the terms error and misconception interchangeably" (p. 33)

In this present study, an error on a single test item is unlikely to be informative enough and, as such, may not be used as an indicator of misconception(s). A single error might be a resultant of a number of factors, including carelessness, motivation, comprehension, and language. This being the case, errors on a cluster of interrelated test items or systematic errors, which are reflected by a pattern of incorrect responses, may form the basis for making inferences on possible misconceptions. Along this line, Nesher (1987) derives misconceptions from the systematic errors and defines them as:

... a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic, unconnected and non-systematic errors (Nesher, cited in Confrey, 1990: 33)

Systematic errors in learners' incorrect responses to post-test items form the basis of the data analysis process aimed at uncovering their possible underlying misconceptions. Categorisation or classification of learners' errors is important for the study of, and understanding the nature of, misconceptions.

According to Mundy (1985):

Error classification schemes which transcend specific content areas, such as those offered by Radatz (1979), and Vinner et al. (1981) are useful in achieving characterization of the error types of a particular population (p. 170)

Radatz (1979) (cited in Confrey, 1990) proposes a categorisation of errors that is based on the information-processing approach. He suggests that:

Various causes of errors that cut across the mathematical content topics can be identified by examining the mechanisms used in obtaining, processing, retaining and reproducing the information in mathematical tasks (Radatz, cited in Confrey, 1990: 32).

According to Confrey (ibid.) the errors identified include:

1. errors due to processing iconic representations,
2. errors due to deficient mastery of pre-requisite skills, facts and concepts
3. errors due to incorrect associations or rigidity of thinking leading to inadequate flexibility in decoding and encoding new information, and
4. errors due to the application of irrelevant rules or strategies (p. 32).

Along these lines, Davis and Vinner (1986) (cited in Bowie, 1998) developed their categories of learners' common causes of errors, from which I have focused more on the following:

1. language in that the everyday meanings of terms used in mathematics provide misleading cues about the concept;
2. mathematical ideas are built gradually in a stage-wise fashion, but at various stages some parts of the representation overshadow others;
3. The influence of specific examples. Over-emphasis and repetition of specific examples will dominate the pupil's image of the concept (Bowie, 1998).

Drawing from Orton (1983), Bowie (ibid.) describes three types of errors that students make: *structural*, *arbitrary* and *executive*. She asserts that *structural* errors are the result of a failure to grasp some fundamental concept, which is a key determinant to the solution. Such errors may also result from a failure to perceive the relationship between elements in the problem. *Arbitrary* errors are those in which the student appears to have acted like disregarding the given information. *Executive* errors are errors that are related to manipulation. Bowie (ibid.) further points out that it might be difficult to make a distinction between arbitrary errors and the executive and structural errors, and that certain errors may be a combination or a resultant of two or more errors.

Bowie further highlights Movshovitz-Hadar et al.'s (1987) descriptive categories of errors common in high school mathematics from which I have distilled the following:

1. Misused data, which involve adding or neglecting data and assigning properties that are not given;

2. Misinterpreted language, which involve translating from English to mathematics incorrectly, using the wrong mathematical symbols and incorrectly interpreting graphical symbols;
3. Logical invalid inferences, which involve making inferences that are logically invalid;
4. Unverified solutions, involving solutions that cannot be easily verified or checked against questions , and
5. Technical errors, which include computational errors and elementary manipulation.

She further suggests that the model can be used across mathematical topics in high school mathematics.

There is visible resonance or significant overlap among the ideas for categorisation of students' common errors and on the causes of the errors developed by various authors (see for example Confrey, 1990, Bowie, 1998). The present study will not focus on the categorising and labelling of learners' errors. However, a background knowledge of the nature and possible sources of errors will inform the data analysis, particularly the analysis of learners' incorrect responses to post-test items for the purpose of uncovering their underlying misconceptions on decimals, percentages and measurement.

From literature on mathematical misconceptions, it can be said that misconceptions are defined or interpreted differently in different contexts. For the purposes of this study, misconceptions are defined as incorrectly conceived concepts. The incorrectly conceived conceptions are reflected through learners' interpretation of mathematical symbols, their computational procedures or their systematic erroneous responses to clusters of similar or topic-related test items. Although the prime purpose of this study is to analyse learners' incorrect responses to the test items and then attempt to make some inferences pertaining to their misconceptions, the study will also explore the suitability of the test in measuring learners' misconceptions.

### 3.6 Test designing or construction

Assessment has assumed an increasingly important role in education, especially in view of the wide spectrum of purposes that it can possibly serve (Webb, 1992). Available research evidence seems to concur on the issue of the fundamental decisions that have to be made before the construction of a test. First, the design of any test should be informed by its intended goals or objectives. Putting it differently, a test is shaped by its purpose.

Clarke (1996) suggests that “Assessment has three distinct fundamental purposes: To model, to monitor and to inform” (p. 328). In a sense, the assessment instrument will be judged on the basis of whether it constitutes not only an effective model of valued performance, but also an effective model of educational practice. Clarke (*ibid.*) further suggests that the test should provide methods that sufficiently monitor the valued performances, and opportunities for all students to adequately demonstrate their capabilities. Further, a test will be judged by the effectiveness with which it informs the actions of all stakeholders and other interested parties.

Considering the process of developing a test, Brown (1970) had this to say: “Although the construction procedure will vary depending upon the type and purpose of the test, certain general steps can be identified” (p. 25). The following steps were identified and regarded as fundamental in the process of test construction:

- specifying the purpose of the test;
- translating the purpose into operational terms;
- constructing the items;
- pre-testing and item analysis;
- assembling the final form of the test;
- standardising, and
- obtaining technical data about the test (Brown, 1970: 25)

Accordingly, Brown (*ibid.*) asserts that the purpose ensures that the test is designed in such a way that it captures accurately the desired information. The process of translating the purpose into operational terms focuses on the most accurate and

feasible method of obtaining the desired information. The process considers the content to be covered and the format of the test. The content should be specified in terms of the topics to be covered and the objectives (for example the knowledge and skills) that need to be achieved. The format refers to the issue of how the content will be presented to the learner. For instance, if a paper-and-pencil test is to be adopted, it has to be established whether it will be entirely multiple-choice or completely free-response or both. Brown (ibid.) suggests that "In all cases the format must be consistent with the purposes..." of a test (p. 25). It is also argued that before the construction of the test items it is important to consider the nature of the learner with respect to age, intellectual level, reading (language) level, socio-economic and cultural background (Brown, 1970).

The process of constructing test items involves consultation of textbooks in use, teachers, experts in test design and other existing standardised tests. However, a single test instrument cannot possibly test every content aspect covered hence sampling of test items has to be conducted. According to Brown (ibid.) sampling of test items from *a universe of potential items* ensures that at least a test is representative of the content area covered.

When the process of item construction is complete, pre-testing can be carried out on a sample of the intended learners. The main purpose for conducting pre-testing is to check whether the test can capture the desired information and check its feasibility potential. Through pre-testing, such attributes as discrimination, item difficulty, test layout can be established, and subsequently reviewed. After pre-testing, a test can then be modified or improved so that it suits the cohort of learners to be tested, and to ensure that the desired information can be captured more accurately and in a more feasible way.

The last but not the least important stage in the process of test construction is standardisation. According to Brown (ibid.) standardisation refers to the process of drawing up a set of specific rules to guide the administration and scoring the test. The major purpose of standardisation is to ensure that the learners write the test and are marked under the same conditions. Therefore standardisation involves drawing up test length with respect to time. For instance, it focuses on the time limit that would allow

say 90% of the learners to finish writing the test. It also involves drawing up directions for responding, that is, test instructions and drawing up clear marking schemes so that consistent scoring of marks is achieved (Brown, 1970). In summary, these are some of the desirable attributes that might be considered in the design or construction of achievement tests. However, depending on the nature and purpose of the test some of the procedures can be omitted, can be carried out at the same time or the order can be changed. The process of test construction, nevertheless, "...uses both statistical and logical reasoning, and balances practical with theoretical considerations" (Brown, 1970: 25).

Closely linked to the purpose of a test, are the contentious issues of validity and reliability of a test. In test design some authors tend to emphasise and focus only on reliability and validity of tests. Marshall and Hales (1971) argue that "Any test that is to be used effectively as a measuring instrument should be (1) reliable, (2) valid, and (3) practical. A gross deficiency in any of these attributes can render a test useless" (p. 8). Gronlund (1982) purports that:

The concept of validity, as used in testing, can be clarified by noting the following general points:

1. Validity refers to the interpretation of test results (not to the test itself).
2. Validity is inferred from available evidence (not measured).
3. Validity is specific to a particular use (selection, placement, evaluation of learning, and so forth).
4. Validity is expressed by degree (for example, high, moderate, or low) (p.126).

In the light of this, Lindquist (1951) defines validity of a test as "... the accuracy with which it measures that which it is intended to measure or the degree to which it approaches infallibility in measuring what it purports to measure" (Lindquist, cited in Brown & Njabili, 1989: 61). In line with this, Shepard (1993) asserts that:

In the early years of testing, validity addressed the question "Does the test measure what it purports to measure?" today, a more appropriate question to guide validity investigations is "Does the test do what it claims to do (p. 443).

The major concern is not only to establish, but also to justify whether a test is capable of sufficiently measuring what it is intended to measure. In other words, does the test convincingly serve its purpose? Parallel with this, Shepard argues that "If a test lacks

validation evidence for a particular purpose, then its use is highly questionable, particularly if critical individual decisions will be based on the test results” (p. 445).

Different authors have suggested different kinds of validity: *inter alia* content validity, predictive validity and construct validity. For the purpose of this study, I will, however, focus on construct validity. Content validity was addressed by the post-test designers insofar as it measured achievement specifically in decimals, percentages and measurement. Meehl (1955) (cited in Brown & Njabili, 1989) defines a construct as “...some postulated attribute assumed to be reflected in a test performance” (p. 61). In resonance with the foregoing, Anastasi (1977) purports that “The construct validity of a test is the extent to which the test may be said to measure a theoretical construct or trait” (Anastasi, cited in Hastings & Stewart, 1983: 702).

In this study, an attempt is made to establish the extent to which learners’ incorrect responses (common errors) to test items reflect their misconceptions. Hence, for this particular study construct validity is based on the premise that a test cannot measure learners’ misconceptions directly (misconception is therefore a theoretical construct) and, as such, misconceptions are to be determined through interpretation of learners’ incorrect answers (or error patterns) in the test. This being the case, such interpretations are somehow value-laden and have consequences and implications that can only be defended by evidence from literature. Accordingly, evidential basis of test interpretation and test use constitute, in part, construct validity (Shepard, 1993).

Hastings and Stewart (1983) suggest that “We feel that a discussion of construct validity is absolutely necessary for paper-and-pencil tests in which students may select correct answers for the wrong reasons” (p. 702). Therefore, in an attempt to reaffirm their assertion on the need to determine construct validity for pencil-and-paper tests like a multiple-choice test where learners can give correct answers for wrong reasons, they cite:

Stewart and Dale (1987) in a study of student problem in genetics, have reported that while most high school students can get correct solutions to monohybrid and dihybrid cross problems, the majority of them do so algorithmically with little conceptual knowledge of meiosis and genetics. To say that these “correct” answers adequately reflect the construct “genetics problem solving knowledge” is to misrepresent what teachers expect students to achieve from instruction (p. 702).

Similarly, through analysis of learners' incorrect responses to the test items, an incorrect response does not necessarily reflect a misconception. On the other hand, a correct response does not necessarily indicate acquisition of conceptual knowledge. Therefore construct validity, which *inter alia* focuses on hidden assumptions and implicit claims, alternative interpretations and meanings, and unintended consequences, has to be established in relation to the extent to which learners' incorrect answers reflect their misconceptions.

Reliability addresses the issue of the replicability of the test scores for the same cohort of learners for the same test, administered under the same conditions over time. In other words reliability answers the question, "Can similar results be obtained when the same test is written by the same learners under similar conditions at a later date?" As such reliability focuses on establishing consistency in test scores for a particular population group and is usually measured in terms of a correlation index, the reliability coefficient which lies between 0 and 1. It is argued from various research reports that for a reliable test instrument, the reliability coefficient should be above 0.9. For the purpose of this study I will focus on reliability coefficient that is based on learners' test results and is referred to as internal-consistency reliability of a test.

### **3.7 Performance or achievement tests**

An important issue, which needs to be considered in test design and analysis, is the establishment of construct validity of a test, designed to measure the learners' mathematical performance. For the purposes of this research study the terms: performance test and achievement test, are used interchangeably. According to Downie (1959) an achievement test is "Any test that measures the attainments or accomplishments of an individual after a period of training or learning..." (p. 113). An achievement test is intended to measure what the learners know (what they can do) and what they do not know (what they cannot do). The major uses of an achievement test include establishing grades, diagnosis and evaluation of teaching. In other words, an achievement test should, to some extent, be diagnostic in nature. Now, isolating the issue of measuring what the learners cannot do, the learners' conceptual knowledge deficiency, should be to some extent reflected by some pattern

in the errors, and therefore evokes the issue of misconceptions. This bridges the issue of construct validity to a fundamental but tacit question of this study, namely to what extent does the performance test measure the learners' misconceptions?

The post-test, which is partly multiple-choice and partly short free-response in structure, to some extent influences the construct validity. Similarly, the nature of language used in the construction of the test items also influences the construct validity. In line with this, Rees and Barr (1984) assert that "In attempting to explain ideas we are often trapped into language which itself can generate for our students the very confusion we are trying to avoid" (p. 74). Parallel to the foregoing, Smith (2000) suggests that assessment, among other things, "... should be accessible, relevant and meaningful to all students so that they can show what they know, understand and can do..." (p. 550). As such, the issue of language use in test construction needs to be accorded due attention and respect. In other words, a test should use language that is easily accessible and meaningful, which students can easily comprehend and make sense from so that incorrect responses will not be attributed to language contamination or deficiencies or distracters.

According to Clements and Ellerton (1995) "Pencil-and-paper tests are widely used to assess mathematical achievement" (p. 184). In line with this, various research studies confirm that multiple-choice tests continue to be widely used throughout the world for assessing the mathematical performance or achievement of students (e.g. Olssen, 1981; Ganet & Mills, 1995, cited in Clements & Ellerton, 1995). Similarly current reviews on assessment consider multiple-choice tests as the most objective form of assessment and other objective tests are regarded as merely trivial variants of multiple choice (Carlson, 1985).

Rees and Barr (1984) assert that if a multiple-choice test is properly constructed "...its distracters, if they are well chosen, they will reflect the errors and misunderstandings that commonly occur" (p. 205). However, contrary to the foregoing, other research studies report on the inadequacy of multiple-choice tests to probe the conceptual understanding of students. In light of this, Cooney (1990) asserts that "... there is growing consensus that traditional pencil-and-paper tests are inadequate in providing useful assessment information" (p. 129). Seemingly, among

researchers, the desire to challenge the previously taken-for-granted performance assessment instruments is slowly gathering momentum. Nevertheless, any form of assessment has its strengths and limitations.

The issue of assessment, the learning process or concept development, and students' misconceptions are intricately interwoven. To appreciate the notion of students' misconceptions evokes the issue of how students develop mathematical concepts, and how they acquire the valued knowledge, and how they should be assessed. This leads to the next chapter, which focuses on the present research design that incorporated assessment as a primary source of data collection to uncover learners' possible underlying misconceptions.

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## Chapter 4

### RESEARCH METHODOLOGY

In this chapter, I will describe the research methodology used to collect and analyse data. Also discussed in this section of the dissertation are the issues of reliability, validity and generalisability of the research findings.

#### 4.1 Data sources

The main research project, an impact evaluation study of a textbook scheme, *Mfa* on teaching and learning in primary school classrooms, adopted a quasi-experimental research method. It focused on 538 Grade 7 learners from fourteen (14) randomly selected ex-DET classrooms at schools, which are broadly representative of predominantly black primary schools in urban areas around Cape Town. Ten (10) of the fourteen classrooms constituted the experimental group, which had been given class sets of the textbook *Mfa*. The remaining four classrooms constituted the control group, and were not given the class sets of *Mfa*.

Since this research study emerged as a component of a larger research project, the issue of the sampling method, and data collection are dealt with in that study (Ensor et al 2001). Nevertheless, the following tables, Table 4.1 and Table 4.2 show an overview of the research questions, the data collection techniques and instruments used in the larger research project.

Table 4.1 Research questions and data collection techniques and instruments used

Aspect of research	Data collection techniques/instruments
What notion of “good practice” does the textbook privilege in the way in which it constructs teachers, learners, and mathematics?	Selection of textbooks and teachers’ guides
How do teachers use “Maths for all”, if at all, in constituting their teaching repertoires?	Classroom observation, interviewing of “experimental” teachers
Do teachers who use “Maths for all” privilege different practices to those who do not?	Classroom observation, interviewing (of all teachers)
How do learners make use of textbooks?	Classroom observation, sampling of learner notebooks
Do learners in classrooms in which the textbook is used achieve significantly better than learners in classrooms where the book is not used?	Pre- and post-tests
Do contextual factors (e.g. school environment, teachers’ experience, home circumstances of children and their attitudes to mathematics) affect the outcome of the above?	School, teacher, learners questionnaires and tests

Source: Ensor et al (2001)

Table 4.2 Data collection instruments used in the study

Instrument	Content of instrument
Initial teacher questionnaire	Details of topics to be covered in first and second term
Teacher questionnaire 1	Confirmation of topics to be taught, curriculum materials to be used, information about attendance of INSET programmes, and organisation of the school day
School questionnaire	Completed by school principal: details of organisation of school day, school roll, resourcing etc.
Teacher questionnaire 2	Personal and professional details of Grade 7 maths teachers, organisation of maths teaching in school
Teacher questionnaire 3 (conducted as an interview)	Details of topic coverage and textbook use
Classroom observation schedule	Systematic observation of classroom teaching and learning
Pre- and post-test	Mathematics content on measurement, percentage and decimals-Grade 7 level
Learner questionnaire 1	Details about conditions at home and school, attitudes to mathematics
Learner questionnaire 2	Attitudes to mathematics
Fieldworker questionnaire	To monitor data collection process, and identify issues which might affect reliability and validity

Source: Ensor et al (2001)

My study focused on re-analysing already collected and captured data through descriptive statistics. The same test was used for both the pre- and post-test; but in the present study only the post-test results were considered for the purpose of identifying learners’ possible underlying misconceptions. The data analysis involved constructing frequency tables of both correct and incorrect responses for each item, and cluster analysis. The cluster analysis of incorrect responses was intended to establish

whether the responses cluster in ways that suggested possible underlying misconceptions.

Chi-square tests were also conducted in order to establish whether there were significant differences between the pre- and post-test results in terms of learners' answer choices or error patterns. In other words, statistical tests were conducted in order to establish whether error patterns in learners responses were persistent or diminished after all the teachers had agreed to teach the topics: decimals, percentages and measurement in the second term of 2000. In addition, the chi-square tests were undertaken to see if there were shifts in performance between pre- and post-test that could shed light on our understanding of misconceptions.

A pertinent issue that needed to be addressed was that the post-test results may not have shown consistent error patterns in part because not all teachers covered the topics decimals, percentages and measurement in the agreed time. Therefore I needed to exclude the possibility that the learners' responses were a result of guessing, and further statistical tests were carried out in order to verify whether multiple-choice items were vulnerable to guessing or not. Table 4.3 below shows the summary of topics that were covered by the teachers from both the experimental and control classrooms before the learners wrote the post-test.

*Table 4.3 Summary of topic coverage by teachers from both the experimental and control classrooms*

Topic		Number and percentage of learners who covered the topics						Total
		Experimental group			Control group			
		Covered	Partly covered	Not covered	Covered	Partly covered	Not covered	
Decimals	Freq. %	147 27%	156 29% (1 or 2 of the 3 ST)	74 14%	51 10%	0 0%	110 20%	538 100%
Percentages	Freq. %	123 23%	24 5% 1 of 2 ST	230 43%	0 0%	0 0%	161 30%	538 100%
Measurement	Freq. %	271 50%	82 15% 3 of 4 ST	24 5%	64 12%	71 13% 3 of 4 ST	26 5%	538 100%

**Note:** ST stands for subtopic; Freq stands for frequency

In summary, Table 4.3 shows that  $(147 + 156 + 51 = 354)$  66% of the learners covered aspects of decimals,  $(123 + 24 = 147)$  28% of the learners covered aspects of percentages and  $(271 + 82 + 64 + 71 = 488)$  90% of the learners covered aspects of measurement.

## 4.2 Statistical methods

The data analysis involved both quantitative and qualitative methods based on the nature of the post-test. The post-test was partly multiple-choice (questions 1-20) and partly free-response (questions 21-31). Although the data analysis incorporated chi-square tests, it was primarily descriptive in nature. Since the post-test was considered as the primary source of data, the data analysis focused more on learners' incorrect responses for each test item. The learners' incorrect responses were regarded as a source of information concerning their possible underlying misconceptions. Since learners' misconceptions could not be measured directly by the post-test, identification of the underlying misconceptions was based upon inferences about patterns in their incorrect responses to similar, topic-related items. The patterns in learners' incorrect responses to a set of similar, topic-related questions were interpreted and discussed. Subsequently, inferences about the learners' possible underlying misconceptions were drawn from the identified patterns in learners' incorrect responses to clusters of related or similar items.

### 4.2.1 Cluster analysis

Cluster analysis formed the basis for the identification of learners' possible underlying misconceptions. Learners' incorrect responses to a single item could not provide sufficient information to make robust inferences about their possible underlying misconceptions. Therefore, clusters of related test items for which some consistency in learners' incorrect responses (systematic patterns in learners' incorrect responses) could be established, had to be identified. Cluster analysis was adopted as a method for uncovering the learners' possible underlying misconceptions.

According to Afifi (1990), cluster analysis is a "...technique which seeks to separate objects into groups such that each object is more like other objects in its group than like objects outside its group" (p. 1). In other words, cluster analysis focuses on

classification of objects based on how close they are to each other in relation to some given attribute. Cluster analysis is a procedure used for grouping together objects or items based on their similarities and differences. Hence, at least a minimum of two distinct clusters can be formed to a maximum number of clusters that is equal to the number of distinct objects under consideration.

The logic behind cluster analysis is that there must be some linking attribute upon which the clustering is based. Along these lines Afifi (ibid.) further asserts that “Clustering methods operate on distance (proximity) measure between objects” (p. 3). For instance, if any two objects are very close to each other with respect to a particular attribute, then the distance between them will be relatively small, as reflected on a *dendrogram*. Nevertheless, subjective judgements have to be made while determining the number of different clusters that can be formed. Furthermore, cluster analysis is not based on *statistical* techniques, that is, it is not based on any probability concept. It is within this context that two distinct methods of clustering were adopted for the purpose of uncovering the learners’ possible underlying misconceptions.

#### **4.2.2 Clustering related test items**

Clustering of related test items was considered a fundamental pre-requisite for the identification of the learners’ underlying misconceptions. However, the basis for clustering test items for the purpose of identifying possible misconceptions seemed narrow, so I focused on two methods for clustering related items.

The first method involved clustering of test items on the basis of whether they were drawn from the same concept, for instance, fractions, percentages, decimals, equivalence between fractions and decimals, equivalence between fractions and percentages, estimation, or area and perimeter. In other words, questions based on the same topic were clustered together. Therefore, identification of possible misconceptions was based on the analysis of learners’ responses to clusters of similar, topic-related items. Clustering of related or similar test items was achieved through analysing the post-test items with respect to the content topics covered.

The second method of clustering was done using the computer software package *Statistica*. This method was based on the proportions of learners who scored correct and incorrect responses per test item. Here, the clustering was a manifestation of the closeness of test items based on level of difficulty. However, the number of items constituting a cluster and the number of clusters that could be formed through this method varied. The number of clusters that could be formed depended on the choice of the linking or the *Euclidean* distance, which was arbitrary. Conversely the number of items in a single cluster depended on the number of clusters formed. The number of clusters ranged from as many clusters as the number of test items, that is, single-item clusters, to as little as two distinct clusters. This method could create clusters ranging in number from two up to the number of distinct test items in the questionnaire.

Since the second method was based on the proportions of correct and incorrect responses to each test item, it appeared to be less appropriate as a means of identifying possible underlying misconceptions. It seemed to provide very little insight to the possible misconceptions held by the learners. Consequently, I adopted a more exploratory method, which was informed by the analysis of the learners' responses to clusters of similar, topic-related test items.

### **4.3 Stages of data analysis**

#### **Stage 1: Tabulation of learners' responses**

For each test item, the frequencies of learners' responses across all possible answer options were established and recorded in a single frequency distribution table. Once the percentage of responses across all possible answer options for each item was recorded, this was followed by the process of determining the proportion of incorrect and correct responses for each test item. The procedure was intended to answer the question: how many learners achieved the correct answers and incorrect ones for each item? Concurrently, the frequency of learners who did not attempt to answer the question was established for all test items.

#### **Stage 2: Ranking of test items**

The ranking of test items in order from the most to the least difficult was conducted. Item difficulty was defined in terms of the number of learners who correctly answered

the question. In other words, in this stage the following question is answered: which test items got the most number of incorrect or the least number of correct responses?

### Stage 3: Comparison of pre- and post-test results

This stage involved the superimposition of bar graphs of the same item from the pre- and post-test results, and the graphs were subsequently compared and contrasted against each other. Statistical significance tests were carried out for the comparison of learners' multiple-choice results in the pre-test and the post-test in order to see if there were any shifts in performance that could shed light on our understanding of learners' possible underlying misconceptions.

In most cases, the McNemar Test is used for  $2 \times 2$  tables, however for larger  $r \times r$  tables the following Chi-square test is used (Bowker, 1948):

$$\chi^2_{Statistic} = \sum_{j=i+1}^{r-1} \sum_{i=1}^j \left( \frac{(B_{ij} - B_{ji})^2}{B_{ij} + B_{ji}} \right)$$

Where:

$B_{ij}$  = Observed frequency in the cell in the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column,

$B_{ji}$  = Observed frequency in the cell in the  $j^{\text{th}}$  row and  $i^{\text{th}}$  column

$r$  = Number of rows = number of columns.

$\chi^2_{Statistic} \sim \chi^2_{r(r-1)/2}$  : This means that  $\chi^2_{Statistic}$  has a  $\chi^2$  distribution with  $r(r-1)/2$  degrees of freedom.

### Stage 4: Analysis of learners' responses to every test item and cluster analysis

Stage 4 involved the analysis of both incorrect and correct learners' responses to clusters or sets of similar, topic-related items. Brief discussions of the learners' responses were subsequently followed by inferences about their possible underlying misconceptions, based upon patterns in their incorrect responses to similar or related test items.

### Stage 5: Hypothesis testing for guessing in the multiple-choice items

Since some teachers from both the experimental and control groups of learners did not cover particular topics or subtopics in the specified time, there was need to establish

whether learners guessed or not when answering the multiple-choice items. In order to establish whether the post-test's multiple-choice items were not susceptible to guessing or the learners' responses were not a result of guessing, a chi-square test was conducted. This involved the chi-square test of goodness-of-fit tests for each multiple-choice item (questions 1-20), ascertaining whether the test design had some flaws with respect to guessing. Nevertheless, the impetus for conducting the statistical tests was that some teachers had partly covered or had not covered the topics: decimals, percentages and measurement in the agreed time, which could have provided the learners with the opportunity to guess.

In summary, the data analysis mainly focused on learners' incorrect responses and attempted to establish possible strategies that could have been used in order to arrive at a particular incorrect answer. The analysis aimed at establishing the basis of the choice of a particular incorrect answer with respect to logic and possible thinking strategies that could have been employed by the learners. This was followed by a consideration of the percentage frequencies of learners' responses for particular incorrect answer choices. This procedure was aimed at establishing why for instance the majority of the learners opted for a particular incorrect answer choice. The aim of the last stage in the analysis of learners' incorrect responses was to establish the trend of errors or the pattern of learners' incorrect responses to clusters of similar, topic-related items, and thus formed the basis for the process of uncovering their possible underlying misconceptions. In addition to analysing potential misconceptions, this research study also aimed at critiquing the post-test as an assessment instrument used to measure the learners' underlying misconceptions.

The issues of validity and reliability of the research findings, in part, should be based on whether the most effective methods for data collection and analysis were adopted and also on the issue of defensibility of the findings. However, more importantly, the reliability and validity of the research findings can largely depend on the recognition of existing theories and the extent to which the findings can fit in those theories. In this context, it is hoped that the research findings based on the data analysis procedures highlighted in this chapter will not only be tenable, but also amenable. The next chapter focuses on data analysis.

## Chapter 5

### ANALYSIS OF TEST RESULTS

This chapter discusses the statistical analysis of post-test results. The analysis involved tabulation of percentage frequencies of learners' correct and incorrect responses for each item, ranking of all items in terms of item difficulty, and cluster analysis. Cluster analysis aimed at uncovering the learners' possible underlying misconceptions based on clustering of their incorrect responses to similar, topic-related items. This was, subsequently followed by a comparison of learners' pre-test and post-test results for the purpose of establishing whether there were significant differences in terms of learners' responses to multiple-choice items.

#### 5.1 Tabulation of post-test results

From the overall summary of statistics for both the pre- and post-test results it was apparent that the majority of learners performed badly. The mean scores for the pre-test and post-test were 20.03% and 26.50% respectively. Only one learner out of 538 passed in both tests, that is, scored more than 50%. Table 5.2 below shows an overall summary of statistics for the pre- and post-test scores

*Table 5.1 Overall summary statistics of pre-and post-test results*

Test	No: of learners	Mean	Min. score	Max. score	Lower quartile	Upper quartile	Range	Standard deviation
Pre-test	538	20.03	0.00	51.61	12.90	25.81	51.61	8.69
Post-test	538	26.50	0.00	67.74	19.35	32.26	67.74	10.28

The frequency distribution of the learners' responses across all possible answer options for each test item is shown in Table 5.2. It will be noted that 7 "options" are reflected on this table, which relate to the coding categories that are discussed below.

##### 5.1.1 Coding categories for learners' responses

The post-test was composed of 20 multiple-choice items (questions 1 – 20) and short free-response items (questions 21 to 31). A coding scheme for the learners' answer choices was developed for both sections of the post-test. All multiple-choice items,

Item 31 demanded high levels of cognition or mathematical reasoning because learners were not only required to provide propositions on how to improve the grid which they had chosen, but were also required to defend or justify their propositions. Further, items 26-31 were a series of questions based on the calculation of area of circles, using grids made up of regular shapes and, as such, a correct answer for item 31 depended largely on whether the learners had not only understood, but also on whether they had correctly answered items 26-30. Similarly, item 25 was problematic in the sense that the correct answer was dependent on the answers obtained for item 23. This implied that if the answer for 23 was incorrect, then the answer for item 25 was also going to be incorrect. It was also observed that more than 15% of the learners did not attempt to answer items 26, 28, 29, 30 and 31. Item 31 got the highest percentage of learners (27%) who did not attempt to answer the question. The above highlighted features could possibly account for the identified item difficulty for items 31, 22, 25 and 21

After the initial analysis of the frequency distribution of learners' responses to all items, which revealed test items that most of the learners found least and most difficult, there was need to establish whether the pattern of learners' responses to such items changed after instruction by comparing the pre- and post-test results.

## **5.2 Comparison of the pre- and post-test results**

Figures 1- 4 (below) show bar charts of learners responses across all possible answer options for both the pre- and post-test multiple-choice items (questions 1-20). The free-response items were not considered for comparison purposes between pre- and post-test results because they did not have a specified number of possible answer options. Considering the fact that the learners wrote the pre-test at the beginning of the term and the post-test at the end of the same term but after being taught the topics: decimals, percentages and measurement, an increase in the percentage of learners who obtained correct answers was expected. Conversely, a decrease in the percentage of learners who obtained incorrect answers was anticipated. However, I focussed more on changes or shifts in frequencies of learners' correct responses and of learners' most frequently appearing incorrect responses.

Figure 1. Bar Charts of Question Choices by Question and test (Item 1-5)

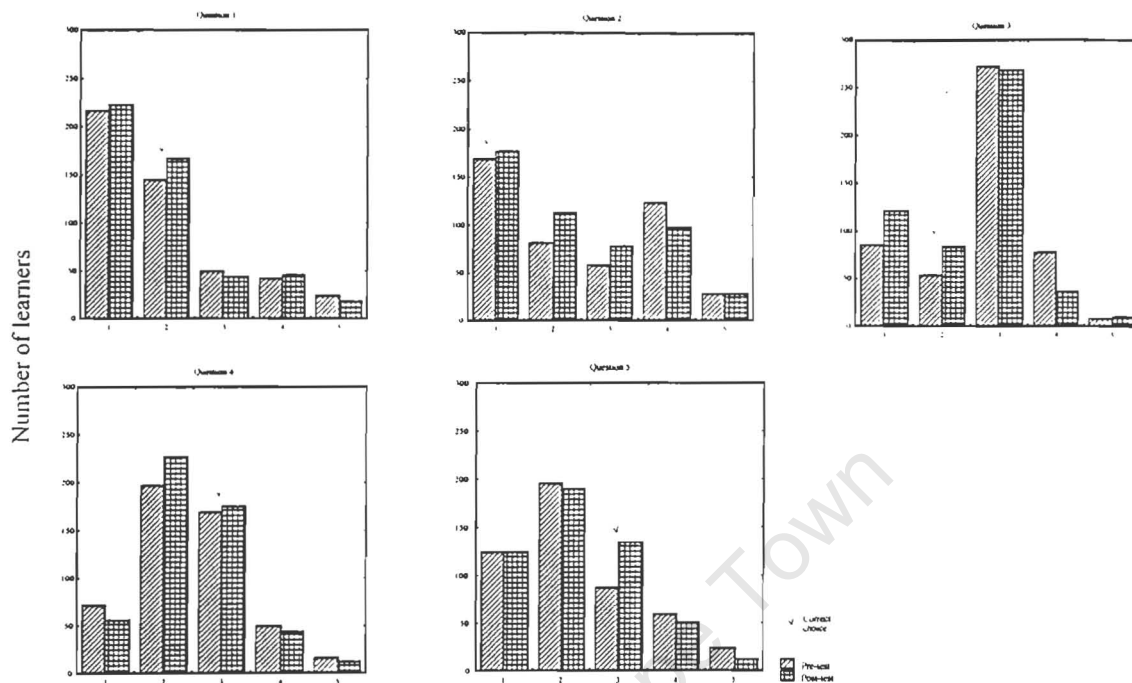
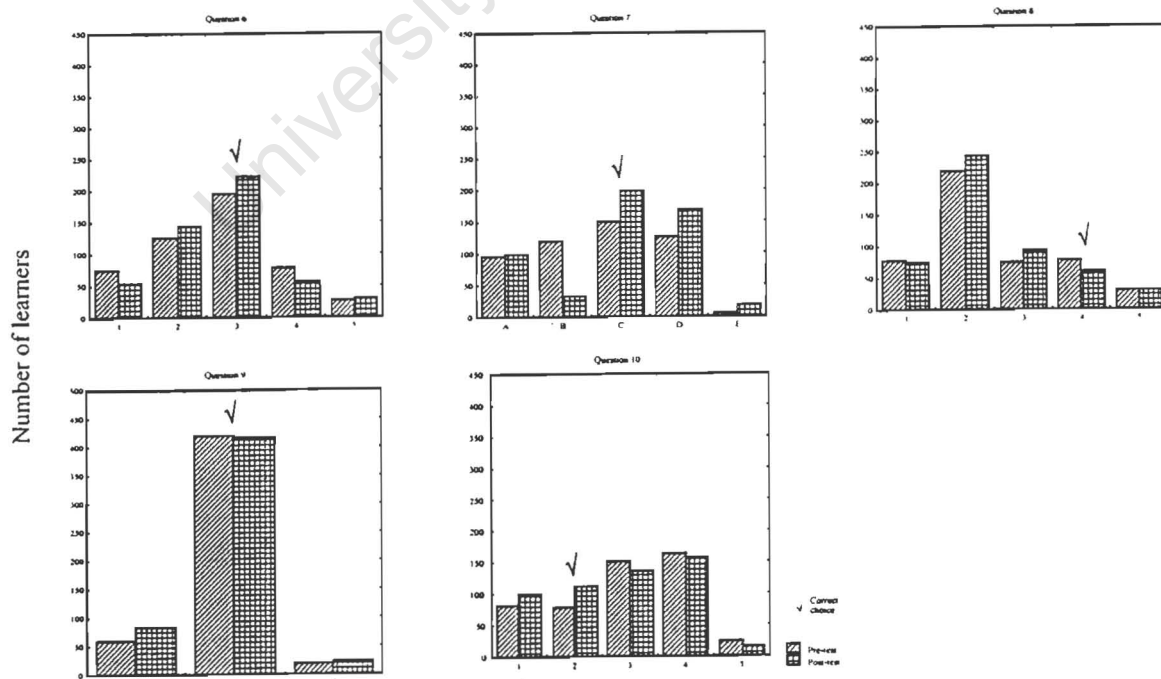


Figure 2. Bar Charts of Question Choices by Question and test (Item 6-10)



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Figure 3. Bar Charts of Question Choices by Question and test (Item 11-15)

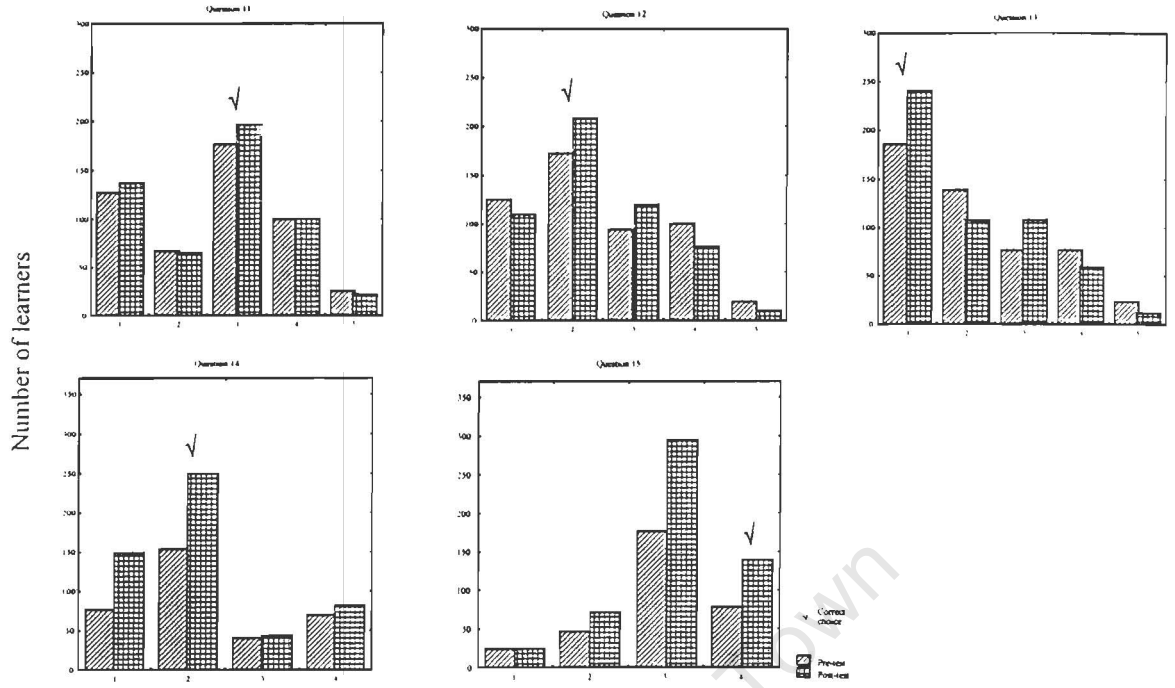
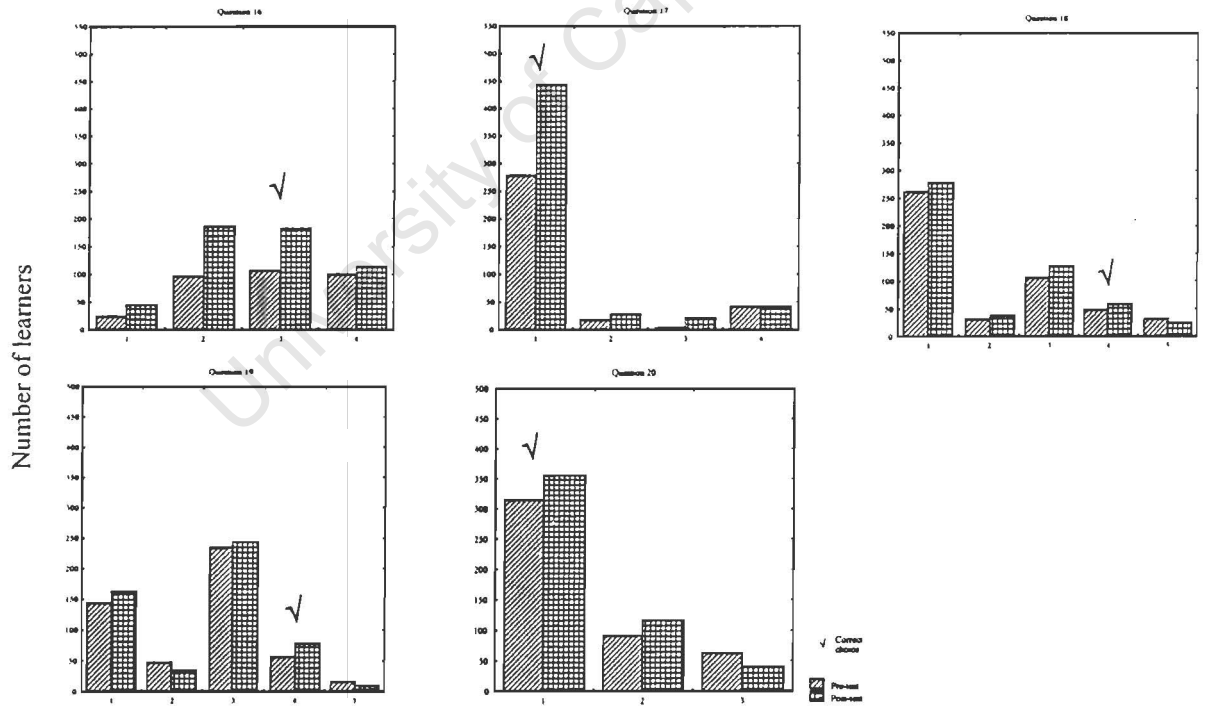


Figure 4. Bar Charts of Question Choices by Question and test (Item 16-20)



### 5.2.1 Differences in pre- and post-test results in terms of answer choice preferences

An analysis of the superimposed bar charts from the pre-test and the post-test results revealed interesting changes (shifts) in patterns of learners' responses to multiple-choice items from the pre-test to the post-test. Through inspection, it was observed that there was a slight increase in frequency of learners' correct responses to items 2, 4 and 18, and a relatively marked increase for all other items except items 8 and 9. For items 8 and 9 there was a decrease in the frequency of learners' correct responses. This, to some extent, suggests that contrary to what is expected to happen after learners go through some instructional process (an increase in correct responses) the opposite happened.

Focusing on the frequencies of learners' most frequently appearing incorrect answer options, it was observed that there were fluctuations in terms of an increase or a decrease in frequencies of learners' responses to different items. Items 3, 8, 18, and 19 were identified as the multiple-choice items which most of the learners found most difficult. For items 3, 5, 18 and 19 there was a slight decrease (almost constant) in, and for items 2, 10, 12 and 13 there was a relatively marked decrease in, frequency of learners' most frequently appearing incorrect answer options. On the contrary, item 11 had a slight increase while items 1, 4, 6, 7, 8, 9, 14, 15, 16 and 20 had a relatively marked increase in frequency of learners' most frequently appearing incorrect answer options. In fact, item 15 seemed to have the largest increase in frequency of learners' most frequently appearing incorrect answer option 3. This suggests that even after instruction more learners opted for particular incorrect answer options.

Since there were observable differences in terms of increases or decreases in frequencies of learners' correct responses and in learners' most frequently appearing incorrect answer options, and at times no differences at all, it became problematic to ascertain whether there were significant differences between the pre- and post-test results. Therefore statistical tests were carried out to establish whether there were significant differences between the pre- and post-test results. This was undertaken in order to establish whether there was some form of reduction in learners' possible misconceptions that could be attributed to instruction during the course of the term.

### 5.2.2 Chi-square tests for differences between pre- and post-test results

To ascertain whether there were significant changes in the results for the pre- and post-test, significance tests based on the chi-square test of symmetry (Bowker, 1948) were conducted on the 20 multiple-choice items and the results are shown in Table 5.4

*Table 5.4 Chi-square test for Pre-Post Distributions of answer choices: testing for similarities (equality) of counts/frequencies pre-post-tests*

Item number	Chi-square value	Degrees of freedom:	P-value	
1	5.947	10	0.8197	P > 0.05
2	17.898	10	0.0570	P > 0.05
3	36.3778	10	0.0005*	P < 0.05
4	11.446	10	0.3238	P > 0.05
5	18.51	10	0.0469*	P < 0.05
6	23.876	10	0.0079*	P < 0.05
7	70.616	10	0.0005*	P < 0.05
8	9.46	10	0.4890	P > 0.05
9	4.350	3	0.2260	P > 0.05
10	15.965	10	0.1006	P > 0.05
11	5.828	10	0.8298	P > 0.05
12	21.867	10	0.0160*	P < 0.05
13	33.54	10	0.0022*	P < 0.05
14	15.501	6	0.0017*	P < 0.05
15	8.333	6	0.2150	P > 0.05
16	5.468	6	0.4850	P > 0.05
17	15.9	6	0.0140*	P < 0.05
18	5.835	10	0.8289	P > 0.05
19	16.172	10	0.0950	P > 0.05
20	6.708	3	0.0082*	P < 0.05

Note: \* indicates p-values that are less than 0.05

There were significant differences between the pre- and post-test results for items 3, 5, 6, 7, 12, 13, 14, 17 and 20 ( $p < 0.05$ ). This perhaps indicates that more learners obtained the correct answers in the post-test than in the pre-test for the above test items. On the other hand, the statistical significant difference between the pre- and post-test results could be attributed to an increase in the percentage of learners who got the incorrect answers. Nevertheless, there was need to establish whether the test items belonged to the same topic-related clusters or not.

The difference in terms of increased frequency of learners, who obtained the correct answers for items 17 and 20, could be attributed to exposure to similar concepts during the course of instruction. For both the pre- and post-test these items got the highest frequencies of correct responses, and the two items were based on the concept of estimation. Although item 9 had a p-value less than 0.05, it is one of the few items the learners found least difficult. For item 9, the frequency of learners' correct responses remained unchanged (stable), but there was an increase in the frequency of learners' most frequently appearing incorrect answer option 1 from the pre-test to the post-test.

It could be suggested that the correct responses were due to familiarity through exposure, conceptual understanding or everyday common knowledge. In fact, item 17 required the learners to estimate the unit of measurement for the distance between two towns. For items 6 and 7, based on the addition and subtraction of decimals, the improvement in the frequency of the correct responses could be attributed to repeated practice, and subsequently the development of the necessary algorithmic skills. Similarly, for items 12 and 13, which involved conversions of units of measurement from one form to another, the difference in the frequency of correct responses could be attributed to exposure and repeated practice. Nevertheless, for multiple-choice items, it was difficult to differentiate the forms of understanding, be it relational or instrumental, both forms would give a correct answer and we could not rule out the possibility of learners having developed relational understanding as well.

For items 3 and 5, which are both linked to the concept of equivalence between decimals and common fractions, there was noticeable improvement in the frequency of the correct responses, for instance the correct answer for item 3:  $\frac{3}{4} = 0.75$ , which could be attributed to either instructional procedures, familiarity, repeated practice, instrumental or relational understanding. None the less, it was observed that the frequency of learners' most frequently appearing incorrect answer option slightly decreased or was almost static for the two items.

In summary, since there were increases, decreases or no differences in frequencies in learners' correct responses and in learners' most frequently appearing incorrect answer options, it became problematic to ascertain whether the statistically significant

differences could be interpreted as an improvement in learner achievement or a decline in learner achievement. Although I attempted to establish the reasons for an increased percentage of learners who got correct answers, it was difficult to account why, for some items, there was a decrease in the percentages of learners who obtained the correct answers. For instance, for item 8 there was a decrease in frequency of correct responses and for item 9 there was no change. In other words, under such circumstances it became difficult to locate the reasons for the significant differences. The significant differences could be attributed to an increase in percentage of learners who got incorrect answers or those who got correct answers. This suggests that learner achievement or misconceptions cannot be influenced by instruction alone. Instruction can either be positive or negative in terms of learner achievement or misconceptions. Nevertheless, in an attempt to uncover the learners' possible misconceptions I focused on clusters of learners' responses to topic-related items.

### 5.3 Topic-related clusters of learners' post-test results

Interesting patterns in pupils' incorrect responses were identified on clusters of topic-related questions, that is, on sets of questions on the same topic. As indicated earlier, post-test items were drawn from four related topics, measurement, percentages, common fractions and decimal fractions. In an attempt to uncover learners' possible underlying misconceptions it was necessary to consider patterns in their incorrect responses to similar or related test items. In this section, I will focus on the analysis of learners' responses to clusters of topic-related items.

I first considered items 1, 3, 8 and 9, on common fractions and equivalence between common fractions and decimals, which are shown below.

1. In which of the following lists of fractions are the fractions arranged from biggest to smallest fraction? Circle your answer.

1.  $\frac{1}{10}, \frac{1}{7}, \frac{1}{4}, \frac{1}{2}$

2.  $\frac{1}{2}, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}$

3.  $\frac{1}{7}, \frac{1}{2}, \frac{1}{10}, \frac{1}{4}$

4.  $\frac{1}{4}, \frac{1}{10}, \frac{1}{7}, \frac{1}{2}$

5. none of these answers

3. The decimal form of  $\frac{3}{4}$  is

1. 0,34
2. 0,75
3. 3,4
4. 4,3
5. none of these answers

Circle your answer

8. Which list shows numbers from smallest to largest. Circle your answer.

1. 0,25;  $\frac{1}{10}$ ;  $\frac{1}{3}$ ; 0,5
2.  $\frac{1}{10}$ ;  $\frac{1}{3}$ ; 0,5; 0,25
3. 0,25;  $\frac{1}{3}$ ;  $\frac{1}{10}$ ; 0,5
4.  $\frac{1}{10}$ ; 0,25;  $\frac{1}{3}$ ; 0,5
5. none of these answers

9. In a maths test, Jabu scored  $\frac{34}{60}$  and Nomsa scored  $\frac{64}{80}$ . Which students' mark was higher? Circle your answer.

1. Jabu
2. Nomsa
3. It is not possible to work this out from the information given.

For item 1, 31% of the learners obtained the correct answer, and 41% of the learners opted for the incorrect answer option 1. This suggests that this incorrect answer could be obtained through the use of an incorrect strategy, namely that the larger the denominator the larger the fraction when comparing a given set of fractions with the same numerator. For item 3, 50% of the learners gave the incorrect answer option 3 and only 15% of the learners obtained the correct answer. The choice of the incorrect answer option 3 possibly means that the majority of the learners could have lacked the appropriate meaning of the symbolic representation of a fraction. The incorrect answer could be obtained if the numerator and denominator were considered as two unrelated whole numbers. For item 8, 11% of the learners obtained the correct answer, while 45% of the learners gave the incorrect answer option 2. This supports the notion that some learners could have used an incorrect strategy once again, namely that common fractions, irrespective of size, are smaller than decimal fractions. However, for decimal fractions, some learners could have considered the decimal

parts as whole numbers, and therefore, 0.25 was regarded as bigger than 0.5. In addition item 9 was identified as one of the items most of the learners found easiest. 78% of the learners obtained the correct answer. Here the incorrect strategy produced correct results.

To augment my assertions in the above paragraph, the most predominant single incorrect answer option in terms of the learners' responses for items 1, 3 and 8 was 1.  $\frac{1}{10}$ ,  $\frac{1}{7}$ ,  $\frac{1}{4}$ ,  $\frac{1}{2}$  (41%), 3. 3.4 (50%) and 2.  $\frac{1}{10}$ ;  $\frac{1}{3}$ ; 0.5; 0.25 (45%) respectively. This could mean that some learners held the misconception that for a fraction  $a/b$ ,  $a$  and  $b$  are taken as separate and unrelated whole numbers. Further, it can be asserted that some learners could have used an incorrect strategy in comparing the sizes of fractions as shown above. In fact, some learners could have held the misconception that the bigger the denominator, the bigger the fraction, if numerators are the same. In a sense, they could have perceived such fractions in a linear configuration, that is, as whole numbers. The likelihood of the use of this wrong strategy was quite evident in errors made to items 1, 8 and 9. It can also be inferred that probably due to the lack of the appropriate meaning of the symbolic representation of fractions, some learners could have confounded the symbols for whole numbers and with fractions, and subsequently, tended to over-generalise their whole number schemas.

Items 3, 4 and 8 were based on equivalence between common fractions and decimal fractions. Interpretations of learners' responses to items 3 and 8 have already been discussed above and item 4 is as follows

4. The common fraction form of 0,125 is

1.  $\frac{12}{5}$
2.  $\frac{125}{100}$
3.  $\frac{125}{1000}$
4.  $\frac{5}{12}$
5. none of these answers

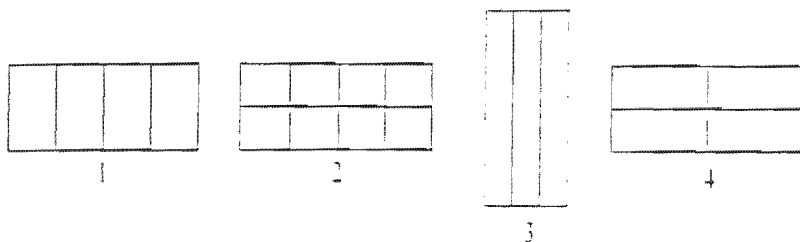
Circle your answer.

For item 4, 32% of the learners obtained the correct answer, and 42% of the learners gave the incorrect answer choice coded 2. Equating 0.125 to 125/100 could mean that some learners lacked the concept of equivalence between common fractions and decimal fractions. The analysis of patterns in learners' incorrect responses to items 3, 4 and 8 supports the notion that some learners could not make sense, and failed to construct the appropriate meaning of the symbolic representation of fractions. As such, they could have regarded fractions as composed of two unrelated whole numbers, that is, the numerator and the denominator as disjoint entities. This misconception can be attributed to the learners' inability to see a fraction as a quantity-quotient relation between two whole numbers. This augments the assertion that some learners could have viewed fractions as 'linear', that is as whole numbers as suggested by the following response:  $\frac{3}{4} = 3.4$ .

Closely linked to the concept of fraction is the notion of equivalence among fractions, decimals and percentages, which appeared to be absent in many learners. From the learners' incorrect responses to items 3, 4 and 8, it can also be said that some of the learners could have regarded  $\frac{3}{4}$ , 0.75 and 75% as different symbols representing different quantities. Some learners seemed to have the misconception that common fractions, regardless of size, are bigger than decimals. This is shown by the learners' most frequently appearing incorrect response for item 8, answer option 2.  $\frac{1}{10}$ ;  $\frac{1}{3}$ ; 0,5; 0,25. On the other hand, focusing on the correct answers to items 1, 3, 4, 8 and 9, the learners who obtained the correct answers could have developed either relational or instrumental understanding of the concept fraction and the concept of equivalence between fractions and decimals. In a sense, for some learners the correct answers could be obtained through use of particular memorized rules or procedures, but without understanding why the rules or procedures work that way.

Item 2 was also based on common fractions, but in this case it was more difficult to link learners' responses to possible underlying misconceptions. Item 2 is as follows:

2. Each of these drawings shows a fraction. Which two drawings shows the same fraction? Circle your answer.



1. 1 and 2
2. 1 and 4
3. 2 and 3
4. 3 and 4
5. none of these answers

For item 2, 21% of the learners gave the incorrect answer option 2, which could mean that some learners ignored the shaded portions of the rectangles and focused on the sub-divisions of the rectangle. Both rectangles are divided into four equal parts. Nevertheless, 33% of the learners obtained the correct answer. Despite the possible lack of the concept of equivalent fractions in some learners, and the difference in the number of sub-divisions in the two rectangles, it is possible to get the correct answer through close observation.

The next cluster of questions on decimal fractions to be considered were items 5, 6, 7 and 11 shown below

5. 756.382 can also be written as:

1.  $700 + 50 + 6 + \frac{3}{10} + \frac{80}{100} + \frac{2}{1000}$
2.  $700 + 50 + 6 + 3 + 8 + 2$
3.  $700 + 50 + 6 + \frac{3}{10} + \frac{8}{100} + \frac{2}{1000}$
4.  $700 + 50 + 6 + \frac{3}{100} + \frac{8}{1000} + \frac{2}{1000}$
5. none of these answers

Circle your answer.

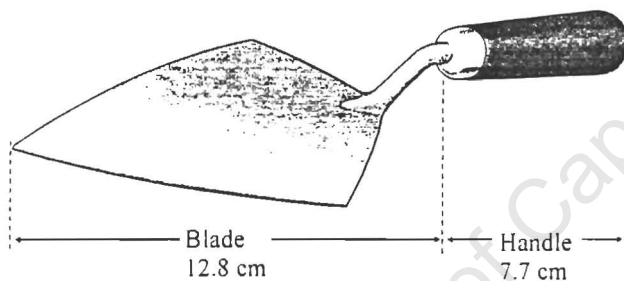
6. Subtract  $2,261 - 0,753$ . Circle your answer.

1. 3,014
2. 1,512
3. 1,508
4. 1,518
5. none of these answers

7. Add  $5,43 + 6,043$ . Circle your answer.

1. 11,86
2. 11,47
3. 11,473
4. 11,086
5. none of these answers

11. What is the total length of this trowel? Circle your answer.



1. 19,5 cm
2. 5,1 cm
3. 20,5 cm
4. 205 cm
5. none of these answers

For item 5, 25% of the learners obtained the correct answer. 35% and 23% of the learners gave the incorrect answers, options 2 and 1 respectively. The learners who opted for the incorrect answer option 2 could have lacked the concept of place-value system and disregarded the decimal point. The second most frequently appearing incorrect answer option suggests that some learners could have made some computational or careless errors, hence they obtained 80/100 instead of 8/100. For item 6, 41% of the learners obtained the correct answer, and 26% of the learners gave the incorrect answer option 2. The incorrect answer choice could mean that some learners separated the whole number part from the decimal part and then considered the decimal parts as whole numbers, and subsequently subtracted the smaller from the

bigger one. This, to some extent, supports the idea that some learners could have regarded the decimal point as a separator of two unrelated whole numbers.

For item 7, 31% and 18% of the learners gave the incorrect answers options 3 and 4 respectively. For those who chose the incorrect answer option 3, it appears that they lacked the conceptual understanding of the place-value system. As a result they could have disregarded the 0 in 6.043 and added the decimal parts as whole numbers. For those who opted for the incorrect answer coded 4, it seems that they considered decimal numbers as whole numbers separated by a comma. 37% of the learners obtained the correct answer. For item 11, 25% and 19% of the learners chose the incorrect answers coded 1 and 4 respectively. In the first instance, the learners could have lacked the concept of place-value system or alternatively the incorrect answer could be arrived at as a result of carelessness. For those who gave the incorrect answer coded 4, it appears that they could have completely disregarded the decimal point. 36% of the learners obtained the correct answer.

From the consistency displayed in learners' incorrect responses to items 5, 6, 7 and 11 together, it could be argued that some learners held the misconception of decimals as composed of two whole numbers separated by a decimal comma. The most frequently appearing incorrect responses for items 5, 6, 7 and 11 were 2.  $700 + 50 + 6 + 3 + 8 + 2$  (35%): 2. 1.512 (27%): 4. 11.086 (31%): and 1. 19.5 (26%) respectively. The pattern of incorrect answers suggests that some learners regarded the decimal comma as a separator of two unrelated whole numbers. This further supports the idea that they perhaps lacked the conceptual understanding of the symbolic representation of a decimal comma, as well as the place-value system. Focusing on the percentages of learners who obtained the correct answers for items 5, 6, 7 and 11 (25%, 41%, 37% and 36% respectively) it suggests that some learners developed the necessary procedural or algorithmic skills on the addition or subtraction of decimals. Some learners could have developed relational understanding of the underlying concepts since in such cases it was difficult to differentiate whether the learner had developed relational or instrumental understanding.

Items 10, 21 and 22 were based on percentages and are as follows.

10. At a clothing sale, clothes are marked "25% off". What will you pay for a jacket on the sale that was originally priced at R240? Circle your answer.

1. R60
2. R180
3. R300
4. R215
5. None of these answers

21. If 30 % of the 700 babies in Pedi village are breast fed, how many babies is this? Write down your answer:

.....

22. If 600 of the 800 babies in the town of Qunu are breast fed, what percentage of the babies is this? Write down your answer:

.....

For item 10, 21% of the learners obtained the correct answer. But, 19%, 25% and 29% of the learners chose incorrect answers options 1, 3 and 4 respectively. For the learners who opted for the incorrect answer option 4, it appears that they subtracted 25 from 240, which suggests that they lacked the concept of percentage and treated the percentage as any other whole number. For those who gave the incorrect answer option 3, it seems they managed to calculate the actual discount R60, but they added the discount to R240 to get R300. This suggests that the learners, to some extent, developed conceptual understanding of percentages but rather encountered some language problems, which culminated in their failure to comprehend the question. Similarly, for the learners who chose the incorrect answer option 1, it seems they could have managed to calculate the discount, but regarded it as the final answer.

For items 21 and 22, 2% and 1% of the learners obtained the correct answers respectively, and these were identified as items that most of the learners found most difficult. Items 21 and 22 were free-response questions, as such the most frequently appearing incorrect answers were coded 3 for both items. The coded 3 represented a wide range of incorrect answers other than 75% and 200% for item 22, and incorrect answers other than 210, 2100 and 670 for item 21. As such, it was not possible to isolate the most frequently appearing single incorrect response because a wide range of learners' incorrect responses were represented or bunched under a single code. Hence it became difficult to uncover the learners' possible underlying misconceptions for items 21 and 22.

For items 10, 21 and 22, it could be asserted that some learners lacked the conceptual understanding of percentages and the appropriate meaning of their symbolic representation. It was observed that some learners could have held the misconception that percentages behave in the same way as whole numbers except that they are masked by the percentage (%) symbol or sign. This is supported by their incorrect responses to items 10)  $R240 - 25[\%] = R215$ , 21)  $\{700 - 30(\%) = 670\}$  and 22)  $[800 - 600] = 200\%$ . It seems that if the learners failed to interpret the symbolic representations of percentages, then, they resorted to their previously developed schemas of whole number concepts.

On linear measurement of length, there were items 19, 23, 24 and 25, which are as follows

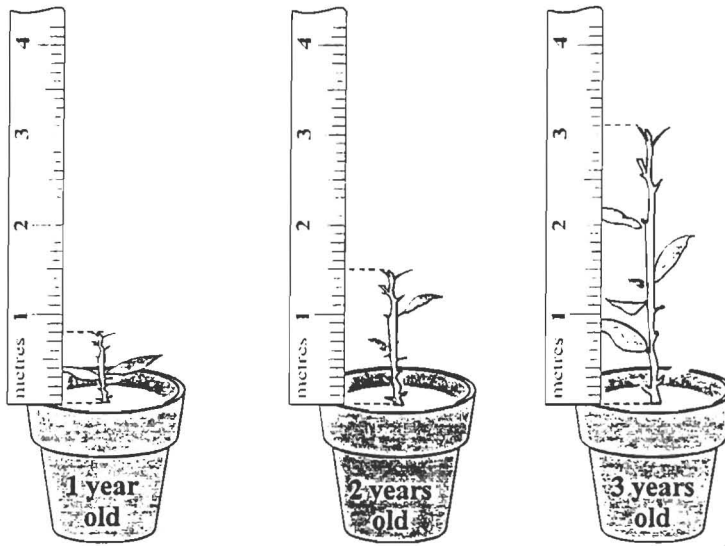
19. What is the length of this pencil (including its rubber)?



Circle your answer.

1. 8cm
2. 8,5 cm
3. 9,0 cm
4. 7 cm
5. none of these answers

These pictures show the same plant at different times.



23. How tall was the plant when it was one year old?

Answer: ..... metres

24. How tall was it when it was 2 years old?

Answer: .....metres

25. How much did it grow between 1 year old and 3 years old?

Answer: .....metres

For item 19, 14% of the learners obtained the correct answer, and 45% of the learners chose the incorrect answer option 3. The choice of the incorrect answer option 3 suggests that the majority of learners took the reading on the ruler at the end point of the pencil, and regarded it as the length of the pencil ignoring the fact that the starting point of the pencil was not at zero point on the ruler. Further, 30% of the learners gave the incorrect answer coded 1, which suggests that the learners obtained the answer by either subtracting 1 from 9 or subtracting 2 from 10. This supports the idea that the learners could have failed to identify or recognise the attribute or the actual length of the object that was being measured. For item 23, 41% of the learners gave some other incorrect answer besides 0.8 and 8. 30% of the learners obtained the

incorrect answer 8, which suggests that the learners correctly counted the divisions or spaces on the ruler and gave their answer in metres, implying that they could not relate the divisions to tenths of a metre. 13% of the learners obtained the correct answer.

Approximately 34% of the learners obtained the correct answer for item 24, which could be largely attributed to the familiarity or over-emphasis of  $\frac{1}{2}$  or 0.5. 30% of the learners gave some other incorrect answer besides 15. 20% gave the incorrect answer 15, which suggests that they could have failed to relate the divisions to tenths of a metre. For item 25, 44% of the learners gave an incorrect answer other than 2.3, 3.1, 31 or 23. 23% of the learners gave the incorrect answer 3.1, which meant that they could have taken the reading on the ruler at the end point of the plant and did not subtract 0.8 as was required. Only 1% of the learners got the correct answer. For one to get the correct answer for item 25 largely depended on whether the learner had obtained a correct answer for item 23.

Drawing from the pattern observed in learners' incorrect responses to items 19, 23, 24 and 25, it appears that some learners held the misconception that the length of an object is obtained by taking the reading of the ruler at the end point of the object being measured disregarding its starting point on the ruler. This was shown particularly by their responses to items 19 and 25. The most frequently appearing incorrect answers for items 19 and 25 were 3.9.0cm (45%) and 2.3.1 (23%) respectively. It was also observed that the learners could not interpret the scale of the ruler. Many learners could have lacked the concept of measurement of length, scale and could not identify the length of the object to be measured. Instead of counting the divisions covering the length being measured, they seemed to take readings from the ruler at the end point of the object.

In addition to the above items on measurement, there were questions 12 and 13 on the equivalence among the different units of measurement of length, which are as follows

---

12. 638 cm can be written as

1. 63,8 metres
2. 6,38 metres
3. 0,638 metres
4. 6380 metres
5. none of these answers

Circle your answer.

---

13. 5,75 metres can be written as

1. 575 cm
2. 57,5 cm
3. 0,575 cm
4. 5750 cm
5. none of these answers

Circle your answer.

---

For item 12, 21% and 22% of the learners chose the incorrect answer options 1 and 3 respectively. For those who gave the incorrect answer option 1, it appears that they assumed that there were 10 cm in a metre. Similarly for the learners who opted for the incorrect answer option 3, it seems that they assumed that there were 1000cm in a metre. Only 38% of the learners obtained the correct answer, which could be obtained through some procedural or algorithmic rules. The correct answer could also be obtained through use of appropriate methods based on conceptual understanding.

For item 13, 45% of the learners obtained the correct answer, and 20% of the learners chose the incorrect answers (options 2 or 3). Some learners could have considered that there were 10cm in metre while others could have considered that there were 10m in a centimetre. Hence from the patterns in learners' incorrect responses to items 12 and 13, the consistency shown in considering that there were 10cm in a metre, can be regarded as a misconception. In both questions, correct answers could be obtained through memorised procedures or algorithmic rules. The rule of counting the number of decimal places either to the right or left depending on the nature of the question is very common for such questions.

Items 14 –17 were based on the estimation of units of measurement of length. The items are as follows.

---

14. Which standard unit of measurement (that is, kilometres, metres, centimetres or millimetres) would be most suitable for measuring the length of the fence around your school?

1. kilometres
2. metres
3. centimetres
4. millimetres

Circle your answer.

15. Which standard unit of measurement would be most suitable for measuring the thickness of a 50 cent coin?

1. kilometres
2. metres
3. centimetres
4. millimetres

Circle your answer.

16. Which standard unit of measurement would be most suitable for measuring the length of a man's belt?

1. kilometres
2. metres
3. centimetres
4. millimetres

Circle your answer.

17. Which standard unit of measurement would be most suitable for measuring the distance between Cape Town and Johannesburg?

1. kilometres
2. metres
3. centimetres
4. millimetres

Circle your answer.

---

For item 14, 47% of the learners obtained the correct answer, which suggests that the majority of the learners were familiar with the appropriate units to measure the perimeter of their schoolyard. 27% of the learners opted for the incorrect answer option 1, which though incorrect seems reasonable and a possible correct answer. For

item 15, 55% of the learners gave the incorrect answer option 3, which suggests that the majority of the learners lacked the necessary practical experience of measuring the thickness of coins. This could also be attributed to limited exposure to practical exercises of measurements involving millimetres. On the other hand, the learners could have confused thickness and diameter of the coin. 26% of the learners obtained the correct answer, which could mean that the learners had grasped the concept of estimation. Approximately, 13% and 5% of the learners chose the incorrect answer options 2 and 1 respectively, which supports the idea that they could have completely lacked the concept of size and estimation.

For item 16, 34% of the learners obtained the correct answer. However, an almost equal number of learners (35%) gave the incorrect answer option 2. The incorrect answer option appeared reasonable and plausible, and could have served as a good distracter. Item 17, has already been discussed and was identified as one of the items that most of the learners found easiest. From the learners' responses to items 14 – 17 it appeared that there were no obvious misconceptions identified. However, it seems, that some learners lacked the concept of estimation. The correct answers for those items could be obtained through familiarity or the use of everyday common knowledge.

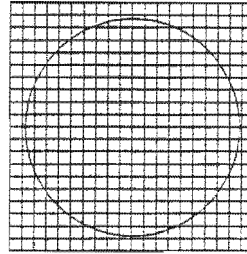
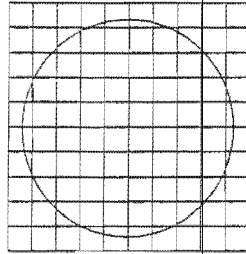
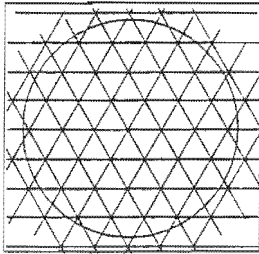
Again on measurement, there were a series of questions on calculating areas of circles using different grids made up of regular shapes, either triangles or squares. The series of questions (items 26 – 31) are as follows:

The same circle is placed on each of the three grids, as shown below.

(1) Triangular grid

(2) Square grid

(3) Square grid



\* Work out the area of each circle, as best as you can, using each of the grid units given.

Answer:

26. Area of circle in drawing (1)..... Triangular grid units  
 27. Area of circle in drawing (2) ..... big square grid units  
 28. Area of circle in drawing (3) ..... Small square grid units

\* The circles on each grid are the same. Why do your answers differ?

29. The answers are different because .....

.....

\* Which grid do you think is the best to work from, if you want the circle area to be as accurate as possible? Why?

30. Grid number ..... is the best because .....

.....

.....

31. How would you improve on the grid you chose?

I would improve on my grid by .....

.....

.....

.....

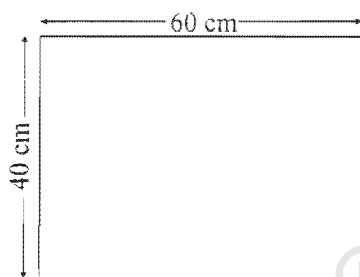
For items 26, 27 and 28 the majority of the learners (47%, 42% and 47% respectively) obtained incorrect answers other than 56-66, 50-60 and 220-245 respectively as approximate areas of the circles. On the other hand, 4%, 10% and 5% of the learners obtained the correct answers for items 26, 27 and 28 respectively. It was also

observed that among the learners' incorrect responses, some were regarded as inappropriate. From the learners' incorrect responses to items 26, 27 and 28 it was found that the majority of the learners could have lacked the conceptual understanding of the spatial representation of area, and had difficulties in measuring area through approximations. However, no obvious or clear misconceptions were identified.

For many learners the concept of area and perimeter seemed to be lacking. This was supported by their responses to item 18, which is as follows

---

18. A rectangular piece of white paper is shown below. What is the perimeter and area of the paper? Circle your answer.



1. 100 cm, 100 cm<sup>2</sup>
2. 100 cm, 2400 cm<sup>2</sup>
3. 200 cm, 240 cm<sup>2</sup>
4. 200 cm, 2400 cm<sup>2</sup>
5. none of these answers

---

52% of the learners gave the incorrect answer option 1. This suggests that the majority of learners added the length and the width of the rectangle to get both the perimeter and the area of the rectangle. Further, this could mean that the learners lacked both the procedural and the conceptual understanding of the concepts perimeter and area of a rectangle. This was supported by the fact that only 11% of the learners obtained the correct answer.

For item 29, 25% of the learners obtained the correct answer, and 34 % of the learners gave answers that were regarded as nonmathematical as shown in Table 5.4. For item 30, 33% and 24% of the learners gave incorrect answers, which were regarded as inappropriate and nonmathematical respectively. Only 4% of the learners obtained the

correct answer. Item 31 was identified as the question, which the majority of the learners found most difficult. This was supported by the fact that no learner obtained a correct answer for item 31. Many of the learners' responses to items 29, 30 and 31 were initially coded as nonmathematical and inappropriate. This resulted in a large number of responses being coded as a single category and it was decided to sub-code these. The analysis of learners' responses to those items is addressed in the next discussion.

### 5.3.1 Qualitative analysis of learners' written (verbal) responses to items 29, 30 and 31

After the initial capture and coding of post-test data, it was observed that the codes used for the free-response items were too broad, particularly for responses to items 29, 30 and 31. This meant that there was a need to expand the coding scheme to cater for the scope of the learners' responses. Sub-categories for codes 2 and 7 were created for items 29, 30 and 31 as shown in Table 5.5 below. The results of the analysis are summarised in Table 5.6.

Table 5.5 Sub-categories for items 29, 30 and 31

Code	Subcategories	Item; and examples of pupils' responses
2(1)	A <i>nonmathematical</i> statement that is possible to understand	30; <i>It makes you understand</i> 30; <i>Easy to count</i> 31; <i>Working hard</i>
2(2)	An ambiguous <i>nonmathematical</i> statement that is unclear and difficult to understand	31; <i>Use would you it is beautifull and it is clean;</i> 31; <i>Drawing it so buitful like putting colour on it</i> 30; <i>It is very beautifll and draw very nice to the other grids</i> 30; <i>It is cool and it is the best</i>
7(1)	An inappropriate <i>mathematical</i> statement that is possible to understand	30; <i>Its square a square has four sides and triangular has three sides</i> 31; <i>By counting the blocks and by looking the area</i>
7(2)	An inappropriate <i>mathematical</i> statement that is ambiguous, unclear and difficult to follow	29; <i>There are not the same fraction</i> 31; <i>By calculating my grid</i> 31; <i>Triangular grid is the best one</i>

The subcategories for coding learners' incorrect responses shown in Table 5.5 above are not shown in the frequency distribution table, Table 5.2, so a separate frequency distribution table (Table 5.6) was created for items 29, 30 and 31 as shown below

*Table 5.6 Frequency distribution for subcategories of learners' responses to items 29, 30 and 31*

Item		Subcategories of learners' responses								Total
		1	2		3	6	7		8	
			1	2			1	2		
29	Freq.	133	0	1	74	78	170	13	69	538
	%	25%	0%	0%	14%	14%	32%	2%	13%	100%
30	Freq.	22	31	94	21	85	169	7	109	538
	%	4%	6%	17%	4%	16%	32%	1%	20%	100%
31	Freq.	0	2	7	6	143	122	207	51	538
	%	0%	0%	1%	1%	27%	23%	38%	10%	100%

After sub-categorisation of learners' responses for options 2 and 7 direct quotations from the learners' scripts were recorded and are shown in the three tables in Appendix C.

Although the further analysis of items 29, 30 and 31 did not reveal explicit misconceptions, it supported the suggestion that some learners lacked conceptual understanding of spatial representation of area, that is, they could not approximate the area of three circles using grids made up of regular shapes such as triangles and squares. The analysis further revealed that some learners could have difficulties with the English language. In fact, it appeared that the majority of the learners lacked the appropriate mathematical language in order to express their ideas explicitly and accurately. In summary, the issues of how best the learners could improve their verbal communication of mathematical ideas was considered essential. The issue of language difficulties could be attributed to their socio-economic background, or to lack of opportunities to communicate mathematical ideas in their classrooms. On the other hand, some learners' written statements for item 31 had aesthetic connotations. Their written verbal responses included such phrases as 'making it more beautiful by colouring it'.

### 5.3.2 Summary of discussions of learners' responses to clusters of topic-related items

From the analysis of learners' incorrect responses to clusters of topic-related questions, several learners' possible underlying misconceptions based on patterns in, or consistency of, incorrect answers were uncovered, particularly on fractions and decimals. It was observed that most of the identified possible misconceptions were linked to symbolic representations of fractions, decimals and percentages. The majority of the learners seemed to have failed to develop the appropriate meanings of the symbols for fractions, decimals and percentages. As a result, for fractions some learners could have used incorrect strategies when comparing fractions with the same numerator, namely that the larger the denominator the bigger the fraction. Similarly when comparing common fractions and decimals some learners could have regarded common fractions, irrespective of size or magnitude, as bigger than decimals.

For decimals, it was observed that some learners seemed to regard the decimal point as a mark or a separator of two unrelated whole numbers. For questions on addition and subtraction of decimals some learners appeared to separate the whole number parts from the decimal parts, and added or subtracted the two categories separately, with both being regarded as whole numbers. When solving problems on percentages some learners seem to have disregarded the percentage symbol and then treated the percentages like any other whole number. Although the analysis of clusters of topic-related items cannot provide conclusive evidence about learners' mathematical misconceptions, a substantial number of their possible underlying misconceptions were uncovered, and will be discussed in detail in the next chapter.

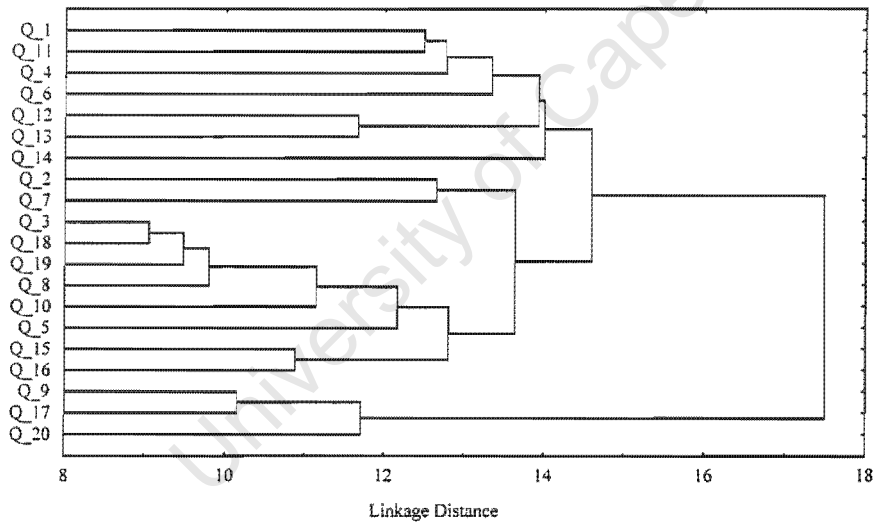
On the other hand, it was observed that for most questions, the correct answers could be obtained through memorised procedures or algorithmic rules. It was observed that a correct answer to a particular test item did not necessarily mean that it could be obtained as a result of conceptual understanding, in as much as failure to identify some kind of misconception could not necessarily mean that there were no misconceptions at all. However, this could not overrule the possibility that some of the learners developed a conceptual understanding of the background concepts. In the next chapter, the research findings will be compared with related literature in order to

reconcile the research findings with both national and international research literatures.

#### 5.4 Cluster analysis

The *Statistica* computer package was used to conduct cluster analysis. Cluster analysis was further considered in order to establish whether there was any additional interesting information that could be obtained with respect to patterns in, or consistency of, learners' incorrect responses to related or similar items. Cluster analysis aimed at expanding the scope of relationships among test items that could be analysed for the purpose of uncovering learners' possible underlying misconceptions. However, it should be noted that the cluster analysis applied to the multiple-choice items (questions 1 – 20) only. Results of cluster analysis are shown in Figure 5 below.

Figure 5. Dendrogram of Post-Test MCQ Items



The number of clusters of related items that could be formed or obtained from the dendrogram depended on the choice of the linkage distance or Euclidean distance. Table 5.6 below shows the different clusters of related items that were formed from the dendrogram.

Table 5.7 Different clusters based on varying the linkage distance/Euclidean distance

Categories	Linkage distance/ Euclidean distance	Clusters	Number of clusters
1	10	3; 18; 19; 8 all other items form single item clusters	17
2	12	12; 13 3; 18; 19; 8; 10 15; 16 9; 17; 20 all other items form single item clusters	12
3	12.5	1; 11 12; 13 3; 18; 19; 8; 10; 5 15; 16 9; 17; 20 all other items form single-item cluster	10
4	13	1; 11; 4 6 12; 13 14 2; 7 3; 18; 19; 8; 10; 5; 15; 16 9; 17; 20	7
5	14	1; 11; 4; 6; 12; 13; 14 2; 7; 3; 18; 19; 8; 10; 5; 15; 18 9; 17; 20	3
6	16	9; 17; 20 all others items form the another cluster	2
7	18	All items belong to a single cluster, hence there is no clustering	0

#### 5.4.1 Analysis of clusters of related items

From the analysis of the various categories of clusters of related items obtained from the dendrogram, it was observed that the items were clustered in terms of the proportions of correct and incorrect responses for each test item. In other words, the items were clustered on the basis of the ratio of correct to incorrect responses for each item rather than on the basis of the topic they were based on. It was further observed that the clusters seemed to agree with the ranking of items in terms of level of item difficulty. For instance, the cluster composed of items: 9, 17 and 20, confirms the rank order, which indicated that the three items got the highest frequencies of correct responses. In other words, items 9, 17 and 20 had the highest proportions of correct responses. Similarly, items 3, 18, 19, and 8 constituted a cluster, which corresponds

with the rank order based on the level of item difficulty. The only disparity between the rank order and the clusters from the dendrograms is that the ranking of the items in Table 5.2 was entirely based on the frequency of correct responses for each item whereas the cluster analysis was based on the proportions of incorrect and correct responses for each item. Hence, for the purpose of uncovering learners' misconceptions, clusters obtained from dendrograms could only be useful if they subsequently fell under the same topic.

It was found that small clusters composed of such items as 12 and 13, 15 and 16 were based on the same topic of measurement and decimals, and estimations on the units of measurement of length respectively. For all other clusters, it appeared that there were no explicit background linking concept(s) so that consistency in learners' incorrect answer choices to similar or related items could be established, and hence deduce their possible underlying misconceptions. This rendered the cluster analysis inappropriate for the purpose of uncovering learners' possible underlying misconceptions. In other words, there were virtually no common clusters from the dendrogram and the identified clusters of topic-related items, since the clustering were based on different notions.

### **5.5 Hypothesis testing for guessing in the multiple-choice questions**

In order to establish whether the post-test's multiple-choice items were not susceptible to guessing a chi-square test was carried out for the 20 items. This was undertaken because not all teachers covered decimals, percentages and measurement in the agreed time and, as such, there was need to exclude the possibility that learners' responses to these items were a result of guessing. Table 5.8 shows hypothesis test results for guessing for each of the 20 multiple-choice questions.

Table 5.8 The test for goodness-of-fit for guessing for MCQ

Item number	Including "none of the above"			Excluding "none of the above"		
	Chi-square Value	Degrees of freedom	P-value	Chi-square Value	Degrees of freedom	P-value
1	328.531	4	P < 0.001	202.269	3	P < 0.0001
2	117.980	4	P < 0.001	46.673	3	P < 0.0001
3	401.071	4	P < 0.001	240.945	3	P < 0.0001
4	335.820	4	P < 0.001	191.392	3	P < 0.0001
5	194.105	4	P < 0.001	77.084	3	P < 0.0001
6	253.529	4	P < 0.001	163.330	3	P < 0.0001
7	248.777	4	P < 0.001	131.558	3	P < 0.0001
8	270.112	4	P < 0.001	179.183	3	P < 0.0001
9	519.496	2	P < 0.001	222.223	1	P < 0.0001
10	113.577	4	P < 0.001	15.135	3	P < 0.0003
11	175.651	4	P < 0.001	77.200	3	P < 0.0001
12	195.346	4	P < 0.001	72.100	3	P < 0.0001
13	279.152	4	P < 0.001	144.030	3	P < 0.0001
14	187.876	3	P < 0.001	N/A	N/A	N/A
15	313.859	3	P < 0.001	N/A	N/A	N/A
16	99.515	3	P < 0.001	N/A	N/A	N/A
17	971.932	3	P < 0.001	N/A	N/A	N/A
18	420.911	4	P < 0.001	37.1901	3	P < 0.0001
19	349.434	4	P < 0.001	198.526	3	P < 0.0001
20	314.254	2	P < 0.001	122.553	1	P < 0.0001

All the p-values for the 20 items were less than 0.05, which indicated that responses to the items did not reveal guessing strategies. Since 'none of the above' was regarded as a poor distracter, nevertheless, there was still need to check for the possibility of guessing for the remaining answer choices. None of the test items (all items except 9, 14, 15, 16, 17 and 20) which included 'none of the above' choice option was found to be susceptible to guessing. Item 10, was the only item which had a p-value close to 0.05. As such, this could be the only test item that would be considered for refinement.

To conclude this chapter, both cluster analysis and the analysis of learners' responses to clusters of topic-related items provided an informed perspective on how learners' possible underlying misconceptions could be explored and uncovered. Although it remains a formidable challenge for researchers to come up with a common criterion for both methods of cluster analysis, several learners' possible underlying misconceptions were uncovered. Chapter 6 focuses on the discussions of the identified possible mathematical misconceptions, limitations of the research study, and the critique of the post-test as an instrument used for measuring learners' possible underlying misconceptions.

## Chapter 6

### DISCUSSION

This chapter reviews and discusses the research findings of this study in relation to findings of similar studies conducted nationally and internationally. In addition, the suitability of the post-test as an assessment instrument used to measure the underlying mathematical misconceptions will be reviewed. Limitations of research findings will also be discussed.

#### 6.1 Identified possible mathematical misconceptions

From the research findings, several learners' possible underlying misconceptions were identified. Most of the identified possible misconceptions seemed to stem from a lack of conceptual understanding of the symbolic representations of fractions, decimals and percentages. The identified possible misconceptions have been reported in other literature on similar research studies conducted in diverse contexts (see for example Hart, 1981; Hiebert & Wearne, 1985; D'Ambrosio & Mewborn, 1994; Baturu & Cooper, 1995; Clements & Ellerton, 1995; Brekke, 1996; Steinle & Stacey, 1998; Newstead & Murray, 1998; Outhred & MacPhail, 2000).

First, from the patterns in learners' incorrect responses to questions related to common fractions, it seemed that some learners held the misconception that for a fraction  $x/y$ ,  $x$  and  $y$  are regarded as separate and unrelated whole numbers. This misconception of regarding a fraction as made up of two disjointed and unrelated whole numbers has been reported in various research studies (see for example D'Ambrosio & Mewborn, 1994; Mack, 1995; Newstead & Murray, 1998). D'Ambrosio and Mewborn (1994) assert that "Children often see little relationship between the numerator and denominator in a fraction representation of a quantity" (p. 153). In other words, this misconception could be attributed to the learners' tendency to interpret fraction symbols at 'face value'. Drawing from this misconception, the research findings further revealed that some learners could have held the misconception that the bigger the denominator the larger the fraction when they compared fractions with the same numerator. In other words, some learners could

have used an incorrect strategy, namely the larger the denominator the bigger the fraction when comparing sizes of fractions with the same numerator. The propensity to choose the fraction with the bigger denominator as the larger one has been reported in other research literature (see for example Newstead & Murray, 1998; Steinle & Stacey, 1998).

According to Newstead and Murray (1998) the misconception of comparing the sizes of fractions with the same numerator by considering the size of the denominator was prevalent with Grade 6 students in certain schools in South Africa. They further argued that the misconception could be derived from the tendency of the students to consider the denominator and the numerator of a fraction as two unrelated whole numbers. This possible misconception could also be interpreted in terms of learners' naïve perception of fractions in a 'linear' configuration as demonstrated by one of their solutions  $\frac{3}{4} = 3.4$ . In other words some learners could have perceived fractions as 'linear', that is, reading fractions from left to right, instead of from top to bottom.

The possible lack of conceptual understanding of equivalence between fractions, decimals and percentages as different representations of the same quantity, was shown through the tendency of some learners to regard fractions, regardless of size, as bigger than decimals. This observation needs further investigation since no research literature has supported such findings.

On decimal fractions, it was observed that some learners could have held the misconception of regarding a decimal number as composed of a pair of unrelated whole numbers separated by a marker, that is, the decimal point. This misconception has been reported in various research studies (see for example Baturu & Cooper 1995; Brekke, 1996; Steinle & Stacey, 1998). According to Brekke (1996)

We regard the assumption that a decimal number is composed of two whole numbers separated by a point to be the most important underlying misconception linked to the conceptual knowledge of decimal numbers (p. 137).

This misconception seemed to be derived from the absence of an appropriate meaning of the symbolic representation of decimals, which subsequently could have led some learners to regard the decimal comma as a mark or separator of two whole numbers.

In line with this, Hiebert and Wearne (1985) assert that “First, many students do not know what the symbols mean. They fail to connect decimal symbols with meaningful referents” (p. 209). Therefore it could be said that the highlighted misconceptions were attributed to the learners’ limited conceptual understanding of the symbolic representations of fractions and decimals. Brekke (1996) attributes this misconception to the learners’ over use of their whole number schemas. In other words, several misconceptions held by some learners, could be a result of the limiting constructions based on the learners’ previously developed whole number schemas (D’Ambrosio & Mewborn, 1994; Mack, 1995).

The lack of a conceptual understanding of mathematical symbols was supported by the way some learners seemed to handle percentages. They could have regarded percentages as natural numbers or as positive whole numbers. Rees and Bar (1984) suggest that misconceptions of percentages might result from a lack of a concrete grasp of the meaning of percentage. This possible misconception needs further investigation since no literature has been located to support its occurrence in a different context.

On measurement of length, some learners seemed to hold the misconception of regarding the length of an object as the reading of the ruler at the end point of the object, disregarding the reading at the starting point of the object. What Hart (1981) regarded as the failure of some students to identify the length of the object being measured support this.

In summary, analysis of the frequency distribution of learners’ responses to clusters of topic-related items revealed patterns in their incorrect answers (error patterns) which suggested several possible underlying misconceptions. Many of the identified possible misconceptions have been reported in other research literature both nationally and internationally. However, some of the identified misconceptions, for instance, the misconception of regarding percentages as natural numbers, need further investigation since no related literature was found to support their occurrence.

It appears from the research findings that many of the identified misconceptions were largely derived from the learners' limited conceptual understanding of the symbolic representations of fractions, decimals and percentages. Some learners seemed to focus on giving the answer without checking or verifying whether it was logical or not. The learners' failure to compare fractions with the same numerator in terms of size indicated that they possibly did not link the concept of fraction to sharing. Most of the learners' answer choices suggested that there was very little bridging between school mathematics and their everyday social activities.

## **6.2 Limitations of research findings**

The challenges inherent in interpreting learners' incorrect responses and making inferences about their perceived misconceptions from the incorrect responses (error patterns) are based on the fact that some learners' incorrect responses could be misinterpreted and misunderstood and, as such, demand further probing through other different methods of data collecting. Research studies of this kind need to consider triangulation as a basis for data collection and analysis, and hence include such methods as clinical interviews and analysis of learner notebooks.

The study focused on ex-DET schools around Cape Town and this consequently limited the demographic range of schools in South Africa. This subsequently limits the generalisability of the research findings to different contexts. Nevertheless, the findings are broadly consistent with research findings both nationally and internationally.

The study also focused on the identification of learners' possible underlying misconceptions assuming that all the learners from the fourteen classrooms had covered all the specified content topics. However, it was reported in the major study (Ensor et al 2001) that only one of the fourteen teachers covered both topics on measurement and decimals. Almost half of the experimental classrooms had not covered such sub-topics on concepts of percentages and their computations and conversions from fractions to decimals (see Table 4.3). The post-test items were topic specific, and as a result, the learners from classrooms that had not covered all the

required content topics were forced to use intuition, everyday common knowledge or some background knowledge in answering particular questions. This phenomenon could influence the decision making process about the learners' possible misconception based on the patterns in their incorrect responses to clusters of topic-related items. This weakened the validity of the identified possible misconceptions.

In summary, the interpretations of learners' incorrect responses to clusters of topic-related items, and the inferences about their possible underlying misconceptions were not conclusive. The study therefore serves as a basis for further investigation, which takes into account the need to expand the individual researchers' repertoire of interpretations and methods. This leads to a discussion on the evaluation of the quasi-experimental research method to sufficiently measure and address the issue of learners' misconceptions.

### **6.3 Short-comings of research methodology**

Strike (1983) suggests that cognitive studies of students' misconceptions need special attention and care in the design of procedures of data collection, and in the interpretation of response data. Although the post-test managed to uncover part of the learners' possible misconceptions, it needed to be augmented by other methods like clinical interviews, group discussions or video taping the learners discussing their solutions to given questions in groups, or by simply observing and listening to the learners. Clinical interviews could have clarified and made the interpretation of the learners' possible misconceptions more explicit and accurate. In the light of this, D'Ambrosio and Mewborn (1994) suggest that

Allowing children to verbalize their thinking, however painful that might be for them, for the teacher, and for the rest of the class, provides irreplaceable opportunities to gain insight into student's thought processes (p. 160).

In other words, the issue of uncovering learners' misconceptions in a more robust context demands triangulation. In this context, the approach of assessing learners' misconceptions from different perspectives forms the basis of triangulation so as to avoid unnecessary skews/biases, distortions and limitations (Harris & Bell, 1994).

Triangulation involves the use of two or more different data collecting techniques or strategies for instance, clinical interviews, observations, tests, questionnaires or group discussions to measure the same variable so as to add value, validity and assertiveness to research results. It further accords multiple analyses, which allow students' misconceptions to be examined through various lenses. This explains why I needed to be cautious in asserting the learners' misconceptions through the use of 'suggested/possible misconceptions' because of the perceived narrowness of the research methodology adopted.

Furthermore, it was also my view that for the purpose of uncovering misconceptions, a different test instrument that was more diagnostic in nature could have been used. In other words, test items could have been designed in such a way as to elicit particular responses from learners, which would then serve as indicators of specific misconceptions. Along this line, Swedosh and Clark (1998) in their study of misconceptions concur:

Each question was designed in such a way that if the student had a particular misconception, this would become apparent when considering the response of that student to the question (p. 3).

Therefore, the methodology, indeed, fell short of in-depth probing of the learners' misconceptions and this ultimately rendered it inadequate to alone measure their possible underlying misconceptions.

#### **6.4 Critique of the post-test**

The research findings based on the analysis of learners' post-test results managed to reveal learners' possible underlying misconceptions although the test was not designed for this purpose. However, as indicated earlier in the literature review, every test instrument has its strengths, weaknesses and limitations. Firstly, its design being partly multiple-choice and partly free-response, the post-test had inherent weaknesses linked to its multiple-choice component as well as its free-response one. The multiple-choice component allows guessing to take place. Although certain precautions can be taken to reduce the probability of guessing, guessing cannot be completely eliminated. On the other hand, although the free-response can allow learners to organise their

ideas and express their answers in written form, the free-response items could not elicit particular answers that could indicate learners' possible underlying misconceptions. Focusing on the weaknesses of the test, since misconceptions were inferred from the analysis of the learners' incorrect responses to post-test items, cognisance on the limitations of such an analysis was taken seriously. Hence the interpretation of an incorrect answer posed considerable dilemmas and, as such, was problematic.

Seemingly, several different interpretations could be attached to an incorrect response. Incorrect responses could imply that the learners held some misconception. They could mean that the learners did not know anything about the question or simply guessed. There could also be a possibility that the incorrect response resulted from carelessness, computational or technical mistakes, misreading, lack of comprehension or language problems. Conversely, a correct response could be interpreted in more than one way. A correct response did not necessarily mean that the learner had acquired adequate background conceptual understanding, since procedural knowledge could not be detected by the correct answer for both sections of the test. Clements and Ellerton (1995) echo this phenomenon when they assert that

It is not widely known, however, that research has generated data which suggest that students who give correct answers to pencil-and-paper mathematics items sometimes have little or no understanding of the mathematical concepts and relationships which the tests were designed to measure...(p. 184).

However, on the contrary, "...education authorities continue to believe that the so-called "valid" and "reliable" pencil-and-paper tests can satisfactorily measure student understanding of mathematical knowledge, concepts, skills, and principles" (Clements & Ellerton, 1995: 184). In a sense, the two vehemently challenged the effectiveness of pencil-and-paper tests to measure learners' misconceptions. In line with this, correct responses based on algorithmic or mechanical skills tended to mask the learners' misconceptions, thus limiting the usefulness of the test to measure or uncover misconceptions. In other words, it was not always possible to identify misconceptions under such circumstances.

Based on the notion that a single error or an incorrect response to a single test item could not sufficiently indicate particular misconceptions, there is then the need to cluster related test items in an attempt to establish consistent incorrect ideas in responses to several related items. The consistency in learners' incorrect responses (errors) to clusters of topic-related items could give an indication of the learners' misconceptions. However, the issue of aggregating related test items for the purpose of uncovering misconceptions was also limited by the fact that misconceptions were not sufficiently incorporated into the design of the test. In other words, test items needed to be constructed and aggregated carefully to assess the range of possible underlying misconceptions.

Along these lines, learners' incorrect responses to test items could also be analysed in terms of connections that have been or have not been formed on related concepts (Hiebert & Carpenter, 1992). As such, for the post-test to sufficiently uncover the learners' possible underlying misconceptions, it needed to be more diagnostic in its design. In other words, it should have been designed to reflect the common ways of incorrect thinking (inconsistencies in learners' thinking), that could have been derived from a comprehensive analysis of the learners' performance in a different pre-test. For the purpose of uncovering learners' misconceptions, the test should have taken into account the identification of the specific sources of students' errors that require remediation.

In summary, a number of deficiencies of the post-test have been identified in relation to its potential as a research instrument that can adequately capture the learners' underlying misconceptions. However, the results from the post-test can be used as a pre-requisite in designing a somewhat diagnostic test that is intended to sufficiently uncover specific misconceptions. According to Gronlund (1982) unlike an achievement test, a "...diagnostic test contains a relatively large number of items for each specific area being tested...(and) attention is focused on the students' responses to specific items or groups of items..." (p. 12)

Further, mathematics has a unique language, which is rich in symbols and demands precision. This unique language of mathematics seems to pose many problems to learners when they attempt to understand what questions demand. Perhaps learners'

language problems or difficulties account in part for their incorrect responses. This could again mask the learners' misconceptions. As such, the test designers needed to consider the extent to which the test assessed language competence or proficiency at the expense of mathematics concepts because it is possible that some learners could have been confused or misled by the language used in the test.

#### 6.4.1 Reliability coefficient of the post-test: Cronbach's Alpha

The Cronbach's alpha coefficient (also called internal-consistency reliability) can measure the overall reliability of a test instrument based on the learners' test scores. Thus the reliability coefficient for the post-test was computed as follows:

$$\alpha = \frac{N}{N-1} \left[ \frac{S^2 - \sum S_i^2}{S^2} \right]$$

where:

N is the number of items on the test

$S^2$  is the sample estimate of the variance of the  $i^{\text{th}}$  test item

$S_i^2$  is the sample estimate of the  $i^{\text{th}}$  test item

The criterion for evaluating whether the test is in fact internally consistent is a Cronbach's alpha coefficient exceeding 0.7 where at least 100 subjects have been evaluated. The Cronbach's alpha coefficient for the post-test scores was found to be 0.5462 for all items and 0.5806 for the multiple-choice items. Therefore the post-test did not meet the requirement for overall reliability, since the coefficient was below the cut-off point of 0.70. However, the reliability coefficient for the multiple-choice component was found to be greater than the reliability coefficient for the entire post-test which suggests that the overall reliability of the post-test was reduced by the reliability coefficient for the free-response component of the post-test. Hence, the multiple-choice component of the post-test was more reliable than the free-response component. Nevertheless, there was the need to seriously consider the reliability of the post-test during the test design process since its reliability coefficient fell below the threshold point.

#### 6.4.2 The post-test as a printed document

Although the post-test question paper, which also served as the answer script, was computer typed, it lacked adequate editing. In item 11, certain answer options had decimal commas whereas other answer options had decimal points. Though seemingly trivial, this could have been confusing for some learners (especially considering that one of the answer options was 'none of these answers').

The language used in the construction of the test items needed to be edited or refined before printing the question paper. For instance, item 21 has a grammatical error in the statement 'how many babies *is* this?' It could have been better framed as 'how many babies *are* represented by this percentage?' Item 18 needed to be more explicit by re-phrasing the question as follows: 'a diagram representing a rectangular piece of paper is shown below' since the diagram is not the actual piece of paper.

Similarly, in item 31, instead of expressing the question as 'how would you improve on the grid you chose?', the test designer could have expressed it as follows: 'what could be done to the grid in order to get a more accurate answer?' From the analysis of the learners' responses to this item, it appeared that quite a substantial number of the learners misinterpreted the question. The language used in the construction of test items should be precise or straightforward so that the possibility of having misinterpretations based on vagueness or ambiguity is reduced.

#### 6.5 How useful is an achievement test?

In the above sections I have attempted to evaluate how useful and appropriate the post- test was in measuring learners' possible underlying misconceptions. However, it should be noted that the post- test was designed as an achievement test. In the light of this, there is also need to assess the suitability (or the usefulness) of the post-test as an instrument to measure learners' understanding of mathematics or achievement and misconceptions, focusing particularly on its design (construction) and the interpretation of test results.

Several research studies on assessment of learners' achievement in mathematics have revealed that there are critical issues that demand serious consideration in both test construction and interpretation of test results. On the one hand, in the construction stage of an achievement test, the major challenge is to address the questions: what is to be tested and how is it to be tested? On the other hand, interpretation of test results in terms of learner achievement or understanding of mathematics and misconceptions is problematic in the sense that the learner's thinking strategies are not always clear. This is supported by Steinle and Stacey (1998) who assert that "A written test cannot distinguish between these various forms of thinking, but it may be straightforward in an interview situation" (p. 2). However, the issues of test construction and interpretation of test results are intricately interwoven. For instance the factors that influence how a learner responds to test items need to be considered in the process of test construction.

What affects or influences how a learner responds to test items (interpretation of test results) therefore influences the construction of tests. Hence the evaluation of the usefulness of a test based on test results is indeed problematic. Along these lines, Ebel (1979) asserts that "The process of assessing achievement in learning is much more complicated than it may seem to be at first glance" (p. 5). In other words, what kind of data analysis of test results can enable valid claims to be made about learners' achievement or understanding?

According to Gronlund (1982) achievement tests can serve dichotomous purposes. He suggests that achievement tests can:

- direct learners' attention either towards or away from the desired objectives of instruction
- reward and reinforce superficial learning or require depth learning (conceptual understanding)
- provide dependable information or provide misleading, distorted or biased information for instructional decisions.

In other words, making inferences about learners' understanding of mathematics or achievement based on test results is problematic. For instance, Cooper et al (1997)

argue that “What the test results cannot tell us, however, is anything about the children’s response strategy and, in particular whether they utilised the given data or drew on their everyday knowledge...” (p. 7). Furthermore, substantial literature (see for example Goldstein, 1990; Cooper et al 1997) has shown that learners’ responses to achievement tests can be influenced by several factors *inter alia*:

- gender
- socio-economic background
- the nature or form of the question, whether ‘realistic’ or esoteric
- familiarity
- cultural background
- contextual factors, which include learner motivation and his or her perception of the purpose of the test.

Goldstein (ibid.) suggests that “The scrutiny of test material for racial and sexual stereotyping is, by now, a standard procedure among test constructors” (p. 73). It has been observed that a particular test format, for instance, multiple-choice tests might disadvantage learners of a particular sex in a group of learners. Similarly, ‘realistic’ assessment may disadvantage girls (Boaler, 1994, cited in Cooper & Dunne, 1998).

The socio-economic background of a learner can also influence the way he or she responds to particular test items, whether ‘realistic’ or ‘esoteric’. ‘Realistic’ questions are questions that refer to items with mathematical operations, which are embedded in everyday life contexts or settings, and ‘esoteric’ questions can be regarded as context-free items. According to Cooper and Dunne (1998)

Working class children may experience more difficulty than others in choosing ‘appropriately’ between using ‘everyday’ knowledge and ‘esoteric’ mathematical knowledge when responding to items...This may lead to underestimation of their mathematical capacities in cases where a rational ‘everyday’ response is ruled out as ‘inappropriate’ by the marking scheme but is ‘chosen’ by the child in place of an alternative ‘esoteric’ response...(p. 2).

In other words, ‘realistic’ and ‘esoteric’ test items may influence learners from a particular socio-economic background to respond in a particular way. Along these lines, Boaler (1993) commenting on her observation of results on gender analysis of learners’ responses to ‘realistic’ and ‘esoteric’ mathematical problems, suggests that

“The reason girls focus on certain aspects of a task and not others should be considered alongside their learning of mathematics and their equality of access to understanding” (p. 66). ‘Realistic’ mathematics or mathematics questions coated in everyday life contexts can either inhibit or enhance learners’ understanding of mathematics and hence construction of achievement tests should take cognisance of such factors.

It has also been observed that a lack of familiarity to particular test items may elicit particular responses from the learners. In most cases unfamiliarity is derived from culturally inappropriate test items. Along these lines, Adler and Reed (2000) argue that “Testing is not simply a matter of ‘carefully designed tasks’ but crucially a function of the testing context, including learners’ familiarity with the tasks” (p. 216). They further argue that

...as is the case with reform anywhere, for some of the learners, the form of the test items was unfamiliar (as was the case with multiple-choice in TIMSS for South African pupil participants). The FDE researchers were present when learners undertook the tests, and noted occasions when a pupil’s inability to respond to an item was due to its unfamiliarity alone. A simple prompt by the researcher enabled a correct response by the learner (Adler & Reed, 2000: 215).

The issue of a single TIMSS test taking care of all diverse cultures of the participating countries was also challenged by Keitel and Kilpatrick (1996). Hence if the construction of an achievement test fails to take cognisance of the above highlighted factors, then evaluation of its usefulness in terms of measuring learners achievement may be rendered problematic.

Furthermore, the test was composed of ‘realistic’ (about 15 items) and ‘esoteric’ items. The form of the questions could have influenced learners, regardless of gender, to respond to such items in a particular way. In addition, the format of the post-test being partly multiple-choice and partly free-response, could have disadvantaged learners of a particular sex in the group. This being the case, the way the post-test was constructed could have ripple effects on the interpretations of test results with respect to learners’ understanding of mathematics or achievement and misconceptions.

In summary, analysis of test results in order to evaluate learners' understanding of mathematics or achievement or misconceptions needs to be conducted with caution. Interpretation of test results can lead to both warranted and unwarranted inferences due to *inter alia*: socio-economic background of learners, gender, unfamiliarity (cultural) and contextual factors. Such factors can influence the learner's response to test items either positively or negatively and hence distort the authentic measurement of learners' understanding of mathematics or achievement and misconceptions. In other words, poor test results cannot be entirely attributed to lack of, or limited, understanding, and possible misconceptions of learners, since such factors as mentioned above can influence learners' responses to test items.

In addition, interpretation of test results does not reflect the strategies the learners used. Therefore an evaluation of learners' understanding of mathematics or achievement and misconceptions, based on a single test cannot be conclusive, because it can either, in part, be informative or misleading. Hence for such kinds of investigations that attempt to establish learner achievement and misconceptions, triangulation needs to be considered to seek clarifications which can either be obtained through clinical interviews or analysis of learner notebooks. Along these lines, Adler and Reed (2000) argue that "Crude analysis of test performance, in the absence of such prompts, will misrepresent learner knowledge" (p. 215).

Although the analysis of the post-test results revealed possible underlying misconceptions held by learners, these need to be considered in the light of the issues raised above. In chapter 7 I focus on making a summary of the research conclusions and make some recommendations based on the research findings.

## Chapter 7

### CONCLUSION AND RECOMMENDATIONS

The related research literature on students' misconceptions seems to focus on the identification of mathematical misconceptions, the identification of sources of misconceptions or identification of teaching methods that can be used to reduce the incidents of students' misconceptions. As far as the literature review is concerned there is no study that has attempted to address these issues from a holistic perspective. This present study attempted to make that particular contribution by identifying learners' possible underlying misconceptions and their possible sources, and then suggesting what teachers can do in terms of designing tests that can be used to uncover mathematical misconceptions, and what teaching strategies can be adopted to reduce the formation of mathematical misconceptions.

#### 7.1 Summary of identified possible underlying misconceptions

The main purpose of this research study was to explore and uncover possible misconceptions of Grade 7 learners on decimals, percentages and measurement (measurement of length, perimeter and area). The investigation of the possible misconceptions was conducted through an analysis of learners' responses to clusters of similar, topic-related post-test items. However, identification of learners' possible underlying misconceptions through an analysis of their responses to test items cannot be verified by statistical methods since the approach adopted was primarily descriptive. In addition, research findings on learners' potential misconceptions based on the analysis of test results alone cannot be conclusive. Nevertheless, in line with other South African research studies (Newstead & Murray, 1998) and other international studies (see for example D'Ambrosio & Mewborn, 1994; Mack, 1995; Brekke, 1996; Baturu & Cooper, 1995; Steinle & Stacey, 1998), this research study uncovered several possible mathematical misconceptions, which included:

- the view of symbolic representations of fractions as meaningless, hence the numerator and the denominator were regarded as unrelated and disjointed whole numbers;

- derived from the above misconception, comparison of the sizes of fractions was based on the notion that the bigger the denominator the larger the fraction, for fractions with the same numerator;
- the fractions were viewed from a 'linear' configuration ( read from left to right) as evidenced by the solution  $\frac{3}{4} = 3.4$ ;
- the regard of a decimal comma as a meaningless mark, separating two unrelated whole numbers;
- derived from the above misconception, the decimal numbers were regarded as composed of two unrelated whole numbers separated by the decimal comma, disregarding the place-value system;
- the view of fractions, irrespective of size, as generally greater than decimals, undermining the concept of equivalence;
- the consideration of the percentage symbol as meaningless, hence the treatment of percentages as natural or positive whole numbers;
- the length of an object was determined by the reading on the ruler at the end point of the object, disregarding the reading of the ruler at the starting point of the object.

It should, nevertheless, be noted that a number of the identified possible misconceptions, were related to fraction, decimal and percentage symbols, and seem to be pervasive in diverse contexts. However, since the misconceptions were based on interpretations or inferences drawn from the analysis of the learners' incorrect responses to clusters of similar, topic-related post-test items, it is not possible to make strong claims about the misconceptions the learners held. This could have needed triangulation through, for example, conducting clinical interviews or analysis learners' notebooks.

## 7.2 Recommendations for test designers

Assessment plays an important role in the identification of students' possible mathematical misconceptions, and precedes identification of the possible sources of misconceptions. Teaching strategies for reducing the incidents of misconceptions can only be developed when both the misconceptions and the possible sources have been identified. However, an assessment instrument such as a post-test cannot serve adequately dual purposes. A performance test cannot adequately measure the students' underlying misconceptions although the results of this study have revealed that a good performance test can give tremendous insight on learners' possible misconceptions.

For the purpose of uncovering students' misconceptions, a test designed for this purpose is required. Such a diagnostic test can be informed by findings or results obtained from a good performance test like the results obtained for this study, which can then be used to construct subsequent test items that are aimed at uncovering specific misconceptions. In line with this, Smith (2000) asserts that "Mathematics was identified as one of the curriculum areas in which the development of diagnostic tests was urgently required" (p. 448).

For any assessment instrument, be it a diagnostic test or a performance test; there are certain rubrics that can be adopted to make the assessment instrument strong. Such factors as the following are paramount and, as such, demand serious consideration in the test design process. These ideas are taken from Brown (1970).

- the purpose of a test should be defined explicitly.
- new or current forms of assessment should be designed in such a way so as to demonstrate what the students are learning and what they can do with their knowledge, in other words, the test should be informed by explicit definitions of what students have to know and be able to do
- a test instrument is required to sufficiently cover all content area in terms of the major or key concepts

- test items should be framed within classroom or real-life situations, integrate process with knowledge so that they elicit actual performance of learners
- standardised tests like the TIMSS (Third International Mathematics and Science Studies) test and experts in test design should be consulted
- the test should be of an acceptable level with respect to grade, age, cognitive levels, socio-economic and cultural background of the learner
- it should be designed in such a way that higher scores represent increased knowledge, in other words, a test should capture accurately the desired information
- the test should be easy to administer with minimum practical and logistical constraints as to ensure that all learners write the test under the same conditions, and marking and scoring of points is uniform
- should suit the learners' language level to foster comprehension, and hence avoid imposing unwarranted ambiguities
- page layout should be designed in such a way that learners can follow easily from question to question, aided by a clear set of instructions on how to answer
- should be diagnostic in nature, and allow both restricted response and free-response items
- should take cognisance of contextual and socio-cultural contingencies
- should have high validity and reliability
- Pre-tests should be conducted to a representative sample of the intended learners so that the test can be modified to suit the intended learners and capture accurately the desired information in a more feasible way.

In *Improving America's School: A Newsletter on Issues in School Reform – Spring 1996* it is stated that

Indeed, with the advent of standards-based reform, researchers, policymakers, and education practitioners agree that methods for assessing student achievement must be revamped in order to better measure what students know and are able to do (p. 1)

Furthermore, in the United States of America, The National Center for Research on Evaluation, Standards, and Student Testing (CRESST) has identified and developed the following criteria for assessment reviews

- Cognitive complexity – the assessment task calls for complex intellectual activity such as problem solving, critical thinking, and reasoning
- Content quality – the assessment calls for students to demonstrate their knowledge of challenging and important subject matter
- Meaningfulness – the assessment tasks are worth students' time and students understand their value
- Language appropriateness – the language demands are clear and appropriate to the assessment tasks and to students
- Transfer and generalizability – successful performance on the assessment task allows valid generalizations about achievement to be made; indicates ability to successfully perform other tasks
- Fairness – student performance is measured in a way that does not give advantage to factors irrelevant to school learning; scoring schemes are similarly equitable
- Reliability – answers to assessment questions can be consistently trusted to represent what students know
- Consequences – the assessment has the desired effects on children, teachers, and the educational system (p. 3)

Although the context of this study which was conducted in South Africa is different from the contexts of other similar studies conducted in United States of America, some research findings from American schools can be incorporated in designing tests for schools in South Africa.

Test construction should take cognisance of various factors that can influence learners' responses to test items like gender, socio-economic background, form of question whether 'realistic' or 'esoteric', familiarity (cultural) and contextual factors. Nevertheless, assessment should be credible in terms of building public support, be practically feasible, and be informative to all stakeholders.

Test results are largely dependent on the strength and goodness of the assessment instruments. Whether the test is good or bad, most test designers tend to agree that what one tests is, in most cases what one gets. Nevertheless, if the test results of a good test are inappropriately interpreted, then the goodness of the test may be undervalued. Mathematics teachers need to be equipped with the appropriate skills of designing strong assessment instruments and analysing test results. An evaluation of test results can provide information about their students' learning difficulties and misconceptions and will enable teachers to evaluate the effectiveness of their instructional strategies. The test format is very important. From *Stanford Diagnostic Mathematics Test, Fourth Edition (SDMT)* it is asserted that "The multiple-choice and free-response components may be used separately or in combination. Using them together will give a more complete picture of students' strengths and needs" (p. 1).

### 7.3 Recommendations for mathematics teachers

From the research findings, which have been supported by both national and international research studies, it can be said that misconceptions on decimals and fractions are quite pervasive in diverse contexts. The sources of students' misconceptions are also diverse, contentious, and debatable. The mathematics teacher has, nevertheless, a crucial role to play in an attempt to reduce the different cases and the frequencies of misconceptions. In an attempt to ameliorate mathematics teaching and learning, and hence reduce the students' misconceptions, there are enormous inertial forces, which militate against the teacher's efforts, and subsequently pose a number of paradoxes and dilemmas.

Hiebert and Carpenter (1992) assert that "One of the implications of research on students' errors is that instruction might be designed to address directly the specific deficits that the error analysis helps us to diagnose" (p. 88). Although I cannot give a prescription for teaching or instructional strategies that reduce the cases and frequencies of students' misconceptions I can, however, offer propositions based on the learning theories discussed in Chapter 2.

The results of this study suggest that most of the learners' underlying misconceptions arise from lack of appropriate meaning constructions of fraction, decimal and percentage symbols. This further suggests that such misconceptions arise from misunderstandings that are based upon incorrect meanings. In other words, some learners failed to make appropriate interpretations of the symbolic representations of fractions, decimals and percentages. Hence, the issue of symbolic representations, as part of the unique mathematics language, needs serious consideration in the teaching and learning of mathematics. Closely linked with the issue of symbols is the issue of language. The importance of language in mathematics discourse cannot be over-emphasised. Therefore mathematics teachers should always provide the learners with the opportunity to communicate their ideas and make them engage in constructive mathematical discourse.

Teaching is largely influenced by ecological or contextual factors (both physical and social) and culturally based ones. I therefore suggest that although teachers can, in part, be informed by research and the current theories of learning, they should, however, reconcile that knowledge with their experiences obtained in specific classroom environments. The mathematics teachers should always attempt to be reflective and reflexive, and nurture abstraction, and consider its richness in terms of promoting interconnectedness among related concepts.

In other words, mathematics teaching should primarily focus on nurturing relationships among related concepts, such as equivalence among fractions, decimal numbers and percentages. This process can also be enhanced by the use of computers, which can offer simultaneous multiple representations of equivalent concepts. It is also argued that computers nurture visual imagery, which promotes the construction of appropriate meanings and is indispensable in the concept formation process. In this context, it is therefore suggested that mathematics teachers should strive to espouse teaching methods that promote both relational and instrumental understanding, since the different forms of understanding seem not to be dichotomous, but rather complement each other. This is aimed at reducing the incidences that promote the formation of mathematical misconceptions in the classroom situation.

It has also been observed that perhaps the apparent divorce between school mathematics and the learner's everyday social life contributes to the formation of mathematical misconceptions. It is here suggested that mathematics teachers should emphasise the utility of mathematics outside the school context and hence bridge the gap between school mathematics and the learner's everyday social life. On the other hand, the view of mathematics as a set of rules, algorithms or procedures can be nurtured by teaching approaches which privilege demonstrations on the chalkboard, repeated practice and memorisation of facts and procedures. Procedural teaching approaches in the classroom situation seem not to be adequate with respect to promoting conceptual understanding of mathematics concepts, and hence learners' possible underlying misconceptions can be greatly reduced if on the one hand, mathematics teachers adopt various teaching methods.

On the other hand, different teaching methods specifically designed in response to students' mathematical misconceptions on topics like decimals, fractions and measurement have been developed on the basis of the ongoing research in mathematics education. Some of the teaching strategies have incorporated computers and have successfully eliminated some misconceptions, however, for particular cohorts of students in specific contexts (see for example Hiebert & Wearne, 1989; Swan, 1990; Irwin, 1997; cited in Irwin, 2000). Similarly, Swedosh and Clark (1998) advocate one such teaching strategy. Drawing from Tirosh (1990) they argue that

There have been various attempts to eliminate students' misconceptions in a wide range of fields. One approach which has met with success is the "conflict teaching approach", based on Piaget's notion of cognitive conflict, in which a teacher and a learner discuss the inconsistencies in the learner's thinking so that the learner realises that conceptions exhibited were inadequate or faulty and needed modification... (Swedosh & Clark, 1999: 1).

In line with the foregoing, Vinner (1990) (cited in Swedosh & Clark, 1999) further argues that "there is no doubt that if inconsistencies in students' thinking are drawn to their attention, it will help some of them to resolve some inconsistencies in a desirable way" (p. 97). In other words, the "conflict teaching approach" focuses on exposing the inconsistencies, which lead to the formation of particular misconceptions to the learner. In short, the method attempts to transform the learners' thinking through challenging and exposing its faults and limitations and concurrently exposing the learner to the expected and correct mode of thinking. Put in a different way, the method challenges the learner's faulty thinking through the use of counter examples, induces some kind of disequilibrium in the learner, forcing her/him to modify and eventually reject the faulty pre-conceptions. Although research is yet to establish whether the approach can work to all different ability groups, and which age groups, and how permanent is the improvement in eliminating misconceptions, I suggest its worthwhile for mathematics teachers to try the method in their different contexts, and subsequently shape it to suit their ecological contingencies.

Focusing on research studies conducted in South Africa, Newstead and Murray (1998), in their study, which was part of MALATI (Mathematics Learning and Teaching Initiative) aimed at curriculum and in-service teacher development, and drawing from the limiting constructions (D'Ambrosio & Mewborn, 1994) they

suggest that in order to enhance conceptual understanding of fractions and thereby reduce or prevent the incidents of misconceptions, teachers should consider seriously the choice of problems, social interaction among students, and students' own representation of fractions. In a series of studies based on the MALATI project, Newstead and Olivier (1999) focusing on the impact learning and teaching materials designed to enhance the conception of common fractions suggest that

A supporting classroom culture is required in which learning takes place via problem solving, discussion and challenge and in which errors and misconceptions are identified and resolved through interaction and reflection. Teachers do not demonstrate solution strategies, but expect students to construct and share their own strategies and thus to gradually develop more powerful strategies (p. 2).

Teaching-learning materials alone, without the teacher's use of appropriate supporting teaching strategies, are not always effective. Lastly, it is also suggested that it seems long overdue that teachers become practitioner researchers. In other words, teachers should begin an inquiry and exploration into their own teaching, that is, they should engage in researching their own teaching in relation to their students' possible misconceptions. I believe that through researching one's own teaching, mathematics teachers can empower themselves and hence contribute more effectively in the mathematics education reform processes. Further, through researching one's own teaching, mathematics teachers get equipped to respond to changing and challenging conditions of which uncertainty prevails, and hence develop sensitivity to mathematical ideas and misconceptions, to pedagogical possibilities or to thinking of the different ways other people think.

In a sense, mathematics teachers can no longer afford to be spectators anymore, that is, they can no longer afford to sit back and wait for the mathematics educators to conduct research in their classrooms on their behalf. In line with this, Hatch and Shu (1998) (cited in Sierpinska & Kilpatrick, 1998) argue that

If all research is carried out by a dedicated body of mathematics educators privileged as researchers, then there is a danger of the isolation of research from teachers and its value cannot be realized (p. 298).

Hence, it is high time mathematics teachers should initiate research projects aimed at uncovering the students' learning difficulties and misconceptions, and explore their

teaching and have direct access to the outcomes on certain aspects of their teaching, their classrooms and their students that might be invisible to others.

However, Pimm (1993) argues that no one has the legitimacy to demand change from teachers, change is the individual teacher's business, however, I believe it is quite hazardous to be left behind by change, more so, change is unavoidable and ubiquitous. Although change is stubborn and slow, we should all strive to manage change, harness and articulate its course and avoid unnecessary delays. Nevertheless, change is sometimes spontaneous, and at times change can be accelerated.

#### **7.4 Recommendations for researchers**

Focusing on the findings of this study, it appears that more questions than solutions have been raised. First, for the purpose of sufficiently uncovering students' misconceptions, not only a test instrument, but also other methods of data collection like clinical interviews, observations and analysis of learners' notebooks are required to ensure triangulation and, subsequently robust research findings. Further, to holistically address the issue of misconceptions, such questions as the following need to be addressed: 1) to what extent does classroom instruction influence the formation of misconceptions? 2). How do misconceptions vary from class to class and from school to school? and, 3) to what extent are misconceptions linked to the socio-economic and cultural background of the students? Researchers need to appreciate that there are no simple answers to questions about how to improve the quality of students thinking and learning. However, Nuthall (1997) suggests that we now need to focus on "Research that embraces the considerable complexity and subjectivity of the multi-layered and multi-dimensional nature of the classroom processes" (p. 758).

#### **7.5 Conclusion**

In summary, although the post-test managed to uncover several learners' possible underlying misconceptions, research findings, such as those for this study, based on an analysis of test results alone cannot be conclusive. Therefore the results obtained

from the analysis of the learners' responses to the post-test items could be used to predict the possible underlying misconceptions and then used as a pre-requisite in the design of a more comprehensive diagnostic test intended to uncover the mathematical misconceptions. However, a more robust conclusion on the possible underlying misconceptions the learners held could have been reached if the research methodology, particularly the data collection procedures had incorporated some form of triangulation. Clarifications on learners' potential misconceptions can be sought through conducting clinical interviews and analysis of learners' notebooks over and above the analysis of test results.

To conclude, this research study identified several possible underlying misconceptions related to common fraction, decimal number and percentage symbols, which have been reported in several literature both nationally and internationally. Looking at the identified possible misconceptions from the perspective of cognitive psychology, it can be said that perhaps the misconceptions arose from the way teachers presented mathematics to the learners in the classrooms. However, Adler and Reed (2000) argue that "More bluntly, test performance on its own is far too limiting to infer anything substantive about teaching..." (p. 217). Nevertheless, the way mathematics is presented in the classroom is one of the possible sources of learners' potential misconceptions and further, something can be done to improve this.

Seemingly, mathematics can be presented as a set of rules, facts and algorithms or procedures. This approach privileges and nurtures instrumental understanding of mathematical concepts. If instrumental understanding is not complemented by relational understanding gaps in the growth of mathematical concepts are likely to be created and subsequently translated into misconceptions. Therefore it is critical that mathematics teachers get equipped with the necessary pedagogical content knowledge and an understanding of how students learn and think. Stacey et al (2001) assert that "...knowledge of students' conceptions and misconceptions is an essential component of pedagogical content knowledge" (p. 207).

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University of Cape Town

APPENDIX A

POST-TEST

University of Cape Town

Name: ..... Age: .....

Name of Teacher: .....

Name of School: .....

*Note to fieldworker:*  
 learner left the classroom during the  
 time .....

*Tick to indicate that the*

*Tick to indicate that the learner arrived late .....*

## MATHEMATICS QUESTIONS

### **GENERAL DIRECTIONS**

Some of the questions in this paper will be followed by 5 possible answers with a number next each answer. For these questions, circle the number next to the answer you think is correct as shown in example 1.

#### **Example 1**

How many days in a week?

1. 10
2. 5
3. 14
4. 7
5. none of these answers

The number "4" has been circled because there are 7 days in the week.

Show your working out in the space next to the questions.

If you are not sure of the answer, circle the answer you think is best and continue with the next question in the test.

If you decide to change your answer to a question, put an "X" over your first choice and then put a circle around your new choice as shown in Example 2.

**Example 2**

2. Which of the following numbers is the largest?
1. 1001
  2. 999
  3. 10
  4. 100
  5. none of these

For other questions you will be asked to write short answers in the space provided. For these questions, you may use words for your answers. For example:

**Example 3**

3. Describe one important difference between a triangle and a square.

Answer:.....  
.....

When you write longer, more detailed answers in the larger space, be sure that your handwriting is clear. Think carefully about each question, and answer as completely as possible. Do not waste time.

## Mathematics problems on measurement and percentages

1. In which of the following lists of fractions are the fractions arranged from biggest to smallest fraction? Circle your answer.

1.  $\frac{1}{10}, \frac{1}{7}, \frac{1}{4}, \frac{1}{2}$

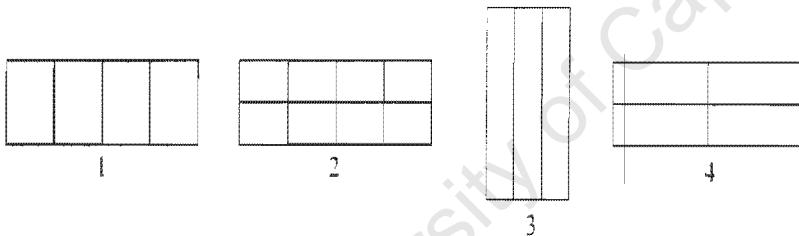
2.  $\frac{1}{2}, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}$

3.  $\frac{1}{7}, \frac{1}{2}, \frac{1}{10}, \frac{1}{4}$

4.  $\frac{1}{4}, \frac{1}{10}, \frac{1}{7}, \frac{1}{2}$

5. none of these answers

2. Each of these drawings shows a fraction.



Which two drawings shows the same fraction? Circle your answer.

1. 1 and 2
2. 1 and 4
3. 2 and 3
4. 3 and 4
5. none of these answers

3. The decimal form of  $\frac{3}{4}$  is

1. 0,34

2. 0,75

3. 3,4

4. 4,3

5. none of these answers

Circle your answer.

---

4. The fraction form of 0,125 is

1.  $\frac{12}{5}$

2.  $\frac{125}{100}$

3.  $\frac{125}{1000}$

4.  $\frac{5}{12}$

5. none of these answers

Circle your answer.

---

5. 756,382 can also be written as:

1.  $700 + 50 + 6 + \frac{3}{10} + \frac{80}{100} + \frac{2}{1000}$

2.  $700 + 50 + 6 + 3 + 8 + 2$

3.  $700 + 50 + 6 + \frac{3}{10} + \frac{8}{100} + \frac{2}{1000}$

4.  $700 + 50 + 6 + \frac{3}{100} + \frac{8}{1000} + \frac{2}{1000}$

5. none of these answers

Circle your answer.

---

6. Subtract  $2,261 - 0,753$ . Circle your answer.

1. 3,014

2. 1,512

3. 1,508

4. 1,518

5. none of these answers

---

7. Add  $5,43 + 6,043$ . Circle your answer.

1. 11,86

2. 11,47

3. 11,473

4. 11,086

5. none of these answers

---

---

8. Which list shows numbers from smallest to largest. Circle your answer.

1.  $0,25; \frac{1}{10}; \frac{1}{3}; 0,5$

2.  $\frac{1}{10}; \frac{1}{3}; 0,5; 0,25$

3.  $0,25; \frac{1}{3}; \frac{1}{10}; 0,5$

4.  $\frac{1}{10}; 0,25; \frac{1}{3}; 0,5$

5. none of these answers

---

9. In a maths test, Jabu scored  $\frac{34}{60}$  and Nomsa scored  $\frac{64}{80}$ . By converting their marks to percentages, decide which students' mark was higher. Circle your answer.

1. Jabu

2. Nomsa

3. It is not possible to work this out from the information given.

---

10. At a clothing sale, clothes are marked "25% off". What will you pay for a jacket on the sale that was originally priced at R240? Circle your answer.

1. R60

2. R180

3. R300

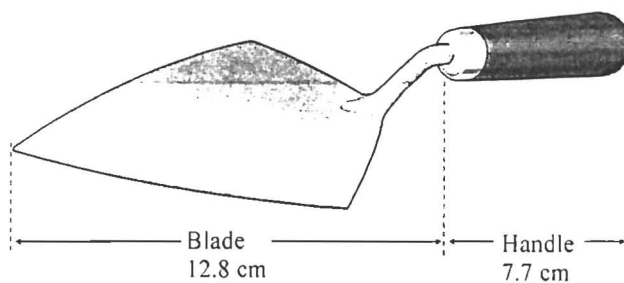
4. R215

5. None of these answers

---

---

11. What is the total length of this trowel? Circle your answer.



1. 19,5 cm
2. 5,1 cm
3. 20,5 cm
4. 205 cm
5. none of these answers

---

12. 638 cm can be written as

1. 63,8 metres
2. 6,38 metres
3. 0,638 metres
4. 6380 metres
5. none of these answers

Circle your answer.

---

13. 5,75 metres can be written as

1. 575 cm
2. 57,5 cm
3. 0,575 cm
4. 5750 cm
5. none of these answers

Circle your answer.

---

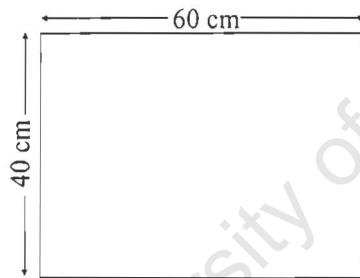
---

14. Which standard unit of measurement (that is, kilometres, metres, centimetres or millimetres) would be most suitable for measuring:

- A. The length of the fence around your school?  
Answer:.....
- B. The thickness of a 50 cent coin? Answer: .....
- C. The length of a man's belt? Answer: .....
- D. The distance between Cape Town and Johannesburg?  
Answer: .....

---

15. A rectangular piece of white paper is shown below.

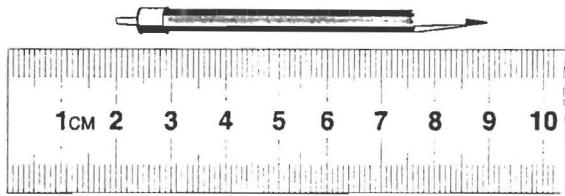


What is the perimeter and area of the paper? Circle your answer.

1. 100 cm, 100 cm<sup>2</sup>
  2. 100 cm, 2400 cm<sup>2</sup>
  3. 200 cm, 240 cm<sup>2</sup>
  4. 200 cm, 2400 cm<sup>2</sup>
  5. none of these answers
-

---

16. What is the length of this pencil?



Circle your answer.

1. 8cm
  2. 8,5 cm
  3. 9,0 cm
  4. 7 cm
  5. none of these answers
- 

17. Shenaaz has a room which is 6,34 metres long and 3,6 metres wide. She is given a carpet which is 5,3 metres long and 3,2 metres wide. **Estimate** which has the bigger area, the floor of the room or the carpet? Circle your answer.

1. the floor of the room
  2. the carpet
  3. impossible to tell because there isn't enough information given.
- 

18. (a) If 30 % of the 700 babies in Pedi village are breast fed, how many babies is this? Write down your answer:

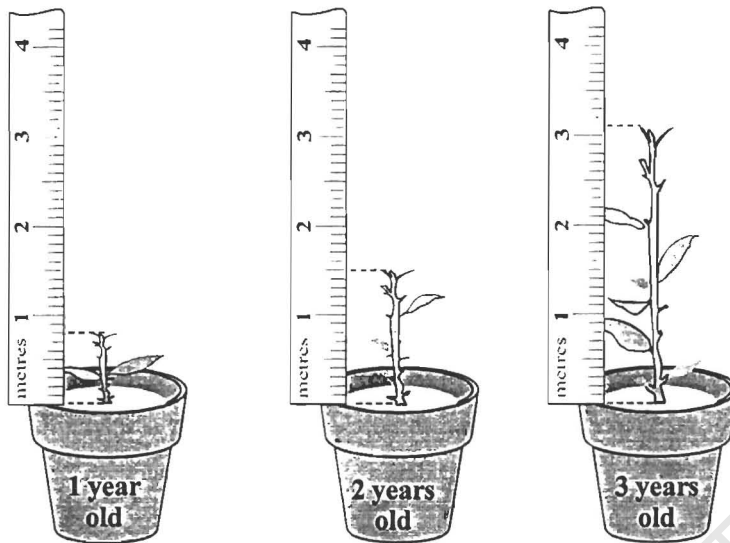
.....

(b) If 600 of the 800 babies in the town of Qunu are breast fed, what percentage of the babies is this? Write down your answer:

.....

---

19. These pictures show the same plant at different times.



a) How tall was the plant when it was one year old?

Answer: .....

b) How tall was it when it was 2 years old?

Answer: .....

c) How much did it grow between 1 year old and 3 years old?

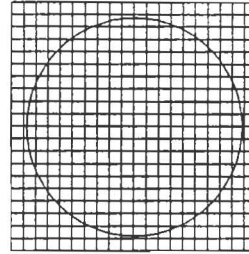
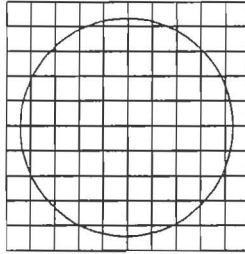
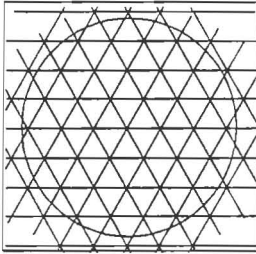
Answer: .....

The same circle is placed on each of the three grids, as shown below.

(1) Triangular grid

(2) Square grid

(3) Square grid



\* Work out the area of each circle, as best as you can, using each of the grid units given.

Answer:

- 26. Area of circle in drawing (1)..... Triangular grid units
- 27. Area of circle in drawing (2) ..... big square grid units
- 28. Area of circle in drawing (3) ..... Small square grid units

\* The circles on each grid are the same. Why do your answers differ?

29. The answers are different because .....

.....

\* Which grid do you think is the best to work from, if you want the circle area to be as accurate as possible? Why?

30. Grid number ..... is the best because .....

.....

.....

31. How would you improve on the grid you chose?

I would improve on my grid by .....

.....

.....

.....

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## APPENDIX B

**CORRECT ANSWER CODES FOR ALL TEST ITEMS AND THE CODING  
SCHEME FOR THE FREE-RESPONSE ITEMS**

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*Correct answer codes for all test items*

<b>Item</b>	<b>Correct answer code</b>
1	2
2	1
3	2
4	3
5	3
6	3
7	3
8	4
9	2
10	2
11	3
12	2
13	1
14	2
15	4
16	3
17	1
18	4
19	4
20	1
21	1
22	1
23	1
24	1
25	1
26	1
27	1
28	1
29	1
30	1
31	1

## Coding scheme for free-response items

Item	Possible answer options	Possible answer codes
21	210	1
	2100	2
	Any other incorrect response	3
	670	4
	No response	6
22	75%	1
	Any other incorrect response	3
	200%	4
	No response	6
23	0.8	1
	8	2
	Any other incorrect response	3
	No response	6
	Inappropriate response	7
24	1.5	1
	15	2
	Any other incorrect response	3
	No response	6
	Inappropriate response	7
25	2.3	1
	3.1	2
	Any other incorrect response	3
	31	4
	No response	6
	23	5
26	50-66	1
	Counted blocks but gave incorrect response	3
	No response	6
	Inappropriate response	7
27	50-60	1
	Any other incorrect response	3
	No response	6
	Inappropriate response	7
28	220-245	1
	Any other incorrect response	3
	No response	6
	Inappropriate response	7
29	Grids no the same	1
	Incorrect response	3
	Nonmathematical	2
	Inappropriate or very difficult to understand	7
	Incomplete-have made an attempt but no full answer e.g. they are not the same	8
30	Grid 3	1
	Nonmathematical response	2
	Inappropriate response	7
31	Mathematical response	1
	Nonmathematical response	2
	Inappropriate response	7

## APPENDIX C

### ANALYSIS OF LEARNERS' WRITTEN VERBAL RESPONSES TO ITEMS 29, 30 AND 31

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**Direct quotations of pupils' responses to items 29, 30 & 31 obtained from pupils' scripts**

*Table 5:7 Learners' incorrect responses to item 29*

<b>Item Number:</b>	<b>Learners' direct responses for sub-category 2(1)</b>	<b>Learner's Code Number:</b>
29	I do not understand this or else I forgot it They use something	C14P44 C04P14
	<b>Learners' direct responses for sub-category 2(2)</b>	
	Is different and is the same thing	C01P06
	<b>Learners' direct responses for sub-category 7(1)</b>	
	The shapes are not the same There are not the same fraction Other one is small	C01P40 C01P38 C04P03
	<b>Learners' direct responses for sub-category 7(2)</b>	
	It have no grid the same sheidet any one smaller I can't count the boxes correctly because I don't understand this Surface is circle It is small and bigger Bigest and smallest grid The square grid is fun or small square The is a trangular and two square The triangular is the serme but not equal Different answers is like not some answers	C04P02 C04P36 C05P25 C12P09 C12P14 C13P06 C13P16 C13P09 C14P44

Table 5:8 Learners' incorrect responses to item 30

Item Number:	Learners' direct responses for sub-category 2(1)	Learner's code Number:
30	It makes you understand	C12P41
	<b>Learners' responses for sub-category 2(2)</b>	
	of its beatiful 26 it my best it we middle is the best It is very beautifull and draw very nice to the other grids It is cool and it is the best I did understand more than the other ones It is very expensive and I love I like to no about grids Is the best of my work I loved I love the number The triangular grid it look like the dice	C02P26 C12P28 C12P52 C06P37 C05P25 C01P34 C09P13 C10P18 C11P03 C12P78 C14P17 C11P19
	<b>Learners' responses for option for sub-category 7(1)</b>	
	Its square a square has four sides and triangular has three sides Squares all sides are equal because when you count square you get 58	C02P27  C04P08 C11P19
	<b>Learners' responses for sub-category 7(2)</b>	
	It is not big and it is not small It can help you when you are counting down 312 u square grid number 2 akaxinena ngo nje ngo square grid number 3 the area of the square grid is opening they I count the area	C03P24 C04P17 C04P11 C05P13  C14P41 C14P17

Table 5.9 Learners' incorrect responses to item 31

Item Number:	Learners' responses for sub-category 2(1)	Learner's code Number:
	Working hard By calculating fast By using ruler and pencil	C02P39 C05P29 C12P53
	<p style="text-align: center;"><b>Learners' responses for sub-category 2(2)</b></p> By knowing it Score high points with it Making it so beautiful like putting colour on it Listening carefully and nice by listening to what the teacher say and learn and one day a can teach some people Looking it so clearly and fockers to eat and consantrait and be sure that I write very good and carefully Working there and have a company of my own there Studing and taught Give you a right answer Working it out every time because fun to work Understanding and simple like abc Use the common sense to improve the grid Colouring it making bigger boxes and trying to ask examples about it is done I don't understand it Drawing a small one beautiful so it can like it Showing that no one tatch it or make it dirty and be in a safe place	C01P36 C03P17 C06P40 C04P26 C04P29 C06P20 C06P33 C07P37 C11P10 C01P17 C02P30 C04P36 C02P11 C11P24
	<b>Learners' responses for sub-category 7(1)</b>	
	By counting the blocks and by looking the area The grid is small Calculating the area and the perimeter Drawing the grid again	C03P20 C04P03 C03P31 C03P23
	<b>Learners' responses for sub-category 7(2)</b>	

By calculating my grid	C01P34
Counting them	C02P44
Calculate the number then you will be improved	C02P04
Working out my grind	C02P37
Calculate it	C03P25
Using formular of AREA L+B wich gives wright answer	C06P41
Drawing it equally	C07P26
Calculate my grid	C09P11
Triangular grid is the best one	C01P07
Looking the squares	C03P28
Triangular grid units is the best number	C11P05
Liking it I like it shape	C11P19
Measurements	C07P18
Triangular grid small square grid biggest	C04P11
Counting the perimetres	C03P19
Units and length	C04P04
Using the squares and my brain	C04P27

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## APPENDIX D

### BAR CHARTS FOR LEARNERS' RESPONSES FOR THE MCQ

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Figure 7. Bar Charts of Question Choices by Question for Post-test (Item 1-5)

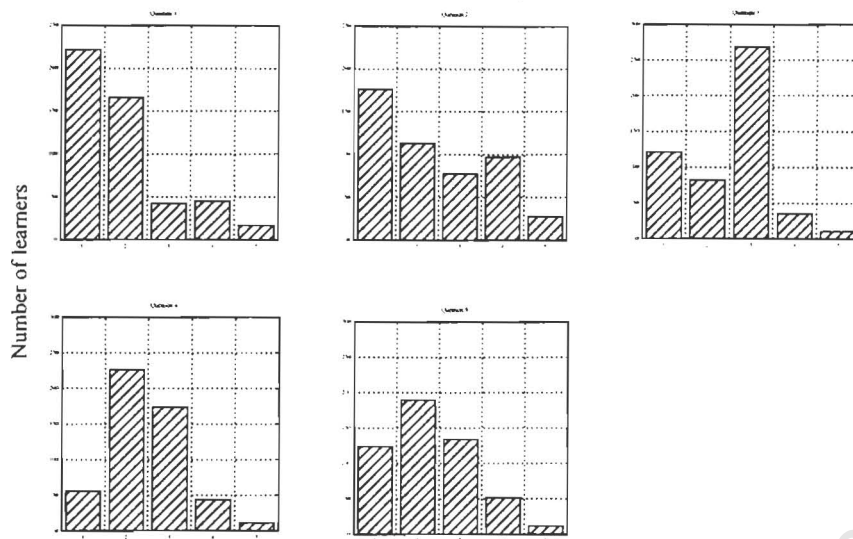


Figure 8. Bar Charts of Question Choices by Question for Post-test (Item 6-10)

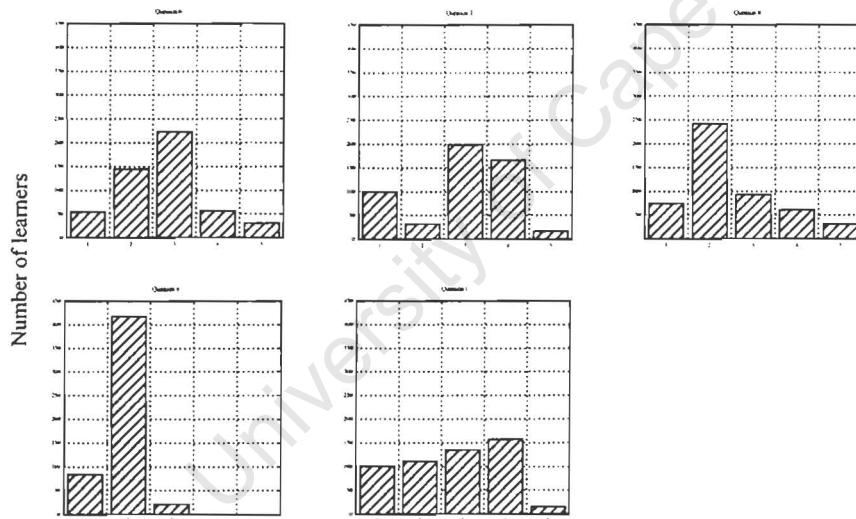


Figure 9. Bar Charts of Question Choices by Question for Post-test (Item 11-15)

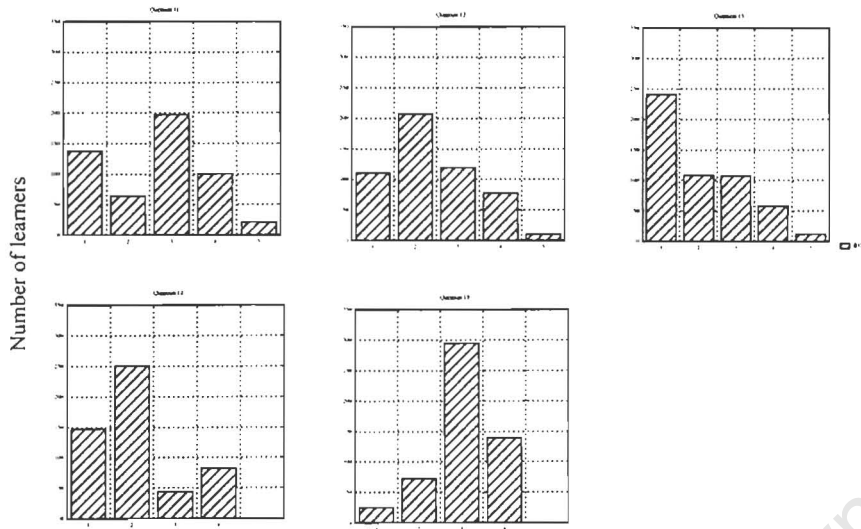


Figure 10. Bar Charts of Question Choices by Question for Post-test (Item 16-20)

