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University of Cape Town

Frontispiece of Rythmomachia by Gustavus Selenus, 1616

University of Cape Town

Department of

Cultural History of Western Europe

RHYTHMOMACHIA

A PROPAEDEUTIC GAME OF THE MIDDLE AGES

Submitted in Fulfilment of the Requirements

for the Degree of Doctor of Philosophy

Margareta Emma Coughtrie

1984

"Omnia in mensura et numero
et pondere constitisti!"

Sapient. XI., 21.
Apocryphal

To George

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CONTENTS

ABSTRACT

Rhythmomachia, a Propaedeutic Game of the Middle Ages.

Summary of a thesis submitted for the degree of Ph.D by Margareta Emma Coughtrie, University of Cape Town, 1984.

This work is an investigation of Rhythmomachia, a medieval number game, and its connection with the validity of two presumptions:

a) that games are a reflection of their culture;
b) that the play-element is particularly strong at the threshold of cultural development.

A brief explanation of the game itself is followed by a discussion of the name.

The connection between chess and Rhythmomachia is sketched and the background of learning in the Middle Ages against which Rhythmomachia found a meaningful setting, is explored.

Medieval manuscripts of Rhythmomachia and their possible authors are discussed, as well as evidence for the existence of an inventor of the game.

Reasons for the re-emergence of Rhythmomachia during the Renaissance and the subsequent disappearance of the game - once so highly regarded by leading scholars

of their time - are suggested. Some of the Renaissance writings on Rhythmomachia are examined.

Copies of some of the earliest manuscripts of Rhythmomachia, otherwise not easily accessible, are placed in the appendix.

INTRODUCTION

Just before 1000 AD, somewhere between Catalonia and Rheims - probably in a Benedictine monastery or a cathedral school - an intricate game of numbers was invented. At that time chess had already become popular with the aristocracy at courts and in religious institutions because it was a war-game reflecting closely the feudal culture of the time. Rhythmomachia, the new number game, was also a war-game, a battle between numbers, but it also reflected scholarly concerns with the liberal arts in general and Boethius' mathematical and musical proportions in particular.

Rhythmomachia was played in Europe for about 600 years. Manuscripts on the rules of the game are found in many important libraries in Europe, sometimes as an appendix to a manuscript on chess, more often in codices on mathematics, computistics and astronomy. The earliest manuscripts are invariably in Latin, fragmentary, undated and unsigned; later ones are by some of the most famous mathematicians of their time; the latest printed versions are in Latin, Italian, French and German and date from the sixteenth or the early seventeenth century. Until at least 1556 equipment for the game was sold in the bookshop of "Jean le Gentil" [1] in Paris.

As far as I am aware, no work has yet been done to place Rhythmomachia against the background of feudal culture and to define its part in the struggle of an emerging Europe in which people used play not only to pass the time, but also to get the measure of their position, to assess the future and the values by which the world proceeds. Europe was beginning to ask itself seriously: what is the world made of and how was it created?

Once present-day medievalists become aware of the importance of Rhythmomachia,more information about it may come to light from the Latin manuscripts as yet unedited and untranslated. Such manuscripts await attention in a number of libraries in Europe which have become the custodians of many thousands of volumes from secularized or dissolved convents, monasteries and abbeys.

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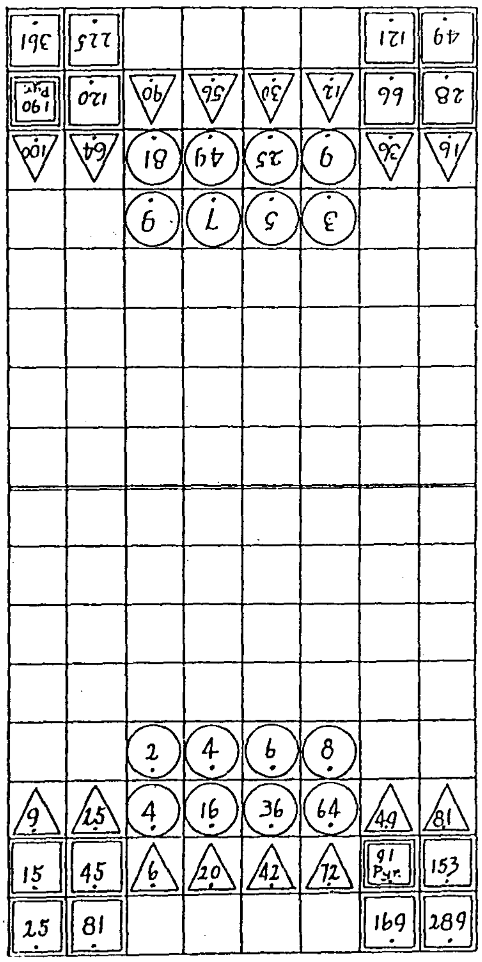
1 On the title page of Claude de Boissière's Latin Rythmomachia, printed by Cavellat in Paris in 1556, we find this advertisement: "Abacus et calculi vaeneunt in Palatio, apud Joannem Gentil".

1

THE GAME OF RHYTHMOMACHIA

AND ITS RULES

Rhythmomachia is played on a double chess-board, 8 by 16 squares. The board can be checkered or uncheckered.



The Setting of the Pieces

Each player has 24 pieces: eight circles, eight triangles, seven squares and one pyramid. Each piece has a number engraved on it and can have a dot placed below the number to avoid confusion between 6 and 9, 81 and 18, etc. One player has the pieces numbered according to proportions arrived at from even numbers; the other player has those arrived at from odd numbers. The base number 1 is omitted.

There are various ways of describing the process involved in determining the number inscribed on the counter. The following is one example (see illustrated layout):

The first row of circles on each of the opposing sides is made up, respectively, of odd and even numbers, less than 10. The second row is obtained by squaring the numbers on the first row of circles. The sum of the two circles in each column gives the corresponding triangle. The other four triangles (the second row) are formed from the first row of triangles by means of a relation known as "superparticularis"; that is, a number in the second row is found by joining the corresponding number in the first row to an aliquot part of it determined by the number in the first circle at the top; e.g. 81 is obtained from 72, by adding $\frac{1}{8}$ of 72, 8 being the number in the circle at the top of that column. Similarly, 42 = 36 + $\frac{1}{6}$ of 36, 20 = 16 + $\frac{1}{4}$ of 16, and 6 = 4 + $\frac{1}{2}$ of 4. The ratio of the aliquot parts added to the numbers at the top are therefore $\frac{3}{4}$ (sesquialtera), $\frac{5}{4}$ (sesquiquarta), etc. The first row of squares is formed by adding the respective triangles

(9 + 6 = 15, 25 + 20 = 45, etc.), but one square on each side, 91 for Even and 190 for Odd is replaced by a pyramid. These pyramids are formed by superimposing squares, thin layers or laminae and are used to call attention to the peculiar construction of the numbers which they represent. Thus $91 = 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1$, and $190 = 8^2 + 7^2 + 6^2 + 5^2 + 4^2$. Since the former contains the squares of all numbers from 6 to 1, it is called a perfect pyramid[1], but since the latter lacks the squares of 3, 2, and 1, it is known as "tricurta", or thrice curtailed.

The lower row of squares is obtained from the upper one by a formula similar to the one used to obtain the lower row of triangles. In the case of squares, if we call the number in the circle at the top n, and the number in the upper square s, then the number in the lower square is $\frac{2n+1}{n+1}$ s. Thus $\frac{2.2+1}{2+1}$ $15 = \frac{5}{3} \cdot 15 = 25$; $\frac{2.4+1}{4+1} \cdot$ $45 + \frac{9}{5} \cdot 45 = 81$, and so on.

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[1] A. Machabey in an article entitled "Notions Scientifiques Disseminées Dans Les Textes Musicologiques Du Moyen Age", in Musica Disciplina V. 17., 1963, p. 13, discusses the preoccupation with the construction of the "perfect" pyramid shown by the musicologist-mathematician pseudo-Odo of Cluny (tenth century). His "perfect" pyramid consists of the numbers 36 - 25 - 16 - 9 - 4 - 1, adding up to 91. Although Machabey does not mention Rhythmomachia, this is of course the pyramid used in Rhythmomachia, and his observation confirms that this concept was known in the Middle Ages.

The setting of the pieces can be left to individual preference but the layout shown in the illustration is considered the most satisfactory.[2]

In the fifteenth and sixteenth centuries it was usual to mark the game-pieces on both sides, the reverse side being given the colour of the opponent but retaining the same number - for reasons explained later.

Every piece can move forward, backward or sideways and in certain cases diagonally. The players move the pieces in turn. A circle moves one space, a triangle two, a square three and the pyramid four spaces. Jumping over occupied squares is not permitted. The objective in the game is to capture the opponent's pieces by different methods, as in a real war. There are four kinds of capturing moves: encounter, assault, ambush and siege.

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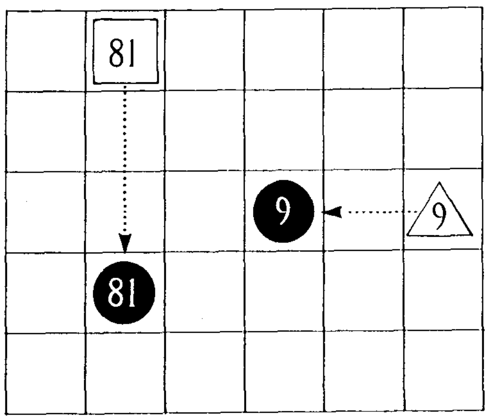
2 "...the position is one of an advancing army, as Polybius has handed down to us in his writings, that the end of the column ought to be less compact, so that there are not as many soldiers stationed at the back as there are at the front; but the space occupied by both should be the same. For if the soldiers in front are weak and are in trouble, they would be able to retreat to the rear part of the army, and from there they would be allowed an opportunity of fighting again. So too, the cavalry with light equipment are like the wings of the column, and this corresponds somewhat to the arrangement of this game..."

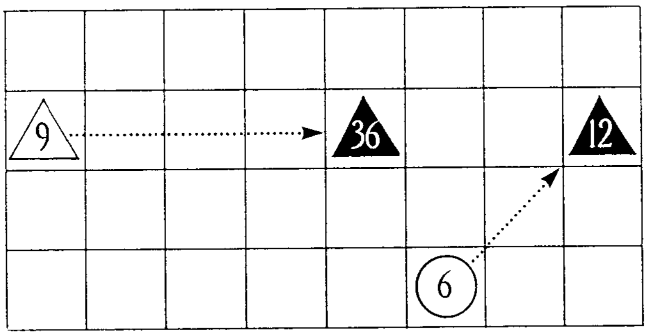
This is a view on strategy of the sixteenth century expressed by Claude de Boissière, in his Rythmomachia, p. 39, translated by J.F. Richards.

In the move known as encounter, if one number finds another of the same value and is able to reach the space it occupies in one move, it can capture that piece and take it off the board, as shown. Seven encounters are possible because there are seven pieces of the same value on each side: 9, 16, 36, 49, 64, 81.



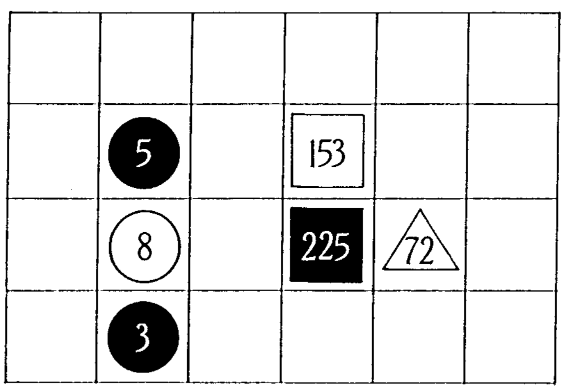
Encounter

A piece can be captured by assault if a smaller number, multiplied by the number of spaces between it and the larger one, (also counting the space it occupies), equals the larger one, e.g. Odd's circle five may take Even's square 45 if there are nine spaces between them. No distinction is made between circles, squares, and triangles and the assault can be made in any direction, even diagonal. Other examples are:



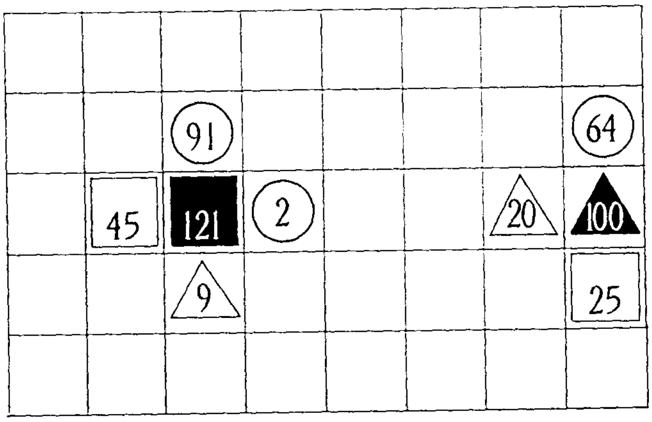
Assault

An ambush is made by two small numbers on a large one if the sum of the smaller numbers equals the larger and if they can move into the spaces directly beside it. For example to capture the triangle 25, the circle nine and triangle 16 must move on either side of it. Further examples to illustrate an ambush are:



Ambush

A siege captures a number by surrounding it on all sides. The value of the pieces in this case makes no difference.



Siege

The pyramid can usually only be taken by siege. As it is made up of several layers of numbers it is considered to be in danger when one of these numbered laminae is attacked by one of the other methods, in which case a ransom is paid.

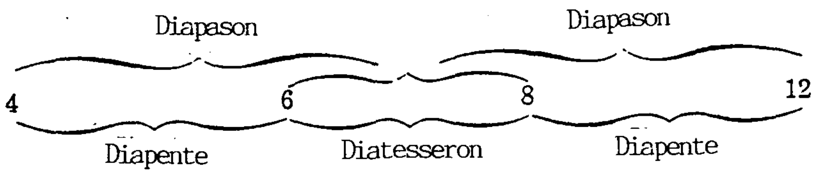
The player makes a capture only if he wishes to, but he may refuse the bait.

Rhythmomachia can be played at various levels of skill. Players come to a mutual agreement about the type of victory for which to aim. There are two major types of victories, the common and the proper. A common victory can be of five kinds:

1. depending on the number of captured pieces;
2. depending on the value of the captured pieces;
3. depending not only on the value of the pieces but on the digits inscribed on them;
4. depending on both the number of the pieces and their value;
5. depending on the number of pieces, their value, and the digits inscribed on them.

The expert player of Rhythmomachia would find the proper victories an appropriate challenge. Here a new rule can come into effect: a piece captured by siege or ambush may be taken on as an auxiliary (by reversing it). There are three kinds of proper victories, known as "Magna", "Major" and "Praestantissima", also called "Excellentissima". The proper victories involve placing your pieces behind the enemy lines so that they form a progression. Three pieces, either your own or your own plus prisoners, must be in arithmetic, geometric or harmonic progression. Next in difficulty is to get four pieces to penetrate and form any two of the three possible progressions. To achieve the most difficult victory, the "Victoria Excellentissima", the player must get four pieces to form all three progressions at once.

There are only six possible solutions to this problem, namely, (2, 3, 4, 6), (4, 6, 8, 12), (7, 8, 9, 12), (4, 6, 9, 12), (3, 5, 15, 25), (12, 15, 16, 20). Of these only one (4, 6, 9, 12) contains four terms in geometric progression, the others containing proportions. For example, in the set (4, 6, 8, 12) the comparison of 12 and 8, or of 6 and 4, is a sesquialtera ($\frac{3}{2}$), corresponding to the fifth in music; 8 and 4, or 12 and 6, have the ratio 2:1, that of the octave or diapason. The ratio 8:6 gives the diatesseron, while 12:4 gives the interval, a twelfth, including both diapason and diapente. The ratio 8 : (6-4) gives the interval of two octaves, or a fifteenth. Expressed in graphic form it would look like this:



The rules of Rhythmomachia were not standardized and through the centuries many variations of the game were played. There may have been some relationship between these variations and the shifts and turns in medieval - and later in Renaissance speculative thinking on music, philosophy, mathematics, science, and metaphysics, and their interaction.

According to Selenus the following additional rules for playing Rhythmomachia were current:[3]

1. An opposing piece can only be taken in the course of the execution of a move, remembering that a circle moves one space, a triangle two spaces and a square three spaces. There are however four exceptions to this rule.
2. If player A has overlooked a possible capture and makes some other move, then player B may make the capture as well as a move.
3. If a move is made by player A which permits immediate capture of the piece by player B, then the capture as well as a move is granted to player B.
4. If player A has overlooked a chance to capture and this has also escaped the notice of player B, then player A on his next turn may make the capture and a move.
5. If two pieces are drawn up in such a way that one can take the other by multiplication or division and player A neglects to carry out the capture but makes some other move, then player B may,on his

[3] Selected and freely translated from Gustavus Selenus' German Rythmomachia, Leipzig, Gros, 1616, p. 475-478 and 488, 489 , following the original numbering.

next turn,capture the piece of his opponent as well as making another move.

6. Players may not jump over their own or the opponent's pieces in order to make a move or a capture.

7. The pyramid can move, capture and be captured according to the different numbers which make up this game-piece. If a segment of the pyramid is captured, this particular number may be replaced either from the bulk of the pieces of the player who is about to lose the segment of the pyramid or from pieces already captured from his opponent. If no suitable piece is available, some other number may be selected by the player who is making the capture.

8. Both players may reverse captured pieces, incorporate them or store them at will. In this way captured pieces (retaining the same number) may be recaptured by their original owner and put to use again by reversing them back to their original colour.

9. If a pyramid has lost one of its segments, e.g. the square, triangle or circle, it may no longer move according to their steps.

10. If a pyramid is under siege, but not otherwise, it may a) rescue itself by moving as a knight in chess, or b) obtain release if one of its fellow

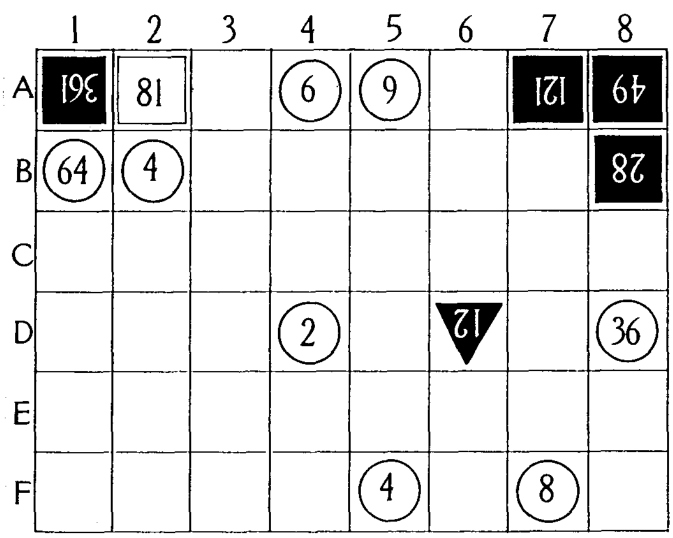
pieces can capture one of the besiegers. If the pyramid can neither rescue itself nor be rescued, it is lost to the opponent who may reverse it and use it as one of his own pieces or hold it in reserve to be used as occasion demands.

Before closing the chapter on the rules for playing Rhythmomachia, the following additional points are enumerated.

1. The most difficult victory, the "Victoria Excellentissima" can only be won after the "Victoria Magna" and "Victoria Major" have already been won.
2. Victory can be achieved by strictly adhering to the rules for the moves of the pieces.
3. Victory must be achieved behind the enemy lines, that is at the end of the board; the player's own as well as captured pieces may be used for the purpose.
4. The pieces used in the highest victory must be placed in close proximity to one another, that is in adjoining squares.
5. Before positioning the last piece for a victory, this intention must be announced to the opponent on the second last move, whether he can prevent it or not.

6. Once the last victory has been announced no more pieces may be removed from the series which accomplishes that victory.
7. Before he announces the last and greatest victory the player has to arrange many of his other pieces in the court of his opponent for his own protection and support, in order to be able to execute the victory without suffering damage or hindrance.

Here is an example of a possible stage of the game that leads to a "Victoria Excellentissima" by Even.



It is Even's turn. He moves 6 (A4) to A3, threatening 12 (D6), which could only move to B6, where it would be captured by 4 from B2, or to F6 where it would be taken by ambush.

Assume that Odd leaves 12 (D6) to be captured by Even's 6. Even places 12 as his own piece (turned) over) in position A6. Odd's three pieces at the corner A8 cannot escape or threaten Even, who has just to move 4 from B2 to A3 (in two moves) and 6 to A4 to complete this victory.

To arrive at this position is far from easy. The complexity of the game will only be appreciated once play is in progress and a major victory is attempted. Circles are the pieces with the greatest freedom of movement and will play an important part in any strategy. Triangles and squares are more restricted and can, for instance, not easily reach the last field of the opponent's side to set up a progression in a sheltered position. The "even" side lacks the number 12, an almost indispensable number for achieving the "Victoria Excellentissima". The capture of certain pieces will have to be planned and executed before a proper victory can be built up. Pyramids are awkward to attack and to dismantle for partial capture. However one must bear in mind that players used to agree on specific rules to suit individual taste before starting to play, and kept by their side lists of progressions (see illustration)[4] for reference and planning of strategy.

1

Aythmetica, Geometrica, Muſica ſimul, D C B A	Aythmetica ſola	Muſica ſola	Geometricaſola
2 3 4 6	2 4 6	2 3 6	2 3 4 6
4 6 8 12	4 8 12	4 6 12	4 6 8 12
6 8 9 12	6 9 12	6 8 12	6 8 9 12
4 6 9 12	6 9 12	4 6 12	4 6 9
3. 5. 15. 25	5 15 25	3 5 15	3 5 15 25
12 15 16 20	12 16 20	12 15 20	12 15 16 20

One of Boissière's Lists of Progressions

[4]Boissière and Selenus use the term "geometric progression" in two different senses. Three numbers, A, B, and C, are in geometric progression if B:A = C:B, while four numbers, a, b, c, d, are in geometric progression if c:a = d:b. In the "Magna" and "Major" victories it is used in the first sense, while in the "Victoria Excellentissima", except in the case 4, 6, 9, 12, it is used in the second (weaker) sense.

2

ETYMOLOGY

"....they say there is divinity
in odd numbers, either in nativity,
chance or death."

Shakespeare, Merry Wives of Windsor,
V. i (3)

Very few modern encyclopaedias and dictonaries list Rhythmomachia, and then not always in the most accurate of ways. The Compact edition of the Oxford English Dictionary deals with Rhythmomachia under the heading of "philosopher", because in England the game was known as the "philosopher's game".

The entry reads: "Ludus philosophorum Arithmomachia, Rythmomachia, an intricate game, played with men of three different forms, round, triangular and square, each marked with a number, on a board resembling two chess-boards united, called also the philosopher's table".[1]

Already in the early days of its existence it was simply referred to as Rhythmomachia, a "pugna numerorum",[2] a battle of numbers. One might expect it to have been called "Arithmomachia" as in the Oxford English Dictionary, but this is not the case. Not from ignorance of Greek, as the nineteenth-century mathematician R. Peiper[3] suggests, but because they wanted to emphasize

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[1] Compact ed. of the English Oxford Dictionary, Oxford, Clarendon Press, 1971, vol. II, p. 2154, 5b.

[2] Codex 2503 (Odo), Vienna.

[3] R. Peiper, "Bemerkungen zur Rythmimachie", Zeitschrift für Mathematik und Physik, Hist. Lit. Abteilung, XXV, (1880), p. 203.

the importance of the numbers being in proportion to one another, and having, or being, a "rhythm". The numbers are fighting each other - odds versus evens - in four pairs of "multiplices", four pairs of "superparticulars" and four pairs of "superpartientes" for the highest victory, the "maxima et perfecta harmonia".[7]

The Greek "rhythmos" itself is defined as "measure", rhythm,[8] beat, proportion of parts, progression, shape and form, hence "Rhythmomachia". "Machia" means fight or battle and contention for the mastery in games. There are several precedents for words formed with "machia" which are connected to the emergence of allegory in post-classical writings.

An early example is the Thebaid of Statius which became a favourite theme in the Middle Ages. Clemens Aurelius Prudentius, 348-410, poet, soldier and advocate, wrote a Psychomachia which describes set combats between vices and virtues; the best is possibly the seventh combat between Luxuria and Sobrietas.

Thomas Blount lists "Psychomachia" in his Glossographia, 1656, and calls it a war between the soul

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[7] These terms will be explained in chapter 3.

[8] E. Jammer "Gregorianischer Rhythmus, was ist das?", Archiv für Musikwissenschaft, 31, 1974, p. 290-311.

and the body.[9] Another source of allegory, this time for the seven liberal arts, is the treatise by Martianus Capella, c 410- c 450, De Nuptiis Philologiae et Mercurii. This is an allegorical marriage between eloquence (the bridegroom Mercury) and learning (the bride Philologiae). During the Middle Ages it was used by scholars as an encyclopaedia for all sorts of information.

These Roman writers must have been familiar with the Greek word "naumachia" denoting a naval battle.[10] It was used by the Romans for a mock sea-fight staged as a public spectacle. The first one on record, given by Julius Caesar in 46 BC on an artificial lake constructed for the purpose on Campus Martius, represented an engagement between Tyrian and Egyptian fleets. Gladiators or criminals dressed in special clothes fought until one side was eliminated.

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[9] T. Blount, Glossographia 1656, Menston, The Scolar (sic) Press, reprint 1969. This is the "bellum intestinum", the "holy war" which C.S. Lewis describes as the root of all allegory, in An Allegory of Love, Oxford University Press, 1959, p. 70. Quintilian, c 37-100, stated that the word allegory derives from etymology: it presents one thing in words and another in meaning. He places simile, riddle and metaphor under allegory. Quintilian, Institutio Oratoria VIII, vi, 44659, as quoted by J. MacQueen, Allegory, London, Methuen, 1970, p. 47.

[10] Encyclopaedia Britannica, 9th ed., Edinburgh, A. & C. Black, 1884, vol. XVII, p. 248.

The Emperors were fond of staging naumachias: Augustus chose a battle between Athenians and Persians; Titus ordered a sea-fight between Corinth and Corcyra; Claudius made a great show on Lake Fucinus, and Nero in the amphitheatre in Rome.[11]

In 1041 Anselm of Besate, chaplain to Henry III, wrote a "Rhetorimachia" in which the author defends his kind of rhetoric against an imaginary opponent.[12] "Tauromachia" is the archaic word for bullfight, still occasionally used.

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[11] "Another great showpiece along the Tiber, the naumachia, built for mock sea-battles, extended north of the mausoleum of Hadrian; but all trace of it seems to be lost." R. Krautheimer, Rome, a profile of a city 312-1308, New Jersey, Princeton Univ. Press, 1980, p. 13. At Taormina in Sicily are well-preserved remains of a Roman naumachia. Because of its narrow shape one may suppose that swimming races or other water-fights might also have been staged there, c fourth century AD. Pauly's Real-Encyclopaedie der klassischen Altertums-wissenschaften, Stuttgart, Metzler, 1927, vol. XIII, col. 1988, mentions a game called a naumachia, but nothing else is known of it. In 1983 in Palermo I attended a performance of excerpts from Orlando Furioso and the Song of Roland by life-size puppets. This was announced as a "pupimachia". Because of language difficulties I was unable to discuss the origin and history of the word "pupimachia" with the manager of the theatre. For information about Opera dei Pupi see Encyclopaedia Dello Spettacolo, Roma, Casa editrice la Maschere, 1961, vol. 8, col. 597-599.

[12] E. Dümmler, Anselm der Peripatetiker, Halle, 1872, p. 20 ff.

"Sciomachia" or "sciamachia" was a word already used in ancient Greece. It means, literally, fighting in the shade,[13] as perhaps when training in the shady part of the gymnasium. It can also mean fighting with shadows, as in shadow-boxing or a sham fight for exercise or practice.

The Romans adopted the word and used it also for performances of moving figures, as perhaps in "Ombres Chinoises", an Eastern custom where cardboard or metal figures are held behind a painted transparent screen, lit from behind and manipulated by wires.[14] In the East a mystical relationship was believed to exist between these figures and the dead, the figure being held up by a "rod of life". The European adaptation concentrated more on throwing shadows, later called silhouettes, of high quality. Performances varied from the most refined rituals to vigorous fighting.

Francesco Colonna, a Venetian Dominican monk,wrote

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[13] The Oxford English Dictionary, Oxford, Clarendon Press, 1933, vol. II, p. 2667.

[14] Encyclopaedia Britannica, 9th ed., vol. XV. See also L. Reiniger, Shadow Theatre and Shadow Films, London, Batsford, 1970. Rabelais was present with a friend in Rome at a sciomachia in February 1549, F.R. Hausmann, François Rabelais, Stuttgart, Metzler, 1979, p. 37.

Hypnerotomachia Poliphili in 1463. An English translation was made in 1592 by "R D" entitled The Strife of Love in a Dream.[15]

Poliphilus dreams of an enigmatic battle between learning and the temptations of the world, which one could perhaps call the "illiberal" arts. He also witnesses a live chess game, where nymphs make love, not war, in an inversion of the usual game, to the strings of an orchestra.

"Rhythmomachia", the battle of rhythms, is spelt in various ways such as Rithmimachie, Rythmimachia, Rithmomachy, Ritmachya and even Ryghtmathye. The mathematical historian D.E. Smith [16] adopted the spelling "Rithmomachia", possibly because of its connection with algorithm and arithmetic, but as the game is a battle of numbers in proportion (rhythm) the appropriate spelling would be Rhythmomachia.

The earliest manuscripts on Rhythmomachia are often fragmentary and anonymous, but the term Rhythmomachia usually appears in one spelling or another at the beginning. An eleventh-century copy of some rules of

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[15] New York, Delmar Scholars' Facsimiles & Reprint, 1973.

[16] D.E. Smith, History of Mathematics, New York, Dover Publications, 1923, reprint 1958, p. 198.

the game by the German Benedictine Abbot of Reichenau, Lake Constance, Hermannus Contractus, 1015-1053, has the title "De conflictu rithmimachie", which was perpetuated by copyists in the eleventh, twelfth and thirteenth centuries.

The word "conflictus" was not in common use at all in the first half of the eleventh century, unlike "pugna", the ordinary word for war. As Hermannus, a Latin, * Greek and Arabic scholar, a philosopher, a mathematician, a creative musical theorist, called the game what amounts to "a conflict of a battle of numbers", one may assume that he wanted to emphasize the contest involved. Among Hermannus' poems is one with the title "Conflictus ovis et linis" which rather suggests that he may have reserved the word "conflictus" for a battle of a spiritual nature and that this was understood by some scholars at least. Soon after him Hildebertus Cenomamensis (born 1055) wrote a "liber de querimonia seu conflictus carnis et animae". In MS Monac. 4613 we find a "Carmen de conflictu virtutum et vitiorum". As model for his "De conflictu rithmimachie" Hermannus may have taken the allegorical debate found in Alcuin (?), "Conflictus Veris et Hiemis".[17]

* See Abbas Spanhemsis, De Scriptoribus Eccle.,Fo. LXIIII

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[17] Monumenta Germaniciae Historica, (Poet. Lat. Med.Aev.) ed. E. Dümmler, vol. I, "Alcuin", p. 270. It proved a popular device for moral teaching. See L. Wallach, Alcuin and Charlemagne: Studies in Carolingian History and Literature, New York, Cornell Univ. Press, 1959, ch. 12.

As Hermannus deliberately used the little known "conflictus" in conjunction with "machia", he may have wanted to underscore the special nature of the game by pointing to the philosophical implication, to the significance of "odd" and "even" numbers in proportion, opposing one another. Rhythmomachia was not just a game for practising arithmetic or the new Arabic-Hindu numerals, and the opposing numbers were not chosen at random.

"Odd" and "even" are opposites. Allegory has throughout the ages inspired this concept of opposition. Opposites are an almost universal idea: the Chinese "Yin"and"Yang"; in Western culture Heraclitus' assumption that conflict between opposites is the basis of all existence, as it was finally expounded in Nicolas of Cusa's <u>De Docta Ignorantia</u>, where he explains the phenomenon as "coincidentia oppositorum".[18]

The Middle Ages was steeped in allegory.[19] After

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[18] For example good-evil, sun-moon, round-square, spiritual-physical. The odd-even principle was taken one step further in Nicomachus and became the deity itself: male-female. Hermes Trismegistus sees the Nous as hermaphroditic. C.G. Jung, <u>Collected works</u>, tr. R.F.C. Hull, 2nd ed., London, Routledge and Kegan Paul, 1953, vol. XII "Psychology and Alchemy", p. 330.

[19] Virgil's <u>Aeneid</u> and the fourth <u>Eclogue</u> already herald the allegories of the Middle Ages. These might be

the time of Prudentius allegories usually had a battle or sequence of encounters at the centre of the action. The object of these battles was salvation at the end of a pilgrimage or a quest. Allegory need not be verbal or written. In medieval times it was often visual, e.g. church buildings and decorations presented history in the form of allegory.

Rhythmomachia too can be seen as an allegory. The players decide beforehand on the severity of the pilgrimages or quest to be undertaken and the victory will be appropriate to the rigours of the battles encountered. The greatest victory, the "Victoria Excellentissima" which is the "maxima et perfecta harmonia", following the hardest and most taxing struggle, might be likened to salvation obtained after life's battles have been fought well.

By calling the game "De conflictu rithmimachie" Hermannus Contractus seems to draw special attention to the allegorical nature of the game and to the opportunities it could provide for philosophical speculation.

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classified as generally narrative, situational prophetic, figurative, alphabetical, numerical, satirical, etc. Rhythmomachia would fall under figurative and numerical allegory.

With regard to the interrelationship of concepts used in allegory, the following additional point can be made. During the Middle Ages, especially in monastic circles where a vow of poverty was taken on entering the brotherhood of monks, there must of necessity have been a shortage of materials and equipment for study as well as for relaxation. Even writing material was often in short supply and highly prized for private use. The counting-board ("exaquier", "Schachbrett", "eschequier" or "chekker") did duty for all sorts of operations, e.g. doing accounts, allocation of monies, the gaming-table in general, chess-board, the board for Rhythmomachia, etc.

Edwin Ripin discusses the use of the word chekker for a musical instrument which was in use until at least the fifteenth century.[20] He produces 31 references where chekker seems to be used in context with other musical instruments. He is satisfied that a chekker was in fact a clavichord. However, Ripin's most important discovery for our purpose is a Paris manuscript[21] written by a cleric, Johannes de Gerson, 1363-1429, in which he describes in Latin and French, and depicts in

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[20] E.M. Ripin, "Towards an Identification of the Chekker", Galpin Musical Society Journal 28, 1975, p. 11-25.

[21] Bibliothèque Nationale Paris, Codex lat. 17487 fo. 226-229.

three diagrams, a chess-board on which vices and virtues are engaged in an allegorical battle.[22]

Johannes de Gerson notes that "in the chekker of our hearts the victory is melodious" and shows crude keys along one side of the board "which are struck by the fingers of meditation".[23] No strings are visible, but according to two captions these were left to the imagination.

The reference to numbers in proportion, that is musical harmonies and consonances, capable of being practised on the chekker seems quite obvious, although unstated by Ripin, who may not have been aware of Rhythmomachia and its cultural implications when he researched the identification of the chekker. Further clarification may be obtained if research were undertaken jointly by cultural historians and experts in various other disciplines.

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[22] Another example of a chess-board used as an enigmatic allegory is illustrated in F. Rabelais: Oeuvres Completes, ed. G. Demerson, Paris, Seul, 1973. In this case abusive terms for women are placed on each square of an inverted chess-board (the left-hand top corner commences with a black square instead of a white one). See appendix A.

[23] "En leschequier de nostre cuer / La victoire est melodieuse" (fo. 228v) "Claves interioris scacordi que percusse digitis meditationis reddunt voces cujuslibet affectionis" (fo. 226); see appendix A.

3

RHYTHMOMACHIA AND LEARNING IN THE MIDDLE AGES: BACKGROUND RELEVANT TO THIS STUDY

> "While Councillors of State sit plotting, and playing their high chess-game, whereof the pawns are Men."
>
> Carlyle, Sartor Resartus, I. iii.

The Function of Play in Culture, as Related to Rhythmomachia

The game of Rhythmomachia is based on the number theory of Nicomachus of Gerasa, c 100 AD, as transmitted by Boethius, c 480 - 524. At this stage of research one may suppose that it was a medieval European invention. No trace of it has been found by mathematical historians in either classical or Arabic writings despite various assertions in Renaissance manuscripts and books that the game was invented by Pythagoras, Boethius or the "Chaldeans". Several major factors seem to have been significant for the development of Rhythmomachia:

1. European culture was on the threshold of new development with the revival of learning in the Carolingian period.
2. There had to exist:
   - i) a familiarity with chess,
   - ii) a relationship to the liberal arts,
   - iii) a working knowledge of Gerbert's new abacus and "apice", [1]
   - iv) a knowledge of Boethius' <u>De Arithmetica</u> and <u>De Musica</u>.

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1 Horn buttons engraved with characters from 1 to 9 were called "apice", see F. Cajori, <u>A History of Mathematics</u>, New York, 3. ed., 1980, p. 68.

The importance of play in the development of culture has been extensively treated by such eminent historians as J. Huizinga[2] and Otto Koenig[3], who believe that culture arises spontaneously through playing. We realize that unrelated play as such does not exist, but one can perform certain actions in a playful way or play something. Play for instance satisfies the imitative instinct through which new skills can be learned; play gives the pleasure of achievement which reinforces a personal feeling of value; play helps to discharge suppressed energy and satisfies a need for competitive relaxation.

Since time immemorial the playing of games has served to establish who is superior in skill with words or actions. Games nearly always have a social character and aid communication. They are intended to be fun but have a serious undertone. The rules must be obeyed. In all higher kinds of play the participants enter the imaginary world of super-reality where they can create

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[2] J. Huizinga, Homo Ludens, London, Temple Smith, 1970.

[3] O. Koenig, Kultur und Verhaltungsforschung, Munich, Deutscher Taschenbuch Verlag, 1970.

a treasure in the mind which can be re-created when so desired. In order to free oneself from "real" life for a period of time, one enters a particular place, a "playground" which helps in the creation of a world of imagined reality.

We call "culture" those activities which have originally arisen through play and developed into religion, philosophy and social life. We measure a culture by its spiritual and artistic achievement, by the quality of its play. Games are a reflection of culture. In early times ceremonies, sacrifices, armed combat and riddle contests were some of the game rituals which served to make man feel more secure in a hostile world where it was important to determine the most able leader, to honour the gods and to secure their protection against enemies.

The desire for sacred knowledge and magical power prompts some enquiring minds to overcome the very real fear of knowing, which brings with it responsibility. Barely glimpsed insights into the phenomena of nature were voiced in myths; hymns and incantations in sombre meter served to placate jealous gods.

Trials of strength in the form of armed combat, sometimes accompanied by games of dice, were used in archaic times to determine the will of the gods. Wars between equals had a cultural function: they were bloody games played strictly according to the rules, but against slaves or barbarians the cultural function as well as the rules fell away in a "total war".

The Greeks particularly understood the necessity of living life as a play. Plato saw philosophy as a noble game and raised it to the level of a search for truth.[4]

Feudal society in Europe also developed play-forms. Its hierarchical structure could be more easily controlled by a stylized mode of existence. Elaborate rules and rituals helped to regulate life, which was largely lived in public, and served to identify at a glance who was who by dress, deportment, skill at games or reading.

The Middle Ages believed in the Church, and the Church and its liturgy and music daily enacted the drama of the mass. Morality and miracle plays were devised and produced in cloister and hall.

Knights were trained in battle-schools for the arts of war. These practical "ludi", modelled on literary descriptions and ancient pictorial illustrations on vases, sarcophagi and floor-tiles, later became known as tournaments or as jousts when single combat was

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[4] Huizinga, Homo Ludens, p. 175.

involved. Spectacular ceremonial dominated the courts of chivalry,and from the eleventh century on, when ladies played a larger part in social life, they became associated with the courts of love.

Sex-games were important,as love was not found in marriage, but out of it.[5] Protection of honour was vital for all concerned, so there had to be secrecy and constancy between lovers. Codes of conduct were elaborate and carefully observed: a language of signs, emblems and tokens was used, and colour and position of ribbons, jewelry, etc. had special meaning.

Later, dancing was important in sex-games, as were mummings and disguises. Feats of constancy were expected and rewarded.

Troubadours, trouvères, minnesingers and troupes of "jongleurs" underscored the importance of the great. Pageantry, public executions and the burning of heretics were all calculated to impress and arouse terror.

The Middle Ages was a time of intense feelings; sudden explosions of hatred or belligerence alternating with repentance; outbursts of joy and gaiety giving way to fear of hell and feelings of guilt. Having

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[5] There is a strong connotation of sex in "spiel" (play, games) e.g. "minnespiel", also "Spielkind", a child born out of wedlock. In Sanskrit "the jewel of games" means copulation. Huizinga, Homo Ludens, p. 63.

tasted all that the world had to offer, many important men and women gave away everything and entered a monastery. Emotions were easily and freely expressed. High churchmen were hardly different from their warlord brothers, whereas monks were enjoined to live a life of poverty and chastity. But they still had to wrestle with these volatile emotions, especially if they had not taken up monastic life from choice or vocation but because of parental vows.

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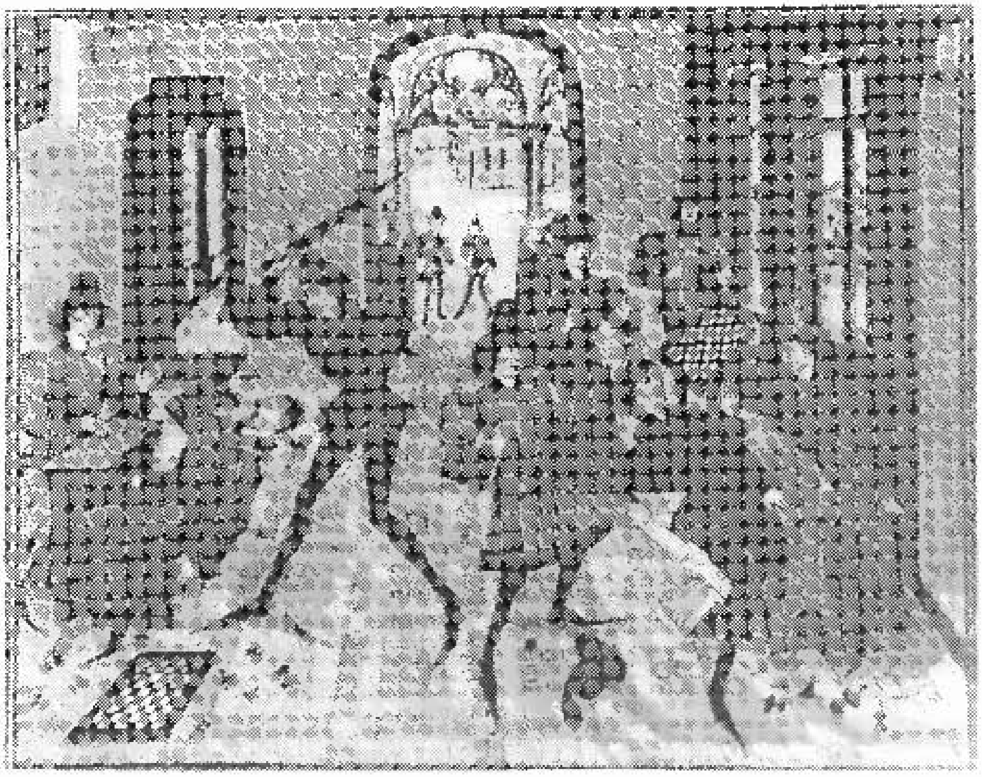

Miniature from a French 15th century manuscript, describing the adventures of Renaud de Montauban.

On the chess-board battles could usually be fought with a degree of safety, although medieval romances recount quite a number of quarrels over the chess-board with fatal results. Not a few noblemen condemned to death are reported to have played chess in prison and insisted on finishing a game before execution; in symbolic revenge before death, as it were, on those with power over life and death.

Medieval men expressed themselves through symbols. Anything found difficult, dangerous or new could be put symbolically. Medieval scholars and scientists ignored nature and hunted for moral and allegorical meanings. They saw reality as immaterial and what their senses told them as only a reflection of reality. They considered the structure of the universe and the nature of matter to be identical. As the greater is somehow always reflected in the lesser, they thought that man, the microcosm, was a close reflection of the universe, the macrocosm. The stars moving regularly in circles could be relied on to control the whole system. Because number symbolism was considered a Greek science which could explain how God created the universe, it affected theology and the liberal arts. Every experience and every object, scripture and nature were examined for hidden meanings and spiritual implications.

Allegory and symbolism was the language used to express the ineffable. In particular, the word of God, the Bible, was subjected to examination. Both Petrarch and Boccaccio still adhered to the idea that symbolical meaning hides in literary guise and that poetry can mask truths too brilliant to look at.

Symbolism had originated in Greece and reached the mind of the Middle Ages despite lack of manuscripts. The neoplatonic thought of St. Augustine, the pseudo-Dionysian beliefs of Macrobius and above all the work of Boethius provided the main sources of symbolism.

Towards the close of the tenth century in Europe we see a situation where chess is being played by the highest echelon at court, but also in church institutions, in a socio-political climate that is war-like and crude but poised for change and improvement. The game of chess appeals to this society because of its allegorical aptness.

Some educators were already at work to raise the general moral standards and continue the reformation of education which had been under way since Charlemagne. For example Odo of Cluny, 879 - 942, set an example to others by his efforts in reforming Benedictine monasteries, which in time became the leading institutions of learning in Europe.

Three examples will help to illustrate that "play" was used deliberately to stimulate an interest in self-improvement and that this was now possible because man's imagination was ready to be engaged in problem-solving. In a general way, a new enquiring spirit seems to have been abroad.

i) Odo's near contemporary, Bishop Wilbold of Cambrai (died 965)tried to propagate the use of a game he invented for the moral education of students. His tract "De Alea Regulari contra Aleam Secularem" is devised to urge players to strive more ardently for all the virtues obtainable by man.[6] The winner of the game was to be called "magister" by the defeated for six days; the vanquished could be addressed as "pupil" by the winner; in the case of a draw the players should greet one another like brothers. One might be justified in suspecting that the efforts of the bishop were hampered both by the utter tameness of the game, and by the use of dice. Dicing was not only connected to gambling, but as a game of chance and a waste of time it was forbidden by the Church.

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[6] The rules for Bishop Wilbold's game are in appendix B.

ii) A nun, Roswitha of Gandersheim, c 935 - c 1003, connected to the royal house of Saxony, produced three volumes in Latin probably between 960 and 970. The first one contained eight legends in hexameters dedicated to her abbess, Gerberga II, niece of Emperor Otto I; the second contained six plays in the manner of Terence, in rhyming and rhythmic prose; the third consisted of two epics, "Emperor Otto I" (Carmen de gestis Oddonis imperatoris) and "The origins of the Convent of Gandersheim".[7]

Roswitha was a woman of the world and wrote for, and associated with, men and women of the world. This is evident from the dedication of her work to the intelligentsia of her surroundings, and from the way she handles the subject of courtly love and scenes at the Imperial Court.

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[7] The Convent of Gandersheim was founded in 881 by Oda, great-grandmother of Henry the Fowler, and was almost habitually headed by ladies from the royal family. Roswitha was educated in the liberal arts by a nun called Rikkarda and the young abbess, Gerberga. Die Dramen der Roswitha von Gandersheim, ed. & tr. O. Piltz, Leipzig, Reclam, Jun. 1823, also The Plays of Roswitha tr. C. St. John, London, Chatto & Windus, 1923.

By modern standards Roswitha's work would not be considered very sophisticated but for the tenth century it was astounding. In her early legends she stays close to the conventional stories and yet manages to treat the sometimes gruesome subject-matter with grace and decorum. The legends and plays were intended to elevate and encourage Christian virtues under adverse circumstances. Two legends deserve particular attention. In "The downfall and conversion of the Vicedomnus Theophilus" Roswitha is the first to treat nothing less than the Faust-problem. In "The suffering of St. Pelagius" she recounts the martyrdom of a Spanish youth at the hands of Calif Abderaman, of which she heard an eye-witness report. This was important to Spanish historians and her poem was used as source material by Spanish and Portuguese writers.[8]

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[8] For example, Jorge Cardoso in Agiolo Lusitano, as quoted by Piltz, Roswitha, p. 13.

To stimulate proper Christian sentiments was only one of Roswitha's aims. For her plays she used the comedies of Terence as a model thereby attracting the attention of those less interested in pious subjects. In addition, in two of her six plays, "The Conversion of Thais" (subtitled "Paphnutius"), and "The Sufferings of the Holy Virgins Fides, Spes and Caritas" (or "Sapientia"), Roswitha involves her spectators or readers[9] in "book-learning". She allows her main characters to deliver lectures on Aristotelian philosophy as presented by Boethius, with special emphasis on musical proportions.

In "Paphnutius", Act I, the hermit of that name talks to his disciples of substance, body and soul[10] and their composition of contrary elements. He explains the quadrivium

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[9] Possibly her plays were read rather than performed.

[10] Based on Boethius' In praedicamenta Aristotelis I, De Substantia, and De Musica I, ch. X - XIX.

as being the basis of all existence and defines music as celestial, human and instrumental. He closes his discourse by telling his disciples that it is better to have a very little idea of these complex things, than to live in total ignorance of them.

In "Sapientia", Act I, the heroine of that name lectures the Emperor Hadrian on the theory of numbers in proportion and the properties of odd and even numbers.[11] This perhaps weakens the drama but was intended to kindle or heighten an interest in learning. By such examples Roswitha shows that learning is open to everyone in her world, even women.

iii) The Latin romance Ruodlieb[12] in leonine hexameters, written probably between 1030 and 1060 by a monk or monks at Tegernsee in Bavaria, has been rediscovered as recently as 1838. The poem suggests in a subtle way

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[11] Based on Boethius' De Arithmetica I, ch. IX, X, XI, XX.

[12] A fragment of 34 leaves of this early epic was recovered in 1838 from the binding of a book which came to the Bayrische Staatsbibliothek Munich from Tegernsee. Edited by F. Seiler, Halle 1882.

how circumstances can be mastered by the hero who is quick to learn. It tells of peace negotiations after a battle, perhaps between King Robert of France and the German Emperor Henry II in 1022. Ruodlieb is sent by the victor with terms of peace. On his return he relates how he spent his time at the other court.

He was well received and was invited to play chess with the High Marshal. Many games were played; the Marshal laboured for victory but could only win when Ruodlieb deliberately allowed him to do so. For five days they played while Ruodlieb waited for an audience with the king. At last he was permitted to deliver his message to the king.

The king then proposed a game of chess to him, as he wanted to see what unfamiliar moves Ruodlieb might make. Both players gave their full attention to the game and three times Ruodlieb managed to win. The king and his many attentive courtiers wagered against Ruodlieb, who was reluctant to play for money until the king explained to him that he should follow the customs of the country in which he found himself.

This is a tale in which the hero is tested in difficult and unfamiliar circumstances. His immediate future is ambiguous, but winning the chess games encourages him to face what may be in store for him - even having to explain his large winnings when he returns.

What has Rhythmomachia Borrowed from Chess?

The invention of Rhythmomachia post-dates the invention of chess by about 450 years. Chess was invented somewhere along the river Ganges in the sixth century AD. It is a war-game played on a counting-board of eight squares by eight. It reflected a highly complex culture in India, ascetic and formalized, obsessed with mathematics but also sensual and exotic. The art of chess playing travelled east and west; Arab conquerors brought it to Spain[1] in the eighth century from where it spread to the rest of Europe. Chess was initially only played by the highest echelon of society. However during the twelfth century chess was played almost universally by the nobility and clergy, and during the thirteenth century a chess craze spread to all levels of society, except the peasants; men, women and children played.[2] The game of chess symbolized the condition of feudal society and every individual's place

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[1] In Catalonia there is written evidence of chess in the form of a will of the Count Urgel, 1010, leaving his chess-pieces to the Convent of St. Giles at Nimes. H.J.R. Murray, A History of Chess, Oxford, Clarendon Press, 1913, pp. 405, 406.

[2] From the nobility the game may first have passed to the knights of fortune and the military mercenary class; its prestige depended to a large extent on association with the upper classes. Chess playing was not a forbidden pastime in medieval town statutes, but was sometimes subject to regulation regarding the size of the stake. Indentures of apprentices prohibited chess

in it, to the extent that chess similes and chess terms have entered everyday language. Even the identification of individuals has in many cases been achieved through the use of chess figures. For example the rook, single and double-headed,is borne on the coats of arms of French, Spanish, German, Swiss and English families, and chess terms occur in surnames, e.g. Rookwood, Roca, Roche, Rogon, etc.

On the metaphysical level, preachers during the Middle Ages used chess allegories in their sermons. These pointed a moral by likening chess-men to men in the world, who share a common birth and death, but occupy different stations during life. In their God-given place they must remain, do their duty and fight for the king who demands their protection and sacrifice. Through allegory and simile the great could also be criticized, e.g. bishops move and take obliquely because nearly every bishop misuses his office through cupidity.

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playing. Conditions of employment for journeymen or masters might stipulate that playing was permitted in moderation, but could be suspended for a specific period if found to interfere with production. Universities usually banned all knightly occupations (listed by Pedro Alfonsi, c 1100, Disciplina Clericalis, Paris, 1824, p. 42: "Probitates vero haec sunt: equitare, saggitare, cestibus certare, aucupare, scacis ludere, versificare"), but allowed chess to be played on public holidays, provided that no money changed hands. Chess was often played in taverns which advertised under the sign of "The Chequer", Murray, A History of Chess, pp 408, 439-441.

The popularity of allegorical interpretations of chess in the Middle Ages led to the compilation of works that were known as chess "moralities".[3] The most important one was a collection of sermons, written down at the request of fellow preachers and friends, by the Dominican Jacobus de Cessolis towards the end of the thirteenth-century.[4] In his Liber de moribus hominum et officiis nobilium Cessolis likens the world to a chess-board which is chequered white and black, showing the colours of the two conditions of life and death, or praise and blame.

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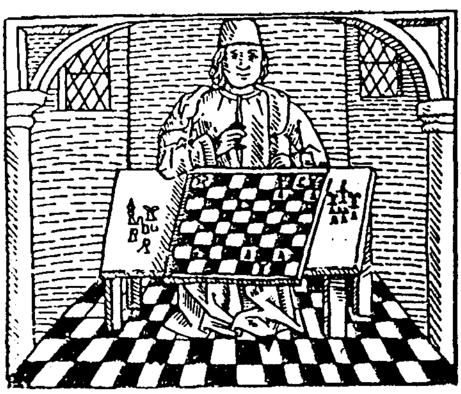

A monk solving a chess problem

A king and a bishop at play

Two wood-engravings from Caxton's chess book of 1474.

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[3] Murray, A History of Chess, "The Moralities", pp. 528-563.

[4] A large number of fourteenth and fifteenth-century manuscripts of this work exist in the original Latin, and in translations in nearly every spoken language of the time. Its popularity must almost have rivalled that of the Bible. William Caxton printed it in English (1474 and c 1480)

Cessolis sets out with sound advice the duties and privileges of men in their various callings based on the Policraticus seu De nugis curicalium et vestigiis philosophorum of John of Salisbury, c 1115 - 1180.[5]

According to medieval romances and poetry, both Christian and Arabic, chess equipment was not only of a utilitarian character but soon came to be used as stores of value and as such turned into heirlooms, princely gifts and bribes.[6] Some chess-boards and pieces were works of art made of semiprecious stones, rock-crystal, gold, silver and ivory and therefore treasured for their intrinsic, symbolic value as well as for their high price. When churches became the recipients of these gifts, they incidentally preserved them at times by converting some chess-boards to serve as liturgical

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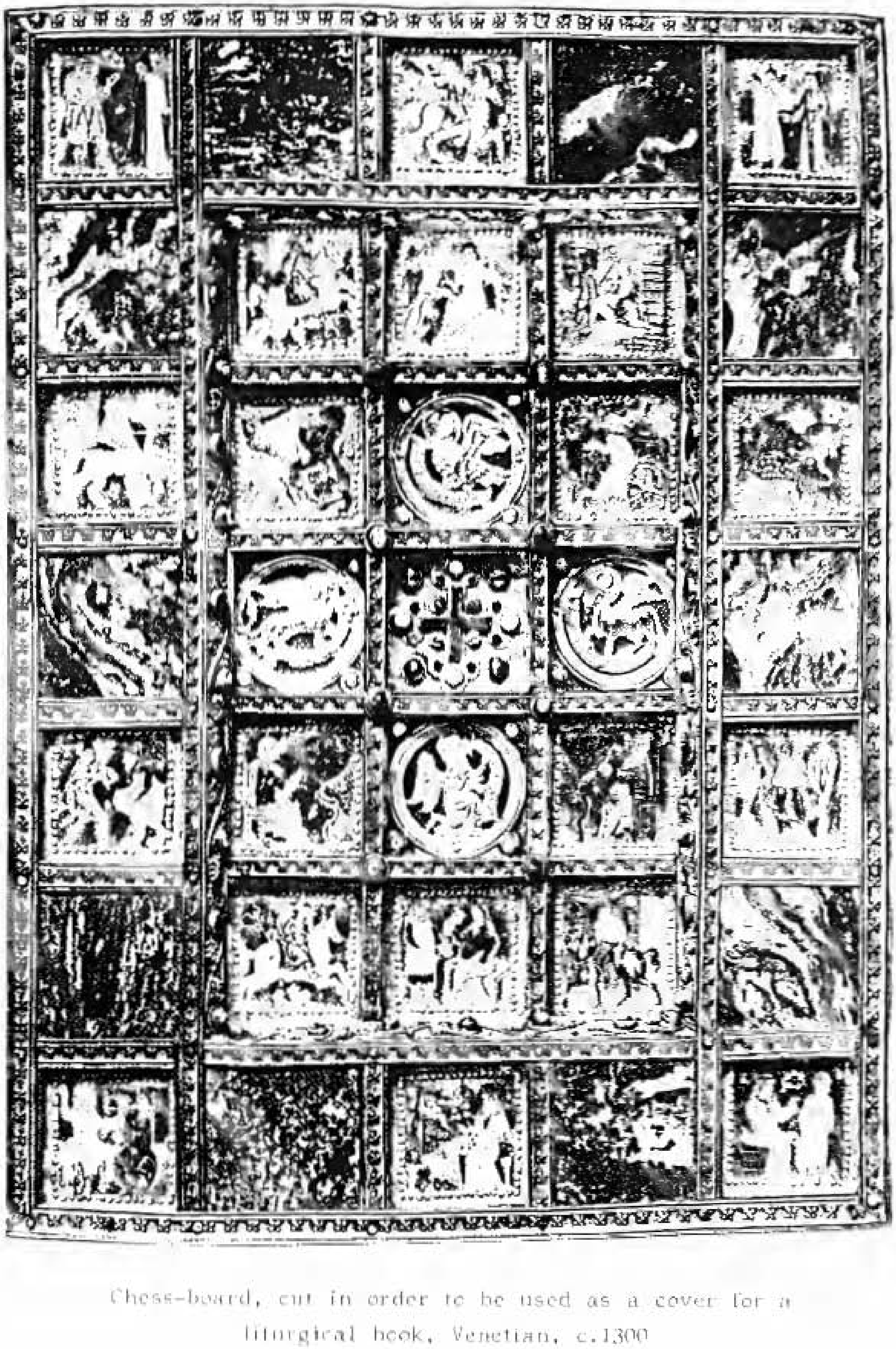
as The game and playe of the chesse, from a French version, Le livre des eschecs moralise en françois (c 1347), by the friar Jehan de Vignay (original dedication to John, Duke of Normandy, printed in folio in 1504); Murray, A History of Chess, pp. 528-563.

5 ibid., p. 540, quoting Prof. Köpke, Iacobus de Cessolis, Brandenburg, 1879.

6 B. Ammār is said by al-Marrākoshī, MS H. (writing 621/1224) to have played chess with the Christian King Alfonso VI of Leon and Castile, c 1078, as quoted by Murray, ibid., p. 203. Legend has it that Alfonso was tempted into a chess game against the finest Arab player, with a magnificent set, while besieging Seville, by its defender Al Mutamid. Alfonso lost, kept the chess set, accepted a handsome tribute, and allowed Al Mutamid continued occupation of Seville until 1090.



Chess-board, cut in order to be used as a cover for a liturgical book, Venetian, c.1300 book-covers,[7] or refashioning the pieces for the decoration of a reliquary or other church regalia.[8]

Chess can be played at various levels of skill. When played as entertainment the moves can be determined by the throw of dice;[9] when played seriously it can teach strategy for warfare. Arab scholars also used the chess-board to examine mathematical problems, e.g.

a) in the "knight's tour" this piece, using only the moves of a knight, visits each space once and ends where it began; a tracing of these moves produces an intricate pattern of geometric shapes;

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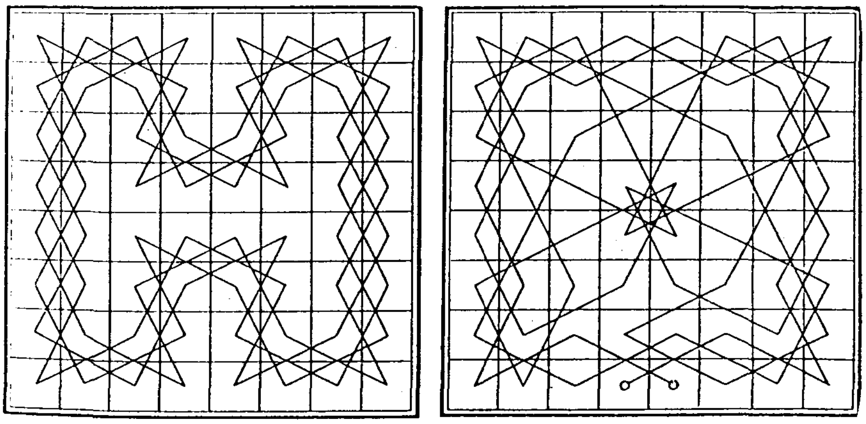
[7] Duke Otto of the Guelphs in penance donated a precious chess-board to a church in Venice to be used as a liturgical book-cover, c 1300. N. von Holst, Creators, Collectors, Connoisseurs, London, Thames and Hudson, 1967.

[8] Crystal chess-men were particularly popular because crystal symbolically stands for the union of opposites - of matter and spirit. Also; stones in general were thought to be containers of the life-force, with all its mystery. The chess-board (squares of multiples of four) symbolizes a union, which is the fulfillment and goal of all religions. It is the union of the soul with God. Rhythmomachia, played on a board of 8 X 16 squares provides each player with his own 8 X 8 square playground. The numbers 4 and 4 X 4 = 16, are particularly strong symbols of wholeness, completeness.

[9] Dicing, but not chess, was forbidden by the church for monks and priests, despite agitation from such purists as St. Damian and St. Bernard. Murray, A History of Chess, p. 408; also H. Faust, "Studie über das Schachspiel in den religiösen Orden im Mittelalter", Rochade-Sonderdruck No. 168. July 1978, p. 1.



Knight's tours: no square is visited twice and the tour ends where it began

5

8	4 mil.	6	6 mil.	4	8 mil.	2	10 mil.
400 tys.	60	600 tys.	40	800 tys.	20	1 mil.	9
600	60 tys.	400	80 tys.	200	100 tys.	90	300 tys.
6 tys.	4 tys.	8 tys.	2 tys.	10 tys.	900	30 tys.	700
40 tys.	800	20 tys.	1 tys.	9 tys.	3 tys.	7 tys.	5 tys.
80	200 tys.	100	90 tys.	300	70 tys.	500	50 tys.
2 mil.	10	900 tys.	30	700 tys.	50	500 tys.	70
1	9 mil.	3	7 mil.	5	5 mil.	7	3 mil.

Arabian chess-board used for counting

b) in the horizontal rows of the squares of the board numbers were placed according to arithmetical progression, in the diagonal ones according to geometrical progression.[10]

The inventor of Rhythmomachia took the uncheckered counting-board[11] on which chess was played and doubled it to 8 X 16 squares. The first four rows of both short sides of the board are taken up by pieces engraved with numbers, reminiscent of Gerbert's "apice". [12] These, like chess-pieces, battle for supremacy. Unlike chess-pieces, they are not hierarchically structured; they only have a numerical preference.

Rhythmomachia shares with chess its versatility. Like chess, Rhythmomachia can by played at various levels of skill. The student uses Rhythmomachia to practise multiplication tables, ratios and proportions of numbers; the master uses his greater skill to find the three progressions of numbers in one row; the advanced scholar, used to the custom of the times of making theology, philosophy and the subjects of the liberal arts occasion for debate, finds it natural to carry the contest of words over onto the board of Rhythmomachia for the contest of numbers. The monk or teacher, like his

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[10] See illustration. The board remained uncheckered until the second half of the eleventh century. These progressions also figure in Rhythmomachia and are explained later.

[11] This counting-board was still called an abacus in the <u>Rythmomachia</u> of 1556 by Claude de Boissière.

[12] Gerbert's "apice" will be explained later.

brother, the nobleman, was geared for combat, which in his case might be, for example, discussing the concept of "unity" and "number"[13] and the symbolic meaning behind the physical shape of the pieces used in play: the circle, triangle, square and pyramid as well as the rectangle of the board.[14] After all, even God, it was believed in the Middle Ages, was fighting the devil for the redemption of man who had fallen booty, because of his sins, to the King of Darkness.

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[13] A favourite subject for disputation in schools was the various definitions of "number". Euclid defined "unity" as that according to which everything which exists is called "one"; and number to be "multitude" composed of unities. By this definition the "1", which indeed is missing from the base numbers of the game, is considered not to be a number at all. Encyclopaedia of Pure Mathematics, London, Griffin, 1847, p. 421.

[14] Thierry of Chartres, one of the most important thinkers of the twelfth century, found the very essence of reality in the mystery of numbers and explained the Trinity by geometrical symbols, e.g. the nature of Christ he saw as a rectangle. F. Heer, The Medieval World, London, Weidenfeld & Nicolson, 1962.

The Seven Liberal Arts

Every culture has at its roots powerful myths expressed as fundamental images and symbols which at one level or another remain with it during its history. Symbolic thinking suffuses human existence; it helps to explain certain aspects of reality which cannot be reached by any other means.

Symbolism of number is an important constituent of Western European culture. Martin Vogel has made an extensive study of the number seven in the speculative theory of music.[1] In this work he traces the significance of the number seven to its cosmic and magical meaning. A brief comment may help to explain medieval concern with number symbolism.

The phases of the moon - that calendar in the sky - first gave the Babylonians the idea of a seven-fold division, which they later connected to the then recognized seven planets; this they related to their seven-note scale of music, which in turn led to the Greek concept of the music of the spheres. The seven-stringed lyre of Apollo became the symbol of the seven planets moving in the heavens.

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[1] M. Vogel, Die Zahl Sieben in der spekulativen Musiktheorie, Inaugural Dissertation, Bonn, 1955.

From the Babylonians number symbolism spread. The Old Testament recorded the seven plagues in Egypt and the seven fat and seven lean years; there are seven virtues and seven deadly sins. In the prayer "Our Father...." we make seven requests. Thebes had seven gates and Rome has seven hills, there are seven wise men in Greek antiquity and seven brave Swabians in the German fairy tale; we are, at this moment, concerned with some of the seven liberal arts.

In Greek antiquity the number seven was holy to Dionysus and Apollo; originally there were only seven muses; seven was the number of immaculate purity, because within the first ten numbers seven neither produces another number nor stems from another number.[2]

The Babylonians proved that seven is the number of perfection because it consists of three and four. Seen geometrically it has three dimensions: length, width, height, and four termini: point, line, surface and solidity.

These are some of the associations of one number only, albeit one described as "rerum omnium fere nodus" in Cicero's <u>Somnium Scipionis</u>.[3]

During the "Dark" Ages the Western world was relatively innumerate. Yet Christianity was influenced by

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[2] Macrobius 497 = VI, 11, in Vogel, <u>Die Zahl Sieben</u>, p. 40.

[3] Macrobius 502 = VI, 34, ibid.

Egypt, Babylon and Greece, the most numerate of cultures, and later by the Islamic world. There was also a certain numerical tradition, especially in the Book of Numbers, Daniel and Revelation, which afforded the pagan art of numbers some respectability.

Philo Judaeus, c 30 BC - 50 AD, was the first important thinker after the birth of Christ to try to reconcile the account of Creation in Genesis with Greek philosophy. He believed the use of allegory would help. He found that Creation needed order, order has to do with number and the number most appropriate

> "to productivity is 6, for if we start with 1 it is the first perfect number, being equal to the product of its factors, 1 X 2 X 3, as well as made up of the sum of them, 1 + 2 + 3, its half being 3, its third part 2, its sixth part 1. We may say that it is in its nature both male and female, and is a result of the distinctive power of either. For among things that are, it is the odd that is male, and the even female. Now of odd numbers 3 is the starting point, and of even numbers 2, and the product of these two is 6. For it was requisite that the world, being most perfect of all things that have come into existence, should be constituted in accordance with a perfect number, namely six; and, in as much as it was to have in itself beings that sprang from a coupling together, should receive the impress of a mixed number, namely the first in which odd and even were combined, one that should contain the essential principle of both the male that sows and the female that receives the seed."[4]

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[4] Philo, De Opificio Mundi, tr. F. H. Colson and G. H. Whitaker, London, 1929, vol. I, p. 13 f.

The most significant writer on mathematics after Philo was Nicomachus of Gerasa, c 100 AD. His "Introduction to Arithmetic" included knowledge and beliefs relating to all the "artes" that had gone before, but emphasized that the aim of the study of mathematics is to arrive at divine number, the true and eternal essence. Martianus Capella, Boethius, Cassiodorus, Isidorus, The Venerable Bede, Alcuin and Hugh of St. Victor all used Nicomachus for their work. Of these writers Boethius proved the most influential for the mind of the Middle Ages.[5]

Since Pythagoras and Plato philosophers have believed in the absolute reality of mathematics.[6] They examined numbers as to their structure, nature, properties, relation, analogies etc., in the hope that a full understanding of numbers would lead them to the understanding of true philosophy. In the process of classifying numbers in every possible way they attached an exaggerated importance to their findings.

Scholars in the Middle Ages debated the various definitions of such concepts as "unity", "multitude"

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5 C. Butler, Number Symbolism, London, Routledge & Kegan Paul, 1970, pp. 32, 33, fn. 22.

6 Ptolemy, Almagestum, i. I, 2, said that theology was incomprehensible, physics dealt with changing things, and therefore only mathematics was certain; as quoted by A. Crombie, Robert Grosseteste, Oxford, Clarendon Press, 1953, p. 129, fn. 3.

and "number" which they had inherited from thinkers of previous ages. The ancients themselves were unable to define or explain adequately a term such as "unity".[7] Many definitions were contradictory and incomprehensible: e.g. it was debated whether "one" is a number at all because Euclid defined "unity" to be "that according to which everything which exists is called one; and number to be 'multitude', composed of unities",[8] therefore one is not a number but a unity. According to another view, number is a quantity that numbers everything that exists and as unity is a quantity it can legitimately number itself, therefore one is a number.

For the understanding of Rhythmomachia it is important to consider some of the older numerical theories in which the essential terminology is already incorporated. An example of how scholars analysed numbers can be taken from Nicomachus, who described numbers as:

a) "perfect" if equal to the sum of its divisors, e.g. 6, 3+2+1=6

b) "deficient" if the sum of the divisors is less than the number e.g. 8, 4+2+1=7

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[7] The concept of "unity" expresses an idea which does not allow a resolution into others more simple than itself.

[8] Encyclopaedia of Pure Mathematics, p. 421.

c) "abundant", or "superabundant" if the sum of the divisors is greater, e.g. 12, 6+4+3+2+1=16 [9]

Nicomachus classified numbers as to their species, for example as odd, even, prime or compound. Even numbers were further divided into those that remain even when divided by an even number and those which have an odd quotient when divided by an even number. They are called respectively:

"pariter pares", as in the series

4, 8, 16, 32, 64; and

"impariter pares", as in the series

6, 10, 14, 18, 22.

The following is an example of how scholars systematically examined and classified ratios and proportions:

1) Ratios could:

a) have equality, be called a "multiple ratio" because the antecedent is a multiple of the consequent;

b) have greater inequality, be called a "superparticular" number, where the antecedent is equal to the consequent and some part of it; dependent on the size of the part, superparticular numbers could be:

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[9] <u>Encyclopaedia of Pure Mathematics</u>, p. 421.

"sesquialteran" (when $\frac{1}{2}$ bigger) 6 9

"sesquitertian" (when $\frac{1}{3}$ bigger) 6 8

"sesquiquartan" (when $\frac{1}{4}$ bigger) 8 10

(These subdivisions could be continued indefinitely);

c) have lesser inequality, be called "superpartient", where the antecedent contains the subsequent once and some multiple of its parts, as in the series 5 3, 7 4, 19 10; [10]

2) Proportions were originally defined by Plato in _Timaeus_.[11] There he expounded the theory that the world soul must embrace all fundamental principles within itself.[12] The mathematical division - by Plato's Demiurge - of the world soul by means of two geometric proportions of four terms each, an odd and an even number series, was expressed in the shape

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[10] The above examples are taken from _The Encyclopaedia of Pure Mathematics_, p. 369-425.

[11] _Timaeus_ was an extremely influential text in the Middle Ages. There was hardly a library which did not have a copy of Calcidius' version and sometimes also a copy of the fragment translated by Cicero. R. Klibansky, _The Continuity of the Platonic tradition_, London, Warburg Institute, 1939, p. 28, 29.

[12] _Plato's Cosmology_, _Timaeus_ 35^{b} - 36^{b}, tr. F.M. Cornford, London, Routledge & Kegan Paul, 1937, p. 66-74.

of a capital lambda:[13]

1

2 3

4 9

8 27

The seven Platonic numbers of the greater tetractys[14] (lambda) were also expressed in geometric figures:

Point

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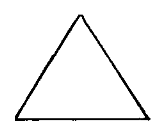
1

Line

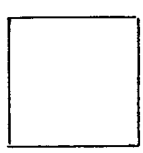
2

Surface



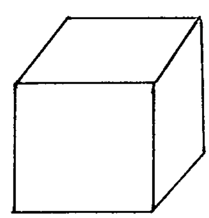
3

Surface (2^2 & 3^2)



4;9

Solid (2^3 & 3^3)



8;27

The Demiurge carried out the division in accordance with arithmetic, geometric and harmonic proportions:

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13 The lambda arrangement brings out the distinction of odd numbers representing the "same", and even numbers representing the "other".

14 From the meaning "fourness" as proposed by Pythagoras.

The progression a, b, c is:

1) arithmetic if $b-a = c-b$.

Examples:

1:2:3	2:4:6	3:6:9
4:8:12	6:12:18	9:18:27
2:3:4	4:6:8	6:9:12

2) geometric if $b/a = c/b$.

Examples:

1:2:4	2:4:8	3:6:12
1:3:9	2:6:18	3:9:27
4:6:9	8:12:18	12:18:27

3) harmonic if $c/a = c-b/b-a$ (i.e. $\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$,

hence also called "inverse" arithmetic).

Examples:

$\frac{1}{3} : \frac{1}{2} : \frac{1}{1}$	2:3:6	4:6:12	6:9:18
$\frac{1}{4} : \frac{1}{3} : \frac{1}{2}$	3:4:6	6:8:12	9:12:18 [15]

These progressions can be generated as follows:

arithmetic a, a+d, a+2d

d = difference

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15 Calcidius, fourth century, first Latin translator of <u>Timaeus</u>. See Jacobus Obrecht, ed. M. van Crevel, <u>Opera Omnia</u>, Introduction, vol. 1, 1959-1964, fasc. p. LVIII.

geometric	$a,\ aq,\ aq^2$	
	q = quotient	
harmonic	$a,\ \frac{2n}{n+1}a,\quad na$	
	(or $a,\quad pa,\ \frac{p}{2-p}a,\quad$ where $p < 2$).	

Nicomachus made a distinction between "numbers" and "quantity":

Numbers:	absolute and per se	=	arithmetic
	in mutual relation	=	music
Quantity:	at rest	=	geometry
	in motion	=	astronomy[16]

These distinctions developed in the Middle Ages into the disciplines of the "quadrivium", a term which is believed to have been used first by Boethius.[17]

Because medieval scholars believed that men in the Golden Age had known the secrets of the universe and its creation,[18] they imagined these secrets might be rediscovered with the aid of mathematics, a privileged and reliable guide for the co-ordination of

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[16] Butler, Number Symbolism, p. 33.

[17] The trivium and quadrivium constitute the seven liberal arts; historically the medieval university courses. The names come from "the place where the three, or the four roads meet". The trivium is made up of: grammar, logic and rhetoric; the quadrivium of: arithmetic, geometry, astronomy and music.

[18] E. Gombrich, "Icones Symbolicae", Journal of the Warburg and Courtauld Institutes, vol. 11, 1949, p. 169.

knowledge. They wrestled with numbers as such and the symbolic meaning that had been attached to them since ancient times. As numbers could be expressed in geometric shapes like the circle, triangle, square, rectangle, pyramid, etc. these shapes took on symbolic meaning and stood for actual things. For example, according to Pythagoras, the number four was justice, but also the pyramid, and the pyramid had produced fire, as the icosahedron had produced water, the octahedron air and the hexahedron earth; from the fifth Platonic solid,[19] the dodecahedron derives the pentagram or triple triangle, a rich source of golden ratios.[20]

Medieval scholars found justification for their particular kind of work with numbers from the ideas expressed by the Church Fathers, especially St. Augustine,who discussed the power and symbolism of numbers for which he in turn saw justification in the Book of Solomon: "Thou hast ordered all things in number and measure and weight."[21]

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[19] This is treated by Euclid in Book XII of his Elements. The pentagram was the badge of recognition of members of the ancient society of Pythagoras and the symbol of health to them. H.E. Huntley, The Divine Proportion, New York, Dover Publ., 1970, p. 28.

[20] Plato deals with these problems in Timaeus 53^c - 57^c, Cornford Cosmology p. 210-238. See also R. Brumbaugh, Plato's Mathematical Imagination, New York, Indiana Univ. Publ., 1968.

[21] Book of Solomon, Wisdom 11 : 21 (Apocryphal).

Carolingian Renaissance revived Latin tradition and in Rome mathematics had not really been respectable.[25]

When King Alfred c 900, encouraged learning, mathematics was not even mentioned. The Church required only a knowledge of "computistic" (from the word for calculate) which meant, between the fifth and eleventh centuries, the art of calendar reckoning for fixing the movable feasts, especially the establishment of the Easter-tables and other liturgical dates of the Church for several years ahead. The Church's attitude to mathematics seems to have been ambivalent. On the one hand they needed to preserve some knowledge of arithmetic, on the other they would not go so far as to teach it at their lay schools, but reserved it for clerical education at the monastic schools.

The Church's interest in music, as also in mathematics, extended only to its own needs. The Church's view on music was based largely on the inherited belief of Greek philosophers and Christian tradition.

Homer believed that the Muse bestowed the gift of song only on the most beloved of men. Hesiod claimed

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praises Alciun's erudition, wit, goodness: "...He brings forth pious lessons from Holy Writ/ and solves the puzzles of numbers with flavouring jest..." tr. G.F. Browne, Alcuin of York, London, Soc. Promoting Christian Knowledge, 1908, p. 245.

[25] Cicero believed the proper study for a man was moral philosophy, from which he should not be distracted by the "vicious" study of the mathematical arts of astrology or geometry. A. Murray, Reason and Society in the Middle Ages, Oxford, Clarendon Press, 1978, p. 154.

he was chosen by the Muse to be a singer and prophet. Plato in Timaeus 47 A ff, says:

> "...the sight of day and night, the months and returning years, the equinoxes and solstices, has caused the invention of number, given us the notion of time, and made us enquire into the nature of the universe; thence we have derived philosophy... we should see the revolutions of intelligence in the heavens and use their untroubled courses to guide the troubled revolutions in our own understanding, which are akin to them... all audible musical sound is given us for the sake of harmony, which has motions akin to the orbits in our soul and which, as anyone who makes intelligent use of the arts knows, is not to be used, as is commonly thought, to give irrational pleasure, but as a heaven-sent ally in reducing to order and harmony any disharmony in the revolutions within us; and rhythm too was given to them for the same reason."[26]

This passage encouraged the belief in the therapeutic properties of music. Even contemplation about music was considered to be good for the soul, producing "harmonia" between the cosmos and the soul.

The common classical belief was that the poet-musician was divinely inspired and actually mad or in a frenzy when he created or reproduced the harmony of the spheres.[27] Music, for the ancient Greeks, had

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[26] Klibansky, The Continuity of the Platonic Tradition, pp. 28. 29.

[27] Socrates said: "... if it were simply that frenzy were an evil...but in fact we receive the greatest benefits through frenzy, that is in so far as it is sent as a divine gift." Phaedrus 244 A.

ethical qualities because it reflected the universe as created by God.

It used to be thought by theologians that the divine gift of speech, which attuned the human soul to spiritual harmony, raised man to the level between the animals and the lowest rank of angels, our personal guardians.[28]

In Hebrew scriptures there are some references to music being of divine origin and Jews believed that music was the gift of God to man. The Church Fathers believed that chanting holy texts in praise of God was infinitely pleasing to Him. Medieval musicians searched for the perfect chord which would find most favour with their maker. In the Middle Ages pleasing God with music had to be balanced against taking too much personal pleasure in music. The Christian life had to be one of abstinence and self-denial, of blind faith and obedience. St. Augustine had felt guilt at experiencing pleasure when hearing music; Pope Gregory and St. Thomas Aquinas were against the study and practice of the liberal arts as they might give too much pleasure.

On the practical level the medieval Church establishment was against any innovation in musical practice for church services because of fear of adverse

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[28] J. Portnoy, Music in the Life of Man, New York, Holt, Rinehart and Winston, 1963, p. 1.

effects on their tranquillity. Besides, their experience was that men who performed music, that is singers and instrumentalists, were apt to be restless and unsettled men who had to be strictly controlled. Employment of such performers and their training could usually not be dispensed with, as monasteries and cathedrals rarely had enough monks and priests with the right kind of voice for the many services.

With the beginning of mass migration in Europe there appeared for the first time a form of music which eventually developed into the medieval and modern art of western music. This survives to the present day as the one-voice chant still in use in certain church services. The Ambrosian rites and the Gregorian chant[29] still represented only a collecting, ordering and regulating of the existing church melodies rather than an introduction of new forms.

Alcuin, 735 - 804, may have been one of the first to organize a course of instruction on independent theory of church music within the framework of teaching the liberal arts.[30] This Greek theory of music, based

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[29] The role of Pope Gregory the Great, 590 - 604, in the repertory of church music has not been fully established. The term Gregorian chant seems acceptable for general use if not interpreted literally. The New Grove Dictionary of Music and Musicians, ed. Stanley Sadie, London, Macmillan, 1980, vol. 7, p. 697, 698.

[30] Not all music historians accept that the teaching manual listed in the ninth century catalogue from

on pre-Carolingian work, and taught in schools from Charlemagne's time, was not classical Greek but vulgar Greek and may have come to Europe with church music from the Orient.

The medieval European system was based on "modes" which in their final form appeared as octave scales built up from the tetrachord (four-note series) D-G. Four modes were called "authentic" and these extended from D, E, F and G respectively upwards; four were "plagal" and descended a tetrachord downwards from the five note series from their corresponding authentic mode:

THE MODES

1st authentic	DEFGabcd	Dorian
1st plagal	ABCDEFGa	Hypodorian
2nd authentic	EFGabcde	Phrygian
2nd plagal	BCDEFGab	Hypophrygian
3rd authentic	FGabcdef	Lydian
3rd plagal	CDEFGabc	Hypolydian
4th authentic	Gabcdefg	Mixolydian
4th plagal	DEFGabcd	(Hypomixolydian)[31]

For the purpose of this study it is not necessary to go into the complicated development of musical theory and notation, suffice it to say that ninth and tenth century harmonic theorists failed to realize

Fulda Abbey was correctly attributed to Alcuin, <u>The New Grove Dictionary</u>, vol. 1, p. 230.

31 G. Reece, <u>Music in the Middle Ages</u>, p. 152 as quoted by A. White, "Boethius in the Medieval Quadrivium", <u>Boethius</u> ed. M. Gibson, Oxford, Blackwell, 1981, p.166.

that their contemporary music was based on principles different from those of classical music. Vulgar Greek terms used for the chants made little sense, and enlightenment was sought in classical texts. These clouded the issue even further by adding metaphysical and arithmetical sophistries.

The desire to arrive at generally applicable rules for a tonal system led some scholars like Berno of Reichenau to delve into philosophy, allegory, mathematics and even observation of nature; e.g. Berno attempts to explain nine combinations of notes used in a single voice chant by the activity of nine parts of the body involved in singing: tongue, four teeth, two lips, the throat and lung.[32] He speculates that for this reason Apollo was given nine muses for company. The number 4 he considers of the utmost importance, in the creation, in the music of the spheres, in singing, in instrumental music as well as in the formation of the cube. From the mystery of the number 4,he says, stem the voices of the four evangelists which carried

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[32] This accords with the belief in the microscopic origin of man and especially the description of First Man, an Iranian myth which was known in Hellenistic times. Mortal man, according to the legend, was created in the likeness of the universe. A. Goetze, "Persische Weisheit in Griechischem Gewande", in Zeitschrift für Indiologie und Iranistik, vol. 2, 1923, p. 6 ff. Also F. Saxl. Lectures, "Macrocosm and Microcosm in Medieval Pictures", London, Peregrin Books, 1970, p. 58.

to the four corners of the earth the praises pleasing to the Lord.[33] To prove the divine influence on the origin of the final tetrachord D E F G he employs every possible philosophical, mathematical and allegorical exegesis of holy scripture.

Musical instruments were thought to have symbolic significance and were therefore employed for appropriate purposes, e.g. allegorical speculation explained the straight sides of the psalter as symbolic of the even way to God, whereas the cithera, because of its curved sound-box, was considered symbolic of the tortuous way to God. The psalter produced sounds on the upper part of the instrument, therefore the psalter epitomised the way of the spirit, going directly upwards. The sound of the cithera goes downwards, therefore represents the way of the flesh, but also the incarnation of Christ, because the sound-box and strings form the sign of the cross.[34]

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[33] Codex 50 Karlsruhe, as quoted by W. Brambach, "Reichenauer Sängerschule", Zentralblatt für Bibliothekwesen, Beiheft ii, 1888, p. 62.

[34] M. F. Bukofzer, "Speculative thinking in Medieval Music" Speculum, vol. XVII, 1942, p. 168.

Gerbert of Aurillac and the Mathematical Revival

While important centres of learning emerged in the seventh and eighth centuries such as Bobbio in Italy, St. Gallen in Switzerland, Reichenau on Lake Constance (724) and Fulda in Hesse (744), these establishments at that time made no serious and critical effort to study ancient, and especially scientific, manuscripts but rather contented themselves with transmitting and copying texts as they found them. Even the great John Scotus Eriugena, c 810 - c 877, and his circle went no further than to speculate on numerical proportions and their importance to the orderly structure of the universe, the nature of celestial harmony, the music of the spheres and the hierarchical structure of the cosmos.[1]

A real mathematical revival in the Middle Ages came about with Gerbert of Aurillac, c 940-1003, later to become Pope Sylvester II. By the fame of his learning and a personality that found itself at home with popes, emperors and kings in the political scene of Europe for more than 20 years he gave impetus to the study of the liberal arts and to cultural development

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[1] Abbot Bovo II of Corvey, (900-916), appears to have been exceptional in giving lectures on Boethius that incorporated the philosopher's source material, i.e. Macrobius and Martianus Capella. M. Manitius, Lat. Lit. I, 526 ff and P. Courcelle,"Les lettres gresques", p. 257 ff. as quoted by H. Homeyer, ed. <u>Hrotsvithae Opera</u>, Munich, Schöningh, 1970, p. 324.

in Europe.

A study of the mathematical revival in the tenth century necessarily needs to centre around Gerbert and those of his contemporaries which historical records have rescued. Unfortunately these records are not very informative as to the extent of mathematical knowledge and practice even in people we can connect with the study of the quadrivium. How much secret study may have gone on in the face of disapproval by clerical superiors cannot even be guessed. The Irish Abbot, and later Bishop, Virgil (died 784) of St. Peter in Salzburg, that cultural alpine centre with its upper class of land-owning "possessores Romani", was strictured for possessing scientific books.[2] This may have been an isolated incident because the impression one gains is that Western Europe was ready for a new step forward and was waiting for a lead. This lead came from Gerbert who had received an education in "philosophy" which at that time meant knowledge of heaven and earth.[3]

According to his contemporary biographer and pupil

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[2] F. Prinz, "Vorbenediktinisches Mönchtum", St. Peter in Salzburg, Salzburg, Amt der Salzburger Landesregierung, 1982, pp. 14-18.

[3] This included theology. One has to be aware that in the terminology of the tenth century "quadrivium" and "mathematics" were used synonymously, as were "astronomy" and "astrology".

Richer[4], Gerbert was instructed in all the liberal arts in Aurillac. Of the trivium Richer mentions particularly only grammar, this comprised at that time not only Latin as a spoken language but also a thorough knowledge of Latin literature. Gerbert's brilliance in rhetoric was first noted when he accompanied Count Borell and Bishop Hatto on a mission to Rome. Lessons in logic he exchanged with G.,[5] Archdeacon of Rheims, whose aqaintance he made in Rome in 970 for instruction in the quadrivium, the subject always nearest to Gerbert's heart.

Count Borell II of Barcelona had made it possible for Gerbert to study the quadrivium, from 967 to 970, under the protection of Bishop Hatto of Vich in Catalonia. There, through contact with Arabic scholars, Gerbert acquired knowledge which was most probably not available anywhere else in Europe at that time. He became particularly interested in globes, armillary spheres and astrolabes for use in astronomy,[6] and a

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[4] Richer incorporated the biography of Gerbert in his Historia Franconia, III.

[5] Nothing further is known of G. except that he was in Rome as emissary from the French king, Lothar. Richer, III, 45, says that G. found mathematics too difficult to learn and disliked music. Gerbert was so keen on lessons in logic from G. that he followed him to Rheims, declining an offer from Emperor Otto I to stay at court as tutor to his son.

[6] Astronomical observations were unknown in Europe in the tenth century. Gerbert constantly encouraged his students to observe the night-sky. Lindgren in

new type of abacus[7] which differed from the antique one by the use of numbered "apice".[8] The new abacus enabled the user to divide and multiply large numbers at greater speed.

6 Armillary sphere

Gerbert's own preference in the study of the quad-

Sudhoffs Achiv, Beiheft 18, p. 10, 11.

[7] S. Gandz, "Did the Arabs know the Abacus?" American Mathematical Monthly, 34, 1927, concluded that they did, p. 308-316.

[8] The "characters" engraved on the "apice" mentioned by Richer may have been Arabic numerals, H. Lattin, "Origin of Notations", Isis, XIX, 1933, pp. 188, 191, 192.

rivium was undoubtedly astronomy, a subject he returned to all his life whenever time permitted or disappointment threatened. As a teacher in Rheims he taught imaginatively by constructing teaching-aids such as the fistula, a long tube without a lense, for observing smaller stars, a globe to demonstrate the constellations and their paths, breaking completely new ground for his time, and the armillary sphere consisting of circular hoops, for demonstrating the paths of the planets according to Ptolemaic theories. To instruct advanced students he used an astrolabe for the projection of the movement of the stars on to a plane. This instrument he had first encountered in Catalonia and continued to use with its Arabic terms. The astrolabe found practical application in cathedrals and monasteries for determining night and day for every meridian and for fixing times for prayers and religious services as well as the movable feasts in the liturgical calendar.

Gerbert's work in geometry showed no Arabic influence but might be called a "scholium" to Euclid with many clear and systematic examples.

In the game of Rhythmomachia specific numbers battle against one another. How these numbers were arrived at will be explained after a closer look at Gerbert's work on arithmetic and music.

1) arithmetic

The Greeks since Plato recognised two kinds of arithmetic:

(a) ἀριθμητική the purely speculative theory of numbers

(b) λογιστική the practical handling of numbers.

Gerbert concerned himself with both kinds. He saw arithmetic as a mental discipline of great value, demanding a high degree of accurate thinking. For his practical work with numbers Gerbert used the new abacus. The old Roman abacus in general use was either one on which beads were threaded on wire or simply a rectangular board on which sand was strewn and unmarked counters placed. Even this instrument had fallen into disuse by Gerbert's time; finger, hand and body-signs were the common aids to arithmetic.[9]

Richer describes the new abacus as a board divided lengthways into 27 sections into which horn discs were placed.[10] Gerbert provided himself with 1000 of these so called "apice" and had them engraved with characters from 1 to 9.[11] Using these he could carry out

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[9] History of Science, "Ancient and Medieval Science from the beginnings to 1450" ed. R. Taton, tr. A. Pomerans, London, Thames and Hudson, 1963, vol. 1, p. 474.

[10] Richer III liv.

[11] These characters may have been Arabic numerals, Lattin in Isis, pp. 181-194. Also S. Gandz, "The Origin of Ghubar Numerals", Isis, XVI, 1931, p. 393-424.

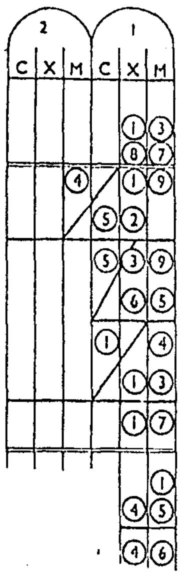
operations of multiplication and division of large figures at great speed. No clearer explanation is given by Richer but he refers to a letter from Gerbert to Constantine, monk of Fleury (c 980) where the rules for using the abacus are explained.[12]

7

DIFFERENCE: $100 - 87 = 13$
DIVISOR: 87
DIVIDEND: 4019
$\frac{4000}{100} = 40;\ 40 \times 13 = 520$
Discard the 4 in 4000; $19 + 520 = 539$
$\frac{500}{100} = 5;\ 5 \times 13 = 65$
Discard the 5 in 500; $39 + 65 = 104$
$\frac{100}{100} = 1;\ 13 \times 1 = 13$
Discard the 1 in 100; thus $13 + 4 = 17$

PARTIAL QUOTIENTS: $40 + 5 + 1$

RESULT: 46



Division by Gerbert's abacus method

When calculating on the abacus with unmarked discs each column that expressed a digit had to contain the number of discs needed to express that figure. The fact that Gerbert's new abacus had 27 columns in groups of three, was already a great advantage when calculating with Roman numerals. It was still only an aid to

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[12] H. Lattin, <u>The Letters of Gerbert with his Papal Privileges as Sylvester II</u>, New York, Columbia Univ. Press, 1961, letter 7. See also N. Buvnov, <u>Gerberti Silvestri II opera mathematica</u>, Berlin 1899.

memory between two operations because the columns were occupied by discs. The use of marked "apice" freed the columns for actual operations of arithmetic, facilitating multiplication and division.

This type of calculation was akin to the written one; it was fast and easy which meant the less intelligent student could learn it as it only needed the execution of elementary steps to lead to the correct results automatically. It had an advantage over the written method as cheap writing material was still not available in Europe.

The question of what characters were engraved on the "apice" must remain an academic one because there exists no proof.* Gerbert almost certainly saw Arabic-Ghubar numerals during his stay in Spain, where they have been found in Christian codices dating back to 972.[13] The type of "apice" in use at the time is not as important as the fact that the new method of calculation spread quickly over Europe and thereby furthered cultural development. It spread not only by written manuscripts, but by word of mouth and by practical example. Students carried their own "apice"

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* So far Roman numerals only have been found in texts dealing with Rhythmomachia up to the twelfth-century except for one table of the game, Oxford, St. Johns College, MS 17, f 56b, written in England, c 1110, which uses Greek arithmetical numerals: G. R. Evans, The Rithmomachia:A Medieval Mathematical Teaching Aid? Janus 63, 1976, p. 260.

[13] Lindgren, Sudhoffs Archiv, Beiheft 18, pp. 28, 29.

and had their work corrected wherever a master was available.[14] Gerbert's great gift was not to become ensnared by inherited views and theorems but to remain aware of wider implications in the general search for logical order. His work with problems of arithmetic logic was not a chasing after random curiosities, but a consistent scholarly progression. For instance he examined how Boethius explained the progression of numerical order from unity as an analogy of the creation of composite things from their elements.[15] This he translated into a mathematical formulae, later known as "saltus Gerberti" or Gerbert's leap, which he explains in a letter to Constantine of Fleury.[16]

Basically it is the method he found for translating sesquiquartal superparticular numbers, as 16, 20, 25 first into sesquitertian, then into sesquialteral and lastly into equal terms.

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[14] Itinerant teachers spread the new arithmetic northward. The conquering Normans took it to England. It was known in Tuscany and Swabia by the 1020's. Lorraine acquired a reputation for its skilled abacists at about the same time. History of Science, Taton, vol. 1, p. 437.

[15] Boethius, De Arithmetica I, 32 and II, 1, where he argues that all inequality, namely plurality, can be reduced back to its source, equality, namely unity. Plurality is made up of unities as words are made up of letters of the alphabet, music is made of sounds, and the elements, earth, air, fire, water make up everything in creation; as quoted by White, Boethius, p. 169.

[16] Lattin, The Letters of Gerbert, letter 3.

$a \quad ar \quad ar^2$

goes to:

$a \quad a(r+1) \quad a(r+1)^2$

Gerbert's example goes as follows:

16 20 25 $r = \frac{5}{4}$

With the first formula we get the ratio $r = \frac{1}{4}$

16 4 1 reversing gives

1	4	16	ratio	r = 4	
1	3	9	"	3	first formula
1	2	4	"	2	applied each
1	1	1	"	1	time

Examples of the other transitions quoted above are:

9 3 1 " $\frac{1}{3}$

With the second formula we get the ratio $\frac{4}{3}$

9 12 16

i.e. $r = \frac{1}{4}$ by second formula, goes to

$r = \frac{5}{4}$ $(= 1 + \frac{1}{4})$

$r = \frac{2}{3}$ goes to $r = \frac{5}{3}$

$r = \frac{5}{4}$ under first formula, goes to

$r = \frac{1}{4}$ [20]

2) <u>music</u>

Gerbert based his work on music chiefly on Boethius' *De Musica*, a work that remained the standard textbook

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[20] This has been translated into modern terms by Prof. H. Schlagbauer, Dept. of Mathematics, University of Cape Town.

throughout the Middle Ages. De Musica, like De Arithmetica dealt with the science of numbers and proportions which afforded the basis for the practice of music and mathematics.

Music, like the other subjects of the quadrivium in the Middle Ages was studied under two aspects: "ars" and "usus"; that is the theory and practice of music, and the latter was often not called music at all but "cantus". Many academics wished their knowledge of the theory of music to remain their exclusive preserve. The practice of music, that is singing and playing an instrument was left to men of less education, the so-called "cantors". The term "musician" was only accorded to those who mastered the theory of music. There is evidence that some learned musicians wished even the title of cantus to be withheld from good singers who had not studied Boethius.

The reason for what seems now such uncharitable behaviour may lie in the extraordinary difficulty and perplexity faced by musicians especially the most able. W. Brambach calls it "ein Jammer",[21] to see the staunchest scholars undertake the task of Sisyphus in bending the apparatus of classical art-language to the completely foreign creation which their contemporary church music was. The best of the independent thinkers

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[21]Brambach, in Zentralblatt für Bibliothekswesen, p. 56.

like Remigius Altisiodorensis and Hucbald, managed the feat of reconciling the contradictions, thereby clouding and complicating the issue. They tore apart and twisted the natural relationship between classical and medieval modes. Other scholars like Aurelianus Reomensis and Regino of Prüm made easier work of it by simply setting classical and contemporary theories side by side without attempting to relate them.

Seen from this perspective it becomes apparent how very exclusive knowledge of the quadrivium was in the ninth to the eleventh century and what hard labour was necessary to master the musical theories current amongst the finest brains. The ability to play Rhythmomachia - the game that aids the understanding of proportions - was a skill which took the scholar or aspirant a long way along the path of learning. Anyone who truly mastered the arts of music and mathematics of the time would have spent years of labour, probably outside "working-hours", giving up what brief periods of relaxation from monastic duties were available.

To come across sharp and unfriendly remarks about mere singers in the writings even of such a mild man as Hermannus Contractus, who, as a cripple would have known at times, hurt from derision for his physical disabilities, is understandable when we realise how hard-won the knowledge of a musician was when compared to the sounds of a God-given voice in one who knew nothing of philosophy or the science behind the melody.

The following examples might give some insight into the feelings of two of the finest musicians of their time: Hermannus Contractus and Guido of Arezzo. Regino of Prüm, though possibly not in the top rank of original musical theorists, provides a third example.

1. Musica Clarissimi Viri Herimanni.

   "......In hoc tamen insto indicio asino inferiores et imperitiores, qui et multo altius resonat et nunquam ruditum mugitu vel alia qualibet voce mutabit...."[22]

2. Regulae rhythmicae, Guido of Arezzo:

   "Musicorum et cantorum magna est distantia,
   Isti dicunt, illi sciunt, quae componit musica.
   Nam qui facit, quod non sapit, difinitur bestia.
   Ceterum tonantis vocis si laudent acumina,
   Superabit philomelam vel vocalis asina." 23

3. Regino of Prüm suggests that being able to sing does not make a singer:

   "Quisquis igitur harmonicae institutionis vim atque rationem penitus ignorat, frustra sibi nomen cantoris usurpat, tametsi cantare optime sciat."[24]

These examples can be balanced by a statement from the wise Abbot Berno of Reichenau, teacher and friend

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[22] This example is taken from Hermanni Contracti, Musica, codex Vindobonensis 51, 139^b - 140^b ed. Wilhelmus Brambach, Lipsiae, 1884.

[23] This example is taken from Brambach, Zentralblatt für Bibliothekwesen, p. 13.

[24] I, p. 246 G, ibid. p. 12.

of Hermannus:

> "Quisquis igitur sibi videtur sine artis huius notitia bene canere, cum interrogatus de numero vel de intervalis acutorum graviumque sonorum nesciat respondere, vultque solummodo aurium sensui credere, non autem rational magistrae, cum utrorumque iudicum sit exquirendum, amplius tamen rationis, quae ipsam veritatem integritatemque ad liquidum in rerum natura, in quantum possibile est, ex munere omnipotentis artificis comprehendit: is, inquam, talis magis lusciniae, quae verno anni tempore ac si numerose et suaviter canat, est comparandus, quam peritus cantor habendus".[25]

Next to nothing is known of Gerbert's ability to play an instrument or sing. He mentions the "organum" several times and says indirectly in a letter to Constantine of Fleury, that the pipes of an organ vary in size.[26]

Gerbert concentrated his studies on Boethius' exhaustive compilation of ancient musical knowledge, where music was divided into:

1. Musica Mundana

   Consonantia coeli et planetarium (harmony of the spheres)
   Consonantia elementorum (harmony of fire, water, earth and air)
   Consonantia temporum (harmony of the seasons)

2. Musica Humana

   Consonantia patrium animae (harmony of spirit and soul)
   Consonantia patrium id est elementorum corporis (harmony of the body)

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[25] II, p. 78 G, Brambach, Zentralblatt für Bibliothekwesen, p. 13.

[26] Lattin, The Letters of Gerbert, letter 2, c 978 (?).

3. Musica Instrumentorum

In intensione (string instruments)
In spiritu (wind instruments)
In percussione (percussion instruments).[27]

According to these divisions by Boethius, Gerbert worked solely on "musica mundana", the musical proportions. He explained Boethius' theory by giving practical examples of numbers in proportion which made it more comprehensible to the serious student or any person desirous of general education, as we have seen from the play "Paphnutius" by Roswitha of Gandersheim. Gerbert used the monochord which he found described in Boethius, recommended it strongly for study purposes and explained its use: it made notes in proportion audible.[28] From the tenth to the thirteenth century the monochord[29] was one of the most important instruments for teaching the notes of a song. The underlying principle is simple:

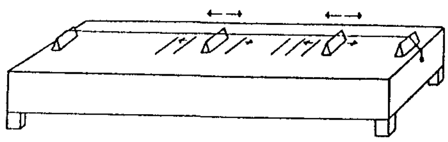
An average ability to sing and play may have been taken for granted by his biographer.

[27] Commentary from the MS "Bibliotheca Vaticana", Rome, Lat. 1005, fo. 3a, as quoted by J. Smits van Waesberghe, "Musikerziehungslehre: Lehre und Theorie der Musik im Mittelalter", Musikgeschichte in Bildern, vol. 3, 3, 1969.

[28] Richer III, as quoted by Lindgren, Sudhoffs Archiv, Beiheft 18, p.22.

[29] No original monochord has survived but is has been described as consisting of a rectangular sound-box over which was stretched a string of gut. A movable bridge helped to fix the intervals between the notes on a piece of parchment. In this way each musical note could be made to ring out clearly. Pythagoras was credited by Boethius with the invention of the theory of music as well as the monochord.

the sound of a musical note is created by the vibration of air. The pitch of a note depends on the number of vibrations per second. For instance if there are 440 vibrations per second we get a note a', at 880 vibrations per second we get a'', one octave higher. The relationship of these two frequencies is two to one (2:1). Any interval could be sounded on the monochord by dividing the string by the bridge in either an "ascending" or a "descending" scale. These divisions were based on the multiple and superparticular proportions of numbers, e.g. the fifth is derived from a sesquialteran superparticular number: this has the ratio $1:1\frac{1}{2}$ or 6:9.

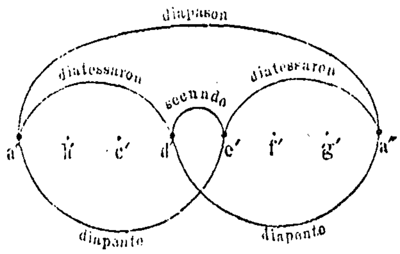


8 Model of a monochord

When dealing with very complex ratios the harmonic theorist would, for practical reasons, dispense with the monochord and work on paper only, thereby widening the scope to smaller intervals and chromatic and enharmonic types of scales.

Theoretically it is possible to divide the octave into many parts but it was found that only simple intervals that give a consonance were musically pleasing.

An illustration based on Boethius explains the position:



If a' and a'' are sounded together we get a diapason, now called octave; the relationship between the frequencies of a'', d' e' a' is 12:9:8:6; between a'' and d', or e' and a' we get a diapente, or fifth; a'' and e' or d' and a' gives a diatesseron or fourth; finally two neighbouring notes, e' and d' give a second.

One of the many difficulties faced by the medieval musician was the lack of serviceable musical notation.[30] The usual way to learn a new melody was to repeat it after a teacher until it remained fixed in the memory. Hucbald of St. Amand, c 840-930, in Flanders, was the first Western musical theorist who thought of marking the division of the monochord with Greek symbols.[31] This allowed the teaching of the rudiments of melody

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[30] For an example of early staff notation see diagram in appendix B.

[31] R. Weakland, "Hucbald as a musician and theorist", Musical Quarterly, 42 , 1956, pp. 66-84.

and established a system of notation that shows the singers the pitch on which each syllable is to be sung.

Hucbald, unlike his contemporaries Regino of Prüm and Remi of Auxerre, eliminates in his treatise De Institutione Harmonica, all speculation on the theory of number and concentrates on the performance of music and practical help for the singer. He acknowledges that his new system was still not able to show variation of speed, "tremula vocis" or other refinements, but it gave sufficient practical help to become a stepping-stone to more sophisticated systems invented by musicians in the next century, e.g. Guido of Arezzo, and Hermannus Contractus.

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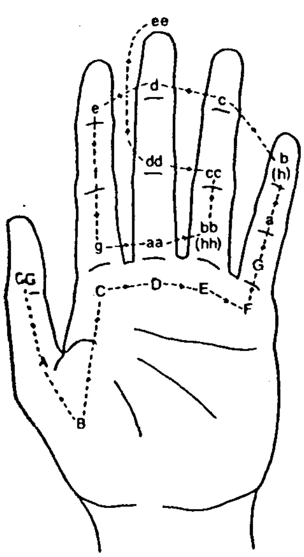
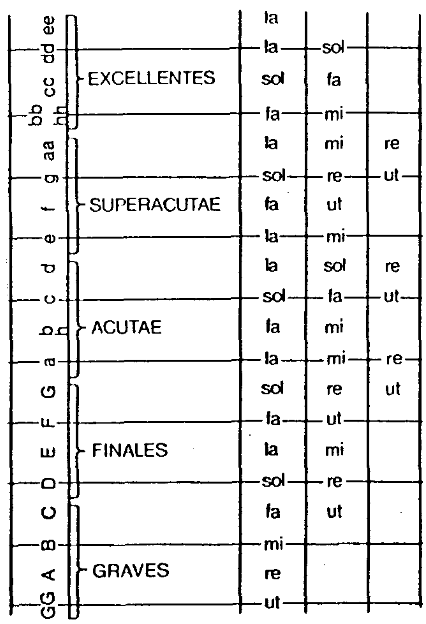


1 "Guido's Hand"



2 The extension of the hand's tone-scale with subdivisions of tetrachords and hexachords

Guido's four-line system became universally adopted in Western Europe, but this was a slow process. For immediate need Guido invented mnemonic devices such as the so-called "Guido's Hand" which helped to speed up the time it took singers to learn a new chant.

Gerbert's contribution to the study of music as far as is known belongs entirely to the abstract knowledge of the quadrivium in which he was unsurpassed in his time; the metaphysical and artistic aspects he left to others. His contribution to the cultural development with regard to the liberal arts, is twofold:

1. Gerbert made Boethius' De Musica and De Arithmetica more readily accessible to scholars by simplifying the language and illuminating the speculative arithmetic involved with practical examples, e.g. a superparticular proportion changes its proportion when it is multiplied by two, because neither the old proportion remains nor is a multiple of it created.[32]
2. By teaching a great number of students all they could absorb about the properties of numbers and their conversions from one ratio into another he made this discipline accessible to a greater

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[32] Lattin, The Letters of Gerbert, letters 4 and 5. Gerbert there explains a passage in Boethius, De Musica II, 4-6, to Constantine of Fleury.

number of people of all ages.[33] This in turn percolated through following generations of students who were able to build on these foundations.

Detailed records of the mathematical activities of Gerbert's contemporaries are scarce but not insignificant.[34] In the first place there are those men who studied with Gerbert at Rheims and went on to high positions in the Church: Herbert, teacher at Chartres, later Abbot of Latigny, famous for his erudition and beautiful voice;[35] Leutheric, later Archbishop of Sens; Richard, later Abbot of Vannes near Verdun; Fulbert, later Bishop of Chartres, praised by his contemporaries as the Socrates of the North. Fulbert wrote several didactic poems with arithmetical and astronomical themes. These rhymes were used for mnemonic purposes. Gerard, a relative of Archbishop Adalbero of Rheims, spent his formative and study years at the Cathedral School at Rheims during Gerbert's time. He later became chaplain to Henry II, and Bishop of Cambrai in 1012. Another student, Ingo of Helgald, was rewarded with the monastery St. Martinus Masciacensis and later with that of St. Germain in recognition of his erudition.

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[33] Gerbert taught at the Cathedral School at Rheims from 972 until 980 when, at the height of his fame as a teacher, he was called to diplomatic duties.

[34] Lindgren, Sudhoffs Archiv, Beiheft 18, pp. 40-68.

[35] ibid., p. 45.

In the second place there are Gerbert's illustrious lay students of a decade or so later: 1. Robert, son of Hugh Capet, King of France and later king himself, was taught by Gerbert intermittently between about 987 and 991. 2. The German Emperor Otto III wrote personally from the imperial residence at Aachen on 21 October 997 to Gerbert inviting him to become his teacher and explain to him "the book on arithmetic"[36] so that he could learn something of the attainments of the ancients. To this Gerbert replied four days later from his place near Sasbach "....For, unless you were not firmly convinced that the power of numbers contained both the origins of all things in itself and explained all from itself you would not be hastening to a full and perfect knowledge of them with such zeal...."[37]

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[36] Lattin, The Letters of Gerbert, letter 230, pp. 294, 295. The book on arithmetic may have been Boethius's De Arithmetica, MS Bamberg Class. 5 (HJ. IV. 12) written on purple parchment in gold and silver letters; it was produced at the scriptorium of Tours for Charles the Bold, c 832 and was sent to Otto by Gerbert as a gift truly fit for a king, footnote 2 to letter 230.

[37] Otto was 14 years of age when this exchange of letters (No. 231) took place. Until Gerbert became Pope in 999 he spent much time with Otto. His last letter extant before his elevation to the throne of St. Peter is a long discussion of the difference between arithmetical and geometrical procedures for finding the area of a triangle, addressed to Adalbero, later Bishop of Utrecht, letter 233, p. 299-302; Adalbero might be called a student by correspondence, he later asked Gerbert for help with finding the volume of a sphere and had previously had copies of manuscripts on surveying, etc. from Gerbert.

The Ottonian Emperors made great efforts to further education in every way. Otto I tried to draw the most learned men to his court.[38] He had offered Gerbert the position of tutor to his son, the future Otto II, at their first meeting in Rome in 971, which was however declined.

Two examples of the preoccupation with learning in general and the theory of numbers in particular in circles closely connected to the court, must suffice. The first one is an allegorical poem by Walter of Speyer, 965-1027, which he wrote on instruction from his bishop, Balderich of Speyer (died 987) in order to furnish proof of his learning.

Balderich himself was educated at the famous Abbey of St. Gallen and opened a similar school when called to the Bishopric of Speyer, which had been founded in the seventh century.

Walter describes his education in the first hexameters of his book, The Life and Martyrdom of St. Christopher.[39] As the following excerpt of lines 148-169

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[38] The scholar Gunzo of Novara travelled with Otto I from Germany to Italy between 956 and 966, taking with him nearly 100 books. Lindgren, in Sudhoffs Archiv, Beiheft 18, p. 48.

[39] Walter's Vita et Passio S. Christophori Martyris is in 5 books, (29 chapters). The 271 hexameters about his education are followed by 1272 lines of verse on the main subject. This manuscript, or possibly a contemporary copy of it, was sent to Salzburg on the death of Walter, who had become Bishop of Speyer and

will show, Walter was not only familiar with Boethius, whom he salutes, but makes plausible his claim that he studied, apart from Boethius, Martianus Capella,[40] one of the most widely-read Latin authors in the Middle Ages, probably because he assisted in the acquisition of knowledge and at the same time provided light relief from the study of too much dry material.

During the tenth century Martianus Capella was quite well known in France[41] and Northern Italy as well as in St. Gallen. Notker Labeo (died 1022) wrote a commentary on De Nuptiis Philologiae et Mercurii of Martianus Capella and translated books I and II into Old High German.[42] Through the St. Gallen connection of Balderich Walter may well have made a special study

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had accompanied Henry II to Rome in 1014 for his coronation as Emperor. Bernhard Pez, librarian of the Stiftsbibliothek, Melk, published the first part of the MS in his Thesaurus anectotorum novissimus, Vienna, 1721-1729, vol. II, p. 29-122, from the MS Munich, Clm 14698 saec. X.

[40] In verses 93-105 Walter lists the ancient authors he claims to have been familiar with, amongst them Homer, Horace, Juvenal, Terence, Lucan and Virgil. In verses 125-147 he introduces each subject of the trivium, in verses 148-223 each subject of the quadrivium.

[41] Gerbert quotes Martianus Capella in letter 161 to Brother Adam, 10 March 989, Lattin, The Letters of Gerbert, pp. 189-191.

[42] W. Stahl, R. Johnson, with L. Bruge, Martianus Capella and the Seven Liberal Arts, vol. I, New York, Columbia Univ. Press, 1971, p. 65.

of Martianus'[43] work and adopted not the extraordinary style, but the device of allegory and "married" eloquence and learning, that is the arts of the trivium and quadrivium to describe his education. Walter uses rhetoric to put forward the universal application of the subjects of the quadrivium, so dear to the heart of the most advanced of the students of the late tenth century.

Walter of Speyer, Vita et Passio S. Christophori Martyris, Excerpt from Book I, Verses 148-168.

"The rhythm of the whole went ahead of the five girls -
Listen, reader, to the terms for the phrases, because you see
That these are not ideal for the dactylic metre:
The first, radiating many metallic beams from her head,
Tells her sister dawdling behind to go faster;
At the same time, the dawdler twists her head round
And says to her companion following her:
"Take the bracelets from my hand with you;
For I entrust this (part of it?) mirror to our companion,
Whom I produced once upon a time situated in our
Proper position". The fifth attendant would stand with her head bare
If the third did not offer her a milky-white head-band.
They went with a circle of followers surrounding each of them,

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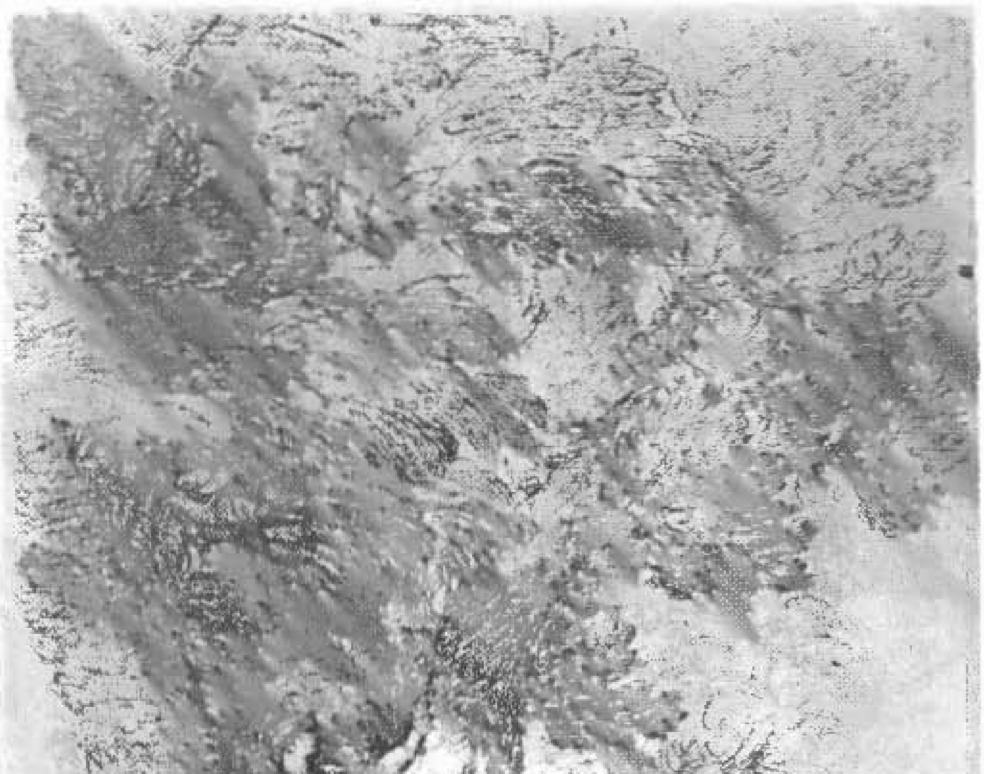
[43] Modern critics (probably unaware of the importance of allegorical and rhetorical culture,as research in that field is relatively recent) are unduly severe on Martianus Capella; e.g. H.J. Rose, A Handbook of Latin Literature, p. 458, says of The Marriage of Philosophy and Mercury: "It is the dullest and poorest stuff imaginable". C.S. Lewis, The Allegory of Love, p. 78: "For this universe, which has produced the bee-orchid and the giraffe, has produced nothing stranger than Martianus Capella".

And after they filed the shapes level as was required
With the measurements of the gaps and spaces duly arranged,
They strove to form cubic shapes and see three dimensions happening.
One distance bind together the contribution of the first dimension;
Equal proportion contains the number of the other;
The mediator rejects both these enclosures from underneath.
Mathematicians rejoiced in these arrangements preserved in due order
Ready to send you consolation, famous Boaet(i)us."[44]

These verses were written in about 980, when Gerbert stopped teaching at Rheims, but when the quadrivium as well as the trivium was studied by an increasing number of students. Walter here, perhaps in a rather wordy and obscure way, refers to the "musica mundana" of Boethius, which includes the harmony of the spheres, the harmony of the elements and the harmony of the seasons. To the four elements, sources of all created things, he adds a fifth, ether, if my interpretation is correct. Now Walter puts the five elements, the five girls, into position. Fire tells her sister to walk faster, not to let the flames scorch Earth. Water gives rain, or dewdrops (bracelets) to Earth and the mirror to Air (water had originally produced air). Air gives rise to Ether which is the substance of heavenly bodies - the milky-white headband - the Milky Way. Walter's elements create geometrical

[44] For the original Latin text see appendix D.



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Pietro da Cortona, Girls inscribing Trees solids instead, like Plato's Demiurge forming the elements out of geometric shapes in the creation.[45] About 600 years later this concept may still have lingered. Pietro da Cortona, 1596-1669, perhaps visualised it in a fine drawing called "Girls inscribing Trees",[46] unless he simply followed Ariosto's tradition of the "nodi d'amore", so much part of the Baroque (amorous) culture, to which he belonged.

A second example comes from the Latin plays of Roswitha of Gandersheim already mentioned in connection with the play-element in the development of culture.

Through the mouth of the hermit Paphnutius in the play of that name Roswitha[47] expounds the Aristotelian philosophy as presented in Boethius and the

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[45] Now at least 104 elements are recognised and can exist as gas, liquid or solids. Using Cesare Ripa's direction in the introduction of his Iconologia, where he develops the theory of allegorical personification in conscious analogy to the Aristotelian theory of definition, we can expect the human figure, the girl, to stand for the substance or essence, the emblem she wears, for its "attributes". Gombrich, Journal of the Warburg and Courtauld Institutes, p. 183.

[46] R. Lee, Names on Trees; Ariosto into Art, New Jersey, Princeton Univ. Press, 1977, p. 84, is puzzled by the title and says: "What is the literary source of this fine drawing? I have found none, and stand ready to believe that it sprang fully armed from the artist's head and hand. But if any learned reader can counter so naive a view, I shall be ready to listen to him." Should one not rename the drawing: "Five Elements creating Trees"? Cesare Ripa's Iconologia, the 1758-60 Hertel ed. with 200 engraved illustrations, tr. Edward A. Maser, New York, Dover Publ., 1971, does not seem to touch on the subject under discussion.

[47] Some historians have suggested that the nun Hazecha

ideas of early scholasticism, based on the concept of "substance". According to this philosophy man carries in him the knowledge that everything he perceives is a different manifestation of one and the same substance. The spiritual substance is God, angels and the soul; corporeal substance is everything perceived by the five senses, that is the four elements which make up, singly or in combination, every other substance. The macrocosm, which Roswitha calls her "big world", is entirely composed of the four elements with God as its soul and obedient to the slightest wish of God, just as every human body obeys the demands of the soul. The microcosm, composed of corporeal substance, the body, and of spiritual substance, the soul, is Roswitha's "little world". Because man is a world in himself and not under the complete control of God he often transgresses God's laws.

Paphnutius discusses the quadrivium in general and music[48] in particular in Boethius' definition and

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may be identical with Hrotsuits, the Old High German for Roswitha, both of Gandersheim, e g. Lindgren, Sudhoffs Archiv, Beiheft 18, p. 50. Walter of Speyer says in Book I that Hazecha had submitted a poem about St. Christopher to the Bishop of Speyer. This work was lost through the carelessness of the librarian and Walter was asked to write on the same subject.

[48] His answer to the student's question why we cannot hear the music of the spheres is that some say our ears no longer note it from constant exposure to it, some that the air is too thick to let the sound through, some that our ears are too small, but others believe

explains three "consonances", the fourth, fifth and second (9:8). The octave he calls a "symphonia". Paphnutius proclaims all is harmony, all is proportion, wonderful are the laws of the Lord. For him, as for his age, mathematics is solely directed towards the contemplation of the universe: "For the more someone realises how God, by a most wonderful law, could order all things in number, measure and weight, the more he burns with love for him".[49]

In the play Sapientia the heroine of that name, about to be martyred with her children, lectures the Emperor Hadrian on "perfect", "deficient" and "abundant" numbers by using as an example of numerical relationships the ages of her daughters.[50] She explains to him "odd" and "even" numbers and their properties and finishes with the confident assertion that the founder of the universe has created the world out of nothing by means of number, measure and weight, but also grants one a human life-span to fathom the wonders of science.

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the music to be so sweet, that if it could be heard, everyone would leave home and duty and follow that sound for ever.

[49] "Quanto enim mirabiliori lege Deum omnia in numero et mensura et pondere posuisse quis agnoscit, tanto in eius amore ardescit". Hrotsvithae Opera, "Pafnutius", ed. Homeyer, p. 334.

[50] Sapientia says the respective ages of her children are: two olympiads (eight years), two lustra (ten years) and three olympiads (twelve years). These numbers she then places into their relative proportions.

Roswitha may have written for a small, educated circle of friends in high places, certainly with the encouragement of her abbess, but these digressions into speculation with numbers and philosophy were probably understood by all of them. As a devout nun she would not include such passages solely to air her own erudition. This shows the preoccupation with learning in the last third of the tenth century in both clerical and lay circles,[51] including nuns and possibly lay women.[52]

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[51] In her introduction to her plays Roswitha addresses herself by inference, to what might be both clerics and laymen and says she wrote at their insistence, but mentions no names.

[52] Only well-educated, high-born ladies could in times of need successfully be recalled to their duty in the world, e.g. the Abbess Mathilda of Quedlinburg took over the regency for her nephew Otto III during his lengthy absences abroad. H. Prutz Geschichte des Mittelalters, II. Teil, p. 257. It seems possible that Queen Adelheid rather than King Hugh Capet insisted on securing Gerbert as tutor to the heir to the throne, Robert. Lindgren in Sudhoffs Archiv, Beiheft 18, p. 40.

The Period after Gerbert

Speculation with numbers as set out above widened in scope and increased in intensity during the eleventh and twelfth centuries, helped by the founding of more cathedral schools and later also universities. At the cathedral school of Chartres and in Paris, especially at the convent of St. Victor, scholars codified various numerologies for biblical exegesis. Men like Thierry of Chartres, his brother Bernard and William of Conches believed in the magical significance of numbers and geometrical shapes, and their usefulness for leading men to a greater understanding of the cosmos.[1] For these scholars, some of them chancellors of Chartres under the patronage of their bishop Godfrey, theology was the highest form of arithmetic. For Thierry, for instance, the equilateral triangle expressed the quality of the Holy Trinity and the square demonstrated the relation between God the Father and God the Son: God the Son is a unity begotten by a unity, just as the

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[1] Thierry of Chartres wrote a cosmographia and a programme for the liberal arts called _Heptateuchon_; William of Conches wrote a _philosophia mundii_. Their metaphysical speculation was dominated by Plato's _Timaeus_ and, in the case of William, to some extent also by Lucretius. A. Crombie, _Robert Grosseteste_, Oxford, Clarendon Press, 1953, p. 12n.

square results from the multiplication of a number by itself. The nature of God the Son, Thierry explains as a rectangle.[2]

Some philosophers endeavoured to construct a system based on numerical relationships in which number, light, music, and architecture combined to prove the nature of God himself. Bernard Silvestris, William of Conches and Alain de Lille, amongst others, used symbolic and allegorical language and the authority of classical writers to widen their speculation into the involvement of "natura" and its role in the regeneration of the cosmos.

At the school of St. Victor, the German Hugh (died 1141) and his pupils Richard and Adam expanded the numerological tradition of allegorical exegesis inherited from St. Augustine,[3] Cassiodorus and others. They also used Pythagoras' ideas of numbers as expressions of symbolic and allegorical functions, rather than

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[2] According to H. Huntley, <u>The Divine Proportion: A Study in Mathematical Beauty</u>, New York, Dover Publ. 1970, p. 61 the "golden rectangle" is truly of divine proportion; it has the property of recreating itself: if one takes a square off the golden rectangle, there remains another golden rectangle; this process can be repeated until a "point rectangle" remains.

[3] St. Augustine's <u>The City of God against the Pagans</u>, tr. G. E. McCracken, London, Heinemann 1957-1972, is probably constructed in a number-symbolic way. It has 22 sections possibly because of the 22 letters of the Hebrew alphabet, and these are subdivided into two groups of five and three groups of four.

as scientific symbols. They incorporated numbers into Christian theology in a logical ordering that could be manipulated by using several methods to arrive at symbolic meanings of considerable complexity.

Because numbers existed for philosophers on different levels,they possessed quite different realities. In order to unify different spheres of knowledge, allegory and metaphor were used, with the inherent danger that only a structure of words might result. As long as it was believed that God had created the universe by numbers, then numbers, if correctly applied, would pin-point the reality of the cosmos and would explain the true meaning of the Bible, the movements of the stars and their resultant heavenly music, and would guide the architect in the construction of an enclosure which would contain all that is holy and divine. Gothic cathedrals, stone edifices in an age of building with wood, constructed by medieval architects by "just means", notably the square and triangle in every variety of combinations, from the ground plan to the smallest detail, were the physical expression of the metaphysical thinking of the age.[4]

John of Salisbury (died c 1180) was one of a number of men who, for all their love of speculative

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[4]On musical consonances in Gothic architecture, see O. von Simson, The Gothic Cathedral, New York, Princeton Univ. Press, 1956, p. 21-58.

In the next century one of the most original thinkers of his time, Robert Grosseteste, Bishop of Lincoln (died 1253),took the veneration of "natura" as practised at Chartres and harnessed it to the Platonic speculation of the light - number principle. His contribution to mathematics and the theory of experimental science was to raise contemporary science above its haphazard empirical techniques.[6] This tradition, practised at Oxford, found its greatest and most imaginative exponent in Roger Bacon (died 1292).[7]

The above concludes my observations on some aspects of the medieval absorption with numbers which influenced philosophy, mathematics, music and architecture in the Middle Ages. It influenced the way scholars wanted to work and liked to amuse themselves.

The game of Rhythmomachia afforded both instruction and relaxation. It satisfied the agonistic instinct as well as the instinct for self-improvement and the mastering of problems. A thousand years after its inception it is not possible to say if Rhythmomachia was invented specifically as a teaching-aid for beginners that later turned into a pastime for mathematicians; or if the greatest masters set out to

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[6] Crombie, Robert Grosseteste, "Mathematical Physics" pp. 91-127, and "Metaphysics of Light", pp. 127-134.

[7] ibid. p. 139.

invent a game for their amusement, that also satisfied their intellectual capacity. No doubt the game was played to suit the individual requirements of lovers of the quadrivium. It made comprehensible and visible the proportions of numbers and their progression, playfully instructing the beginner and delighting the expert. Students of music who also needed to learn proportions for their theoretical work could do so more quickly with the help of Rhythmomachia than by studying all of Boethius' De Musica.

Rhythmomachia was an exclusive game for a smaller minority than chess. During the eleventh century chess was played by almost all the nobility and clergy, including many who had become monks and nuns.[8] During the thirteenth century the chess-craze spread to all levels of society except the peasants. This did not happen to Rhythmomachia; it stayed, as far as is known, in clerical and monastic circles, especially amongst the Benedictines, as long as Neo-Pythagorean and Neoplatonic philosophical speculation remained dominant.

The Aristotelian view of the world provided by

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[8] The monk Honorius Augustodunensis, c 1120, in his teaching manual De artibus lists chess as part of the quadrivium. Chess was particularly cultivated in aristocratic foundations, such as the Benedictines, especially at Einsiedeln (Switzerland) and Tegernsee (Bavaria). The Dominicans, leaders in theology and canonical law, used chess similes, moralities and allegories in their theological and ethical instruction and preaching. H. Faust, in Rochade-Sonderdruck, 168, Juli 1978, p. 2.

Thomas Aquinas brought about a change in outlook. Certain circles, however, such as the Franciscans at Oxford, maintained their interest in Platonic philosophy and possibly also an interest in Rhythmomachia. During the Renaissance some lay and clerical scholars, especially cultured men and mathematicians, took an interest once again in Rhythmomachia.

Manuscripts of the rules of Rhythmomachia from the eleventh to the sixteenth century are not at all rare, but evidence of how, where, when and by whom the game was played is a little harder to find.

In the next chapter six of the earliest manuscripts of Rhythmomachia will be discussed. One of these is attributed to the Benedictine abbot and musicologist Fortolfus[9] (died 1103) of St. Michelsberg at Bamberg. His Rythmimachia[10] is of exceptional interest because Fortolfus was an important musician, the author of Breviarium de Musica and Tonarus.[11] In these works he incorporates his own studies of the

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[9] The spelling of Fortolfus varied considerably: Frutolf, Frotolfus, Rudolf or Frodulf. G. Sarton, Introduction to the History of Science, Baltimore, Williams & Wilkins, 1927, vol. I, p. 763 says Abbot Fortolfus of Bamberg was probably the author of this Rythmimachia.

[10] The original manuscript of the Rythmimachia is in Wrocław, Biblioteka Uniwersytecka (formerly Breslau, Stadtbibliothek) Rehdig. n 54, f 86a-94b.

[11] The original is in the Bayrische Staatsbibl. Munich, Clm 14965b (ex. Kat. S. Em. 965 y 3).

theory of music with the work of other writers, and produces a harmonious whole at a time when a systematic treatment of all the available material on the theory of music was still lacking. Fortolfus set down, in a logically ordered form, the complete theory (at that time) of one-voice church music. In the introduction to the Breviarium, Fortolfus calls his treatise a schoolbook or handbook for oral teaching and gives a definition of the word "music", which is identical with that given by his predecessors. Music is the "scientia bene modulanti", the theory of music, not the practice. "Modulari" to the teacher of music means essentially composing. The composer has to be aware of the musical-philosophical implications when selecting a melody and its important end-chord.

Fortolfus' Breviarium begins with the study of the monochord which helps to demonstrate musical intervals.[12] When dealing with musical consonances Fortolfus, like almost every medieval music theorist, follows Boethius and through him Pythagoras, who decreed that not the ear but mathematics decides what is a consonance. The ear may only be trusted in so far as it does not contradict arithmetical proportions.[13]

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[12] C. Vivell, "Das Breviarium de Musica des Frutolf von Michelsberg", in Studien und Mitteilungen zur Geschichte des Benediktiner-Ordens und seiner Zweige; Neue Folge Jahrgang 3, 1913, p. 413-423.

[13] In this they contradicted their own theory by decla-

In the manuscript with which we are concerned, Fortolfus carefully sets out, in detail, the rules of Rhythmomachia (in Latin),[14] but also finds it necessary to defend the game in a "Prologus in Rithmimachiam" against attacks by the ignorant who judge by appearances. He calls Rhythmomachia an art but classifies it as a natural science because "numbers occur by nature".[15]

He declares that a proper knowledge of the game is useful and also brings pleasure, and he quotes Horace in praise of that which instructs and amuses at the same time. Far from being "irreligious and worthless",[16] it in fact employs number, provided by nature, and used by God himself in the creation of the

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ring that the "proportio multiplex" 2:1, 3:1, 4:1 and the "proportio superparticularis" 3:2, 4:3, 9:8 produced a consonance. But they refused to recognise the full tone, the second 9:8, because its dissonance offends the ear. In actual musical practice they allowed the ear to be the judge, thereby following the pupil of Pythagoras and his protagonist, Aristoxenos, whom they condemned for his theory that the ear alone must be the judge of consonances. For the same reason musicians also condemned the third, 5:4, a conconance, despite its pleasing sound.

C. Vivell, <u>Studien und Mitteilungen</u>, p. 420.

14 Fortolfus, <u>Rythmimachia</u>, MS n 54, Wrocław. Refences here are to ed. R. Peiper in <u>Abhandlungen zur Geschichte der Mathematik</u>, 3, <u>Supplement zur historisch- literarischen Abteilung der Zeitschrift für Mathematik und Physik,</u> Band 25, 1880, p. 203.

15 ibid., Book I, ch. V.

16 ibid., Prologus p. 169.

universe by "measurement, number and weight".[17]

Number, says Fortolfus, is concerned with the elements, the seasons and the movement of stars, but the ignorant engaged in idle merry-making decry, from envy, that which they do not understand. The art of Rhythmomachia is worth knowing because those trained in arithmetic quickly penetrate to its depth, but the untrained can also enter, if at a slow and more laborious pace.

In Book I, chapter III, Fortolfus declares that Rhythmomachia, without boring the less well educated, affords them access to arithmetic in a summarised form. It teaches the three progressions (arithmetical, geometrical and harmonic) and helps to make visual what Boethius describes at length in his De Arithmetica. Rhythmomachia, says Fortolfus, springs from the fountain-head of all seven liberal arts, namely arithmetic. Geometry, music, astronomy would not exist if arithmetic did not exist first - without it there would be no square, or triangle, diatesseron, diapente or diapason. The substance of number will endure if the other arts are removed, but if arithmetic is removed nothing endures (chapter V).

In chapter VIII Fortolfus poses the question of

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[17] Fortolfus, Abhandlungen, Prologus, p. 169.

the origin of numbers themselves, and answers by quoting Boethius' views on "good" and "bad" which can be likened to "even" and "odd". Evenness, which is the root of everything, also produces oddness which is a straying from evenness.

Fortolfus shows the several levels on which Rhythmomachia could be played:

1) to practise arithmetic with its proportions and progressions illustrated by diagrams.[18]

2) to use the game for philosophical speculation on the origin and properties of numbers and God's use of them in the creation of the universe.

For Neoplatonic philosophers number existed in nature from the beginning of the world and God naturally arranged the world in its pattern: immanence is stressed rather than transcendence. Number takes on an almost magical significance and the play-element in these speculations can hardly be doubted. From the playground for numbers grew a discipline adopted by architects steeped in this philosophy: number, circle, triangle and square "engineered" Gothic architecture. God the Father was pictured in the Renaissance as the

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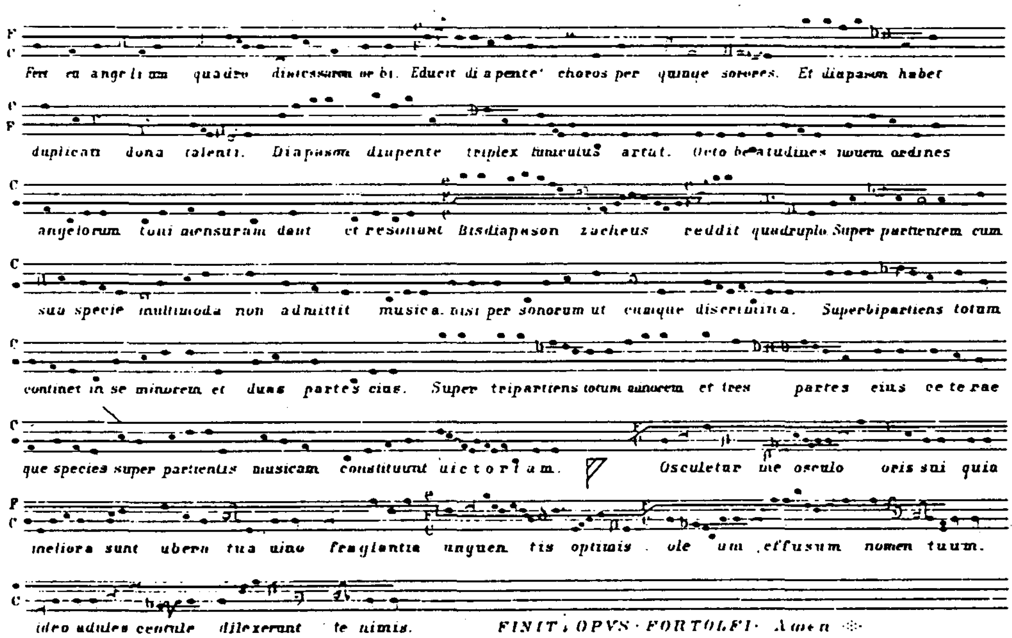
[18]Fortolfus, Abhandlungen, pp. 190-195. See appendix C for an English translation and summary of the work.

architect of the world, holding the tools of an architect in his hands.[19]

Fortolfus ends his manuscript with a poem set to music. The notation is that of the eleventh century; the words are allegorical.



11 Last MS page of Fortolfus' <u>Rythmimachia</u>[20]

1 The Gospel carries the fourth to the four corners of the world.

2 The fifth starts dances throughout the five sisters.

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[19] Taking as biblical reference Proverbs VIII, 27 : "When he prepared the heavens I was there: when he set a compass upon the deep".

[20] Translation in collaboration with Prof. H. Schlagbauer and Dr. C. Coleman.

3 And the octave is endowed with the double talent.

4 A triple tie links the octave and the fifth.

5 The eight beatitudes sound and give the nine ranks of the angels a tonal measure. The zacheus quadruples the double octave.

6 Superpartients [for example 4:5] in all their forms cannot be admitted as music, unless the sound decides otherwise.

7 In the superbipartients the consequent contains the antecedent once and two of its parts.

8 Supertripartients, in which the consequent contains the antecedent once and three of its parts, and the remaining types of superpartients, constitute a victory of music.

9 Let him kiss me with a kiss from his lips (she turns to the men) your fragrant wine-filled kisses are better than the best perfumes and your name is redolent with ointment. That is why the young girls have admired you exceedingly.

Analysis of the Fortolfus Song.

An analysis of the Fortolfus song can be derived from the number symbolism practised[21] by the outstanding theoretical musicians Berno (died 1048) and Hermannus Contractus in Reichenau, one of the most famous musical

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[21] Brambach, in <u>Zentralblatt für Bibliothekswesen</u>, p. 48.

schools of the time. They believed that music is number in mutual relation.

Line 1 It was believed that through divine inspiration originally the end tetrachord D E F G was found. The number "four" was given a major part in Creation, such as in the music of the spheres, the human voice, instrumental music, the formation of the first cube, elements and the seasons.

Influenced by the mystery of this number the harmonic voice of the four Evangelists went forth to the four ends of the world. The Psalmist knew that such harmony pleases the Lord; he calls four times to sing praise : Psallite Deo nostro, psallite: psallite ergi nostro, psallite sapienter.

Line 2 The fifth is also a consonance, (quint or diapente). Its sound moves the five elements: air, water, earth, fire and ether, which are involved in Creation; like the five girls in Walter of Speyer's poem.

Line 3 The octave or diapason contains a "double diapente" (4:2:3), the double talent.

Line 4 A triple proportion makes up a diapason (octave) and diapente (fifth), for example 27 includes the proportions 27:18:9.

Line 5 Matthew 5:3-11 names eight beatitudes. Here they make the ninefold angelic celestial hierarchy dance (guardian angels belong to the lowest order).
"Zacheus" refers to a chess-piece. The earliest name given to it in Europe was "scacus", which acquired a wider meaning of "game-piece" or "man". Quadrupling the octave gives 64, the number of squares of the chess-board.

Lines 6-8 Lines 6, 7 and 8 simply explain the meaning of the various types of superpartients. In line 6 it is stated that the ear, not theory, must decide what is a musical sound. Line 8 describes supertripartients and states, again, that music, (that is sound), is victorious - over mathematical musical proportions.[22]

Line 9 is from the "Song of Solomon", 1:1-3 and 4:10.

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[22] This was controversial at the time. Some musicians would have insisted that theory must decide what is a musical sound because they adhered strictly to Pythagoras and Plato who "restricted consonances to ratios made out of their tetractys (1.2.3.4.) and

As Fortolfus places this poem, set to music as a finale to his Rythmimachia, he may have wished to say that the study of the quadrivium and its teaching aid, Rhythmomachia, was a means not only to education, but to spiritual perfection.[23]

therefore failed to include thirds and sixths..." D.P. Walker, "Kepler's Celestial Music", Journal of the Warburg and Courtauld Institutes, vol. 30, 1967, p. 235.

[23] Bukofzer, "Speculative Thinking in Medieval Music", Speculum, vol. XVII, p. 166.

4

MANUSCRIPTS OF RHYTHMOMACHIA IN THE MIDDLE AGES AND THEIR POSSIBLE AUTHORS

One of the difficulties in dealing with medieval manuscripts is identification.[1] At that time only Christian names were used, sometimes with sobriquets, sometimes with place names. Vanity quite often prompted simple monks to claim the authorship of work done by others, for the greater glory of their particular establishment. Even in well-documented lives some confusion may never be resolved. For instance John of Salisbury left an autobiography and many letters and yet it is still not possible to establish exactly who his teacher "Master Gilbert" was.

Research into manuscripts of Rhythmomachia has led to the conclusion that no entirely original work on the game has yet been found, if it ever existed. Just as the new reckoning spread by word of mouth initially rather than through the production of tracts, so Rhythmomachia may have been propagated at first by playing and may even have incurred disapproval from certain conservative sections always reluctant to examine new ways. As time went on particular points more difficult to remember may have been written down. The

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[1] Library catalogues of manuscripts that concern us here usually date from the middle of the nineteenth century; e.g. the catalogue of the Handschriftensammlung, Vienna, vol. II is dated 1868, also Munich, vol. IV, 1876. Attribution of texts on Rhythmomachia has

extant manuscripts of the eleventh century which are the earliest that have survived point very strongly in this direction.

Our concern for the moment is with six early medieval codices of which copies from the eleventh, twelfth and thirteenth century exist in various libraries, for instance in Vienna, Munich, Paris, Brussels, Montpellier Avranches and Dresden.

These manuscripts on Rhythmomachia are either anonymous or attributions; of the writers, only one can be identified with reasonable certainty. This leaves open the question of the identity of the inventor of Rhythmomachia - if there was <u>one</u> inventor.

At best one can say that if Rhythmomachia was not devised by a group of scholars, but was the brain-child of a single man, prominent in his time (not someone unknown who had one good idea), then this inventor must have fulfilled certain requirements:

1. He must have lived not earlier than 980.[2]

2. He must have been a chess player, which at

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in most cases been accepted in good faith and was allowed to remain unquestioned for several hundred years. Examination by experts in palaeography of all the relevant MSS could clear the picture a little. However, the common usage of scribes or scriptoria can still leave the identity of the originator of an invention in some considerable doubt.

[2] This is the terminus post quem deduced from the manuscripts consulted.

that time places him amongst the highest in rank at court or in a cathedral.

3. He must have been very well educated in the liberal arts with emphasis on the quadrivium - the latter was quite exceptional in the tenth century - and he must have been familiar with Gerbert's "apice".
4. He must have had the leisure time for such intensive study and at the same time have shared his interests with like-minded men.

Suggestions can only be put forward after an examination of a selection of representative manuscripts which date from the tenth to the twelfth-century:

a) Odo, Vienna, Österreichische Nat. Bibl., Handschriftensammlung, codex 2503

b) Hermannus Contractus, Munich, Bayrische Staatsbibl., Clm 14836

c) Gerbert, Paris, Bibl. Nat. lat., codex 7185

d) Asilo, Rome, Vat. lat., codex 3101

e) Fortolfus, Wrocław, Univ., n 54

f) Regule S. Benedicti (insertion). Munich, Bayrische Staatsbibl., Clm 28118.

a)

The <u>Tabulae Codicum Manu Scriptorum in Bibliotheca Palatina Vindobonensi</u>,[3] lists the contents of codex 2503 as follows:

"(Univ. 643) m XIII. 57. 8°.

1) 1^a - 17^a. Guido de Aretio, Micrologus de musica.

2) 17^a - 23^a. Idem, Regulae musicae rhythmicae.

3) 23^a - 32^a. Idem, Regulae musicae de ignoto cantu.

4) 32^a - 37^a. Isidorus Hispalensis, Sententiae ad Braulionem episcopum de musica et Excerpta de rhythmimachia.

5) 37^a - 42^a. Oddo abbas Cluniacensis, Dialogus de arte musica. Incip.: 'Quid est musica? M. Veraciter canendi scientia...' Expl.: 'Et plaga tetrardi modum VII et octauum.'

6) 42^a. Idem, Quomodo organistrum construatur et de fistulis.

7) 42^a - 43^a. Gerlandus Vesontinus, De fistulis et nolis.

8) 43^a - 48^b. Oddo Cluniacensis, Regulae super abacum.

9) 48^b. Gerlandus Vesontinus, Nomina characterum. Incip.: 'Hec sunt nomina karacterum cum figuris...' Expl.: 'Sicilicus.'

10) 49^a - 57^b. Oddo Cluniacensis, Regulae de rhythmimachia.

11) 57^b. Gerlandus Vesontinus, Ad pectus purgandum apozimata et electuarium ed vocem clarificandam.

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[3] vol. I-II, Cod. 1-3500, Graz-Austria, reprint 1965.

Cum notis marginalibus."

In the above, item (4) contains "Excerpta de rhythmimachia", comprising 22 lines in all, by Isidorus (of Seville c 570-636) Hispalensis. Similar material is also in Munich, Clm 6369, fo.66a, but there listed as anonymous.

The Munich manuscript originally came from the Dombibliothek in Freising and bears in the right-hand margin faintly, by a later hand, the words "Gerbert, SS. eccl. de mus. I. 25. Excerptum de Rythmimachia".[4]

Abbot Martin Gerbert of Hornau, 1720-1793, a Benedictine music historian, who spent most of his working life transcribing and editing medieval musical manuscripts,[5] copied item (10) 49^a - 57^b Odo of Cluny, "Regulae de rhythmimachia" and published it in Latin without a translation in <u>Scriptores ecclesiastici de musica</u> in 1784.[6] Possibly because codex 2503 contains other work by Odo of Cluny, 879-942, Gerbert of Hornau attributed the manuscript about Rhythmomachia also to Odo.

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[4] The attribution of item (4) to Isidorus of Seville is erroneous as he belongs to the seventh century. The marginal remark on the "Excerpta de rhythmimachia" refers to Gerbert of Hornau. For a copy of the relevant page see appendix A.

[5] Some editorial notes have been appended at the end of four out of ten pages. For a copy of the Rhythmomachia of codex 2503 see appendix A.

[6] Martin Gerbert, Abbot of Hornau, <u>Scriptores ecclesiastici de musica sacra potissimum</u>, San Blasianis, vol. I, pp. 285-295.

Modern critical research into the authorship of medieval tracts on music is constantly contributing to the compilation of a correct register of extant manuscripts.[7] For instance an attribution by Abbot Martin Gerbert of Hornau of a manuscript, entitled "Musica Enchiriades", Paris, Bibl. Nat. Fonds lat.7202, to Hucbald of St. Amand, in Scriptores ecclesiastici de musica sacra potissimum, 1784, I, p. 152-212, has been proved erroneous according to Hans Müller, "Hucbalds echte und unechte Schriften über Musik", 1884, as quoted by R. Weakland.[8]

The Rhythmomachia under discussion, attributed by Martin Gerbert to Odo of Cluny, has not yet been subjected to examination by experts. This attribution rests solely on Abbot Martin Gerbert. The handwriting of the manuscript is Carolingian, showing rounded characters, and possibly dates from around 1200. Its place of origin could be northern Italy, but the handwriting suggests no particular scriptorium.[9]

The history of codex 2503 is only known since the sixteenth century when it passed from the possession

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[7] M. Huglo, "Der Prolog des Odo zugeschriebenen 'Dialogus de Musica'", Archiv für Musikwissenschaft, vol. 28, 1971, p. 134.

[8] R. Weakland, "Hucbald as Musician and Theorist", in The Musical Quarterly, vol. 42, 1956, p. 66.

[9] Dr. E. Irbich of the Handschritensammlung, Österr.

of Alexander Brassican to Bishop Johannes Fabri, who left his library to the St. Nikolaus-collegium in Vienna in 1540. From there it went to the old Vienna University Library and then arrived in the Imperial Court library in 1756, now the Österreichische Nationalbibliothek. The "Regulae de rhythmimachia" in codex 2503, 49^a - 57^b, attributed to Odo of Cluny (hereafter called the Odo MS) starts with proportions, interrupts that with "De tabula", continues on the pyramid and then ends abruptly. It may possibly be a later copy of some of the earliest rules of Rhythmomachia found so far. The writer says on fo. 51^a that he has placed the battling numbers in memory of Boethius' three progressions: arithmetic, geometric and harmonic. This is followed immediately, fo. 54^a - 57^b, by what seems to be the comment of a scribe who calls the game a "numerorum pugna" and himself an ignoramus (fo. 55^b). Rather fatuously he goes on to describe himself as gathering the flowers, dripping with honey, from the meadow of the work, which is good for his ignorance. The boredom this scribe seems to have felt for the work in hand is indicated by the repetitive doodles that decorate the bottom of every page.[10]

Nat. Bibl., Vienna, gave this as her considered opinion of codex 2503, 49^a - 57^b.

[10] A copy of the original MS of the Rhythmomachia attributed to Odo, codex 2503, is in the appendix A.

The Odo MS is fragmentary, whether from the loss of pages or the carelessness of scribes cannot be determined. It is bound in codex 2503 with other work by Odo of Cluny but has itself no identification.

b)

Codex Clm 14836, Bayrische Staatsbibliothek, Munich, contains a collection of eleventh-century manuscripts, 162 pages in all. These pages are only 137 mm high and 107 mm wide. The codex previously belonged to the Convent of St. Emmeram (K.6.). The binding dates perhaps from the fifteenth century, as does the list of contents on the front page. The pages are numbered twice, in Roman and Arabic numerals, which do not always coincide. The binders proceeded according to the Arabic numbering.

List of contents of the miscellanea:

"Cilindrus
Rythmomachine (!) siue pugna numerorum
ac scacus mathematicus Wirzibergen
Commentum in rationem (?) Boethii
Hermani Astrolabium
Geometria Gerberti
Ludus laterculorum siue scacus
Coniunctio numerorum Wircebergensium
Geometrica Musica
Mensura limitum
Pondera mensure
Astrologica Musica
Adelpoldi Geometrica
Pondera mensure rursus
Astrolabium".

The second item in the list of contents, fo. 3b -4b is the "De conflictu rithmimachie" of Hermannus Contractus, 1013-1054. This is followed immediately (continuing on the same line, ever conscious of the scarcity of writing material) fo. 4b -6b, by excerpts from Asilo'a "Itē de rithm̄", which is complete in Vat. Lat. 3101.[11]

Other interesting items listed include two mathematical comments on chess. One of these is by "Wirzeburgensi" who also wrote a "coniunctio numerorum". There is also a treatise linking music with geometry and another one linking music with astrology.

The contents of codex 14836 provide a very good example of the close connection and interrelationship between all the subjects of the quadrivium to which, in the Middle Ages, also belonged chess and Rhythmomachia. These "games" helped medieval scholars to undertake the difficult and unpopular task of thinking original thoughts.

The first known attribution of "De conflictu rithmimachie" to Hermannus was made by Jacob Philip of Bergamo, 1434-1520, in his catalogus.[12] The German Benedictine Abbot Trithemius of Spanheim,[13] 1462-1516, also lists a "De conflictu rythmimachiae" (his spelling)

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[11] For a copy of fo. 3b -6b, "De conflictu rithmimachie" and "Itē de rithm̄" see appendix A.

[12] Giacomo Filippo Foresti (usually Jacobus Philippus Bergomensis, of the noble house of Solto near Bergamo), Supplementi Chronicarum ab ipso mundi Exordio usque ad redemptionis nostrae Annum MCCCCCX. Venice 1513, p. 208.

[13] Catalogus scriptorum ecclesiasticorum, Coloniae, 1531, p. 64.

among the works of Hermannus who, he informs us, knew Greek, Latin and Arabic.

Hermannus' Rhythmomachia is fragmentary and probably a mid-eleventh century copy of an earlier work. There was a large scriptorium at Reichenau where this copy of a Rhythmomachia could have been made and the script is Bavarian and consistent with their output.[14] As was discussed in the chapter on etymology, Hermannus gave the game a special emphasis by using the relatively uncommon word "conflictu" in conjunction with "battle of numbers", the "rithmimachie".

[14] This is the opinion of Dr. H. Hauke of the Bayrische Staatsbibliothek, Munich (Handschriftensammlung).

c)

A copy of "De conflictu rithmimachie" and "Itē de rithm̄" with an attribution to Gerbert of Aurillac is in the Bibliotheque Nationale, Paris, codex 7185.[15] This work of 69 lines (the first four lines of the Vatican Library's "Itē de rithm̄" are missing) is also at the Bibliothèque de l' Arsenal sciences et arts, codex 830[3], the city library of Avranches, 235 mbr. 8. sXII and Montpellier bibliothèque de l'école de medicine, codex 366[3], where it is listed as anonymous. These manuscripts date from between the twelfth and fourteenth-centuries.

The Paris codex 7185, fo. 107a - 108b, starts: "Ludus qui dicitur rithmimachia". Apart from the fact that the first four lines from Asilo's work are missing, the differences in the text between the Paris and Munich manuscripts are negligible.

Codex 7185 used to be in the Colbert library, 4001, and before that in the collection of Jacques-Auguste

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[15] No clarity could be obtained from the Bibliothèque Nationale as to the reasons for this attribution. Archiv der Gesellschaft für ältere deutsche Geschichtkunde, ed. G. Pertz, Hanover, Hahn 1874. vol. XII, p. 383. Pertz publishes a disclaimer to the presumed authorship of Gerbert in the form of a letter by the mathematical historian M. Chasles of Chartres: "Je doute que la rithmomachia soit de Gerbert; je ne vois pas sur quelle autorité on la lui attribue; je

De Thou. The handwriting of both Rhythmomachias, that in Munich attributed to Hermannus, and that in Paris attributed to Gerbert would, on palaeographical evidence place their origin in the middle of the eleventh century. Both of these manuscripts are most probably copies an earlier work.[16]

l'ai trouvé dans beaucoup de mss. et jamais je n'y ai vu le nom de Gerbert".

For a copy of codex 7185 see appendix A.

[16] For this view I am indebted to Dr. H. Hauke, Staatsbibl. Munich.

d)

The Rhythmomachia of Asilo in Rome, Vat. lat. 3101, "Descriptio Rithmachyae', is listed in Pertz as "im Jahre 1077 von Benedictus accolytus mon. S. Arsacii (sic) in Süddeutschland geschrieben".[17] This codex in the Vatican library is written by the same hand throughout,[18] in black and red ink on parchment and was rebound in red leather in the sixteenth or seventeenth-century. The Rhythmomachia is on f 2a and 2b followed by a miscellany of work by Hermannus Contractus and others, amongst it counting with Arabic figures and several calendarii, one with necrological notes. It continues with multiplication tables, work on computistic, arithmetic, how to mix colours, etc. This is a collection of very specific knowledge treasured at the time and known only to a privileged élite.

The heading: "Descriptio Ritmachyae" is followed by "Quinque genera inaequalitatis ex aequalitate procedere manifestum est". On line six we read "Huiusmodi conflictum quidam ex clero Wirzeburgensi, si periti iudicent, dabit posteritati...."

A Rhythmomachia in Paris, codex 7377 C may also

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[17]Pertz Archiv, vol. 12, p. 232. The name of the cleric from Würzburg, "Asilo", is given in Paris codex 7377C.

[18]A specialist in medieval manuscripts at the Vatican library was unable to make any further comments as to the possible date and origin of codex 3101.

have been based on the work of the "cleric of Würzburg". The reference there reads: "Huiusmodi conflictum quidam ex clero Wirzeburgensi nomine Asilo si periti iudicent dabit posteritati". In the margin is the verse:

> "Nomen id expelle, quod dicis cesar Aselle,
> Asilo dicor ego, cui si tria grammata tollo,
> A remanebit et O; quid erit praestantius illo?"

This might read in a very free translation:

> Caesar, if you take with one stroke of the pen
> the middle piece of my name,
> Asilo I am called,
> Then would remain A and O
> Which of the two would be more important,
> The one or the other?

A cleric from Würzburg called Asilo on familiar terms with the Emperor,living in the eleventh century can be found, if we note that Asilo,was a hypocorism for Adalbert and Adalbero.[19] Adalbero, Count of Lambach, member of the chapter of the Cathedral of Würzburg, educated in Paris, became Bishop of Würzburg by favour of the Emperor Henry III in 1045 (died 1090).[20] Henry was considered to be a highly educated man and was

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[19]F. Stark, Die Kosenamen der Germanen, Vienna, Hof, 1866, p. 92 f., and E. W. Förstemann, Altdeutsches Namenbuch, Munich, Fink, 1966-67 (reprint of 1900-16), vol. I, p. 192. Ascelin, Azilin, Aselle, the spelling varied. Archbishop Adalbero of Rheims (died 989) was called Ascelin by his friends. Lattin, The Letters of Gerbert, p. 10. Bishop Adalbero of Laon, 977-1030, was also called Ascelin, Peiper, in Zeitschrift für Mathematik und Physik, p. 215.

[20]Allgemeine Deutsche Biographie, Königliche Akademie der Wissenschaften, Leipzig, 1896, vol I.

much praised for his efforts in encouraging the study of the liberal arts. Asilo would presumably have written the Rhythmomachia before he became bishop in 1045 and,as he uses the word "conflictu",he most probably based his work on Hermannus' "De conflictu rithmimachie".

Whatever the identity of Asilo, his Rhythmomachia was carefully copied and incorporated with other material relating to the arts of the quadrivium in codex 3101 by the acolyte Benedictus at the monastery of St. Arsacius before 1077.

According to legend Arsacius, a Persian monk, was martyred in the fourth century. The influential Huosier family of Tegernsee founded a Benedictine monastery at Ilmmünster in Upper Bavaria in 746 for a Ijo or Uto, who became its first Abbot. The founders are reputed to have transferred the reliquiae of St. Arsacius from Milan to Ilmmünster and dedicated the monastery to the saint.[21] In 921 the Cella Ilmina, as

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[21] The history of Arsacius is somewhat controversial. Until stonemasons from Milan erected the collegiate church in the early thirteenth century Arsacius (Arsatius) was revered as a confessor, not a saint. Ijo is reputed to have brought the remains, or part of them, with him from a pilgrimage to Rome as a gift from Pope Zacharias. The Roman hagiographia does not mention St. Arsacius of Ilmmünster, neither does a Tegernsee calendarium of the eleventh century. On the other hand an eighth century Anglo-Saxon calendarium of Ilmmünster notes: "dedicatio basiliae sancti arsacii ad ilmina", 11th May. R. Bauerreiss, "Ein angelsächsisches Kalenderfragment des Bairischen Hauptarchivs", Munich, 51, 1933, p. 178.

it used to be called, was devastated by the Huns. Between 1060 and 1068 it was revived as a seat for canons of the chapter by the Margrave Ernst. The monastery of St. Arsacius must have acquired a considerable reputation for learning as Conradin I, King of Jerusalem and Sicily from 1266-1268, grandson of the sophisticated and highly-cultured Frederick II, was educated there in about 1260.

Similar texts to the one produced at St. Arsacius are in Rochester N.Y. and Hereford. These have been discovered only fairly recently and have been discussed by J.F.C. Richards and G.R. Evans.*

In 1495 with permission from Pope Alexander VI, but against the wishes of the Bishop of Freising and the Prior of Ilmmünster, by order of Duke Albrecht IV the reliquiae of Arsacius were transferred to the Frauenkirche in Munich, but were returned to Ilmmüster in the nineteenth century. Cf. W. Hotzelt, "Translationen von Martyrer-reliquien: Arsatius von Ilmmünster", in Studien und Mitteilungen zur Geschichte des Benediktinerordens und seiner Zweige, 53, 1935, p. 303-305; Germain Morin OSB, St. Bonifaz, Munich, "Qui est saint Arsacius honoré à Milan et en Bavière?" ibid., 54, 1936, 1-6, Lexikon für Theologie und Kirche, ed. M. Buchberger, Freiburg, Herder, 1960.

*John F.C. Richards, "A New Manuscript of a Rithmomachia", Scripta Mathematica, vol. 9, 1943,pp. 87-99, 169-183, 256-264 and Gillian R. Evans,"The Rithmomachia: A Medieval Mathematical Teaching Aid?" Janus, 63, 1976, pp. 257-273.

e)

The Fortolfus manuscript of Rythmimachia, Wrocław, Biblioteka Uniwersytecka n 54 was written by a monk.[22] He says in the prologue to his second book: "fraterna Karitas extorsit opus". It is obvious that the writer was well-educated in the quadrivium and had studied Boethius, whom he quotes frequently. There is no doubt about the monk's name. He closes his work with the words: "Explicit opus Fortolfi".[23]

The neatness of execution of the manuscript and the absence of mistakes together with the excellence of the writing suggest that the author perhaps also wrote the manuscript himself[*] or closely supervised a skilled scribe. It is of course possible, though unlikely, that Fortolfus made use of the services of a professional copyist. A more reasonable supposition might be that he managed to obtain the goodwill of a sympathetic abbess who gave the task to her diligent and careful nuns. The book might perhaps have been

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[22] Ex Rhedigerianus s I 5, 5 membr, f 86a - 94b. A copy of this MS (formerly Bourgogne) is in the Bibliothèque Royale, Brussels, n 927-940.

[23] This was not the usual way at that time to state the authorship of a tract, but it is not unique. The poems of Alcimus Avitus finish (Book 6) with the words "Explicit opus docti Alcimi". Vienna n 391.

[*] I am indebted to Prof. Dr. M. Folkerts of the Institut

intended as a gift to a particularly keen collector of high standing.[24]

Speculation about the possibility of the Fortolfus Rhythmomachia being intended as a "presentation copy" is reasonable because the quarterino pages of double column were united with a particularly good copy - also in German handwriting - of Boethius' De arithmetica f 1-85. A fourteenth-century table of contents in the codex, on f 1a reads:

> "Item duo libri Boecij de arismetrica
> Item liber qui dicitur Rithmimachia".

Unfortunately nothing else can be gleaned from codex n 54.

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für Geschichte der Naturwissenschaften d. Univ. Munich, for his opinion that the argument, which suggests that the author, Fortolfus, is also the writer of codex n 54, is not at all convincing. Cf Richards, Scripta Mathematica, vol. 9, pp. 88, 89 discussing L. Ellenwood's dating and writer of a Rochester MS.

[24] As an example of nuns copying manuscripts for monks can be cited: the Englishman (St.) Boniface, 'Apostel der Deutschen', killed in Friesland in 754, always starved of books (and passionately desiring them), writes to the Abbess Eadburge full of gratitude for the present of books which her nuns had copied for him. He sends her a silver graphium and begs for more. F. Merryweather, Bibliomania in the Middle Ages, 1849, rev. ed., London, Woodstock Press, 1933, p. 166, 167.

f)

The Bayrische Staatsbibliothek in Munich has a ninth-century manuscript of the rules of Benedict of Aniane, Abbot of Kornelimünster, codex 28118. It is a large book, 415 mm by 325 mm, containing 216 pages of parchment. The lettering is black with many red capitals; the script is Carolingian and by three distinct hands. The volume was rebound in pigskin in 1559. This manuscript is the oldest extant collection of the rules of Benedict of Aniane who died in Kornelimünster in 821. His friend Helisachas of St. Maximin in Trier was present at his deathbed and may have removed the rule-book to Trier where it remained until the secularization of the monastery in 1802.[25]

In this beautifully preserved manuscript an addition was made in the eleventh century: folio 18 vb - 19r contains a fragment of some rules of Rhythmomachia: "II ab hostibus undique circumuentus captivatur...", f 18vb, and the field of play without the shapes of the pieces; the numbers are given in Roman numerals (19r).

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[25] The publisher Joseph von Gorres acquired the codex and left it to his heirs, who sold it to Professors Grauert and Traube of Munich from whom the Bayrische Staatsbibl. (BSB) Munich purchased it in 1902. See O. Seebass, "Über das Regelbuch Benedikts von Aniane", in Zeitschrift für Kirchengeschichte, 15, 1895, p. 244-260.

This information was obtained from a preview Dr. Hauke of BSB gave me of the catalogue of manuscripts before

Two mistakes can be observed on the "even" side: the number for the pyramid should be 91, and "basis" should be one square higher.

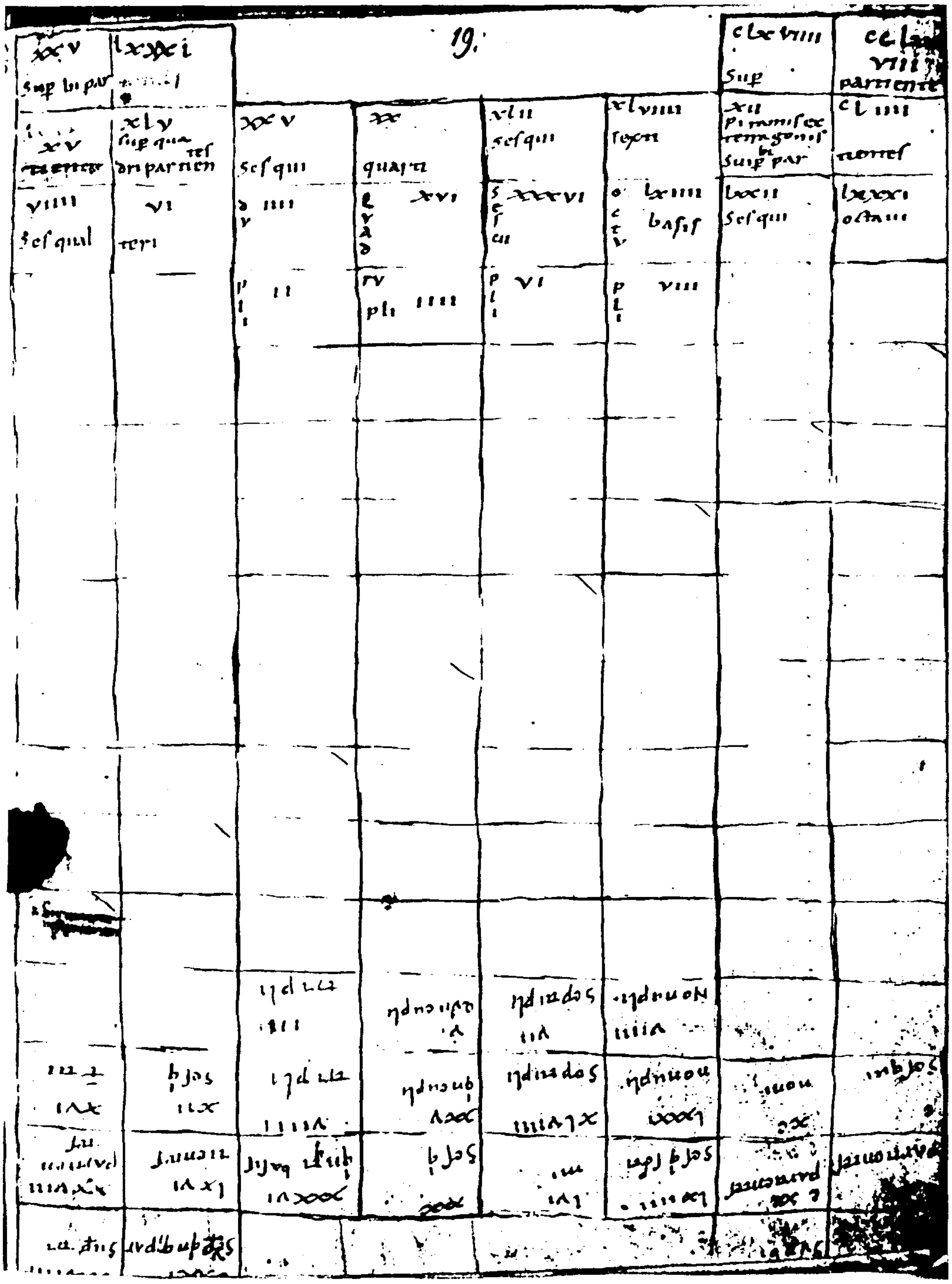
If the rule-book was taken to Trier in 821, the addition of the Rhythmomachia, in the handwriting of the eleventh century, was made at Trier.[26]

To find a Rhythmomachia recorded in a highly treasured and revered rule-book is rather curious. It indicates either a great love of learning and the desire to record the practice of the quadrivium and Platonic philosophy in the abbey's most important book after the Bible, or else disapproval and suppression. Could it be that a safe hiding-place had to be found for Rhythmomachia when superiors less well educated and therefore less sympathetic to the liberal arts frowned upon playing a game, even one like Rhythmomachia? The man who recorded the Rhythmomachia in the rule-book seems to have been anxious to give it minimal exposure. He refrained from sketching the shape of the pieces, the circle, square, triangle and pyramid, but confined himself to the proportions and numbers.

During the eleventh and particularly the twelfth-

1000 AD at the BSB, which is in preparation for publication.

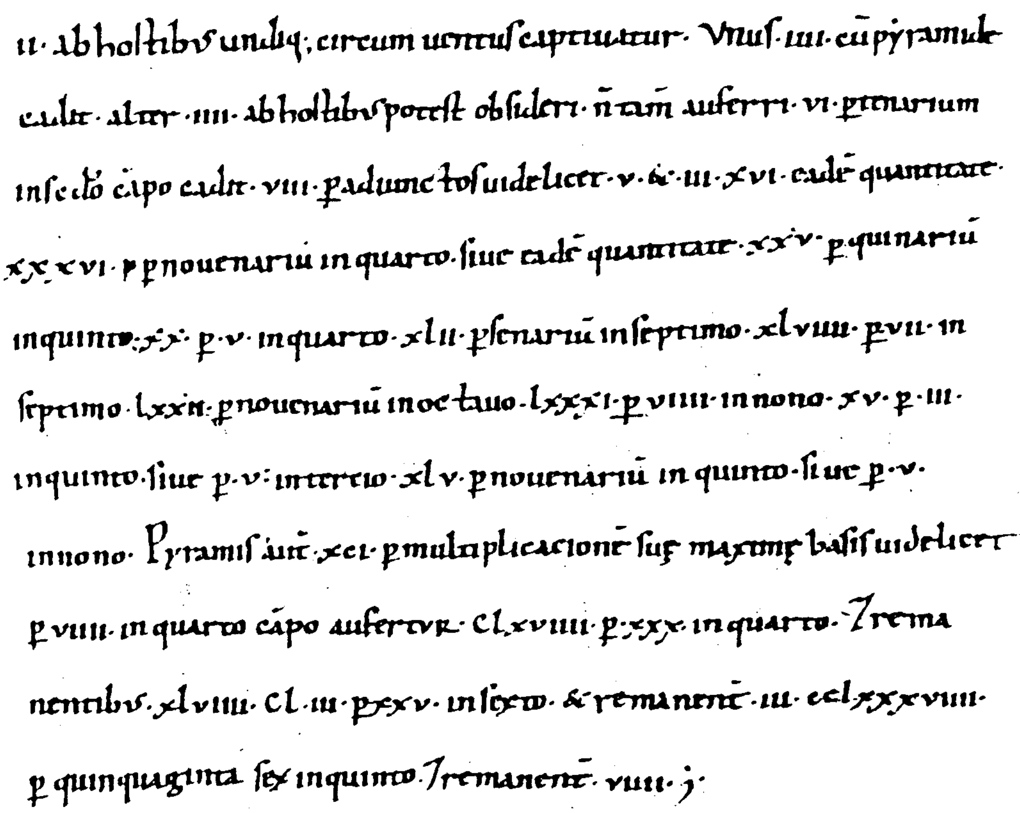
[26] Trier had a famous scriptorium in the ninth century and would have welcomed this beautiful rule-book of Benedict of Aniane.



12

Insertion in Regule S. Benedicti

Text of insertion in Regule S. Benedicti


ii· ab hostibus undiq; circum uentus capitur· Vnus· iiii· cum pyramide
cadit· alter ·iiii· ab hostibus potest obsideri· non tamen auferri· vi· per ternarium
in secundo campo cadit· viii· per adiunctos suos uidelicet· v· & · iii· xvi· cadit quantitate·
xxxvi· per nouenarium in quarto· siue eadem quantitate· xxv· per quinarium
in quinto· xx· p· v· in quarto· xlii· per senarium in septimo· xlviiii· per vii· in
septimo· lxxii· per nouenarium in octauo· lxxxi· per viiii· in nono· xv· p· iii·
in quinto· siue p· v· in tertio· xlv· per nouenarium in quinto· siue p· v·
in nono· Pyramis autem· xci· per multiplicationem suae maximae basis uidelicet
per viiii· in quarto campo aufertur· clxviiii· p· xxx· in quarto· et rema
nentibus· xlviiii· cliii· p· xxv· in sexto· & remanente· iii· cclxxxviiii·
per quinquaginta sex in quinto et remanente· viiii· ;·

(Attack of Even side by Odd)

The 2 is captured when completely surrounded by enemy pieces. One 4 falls with the pyramid; the other 4 can be attacked by enemy pieces, but not taken away. A 6 falls by the 3 on the second square, the 8 by its complements, that is the 5 and the 3. The 16 by the same amount (i.e. by the enemy's 16). The* 36 by a 9 on the fourth square, or by the same amount. A 25 by the 5 on the fifth, the 20 by the 5 on the fourth, the 42 by the 6 on the seventh (error for 'by the 7 on the sixth'?), the 49 by the 7 on the seventh, the 72 by the 9 on the eighth, an 81 by a 9 on the ninth, the 15 by the 3 on the fifth or by the 5 on the third, the 45 by a 9 on the fifth or by the 5 on the ninth, But the pyramid, the 91, is taken away by multiplication of its largest base, that is by a 9 on the fourth square. The 169 by the 30 on the fourth, and with the 49 remaining. The 153 by the 25 on the sixth and with the 3 remaining. The 289 by the 56 on the fifth and with a 9 remaining.

* He omits the 64.

Translation of text with Arabic numerals replacing the Roman numerals of the original by B.S. Lee, Department of English, University of Cape Town.

century there developed an almost excessive preoccupation with mathematics and even liberal scholars like John of Salisbury found it necessary to sound a warning.

Benedict of Aniane had himself been less keen than his friend Charlemagne on general education and restricted monastic teaching to their own intake of postulants.[27] Helisachas of St. Maximin[28] may have shared this view and continued the tradition at Trier, unlike Tegernsee,[29] Reichenau and Einsiedeln for instance where chess was considered part of the quadrivium.

If lovers of Rhythmomachia at Trier in the eleventh century needed to conceal their secret passion, what better hiding-place for the game? Supplied with "apice", two monks might retire to their cell on a wet winter's day and pore over their "rule book".

These earliest Rhythmomachias just discussed have been attributed to Odo, Hermannus Contractus, Gerbert,

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[27] Abbot Benedict of Aniane, son of Count Maguelone in Lower Languedoc, was a courtier friend of Charlemagne who accompanied him to Italy in 773. On his return he entered the monastery of St. Seine near Dijon, and moved in 780 to the Abbey of the Holy Saviour founded by Witiza at Aniane. He became a friend of Alcuin and with the help of Charlemagne built Kornelimünster near Aachen in 815/6. He was head of all monasteries (First Benedictine reform). Neue Deutsche Biographie, Berlin, Duncker & Humblot, 1957, vol. 2, pp. 43, 44.

[28] St. Maximin helped to found Echternach, Tegernsee, Maria Laach and Brauweiler. New Catholic Encyclopaedia, vol. XIV, p. 287.

[29] In 1803 with the dissolution of Tegernsee 60,000 books were moved to the Bayrische Staatsbibliothek, Munich.

Asilo and Fortolfus. Can any one of these men have also been responsible for the invention of the game?

The Abbot Odo of Cluny, 879-942, fulfills most of the requirements to qualify as the possible inventor of Rhythmomachia. As the son of Ebbo I, Lord of Déols, he was brought up at the court of Duke William of Aquitaine where he may have become familiar with chess. He studied at Tours and at Paris under Remigius of Auxerre and for 14 years was rector of a school in Burgundy. He was a musician and composer, though his overriding interest was the restoration of monasteries and the Holy Rule. However, Odo died in 942, far too early to have seen Gerbert's "apice" and it can hardly be supposed that Odo invented them also, independently of the abacus and Gerbert.

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Fifty years later there was another Odo at Cluny, a simple monk, "Odo levita", deacon and "schola cantorum magister".[30] He lived at the right time but nothing else is known of him.

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Another Odo who can be considered as a possible inventor of Rhythmomachia was the Benedictine Abbot Odo of Arezzo. Little is known of his personal life.

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[30] A. Bruel, ed. Recueil des chartes de l'abbaye de Cluny, iii, Paris 1884, p. 145 as quoted in The New Grove Dictionary, vol. 5, p. 503.

He was a musical theorist, therefore well educated in the quadrivium, and he wrote an influential tonary and a treatise to provide the theoretical and practical foundation for learning chants.[31]

Guido of Arezzo used Odo's work as source material for his own studies and, with his greater fame, somewhat eclipsed Odo who belonged to the previous generation of musicians.[32] There is nothing about the Odo manuscript that suggests it could not have come from a scriptorium in or near Arezzo; there is also no proof that it did. If an "Odo" invented Rhythmomachia, then evidence could be in favour of Odo of Arezzo. This evidence is, however, only circumstantial.[33]

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Hermannus Contractus qualifies as a possible inventor of the game.[34] He lived at about the right

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[31] The New Grove Dictionary, vol. 5, p. 503 . These treatises were compiled in the late tenth century.

[32] Huglo, in Archiv für Musikwissenschaft, pp. 134-146.

[33] U. Chevalier Rèpertoire des Sources Historiques du Moyen Age, New York, Kraus repr. 1960, lists no Odo who might be more likely as the inventor of Rhythmomachia.

[34] Hermannus was the spastic son of a Count of Vehringen (in Swabia) and entered the Benedictine monastery at the then usual age of seven. There he spent the rest of his life, first under Abbot Berno, who built up the "Reichenauer Sängerschule" and later as abbot. He was one of the great versatile intellectuals of his day. He wrote a history of the Germans and propagated the scientific work of Gerbert of Aurillac, particularly the use of the astrolabe. His greatest work was done in the field of music. He invented a new musical

time, came from the right background to know chess, had the best education available and was particularly interested in the work of Gerbert of Aurillac, which would include reckoning with "apice". He had the time to devise the game, and the opportunity and company to play it with kindred spirits at Reichenau, famous for its high standard of learning and music.

It seems reasonably certain that the Rhythmomachia attributed to Hermannus goes back to rules he used or recorded. There is no proof however, that he also invented the game.

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The greatest of all the scholars at the turn of the tenth to the eleventh century was almost certainly Gerbert of Aurillac. He possessed all the characteristics required of the inventor of Rhythmomachia. Gerbert had the scholarship: he studied mathematics in Spain, he invented the new reckoning with "apice", he moved in the highest circles of church and state where chess was played, he invented and encouraged the use of other teaching aids to help popularize the study

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notation independent of that of Guido of Arezzo and encouraged the use of "Guido's Hand" and the monochord, which helped to show the diatonic division of scales based on their multiple and "superparticular" proportions. Hermannus, like Gerbert of Aurillac and Guido of Arezzo, seems to have been keen to share his knowledge. This was not common at the time, as some well educated men were not free of a certain intellectual superiority and desire to remain exclusive.

of mathematics and music, he taught for many years in the cathedral school of Rheims and associated with intellectuals who were eager to learn.

If Gerbert was the inventor, why does he not say so in one of his letters and why is his biographer Richer silent on this point? Many of Gerbert's letters are lost. Only during the latter part of his life when he corresponded with kings and was active in politics, did he order copies of his letters to be kept.

Richer may not have mentioned Rhythmomachia for various reasons. As a monk he may have taken his vows very seriously and not played any games at all and therefore not known about this game; he may have known about it and disapproved of others playing and tried to protect Gerbert whom he admired; he may not have found Rhythmomachia worth mentioning as every one in their circle knew about it; or it may simply have been an oversight.[35]

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[35] Richer was in fact a somewhat naive biographer as far as Gerbert's mathematics is concerned and a careless one as far as Gerbert's life history is concerned. Gerbert spent three years as Abbot of St. Columban in Bobbio in Italy (a royal appointment by Otto II, 980-983). His energetic land reforms there caused an uproar among the local nobles. After the sudden death of Otto II (7. 12. 983) Gerbert found his position untenable and fled to the Imperial Court of the Dowager Empress Adelaide at Pavia, returning afterwards to Rheims. Richer completely ignores these three eventful years in Gerbert's life. If Gerbert ever was a monk, which is doubtful, he may have had dispensation from his vows from Pope John XIII. He was ordained to the priesthood by Archbishop Adalbero of Rheims soon after 972. Concilium Mosomense in Olleris, p. 246

Rhythmomachia is a game in the very spirit of Gerbert: he tried stretching the mind to grasp that which had not yet been thought of, to push out the frontiers of human endeavour in matters of mental activity. Rhythmomachia for Gerbert might have been the equivalent of the modern "think-tank", or "Scrabble" with numbers.

There is no firm evidence that Gerbert invented the game, yet he was the person nearest to its creation: he was intimately involved with counting-boards and numbered "apice". During his teaching years at Rheims (972-980) he would spend leisure hours as well as working days amongst the keenest students of the quadrivium. A game of chess for relaxation could, in a moment of inspiration have led to two chess-boards being joined together (possibly a counting-board of 8 X 16 squares existed), and with counters at hand a new mathematical game was devised. All this however must remain conjecture.

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Asilo, if we are correct in surmising that we are talking of Adalbero, Bishop of Würzburg, died in 1090. He figures not so much as the possible inventor of Rhythmomachia, for which he lived too late, but as the man who perhaps brought the game with coherent rules

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as quoted by Lattin, <u>The Letters of Gerbert</u>, p. 4, fn. 5.

to Germany. During his years of study in Paris he may well have become acquainted with Rhythmomachia and wished his friends at home also to profit from it.

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In the Middle Ages many monks, especially those of the order of St. Benedict, had a great love of learning; some of them were very keen bibliophiles, not to say bibliomaniacs, who copied all available material, thereby building up their own private collections and also enriching monastery libraries with bequests of books. The gift of a book, especially a rare one, to a monk or a lay collector would be highly welcomed.

The identity of the monk Fortolfus or Frutolfus can only be established by inference. Palaeographically the manuscript belongs to the twelfth-century. Only one Fortolfus, a German Benedictine theorist of music who lived at St. Michelsberg Abbey in Bamberg, made his mark on history towards the end of the eleventh-century and died in 1103. This Fortolfus wrote a Breviarium de musica. In the foreword he gives a definition of the work "music": musica is the "scientia bene modulandi", the theory of music practised by philosophers. The theoretically educated practitioner of music, that is the singer or instrumentalist, he calls a "cantor musicus".[36]

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[36]Codex Bruxellensis, Fètis (= Cbrf) 5266, as quoted in

Fortolfus also wrote a Tonarius and completed the theory of Gregorian music begun by Guido of Arezzo. The writer of this beautiful Rhythmomachia would most probably have been that kind of man: a philosopher of great accomplishment. No other such Fortolfus is known and a man of such gifts would hardly have remained unknown.

Fortolfus[37] of St. Michelsberg, the presumed author of the Rhythmomachia, makes it clear that he is not the inventor of the game. In the prologue to Book II he says he has built on what others have written.[38] Fortolfus is the only writer of the eleventh century to set Rhythmomachia into its cultural background. This and his defence of the game against the Philistines has already been discussed in the previous chapter.

Four of the early manuscripts of Rhythmomachia

Cölestin Vivell, Das Breviarium de musica des Mönches Frutolf von Michelsberg, Studien und Mitteilungen zur Geschichte des Benediktiner-Ordens und seiner Zweige, Neue Folge, 3 (Band 34), Salzburg, 1913, pp. 413-423.

[37] The name of Fortolfus or Furtolf, Fortolf or Frotolf does not very commonly appear in writing. Pertz's Monumenta does not mention it at all. G. Sarton, Introduction to the History of Science, p. 763 says under Frutolf..."He is probably the author of the Rythmimachia...".

[38] See appendix C for a translation of Prologue I, II and a summary of the text. Peiper edited and published the Fortolfus Rythmimachia in Latin without a translation in Abhandlungen zur Geschichte der Mathematik, 3, Supplement zur historisch- literarischen Abteilung der Zeitschrift für Mathematik und Physik, Band 25, 1880, p. 167-227.

just discussed, those attributed to Odo, Hermannus, Asilo and Fortolfus, have certain passages, phrases or sentences in common. Copying of manuscripts was done in many scriptoria; parts of Asilo's work were at some stage joined to that of Hermannus, and both had perhaps taken from Odo; Fortolfus incorporated into his own work what had been written before him. These four manuscripts of Rhythmomachia were copied for the next few hundred years and also provided the basis for new and improved versions of the rules.[39] Not until the Renaissance do we again find someone like Fortolfus to set Rhythmomachia against the cultural background of its time.

Indirect evidence that Rhythmomachia was played by well-known men in the Middle Ages can be obtained from letters and reports. Abbot Rupertus of Reichenbach in Bavaria asked Wilhelm of Tegernsee, to send on the rules of the game as he had promised[40]. John of Salisbury,

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[39] Abraham Riese, son of the famous sixteenth-century German mathematician Adam Riese, produced a German Rythmomachia for Duke August of Saxony in 1562, : cod. Dresden, C 433 s. xvi., which starts: "Das die funff genera Proportionis inaequalitatis, aus der proportion aequalitatis ihren ursprung haben, Darum ist genugsam in Arithmetica Speclatiua gehandelt. Die funff genera aber Proportionis inaequalitatis seind, Multiplex, Superparticulare, Superpartiens Multiplex superparticulare, und Multiplex Superpartiens. Under diesen seind, drei genera, als Multiplex, Superparticulare, und Superpartiens. Zwischen welchen ein Kampf, schlacht und Streich aus Zwiespal, von wegen der geraden und ungeraden tzalen sich ehrheben, Und dieser Kampff wurdt Arithmomachia genannt".

[40] B. Pez, Thesaurus anectotorum novissimus, Aug. Vind.,

c 1115-1180, uses Rhythmomachia as a simile in a letter to Baldwin, archdeacon of Totnes, c May 1168. He writes in a way that leaves no doubt in our mind that John expects Baldwin to be perfectly familiar with the game:

> "...What remains for us to look for from the Lord, save that his flanks may perish with him, and those who were his comrades in evil may be with him too in his fall? For this is suggested by the game of rithmachia: whenever the pyramid is captured its sides fall in with it. Do not the powers of this world have a likeness to these pyramids, and great houses too, as in a body the limbs crash along with the head - those, that is, who on the head's strength had risen against the Lord and were raging against Christ's poor?"[41]

Further evidence of Rhythmomachia among the famous comes from Richard Fournival, c 1200-1250, Chancellor of Amiens, the presumed author of the Latin pseudo-Ovidian romance "The Vetula". Fournival included in

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1721, vol. VI, p. 2. The Abbey of Tegernsee had been famous for its learning from early days. The "Ruodlieb" with its plot that hinges on chess was written there c 1030. The Abbey continued as one of the great teaching institutions,and cared for one of the finest collections of books in the Middle Ages.

41 "...Quid ergo superest a Domino expectandum, nisi ut ei compereant latera sua, et quos complices habuit in errore comites habeat in ruina? Nam et in rith-machia ludentium hoc indicat iocus, ubi quociens aufertur piramis intercepta, totiens concidunt latera eius. Nonne harum piramidium instar habent saeculi potestates, et quaelibet amplea domus, ubi quasi corporis membra capiti suo concorruunt, qui de illius uiribus contra Dominum intumuerant et in Christi pauperes saeuiebans?" The Letters of John of Salisbury, ed. W. J. Millor, and C. Brooke, Oxford, Clarendon Press, 1979, vol. 2. "The Later Letters"(1163-1180) pp. 570-573, rendering "rithmachia ludentium" as "the game of rithmachia" instead of Millor's "game

his didactic poem a digression on games and a chess morality, and ended with a description of Rhythmomachia in verse, explaining the form of the pieces and their numerical relationship. He praised the excellence of the game and closed with the words: "O ultinam multis numerorum pugna placeret...."[42]

The Franciscan Roger Bacon, c 1214-1294, elevated Rhythmomachia to the seventh science and listed it after Pythagorean tables, astronomical tables, weights, measures, practical geometry and commerce.[43] This indicates that the game was known at Oxford in the thirteenth century, when Oxford was becoming distinguished

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of spillikins" and "piramis intercepta" as "(whenever) the pyramid is captured" instead of "the pyramid is broken".

[42] De Vetula was translated into French in the fourteenth-century by Johan Lefèvre, as "La Vieille", ed. Cicheri, Paris, 1861. See also: Paul Klopsch, Pseudo-Ovidius De Vetula, (Mittellateinische Studien und Texte), Leiden und Köln, Brill, 1967. For the relevant verses see appendix D.

[43] Septima docet propter tedium amovendum et in solacium hominum ludos Pharaonicos constituere, cujusmodi sunt ludi Rithmimachie. Hec enim pars constituit duos reges in numeris et duas numerorum acies ingerens tabellis (aliter caballis) et congredientes adinvicem ad modum scacorum ludi, in quibus numerorum pugna mirabili subtilitate et jocumditate conseritur numerus uterque acierum hinc inde se aggredientium ad similitudinem militum in excercitibus et bellis, et ideo hic ludus sapiencie vocatur Rithmimachia, id est, numerorum pugna, nam ad 'rithmo' Grece 'numerus' est Latine, et 'machia', media producta, 'pugna' dicitur in Latino, et hec traduntur in libris propriis per singulas practicas nominatis, ut in libro Rithmimachie et in Algorismo completo in integris et fraccionibus, et in Algebra que est 'negociacio', et in Almagabale que est 'census', et in libro Abbaci, et in aliis practicis Arismetice, licet non isto ordine quem inter

as a centre for the study of Platonic cosmology and research into fundamental mathematics.[44] Scholars at Oxford managed to combine empirical investigation with metaphysical speculation, e.g. Grosseteste's preoccupation with the properties of light in physics and the highly conjectural notion that light was the agent through which the soul worked on the body and illuminated the beauty of creation, an idea expressed by the architects of Gothic cathedrals.

Already in the twelth century Oxford attracted students of the quadrivium in particular, such as the widely travelled mathematician and linguist Robertus Cestrensis,[45] who translated the Koran from Arabic into Latin (1143) at the suggestion of Peter the Venerable (died 1156), Abbot of Cluny. Robertus wrote in his introductory letter to this work that from then on his life would be dedicated only to mathematics, astronomy and astrology for the greater glory of God. According

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partes has nunc michi placuit observare. "Communia Mathematicae", Opera hactenus, inedita Rogeri Baconi, ed. R. Steele, Oxford, Clarendon Press, vol. 16, (1940), pp. 48, 49.

[44] Roger Bacon not only enjoyed the patronage of Pope Clement IV, the former Provençal soldier, lawyer and secretary to St. Louis of France, but was also instructed by Clement to send directly to him reports of his scientific discoveries, to the displeasure of his superiors. One would like to think that the game of Rhythmomachia was amongst the scientific discoveries sent to Rome. J. J. Bridges, The Life and Work of Roger Bacon, London, Williams & Norgate, 1914, p 27.

[45] Robertus, also called Katene, is said to have travelled

to a Florentine manuscript[46] he wrote a Rhythmomachia and may have introduced the game to England when he settled there in 1150.

Another famous Oxonian, professor and scientist, the Augustinian, Thomas Bradwardine, 1290-1349, later Archbishop of Canterbury, confessor to Edward III of England, "Doctor Profundus", wrote on numerical proportions and a small treatise on Rhythmomachia. At about the same time the mathematician and Bishop of Lisieux, Nicole d'Oresme, 1325-1382, was occupied with similar thoughts. He included a Rhythmomachia in his "De proportionibus proportiorum", which was published in Venice in 1505 with works of Bradwardine and others on the same subject.[47]

A few verses from John Lydgate's chess-allegory <u>Reson and Sensuallyte</u>[48] might perhaps be a fitting

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in Greece and Asia Minor. He settled in Barcelona in 1136 and was Archdeacon of Pamplona for several years. He translated Morenius into Latin (1144). His arrival in England must have encouraged those students of the quadrivium who could not arrange a sojourn in Spain for lectures by advanced mathematicians. <u>Dictionary of National Biography</u>, ed. L. Stephen and S. Lee, London, Smith, Elder, 1908, vol. 48, pp. 362-364.

[46] Florence, Biblioteca Nazionale Centrale, MS II., II., 278, p. 40 v.

[47] A. Chicco, La Rithmomachia, <u>Bonus Socius</u>, 's Gravenhage, 1977, p. 85.

[48] Lydgate (1370-1451) was a Benedictine monk and court poet. He translated the anonymous French didactic poem of the thirteenth century "Les Eschez Amoreuz", which itself owed much to the "Roman de la Rose", and

conclusion to some of the evidence of Rhythmomachia in the Middle Ages. Lydgate links chess with Rhythmomachia in the list of games at which Deduit, the son of Venus, excels. Dull brains, he says, cannot understand, but Deduit as God of Games (line 2376 ff) is willing to share his knowledge. Lydgate expands on man's passions and their playground, using chess perhaps as the secular metaphor and Rhythmomachia as the theological metaphor. Here is the relevant text:

		lines
	"..In al the crafte and melody	2399
music,	Of musyke and of Armony,	
	What tyme that hit shal be do,	
	He ys expert; and eke also	
	At al(le) pleyes delytables:	
Dice,	At mereles, dees and tables	2404
	He kan pley(en) passyngly;	
	But best and most specially	
Chess,	At the Chesse he dooth excelle	2407
	The philomestor, soth to telle	iste philosophus
	For to make comparyson,	secundum quosdam
	Ne was nat lyke him of renoun,	innenit ludum
	That first founde this play notable,	Scacorum
	With him to play(e) was not able.	2412

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expanded the first 4873 lines into 7042, breaking off at the 4th pawn of the lover's side. MS Fairfax 16, Bodleian, 22 ff.

	And I dar also specifie,	
and the	The play he kan of Ryghtmathye	Rihtmachia est
game of	Which dulle wittis doth encombre,	ludus philosophorum
arithmetic	For thys play stant al by noumbre,	et consistit in
	And hath al his conclusions	arsmetica et pro-
(leaf 235,	Chefly in proporsions	porcionibus
bk.)	By so sotil ordynance,	numerorum
	As hyt ys put in rememberaunce	
	By thise Philosophurs olde...."	[49]

[49] Lydgate's Reson and Sensuallyte, MS 16 Bodleian, ed. E. Sieper, London, Kegan Paul, 1903.

5

RHYTHMOMACHIA AND THE IMPORTANCE OF NUMBERS IN THE RENAISSANCE

The Cultural Background

During the Renaissance a number of famous scholars and cultured men devoted themselves to the study of the quadrivium and incidentally of Rhythmomachia,which was associated with it. They believed that Pythagoras, the Chaldeans or Boethius had invented the game because it is suffused with the spirit of Neoplatonism. It also reflects Pythagoras' view of the creator which Boethius expressed when, in the ninth hymn of the third book of _De consolatione Philosophiae_,God is addressed as "Tu numeris ligas", a theme used by Dante in _Paradiso_ XXXIII. Some mathematicians may have found the game helpful in teaching arithmetic and proportions, others may have looked for a mathematical curiosity that satisfied their interest in number and proportion and at the same time could entertain and amuse some of their noble patrons.

New treatises on Rhythmomachia were written during the Renaissance and put into circulation in greater numbers and with greater ease than before, because of the recent invention of printing. A brief discussion of the cultural background against which Rhythmomachia experienced this revival, as far as is relevant to this study, will be followed by a review of the more important manuscripts, books and references.

Just as the prologues of the Fortolfus manuscript provided some information about Rhythmomachia in the daily lives of the players of the eleventh century, so, fortunately, an early sixteenth-century manuscript of Florence shows that the philosophy of number and proportion and the directly related game of Rhythmomachia could interest and even excite a group of highly-educated men in the Renaissance.

These men may have hoped that Rhythmomachia would help to explain and justify, as well as defend, the ancient idea of the underlying harmony between God, the world, man and matter, especially at a time when political upheaval and disturbing new discoveries threatened from all sides.

During the Middle Ages scholars had relatively few pagan classical sources available for study. Such texts as they had they managed to reconcile with Christian teaching without great difficulty. During the Renaissance scholars still relied on the classical texts used earlier, but also had to cope with a wealth of new material as more and more Greek literature and philosophy and writings on science, medicine, etc. were discovered and translated into Latin.

Sometimes reverence for antiquity led to the indiscriminate use of material, some of doubtful value and authenticity. For example Hermes Trismegistus (the Greek name for the Egyptian god Thot) was credited with having inspired about 42 Greek, Latin and Arabic works, the so-called Corpus Hermeticum,[1] which explained the origin of the world and claimed to show the way to salvation. This source was used for much of the number symbolism current in the Renaissance, especially for astrological work.[2]

Renaissance philosophers did not set out to produce a new philosophical system but rather tried to adapt the old one to the new material. They still believed that the universe was numerically ordered and that numerology would reveal the secrets of the whole creation. New methods of applying numerology were carefully guarded as not fit for the uninitiated.

Only the initiated, it was felt, could grasp the splendour and radiance of truth, which they perceived as synonymous with beauty. The attributes of beauty were consonance of parts, proportion and light. Medi-

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[1] Isaac Casaubon in 1614 dated the Hermetica as second or third century A.D. gnostic versions of Greek philosophy. For an in-depth study see J. Festugière, Le Révélation d'Hermes Trismegistus, Paris, Lecoffre, 1949-1954.

[2] Another source was the cabbala, e.g. the number 32 (multiple of four, half the number of squares on the chess-board). Sepher Yetsirah (1:1): "Jehova, the Lord of Hosts, the God of Israel, the living God, King of the world... has graven His name in 32 mysterious paths of wisdom", these consist of 10 self-contained numbers (Sephiroth) and 22 basic letters (1:2). The meaning of the 10 numbers: 1: the spirit of the living God; 2: spirit from spirit; 3: water from spirit; 4: fire from water; 5-10: height, depth, East, West, South, North. Jung, Collected Works, vol. XII "Psychology and Alchemy", tr. R. Hull, 2nd ed., London, Routledge &

eval philosophers had laboured to create the conditions which would afford an insight into perfection: the Gothic cathedral, built in "perfect" ratios, flooded with light - the closest approximation to pure form - echoing with music written in "perfect" concord. Their guides had been the principles of Platonic aesthetics with the blessings of St. Augustine.

This seems to have been understood almost instinctively and acted upon by Abbot Suger on the rebuilding of the church of St. Denis. As a devotee of Dionysius the Areopagite, whose works Suger studied in the translation of John Scotus, he saw "the Lord" as the "super-essential light"; God the Father as the "Father of lights" (Pater luminum) and Christ as the "first radiance".

As Dionysius put it in De Coelesti Hierarchia: "Every creature, visible or invisible, is a light brought into being by the Father of the lights... This stone or that piece of wood is a light to me...For I perceive that it is good and beautiful; that it exists according to its proper rules of proportion; that it differs in kind and species from other kinds and species; that it is defined by its number, by virtue of which it is 'one' thing; that it does not transgress its order; that it seeks its place according to its specific gravity. As I perceive such and similar

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Kegan Paul, 1935, p. 205.

things in this stone they become lights to me, that is to say, they enlighten me (me illuminant). For I begin to think whence the stone is invested with such properties...; and soon, under the guidance of reason, I am led through all things to that cause of all things which endows them with place and order, with number, species and kind, with goodness and beauty and essence, and with all other grants and gifts."[3]

In this way the human mind is guided upwards by myriads of lights to partake of the radiance and harmony which is God.

Humanists believed in order, hierarchy and universal correspondences. Their example for order came from the belief that God had created everything according to a plan or pattern, by using number. By hierarchy they understood the three worlds:

1) the world of the senses, the everyday world,
2) the intelligible world, a world free and yet not free of sense experience (e.g. on this level science could be experienced),
3) the super-sensible world, the realm of God and his angels. The universal correspondences included man, who had the duty to explain the interrelationship; therein lay his wisdom and dignity.

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[3]Quoted by E. Panofsky, Introduction to Abbot Suger on the Abbey Church of St. Denis, and its Art Treasures, New Jersey, Princeton Univ. Press, 1946, p. 19, 20.

Renaissance thinkers believed, with Plato, that numerical structure underlies all that exists: the cosmos, the human figure, the pleasing sound in music, and all that is good in art and architecture. Their quest was for ideal proportions, with which, it was hoped, the artist would be able to recreate on earth, in imitation of God, the harmony and beauty of the cosmos. Through the knowledge of the liberal arts and especially music and its mathematical connotations many excursions could be made into the world of the fine arts. When painting, sculpture or architecture are particularly concerned with proportions and where correspondences are drawn between the arts and music, reference to Rhythmomachia seems justified though it is only indirectly related.

L. B. Alberti had laid down guide-lines for the artist and philosopher:

1) that works of art give greater pleasure when constructed on the most pleasing proportions,
2) that these proportions can be found in nature and are therefore God-given,
3) that these proportions, as recognised by Pythagoras and Plato, originated from music but can also be applied in other disciplines, because as music is only pleasing to the ear when sung or played in harmony, so painting or architecture will be more pleasing when organized on principles of

harmonic proportions.[4]

He told painters that they must first be familiar with geometry so that they could understand perspective. Proportion and symmetry were for him objective standards and a guarantee of beauty, which he defined as a certain regular harmony of all parts of a thing of such a kind that nothing could be added, taken away or altered without making it less pleasing.[5]

In painting guilds apprentices were taught about materials, methods of painting and perspective after taking a vow of secrecy. Greater secrets, reserved for masters, were those of the composition of paintings: the magic significance of the circle, the square and the triangle and that most divine of all ratios, the golden section.[6] Only the most exclusive artistic

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[4] L. B. Alberti, Ten Books on Architecture, tr. into English L. Leoni, 1726, ed. J. Rykwert, London, Tiranti, 1955, Book IX, ch. IV, V.

[5] ibid. Book VI, ch. 2.

[6] The golden section was defined as that division of a line in which the smaller part is in the same proportion to the greater part, as that greater part is to the whole. Luca Pacioli wrote a treatise about it, which was illustrated by Leonardo. In this he claimed that the golden proportion was divine because, like God, it was unique. It was in addition like the Trinity, three in one: three points united in a continuous harmony. After this publication the secret became more or less common knowledge in artists' workshops. (See diagram overleaf).

circles of the time knew the well-guarded secret of translating musical proportions into the structure and composition of a painting or a sacred edifice.

Examples where the painter has used numerical structure, mainly circles and triangles, in the composition of his painting are the "Crucifixion", by Raphael, and Tintoretto's "The Origin of the Milky Way",[7] c 1578, both now in the National Gallery, London.

In the schools for medieval masons the skills needed by architects were taught and the true meaning of religious allegory and symbolism were explained according to the degree of initiation. On the ordinary level the secret of how to build allowing for the pressure and counterpressure of the complicated arches that made up Gothic cathedrals could be acquired. At "inner" schools deeper insights could be gained by meditation and from the theoretical teaching of architects, who, like musicians, followed the authority of

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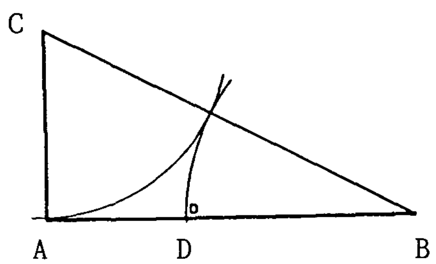

Construction of the golden proportion

$AC = \frac{1}{2} AB$

$AD : DB = DB : AB$

[7] Fred Gettings, The Hidden Art, A Study of Occult Symbolism in Art, London, Cassel, 1978, p. 98-106.

St. Augustine.[8] He had believed in the close relationship between architecture and music; architecture providing the mirror for eternal harmony and music its echo. Masons were also taught the true relationship between God, man and the universe, encompassing proportion, ratio, harmony and the secret of creation by number. From these schools the Society of Freemasons developed.[9]

An example of an architectural design based on numerical proportion derived from music, and therefore considered capable of re-creating the true relationship between God, man and matter during worship, is the plan for a church in Venice, San Francesco della Vigna. This was commissioned by the Doge Andrea Gritti in 1534 from the architect Francesco di Giorgio. The latter constructed the plan on musical proportions.[10]

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[8] In Retractatio, the work in which, towards the end of his life, St. Augustine examined his literary production, he reiterated his belief that number could guide the intellect to the visible truth, in God, via the understanding of all created things. Retractatio, I, ii (PL, XXXII, 600ff).

[9] R. Gould, A History of Freemasonry, London, Caxton, undated subscription edition.

[10] In the Renaissance the proportions of the human body were seen as a visual realization of musical harmony. E. Panofsky, Meaning in the Visual Arts, New York, Doubleday Anchor Books, 1955, p. 91. Panofsky also quotes Giorgio and Ghiberti transferring the centre of the human figure to the crotch, when this figure is inscribed in a circle, ibid., n65. Giorgio revered Pico della Mirandola, whose cabbalistic theses he trans-

The width of the nave was to be nine paces, which is the square of three, the first and divine number (1:1:1); the length of the nave was to be 27 paces, a triple proportion which makes a diapason and a diapente (octave and fifth):

9:18:27

9:18 (1:2) for the diapason

18:27 (2:3) for the diapente.

These proportions were considered to be the microscopic reflection of the macrocosm.[11] The humanist Fortunato Spira, the painter Titian, and the architect Sebastiano Serlio, a pupil of Alberti, were consulted about the design and expressed approval.

Giorgio justifies his plans: "this mysterious harmony is such that when Plato in the Timaeus wished to describe the wonderful consonance of the parts and fabric of the world, he took this as the first foundation of his description...", and again "...we being desirous of building the church, have thought it necessary, and most appropriate to follow the order of which

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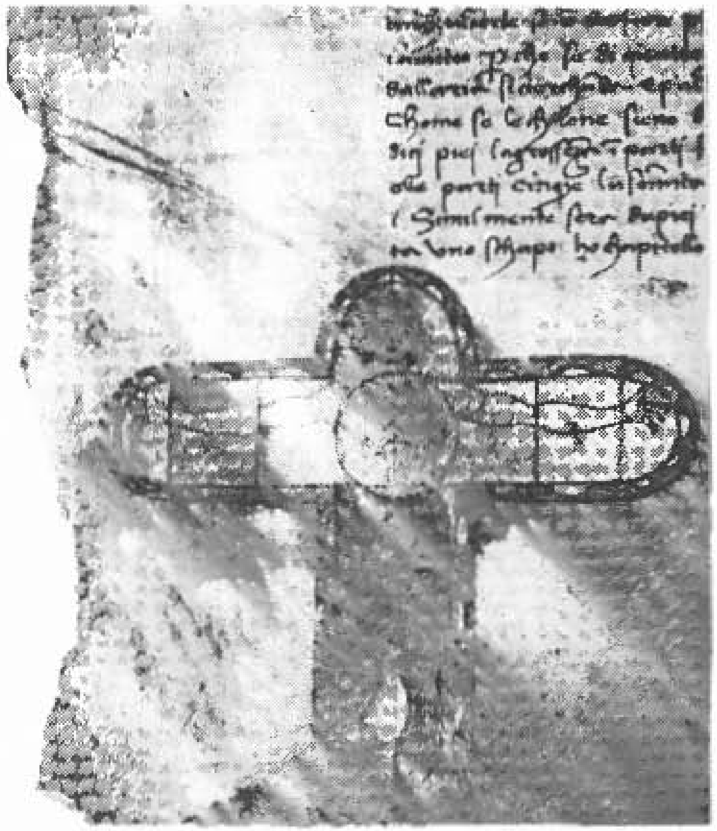
mitted to his pupil Arcangelo da Borgonuovo, who wrote a defence and explanation of this work of Pico, E. Wind, Pagan Mysteries in the Renaissance, Oxford Univ. Press, 1980. p. 226 (n24).

[11] R. Wittkower, Architectural Principles in the Age of Humanism, 3rd revised ed., London, Tiranto, 1962, deals with aesthetic proportions in a number of Italian churches built according to secret number symbolism. The rules for the moves of Rhythmomachia are laid down according to this theory of proportions.



13

Francesco di Giorgio. Ground plan of a church corresponding to the proportions of the human figure

God, the greatest architect, is master and author".[12]

With these plans for the church Giorgio not only put into practice his own philosophy, but acted like a man well-educated in the importance and interrelationship of all the liberal arts.[13] He epitomizes the highly-cultured man of his time.

Giorgio wrote a Platonic account of the creation,

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[12] Francesco di Giorgio, De harmonia mundi totius cantica tria, Paris, 1545, tr. Guy LeFèvre de la Boderie, L'Harmonie du Monde, Paris, 1579 , as quoted by Butler, Number Symbolism, p. 107.

[13] Modern in-depth study of music in the Renaissance has

Harmonia mundi totius (1525) in which he attempted to show the beauty and harmony of the created world. He used musical terms like consonance and accord in number symbolism in order to unite all knowledge of cosmology and theology into a work of art. By doing so he hoped to mirror in his opus God's creation of the harmonies of the universe. He used not only Neoplatonic but also cabbalistic-hermetic ideas and worked out exact numerical correspondences between the signs of the zodiac, the twelve tribes of Israel and the twelve apostles. There were others who shared his interests, for example, influenced by Alberti and Ficino, Garfurius became one of the musical authorities consulted by Renaissance artists who desired to structure their work on musical proportions.[14]

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revealed the enormous part mathematics played in the development of the system of musical notation in the fourteenth and fifteenth centuries. Johannes de Muris (Jehan des Murs) c 1300-1350, Nicole d'Oresme, who wrote a treatise on Rhythmomachia, and Philip of Vitry (Vitriaco), 1291-1361, in co-operation with the Jewish scholar Levi ben Gersonides, 1288-1344, (living in Avignon under the protection of the popes) paved the way for the new rhythmic and poly-rhythmic music. See Eric Werner, "The Mathematical Foundation of Philippe de Vitri's Ars nova", in _The Journal of the American Musicological Society_, vol. IX 1956, p. 128-132. The usual spelling of Vitry is converted by Werner to Vitri. Edward Lowinsky, "Music in the Culture of the Renaissance", in _Journal of the History of Ideas_, XV, 1954, p. 543 ff. Joseph Carlebach, _Levi ben Gerson als Mathematiker_ (De numeris harmonicis, based on Col. Basiliensis F II 33) Berlin, 1910.

[14] F. Garfurius, _Practicae musicae_, 1480, quoted by Gettings, _The Hidden Art_, p. 58, n 15.

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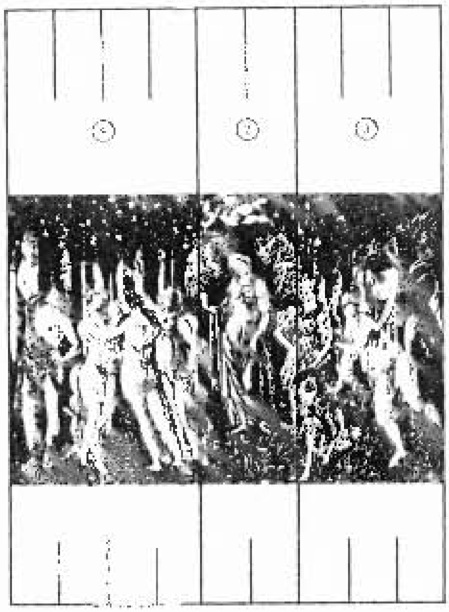
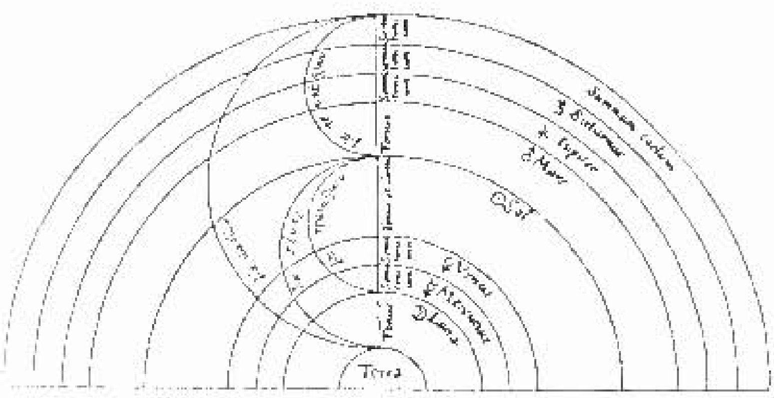


Analysis of the musical ratio 4:2:3 time on which the Primavera is constructed

15



Pythagorean musical ratios, related to the planetary intervals

An example of a painting dealing with musical proportions is Botticelli's "Primavera",[15] 1477. The

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[15] The name comes from a remark by Vasari who thought certain figures might symbolize spring. F. Garfurius, Practicae musicae, 1480, quoted by Cottings, The Hidden Art, pp. 57-63 and n 13. Also: E.H. Gombrich, "Botticelli's Mythologies", Journal of the Warburg and Courtauld Inst., vol. 8, 1945, pp. 7-60.

placing of the figures follows the ratio of 4:2:3, known as the double diapente (double fifth). This is an appropriate choice of number symbolism as the goddess Venus was patroness of music and dance. Gettings suggests that the real subject of the painting is the nature of the arts and the nature of love.

Another example perhaps is a painting of a chess game on a panel of wood, probably part of an Italian "cassone",[16] a marriage chest, now in the Metropolitan Museum of Art, New York.[17]

The painting is called "The Chess Players";[18] it is 350 mm high and 420 mm wide and is attributed by the Metropolitan Museum of Art to Girolamo da Cremona, 1442-1500.[19] Amongst art historians and connoisseurs the painting has been well known since it was first published by P. Schubring in 1927.[20]

A lesser known marriage chest panel, unpublished until 1940, was in the private collection of Bernard Berenson at Settignano, now the Harvard Center for

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[16] P. Schubring, Cassoni, Leipzig, Hiersemann, 1916, 2nd. ed. 1923, is an exhaustive study of the subject.

[17] From the collection of Maitland Fuller Griggs, New York, 1943, "The Chess Players" (43.98.8.)

[18] "The Chess Players" has been reproduced in some modern publications on the history of chess, e.g. The World of Chess by A. Saidy and N. Lessing, London, Collins, 1974.

[19] Personal correspondence, 28 January 1982.

[20] P. Schubring, Apollo, V, 1927, p. 156 attributed it

Renaissance Studies I Tatti. Berenson's panel, called "Group of Youths", is 345 mm high and 280 mm wide and is attributed to Francesco di Giorgio,the architect of San Francesco della Vigna.[21]

16 The Chess Players

Attribution of "The Chess Players" and "Group of Youths" to Francesco di Giorgio was accepted by Harry B. Wehle, Curator of paintings, in an article written

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to Matteo di Giovanni. F.M. Perkins in Art in America, XVI, 1928, p. 68 credited Francesco di Giorgio with its creation. Since then it has been published in various art magazines and exhibited on several occasions.

[21] Berenson attributed "The Chess Players" to Francesco di Giorgio, I Pittori italiani del Rinascimento, Milan, 1936, p. 146.

by him for the Metropolitan Museum of Art Bulletin.[22]

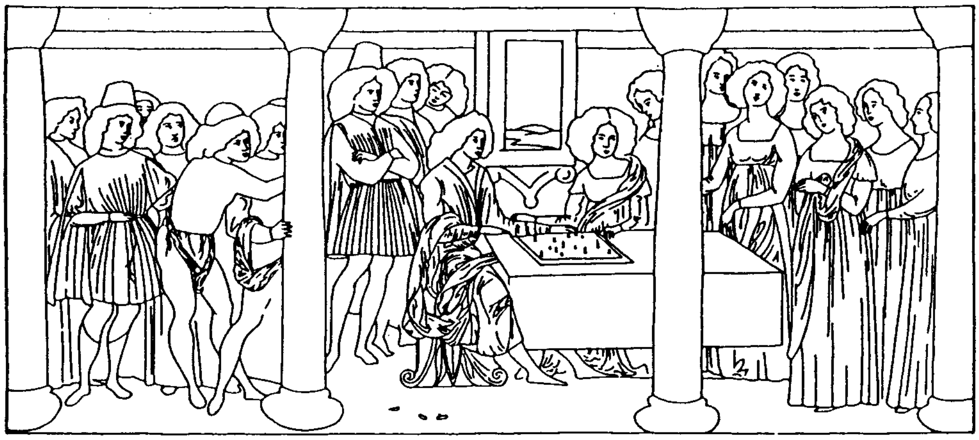
Allen Weller has made a study of "The Chess Players" (1) and "Group of Youths" (2).[23] These two panels, Weller shows, fit together perfectly: one supplies the rest of a split (green) vertical column of the other panel and shows the continuation of the interior and the group of youths on the side of the male chess player. More than that, the composition and style of painting, the colour scheme, the similarity of costumes and hair styles as well as the surface wear and tear imply that these two panels once formed a continuous whole.

Pictorial reproduction and the assembly of panel (1) and panel (2) suggested to Weller that the composition originally must have continued to the right side of the woman chess player to include additional attendants and another column to balance the composition.

Weller made an imaginative reconstruction of the still missing portion of the original "cassone" panel by adapting a similar group of ladies from a marriage chest panel by Giorgio which is in the Stibbert Museum

[22] H. Wehle, "The Chess Players by Francesco di Giorgio" in The Metropolitan Museum of Art Bulletin, February 1947.

[23] A.S. Weller, "A reconstruction of Francesco di Giorgio's chess game", in Art Quarterly, vol. VIII, 1940, p. 162-172. Previously only one scholar had mentioned Berenson's "Group of Youths", A. McComb, Art Studies, II, 1924, p. 27, although the painting was listed by Berenson in I Pittori italiani del Rinascimento.



Francesco di Giorgio, Chess Game
Reconstruction of original painting

17

Francesco di Giorgio, Chess Game
Composite photograph of the Berenson
and Griggs paintings

in Florence, No. 4098.

According to Weller the literary source of this serious chess game has eluded researchers, although P. Schubring thinks it might be an illustration of a Boccaccio story from the Decameron.[24] Weller feels the task might be a hopeless one as Prof. Grandgent said he cannot identify the source. Weller cites Courtney Conover, who studied the game itself and believes a conventionalized game is shown and not a situation resulting from actual play. John Pope-Hennessy[25] repeats, without much conviction, a suggestion already made by H. B. Wehle[26] thirty-three years earlier that the painting has been thought to represent the story of Duke Huon of Bordeaux who plays chess for the hand of a king's daughter, known to be invincible at chess. She deliberately loses because she had instantly become infatuated with Huon, who however releases her from the king's promise.

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[24] P. Schubring, Apollo V, 1927, p. 156. In fact no Boccaccio story fits the illustration, although Boccaccio mentions chess five times in the Decameron: introduction to the first, third and sixth days and the conclusion of the third and sixth days. G. Boccaccio, The Decameron, tr. G.H. McWilliam, Penguin, Books, 1972, reprint 1981.

[25] J. Pope-Hennessy and K. Christiansen, "Secular Painting in fifteenth-century Tuscany", in The Metropolitan Museum of Art Bulletin, vol. 38, 1980 to 1981, p. 17.

[26] Wehle, in The Metropolitan Museum of Art Bulletin, p. 154.

Apart from remarks on colour in the painting: the costumes of golden yellow, crimson and dark green, and the blond hair of many of the youths, no further analysis of the painting has been attempted.

Accepting Weller's reconstruction of the panel depicting chess players as valid,[27] additional points can be made.

Renaissance thinkers had confidence in the power of number; they expected number to lead to the understanding not only of creation (by God) but also of the secret of re-creation using numbers. Number, they believed, would teach the artist, architect, musician and scientist to create "correctly" in their own world of the senses. Man's position in the interrelationship of all creation was not questioned but had constantly to be redefined to assure a healthy life. Horoscopes were thought to provide one guide-line to understanding the effects of external forces (i.e. the planets) on the internal ones (i.e. the humours). Horoscopes were now cast almost as a matter of course among the nobility. Most rulers and some popes employed astrologers or were astrologers themselves.

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[27] Wehle, in The Metropolitan Museum of Art Bulletin, pp. 153-156, recognises Weller's findings regarding the two panels as correct. He also mentions another curious panel as coming from either the same or a matching "cassone" (p. 156). On this panel, which has so far defied interpretation, the protagonists are clearly the same as those that were involved in the chess game. Wehle gives no information as to the

An astrological image, or talisman, was used by those desiring protection or wanting to influence the stars. Almost anything could be a talisman for channelling the effluvia that constantly poured (it was believed) from the heavens, e.g. a talisman in the shape of a magic square was a popular protection against the influence of Saturn, which was usually considered bad but could also be beneficial because Saturn encouraged scientific endeavour and helped to create energy.

Dürer's "Melencolia I", shows the magic square of Jupiter[28] thought to counteract the melancholic temperament which falls under Saturn. It also shows Melencolia's creative ability, e.g. the geometric planes of the stone that the builders rejected but which Dürer uses to understand perspective, that is proportion.[29]

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whereabouts of this panel, but Pope-Hennessy says it was "formerly in the Wauters collection in Belgium", ibid, vol. 38, p. 53.

[28]This is the magic square of the numeral four, also showing the date of the engraving, 1514 (bottom line). The numbers add up to 34 in every direction, horizontally, vertically and diagonally.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

[29]Erwin Panofsky deals with many aspects of "Melencolia I" Albrecht Dürer, 2nd ed. New Jersey, Princeton Univ. Press, 1945, vol. I.

In Renaissance Italy "cassoni", often given in pairs as dowries, were not only important pieces of furniture and valued possessions used as strong-boxes, but also provided an aid to education: their painted panels usually showed scenes from Homer, Virgil, Ovid, Livy, Petrarch or the Bible and could be read like "strip cartoons". Depending on the affluence of the giver, great efforts were made to secure chests decorated by the finest painters.[30] Only painters in possession of all available hermetic knowledge were capable of constructing the paintings on musical proportions and at the same time tailoring the subject matter to the recipient in such a way that the gift could also serve as a talisman to assure a happy future for the couple. The following points relate to the "cassone" panel reconstructed by Weller.

The reconstructed panel suggests that the "cassone" was a gift of the finest kind: it combined usefulness and beauty with the protective power of a talisman.

As a talisman the painting fulfils the require-

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[30] Such as Uccello and Botticelli: The Oxford Companion of Art, ed. H. Osborne, Oxford Univ. Press, 1970, pp. 210, 211. Painted "cassoni" were particularly fashionable in Florence in the fifteenth century, although from the 1460s onwards high-class "cassone" panels were produced in a workshop in Siena, jointly run for a time by Francesco di Giorgio and Neroccio. One of the largest Florentine workshops produced twenty-three pairs of marriage chests for the top families of the town. Pope-Hennessy, The Metropolitan Museum of Art Bulletin, pp. 12, 15.

ments recommended by Marcilio Ficino,[31] 1433-1499, who considered melancholy a natural condition of the soul in the body and suggests that those plagued by Saturn's dry and depressing influence turn to Jove for help, i.e. to make their lives a bit more "jovial". This can be done by associating with "solar" people, namely blonds. The yellow hair or wigs of the young people hint at the specific intention to combat melancholy, something these sombre chess players seem to need. Ficino also recommends walks in green fields, such as can be seen through the window in the painting, and the contemplation of suitably painted ceilings in bedrooms[32] - contemplation of suitably painted "cassoni" should be just as beneficial. Crimson, green and gold-yellow, the colours worn by the young people in the painting, were colours recommended to "temper" the Saturnian tendency to sombre extremes. Furthermore, construc-

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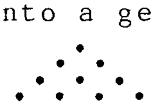
[31] M. Ficino, Opera Omnia, Basle, 1573, consulted in the translation by C. Boer, The Book of Life, Irving Texas, Spring Publ., Univ. Dallas, 1980, and R. Klibansky, E. Panofsky and F. Saxl, Saturn and Melancholy, London, Nelson, 1964, pp. 254-274.

[32] Other recommendations are the use of sympathetic herbs; for rejuvenation the collecting and drinking of dew, the heavenly effusion, and the wearing and drinking of gold as it contains the spirit of life. Ficino paid special attention to the imbalance of the four humours: blood, phlegm, black bile (melancholy) and yellow bile (choler), prompted possibly by the gout-suffering Medicis. He dealt with the psychology of the Renaissance, in this lay his originality: bringing together the body and soul with diet, herbal and alchemical practice.

ting the painting on musical proportions would, it was believed, ensure the proper interrelationship between the correspondences.

Looking at the composition of the reconstructed and extended panels we see three groups of six figures: these are clearly divided by pillars. Six men stand to the left of the chess players and six women to their right. The centre group is made up of four men and two women; a multiple ratio, here perhaps used to symbolize domestic harmony. Altogether there are eight men. The number ten has always been of great importance because it is built up of a number series that fits into a geometrical shape, it has a "triangular" nature:



The relationship of eight women to ten men is one of greater inequality, it is a sesquiquartan superparticular number (The antecedent is equal to the consequent and some part of it, namely $\frac{1}{4}$, 8, 10). If we grant the octave 12 semi-tones, the overall number of semitones is 18, one for each of the 18 figures in the painting, $1\frac{1}{2}$ octaves; or, considering six figures in each section, three groups of six, each group a pleasant consonance.[33]

The chess-board in the painting is made up of

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[33] Ficino believed all creation to be divine. Man creating a work of art or song, acts in God's image. The general view of Renaissance scholars was that the artist was divinely inspired, metaphorically speaking. In our case it may simply indicate that musical harmony is mathenatics (geometry) translated into sound. See Klibansky, <u>Saturn and Melancholy</u>, p. 250.

7 X 14 squares, a composite board between that of chess and Rhythmomachia.[34] The players are not engaged in a realistic game, the implication rather seems to be that of an occupation involving considerable mental effort, as for example practising the discipline of the quadrivium, which, it seems to be suggested, will bring true happiness and lift the spirits of the players.

This "cassone" may have been a gift intended for a scholar, scientist or mathematician. The broken object on the ground, the potsherds, might refer to the consummated marriage. This would accord with the curious panel in a private Belgian collection which might come from the same, or possibly from the other one of the pair of matched "cassoni". In it a young woman signals to a youth who sits on some rocks or she points to herself; again the same kind of potsherds lie on the ground.

Whether the "cassoni" panels were painted by Francesco di Giorgio as suggested by Berenson, or Girolamo da Cremona, as suggested by the Metropolitan Museum of Art, is not of importance. Both painters had the training and classical education necessary to understand the underlying symbolism of the paintings. Giorgio was in the forefront of theoretical thinking in the Renaissance and more prominent than Girolamo

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[34] Boards often varied in size and shape.

da Cremona. Bernard Berenson calls Girolamo the most intellectual, imaginative and accomplished of Italian miniaturists.[35] As the illustrator of the "most beautiful book in the world",[36] a Latin edition of the works of Aristotle in two volumes, he had access to Florentine education and the Neoplatonic thought of the time. His intelligence and sensitivity would have enabled him to express in his painting the various layers of numerological and philosophical thought.

The occult sciences also reveal a certain relationship to Rhythmomachia. During the Renaissance number symbolism became fused with the occult: both were kept secret from the uninitiated. An influential scholar in that field in the fifteenth century was the Benedictine abbot Trithemius of Spanheim, 1462-1516, who devoted his life to numerology and astrological calculations, used cryptography and apparently devised a method for sending messages via angels.

In the sixteenth century less mysterious and more easily understood works on numerology started to appear and were assimilated and accepted by most literate people.

Giorgio's belief in the harmonious interrelation-

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[35] Berenson, The North Italian Painters of the Renaissance, p. 169-170.

[36] Printed by Andrea dei Torresani, Venice 1483, H. Yates Thompson, The Burlington Magazine, IX, 1906.

ship of all creation has already been touched on. The Protestant view of the world, of great influence on writers such as Milton, was given by the Huguenot Salluste du Bartas, 1544-1590, in accounts of the Creation, Septmaine, 1578, and Second Septmaine, 1584. For him too, God is the great mathematician or geometrician. He discusses biblical events, astrology and astronomy, incidentally giving encyclopaedic information. He believes Hebrew to be the most important language because of its numerical properties and goes on to comment briefly on each number, e.g.

One: "... the right
Root of all Number; and of Infinite,
...
No number, but more than a Number yet
Potentially in all, and all in it. ..."[37]

He discusses the liberal sciences, speaking of music, e.g.

"... sacred harmony
And Numb'ry Law, which did accompany
The Almighty-most, when first his Ordinance
Appointed Earth to rest, and Heaven to dance..."[38]

Salluste du Bartas' essential belief was that the microcosm is a reflection of the macrocosm; therefore what happens to man on earth has its source somewhere in heaven. He, like Giorgio, Ficino, Pico della Mirandola, etc. tried to stay clear of religious prohibitions in

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[37] Salluste du Bartas, Divine Weekes and Workes, tr. Joshua Sylvester, London, 1613, as quoted by Butler, Number Symbolism, p. 63.

[38] ibid. p. 64.

his preoccupation with the occult. Some Renaissance "magicians", that is men involved in a quest for understanding, for an ultimate transformation of themselves as the "enlightened", went further.

Agrippa[39] and Paracelsus[40] no longer tried to incorporate their occult practices into the general doctrines of the Church. According to Agrippa, number is the most powerful magic imaginable, because by its structure every "thing" is "number". Since the time of Pythagoras many philosophers believed that "number" is of itself more powerful in action than any other thing. Agrippa explained the various types of number symbolism that had come to his notice and attested the virtues of various numbers.

As Butler states,

> "in discussing the number three (II,v) he (Agrippa) shows how the number has a scale of meaning on different levels. The highest or archetypal meaning of three is the three-lettered name of God (given in Hebrew, God's own language), which signifies the Christian Trinity of the Father, Son and Holy Ghost. In the intellectual (ultramondane) world it signifies the three hierarchies of angels, that is the nine pseudo-Dionysian orders of angels grouped in threes to represent the Trinity. In the celestial world, three represents the three quarternions of the zodiacal signs of the Houses used in casting a horoscope. Agrippa divides the sublunary

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[39] Cornelius Agrippa of Nettesheim, +1553, wrote De Occulta Philosophia (3 vol.) and De Vanitate Scientiarum, C. Wechelus, 1531.

[40] Theophrast von Hohenheim, known as Paracelsus, (+1541) Sämtliche Werke, ed. K. Sudhoff, Munich, R. Oldenber, 1929.

> world into two, the elemental (physical) world in which three represents the three degrees of the elements, and the minor, or microcosmic world of man, in which three represents the three main parts of a man's body, the head, breast and belly. He also, as a 'Negromantike', finds triplicities in the infernal world, i.e. the three infernal Furies, the three judges and the three degrees of the damned".[41]

On music Agrippa says that the planets emit sounds and he assigns musical modes to the stars (II, XXVI). He comments on Giorgio's dissertation "Of the Composition and Harmony of the human soul", e.g. "Reason to concupiscence hath the proportion diapason (octave); but to Anger diatesseron (fourth).[42]

Agrippa obtained information from many sources, including Arabic. He seems to have been the originator of symbols of a cabbalistic nature which frequently appear on astrological medallions.[43] These "signacula" are related to various kinds of magic squares. The magic squares in their turn are assigned to the seven planets in the order of their velocity.

Lastly, Pietro Bongo's De Numerorum Mysteria (ed. 1618) must be mentioned as a monumental encyclopaedic

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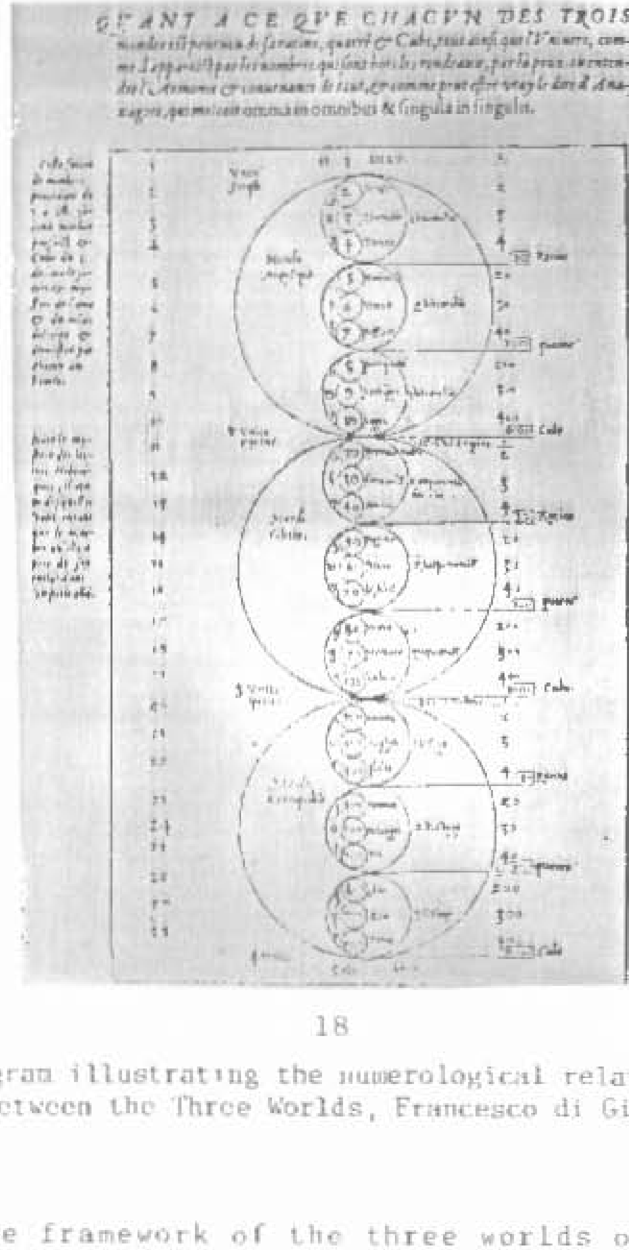
[41] Butler, Number Symbolism, p. 68, 69.

[42] ibid., p. 70.

[43] For his work on seals see K.A. Nowotny, "The Construction of Certain Seals and Characters in the work of Agrippa of Nettesheim", in Journal of the Warburg and Courtauld Inst., vol. 12, 1949, p. 46-57.

work on number symbolism relating to biblical exegesis.



18

Diagram illustrating the numerological relationship between the Three Worlds, Francesco di Giorgio

Within the framework of the three worlds of Giorgio and Agrippa and the numerical correspondences intended by the mathematician-Creator, he systematically expounds hierarchical orders in their triplicities. For example, mindful of the correspondences of the celestial and sub-lunary worlds, the seven planets rule over and are assigned to the seven organs of the body, seven metals and seven archangels, who presumably rule over them.[44] Man, it was thought develops in cycles of seven.

This search for a harmonic relationship of numbers was incorporated by astronomers like Copernicus (died 1543) and Kepler (died 1630) into their field of empirical observation. Kepler was able to differentiate between quantitative mathematics and hermetic numerological mathematics. His investigation ran on two lines; one aim was a religious one, "to illustrate the glory of the world and of God the architect".[45] The other was to break away from the idea of the universe as a sphere and the paths of the planets being circular.[46] For him a mathematical correspondence existed between the stars and the soul,[47] creating harmony between them for the aesthetic contentment of man.

In England some Renaissance buildings were inspired by numerology, e.g. the Triangular Lodge built in 1593

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[44] Butler, Number Symbolism, p. 74.

[45] ibid., p. 89.

[46] From the time of Plato it was believed that God created the universe as a sphere: Cardinal Nicholas of Cusa took the sphere as the symbol of the Trinity, the centre corresponding to the Father, the surface to the Son (the visible body), and the ether-filled intermediate region to the Holy Ghost. Kepler showed the elliptical movement of the planets round the sun. He also worked out the exact musical chord emitted by the planetary spheres.

[47] In this way astrological influences on man can be explained.

for Sir Thomas Tresham.[48] However, in England, poets rather than artists and architects structured their work on number symbols and allegorical lines.[49]

The critic in this relatively new field labours under the difficulty of the esoteric allegory involved, the scarcity of clues and the lack of any explicit information about the numerological construction of poems.

Alistair Fowler in a study of the "Faerie Queene, Spenser and the Numbers of Time" says the poem:

> "is in fact an astonishingly complex web of interlocking numerical patterns of many different kinds. We find numerological patterns in line-, stanza-, canto-, and book- totals; in the location of these units; and even in the numbers of characters mentioned in each episode. Pythagorean number symbolism, astronomical number symbolism based on orbital period figures and on Ptolemaic star catalogue totals, medieval theological number symbolism: all these strands, and more besides, are worked together into what - in this respect at least - must be one of the most intricate poetic textures ever devised."[50]

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[48] The three walls are each 33'4" long, which adds up to 100', (multiples of ten were thought to reduce all numbers to unity, the monad of God); each wall represented one person of the Trinity, mystically united. Butler, Number Symbolism, p. 108-9.

[49] For example only fairly recently has Renaissance poetry been subjected to critical analysis in an attempt to discover its hidden meaning, by Alistair Fowler, Maren-Sofie Røstvig, Christopher Butler and A. Kent Hiett.

[50] A. Fowler, Faerie Queene, Spenser and the Numbers of Time, p. 4, as quoted by Butler, Number Symbolism, pp. 132, 133.

This indicates the complexity involved in the interpretation of poetry which might by structure and content contain any one of the various number symbolisms: cosmological, astrological, musical etc. The greatest Renaissance poets, e.g. Milton, were able to manifest the

> " ... secret power
> Of harmony in tones and numbers hit
> By voice or hand, in various measured verse, ..." [51]

Creating poetry of this calibre allowed the soul - in Ficino's terms - to participate in the harmony of the Divine mind.

Another way of expressing the seemingly inexpressible appears to be epitomized by the Hypnerotomachia Poliphili.[52] This highly enigmatic and seminal work was published in Venice in 1463 by the Dominican Francesco Colonna, (died 1527). He used Greek, Arabic, Latin and Italian to recount the struggles of the hero Poliphilus to gain learning and insight. This is represented as a dream analogy. Poliphilus is led from

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[51] John Milton, Paradise Regained, London, Frowde, 1904, Book IV, lines 254-256.

[52] 1592 edition translated by "RD" as Strife of Love in a Dream, Scholars Facsimiles and Reprints, 1973. Aldus Manutius published it in 1499 with many illustrations. The English edition was dedicated to the humanists Sir Philip Sidney and Robert Earl of Essex.

the temptations of the world by five nymphs, corresponding possibly to the five girls who created trees in Pietro da Cortona's drawing and personified the five elements we met in Walter of Speyer's poem. The hero arrives at the magnificent Palace of Liberty, over the entrance of which is written in Greek "The Wealth of Nature", perhaps hinting at the view that natural elements and mathematics are the basis of creation, and that knowledge frees the spirit of man.

Queen Elenterilda entertains the hero with a very special game of chess, the war-game inverted to a love-game. Nymphs instead of pawns, dressed in gold and silver, with flowers in their long curls, move in a graceful dance or ballet to the music of a threefold measure, and kiss their captors before going off the board. Chess, it seems to be implied, teaches not only strategy and the hierarchical ordering of society but also mathematics, the basis of creation, (i.e. the elements are moved to creation by the music of the spheres).

Colonna's illustrations strongly reflect his predeliction for the writings of Hermes Trismegistus and the ancient wisdom of elephant and obelisk[53] embellished

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[53]A Baroque fountain outside the Cathedral in Catania, Sicily, is in the shape of a black lava elephant, bearing a white marble obelisk on its back.
F. Rabelais (died 1553) a hundred years or so later used the chess allegory in The Histories of Gargantua and Pantagruel (ed. J.M. Cohen, Penguin Books, 1979)

with hieroglyphics, but also his interest in shapes and forms. These stimulated ideas in contemporary and later artists and provided a model for fine pottery, attributed to Niccolo da Urbino, c 1515. Generally it helped to create an interest in classical learning and classical ruins.

The cultural background of Renaissance thinking in Italy sketched above is that of the Medici, Strozzi and Rucellai families of Florence. These families were related by marriage and during the political upheaval between about 1530 and 1540 went into (partly) voluntary exile in Padua, taking many of their friends with them.[54] These men were well educated, interested in the liberal arts and, as humanists, studied Greek, discussed Dante, the meaning of the Divine Comedy and artistic problems. Boards, pieces and rules for Rhythmomachia came into their hands; some good players are mentioned,e.g. Don Mauro, Father Giuliano del Carmine, Papi Tedaldi and Cristophano Boniai. This game appealed to the young men because it taught mathematical proportions, a subject much discussed at the time. Carlo

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Book V, ch. 24-25 (I am aware of the controversy about the authenticity of Book V, published only in 1564; it does however not affect the topicality of the allegory).

[54] Cosimo Medici, 1389-1464, founded the Platonic Academy. Cosimo I, later Duke of Florence, 1519-1574, was a young man during these political upheavals. Florence was the centre of cultural activity during the Renaissance.

Strozzi describes a visit to the gardens of Cosimo Rucellai in Padua, where he found his friend discussing proportions with Jacopo Vettori. After some time had passed, the board and pieces for Rhythmomachia were sent for and the friends settled down to a game of proportions.[55]

Benedetto Varchi, 1503-1565, one of these exiles, wrote a treatise on proportions for his friends which he dedicated to Luca Martini, who in turn prepared for them an exhaustive work on Rhythmomachia.

In December 1539 Varchi wrote to his friend Luca Martini, reporting that nearly every day was spent in playing Rhythmomachia with their friends Albertaccio and Ugolino. The latter, he goes on to say normally dislikes and abominates all other games.[56] Varchi goes on to suggest that a pair of boards should be made and the equipment and rules of the game should be sent to Nicolo Alamanni[57] who lived in Rome at that time.

Varchi was a leading literary figure of this period, reviving the classical concept of "Ut pictura poesis"

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[55] Il Manoscritto di Carlo Strozzi, Florence, II. II. 278 contains the treatise on Rhythmomachia, a work on proportions, a description of the visit to the garden, and of how Rhythmomachia came to their knowledge.

[56] For a copy of the letter from Varchi to Martini, 31 December 1539 see appendix

[57] This was the son of Luigi Alamanni, + 1556, the Florentine democratic politician and poet. Other humanists involved were Daniel Barbaro and Sperone Speroni.

for the Renaissance and arguing whether sculpture or painting had the primacy in art.[58] His many friends included the cardinal of Nicasto and Pope-elect Marcello Cervini, 1501-1555, and the Papal Nuncio, Lorenzo Lenzi, (died 1571). His uncle Cardinal Gaddi kept open house in Padua, and apart from politics there may have been many wide-ranging discussions of interest to humanists.

For these cultured circles artists, architects, musicians and poets created works that were commissioned and understood, e.g. the Primavera and the cassoni; at the Florentine academy[59] Ficino translated Plato and Hermes Trismegistus from the Greek into Latin for Cosimo Medici, and after his death continued to work for his son Piero and educated the grandsons Lorenzo and Giuliano. There he wrote a commentary on Plato's Symposium and his most famous book, the Theologia Platonica.[60]

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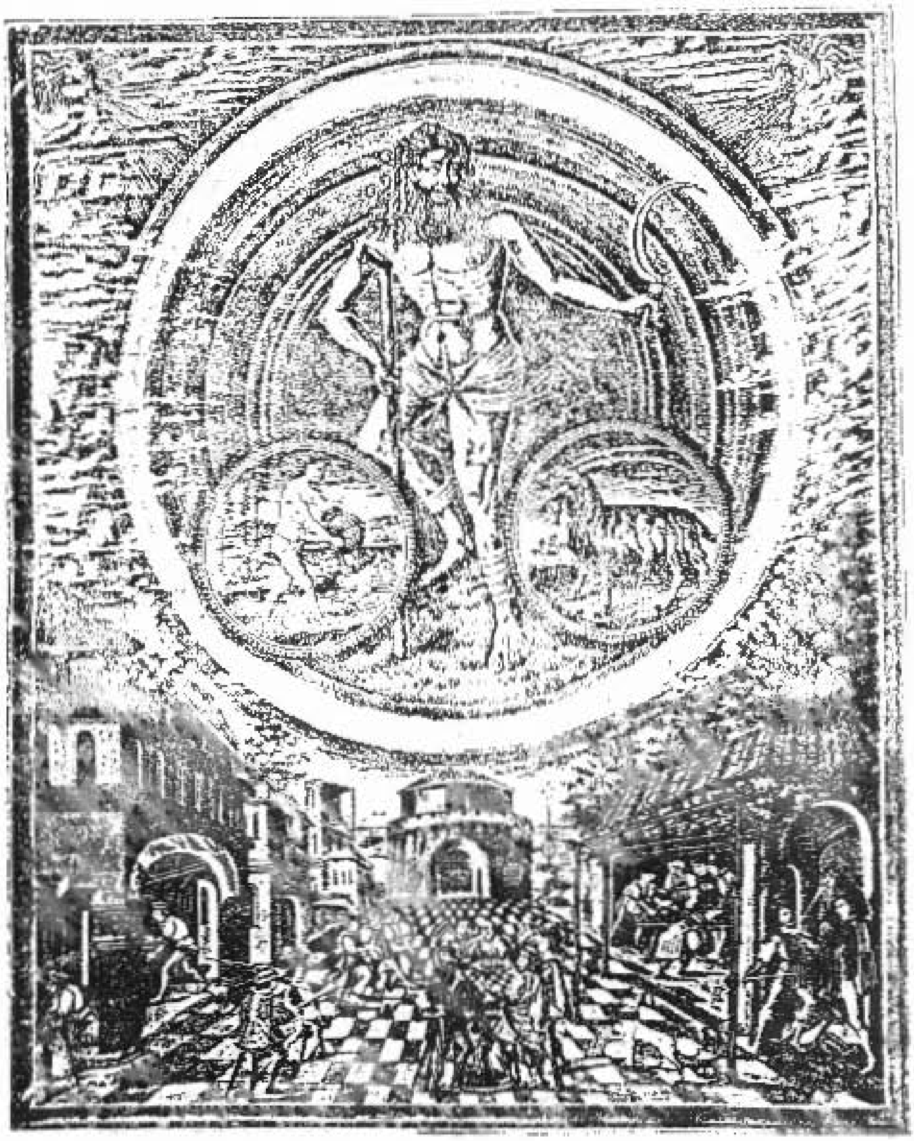
[58] Varchi was also the author of a political history of Florence, Storia Florentina, 1528-1538.

[59] Cardinal Ippolito de' Medici founded a club in Rome for the study of Vitruvius, the famous Roman architect of the first century A.D., whose work, a ten volume architectura dealing with everything related to building from town planning to interior-decoration, was discovered in the monastery of St. Gallen in 1414 and printed for the first time in 1484, reprinted with illustrations in 1511. It exerted an enormous influence on Renaissance ideas of architectural beauty, harmony and proportions. During the sixteenth century academies were founded in many Italian towns.

[60] The genius of Ficino was recognised during his lifetime

·SATVRNVS·



20

Saturn
from an Italian Book of Days

Artists, philosophers, merchant-bankers and scholars thought of themselves as Saturnians, with a tendency to melancholia, a condition to which scholars were thought to be particularly prone. One of their problems was how to live with their restless souls in a restless age. However, Saturn did not come without "gifts": he brought men creative power. These men saw no contradiction between astrology, science and empirical observation; on the contrary they seem to have welcomed the rule of heavenly bodies over earthly concerns as comforting and natural, guaranteeing continuity and stability.

An illustration from an Italian Book of Days shows Saturn's "gift" to men: farming (Saturn the Reaper), busy workshops in a well-planned city and a fighting spirit (but also private strife). Central in the picture is the chess-board, surrounded by six men, more than are needed for a mere game. This board may refer to what it can teach; obviously strategy, the hierarchical ordering of society and chess-moralities,

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and proved an inspiration to many of the greatest philosophers, poets and artists of the Renaissance. Ficino's Liber de Vita was published in 1489 with endorsements from powerful friends at the Academy but against the disapproval and attack by various clerical enemies. It was dedicated to Lorenzo de Medici, Philippo Valori and the King of Hungary. Michelangelo frequently stayed at the villa and would have heard Ficino discuss his ideas about the importance of the body; it may have influenced the creation of his magnificent figures.

but also mathematics. The chess-board itself could be considered a magic square, connected to mathematics.[61]

The importance of chess as a metaphysical concept, an allegory or a symbol can be seen from the example of a political satire. The play A Game at Chess[62] by the prolific playwright Thomas Middleton, 1580-1627, caused a furore in London and may have led to the imprisonment of the author. T.S. Eliot called it "a perfect piece of literary political art".[63]

The subject of the play is the ill-conceived visit to Spain by Prince Charles and the Duke of Buckingham in 1623 to settle a marriage. Charles and

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[61] Magic squares were known from ancient as well as Roman times. One of the earliest western MS on the subject is by Emmanuel Moschopoulus of Constantinople, Bibl. Nat. Paris, cod. 2428, c 1300.
In medieval romances magic boards are not uncommon. Alfonso X of Spain, El Sabio, ordered a treatise on magic and magic squares, the Picatrix, to be translated from Arabic into Latin, 1276.
See also David Pingree, "Between the Ghāya and Picatrix, I: The Spanish Version". Journal of the Warburg and Courtauld Inst., vol. 44, 1981, pp. 27-56.
In the collection of games made for Alfonso X, in Spanish, Escorial, siglo T-I-6, Libro de ajedrez, several versions of chess are recorded: one board is in the shape of the sign of the zodiac, seven concentric rings, divided into 12 parts, each showing one of the constellations. The game was played by seven people with a seven-faced die.
In another variation four players battled with the four humours, the four colours (green, red, white, black), the four elements and the four seasons.

[62] ed. J.W. Harper, London, Ernst Benn, 1966.

[63] T.S. Eliot, "Thomas Middleton", Selected Essays, 1917-1932, London, Faber & Faber, 1932, p. 166.

Buckingham are the "white knights" in the game of chess. The other players in the great catholic chess game are trying to convert Charles. The satirical attack was focussed on Count Godomar, the "black knight", Spanish Ambassador to England until 1622.

An example of the recognition of the importance of games as serious intellectual teaching aids in the Renaissance, which may have come to the notice of Florentine scholars, is Thomas Murner's use of special playing cards. Some of these incorporate chess situations.

19

Two of Thomas Murner's playing-cards

[64]The Franciscan monk, teacher and humanist, best known for his satyrical writing, Thomas Murner, 1475-1537, studied mathematics and mnemotechnique with Lefèvre d'Etaples in Paris, travelled in Italy; he published his highly successful teaching aid of playing cards in book form in Cracovie, Brussels and Strasbourg between 1507 and 1509 (reprinted in Paris as late as 1842). Thomas Murner, Logica memorativa ...Joannes Gruninger, Strasbourg, 1509, NB Vienna, sign. 80. E. 66. Murner devised yet another card game for teaching at the Institute of Justinian, which also came out in book form, Thomas Murner, Chartiludium Institute summarie... , Johannes Prus, Strasbourg, 1518, NB Vienna, 33. H. 14.

Selected Manuscripts and Books

During the fifteenth century chess remained as popular as it had been during the Middle Ages. Many keen players copied chess problems ("mansubat"), others transcribed chess manuals which were always in demand. In about the middle of the fifteenth century there spread from Spain what came to be known as the "new game" in which, amongst other minor alterations, the queen and bishop exchanged their medieval privileges for the moves which they still retain. Thus the queen became twice as strong as the rook, formerly the strongest piece.[1]

Within the next fifty years or so the new game became known throughout Europe,and with it came the demand for new books of rules. These later also became available in printed form. Some of the manuscripts and printed books on chess included a treatment of Rhythmomachia.

An example of a handwritten chess manual with 20 pages on Rhythmomachia bound in the same volume is in the Ashmolean Library, dated c 1470. It consists of 61 vellum leaves and on fo. 3 Roger Hartwell claimed ownership of it in 1529. The chess book is written in an English North Country dialect and shows much use,

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[1] Murray, A History of Chess, pp. 776-777.

with annotations and corrections.[2] By contrast the Latin Rhythmomachia in this case seems to have been largely unused.

Towards the end of the fifteenth century the first printed treatises on Rhythmomachia started to appear. In 1482 John Shirwood, Bishop of Durham (died 1494) published an account of Rhythmomachia[3] which he had started to compile for his patron George Neville, Archbishop of York. On Neville's premature death in 1476 before the completion of the book, Shirwood found a new patron in the Venetian nobleman Marco Barbo, Bishop of Vicenca, later Bishop of Palestrina.

The British Museum has a copy of the 28 page book by Shirwood. No publisher is mentioned but the volume is believed to have come from the printing shop of Ulrich Han. In the margin someone stated that this treatise on Rhythmomachia is of great rarity. The writer of manuscript Casanatense 791, dated 1511 seems to have plagiarized Shirwood's work by copying it exactly but eliminating any attribution to the author.[4]

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[2] Murray, A History of Chess, p. 601.

[3] The full title reads: "Ad reverendissimum religiosissimumque in Christo patrem ac amplissimum dominum Marcum cardinalem Sancti Marci vulgariter nuncupatum Johannis Shirvuod (sic) quod latine interpretatur Limpida Silva sedis Apostolicae protonotarii Anglici, praefatio in Eptinomen de ludo arithmomachiae feliciter incipit".

[4] Chicco, Bonus Socius, p. 86.

A second account of Rhythmomachia was published towards the end of the fifteenth century in Paris by the famous French scholar and humanist Jacques Lefèvre d'Etaples, [5] 1450-1536.

This four and a half page "Rithmimachie" was included in 72 unnumbered pages with three other works: Lefèvre's commentary on Jordanus' "elementa arithmetica", a treatise on music, and his commentary and "epitome" of Boethius. The Rhythmomachia carries a dedication to Bernardo Vencario, a doctor of medicine and "lover of numbers".[6] Here Lefèvre mentions that his friend has advised him to relieve his studies with a game; this he does, with a recommendation of Rhythmomachia, which makes the study of arithmetic easier for those who find it a difficult discipline.

The first edition of this work came out in Paris on 22 July 1496.[7] It must have been a fairly large one as many libraries in France, England, Germany and Italy have copies of it. A second edition followed in 1507 and a third in 1514. In Germany the "Lefèvre Rithmimachie" was referred to by Gustavus Selenus in the

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[5] The Latinized name is Jacobus Faber Stapulensis.

[6] See appendix A for a copy of this "Rithmimachie", Wolfenbüttel Library, sign. A I, Arithmetica: in hoc opere contenta: Arithmetica decem libris demonstrata quatuor (jordani), musica libris, Epitome in libros/ Arithmeticos diui Seuerini Boetii, Jacobi Fabri, Rithmimachie ludus qui et Pugna numerorum appellatur.

[7] Published by David Lauxius of Edinburgh, then working

introduction to his own German Rythmomachia of about one hundred years later.

This "Rithmimachie" by Lefèvre d'Etaples, was brought back from Paris to Florence and Padua by a friend of Pella Rucellai (who calls Lefèvre "Jacopo Fabro"). Pella's son Cosimo and other political exiles became enthusiastic players of the game because it fitted in with their interests in Platonic and Neo-platonic studies of harmony and proportion and the general preoccupation of humanists of that time.[8]

Luca Martini was prevailed upon to write a treatise for his friends. His manuscript,[9] which is in Italian and was never published, extends over some 74

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in Paris, printed by John Higman and Wolfgang Hopilus.

[8] Cosimo Rucellai's grandfather was Bernardo Rucellai, a Florentine nobleman, humanist, historian and the greatest patron of the arts after Lorenzo il Magnifico whose brother-in-law he was. Cosimo continued to host the famous Orti Oricellari meetings in the Rucellai gardens in the Via della Scala in Machiavelli's time; Machiavelli dedicated his Discorsi to Cosimo Rucellai and Bondelmonte. These intellectuals from a merchant-banking background no longer devoted most of their time to money-making, although large fortunes were sometimes needed for their intellectual pursuits. Ficino's Platonic movement had the effect of toning down the traditional political problems of Florentine humanists and partly replaced them with philosophical problems of a general character: e.g. discussions of abstract concepts, love, beauty, etc. Felix Gilbert, "Bernardo Rucellai and the Orti Oricellari" Journal of the Warburg and Courtauld Inst., vol. 12, 1949, p. 103-131.

[9] "Strozzi MS" II, II, 278, Biblioteca Nationale, Florence.

first French Rhythmomachia, in 1554. Two years later, in 1556, he brought out one in Latin with the title:

> Nobilissimus et Antiquissimus ludus Pythagoreus (qui Rythmomachia nominatur) in utilitatem & relaxationem studiosorum comparatus ad veram & facilem proprietatem & rationem numerorum assequendam, nunc tandem per Claudium Buxerium Delphinatem illustratus

printed by Guillaume Cavellat, "at the sign of the Fat Hen in front of the college of Cambray". The sale of the book, board and pieces is advertised on the front page, for the bookseller John the Gentile near the Palace (see introduction).

Boissière was an influential scholar and successful author.[12] He published "Art Poetique" and "Art d'Arithmetique" with Annet Brière in 1554, followed by treatises on geometry and astronomy and a comment on practical astrology by Jean Bullant. In 1557 he published "La propriété et l'usage des quadrans". Boissière mentions a few of his contemporaries who played Rhythmomachia: Oronce Fine (Orontius Finaeus, 1494-1555), a mathematiciam with interests in the hermetic arts, and Jacques Lefèvre d'Etaples. As earlier players he names Gilbertus Papa, Hermannus Contractus, Cestrensis and Nicolaus Orestinus.

Boissière dedicated his Rythmomachia (1 June 1556) to Antoine Escalin des Aimars, baron de Lagarde,

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[12] A. Rochas, Biographie du Dauphiné, Genève 1971, reprint of Paris ed., 1856-1860, vol. I, p. 157.

1498-1578, who led a naval campaign against Henry VIII of England under the high command of Claude d'Aimebaut, fought against Emperor Charles V and was sometime ambassador to Venice for Francis I.[13] In a lengthy discourse Boissière emphasizes that turmoil and adversity are better sustained when one is employed on some noble task, such as the study of mathematics, and he cites as one example amongst several, Julius Caesar, who in "the midst of storms and tempests never gave up the noble study of the mathematical arts".

Boissière closes the dedication by explaining that Rhythmomachia takes its origin from "harmonies and proportions, ratios of the most subtle and ingenious Pythagoras" and promises that after playing "...not only men who are sad and gloomy come out cheerful and gay but also some fruitful result will accrue to those who pursue excellence: since they will be more ready for the investigation of principles and harmonies: acuteness in which and understanding of which will be of the greatest assistance to us for the contemplation of the quantity of all things - the quantity which this so great and so wonderful world embodies and embraces".[14]

In the main body of the work, after setting out

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[13] Escalin seems to have fallen out of favour in the Spanish war and became, like so many aristocrats, politically suspect. See dedication of Boissière's Rythmomachia in appendix C.

[14] For a fuller translation see appendix C.

how one arrives at the particular numbers which do battle in the game, and before he explains procedure, Boissière gives an allegorical account of the numbers and pyramids. He likens the odd and even numbers to a marriage; male and female will produce offspring, again male and female, who will in turn combine and reproduce until the numbers to be used in the battle are arrived at. The pyramids are constructed by superimposing decreasing layers of squared numbers starting with six as the base for pyramid (a) and eight as a base for pyramid (b). These layered pyramids Boissière

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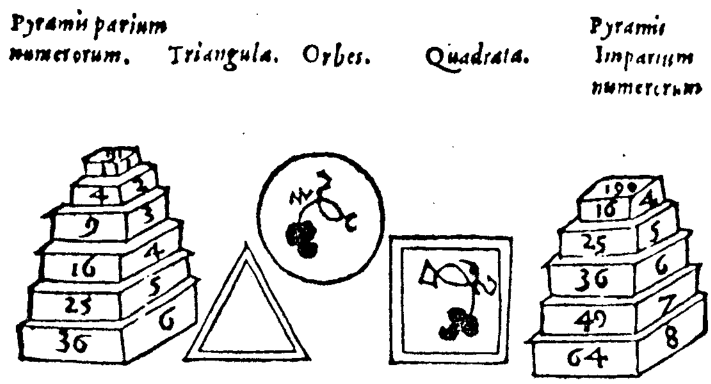

[15]

likens to friendship. The layers are friends who cleave together, fight together and, because under certain rules they can be broken up under attack, these friends can sacrifice themselves for one another.[16]

[15] Boissière, Rythmomachia, p. 12.

[16] ibid, p. 43-45.

Rhythmomachia was known and played during the Renaissance not only in France and Italy but also in England and Germany. In England an alternative way of playing was sometimes practised. This variation was, to an extent, an imitation of chess, played on the single chess-board with hierarchically graded pieces,e.g. the circles doing duty as pawns, the pyramids as kings, and so on. Boissière included a description of this version in his book on Rhythmomachia under the heading: "Altera Chaleorum ludendi ratio". In the introduction he relates that two Englishmen acquainted him with this variant of Rhythmomachia: Thomas Randolphus, who studied in Paris, possibly mathematics at the Sorbonne under Boissière, and Thomas Topcliphus, one of the most erudite and highly-cultured men amongst the English.

Thomas Randolphus was a noted English Protestant politician, for some time hiding in Paris from persecution under the reign of Mary Stuart. Thomas Topclyphus, probably a member of a prominent Yorkshire family, was particularly "distinguished, pre-eminent and excellent in astronomy, algebra and scenography".[17]

From the presumption that Rhythmomachia was invented by Pythagoras because it demonstrates Pythagorean proportions, Boissière is led to the erroneous supposition that chess was an adaptation of the much older

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[17] Boissière, Rythmomachia, p. 83.

Rhythmomachia. On the contrary, what we see here is the closing of the circle: Rhythmomachia originally, about 1000 AD, borrowed from chess (a game only recently imported from the East) the board, or rather the idea of using a counting-board (8 X 16 uncheckered squares) and the idea of opposing "armies" engaged in battle. More than 500 years later, when metaphysical thinking about the world was radically changing, Rhythmomachia collapsed into itself as it were and shrank to nothing more than "chess by numbers".

Soon after Boissière published the "alternative" Rhythmomachia, William Fulke in England, in 1561, published a Rhythmomachia under the pseudonym "W.F." with a dedication to Lord Robert Dudley: "The most noble and learned play called the Philosopher's game, invented for the honest student and other sober persons...".

Fulke was something of a busybody, interested in the arcane. The work he published under "W.F." was in fact written by Ralph Lever, also a mathematician with similar interests.[18] In imitation of Boissière "W.F." also advertised the sale of his Rhythmomachia, board and pieces with good results:

> "All this belonging to this game
> for reason you may bye:
> at the booke shop under Bochurch
> in Chepesyde redilye".

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[18] W. Fulke also published astronomical and geometric games. Dictionary of National Biography, L. Stephen & S. Lee, London 1908, vol. VI, p. 745 f.

Fifty years earlier Sir Thomas More would have known and probably understood the uncorrupted Rhythmomachia - dare one speculate that he may have discussed and played it with Erasmus? More was sufficiently impressed with it to declare in his Utopia[19], V, (1516) that the inhabitants are opposed to dicing for recreation, but interested in the "battle of numbers, in which one number plunders another".[20]

A century later Robert Burton in his Anatomy of Melancholy lists as "ordinary recreations which we have in winter, and in most solitary times busy our minds with, are cards, tables and dice, shouelboard, Chesse Play, the Philosopher's game, Billiards and Musicke".[21] Joseph Strutt gives a similar list of games in his Sports and Pastimes of the people of England.[22]

The most prominent man in Italy to publish a Rhythmomachia in the sixteenth century was the Venetian nobleman Francesco Barozzi, 1537-1604. Il nobilissimo et antiquissimo giuoco pythagoreo nominato rythmomachia cioè battaglia de consonantie de numeri,[23] Venice 1572,

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[19] Sir Thomas More, Utopia, 1516, ed. Edward Surtz, S.J., New Haven, Yale Univ, Press, 1964.

[20] On p. 71 the editor identifies the game in a footnote as that described by Jacques Lefèvre d'Etaples.

[21] 1621, pp. 345, 346.

[22] London, Tegg, 1841, vol. IV ch. II, p. 305-316.

[23] In another edition printed by Gratioso Perchacino it is entitled Il guico pittagorico nominato Ritmomacchia per Franc. Barrozi. Venet. 1572.

is dedicated to Cardinal Caillo Paleotti.

Barozzi was a humanist, classical scholar and antiquarian, who published works on mathematics and geometry. His treatise on Rhythmomachia is clear and easy to follow and includes the alternative way of playing, more similar to chess. It was well known beyond the borders of Italy and, with Claude de Boissière's work, formed the basis of the most important German book on Rhythmomachia, that by Gustavus Selenus, 1579-1666. Gustavus Selenus (pseudonym for Duke August of Braunschweig-Wolfenbüttel, a great scholar and bibliophile) included the Rhythmomachia in a treatise on chess entitled: Das Schach oder König-Spiel...Diesem ist zu Ende angefüget ein sehr altes Spiel genannt Rythmo-Machia. The Rhythmomachia commences on p. 444 as: "Ein vortrefflich und uhraltes Spiel dess Pythagoras", presumably because the numbers are derived from Pythagorean principles of proportion and harmony.

This combined volume of chess and Rhythmomachia, lucid and well-written, with helpful illustrations, was printed in considerable numbers by Henning Gros, Jün. The National library in Vienna has two copies; there is a copy in the Beinecke Library at Yale University, New Haven; and the illustrations reproduced in this study come from a copy in the Wolfenbüttel library. Selenus says in the introduction that the game is worthy of its inventor Pythagoras - as Selenus believes - who called it in his "mother-tongue" "Rythmomachia", which means a battle of numbers in proportion. This game, says Selenus, helps one to understand the art of numbers and the art of singing, which have their roots in it. It diverts and refreshes those who work with their heads and leads to higher and more serious things.

The Rhythmomachia of Selenus takes up 53 pages, and has many illustrations, elaborate headings and capital letters at the beginning of each chapter. The whole book is much more baroque in appearance than the publications of Boissière of half a century earlier.

Selenus also made some changes: he no longer uses square laminae only for building his pyramids but models his "towers",as he calls them, on those demonstrated by Boissière in his supplement, where he discusses the alternative, the so-called English way of playing Rhythmomachia (p. 86). Selenus refrains from explaining how the numbers 91 for pyramid (a) and 190 for pyramid (b) are arrived at, but says in effect that it is difficult enough to learn to play the game and not necessary to burden his readers with the science of numbers necessary for the construction of the pyramids.[24]

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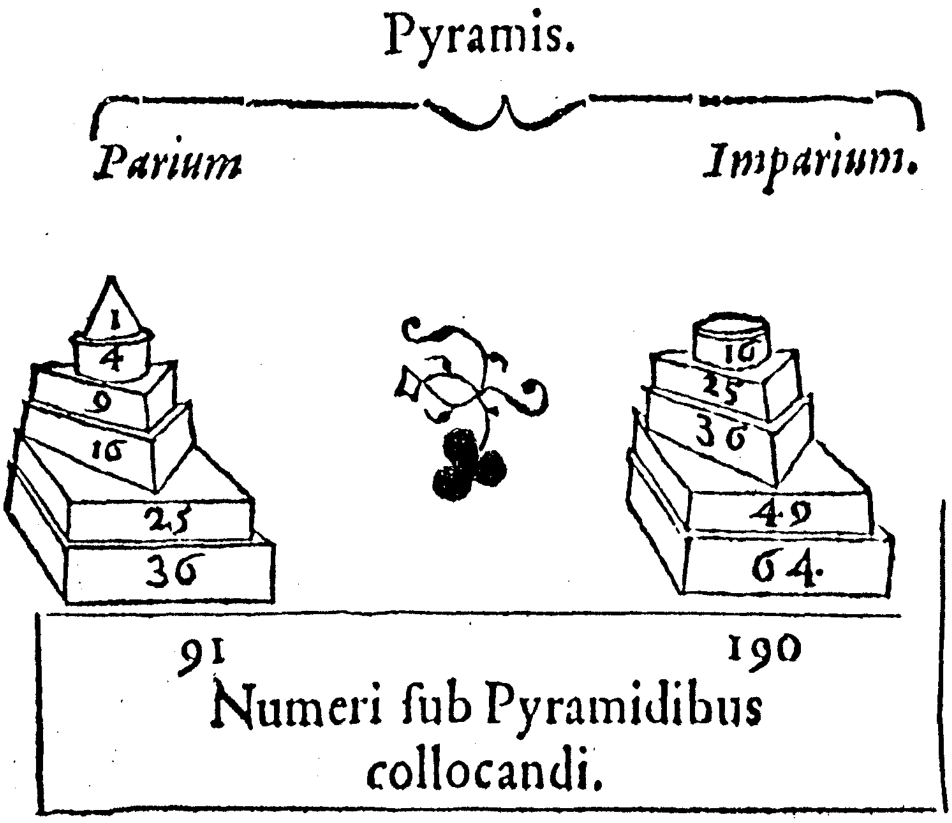
[24] "... Diese oberwehnte Zahle/ob sie wol/durch ein sonderbares künstliches nachdenken/von dem ersten Erfinder dieses Spieles/erdacht/und aus den Abtheilungen/der Zahl-Kunst/genommen worden: So habe ich doch/ dieselbe Abtheilungen/ mit sonderem Fleiss/

Boissière's pyramids for the alternative way of playing



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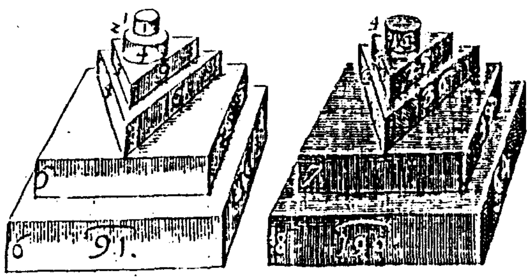

Die Zahle der Stäflen / de. beeden Thürne / welche man / an stat zweyer Vierecketen Steinen / (alß dessen / welcher / in der Weissen Steinen / hauffen / mit 91: und dessen / welcher / in der Schwartzen / hauffen / mit 190, gezeichnet ist) gebrauchet / werden /

Gustav Selenus' so-called "towers"

As far as can be established now, Shirwood was the first man in the Renaissance to suggest alterations in the rules of Rhythmomachia, which started a hierarchical grading of the pieces that culminated in the use of the pyramid as "king" in the English alternative way of playing. Shirwood's suggestion was that no game could be considered as won unless the opposing pyramid had been taken.

The Wolfenbüttel library preserves a letter, II $\frac{1}{692}$, written on behalf of the Emperor Mathias on 5 September 1615 granting permission to the publisher Henning Gros, der Jüngere, Leipzig, to bring out Gustav Selenus' Schach oder Königspiel and the "age-old" game of Rhythmomachia and sell the book for six years provided he also delivered free of charge to the Hradschin in Prague three copies of the same book. There is

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/stillschweigend vorbey gegangen/und meinem Bericht/viel lieber dahin/wie man gedachte Zahle/ehe durch den Gebrauch/dan durch die blosse wissenschaft/erfinden möge/richten wollen. In erwegung/die gemühter und gedanken der jenigen/welche dieses Spiel/ohne sonderliche mühe/zu erlernen begierig/mit diesen schweren Abtheilungen/sondern verwirret würden: So könten auch die jehnigen/welche/von diesen Abtheilungen/gantz keine wissenschaft tragen/dieselben/so leidlich/nicht fassen/noch erlernen..." p. 461.

[25] Shirwood was also interested in chess, an extremely popular game, and seems to have wanted to bring the two games closer together. He used the medieval move of the bishop to illustrate the move of certain pieces in Rhythmomachia. "Trianguli autem in tercium locum, non quidem ut Miles in scacho, trahuntur, sed uel directe dextrorsum sinistrorsum ante seu retro, uel omnino angulariter quem ad modum in ludo scaccorum Alphinus". MS Casanatense (C), vii a. as quoted by Murray, A History of Chess, p. 776.

unfortunately no information as to who made use of these books. The most probable explanation is that they were ordered for the Kunst-und-Wunderkammer library which the scholarly emperor Rudolf II had started in his castle, the Hradschin of Prague, in an attempt to assemble an encyclopaedia of the visible world. This idea was completely in the spirit of the Renaissance and was widely imitated.

Gustavus Selenus may have come across Rhythmomachia in the library of his kinsman Duke August of Saxony. Abraham Riese had produced a German Rhythmomachia for Duke August : "Endliche erclerung Churfürstlich Sexischer Arithmomachie durch Adam Riese Anno 1562.[26]

A publication that might also have come to the notice of Selenus is that by G. Baumann: Rithmomachie oder arithmomachia, Beschreibung eines kunstreichen Spiels mit Zahlen aus dem Latein verdeutscht.[27]

The last book exclusively on the subject of Rhythmomachia was produced in Görlitz in 1705 by H.A.V.W. and paid for by Gottlob Laurentius, but this was no more than an elaboration of the Erfurt publication of 128 years earlier.

With the political troubles unleashed in Europe

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[26] Dresden, Sächsische Landesbiblothek, Codex C 433.

[27] Erfurt, 1577, translated from Latin by Gottschalk Everback. The public library in Cleveland, USA, has a copy of this rare book.

by the Reformation and Counter-Reformation the interest in Rhythmomachia, as in culture in general, seems to have come to an end. Mention of the game lingered a little longer in volumes of games, but the real purpose of it was no longer understood. The last Italian version by Marco Aurelio Severino, La Filosofia ovvero il perchè de gli scacchi appeared in Naples in 1690, published by Bulifon, with incorrect illustrations of the pieces. There was probably nobody left to remember correctly, Severino having died in 1620.

On the whole, evidence of Rhythmomachia has not survived as well as one would wish. It was of course, essentially, a game only for scholars, a very small part of the total population. Manuscripts and books of the rules were often small in size and would perish easily. Treatises which have survived did so because they were included in codices of miscellaneous mathematical material, or as we have seen, even hidden in a "Holy Rule" book. Collectors of games largely ignored Rhythmomachia because of the complexity of the rules and a misunderstanding of its purpose.

CONCLUSION

"Direct thinking in symbols, like a game of chess, with its king and queen and pawns, is characteristic of those men who see power as the great desideratum - and they are the majority".

D.H. Lawrence, _Apocalypse_, p. 68.

At the threshold of cultural development the play-element seems to be particularly important. During the Middle Ages two games (not in rivalry with one another), namely chess, reflecting a secular and feudal culture, and Rhythmomachia, reflecting clerkly, spiritual concerns, had an important part in the development of Europe. While we know that chess was universally played in lay and church circles, there is no firm evidence that Rhythmomachia was played by laymen except in the sixteenth century.

At the turn of the fifteenth century chess became a more scientific and faster game when new rules were adopted by serious players:[1] at the same time it lost much of its popular appeal at the parlour game level.[2]

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[1] Today the popularity of chess as a serious and scientific game is undiminished and world-wide.

[2] Chess was perhaps in part supplanted as light entertainment by a new parlour game, probably invented by the Venetians in the fifteenth century. This was the game of Tarot or Tarocchi which could be used for the purpose of divination or fortune-telling. Isolated playing-cards had come to Europe earlier, probably via the Arabs, perhaps also via Marco Polo and other merchant travellers. The Venetians augmented the normal pack of numbered cards from 56 to 78 by "trumps", attuti or naibes, numbered consecutively from 1 to 21. These allegorically represent material forces, virtues and vices with the addition of a "fool" which could be numbered or carry the number 0 or the number 22, the number of letters in the Hebrew alphabet . "Three packs of Italian Tarocco cards" written by the Count Emiliano di Parravacino (sic), The Burlington Magazine, vol. 3, October-December, pp. 237-248.

Amongst the affluent, chess equipment continued to represent a store of value and give expression to artistic invention.[3] On the whole, the advent of printing affected the activities of individuals and groups by allowing them increasingly to devote their leisure hours to acquiring literacy and education.

By the second half of the seventeenth century Rhythmomachia, unlike chess, was all but completely forgotten. It had always been a slow and complicated game for well-educated adults, that is monks and clerics, who were supposed to avoid sloth and spend their entire lives in self-improvement. Rhythmomachia therefore, one might presume, would be less directly affected by the appearance of printed books than by the changes which had taken place in philosophical and scientific thinking over almost six hundred years. Other probable reasons for the decline in the popularity of Rhythmomachia may have been the expansion of Europe in the age of exploration, which drew monks and priests to serve overseas, the disruptive conditions of life in monasteries due to the Reformation and its aftermath, and the changes in education brought about by the Jesuits during the Counter-Reformation.

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[3] Murray, A History of Chess, pp. 447-451, lists some of the many references to chess equipment he has collected from inventories, wills and accounts of the period 1100 to 1600. A fine collection of chess sets and tables can be seen at Ambrass Castle near Innsbruck.

The chief factor in the demise of Rhythmomachia as a propaedeutic game was undoubtedly its irrelevancy in the dawning age of reason. Until the twelfth century mathematics[4] served as an aid to metaphysical speculation and the contemplation of God and His creation. Rhythmomachia was a mathematical game that assisted in this activity, but Rhythmomachia was also a scientific game in the age of monasticism and scholasticism and, as such, was of importance to those individuals who found themselves interested in the science of numbers for its own sake. Since the tenth century there had always existed a small band of scholars who wished to travel on the road that led to "more certain things of intelligible knowledge".[5]

Medieval science of number, needed by mathematicians and musicians, concerned itself chiefly with proportions, and Rhythmomachia provided the training ground for learning about numbers, digits and relationships, even for scholars without an in-depth knowledge of Pythagorean and Platonic theories as transmitted by

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[4] Mathematicians of Alexandria, e.g. Archimedes, evolved practical and useful mathematics (with which we are not concerned here) in contrast to classical Greek contemplative mathematical theory; a tradition a little later developed by the Hindus and Arabs. There were no European mathematicians between c 500 AD and c 1500 AD who equalled these achievements.

[5] White, Boethius, p. 181.

Boethius.

Gerbert of Aurillac in the 970s started "modern" investigation into the properties of numbers; two hundred years later Oxford had become the centre of scientific learning under Robert Grosseteste[6] and his pupil Roger Bacon. At Oxford, scholars like John Pecham, Duns Scotus, William Ockham, John Bradwardine and others combined the study of Platonic cosmology and empirical research into fundamental mathematics; a study soon to be taken up in France and later in Italy and Germany.

While Grosseteste and the school of Oxford realized the importance of mathematics, their mathematical concepts were still rather primitive and difficult to translate into practice. The most significent breakthrough in the mid-thirteenth century was perhaps made in the field of the methodology of science, helping Nicole Oresme to his graphing system. This in turn anticipated the calculus,[7] which was to become of such great importance in the age of Newton.

The late sixteenth century saw the invention of

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[6] Grossesteste's contribution to the theory of experimental science was to raise contemporary science above its haphazard empirical techniques. Crombie, Grosseteste, pp. 132-134.

[7] Isaac Newton invented the calculus in the 1660s but published his findings only in 1704; Gottfried Leibniz independently came to the same conculsions in the 1670s and published his results in 1696.

symbols for mathematical operations such as addition (+), subtraction (-), multiplication (x), brackets, and others, which helped to raise algebra to a symbolic science of abstraction.[8] The invention of logarithms[9] helped with the increasingly more complicated astronomical calculations.

It may however have been the introduction of the decimal system which finally relegated Rhythmomachia to no more than a game of mathematical curiosity. By the seventeenth century mathematics as a science had left medieval work with numbers in proportion far behind. Johannes Kepler (died 1630), for instance, derived his musical intervals no longer arithmetically from simple numerical proportions as was done in the Middle Ages, but centred his musica mundana on the sun; his celestial harmony is soundless, unlike medieval (and Greek) music, which was either metaphorical or audible only to certain people or in certain circumstances. Kepler's music is polyphonic, whereas earlier music consisted of scales. His harmonies are in just intonation, including consonant thirds and sixths, whereas formerly the smallest consonant was a fourth.[10]

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[8] It was largely Francois Viète (died 1603) who in his work on algebra rationalized and stabilized the symbols.

[9] John Napier published his discovery of logarithms in 1614; this was taken up and augmented by Henry Briggs and published as Arithmetica Logarithmica in 1624.

[10] D.P. Walker, "Kepler's Celestial Music", The Journal of

Rhythmomachia could no longer help the aspiring mathematician and musician, or provide appropriate aid for metaphysical speculation as practised in the Middle Ages. A resurgence of Platonism during the Renaissance indeed kindled and renewed interest in the game among humanists, but the conditions for metaphysical speculations had changed over the preceding centuries. Philosophers no longer looked solely to authority, that is the Church and the works of the ancient Greeks, to confirm their reasoning. The advances in scientific investigation had significantly affected the view of the universe and man's place in it, held not only by philosophers but also by artists.

The greatest thinkers of the seventeenth century tackled the task of constructing a new all-embracing philosophical system in the light of scientific advances; René Descartes (died 1650), Benedict de Spinoza (died 1677) and Gottfried Leibniz (died 1648) applied strict deductive reasoning, whereas John Locke (died 1704) used the empirical approach of examining and comparing ideas and experiences.

In contrast to these men who were to become preeminent in their influence on later generations, Robert Fludd (died 1637) tried to revive a lost philosophy by advocating a reformation of the older sciences. He

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<u>the Warburg and Courtauld Inst.</u>, vol. 30, 1967, pp. 228-250.

tried to show that the right philosophy was once known, for instance to the prophets of the Old Testament, in particular to Moses, and to some Greek thinkers, such as Plato. If man could only bring back this philosophy, Fludd argued, he would finally attain a mystical knowledge of God. The way to achieve this goal, Fludd believed, was through the revival of the sciences, particularly through musica mundana, musica humana and musica instrumentalis.[11] Fludd turned back to the medieval ideas of the creation of the world, to the separation of the opposite principles of light and darkness, which he represented as two opposite pyramids. He likened the world to a monochord (consisting of proportions) and called God "pulsator monochordii". The hierarchy of the monochord of the world, for him, reflected the grades of all material and spiritual things, like a mirror.[12] Despite the ingeniousness of Fludd's system, he could not materially influence the mainstream of philosophical thinking.

Unlike chess, Rhythmomachia left no physical traces except in writing. The simple counting-board

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[11] The views expressed by Fludd in his great history of the macrocosm and microcosm, Utriusque cosmi... historia, Oppenheim, 1617 onwards, involved him in written controversy with Kepler and Marin Mersenne. P. Ammann, "The Musical Theory and Philosophy of Robert Fludd", The Journal of the Warburg and Courtauld Inst., vol. 30, 1967, 198-227.

[12] ibid.

and the numbered "apice"[13] represented no store of value and were, therefore, easily disregarded and lost. As far as is known no museum or collection has found or recognised equipment for the game of Rhythmomachia.[14]

Written evidence allows us to connect Rhythmomachia with some of the finest brains of Europe over a span of about 600 years. This alone speaks of the success of this propaedeutic game. The fact that it became obsolete can only be proof of the changes made in philosophy and science; each generation of scholars saw something meaningful in play with numbers, which led them on to greater insights and experiments.

In the seventeenth century a new mechanistic-dualistic system of philosophy helped to split medieval "philosophy" into separate disciplines.

Modern mathematics however, could not completely eclipse the medieval and Renaissance tradition of numerology. Some poets, musicians, philosophers,

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[13] When Gerbert thought of numbering the counters, he charged a shieldmaker with the preparation of 1000 "apice", numbered from 1 to 9. Richer, III, 54. Boissière and Gustavus Selenus describe how the counters for Rhythmomachia should be made but say nothing about the material to be used, precious or otherwise.

[14] Murray, A History of Chess, p. 450, lists a wardrobe account for Henry VIII (MS Harl. 1419): "2 payre of playing tables of bone" and comments: "The 'pairs of tables' are pretty obviously folding chessboards..." They may however equally well have been two pairs of counting-boards for playing Rhythmomachia. After all Sir Thomas More strongly recommended the game in his Utopia V.

architects and painters continued to believe in the aesthetics of proportions and adapted number-symbolism to modern needs.[15] These needs are, as they always have been, to create work of greatest possible harmony: of balance, unity and proportion.

Today's cultural historian attempts to reconcile "art" and "science", hoping to heal the split between the "consonances" as understood by the medieval philosopher.[16] The philosopher's game of Rhythmomachia may serve as an example. Nineteenth-century mathematical historians like Wappler, Peiper, Friedlein and later Smith rediscovered Rhythmomachia during their search for the early beginnings of European arithmetic. They discussed how the game was played, without apparently realising the meaning and importance of numbers in proportion to the medieval philosopher. Chess historians of the nineteenth century like A. van der Linde and H.J.R. Murray looked briefly at Rhythmomachia as a game that had some small similarity to chess and was sometimes found in manuscripts on chess. As far as is

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[15] For instance W.B. Yeats, James Joyce, Alban Berg, C.G. Jung, Le Corbusier, Piet Mondrian and others used numerology to structure some of their work.

[16] Modern cosmologists still ask the question: can the cosmos be measured? It has been established that to answer this question strict mathematical formalism on the one hand and philosophical speculation on the other have to work closer together than in any other discipline. See for example Peter C. Aichelburg, "Ist der Kosmos berechenbar?" Mathematischer Formalismus und philosophische Spekulation gehen Hand in Hand. "Spectrum", Die Presse, 12.7. 1981.

known no medieval Latinist or music historian has commented on Rhythmomachia.

Interestingly a reference to the "philosopher's game" - under which it is listed in the Oxford English Dictionary[17] - occurs in a letter of John of Salisbury to Baldwin. But the editors of his letters, W.J. Millor and C. Brook, say of "rithmachia ludentium": "The game sounds like the knocking down of a card-house or pyramid of children's bricks, presumably John was thinking of some kind of spillikins".[18]

In the twentieth century Rhythmomachia has very occasionally aroused interest as a mathematical-scientific game of considerable complexity. The very difficulty involved in playing it seems however to work against the game being more commonly popular now.

During the Middle Ages and the Renaissance a limited number of highly-educated men had enough leisure to occupy themselves with this challenging game which also provided an allegorical playground for philosophic contemplation of the world and man's place in it.

Any review of the history of Rhythmomachia is of necessity conjectural and subjective. Any synthesis attempted by the cultural historian must be provisional

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[17] Oxford, Clarendon press, 1933, vol. 7, p. 780.

[18] Millor and Brook, The Letters of John of Salisbury, vol. 2, pp. 572, 573, fn. 5.

because, as the field of research develops, new material is uncovered and has to be incorporated. From the material available at the moment, it seems possible to draw the conclusions set out above.

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APPENDIX A

Rhythmomachia by Hermannus Contractus and Asilo
MS 14836, Munich

QUI CONFLICTU RITHMIMACHIE

Qui peritus arithmetice huius inventionis
noticiam curet habere · certus sit omnes species
trium generum · multiplicis · superparticularis ·
superpartientis · usque ad decuplam proportionem in hoc
conflictu reperiri · ita ut estimant ex alia parte
tabule que denominantur ex pari · ex alia que ex inpari ·
Species multiplicis sequenti modo tractus habeant in ante
retro · dextrorsum · sinistrorsum · angulariter in campum secundum · superpar
ticularis in tercium · superpartiens in quartum · Quicumque
numerus ex altera parte alium numerum eiusdem quantitatis
per hos tractus offendat · auferat · Et si numerus ex altera
parte ceterum ponatur partibus quibus multiplicatur aut
... efficiant eidem summam auferat · aut si quantitate
campos intiacentium alienum ... cum ... multi
plicata alieni efficiant summam auferat · In illa
parte tabule ubi omnes species denominantur ex pari po
sita est pyramis ... Quam si offendat sua basis · xxxvi · aut
numeri qui eandem quantitatem intiacentium camporum basim efficiant

Rythmomachia s[ecundu]m Forg[?] math[?]

n[on] solu[m] pyramide · si om[ne]s tetragono sunt de existit
piramide
au[?]ferat · Id[est] fiat de pyramide e[ss]e tranie partis ex[?]
eni basis e[st] lx[?]iiii · Qui tendat ad uictoria[m] in alter
t[?] motis
utra parte omib[us] modia p[ri]me positionis locis. stu
deat in parte e[st] tranie medietates efficere · armo
tri ar
nica[m] arith[m]ethica[m] · Quaru[m] utraq[ue] in trib[us] t[er]mi[n]is e[st] stan[do] · n[?]s
minimo
maximo · medio · [illegible] · tali diligentia ponenda
e[st] · ut nullus ex alienis t[er]minis possit interserere · Ex[?]
t[?] exim[?]gis utraq[ue]
p[ri]m[?] exim[?] ponit[ur] in dicet[ur] aduersario · In altera
parte copia e[st] arith[m]etica[m] cu[m] p[ro]p[r]iis efficere · Ad
armonica[m] necesse e[st] ac q[ue]rere unu[m] p[re]da · ut eu
in altera parte sint armonice · ysn · &vi · ex alt[er]a[?] · x[?]
xii · sit [?] Id e[st] ad xv · et xx & xxx ad xxv · & xlv[?]
xxv · ad[?] · & xii & liii · ad xc · & xxx · xlv · Qui[?] om[n]es[?]
p[ro]p[or]tiones e[st] · ut sic maximi e[st] ad minimu[m] · ita diffa[?] s[?] maxi
mi & medii ad differentiam medii & minimi · Arith[m]e
tice[?] aute[m] q[uo]m facilior inuenitur e[st] unu[m] pono exem
p[l]u[m] · ut [illegible] vi · xvi · Qui[?] p[ro]p[or]tiones[?] e[st] ut diffa[?] gesi
unt maximu[m] & mediu[m] · sic [?] mediu[m] & minimu[m].

Ceterum si inuestigetur si quis his diligentiā adhibe-
re delectat. Qui sic n̄ possit uenire aduic-
toriā pfectā. & maximā conet̄ ponere armo-
niā q. iiii. existens ī minimis. xii. viii. viiii. vi. ī ma-
ximis cunet medietates. & insup omniū musicarū ppor-
tiones simphoniarū. ut lxv. viii. v. lxxxi. viiii. & viii.
xv. iii. & v. xc. vi. & xc. c. xxv. & iiii. xvi. iiii. & iiii. c.
xxvi. & xx. viiii. v. & iii. xvi. viiii. & vii. lxxxi. lvi. xxxiii.
& cx. xxviii. xx & viii. Pyramis xci. ē ft̄ a tetrago-
nis. xx. xvi. xxv. xvi. viiii. iiii. i. & uocat pfecta. cui
latus ē. ccxvi. Basi lata viii. & iii. Pyramis. cxc. ē a
a tetragonis lxiiii. xlviii. viiii. xxxvi. xxv. [illegible] xvi.
[illegible] tercia uocat. Cuius basis ē lxiiii. basi lata viii. & xi.

Quinq; gna inequalitatis ex equalitate. DE RITHMI-
tate pcedere manifestū ē ex libris arithmetice.
multiplex. sup particulare. sup partiens. [illegible]
duob; copositis. ex trib; simplicib; hui[illegible]

B. q dā ex clero Wirtzburgensi. si [illegible]
bre posteritati. S tabula ad longitudinem [illegible]

[illegible] multiplex [illegible] sup particulare. [illegible] sup partiens.

Ex [illegible]

ordine distincta capias. sup qui ex altera parte
disponant usque ad decupla proportione. omnis triu
gium predictos species. Hinc viii. albi minores ex
pari denominatas habentes proportiones. dupla
ut iiii. ad ii. quadrupla ut xvi. ad iiii. sescupla ut xxxvi.
ad vi. octupla ut lxiiii. ad viii. His opponantur
eidem generis viii. nigri minores ex impari deno
minatas habentes proportiones. tripla ut viiii. ad iii.
quincupla ut xxv. ad v. septupla ut xlviiii. ad vii.
nonupla ut lxxxi. ad viiii. Retro albos viii. ex [illegible]
rubri ex genere superparticulari. ut sesqualtera iunc
ti sint duplis. sesquiquarta quadruplis. sesquisexta
sescuplis. sesquioctava octuplis. Item retro nigros
ex eodem genere viii. maiores existant albi. ut sesquiter
tia iii iuncta sint triplis. sesquiquinta quincuplis. sesqui
septima septuplis. sesquinona nonuplis. Retro
rubros viiii. existant maiores nigri ex genere sup
partiente. ut superbipartientes sesquiquartis. supertri
partientes sesquisextis. super octo partientes sesquioc
partientes
iuncta sint sesqualteris. super quar[illegible]

[illegible]

rauit: late retro albos maiores · viii · existant exceo
nē quīre coloris & uiridis sunt super parcientes nunc
ri sint sesq̄ tercius · sup̄ quinq̄ parcientes sesq̄ quintis · sup̄ sep
tē parcientes sesq̄ septimis · sesq̄ nonis · sup̄ viiii ·
parcientes · His ita dispositis ex alterutra parte
altĕnati trahuntur omnis species multiplicis · inante ·
retro · dextrorsū sinistrorsū · angulariter · in eā
pū sedm · superparticularis in tertiā superparti
entis in q̄rtā · Et si per hos legitimos tractus aliquē
cōtrarię partis numerū ita offendant · ut q̄nti
tatis inter iacentium cāporū p illos ducta eunde
essi ciat auferant · aut si contrari numeri s in angulis
aut in lateribus circūponatur his partibus q in se mul
tiplicatę aut iuncte reddant ei de summā auferatur ·
Quicūq; numeris in suo legitimo tractu aliū eidē quan
titatis offendat auferat · In illa parte ubi denomi
nantur proportiones omnis ex pari · posita est xcī · pyramis
perfecta · Quā si xxxvi · offenderit sua basis · q multi
plicat in aduersis castris · per legitimos · non solū ipsam

quib; idē

piramidē auferat · si omnes tetragonos quadratis idē fiat

de pyramide ex eo · ē(st) tertię partis similiter et tetra

gonis cō(m)posita · &(que) curta nominata · cui basis ē(st)

·IIII·
lxiiii · Non solum in his basib; lxiiii · & xxxvi · pyrami

des auferant · si q(ui)c(un)q(ue) n(umer)i cū q(uan)titate spacii gemi

na pliciter easdē bases efficiant · pyramides auferant

Tali (quo)q(ue) p(ar)de subiaceant omnes partiū pares · l(ue)l parium im

pares · l(ue)l impariū pares · l(ue)l sedī et cō(m)positi · soli p(ri)

mi et inpositi uagent(ur) tim · n(on) ita undiq(ue) sint cir

cū septi aduersariis · ut p(er) legitimū tractū n(on)

possint euadere · Quocies h(oc) eueniat · tocies au

ferant · Tali altercatione altnorū tractatui omnib(us)

n(umer)is lis a p(ri)mę positionis locis · q(ui) uictoriā deside

rat · in cāpis aduersariū festinet medietates po

nere · arithmeticā armonicā · q(uo)r(um) utraq(ue) ex trib;

t(er)minis ē(st) stans · maximo · medio · minimo · siue fiat

p(er) angulos · siue in directū · n(on) ualet uictorię · dū ab eis

aliquis t(er)minos earū possit intercipere · Qui p(ri)mus in

inq(ue)ing(ue) tuto
ii ii q(ue) ponat · indicet aduersario · Illū (con)uincat

per deceni numerus in singuli arcu num . . .

et in ipso per debito deceni debet diuidi. ut adhuc ex ipso in quolibet ulteriore arcu . . . trahi. et in illo per debito deceni arcus diuidi.

posita ex illo loco trahi. nec ab aduersario au
ferri. Ex utraque parte complures inueniuntur idonei
ad omnis . . . arithmeticos. Armonici autem nec
altera parte inueniuntur omnis. Sed breuius proponenda ue
bet . . . Nemo arbitretur esse . . . inordinate
numeros hos positos esse. si memor tamen praeceptorum hor
um quibus omnem inequalitatem ex equalitate indicat
nasci. certius sit omnem hanc monadicam (vel monadiam) superficiem
ex . . . unitatibus . . . praecepto . . .
neglecto. Quod desiderat superficiem binariam dupli
cet monadicam. si ternariam triplicet. si quaternariam
quadruplicet. si quinariam quincuplicet. si senariam
sextuplicet. DE SIMPLICI DIVISIONE ET DIFFE
RENTIA. Simplex diuisio dicitur ubi unus (cum differentia) diuisor
est. unus quoque uel plures diuidendi. Huic ergo diuisori
tale semper subponas differentiam ut denarium numerum
suppleat . . . et diuidendum semper . . .
. . . lineas ad denominationem . . . retrahas. con . . .
. . . ut si diuisor singularis sit. diuidendus . . .

Odo Rhythmomachia MS 2503 f 49a-57b, Vienna

Sesquialtera. Regule domni Odonis de rithmimachia
Sesquialtera proportio est quando numerus maior continet
in se totum minorem numerum et eius alteram partem
ut vi ad viiii. Sesquiquarta proportio est quando maior
numerus continet in se totum minorem numerum et eius
iiii partem ut xx ad xxv. Sesquisexta proportio est
quando maior numerus continet in se totum minorem numerum
et eius vi partem ut triginta ad xxxv. Sesquioctava proportio
est quando maior numerus totum in se continet minorem
numerum et eius octavam partem ut lxiiii ad lxxii.
Sesquitercia proportio est quando maior numerus continet in se
totum minorem et eius terciam partem ut sedecim ad xii.
Sesquiquinta proportio est quando maior numerus con
tinet in se totum minorem et eius quintam partem ut
xxxvi ad xxx. Sesquiseptima proportio est quando maior
numerus continet in se totum minorem et eius septimam
partem ut lxiiii ad lvi. Sesquinona proportio est
quando maior numerus continet in se totum minorem
et eius nonam partem ut c ad xc. De tabula ad ro

Sit tabula ad latitudinem longitudine distinc
ta campis super quam exaltatur parte disponantur
invicem loco omnes species trium generum multiplicis
superparticularis et superpartientis usque ad decuplam
proportionem ita ut hinc appareant que denominationem
habent ex pari et hinc que ex impari. Primo enim ex genere
minores albi ex genere proponantur multiplici dupli
quadrupli sexcupli octupli. His opponantur octo

VII

minores nigri · ei de gneri · tpli · qncupli · septupli · no
nupli / Ret minores albos · viii excitat rubri · ex gne
sup particulari · ita ut sesqalteri adhereant dupli ·
sesq qrti · qdrupli · sesqsexti sexcupli · sesqoctaui
octupli · Itē ret minores nigros · viii excitant m.i
nores · albi ex genere sup particulari · ut sesqterci
iuncti sint tpli · sesqquinti quincupli · sesqui
septimi septupli · sesquinoni nonupli / Retro ru
bros viii · excitant minores nig ex gene sup parti
enti · ut sup bipcientes iuncti sint sesq ltertis · sup
quadri partientes sesqquarti · supsexpartien-
tes sesquisexti · sup octipartientes sesqui octa-
uis · Item ret albi minores viii excitant ex gene
sup partienti · ut sup tripartientes · adhereant sesq
tertiis · sup v partientes sesqqntis · sup septipartientes ·
sesqseptimi · sup nonipartientes sesqnoni · His ita dis
Tur ... ¶ positis legitimi fiant tēr exaltat parte alternatim ·
ut multiplices tēant inscdm ante ret dextrorsu · sini
trorsu · angulari / superparticulares interciu · sup par
Nog Rapudi ¶ cientes maim · Quicuq numus cotrie partes numu
1 isuo legiptimo cursu offendit · illu qui sit ei dem
2 · qntitatis auferat · Si nus circuponit ctrie parti qui
multiplicati eposti efficiat circupositu auferatur
3 quicuq ctrarie parti num sic offendit · ut qntitas
int iacenciu cipors ise multiplicata cotru nui reddat
suma auferat · Tali pde subiacent omes pari partes
4 · aut pari impares · ul soli 7 coposti; Si pim 7 coposita

uagent͛ cam· n̄ ita sint aduersarius circumsepta· ut per
legitimos cursus n̄ possint euadere. Hae sunt arithme
metice incidentes auferantur. In nulla parte u̾ ex pari
hn̄t denominatas proporciones ē xci· piramis· Quā si sua
basis xxxvi· q̄ militat in aduersarius castris p legiti-
mū tractū aut basis latera cū q̄ntitate capitis in trā
cenciū multiplicata basim efficientia offendant aufe-
rār. Ita fiat de cxc· piramide tecurta cuius basis lxiiii·
Non solū h͛ basib͛ xxxvi· & lxiiii· piramides auferantur
s; q̄cūq; n̄ri· cū q̄ntitate spacia multiplicata easdē
bases efficiant· auferant piramides· & simul omnes tet
gonos q̄b͛ cōstant. Qui tendit ad uictoriā suo pere
studeat in castris aduersarii maximā & pfectā armoniā cō
stituere· q iii· cōstans tminis· ceteras in se cōtinet iii·
geometicā arithmeticā armo͞nicā· & insup proporciones
oīū musicarū simphoniarū· Si hec fieri n̄ possit termi
ni ei ppdā n̄ adq̄sita aut p incuriā pditi sufficiat ad ui
toriā arithmeticā & armonicā. Quarū utraq; ex trib͛ cō
stans tminis· maximo· medio· minimo· siue p angulos
fiat indirectū· n̄ prius uictorie titulet dū alien͛ aliqs
tminos eius possit inuadere tru̾pe. Qui p̄m͛ ponit
indicet adusario· illū nec licere ex illo loco tm̄ nec
ab adusario auferri· Inuiq; tmini abundant arith
metica / armonicarū ppdā uni adq̄rendi est. Quem
p̄mū pones ex nomine indices adusario in talicatione
cione pones· ut illū adhuc nullus ponendos possit
intre _ pe. Nemo existimet me incōfuse & iordi

nare hos calculos p[er]uidisse: i[d est] memor p[re]ceptor[um] iii.
boetii q[ui]b[us] h[ec] c[on]flictu[m] s[e]c[un]d[u]m eade[m] me potuisse: m. ne
glecto; p[ro]pt[er] uitandu[m] numositate[m]: Monadu[m]. diadu[m]
triadu[m] et ceteras genituras. Qui quere[n]t binariu[m] a
monade. Si t[er]nariu[m] t[ri]plicet. Si q[ua]t[er]nariu[m] q[ua]drupli
cet. Si q[ui]nariu[m] q[ui]ncuplicet. Si senariu[m] sexcuplicet.
b hostib[us] undiq[ue] circuuenit[ur] captiuat[ur] un[us]. iiii. cu[m]
piramide cadunt. alt[er] iiii. obsideri pot[est]. Senarius
p[er] t[er]nariu[m] in s[e]c[un]do capo cadit. Octonari[us] p[er] adiunctos
uidelicet. v. - iii; lvi. p[er] noue[m] - vii; xxxvi. p[er] nouena
riu[m] in q[ua]rto. siue eade[m] quantitate. lxiiii. eade[m] q[uan]ti
tate siue p[er] xvi in iiii. Nouenari[us] p[er] t[er]nariu[m] in iii.
siue eade[m] q[uan]titate: xxv. p[er] v. in q[ui]nto; xv. p[er] q[ui]nariu[m]
in iii; xlii. p[er] vii. in sexto: xlviii. in vii. lxxii. p[er]
nouenariu[m] in octauo. lxxxi. p[er] viiii. in nono; xv.
p[er] iii. in v. siue p[er] q[ui]nariu[m] in iii; xlv. p[er] nouenari
um in v. clxviii p[er] tricesimu[m] i[n] iiii. capo. et rema
ne[n]t xlviii. cliii. p[er] xxv. in vi. adiuncto terna
rio. cclxxviii. p[er] lvi. in v. et remanet nouem
Binarius q[ua]rt[us] vii. ab ostib[us] circu[m]uentu[s] du[m] euadere
nequeat captiuant[ur]. Nouenari[us] viiii. in suo legi[ti]
mo cursu. lxxxi. p[er] nouenariu[m] in viiii. xvi. p[er] iiii.
in iiii. capo. siue p[er] octonariu[m] — duodecim p[er] ii.
in vi. siue p[er] iiii. in iiii; xxxvi. p[er] vi in vi. xxv iv.
xxviiii. lvi. p[er] octonariu[m] in vii. lxiiii. i[n] viii. p[er] octo
nariu[m] siue eade[m] q[uan]titate; xc. p[er] viiii. in x. capo;
c. p[er] uicenariu[m] in v; xxviii. p[er] iiii. in vii; lxviii.

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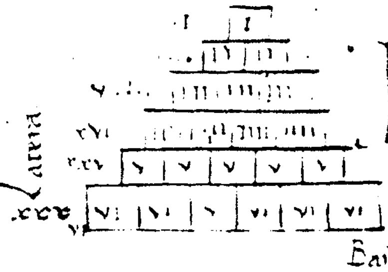
xl noue. xlviiii. in suo [illegible] siue per adiunc
tos xlv. et iiii. levi. per adiunctos levini. et ii. siue
per quindeni [illegible] remanente binario. cxvi. per xvi.
in vii. adiuncto viiii. lc xx. per xx in vi. lcc xxv. per xxv
in nono. Sunt numeri qui consonantias creant. per quos ipse dis
cernuntur consonantie tantum vi. id est epitritus. hemolius
duplaris. triplaris. quadruplaris. et epocdous. Epitritus autem
est cum de duobus numeris maior habet totum minorem et in
super eius terciam partem ut sunt iiii. ad iii. Nam in quatuor
sunt iii. et tercia pars trium .i. unitas. De hoc nascitur sim
phonia que in musica appellatur diapason. Hemiolius est
cum duobus numeris maior habet minorem et insuper eius
medietatem. ut sunt iii. ad ii. Nam in tribus sunt ii. et media par
[illegible] appellatur hic numerus in arithmetica sesquialtera
[illegible] diapente simphonia vocatur in musica. Duplaris numerus
est cum de duobus numeris minor in maiore bis numeratur
ut sunt iiii. ad ii. Ex hoc duplari nascitur simphoni
a cui nomen est diapason. Triplaris autem est cum de
duobus numeris minor ter in maiore numeratur ut sunt
iii. ad i. Ex hoc triplari nascitur simphonia que dicitur dia
pason et diapente. Quadruplaris vero numerus est cum
de duobus numeris minor in maiore quater numeratur.
ut sunt iiii. ad unum. Qui numerus facit simphoniam
que dicitur bis diapason. Epogdous numerus est qui intra
se habet numerum minorem et eius octavam partem
ut sunt viiii. ad viii. quia in novenario sunt octo et
insuper octava pars .i. unitas. Sonum vero tonum nomine
rem

que ueteres semit. uocant. d. n. ita accipiendu est. ut
dimidius toni computet. q. nec simiuocale in litteris
p medietate uocal accipim. Deniq. toni. p natura
sui in duo eq. diuidi n pot. Su cu ex nouenario nu
mero constet. noue n. nuq. eq. lr diuiduntr. tonus
diuidi in ii medietates n ualuit. d. semitonium.
uocauer. sonu atono minore. que tu paruo dista
re cophensu est. q tu hi numi inter se distant. i. cc
lui. 7 ducenti l vi. Boecius et pitagorici qde uete
res diesim nominabat. subseqntes sonu semiorem
diesi. costituit nominandu. Plato semitoniu lim
ma uocitauit. He st partes in qb omnis musica resol
uit. St q. v. simplic. Diapason. Diatessaron. Diapte
Diatess heec diapente. bis diapason. Costitunt ita
q; os musice consonantie aut in duplici aut in tri
aut in quadrupla. aut in sesqaltera. aut in sesqtercia
numerorum pporcione. Que aut uocat in numis sesquiter
tia diatessaron i melis. Que aut in numeris sesql
tera diapente uocat in uocib. Que dupla i numeris
diapason i consonantiis. Tripla u diapente ac di
apason. Quadrupla aut bis diapason. Agnoscat aut
diligens lector qd consonantie consonantiis iuncte pos
te alia qde consonantia efficer. Sda diapente
7 diatess iuncte. diapason creat

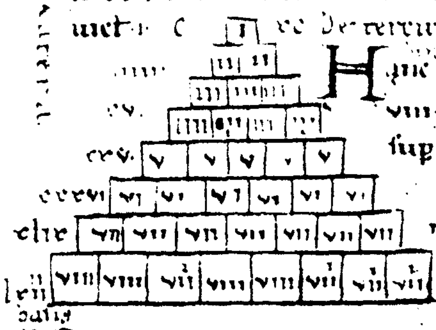
Qui uelit diffinitiua uictoria adquirere ita
ut numi uniuersita n possit diutius rebella
re. studeat modis oib aliq armonica medietate

facere ultra directorum linea .i. in regno aduersarii. Quam in quo loco agere proposueris. cautissima loca in quibus tuos qui debeant triumphare ponas. prius ut aliquibus tuis obsideas. aut aliis tamen minoris obsessa prouideas ut nullus excaduitanis possit intrumpere. nam debes contrario indicare illum quam primum ad faciendam uictoriam ponas. utrum sit minimus aut medius aut maximus & illum ex eo loco numquam tenendi licentiam habeas. nisi alios duos quo uictori possis apponere. Quid sit medietas in uno dicam. Tres numeros debes coadunare tali proportione ut eadem differentia sit inter medium & maximum quae est inter minimum & medium ut exempli causa est: inter iiii. vi. & viii. Nam inter viii. & vi. inter eum ii. sunt sicut inter vi. & iiii. Inter noue autem & xii. & xv. e differentia tria.

Vbicumque tuos propositos non habeas tamen non possis aliquam medietatem facere prima per rapinam aliquam adquire. quem ponas in locum sunt et qui tibi de sit. Verbi gratia Si uelis ex paribus medietatem facere illam in quo tibi xii. sit necessarius rape eum prius per vi. aut per quattuor aut per ii. & serua eum donec indigeas eo & cum posuisti eis viii. & xvi. dimitte istum locum medium. & pone xii. illuc tu potestatiue ac si sit tuus. xii piramis grecta. hec piramis rapitur per xii. super iii. campos aut per viiii super iiii. campos. aut per xxx sex super tres campos .i. in suo legitimo tractu. quia comes est & eius dem basis est



Piramis ex parte parua
.i. xci. tollenda ẽ p(er) xu
sup capanos .iu. ter cu
.xii. faciũt .xxxvi. q(ue) ẽ la
sis .xci. aut lapit p(er) .viiii.
sup iiii capas. Quater .viiii.
faciũt .xxxvi. Si ũ en ter .xii. or piramidẽ .xci. donec
in capos adipsũ habeas dic ter duodeci faciũt
.xxxvi. aũt in p(ri)mis piramidẽ .xci. eadem basi
et .xxxvi. lata basi .i. oms nũos numeros qui
infra senariũ nũm p(er) semet ipsos multiplicauit
id .xxv. .xvi. .viiii. .iiii. .i. Victoria piramidũ ex
uiet .i. C ... .xc De tercurta Victoria fit.



Hanc piramidẽ .cxc. capit
.viii. p(er) .viii. capos aut p(er) .x.
sup .iiii. campos. l basis ei
.lxiiii. cap ipsam pira
midẽ a primo campo
q(uae) est pes piramidis
tercurta .cxc.

Piramis ex impari .i. .cxc. debet auferri cu .viii.
sup octo campos. Octies en .viii. faciũt .lxiiii. q(uo)d
est basis eidẽ piramidis. aut capitẽ cu .xvi. sup
.iiii. capos. nãquater .xvi. facit .lxiiii. Sic co
pia fiat. ita .xvi. aut .viii. cot piramidẽ posse
trahere. ut cu illis posit hac multiplicatione
facere in piramidẽ. autem ipsã piramidẽ et

partes unaq(ue)q(ue) pars erit viii. Quar(um) unu(m) viii. abice (et) remanet xxxvi. Que xxxvi ad xlv. addideris. lxxxi. faciunt. Superpartiens est clxviii. ad xci. Nam si diuidas xci. in vii unaq(ue)q(ue) pars erit xiii. Quarum partium xiii. una abice. (et) remanent lxxviii. Que sup(er)pone illis xci. (et) faciunt clxviii. Sup(er)octo part(iens) est. cclxxxix ad cliii. Que cliii. si in viiii diuidis unicuiq(ue) partis erit xvii. Tunc omitte unu(m) xvii. (et) remanet cxxxvi. Hos adde ad cliii. (et) faciunt cclxxxviiii. Ex parte inparium sunt pediti isti numeri.

Du	pli	Quin	cupli	Septu	pli	nonu	pli
viiii	iii	xxv	v	xl ix	vii	lxxxi	viiii
Sup(er)bi	parti(ens)	[illegible]	parti(ens)	sup(er)sex	q(ui)partient	sup(er)oc	taui par(tiens)
xxv	xv	xxvii	xlv	clx ix	xci	cclxxxix	cliii
Sesq(ui)	terti	Sesq(ui) quin	ti	Sesqui	septimi	Ses	q(ui) noni
xvi	xii	xxx	xxv	lxiiii	lvi	c	xc

Super particulares sunt isti numeri maiores comites.

Sup(er) parcientes sunt isti ... medii et maxime qui ducis.

Sup(er)tripartiens est xlviii. ad xxviii. Quater enim vii. fac(iunt) xxviii. Quarum vii unu(m) dimitte (et) remanet xxi. Quos sup(er)pone illis xxviii. (et) faciunt xlviii. Sup(er)quin(que)part(iens) e(st) cxxi ad lxvi. Nam lxvi si diuiseris in eius partes. unaq(ue)q(ue) pars erit xi. Quarum partium vi. xi tolle (et) remanent lv. Quos si addideris ad lxvi. faciunt cxxi. Sup(er)part(iens) est. cxxxv ad cxx. Nam si cxx in viii. partiris unaq(ue)q(ue) pars erit. xv. Quarum unum xv. omitte. (et) remanet cv. Quos si ad cxx apposueris erunt ccxxv. Sup(er)noni part(iens) est cccl xi. ad cxc. Nam cxc diuidas in partes. cuiq(ue)

paris erit xviii. Si unu x. 7 viii. abiecerisi. rema
net clxxx. Quos si addas ad ccc efficiunt ccclxi.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Sextu par	Sup [illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	ccx	ccc	ccclxi

R itmimachia grece. numerorum pugna expo nitur latine. Inde autem ritmimachia dicitur: quod instar ge mine aciei invicem bello certantis in tabula ad hoc apta videtur in qua dam campi planicie. par inparque numerus quasi se invicem dissentiunt. et singuli cuiusque trium gene rum speciebus. scilicet multiplices. superparticulares. et superpartientes usque ad decuple proportionis summam omnis partibus perpulsi perpetuis electibus confligunt. Sed quisquis scire velit quod iste arimeticalis contineat pagina inveniet huius ispi ex in tabula ita certe invenies argumentum maximam inter arimetice continentie fructum. Recte namque debet dici argumentum quod 7 auribus sonat. et ipsa re veritatem probatur arguere. Mirus inventum. ubi una eademque numerorum compositione pariis videlicet inparisque invicem concordis discordan tis subtili delectamur certamine. Si enim diligentius inspiciamus intimo mentis intuitu. quod nobis utile lateat numeri multiplicatione. clarebit perlucide in his maxima ut superius dixi arithmetice disciplinam fructificare. in his est difficili proportionum ad se invicem habitudine omnem monocordi mensuram. vel 7 os cuiuscumque musici instrumenti simphonias consonare. in his quoque rationes individuae lectionis expositione multimodis utiles constare. Nos vero velut rudes intellectu quod huius novelle plantacio

Finis

ritmimachie sunt numerorum pugna expositio

[illegible]

1 2 3 4

Tertiū quere in sequenti pagina ad signū [illegible]
Hec est tabula [illegible] [illegible] computis [illegible]
[illegible] [illegible]

[illegible]	[illegible]					[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]		
		viii	vi	iiii	ii		
i lxxx	ii lxx	iiii lx	vi xxx	xvi	iiii	vi	viii
cliii	cx	ix xl	xlii	xx	xxv	xlv	xv
ccl xxx viiii	clx viiii					i lxxx	v xxv

x. 5.5

[illegible] sanam fructum ipsius tamen pomi dulce flagrantis per ipsius exteriorem non dulcedinem interiorem palati adhuc esurientis summatim plibaturum gustu. tamen favi mel interius latens. tante scilicet artis subtilitatem non suppetit facultas singillatim exponere. apud enim repetiliam acu leis. ne desiderata valeam percipere ipsius videlicet rithmimachie divisas proportionum genera rete in condiloma te. ne ad huius scientie secretiora possit pertingere. Exceptem salter leviora quibus haud posse subesse presens discutere difficiliora. Nec non usu temerario ea que ipse huius artis pandator studiose investigata ut omnium liberalium artium imbutus scientia notitiam futuros stilo haud parvi pendendo paratiter reperta in Sed salva ipsius pro ratione auctoritate ex eius de conceptio nis prato flosculos mellifluos legentes nostre ignorantie utiles recondam. Ceterum vero rithmimachie normas ibidem plenius subtitulata memorie non subtraham. Liber namque plibatum conflictus certamen siquis libeat potius cognoscere ibi quod generum species huius in paribus distinguantur valebit inspicere. et que campos spatii et quos in partes singulas liceat species producere qui minoris forme quique rotunde. que quadrate debeant existere. qualiter et singuli numeri adversus alterius insidiantes se ipsos metiantur cum camporum intercapedine. ad ultimum quomodo quis advincere tendens in ipsa numerorum constitucione debeat constituere sicque ad perfectam victoriam tendere. Hii itaque omnibus in prelata lectione certa sede. ecco motu.

certa uictoria. tinc. legatine dispositis detero
siquid restare cernitur initiatum ut nobis mini pri
cium. ique non tribuet affectus in hoc enucleando
lucubratione nostrum acuet effectus De iii. genere
spiritus nostra sollertia uideatur quorum prima sui multi
plicatione crescat. penultima quam adiectione habitu
dine nouo numerum coget. ultima quorum serie proporti
naliter augeat. et singulis plus reliquis sibi aliquid super ad
dendo sui dignitate et numerositate crescit super ex

S ecundum arithmeticam multiplex S ecular
genus est. cum numerus numero comparatur illi cui comparatur
est inest bis plus quam semel. Huius generis numeri sunt a pari. ii.
iiii. vi. viii. et totidem inmitum numeri. quorum ii. duplo
iiii quadruplo. vi sexcuplo viii. octuplo in serie
multiplicatos quasi sociali copulatione sacieitur
Hic notandum quod unitas totius pluralitatis genitrix natura
aliter singularis nulla recipiat paritatem. et imparitatis
sectione qua paritate si propria actu principalis extans
sui dignitate minorari et augeri nesciens unitas perse
uerat. Ceterum ad proposita redeamus. et super prolatum exempli
causa ipsius multiplicationis opere aperte exprimendo discu
tiamus sic. Bis ii. sunt iiii. Ecce habes duplas. i. numeros ad ii.
Hi enim iiii. ii. duplo inse. iiii. quater sunt xvi. Ecce quadru
plos. i. xvi ad iiii. Continet namque xvi quater iiii in se. Sexies vi.
sunt xxxvi. Ecce sexcuplos. i. xxxvi ad vi. Habet enim xxxvi
in se sexies sex. Octies viii sunt lxiiii. Ecce habes octuplos
i. lxiiii ad viii. Continet namque lxiiii. in se octies octo In con

[S]ituatione aūt tabule hor sep̄ binos multiplicatorem
ꝯ multiplicatū ꝯiunctū capit s institute· ut ubi g̃ra
iiii· cū ii xvi· cū iiii xxxvi· cū vi lxiiii· cū octo corr
concordantes studeas ꝯstituere· h̄ ex bitter memorialit
sufficiat· de gen̄e multiplici iā aliq̄d enucleare co
nem̄ de sup particulari·

Sup particularis dr̄ nūs nūo ꝯparat̄· q̄tiēs maior
cōtinet in se totū minorē̄ ⁊ sup ei aliq̄ partem.
Si eñ maior nūs minorē totū habeat & sup ei medium
emiolius i· sesq̄lt̄ uocat̄· Si ei tciā sesq̄tcia· Similit̄ p
oms nūos usq; dū habeat ei nonā partē q̄ uocat̄ ses
quinon· Vt aūt oīū iiii ꝓporcionū a partib' pcedenti
um pores adq̄ra numos p ipsos pares sup multi
plices adiacentes ei multiplica numos· h̄ ē p ii· iii·
iiii· v· vi· vii p viii noue h̄ m̄· Bis tres faciunt vi· iste
est por sesq̄lte ꝓporcionis nūs· Q̄ter v sīt xx· Et hic por.
sesq̄q̄rte· Sexies vii xlii· hic ē prior sesq̄sexte· Octies
viiii fiūt lxxii· Et hic por sesq̄octaue ꝓporcionis nūs
Siu facilit ꝯ natalit eosdē numos uel repire· oms pares
quos sup in multiplici gen̄e singlos i se ie multiplica
h̄ multiplicatorē & multiplicatū ꝯpone sic· Ad sesq̄ltā
ii ⁊ iiii· iunge: ⁊ fiūt vi· Ad sesq̄q̄rtā iiii· ⁊ xvi et erūt
xx· Ad sesq̄sextā vi & xxxvi· ꝯ fiūt q̄dragīta ii· Ad ses
q̄octauā· viii ⁊ lxiiii· fiūt lxxii· Et q̄a ipsas ꝓporcionū
pores ut ita dicā sillās adept̄ es· oportet ꝯ ut earū
t aq̄ra ꝓporcionales· Isti g̃ q̄s t adq̄sisti numis singlis
totā partē sui adiungere memito· q̄ta fuerit ipsa de

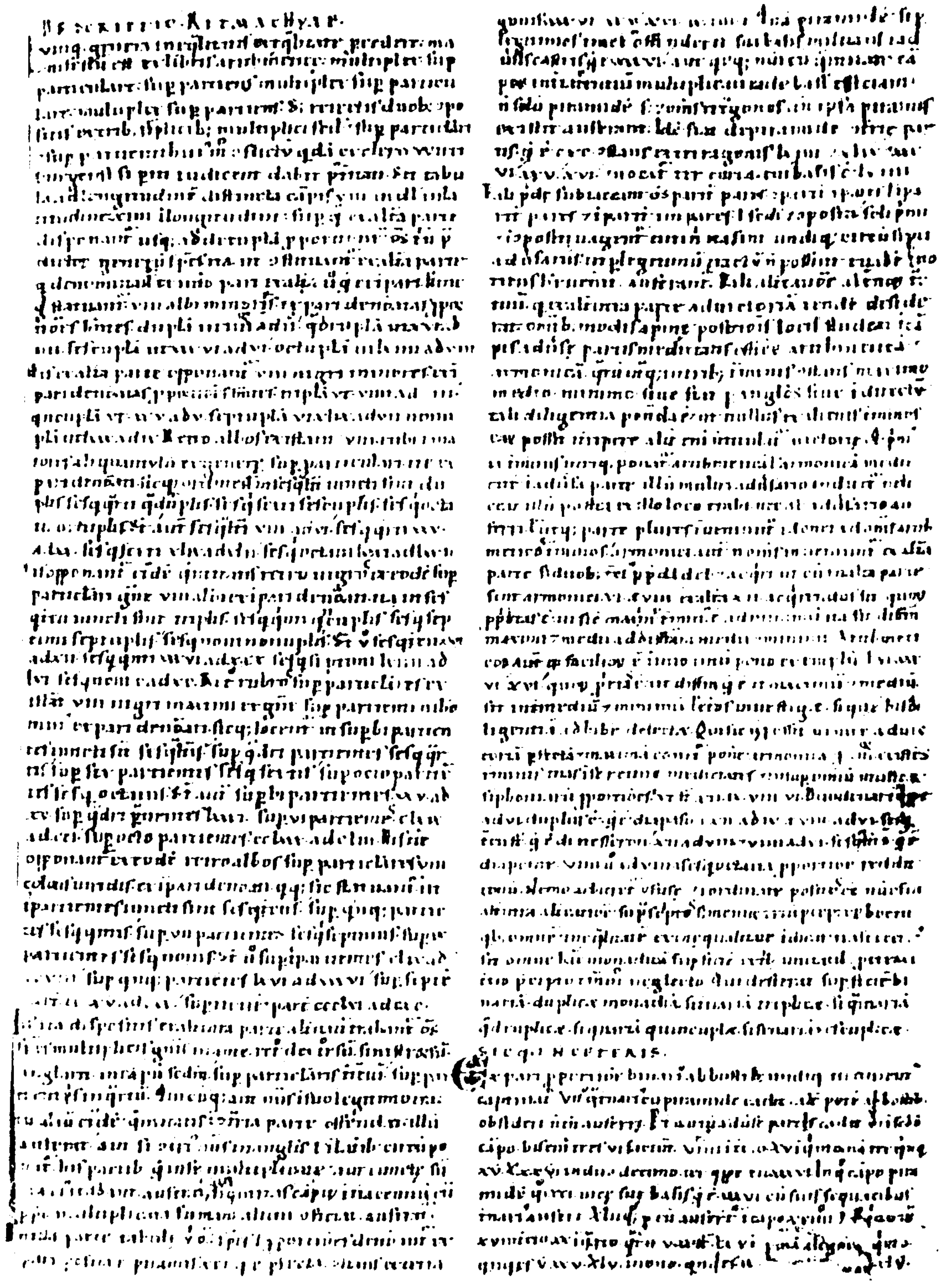
qua agit͞, pporcio ubi quarta. Si agitur de sesquialtera. ali[illegible]
i medietatem partem suam adiunge. Si de sesquiquarta. iiii. Si de[illegible]
quinta sexta. vi. Si de sesquioctaua. octauam. Hoc autem quod dic[illegible]
sic fiet. vi que sesquitertię pporcionis portem addere medieta
tem suam i. tres. adice. et fiunt viiii. que est pporcionalis
viiii. cum hec totum viiii. et medietatem i. iii. Ecce habes
perfectam sesquiteriam pporcionem i. viiii. ad vi. Similiter ad eum
quem sesquiquarte portem ad quintam adiunge quartam partem sua
i. v. fiunt xxv. que est pporcionalis. hec enim xxv. continet xx.
insuper et iiii partem i. v. Ecce est perfecta sesquiquarta pporcio
i. xxv. ad xx. Sic de aliis. Illuc ad superparcientes numeros
transeamus. Constat enim quia maior numerus minorem duabus tertiis
superesset cum dicitur superbiparciens. subaudiri tertias ne
cesse est, superbiparciens subaudi tertias. Quin ergo a binario oritur
tur subsuperbipartiens subaudi tertias ii. iii. per inde subaudi
ri quartarum multiplicare quasi summa in ea creuerit. por
numus erit in pporcione ipsa. Quod ex parte libuerit quem in
geminet. a superparticularibus nil refert quin idem crescat
cat numerus. Sesquialteram pporcionem i. vi. et viiii. compone
fiunt xv. que est por superparcienti numerus. Huic iii
partem suam i. v. bis augebis. et fiunt xxv. hec superbipartiens
pporcio i. xxv ad xv. Sesquiquarta xxv. et xx. insimul
pone. fiunt xlv. si idem suas v. partes adibes. erunt lxxx.
Sic de aliis facies iungendo sex quinque sexta. et viii. inue
nies. Sunt ergo inpares multiplices generis. iii. v. vii. viiii.
hic suos pporcionales educunt sic ter. iii. sunt viiii. isti
sunt tripli Quinquies v. sunt xxv. isti sunt quincupli hoc

Septies vii sunt xlviiii. hi sunt septu
viii ad vii. Novies novem lxxxi. hi sunt
.i. lxxxi ad viiii. Declaratis tam figuris que
admodum sunt disponuntur piramides tam et bases.
basi lata. terra. in rithmimachia quam in numero positione fiat
oportunum videtur depingere. Disponuntur itaque in rithmima
chia piramides ii. una a paribus altera ab imparibus. quarum pri
or perfecta altera vocatur ter curta. Perfecta piramis est xci.
autem a xxxvi. qui numerus idcirco eius basis dicitur eo quod ipsa
piramis quasi columna basi imposita super basim innititur.
Erigitur autem ipsa piramis super basi xxxvi usque ad unitatem
singulis numeris in se multiplicatis hoc modo. Sexies vi xxxvi.
quinquies v. xxv. quater quatuor xvi. ter iii sunt viiii. bis ii.
sunt iiii. Semel unum est unum. Hi omnes numeri in summa coacti
fiunt xci. Hec perfecta piramis. Et ut boetius ait iste nu
merus qui hoc coniugatione numerorum est maior vultum. Necesse
est enim ut huius piramidis sit basis .i. xxxvi. Et quia eadem
crescit a vi. ipse senarius vel omnes numeri qui eius
plent summam basi lata vocantur. Tercurta autem
que a sua basi .i. ccxlviiii efficitur. ab viiii usque
singulis numeris in se multiplicatis sic enim
bis viii lxiiii. Septies septem xlviiii. Omnis sexies vi xxxvi
meri in summa collectus efficit tercurtam pira quinquies v xxv
ccc. Et quia a quaternario ultra non erigitur quater iiii xvi
tres numeri .i. et tria. bis ii. semel unus. Ideo
arimetica tercurta vocatur. Sunt autem ipsius basi la
omnes illi qui basis illius efficiunt quantitatem
lata tot unitates in se quod unitates continent

in se singule piramides in latere. fi[illegible]
puncti i dii et ad model anticandi [illegible]

BIBLIOTHECA PALAT. VINDOBONENSIS.

Rhythmomachia by Asilo, MS 3101, Rome



[illegible]

A	B	C	d	E		Z	H	Θ
Alpha	Beta	Gama	Delta	E breuis	Episimon	Zeta	Eta	Theta
i	ii	iii	iiii	v	vi	vii	viii	viiii
Iota	Kappa	Lauda	my	ny		xi		o
x	xx	xxx	xl	l		lx		lxx
Pi		coppe	ro		simma	tau	yi	fi
lxxx	lxxxx		c		cc	ccc	cccc	d
[illegible]	quingenta		[illegible]	[illegible]				
chi		psi		elonga				
dc		dcc		dccc	dcccc			
[illegible]	[illegible]		[illegible]	[illegible]	[illegible]			

BIBLIOTHECA APOSTOLICA VATICANA

Fragments of Rhythmomachia, MS 6369, Munich

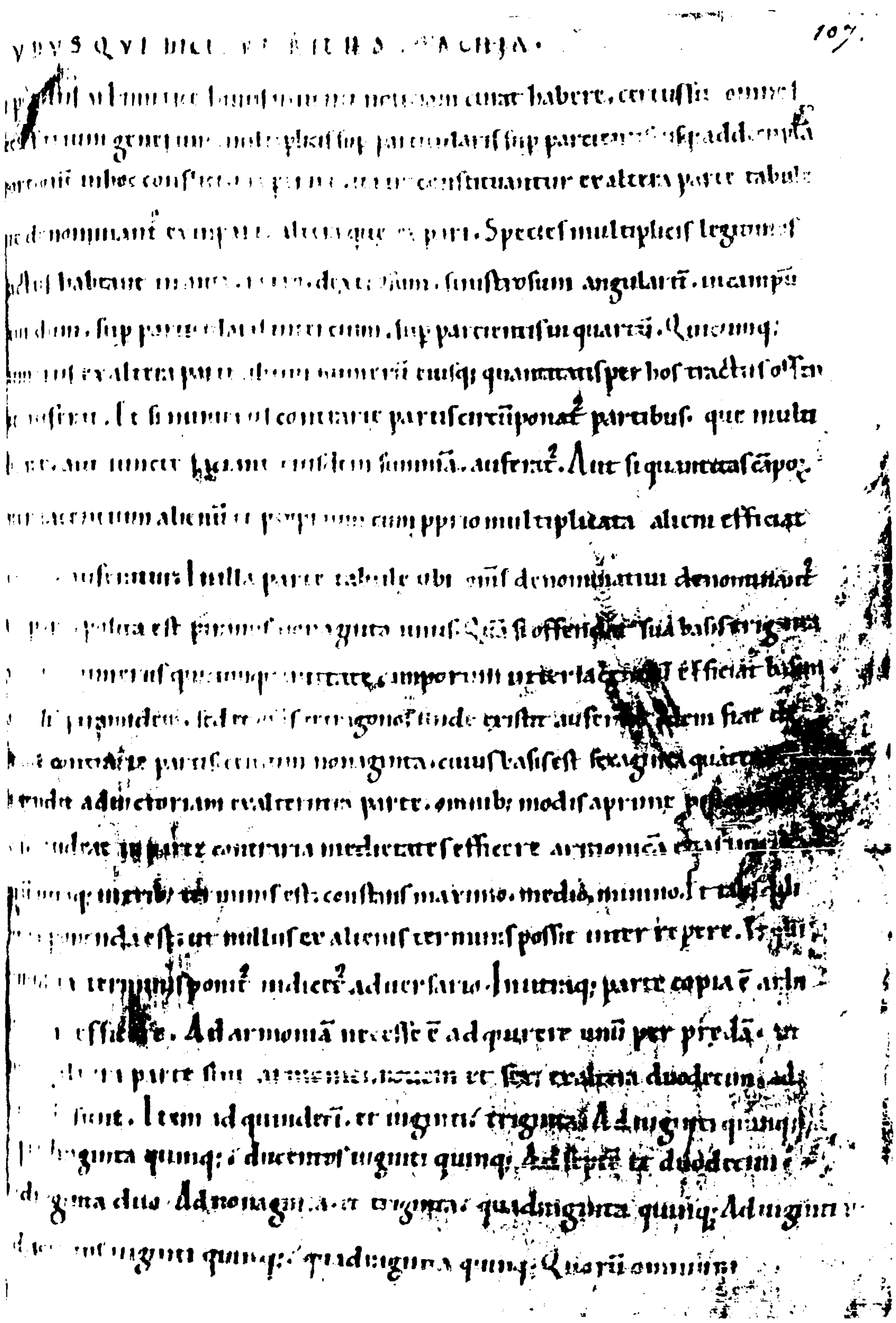
uocari. Iam non de tribus ossibus ut ante. hinc et tibicen quasi tibiarum cantor. Fistula dicta est quod uocem emittat. nam grece fos. uox. stola. missa appellatur. Lyra dicta apo tou lerin. id est a uarietate uocum quod diuersos sonos efficiat. Tympanum est pellis uel corium ligno ex una parte extensum. est enim pars media symphoniae in similitudinem cribri. Tympanum autem ut simphonia cum uirgula. Symphonia uulgo appellatur lignum cauum ex utraque parte pelle extenta. quam uirgulis hinc et inde musici feriunt. Fitque in ea ex concordia graui et acuta suauissimus cantus. Cymbala acitabula quaedam sunt. quae percussa inuicem se tangunt et sonum faciunt.

Ex innumera uarietate numerorum pauci et numerabiles inuenti sunt. qui sibi ad efficiendam musicam conuenirent. Sunt autem omnes sex. Epitritus. Emiolius. Duplaris. Triplaris. Quadruplaris. Epogdous. Est epitritus cum de duobus numeris maior habet totum minorem [illegible] et insuper eius tertiam partem. ut sunt. IIII. ad III. nam in. IIII. sunt. III. et tertia pars trium. idcirco et hic numerus uocatur epitritus. de quo nascitur simphonia quae appellatur diatessaron. Emiolius est cum de duobus numeris maior habet totum minorem et insuper eius medietatem [illegible] ut sunt. III. ad II. nam in. III. sunt. II. et media pars eorum id est I. ex quo nascitur simphonia quae appellatur diapente. Duplaris numerus est cum de duobus numeris minor bis in maiore numeratur. ut sunt. IIII. ad II. et ex hoc nascitur simphonia cui nomen est diapason. Triplaris autem cum de duobus numeris minor ter in maiore numeratur. ut sunt. III. ad I. et ex hoc procedit simphonia quae dicitur diapason kai diapente. Quadruplaris est cum de duobus numeris minor quater in maiore numeratur. ut sunt. IIII. ad I. qui numerus facit simphoniam quam dicunt bis diapason. Epogdous est qui in se habet minorem et insuper eius octauam partem. ut sunt. VIIII. ad VIII. quia in VIIII. VIII. sunt et insuper octaua pars eorum id est unum. hic numerus sonum parit. quem tonum musici uocauerunt. Sunt igitur simphoniae VI. id est. Diatessaron. Diapente. Diapason. Diapason kai diapente. Bis diapason. Epogdous. Sed hic numerus simphoniarum ad musicam pertinet. quem uel flatus humanus intendere uel capere potest humanus auditus. Symphonia diatessaron constat ex duobus tonis et semitonio. id est ex epitrito. Diapente autem ex tribus tonis et semitonio id est de emiolio. Diapason constat ex quinque tonis et duobus semitoniis minoribus.

ilasi (?) triplari. Diapason & diapente ex octo tonis et tribus semitoniis minoribus
et fit duplici triplari. Diapason continet tonos .x. et quattuor semitonia minora.
Epogdous continet tonum. Conuersatio. uita. obitus sci presulis ...

Celestis eterna perennitas dulci reuelata facundia erga uiros fidelium cotidie crescit. Nullius
enim sermonis inbecillitas sicca ualet enarrare mysteria. nisi fuerit superno magisterio eruditus.
dicente sapientissimo salomone. Dns dat sapientiam et ex ore ipsius scientia et prudentia.
Quod et beatus iacobus apostolus adfirmat dicens. Omne datum optimum et omne donum perfectum desursum
est descendens a patre luminum. His itaque ueritatis assertionibus qualiter ticinensis urbs que pro
papia nomen uocitatur. diuine respectu misericordie et instatione superne gratie sit inluminata ut in
quibus diebus quibusque predicantibus intimetur. Nam ut a primo statu crescentis ecclesie et apostolorum uicinia
ticinensis ciuitatis apostolicis ad dominum sit fidem conuersa. ac ubi beatissimorum syri atque iuuentii
qui huius urbis primi predicatores ac presules extiterunt scripta informant. Redemptore
etenim nostro peractis quibus quibus uenerat carnis passionis mysteriis remeante ad celos.
reparato discipulis duodenario numero ac sci spiritus aduentu glorificati uniuersum mundum sortes
mittentes diuiserunt. atque unicuique sors dedit per quas prouincias susceperunt. Quibus etiam
incarnationis dominice predicantibus crescebat numerus fidelium. ita ut relicto infidelitatis
errore. sub eorum magisterio sonarent. Unde factum est ut adiutores et successores apostolorum
existerent per quos xpi ecclesia ueris informata assertionibus ad fidem xpi et adcultum celeste
proficeret. Quo in tempore fundata beatissimus petrus xpi apostolus antiochenam ecclesiam. in qua
primis nouelle gentis xpiani nominis exortum est. Edificaturus ecclesiam romanam ad romanam urbem transiuit; ibique predicans xpm crucifixum. omnisque uirtutes atque miracula
que eorum conspectibus iudeorum fecerat multos ad credulitatem sui fecit oraculi.
Quibus in diebus marcus euangelista adiutor et in scriptis effectus de omnibus que ex ore illius
didicerat mirabilibus saluatoris nostri ihu xpi euangelium edidit. Quod plurimi etiam
beati petri esse confirmant. eo quod ex documento eruditus euangelista marcus eum
conscripsisset. sicut et eius sit auctoritate firmatum atque ad legendum ecclesie traditum.
Marcus uero euangelista post hec digressus roma. egyptum petiit. atque alexandrinam ecclesiam
ex doctrina magistri operatius ueritatis. mira exordia et sanctissima est ...

Rhythmomachia, MS 7185, Paris, attributed to Gerbert



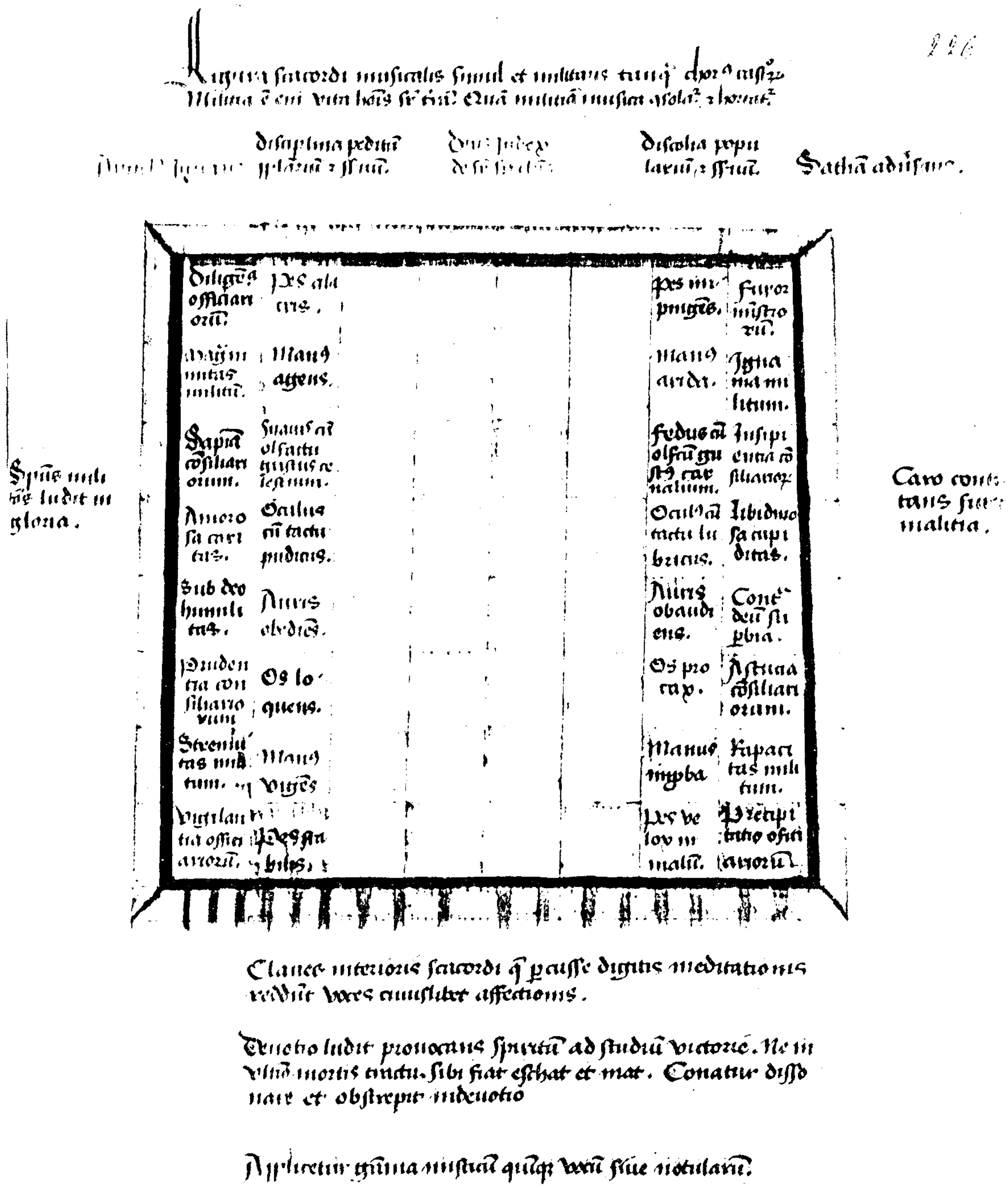
[illegible] ut sicut maximus est ad maximum, ita differentia sit maximi et m[illegible]
ad differentiam maximi et minimi. Arbitrantur autem qua facilior in-
[illegible] pono exemplum ut quinquaginta sex. triginta sex. sedecim; quorum p-
est ut differentia que est inter maximum et medium sit inter medium et min-
Ceteros in intelligi si quem huius diligentiam adhibere delectet. Qui sic non po-
uenire adinuentam perfectam et maximam omnem ponere armoniam. que quam
existens in terminis duodecim. nouem. septem. sex. [illegible] recom
[illegible] medietates. et insuper omnium musicarum symphoniarum proportiones. ut
[illegible] diaginta quinque. nouem [illegible] quinque. octoginta unus. nouem. et nou-
[illegible] decim. [illegible] et quinque [illegible] ginta. sex. et quindecim. centum uigi-
[illegible] quatuor. [illegible] quatuor centum uiginti. sex. uiginti. du-
uiginti quinque. nouem. duodecim [illegible] nouem. quinque et quatuor. sede-
nouem [illegible] octoginta unus [illegible] quinquaginta. sex. et uiginti
quinquaginta sex. et uiginti quinque. quinquaginta sex. trigin[illegible]
[illegible] uiginti octo. uiginti et nouem. [illegible] nonaginta [illegible]
[illegible] triginta sex. uiginti. quindecim. nouem. quatuor [illegible]
[illegible] perfecta. cuius basis est triginta sex. basis latera nouem. [illegible]
[illegible] centum nonaginta constat [illegible] sexaginta nouem. [illegible]
[illegible] quinque. sedecim. et [illegible] cuius basis est sex [illegible]
[illegible] latera octo. et sedecim [illegible]

[illegible] tabula ad latitudinem et longitudinem distincta campis super q[illegible]
alterutra parte disponantur. multiplicis species [illegible]
multiplicis superparticularis. et superpartientis. usque ad decupl[illegible]
[illegible] ut hinc appareant. qui denominationem habent ex par[illegible]
[illegible] Octo enim minores albi ex genere proponantur multipl[illegible] du-
quadrupli. sextupli. octupli his obponantur octo minores [illegible]
tripli. quincupli. septupli. nonupli. Retro minores albos. octo [illegible]

ri ex genere supparticulari. ita ut sexquialteri adhereant dupli.
quarti quadrupli. sexquisexti sextupli. sexquioctavi octupli. Retro minores
recto existant maiores albi. ex genere supparticulari. ut sexquiterci
sunt tripli. sexquiquinti quincupli. sexquiseptimi septupli. sexquinoni
pli. Retro rubri recto existant maiores nigri. ex genere supparcienti.
superparcientes iuncti sunt sexqualteris. superquadriparcientes sexqui
superquinqueparcientes sexquisextis. superoctiparcientes sexquioctavis.
albos maiores recto existant. ex genere supparcienti. ut supertripar
adhereant sexquitercis. superquinqueparcientes sexquiquintis. supersepti
sexquiseptimis. supernoniparcientes sexquinonis. His ita dispositis
tractus ex alterutra parte alternati. ut multiplices trahantur
capitum manente recto. dextrorsum. sinistrorsum. et angulariter.
angulares interius. supparcientes in quarti. Quicumque numerus contra
numerum in suo legitimo cursu offendat. qui sic quantitas
numerus cuius positione contrarie parti numerus qui multiplicati
circuli positum efficiant. auferantur. Et quicumque numeri contrarii
numerum sic offendat. ut quantitas intiacentium caporum in se multipli
numeri reddat summam auferat. Tali pede subiacent omnes
pares aut pariter impares. vel secundi et compositi. Soli primi et compositi.
tum. nisi sunt adversarius cursus ut plegitimos cursus non
evadere fontem arithmeticam incidentes auferant etiam
parte ubi ex pari habent denominatas proportiones. Sit nonagesima
piramis. Quia si sit basis triginta plegitimum tractum aut basis latus
unitate caporum multiplicat. basim efficientia offendunt.
tam nonaginta piramide ter curta. cuius basis
Qui vero ad victoriam summopere studeat in
adversarii armoniam construere. que quatuor

constans terminis ceteras in se continet tres. geometricā. arithmeticam. armonicā. et insup proportiones omniū musicarū symphoniarū. Si hec non possit fieri. terminus eius per p̄dā non adquisitas aut pincuriā p̄dō sufficiam adiutoriā arithmetica. et armonica. quarū utraq; in tri constat t̄minis. maximo. medio. minimo. In utraq; parte t̄mini habiti arithmetici armonicorū p̄ p̄dam unus adquirendus est. Quē primum pones ex nomine in dicēs aduersario. Et tali cautione pones. ut illum adhuc ponendos nullus ex aliēnis possit interripere. Nemo existimet me confuse et inordinate hos calculos posuisse. sed memor p̄ceptorum tuii boetii quibȝ nos instruit ex equalitate inequalitatē nasci. sciat et intellegat totū hunc conflictū secundū eundem posuisse. iii neglecto p̄pter uitandam numerositatē monadū. diadū. triadum et ceterarū geminarū. Qui quęrit supficiem binariā. duplicet me. Si ternariā triplicet. Si quaternariā quadruplicet. Si quinariam quincuplicet. Si senariā. sescuplicet.

Illustration of the "chekker" from the Johannes de Gerson MS Paris, Bibl. Nat. lat. 17487, f 226-229.

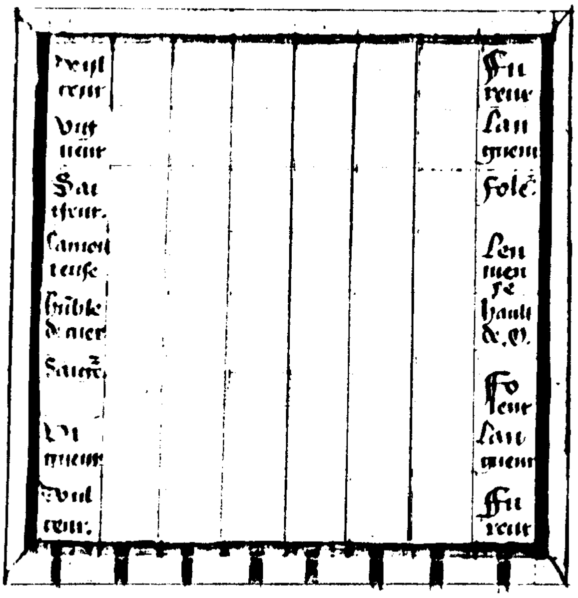


Caro concupiscit adversus spiritum. Sathan.

Chastai bon pion a nos Gaigneur.

Dieu regarde et juge du jeu.

Chastai mal pion a nos Tricheur.



en la char. Mordant plaisir vicieux.

Les clefs qui font sonner les cordes par dedans leschequier

Devocion joue du senicorde musical qui est enclos dedans pour animer franc vouloir vertueux qui le garde ou derrain trait de la mort. Deschac et mat.

Pour entendre a plain la musique morale de leschequier convient savoir la doctrine du canticorde ou chastecuer. Comment devocion par les dois de saincte meditation touchans les clefs de consentement fait sonner diverses affections. et en especial .v. selon les .v. voyeulx. A. joie. E. espoir. I. pitie ou compassion. O. paour. V. douleur.

Militia est vita hominis super terram.

En leschequier de nostre cuer.
La victoire est melodieuse.

Humble de cuer vaint hault de cuer.

Lamoureuse vaint lenmeuse.

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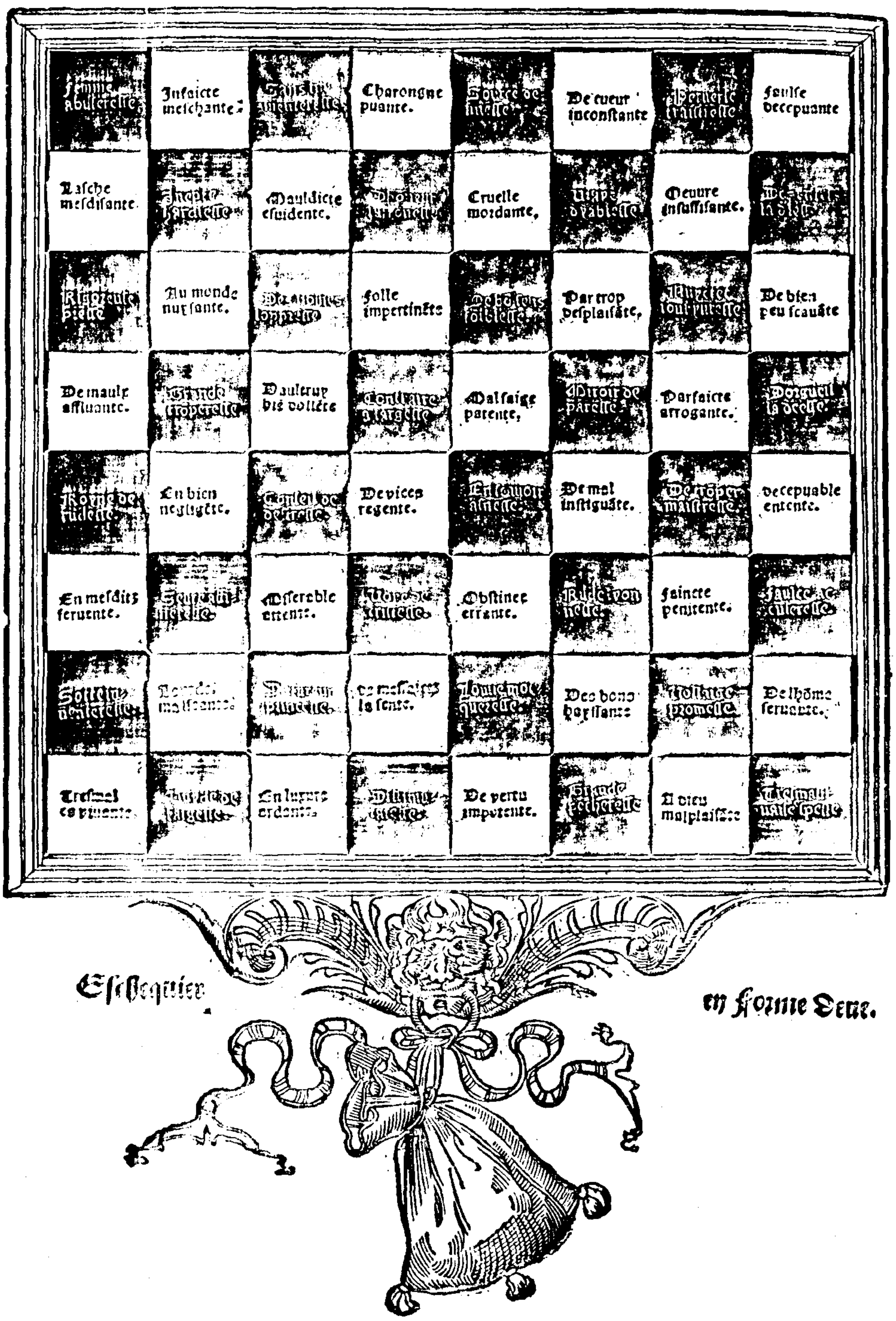
Multa quippe sunt que potest viva vox vel tua vel sui confessionis aperire. Inductus deus si hic detur consilia fructus otii mei. quo sub ipso verte et per ipsum concedit mundi deus xti. Cum dicendum occurrit illud de bucolicis. Deus nobis hec otia fecit. Ille meas errare boves ut cernis Et ipsum que vellem ludere calamo permisit agresti. Postremum rogandus est deus monens id ipsum per os prophete. Ut detur salus regi nostro qui in ipso pendet pax. pendet salus nostra. Impleat orationem nostram qui iubet illam. In quo bene vale.

	Ce sont les vertus de raison par dedans.	Ces pions sont preudommes gaignans selon les .v. sens.					Ces pions sont felons tricheurs selon les .v. sens.	Ce sont les vices de lame par dedans.	
Roy officier en temporel.	Le diligent	Ferme de pie.					Pie trebuchant	Le trop hatif.	
Chevalier en temporel.	Le convaincus	Lactif de bras.					Main vains fant.	Le despiteux.	
Alphin conseiller en temporel.	Le prudent:	Langue eloquent					Bouche mentant	Le mal engin.	
Lesprit joue. Roy.	Humble de cuer.	Le cler oiant q' olerst.					La sourde oreille.	Le hault de cuer.	La char contraire.
Royne.	Bonne amoureuse.	Le simple oeil o le tast aussi					Lubrique oeil et vil toucher	Fole amoureuse.	
Alphin conseiller en lespirituel.	Sage desire rit.	Le doulx fleurer et savourer.					Tost fleurer et savourer.	Sauvage charnel	
Chevalier pour lespirituel.	Le victorieux.	Le fort de bras.					Lasche de bras.	Le vert doneur.	
Roy officier pour lespirituel.	Le veillant.	Ly fuel de pie.					Retif de pie.	Le sommeilleux	

Ce sont les clefs de leschiquier par dedans.

Devotion fait sonner les cordes de leschiquier par dedans selon diverses affections.

Chess-board, F. Rabelais, Oeuvres Completes,
ed. G. Demerson, Paris, Seul, 1973



Epitome

¶ Di uus Se ue ri nus Bo e ti us. ¶ Jor da nus.

Numerorum proprietates.	Caput.	Liber.	Propositio.	Liber.
Medietas musice.				
1	45	2	34	10
2	47	2	37	10
3	50	2	40	10
4 ex musica	48	2	32	3
Quarta medietas.				
1	51	2	7	2
Quinta medietas				
1	51	2	7	2

¶ Epitomes librorum Arithmeticorum Boetii finis.

Jaques Lefèvre d'Etaples' Rithmimachie

¶ Iacobus Stapulensis Bernardo Uentarlo doctori medico numerorum amatori.

Considerasti mi Bernarde/ omnes disciplinas ad quas generoso spiritu sit annitendū difficiles esse: siquidem virtus ois circa difficile versetur. verum hanc difficultatem nullos vnq̃ absterrere: preter eos solos qui de re litteraria male meriti quod capere diffidant ceteris dissuadēt, aut qr ipsis inuideant velut felicioribus/ aut potius qr eos ipsos nescire pudibundū sit. At scis q̃ procul p̄sertim ab hoc preclarissimo Parhisiensi studio liuor omnis absit/ q̃ ignorātia pellatur/ q̃ qȝ probent Clitomachū Cartaginensem qui quadragesimū annum natus: primum studia litterarū petiit. in quibus adeo profecit: vt Carneadi sederit successor. quominus dubitandū est hunc locum/ tutam esse disciplinarū stationē/ securūq; portum: et vndecūq; aduenerint benigno excipi fauore/ peramicoq; hospitio. Quapropter cum Arithmetice et Musice superioribus diebus in studentium fauorē/ huic loco tanq̃ asylo cōmitterētur/ voluisti Rithmimachiā simul formari: ludum quidem numerorū non illiberalem/ sed quē deceat studiosos adolescentes cognoscere/ ne nimium tetrice videantur aduentasse discipline/ et quo interdum studio defessi primi earum Tyrones solentur animū et cum vtili ocio/ tum honesto/ vires custodiant incolumes. tale profecto cōsilium medicū decuit. Et si qui hanc ludi honestatē amabūt: gaudebunt tua opera post serias disciplinas hic esse insertū. Introducūtur Alcmeon mathematicus Pythagore discipulus: et Brontinus et Bathillus eius tēporis adolescentes. Vale.

Rithmimachia

Bront. quid est Alemeon q̃ calculos in officinis extornantiũ multicolores sepenumero egoipse aspexerim? Alc. neq̃ absurde queris Brontine. hoc ad acierum discrimen existimãt: sed duo colores michi satis esse videntur / vt acies parium sit nigricans q̃ pares ad mundũ pertineant sensibilem. acies autem imparium niuea sit et candicans: q̃ impares / masculi sint et indiuidue semperq̃ eiusdẽ substantie exprimant similitudinẽ. Pyramides tamen ob excellentiã cum sue nature tum diuinitatis quadratorum substãtie ex quorum coaceruatione surgunt coalescuntq̃ nobili quodã par est insignire colore. vt perfectam pyramida rubro et tricurtã ceruleo. Bat. Perfectiorẽ colorem pyramidi parium tribuis q̃ ei que presidet acici impariũ. Alc. Nec iniuria o Bathille: nam et eadẽ impar est et in eius mucrone ac vertice summo residet diuinitas. et latus sue basis infime: primus est a monade perfectus numerus / et suarũ basium numerus eiusdem sue basis latus est / numerusq̃ perfectus. et illa eadẽ sua basis primus est sensibilium / rerũq̃ mobilium circulus: verũtamen in ordine circulorum tertius. primus enim: simplicissimus est atq̃ plenissimus cuius centrũ vbiq̃ est et circũferentia nusq̃. secũdus: est eternitatis horizon atq̃ supra tempus. tertius: est horizon cui at cum tempore indefessa vertigine reuolutus. primus: est nostre pyramidis summus mucro / summusq̃ vertex. secundus: secunda basis. tertius: prima / infimaq̃ basis. Bat. Paradigmata fingis Alemeon: ludum querimus non symbola. Alc. Recte ammones Bathille. verum ego memetipsum negligebã: et a suscepto officio decrrabam longius. colores ergo vestri arbitrij sunto. nam cum et alijs vsi fueritis nichil refert. verum quod superest his paucis deprehendetis regulis: et quo in re parua modicus vobis satis sit sermo / regularum Prima sit hec.

¶ Calculi vndecũq̃ in vacuos dimoueãtur campos: rotundi in secundũ. trigoni in tertiũ. tetragoni cum sua pyramide in quartum. Bat. Dimoueri in secũdum intelligis forte in proximum cãpum. Alc. Ita volunt. est enim secũdus a se: sed post se primus. 1

¶ Secũda. Calculus oĩs quocũq̃ recto incessu calculũ tollere potest: sed angulari atq̃ obliquo nunq̃. 2

¶ Tertia. Numerus numerum partis aduerse et quãtitatis eiusdẽ suo recto cursu offendens: eũdem tollit. 3

¶ Quarta. Si duo numeri eiusdem partis / numero partis aduerse quem iuncti cõstituũt / circũponantur: clausum auferunt. hinc fit vt ternarius et quinarius clausum octonarium partis aduerse tollant: et quaternarius et octonarius duodenarium. et hoc pacto de similibus. 4

¶ Quinta. Si inter duos aduerse partis numeros vacui interiacentes campi per minorem numerũ multiplicati maiorem efficiant: numerus tollitur maior. hinc euenit vt binarius duodenariũ sex interiacentibus campis auferat / et sextumdenarium octo / et ternarius senariũ interiacentibus duobus / et nouenarium tribus: et quintumdenarium quinq̃. 5

¶ Sexta. Quicũq̃ numerus: cui vt recto calle dimoueatur oĩs adẽpta est potestas / tollitur. 6

¶ Septima. Si maxima pyramidis basis offendit pyramidem: eam tollit. si offenditur: tollitur. 7

¶ Octaua. Si numerus per interceptos cãpos multiplicatus basim pyramidis maximam restituat: eandem tollit. 8

¶ Nona. Si alie bases pyramidem offendant / cõsimilis in campis (si adest) auferat: et idem eueniat si numerus per interualla multiplicatus aliquã illarũ basium efficiat. si offendãtur: eedem tollantur. 9

¶ Decima. Victoria parta est si in aduersis castris maxima constituitur harmonia. itidẽ sed inferior ignobiliorq̃: si constituitur harmonia minor. 10

¶ Undecima. Calculi acquisiti ad complendam harmoniam aduerse parti auxiliares accipiuntur. 11

¶ Duodecima. Cum maxima harmonia paratur: calculus ad complẽdam harmoniam acquisitus / a suo loco non dimouetur. 12

Maxime	D	C	B	A				
Victorie	2	3	4	6	2.4.6 Medictas arith.	2.3.4.6 Geo.	2.3.6 har.	1
Pro acie	4	6	8	12	4.8.12 Medictas arith.	4.6.8.12 Geo.	4.6.12 har.	2
Parium	6	8	9	12	6.9.12 Medictes arith.	6.8.9.12 Geo.	6.8.12 har.	3
	4	6	9	12	6.9.12 Medictas arith.	4.6.9 Geo.	4.6.12 har.	4
	2	9	16	72	2.9.16 Medictas arith.	2.9.16.72 Geo.	9.16.72 har.	5
Maxime	3	5	15	25	5.15.25 Medictas arith.	3.5.15.25 Geo.	3.5.15 har.	1
Pro acie	5	9	45	81	9.45.81 Medictas arith.	5.9.45.81 Geo.	5.9.45. har.	2
Impariũ	5	25	45	225	5.25.45 Medictes arith.	5.25.45.225. Geo.	25.45.225. har.	3
Minores	5	15	25	45	5.25.45 Medictas arith.	5.15.45. Geometrica.		1
Victorie	12	15	16	20	12.16.20 Medictas arith.	12.15.16.20 Geometrica.		2

Prime maxime victorie
consonantie musice
A b diapente
B c diatessaron
A c diapason
A d dispason ac diapente
B ad dxiam c d bis diapason

Secunde maxime victorie
consonantie musice
A b diapente
B c diatessaron
A c diapason
A d diapason ac diapente
B ad dxiam c d bis diapason

Tertie maxime victorie
consonantie musice.
A b diatessaron
B c tonus cõsonantiarũ principiũ.
A c diapente
A d diapason
A ad dxiam a c diapasõ ac diapẽte
C ad dxiam c d bis diapason

Bathillus/Alcmeon/Brontinus.

O Alcmeon q̃ tempestiuus ades. Alc. quid hoc o adolescentes. Bat. quia nõnihil abs te ego & Brontinus perdiscere cupim⁹. Alc. quidnam id o Bathille. Bat. Priusq̃ in secentorum numero Pythagoram cõgrediamur: cupimus in rudib⁹ aliq̃tulum formari. Et cũ iam e nocturno cõgressu redeũt Pythagore discipl'i: sciscitamur leuiora quedã que silenda non tradiderit. Atq̃ nosipsi iam Abacum atq̃ mensulã perdidicimus. nũc autem ludũ quendã querimus inter vos cõsuetum: plerũq̃ vestras curas post seria studia leuantem / et quem vos ipsi in numeris vt & pleraq̃ alia exercere soletis. Alc. Rithmimachiã o Bathille forsitan itelligis. Bat. Eã ipsam Alcmeon. Alc. Dic age igitur Bathille quot numerorũ simplices inequalitates? Bat. Tres: multiplex / superparticularis / et superpartiens. Alc. Maioris scʒ habitudinis ad minorẽ. et in his tribus generibus omo consistit ludus a te & Brõtino nunc a me petitus. Quid maxima harmonia: et quid harmonia minor? Bat. Maxima harmonia est cum quattuor numerorum in geometrica medietate cõstitutorum: extremi ad vnum mediorũ Arithmeticam / ad alterũ vero Musicam seruãt medietatẽ. Et harmonia minor solũ in quattuor terminis duas seruat medietates. Et quid medietas Arithmetica / quid Geometrica / et quid Musica vocetur: dudum percunsiuimus / atq̃ satis quid sint nos ipsi nobis tenere persuademus. Alc. Recte multa tenes o Bathille. attamen paulo latius hoc in loco noie vtuntur maxime harmonie: vt eaipsa dicatur quoties in quattuor terminis minori⁹ us quidẽ maiores gradatim sequẽtibus / he tres reperiũtur proportionum medietates. mox disponunt primo duobus ordinibus multiplices ex quattuor primis paribus ortos atq̃ ab ipsis denomiatos: duplos / quadruplos / sescuplos / octuplos in leui / planaq̃ area que sit sexaginta quattuor quadris spacijs diremptã / alternis se intersecantibus lineis. post quos itidem duobus ordinibus consimiles superparticulares locant: sesqualteros / sesquiquartos / sesquisextos / atq̃ sesquioctauos. dehinc consimiliter superpartientes illis quidem affines / atq̃ congeneos: superbipartientes / superquadripartientes / supersextupartientes / atq̃ superoctupartientes. et spacia illa campos vocant et ordinates in ipsis numeros parium aciem. Et e regione in totidem campis / similiq̃ processu multiplices ordinant a quattuor primis iparibus ductos atq̃ ab eisdem denominatos: triplos / quincuplos / septuplos / nõcuplos. post quos cõsimiles superparticulares. post supparticulares cõsimilis affinitatis suppartientes / structeq̃ erunt e regione duobus ordinibus hic imparium / illic vero imparium acies hoc pacto.

Acies Parium in Rithmimachia.

Duces superpartientium.			25	81	169	289			Calculi quadrã-
Comites suppartientium.			15	45	91	153			tales.
Duces supparticularium			9	25	49	81			Triquetri.
Comites supparticulariũ.			6	20	42	72			Triquetri.
Duces multiplicium.			4	16	36	64			Rotundi.
Comites multiplicium.			2	4	6	8			Rotundi.
Primi campi prime arce.									
Primi campi secũde arce.									
Comites multiplicium.			3	5	7	9			Calculi rotundi.
Duces multiplicium.			9	25	49	81			Rotundi.
Comites supparticulariũ.			12	30	56	90			Triquetri.
Duces supparticularium.			16	36	64	100			Triquetri.
Comites suppartientium.			28	66	120	190			Quadrãtales.
Duces superpartientium.			49	121	225	361			Quadrãtales.

Acies Imparium in Rithmimachia.

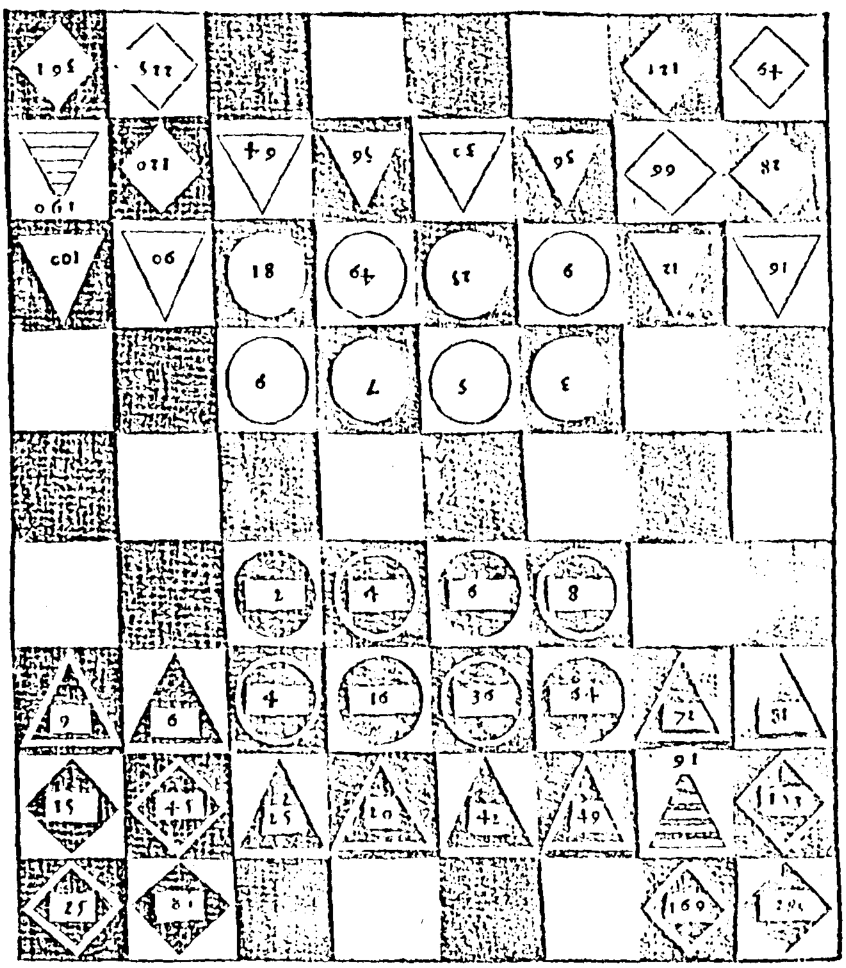
Bat. Satis hanc numerorum assituationẽ apprehendo. Alc. Vbiq̃ consimiliũ responsus vt campi sibi inuicẽ subiecti duplos / sesqualteros / mox supbipartiẽtes tenẽt: et iterũ sibi mutuo subiecti quadruplos / hinc sesquiquartos / atq̃ supquadripartiẽtes. Et hoc pacto de reliquis. Atq̃ assignati numeri designatos sibi vendicant calculos eisdẽ numeris inscriptos; multiplices quidẽ rotundos / supparticulares trigonos / suppartientes tetragonos: excepto deciminoni loci pariũ numero qui tetragona Pyramis est sex primis tetragonis orta: & hinc sex basium / sexq̃ facierũ Pyramis dicit. Huic ergo numero mete Pyramidisq̃ forma debita est: que primos sex quadratos minorib⁹ quidẽ maioribus suppositis / habeat inscriptos. & hec pfecta Pyramis nũcupatur: q̃ citra vnitatem eius Pyramidis mucro atq̃ vertex non deficiat. Cõsimili quoq̃ modo vicesimi loci impariũ numero impfecte Pyramidis tetragonice forma debita est: cui Pyramidi tres primi tetragoni ad cõplementũ desint / atq̃ ideirco tricurta dicitur. & quia ex quinq̃ sequentibus tetragonis surgit: ideirco quinq̃ basium / facierũq̃ eadem Pyramis nũcupatur. et illi quinq̃ tetragoni suis lateribus suo modo inscribũtur.

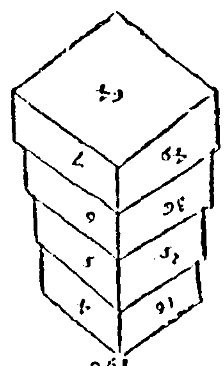
Minores victorias perpaucas posui: ut vos venatoribus similes relinquam: qui magis fere sumptione quam oblata sine venatione gaudere solent. Ex tribus tamen Mathille preceptis minorum victoriarum cognitio pendet.

Primum. Datos duos camporum numeros iunge: iunctorum dimidium est inter eos in Arithmetica medietate medius numerus.

Secundum. Datis itidem duobus numeris/ duc primum in secundum: et producti accipe latus tetragonicum. et illud est eorum rationis geometrice medius numerus.

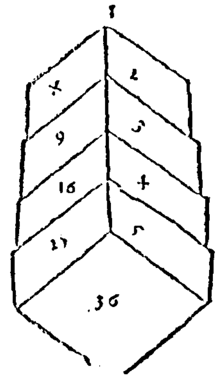
Tertium. Datos duos numeros aggrega: et qui aggregatus est serva. duc primum in secundum/ et productum dupla/ et duplatum divide per servatum aggregatum: et qui provenit est illorum numerorum harmonice proportionalitatis medius. Et si hoc pacto quemadmodum hec tria docent precepta integros numeros non reperias: numeri dati nullos habebunt in illa ratione/ proportionalitateque medios. et non modo minores victorias: seu sint ex Arithmetica et Geometrica/ seu ex Arithmetica et Harmonica/ seu ex Geometrica et Harmonica repertas: sed et si volueris maximas. Verum solent o Mathille aliter campos/ acierum duces/ comitesque ordinare. ut area sit camporum sexaginta quattuor. aut limite camporum medio aciem utramque dirimente superaddicto/ duorum atque septuaginta.





Pyramidis summa. 190

Pyramidis summa. 91



Et campis hoc modo digestis/ rudes adhuc et quos ad mysteria ipse certus silentii candidissimus pater nondum admittit: ex superioribus preceptis inter se concertant. mox vero puerilibus actis/ tociesque reiectis/ ad silentia hymnosque allicefacti: presentia preceptoris tamquam numinis cumulata fruantur/ solan=

tur/et vitam sine crimine transigunt innocuam/labeque carentem. hoc illis studium/hoc solamen/hec vigilantissima cura:

Ut sese totos rimentur ad unguem
Ne quid hiet/ne quis perturbet se angulus/equis
Partibus ut coeat/nil ut declinet amussis.
Nec prius in dulcem declinant lumina somnum:
Omnia quam longi reputarint acta diei.
Quo pretergressi/quid gestum in tempore/quid non.
Officii prauis: dant palmam et premia rectis.

¶ Talesque condidit discipulos decet esse Pythagore. Ergo meum consilium est o Bathille et Brontine generosi adolescentes: ut tantisper ludo indulgeatis dum tenerior etas vos ad altiora conscendere non sinit/dum tardat aditum. mox autem plusculo robustiores animo facti: nostrum silentium querite/et vos nostro cetui comites asciscite/vitam semper que frugi sit querentes innocuam. et me potius magistro (aut si mauultis vobiscum condiscente) doctrinam/moresque quam iocos discite. vos autem dum etas teneriuscula patitur: a maioribus que petiti ludi supersunt vel quam facillime discetis/nuncque ambo recte valete. Bat. Nos quam valemus/quamque possumus o Alcmeon ego et Brontinus: bonas pro tue humanitatis officio habemus nunc/habebimusque semper gratias. et cognoscimus te preter dignitatem hec minutula percontatos esse. que vero superest nobis facile vendicabimus. atque tu interim dum te ad seria magis erige nos utemur preceptoris opera: nostri semper memor felicissimus valeto.

Rithmimachie Finis.

¶ Has duas Quadriuii partes et artium liberalium precipuas atque duces cum quibusdam amminiculariis adiectis: curauit ex secunda recognitione una formulis emendatissime mandari ad studiorum utilitatem Henricus Stephanus suo grauissimo labore et sumptu Parhisiis Anno salutis domini: qui omnia in numero atque harmonia formauit 1514. absolutumque reddidit eodem anno: die septima Septembris/suum laborem ubicunque valet semper studiosis deuouens.

BIBLIOTHEK ZU WOLFENBÜTTEL

G. Gonterius Cabilonensis: in laudem Arithmetices et Musices.

Tempore iam multo docte latuere sorores:
Quas retinet comites flaua minerua suas.
Nunc placide terras post tempora multa reuisunt:
Grataque quoque ante alias Gallica terra placet.
His olim celebris fuit omnis Achaica tellus:
Pythagora patriam diffugiente samon.
Hellade nunc linquunt/et doctas palladis urbes:
Sequanicosque petunt/parhisiosque lares.
Hec venit omnimoda numerorum cincta caterua:
Atque docet numeris quicquid in orbe situm est.
Altera dulcisono cantu/fidibusque canoris
Edomuisse viros traditur atque feras:
Que sua Pieriis tenet unica nomina musis/
Et nichil hac musis gratius esse solet.
Sit tamen artificem Stepule misere marine:
Qui lete hoc studiis fronte dicaret opus.
Hoc solum studium atque hec illi cura: iuuare/
Irritus et ne sit/dispereatque labor.

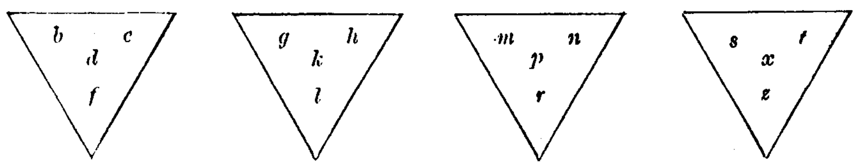
APPENDIX B

De Alea Regulari contra Aleam Secularem.

Bishop Wilbold adapted three dice, 1-6 points on each side, 56 different throws could give the name of a "virtue", led by Caritas (1 1 1), followed by Fides, (1 1 2), Spes, (1 1 3), lastly Humilitas. A list records the throws' meaning, the points of the dice are replaced by letters.

I	a	ei	ona	cion	aeion	aciona
II	c	ia	uae	iona	eiona	eionae
III	i	ou	aei	anae	ionae	ionaei

Further, a four-sided pyramid (tetrahedron) is used to record consonants.



Dice and pyramid were thrown together to arrive at the name of a virtue. The winner is he who collects the greatest number of virtues. He is then encouraged to strive for these virtues in himself and set an example for six days to all those under his sway to do likewise. The loser should regard the winner as "magister"; if there is a draw, both should greet one another as "brothers".[1]

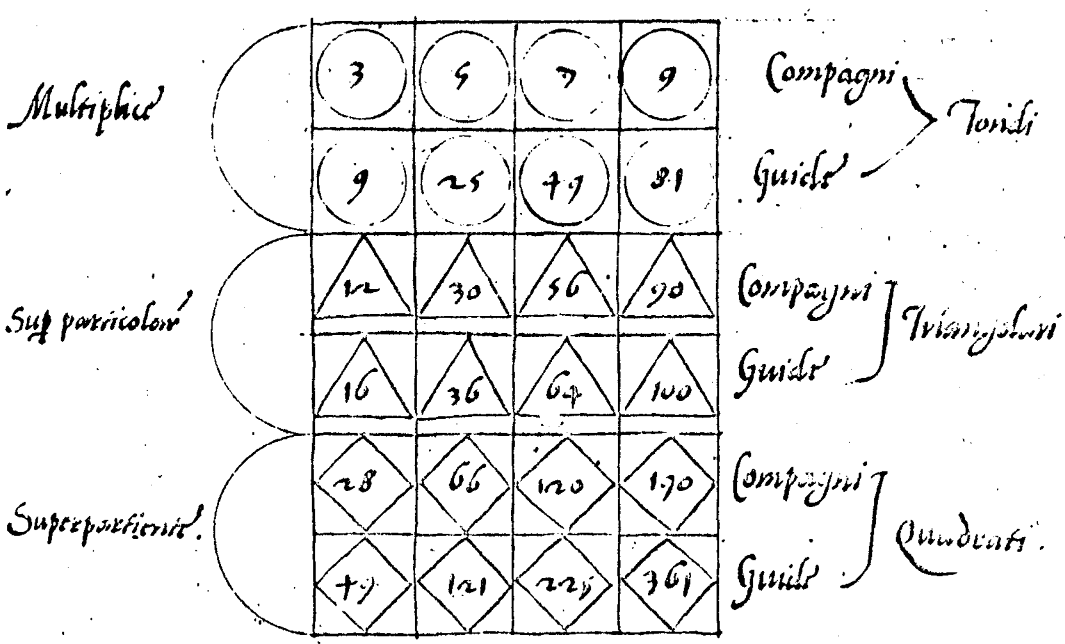
[1] Peiper, Zeitschrift für Mathematiks und Physik, Historischer Anhang, p. 198-227.

Excerpts from the Strozzi MS II, II 278, Florence
Bibl. Nat.: proportions and progressions.

75

della proportione superparticulare cioe sesquiterza sesquiquinta, sesquisettima, sesquinona, — I quadrati fanno le prime quattro sotto cassa della superpartiente quarte, superquinquepartiente sexte; supersettimapartiente ottave supernone partiente decime come potete vedere qui in questi sei righi. —

Squadra bianca de Cassi.



Car. Io gli veggio benissimo ma vorrei che procedessi piu oltre ovvero sapere s'e' nato a caso o come mi pare o pure si fatto a arte che 'l primo rigo del secondo, et il sesto, cioe l'ultimo sono tutti numeri cassi et gli altri tre cioe il terzo quarto e c 46

Vittorie Prime della squadra de Pari

	Arithmetiche.	Geometriche.	Armoniche.
2 · 3 · 4 · 6	2 · 4 · 6	2 · 3 · 4 · 6	2 · 3 · 6
4 · 6 · 8 · 12	4 · 8 · 12	4 · 6 · 8 · 12	4 · 6 · 12
6 · 8 · 9 · 12	6 · 9 · 12	6 · 8 · 9 · 12	6 · 8 · 12
4 · 6 · 9 · 12	6 · 9 · 12	4 · 6 · 9	4 · 6 · 12
2 · 9 · 16 · 72	2 · 9 · 16	2 · 9 · 16 · 72	9 · 16 · 72
2 · 4 · 6 · 12	2 · 4 · 6	2 · 4 · 6 · 12	4 · 6 · 12
3 · 4 · 6 · 8	4 · 6 · 8	3 · 6 · 4 · 8	3 · 4 · 6

Vittorie Prime della squadra de Cassi

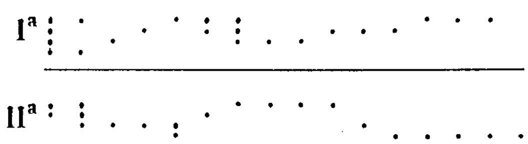
	Arithmetiche.	Geometriche.	Armoniche.
3 · 5 · 15 · 25	5 · 15 · 25	3 · 5 · 15 · 25	3 · 5 · 15
5 · 9 · 45 · 81	9 · 45 · 81	5 · 9 · 45 · 81	5 · 9 · 45
5 · 25 · 45 · 225	5 · 25 · 45	5 · 25 · 45 · 225	25 · 45 · 225

Vittorie seconde, & Minori

	Arithmetiche.	Geometriche.	
5 · 15 · 25 · 45	5 · 25 · 45	5 · 15 · 45	
12 · 15 · 16 · 20	12 · 16 · 20	12 · 15 · 16 · 20	

Rudimentary form of staff notation

SPECIES DIATESSARON



SPECIES DIATESSARON CONSONANTIAE

Nil

sua	dul	
ui	ce	ka
us	di	ri
	ne	ta
		tis

[Nothing tastes better/
than the sweetness/
of charity.]

The types of fourth

(SPECIES DIAPENTE)

Qui

non	fra		
di	trem	in	
li	quem	ui	non
git	ui	si	a
	det	bi	mat
		lem	de
			um

[He who does not love/his
brother whom he sees/
does not love God/who is
invisible (*cf.* 1 John 4:20).]

The types of fifth

MS Paris, Bibl. Nat. lat. 10275, f. 64 - 65,
as quoted by White, Boethius, p. 176.

APPENDIX C

Fortolfus' RYTHMIMACHIA, Wrocław, BU, n 54, f 86a-96b.

PROLOGUE

Seeing indeed that knowledge of this art is scorned by those who do not know it and consigned more to the realms of frivolousness than usefulness, because its divisions into fields and its setting out into squares in different directions seem to give it the appearance of a game of hazard, I think it a good idea to show many people how far removed from frivolousness it is, and how much usefulness a proper knowledge of it brings.

There is in this art a double cause for admiration, namely both its pleasant usefulness (iocunda utilitas) and it useful pleasantness (utilis iocunditas), which not only do not bring boredom, but rather take boredom away, and usefully keep busy (utiliter occupent) the man uselessly idle (inutiliter exoccupatum), and on the other hand, usefully divert (utiliter exoccupent) the man uselessly busy (inutiliter occupatum), while as a rule such employment, because it is both useful and pleasant, can relieve a mind weighed down with cares after business matters have been settled, and the grandeur of this beautiful theory can lift up spirits oppressed by bitterness.

Indeed in skills of every sort the exercise is more to be praised if it both instructs and amuses, like that saying of Horace: "He has carried the vote who has mingled the useful and the sweet".

Therefore let those who consider knowledge of this sort worthless and irreligious say what worthlessness it has or how it detracts from religion to research such things as reason has promulgated under the leadership of nature. Indeed nature has provided number which even God the founder of nature Himself used in his creation of the universe. This is said about Him because "You based everything on measurement, number and weight". For this, as Boethius said, was the chief pattern in the mind of the Creator.

From here was borrowed the number of the four elements, the changes of the seasons, the movements of the stars, the revolving of the heavens. So those who call the knowledge of natural phenomena worthless are doing an injustice to the creator of those very things in not directing their attention to them, because they themselves rather are triflers, either sluggish with idleness or engaged in merrymaking or other earthly vanities by which God is more offended. But because nothing is blessed in every respect, let us leave to themselves those who find fault most greatly with what they do not know because they are driven to it by envy which tortures them - for Sicilian tyrants did not find a greater torture than envy - and let us pass the voices of the Sirens with deaf ears.

Let us leave this unsophisticated effort of ours to ourselves and those like us. For here we do not set forth any parade of words, but in a simple way we are ready to share with well-wishers whatever we can explore in this art, by reason of good will.

This art is worth knowing because it has flowed forth from the fountain of Arithmetic like a stream and for that reason offers to those with training in Arithmetic a very easy and quick way to itself, to those not so versed it gives a slower entry and one not without effort.

CHAPTERS OF BOOK I

1. The derivation of the name: it is made up of two Greek words - number and battle. The numbers are like soldiers ranging over a plain, moving backwards and forwards, right and left and diagonally, regularly changing position to so nice a position of victory for one of the two sides that it is like a song arranged in harmony.

2. Of what the game is composed: number, but not random; not only woven in natural order like a house composed of walls, ceiling, beams, roof, made of wood or stone, all making an entity: so here the various manifestations (species) of number make one art.

3. What is the aim? (intentio): the aim of making up this game was, in my opinion, that because everyone cannot range the whole breadth of the field of Arithmetic, they can train themselves by the very conciseness of this game, and in training amuse themselves. What they achieve by a long programme of reading there, they can assemble here as if in a summarized version. The game is so logically set out that it instructs and yet does not bore.

4. Its usefulness (utilitas): not small, for it is both honourable and necessary. It brings knowledge of multiplying and learning the behaviour of numbers themselves. Also progressions of pyramids

and the distinctions between the three means and many other noteworthy things that anyone alert and a curious observer will be able to dig out, so that he can visualize briefly things that Boethius expands on at length in his Arithmetic.

5. <u>What branch of Philosophy is it to be put under</u>?: Physical, i.e. Natural Science. For the property of number is natural and,existing naturally from the very beginning of the world,it is in all its natural manifestations distinguished by natural pattern. Indeed the science of Arithmetic, from the fountainhead of which this game we have before us was derived, is, as Boethius said, fundamental to all the arts, not only because God the creator of the universe had it as the primary pattern of his reasoning, and arranged everything in accordance with it, and with this pattern operating, men have found a harmony throughout the numbers of the allotted order. But Arithmetic is claimed to be fundamental for the following reason too. Whatsoever things are primary (priora) in nature, if you take them away, what follows (posteria) is removed at the same time; if however the later things are destroyed, there is no change from the state of the original (prioris) substance. Thus if you take away Arithmetic the other arts too, i.e. Geometry, Music, Astronomy, which exist by virtue of numbers, are removed. If these however are removed, the substance of number is not taken away at the same time. For, if you take away numbers there will no longer be a triangle or square, or anything that exists in Geometry for they all derive their names from numbers. But if you take away the square and the triangle and the whole of Geometry is destroyed, "three" and "four" and the appellations of the other numbers will not perish. Similarly in Music if diatessaron, diapente, diapason, which are named from numbers are removed, the substance of numbers will still endure. The same is to be perceived too as regards the rest of the arts which are based on numbers.

6. <u>How the board is to be assembled</u>.

7. <u>The placing of the numbers</u>.

8. <u>How and from where the numbers themselves arise</u>: In a passage in his Arithmetic where he discusses evenness and oddness and says that all oddness arises from evenness, Boethius puts forward the similarity of the relation of good and bad and even and odd. He says there is great gain if someone is not ignorant because good is defined

PROLOGUE TO BOOK II

About winning at Rithmimachia

They seem to offend many, those who dare to add something to someone else's work already published and accepted as authoritative - something which they have worked out because of their own enthusiasm, as though this were put forward with no claim to truth, although they are able to prove it with the aid of logical reasoning. Surely a good craftman or painter or trained pursuer of any art does not overlook what reason puts his way, or the equipment of his work demands, because he did not learn it from a master? But rather he joyfully embraces it because he is full of wonder at having found it, under the tutelage of reason. For this is not dismantling art but promoting it and making it more gratifying and leading on the mind to the exercise of greater research. Pythagoras devised music from 4 little hammers, about which others carrying the argument far afield filled more books, and many others either increased or clarified with their own more detailed writings inventions and researches and discoveries of philosophers. Therefore I must not be blamed, I reckon, if I propose some ideas about the way to win at Rithmimachia in this booklet of mine that have not been perceived before. Because brotherly affection has wrung it from me, although I have collected them not from my own ingenuity but rather to a large extent from the writing of our forbears.

I do this to be sure, not from the fault of boastfulness, but for the sake of common usefulness and enhancing this game and so that for its enthusiasts the playing of it may become the more gratifying the more pleasant it is as a pastime.

CHAPTERS OF BOOK II

4. <u>How they can be found by another method</u>.

5. <u>Their differences and properties</u>.

6. <u>Geometric investigation (speculatio) to secure victory through using these means</u>.

7. <u>Another</u>

8. <u>Another</u>

9. <u>Another</u>

10. <u>How victory ought to take place</u>.

11. <u>What numbers are harmonic</u>.

12. <u>About geometric harmony</u>.

13. <u>About the greatest and perfect harmony</u>.

Translated from Latin by Mrs. A.C. Thompson.

Translation of the beginning of Boissière's Latin <u>Rythmomachia</u> 1556

The most noble and ancient Pythagorean game (which is named Rythmomacia), ordained for the use and relaxation of studious men with a view of acquiring a true and easy grasp and theory of numbers, has now at last been described by Claudius Buxerius Delphinas.

Paris 1556.

INTRODUCTION

To the most Illustrious Hero

<u>Antonius Scalinus an Eymaribus, Lord of Gardia</u>

Knight of the purple-weavers, Prefect of the Royal Fleet in both seas

from Claudius Buxerius Delphinas, Professor of Mathematics, Greetings.

You are not unaware, most illustrious Sir, how much usefulness and relaxation is conveyed to the minds of men by an understanding of numbers, and on the contrary how much darkness and blindness engulfs the same through ignorance of them for neither to dispose battle-lines nor direct columns, nor carry through and complete innumerable deeds (if they are not helped by arithmetic and geometry) do they avail.

Nothing here do I say about the principles of numbers: from which that admirable sweetness of voices and the making of musical harmonies has emanated. Wherefore I shall readily conclude with Aristotle, that everything that is without share in quantity is nothing: and very little different from this is the judgement of divine Plato, who holds this: that all those who have a natural propensity for arithmetic also have a natural aptitude for the other noble disciplines: And thus it was not unwisely that he ordered to be inscribed on the gates of his academy: No-one who is ignorant of Geometry may enter here. However he understood Arithmetic by the name of Geometry. Moved therefore by the authority of so great a philosopher, after I had devoted myself for many days to the art of numeration, there fell into my hands a certain excellent combination and structure of number culled from Pythagorean principles and harmonies: which, when they wished to give themselves over to enjoyment, was set forth by those ancient Philosophers in order that their troubles might be sweetened for them and their intellects, wearied by great length of studies, might be refreshed by the virtue of this game. But if Lucan and other historians were willing to ennoble Julius Caesar in the ranks of those Philosophers exactly because he in the midst of the storms and tempests of civil wars never allowed the so noble study of the mathematical arts to pass from his hands, what place, pray, shall we assign to him who not only by winning the praises of war but also by innumerable other deeds even among foreign and barbarous nations has rendered his name worthy of immortality and who, when he was crushed by the greatest jealousy of his own people, in the midst of the whirlpool of adversaries not only acquired an understanding of the humanities but also acquired experience of the divine arts by a most burning zeal for learning? Who, I say, not only among the ancients but among men of our own time could strive to exceed you in this respect? Could it be Aristides or Themistocles? Surely Cato did not sustain adversity with more constant or braver mind than you. Surely Scipio did not approach ambassadorship both domestic and external with more careful sagacity or sharper wits and prudence than you. Surely Pompey did not suppress the furious attacks of enemies by land and sea more vigorously than you have done and continue to do again and again as the days go by, that you may defend and preserve by far the most flourishing kingdom of the Gauls. Since therefore I am more than persuaded that your deeds are so illustrious that they irradiate the whole world with your brilliance, I think it better for me to be silent and to commit these things to the judgements of men (for whom they will be no ordinary source of admiration) rather than to enclose them in such narrow limits and confines, in the hope that there will be no inadequacy on the part of those who, in adding the same to their own great annals and histories, may adorn them and polish them with greater accuracy. In the meantime then, since there are always before my eyes considerations of your superabundant mass of virtues, so many and so great and the perfect kindness with which you favoured me more and more, I desired to declare and bear witness to my regard for you to be something neither ungrateful nor displeasing. But since last year I had published this Philosophers' game, whose name is Rythmomachia, for the relaxation of your mind, in the French tongue, I judged that you would not take exception if the same (game) now again done in Latin were to appear before the sight of all men distinguished by the brilliance of your eminence.

Moreover Ioannes Morellus Ebredunensis, a man both of the greatest erudition and devotion to your great name (I add also that he is most dear to you) added spurs to my galloping hopes and gave his opinion that I should not hesitate to publish this game under the auspices of your name. Wherefore most illustrious of men, let this my slight little work be accepted with an eager and cheerful spirit: so that you may supply me with greater strength for meditating other things even more diligently which hasten to the light of day under the felicity of your glorious name as being worthy of your person.

Farewell.

Paris, 1st June, 1556

Boissière continues:

For the purpose of raising up minds that have been depressed by the weight of labours and studies, very many wise men have invented certain most fine games which have perished by the lack of care of certain men and the ravages of time. For since it was not unknown to those men that these equally empty and useless games (which men thought of as affording the greatest recreation and therefore as being worthy of wasting years of their lives on) were not only a relaxation for them but rather a source of the greatest distress and sadness on account of the time which they consumed in the playing of them, they brought out others and deceived men by giving them the appearance of games. Imitating this precedent of the wise men and desiring at once to satisfy a human weakness and consult the interests of those who are given over to excellence, I decided to pick out some game that would feed and delight men by its own pleasure without wasting time or skill. Now although this method of playing (which takes its origin from the harmonies and proportions, ratios of the most subtle and ingenious Pythagoras) acquired for itself the greatest respect among the ancients, today nevertheless it is almost without honour or appreciation, for the reason that it is thought to contain enigmas, (puzzles, riddles, mysteries) that defy the understanding. But when I considered quietly how much advantage can be won from this, I was not able to restrain myself from descending to a description and explanation of it, the which I shall set out with such ease and brevity, that not only men who are sad and gloomy come out cheerful and gay but also some fruitful result will accrue to those who pursue excellence: since they will be more ready for the investigation of principles and harmonies: acuteness in which and understanding of which will be of the greatest assistance to us for the contemplation of the quantity of all things, the quantity which this so great and so wonderful world embodies and embraces.

This paragraph is followed immediately by:

Definitio Rythmamachiae

On page 84 the alternative method of playing (after the manner of chess) is given with the following introduction, not translated by Richards:

Alternative Method of Playing of the Chaldeans

Since there often came into my mind those things which Thomas Randolph, a man most zealous in good letters, used to tell me, when he was studying in Paris, concerning a variant method of playing Rythmomachia in which the English take great delight, nothing was of greater concern to me than to make diligent search for an exact understanding of this thing: so that this game might come to studious men perfect in all parts and numbers: then indeed shall I consider I have done sufficiently well if I shall have passed the whole time and study of my life for their advantage. But when I was enquiring more diligently into this, there presented himself to me that most cultured man Thomas Topcliphus (in Yorkshire?) of more excellent erudition and virtue than anyone could scarce persuade himself possible for an Englishman: not that I wish this expression to make anyone think that I wanted to devalue the majesty of the English name (for I judge the English to be not inferior to other men) but because the absolute perfection of the man is the greatest source of my admiration: for whether you look at the integrity of his life, or the sweetness of his character, or finally the dignity of his body, you will find nothing to be desired: and although he is distinguished in all disciplines he is particularly prominent and excellent in Astronomy, Algebra and Scenography (Drawing in Perspective). Such is his culture that he gave me the advantage of a certain book which, (as he related) has been culled from a certain Chaldean book and translated into the English tongue. Now this book contains a certain method of playing Rythmomachia slightly different from the former one. And when I had recovered the method to the best of my ability, I did not wish to leave it out of this book of mine, especially since I am of the opinion that this Style of progression (which corresponds to the Schachian) is the perfection of our game which the ancients pursued in playing: wherefore I conjecture that the progression and attack of the Schachians derived their origin from the same source. Hence it is sufficiently valid to observe how severely and stoically this game has been described by certain ancients: who, while they strained their minds in the contemplation of the game,despised that which in the exercise of the game, had not been displeasing or thankless to the players: These indeed are Gilbertus Papa, Hermanus Contractus, sic., Castrensis, Nicolaus Crestinus: who have been initiated in living memory by Orentius Finaeus Delphinas, Jacobus Faber Stapulensis: nor have I departed thus far from their tracks. But since I saw that this contemplation separated from recreation is sometimes wearisome I greatly rejoiced to have recovered that which had been formerly cut off and separated, which seemed to restore the absolute perfection of the game. Yet in the meantime, lest we infringe the authority of such great writers, we added this pleasing and absolute method of playing as an appendix, nor did we detract at all from what went before but sent it away whole, in order that anyone who wishes may use it. However, let us attempt the explanation of the game.

Translated by Dr. P.A.L. Greenhalgh

APPENDIX D

Estiue medio dici tempore
umbrosa recubans sub Iouis arbore
astantis uideo formam Pictagore,
deus scit, nescio, utrum in corpore.

Ipsam Pictagore formam inspicio
inscriptam artium scemate uario.
an extra corpus sit hec reuelatio,
utrum in corpore, deus scit, nescio.

In fronte nituit ars astrologica;
dentium seriem regit grammatica;
in lingua pulcrius uernat rethorica;
concussis estuat in labris logica.

Est arismetica digitis socia;
in caua musica ludit arteria;
pallens in oculis stat geometria,
quelibet artium uernat ui propria.

Est ante ratio totius ethice;
in tergo scripte sunt artes mechanice.
qui totum explicans corpus pro codice
uolam exposuit et dixit: „Respice!"

Manus aperuit secreta dextere,
cumque prospexeram coepique legere,
inscriptum reperi fusco caractere:
DVX EGO PREVIVS ET TV ME SEQVERE.

From Apocalypse
Die zehn Gedichte des Walther von Lille genannt Chatillon, n. IV, v. 5-22.
F.A.W. Müldener, Hannover, 1859

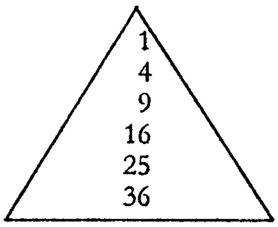
Rhythmica summarum praecessit quinque puellas:-
Quae circumscriptis intende uocabula, lector,
haec quia dactylico non cernis idonea metro;-
primula multiplici caput irradiata metallo
tardantem retro citius iubet ire sororem,
quae simul ad sociam conuersa fronte sequentem
inquit: Habeto meae tecum dextralia palmae;
hoc etenim speculi nostrae commendo sodali,
quam genui patria quondam statione locata.
Staret inormatis famularum quinta cappilis,
ni sibi lacteolam praeberet tertia uittam.
ibant quamque sua comitum stipante corona,
et postquam planas limabant rite figuras
interruallorum mensuris et spatiorum
ordine compositis, cybicas effingere formas
nituntur mediumque uident incurrere triplum:
collatum primi distantia colligat una;
alterius numeros proportio continet aequa;
respuit haec ambo mediatrix clausa sub imo.
ordinibus mathesis gaudebat rite paratis
haec missura tibi solatia, clare Boaeti.

Walter of Speyer, *Vita et Passio S. Christophori Martyris*, first published by B. Pez, Anecd. T. II P. 3 p. 37 sqq, also Dr. W. Harster, *Beigabe zum Jahresbericht 1877/78 der k. Studienanstalt Speier*, München, 1878, 8°.

De Vetula

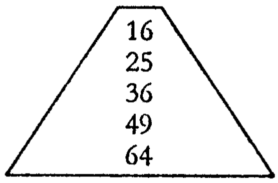
I 649 O utinam ludus sciretur Rythmimachiae!
Ludus Arithmeticae folium, flos fructus et eius
gloria laus et honor, quia totum colligit in se
ludus, ubi bellum disponitur ordine miro.
Campis in geminis congressio fet numerorum
quattuor impatrium, qui sunt in limine primo,
cum totidem paribus, qui limite sunt in codem,
principio numeri numeris non commumerato,
octoque sunt isti patres utriusque cohortis;
auxiliatores nam parti dantur utrique.
primo multiplices, quia ducto quolibet in se
quadrati subduntur eis, quibus ordine bino
subsunt supraparticulares adicientes
toti particulam dictam patris a quotitate.
His alii subsunt, qui particulas superaddunt
dictas a numero uincente patris quotitatem
uno, sed numero, patris acquales quotitati,
ordoque binus eis. Numeros hinc inde tabellae
seu Scaci portant, et sunt acies bicolores
ad discernendum, praesortim cum paritas et
imparitas mixtae sibi sint in utraque cohorte.
distinguuntur item Scaci tabulaeue figuris:
hi trigonis, hi tetragonis, illique rotundis;
scilecet ut Scaci numeros utrinque rotundi
primos octo ferant, trigoni sint octo sequentes,
tetragoni reliqui, nisi quod duo sunt ibi Reges
pyramidalibus ex numeris. Ideo quoque Scaci
pyramidales sunt: et habet pars utraque Regem.
In castris parium nonus decimus locus unam
perfectam dat pyramidem: senarius in se
ductus pyramidi basim producit eidem
totaque pyramis est nonagenarius unus.
At locus imparium decimus bis pyramidem dat
tercurtem, cuius basim octonarius in se
ductus producit, quam pyramidem coadunant
centenarius et nonagenarius una.
Istae pyramides sunt Reges his aciebus
et sunt ex numeris quadratis omnibus ambae,
quod petes ex tabula subiecta noscere plane:

acies parium
2 4 6 8
multiplices
4 16 36 64
superparticulares
6 20 42 72
superparticulares
9 25 49 81
superpartientes
15 45 91 153
superpartientes
25 81 169 249
pyramis perfecta



basis cuius 6 est radix

acies imparium
3 5 7 9
multiplices
9 25 49 81
superparticulares
12 30 56 90
superparticulares
16 36 64 100
superpartientes
28 66 130 190
superpartientes
49 121 215 361
pyramis tercurta
desunt enim 1 4 9



basis cuius 8 est radix

I 688 O utinam multis numerorum pugna placeret!
quae si sciretur, placitam se redderet ultro.
Sed Mathesis uix inueniet qui iam uelit ipsam, etc

III 19 Adiciamque iocos dociles Mathesisque sequaces,
sumptibus exiguis aliquatenus aedifacabo
concernens ad materiam geometrica quaedam,
sic abstracta quidem, quod non sine materia sint,
Algebraeque memor, qui ludus arithmeticorum.
admittam ludum, qui Rythmomachia uocatur. . . .

P. Klopsch, Pseudo-Ovidius De Vetula, pp. 217-219, 250.

APPENDIX E

The letters of Gerbert of Aurillac, translated by H. Lattin.

3. RHEIMS. 978-980?

Gerbert to Constantine (?) of Fleury, explaining a passage in the Boethius De arithmetica referring to the changing of sesquiquartal numbers

(GERBERT TO CONSTANTINE (?))

This passage, which some persons think is insolvable, is solved thus. Take sesquiquartal superparticular numbers, as 16, 20, 25. If you wish to know how these sesquiquartal numbers are resolved first into sesquitertian, then into sesquialteral, and lastly into three equal terms, arrange them thus: 16, 20, 25. Take the lesser number from the middle and make it the first, i.e., take 16 from 20 and place this 16 first, and the remainder from 20 is 4: place this in second place. From the third term, then, i.e., from 25, take one first term, i.e., 16, and twice the second term, i.e., two 4s, which are 8, and 1 remains. Place this unity as the third term, and the ratios are: 16, 4, 1. You see how sesquiquartas are changed into quadruple ratios and from whence the numbers came.

But this resolution ought not to be done in a confused and disordered manner, i.e., they ought not to be resolved into sesquitertias suddenly, but in orderly fashion. Thus, take these quadruple numbers - 16, 4, 1. Reserve them and arrange them thus: 1, 4, 16. Therefore, take away the lesser from the middle, i.e., 1 from 4, and this 1 place first and the 3, which is left from the 4, place second. From the third term, i.e., from 16, take away the one first and two second (terms), i.e., 1 and twice 3, and what is left from 16, i.e., 9, place as a third term and arrange thus: 1, 3, 9.
You see, therefore, how quadruple ratios may be changed into triple, and after these same quadruple ratios have been changed, from what they originated. With this triple ratios, i.e., 1, 3, 9, reversed and arranged thus: 9, 3, 1, if, mindful of the rules of Boethius, you make the first equal to the first, i.e., 9; the second equal to the first and second, i.e., 12; the third to the first and the two seconds and the third, i.e., 16, the resolution of the sesquiquartal superparticular has been accomplished first into a sesquitertian, as Boethius teaches by the rules which he lays down so astutely, not confusedly, but in orderly

fashion, just as numbers were procreated from the beginning of number.

If, therefore, you wish to know secondly how these sesquiquartas may be changed to sesquialteras, (take) these triple converted ratios, i.e., 9, 3, 1, and change them about by arranging them thus: 1, 3, 9, and take away the lesser from the middle, i.e., 1 from 3, and place it as the first term and what remains from 3, place as the second term. Thus, from the third, i.e., 9, take away the first, i.e., 1, and two second numbers, i.e., twice the second and what remains from 9, i.e., 4, place as the third term, and these numbers are then arranged thus: 1, 2, 4. Therefore, you see how triple ratios are turned back into duples, whence they were procreated.

If indeed, these duples are reversed and arranged thus: 4, 2, 1, and if, as above, you make the first equal to the first, i.e., 4; and the second to the first and second, i.e., 6; the third to the first and two seconds and to the third, i.e., 9, they will be 4, 6, 9. The resolution of the sesquiquarta into a sesquialter has been accomplished as Boethius points out, not in one step but in two, not confusedly but methodically.

If you still wish to know how the sesquiquarta may finally be resolved into three equal terms, reverse these converted duples, to wit: 4, 2, 1, and arrange them thus: 1, 2, 4. Therefore take away the lesser from the middle, i.e., 1 from two, and place this 1 as the first term, and the remainder place second, i.e., 1. From the third term, i.e., from 4, take away unity, i.e., 1, and twice the second term, i.e., two unities, and the remainder will be for you one unity, and it will be 1, 1, 1. Therefore, you see how the whole quantity of the sesquiquarta has been changed into three equal terms, i.e., unities: 1, 1, 1, not confusedly but in definite order, just as it was procreated in the beginning. This, therefore, is the true nature of numbers.

4. RHEIMS. 978-80?

Gerbert explains to Constantine of Fleury a passage in Boethius <u>De musica</u>, relative to superparticular numbers

GERBERT, THE TEACHER, TO HIS CONSTANTINE

The theory of superparticular numbers, briefly discussed in the second book (iv-xi) of <u>De musica</u>, is expounded more fully and by examples in the second chapter of the fourth book. We shall copy these words here and proceed forthwith to discuss them: "If a superparticular ratio is multiplied by two, the result is neither a superparticular nor a multiple". The ratio is said to be multiplied by two when the ratio is doubled so that as the first ratio is, so is the second, that is, the first term's ratio to the second will be the same as the second term's ratio to the third.

Let there be the superparticular ratio four to six. Because this is one (ratio), let it be multiplied by two. For twice one are two. It is necessary that as four is to six, six is to some other number; this is nine. I say that the ratio of nine to four is neither that of a multiple number to its root, nor that of a number to one of its aliquot parts. But, if the result of such multiplication is neither a multiple number nor a superparticular one, then the result when it is multiplied by two, whether superparticular or any other kind of number, will not be a multiple number.

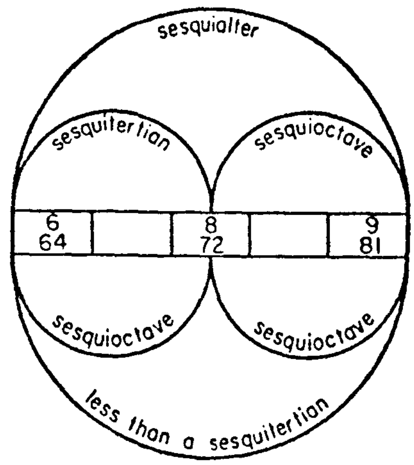
The result of the first multiplication will be a double sesquiquartal ratio, as nine to four, which is neither multiple, nor superparticular, but multiple superparticular. However, the result is not a multiple number because it was multiplied by two but a superparticular of another sort. But here the question is not one of a multiple nor of any other, but definitely of a superparticular. For it is a multiplied sesquialter ratio.

5. RHEIMS. 978-80?

Gerbert explains to Constantine of Fleury another passage in Boethius De musica, relative to superparticular numbers.

GERBERT, THE TEACHER, TO HIS CONSTANTINE

In Chapter 21 of the Second Book of De musica, Boethius states as follows: "If from every superparticular, one subtracts a continuous superparticular ratio, that which is left, which is indeed less, is less than half of the ratio which has been subtracted: as in a sesquialtera and sesquitertia. Because the sesquialtera is larger, let us subtract the sesquitertia from the sesquialtera. There remains a sesquioctaval ratio, which, when doubled will not make a whole sesquitertian ratio, but is less by the distance that is found in a semitone. But if the doubled sesquioctaval relationship is not a whole sesquitertia, a simple sesquioctave is not a whole half of a sesquitertian ratio".



Let there be assumed one and the same number on which the sesquialteral and the sesquitertian ratio is constructed, namely 6, to which 9 is the sesquialter, and 8, indeed, is the sesquitertia. Let these be arranged in this manner: 6, 8, 9. And because these two ratios are continuous superparticulars arranged in three terms: 6, 8, 9, let us take away the first term to which 8 is the sesquitertia; 9 the sesquialtera. There will remain 8 and 9, which are a sesquioctave. But the sesquioctaval ratio is not a half of the smaller ratio, that is the sesquitertian, because when doubled it will not make it, but is less. Therefore, let us double the sesquioctaval ratio, and the three numbers will be arranged in such a way that they do not depart from the ratio 8 to 9, and there will result 8 times 8, and 8 times 9, and 9 times 9, i.e., 64, 72, 81.

I say that the first to the second and the second to the third maintain the sesquioctaval relationship, but the third to the first is less than a sesquitertia, and therefore the sesquioctave is not one half of a sesquitertia. And this will be observed generally in all continuous superparticulars because, if the lesser is subtracted from the larger, that which remains is less than half of the subtracted ratio, because doubled it is not equal to it, as the accompanying diagram demonstrates.

6. RHEIMS. 979?

Gerbert refers to the argument over the propriety of studying numbers and promises Constantine a more complete discussion

(GERBERT TO CONSTANTINE)

O Constantine, sweet solace of my labors, we are entrusting your sagacity, which has always flourished in the freest honesty of studies, with a prepublication of these axioms designed for the utmost exercise of the mind. By using them, the way for grasping these ideals immediately is opened to those persons of less comprehension who, because this pattern of thinking has either been neglected or completely unknown, exasperate every one of the skilled masters of the subject, moreover, by their habitual loquacity, replete with fallacies.

For, if, by considering carefully the nature of each statement, they preserved the maxims for observing the rules of health equally derived from the ancients, then they are in nowise to be so vigorously censured for diligently studying their works through arguments and disputations. Thus, when they are about to discuss numbers standing either by themselves or in chance relationship with each other, they will be able to say what are the digits, what also the articles, since the nature of all number revolves only about the change of such relationships.

But for the sake of our aforesaid friendship I shall, if life continues, explain these matters (to you) more clearly, as much as is necessary (for you) to attain fullest understanding.

7. RHEIMS. First half of 980

Gerbert sends to Constantine, monk of Fleury, rules for using the abacus and indicates their usefulness

GERBERT, THE TEACHER, TO HIS CONSTANTINE

Only the compulsion of friendship reduces the nearly impossible to the possible. Otherwise, how could we strive to explain the rules of the abacus unless urged by you, O Constantine, sweet solace of my labors? Since it has now been some years since we have had either a book or any practice in this sort of thing, we can offer you only certain rules repeated from memory, sometimes in the same words, sometimes in the self-same sentences.

Do not let any half-educated philosopher think they are contrary to any of the arts or to philosophy.

For, who can say which are digits, which are articles, which the lesser numbers of divisors, if he disdains sitting at the feet of the ancients? Although really still a learner along with me, he pretends that only he has knowledge of it, as Horace says. How can the same number be considered in one case simple, in another composite, now a digit, now an article?

Here in this letter, diligent researcher, you now have the rational method, briefly expressed in words, 'tis true, but extensive in meaning, for the multiplication and division of the columns (of the abacus) with actual numbers resulting from measurements determined by the inclination and erection on the geometrical radius, as well as for comparing with true fidelity the theoretical and actual measurement of the sky and of the earth.

230. AACHEN. October 21, 997

Emperor Otto III writes a letter for himself inviting Gerbert to become his teacher and enclosing an original verse

EMPEROR OTTO TO GERBERT, HIS TEACHER

Otto himself writes Gerbert, most skilled of masters and crowned in the three branches of philosophy.

We wish to attach to our person the excellence of your very loving self, so revered by all, and we seek to affiliate with ourself the perennial steadfastness of such a patron because the extent of your philosophical knowledge has always been for Our Simplicity an authority not to be scorned. Not to be ambiguous but to enjoy plain speaking with you, we have firmly resolved and arranged that this letter shall make clear to you our desire as to the extent of our choice and the singleness of our request in order that your expert knowledge may be zealous in correcting us, though not more than usual, unlearned and badly educated as we are, both in writing and speaking, and that with respect to the commonwealth you may offer advice of the highest trustworthiness.

We desire you to show your aversion to Saxon ignorance by not refusing this suggestion of our wishes, but even more we desire you to stimulate Our Greek Subtlety to zeal for study, because if there is anyone who will arouse it, he will find some shred of the diligence of the Greeks in it. Thanks to this, we humbly ask that the flame of your knowledge may sufficiently fan our spirit until, with God's aid, you cause the lively genius of the Greeks to shine forth.

Pray explain to us the book on arithmetic so that when fully taught by its lessons we may learn something of the attainments of the ancients.

Whether it pleases you to act upon this invitation, or displeases you, may Your Paternity not postpone making a reply to us by letter.

Farewell.

> Verses have I never made
> Nor in such study ever stayed.
> When to its practice myself I apply
> And can write successfully,
> As many men as has Lorraine,
> To you, then, songs I'll send the same.

231. NEAR SASBACH. October 25, 997

Gerbert accepts Otto III's invitation to join his court as a teacher

GERBERT TO OTTO CAESAR

Gerbert, archbishop of Rheims, by the grace of God, (sends) whatever is worthy so great an emperor to the ever august glorious Lord Otto.

Not because of our merits, though perchance because of solemn vows, are we able to make answer to your surpassing kindness that deems us worthy of perpetual obedience to you. If we are aglow with the slightest spark of knowledge, it redounds to your glory through the excellence of your father who nourished it and the magnificence of your grandfather who matched it.

What shall I say? We are not bringing our own treasures to yours, but rather are giving back what we once received, some of which you have enjoyed already, some of which you are very soon to enjoy as is evidenced by the honest and useful invitation, so worthy of Your Majesty. For, unless you were not firmly convinced that the power of numbers contained both the origins of all things in itself and explained all from itself, you would not be hastening to a full and perfect knowledge of them with such zeal. Furthermore, unless you were embracing the seriousness of moral philosophy, humility, the guardian of all virtues, would not thus be impressed upon your words.

Not silent, moreover, is the subtlety of a mind conscious of itself since, as I might say, oratorically you have shown its oratorical capabilities as flowing from itself and its Greek fountain. I do not know what more evidence of the divine there can be than that a man, Greek by birth, Roman by empire, as if by hereditary right seeks to recapture for himself the treasures of Greek and Roman wisdom.

Therefore, Caesar, we obey the imperial edicts not only in this, but also in all things whatsoever Your Divine Majesty has decreed. For we who consider nothing sweeter among human affairs than your command cannot fail in obedience to you.

APPENDIX F

Selective Index of Manuscripts and Books of Rhythmomachia

Hermannus

Staats Bibl. Munich Clm 14836 XI f 3b-4b

Bibl. Nat. Paris lat. 7185 XII f 107a-108b

Bibl. Avranches 235 (145) XII

Bibl. de l'Arsenal Paris 830 (55. S. A.L.) XV

Bibl. med. Montpellier 366 XV

Vat. Reg. lat. Rome 598

Asilo

Vat. lat. Rome 3101 XI f2a-2b

Staats Bibl. Munich Clm 14836 XI f 4b-6b

Staats. Bibl. Munich Clm 6369 XI f 66a-66b

Sibley Music Libr. Rochester NY 1, Acc. 149667 XI f 16a-16b

Bibl. Nat. Paris lat. 7377C XI

Bibl. Nat. Paris lat. 7185 XII

Cathedral Libr, Hereford O.I. vi, XII f 76-78 XII

Bodleian Libr. Oxford Auct. F. 1. 9. XII f 1a-1b

Bodleian Libr. Oxford Rawlinson C 270 XII f 20a-22b

St. John Col. Oxford 17 XII f56b

Bibl. Amploniana Erfurt Q 325 XIV

Bibl. de l'Arsenal Paris 830 XV

Nat. Bibl. Vienna 5216 XV

SLB Dresden C 80 XV 258a-258b

Odo

Nat. Bibl. Vienna 2503 X f 49a-57b

Fortolfus

Univ. Wrocław n54 XII f 86a-94b
B R Brussels 927-940 XV

anonymous

S L B Dresden C 19 3 XV
Catal. G. Libri [1] (1859) 483 n^{c} "In cipit Rithmachia..."
Bibl. Casanatense Rome 791 1511

Abraham Riese

S L B Dresden C 433 XVI

Strozzi MS

Bibl. Nat. Florence II., II., 278 1539

Vetula MS

Vetula printed in Wolfenbüttel, ed. 1662 typis Sterniis

Insertion XI

Rhythmomachia in the Rulebook of St. Benedict of Aniane
Clm 28118 Munich 18vb-19r

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[1] Guglielmo Libri, eminent collector of London, disposed of his library treasures in 1859; the Rhythmomachia is known only from this rare catalogue, p. 103 f. Peiper, Abhandlungen, p. 213.

Some Printed Books on Rhythmomachia

Shirwood 1470 London

Jacques Lefèvre d'Etaples 1496, 1507, 1514 Paris

Boissière 1554, 1556 Paris

"W F", Fulke 1563 London

Barozzi 1572 Venice

Gustavus Selenus 1616 Leipzig

Baumann 1577 Erfurt

Severino 1620 Naples

Rhythmomachia in Journals

Peiper 1880

Wappler 1882

Smith 1911

Richards 1943, 1946

Leete 1960

Breidert 1973

Evans 1976

Chicco 1977

Math-Jeunes 1981

Rhythmomachia in the Twentieth Century

John F. C. Richards published two articles about the game of Rhythmomachia in Sripta Mathematica. The first cne, "A new Manuscript of a Rithmomachia", came out in 1943 and was followed three years later by "Boissière's Pythagorean Game".[1]

The new manuscript in question is in the Sibley Music Library, No. 1, Acc. 14667, the Eastman School of Music of the University of Rochester, N. Y. The text is similar to the Asilo MS Vatican lat. 3101, dates probably from the second half of the eleventh-century and is in German Carolingian script found in Bavaria.

Richards edited the Latin text and gave a translation. He also examined parallel passages from various manuscripts which were based on a common source.

In the second article Richards published a translation of Boissière's Latin "Rythmomachia" of 1556. This is of considerable help to anyone interested in playing the game. Boissière's voluminous prose (using two words rather than one whenever possible) is rather a trial to the modern reader. Richards however omitted to translate certain passages, such as the dedication and preface and a digression into an allegorical

[1]John F. C. Richards, "A new manuscript of a Rithmomachia", Scripta Mathematica, vol. 9, 1943, pp. 87-99, 169-183, 256-264, and Scripta Mathematica, vol. 12, 1946, pp. 177-217, "Boissière's Pythagorean Game".

account of the numbered pieces and pyramids. As these are of particular interest to the cultural historian, a translation has been provided in appendix C, the allegorical account can be found on pp. 204, 205.

Charles Leete's "Ye Olde Gayme of Rithmomachy", [2] based on Boissière, appeared in _Engineering and Science Review_, 1960. It gives a very brief but correct explanation of how to play the game.

W. Breidert, in "Rhythmomachie und Globusspiel" [3] compares Rhythmomachia with the so called "Globe Game" and gives a number of details on printed works of Rhythmomachia.

G. R. Evans' article in _Janus_: "The Rithmomachia: A Medieval Mathematical Teaching Aid?" [4] discusses the interesting point that early medieval manuscripts do not give a sufficiently clear explanation of the game for the uninitiated to learn how to play without additional instruction by a master. The sketchy "rules" given in these manuscripts would therefore presumably have done duty only as an extra educational aid.

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[2] Charles Leete, "Ye Olde Gayme of Rithmomachy", in _Engineering and Science Review_, January 1960, pp. 18-20.

[3] W. Breidert, "Rhythmomachie und Globusspiel". Bemerkungen zu zwei mittelalterlichen Lehrspielen, _Mitteilungen und Forschungsbeiträge der Cusanus- Gesellschaft_ vol. 10 (1973) pp. 155-171.

[4] Gillian R. Evans, "The Rithmomachia: A Medieval Mathematical Teaching Aid?" _Janus 63_, 1976, pp. 257-273.

Evans also draws attention to four short English manuscripts which belong to the first half of the twelfth-century and which in her opinion may have been used in "specialist" mathematical schools. These are: Hereford Cathedral Library MS O. I. vi, fo. 76-78 (a copy of the Rithmomachia Wirceburgensis), Oxford, Bodleian Library Auct- F. 1. 9, fo. 1-1b, Oxford, Bodleian Library MS Rawlinson C 270, fo. 20-22b, Oxford, St. John's College MS 17, fo. 56b. These manuscripts have so far largely been overlooked.

Evans points out that the ability to play Rhythmomachia helped the student of mathematical theory to understand his Boethius better. It is for this reason that so many manuscripts of Rhythmomachia have been found in collections of other mathematical material. In the Hereford text the game is followed immediately by another mathematical teaching aid, namely a set of verses for remembering the names of Arabic numerals.[5]

Adriano Chicco contributed a scholarly article[6] in Italian on the history of Rhythmomachia for the

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[5] These verses are discussed by Hope Hannyngton in "Notes on a Twelfth Century Mathematical Manuscript from Cirencester", Transactions of the Woolhope Club (1955) pp. 61-64.

[6] Adriano Chicco, Bonus Socius, "Bijdragen tot de cultuurgeschiedenis van het schaakspel en andere bordsspelen. Jubileumuitgave voor Meinert Niemeijer ter gelegenheid van zijn 75ste verjaardag". 's-Gravenhage, 1977.

occasion of a 75th birthday celebration of Meindert Niemeijer, a chess enthusiast. Chicco's interest in the game stems from his involvement with so-called Fairy Chess, a term invented by an Australian, H. Tate, in 1913 for all unorthodox chess games, including puzzles (mansubat), expanded, four-handed, single-handed chess and many other varieties. Dickens[7] in fact states that a full collection of Fairy Chess must include the publications on Rhythmomachia by Faber Stapulensis, Boissière and "W.F."

The most recent publications of how to play Rhythmomachia appeared in a French schoolboy's magazine, "Soyez 'Vieus Jeux', Apprenez la Rithmomachie".[8] It claims to have taken the rules from a book dated 1554, without naming the author, who must have been Boissière as his French Rythmomachia was published in 1554, two years before the Latin version.

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[7] The Royal Library, The Hague, Holland, has the largest collection of Fairy Chess books. Anthony Dickens, editor of the Catalogue of Fairy Chess, Kew Gardens, The Q Press, 1973, lists Chicco as a contributor to the Fairy Chess group of interest: A. Chicco and G. Porreca, Dicionario degli Enciclopedico degli scacchi and A. Chicco, Contributi Alla Storia dei Problemi di Scacchi (Gilio de' Zelati e Ercole des Rio) L'Italia Scacchistica, Milan, 1950, pp. 1-3.

[8] Math-Jeunes, Sept./Oct. 1981, pp. 11-14.

ILLUSTRATIONS

11 Last page of Fortolfus' Rythmimachia, MS n^{c} 84 s. XII, Breslau.

12 Inclusion of Rhythmomachia in the Regule S. Benedicti, MS 28118, Munich.

13 Groundplan of a church corresponding to the proportions of a human figure, Francesco di Giorgio, MS Ashb. 361, c 10 b, Biblioteca Laurenziana, Florence; German Institute of Art History, Florence.

14 Analysis of the musical ratio of 4:2:3 time on which the composition of the Primavera is based, after C. Bouleau, The Painter's Secret Geometry, 1963; from F. Gettings, The Hidden Art, London, Cassel, 1978.

15 The Pythagorean musical ratios, related to the planetary intervals, from Stanley's The History of Philosophy, 1660, vol. 3, as quoted by F. Gettings, The Hidden Art, London, Cassell, 1978.

16 The Chess Players, Gerolamo da Cremona or Francesco di Giorgio, Metropolitan Museum of Art, New York.

17 Reconstruction of "The Chess Game" and "Group of Youths", A.S. Weller, Art Quarterly, vol. VIII, 1940.

18 Diagram illustrating the numerological relationship between the Three Worlds, by Francesco di Giorgio, L'Harmonie du Monde, Paris 1578 (tr. Nicolas Lefèvre de la Boderie) as quoted by F.A. Yates, Occult Philosophy in the Elizabethan Age, London, Routledge & Kegan Paul, 1979.

19 Playing-cards from Thomas Murner's Logica memorativa J. Gruninger, Strasbourg, 1509, from Tractatus Applicatio Octavo and Sexta.

20 Saturn, from an Italian Book of Days, published by the Italian Tourist Department, Rome, 1959, MS Historical Institute of Medicine, University, Rome.

21 Pyramids from Claude de Boissière's Rythmomachia, Paris, 1556.

22 Pyramids from Claude de Boissière Rythmomachia, Paris, 1556, the alternative way of playing and Gustavus Selenus' so-called towers, Rythmomachia, Leipzig, H. Gros, 1616.

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