Students, Texts and Mathematics: an analysis of mathematics texts and the construction of mathematics knowledge

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I hereby declare that this minor dissertation is my own unaided work and that all sources of reference have been acknowledged. The dissertation is submitted for the degree of Master of Education at the University of Cape Town. It has not been submitted before for any degree or examination at another university.

Signed Signed by candidate

Ferial Allie-Ebrahim

this 14th day of February 2001

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Abstract

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This study deals with a systematic description of student production of mathematics texts and focused on written texts that appeared to be legitimate yet could not be upheld by a principled verbal description.

A search of the literature on the analysis of student texts revealed that semiotic analysis, was not only scarce, but ideally suited to examining the social organisation of school mathematics practice.

This study examines how student texts produced in response to typical school mathematics problems can, via a systematic analysis of texts, index the construction of mathematics knowledge. It outlines Dowling's model of Social Activity Theory (1998) to produce a textual analysis which focuses on textual strategies to distribute message. These strategies and the message underpin the analysis. Practices that establish the message distributed indexes mathematics knowledge and curriculum practices. The notion of a mathematising gaze informing school practice was explored and was related to the construction of existing and pre-existing mathematics knowledge. To locate the effects of a mathematics gaze that could produce texts that lacked discursive elaboration in verbal descriptions, a comprehensive list of ideal types were developed to act as an interface between the empirical text produced that acted as a reading for constructive description of the theoretical terrain.

A significant part of this analysis is qualitative, but also includes some quantitative content analysis. The quantitative component focuses on the occurrences of distributing strategies and ideal types. The results of the analysis are used to comment on school mathematics practices as well as pedagogic practices. These are then used to discuss the limitations of the study and possibilities for future research. The concluding section points to implications of textual analysis reproducing students, teachers and social relations between them or in establishing equilibration to this disturbance in education practice.

CHAPTER 1 INTRODUCTION

1.1 Locating the study

This research project was born out of my experiences teaching mathematics to senior high school students at a Western Cape high school. I focused on the students' inability to synthesise their mathematics knowledge to produce solutions when encountering problems in a new section of work unless specific instruction with regard to techniques and concepts is given. This is particularly intriguing since contents, principles and techniques that are encountered in grade 8 and grade 9 of junior high school mathematics form the basis of procedures that are required in the more senior grades.

I noticed that students were able to perform proficiently in the provincial matric examination at the end of grade 12 even though I knew they were not always able to describe and justify their actions when producing a solution to a mathematics problem. It seemed that matric exam performance was a poor indicator of mathematics "understanding". This observation cannot be supported by formal statistics and is rather based on my experience. Against this backdrop of disjuncture between students' production of correct mathematics solutions and principled mathematics knowledge, the impetus for this study emerged.

A students' compulsory (state) mathematics education in South Africa extends for more than 1500 days. During most of their mathematics learning career from 6 to 14 years and beyond, students work on textual or symbolically represented tasks. Ernest (1997) produces a rough estimate of compulsory state schooling in Britain as more than 2000 days. He suggests that the repetitive nature of mathematics activity is under-accommodated in many current accounts of mathematics learning, where the emphasis is more often on the construction of meaning. Any analysis of mathematics activity, he argues, must acknowledge the way in which the social organisation of the classroom presents patterns of authority and modes of communication, for example. A mathematics task in its final presentation, he argues, is a representation of the task and its requirements as interpreted by the learner. These requirements appear to be related to the organised form of social life in the mathematics classroom and is presented in many forms, like patterns of authority for example. Ernest describes it as "rhetorical requirements". The rhetorical requirements of the organised social form of the mathematics classroom determine the form that the students' presentation and representation takes. This suggests that student representations manifested in their productions are informed by the complex form of organised social life (who can say and do what) while simultaneously confirming this form of the social.

This study will attempt to uncover the way in which students have interpreted mathematics problems and tasks and will explore the reasons and limitations of their interpretation during their production of a text. The relation between interpretations, implicit in classroom practice, and the extent of their mathematics knowledge will also be explored. The mathematics content considered will be based on the mathematics syllabus pertinent to these students. Analytic geometry was chosen.

Analytic geometry is presented to grade 12 students as a new topic but is in fact an accumulation of techniques encountered in grades 8 to 11. Evidence of the repetition of mathematics content, skills and techniques based on the 1996 Western Cape Education Department Mathematics syllabus (valid at the time of this research) is detailed in Table 1.1.

Topic	Grade 8	Grade 9	Grade 10	Grade 11
Algebra	* Factorisation * Plotting points	* Factorisation * The Cartesian plane and point plotting. * Graph of the straight line	* Factorisation * Graph of the straight line * Graph of the circle	* Factorisation * Graph of the straight line * Graph of the circle
Geometry	* Theorem of Pythagoras * Angles on a straight line * Intersecting straight lines	* Theorem of Pythagoras * Angles on a straight line * Intersecting straight lines	* Theorem of Pythagoras * Straight lines * Quadrilaterals	* Theorem of Pythagoras * Straight lines * Quadrilaterals * Circles
Trigonometry			* Basic trig ratios * Interpretive trig problems (Word sums) * Positioning and location of points on the Cartesian plane	* Basic trig ratios * Interpretive trig problems (Word sums) * Positioning and location of points on the Cartesian plane

Table 1.1: Content, skills and techniques related to Grade 12 analytic geometry

Clearly, the mathematics syllabus displays cumulative organisation. Plotting points, for example, is a notational convention that is taught in grade 8 and is crucial to the later development and understanding of graphs in grades 9, 10 and 11. Similarly, the theorem of Pythagoras, which is a core concept in geometry in all grades, is also central to the understanding of basic trigonometric ratios. The cumulative organisation of the mathematics syllabus thus presents concepts in the lower grades and later establishes relational links between them. On these grounds analytic geometry can be seen as an alternative presentation of mathematics content. Included in the alternative presentations are skills and techniques like plotting points on the Cartesian plane; applications of the theorem of Pythagoras; applications of the geometric properties of the straight line, line segments and quadrilaterals. It is therefore plausible to suggest that students at the end of grade 11 should be able to engage with questions posed in analytic geometry without having to be taught it as a new topic. However, this rarely occurs in my experience.

It is from this initial curiosity that an important factor of this research project emerges. Why do students appear to learn mathematics in isolated segments rather than in a way that synthesises solutions from the mathematics learned? Why does the mathematics learned not form an ever increasing resource pool? Why does this lack of synthesis manifest itself at the end of school mathematics? Why do student textual productions not reflect this lack of synthesis?

The answers to these questions are complex and multi-faceted, and can be approached from different positions, drawing on different resources. Student textual productions are central to this account. Typically student textual productions are a response to questions presented by a teacher or some other mathematics authority and index principled mathematics knowledge. The theorisation of student textual production draws on Social Activity Theory (Dowling, 1998) and semiotic analyses of mathematics (e.g. Rotman, 1988) as well as cognitive science approaches. Mathematics education sources include Skemp (1989), Ernest (1997) and Dowling (1998).

1.2 The research question

A considerable amount of research has been conducted in the USA and the United Kingdom on the way in which students think about mathematics, the way in which they work in mathematics and the way in which school mathematics knowledge develops. The research can broadly be categorised into two groups. The first group considers the aims of mathematics: how and why phenomena occur. The second group considers the mathematics profession and makes recommendations on how to achieve, improve and change the learning of mathematics and instruction given in mathematics.

It will be argued that the attention of most research efforts have presented arguments that focus on the ability of students to synthesise solutions in a particular way, a way that focuses on the difference between those who are able to synthesise existing mathematics concepts and those who are unable to do so. The notion of difference between students in this research group is viewed as given and measurable rather than viewed as reflections of the social context.

The work of Dowling (1993, 1998, 1999) highlights how much of the research on differences between students' ability on knowing what to do and why to do it, does not consider the way in which school mathematics teaching and learning practices produce and reproduce these differences in students. The bias towards measured comparisons between these two groups prioritizes difference at the expense of studies that explore why these differences exist.

This study wishes move away from research that focuses on differences in performance through an examination of cognition only. A consideration of abilities, attitudes and beliefs of students, largely ignores the effects of school mathematics practices and the way in which it constructs learners. When students produce mathematics texts the mode of representation (such as a figure or a formula) is a response to the question posed, or the task given. Transformations of any initial response made in the text are intended to satisfy the task as interpreted by the student. This indexes a certain amount of mathematics knowledge and the way in which it has been constructed by the social practice of school mathematics.

I will thus attempt to answer the following research question: How does the student recruit his/her mathematics knowledge during the production of mathematics texts and how does this production of mathematics texts recruit the student's mathematics knowledge?

knowledge develops and the factors that shape its development. The present study tries to move beyond this. It was for this reason that the development of mathematics knowledge was explored via student production of mathematics texts (Corran & Walkerdine 1981).

Thus in summary, this study focuses on the following three questions:

1. Why is access to specific instruction a prerequisite for students tackling problems that are seemingly new?
2. Why are students able to produce texts that are seemingly correct yet cannot be mathematically justified?
3. To what extent do the strategies applied during the production of text reflect school mathematics practice?

Chapter 2 considers the literature in the field of mathematics texts, their use and their development as resource tools. It will consider how student texts have been theorised and investigated in previous research. Research on structures in mathematics for understanding the development of meaning will also be surveyed.

Chapter 3 provides a general methodology and analytic framework for considering the empirical text while simultaneously providing an analysis of school practices.

Chapter 4 deals with generating and reading the text. Ideal types are generated to act as an interface between the general methodology and the empirical data thus facilitating further categorisation of textual strategies described.

Chapter 5 analyses the results and provides a discussion on the way in which students synthesise mathematics based on their textual strategies as well as the discursive content and expression of the mathematics texts produced.

Chapter 6 presents the concluding remarks and considers possible limitations of the study.

CHAPTER 2 Literature review

The central concern of this literature review is to position my empirical focus on student mathematics texts in relation to associated research literature. In this chapter I will give some consideration to research that has focused on student textual production.

The way in which relevant research is constituted will be viewed in terms of its possible sources as well its motivation. In my preparation, I generated a series of simple questions as a simple means of categorisation: "How?", "Why?" and "How to?" were posed with reference to the recruitment of mathematics knowledge. "How?" and "Why?" focus specifically on students. The question, "How to?", is more general and considers recommendations made on what ought to occur during learning while simultaneously considering the production of student texts as a school mathematics practice. The literature reviewed thus considered research on empirical texts that was able to generalise features across texts in a way that goes beyond the confines of an individual or specific text.

Most of the initial literature considered were from journals in mathematics education: *Educational Studies in Mathematics*; *Arithmetic Teacher*; *For the Learning of Mathematics*; *Philosophy of Mathematics Education*; *Psychology of Mathematics Education*; *Mathematics in School*; *Journal of Mathematical Behaviour*. A body of research focusing largely on establishing meaning rather than distinguishing what makes meaning possible, was found. A noticeable feature was the way this research ignored the pre-existing system of mathematics implied in the (re)production[1] of school mathematics. The search revealed very little that could be described as sociological or as semiotic. There appeared to be a focus on mathematics form at the expense of the structure of mathematics content, a focus on the development of the learning of mathematics via processes of causation. In an attempt to address this concern, I extended the range of my review beyond the confines of mathematics research. The journals considered were:

Teaching and Teacher Education; *Journal of Reading*; *Linguistics and Education*; *British Journal of Sociology of Education*; *Preventing School Failure*; *Journal of Reading Behaviour*. This still revealed very little of direct interest. I found that education research journals appear to largely focus on cognition in order to generate prescriptions for "good practice". Very little on the syntax of mathematics texts exists in the literature. Even less exists on the way in which student texts can be analysed. In most instances one could infer that mathematics tasks analysed had to be judged or assessed by students' verbal and/or oral productions, regardless of the perspective adopted.

There does, however, appear to be an increasing interest in semiotics in mathematics education. A survey of recent articles published in *Pythagoras*, a journal of the Association for Mathematics Education in South Africa, from August 1998 to August 1999 reveals two research reports (Ernest, 1998 and Vile, 1998) related to this area. Interest in language and other general socio-cultural issues have been considered as part of this budding interest even though they could be regarded as part of semiotics (Vile, 1998).

My research study also considers the study of semiotics as related to mathematics. Here I refer to the use of signs, the relations between signifiers and signifieds and their influence on mathematics knowledge. This study's concern with student production of mathematics texts and pedagogic practices implicated in the (re)production of school mathematics foregrounds reflection on the pedagogic situation and subsequent pedagogic action with respect to transmission strategies. The review of the literature thus seeks to interrogate previous research on student productions.

This chapter will be organised in the following way. First, I will consider different ways in which students learn mathematics. These are not learning theories. They can rather be viewed as general descriptions of mathematics practices. This will be followed by reports on interrogative structures used to interpret mathematical meaning and knowledge; on the

[1] My use of (re)described corresponds to Dowling (1998) which illustrates the simultaneous production and

use of language in formalising mathematics intuition and orientation to meaning; on semiotic analyses of mathematics texts; on investigations of student texts. Finally, a discussion on the development of mathematical argument will follow.

2.1 Ways in which students learn mathematics

The starting point in this literature survey is research that focuses on the way in which students learn and understand mathematics. This is not presented as a learning theory that attempts to explain why it is that learners proceed in the way they do. It rather presents a way for describing mathematics cognition by presenting them in contrasts with cognition referenced in terms of understanding or characterising components of mathematics knowledge.

The distinguishing factor in this research lies in the differentiation between mathematical competency and mathematics understanding. Consensus on what constitutes mathematics understanding exists in the literature. Mathematics understanding is based on a thorough and principled understanding of all theorems and mathematics procedures that can be consistently applied to different problems and different contexts. Mathematics competency on the other hand emphasises problem solving techniques and procedures with scant attention paid to whether the solutions are transferable to other contexts, mathematical and non-mathematical.

This dichotomous relation is apparent in Skemp (1989) in which he presents two possible ways of describing the way in which students understand mathematics concepts: ***instrumental*** and ***relational*** understanding. Relational understanding refers to students being able to think deductively and at high levels of abstraction so that the knowledge learned can be related or transferred from one context to another. For this to occur, the principles which underpin it have to be known. Instrumental understanding, on the other hand, is largely algorithmic and analogic. It is often independent of underlying theory and

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reproduction of that which underpins school mathematics practice.

is largely based on exemplars: hence the emphasis on competency. Skemp regards the use of principles with general application as the main differentiating criterion between the two kinds of understanding. Similar findings can be found in Mansoor (1994), Halmos (1985), Anderson (1976) and White *et al.* (1996) who all report that courses emphasising manipulations have difficulty in addressing the application of problems that require conceptual or theoretical knowledge. This is described by White *et al.* (1996) as "abstract-apart" understanding. This term is endorsed by the operation on symbols without any regard for their possible contextual meaning. The absence of integration between symbols and their contextual meaning is also addressed by Tall (1991). This research terms the accumulation of new rules, learned by rote and added to existing knowledge without any attempt to integrate the rules with the old ideas, "disjunctive realisation". Terms like "abstract-apart", "disjunctive realisation" and "instrumental understanding" all reflect an impoverished view of mathematics which cannot establish relations between various mathematical segments.

Mansoor (1994) refers to "algorithmic understanding" and "conceptual understanding". His study primarily shows how improved understanding can evolve through the use of conjecture, refutations and reformulations. Mansoor regards "algorithmic understanding" as part of a series of models which facilitate "conceptual understanding" and subsequent conceptual development. This work is clearly a recommendation of what school mathematics practice ought to do. Unlike Skemp, "understanding" for Mansoor does not form the basis of the contrast between the two. Algorithmic understanding can rather be regarded as the first stage in the development of a broader conceptual understanding. The emphasis and progression towards the incorporation of explicit principles into understanding, he argues, does not necessarily guarantee that conceptual understanding develops. An acknowledgment of this possibility is significant but does not assist in describing different kinds of understanding evidenced by students in their texts.

Other studies have also looked at the different kinds of knowledge that can develop in a way that highlights a similar distinction between the algorithmic and the conceptual.

Halmos (1985) talks about "abstract" and "algorithmic components", while Anderson (1976) suggests "declarative" and "procedural components" respectively. Their work suggests that algorithmic and procedural are similar and that declarative and abstract components can be grouped together. Although both Halmos and Anderson produce extensive descriptions of each type, their focus is on the components of knowledge rather than the way in which this knowledge manifests itself in texts. In other words, they provide characterising features of the two knowledge types without explicitly pointing to student strategies that result in this kind of knowledge. Declarative and abstract components both operate at a higher level of abstraction than algorithmic and procedural components. This difference suggests that the latter operates segmentally while the former establishes relations between segments.

Dual contrasts of this nature are also present in the work of Vergnaud (1982) who considers the different kinds of mathematics knowledge associated with the different kinds of mathematics skills by focusing on "practical" and "theoretical" knowledge. The study contends that there is no conflict between practical knowledge and theoretical knowledge by suggesting that most practical competencies in mathematics refer to some theoretical view. Competencies are thus always related to conceptions, however weak or incorrect the conception. He cites algorithmic use as an example of this. No algorithm or procedure can be developed free of any idea of the relationships involved. The lack of differentiation, he argues, between practical and theoretical knowledge implies that practical knowledge has the potential to become more theoretically based. Even though knowledge is polarised into opposition, it is clear that theoretical knowledge embraces practical knowledge. This element is absent in the work of Skemp (1989), Halmos (1985) and Anderson (1976) who regard their dual contrasts as two separate entities.

The studies reported on can be summarised in the following table which looks at the two forms of understanding for the different authors.

Author	Different mathematical forms.	
Skemp (1989)	instrumental understanding	relational understanding
Halmos (1985)	algorithmic component	abstract component
Anderson (1976)	procedural component	declarative component
Vergnaud (1982)	practical knowledge	theoretical knowledge
Mansoor (1994)	algorithmic understanding	conceptual understanding

Table 2.1: Summary of different forms of understanding

All these studies suggest that understanding (knowing what to do and why) underpins mathematics practices. Whether or not algorithmic use can be upheld by mathematics principles is an important criterion in establishing a contrast between practices based on competency and practices based on principles.

Skemp's assertion that generalisable principles applied in other contexts is an important criterion for viewing the way in which mathematics knowledge is regarded. Differences between relational and instrumental; between the algorithmic and the conceptual; between the declarative and the procedural; all fixed in opposition, all versions of mathematics knowledge, are polarised into opposition. The explicit descriptions of instrumental and relational understanding, for example, in terms of examinations, syllabi assessment and the role of the teacher, suggest that the theoretical formulation for learning relational mathematics centres on the development of a conceptual structure. Herein lies the rub. The formulation of relational understanding, while acknowledging the development of a conceptual structure, cannot stand alone since it does not account for the possibility of underdevelopment of theory. Fragments of developed theory and concepts may exist segmentally without any integration. Dichotomous contrasts present two extreme possibilities: the one or the other. A combination of the two contrasts is not allowed.

Furthermore, it is implicitly suggested that the generalisable type is more desirable and advantageous in the context of school mathematics. While it is accepted that understanding which is consistently principled can be transferred between different contexts and problem types, the constructions that have to take place to transform the instrumental (non-generalisable type) into the relational (generalisable type) are not clear. No interrogative framework exists for explaining how the text that students produce can be described as being generalising or non-generalising.

2.2 *Structures in mathematics for understanding mathematical meaning*

An understanding of what constitutes mathematical meaning is an important consideration when trying to shed light on how it is that students recruit their mathematics knowledge. As their mathematics learning career progresses, learners develop an ever-increasing pool of mathematics knowledge. Any recruitment of this mathematics knowledge when answering a particular problem, is a consideration of when to do what and how to do it. The following readings attempt a consideration of ideas of how student knowledge is constructed during the production of mathematically legitimate texts. The readings chosen provide a framework or specific criteria for gauging mathematics knowledge.

Schoenfeld's (1985, 1992, 1994) research on problem solving explores how well people use the knowledge potentially at their disposal, and how the degree of efficiency of their knowledge applied affects their success or failure during problem solving. This focus produces a list of requirements that need to ensure success at problem solving. The claim being made is that mathematics content alone is not sufficient to ensure success at problem solving. Knowledge of what to do with any mathematical content they may have is significant. The book *Mathematical Problem Solving* supports this claim by providing a description of problem solving largely based on comparative analyses of its successes and failures. Improved performance in students who were provided with strategies for using what they did know was illustrated. Another phase of Schoenfeld's research revealed that the context in which mathematically competent students were asked to solve a problem impacted significantly on problem solving. This was illustrated when a maximum-minimum problem was not included in the context of a calculus course, and was phrased differently: "What is the largest .... ?" rather than "Find the maximum value ..... ?". This illustrates how the absence of contextual cues may result in incorrect problem solving.

The research undertaken by Schoenfeld suggests that communicating meaningfully with students (teacher-pupil dialogue) will have a significant impact developing an understanding of the ways in which students understand things, and finding a way to make connections to those understandings (whether correct or incorrect). This form of communication does not acknowledge typical asymmetry of teacher-pupil dialogue within classroom practice. The teacher may, for example, respond to a student communication (written or verbal) by shaping developing mathematics knowledge that could reinforce a particular mathematics practice.

> The details of knowledge and behaviour necessary for an adequate characterisation of mathematical problem-solving performance appear below. Knowledge and Behaviour Necessary for an Adequate Characterisation of Mathematical Problem-Solving performance.
>
> ---
>
> *Resources*: Mathematical knowledge possessed by the individual that can be brought to bear on the problem at hand.
> - Intuitions and formal knowledge regarding the domain
> - Facts
> - Algorithmic procedures
> - "Routine" non-algorithmic procedures
> - Understandings (Propositional knowledge) about the agreed upon rules for working in the domain
>
> *Heuristics*: Strategies and techniques for making progress on unfamiliar or nonstandard problems; the rules of thumb for effective problem solving, including
> - Drawing figures; introducing suitable notation
> - Exploiting related problems
> - Reformulating problems; working backwards
> - Testing and verification procedures
>
> *Control*: Global decisions regarding the selection and implementation of resources and strategies.
> - Planning
> - Monitoring and assessment
> - Decision-making
> - Conscious metacognitive acts
>
> *Belief systems*: One's "mathematical world view", the set of (not necessarily conscious) determinants of an individual's behaviour
> - About self
> - About the environment
> - About the topic
> - About mathematics
>
> (Schoenfeld, 1985, 15; italics in the original)

There appears to be a strong attempt to illustrate exactly what is needed for students to maximise what can be learnt from mathematics instruction with no consideration of how knowledge was constructed. Modes of communication, for example, between the teacher and the students may be skewed by the stylized way a teacher responds to the content and form of written work students submit. Schoenfeld argues that his view or approach is different to a more traditional approach where students have to engage and master a fixed body of knowledge. Although an attempt is made to acknowledge social variables that affect one's belief system in mathematics problem solving, performance is largely dependent on ways of enhancing understanding of mathematical thinking, learning and teaching. Schoenfeld (1985) makes it clear that mastery of mathematics content has to be coupled to the ability to communicate using mathematical language. The extent to which this can be linked to the kinds of mathematics instruction that students have received and their subsequent mathematical behaviour is open to question, he argues.

The work of Schoenfeld focuses on what the individual is capable of bringing to bear on a particular problem. Furthermore, it is underpinned by an inventory of what the individual knows, believes or suspects to be true. The way in which this belief system arose is largely ignored. The way in which it regulates school mathematics practice and the way in which school mathematics practice sustains this belief system are also largely ignored. The prime objective is to show that when instruction focuses almost exclusively on mastery of facts and procedures, students are not likely to develop some of the higher order skills necessary for using mathematics.

Background assumptions of knowledge and behaviour necessary during problem solving are of interest to the present research since they affect the mode of expression (verbal and written) and the degree of control over knowledge. Knowledge centering on the provision of contextual cues or certain classroom practices has significant implications for the control and authorisation of knowledge. Data illustrating this will be presented and discussed.

The work of Corran and Walkerdine (1981) and Walkerdine (1988) comment on the construction of mathematics knowledge through a consideration of the contextual nature of children's interpretations. Their work regards mathematics as a discourse and its development is considered through its application in non-mathematics practices as well as the relation between knowledge and discourse formats. The study is based on the differences between strictly controlled experimental educational settings and everyday situations in which many variables interact simultaneously in the production of any cognitive performance. Walkerdine foregrounds the social and questions the (re)creations of reasoning within the context of school mathematics. It is argued that understanding the relationship between one practice and discourse and another practice and discourse is what underpins more effective teaching. She further argues that an awareness of discursive relations will facilitate sensemaking in children that takes more than the individual student or a specific context into account.

Her analysis depends on linguistic tools like metaphor, signifier, signified for identifying discursive elements. For example, words that are uttered or diagrams that are drawn are referred to as 'signifiers' for physical actions, which are the corresponding 'signifieds'. This relation is regarded as metaphorical. Mathematics, she claims, suppresses metaphoric relations in order to produce generalised statements. The transformation of a language sentence using words to a mathematical sentence using mainly mathematical signs possibly entails, but cannot detail, the metaphors present in language. The absence of metaphoric content remains for the student as signifieds to be united with mathematical signifiers (signifier refers to concepts already formed via students' own experience). The connection between signifieds and signifiers establishes a signifying chain, which allows learners to make mathematical sense. The way in which this arises is when metaphors, initially related to objects or a particular situation, are used to produce a correct mathematics statement. This can only occur if learners are able to keep the chain of signification in mind. The significance of the practical example lies in its access to mathematics discourse since this determines the development of mathematics knowledge.

> Without the relation with signifieds the discourse of mathematics is inert: it has to be used. Therefore each signifier gets its value and significance both from its metaphoric position in the act of its semiotic relation with the chain of signifiers/signifieds and from its metonymic place, its relation to, and positioning with, other signifiers in the [mathematics] statement.
>
> Corran and Walkerdine, 1981, 152-153.

The power of mathematical statements lies in the ability to act as an application of many different practices. Everyday practices such as shopping can be described by the meanings and metaphors of practical knowledge. They can also be described mathematically. However, the discourse associated with shopping is unlikely to be appropriately described as regulated to the same extent as the discourse associated with school mathematics for example. This distinction is not clearly made in Walkerdine's work.

Within classroom practice, certain mathematical descriptions connote "real understanding". The distinction between what students can do and their level of understanding is strongly correlated to the discourse associated with school mathematics. The distinction between the discursive and the non-discursive is thus important. The blurring of this difference is a criticism of this work (Dowling, 1998). This concern is addressed in the chapter dealing with methodological issues.

In the mathematics classroom, discursive relations have to include the use of symbols and symbolic relations and this is constituted in a very specific way giving social and communicative meaning to letters, signs and diagrams. Social interaction in the mathematics classroom is based on symbols that are interpreted according to particular conventions and rules. A teacher can reformulate and reinterpret student responses governed by the rule. The use of language is important since there is no simple transmission of meaning through language.

Chapman (1995) acknowledges that students may learn to say by routine what they are expected to say in certain defined situations and addresses the generation of meaning in school mathematics bearing this phenomenon in mind. This research explores the way in which shared meanings are constructed by looking at language practices. The characteristic principles of different bodies of text are considered so that commonality can be made explicit. The relations between the texts, termed its intertextuality, are based on relations between word clusters in the text and the themes identifiable in the text. The relation between the two is then reconstructed so that a pattern of thematic relations can be identified.

A similar viewpoint is held by Steinbring (1997):

> Meanings of mathematical concepts emerge in the interplay between sign/symbol systems and reference contexts or object domains. [...] This relationship between the signs for coding the knowledge and the reference contexts for establishing the means of the knowledge can be structured in the epistemological triangle.
>
> Steinbring, 1997: 50.

Steinbring designs an epistemological triangle, a triadic scheme that tries to explain the problem of understanding relationships between symbols and referents in mathematics. In this scheme a semiotic sign is equivalent to symbols in mathematics and it is linked to an object or reference context as well as the concept forming the three points of the triangle. The reference context (like a die used in probability studies, for example) can originally be regarded as a concrete object, but this changes more into an abstract relational structure as the mathematics develops. The initial mathematics object evolves during mathematics development into a new relational connection. The resultant change in the status of the original reference context's mathematical meaning is now determined by the interplay between the reference context[2] (the die) and the sign system associated with more general mathematics discourse (probability theory). The growth of knowledge and mathematical meaning emerges from the re-interpretation of mathematics contexts. This constant re-evaluation underpins the development of mathematics meaning and led to the revision of the epistemological triangle. Adaptation of the triadic relationship was redescribed as a spiraling epistemological circle to better illustrate the emergence of a new context from an old context. Both structures provide an interrogative framework for

[2] The reference context referred to corresponds to Steinbring's language.

analysing the interplay between the symbol system and the reference context in the learning environment. Like Chapman, Steinbring considers the relational linkage between an existing sign system and referential contexts, but in a more structured manner.

Steinbring's approach does not explicitly acknowledge the structure of the language used as a means of interpreting mathematics meaning. Although signs and symbols are significant in the epistemological triangle, his work cannot be described as a semiotic study of signs. It nevertheless points to the importance of the context in which mathematics is taught since this will be reflected in the texts analysed. While this work points to the development of mathematics meaning, it also regards the text as a means of evidencing this meaning. However, no explicit statement is made about the way in which textual strategies are deployed to facilitate the development of this meaning.

A closer examination of the signs employed and their use in the mathematics system is taken up by Ernest (1998). His primary focus is on mathematics texts and signs. He uses semiotics to establish mathematics meaning by examining the relationship between the signifier and the signified of a sign in establishing a signifying chain. When all signifying relations are made explicit mathematics knowledge is enhanced. The signifying chain can also be used to explain any mathematics transformations that may occur. This research derives a structured framework for looking at mathematics activity by looking at the mutual interaction between the signifiers and the signified where the task is regarded as the sign and the student's text as the signifier. This semiotic analysis considers the way in which texts signify relations thereby determining the way in which mathematics meaning is created. The text is regarded first as a sign for mathematics that points to a specific signified. The signified in turn can be regarded as a sign for a different signified. In this way a sequence of infinitely many sequences of signifier/signifieds become possible. This kind of analysis reveals a systematic way of looking at how mathematics signs can be transformed during the production of a text.

Pimm (1995), too, is concerned with the use of symbols in mathematics and the notion of meaning. He regards the fluent use of symbols as a primary source of mathematical understanding. His discussion on fluency and understanding, however, does not explicitly show how an expression using symbols is related to understanding. Pimm's work suggests that too large a focus on the symbols negatively affects the development of meaning. This forms one of the themes in this study and will be used to shed light on the research question, which attempts to uncover textual strategies.

Brown (1996) argues that categories implicit in the use of language itself reveal much about the practice of classroom mathematics and the research process itself. He too attempts to understand the development of meaning by understanding how individual words inter-relate. The model used to provide a framework of understanding is based on an analytic series of successive writings as a means of highlighting changes in practice. The analysis' dependence on a series of writing does not provide an explicit, comprehensive plan for identifying the structures inherent in the writing. The absence of these structures is taken up by Catterson (1990) who points out that themes in writing, termed sub-structures, cannot be fully understood without knowing about a text's main topics and its sub-topics: its macro structure. Failure to do this results in analysis occurring in isolation of the broader language of the field.

The present study concurs with these ideas: relations between different contents and expressions, the use of symbols and the different modes of expressions form part of the strategies employed during textual production. Data illustrating and supporting this will be presented in a later chapter.

Hayes (1987) considers the potential for directed study in combined reading and writing activities that focus on paraphrasing related texts. By looking at the way a text is encoded, he argues that a construction of generative semantics, which underpins the text, develops. His regard for reading as an inferential process which will reflect the extent to which students are engaged in the activity is significant because it shows that the text is a valuable resource in developing an understanding of the text that students can generate. Unfortunately, the absence of a comprehensive interrogative framework reduces the generalisabilty of this work. Furthermore, the principles of comparison based on the way in which the content of texts is organized in unified ways, congruent with experience and expectations, are not readily transferable to other activities. The use of paraphrasing nevertheless is an important consideration when considering the way in which texts are structured and the way in which learners have understood the texts. It is via paraphrasing that the discursive element in language can be gauged. The link with the present study lies in the way in which students talk about their written productions.

Research that focuses on the use of language specifically in mathematics not related to the semiotic approach is not significantly reported in mathematics education journals. Their significance lies in the emerging awareness of the extent to which texts can reveal the way in which students understand mathematics.

These readings have attempted to describe structures used in previous research that could inform strategies in the text. Strong links with language have been highlighted in this regard.

2.3 Language

The language of school mathematics includes the content of school mathematics (the symbols used, concepts, conventions, definitions and symbolic procedures of mathematics knowledge) and the mode of communication (written and oral modes).

Many research studies regard language as a significant factor in determining the way in which mathematics develops for students. Language serves as a means to link representation of an idea to its verbal and symbolic representation. Different linguistic interpretations impact on how they acquire knowledge, when to invoke one meaning rather than another (Pimm, 1987, 77; Rotman, 1988, 24). Associated research has language, together with the words and structures which express these meanings". This means that students may adopt a certain register on certain occasions and not on others depending on their interpretation of the mathematical context.

Similar regard for mathematics as language is held by Richards (1990), Hunt (1993), Sierpinska (1997), Sfard (1991; 1997), Bruner (1985) and Laborde (1996). These researchers equate the learning of mathematics with the learning of a language. The focus of all these studies is the correct use of language and terminology. The present study explores the legitimate use of language (verbal and written).

While language can be used to formalise mathematics, it can also dictate the way in which mathematics language ought to develop. This is an important consideration in the development of mathematics meaning and understanding. This is presented as another group of research which pertains to language use in mathematics.

2.3.2 The orientation of mathematics meaning with regard to language used in mathematics

Sfard (1991) looks at the way in which language structures the development of mathematics concepts. She argues that mathematics concepts can be conceived structurally and operationally. Structural is used as a verbal description of a mathematics concept. For example, a circle has a structural description of "the locus of all points equidistant from a given point" as well as an operational definition of "a curve obtained by rotating a compass around a fixed point" (ibid., 5). They are necessary parts of a whole since the one provides verbal coding while the other provides visual coding though not necessarily in a one-to-one correspondence. Mathematical understanding, according to Sfard, is related to the ability to successfully capture the structural and the operational concepts. The extent to which this is possible is in turn related to the learners' language use in mathematics.

A different perspective is adopted by Bruner (1985). He argues that the path to mathematical understanding is achieved by getting clued into the language of mathematical discourse: knowing what the intentions are behind certain typical expressions. Even though Bruner acknowledges that language is an expression of the way in which students think mathematically, his position on mathematics language differs somewhat. Language in mathematics, he argues, is learned by interacting with those who legitimately set standards of mathematics discourse (usually teachers). These interactions determine boundaries, what to say, how to say it, when to say it, and who says what in different circumstances. This formalisation of language has been termed the "format of interaction" concept (Ibid.).

Bruner's "format of interaction" concept employed in language acquisition and in understanding mathematics augmentation and development is applied by Sierpinska (1997). This research shows that articulation and syntax met by approval of significant adults like teachers and tutors provides positive reinforcement for students. A similar view is held by Richards (1990) who considers the development of mathematics and language to be simultaneous.

The focus on mathematics being the basis for formalising mathematics is approached differently by Sfard (1997), Hunt (1993) and Laborde (1996). They argue that mathematics language is by and large non-negotiable and is presented in standard formats. Sfard (1997) regards this positively when she focuses on linguistically induced expectations. She argues that linguistic associations of new symbols which may evoke old meanings and their corresponding ways of use can give rise to notions and expectations of how they ought to be used. These notions are then tested or refuted. Laborde (1996) and Hunt (1993), on the other hand, highlight negative aspects of the language in mathematics. For Hunt, worksheets, structured scheme books, and all other written tools of mathematics seldom encourage children to use their own language in order to create or to share mathematical ideas. Students are thus positioned as receivers of mathematics language rather than as active participants in its construction and use, making it a possible source of conflict in mathematics learning. Laborde's research (1996) focuses on the systematic use of signs as a means of hampering the ability of students to solve problems.

Thus far two ways in which the language of mathematics can be regarded has been reported on. On the one hand mathematics has to be put into language, while on the other hand students' language has to be put into mathematics. Another view of language in the mathematics classroom exists: language as an indicator of the socialisation of thinking.

2.3.3 Language as an indicator of the socialisation of thinking

School mathematics is a complex organised form of social life. The belief that students think and act within socio-cultural contexts that are mediated by cultural tools has highlighted how different workplace mathematics is from that taught in school. Different mathematisations to try and bridge the gap between one school of thought that sees any activity in mathematics as inherently mathematical, and another which searches for specific signs and conventions associated with established mathematics practices. The accepted signs and conventions characterise legitimate mathematics texts and the way in which these are used during the production of texts is important in answering the research question: how does the student recruit his/her mathematics knowledge during the production of mathematics texts.

The practice of mathematics, according to Nunes (1996) involves a wide variety of signs and shows how actions depend on their material base and social circumstance. Available everyday resources shape mathematical thinking. Her references to associated signs and conventions as well as her regard for a mathematical system of signs representing signifieds, in particular conventional ways, suggest a semiotic approach to the learning of mathematics. Negotiating meanings represented by signifiers in the system of signs then becomes the learner's responsibility. For Nunes students are participants in linguistically created realities in which they have to think of implications rather than the actual consequences of a mathematical problem. An everyday activity like trading in the market requires a degree of forethought before a business transaction is completed. This is a crucial tool in the learning of mathematics and is often absent from classroom practice.

Allusions to the semiotic approach are also made by Puig (1996). In this research it is argued that a mathematical sign system for arithmetic can have segments with the same matter of expression as the vernacular tongue. The difference in the use of similar words lies in its grammar, semantics and pragmatics which are all characteristic divisions or fields of semiotics.

The reasoning of Puig (1996) and Nunes (1996) is that the grammar which connects the terminology, manifests itself in students' articulations and facilitates linguistic associations with their corresponding uses. However, it is unclear as to how everyday practices are refined so that they can be absorbed into the realm of mathematics or using Walkerdine's (1981) language, how one discourse can be formatted in terms of the discourse of mathematics. This appears to be a recommendation on curriculum and classroom practice. The imposition of a mathematics discourse on any practice besides mathematics is well and good. Inappropriate and incomplete formatting, as well as formatting that cannot be justified mathematically is an important theme in this research. Data will be presented that will facilitate a detailed analysis of this phenomenon.

Morgan (1996) points out that very little exists on grammatical structure or form of mathematical texts. This is addressed in semiotic analysis of mathematics texts.

2.4 Semiotic analysis of mathematics texts

Rotman (1988) explores a semiotic approach to mathematics in a way which allows mathematicians to explore semiotics without sacrificing the familiar language of mathematics. He argues that mathematics is about counting and thus the study of natural numbers: semiotics is concerned with the study of number and the sign activity associated with numerals. Semiotics is can thus be seen to be the study of formal written texts between mathematicians or of an informal signifying activity. He argues that reading a written text is the first step in decoding what mathematicians say, and recognising what has been written is the second step. This view directly addresses one of the prime concerns of this study: the production of mathematics students who fail to decode and recognise the mathematics in an activity.

Furthermore, he argues that mathematics writing is structured like grammar and has an object and a verb. The verb points to the action of a mathematics activity and two types are identified: the indicative and the imperative. The indicative refers to the interrogative case and questions, makes assumptions and in short asks for mathematics knowledge. The imperative, on the other hand, expects an action and presents texts as a sequence of instructions. Differentiation between the indicative and the imperative produces actors and agents respectively. Actors (mathematicians) are imbued with the ability to create future mathematicians while simultaneously securing their positions as mathematicians by the semiotic analysis of their mathematics activity. Definitive descriptions of agents and actors suggest that an agent can not be seen as a developing model of an actor because of the absence of mathematics theory and discourse. Mathematicians are rather constructed via their use of language and this dictates the extent of the mathematics they can engage with. Do they follow the imperative or will they give the imperative? Ways of developing and answer to this question will be further explored in the chapter dealing with the methodology.

Winslow (1997), like Rotman (1998) acknowledges the structure of mathematics texts semiotically by considering the extensive use of mathematics symbols. While Ernest (1998) focused on the text and sign relations in mathematics, Winslow (1997) considers the surface and deep structures of the text. In accordance with the semiotic approach, surface and deep are linguistic tools. To argue that two texts have the same deep structure is to suggest that they make explicit the underlying principles of the text. It seems as if Winslow's work addresses the chain of signification. Access to the complete chain of signification is a strongly related to the "depth" of the text.

The perspective that Ernest, Rotman and Winslow adopt in mathematics education acknowledges linguistic, cognitive, social and cultural perspectives. The extent to which this occurs does not fully emerge within the parameters of the literature review. The extent to which a structured interrogative framework has been produced has been highlighted. Research that addresses how writing produced in mathematics can be scrutinised is an integral part of mathematics practice. The nature of teaching practice and existing knowledge is necessarily reflected in student writing. This view supports this study's argument that there is a dialectical relationship in the way in which students position their mathematics and are positioned by it.

All the studies reported above indicate that the semiotic structure of the language points to the extent to which mathematics discourse can be realised in language. If the grammatical structure is underpinned by indicative verbs, then information connected to mathematics discourse must exist.

2.5 Investigations of student texts in mathematics

The classroom is an important arena in which mathematics learning takes place. The studies discussed below address the way in which mathematics texts have been deployed in research thus far.

Shield and Galbraith (1998) examined student writing so that factors that impact significantly on its format could be considered. The texts that they studied were called "writing products". These products were analysed according to a coding scheme that the researchers developed. The analysis revealed that the writing of students closely resembled the style of the mathematics textbook used. This phenomenon is not unexpected, since students learn by modeling their work on formats that are privileged in the mathematics classroom. The point rather is about the way students are able to produce "writing products" that are mathematically correct. Whether this is matched by an appropriate degree of mathematics understanding is not clear and forms a significant theme in what this study hopes to address.

Students' texts as a means of explaining mathematics understanding were the focus of the research undertaken by Borasi *et al.* (1989). This work emphasises that student texts is an under utilised tool. The texts referred to are student writings in journals. This writing reflects student perceptions of understanding directly and makes it difficult for other influences like text book writing to shape the text. The focus is on their own contribution to their journal. The structure which this analysis takes is not a major concern. The recommendation for student writing as a means of accessing student understanding is being highlighted.

The study that Clarke (1993) undertook probed the structure of mathematics writing as a means of unveiling the way in which students construct mathematics. The scheme they developed described student writing consisting of three main categories: recount, summary and dialogue. Dialogue is the most mathematically developed category while summary is the least developed. This was used to show how students progressed when constructing mathematics. Van Dormolen (1985) considers the textual analysis of mathematics texts by looking at the level of language used as a means of understanding its depth. He speaks of "exemplary" and "relative" levels of language and they differ with regard to the use of a specific example: exemplary level of language employs a specific example while relative level does not. This emphasis is significant because it points to the text as being a tool which facilitates insights into the development of mathematical argument via the use of the language used, the format which the text takes and the way in which students' existing knowledge has been constructed.

This group of research gives little information on the overall structure of the text. Factors that impact on the text produced, and factors concerned with classroom practice and its influence in determining the kinds of knowledge that exists, are largely ignored. This deficiency can be linked to the primary focus of this research: the production of texts that cannot be sustained by mathematics content and knowledge.

The development of mathematics argument will be considered as a separate category.

2.6 The development of mathematical argument

Sfard's (1991) study showed that students are able to solve linear equations, for example, without a broader understanding of the necessary vector space theory. The subsequent impoverished understanding is due the absence of processes able to deal with the theory. A similar point is made by Tall (1991) who points out that formal mathematics definitions often do not match the simplistic understanding that learners have. These definitions, which operate at a high level of abstraction, are a potential source of conflict and are manifested in the way in which understanding develops. This view is supported by Tall and Thomas' study (1991) that points to the need for an understanding that facilitates abstraction if future development in mathematics is to be sustained.

The need for operation at higher levels of abstraction is considered in Bromme *et al*'s study (1994), which explored the role that students' early conceptions of sets and functions play in relation to more advanced group concepts. Mathematical meaning was found to be dependent on consistency in presentation of mathematical symbols and the referential meanings of these symbols with respect to tasks given. A similar study by Dubinsky *et al.* (1994) showed that success at abstract algebra depends on the student's ability to transform existing processes by another action so that a more sophisticated process of "encapsulation" emerges. Existing knowledge and the processes associated with it are significant contributors to and need to be incorporated in the development of more advanced processes. This idea of "encapsulation" was already expounded in Dubinsky (1992) and Beidenbach (1992). These studies clearly suggest that all mathematics concepts are related. The possibility that knowledge can exist in well insulated segments that bear little relation to each other is not sufficiently addressed. It is precisely this deficiency that this study wishes to explore.

Levels of abstraction are also related to the way in which students are able to navigate core structural concepts in mathematics and its associated operations. Any disjuncture between the structural and operational can be detected by the understanding and use of mathematics language employed in definitions and proof for example. The imbalance between the structural and operational can be disguised by strategies that students deploy. Tall (1991) suggests that students who have difficulty in concepts, hide their difficulties by resorting to "routine activities to obtain correct answers and gain approval". This strategy quickly degenerates into activities, which become increasingly instrumental and are in fact detrimental to future mathematical development.

The link between mathematics argument and texts in all the studies reported is not explicit. They rather point to possible reasons for the absence of mathematics argument.

Summary of the chapter

This literature survey has been largely concerned with the recruitment of mathematics knowledge by students and the way in which this manifests itself in their productions. The research reported clearly suggests that students do not always learn mathematics in a way that encourages them to think relationally. An analysis of reports on mathematics practices showed dichotomous polarisation: one that is characterised by relational thinking and another that appears to focus on competency without establishing relational links. It appears that competency and the ability to produce responses that are correct can exist in isolation from a deeper mathematics understanding and discourse.

The way in which language was used in the development of mathematics understanding was significant. The following reasons were cited for this.

* Language is the basis for formalising intuitive mathematics understanding.

* The language of (school) mathematics shapes the way in which students are able to express their mathematics thoughts.
* language used is an indicator of the socialisation of thinking.

Analytical schemas for examining the mathematics meaning and understanding in a verbal and a written text were explored. The structural frameworks indexed triadic relations for understanding possible relationships between the signifier and the signified. A focus on the individual restricts the any possible analysis of classroom practice and pedagogy to the individual rather than the social. One could argue that this discrepancy between the individual and the social depends of the perspective of the individual and the social. I would like to suggest that the studies that acknowledge the social would significantly alter any findings in research studies.

My task is to try and explain why students who have not been given mathematics instruction are reluctant to engage with mathematics problems from a topic they have yet to be taught. In Rotman's language, it seems as if students need to be given an imperative or realisation rule. This strongly suggests that students need to be nudged in a particular direction so that they can behave according to an expected pattern.

Research that specifically considered investigations of mathematics texts was examined. It became apparent that the use of mathematics texts as a research object in the practice of school mathematics was not extensive. This pointed to a need for more research that was informed by student texts. Within this group of research, note was taken of mathematics texts that were semiotically analysed.

The way in which students recruit their mathematics knowledge, in terms of the literature reviewed, is indexed by the dichotomous forms of knowledge or understanding. Impoverished mathematics knowledge is recruited when students are unable to establish relational links between different mathematics topics. The polarisation of mathematics understanding presents trends: one which is based on memorisation and rote learning without a principled understanding and one which is based on principled recognition and transformations. Structures for understanding mathematical meaning may be focused on the individual or they may be focused on the social.. When mathematical meaning is focused on the individual an emphasis on what students ought to know exists and is underpinned by an inventory of individual's perception of the way in which mathematics is constituted. Acknowledgment of the social facilitates a consideration of how an individual's view of mathematics arose, how it regulates school mathematics practice, and the way in which school mathematics practice sustains this belief system.

The use of language in the students' recruitment of mathematics knowledge is also an important consideration. The literature considered suggests that a strong relationship exists between language and the formalisation of mathematics intuition and ideas. It became apparent that regard for mathematics is either one in which mathematics has to be put in the language of students or one where the students' language has to be transformed into mathematics. Furthermore, student articulations and linguistic associations with the vernacular inform mathematics practices.

The reports on mathematics knowledge and mathematics language all impact on student textual productions, a primary consideration of this study, were indexed by the grammatical structure or form of mathematics texts. Semiotic analysis of mathematics texts in the literature pointed to the extent of which mathematics discourse can be realised in language. The reflection of existing knowledge and school mathematics practices in student textual productions has informed this study in a meaningful way.

The issues of mathematics knowledge, mathematics language and student texts highlighted thus far has significantly informed the research question on how students recruit their mathematics knowledge during the production of mathematics texts and how this production recruits students mathematics knowledge.

The next chapter will focus on the methodological issues concerned with unveiling the strategies deployed during the production of student texts.

CHAPTER 3 General Methodology and Analytical Framework

The intention of the present chapter is to set out the theoretical framework in terms of which the present study was undertaken. The initial interest was a consideration of the generation of written responses within the context of normal classroom practice that appeared to be correct but could not be supported by a principled verbal explanation. This stemmed from the way in which the production of written responses to mathematics questions could disguise the extent of mathematics understanding.

Comment on the practice of school mathematics and the pedagogic situation will be derived from the analysis of the student productions. Student productions, both verbal and written, are to be regarded as text. The analysis will address the question: "How does the student recruit his/her mathematics knowledge during the production of mathematics texts and how does this production of mathematics text recruit the students' mathematics knowledge."

3.1 Mathematics Education - knowledge and pedagogy

Mathematics education is concerned with the transmission and acquisition of mathematics. I will attempt to produce a reading of the way in which students appropriate pedagogic discourse in school mathematics during their description of the texts that they produce. At the heart of pedagogic discourse is the way in which selected groupings of different discourses and their applications are redescribed so that they can be communicated via transmission methods to learners. The subsequent acquisition by learners is a crucial factor in an understanding of pedagogic discourse. This occurs in a framework which determines who can say what and how it can be said on the part of those who transmit and those who receive the transmission.

Consider Bernstein's description of pedagogic discourse as a means of illustrating the way in which school mathematics is constituted.

> *Pedagogic discourse is a principle for appropriating other discourses and bringing them into a special relation with each other for the purposes of their selective transmission and acquisition.*
>
> Bernstein (1990: 183-184)
> (Italics in original)

School mathematics is constituted by selections from, at least, formal mathematics and developmental psychology. Of particular interest in Bernstein's definition is that student productions can be seen as instances of "selective transmission and acquisition". The production of a text by students is thus determined by the way in which mathematics activity can be described (using language) in terms of the mathematics discourse.

The notion of activity is taken from Dowling's social activity theory[3]. He argues that school mathematics activity is established as a set of practices and considers how this affects divisions and distributions within mathematics and between mathematics and other practices. This is done via a consideration of what constitutes a mathematics utterance, the language and register of school mathematics as well as a textual analysis of mathematics texts. A notable feature in Dowling's work is his clear distinction between mathematics practices and non-mathematics practices in terms of language, grammar, expression and content. Mathematics is able to constitute principles of recognition and realisation, which allows it to cast a gaze on non-mathematical practices and systematically redescribe it in its own terms. Mathematics activity regulates who can say, do or, mean what and these are (re)produced in texts. Pedagogic texts "construct authors as transmitters and readers as acquirers" (Dowling, 1998: 131). An activity is thus (re)produced by texts, and in particular pedagogic texts. By this definition, student texts cannot be pedagogic texts. Student texts or student productions can rather be seen as a product of the social within the practice of school mathematics.

[3] Dowling (1998: 131) regards activity as the contextualising basis of social practice: an activity regulates who can say or do or mean what. Practices and positions of an *activity* are realised in texts which he defines as an utterance or set of utterances made within the context of an activity.

The high degree of explicitness and grammar that needs to be manifested in a student text for it to be described as mathematically legitimate is an important consideration. Do students need to have access to this grammar before they are able to produce legitimate texts, or can this occur regardless? The (re)production of a mathematics algorithm, for example, would illustrate this very clearly. The algorithm itself is principled with strong mathematics grammar but use of the algorithm is not necessarily principled. It thus becomes possible to produce a written text that is legitimate but cannot be sustained by constructively principled verbal explanation.

A legitimate text must be characterised by the correct form as well as a principled verbal description. This will differentiate between legitimate and non-legitimate texts.

Bernstein (1990; 1996) speaks of *restricted* and *elaborated codes*. These are presented in dichotomous opposition.

> *The simpler the social division of labour, and the more specific and local the relation between an agent and its material base, the more direct the relation between meanings and a specific material base, and the greater the probability of a restricted coding orientation. The more complex the social division of labour, the less specific and local the relation between an agent and its material base, and the greater the probability of an elaborated coding orientation.*
>
> Bernstein (1990: 20)
> (Italics in original)

Bernstein's work indexes different forms of the division of labour in society. Dependence on the material base is the differentiating factor between restricted and elaborate coding orientation.

An orientation towards restricted code refers to meanings that are dependent on a specific context. This suggests that meaning and understanding are only realised in a very specific "enactment of the practice within which it occurs" (Dowling, 1993: 65) and is not transferable. A restricted code is thus localised while an elaborate code is more generalised. It thus becomes possible to describe an activity like school mathematics by considering the opposition between restricted and elaborate coding.

Dowling divides *activity* into three analytic levels: structural, textual, and resource levels.

> At one level, activity constructs positions via the distribution of practices to a range of positions. At this level activity is (re)produced by human subjectivities which may articulate multiple positions. [...] At the second level, activity is reproduced by texts, and in particular pedagogic texts. Pedagogic texts distribute message over a range of voices and so (re)produce the practices and positions of activity. Activity and human subjectivity comprise the structural level; text constitutes the level of events. [...] The relation between the two corresponds to that between langue and parole, where this relationship is seen as dialectical.
>
> (Ibid.: 131-132)

The relationship between the structural and the textual levels is said to be dialectical since the structural level is accessible only via the textual level. In other words, practices and positions are to be inferred from the analysis of texts. He argues further that text is the material instance of the activity and that text and activity can be regarded as theoretical objects in his work.

At the third level of Dowling's model, textual strategies evidenced in pedagogic texts are realised through resources. It is impossible to derive an explicit constitution for resource since, according to Dowling, "there is not *a priori* limitation on what can count as resource" (Ibid.: 150). This suggests that it is difficult to say how textual strategies are deployed during the recruitment of resources. Their recruitment points to the need to acknowledge this level. This can be hierarchically summarised in the sketch below.

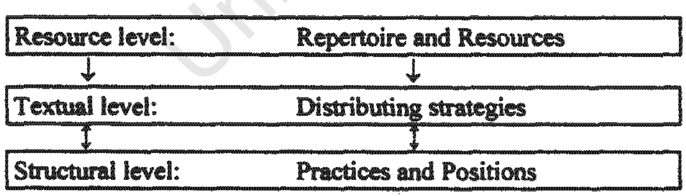

Figure 3.1: The levels of an activity

He argues further that resources are arbitrary and that their arbitrariness is theoretical. He points out that a text has to be constituted by a "repertoire of resources" which has been selected from a "reservoir of resources" (Ibid.: 151). Furthermore, differential selection of resources determines the textual strategy.

Eco uses the notions of *open* and *closed* (1979) to talk about text in relation to the reader. These terms are commonly used in a way which is opposite to what he suggests. A closed text does not refer to closure used in the normal sense of the word. The texts that Eco considers are mainly literary works: novels, poems, comics, etc.

Consider the definition of a closed text:

> Those texts that obsessively aim at arousing a precise response on the part of more or less precise empirical readers [...] are in fact open to any 'aberrant' decoding. A text so immoderately 'open' to every possible interpretation will be called a *closed* one.
>
> (Eco, 1979: 8)
> (italics in the original)

The terms open and closed have been recontextualised[4] for my purposes. In open texts the principles underlying the construction of the text are made explicit. This means that all interpretations or recontextualisations are governed by principles and rules so that texts produced are limited to an orientation that reflects them. Closed texts, on the other hand, do not exhibit the underlying principles. These texts can be interpreted in any way.

My own study has been significantly informed by Eco's notions of the open and closed as a means of identifying the extent to which students are able to (re)produce their knowledge in their own productions. Dowling's model will largely be used to identify certain aspects of the structural level and the textual level while simultaneously describing the way in which the textual level is informed by the resource level. I will attempt to produce a reading of the way in which pedagogic discourse in school mathematics is redescribed by students during their description of the texts they produce. These texts are to be regarded as the empirical text. The verbal description of the written text will form part of the empirical text be analysed using language of mathematics discourse.

[4] My use of recontextualisation corresponds to Dowling's recontextualisation.

Theoretical resources, which I have drawn on, are based in sociology and recruits theoretical concepts derived from the language of description of Dowling and the work of Bernstein.

The remainder of the chapter will be organised as follows: A discussion on the composition of the structural level of the language of an activity and its practices. Then I will consider the language at the textual level and its distributing strategies, which will be followed by a brief discussion of the resource level

3.2 The structural level

3.2.1 Practices: Domains of Practice

The domains of practice of school mathematics can be described in terms of variations in form of expression and content. Dowling (Ibid.) identifies four domains of practice: the *esoteric*, the *expressive*, the *descriptive* and the *public domain*. The principles of recognition and realisation of the mathematics gaze are manifested in varying degrees in the domains. Strong classification of expression and content characterises the esoteric while weak classification of expression and content characterises the public domain. The public domain, which "has the appearance of a non-specialised practice", is "subject to the regulative principles of the esoteric domain". These regulative principles cannot be adequately expressed in this domain. The significance of the public domain lies in it being the point of entry of a novice. The action of the mathematics gaze, in Dowling's language, will prioritise specialised denotations and connotations expressed within the esoteric domain facilitating the transformation from novice to adept.

Possible inconsistencies between the notion of 'text' and 'domains' exist. Texts described are pedagogic texts and the description is not readily transferable to student textual productions. When using student textual productions it becomes possible to access students' written texts as well as the corresponding verbal descriptions. The verbal description assesses the written text in terms of mathematics knowledge that underpins denotations and connotations. The transformation from novice (public domain) to adept does not always occur. It is thus possible that a mathematising gaze may transform the novice so that specialised denotations and connotations, although prioritised, can be (re)produced yet can not be adequately expressed verbally.

Inconsistencies in 'domains' also exist in Dowling's work. His work emphasises that the public domain is not external to the practice of school mathematics and that it "comprises practices which have been or are being recontextualised from other activities" (Ibid.: 136). He argues that recontextulaised practices correspond to text-as-text while pre-recontextualised practices correspond to text-as-work.[5] The induction of text-as-work into text-as-text can be correlated to an everyday practice being inducted into the public domain through weakly classified mathematics knowledge. The induction of the public domain to the esoteric domain cannot assume that the presentation of strongly classified mathematics texts reflect the same degree of mathematics knowledge. The possibility of students rehearsing or mimicking different forms of expression by rote is not be described in terms of Dowling's domains of practice.

In this study the domain of practice is restricted to the esoteric because of the specialised form of expression and content of the texts produced as data. In this research the notion of text is not restricted to the written text only but also acknowledges the respective authors' verbal description of the text. The possibility of student production of texts prioritising the necessary denotations and connotations of the esoteric domain (through rehearsal, for example) without exhibiting the necessary principles that regulate expression and content will produce textual hybrids in the esoteric domain. The main criteria for differentiation between these hybrids will be the extent to which principled redescriptions are made available. These will be gauged by the number of ways that the

[5] The distinction in the empirical text as 'work' or as 'text' is based on different levels of analysis . At the lowest level is the work which is produced as a text in its reading. At the next level is the text which is understood as that which is produced by the reading. (Dowling, 1998: 127)

text can be redescribed in language. Here I will differentiate between single and multiple redescriptions.

The distinction between single and multiple redescriptions is related to the linguistic term, *isotopy*. An understanding of the term isotopy is best explained by looking at the contrast between *topic* and isotopy (Eco, 1979, 23). *Topic* is governed by the semantic relations that assume and expect certain things to be taken into account during the reading of a text. Isotopy is the actual textual verification of the semantic relations marked in the text. Multiple forms of realisation of texts produced can be regarded as paraphrases, if the underlying principle prevails.

In the preceding chapter references were made to research that explored paraphrasing of texts. Paraphrase and redescribe have the same meaning. The term to be used for the remainder of this report is redescriptions.

Dowling uses the term *topic* differently although not in an incompatible way. He regards topics as sub-regions of the esoteric domain.

> School mathematics, for example, consists of topics such as algebra, statistics and probability, arithmetic, Euclidean geometry, transformation geometry, and so on. Topics exhibit a certain degree of positivity, but are multiply interconnected (otherwise the activity itself could not exhibit positivity). Because ambiguity is minimised in the esoteric domain, specialised denotations and connotations are always prioritised. It is, therefore, only within this domain that the principles which regulate the practices of the activity can attain their full expression.
>
> (Dowling, 1998: 135)

Relations in different topics can be established through the transformation of a particular principle from one topic to another. For example, the theorem of Pythagoras appears as a direct application of right-angled triangles in Grade 9 mathematics. A transformation of the theorem of Pythagoras occurs in grade 12 when it is used to derive the length of the line between two points in a Cartesian plane. The generalising principle is consistent by virtue of their location in mathematics discourse. The coherence between different topics is significant since it suggests that the privileged selection of mathematics knowledge is underpinned by the transformation of existing knowledge.

The regulating principles of the production of school mathematics texts are distinguished by their syntactic and semantic levels. The semantic level signals the meaning of what has been written down while the syntactic level refers to the rules that govern what has been written. The different expressions in mathematics do not show much variation so that they appear to be syntactically similar at a general level, but different at a semantic level. Differences in the syntactic level form the basis of this study rather than the semantic level.

It has been previously mentioned that the domain of practice in this study will be the esoteric domain. All student texts produced are a response to standard classroom type questions. Legitimate texts will conform semantically because of the esoteric location. It is thus not possible to have syntactic and semantic differences given the limited range of possible legitimate answers. Instead differences between texts would be syntactic only.

The way in which topic influences the texts produced can be illustrated in a typical mathematics problem which was used in this study.

> Given the circle with equation $x^2 + y^2 = 10$.
> The midpoint of the chord AB in the third quadrant is D(-2;-1). What are the co-ordinates of A and B?

The problem is extracted from a section in school mathematics known as analytic geometry. It is specialised in terms of its mathematics expression and mathematics content. This is illustrated through the use of the formula, as well as the terms "equation", "chord" and "quadrant". In asking the questions "What is prioritised and how is this done?", it became apparent that an emphasis on expression (syntactic level) and content (semantic level) targets those who have sufficient mathematics knowledge to engage with the problem in a significant way. The fact that a circle is described in terms of an algebraic equation illustrates this.

The assertion being made is that student productions of text can be described as being either open or closed. Each textual (re)production can be considered as a response to mathematical typographic conventions like "the circle with equation" as well as a consideration of specialised connotations relevant to the question posed. This suggests that the forms of expression may be single or multiple. This corresponds to restricted and elaborate coding respectively.

Mathematically, there can only be one response that will satisfy the parameters of the question posed. The correct answer is not of importance. The point is whether or not student productions can be justified by mathematics semantics suggested, implied or referred to outside the immediacy of the question posed. The extent to which this occurs can be described as the isotopy of the text. A further descriptive category can be introduced at this point: *topic dependence* or *topic independence*. Topic independence marks texts which have multiple forms of realisation in addition to explicit redescribing principles. Redescribing principles are related to the initial context of acquisition (which occurs in a particular topic). When the same concept is recognised, albeit in a transformed way, in a different topic, topic independence is illustrated.

For Sfard (1991) mathematics concepts can be conceived *structurally* and/or *operationally*. She suggests that a circle, for example, has a structural description of "the locus of all points equidistant from a given point" and an operational description of "a curve obtained by rotating a compass around a fixed point". Clearly the operational structure is dependent on the topic at hand: the construction of a circle using a compass. If the structural description is absent the concept is not fully developed and can not sustain principled (re)descriptions. This implies that topic dependence exists. Topic independence, on the other hand, occurs when a mathematics concept is developed structurally and operationally.

It is also the case that singular forms of realisation can be represented by texts that may appear to be topic independent in "conventional" responses. The following extracts are taken from the transcript of a student's description of her text.

When asked to locate the co-ordinates of a chord of a particular circle given the midpoint of the chord, M, one of the student participants in this study, drew a sketch and tried to use the midpoint to generate an answer. When this was unsuccessful, a second sketch displaying the given information, was completed so that an approach using the gradient of the given chord could be developed. This too produced equations that appeared to be unproductive. At this point the following exchange took place.

F:	Can you just recap what you have done. And explain to me why you think it is not working.
M:	I found the gradient of OD which is, ... umm. The gradient of OD. And I determined the gradient of A. Because it will be perpendicular, it will be minus one. And I found the gradient of OD, which is a half. Therefore the gradient of AB would be minus two.
F:	Why do you make this nought and nought?
M:	Because of the general solution of a circle which is *x* minus *a* squared plus *y* minus *b* squared. It's that. ((M refers to the general equation which she has written down)) So it's that. But over here, we don't have *a* and *b* so we say that it is nought, the centre.
F:	Right. Sorry, you said that the gradient of BA would be minus two. And then?
M:	And then I found AD, the gradient of AD which would be minus two. And then I got that equation. And then I found the gradient of DB which is also equal to minus two which is the same. Which is equal to that.
F:	What about this equation, here now? ((F refers to the last equation generated on the page asked for))
M:	So I could substitute that into that and I could find. ((M starts applying this strategy))

Extract 3.1: Transcript 2 lines 627 - 646

It is significant that the privileged selection constituting the repertoire appropriated for this problem, appears to be based on the ability to recognise procedures and algorithms previously encountered. When the first attempt is unsuccessful another attempt is made. The knowledge connecting these attempts appears to be fragmented and not related conceptually. M recognises that the equation characterising the circle must have its origin as (0;0) based on a working knowledge of the general equation of a circle $(x-a)^2+(y-b)^2=r^2$ where $(a;b)$ are the co-ordinates of the origin. Contained within this equation is a latent wealth of information relating to a transformation of the theorem of Pythagoras as well as understanding and recognition of the format of the equation. Evidently proficient use of the formula does not need to incorporate all this knowledge. This illustrates the way in which a topic independent statement like a general formula can become topic dependent because of the way in which students are able to describe it in language.

The following transcript can further support this. A somewhat different application of the equation of a circle is illustrated.

F:	Do you know how that was generated? ((F refers to the final answer offered as a solution.))
M:	The general formula?
F:	Yes. *x* minus *a* all squared plus *y* minus *b* all squared.
M:	No I don't know. I just know the formula.

Extract 3.2: Transcript 2, lines 452 - 456.

The way in which students understand and apply formulae can be obtained from a description of their production. Only when underlying principles can be made explicit can principled understanding exist. In the absence of a principled understanding, the text is a rich source for realising other features of the way in which such texts are produced.

Firstly, the way in which the formula is used indexes a privileged way of teaching school mathematics. In the above extract, it becomes apparent that the formula functions only as a manipulative tool rather than being an application of a transformation of the theorem of Pythagoras. Secondly, the questions posed by the interviewer allowed access to the way in which students, when confronted with a problem, attempt problem solving. The question presents information about a circle. No explicit clues are given about how the problem is to be solved. In this study, students unable to engage with the problem are given a prepared selection of questions, which allows them to develop insights about the problem. This would allow them to transform the information and in this way possibly increase, or bring into sharper focus, the privileged selection of their repertoire. In this way students are forced to verbalise the extent of their repertoire.

Further insights into classroom practice and pedagogy are marked by the way in which problems are solved. Formal tuition in this topic increased the likelihood of students being familiar with questions of this nature. If this was not the case, students had to explore their resources more generally and adopt and transform existing knowledge.

3.2.2 Practices: Discursive Saturation

Dowling (1993, 1994, 1998) uses *discursive saturation* to describe the way in which language and discourse can be used to describe texts. Dowling's (1998: 62) discussion on the modality of the discursive stems from the discursive/non-discursive distinction indexed by Michel Foucault. He argues that Foucault's lack of distinction between the two is paradoxical: a distinction constrains the discursive within the discourse or renders the non-discursive discursive. This paradox is not a feature in *discursive saturation.* All practices must have an element of the discursive, an element which is realisable in language. The extent to which this occurs describes practices with either a discursive excess or a non-discursive excess. These modes of practice are described in high or low discursive saturation.

Dowling uses Foucault to establish his distinction between the discursive and the non-discursive. However, his use of Foucault is metaphoric and appears to be an attempt to justify the distinction he wishes to make using Foucault as an authority. Foucault's discussion of the discursive and the non-discursive implies that there will always be an element of the non-discursive.

Description of texts in this manner provides a way of highlighting another aspect of the structural level of activity. Consider the extract below, which illustrates the relation between activity and discursive saturation.

> Activities vary internally and from one-another according to the extent of the saturation of material practice by discourse, that is, their discursive saturation may be high (DS^+) or low (DS^-). Discursive saturation may never be total. [...] An activity is a relational totality which exhibits principles, discursively and/or non-discursively (depending on their discursive saturation). In discursive terms activities exhibiting DS^+ can give rise to relatively generalisable utterance. DS^- activities can generate only localised utterances.
>
> (Dowling, 1994: 129)

Empirical texts can be measured in terms of the extent to which the principles of an activity can be realised discursively. These empirical texts Dowling refers to are pedagogic texts. In this study empirical texts are (re)productions of mathematics activity. For Dowling pedagogic texts are generalisable when discursive language is used to describe the activity. The (re)production of activity by students is my primary concern.

Dowling's work is concerned with the way in which school mathematics is established as a set of practices and the divisions and distributions within mathematics. To distinguish between mathematics and other practices he analyses two different textbook series specifically designed for low and high ability students. Analysis of these pedagogic texts is used to comment on the nature of school mathematics activity.

The methodology Dowling uses explains the (re)production of differential knowledge but cannot always explain how activity is constituted or reproduced. The empirical texts used in this study are texts that students produce and all are located in the esoteric domain. If, hypothetically, all students produce texts that display a high level of the discursive, does this mean that they all share the depth of mathematics understanding with regard to the underpinning mathematics principles? This is precisely what this study wishes to explore. However, Dowling's theory is not fine-grained enough to make this distinction. This shortcoming will now be addressed.

At the level of school mathematics, Dowling is judging texts and their utterances. The domains of practice provide a framework for structuring the way in which texts can be described. The four domains of practice can be described with reference to the esoteric domain and the specialised denotations and connotations associated with it. Only within the esoteric domain can an activity be described as mathematical. The syntactic and semantic levels of the esoteric domain index specialised forms of expression and content. It is only within the esoteric domain that the principles, which regulate school mathematics, can attain its full expression. To develop this full expression a mathematising gaze is cast beyond the esoteric domain in order to create and recruit apprentices. Herein lies the rub. Dowling's domains of practice assumes apprenticeship and the transformation from novice to adept, from apprentice to master. The domains of practice are thus underpinned by the moral and ethical judgment being made of school mathematics, one that assumes that all mathematics students, as apprentices of mathematics, have access to the development of mathematics principles and thus principled mathematics knowledge. It is precisely this assumption, which is examined in this study. Consider the research question "How does the student recruit his/her mathematics knowledge during the production of mathematics texts and how does this production of mathematics texts recruit the student's mathematics knowledge?". It clearly attempts to address the recruitment of mathematics knowledge in a way that moves away from Dowling's assumption and also explores how the social organisation of school accedes to the creation of mathematics knowledge that is not always principled.

Nevertheless the description of practices of an activity in terms of domain and gaze, and discursive saturation is useful for analysing mathematics texts since it is a measure of highly systematised discursive structures. Dowling argues that "the high degree of discursive organisation of the esoteric domain of a DS^+ activity facilitates the generation of languages of description having highly explicit realization principles" (Ibid., 138). In this study, at the structural level, activity is DS^+. However, when we consider empirical utterances, they might well be DS^+ or DS^-. Descriptions classified as DS^+ frequently have multiple ways of describing realisation principles. They can also be described in a highly principled way. The extent to which their principles can be realised in discursive form is the distinguishing feature rather than the number of descriptions. In DS^- descriptions, the description of the text is frequently singular. However, multiple, unprincipled and incorrect responses that cannot be realised in discursive form are also possible. Clearly, the number of responses is not a good indicator of whether or not texts can be described as either DS^+ or as DS^-.

Considering the dependence of descriptions on the topic enhances the differentiation between DS^- and DS^+ descriptions. Texts that can be realised in multiple ways demonstrate topic independence (TI) due to the number of variations in principled (re)descriptions. Texts that are realised in only one way demonstrate topic dependence (TD). The singular form of TD texts does not allow for principled (re)descriptions. Discursive saturation gauges the extent to which the principles of redescription are made explicit.

The question that needs to be asked is how Bernstein's definition of pedagogic discourse relates to discourse in discursive saturation.

Bernstein uses the notion of discourse as a means of describing social relations and cultural reproduction via selective transmission and acquisition while Dowling (1998: 146) uses it as a tool for describing the extent to which principles of activity can be realised in discursive form. This facilitates a generalised focus on context dependence and context independence as a means of analysing texts (adapted to topic dependence and topic independence in this study). On a more general level, the language used to describe the text represents the distribution of discursive practices in school. Language, organised as text, informs and is informed by the kind of distribution strategy employed. The point is that a consideration of social practice by Bernstein is described in terms of selective transmission and acquisition while Dowling uses discourse to describe texts as an indicator of social practice. While Bernstein's understanding has helped me shape the development of an analytical tool for interrogating text, it is Dowling's notion of discourse that will be employed in the actual analysis.

Activities like school mathematics are sites where subjects are positioned and practices distributed. The distribution of specialised practices is what constitutes school mathematics. Student productions of text (verbal and written) will index the level of discourse. Differential use of discourse can thus legitimate only certain practices.

3.3 The textual level

In Dowling (1998) school mathematics is the activity and the texts are pedagogic texts. The content of school mathematics is used in these pedagogic texts to constitute message and voice. He then produces an analysis of school mathematics texts to show how activity regulates who can say or do what via positions and practices, respectively. The empirical texts in this study are student rather than pedagogic texts. Mathematics content, pedagogic contexts as well as curriculum features constitute message and voice in student texts. This means that message in student texts centres on discourses in teaching and learning of school mathematics rather than school mathematics learning contents.

Practices at the structural level of an activity correspond to message at the textual level. Dowling argues (op.cit.: 142) that the use of different terms for the two levels points to the difference in theoretical status. Message is an immediate consequence of textual analysis and is determined by the production of text-as-text via a reading of the text-as-work (the student texts). Dowling suggests that the reading of the text-as-work has an element of uncertainty in it since "it cannot be asserted that the reading presented is the only reading that could have been generated" (Ibid.). The validity and reliability of the text corresponds to the stability of the reading. Validity is related to the explicitness in the descriptive language transforming text-as-text to text-as-work. Reliability is related to the sampling of the text. Analysis in this study is largely quantitative. Reliability and validity can thus not be measured. They can rather be viewed as "arenas in which claims are to be supported and contested" (Ibid.).

The practices of an activity as evidenced in the structural level are inferred from an analysis of the text.

> The practices [...] of an activity are structuring resources in the production of texts, which, themselves, are produced and reproduced by texts; the relationship is dialectical. Activities are thus (re)produced by texts and, in particular, by pedagogic texts. Pedagogic texts must therefore (re)produce the various features of an activity described [...]. I shall denote the various mechanisms by which they achieve this as *textual strategies.* [...] In my language of description, a pedagogic text is a weaving of textual strategies which position voices and distribute message. It is these strategies and the patterns of voices that must be identified in and by the analysis.
>
> Dowling (1998: 142-143)
> emphasis in the original

The structural level for Dowling is clearly underpinned by strategies that position voice and distribute message. In this study the structural level is underpinned by the practices of an activity: its domain and discursive saturation. The production of text varies according to the forms of possible expressions and the extent of the discursive saturation. The degree to which this occurs describes the distribution of textual strategies.

The notions metaphor and metonymy as well as message, terms, which will be used to describe the distribution of textual strategies, will now, be defined.

3.3.1 Metaphor and metonymy

Metaphor and metonymy are linguistic modes that examine the structure of language. In traditional usage, metonymy is a figure of speech where the name of one item is given to another item associated by proximity to it. Subjacent metonymic chains facilitate the production of metaphor. In the following sentence, "An equation in which 5 is added to both sides of an equation keeps the equality or balance the same" works by metonymy: "equation " and "balance" stand for equalising activities with which they are closely associated. Metaphor on the other hand relies upon similarity rather than proximity. An example would be "An equation is a balance". Eco (1984) suggests that each metaphor can be traced back to a subjacent chain of metonymic connections, which constitute the framework of the code. Metonymy relies on accepted interdependence if the metonym is to make sense. This is illustrated in the following metonymic links:

Equation →add 5 →both sides→equality→balance.

Absence of the metonymic links results in the metaphor "An equation is a balance". This suggests that there is a similarity between an equation and a balance.

Metonymic chains are always produced. Whether or not an awareness of the privileged metaphor exists is what will be explored during the analysis of text.

3.3.2 Message

Message is described by Jakobson (1960) as one of the functions of language.

> The *addresser* sends a *message* to the *addressee*. To be operative the message requires a context referred to ('referent' in another, somewhat ambiguous, nomenclature), seizable by the addressee, and either verbal or capable of being verbalised
>
> (Jakobson, 1960: 353)

Message is understood to be a form of communication transmitted from one person to another. Dowling (1995: 3) offers an alternate view of message.

> [...] mathematical knowledge - which, in its textualised form, I want to refer as *message* - is distributed to the voices such that esoteric domain message and public domain message are associated, respectively, with superordinate and subaltern voices.
>
> (Dowling, 1995:3)

In terms of Dowling's language, the activity of school mathematics regulates who can say and do what. This means that it distributes its practices across a range of subject positions. Subject positions and practices are realised in text as voice and message.

Voices are positioned and message is distributed. A thread of this language, the distribution of message will be used in this study.

3.3.3 Message and distributing strategies

Interrogation of student texts will be based on the way in which the text distributes the metonymic chains of possible metaphors. An analysis of the message transmitted in the text will give insights to the ways in which strategies are distributed.

Dowling (1998) provides a way of describing strategies in terms of the extent of discourse and range, simultaneously. Each classification is presented as a dichotomous relation. Discourse may be "abstracting" or "particularising" while range may be "expanding" or "limiting". This will be related to topic independence and topic dependence respectively.

Abstracting strategies can be described as those for which practices are topic independent with language that is discursively regulated. The principled connectivity of the discourse is potentially available.

Particularising strategies can be regarded as topic dependent with a more limiting range. Articulations are not related by discursive principles. The absence of discursive links in language renders this strategy topic dependent. By considering the distributing strategy and their topic dependence or topic independence, further insights can be gained into the repertoire used.

Strategies are distributed during the production of the text. The distinction between proceduralising and principling, instances of abstracting and particularising respectively, is now quoted.

> The inverse of proceduralizing is principling. Here, the use of definitions and taxonomic classifications etc., facilitate the expression of the regulating principles of a DS+ practice, such as school mathematics. [...] Where exemplars are used, their abstractive properties will be made explicitly available. In this way, the metaphorical relationships between exemplars are reduced metonymically, so that the context dependency of the message is reduced. [...] I should note, here, that the move from principles and definitions to exemplars as metaphors for the principles and definitions is a particularizing of message .
>
> (Ibid.: 146-149)

The use of linguistic terms like metaphor and metonymy significantly influence the way in which distributing strategies are described. The reference to the reduction in metonymy in the relation between exemplars implies that the internal structure of the metaphor is absent. I will describe student textual production as the construction of metonymic chains between the privileged selection of the repertoire and the problem posed. An awareness of metaphor suggests that students are able to retrace their steps and show how the various components of the repertoire "hang together".

The relations between the distributing strategies have been organised within a two dimensional space (Dowling, 1998: 147). The first dimension is scaled according to the expanded or limited range of the message. The second dimension is scaled according to the abstracting or particularising aspects of discourse. These are arranged to obtain a Cartesian product shown in Figure 3.2.

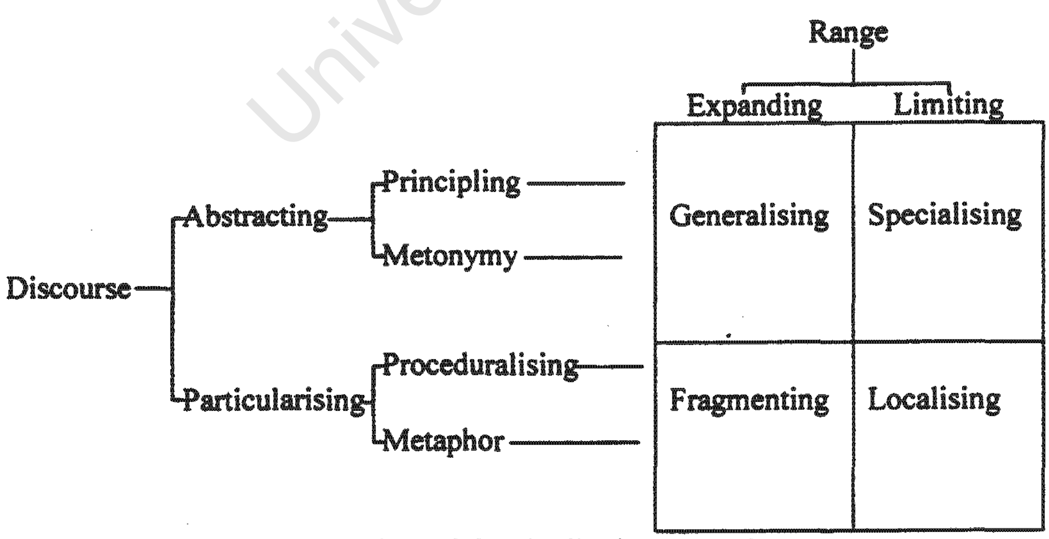

Figure 3.2: Distributing strategies

A *localising* strategy presents the textual production, as limiting and proceduralising with a dependence on metaphorical rather than metonymic links. The absence of these intermediary links compromises the mathematical, reducing the text to topic dependency. There is a subsequent lack of articulation between topics (Dowling's "topic") in mathematics. *Specialising* and *fragmenting* strategies differ in terms of their range as well as the level of abstraction. Fragmenting strategies have a more expanded range than localising and present the range of topics as segmental rather than integrated. Specialising strategies, like localising strategies, also have a limited message range. *Generalising* strategies are the construction of abstract message with respect to a specific topic. The language of the strategy is easily transformed into the language of the discourse so that the text can be integrated into the discourse.

A localised strategy is particularising which limits it to the topic in which it occurred. Consider, for example, a calculation in a student text that is based on the use of a general formula. Subsequent redescription can be realised in only one way. The following extract illustrates the strategy described.

M:	I said that why, why do they intersect? They do. Why do they intersect? Because they cross one another. I don't know [....] If they do, then where, why? I don't know that answer. Why do they intersect at that point?
F:	Why two and one and not seven and six? [...]
M:	It means that two points cross at that point. That's all I know about that point of intersection. It is where they cross.

Extract 3.3: Transcript 2, lines 176 - 191.

It would appear that the student has little recourse to a specialist language in describing the textual strategy. The opening line in the above extract demonstrates the dependence on metaphor. "The lines intersect because they cross one another". The absence of mathematics justification shows the lack of the discursive in the language used. Such a text is dependent on the algorithm employed, for example, where a DS^+ text is presented as DS⁻. The student is not able to discursively articulate why it is that lines intersect. The distributing strategy is thus localised and is further characterised by the single realisation pathway.

A generalising strategy is not specific to context and has an expanding range. It realises the esoteric domain as articulated and embraces many localised instances in terms of the redescribing principles. Multiple realisation pathways therefore exist based on specific definitions. It thus derives its power from working with a specific definition or rule rather than with exemplars and algorithms. Principling strategies can be realised as having either an expanding or a limiting range constituted by generalising and specialising respectively. They can all be described in terms of their context independence, strong grammar and articulation of the esoteric domain. All texts are thus presented as DS^+.

A *proceduralising* strategy (like fragmenting and localising) proceeds by "the substitution of algorithms or procedures for principles" (Dowling, 1998: 146). A procedure is an application of a particular principle. A text is proceduralising when these principles cannot be redescribed. The opposite of this is principling. Although both proceduralising and principling can have an algorithm in common, they differ with regard to their articulation of expression. Proceduralising subsequently rests on recognition of procedure without any other comparable description, transformation or access to discourse. This re-establishes the topic dependency of proceduralising strategies.

Particularising strategies particularise mathematics knowledge by reducing abstraction. This minimising of connections between mathematics relations, results in the formation and, indeed, dependence on algorithms. This implies an increased use of exemplars that represent insulated units of mathematics knowledge. Analogic recognition becomes important, since it determines the choice of algorithm.

Principling strategies can be described in the exact opposite way: it maximises relations and encourages abstraction. Exemplars are used differently since abstractions and principles are made explicit, and as a result, there is a reduced topic and algorithmic dependency.

Another useful distinction between principling and proceduralising, not indicated in Figure 3.2, lies in the number of principled ways the strategy can be realised. Proceduralising strategies are the result of single realisation pathways hence the focus on a very specific process or algorithm. Principling strategies are all-encompassing and this manifests itself in the multiple realisation pathways.

Abstracting and *particularising* refer to the most general level of the text. An abstracting strategy is based on (multiple) connections made at a higher discursive level. These connections are not necessarily reflected in the written text. They become explicit in student descriptions of the text. The comparable use of the principles of description characterise all articulations. Particularising strategies are not as flexible and are based on specific definitions.

Thus far I have described distributing strategies in nested levels with generalising, localising fragmenting and specialising being the most basic. Each one of these could be further described in terms of grammar, context and recontextualising principles

3.4 The resource level

This level of activity cannot be directly related to the empirical. Thus far the structural level with its associated practices and the message and distributing strategies of the textual level have all been derived from recognition and realisation principles of an empirical object. However, the empirical object can never be completely described. For example, the esoteric domain with its strong classification of content and expression must include the public domain via the operation of recontextualising principles (Ibid.). The way in which this occurs is derived from the message and distributing strategies of the empirical object and is the focus of textual analysis. This cannot be anticipated or predicted at the resource level.

In terms of the analytical tool developed thus far, the resource level shapes a particular text. A pool of resources must exist from which a repertoire can be selected for suitable appropriation. The way in which this occurs manifests itself in the text. Texts are thus constituted by privileged selections from the repertoire of the student. The privileged selections are adapted or redescribed from one production to another. The factors which influence the selection are significant since they underpin the text that is to be produced.

My intention is to use the methodology to foreground strategies in the text. The textual strategies will be summarised in terms of student redescriptions of mathematics knowledge. Pedagogic prescriptions and practices will be inferred from here.

This can be illustrated by a topic which informs texts so that there are legitimate utterances. There is a subsequent orientation of interpretation, recognition and response of those who produce texts. For example, "A(3;4)" effects a response in learners that allows them to assign special mathematical meaning in a very specific location on the Cartesian plane. The topic is realised in the ensuing response and the subsequent utterances. The regulating principles of the topic then operate in the production of text in its exact or in a hybrid form. This suggests that the regulating principles of the topic are appropriated as part of the resources employed suggesting that the regulating principles of the topic are embedded in the resource level.

Another way in which the privileged repertoire may be affected is via the regulation of teacher-pupil communication and the deployment of forms of assessment (Ensor, 1999). A particular emphasis in either the privileged repertoire or in the privileged regulation of communication is manifested in the text produced. Once again, it is the textual analysis conducted that will point to the repertoire of resources incorporated.

3.5 Relations between general methodology and empirical texts

Student production of mathematics texts centres round the production of highly articulate, highly developed and highly systematised discursive structures. This claim resonates with Foucalt's assertion that '[m]athematics has certainly served as a model for most scientific discourses in their efforts to attain formal rigor and demonstrativity' (1972: 88-89). Dowling (1998: 138) has re-theorised Foucalt's view of mathematics: he argues that a high level of discursive saturation in mathematics topics like arithmetic, trigonometry, calculus, etc. allows one to think of school mathematics related to this content as constituting a specialised esoteric domain.

The problematic here relates to the location of an analysis of student textual production, in terms of their practice, within the field of curriculum (re)production. Research studies that explore the recruitment of mathematics knowledge during textual reproduction focuses largely on readability and evaluation. One study in particular, by Ernest (1997), which produces a semiotic analysis of mathematics activity has shaped this case study. Ernest's regard for a mathematics tasks, presented by an authority figure (like a teacher), views the task as a series of transformations of mathematics signs. He concluded that there is a multi-leveled complexity involved in "semi-routine mathematics tasks" that address the rhetorical demands of written mathematics in a student's social (school) context.

The student textual productions to be analysed here focus on the construction of knowledge in school mathematics. Not only do they reflect teaching and learning in the classroom, they also index syllabus construction, curriculum implementation, teachers and students. They also privilege particular views of mathematics knowledge and they differentiate and construct mathematical knowledge, teachers and students in particular ways. This study will produce an analysis on how student texts recruit mathematics knowledge and how the texts (re)produce school mathematics and students' mathematics knowledge. The methodological issue is concerned with how they are to be analysed.

Social activity theory was introduced to describe how school mathematics activity can be (re)produced through student texts. Three analytic levels to activity were introduced: the structural, the textual and the resource levels. Using Dowling's 1998 model students' texts will be viewed as material instances of the activity of school mathematics. In the textual analysis mathematicians, teachers and students will be described as *voices* in the text and they are constructed by social practices. Their relations to each other and to the activity will be analysed in terms of their *distributing strategies*, which distribute message. Knowledge and practices that constitute message in student texts index mathematics knowledge as well as curriculum practices.

Following this methodology, the analysis will set out to establish the voice in the text by first examining texts and then looking at message and voice across the texts. The central feature of differentiation between texts lies in the access to discourse and the way in which descriptions of the text reflect the discourse principally.

However, this methodology cannot describe a written text, exhibiting a high discursive level and mathematically correct, that is not accompanied by a principled verbal description. Dowling's analytical tool is not fine grained enough. This concern is addressed in the following chapter where an attempt is made to describe empirical texts using ideal types.

Summary

The main findings of this chapter will now be summarised.

The general methodology provides a theoretical framework for my analysis. In the analysis of texts, textual strategies are realised as voice, distributing strategies and message. Textual strategies distribute message: this is the metonymic chain of possible metaphors. Interrogation of students' texts will describe the strategy in terms of the range (expanding or limited) and discourse (abstracting or particularising). Discursive saturation is indexed not only by the written text, but also by the verbal description of the texts so that the students' mathematics knowledge can be accessed. Textual strategies are achieved via the recruitment of the resource level. The privileged selection from the resource pool constitutes the repertoire deployed by the textual strategy.

Although all the texts are located in the esoteric domain, the written texts cannot always be upheld discursively. Different nuances in the esoteric domain are needed and this is achieved by the construction of ideal types. Dowling's distribution strategies are just not fine grained enough. It is the strategy of addressing illicit divides (via ideal types) in Dowling's distributing textual strategies that opens up the way for a more comprehensive notion of the empirical student text.

Chapter 4 Generating and Reading the Text

This chapter will describe the research design and the methodology chosen to collect material that would shed the most light on the research question. The general methodology of the previous chapter will be the lens through which empirical students' text, that is the material produced, will be considered. I will now clarify how the empirical text (the information gathered) is to be realised as data.

4.1 The production of data

Information gathered only becomes data once it has been read in terms of the theoretical, interpretive framework presented earlier. There needs to be a transformation from the empirical information to the data. This development according to Dowling (1998: 89) is constructive description based on equilibration. He cites Piagetian developmental theory to support the human phenomenon of autoregulation.

> Disequilibrium occurs constantly out of conflict arising between cognitive structures, out of personal experience, and out of new internalised actions. Equilibration, then, is the process whereby cognitive structures develop hierarchically in the resolution of such conflict
>
> Dowling, 1998: 128

The disequilibrium for Dowling, and indeed this study, stems from the difference between the empirical world and the constructive description of the theoretical approach. The difference between the two is resolved by the induction of the empirical text-as-work into a description of the empirical as text-as-text. The reverse deductive process from text-as-text to text-as-work can also occur.

Before engaging with the transformation of knowledge into data the empirical setting needs to be clearly defined.

4.1.1 The empirical setting

The initial interest stemmed from my heuristic observation that students at the end of Grade 12 were able to produce written work that was technically and mathematically correct that they could not explain or justify principally. This was a general observation but for the purpose of research I decided to focus on the topic analytic geometry. This was chosen because this work is presented as a new topic in grade 12. The title "analytic geometry" is new but the work encountered is not. It is concerned with plotting points, determining the length and positioning of lines, quadrilaterals and circles on the Cartesian plane all of which have been encountered in previous grades according to the Western Cape Education Department 1996/7 Interim Syllabus. It combines algebra and Euclidean geometry so that an algebraic equation could now be used to describe a circle, for example.

In designing the research project, I wanted to obtain a reliable way of ensuring that comments on the way in which textual production could be used as a specific instance to describe the more general practice of school mathematics. Sample selection had to facilitate this while also accommodating significant contributors to pedagogic practice: teachers, students and pedagogic material.

Brown and Dowling (1997: 29-30) suggest that the selection of the empirical setting for an educational researcher often results in the setting being the school at which the researcher is involved or has easy access to. This is termed opportunity sampling. As Brown and Dowling comment, care must be taken to prevent "a gloss of deliberation (of) an opportunity sample by referring to them as case studies" as a means of magnifying or blurring the issues of validity and the more general theoretical field. Sampling only can not support construction of an interesting idea or phenomenon into research. All research has to be linked to the more general theoretical field.

I will now describe the actual sample very briefly to illustrate how it can provide the information necessary to deal with my hypothesis that learners are able to produce mathematically correct texts without demonstrating principled mathematics knowledge.

The most competent grade 11 and grade 12 learners were selected on the basis of their results achieved in their final exams. To ensure that the texts produced in the study would mirror texts learners would normally produce, careful attention had to be given to the design of the questions. This was achieved by developing questions which were very similar to those usually encountered in analytic geometry during teaching/learning, in tests, class exercises, text books and examinations.

At a more general level, I hoped to address the research question, which is concerned with the way in which students recruited mathematics knowledge. This could be achieved by considering the difference between Grade 12 students who had completed the course in analytic geometry and Grade 11 students who had not. Difference is not the focus of the research. It rather opens the discussion on the general practice of school mathematics.

Support for this decision was found in the work of Leron and Hazzan (1997) who report that when students are required to answer a non-routine task, this tends to disequilibrate the student who then strives to relieve the pressure by producing a solution. They argue that this applies to novices and experts alike. Differentiation between a novice and an expert mathematics student lies in the conjectures they make. Where experts are able to produce many solutions and discard most of them on validation, novices take the first conjecture that fits their expectation without further examination. If this conjecture is correct then, according to Skemp (1976), students will not take kindly to suggestions that they should try to explore their conjecture even further. This view is used to make a further point: while teachers may make an effort for students to understand proofs of theorems, for example, students may only learn the techniques of the solutions of some type of problems, if this is what they believe would be sufficient during assessment.

Pimm (1995: 74-75) comments on pencil-and-paper computations that significantly shaped the design of the research. He claims that in school mathematics practices symbols start being used as if they were objects themselves. This is particularly noticeable in the language of algorithms since algorithms are frequently explained and taught in terms of operations to be carried out on the symbols. For example, "take it over to the other side and change the sign".

Other studies that have been instrumental in the way in initial choices made in the research design will now be referred to. This will be presented in a way that allows me to first justify the sampling choices made. Then I will consider research that influenced the design of the questionnaire.

My choice of sampling was affected by the research indexing the way in which students learn. Dubinsky *et al.* (1994) showed that success at abstract algebra depends on the student's ability to transform existing processes by another action so that a more sophisticated process of "encapsulation" can emerge. Existing knowledge and the processes associated with it are significant contributors to and need to be incorporated in the development of more advanced processes. Encapsulation appears to be a recommendation in mathematics practice. Reasons for its presence or absence can be offered by focusing on students who have completed the work in components in mathematics other than analytic geometry. Similar reasoning is found in the work of Sfard (1991) who argues that difficulties experienced by learners are a result of a deficiency in their empirical understanding of sets and functions. This impoverished understanding does not match the abstract way in which they are expected to understand it. This suggests that an impoverished resource pool disadvantages the development of new knowledge.

Tall (1991) also makes this point with regard to existing knowledge and formal mathematics definitions. Formal definitions encountered often do not match the simplistic understanding learners have. A definition, which invariably operates at a higher level of abstraction, is a potential source of conflict and manifests itself in the way in which mathematical meaning develops. Tall (ibid.) suggests that students who have difficulty in understanding concepts, hide their difficulties by resorting "to routine activities to obtain correct answers and gain approval". This approach quickly degenerates into activities that become increasingly instrumental which and are detrimental to future mathematical development.

A brief review of the literature surveyed will now be used to justify the way in which visual representation impacts on the design of the questionnaire.

Sfard (1991) establishes a relation between verbal and visual coding with her structural and visual components respectively. She suggests that a mismatch between visual and verbal often results in misunderstanding and misrecognition of mathematics concepts. A similar relation between verbal and visual was made by Zazkis *et al.* (1996) who found that verbal and visual analytic models were mutually independent and that they compliment each other.

Not all studies regard a link between the visual and written mathematics as desirable. Aspinwall *et al.* (1997) argues that the use of mathematical visualisation can lead to uncontrollable visual imagery that detracts from the mathematics by biasing thought processes and creating a hindrance during learning. The work of Wheately and Brown (1994) introduces another nuance to the relationship between the visual and the non-visual. They claim that images are constructed, represented and transformed, and ultimately maintained during an activity regardless of whether it is correct or not. This means that the visual image is always there.

While the studies above acknowledge a link between the visual and other general expressions, there are studies that indicate that this link does not always exist. Here I refer to the work of Shamma (1994) and Skemp (1989).

Shamma (1994) found that visual strategies, existing alongside other strategies, do not necessarily provide a significant change in student performance. This is because other strategies can exist independently of the visual. Alternative strategies do not establish links with the visual, minimising its importance and significance. Misconceptions that exist in the visual subsequently have no significant effect. This poses questions about strategies learned during instructional practices being well insulated. Skemp (1989: 39-41) suggests that inappropriate assimilation into schema (cognitive maps, knowledge structures and mental models) constructed by students may result in the absence of a unified knowledge structure.

The studies reported point to a possible commutative relationship between the visual and the non-visual, which has the potential to develop into modeling. An enriched understanding of mathematics may emerge. This only occurs if there is a principled transfer of mathematics between the visual and written text. In an absence of connections between the two, the visual may retard the development of understanding and concepts and this will be reflected in the texts that are produced by students. The final questionnaire thus had no diagrams or illustrations so that information gained could potentially be richer.

The interest in analytic geometry generated two main concerns which had to be addressed. Firstly, the topic "Analytic Geometry" serves as a case that illuminates how students solve problems because, even though students have been introduced to all the necessary mathematics prior to encountering this topic, they generally appear unable to synthesise solutions in a principled manner. Secondly, textual strategies deployed during the production of text will be unveiled. The first concern was directly connected to the empirical texts based on the analysis of learner engagement in a mathematical task. This developed insights as to how learners deal with signs, symbols, abstract diagrams and relations during their production of text. The second concern, also related to the empirical object, facilitates deductive working through of theory. This involves an analysis of the way in which redescriptions tend to realise principles of the activity in discursive form.

Texts thus become the lens through which specific instances could be related to the more general theoretical framework outlined earlier.

4.1.2 An overview of the study

This research takes the form of a quasi-longitudinal study. Instead of studying change in the way in which students approach problems as they proceed from grade 11 to grade 12, a comparison has been made between students in grade 11 and 12 by selecting two of the most competent students in each grade. The distinguishing feature between the two grades is that analytic geometry is taught only to grade 12s. It is understood that an exact match in responses between the current grade 11s and those who had been in grade 11 the previous year is not possible. This does not matter since the pedagogic discourse can be regarded as relatively stable.

It must be noted that grade 12s were not subjected to any "special treatment" when being taught. Mathematics instruction proceeded as it normally would at the school. This confirms the stability of pedagogic discourse.

The teaching methodology in analytic geometry can be regarded as the dependent variable[6], which will be assessed in terms of the presence or absence of it being formally taught. The correlation between the two will be linked to the theoretical constructs.

A questionnaire was designed based on typical questions that appear in the prescribed textbook used in a local WCED school and in the external examination at the end of secondary school (See Appendix 1). In a very broad sense, the questions were designed to reveal whether learners understand the strategy selected to solve the problem by looking at how they solved problems. This was then used in a pilot study which was video recorded.

[6] Rose (1982) uses the term variable to refer to the empirical world only. However, variable and indicator are used interchangeably as a means of relating the empirical level to the theoretical level of concepts. The same applies to Brown and Dowling (1998: 41)

Initially the questionnaire was designed with the problem stated in words as well as a visual illustration thereof, so that an indication of the student's interpretation of the problem could be obtained (See Appendix 5). The student used was the top mathematics pupil at a different school. The text produced (See Appendix 6) seemed to indicate that richer data could be obtained if no sketch was given so that their visual perception of the mathematics problem could be explored. The text also pointed to the need for the researcher to have alternative options if the student was unable to immediately answer the question and would help them to explore their own mathematics knowledge.

The interviewer's options (See Appendix 2) were based on the mathematics questions given and would attempt to lead students through different mathematics propositions so that the structure of their mathematics knowledge could be indexed. The decision to exclude the diagram is not a technical issue. It rather relates back to the theoretical framework, which hopes to explore the development of mathematics knowledge in students (assessed as mathematically strong) using language and access to mathematics discourse. The work of Luria (1976)[7] influenced my decision.

The final questionnaire required that the student explain, at the end of each question, what he/she produced during a semi-structured interview[8]. The interview was transcribed so that the text (written and verbal) produced could be considered as source of data. The pilot study thus informed the research on two levels: how to restructure the questionnaire and how to structure the interview.

[7] Luria's work explores the cultural and social foundations of cognitive practices in the Soviet Union in the 1930s. He looked at differentiation in people's literacy, their cultural relations and their thinking. This study does not focus on mathematics literacy since all students must be literate to even be considered as candidates in this case study. It is the extent of mathematics knowledge that will be explored, in a manner similar to the way in which Luria explores productive relations of cognitive practice, for example.

[8] Brown and Dowling (1997: 52) regard all interviews as being structured. They argue that no interviewer comes to the interviewing process without some kind of agenda, about some plan of the kinds of questions

4.1.3 The context of the study

This study took place in a Western Cape high school where the researcher had been engaged as a mathematics teacher for the preceding 12 years. The school is situated in a middle-class residential area, and most of the learners come from "coloured" townships in the nearby and outlying vicinity. Some of these areas are deprived with known socio-economic problems.

The school accommodates approximately 250 grade 11 and 12 mathematics learners, of which 28 are grade 11 and 20 are grade 12 higher grade mathematics students[9]. Students are taught separately from the standard grade group and have been streamed according to their performance. The school is a dual medium school: mathematics is taught in both English and Afrikaans. Lack of insufficient teachers meant that the mathematics higher grade class was a dual medium class, with the lessons taught predominantly in English. The Afrikaans first language pupils represented 10% of the grade 12 group and 35% of the grade 11 group.

It was found that the number of higher grade students decreases when students move to grade 12. The higher grade mathematics course, not surprisingly, has produced a 100% pass rate at the end of grade 12 consistently for the past 5 years. It is usual that students who battle with the course in grade 11 are allowed to change to standard grade in grade 12. This kind of screening in terms of academic performance means that all grade 12 mathematics learners can be regarded as very competent and capable of succeeding in the course. This conforms to the overwhelming perception of higher grade pupils being good mathematics students, amongst students and teachers alike. There generally appears to be strong correlation between top students in grade 11 and grade 12 in terms of their performance as indicated by the various forms of assessment.

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they want answered. The form of the questioning is what distinguishes the semi-structured from the structured interview.

Students participating in the study have to be positioned within this mathematics background. Top performing and proficient students were selected in the hope that they would have sufficient mathematics knowledge to do justice to the questionnaire by producing information at an acceptable mathematics level.

Gender difference was not a focus of my research. I thus chose a sample that was balanced with respect to the sexes. The top girl and boy from grade 11 and 12 were chosen. In this instance the top boy and girl in each grade came from the top three learners in each grade, and their performance was relatively similar. Not one of them outshone the other to any large degree minimising difference between the top boy and the top girl for each grade with respect to performance in examinations. The Grade 12 student was a girl while the top grade 11 student was a boy. Furthermore, the size of the sample does not allow for any significant comments on gender differences within mathematics.

4.1.4 The research project

A substantive part of the research study was based on the textual analysis of student texts, both written and verbal. The transcripts of the unstructured interview coupled with the "answers" that the questionnaire produced, provided information that could be transformed into data.

There were advantages and disadvantages drawing research subjects from the school where the researcher taught since all the learners participating in the study had been taught by her at some stage of their high school career. The grade 11s had been taught Physical Science by the researcher for two years while the grade 12s had been taught mathematics by the researcher for the last two years of their schooling. Students were therefore familiar with communicative practices employed by the teacher. This was magnified in the case of grade 12 learners who had a fair idea of what was privileged during transmission.

[9] **At a senior level school mathematics is streamed into higher grade and standard grade. They differ**

One could argue that an existing relationship between the teacher and the learner could impact negatively on the research. However, it is precisely this familiarity that needed to be exploited. Students will thus proceed to solve the problems in the usual way, the way that they are accustomed to. The disadvantage is the "authority relations" (Brown and Dowling, 1998: 74) that come into play. In teacher-student relations, the teacher is generally positioned as the authority. This can have a negative impact on the interview because learners position themselves according to the perceived views of the teacher. Ensor (1997: 54-55) makes a similar point about interviewing which she regards as a context in which subjects position themselves in relation to the researcher. In doing this they recruit language which is not used in normal conversation to say things they will not normally say. Learners thus position themselves on two levels: what they must/ought to say and how they can/ought to say this.

It is certainly possible that the interview may be compromised in this manner, but it is not likely given the circumstances under which the interview takes place. The interview is based on the text produced in response to the problems. The interview is thus not completely open-ended. Hammersley (1986: 191) suggest that interviewers need to be aware of the way in which learners may position themselves according to the interviewer and make a conscious attempt to ensure that the interview is not affected.

December was the most feasible time to conduct the research in terms of guaranteed free time for the researcher as well as the students. By then Grade 11 and Grade 12 students had completed their final examinations. This was useful in the sense that learners were relatively well prepared in terms of their mathematics knowledge.

The top girl and the top boy chosen from each grade because the results obtained in the midyear exam and the full scale, "mock-matric" September exam for the Grade 12s. For the grade 11s the choice was determined by the June exam and the performance report for September. Only the scores obtained were looked at and no consideration was given to

significantly with regard to content, style of questions as well as assessment.

factors, which may have affected the performance of other students. Learners who had the highest scores were treated as the top students. The grade 11 students chosen were Daphne and George. The grade 12 students chosen were Mandy and Kurt. (Their names have been changed here in order to preserve their anonymity. They are referred to as D, G, M, and K in the transcripts.)

The students asked to participate in the study did so willingly in their free time. The interviews were conducted at the University of Cape Town because of easier access to video recording equipment. During the interview, the camera was trained on the actual text being produced rather that on the student. The result was that a visual as well as an audio recording of the text being produced was made. In addition to this the interviewer, as an external observer to the production of the written text, made field notes on the way in which the text was being produced. The venue provided a setting which was conducive to privacy.

Students answered the questionnaire and were interviewed individually with no discussion between them to ensure that the interview was approached without any knowledge of the actual questionnaire items and interview questions.

There was no time limit set as to how long learners should take to produce their text. This meant that probing could continue until learners were no longer able to produce any different articulations.

4.2 Ideal Types

The theoretical ideas of discourse and Social Activity Theory are not completely suited to the empirical information. I will briefly recap for the sake of continuity.

In the previous chapter dealing with the theoretical interpretive framework, I pointed to the textual strategies in Dowling's social activity theory (1998) used to describe pedagogic texts: specifically two textbooks in a series designed for low ability and high ability students. Although Dowling's social activity theory is presented as an analysis of school mathematics practices, he argues that it has a more general applicability in sociology. However, it is not able to describe highly discursive written student texts that can not be supported by a principled verbal description.

To overcome this, ideal types will be introduced to act as an interface between the text-as-work and the text-as-text. These ideal types are conceptually coherent and complement the general methodology previously presented.

The method of ideal types was introduced by Max Weber (1964) who abstracted ideal types from non-quantitative research and used them in a "non-quantitative fashion to support his major comparative analyses of the meanings and values of religious or political movements" (Riley, 1963: 336). It provides a way of generating contrasting types which originate from the data and which are conceptually coherent. Weber's ideal type is not an exact description of a concrete case. It might never exist in reality. At the theoretical level, it is a carefully defined set of related categories. At the research level, it is to be used as a kind of standard or model against which actual cases can be compared, and found to be similar or different. Thus, adopting a particular interpretive scheme facilitates a common understanding of whatever facts were compared to it.

Bernstein, has reservations about ideal types:

> Classically the ideal type is constructed by assembling in a model a number of features abstracted from a phenomenon in such a way as to provide a means of identifying the presence or absence of the phenomenon, and a means of identifying the 'workings' of the phenomenon from an analysis of the assembly of its features. Ideal types constructed in this way cannot generate other than themselves. They are not constructed by a principle which generates a set of relations of which any one form may be only one of the forms the principle may regulate.
>
> (Bernstein, 1996: 126-7)

The range of possibilities defined by ideal types is limited. Bernstein is suggesting that the ideal types abstracted from a phenomenon are internal to that phenomenon and are themselves constrained by, and in turn regulate, the phenomenon. A collective assembly of these features, no matter how comprehensive, is restricted by possible characterising features of the phenomenon. The construction of the phenomenon observed it not possible because of the absence of principles and rules that underpin the phenomena.

Dowling has a different view of ideal types, and it is to this view that this study subscribes.

> Ideal types are categories that originate from observation, but which have been made conceptually coherent. In referring back to the empirical, it is important to remember that concrete instances are likely to combine elements of more than one ideal type, so that the latter are unlikely to be found in their pure form. [...] the method of ideal types is meant to be a guide to empirical research, not a substitute for it. Crucially, the generation of ideal types may be considered as a step in the dialogue between a developing theory and an empirical observation.
>
> Dowling 1998: 20

It is important to bear in mind that an "ideal-type" model is constructed as a means of accessing the empirical in a way that will allow me to discuss text before further describing it in terms of the theoretical framework. However, one should be wary of the dangers posed by the interaction of ideal-types. Schütz (1967) termed this "typification" and it involves the possible creation of stereotyped categories which may sacrifice the interpretive processes through the inability to provide a perspective on other likely modes of conduct. Nevertheless a model of this sort provides a way of signaling and interpreting, drawing from stocks that individuals have acquired and recontextualised.

In order to contrast the real situation with the ideal type so that possible biases may be analysed, and so that the problems of generalisation of this sample to the broader population can be assessed realistically, the ideal type must be constructed and defined unambiguously (Ritzer, 1992).

4.2.1 The Construction of Ideal- types

The construction of a group of ideal types that is conceptually coherent will facilitate inductive and deductive movement between the empirical analysis and the theoretical framework. This will be achieved by considering the empirical text in terms of the extent to which underlying principles are made available in discursive form together with the range of the activity.

When reading through the transcripts, I looked for markers reflecting the possible strategies used, to arrive at an exhaustive collection of ideal types. Three distinct strategies emerged. The simplest one occurred when information in the question was transformed into an accurate representation so that a solution based on measuring could be obtained. This ideal type was termed *empirical*. A more advanced strategy was based primarily on the use of algorithms and invariably centred on substitution into a formula. This ideal type was termed *procedural*. The most complex solution strategy that emerged was one in which utterances indexed the underlying principles of the text produced. This ideal type was termed *grammatical*.

I will now define each of the core ideal types (*empirical* (E), *procedural* (P) and *grammatical* (G)) as succinctly as possible. *Empirical* will be largely dependent on measuring and has no connection to any general theorems in mathematics. This is a single form of realisation. *Procedural* is largely dependent on algorithms and exemplars. This form of realisation describes the "how" of a procedure but not why it exists, centering realisation around the details of the procedure, rendering them topic specific. *Grammatical* indicates access to the discursive principles of the text: the student can explicitly state the relation to theory and other symbolic connections. This type is further characterised by multiple forms of expression reflecting the same underlying principle.

Ideal types can be hierarchised with empirical occupying the lowest level and grammatical occupying the highest level. This can be done based on Piaget's theory of cognitive development. Before arranging the core ideal types in a systematic manner, I will justify the hierarchical arrangement by briefly outlining Piaget's approach to cognition and the paradoxical role of language in his theory.

Piaget's learning theory describes the development of cognition in stages which proceed from a primitive to more advanced levels. These stages have been recontextualised in the field of mathematics education as a theory of learning which asserts that cognitive development proceeds from a primitive form employing concrete apparatus to a stage where students are able to operate at a high level of abstraction that does not require any apparatus. In terms of these levels students beginning secondary school are beginning to think in formal and more abstract terms. This manifests itself in classroom practice via decreased dependence on concrete apparatus. Students in a senior secondary mathematics classroom are expected to function at a higher level of abstraction than those at junior levels. Dowling (1996) uses this to explain how problem solving strategies are transformed from the junior secondary phase to the senior secondary phase where activities requiring physical apparatus are considerably diminished due to the expected increase in cognitive and abstraction skills.

The *empirical*, which incorporates accurate measurement, requires the use of calibrated apparatus like a ruler, protractor, etc. and can be likened to Piaget's first level of cognitive development. The *grammatical* can be likened to Piaget's formal operational level. This supports their hierarchical positioning in which the grammatical is positioned at a level higher than the empirical. The paradoxical role of language in Piaget's theory is significant at this point. Movement beyond sensori-motor activity (associated with the first level of cognitive development) without the means of representing the concrete actions mentally and without the existence of the semiotic function is not possible. If the representation of actions is to be sustained through the use of signs, symbols and language, development of the semiotic function is necessary.

The three core ideal types can be grouped in a systematic manner and are concisely described in the Figure 4.1.

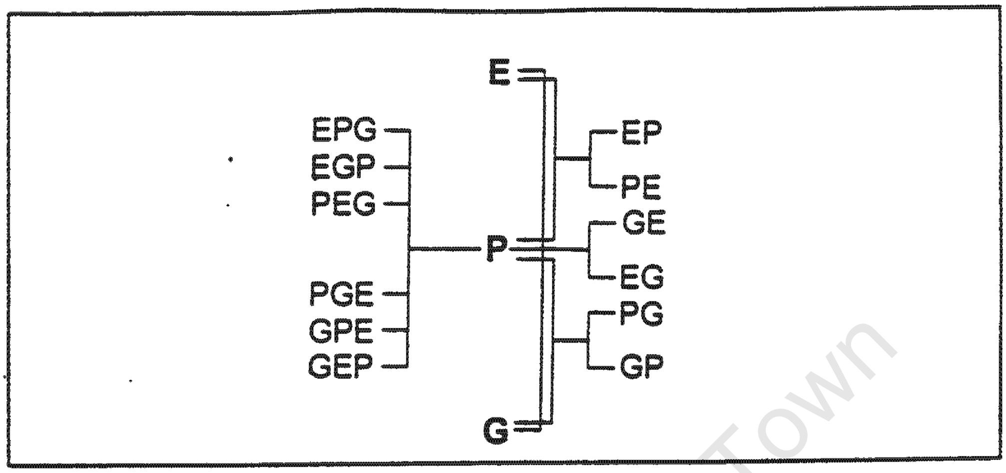


Figure 4.1 : Ideal strategy types

Different combinations of the primary ideal types yield the following possibilities: E, P, G, EP, EG, PG, PE, GP, GE, EPG, GPE, PEG, PGE, GEP, EGP. There are 15 ideal types altogether and they will be used to illustrate differences within the distributing strategies.

The code employed in the subtypes is sequentially organised. *Empirical-procedural* (EP) is not the same as *procedural-empirical* (PE). The EP subtype first employs the *empirical*, which is then followed by the *procedural*. The *procedural* is based on, or incorporates the *empirical*. In this way the *empirical* legitimates the *procedural*. This can also be illustrated in the *empirical-grammatical* (EG) sub-type. Here empirical measurement legitimates the grammatical. This suggests that a student understands the underlying principles and grammar of his/her production, yet begins with an accurate solution.

Consider EPG. Utterances in these student descriptions index mastery of all three types. In this instance strategy develops from *empirical* to the more abstracting *grammatical*. Similarly, mastery of E, P and G is implied in GPE, PEG, PGE, GEP and EGP. They merely differ with respect to the sequencing of types.

A key distinction between the ideal types will be its discursive saturation. It can be argued that both *empirical* and *grammatical* are ideal types that can be described in a principled fashion despite the differences in the mode of operation. This can be contrasted to the *procedural* in which an appeal is made to memory for the selection of the correct algorithm. The limited extent of the discursive in language in texts described as *procedural* type separates it from the other two core ideal types. This suggests that *procedural* should occupy the lowest hierarchical position with *empirical* and *grammatical* above it.

I will now establish reasons for equivalence between *empirical* and *grammatical* not being feasible in terms of the interpretive framework of the general methodology. They differ with the extent to which the principles of their productions can be realised in discursive form: their discursive saturation. This differentiation occurs at the resource, textual and structural level of mathematics activity. At the resource level, the extent to which student redescriptions of the privileged repertoire can be expressed in discursive form is manifested during the production of texts: that is the extent to which the regulating principles are made explicit. Dowling (1998: 150) suggests that there is no "a priori limitation on what can count as a resource". The focus thus lies on the extent discursive saturation of ideal types. The discursive form manifests itself in the *empirical* to such a small degree, rendering it topic dependent producing a low discursive saturation practice (DS^-). The amount of the discursive in the *grammatical* renders it topic independent, producing a high discursive saturation practice.

The distinction between topic dependence/independence is an important distinction in ideal types. Topic dependence implies suitability for only that purpose. P, characterised by algorithmic use, is topic dependent and has a high degree of fit with its intended purpose. It can not make explicit principles regulating the intended purpose. G, which is topic independent, displays a high degree of fit with the algorithm and the principles regulating the purpose.

E, P and G can thus be arranged hierarchically with E displaying the most primitive level and G the most advanced level of the discursive, realisable in language.

4.2.2 Relation between ideal types and the analytic tool

It is significant that grammar is only used as a descriptor of ideal types and is not used in Dowling's textual distributing strategies. Dowling (1998) argues that all mathematics pedagogic texts have strong grammar due to their strong classification of expression and content. My focus on student text acknowledges the grammar in the ideal type but does so in a way that is compatible with Dowling's distributing strategies.

The types in which *grammatical* is absent (E, P, EP, PE) can be described as fragmenting or localising. Fragmenting is an example of a proceduralising strategy. According to Dowling "fragmenting realises the esoteric domain as segmental rather than articulated" (Ibid.: 149). The use of the term segmental suggests that disconnected fragments within an extended range of topics exist with a lack of discursive articulation connecting fragments. The discursive is thus particularised rather than abstracted. The subsequent context dependency does not readily allow relational connections between fragments. It also suggests that segments can be transferred across contexts only on the basis of analogic recognition and not on the basis of discursive redescribing principles.

Dowling suggests that specialising can be seen as "a career pacing" strategy (Ibid.: 149). Consider specialist branches of school mathematics, like *functional mathematics* and *commercial mathematics*, which have a limited range of topics. Activities in the specialist branches explain only certain specialisms. The specialisms accompanying these branches of mathematics could possibly manifest itself in school mathematics as well. This will be as a result of an individual students interest that is developed outside classroom practice rather than being part of "ordinary" school mathematics. Description of the texts in this study would be confined to articulations available within the limited range of school mathematics. Any ideal type that includes G will be valid.

Generalising strategies (Ibid.,) are described as "an articulation between topics within the more general discourse". Since recontextualising principles will make the general discourse available, the texts will be translatable and topic independent. Once again, all ideal types that include G will be valid.

The diagram representing the analytic tool can be further adapted to accommodate the ideal types below. The ideal types have been arranged so that proceduralising strategies can be represented by E, P, EP and PE. This implies that these 4 ideal types are possible for fragmenting as well as localising strategies. Similarly, all ideal types with G represent generalising and specialising strategies.

The ideal types allow an extra description of abstracting and particularising strategies significant to this study.

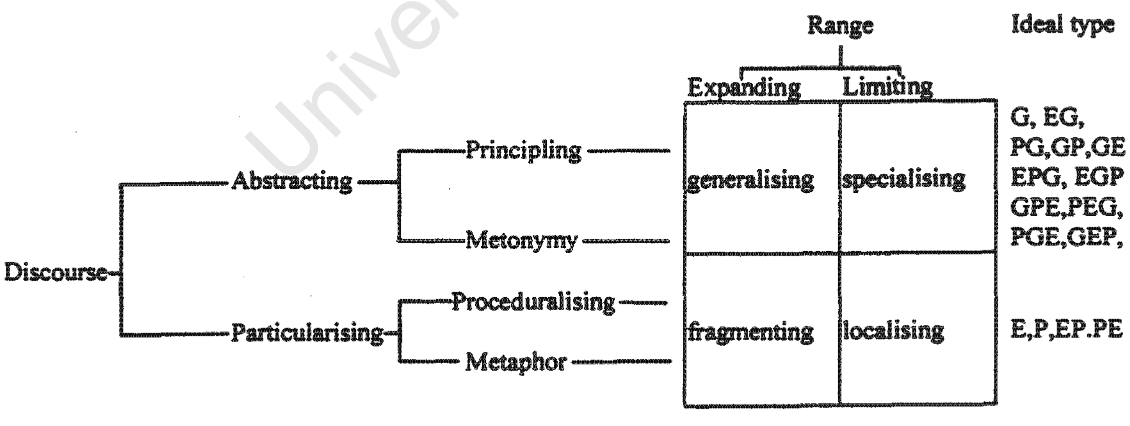


Fig 4.2: Textual Distributing Strategy and Ideal Type

4.2.3 Discussion of ideal types pertaining to this study

The *empirical* in this study manifests itself as the construction of a Cartesian axes system with accurate calibrations which may be marked or unmarked. Points are accurately positioned and information is accurately interpreted with regard to the Cartesian plane. No theoretical constructs like theorems are used. When asked to determine the length of the line between two points, students accurately locate the points, and then determine the solution by measuring the length of a line constructed between the points. The *empirical* also takes on the form of visual representation that is not accompanied by a written text. Its determination by the verbal text only, which uses metaphor, particularises the discursive.

The second ideal type *procedural* (P) focuses on the use of procedures and algorithms. Algorithms are regarded as tools that can produce a correct answer. Its existence is not affected by its origin. In this pure form, discursive justification for the procedure is markedly absent. It would seem as if the mathematics has been reduced to a choice between different procedures for different questions. This suggests that mathematics is seen as a finite, narrowly circumscribed, predictable formatted set of procedures. The challenge for students would be to recognise appropriate procedures from the repertoire of procedures available to them. The resource level thus informs textual strategies. Recognition of the procedure embedded in the algorithm is the pivotal point and as such deserves further consideration.

Different skills are required for the different ideal types. I will consider the core ideal types separately.

The *empirical*, E, requires measurement and construction. The level of the measurement skills can progress from basic line measurement to more advanced sketches. The constraint would be competence with measuring/construction tools (compass; ruler; protractor). For example, measuring the length of a line between two co-ordinates on the

Cartesian plane only requires the accurate use of a ruler. Clearly, access to mathematics discourse is not a prerequisite. This type is characterised by the faith that students appear to have in its visibility and concreteness. Authorisation thus lies in the self, that is, with the student.

The *procedural*, P, requires a detailed list of possibilities for different problems. The procedure characterising P exists in isolation since it is not supported by knowing why it takes the form that it does. Its outstanding feature is the absence of any discursive abstraction and mathematics knowledge takes the form of extensive list of procedures with little or no links between procedures. The use of algorithms and exemplars is marked to such an extent that it becomes prioritised and needing to know why becomes secondary. The order and number of steps to be executed is a function of memory. Regarding communication and language skills: appropriate vocabulary and words appear to exist in this type. Correct grammatical structure in the written form may even be displayed. Without a principled knowledge of mathematics and without access to the discursive language, solving a problem becomes an exercise in memorisation and analogic recognition. Students then have to commit lists of algorithms and significant words to memory. Any acquisitions of words and necessary procedures or algorithms are necessarily cumulative, with no generalising mathematics rules. Instead, words and algorithms are associated with certain types of problems, so that students learn to become proficient and even achieve success. Using a linguistic perspective, the appearance of known signs (words, word clusters, diagrams, etc.) may allow meaningful action in an (un)known problem. An impoverished mathematics knowledge generally associated with signs, may or may not evoke prior associations, which may or may not be useful. Memorising which problems a method works for and being able to navigate the list of possibilities so that specific methods are known for specific problems thus determine success. This works easily when the number of procedures committed to memory is small and becomes increasingly difficult as the number increases. The actual solution to a specific problem will be determined by recognition of all possibilities from a range of non-related, segmental possibilities rather an understanding of any rules or principles.

Recognition in the *procedural* depends on the topic in which it occurs. Familiarity with the topic will significantly influence recognition. The authority of any selection made to solve a problem does not lie in the recognition of the algorithm. It rather resides in the deferred authority of an external community of mathematicians who have access to the generalising rules of the algorithm.

Within the *procedural*, two possibilities exist. Firstly, there may be complete recognition of the procedure given the information in the question so that it can stand independently of the production of a sketch. It may be accompanied by the use of a formula. Here graphical representation does not facilitate the solution procedure at all. The second possibility combines the procedure with the graphical representation in a mutually dependent relationship. In this instance graphical representation provides a concise way of representing all the information given in the problem so that learners can engage with the key information.

In the *grammatical* (G) principled redescriptions are made in a coherent and systematic fashion. Construction of an algorithm based on a few select rules is possible and mathematics knowledge of how the grammar works. The explicit way in which these redescriptions operate makes principled paraphrasing possible. Multiple solutions are thus possible provided they conform to a specific rule. The rules of mathematics inflexions are understood so that relations between writing and verbal discussion of the writing are recognisable by the shared principle. Paraphrasings may be accompanied by visual representation. The *grammatical* may or may not emphasize the computational stage in the written text: that is the algorithm. Successful redescription of the algorithm manifests itself in all student discussions of their own productions. Authorisation of G lies in the rules of mathematics discourse and its grammar.

It is important that the general distinction between the *grammatical* and the *procedural* types remain clear with regard to recontextualising principles and the implicit principles which underpin the written text. The way in which language is used is the differentiating criterion. The use of phrases and terminology appearing in a seemingly appropriate way cannot always be supported by strong discursive argument. This is illustrated, in the extract from Transcript 1.

K:	When I equated. I think when I equated the two lines I could. **I got an equation that represents a point. But it is in the form of a circle.**
F:	This equation represents a point in the form of a circle. Right?
K:	Yah.
F:	What does that mean?
K:	That means that the point can be anywhere on that circle. ((K refers to a sketch drawn on the side of the page in front of him)) And that's how I (couldn't) get the co-ordinate from there. Then I equated. I said that AC would also be equal to BD. But then I got another circle. Which is, the same thing applies. And then, I do not know how to get the point out of that.

Extract 4.1: Transcript 1, lines 215-227, my emphasis

According to the transcript, the format of the equation given suggests that it is an equation of the circle. This particular concept is frequently encountered in a unit in analytical geometry which deals with the *locus of a point under given conditions* (Laridon et.al., 1985: 145). *Locus* is defined as a variable point. It seems as if K is suggesting that the equation generated represents the locus for the unknown point. However, the unknown vertex of a parallelogram cannot be variable if the geometric properties of the parallelogram are to be maintained. Only one solution for the fourth vertex of the parallelogram is possible. If the procedure can not be redescribed discursively then the classifying type can only be regarded as *procedural*. Low discursive saturation exists. Clearly K's discussion of his production provides evidence of a disjuncture between the written production and the accompanying verbal description. A (seemingly) correct production of text is not sufficient reason for a text to be classified as grammatical. It is the access to underlying principles and the extent to which the principles can be realised in discursive form which are the defining criteria.

Based on the researcher's experience, a minimal occurrence of EG and EPG in the entire sample seems likely. This is because these ideal types are not a focused form of production within the context of school mathematics. It has been established that this would be acceptable based on Piaget's expectation that knowledge develops from an initial use of concrete tools, in this case measuring. This suggests that the particular is used as a starting point for the development of a procedure and ultimately a generalisation which needs to be tested.

In the type EP, the empirical prefaces the procedural. One of the Grade 11 students participating in the study was unable to recognise the transformation of the Theorem of Pythagoras in the "distance formula" to a revised application in the next question. The theorem of Pythagoras is only recognisable in its standard orientation that occurs in formal textual representations like the textbooks that the students used. This illustrates the topic dependency of empirical and procedural ideal types.

Skemp's (1976) reference to a high level and a more basic level has already been referred to with his respective relational and instrumental understanding. The E, P and G categories can clearly enhance the distinction between the two. The empirical resonates with instrumental while the grammatical resonates with relational.

The theoretical framework, which includes Dowling's textual distributing strategies, will enable a structured analysis that can be enhanced by the appropriate ideal types.

The characteristic differences between E, P and G lies is summarised in table 4.1

Descriptive criteria	*Empirical* (E)	*Procedural* (P)	*Grammatical* (G)
1. Skill required	measurement	navigation of selection procedures	construction of algorithm
2. Hierarchical positioning	lowest	middle	highest
3. Principled (re)descriptions	absent	absent	present
4. Discursive saturation	low	low	high
5. Location of Authority	self	third party	mathematics discourse

Table 4.1: Descriptive Criteria of Ideal Types

Summary

So far I have shown that the development of ideal types is central to any discussion that can be generated with regard to the kinds of textual strategies described in the general methodology. The three ideal types that emerged could be associated with each other to create subtypes. The range of possibilities are E, P, G, EP, PE, EG, GE, PG, GP, EPG, GPE, PEG, GEP, PGE, EGP.

The texts produced are to be regarded as instances of pedagogic practice and the range of the textual distributing strategies and the extent to which mathematics discourse is realised in language will be examined. The pedagogic ideal types which were generated act as an interface with the empirical text-as-work. The text-as-text is produced as a reading of the text-as-work. This was done so that the interpretive framework of the general methodology can be applied.

The following chapter applies the ideal types and textual strategies so that the more general interpretive framework can be accessed.

CHAPTER 5 Analysis of student texts

In the analysis to be presented I will illustrate the textual level of social activity theory and the way it is informed by the structural and resource levels. The structure of an activity is (re)produced through its texts in the (re)production and distribution of message. The analysis will be largely semiotic and will be mainly concerned with denotations and connotations of the text.

Textual analysis is concerned with a comparison in the texts produced by grade 11 and grade 12 mathematics students. Each text produced was analysed to produce an element of the percentages of the structural level.

5.1 The structural level

I need to announce that all student textual productions belong to the esoteric domain, one of four domains of practice that Dowling (1993; 1998) presents. The domains are used to describe the practice evident in pedagogic texts which are presented in the written form only. Briefly, the esoteric domain in mathematics is one which presents texts with the highest discursive element in relation to the other domains of practice.

Evidence of the discursive element will now be given. The text produced was a response to typical school questions (See Appendix 1). Denotations and connotations in the questionnaire evoked signifiers and signifieds that presented themselves in the student text. It was apparent that the question determined the text to be produced as evidenced in the extract from Transcript 2, lines 123 - 124.

M:	I started over here **but then I saw quadrant so I knew I had to draw the umm, Cartesian plane.** So I drew that and so I plotted the points.

Extract 5.1.1: Transcript 2, lines 123 - 124. (My emphasis)

The topic with which the questions can be associated is apparent to students: plotting points on the Cartesian plane; Pythagoras' theorem, analytic geometry. However, in the absence of knowledge of the topic, engagement in the task is reluctant as illustrated in Extract 5.1.2.

G:	We didn't do these equations yet.
F:	OK. If you draw a circle, what will be the relationship between C and P?
G:	CP. Umm. That equation will be equal CP something.
F:	What is CP. What can we call the line CP? What is C? Is C the centre of the circle?
G:	C is the centre of the circle, yes.
F:	And P is a point.
G:	On the circumference.
F:	On the circumference. What kind of line will we get from the centre of the circle to the circumference?
G:	The radius.
F:	OK. Can you determine the length of that radius?
G:	It will be five.
F:	Five? How did you get that?
G:	The y's is three and two.
F:	Did you add it? Three plus two?
G:	((No answer))

Extract 5.1.2: Transcript 4, lines 488 - 563.

It is apparent that students are reluctant to engage with work they have not yet been taught. Whether or not they are able to do it is not a concern. The point being made is about the response that is elicited in accordance with any associations made with these topics. Textual production in this study was based on algebra and/or Cartesian plane geometry. The topics in the questionnaire thus evoked a specific range of responses. The extent to which utterances are shaped by the topic will be discussed in terms of the format of the solution offered, presentation of information and orientation of the problem.

5.1.1 Format of the text offered

Acceptance into a community of mathematicians determines the legitimacy of the text and subsequently the format that the text will take. Intuitive solutions to mathematics problems often need to be developed to the point where they conform to other mathematics texts that are regarded as legitimate. Intuitive solutions are reluctantly offered when no other alternative existed. Many possible reasons for this exist, but central to all of them is the issue of readership. The student, or author of the text, needs some measure of confidence that his/her textual production is legitimate and will meet the approval of the mathematics reader. This requires that the message of the text have nuances of authority, which will be evidenced by the voice of the text. The discursive rules, or a viable alternative, thus need to be available to the author/student if they wish to speak with any appreciative authority. The absence of these rules constitutes a "spurious or displaced authorial voice" (Dowling 1998: 143) and dismisses the generalisability of the text.

Furthermore, students are apprenticed in school mathematics practices that prioritise a formal, axiomatic text. Intuitive understanding, although encouraged during the junior phases, has a fragile position if it cannot be extended to a higher level of abstraction in the more senior phases. This emphasis is reflected in textual validation. Assessment, formal as well as informal, legitimates appropriate texts thereby shaping the text. The existence of national, external assessment at the end of Grade 12 magnifies the view of what counts as legitimate texts.

5.1.2 Presentation of information and the production of texts

The way in which the question is posed, triggers a particular orientation, which is reflected in the student text. Using the transcripts as a guide, two categories in strategy emerged: measurement and calculation. Students who had not completed the senior phase tried to solve the question with empirical measurement. Their main resource was their knowledge of graphs. Since the nature of the questions often went beyond this, they were required to make links with Euclidean properties and theorems as well. Students who had completed the senior course depended largely on their knowledge of known procedures and formulae.

5.1.3 Orientation of the problem

The best method for solving the problem in the first question, for example, increased the possibility of a similar approach in questions that follow. The first question employs the theorem of Pythagoras. The possibility that subsequent questions will follow suit is magnified within the minds of students. This conclusion is based on the researcher's knowledge of the way in which class exercises are set out in text books - for example, an example with a few related problems. A similar pattern is followed in examinations, particularly the final, external, national examination in which questions are generally organised according to related sections of work.

All student textual productions thus present a specialised mathematics form. A detailed discussion on the resource level will be offered after the discussion on textual strategies below.

5.2 The textual level

In this section the message and distributing strategies described in the general methodology will be presented as emerging from the text. This dialogic process involves moving between the empirical text and the theoretical field.

A brief glance at the texts produced showed that one group proceeds by means of measurement or some related procedure while the other group uses generalisable formulae and related procedures. This translates into one group showing a dependence on the topic presented in the question while the other group shows topic independence through the use of calculations. These observations have been summarised in the network below.

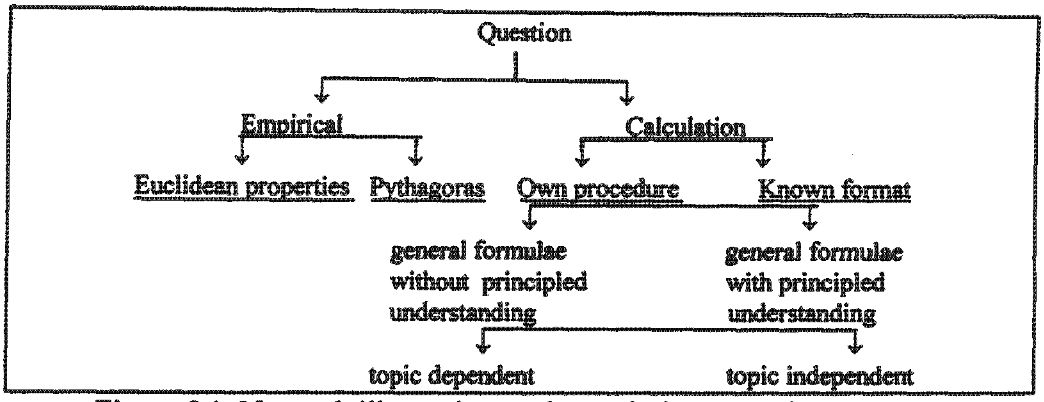


Figure 5.1: Network illustrating student solution procedures

This study proposes that students who have been taught analytic geometry are able to produce strategies dominated by fragmenting strategies, whilst those who have not been taught produce strategies that are more localised. The degree of truth in this proposition can only be fully described by semiotic analysis. However, the extensive elaboration that this requires is only possible with small quantity of information. Strategies have thus been operationalised to allow a comprehensive quantitative analysis and this underpins the content analysis that follows.

5.2.1 Content Analysis

The first part of the analysis is concerned with applying the map for distributing strategies and corresponding ideal type. All texts will be categorised as being *localising*, *fragmenting*, *generalising*, or *specialising*. The initial proposal centered on fragmenting and localising but Dowling's social activity theory provides generalising and specialising strategies that allow for scaled differences with respect to the range and the level of discourse.

Question Number	Distributing Strategies and corresponding Ideal Type			
	Grade 12		Grade 11	
	K	M	D	G
1	Generalising; PG	Fragmenting; P	Fragmenting; EP	Fragmenting; EP
2	Fragmenting; PG	Fragmenting; PG	Fragmenting; E	Fragmenting; EP
3	Generalising; PG	Fragmenting; P	Fragmenting; EP	Generalising; EPG
4	Fragmenting; P	Fragmenting; P	Localising ; E	Fragmenting; EP
5	Fragmenting; P	Fragmenting; P	Localising ; E	Localising ; E
6	Fragmenting; P	Localising ; P	Fragmenting; E	Localising ; E
7	Fragmenting; P	Fragmenting; P	Localising ; E	Fragmenting ; E

Table 5.1: Distributing Strategies and Ideal Types.

The corresponding frequency for the distributing strategies appear below grouped according to their grade.

Distributing Strategies			Ideal Types		
	Grade 12	Grade 11		Grade 12	Grade 11
Fragmenting	79 %	57 %	E	0 %	57 %
Generalising	14 %	7 %	EP	0 %	36 %
Localising	7 %	36 %	P	71 %	0 %
Specialising	0 %	0 %	PG	29 %	0 %
			EPG	0 %	7 %

Table 5.2: Percentage Distributing Strategies and Percentage Ideal Types

A discussion of the above summaries will follow.

The percentage strategies clearly reveal that fragmenting strategies are most frequent throughout the entire sample increasing from 57% to 79% from Grade 11 to Grade 12. This suggests that student exposure to teaching in analytic geometry facilitates the increases. An increase from 7% to 14% in generalising strategies as well as a decrease in localising strategies from 36% to 14% occurs from grade 11 to grade 12. The abstracting potential in the grade 12s thus exceeds that of the grade 11s. However, the particularising potential increases in the Grade 12s and significantly exceeds that of the grade 11s.

The portion of the sample that has the procedural (present in E, EP and PG) as a distributing strategy is not evenly distributed between the two grades. The information in table 5.2 shows that 57% less procedural based ideal types (P, EP, PG and EPG) occur in grade 11. This marks the procedural ideal type as a possible criterion for polarisation between the two groups. The high percentage of the procedural marks it out as a key type with regard to the form of the expressions of the esoteric domain. We might infer from this that procedural ideal types are highly visible in grade 12 and that this is realised by the gaze of the esoteric domain. This is important. Analytic geometry may be presented as a topic that concludes and includes all related segments in school mathematics (geometry, algebra and trigonometry) and can be described in terms of the production of new formulae and procedures. In other words, analytic geometry may be described as a higher level of abstraction, a level that develops from further mathematics knowledge based on existing concepts.[10] It is thus possible that being taught the topic analytic geometry should actually enhance mathematics knowledge by tying together any other mathematics knowledge that may have existed in a non-related, segmental way. Evidently this does not occur. In grade 12 ideal types pointing to the grammatical only represents $\frac{4}{14}$ instances, i.e. 29%.

Localising strategies are described by either E or P. Fragmenting strategies occur as E, P and EP. E significantly features in grade 11 and P in grade 12. The overlap of E and P in localising and fragmenting is not problematic. It rather shows the ease with which the ideal types acts as an interface between the empirical text-as-work and its induction into the empirical text-as-work via the theoretical field and methodology.

Fragmenting and localising strategies can both be described as particularising strategies. The range in fragmenting is more expanding although they both show a particularising of the discourse. Fragmenting strategies in grade 12 are all based on P while fragmenting strategies in grade 11 are all based on E. They merely differ with regard to problem solving. Grade 11's employ their mathematics knowledge to engage in construction and measurement while the grade 12s use it to engage in a selection of procedures.

[10] One could argue that Piagetian cognitive development demands this progression from primitive to more abstract and that this situation is not unique to mathematics pedagogic practices. The point rather is that pedagogic action does not ensure this development.

The difference between E in fragmenting and E in localising lies in the range of the text with fragmenting strategies having a more expanded range. This conforms to the theoretical map of distributing strategies described in the previous chapter. Associated with the particularising distributing strategy is the use of metaphor to particularise discourse. This is reflected in procedural strategies and also displays topic dependence which can only be realised in a singular way. In Eco's terms a closed text does not make the principles of realisation available. The discursive saturation by definition will be low with an impoverished interpretation of mathematics discourse.

This quantitative analysis has limitations in terms of validity and reliability since it is removed from the substance of the text. The resultant distance ignores subtleties and nuances in the text. Nevertheless it reveals evidence of the different priorities in different grades when recruiting mathematics knowledge. These distinctions are consistent with the claims that grade 12 students are able to produce written mathematics texts without having a principled understanding of the mathematics.

This quantitative analysis has also highlighted assessment-related criteria. The production of strategies appears to place an emphasis on grammatically correct procedure rather than a more generalised elaboration entailing the grammar of mathematics. The grade 12's can clearly manage adequately without it since they are able to produce texts that display correct mathematics grammar without being able to elaborate in a corresponding way.

The content analysis offered is not fine-grained. Instead, it offers an initial preview to the induction into the interpretive, theoretic framework described in the general methodology. It has pointed to the need for a more detailed analysis that closely studies the texts while simultaneously providing a comparison between the texts. Subsequent analysis will be shaped by the structure of the theoretic framework rather than the structure of the text.

Strategies based on the empirical (E) are prevalent in grade 11s and strategies based on procedural (P) are prevalent in grade 12s. All these texts are strongly embedded in the esoteric domain. Principled knowledge was cursory since the texts appeared to focus on the production of a correct answer rather than structured relations between different components of mathematics knowledge. School mathematics is often concerned with the production of mathematics procedures and this often incorporates the manipulation of equations and their solutions. This emphasis does not appear in the Grade 11s but in the grade 12s. Grade 11s rather favour strategies that require measurement and construction. The treatment of analytic geometry in grade 11 favours measuring based strategies, which tends to be localised, and fragmenting while the grade 12s favour procedurally based strategies, which tends to be largely fragmenting. Further textual analysis will be concerned with the language of mathematics in analytic geometry.

5.2.2 The language of Analytic Geometry

Textual productions highlight the preference and the prevalence of the symbolic mode suggesting that it is a privileged emphasis. The topic analytic geometry generates texts that conform as far as possible to selective associations made and texts generally associated as belonging to this mathematics topic. Symbols used are those that generally occur in this topic. For example, "Σ" in school mathematics is generally associated with the topic "sequences and series". It does not readily occur in analytic geometry, for example. The symbolic mode revealed the use of signifiers (mathematics variables) like $x, y, a, b, x_a, x_b, y_a, y_b, x_1, x_2, y_1, y_2$, etc. and mathematics symbols like $\perp$, $\surd$, $\rightarrow$, $\times$, $=$, $+$, $\div$, Δ, ^{a}, etc. in complete accordance with the topic.

Texts are thus reflections of esoteric competence. It is implicitly understood that the production of legitimate texts needs to reflect a privileged selection of symbols pertaining to this topic if it is to be accepted by an international community of mathematicians. K illustrates this in extract 5.2.1

F:	Explain to me the significance of this. ((F points to the formulae which K has written down))
K:	This is **the distance formula.**
F:	Yes.
K:	So?
F:	So what does this represent? Which distance would this represent?
K:	The distance PQ. From that line to Q. From that value P to Q and from co-ordinate P to R as well.

<u>Extract 5.2.1: Transcript 1, lines 57 - 81, my emphasis</u>

Clearly K's use of the term "distance formula" is meant to address all mathematicians. The assumption is made that the term alone signifies all that needs to be signified, that the language of mathematics discourse is shared between K and F. His initial explanation is staccato and the specifics of his explanation have to be drawn out. This seems to suggest that explanations of this kind are not part of normal classroom practice. His interpretation and explanation of the distance formula amounts to little more than a verbal account of what he has written. No indication is given of the principles and mathematics law that underpin the distance formula.

A similar argument holds for the use of specialised terms. The use of these terms is determined mainly by their acceptance in the topic rather than an awareness of the mathematics rule or principle. This can be illustrated in the extract 5.2.2 (related to question 2) and extract 5.2.3 (related to question 5) which both index the word "locus", a specialised term.

K:	What I did was, umm. I found that there are three points. **And one of these three points is a locus. So to find the equation of a locus, or to find the locus, you equate the two lines.** And they tell you that the line of PQ will equal the line of QR so you know that you can form an equation by saying that the one line will equal the other line. And therefore you, you isolate where "a" can be.
F:	OK.
K:	And it can either be in the second or the first quadrant and that is how I eventually got to the values for "a". Because it can be on a line. Either the first or the second.

<u>Extract 5.2.2: Transcript 1, lines 48 - 56, my emphasis.</u>

The extract above will be contrasted with a response to question 6, which also references "locus".

K:	They start off by saying that point P will be equidistant from A and B. So wherever P is, the length of P, the length from P to B will equal the length from A to B. So: **it could be either that way or this way. So it lies anywhere on a certain..., on a locus. The equation, the way you would get the equation for a locus would be by saying, you would obtain by saying BP equals AP.** And using the equation, or the distance formula saying that the two distance formulas you would arrive by the equation y equals three x plus five. And that represents the line where P of x, y would lie on. [...]

Extract 5.2.3a: Transcript 1, lines 380-387, my emphasis

And elsewhere,

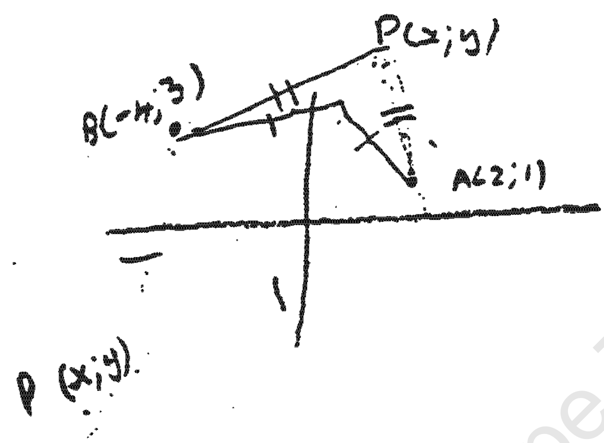
K:	That umm, that is the equation of where the point can lie. The set of points. The locus.
F:	And let me confirm. Two points will satisfy that equation. Just that one and that one.
K:	Yah

Extract 5.2.3b Transcript 1, lines 439-443

Clearly there is a specialised understanding of the term locus and how this term is presented in a question. Extract 5.2.3b refers to a text generated in a response to question 6, which was designed to mirror the exemplars used in text books to illustrate application of the locus principle. The initial written text shows that K knows what to do and when to do it.

Question 6:

The point P(x;y) is equidistant from A(2;1) and B(-4;3).
What will the equation of the set of points equidistant from A and B be?



BP = AP

$$(x+4)^2 + (y-3)^2 = (x-2)^2 + (y-1)^2$$

$$x^2 + 8x + 16 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 - 2y + 1$$

$$8x - 6y + 25 = -4x - 2y + 5$$

$$12x + 20 = 4y$$

$$4y = 12x + 20$$

$$y = 3x + 5$$

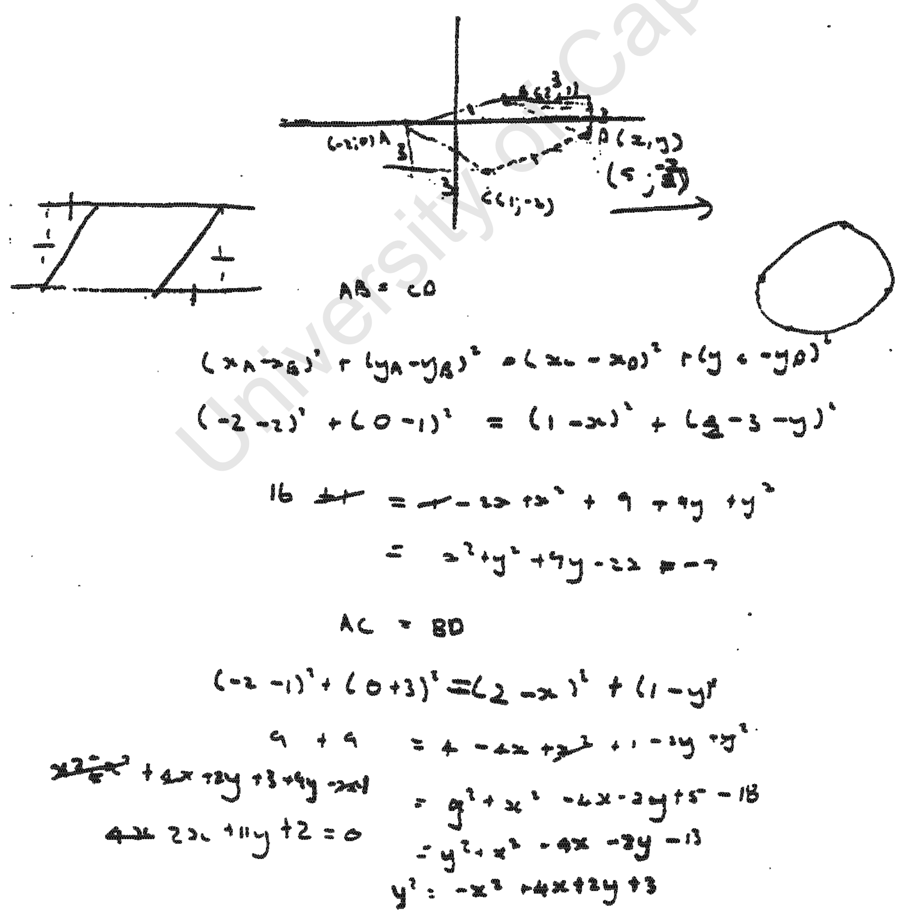
However, discrepancies creep in when K does not understand that there can be infinitely many points on the equation generated, instead of only two possibilities. Interestingly, the specialism exists to the point of generating the necessary equation. Contextualising the equation in terms of the number of points possible cannot be done, suggesting that a principled understanding of the language and terminology does not exist.

Thus far I have shown that phrases and terminology can be used in the absence of the grammar of the text. In these instances, the indiscriminate use of phrases and terminology are used in a seemingly appropriate way, but actually contradicts mathematics principles. This is illustrated in the extract below.

Question 4:

If the points A(-2;0), B(2;1), C(1;-3), D(x;y) form a parallelogram, what are the coordinates of D?

Can you suggest another way of arriving at the same solution?



K:	When I equated. I think when I equated the two lines I could . I got an equation that represents a point. But it is in the form of a circle.
F:	This equation represents a point in the form of a circle. Right?
K:	Yah.
F:	What does that mean?
K:	That means that that point could be anywhere on that circle.
F:	Can you draw it for me?
K:	If this was the circle, the point can be anywhere on the circle.
((K refers to a sketch drawn on the side of the page in front of him))	
	And that was how I couldn't get the co-ordinate from there. Then I equated, I said that AC will also be equal to BD. But then I got another circle. Which is, the same thing applies. And then , then I did not know how to get the point out of that.

Extract 5.2.4: Transcript 1, lines 215 - 227,(my emphasis).

Evidently, the format of the equation given suggests that it is an equation of the circle. This particular concept is frequently encountered in analytical geometry which deals with the *locus of a point under given conditions* (Laridon et.al., 1985: 145). *Locus* is defined as a variable point. K is suggesting that the equation generated represents the locus for the unknown point. However, the unknown vertex of a parallelogram cannot be variable if the geometric properties of the parallelogram are to be maintained. There can only be one solution for the fourth vertex of the parallelogram. Clearly, knowledge about the procedure that should be followed to develop the solution exists. This cannot be realised in language. There is a clear attempt by students to use the appropriate language. K's discussion of his production evidences a written text which obscures the incorrect, comparable use of language.

The language used in the text shows how the language of analytic geometry is relationally linked to other mathematics knowledge. This can be illustrated in M's responses to question 7. M's first choice in her text is related to the word midpoint and the strategy she follows is based on the "midpoint formula". Language-based association between strategy and specialised terms exists. Failure to generate a solution may be indicative of a segmentally organised mathematics knowledge. The element of uncertainty that pervades the text is clear particularly when M contradicts herself when considering whether or not lines are perpendicular. The initial suggestion that the lines are perpendicular is mathematically correct. However, this cue cannot induce a strategy. This supports the view that the understanding of mathematics knowledge is instrumental (Skemp, 1979) and is dependent on memory recall and associations mainly.

F:	But you don't think it is going to get you anywhere. Do you think there is any geometrical relationship between this here and this here?
M:	**I was thinking, maybe they were perpendicular.**
F:	Will they be perpendicular?
M:	**I'm not sure. I think so.** In a way I think so because if they divide the line equally. Because I think D divides the line equally because it is a midpoint. So this would be ninety degrees if that were equal.
F:	If that were ninety degrees what information will that give us which will be of use to us?
M:	We could say that AD would be equal to DB squared, by using the length of the line.
F:	What else could we say?
M:	(3)
F:	Could we say that the lines are perpendicular?
M:	Which line? This line? **Are they perpendicular? I never said that. I didn't say that. But the gradient of this would be. The gradient of OD would be equal to the gradient. NO would be perpendicular the gradient of AB. Which means that MOD multiplied by MAB is equal to minus one.**
F:	Is that useful?
M:	I don't think so.

Extract 5.2.5: Transcript 2, lines 576 - 595. (My emphasis)

Limited planning skills are evidenced in extract 5.2.5 this suggests that the mathematics knowledge is fragmented; relations between different links have not been established.

The comparison between the two strategies in extract 5.2.5 suggests that the level of significance assigned to a strategy may be an indication of the mathematics knowledge and whether or not relational links exist between the mathematics knowledge and the strategy. Focussing on relational mathematics knowledge allows an evaluation of texts in terms of localising or generalising.

5.3 The Resource level

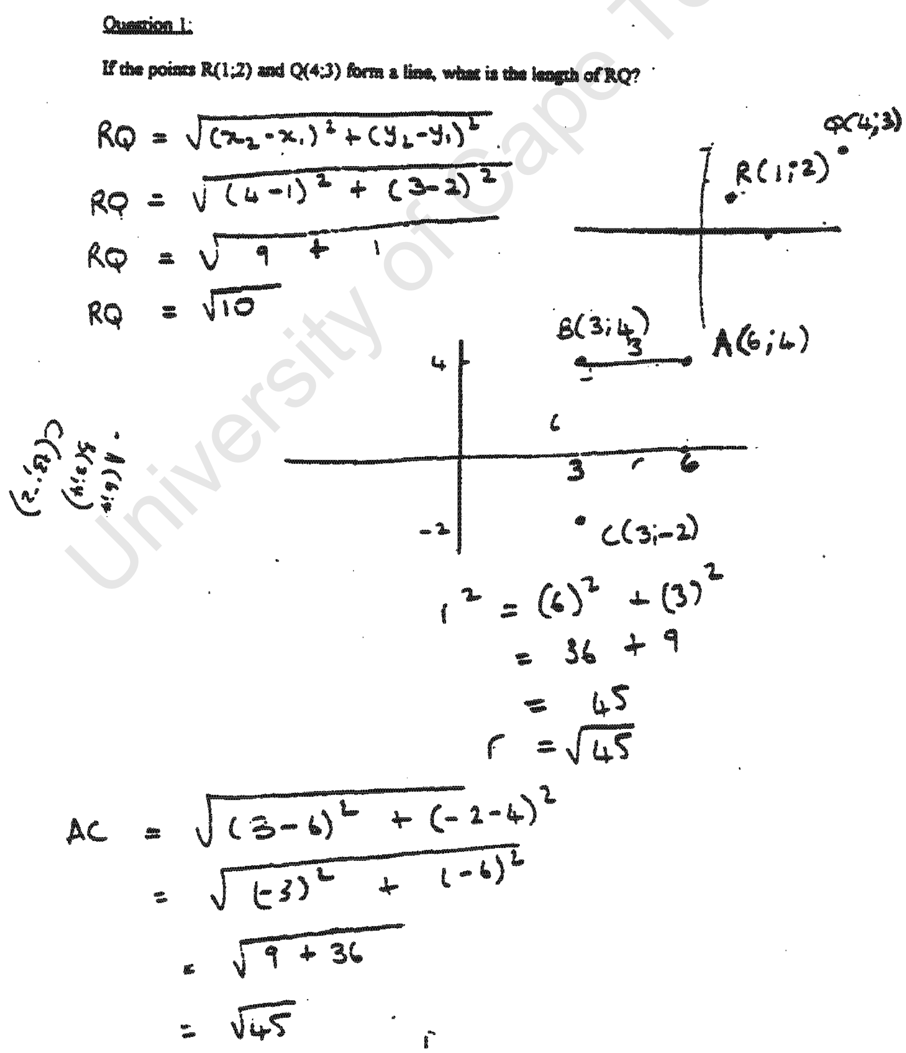
The signifying mode in Dowling's terms (1998: 151) describes the relation between the form of expression and the content of the text that is implicated in sign production. "Signifying mode" specifically refers to the repertoire of resources associated with generalising and localising strategies. Although Dowling identifies three signifying modes: the iconic (pictures, photographs), the indexical (diagrams, graphs) and the symbolic (mathematical signs, variables). The texts examined strongly evidence the indexical and symbolic modes. I would like to include the use of algorithms in the symbolic mode. The elements of the signifying mode to be considered will be the algorithmic mode and the diagrammatic mode.

5.3.1 Algorithmic mode

Algorithms in mathematics can be understood to be a mechanical procedure for determining a procedure for the value of a function for any argument in a specified domain (Oliver in Honderich, 1995: 21). The mechanical procedure can be can be understood to be a finite set of instructions which are completed step-wise without resort to any random processes or proficiency. It is important to bear in mind that the algorithm itself displays a strong grammar based on mathematics rules and principles. Its use, however, does not necessarily display the grammar implicit in the steps of the algorithm. For example, determining the length of a line on a Cartesian plane can be determined by using an algorithm based on the "distance formula". Answers generated by the algorithm, when properly used, are correct testifying to its reliability. Ideally, mathematics learning should be underpinned by getting clued into the mathematics discourse: knowing what the intentions are behind certain typical expressions. Thus when students are asked to determine the length of a line given specific co-ordinates, they know that the distance formula can be used. It can be argued that an assumption is being made about mathematics knowledge underpinning and related to this algorithm. However, knowledge of the way in which the algorithm operates is all that is needed to generate a correct and accurate answer. Mathematics knowledge of the rules underpinning the algorithm can be superfluous to the use of the algorithm. Skemp (1989) terms this instrumental understanding which he argues is underpinned by memorising which problems a method works for and which not. Often, he argues, this involves learning a different method for a different type of problem.

Algorithmic use, which occurs in a way that which reflects instrumental understanding, positions students as consumers of rules governing the algebraic code. Although access to the symbolic system exists, it is not supported by algebraic syntax. The point being made is that the discourse of the mathematics text is not always accurately reflected in the written text: that is the written student production. Strategies underpinned by the use of algorithms format the written text in a way which conceals the students' knowledge of algebraic syntax.

The algorithm, used procedurally without knowledge of its grammar, is illustrated in the following extract.



Question 1:

If the points R(1;2) and Q(4;3) form a line, what is the length of RQ?

$RQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$RQ = \sqrt{(4-1)^2 + (3-2)^2}$

$RQ = \sqrt{9 + 1}$

$RQ = \sqrt{10}$

$r^2 = (6)^2 + (3)^2$

$= 36 + 9$

$= 45$

$r = \sqrt{45}$

$AC = \sqrt{(3-6)^2 + (-2-4)^2}$

$= \sqrt{(-3)^2 + (-6)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

F:	Can you just explain to me what it is that you've done?
M:	I used the distance formula.
F:	Umm.
M:	Because they say, umm, ... **they want to find the length. So we have to use the distance formula. I used the formula.** And I substituted the x- and y-values into, *into* it. And I got that.
F:	OK. Umm. Do you know where these things come from? Where x two, x one, y two, y one come from?
M:	Umm.
F:	What does it mean? Can you explain to me where it comes from?
M:	Umm. Explain where it comes from?
F:	What does this mean?
M:	R, Q. It means the co-ordinates of the points of a line.
F:	Can you show me what it means on a drawing?
M:	OK. ((M draws graphic interpretation)) Those points.
F:	OK. What is the significance of this here? ((F points to part of the formula))
M:	Umm. Of?
F:	In terms of your sketch.
M:	I don't understand the question.
F:	**This is a formula which you've used, right? And here you've drawn a picture for me, with this here. Now I'm asking, in terms of this picture which you've drawn, where does four minus one fit in? Where does three minus two fit in? In other words, does this appear on your sketch?**
M:	I don't know. No, I don't think so. If you draw this line like that, I *think* that it would fit in. Something minus that. I don't know.
F:	What minus?
M:	Four minus one. There that one. (4)
F:	Don't know?
M:	**I don't know.**
F:	And three minus two?
M:	The y-values?
F:	umm. Wait. **Are these** y-values?
M:	**Yes.** Y two minus =
F:	= And this here?
M:	**x-values.**
F:	So, are there y-values on the sketch?
M:	Two and three are the y-values because of that point and that. And x
F:	Where are the x-values?
M:	X is over here and over there.
F:	So in terms of that. Umm, is this the, is this the length of a line.
M:	This? Only this?
F:	Yes, x two minus x one.
M:	No, because they add it. So, **I don't know**

Extract 5.3.1: Transcript 2, lines 7 - 53 (my emphasis)

Lines 10 - 12 show that there is a strong link between "length" and "distance formula". The formula used is a general one and has the following form:

$$\text{Distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\,.$$

The use of x_1 and x_2 clearly index the generalisability since the subscripts "1" and "2" do not appear as part of the question. In line 19 it is clear that it is understood that the subscripts reference the co-ordinates of R and Q respectively. However, the links between the graphic representation of the information given and the elements of the formula like "$(x_1 - x_2)$" and "$(y_1 - y_2)$" cannot be made. Even when the question is rephrased in a more specific manner, a way which considers the second line of the written text produced, a line determined by substitution into the formula, M says that she does not know. The text to this point indicates proficiency with the formula but no understanding of the theorem of Pythagoras, which underpins the formula. To overcome this impasse F tries to redirect the mathematics knowledge in a way that attempts to trace the development of the distance formula by offering two points *A*(6;4) and *B*(3;4) which share the same *y* co-ordinate. M obtains this solution by subtracting (transcript 2, line 70) and this mirrors the development of $(x_1 - x_2)$. *C*(3;-2) is then introduced vertically below *B*. Once again M is able to determine the length of this line since, like the line $\overline{AB}$, it shares a common coordinate. $\overline{AB}$ is also determined by subtraction. It was hoped that this would establish relations with $(y_1 - y_2)$ in the formula. To find the length of the line $\overline{AC}$, M suggests the theorem of Pythagoras and is able to arrive at the correct solution. However, links with the original distance formula cannot be established. Evidently the mathematics knowledge exists in insular fragments with no relational links between them.

In the above I have attempted to illustrate that proficient use of a formula is not necessarily accompanied by the appropriate mathematics knowledge.

Use of the algorithm related to distributing strategies is displayed in the table. M6 refers to the text generated by M in question 1. Different responses with the same ideal type from the same student have been arranged below each other. Responses have also been horizontally organised according to the question where necessary.

Distributing Strategy	Ideal Types						
	E	P	G	EP	PE	PG	EPG
Localising		M6					
Fragmenting		**M1** M3 M4; K4 M5; K5 K6 M7; K7		**D1; G1** G2 D3; G4		 M2; K2	
Generalising						**K1** K3	G3

Table 5.3: Algorithmic use related to distributing strategy and ideal types

Extract 5.3.1 comes from M1 and is one of 4 extracts (highlighted in Table 5.3) that will be considered regarding algorithmic use. The extract is an illustrative example for procedural use of the algorithm in which fragmented mathematics knowledge exists. The range is more expanded than a localising strategy. The written texts here appear to have access to mathematics discourse but the absence of principled justification shows that this is not so.

I will point to possible similarities and differences between first D1 and then K1. To avoid duplication of the above discussion, similarities and differences will be presented in Table 5.4. Since D1 and G1 have the same ideal type, only D1 will be considered.

Characteristic	M1	D1	K1
Format of formula	generalising with subscripts 2 and 1.	specific with values pertaining to the question.	generalising with subscripts 2 and 1.
Position of formula in the written text	Beginning of written text; own response.	End of written text; cued response.	Beginning of written text; own response.
Relational links with the Pythagorean theorem.	Not established.	Segmental relations	Established.
Disjuncture between written text and verbal description	Complete disjuncture	No disjuncture	No disjuncture
Relation with diagrammatic illustration	No relation	Strong relation	Strong relation

Table 5.4: Summary of algorithmic use

Different algorithms are used in grade 11 and grade 12. Grade 12s use the distance formula, a transformation of the theorem of Pythagoras, while the formula used in grade 11 is an original application of the Pythagorean theorem. This is evidenced in the form that the formula takes. The form of the equations the grade 12s use is the privileged form of their learning. In the grade 11s the algorithm has to be developed based on their previous mathematics knowledge and this can only be established once students see the relation between the accurate sketch that they construct and their existing algebraic knowledge.

I am attempting to show how algorithmic use is related to any diagrammatic representation since the formula indexes mathematics signs and symbols that only make sense when presented graphically or diagrammatically. I am arguing that when students understand the principles underpinning the algorithm they are able to establish discursive links with the sketch and that no disjuncture will exist between the two. When students do not understand mathematics principles the algebraic syntax stands alone and is not related to the sketch. The text produced will have low discursive saturation. Extract 5.6 illustrates this when M produces the phrase "I don't know" four times when questioned on the written text produced. A strong dependence on metaphor is demonstrated with no metonymic links to support statements made. Strategy, which uses an algorithm in this way, is localising.

When students recognise the suitability of the Theorem of Pythagoras in determining the length of the line, an algorithm begins to develop. However, initial forays of this nature are not enough to sustain any generalisability meaning that transformations of the theorem of Pythagoras are not always consistent. Relational links are not well established between different knowledge components pointing to segmental organisation thus making the distributing strategy fragmenting.

Generalisation occurs when discursive relations can be established with diagrammatic representation, the written text as well as mathematics rules and principles. For K use of the distance formula is the primary focal point of the solution. Its use is not isolated from the mathematics since he is able to explain the origin of signs and symbols of the formula by using a diagram. The relations established between the symbolic system and the formula, coupled with the principled understanding of the Pythagorean transformation, makes all metonymic links explicit. This indexes a generalising distributing strategy.

The differential use of the algorithm points to differential authorisation of the text. Transcript 2 for K5 shows deferred authorisation on two levels. In line 337 -338 "they" refers to a group of mathematicians who can be positioned as experts who are questioning/testing mathematics students. Mathematicians ("They") tell you and you have to infer the appropriate solution ("So you can find ..."). This coupled with the classification of K5 as a fragmenting strategy of ideal type P permits the following claim regarding mathematics knowledge: knowledge dependent on procedures and algorithms necessitates memorising which problems a method works for and which not. This invariably results in the unique meanings implied in mathematics being side-stepped.

Line 366 in K5's transcript also illustrates transferred authorisation for a mathematics formula. Authorisation lies with other mathematicians in general ("So what they do is...").

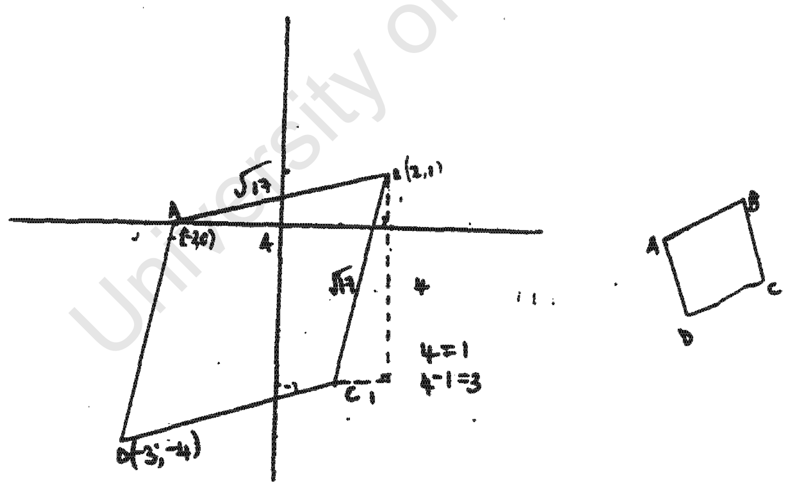
The view that begins to emerge is that mathematicians develop questions that students have to answer. Mathematicians also derive formulae which students use. This provides insights into the relation between mathematicians and students as the authors of the texts. Expressions of certainty and authority ("What they do ...") index the undisputed authority of a community of mathematicians.

Of the 28 questions answered all together by all the students $\frac{20}{28}$ of the texts produce use algorithms. This strongly suggests that that the use of algorithms is a privileged mathematics practice for these students. The preference for algorithms and its possible privileged status can be found in Extract 5.7 referencing Transcript 4, G4.

<u>Question 4:</u>

If the points A(-2;0), B(2;1), C(1;-3), D(x;y) form a parallelogram, what are the coordinates of D?

Can you suggest another way of arriving at the same solution?



F:	Is there another way? That was very creative. Is there another way you could have obtained the same answer? Can you think of another way? **Do you think there is another way?**
G:	**Yes, this is the right way. This is the wrong way =**
F:	**= This is the wrong way?**
G:	**This is the wrong way.**
F:	**This is the wrong way with the right answer?**
G:	**The wrong method.**
F:	Why the wrong method?
G:	**Because it does not always work.**
F:	**Why do you think it won't always work.**
G:	**Because this I made up.((G giggles))**
F:	**So, why don't you trust your own brain?**
G:	((no answer))
F:	OK. What are the properties of a parm?
G:	The opposite sides are equal. And it is parallel.
F:	Is that constant. Is that always true?
G:	yes. The angles also.

Extract 5.3.2: Transcript 4, lines 416 - 433, my emphasis

The initial response to the question produced an answer through the accurate plotting of points. An alternative solution based on the symmetry of a parallelogram was then generated. However, there was no written text to support this answer. This solution was discarded even though it also generated the correct answer. After acknowledging that it is correct he clearly discards this method, doubting its generalisability. This appears to indicate a preference for a method not dependent on measuring or some related procedure. An implicit understanding that this concrete approach is not abstract enough at a senior school mathematics level exists. The links that appear to exist with Euclidean properties of a parallelogram, even if they are fragmented, suggest a legitimate text not dependent on measuring exists. The point is that a procedure exists and G believes that he is not clued up to deal with this question. This preference strongly suggests that a formal written text using algebraic signs and symbols is prioritised in school mathematics practices.

In this section on algorithms I have attempted to show how extensive the use of algorithms is in school mathematics and how its assists in the production of legitimate texts.

5.3.2 Diagrammatic mode of expression

The discussion on algorithms has pointed to the use of diagrams to support mathematics argument. In this section I will show how the use of diagrams, accurate or rough, are deployed and how they reflect mathematics knowledge.

First I will account for the size of the drawing as a percentage related to the size of the written text. A rectangular block was placed around all the text besides the drawing. A separate block was place around the diagram and this area was presented as a percentage of the sum of the areas. A discussion on mathematics knowledge will follow.

Question	Grade 12		Grade 11	
	K	M	D	G
1	42 %	42 %	64 %	98 %
2	22 %	44 %	91 %	81 %
3	29 %	21 %	68 %	63 %
4	33 %	6 %	100 %	100 %
5	33 %	15 %	92 %	100 %
6	33 %	47 %	84 %	100 %
7	16 %	30 %	100 %	97 %
Average	29, 7 %	29,3 %	85,6 %	91,3 %

Table 5.5: Percentage of text presented diagrammatically

Table 5.5 evidences the dependence of grade 11s on diagrams in developing a solution procedure. This affirms the expectation that students who have not been taught a section in mathematics, will attempt to solve it based on first principles: accurate measurement. The concrete approach uses a measuring device (a ruler) and may assist the development of possible generalisations. This argument has its origin in Piagetian developmental theory referenced in the general methodology and supports the significant decrease of diagrams in grade 12 students who have access to other methodologies and procedures after having been taught.

Limited sample size means that no conclusions regarding students generally can be drawn from this information. Nevertheless, the table highlights the way in which students resort to more primitive methods in an attempts to develop a solution. It also suggests that minimal abstractive powers exist in students since theoretically they ought to have access to mathematics knowledge that allows them to engage with the problem at a more advanced level.

I will attempt to address the purpose that the sketch serves. The information has been summarised in table 5, which details the corresponding distributing strategy and ideal type. A brief discussion will then follow:

Question	Grade 12		Grade 11	
	K	M	D	G
1	42 %; PG generalising	42 %; P fragmenting	64 %; EP fragmenting	98 %; EP fragmenting
2	22 %; PG fragmenting	44 %; PG fragmenting	91 %; E fragmenting	81 %; EP fragmenting
3	29 %; PG generalising	21 %; P fragmenting	68 %; EP fragmenting	63 %; EPG generalising
4	33 %; P fragmenting	6 %; P fragmenting	100 %; E localising	100 %; EP fragmenting
5	33 %; P fragmenting	15 %; P fragmenting	92 %; E localising	100 %; E localising
6	33 %; P fragmenting	47 %; P localising	84 %; E fragmenting	100 %; E localising
7	16 %; P fragmenting	30 %; P fragmenting	100 %; E localising	97 %; E fragmenting
Average	29, 7 %	29,3 %	85,6 %	91,3 %

Table 5.6: Diagram percentage of text with matching ideal type and distributing strategy

Evidently the grade 11 percentages of diagrams in the texts produced are approximately three times as much as those of the grade 12s. This is because their problem solving methodology was based first on sketches with accurate calibrations, plotting points and measurement. More advanced problem solving skills may follow depending on the whether the mathematics knowledge was segmentally accumulated or whether relational links existed in their mathematics. Segmental knowledge produced a fragmenting distributing strategy with ideal types P and EP. If the knowledge employed centres around construction, the ideal type is E with a localised distributing strategy. Other localising strategies have ideal type P and like E the text is singular since it makes no reference to mathematics knowledge outside itself.

This dependence of Grade 11s on an accurate sketch suggests that the sketch is a possible stepping stone to any possible generalisations or algorithmic development. Accurate constructions are offered as the most primitive level of mathematics similar to the Piagetian sensori-motor stage in developmental psychology. Further developments may or may not occur depending on students' mathematics knowledge and the way in which it can be recruited. The popular textual strategy is fragmenting in grade 11 and 12. The difference between the two is that the grade 11s favour localising while grade 12s do not.

The decreased percentages for the use of diagrams as part of the text in grade 12 suggests that mathematics knowledge is more developed if Piagetian developmental psychologists are to be followed. This is clearly not the case when the extent of the much smaller percentages for diagrams as part of the sketch in the grade 12s also shows marked fragmenting distributing strategies suggesting the following. Firstly, sketches are not needed in algebraic methods learnt and necessary to answer questions. Secondly, assessment criteria position sketches as extraneous.

The textual strategies implicated in the two groups (re)produce differences and similarity in message. The fragmented backgrounds in both groups render the mathematics gaze of their redescriptions invisible. Texts rarely incorporate the procedural to evolve into a more generalising strategy. Although the written text is firmly embedded in the esoteric domain, there is no room for the unveiling of idiosyncratic mathematisations of the student voice. The written text conforms to the format of legitimate texts. The narrative is closed showing that the student voice is dependent on the immediacy of the topic and cannot unveil that implied and invisible mathematics discourse in the written text. This, coupled with the presence of localised message of grade 11s, achieved via proceduralising and limiting strategies displaying extensive use of accurate diagrams, is an indicator of the way in which students' mathematics knowledge is recruited. Students practice the production of legitimate mathematics texts and use them as exemplars for their own textual productions. It appears as if the recruitment of these exemplars can occur without the recruitment of the necessary mathematics knowledge. In the grade 12s the more procedural approach foregrounds mathematics taught in the analytic geometry section. The grade 11s on the other foreground construction and diagrams and use this to attempt establishing relations with other mathematics knowledge they have.

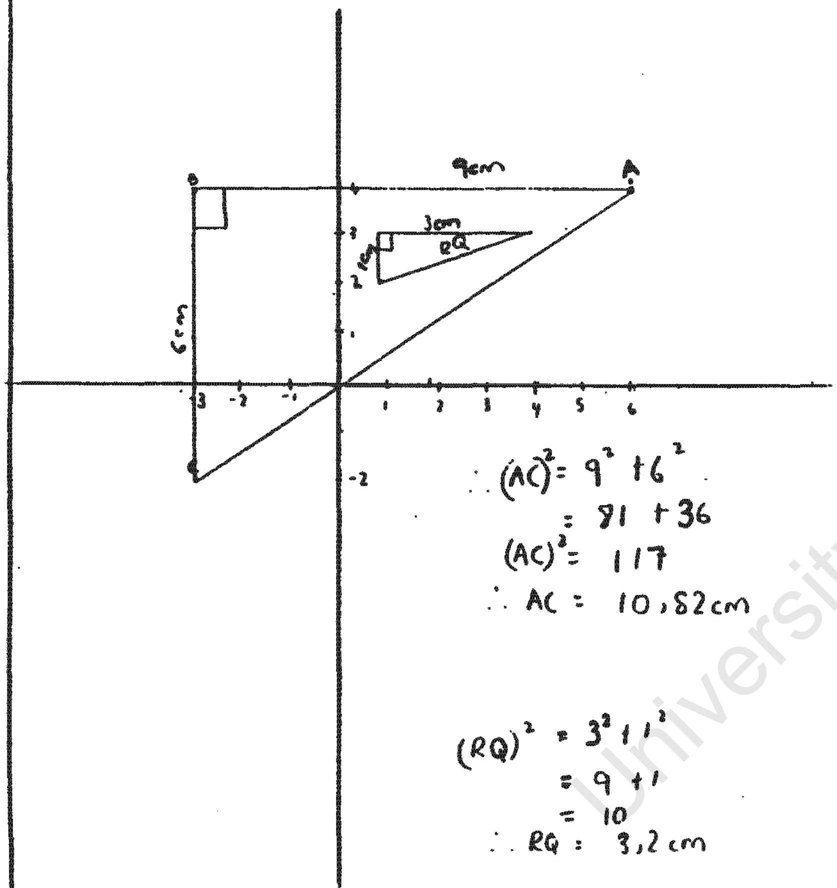
5.3.3 Ad hoc recruitment of signifying modes

This section will briefly describe the way in which certain signifying modes are recruited in an attempt to justify the texts produced. Many of the instances have already been identified previously. However, special mention needs to be made of the manner of recruitment based on the use of diagrams that are not immediately related to the written text. I refer specifically to diagrammatic arguments that do conform to the argument being presented and appear in a standard textbook orientation.

A comparison of the orientation of the basic theorem of Pythagoras transformation in determining the length of a line will be offered in transcript 3, questions 1 and 2. The texts produced appear below.

Question 1:

If the points R(1;2) and Q(4;3) form a line, what is the length of RQ? 3,3cm



$\therefore (AC)^2 = 9^2 + 6^2$

$= 81 + 36$

$(AC)^2 = 117$

$\therefore AC = 10,82cm$

$(RQ)^2 = 3^2 + 1^2$

$= 9 + 1$

$= 10$

$\therefore RQ = 3,2\ cm$

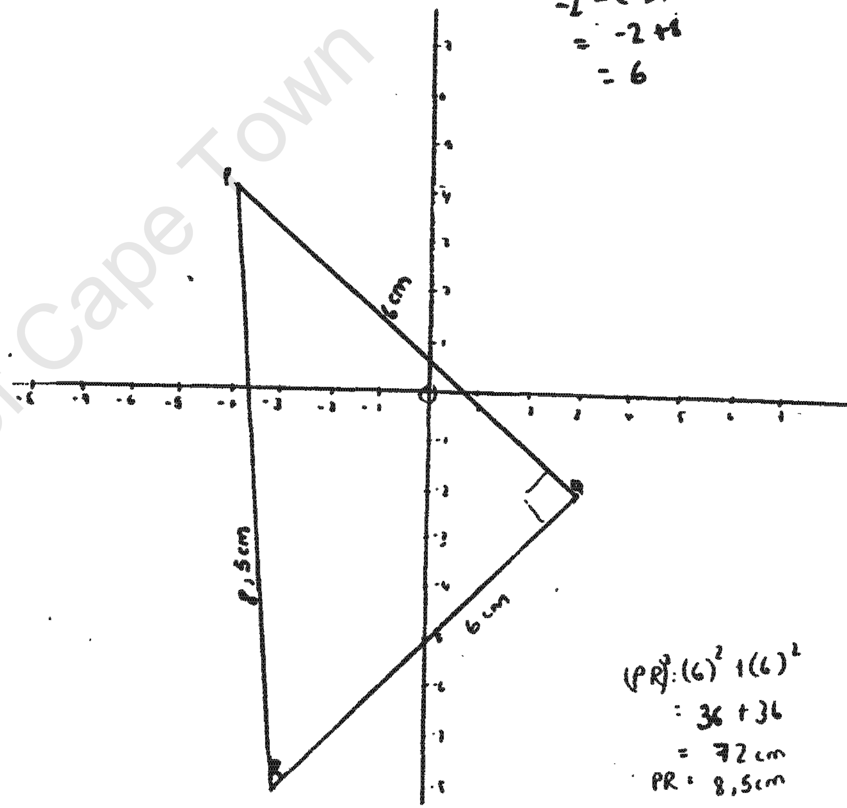
Question 2:

Consider the points P, Q(3;-2) and R(-3;-8). P is in (a;4) in the second quadrant. If PQ = QR, find a.

$-2 - (-8)$

$= -2 + 8$

$= 6$



$(PR)^2 : (6)^2 + (6)^2$

$: 36 + 36$

$= 72\ cm$

$PR : 8,5cm$

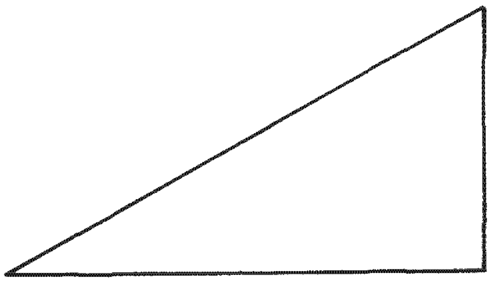
For question 1, D first obtains an answer using an accurate sketch (only $\overline{RQ}$ is drawn). Questions posed result in the bigger outer triangle being drawn, which facilitates the recognition of the Pythagorean theorem. The format of this triangle is superimposed on the line RQ resulting in a triangle, which makes it easy to offer the exact application of the theorem of Pythagoras. In question 2, however, the length of two lines is needed to develop a solution. To apply the theorem of Pythagoras to determine the length of the line, the previous application is used as an exemplar to model the new answer.

F:	Right. How did we say or how can we determine the length of the line?
D:	By using Pythagoras.
F:	Can you try that?
((D draws in PQ, PR and QR such that angle Q in the triangle PQR is right angled)) (29)	
D:	I don't know how to go further.
F:	OK. Are you happy with your answer?
D:	Yes, so far.
F:	Let's look at the first one. Let's look at the line PQ. Let's not confuse it with the other answers that we have. **If you wanted to find PQ, what did we need to do?**
D:	**That corresponds with the y-axis.**
F:	**What corresponds with the y-axis?**
D:	**That line PQ. When it is turned over it will be parallel to the y-axis.**
F:	**Can we do that?**
D:	**You can do it in your mind.**
F:	OK.
D:	And this will correspond with the x- axis. So to get the value of PQ you say, you get the two minus negative eight, so you get positive six.
F:	Right.
D:	And over here you say three minus negative three. You get six. And they say PQ is equal to QR and that corresponds with what I have here.
F:	Do you know that. How do you know that this is equal to ninety degrees?
D:	Umm ((Contemplating an answer))
F:	I understand that you're turning it in your mind, but how do you know that it will be ninety and not seventy?
D:	Because this line over here, is parallel to the y-axis and this is parallel to the x-axis. And these are both straight lines. So that has to be ninety degrees, if it intersects like that. Then this will be ninety.

Extract 5.3.3: Transcript 3, lines 116 - 143, my emphasis

When the triangle drawn does not have the same orientation as question 1, D "turns the triangle in her mind". Geometrically this is possible considering that her drawing is meant to be an accurate one but is not simple considering the effects on co-ordinates in the Cartesian plane. Also $\overline{PQ}$ is longer than $\overline{RQ}$ which is not 6 cm long. The understanding of the application of the theorem of Pythagoras appears to be one dimensional and not communicable in different contexts. The failure to redescribe the theorem in any orientation besides the one suggested, is an indication of the segmental nature of the privileged selection of the student repertoire.

A right-angled triangle is generally presented in the following form:



A selection of grade 12 mathematics textbooks were examined for their orientation of the right angled triangle and the subsequent application of the theorem of Pythagoras, development of the distance formula and the format of examples illustrating the calculation of the length of a line given two co-ordinates. A summary of this is presented in table 5.5.

Textbook	Presentation of Pythagorean theorem	Relation between distance formula and sketch	Relations between exemplars, diagrams and distance formula
1. New Dimensions in Mathematics, Hay *et. al.* (1996)	Use of Pythagoras in terms of standard sketch on Cartesian Plane.	Sketch used to produce formula	Based on application of formula. No sketch given.
2. Just Mathematics 12, Fitton *et. al* (1998)	Use of Pythagoras in terms of standard sketch on Cartesian Plane.	Student must infer that $\overline{BC} = (x_A - x_B)$. $(x_A - x_B)$ not marked on sketch.	Direct application of formula without sketch.
3. Mathematics Plus 10, Ladewig *et. al* (1998)	Use of Pythagoras in terms of standard sketch not on Cartesian plane	Markings on sketch specifically related to formula.	Based on formula only without sketch.
4. Classroom Mathematics Std	Use of Pythagoras in terms of standard	Markings on sketch specifically related	Examples explicitly state

10/Grade 12, Laridon *et. al* (1996)	sketch on Cartesian Plane.	to formula	that a sketch must be drawn, first accurately, then to present information.
5. Pass your matric maths easily, Gibson, *et. al*, (1993)	No reference to Pythagoras nor the Cartesian plane	No relation.	Application of formula.
6. Passport to geometry and trigonometry, ESST, (1994)	Use of Pythagoras in terms of standard sketch.	Markings on sketch match the Pythagorean explanation.	Application of formula without sketch.

Table 5.7: Textbooks and their presentation of the length of the line between 2 points

The textbooks above have recruited the signifying modes differently. The indexical mode presents a standard orientation of the Pythagorean triangle in $\frac{4}{6}$ books. The same textbooks establish relational links with the distance formula. However, only one book "Classroom Mathematics" recommends that diagrams always be used, especially to confirm solutions. All other exemplars present an application of the formula only.

The texts in Table 5.7 authorise the use of the formula without diagrammatic referencing thus authorising a procedural approach. This does not suggest that a more principling strategy is impossible. It rather sets the precedents for this particular application. Students frequently model their solution strategies on exemplars. If the request for checking and reflection on the solution generated is not consistently made, then this form of authorisation is diminished. Instead, authorisation is given to the application of formula and specific procedures so that students feel disadvantaged when this exposure has not occurred. This is evidenced in both grade 11 students who are reluctant to attempt questions 5, and 7. (Transcript 3: 415, 512; Transcript 4: 488).

The point about "*ad hoc* recruitment of the signifying mode" centres on authorisation of student texts in school mathematics practices. Authorisation is derived from all mathematics texts used and consulted, the teacher and forms of assessment. In the student texts generated, students are not always able or willing to engage with questions which require that they function in a mode that is outside a form of authorisation they expect.

Popular modes of creating legitimate mathematics texts thus authorise the texts that students produce.

Summary

In the analysis of student texts thus far I have discussed the way in which the three levels of social activity theory have impacted on the texts produced. The description of student texts suggests that different solution strategies are followed for different groups of students. Grade 11 students favour an empirical approach to work that has not yet been taught in the classroom. Content analysis of the texts showed that grade 11s were generally not able to transform existing knowledge in new and different problems. Grade 12s used procedures taught during problem solving. Despite this difference in problem solving methodology, both groups of students were unable to consistently produce a principled description of their textual productions. This suggests that students and their school mathematics practices do not privilege a principled view of mathematics knowledge, even when they are able to produce texts that appeared to be mathematically legitimate. In other words, student textual production is derived from practices which students interpret as being privileged.

All student textual productions favoured specialised signs and symbols that were presented in a very specific format. Verbal descriptions of textual productions required specialised language. School mathematics practice privileges a specific textual format which may be produced and reproduced without the necessary principled mathematics knowledge. This can be achieved via the use of algorithms and similar procedural techniques.

In the rest of the analysis I will discuss the role of assessment as well as the textual strategies of each of the four students. Although synopses of textual strategies have been given, a discussion on the mathematics knowledge of each student need to be made. Within each group of students I will point to similarities and I will point to differences between the learners. After this I will compare the recruitment of knowledge between grades.

Chapter 6 Modes of textual productions

In chapter 5 the textual distributing strategies of two different groups of students were produced. I argued that grade 11 students tended to concentrate on constructions while grade 12s concentrated on procedures. In this chapter I want to consider the mode of textual production for each of the students.

Generally students focus on what they perceive to be privileged and these perceptions will influence the format of the actual text. This will be reinforced by assessment.

This chapter will be organised in the following way. First, I will consider the way in which student textual productions are shaped by assessment. A discussion on all the texts produced, by each of the four students used in this case study student, will follow. Finally, a discussion on texts produced by grade 11s and grade 12s will be presented.

6.1 The role of assessment

High and low discursive saturation indicate the degree of the discursive in the complete text, verbal and written. All texts exhibiting a generalising strategy are DS^+ while fragmenting strategies are DS^-. Dowling (1998: 30) suggests that DS^+ and DS^- are good indicators of performances and are reflected in the relationship between novices and adepts[11]. In the chapter dealing with the general methodology I have indicated that this suits Dowling's description of pedagogic texts and does not suit the analysis of student texts.

Dowling claims that students' performances are economised via assessment. This means that the basis of comparison between students by standardisation, schemes of assessment and external national examinations, determined by their performances, objectifies the

[11] In Dowling's language of description, a novice is the object of pedagogic action where the adept is described as the *subject* of the practice which the novice is to acquire. The adept is subjected to the

student as a product. The belief that all texts exhibiting esoteric competence can be independent of their material product does not reflect school mathematics practices. This does not make allowances for the rehearsal of esoteric competence. Dowling's suggestion that pedagogising action presents school students with simplified practices best described as DS$^-$, nevertheless makes the assumption that students are able to produce texts that are generalising by virtue of their esoteric competence.

This view of texts in the esoteric domain fails to recognise the way in which standardised forms of assessment encourages a hybridisation of the principled, regulating rules of school mathematics. When assessment of mathematics activity is recontextualised[12] by students, the privileging of its outcome occurs. For example, the focus of assessment in the national examination at the end of grade 12, reinforced by mathematics subject advisers and teachers, (re)produces a very specific form of student mathematics texts. It is this form that then becomes the objective of school mathematics practice.

Mathematics practice involves written language and it can be defined in terms of competency. The result of this is students' rehearsal of legitimate texts. This collapses Bernstein's definition of pedagogic discourse as the principle of other discourses as well as Dowling's description (using language) of mathematics activity in terms of mathematics discourse. School mathematics as a structured symbolic practice, distinct from other practices is thus corroded.

NB

6.2 Textual production in the four students

6.2.1 Mathematics texts produced by K

An interesting question, "Did I have to study for this?" was posed by K before the interview and does not form part of the video recording, nor of the transcript. The

regulative principles of the practice and the novice has to be subjected to the regulative rules during the apprenticeship.

[12] Recontextualisation corresponds to Dowling's recontextualisation rather than Bernstein's.

question was not asked in a frivolous tone and says something about the way in which K regards mathematics. The comment suggests that mathematics requires some form of study, preparation or rehearsal before being tackled. This idea makes sense in terms of the kinds of assessment that K has experienced at school.

I will briefly repeat the summary of the distributing strategies and matching ideal types for K.

Question Number	Distributing strategy	Ideal Type
1	Generalising	PG
2	Fragmenting	PG
3	Generalising	PG
4	Fragmenting	P
5	Fragmenting	P
6	Fragmenting	P
7	Fragmenting	P

Table 6.1: The Distributing Strategies and Ideal Types for K

All the distributing strategies have an expanded range and differ with regard to the abstracting and particularising of mathematics discourse. Recall that a generalising strategy is abstracting while a fragmenting strategy is particularising.

Generalising strategies occur in $\frac{2}{7}$ instances. Question 1 involves the determination of the length of a line while question 3 involves intersecting lines suggesting that the mathematics knowledge regarding lines is relational. In an earlier chapter I have shown that the mathematics syllabus is arranged in a cumulative way and that it possible for students to develop their mathematics knowledge so that stronger mathematics relations and discursive links can be formed between different segments of mathematics knowledge. The fragmenting strategy in question 2, which also involves equal lines, is fragmenting. The difference in this question is that it explores the knowledge of the straight lines as well as the corresponding connection with algebraic equations as they pertain to co-ordinate geometry. According to the syllabus this explicitly occurs in the grade 12 mathematics syllabus. It thus seems as if development of the rules, which underpin the algebra and geometry of the straight line, is incomplete[13]. The role of assessment in not requiring further relational links between the different mathematics knowledge has already been noted, although the possibility of the individual student being unable to establish relational links also exists. The individual understanding of one student is not the point. The point is rather that the student interprets mathematics pedagogy in a way which constructs him/her as the consumer of mathematics rules. At a more general level, pedagogic content emerges and occurs as a result of an interpretation of the pedagogic action that the student is exposed to.

Mathematics curriculum content is governed by syllabus requirements and is officially regulated. The provincial curriculum attempts to standardise the external examination syllabus. Regulation of the curriculum imposed constitutes a regulation of the teacher, and according to Dowling (1998: 46), the teacher can act as a relay conveying mathematics or as one who apprentices students into mathematics principles. The distributing strategies presented for K thus far suggests that students, as producers of mathematics texts, have developed evaluative rules for the mathematics curriculum which are not explicitly transmitted to the student. The form of presentation of student mathematics texts omits the mathematics principles of their construction. The pedagogic action is thus not one of apprenticing students into mathematics discourse, but one in which mathematics content is relayed.

The occurrence of proceduralising mathematics strategies in questions 4 to 7 strongly confirms the relaying pedagogic action. Texts produced can thus be described as having a restricted orientation since the mathematics principles that appear to be present in the esoteric textual production cannot be made explicit in language. This is not meant to be an indication of mathematics ability. It rather points to the mode of the pedagogic action.

[13] No form of mathematics knowledge can be complete. The point is rather than the knowledge that the student has been exposed to during school mathematics practices has not been internalised that allows a mathematising gaze to function completely.

6.2.2 Mathematics texts produced by M

The summary of the distributing strategies and matching ideal types for M will be repeated.

Question Number	Distributing strategy	Ideal Type
1	Fragmenting	P
2	Fragmenting	PG
3	Fragmenting	P
4	Fragmenting	P
5	Fragmenting	P
6	Localising	P
7	Fragmenting	P

Table 6.2: The Distributing Strategies and Ideal Types for M

The overwhelming distributing strategy once again is fragmenting, representing $\frac{6}{7}$ instances with the same number of procedural (P) ideal types. Only one of the ideal types evidences the grammatical mode and appears in question 2, which has been described in exactly the same way in the text produced by K. This suggests that some commonality exists in the repertoire of the resources considered and subsequent fragmentation. In the school mathematics syllabus a question like this one only appears at grade 12 according to syllabus design and the textbooks consulted, pointing to the way in which similar exemplars can be rehearsed. This seems to be confirmed when both K and M proceed to produce a written text confidently using a familiar algorithm. The shared fragmenting and ideal type P in questions 4, 5 and 7 also appears to strengthen the contention that the relaying pedagogic action is privileged amongst grade 12s. The claim that an understanding of the principles that underpin mathematics knowledge is not needed for positive, successful assessment favours a pedagogising action that makes it possible for students to engage with a recontextualised view of the importance of mathematics principles.

The possibility of the procedural ideal type in a localising strategy presents texts with a more limited range. Clearly, procedures can be carried out in a way that does not even acknowledge any mathematics knowledge besides the immediacy of the mathematics topic being examined, resulting in topic dependency. When students use a formula or algorithm in this way, authorisation is external to the student and lies with an unknown mathematics authority.

Fragmenting and localising textual strategies are not an indication of whether or not students are able to understand or engage with the regulating rules and principles of school mathematics. It is rather a comment of the pedagogic action and how this recruits mathematics knowledge and is recruited by school mathematics. The mode of pedagogic action in M's textual productions results in distributing strategies that are largely fragmenting.

6.2.3 Mathematics texts produced by D

I will briefly repeat the summary of the distributing strategies and matching ideal types for D.

Question Number	Distributing strategy	Ideal Type
1	Fragmenting	EP
2	Fragmenting	E
3	Fragmenting	EP
4	Localising	E
5	Localising	E
6	Fragmenting	E
7	Localising	E

Table 6.3: The Distributing Strategies and Ideal Types for D

Clearly, all the ideal types revolve round the empirical showing a dependence on measuring. When the ideal type is only E, then only measuring has occurred and no other mathematics knowledge is indexed, creating a localised distributing strategy that is topic dependent. When other mathematics knowledge or procedure is indexed in addition to measuring in a non-relational manner, the distributing strategy is fragmenting.

Three possible reasons for the dependence on measurement exist. A calibrated ruler, along with pencil, eraser and calculator, were given making accurate construction a possibility. Secondly, the possibility existed that the repertoire of the resource pool is impoverished in terms of principled mathematics knowledge, making accurate construction the only viable solution procedure. Finally, accurate measurement gives students the opportunity to speculate and explore their repertoire further.

The last 2 possibilities are closely linked but it becomes possible to distinguish between the two by using the textual distributing strategies. Texts which do not reveal the underlying principles are either localising or fragmenting. If any other related mathematics knowledge is indexed in a non-relational way, the text is fragmenting. If not it is localising.

The texts produced for questions 4, 5 and 7 are localising with the *empirical* ideal type. In each of these instances D makes it clear that she is engaging with mathematics problems that she is not used to and that no pedagogic action has occurred to facilitate this. When asked whether she would be able to engage with a new and different algebraic method to solve question 4 (Transcript 3: 406 - 411), the reply "If I were used to it" is made. This supports the suggestion previously made that the mode of pedagogic action makes students used to many different things. To be informed or presented with an algorithm certainly forms part of pedagogic practice but textual strategies presented seem to indicate that this has not occurred in a way that allows the student to make mathematics principles explicit given the existing mathematics knowledge.

The discursive element in language is simple, further supporting the claim that localising and fragmenting textual strategies recruit mathematics knowledge disjointedly and this insulated segmental recruitment makes it difficult to establish principled relations.

6.2.4 Mathematics texts produced by G

I will briefly repeat the summary of the distributing strategies and matching ideal types for G.

Question Number	Distributing strategy	Ideal Type
1	Fragmenting	EP
2	Fragmenting	EP
3	Generalising	EPG
4	Fragmenting	EP
5	Localising	E
6	Localising	E
7	Fragmenting	E

Table 6.4: The Distributing Strategies and Ideal Types for G

Earlier it had been reported that three textual strategies occurred in this study: localising, fragmenting, and generalising and all of them are in the texts that G has produced. This does not occur for any of the other students. Before further comparisons are made I will first discuss G's texts.

The two localising strategies with ideal type E are completely dependent on construction. The fragmenting strategy in question 7 also has ideal type E. This question is different to questions 5 and 6 since it indexes other mathematics knowledge but not well enough to be classified as EP, although the possibility exists that a tentative procedure exists and that G refuses to offer it because of uncertainty around its mathematics validity. Classification of distributing strategy can only occur on the basis of textual evidence, hence the E ideal type. The student repertoire in this instance comprises loosely held associative mathematics knowledge which cannot be significantly indexed. The remaining texts (questions 1, 2 and 4) that are fragmenting have ideal type EP and are more established procedurally.

The point about the different ideal types in fragmenting strategies centres on authorisation and the mathematics knowledge that is recruited in this process. The degree of mathematics authorisation is far more in the EP texts. Earlier I alluded to the relaying pedagogic action not apprenticing students into mathematics principles when commenting on the extent of P as a subtype for the grade 12s, K and M. It seems that the perception that mathematics validity has to be manifested in writing is prioritised. The writing is very specific and is concise and brief like an algebraic formula or algorithm, for example. There are no long sentences and the permissible notation within this topic is very specific. This, coupled to the positive assessment of text presented in this way, reinforces their legitimacy. The way in which legitimate texts are constituted once again seems to suggest that the pedagogic action significantly emphasises the production of these texts.

The generalisng strategy has ideal type EPG, and this ideal type only occurs once in all the texts produced. (The two other generalising strategies were PG). The actual written text appears below.

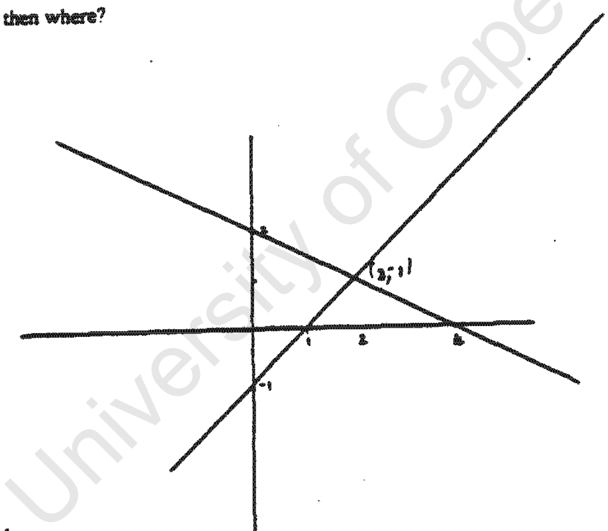
Question 3:

Do the lines $y = x - 1$ and $y = -½x + 2$ intersect?

Why?

If they do, then where?

Why?



Yes, intersect.

$x - 1 = -\frac{1}{2}x + 2$
$x + \frac{1}{2}x = 2 + 1$
$\frac{3}{2}x = 3$
$x = \frac{3}{1} \cdot \frac{2}{3}$
$x = 2$

$y = 2 + (-1)$
$= 1$

Intersect at because the 2 equations can be equated

The first part of the text produced is a sketch of the axes system which initially had no markings. The values later inserted are specific since they represent the *x*- and *y*-intercepts for each of the lines given. No indication of how this calculation was made is given. Once the lines are drawn, the *x*- and *y*- coordinates of the point of intersection are marked on their respective axes. The point of intersection is then marked. Interestingly, the absence of certain markings like the accurate calibrations, identification of the *x*- and *y*-axes, arrows at both ends of the axes and lines as well as equations identifying which line is which suggests that the diagram is primarily meant to provide an answer rather than reflect all the signs and symbols needed to produce a complete mathematics sketch. Once the answer is generated, he proceeds with an algebraic algorithm in which the two equations are equated. The answer obtained matches the answered generated in the sketch. This I believe was the main reason for the sketch. It was a concrete application used to ratify the principles of the algorithm he used.

F:	Can you explain to me what it is that you've done?
G:	They will intersect at a co-ordinate. So I let the two equal each other.
F:	Why?
G:	**Because this line is a negative, so it goes this way.**
((G points in a westerly direction))	
	This one **to the left** and this one **to the right**. So they must pass each other. So I equate the two. They will equal each other at one point. So that y the x is two. The y will also equal each other. The y's are also going to pass each other.
F:	If you had not done this calculation, why can you say that the lines intersect?
G:	Because of the **negative** and the **positive**.
F:	What does that mean?
G:	**The direction of the graph**, that line
F:	So if one is negative and one is positive [they
G:	[It must cross at some point.
F:	How many ways can you think of to find where that point is where they cross?
G:	You can't always draw it because sometimes some graphs, they meet, and sometimes they pass the page.
F:	OK. So you can draw it and it might work, or. And what is the other way?
G:	You must use the equation.
F:	And when you use the equation, what is it that you're doing?
G:	I equate the two y's. The two line equations.
F:	What is the significance of this point here which you've marked as two and one? Could the point three and one have been a point of intersection?
G:	It can only be this one.
F:	What is special about two and one?
G:	Because that is where they lie on each other. Where they intersect.
F:	Is there anything else that is special?
G:	You can use it also to find the **gradients** of the graph.
F:	What do you mean?

G:	**About the minus. If you times that gradient by that gradient, you get minus one.**
F:	Where is minus one? I don't understand now.
G:	**No, that's wrong. It's stupid.**
F:	Let me decide if it is stupid.
G:	**I can't. I forgot how.**
F:	What do you mean?
G:	You can use this point to find the line equation. The **negative gradient**.
F:	Is there any other reason that makes the point two and one the only solution?
G:	It must be a solution because that is where the lines cross. And they ask you to find that point.
F:	OK. Can you this approach for any figures that cut? Can you just equate the y's. Or is it only for lines that you can do this?
G:	Because it is different. You can only use it for lines. I don't know but you can use if for a parabola also, I think and other lines and equations.
F:	OK. Right. Is there anything else you wanted to add.
((G shakes his head))	

Extract 6.1: Transcript 4, lines 357 - 400, my emphasis

The strong discursive element in the language used in the text referred to in the extract above does not show any specialised terms. For example, the term gradient is never used mathematically to describe the slope of the line. Instead, terms like *negative* and *positive* are used to signify the direction of the line. Mathematically this is correct. The term gradient, a mathematics term with very definite associations, could also be used to describe the direction of a line, but more than that its numerical value indicates the degree of slope. All of this is implicit in his use of "gradients" in line 384. It is also mathematically correct to say that the product of two lines pointing in opposite directions will be negative. It is incorrect to say that the product will be minus one. This indexes perpendicular lines. G is able to recognise that this proposal is incorrect but is not able to say why. "No that's wrong. It is stupid" is discursively weak. Despite this, the mathematics grammar of two intersecting lines appears in tact and is evidenced by his metaphoric explanation.

6.3 Synopsis of textual strategies in grade 11 and grade 12

The cumulative organisation of school mathematics rarely evidences itself in any of the texts. Instead of different knowledge segments establishing themselves in the junior grades of high school and then developing principled connections between them, insulation occurs. This segmental organisation produces a random organisation that cannot be structured and fails to produce a unifying pattern. It is characterised by a distinct lack of abstraction and this has important consequences for the discursive practices of school mathematics.

The segmental organisation of mathematics knowledge results in textual strategies that are exclusive to a particular segment. Principled relations between different segments do not occur. In this analysis different segments are regarded as topics. This can be illustrated in question 7 which indexes Euclidean circle geometry, the graph of a circle, perpendicular lines, as well as the theorem of Pythagoras. The grammar of the texts produced was not principally understood by any of the students. The grade 12s are able to produce texts procedurally while grade 11s do so via construction.

Although students were able to communicate to the interviewer what it was they doing even if it only meant reading the text produced, the question that needs to be asked is "Are the texts produced communicable in the broader mathematics field?".

For the grade 12s the assessment of texts produces ideal exemplars of many different varieties. It becomes possible to master and reproduce these exemplars without access to regulating principles in mathematics, using memorisation techniques. A focus on competency in answering an expected type of question develops so that students operate on a "need-to-know" basis.

Grade 11s are generally not able to use the existing knowledge that they have to answer new questions because they have not been exposed to the process of induction into abstraction and principling - that is, mathematics discourse. This is exhibited not only by the separate knowledges they have, but also by the dependence on accurate mathematics construction.

Texts produced in this instrumental way have to reflect the strong grammar associated with legitimate texts. It is not plausible to say that the reasoning followed is purely analogic because students are sometimes able to re-describe the features of their text in an appropriate way. An under-elaborated feature of mathematics rules and principles exists and it takes the form of a segmental knowledge structure in which the written text has strong grammar and the verbal text has a much more simple grammar. A disjuncture in the discursive element exists between the two.

The extent to which texts exhibiting this disjuncture is communicable is determined by the forms of assessment. If students are never required to display their knowledge of mathematics discourse in forms other than written tests and examinations, mimetic learning of mathematics becomes privileged.

Mimetic, unprincipled and particularised learning of mathematics is generated as an intelligible and legitimate form within that style of reasoning and to understand requires an induction into style of reasoning - which, in Dowling's language, means having been completely inducted into mathematics discourse via principles of recontextualisation. The incomplete induction that occurs must be linked to the pedagogising action that occurs.

The pedagogising action positions students as the consumers of rules and principles rather than inducting them into mathematics discourse. Students know how to organise signs and symbols but do not know why it has to be so. The teacher's role as the transmitter of information does not contribute to the ongoing development of signs thus diminishing the competence in the sign system that is mathematics.

6.4 Concluding remarks

In the data analysis I have attempted to unveil the textual strategies of student texts by exploring distributing strategies of the text and its message. I have also explored the way in which the structural level of the text, informed by the format of the question offered, skews the kinds of texts that can be produced given the emphasis on the formal, axiomatic approach senior school mathematics. I have also pointed to the way in which the text of one question possibly influences forthcoming texts.

The textual analysis highlights two approaches to textual production: measuring for grade 11s and calculation for grade 12s. In each of these methodologies students were generally not able to argue discursively when presenting their texts. Subsequent content analysis of the distributing strategies coupled with the identification of ideal types pointed to largely fragmenting strategies in both grades suggesting that students have segmental mathematics knowledge even if the written text appeared to be DS^+.

The language of all the texts produced displayed esoteric competence and made use of specialised mathematics terms which could not always be discursively articulated. A dialectical relationship is evident in the way that students are recruited by the mathematics knowledge (in the questions given to them) and the mathematics knowledge that they recruit. This is reflected in the repertoire of their resources.

The algorithmic mode privileges formula and is positioned as an authorial mode for the production of legitimate texts. I have shown that proficient use of an algorithm does not mean that underlying principles are understood. Algorithmic use often revealed a disjuncture between written texts and the level of the discursive in the verbal description. Procedural use of the algorithm frequently resulted in weak mathematics relations with a diagrammatic signifying mode. Topic-dependent algorithms often do not unveil the abstracting strategies that underpin them by failing to make the metonymic links explicit. This backgrounding of metaphor is achieved by closed narratives and signifying modes that impoverishes the esoteric domain setting of the written text by fragmenting the message and particularising a seemingly principled text.

Students with limited access to mathematics discourse recruited fragments of mathematics knowledge on an "ad hoc" basis in attempt to justify the initial markings made as part of the text contributing to the ways in which textual distributing strategies largely fragment the esoteric domain setting. The mathematising gaze of the esoteric domain presents student texts that cannot be articulated in a discursive fashion. The disjuncture between the written text and the principles underpinning the signifying mode presented in the text appears to be a strong modality of student mathematics texts.

The concluding chapter will discuss some of the limitations of textual analysis of student productions of mathematics texts. Included would be further comments on pedagogic and curriculum practices and the way in which they recruit mathematics knowledge and resources. Of particular interest is the way in which knowledge is produced and reproduced within school mathematics constitutes a limiting pedagogic action.

CHAPTER 7 Discussion and conclusion

The textual analysis set out in the two previous chapters provides a framework for engaging in a systematic analysis of the present school mathematics curriculum. This is not what I wish to do. The central concern of this chapter is to consider outcomes of this research and allow me to comment on the general field of educational practice and attempt to draw the themes together in a conerent way.

Student production of text is a reflection of pedagogic practices and the way in which it has been regulated. The constraints of the agents of pedagogic practices like the teacher, the student as well as curriculum practices dictating forms of assessment and the creation of legitimate texts, will be explored. These constraints form the backdrop for understanding the way in which pedagogic practices recruits and is recruited by its agents. I will also point to the limitations of the study in terms of the limited evidence produced for a comprehensive view on mathematics discourse.

7.1 Adopting a theoretical orientation and methodology

The textual analysis produced references mathematics discourse as they are related to textual distributing strategies. Although the linguistic tools metaphor and metonymy are indexed, the extent to which discourse is exhibited in language is an important focus of the analysis.

The chapter dealing with the literature review presented research articles that could be organised into one of two groups. The first group tends to focus on analyses of students' mathematics knowledge in a way that emphasises what the individual ought to know. The second group considered the way in which the social organisation of mathematics impacts on the construction of mathematics knowledge. Research studies tended to ignore the way in which textual analyses could be used to show how mathematics knowledge was constructed and how textual analyses reflected the social organisation of school mathematics practices. Studies that explored semiotic analyses of student textual productions were rare. The survey illustrated that the production of school knowledge is a contested arena and that social interests are served in ways through the legitimation of what counts as school knowledge. My interest stemmed from the way in which students' texts interpreted what counts as legitimate texts and the way in which these texts were produced without the principled mathematics knowledge which ought to underpin the text. Since this was my interest, I explored the construction of mathematics knowledge in an attempt to develop a theoretical orientation to analysing the texts.

The literature review which informed the theoretical orientation, which underpins this study, will now be presented. Firstly, there was the perspective that mathematics knowledge could be presented as a dichotomous relationship which was polarised in terms of principled access to mathematics laws and principled recognition of mathematics. Further views on mathematics knowledge production and reproduction saw school knowledge as the production of discourse.

Dowling (1998) considers the sociology of mathematics education through his focus on the semiotic analysis of mathematics texts. He explores the sociology of interpersonal relations and class relations via the mathematics text. His work is underpinned by the theoretical notion that a mathematising gaze appropriates all that it surveys and all descriptions of texts are judged via this gaze that sanctions discursive practices (like mathematics) rather than manual and everyday practices. Mathematics brings principles of recognition and realisation which enables it to project a mathematising gaze on non-mathematics practices and redescribe them in terms of mathematics. This is achieved via his *social activity theory*. Although the texts he uses are pedagogic texts (from two textbook series in the secondary school mathematics scheme (*SMP 11-16*)), the notion that practices of an activity are realised in texts was adopted for this analysis by pointing it towards student texts.

This model provided a way of reading texts as material instances of an activity in which specialised domains practices structured by the activity are (re)produced as message and voice in texts. Student texts as the activity at the structural level of activity were viewed as pedagogic discourse around the teaching and learning of mathematics. Voices in the text are the student, the teacher, and mathematicians. The resources recruited to establish textual strategies entailed existing mathematics knowledge indexed by students' verbal descriptions of the texts they produced. The analysis thus considered the actual written text and explored how much of that production could be justified mathematically.

Dowling's framework for the analysis of textual strategies was not fine grained enough to consider the possibility that legitimate mathematics texts may be produced without the necessary mathematics knowledge. A comprehensive list of ideal types, based on the range of possibilities of how students could produce texts, were created to act as an interface between the empirical text and Dowling's analytic framework.

My use of Dowling (1998) has restricted this analysis to considering mainly textual distributing strategies and message in the texts. This necessarily excludes many other aspects related to school mathematics practices and methodological issues, which have not been explored. For example, I did not significantly explore issues relating to the assessment criteria in school mathematics. The analysis is thus selective and cannot be regarded as exhaustive.

7.2 Discussion of textual analysis of student texts: constraints and limitations

It is important to bear in mind that this analysis regards student texts as closed. This means that the immediacy of the research situation with regard to the students does not index specific social relations of a particular context. It rather represents an instance of school mathematics practice.

The analysis of student mathematics texts has highlighted a number of features. The most striking feature is not that students are incapable of engaging with mathematics knowledge and discourse , but that the need for mathematics discourse is not crucial to the production of texts. The discussion will focus on the divisions and distributions in mathematics that are manifested in a proficient mathematics knowledge which cannot always establish relational links with texts produced. In chapter 6 I argued that the pedagogising action of school mathematics practices positions students as the consumers of mathematics rules and principles rather than inducting them into mathematics discourse.

Rotman distinguishes three aspects of discourse.

> There is the referential aspect, which concerns itself with the code's secondarity, with the objects of discourse, the things that are supposedly talked about and referred to by the signs of the code; the formal aspect, whose focus is on the manner and form of the material means through which the discourse operates, its physical manifestation as a medium; and the psychological aspect, whose interest is primarily in those interior meanings which the signs of the code answer to or invoke
>
> Rotman (1988: 3-4)

The three aspects of discourse, Rotman argues, do not readily lend itself to a semiotic account of mathematics. Nevertheless he argues that these aspects of discourse can be put in perspective via a consideration of what constitutes mathematics. Firstly, it may be considered (by the formalist) as written marks which can be transformed by explicit formal rules. Secondly it may be considered as a mental construction of an individual. Finally, it may also be considered as a discipline concerned with discovering and validating propositions.

The relevance of these accounts in this chapter demonstrates the need for mathematical activity to encapsulate all three accounts of what constitutes discourse.

The analysis of texts has pointed to a fundamental difference between the two groups of students (Grade 11s and Grade 12s). The grade 12s, who have been taught analytic geometry, favour the use of formula and procedures. As the analysis in chapters 5 and 6 indicate, the use of formulae and procedures is increasingly foregrounded as the level of complexity in the mathematics problem posed increases. This illustrates the particularising of mathematics discourse.

Analysis of the grade 11 texts, on the other hand, show a progressive dependence on localising, empirical strategies like measurement as well as an increased reluctance to produce formal written texts.

The pedagogising action differs in the two groups. For the grade 12s it seems as if pedagogising action focuses mainly on the relaying of procedures and mathematics knowledge. The extent of the focus on procedure as opposed to a focus, which prioritises abstraction of mathematics, is speculative since no empirical evidence exists. Nevertheless, textual analysis produces very little evidence of this and this seems to suggest that more developed principling skills are not deemed necessary for the production of legitimate texts.

The absence of principled mathematics strategies in the grade 11s suggest a trend in school mathematics practices. The grade 11s have encountered the necessary mathematics knowledge elsewhere in school mathematics. The fact that they are unable to establish relations between existing knowledge and seemingly new mathematics problems poses serious questions about the way in which their knowledge has been constructed. The tendency of school mathematics practices to produce texts that are insular and related to mathematics in terms of different topics points to a segmental rather than an integrated orientation in mathematics knowledge.

Discursive practices like textbooks, teaching methods and examinations can only be inferred because no empirical evidence exists.

Nevertheless texts were the prime focus of this research particularly since students were able to produce them without having to base their productions on mathematics rules and principles. In the absence of generalising principles to generate texts, the only other means available to students was memory. This suggests that segments are insular and this reinforces the suggestion that segmental organisation and compartmentalization of mathematics knowledge is pervasive. Textual production invariably involved mimetic learning and extensive applications of algorithms that were not principally understood as well as extensive use of exemplars.

Although physical evidence of different exemplars is not given, the questionnaires distributed to students were based on typical questions that appeared in textbooks, tests and examinations. The principles of meaningful organisation of algorithms and exemplars are not determined by a system of connections established by the canons of mathematics. Written texts based on the pragmatic use of the algorithm taints the constitution of mathematics texts at senior high school level. Unlike Dowling (1993, 1998) who completed a textual analysis of two textbooks in the secondary school mathematics scheme (*SMP 11-16*) for high and low ability students, the texts produced are a response to very specific questions and all students selected were high ability students.

Esoteric domain message of the written text produced appear to be DS^+ since they appear to have appropriated the mathematising gaze of the esoteric domain. Esoteric domain message is generally discursive. However, location of the texts in the esoteric domain cannot make assumptions about the expertise of students as authors of texts. The accompanying verbal description and its level of discursive is a valid indicator of whether or not principled mathematics knowledge underpins the text.

The textual analysis of student texts has shown that texts with a generalising distributing strategy hardly ever occur. Its absence makes it seem as if the message of the text is abstracted from the esoteric domain in a way which fools the reader into thinking that the author of the text has been mathematised by the by the gaze of the esoteric. The methodology used to derive the text functions only in the parameters of the problem since the text is not readily transferable.

Student texts use signs that connote generalised mathematics knowledge, like equations for example. However, fragmenting and localising strategies particularise mathematics discourse. While the range of the text is expanding in fragmenting and limiting in localising strategies, authorisation of the text tends to reside with an external group of mathematics experts, like teachers and the community of mathematicians.

It thus appears that the distributing strategies evident in student text distribute message in a way that positions students as dependents so that they have little to say in terms of the regulation of mathematics activity.

Student texts appear to present an element of a formalist approach to mathematics as a series of transformations according to explicit rules. The textual analysis shows that attempts to develop any form of signification are markedly absent. Rotman (1998: 4-5) likens this view of mathematics to a game in chess where the moves that may be made to carry significance, actually function independently of this significance. In Rotman's language "formalism reduces mathematical signs to material signifiers which are, in principle, without signifieds". Textual analysis of student texts clearly shows that this formalism is part of mathematics activity. Engagement with material signifiers like formulae is an important part of mathematics texts. As important is the recognition of any prior knowledge in establishing relational links. This too is part of mathematics activity. In Dowling's language (1998) recognition of prior knowledge is part of the resource level of Social activity.

The textual analysis suggests that curriculum practice shapes mathematics activity in a way that impoverishes the construction of mathematics knowledge. This is contrary to the principles of the Western Cape Education Department's (WCED) mathematics syllabus in which students are described as participants in the learning situation.

> The pupil is not a passive receiver of knowledge, but an active participant who learns by re-organising and restructuring his/her present knowledge structures, and this can only be done by the pupil himself or herself.
>
> WCED, 1997: 2

Clearly, the pedagogising action of school mathematics practices does not realise this WCED principle. Instead of active participation, the texts suggest that the pedagogising action is one in which mathematics knowledge has been relayed with very little participation. Students as authors of texts emerge as a consequence of interactional situations of classroom practice. While students are ideally described as active participants rather than passive receivers, students' texts do not exhibit this ideal.

One of the concerns of this study was the way in which students learn mathematics in what appeared to be isolated segments and more interestingly the way in which this kind of learning culminates in what appears to be proficient mathematics learning at the end of high school mathematics. Out of this emerged the primary consideration of the way in which the cumulative organisation of school mathematics (re)produces students with an underdeveloped view of mathematics discourse.

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The language used in the description of student texts did not exhibit a high level of discursive. The hypothesis that students recruit and are recruited by mathematics knowledge associated with school was put forward. This extent to which mathematics knowledge is recruited by school is evidenced by the forms of assessment that are privileged. Implications for this hypothesis are that assessment can skew the focus of developing mathematics knowledge. Assessment is the yardstick by which students and teachers pace themselves to ensure that the students are adequately prepared. The practice of evaluation thus forms a significant part of the pedagogic practice.

The pedagogic relationship in school mathematics is not one which centres on the broader mathematics field. Students themselves are not being trained to be mathematicians. Instead they are objectified by the mathematics curriculum in a way that measures their ability. This comment is justified by the way in which an external examination at the end of high school regards students' performance as an indicator of mathematics ability and suitability for further study or employment. This further illustrates the way in which pedagogic relations and subsequent pedagogic action may develop a focus on competence and performance rather than mathematics subjects.

Relations in pedagogic practices are also influenced by the discursive regulations of the teacher as well as published textbooks. These factors do not form part of this study. McBride (1989) identifies four discursive practices in mathematics based on her Foucauldian analysis of mathematics discourse. These are textbooks, teaching methods, examinations and use of space. This study does not address any of these categories singularly. The role of the textbook, teaching methods and assessment are implicated but are not addressed in the study.

School practices were recognised to be a complex form of social life that that could structure and dictate the learning of mathematics in different ways. This study cannot, of course, address all the complex forms of social life. What it can do is highlight the findings of the study as a particular instance of the social while simultaneously acknowledge its limitations.

The mode of textual interrogation describes the way in which students of mathematics are inducted into the mathematics field based on their performance rather than their competence. The authority of the student as the author of the text reflects the dependency on the teacher in delivering mathematics content. Very little can be said about pupil teacher-roles and modes of interaction. Very little can also be said about material resources that students may have had access to. Support can rather be found in the texts representing school mathematics knowledge (official mathematics syllabus and a selection of text books); the content of school mathematics; the symbols concepts, conventions, symbolic procedures, and linguistic presentations of mathematical knowledge.

A question that remains unanswered is "To what extent must the structure of mathematics be understood in order for it to be used effectively during the production of mathematics texts?" Evidently the rehearsing of a selection of privileged texts is not sufficient.

To sum up briefly, student texts are indicators of pedagogic practices as well as an indicator of how it is that students recruit and are recruited by mathematics knowledge. Assessment plays a significant role in skewing the way in which this occurs. The way in which students position themselves as authors of the texts is related to the textual strategies which underpin their production. The emphasis on fragmenting strategies is a cause for concern because of the underdeveloped abstracting skills necessary in mathematics. Student textual productions should thus be regarded as an independent entity. They should rather be viewed as yardsticks for gauging school mathematics practices.

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4A

QUESTIONNAIRE

Question 1:

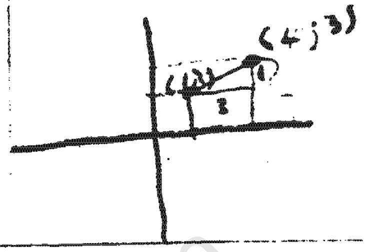
If the points R(1;2) and Q(4;3) form a line, what is the length of RQ?

$RQ = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$\therefore RQ^2 = (1-4)^2 + (2-3)^2$

$= (9) + (1)^2$

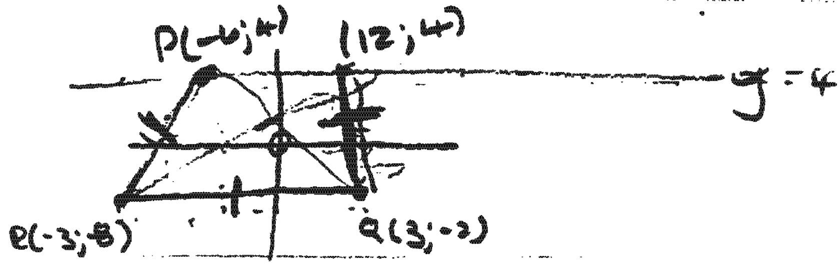
$\therefore RQ = \sqrt{10}$



7x

Question 2:

Consider the points P, Q(3;-2) and R(-3;-8). P is in (a;4) in the second quadrant. If PQ = QR, find a.



PQ = QR

$\sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2} = \sqrt{(x_Q - x_R)^2 + (y_Q - y_R)^2}$

$(a - 3)^2 + (4 + 2)^2 = (3$ +3 [redacted]$)^2 + (-2 + 8)^2$

$a^2 - 6a + 9 +$ 36 [redacted] $= 81 + 36$

$a^2 - 6a$ [redacted] $-72 = 0$

$(a - 12)(a + 6) = 0$

$\therefore a = 12 \; ; \; a = -6$

a is II quad

$\therefore a =$ [redacted] 12

Question 3:

Do the lines y = x – 1 and y = -½x + 2 intersect? Yes

Why? There gradients are not equal

If they do, then where? at [redacted] $x = 2$; $y = 1$

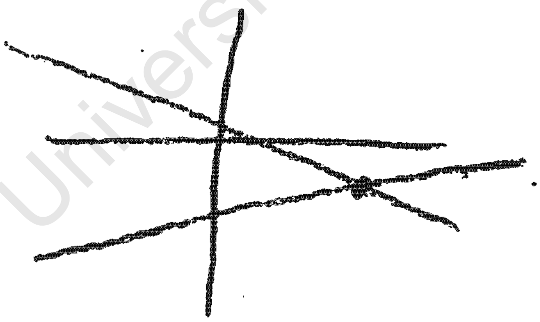
Why? This is where the 2 lines meet ie - the pt that satisfies both equns.

$y = x - 1$ $\qquad$ $y = -\frac{1}{2}x + 2$

$x - 1 = -\frac{1}{2}x + 2$

$1\frac{1}{2}x = 3$

$x = 2$

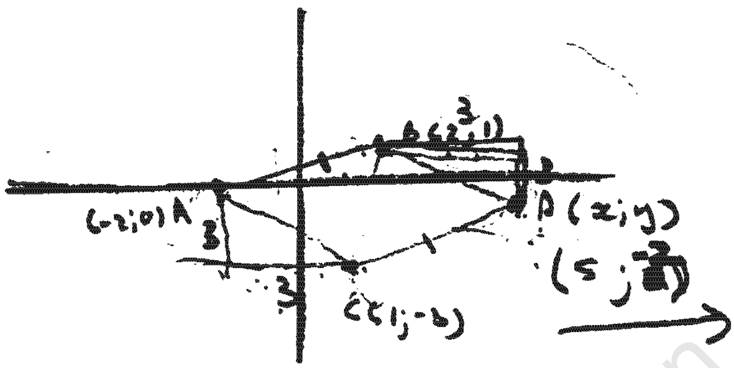


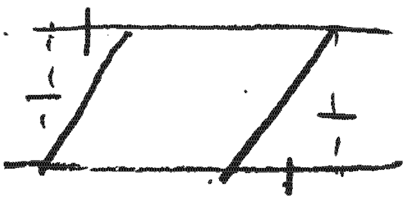
Ex. V

Question 4:

If the points A(-2;0), B(2;1), C(1;-3), D(x;y) form a parallelogram, what are the coordinates of D?

Can you suggest another way of arriving at the same solution?





AB = CD

$(x_A - x_B)^2 + (y_A - y_B)^2 = (x_C - x_D)^2 + (y_C - y_D)^2$

$(-2-2)^2 + (0-1)^2 = (1-x)^2 + (-3-y)^2$

$16 = -2x + x^2 + 9 + 4y + y^2$

$= x^2 + y^2 + 4y - 2x = -7$

AC = BD

$(-2-1)^2 + (0+3)^2 = (2-x)^2 + (1-y)^2$

$9 + 9 = 4 - 4x + x^2 + 1 - 2y + y^2$

$+4x + 2y + 3 + 4y - 2x - 7$ $= y^2 + x^2 - 4x - 2y + 5 - 18$

$4x\ 2x + 11y + 2 = 0$ $= y^2 + x^2 - 4x - 2y - 13$

$y^2 = -x^2 + 4x + 2y + 3$

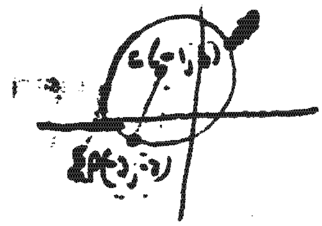
Incorrect

4A

Question 5:

A circle with centre (-1;3) passes through P(-3;-2). What will the equation of the circle be?

$\therefore x^2+y^2=r^2$



$CP = \sqrt{(x_c - x_p)^2+(y_c-y_p)^2}$

$CP = \sqrt{(-1+3)^2+(3+2)^2}$

$CP = \sqrt{(4)+(25)}$

$CP = \sqrt{29}$

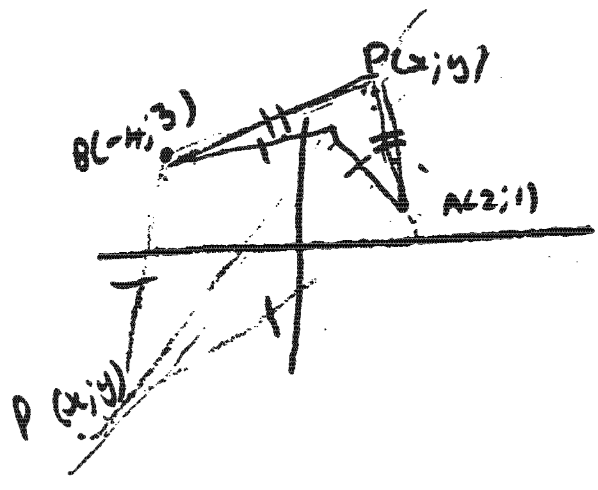
$\therefore x^2+y^2 = \sqrt{29}$

Incorrect.

4A

Question 6:

The point P(x;y) is equidistant from A(2;1) and B(-4;3).
What will the equation of the set of points equidistant from A and B be?



BP = AP

$(x+4)^2 + (y-3)^2 = (x-2)^2 + (y-1)^2$

$~~x^2~~ + 8x + 16 + ~~y^2~~ - 6y + 9 = ~~x^2~~ - 4x + 4 ~~+y^2~~ - 2y + 1$

$8x - 6y + 25 = -4x - 2y + 5$

$12x + 20 = 4y$

$4y = 12x + 20$

$y = 3x + 5$.

correct

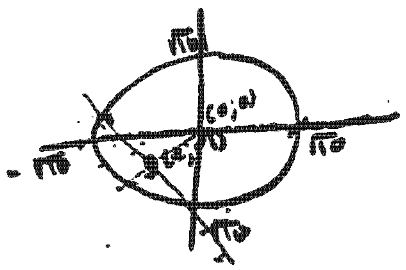
4A

Question 7:

Given the circle with equation $x^2 + y^2 = 10$.
The midpoint of chord AB in the third quadrant is D(-2;-1). What are the co-ordinates of A and B?

$(x-0)^2 + (y-0)^2$

$x^2 \quad y^2$



$y = mx + c$

$y = -2x + c$

$(-2;-1)$ $\quad -1 = -2(-2) + c$

$-1 = +4 + c$

$c = -5$

$y = -2x - 5$

$m = \frac{y_1 - y_2}{x_1 - x_2}$

$= \frac{0+1}{0+2}$

$= \frac{1}{2}$

$\therefore m_1 = -2$

$y = -2(-3) - 5$

$y = 1$

$y = -2(-1) - 5$

$y = -3$

$x^2 + y^2 = 10$

$x^2 + (-2x-5)^2 = 10$

$x^2 + 4x^2 + 20x + 25 = 10$

$5x^2 + 20x + 15 = 0$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$\therefore x = -3; \; x = -1$

A(-1; -3) $\quad$ B(-3; 1)

$x^2 + y^2 = 10$

$(-3)^2 + y^2 = 10$

$9 + y^2 = 10$

$y^2 = 1$

$y = \pm 1$

$x^2 + y^2 = 0$

$(-1)^2 + (y)^2 = 10$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

QUESTIONNAIRE

Question 1:

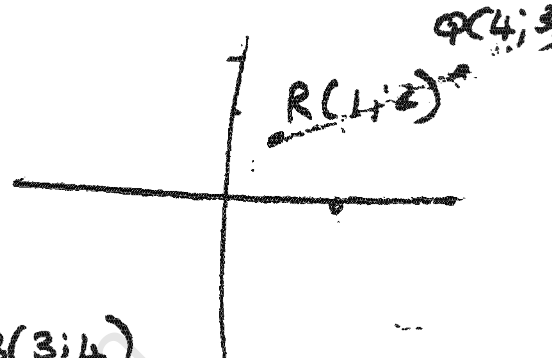
If the points R(1;2) and Q(4;3) form a line, what is the length of RQ?

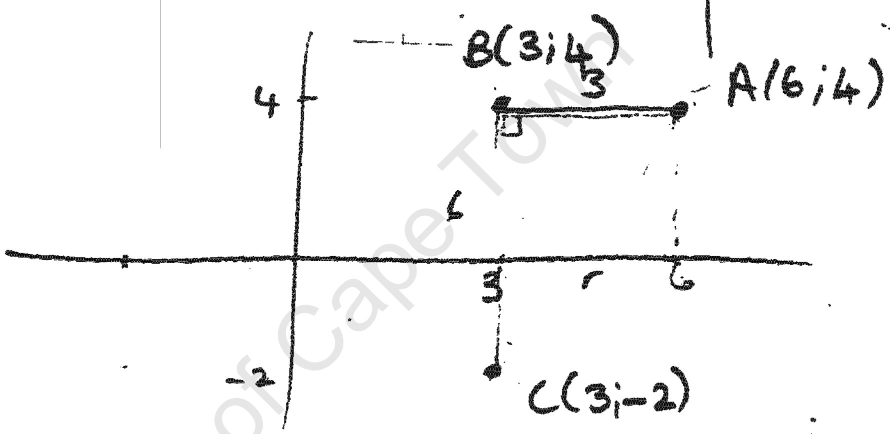
$RQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$RQ = \sqrt{(4-1)^2 + (3-2)^2}$

$RQ = \sqrt{9 + 1}$

$RQ = \sqrt{10}$





$r^2 = (6)^2 + (3)^2$

$= 36 + 9$

$= 45$

$r = \sqrt{45}$

$AC = \sqrt{(3-6)^2 + (-2-4)^2}$

$= \sqrt{(-3)^2 + (-6)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

Question 2:

Consider the points P, Q(3;-2) and R(-3;-8). P is in (a;4) in the second quadrant.
If PQ = QR, find a.

P(a;4)

$$QR = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$QR = \sqrt{(-3-3)^2 + (-8+2)^2}$$

$$QR = \sqrt{(-9)^2 + (-6)^2}$$

$$= \sqrt{81 + 36}$$

$$QR = \sqrt{117}$$

$$QP^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$$\sqrt{117} =$$

$$117 = (a-3)^2 + (4+2)^2$$

$$117 = a^2 - 6a + 9 + 36$$

$$0 = a^2 - 6a + 45 - 117$$

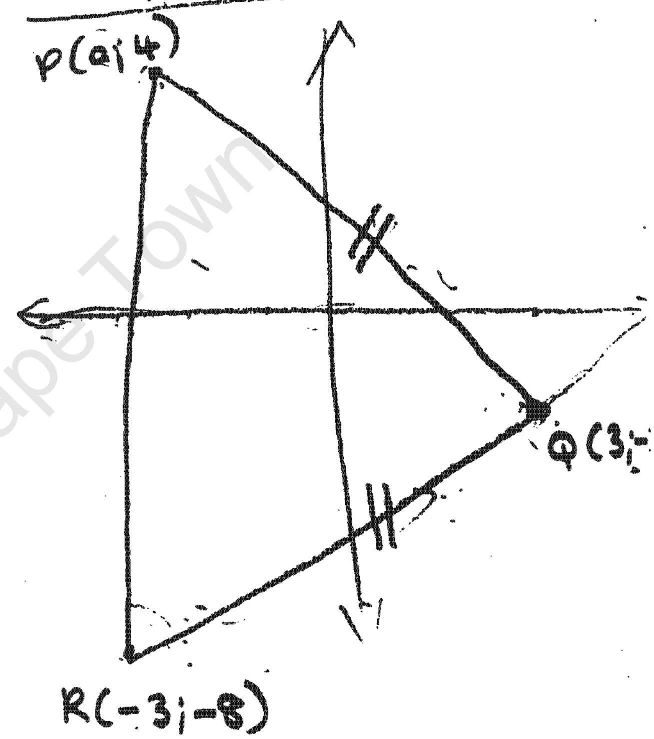
$$0 = a^2 - 6a - 72$$

$$0 = (a - 12)(a + 6)$$

$$a = 12 \; ; \; a = -6$$

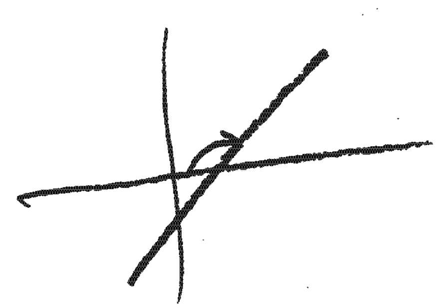
$$a = -6$$

Q(3;-2) R(-3;-8)



8
9

$\tan\theta = m$



Question 3:

Do the lines y = x – 1 and y = -½x + 2 intersect?

Why? cross each other

If they do, then where? (2;1)

Why?

intersect

$$x - 1 = -\frac{1}{2}x + 2$$

$$x + \frac{1}{2}x = 3$$

$$\frac{3}{2}x = 3$$

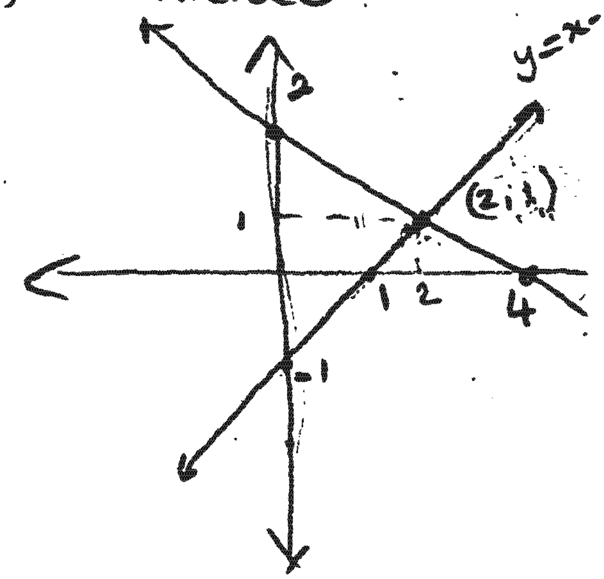
$$x = 2$$

$$\frac{3}{\frac{3}{2}} = \frac{3}{1} \times \frac{2}{3} = \frac{6}{3} = 2$$

$y = 2 - 1$
$y = 1$

(2;1)

intersect



$1 = 2 - 1$
$1 = 1$

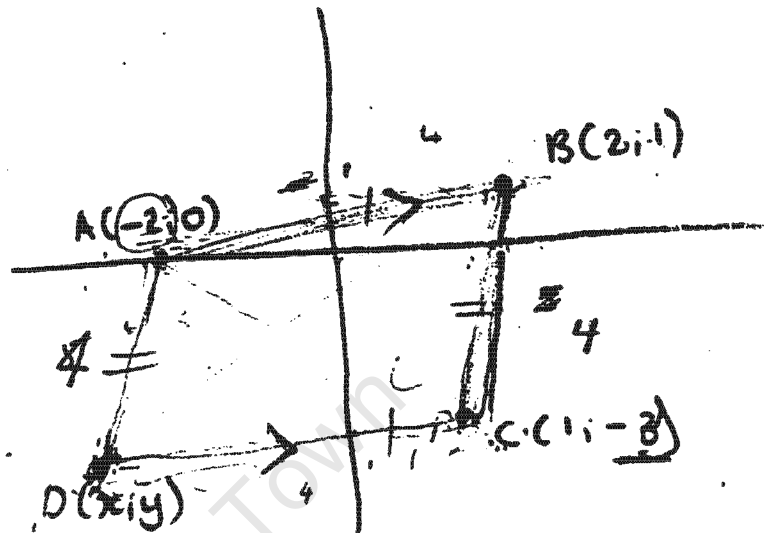
$1 = -\frac{1}{2}(2) + 2$
$1 = -1 + 2$
$1 = 1$

y=
$0 = -\frac{1}{2}x + 2$
$\frac{1}{2}x = 2$
$x = 4$

<u>Question 4:</u> ①

If the points A(-2;0), B(2;1), C(1;-3), D(x;y) form a parallelogram, what are the coordinates of D?

Can you suggest another way of arriving at the same solution?



$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$AB = \sqrt{(2+2)^2 + (1-0)^2}$

$AB = \sqrt{16+1}$

$= \sqrt{17}$

$DC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$DC = \sqrt{(1-x)^2 + (-3-y)^2}$

$17 = 1 - 2x + x^2 + 9 + 6y + y$

$0 = x^2 - 2x + y^2 + 6y - 7$

$y = 4$

$y = -4$

$x = -2$

$BC = \sqrt{(1-2)^2 + (-3-1)^2}$

$= \sqrt{(-1)^2 + (-4)^2}$

$= \sqrt{1+16}$

$= \sqrt{17}$

$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$

$= \frac{1-0}{2+2}$

$= \frac{1}{4}$

$m_{DC} = \frac{y_C - y_D}{x_C - x_D}$

$\frac{1}{4} = \frac{-3-y}{1-x}$

$1 - x = 4(-3-y)$

$1 - x = -12 - 4y$

$0 = -13 - 4y + x$

$x =$

②

48

Question 4:

If the points A(-2;0), B(2;1), C(1;-3), D(x;y) form a parallelogram, what are the coordinates of D?

Can you suggest another way of arriving at the same solution?

$(4y+13)(4y+13)$

2

$x = 4y + 13$ ⟶

$0 = (4y+13)^2 - 2(4y+13) + y^2 + 6y - 7$

$0 = 16y^2 + 104y + 169 - 8y - 26 + y^2 + 6y - 7$

$0 = 17y^2 + 102y + 136$

$0 = (17y + 68)(y + 2)$

17y 17y 68
y 2

$y = -\frac{68}{17}$; $y = -2$

$y = -4$; $y = -2$

68
2

$x = 4(-4) + 13$

$x = -16 + 13$

$x = -3$

⟶

$x = 4(-2) + 13$

$x = -8 + 13$

$x = 5$

⟶

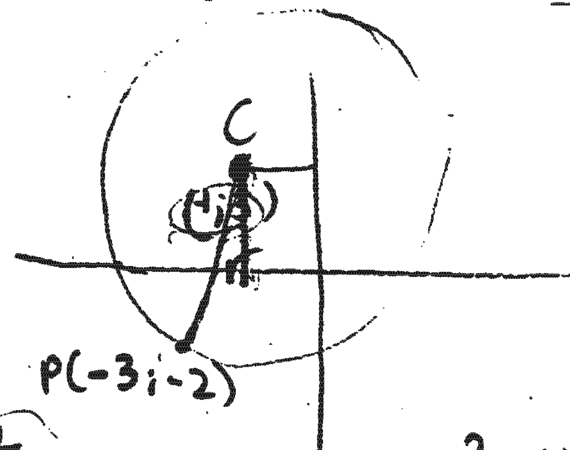
$(-3; -4)$

⟶

$(-2; 5)$

Question 5:

A circle with centre (-1;3) passes through P(-3;-2). What will the equation of the circle be?



$(x+1)^2 + (y-3)^2 = r^2$ $\qquad (x-a)^2+(y-b)^2$

$(a;b)$

$CP^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (-3+1)^2 + (-2-3)^2$

$= (-2)^2 + (-5)^2$

$= 4 + 25$

$= 29$

$r^2 = 29$

$r = \sqrt{29}$

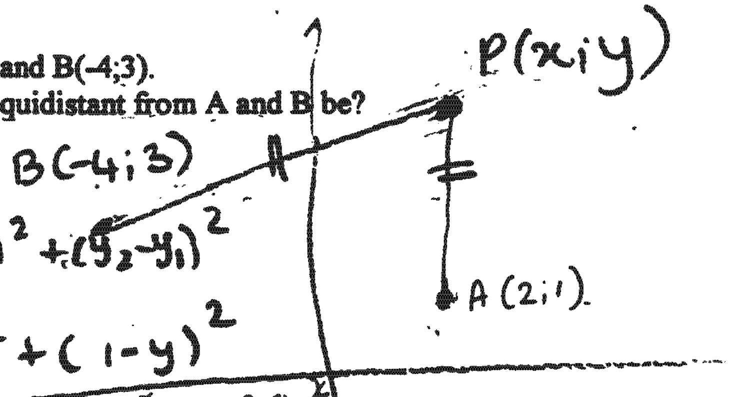
$(x+1)^2 + (y-3)^2 = 29$

$x^2 + 2x + 1 + y^2 - 6y + 9 - 29 = 0$

$x^2 + 2x + y^2 - 6y - 19 = 0$

Question 6:

The point P(x;y) is equidistant from A(2;1) and B(-4;3).
What will the equation of the set of points equidistant from A and B be?



$PB^2 = PA^2$

$(x_2-x_1)^2 + (y_2-y_1)^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$(-4-x)^2 + (3-y)^2 = (2-x)^2 + (1-y)^2$

$16 + 8x + x^2 + 9 - 6y + y^2 = 4 - 4x + x^2 + 1 - 2y + y^2$

$8x + 4x - 6y + 2y - 5 + 25 = 0$

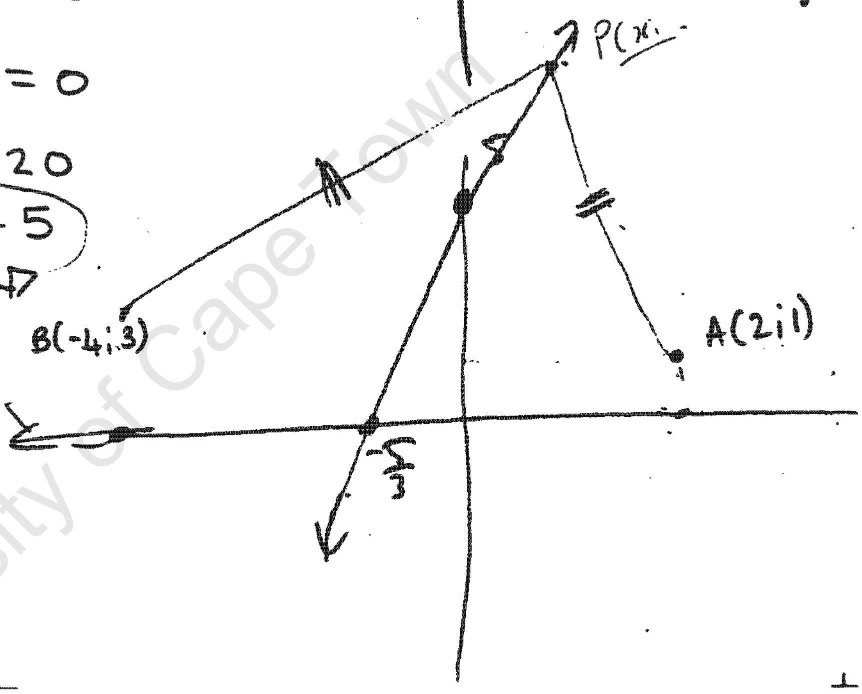
$12x - 4y + 20 = 0$

$4y = 12x + 20$

$y = 3x + 5$

$1 = 3(2) + 5$

$1 = 11$



(1½)

$0 = 3x + 5$

$3x = -5$

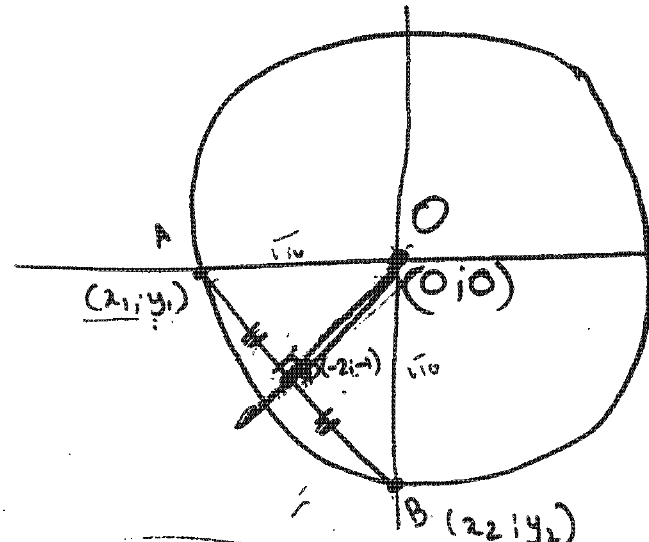
$x = -\frac{5}{3}$

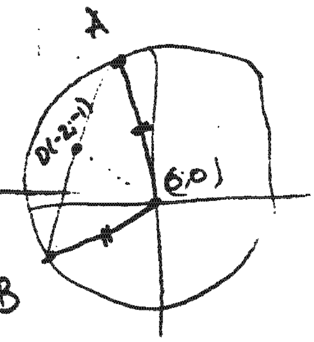
48 $(x-a)^2+(y-b)^2=r^2$

$(a;b)$ $(0;0)$

Question 7:

Given the circle with equation $x^2 + y^2 = 10$.
The midpoint of chord AB in the third quadrant is D(-2;-1). What are the co-ordinates of A and B?





Mid $D_x = \frac{x_1+x_2}{2}$

$-\frac{2}{1} = \frac{x_1+x_2}{2}$

$x_1+x_2 = -4$

Mid $D_y = \frac{y_1+y_2}{2}$

$-\frac{1}{1} = \frac{y_1+y_2}{2}$

$y_1+y_2 = -2$

$AD^2 = DB^2$

$m_{OD} \perp m_{AB}$

$m_{OD} \cdot m_{AB} = -1$

$m_{OD} = \frac{-1-0}{-2-0}$

$= \frac{1}{2}$

$m_{AB} = -2$

$m_{AB} = \frac{-1-y}{-2-x}$

$\frac{-2}{1} = \frac{-1-y}{-2-x}$

$-2(-2-x) = -1-y$

$4+2x = -1-y$

$y = -2x-5$

$m_{DB} = \frac{y+1}{x+2}$

$\frac{-2}{1} = \frac{y+1}{x+2}$

$-2(x+2) = y+1$

$-2x-4 = y+1$

$-2x-5 = y$

7

Question 7:

Given the circle with equation $x^2 + y^2 = 10$.
The midpoint of chord AB in the third quadrant is D(-2;-1). What are the co-ordinates of A and B?

$$\frac{-2}{1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y_2 - y_1 = -2x_2 + 2x_1$$

$$x^2 + (-2x - 5)(-2x - 5) = 10$$

$$x^2 + 4x^2 + 20x + 25 - 10 = 0$$

$$5x^2 + 20x + 15 = 0$$

$$x^2 + 4x + 3 = 0$$

$$(x + 3)(x + 1) = 0$$

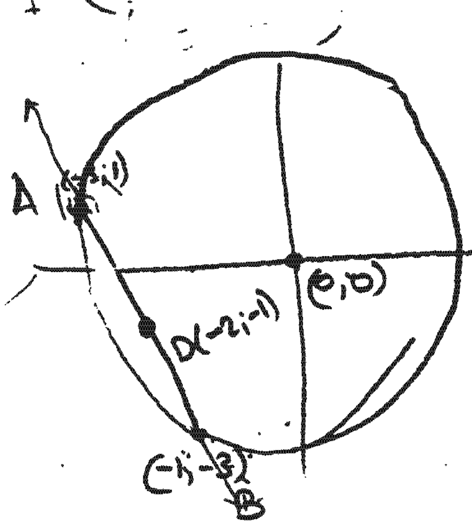
$$x = -3; \quad x = -1$$

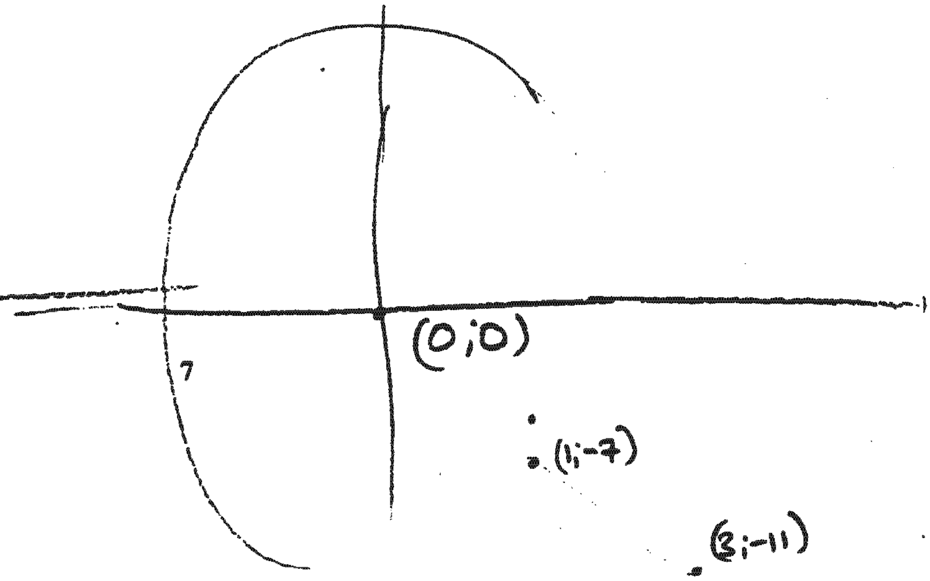
$$y = -2(-3) - 5$$
$$y = +6 - 5$$
$$y = -1$$

$$y = -2(-1) - 5$$
$$y = +2 - 5$$
$$y = -3$$

$(-1;-3)$

$(-3;1)$



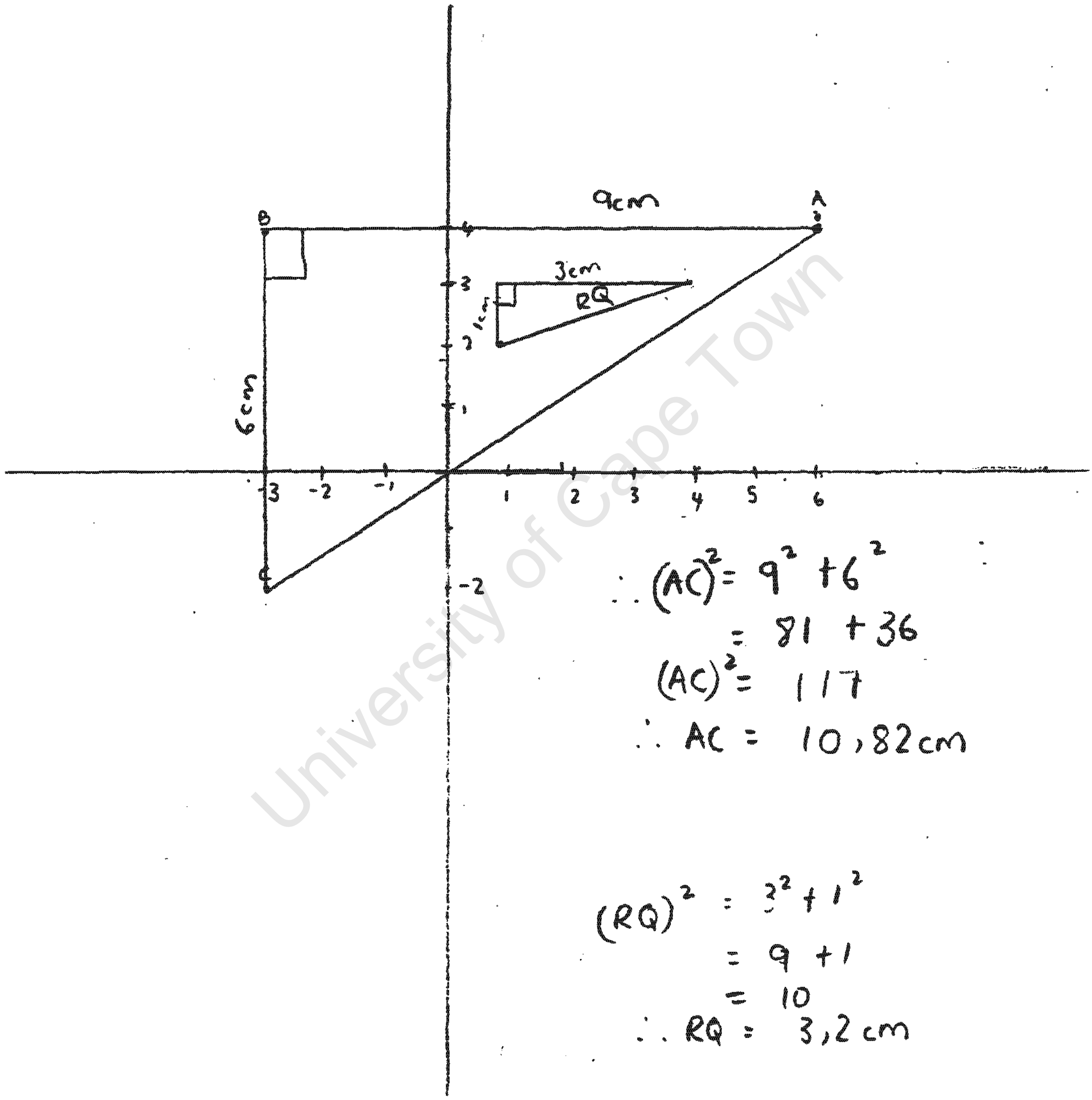


QUESTIONNAIRE

4c

Question 1:

If the points R(1;2) and Q(4;3) form a line, what is the length of RQ? 3,3cm



$\therefore (AC)^2 = 9^2 + 6^2$

$= 81 + 36$

$(AC)^2 = 117$

$\therefore AC = 10{,}82\text{cm}$

$(RQ)^2 = 3^2 + 1^2$

$= 9 + 1$

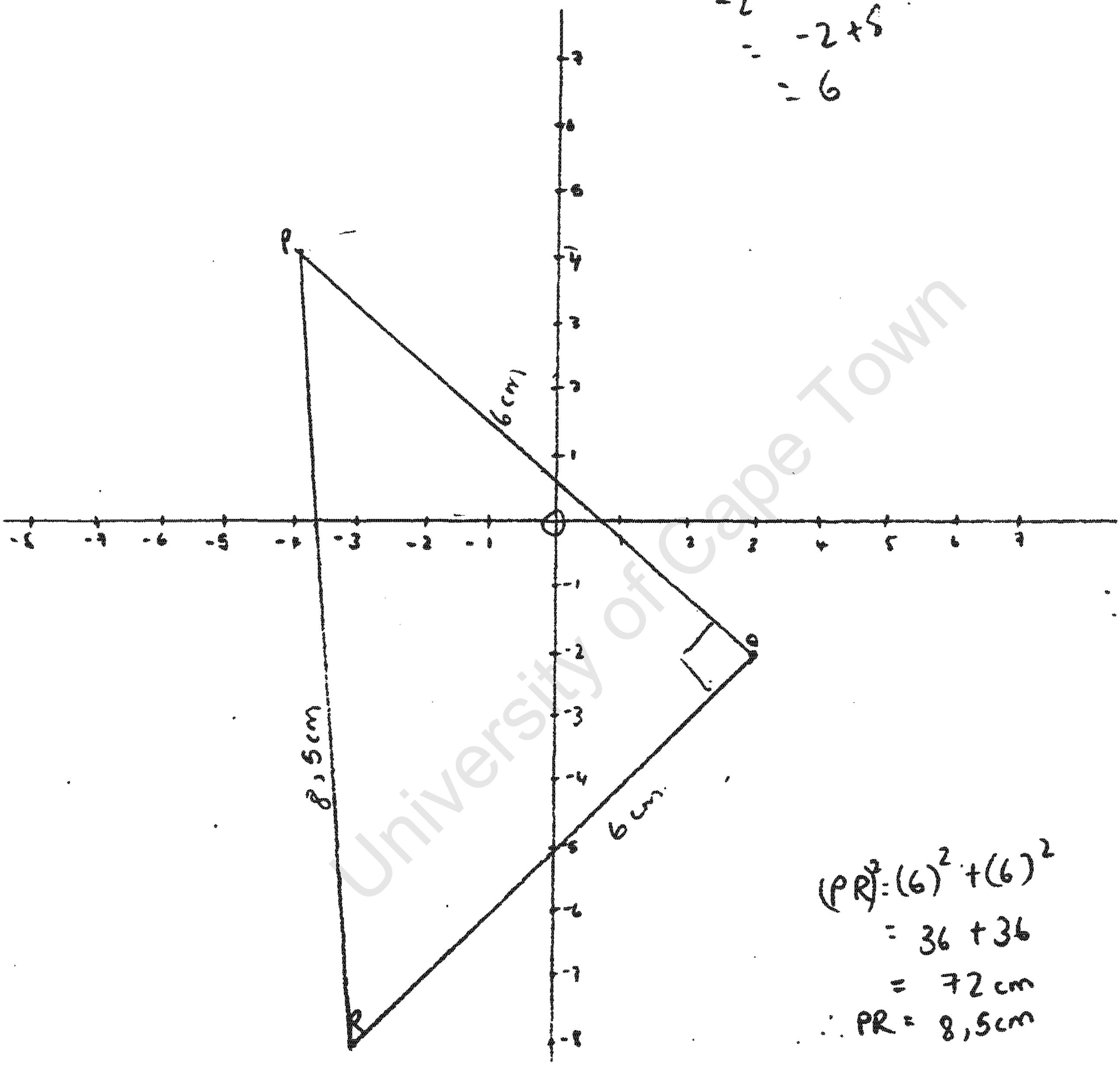
$= 10$

$\therefore RQ = 3{,}2\text{ cm}$

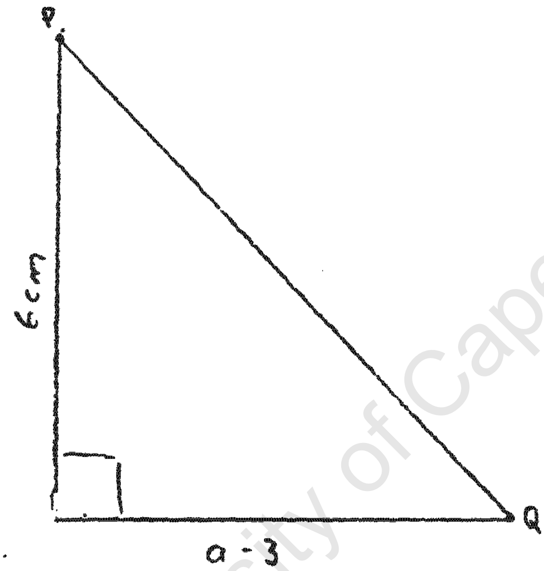
Question 2: 4c

Consider the points P, Q(3;-2) and R(-3;-8). P is in (a;4) in the second quadrant.
If PQ = QR, find a.

$-2-(-8)$
$= -2+8$
$= 6$



$(PR)^2 = (6)^2 + (6)^2$
$= 36 + 36$
$= 72$ cm
$\therefore PR = 8,5$cm



$$(PQ)^2 = (6)^2 + (a-3)^2$$
$$(PQ)^2 = 36 + a^2 - 6a + 9$$
$$= a^2 - 6a + 45$$

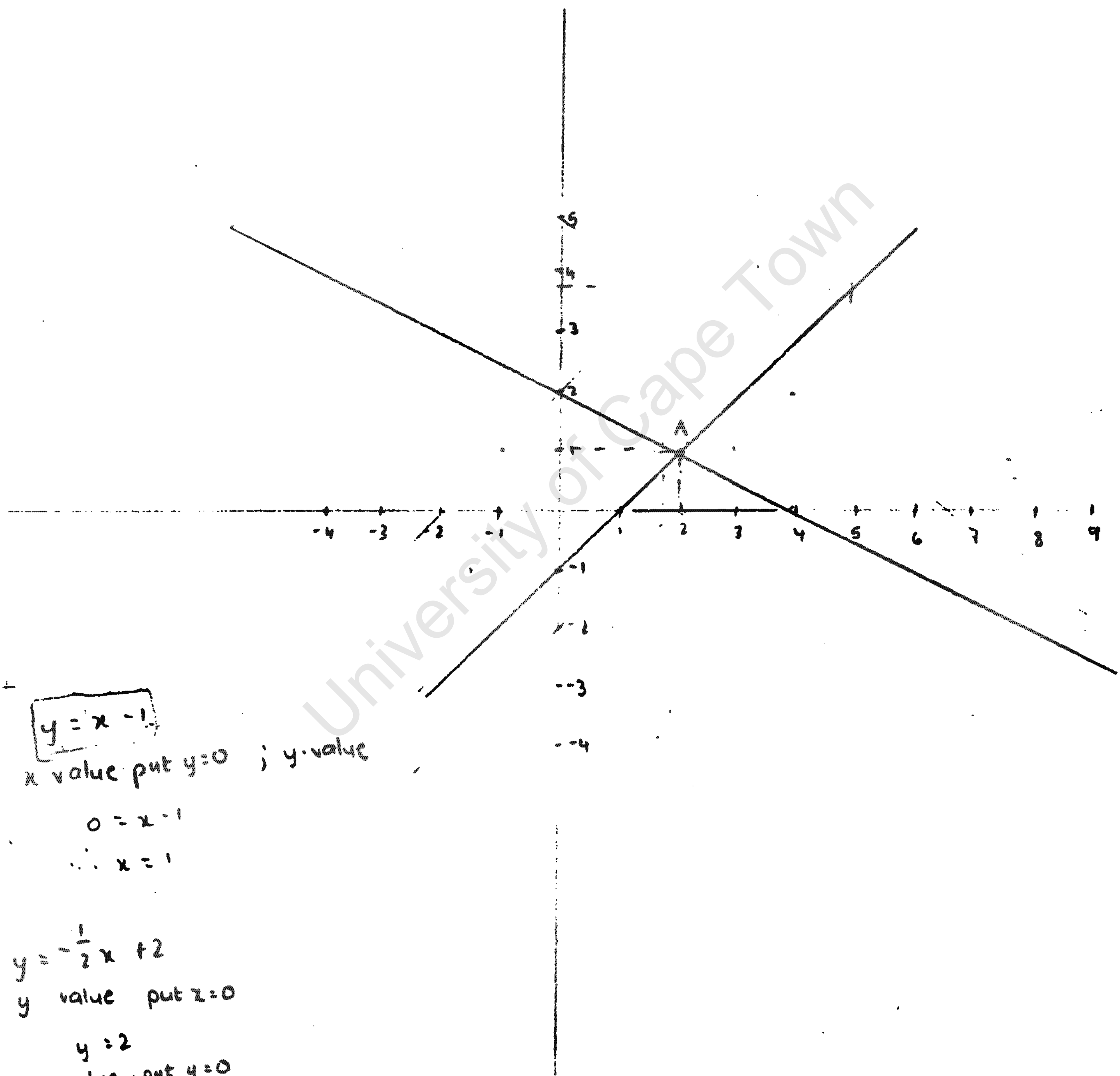
Question 3:

Do the lines $y = x - 1$ and $y = -\frac{1}{2}x + 2$ intersect? yes

Why? The lines run in opp. directions
$y = x - 1$ being ⊕ ; $y = \frac{1}{2}x + 2$ being ⊖

If they do, then where? 2 ; 1

Why?



$y = x - 1$
x value put y=0 ; y-value

$0 = x - 1$
$\therefore x = 1$

$y = -\frac{1}{2}x + 2$
y value put x=0

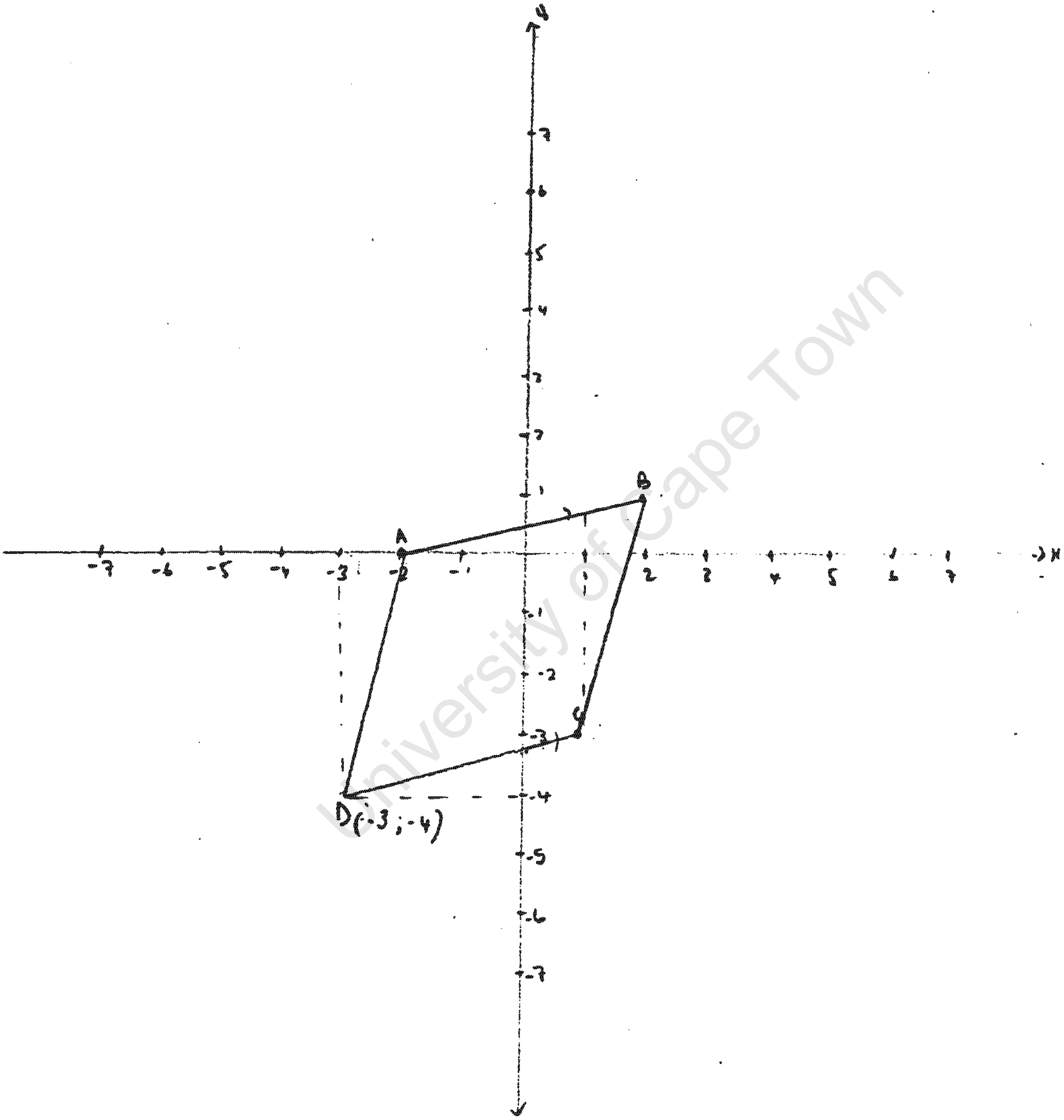
$y = 2$
x value : put y=0
$0 = -\frac{1}{2}x + 2$
$+\frac{1}{2}x = 2$
$x = \frac{2}{\frac{1}{2}}$
$= 2 \times 2 = 4$

4c

Question 4:

If the points A(-2;0), B(2;1), C(1;-3), D(x;y) form a parallelogram, what are the coordinates of D? D = (-3 ; -4)

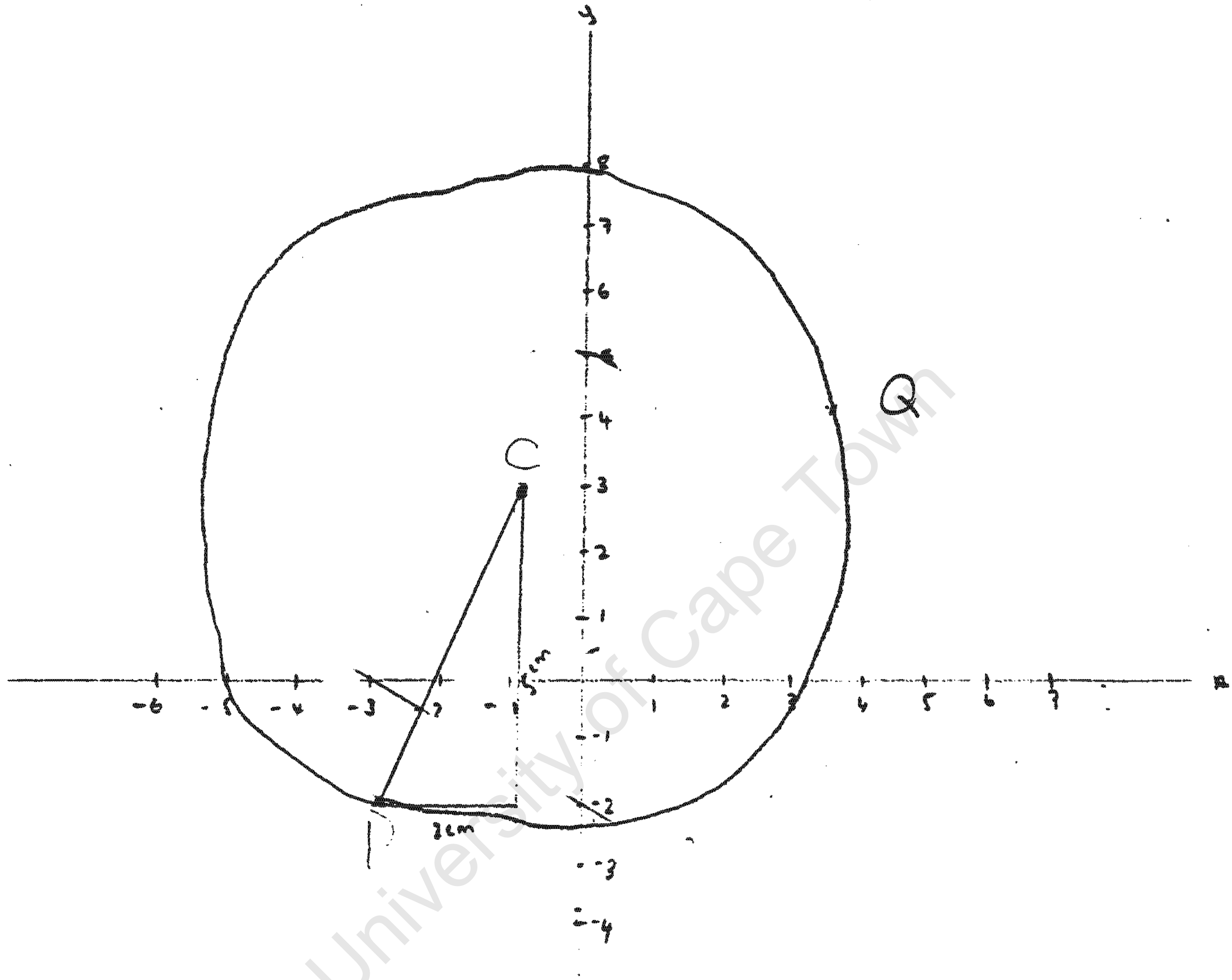
Can you suggest another way of arriving at the same solution?



4c

<u>Question 5:</u>

A circle with centre (-1;3) passes through P(-3;-2). What will the equation of the circle be?

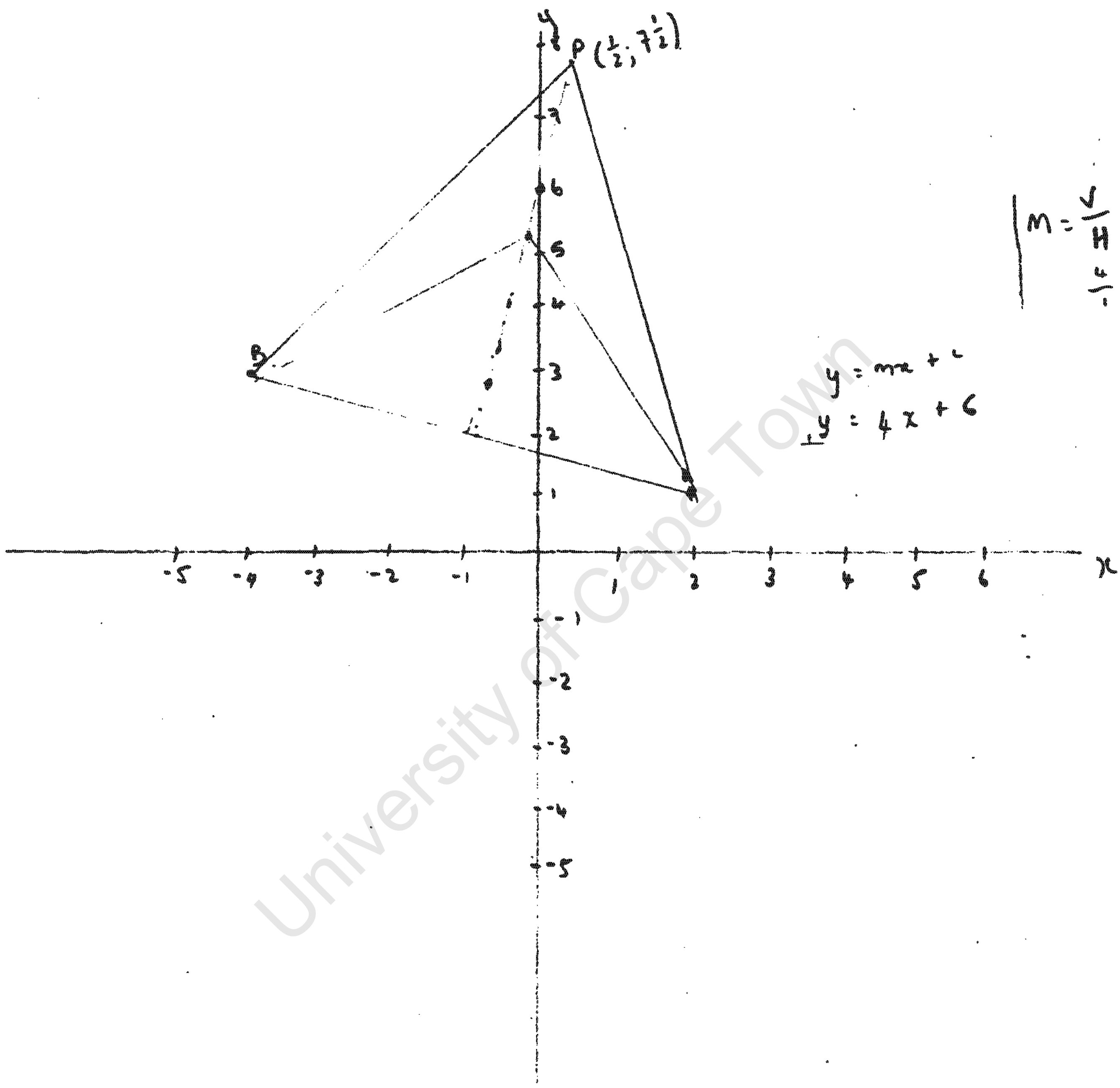


$$(CP)^2 = 5^2 + 2^2$$
$$= 29$$
$$\therefore CP = 5{,}4\text{ cm}$$

4c

Question 6:

The point P(x;y) is equidistant from A(2;1) and B(-4;3).
What will the equation of the set of points equidistant from A and B be?



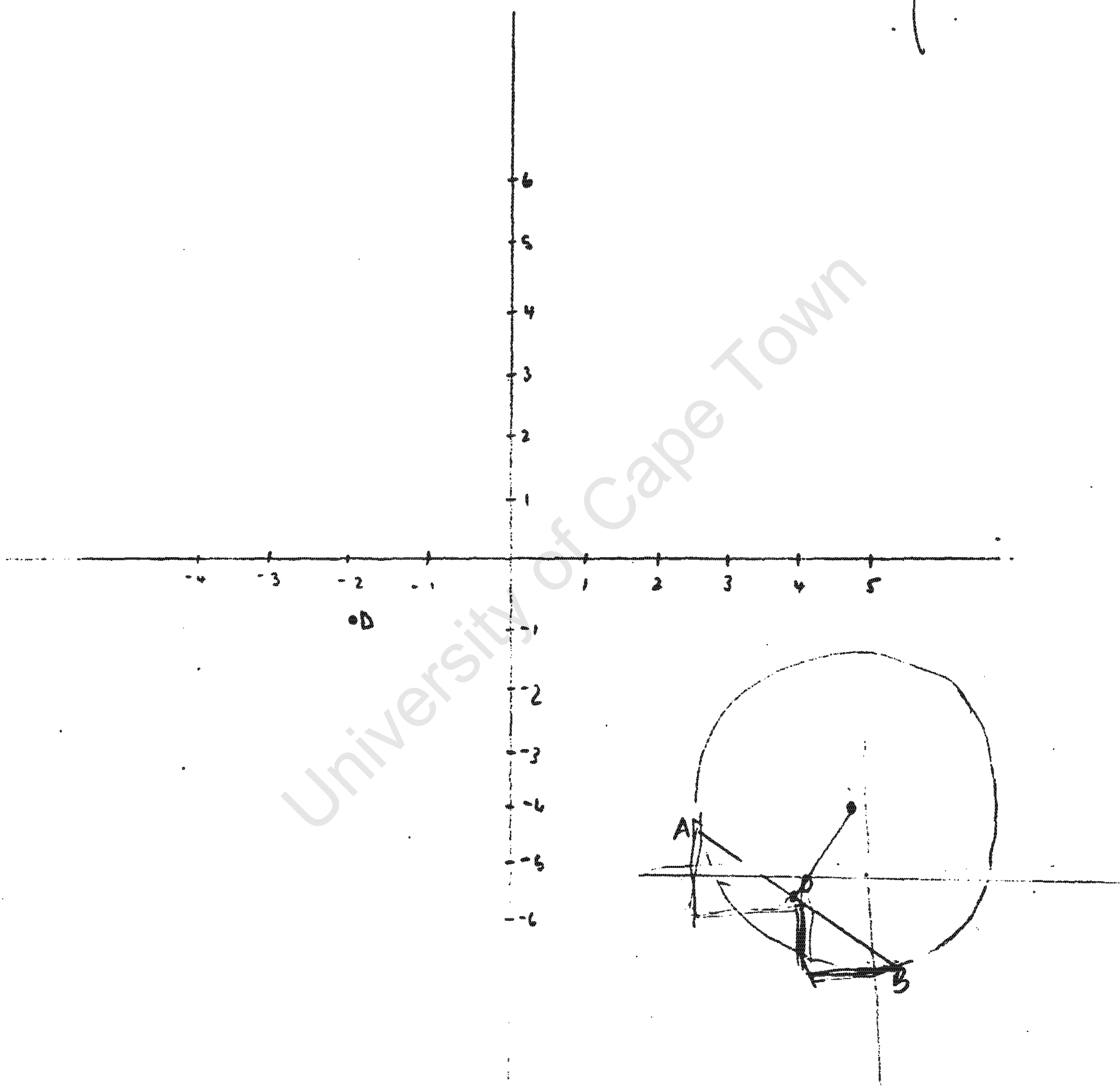
y = mx + c
y = 4x + 6

M = V/H
= 4/1

4c

Question 7:

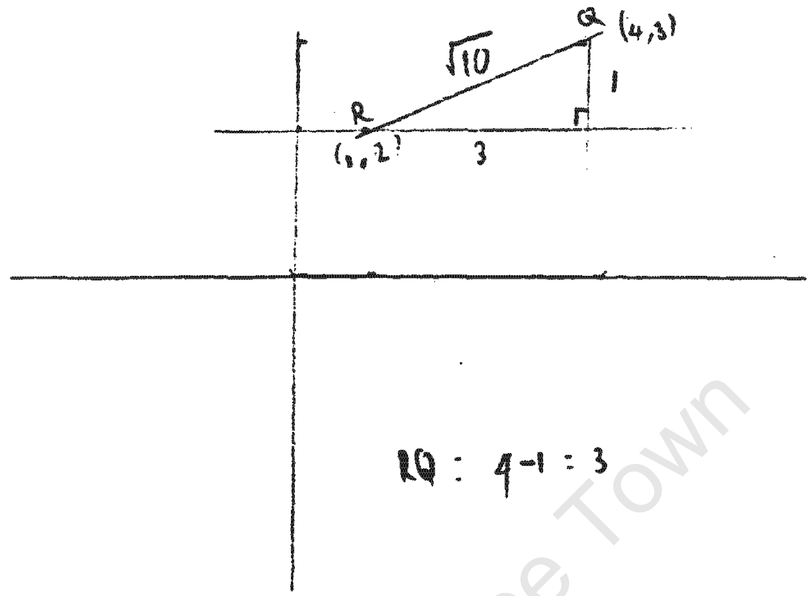
Given the circle with equation $x^2 + y^2 = 10$.
The midpoint of chord AB in the third quadrant is D(-2;-1). What are the co-ordinates of A and B?



QUESTIONNAIRE

Question 1:

If the points R(1;2) and Q(4;3) form a line, what is the length of RQ?

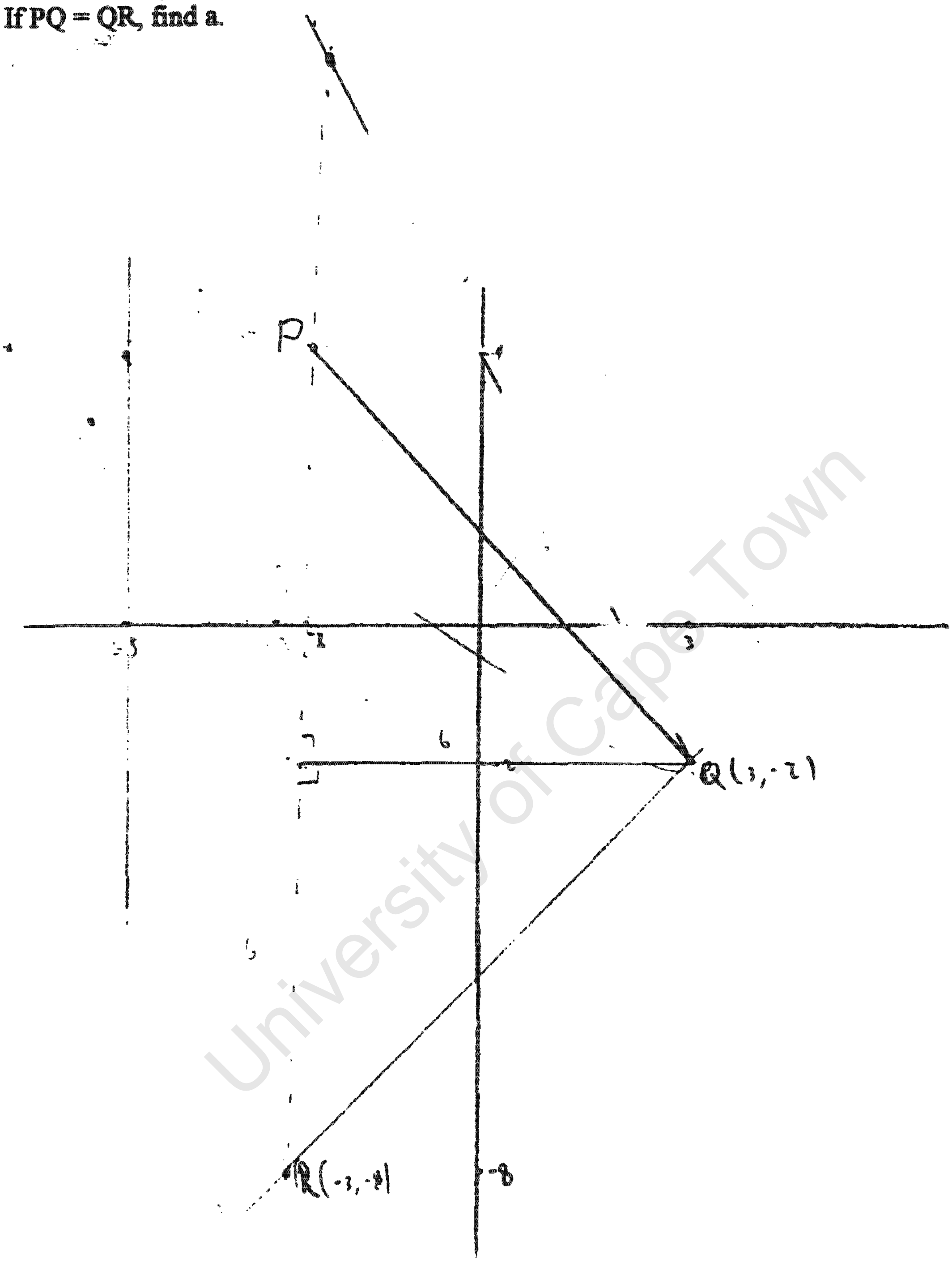


RQ : 4-1 : 3

40

Question 2:

Consider the points P, Q(3;-2) and R(-3;-8). P is in (a;4) in the second quadrant. If PQ = QR, find a.



PQ = QR

P(+3,-2) = (+3;-2) + (-3-8)

= -10

= -10

= -5

RQ = 6^2+6^2

= 36+36

RQ = $\sqrt{72}$

2

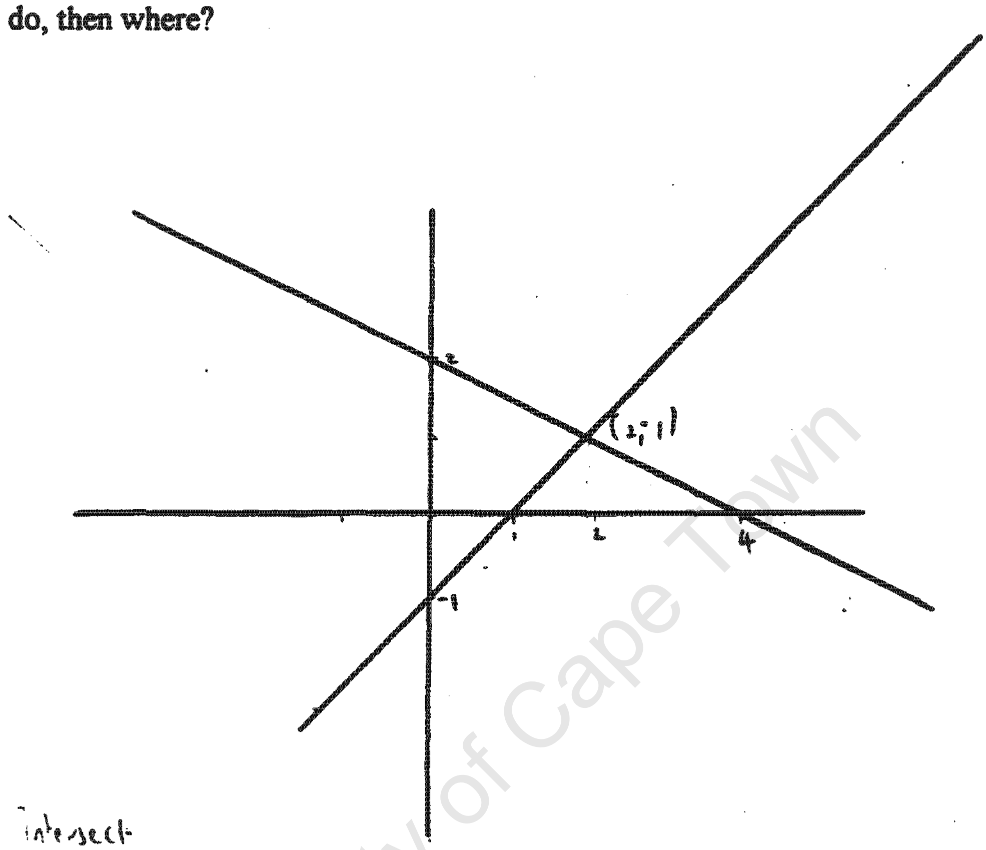
Question 3:

Do the lines y = x – 1 and y = -½x + 2 intersect?

Why?

If they do, then where?

Why?



Yes, intersect

$x-1 = -\frac{1}{2}x + 2$

$x + \frac{1}{2}x = 2+1$

$\frac{3}{2}x = 3$

$x = \frac{3}{1} \times \frac{2}{3}$

$x = 2$

$y = 2+(-1)$

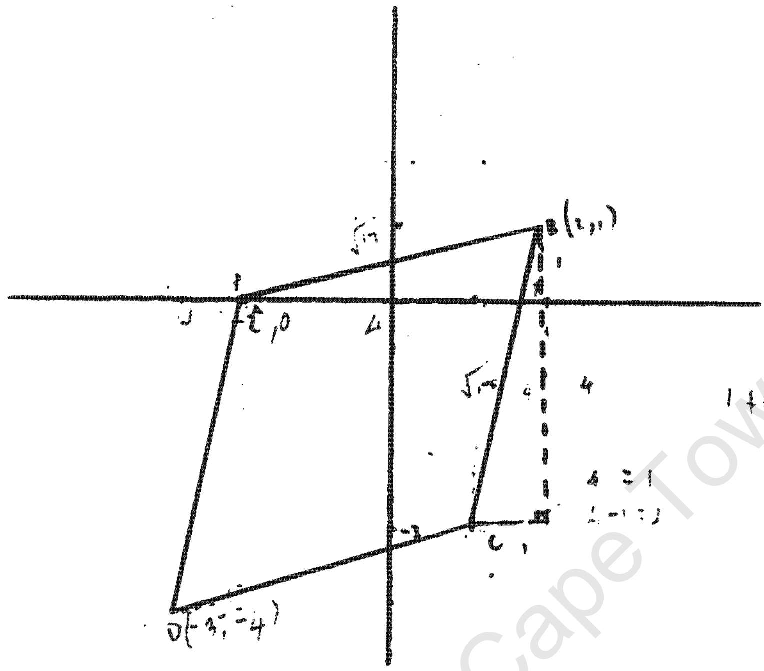
$= 1$

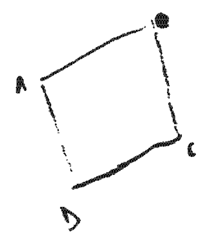
Intersect at because the 2 equations can be equated

Question 4:

If the points A(-2;0), B(2;1), C(1;-3), D(x;y) form a parallelogram, what are the coordinates of D?

Can you suggest another way of arriving at the same solution?





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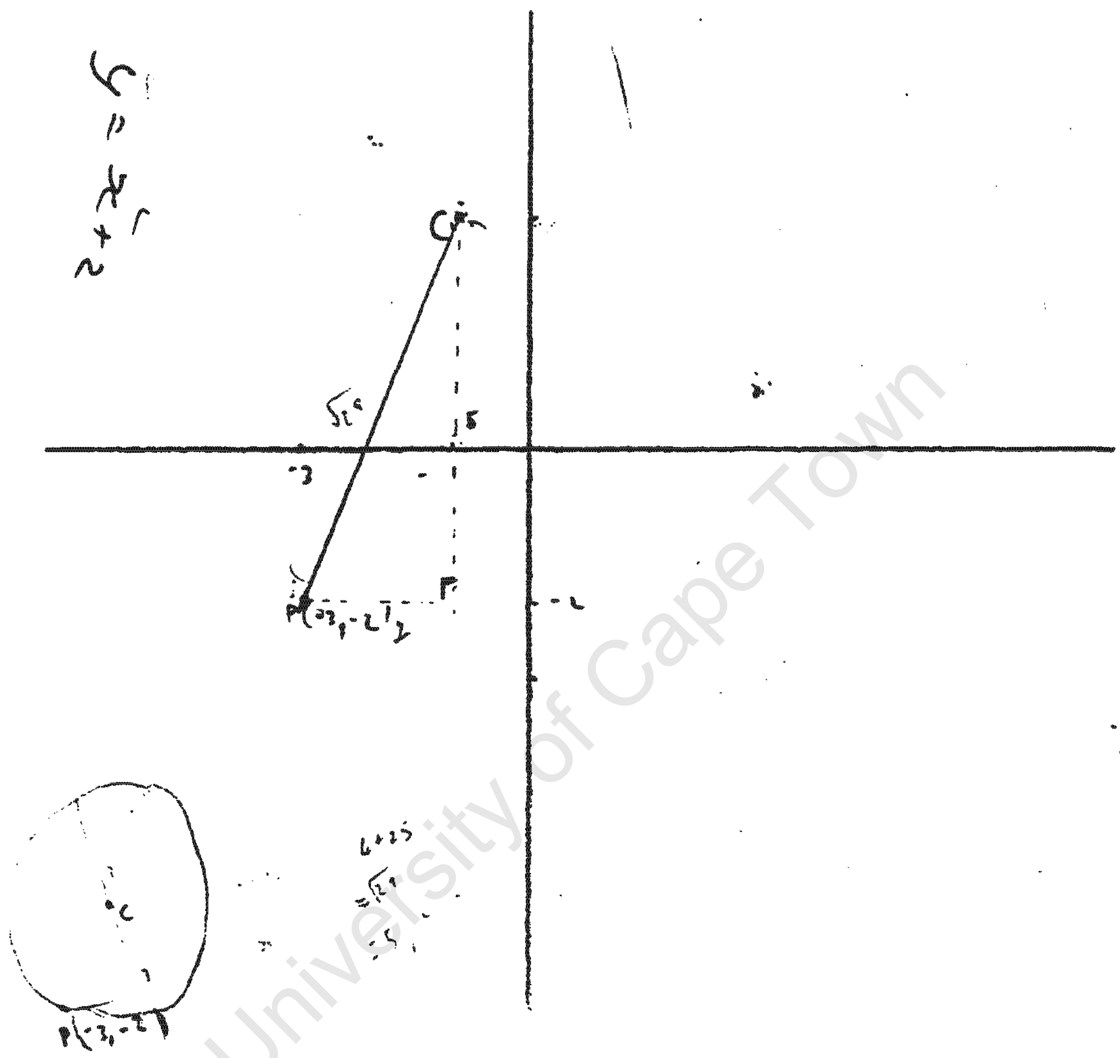
<u>Question 5:</u>

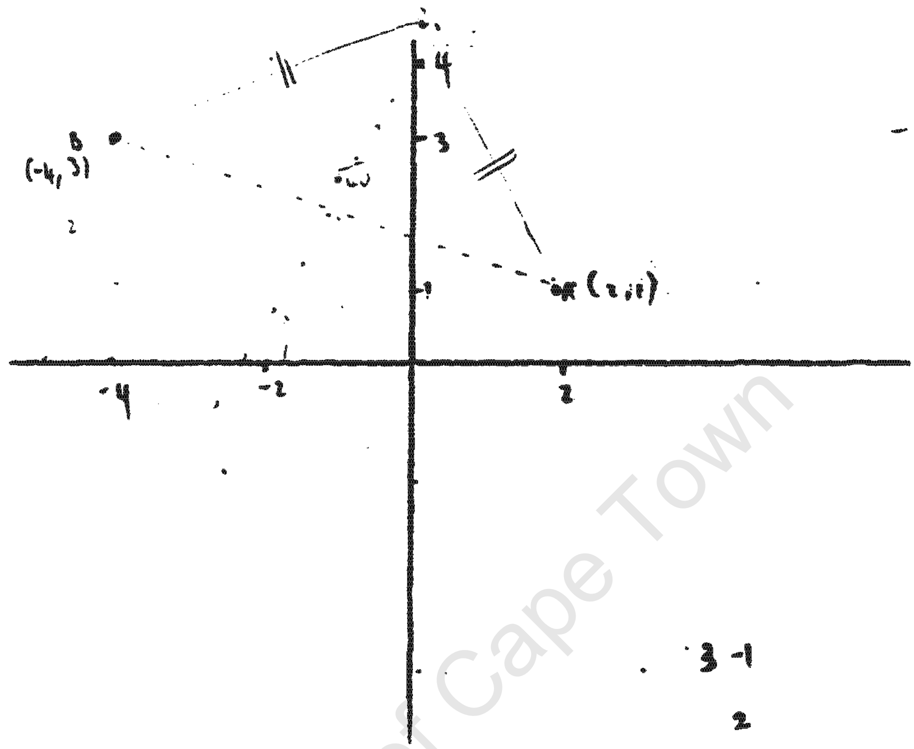
A circle with centre (-1;3) passes through P(-3;-2). What will the equation of the circle be?



4D

Question 6:

The point P(x;y) is equidistant from A(2;1) and B(-4;3).
What will the equation of the set of points equidistant from A and B be?



4D

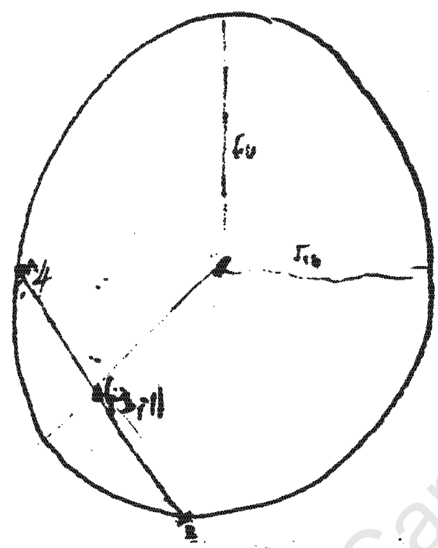
Question 7:

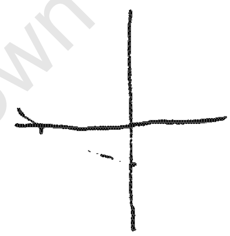
Given the circle with equation $x^2 + y^2 = 10$.
The midpoint of chord AB in the third quadrant is D(-2;-1). What are the co-ordinates of A and B?

$10 = x^2 + y^2$

$x = \sqrt{10}$

$y = \sqrt{10}$



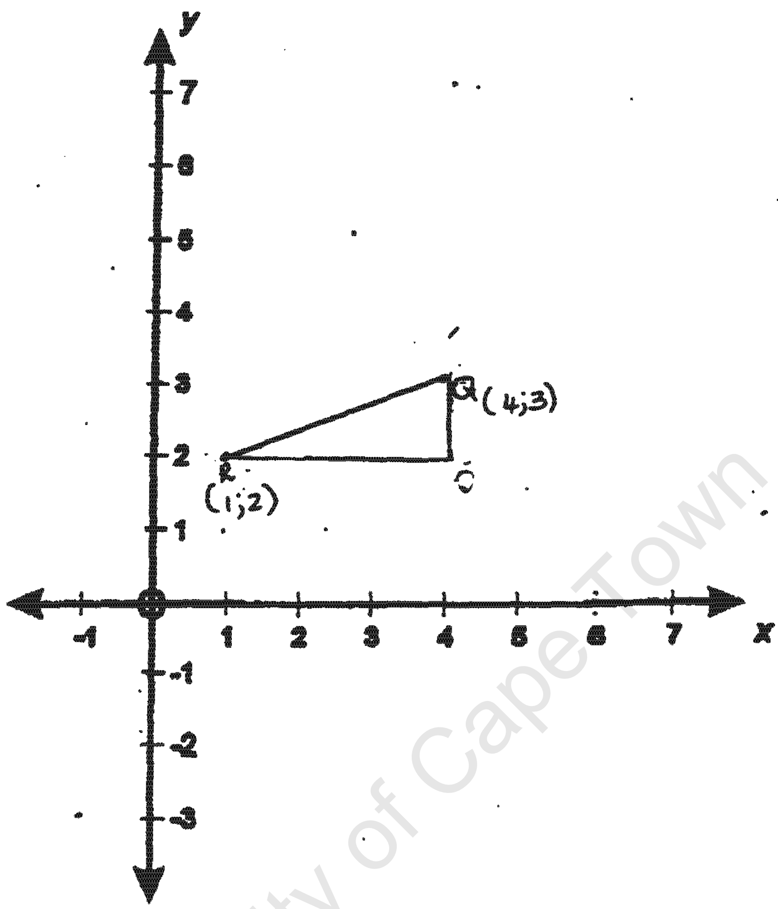


Appendix 5

Pilot study: questions and textual production

QUESTIONNAIRE

Question 1:

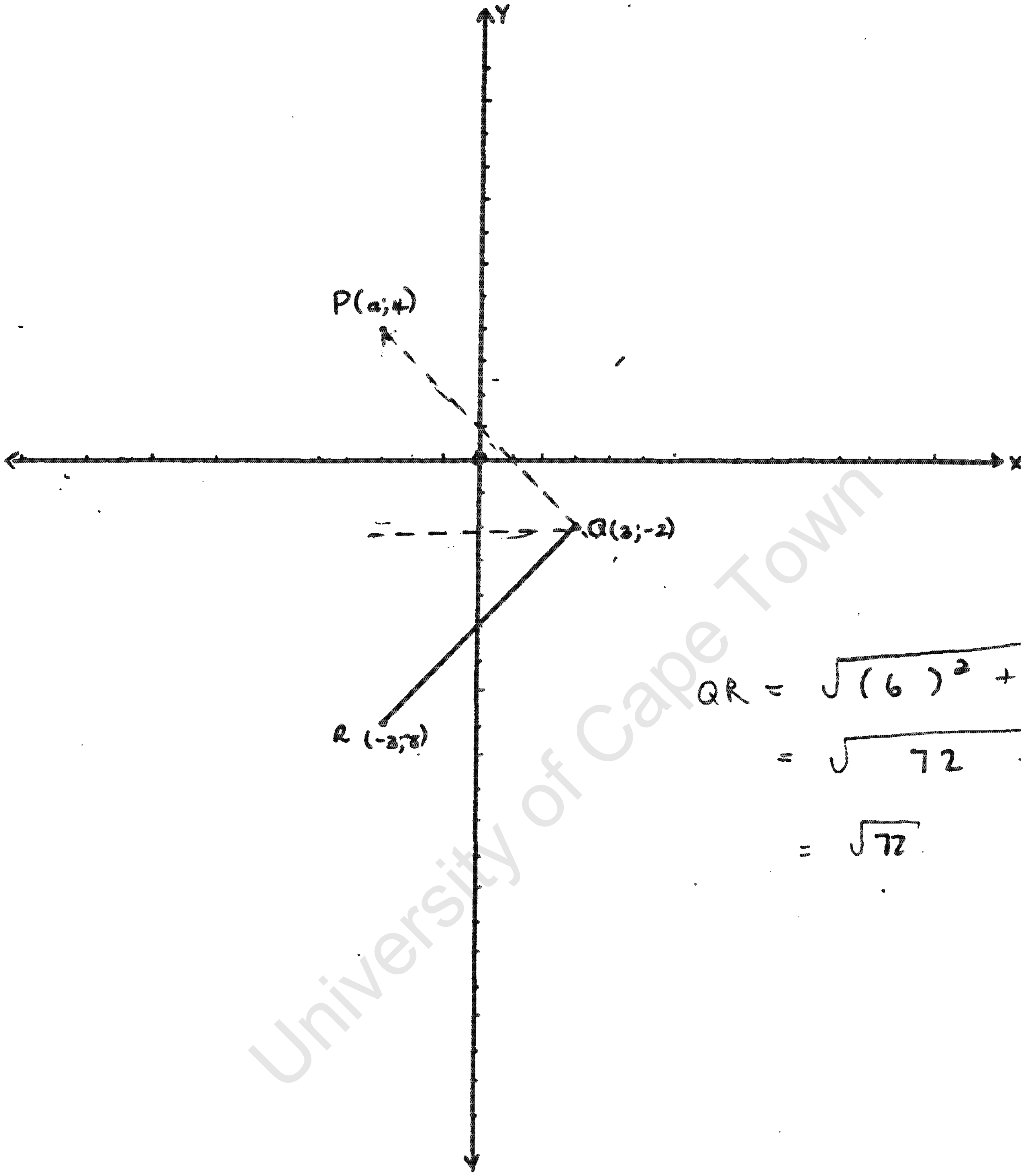


What is the length of RQ?

$$RQ = \sqrt{(4-1)^2 + (3-2)^2}$$
$$= \sqrt{9 + 1}$$
$$= \sqrt{10}$$

Question 2:

Consider the point P(a;4) in the second quadrant. If PQ = QR , find a.



$QR = \sqrt{(6)^2 + (6)^2}$

$= \sqrt{72}$

$= \sqrt{72}$

$\sqrt{(3-a)^2 + (-2-4)^2} = \sqrt{72}$

$\sqrt{(9-6a+a^2)+(36)} = \sqrt{72}$

$a^2 - 6a + 45 = 72$

$a^2 - 6a - 27 = 0$

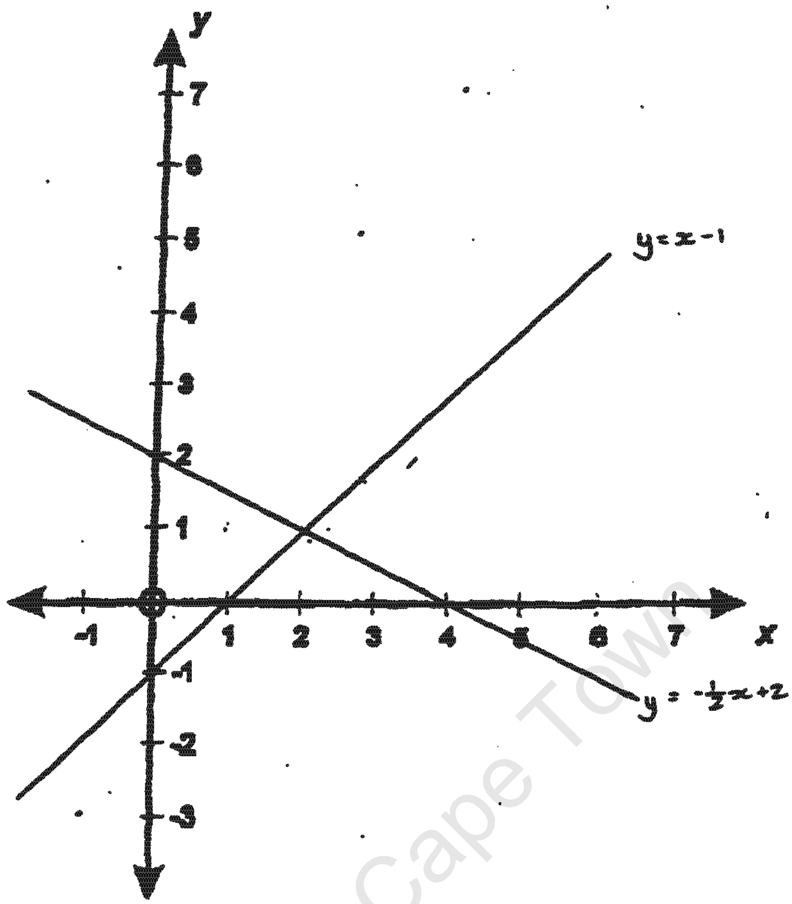
$(a-9)(a+3) = 0$

$a = 9$ or $a = -3$

But P is in 2nd Quad.

So $a = -3$

Question 3:



How can one find the point at which the lines y = x – 1 and y = -½x + 2 intersect?

$x - 1 = -\frac{1}{2}x + 2$

$\frac{2}{2}x + \frac{1}{2}x = 3$

$\frac{3x}{2} = 3$

$3x = 6$

$x = 2.$

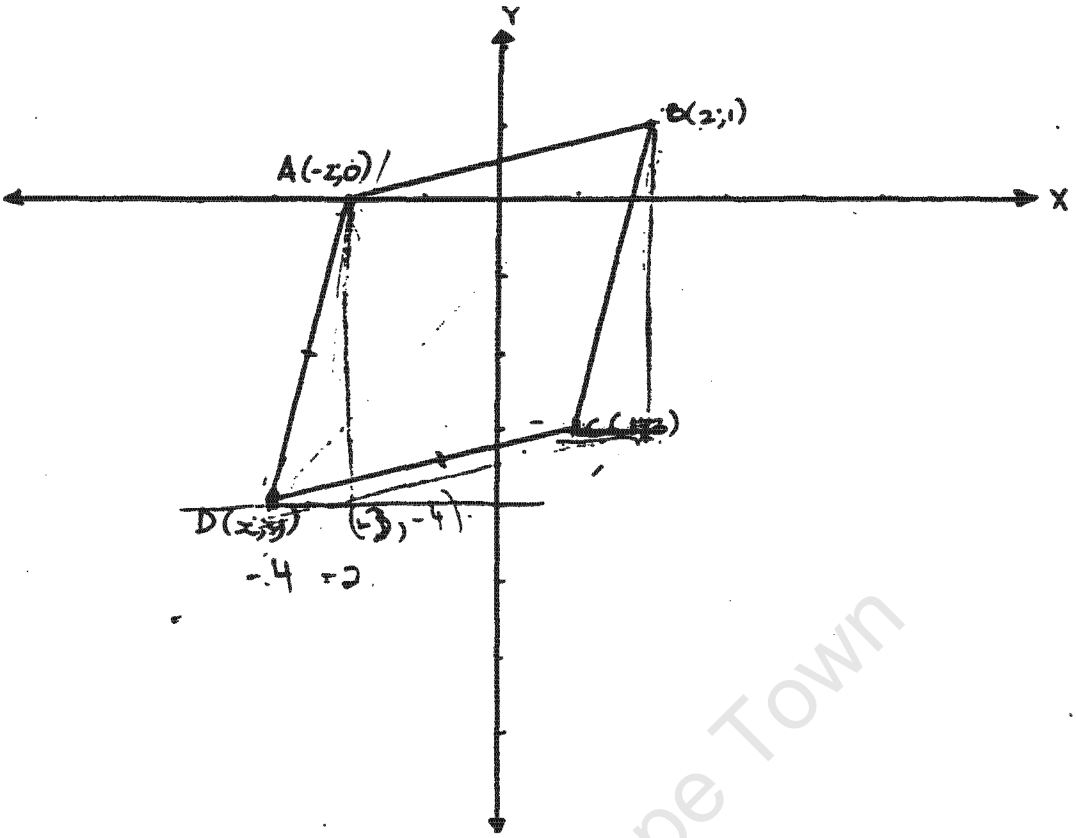
sub. (x) into $y = x - 1$

$= 2 - 1$

$= 1$

pt. inter. $(2 ; 1)$

Question 4:



ABCD is a parallelogram. How could we determine the co-ordinates of D?

$m_{AB} = \frac{1}{4}$

$= \frac{1}{4}$

$y = \frac{1}{4}x + c$

$\frac{y_1 - (-3)}{x_1 - 1} = \frac{1}{4}$

$\frac{y + 3}{x - 1} = \frac{1}{4}$

$y + 3 = \frac{1}{4}x - \frac{1}{4}$

$y = \frac{1}{4}x - 3{,}25$

$y = \frac{1}{4}x - 3\frac{1}{4}$

$M_{BC} = \frac{\ldots}{1}$

$= 4$

EQN AD : $y = 4x + c$

$\frac{y - 0}{x + 2} = 4$

$y = 4x + 8$

$\frac{1}{4}x - 3\frac{1}{4} = 4x + 8$

$\frac{1}{4}x - 4x = 8 + 3\frac{1}{4}$

$-3{,}75x = 11\frac{1}{4}$

$x = -3$

sub (x) into $y = 4x + 8$

$= -12 + 8$

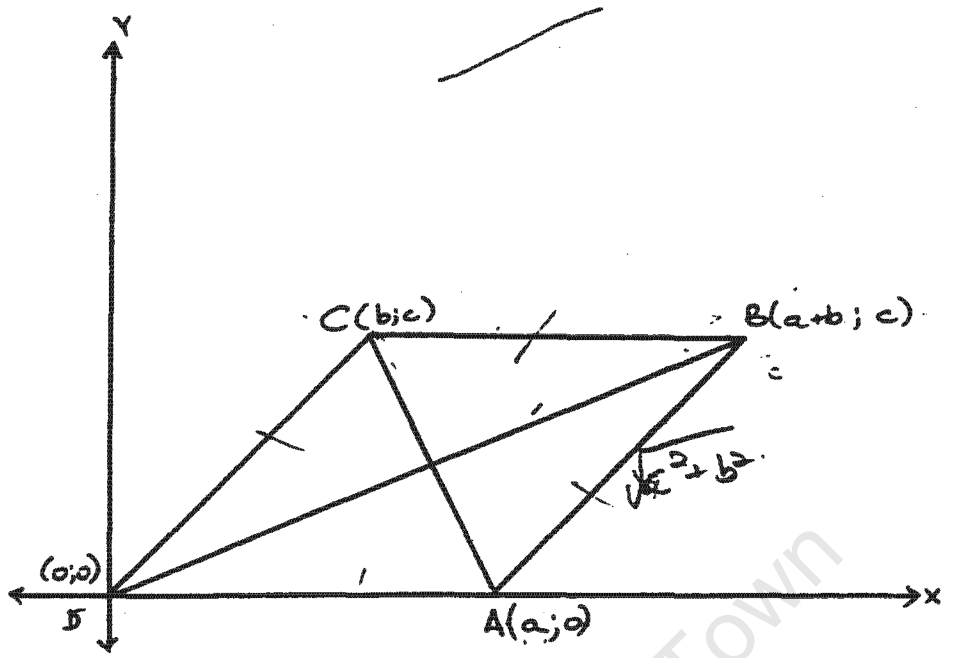
$= ~~-20~~ -4$

D $(-3; -4)$

Solution to Q4 continued

$a^2 = b^2 + c^2$

Question 5:



ABCD is a parallelogram.
Do the diagonals bisect each other?

midpt AC $\left(\frac{a+b}{2}; \frac{c}{2}\right)$

midpt DB $\left(\frac{a+b}{2}; \frac{c}{2}\right)$

midpt AC = midpt DB

$\therefore \frac{a+b}{2} = \frac{a+b}{2}$

the diagonals bisect (they have the same midpt)

$a^2 = b^2 + c^2$.

$CB = AB$

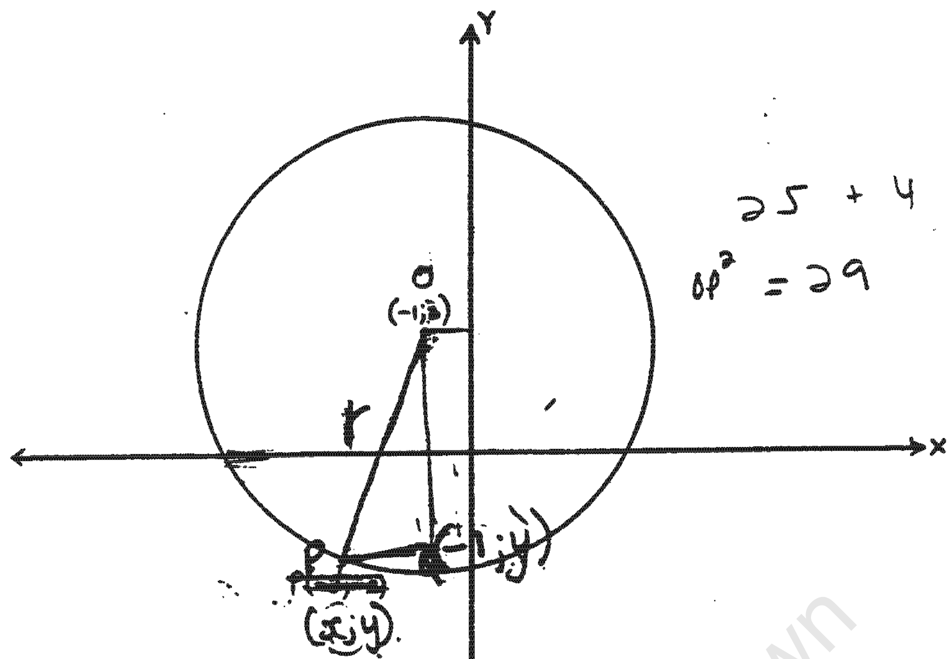
$CB^2 = AB^2$.

$a^2 = \left(\sqrt{c^2 + b^2}\right)^2$.

$a^2 = c^2 + b^2$.

Question 6:

A circle centre (-1;3) passes through P(-3;-2). Find the equation.



25 + 4

$OP^2 = 29$

$(x-a)^2 + (y-b)^2 = r^2$ (a;b) coordinate of centre

$(x+1)^2 + (y-3)^2 = 29$.

$(x+1)^2 + (y-3)^2 = 29$

Question 7:

Give the centre and radius of a circle represented by the equation $x^2 - 6x + y^2 + 4y + 8 = 0$.

$$x^2 - 6x + y^2 + 4y = -8$$

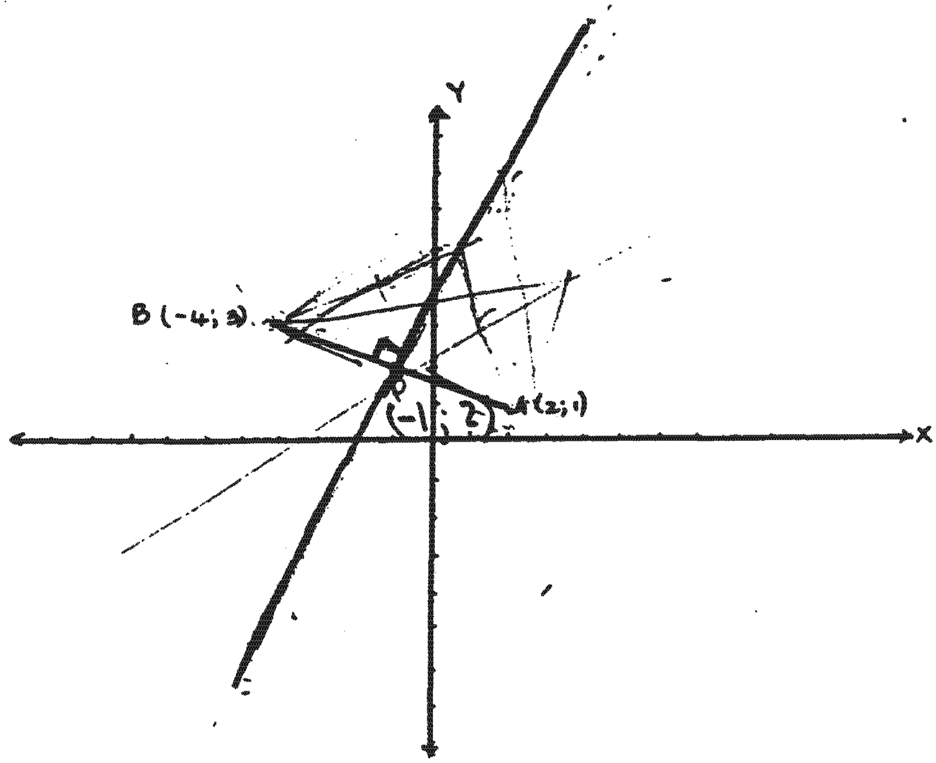
$$x^2 - 6x + 9 + y^2 + 4y + 4 = -8 + 9 + 4$$

$$(x-3)^2 + (y+2)^2 = 5$$

centre $(+3 ; -2)$

radius $= \sqrt{5}$

Question 8:



Determine the equation of the locus of P(x;y) if the point is equidistant from A(2;1) and B(-4;3)

$M_{AB} = \frac{2}{-6}$

$= -\frac{1}{3}$

$M_{AB} \times M_{P} = -1$

$-\frac{1}{3} \times 3 = -1$

$M_P = 3$

$\frac{y-2}{x+1} = 3$

$y - 2 = 3x + 3$

$y = 3x + 5$

(3; 1)

$1 = 3(3) + 5$

$1 \neq 9 + 5$

$1 \neq 14$

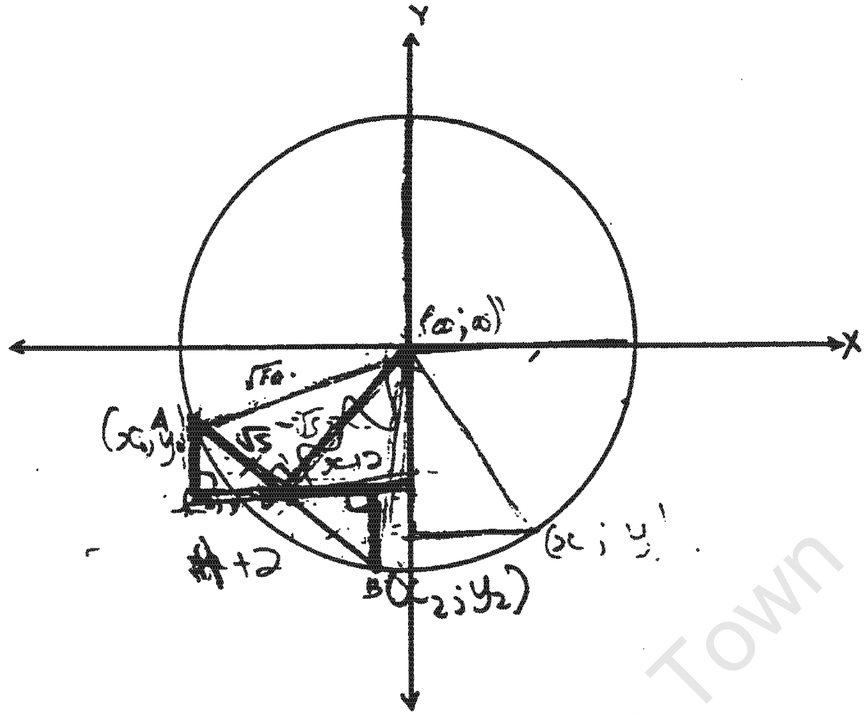
so (3; 1) is not a locus.

(-1; ?)

(-1; 2)

Question 9:

Given the circle with equation $x^2 + y^2 = 10$ in the figure below.
The midpoint of chord AB is D(-2;-1). What are the co-ordinates of A and B?



$$(x - a)^2 + (y - b)^2 = r^2$$

$$x^2 + y^2 = 10$$

$$(x - 0)^2 + (y - 0)^2 = 10$$

~~$(x+2)^2 + (y+1)^2 = 5$~~

A $(-4;0)$

B $(0;-2)$

$m_{OD} \times m_{AB} = -1$

$+\frac{1}{2} \times -2 = -1$

$m_{AB} = -2$

$\frac{y+1}{x+2} = -2$

$y + 1 = -2x - 4$

$y = -2x - 5$ ①

$y^2 = 4x^2 + 20x + 25$

$x^2 + y^2 = 10$

$y^2 = 10 - x^2$

$4x^2 + 20x + 25 = 10 - x^2$

$5x^2 + 20x + 15 = 0$

$x^2 + 4x + 3 = 0$

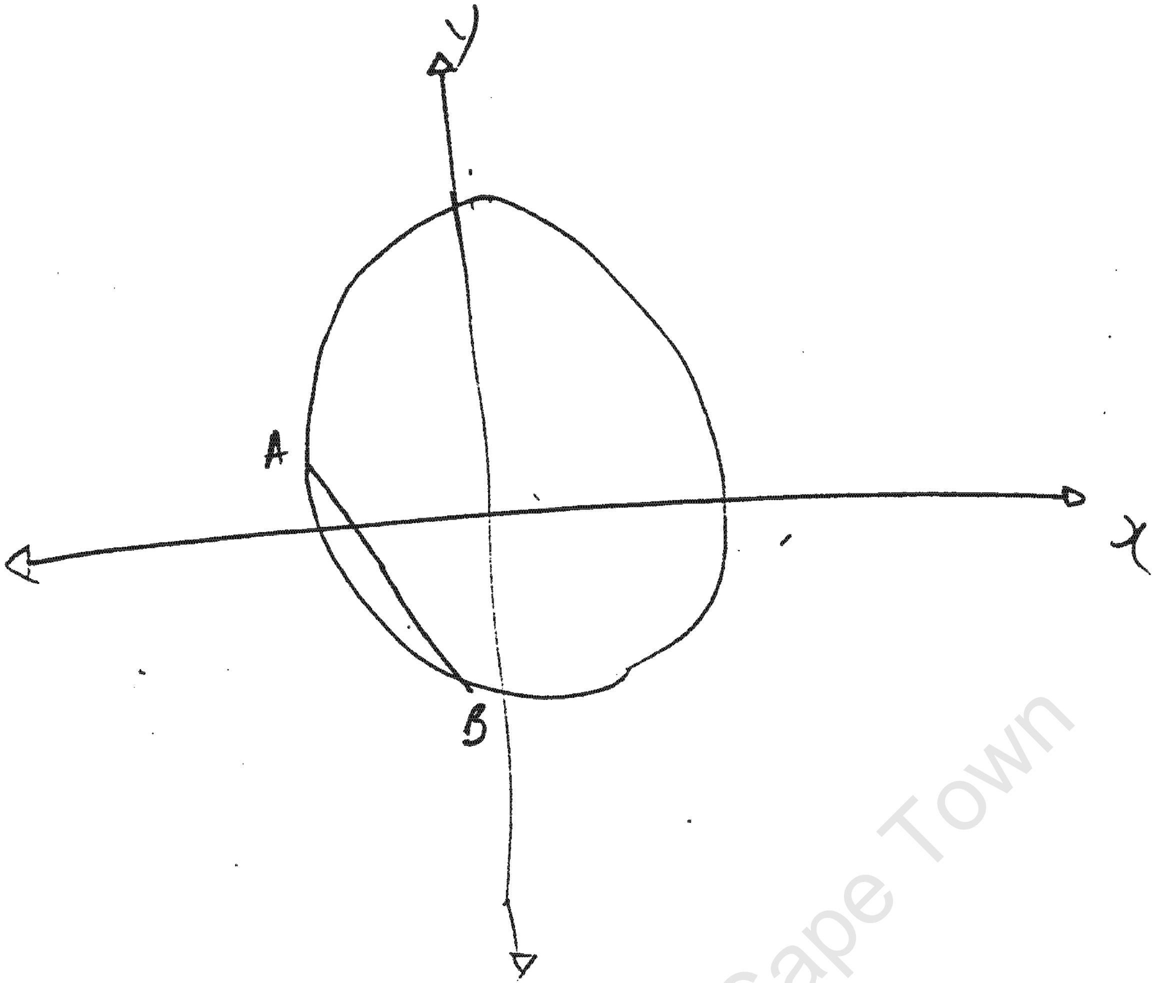
$(x+1)(x+3) = 0$

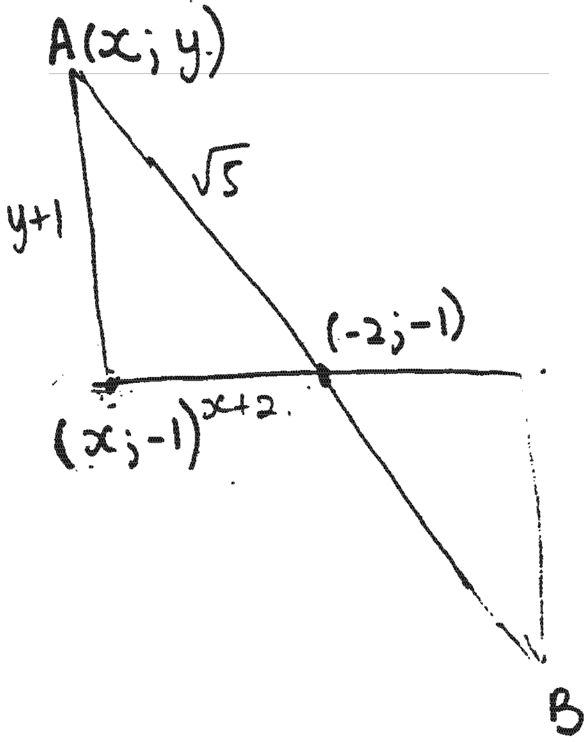
$x = -1$ or $x = -3$

sub (x)values into ①

$y = -3$ or $y = 1$

$B(-1; -3)$ $A(-3; 1)$


y
x
A
B



$(y+1)^2 + (x+2)^2 = 5$

$y^2 + 2y + 1 + x^2 + 4x + 4 = 5$

$y^2 + 2y + x^2 + 4x = 0$

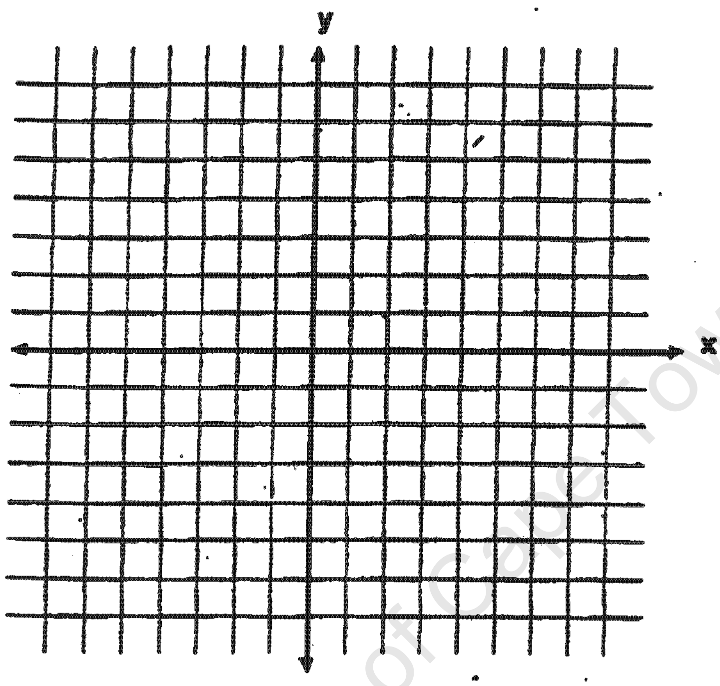
EQN OF CIRCLE

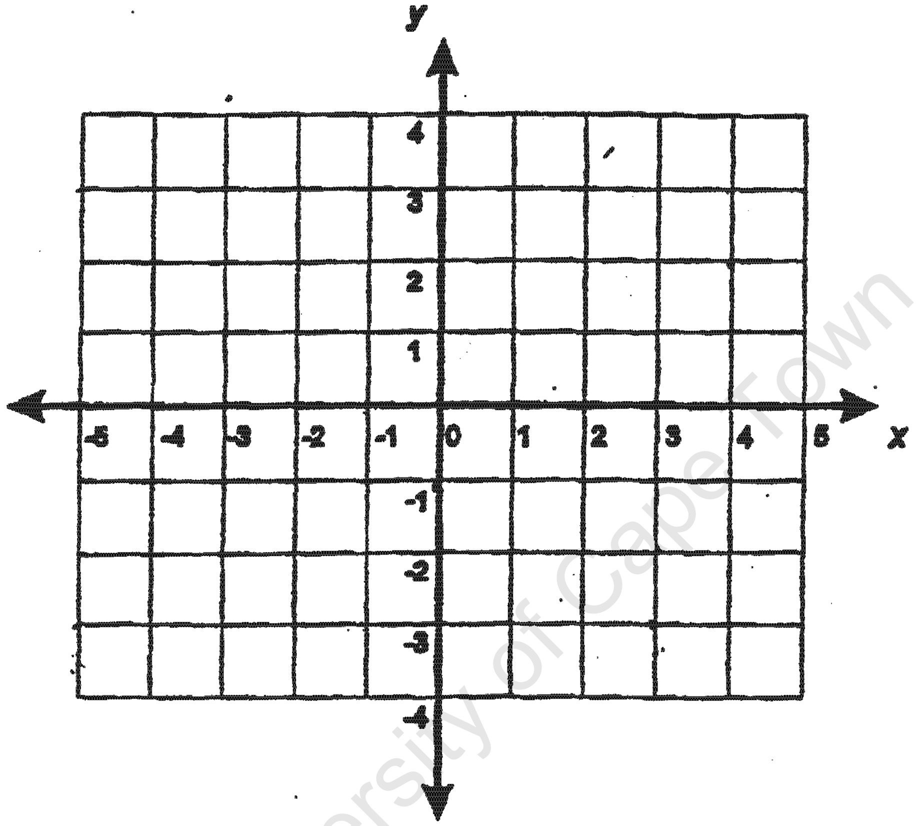
$y^2 = 10 - x^2$

$y^2 = -x^2 - 4x - 2y$

$y^2 + 2y = -x^2 - 4x$

$y(y+2) = -x^2 - 4x$


y
x


y
4
3
2
1
-5
-4
-3
-2
-1
0
1
2
3
4
5
x
-1
-2
-3
-4

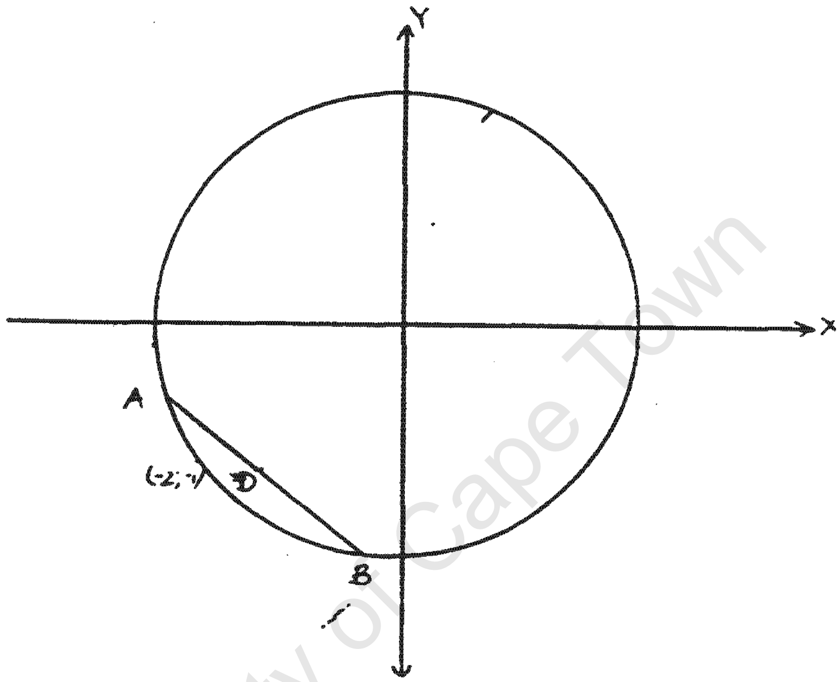
Appendix 6

Pilot study: Transcript of question 9 only

PILOT STUDY

TRANSCRIPT OF QUESTION 9:

S: ***(Reads the question out loud)***
Given the circle with the equation $x^2+y^2 = 10$ in the figure below.
The midpoint of chord AB is D(-2;-1). What are the co-ordinates of points A and B?
Chord AB is (-2;-1). OK
We need to find the co-ordinates of A and B.
And we know....*(looks at the sketch below)*

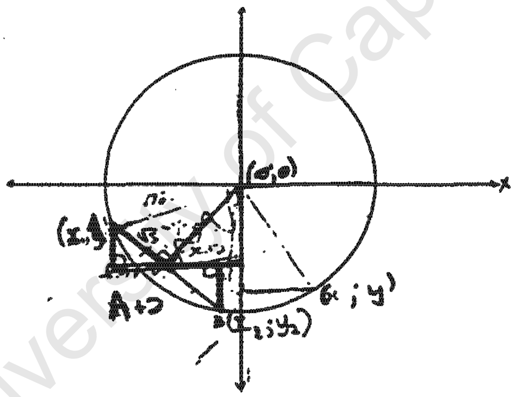


OK That is the centre.
OK. That line. That is a radius. That's going to be..
Let me just see now...
Umm....
That's minus 2 and minus 1.
Umm. T t t t t t t. OK let me just see.
I've got one method in mind.
That's point (0;0).*(Refers to the centre of the circle)*
That's 10. *(Refers to the radius OA)*
So lets say that is x and y.
So, $(x + 2)^2 + (y + 1)^2 = 10$. 'Cause that's the length of that line.
Its the length of that line.
I'm confusing myself here now. Ummm
x - 2. X-1.
Umm
Lets find the x value.
is equal to 10.
And that is -1.
Umm.
Umm.
The length of that is $-2^2 + -1^2 = 5$.

So,oo. Umm.
Let me just see how far I've progressed.
I know that this is 10;r^2 from the standard form of equation*(Refers to the standard form of the circle which was written down).*

$$(x-a)^2 + (y-b)^2 = r^2.$$

I: OK.
S: OK. That line there must be the root of 10.*(Refers to OA)*
Then I took that point and that point which is there and there and I found the length to be the root of 5. OK. *(Refers to the co-ordinates of D)*
I: How did you do that?
S: That's from that old equation which I have been using all the time. *(Refers to an equation used to determine the distance between two points - based on a transformation of Pythagoras).*
I: No. No. This I understand is the old equation. How did you find that to be the root of 5? Did you use a particular triangle.
S: Yah. I did not use any of the triangles that you see now.
I: Oh. OK. Sorry.
S: I used that triangle there.*(See marked triangle in sketch).*



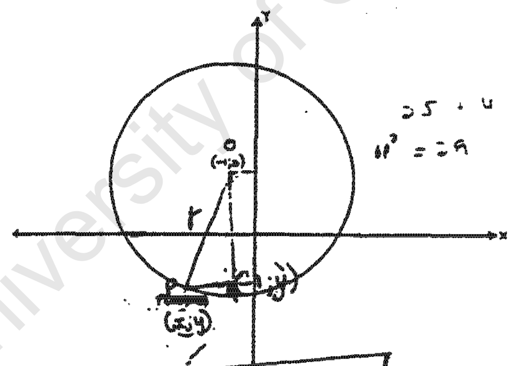
I: I'm just keen to know why you say this particular.. why this origin is (0;0).
S: Umm. Because the centre. Why the centre?
I: Why the centre?
S: You want to know why I've chosen the centre (0;0).
Umm. I just know that it is zero and zero at the moment.
The, umm. That equation tells us the centre is zero and zero.
(Refers to the given equation $x^2 + y^2 = 10$).
That line there is going to be equal to that line there.*(Refers to OA and OB).*
That still does not show that the center is zero zero.
That is the centre from the sketch. But...*(Uses the sketch for visual confirmation).*
I: It seems to me as if.. The fact that this is going to be root 5, *(Refers to OD);*
zero and zero is going to be crucial.
So I'd like to know ...

S: Why I have chosen zero and zero before I go on to show you the rest of the proof.
I: Er. *(Nodding her head)*.
S: Umm. I think basically, umm.
This equation tells us that the circle has the centre the origin.*(Refers to standard equation of the circle)*.
That equation there.

$$x^2 + y^2 = 10$$

$$(x-0)^2 + (y-0)^2 = 10$$

I: How does it do that?
S: Because, Umm..... , OK. How can I say this now?
Umm.
The radius. OK lets just use that there.
The radius is here. OK.*(Uncertain of the above equation as confirmation for thoughts)*
And in the other. Lets just look back... Hold on.
(Starts flipping through pages to access a previous solution which uses the equation he is currently referring to).
Where is that circle that we were looking at? Is it that one?
(Continues to flip through the pages)
It's not this one here. OK here it is.
That's x+1 and that is y - 3.*(Part of the solution he is referring to is illustrated below. This is then used as an exemplar in the current problem)*.



$$(x-a)^2 + (y-b)^2 = r^2$$ $(a;b)$ coordinate of centre

$$(x+1)^2 + (y-3)^2 = 29$$

$$(x+1)^2 + (y-3)^2 = 29$$

In that equation we looked at the radius and we looked at the points in there. OK
So if we look at the radius in here and we take the height like that. For any radius you're going to get a height that is on the Y-axis and the x-axis.

And one of the points there.. OK. So one of the points
That's point (x;y).*(Refers to point A).*
Umm.
The , the umm the length of the ...... .OK.
But that still does not prove why the centre is (0;0). OK
Let me see.

I: Why couldn't it have been (-1;3).
S: The centre?
I: umm.
S: OK.
I: Why (0;0) as opposed to.
S: *(interrupts and does not allow I to continue. It seems as if I provided a cue which triggered an appropriate strategy).*
I explained the standard form of the equation . So can I use that to...
I: OK.
S: We said that $(x-a)^2+(x-b)^2=r^2$.
If we take that as the primary.. . OK. *(Explains the origin of the equation).*
We understand why it is -a and -b. It has been explained already*(A previous question, not part of the transcript discusses this).*
I: Umm Hmm*(affirmative).*
S: So if we take that equation and we say:
$(x-a)^2+(y-b)^2=r^2$.
And this is the equation that they have given us.
I: Umm.*(affirmative)*
S: $x^2+y^2=10$. And they say that that's the equation of the circle.
I: umm.*(affirmative)*
S: Then if we want to punt it into standard form like that is *(pointing towards the standard form written on his answer sheet)*then $(x-0)^2+(y-0)^2=10$.
And from that.
And from that we said that we can read off the centre of the circle a and b.
I: Umm.*(affirmative)*
S: From that we can read off the centre as 0 and 0
OK.
I: *(nods head).*
S: Does that satisfy you?
I: Somewhat. OK, its fine.
Have you figured out how to do it yet?
S: yah, I have.
I don't know whether it is a short way or not, but in any case.
(Making pencil marks on the answer sheet).

A (-4; 0)

B (0; -2)

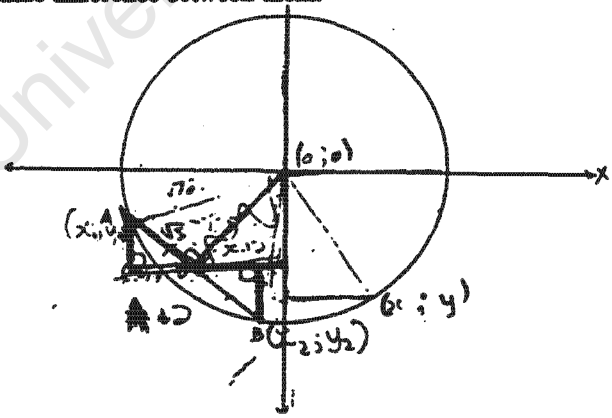
(Stares at what he has written)
I just want to see.
That is 2 and that is 1. *(Points to these lengths on his sketch).*
That x value is minus and 2 point to this side*(points to the LHS of his sketch)*

This x-value is 2 points to that side*(points to the RHS).*
That's symmetry again.*(A previous question could be solved using symmetry. This provided a cue for a similar approach).*
2 points to this side*(points towards the LHS)*, so that point A is going to be....
(refers to a point A which S has marked on the sketch).
-2 minus 2 which is minus 4.
The point y is going to be 1 up which is -1 + 1, which is 0.
That does not make sense.
Umm.
Stares at the sketch.
The y-value is going to be...
Refers back to the sketch
y + 1 because we're going up.
That's -1.
-1+1=0. That's what I get.
Let me just see this one here.*(Refers to point A on the sketch)*
It going to be a point below.
Umm.
This x-value is going to be 2 points that way*(refers to B on the sketch and points to the LHS)*
So its -2 +2 which gives 0 again.
And the y-value is -1-1 which gives -2.
I don't know if that is right.. if I'm using the right logic.

I: I understand that this generates, that this shows that the origin is zero and zero.
Then how did you get that.*(Refers to an equation on the answer sheet).*

$$(x+2)^2 + (y+1)^2 = 5$$

S: No, I was just, I was just playing around.*(S disregards this solution as inappropriate).*
I: OK
S: OK. So that not part of that.*(pointing to A and B in the solution)*
What I was trying to do.
I was just trying to figure out .
I took that circle, that triangle there and that triangle there.
That is going to give you two congruent triangles.
They have the same difference between them.



I: OK
S: That must be the same and that must be the same.
I: OK
S: And that length*(pauses and considers the sketch)*
is... Let me just see.
That would be wrong. I'm not sure that that would be right.
I: Lets hold that thought for while. Umm.

When you did this sketch, I see that you filled in a value here which I assume is the length of the line.*(Refers to the values on the sketch: OA = √10 ; OD = √5 ; AD = √5.).*

S: Umm Hmm.*(affirmative).*

I: And that line and that bit over there.*(Refers to AD and DB).*
Is there any relationship between OD and AB.

S: They are perpendicular.*(Traces the lines on the sketch to confirm this).* Like that.
And it cuts it in half.

I: Could that information be useful?

S: Yes. Definitely.*(Nods head and stares at the sketch).*
Let me just see.*(Unable to see how this information will be useful).*

I: You said this line is perpendicular to this line. Is there some kind of algebraic manifestation of that relationship.

S: In terms of finding their equations?

I: In terms of just a relationship.

S: Just a relationship.
Well you know that the gradient of that line times the gradient of that line is -1.
So m_{OD} x m_{AB} is -1.
Then you also know that *(stares at sketch)*
What else do you know

I: Can you find m of A.. Can you find either one of these.?*(Referring to the gradients of the lines mentioned).*

S: Umm. You can. You can find m of OD which is 0 - 1, minus 1 over minus 2 which is minus a half. And then..*(Able to do this without understanding the need for it).*

I: Minus 1 over minus 2.

S: Oh! Plus a half.
Then that will be minus 2.*(points to m_{AB})*, which is equal to -1.*(points to the equation on answer sheet).*
And then you've got that one point on there and then you can find the equation of that line AB.
The point is (-2;-1).*(This procedure triggers an appropriate strategy which allows S to progress)*
So its x- minus 2, which is x+2; and that's y+1 ; which is equal to -2.
So that's y+1 = -2x -4.
y = -2x - 5.
And then you know at this point this equation is equal to this.*(writes down the given equation of the circle, $x^2+y^2=10$).*
OK. Wait... $y^2= 10 - x^2$
We would have to square. I'm just going to put that equation in a block.*(Blocks off the equation = -2x+5).* We'd have to square that equation which gives us.
Square both sides. The square of that whole side;$4x^2$; that's -2 x -5 which is 10 x 2 which is 20x; + 25.
And then we'd set both equations equal to one another.
Which is equal to $10 - x^2$.
And its $5x^2+20x+15=0$.
You can take out 5.
Which is $x^2+4x+3=0$.
$(x+1)(x+3)=0$
x=-1 or x=-3. OK
Then we would just substitute both those values into say that equation there. Not that equation, this equation over here.*(refers to equation on the answer sheet which had been blocked off rather than the equation which had been given).*
And that would give us.
Sub x into -1 into 1*(marks blocked equation as 1).*
That's -2.-1; which is 2:-5 is -3
And -2 times -3 gives us positive 6; minus 5 which is 1. y =1
But that does not make sense.*(Refers to the original sketch).*
Let me just see if I've done that properly.

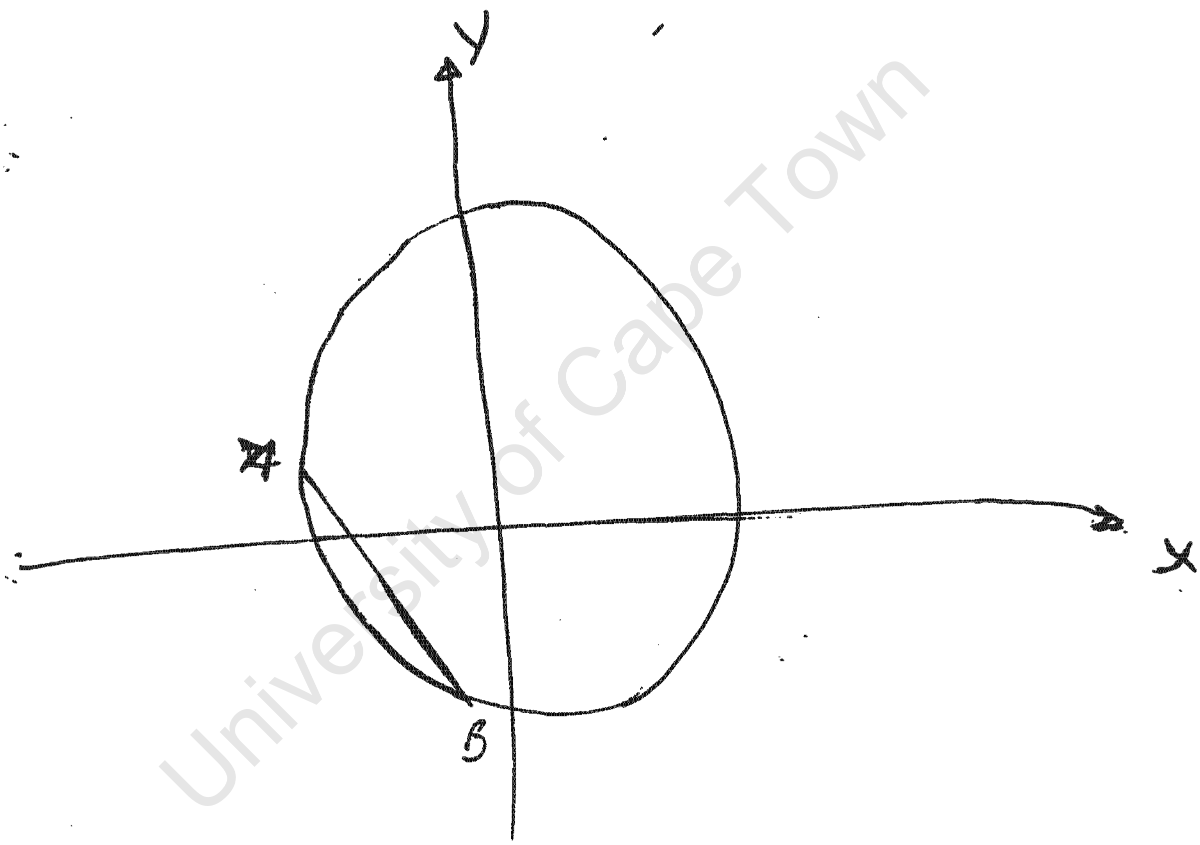
-3 times -2 is positive 6; minus 5 is 1.
I think that that's right.
But from the sketch it does not seem right.
That's A I would say. No. No!. That's B. *(Writes down B(-1;-3).)*
And A would be -3 and 1.

I: So if it does not appear this way on the sketch, can you show me what the drawing should have looked like?

S: What the drawing should have looked like? The sketch?

I: Ja.

S: The sketch should have looked something like...
OK
The circle would be there, it does not look like its in the centre.
And AB should have been something like that.*(Refers to own sketch).*



Algebraically that's what it is saying, but what we see from the drawing, it does not seem as if the answer is write.

I: Why did you say that OD is perpendicular to AB.

S: hat is a line that I drew in.*(Refers to the line OD).*
We know that AD = DB because D is the midpoint.
From O to D we would have a perpendicular.
Not the Tan-chord, the .. umm the theorem about the radius and the chord. OK.
Midpoint theorem I think it is.
You know that OD is perpendicular to AB if AD= DB. That's the converse of the midpoint theorem.

I: OK.
Coming back to this over here. I'm interested to know what you did.

S: What I did there?
I was just being stupid. *(It is not clear whether he is referring to the actual equation which happens to be correct; or whether he is referring to a strategy which employs the equation)*

OK I actually was trying to go about it in a long way*(This suggests that he is referring to the strategy)*
And that was that way there.
I said that $(x+2)^2+(y+1)^2 = 5$

I: What exactly were you doing over here?

S: OK. There I was finding the length of that line over here.
I found the length of that line to be root 5. Cause that's root 10 and that's root 5.*(Refers to AD on the sketch)*.
And that is root 5. I showed you why earlier on.*(Refers to DB)*.
I used that triangle there *(Refers to triangle made by OD and the y-axis)*.
And so on

I: OK. Right.

S: So that line AD would be the square of that minus the square of that, which would be the root of 5.*(Refers to AO with length $\sqrt{10}$ and OD with length $\sqrt{5}$)*.
You find the length $(x-2)^2 + (y-1)^2 = (\sqrt{5})^2$ which is 5. OK.

I: OK.

S: Where I got that, I think that I was being really stupid.
How I did that. Lets see.
I tried to use the symmetry thing we used earlier on which was incorrect.

I: Why was it incorrect?

S: Because we did not know that that was the difference of 2.
But we do know that it is the difference of 2.
That's what I'm trying to.. OK
From that point to that point there we know that that OK.
We know that that line is –2 which is + 2 units. That line will also be +2 units
And that line there is going to be y+1 units and so on

I: OK

S: What I should have done is. I should have said That the x value is -2 + (x+2) which would have given me that line.
That point there for x.
+ x + 2.
-2 + (x+2) is x which is the same thing
I was trying t do that but I made the mistake of leaving out the x when finding the units of the line.

I: So would that equation generate the same answer as this one.

S: It would but it would be much longer. I think.
You would get two equations.

I: But you've got two equations there as well.

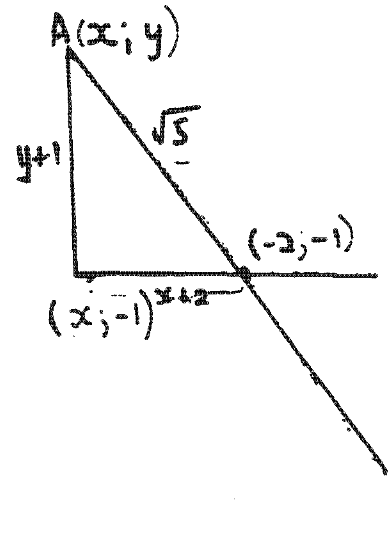
S: You've got two equations.
Here you cancel out the Y's; and here you.
OK but then you'd get the same thing I suppose.

I: OK.

S: But the difference in approach would be in this one..
Tell me what would be the gist of the difference between the two approaches.
That one would be looking more at the drawing. Establishing.. Getting your information a lot from the drawing and umm.
There would be slightly less algebra because you wouldn't have to worry about finding the gradient of the one line; and then the gradient of the other line; finding the equation of the line; and setting the equations equal to one another.
You would not have to use that and you would just work with those two triangles.
(Refers to a sketch drawn on the given sketch).

I: Do you want to see if it works?

S: Yes. I just want to draw it over again.



I: That's all right.
S: OK. And we've got there and there.
We've got this point here which is (-2;-1).
We want to find A and B, which are both point on the circle. OK
We know that this point here is going to be equal to x and -1. OK
We let this point be x and y. OK
That point is going to be (x;-1.)
The length of that line, of that line is (y–1) which is (y+1)
And the length of this line is going to be(x–2) which is (x+2) units. OK.
And then we know that $(y+1)^2 + (x+2)^2=5$
I: OK.
S: OK. And then we get $y^2 + 2y + 1 + x^2 + 4x + 4 = 5$
That's $y^2 + x^2$.
y^2+x^2.
OK wait. $y^2+ x^2$.
$y^2+ 2y + x^2 + 4x = 0$
That's - 5 which is 0. 0. Right.
Now from this drawing here, we know that this x is x and this is x-1. So its y
So the x-value is going to be the same.
What is the y-value going to be?
OK. Now you've got that
Well, you've got that equation, if we can set that equal o the equation of the circle.
I'm just going to use the y one there.*(Refers to the given equation which was also blocked off in the previous solution)*
Lets just write it down. And I will put it in a block again.
Let's say the equation of the circle.
$y^2= 10-x^2$.
That one down there.*(draws an arrow from the last equation in the current solution to the one which was given the original question)*
$y^2 = -x^2 - 4x - 2y$.
But we've still got a y which is a problem.
Umm.
Lets just try to keep the Y's on this side.*(Refers to the LHS of the = sign).*
$y^2 + 2y = -x^2 -4x$.
$Y(y+2)= -x^2- 4x$. UMM.
I think that this is going to be a very much longer equation.
Oh wait. We have y^2 and we have x^2
hmm umm
I don't think that you can solve it in a quick way using this method.
I: Is there anything you wanted to add?
S: To this?
I: Or generally.
S: Generally.

I think you could have if they had given you a y-value for that point.
OK. Lets just see.
The y-value is going to be Y+1 units up from that point.

I: All right

S: I can't see. *(Student decides that he cannot solve using this approach).*

Synopsis of strategies employed by the student.

- Selects a strategy which uses equations to determine length of lines:
  Possible reasons for this:
  1. The equation supplied gives the length of the radius.
  2. The radii in the sketch thus have values.
  3. The length OD can be determined if the origin of the circle is (0;0).

- Rationale for using (0;0) as the origin of the circle
  Possible reasons for this:
  1. Uses the given sketch as confirmation but is unable to do this algebraically.
  2. A previous solution is used as an exemplar for providing a solution.

- Constructs symmetrical triangles which will use the known co-ordinates to quickly access the co-ordinates of A and B
  Possible reason for this:
  1. A previous question in the pilot study allows the fourth vertex of a parallelogram to be determined by inserting symmetric triangles on the sides of the figure. This method avoided algebra and provided a quick solution. The student had not been aware of this short-cut.

- Relationship between AD and AB
  Possible reason for this:
  1. The interviewer provided the cue which focused the student on these lines. The student, however does not see how this information will assist in providing a solution to the problem.
  2. The cue for an algebraic relationship between these lines, enables student to proceed mechanically even though the rationale for it is not clearly understood. This triggers an awareness of an appropriate strategy.
  3. An equation for AB is generated. Student realizes that the given equation and the derived equation can be solved simultaneously to yield the points of intersection.

- The validity of the given sketch
  Possible reasons for this:
  1. Algebraic solution is not congruent with the sketch supplied.
  2. When the algebra appears correct, student redraws the sketch.

- Rationale for generating a correct algebraic solution.
  Possible reasons for this:
  1. Once a correct solution has been found to the problem, the student tries to justify it from a selection of known Euclidean theorems.
  2. The appropriate theorem is not immediately recognized. Student is able to discern that the first choice is not correct. The correct second choice accesses the details of the theorem, and the information which needs to be known in order to make this selection.

- Rationale for initial strategy

<u>Possible reasons for this:</u>

1. Interviewer insisting on a reason; student not to keen since a correct solution has already been found.
2. Decides that symmetry as a solution is incorrect. When trying to justify this, reasons support the validity of this approach. This forces the student to engage with the problem even more.
3. Generates an equation for the length of the line and equates this with the equation supplied for the circle. Student is unable to solve this.