

## Further 2019 horse mackerel assessments and projections

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### Summary

This document reports further updated 2019 horse mackerel assessments, along with constant catch projections for the base case model, in accordance with requests made the Demersal Working Group.

### INTRODUCTION

Following FISHERIES/2019/OCT/SWG-DEM/25, this document reports further 2019 updated horse mackerel assessments, along with constant catch projections for the base case model. The input data, which include updates to the catch, survey and CPUE series, are reported in Fairweather (2019) and are duplicated here in Appendix 1. Note that the Autumn 2019 survey result, whilst now available, is not used in these assessments as there are no corresponding catches available as yet for the ongoing 2019 season so that computations cannot yet be extended to include this year. The full model specifications are provided in Appendix 2.

### OPERATING MODELS

Following presentation of FISHERIES/2019/OCT/SWG-DM/25 the SWG recommended that the following (Base Case – BC) model be explored further.

**Model 3b BC:**  $q = q_1$  for years up to and including 2013,

$q = q_2$  for year 2014,

$q = q_1$  for year 2017+ ,

$q$  is linearly interpolated between  $q_2$  (in 2014) and  $q_1$  (in 2017) to obtain the  $q$  values for 2015 and 2016 (although note that the observed 2015 CPUE value is omitted in the model fit, as decided earlier by the DWG).

This model thus assumes that recent (2014-2016) low CPUE values are a result of reduced fishing catchability.

**Model 3b\_V1** Model 3b BC but the DD and DR CPUE are down-weighted in likelihood by a multiplicative factor of 0.33.

**Model 3b\_V2** Model 3b BC but the DD and DR CPUE are down-weighted in likelihood by a multiplicative factor of 0.1.

**Model 3b\_V3** Model 3b BC but the 2015 and 2016 stock recruit residuals set equal to zero.

**Model 3b\_V4** Model 3b BC but the demersal (and survey) selectivity is made time invariant.

**PROJECTIONS**

Projections are reported for Model 3b BC. For each projection scenario, the resource is projected ahead for 10 years and the projections are repeated 1000 times with noise added to the future recruitment and incorporating uncertainty about future CPUE estimates.

The rules to compute future simulated catches under various management approaches are set out below.

**1) Pelagic bycatches**

Figure 1 below plots pelagic bycatches (in 1000 MT) against annual horse mackerel recruitment (in billions).

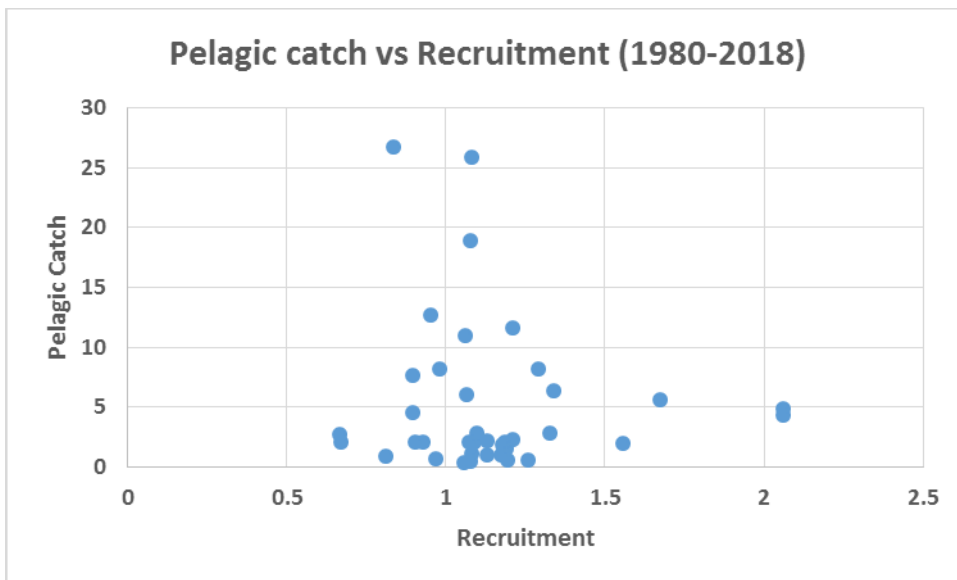


Figure 1: Pelagic catches versus recruitment (Model 3b BC).

- Note that there is no clear relationship between pelagic bycatches and recruitment.
- Hence future (2019+) pelagic bycatches are set by drawing at random with replacement from the set of pelagic bycatches for the period 2000-2018, except that a value generated which is in excess of  $PUCL_{y+1}$  below is reduced to  $PUCL_{y+1}$ , where:

$$PUCL_{y+1} = 12\,000 - C_y^{pel} - C_{y-1}^{pel}. \quad (\text{Units: MT})$$

Note 12 000 (previously called PULC<sub>3</sub>) is the total amount in MT that may be caught over a three-year period (see FISHERIES/2015/MAR/SWG-DEM/03).

## 2) Incidental trawl/Demersal bycatches – constant proportion of HM biomass

As recommended in FISHERIES/2016/OCT/SWG-DEM/79, the average reported incidental bycatches for the period 2000-2018 should be considered in the averaging used in order to produce a more representative  $\bar{F}_{trawl}$  exploitation rate value. Table 1 below reports the demersal bycatches, Model 3b estimated horse mackerel biomass values, and the resultant exploitation proportion  $F=C/B$ . The median and upper 95<sup>th</sup> percentile of the F values over the years calculated (assuming a normal distribution) are reported. It was agreed (in 2018) that the upper 95<sup>th</sup> percentile (to allow for catchability fluctuations) of the 2000-2017 (now 2000-2018) F values (which turns out to be 0.0347) would be used as the  $\bar{F}_{trawl}$  value in future equations to calculate the future demersal bycatches, i.e.:

$$\text{Future demersal bycatches} = \bar{F}_{trawl} * B_{exp}^{dem}$$

## 3) Midwater directed catches

Projections for a series of future constant annual catches of 0, 10, 20, 30 and 40 thousand MT are reported.

## Results

Table 1 provides a summary of results for the different assessments. Figure 2a compares the model fits to the *Desert Diamond* (DD) CPUE values. Figure 2b compares the model fits to the Dual Rights vessels' CPUE values. Figures 3a and b compare the model fits to the Autumn and Spring survey biomass estimates respectively. Figures 4a and b plot the spawning biomass estimates and the spawning biomass estimates relative to pristine for the three models. Figure 4c plots the Spawning biomass (B<sub>sp</sub>), exploitable demersal (B<sub>exp\_d</sub>) and exploitable midwater (B<sub>exp\_m</sub>) biomass trajectories for the Base Case Model 3b BC.

Figure 5a plots the estimated stock-recruit residuals for each assessment, with Figure 5b providing stock-recruit and recruitment over time plots. Figure 6a reports the estimated (and input) selectivity function for Model 3b BC. Figure 6b shows the maturity at age (which applies in the calculation of spawning biomass).

Figure 7 shows projection results for Model 3b BC. Results are shown for various projected levels of constant annual midwater catches. Plots of median and lower 5 %ile B<sub>sp</sub>/K, median CPUE and median midwater catches are shown.

## Discussion

Some points worth noting about the results are as follows.

- The impact of down-weighting the contribution of the CPUE data to the likelihood is relatively small (Table 1 and Figures 2-5). To avoid clutter, only the variant with the heavier down-weighting (Model 3b\_V2) is shown in the Figures.
- Rendering the demersal selectivity time-invariant leads to a much worse fit to the data, particularly to the Autumn survey abundance index and to the CAL data for both sets of surveys (Table 1 and Figure 3a). Consequently, this assessment variant is not considered defensible, but the need to assume a survey selectivity which changes over time is somewhat concerning.
- Recent estimates of trends in different components of the biomass (Figure 4c) differ quite substantially, leading or lagging each other and reflecting differing extents of increase. The reason lies in the rather different selectivity functions to which each corresponds (see Figures 6a and 6b).

Table 1: Summary of results for different assessment models. All variants fix  $q_{aut} = 0.75$  and  $h = 0.75$ . "SR" and "CAL" refer to stock-recruitment and catch-at-length contributions respectively. Biomass units are thousand MT.

	<b>Model 3b</b> BC	<b>Model 3b_V1</b> DD and DR CPUE *0.33	<b>Model 3b_V2</b> DD and DR CPUE *0.1	<b>Model 3b_V3</b> 2015 and 2016 SR residuals fix=0	<b>Model 3b_V4</b> Demersal selectivity time invariant
# parameters	46	46	46	46	40
-ln L :Total	-260.350	-259.062	-245.536	-260.185	-213.401
-ln L :Spr survey	1.162	0.937	0.768	1.184	1.189
-ln L :Aut survey	-8.048	-8.489	-8.729	-8.041	-0.486
-ln L :CPUE	-10.595	-9.424 (*0.33)	-8.701 (*0.1)	-11.029	-10.038
-lnL Dual Rights	-6.962	-6.448 (*0.33)	-6.012 (*0.1)	-6.825	-6.510
-ln L :CAL Spr survey	-48.506	-48.920	-48.970	-47.811	-37.667
-ln L :CAL Aut survey	-87.324	-87.778	-87.937	-87.023	-77.913
-ln L :CAL commercial	-81.810	-81.395	-81.165	-82.180	-66.660
-ln L :SR residuals	-18.267	-18.180	-18.032	-18.457	-15.332
$K^{SP}$ (KT)	789	787	785	798	773
$B_{2018}^{SP}$ (KT)	<b>512</b>	<b>487</b>	<b>470</b>	<b>535</b>	<b>428</b>
$MSYL^{SP}$ (KT)	193	193	192	195	187
$MSY$ (KT)	58	57	57	58	66
$B_{2018}^{SP}/K^{SP}$	<b>0.649</b>	<b>0.619</b>	<b>0.599</b>	<b>0.668</b>	<b>0.553</b>
$q$ : Spr survey	0.790	0.780	0.773	0.785	0.737
$q$ : CPUE ( $\times 10^{-6}$ )	1.950	1.940	1.941	1.940	2.090
$q_2$ (applies to 2014)	$0.265 * q_{CPUE}$	$0.258 * q_{CPUE}$	$0.258 * q_{CPUE}$	$0.267 * q_{CPUE}$	$0.237 * q_{CPUE}$
	M3b.tpl	M3bk.tpl	M3bkk.tpl	M3bj.tpl	M3bv3.tpl

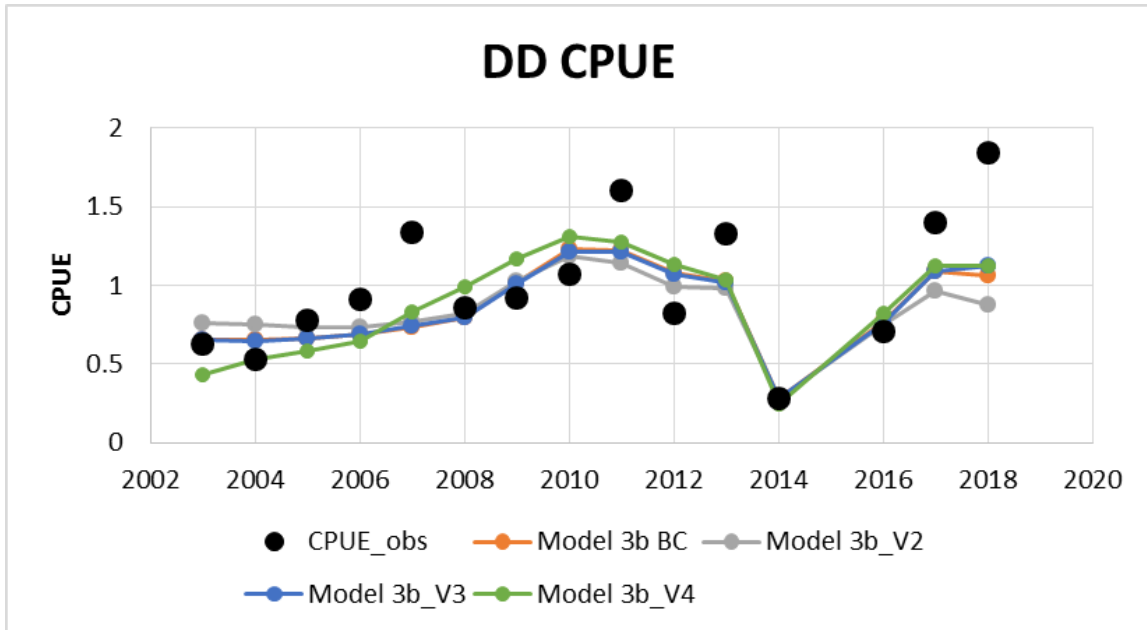


Figure 2a: Comparisons amongst the model fits to the *Desert Diamond* (DD) CPUE values.

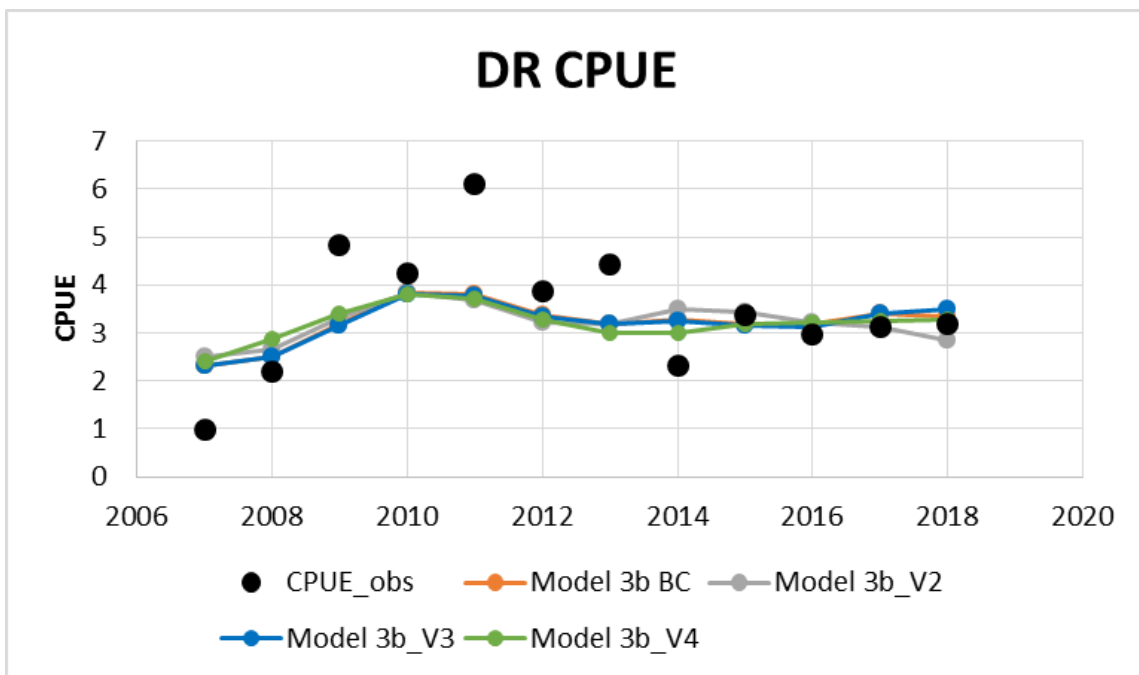


Figure 2b: Comparisons amongst the model fits to the Dual Rights vessels' CPUE values.

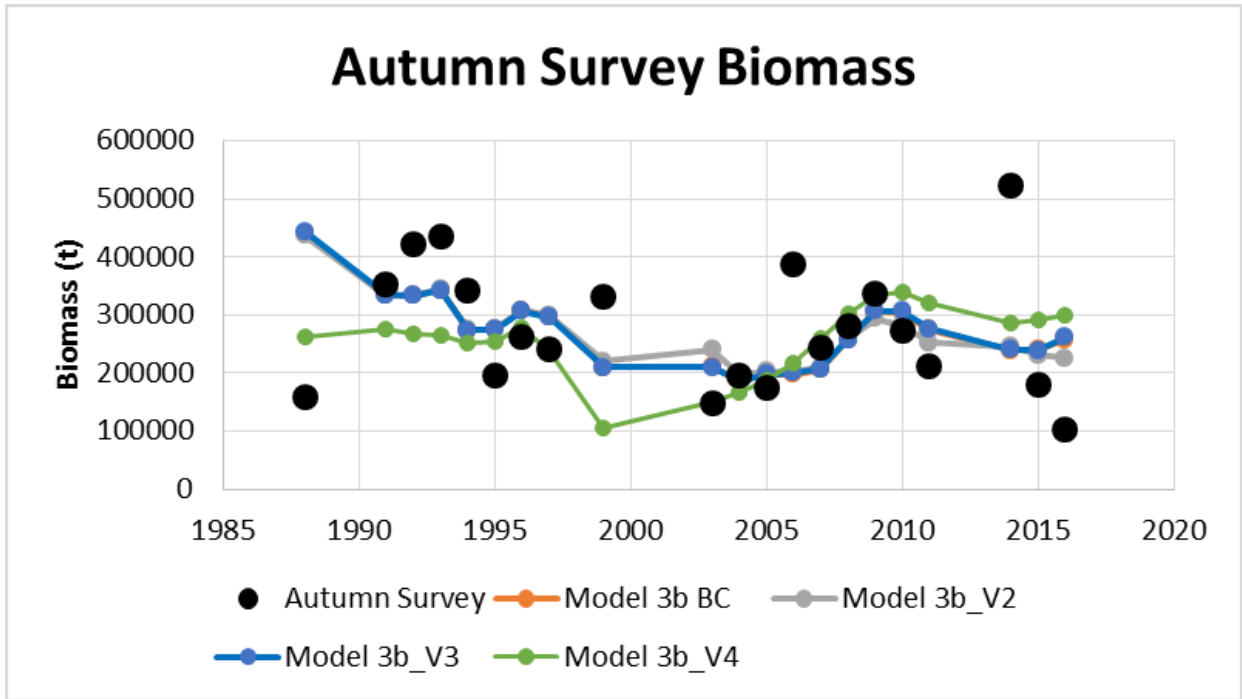


Figure 3a: Model fits to the Autumn survey biomass estimates.

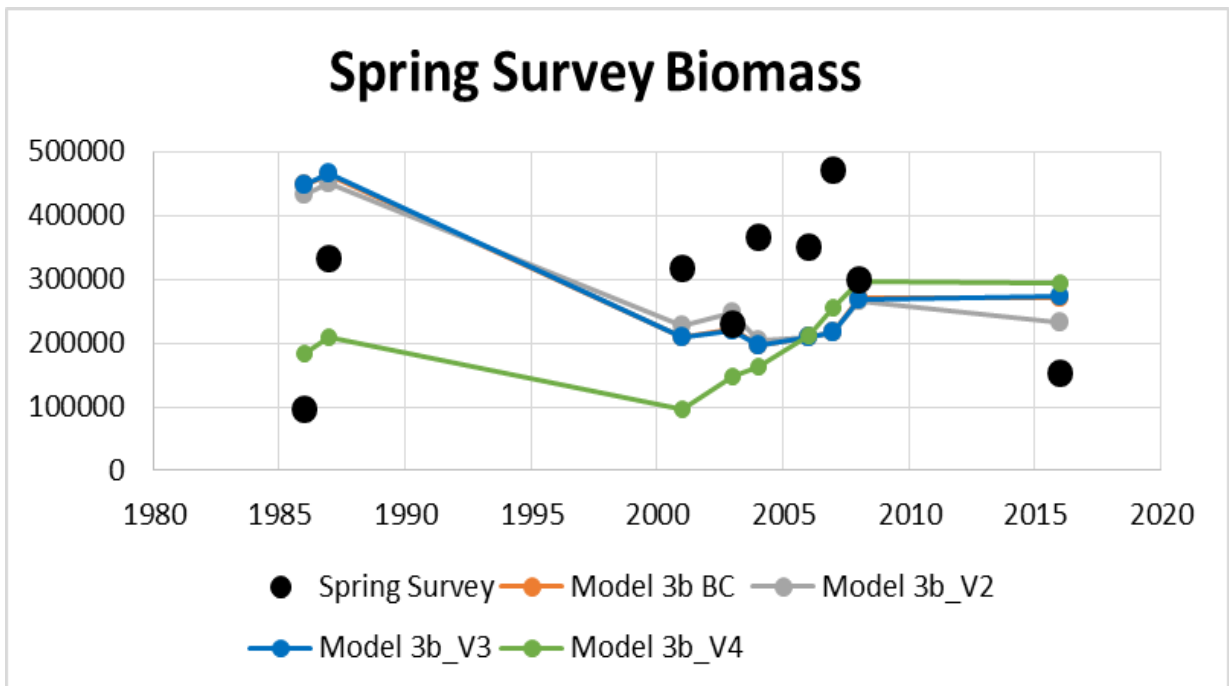


Figure 3b: Model fits to the Spring survey biomass estimates.

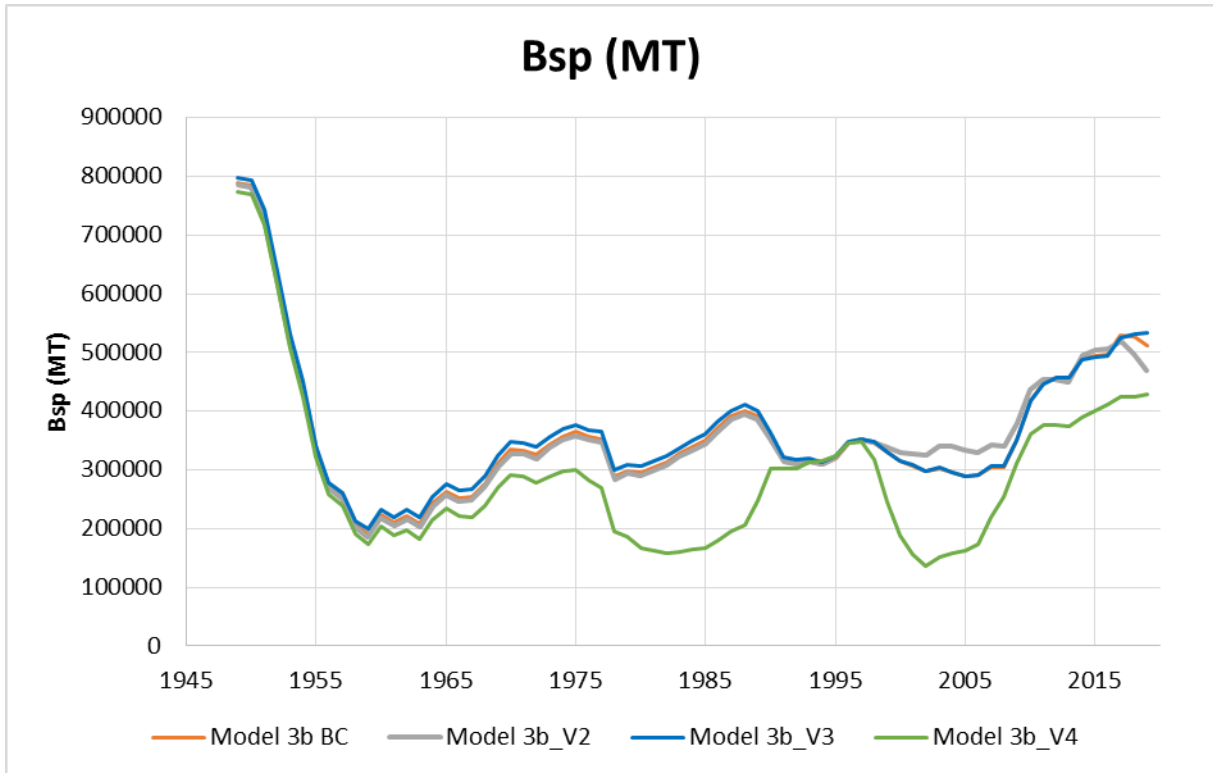


Figure 4a: Spawning biomass estimates for four of the assessment models.

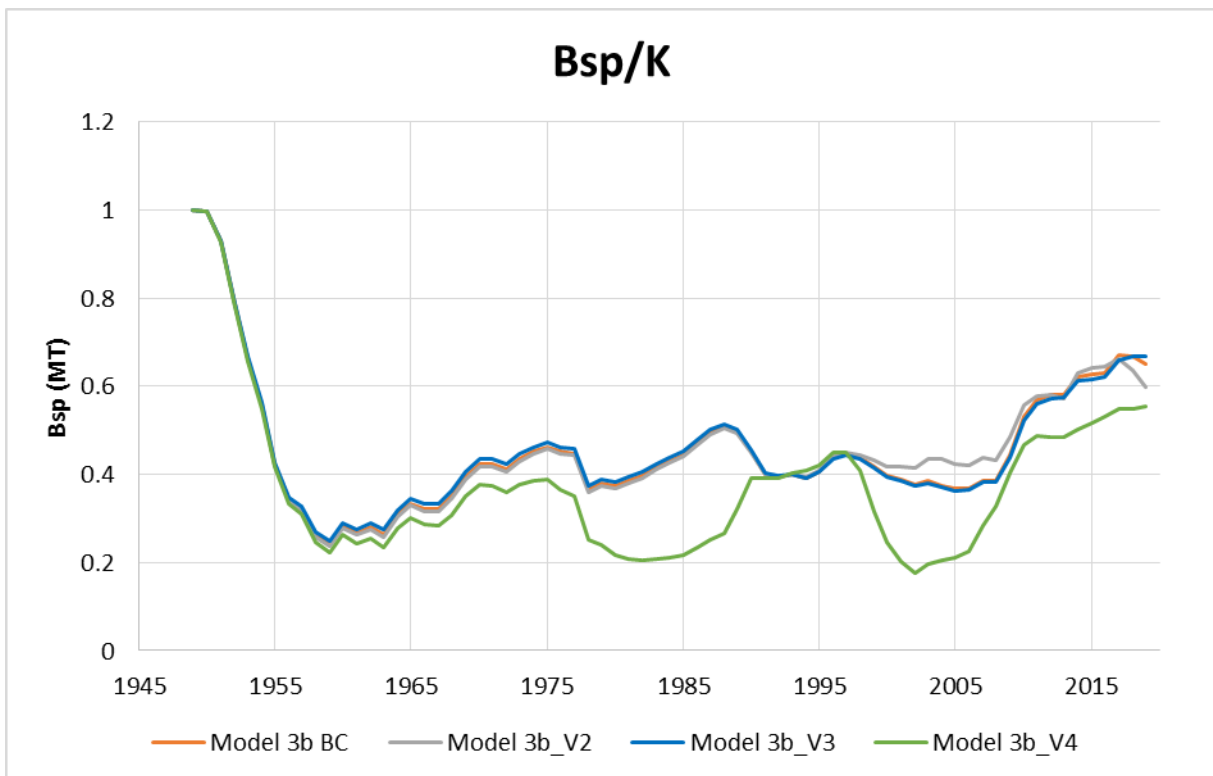


Figure 4b: Spawning biomass relative to K estimates for four of the assessment models.

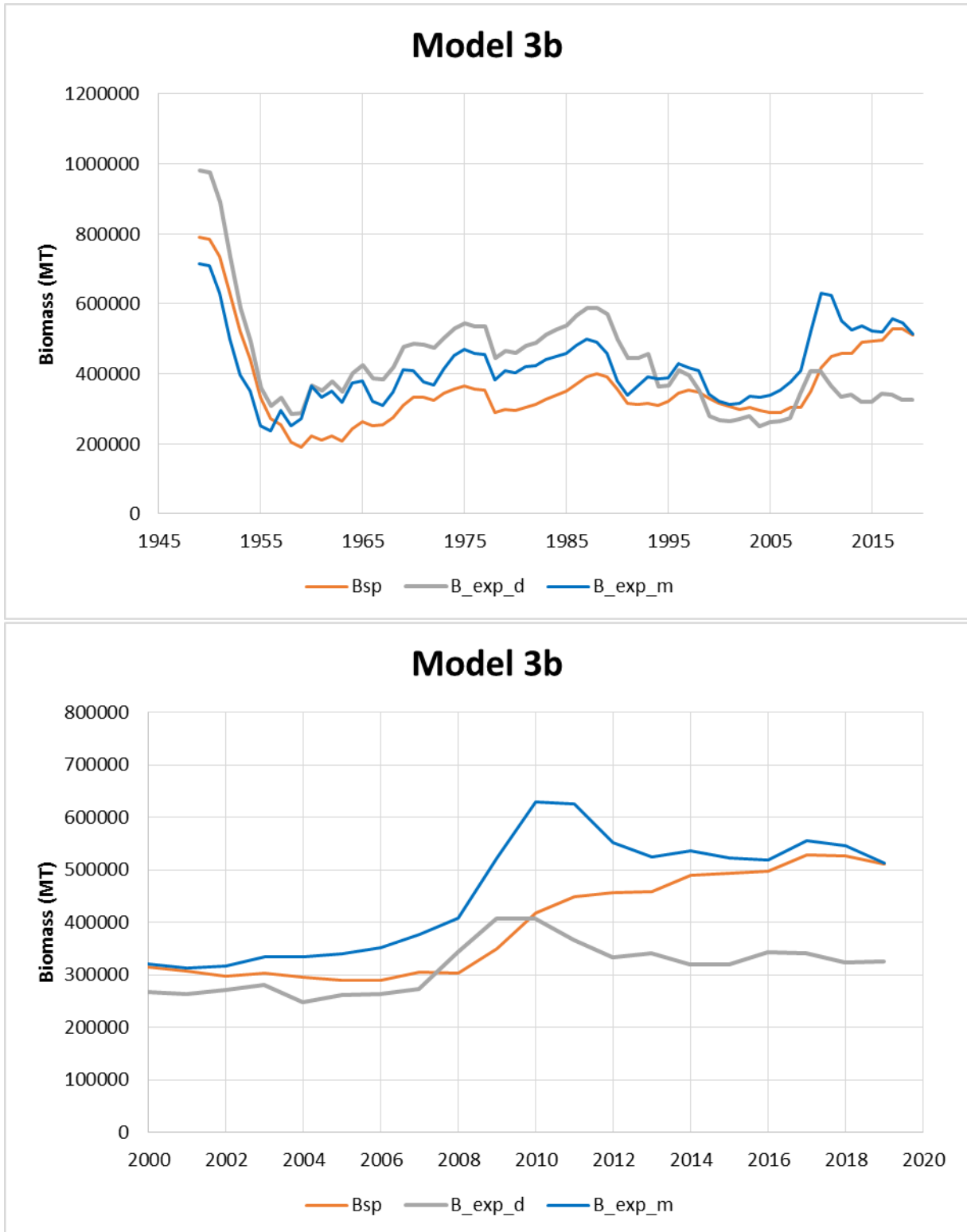


Figure 4c: Spawning biomass (Bsp), exploitable demersal (B\_exp\_d – related also to surveys) and exploitable midwater (B\_exp\_m – pertinent to the Desert Diamond) biomass trajectories for Model 3b BC.

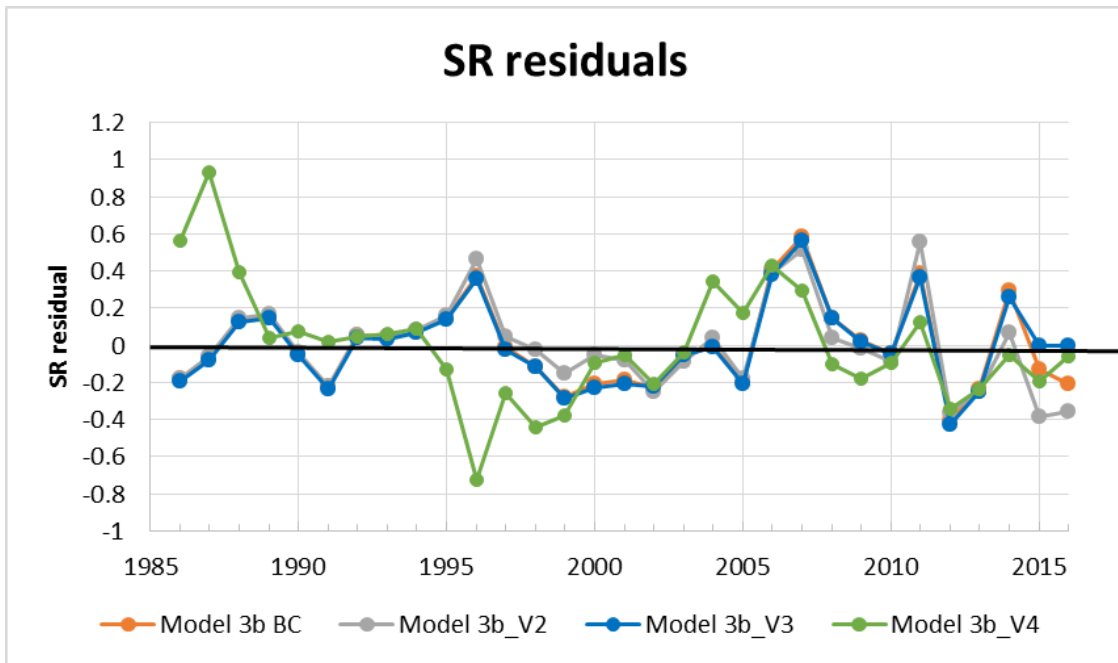


Figure 5a: Estimated stock-recruit residuals.

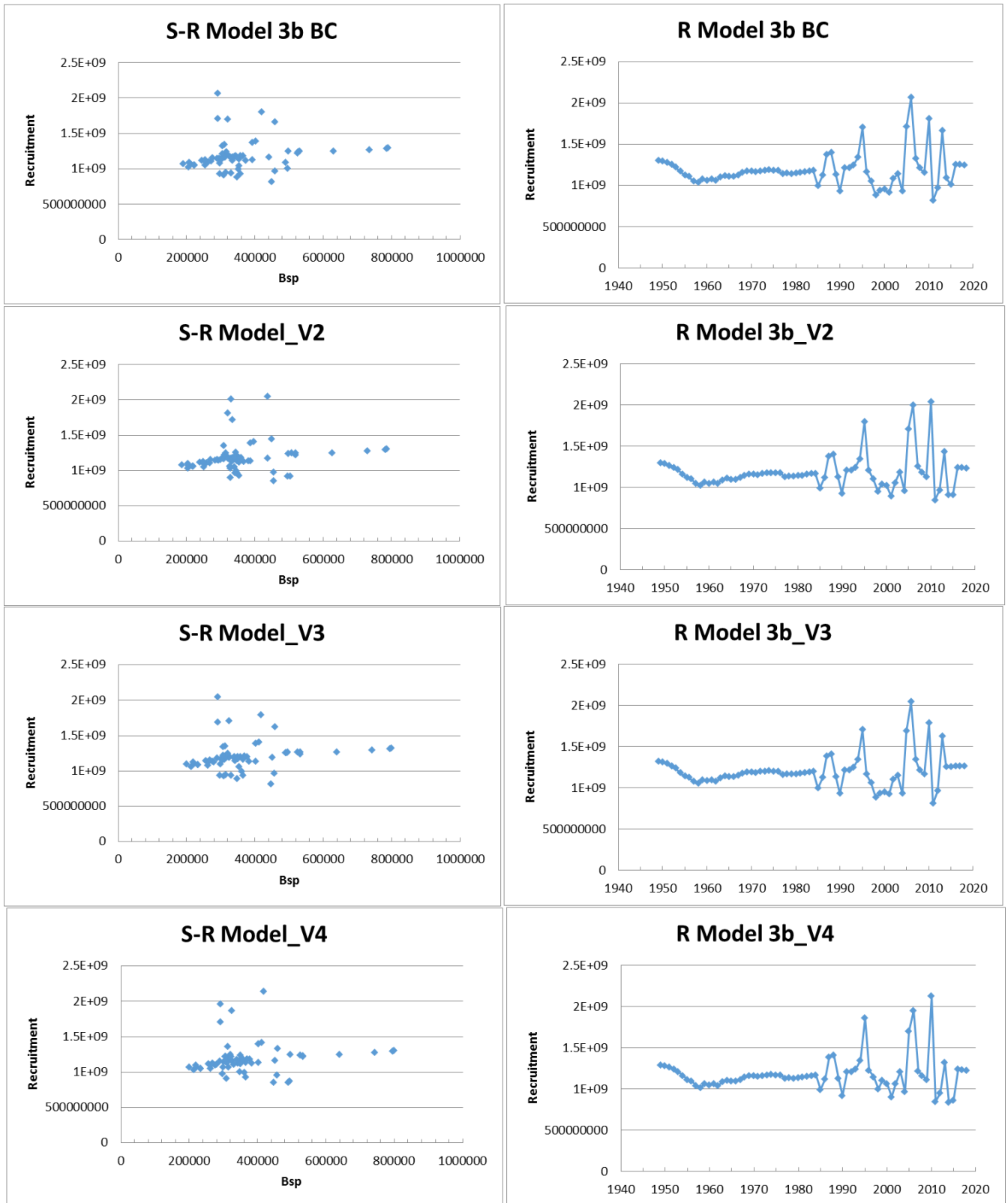


Figure 5b: Stock-recruit (left) and Recruitment over time (right) plots.

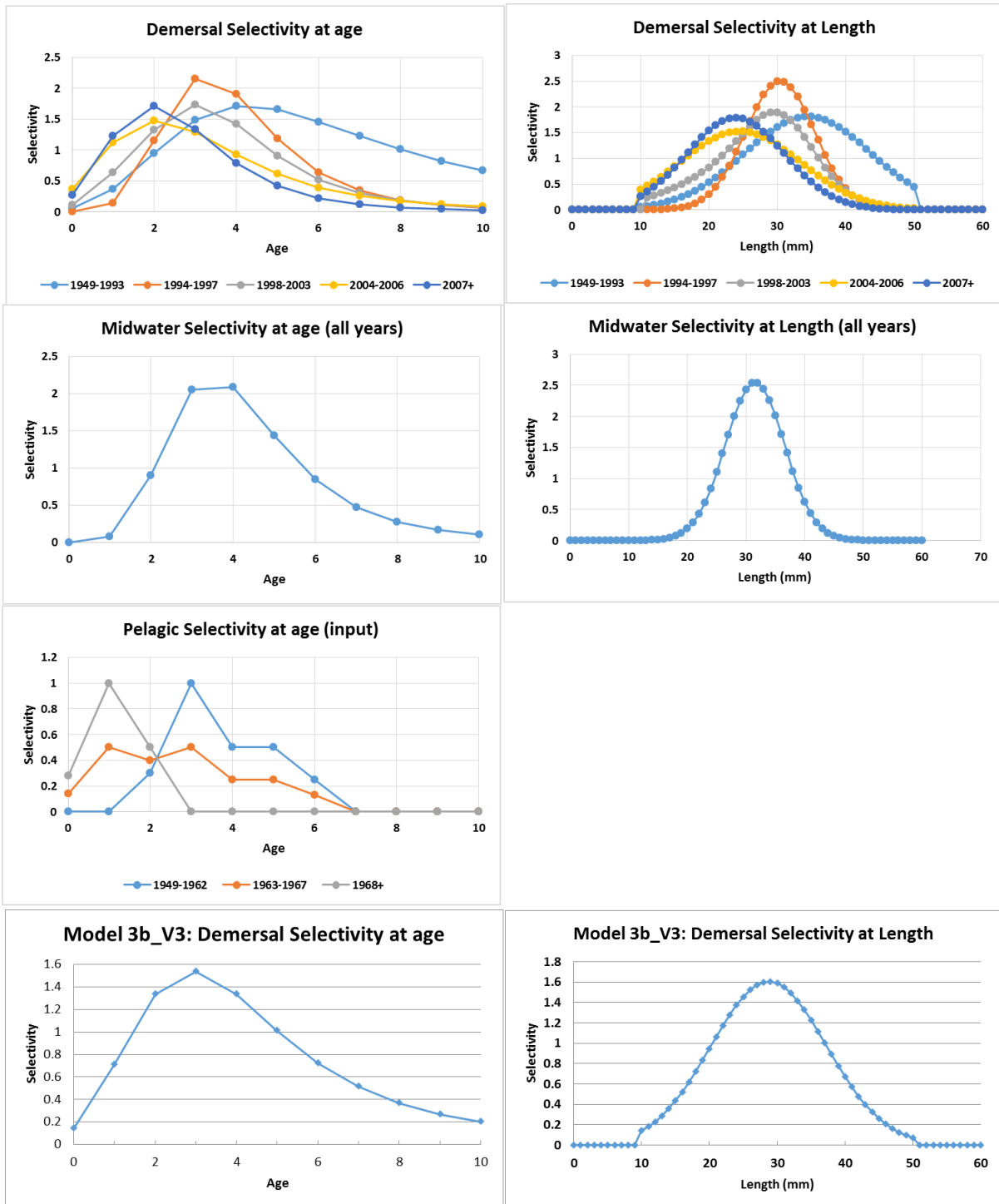


Figure 6a: Selectivity functions (Model 3b BC). The demersal selectivity plots apply also to the surveys. The bottom two plots show the time-invariant demersal selectivity for Model 3b\_V3. Age 10 as indicated reflects a plus group of ages 10+.

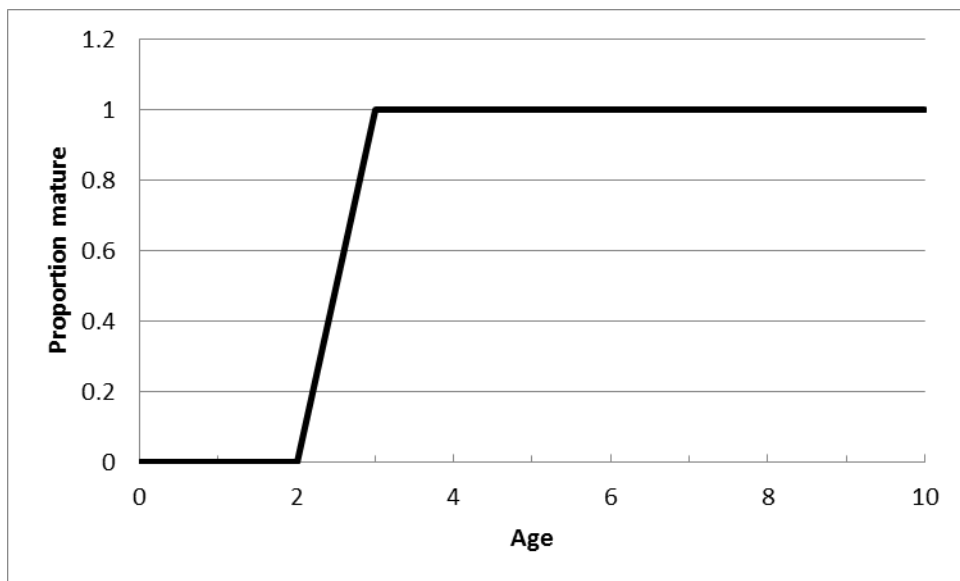


Figure 6b: Maturity at age. Age 10 as indicated reflects a plus group of ages 10+.

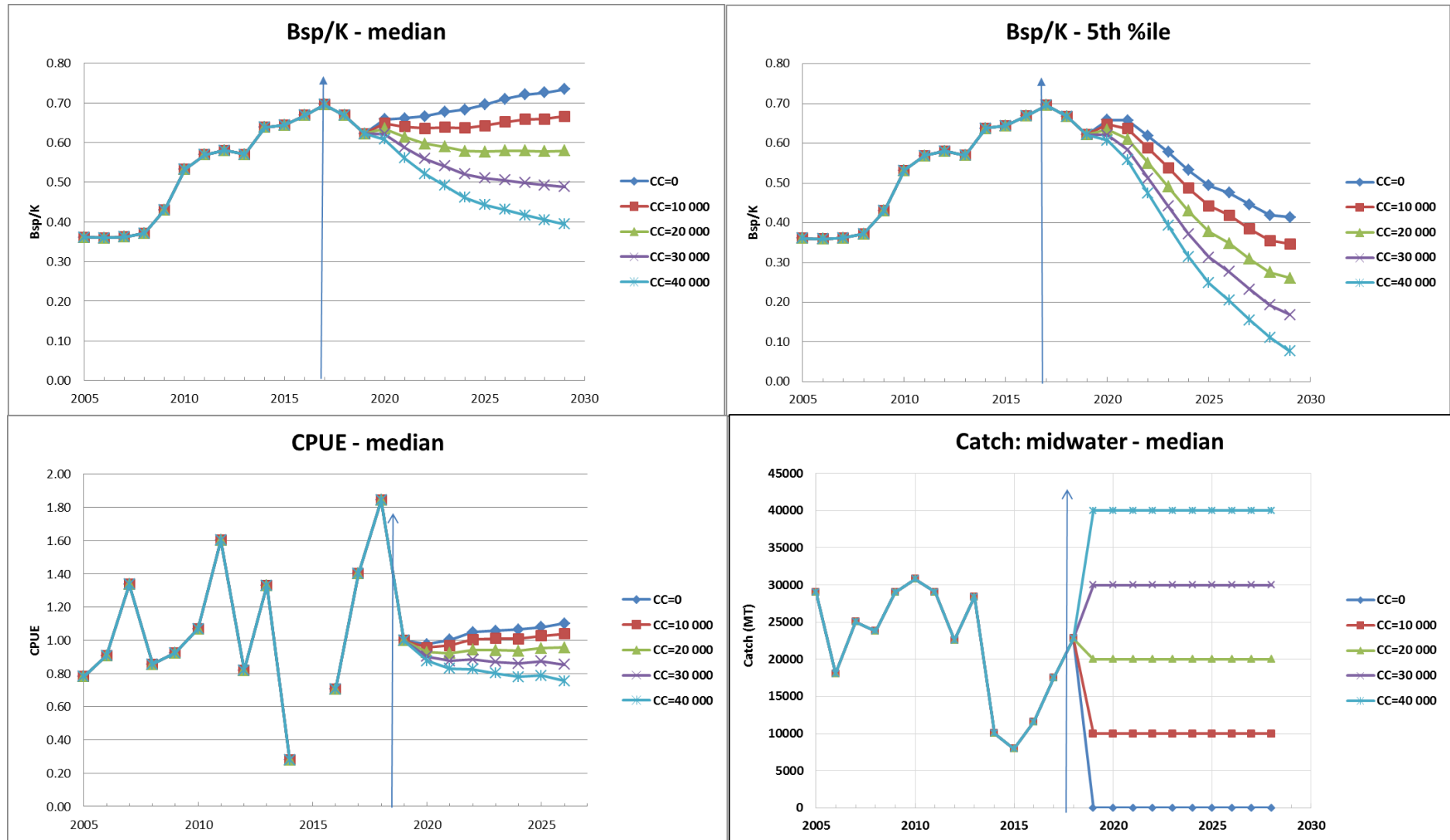


Figure 7: Model 3b BC projections for different constant future levels of annual midwater catch.

### Appendix 1: Input data used for the 2019 Horse mackerel assessments

Table A1: Horse Mackerel catch data for the three different fleets (values in MT) (data provided by DEFF).

Year	Pelagic catch	Demersal catch	Midwater catch
1949	3360	0.00001	0.00001
1950	49900	445	0.00001
1951	98900	1105	0.00001
1952	102600	1226	0.00001
1953	85200	1456	0.00001
1954	118100	2550	0.00001
1955	78800	1926	0.00001
1956	45800	1334	0.00001
1957	84600	959	0.00001
1958	56400	2073	0.00001
1959	17700	2075	0.00001
1960	62900	3712	0.00001
1961	38900	3627	0.00001
1962	66700	3079	0.00001
1963	23300	1401	0.00001
1964	24400	9522	0.00001
1965	55000	7017	0.00001
1966	26300	7596	0.00001
1967	8800	6189	0.00001
1968	1400	9116	0.00001
1969	26800	12252	0.00001
1970	7900	17872	0.00001
1971	2200	33329	0.00001
1972	1300	20560	0.00001
1973	1600	33900	0.00001
1974	2500	38391	0.00001
1975	1600	55459	0.00001
1976	400	50981	0.00001
1977	1900	116400	0.00001
1978	3600	37290	0.00001
1979	4300	53584.5	0.00001
1980	400	39187.5	0.00001
1981	6100	41215	0.00001
1982	1100	32176	0.00001

Year	Pelagic catch	Demersal catch	Midwater catch
1983	2100	38332	0.00001
1984	2800	37969	0.00001
1985	700	27278	0.00001
1986	500	31378	0.00001
1987	2834	38571	0.00001
1988	6403	41482	0.00001
1989	25872	58205.5	0.00001
1990	7645	56721.3	0.00001
1992	2057	37207.53	0.00001
1993	11651	35998	0.00001
1994	8207	20029.5	0.00001
1995	1986	10790	0.00001
1996	18920	31846	0.00001
1997	12654	34670.5	0.00001
1998	26680	36278.8	15769.8
1999	2057	21579.73	2160.77
2000	4503	9228.977	15375.74
2001	915	8813.736	19220.38
2002	8148	4863.111	11098.47
2003	1012	3562.168	25290.98
2004	2048	4933.367	27154.31
2005	5627	5280.164	29005.21
2006	4824	4132.990	18068.35
2007	1903	4811.698	24251.18
2008	2280	4449.295	23774.56
2009	2087	4128.813	29021.42
2010	4385	5595.850	23479.62
2011	10990	5228.260	29048.46
2012	2199	4941.442	22616.49
2013	596	2695.003	28480.64
2014	2760	3087.010	10053.03
2015	2040	4747.106	7975.594
2016	1588	5230.374	11612.686
2017	1466	5703.439	17545.203
2018	967	4625.880	22774.618

Table A2: GLM standardised CPUE (for the *Desert Diamond*), the Dual rights CPUE and survey abundance estimates for South African horse mackerel for the period 1986-2018. Data were provided by Coetzee, Fairweather and Singh (DEFF, *pers. commn*).

Year	Desert Diamond CPUE	Dual rights CPUE	Autumn demersal survey		Spring demersal survey	
			Biomass (KT)	CV	Biomass (KT)	CV
1986					97.36	0.13
1987					332.97	0.14
1988			159.07	0.29		
1989						
1990						
1991			352.19	0.23		
1992			422.21	0.23		
1993			435.28	0.20		
1994			340.72	0.26		
1995			195.13	0.24		
1996			261.77	0.23		
1997			241.02	0.23		
1998						
1999			330.63	0.24		
2000						
2001					316.72	0.18
2002						
2003	0.622		146.72	0.24	231.36*	0.20*
2004	0.525		195.73*	0.32*	366.50*	0.19*
2005	0.781		175.04*	0.21*		
2006	0.907		386.57	0.20	350.28	0.19
2007	1.336	0.994	243.58*	0.40*	473.22*	0.19*
2008	0.856	2.202	279.86*	0.27*	300.00*	0.17*
2009	0.923	4.820	337.16*	0.24*		
2010	1.068	4.254	271.79	0.37		
2011	1.602	6.098	213.09*	0.22*		
2012	0.820	3.854				
2013	1.331	4.432				
2014	0.280	2.311	522.69*	0.28*		
2015		3.381	180.08*	0.17*		
2016	0.706	2.960	104.00*	0.43*	153.32*	0.25*
2017	1.401 <sup>#</sup>	3.138				
2018	1.843	3.193				

\*These values correspond to surveys that used the new trawl net, which was introduced in September 2003.

<sup>#</sup> The 2017 DD CPUE has recently been corrected for an error made in its computation in 2018. Table A3a: **Spring** demersal survey catch-at-length for South African horse mackerel (shown as proportions of numbers each year) as used in the assessment models. Provided by Fairweather (DEFF, *pers. commn*).

Table A3a: Spring demersal survey catch-at-length for South African horse mackerel (shown as proportions of numbers each year) as used in the assessment model. Provided by Fairweather (DEFF, *pers. commn*).

Year	Total length (cm)								
	0–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45+
1986	0.0000	0.0000	0.0020	0.0900	0.2380	0.1640	0.1690	0.2310	0.1050
1987	0.0000	0.0000	0.1160	0.2230	0.1600	0.2060	0.1240	0.1290	0.0430
2001	0.0020	0.0150	0.3750	0.2550	0.1240	0.1360	0.0750	0.0150	0.0040
2003	0.0000	0.0500	0.0680	0.3760	0.3670	0.0910	0.0400	0.0080	0.0010
2004	0.0010	0.2380	0.2560	0.1610	0.2260	0.0740	0.0350	0.0080	0.0010
2006	0.0080	0.2670	0.2430	0.2880	0.1440	0.0410	0.0080	0.0010	0.0000
2007	0.0000	0.2230	0.6340	0.0950	0.0440	0.0030	0.0010	0.0000	0.0000
2008	0.0010	0.0270	0.4580	0.4290	0.0680	0.0100	0.0050	0.0020	0.0000
2016	0.0001	0.0263	0.2914	0.5157	0.1325	0.0223	0.0099	0.0008	0.0010

Table 3b: **Autumn** demersal survey catch-at-length for South African horse mackerel (shown as proportions of numbers each year) as used in the assessment models. Provided by Fairweather (DEFF, *pers. commn*).

Year	Total length (cm)								
	0–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45+
1988	0.0000	0.0150	0.0510	0.0140	0.1560	0.1660	0.1800	0.2910	0.1270
1992	0.0000	0.0720	0.0460	0.1050	0.3740	0.2730	0.0560	0.0430	0.0300
1993	0.0000	0.0920	0.3530	0.0750	0.1980	0.1180	0.0760	0.0650	0.0230
1994	0.0000	0.0270	0.1570	0.2200	0.2980	0.2540	0.0290	0.0100	0.0040
1995	0.0000	0.0000	0.0230	0.1090	0.4600	0.2710	0.0920	0.0330	0.0110
1996	0.0000	0.0000	0.0010	0.0230	0.5420	0.3080	0.1110	0.0130	0.0020
1997	0.0000	0.0030	0.0240	0.0050	0.4680	0.4010	0.0790	0.0160	0.0050
1999	0.0000	0.0100	0.1690	0.0630	0.0820	0.5220	0.1140	0.0330	0.0060
2003	0.0000	0.0010	0.3930	0.3290	0.1200	0.0600	0.0820	0.0150	0.0010
2004	0.0220	0.1420	0.4320	0.0550	0.1860	0.1000	0.0530	0.0080	0.0010
2005	0.0000	0.3540	0.1980	0.1480	0.1860	0.0570	0.0500	0.0070	0.0000
2006	0.0010	0.0330	0.2390	0.3450	0.2820	0.0630	0.0300	0.0060	0.0000
2007	0.1080	0.4630	0.3190	0.0880	0.0160	0.0040	0.0020	0.0010	0.0000
2008	0.0010	0.0710	0.3820	0.3840	0.1500	0.0090	0.0010	0.0020	0.0000
2009	0.0000	0.0680	0.1550	0.5250	0.2200	0.0280	0.0020	0.0010	0.0000
2010	0.0000	0.0560	0.0680	0.5270	0.2940	0.0440	0.0030	0.0060	0.0010
2011	0.1410	0.7700	0.0320	0.0330	0.0220	0.0010	0.0000	0.0000	0.0000
2014	0.0011	0.2538	0.3791	0.3062	0.0410	0.0132	0.0043	0.0007	0.0005
2015	0.0003	0.0550	0.3614	0.4436	0.0902	0.0350	0.0078	0.0023	0.0044
2016	0.0000	0.0678	0.1958	0.3441	0.1749	0.1353	0.0490	0.0313	0.0017

Table A3c: Commercial midwater catch-at-length for South African horse mackerel (shown as proportions of numbers each year) as used in the assessment models. Provided by Singh (DEFF, *pers. commn*).

Year	Total length (cm)								
	0–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45+
2003	0	0	0	0.0010	0.1350	0.2560	0.5050	0.1020	0.0010
2004	0	0	0	0.0120	0.2410	0.3820	0.3280	0.0360	0.0010
2005	0	0	0.0040	0.0790	0.2880	0.3880	0.1900	0.0350	0.0160
2006	0	0	0.0060	0.1130	0.3390	0.4030	0.1260	0.0100	0.0030
2007	0	0	0.0030	0.0900	0.2930	0.3590	0.1870	0.0540	0.0140
2008	0	0.0010	0.0430	0.2560	0.3280	0.2460	0.1110	0.0140	0.0010
2009	0	0	0.0010	0.0880	0.3860	0.3180	0.1700	0.0340	0.0020
2010	0	0	0.0180	0.2200	0.3780	0.2550	0.1000	0.0260	0.0030
2011	0	0	0.0052	0.0482	0.3945	0.1932	0.1272	0.1077	0.1240
2012	0	0	0.1175	0.1337	0.3229	0.2901	0.1027	0.0306	0.0024
2013	0	0.0001	0.4181	0.2915	0.0893	0.1555	0.0395	0.0047	0.0013
2014	0	0	0.0002	0.0414	0.1093	0.5491	0.2703	0.0273	0.0024
2016	0	0	0.0010	0.1707	0.5813	0.1906	0.0430	0.0111	0.0022
2017	0	0	0.0004	0.1868	0.5711	0.2040	0.0269	0.0089	0.0019
2018	0	0	0.0003	0.0818	0.4975	0.3481	0.0585	0.0096	0.0042

## Appendix 2: The Horse Mackerel Assessment Model

Furman (2014) developed and described the horse mackerel assessment model for his MSc thesis. This Appendix extracts pertinent sections from his thesis which describe the population assessment model, and provides a description of assumptions made for projecting the resource into the future.

### 1.1 Population Model

An age-structured production model (ASPM) is used as the underlying assessment model for the Horse Mackerel. This model is able to fit to CPUE indices as well as catch-at-length data to allow for past recruitment fluctuations to be estimated. The assessment process involves developing a model of the resource dynamics and conditioning its output to the available data by minimising a log-likelihood function. A single-stock model is used which is based largely on the assessment model by Johnston and Butterworth (2007), except that a midwater CPUE series, time-varying selectivity, catch-at-length data and recruitment fluctuations are incorporated. Important features of the model are described below, with full specifications given below.

#### 1.1.1 Dynamics

The ASPM reflects the dynamics of the resource over the period 1949-2018. The resource is managed as a single stock (since 2001). It is assumed that the population was in equilibrium at its carrying capacity in 1949. In reality, horse mackerel catches have been taken as bycatch in other fisheries since the 1900s but these catches were recorded only from 1949 shortly after substantial development of the pelagic fishery commenced. Nevertheless, the cumulative catch before 1949 is unlikely to have been high.

Pope's approximation to the Baranov equations is used to determine fishing mortality (Pope 1972). It assumes that all catches are taken as a pulse in the middle of the fishing season, instead of continuously throughout. The number of recruits at the start of a new year is related to the biomass of the mature component of the population (i.e. spawning biomass) of the previous year by a stock-recruitment relationship. A Beverton-Holt form is assumed. Additionally, stock-recruitment residuals that reflect natural fluctuations about expected recruitment are estimated for the years 1986-2016. Despite this variability in recruitment, the model assumes that at the start of the fishery in 1949, the population is stable at its unexploited equilibrium. Selectivity functions for each fleet are estimated during the fitting procedure, and are assumed to have a Gaussian dependence on length. Demersal fishing selectivity is additionally assumed to vary over time, because the corresponding catch-at-length data show distinct patterns over the years. However, the confounding between time-varying selectivity and catchability introduces difficulties which are addressed by normalising under the assumption that the demersal survey catchability remains constant over time. Further details are provided below.

#### 1.1.2 Likelihoods

The assessment model is conditioned on survey abundance and catch-at-length data, and on commercial CPUE and catch-at-length data. Additional contributions to the negative of the (penalised) log-likelihood come from the stock-recruitment residuals and various penalty functions which are discussed below.

The midwater CPUE and demersal survey time-series are both considered to be relative indices of abundance, each proportional to the biomass available to their respective fleets at midyear. However, without any estimates of biomass in absolute terms, the model is unable to estimate catchability coefficients for these indices reliably. The autumn survey is therefore treated as an absolute index by fixing its catchability to one of two values considered to be reasonable. The value of this catchability parameter is a key uncertainty of the model. Likelihoods are calculated by assuming that observed indices are log-normally distributed about their expected values. Although estimates of sampling variability are given for each demersal survey, the model estimates additional variance because there are likely to be other sources of variability; otherwise unrealistically high precision, and hence weight in the fitting procedure, would be accorded to these indices.

Because the assessment model is age-structured, catch-at-age estimates must be transformed into catch-at-length estimates before they can be compared to the observed catch-at-length data. This is done via an age-length matrix that is based on an input von Bertalanffy growth curve. The likelihood contributions are then calculated by comparing the model-predicted length distribution of horse mackerel catches with empirical data. Errors are assumed to be log-normally distributed.

The stock-recruitment residuals are also assumed to be log-normally distributed with no auto-correlation. Unfortunately, their variability cannot be estimated within the maximum likelihood framework used in this assessment, because the penalised likelihood function will always yield a minimum in the limit of the extent of this variability approaching zero. This issue is somewhat problematic, because recruitment fluctuations are of particular importance to the testing of pelagic MPs. While it could be dealt with by adopting a fully Bayesian methodology, it is simpler and adequate for present purposes to input the standard deviation for those residuals as a fixed value.

Finally, there are contributions to the negative log-likelihood from penalty functions. These do not correspond to any particular observed data or prior knowledge, but are instead included to discourage the optimisation from moving into unrealistic regions of parameter space, such as those resulting in negative population counts or fishing mortality. The models presented thus far for horse mackerel achieve convergence without triggering those penalty functions.

### 1.1.3 Parameters

#### Estimable parameters

A complete list of the 46 parameters estimated by the model fitting procedure is given below.

$K^{sp}$  is the pre-exploitation spawning biomass of horse mackerel;

$q_{spr}$  is the catchability coefficient for the spring demersal survey abundance index;

$\zeta_y$  is the fluctuation about the expected recruitment for year  $y$ , which is estimated for years 1986-2016;

$\mu^m$  is the centre of the Gaussian selectivity-at-length curve for the midwater fleet;

$\lambda^m$  controls the width of the Gaussian selectivity-at-length curve for the midwater fleet;

$\mu_{y1-y2}^d$  is the centre of the Gaussian selectivity curve for the demersal fleet for years  $y1$ - $y2$ , and is

estimated for periods 1949-1993, 1994-1997, 2004-2006 and 2007+;

$\lambda_{y1-y2}^d$  controls the width of the Gaussian selectivity-at-length curve for the demersal fleet for years  $y1-y2$ , and is estimated for the periods 1949-1993, 1994-1997, 2004-2006 and 2007+; and

$\sigma_{add}^s$  is the square root of the additional variance for the survey abundance index  $s$  ( $s$  is either *aut* for the autumn survey of *spr* for the spring survey), and reflects variability not included in the corresponding survey CVs.

### Input parameters

Some parameters cannot be estimated by the model, or are adequately specified by other studies and need not be estimated. They are therefore input with fixed values. The following is a list of these parameters:

$q_{aut}$  is the catchability coefficient for the autumn demersal survey abundance index, and is assumed to be 0.75;

$h$  is the “steepness” of the Beverton-Holt stock-recruitment function, and is assumed to be 0.75;

$M$  is the natural mortality rate of horse mackerel, and is fixed at  $0.3 \text{ yr}^{-1}$ ; although this choice is somewhat arbitrary (Johnston and Butterworth 2007), Horsten (1999a) found key ASPM results to be fairly robust to alternative assumptions regarding this value.

$a_m$  is the age-at-maturity for South African horse mackerel, and is described by a knife-edge function of age with 100% of the population being sexually mature at 3 years (Butterworth and Clark 1996; Hecht 1990);

$l_a$  is the expected length of a fish at age  $a$  in centimetres, and is based on the von Bertalanffy growth function given by Equation A.1 and the growth parameters reported in Table A1 below;

$w_a$  is the weight in metric tonnes of a fish at age  $a$ , and is based on the length-at-age relationship described above, in combination with the mass-at-length function given by Equation A.2 and the growth parameters reported in Table A1;

$S_{a,y1-y2}^p$  is the fishing selectivity for the pelagic fleet for a fish at age  $a$  for years  $y1-y2$ , and is listed in Table A2 for the periods 1949-1962, 1963-1967 and 1968+;

$\sigma_R$  is the standard deviation of the stock-recruitment log-residuals, and is assumed to be equal to 0.5, which is roughly typical for a species like horse mackerel;

$\gamma$  is the CV of the length distribution of horse mackerel at any given age, and is assumed to be equal to 0.09 because this value provides good fits to catch-at-length data and lies within the expected range for a species like horse mackerel; and

$w_{cal}$  is the weighting of the catch-at-length likelihood contributions, and is fixed at 0.35 (a weighting of 1 is equivalent of being “unweighted”).

### Growth

The Cape horse mackerel has a maximum reported (fork) length of 60cm and may live to more than ten years of age (Bianchi *et al.* 1999). The length-at-age relationship used in the work presented in this thesis is taken from Kerstan (pers. commn) as quoted in Horsten (1999b). This relationship takes the form of a von Bertalanffy growth curve:

$$l_a = l_\infty(1 - e^{-\kappa(a-t_0)}) \quad (\text{A.1})$$

where

$l_a$  is the expected total length of a fish of age  $a$  in years in centimetres;

$l_\infty$  is the asymptotic total length in centimetres;

$\kappa$ , the Brody growth coefficient, is a growth rate parameter; and

$t_0$  is the theoretical age at which length would be zero.

The mass-at-length relationship used for Cape horse mackerel is from Naish *et al.* (1991). It is provided by the power model:

$$w = \alpha(l)^\beta \quad (\text{A.2})$$

where

$w$  is the expected weight in grams of fish;

$l$  is the total length of the fish in centimetres; and

$\alpha$  and  $\beta$  are growth parameters.

Estimates for the parameters of these growth equations are reported in Table A1. Hecht (1990) found no difference between the mean length-at-age of males and females. This provides further support for a sex-aggregated model.

Given the limited data available at present, the assessment model is unable to reliably estimate the parameters  $q_{aut}$  (autumn survey catchability) and  $h$  (stock-recruitment steepness). Hence they must be set externally. Note that  $q_{aut}$  can be thought of as a measure of the bias in the survey absolute biomass estimates. For example, a value of 0.5 means that actual biomass is twice as large as the swept-area estimate from the surveys, whereas a value of 1 would mean that these surveys provide unbiased results.  $h$  determines the productivity of the resource, with a larger  $h$  corresponding to

greater productivity. The Base Case model, which is used in assessments and MP testing since 2014 assumes  $q_{aut} = 0.75$ ;  $h = 0.75$ .

## 1.2 Projections

Projections are simulations of the future state of a fishery given present understanding of the resource dynamics as represented by an assessment model. By providing a basis to calculate fishery performance statistics, they give means of testing candidate MPs and enable stake-holders to make informed decisions about trade-offs. In this section we look at projections under the assumption of constant future catches for both the future pelagic catches and future midwater catches. The horse mackerel resource is projected 30 years into the future. Because there are stochastic elements in the model dynamics, 1000 projections, each using different random numbers, are simulated for each future catch scenario as explained below. This allows for realistic estimates of performance statistics. Additionally, the random number generator is seeded with the same value at the start of each set of 1000 projections in order to eliminate the variability that would result from using different seeds; this allows for readier comparisons between scenarios.

To simplify projections, the time-varying fishing selectivities for the pelagic and demersal fleets are assumed to remain in the future at their 2012 values. Future stock-assessment residuals are drawn randomly from a normal distribution with a standard deviation of  $\sigma_R$ . Additionally, they are assumed to be serially correlated, with a Pearson correlation coefficient of 0.47. This  $r$  value is taken from the serial correlation of the mode-estimated residuals.

Future “observed” midwater CPUE and autumn demersal and pelagic survey biomass estimates are generated during projections, because these indices of abundance are potentially useful as inputs to many MPs. Realistic observation errors are added to the expected values of these abundance indices by drawing them at random from the same log-normal distribution assumed in the assessment model (Equations A.26, A.28, A.30 and A.31 below). The variance of the error distribution for the CPUE and pelagic survey indices are estimated in the assessment (Equation A.32 below), while the variance for the autumn demersal survey abundance estimate is a combination of the estimated  $\sigma_{add}^{aut}$  (additional variance) and a CV (Equation A.29 below). Future CVs are drawn randomly with replacement from historic autumn survey CVs.

Table A1: Parameter values for the von Bertalanffy growth curve (Equation A.1) and mass-at-length relationship (Equation A.2) for Cape horse mackerel. Values reported are taken from Kerstan *pers. commn*) as quoted in Horsten (1999b) and from Naish *et al.* (1991) respectively.

Parameter	Value
$l_{\infty}$ (cm)	54.56
$\kappa$ ( $\text{yr}^{-1}$ )	0.183
$t_0$ (yr)	-0.654
$\alpha$ ( $\text{g}/\text{cm}^{\beta}$ )	0.0078
$\beta$	3.011

Table A2: Selectivity-at-age vectors assumed for the pelagic fleet over three different periods (Johnston and Butterworth 2007).

Age (yr)	Period		
	1949-1962	1963-1967	1968+
0	0.00	0.14	0.28
1	0.00	0.50	1.00
2	0.30	0.40	0.50
3	1.00	0.50	0.00
4	0.50	0.25	0.00
5	0.50	0.25	0.00
6	0.25	0.13	0.00
7	0.00	0.00	0.00
8	0.00	0.00	0.00
9	0.00	0.00	0.00
10+	0.00	0.00	0.00

## Mathematical details of the ASPM

### A.1 Dynamics

The population dynamics are described by the following equations:

$$N_{y+1,0} = R_{y+1} \quad (\text{A.3})$$

$$N_{y+1,a+1} = (N_{y,a}e^{-M/2} - C_{y,a})e^{-M/2} \quad 0 \leq a \leq m-2 \quad (\text{A.4})$$

$$N_{y+1,m} = \left(N_{y,m}e^{-\frac{M}{2}} - C_{y,m}\right)e^{-\frac{M}{2}} + (N_{y,m-1}e^{-\frac{M}{2}} - C_{y,m-1})e^{-M/2} \quad (\text{A.5})$$

where

$N_{y,a}$  is the number of horse mackerel of age  $a$  at the start of year  $y$ ;

$C_{y,a}$  is the total number of horse mackerel of age  $a$  taken in year  $y$  by the pelagic, midwater and demersal fleets combined;

$R_y$  is the number of recruits (0-year olds) at the start of year  $y$ ;

$M$  is the natural mortality rate for horse mackerel; and

$m$  is the minimum age within the plus-group and is set here to ten years old.

The approximation of the fishery as a pulse catch in the middle of the season is considered of sufficient accuracy for present purposes. Note that the model also assumes that recruitment to the population occurs at the start of a new year (Equation A.3), even though in reality there are two pawning peaks roughly two months apart.

The total number of horse mackerel of age  $a$  caught each year is given by:

$$C_{y,a} = \sum_f C_{y,a}^f \quad (\text{A.6})$$

where  $f$  indicates the fishery concerned and in this case is either  $p$  for pelagic,  $d$  for demersal or  $m$  for midwater.

The annual catch by mass for fleet  $f$  is given by:

$$\begin{aligned} C_y^f &= \sum_{a=0}^m w_{y,a}^f C_{y,a}^f \\ &= \sum_{a=0}^m w_{y,a}^f S_{y,a}^f F_y^f N_{y,a} e^{-M/2} \end{aligned} \quad (\text{A.7})$$

where

$S_{y,a}^f$  is the fishing selectivity-at-age for fleet  $f$  for fish of age  $a$  in year  $y$ ;

$F_y^f$  is the fleet-specific fishing mortality for a fully selected age class in year  $y$ ; and

$w_{y,a}^f$  is the effective weight of a horse mackerel of age  $a$  for fleet  $f$  in year  $y$ .

Fishing selectivity for the pelagic fleet is described by a selectivity-at-age function; therefore that fleet's effective weight-at-age (in gm) is simply given by a combination of the length-at-age (in cm) and weight-at-length relationships discussed above:

$$l_a = 54.56[1 - e^{-0.183(a+0.654)}] \quad (\text{A.8})$$

$$w_a^p = 0.0078 l_{a+1/2}^{3.011} \times 10^{-6} \quad (\text{A.9})$$

Because the fishing selectivities of the midwater and demersal fleets are modelled by selectivity-at-length functions, their effective weights-at-age must be calculated differently:

$$w_{y,a}^f = \frac{\sum_l w_l S_{y,l}^f A_{l,a}}{\sum_l S_{y,l}^f A_{l,a}} \quad (\text{A.10})$$

where

$w_l$  is the weight of a horse mackerel of length  $l$  (Equation A.2);

$S_{y,l}^f$  is the fishing selectivity for fleet  $f$  for fish of length  $l$  in year  $y$ ; and

$A_{l,a}$  is the age-length key, which gives the proportion of fish of age  $a$  that are of length  $l$  (detailed later in Equation A.18).

Note that fishing selectivity for the midwater fleet is assumed to be time-invariant; therefore the  $y$  subscript may be dropped when determining the effective weight-at-age for that fleet.

The fleet-specific exploitable component of abundance is taken to be given by exploitable biomass at midyear:

$$B_y^f = \sum_{a=0}^m w_{y,a}^f S_{y,a}^f N_{y,a} e^{-M/2} \quad (\text{A.11})$$

or in terms of numbers of individuals:

$$N_y^f = \sum_{a=0}^m S_{y,a}^f N_{y,a} e^{-M/2} \quad (\text{A.12})$$

The proportion of the resource harvested each year by fleet  $y$  is therefore given by:

$$F_y^f = \frac{C_y^f}{B_y^f} \quad (\text{A.13})$$

and

$$C_{y,a}^f = S_{y,a}^f F_y^f N_{y,a} e^{-M/2} \quad (\text{A.14})$$

Note that in terms of Equations A.13 and A.14 the model assumes the same fishing selectivity for the commercial demersal fleet and both demersal surveys. This simplifying assumption has been made because there are no catch-at-length data available to estimate selectivity functions for the commercial fleet.

### Fishing selectivities

Selectivity-at-age for the pelagic fleet is input and assumed to change with time. The same values are used as for the 2007 assessment model (Johnston and Butterworth 2007). Essentially there is one selectivity function for the pre-1993 period and another for the post-1967 period, while for the period between (1963-1967) the average of those two selectivity functions is used.

In contrast, selectivity-at-length is estimated for both the midwater and demersal fleets. These are assumed to have a Gaussian form with length:

$$S_{y,l}^f = \begin{cases} \frac{e^{-(l-\mu_y^f)^2}}{2(\lambda_y^f)^2} & \text{if } l_{min}^f \leq l \leq l_{max}^f \text{ or} \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.15})$$

where

$\mu_y^f$  is an estimated selectivity parameter that determines the centre of the Gaussian for fleet  $f$  in year  $y$ ;

$\lambda_y^f$  is an estimated parameter that determines the width of the Gaussian for fleet  $f$  in year  $y$ ;

$l_{min}^f$  is a fixed selectivity parameter that determines the smallest length class with non-zero selectivity for fleet  $f$ , and is set equal to 10cm for both the demersal or the midwater fleets, and

$l_{max}^f$  is a fixed selectivity parameter that determines the largest length class with non-zero selectivity for fleet  $f$ , and is set equal to 50cm or 60cm for the demersal or midwater fleets respectively.

Note again that the  $y$  subscript may be dropped when dealing with selectivity for the midwater fleet because it is time-invariant. Selectivity-at-length is then normalised according to:

$$S_{y,l}^f \rightarrow S_{y,l}^{*,f} = \frac{S_{y,l}^f}{\sum_{l'=l_1}^{l_2} \frac{S_{y,l'}^f}{l_2-l_1+1}} \quad (\text{A.16})$$

In other words, the selectivity function is scaled by the inverse of its average value over a certain length range.  $l_1$  and  $l_2$  are the same for both midwater and the demersal fleets and are set equal to 10cm and 40cm respectively.

Because the model is age-structured, selectivity-at-length must be transformed into selectivity-at-age using an age-length relationship:

$$S_{y,a}^f = \sum_l A_{l,a} S_{y,l}^f \quad (\text{A.17})$$

It is assumed that the length distribution for horse mackerel of age  $a$  is described by a normal distribution with mean which is given by the von Bertalanffy growth curve input, and with a standard deviation that is proportional to this mean. Consequently, with length classes of 1cm,  $A_{l,a}$  is computed according to:

$$A_{l,a} = \frac{1}{2} \left[ \text{erf} \left( \frac{l+0.5-l_{a+0.5}}{\sqrt{2}(\gamma l_{a+0.5})} \right) - \text{erf} \left( \frac{l-0.5-l_{a+0.5}}{\sqrt{2}(\gamma l_{a+0.5})} \right) \right] \quad (\text{A.18})$$

where

erf is the error function;

$l_{a+0.5}$  is the expected midyear length for a horse mackerel of age  $a$ , which is calculated using the

input von Bertalanffy growth curve given by Equation A.1; and

$\gamma$  is the CV of the length-at-age distribution, which is fixed at 0.9.

### Stock-recruitment relationship

The spawning biomass in year  $y$  is given by:

$$B_y^{sp} = \sum_{a=a_m}^m w_a N_{y,a} \quad (\text{A.19})$$

where

$a_m$  is the age corresponding to 100% sexual maturity, which is assumed here to be described by a

knife-edge function of age; and

$w_a$  is the mass of a horse mackerel of age  $a$  at the start of the year.

The number of recruits at the start of fishing year  $y$  is related to the spawner stock size by a Beverton-Holt stock-recruitment relationship:

$$R(B_y^{sp}) = \frac{\alpha B_y^{sp}}{\beta + B_y^{sp}} e^{\zeta y} \quad (\text{A.20})$$

where

$\alpha$  and  $\beta$  are stock-recruitment parameters; and

$\zeta_y$  are stock-recruitment residuals reflecting fluctuations about expected recruitment in year  $y$ .

In order to work with estimable parameters that are more biologically meaningful than  $\alpha$  and  $\beta$ , the stock-recruitment relationship is re-parameterised in terms of pre-exploitation equilibrium spawning biomass,  $K^{sp}$ , and the steepness of the stock-recruitment relationship,  $h$ , where steepness is the fraction of pristine recruitment,  $R_0$ , that results when spawning biomass drops to 20% of its pristine level:

$$hR_0 = R(0.2K^{sp}) \quad (\text{A.21})$$

from which it follows that:

$$h = \frac{0.2(\beta + K^{sp})}{\beta + 0.2K^{sp}} \quad (\text{A.22})$$

and hence:

$$\alpha = \frac{4hR_0}{5h-1} \quad (\text{A.23})$$

and

$$\beta = \frac{K^{sp(1-h)}}{5h-1} \quad (\text{A.24})$$

Given a value for the pre-exploitation spawning biomass  $K^{sp}$  of horse mackerel, together with the assumption of an initial equilibrium age-structure, pristine recruitment can be determined from:

$$R_0 = \frac{K_{sp}}{[\sum_{a=am}^{m-1} w_a e^{-aM} + w_m e^{-mM} / (1 - e^{-M})]} \quad (\text{A.25})$$

## A.2 Likelihood functions

The model is fitted to three biomass indices and three sets of catch-at-length data. Stock recruitment residuals also contribute to the penalised negative log-likelihood that is minimised in the fitting process.

### Abundance indices

The assessment model is ordinarily fitted to three abundance indices: spring and autumn demersal biomass estimates, and a commercial midwater CPUE series. The associated likelihood contribution are calculated by assuming that the observed abundance index is log-normally distributed about its expected value:

$$I_y^s = \hat{I}_y^s e^{\epsilon_y^s} \text{ or } \epsilon_y^s = \ln(I_y^s) - \ln(\hat{I}_y^s) \quad (\text{A.26})$$

where

$s$  indicates the abundance index concerned and is either *aut* for the autumn survey, or *spr* for the spring survey, *cpue* for CPUE or *pel* for the pelagic index;

$I_y^s$  is the observed value of index  $s$  in year  $y$ ;

$\hat{I}_y^s$  is the model predicted value of  $s$  in year  $y$ .

The negative of the log-likelihood function (after removal of the constant) is then given by:

$$-\ln L = \sum_s \sum_y [\ln \sigma_y^s + (\epsilon_y^s)^2 / 2(\sigma_y^s)^2] \quad (\text{A.27})$$

The spring and autumn demersal survey biomass estimates are assumed to reflect demersal exploitable biomass:

$$\hat{I}_y^s = q_s B_y^d \quad (\text{A.28})$$

where  $q_s$  is the catchability coefficient corresponding to index  $s$ . Note that the same demersal exploitable biomass  $B_y^d$  is used to fit both the autumn and spring demersal surveys even though they occur several months apart. Because a mid-year pulse catch assumption is made (Equation A.13), this exploitable biomass does not account for fishing mortality that may occur between the surveys. For these series, reliable estimates of sampling variability and additional variance are available; therefore the standard deviations are calculated according to the following formula:

$$\sigma_y^s = \sqrt{\ln[1 + (CV_y^s)^2] + (\sigma_{add}^s)^2} \quad (\text{A.29})$$

where

$CV_y^s$  is the CV for survey  $s$  in year  $y$ , which is given in Table A2, and

$\sigma_{add}^s$  is the model estimated additional variance for survey abundance index  $s$ .

The midwater CPUE index is assumed to reflect the midwater exploitable biomass:

$$\hat{I}_y^{cpue} = q_{cpue} B_y^m \quad (\text{A.30})$$

and the pelagic hydro-acoustic survey index from November of year  $y$  is assumed to reflect recruitment in year  $y+1$ :

$$\hat{I}_y^{pel} = q_{pel} R_{y+1} \quad (\text{A.31})$$

Reliable estimates of CVs and catchability are unavailable for the CPUE and pelagic abundance indexes. Therefore, they are set to their maximum likelihood estimates:

$$\sigma^s = \sqrt{1/n \sum_y (\epsilon_y^s)^2} \quad (\text{A.32})$$

$$\ln q_s = 1/n \sum_y \epsilon_y^s \quad (\text{A.33})$$

### Catch-at-length

Model estimated catch-at-length proportions are fitted to spring and autumn demersal survey length-frequency data, and commercial midwater length frequency data.

Catch-at-age estimates (Equation A.14) are transformed into catch-at-length estimates using the age-length relationship  $A_{l,a}$  (Equation A.18):

$$C_{y,l}^f = \sum_{a=0}^m A_{l,a} C_{y,a}^f \quad (\text{A.34})$$

where  $C_{y,l}^f$  is the total number of horse mackerel of length  $l$  caught in year  $y$ .

The contribution of catch-at-length data to the negative log-likelihood function is then given by:

$$-\ln L = w_{cal} \sum_s \sum_y \sum_l [\ln \sigma_{cal}^s + \left( \sqrt{p_{y,l}^s} - \sqrt{\hat{p}_{y,l}^s} \right)^2 / 2(\sigma_{cal}^s)^2] \quad (\text{A.35})$$

where

$w_{cal}$  is a weighting for this likelihood contribution, and is fixed at 0.35;

$p_{y,l}^s$  is the observed proportion of fish caught in year  $y$  that are of length  $l$  for dataset  $s$ ;

$\hat{p}_{y,l}^s$  is equal to  $C_{y,l}^f / \sum_l C_{y,l}^f$  and is the model predicted proportion of fish caught in year  $y$  that are of length  $l$  in dataset  $s$ , where  $f$  is the appropriate fleet; and

$\sigma_{cal}^s$  is the standard deviation associated with catch-at-length dataset  $s$ , which is estimated in the fitting procedure by:

$$\sigma_{cal}^s = \sqrt{\sum_y \sum_l (\sqrt{p_{y,l}^s} - \sqrt{\hat{p}_{y,l}^s})^2 / \sum_y \sum_l 1} \quad (\text{A.36})$$

Note that allowance is made for a minus group (fish smaller than 10 cm) and a plus group (fish 46 cm and larger). Length classes are specified with intervals of 5 cm.

### Stock-recruitment residuals

It is assumed that these residuals are log-normally distributed and are not serially correlated. Therefore, their contribution to the penalised negative log-likelihood is given by:

$$-\ln L = \sum_y \frac{\zeta_y^2}{2\sigma_R^2} \quad (\text{A.37})$$

where

$\zeta_y$  is the estimated stock-recruitment residual for year  $y$ ; and

$\sigma_R$  is the input standard deviation of the log-residuals, which is assumed to be equal to 0.5.

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