Risk-Return Portfolio Modelling

A thesis presented

by

Emmeleen Ulita Gilbert

to

The Department of Statistical Science

in partial fulfillment of the requirements

for the degree of

Master in Science

in the subject of

Mathematics of Finance

University of Cape Town

December 2007

Abstract

Markowitz introduced the concept of modelling the risk associated with a given security as the variance of the expected return and showed how under certain conditions an investors portfolio can be managed by balancing the expected return of the portfolio and its variance. Building on Markowitz original framework, William Sharpe, extended these ideas by connecting a portfolio to a risky asset. This extension became known as the Sharpe Index Model. There are number of assumptions governing the residuals of the Sharpe index model, one being that the error terms of the stocks are uncorrelated. The Troskie-Hossain innovation to the Sharpe Index model relaxes this assumption. We evaluate the Troskie-Hossain model relative to the Sharpe Index Model and Markowitz portfolio, and find that the Troskie-Hossain model approximates the Markowitz efficient frontier and optimal portfolio very closely.

Further examining the residuals, we find evidence of autocorrelation and heteroskedasticity. Using ARMA to model the autocorrelation of the residuals has very little impact on the efficient frontier when working with log returns. However when working with simple returns the ARMA shifts the efficient frontier to the left. We find that GARCH(1,1) models capture most of the autocorrelation in the squared residuals for both simple returns and log returns and shifts the efficient frontier to the left. Mod-

elling a non-constant conditional mean and non-constant conditional variance (ARMA and GARCH) has proven difficult. The more complex a model becomes the more difficult the estimation.

We investigate the effects of dividend yields on the efficient frontier, as well as using simple returns vs log returns in portfolio construction. Including dividend yields in our return data shifts the efficient frontier upwards. However only the α's are increased, and the β's and β t-statistics of the shares remain the same. This shift effect of dividends has no impact on the time series or heteroskedastic models. The simple returns efficient frontier lies above that of the log returns efficient frontier. The α's for simple returns are very different to those of log returns, however the β's lie in a similar region to those of log returns.

Acknowledgments

I would like to expression my sincere gratitude to the following people and organisations

- My supervisor Professor C.G Troskie
- My supervisor Mr A Clark
- The National Research Foundation
- UCT Postgraduate funding office and the Ernest and Ethel Eriksen Trust

Contents

Chapter 1
Introduction

1.1 Background to the Problem

In 1952 Harry Markowitz's paper entitled "Portfolio Selection" laid the foundation for Portfolio Theory and Portfolio Management as we know it today. Markowitz introduced the concept of modelling the risk associated with a given security as the variance of the expected return and showed how under certain conditions an investors portfolio can be managed by subjectively balancing the expected return of the portfolio and its variance. Building on Markowitz original framework, William Sharpe (Sharpe, 1970) extended these ideas by connecting a portfolio to a single risky asset. This model became known as the Sharpe Index Model and can be formally written as:

$$\mathbf{R}_t = \alpha + \beta \mathbf{I}_t + \mathbf{e}_t, \quad \mathbf{t} = 1, \ldots, \mathbf{N} \tag{1.1}$$

where R_t represents the return of a particular stock at time t.

I_t represents the return of the market proxy at time t.

e_t a random shock to the system at time t.

Sharpe made the following assumptions with regard to the error term e_t

$$E(e_{it}^2) = \sigma_{ei}^2 \quad (1.2)$$

$$E(e_{it}e_{is}) = 0, \quad t \neq s = 1, \ldots, N \quad (1.3)$$

$$E(e_{it}I_t) = 0, \quad t = 1, \ldots, N \quad (1.4)$$

$$E(e_{it}e_{jt}) = 0, \quad t = 1, \ldots, N \quad (1.5)$$

Equation 1.5 assumes that the error terms of the stocks are uncorrelated, that is the stocks are only related through their mutual relationship with the index. Troskie (2001) has shown that this assumption is not valid on the JSE. In this study we build on Troskie's formulation and further explore assumptions 1.3 and 1.2, i.e. no serial correlation in the residuals and homoskedasticity, respectively.

When calculating expected returns a number of questions arise. What is the impact of dividends yields? How does using simple returns versus log returns impact one's portfolio. This thesis further investigates the effect of dividend yields on the efficient frontier and the impact of using simple returns versus log returns in portfolio construction.

1.2 Objectives of the Study

- Evaluate the Troskie-Hossain Index Models (Hossain, 2006) relative to the Markowitz and Sharpe Index Models.

- Model the serial correlation of the residuals from the Index Models (i.e. under non-constant conditional mean and constant variance)
- Model the heteroskedasticity of the residuals from the Index Models (i.e. under constant mean and non-constant conditional variance)
- Model the serial correlation and heteroskedasticity of the residuals from the Index Models (under non-constant conditional mean and non-constant conditional variance).
- Determine the impact of dividend yields on the efficient frontier and the above models.
- Compare portfolios generated using log returns and simple returns for the above models.

1.3 Scope and Limitations of the Study

15 shares with large to mid market capitalisations on the Johannesburg Stock Exchange (JSE) have been chosen for the purposes of this study. These shares have been chosen so as to avoid the problem of thin trading. For the purposes of this study, we assume that monthly log returns of the selected shares are distributed as Gaussian (refered to as the 'normal' distribution). Our study of heteroskedastic models is limited to GARCH(1,1) models and low order ARCH models.

1.4 Plan of Development of the thesis

This study begins by examining the basic Markowitz portfolio in Chapter 2. In Chapter 3 the Sharpe Index models are introduced, and the Troskie-Hossain model is evaluated relative to the Sharpe Index models and Markowitz portfolio. Chapter 4 further examines the assumptions of no serial correlation and homoskedasticity of the residuals for the index models. ARMA models are used to model the serial correlation of the residuals. GARCH models are used to model the serial correlation in the squared residuals. ARMA&GARCH models are used to model the serial correlation and heteroskedasticity of the residuals. Chapter 5 examines the effects of dividends on the efficient frontier. The exercises of chapter 4 are repeated, this time including dividend yields in the returns. Chapter 6 explores the impact of using simple returns vs log returns in portfolio construction. Once again the exercises of chapter 4 are repeated using simple returns.

Chapter 2
Markowitz Portfolio Theory

2.1 Introduction

To many, Harry Markowitz is considered the father of modern portfolio theory. His 1952 paper entitled "Portfolio Selection" and his book entitled Portfolio Selection: Efficient Diversification (1959), laid the foundation for Portfolio Theory and Portfolio Management as we know it today.

Markowitz introduced the concept of modelling the risk associated with a given security as the variance of the expected return and showed how under certain conditions an investors portfolio can be managed by balancing the expected return of the portfolio and its variance. The risk of a portfolio measured as variance depends on the individual variances of the assets as well as the pairwise covariance of assets. Markowitz also demonstrated how an investor can reduce this portfolio variance/risk through diversification. For his contribution to Portfolio Theory, Markowitz was awarded the Nobel Prize in Economics in 1990. The following discussion for the Markowitz Formulation is revised from Troskie (2001).

2.2 The Markowitz Formulation

Consider a portfolio consisting of p stocks. Let R_i be the return of share i, then the vector of stock returns for the portfolio is

$$\mathbf{R} = \begin{pmatrix} R_1 \\ \vdots \\ Rp \end{pmatrix} \tag{2.1}$$

with the expected return

$$E(\mathbf{R}) = \boldsymbol{\mu} \tag{2.2}$$

and covariance matrix

$$\boldsymbol{\Sigma} = E(\mathbf{R} - \boldsymbol{\mu})(\mathbf{R} - \boldsymbol{\mu})' \tag{2.3}$$

Further assume that

$$\mathbf{R} \backsim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{2.4}$$

is the multivariate normal. A portfolio of stocks can be thought of as a cash investment, of say w_i in each stock such that

$$\sum_{i=1}^{p} w_i = 1 \tag{2.5}$$

Let

$$\mathbf{W} = \begin{pmatrix} w_1 \\ \vdots \\ w_p \end{pmatrix} \tag{2.6}$$

The objective is to maximize the return of the portfolio P, whilst minimizing the variance. The Markowitz portfolio problem is then formally

$$\max_{w_i} E(P) = \mathbf{W}'\boldsymbol{\mu} = \sum_{i=1}^{p} w_i \mu_i \tag{2.7}$$

$$\min_{w_i} \sigma_p^2 = \mathbf{W}'\Sigma\mathbf{W} = \sum_{i=1}^{p} w_i w_j \sigma_{ij} \tag{2.8}$$

$$\text{subject to } \sum_{i=1}^{p} w_i = 1 \tag{2.9}$$

Markowitz constrained the weights such that

$$0 \leq w_i \leq 1 \qquad for\ i = 1, .., p \tag{2.10}$$

and added the concept of an Efficient Frontier.

2.3 The Efficient Frontier

Definition 1 A portfolio is called efficient if:

1. For a given amount of risk (σ_p^2) the expected return (E_k) is maximized, or for a given amount of return, the expected risk is minimized.
2. The portfolio is legitimate (no short sales).

Definition 2 Let σ_p^2 represent the x-axis and E_k represent the y-axis on a Cartesian Plane. The plot of all efficient portfolios on the risk and return axes is called the efficient frontier.

It is now no longer necessary to restrict portfolios to be legitimate, as short sales (negative w_i) are now common practice in markets around the world

2.4 The Capital Market Line

The Capital Market Line (CML) was derived by Linter and gives a satisfactory solution of how to use the Efficient Frontier once it has been generated. Refer to Figure 2.1. Suppose it is possible to borrow or lend money at the fixed interest rate R_f. The CML follows by drawing a straight line from the riskfree rate R_f, tangent to the Efficient Frontier. We shall refer to this point as M. The portfolio derived using the stocks that make up point M is called the Optimal Portfolio. Points between R_f and M represent lending portfolios and points that lie above M, represents borrowing portfolios. Their creation requires borrowing money at rate R_f to increase total investable capital. The total investable capital is then invested in M,which means that both return and risk (variance) are increased along the CML, relative to other points on the efficient frontier.

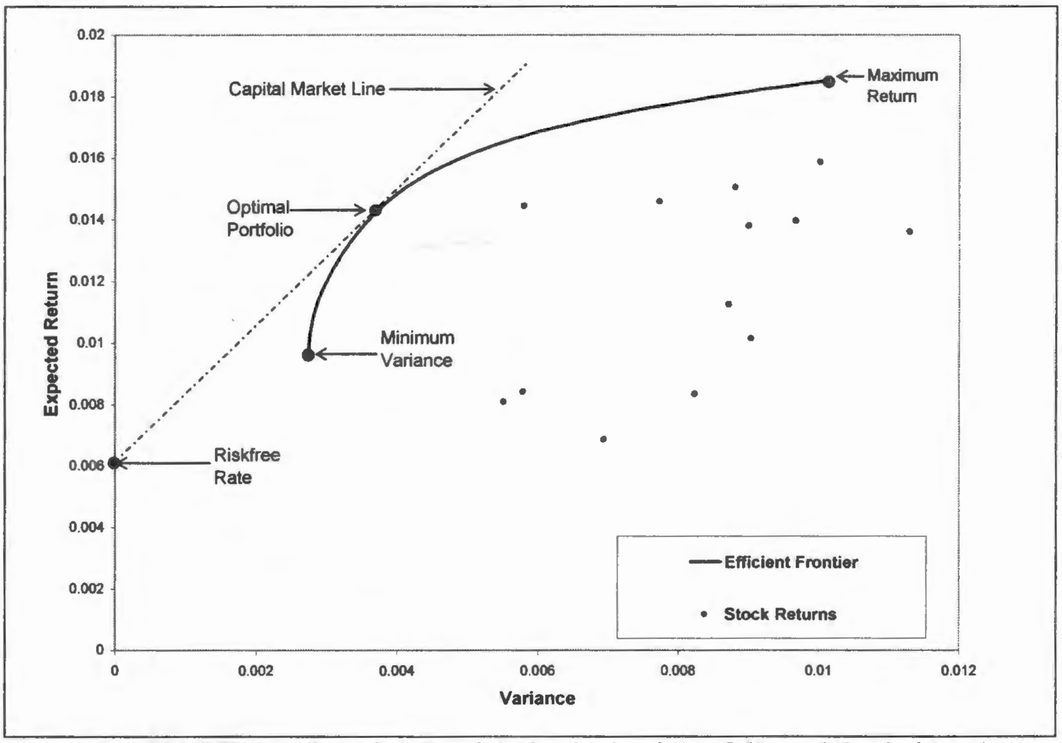

Figure 2.1 The Efficient Frontier showing the Optimal Portfolio and Capital Market Line

2.5 Markowitz Model Empirical Study

2.5.1 Study Objectives

To construct the Efficient Frontier for the 15 stocks in our portfolio using Markowitz mean-variance optimisation and to identify the optimal portfolio.

2.5.2 The Data

Monthly closing prices for the 15 shares listed in Table 2.1 are used in this study. These shares have large to mid market capitalisations on the Johannesburg Stock Exchange (JSE) and have been chosen so as to avoid the problem of thin trading. Thin trading is a problem that afflicts a large number of shares on the JSE. For more information on thin trading the interested reader is referred to Bowie (1994). This study spans the period 30 June 1995 to 28 February 2007. Monthly log returns have been computed for all 15 shares over 140 months. From this point forward "return" shall refer to the "log return of the share" unless otherwise specifically stated. This dataset will also be referred to as the *"ret"* dataset. The Banks Acceptance as of February 2007 is used as the Riskfree rate. This rate is a simple yearly rate.

Table 2.1: List of Shares in Markowitz Portfolio.

	Share Name	JSE Code
R1	Anglo American	AGL
R2	Bidvest	BVT
R3	First Rand	FSR
R4	Goldfields	GFI
R5	Pick 'n Pay	PIK
R6	Pretoria Portland Cement	PPC
R7	SAB Miller	SAB
R8	Standard Bank	SBK
R9	Sasol	SOL
R10	Sun International	SUI
R11	Tigerbrands	TBS
R12	Santam	SNT
R13	Afrox	AFX
R14	Illovo	ILV
R15	Shoprite	SHP
	Riskfree Rate	
	Banks Acceptance Rate	7.5% p.a

We compute the log return of a share using the following formulation:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{2.11}$$

where

r_t is the return of the share at time t.

P_t is the price of the share at time t

2.5.3 Methodology

The means, variances and covariances for the 15 shares in our portfolio have been calculated using Eviews6. The optimisation program used to compute the efficient frontiers has been written in Fortran by Professor Caspar Troskie of the Department of Statistical Sciences at the University of Cape Town, and utilises the Sharpe corner portfolio algorithm (Sharpe 1970). The bounds in this case are $0 \leq w_i \leq 1$ so that the first stock that enters the efficient frontier at the right hand extremity is the one with the largest return.

The Bankers Acceptance Rate is a simple rate per annum and must be converted into a monthly simple rate. This monthly rate must also be converted to a log rate, as we are using monthly log returns. Many studies make the mistake of using log returns when computing the efficient frontier but fail to convert the riskfree rate to a log rate as well.

$$R_{f,pa} + 1 = (R_{f,m} + 1)^{12} \tag{2.12}$$

$$r_f = \ln(R_{f,m} + 1) \tag{2.13}$$

$$r_f = \frac{\ln(R_{f,pa} + 1)}{12} \tag{2.14}$$

where $R_{f,pa}$ is the simple riskfree rate per annum

$R_{f,m}$ is the simple riskfree rate per month

r_f is the monthly log simple riskfree rate

2.5.4 Primary Findings

Figure 2.2 shows the plot of the Markowitz Efficient Frontier. We can see that for any given level of variance (risk), the expected return is higher on the efficient frontier than for any one stock.

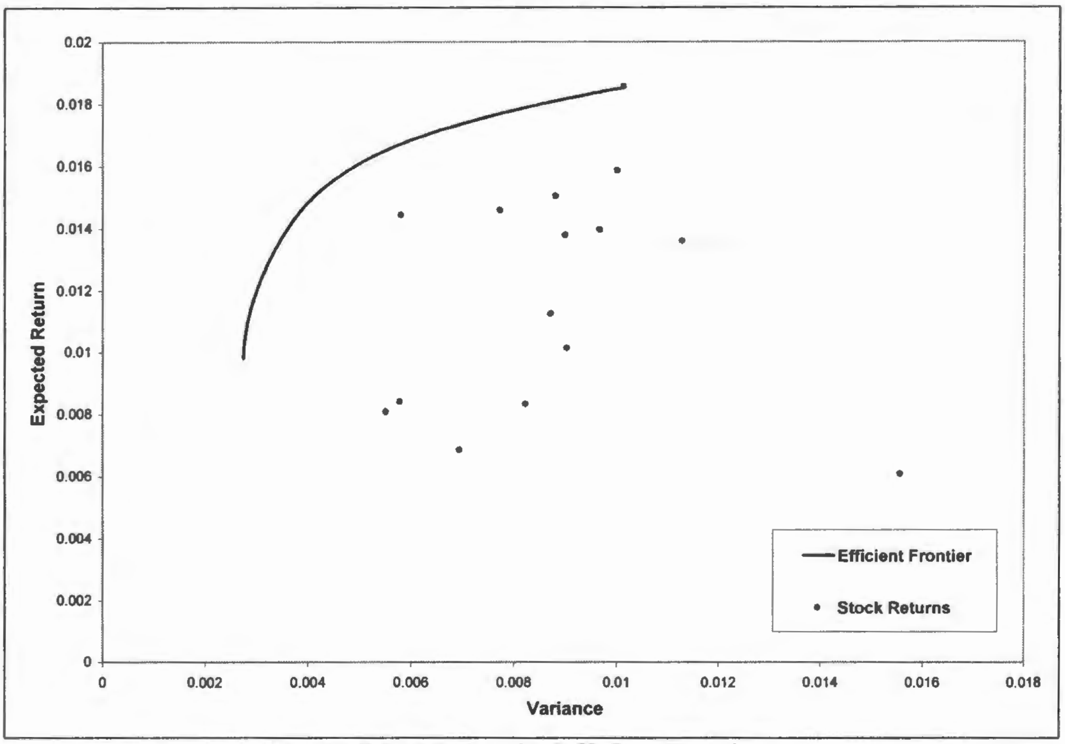

Figure 2.2: Markowitz Efficient Frontier

Table 2.2: The Markowitz Optimal Portfolio

Share Name	**Percent in Portfolio**
Anglo American	7.87 %
Bidvest	8.28 %
First Rand	35.96 %
Pick 'n Pay	12.07 %
Pretoria Portland Cement	1.99 %
Sasol	17.02 %
Shoprite	16.82 %
Expected Return (%p.a)	19.10 %
Portfolio Variance (%p.a)	5.78 %

2.5.5 Conclusion

It is evident from the Markowitz efficient frontier, that for any given level of risk/variance, the return of a portfolio on the efficient frontier is higher than that of any single stock. Using the capital market line, we are able to compute the optimal portfolio as in Figure 2.1.

Chapter 3
The Index Models

3.1 Introduction

In 1963, William Sharpe, a PhD student under the guidance of Markowitz published his dissertation. "A Simplified Model of Portfolio Analysis". Building on Markowitz original framework, Sharpe extended these ideas by connecting a portfolio to a risky asset. For his contribution to modern portfolio theory, William Sharpe was awarded the Nobel prize in Economics in 1990. The market model can formally be written as

$$\mathbf{R}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{I}_t + \mathrm{e}_t, \quad t = 1, \ldots, N \tag{3.1}$$

where R_t represents the return of a particular stock at time t.

I_t represents the return of the market proxy at time t

e_t represents a random shock to the system at time t.

The parameter β is often used as a measure of the volatility of a security relative to a market proxy. It has been used extensively in theory and practice to construct and analyze market portfolios. If β is greater than one, then when the market rises, the return of a security will rise more rapidly than the return on the market. Similarly, if the market falls, the return on the security will fall more rapidly than the return on the market. If β is less then one then the converse is true.

One of the fundamental assumptions of the Sharpe index model is that the error terms of the stocks are uncorrelated. Troskie (2001) has shown that this is not the case on the Johannesburg Stock Exchange and has proposed a modified version of the Sharpe index models, which takes into consideration this correlation of the error terms. The discussions to follow, for the Single Index Models and Multiple Index Models, are revised from Troskie (2001)

3.2 The Sharpe Single Index Model

We assume that the return R_t of a stock follows a Gaussian $N(\mu_r, \sigma_r^2)$ distribution and the return of the market proxy I_t follows a Gaussian $N(\mu_I, \sigma_I^2)$ distribution. We then have that

$$\begin{pmatrix} R_t \\ I_t \end{pmatrix} \backsim N\left[\begin{pmatrix} \mu_r \\ \mu_I \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & \sigma_{rI} \\ \sigma_{Ir} & \sigma_I^2 \end{pmatrix}\right] \tag{3.2}$$

has a bivariate normal distribution. From the properties of the bivariate normal distribution

$$E(R_t \mid I_t) = \alpha + \beta I_t \tag{3.3}$$

and

$$\begin{aligned} var(R_t/I_t) &= \sigma_{r.I}^2 = \sigma_I^2(1-\rho^2) \\ &= \sigma^2 \end{aligned} \tag{3.4}$$

The market model can be formally written as

$$R_{it} = \alpha_i + \beta_i I_t + e_{it} \ , \quad i = 1, \ldots, p; \quad t = 1, \ldots, N \tag{3.5}$$

where all stocks are regressed on the same single index I. The term e_t is refered to as white noise is statistics and follows a normal $N(0, \sigma^2)$ distribution. The Sharpe Single Index Model makes the following assumptions:

$$E(e_{it}^2) = \sigma_{ei}^2 \tag{3.6}$$

$$E(e_{it}e_{is}) = 0, \quad t \neq s = 1, \ldots, N \tag{3.7}$$

$$E(e_{it}I_t) = 0, \quad t = 1, \ldots, N \tag{3.8}$$

$$E(e_{it}e_{jt}) = 0, \quad t = 1, \ldots, N \tag{3.9}$$

- Equation 3.6 assumes each stock has its own variance for the error term.
- Equation 3.7 assumes that the error terms of each stock are independent over time i.e. no autocorrelation - no time series.
- Equation 3.8 assumes that the errors of each stock and the explanatory variable I are uncorrelated.
- Equation 3.9 assumes that the error terms of the stocks are uncorrelated, that is the stocks are only related through their mutual relationship with the index I.

Let

$$E(I) = \mu_I \text{ and } var(I) = \sigma_I^2 \tag{3.10}$$

be the mean and variance of the index. The return for stock i, is then

$$E_i = E(R_i) = \alpha_i + \beta_i \mu_I \tag{3.11}$$

with variance

$$\begin{aligned} var(R_i) &= \beta_i^2 \sigma_I^2 + \sigma_{ei}^2 \\ &= \sigma_{ii} = \sigma_i^2 \end{aligned} \tag{3.12}$$

and covariance

$$cov(R_i R_j) = \beta_i \beta_j \sigma_I^2 \tag{3.13}$$

Our standard portfolio problem then becomes

$$\begin{aligned} Max\ Z &= \phi \mu_p - \sigma_p^2 \\ &= \phi \mathbf{W}' \boldsymbol{\mu} - \mathbf{W}' \Sigma \mathbf{W} \quad \text{subject to} \\ & \sum_{i=1}^{p} w_i = 1 \end{aligned}$$

where

$$\begin{aligned} \mu_p &= \sum_{i=1}^{p} w_i E_i & (3.14) \\ &= \sum_{i=1}^{p} w_i (\alpha_i + \beta_i \mu_I) & (3.15) \end{aligned}$$

and

$$\begin{aligned}\sigma_p^2 &= \mathbf{W}'\mathbf{\Sigma}\mathbf{W} \\ &= \sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \sigma_{ij}\end{aligned}$$

with

$$\sigma_{ij} = \begin{cases} \beta_i^2\sigma_I^2 + \sigma_{ei}^2 & for\ i = j \\ \beta_i\beta_j\sigma_I^2 & for\ i \neq j \end{cases} \tag{3.16}$$

3.3 The Troskie-Hossain Single Index Model Innovation

The Troskie-Hossain Single Index Model (Hossain 2006) builds on the Sharpe Single Index Model. Whereas Sharpe assumes that the error terms of the stocks are uncorrelated, that is the stocks are only related through their mutual relationship with the index, Troskie (2001) has shown that this condition is not satisfied by the JSE. The Sharpe single index model is formulated as

$$R_{it} = \alpha_i + \beta_i I_t + e_{it}\ ,\quad i = 1, \ldots, p;\quad t = 1, \ldots, N \tag{3.17}$$

$$E(e_{it}^2) = \sigma_{ei}^2 \tag{3.18}$$

$$E(e_{it}e_{is}) = 0,\quad t \neq s = 1, \ldots, N \tag{3.19}$$

$$E(e_{it}I_t) = 0,\quad t = 1, \ldots, N \tag{3.20}$$

$$E(e_{it}e_{jt}) = 0,\quad t = 1, \ldots, N \tag{3.21}$$

This formulation implies that

$$cov(\mathbf{R}) = \sigma_I^2 \beta\beta' + \begin{pmatrix} \sigma_{e1}^2 & 0 & \ldots & 0 \\ 0 & \sigma_{e2}^2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & \ldots & \sigma_{ep}^2 \end{pmatrix}. \tag{3.22}$$

Here we have assumed that all $e_{it},\ i = 1, \ldots, p,\ t = 1, \ldots, N$ are independent. The Troskie-Hossain model now relaxes this assumption by assuming that the disturbances e_{it} of the different stocks are dependent (correlated).

Thus

$$E(e_{it}e_{jt}) = \begin{cases} \sigma_{ij}, & i \neq j \\ \sigma^2_{ei}, & i = j \end{cases} \tag{3.23}$$

or

$$E(\mathbf{ee}') = \mathbf{\Omega} = \begin{pmatrix} \sigma^2_{e1} & \sigma_{12} & \dots & \sigma_{1p} \\ \sigma_{21} & \sigma^2_{e2} & \dots & \sigma_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{p1} & \dots & \dots & \sigma^2_{ep} \end{pmatrix} \tag{3.24}$$

with

$$cov(\mathbf{R}) = \sigma^2_I \boldsymbol{\beta}\boldsymbol{\beta}' + \mathbf{\Omega} = \mathbf{\Phi} \tag{3.25}$$

For portfolio $P = \mathbf{W}'\mathbf{R}$ we now have

$$E(P) = \mathbf{W}'(\boldsymbol{\alpha} + \boldsymbol{\beta}\mu_I) = \mu_p \tag{3.26}$$

and

$$var(P) = \mathbf{W}'(\sigma^2_I \boldsymbol{\beta}\boldsymbol{\beta}' + \mathbf{\Omega})\mathbf{W} \tag{3.27}$$

$$= \mathbf{W}'\mathbf{\Phi}\mathbf{W} = \sigma^2_p \tag{3.28}$$

To estimate Ω let

$$\hat{\mathbf{E}} = \begin{pmatrix} \hat{e}_{11} & \hat{e}_{12} & \dots & \hat{e}_{1N} \\ \hat{e}_{21} & \hat{e}_{22} & \dots & \hat{e}_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{e}_{p1} & \dots & \dots & \hat{e}_{pN} \end{pmatrix} \tag{3.29}$$

then the least squares estimate of Ω is

$$\hat{\mathbf{\Omega}} = \frac{1}{N-1}\hat{\mathbf{E}}\hat{\mathbf{E}}' \tag{3.30}$$

and

$$\hat{\mathbf{\Phi}} = \hat{\sigma}_I^2\hat{\boldsymbol{\beta}}\hat{\boldsymbol{\beta}}' + \hat{\mathbf{\Omega}} \tag{3.31}$$

3.4 Single Index Model Empirical Study

3.4.1 Study Objectives

To evaluate the Sharpe Single Index Model and the Troskie-Hossain Single Index Innovation.

3.4.2 Data

This study spans the period 30 June 1995 to 28 February 2007. Monthly closing prices for the 15 shares in Table 3.1 on page 24 are used in this study. The shares have been chosen so as to avoid the problem of thin trading. The JSE All Share Index has been chosen as the market proxy for the Single Index study. Monthly log returns have been computed for all 15 shares as well as the JSE All Share Index. We refer to the "log return" of the share as "return" unless otherwise specifically stated. This data set will also be referred to as the *"singleret"* dataset. The Banks Acceptance Rate as of February 2007 is used as the Riskfree rate. This rate is a simple yearly rate.

3.4.3 Methodology

The return data for each of the 15 shares in our portfolio is regressed against the returns of the JSE All Share Index and Ordinary Least Squares (OLS) is used to estimate α and β for each share. These regressions are run in Eviews6. We compute

Table 3.1: List of Shares and Market Proxy for the Single Index Model

No.	Share Name	JSE Code
R1	Anglo American	AGL
R2	Bidvest	BVT
R3	First Rand	FSR
R4	Goldfields	GFI
R5	Pick 'n Pay	PIK
R6	Pretoria Portland Cement	PPC
R7	SAB Miller	SAB
R8	Standard Bank	SBK
R9	Sasol	SOL
R10	Sun International	SUI
R11	Tigerbrands	TBS
R12	Santam	SNT
R13	Afrox	AFX
R14	Illovo	ILV
R15	Shoprite	SHP

No.	Market Proxy	JSE Code
X1	JSE All Share	ALSI
	Riskfree Rate	
	Banks Acceptance Rate	7.5% p.a

the means for our portfolio using equation 3.11. For the Sharpe Single Index model we use equation 3.13 to estimate the covariance matrix. In the case of the Troskie-Hossain Single Index innovation we use equation 3.27 to estimate the covariance matrix. The optimisation programme used to compute the efficient frontiers has been written in Fortran by Professor Caspar Troskie. We use the following bounds : $0 \leq w_i \leq 1$.

3.4.4 Primary Findings

Table 3.2 shows the alphas and betas computed for each share. It must be noted that these parameters are the same for both the Sharpe and Troskie-Hossain single index models. The t-statistics for the $\hat{\beta}$ are all very large and positive. Figure 3.1 on page 26 shows the efficient frontiers for the Sharpe single index model, the Troskie-Hossain single index innovation and the Markowitz portfolio. The Troskie-Hossain Efficient frontier lies on the Efficient Frontier generated by the Markowitz portfolio.

The Sharpe index model assumes that the covariances between the residuals are zero. If positive correlations are dominant in the residual matrix, then the Sharpe model under-estimates risk, likewise if negative correlations are dominant in the residual matrix, then the Sharpe model over-estimates risk. From Figure 3.2 we can see that there are both positive and negative correlations in our residual matrix. The Troskie -Hossain innovation takes into consideration these residual covariances. As is evident from Figure 3.1 using the Troskie-Hossain innovation to the Sharpe single index model has shifted our single index efficient frontier to the right. This shift is due to the dominance of positive correlations in the residuals.

Table 3.2: Regression statistics for the Single Index Model

Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\sigma}_e^2$	R^2	Schwartz
AGL	-0.0004	-0.08	1.2885	15.66	0.0035	0.6400	-2.76
BVT	0.0054	1.08	0.8066	9.95	0.0034	0.4175	-2.79
FSR	0.0070	1.03	1.0351	9.47	0.0062	0.3940	-2.19
GFI	-0.0039	-0.40	0.8917	5.70	0.0127	0.1905	-1.47
PIK	0.0077	0.99	0.7328	5.87	0.0081	0.1998	-1.92
PPC	0.0054	0.72	0.5193	4.24	0.0078	0.1154	-1.96
SAB	-0.0018	-0.40	0.8839	12.42	0.0026	0.5278	-3.05
SBK	0.0038	0.68	0.9610	10.53	0.0043	0.4453	-2.55
SOL	0.0014	0.20	1.0887	9.42	0.0069	0.3914	-2.08
SUI	0.0019	0.26	0.7379	6.32	0.0071	0.2247	-2.06
TBS	0.0008	0.14	0.6848	7.73	0.0041	0.3019	-2.61
SNT	0.0035	0.54	0.9178	8.60	0.0059	0.3489	-2.24
AFX	0.0006	-0.09	0.6637	6.54	0.0053	0.2366	-2.34
ILV	-0.0004	-0.06	0.7813	7.26	0.0060	0.2764	-2.22
SHP	0.0082	1.11	0.6074	5.05	0.0075	0.1562	-2.00

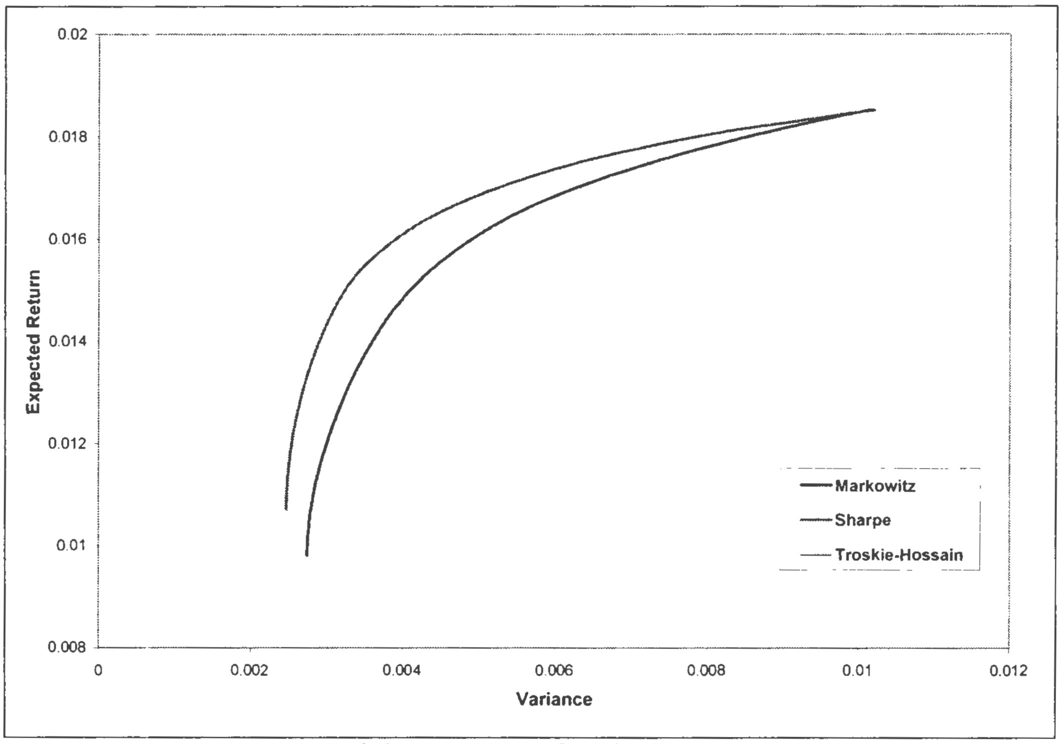

Figure 3.1: Efficient Frontiers for Single Index Models

1.0000	-0.3021	-0.4252	0.0321	-0.2475	-0.0355	-0.1015	-0.5065	0.3500	-0.1658	-0.1946	-0.2300	-0.1266	-0.0188	-0.2657
-0.3021	1.0000	0.4240	-0.2214	0.4121	0.2115	0.0778	0.2120	-0.3016	0.2040	0.3153	0.2872	0.1263	0.0537	0.4249
-0.4252	0.4240	1.0000	-0.1791	0.3652	0.0618	0.1029	0.4854	-0.3576	0.0845	0.3032	0.2745	-0.0242	0.0476	0.3353
0.0321	-0.2214	-0.1791	1.0000	-0.1004	-0.1239	-0.0904	-0.3587	0.0820	-0.1623	-0.1450	-0.0397	-0.0311	0.1850	-0.0743
-0.2475	0.4121	0.3652	-0.1004	1.0000	0.3462	0.1976	0.3914	-0.2199	-0.0602	0.3397	0.1654	0.0095	-0.2143	0.3847
-0.0355	0.2115	0.0618	-0.1239	0.3462	1.0000	0.1214	0.0779	-0.1116	0.0498	0.2040	0.1310	0.1990	-0.1701	0.1082
-0.1015	0.0778	0.1029	-0.0904	0.1976	0.1214	1.0000	0.2002	-0.1732	0.0897	0.2708	0.1512	0.0809	-0.0910	0.0514
-0.5065	0.2120	0.4854	-0.3587	0.3914	0.0779	0.2002	1.0000	-0.1778	0.1088	0.1922	0.1681	0.1279	-0.0466	0.1283
0.3500	-0.3016	-0.3576	0.0820	-0.2199	-0.1116	-0.1732	-0.1778	1.0000	-0.1671	-0.3104	-0.1700	-0.1358	0.0397	-0.1970
-0.1658	0.2040	0.0845	-0.1623	-0.0602	0.0498	0.0897	0.1088	-0.1671	1.0000	-0.0378	0.3117	0.1049	0.1114	0.0454
-0.1946	0.3153	0.3032	-0.1450	0.3397	0.2040	0.2708	0.1922	-0.3104	-0.0378	1.0000	0.0372	0.0040	-0.0137	0.3190
-0.2300	0.2872	0.2745	-0.0397	0.1654	0.1310	0.1512	0.1681	-0.1700	0.3117	0.0372	1.0000	0.2782	0.0774	0.1192
-0.1266	0.1263	-0.0242	-0.0311	0.0095	0.1990	0.0809	0.1279	-0.1358	0.1049	0.0040	0.2782	1.0000	0.0394	-0.1665
-0.0188	0.0537	0.0476	0.1850	-0.2143	-0.1701	-0.0910	-0.0466	0.0397	0.1114	-0.0137	0.0774	0.0394	1.0000	0.0812
-0.2657	0.4249	0.3353	-0.0743	0.3847	0.1082	0.0514	0.1283	-0.1970	0.0454	0.3190	0.1192	-0.1665	0.0812	1.0000

Figure 3.2: Residual Correlation Matrix

Table 3.3: Optimal Portfolio's for the Single Index Models

Share Name	Markowitz	Sharpe	Troskie-Hossain
Anglo American	7.87 %	-	8.03 %
Bidvest	8.28 %	21.08 %	8.36 %
First Rand	35.96 %	25.59 %	35.84 %
Pick 'n Pay	12.07 %	18.32 %	12.00 %
Pretoria Portland Cement	1.99 %	4.89 %	2.02 %
Standard Bank	-	7.10 %	-
Sasol	17.02 %	-	17.00 %
Santam	-	2.34 %	-
Shoprite	16.82 %	20.68 %	16.74 %
Expected Return (%p.a)	19.10 %	18.83 %	19.09 %
Portfolio Variance (%p.a)	5.78 %	4.37 %	5.77 %

The optimal portfolio generated by the Troskie-Hossain Model is very close to that generated by the Markowitz portfolio. The optimal portfolio generated by the Sharpe single index model is very different to both that of the Troskie-Hossain model and Markowitz portfolio. We can see that a number of shares included in the Markowitz optimal portfolio do not feature in the Sharpe optimal portfolio and *vice versa*. The portfolio weights are also extremely different. From the above it is evident that the correlations amongst the residuals have a significant impact on the efficient frontier and optimal portfolio.

3.4.5 Conclusions

The Sharpe single index model assumes that the residuals from the regressions are uncorrelated. We have shown that this is infact not the case, and that the correlation amongst the residuals have a significant impact on the efficient frontier and optimal portfolio. If positive correlations are dominant in the residuals, then the Sharpe index model underestimates risk, likewise if there are negative correlations in the residuals, the Sharpe index model over estimates risk. Our single index empirical study has shown that the Troskie-Hossain innovation, approximates the Markowitz efficient frontier and optimal portfolio very closely. For all further single index models we shall use the Troskie-Hossain innovation.

3.5 The Sharpe Multiple Index Model.

The multiple index model can be written as

$$R_{it} = \alpha_i + \beta_{i1}I_1 + \beta_{i2}I_2 + \ldots + \beta_{iM}I_M + e_{it} \tag{3.32}$$

$$i = 1, \ldots, p, \; t = 1, \ldots, N$$

with the following assumptions

$$E(e_{it}^2) = \sigma_{ei}^2 \tag{3.33}$$

$$E(e_{it}e_{is}) = 0, \quad t \neq s = 1, \ldots, N \tag{3.34}$$

$$E(e_{it}I_{jt}) = 0, \quad j = 1, \ldots, M, \; t = 1, \ldots, N \tag{3.35}$$

$$E(e_{it}e_{jt}) = 0, \quad i \neq j, \; t = 1, \ldots, N \tag{3.36}$$

$$E(I_{jt}I_{kt}) = c_{jk}, \; j, k = 1, \ldots, M \tag{3.37}$$

These assumptions are identical to the Single Index Model where in equation 3.8 we assume that the disturbance term e_{it} is also independent of the Indices $I_j, \; j = 1, \ldots, M$. We further assume that the Indices are dependent with covariances given by c_{jk}. Let u_{I_i} be the observed return of Index I_i.

Then

$$E(R_i) = \alpha_i + \beta_{i1}u_{I_1} + \ldots + \beta_{iM}u_{I_M} \tag{3.38}$$
$$i = 1, \ldots, p$$

The expected return of our portfolio P is then

$$\mu_p = \sum_{i=1}^{p} w_i E(R_i) \tag{3.39}$$

with variance

$$\sigma_p^2 = \sum_{i=1}^{p} w_i w_j \sigma_{ij} \tag{3.40}$$
$$= \sum_{i=1}^{p} w_i^2 \sigma_{ei}^2 + \sum_{k} \sum_{l} \beta_{pk} \beta_{pl} c_{kl} \tag{3.41}$$

where

$$\beta_{pk} = \sum_{i=1}^{p} w_i \beta_{ik}, k = 1, \ldots, M$$
$$\beta_{pl} = \sum_{j=1}^{p} w_j \beta_{jl}, l = 1, \ldots, M$$

Our portfolio problem is then

$$Min \;\; Z = -\Lambda \mu_p + \sigma_p^2$$

subject to

$$
\begin{aligned}
\beta_{p1} &= \sum_{i=1}^{p} w_i \beta_{i1} \\
\beta_{p2} &= \sum_{i=1}^{p} w_i \beta_{i2} \\
&\vdots \\
\beta_{pM} &= \sum_{i=1}^{p} w_i \beta_{iM}
\end{aligned} \tag{3.42}
$$

$$
\sum_{i=1}^{p} w_i = 1, \tag{3.43}
$$

and possibly any other equality, in-equality constraints or bounds.

3.6 The Troskie-Hossain Multiple Index Model Innovation

The Troskie-Hossain Multiple Index Model (Hossain 2006) builds on the Sharpe Multiple Index Model. Whereas Sharpe assumes that the error terms of the stocks are uncorrelated, that is the stocks are only related through their mutual relationship with the index, Troskie (2001) has shown that this assumption is not valid on the JSE. The Troskie-Hossain Multiple Index model can be written as

$$
\begin{aligned}
R_{it} &= \alpha_i + \beta_{i1} I_{1t} + \beta_{i2} I_{2t} + \ldots + \beta_{iM} I_{Mt} + e_{it} \\
i &= 1, \ldots, p, \; t = 1, \ldots, N
\end{aligned} \tag{3.44}
$$

or in vector notation

$$\mathbf{R}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{I}_t + \mathbf{e}_t \tag{3.45}$$

with the following assumptions

$$E(e_{it}^2) = \sigma_{ei}^2 \tag{3.46}$$

$$E(e_{it}e_{jt}) = \sigma_{ij}, \quad i \neq j, \ t = 1, \dots, N \tag{3.47}$$

$$E(e_{it}e_{is}) = 0, \quad t \neq s = 1, \dots, N \tag{3.48}$$

$$E(e_{it}I_{jt}) = 0, \quad j = 1, \dots, M, \ t = 1, \dots, N \tag{3.49}$$

$$E(I_{jt}I_{kt}) = c_{jk}, \ j, k = 1, \dots, M \tag{3.50}$$

The expected return of $\mathbf{R}_t$ is then

$$E(\mathbf{R}_t) = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_{\mathbf{I}} \tag{3.51}$$

with

$$\boldsymbol{\mu}_{\mathbf{I}} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_M \end{pmatrix} \tag{3.52}$$

and covariance

$$cov(R_t) = \boldsymbol{\beta}\mathbf{C}\boldsymbol{\beta}' + \boldsymbol{\Omega} = \boldsymbol{\Phi} \tag{3.53}$$

To estimate $\boldsymbol{\Phi}$, we have

$$\hat{\mathbf{\Phi}} = \hat{\boldsymbol{\beta}}\hat{\mathbf{C}}\hat{\boldsymbol{\beta}}' + \hat{\Omega} \tag{3.54}$$

The matrix $\hat{C}$ is the estimated covariance matrix of the M Indices and the least squares estimate of $\hat{\Omega}$ is given by

$$\hat{\Omega} = \frac{1}{N - M - 1}\hat{\mathbf{E}}\hat{\mathbf{E}}' \tag{3.55}$$

with

$$\hat{\mathbf{E}} = \begin{pmatrix} \hat{e}_{11} & \hat{e}_{12} & \dots & \hat{e}_{1N} \\ \hat{e}_{21} & \hat{e}_{22} & \dots & \hat{e}_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{e}_{p1} & \dots & \dots & \hat{e}_{pN} \end{pmatrix} \tag{3.56}$$

The expected return of our portfolio P is then

$$E_p = E(P) = \mathbf{W}'(\boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_{\mathbf{I}}) \tag{3.57}$$

with variance

$$\sigma_p^2 = var(P) = \mathbf{W}'\mathbf{\Phi}\mathbf{W} \tag{3.58}$$

3.7 Multiple Index Model Empirical Study

3.7.1 Study Objectives

To evaluate the Sharpe Multiple Index Model and the Troskie-Hossain Multiple Index Innovation.

3.7.2 Data

This study spans the period 30 June 1995 to 28 February 2007. Monthly closing prices for the following 15 shares listed on the JSE and the 8 selected indices in Table 3.4 on page 35 are used in this study. The shares have been chosen so as to avoid the problem of thin trading. Monthly log returns have been computed for all 15 shares as well as the Indices. We refer to the "log return" of the share as "return" unless otherwise specifically stated. This data set will also be referred to as the *"multiret"* dataset. The Banks Acceptance as of February 2007 is used as the Riskfree rate. This rate is a simple yearly rate.

AngloGold has been chosen as a proxy for the gold index, Implats has been chosen as a proxy for the platinum index and Palamin has been chosen as a proxy for the palladium index.

Table 3.4: List of Shares and Explanatory Variables for the Multiple Index Model

No.	Share Name	JSE Code
R1	Anglo American	AGL
R2	Bidvest	BVT
R3	First Rand	FSR
R4	Goldfields	GFI
R5	Pick 'n Pay	PIK
R6	Pretoria Portland Cement	PPC
R7	SAB Miller	SAB
R8	Standard Bank	SBK
R9	Sasol	SOL
R10	Sun International	SUI
R11	Tigerbrands	TBS
R12	Santam	SNT
R13	Afrox	AFX
R14	Illovo	ILV
R15	Shoprite	SHP

No	Explanatory Variables	JSE Code
X1	JSE All share	ALSI
X2	R/$ exchange rate	R/$
X3	Gold Price (Rands)	Gold/R
X4	Dow Jones Transport	DJTrans
X5	Palamin	PLM
X6	Richemont	RCH
X7	AngloGold	ANG
X8	Implats	IMP
	Riskfree Rate	
	Banks Acceptance Rate	7.5% p.a

3.7.3 Methodology

We first need to find an adequate set of explanatory variables for the multiple index models. We use the Backward Elimination procedure to select a minimum of 3 and a maximum of 4 explanatory variables for our portfolio of shares. We commence by fitting all 8 indices as explanatory variables to each of the 15 shares. A t-statistic with absolute value larger than 1.96 is considered significant at the 5% significance level. We then delete the index with lowest explanatory power amongst the 15 shares, and refit our models with the remaining 7 indices. The R^2, *adjusted* R^2 and *Schwartz Criterion* are used as guidelines in the variable selection process. We repeat this process until we find a common subset of 3-4 indices that explains a larger percentage of variation in the dependent variables, as opposed to that percentage explained by all 8 indices. Refer to Appendix: *Index Selection for Multiple Index Model* for detailed selection of explanatory variables for the multiple index model.

The multiple index model to be fitted to our portfolio of shares is

$$R_i = \alpha_i + \beta_{i1}(\mathbf{JSE}) + \beta_{i2}(\mathbf{R/\$}) + \beta_{i3}(\mathbf{Gold\ Proxy}) + e_i \tag{3.59}$$

These regressions are run in Eviews6 and ordinary least squares is used to estimate the parameters in our regression models. The estimated means and covariances are computed for the Sharpe multiple index model and the Troskie-Hossain multiple index models. The optimisation programme used to compute the efficient frontiers has been written in Fortran by Professor Caspar Troskie. We use the following bounds: $0 \leq w_i \leq 1$

3.7.4 Primary Findings

Tables 3.5 to 3.6 on page 38 show the α's and β's computed for each share. It must be noted that these parameters are the same for both the Sharpe and Troskie-Hossain multiple index models. Only the covariance structure differs between both models. The t-statistics for $\hat{\beta}_1$ *(JSE All Share Index)* are all very large and positive. This set of large t-values is due to the fact that the shares in our portfolio have large market capitalisations and as such have large weightings in the JSE All Share Index. Anglo American which has the largest market capitalisation on the JSE has the highest t-statistic.

11 of the 15 t-statistics for $\hat{\beta}_2$ *(R/\$ exchange rate)* are significant at the 5% and 10% significant levels, implying that the R/\$ exchange rate is a good explanatory

variable for our multiple index model. $\hat{\beta}_3$ *(AngloGold)* however is significant at the 5% and 10% significance level in only half of our regression models.

Table 3.5: Regression statistics for the Multiple Index Model

Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\sigma}_e^2$	R^2	Schwartz
AGL	-0.0014	-0.29	0.0034	0.6804	-2.81
BVT	0.0058	1.20	0.0031	0.4738	-2.82
FSR	0.0078	1.23	0.0054	0.4777	-2.27
GFI	-0.0008	-0.09	0.0099	0.3793	-1.67
PIK	0.0080	1.06	0.0075	0.2647	-1.94
PPC	0.0076	1.04	0.0070	0.2108	-2.01
SAB	-0.0018	-0.41	0.0027	0.5296	-2.98
SBK	0.0040	0.76	0.0037	0.5377	-2.66
SOL	-0.0006	-0.08	0.0067	0.4216	-2.06
SUI	0.0013	0.18	0.0069	0.2507	-2.02
TBS	0.0033	0.63	0.0036	0.3852	-2.66
SNT	0.0053	0.82	0.0056	0.3959	-2.24
AFX	0.0006	0.09	0.0053	0.2585	-2.30
ILV	-0.0001	-0.02	0.0060	0.2849	-2.16
SHP	0.0111	1.56	0.0067	0.2526	-2.05

Table 3.6: Share Beta's for the Multiple Index Model

Code	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat
AGL	1.2006	13.69	0.3497	3.31	0.1035	2.43
BVT	0.9109	10.45	-0.2664	-2.54	-0.1188	-2.80
FSR	1.1938	10.41	-0.4530	-3.28	-0.1821	-3.26
GFI	0.4491	2.90	0.1584	0.85	0.4794	6.36
PIK	0.8911	6.58	-0.3420	-2.10	-0.1788	-2.71
PPC	0.6024	4.61	-0.5793	-3.68	-0.1042	-1.64
SAB	0.9063	11.28	-0.0329	-0.34	-0.0248	-0.64
SBK	1.1334	12.02	-0.3373	-2.97	-0.1939	-4.23
SOL	1.0878	8.54	0.4075	2.66	0.0114	0.18
SUI	0.8575	6.61	-0.0956	-0.61	-0.1309	-2.07
TBS	0.6726	7.15	-0.4862	-4.29	0.0006	0.01
SNT	0.9599	8.26	-0.4310	-3.08	-0.0563	-1.00
AFX	0.6847	6.05	-0.2620	-1.92	-0.0293	-0.53
ILV	0.7173	5.93	0.0613	0.42	0.0702	1.19
SHP	0.6411	5.01	-0.6297	-4.09	-0.0524	-0.84

From Figure 3.3 we can see that the Troskie-Hossain efficient frontiers lies on the Markowitz efficient frontier. The Sharpe index model assumes that the covariances between the residuals are zero. If positive correlations are dominant in the residual matrix, then the Sharpe model under-estimates risk, likewise if negative correlations are dominant in the residual matrix, then the Sharpe model over-estimates risk. As is evident from Figure 3.3 using the Troskie-Hossain innovation has shifted our Sharpe multiple index efficient frontier to the right. This illustrates the significant impact the correlation of the residuals has on portfolio selection and the optimal portfolio.

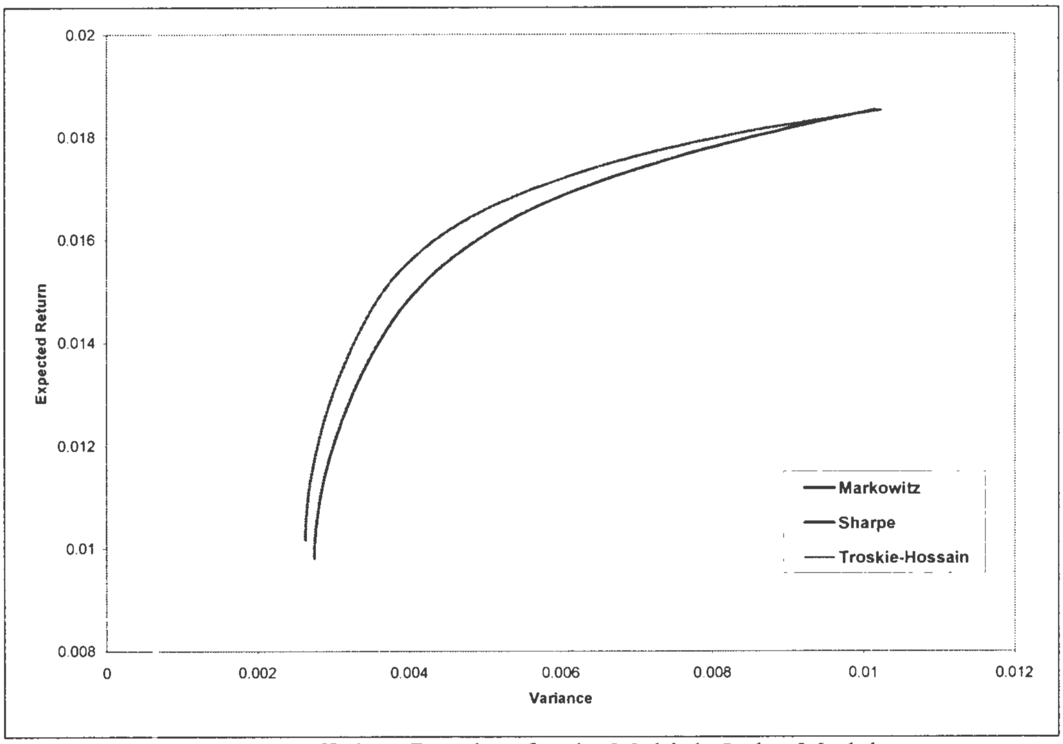

Figure 3.3: Efficient Frontiers for the Multiple Index Model

Table 3.7: Optimal Portfolio's for the Multiple Index Models

Share Name	Markowitz	Sharpe	Troskie
Anglo American	7.87 %	-	8.10 %
Bidvest	8.28 %	23.09 %	8.40 %
First Rand	35.96 %	28.80 %	35.84 %
Pick 'n Pay	12.07 %	18.73 %	11.95 %
Pretoria Portland Cement	1.99 %	-	2.06 %
Standard Bank	-	3.44 %	-
Sasol	17.02 %	3.65 %	16.98 %
Santam	-	1.21 %	-
Shoprite	16.82 %	21.08 %	16.68 %
Expected Return (%p.a)	19.10 %	19.15 %	19.09 %
Portfolio Variance (%p.a)	5.78 %	5.16 %	5.75 %

As is the case in the single index model, the optimal portfolio generated by the Troskie-Hossain innovation is very close to that generated by the Markowitz portfolio. The optimal portfolio generated by the Sharpe multiple index model is very different to both that of the Troskie-Hossain model and Markowitz portfolio. We can see that a number of shares included in the Markowitz optimal portfolio do not feature in the Sharpe optimal portfolio and vice versa. The portfolio weights are also extremely different.

3.7.5 Conclusions

The Sharpe multiple index model assumes that the residuals from the regressions are uncorrelated. We have shown that when this assumption is false (e.g JSE), the correlation amongst the residuals has a significant impact on the efficient frontier and optimal portfolio. If positive correlations are dominant in the residuals, then the Sharpe index model underestimates risk, likewise if there are negative correlations in the residuals, the Sharpe index model over estimates risk.

Our multiple index empirical study has shown that the Troskie-Hossain innovation, approximates the Markowitz efficient frontier and optimal portfolio very closely. For all further multiple index models we will use the Troskie-Hossain innovation.

3.A Index Selection for Multiple Index Model

We require a maximum of 4 and a minimum of 3 explanatory variables for the multiple index model. To commence, we fit all 8 explanatory variables. A t-statistic with absolute value larger than 1.96 is considered significant at the 5% significance level. We find that X6 and X4 is significant for only R6. X4 has a higher t-statistic than X6. Hence we delete X6 and refit our models, with the remaining 7 indices. From Table 3.9 we can see that by deleting X6 the R^2 statistics have not changed very much. Overall the Schwartz statistics have improved. Even for share R6 the Schwartz criterion remains unchanged. The next variable we delete is X4 and refit our models with the remaining 6 variables. From Table 3.10 on page 44 we can see that by deleting X4 the R^2 statistics have not changed very much. Overall the Schwartz criterion have improved. For share R6 the R^2 has decreased slightly. The next variable we delete is X3 and refit our models with the remaining 5 variables. In most instances the Schwartz criterion has improved after deleting X3. The R^2 are slightly lower in some cases, but overall there is not much change. Refer to Table 3.11 on page 45. We now delete X5 and refit our model with the remaining 4 variables. Deleting X5 has improved our Schwartz criterion overall and has not had any dramatic impact on the R^2 of our models. We are now down to 4 variables as required. We proceed by fitting two different models, the first with X1 X2 X7 as explanatory variables and the 2nd model with X1 X2 X8 as explanatory variables. We see in this instance that fitting 3 explanatory variables is better than fitting 4 explanatory variables. Both models

X1 X2 X7 and X1 X2 X8 have similar standard errors of regression and Schwartz criterion. We have however chosen X1 X2 X7 as explanatory variables. Had we chosen X1 X2 X8 as explanatory variables, R4 and R10 would only have one significant variable

Table 3.8: t-statistics for Variable Selection Process: 8 Explanatory Variables

Code	X1	X2	X3	X4	X5	X6	X7	X8	R^2	Schwartz
R1	8.66	2.52	-0.56	1.85	2.41	-0.6	1.9	2.91	0.7223	-2.77
R2	8.08	-0.95	-1.71	-0.17	-1.73	0.46	-1.91	-1.1	0.5083	-2.71
R3	9.02	-1.87	-0.41	0.21	-1.73	-1.3	-2.51	-1.55	0.5087	-2.15
R4	4.00	-0.55	3.23	-1.57	-1.71	-1.83	5.17	-0.39	0.4634	-1.64
R5	7.02	-0.29	-0.85	0.61	0.83	-1.93	-1.17	-3.8	0.3684	-1.91
R6	5.09	-1.33	-2.42	-3.16	1.7	-2.08	-1.06	0.05	0.3122	-1.97
R7	9.43	0.4	-0.16	0.89	-0.06	-1.85	0.00	-1.56	0.5546	-2.86
R8	10.4	-1.86	0.28	-0.35	-0.61	-0.2	-3.07	-3.61	0.5829	-2.59
R9	6.56	2.61	-0.38	-1.7	3.32	-0.3	0.58	-0.89	0.4789	-1.99
R10	4.06	-0.99	0.24	1.28	-2.35	0.73	-2.4	1.27	0.2999	-1.91
R11	6.11	-2.19	-1.59	0.24	-1.01	-0.26	0.99	-1.91	0.4249	-2.56
R12	6.81	-2.73	1.27	0.68	-0.71	-0.87	-0.81	-1.47	0.4187	-2.1
R13	4.74	-0.67	-1.48	-1.82	1.22	-0.53	-0.26	0.37	0.2909	-2.16
R14	3.7	-0.88	1.96	0.22	0.26	0.63	0.54	0.11	0.3089	-2.02
R15	4.6	-2.85	0.12	1.41	-0.83	-0.26	0.11	-2.93	0.3145	-1.96

Table 3.9: t-statistics for Variable Selection Process: 7 Explanatory Variables

Code	X1	X2	X3	X4	X5	X7	X8	R^2	Schwartz
R1	9.90	2.46	-0.54	1.87	2.45	1.91	2.88	0.7215	-2.80
R2	9.89	-0.88	-1.73	-0.18	-1.76	-1.92	-1.07	0.5075	-2.74
R3	9.83	-2.14	-0.36	0.25	-1.65	-2.49	-1.65	0.5023	-2.17
R4	3.55	-0.90	3.27	-1.51	-1.60	5.15	-0.53	0.4497	-1.65
R5	7.02	-0.65	-0.77	0.65	0.93	-1.14	-3.93	0.3503	-1.92
R6	4.65	-1.72	-2.31	-3.07	1.79	-1.02	-0.11	0.2894	-1.97
R7	9.91	0.06	-0.09	0.93	0.05	0.02	-1.70	0.5429	-2.87
R8	12.24	-1.94	0.29	-0.34	-0.60	-3.08	-3.66	0.5828	-2.62
R9	7.61	2.61	-0.37	-1.69	3.35	0.59	-0.92	0.4785	-2.02
R10	5.28	-0.87	0.21	1.27	-2.40	-2.41	1.34	0.2970	-1.94
R11	7.11	-2.28	-1.59	0.24	-1.00	1.00	-1.94	0.4247	-2.59
R12	7.52	-2.95	1.30	0.71	-0.66	-0.80	-1.55	0.4153	-2.13
R13	5.30	-0.78	-1.46	-1.81	1.25	-0.26	0.33	0.2894	-2.20
R14	4.80	-0.78	1.94	0.20	0.23	0.53	0.16	0.3068	-2.05
R15	5.30	-2.96	0.13	1.42	-0.82	0.11	-2.97	0.3142	-2.00

Table 3.10: t-statistics for Variable Selection Process: 6 Explanatory Variables

Code	X1	X2	X3	X5	X7	X8	R^2	Schwartz
R1	10.83	2.73	-0.91	2.50	1.85	2.89	0.7141	-2.81
R2	10.33	-0.92	-1.74	-1.77	-1.92	-1.08	0.5074	-2.78
R3	10.40	-2.14	-0.42	-1.65	-2.51	-1.65	0.5021	-2.21
R4	3.23	-1.12	3.61	-1.65	5.16	-0.56	0.4401	-1.66
R5	7.56	-0.57	-0.92	0.95	-1.16	-3.92	0.3483	-1.95
R6	3.80	-2.11	-1.70	1.63	-0.92	-0.16	0.2387	-1.94
R7	10.65	0.20	-0.28	0.08	-0.01	-1.68	0.5400	-2.90
R8	12.74	-2.02	0.36	-0.61	-3.08	-3.67	0.5824	-2.66
R9	7.39	2.37	-0.04	3.27	0.63	-0.94	0.4672	-2.03
R10	5.90	-0.70	-0.03	-2.35	-2.44	1.35	0.2884	-1.97
R11	7.53	-2.28	-1.67	-1.00	0.99	-1.94	0.4244	-2.62
R12	8.10	-2.88	1.19	-0.64	-0.82	-1.54	0.4131	-2.16
R13	4.94	-1.04	-1.12	1.18	-0.21	0.30	0.2717	-2.21
R14	5.10	-0.76	1.94	0.23	0.53	0.16	0.3066	-2.09
R15	5.96	-2.78	-0.14	-0.77	0.07	-2.93	0.3037	-2.02

Table 3.11: t-statistics for Variable Selection Process: 5 Explanatory Variables

Code	X1	X2	X5	X7	X8	R^2	Schwartz
R1	10.89	2.66	2.39	1.67	2.83	0.7123	-2.84
R2	10.34	-2.18	-2.03	-2.44	-1.19	0.4962	-2.79
R3	10.46	-2.81	-1.74	-2.72	-1.69	0.5014	-2.24
R4	2.93	0.92	-1.08	6.06	-0.30	0.3853	-1.61
R5	7.62	-1.26	0.83	-1.45	-4.00	0.3442	-1.98
R6	3.86	-3.56	1.39	-1.40	-0.27	0.2222	-1.95
R7	10.71	0.06	0.04	-0.08	-1.71	0.5397	-2.93
R8	12.78	-2.16	-0.57	-3.10	-3.67	0.5820	-2.69
R9	7.43	2.79	3.31	0.64	-0.95	0.4672	-2.07
R10	5.93	-0.85	-2.39	-2.54	1.36	0.2884	-2.00
R11	7.57	-3.74	-1.25	0.57	-2.05	0.4123	-2.64
R12	8.04	-2.65	-0.47	-0.53	-1.46	0.4069	-2.19
R13	5.00	-1.95	1.02	-0.52	0.22	0.2647	-2.23
R14	4.96	0.33	0.52	1.06	0.29	0.2869	-2.09
R15	6.00	-3.39	-0.80	0.04	-2.96	0.3036	-2.05

Table 3.12: t-statistics for Variable Selection Process: 4 Explanatory Variables

Code	X1	X2	X7	X8	R^2	Schwartz
R1	11.41	2.64	1.47	2.98	0.7000	-2.83
R2	10.02	-2.17	-2.27	-1.33	0.4807	-2.80
R3	10.25	-2.80	-2.58	-1.82	0.4902	-2.26
R4	2.77	0.91	6.15	-0.39	0.3800	-1.63
R5	7.96	-1.25	-1.52	-3.95	0.3408	-2.01
R6	4.21	-3.53	-1.50	-0.17	0.2110	-1.97
R7	10.99	0.06	-0.09	-1.72	0.5397	-2.97
R8	12.96	-2.18	-3.07	-3.73	0.5810	-2.72
R9	7.97	2.73	0.38	-0.67	0.4236	-2.03
R10	5.47	-0.86	-2.33	1.16	0.2581	-2.00
R11	7.46	-3.74	0.67	-2.14	0.4054	-2.66
R12	8.13	-2.67	-0.49	-1.50	0.4059	-2.22
R13	5.31	-1.93	-0.60	0.30	0.2590	-2.26
R14	5.19	0.33	1.03	0.33	0.2855	-2.13
R15	5.96	-3.41	0.10	-3.03	0.3002	-2.08

Table 3.13: t-statistics for Variable Selection Process: 3 Explanatory Variables

Code	X1	X2	X7	R^2	Schwartz	σ_e^2
R1	13.69	3.31	2.43	0.6804	-2.81	0.0032
R2	10.45	-2.54	-2.80	0.4738	-2.82	0.0031
R3	10.41	-3.28	-3.26	0.4777	-2.27	0.0054
R4	2.90	0.85	6.36	0.3793	-1.67	0.0099
R5	6.58	-2.10	-2.71	0.2647	-1.94	0.0075
R6	4.61	-3.68	-1.64	0.2108	-2.01	0.0070
R7	11.28	-0.34	-0.64	0.5296	-2.98	0.0027
R8	12.02	-2.97	-4.23	0.5377	-2.66	0.0037
R9	8.54	2.66	0.18	0.4216	-2.06	0.0067
R10	6.61	-0.61	-2.07	0.2507	-2.02	0.0069
R11	7.15	-4.29	0.01	0.3852	-2.66	0.0036
R12	8.26	-3.08	-1.00	0.3959	-2.24	0.0056
R13	6.05	-1.92	-0.53	0.2585	-2.30	0.0053
R14	5.93	0.42	1.19	0.2849	-2.16	0.0060
R15	5.01	-4.09	-0.84	0.2526	-2.05	0.0067

Table 3.14: t-statistics for Variable Selection Process: 3 Explanatory Variables

Code	X1	X2	X8	R^2	Schwartz	σ_e^2
R1	12.15	2.55	3.58	0.6952	-2.85	0.0030
R2	9.62	-2.03	-2.10	0.4609	-2.79	0.0032
R3	9.72	-2.62	-2.68	0.4651	-2.24	0.0055
R4	4.00	0.53	1.39-	0.2061	-1.42	0.0126
R5	7.79	-1.17	-4.61	0.3296	-2.03	0.0069
R6	3.94	-3.44	-0.65	0.1978	-1.99	0.0072
R7	11.39	0.06	-1.84	0.5397	-3.00	0.0026
R8	12.22	-1.96	-4.76	0.5517	-2.69	0.0035
R9	8.38	2.73	-0.58	0.4229	-2.06	0.0067
R10	4.96	-0.73	0.46	0.2282	-1.99	0.0071
R11	7.91	-3.79	-2.04	0.4035	-2.70	0.0035
R12	8.31	-2.65	-1.74	0.4048	-2.26	0.0055
R13	5.35	-1.91	0.13	0.2570	-2.29	0.0053
R14	5.64	0.28	0.68	0.2799	-2.15	0.0061
R15	6.22	-3.43	-3.16	0.3002	-2.12	0.0063

Chapter 4
Time Series Errors and Conditional Heteroskedastic Models

4.1 Introduction

There are 4 fundamental assumptions underlying the error terms in the Sharpe index models.

$$E(e_{it}^2) = \sigma_{ei}^2 \tag{4.1}$$

$$E(e_{it}e_{is}) = 0, \quad t \neq s = 1, \ldots, N \tag{4.2}$$

$$E(e_{it}I_t) = 0, \quad t = 1, \ldots, N \tag{4.3}$$

$$E(e_{it}e_{jt}) = 0, \quad t = 1, \ldots, N \tag{4.4}$$

In previous chapters we have demonstrated the effect assumption 4.4 has on the efficient frontier and have noted that this assumption does not hold on the JSE. In this chapter we further examine assumptions 4.2 and 4.1 : the assumptions of no serial correlation in the residuals and homoskedasticity.

4.2 Time Series Errors

"Regression models with time series errors is widely applicable in economics and finance, but it is one of the most commonly misused econometric models because the serial dependence in e_t is often overlooked. It pays to study the model carefully." (Tsay 2002). Time Series analysis is an area well understood by statisticians and is standard reading in most statistical science textbooks. We introduce a few basic concepts. There are two basic types of serial correlation of residuals, the third being a combination of the two basic types of serial correlation.

- 1. An autoregressive process of order p, $AR(p)$:

$$e_t = \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots + \theta_p e_{t-p} + v_t \tag{4.5}$$

- 2. A moving average process of order q, $MA(q)$:

$$e_t = v_t + \alpha_1 v_{t-1} + \alpha_2 v_{t-2} + \ldots + \alpha_q v_{t-q} \tag{4.6}$$

- 3. An autoregressive moving average process of

  order (p, q), $ARMA(p, q)$:

$$\begin{aligned} e_t &= \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots + \theta_p e_{t-p} + \\ & \quad v_t + \alpha_1 v_{t-1} + \alpha_2 v_{t-2} + \ldots + \alpha_q v_{t-q} \end{aligned} \tag{4.7}$$

  where we further assume

$$E(v_t) = 0, \quad E(v_t^2) = \sigma_v^2, \quad E(v_t v_s) = 0 \text{ for } t \neq s \tag{4.8}$$

ARMA processes have non-constant conditional mean and constant variance.

4.3 Heteroskedatiscity

Volatility models were established to model the time varying nature of the conditional variance of time series. For the purposes of this study we limit ourselves to the GARCH(1,1) models. Let e_t be the residual from our regression at time t, and u_t and σ_t the conditional mean and variance at time t. Then a_t is the mean adjusted residual from our regression at time t. We model the variance of the residuals from our regressions as :

$$a_t = e_t - u_t \tag{4.9}$$

$$a_t = \sigma_t \varepsilon_t \quad \text{where} \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{4.10}$$

where we assume that $\varepsilon_t \sim N(0,1)$. Note however that since σ_t depends on the first lagged conditional variance one requires an estimate of σ_1. The unconditional sample estimate is often used as a simple approximation (Bollerslev 1986). σ_1 could also be set equal to zero. GARCH models have non-constant conditional variance and zero conditional mean. The coefficients of GARCH models must be constrained so as to all be strictly positive.

Enders (2004: p135) states that caution must be taken when using the Akaike and Schwartz Criterion to assess the adequacy of a GARCH model, as the Akaike and Schwartz criterion measure squared deviations of the model of the mean. "*...their statistical properties in the ARCH context are largely unknown.*" (Bollerslev, Engle and Nelson 1994)

4.4 Time Series Errors and Heteroskedasticity

An ARMA and GARCH model has both a non-constant conditional mean and non-constant conditional variance. The GARCH processes can be used as the "noise" term of an ARMA process.

4.5 Study Objectives

To examine the effects of the following assumptions for the residuals in the single index model and multiple index model and their impact on the efficient frontier

(i) no serial correlation

(ii) homoskedasticity

4.6 Limitations of Study

The accuracy and consistency of our models depends on the algorithms utilised by Eviews6. In the interest of accuracy and consistency, we have verified ARMA model fits in Eviews6 to those in RATS. Both software packages have similar estimates for ARMA model coefficients and t-statistics. GARCH model estimates in Eviews6 were compared to those in Matlab7. Of concern were the negative coefficients obtained in Eviews6 for some GARCH models. These models were rerun in Matlab7.When a coefficient becomes negative, Matlab sets this parameter to zero and re-estimates the model, Eviews6 reports the negative coefficient. The Berndt-Hall-Hall-Hausman algorithm in Eviews6 yields results similar to those of the GARCH toolbox in Matlab, with exception of the previously mentioned points. Estimation of parameters for GARCH models higher than GARCH(1,1) has proven difficult in both Matlab and Eviews6.

4.7 Data

The datasets *"singleret"* and *"multiret"* are used for the single index and multiple index empirical studies respectively.

4.8 Methodology

4.8.1 Serial Correlation

Having run the regressions for the single index model and multiple index model in chapter 3, we now examine the residuals from these regressions for each share. We use the procedure outlined by Troskie (2001) to check for serial correlation. This method is similar to the variable selection process, as we only fit a model with significant t-statistics in the time series process. A t-statistic with absolute value larger than 1.5 is considered significant in regression analysis (Troskie).

For the purposes of our study, an AR(p) model is of the form

$$e_t = \theta_p e_{t-p} + v_t \tag{4.11}$$

By inspecting the partial autocorrelations we determine lags exhibiting significant autocorrelation. We fit the simplest model (lowest AR(p) term or MA(q)) and then rerun the regression in Eviews6. Eviews 6 uses nonlinear least square to estimate the coefficients of a regression with AR terms. If the model is adequate then the residuals should resemble white noise. We use the autocorrelations function (ACF) and Ljung-Box statistics (Q-statistic) of the residuals at the 5% significance level to check the closeness of $\hat{e}_t$ to white noise. For an AR(p) model, the Q-statistic follows asymptotically a chi-squared distribution with $m - p$ degrees of freedom. We then repeat the process until the residuals resemble white noise. The Schwartz criterion is used as a guideline in the model building process.

4.8.2 Heteroskedasticity

We examine the graphs, correlograms and Q-statsistics of the squared residuals from our OLS regressions for heteroskedasticity. If there is correlation in the squared residuals, we fit the simplest GARCH model so as to eliminate or reduce this correlation. Troskie advocates the use of GARCH(1,1) to model the volatility of financial data. Bollerslev recommends fitting a lower order GARCH model rather than a higher order ARCH model. Eviews6 uses maximum likelihood to estimate the coefficients of a regression with autocorrelation and GARCH terms or with GARCH terms only.

4.8.3 Serial correlation and Heteroskedasticity

We first establish time series models for the residuals to eliminate any serial correlation. We then examine the squared residuals for correlation and fit the simplest GARCH model so as to reduce or eliminate this correlation between variances. Parameters are re-estimated and checked for significance. The models are then refined. Eviews6 uses maximum likelihood to estimate the coefficients of a regression with autocorrelation and GARCH terms or with GARCH terms only.

4.8.4 Backcasting of residuals

For an AR(4) model, one loses 4 observations. As some of the models exhibited serial correlation at higher lags, and to avoid loosing too many observation, we backcast the residuals lost by using the new estimates of α and β. For example when using an AR(4) process, 4 residuals are lost. We then re-estimate residuals for t =1 to 4 by

$$\hat{e}_t = Y_t - \hat{Y}_t \tag{4.12}$$

$$= Y_t - (\hat{\alpha}_{new} + X_t\hat{\beta}_{new}) \tag{4.13}$$

4.8.5 Troskie-Hossain Model

Finally we use the Troskie-Hossain model to estimate the means and covariance ($\hat{\mathbf{\Phi}}$) structure for our portfolio of shares.

$$\hat{\mathbf{\Phi}} = \hat{\sigma}_I^2\hat{\beta}\hat{\beta}' + \hat{\Omega} \tag{4.14}$$

To estimate Ω let

$$\hat{\mathbf{E}} = \begin{pmatrix} \hat{e}_{11} & \hat{e}_{12} & \dots & \hat{e}_{1N} \\ \hat{e}_{21} & \hat{e}_{22} & \dots & \hat{e}_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{e}_{p1} & \dots & \dots & \hat{e}_{pN} \end{pmatrix} \tag{4.15}$$

then the nonlinear least squares and maximum likelihood estimate of Ω is

$$\hat{\Omega} = \frac{1}{N}\hat{\mathbf{E}}\hat{\mathbf{E}}' \tag{4.16}$$

The suitability of this estimate follows from the properties of non-linear least squares:

- *"Nonlinear least squares estimates are asymptotically equivalent to maximum likelihood estimates and are asymptotically efficient."* (Davidson and Mackinnon,1993), (Greene 1997).

- *"There is no need to correct for bias since what is of greatest interest is rapid convergence of* $\hat{\Sigma}$ *to* Σ*."* (Press J, 1972)

Table 4.1: Single Index Optimal Portfolio's

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	8.03 %	4.79 %	4.60 %	6.04 %
Bidvest	8.36 %	6.36 %	2.55 %	1.36 %
First Rand	35.84 %	36.86 %	36.85 %	40.14 %
Pick 'n Pay	12.00 %	12.86 %	15.81 %	22.88 %
Pretoria Portland Cement	2.02 %	6.36 %	-	-
Sasol	17.00 %	17.56 %	18.07 %	17.58 %
Santam	-	-	5.68 %	-
Shoprite	16.74 %	15.21 %	16.44 %	12.00 %
Expected Return (%p.a)	19.09 %	19.00 %	19.25 %	19.54 %
Portfolio Variance (%p.a)	5.77 %	5.68 %	5.56 %	5.80 %

4.9 Single Index Model

The "least squares model" refers to the Troskie-Hossain index model estimated using least squares and without any time series or heteroskedastic models. Having established the effect of residual correlation on the efficient frontier, we use this model as a base to compare our other models with time series errors and heteroskedasticity.

4.9.1 Effects of Serial Correlation

There is evidence of autocorrelation in the residuals of 11 of the 15 shares. AR models have been fitted to these shares. No significant MA models were found. Table 4.2 on page 57 shows the ARMA models fitted to the residuals of each single index model regression. From Figure 4.1 on page 58 we can see that modelling the serial correlation in the residuals has had very little impact on the efficient frontier. The ARMA efficient frontier lies very close to that of the least squares efficient frontier. However at the left tail end, the ARMA efficient frontier is steeper and lies below the least squares efficient frontier. Examining the optimal portfolio's in Table 4.1 we see that the least squares and ARMA optimal portfolios have the same shares but in slightly different weightings.

Table 4.2: Single Index: ARMA Models

Code	AR term	t-statistic	AR term	t-statistic	Comments
AGL	AR(2)	-2.12			
BVT					No significant autocorrelation
FSR					No significant autocorrelation
GFI					No significant autocorrelation
PIK	AR(1)	-3.83	AR(5)	-2.05	
PPC	AR(2)	-2.78	AR(6)	3.02	
SAB	AR(1)	-2.59	AR(4)	-2.04	
SBK	AR(5)	-1.87	AR(8)	-1.72	
SOL					No significant autocorrelation
SUI	AR(8)	-1.91			
TBS	AR(5)	-2.58	AR(6)	-2.54	
SNT	AR(1)	-1.65			
AFX	AR(1)	-2.60			
ILV	AR(2)	-2.22			
SHP	AR(1)	-1.59			

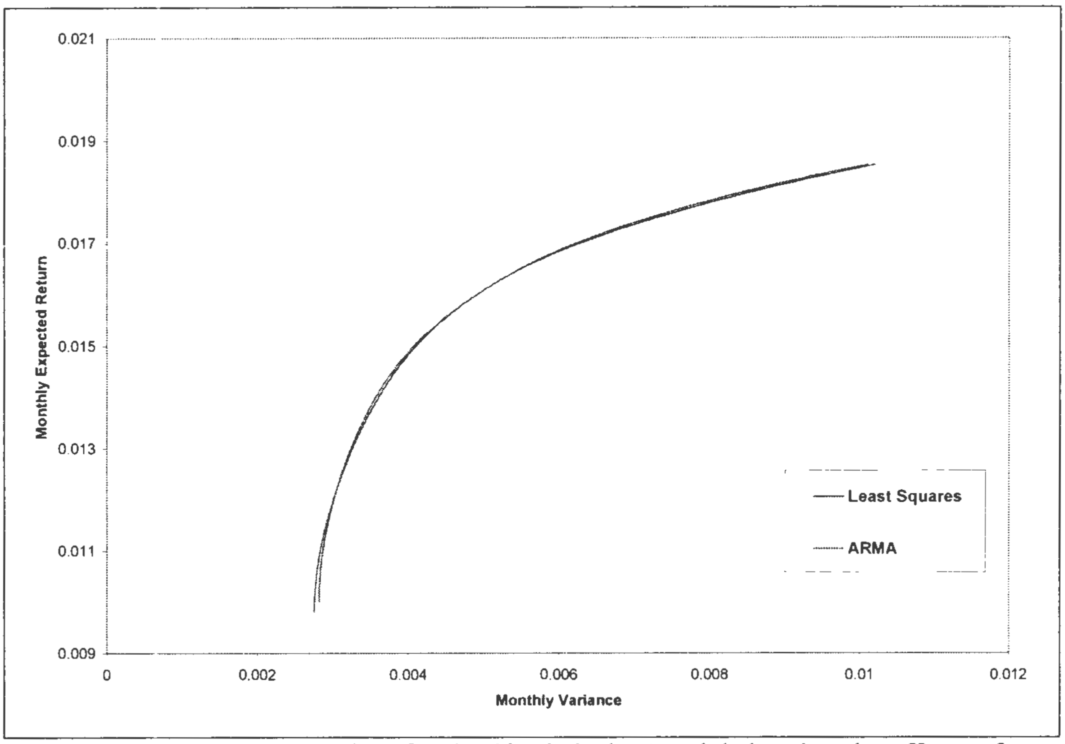

Figure 4.1: Efficient Frontiers for the Single Index Model showing the effects of ARMA.

4.9.2 Effects of Heteroskedasticity

Examining the graphs and correlograms of the squared residuals from the least squares regressions, we find evidence of heteroskedasticity. The GARCH(1,1) models capture most of the higher order serial correlation in the squared residuals. Refer to Table 4.3 for GARCH models fitted to the residuals from the single index models. Modelling a constant mean and non-constant conditional variance has shifted the efficient frontier to the left. From Table 4.1 on page 56 we can see that the composition and weightings of the optimal portfolio have also changed.

Table 4.3: Single Index: GARCH Models

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL	0.0002	1.57	0.0831	1.54	0.8688	11.13	
BVT							Autocorrelation. Model inadequate
FSR	0.0001	0.36	0.0846	1.82	0.9026	15.99	
GFI							No significant autocorrelation
PIK	0.0003	1.41	0.1626	4.06	0.8143	16.40	
PPC	0.0051	4.65	0.3209	1.95	-	-	
SAB	0.0018	5.13	0.3315	1.76	-	-	
SBK	0.0009	1.62	0.3105	1.99	0.4577	2.12	
SOL	0.0000	0.10	0.0423	0.97	0.9483	11.45	
SUI	0.0003	0.63	0.0595	1.04	0.8960	8.18	
TBS	0.0002	0.60	0.0446	0.99	0.9103	8.91	
SNT	0.0018	1.52	0.2017	1.58	0.5029	1.99	
AFX	0.0005	1.70	0.0910	1.56	0.8219	8.87	
ILV							No significant autocorrelation
SHP	0.0001	0.57	0.1015	1.60	0.8826	12.23	

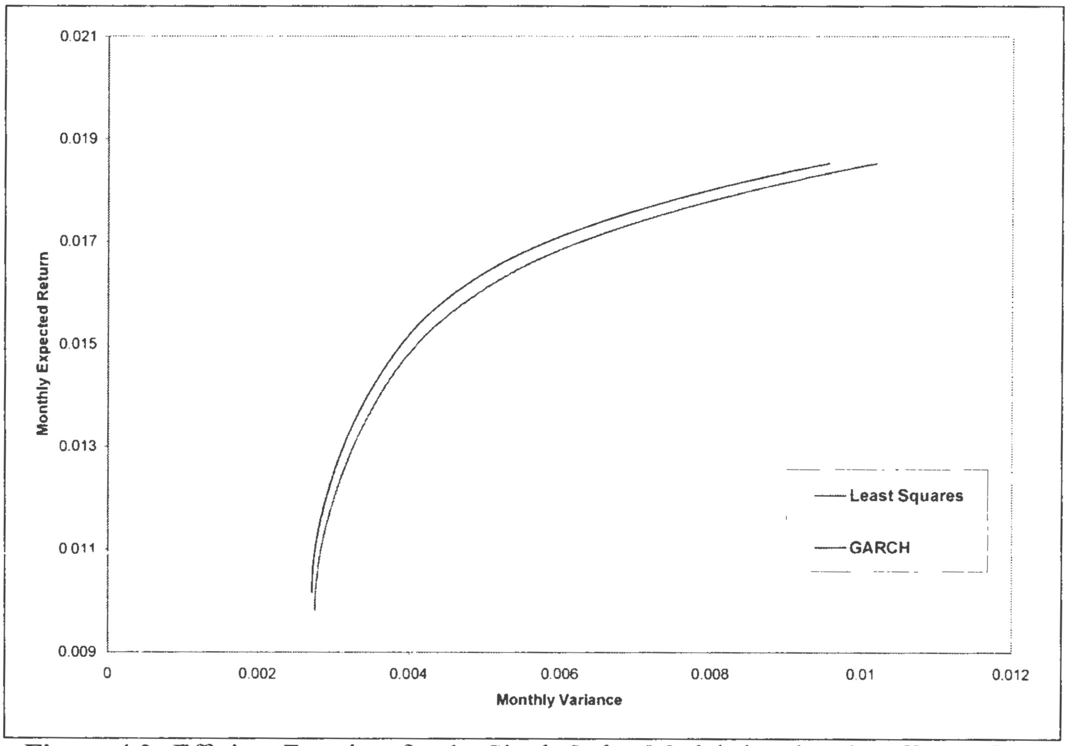

Figure 4.2: Efficient Frontiers for the Single Index Model showing the effects of GARCH

Table 4.4: Single Index: ARMA&GARCH Models

Code	Autocorrelation	Variance Model	Variance Model comments
AGL	AR(2)	-	Evidence of autocorrelation. Model inadequate
BVT	-		No significant autocorrelation.
FSR	-	GARCH(1,1)	
GFI	-	-	No significant autocorrelation.
PIK	AR(1) , AR(5)	GARCH(1,1)	
PPC	AR(2) , AR(6)	GARCH(1,0)	
SAB	AR(1) , AR(4)	GARCH(1,0)	
SBK	AR(5) , AR(8)	GARCH(1,1)	
SOL	-	GARCH(1,1)	
SUI	AR(8)	GARCH(1,1)	
TBS	AR(5) , AR(6)	-	Evidence of autocorrelation. Model inadequate
SNT	AR(1)	GARCH(1,1)	
AFX	AR(1)	GARCH(0,1)	
ILV	AR(2)	-	Evidence of autocorrelation. Model inadequate
SHP	AR(1)	-	Evidence of autocorrelation. Model inadequate

4.9.3 Effects of Time Series Errors and Heteroskedasticity

Modelling a non-constant conditional mean and non-constant conditional variance has proven difficult. In a number of cases the GARCH models have not captured any of the serial correlation in the squared residuals. Estimation of parameters has proven far more complex than estimating a model with constant mean and non-constant conditional variance or non-constant conditional mean and constant conditional variance. Table 4.4 summarises the ARMA&GARCH models fitted to the residuals from the single index models.

For most part, in Figure 4.3 on page 61 the ARMA&GARCH efficient frontier lies very close to the GARCH efficient frontier. However the left tail end of the ARMA&GARCH efficient frontier lies very close to and resembles the ARMA efficient frontier.

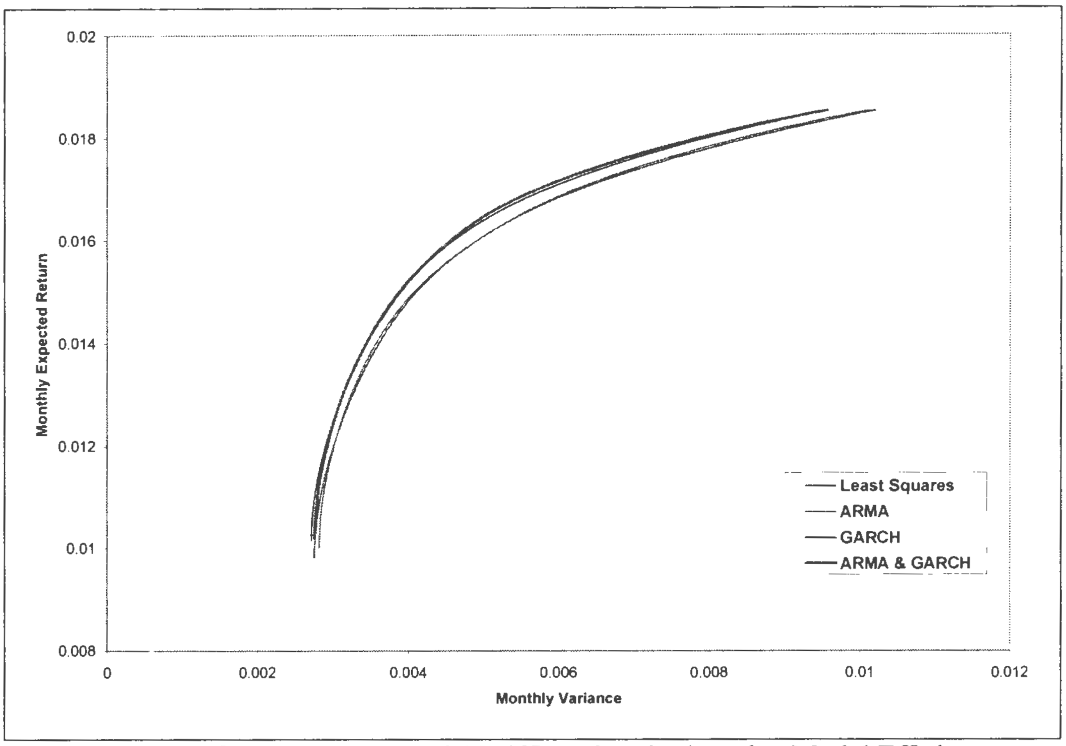

Figure 4.3: Effects of ARMA&GARCH on the Single Index Model Efficient Frontier

Table 4.5: Multiple Index Optimal Portfolio's

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	8.10 %	4.80 %	3.55 %	3.88 %
Bidvest	8.40 %	8.30 %	10.26 %	7.03 %
First Rand	35.84 %	36.24 %	34.62 %	34.13 %
Pick 'n Pay	11.95 %	10.12 %	8.89 %	12.95 %
Pretoria Portland Cement	2.06 %	7.15 %	-	-
Standard Bank	-	-	5.49 %	4.42 %
Sasol	16.98 %	17.62 %	19.92 %	19.91 %
Shoprite	16.68 %	15.78 %	17.26 %	18.19 %
Expected Return (%p.a)	19.09 %	18.89 %	19.07 %	19.12 %
Portfolio Variance (%p.a)	5.75 %	5.58 %	5.59 %	5.55 %

4.10 Multiple Index Model

In this section the "least squares model" refers to the Troskie-Hossain multiple index model estimated using least squares and without any time series or heteroskedastic models. Having established the effect of residual correlation on the efficient frontier, we use this model as a base to compare our other multiple index models with time series errors and heteroskedasticity.

4.10.1 Effects of Serial Correlation

We have found evidence of autocorrelation in 11 of the 15 shares. Table 4.6 summarises the ARMA models fitted to the residuals from the multiple index model. From Figure 4.4 we can see that the ARMA models have very little impact on the efficient frontier. The ARMA efficient frontier lies very close to the least squares efficient frontier, however the left tail end is slightly steeper than the least squares efficient frontier. This have very little impact on the optimal portfolio's in Table 4.5.

Table 4.6: Multiple Index: ARMA Models

Code	AR term	t-statistic	AR term	t-statistic	AR term	t-statistic	Comments
AGL	AR(2)	-2.46					
BVT							No significant autocorrelation
FSR							No significant autocorrelation
GFI							No significant autocorrelation
PIK	AR(1)	-5.12	AR(2)	-1.76			
PPC	AR(2)	-3.01	AR(6)	2.66			
SAB	AR(1)	-2.56	AR(4)	-1.97			
SBK	AR(5)	-1.97	AR(8)	-2.27			
SOL							No significant autocorrelation
SUI	AR(8)	-2.06					
TBS	AR(1)	-2.12	AR(5)	-2.74	AR(6)	-3.03	
SNT	AR(1)	-2.07					
AFX	AR(1)	-3.96	AR(2)	-2.13			
ILV	AR(2)	-2.14					
SHP	AR(5)	-1.92					

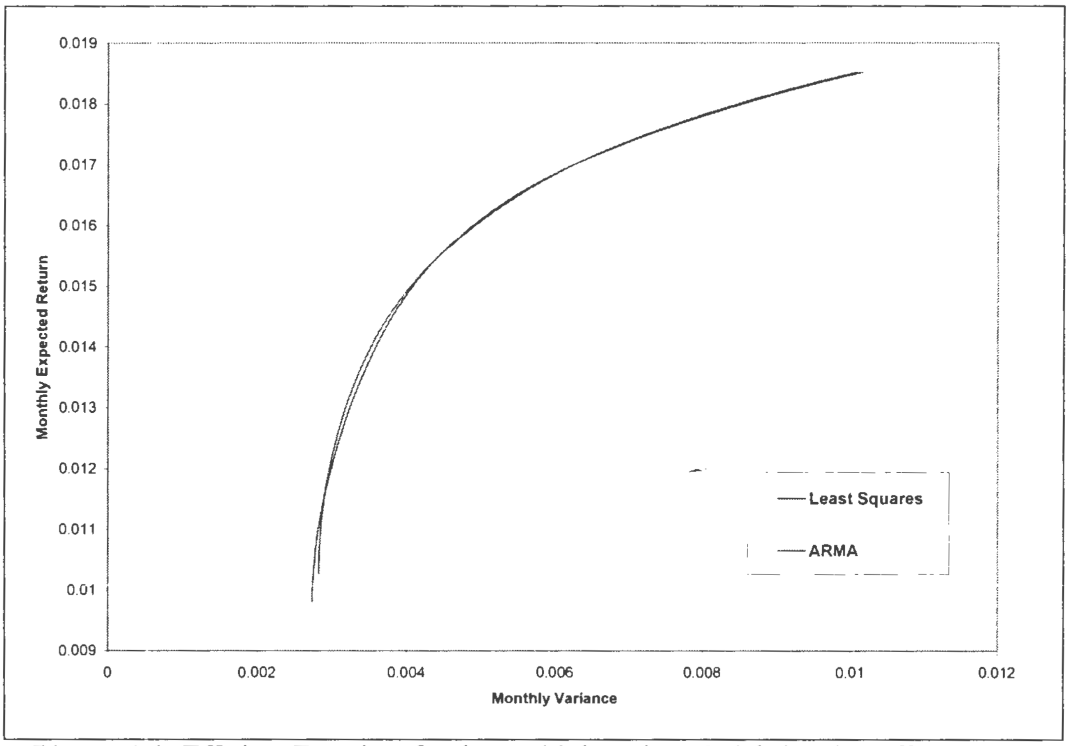

Figure 4.4: Efficient Frontiers for the Multiple Index Model showing effects of ARMA

4.10.2 Effects of Heteroskedasticity

Examining the graphs and correlograms of the squared residuals from the multiple index least squares regressions, we find evidence of heteroskedasticity. The GARCH(1,1) models capture most of the serial correlation in the squared residuals. Refer to Table 4.7 for the Multiple Index GARCH models. From Figure 4.5 on page 65 we can see that using GARCH to model the variance of the residuals from the multiple index model shifts the efficient frontier slightly to the left. This left shift in the multiple index model is not as marked as that in the single index model. In the multiple index model, the GARCH efficient frontier lies to the left, but very close to the least squares efficient frontier.

Table 4.7: Multiple Index: GARCH Models

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL	0.0001	1.33	0.1126	1.65	0.8619	11.68	
BVT							Evidence of autocorrelation. Model inadequate.
FSR	0.0000	0.00	0.0788	1.62	0.9137	17.67	
GFI							No significant autocorrelation
PIK	0.0001	0.59	0.1439	3.44	0.8536	18.91	
PPC	0.0046	5.33	0.3138	2.23	-	-	
SAB	0.0017	4.64	0.3673	1.82	-	-	
SBK	0.0010	1.42	0.3582	1.60	0.3444	0.97	
SOL	0.0002	0.53	0.0601	1.06	0.9107	9.42	
SUI	0.0001	0.39	0.0623	1.16	0.9149	9.89	
TBS	0.0002	0.44	0.0555	0.77	0.8960	5.19	
SNT							No significant autocorrelation
AFX	0.0004	1.79	0.1056	1.66	0.8116	8.64	
ILV							No significant autocorrelation
SHP	0.0002	0.83	0.1219	1.48	0.8553	9.41	

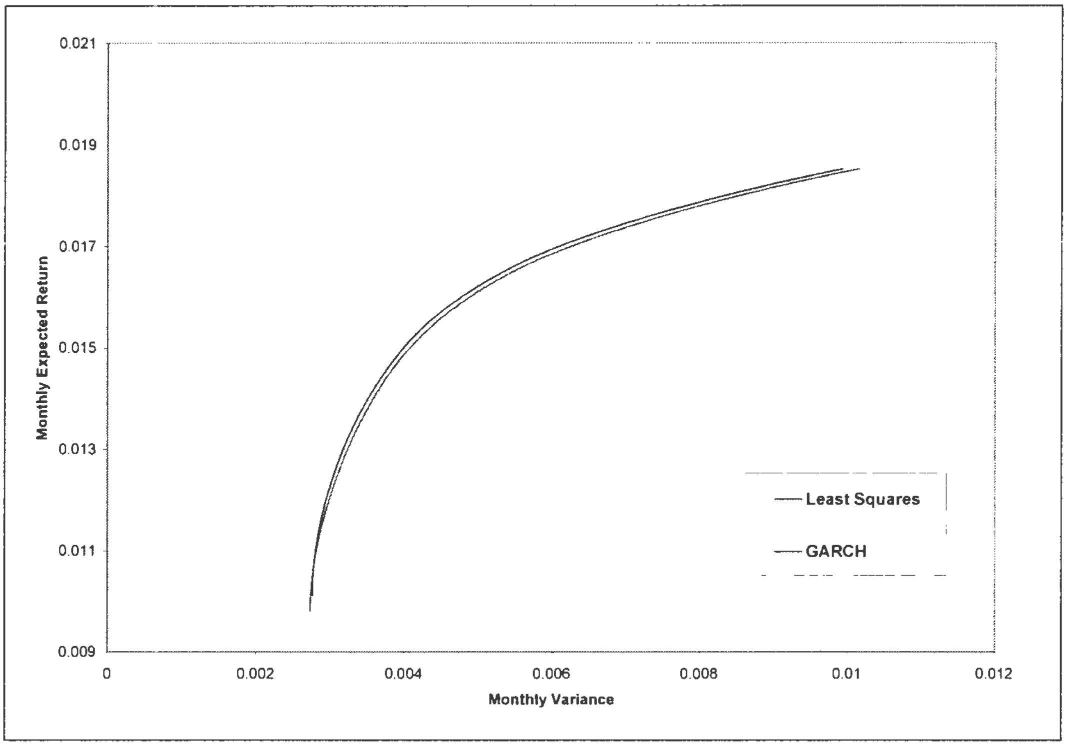

Figure 4.5: Efficient Frontiers for the Multiple Index showing the effects of GARCH

4.10.3 Effects of Time Series Errors and Heteroskedasticity

An ARMA&GARCH process has a non-constant conditional mean and non-constant conditional variance. The conditional variance is dependent on the conditional mean. Estimation of parameters for a model with a non-constant conditional mean and non-constant conditional variance has proven more complex than estimating a model with constant mean and non-constant conditional variance or vice-versa.

Table 4.8: Multiple Index: ARMA&GARCH Models

Code	Autocorrelation	Variance	Variance Model Comments
AGL	AR(2)		Evidence of autocorrelation. Model inadequate
BVT	-		Evidence of autocorrelation. Model inadequate
FSR	-	GARCH(1,1)	
GFI	-		No significant autocorrelation
PIK	AR(1) , AR(2)	GARCH(1,1)	
PPC	AR(2) , AR(6)	GARCH(1,0)	
SAB	AR(1) , AR(4)	GARCH(1,0)	
SBK	AR(5) , AR(8)	GARCH(1,1)	
SOL	-	GARCH(1,1)	
SUI	AR(8)	GARCH(1,1)	
TBS	AR(1) , AR(5) , AR(6)		Evidence of autocorrelation. Model inadequate
SNT	AR(1)		No significant autocorrelation
AFX	AR(1) , AR(2)	GARCH(1,1)	
ILV	AR(2)		Evidence of autocorrelation. Model inadequate
SHP	AR(1)	GARCH(1,1)	

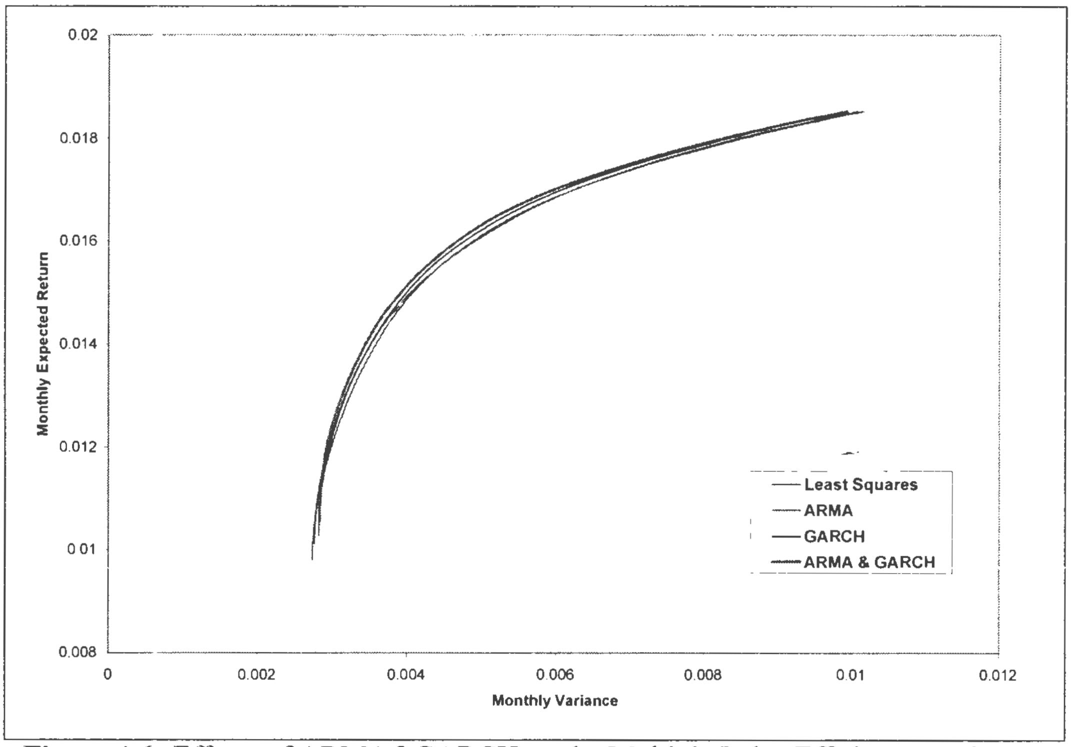

Figure 4.6: Effects of ARMA&GARCH on the Multiple Index Efficient Frontier

From Table 4.8 on page 66 we can see that for a number of shares the GARCH models have been inadequate. For these shares there is evidence of serial correlation in the squared residuals, however the variance model did not reduce this serial correlation. Examining the efficient frontiers in Figure 4.6 on page 66, we see that the ARMA&GARCH efficient frontier lies slightly above the GARCH efficient frontier. However the left tail end of the ARMA&GARCH efficient frontier lies very close to and resembles the ARMA efficient frontier.

4.11 Conclusions

- For the single index and multiple index models we find evidence of serial correlation and heteroskedasticity in the chosen data set.

- The ARMA efficient frontiers which model the autocorrelation in the residuals, lie very close to the Least Squares efficient frontier. Both optimal portfolios have the same shares but in slightly different weightings. This similarity suggests that the autocorrelation of the residuals has a small impact on the efficient frontier for this dataset and possibly more generally.

- The GARCH(1,1) volatility model tends to capture most of the serial correlation in the squared residuals. Modelling the heteroskedasticity of the residuals, shifts the efficient frontier to the left. This shift suggests that the assumption of homoskedasticity of the residuals in the single index model has an adverse effect on the efficient frontier and portfolio selection. However this left shift is not as pronounced in the multiple index model as in the single index model.

- Modelling both the serial correlation and homoskedasticity (non-constant mean and non-constant conditional variance) of the residuals has proven difficult. The more complex the model, the more difficult parameter estimation becomes. Also in some instances the variance models failed to capture the serial correlation in the squared residuals.

4.A Single Index Appendices

4.A.1 Single Index Model Alpha's and Beta's

	Least Squares		ARMA		GARCH		ARMA & GARCH	
Code	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ z-stat	$\hat{\beta}$	$\hat{\beta}$ z-stat
AGL	1.2885	15.66	1.3068	15.87	1.3570	28.70	1.3068	15.87
BVT	0.8066	9.95	0.8066	9.95	0.8066	9.95	0.8066	9.95
FSR	1.0351	9.47	1.0351	9.47	0.9565	9.89	0.9565	9.89
GFI	0.8917	5.70	0.8917	5.70	0.8917	5.70	0.8917	5.70
PIK	0.7328	5.87	0.7939	6.87	0.6344	4.78	0.6387	4.67
PPC	0.5193	4.24	0.4815	4.15	0.6751	6.39	0.6352	5.24
SAB	0.8839	12.42	0.9226	13.35	0.8997	12.33	0.9188	12.67
SBK	0.9610	10.53	0.9864	10.88	0.8420	11.36	0.8890	12.44
SOL	1.0887	9.42	1.0887	9.42	1.1063	8.52	1.1063	8.52
SUI	0.7379	6.32	0.7276	6.34	0.7172	5.61	0.6945	5.41
TBS	0.6848	7.73	0.7736	9.16	0.7060	7.13	0.7736	9.16
SNT	0.9178	8.60	0.9745	8.94	0.7706	7.35	0.9587	10.59
AFX	0.6637	6.54	0.7041	7.09	0.5958	5.60	0.6295	5.96
ILV	0.7813	7.26	0.8097	7.66	0.7813	7.26	0.8097	7.66
SHP	0.6074	5.05	0.6412	5.36	0.5689	4.41	0.6413	5.36

	Least Squares		ARMA		GARCH		ARMA & GARCH	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ z-stat	$\hat{\alpha}$	$\hat{\alpha}$ z-stat
AGL	-0.0004	-0.08	-0.0004	-0.09	-0.0036	-0.70	-0.0004	-0.09
BVT	0.0054	1.08	0.0054	1.08	0.0054	1.08	0.0054	1.08
FSR	0.0070	1.03	0.0070	1.03	0.0042	0.67	0.0042	0.67
GFI	-0.0039	-0.40	-0.0039	-0.40	-0.0039	-0.40	-0.0039	-0.40
PIK	0.0077	0.99	0.0062	1.21	0.0080	1.16	0.0067	1.36
PPC	0.0054	0.72	0.0052	0.69	0.0038	0.56	0.0036	0.53
SAB	-0.0018	-0.40	-0.0028	-0.87	-0.0016	-0.39	-0.0025	-0.69
SBK	0.0038	0.68	0.0034	0.77	0.0096	1.95	0.0083	2.03
SOL	0.0014	0.20	0.0014	0.20	0.0007	0.11	0.0007	0.11
SUI	0.0019	0.26	0.0006	0.10	0.0062	0.87	0.0040	0.64
TBS	0.0008	0.14	-0.0006	-0.15	0.0015	0.27	-0.0006	-0.15
SNT	0.0035	0.54	0.0028	0.48	0.0083	1.20	0.0011	0.18
AFX	0.0006	-0.09	-0.0011	-0.22	0.0030	0.49	0.0022	0.42
ILV	-0.0004	-0.06	-0.0011	-0.19	-0.0004	-0.06	-0.0011	-0.19
SHP	0.0082	1.11	0.0081	1.23	0.0112	1.58	0.0081	1.23

4.A.2 Single Index Model R^2

Code	Least Squares	ARMA	GARCH	ARMA & GARCH
AGL	0.6400	0.6524	0.6376	0.6524
BVT	0.4175	0.4175	0.4175	0.4175
FSR	0.3940	0.3940	0.3904	0.3904
GFI	0.1905	0.1905	0.1905	0.1905
PIK	0.1998	0.3062	0.1962	0.2964
PPC	0.1154	0.2122	0.1050	0.1989
SAB	0.5278	0.5661	0.5276	0.5615
SBK	0.4453	0.4802	0.4360	0.4725
SOL	0.3914	0.3914	0.3913	0.3913
SUI	0.2247	0.2402	0.2226	0.2377
TBS	0.3019	0.3637	0.3015	0.3637
SNT	0.3489	0.3613	0.3388	0.3606
AFX	0.2366	0.2734	0.2330	0.2687
ILV	0.2764	0.3021	0.2764	0.3021
SHP	0.1562	0.1711	0.1548	0.1711

4.A.3 Single Index Model $\hat{\sigma}_e^2$

Code	Least Squares	ARMA	GARCH	ARMA & GARCH
AGL	0.0035	0.0035	0.0036	0.0035
BVT	0.0034	0.0034	0.0034	0.0034
FSR	0.0062	0.0062	0.0064	0.0064
GFI	0.0127	0.0127	0.0127	0.0127
PIK	0.0081	0.0072	0.0083	0.0075
PPC	0.0078	0.0072	0.0080	0.0074
SAB	0.0026	0.0025	0.0027	0.0026
SBK	0.0043	0.0042	0.0045	0.0044
SOL	0.0069	0.0069	0.0071	0.0071
SUI	0.0071	0.0070	0.0072	0.0071
TBS	0.0041	0.0038	0.0042	0.0038
SNT	0.0059	0.0059	0.0061	0.0060
AFX	0.0053	0.0052	0.0055	0.0053
ILV	0.0060	0.0059	0.0060	0.0059
SHP	0.0075	0.0075	0.0077	0.0075

4.B Multiple Index Appendices

4.B.1 Multiple Index Model Beta's

	Least Squares		ARMA		GARCH		ARMA & GARCH	
Code	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ z-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ z-stat
AGL	1.2006	13.69	1.2379	14.12	1.2599	20.07	1.2379	14.12
BVT	0.9109	10.45	0.9109	10.45	0.9109	10.45	0.9109	10.45
FSR	1.1938	10.41	1.1938	10.41	1.1110	9.78	1.1110	9.78
GFI	0.4491	2.90	0.4491	2.90	0.4491	2.90	0.4491	2.90
PIK	0.8911	6.58	0.9922	8.53	0.8201	6.51	0.8602	6.48
PPC	0.6024	4.61	0.5632	4.50	0.7868	6.44	0.7223	5.35
SAB	0.9063	11.28	0.9322	11.93	0.9014	11.00	0.9288	11.95
SBK	1.1334	12.02	1.1364	12.68	0.9245	11.86	0.9479	12.81
SOL	1.0878	8.54	1.0878	8.54	1.0766	7.08	1.0766	7.08
SUI	0.8575	6.61	0.8346	6.73	0.8724	6.25	0.8716	6.37
TBS	0.6726	7.15	0.7922	8.89	0.7068	8.09	0.7922	8.89
SNT	0.9599	8.26	1.0651	9.32	0.9599	8.26	1.0651	9.32
AFX	0.6847	6.05	0.7770	7.56	0.6414	5.67	0.6996	6.15
ILV	0.7173	5.93	0.7487	6.25	0.7173	5.93	0.7487	6.25
SHP	0.6411	5.01	0.6540	5.06	0.5982	3.70	0.5727	3.98

	Least Squares		ARMA		GARCH		ARMA & GARCH	
Code	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ z-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ z-stat
AGL	0.3497	3.31	0.3669	3.67	0.4016	4.12	0.3669	3.67
BVT	-0.2664	-2.54	-0.2664	-2.54	-0.2664	-2.54	-0.2664	-2.54
FSR	-0.4530	-3.28	-0.4530	-3.28	-0.5297	-4.96	-0.5297	-4.96
GFI	0.1584	0.85	0.1584	0.85	0.1584	0.85	0.1584	0.85
PIK	-0.3420	-2.10	-0.3705	-2.71	-0.3842	-3.07	-0.3748	-3.22
PPC	-0.5793	-3.68	-0.4862	-3.47	-0.5074	-3.75	-0.4985	-3.62
SAB	-0.0329	-0.34	-0.0205	-0.22	-0.0872	-1.02	-0.0785	-0.94
SBK	-0.3373	-2.97	-0.3230	-2.98	-0.3986	-4.42	-0.3618	-3.67
SOL	0.4075	2.66	0.4075	2.66	0.4260	2.36	0.4260	2.36
SUI	-0.0956	-0.61	-0.1526	-1.00	-0.0833	-0.51	-0.2140	-1.48
TBS	-0.4862	-4.29	-0.4489	-4.27	-0.4680	-3.91	-0.4489	-4.27
SNT	-0.4310	-3.08	-0.4827	-3.59	-0.4310	-3.08	-0.4827	-3.59
AFX	-0.2620	-1.92	-0.3812	-3.14	-0.2633	-1.76	-0.3699	-2.66
ILV	0.0613	0.42	0.0271	0.19	0.0613	0.42	0.0271	0.19
SHP	-0.6297	-4.09	-0.6578	-4.26	-0.5997	-4.02	-0.5698	-4.06

	Least Squares		ARMA		GARCH		ARMA & GARCH	
Code	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$z-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ z-stat
AGL	0.1035	2.43	0.0834	1.99	0.0877	2.45	0.0834	1.99
BVT	-0.1188	-2.80	-0.1188	-2.80	-0.1188	-2.80	-0.1188	-2.80
FSR	-0.1821	-3.26	-0.1821	-3.26	-0.1215	-3.05	-0.1215	-3.05
GFI	0.4794	6.36	0.4794	6.36	0.4794	6.36	0.4794	6.36
PIK	-0.1788	-2.71	-0.2002	-3.46	-0.0496	-0.97	-0.0851	-1.60
PPC	-0.1042	-1.64	-0.0840	-1.39	-0.1028	-1.77	-0.0734	-1.19
SAB	-0.0248	-0.64	-0.0103	-0.27	-0.0080	-0.23	-0.0025	-0.07
SBK	-0.1939	-4.23	-0.1836	-3.98	-0.0910	-2.70	-0.0879	-2.30
SOL	0.0114	0.18	0.0114	0.18	-0.0181	-0.23	-0.0181	-0.23
SUI	-0.1309	-2.07	-0.1367	-2.09	-0.1635	-2.84	-0.1960	-3.36
TBS	0.0006	0.01	-0.0108	-0.26	0.0120	0.24	-0.0108	-0.26
SNT	-0.0563	-1.00	–0.0805	-1.46	-0.0563	-1.00	-0.0805	-1.46
AFX	-0.0293	-0.53	-0.0392	-0.77	-0.0287	-0.45	-0.0226	-0.34
ILV	0.0702	1.19	0.0642	1.09	0.0702	1.19	0.0642	1.09
SHP	-0.0524	-0.84	-0.0537	-0.86	-0.0113	-0.24	-0.0237	-0.48

4.B.2 Multiple Index Model Alpha's

	Least Squares		ARMA		GARCH		ARMA & GARCH	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ z-stat	$\hat{\alpha}$	$\hat{\alpha}$ z-stat
AGL	-0.0014	-0.29	-0.0016	-0.39	-0.0053	-1.19	-0.0016	-0.39
BVT	0.0058	1.20	0.0058	1.20	0.0058	1.20	0.0058	1.20
FSR	0.0078	1.23	0.0078	1.23	0.0015	0.28	0.0015	0.28
GFI	-0.0008	-0.09	-0.0008	-0.09	-0.0008	-0.09	-0.0008	-0.09
PIK	0.0080	1.06	0.0072	1.61	0.0023	0.38	0.0028	0.65
PPC	0.0076	1.04	0.0071	1.02	0.0063	0.93	0.0063	1.00
SAB	-0.0018	-0.41	-0.0027	-0.84	-0.0013	-0.31	-0.0023	-0.65
SBK	0.0040	0.76	0.0034	0.85	0.0080	1.82	0.0073	2.06
SOL	-0.0006	-0.08	-0.0006	-0.08	0.0006	0.08	0.0006	0.08
SUI	0.0013	0.18	0.0002	0.03	0.0052	0.73	0.0048	0.82
TBS	0.0033	0.63	0.0015	0.48	0.0029	0.56	0.0015	0.48
SNT	0.0053	0.82	0.0043	0.82	0.0053	0.82	0.0043	0.82
AFX	0.0006	0.09	-0.0001	-0.02	0.0034	0.55	0.0027	0.57
ILV	-0.0001	-0.02	-0.0006	-0.11	-0.0001	-0.02	-0.0006	-0.11
SHP	0.0111	1.56	0.0112	1.78	0.0117	1.72	0.0099	1.53

4.B.3 Multiple Index Model $\hat{\sigma}_e^2$

Code	Least Squares	ARMA	GARCH	ARMA & GARCH
AGL	0.0034	0.0031	0.0033	0.0031
BVT	0.0031	0.0031	0.0031	0.0031
FSR	0.0054	0.0054	0.0057	0.0057
GFI	0.0099	0.0099	0.0099	0.0099
PIK	0.0075	0.0064	0.0080	0.0068
PPC	0.0070	0.0066	0.0073	0.0068
SAB	0.0027	0.0026	0.0027	0.0026
SBK	0.0037	0.0035	0.0039	0.0038
SOL	0.0067	0.0067	0.0069	0.0069
SUI	0.0069	0.0068	0.0071	0.0070
TBS	0.0036	0.0033	0.0037	0.0033
SNT	0.0056	0.0054	0.0056	0.0054
AFX	0.0053	0.0048	0.0054	0.0050
ILV	0.0060	0.0059	0.0060	0.0059
SHP	0.0067	0.0068	0.0069	0.0070

4.B.4 Multiple Index Model R^2

Code	Least Squares	ARMA	GARCH	ARMA & GARCH
AGL	0.6804	0.6947	0.6778	0.6947
BVT	0.47.8	0.4738	0.4738	0.4738
FSR	0.4777	0.4777	0.4663	0.4663
GFI	0.3793	0.3793	0.3793	0.3793
PIK	0.2647	0.3905	0.2386	0.3632
PPC	0.2108	0.2931	0.1950	0.2795
SAB	0.5296	0.5665	0.5278	0.5612
SBK	0.5377	0.5691	0.5134	0.5463
SOL	0.4216	0.4216	0.4200	0.4200
SUI	0.2507	0.2725	0.2473	0.2620
TBS	0.3852	0.4583	0.3835	0.4583
SNT	0.3959	0.4248	0.3959	0.4248
AFX	0.2585	0.3373	0.2567	0.3329
ILV	0.2849	0.3088	0.2849	0.3088
SHP	0.2526	0.2734	0.2499	0.2661

Chapter 5
The Effects of Dividends on the Efficient Frontier

5.1 Introduction

The foundation of the Markowitz portfolio theory and the index models, is mean-variance optimisation. We model the risk of the portfolio as variance and maximize the expected return with respect to this risk (variance). Up until this point we have modelled share returns as monthly incremental (logarithmic) changes in the share price. But what about dividends? Do dividends also constitute the return of a share? Yes, but a very small part of the return. Can they be ignored? In this chapter we examine the effects of dividends on the efficient frontier and portfolio selection, using the models developed in previous chapters.

5.2 Study Objectives

To determine the effects of dividends on the efficient frontier and portfolio selection.

Table 5.1: Average Dividends Yields per Share

Share Name	Average Dividend Yield (% pa)
Illovo	5.15 %
PPC	5.05 %
Santam	3.99 %
Sasol	3.59 %
Pick 'n Pay	3.56 %
Sun International	3.49 %
Afrox	3.48 %
Tigerbrands	3.11 %
Standard Bank	2.90 %
Anglo American	2.80 %
SAB Miller	2.80 %
First Rand	2.66 %
Gold Fields	2.59 %
Bidvest	2.37 %
Shoprite	2.28 %

5.3 Data

We use the datasets *"singleret"* and *"multiret"*. Monthly dividend yields (quoted as % per annum) for the shares are obtained from INET Bridge for the 140 month period 30 June 1995 to 28 February 2007. Table 5.1 shows the average dividends yields for this period and the shares are ranked from highest to lowest dividend yield

5.4 Methodology

Share returns are adjusted for dividends using the following formulation, when the timing of dividends is unknown. (G.D.I Barr)

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) + \frac{DY_t}{1200} \tag{5.1}$$

where

r_t is the return of the share at time t.

P_t is the price of the share at time t.

DY_t is the dividend yield (% p.a) at time t.

The single index model dataset with dividends shall by refered to as *"divretsingle"*.The multiple index model dataset with dividends shall be refered to as *"divretmulti"*. Dividend yields have not been added to the indices (explanatory variables) as indices on the JSE do not pay out dividends (Troskie). In our empirical study returns with dividends shall be refered to as *"dividends"* and returns without dividends as *"returns"*. We follow the procedures outlined in Chapters 3 and 4 to fit the Markowitz model and Troskie-Hossain index models to our datasets with dividends. We repeat the analysis of Chapter 5, modelling time series errors and heteroskedasticity using the dividends datasets.

5.5 The Markowitz Portfolio

Figure 5.1 illustrates the effect of including dividends in the calculation of share returns. As is evident from the graph, including dividend yields in the share returns has shifted the Markowitz efficient frontier upwards. Professor Troskie has always considered the dividend a constant. His initial expectation was that adding dividend yields to the returns would change the mean and as such shift the efficient frontier slightly above that of the returns portfolio. However the amount of upward shift is quite astonishing. Professor Troskie considers this of major significance and a new insight into portfolio theory.

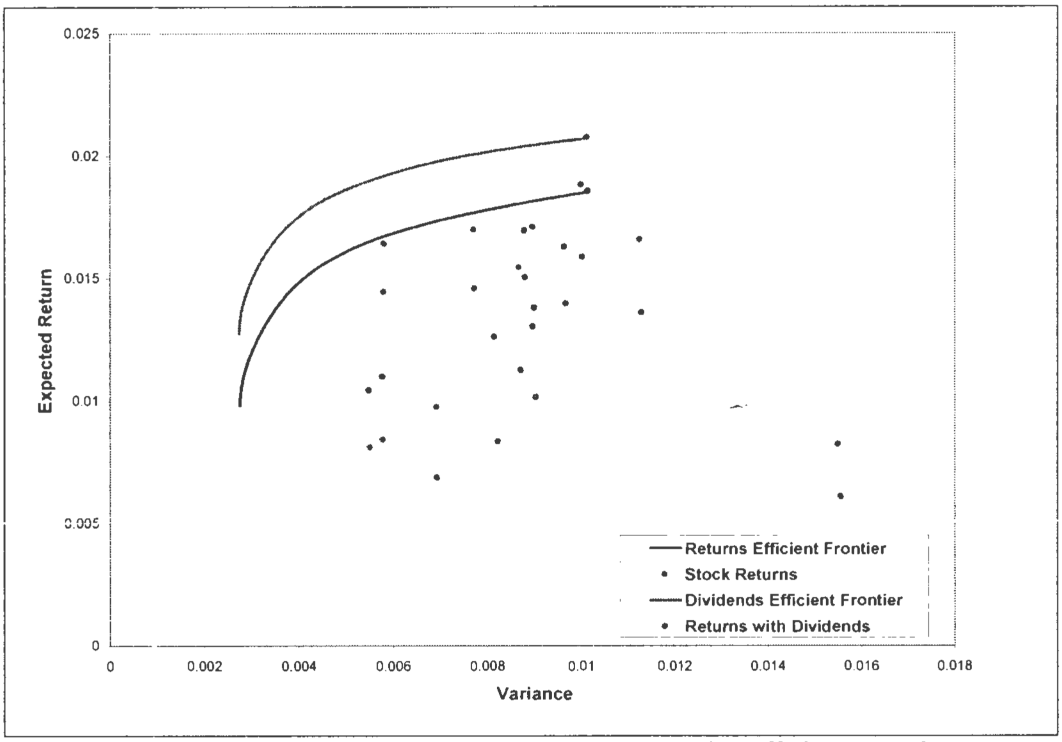

Figure 5.1: The Effect of Dividends on the Markowitz Efficient Frontier

Table 5.2: Markowitz Optimal Portfolio's for Returns and Dividends Datasets

Share Name	Average Dividend Yield (%pa)	Percent in Portfolio	
		Returns	Dividends
Anglo American	2.80 %	7.87 %	3.14 %
Bidvest	2.37 %	8.28 %	2.70 %
First Rand	2.66 %	35.96 %	25.88 %
Pick 'n Pay	3.56 %	12.07 %	11.09 %
Pretoria Portland Cement	5.05 %	1.99 %	15.71 %
Sasol	3.59 %	17.02 %	20.38 %
Sun International	3.49 %	-	0.04 %
Santam	3.99 %	-	6.50 %
Shoprite	2.28 %	16.82 %	14.57 %
Expected Return (%p.a)		19.10 %	21.32 %
Portfolio Variance (%p.a)		5.78 %	5.00 %

We can see that the composition and weighting of shares in the Markowitz optimal portfolio has changed quite significantly with the inclusion of dividend yields. Shares such as Santam and Sun International which did not feature in the returns optimal portfolio, are in the dividends optimal portfolio. The weighting of PPC (which has a large dividend yield) has increased significantly in our new portfolio, and shares such as Anglo American and First Rand (with smaller dividend yields) have been down weighted.

5.6 Single Index Model

From Figure 5.2 and Table 5.3 on page 79 we can see that including dividend yields has shifted the single index model efficient frontier quite markedly and changed the composition and weightings of the optimal portfolio.

Table 5.3: Single Index: Optimal Portfolio's for Returns and Dividends Datasets

Share Name	Returns	Dividends
Anglo American	8.03 %	3.30 %
Bidvest	8.36 %	2.74 %
First Rand	35.84 %	25.85 %
Pick 'n Pay	12.00 %	11.05 %
Pretoria Portland Cement	2.02 %	15.63 %
Sasol	17.00 %	20.35 %
Sun International	-	0.07 %
Santam	-	6.50 %
Shoprite	16.74 %	14.50 %
Expected Return (%p.a)	19.09 %	21.32 %
Portfolio Variance (%p.a)	5.77 %	5.00 %

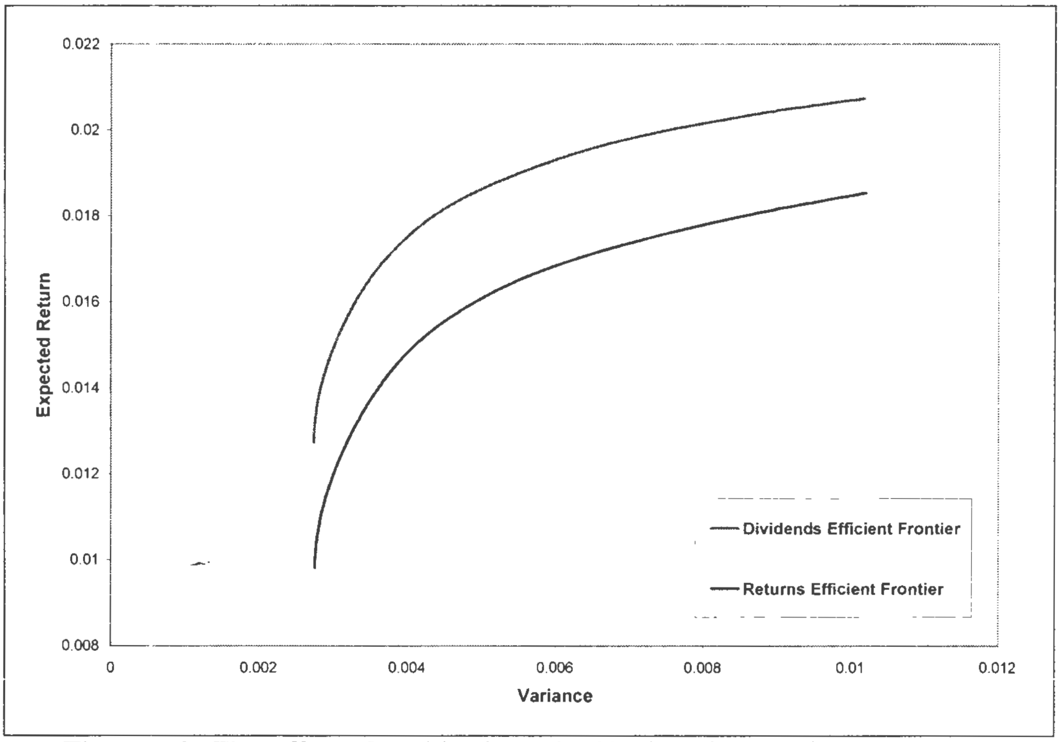

Figure 5.2: The Effect of Dividends on the Single Index Efficient Frontier

Table 5.4: Single Index: Alpha's and Beta's for Returns and Dividends Datasets

	Returns		Dividends		Returns		Dividends	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat
AGL	-0.0014	-0.29	0.0019	0.38	1.2885	15.66	1.2850	15.63
BVT	0.0058	1.20	0.0074	1.47	0.8066	9.95	0.8077	9.96
FSR	0.0078	1.23	0.0092	1.36	1.0351	9.47	1.0353	9.49
GFI	-0.0008	-0.09	-0.0017	-0.18	0.8917	5.70	0.8890	5.69
PIK	0.0080	1.06	0.0106	1.38	0.7328	5.87	0.7316	5.87
PPC	0.0076	1.04	0.0097	1.28	0.5193	4.24	0.5162	4.22
SAB	-0.0018	-0.41	0.0006	0.13	0.8839	12.42	0.8815	12.41
SBK	0.0040	0.76	0.0063	1.11	0.9610	10.53	0.9602	10.53
SOL	-0.0006	-0.08	0.0044	0.62	1.0887	9.42	1.0864	9.42
SUI	0.0013	0.18	0.0048	0.67	0.7379	6.32	0.7339	6.31
TBS	0.0033	0.63	0.0033	0.61	0.6848	7.73	0.6840	7.72
SNT	0.0053	0.82	0.0069	1.04	0.9178	8.60	0.9151	8.59
AFX	0.0006	0.09	0.0023	0.37	0.6637	6.54	0.6618	6.53
ILV	-0.0001	-0.02	0.0039	0.59	0.7813	7.26	0.7767	7.25
SHP	0.0111	1.56	0.0101	1.37	0.6074	5.05	0.6074	5.06

On further examining the β's we see that including dividends in our return data has had for all practical purposes no impact on the β's or β t-statistics. The α's on the hand have increased. Although the α's are only a constant in the regression, there is an area of portfolio management known as "Active Management" based on the interpretation of α.

5.6.1 Time Series Errors and Conditional Heteroskedastic Models

We use the Troskie-Hossain single model with parameters estimated using least squares, as a base for which to compare our other models. This model will be refered to as the Least Squares Model.

Table 5.5: Single Index: Optimal Portfolio's for Returns Dataset

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	8.03 %	4.79 %	4.60 %	6.04 %
Bidvest	8.36 %	6.36 %	2.55 %	1.36 %
First Rand	35.84 %	36.86 %	36.85 %	40.14 %
Pick 'n Pay	12.00 %	12.86 %	15.81 %	22.88 %
Pretoria Portland Cement	2.02 %	6.36 %	-	-
Sasol	17.00 %	17.56 %	18.07 %	17.58 %
Santam	-	-	5.68 %	-
Shoprite	16.74 %	15.21 %	16.44 %	12.00 %
Expected Return (%p.a)	19.09 %	19.00 %	19.25 %	19.54 %
Portfolio Variance (%p.a)	5.77 %	5.68 %	5.56 %	5.80 %

Table 5.6: Single Index: Optimal Portfolio's for Dividends Dataset

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	3.30 %	1.63 %	0.86 %	1.91 %
Bidvest	2.74 %	1.50 %	-	-
First Rand	25.85 %	27.26 %	24.89 %	29.58 %
Pick 'n Pay	11.05 %	12.63 %	16.29 %	23.46 %
Pretoria Portland Cement	15.63 %	20.64 %	6.73 %	9.66 %
Standard Bank			2.10 %	
Sasol	20.35 %	19.90 %	20.80 %	20.45 %
Sun International	0.07 %	-	-	4.37 %
Santam	6.50 %	2.22 %	14.22 %	1.16 %
Shoprite	14.50 %	12.99 %	14.11 %	9.41 %
Expected Return (%p.a)	21.32 %	21.23 %	21.61 %	21.70 %
Portfolio Variance (%p.a)	5.00 %	4.89 %	4.94 %	5.01 %

Effects of Serial Correlation

We fit ARMA models independently to the returns and dividends datasets. For any share from the returns and dividends datasets, the correlograms of the residuals are very similar. Both exhibit serial correlation at the same lags, and the coefficients of the ARMA terms and t-statistics are almost the same.

Table 5.7: Single Index: ARMA Models for Returns Dataset

Code	AR term	Coefficient	t-statistic	AR term	Coefficient	t-statistic	Comments
AGL	AR(2)	-0.1831	-2.12	-	-	-	
BVT							No significant autocorrelation
FSR							No significant autocorrelation
GFI							No significant autocorrelation
PIK	AR(1)	-0.3138	-3.83	AR(5)	-0.1653	-2.05	
PPC	AR(2)	-0.2348	-2.78	AR(6)	0.2520	3.02	
SAB	AR(1)	-0.2180	-2.59	AR(4)	-0.1710	-2.04	
SBK	AR(5)	-0.1587	-1.87	AR(8)	-0.1459	-1.72	
SOL							No significant autocorrelation
SUI	AR(8)	-0.1630	-1.91	-	-	-	
TBS	AR(5)	-0.2142	-2.58	AR(6)	-0.2146	-2.54	
SNT	AR(1)	-0.1451	-1.65	-	-	-	
AFX	AR(1)	-0.2217	-2.60	-	-	-	
ILV	AR(2)	-0.1886	-2.22	-	-	-	
SHP	AR(1)	-0.1354	-1.59	-	-	-	

Table 5.8: Single Index: ARMA Models for Dividends Dataset

Code	AR term	Coefficient	t-statistic	AR term	Coefficient	t-statistic	Comments
AGL	AR(2)	-0.1816	-2.10				
BVT							No significant autocorrelation
FSR							No significant autocorrelation
GFI							No significant autocorrelation
PIK	AR(1)	-0.3135	-3.82	AR(5)	-0.1659	-2.06	
PPC	AR(2)	-0.2342	-2.77	AR(6)	0.2525	3.03	
SAB	AR(1)	-0.2180	-2.59	AR(4)	-0.1732	-2.07	
SBK	AR(5)	-0.1601	-1.89	AR(8)	-0.1466	-1.73	
SOL							No significant autocorrelation
SUI	AR(8)	-0.1678	-1.97				
TBS	AR(5)	-0.2143	-2.58	AR(6)	-0.2142	-2.54	
SNT	AR(1)	-0.1451	-1.65				
AFX	AR(1)	-0.2198	-2.58				
ILV	AR(2)	-0.1935	-2.28				
SHP	AR(1)	-0.1365	-1.60				

From Table 5.7 and Table 5.8 it is evident that including dividend yields in our return data has had no impact on the ARMA models. This can be expected as adding dividend yields to returns, changes the mean (α) and not the β of the share. We have established the relationship for the returns dataset for the least squares and ARMA single index efficient frontiers. From Figure 5.3 we see that the dividends efficient frontiers for least squares and ARMA follow a very similar relationship. However including dividends in our returns data has shifted our efficient frontier up.

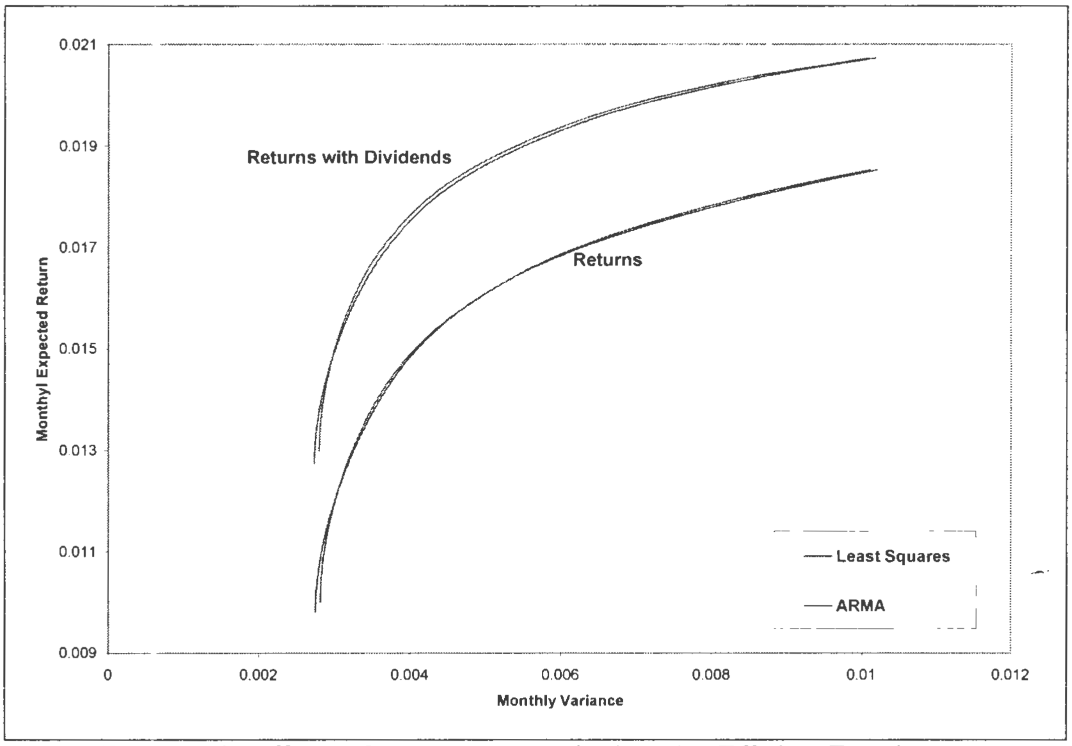

Figure 5.3: Effects of ARMA on the Single Index Efficient Frontier

Effects of Heteroskedasticity

From Tables 5.9 and 5.10 on page 85 we can see that for any share from the returns and dividends datasets, that the coefficients and t-statistics of the GARCH models are for all practical purposes the same. It is evident that including dividend yields in our return data does not impact our GARCH models. This outcome makes sense as adding dividend yields to share returns does not change the volatility. From Figure 5.4 below we see that the dividends efficient frontiers for least squares and GARCH follow a very similar relationship to those of the returns efficient frontiers for least squares and GARCH.

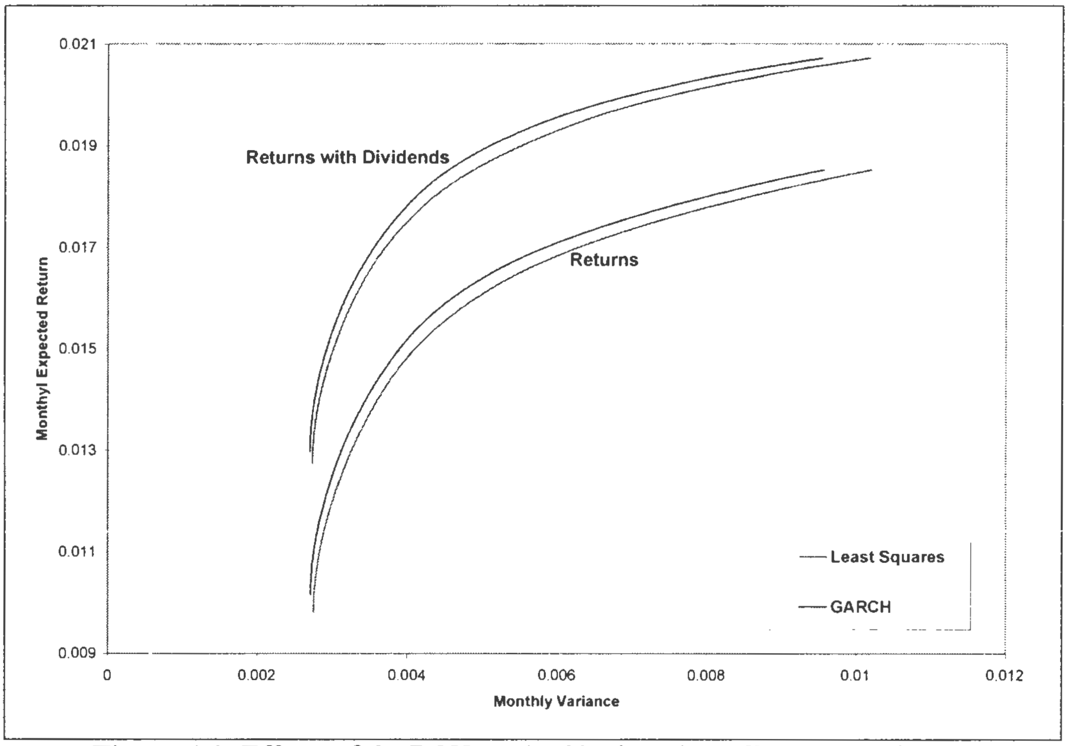

Figure 5.4: Effects of GARCH on the Single Index Efficient Frontier

Table 5.9: Single Index: GARCH Models for Returns Dataset

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL	0.0002	1.57	0.0831	1.54	0.8688	11.13	
BVT							Autocorrelation. Model inadequate
FSR	0.0001	0.36	0.0846	1.82	0.9026	15.99	
GFI							No Significant autocorrelation
PIK	0.0003	1.41	0.1626	4.06	0.8143	16.40	
PPC	0.0051	4.65	0.3209	1.95	-	-	
SAB	0.0018	5.13	0.3315	1.76	-	-	
SBK	0.0009	1.62	0.3105	1.99	0.4577	2.12	
SOL	0.0000	0.10	0.0423	0.97	0.9483	11.45	
SUI	0.0003	0.63	0.0595	1.04	0.8960	8.18	
TBS	0.0002	0.60	0.0446	0.99	0.9103	8.91	
SNT	0.0018	1.52	0.2017	1.58	0.5029	1.99	
AFX	0.0005	1.70	0.0910	1.56	0.8219	8.87	
ILV							No Significant autocorrelation
SHP	0.0001	0.57	0.1015	1.60	0.8826	12.23	

Table 5.10: Single Index: GARCH Models for Dividends Dataset

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL	0.0002	1.57	0.0840	1.54	0.8685	11.10	
BVT							Autocorrelation. Model inadequate
FSR	0.0001	0.36	0.0844	1.83	0.9030	16.08	
GFI							No Significant autocorrelation
PIK	0.0003	1.40	0.1623	4.06	0.8144	16.35	
PPC	0.0051	4.66	0.3192	1.94	-	-	
SAB	0.0018	5.19	0.3301	1.77	-	-	
SBK	0.0009	1.62	0.3079	2.00	0.4619	2.15	
SOL	0.0000	0.11	0.0420	0.96	0.9481	11.31	
SUI	0.0003	0.62	0.0593	1.02	0.8959	8.01	
TBS	0.0002	0.60	0.0446	0.99	0.9104	8.95	
SNT	0.0018	1.53	0.2026	1.58	0.4977	1.95	
AFX	0.0005	1.71	0.0897	1.56	0.8232	8.92	
ILV							No Significant autocorrelation
SHP	0.0001	0.55	0.1016	1.62	0.8835	12.35	

Effects of Time Series Errors and Heteroskedasticity

Including dividend yields in the return data has had no impact on the models of the conditional mean and conditional variance. From Figure 5.5, we can see that the relationships between the Least Squares, ARMA, GARCH and ARMA&GARCH models for returns with dividends follow a very similar relationship to those without dividends.

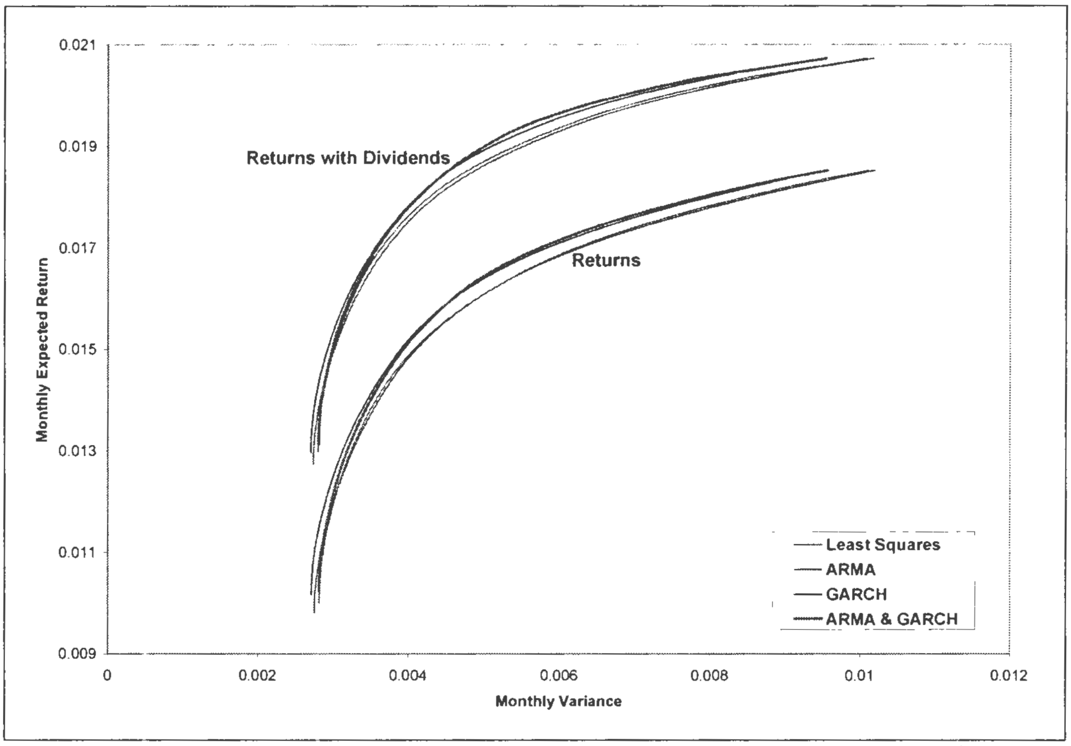

Figure 5.5: Effects of ARMA&GARCH on the Single Index Efficient Frontier

Table 5.11: Single Index: ARMA&GARCH Models for Dividends Dataset

Code	Autocorrelation	Variance Model	Variance Model comments
AGL	AR(2)	-	Evidence of autocorrelation. Model inadequate
BVT	-		No significant autocorrelation.
FSR	-	GARCH(1,1)	Model captures most of the autocorrelation.
GFI	-	-	No significant autocorrelation
PIK	AR(1) , AR(5)	GARCH(1,1)	
PPC	AR(2) , AR(6)	GARCH(1,0)	
SAB	AR(1) , AR(4)	GARCH(1,0)	
SBK	AR(5) , AR(8)	GARCH(1,1)	
SOL	-	GARCH(1,1)	
SUI	AR(8)	GARCH(1,1)	Model reduces autocorrelation significantly.
TBS	AR(5) , AR(6)	-	Model does not reduce autocorrelation
SNT	AR(1)	GARCH(1,1)	
AFX	AR(1)	GARCH(0,1)	
ILV	AR(2)	-	Evidence of autocorrelation. Model inadequate
SHP	AR(1)	-	Evidence of autocorrelation. Model inadequate

Table 5.12: Multiple Index: Optimal Portfolio's

Share Name	Returns	Dividends
Anglo American	8.10 %	3.37 %
Bidvest	8.40 %	2.76 %
First Rand	35.84 %	25.88 %
Pick 'n Pay	11.95 %	11.01 %
Pretoria Portland Cement	2.06 %	15.60 %
Sasol	16.98 %	20.30 %
Sun International	-	0.09 %
Santam	-	6.53 %
Shoprite	16.68 %	14.46 %
Expected Return (%p.a)	19.09 %	21.32 %
Portfolio Variance (%p.a)	5.75 %	4.97 %

5.7 Multiple Index Model

For the Troskie-Hossain Multiple Index Model in Figure 5.6 we can see that including dividend yields in our return data has shifted the efficient frontier up, changing the compositions and weightings of the optimal portfolio's. From Tables 5.13 and 5.14 we can see that the α's for the multiple index model increase with the addition of dividend yields, however this has no impact on the β's. The fact that the α's increase and the β's remain unchanged, implies that the change is in the mean and not the variance.

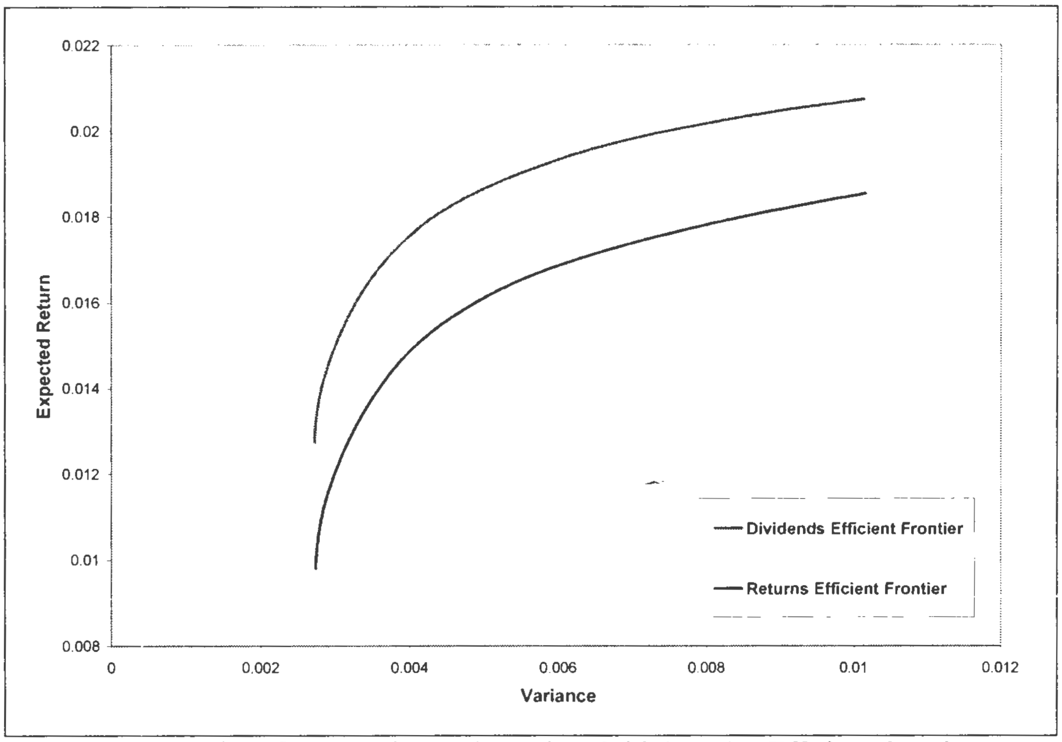

Figure 5.6: The Effect of Dividends on the Multiple Index Efficient Frontier

Table 5.13: Multiple Index: Alpha's and Beta's for Returns Dataset

Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat
AGL	-0.0014	-0.29	1.2006	13.69	0.3497	3.31	0.1035	2.43
BVT	0.0058	1.20	0.9109	10.45	-0.2664	-2.54	-0.1188	-2.80
FSR	0.0078	1.23	1.1938	10.41	-0.4530	-3.28	-0.1821	-3.26
GFI	-0.0008	-0.09	0.4491	2.90	0.1584	0.85	0.4794	6.36
PIK	0.0080	1.06	0.8911	6.58	-0.3420	-2.10	-0.1788	-2.71
PPC	0.0076	1.04	0.6024	4.61	-0.5793	-3.68	-0.1042	-1.64
SAB	-0.0018	-0.41	0.9063	11.28	-0.0329	-0.34	-0.0248	-0.64
SBK	0.0040	0.76	1.1334	12.02	-0.3373	-2.97	-0.1939	-4.23
SOL	-0.0006	-0.08	1.0878	8.54	0.4075	2.66	0.0114	0.18
SUI	0.0013	0.18	0.8575	6.61	-0.0956	-0.61	-0.1309	-2.07
TBS	0.0033	0.63	0.6726	7.15	-0.4862	-4.29	0.0006	0.01
SNT	0.0053	0.82	0.9599	8.26	-0.4310	-3.08	-0.0563	-1.00
AFX	0.0006	0.09	0.6847	6.05	-0.2620	-1.92	-0.0293	-0.53
ILV	-0.0001	-0.02	0.7173	5.93	0.0613	0.42	0.0702	1.19
SHP	0.0111	1.56	0.6411	5.01	-0.6297	-4.09	-0.0524	-0.84

Table 5.14: Multiple Index: Alpha's and Beta's for Dividends Dataset

Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat
AGL	0.0010	0.20	1.1975	13.66	0.3474	3.29	0.1030	2.41
BVT	0.0078	1.61	0.9130	10.50	-0.2727	-2.60	-0.1202	-2.84
FSR	0.0101	1.58	1.1942	10.44	-0.4566	-3.31	-0.1824	-3.28
GFI	0.0014	0.16	0.4463	2.89	0.1587	0.85	0.4796	6.38
PIK	0.0110	1.46	0.8894	6.58	-0.3444	-2.12	-0.1783	-2.71
PPC	0.0118	1.63	0.5984	4.59	-0.5803	-3.69	-0.1033	-1.63
SAB	0.0005	0.12	0.9041	11.27	-0.0349	-0.36	-0.0251	-0.64
SBK	0.0064	1.23	1.1325	12.04	-0.3399	-3.00	-0.1938	-4.23
SOL	0.0024	0.34	1.0857	8.54	0.4048	2.64	0.0111	0.18
SUI	0.0042	0.59	0.8529	6.59	-0.0920	-0.59	-0.1302	-2.07
TBS	0.0059	1.12	0.6719	7.15	-0.4880	-4.31	0.0004	0.01
SNT	0.0086	1.34	0.9576	8.25	-0.4320	-3.09	-0.0568	-1.01
AFX	0.0035	0.55	0.6828	6.04	-0.2644	-1.94	-0.0293	-0.53
ILV	0.0042	0.63	0.7130	5.92	0.0551	0.38	0.0698	1.19
SHP	0.0130	1.83	0.6412	5.02	-0.6329	-4.12	-0.0527	-0.85

5.7.1 Time Series Errors and Conditional Heteroskedastic Models

We use the Troskie-Hossain multiple index model with parameters estimated using least squares, as a base for which to compare our other models. This model shall be refered to as the Least Squares Model.

Table 5.15: Multiple Index: Optimal Portfolio's for Returns Dataset

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	8.10 %	4.80 %	3.55 %	3.88 %
Bidvest	8.40 %	8.30 %	10.26 %	7.03 %
First Rand	35.84 %	36.24 %	34.62 %	34.13 %
Pick 'n Pay	11.95 %	10.12 %	8.89 %	12.95 %
Pretoria Portland Cement	2.06 %	7.15 %	-	-
Standard Bank	-	-	5.49 %	4.42 %
Sasol	16.98 %	17.62 %	19.92 %	19.91 %
Shoprite	16.68 %	15.78 %	17.26 %	18.19 %
Expected Return (%p.a)	19.09 %	18.89 %	19.07 %	19.12 %
Portfolio Variance (%p.a)	5.75 %	5.58 %	5.59 %	5.55 %

Table 5.16: Multiple Index: Optimal Portfolio's for Dividends Dataset

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	3.37 %	1.32 %	-	0.11 %
Bidvest	2.76 %	3.25 %	5.07 %	2.06 %
First Rand	25.88 %	27.06 %	23.78 %	24.36 %
Pick 'n Pay	11.01 %	10.36 %	9.63 %	14.87 %
Pretoria Portland Cement	15.60 %	21.37 %	8.58 %	10.99 %
Standard Bank	-	-	8.40 %	6.08 %
Sasol	20.30 %	20.16 %	22.50 %	21.95 %
Sun International	0.09 %	2.63 %	0.43 %	3.97 %
Santam	6.53 %	-	6.03 %	-
Shoprite	14.46 %	13.85 %	15.58 %	15.61 %
Expected Return (%p.a)	21.32 %	21.14 %	21.31 %	21.25 %
Portfolio Variance (%p.a)	4.97 %	4.77 %	4.97 %	4.82 %

Effects of Serial Correlation

In Tables 5.17 and 5.18 on page 92 we can see that the ARMA models for the dividends dataset and returns dataset are identical. From this observation we can conclude that including dividend yields in the return data has no impact on serial correlation. This can be expected as adding dividend yields to returns, changes the mean (α) and not the β of the share. In Figure 5.7 we can see that the ARMA and least squares efficient frontiers for the dividends dataset follows a similar relationship to those of the returns dataset.

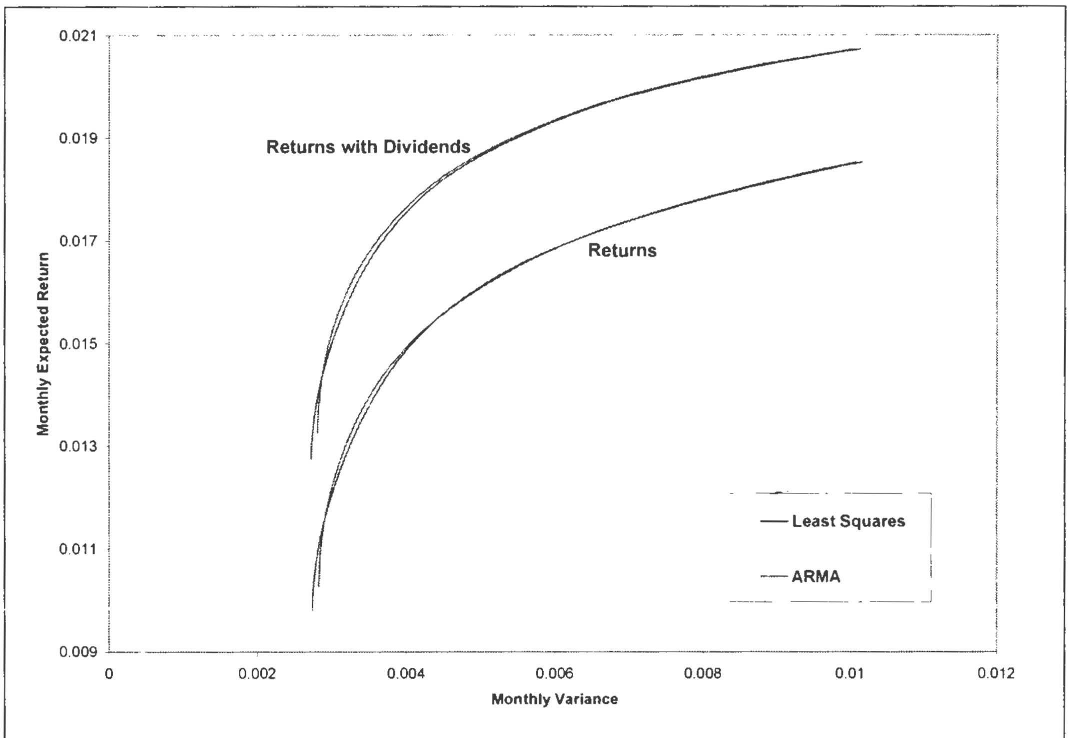

Figure 5.7: The Effects of ARMA on the Multiple Index Efficient Frontier

Table 5.17: Multiple Index: ARMA Models for Returns Dataset

Code	AR term	Coefficient	t-statistic	AR term	Coefficient	t-statistic	AR term	Coefficient	t-statistic
AGL	AR(2)	-0.2138	-2.46						
BVT	No significant autocorrelation								
FSR	No significant autocorrelation								
GFI	No significant autocorrelation								
PIK	AR(1)	-0.4545	-5.12	AR(2)	-0.1567	-1.76			
PPC	AR(2)	-0.2630	-3.01	AR(6)	0.2246	2.66			
SAB	AR(1)	-0.2179	-2.56	AR(4)	-0.1670	-1.97			
SBK	AR(5)	-0.1691	-1.97	AR(8)	-0.1939	-2.27			
SOL	No significant autocorrelation								
SUI	AR(8)	-0.1783	-2.06						
TBS	AR(1)	-0.1825	-2.12	AR(5)	-0.2297	-2.74	AR(6)	-0.2592	-3.03
SNT	AR(1)	-0.2339	-2.07						
AFX	AR(1)	-0.3447	-3.96	AR(2)	-0.1846	-2.13			
ILV	AR(2)	-0.1853	-2.14						
SHP	AR(5)	-0.1671	-1.92						

Table 5.18: Multiple Index: ARMA Models for Dividends Dataset

Code	AR term	Coefficient	t-statistic	AR term	Coefficient	t-statistic	AR term	Coefficient	t-statistic
AGL	AR(2)	-0.2105	-2.42						
BVT	No significant autocorrelation								
FSR	No significant autocorrelation								
GFI	No significant autocorrelation								
PIK	AR(1)	-0.4555	-5.21	AR(2)	-0.1576	-1.77			
PPC	AR(2)	-0.2623	-3.00	AR(6)	0.2254	2.67			
SAB	AR(1)	-0.2184	-2.57	AR(4)	-0.1686	-1.99			
SBK	AR(5)	-0.1723	-2.01	AR(8)	-0.1962	-2.30			
SOL	No significant autocorrelation								
SUI	AR(8)	-0.1820	-2.11						
TBS	AR(1)	-0.1842	-2.14	AR(5)	-0.2319	-2.77	AR(6)	-0.2609	-3.05
SNT	AR(1)	-0.2352	-2.72						
AFX	AR(1)	-0.3437	-3.94	AR(2)	-0.1833	-2.12			
ILV	AR(2)	-0.1906	-2.20						
SHP	AR(5)	-0.1705	-1.95						

Effects of Heteroskedasticity

In Tables 5.19 and 5.20 on page 94 we can see that the coefficients and t-statistics of the GARCH models for the returns and dividends datasets are for all practical purposes the same. It is evident that including dividend yields in our return data does not change our multiple index GARCH models. From Figure 5.8 we see that the multiple index dividends efficient frontiers for least squares and GARCH follows very similar relationship to those of the returns dataset.

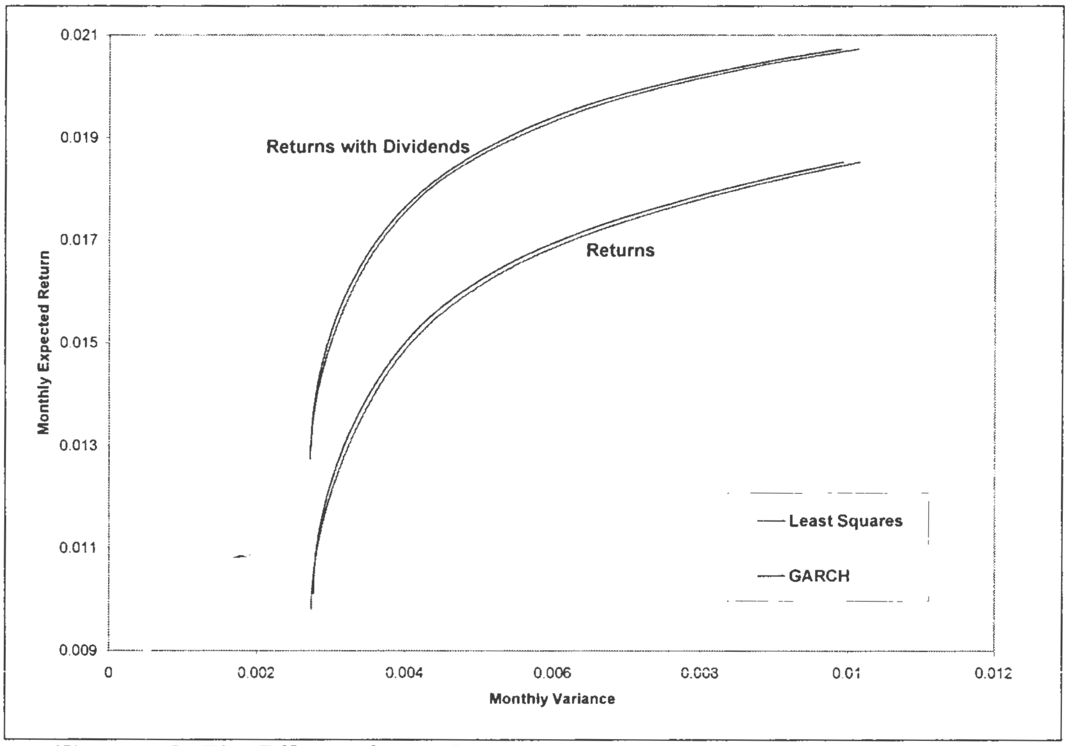

Figure 5.8: The Effects of GARCH on the Multiple Index Efficient Frontier

Table 5.19: Multiple Index: GARCH Models for Returns Dataset

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL	0.0001	1.33	0.1126	1.65	0.8619	11.68	
BVT							Autocorrelation. Model inadequate.
FSR	0.0000	0.00	0.0788	1.62	0.9137	17.67	
GFI							No significant autocorrelation
PIK	0.0001	0.59	0.1439	3.44	0.8536	18.91	
PPC	0.0046	5.33	0.3138	2.23	-	-	
SAB	0.0017	4.64	0.3673	1.82	-	-	
SBK	0.0010	1.42	0.3582	1.60	0.3444	0.97	
SOL	0.0002	0.53	0.0601	1.06	0.9107	9.42	
SUI	0.0001	0.39	0.0623	1.16	0.9149	9.89	
TBS	0.0002	0.44	0.0555	0.77	0.8960	5.19	
SNT							No significant autocorrelation
AFX	0.0004	1.79	0.1056	1.66	0.8116	8.64	
ILV							No significant autocorrelation
SHP	0.0002	0.83	0.1219	1.48	0.8553	9.41	

Table 5.20: Multiple Index: GARCH Models for Dividends Dataset

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL	0.0001	1.33	0.1135	1.66	0.8613	11.65	
BVT							Autocorrelation. Model inadequate.
FSR	0.0000	-0.01	0.0788	1.62	0.9139	17.69	
GFI							No significant autocorrelation
PIK	0.0001	0.60	0.1435	3.45	0.8537	18.87	
PPC	0.0046	5.32	0.3118	2.22	-	-	
SAB	0.0017	4.70	0.3659	1.82	-	-	
SBK	0.0010	1.42	0.3620	1.61	0.3401	0.95	
SOL	0.0002	0.54	0.0601	1.05	0.9102	9.37	
SUI	0.0001	0.38	0.0615	1.15	0.9162	9.89	
TBS	0.0002	0.44	0.0560	0.78	0.8951	5.19	
SNT							No significant autocorrelation
AFX	0.0004	1.80	0.1044	1.66	0.8127	8.70	
ILV							No significant autocorrelation
SHP	0.0002	0.81	0.1215	1.48	0.8563	9.50	

Effects of Time Series Errors and Heteroskedasticity

Including dividend yields in the return data has had no impact on the models of the conditional mean and conditional variance. From Figure 5.9, we can see that the relationships between the Least Squares, ARMA, GARCH and ARMA&GARCH models for returns with dividends follow a very similar relationship to that between returns without dividends.

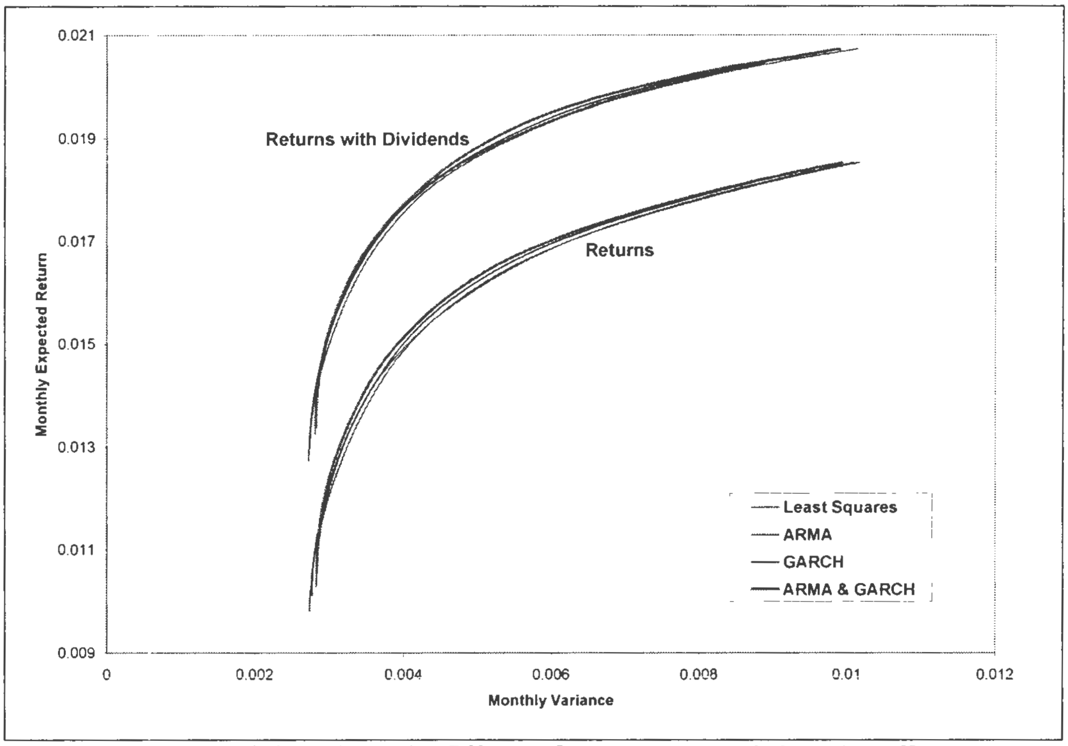

Figure 5.9: Multiple Index: The Effects of ARMA&GARCH on the Efficient Frontier

Table 5.21: Multiple Index: ARMA&GARCH Models for Dividends Dataset

Code	Autocorrelation	Variance	Variance Model Comments
AGL	AR(2)		Model does not capture autocorrelation in residuals.
BVT	-		Autocorrelation. Model inadequate.
FSR	-	GARCH(1,1)	
GFI	-		No significant autocorrelation
PIK	AR(1) , AR(2)	GARCH(1,1)	
PPC	AR(2) , AR(6)	GARCH(1,0)	
SAB	AR(1) , AR(4)	GARCH(1,0)	
SBK	AR(5) , AR(8)	GARCH(1,1)	
SOL	-	GARCH(1,1)	
SUI	AR(8)	GARCH(1,1)	
TBS	AR(1) , AR(5) , AR(6)		Model does not capture autocorrelation in residuals.
SNT	AR(1)		No significant autocorrelation.
AFX	AR(1) , AR(2)	GARCH(1,1)	
ILV	AR(2)		Model does not capture autocorrelation in residuals.
SHP	AR(1)	GARCH(1,1)	

5.8 Conclusions

- Including dividend yields in share returns has shifted the efficient frontier upward, changing both the composition and weightings of the optimal portfolio.
- The β's and β t-statistics remain the same, even with the inclusion of dividend yields.
- The α's and α t-statistics increase with the inclusion of dividend yields.
- The ARMA, GARCH and ARMA&GARCH models fitted to both the dividends and returns datasets for both the single index models and multiple index models are identical. From this observation we can conclude that

including dividend yields in our returns has no impact on the serial correlation or heteroskedasticity of the residuals.

5.A Single Index Model Appendices

5.A.1 Single Index Alpha's for Dividends and Returns Datasets

	Least Squares				ARMA			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat
AGL	-0.0014	-0.29	0.0019	0.38	-0.0004	-0.0948	0.0020	0.45
BVT	0.0058	1.20	0.0074	1.47	0.0054	1.0803	0.0074	1.47
FSR	0.0078	1.23	0.0092	1.36	0.0070	1.0338	0.0092	1.36
GFI	-0.0008	-0.09	-0.0017	-0.18	-0.0039	-0.4004	-0.0017	-0.18
PIK	0.0080	1.06	0.0106	1.38	0.0062	1.2136	0.0092	1.80
PPC	0.0076	1.04	0.0097	1.28	0.0052	0.6930	0.0095	1.26
SAB	-0.0018	-0.41	0.0006	0.13	-0.0028	-0.8664	-0.0004	-0.12
SBK	0.0040	0.76	0.0063	1.11	0.0034	0.7677	0.0058	1.32
SOL	-0.0006	-0.08	0.0044	0.62	0.0014	0.2013	0.0044	0.62
SUI	0.0013	0.18	0.0048	0.67	0.0006	0.0999	0.0035	0.56
TBS	0.0033	0.63	0.0033	0.61	-0.0006	-0.1487	0.0020	0.53
SNT	0.0053	0.82	0.0069	1.04	0.0028	0.4754	0.0061	1.05
AFX	0.0006	0.09	0.0023	0.37	-0.0011	-0.2209	0.0018	0.35
ILV	-0.0001	-0.02	0.0039	0.59	-0.0011	-0.1887	0.0033	0.59
SHP	0.0111	1.56	0.0101	1.37	0.0081	1.2335	0.0100	1.53

	GARCH				ARMA & GARCH			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat
AGL	-0.0036	-0.70	-0.0014	-0.27	-0.0004	-0.09	0.0020	0.45
BVT	0.0054	1.08	0.0074	1.47	0.0054	1.08	0.0074	1.47
FSR	0.0042	0.67	0.0068	1.10	0.0042	0.67	0.0068	1.10
GFI	-0.0039	-0.40	-0.0017	-0.18	-0.0039	-0.40	-0.0017	-0.18
PIK	0.0080	1.16	0.0112	1.61	0.0067	1.36	0.0098	1.99
PPC	0.0038	0.56	0.0081	1.20	0.0036	0.53	0.0081	1.19
SAB	-0.0016	-0.39	0.0007	0.17	-0.0025	-0.69	-0.0002	-0.05
SBK	0.0096	1.95	0.0120	2.45	0.0083	2.03	0.0107	2.64
SOL	0.0007	0.11	0.0037	0.54	0.0007	0.11	0.0037	0.54
SUI	0.0062	0.87	0.0087	1.23	0.0040	0.64	0.0070	1.10
TBS	0.0015	0.27	0.0041	0.73	-0.0006	-0.15	0.0020	0.53
SNT	0.0083	1.20	0.0118	1.70	0.0011	0.18	0.0048	0.84
AFX	0.0030	0.49	0.0059	0.96	0.0022	0.42	0.0052	0.96
ILV	-0.0004	-0.06	0.0039	0.59	-0.0011	-0.19	0.0033	0.59
SHP	0.0112	1.58	0.0133	1.91	0.0081	1.23	0.0100	1.53

5.A.2 Single Index Beta's for Dividends and Returns Datasets

	Least Squares				ARMA			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat
AGL	1.2885	15.66	1.2850	15.63	1.3068	15.87	1.3025	15.82
BVT	0.8066	9.95	0.8077	9.96	0.8066	9.95	0.8077	9.96
FSR	1.0351	9.47	1.0353	9.49	1.0351	9.47	1.0353	9.49
GFI	0.8917	5.70	0.8890	5.69	0.8917	5.70	0.8890	5.69
PIK	0.7328	5.87	0.7316	5.87	0.7939	6.87	0.7926	6.86
PPC	0.5193	4.24	0.5162	4.22	0.4815	4.15	0.4784	4.13
SAB	0.8839	12.42	0.8815	12.41	0.9226	13.35	0.9194	13.33
SBK	0.9610	10.53	0.9602	10.53	0.9864	10.88	0.9864	10.90
SOL	1.0887	9.42	1.0864	9.42	1.0887	9.42	1.0864	9.42
SUI	0.7379	6.32	0.7339	6.31	0.7276	6.34	0.7235	6.33
TBS	0.6848	7.73	0.6840	7.72	0.7736	9.16	0.7734	9.16
SNT	0.9178	8.60	0.9151	8.59	0.9745	8.94	0.9715	8.92
AFX	0.6637	6.54	0.6618	6.53	0.7041	7.09	0.7016	7.06
ILV	0.7813	7.26	0.7767	7.25	0.8097	7.66	0.8047	7.66
SHP	0.6074	5.05	0.6074	5.06	0.6412	5.36	0.6416	5.37

	GARCH				ARMA & GARCH			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat
AGL	1.3570	28.70	1.3535	28.62	1.3068	15.87	1.3025	15.82
BVT	0.8066	9.95	0.8077	9.96	0.8066	9.95	0.8077	9.96
FSR	0.9565	9.89	0.9569	9.95	0.9565	9.89	0.9569	9.95
GFI	0.8917	5.70	0.8890	5.69	0.8917	5.70	0.8890	5.69
PIK	0.6344	4.78	0.6347	4.78	0.6387	4.67	0.6397	4.66
PPC	0.6751	6.39	0.6744	6.40	0.6352	5.24	0.6323	5.24
SAB	0.8997	12.33	0.8971	12.30	0.9188	12.67	0.9163	12.63
SBK	0.8420	11.36	0.8401	11.36	0.8890	12.44	0.8882	12.42
SOL	1.1063	8.52	1.1033	8.50	1.1063	8.52	1.1033	8.50
SUI	0.7172	5.61	0.7162	5.55	0.6945	5.41	0.6931	5.25
TBS	0.7060	7.13	0.7051	7.10	0.7736	9.16	0.7734	9.16
SNT	0.7706	7.35	0.7673	7.33	0.9587	10.59	0.9401	10.19
AFX	0.5958	5.60	0.5954	5.57	0.6295	5.96	0.6283	5.92
ILV	0.7813	7.26	0.7767	7.25	0.8097	7.66	0.8047	7.66
SHP	0.5689	4.41	0.5694	4.41	0.6413	5.36	0.6416	5.37

5.B Multiple Index Appendices

5.B.1 Multiple Index Alpha's for Dividends and Returns Datasets

	Least Squares				ARMA			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat
AGL	-0.0014	-0.29	0.0010	0.20	-0.0016	-0.39	0.0008	0.20
BVT	0.0058	1.20	0.0078	1.61	0.0058	1.20	0.0078	1.61
FSR	0.0078	1.23	0.0101	1.58	0.0078	1.23	0.0101	1.58
GFI	-0.0008	-0.09	0.0014	0.16	-0.0008	-0.09	0.0014	0.16
PIK	0.0080	1.06	0.0110	1.46	0.0072	1.61	0.0102	2.29
PPC	0.0076	1.04	0.0118	1.63	0.0071	1.02	0.0114	1.65
SAB	-0.0018	-0.41	0.0005	0.12	-0.0027	-0.84	-0.0003	-0.10
SBK	0.0040	0.76	0.0064	1.23	0.0034	0.85	0.0058	1.49
SOL	-0.0006	-0.08	0.0024	0.34	-0.0006	-0.08	0.0024	0.34
SUI	0.0013	0.18	0.0042	0.59	0.0002	0.03	0.0031	0.49
TBS	0.0033	0.63	0.0059	1.12	0.0015	0.48	0.0041	1.31
SNT	0.0053	0.82	0.0086	1.34	0.0043	0.82	0.0076	1.46
AFX	0.0006	0.09	0.0035	0.55	-0.0001	-0.02	0.0029	0.71
ILV	-0.0001	-0.02	0.0042	0.63	-0.0006	-0.11	0.0038	0.66
SHP	0.0111	1.56	0.0130	1.83	0.0112	1.78	0.0131	2.10

	GARCH				ARMA & GARCH			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat
AGL	-0.0053	-1.19	-0.0032	-0.71	-0.0016	-0.39	0.0008	0.20
BVT	0.0058	1.20	0.0078	1.61	0.0058	1.20	0.0078	1.61
FSR	0.0015	0.28	0.0042	0.78	0.0015	0.28	0.0042	0.78
GFI	-0.0008	-0.09	0.0014	0.16	-0.0008	-0.09	0.0014	0.16
PIK	0.0023	0.38	0.0055	0.91	0.0028	0.65	0.0060	1.38
PPC	0.0063	0.93	0.0105	1.56	0.0063	1.00	0.0108	1.72
SAB	-0.0013	-0.31	0.0010	0.25	-0.0023	-0.65	0.0000	0.00
SBK	0.0080	1.82	0.0104	2.39	0.0073	2.06	0.0098	2.78
SOL	0.0006	0.08	0.0036	0.54	0.0006	0.08	0.0036	0.54
SUI	0.0052	0.73	0.0075	1.06	0.0048	0.82	0.0067	1.08
TBS	0.0029	0.56	0.0056	1.06	0.0015	0.48	0.0041	1.31
SNT	0.0053	0.82	0.0086	1.34	0.0043	0.82	0.0076	1.46
AFX	0.0034	0.55	0.0064	1.02	0.0027	0.57	0.0058	1.22
ILV	-0.0001	-0.02	0.0042	0.63	-0.0006	-0.11	0.0038	0.66
SHP	0.0117	1.72	0.0138	2.06	0.0099	1.53	0.0124	2.16

5.B.2 Multiple Index Beta's for Dividends and Returns Dataset

	Least Squares				ARMA			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat
AGL	1.2006	13.69	1.1975	13.66	1.2379	14.12	1.2336	14.05
BVT	0.9109	10.45	0.9130	10.50	0.9109	10.45	0.9130	10.50
FSR	1.1938	10.41	1.1942	10.44	1.1938	10.41	1.1942	10.44
GFI	0.4491	2.90	0.4463	2.89	0.4491	2.90	0.4463	2.89
PIK	0.8911	6.58	0.8894	6.58	0.9922	8.53	0.9899	8.52
PPC	0.6024	4.61	0.5984	4.59	0.5632	4.50	0.5590	4.47
SAB	0.9063	11.28	0.9041	11.27	0.9322	11.93	0.9296	11.92
SBK	1.1334	12.02	1.1325	12.04	1.1364	12.68	1.1363	12.73
SOL	1.0878	8.54	1.0857	8.54	1.0878	8.54	1.0857	8.54
SUI	0.8575	6.61	0.8529	6.59	0.8346	6.73	0.8293	6.72
TBS	0.6726	7.15	0.6719	7.15	0.7922	8.89	0.7935	8.92
SNT	0.9599	8.26	0.9576	8.25	1.0651	9.32	1.0631	9.33
AFX	0.6847	6.05	0.6828	6.04	0.7770	7.56	0.7734	7.52
ILV	0.7173	5.93	0.7130	5.92	0.7487	6.25	0.7440	6.25
SHP	0.6411	5.01	0.6412	5.02	0.6540	5.06	0.6549	5.08

	GARCH				ARMA & GARCH			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat	$\hat{\beta}_1$	$\hat{\beta}_1$ t-stat
AGL	1.2599	20.07	1.2578	20.05	1.2379	14.12	1.2336	14.05
BVT	0.9109	10.45	0.9130	10.50	0.9109	10.45	0.9130	10.50
FSR	1.1110	9.78	1.1113	9.84	1.1110	9.78	1.1113	9.84
GFI	0.4491	2.90	0.4463	2.89	0.4491	2.90	0.4463	2.89
PIK	0.8201	6.51	0.8191	6.49	0.8602	6.48	0.8480	6.29
PPC	0.7868	6.44	0.7840	6.42	0.7223	5.35	0.7167	5.34
SAB	0.9014	11.00	0.8988	10.95	0.9288	11.95	0.9260	11.88
SBK	0.9245	11.86	0.9222	11.83	0.9479	12.81	0.9506	12.84
SOL	1.0766	7.08	1.0746	7.06	1.0766	7.08	1.0746	7.06
SUI	0.8724	6.25	0.8711	6.22	0.8716	6.37	0.8603	6.03
TBS	0.7068	8.09	0.7064	8.10	0.7922	8.89	0.7935	8.92
SNT	0.9599	8.26	0.9576	8.25	1.0651	9.32	1.0631	9.33
AFX	0.6414	5.67	0.6413	5.66	0.6996	6.15	0.6971	6.15
ILV	0.7173	5.93	0.7130	5.92	0.7487	6.25	0.7440	6.25
SHP	0.5982	3.70	0.5981	3.71	0.5727	3.98	0.5840	4.38

	Least Squares				ARMA			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat
AGL	0.3497	3.31	0.3474	3.29	0.3669	3.67	0.3637	3.63
BVT	-0.2664	-2.54	-0.2727	-2.60	-0.2664	-2.54	-0.2727	-2.60
FSR	-0.4530	-3.28	-0.4566	-3.31	-0.4530	-3.28	-0.4566	-3.31
GFI	0.1584	0.85	0.1587	0.85	0.1584	0.85	0.1587	0.85
PIK	-0.3420	-2.10	-0.3444	-2.12	-0.3705	-2.71	-0.3752	-2.75
PPC	-0.5793	-3.68	-0.5803	-3.69	-0.4862	-3.47	-0.4875	-3.49
SAB	-0.0329	-0.34	-0.0349	-0.36	-0.0205	-0.22	-0.0236	-0.26
SBK	-0.3373	-2.97	-0.3399	-3.00	-0.3230	-2.98	-0.3270	-3.02
SOL	0.4075	2.66	0.4048	2.64	0.4075	2.66	0.4048	2.64
SUI	-0.0956	-0.61	-0.0920	-0.59	-0.1526	-1.00	-0.1487	-0.98
TBS	-0.4862	-4.29	-0.4880	-4.31	-0.4489	-4.27	-0.4532	-4.33
SNT	-0.4310	-3.08	-0.4320	-3.09	-0.4827	-3.59	-0.4845	-3.61
AFX	-0.2620	-1.92	-0.2644	-1.94	-0.3812	-3.14	-0.3858	-3.18
ILV	0.0613	0.42	0.0551	0.38	0.0271	0.19	0.0180	0.13
SHP	-0.6297	-4.09	-0.6329	-4.12	-0.6578	-4.26	-0.6630	-4.31

	GARCH				ARMA & GARCH			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat	$\hat{\beta}_2$	$\hat{\beta}_2$ t-stat
AGL	0.4016	4.12	0.3996	4.10	0.3669	3.67	0.3637	3.63
BVT	-0.2664	-2.54	-0.2727	-2.60	-0.2664	-2.54	-0.2727	-2.60
FSR	-0.5297	-4.96	-0.5316	-4.98	-0.5297	-4.96	-0.5316	-4.99
GFI	0.1584	0.85	0.1587	0.85	0.1584	0.85	0.1587	0.85
PIK	-0.3842	-3.07	-0.3849	-3.08	-0.3748	-3.22	-0.3795	-3.29
PPC	-0.5074	-3.75	-0.5117	-3.76	-0.4985	-3.62	-0.5043	-3.65
SAB	-0.0872	-1.02	-0.0888	-1.04	-0.0785	-0.94	-0.0810	-0.97
SBK	-0.3986	-4.42	-0.4021	-4.48	-0.3618	-3.67	-0.3674	-3.76
SOL	0.4260	2.36	0.4234	2.35	0.4260	2.36	0.4234	2.35
SUI	-0.0833	-0.51	-0.0802	-0.50	-0.2140	-1.48	-0.1696	-1.08
TBS	-0.4680	-3.91	-0.4701	-3.93	-0.4489	-4.27	-0.4532	-4.33
SNT	-0.4310	-3.08	-0.4320	-3.09	-0.4827	-3.59	-0.4845	-3.61
AFX	-0.2633	-1.76	-0.2661	-1.78	-0.3699	-2.66	-0.3748	-2.70
ILV	0.0613	0.42	0.0551	0.38	0.0271	0.19	0.0180	0.13
SHP	-0.5997	-4.02	-0.6024	-4.06	-0.5698	-4.06	-0.5837	-4.41

	Least Squares				ARMA			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat
AGL	0.1035	2.43	0.1030	2.41	0.0834	1.99	0.0833	1.99
BVT	-0.1188	-2.80	-0.1202	-2.84	-0.1188	-2.80	-0.1202	-2.84
FSR	-0.1821	-3.26	-0.1824	-3.28	-0.1821	-3.26	-0.1824	-3.28
GFI	0.4794	6.36	0.4796	6.38	0.4794	6.36	0.4796	6.38
PIK	-0.1788	-2.71	-0.1783	-2.71	-0.2002	-3.46	-0.1994	-3.45
PPC	-0.1042	-1.64	-0.1033	-1.63	-0.0840	-1.39	-0.0830	-1.38
SAB	-0.0248	-0.64	-0.0251	-0.64	-0.0103	-0.27	-0.0109	-0.29
SBK	-0.1939	-4.23	-0.1938	-4.23	-0.1836	-3.98	-0.1831	-3.98
SOL	0.0114	0.18	0.0111	0.18	0.0114	0.18	0.0111	0.18
SUI	-0.1309	-2.07	-0.1302	-2.07	-0.1367	-2.09	-0.1356	-2.09
TBS	0.0006	0.01	0.0004	0.01	-0.0108	-0.26	-0.0114	-0.27
SNT	-0.0563	-1.00	-0.0568	-1.01	–0.0805	-1.46	-0.0813	-1.48
AFX	-0.0293	-0.53	-0.0293	-0.53	-0.0392	-0.77	-0.0388	-0.76
ILV	0.0702	1.19	0.0698	1.19	0.0642	1.09	0.0637	1.09
SHP	-0.0524	-0.84	-0.0527	-0.85	-0.0537	-0.86	-0.0542	-0.88

	GARCH				ARMA & GARCH			
	Returns		Dividends		Returns		Dividends	
Code	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat	$\hat{\beta}_3$	$\hat{\beta}_3$ t-stat
AGL	0.0877	2.45	0.0870	2.44	0.0834	1.99	0.0833	1.99
BVT	-0.1188	-2.80	-0.1202	-2.84	-0.1188	-2.80	-0.1202	-2.84
FSR	-0.1215	-3.05	-0.1221	-3.06	-0.1215	-3.05	-0.1221	-3.06
GFI	0.4794	6.36	0.4796	6.38	0.4794	6.36	0.4796	6.38
PIK	-0.0496	-0.97	-0.0498	-0.98	-0.0851	-1.60	-0.0787	-1.47
PPC	-0.1028	-1.77	-0.1019	-1.75	-0.0734	-1.19	-0.0723	-1.17
SAB	-0.0080	-0.23	-0.0080	-0.23	-0.0025	-0.07	-0.0027	-0.07
SBK	-0.0910	-2.70	-0.0909	-2.71	-0.0879	-2.30	-0.0889	-2.33
SOL	-0.0181	-0.23	-0.0188	-0.24	-0.0181	-0.23	-0.0188	-0.24
SUI	-0.1635	-2.84	-0.1627	-2.85	-0.1960	-3.36	-0.1863	-3.10
TBS	0.0120	0.24	0.0118	0.23	-0.0108	-0.26	-0.0114	-0.27
SNT	-0.0563	-1.00	-0.0568	-1.01	-0.0805	-1.46	-0.0813	-1.48
AFX	-0.0287	-0.45	-0.0288	-0.45	-0.0226	-0.34	-0.0224	-0.34
ILV	0.0702	1.19	0.0698	1.19	0.0642	1.09	0.0637	1.09
SHP	-0.0113	-0.24	-0.0112	-0.24	-0.0237	-0.48	-0.0271	-0.59

Chapter 6
Portfolio Construction: Simple vs Log Returns

6.1 Introduction

When working with asset returns, statisticians and econometricians prefer log returns due to the powerful underlying statistical theory when it comes to statistical inference. However in practice, more often than not simple returns are used. In this chapter we explore the differences between portfolio's generated using simple returns and log returns utilising the modelling techniques developed in previous chapters.

6.2 Study Objectives

To determine the impact of using simple returns versus log returns in portfolio construction.

6.3 Data

We use the datasets *"singleret"* and *"multiret"*.

6.4 Methodology

We calculate the log returns r_t and simple returns R_t for our datasets as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{6.1}$$

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{6.2}$$

where

r_t is the log return of the share at time t.

R_t is the simple return of the share at time t

P_t is the price of the share at time t

We follow the procedures outlined in Chapters 3 and 4 to fit the Markowitz model and Troskie-Hossain index models to our datasets. We repeat the exercises of chapter 5, modelling time series errors and heteroskedasticity using simple returns. For the optimal portfolio for our efficient frontiers using simple returns we convert the Bankers Acceptance rate $R_{f,pa}$ which is a simple yearly rate to a simple monthly rate $R_{f,m}$. To calculate the optimal portfolio for our efficient frontiers using log returns we convert the Bankers Acceptance Rate $R_{f,pa}$ to a monthly log rate r_f.

$$
\begin{aligned}
R_{f,pa} + 1 &= (R_{f,m} + 1)^{12} && (6.3) \\
r_f &= \ln(R_{f,m} + 1) && (6.4) \\
r_f &= \frac{\ln(R_{f,pa} + 1)}{12} && (6.5)
\end{aligned}
$$

where $R_{f,pa}$ is the simple riskfree rate per annum

$R_{f,m}$ is the simple riskfree rate per month

r_f is the monthly log simple riskfree rate

6.5 Distributional Properties of Log Returns and Simple Returns

Examining the distributional properties of log returns and simple returns, in Tables 6.1 and 6.2 on page 107, we see that for AGL, PIK, SOL and explanatory variables R/$ exchange rate and AngloGold, both the skewness and excess kurtosis of the log returns are lower than those of the corresponding simple returns. However, strangely enough simple returns for 9 of the 15 shares, BVT, FSR, PPC, SAB, SBK, SUI, TBS, SNT, AFX and the explanatory variable ALSI, have lower skewness and lower excess kurtosis than those of the corresponding log returns.

Table 6.1: Distributional Properties of Log Returns

Code	Mean	Std. Deviation	Skewness	Excess Kurtosis	Min.	Max
AGL	0.0139	0.0984	-0.06	1.88	-0.3071	0.3847
BVT	0.0144	0.0763	-0.58	1.08	-0.2894	0.1857
FSR	0.0185	0.1008	-0.57	5.34	-0.5184	0.3203
GFI	0.0061	0.1248	0.085	-0.35	-0.2850	0.3109
PIK	0.0158	0.1002	1.15	7.07	-0.2744	0.5892
PPC	0.0112	0.0934	-0.53	0.74	-0.3177	0.2348
SAB	0.0081	0.0743	-0.58	1.83	-0.2921	0.2054
SBK	0.0146	0.0880	-1.50	12.16	-0.5543	0.3341
SOL	0.0136	0.1063	-0.07	0.62	-0.3483	0.3435
SUI	0.0101	0.0951	-0.47	0.81	-0.3321	0.2735
TBS	0.0084	0.0761	-0.30	1.04	-0.2688	0.2375
SNT	0.0138	0.0949	-0.89	4.87	-0.4893	0.2739
AFX	0.0068	0.0834	-0.38	1.58	-0.2763	0.2676
ILV	0.0083	0.0908	-0.14	2.09	-0.3677	0.2987
SHP	0.0150	0.0939	-0.50	0.9	-0.3104	0.2147
Explanatory Variables						
ALSI	0.0111	0.0611	-1.57	7.95	-0.3548	0.1338
R/$	0.0049	0.0452	0.33	0.8	-0.1087	0.1552
ANG	0.0022	0.1255	-0.10	2.27	-0.5167	0.4658

Table 6.2: Distributional Properties of Simple Returns

Code	Mean	Std. Deviation	Skewness	Excess Kurtosis	Min.	Max
AGL	0.0189	0.1008	0.54	2.67	-0.2644	0.4692
BVT	0.0174	0.0761	-0.29	0.39	-0.2513	0.2041
FSR	0.0238	0.1018	0.32	2.95	-0.4045	0.3776
GFI	0.0139	0.1275	0.39	-0.15	-0.2480	0.3646
PIK	0.0213	0.1107	2.54	16.58	-0.2400	0.8026
PPC	0.0156	0.0928	-0.20	0.33	-0.2722	0.2647
SAB	0.0108	0.0738	-0.21	1.27	-0.2533	0.2280
SBK	0.0184	0.0858	-0.15	6.53	-0.4255	0.3967
SOL	0.0194	0.1085	0.34	0.74	-0.2941	0.4098
SUI	0.0147	0.0947	-0.11	0.5	-0.2826	0.3145
TBS	0.0113	0.0764	0.03	0.87	-0.2357	0.2681
SNT	0.0183	0.0938	-0.15	2.34	-0.3869	0.3151
AFX	0.0103	0.0833	0.04	1.39	-0.2414	0.3068
ILV	0.0125	0.0919	0.38	1.75	-0.3077	0.3481
SHP	0.0195	0.0939	-0.16	0.29	-0.2668	0.2394
Explanatory Variables						
ALSI	0.0130	0.0594	-0.98	4.34	-0.2987	0.1432
R/$	0.0060	0.0459	0.52	1.07	-0.1030	0.1678
ANG	0.0100	0.1277	0.69	2.98	-0.4035	0.5932

6.6 The Markowitz Portfolio

The log returns and simple returns Markowitz efficient frontiers in Figure 6.1 on page 109 are extremely different. The simple returns efficient frontier is higher than that of the log returns efficient frontier. This can easily be explained. It is well known that taking log returns will generally make the distribution more symmetrical and a better approximation to the normal distribution. Taking log returns transforms positive returns to smaller positive returns and negative returns to larger negative returns. This transformation shifts the mean of the share to the left. However the amount of shift is astonishing. Comparing the distributional properties of simple returns to the log returns, we see that the shift is clearly in the mean, as the variances of the shares are similar. Comparing optimal portfolio's in Table 6.3 we see that the composition and weighting of the simple returns and log returns optimal portfolio's are very different.

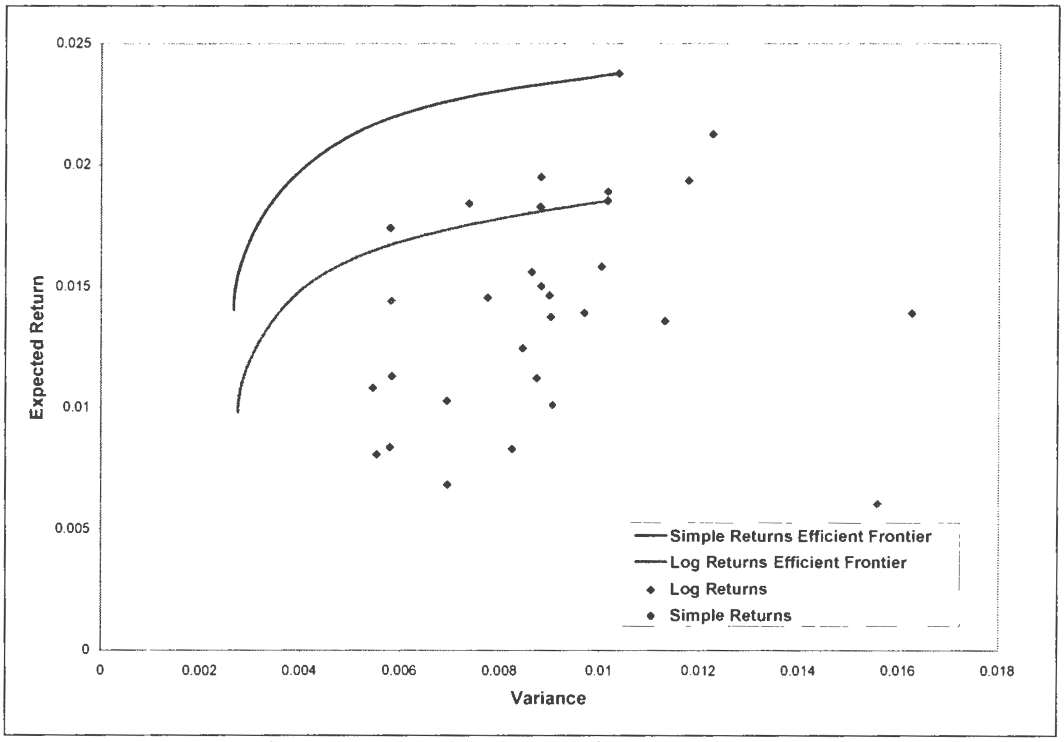

Figure 6.1: Markowitz Efficient Frontiers.

Table 6.3: Markowitz Optimal Portfolio's: Simple Returns vs Log Returns

Share Name	Log Returns	Simple Returns
Anglo American	7.87 %	5.38 %
Bidvest	8.28 %	-
First Rand	35.96 %	27.95 %
GoldFields	-	0.72 %
Pick 'n Pay	12.07 %	6.47 %
Pretoria Portland Cement	1.99 %	8.48 %
Sasol	17.02 %	22.03 %
Sun International	-	3.67 %
Santam	-	5.77 %
Shoprite	16.82 %	19.53 %
Expected Return (%p.a)	19.10 %	24.16 %
Portfolio Variance (%p.a)	5.78 %	5.04 %

6.7 Single Index Model

Figure 6.2 shows the Troskie-Hossain single index efficient frontiers for log returns and simple returns. The simple returns efficient frontier lies above that of the log returns efficient frontier. From Table 6.5 on page 111 we can see that the optimal portfolio's for simple returns and log returns have different compositions and weightings. Examining the α's and β's for the single index model in Table 6.4 on page 111 we see that α's for the simple returns are different to those obtained by using log returns. Although the simple return β's are different to those of log returns, the β values and β t-statistics are in a region of similarity.

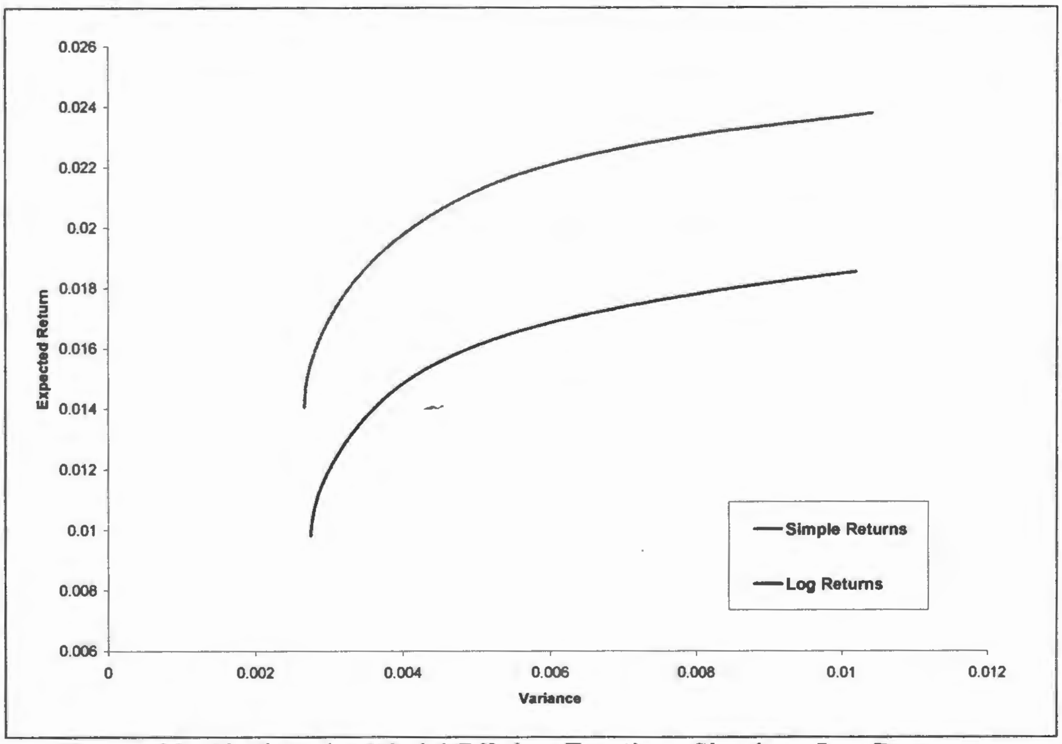

Figure 6.2: Single Index Model Efficient Frontiers: Simple vs Log Returns

Table 6.4: Single Index statistics: Simple Returns vs Log Returns

	Log Returns		**Simple Returns**		**Log Returns**		**Simple Returns**	
Code	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\alpha}$	$\hat{\alpha}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat	$\hat{\beta}$	$\hat{\beta}$ t-stat
AGL	-0.0014	-0.29	0.0015	0.27	1.2885	15.66	1.3387	15.08
BVT	0.0058	1.20	0.0068	1.34	0.8066	9.95	0.8126	9.63
FSR	0.0078	1.23	0.0105	1.48	1.0351	9.47	1.0150	8.63
GFI	-0.0008	-0.09	0.0017	0.17	0.8917	5.70	0.9346	5.68
PIK	0.0080	1.06	0.0107	1.23	0.7328	5.87	0.8121	5.69
PPC	0.0076	1.04	0.0083	1.10	0.5193	4.24	0.5616	4.52
SAB	-0.0018	-0.41	-0.0008	-0.17	0.8839	12.42	0.8898	12.05
SBK	0.0040	0.76	0.0067	1.15	0.9610	10.53	0.9011	9.37
SOL	-0.0006	-0.08	0.0048	0.64	1.0887	9.42	1.1208	9.13
SUI	0.0013	0.18	0.0053	0.72	0.7379	6.32	0.7186	5.93
TBS	0.0033	0.63	0.0022	0.39	0.6848	7.73	0.7014	7.65
SNT	0.0053	0.82	0.0070	1.03	0.9178	8.60	0.8688	7.74
AFX	0.0006	0.09	0.0017	0.27	0.6637	6.54	0.6601	6.27
ILV	-0.0001	-0.02	0.0023	0.34	0.7813	7.26	0.7783	6.83
SHP	0.0111	1.56	0.0117	1.56	0.6074	5.05	0.5961	4.79

Table 6.5: Single Index Optimal Portfolio's: Simple Returns vs Log Returns

Share Name	**Log Returns**	**Simple Returns**
Anglo American	8.03 %	5.52 %
Bidvest	8.36 %	-
First Rand	35.84 %	27.86 %
Goldfields	-	0.77 %
Pick 'n Pay	12.00 %	6.45 %
Pretoria Portland Cement	-	8.47 %
Sasol	17.00 %	21.96 %
Sun International	-	3.74 %
Santam	-	5.77 %
Shoprite	16.74 %	19.45 %
Expected Return (%p.a)	19.09 %	24.14 %
Portfolio Variance (%p.a)	5.77 %	5.04 %

Table 6.6: Single Index Share Beta rankings: Simple Returns vs Log Returns

	Log Returns		**Simple Returns**	
	β Ranking	β	β Ranking	β
1	AGL	1.2885	AGL	1.3387
2	SOL	1.0887	SOL	1.1208
3	FSR	1.0351	FSR	1.015
4	SBK	0.961	GFI	0.9346
5	SNT	0.9178	SBK	0.9011
6	GFI	0.8917	SAB	0.8898
7	SAB	0.8839	SNT	0.8688
8	BVT	0.8066	BVT	0.8126
9	ILV	0.7813	PIK	0.8121
10	SUI	0.7379	ILV	0.7783
11	PIK	0.7328	SUI	0.7186
12	TBS	0.6848	TBS	0.7014
13	AFX	0.6637	AFX	0.6601
14	SHP	0.6074	SHP	0.5961
15	PPC	0.5193	PPC	0.5616

When ranking the shares with respect to β from highest to lowest, we see that due to the slight difference in β obtained using simple returns rather than log returns, share rankings change.

6.7.1 Time Series Errors and Conditional Heteroskedastic Models

We use the Troskie-Hossain single index model with parameters estimated using least squares, as a base for which to compare our other models. This model will be referred to as the Least Squares Model. Tables 6.7 and 6.8 on page 113 summarise the optimal portfolio's for simple returns and log returns for the single index model with time series errors and heteroskedastic models

Table 6.7: Single Index Optimal Portfolio's: Log Returns

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	8.03 %	4.79 %	4.60 %	6.04 %
Bidvest	8.36 %	6.36 %	2.55 %	1.36 %
First Rand	35.84 %	36.86 %	36.85 %	40.14 %
Pick 'n Pay	12.00 %	12.86 %	15.81 %	22.88 %
Pretoria Portland Cement	2.02 %	6.36 %	-	-
Sasol	17.00 %	17.56 %	18.07 %	17.58 %
Santam	-	-	5.68 %	-
Shoprite	16.74 %	15.21 %	16.44 %	12.00 %
Expected Return (%p.a)	19.09 %	19.00 %	19.25 %	19.54 %
Portfolio Variance (%p.a)	5.77 %	5.68 %	5.56 %	5.80 %

Table 6.8: Single Index Optimal Portfolio's: Simple Returns

Share Name	Least Squares	ARMA	GARCH	ARMA & GARCH
Anglo American	5.52 %	2.09 %	2.05 %	4.00 %
First Rand	27.86 %	27.26 %	26.08 %	23.38 %
Goldfields	0.77 %	2.44 %	-	2.61 %
Pick 'n Pay	6.45 %	5.84 %	14.23 %	5.87 %
Pretoria Portland Cement	8.47 %	11.49 %	1.20 %	6.20 %
Standard Bank	-	3.48 %	1.39 %	8.84 %
Sasol	21.96 %	20.56 %	23.61 %	18.82 %
Sun International	3.74 %	5.04 %	4.50 %	6.44 %
Santam	5.77 %	0.52 %	9.96 %	0.26 %
Shoprite	19.45 %	21.28 %	16.98 %	23.58 %
Expected Return (%p.a)	24.14 %	23.82 %	24.50 %	23.71 %
Portfolio Variance (%p.a)	5.04 %	4.53 %	4.90 %	4.53 %

Effects of Serial Correlation

Table 6.9 on page 114 and Table 6.10 on page 115 show the ARMA models fitted to the log returns and simple returns datasets. Some shares such as PIK, PPC and SAB have required the same ARMA models for the simple returns, as in the case of the log returns. The AR coefficients and t-statistics for these shares are similar for both log returns and simple returns. Log returns and simple returns of BVT, GFI and SOL show no significant autocorrelation. Examining the distributional properties of the simple returns and log returns of PIK, PPC, SAB, BVT, GFI and SOL we find no evidence of similarities, except as previously noted that the variances for all shares in the portfolio using simple returns are similar to those using log returns. FSR whose log returns showed no sign of significant autocorrelation requires an AR(8) model when using simple returns. Simple Returns of AFX, SUI and AGL require additional AR terms to those of log returns, to correct for autocorrelation. Overall the simple return models required more and higher order AR terms

Figure 6.3 on page 115 shows the ARMA and least squares efficient frontiers for simple returns and log returns. The ARMA models have had almost no effect on the log returns efficient frontiers and optimal portfolio's. Refer to Tables 6.7 and 6.8 on page 113 for optimal portfolio's. However when using simple returns the ARMA models have shifted the efficient frontier quite significantly to the left and changed the composition and weightings of the optimal portfolio.

Table 6.9: Single Index: ARMA models for Log Returns

Code	AR term	t-statistic	AR term	t-statistic	Comments
AGL	AR(2)	-2.12			
BVT					No significant autocorrelation
FSR					No significant autocorrelation
GFI					No significant autocorrelation
PIK	AR(1)	-3.83	AR(5)	-2.05	
PPC	AR(2)	-2.78	AR(6)	3.02	
SAB	AR(1)	-2.59	AR(4)	-2.04	
SBK	AR(5)	-1.87	AR(8)	-1.72	
SOL					No significant autocorrelation
SUI	AR(8)	-1.91			
TBS	AR(5)	-2.58	AR(6)	-2.54	
SNT	AR(1)	-1.65			
AFX	AR(1)	-2.60			
ILV	AR(2)	-2.22			
SHP	AR(1)	-1.59			

Table 6.10: Single Index: ARMA models for Simple Returns

Code	AR term	t-statistic	AR term	t-statistic	AR term	t-statistic	Comments
AGL	AR(2)	-1.82	AR(6)	-1.97			
BVT							No significant autocorrelation
FSR	AR(8)	2.49					
GFI							No significant autocorrelation
PIK	AR(1)	-3.49	AR(5)	-2.02			
PPC	AR(2)	-2.75	AR(6)	2.94			
SAB	AR(1)	-2.75	AR(4)	-2.25			
SBK	AR(4)	-1.80	AR(5)	-2.18			
SOL							No significant autocorrelation
SUI	AR(7)	1.67	AR(8)	-1.83			
TBS	AR(1)	-1.87	AR(5)	-2.84	AR(6)	-3.01	
SNT	AR(1)	-1.63					
AFX	AR(1)	-3.05	AR(2)	-1.56			
ILV	AR(2)	-2.13					
SHP	AR(11)	-2.28					

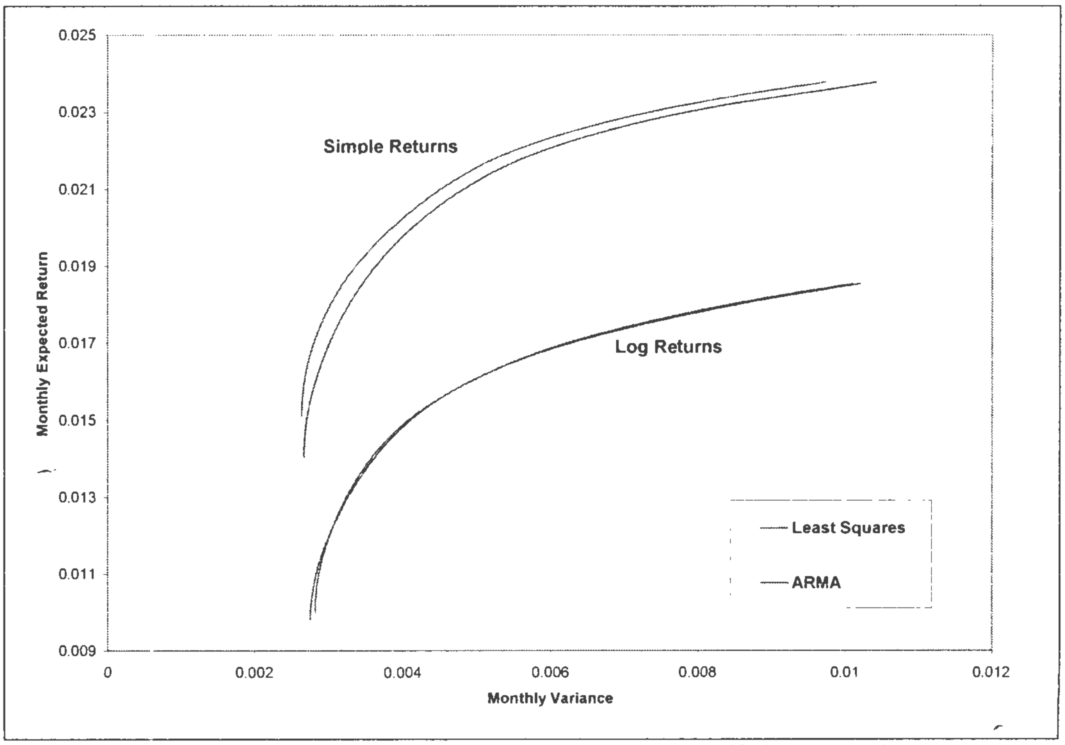

Figure 6.3: Effects of ARMA on the Single Index Efficient Frontier.

Effects of Heteroskedasticity

Table 6.11 and Table 6.12 on page 117 show the single index GARCH models for log returns and simple returns. The squared residuals of GFI exhibit no serial correlation when working with log returns, however this is not the case when working with simple returns. Likewise the squared residuals of PIK and SNT exhibit no serial correlation when working with simple returns, however the log returns exhibit serial correlation. The GARCH models proved inadequate when working with simple returns of AGL and simple and log returns of BVT, as the models did not capture any serial correlation in the residuals. Overall GARCH models provided a good fit to both simple returns and log returns for the single index model. From Figure 6.4 on page 117. we see that the GARCH shifts the simple returns and log returns single index efficient frontiers to the left.

Table 6.11: Single Index: GARCH models for Log Returns

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL	0.0002	1.57	0.0831	1.54	0.8688	11.13	
BVT							Autocorrelation. Model inadequate
FSR	0.0001	0.36	0.0846	1.82	0.9026	15.99	
GFI							No autocorrelation
PIK	0.0003	1.41	0.1626	4.06	0.8143	16.40	
PPC	0.0051	4.65	0.3209	1.95	-	-	
SAB	0.0018	5.13	0.3315	1.76	-	-	
SBK	0.0009	1.62	0.3105	1.99	0.4577	2.12	
SOL	0.0000	0.10	0.0423	0.97	0.9483	11.45	
SUI	0.0003	0.63	0.0595	1.04	0.8960	8.18	
TBS	0.0002	0.60	0.0446	0.99	0.9103	8.91	
SNT	0.0018	1.52	0.2017	1.58	0.5029	1.99	
AFX	0.0005	1.70	0.0910	1.56	0.8219	8.87	
ILV							No autocorrelation
SHP	0.0001	0.57	0.1015	1.60	0.8826	12.23	

Table 6.12: Single Index: GARCH models for Simple Returns

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where $a_t = e_t - u_t$							
Code	α_0	**z-statistic**	α_1	**z-statistic**	β_1	**z-statistic**	**Comments**
AGL							Autocorrelation. Model inadequate
BVT							Autocorrelation. Model inadequate
FSR	0.0000	0.22	0.0717	1.98	0.9194	20.33	
GFI	0.0104	5.35	0.2107	1.66	-	-	
PIK							No autocorrelation
PPC	0.0052	4.77	0.2944	1.79	-	-	
SAB	0.0018	5.05	0.3218	1.78	-	-	
SBK	0.0009	1.61	0.2860	2.03	0.4862	2.25	
SOL	0.0001	0.19	0.0504	1.15	0.9368	11.33	
SUI	0.0004	0.72	0.0671	1.09	0.8714	6.93	
TBS	0.0002	0.56	0.0417	0.89	0.9146	8.52	
SNT							No autocorrelation
AFX	0.0004	1.68	0.0929	1.42	0.8267	8.53	
ILV							No autocorrelation
SHP	0.0002	0.65	0.0973	1.45	0.8764	10.25	

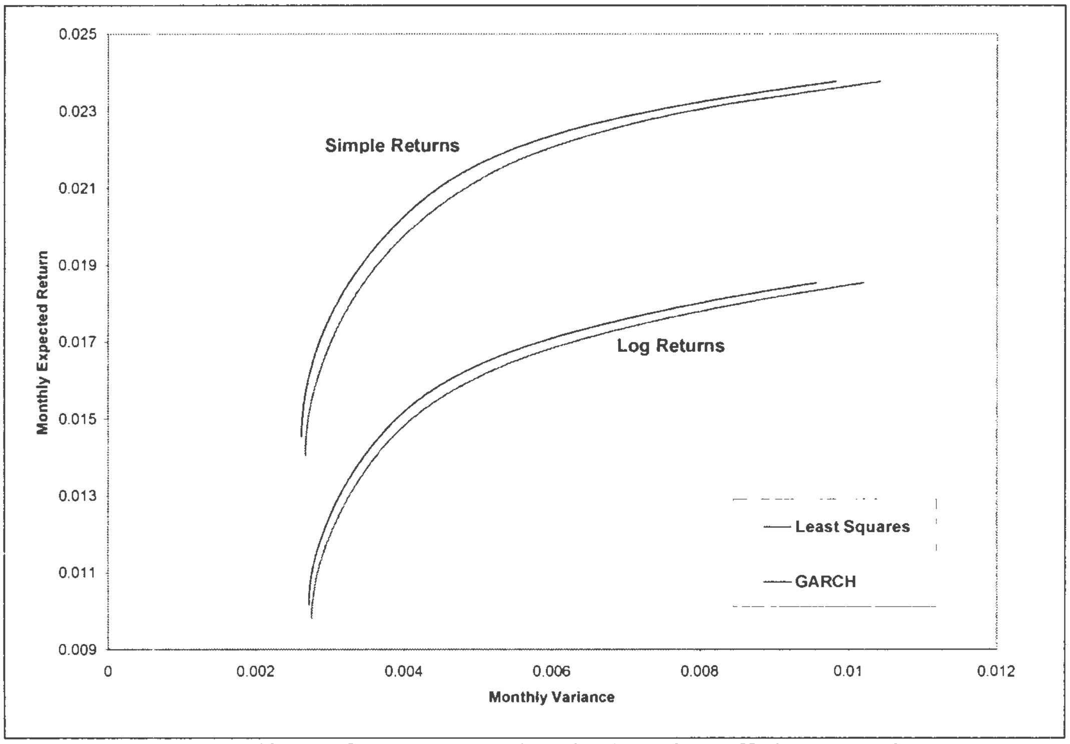

Figure 6.4: Effects of GARCH on the Single Index Efficient Frontier.

Effects of Time Series Errors and Heteroskedasticity

For the ARMA&GARCH models, we have encountered the modelling problems mentioned in previous chapters. Tables 6.13 and 6.14 show the ARMA&GARCH models fitted to log returns and simple returns. The conditional variance model is dependent on the model of the conditional mean. We can see that for a number of cases for both simple returns and log returns the variance models have proved inadequate, as the models did not reduce the serial correlation in the squared residuals. GARCH models for log returns of AGL, TBS, ILV and SHP and simple returns of AGL, BVT, PIK and TBS proved inadequate. Even after fitting GARCH models to simple returns of FSR, SAB and ILV there was evidence of serial correlation in the squared residuals.

Figure 6.5 shows the ARMA&GARCH single index efficient frontiers for simple returns and log returns. For simple returns the ARMA efficient frontier lies very close to the GARCH efficient frontier, with the ARMA efficient frontier bulging higher up at the left tail end. The simple returns ARMA&GARCH efficient frontier follows a similar trend, bulging higher up than both the ARMA and GARCH efficient frontiers at the left tail end, then becoming less steep and running slightly below the ARMA and GARCH efficient frontiers.