SOME CONTRIBUTIONS TO THE ANALYSIS AND CONSTRUCTION OF FUNDS IN SOUTH AFRICA

By

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OVERVIEW

Following international trends, the South African unit trust industry has become one of the fastest growing forms of investment in our financial market. Since the first fund was established in 1965, the industry has grown to over 100 funds with more than 20 companies managing these funds. Since 1990 there has been particularly rapid growth in 'Specialist Equity Funds' with more than 30 new 'specialist' unit trusts emerging. Specialist equity fund managers usually concentrate their investments on a particular sector of the economy or alternately aim to satisfy specific characteristic investment objectives. Two classes of specialist equity funds, namely Index funds and International funds, have emerged recently in our unit trust industry and are receiving increasing attention from the investment community. Much attention therefore is given to these funds in this thesis.

The growing importance of the unit trust industry has heightened the need to effectively and accurately measure the performance of managed funds. A wealth of literature exists in this field and a number of models have been developed to measure the performance of managed funds and the fund managers themselves. This thesis reviews and demonstrates the implementation of these various measures with the emphasis on providing a practical interpretation of each measure.

Although the recent development of Index funds and International funds has received considerable attention in the financial media, little attention has been paid to the technical aspects of the construction of these funds in the academic literature. To the authors knowledge there has been no published research on the construction of Index funds or International funds in South Africa. This thesis examines approaches to constructing Index funds and International funds and empirically assesses these approaches on the Johannesburg Stock Exchange (JSE).

Chapter 1 gives a brief outline of Modern Portfolio Theory. In Chapter 2 an empirical analysis is conducted which gives support for the promotion of investment in local equities via, for example, unit trusts. Chapter 3 reviews the various performance measures and empirically demonstrates their application to a sample of South African Unit Trusts. Chapter 4 examines two approaches to constructing Index funds and empirically tests these methods on the JSE. Chapter 5 focuses on portfolio construction in markets, such as the JSE, where legislation restricts investors from investing abroad. Final conclusions and proposals for future research are put forward in Chapter 6.

Although it is customary to review all the relevant literature at the outset of a thesis, the approach taken here is slightly different. The literature relating to performance measurement, Index funds and International funds is reviewed at the beginning of each respective chapter. Since these are three distinct, albeit related, areas of investigation, it was felt that such an approach would better facilitate the flow of the discussions.

A more detailed summary of the content of each chapter follows :

Chapter 1 Background and Review

This chapter gives a brief outline of Modern Portfolio Theory in order to introduce the concepts and models used and referred to in later chapters. Readers familiar with these ideas may wish to skip to Chapter 2.

Chapter 2 Promotion of Investment in Local Equities

The major challenge facing investors has always been the maximisation of wealth in a world of uncertainty. Investors are concerned about the reward or return earned from an investment and the risk borne in earning that return. This chapter focuses on investment on the JSE, specifically examining the returns earned and the risk borne. The aim of this chapter is to promote investment in local equities. Firstly, the attractiveness of the returns on the JSE relative to alternative investments are highlighted. Secondly, and more importantly, the risk of longer term investment on the JSE is examined. This chapter contends that longer term risk on the JSE is generally lower than perceived and tentatively promotes asset allocation choice in Unit Trusts.

Chapter 3 Performance measurement

The performance of managed funds has attracted considerable attention amongst both practitioners and academics. The focus of this chapter is to review the various performance measures focusing not only on the fund per se but on the manger as well. An emphasis on providing a practical interpretation of each measure is also given in the chapter and the limitations and differences between each measure are highlighted. A short empirical investigation into the performance of 13 South African Unit Trusts is conducted utilising some of these measures. Although the performance of South African Unit Trusts is of interest, the focus in this thesis is not on the relative performance of unit trusts, but rather on the practical interpretations of the various measures.

Chapter 4 Index Fund Construction

This chapter assesses two sampling approaches to index fund construction on the JSE. The two methods, namely stratification and optimisation are empirically tested on the JSE and the results are contrasted with studies conducted on other exchanges. The results reveal that the stratification approach produces superior results on the JSE. Explanations for why this result differs to results on other international exchanges are offered. The suitability of the various statistics employed to measure tracking ability is also considered.

Chapter 5 International Fund Construction

This chapter focuses on portfolio construction in markets where legislation restricts investors from investing in international markets. A construction technique is adopted where international diversification is "mimicked" using local securities. Here an extended market model is used to obtain a more detailed decomposition of the risk of local securities, which allows for a component of foreign market risk to be estimated. In the first part of the chapter the decomposition of the risk of securities on the JSE is empirically demonstrated. In the second part a portfolio construction methodology using an automated technique for searching/sweeping across the JSE for securities having significant foreign risk components is empirically tested. The results confirm there is potential for improving the performance of existing "International" funds on the JSE using more rigorous quantitative approaches such as the one proposed here.

Chapter 6 Conclusion

This chapter summarises the main findings of this thesis and suggests areas for future research.

CONTENTS

CHAPTER 1

BACKGROUND

This chapter presents a brief outline of Modern Portfolio Theory in order to introduce the concepts and models that are referred to in the subsequent chapters of this thesis. Readers familiar with these concepts may wish to skip to Chapter 2.

Modern Portfolio Theory

The major challenge facing investors has always been the maximisation of wealth in a world of uncertainty. Modern work on the study of portfolio selection commenced with the seminal paper by Harry Markowitz (1952). The Markowitz approach commences with the proposition that a rational investor will seek an optimal combination of risk and return from his/her investments; i.e. the higher the level of risk to be accepted, the greater the required expectation of return.

Risk

The risk associated with a security or a portfolio can be thought of as a measure of the uncertainty of the expected return. Numerous quantitative methods of measuring risk have been used, but Markowitz's definition of total risk as the variance of expected return is the most generally used , i.e.

$$\text{Total Risk} = \text{Var (Expected Return)}$$

Markowitz, considering the concept of risk, proposed a linear relationship between the returns on each individual security and the return on the market. This relationship has been extended and developed by numerous researchers into the so called "market model". The market model is a descriptive model relating the returns on a security to the returns on the market.

The Market Model

The model can be briefly summarised as follows:

Symbolically the relationship between the return on the i^{th} security and the return on the market can be written as

$$r_{it} = \alpha_i + \beta_i r_{mt} + e_{it} \qquad (1.1)$$

where

r_{it} is the return on the i^{th} security in the t^{th} period.

α_i and β_i are the parameters unique to security i

r_{mt} is the return on the market in the t^{th} period.

e_{it} is the disturbance or error term for security i and is assumed to have zero expectation and to be independent of all r_{mt} for all t.

The α and β parameters can be estimated using regression analysis procedures. The return on the market is generally computed using some overall market index.

Beta

The β parameter has been used extensively as a measure of risk of the specific security in relation to the market. It can be shown that the ordinary least squares estimate of β_I is

$$\beta_i = \sigma_{im}/\sigma^2_m \qquad (1.2)$$

where β_i is the beta coefficient for share i;

σ_{im} is the covariance between the returns on share i and the market return

σ^2_m is the variance of the return on the market index.

The value of β_i indicates the volatility of security i's rate of return by comparison with the market.

If a security's β is greater than one, then when the market rises, the return on the security will, on average, rise more rapidly than the return on the market. On the other hand, if the market falls, the return on the security will fall more rapidly than the return on the market. However if a security's β is less than one, then in a rising market the security will rise more slowly than the market, and in a falling market will fall less than the market. Securities having β's greater than one are therefore regarded as being more risky than the market. While securities having β's less than one are regarded as being less risky than the market.

Unique and Market risk

Inspection of equation (1.1) shows that the variance of a security's returns stems from two sources:

(i) the variance of the returns on the market index, r_m

(ii) the variance of the random error term e_i

These two elements of risk are commonly known as market or systematic risk and unique, unsystematic or residual risk respectively.

Mathematical expressions for the above two components of risk can be found by considering

$$var(r_i) = var(\alpha_i + \beta_i r_m + e_i) \quad (1.3)$$

which after some simple manipulation can be expressed as

$$\sigma^2_i = \beta^2_i \sigma^2_m + \sigma^2_{ei} \quad (1.4)$$

where

σ^2_i is the variance of the returns on share i

σ^2_m is the variance of returns on the market index

σ^2_{ei} is the residual variance of share i

Verbally, equation (1.4) can be written as

Total risk = Market risk + Unique risk

Brealey and Myers (1991) interpret the two components of total risk as follows. Unique risk stems from the fact that many of the perils that surround an individual company are peculiar to that company and perhaps its immediate competitors. Market risk stems from the fact that there are other economy wide perils which threaten all businesses. That is why stocks have a tendency to 'move together'.

Clearly unique risk is the risk embodied in individual companies, for example, local strikes, bad management and setbacks affecting production, while market risk is the risk of a share which is associated with general economic conditions affecting the market as a whole.

Mathematical expressions for the market and unique risk of a portfolio can be found by considering

$$\sigma^2_p = \mathrm{var}(\Sigma^n_{i=1} X_i r_i) \qquad (1.5)$$

which after some simple manipulation within the context of the market model can be expressed as

$$\sigma^2_p = (\Sigma^n_{i=1} X_i \beta_i)^2 \sigma^2_m + \Sigma^n_{i=1} X_i^2 \sigma^2_{ei} \qquad (1.6)$$

where is σ^2_p the variance of the returns of the portfolio n is the number of shares in the portfolio

X_i is the proportion of the capital invested in share i, $0 \leq X_i \geq 1$

and the other terms have been previously defined.

Verbally equation (1.6) can be expressed as

Total portfolio risk = Market risk of the portfolio + Unique risk of the portfolio

It is argued that in the context of measuring portfolio risk, the market risk of a well diversified portfolio is the only risk that needs to be considered. This argument can be justified by considering equation (1.6). For ease of explanation, assume that an equal proportion of funds is invested in each share, that is the proportion X_i equals 1/n for each of the n securities. Now as n becomes large the value of X_i will become smaller and the value of X^2_i will become even smaller. Since the values of e_i will not generally vary a great deal, the expression for unique risk in equation (1.6) becomes negligible. Hence the market risk becomes dominant. Clearly the beta coefficient is central to the computation of market risk (as can be seen from equations 1.4 and 1.6).

Empirical studies have found evidence that diversification can provide a substantial reduction in variability. Brealey and Myers (1991) argue that most of the benefit in risk reduction can be achieved by relatively few shares and that the improvement in risk reduction is slight when the number of securities is increased to more than twenty.

Markowitz's Portfolio Selection Model

Markowitz went on to work out the basic principles of portfolio construction. He developed his portfolio selection model based on the following assumptions:

1. Investors have probability distributions about the future performance of shares.
2. These probability distributions have finite means and variances.
3. There are decreasing returns to risk bearing beyond some point.
4. An individual's preferences are a function of portfolio return and variance only.
5. For any given expected return on a portfolio, the portfolio with the smallest variance is preferred to all others, and for any given portfolio variance, the portfolio with the maximum expected return is preferred to all others.

Assumption 5, which Markowitz called the mean-variance criterion, was a significant insight. The problem was to find a portfolio with the lowest possible portfolio variance subject to a given level of portfolio return. Figure 1.1 below graphically depicts Markowitz's mean-variance theory.

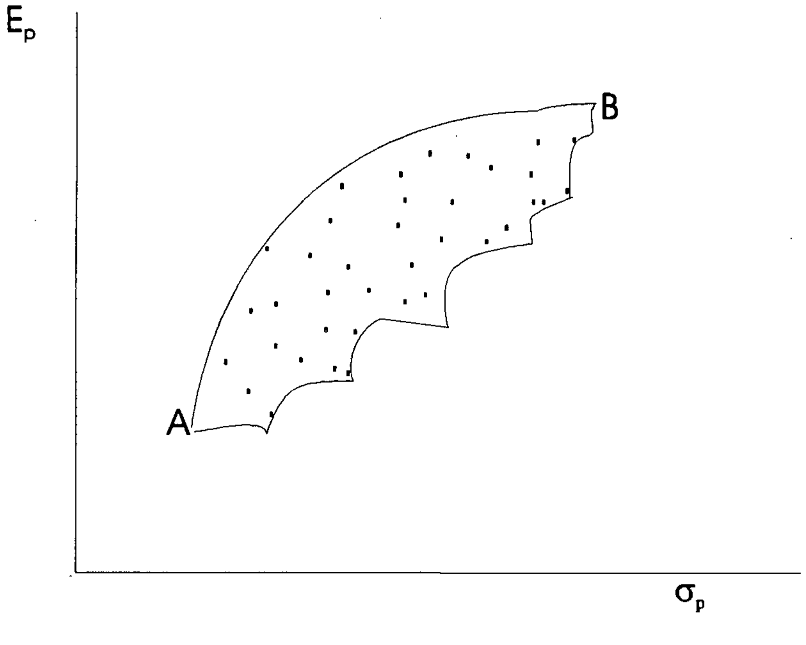

FIGURE 1.1 Graphical depiction of Markowitz's mean-variance theory

The horizontal axis shows the standard deviation of returns, while the vertical axis shows expected portfolio returns. The shaded area shows the feasible region of all the different possible combinations of risk and return that one could attain from investing in risky shares. However, only those portfolios lying on the curve AB represent the set of mean-variance efficient portfolios. AB is known as the efficient frontier. A portfolio is efficient if it is impossible to find a portfolio which has a greater expected return without incurring increased risk, while at the same time, one cannot achieve a smaller level of risk without decreasing one's return. To accommodate all investor's preferences, the entire set of efficient portfolios must be drawn, resulting in the efficient frontier AB. To maximise their expected utility an investor would select the portfolio at the point of tangency between the efficient frontier and the highest indifference curve. Note that an indifference curve captures an investors attitude to the trade-off between risk and return. Consequently indifference curves differ for different individuals; as individuals typically have different attitudes towards risk-return trade-offs.

The introduction of a risk-free asset greatly simplifies the portfolio selection problem. All combinations of a risky asset and the risk-free asset are found on the straight line between the risk-free asset and the risky asset. Portfolios along any line from the risk-free asset to each risky asset are possible. The tangent from the risk-free asset to the efficient frontier however, dominates all other lines. Figure 1.2 shows that if investors can borrow or lend to the market at the same rate, R_f, then all investors would invest their funds at risk in portfolio S (the point of tangency from the risk-free asset to the efficient frontier).

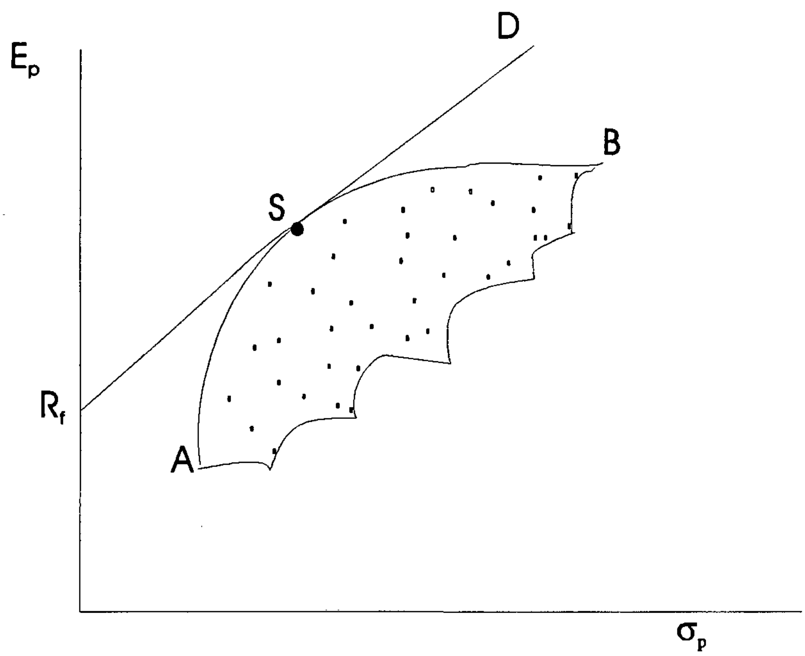

FIGURE 1.2 Graphical depiction of the Separation Theorem

Thus all an investor needs to know is the combination of assets that makes up portfolio S in Figure 1.2 as well as the return on the risk-free asset. This concept is known as the Separation Theorem.

If an investor is willing to lend a portion of his/her funds to the market he/she would invest the remainder of the funds in portfolio S and his/her preference would be located along the R_fS segment. An investor who borrows and invests the total in S would have his/her preference located on the SD segment depending on the proportion of funds borrowed.

Portfolio S, called the optimal combination of risky securities, is thus extremely important and under the above assumptions the optimal investment choice for all investors should be to identify portfolio S and combine this with a degree of borrowing from or lending to the market. Thus the individual investor need only consider how much he/she should borrow or lend in accordance with where his preference would be located along R_fSD.

It is argued that under market equilibrium S would consist of all securities in the market and the proportion of each security must equal its proportionate value in the market as a whole. Hence portfolio S is often called the 'market' portfolio.

The Capital Asset Pricing Model

Although Markowitz's portfolio selection model theorised asset *choice* under conditions of risk, it did not directly consider how individual assets are actually *priced* in the market place. However, subsequent to the Markowitz portfolio selection model being completed, much work was written on the pricing of assets, culminating in the Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM).

As in all financial theories, a number of assumptions were made in the development of the CAPM. These assumptions as outlined in Copeland and Weston (1992) are summarised below:

1. Investors are risk-averse individuals who maximise the expected utility of their end-of-period wealth.
2. Investors are price takers and have homogenous expectations about asset returns that have a joint normal distribution.
3. There exists a risk-free asset such that investors may borrow or lend unlimited amounts at the risk-free rate.
4. The quantities of assets are fixed. Also, all assets are marketable and perfectly divisible.
5. Asset markets are frictionless and information is costless and simultaneously available to all investors.

6. There are no market imperfections such as taxes, regulations, or restrictions on short selling.

The CAPM is a simple linear model that is expressed in terms of expected returns and expected risk. In its *ex ante* form the model can be written as

$$E(R_i) = R_f + \beta_i \, [E(R_m) - R_f] \qquad (1.7)$$

where $E(R_i)$ is the expected return on the ith security

$E(R_m)$ is the expected return on the market of all assets

R_f is the risk free rate

β_i is the beta of the ith security

Equation 1.7, also known as the Security Market Line, is shown graphically in Figure 1.3.

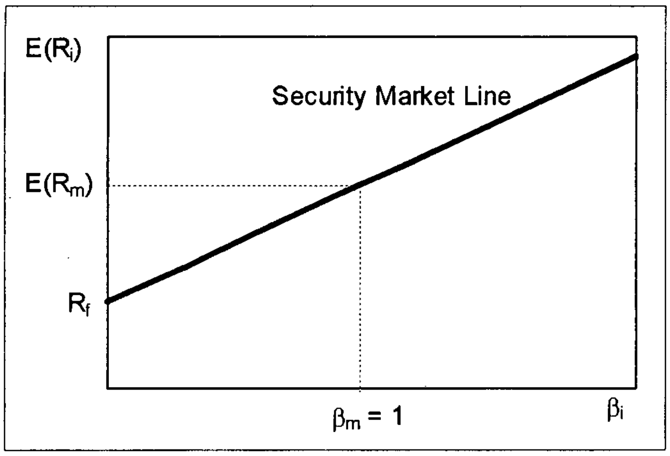

FIGURE 1.3 Security Market Line

The most important implication of the model is that investors can only expect to be compensated for bearing systematic or market related risk. Consequently any unsystematic or firm specific risk would not be priced in the market for all assets.

The 'market-risk premium' is the return that investors require over and above the risk-free rate for assuming an additional amount of risk. The CAPM therefore implies that in a competitive market, the expected risk premium on a share varies in direct proportion to the share's beta.

Numerous empirical tests have been conducted to test how well the CAPM fits the data. In order to empirically test the theoretical CAPM it is necessary to transform it from an expectations or *ex ante* form (since expectations cannot be measured) into a form that uses observed data. Copeland and Weston (1992) show that the *ex post* form of the CAPM, which has come to be know as the Empirical Market Line, can be written as:

$$R_{it} - R_{ft} = \beta_i (R_{mt} - R_{ft}) + e_{it} \qquad (1.8)$$

where e_{it} is a random error term and $E(e_{it}) = 0$.

The residual term, e_{it}, is often interpreted as 'abnormal' since it represents returns in excess of or below that predicted by the Security Market Line. This is shown graphically in Figure 1.4 below.

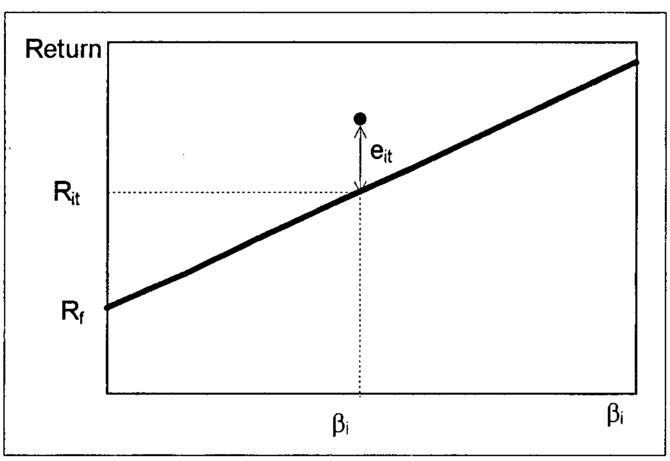

FIGURE 1.4 Empirical Market Line

When the CAPM is empirically tested or the Empirical Market Line used as a benchmark for security performance, it is usually written in the form :

$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + e_{it}$ (1.9)

Equation 1.9 is the same as equation 1.8 expect that a constant term, α_i, has been added.

Arbitrage Pricing Theory

Ross's (1977) Arbitrage Pricing Theory (APT) is an alternative theory of how assets are priced in the market place. The CAPM predicts that the return on a security is linearly related to the return on the market portfolio. The APT is more general in that it assumes that the rate of return on a security is linearly related to k factors, i.e

$\text{Return} = a + b_1(r_{factor1}) + b_2(r_{factor2}) + b_3(r_{factor3}) + \ldots + b_k(r_{factork}) + \text{noise}$

The APT assumes that each stock's return depends partly on pervasive macroeconomic influences or "factors" and partly on "noise" - events that are unique to that company. The theory doesn't state what the factors are. There could be an interest rate factor, an exchange rate factor, and so on. The return on the market portfolio *might* serve as one factor. The CAPM may be viewed as a special case of the APT when there is only a single factor, the return on the market portfolio.

For any individual stock there are two sources of risk. Brealey and Meyers (1991) interpret these two sources of risk as follows. First is the risk that stems from the pervasive macroeconomic factors which cannot be eliminated by diversification. Second is the risk arising from possible events that are unique to the company. Diversification *does* eliminate unique risk, and diversified investors can therefore ignore it when deciding whether to buy or sell a stock. The expected risk premium on a stock is affected by "factor" or "macroeconomic" risk; it is *not* affected by unique risk.

Arbitrage pricing theory states that the expected risk premium on a stock should depend on the expected risk premium associated with each factor and the stock's sensitivity to each of the factors (b_1, b_2, b_3, etc.) Thus the formula is

Expected risk premium on investment $= r - r_f$

$$= b_1(r_{factor1}-r_f) + b_2(r_{factor2} - r_f) + \ldots$$

This chapter has served as a brief overview of the main concepts and models of Modern Portfolio Theory. These concepts and models were introduced here, as they are referred to and applied in subsequent chapters.

In the next chapter the focus is on investment in equities on the JSE. The returns and risks associated with investment on the JSE are examined with the aim of promoting investment in our local equity.

CHAPTER 2

INVESTIGATION INTO INVESTMENT IN LOCAL EQUITIES

2.1 INTRODUCTION

"October. This is one of the peculiarly dangerous months to speculate in stocks. The others are July, January, September, April, November, May, March, June, December, August and February."

Mark Twain

The above quotation by Mark Twain typifies the attitude that 'seasoned' investors have to investing on the stock exchange. Investors are concerned not only about the return on an investment but also the risk associated with that investment.

The main focus of this thesis is on the performance and construction of equity portfolios in the context of the JSE. This brief chapter's primary aim is to promote investment in equity by discussing the returns and risks associated with equity investment on the JSE. Firstly the returns on the JSE are contrasted to alternative investments. Secondly, and more interestingly, the risk of investment on the JSE is examined and projections for the longer term assessment of risk are critically examined.

2.2 RETURN ON THE JSE

In order to assess the attractiveness of investment on the JSE, the returns on equity are compared with the returns earned on alternative investments. Table 2.1 below shows the annual returns on various investments over the period January 1985 to January 1996. The JSE Actuaries All Share Index is used to represent the return on our equities. Dividends are included in these annual returns. The commonly used measure of inflation, i.e. the Consumer Price Index (CPI) is used here to proxy inflation. The 91-day Treasury Bill Rate is used as a proxy for the return earned on a risk free investment. Together with the above mentioned series, the returns for R150s, the RSA long-term government stock, is shown in Table 2.1. The measured returns on the R150 are the capital returns on the bond plus occasional interest receipts. The final row in the table shows the average annual return on each of these investments.

TABLE 2.1 Annual returns on investments (January to January)

YEAR	JSE INDEX	CPI	TREASURY BILL	R150
1985	7.39	13.88	21.79	-0.90
1986	52.32	20.72	12.05	13.33
1987	55.74	16.11	8.77	30.31
1988	-21.08	14.19	9.49	9.74
1989	41.73	13.31	15.08	13.59
1990	51.31	15.10	18.00	22.65
1991	-15.67	14.30	17.33	12.76
1992	43.84	16.15	16.02	12.67
1993	-1.37	9.67	11.54	27.45
1994	41.01	9.91	10.61	28.23
1995	8.79	9.61	12.62	-12.61
1996	38.05	6.89	13.93	34.10
Average	25.17	13.32	13.94	15.94

From the above table it is clear that the average annual return on the market, as proxied by the JSE Actuaries All Share Index, over the period 1985 to 1996 was higher than the inflation rate, the rate of return on a riskless investment and the return on the bond market. Typically the Unit Trust industry recommends that investments in unit trusts should be seen as at least a three to five year investment. The following two graphs allow one to obtain a visual feel for the differences in returns represented in Table 2.1 over periods longer than a year. Each graph depicts the time series over the period 1985 to 1996 of the value of R100 invested in the market in 1985 and R100 invested in one of the alternative investments.

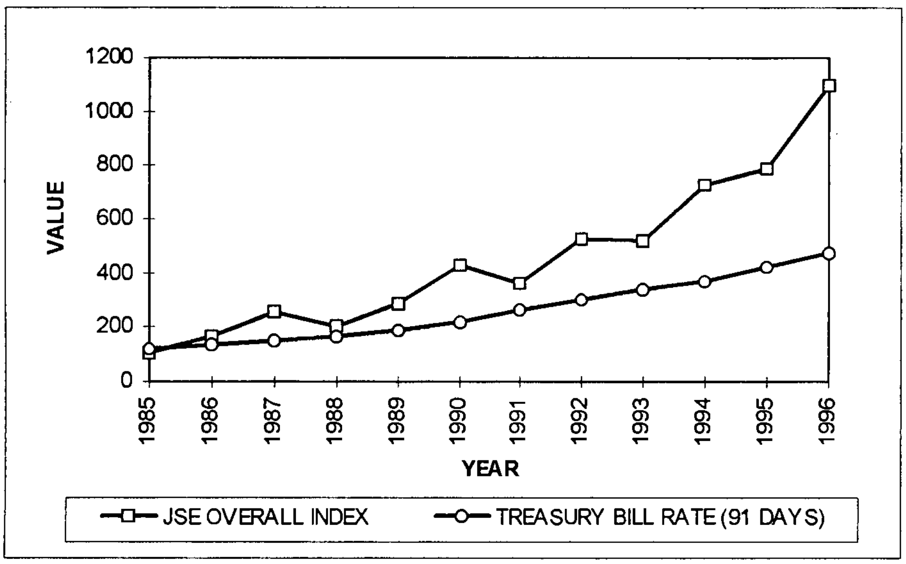

FIGURE 2.1 : Value of R100 invested in 1985

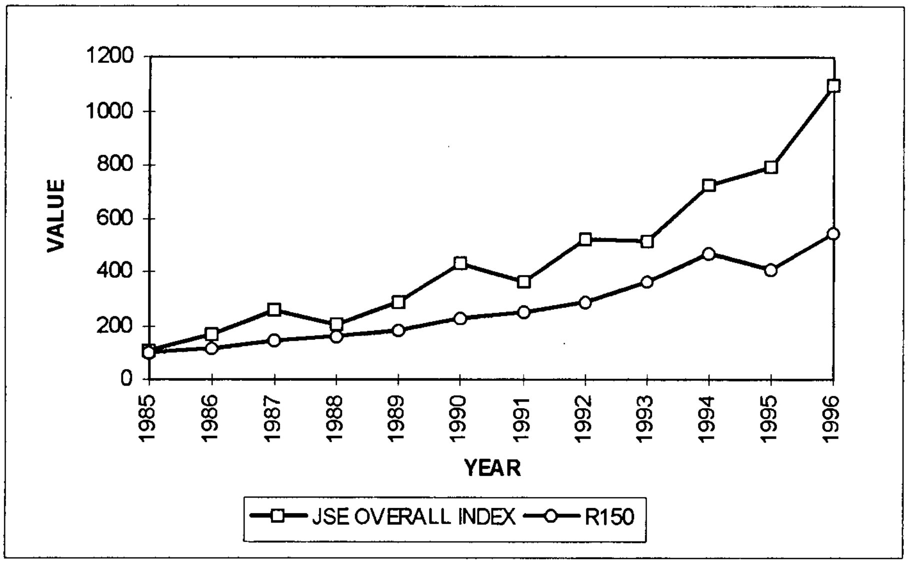

FIGURE 2.2 : Value of R100 invested in 1985

The superior return on the market portfolio over longer periods is graphically evident. An investment in equities convincingly outperforms a risk-free investment and investment in the bond market over the period 1985 to 1996.

Investors are clearly, however, not only interested in return. They are also concerned with the risk of an investment. Although the returns on the JSE Actuaries All Share Index are on average higher than those on the bond market for example, they are also much more variable as can be seen from Figure 2.2. The next section examines the risk of investing in equity on the JSE.

2.3 RISK OF THE JSE

2.3.1 Introduction

This section discusses the risk associated with longer term investment, typically three to five years, on the JSE. Central to these discussions is the use of the statistical measure of variance as a proxy for risk. Traditionally the variance of the returns on an investment has been used to measure the risk of the investment. In an efficient market, time series of returns are considered to be random, i.e. returns are not correlated with their prior values. If longer term returns are not random then the returns exhibit serial dependence. This section examines (amongst other issues) the implications of the presence of serial dependence when measuring risk.

While academics generally accept evidence that market returns are random when intervals of returns are less than one month, there is persuasive evidence that for longer measurement intervals, returns display some non-random behaviour (see Cuthbertson (1996)). If asset returns are non-random then their variance will depend on the interval over which they are measured. Instead of varying proportionally with the time interval, the variance of returns will vary disproportionately. This means that one cannot, for example, use the variance of monthly returns to estimate yearly returns, or yearly returns to estimate the variance of five-yearly returns since they are not linearly related.

Additionally, if serial dependence is present, then caution is advised when using the variance of shorter term return intervals to make judgements on the risk of investments over longer time horizons. To substantiate this cautionary advice it is worth noting that two common patterns of serial dependence, known as mean reversion and trending (mean aversion), have been the focus of recent scrutiny regarding the behaviour of stock market returns (see Kritzman (1994)). Mean reversion refers to the notion that returns revert to an average value. A trending pattern (mean aversion) suggests a positive return is more likely to be followed by another positive return than a reversal and vice-versa.

Consider Figure 2.3 below which depicts the prices of two hypothetical assets, and Figure 2.4 which depicts the returns on these two assets. The returns on asset one are characterised by mean reversion while asset two exhibits a trending pattern. On examination of Figure 2.3 it appears as if asset one is less risky because a decline is always immediately followed by an incline in price, whereas with asset two a negative return is more likely to be followed by another negative return. In our hypothetical example the variance of return of asset one turns out to be 0.001297 which is approximately three times higher than the variance of asset two, which is 0.000424. Traditionally this implies that asset one is riskier. Certainly many investors may well feel reassured with the knowledge that any decline in the value of an asset will immediately be followed by an incline, i.e. reverting to an average return. Consequently they may well see this as a lower risk than that advocated by the variance of the series. From this viewpoint it may be problematic using variance as a measure of risk when returns don't occur randomly. In sum, series of returns characterised by mean reversion will exhibit a higher variance than series of returns characterised by trending.

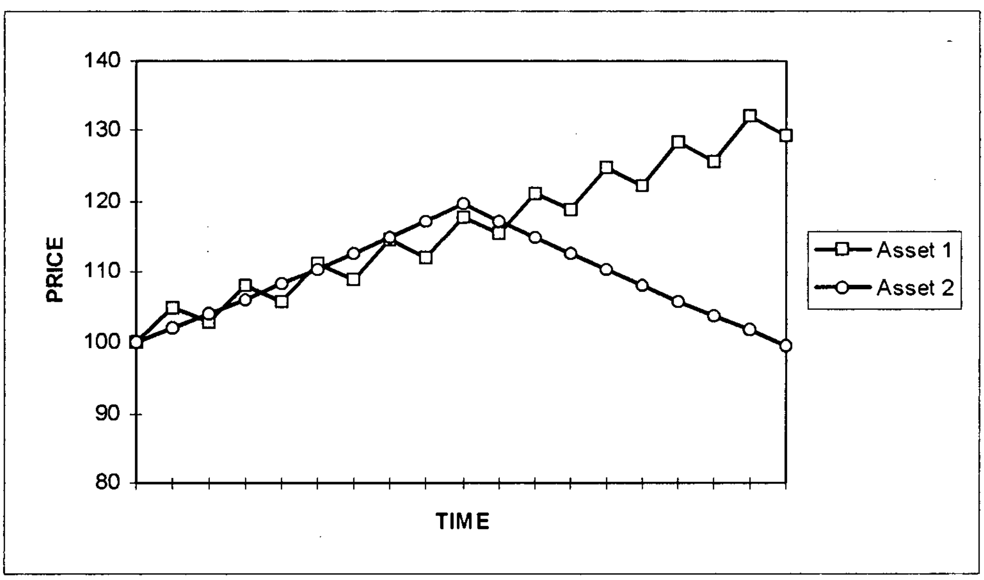

FIGURE 2.3 Price time series of two hypothetical assets

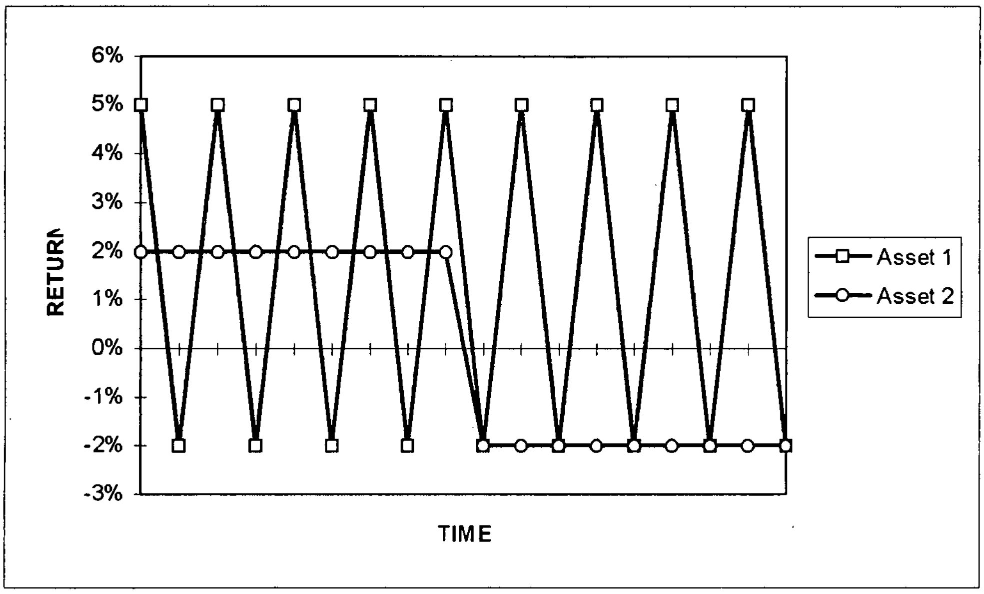

FIGURE 2.4 Returns time series of two hypothetical assets

2.3.2 Graphical Evidence

Turning to the situation for real data: Figures 2.5 and 2.6 below depict the level and returns on the JSE Actuaries All Share Index at the end of January over the period 1980 to 1996. From Figure 2.6 it is visually clear that the returns are characterised by mean reversion. The implication is that annual returns are not completely random (notice that a negative return is always followed by a reversal). In fact it is evident from Figure 2.6 that if returns were to be measured in two-yearly intervals all of the two-yearly returns over the entire period would be positive. This implies that over the last 16 years, never have there been two consecutive years in which the stock market has declined! This has very positive implications regarding the assessment of the “riskiness” of the JSE for long term investment.

A further interesting point is that not only does it appear that the long term riskiness of the JSE is less than generally perceived, but in Figure 2.6 it seems as if the variability of returns on the index is additionally decreasing. This implies our market seems to be becoming less risky through time. This is confirmed quantitatively since the variance of the returns up to 1990 is 0.086526 while the variance is less then half the prior value, i.e. 0.041326 in the period 1990 to 1996. (This difference is not statistically significant but the power of the F-test is very low due to the small sample size.)

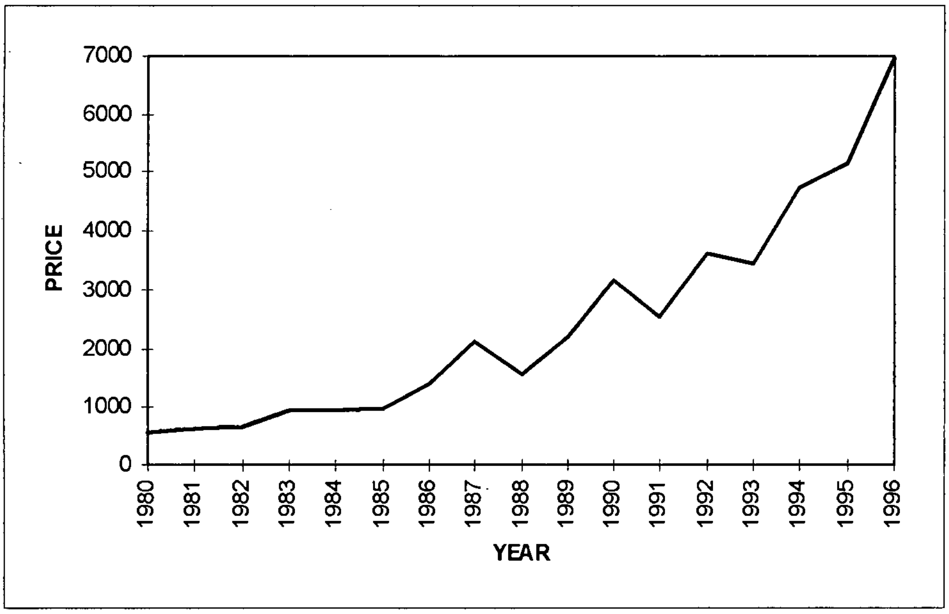

FIGURE 2.5 JSE Actuaries All Share Index : Price series

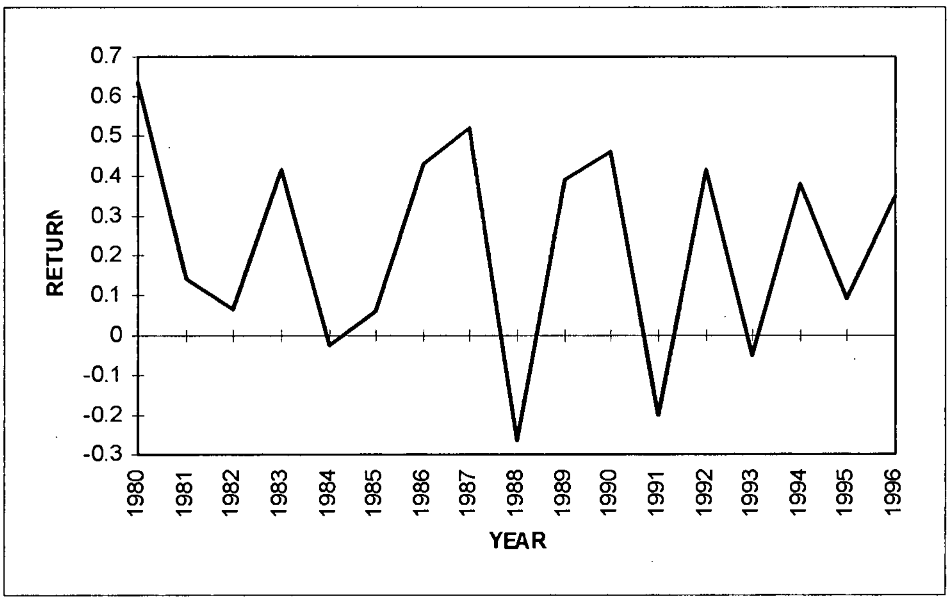

FIGURE 2.6 JSE Actuaries All Share Index : Returns Series

The market premium is the difference between the return on the market portfolio and a risk-free investment. The market premium thus represents the additional return that investors require from the market to be compensated for the risk they bear. If the market is becoming less risky through time the market premium would be expected to drop.

An examination of the market premium on the JSE over the period 1985 to 1996 is therefore conducted briefly below to examine the above assertion. The JSE Actuaries All Share Index was used as a proxy for the market portfolio while the 91-day Treasury Bill Rate was used as a proxy for the risk-free rate. Figure 2.7 below shows the time series of the annual returns on the JSE Actuaries All Share Index and the Treasury Bill Rate. The average annual market premium (i.e. the difference in return on the JSE Actuaries All Share Index and the Treasury Bill Rate) over the period 1985 to 1996 was 11.23%. The average market premium was 17.04% for the period 1985 to 1990 and 5.45% for the period 1991 to 1996. The decrease in the market premium serves as further confirmation that investors perceive our market as being relatively less risky of late. (The difference in the market premium over the two periods is not statistically significant but the power of the t-test is very low due to the small sample size.) One plausible explanation for this is that there are more longer term investors in the market of late who are typically exposed to less risk (as a consequence of the relatively low long term volatility of returns on the JSE).

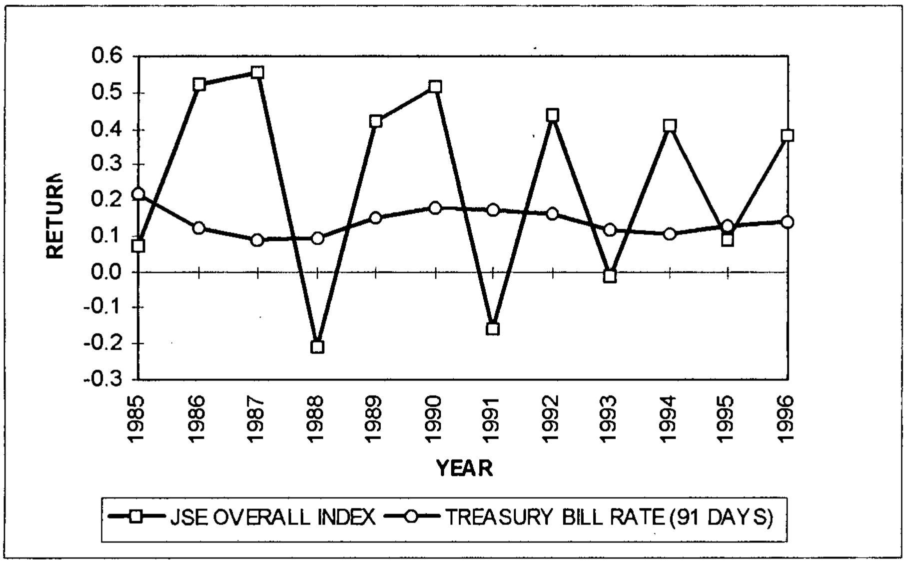

FIGURE 2.7 JSE Actuaries All Share Index Returns and 91-Day Treasury Bill Rate

2.3.3 Quantitative Evidence

Returning to measurement of risk in the presence of serial dependence, in addition to graphical examinations there are a number of tests which can be used to detect serial dependence and give some insights into the behavioural patterns of returns. Three of these tests are discussed and applied to the JSE All Share Index below.

There are several ways to detect serial dependence. The simplest is probably the non-parametric runs test.

2.3.3.1 Runs test:

In order to perform a runs test, the average value of the series is first computed. Then every value that is above the mean is designated as positive and every value that is below the mean as negative. Next the number of runs in the series is computed. A run is defined as an uninterrupted sequence of positive or negative values. If more runs than expected are observed, the duration of the series' typical run is shorter than one would expect for a random series. The series is then characterised by mean reversion. If fewer runs than expected are observed, then the duration of the series' typical run is longer than one would expect for a random series and the series is characterised by trending. Runs tests for annual, quarterly and monthly returns over the period 1986 to 1996 were conducted. Table 2.2 below gives the observed and expected number of runs. The normal deviate and the p-value are also given. The difference between the observed and expected number of runs was only significant for annual returns. For the annual returns more runs were observed than expected, implying that the series of annual returns is characterised by mean reversion. This supports the graphic evidence of mean reversion in annual returns in Figure 2.6.

TABLE 2.2 Runs tests

	Monthly	**Quarterly**	**Annually**
Expected runs	61.17	21.39	6.09
Observed runs	61	17	9
Normal deviate	0.03	1.40	2.02
P-value	P>0.20	P<0.20	P<0.05

A runs test is limited because it deals only with direction. It depends only on whether an observation is above or below average and not on the extent to which the observation differs from the average. The next two tests rely on the magnitude of the observations instead of just their rank.

2.3.3.2 Autocorrelation test:

For the autocorrelation test the series is regressed on its immediately prior values. A significantly negative correlation coefficient implies that the series is mean reverting.

A significantly positive correlation coefficient implies that the series is trending. Table 2.3 below gives the autocorrelation coefficients at one lag for annual, quarterly and monthly returns. The normal deviate and p-values are also given. The correlation coefficient was not significant for annual, quarterly or monthly returns. The correlation coefficient for annual returns was, however, nearly significant and negative, supporting the notion that annual returns mean revert.

TABLE 2.3 Autocorrelation tests

	Monthly	Quarterly	Annually
Autocorrelation coefficient	0.098	0.134	-0.540
Normal deviate	1.077	0.887	-1.709
P-value	P>0.20	P>0.20	P<0.10

2.3.3.3 Variance ratio test:

If a sequence of returns is random and several estimates of the variance based on increasing return intervals are computed, the estimates should increase in proportion to the length of the interval. The variance ratio is computed by dividing the variance of returns estimated from the longer interval by the variance of returns estimated from the shorter interval and then normalising this value by dividing it by the ratio of the longer interval to the shorter interval. A variance ratio of less than one suggests that the shorter-interval returns tend toward mean reversion within the duration of the longer interval. By contrast, a variance ratio that exceeds one suggests that the shorter-interval returns are inclined to trend within the duration of the longer interval. Table 2.4 gives variance ratios for several different combinations of intervals. The normal deviates and the p-values are also given. The variance ratio of yearly to monthly returns is significantly greater than one, implying that the months trend within the one year periods. The variance ratio of three yearly to yearly returns and the ratio of five yearly to yearly returns are both significantly less than one. The yearly returns therefore mean revert within three years and within five years.

TABLE 2.4 Variance Ratio tests

	Quarter/ Month	Year/ Quarter	Year/ Month	3 Year/ Year	5 Year/ Year
Variance Ratio	1.257	1.518	1.908	0.287	0.545
Normal deviate	1.898	1.772	2.664	-5.264	-2.285
P-value	P<0.10	P<0.10	P<0.01	P<0.0001	P<0.05

The graphical evidence together with the quantitative results of the above tests imply that annual returns on the JSE mean revert. This evidence is surely reassuring to the long-term investor, knowledge that any downturn in the market will immediately revert to some positive mean value is indeed encouraging.

All the above discussions confirm that when investors have longer term horizons (longer than a year say) using the measure of variance (even over one year intervals) gives an unintuitive (and upward biased) measure of risk. An alternative approach to assess risk over the longer term is to take longer intervals of return - if sufficient data is available. Assuming one has a three year time horizon say, i.e. one is only interested in assessing the behaviour of the market at three year intervals of time, then Figure 2.8 below shows the behaviour of the JSE Index over the period 1979 to 1994 assessed at three year intervals.

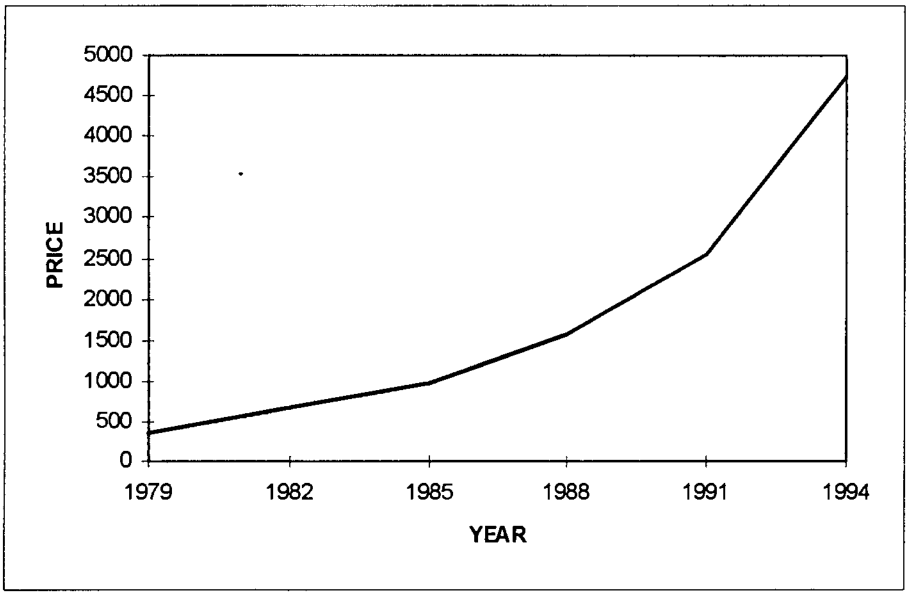

FIGURE 2.8 JSE Actuaries All Share Index : Prices at three year intervals

Two interesting points emerge : no market declines and very little variability in price. Analysing this information quantitatively, the variance of three year returns is 0.045241 over the 1979-1994 period. Contrasting this variance to the variance of returns measured at annual intervals, surprisingly the variance of annual returns is larger (i.e. 0.069854) than the variance of three year returns even without adjustment for interval length. If the series of returns was random the variance of three year returns should be three times the variance of annual returns, i.e. 3 x 0.069854 = 0.209562. The variance of three year returns is less than one-quarter of this estimate (The low variance ratio in Table 2.4 confirms this)! This evidence confirms that the variance of three yearly returns is far lower than expected if the variance of annual returns is used as a proxy for risk.

2.4 CONCLUSION

This chapter examined the returns and risks associated with longer term investment on the JSE[1]. The superior returns on the JSE relative to alternative investments were highlighted. The risk of longer term investment on the JSE was shown to be less than generally estimated by using an extension of risks in the shorter term. This chapter therefore promotes investment in our local equity through, for example, Unit Trusts, by arguing longer term risk, as measured by variance, is probably less than perceived. Hence from an asset allocation perspective, if longer term risks are indeed overestimated, there is a case for larger proportions of funds to be allocated towards longer term investment in equity, as opposed to alternative investments.

[1] Since the examination was based on the last sixteen years only this limits to some extent the conclusions or assertions made.

CHAPTER 3

PERFORMANCE MEASUREMENT

3.1 INTRODUCTION

This chapter reviews several portfolio performance measures and empirically demonstrates their application using a sample of South African Unit Trusts. The primary aim of this chapter is to provide a practical interpretation of each performance measure.

The performance of managed funds has attracted considerable attention amongst both practitioners and academics. Although the ex post return on a portfolio has been one of the most important performance measures considered by the investment community, financial academics have argued that the risk investors are exposed to is a vital consideration as well. Initially "risk adjusted" measures of portfolio performance were proposed by Sharpe (1966) and Treynor(1965). Sharpe's measure adjusts the portfolio return by the "total" risk exposure of the portfolio, while Treynor's measure adjusts return by only the "market" risk exposure of the portfolio.

Subsequently a variety of statistical techniques that focus on the performance of the fund manager *per se* were proposed. Underpinning these techniques is the concept that portfolio managers may only be able to achieve superior (risk-adjusted) portfolio returns (to that of the market portfolio) in two possible ways: (i) by selecting shares having abnormal returns (henceforth referred to as selection ability or microforecasting); (ii) or by successfully timing when to trade in the market (henceforth referred to as timing ability or macroforecasting). Jensen (1968,1969) proposed a model based on the *ex post* form of the CAPM, or the Empirical Market Line, to measure the selection aspect of the performance of managed portfolios. Treynor and Mazuy (1966) extended the Empirical Market Line by adding a quadratic term in an attempt to include a measure of the timing ability of a manager. A similar model was developed by Jensen (1972) which measures both selection and timing ability. Merton (1981) and Henriksson and Merton (1981) attempted to overcome some of the practical problems in estimation precision in Jensen's (1972) theoretical structure. However Zimmermann and Zogg-Wetter (1992) questioned the ability of this technique to capture true market timing ability alone . They thus conducted an empirical study on stock market indices and found that significant market timing statistics were likely to be recorded even for totally passive portfolios. Hence results of this study casts serious doubt on the ability of this method in detecting authentic timing ability. In order to overcome this problem Bhattacharya and Pfleiderer (1983) developed a more sophisticated model by correcting an error made by Jensen (1972). Their model not only permits the separation of the selection and timing skills of fund managers, but is the first model to analyse the error term to identify a manager's forecasting ability.

All the timing and selectivity measures are based on the *ex post* form of the CAPM. Copeland and Weston (1992) argue that these measures are therefore subject to Roll's (1977) general criticisms of empirical tests of the CAPM. Furthermore these measures all compare returns of managed portfolios to the returns of a benchmark portfolio. Roll (1977) argued that it was impossible to choose a 'benchmark' portfolio that accurately portrayed the 'market' portfolio. In an attempt to avoid this concern

Grinblatt and Titman (1993) proposed a measure that does not require the use of a benchmark portfolio. Furthermore their technique utilises information about the composition of the portfolios. Performance measures that are based on the APT also avoid Roll's criticisms as they do not depend on correct measurement of the 'market' portfolio. The disadvantage of the theory, however, is that the number and nature of factors that can influence the return relationship among securities are not specified.

The focus of this chapter is on reviewing the various portfolio performance measures. The first Section takes the form of a theoretical review on each of the above-mentioned measures. Additionally an attempt is made to provide a less theoretical interpretation of each measure to assist practitioners. Furthermore the limitations of and differences between the measures are highlighted.

Although the focus of this chapter is on the implementation and interpretation of the various measures themselves, (rather than on the assessment of the performance of the South African Unit Trust Industry) an empirical investigation nevertheless follows in Section 3.4 with the aim once more of demonstrating the application of these measurement techniques.

3.2 PERFORMANCE MEASURES

The first risk adjusted performance measures were proposed in 1965 and 1966 by Treynor and Sharpe. Subsequently a variety of more sophisticated techniques that focus on the performance of the fund manager were developed. A review of these existing portfolio performance measures follows.

3.2.1 Sharpe's Risk Adjusted Performance Measure

It is evident from the expression given below that Sharpe's (1966) measure adjusts the portfolio return by the "total" risk exposure of the portfolio.

$$\text{Sharpe's measure} = \frac{\text{average excess return}}{\text{total risk}}$$

$$= \frac{\overline{R_{pt} - R_{ft}}}{\sigma_p} \qquad (3.1)$$

where R_{pt} = return on portfolio at time t;

R_{ft} = risk-free rate at time t and

σ_p = standard deviation of R_p (variability of portfolio return).

Intuitively, Sharpe's risk adjusted performance measure is easily understood as a reward to risk ratio. Sharpe defines risk as the variability or standard deviation of return. The standard deviation of return is commonly interpreted as the "total" risk of a portfolio. As reviewed in Chapter 1 this "total" risk is composed of market risk and unique risk. Sharpe's measure will thus penalise a fund that is exposed to both market risk and unique risk.

An intuition for Sharpe's measure is easily understood by a graphical depiction of portfolios in the usual Markowitz (σ_p ; R_p-R_f) space. In Figure 3.1 the standard deviations and excess returns of two hypothetical portfolios A and B are shown. Sharpe's measure can be graphically interpreted as the slope of the line from the origin to the position of the portfolio. The steeper the slope the higher the reward to risk ratio (i.e. the larger Sharpe's measure). An additional insight worth mentioning is that if the notion of borrowing and lending is introduced (at a constant rate R_f) any given σ_p can be attained by the separation of investment funds between R_f and A or B (i.e. one can slide up or down the ray OA or OB). Clearly for any given σ_p position, R_p-R_f will always be higher on ray OA. Hence it can be concluded that investment in A is always superior to B no matter what level of risk is preferred.

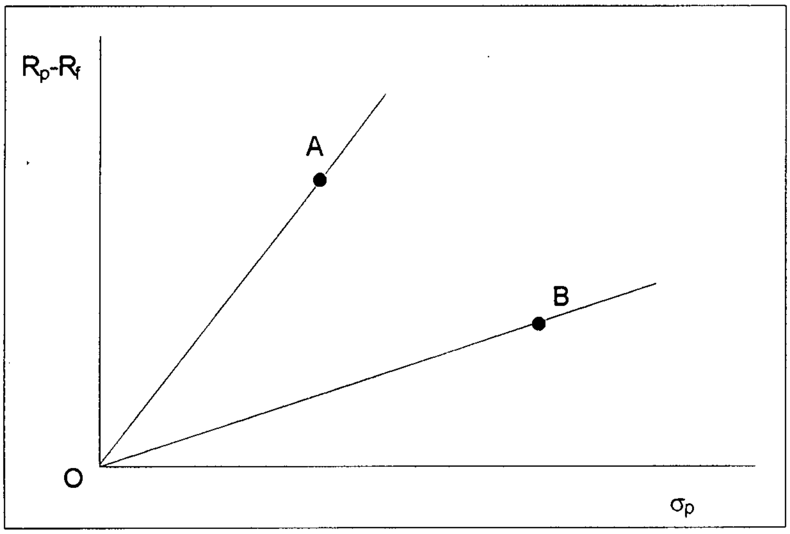

FIGURE 3.1 Graphical depiction of Sharpe's measure

PRACTICAL INTERPRETATION:

- Sharpe's measure is the reward to total risk ratio

ADVANTAGES:

- Easy to understand
- Simple to implement
- No need to identify an appropriate benchmark as ratios for each fund are understood relative to ratios for other funds
- Penalises managers who do not diversify fully

DISADVANTAGES:

- Sharpe's measure penalises managers who do not diversify fully. Thus comparisons of general equity funds with specialist funds using this measure are inappropriate due to the nature of specialist funds, which are by construction not fully diversified.

3.2.2 Treynor's Risk Adjusted Performance Measure

Treynor's (1965) measure risk adjusts return by only the "market" or systematic risk exposure of the portfolio.

$$\text{Treynor's measure} = \frac{\text{average excess return}}{\text{market risk}}$$

$$= \frac{\overline{R_{pt} - R_{ft}}}{\beta_p} \qquad (3.2)$$

where R_{pt} = return on portfolio at time t;

R_{ft} = risk free rate at time t and

β_p = portfolio beta (sensitivity of the portfolio to market return).

Clearly if the portfolios under scrutiny are fully diversified (i.e. have no unique risk) then the Sharpe and Treynor measures are essentially equivalent. If, however, a portfolio is exposed to unique risk Treynor's measure will not penalise the manager for not diversifying fully. In the context of specialist funds where the investment objective necessitates that the portfolio is not fully diversified then the Treynor measure can be considered a more useful performance measure. Treynor's measure requires the use of a benchmark for the 'market' portfolio, as the portfolio beta is calculated relative to this 'market' portfolio.

One can easily depict Treynor's measure graphically in (β_p ; R_p-R_f) space by plotting beta along the horizontal axis and the excess returns along the vertical axis. In Figure 3.2 the beta and excess returns of two hypothetical portfolios A and B are depicted. Treynor's measure can be graphically interpreted as the slope of the line from the origin to the position of the portfolio. The steeper the slope the better the risk to reward ratio. In this example portfolio A outperforms portfolio B.

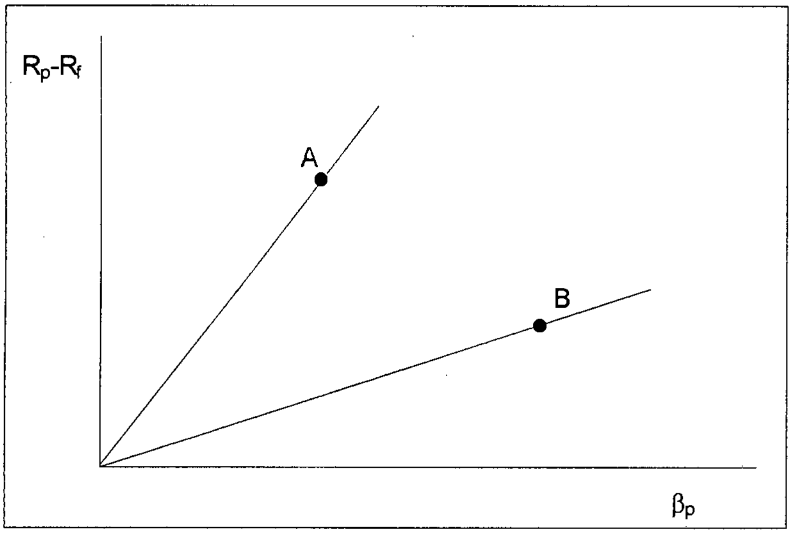

FIGURE 3.2 Graphical depiction of Treynor's measure

PRACTICAL INTERPRETATION:

- Treynor's measure is the reward to market risk ratio

ADVANTAGES:

- Easy to understand
- Simple to implement
- Suitable for evaluating specialist funds where the investment objective necessitates that the fund is not fully diversified

DISADVANTAGES:

- Treynor's measure will not penalise a portfolio that is exposed to unique risk
- A suitable benchmark or market portfolio needs to be identified and the portfolio beta needs to be estimated

3.2.3 Jensen's alpha

One of the first measures that focused on the manager *per se* was developed by Jensen (1968, 1969). Jensen proposed to measure the selection aspect of performance of managed portfolios by employing the CAPM (see Copeland and Weston (1992)). The CAPM in its *ex post* form (see Chapter 1, equation 1.9) can be written as :

$$(R_{pt} - R_{ft}) = \alpha_p + \beta_p(R_{mt} - R_{ft}) + e_{pt} \qquad (3.3)$$

where

α_p is a measure of the selection ability

R_{pt} is the return on the fund/portfolio at time t

R_{mt} is the return on the market portfolio at time t

R_{ft} is risk free return at time t

β_p is the sensitivity of the portfolio to market return

e_{pt} is random error which has an expected value of zero.

Jensen's alpha (α_p) is the intercept estimated from a regression of the excess return of the managed portfolio on the excess return on the benchmark portfolio. Estimation of the coefficients of the above model using OLS regression suggests α_p can be interpreted as the average excess return earned on the portfolio over and above/below a benchmark portfolio with the same systematic risk. That is, a positive alpha represents 'abnormally' good performance. On the other hand a negative alpha implies that the fund failed to adequately compensate the investor for the given level of systematic risk.

Figure 3.3 below is a graphical illustration of 'abnormal' performance. It is a scatter plot of the excess returns on the market and the excess returns on a hypothetical portfolio. The regression line is estimated using equation 3.3. Jensen's alpha is the intercept of this line with the R_p - R_f axis. In this instance the alpha is greater than zero indicating positive 'abnormal' performance.

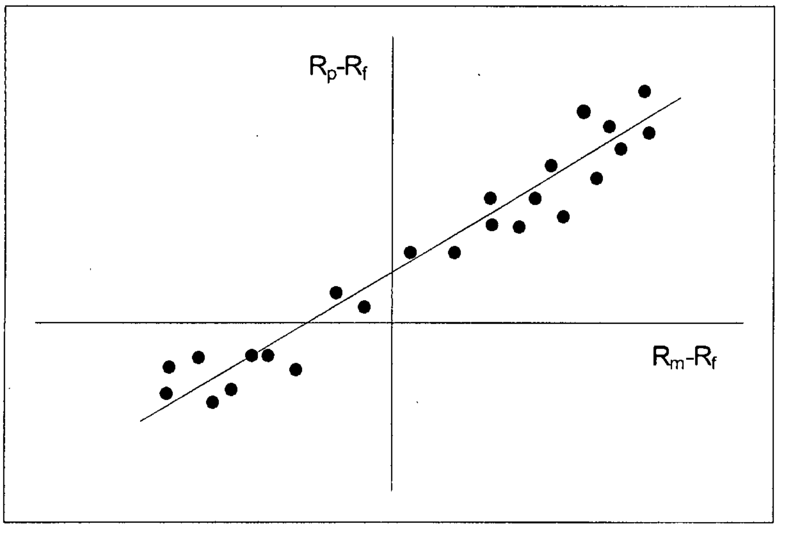

FIGURE 3.3 Graphical depiction of Jensen's alpha

Like Treynor's measure Jensen's alpha only examines systematic risk and gives no indication of how well diversified the trust being measured is. An advantage of Jensen's measure is that it allows statistical tests of significance to be conducted on the results whereas the Sharpe and Treynor measures make no attempt to identify bias which may be introduced by parameter estimation errors.

Since the β_p is fixed over the measurement period the model implicitly assumes the systematic risk of the portfolio is stationary throughout the measurement period and as a consequence the fund manager's ability to shift the systematic risk (i.e. β_p) of the portfolio, to take advantage of anticipated market movements, is ignored. Moreover there is the possibility that errors in inference may arise when the fund manger is a market timer. In particular, the Jensen measure may indicate poor performance when the manager possesses and utilises superior timing information. Grant (1977) explains that market-timing ability will cause the regression estimate of

α_p in equation 3.3 to be biased downward. Admati and Ross (1985) and Dybvig and Ross (1985) also show that, when a manager has superior timing information, the Jensen measure can be negative.

PRACTICAL INTERPRETATION:

- Jensen's alpha is the average return earned on the portfolio over and above a benchmark portfolio with the same systematic risk.

ADVANTAGES:

- Relatively easy to understand and implement.
- Focuses on the fund manager
- Intended to identify selection ability
- Tests of statistical significance can be conducted on results

DISADVANTAGES:

- Ignores exposure to unique risk
- Requires a benchmark
- Assumes β_p is fixed and so ignores market timing skill of managers
- Jensen's alpha will be downward biased if market timing ability is present

Market timing

The next three measures discussed below attempt to test for both selection ability and market timing ability. It is argued that a fund manager with market timing ability will shift the beta of his/her portfolio to take advantages of anticipated market movements. A fund manager having market timing ability is therefore likely, on average, to increase the equity proportion of the managed portfolio when equity returns are high and reduce it when equity returns are low. Figure 3.4 is a graphical depiction of a hypothetical portfolio whose manager exhibits market timing ability. The regression line estimated using equation 3.3 is included in the figure. The excess returns on the portfolio are seen to be higher than predicted by the regression line when the excess returns on the market are high. The observed values are also higher than predicted when the excess return on the market is negative. This figure highlights the shortcomings of using Jensen's measure (which assumes a constant slope) when market timing skills are evident.

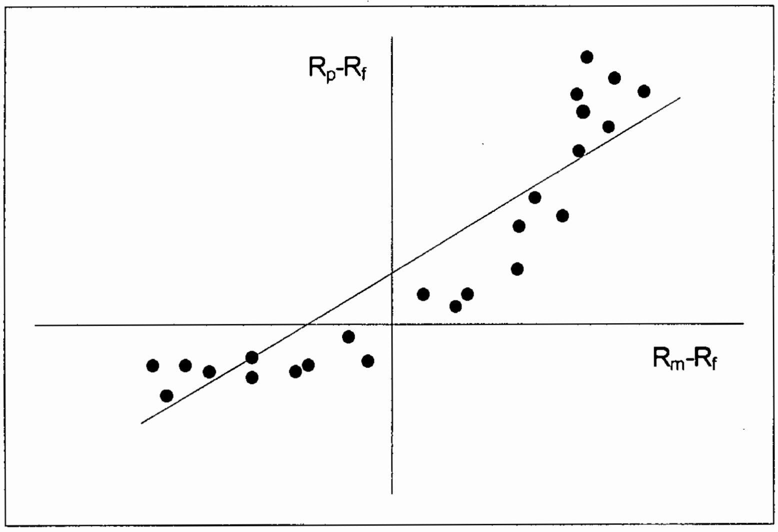

FIGURE 3.4 Graphical depiction of market timing ability

One can however allow for the existence of timing ability in equation 3.3 by permitting the beta coefficient β_p to be stochastic. Market-timing ability will therefore be present where β_p and $(R_m - R_f)$ are positively correlated.

3.2.4 Treynor and Mazuy

Treynor and Mazuy (1966) added a quadratic term to equation 3.3 to test for market-timing ability. In the standard CAPM regression equation, a portfolio's return is a linear function of the market return. However, Treynor and Mazuy argue that if the manager can forecast market returns, he/she will hold a greater proportion of the market portfolio when the return on the market is high and a smaller proportion when the return on the market is low. Thus, the portfolio return will be a convex function of market return. Thus

$$(R_{pt} - R_{ft}) = \alpha_p + \beta_p(R_{mt} - R_{ft}) + \psi_p (R_{mt} - R_{ft})^2 + e_{pt} \qquad (3.4)$$

The ψ_p term thus measures the managers market timing skill. One would expect to find $\psi_p > 0$ if the manager possesses and utilises superior information on the returns on market indices. As with Jensen's measure, α_p represents selection ability.

Figure 3.5 below depicts the same hypothetical portfolio depicted in figure 3.4. The fitted line is estimated using equation 3.4. This quadratic function provides a better fit under the circumstances than a linear function as it allows for market timing ability.

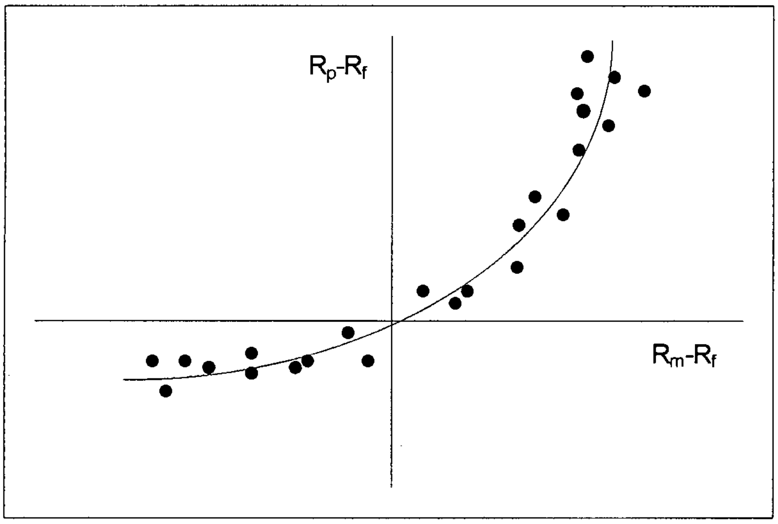

FIGURE 3.5 Graphical depiction of Treynor-Mazuy measure

PRACTICAL INTERPRETATION:

- Selection ability, α_p, measures the average return earned on the portfolio over and above a benchmark portfolio with the same systematic risk

- Market timing ability, ψ_p, measures the ability of the manager to hold a higher beta portfolio when returns on the market portfolio are high and a lower beta portfolio when returns on the market are low.

ADVANTAGES:

- Identifies timing and selection ability

DISADVANTAGES:

- The interpretation of results is slightly more complex than the less sophisticated preceding measures
- Calculation of the measure involves multivariate regression (which for some practitioners amounts to an added sophistication)

3.2.5 Henriksson and Merton

A similar model to Treynor and Mazuy was developed by Jensen (1972) which measures both selection and timing ability. Merton (1981) and Henriksson and Merton (1981) attempted to overcome some of the practical problems in estimation precision in Jensen's (1972) theoretical structure. Henriksson and Merton (1981) reason that managers who possess timing ability will be able to correctly forecast when the market returns will exceed the risk-free rate. Thus

$$(R_{pt} - R_{ft}) = \alpha_p + \beta_p(R_{mt} - R_{ft}) + \psi_p (R_{mt} - R_{ft})^+ + e_{pt} \qquad (3.5)$$

where $(R_{mt} - R_{ft})^+$ is $\max[0; R_{mt} - R_{ft}]$ and

α_p measures selection ability.

A positive ψ_p indicates market timing ability on the part of the portfolio manager.

Figure 3.6 below shows the hypothetical portfolio depicted in Figure 3.5. The regression line estimated from Equation 3.5 is seen to capture the market timing ability of the portfolio as the slope of the line increases as the excess returns on the market increase.

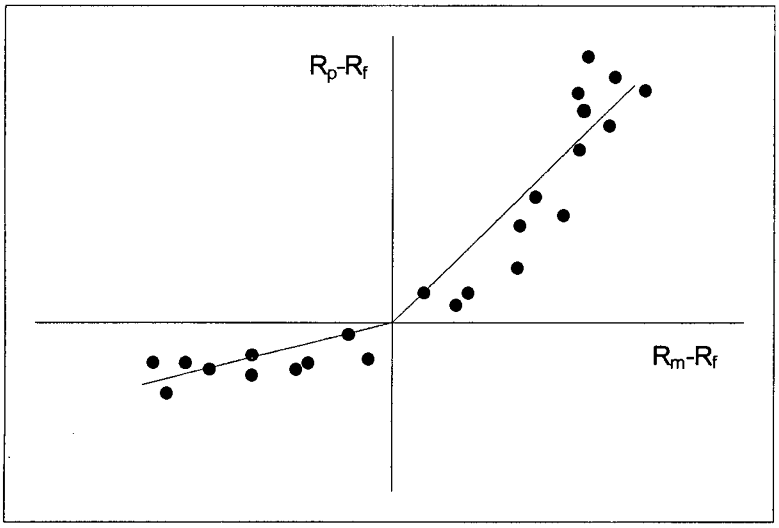

FIGURE 3.6 Graphical depiction of Henriksson-Merton measure

Although the Henriksson and Merton (1981) model permits separation of selection and timing skills of mangers, Dybvig and Ross (1985) point out that this model only tests whether the manager has special information, and not whether the manager uses the information correctly. Zimmermann and Zogg-Wetter (1992) cast doubt on the usefulness of this model by demonstrating that significant market timing coefficients were likely to be recorded even for totally passive portfolios.

PRACTICAL INTERPRETATION:

- Selection ability, α_p, measures the average return earned on the portfolio over and above a benchmark portfolio with the same systematic risk
- Market timing ability, ψ_p, measures the ability of the manager to hold a higher beta portfolio when returns on the market portfolio exceed the risk free rate and a lower beta portfolio when the returns on the market are below the risk free rate.

ADVANTAGES:

- Identifies timing and selection ability

DISADVANTAGES:

- The interpretation of results is slightly more complex than the less sophisticated preceding measures
- Calculation of the measure involves multivariate regression (which for some practitioners amounts to an added sophistication)
- It only examines whether managers can forecast whether the return on the market will exceed the risk free rate and not by how much it will exceed the risk free rate
- Dyvbig and Ross (1985) point out that this model only tests whether the manager has special information, and not whether the manager uses the information correctly
- Zimmermann and Zogg-Wetter (1992) cast doubt on the usefulness of this model by demonstrating that significant market timing coefficients were likely to be recorded even for totally passive portfolios.

3.2.6 Bhattacharya and Pfleiderer

Bhattacharya and Pfleiderer (1983) developed a more sophisticated model by correcting an error made in Jensen (1972). In essence Bhattacharya and Pfleiderer assume that the manager adjusts forecasts to minimise the variance of the forecast error while Jensen assumed that the manager uses the unadjusted forecast of the market return in the timing decision. The Bhattacharya and Pfleiderer model not only permits the separation of the selection and timing skills of fund managers, but is the first model to analyse the error term to identify a manager's forecasting ability. Lee and Rahman (1990) implemented the Bhattacharya and Pfleiderer model in an empirical study on American mutual fund performance. They argued that the above-mentioned refinements should make the model more useful than prior models. The model is couched in terms of observable variables as follows:

$(R_{pt}-R_{ft}) = \alpha_p + \theta E[(R_m-R_f)](1-\psi)(R_{mt}-R_{ft}) + \psi\theta(R_{mt}-R_{ft})^2 + \psi\theta e_t(R_{mt}-R_{ft}) + u_{pt}$ (3.6)

where

α_p is a measure of selection ability;

θ is the fund manager's response to information;

ψ is the coefficient of determination between the manager's forecast and the excess return on the market;

e_t is the error of the manager's forecast.

The empirical analog of (3.6) is:

$(R_{pt} - R_{ft}) = \eta_0 + \eta_1(R_{mt} - R_{ft}) + \eta_2(R_{mt} - R_{ft})^2 + w_t$ (3.7)

where the η_i's are the estimated coefficients and are used to estimate the parameters below. These estimates are couched in terms of probability limits (plim).

$\text{plim } \eta_0 = \alpha_p$ (3.8)

$\text{plim } \eta_1 = \theta\, E[(R_m - R_f)](1 - \psi)$ (3.9)

$\text{plim } \eta_2 = \psi\theta$ (3.10)

The disturbance term in equation (3.7):

$w_t = \psi\theta e_t(R_{mt} - R_{ft}) + u_{pt}$ (3.11)

captures the information needed to measure the manager's timing skill. This information can be estimated further by regressing $(w_t)^2$ on $(R_{mt} - R_{ft})^2$, that is:

$(w_t)^2 = \psi^2\theta^2\sigma_e^2(R_{mt} - R_{ft})^2 + \zeta_t$ (3.12)

where

$\zeta_t = \psi^2\theta^2(R_{mt} - R_{ft})^2[e_t^2 - \sigma_e^2] + (u_{pt})^2 + 2\theta\psi(R_{mt} - R_{ft})e_t u_{pt}$ (3.13)

According to Lee and Rahman (1990) the regression of (3.12) yields a consistent estimator of $\psi^2\theta^2\sigma_e^2$, where σ_e^2 is the variance of the manager's forecast error. Furthermore from equation (3.6) the consistent estimator of $\theta\psi$ enables the recovery of σ_e^2. The final estimate of selection ability first requires estimation of σ_π^2 (the variance of the excess returns on the market).

Based on Merton's assumption that π_t follows a Wiener process, Bhattacharya and Pfleiderer (1983) suggest the following as an estimate for σ_π^2 (the estimate is biased but this bias is insignificant in large samples according to Bhattacharya and Pfleiderer(1983)).

$$\sigma_\pi^2 = \{\Sigma[\ln(1 + (R_{mt} - R_{ft}))]^2\}/n \qquad (3.14)$$

Together with knowledge of σ_π^2 enables finally an estimate of

$$\psi = \sigma_\pi^2 / [\sigma_\pi^2 + \sigma_e^2] \qquad (3.15)$$

$$= \rho^2$$

to be obtained, where ρ is the correlation between the manager's forecast and excess return on the market and finally measures the quality of the manager's timing information. In Lee and Rahman (1990)'s study on American mutual fund performance, they chose to ignore negative correlation. They point out that Bhattacharya and Pfleiderer (1983) argued that negative correlation between the prediction and the realisation of the average excess returns on the market would imply that the fund manager possessed timing information that had positive value but that the manager was misguided by its application. Lee and Rahman (1990) rule out the possibility that there exist managers who are both well informed and foolish. Coggin, Fabozzi and Rahman (1993), however, hypothesised that managers could exhibit negative *ex post* timing skill. They argued that in the Bhattacharya and Pfleiderer model, this is indicative of a negative correlation between the manager's beta and the market return. They add that such a result could be due to the inability

of managers to correctly forecast the expected return on the market portfolio. They conclude that these managers would forecast the market return to be high when it is actually low and vice versa. Coggin, Fabozzi and Rahman (1993) suggest that one examine the sign of the coefficient of
$(R_{mt} - R_{ft})^2$ in equation 3.7. They argue that intuitively, in the spirit of the Treynor and Mazuy model, the sign of this coefficient will be indicative of the direction of timing skill. If the estimated value of this coefficient is negative, they designate timing skill (given by ρ) to be poor (negative).

Because of the potential presence of heteroscedasticity in the disturbance terms (they are related to the explanatory variables used in the model), the above system of regressions will result in inefficient parameter or coefficient estimates. Adjustments to take this heteroscedasticity into account must therefore be made. Lee and Rahman (1990) outlined a regression approach to correct for the heteroscedasticity. They showed that equations 3.7 and 3.12 must be divided by the variance of each respective error term. The variance of the error term for equation 3.7 is given by:

$$(\sigma_w)^2 = \psi^2\theta^2\sigma_e^2(R_{mt} - R_{ft})^2 + (\sigma_u)^2$$

and the variance of the error term for equation 3.12 by:

$$(\sigma_\zeta)^2 = 2\psi^4\theta^4(R_{mt} - R_{ft})^4\,\sigma_e^4 + 2\sigma_u^4 + 4\theta^2\psi^2(R_{mt} - R_{ft})^2\sigma_e^2\sigma_u^2.$$

Lee and Rahman (1990) argue that performing ordinary least squares regression on the transformed variables will provide more efficient estimates of the coefficients. This procedure used to adjust for heteroscedasticity is known as generalised least squares.

PRACTICAL INTERPRETATION:

- Selection ability, α_p, measures the average return earned on the portfolio over and above a benchmark portfolio with the same systematic risk
- Market timing ability, ρ, measures the correlation between the manager's forecast and excess return on the market.

ADVANTAGES:

- Identifies timing and selection ability
- First model to analyse the error term to identify a manager's forecasting ability

DISADVANTAGES:

- The interpretation of results is complex
- Calculation of the measure involves a number of multivariate regressions and requires a generalised least squares approach to correct for heteroscedasticity.
- Bhattacharya and Pfleiderer (1983) ignored negative timing ability (this was however corrected by Coggin, Fabozzi and Rahman (1993))

Although the above three measures overcome the criticisms of Jensen's alpha by including a measure of timing ability, they are, like Jensen's measure, all based on the *ex post* form of the CAPM (see Copeland and Weston (1992)) and express performance relative to a benchmark or 'market' portfolio. Copeland and Weston (1992) argue that measures based on the CAPM are subject to Roll's (1977) general criticisms of empirical tests of the CAPM. One of Roll's major criticisms of empirical tests of the CAPM revolves around the need to proxy the 'market' portfolio. He argues that it is impossible to choose a 'benchmark' portfolio to represent the 'market'. Neither of the following two measures are based on the CAPM and therefore the need to proxy the 'market' portfolio is eliminated. The next measure also differs from the preceding measures in that it utilises information about the composition of the portfolios.

3.2.7 Grinblatt and Titman

Grinblatt and Titman (1993) noted that none of the traditional measures of portfolio performance utilised information on the composition of the evaluated portfolio. They further noted that when the composition of the portfolio under consideration was used, the need to compare returns to a benchmark portfolio was eliminated.

Grinblatt and Titman (1993) developed a measure that allows one to compare the relative performance of funds by comparing the 'value-added' to the fund's total return through superior fund management. Their Portfolio Change Measure (PCM) measures the 'value-added' to the overall return of the fund through the portfolio manager making net positive adjustments to the overall composition of the shares making up the fund under his/her control.

The fundamental premise of the Grinblatt and Titman methodology is that, for 'informed' managers, the expected returns on assets change over time. By successfully predicting when individual assets have higher (lower) than average returns, a fund manager can increase (decrease) the percentage weightings of those assets in his portfolio. Therefore, for 'informed' managers, asset weightings over time, should be positively correlated with their associated conditional expected returns. For 'uninformed' managers, expected asset returns are perceived as constant over time; therefore the percentage holdings of the portfolio's individual assets should be uncorrelated with their subsequent returns. According to Grinblatt and Titman , a convenient measure which permits the evaluation of a portfolio's performance based on these assumptions is the aggregation of the covariances between individual asset returns and their portfolio weights:

$cov = \sum (E[w_i r_i]-E[w_i]E[r_i])$ (3.16)

where w_i = portfolio weight of the i^{th} share and;

r_i = the return of the i^{th} share.

This measure of covariance can be thought of as the actual expected return of a managed portfolio where asset weights and returns are correlated, minus the expected return on a portfolio where the weights and returns are uncorrelated. In addition, the second term inside the brackets can be seen as the appropriate adjustment for risk since it represents the expected return of a constant weight portfolio with the same average risk as the evaluated portfolio. For the 'uninformed' manager, the first and second terms should be approximately equal so that the covariance is equal to zero. For informed managers who are able to successfully adjust their portfolios so as to take advantage of changing returns on various assets, this covariance term should be significantly positive.

Equation 3.16 can be rewritten in the following two ways:

$cov = \sum E(w_i(r_i-E[r_i]))$ (3.17)

$cov = \sum E([w_i-E[w_i]]r_i)$ (3.18)

Grinblatt and Titman refer to equation 3.18 as the foundation for their new measure. Equation 3.18 indicates how to calculate a covariance between portfolio weights and the return of a *single* asset. To calculate the PCM, one is required to aggregate the covariance measures from equation 3.18 for *each* asset over the particular sample period. The PCM is thus equal to:

$PCM = \sum\sum (r_{it}[w_{it} - w_{i(t-1)}])$ (3.19)

where w_{it} = weighting of the i^{th} share in period t and;

r_i = the return of the i^{th} share in period t.

In equation 3.19, $w_{i(t-1)}$ gives one an estimate of asset i's expected portfolio weight at time t, and Grinblatt and Titman assume that one can use the actual returns in period t+1 as a proxy for the expected return in period t. Under the null hypothesis of no 'superior' information, current and past asset holdings are uncorrelated with current asset returns. This will cause equation 3.19 to converge towards zero. Should the manager be 'informed', changes in asset weightings will be positively correlated to returns, and the PCM have a net positive outcome.

On an intuitive level the PCM is easily understood as follows:

1. If the return (r_{it}) on a particular share is positive over period t and the weighting in the share (w_{it} - $w_{i(t-1)}$) was increased in period t, then a positive adjustment was made to the fund.
2. If the return (r_{it}) on a particular share is negative over period t and the weighting in the share (w_{it} - $w_{i(t-1)}$) was decreased in period t, then a positive adjustment was made to the fund.
3. If the return (r_{it}) on a particular share is positive over period t and the weighting in the share (w_{it} - $w_{i(t-1)}$) was decreased in period t, then a negative adjustment was made to the fund.
4. If the return (r_{it}) on a particular share is negative over period t and the weighting in the share (w_{it} - $w_{i(t-1)}$) was increased in period t, then a negative adjustment was made to the fund.

The PCM is clearly just the sum of all of these adjustments.

The PCM is able to avoid all the shortcomings of employing a benchmark portfolio, as portfolio risk is never directly quantified. Rather, the key assumption is that mean asset returns are stationary over the relevant sample period. Should this assumption be violated, then even the uninformed manager can achieve an overall positive performance result.

In the practical application of this measure, it is possible for the PCM to be positive even if no changes to the portfolio are made, i.e. a buy and hold strategy could be interpreted as 'adding value'.

PRACTICAL INTERPRETATION:

- The PCM measures 'value-added' to the fund's total return through the portfolio manager making net positive adjustments to the overall composition of the shares in the fund.

ADVANTAGES:

- The need to identify a benchmark portfolio is eliminated
- Information on the composition of the evaluated portfolio is utilised
- Easy to interpret
- It does not involve fitted values or estimation

DISADVANTAGES:

- The data requirements are enormous : the exact weighting and composition of the portfolio at various intervals, the returns and dividends on each share held over each interval and the relative weighting of the liquid assets at each interval
- It is possible for the PCM to be non-zero for a buy-and-hold approach
- The underlying performance of the original portfolio is not measured, i.e. the PCM measures only the changes to the portfolio

3.2.8 Performance measurement in the framework of the APT

Most of the research on the performance of mutual funds has been conducted under the framework of the CAPM (see Copeland and Weston (1992)). The literature on mutual fund performance is rich with debate as to the proper measure of the "market portfolio", the validity of the assumptions of the CAPM, and the validity of empirical studies which are based on the CAPM. The APT provides an alternative model of the securities valuation process and empirical tests using the APT do not depend upon "correct measurement" of the market portfolio. The disadvantage of this theory is that the number and nature of factors that can influence the return relationship among securities are not specified; they must be determined empirically before a portfolio performance measure can be obtained. Biger and Page (1993) employed the APT to assess the performance of 25 South African Unit Trusts. Their study utilises a one-parameter, risk adjusted performance appraisal methodology in the multi-factor APT framework. In this sense, their study employs a measure of performance which is equivalent to Jensen's measure, namely the intercept term of the regression coefficient, where the excess rate of return on the shares of each unit trust are regressed against a set of independent variables. Clearly this performance measure is subject to the same criticisms as Jensen's alpha in that it ignores factor timing ability. If it was specified that the square of each independent variable be included as additional factors, a measure in the spirit of the Treynor-Mazuy model could be applied.

PRACTICAL INTERPRETATION:

- The APT is a pricing model and not a performance measure. The practical interpretation would depend on the performance measure applied

ADVANTAGES:

- The identification of a benchmark portfolio is not required

DISADVANTAGES:

- The number and nature of the factors are not specified and need to be empirically determined
- Depending on the measure applied many of the criticisms of the preceding measures would be valid

In order to consolidate the preceding discussion a brief empirical investigation into the performance of South African Unit Trusts was conducted. The methodology and results of this investigation are presented in Section 3.4. A summary of the findings of prior studies of the application of these performance measures to the South African Unit Trust industry follows.

3.3 SOUTH AFRICAN UNIT TRUST PERFORMANCE STUDIES

Since their initial introduction in 1965 there has been extensive investigation of the performance of South African Unit Trust funds. The early research conducted on these unit trust funds concentrated on making use of the Sharpe, Treynor and Jensen performance measures. Taylor (1977) analysed ten unit trust funds over a six year period covering 1970 to 1976. He made substantial use of the Sharpe, Treynor and Jensen performance measures and calculated, after risk-adjustment, that the ten funds made on average about 2.4% less return per annum when compared to the market. However this difference was not significant at the 5% significance level.

Gilbertson (1976) investigated eleven unit trusts for the period 1970 - 1976 in the context of testing the strong form of the Efficient Markets Hypothesis (EMH). Gilbertson found that the funds under investigation on average earned 1% less per annum than the market when adjusted for risk. Out of the eleven funds employed in this study, two funds seemed to show consistently higher performance over certain periods, but these returns were not found to be statistically significant at the 5% level. Gilbertson nevertheless concluded that the strong form of the EMH could be said to hold.

Gilbertson and Vermaak (1982) followed up Gilbertson's previous work with another study in which they investigated eleven funds over the period 1974-1981. In contrast to Gilbertson's earlier findings they concluded that the majority of the mutual funds outperformed the three market indices which they had chosen as 'benchmark' portfolios and against which they measured performance. For example, they found that 10 of the 11 Unit Trusts had higher Sharpe ratios than the All Share Index and 6 of the 11 Unit Trusts had higher Treynor ratios. Although all the Jensen's alphas were positive only one was significantly greater than zero at the 5% level.

Knight and Firer (1989) applied Sharpe, Treynor and Jensen's measures to eleven unit trusts over the period 1977 to 1986. They found that all eleven funds outperformed the market based on Sharpe and Treynor's measures. Five funds were found to outperform the market significantly based on Jensen's alpha.

More recently studies examining the selection and timing ability of portfolio managers have been applied to South African Unit Trusts. Du P Smith and Chapman (1994) applied the Treynor and Mazuy measure to 28 funds over the period March 1973 to December 1992. They use one, three, five and seven year measurement periods. They found that only one fund exhibited significant timing ability over the one and three year periods and no funds exhibited significant timing ability over the five and seven year periods. Significant selection ability was exhibited by no funds over the one year period, two funds over the three and five year periods and four funds over the seven year period. They concluded that these results showed little evidence of market timing or selection ability.

Garvin (1995) applied Grinblatt and Titman's Portfolio Change Measure to 32 unit trusts. The period of study ranged from June 1970 to December 1992. He found that, based on the PCM, only one unit trust exhibited significantly superior performance.

Biger and Page (1993) employed the APT model in order to assess the performance of twenty-five unit trusts during the four-year period from February 1988 to March 1992. They developed two single-factor models, two three-factor models and two five-factor models. They found that the unit trusts seem to have outperformed a buy-and-hold strategy when measured within the framework of a single-factor model. However, for three and five factor economies the unit trusts as a group actually returned inferior performance.

In order to consolidate both the preceding theoretical discussion and the evidence from the various prior studies, an empirical investigation into the performance of South African Unit Trusts over the ten-year period June 1985 to June 1995 ensues.

3.4 EMPIRICAL INVESTIGATION

The primary purpose of this investigation is to demonstrate the practical implementation of the various performance measures on South African Unit Trusts. The actual relative performance of the various Unit Trusts, although interesting, was a secondary aim. As summarised in the previous Section, many studies have been conducted on South African Unit Trusts. However this study differs from previous studies in that it uses a more comprehensive array of the performance measures in the literature. Six of the preceding measures are implemented and the results contrasted in Section 3.4.2.

3.4.1 Data and Methodology

Although Grinblatt and Titman's (1993) measure and the measures based on the APT have theoretical appeal, their practical implementation is difficult. Grinblatt and Titman (1993) require not only the quarterly composition of each portfolio but the time series of prices and dividends for each share in each portfolio. The disadvantage of the APT is that the number and nature of factors that affect security prices are not specified and have to be determined empirically. It was therefore decided for the purposes of this study to implement and compare the first six measures presented in Section 3.2 and to omit the above-mentioned two measures.

Since most unit trust management companies advise that unit trusts should be seen as at least a three to five year investment, it was decided that five years would be the most relevant period over which to apply the measures. In order to be able to investigate whether there is any persistence in performance, it was necessary to calculate the measures for two non-overlapping five year performance periods. The unit trusts included in the sample are therefore the 13 unit trusts in existence for the ten year period June 1985 to June 1995 (In the results that follow abbreviations are used for these unit trusts - the full names of each trust and the fund type are included in the appendix in Section A3.1). The two non-overlapping measurement periods were June 1985 to June 1990 and June 1990 to June 1995.

Repurchase prices and dividend information were obtained from the University of Pretoria. For the purposes of this study transaction costs were ignored and the repurchase price of the unit trusts were used in the analysis[1]. Monthly rates of return were calculated using equation 3.20 which includes the dividends in the month of payment.

$$R_{pt} = [P_t - P_{t-1} + D_t]/P_{t-1} \qquad (3.20)$$

The JSE Actuaries All Share Index was chosen to represent the return on the market (R_m). Monthly rates of return for the market were calculated including the dividend yield. The 90 day Treasury Bill rate was used to represent the risk-free rate of return (R_f).

Since each of the six performance measures employed were discussed in Section 3.1, only a brief summary of each measure is presented below:

1. Sharpe's measure

The average excess return per unit of "total" risk

$$= \frac{\text{average excess return}}{\text{total risk}}$$

$$= \frac{\overline{R_{pt} - R_{ft}}}{\sigma_p}$$

[1] Since the costs that management companies charge are all very similar, it was felt that inclusion of transaction costs would not alter the relative comparison of the unit trusts'

2. Treynor's measure

The average excess return per unit of "market" risk

$$= \frac{\text{average excess return}}{\text{market risk}}$$

$$= \frac{\overline{R_{pt} - R_{ft}}}{\beta_p}$$

3. Jensen's alpha

The average return earned on the portfolio over and above a benchmark portfolio with the same systematic risk. It is a measure of the selection ability of the fund manager. Jensen's alpha (α_p) is the intercept from a regression of the excess return of the managed portfolio on the excess return of a benchmark portfolio.

$$(R_{pt} - R_{ft}) = \alpha_p + \beta_p(R_{mt} - R_{ft}) + e_{pt}$$

4. Treynor - Mazuy

This measure identifies both the selection ability (α_p) and the timing ability (ψ_p)of a manager.

$$(R_{pt} - R_{ft}) = \alpha_p + \beta_p(R_{mt} - R_{ft}) + \psi_p (R_{mt} - R_{ft})^2 + e_{pt}$$

5. Henriksson - Merton

This similar measure also identifies both the selection ability (α_p) and the timing ability (ψ_p)of a manager.

$$(R_{pt} - R_{ft}) = \alpha_p + \beta_p(R_{mt} - R_{ft}) + \psi_p (R_{mt} - R_{ft})^+ + e_{pt}$$

performance. Furthermore returns are compared with indices which themselves do not include transaction charges.

6. Bhattacharya and Pfleiderer

This model permits the separation of the selection and timing skills of fund managers, and is the first model to analyse the error term to identify a manager's forecasting ability.

A brief step by step procedure of the estimation process is outlined below:

Step 1: Run the regression specified in (3.7).

Step 2: Obtain from (3.7) the estimate of selection ability, α_p shown in (3.8).

Step 3: Run the regression in (3.12) using the squared residuals, w_t^2, obtained in (3.7).

Step 4: Factor out σ_e^2 by dividing the slope coefficient of (3.12) by $(\eta_2)^2$ (obtained from (3.7)).

Step 5: Estimate σ_π^2 using (3.14).

Step 6: Compute ρ, the final measure of market timing ability from (3.15).

Micro TSP version 6.54 was used to run the various required regressions for the above models. The generalised least square function of TSP was used to adjust for the heteroscedasticity in the Bhattacharya and Pfleiderer model. The TSP batch program is included in the appendix in Section A3.2. Microsoft Excel was used for all other calculations.

The two non-overlapping measurement periods (i.e. June 1985 to June 1990 and June 1990 to June 1995) were used in order to assess whether there was persistence in performance, i.e. whether the top performing funds in one period remain superior in the subsequent period. For both periods and each measure, funds were classified as "winners" or "losers". Winners were funds whose performance measure was greater than the average. Losers were funds whose performance measure was lower than the average. Contingency tables were formed on the basis of this classification and chi-squared tests of association were applied.

3.4.2 Results

The following six tables summarise the results for the more recent measurement period, June 1990 to June 1995 (The equivalent tables for the first five year period, June 1985 to June 1990 are included in the appendix in Section A3.3). Table 3.1a presents the results for Sharpe and Treynor's risk adjusted measures. Table 3.1b presents the Spearman's rank correlation coefficients for these measures. Tables 3.2a and 3.2b present the results and rank correlations for the various measures of selection ability while Table 3.3a and 3.3b present the results and rank correlations for the various measures of timing ability. The rank is presented next to each measure in italics. Figures in bold represent measures which are significant at the 5% level.

Table 3.1a below presents the most basic risk adjusted statistics for the 13 unit trusts and the JSE Actuaries All Share Index. The final row gives the averages excluding the index. The first column gives the average excess return for each unit trust over the period June 1990 to June 1995. The second column is the standard deviation of the excess return. The beta coefficient for each unit trust is given in the third column. The last two columns give the Sharpe and Treynor measures.

Six unit trusts have average excess returns above that of the index. Only three funds, Sanlam Mining Trust, UAL Mining and Resources and Standard Bank Gold, have higher standard deviations of return than the index. The same three unit trusts had higher standard deviations of return than the index in the previous period of analysis, June 1985 to June 1990. This reflects the riskiness of the mining sector. The same three unit trusts have betas greater than the index (i.e. greater than unity), another indication of the riskiness of the mining sector. All the general funds have betas less than unity. One would expect these funds to have lower betas than the index due to the cash component they are forced to hold. All but one of the beta coefficients are significant at the 5% level. Standard Bank Extra Income has a beta of 0.0148 which is not significant. One would not expect a high income fund to have a high or significant beta as the equity portion of this fund would be insignificant. One should bear this in mind when using measures that utilise beta.

Based on the two risk adjusted measures, six unit trusts outperformed the index. The rankings for both measures are identical for the top ten unit trusts. Since most funds in the sample are general equity funds these two measures would be expected to be nearly equivalent. Looking at Table 3.1b the Spearman's rank correlations between Sharpe and Treynor's measures and the average return are seen to be highly significant.

Comparing these results to the results of the previous period of analysis June 1985 to June 1990 (tables included in the appendix) we find that three funds had average excess returns above that of the index, five funds outperformed the index based on Sharpe's risk-adjusted measure and seven funds outperformed the index based on Treynor's measure.

TABLE 3.1a Risk Adjusted Performance Measures

Unit Trust	Average excess return	Standard deviation	Beta	Sharpe[a]	Treynor[b]
GUARD	0.003849 *3*	0.045334 *8*	**0.76109** *8*	0.084913 *4*	0.005058 *4*
OMIF	0.002735 *5*	0.047111 *6*	**0.84814** *6*	0.058055 *5*	0.003225 *5*
SAGE	0.002156 *6*	0.042062 *11*	**0.74300** *9*	0.051255 *6*	0.002902 *6*
SNI	0.000550 *10*	0.048030 *5*	**0.85986** *5*	0.011456 *10*	0.000640 *10*
SNPG	0.005404 *2*	0.041504 *12*	**0.63552** *11*	0.130210 *2*	0.008504 *2*
SNT	0.000752 *9*	0.042967 *10*	**0.73368** *10*	0.017502 *9*	0.001025 *9*
SBM	0.003355 *4*	0.037023 *13*	**0.56582** *13*	0.090620 *3*	0.005930 *3*
UAL	0.001313 *8*	0.043593 *9*	**0.76624** *7*	0.030110 *8*	0.001713 *8*
SNMT	-0.006747 *13*	0.061599 *2*	**1.02432** *1*	-0.109535 *14*	-0.006587 *12*
UALMR	-0.001530 *12*	0.056685 *3*	**1.00124** *3*	-0.026998 *12*	-0.001528 *11*
SBG	-0.007180 *14*	0.078126 *1*	**1.00770** *2*	-0.091901 *13*	-0.007125 *13*
SNIT	0.006410 *1*	0.045817 *7*	**0.62081** *12*	0.139900 *1*	0.010325 *1*
SBEI	-0.000683 *11*	0.028515 *14*	0.01478 *14*	-0.023954 *11*	-0.046206 *14*
JSE	0.001995 *7*	0.051688 *4*	**1.00000** *4*	0.038598 *7*	0.001995 *7*
Average*	0.000799	0.047567	0.73709	0.027818	-0.00170

*excluding JSE Index

Figures in bold are significant at 5% level of significance

Ranks presented next to each measure in italics

a Sharpe's measure calculated according to equation (3.1)

b Treynor's measure calculated according to equation (3.2)

TABLE 3.1b Spearman's rank correlation

	Sharpe	Treynor
Average excess return	**0.989011**	**0.961538**
Sharpe		**0.961538**

Figures in bold are significant at 5% level of significance

Spearman's rank correlation shows the correlation between the ranks in Table 3.1a

Table 3.2a below presents the statistics which measure the selection ability of fund managers. Only six unit trusts had positive Jensen's alphas and none of these were statistically significant at the 5% level. Sanlam Mining Trust had a significantly negative Jensen's alpha. This implies that the manager had negative stock selection ability. The average Jensen's alpha was negative (-0.000672). None of the unit trusts had significant selection ability according to the Treynor-Mazuy, Henriksson-Merton or Bhattacharya-Pfleiderer measures. Notice that the average alphas for these three measures are positive while the average Jensen alpha was negative. This supports Grant's (1977) contention that market-timing ability will cause Jensen's alpha to be downward biased. The rankings seem to be fairly consistent across the four measures as can be seen from the significant high rank correlations in Table 3.2b. However, given that the alphas are not significant it is unlikely that the differences between them would be significant so one should exercise caution when interpreting the ranks.

TABLE 3.2a Measures of selection ability

Unit Trust	Jensen[a]		Treynor - Mazuy[b]		Henriksson - Merton[c]		Bhattacharya - Pfleiderer[d]	
GUARD	0.002331	*3*	0.003541	*4*	0.005794	*2*	0.003597	*3*
OMIF	0.001043	*5*	0.000283	*7*	0.001434	*9*	-0.001352	*8*
SAGE	0.000674	*6*	0.002504	*6*	0.004528	*6*	0.002320	*6*
SNI	-0.001165	*10*	-0.001283	*10*	-0.000214	*12*	-0.001860	*10*
SNPG	0.004136	*2*	0.004614	*1*	0.005781	*3*	0.005441	*1*
SNT	-0.000712	*8*	-0.000507	*9*	0.000138	*10*	-0.000018	*7*
SBM	0.002226	*4*	0.003747	*3*	0.006322	*1*	0.002930	*4*
UAL	-0.000216	*7*	0.002520	*5*	0.005601	*4*	0.002384	*5*
SNMT	**-0.008800**	*12*	-0.005133	*13*	-0.001649	*13*	-0.005426	*13*
UALMR	-0.003528	*11*	-0.002331	*12*	-0.000133	*11*	-0.003487	*12*
SBG	-0.009190	*13*	-0.002055	*11*	0.003451	*7*	-0.001381	*9*
SNIT	0.005171	*1*	0.004512	*2*	0.005538	*5*	0.004293	*2*
SBEI	-0.000713	*9*	-0.000391	*8*	0.002592	*8*	-0.002466	*11*
Average	-0.000672		0.000771		0.003014		0.000383	

Figures in bold are significant at 5% level of significance

Ranks presented next to each measure in italics

a Jensen's α estimated according to equation (3.3)

b Treynor-Mazuy's α estimated according to equation (3.4)

c Henriksson-Merton's α estimated according to equation (3.5)

d Bhattacharya-Pfleiderer's α estimated according to equation (3.7)

TABLE 3.2b Spearman's rank correlation

	Treynor - Mazuy	Henriksson - Merton	Bhattacharya - Pfleiderer
Jensen	**0.945055**	**0.730769**	**0.895604**
Treynor - Mazuy		**0.868132**	**0.945055**
Henriksson - Merton			**0.857143**

Figures in bold are significant at 5% level of significance

Spearman's rank correlation shows the correlation between the ranks in Table 3.2a

For the previous period of analysis, June 1985 to June 1990, seven funds had positive Jensen's alphas and again none of these were statistically significant at the 5% level. In contrast to the later period one fund, Old Mutual Industrial Fund, had significant selection ability according to the Treynor-Mazuy, Henriksson-Merton and Bhattacharya-Pfleiderer measures. Further five other funds had significant selection ability according to the Henriksson-Merton measure. Standard Bank Gold Fund's manager had significant negative selection ability according to the Bhattacharya-Pfleiderer measure. Note that this fund had the lowest ranking for all four measures.

Table 3.3a below presents the statistics which measure the timing ability of fund managers. Only UAL Unit Trust had significant timing ability according to the Treynor-Mazuy and Henriksson-Merton measures. In both cases the unit trust exhibited significant negative timing ability. Seven unit trusts exhibited significant timing ability according to the Bhattacharya-Pfleiderer measure. Only one of these, namely Old Mutual Investor's Fund, exhibited positive timing ability. Note that although the results are not significant, Old Mutual Investor's Fund was ranked first and second according to the Treynor-Mazuy and Henriksson-Merton measures respectively. According to the Bhattacharya-Pfleiderer measure the highest correlation for negative timing ability was for UAL Unit Trust which is consistent with the results of the other two timing measures. (Note that if one had ignored negative timing ability as Lee and Rahman (1990) did in their study on American mutual funds one would have concluded that seven out of the thirteen funds in our sample exhibited significant positive timing ability, when in fact six funds exhibited significant negative timing ability.) Note the high rank correlations between all three measures of timing ability in Table 3.3b.

TABLE 3.3a Measures of Timing Ability

Unit Trust	Treynor – Mazuy[a]		Henriksson - Merton[b]		Bhattacharya - Pfleiderer[c]	
GUARD	-0.460317	*8*	-0.172941	*8*	**-0.380900**	*8*
OMIF	0.289415	*1*	-0.019532	*2*	**0.484770**	*1*
SAGE	-0.696552	*10*	-0.192478	*9*	**-0.679950**	*11*
SNI	0.044723	*3*	-0.047497	*4*	0.161212	*3*
SNPG	-0.181576	*6*	-0.082128	*5*	-0.206282	*7*
SNT	-0.078013	*4*	-0.042429	*3*	-0.134682	*5*
SBM	-0.578723	*9*	-0.204517	*10*	**-0.400530**	*9*
UAL	**-1.041300**	*11*	**-0.290500**	*11*	**-0.723730**	*13*
SNMT	-1.391972	*12*	-0.356645	*12*	**-0.527000**	*10*
UALMR	-0.455452	*7*	-0.169550	*7*	-0.171503	*6*
SBG	-2.715375	*13*	-0.631306	*13*	**-0.686190**	*12*
SNIT	0.251009	*2*	-0.018300	*1*	0.215524	*2*
SBEI	-0.122325	*5*	-0.165009	*6*	-0.123342	*4*
Average	-0.548957		-0.184064		-0.244046	

Figures in bold are significant at 5% level of significance

Ranks presented next to each measure in italics

a Treynor-Mazuy's ψ estimated according to equation (3.4)

b Henriksson-Merton's ψ estimated according to equation (3.5)

c Bhattacharya-Pfleiderer's ρ estimated according to steps set out in Section 3.4.1

TABLE 3.3b Spearman's Rank Correlation

	Henriksson - Merton	Bhattacharya - Pfleiderer
Treynor - Mazuy	**0.978022**	**0.961538**
Henriksson - Merton		**0.917582**

Figures in bold are significant at 5% level of significance

Spearman's rank correlation shows the correlation between the ranks in Table 3.3a

Comparing the results to the results of the previous period of analysis some interesting contrasts emerge. For all three measures Standard Bank Gold fund was the only fund with positive timing ability. This positive timing ability was significant at the 5% level only for the Bhattacharya-Pfleiderer measure. Five funds had significant negative timing ability according to the Treynor-Mazuy measure, seven according to the Henriksson-Merton measure and twelve according to the Bhattacharya-Pfleiderer measure. These results imply that the majority of unit trust managers had negative timing ability over the period June 1985 to June 1990. This may be a consequence of the confounding influence of the 1987 market crash.

Persistence

For both periods funds were classified as winners or losers. Two-way contingency tables (included in the appendix) for each measure were formed and Chi squared tests of association were conducted in order to assess whether there was any evidence of persistence in performance. The four measures which exhibited significant chi-squared statistics are tabulated below. The first column gives the chi-squared statistic and the second column the p-value. One should interpret the results below with caution due to small observed cell frequencies in some of the chi-squared tests.

TABLE 3.4 Chi-squared tests of association

Measure	χ^2	P-value
Sharpe	3.75	0.00530
Treynor - Mazuy Selection	3.90	0.0483
Henriksson - Merton Selection	3.90	0.0483
Bhattacharya - Pfleiderer Selection	4.95	0.0261

According to the Sharpe ratio, Treynor-Mazuy selection measure, Henriksson-Merton selection measure and the Bhattacharya-Pfleiderer selection measure there was found to be a significant positive relationship in performance between one measurement period and the next. That is, the winners in the first period tended to be winners in the second period. Note that three of the four selection ability measures indicated persistence while no timing ability measures indicated persistence. Treynor's measure showed no indication of persistence.

3.4.3 Conclusions

On the basis of this empirical investigation the relative performance of the thirteen South African Unit Trusts included in the sample can be summarised as follows :

Risk-adjusted measures : Six of the thirteen unit trusts in the sample outperformed the JSE All Share Index over the period June 1990 to June 1995 on a risk-adjusted basis according to both the Sharpe and Treynor measures. One must be cautious in making inferences on the basis of these results as no tests of statistical significance can be conducted on the results of these two measures.

Selection ability : According to all four measures of selection ability, no fund was found to exhibit significant positive selection ability, implying, at face value[2], that the managers of these unit trusts are unable to consistently select shares having abnormally high returns. One fund, Sanlam Mining Trust exhibited significant negative selection ability according to Jensen's alpha.

Timing ability : Only one fund, namely Old Mutual Investor's Fund, exhibited significant positive timing ability according to Bhattacharya and Pfleiderer's measure of timing ability. Note that although the results are not significant, Old Mutual Investor's Fund was ranked first and second according to the Treynor-Mazuy and Henriksson-Merton measures respectively. According to the Treynor-Mazuy and Henriksson-Merton timing ability measures no funds exhibited significant positive timing ability[3] and one fund, namely UAL Unit Trust, exhibited significant negative timing ability. UAL Unit Trust also exhibited the highest correlation for negative timing ability according to the Bhattacharya and Pfleiderer model. Altogether six unit trusts had significant negative timing ability according to the Bhattacharya and Pfleiderer model.

Persistence of fund performance : The results of this empirical investigation offer no clear support for or against the persistence argument. According to Sharpe's measure and three of the four selection ability measures there is persistence in fund performance from one period to the next. That is, the "winners" in the period June 1985 to June 1990 tended to be "winners" in the period June 1990 to June 1995 and vice versa. According to Treynor's measure, Jensen's alpha and the three

[2] This conclusion amongst other things takes no account of the realism of any underlying assumptions of the models - or indeed the power of the test!
[3] Again this conclusion amongst other things takes no account of the realism of any underlying assumptions of the models - or indeed the power of the test!

timing ability measures, however, there is no evidence of persistence in performance.

According to these measures South African managers have not shown any statistically significant skills to perhaps justify additional costs investors forgo for investing in such funds.

Although the actual relative performance of the various Unit Trusts is interesting, the primary aim of this empirical investigation was to demonstrate the practical implementation of the various performance measures. Insights gained from this empirical study are discussed below.

Insights from empirical study

The Spearman's rank correlations in Tables 3.1b, 3.2b and 3.3b are all positive and significant implying that the rankings of the various Unit Trusts are therefore similar within each category of performance measurement. In order to investigate the similarity of the rankings based on these measures and the naïve measure of average excess returns, the rank correlations were examined. Table 3.5 below shows the rank correlation of each measure with average excess returns. From Table 3.5 below it can be seen that apart from the three measures of timing ability, all the measures have highly significant positive rank correlations with average excess returns.

TABLE 3.5 Spearman's Rank Correlation with Average Excess Return

	Average excess return
Sharpe	**0.989011**
Treynor	**0.961538**
Jensen's alpha	**0.994505**
Treynor - Mazuy Selection	**0.934066**
Henriksson - Merton Selection	**0.708791**
Bhattacharya - Pfleiderer Selection	**0.901099**
Treynor - Mazuy Timing	0.428571
Henriksson - Merton Timing	0.489011
Bhattacharya - Pfleiderer Timing	0.274725

Figures in bold are significant at 5% level of significance

Spearman's rank correlation shows the correlation between the ranks according to average excess return and the various measures

For the purposes of ranking funds relative to each other, it may be argued that as the measures become more sophisticated, not much is gained at the expense of ease of interpretation. Average excess return and measures such as Sharpe and Treynor's are easy to implement and make intuitive sense. The more sophisticated measures are useful, however, in that they enable one to obtain a more specific breakdown of performance. Not only is one able to examine the performance of the fund manager, but one is also able to distinguish between the ability to select shares with abnormal performance and the ability to time the market.

Two additional insights from the empirical investigation are worth mentioning here. Firstly, in Table 3.2a the average alphas for the Treynor-Mazuy, Henriksson-Merton and Bhattacharya-Pfleiderer measures are positive while the average Jensen's alpha is negative. This supports Grant's (1977) contention that market-timing ability will cause Jensen's alpha to be downward biased. Secondly, if negative timing ability had been ignored with the Bhattacharya and Pfleiderer measure one would have come to the erroneous conclusion that seven out of the thirteen funds in the sample exhibited significant positive timing ability. In fact, six of these funds exhibited significant negative timing ability.

3.5 CONCLUSIONS

In this chapter the various portfolio performance measures were reviewed with the aim of providing a practical interpretation of each measure. Each measure was discussed and the practical interpretation, advantages and disadvantages were summarised. Although the focus of this chapter is on reviewing the measures themselves a brief empirical investigation of a sample of South African Unit Trusts was conducted in order to demonstrate the application of these measures. The results of the empirical investigation showed little evidence of selection or market timing ability.

CHAPTER 4

INDEX FUND CONSTRUCTION

4.1 INTRODUCTION

Since the first index fund was established in the early 1970s in the US, vast sums of capital have been attracted to this form of investment. Bishop (1990) estimates that between 25% to 30% of all pension fund assets in the US, and 10% of all pension fund assets in the UK are managed on an index fund basis. In South Africa index funds have emerged relatively recently, with the first, Composite Unit Trust Manager's All Share Index Fund being launched only in mid 1993. Subsequently Investec, Standard and Fedgro have also established local index funds.

What is an index fund ?

An index fund is a managed portfolio of shares which has been specifically designed to track, as closely as possible, the performance of a particular index. The purpose of an index fund is not, in any sense, to outperform a given index but to perform, as closely as possible in line with it.

Rationale for indexation

The motivation for indexation is supported by the joint hypothesis that the validity of the CAPM and the expected optimality of the "market" portfolio are theoretically equivalent assertions. In other words, in a CAPM world, the market portfolio is expected to provide the highest level of return per unit of risk.

Hence the implication is that investors should simply hold the "market portfolio". A broad index is possibly the most suitable proxy to the "market portfolio". Hence an index fund, which attempts to mimic a broad index, is expected to be the most efficient portfolio in terms of risk-return trade-offs.

Active managers may argue that tracking the index is guaranteed mediocrity but, Bishop (1990) points out that there has been consistent evidence that the majority of actively managed funds in the UK and the US have not produced consistently higher returns than the indices they are aiming to beat. In South Africa institutional portfolios have performed fairly well in relation to the index. According to Braun (1993), only one unit trust failed to beat the All Share Index over the three year period June 1990 to June 1993. In June 1993, over the past five years 55% of the unit trusts beat the All Share Index; over the last ten years 63% beat the index and over the last twenty years 29% beat the index. According to the empirical investigation in the previous chapter 46% of the unit trusts had higher average returns than the All Share Index over the period June 1990 to June 1995. The same unit trusts outperformed the index according to Sharpe and Treynor's risk adjusted measures (see Section 3.4.2). According to Herman Steyn[1] and Adrian Baskir[2], one of the reasons why unit trust managers have beaten the All Share Index in recent years is that they have been underweight in mining shares, which have been sub-performers over the last few years. However, either the mining sector will continue to underperform and will thus begin to constitute a smaller part of the index (since it is capitalisation weighted) or the mining sector will turn and institutional portfolios will be caught underweight in mining shares.

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[1]Herman Steyn is currently Composite Unit Trust Management's All Share Index Fund portfolio manager.
[2]Adrian Baskir is currently Old Mutual's Investment Development Manager.

A further argument for index funds is the low costs associated with running them. Since maintenance is simple and turnover is low, management fees and transaction costs are traditionally much lower than in an actively managed portfolio. For example, although the initial fees are the same as their other unit trusts, Standard Bank does not charge an annual service fee on its index fund[3].

Evaluation of index funds is straightforward as they have a specific objective, that of tracking the index, and a clearly defined benchmark, the index they are tracking.

Methods of construction

Index funds can be constructed using a variety of techniques. Broadly speaking, however, there are two basic approaches to index fund construction. The first approach involves including in the fund all the components which comprise the index in precisely the same proportion, such funds are commonly referred to as "fully replicated funds". By contrast the second approach involves selecting a smaller sample of stocks to proxy the index, commonly termed "sampled funds". Briefly the objective of a sampling approach is simply to select a smaller set of shares than those represented in the index with the goal of mimicking the performance of the index as closely as possible.

Although fully replicated funds are theoretically appealing there are logistical problems in establishing and maintaining such funds. The costs of setting up a fully replicated index fund are substantial. As the entire index must be purchased, it will be necessary to deal in many small stocks. The result is that the fund can have acute liquidity problems which are exacerbated in thinly traded environments such

[3] Source : Lambrechts, H (1996) 96 Unit Trusts Handbook

as the JSE. As so many shares need to be held maintenance is expensive and time consuming. In theory all new cash flows must be divided proportionally between all the stocks in the index, resulting in a high number of tiny trades. Since the structure of the index is constantly changing, fully replicated index funds need to be rebalanced frequently. In this chapter attention is therefore focused only on assessing the various approaches for constructing "sampled funds".

Any method other than full replication necessarily exposes the portfolio to unique risk. Since the objective of index funds is not only to match the returns on the index, but also to achieve this at minimum cost, sampled funds enable one to overcome the trading difficulties and cost associated with full replication, albeit, at a lower degree of accuracy. In the literature dealing with "sampled index funds", two basic techniques have emerged, these techniques are generally referred to as "optimisation" and "stratification". The term "stratified sampling" generally refers to techniques which attempt to proxy both the characteristics of market capitalisation and sector distribution inherent in the index being mimicked. In essence optimisation can be thought of as a search for a combination of shares that in aggregate has similar characteristic features as the index being mimicked.

Monitoring performance

Central to the area of index fund construction is the necessity to establish a criterion for adjudging the proximity , or congruence, of the index fund to the index being mimicked. There are numerous measures used to monitor the performance/proximity of an index fund relative to the index. Although the objective of an index fund is clearly to track the chosen index as closely as possible, how one measures the fulfilment of this objective is not so clear. Three basic measures commonly used to monitor the performance of an index fund have emerged in the literature, namely tracking error, correlation between the returns on the index and the index fund and the unique risk of the index fund. Attention is also given to these measures in this chapter.

This chapter is organised as follows:

The ensuing Section (Section 4.2) provides a background discussion of the methodology used in the empirical investigation (in Section 4.3). In particular in Section 4.2 the sampling techniques of stratification and optimisation are introduced and the technical details of their construction are discussed. The various statistics used to monitor performance are also reviewed in Section 4.2.

Section 4.3 presents the results of the empirical assessment on the JSE. In the empirical assessment both sampling techniques are implemented and tested over a three and a half year period. Additionally particular attention is given to the impact of (i) changing the number of shares in the fund as well as (ii) the trade-off between frequency of revision and transaction costs incurred. Conclusions based on the empirical investigation are cited in the final Section (Section 4.4).

4.2 REVIEW AND METHODOLOGY

Index funds have received increasing attention in the financial media since they were first formed in the early 1970s. Most of the discussions are centred around the merits of index funds, their different forms, their performance relative to actively managed portfolios and the different methods of constructing them. Although the two central sampling techniques, employed to proxy the market index, of optimisation and stratification are widely known, little has been written on the technical details of these methods of construction in the academic literature. In a book on index funds edited by Bishop (1990), Liesching and Manchanda (1990) give a fairly detailed discussion about stratification and optimisation, but do not elaborate on the technical aspects of the construction of these sampled index funds. To the author's knowledge, only two articles by Rudd (1980) and Meade and Salkin (1989) actually propose methods of constructing sampled index funds and empirically test their performance on the New York Stock Exchange and the Tokyo Stock Market respectively. In the ensuing discussion (Section 4.2.1) the sampling techniques implemented in the two above-mentioned articles, namely stratified sampling and optimisation, are reviewed. The two selection techniques are outlined in sections 4.2.1.1 and 4.2.1.2. The results of Rudd (1980) and Meade and Salkin's (1989) empirical studies are summarised in Section 4.2.2.

There are numerous measures used to monitor the performance of an index fund. Although the objective of an index fund is clearly to track the chosen index as closely as possible, how one measures the fulfilment of this objective is not so clear. There are three basic measures commonly used to monitor the performance of an index fund, namely tracking error, correlation between the returns on the index and the index fund and the unique risk of the index fund. If the beta of the index fund is one, these three measures can be shown to be equivalent. However, the fund beta is typically not one during the test period and these measures are no longer equivalent. Some interesting implications, discussed in sections 4.2.3.2 and 4.2.3.3, for the use of these statistics in monitoring the performance of an index fund emerge due to this lack of equivalency. Tracking error is usually some cumulative/aggregate measure of the difference between the return on the fund and the index being proxied. Various methods of measuring tracking error are proposed by Bishop (1990), Meade and Salkin (1989), Rudd(1980) and Roll (1992). The implications of these various measures of tracking error are considered in Section 4.2.3.1.

4.2.1 Sampling Techniques

In this Section a more detailed discussion on the two methods of constructing sampled index funds, namely stratification and optimisation, is presented.

4.2.1.1 Stratification

The term "stratified sampling" generally refers to techniques which attempt to proxy both the characteristics of market capitalisation and sector distribution inherent in the index being mimicked. Thus, with stratified sampling all companies above a minimum pre-specified size are selected. The weighting of each company is based on their relative capitalisation weighting in the index. The balance of the portfolio is made up of smaller capitalisation companies, which are selected in order to attain similar industry/sector weightings as the index being mimicked.

Rudd (1980) proposes the following steps for index fund construction based on stratification:

1. Specify the total dollar value, D, of the fund and a minimum investment proportion, h, of the total value of the fund. The number of "units" in the fund is 1/h and the dollar value of each unit is $ h x D.
2. A list of companies with total value D is then specified from all the constituents of the index being mimicked. The weight of each company in the list is determined by its relative capitalisation- weighting. These weights are then ranked from

smallest to largest. All companies that have a weighting in this list of h or more are then selected. The index fund is subsequently formed by purchasing these companies in "units" of dollar value $ h x D. The number of "units" of each company in the index fund is determined so that the dollar value of the weight of the company in the index fund is as close as possible to the dollar value of the company in the capitalisation-weighted list.

3. If all purchases based on capitalisation-weighting are completed and there are still "units" remaining, the balance of the fund is formed by matching additional purchases to sector weightings. These additional purchases start with the sector in which the index fund is most underweight and "units" of shares are bought from the sector that has not yet been purchased. Once the weighting of that sector in the index fund and the index is now matched, to the nearest "unit", shares are purchased from the sector in which the index fund is now most underweight. This process continues until all "units" are exhausted.

The stratified sampling technique is fairly unsophisticated in the sense that it only attempts to match the fund and index holdings along the two dimensions of capitalisation and industry groups. The second sampling technique discussed below, that of optimisation, by contrast is mathematically more sophisticated.

4.2.1.2 Optimisation

In the area of sampled index funds, optimisation can generally be described as a mathematical process which identifies the best available solution subject to specified constraints. In essence optimisation can be thought of as a search for a combination of shares that in aggregate has similar characteristic features as the index being mimicked. An objective function is usually specified in terms of minimising some measure of dissimilarity between the fund and the index. In the literature the measures of dissimilarity that have typically been used in the objective function are tracking error and unique variance[4].

A critical consideration underpinning the success of the optimisation technique is that the historical statistical relationships between share prices and index values persist, or more simply put, consideration must be given to the fact that although the approach yields an optimal solution using historical data, these optimal results will not necessarily persist on "unseen" or "out-of-period" data.

Rudd (1980) argues that an index fund should theoretically have the same exposure to aggregate economic events as the index. In the context of the market model this exposure translates to the index fund being constrained to having a beta of unity. Furthermore an obvious consequence of sampled funds is that perfect diversification cannot be achieved with fewer shares than the index comprises. As a consequence sampled index funds will thus exhibit random variability around the index. This random variability should clearly be as small as possible if an index fund is to successfully mimic the market index. The reduction of this variability involves minimising unique risk.

Rudd (1980) formulates the optimisation problem as such :

Min σ^2_{ep}

subject to: (4.1)

$$\beta_p = \Sigma^n_{i=1} x_i\beta_i = 1$$

$$\Sigma^n_{i=1} x_i = 1; \qquad x_i \geq 0$$

where σ^2_{ep} is portfolio residual variance

[4]See for example Rudd(1980) and Meade and Salkin (1989).

β_p is the portfolio beta, defined as the weighted sum of the asset betas, where the weights are the asset holdings; and

x_i is the revised holding of the i^{th} asset in the portfolio, assumed non-negative (i.e. no short sales are allowed).

Rudd (1980) develops his optimisation model (4.1) further to include transaction costs as rebalancing of the fund should only take place when transaction costs are not greater than the increase in utility arising from either maintaining a beta closer to unity or decreasing residual risk. Rudd (1980) incorporates transaction costs directly into the objective function which permits the effect of adding or deleting a stock to be quantified so that the cost of transaction can be compared directly with the benefit of the trade. Realistic modelling of transaction costs is computationally unfeasible as it includes both a set-up cost and is cost dependent on the amount traded. Rudd (1980) therefore assumes that costs are proportional to the transaction and introduces two parameters into the objective function which define the after-transaction cost risk-returns trade-off. He does not, however, elaborate on how to estimate these parameters. Under this model the only trades undertaken are where the benefit is greater than the cost.

Meade and Salkin (1989) measure tracking error as the root mean square deviation between the returns on the index and on the index fund. They formulated their optimisation technique as unconstrained minimisation of tracking error.

Meade and Salkin (1989) then identified two desirable properties that a fund manager may wish an index fund to possess:

i. the fund should have a portion invested in each industrial sector similar to that in the index.

ii. the representation of a share in the index fund should be proportional to the valuation of the company.

The optimisation technique of minimising tracking error was reapplied incorporating these properties as constraints. The optimisation problem was formulated subject to each constraint separately and then with both properties combined into one constraint.

The four versions of the optimisation problem can be summarised as follows :

a. unconstrained minimisation of tracking error

b. minimisation of tracking error subject to the constraint that the index fund has the same proportion invested in each industrial sector as the index

c. minimisation of tracking error, where the proportion of each share that is chosen is predetermined by its capitalisation in the underlying index

d. minimisation of tracking error subject to both the constraints.

Below it is demonstrated that the optimisation techniques proposed by Rudd (1980) and Meade and Salkin (1989) are equivalent.

From the market model unique risk can be expressed as

$$\begin{aligned} var(e_p) &= var(R_p - (\alpha_p + \beta_p R_m)) \\ &= var(R_p - \beta_p R_m) \end{aligned}$$

where

e_p is the error term for the portfolio

R_p is the return on the portfolio

α_p is the alpha of the portfolio

β_p is the beta of the portfolio

R_m is the return on the market

If $\beta_p = 1$

this becomes

$$var(e_p) = var(R_p - R_m)$$

Hence minimisation of $var(R_p - R_m)$ is equivalent to minimising $var(e_p)$ subject to β_p =1. That is, minimisation of the variance of tracking error is equivalent to minimising unique risk subject to a fund beta of one.

The above two optimisation techniques can also be shown to be equivalent to minimising total risk and maximising correlation between the returns on the fund and the index subject to a fund beta of one.

Firstly consider the total risk of the portfolio. From the market model

$$\sigma^2_p = \beta^2_p \sigma^2_m + \sigma^2_{ep}$$

Now if β_p =1

$$\sigma^2_p = \sigma^2_m + \sigma^2_{ep}$$

$$= \text{constant} + \sigma^2_{ep}$$

So clearly minimisation of σ^2_{ep} subject to β_p =1 is equivalent to minimising σ^2_p subject to β_p =1. That is, minimisation of unique risk subject to a fund beta of unity is equivalent to minimising the total risk of the portfolio subject to a fund beta of unity.

Secondly consider the correlation between the returns on the index and the portfolio.

$$\beta_p = \sigma_{pm}/\sigma^2_m$$

$$= \rho_{pm}\,\sigma_p\sigma_m / \sigma^2_m$$

$$= \rho_{pm}\,\sigma_p / \sigma_m$$

Now if β_p =1

then rearranging the above equation yields

$$\rho_{pm} = \sigma_m / \sigma_p$$

$$= \quad \text{constant} / \sigma_p$$

Hence minimisation of σ_p subject to $\beta_p = 1$ is equivalent to maximising ρ_{pm} subject to $\beta_p = 1$. That is, minimisation of the standard deviation of the returns on the portfolio (i.e. the square root of total risk) subject to a fund beta of one is equivalent to maximising the correlation between the returns on the index and the portfolio subject to a fund beta of one.

It has been demonstrated above that the following four optimisation techniques are equivalent

(i) minimisation of the unique risk of the fund subject to the fund having a beta of one

(ii) minimisation of the variance of tracking error, where tracking error is the difference in the returns on the index and the fund

(iii) minimisation of the total risk of the fund subject to the fund having a beta of unity

(iv) maximisation of the correlation between the returns on the index and the fund subject to the fund having a beta of unity.

Calculation of the unique risk of a fund involves more computation than calculation of the total risk of a fund. Since minimisation of the unique risk and the total risk of the fund are equivalent, if the fund has a beta of unity, it is computationally more efficient to formulate the objective function of the optimisation technique in terms of the total risk of the fund. The third technique outlined above is thus adopted in the empirical investigation in Section 4.3. This optimisation procedure is implemented and tested over 42 months on the JSE. The results of the analysis are given in Section 4.3.2.

4.2.1.2.1 Estimation of parameters for optimisation

Empirical assessment of estimation period for optimisation

The stratification procedure outlined in Section 4.2.1.1 is completed when all funds are invested. The resultant index fund is in no sense optimal. By contrast an optimised index fund is optimal over the formation period. In order for the optimisation technique to maintain this advantage out of the sample, or test, period, the estimates of the parameters used need to be both as accurate/reliable and as stable as possible.

In choosing the length of the estimation period for parameters, such as beta, trade-offs need to be made between the desire to maximise the number of data points used, the stability of the underlying statistics, and the desire for the estimates to reflect recent changes. Bowie and Bradfield (1993) identify 60 months of data to have traditionally been considered a reasonable compromise between the stability of the underlying betas and a sufficient number of data points for efficient estimation. In their Financial Risk Service, Bradfield and Bowie use monthly data for their estimates as they argue that using daily, or even weekly, prices in order to maximise the number of data points is futile as it introduces considerable noise (error) which can bias the estimates. Meade and Salkin (1989), by contrast, use monthly data from a two year estimation period in their simulated tests of their sampling techniques. This shorter estimation period would clearly allow recent changes to have a greater impact on the estimates. If, for example, a firm changes its underlying systematic risk structure its beta will change.

In order to assess the impact of a shorter estimation period versus the 60 month period traditionally used on index fund performance, a brief empirical analysis on the JSE ensues. The analysis was conducted for four optimised index funds using both a two year and a five year formation period. The four index funds were based on four different constraints on the number of shares held in order to assess whether the size of the fund had an impact on this assessment.

Data and Methodology

The universe of shares was the top 30 market capitalisation shares, for which there was a suitable set of data, on the JSE at January 1992. The optimisation technique used is described in Section 4.2.1.2. The objective function was to minimise total variance of the fund's returns, subject to a beta of one. The optimisation problem was run four times, each time reducing the maximum number of shares in the index fund. As the only criterion being assessed was a suitable length for the estimation period, both transaction costs and portfolio rebalancing/revision were not considered. The five year estimation period ranged from January 1987 to December 1991, while the two year period ranged from January 1990 to December 1991. The three monitoring statistics used were correlation between the returns on the index and the fund, the variance of the monthly tracking errors and the unique risk of the fund. Later, in Section 4.2.3, a more complete discussion of monitoring performance ensues.

Results

Tables 4.1 through to 4.4 show the correlation between the returns on the index and the index fund, the variance of tracking error and the unique risk of the index fund over the period January 1992 to June 1995, for the four different sizes of index funds separately.

TABLE 4.1 Comparison of estimation periods

No restriction on the number of shares

	2 YEARS	5 YEARS
CORRELATION	0.95394	0.97375
VARIANCE OF TRACKING ERROR	0.00024	0.00022
UNIQUE RISK	0.00022	0.00017

TABLE 4.2 Comparison of estimation periods

15 Shares only

	2 YEARS	5 YEARS
CORRELATION	0.93702	0.96720
VARIANCE OF TRACKING ERROR	0.00031	0.00030
UNIQUE RISK	0.00029	0.00022

TABLE 4.3 Comparison of estimation periods

10 Shares only

	2 YEARS	5 YEARS
CORRELATION	0.90977	0.96008
VARIANCE OF TRACKING ERROR	0.00046	0.00038
UNIQUE RISK	0.00044	0.00028

TABLE 4.4 Comparison of estimation periods

5 Shares only

	2 YEARS	5 YEARS
CORRELATION	0.85838	0.93500
VARIANCE OF TRACKING ERROR	0.00090	0.00050
UNIQUE RISK	0.00084	0.00044

For each variation of the optimisation problem, the five year estimation period resulted in a higher correlation and a lower variance of tracking error and unique risk in the test period. From this empirical investigation it appears that the desire to reflect recent information in the estimates produces poorer performance as adjudged by the three monitoring statistics shown in the above tables. Although the differences between these monitoring statistics are not tested for significance they are all consistently pointing in the same direction (i.e. a five year estimation period produces better results than a two year period). Clearly the shorter estimation period yields less reliable/stable estimates of the true parameters. Portfolios formed on the basis of these unstable estimates shift further away from the index out of period. The empirical study in Section 4.3, therefore uses a five year estimation period.

Thin trading

Another important consideration in the accuracy of estimates is the phenomenon of thin trading. Thinly traded shares are shares which are not traded every day. Bradfield (1989a) found that there was a high degree of thin trading on the JSE. When a share is not traded, the price that is recorded is the price at the last trade. This recorded price is thus not necessarily equal to the share's underlying theoretical value. Market movements over the period that it is not traded are not reflected in the price. Bowie and Bradfield (1993) found that when using Ordinary Least Squares (OLS) regression to estimate beta thin trading caused a downward bias in the estimates. They showed in an empirical study that a trade to trade approach produces the most accurate estimates of betas of thinly traded shares. In the empirical study in Section 4.3 the universe of shares was restricted to the top 30 market capitalisation shares. Since these shares do not suffer from significant thin trading problems, OLS regression was used in the estimates of beta. If, however, one wanted to chose a broader universe of shares, one would be well advised to use betas which have been corrected for thin trading[5].

4.2.2 Results of empirical studies in the literature

Both Rudd (1980) and Meade and Salkin (1989) tested their sampling techniques empirically in order to compare their performances.

The index that Rudd (1980) chose to track was the SP500 on the NYSE. He tested his portfolios over the period January 1988 to June 1989. For his optimised index funds he included transaction costs and revised the portfolios every three months. A risk analysis of the resulting portfolios was conducted. Annual residual standard deviation was used as the monitoring statistic. In all cases he found that optimisation produced portfolios with lower annual residual standard deviations than stratification.

Meade and Salkin (1989) tested their optimisation techniques on the Tokyo stock exchange. They restricted their universe of shares to the top 100 largest market capitalisation companies. They tested their portfolios from January to September 1987. They did not revise their portfolios in this period. Tracking error, measured as the root mean square deviation between the returns on the index and the fund, was used to monitor the performance of the funds in the test period. Correlation between coincident returns on the index and the fund and correlation between coincident values of the index fund and the index were also considered as performance

[5] The Financial Risk Service produced by Bradfield and Bowie gives beta estimates which

indicators. They found that unconstrained minimisation of tracking error produced the best results. The imposition of the additional constraints of stratification and capitalisation led to worse performance, with stratification carrying a higher penalty than capitalisation. The imposition of both constraints led to the worst performance.

4.2.3 Monitoring performance

Since the objective of an index fund is to track the chosen index as closely as possible, the statistics used to monitor the performance of the fund in the test period should reflect the congruence or similarity of the fund with the index. As mentioned previously there are three basic measures used to monitor performance of index funds, namely tracking error, the correlation between the returns on the fund and the index and the unique risk of the fund. In Section 4.2.1.2 these three measures were shown to be equivalent when the fund beta is one. Typically after the fund has been formed the beta of a fund may well shift (off unity), implying the three measures of performance will no longer be equivalent. (In any event forming funds using the stratification technique does not require the fund beta to be unity.) Hence in the empirical analysis that follows in testing and monitoring index funds "out of period" consideration is given to all three basic performance measures.

4.2.3.1 Tracking Error

Tracking error is one statistic which indicates how closely an index fund tracks the chosen index. There is broad agreement that tracking error is a function of the difference between the return on the index fund and the return on the index being proxied. The exact formulation of this difference, however, differs from author to author. Meade and Salkin (1989) define tracking error to be the mean square

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have been corrected for thin trading.

deviation between the returns of the index and the index fund over a period. Their definition is dependent on both the frequency of observations and the length of the period. Bishop (1990) and Rudd (1980) define tracking error as the percentage difference in returns of the index and the index fund. Bishop (1990) states that this difference is usually expressed annually. Roll (1992) by contrast defines tracking error to be the monthly difference between the portfolio and index returns and examines both the mean and the variance of this tracking error.

These different approaches to measuring tracking error can lead to confusion when trying to compare the performance of different index funds. As an illustration consider the three figures below. Figure 4.1 depicts the comparison of variances of the monthly tracking error for two index funds A and B on the JSE over the period January 1992 to June 1995 (these index funds were constructed in the empirical study in Section 4.3). Figure 4.2 depicts the mean monthly tracking error and Figure 4.3 depicts the forty-two month difference in the return on the index and the index fund, i.e. using only the index and fund values at the beginning and the end of the period. If one chose to measure tracking error by its variance, one would prefer portfolio A over portfolio B. On the other hand, if one was concerned with how close the mean tracking error was to zero, or how close the difference in returns between the index and the index fund were over a long period, one would prefer portfolio B to portfolio A.

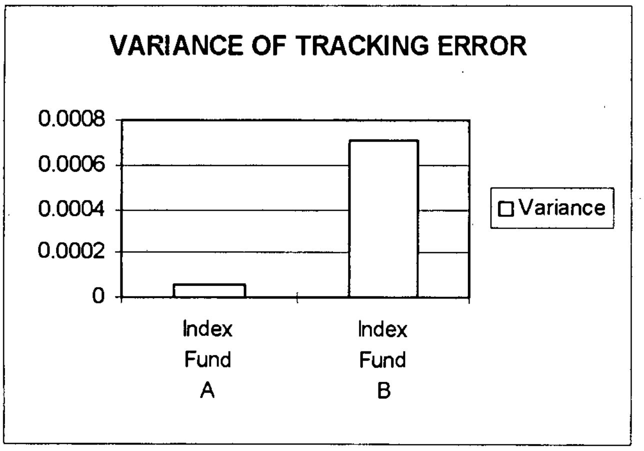

FIGURE 4.1 Comparison of two index funds - January 1992 to June 1995

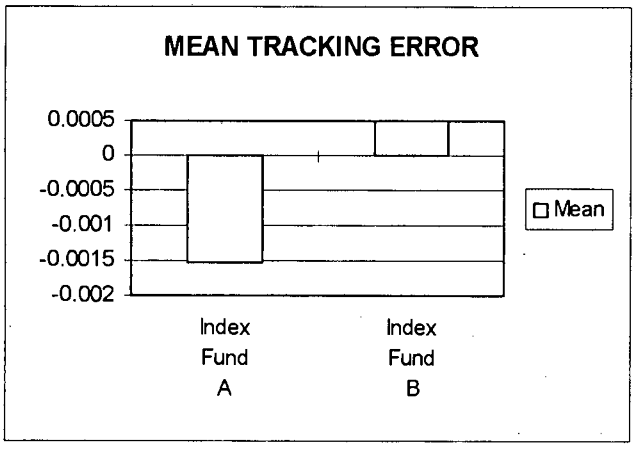

FIGURE 4.2 Comparison of two index funds - January 1992 to June 1995

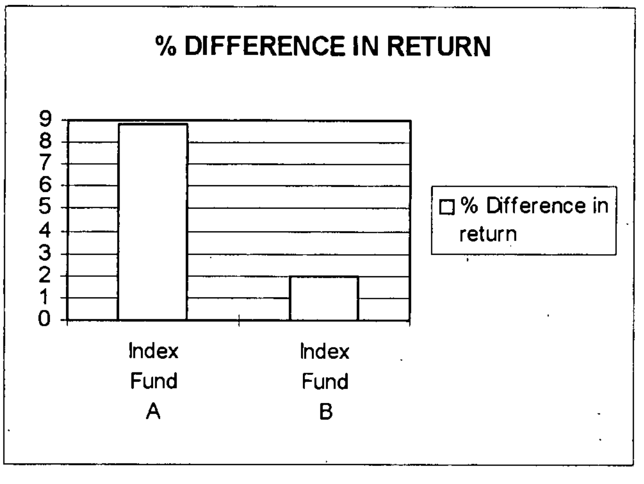

FIGURE 4.3 Comparison of two index funds - January 1992 to June 1995

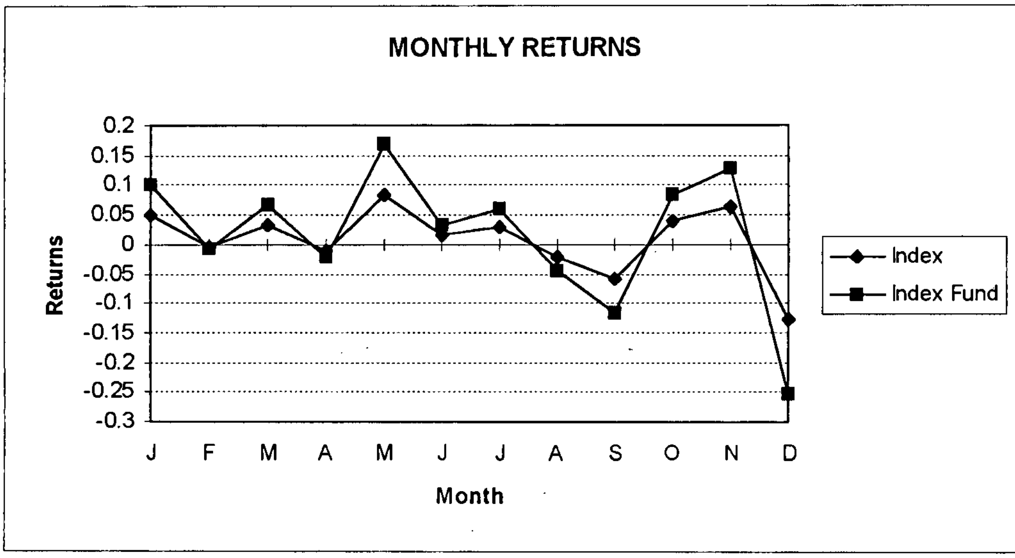

FIGURE 4.7 Monthly returns of two hypothetical funds

Portfolios with a larger total variance, will also generally have larger correlations with the index. To see this consider the following equation relating the square of the correlation coefficient to the various sources of variance:

$$R^2 = SSR/SST \qquad (4.2)$$

$$= 1 - SSE/SST$$

where

SSE is the sum of squares due to error ,

SSR is the sum of squares due to regression and

SST is the total sum of squares .

In terms of the market model, equation (4.2) can be expressed as follows :

$$\rho^2_{pm} = \beta^2_p \sigma^2_m / \sigma^2_p$$

$$= 1 - \sigma^2_{ep} / \sigma^2_p$$

where

ρ_{pm} is the correlation between the returns on the portfolio and the market index,

β_p is the beta of the portfolio,

σ^2_m is the variance of the returns on the market index,

σ^2_p is the variance of the returns on the portfolio,

σ^2_{ep} is the residual variance or unique risk of the portfolio.

Clearly if σ^2_p is large, the correlation will be high (as long as σ^2_{ep} is not proportionately larger than σ^2_p). Since the rationale behind index funds is to hold a well diversified low risk portfolio, one should be cautious in using correlation as a measure of performance.

4.2.3.3 Unique risk of the index fund

Unique risk is a useful measure because with an index fund one hopes to be as well diversified as possible. A theoretically perfect index fund should only exhibit market risk. One would want an index fund to be exposed to as little unique risk as possible. However, one also wants the index fund to have the same exposure to aggregate economic events as possible. For example, a portfolio with a beta of 0.5 may have a lower unique risk than one with a beta of one. The portfolio with a beta of 0.5 will not, however, respond to aggregate economic events with the same magnitude as the index. One should thus desire an index fund to have minimal unique risk jointly with a beta as close to unity as possible.

4.3 EMPIRICAL STUDY OF THE PERFORMANCE OF THE SAMPLING TECHNIQUES

In this Section the results of an empirical study conducted on the JSE are presented. The central aim of the analysis is to assess the suitability of the various sampling techniques for constructing and running index funds on the JSE. The stratification and optimisation sampling techniques are specifically focused on.

Some secondary aims are (i) to assess the impact of decreasing the number of constituents of the index funds, (ii) to gain insight as to the suitability of the various measures of monitoring performance and (iii) to gain insight into the extent of revision required and the effect of transaction costs.

4.3.1 Data and Methodology

The data used in the analysis consisted of month-end price series ranging from January 1987 to June 1995. The index which formed the basis of the analysis was the JSE Actuaries Overall Index. This index (by construction) incorporates 80% (of the market capitalisation) of the shares in each sector.[6] In the ensuing analysis all the sampled funds were tested over the period January 1992 to June 1995, using monthly data. Furthermore all the funds were revised/rebalanced at the end of each year. Since the central purpose of the analysis was to compare the two above mentioned sampling techniques, a number of assumptions were made to simplify the study. Estimated transaction costs of 2.89% of the amount sold and purchased were included[7]. Dividends were ignored[8] and it was assumed that there were no new cash flows into the fund. The prices used when estimating transaction costs and calculating returns were the last recorded prices at each month-end. Additionally the analysis was repeated for various constraining scenarios on the number of shares in the funds.

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[6]Source: JSE Actuaries Indices. In March 1995 the index was changed. The new index is comprised of 100% of the shares. Although the index being tracked for the last four months of the study was different, it is unlikely to have had any impact on the outcome of the analysis.

[7]Transaction costs are calculated on a sliding scale basis, the percentage to be paid depends on the amount sold/bought. In order to avoid constraining the analysis to funds of certain monetary values, on the advice of brokers at Simpson McKie Inc we used 2.89% for a round trip estimate of transaction costs.

[8]The price time series of the index being proxied clearly excluded dividends as well!

Stratification

The stratified sampling technique attempts to match the characteristics of the size (i.e. market-capitalisation) of companies and the spread across sectors to that of the index being mimicked. The reader is referred to the method outlined in Section 4.2.1.1 for details of the construction methodology. The opportunity set of shares considered comprised all the constituents of the JSE Actuaries Overall Index at the beginning of each year. The analysis was also re-run several times for a variety of minimum investment size constraints in order to assess the impact of reducing the number of shares held in the fund.

Optimisation

As stated previously the optimisation approach in essence involves a search for a combination of shares that have similar characteristic features to the index being mimicked. The reader is referred to the method outlined in Section 4.2.1.2 for the details of the construction methodology. The opportunity set of shares was restricted to the largest 30 (by market capitalisation) companies reviewed at year-end. Based on the results of the empirical investigation conducted in Section 4.2.1.2.1, a five year estimation period was used for the computation of beta and of portfolio variance. Furthermore the portfolio betas were estimated using OLS and were not corrected for thin trading[9]. Additionally the optimisation analysis was re-run several times using different constraints on the maximum number of shares in the fund. The optimisation problem was solved using a Microsoft Excel algorithm.

4.3.2 Results

The results of the empirical study are summarised by statistics which reflect the congruence or similarity between the fund and the index. All three performance measures discussed in the previous Section, namely tracking error, correlation between the returns on the index and the fund and the unique risk of the fund, are presented. Tracking error was calculated using monthly intervals and both summary statistics of tracking error i.e. mean and variance are presented. Additionally the unique risk and beta of the funds are presented. Since transaction costs are an important consideration the average percentage of the portfolio sold at each revision is also included.

The following tables compare the summarised "out-of-period" performance of the sampling techniques, stratification and optimisation over the 1992 to 1995 test period. Since the number of shares in the fund could be controlled in each method, the results for a 5, 10 and 15 share fund are shown in Tables 4.5, 4.6 and 4.7 respectively (The actual composition of the portfolios and the graphs of the time series of returns on the JSE All Share Index and the portfolio are included in the appendix).

TABLE 4.5 "Out of period" performance January 1992 to June 1995 - 5 Shares

	STRATIFICATION	OPTIMISATION
CORRELATION	0.91287	0.92579
BETA	1.01711	1.22068
UNIQUE RISK	0.00049	0.00059
AVERAGE % OF PORTFOLIO SOLD AT EACH REVISION	26.0	57.3
VARIANCE OF TRACKING ERROR	0.000495	0.00071
MEAN TRACKING ERROR	0.00005	0.00050

[9]The top thirty market capitalisation shares do not suffer from significant thin trading

TABLE 4.6 "Out of period" performance January 1992 to June 1995 - 10 Shares

	STRATIFICATION	OPTIMISATION
CORRELATION	0.93601	0.94894
BETA	0.88249	1.27905
UNIQUE RISK	0.00026	0.00043
AVERAGE % OF PORTFOLIO SOLD AT EACH REVISION	19.5	47.43
VARIANCE OF TRACKING ERROR	0.000296	0.00062
MEAN TRACKING ERROR	-0.00077	-0.00029

TABLE 4.7 "Out of period" performance January 1992 to June 1995 - 15 Shares

	STRATIFICATION	OPTIMISATION
CORRELATION	0.94178	0.95540
BETA	0.87868	1.23411
UNIQUE RISK	0.00023	0.00035
AVERAGE % OF PORTFOLIO SOLD AT EACH REVISION	16.35	44.03
VARIANCE OF TRACKING ERROR	0.00027	0.00048
MEAN TRACKING ERROR	0.00191	0.00079

Several important insights emerge from comparison of the above tables. A summary of the pertinent features of the above-mentioned tables follows. In order to keep the discussion on the above important tables focused the various summary statistics are discussed under the ensuing headings.

problems.

Number of shares

From the above tables it is evident that as the number of shares in the funds increase (i.e. comparing the statistics across the tables) the correlation between the returns on the funds and the index increase for both sampling techniques (clearly as a consequence of the fund having less components of the index being tracked). With the optimisation technique for example, the correlation increases from 0.92579 for a 5 share fund to 0.95540 for a 15 share fund. The improvement in correlation is similar for the stratified index funds.

Furthermore, with both sampling techniques, increasing the number of shares in the funds results in decreases in both the unique risk, the variance of tracking error and the percentage of the portfolio sold at revision. For the stratification technique, for example, the unique risk decreases from 0.00049 to 0.00023 and the variance of tracking error decreases from 0.000495 to 0.00027. The percentage of the portfolio sold at revision for the optimised funds is 57.3% for the 5 share fund by contrast to 44.03% for the 15 share fund.

There seems to be no obvious relationship between the number of shares in the optimised funds and the absolute value of mean tracking error. The mean tracking error decreases from 0.00050 to 0.00029 and then increases again to 0.00079 as the number of shares increases. With the stratification technique the absolute value of mean tracking error increases from 0.00005 for the 5 share fund to 0.00191 for the 15 share fund. If one were to use the mean tracking error as an indication of performance, one deduces that the tracking ability of the fund improves as the number of shares in the fund decreases. This is clearly counterintuitive and highlights the shortcoming of using the mean tracking error as a monitoring statistic for index funds.

Tracking error

Variance of tracking error

From the tables it is evident that the variance of tracking error is consistently lowest for stratified sampling and highest for optimisation. For example, the variance of tracking error of the 15 share stratified index fund shown in Table 4.7 is only 0.00027, while the optimised fund with the same number of shares has a tracking error variance of 0.00048.

Mean tracking error

From the above tables there is no clear or obvious relationship between the absolute value of the mean tracking error and the sampling technique used. The mean tracking error for the 15 share optimised fund (0.00079) is lower than that for the 15 share stratified fund (0.00191). With the 5 share funds, however, the mean tracking error for the optimised fund (0.00050) is higher than that for the stratified fund (0.00005). Furthermore, it has already been noted that there is no obvious relationship between the number of shares in the optimised funds and this performance measure. With the stratified funds the counterintuitive result that mean tracking error decreases as the number of shares decreases was noted. Clearly by averaging over positive and negative signs information on the departure from the index is destroyed. Mean tracking error could however be used to detect if any significant under/over performing bias is evident.

The author therefore tentatively concludes for the purpose of monitoring tracking ability that, of the two stand alone measures, variance of tracking error appears to be the more appropriate measure to monitor index funds. Henceforth the mean tracking error will be excluded in the summarised results that follow.

Correlation

The correlation between the sampled funds and the index being mimicked is seen to be marginally higher for the optimisation technique for the 5, 10 and 15 share funds. The correlations are however, fairly similar for both techniques with the largest difference being only 1.4 percent. Note that with the 15 share funds, for example, although the variance of tracking error and unique risk are lower for the stratified fund the correlation is higher for the optimised fund. This observation is consistent with the discussion in the previous Section where it was pointed out that correlation is not an equivalent measure to tracking error when the fund beta is not unity. Furthermore it is worth re-emphasising that in Section 4.2.3.2 it was pointed out that a perfect correlation (i.e. of unity) does not imply perfect tracking ability.

Unique risk and beta

From the tables it is evident that, across all fund sizes, the unique risk is lowest and the betas are closest to unity for the stratified funds. For example, with the 10 share funds the stratification technique results in a fund having a unique risk of 0.00026 and a beta of 0.88249 by contrast to a unique risk of 0.00043 and a beta of 1.27905 for the optimisation technique. Interestingly, although only the optimisation technique requires the fund beta to be unity at formation, the betas for the optimised funds are consistently further from unity in the test period than the stratified funds.

Average percentage of the portfolio sold at each revision

The percentage of the funds sold at each revision was lowest for stratification and highest for optimisation. For example, for the 10 share funds, on average 19.5% of the stratified fund was sold by contrast to the average 47.43% of the optimised fund sold.

Some preliminary conclusions

It should be borne in mind that by construction the optimisation index funds are optimal during the formation period. However, they will only remain optimal in the test period if the statistical relationships which affected the index values and the share prices in the formation period persist. The success of the optimisation sampling technique thus relies on the accuracy and stability of the estimates used in the formation period. From the above results the author concludes that the estimates on which the optimisation technique is based are not sufficiently stable for the optimisation technique to yield superior results. Rudd (1980) by contrast found, in his empirical study in the US, that the optimisation technique produced superior results to the stratification technique. Although Liesching and Manchanda (1990) do not refer to any empirical investigations to support their claim, they state that optimisation "offers the greatest sophistication in terms of the trade-off between costs and accuracy of tracking".

The results of this empirical study on the JSE are thus not consistent with their findings. Rudd's (1980) investigation was conducted on the New York Stock Exchange and Liesching and Manchanda's (1990) discussions are mostly centred around index funds in the US and the UK. A plausible explanation for our contrary results is that the South African market is generally more volatile than those in the US and the UK which clearly impacts on the stability/reliability of estimates. For example, Bradfield (1989b) shows that the standard deviations of estimates, such as beta, in South Africa are typically larger than the equivalent estimates in the US and the UK. From the results of this empirical study it is evident that due to the instability/unreliability of estimates used in the optimisation procedure, optimisation does not appear to be a superior technique to stratification on the JSE.

4.3.3 Frequency of revision and transaction costs

A further important/practical consideration which was not considered empirically by Rudd (1980) or Meade and Salkin (1989) is the impact of changing the frequency of revision of the sampled funds on, for example, transaction costs. Rudd (1980) revised the index funds every three months and Meade and Salkin (1989) did not revise their portfolios in their nine month test period. Clearly in the absence of transaction costs, one should revise the fund as frequently as possible. However there is an obvious a trade-off between the frequency of revision and the effects of transaction costs on tracking ability. An extreme case is examined below in order to gain insights into the relationship between revision, transaction costs and tracking ability.

Revision, in the absence of transaction costs, clearly results in a fund that mimics the chosen index more closely than an unrevised fund. However, the practitioner cannot ignore transaction costs. So the purpose of this exercise was to ascertain the effect that transactions costs have on the funds. In the following tables the annually revised funds are compared to funds formed in January 1992, which remain unchanged until December 1995. The effect of transaction costs on the revised funds is fairly dramatic.

TABLE 4.8 Comparison between revised and unrevised funds - Stratification

	Revised	Unrevised	Revised	Unrevised	Revised	Unrevised
Number of shares	**15 SHARES**		**10 SHARES**		**5 SHARES**	
CORRELATION	0.94178	0.96058	0.93601	0.96098	0.91287	0.91283
BETA	0.87868	0.92083	0.88249	0.94595	1.01711	1.02805
UNIQUE RISK	0.00023	0.00016	0.00026	0.00017	0.00049	0.00049
VARIANCE OF TRACKING ERROR	0.00027	0.000182	0.000296	0.000185	0.000495	0.000504

TABLE 4.9 Comparison between revised and unrevised funds - Optimisation

	Revised	Unrevised	Revised	Unrevised	Revised	Unrevised
Number of shares	**15 SHARES**		**10 SHARES**		**5 SHARES**	
CORRELATION	0.95540	0.95862	0.94894	0.95118	0.92579	0.93470
BETA	1.23411	1.19263	1.27905	1.21679	1.22068	1.20095
UNIQUE RISK	0.00035	0.00029	0.00043	0.00036	0.00059	0.00049
VARIANCE OF TRACKING ERROR	0.00048	0.00026	0.00062	0.00028	0.00071	0.00044

The results in the preceding tables are initially discussed for each sampling technique and then followed by some general conclusions.

Stratification

Table 4.8 summarises the out of period performance results of the revised and unrevised stratified funds. Apart from the funds with 5 shares, the unrevised funds have higher correlations between the returns on the index and the funds, betas close to unity, lower unique risks and lower variance of tracking error.

Optimisation

Table 4.9 summarises the performance results of the revised and unrevised optimised funds. The unrevised funds have higher correlations between the returns on the index and the funds, betas closer to one, lower unique risk and lower variance of tracking error than the revised funds.

From the above results, one comes to the bizarre conclusion that funds that were not revised for three and a half years had better tracking ability than annually revised funds. The percentage of the portfolio that needs to be sold at each revision with the optimisation technique is very high, with over 50% of the 5 share funds being sold each year. The impact of transaction costs on tracking ability is severe.

Some conclusions

In both cases above, the unrevised funds performed fairly well in relation to the revised funds, and with the optimisation technique the unrevised funds clearly outperformed their revised counterparts. The gain in accuracy by rebalancing the fund is completely offset by the transaction costs. Since the percentage of the portfolio that needed to be sold at each revision was highest for optimised index funds, their tracking ability was most detrimentally affected by transaction costs.

Since the objective of index funds is not only to track the index as closely as possible but to do this at the minimum cost, ways of reducing transaction costs must be seriously considered. One suggestion given by Rudd (1980) is that transaction costs be included in the objective function of the optimisation technique, so that trades are only undertaken when the benefit is greater than the cost. With both sampling techniques, one should attempt to sell shares as seldom as possible, and rebalance the fund solely with purchases. Implementation of this strategy would, however, require new cash flows. Index futures should be used to build up cash flows so that transactions are conducted only when most efficient.

4.4 CONCLUSIONS

The main focus of this chapter was to assess and compare two sampling techniques, namely stratification and optimisation, for constructing and running index funds on the JSE. In the empirical investigation attention was also given to the effect of reducing the number of shares in the fund and to the trade-off between frequency of revision and transaction costs incurred. Additionally the suitability of the various measures of monitoring performance were investigated.

In considering the various measures of performance, their ability to reflect the congruence or similarity of the funds with the index was evaluated. The theoretical problems with correlation and unique risk were highlighted in Section 4.2.3 and in the empirical investigation mean tracking error produced counterintuitive results. The author therefore tentatively concludes that variance of tracking error appears to be the most appropriate measure for monitoring the tracking ability of index funds.

Insights into the trade-off between frequency of revision and transaction costs incurred, and the effect of decreasing the number of shares in the fund were gained from the empirical investigation. The impact of transaction costs on tracking ability was found to be severe, with the advantages of revision largely offset by the transaction costs incurred. The optimised funds were particularly affected by transaction costs as larger proportions needed to be sold at rebalancing. As expected decreasing the number of shares in the fund resulted in a deterioration of the tracking ability of the funds.

The most important conclusion of this chapter is that, in contrast to studies conducted on the New York and Tokyo Stock Exchanges, stratification appears to be the superior sampling technique on the JSE. The superiority of stratified funds was based on the criteria that stratified funds had lower variance of tracking error, lower unique risk and betas closer to unity than their optimised counterparts.

At the risk of being repetitious, the findings of this chapter are summarised in the form of advice to practitioners who may be interested in setting up index funds and investors who may be considering index funds as an investment vehicle.

1. Caution should be exercised when using statistics such as correlation and unique risk to monitor tracking ability. Variance of tracking error seems to be the most suitable measure of tracking ability.
2. Since transaction costs are so detrimental to tracking ability, the fund should be run in such a way as to minimise these. The frequency of revision and approaches to trading, such as avoidance of selling and the use of index futures, should be investigated so that an optimal strategy can be adopted.
3. On the basis of the results of this empirical investigation it is recommended that stratification, and not optimisation, be used as a technique for constructing and running index funds on the JSE.

CHAPTER 5

INTERNATIONAL FUND CONSTRUCTION

5.1 INTRODUCTION

The interest in "international" or "global" unit trusts was recently highlighted with the growth of these funds during March to June 1996 when the assets of international unit trusts grew 46,7% from R461.8 million to R677,4 million. Although these unit trusts are labelled "international", local legislation currently permits only 10% of the funds to be held offshore via an asset swap. Hence they are not able to share fully in the benefits of International Diversification and consequently attempt to make up the balance (90%) from local investment. In most instances it is apparent that the local investments that make up this 90% typically consist of rand hedge shares i.e. shares of companies that operate off-shore or earn revenue off-shore.

The major aim of this chapter is to suggest a quantitative approach to constructing "international funds" from local shares which to some extent mimic international diversification. A model proposed by Bradfield (1990) is used to decompose the risk of "local" shares into both international and local components. In this way "local" shares which are linked to the performance of foreign/international markets can be identified. The next step is to form portfolios of these shares based on the strength of these foreign associations which, it is hoped will mimic some of the benefits of International Diversification. Although there may be several quantitative approaches the author prefers to use the model developed by Bradfield as this model remains within the "risk-return" framework in which most of Capital Market Theory has evolved.

The well known market model, in essence, makes a simple statement concerning the relationship between the returns on a given security and the returns on some market index. The coefficients of the model are of considerable importance to financial analysts and researchers alike. Estimation of the coefficients of the market model requires, inter alia, a series of returns of the given security, and a series of returns on some market index. Usually the market index is constructed from some aggregate of value-weighted securities of the local stock market in question. It is well known that indices constructed in this manner embody movements caused by factors which influence the market as a whole, for example local rates, inflation, the business cycle etc. It is also likely however, that movements of overseas markets may have an impact on local markets. Consider for example the events of October 1987 (and to a lesser extent October 1989) when the prices of shares listed on the New York Stock Exchange (NYSE) fell dramatically. Prices on all stock exchanges world-wide including Belgium, Frankfurt, London, Paris, Tokyo, Hong Kong, Sydney and the Johannesburg Stock Exchange fell in unison. The crash illustrated that events occurring on international markets such as the NYSE can affect stock markets world-wide.

In order to investigate the relationship between international stock markets, the correlation between the NYSE, the London Stock Exchange (LSE), the Tokyo Stock Exchange (TSE) and the Johannesburg Stock Exchange (JSE) was estimated using monthly data from January 1990 to July 1996. The indices used to represent these markets were the Standard and Poor's 500, the Financial Times 100, the Nikkei-

Dow and the JSE-Actuaries Overall Index respectively. In addition the correlation between all these markets and a "world market" was estimated. The Morgan Stanley World Index was used as a proxy for the "world market". Table 5.1 shows the correlation coefficients between the rand returns on these indices.

Table 5.1 reveals that in all cases the correlations are positive and significant at the 5 percent level of significance, confirming that significant relationships between stock markets do exist. The overseas market that has the highest correlation with the JSE is the LSE with a correlation coefficient of 0.336. The highest correlation of 0.736 in Table 5.1 is between the Japanese and World market index.

TABLE 5.1 Correlation coefficients between rand returns on international markets (January 1990 - July 1996)

	JSE	NYSE	LSE	TSE	WORLD
JSE	1.000				
NYSE	0.244	1.000			
LSE	0.336	0.492	1.000		
TSE	0.265	0.375	0.445	1.000	
WORLD	0.192	0.572	0.631	0.736	1.000

Lessard (1974) conducted a study on the world-wide influences on stock returns. His study included 16 major stock exchanges and a constructed "world index". The study examined the international diversification benefits from the viewpoint of the investor with dollars to invest. Lessard (1974) showed that, on average, 22 percent of the variation in the market indices could be explained by the "world index".

Erunza and Losq (1985) by contrast, have focused on the debate on whether international markets can be thought of as being either segmented or a perfectly integrated single market. Errunza and Losq (1985) conducted a study incorporating the US market as well as nine lesser developed countries[1] (LDC's). On the basis of their results they find tentative support for a mild segmentation hypothesis. This hypothesis assumes that the world's various capital markets do not behave as if they were a perfectly integrated efficient single market, and that this behaviour is caused by the fact that many non-USA countries restrict free access to capital markets. Various other aspects of the international segmentation-integration issue have been investigated by Solnik (1974), Black (1974), Adler and Dumas (1975), Grauer *et al* (1976), Glenn (1976), Stehle (1977), Stapleton and Sabrahmanyam (1977), Stulz (1981a,1981b) and Bradfield *et al.* (1988).

Most of the above researchers however have focused on various aspects of asset pricing under conditions of market equilibrium. In this chapter a more modest objective is pursued; here the focus is concerned primarily with estimating more detailed risk diagnostics for individual securities in order to identify securities with significant international risk components. In the following section a "multi-market" model, proposed by Bradfield (1990), in order to estimate a more detailed breakdown of risk than that of the traditional market model is presented. In Section 5.3 an empirical study is presented which demonstrates how the model can be used to estimate the risk components across all shares listed on the JSE. In Section 5.4 a portfolio construction technique based on the selection of shares with foreign risks is presented and empirically tested on the JSE. In the final section concluding remarks are made.

[1] The nine LDC's are: Argentina, Brazil, Chile, Greece, India, Korea, Mexico, Thailand and Zimbabwe.

5.2 THEORETICAL DISCUSSION

The notion of risk was originally dealt with in two main conceptual frameworks, the state-preference framework developed by Arrow (1951)and later Debreu (1959), and the mean-variance framework developed by Markowitz (1952). The state-preference framework approach assumes that objects of choice yield payoffs offered in different states of nature. While this framework is useful for investigating theoretical issues, it lacks empirical content due to the difficulty in quantifying all the payoffs offered in different states of nature. The pioneering work of Markowitz (1952,1959) on portfolio selection in the mean-variance framework however, is the framework that subsequently captured the interest of researchers in the field of financial economics and paved the way for the development of Capital Market Theory. It is within this framework that the variance of a series of returns has been well-entrenched in literature as a measure of total risk. One of the advantages of the model presented below therefore is that it addresses the notion of risk in the mean-variance framework within which the bulk of Capital Market Theory has evolved.

The model is essentially similar to the model used by Errunza and Losq (1985) to investigate the notion of segmentation/integration of world markets and is also similar in spirit to the models used by Lessard (1973,1974), Solnik (1974) and Stehle (1977) who also investigated the risk across world markets.

The model outlined below does not require any assumption about its process generating returns (see Stehle (1977)). This model, in essence, relates the return of a security listed on some local market to the return on a local market index, plus the return on a world market index. In order to obtain tractable expressions for the risk components, i.e. local market risk, world market risk and unique risk, the vector of local and world market returns are orthogonalised. This amounts to removing the effect of the world index from the returns of the local index. This can be simply achieved by regressing the returns of the local index on the returns of the world index, and using the resultant residuals to represent the local index with the effects of the world index removed.

The model, henceforth referred to as the multi-market model can be written as

$$R_{it} = \alpha_i + \beta_i^{LOC\text{-}WO} R_{mt}^{LOC\text{-}WO} + \beta_i^{WO} R_{mt}^{WO} + e_{it} \qquad (5.1)$$

where

R_{it} is the return on share i at time t ;

α_i ,$\beta_i^{LOC\text{-}WO}$, β_i^{WO} are coefficients unique to share i ;

R_{mt}^{WO} is the return on the world market index at time t ;

$R_{mt}^{LOC\text{-}WO}$ is the residual local market index return at time t, obtained by regressing the returns of the local market index on the world market index returns; and the following assumptions regarding the e_{it} are made :

$E(e_{it}) = 0$

$COV(e_{it} ; e_{is}) = 0$ for $t \neq s$

$COV(R_{mt}^{WO} ; e_{it}) = 0$ for all t

$COV(R_{mt}^{LOC\text{-}WO} ; e_{it}) = 0$ for all t .

The components of risk for security i can be obtained by considering the expression for the variance of security i 's returns, i.e.

$$Var(R_{it})$$

$$= Var(\alpha_i + \beta_i^{LOC\text{-}WO} R_{mt}^{LOC\text{-}WO} + \beta_i^{WO} R_{mt}^{WO} + e_{it})$$

$$= Var(\alpha_i) + \beta_i^{LOC\text{-}WO^2} Var(R_{mt}^{LOC\text{-}WO}) + \beta_i^{WO^2} Var(R_{mt}^{WO}) + 2\beta_i^{LOC\text{-}WO} \beta_i^{WO} COV[R_{mt}^{LOC\text{-}WO}; R_{mt}^{WO}] + Var(e_{it})$$

Since α_i is a constant Var (α_i) = 0 and; by construction $COV[R_{mt}^{LOC\text{-}WO}; R_{mt}^{WO}] = 0$; the above expression simplifies to :

$$Var(R_{it}) = \beta_i^{LOC\text{-}WO^2} Var(R_{mt}^{LOC\text{-}WO}) + \beta_i^{WO^2} Var(R_{mt}^{WO}) + Var(e_{it}) \quad (5.2)$$

Thus the above expression can be interpreted as:

Total risk = Local market risk only + world market risk + unique (diversifiable) risk.

It is clear that model (5.1) can easily be extended to include a more comprehensive explanation of an individual shares total risk by including several other specific overseas market indices in the model as well. In order to obtain tractable expressions for these risk components however, it follows that *all* independent variables are once more orthogonalised. To avoid over elaborate interpretation and estimation procedures, the three specific “other” markets chosen, i.e. the NYSE, the LSE and TSE, are considered separately in the extended models shown below:

$$R_{it} = \alpha_i + \beta_i^{LOC\text{-}WO\text{-}USA} R_{mt}^{LOC\text{-}WO\text{-}USA} + \beta_i^{USA\text{-}WO} R_{mt}^{USA\text{-}WO} + \beta_i^{WO} R_{mt}^{WO} + e_{it} \qquad (5.3)$$

$$R_{it} = \alpha_i + \beta_i^{LOC\text{-}WO\text{-}UK} R_{mt}^{LOC\text{-}WO\text{-}UK} + \beta_i^{UK\text{-}WO} R_{mt}^{UK\text{-}WO} + \beta_i^{WO} R_{mt}^{WO} + e_{it} \qquad (5.4)$$

$$R_{it} = \alpha_i + \beta_i^{LOC\text{-}WO\text{-}JAP} R_{mt}^{LOC\text{-}WO\text{-}JAP} + \beta_i^{JAP\text{-}WO} R_{mt}^{JAP\text{-}WO} + \beta_i^{WO} R_{mt}^{WO} + e_{it} \qquad (5.5)$$

where

R_{it} is the return on share i at time t ;

α_i ,$\beta_i^{LOC\text{-}WO\text{-}USA}$, $\beta_i^{LOC\text{-}WO\text{-}UK}$, $\beta_i^{LOC\text{-}WO\text{-}JAP}$, $\beta_i^{USA\text{-}WO}$, $\beta_i^{UK\text{-}WO}$, $\beta_i^{JAP\text{-}WO}$, β_i^{WO} are coefficients unique to share i ;

R_{mt}^{WO} is the return on the world market index at time t ;

$R_{mt}^{USA\text{-}WO}$ is the residual USA market index return at time t (with the effect of the world return market removed);

$R_{mt}^{UK\text{-}WO}$ is the residual UK market index return at time t (with the effect of the world market return removed);

$R_{mt}^{JAP\text{-}WO}$ is the residual JAP market index return at time t (with the effect of the world market return removed);

$R_{mt}^{LOC\text{-}WO\text{-}USA}$ is the residual local market index return at time t (with the effects of both the world and US market returns removed);

$R_{mt}^{LOC\text{-}WO\text{-}UK}$ is the residual local market index return at time t (with the effects of both the world and UK market returns removed);

$R_{mt}^{LOC\text{-}WO\text{-}JAP}$ is the residual local market index return at time t (with the effects of both the world and JAP market returns removed);

It can easily be shown that the variance of the dependent variable, R_i, can be decomposed into several components using either model (5.3), (5.4) or (5.5). From model (5.3) :

$$Var(R_{it}) = \beta_i^{LOC\text{-}WO\text{-}USA^2} Var(R_{mt}^{LOC\text{-}WO\text{-}USA}) + \beta_i^{USA\text{-}WO^2} Var(R_{mt}^{USA\text{-}WO}) + \beta_i^{WO^2} Var(R_{mt}^{WO}) + Var(e_{it}) \quad (5.6)$$

which can be interpreted as :

Total risk = Local market risk only + USA market risk + World market risk + unique (diversifiable) risk.

Similarly from models (5.4) and (5.5) the risk can be decomposed into several components.

5.3 THE COMPONENTS OF RISK: AN EMPIRICAL DEMONSTRATION

Copeland and Weston (1992) state that the market model is not supported by any theory but only assumes that the slope and intercept terms are constant over the time period during which the model is fit to available data. Since the model presented in the previous section is similar in spirit to the market model the focus of the empirical analysis therefore is concerned more with *demonstrating* the use of the extended model rather than on *testing* it.

Data and methodology

The data used in the empirical demonstration consisted of the monthly[2] return series of all securities on the JSE with a five year price history as well as the JSE-Actuaries Overall Index (representing the local market index)and the Morgan Stanley World Index (representing the world market index). The period of study ranged from January 1990 to July 1996.

The multi-market model (5.1) and the extended multi-market models (5.3), (5.4) and (5.5) were estimated using the return data of each of the securities over the period January 1990 to December 1994[3]. Fortran programs, included in the appendix, were written to estimate the coefficients of the models and decompose the risk for each share.

The resulting beta coefficients, for shares with significant "world" or foreign betas, obtained from running the regressions (5.1), (5.3), (5.4) and (5.5) on each of the securities are shown in the Appendix.

Results

Instead of focusing on the coefficients of these models the focus is on the decomposition of total risk into the various components of risk so that the influence

[2] Monthly data is traditionally used to estimate the coefficients of the market model.
[3] Although the period of study was up until July 1996, the model was fitted for this period in order that there would be some historical "unseen" data on which to test the effectiveness of the portfolio construction technique in the following section.

of the world market and particular foreign exchanges on the risk of local stocks can more easily be examined. The detailed breakdown of the components of risk are presented separately for each of the models in the following four tables.

Table 5.2 gives a detailed breakdown of the components of risk expressed as a percentage of total risk for all the shares on the JSE which exhibited significant foreign betas over the period January 1990 to December 1994. The shares in Table 5.2 are ranked according to the percentages of each share's "world" market risk (relative to total risk).

The risk components derived from Model (5.1) contain useful information on the risk characteristics of individual securities and as such Table 5.2 reveals several interesting features:

In particular, after searching across the population of shares on the JSE only 36 securities on the JSE had significant world betas. Furthermore the percentage of world market risk was found to be greater than 10 percent for 8 of these 36 shares, with Fedsure having the largest component, making up 15.2 percent of total risk. The average world risk component was 8.1 percent.

TABLE 5.2 Percentage of total risk attributable to each of the components of risk (as at 31/12/1994)

Share Name	World Risk[a] Component %	JSE Risk[b] Component %	Unique Risk[c] Component %
FEDSURE	15.2	8.2	76.6
FINTECH	14.5	7.5	78.1
INVICTA	13.8	0.7	85.5
CHARTER	13.2	1.8	85.1
OVBEL	12.1	2.4	85.5
PROPFIN	11.8	2.6	85.6
JADE	11.7	1.2	87.1
W-R-CONS	10.9	4.4	84.7
SA-BREWS	9.5	27.1	63.4
BERTRAD	9.4	6.1	84.5
ALLWEAR	9.3	3.9	86.8
BLYVOOR	9.0	6.7	84.4
CROOKES	8.2	7.7	84.1
MALBAK	8.2	18.8	73.0
WALTONS	7.9	26.9	65.2
IBJOFFE	7.8	2.4	89.8
HYPROP	7.5	4.7	87.8
METCASH	7.5	7.8	84.7
SPECLTY	7.3	5.2	87.5
DA-GAMA	7.1	8.3	84.7
BEVCON	6.9	20.6	72.5
TAMBOTI	6.9	1.8	91.3
FIT	6.8	9.6	83.6
SILTEK	6.4	12.2	81.4
LASER	6.4	0.8	92.8
PUTCO	6.2	2.4	91.4
JASCO	6.2	4.9	89.0
CBD-FUND	6.1	1.6	92.3
AECI	5.8	11.9	82.3
RICHEMONT	5.7	21.9	72.3
LEPLAT	5.2	23.4	71.4
AMIC	5.1	33.9	61.0
WALHOLD	5.0	23.6	71.3
SAPPI	5.0	21.0	74.1
RUSPLAT	4.3	43.2	52.5
ANGLO-AM	2.1	71.4	26.5
Average	8.1	12.7	79.2

All underlying world beta coefficients significant at the 5% level

a Percentage of risk computed using: $\beta^{WO2}_{i\,LOC\text{-}WO2}$ Var($R^{WO}_{mt\,LOC\text{-}WO}$) * 100 / Var($R_{it}$)

b Percentage of risk computed using: β_i Var (R_{mt}) * 100 / Var(R_{it})

c Or diversifiable risk i.e.: $(1-R^2) * 100$

Table 5.3 gives a detailed breakdown of the risk components as decomposed in (5.6) for the period January 1990 to December 1994. Once more these are expressed as a percentage of total risk and the individual shares are ranked according to their percentage of USA market risk.

TABLE 5.3 Percentage of total risk attributable to each of the components of risk (as at 31/12/1994)

Share Name	World Risk[a] Component %	USA Risk[b] Component %	JSE Risk[c] Component %	Unique Risk Component %
LIB-HOLD	0.2	26.0	33.2	40.5
CGS-FOOD	1.4	21.8	5.7	71.2
ALLWEAR	9.3	21.5	0.5	68.7
RICHEMONT	5.7	20.8	12.5	61.1
STANPRO	1.5	19.5	5.1	73.8
LIBERTY	0.8	18.5	32.6	48.1
ANAMINT	2.8	17.8	35.3	44.1
CGSMITH	0.0	16.8	19.6	63.5
DEBEERS	2.5	14.7	39.1	43.6
LENCO	0.2	13.7	13.0	73.1
TOCO	1.0	13.5	4.9	80.6
GRINCOR	0.2	13.4	6.8	79.6
TEMPORA	0.3	13.2	0.7	85.8
FR	2.3	13.1	0.1	84.5
JOHNNIC	0.3	12.4	51.4	35.8
SUNCRUSH	5.1	12.3	2.7	79.9
TIB	0.6	12.2	39.3	47.9
POWTECH	4.0	12.2	12.3	71.5
CTP	0.0	11.8	0.2	87.9
REMGRO	3.7	11.7	27.1	57.5
CBD-FUND	6.1	11.6	0.1	82.1
CULLINAN	0.0	11.3	0.0	88.7
REMBR-BEH	2.2	11.1	29.6	57.1
METPOL	0.8	11.0	13.7	74.5
MID-WITS	0.2	10.7	29.0	60.0
TRNSHEX	3.6	10.6	0.3	85.5
KWV-BEL	0.0	10.5	2.6	86.9
FOODCRP	2.7	10.5	2.9	83.9
FIRSTBK	2.3	10.4	9.7	77.6
INTELES	0.1	10.3	11.9	77.8
COPI	0.3	10.1	4.2	85.4
ALTRON	4.0	10.0	3.7	82.3
AFROX	0.8	9.8	12.3	77.0
UMDONI	1.0	9.7	1.6	87.7
METPROP	3.3	9.2	1.3	86.2
DELTA	0.6	9.0	13.5	76.9
ELLERINE	0.2	9.0	3.6	87.2
FEDSURE	15.2	8.9	4.4	71.5
PALAMIN	3.3	8.6	4.5	83.6
FRALEX	0.1	8.5	5.1	86.3
PPC	3.1	8.4	7.4	81.2

USKO	0.0	8.3	5.6	86.1
MCPHAIL	1.1	8.0	4.8	86.2
INHOLD	0.6	8.0	9.6	81.7
FINTECH	14.5	7.9	4.1	73.6
TIGR-OATS	2.3	7.8	16.3	73.6
CHOICE	3.0	7.6	0.6	88.9
PANPROP	0.2	7.6	5.2	87.0
NAMPAK	3.6	7.4	29.8	59.2
GENTYRE-B	0.3	7.3	1.7	90.7
CENMAG	0.7	7.0	3.6	88.7
I-G-I	0.5	6.9	3.5	89.1
DALYS	4.9	6.7	0.7	87.6
METCASH	7.5	6.6	4.6	81.3
COASTAL	0.1	6.5	1.5	91.9
TEGKOR	0.7	6.5	18.6	74.3
Q-DATA	0.0	6.4	1.3	92.3
WALHOLD	5.0	6.0	18.9	70.1
FDCRP-7CP	1.6	5.9	3.9	88.5
CASHBIL	0.0	5.9	5.8	88.3
EDGARS	0.8	5.9	4.9	88.4
SCHARIG	3.9	5.6	11.7	78.8
SA-BREWS	9.5	4.5	23.0	63.0
WALTONS	7.9	4.4	23.0	64.8
ISCOR	3.6	4.2	46.8	45.4
ANGLO-AM	2.1	3.5	68.2	26.2
Average	2.5	10.4	12.4	74.6

All underlying USA beta coefficients significant at the 5% level

a Percentage of risk computed using: $\beta^{WO2}_{i\,USA\text{-}WO2}\ Var(R^{WO}_{mt\,USA\text{-}WO})$ * 100 / Var(R_{it})

b Percentage of risk computed using: $\beta_{i\,LOC\text{-}WO\text{-}USA2}\ Var(R_{mt\,LOC\text{-}WO\text{-}USA})$ * 100 / Var(R_{it})

c Percentage of risk computed using: $\beta_{i}\ Var(R_{mt})$ * 100 / Var(R_{it})

It is evident from Table 5.3 that only 66 securities on the JSE has significant USA risk components.

Table 5.4 and Table 5.5 show the breakdown of risk components where the other major markets considered are the LSE and the TSE. These risk components(expressed as a percentage of total risk)are computed using an equation similar to (5.6) and the individual shares are ranked according to their percentage of UK market risk (Table 5.4) and Japanese market risk (Table 5.5) respectively.

TABLE 5.4 Percentage of total risk attributable to each of the components of risk (as at 31/12/1994)

Share Name	World risk[a] Component %	UK Risk[b] Component %	JSE Risk[c] Component %	Unique Risk Component %
FINTECH	14.5	19.1	2.0	64.4
BEVCON	6.9	15.6	12.0	65.4
DEBEERS	2.5	15.5	37.9	44.0
CGS-FOOD	1.4	14.1	6.5	78.0
REMBR-BEH	2.2	14.1	27.5	56.2
ANGLO-AM	2.1	12.4	59.8	25.7
GENCOR	3.4	12.3	27.1	57.2
CHARTER	13.2	11.2	0.1	75.6
RICHEMONT	5.7	10.7	14.8	68.8
SAMANCOR	0.7	10.5	22.6	66.2
MAST	4.0	10.3	0.2	85.6
REMGRO	3.7	9.9	27.4	59.0
DELTA	0.6	9.9	12.7	76.8
ANAMINT	2.8	9.7	38.9	48.7
CTP-6%-PP	4.5	9.5	0.6	85.4
TRANS-NTL	0.1	9.3	14.2	76.4
PREM-GRP	2.3	8.7	3.8	85.1
CLYDE	1.4	8.7	1.1	88.7
SA-BREWS	9.5	8.7	20.3	61.6
A-V-I	0.6	8.1	4.0	87.3
LIBERTY	0.8	7.9	37.5	53.8
LASER	6.4	7.7	0.0	86.0
SAFREN	0.3	7.7	7.3	84.7
CGSMITH	0.0	7.5	23.0	69.5
TIB	0.6	7.4	42.0	50.1
LIB-HOLD	0.2	7.4	41.7	50.7
BARLOWS	0.0	7.3	21.0	71.7
CNAGALO	0.3	7.2	3.6	89.0
AUTOQIP	0.6	6.5	2.4	90.4
KERSAF	4.6	6.4	6.5	82.6
RUSPLAT	4.3	6.3	37.1	52.3
SAPPI	5.0	5.9	16.1	73.0
CROOKES	8.2	5.7	4.5	81.6
AMIC	5.1	5.5	28.7	60.7
TEGKOR	0.7	5.2	19.0	75.1
NAMPAK	3.6	5.2	31.1	60.2
JOHNNIC	0.3	4.8	57.3	37.5
Average	3.3	9.2	19.3	68.2

All underlying UK beta coefficients significant at the 5% level

a Percentage of risk computed using: $\beta_i^{WO2}\ Var(R_{mt}^{WO})$ * 100 / $Var(R_{it})$

b Percentage of risk computed using: $\beta_i^{UK\text{-}WO2}\ Var(R_{mt}^{UK\text{-}WO})$ * 100 / $Var(R_{it})$

c Percentage of risk computed using: $\beta_i^{LOC\text{-}WO\text{-}UK2}\ Var(R_{mt}^{LOC\text{-}WO\text{-}UK})$ * 100 / $Var(R_{it})$

TABLE 5.5 Percentage of total risk attributable to each of the components of risk (as at 31/12/1994)

Share Name	World Risk[a] Component %	JAP Risk[b] Component %	JSE Risk[c] Component %	Unique Risk Component %
ICH	3.7	16.0	9.3	71.0
PREMPHARM	3.5	13.0	0.1	83.5
KLOOF	0.3	11.3	27.5	60.9
E-DAGGA	1.1	11.2	14.4	73.3
DELCORP	5.4	10.9	0.9	82.8
ALEXWYT	1.9	10.6	2.6	85.0
AGA	2.4	9.8	13.1	74.7
BOLWEAR	0.8	9.4	2.2	87.7
AIDA	0.0	9.1	0.0	90.8
TWEEFONTN	0.2	9.0	8.8	82.0
REUNERT	0.0	8.9	29.9	61.2
PORT	1.9	8.9	2.5	86.7
DUIKERS	0.0	8.7	10.9	80.4
WIT-NIGEL	0.2	8.6	0.2	91.1
SPANJRD	0.4	8.3	1.0	90.3
NAMFISH	0.9	7.4	0.6	91.0
DEELKRL	0.4	7.4	18.5	73.7
FORIM	4.7	7.3	3.4	84.5
VOGELS	0.4	7.3	16.5	75.8
GFSA	0.3	7.3	33.3	59.2
KETTER	0.1	7.1	1.7	91.1
BENONI	1.0	7.0	2.6	89.4
MESSINA	3.7	6.9	3.5	85.9
GENCOR	3.4	6.9	32.7	57.1
DA-GAMA	7.1	6.6	6.3	80.0
INMINS	2.1	6.6	2.2	89.1
PROSURE	0.0	6.5	5.8	87.6
COROHLD	0.9	6.2	1.2	91.6
HARTIES	0.2	6.2	17.1	76.5
TECFIN	0.0	6.2	0.0	93.8
TEGKOR	0.7	6.0	20.6	72.7
LORAINE	0.5	6.0	18.5	75.0
NINIAN	2.4	6.0	5.9	85.7
BENCO	1.9	5.9	11.4	80.7
FINTECH	14.5	5.8	5.7	74.0
ZANDPAN	0.2	5.8	16.4	77.6
E-T-CONS	0.7	5.7	17.5	76.0
IMPLATS	2.6	5.5	32.0	59.9
MINORCO	0.0	5.5	11.5	82.9
ANG-ALPHA	0.2	5.4	17.8	76.6
DRIES	3.1	5.1	29.0	62.8
RANDFONTN	0.1	4.8	23.4	71.7
GENBEL	2.5	4.2	43.4	49.9
VAAL-REEF	0.4	4.0	36.3	59.3

LIB-HOLD	0.2	3.7	45.8	50.3
RUSPLAT	4.3	3.6	40.4	51.7
JOHNNIC	0.3	2.7	59.5	37.4
Average	1.7	7.3	15.0	76.0

All underlying Japanese beta coefficients significant at the 5% level

a Amount of risk computed using: $\beta_i^{WO2} \; Var(R_{mt}^{WO}) * 100 / Var(R_{it})$

b Amount of risk computed using: $\beta_i^{JAP\text{-}WO2} \; Var(R_{mt}^{JAP\text{-}WO}) * 100 / Var(R_{it})$

c Amount of risk computed using: $\beta_i^{LOC\text{-}WO\text{-}JAP2} \; Var(R_{mt}^{LOC\text{-}WO\text{-}JAP}) * 100 / Var(R_{it})$

Once more it is evident from Table 5.4 and Table 5.5 that only a small proportion of shares on the JSE are significantly related to movements of the UK and Japanese markets.

The multi-market model and the extended multi-market models are useful, as they provide ways of identifying the proportion of a local share's total risk that is related to movements in world markets and in specific foreign markets. These results may be of particular interest to South African (and other LDCs) investors who are restricted by law from fully reaping the benefits of international diversification. The ability to identify and select individual shares which are significantly related to movements in the international market or particular overseas markets based on quantitative information could clearly be a useful aid to portfolio design. In the next section of this chapter a portfolio construction technique based on the resulting estimates from running this model is presented and empirically examined and contrasted to some relevant benchmarks.

It would be appropriate at this point to allude to two potential weaknesses in this study :

1. This proposed approach is an automated procedure which takes no account of underlying fundamentals (*vis-a-vis* operations abroad) of the individual companies.

However it is encouraging to note that companies with major off-shore interests such as Charter and Richemont tend to be selected by the procedure.

2. Although the test period used in the following section is an "out of period" test period, the stability/persistence of the foreign risk components of the individual companies was not assessed.

5.4 PORTFOLIO DESIGN

In this section the construction of portfolios whose constituents consist of those shares identified as having significant links with overseas markets is considered. The rationale behind such a consideration is that such a fund would, to some extent, mimic the benefits of international diversification (from which South African investors are restricted by regulation). In the preceding section the focus was on identifying shares with significant foreign risk components. In this section a portfolio construction technique based on the previous analysis is examined/assessed. Portfolios are formed using this technique and their performance is examined and contrasted to two South African "international" unit trusts[4].

Figure 5.4 below depicts the price time series of Standard International Unit Trust, Absa International Unit Trust, the Morgan Stanley World Index in rands and the JSE Actuaries All Share Index. The Morgan Stanley World Index clearly outperforms both the JSE Index and the two international funds. Since these funds can only hold a maximum of 10% of their assets off-shore, they are composed mainly of shares listed on the JSE. Their performance is therefore more in line with the JSE Index

[4] Absa International and Standard International were the only two unit trusts in existence over the entire test period

than the MS World Index. It is interesting to note the improved performance of these two funds in early 1996 when the rand fell dramatically. This can be explained by the investment strategy of these funds. It is evident that their constituent holdings comprise a significant weighting of rand-hedge shares on the JSE.

There are two considerations or objectives of "international" funds, i.e.

1. hedging against currency risk
2. diversifying into international market action

Examining both the holdings and the performance of these unit trusts it is evident that although the funds are heavily weighted with rand-hedge shares, they do not appear to have a significant component for capturing "international diversification".

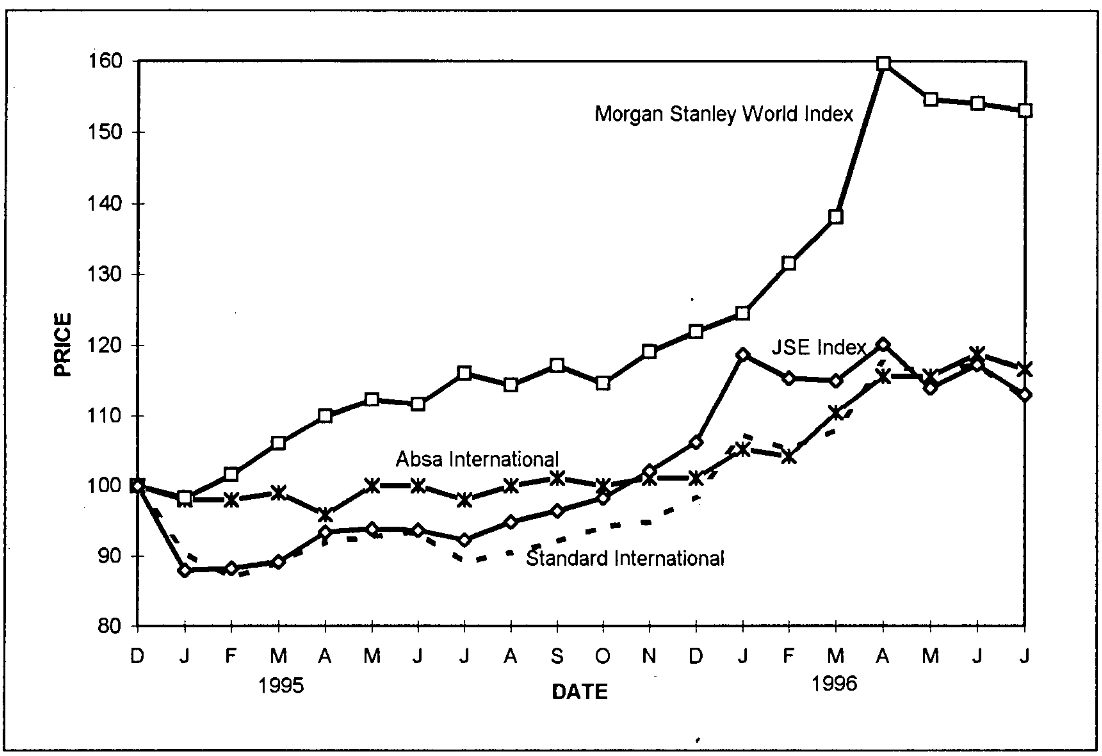

FIGURE 5.4 Performance of Unit Trusts and Indices

The aim here is to suggest a more quantitative approach to selecting shares which it is hoped will mimic, to some extent, the benefits of international diversification. The proposed portfolio construction technique involves running the regressions presented in the previous section (Model (5.1)) and identifying shares with significant positive foreign betas. Portfolios are then formed from these shares. The weighting of each share in the portfolio was determined by dividing the share's foreign beta by the sum of the foreign betas of all the selected shares, i.e.

$$W_i = \beta_i / \Sigma\beta_j$$

where W_i is the weight of share i

β_i is the shares foreign beta

$\Sigma\beta_j$ is the sum of the foreign betas of all the selected shares.

In this way, shares with high foreign betas were more heavily weighted in the portfolio.

Model (5.1) was run over the period January 1990 to December 1994 and a portfolio was formed using the above construction technique. The performance of this portfolio (henceforth referred to as the "Global" portfolio) was then tested in the "unseen" period January 1995 to December 1995. The model was then run again over the period January 1991 to December 1995 and a new portfolio was formed. The performance of this portfolio was then examined over the "unseen" period January 1996 to July 1996 (The actual composition of this portfolio is given in the appendix). For the sake of simplicity dividends and transaction costs were ignored.

Figure 5.5 below depicts the time series of prices for this portfolio over the test period, January 1995 to July 1996. For comparative purposes the time series of prices for the Morgan Stanley World Index (in rands) and the JSE Actuaries All

Share Index are also presented. It is clear that the constructed "Global" portfolio manages to substantially outperform the JSE Index. It must be kept in mind that all the constituent shares of the "Global" portfolio are from the JSE. The performance of this portfolio is more similar to that of the MS World Index than that of the JSE Index.

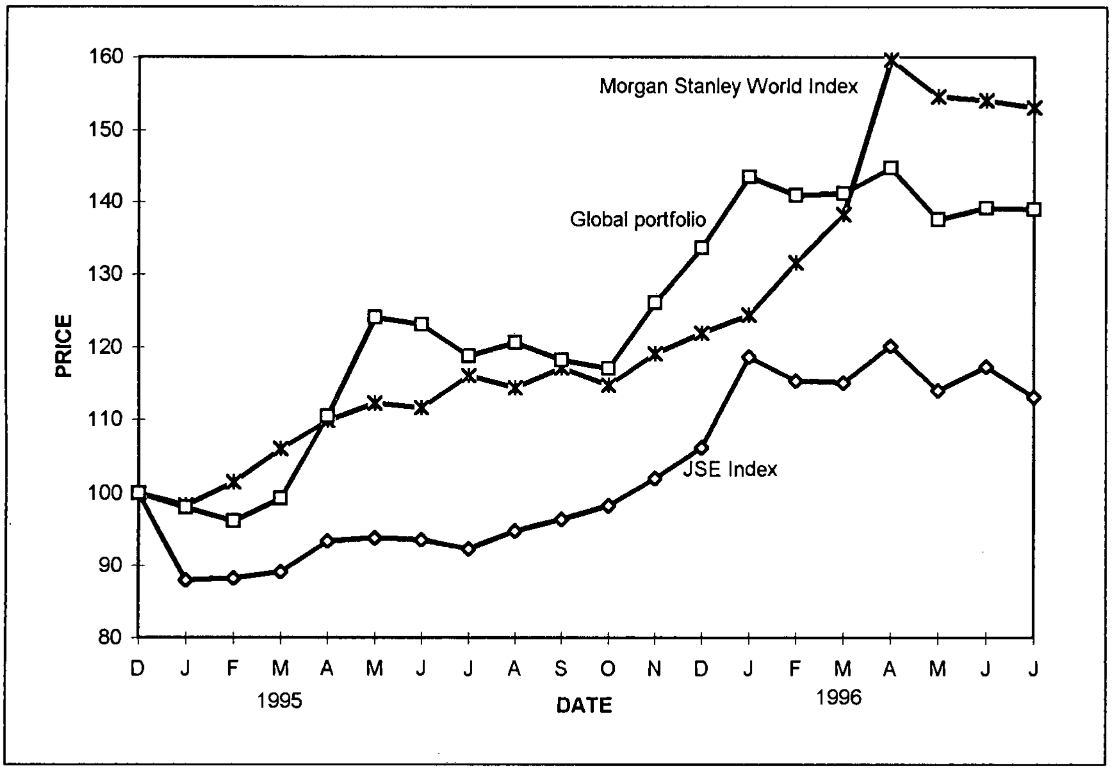

FIGURE 5.5 Performance of Indices and Global Portfolio

Figure 5.6 below depicts the prices of the two unit trusts and the "Global" portfolio. Over the test period the "Global" portfolio clearly outperforms these two unit trusts. Note that in early 1996 the performance of the two unit trusts improves considerably due to the decline in the rand. The "Global" portfolio does not improve over this period. [5] At this point it is worth noting that the suggested "Global" portfolio in

[5] Our construction technique, however, was not specifically aimed at creating rand-hedge portfolios but could be fine tuned to hedge against currency risks as well as mimicking international diversification.

particular would obviously be combined with the maximum allowable funds invested off-shore. This off-shore component would clearly have enhanced the performance of the combined portfolio over the above-mentioned period.

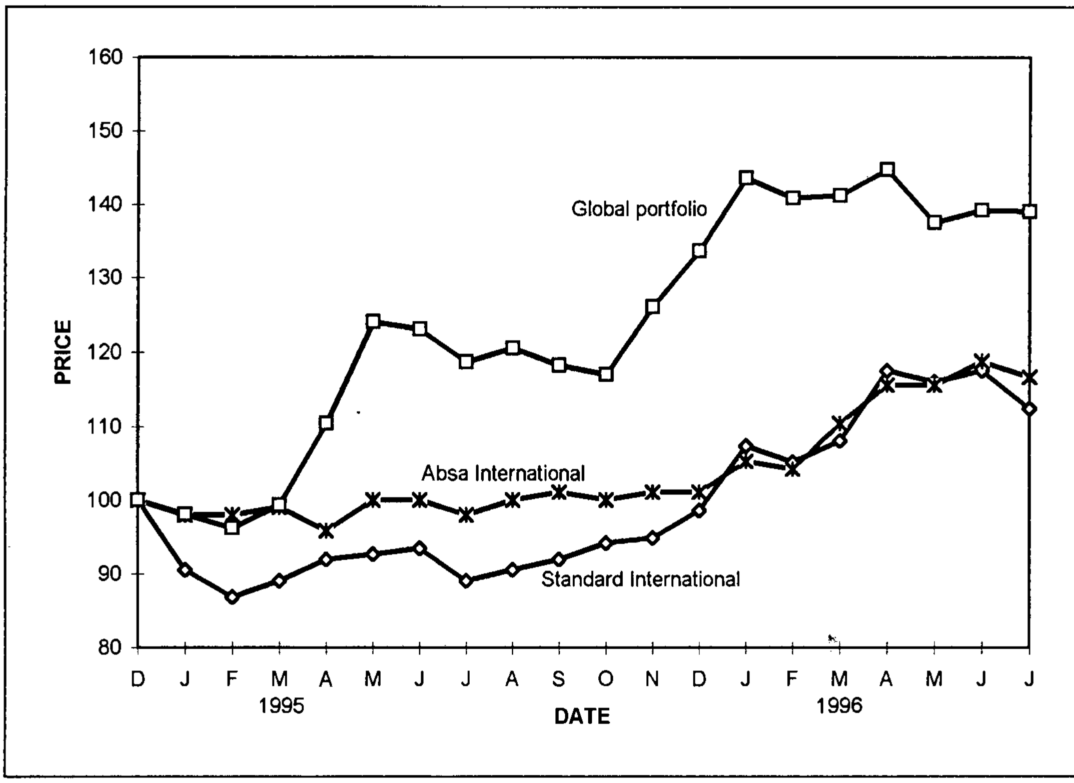

FIGURE 5.6 Performance of Unit Trusts and Global Portfolio

The performance of the unit trusts, indices and the "Global" portfolio are summarised quantitatively in Table 5.6 and Figure 5.7 below.

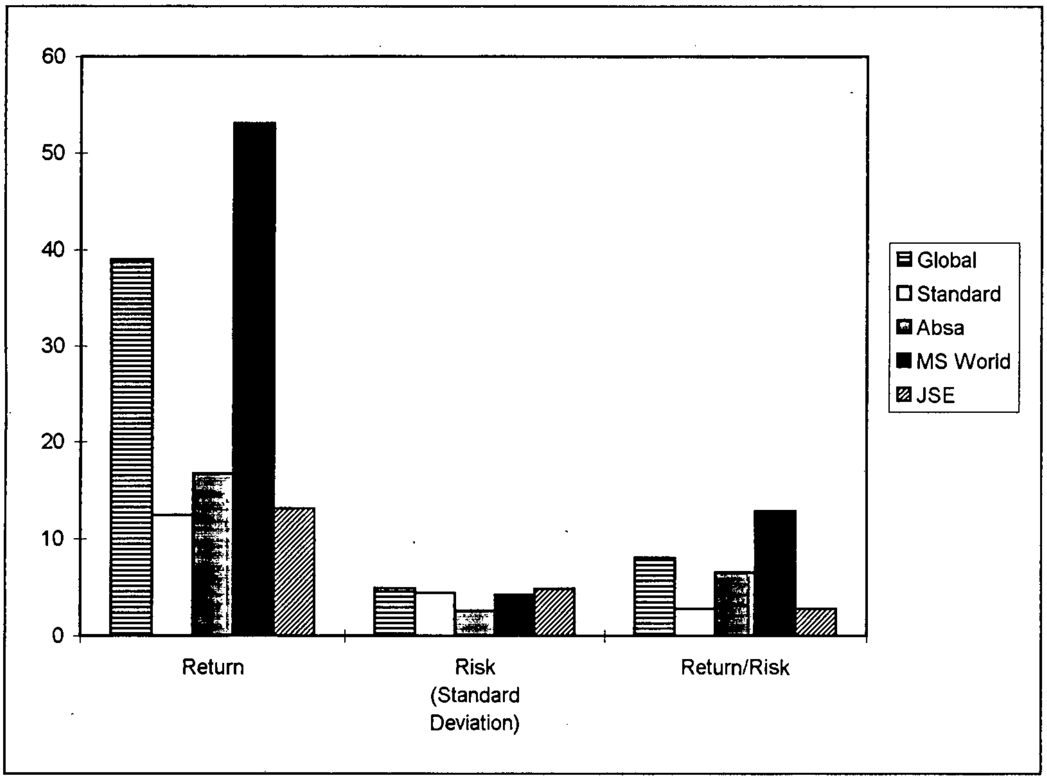

FIGURE 5.7 Quantitative performance summary

TABLE 5.6 Quantitative performance summary

	Return	Risk (Standard Deviation)	Risk adjusted return
Global	39.0	4.9	8.0
Standard	12.4	4.4	2.8
Absa	16.7	2.6	6.5
Morgan Stanley	53.0	4.2	12.8
JSE	13.1	4.8	2.8

The above table and graph present summary statistics for the "Global" portfolio, the two unit trusts and the indices for the period January 1995 to July 1996. The first column is the return over the period. The second column is the standard deviation of the monthly returns over the period and represents the risk of the portfolios. The final column gives a simple risk-adjusted performance measure, the ratio of the return and standard deviation[6]. Clearly the "Global" portfolio not only has a higher return than the two unit trusts and the JSE Index but also better risk adjusted performance. Although the "Global" portfolio underperformed the Morgan Stanley World Index its performance was closer than both the JSE Index and the unit trusts. The risk adjusted performance measure of Absa International fund is fairly close to that of the "Global" fund. This is, however, largely attributable to the low risk or standard deviation.

INDIVIDUAL COUNTRY PORTFOLIOS

One may also wish to consider portfolios that mimic specific international markets. Versions of Model (5.1) where the Morgan Stanley World Index is replaced with the SP500 (NYSE), FT-100 (LSE) and NIKKEI (TSE) were run to obtain a US Fund, a UK Fund and a Japanese Fund. As with the "Global" portfolio the funds consist of all shares with significant foreign betas. The weighting of the shares in the funds is again based on the magnitude of their foreign betas. Figure 5.8 below depicts the performance of these "Country" Funds over the period January 1995 to July 1996 (The actual composition of these funds is included in the appendix).

[6] Note that this is essentially equivalent to Sharpe's Measure (see Chapter 3), except that the return is not excess return. Since we are dealing with international indices and local funds it was felt that subtracting our risk free rate would not be appropriate. This would also not affect the relative performance of the portfolios.

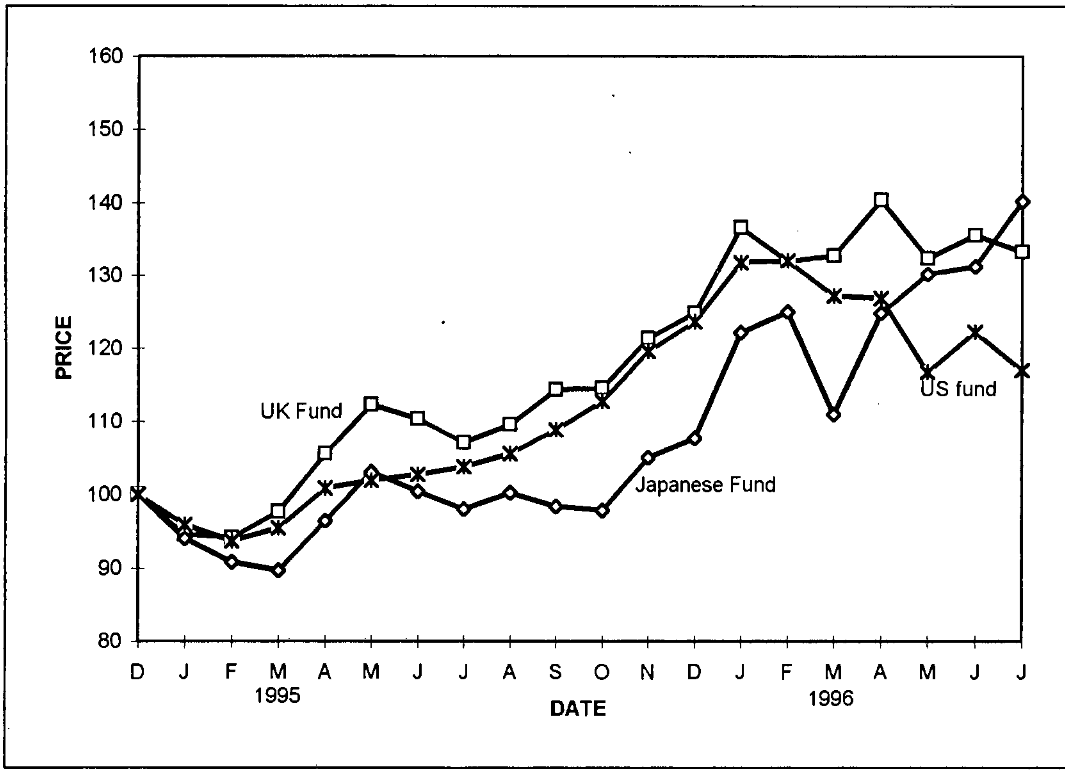

FIGURE 5.8 Performance of Country Funds

It is interesting to note that in the second quarter of 1996 while the US and UK Funds are declining the Japanese Fund performs well. This again highlights the benefits of international diversification. A combined fund was created by selecting all shares with significant foreign betas in the US, UK and Japanese models. If a specific share had a significant foreign beta in more than one model the share's weighting was determined by adding all its foreign betas. Figure 5.9 below depicts the performance of this fund relative to the Morgan Stanley World Index and the JSE Index.

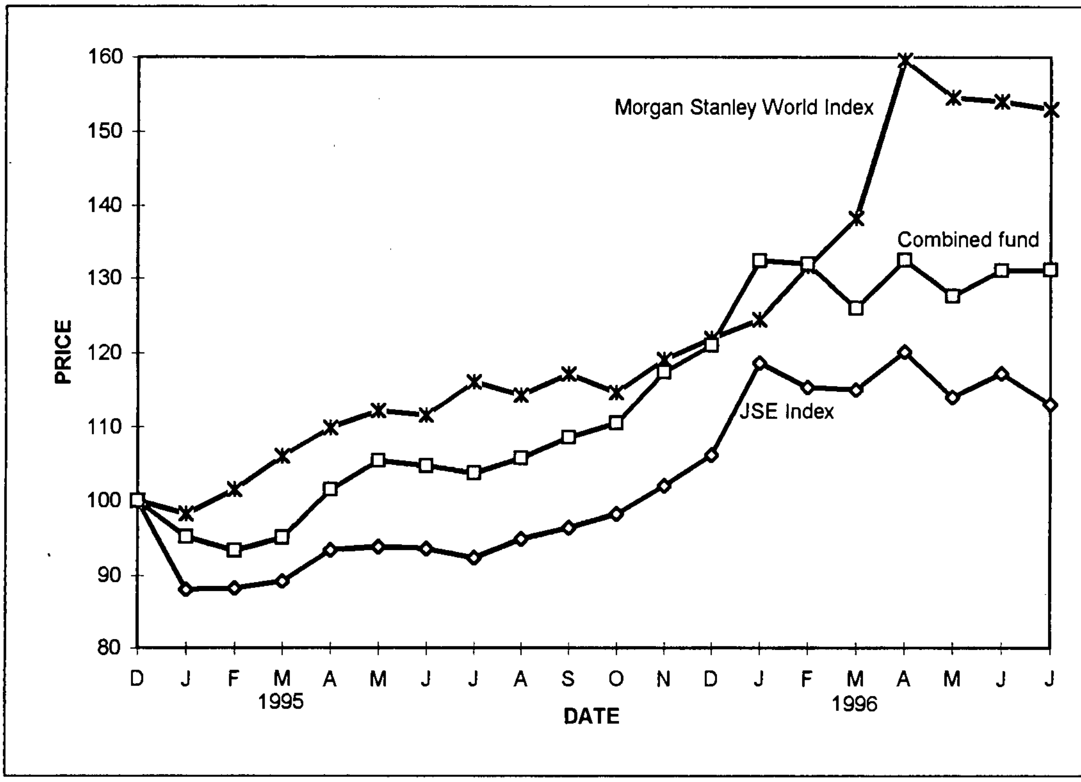

FIGURE 5.9 Performance of Combined Fund and Indices

The Combined Fund managed to outperform the JSE Index over the period January 1990 to July 1996. It this way it seemed to mimic, to some extent, the performance of the Morgan Stanley Index.

Figure 5.10 below depicts the performance of the two unit trusts and the Combined fund. The Combined fund clearly outperforms the two unit trusts. Note that in the last month of the test period, when the price of the JSE Index and both unit trusts declines, the price of the Combined fund remains stable. This can be attributed to the strong positive performance of the Japanese fund over that period.

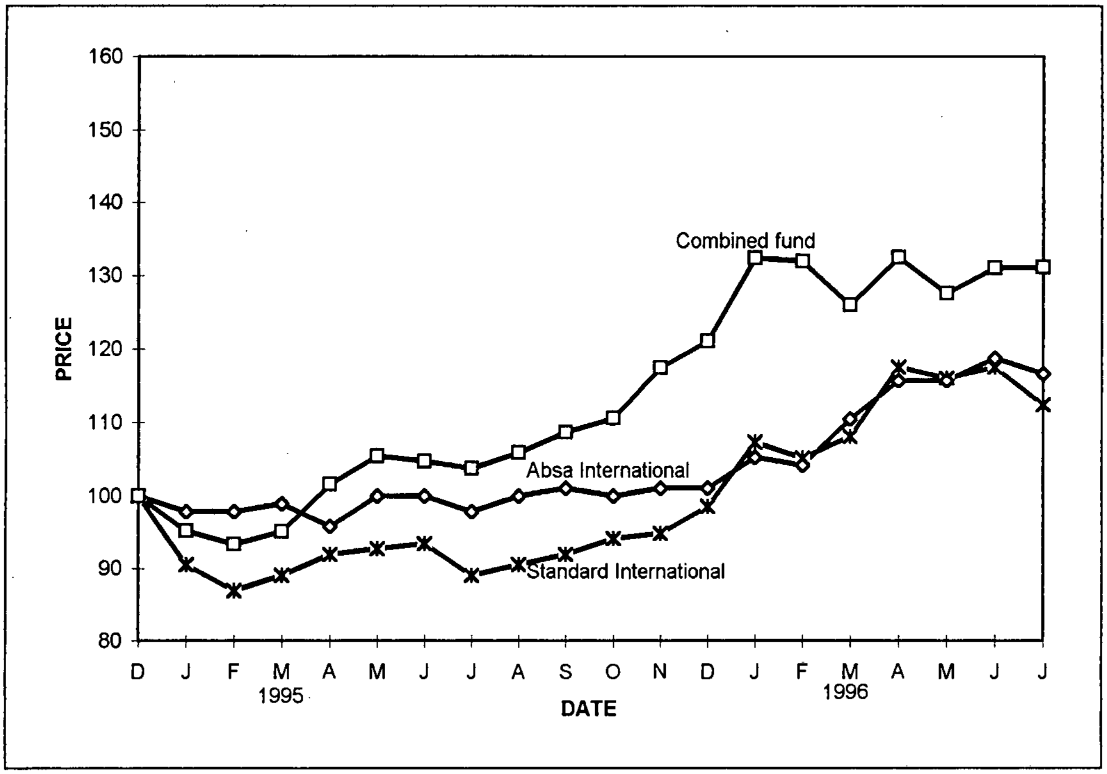

FIGURE 5.10 Performance of Unit Trusts and Combined Fund

Table 5.7 below summarises the performance of the Country Funds and the Combined fund over the period January 1995 to July 1996. For comparative purposes the performance of the unit trusts and indices are also included.

TABLE 5.7 Quantitative performance summary

	Return	Risk (Standard Deviation)	Risk Adjusted Return
US Fund	17.0	3.9	4.3
UK Fund	33.3	4.4	7.5
Japanese Fund	40.3	6.2	6.5
Combined Fund	31.1	3.8	8.1
Standard	12.4	4.4	2.8
Absa	16.7	2.6	6.5
Morgan Stanley	53.0	4.2	12.8
JSE	13.1	4.8	2.8
NYSE	37.0	2.9	16.3
LSE	20.2	1.9	12.6
TSE	5.7	6.3	0.08

The return on all of the constructed portfolios is higher than that of both unit trusts. Absa International has a better risk-adjusted performance than the US Fund. As stated before this is largely attributable to the low risk of the Absa International Unit Trust. Furthermore all other constructed portfolios have better risk-adjusted performance than Absa International. All the constructed portfolios outperform Standard International and the JSE Index based on both raw returns and risk-adjusted returns. Apart from the Japanese Index which performed poorly over this period, the world and specific international indices performed notably better than our index, unit trusts and constructed portfolios, especially on a risk-adjusted basis. Note, however, that the constructed portfolios came closest to capturing that performance.

5.5 CONCLUSION

In this chapter it has been demonstrated how local securities with significant foreign risk can be identified using a "multi-market" model. It is the author's contention that this information may be of use for portfolio construction. The empirical demonstration which contrasts the results of this particular portfolio construction methodology with some existing benchmarks gives support to the contention that portfolios can be enhanced using this knowledge[7]. The portfolio construction technique presented in this chapter is intended as an aid to portfolio design. The author is not advocating rigid adherence to this quantitative selection procedure but rather presents it as a tool to enhance portfolio design. There are several variants of the model that could be run to accommodate any tuning or hedging against specific risks, for example rand-hedging. The general form of this model and the use of summarised quantitative information is the essence of what this chapter attempts to promote.

[7] Note that although this model was tested 'out-of-period' with studies of this nature there can be no guarantees that this superior performance will persist.

CHAPTER 6

CONCLUSION

This chapter summarises the main findings of this thesis and suggests areas for future research.

In Chapter 2 it was argued that from the aspect of return, investment in equity on the JSE is certainly attractive in comparison to alternative investments. For example the average annual return on the JSE Actuaries All Share Index over the period 1985 to 1996 was 25.17% compared with returns of 13.94% on a risk free investment and 15.94% on the R150 long-term government stock. Over this period the average annual inflation rate, as measured by the CPI, was 13.32%. Clearly equities on the JSE provided an inflation-beating investment. The major focus of Chapter 2 however was on the estimation of the longer term risk of investment in equity on the JSE. In this chapter it was argued that the risk of longer term investment, typically three to five years, is less than projected using an extension of risks in the shorter term (e.g. annual or shorter). There is also strong graphical and statistical evidence that annual returns on the JSE mean revert, i.e. a decline is more likely to be followed by an incline in annual returns. This finding has direct implications for the assessment of risks in the longer term. In particular it was shown that if one had used an extrapolation of the variance of annual returns to estimate the variance of three-yearly returns, one would have overestimated the risk of three yearly returns on the JSE by a factor of over four times! As a consequence the point was made that if longer term risk was indeed overestimated, there is a case for larger proportions of funds to be allocated towards longer term investment in equity, as opposed to alternative investments.

In Chapter 3 the various portfolio performance measures were reviewed and a practical interpretation of each measure was given. It was emphasised that the measures enable one not only to assess the performance of the funds on a risk-adjusted basis, but also to assess the abilities of the fund managers. Furthermore more refined measures were discussed which enabled the separation between a manager's selection ability and a manager's timing ability to be assessed. In order to demonstrate the practical implementation of each of these measures, the majority of the measures were applied to a sample of 13 South African Unit Trusts. On the basis of the empirical demonstration (over the period June 1990 to June 1995), 46% of these funds were found to outperform the JSE All Share Index on a risk-adjusted basis. On analysing the abilities of the managers, no evidence of selection ability was found using any of the measures and only one fund was found to exhibit significant timing ability. Interestingly, six funds out of the thirteen funds exhibited significant negative timing ability.

The central theme of Chapter 4 was on the construction of index funds. The focus was on the construction and assessment of two sampling techniques, namely stratification and optimisation, on the JSE. Attention was also given to the trade-off between frequency of revision and transaction costs incurred. Additionally the suitability of the various measures of monitoring performance were investigated. In contrast to studies conducted on the New York and Tokyo Stock Exchanges, stratification appears to be the superior sampling technique on the JSE. Therefore on the basis of the results of the empirical analysis it is recommended that stratification, and not optimisation, be used as a technique for constructing and running index funds on the JSE. The effect of transaction costs on tracking ability was found to be severe. It was therefore recommended that the frequency of revision and approaches to trading such as avoidance of selling and the use of index futures should be investigated so that an optimal strategy can be adopted. Ways of taking account of transaction costs into the decision process of the stratification technique so that trades are undertaken only when the benefit is greater than the costs should be investigated. A further issue worth mentioning is that caution should be exercised when using statistics such as correlation and unique risk to monitor tracking ability. On the basis of the theoretical discussion and the empirical investigation variance of tracking error seems to be the most suitable measure of tracking ability. Variance of tracking error was however shown to be an imperfect measure and so novel ways of measuring tracking ability is an area of future research.

In Chapter 5 it was demonstrated how local securities with significant foreign risk components can be identified using an extension of the market model. A portfolio construction technique utilising this model to select and weight shares was presented. Portfolios were formed using this particular construction methodology and their performance was contrasted with the performance of two South African international unit trusts over the period January 1995 to July 1996. The portfolios constructed using this technique and monitored on "unseen" data were found to achieve superior performance to the existing unit trusts over this period. The quantitative selection procedure proposed is recommended as a tool to enhance portfolio design. Additionally the model could be varied to accommodate any tuning or hedging against specific risks, for example rand-hedging. Currently the government appears committed to gradually lifting exchange controls. Until exchange controls are completely abolished the proposed technique will still remain useful for selecting shares for that portion of the portfolio which still needs to be comprised of South African shares.

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APPENDIX

APPENDIX FOR CHAPTER 3

A3.1 Table of South African Unit Trusts used in sample

TABLE 1 UNIT TRUSTS

Abbreviation	Name
GENERAL EQUITY FUNDS	
GUARD	Guardbank Growth Fund
OMIF	Old Mutual Investor's Fund
SAGE	Sage Fund
SNI	Sanlam Index Trust
SNPG	Sanlam Prime Growth Fund
SNT	Sanlam Trust
SBM	Standard Bank Mutual
UAL	UAL Unit Trust
SPECIALIST EQUITY FUNDS	
Mining and Resources Funds	
SNMT	Sanlam Mining Trust
UALMR	UAL Mining and Resources
Gold Funds	
SBG	Standard Bank Gold
Industrial Funds	
SNIT	Sanlam Industrial Trust
HIGH INCOME FUNDS	
SBEI	Standard Bank Extra Income

A3.2 TSP Batch Program and Example Output

The TSP batch program used to produce the various regressions was the following:

```
1.   OPTION OUTPUT %0.BNP
2.   PON
3.   LS %0 C JSE
4.   GENR C(60)=@VAR(RESID)
5.   LS %0 C JSE SJSE
6.   GENR WRES = RESID^2
7.   LS WRES SJSE
8.   GENR C(61) = COEF(1)
9.   GENR SIGW = C(61)*SJSE+C(60)
10.  GENR SIGD = 2*(C(61)^2)*(SJSE)^2 + 2*(C(60))^2 + 4*C(61)*SJSE*C(60)
11.  LS (W=SIGW) %0 C JSE SJSE
12.  GENR WH = RESID^2
13.  LS (W=SIGD) WH SJSE
14.  LS %0 C JSE PJSE
```

An example of the output generated follows

In the example the series GUARD is the series of excess returns on Guardbank Growth Fund, JSE is the series of excess returns on the JSE Actuaries All Share Index, SJSE is the square of the series JSE and PJSE is a series of Max(0,JSE).

Lines 1 and 2 direct the output to a file.

Line 3 produces parameter estimates used for Treynor's and Jensen's measures. The output for this particular example is:

```
LS // Dependent Variable is GUARD
    Date: 2-07-1997 / Time: 0:05
    SMPL range:   1 -   60
    Number of observations: 60

==================================================================
       VARIABLE   COEFFICIENT   STD. ERROR     T-STAT.   2-TAIL SIG.

==================================================================
          C       0.0029725    0.0038971    0.7627439    0.449
          JSE       0:7555941    0.0601737    12.556878    0.000

==================================================================
    R-squared              0.731077    Mean of dependent var   0.011872
    Adjusted R-squared     0.726440    S.D. of dependent var   0.056753
    S.E. of regression     0.029683    Sum of squared resid    0.051104
    Durbin-Watson stat     2.405380    F-statistic             157.6752
    Log likelihood         126.9108

==================================================================
```

An estimate for Jensen's alpha would be the intercept obtained from this regression. The beta value for Treynor's measure would be the JSE coefficient with the relevant R^2 appearing below it.

Line 4 stores the variance of the residual obtained from the regression of line 3, that is needed for calculation of the weights used to adjust for heteroscedasticity in the Bhattacharya and Pfleiderer model.

Line 5 produces parameter estimates for the Treynor-Mazuy measure. The output for this example is:

```
LS // Dependent Variable is GUARD
Date: 2-07-1997 / Time: 0:05
SMPL range:   1 -   60
Number of observations: 60

==========================================================================
    VARIABLE   COEFFICIENT   STD. ERROR    T-STAT.   2-TAIL SIG.

==========================================================================
       C       0.0084809    0.0043179    1.9641480    0.054
      JSE      0.7061309    0.0607931    11.615306    0.000
      SJSE    -1.1743850    0.4645564   -2.5279708    0.014

==========================================================================
R-squared              0.758188   Mean of dependent var   0.011872
Adjusted R-squared     0.749704   S.D. of dependent var   0.056753
S.E. of regression     0.028393   Sum of squared resid    0.045952
Durbin-Watson stat     2.518433   F-statistic             89.36024
Log likelihood         130.0987

==========================================================================
```

The estimate of selection ability would be the intercept obtained from this regression. The coefficient of SJSE is an estimate of timing ability.

Line 6 stores the square of the residual in a new series WRES. This is required for equation 3.12 in the Bhattacharya and Pfleiderer model.

Line 7 produces parameter estimates for equation 3.12 in the Bhattacharya and Pfleiderer model (Note that this regression is OLS and is run merely to obtain the coefficient needed for the weightings to correct for heteroscedasticity). The output for this example is:

```
LS // Dependent Variable is WRES
Date: 2-07-1997 / Time: 0:05
SMPL range:   1 -   60
Number of observations: 60

==========================================================================
    VARIABLE   COEFFICIENT   STD. ERROR    T-STAT.   2-TAIL SIG.

==========================================================================
      SJSE     0.0268707    0.0180096    1.4920187    0.141

==========================================================================
R-squared             -0.458562   Mean of dependent var   0.000766
Adjusted R-squared    -0.458562   S.D. of dependent var   0.001078
S.E. of regression     0.001302   Sum of squared resid    9.99E-05
Durbin-Watson stat     1.505438   Log likelihood          314.0206

==========================================================================
```

Line 8 stores the coefficient of SJE from the above regression which is required for calculating the transforming weights to correct for heteroscedasticity in the Bhattacharya and Pfleiderer model.

Lines 9 and 10 generate the weightings required for the generalised least squares regression used in lines 11 and 13.

Line 11 produces estimates for the Bhattacharya and Pfleiderer model. The output for this example is:

```
LS // Dependent Variable is GUARD
Date: 2-07-1997 / Time: 0:05
SMPL range:   1 -   60
Number of observations: 60
Weighting series: SIGW

===================================================================
   VARIABLE  COEFFICIENT  STD. ERROR    T-STAT.  2-TAIL SIG.

===================================================================
      C      0.0086237    0.0043544    1.9804732   0.052
     JSE     0.6941821    0.0485882    14.287064   0.000
     SJSE   -1.1827872    0.2844391   -4.1583138   0.000

===================================================================
                   Weighted Statistics

===================================================================
 R-squared           0.912103   Mean of dependent var   0.006239
 Adjusted R-squared    0.909019   S.D. of dependent var   0.091493
 S.E. of regression    0.027597   Sum of squared resid    0.043411
 Durbin-Watson stat    2.447813   F-statistic            295.7427
 Log likelihood       131.8053

===================================================================
                   Unweighted Statistics

===================================================================
 R-squared           0.758014   Mean of dependent var   0.011872
 Adjusted R-squared    0.749524   S.D. of dependent var   0.056753
 S.E. of regression    0.028403   Sum of squared resid    0.045985
 Durbin-Watson stat    2.515294

===================================================================
```

An estimate of the Bhattacharya and Pfleiderer measure of selection ability is the intercept from this regression. The coefficient of SJSE is an estimate of η_2.

Line 12 stores the square of the residual in a new series WH. This is required for equation 3.12 of the Bhattacharya and Pfleiderer model. Estimates for this equation are produced by line 13. The output for this example is:

```
LS // Dependent Variable is WH
Date: 2-07-1997 / Time: 0:05
SMPL range:   1 -   60
Number of observations: 60
Weighting series: SIGD
```

VARIABLE	COEFFICIENT	STD. ERROR	T-STAT.	2-TAIL SIG.
SJSE	0.0014868	0.0033781	0.4401405	0.661

Weighted Statistics

R-squared	-0.492334	Mean of dependent var	0.000687
Adjusted R-squared	-0.492334	S.D. of dependent var	0.000982
S.E. of regression	0.001199	Sum of squared resid	8.49E-05
Durbin-Watson stat	1.320089	Log likelihood	318.9265

Unweighted Statistics

R-squared	-0.496907	Mean of dependent var	0.000766
Adjusted R-squared	-0.496907	S.D. of dependent var	0.001090
S.E. of regression	0.001334	Sum of squared resid	0.000105
Durbin-Watson stat	1.262103		

The coefficient of SJSE is an estimate of η_3. The timing component of the Bhattacharya and Pfleiderer model is calculated using equation 3.15. The estimate of the variance of π obtained using equation 3.14 was 0.0044095. For this example, the timing component was therefore:

$$\sqrt{\frac{0.0044095}{0.0044095 + \frac{\eta 3}{\eta 2^2}}}$$

= 0.897657

Note that since the coefficient of SJSE in the regression produced by line 11 is negative, the timing ability here should be -0.897657.

Line 14 produces estimates for the Henriksson-Merton measure. The output for this example is:

LS // Dependent Variable is GUARD
Date: 2-07-1997 / Time: 0:05
SMPL range: 1 - 60
Number of observations: 60

VARIABLE	COEFFICIENT	STD. ERROR	T-STAT.	2-TAIL SIG.
C	0.0158998	0.0057957	2.7434031	0.008
JSE	0.9900677	0.0991407	9.9864918	0.000
PJSE	-0.5085048	0.1763658	-2.8832390	0.006

R-squared	0.765306	Mean of dependent var	0.011872

Adjusted R-squared	0.757071	S.D. of dependent var	0.056753
S.E. of regression	0.027972	Sum of squared resid	0.044600
Durbin-Watson stat	2.428236	F-statistic	92.93455
Log likelihood	130.9950		

The intercept from this regression is an estimate of the Henriksson-Merton measure of selection ability. The coefficient of PJSE is an estimate of timing ability.

A3.3 Performance measurement tables for period 1985 to 1990

TABLE 2 Risk Adjusted Performance Measures (1985 - 1990)

Unit Trust	Average excess return		Standard deviation		Beta		Sharpe		Treynor	
GUARD	0.01187	*3*	0.05675	*10*	**0.75559**	*9*	0.20919	*2*	0.01571	*3*
OMIF	0.01490	*1*	0.05995	*9*	**0.80884**	*8*	0.24847	*1*	0.01842	*1*
SAGE	0.01023	*8*	0.05385	*12*	**0.72335**	*10*	0.19005	*5*	0.01415	*6*
SNI	0.01251	*2*	0.06158	*7*	**0.88278**	*5*	0.20312	*4*	0.01417	*5*
SNPG	0.00617	*12*	0.06255	*5*	**0.72223**	*11*	0.09867	*12*	0.00855	*13*
SNT	0.00894	*10*	0.06083	*8*	**0.83151**	*7*	0.14693	*11*	0.01075	*12*
SBM	0.01056	*6*	0.05140	*13*	**0.68069**	*12*	0.20539	*3*	0.01551	*4*
UAL	0.01014	*9*	0.06230	*6*	**0.87762**	*6*	0.16281	*8*	0.01156	*9*
SNMT	0.01076	*5*	0.06922	*2*	**0.95407**	*4*	0.15542	*9*	0.01128	*10*
UALMR	0.01048	*7*	0.06785	*3*	**0.97397**	*3*	0.15439	*10*	0.01076	*11*
SBG	0.00608	*13*	0.07759	*1*	**1.02239**	*1*	0.07831	*13*	0.00594	*14*
SNIT	0.00887	*11*	0.05391	*11*	**0.64375**	*13*	0.16451	*7*	0.01378	*7*
SBEI	0.00193	*14*	0.03234	*14*	0.11752	*14*	0.05957	*14*	0.01639	*2*
JSE	0.01178	*4*	0.06422	*4*	**1.00000**	*2*	0.18340	*6*	0.01178	*8*
Average*	0.00949		0.05924		0.76879		0.15976		0.01284	

* excluding JSE Index

TABLE 3 Measures of Selection Ability (1985 - 1990)

Unit Trust	Jensen		Treynor - Mazuy		Henriksson - Merton		Bhattacharya - Pfleiderer	
GUARD	0.00297	*2*	0.00848	*2*	**0.01590**	*2*	0.00862	*3*
OMIF	0.00537	*1*	**0.00948**	*1*	**0.01725**	*1*	**0.00949**	*2*
SAGE	0.00171	*5*	0.00710	*4*	**0.01392**	*3*	0.00702	*6*
SNI	0.00211	*4*	0.00470	*7*	0.00932	*7*	0.00462	*8*
SNPG	-0.00233	*12*	0.00324	*8*	0.00931	*8*	0.00437	*9*
SNT	-0.00086	*10*	0.00305	*10*	0.00699	*12*	0.00497	*7*
SBM	0.00254	*3*	0.00705	*5*	**0.01273**	*5*	0.00727	*5*
UAL	-0.00019	*8*	0.00594	*6*	**0.01217**	*6*	0.00794	*4*
SNMT	-0.00048	*9*	0.00316	*9*	0.00883	*9*	0.00223	*10*
UALMR	-0.00100	*11*	0.00231	*12*	0.00817	*10*	0.00110	*12*
SBG	-0.00597	*13*	-0.00870	*13*	-0.00835	*13*	**-0.01400**	*13*
SNIT	0.00129	*6*	0.00739	*3*	**0.01392**	*4*	0.00990	*1*
SBEI	0.00054	*7*	0.00263	*11*	0.00719	*11*	0.00144	*11*
Average	0.00044		0.00429		0.00980		0.00423	

TABLE 4 Measures of Timing Ability (1985 - 1990)

Unit Trust	Treynor - Mazuy		Henriksson - Merton		Bhattacharya - Pfleiderer	
GUARD	**-1.17439**	*10*	**-0.508505**	*13*	**-0.897657**	*12*

OMIF	-0.87632	*7*	**-0.467219**	*9*	**-0.791445**	*8*
SAGE	**-1.14842**	*9*	**-0.480300**	*10*	**-0.891802**	*11*
SNI	-0.55243	*3*	-0.283414	*3*	**-0.756540**	*4*
SNPG	-1.18852	*11*	-0.458025	*8*	**-0.781285**	*6*
SNT	-0.83242	*6*	-0.308513	*4*	**-0.791090**	*7*
SBM	**-0.96235**	*8*	**-0.400975**	*7*	**-0.843871**	*10*
UAL	**-1.30780**	*13*	**-0.486541**	*11*	**-0.906023**	*13*
SNMT	-0.77618	*5*	-0.366202	*6*	**-0.767959**	*5*
UALMR	-0.70406	*4*	**-0.360461**	*5*	**-0.716741**	*3*
SBG	0.58366	*1*	0.093639	*1*	**0.779901**	*1*
SNIT	**-1.30064**	*12*	**-0.496998**	*12*	**-0.829322**	*9*
SBEI	-0.44503	*2*	-0.261301	*2*	**-0.420071**	*2*
Average	-0.82191		-0.368063		-0.662608	

A3.4 Contingency Tables

SHARPE MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	6	2
1995	Loser	1	4

Value of χ^2 test of association : 3.75
p-value : 0.0530

TREYNOR MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	6	4
1995	Loser	1	2

Value of χ^2 test of association : 0.66
p-value : 0.4165

JENSEN'S ALPHA

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	5	2
1995	Loser	2	4

Value of χ^2 test of association : 1.89
p-value : 0.1696

TREYNOR-MAZUY SELECTION MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	5	1
1995	Loser	2	5

Value of χ^2 test of association : 3.90
p-value : 0.0483

TREYNOR-MAZUY TIMING MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	3	5
1995	Loser	2	3

Value of χ^2 test of association : 0.01
p-value : 0.9282

HENRIKSSON-MERTON SELECTION MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	5	2
1995	Loser	1	5

Value of χ^2 test of association : 3.90
p-value : 0.0483

HENRIKSSON-MERTON TIMING MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	4	4
1995	Loser	2	3

Value of χ^2 test of association : 0.12
p-value : 0.7249

BHATTACHARYA-PFLEIDERER SELECTION MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	6	0
1995	Loser	3	4

Value of χ^2 test of association : 4.95
p-value : 0.0261

BHATTACHARYA-PFLEIDERER TIMING MEASURE

		1985 - 1990 Winner	1985 - 1990 Loser
1990 -	Winner	1	6
1995	Loser	1	5

Value of χ^2 test of association : 0.01
p-value : 0.9056

APPENDIX FOR CHAPTER 4

A4.1 Composition of Stratified Funds

Table 1 Composition of Stratified Funds 1992 - 1995

1992

SHARE	INDEX%	SECTOR	Minimum investment size (% of fund) 7.14%	10%	20%
SBIC	1.54	Banks and Financial Services	7.14	10	20
SA-BREW	4.38	Beverages, Hotels and Leisure	7.14	10	20
SASOL	2.96	Chemicals and Oils	7.14	10	
DE BEERS	10.04	Diamonds	7.14	10	20
TIGER OATS	1.59	Food	7.14		
DRIES	2.22	Gold - Witwatersrand	7.14	10	
BARLOWS	2.81	Industrial Holding	7.14		
RICHEMO	5.07	Industrial Holding	7.14	10	20
LIBERTY	2.41	Insurance	7.14	10	
MINORCO	2.25	Mining Holding	7.14	10	
ANGLOS	8.36	Mining Houses	7.14	10	20
GENCOR	4.16	Mining Houses	7.14		
EDGARS	0.82	Retailers and Wholesalers	7.14		
REMGRO	3.89	Tobacco and Match	7.14	10	

1993

SHARE	INDEX%	SECTOR	Minimum investment size (% of fund) 7.14%	10%	20%
SBIC	2.6	Banks and Financial Services	7.14	10	20
SA-BREW	4.77	Beverages, Hotels and Leisure	7.14	10	
SASOL	2.78	Chemicals and Oils	7.14	10	
DE BEERS	6.54	Diamonds	7.14	10	20
TIGER OATS	2.02	Food	7.14	10	
BARLOWS	2.57	Industrial Holding	7.14		
RICHEMO	5.72	Industrial Holding	7.14	10	20
LIBERTY	4.02	Insurance	7.14	10	20
MINORCO	3.21	Mining Holding	7.14	10	
ANGLOS	5.99	Mining Houses	7.14	10	20
GENCOR	4.01	Mining Houses	7.14		
SAPPI	1.46	Paper and Packaging	7.14		
EDGARS	0.87	Retailers and Wholesalers	7.14		
REMGRO	4.16	Tobacco and Match	7.14	10	

1994

SHARE	INDEX%	SECTOR	Minimum investment size (% of fund) 7.14%	10%	20%
SBIC	2.53	Banks and Financial Services	7.14	10	20
SA-BREW	4.86	Beverages, Hotels and Leisure	7.14	10	20
SASOL	2.08	Chemicals and Oils	7.14		
DE BEERS	8.11	Diamonds	7.14	10	20
TIGER OATS	1.52	Food	7.14	10	
VAAL-REEF	1.64	Gold - Klerksdorp	7.14		
DRIES	2.17	Gold - Witwatersrand	7.14	10	
RICHEMO	4.16	Industrial Holding	14.28	10	20
LIBERTY	3.54	Insurance	7.14	10	
MINORCO	3.03	Mining Holding	7.14	10	

ANGLOS	9.31	Mining Houses	7.14	10	20
JOHNNIC	2.52	Mining Houses	7.14		
EDGARS	1.12	Retailers and Wholesalers	7.14		
REMGRO	3.51	Tobacco and Match		10	

1995

			Minimum investment size (% of fund)		
SHARE	**INDEX%**	**SECTOR**	**7.14%**	**10%**	**20%**
SBIC	2.19	Banks and Financial Services	7.14	10	20
SA-BREW	4.17	Beverages, Hotels and Leisure	7.14	10	
SASOL	3.09	Chemicals and Oils	7.14	10	
DE BEERS	5.6	Diamonds	7.14	10	20
TIGER OATS	1.18	Food	7.14		
DRIES	1.78	Gold - Witwatersrand	7.14		
RICHEMO	3.41	Industrial Holding	7.14	10	20
LIBERTY	3.56	Insurance	7.14	10	20
MINORCO	3.25	Mining Holding	7.14	10	
ANGLOS	8.09	Mining Houses	7.14	10	20
GENCOR	3.01	Mining Houses	7.14		
SAPPI	1.7	Paper and Packaging	7.14		
EDGARS	1.09	Retailers and Wholesalers	7.14	10	
REMGRO	2.24	Tobacco and Match	7.14	10	

A4.2 Composition of Optimised Funds

Table 2 Composition of Optimised Funds 1992 - 1995

1992 **Additional constraints**

SHARE	15 SHARES	10 SHARES	5 SHARES
ABSA	4%	7%	
AMCOAL			10%
AMIC	6%	8%	
ANGLOS	10%	12%	25%
BARLOWS	7%	9%	
DE BEERS	14%	15%	
DRIES	15%	13%	26%
GENBEL	2%		10%
GENCOR	6%	7%	
RUSPLAT	4%		
SA-BREW	9%	11%	29%
SAFREN	2%		
SBIC	4%		
SOTHERN	5%	8%	
TIGER OATS	3%		
VAAL-REEF	8%	9%	

1993 **Additional constraints**

SHARE	15 SHARES	10 SHARES	5 SHARES
AMGOLD	10%	6%	
AMIC	7%	6%	
ANGLOS	12%	13%	15%
BARLOWS	6%	8%	
DE BEERS	8%	12%	17%
DRIES	9%	19%	26%
GENCOR	5%	6%	
KLOOF	5%		
MALBAK	4%		
MINORCO	4%		
NAMPAK	6%	8%	
RUSPLAT	4%		
SA-BREW	10%	10%	20%
SOTHERN	8%	12%	22%
TIGER OATS	3%		

1994 **Additional constraints**

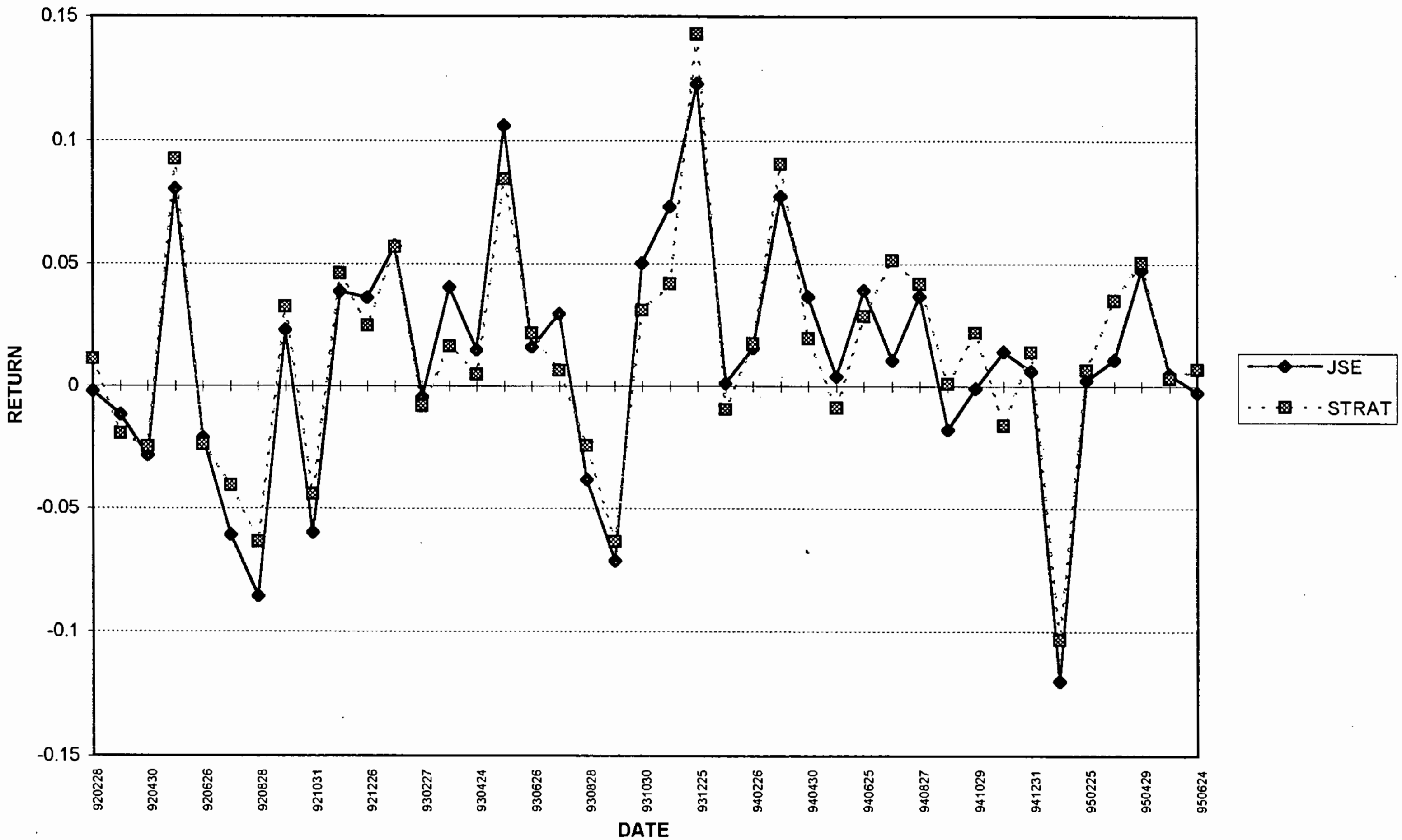
SHARE	15 SHARES	10 SHARES	5 SHARES
AMGOLD	9%	5%	
ANGLOS	12%	17%	18%
BARLOWS	5%		
DE BEERS	4%		
DRIES	6%	9%	18%
FREGOLD	4%		
GENCOR	4%		

LIBERTY	7%	11%	17%
MALBAK	6%	7%	
MINORCO	5%	7%	
NAMPAK	8%	14%	30%
RICHEMO	8%	8%	
RUSPLAT	7%	8%	
SASOL	11%	14%	17%
TIGER OATS	5%		

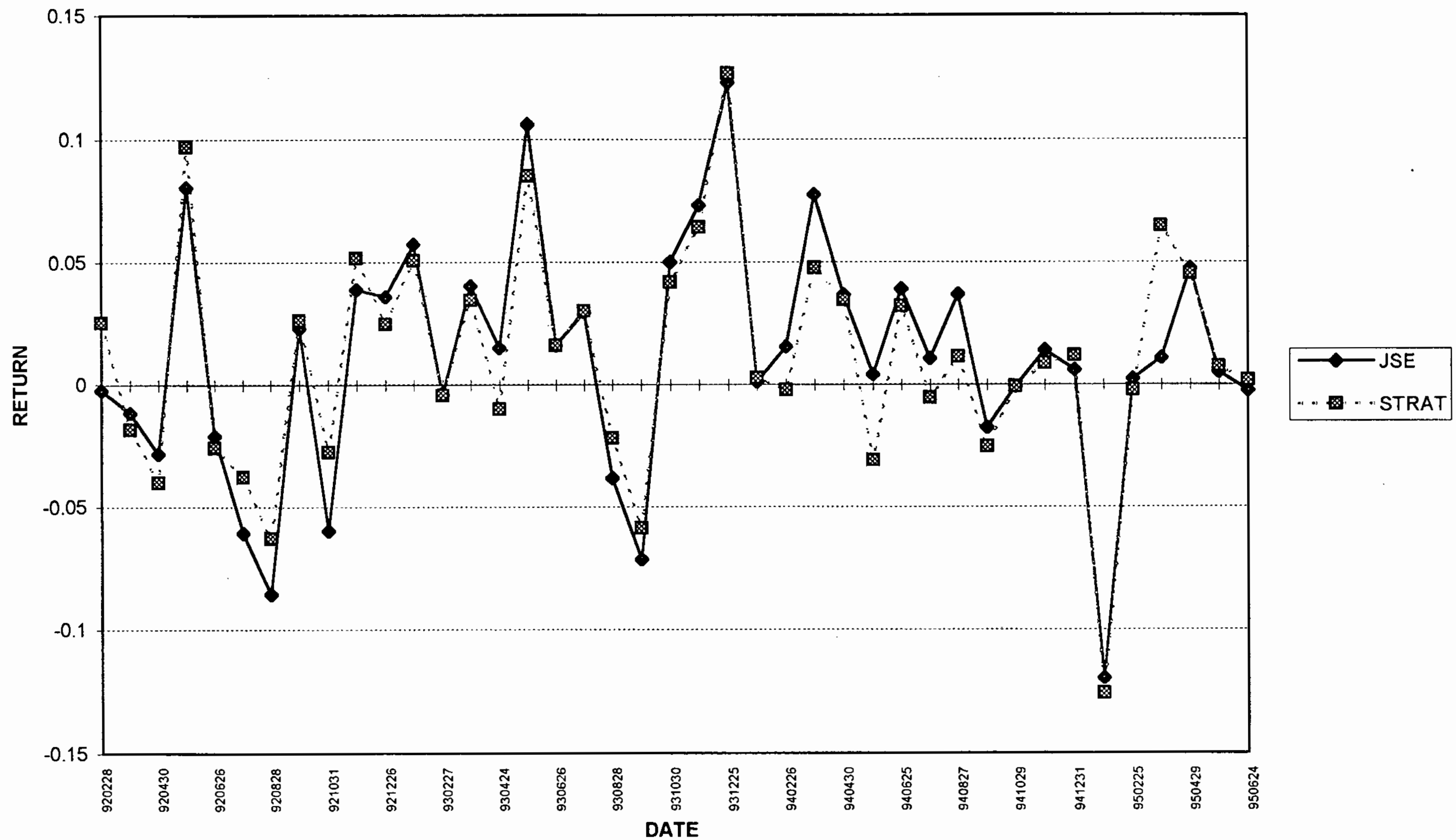
1995 **Additional constraints**

SHARE	15 SHARES	10 SHARES	5 SHARES
AMGOLD	9%	7%	
AMIC	5%		
ANGLOS	6%	14%	22%
CGSMITH	11%	12%	23%
DE BEERS	5%		
DRIES	5%	7%	
GENCOR	7%	10%	12%
KLOOF	2%		
LIBERTY	9%	12%	23%
M&R HOLD	7%	8%	
MINORCO	9%	10%	19%
NEDCOR	7%	9%	
RUSPLAT	4%		
SA-BREW	9%	10%	
TIGER OATS	5%		

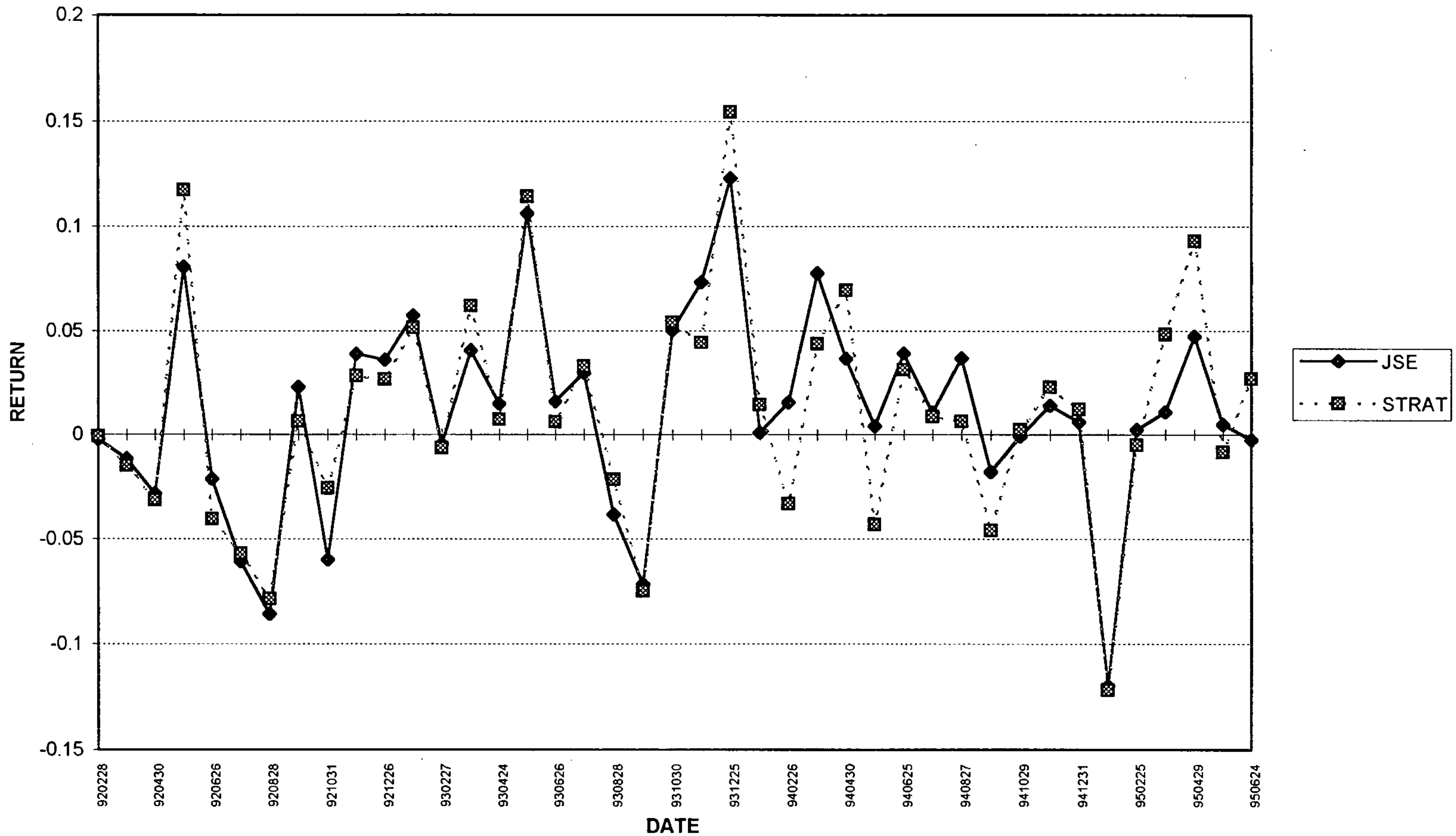
A4.3.1 STRATIFIED FUND - 15 SHARES



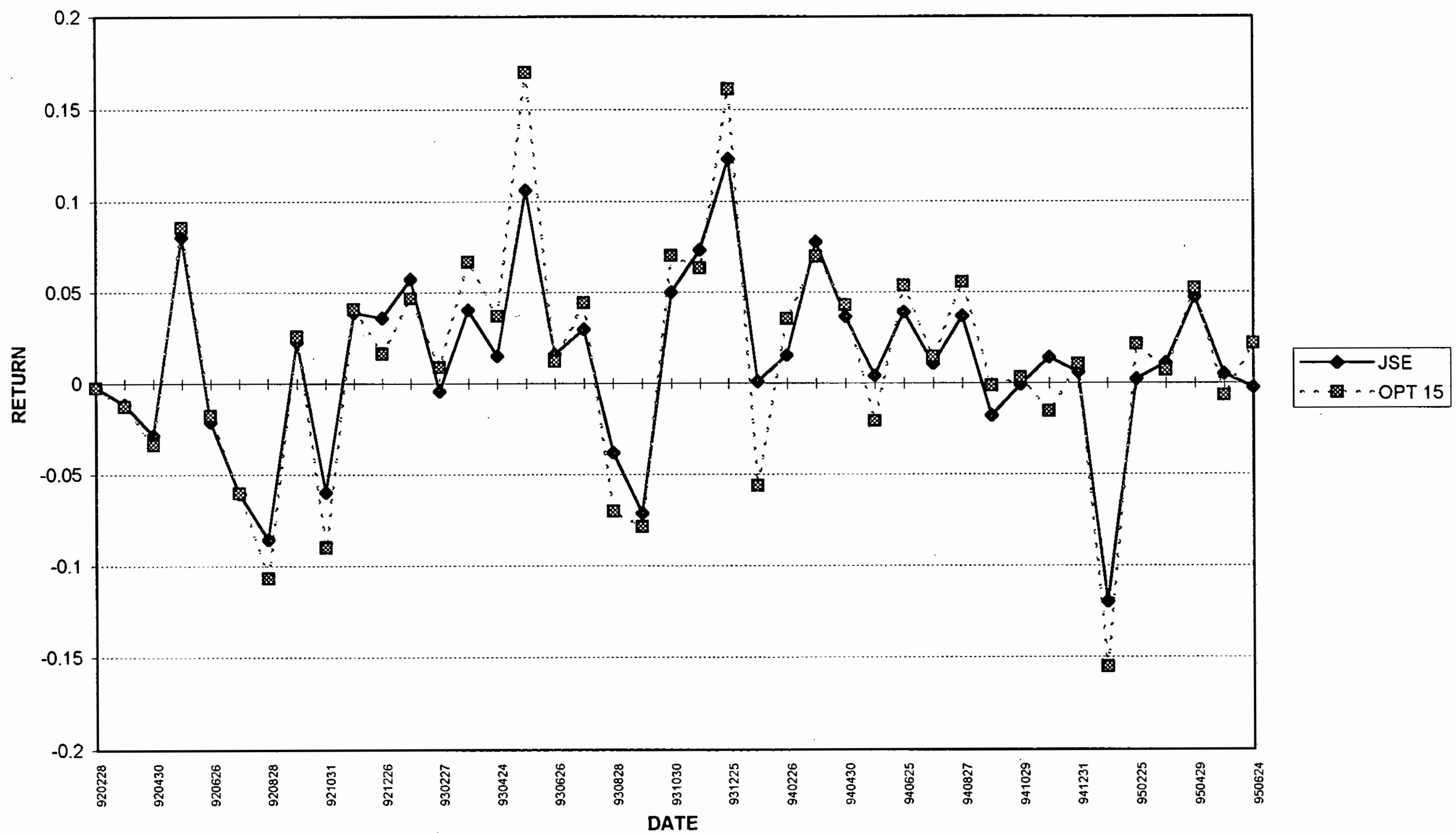
A4.3.2 STRATIFIED FUND - 10 SHARES



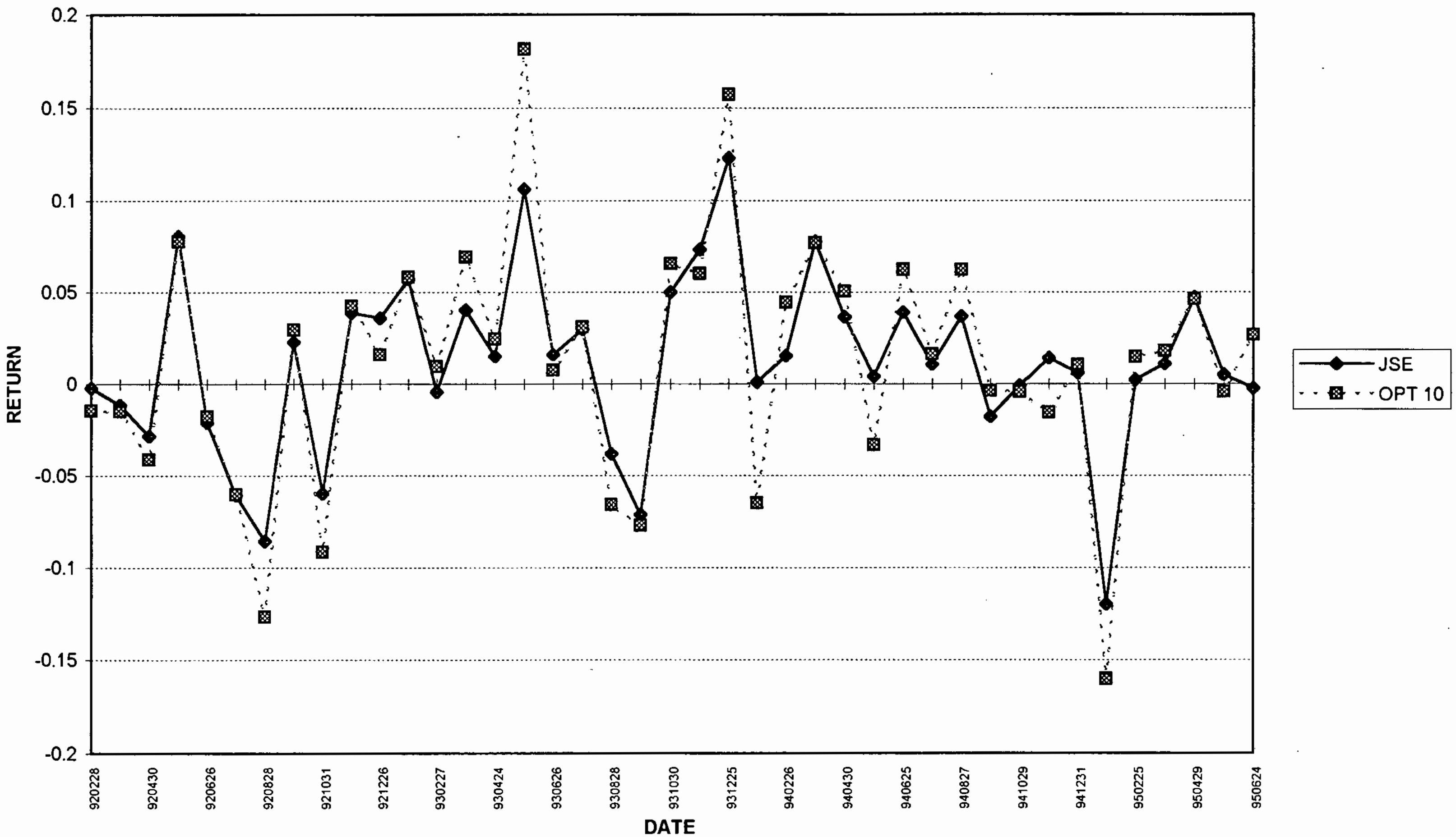
A4.3.3 STRATIFIED FUND - 5 SHARES



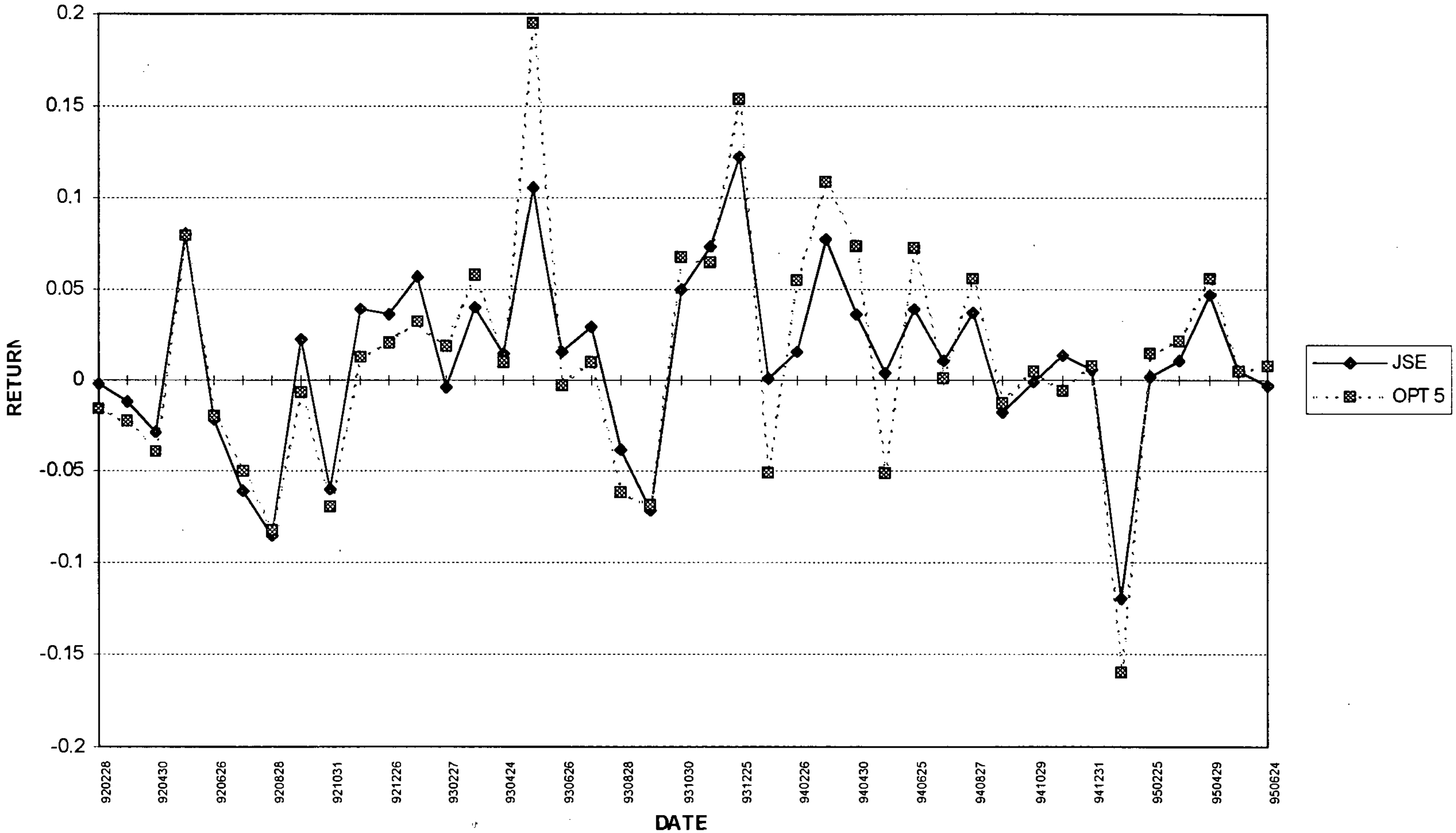
A4.3.4 OPTIMISED FUND - 15 SHARES



A4.3.5 OPTIMISED FUND - 10 SHARES



A4.3.6 OPTIMISED FUND - 5 SHARES



APPENDIX FOR CHAPTER 5

A5.1 Parameters of the World Model

Table 1 Parameters of the World Model (as at 31/12/1994) ranked according to World beta

Share Name	alpha		World beta		SA beta	
PUTCO	0.043	*0.952*	1.873	*1.951*	1.17	*1.217*
INVICTA	0.012	*0.514*	1.556	*3.005*	0.346	*0.668*
FINTECH	0.035	*1.683*	1.438	*3.22*	1.035	*2.315*
LEPLAT	-0.001	*-0.032*	1.132	*2.019*	2.405	*4.285*
JADE	0	*-0.016*	1.059	*2.745*	0.343	*0.888*
DA-GAMA	-0.014	*-0.752*	0.841	*2.163*	0.91	*2.338*
SPECLTY	0.039	*2.182*	0.818	*2.165*	0.69	*1.825*
BERTRAD	-0.019	*-1.312*	0.758	*2.494*	0.614	*2.019*
LASER	0.019	*1.039*	0.75	*1.958*	0.269	*0.7*
JASCO	0.035	*1.993*	0.749	*1.974*	0.663	*1.748*
WALTONS	0.008	*0.644*	0.715	*2.605*	1.322	*4.81*
METCASH	0.011	*0.846*	0.644	*2.223*	0.659	*2.272*
FEDSURE	0.023	*2.534*	0.64	*3.332*	0.471	*2.451*
MALBAK	0.017	*1.428*	0.626	*2.504*	0.951	*3.8*
CROOKES	0	*-0.028*	0.597	*2.333*	0.579	*2.261*
WALHOLD	0.01	*0.695*	0.581	*1.987*	1.26	*4.307*
AECI	0.009	*0.667*	0.546	*1.988*	0.782	*2.844*
FIT	0.005	*0.445*	0.542	*2.137*	0.645	*2.54*
CHARTER	0.009	*1.106*	0.534	*2.943*	0.195	*1.075*
OVBEL	-0.009	*-1.006*	0.511	*2.818*	-0.226	*-1.243*
SAPPI	0.01	*0.901*	0.48	*1.942*	0.986	*3.981*
CBD-FUND	-0.007	*-0.683*	0.442	*1.927*	0.228	*0.991*
AMIC	0.013	*1.324*	0.441	*2.173*	1.133	*5.582*
SA-BREWS	0.017	*2.354*	0.438	*2.888*	0.742	*4.891*
RUSPLAT	0.006	*0.623*	0.427	*2.152*	1.35	*6.791*
BEVCON	0.019	*2.22*	0.426	*2.313*	0.735	*3.99*
TAMBOTI	-0.002	*-0.195*	0.387	*2.053*	0.198	*1.052*
RICHEMONT	0.014	*1.681*	0.37	*2.103*	0.725	*4.121*
IBJOFFE	-0.005	*-0.73*	0.343	*2.2*	-0.19	*-1.22*
HYPROP	-0.001	*-0.224*	0.294	*2.191*	0.233	*1.734*
ANGLO-AM	0.011	*1.817*	0.262	*2.097*	1.534	*12.289*
PROPFIN	0.005	*1.184*	-0.261	*-2.781*	0.122	*1.297*
SILTEK	0.017	*1.562*	-0.499	*-2.096*	0.689	*2.892*
W-R-CONS	0.01	*0.536*	-1.018	*-2.688*	0.646	*1.704*
BLYVOOR	0.007	*0.316*	-1.118	*-2.442*	0.964	*2.103*
ALLWEAR	0.137	*1.721*	-4.176	*-2.455*	2.703	*1.588*

t statistics of each estimate given in italics in adjacent column

A5.2 Parameters of USA (World) Model

Table 2 Parameters of the USA (World) Model (as at 31/12/1994) ranked according to USA beta

Share Name	alpha		World beta		USA beta		SA beta	
ALLWEAR	0.137	1.917	-4.176	-2.734	8.823	4.149	0.972	0.609
USKO	0.017	0.403	0.019	0.022	2.813	2.301	1.732	1.888
FINTECH	0.035	1.718	1.438	3.287	1.479	2.428	0.796	1.743
CULLINAN	-0.003	-0.139	0.02	0.049	1.478	2.646	-0.061	-0.146
ANAMINT	0.008	0.854	0.362	1.852	1.28	4.705	1.355	6.637
I-G-I	-0.024	-1.179	0.244	0.573	1.224	2.063	0.65	1.46
TOCO	0.035	2.612	-0.236	-0.818	1.218	3.035	0.553	1.836
FOODCRP	0.024	1.703	0.402	1.324	1.11	2.626	0.441	1.391
COASTAL	-0.019	-1.02	0.092	0.231	1.094	1.973	0.39	0.937
LENCO	0.036	3.125	0.092	0.38	1.084	3.213	0.793	3.13
GRINCOR	0.023	1.964	-0.096	-0.376	1.078	3.041	0.576	2.167
CGS-FOOD	0.014	1.644	-0.194	-1.033	1.069	4.099	0.409	2.092
DEBEERS	0.009	1.066	0.317	1.784	1.066	4.311	1.304	7.025
POWTECH	0.024	2.073	0.437	1.751	1.062	3.057	0.802	3.08
FRALEX	0.012	0.823	-0.085	-0.268	1.038	2.333	0.6	1.797
INTELES	0.018	1.393	0.053	0.193	1.035	2.692	0.835	2.895
LIB-HOLD	0.029	5.056	0.067	0.55	1.011	5.947	0.857	6.72
SCHARIG	0.047	2.722	-0.604	-1.652	1.009	1.982	1.091	2.859
RICHEMONT	0.014	1.814	0.37	2.269	0.981	4.325	0.57	3.351
TEMPORA	0.021	1.814	-0.108	-0.444	0.98	2.906	0.168	0.662
INHOLD	0.055	3.867	0.195	0.647	0.975	2.323	0.802	2.546
ELLERINE	0.046	3.35	0.103	0.35	0.968	2.376	0.462	1.511
MID-WITS	0.012	1.198	-0.095	-0.433	0.958	3.138	1.182	5.159
CASHBIL	0.046	2.737	-0.043	-0.12	0.958	1.917	0.712	1.9
GENTYRE-B	0.008	0.534	0.143	0.446	0.939	2.107	0.339	1.013
Q-DATA	0.052	3.237	-0.011	-0.032	0.934	1.949	0.317	0.883
MCPHAIL	0.014	1.044	0.242	0.821	0.929	2.263	0.537	1.743
SUNCRUSH	0.034	3.162	-0.428	-1.88	0.923	2.913	0.322	1.357
CENMAG	-0.006	-0.405	0.21	0.672	0.907	2.086	-0.488	-1.496
JOHNNIC	0.017	2.401	-0.108	-0.725	0.905	4.371	1.38	8.879
TRNSHEX	0.009	0.8	-0.371	-1.511	0.891	2.607	0.121	0.474
WALHOLD	0.01	0.695	0.581	1.987	0.883	2.17	1.174	3.846
STANPRO	0.005	0.691	0.177	1.073	0.877	3.809	0.338	1.955
METPOL	0.032	3.142	0.169	0.772	0.867	2.847	0.727	3.184
KWV-BEL	0.02	1.779	0.031	0.131	0.855	2.58	0.318	1.279
CBD-FUND	-0.007	-0.718	0.442	2.024	0.849	2.792	0.058	0.255
LIBERTY	0.027	4.348	0.127	0.955	0.847	4.595	0.845	6.108
METCASH	0.011	0.856	0.644	2.249	0.844	2.119	0.528	1.767
CGSMITH	0.018	2.486	-0.003	-0.017	0.805	3.815	0.653	4.122
PALAMIN	0.007	0.619	0.338	1.467	0.763	2.378	0.416	1.727
PPC	0.027	2.521	0.331	1.443	0.761	2.38	0.537	2.237
WALTONS	0.008	0.64	0.715	2.59	0.742	1.93	1.273	4.415
DALYS	0.034	2.779	-0.453	-1.751	0.741	2.055	0.185	0.683
ALTRON	0.018	1.858	0.333	1.633	0.736	2.591	0.334	1.566
REMBR-BEH	0.013	1.671	0.233	1.451	0.73	3.27	0.893	5.334
REMGRO	0.012	1.716	0.294	1.894	0.725	3.351	0.826	5.089
CTP	0.03	3.361	-0.015	-0.08	0.719	2.723	0.075	0.38
FEDSURE	0.023	2.6	0.64	3.418	0.683	2.621	0.36	1.844
AFROX	0.025	2.866	-0.143	-0.778	0.681	2.651	0.57	2.96

TIB	0.013	*2.19*	-0.104	*-0.802*	0.675	*3.744*	0.908	*6.717*
FIRSTBK	0.027	*3.245*	0.224	*1.271*	0.667	*2.713*	0.483	*2.619*
DELTA	0.02	*2.269*	0.122	*0.663*	0.648	*2.536*	0.595	*3.104*
TIGR-OATS	0.011	*1.228*	0.244	*1.315*	0.626	*2.42*	0.676	*3.489*
FDCRP-7CP	0.024	*2.217*	0.234	*1.008*	0.62	*1.923*	0.378	*1.563*
EDGARS	0.026	*2.475*	0.161	*0.708*	0.606	*1.908*	0.416	*1.748*
PANPROP	0.001	*0.125*	0.06	*0.327*	0.563	*2.189*	0.351	*1.82*
NAMPAK	0.02	*2.869*	0.277	*1.827*	0.552	*2.617*	0.832	*5.26*
FR	0.004	*0.659*	-0.163	*-1.236*	0.536	*2.92*	-0.027	*-0.199*
COPI	0.012	*1.709*	-0.069	*-0.466*	0.53	*2.555*	0.255	*1.637*
METPROP	-0.005	*-0.642*	0.223	*1.449*	0.519	*2.426*	0.144	*0.897*
TEGKOR	0.011	*1.439*	0.119	*0.7*	0.518	*2.186*	0.66	*3.714*
UMDONI	0.006	*0.916*	0.119	*0.811*	0.505	*2.468*	0.153	*0.995*
ANGLO-AM	0.011	*1.811*	0.262	*2.091*	0.474	*2.724*	1.564	*11.971*
SA-BREWS	0.017	*2.341*	0.438	*2.872*	0.42	*1.98*	0.714	*4.486*
ISCOR	0.013	*1.147*	0.497	*2.099*	-0.739	*-2.243*	1.861	*7.532*
CHOICE	0.074	*1.462*	1.468	*1.352*	-3.28	*-2.169*	0.673	*0.593*

t statistics of each estimate given in italics in adjacent column

A5.3 Parameters of the UK (World) Model

Table 3 Parameters of the UK (World) Model (as at 31/12/1994) ranked according to UK beta

Share Name	alpha		World beta		UK beta		SA beta	
FINTECH	0.035	*1.836*	1.438	*3.513*	2.029	*4.038*	0.567	*1.311*
CTP-6%-PP	0.016	*0.679*	0.866	*1.701*	1.543	*2.468*	-0.339	*-0.631*
AUTOQIP	0.017	*0.765*	0.297	*0.609*	1.193	*1.993*	0.626	*1.216*
LASER	0.019	*1.07*	0.75	*2.016*	1.012	*2.217*	0.007	*0.018*
DEBEERS	0.009	*1.061*	0.317	*1.776*	0.965	*4.404*	1.297	*6.884*
TRANS-NTL	0.03	*2.158*	0.071	*0.243*	0.929	*2.587*	0.986	*3.195*
SAMANCOR	0.023	*1.925*	0.2	*0.782*	0.927	*2.954*	1.17	*4.334*
MAST	0.01	*0.795*	-0.436	*-1.594*	0.862	*2.568*	0.114	*0.394*
GENCOR	0.004	*0.432*	0.355	*1.796*	0.836	*3.44*	1.067	*5.106*
ANAMINT	0.008	*0.813*	0.362	*1.764*	0.834	*3.311*	1.436	*6.627*
BEVCON	0.019	*2.315*	0.426	*2.412*	0.786	*3.626*	0.592	*3.174*
ANGLO-AM	0.011	*1.828*	0.262	*2.11*	0.784	*5.153*	1.48	*11.312*
CGS-FOOD	0.014	*1.571*	-0.194	*-0.987*	0.76	*3.158*	0.443	*2.142*
REMBR-BEH	0.013	*1.685*	0.233	*1.463*	0.725	*3.712*	0.871	*5.183*
SAFREN	0.023	*2.009*	0.108	*0.436*	0.681	*2.229*	0.573	*2.181*
BARLOWS	0.001	*0.133*	0.024	*0.102*	0.678	*2.36*	0.992	*4.016*
PREM-GRP	0.02	*1.821*	0.284	*1.22*	0.678	*2.375*	0.386	*1.574*
SAPPI	0.01	*0.9*	0.48	*1.938*	0.642	*2.11*	0.911	*3.484*
RUSPLAT	0.006	*0.619*	0.427	*2.136*	0.632	*2.573*	1.319	*6.249*
RICHEMONT	0.014	*1.709*	0.37	*2.137*	0.621	*2.926*	0.628	*3.437*
A-V-I	0.026	*2.538*	0.136	*0.612*	0.617	*2.265*	0.37	*1.58*
CROOKES	0	*-0.028*	0.597	*2.348*	0.612	*1.962*	0.468	*1.743*
CHARTER	0.009	*1.163*	0.534	*3.094*	0.603	*2.848*	0.043	*0.238*
DELTA	0.02	*2.27*	0.122	*0.663*	0.598	*2.657*	0.583	*3.012*
KERSAF	0.01	*0.895*	0.406	*1.749*	0.588	*2.065*	0.508	*2.074*
REMGRO	0.012	*1.694*	0.294	*1.869*	0.586	*3.034*	0.84	*5.057*
AMIC	0.013	*1.315*	0.441	*2.158*	0.56	*2.233*	1.099	*5.099*
CNAGALO	0.026	*2.619*	0.085	*0.394*	0.555	*2.107*	0.338	*1.491*
SA-BREWS	0.017	*2.368*	0.438	*2.905*	0.515	*2.788*	0.677	*4.255*
JOHNNIC	0.017	*2.346*	-0.108	*-0.709*	0.497	*2.658*	1.473	*9.166*
LIBERTY	0.027	*4.112*	0.127	*0.903*	0.488	*2.839*	0.916	*6.196*
LIB-HOLD	0.029	*4.516*	0.067	*0.491*	0.475	*2.828*	0.97	*6.719*
CGSMITH	0.018	*2.377*	-0.003	*-0.016*	0.474	*2.437*	0.714	*4.266*
TIB	0.013	*2.142*	-0.104	*-0.784*	0.463	*2.848*	0.948	*6.789*
TEGKOR	0.011	*1.431*	0.119	*0.696*	0.41	*1.952*	0.675	*3.734*
NAMPAK	0.02	*2.847*	0.277	*1.813*	0.407	*2.173*	0.859	*5.328*
CLYDE	0.027	*1.656*	0.324	*0.946*	-0.977	*-2.324*	0.301	*0.832*

t statistics of each estimate given in italics in adjacent column

A5.4 Parameters of Japanese (World) Model

Table 4 Parameters of the Japanese (World) Model (as at 31/12/1994) ranked according to Japanese beta

Share Name	alpha		World beta		JAP beta		SA beta	
INMINS	0.083	*1.162*	1.748	*1.148*	2.414	*2.024*	-1.788	*-1.159*
FORIM	0.049	*0.803*	2.278	*1.751*	2.227	*2.185*	-1.958	*-1.486*
WIT-NIGEL	0.031	*0.905*	0.232	*0.318*	1.303	*2.28*	-0.223	*-0.302*
LORAINE	0.043	*1.224*	-0.471	*-0.632*	1.225	*2.099*	2.778	*3.681*
BENONI	-0.002	*-0.077*	0.518	*0.78*	1.078	*2.075*	0.849	*1.264*
COROHLD	0.056	*1.68*	0.529	*0.746*	1.075	*1.934*	0.607	*0.845*
BENCO	0.056	*1.856*	-0.734	*-1.15*	1.005	*2.008*	1.802	*2.786*
AIDA	0.02	*0.842*	-0.043	*-0.086*	0.921	*2.354*	-0.065	*-0.129*
DEELKRL	-0.003	*-0.126*	-0.271	*-0.578*	0.862	*2.35*	1.761	*3.712*
E-DAGGA	-0.005	*-0.308*	0.338	*0.923*	0.831	*2.895*	1.22	*3.286*
DUIKERS	0.043	*2.13*	-0.006	*-0.015*	0.816	*2.441*	1.18	*2.73*
ALEXWYT	0.018	*1.012*	0.425	*1.101*	0.791	*2.618*	-0.502	*-1.286*
PREMPHARM	0.042	*2.625*	0.519	*1.512*	0.786	*2.925*	0.085	*0.244*
MESSINA	-0.027	*-1.311*	0.689	*1.542*	0.737	*2.106*	0.676	*1.495*
TWEEFONTN	0.039	*2.215*	-0.142	*-0.376*	0.728	*2.453*	0.934	*2.433*
FINTECH	0.035	*1.713*	1.438	*3.277*	0.713	*2.073*	0.916	*2.061*
SPANJRD	-0.003	*-0.174*	0.18	*0.473*	0.672	*2.249*	-0.303	*-0.785*
HARTIES	-0.001	*-0.062*	0.149	*0.368*	0.671	*2.111*	1.44	*3.505*
KLOOF	0.012	*0.992*	-0.138	*-0.519*	0.669	*3.201*	1.347	*4.983*
NAMFISH	0.012	*0.645*	-0.295	*-0.74*	0.661	*2.12*	0.248	*0.616*
E-T-CONS	0.002	*0.089*	0.298	*0.73*	0.651	*2.039*	1.471	*3.563*
DA-GAMA	-0.014	*-0.766*	0.841	*2.205*	0.638	*2.136*	0.803	*2.079*
KETTER	0.019	*1.047*	0.101	*0.261*	0.624	*2.064*	0.395	*1.011*
ZANDPAN	0.001	*0.042*	-0.125	*-0.341*	0.583	*2.021*	1.273	*3.413*
VOGELS	0.015	*1.05*	0.171	*0.546*	0.565	*2.295*	1.103	*3.463*
RANDFONTN	0.017	*0.986*	-0.126	*-0.338*	0.561	*1.919*	1.603	*4.237*
IMPLATS	0.008	*0.582*	0.444	*1.532*	0.512	*2.254*	1.593	*5.423*
PROSURE	0.022	*1.502*	-0.042	*-0.135*	0.487	*2.027*	0.593	*1.908*
GFSA	0.009	*0.876*	-0.107	*-0.501*	0.437	*2.597*	1.21	*5.566*
NINIAN	0.008	*0.638*	-0.33	*-1.241*	0.409	*1.961*	0.526	*1.953*
VAAL-REEF	0.005	*0.406*	-0.166	*-0.628*	0.401	*1.932*	1.555	*5.802*
GENCOR	0.004	*0.432*	0.355	*1.798*	0.399	*2.574*	1.124	*5.61*
DRIES	0.01	*0.886*	-0.392	*-1.637*	0.397	*2.12*	1.22	*5.036*
RUSPLAT	0.006	*0.622*	0.427	*2.149*	0.306	*1.963*	1.321	*6.558*
GENBEL	0.006	*0.73*	0.286	*1.65*	0.294	*2.165*	1.216	*6.922*
MINORCO	0.012	*1.367*	-0.029	*-0.152*	0.288	*1.914*	0.537	*2.764*
TEGKOR	0.011	*1.454*	0.119	*0.707*	0.281	*2.132*	0.673	*3.945*
JOHNNIC	0.017	*2.349*	-0.108	*-0.71*	0.237	*1.987*	1.441	*9.351*
LIB-HOLD	0.029	*4.536*	0.067	*0.494*	0.213	*1.999*	0.976	*7.077*
ANG-ALPHA	0.032	*2.713*	0.106	*0.418*	-0.39	*-1.963*	0.919	*3.571*
AGA	0.016	*1.622*	0.275	*1.331*	-0.434	*-2.682*	0.651	*3.107*
PORT	0.011	*0.951*	-0.271	*-1.089*	-0.462	*-2.372*	0.318	*1.263*
REUNERT	0.031	*3.11*	-0.022	*-0.105*	-0.47	*-2.828*	1.115	*5.189*
BOLWEAR	0.007	*0.443*	-0.243	*-0.697*	-0.662	*-2.427*	0.412	*1.167*
DELCORP	0.028	*1.636*	0.697	*1.887*	-0.78	*-2.695*	0.284	*0.759*
ICH	0.023	*1.359*	0.614	*1.685*	-1.005	*-3.52*	0.991	*2.686*
TECFIN	0.029	*0.691*	-0.08	*-0.09*	-1.332	*-1.903*	0.138	*0.152*

t statistics of each estimate given in italics in adjacent column

A5.5 Composition of Global Portfolio as at 31/12/94

Table 5

Share Name	%
PUTCO	8.826579
INVICTA	7.332705
FINTECH	6.776626
LEPLAT	5.33459
JADE	4.990575
DA-GAMA	3.963242
SPECLTY	3.854854
BERTRAD	3.572102
LASER	3.534402
JASCO	3.529689
WALTONS	3.369463
METCASH	3.034873
FEDSURE	3.016023
MALBAK	2.950047
CROOKES	2.813384
WALHOLD	2.737983
AECI	2.573044
FIT	2.554194
CHARTER	2.516494
OVBEL	2.408106
SAPPI	2.262017
CBD-FUND	2.082941
AMIC	2.078228
SA-BREWS	2.06409
RUSPLAT	2.012253
BEVCON	2.00754
TAMBOTI	1.823751
RICHEMONT	1.743638
IBJOFFE	1.6164
HYPROP	1.385485
ANGLO-AM	1.234684

A5.6 Composition of Global Portfolio as at 31/12/95

Table 6

Share Name	**%**
CAXTON	6.23869
DA-GAMA	5.609844
BOWCALF	5.329352
FINTECH	4.858849
FIT	4.723127
FEDSURE	4.234528
SPUR	4.017372
EDGARS	3.809265
CBD-FUND	3.764025
MALBAK	3.759501
MARCONS	3.655447
BEVCON	3.574014
SBIC	3.46996
WOOLTRU	3.062794
CHEMSERVE	2.949692
ANAMINT	2.854687
HISTONE	2.854687
AMAPROP	2.750633
DEBEERS	2.633008
SA-BREWS	2.578719
OCTODEC	2.542526
INVSTEC	2.452045
CHARTER	2.25751
LIB-HOLD	2.198697
LIBVEST	2.198697
GENBEL	2.117264
STANPRO	2.099168
METPROP	2.031307
AF-OVR6PP	1.886536
RICHEMONT	1.791531
LIBERTY	1.696526

A5.7 Composition of UK Portfolio as at 31/12/94

Table 7

Share Name	%
FINTECH	5.984949
RHOEX	4.4989
CTP-6%-PP	4.197343
LASER	3.037301
INVICTA	2.906898
EVERITE	2.784645
AUTOQIP	2.665109
DEBEERS	2.276617
CARGO	2.170665
BEVCON	2.113614
ANAMINT	2.105463
GENCOR	2.097313
CROOKES	2.059279
SAMANCOR	2.023962
BTRDUN	1.977777
CHARTER	1.94246
SAPPI	1.931593
ANGLO-AM	1.855524
WALTONS	1.839224
RUSPLAT	1.831074
TRANS-NTL	1.82564
MALBAK	1.79304
BERTRAD	1.779456
RICHEMONT	1.722405
AMIC	1.719688
KERSAF	1.719688
REMBR-BEH	1.700671
PREM-GRP	1.692521
FOODCRP	1.665354
SA-BREWS	1.632753
WALHOLD	1.600152
FEDSURE	1.594719
ISCOR	1.562118
REMGRO	1.540384
SAFREN	1.426282
VENTRON	1.404548
WOOLTRU	1.390964
BIDVEST	1.380097
SANTAM	1.366514
TAMBOTI	1.358363
A-V-I	1.350213
DELTA	1.293162
BARLOWS	1.287729
GENBEL	1.189926
NAMPAK	1.184493
CNAGALO	1.157326
TIGR-OATS	1.149175
ALTRON	1.116575
CGS-FOOD	1.102991
FIRSTBK	1.100274

LIBERTY	1.097558
LIB-HOLD	0.980738
TEGKOR	0.942704
CGSMITH	0.872069

A5.8 Composition of UK Portfolio as at 31/12/95

Table 8

Share Name	%
RHOEX	8.356918
FINTECH	5.971537
LASER	5.59958
EVERITE	5.086116
CTP-6%-PP	4.516051
TRANS-NTL	3.54977
DEBEERS	3.311232
BEVCON	3.064607
GENCOR	2.967575
DELCORP	2.951403
WOOLTRU	2.890758
FOODCRP	2.874586
ANAMINT	2.785639
SAPPI	2.55923
ANGLO-AM	2.555187
A-V-I	2.526886
MALBAK	2.498585
RUSPLAT	2.470284
CHARTER	2.381337
CHEMSERVE	2.191316
CNAGALO	2.150885
SA-BREWS	2.134713
EDGARS	2.013423
AMIC	2.001294
PREM-GRP	1.993208
FDCRP13CD	1.985122
FEDSURE	1.977036
REMBR-BEH	1.928519
OCTODEC	1.871917
DELTA	1.815315
CENPROP	1.803186
LIB-HOLD	1.576777
REMGRO	1.455486
AF-OVR6PP	1.431228
RICHEMONT	1.415056
LIBERTY	1.338239

A5.9 Composition of Japanese Portfolio as at 31/12/94

Table 9

Share Name	%
FORIM	9.021868
INMINS	8.486595
PUTCO	5.284107
FINTECH	4.15866
GEFCO	3.861286
INVICTA	3.591362
COROHLD	3.335163
BENONI	3.316863
LEPLAT	3.092689
EGOLI	3.051514
DA-GAMA	2.91884
MESSINA	2.86394
PREMPHARM	2.676366
ALEXWYT	2.516241
E-DAGGA	2.447616
MSAULI	2.324092
E-T-CONS	1.976393
IMPLATS	1.930643
SPANJRD	1.811694
FEDSURE	1.770519
CARGO	1.747644
POWTECH	1.610394
BERTRAD	1.578369
VOGELS	1.56007
PLATE-GL	1.528045
GENCOR	1.518895
RUSPLAT	1.445695
R-M-PROPS	1.43197
WALTONS	1.427395
CROOKES	1.42282
FIT	1.308445
SA-DRUG	1.308445
TAMBOTI	1.290145
SANTAM	1.271846
ENSIGN	1.235246
RICHEMONT	1.180346
GENBEL	1.166621
BIDVEST	1.152896
CHARTER	1.038521
REMGRO	1.011071
TEGKOR	0.837222
NAMPAK	0.805197
ANGLO-AM	0.686248

A5.10 Composition of Japanese Portfolio as at 31/12/95

Table 10

Share Name	%
INMINS	13.38594
COROHLD	5.186932
WES-AREAS	5.052206
EGOLI	4.898234
WIT-NIGEL	4.566232
MESSINA	4.311216
E-T-CONS	3.87817
HARTIES	3.541356
DEELKRL	3.194919
E-DAGGA	3.012077
IMPLATS	3.012077
PREMPHARM	2.853294
BOWCALF	2.728191
KLOOF	2.646394
DA-GAMA	2.511668
WSTN-DEEP	2.482798
ERGO	2.473175
RUSPLAT	2.357696
FEDSURE	2.30958
ALEXWYT	2.304768
MCPHAIL	2.011259
SAAMBOU	2.001636
GENCOR	1.943896
FIT	1.92465
DRIES	1.886157
LIB-HOLD	1.583025
AMGOLD	1.515662
HARWILL	1.453111
GENBEL	1.438676
GFSA	1.385748
OCTODEC	1.279892
LIBVEST	1.236588
AF-OVR6PP	1.149978
RICHEMONT	1.010441
LIBERTY	1.000818
KR-HALF	0.471539

A5.11 Composition of US Portfolio as at 31/12/94

Table 11

Share Name	%
ALLWEAR	7.206994
USKO	3.650845
FINTECH	3.348158
EUREKA	2.516191
INVICTA	2.34371
ANAMINT	2.013968
CULLINAN	1.926036
FOODCRP	1.834723
I-G-I	1.824577
POWTECH	1.807667
METCASH	1.733263
WALHOLD	1.719736
DEBEERS	1.69268
WALTONS	1.674079
RICHEMONT	1.636877
SPECLTY	1.62504
CBD-FUND	1.5388
FEDSURE	1.52189
LENCO	1.491452
INHOLD	1.45425
MCPHAIL	1.440722
CRULIFE	1.393375
INTELES	1.389993
CENMAG	1.381538
LIB-HOLD	1.373083
GENTYRE-B	1.354482
ELLERINE	1.352791
TOCO	1.335881
PALAMIN	1.324044
PPC	1.313898
STANPRO	1.308825
GRINCOR	1.295297
METPOL	1.288533
ALTRON	1.28346
OZZ	1.259787
REMGRO	1.229349
LIBERTY	1.220894
CGS-FOOD	1.187074
REMBR-BEH	1.175237
MALBAK	1.165091
PLATE-GL	1.160018
TEMPORA	1.156636
MID-WITS	1.141417
KWV-BEL	1.134653
ENSIGN	1.126198
SENTRCHEM	1.087306
FIRSTBK	1.085615
JOHNNIC	1.06025
TIGR-OATS	1.051795
CGSMITH	1.036576

FDCRP-7CP	1.034885
CROOKES	1.024739
SOTHERN	1.014593
SANTAM	0.990919
NAMPAK	0.989228
SA-BREWS	0.980773
DELTA	0.9571
EDGARS	0.943572
CTP	0.913134
METPROP	0.892842
BEVCON	0.875932
ANGLO-AM	0.874241
GENBEL	0.808293
RUSPLAT	0.801529
TEGKOR	0.788001
PANPROP	0.78631
UMDONI	0.771091
TIB	0.767709
NEDCOR	0.75249
CHARTER	0.744035
COPI	0.613829

A5.12 Composition of US Portfolio as at 31/12/95

Table 12

Share Name	%
USKO	8.793262
GENTECH	5.624017
FINTECH	4.575249
FOODCRP	4.381883
DA-GAMA	3.98204
GENTYRE-B	3.552701
Q-DATA	3.513372
POWTECH	3.224961
STANPRO	2.988988
INTELES	2.988988
CENMAG	2.874279
ELLERINE	2.867724
CBD-FUND	2.841505
EDGARS	2.68419
FDCRP-7CP	2.654693
METCASH	2.579313
ADCOCK	2.556371
SBIC	2.484268
TEMPORA	2.366282
RICHEMONT	2.366282
LIB-HOLD	2.323676
CHEMSERVE	2.195857
FDCRP13CD	2.189303
METPROP	2.018878
FEDSURE	1.999213
BEVCON	1.972994
METPOL	1.966439
SA-BREWS	1.874672
ABI	1.828789
AFROX	1.818956
ATLAS	1.78946
REMGRO	1.756686
OCTODEC	1.743576
DELTA	1.727189
UMDONI	1.560042
LIBERTY	1.333901

A5.13 Program to calculate parameters of extended market model - World Index only

```
C****************************************************************
C PROGRAM TO CALCULATE PARAMETERS OF
C EXTENDED MARKET MODEL
C
C WRITTEN BY : CALLY ARDINGTON 1996
C****************************************************************

C              VARIABLE DEFINITIONS

      CHARACTER*11 SH(-2:1520),INDEX,CURRENCY
      CHARACTER*9 INAME,ONAME,PNAME
      INTEGER*4 ClcDate,N1,PERIOD,NMBR(1520)
      REAL*4 RET(1520,60),RESID(60),LASTPR(1520,60),TMP(60)

C****************************************************************
C READ IN
C 1. THE NAME OF FILE CONTAINING SHARE NAMES
C 2. THE DATE AT WHICH TO FIT THE MODEL
C 3. THE LENGTH OF THE ESTIMATION PERIOD
C****************************************************************

      WRITE(*,900)
900   FORMAT(' INPUT FILE NAME FOR SHARE NAMES ')
      READ(*,910) INAME
910   FORMAT(A20)
      write(*,*) 'Rand model - two indices'
908   write(*,*) 'Enter date for which to fit model:'
909   write(*,918) 'enter as YYMMDD, 0 for latest date '
      Read(*,*) ClcDate
918   Format(10X,A)
      WRITE(*,*) 'Enter length of period '
      READ(*,*) PERIOD



C****************************************************************
C THE SUBROUTINE SHARE READS IN THE SHARE
C NAMES FROM THE FILE INAME
C****************************************************************
      CALL SHARE(INAME,SH,N1)


C****************************************************************
C THE SUBROUTINE GETPR RETURNS A MATRIX
C OF RETURNS FOR THE INDICES AND EACH OF
C THE SHARES SPECIFIED IN THE FILE INAME
C****************************************************************
      CALL GETPR(SH,N1,LASTPR,RET,CLCDATE,PERIOD,NMBR,TMP)


C****************************************************************
C THE SUBROUTINE INTERNAT CALCULATES
C THE PARAMETERS FOR THE EXTENDED
C MARKET MODEL AND WRITES THE OUTPUT
C TO FILE
```

```
C*******************************************************************
     CALL INTERNAT(SH,N1,RET,PERIOD,NMBR)


     STOP

     END


C*******************************************************************
C END OF THE MAIN PROGRAM
C*******************************************************************


      SUBROUTINE SHARE(INAME,SH,N)
C*******************************************************************
C This routine is used simply to read in all the share names
C specified in the file INAME
C*******************************************************************
        CHARACTER*11 SH(-2:1520)
        CHARACTER*9 INAME
         OPEN(1,FILE=INAME,STATUS='OLD')
       SH(-2)='WORLD     '
       SH(-1)='RAND      '
       SH(0)='JSE-OVER  '
         I= 0
930     I=I+1
         READ(1,35,END=200) SH(I)
35      FORMAT(4X,A11)
         GOTO 930
200     N=I-1
         RETURN
         END


        SUBROUTINE CALCDATE(VALUE,YR,MT,DY)
C*******************************************************************
C This routine computes the day, month and year of a
C given date.
C*******************************************************************
        INTEGER*4 VALUE,Mant,YR,MT,DY
         YR = INT(VALUE/10000)
         Mant = VALUE - YR * 10000
         MT = INT(Mant/100)
         Mant = Mant - MT * 100
         DY = INT(Mant)
        RETURN
        END


        SUBROUTINE CalcReturns(I,NM,Y,R)
C*******************************************************************
C Calculates monthly returns
C*******************************************************************
      INTEGER*4 I,NM,M
        REAL*4 R(1520,60),Y(1520,60)
         DO 433 M=2,NM
           R(I,M)=(Y(I,NM+1-M)-Y(I,NM+2-M))/Y(I,NM+2-M)
433     CONTINUE
```

```
      RETURN
      END



    SUBROUTINE REGRESS(X,Y,N,ALPHA,BETA,RSQ,RESID)
C*****************************************************************
C     Linear Regression
C*****************************************************************
     INTEGER*4 N
    REAL*4 X(N),Y(N),ALPHA,BETA,RESID(N)
    XSUM=0
     YSUM=0
     XYSUM=0
     XSQ=0
     YSQ=0
     SSXX=0
     SSYY=0
     SSXY=0
     DO 500 G=1,N
          XSUM=XSUM + X(G)
             YSUM=YSUM + Y(G)
             XSQ=XSQ + X(G)**2
             YSQ=YSQ + Y(G)**2
             XYSUM=XYSUM + X(G)*Y(G)
500          CONTINUE
     SSXX=XSQ-XSUM*XSUM/FLOAT(N)
     SSYY=YSQ-YSUM*YSUM/FLOAT(N)
     SSXY=XYSUM-XSUM*YSUM/FLOAT(N)
     BETA=SSXY/SSXX
     ALPHA=YSUM/FLOAT(N)-BETA*XSUM/FLOAT(N)
     RSQ=SSXY**2/(SSXX*SSYY)
    DO 501 G=1,N
       RESID(G)=Y(G)-ALPHA-BETA*X(G)
501   CONTINUE
     RETURN
     END



    SUBROUTINE GETPR(SH,N1,LASTPR,RET,CLCDATE,
   &     PERIOD,NMBR,TMP)
C***********************************************************
C READS IN PRICES AND CALCULATES
C RETURNS FOR ALL SHARES AND INDICES
C*******************************************************

    PARAMETER (MM=4,MAX=1200,NZ=1520)
   STRUCTURE /FILE_REC_STRUCT/          !
    CHARACTER*11 SHARE                  !
    INTEGER*4 KNUM                !STRUCTURE OF THE
    INTEGER*4 KHIST(10)            !  DATA FILE
    REAL PRICE(10)                !CONTAINING SHARE
    INTEGER*4 DATE(MAX)              !   PRICES
    INTEGER*4 LAST(MAX)              !i.e. DCBALL.DAT
    INTEGER*4 HI(MAX)              !
    INTEGER*4 LO(MAX)              !
    INTEGER*4 VO(MAX)              !
   END STRUCTURE                   !
```

```
      RECORD /FILE_REC_STRUCT/ FILE_REC, WEEK_REC
      CHARACTER*11 SH(-2:NZ)
      INTEGER*4 Jy1,Jy2,Jm1,Jm2,PERIOD,OBS,Z
      INTEGER*4 Jd1,Jd2,WeekNo,N1,NMBR(1520),G3
      INTEGER*4 ClcDate,WDATE
      REAL*4 LASTPR(1520,60),TMP(60),WPRICE
      REAL*4 Ret(1520,60)
      LOGICAL MoreData,ZeroPr,mistake
      COMMON /CMNBLK/ FILE_REC, WEEK_REC

C***************************************************************
C OPEN WORLD DATA FILE AND READ IN VALUES
C***************************************************************
       OPEN(2,FILE='WORLD.INP',STATUS='OLD')
         DO 777 J=1,60
905        READ(2,56) WDATE, WPRICE
           IF (WDATE .GT. CLCDATE) GOTO 905
           LASTPR(-2,J)=WPRICE
777        CONTINUE
56       FORMAT(I6,1X,F7.3)

C***************************************************************
C OPEN THE FILE CONTAINING THE SHARE DATA
C (DCBALL.DAT)
C***************************************************************
      OPEN(11,FILE='STA:[CGTROS.DOOL]DCBALL.DAT',
     +    ORGANIZATION='INDEXED',STATUS='OLD',
     1    ACCESS='KEYED',RECORDTYPE='VARIABLE',
     2    FORM='UNFORMATTED',RECL=8000,
     3    KEY=(1:11:CHARACTER),
     4    IOSTAT=IOS,ERR=999)

      OBS=NINT(FLOAT(PERIOD)/52*12)
      NUMMTHS = 60
      DO 5555 I=-1,N1
           write(*,*) 'Computing statistics for ',SH(I)

C***************************************************************
C READ IN THE MONTH-END PRICES OVER THE
C PAST 5 YEARS.
C***************************************************************

           Read(11,Key=SH(I),KeyID=0,IOStat=IOS,ERR=997)
     &      Week_Rec.Share,Week_Rec.KNum,
     1      (Week_Rec.KHist(J),J=1,10),
     2      (Week_Rec.Price(J),J=1,10),
     3      (Week_Rec.Date(J),Week_Rec.Last(J),
     4       Week_Rec.Hi(J),Week_Rec.Lo(J),
     5       Week_Rec.Vo(J),J=1,Week_Rec.KNum)

           MaxWeeks = Week_Rec.KNum  ! # OF WEEKS DATA AVAILABLE
           WeekNo = MaxWeeks
           IF (ClcDate .NE. 0) THEN
             DO WHILE ( Week_Rec.Date(WeekNo) .GT. ClcDate )
              WeekNo = WeekNo - 1  ! GO TO THE PRICE AS AT ClcDate
             END DO
           END IF
```

```
      IF (WeekNo .GT. PERIOD) THEN  ! Share must have at least
         NumMths = 1                ! PERIOD weeks data
         MoreData = .TRUE.
         LastPr(I,NumMths) = Week_Rec.Last(WeekNo)
         IF (LastPr(I,1) .NE. 0) THEN
            ZeroPr = .FALSE.
         ELSE
            ZeroPr = .TRUE.
         END IF
         DO WHILE ((NumMths .LT. OBS) .AND. (MoreData))

C*****************************************************************
C CHECK TO SEE IF NEW MONTH YET, AND IF SO
C READ THE PRICE AT THE END OF THE
C PREVIOUS MONTH
C*****************************************************************
         CALL CALCDATE(Week_Rec.Date(WeekNo),Jy1,Jm1,Jd1)
         CALL CALCDATE(Week_Rec.Date(WeekNo-1),Jy2,Jm2,Jd2)
          IF (Jm1 .NE. Jm2) THEN
            NumMths = NumMths + 1
            LastPr(I,NumMths) = Week_Rec.Last(WeekNo - 1)
            IF (LastPr(I,NumMths) .EQ. 0) ZeroPr = .TRUE.
          END IF

          WeekNo = WeekNo - 1      !go back a week
          IF (WeekNo.LE.1) MoreData=.FALSE.  !start of file
         END DO

         IF (ZeroPr) THEN
           write(*,443) SH(I)
           goto 5555          !if there is a zero price then
         END IF               !skip to next share
443     FORMAT(4X,A11,'*** NOTE: ZERO PRICE IN DATA***')


C*****************************************************************
C CONVERT WORLD INDEX TO RANDS BY
C MULTIPLYING INDEX LEVEL BY EXCHANGE RATE
C*****************************************************************

      IF (I.EQ.-1) THEN
      DO 222 Z=1,NUMMTHS
         LASTPR(-1,Z)=LASTPR(-1,Z)*LASTPR(-2,Z)
222     CONTINUE
      END IF


C*****************************************************************
C CALCULATE RETURNS FOR INDEX AND ALL SHARES
C*****************************************************************
539     CALL CalcReturns(I,NumMths,LastPr,Ret)


         NMBR(I) = NUMMTHS
        ELSE
         WRITE(*,*) '*** NOT ENOUGH DATA FOR ',SH(I),' ***'
        END IF   !enough weeks for at least twelve months

5555  CONTINUE        !with next share
```

```
999   WRITE(*,*)'IOSTAT = ',IOS
997   WRITE(*,*)'NO WEEKLY DATA'
     RETURN
     END



      SUBROUTINE INTERNAT(SH,N1,RET,PERIOD,NMBR)
C************************************************************
C CALCULATES THE PARAMTERS OF THE
C EXTENDED MARKET MODEL AND WRITES
C THE OUTPUT TO A FILE
C************************************************************
      CHARACTER*11 SH(-2:1520)
      INTEGER*4 N1,PERIOD,NMBR(1520),OBS,P
      REAL*4 RET(1520,60),X(60),Y(60),RESID(60)
      REAL*4 ALPHA,BETA,RSQ,YY(60,1),XX(60,20),S2
      REAL*4 BET(20,1),RSS,TSTAT(20,1),V(60),SV(20,1)

      OPEN(8,FILE='MODEL.DAT',STATUS='NEW')
      OPEN(9,FILE='BETA.DAT',STATUS='NEW')
      OPEN(10,FILE='RISK.DAT',STATUS='NEW')

       OBS=NINT(FLOAT(PERIOD)/52*12)
       OBS=OBS-1
       P=3
      DO 555 I=1,OBS
       X(I)=RET(-1,I+1)
       Y(I)=RET(0,I+1)
555    CONTINUE

      CALL REGRESS(X,Y,OBS,ALPHA,BETA,RSQ,RESID)

      DO 505 J=1,OBS
        XX(J,1)=1.0
        XX(J,2)=RET(-1,J+1)
        XX(J,3)=RESID(J)
505     CONTINUE
      DO 5506 Z=1,N1
        IF (NMBR(Z) .GT. 0) THEN
          DO 5507 J=1,OBS
            YY(J,1)=RET(Z,J+1)
5507         CONTINUE

          CALL MREGRESS(XX,YY,OBS,P,BET,TSTAT,RSS)
          DO 220 K=1,OBS
            V(K)=YY(K,1)
220         CONTINUE
          CALL SVAR(V,OBS,S2)
          SV(1,1)=S2
          DO 221 I=2,P
            DO 222 K=1,OBS
              V(K)=XX(K,I)
222           CONTINUE
            CALL SVAR(V,OBS,S2)
            SV(I,1)=S2*BET(I,1)**2/SV(1,1)*100
221         CONTINUE
           SV(4,1)=100.0-SV(2,1)-SV(3,1)
```

```
       IF (ABS(TSTAT(2,1)) .GE. 1.90) THEN
       WRITE(8,676) SH(Z),BET(1,1),TSTAT(1,1),BET(2,1),
   &     TSTAT(2,1),BET(3,1),TSTAT(3,1)
676      FORMAT(1X,A11,2X,F7.3,2X,F7.3,2X,F7.3,2X,F7.3,2X,
   &     F7.3,2X,F7.3)
       IF (BET(2,1) .GT. 0.0) WRITE(9,677) SH(Z),BET(2,1)
677      FORMAT(4X,A11,1X,F6.2)
       WRITE(10,678) SH(Z),SV(1,1),SV(2,1),SV(3,1),SV(4,1)
678      FORMAT(1X,A11,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4)
       END IF
        ENDIF
5506    CONTINUE

      RETURN
      END



      SUBROUTINE SVAR(V,OBS,S2)
C*************************************************************
C  Finds the variance of v(OBS). Output is S2. OBS <=60
C*************************************************************
      REAL*4 V(60),S2,SUM,SUMSQ
      INTEGER*4 OBS
      SUM=0
      SUMSQ=0
      DO 444 I=1,OBS
        SUM=SUM+V(I)
        SUMSQ=SUMSQ+V(I)**2
444     CONTINUE
      S2=(1.0/FLOAT(OBS-1))*(SUMSQ-(1.0/FLOAT(OBS))*SUM*SUM)
      RETURN
      END

      SUBROUTINE TRANP(MATIN,A,B,C,D,MATOUT)
C**************************************************************
C Finds the transpose of MATIN (R1xC1). The result
C  is MATOUT. DIM1XDIM2 is the size of the array.
C**************************************************************
      INTEGER*4 A,B,C,D
      REAL*4 MATIN(C,D),MATOUT(D,C)

      DO 8000 I=1,A
      DO 8000 J=1,B
          MATOUT(J,I)=MATIN(I,J)
8000    CONTINUE
      RETURN
      END


      SUBROUTINE MULT(MAT1,R1,C1,D1,D2,MAT2,R2,C2,D21,D22,PROD)
C**************************************************************
C Multiplies MAT1 (R1xC1) by MAT2 (R2xC2). The result
C  is PROD (R1xC2). Array size: MAT1(D1xD2),MAT2(D21xD22)
C**************************************************************
      INTEGER*4 R1,C1,D1,D2,R2,C2,D21,D22
      REAL*4 MAT1(D1,D2),MAT2(D21,D22),PROD(D1,D22)
     DO 7000 I=1,R1
          DO 705 J=1,C2
            PROD(I,J)=0.0
```

```
            DO 7010 K=1,C1
              PROD(I,J)=PROD(I,J)+MAT1(I,K)*MAT2(K,J)
7010          CONTINUE
705         CONTINUE
7000    CONTINUE
      RETURN
      END


       SUBROUTINE INVER(MAT1,MATIN,DIM,R)
C*****************************************************************
C  Inverts MATIN (DIMxDIM). The array is MATIN(RxR)
C*****************************************************************
       INTEGER*4 DIM,R
        REAL*4 MATIN(20,20)
        REAL*4 MAT1(20,20)
        REAL*4 INV(20,20)
        REAL*4 TEST(20)
      DO 33 J=1,DIM
        DO 34 I=1,DIM
         MATIN(I,J)=MAT1(I,J)
34       CONTINUE
33     CONTINUE
      K=0
20     K=K+1
      TEST(K)=1.0/MATIN(K,K)
      DO 40 J=1,DIM
         DO 30 I=1,DIM
           IF (J .EQ. K .AND. I .EQ. K) THEN
            INV(I,J)=(-1.0)*TEST(K)
           ELSEIF (J.EQ.K .AND. I.NE.K) THEN
            INV(I,J)=MATIN(I,J)*TEST(K)
           ELSEIF (I.EQ.K .AND. J.NE.K) THEN
            INV(I,J)=MATIN(I,J)*TEST(K)
           ELSE
            INV(I,J)=MATIN(I,J)-((MATIN(I,K)*MATIN(K,J))*TEST(K))
           ENDIF
30        CONTINUE
40     CONTINUE.
      CALL COPY (INV,DIM,DIM,R,R,MATIN)
      IF (K.LT.DIM) GO TO 20
       DO 31 J=1,DIM
         DO 32 I=1,DIM
          MATIN(I,J)=(-1.0)*MATIN(I,J)
32        CONTINUE
31      CONTINUE

       RETURN
       END


       SUBROUTINE COPY(MAT1,R1,C1,DIM1,DIM2,MAT2)
C***************************************************************************
C  Copies MAT1(R1xC1) to MAT2(R1xC1). The array is MAT1(DIM1,DIM2).
C***************************************************************************
        INTEGER*4 R1,C1,DIM1,DIM2
       REAL*4 MAT1(DIM1,DIM2),MAT2(DIM1,DIM2)
       DO 1002 I=1,DIM1
         DO 1002 J=1,DIM2
```

```
          MAT2(I,J)=0.0
1002    CONTINUE

        DO 1000 I=1,R1
         DO 1000 J=1,C1
          MAT2(I,J)=MAT1(I,J)
1000    CONTINUE
        RETURN
        END

      SUBROUTINE MREGRESS(XX,YY,OBS,P,BET,TSTAT,RSS)
C***************************************************************
C   Multiple regression of YY on XX. OBS is number of
C   observations. P is number of independent variables
C   including the mean. Output is BET(P) - the regression
C   coefficients, TSTAT(P) - the t-stats of these
C   estimates, RSS - the multiple R sqaured. OBS has
C   a maximum value of 60. P has a maximum value of
C   20.
C***************************************************************
       INTEGER*4 OBS,P
       REAL*4 C(20,20)
       REAL*4 YY(60,1),XX(60,20)
       REAL*4 XXT(20,60),XXTXX(20,20),XXTYY(20,1),BET(20,1)
       REAL*4 BETT(1,20),YYT(1,60),YYTYY(1,1)
       REAL*4 BTXTY(1,1),SSE,SBET(20,1),YSUM,SST,RSS
       REAL*4 TSTAT(20,1)

        CALL TRANP(XX,OBS,P,60,20,XXT)
        CALL MULT(XXT,P,OBS,20,60,XX,OBS,P,60,20,XXTXX)
        CALL INVER(XXTXX,C,P,20)
        CALL MULT(XXT,P,OBS,20,60,YY,OBS,1,60,1,XXTYY)
        CALL MULT(C,P,P,20,20,XXTYY,P,1,20,1,BET)
        CALL TRANP(YY,OBS,1,60,1,YYT)
        CALL TRANP(BET,P,1,20,1,BETT)
        CALL MULT(YYT,1,OBS,1,60,YY,OBS,1,60,1,YYTYY)
        CALL MULT(BETT,1,P,1,20,XXTYY,P,1,20,1,BTXTY)
        SSE=YYTYY(1,1)-BTXTY(1,1)
        IF((SSE.LT.0.0000001).OR.(SSE.LT.0.0))SSE=0.0000001
        YSUM=0
        DO 626 I=1,OBS
         YSUM=YSUM+YY(I,1)
626     CONTINUE
        DO 627 I=1,P
         SBET(I,1)= SQRT(SSE)/SQRT(FLOAT(OBS-P))*SQRT(C(I,I))
         TSTAT(I,1)=BET(I,1)/SBET(I,1)
627     CONTINUE
        SST=YYTYY(1,1)-(YSUM*YSUM)/OBS
        RSS=(SST-SSE)/SST
        RETURN
       END
```

A5.14 Program to calculate parameters of extended market model - World and Country Indices

```
C*****************************************************************
C PROGRAM TO CALCULATE PARAMETERS OF
C EXTENDED MARKET MODEL WITH WORLD
C INDEX AND INDEX FOR PARTICULAR FOREIGN
C MARKET
C
C WRITTEN BY : CALLY ARDINGTON 1996
C*****************************************************************

C              VARIABLE DEFINITIONS

    CHARACTER*11 SH(-4:1520),INDEX,CURRENCY
    CHARACTER*9 INAME,ONAME,PNAME
    INTEGER*4 ClcDate,N1,PERIOD,NMBR(1520)
    REAL*4 RET(1520,60),RESID(60),LASTPR(1520,60),TMP(60)

C*****************************************************************
C READ IN
C 1. THE NAME OF FILE CONTAINING SHARE NAMES
C 2. THE DATE AT WHICH TO FIT THE MODEL
C 3. THE LENGTH OF THE ESTIMATION PERIOD
C 4. THE INTERNATIONAL INDEX AND CURRENCY
C*****************************************************************

    WRITE(*,900)
900  FORMAT(' INPUT FILE NAME FOR SHARE NAMES ')
    READ(*,910) INAME

910  FORMAT(A20)
    WRITE(*,*) 'Rand model - three indices'
908  write(*,*) 'Enter date for which to fit model:'
909  write(*,918) 'enter as YYMMDD, 0 for latest date '
    Read(*,*) ClcDate
918  Format(10X,A)
    WRITE(*,*) 'Enter length of period '
    READ(*,*) PERIOD
     WRITE(*,*) 'Enter non-US foreign index to use '
     READ(*,917) INDEX
     WRITE(*,*) 'Enter currency to use'
     READ(*,917) CURRENCY
917  FORMAT(A11)


C*****************************************************************
C THE SUBROUTINE SHARE READS IN THE SHARE
C NAMES FROM THE FILE INAME
C*****************************************************************
CALL SHARE(INAME,SH,N1,INDEX,CURRENCY)     ! READ IN THE SHARE NAMES


C*****************************************************************
C THE SUBROUTINE GETPR RETURNS A MATRIX
C OF RETURNS FOR THE INDICES AND EACH OF
```

```
C THE SHARES SPECIFIED IN THE FILE INAME
C*****************************************************************
   CALL GETPR(SH,N1,LASTPR,RET,CLCDATE,PERIOD,NMBR,TMP)


C*****************************************************************
C THE SUBROUTINE INTERNAT CALCULATES
C THE PARAMETERS FOR THE EXTENDED
C MARKET MODEL AND WRITES THE OUTPUT
C TO FILE
C*****************************************************************
   CALL INTERNAT(SH,N1,RET,PERIOD,NMBR)


   STOP

   END



C*****************************************************************
C END OF THE MAIN PROGRAM
C*****************************************************************



    SUBROUTINE SHARE(INAME,SH,N,INDEX,CURRENCY)
C*****************************************************************
C This routine is used simply to read in all the share names
C specified in the file INAME
C*****************************************************************
       CHARACTER*11 SH(-4:1520),INDEX,CURRENCY
       CHARACTER*9 INAME
        OPEN(1,FILE=INAME,STATUS='OLD')
      SH(-4)='WORLD      '
      SH(-3)=CURRENCY
      SH(-2)='RAND       '
      SH(-1)=INDEX
      SH(0)='JSE-OVER   '
        I= 0
930    I=I+1
        READ(1,35,END=200) SH(I)
35     FORMAT(4X,A11)
        GOTO 930
200    N=I-1
        RETURN
        END


       SUBROUTINE CALCDATE(VALUE,YR,MT,DY)
C*****************************************************************
C This routine computes the day, month and year of a
C given date
C*****************************************************************
       INTEGER*4 VALUE,Mant,YR,MT,DY
        YR = INT(VALUE/10000)
        Mant = VALUE - YR * 10000
        MT = INT(Mant/100)
        Mant = Mant - MT * 100
        DY = INT(Mant)
```

```
        RETURN
        END

        SUBROUTINE CalcReturns(I,NM,Y,R)
C*************************************************************
C Calculates monthly returns
C*************************************************************
     INTEGER*4 I,NM,M
        REAL*4 R(1520,60),Y(1520,60)
         DO 433 M=2,NM
           R(I,M)=(Y(I,NM+1-M)-Y(I,NM+2-M))/Y(I,NM+2-M)
433     CONTINUE
        RETURN
        END


     SUBROUTINE REGRESS(X,Y,N,ALPHA,BETA,RSQ,RESID)
C*************************************************************
C       Linear Regression
C*************************************************************
        INTEGER*4 N
     REAL*4 X(N),Y(N),ALPHA,BETA,RESID(N)
     XSUM=0
        YSUM=0
        XYSUM=0
        XSQ=0
        YSQ=0
        SSXX=0
        SSYY=0
        SSXY=0
        DO 500 G=1,N
            XSUM=XSUM + X(G)
              YSUM=YSUM + Y(G)
              XSQ=XSQ + X(G)**2
              YSQ=YSQ + Y(G)**2
              XYSUM=XYSUM + X(G)*Y(G)
500           CONTINUE
        SSXX=XSQ-XSUM*XSUM/FLOAT(N)
        SSYY=YSQ-YSUM*YSUM/FLOAT(N)
        SSXY=XYSUM-XSUM*YSUM/FLOAT(N)
        BETA=SSXY/SSXX
        ALPHA=YSUM/FLOAT(N)-BETA*XSUM/FLOAT(N)
        RSQ=SSXY**2/(SSXX*SSYY)
     DO 501 G=1,N
          RESID(G)=Y(G)-ALPHA-BETA*X(G)
501    CONTINUE
        RETURN
        END



    SUBROUTINE GETPR(SH,N1,LASTPR,RET,CLCDATE,
   &     PERIOD,NMBR,TMP)
C*******************************************************
C READ IN PRICES AND CALCULATES RETURNS
C FOR ALL SHARES
C*******************************************************
```

```
      PARAMETER (MM=4,MAX=1200,NZ=1520)
     STRUCTURE /FILE_REC_STRUCT/          !
      CHARACTER*11 SHARE                !
      INTEGER*4 KNUM                  !STRUCTURE OF THE
      INTEGER*4 KHIST(10)              !  DATA FILE
      REAL PRICE(10)                 !CONTAINING SHARE
      INTEGER*4 DATE(MAX)               !   PRICES
      INTEGER*4 LAST(MAX)              !i.e. DCBALL.DAT
      INTEGER*4 HI(MAX)               !
      INTEGER*4 LO(MAX)               !
      INTEGER*4 VO(MAX)               !
     END STRUCTURE                    !


     RECORD /FILE_REC_STRUCT/ FILE_REC, WEEK_REC
     CHARACTER*11 SH(-4:NZ)
     INTEGER*4 Jy1,Jy2,Jm1,Jm2,PERIOD,OBS,Z
     INTEGER*4 Jd1,Jd2,WeekNo,N1,NMBR(1520),G3
     INTEGER*4 ClcDate,WDATE
     REAL*4 GG,G1,G2,LASTPR(1520,60),TMP(60)
     REAL*4 Ret(1520,60),WPRICE
     LOGICAL MoreData,ZeroPr,mistake
     COMMON /CMNBLK/ FILE_REC, WEEK_REC


C*****************************************************************
C OPEN WORLD DATA FILE AND READ IN VALUES
C****************************************************************
      OPEN(2,FILE='WORLD.INP',STATUS='OLD')
      DO 777 J=1,60
905     READ(2,56) WDATE, WPRICE
      IF (WDATE .GT. CLCDATE) GOTO 905
      LASTPR(-4,J)=WPRICE
777   CONTINUE
56    FORMAT(I6,1X,F7.3)

C*****************************************************************
C OPEN THE FILE CONTAINING THE SHARE DATA
C (DCBALL.DAT)
C*****************************************************************
     OPEN(11,FILE='STA:[CGTROS.DOOL]DCBALL.DAT',
    +    ORGANIZATION='INDEXED',STATUS='OLD',
    1    ACCESS='KEYED',RECORDTYPE='VARIABLE',
    2    FORM='UNFORMATTED',RECL=8000,
    3    KEY=(1:11:CHARACTER),
    4    IOSTAT=IOS,ERR=999)

     OBS=NINT(FLOAT(PERIOD)/52*12)
     NUMMTHS = 60
     DO 5555 I=-3,N1
         write(*,*) 'Computing stats for ',SH(I)

C*****************************************************************
C READ IN THE MONTH-END PRICES OVER THE
C PAST 5 YEARS.
C*****************************************************************

         Read(11,Key=SH(I),KeyID=0,IOStat=IOS,ERR=997)
    &     Week_Rec.Share,Week_Rec.KNum,
    1     (Week_Rec.KHist(J),J=1,10),
```

```
     2       (Week_Rec.Price(J),J=1,10),
     3       (Week_Rec.Date(J),Week_Rec.Last(J),
     4        Week_Rec.Hi(J),Week_Rec.Lo(J),
     5        Week_Rec.Vo(J),J=1,Week_Rec.KNum)

        MaxWeeks = Week_Rec.KNum  ! # OF WEEKS DATA AVAILABLE
        WeekNo = MaxWeeks
        IF (ClcDate .NE. 0) THEN
          DO WHILE ( Week_Rec.Date(WeekNo) .GT. ClcDate )
           WeekNo = WeekNo - 1  ! GO TO THE PRICE AS AT
          END DO              ! ClcDate
        END IF

      IF (WeekNo .GT. PERIOD) THEN  ! Share must have at least
         NumMths = 1                 ! PERIOD weeks data
         MoreData = .TRUE.
         LastPr(I,NumMths) = Week_Rec.Last(WeekNo)
         IF (LastPr(I,1) .NE. 0) THEN
            ZeroPr = .FALSE.
         ELSE
            ZeroPr = .TRUE.
         END IF
         DO WHILE ((NumMths .LT. OBS) .AND. (MoreData))

C*************************************************************
C CHECK TO SEE IF NEW MONTH YET, AND IF SO
C READ THE PRICE AT THE END OF THE
C PREVIOUS MONTH
C*************************************************************
         CALL CALCDATE(Week_Rec.Date(WeekNo),Jy1,Jm1,Jd1)
         CALL CALCDATE(Week_Rec.Date(WeekNo-1),Jy2,Jm2,Jd2)
           IF (Jm1 .NE. Jm2) THEN
            NumMths = NumMths + 1
            LastPr(I,NumMths) = Week_Rec.Last(WeekNo - 1)
            IF (LastPr(I,NumMths) .EQ. 0) ZeroPr = .TRUE.
           END IF

           WeekNo = WeekNo - 1      !go back a week
           IF (WeekNo.LE.1) MoreData=.FALSE.  !start of file
          END DO

         IF (ZeroPr) THEN
           write(*,443) SH(I)
           goto 5555         !if there is a zero price then
         END IF              !skip to next share
443     FORMAT(4X,A11,'*** NOTE: ZERO PRICE IN DATA***')


C*************************************************************
C CONVERT INTERNATIONAL INDEX INTO RANDS
C BY MULTIPLYING BY EXCHANGE RATE
C NB: CHECK IF INDEX IS NIKKEI - IF SO DIVIDE
C BY EXCHANGE RATE
C*************************************************************
       IF (I.EQ.-2) THEN
       DO 222 Z=1,NUMMTHS
         IF (SH(I) .EQ. 'NIKKEI') THEN
        LASTPR(-2,Z)=LASTPR(-2,Z)/LASTPR(-4,Z)
        ELSE
         LASTPR(-2,Z)=LASTPR(-2,Z)*LASTPR(-4,Z)
```

```
      END IF
222     CONTINUE
      END IF


C*******************************************************************
C CONVERT WORLD INDEX TO RANDS BY
C MULTIPLYING INDEX LEVEL BY EXCHANGE RATE
C*******************************************************************
      IF (I .EQ. -1) THEN
      DO 232 Z=1,NUMMTHS
         LASTPR(-1,Z)=LASTPR(-1,Z)*LASTPR(-3,Z)
232     CONTINUE
      ENDIF

C*******************************************************************
C CALCULATE RETURNS FOR INDEX AND ALL SHARES
C*******************************************************************
539     CALL CalcReturns(I,NumMths,LastPr,Ret)


         NMBR(I) = NUMMTHS
        ELSE
          WRITE(*,*) '*** NOT ENOUGH DATA FOR ',SH(I),' ***'
        END IF    !enough weeks for at least twelve months

5555   CONTINUE         !with next share


999   WRITE(*,*)'IOSTAT = ',IOS
997   WRITE(*,*)'NO WEEKLY DATA'
     RETURN
     END


      SUBROUTINE INTERNAT(SH,N1,RET,PERIOD,NMBR)
C***************************************************************
C CALCULATES THE PARAMETERS OF THE
C EXTENDED MARKET MODEL FOR A PARTICULAR
C FOREIGN INDEX AND WRITES THE OUTPUT
C TO A FILE
C***************************************************************
      CHARACTER*11 SH(-4:1520)
      INTEGER*4 N1,PERIOD,NMBR(1520),OBS,P
      REAL*4 RET(1520,60),X(60),Y(60),RESID(60)
      REAL*4 ALPHA,BETA,RSQ,YY(60,1),XX(60,20),S2
      REAL*4 BET(20,1),RSS,TSTAT(20,1),V(60),SV(20,1)

      OPEN(8,FILE='MODEL.DAT',STATUS='NEW')
      OPEN(9,FILE='BETA.DAT',STATUS='NEW')
      OPEN(10,FILE='RISK.DAT',STATUS='NEW')

       OBS=NINT(FLOAT(PERIOD)/52*12)
       OBS=OBS-1
       P=4
      DO 555 I=1,OBS
       X(I)=RET(-2,I+1)
       Y(I)=RET(-1,I+1)
555    CONTINUE
```

```
      CALL REGRESS(X,Y,OBS,ALPHA,BETA,RSQ,RESID)

      DO 505 J=1,OBS
        XX(J,1)=1.0
        XX(J,2)=RET(-2,J+1)
        XX(J,3)=RESID(J)
        YY(J,1)=RET(0,J+1)
505     CONTINUE

      CALL MREGRESS(XX,YY,OBS,3,BET,TSTAT,RSS)
      DO 506 J=1,OBS
        XX(J,4)=RET(0,J+1)-BET(1,1)-BET(2,1)*XX(J,2)
   &      -BET(3,1)*XX(J,3)
506     CONTINUE
      DO 5506 Z=1,N1
        IF (NMBR(Z) .GT. 0) THEN
          DO 5507 J=1,OBS
            YY(J,1)=RET(Z,J+1)
5507        CONTINUE

          CALL MREGRESS(XX,YY,OBS,P,BET,TSTAT,RSS)

          DO 220 K=1,OBS
            V(K)=YY(K,1)
220         CONTINUE
          CALL SVAR(V,OBS,S2)
          SV(1,1)=S2
          DO 221 I=2,P
            DO 222 K=1,OBS
              V(K)=XX(K,I)
222           CONTINUE
            CALL SVAR(V,OBS,S2)
            SV(I,1)=S2*BET(I,1)**2/SV(1,1)*100
221         CONTINUE
          SV(5,1)=100.0-SV(2,1)-SV(3,1)-SV(4,1)

       IF (ABS(TSTAT(3,1)) .GE. 1.90) THEN
       WRITE(8,676) SH(Z),BET(1,1),TSTAT(1,1),BET(2,1),
   &    TSTAT(2,1),BET(3,1),TSTAT(3,1),BET(4,1),TSTAT(4,1)
676     FORMAT(1X,A11,1X,F6.3,1X,F7.3,1X,F6.3,1X,F7.3,1X,
   &     F6.3,1X,F7.3,1X,F6.3,1X,F7.3)
       IF (BET(3,1) .GT. 0.0) WRITE(9,677) SH(Z),BET(3,1)
677     FORMAT(4X,A11,1X,F6.2)
       WRITE(10,678) SH(Z),(SV(I,1), I=1,5)
678     FORMAT(1X,A11,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4)
       END IF
        ENDIF
5506    CONTINUE

      RETURN
      END


      SUBROUTINE SVAR(V,OBS,S2)
C**************************************************************
C  Finds the variance of v(OBS). Output is S2. OBS <=60
C**************************************************************
      REAL*4 V(60),S2,SUM,SUMSQ
      INTEGER*4 OBS
```

```
      SUM=0
      SUMSQ=0
      DO 444 I=1,OBS
        SUM=SUM+V(I)
        SUMSQ=SUMSQ+V(I)**2
444     CONTINUE
      S2=(1.0/FLOAT(OBS-1))*(SUMSQ-(1.0/FLOAT(OBS))*SUM*SUM)
      RETURN
      END

      SUBROUTINE TRANP(MATIN,A,B,C,D,MATOUT)
C*****************************************************************
C Finds the transpose of MATIN (R1xC1). The result
C  is MATOUT. DIM1XDIM2 is the size of the array.
C*****************************************************************
      INTEGER*4 A,B,C,D
      REAL*4 MATIN(C,D),MATOUT(D,C)

      DO 8000 I=1,A
      DO 8000 J=1,B
          MATOUT(J,I)=MATIN(I,J)
8000    CONTINUE
      RETURN
      END


      SUBROUTINE MULT(MAT1,R1,C1,D1,D2,MAT2,R2,C2,D21,D22,PROD)
C*****************************************************************
C Multiplies MAT1 (R1xC1) by MAT2 (R2xC2). The result
C  is PROD (R1xC2). Array size: MAT1(D1xD2),MAT2(D21xD22)
C*****************************************************************
      INTEGER*4 R1,C1,D1,D2,R2,C2,D21,D22
      REAL*4 MAT1(D1,D2),MAT2(D21,D22),PROD(D1,D22)
     DO 7000 I=1,R1
          DO 705 J=1,C2
            PROD(I,J)=0.0
            DO 7010 K=1,C1
              PROD(I,J)=PROD(I,J)+MAT1(I,K)*MAT2(K,J)
7010          CONTINUE
705         CONTINUE
7000    CONTINUE
      RETURN
      END



        SUBROUTINE INVER(MAT1,MATIN,DIM,R)
C*****************************************************************
C  Inverts MATIN (DIMxDIM). The array is MATIN(RxR)
C*****************************************************************
        INTEGER*4 DIM,R
         REAL*4 MATIN(20,20)
         REAL*4 MAT1(20,20)
         REAL*4 INV(20,20)
         REAL*4 TEST(20)
       DO 33 J=1,DIM
         DO 34 I=1,DIM
           MATIN(I,J)=MAT1(I,J)
34         CONTINUE
33      CONTINUE
```

```
      K=0
20     K=K+1
      TEST(K)=1.0/MATIN(K,K)
      DO 40 J=1,DIM
         DO 30 I=1,DIM
           IF (J .EQ. K .AND. I .EQ. K) THEN
             INV(I,J)=(-1.0)*TEST(K)
           ELSEIF (J.EQ.K .AND. I.NE.K) THEN
             INV(I,J)=MATIN(I,J)*TEST(K)
           ELSEIF (I.EQ.K .AND. J.NE.K) THEN
             INV(I,J)=MATIN(I,J)*TEST(K)
           ELSE
             INV(I,J)=MATIN(I,J)-((MATIN(I,K)*MATIN(K,J))*TEST(K))
           ENDIF
30        CONTINUE
40     CONTINUE
      CALL COPY (INV,DIM,DIM,R,R,MATIN)
      IF (K.LT.DIM) GO TO 20
       DO 31 J=1,DIM
         DO 32 I=1,DIM
           MATIN(I,J)=(-1.0)*MATIN(I,J)
32        CONTINUE
31      CONTINUE

       RETURN
       END


       SUBROUTINE COPY(MAT1,R1,C1,DIM1,DIM2,MAT2)
C*************************************************************************
C  Copies MAT1(R1xC1) to MAT2(R1xC1). The array is MAT1(DIM1,DIM2).
C*************************************************************************
        INTEGER*4 R1,C1,DIM1,DIM2
       REAL*4 MAT1(DIM1,DIM2),MAT2(DIM1,DIM2)
       DO 1002 I=1,DIM1
         DO 1002 J=1,DIM2
           MAT2(I,J)=0.0
1002    CONTINUE

       DO 1000 I=1,R1
         DO 1000 J=1,C1
           MAT2(I,J)=MAT1(I,J)
1000    CONTINUE
       RETURN
       END

      SUBROUTINE MREGRESS(XX,YY,OBS,P,BET,TSTAT,RSS)
C*************************************************************
C  Multiple regression of YY on XX. OBS is number of
C  observations. P is number of independent variables
C  including the mean. Output is BET(P) - the regression
C  coefficients, TSTAT(P) - the t-stats of these
C  estimates, RSS - the multiple R sqaured. OBS has
C  a maximum value of 60. P has a maximum value of
C  20.
C*************************************************************
      INTEGER*4 OBS,P
      REAL*4 C(20,20)
      REAL*4 YY(60,1),XX(60,20)
      REAL*4 XXT(20,60),XXTXX(20,20),XXTYY(20,1),BET(20,1)
```

```
      REAL*4 BETT(1,20),YYT(1,60),YYTYY(1,1)
      REAL*4 BTXTY(1,1),SSE,SBET(20,1),YSUM,SST,RSS
      REAL*4 TSTAT(20,1)

      CALL TRANP(XX,OBS,P,60,20,XXT)
      CALL MULT(XXT,P,OBS,20,60,XX,OBS,P,60,20,XXTXX)
      CALL INVER(XXTXX,C,P,20)
      CALL MULT(XXT,P,OBS,20,60,YY,OBS,1,60,1,XXTYY)
      CALL MULT(C,P,P,20,20,XXTYY,P,1,20,1,BET)
      CALL TRANP(YY,OBS,1,60,1,YYT)
      CALL TRANP(BET,P,1,20,1,BETT)
      CALL MULT(YYT,1,OBS,1,60,YY,OBS,1,60,1,YYTYY)
      CALL MULT(BETT,1,P,1,20,XXTYY,P,1,20,1,BTXTY)
      SSE=YYTYY(1,1)-BTXTY(1,1)
      IF((SSE.LT.0.0000001).OR.(SSE.LT.0.0))SSE=0.0000001
      YSUM=0
      DO 626 I=1,OBS
       YSUM=YSUM+YY(I,1)
626    CONTINUE
      DO 627 I=1,P
       SBET(I,1)= SQRT(SSE)/SQRT(FLOAT(OBS-P))*SQRT(C(I,I))
       TSTAT(I,1)=BET(I,1)/SBET(I,1)
627    CONTINUE
      SST=YYTYY(1,1)-(YSUM*YSUM)/OBS
      RSS=(SST-SSE)/SST
      RETURN
      END
```

A5.15 Program to calculate parameters of extended market model - Country Index only

```
C***************************************************************
C PROGRAM TO CALCULATE PARAMETERS OF
C EXTENDED MARKET MODEL - WITH A SPECIFIC
C FOREIGN MARKET FOR THE MAIN INDEX
C
C WRITTEN BY : CALLY ARDINGTON 1996
C***************************************************************

C           VARIABLE DEFINITIONS

      CHARACTER*11 SH(-2:1520),INDEX,CURRENCY
      CHARACTER*9 INAME,ONAME,PNAME
      INTEGER*4 ClcDate,N1,PERIOD,NMBR(1520)
      REAL*4 RET(1520,60),RESID(60),LASTPR(1520,60),TMP(60)

C***************************************************************
C READ IN
C 1. THE NAME OF THE FILE CONTAINING SHARE NAMES
C 2. THE DATE AT WHICH TO FIT THE MODEL
C 3. THE LENGTH OF THE ESTIMATION PERIOD
C 4. THE INTERNATIONAL INDEX AND CURRENCY
C***************************************************************

      WRITE(*,900)
900   FORMAT(' INPUT FILE NAME FOR SHARE NAMES ')
      READ(*,910) INAME
910   FORMAT(A20)
      write(*,*) 'Rand model - two indices'
908   write(*,*) 'Enter date for which to fit model:'
909   write(*,918) 'enter as YYMMDD, 0 for latest date '
      Read(*,*) ClcDate
918   Format(10X,A)
      WRITE(*,*) 'Enter length of period '
      READ(*,*) PERIOD
       WRITE(*,*) 'Enter foreign index to use '
       READ(*,917) INDEX
       WRITE(*,*) 'Enter foreign currency to use '
       READ(*,917) CURRENCY
917   FORMAT(A11)


C***************************************************************
C THE SUBROUTINE SHARE READS IN THE SHARE
C NAMES FROM THE FILE INAME
C***************************************************************
      CALL SHARE(INAME,SH,N1,INDEX,CURRENCY)    ! READ IN THE SHARE NAMES


C***************************************************************
C THE SUBROUTINE GETPR RETURNS A MATRIX
C OF RETURNS FOR THE INDICES AND EACH OF
C THE SHARES SPECIFIED IN INAME
C***************************************************************
      CALL GETPR(SH,N1,LASTPR,RET,CLCDATE,PERIOD,NMBR,TMP)
```

```
C*****************************************************************
C THE SUBROUTINE INTERNAT CALCULATES
C THE PARAMETERS FOR THE EXTENDED
C MARKET MODEL AND WRITES THE OUTPUT
C TO FILE
C*****************************************************************
   CALL INTERNAT(SH,N1,RET,PERIOD,NMBR)


   STOP
   END

C*****************************************************************
C END OF THE MAIN PROGRAM
C*****************************************************************




    SUBROUTINE SHARE(INAME,SH,N,INDEX,CURRENCY)
C*****************************************************************
C This routine is used simply to read in all the share names
C specified in the file INAME
C*****************************************************************
        CHARACTER*11 SH(-2:1520),INDEX,CURRENCY
        CHARACTER*9 INAME
         OPEN(1,FILE=INAME,STATUS='OLD')
      SH(-2)=CURRENCY
      SH(-1)=INDEX
      SH(0)='JSE-OVER   '
         I= 0
930      I=I+1
         READ(1,35,END=200) SH(I)
35      FORMAT(4X,A11)
         GOTO 930
200      N=I-1
        RETURN
        END


        SUBROUTINE CALCDATE(VALUE,YR,MT,DY)
C*****************************************************************
C This routine computes the day, month and year of a
C given date
C*****************************************************************
        INTEGER*4 VALUE,Mant,YR,MT,DY
         YR = INT(VALUE/10000)
         Mant = VALUE - YR * 10000
         MT = INT(Mant/100)
         Mant = Mant - MT * 100
         DY = INT(Mant)
        RETURN
        END

        SUBROUTINE CalcReturns(I,NM,Y,R)
C*****************************************************************
```

```
C Calculates monthly returns
C****************************************************************
      INTEGER*4 I,NM,M
         REAL*4 R(1520,60),Y(1520,60)
          DO 433 M=2,NM
            R(I,M)=(Y(I,NM+1-M)-Y(I,NM+2-M))/Y(I,NM+2-M)
433      CONTINUE
          RETURN
          END


      SUBROUTINE REGRESS(X,Y,N,ALPHA,BETA,RSQ,RESID)
C***************************************************************
C Linear Regression
C***************************************************************
         INTEGER*4 N
      REAL*4 X(N),Y(N),ALPHA,BETA,RESID(N)
      XSUM=0
         YSUM=0
         XYSUM=0
         XSQ=0
         YSQ=0
         SSXX=0
         SSYY=0
         SSXY=0
         DO 500 G=1,N
              XSUM=XSUM + X(G)
                 YSUM=YSUM + Y(G)
                 XSQ=XSQ + X(G)**2
                 YSQ=YSQ + Y(G)**2
                 XYSUM=XYSUM + X(G)*Y(G)
500              CONTINUE
         SSXX=XSQ-XSUM*XSUM/FLOAT(N)
         SSYY=YSQ-YSUM*YSUM/FLOAT(N)
         SSXY=XYSUM-XSUM*YSUM/FLOAT(N)
         BETA=SSXY/SSXX
         ALPHA=YSUM/FLOAT(N)-BETA*XSUM/FLOAT(N)
         RSQ=SSXY**2/(SSXX*SSYY)
      DO 501 G=1,N
           RESID(G)=Y(G)-ALPHA-BETA*X(G)
501     CONTINUE
         RETURN
         END


      SUBROUTINE GETPR(SH,N1,LASTPR,RET,CLCDATE,
     &    PERIOD,NMBR,TMP)
C******************************************************
C READ IN PRICES AND CALCULATE RETURNS
C FOR ALL SHARES AND INDICES
C******************************************************

      PARAMETER (MM=4,MAX=1200,NZ=1520)
     STRUCTURE /FILE_REC_STRUCT/          !
      CHARACTER*11 SHARE                  !
      INTEGER*4 KNUM                  !STRUCTURE OF THE
      INTEGER*4 KHIST(10)              !  DATA FILE
```

```
 REAL PRICE(10)               !CONTAINING SHARE
 INTEGER*4 DATE(MAX)              !   PRICES
 INTEGER*4 LAST(MAX)             !i.e. DCBALL.DAT
 INTEGER*4 HI(MAX)             !
 INTEGER*4 LO(MAX)              !
 INTEGER*4 VO(MAX)              !
END STRUCTURE                  !


RECORD /FILE_REC_STRUCT/ FILE_REC, WEEK_REC
CHARACTER*11 SH(-2:NZ)
INTEGER*4 Jy1,Jy2,Jm1,Jm2,PERIOD,OBS,Z
INTEGER*4 Jd1,Jd2,WeekNo,N1,NMBR(1520),G3
INTEGER*4 ClcDate
REAL*4 LASTPR(1520,60),TMP(60)
REAL*4 Ret(1520,60)
LOGICAL MoreData,ZeroPr,mistake
COMMON /CMNBLK/ FILE_REC, WEEK_REC



C*****************************************************************
C OPEN THE FILE CONTAINING THE SHARE DATA
C (DCBALL.DAT)
C*****************************************************************
  OPEN(11,FILE='STA:[CGTROS.DOOL]DCBALL.DAT',
 +    ORGANIZATION='INDEXED',STATUS='OLD',
 1    ACCESS='KEYED',RECORDTYPE='VARIABLE',
 2    FORM='UNFORMATTED',RECL=8000,
 3    KEY=(1:11:CHARACTER),
 4    IOSTAT=IOS,ERR=999)

  OBS=NINT(FLOAT(PERIOD)/52*12)
  NUMMTHS = 60
  DO 5555 I=-2,N1
      write(*,*) 'Computing stats for ',SH(I)

C*****************************************************************
C READ IN THE MONTH-END PRICES OVER THE
C PAST 5 YEARS.
C*****************************************************************

      Read(11,Key=SH(I),KeyID=0,IOStat=IOS,ERR=997)
 &     Week_Rec.Share,Week_Rec.KNum,
 1     (Week_Rec.KHist(J),J=1,10),
 2     (Week_Rec.Price(J),J=1,10),
 3     (Week_Rec.Date(J),Week_Rec.Last(J),
 4      Week_Rec.Hi(J),Week_Rec.Lo(J),
 5      Week_Rec.Vo(J),J=1,Week_Rec.KNum)

      MaxWeeks = Week_Rec.KNum  ! # OF WEEKS DATA AVAILABLE
      WeekNo = MaxWeeks
      IF (ClcDate .NE. 0) THEN
        DO WHILE ( Week_Rec.Date(WeekNo) .GT. ClcDate )
          WeekNo = WeekNo - 1  ! GO TO THE PRICE AS AT
        END DO              ! ClcDate
      END IF

   IF (WeekNo .GT. PERIOD) THEN  ! Share must have at least
       NumMths = 1            ! PERIOD weeks data
```

```
          MoreData = .TRUE.
          LastPr(I,NumMths) = Week_Rec.Last(WeekNo)
          IF (LastPr(I,1) .NE. 0) THEN
             ZeroPr = .FALSE.
          ELSE
             ZeroPr = .TRUE.
          END IF
          DO WHILE ((NumMths .LT. OBS) .AND. (MoreData))

C*************************************************************
C CHECK TO SEE IF NEW MONTH YET, AND IF SO
C READ THE PRICE AT THE END OF THE
C PREVIOUS MONTH
C*************************************************************
          CALL CALCDATE(Week_Rec.Date(WeekNo),Jy1,Jm1,Jd1)
          CALL CALCDATE(Week_Rec.Date(WeekNo-1),Jy2,Jm2,Jd2)
           IF (Jm1 .NE. Jm2) THEN
            NumMths = NumMths + 1
            LastPr(I,NumMths) = Week_Rec.Last(WeekNo - 1)
            IF (LastPr(I,NumMths) .EQ. 0) ZeroPr = .TRUE.
           END IF

           WeekNo = WeekNo - 1      !go back a week
           IF (WeekNo.LE.1) MoreData=.FALSE.  !start of file
          END DO

          IF (ZeroPr) THEN
            write(*,443) SH(I)
            goto 5555          !if there is a zero price then
          END IF               !skip to next share
443      FORMAT(4X,A11,'*** NOTE: ZERO PRICE IN DATA***')


C*************************************************************
C CONVERT INTERNATIONAL INDEX INTO RANDS
C BY MULTIPLYING BY EXCHANGE RATE
C NB : CHECK TO SEE IF INDEX IS NIKKEI - IF SO
C DIVIDE BY EXCHANGE RATE
C*************************************************************
        IF (I.EQ.-1) THEN
        DO 222 Z=1,NUMMTHS
         IF (SH(I) .EQ. 'NIKKEI') THEN
         LASTPR(-1,Z)=LASTPR(-1,Z)/LASTPR(-2,Z)
         ELSE
         LASTPR(-1,Z)=LASTPR(-1,Z)*LASTPR(-2,Z)
         END IF
222      CONTINUE
        END IF


C*************************************************************
C CALCULATE RETURNS FOR INDICES AND ALL SHARES
C*************************************************************
539      CALL CalcReturns(I,NumMths,LastPr,Ret)


          NMBR(I) = NUMMTHS
         ELSE
           WRITE(*,*) '*** NOT ENOUGH DATA FOR ',SH(I),' ***'
```

```
      END IF   !enough weeks for at least twelve months

5555  CONTINUE          !with next share


999   WRITE(*,*)'IOSTAT = ',IOS
997   WRITE(*,*)'NO WEEKLY DATA'
     RETURN
     END


      SUBROUTINE INTERNAT(SH,N1,RET,PERIOD,NMBR)
C**************************************************************
C CALCULATES THE PARAMETERS OF THE
C EXTENDED MARKET MODEL WITH A SPECIFIC
C INTERNATIONAL MARKET AS THE MAIN INDEX
C WRITES OUTPUT TO FILE
C**************************************************************
      CHARACTER*11 SH(-2:1520)
      INTEGER*4 N1,PERIOD,NMBR(1520),OBS,P
      REAL*4 RET(1520,60),X(60),Y(60),RESID(60)
      REAL*4 ALPHA,BETA,RSQ,YY(60,1),XX(60,20),S2
      REAL*4 BET(20,1),RSS,TSTAT(20,1),V(60),SV(20,1)

      OPEN(8,FILE='MODEL.DAT',STATUS='NEW')
      OPEN(9,FILE='BETA.DAT',STATUS='NEW')
      OPEN(10,FILE='RISK.DAT',STATUS='NEW')

       OBS=NINT(FLOAT(PERIOD)/52*12)
       OBS=OBS-1
       P=3
      DO 555 I=1,OBS
       X(I)=RET(-1,I+1)
       Y(I)=RET(0,I+1)
555    CONTINUE

      CALL REGRESS(X,Y,OBS,ALPHA,BETA,RSQ,RESID)

      DO 505 J=1,OBS
        XX(J,1)=1.0
        XX(J,2)=RET(-1,J+1)
        XX(J,3)=RESID(J)
505     CONTINUE
      DO 5506 Z=1,N1
        IF (NMBR(Z) .GT. 0) THEN
         DO 5507 J=1,OBS
           YY(J,1)=RET(Z,J+1)
5507       CONTINUE

         CALL MREGRESS(XX,YY,OBS,P,BET,TSTAT,RSS)
         DO 220 K=1,OBS
          V(K)=YY(K,1)
220       CONTINUE
         CALL SVAR(V,OBS,S2)
         SV(1,1)=S2
         DO 221 I=2,P
          DO 222 K=1,OBS
           V(K)=XX(K,I)
222        CONTINUE
          CALL SVAR(V,OBS,S2)
```

```
          SV(I,1)=S2*BET(I,1)**2/SV(1,1)*100
221        CONTINUE
          SV(4,1)=100.0-SV(2,1)-SV(3,1)

      IF (ABS(TSTAT(2,1)) .GE. 1.90) THEN
      WRITE(8,676) SH(Z),BET(1,1),TSTAT(1,1),BET(2,1),
   &    TSTAT(2,1),BET(3,1),TSTAT(3,1)
676     FORMAT(1X,A11,2X,F7.3,2X,F7.3,2X,F7.3,2X,F7.3,2X,
   &     F7.3,2X,F7.3)
      IF (BET(2,1) .GT. 0.0) WRITE(9,677) SH(Z),BET(2,1)
677     FORMAT(4X,A11,1X,F6.2)
      WRITE(10,678) SH(Z),SV(1,1),SV(2,1),SV(3,1),SV(4,1)
678     FORMAT(1X,A11,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4)
      END IF
       ENDIF
5506    CONTINUE

      RETURN
      END


      SUBROUTINE SVAR(V,OBS,S2)
C***************************************************************
C  Finds the variance of v(OBS). Output is S2. OBS <=60
C***************************************************************
      REAL*4 V(60),S2,SUM,SUMSQ
      INTEGER*4 OBS
      SUM=0
      SUMSQ=0
      DO 444 I=1,OBS
        SUM=SUM+V(I)
        SUMSQ=SUMSQ+V(I)**2
444     CONTINUE
      S2=(1.0/FLOAT(OBS-1))*(SUMSQ-(1.0/FLOAT(OBS))*SUM*SUM)
      RETURN
      END

      SUBROUTINE TRANP(MATIN,A,B,C,D,MATOUT)
C*****************************************************************
C Finds the transpose of MATIN (R1xC1). The result
C  is MATOUT. DIM1XDIM2 is the size of the array.
C*****************************************************************
      INTEGER*4 A,B,C,D
      REAL*4 MATIN(C,D),MATOUT(D,C)

      DO 8000 I=1,A
      DO 8000 J=1,B
          MATOUT(J,I)=MATIN(I,J)
8000    CONTINUE
      RETURN
      END


      SUBROUTINE MULT(MAT1,R1,C1,D1,D2,MAT2,R2,C2,D21,D22,PROD)
C*****************************************************************
C Multiplies MAT1 (R1xC1) by MAT2 (R2xC2). The result
C  is PROD (R1xC2). Array size: MAT1(D1xD2),MAT2(D21xD22)
C*****************************************************************
      INTEGER*4 R1,C1,D1,D2,R2,C2,D21,D22
```

```
      REAL*4 MAT1(D1,D2),MAT2(D21,D22),PROD(D1,D22)
      DO 7000 I=1,R1
         DO 705 J=1,C2
           PROD(I,J)=0.0
           DO 7010 K=1,C1
             PROD(I,J)=PROD(I,J)+MAT1(I,K)*MAT2(K,J)
7010         CONTINUE
705        CONTINUE
7000    CONTINUE
      RETURN
      END



        SUBROUTINE INVER(MAT1,MATIN,DIM,R)
C*******************************************************************
C  Inverts MATIN (DIMxDIM). The array is MATIN(RxR)
C*******************************************************************
        INTEGER*4 DIM,R
         REAL*4 MATIN(20,20)
         REAL*4 MAT1(20,20)
         REAL*4 INV(20,20)
         REAL*4 TEST(20)
       DO 33 J=1,DIM
         DO 34 I=1,DIM
          MATIN(I,J)=MAT1(I,J)
34        CONTINUE
33      CONTINUE
       K=0
20      K=K+1
       TEST(K)=1.0/MATIN(K,K)
       DO 40 J=1,DIM
          DO 30 I=1,DIM
            IF (J .EQ. K .AND. I .EQ. K) THEN
              INV(I,J)=(-1.0)*TEST(K)
            ELSEIF (J.EQ.K .AND. I.NE.K) THEN
              INV(I,J)=MATIN(I,J)*TEST(K)
            ELSEIF (I.EQ.K .AND. J.NE.K) THEN
              INV(I,J)=MATIN(I,J)*TEST(K)
            ELSE
              INV(I,J)=MATIN(I,J)-((MATIN(I,K)*MATIN(K,J))*TEST(K))
            ENDIF
30         CONTINUE
40      CONTINUE
       CALL COPY (INV,DIM,DIM,R,R,MATIN)
       IF (K.LT.DIM) GO TO 20
        DO 31 J=1,DIM
          DO 32 I=1,DIM
           MATIN(I,J)=(-1.0)*MATIN(I,J)
32         CONTINUE
31       CONTINUE

        RETURN
        END


        SUBROUTINE COPY(MAT1,R1,C1,DIM1,DIM2,MAT2)
C******************************************************************************
C   Copies MAT1(R1xC1) to MAT2(R1xC1). The array is MAT1(DIM1,DIM2).
C******************************************************************************
```

```
      INTEGER*4 R1,C1,DIM1,DIM2
      REAL*4 MAT1(DIM1,DIM2),MAT2(DIM1,DIM2)
      DO 1002 I=1,DIM1
       DO 1002 J=1,DIM2
         MAT2(I,J)=0.0
1002   CONTINUE

      DO 1000 I=1,R1
       DO 1000 J=1,C1
         MAT2(I,J)=MAT1(I,J)
1000   CONTINUE
      RETURN
      END

      SUBROUTINE MREGRESS(XX,YY,OBS,P,BET,TSTAT,RSS)
C***************************************************************
C   Multiple regression of YY on XX. OBS is number of
C   observations. P is number of independent variables
C   including the mean. Output is BET(P) - the regression
C   coefficients, TSTAT(P) - the t-stats of these
C   estimates, RSS - the multiple R sqaured. OBS has
C   a maximum value of 60. P has a maximum value of
C   20.
C***************************************************************
      INTEGER*4 OBS,P
      REAL*4 C(20,20)
      REAL*4 YY(60,1),XX(60,20)
      REAL*4 XXT(20,60),XXTXX(20,20),XXTYY(20,1),BET(20,1)
      REAL*4 BETT(1,20),YYT(1,60),YYTYY(1,1)
      REAL*4 BTXTY(1,1),SSE,SBET(20,1),YSUM,SST,RSS
      REAL*4 TSTAT(20,1)

      CALL TRANP(XX,OBS,P,60,20,XXT)
      CALL MULT(XXT,P,OBS,20,60,XX,OBS,P,60,20,XXTXX)
      CALL INVER(XXTXX,C,P,20)
      CALL MULT(XXT,P,OBS,20,60,YY,OBS,1,60,1,XXTYY)
      CALL MULT(C,P,P,20,20,XXTYY,P,1,20,1,BET)
      CALL TRANP(YY,OBS,1,60,1,YYT)
      CALL TRANP(BET,P,1,20,1,BETT)
      CALL MULT(YYT,1,OBS,1,60,YY,OBS,1,60,1,YYTYY)
      CALL MULT(BETT,1,P,1,20,XXTYY,P,1,20,1,BTXTY)
      SSE=YYTYY(1,1)-BTXTY(1,1)
      IF((SSE.LT.0.0000001).OR.(SSE.LT.0.0))SSE=0.0000001
      YSUM=0
      DO 626 I=1,OBS
       YSUM=YSUM+YY(I,1)
626    CONTINUE
      DO 627 I=1,P
       SBET(I,1)= SQRT(SSE)/SQRT(FLOAT(OBS-P))*SQRT(C(I,I))
       TSTAT(I,1)=BET(I,1)/SBET(I,1)
627    CONTINUE
      SST=YYTYY(1,1)-(YSUM*YSUM)/OBS
      RSS=(SST-SSE)/SST
      RETURN
      END
```