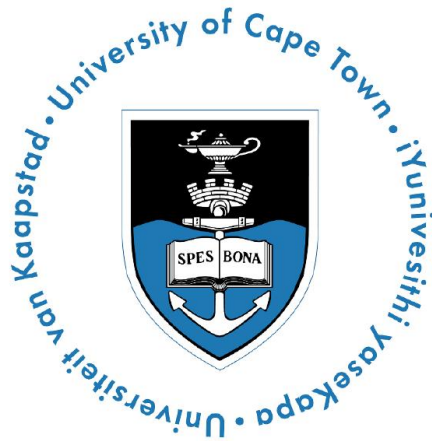


The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.



# Empirical Essays in Financial Economics

Ali Ahmed Endi

Supervised by:

Professor Haim Abraham

Thesis Presented for the Degree of

DOCTOR OF PHILOSOPHY

in the School of Economics

UNIVERSITY OF CAPE TOWN

August 2010

This thesis is submitted to the University of Cape Town in fulfillment of the academic requirements for the Degree of Doctor of Philosophy in Economics

## DECLARATION

I declare that this thesis is my own work, and has not been submitted for any degree or examination in any other university. All sources I have used or quoted have been indicated and acknowledged by complete references.

Full name: ...**Ali Ahmed Endi**..... Date.....**2011/09/06**.....

Signed: .....

University of Cape Town

# Abstract

## Empirical Essays in Financial Economics

by

Ali Ahmed Endi

University of Cape Town, August 2010

This thesis is a collection of four papers related to financial economics that appear as separate chapters: Paper one focuses on implied volatility estimation and investigates the volatility smile in a South African context with fourteen stocks listed on JSE Limited and fifty-nine options on these underlying stocks for the period between April 4, 2002 and November 8, 2008. Evidence of the existence of a South African volatility smile is shown. A possible reason for the existence of the smile could be the nonconstant variance of the underlying asset, which violates one of the assumptions of the Black-Scholes model. Furthermore, Value at Risk (hereafter referred to as VaR) is proposed to examine underlying stock volatilities and their forecasts. In this section, two different methods are used to calculate VaR: Historical simulation and Monte Carlo simulation, as well as conditional VaR. In the forecasting of volatility, these VaR approaches are found to be a significant forecasting tool for future underlying stocks' volatility; and therefore their significance could be confirmed for implied volatility forecasting.

Paper 2 uses an empirical approach, based on the CAPM model, to study the risk and return relationships of *A* shares (available for domestic investors) and *B* shares (available for foreign investors) in the Shanghai Stock Exchange (SSE). I utilize CAPM models to directly estimate the betas of *A* and *B* shares listed on SEE for the period between May 1, 1998 and May 1, 2008. Findings were that domestic and foreign investors do not price *A* shares and *B* shares differently. Moreover, it was discovered that the standard risk and return relationships

implied by CAPM models comply with *A* shares and therefore, domestic investors price asset risk as predicted by CAPM models in China. However, the study also revealed that the standard risk and return relationships implied by CAPM models also comply with *B* shares and foreign investors price asset risk as predicted by CAPM models in the US. Thus, price differences between domestic *A* shares and foreign *B* shares for the same company could be explained by the different systematic risks in China and the US. The investment opportunities and market portfolios are different for Chinese and foreign investors. Hence, *A* and *B* shares will be valued differently by these two segmented groups of investors.

Paper 3 takes an empirical approach to examine and compare three different methods for measuring the trade-off between the risk and the return of trading stocks in both South Africa and China. This work is organized into two main parts: the first compares the performance of two different methods, the Market price of risk and the Sharpe ratio, for measuring the risk and return of trading stocks on the JSE during 2006 and 2007, and the second investigates whether the Chinese *A* and *B* shares listed on the SSE have a different market price of risk. In South Africa the study finds the Sharpe ratio is as efficient a measure as the market price of risk. However, in China findings are that the *A* to *B* share price premium can be explained by the higher volatility of the *A* shares. Here the market price of risk for *A* shares and *B* shares is almost identical.

Paper 4 suggests an empirical framework as a possible mechanism to describe asset-price bubbles. The relationship between stock market value based on market fundamentals and the stock market price are analyzed in the context of rational speculative bubble theory.

The theory of speculative bubbles predicts that stock market prices fluctuate around a fundamental value path and price bubbles develop as a series of small persistent steps away from their path. Any sudden movement back to the fundamental path is the bursting of the bubble. This theory is tested by attempting to capture these characteristics through employing different VaR techniques. The Monte Carlo simulation model and historical simulation method were employed to compute VaR. The data used in this paper are from the JSE-All

Share Index for the period between July10, 1995 and March10, 2009 and the NASDAQ-100 Index for the period July10, 1994 to July10, 2009. A bubble burst is defended as a violation of VaR. We found VaR has successfully detected all crashes that occurred in the NASDAQ and JSE markets during the period covered by our data, which include the dot-com bubble burst.

The conclusion of this study is that the “predictability” of long horizon stock returns is likely driven by the economic agent’s rational asset pricing behaviour which requires compensation for the expected risk in holding stocks, and not driven by “animal spirits”.

University of Cape Town

# Acknowledgments

I am grateful to God, the most high, for his generosity in granting me many blessings and bounties. In view of my modest achievements in the past, in financial economics and financial studies, this work on financial risks and reducing the potential for financial crises would have been simply a dream. I also gratefully acknowledge Professor. Haim Abraham, my supervisor, for the advice, guidance and support provided to me throughout my studies, all of which, have been instrumental in realising this dream.

I also wish to express my sincere thanks to the University of Cape Town community. This includes members of the UCT library staff who assisted me in accessing relevant literature on the topic, the most congenial staff in the School of Economics, for their support and concern throughout my stay in South Africa and all the people at the University of Cape Town for their excellent hospitality towards me throughout my studies in this country.

I am also grateful to my wife, Fatima, for her constant and unfailing support.

I Dedicate This Work

to:

My Parents, My wife Fatima, my daughters Maryam and Asia, my Country “Free Libya”

and all the Members of my Family

University of Cape Town

# Table of Contents

Declaration .....	ii
Abstract .....	iii
Acknowledgements .....	vi
Dedication .....	vii
Table of Contents .....	viii
List of Tables .....	viii
List of Figures .....	viii
List of Flowcharts .....	xv
Chapter 1	
Introduction .....	1
1.1 Research Background and Aims .....	1
1.2 Published Work .....	7
Chapter 2	
Literature Overview .....	8
2.1 Introduction .....	8
2.2 Finance Background .....	11
2.3 Option Pricing Model Prior to Black-Scholes .....	12
2.3.1 Bachelier 1900 .....	12
2.3.2 Sprenkle 1964 .....	14
2.3.3 Boness 1964 .....	15
2.3.4 Samuelson 1965 .....	15
2.4 The Black-Scholes Model .....	17
2.5 Implied Volatility .....	20
2.5.1 Volatility Smiles .....	21
2.6 Volatility Smiles and Risk Management .....	24
2.6.1 Value-at-Risk .....	26
2.6.1.1 VaR Models Comparison .....	27
2.7 Summary .....	31
Chapter 3	
The Volatility Smile of South African Stock Options: Estimations and Tests .....	33

3.1	Introduction.....	33
3.2	Black-Scholes Constant Volatility Model.....	34
3.3	Measures of Extreme Risks: VaR and Expected Shortfall (ES).....	36
3.3.1	Historical Simulation Method.....	37
3.3.2	Monte Carlo Simulation Method.....	38
3.3.3	Expected Shortfall Method.....	39
3.3.4	Backtesting.....	40
3.4	Data Description.....	40
3.5	Empirical Results.....	42
3.5.1	Sensitivity of Price Error to the Implied Volatility Estimation.....	42
3.5.2	VaR Estimation and Backtesting Results.....	45
3.6	Conclusions.....	48
3.7	Appendix A.....	50
3.7.1	A.1 Data Outline.....	50
3.7.2	A.2 Computing Implied Volatilities.....	51
3.7.3	A.3 Comparison of Value-at-Risk Methodologies.....	53

## Chapter 4

	Market Segmentation, Relative Supply of Shares and Share Price Premium: Evidence from the Chinese Market.....	59
4.1	Introduction.....	59
4.2	Institutional Facts about the Chinese Stock Industry.....	61
4.2.1	Stock Market Structure.....	61
4.2.2	Share Structure.....	62
4.2.3	Investors.....	63
4.3	The Puzzle of China's A-Share and B-Share Price Disparities.....	64
4.3.1	The Price Gap Between A and B Shares Narrows Rapidly.....	64
4.3.2	The Relationship Between A and B Shares Prices.....	65
4.3.3	The Information Efficiency of China's Stock Market.....	68
4.4	Asset Pricing: A and B Shares Prices.....	69
4.5	Methodology and Tests.....	70
4.5.1	Historical Analysis Test.....	70
4.5.2	The Capital Asset Pricing Model (CAPM).....	71
4.5.3	Tests of Information Efficiency of China's Stock Market.....	74
4.6	Data, Variables and Sample Characteristics.....	76
4.7	Results and Explanations.....	77

4.7.1	Results of the Historical Analysis .....	77
4.7.2	The Risk and Return Relationship for <i>A</i> and <i>B</i> Shares .....	80
4.7.3	Information Efficiency of China's Stock Market .....	81
4.8	Conclusion .....	82
4.9	Appendix B .....	84
4.9.1	B.1 Examination of Historical <i>A</i> and <i>B</i> Share Prices .....	84
4.9.2	B.2 Regression Result of <i>A</i> and <i>B</i> Shares for CAPM Model.....	88

## Chapter 5

Incorporating the Sharpe Ratio and the Market Price of Risk into Asset Allocation.....		90
5.1	Introduction.....	90
5.2	Risk and Return in the Equity Markets .....	93
5.2.1	The Risk-Return Relationship .....	93
5.2.2	Composite Risk and Returns.....	96
5.3	Methodology.....	99
5.3.1	The Sharpe Ratio .....	99
5.3.2	The Sharpe Ratio for Chinese <i>A</i> and <i>B</i> Shares.....	100
5.3.3	The Market Price of Risk .....	101
5.3.4	The Market Price of Risk for Chinese <i>A</i> and <i>B</i> Shares .....	102
5.4	Data Description .....	103
5.4.1	The JSE .....	103
5.4.2	The SSE.....	105
5.5	Study Results .....	107
5.5.1	The JSE Results and Analysis .....	107
5.5.2	The SSE Results and Analysis.....	109
5.5.2.1	Constant Expected Return and Volatility .....	109
5.5.2.2	Market Price of Risk with Constant Volatility.....	111
5.5.2.3	The Sharpe Ratio.....	112
5.6	Conclusion .....	113
5.7	Appendix C .....	115
5.7.1	C.1 The Probability Density Function .....	115
5.7.2	C.2 Sharpe Ratio and Market Price of Risk Estimation for JSE .....	116
5.7.3	C.3 Market Price of Risk Estimation for SSE .....	117
5.7.4	C.4 Sharpe Ratio Estimation for SSE.....	118

## Chapter 6

Bubbles: Econometric Analysis and Empirical Evidence.....	119
6.1 Introduction.....	119
6.1.1 The 1990s US Stock Market Bubble.....	119
6.1.2 Measuring Stock Market Bubbles.....	120
6.2 Asset-Price Bubbles: Theoretical Background.....	122
6.2.1 Review of Research on Stock Market Bubbles.....	122
6.2.2 Rational Bubbles: Theoretical Perspective and Empirical Studies.....	123
6.2.3 Bubbles in Behavioral Studies.....	129
6.3 Defining a Bubble: Empirical Framework.....	132
6.4 Empirical Method for Testing Bubbles in Two Countries.....	135
6.4.1 Date Description.....	138
6.4.2 Empirical Estimation and Results.....	139
6.4.2.1 Efficiency in the NASDAQ and JSE.....	139
6.4.2.2 VaR and Bubble Detection.....	142
6.5 Conclusion.....	144
6.6 Appendix D.....	145
6.6.1 D.1 Indexes Performances and VaR Historical Simulation Approach.....	145
6.6.2 D.2 Indexes Performances and VaR Monte Carlo Simulation Approach.....	147

## Chapter 7

Conclusion and Future Work.....	149
7.1 Conclusion.....	149
7.2 Future Study.....	155
Bibliography.....	156

# List of Tables

Table 3.1	Data Outline: fourteen stocks listed on JSE Limited and fifty-nine options .	50
Table 3.2	Reporting a summary of our empirical results, each percentage shows the degree to which the VaR is greater than the actual price changes.....	46
Table 4.1	SSE Trading Summary for 2007 .....	62
Table 4.2	Regression Result of Chinese <i>A</i> and <i>B</i> Shares for CAPM Model .....	88
Table 5.1	Thirteen companies listed on JSE used in our analysis.....	104
Table 5.2:	Fifteen companies listed on the SSE which issue both <i>A</i> and <i>B</i> shares used in our analysis .....	106
Table 5.3	Sharpe Ratio and Market Price of Risk Estimation for JSE .....	116
Table 5.4	Market price of risk estimation for fifteen companies listed on SSE.....	117
Table 5.5	Sharpe Ratio estimation for fifteen companies listed on SSE.....	118

# List of Figures

Figure 2.1	Volatility Smile .....	22
Figure 2.2	The P&L density and Value-at-risk .....	27
Figure 3.1	Reports the slopes and the curves resulted from plotting different strike prices and implied volatilities.....	51
Figure 3.2	The actual stock price changes and 1-day 99% VaRs obtained through Historical Simulation method using 500 and 250 simulations .....	58
Figure 3.3	The actual stock price changes and 1-day 99% VaRs obtained through Monte Carlo Simulation method using 5,000 and 10,000 simulations .....	58
Figure 4.1	Three stages in the removing of restrictions.....	71
Figure 4.2	Real S&P Composite Stock Price Index and ex-post Rational Price.....	76
Figure 4.3	Prices of <i>A</i> and <i>B</i> Shares .....	84
Figure 4.4	Variances of <i>A</i> and <i>B</i> share in the first period from May 1 <sup>st</sup> 1998 to February, 16 <sup>th</sup> 2001, just before the lifting of the restrictions on <i>B</i> shares.....	79
Figure 4.5	Variances of <i>A</i> and <i>B</i> share in the second period from February, 20 <sup>th</sup> 2001 to November, 29 <sup>th</sup> 2002, just before the lifting of the restrictions on <i>A</i> shares ....	79
Figure 4.6	Variances of <i>A</i> and <i>B</i> share in the third period from December, 2 <sup>nd</sup> 2002 to May, 1 <sup>st</sup> 2008 after which there were no restrictions .....	80
Figure 4.7	Shanghai <i>A</i> share index (solid line <i>p</i> ) and ex-post rational price (dotted line <i>p</i> *), 1992- 2008. ....	82
Figure 5.1	Drift terms obtained by Sharpe ration equation (Solid line) and drift terms obtained by market price of risk equation (dotted line) .....	107
Figure 5.2	Volatility terms obtained by Sharpe ration equation (Solid line) and volatility terms obtained by market price of risk equation (dotted line).....	108
Figure 5.3	Sharpe ration (Solid line) and market price of risk (dotted line) .....	108
Figure 5.4	The drift terms of <i>A</i> shares (Solid line) and the corresponding <i>B</i> shares (dotted line).....	109
Figure 5.5	The Volatility terms of <i>A</i> shares (Solid line) and the corresponding <i>B</i> shares (dotted line).....	110

## List of Figures (Continued)

Figure 5.6	The Market price of risk terms of <i>A</i> shares (Solid line) and the corresponding <i>B</i> shares (dotted line).....	112
Figure 5.7	The Sharpe ratio terms of <i>A</i> shares (Solid line) and the corresponding <i>B</i> shares (dotted line) .....	113
Figure 6.1	Presents 3 bubble's states and in which state VaR could detect the bubble....	134
Figure 6.2	The NASDAQ 100 Stock Price Index (solid line <i>p</i> ) and ex-post rational price (dotted line <i>p</i> <sup>*</sup> ).....	140
Figure 6.3	The JSE All Stock Price Index (solid line <i>p</i> ) and ex-post rational price (dotted line <i>p</i> <sup>*</sup> ) .....	141
Figure 6.4	Fifteen year graph of the NASDAQ-100 Index .....	145
Figure 6.5	Reports a comparison of NASDAQ-100 price index fluctuations with 90% and 95% VaR, obtained via historical simulation method with 100 simulations.....	145
Figure 6.6	Fifteen year graph of the NASDAQ-100 Index.....	147
Figure 6.7	Reports a comparison of NASDAQ 100 price index fluctuations with 90% and 95% VaR, obtained via a Monte Carlo simulation method with 5,000 simulations. ....	147
Figure 6.8	Six years graph of JSE All Share Prices Index .....	146
Figure 6.9	Reports a comparison of JSE All Share price fluctuations with 90% and 95% VaR, obtained via a historical simulation method using 100 simulations .....	146
Figure 6.10	Six years graph of JSE All Share Prices Index .....	148
Figure 6.11	Reports a comparison of JSE All Share price fluctuations with 90% and 95% VaR, obtained via a Monte Carlo simulation method with 5,000 simulations .....	148

# List of Flowcharts

Flowchart 2.1	The major development in the field of option valuation throughout the past three decades .....	10
---------------	--	----

University of Cape Town

# Chapter 1

## Introduction

### 1.1: Research Background and Aims

This dissertation consists of four papers in financial economics that appear as separate chapters. The first chapter is entitled “The volatility smile of South African stock options: estimations and tests”. In this chapter, I study the volatility smile derived from options traded on the JSE and the possibility of forecast this volatility using different Value at Risk (VaR) approaches.

Since Black and Scholes (1973) published their seminal article on option pricing, numerous empirical studies have found that this famous model results in systematic biases across moneyness and maturity. It is well known that since the October 1987 crash, the implied volatility computed from options on the stock index in the US market inferred by the Black-Scholes model appears to be different across exercise prices. This is the so-called “volatility smile”. Given the Black-Scholes assumptions, all option prices on the same underlying security with the same expiration date but different exercise should have the same implied volatility. However, the “volatility smile” pattern suggests that the Black-Scholes model tends to misprice deep-in-the-money (ITM) and deep out-of-the-money (OTM) options.

In the last two decades, option pricing has experienced an explosion of new models that relax some of the restrictive Black-Scholes assumptions. Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), Melino and Turnbull (1990, 1995), Stein and Stein (1991) and Heston (1993) suggest a continuous-time stochastic volatility model. Merton (1996), Bates (1991) and Naik and Lee (1990) propose a jump-diffusion model. Duan (1995)

and Heston and Nandi (2000) develop an option pricing model based on the GARCH process. Recently, Madan et al. (1998) used a three-parameter stochastic process, termed the variance gamma process, as an alternative model for the dynamics of log stock prices.

Numerical empirical results, however, show that most of the above mentioned alternative models perform worse than the Black-Scholes Model (e.g. Jackwerth, 1999; Alexander and Nogueira, 2004). Hybrid models are rich, but they are difficult to implement in reality.

Although efforts have been devoted to explaining the volatility smile or skew phenomenon and to seek alternative models, currently there is no overwhelming consensus on a suitable alternative model to the Black-Scholes model. So far, the existing literature has not been conclusive either on whether the Black-Scholes model is ineffective, or the option market is inefficient. Indeed, there is no evidence that the persistent pricing biases represent risk-free arbitrage opportunity.

To the best of the researcher's knowledge, the volatility smile has not been studied in the South African context. Whether there is a South African volatility smile remains an empirical question. If the smile exists, the possibility of predicting this smile becomes an important area of interest.

The first objective in this chapter is to provide evidence of a South African volatility smile and confirm the mis-pricing of the Black-Scholes (1973) model in this market. The second objective of this chapter is to attempt to use different VaR approaches to predict the movement in the underlying stocks, and accordingly predict if the underlying stock's volatility exhibits a smile or skew effect. In this section, two different methods are applied to calculate VaR: Historical simulation and Monte Carlo simulation as well as conditional VaR.

The second chapter is entitled "Market segmentation, relative supply of shares and share price premium: evidence from the Chinese market". In this chapter, I study the risk and return relationships observed in the Shanghai Stock Exchange (SSE) with standard CAPM models.

The establishment of the Shanghai (1990) and Shenzhen (1990) stock exchanges marked an important step towards building a functioning capital market in the history of economic reforms started in 1978 in China. Like other newly established emerging capital markets, China's security markets exhibit many unique and constraining institutional features that have no counterparts in developed nations. Attracting foreign capital is one of the priorities for many emerging economies which open their security markets to foreign investors (with different forms of restrictions on foreign share ownership). But do foreign and domestic investors price assets in these markets the same way? In such an environment with more instability and unique institutional constraints, can we still expect the same risk and return relationships predicted by standard CAPM models? These are the questions addressed in this chapter. Focusing on the shares listed on the SSE.

The third chapter is entitled "Incorporating the Sharpe ratio and the market price of risk into asset allocation". In this chapter, I empirically examine and compare the performance of three different methods of measuring the trade-off between the risk and the return of trading stocks in both South Africa and China.

In portfolio theory, it is assumed that investors always prefer higher returns to lower returns for a given level of risk; likewise for a given level of return, one prefers a lower risk to a higher risk. As return and risk are two important entities in measuring the performance of an investment, it is crucial to consider both return and risk in the selection of assets. The Sharpe Ratio is one of the most popular performance measures. It is defined as the ratio of the expected return to the standard deviation of the returns. It captures both return and risk. By the generalized Sharpe rule, a new asset with a higher Sharpe ratio has a higher probability of being selected (e.g. Sharpe, 1966, 1975, 1994; Dowd, 1999, 2000; Hodges, 1998 and Amin and Kat, 2002).

This chapter is organized into two main parts: the first comparing the performance of the Market price of risk and the Sharpe ratio of trading stocks on the JSE. In obtaining the market price of the risk, it was assumed that stock prices behave according to Geometric Brownian

Motion (GBM). Firstly, the constant drift and the volatility is estimated; then the drift term is decomposed into a risk-free rate and the market price of risk-multiplying volatility. The market price of risk is assumed to be constant and independent of time. The Maximum Likelihood Method is adopted to estimate the parameters. However, in obtaining the Sharpe ratio, the Maximum Likelihood Estimation Method is adopted to estimate the parameters in the Sharpe ratio equation developed by Sharpe in 1966. In this way the Sharpe Ratio becomes a forward-looking risk measurement tool instead of a backward-looking risk measurement tool. By this method a basis is found from which to compare the two risk measurements, as both of them are forward-looking risk-measurement tools. t-statistics are provided to show the significance of the difference between the market price of risk and the Sharpe ratio.

The second part of the chapter investigates whether Chinese domestic and foreign shares listed on the SSE have a different market price of risk. We know that market price of risk measures the trade-off between risk and return of an asset, that is, the increase of expected returns demanded per additional unit of risk. Basak (2005) argues that investors holding heterogeneous beliefs will have different market price of risk even for the same investment. Since Chinese domestic shares and foreign shares have the same payoff streams but are held by different investors, we can test their market price of risk to see whether an investor's belief matters for the price difference.

Domestic and foreign shares are issued by the same company and have virtually the same voting rights and dividends. The difference that exists may be caused by different risk-free rates or different volatilities. In other words, this study aims to test whether they have the same market price of risk. The procedure in this test is exactly as it is in the first part of this chapter. However, the data we use for this test are the daily closing prices of domestic and foreign shares of the companies who issue both classes of shares listed on the SSE. The theory behind the analysis is straightforward since the corresponding domestic and foreign shares are issued by the same company and have identical voting policies and dividend rights, if we take the company-specific fundamentals as given and assume that the prices of the corresponding domestic and foreign shares are derived from the fundamentals, then their

market price of risk should be identical since they share the same company-specific risk. If investors view the firm-specific risk as the only risk they bear, then they should have the same market price of risk.

On the other hand, if the market price of risk is different, this indicates that although sharing the same firm-specific risk, domestic and foreign shares are considered to be at different market-risk levels and thus are expected to have different excess returns for investors.

Furthermore, besides the comparison of the market price of risks for individual domestic and foreign shares, we can also stack all domestic shares or foreign shares' returns and test the average market price of the risks for the two groups. This test is reliable insofar as it relates to the individual results since it takes the average of the individual estimators and thus provides one with more intuitive results for domestic and foreign shares as a whole. Nevertheless, this chapter also estimates and compares the Sharpe ratios of domestic shares and foreign shares based historical-return data.

The fourth chapter, entitled "Bubbles: econometric analysis and empirical evidence", examines the presence of rational speculative bubbles in the NASDAQ and JSE stock markets. This chapter attempts to develop an empirical framework on a possible mechanism to describe asset-price bubbles.

"Bubble" is not a word specific to the stock market. Initial opinions about so-called price bubbles referred to various kinds of assets, such as foreign exchange, gold, real estate, and stock. Bubbles have been concerned with driving up all these asset prices and causing them to grow rapidly. Following Blanchard and Watson (1982) who stated that bubbles are more likely to exist in the price of an asset with difficult to understand fundamental values, it is expected that bubbles hardly exist if the fundamental value of an asset is easily identified. With this idea in mind, it is expected that the research of bubbles is best conducted in stock markets where the fundamental values of stocks are unclear.

In this chapter the relationship between stock market value based on market fundamentals and the stock market price will be analyzed in the context of rational speculative bubble theory. The theory of speculative bubbles predicts that stock market prices fluctuate around a fundamental value path and price bubbles develop as a series of small persistent steps away from their path. Any sudden movement back to the fundamental path is the bursting of the bubble. I test this theory by attempting to capture these characteristics by employing different VaR techniques.

This work follows rational bubbles studies and stems from two basic opinions: firstly, bubbles persist in stock markets since they result from optimistic beliefs and speculative behaviours which dominate the market always. Accordingly, this work follows Fukuta (1998) which presents a class of rational bubbles named “incompletely bursting bubbles”, which has three states: a large bubble state, a small bubble state and an incomplete burst state. Only in the incomplete burst state is the expected bubble tomorrow less than the bubble today. Secondly, we can detect the incomplete burst bubble state where the expected bubble tomorrow is less than the bubble today using different VaR approaches as long as VaR can be used to estimate the potential loss from adverse price movements. Thus, we can use VaR to detect the downside (burst) of the bubble.

In this work VaR is used as a benchmark for a bubble. We will recognize any violation to this benchmark as result of bubble burst. The data used in this chapter is from two different markets, namely the NASAQ and JSE.

Finally, Chapter seven provides a summary of the thesis and its main conclusions.

## 1.2: Published Work

Some original work that I present in this thesis has been presented at conferences and published. These are

[1] Endi, A (2006): “New Approaches to Risk Management in Financial Optimization”  
Quantitative Methods in Finance 2006 Conference, Sydney.

[2] Endi, A (2009): “An examination of the value-at-risk volatility relationship in the options market: New evidence on mispricing in the Black-Scholes model”, *Studies in Economics and Econometrics*, 33(1), 85-103.

# Chapter 2

## Literature Overview

### 2.1: Introduction

The work of Black and Scholes (1973) on the valuation of options on assets resulted in a formula which calculates the price of a stock option based on the price and volatility of the stock, the time horizon of the option and the current risk-free interest rate. Within the Black-Scholes formula all the independent variables are observable with the exception of the stock's volatility. However, it is possible to observe the market price of the stock options, invert the option pricing formula and solve for the volatility (Latane and Rendleman, 1976). The resulting number is known as the implied volatility of the option, so called because it refers to the volatility implied by the Black-Scholes option pricing model.

The implied volatility is regularly used by option traders as an index of how the market is pricing an option. Traders observe the implied volatility, modify the volatility estimate with their own forecast and re-calculate the Black-Scholes prices to determine appropriate buy and sell actions (Figlewski, 1989). However, using the Black-Scholes formula for calculating the implied volatility is problematic. The Black-Scholes model has been confirmed by extensive empirical testing to be consistent with market prices for at-the-money call options, however, it has been empirically demonstrated to have systematic biases for options which are in-the-money, out-of-the-money and with differing term structures (time to expiration) (MacBeth and Merville, 1979).

In this chapter, we summarize the major developments in the field of option valuation in the past three decades. The first section in this chapter is a survey of the literature on option valuation from the early literature to the original contributions of Black and Scholes (1973)

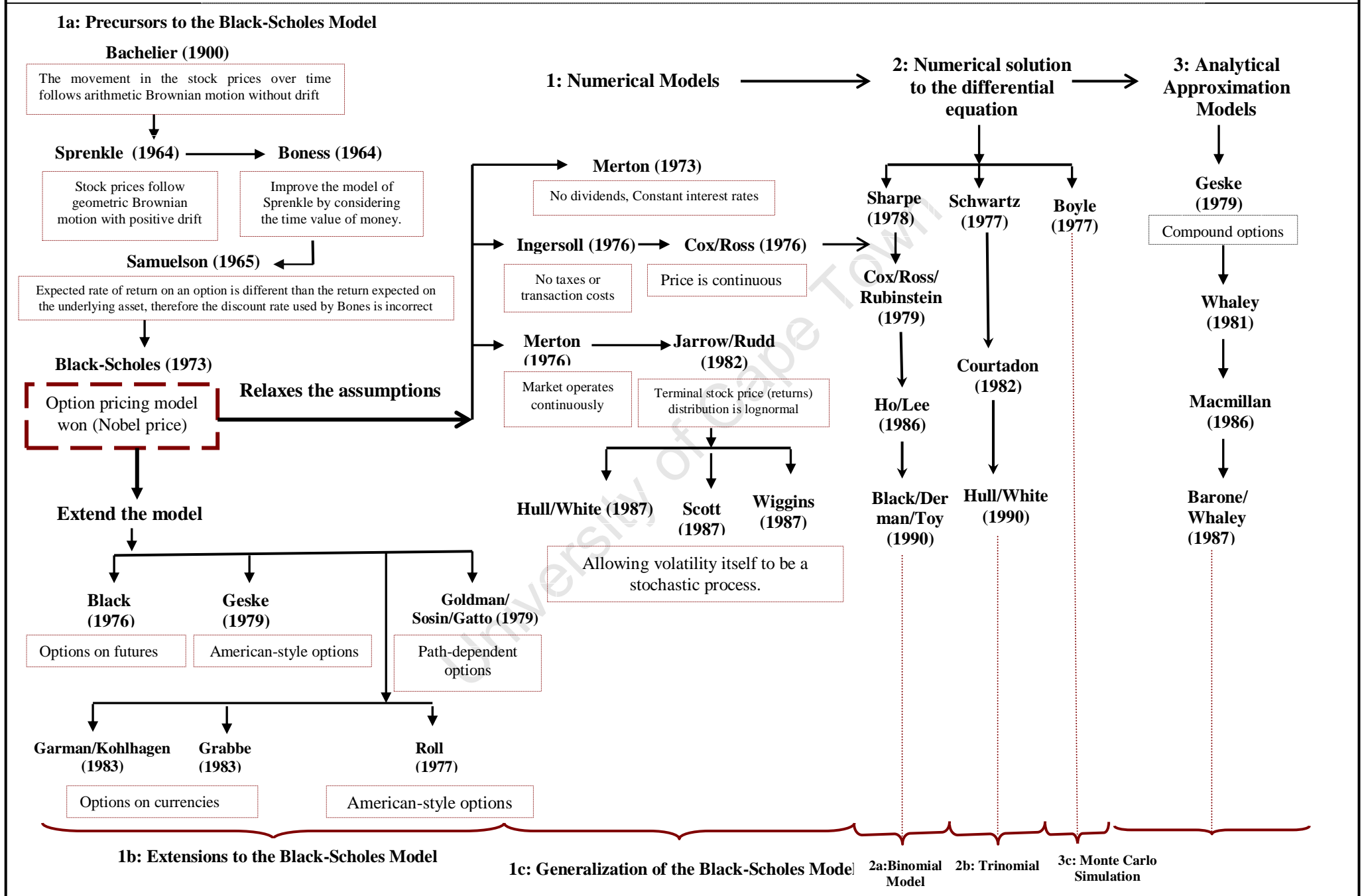
pricing model. Hence it is most helpful to start this chapter by considering the Black-Scholes model in the context of a family tree of option pricing models. Using this family tree analogy, it is possible to identify three major branches within the family of option pricing models; analytical, numerical and analytic approximation.

The analytical branch can itself be divided into three distinct lineages; precursors to the Black-Scholes model, generalisations of the Black-Scholes model, and extensions of the Black-Scholes model. However, within the branch of numerical models, there are three lineages. The best known of the three is the binomial model line; the others are the trinomial model and the Monte Carlo simulation. The analytic approximation models branch represents a reunification of the other two branches. A full detail of the family tree of option pricing models which is developed here is presented in flowchart 2.1. The flowchart is designed to provide a comprehensive picture of the option pricing models on a single page.

This chapter also focuses on implied volatility estimation and on the well known volatility smile phenomenon. Numerous explanations for the volatility smile phenomenon and extensions as well as alternatives to the Black-Scholes model will be offered in this literature. In the second section, however, the focus is on volatility forecasting and capturing the skewness and kurtosis of underlying risk factors by using the VaR framework. VaR models predict tail fatness and negative autocorrelation of large returns and thus can predict an option implied volatility smile.

The layout of this chapter is as follows: Section 2.3 provides option pricing models prior to that of Black-Scholes. Section 2.4 demonstrates the Black Scholes model. Section 2.5 provides alternative option pricing models and volatility models. Section 2.6 demonstrates the Greeks under the Black-Scholes model and provides an overview of developments, methodologies, and applications of VaR. Various key methodologies of VaR estimation and evaluation are discussed and compared in this section. Section 2.7 provides a summary to this literature overview.

**Flowchart 2.1: The major developments in the field of option valuation throughout the past three decades**



## 2.2: Finance Background

We begin by presenting a brief overview of the financial terminology used throughout the dissertation. A derivative security is a financial asset whose value is derived in part from the value and characteristics of an underlying asset(s). Common types of derivative securities are options, forwards, futures, and interest rate swaps. This dissertation focuses on options.

A stock option is a derivative which gives its holder the right, but not the obligation, to engage in some future transaction involving the underlying stock. A call (put) option gives its holder the option to purchase (sell) a share of the underlying stock for a pre-specified price, known as the strike price. Option contracts also have an expiration date. The holder of a European option has the right to exercise the option only on the option's expiration date. The holder of an American option has the right to exercise the option on or at any time before the expiration date. Of course, rather than exercising an option, the holder may instead choose to simply sell the option at any time before its expiration date.

The value of an option contract depends on several parameters including the value of shares of the underlying stock, the strike price of the option, and the amount of time remaining until expiration. An option's value also depends on characteristics of the underlying asset price dynamic, including the volatility and the risk-free interest rate. A rational pricing theory dictates that the value of a call (put) option increases as the value of the underlying asset increases (decreases). The value of a call (put) option decreases as the strike price increases (decreases). For non-dividend paying stocks, the value of all call options increase as the amount of time to expiration increases. The value of a European put as a function of time remaining until expiration increases from zero to a time corresponding to a maximum value and then decreases beyond this time. The value of any option increases as the volatility of the underlying asset increases. The value of a call (put) option increases (decreases) as the risk-free interest rate increases. We relate changes in the values of options to changes in these underlying variables more formally in later sections.

## 2.3: Option Pricing Models Prior to Black-Scholes

The valuation of option pricing has a long and renowned history, dating back to the late nineteenth century. Most of the early literature on the valuation of options is concerned with warrants. However more issues were introduced when the valuation of options underwent revolutionary changes in the 1970's. The first contribution to the theory of option pricing was from Castelli (1877). Castelli presented the public with the hedging and speculation features of options. His work did not, however, provide any major theoretical basis for the valuation of options. Bachelier (1900) offered the first significant literature on option valuations in his dissertation "Theorie de la Speculation".

### 2.3.1: Bachelier -1900

The first attempt at creating a model for the pricing of options that can be found in today's finance literature was provided by Louis Bachelier, in 1900.

The central assumption in deriving a theoretical model for the pricing of options concerns the statistical process which describes the behaviour of the underlying asset over time<sup>1</sup>. Bachelier believed that transactions in the stock market are a result of buyers and sellers having different expectations of the future stock price, and that it would be unreasonable to presume that, on average, one or the other could consistently make better predictions. From these postulations, he conjectured that at any moment, the market as a whole cannot be expected to go either up or down. Positive and negative expectations cancel each other out when one considers the aggregate of market participants at any moment in time. On average then, the expected price change per unit of time is assumed to be zero (Bernstein, 1992:20)<sup>2</sup>.

---

<sup>1</sup> The underlying asset in the models described in this section is a share of common stock.

<sup>2</sup> It should be clear that Bachelier did not ignore the fact that what motivates investors towards buying stocks is the expectation of gain from future appreciation in the price of the stocks. He simply states that the reason for people to be able to do so, is that others believe such appreciation will not materialize, otherwise prices would keep rising indefinitely. The net effect of buying and selling at the time, having these different views of the likely future behaviour of prices is, according to Bachelier, that the expected change in the stock price will, on average, be equal to zero. (Unless buyers and sellers simultaneously receive new information that causes their views on the future price to converge.)

Bachelier also noted that stock price volatility increases as the time horizon over which the volatility is measured is increased. This led him to investigate the mathematical properties of the behaviour of particles subject to random shocks as they move in space, known in physics as the Brownian motion<sup>3</sup>; Bachelier showed that the standard deviation of stock prices increases in proportion to the squared root of time.

He then assumed that the movement of stock prices over time follows arithmetic Brownian motion<sup>4</sup> without drift, and with variance of  $\sigma^2$  per unit of time. (If the statistical process that describes the movement of the stock price over time has no drift, then on average the stock price will not tend to move either up and down.) This assumption implies that the stock price is a normally distributed random variable, generated by a statistical process comparable to successive flips of a fair coin, and there is an equal probability of getting either outcome, and each outcome is independent of the previous outcome. But more specifically, if the stock price follows arithmetic Brownian motion without drift, the probability of the price rising or falling by one absolute unit, (e.g. one Rand) is equal, irrespective of the price level (Smith, 1976:15). Bachelier also assumed the expected return on a call option to be equal to the expected change in the value of the underlying asset, which, as stated previously, was assumed to be equal to zero.

Arithmetic Brownian motion is not an adequate representation of the statistical process followed by stock prices. First, investors are concerned with the proportional, not absolute change in price. Second, the assumption of arithmetic Brownian motion and normally distributed prices ignores the fact that stocks represent shares of the equity of public companies, the owners of which cannot be held liable for more than the value of their shareholdings. The value of such stocks cannot be negative. The distribution of the stock prices cannot therefore be symmetrical, although in theory they can rise by any percentage,

---

<sup>3</sup> The mathematical representation of Brownian Motion, for a long time credited to the physicist Albert Einstein, was apparently first discovered by Bachelier as he investigated the behaviour of stock prices (Bernstein, 1992:18-122 and Ingersoll, 1989:202).

<sup>4</sup> This process is commonly known today as “Random Walk”

stock prices cannot fall by more than 100%. The assumption of arithmetic Brownian motion and normally distributed prices implies that stock prices can become negative.

Assuming a mean expected change in stock prices of zero implies that economic agents are risk-neutral, and ignores the time value of money (the expected terminal value of the option was not discounted to obtain the present value). If interest rates are positive and investors are risk-averse (that is, they prefer a certain outcome to an unexpected outcome), stocks will be expected, at any moment and on average, to yield a return at least equal to the rate of interest on a bond with no probability of default (the risk-free rate). Also, differences in the variability of the price of an option and the underlying stock were not investigated.

Despite what is today acknowledged as an enormous contribution to the fields of financial economics and mathematical statistics, Bachelier's work went unnoticed for more than half a century until it was accidentally found in the University of Chicago's library and circulated among established economists. Clearly rooted in Bachelier's work, the next important option-pricing model formulated within a probabilistic framework is that of Sprenkle, published in 1964.

### 2.3.2: Sprenkle -1964

Sprenkle in 1964 noted that it is the percentage and not absolute change in the stock price that matters. Sprenkle assumes that stock prices follow geometric Brownian motion with positive drift<sup>5</sup>. This means that the probability of a one percent increase in the stock price equals the probability of one percent decrease, irrespective of the stock price level (Smith, 1976:15). Positive drift means that the random walk followed by the stock price has an upward trend. The drift factor can be interpreted as the mean rate of return on the stock. The mean rate of return and the variance of the stock price are assumed to be constant. The model thus allows for the existence of positive interest rates and risk-averse behaviour.

---

<sup>5</sup> Although the technical aspects of this process are beyond the scope of this thesis, geometric Brownian motion is looked at in more detail in chapter 3.

The assumption of a stock price generated by geometric Brownian motion is consistent with a log-normal distribution of the possible stock price, at the end of a finite time interval, such as the life of an option (Sprengle, 1964: 428). It is the natural logarithm of the stock price, and not the stock price itself, which can be assumed to be normally distributed. The log-normal distribution, unlike the normal distribution which is bell-shaped, is skewed to the right, that is, the right rail is fatter than the tail on the left-hand side of the distribution. It serves as a better approximation of the actual distribution of stock prices, reflecting the fact that while stock prices can increase by any amount, they cannot become negative (note that natural logs are always positive). This seems to be about as far as Sprengle's model differs from Bachelier's. Sprengle ignores the time value of money in his model, despite the assumption of a positive drift in the stochastic process.

### 2.3.3: Boness -1964

Boness's model is very similar to Sprengle's. The crucial difference is that Boness does not ignore the time value of money. He also recognizes the importance of risk, but assumes that all stocks on which options are traded have the same risk profile, and, for simplicity, that investors are risk-neutral. Accordingly, no distinction is made between the expected rates of return or risk levels of options and the underlying stocks. The expected return on the stock is used as the discount rate in calculating the present value of the expected terminal value of the option (Ingersoll, 1989:203, and Smith, 1976:17).

### 2.3.4: Samuelson-1965

Samuelson (1965) provided a rigorous review of option valuation theory and pointed out that an option may have a different level of risk when compared with a stock, and therefore the discount rate used by Boness (1964) is incorrect. Samuelson assumes, as Sprengle and Boness did, that the distribution of stock prices is lognormal, and that stock prices follow

geometric Brownian motion with positive drift (Smith, 1976:18). The value of a call option is posited as given by the expected value of the option at maturity, discounted by the expected rate of return on the option. Samuelson does not suggest a procedure for estimating the expected rate of return on the option and admits this to be a weakness of the model (Smith, 1976:20).

It is clear that not only the expected rate of return on an option differs from the return expected on the underlying asset, as explicitly recognized in Samuelson's model<sup>6</sup>, but also, the volatility (and therefore expected return if investors are risk-averse) of an option varies as the stock price changes, if it follows a random walk. Especially, "the higher the stock prices relative to the exercise price, the safer the option, although the option is always riskier than the stock. The option's risk changes every time the stock price changes" (Brealey and Myers, 1996:573).

Recognizing the fact that the expected return (or riskiness) of an option is not constant puts a severe limitation on the theoretical validity of any model derived by simply estimating the expected value of the option at maturity (in itself not likely to be an easy task), and using the expected return on the option to discount this expectation to the present. The procedure would only be satisfactory if it were possible to specify one expected rate of return on the option that could serve as an appropriate discounting factor, that is, if the option's risk could be assumed to be constant.

Finally in 1973, Black and Scholes developed the basic framework for the currently well-accepted Black-Scholes option-pricing model. The only difference between the Black-Scholes model and the model derived by Boness (1964) is the use of the risk-free rate as the rate of return on the option instead of the rate of return on the underlying asset. "If Boness had carried his assumption that investors are indifferent to risk to its logical conclusion (that the expected rate of return on the stock is equal to the risk-free rate), he would have derived

---

<sup>6</sup> In 1955 Professor Paul Samuelson, wrote an unpublished paper entitled "Brownian Motion in the Stock Market", during that same year, Richard Krueger, one of Samuelson's students, cited Bachelier's work in his dissertation entitled "Put and Call Option: Theoretical and market Analysis".

the Black-Scholes equation” (Ingersoll, 1989:204). However, it is clear that the derivation of the two models differs substantially. Boness did not formulate his model within the context of capital equilibrium. He failed to demonstrate that a risk-free portfolio could be constructed. This would result in risk neutrality being acceptable as a basis to calculate the price of an option. Furthermore, although the Black-Scholes formula was obtained within a risk neutral environment, it is never assumed that investors are risk neutral. The formula is valid irrespective of risk preferences.

The following section is a review of the literature concerning implied volatility and volatility smile from the development of the Black and Scholes model (1973) to the present day.

## 2.4: The Black-Scholes Model

In deriving their valuation model for a European call option on an underlying stock with no dividend, Black and Scholes (1973) assume that the variance of stock return is constant over the life of the option, and known by market participants. They also assume the short term interest rate is known and constant throughout the life of the option and that this rate is the borrowing and lending rate for market participants. Other assumptions made in the model are that the underlying stock price follows a geometric Brownian motion with constant volatility, and that capital markets are frictionless in the sense that there are no transaction costs, no limitations on the use of short selling proceeds, and unlimited borrowing and lending (After the development of their model, there was a wave of option models that relaxed many of the simplifying assumptions made by Black and Scholes, and these models are shown in flowchart 2.1).

Given these assumptions, the value of an option is only a function of the underlying stock price and time. This means that the option and its underlying stock have a common stochastic diffusion component, and option traders can construct a riskless hedge portfolio by establishing a long (short) position in the underlying stock on which the option is written

(bought). The appropriate position in the underlying stock in the hedge portfolio is determined by the first partial derivative of the Black and Scholes option pricing model with respect to the asset price. It should be noted that the portfolio is riskless only for an infinitesimally short period of time. As the price of the underlying stock and time to maturity change, the first partial derivative of the Black-Scholes option pricing model also changes. To keep the portfolio riskless, it is therefore necessary for option traders to continuously rebalance the relative proportions of the derivative and the stock in the hedge portfolio. In other words, if an option were to become mispriced, a riskless arbitrage situation would exist causing an immediate return to the correct value. In an ideal market, if the option's price were to differ from the correct price as indicated by the Black-Scholes formula, an arbitrageur would trade stock options against the actual stock and produce a riskless profit situation. As a result, the price of the option would immediately change to reflect the profit neutral arbitrage position.

Their model generates the theoretically correct value of an option based on the following independent variables:

- 1) The price of the underlying stock
- 2) The time until expiration
- 3) The difference between strike and market price
- 4) The stock's volatility
- 5) The risk-free rate of return

The variable that is the hardest to estimate is the stock's volatility as it is the only variable that cannot be directly observed. Likewise, it is the variable of most interest to purchasers of options (Feinstein, 1989). Historical stock volatility was used for this variable by Black and Scholes (1972) in an empirical study of their model<sup>7</sup>, as well as by other early empirical studies of option pricing models<sup>8</sup>. The simplest method to obtain the historical stock volatility is to calculate the conventional standard deviation of the rate of return on the underlying

---

<sup>7</sup> Black and Scholes's seminal paper was published in 1973. Their test of the model was published before the model itself. The original theoretical paper had however been submitted for publication before the empirical study, but it was not accepted and had to be re-submitted, after the intervention of more well known economists (Bernstein, 1992).

<sup>8</sup> See, for example, the study by Galai (1977).

stock, using historical stock price data. However, an historical measure of volatility may fail to reflect the market's view, at a point in time, of the future volatility of the stock. We can expect this to be the case wherever a shock hits the stock market. Since the market's assessment of future volatility will determine the volatility input when options are valued, a model that uses only past data to estimate this parameter will fail to price them correctly (Schmalensee and Trippi, 1978).

To understand the important relationship between the volatility and the pricing of the option, an example of two stocks having the same price of R90 but different volatilities is considered. It is possible to buy both a call and a put on the same stock, taking the position that if the stock moves sufficiently from its current price, the position will be profitable. In this case, if the stock moves either above R100 or below R80 the holder of the position will make money. On the other hand, if the stock price stays within the range of R80-R100 then the position will not make money and the put and call will expire worthless. Clearly, options on stock 1 with the greater volatility should have the higher price because its volatility will drive the stock into the profitable range. Interestingly, profitability doesn't depend on whether the stock moves up or down in price. It is the volatility that drives the price of the option.

As shown by Black and Scholes (1973), if an investor were able to perfectly forecast volatility, large profits could be made. As in the example shown above, knowing the exact volatility would enable an investor to purchase only those options that would be profitable and avoid those that would be unprofitable. Using the correct volatility estimate is the key to accurate Black-Scholes option pricing.

The Black-Scholes model has been confirmed by extensive empirical testing to be consistent with market prices for at-the-money call options. However, Black (1975) noted and the model has been shown to differ systematically from market prices for in-the-money, out-the-money and for differing time to expiration (term structures) of call options (MacBeth and Merville, 1979; Rubenstein, 1985; Butler and Schacter, 1986).

One of the causes of this problem is the distinction between model price and market price (Figlewski, 1989). The model price assumes an ideal market while the market price is the result of supply and demand pressures of investors and the market environment. Investors may have irrational expectations and be willing to pay for these expectations. The market environment includes many variables such as hours open for trading, transaction costs, taxes and economic conditions. None of these variables are included in the model but can have a potentially significant impact on the pricing of an option.

In the case of transactions costs, investors would only be expected to execute the arbitrage whenever the expected profit exceeds the transaction costs. However, these upper and lower bounds can be very large. Figlewski (1989) simulated the riskless arbitrage position including transaction costs and found the impact of these costs to be significant. The price of an at-the-money call option with a one month expiration date and a Black-Scholes value of \$2.05 for example, would have to be outside the bounds of \$1.74 to \$2.35 to generate a riskless arbitrage situation including transaction costs (Figlewski, 1989).

## 2.5: Implied Volatility

Each of the six variables in the Black-Scholes formula (stock price, exercise price, time to expiration, volatility, dividend, and risk-free rate) are observable, except for the volatility of the underlying stock. Latane and Rendleman (1976) demonstrated that the Black-Scholes formula can be inverted, using numerical techniques, to cater for volatility. The resulting value has been termed Black-Scholes implied volatility. Given that the implied volatility is the only unobservable variable in the Black-Scholes formula and given the accuracy of the Black-Scholes model, the implied volatility should be the market's expectation of the future volatility of the underlying stock.

Because the model values options in an ideal market and the market prices options according to supply and demand, there may be mathematically consistent differences between the

model generated values and the actual prices. Because of these additional market factors which are not part of the Black-Scholes formula, the implied volatility produced by inverting the formula may include more information than merely the implied volatility. In particular, because the implied volatility is the only unobservable variable in the formula it includes the implied volatility and all other information such as the effect of transaction costs, and other market environment factors of supply/demand and irrational investors.

If the Black-Scholes model were precisely accurate, all of the implied volatilities on the same underlying stock would be the same. Empirically, however, it has been demonstrated that implied volatilities are systematically biased across strike prices and across expiration (Rubenstein, 1985).

MacBeth and Merville (1979), for example, find that implied volatility recovered from the Black-Scholes model higher (lower) for in-the-money (out-of-the-money) options than at-the-money options. This was a dominant pattern before the October 1987 market crash. After the crash, the implied volatility decreases monotonically as the options are out-of-the-money (Dumas, Fleming, and Whaley, 1998). Option professionals often refer to this effect as a volatility “smile” or “skew.”

### 2.5.1: Volatility Smiles

The volatility smile is the term used to describe the systematic bias in the Black-Scholes formula for options that are in-the-money or out-of-the-money. The term is derived from the empirical evidence showing that the option price (and implied volatility) tends to be higher than the Black-Scholes value when the option is deep-in-the-money or deep-out-of-the-money (Hull and White, 1987). When charted, the resulting graph of implied volatilities across strike prices has the look of a smile. Figure 2.1 is typical volatility smile for the March 1996 S&P 500 futures options which were traded on March 8, 1996 with a settlement price of

632. The smile illustrates the systematic bias in the Black-Scholes formula for options that are in-the-money or out-of-the-money.

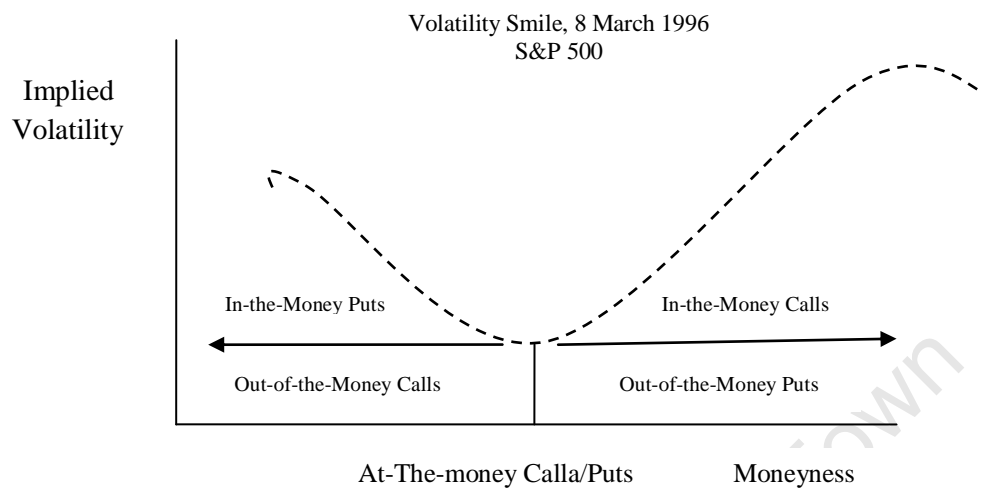


Figure 2.1: Volatility Smile Graph<sup>9</sup>.

The literature is consistent in the finding that there exists significant bias in the Black-Scholes formula (Ncube and Satchell, 1997). However, the direction of the bias has not been so consistently reported.

Black (1975) reported that the Black-Scholes model systematically underprices deep out-of-the-money calls and overprices deep in-the-money calls. Merton (1976) suggested that practitioners observe Black-Scholes model prices to be less than market prices for deep in-the-money as well as deep out-of-the-money options. MacBeth and Merville (1979) report the striking price bias, but the direction is opposite to that reported by Black (1975) and Merton (1976). Galai (1983) confirmed this work and reported that the Black-Scholes model undervalues in-the-money and overvalues out-of-the-money options. Rubinstein (1985) found both directions of bias depending on the period of the sample. Sheikh (1991) documented the

<sup>9</sup> "Volatility project". Finance622volatility. 15 March. 2009 <<http://finance622volatility.wikidot.com/volatility-p>>.

smile effect on call options using transaction data between 1983 and 1987 and concluded that the Black-Scholes model was unable to correct for these systematic biases.

The persistence of the volatility smile or skew phenomenon has led several scholars to offer alternative models to the Black-Scholes model. Most alternative models fall into one of the following four categories: first, jump-diffusion models which augment the Black-Scholes returns distribution with a Poisson driven jump process (Jarrow and Rosefeld,1984; Ball and Torous,1985; Ahn,1992; Amin,1993; Bates,1991,1996; Das and Foresi,1996); second, stochastic volatility models which extend the Black-Scholes model by allowing the volatility of the return process to evolve randomly over time (Merton,1976; Hull and White,1987; Melino and Turnbull,1990; Stein and Stein,1991; Amin and Ng,1993; Heston,1993; Nabdi,1998); third, local volatility models, also called deterministic volatility function models or implied tree models, which assume that future volatility is a deterministic function of the underlying stock value and time, and this function is implied by the current volatility smile and can be fully reflected in a suitably calibrated binomial or trinomial tree (Dupire,1994; Derman and Kani,1994; and Rubinstein,1994); fourth, mixed distribution models which model the underlying stock value with a mixture of distributions (Brigo and Mercurio, 2000). Other models are hybrids and combine aspects of those described above (Bates, 1996; Britten-Jones and Neuberger, 2000). Numerical empirical results, however, show that most of the above mentioned alternative models do not fit the observed option prices either, and some even perform worse than the Black-Scholes Model (Jackwerth, 1999; Alexander and Nogueira, 2004). Hybrid models are rich, but they are difficult to implement in reality.

Although efforts have been devoted to explaining the volatility smile or skew phenomenon and to seek alternative models, currently there is no overwhelming consensus on a suitable alternative model to the Black-Scholes model. So far, the existing literature has not been conclusive either on whether the Black-Scholes model is ineffective, or the options market is inefficient. Indeed, there is no evidence that the persistent pricing biases represent risk-free arbitrage opportunity.

The bottom line is that although there are multiple options written on a given stock, by definition there is only one volatility for a given underlying stock. A natural question arises in the presence of the volatility smile phenomenon: which implied volatility or combination of implied volatilities provides the best measure of the volatility of the underlying stock over the remaining life of the options? A possible answer to this question is the idea of using a single, uniform measure of market risk, such as VaR, in predicting the changes in the underlying stock prices and their expected volatility due to price changes of the underlying stocks. The rest of this literature review attempts to find out more about how the criteria and testing procedures developed for evaluating VaR models can also be adopted for evaluating option pricing models and predicting volatility smiles or skew phenomena.

## 2.6: Volatility Smiles and Risk Management

In the derivation of the Black-Scholes equation, the elimination of the randomness in the option pricing process is employed to derive the deterministic Black-Scholes equation. The quantity that eliminates the main contribution to randomness in this model,  $\Delta = \frac{\partial C}{\partial S}$  is one of the important parameters in option pricing. It is the rate of change of the option price with respect to the price of the underlying asset. It indicates the number of stocks that should be kept with each option issued in order to cope with a loss in the case of exercise.

Another important parameter is called  $\Gamma$ . It is defined by the rate of change of the portfolio's  $\Delta$  with respect of the price of the underlying asset.  $\Gamma$  is an indication of the sensitivity of  $\Delta$ . If  $\Gamma$  is low, it is sometimes necessary to change the portfolio. If it is high, the portfolio under consideration represents, for a very short period of time, a risk-less scenario. These parameters are known as the Greeks (because they are often denoted by Greek letters such as delta, gamma, theta, vega, rho).

Professionals use the Option Greeks to measure exactly how much they need to hedge their portfolio and to remove specific risk factors from their portfolio. The Option Greeks also enable the measurement of how much risk the portfolio is exposed to, and where that risk lies (for example, with movements in interest rates or volatility). However, due to the fact that the Greeks rely on assumptions made in the classical Black-Scholes formulas, which usually suggest that asset returns have a lognormal distribution and financial time series follow a stochastic process of a geometric Brownian motion. Those assumptions are questionable and overwhelmingly rejected by empirical evidence as shown in this literature overview which can be summarized by the following points.

- 1) The Black-Scholes Model and its Greeks show trends based on past performance. However, it is not guaranteed that the future performance of the stock will behave according to the historical numbers. These trends can change drastically based on new stock performance.
- 2) The Black-Scholes Model assumes that the risk-free rate and the stock's volatility are constant. Furthermore, the model assumes that stock prices are continuous and that large changes (such as the October 1987 market crash) don't occur.
- 3) With the model analysts can only estimate a stock's volatility instead of directly observing it, as they can for the other inputs.
- 4) The Black-Scholes Model tends to overvalue deep out-of-the-money calls and undervalue deep in-the-money calls, which leads to arbitrage possibilities opposite to the Black-Scholes assumption of no arbitrage possibilities.

This work has to therefore eliminate the use of the Greeks in predicted risks associated with the volatility smile in the empirical work in the next chapter. VaR is a single, uniform measure of market risk and is also a category of risk measures. However, unlike market risk metrics such as the Greeks, duration and convexity, or beta, which are applicable only to certain asset categories or certain sources of market risk, VaR can be applied to all asset categories and can cover all sources of market risk. Thus, VaR could have the ability to predict risks associated with the volatility smile.

## 2.6.1: Value-at-Risk

The field of risk management has developed enormously in the last two and a half decades, both in theory and practice (Dowd 1998:4). In theory, risk management analysis focuses on the extreme values related to the tail of the underlying distribution. This is desirable since many studies suggest that most financial time series have fat-tailed and asymmetric distributions. One definition of fat-tailness (heavy-tailness), given by Gencay and Selcuk (2004), is that a distribution is fat-tailed if a power decay of the density function is observed in the tail. In practice, the development of VaR opens up a radically new approach to firm-wide risk management (Fletcher, 1981). VaR is defined as the maximum expected loss over a given horizon period, at a given level of confidence. The concept of VaR involves two arbitrarily chosen parameters: the horizon period, which might be daily, weekly, monthly, or other time frequencies, and the level of confidence, usually 95% or above. Summarizing the overall market risk in a single number, VaR has become a way of quantifying market risk, assessing the risk of investment, and reporting the level of risk to investors.

Supporters of this methodology argue that VaR is sound as a risk measure for several reasons: it represents a summary measure of aggregated portfolio risk expressed in a currency unit, combined with a probability statement. VaR is much broader than any single measure of sensitivity, because it comprises many sources of risk (for example volatility, interest rate, transactions cost.) and also accounts for correlations and leverage, which becomes very important when dealing with large portfolios.

VaR is also important in portfolio management. Traditional, active portfolio management involved attempts to outperform benchmarks by a given percentage subject to a limit on tracking error volatility (TEV)<sup>10</sup>. As this practice resulted in selecting inefficient portfolios, alternative methods have been proposed, such as mean VaR optimization, which was spurred on by the early studies on shortfall constraints (Roy, 1952; Telser, 1956 and Kataoka, 1963).

---

<sup>10</sup>The volatility of the difference between the portfolio and the benchmark returns

More recently, studies such as those of Basak and Shapiro (2001) or Alexander and Baptista (2002, 2004) address the use of VaR in the context of portfolio optimization.

### 2.6.1.1: VaR Models Comparison

In its most general form, VaR<sup>11</sup> can be computed from the probability distribution of the future portfolio value  $\tau_t$ :

$$1 - \theta = \int_{VaR}^{\infty} f_r dr \quad (2.1)$$

Where  $f_r$  is the probability density function of portfolio returns and VaR is the worst possible return realized with  $\theta$  percent probability of being exceeded. For example, if  $\theta = 5$  percent VaR is the cutoff quantile that will only be exceeded in 5 percent of the cases as shown in figure 2.2.

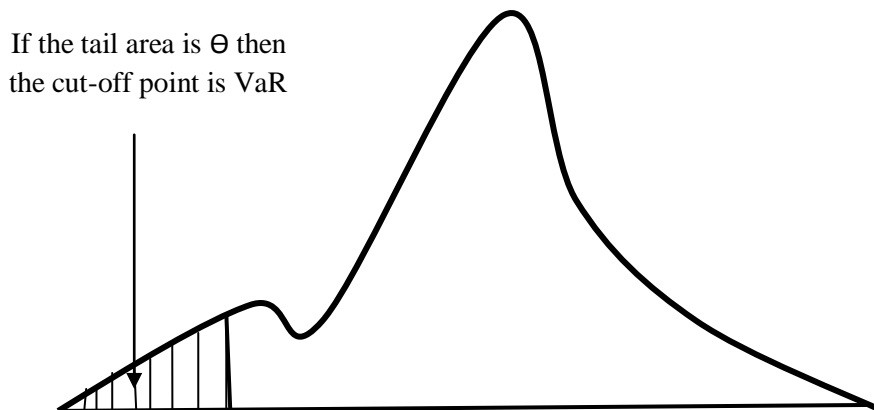


Figure 2.2: The P&L density and Value-at-risk (Alexander, 2001).

<sup>11</sup> Although VaR is ultimately expressed in an absolute currency for example Rand terms in a statement such as “Tomorrow’s Value-at-Risk for the entire portfolio is 100 million Rand”, VaR is technically a rate of return forecast. More specifically, a negative rate of return.

Conversely, VaR can be computed as

$$\theta = \int_{-\infty}^{VaR} f_r dr = Prob(r_t \leq VaR) \quad (2.2)$$

For full details refer to Jorion (2000).

Overall, three classes of models are used to estimate VaR: parametric (GARCH, RiskMetrics, and higher-order moments), nonparametric (historical simulation, Monte Carlo simulation and the hybrid approach), and semiparametric (Extreme Value Theory, Conditional Value-at-Risk and Quasi-Maximum Likelihood GARCH). Thus far, the focus in VaR research has been model development rather than model testing, despite plenty of evidence suggesting results differing by a factor as large as 14 when using different VaR methods (Beder, 1995).

All VaR methods have advantages and disadvantages. Most of the shortcoming of parametric VaR methods arise from the assumption that stock returns are normally distributed, which is different from the empirical reality. To overcome this drawback, conditional normality is used extensively. RiskMetrics, for example, computes the variance conditional on the most recent information, using an exponentially-weighted approach. ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity) methods, introduced by Engle (1982) and Bollerslev (1986), conditionally model the variance as a function of past squared and variances. Both RiskMetrics and GARCH have been found to underestimate VaR (Engle and Manganelli, 2001). Since the VaR is based on the tails behaviour of the financial returns, specifically on the left hand side, it is obvious that the issue of fat tails is become basic to our investigation.

There has been a significant amount of empirical research on this topic in the last twenty-five years. However it is only recently, since the introduction of VaR as a market risk measure, that important contributions are being made, not only at a theoretical level, but also in terms of empirical research, with an open field of research for the future. Therefore, in addition to

the traditional empirical studies which confirm the proven aspects of the original studies by Mandelbrot (1963) and Fama (1965) about the non-normal features of financial returns (fat tails and excess kurtosis; asymmetry, volatility smile). Among the empirical studies that have proven that the returns have the aforementioned characteristics and not exactly those of a normal distribution, it is necessary to refer to: Praetz (1972), Blattburg and Gonedes (1974), Kon (1984), Jorion (1988), Jansen and de Vries (1991), Tucker (1992), Kim and Kon (1994); Danielsson and de Vries (1997b), Klupelberg et al. (1998), McNell (1998) and Huisman et al. (1998) for stock prices.

The above are the most recent studies on the subject of fat tails and VaR estimation for stock prices. These studies identify how estimates of VaR based on an assumed normal distribution of portfolio returns could underestimate (and overestimate) “true” VaR, and therefore, the capital required to cover the losses derived from market risk, leading to massive financial losses. This has led many authors to propose (in order to describe the behaviour of returns) other distributions with fatter tails than the normal distribution (for example, Pareto’s stable distribution, student’s t distribution, the normal mixture approach, or the generalised error distribution); thereby accommodating the modelling of the biggest movements in order to avoid erroneous VaR estimates. Even the most recent studies propose distributions that do not describe all returns behaviour but outline only the behaviour of the extreme returns, that is to say, those based on the Extreme Value Theory.

However, due to the complexity of estimating the VaR for non-linear positions, some authors choose to value such positions in a partial way, through the Taylor Theorem (delta or delta-gamma valuation). Nevertheless, Estrella (1996) warns about the necessity of applying Taylor approximations in the case of the options valuation with some caution. Estrella shows how a Taylor approximation, applied to the Black-Scholes options valuation model in terms of prices regarding the underlying stock, changes as a function of such prices. On the other hand, if the approximation is used in terms of the logarithmic prices, no divergence will occur.

According to Estrella (1996: 375): “In risk management applications involving a preponderance of relatively small moves, it may be feasible though sometimes risky to use Taylor approximations. For moves no larger than one standard deviation, the accuracy of gamma approximations seems generally adequate. Problems may arise however, if attention is focused on the tails of the distribution as is often the case in risk management applications. Special care should be used when approximating the values of highly nonlinear options, such as near-the-money short maturity options”.

All things considered, the problem is what to do when returns are non-linear functions of the risk factors (as is the case with options) or when the risk variables themselves are non-normal. This is precisely the point of our focus here. According to Minnich (1998:41), “One of the most difficult aspects of calculating VaR is selecting among the many types of VaR methodologies and their associated assumptions”.

Nonparametric methods, such as Historical simulation, Monte Carlo simulation and the hybrid approach (Boudoukh, Richardson and Whitelaw, 1998), totally drop any assumption about the distribution shape. These methods compute VaR directly from the empirical distribution of returns, and consequently, exclude the necessity of establishing approximations (such as those based on Taylor), which produce inaccuracy in calculations. Thus, they can be applied to all kinds of instruments, both linear and non-linear. Furthermore, these methods allow capturing fat tails; this is not the case in the variance-covariance matrix approach and Black-Scholes's Greeks.

Lastly, semiparametric methods cover Conditional VaR (C-VaR), Quasi Maximum Likelihood (QML) GARCH, and Extreme Value Theory based models. These approaches are not fully “parametric” in that they do not attempt to parameterize the entire return distribution (Manganelli and Engle, 2001). C-VaR attempts to model directly the evolution of the quantile of interest over time, using a special type of autoregressive process (Engle and Manganelli: 1999, 2001, 2004). QML GARCH derives VaR from the distribution of the standardized residuals. Extreme Value Theory (EVT) is based on the work of Fisher and Tippett (1928)

who found that, for appropriately normalized maxima, the distribution of the tails can have one of three forms: Gumbell, Frechet, or Weibull. Unfortunately, EVT seems to work best only for low probability levels, and only asymptotically. The extensions that have been proposed by Leadbetter, Lindgren, and Rootzen (1983), among others, rule out important empirical features of financial data, such as volatility clustering.

## 2.7: Summary

This chapter summarized the major developments in the field of option valuation in the past three decades, up to the original contributions of Black and Scholes (1973), and Merton (1973). The chapter then focused on implied volatility estimation and on the well known volatility smile phenomenon. Numerous explanations for the volatility smile phenomenon and extensions as well as alternatives to the Black-Scholes model were offered in the chapter.

This chapter also provided an overview of developments, methodologies, and applications of VaR. Various key methodologies of VaR estimation and evaluation were discussed and compared.

In conclusion, the nonparametric methods of VaR, such as Historical simulation, Monte Carlo simulation and hybrid approach totally drop any assumption about the distribution shape, these methods compute VaR directly from the empirical distribution of returns, and consequently, exclude the necessity of establishing approximations, which produce inaccuracy in calculations. Thus, they can be applied to all kinds of instruments, both linear and non-linear. Furthermore, these methods allow capturing fat tails; this is not the case in the variance-covariance matrix approach as well as Black-Scholes's Greeks. Thus nonparametric methods seems to be perfect candidates to predicate options implied volatility smile.

We close this second chapter with an outline of the next chapter. Chapter 3 will investigate the volatility smile derived from options traded on the Johannesburg Stock Exchange (JSE)

and the possibility of forecast this volatility using different VaR approaches. My first objective in this chapter is to provide evidence of a South African volatility smile and thus confirm the mis-pricing of the Black-Scholes model in this market. The second objective, will use VaR to predict the movement in the underlying stocks, and accordingly predict if the underlying stock's volatility exhibits a smile or skew effect. In this section, two different methods are applied to calculate VaR - Historical simulation and Monte Carlo simulation - as well as conditional VaR and backtesting.

University of Cape Town

# Chapter 3

## The Volatility Smile of South African Stock Options: Estimations and Tests<sup>12</sup>

### 3.1: Introduction

The implied volatility given by the Black-Scholes (1973) model is assumed to be constant. However, empirical studies on S&P 500 index options have shown that the implied volatility changes with the ratio of the strike price to the index price. When the implied volatility is plotted against the moneyness (ratio of the underlying asset spot price to the strike price) of the option, an upward sloping curve is observed. This known as the volatility smile. A possible reason for the existence of this smile is that the theoretical valuation model of Black-Scholes cannot take into account imperfections in the marketplace caused by supply and demand. As a result, using the Black-Scholes formula for calculating implied volatility is problematic. In particular, the theoretical formula uses actual market values to calculate the implied volatility. Consequently, the resulting implied volatility includes not only the market's expectation of future volatility, but also includes any error between the theoretical price and the actual price. In fact, every other variable that is not in the formula, but that is a causal variable in determining the market price of the option, is included in the resulting implied standard deviation.

The present study is an attempt to empirically test the validity of the Black-Scholes formula for determining equity warrants<sup>13</sup> listed on the JSE. However, if the results demonstrate any systematic biases towards options which are in-the-money, out-of-the-money or with

---

<sup>12</sup> This chapter is a published paper: Endi, A (2009): "An examination of the value-at-risk volatility relationship in the options market: new evidence on mispricing in the Black-Scholes model", *Studies in Economics and Econometrics*, 33(1), 85-103.

<sup>13</sup> In South Africa and Europe, a warrant is an instrument issued by a third party on an underlying share. The issuer does not call on the underlying company to issue more shares. Therefore, the shareholders' funds are not diluted in the process. Thus, fundamentally a South African warrant is an option. Warrants will be hereafter referred to as options.

differing term structures, then, there is clearly something wrong with the use of the Black-Scholes model for this purpose. It follows that we should look at alternative methods of measuring risk. The second section will use VaR as a means of predicting any movement in the underlying stocks, and accordingly to predict whether the underlying stock's volatility exhibits a smile or a skew effect. In this section, two different methods are used to calculate VaR - Historical simulation and Monte Carlo simulation - as well as conditional VaR and backtesting.

The remaining part of the chapter is organized as follows: section 2 briefly reviews option pricing models being tested in this chapter. Section 3.3 provides a detailed description of VaR methodologies and related models. In section 3.4 the data used for this analysis are described. Section 3.5 outlines some empirical findings on evaluating the pricing performance of alternative models. Section 3.6 summarizes the results and reviews the conclusions.

## 3.2: Black-Scholes Constant Volatility Model

In deriving the valuation model for a European call option on an underlying stock with no dividend, Black and Scholes (1973) assumed that the variance of stock return is constant over the life of the option and is known by the market participants. They also assumed that the short-term interest rate is known and constant throughout the life of the option, and that this rate is the borrowing and lending rate for market participants. The underlying stock price follows a geometric Brownian motion; capital markets are frictionless, in the sense that there are no transaction costs; there are also no limitations on the use of short-selling proceeds, and there can be unlimited borrowing and lending. Given these assumptions, the value of an option is only a function of the underlying stock's price and time. This means that the option and its underlying stock have a common stochastic diffusion component. Option traders can then construct a riskless hedge portfolio by establishing a long (or short) position on the underlying stock on which the option is written (bought). The appropriate position in the underlying stock in the hedge portfolio is determined by the first partial derivative of the

Black and Scholes option-pricing model with respect to the asset price. The Black-Scholes formula is given by:

$$C(t, S_t, X, \sigma) = S_t e^{-q(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2) \quad (3.1)$$

where,

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sqrt{T-t} \quad (3.2)$$

$N(\cdot)$  is the cumulative probability distribution for a variable that is normally distributed with a mean of zero and a standard deviation of one. In particular,  $N(d_2)$  is the probability that the option will be exercised in a risk-neutral world so that  $XN(d_2)$  is the strike price times the probability that the strike price will be paid. The expression  $S_t N(d_1) e^{r(T-t)}$  is the expected value equal to  $S_T$  if  $S_T > X$  and zero otherwise in a risk-neutral world. This interpretation of the terms shows that the formula is consistent with risk-neutral valuation (Hull, 1997).

The above equation also gives the value of an American call option that does not pay any dividends (Merton, 1973; Roll, 1977; Geske, 1979 and Whaley, 1981).

However, equation 3.2 (above) implies that there is a one-to-one correspondence between the option price and the volatility. As a result, the implied volatility can be computed by solving for the volatility that equates the model price with the observed market price. Under the assumptions of the Black-Scholes model, implied volatilities should be the same for options on the same asset with different strikes. This assumption will be our measurement of the model validity in our empirical section.

### 3.3: Measures of Extreme Risks: VaR and Expected Shortfall (ES)

Two parameters are very important in VaR estimation; the confidence level and the length of the time horizon. If VaR is used to compare different markets (for example, equities, fixed income, commodities, currencies), the choice of the confidence level and time horizon does not matter much as long as consistency is maintained, as the goal is to compare risk across trading units. However, if VaR is used as a potential loss measure, risk aversion and the nature of the portfolio dictate the confidence level and the time horizon. Due to the liquid nature of and the rapid turnover in their portfolios, commercial banks report VaR on a daily basis. On the other hand, since pension funds generally make fewer liquid investments and adjust their risk profiles slowly, a longer horizon is more appropriate.

The choice of these two parameters is most crucial when VaR is used to set capital requirements: in the case of erroneous estimates, losses exceeding the VaR could totally wipe out the equity, resulting in bankruptcy. Thus, selection of the time horizon should appropriately reflect the time needed for corrective action as losses start to build up, whereas selection of the confidence level should relate to the extent of risk aversion and the cost of loss exceeding VaR. A higher cost of exceeding the VaR and a higher degree of risk aversion imply a larger amount of capital needed to cover possible losses, which leads to employing a higher confidence level.

Time horizon and the confidence level are also important for testing model validity, which involves comparing the estimated VaR with the subsequently realized loss. High confidence levels and longer horizons diminish the power of the tests<sup>14</sup>. For example, as opposed to a 95 percent confidence level, where we should expect on average a loss exceeding the forecasted VaR in 1 day out of every 20 days, at 99 percent confidence we need on average of 100 days to test a model. Also, using a two-week horizon VaR will result in only 26 independent

---

<sup>14</sup> The power of a test is formally as 1-probability of a type II error, which is the probability of failing to reject the null hypothesis when the null should be rejected.

observations per year, in contrast to a daily VaR which will result in a sample of 252 independent observations over the same year.

The focus of this chapter will be on the non-parametric model. This is because the non-parametric model includes the historical simulation and Monte Carlo approaches drop any assumption about the distribution shape, and compute VaR directly from the empirical distribution of returns, and consequently, exclude the necessity of establishing approximations (such as those based on the work of Taylor), which produce inaccuracy in calculations. For more details on different VaR modes refer to the previous chapter.

All VaR measurement approaches use a similar scheme: (1) Selection of basic parameters (time horizon, confidence level, time of measurement); (2) Selection of relevant market factors; and (3) VaR calculation.

For step (1) in this research the relevant parameters are defined according to goals and resources. The next step, (2) involves the assumption of some kind of model, either just a set of relevant factors or a detailed pricing model. In any case the relatively small set of relevant parameters should be defined, and some method for portfolio valuation based on this set should be established. Step (3) includes the calculation itself. This step can be very time consuming, especially when Monte Carlo methods are used. There are numerous techniques for speeding up the calculation. The following are the different types of techniques to calculate VaR.

### 3.3.1: Historical Simulation method

Historical simulation is the most transparent method of calculation. This involves running the current portfolio across a set of historical price changes to yield a distribution of changes in stock value, and computing a percentile (the VaR). The benefit of this method is that it is

simple to implement, and does not assume a normal distribution of stock returns. Drawbacks are the requirement for a large market database and the computationally intensive calculation.

Historical simulation to perform analysis on a single instrument portfolio can be described in five steps:

1. Identification of the basic risk factors.
2. Obtaining of historical values of the risk factors for the last  $N$  periods.
3. Taking into account the current stock price to the changes in market rates and prices experienced on each of the most recent 250 or 500 business days; calculation of the daily profits and losses that would occur if comparable daily changes in the risk factors are experienced.
4. Ordering of the profits and losses from the largest profit to the largest loss.
5. Selection of the loss which is equalled or exceeded five percent of the time. This loss is the value at risk at 95% confidence level (assuming that a 95% confidence interval is being used).

### 3.3.2: Monte Carlo Simulation method

The Monte Carlo simulation method has a number of similarities to historical simulation. The main difference is that rather than carrying out the simulation using observed changes in the market factors over the last  $N$  periods to generate  $N$  hypothetical portfolio profits and losses, a statistical distribution is chosen that is believed to adequately capture or approximate possible changes in the market factors. Then, a pseudo-random number generator is used to generate thousands or tens of thousands of hypothetical changes in the market factors. These are then used to construct thousands of hypothetical portfolio profits and losses on the current portfolio, and the distribution of possible portfolio profits or losses. Finally, the value at risk is determined from the distribution.

Similar to historical simulation, the Monte Carlo simulation can be described in five steps:

1. Identification of the basic risk factors.
2. Determination or assumption of a specific distribution for changes in the basic market factors, and estimation of the parameters of that distribution. The ability to choose the distribution is the feature that distinguishes the Monte Carlo simulation from other approaches, as in the other methods the distribution of changes in the market factors is specified as part of the method. The designers of the risk management system are free to choose any distribution that they think reasonably describes possible future changes in the market factors.
3. Use of a pseudo-random generator to generate  $N$  hypothetical values of changes in the risk factors based on the selected distribution, where  $N$  is almost certainly greater than 1,000 and perhaps greater than 10,000. These hypothetical market factors are then used to calculate  $N$  hypothetical market-to-market portfolio values. From each of the hypothetical portfolio values it is necessary to subtract the actual market-to-market portfolio value to obtain  $N$  hypothetical daily profits and losses.

Steps 4 and 5 are the same as in historical simulation. The distribution of profits and losses are ordered from the largest loss to the largest profit, and the value at risk is defined as the loss which is equalled or exceeded five percent of the time.

### 3.3.3: Expected Shortfall method

Another VaR metric is expected shortfall (ES), which is sometimes also known as, amongst other names, expected tail loss (ETL), conditional VaR (C-VaR), and worst conditional expectation.

The ETL is the expected value of losses,  $L$ , if the loss exceeds VaR:  $ETL = E[L / L > VaR]$ .

The VaR indicates the most that can be lost if a bad (that is, tail) event does not occur, and the ETL indicates what can be lost if a tail event does occur. For example, a 90% ETL VaR metric indicates the expected loss conditional on that loss exceeding its own .90-quantile (Hull, 1997).

### 3.3.4: Backtesting

The Basel standard requirement of backtesting is a procedure whereby one checks (a posteriori) how often the actual losses have exceeded the level predicted by VaR. As soon as a 99% confidence interval and a 10 day time horizon are used, there should not be too many cases in which the actual losses are greater than the predicted ones.

Results are in three zones. If during the last year (approximately 250 business days) there are four or fewer exceptions (losses that exceed the VaR level), the model is said to be in a green zone, and it is acceptable. If there are five to nine exceptions the model is in the yellow zone and certain actions (such as an increase of the safety multiplier from 3 to 3.65) are recommended (Gerhard, 1997). When there are 10 or more exceptions the whole model should be revised. This mechanism prevents banks from setting the VaR too low (Hull, 1997).

## 3.4: Data Description

Stock options make up the largest portion of options listed on the JSE and these are the options that will be the focus of this study. These options are usually issued over the shares of a single company listed on the JSE. They are currently the most popular type of options traded. The options can be American or European in style. The data used in this study consists of fourteen companies' stock, listed on the JSE and fifty-nine Call and Put options with different styles, exercise prices and times of expiration, all traded on the JSE. An outline of this data is presented in Table 3.1 in appendix A.1. The dataset for my empirical analysis contains an options expiration date, the exercise price, cover ratios, the risk-free rate, the daily closing premium, and finally, the dividend yield. The daily closing share price of the underlying stock is also needed. All of these data were obtained from the Inet-Bridge and the DataStream financial databases in the University of Cape Town's library.

The time-to-maturity of an option is measured by the number of calendar days between the valuation and the expiration dates. South African 91-day T-Bill rates were used for the risk-free interest rate determination.

The following are the criteria the data needs to satisfy in order to be applied in the Black-Scholes formula: (1) One of the fundamental assumptions underlying the theory of option pricing, as set out in the Black-Scholes pricing formula, is that the natural logarithms of the spot price of the underlying stock are normally distributed, that is, they follow a log-normal distribution. Hence, the natural logarithms of the underlying stocks were tested for normality using the Jarque-Bera normality test (1987)<sup>15</sup>. Some underlying stocks were eliminated from the analysis as they were rejected by the Jarque-Bera normality test.

(2) The Black-Scholes formula in Black and Scholes (1973) and Merton (1973) is based on the assumption that stocks pay no dividends during an option's life. Some of the researcher's sample stocks paid dividends to their shareholders, so this might seem a serious limitation to the model, considering the observation that higher dividend yields elicit lower call premiums. In this case the common method of adjusting the model for this situation was followed, which involved subtracting the discounted value of a futures dividend from the stock price using the risk-free rate (Gua and Su, 2006).

For the purpose of this analysis, 365 days of the year were assumed and a continuous risk-free rate was used throughout the analysis. However, for VaR analysis three parameters had to be identified: (1) the time horizon (period); that is, the length of time over which one plans to hold the stock in the portfolio, also called the "holding period". In this work a 1-day holding period has been considered. (2) The confidence level at which one plans to make the

---

<sup>15</sup>The Jarque-Bera test is a goodness-of-fit, measure of departure from normality, based on the sample Kurtosis and Skewness. The test statistic JB is defined as

$$JB = \frac{n}{6} \left( S^2 + \frac{K^2}{4} \right)$$

Where S is the skewness, K is the Kurtosis, and n is the number of observations. Given a level of significance 0.05 ( $\chi^2_{0.95} = 5.99$  from  $\chi^2(2)$ ), if  $JB > 5.99$ , then the Null hypothesis of asymptotic normality is rejected. On the other hand, if  $JB \leq 5.99$ , then normality cannot be rejected.

estimate. This work attempts to use the popular confidence levels of 99% and 95%. (3) The unit of the currency which will be used to denominate the Value at Risk. In this work the South African Rand was used.

## 3.5: Empirical results

### 3.5.1: Sensitivity of Price Error to the Implied Volatility Estimation

The Black-Scholes model states that the price of an option is a function of the stock price, the exercise price, a risk-free rate, the time to expiration, the volatility, and any dividends on the stock over the life of the option. Of these six variables, the stock price, the exercise price, and the time to expiration are easily observable and can be measured without any appreciable error.

The risk-free rate is largely observable, and its impact is small<sup>16</sup>. The dividends are not observable, but they are not too difficult to measure accurately<sup>17</sup>. The volatility, is almost completely unobservable. Even so, all options on the same underlying stock should be priced using the same volatility. It is, after all, the volatility of the underlying stock with which one is concerned. The stock cannot have more than one volatility. Suppose the Black-Scholes model is employed to infer the volatility used by option traders to price an option. The volatility is identified that makes the model provide an option price that corresponds to the market price. The model can then be said to imply a volatility that gives rise to the concept of the implied volatility. But again, would each option on the same underlying stock not imply the same volatility? This is considering option quote prices are available and the only

---

<sup>16</sup>Each option on the same underlying stock would have a risk-free rate derived from a risk-free investment maturing at the time of the option expiration. Therefore, an option which expires in March and an option that expires in February could have different prices because of different risk-free rates. But it is well known that the risk-free rate has only a minor effect on the price of an option.

<sup>17</sup>Dividends affect option prices on the same underlying stock, but do so by effectively lowering the stock price by the present value of the dividends. Since none of the options differ by the dividends on the underlying stock, dividends cannot explain which option is more expensive.

unknown parameter is the volatility. If the Black-Scholes assumptions were true and reliable, one could find the volatility of the stock by simply inverting the Black-Scholes formula. A volatility thus obtained is called the implied volatility. Therefore the implied volatility for all underlying stocks and options in the dataset has been calculated. These results are shown graphically in figure 3.1 in appendix A.2.

The empirical findings show that the implied Black-Scholes volatilities vary systematically with strikes, showing the volatility smile. In the equity market the implied volatilities for options with the same maturity usually decrease as the strikes increase. In other words, the Black-Scholes model underprices deep out-of-the-money put options and overprices deep out-of-the-money call options. This volatility pattern is particularly noticeable since the 1987 market crash (Rubinstein, 1994).

When first observed, the implied volatilities were u-shaped, giving the appearance of a smile<sup>18</sup>. Hence this relation was named the volatility smile. In more recent years, the smile has mostly disappeared, and the relationship has sometimes been referred to as a skew or even a smirk, which might well describe some of our graphs. Also different implied volatilities are obtained depending on whether calls or puts are being examined. There is no reason why puts and calls should have different implied volatilities. However, it is clear that they do.

The existence of multiple implied volatilities, regardless of whether they arrange themselves in a smile-like pattern, should be somewhat disconcerting. How can the market tell one that there is more than one volatility for a stock. Clearly, there is something wrong with the Black-Scholes model. This is that it fails to consider all of the factors that enter into the pricing of an option. It accounts for the stock price, the exercise price, the time to expiration, the dividends, and the risk-free rate, but there must be some factors that it overlooks. The

---

<sup>18</sup>Breeden and Litzenberger (1978) were the first to show that the implied risk-neutral distribution function could be derived from option prices: the probabilities are equal to the second order derivatives of option prices with respect to the strike price. Shimko (1993) offers a practical application of this general idea. He proposes to model the volatility smile as a quadratic function of moneyness, and then to calculate the second order derivative numerically. Other methods construct implied binomial (Rubinstein, 1994) or trinomial trees (Nagot and Trommsdorff, 1999), or estimate the end-of-term distribution non-parametrically (Jackwerth and Rubinstein, 1996).

implied volatility is more or less a catch-all term, capturing whatever variables are missing, as well as the possibility that the model is improperly specified or blatantly wrong.

When the volatility smile was first observed<sup>19</sup>, some researchers believed that the explanation was the liquidity of underlying items. The true “smile appearance” meant that out-of-the-money options had the highest implied volatilities. These options were also the least liquid. Hence, it was argued that the prices observed for these options of low liquidity reflected the thinness of their markets. But this explanation would suggest that highly liquid options, typically those trading nearly at-the-money, would have the same implied volatilities. In fact, they do not and never did have, as is shown in Figure 3.1, the results of the analysis.

Moreover, when the smile turned into a skew, the moneyness (in the money) argument fell by the wayside.

Other researchers, like Hull and White (1987), Heston (1993), Scott (1987), Stein and Stein (1991) and Wiggins (1987), believed that the smile reflects stochastic volatility. Volatility is surely not a constant, therefore as is assumed in the Black-Scholes model. If volatility were stochastic, these researchers argue, the smile would reflect the failure of the Black-Scholes model to capture the random nature of volatility. Others argue that the Black-Scholes model, which assumes that stock prices fluctuate in a smooth and continuous manner, fails to capture the true nature of stock price movements, which are observed to have discrete jumps (Merton, 1979).

The arguments of stochastic volatility and jumps have a great deal of appeal, because they preserve many of the essential features of the Black-Scholes model. These arguments do not require the model to motivate the holding of options and the preference for some investors for certain options over others. They essentially argue that if the Black-Scholes model were re-derived under looser assumptions, the smile would go away. Unfortunately, once these looser assumptions are introduced, the mathematical tractability of the model is lost, and the process of pricing an option becomes one of making other fearless assumptions or imposing severe

---

<sup>19</sup>See Dupire (1994) and Derman and Kani (1994).

computational demands on the model<sup>20</sup>. It is fair to say that mathematicians have devoted excessive hours of human and mechanical time in researching the smile, with little if any explanation for why the smile still exists<sup>21</sup>.

In the conclusion of this section, the whole notion of implied volatility and the existence of the volatility smile are suggested to be the result of using a model that does not capture everything that affects the price of options. Practitioners and academics largely accept the limitations of the model and consider the smile a means of forcing the model to reveal information it is not designed to reveal. However flawed the model may be, the advantages of the Black-Scholes model, even with its attendant defects, may outweigh the disadvantages of other more complex models.

### 3.5.2: VaR Estimation and Backtesting Results

The Table below summarises all the VaRs' results as a percentage of the actual changes in the underlying stocks. First, we start with the Historical simulation to predict the underlying stocks prices' VaR at 99% and 95% levels of confidence. In the first step a database is created of the daily movements in the stock prices over 250 and 500 days. The first simulation trial assumes that the percentage change in the market variable (stock price) was the same as it was on the first day covered by the database. The second simulation trial assumes that the percentage change was the same as that on the second day; and so on, to build up a probability distribution of the percentage changes in the market variable (stock price). The VaR is calculated as the appropriate percentile of the probability distribution of the percentage change in the stock prices. The results of these VaRs' quantification, together with the actual price changes are shown in Figure 3.2 in appendix A.3. As can be seen in these Figures, which are summarised in Table 3.2 below, a 99% VaR that is obtained using

---

<sup>20</sup>For example, one assumption is that the risk arising from stochastic volatility is a non-priced risk. That is, the risk associated with uncertain volatility is a risk that does not concern investors. They are either neutral towards that risk or the risk is uncorrelated with their other holdings, meaning that the risk is diversifiable. Once these assumptions are invoked, the financial economics are lost and what remains is simply an exercise in computational finance.

<sup>21</sup>Perhaps if these reasons were found, the mathematicians would be out of work.

500 simulations, is greater than the actual price changes obtained when using 100% for the forecasted period (see Table 3.2).

Table 3.2 reporting a summary of our empirical results, each percentage shows the degree to which the VaR is greater than the actual price changes.

VaR methodology	Historical simulation				Monte Carlo Simulation			
	500		250		5,000		100,000	
Simulations	99%	95%	99%	95%	99%	95%	99%	95%
<b>African Bank</b>	100%	97%	100%	97%	93%	83%	93%	83%
<b>BHP Billiton</b>	100%	100%	100%	97%	100%	97%	100%	97%
<b>Firstrand</b>	100%	97%	100%	97%	97%	87%	97%	83%
<b>Gold Fields</b>	100%	100%	100%	100%	100%	97%	100%	97%
<b>Harmony Gold</b>	100%	100%	100%	100%	100%	97%	100%	97%
<b>Impala Platinum</b>	100%	97%	100%	100%	100%	97%	100%	97%
<b>Mittal Steel</b>	100%	93%	93%	90%	90%	83%	90%	87%
<b>MTN Group</b>	100%	97%	100%	97%	97%	93%	97%	93%
<b>Pick'n Pay</b>	100%	100%	100%	100%	97%	83%	93%	83%
<b>Sappi</b>	100%	93%	100%	93%	100%	90%	100%	90%
<b>Sasol</b>	100%	93%	100%	93%	93%	87%	97%	93%
<b>Standard Bank</b>	100%	97%	100%	100%	97%	93%	97%	90%
<b>Steinhoff</b>	100%	100%	100%	100%	97%	97%	97%	97%
<b>Telkom</b>	100%	97%	100%	97%	97%	93%	97%	93%
<b>Average</b>	<b>100%</b>	<b>97%</b>	<b>100%</b>	<b>97%</b>	<b>97%</b>	<b>91%</b>	<b>97%</b>	<b>91%</b>

As shown, a 1-day 99% VaR using 500 simulations is more efficient when historical simulation is used. Historical simulation puts the same weight on all the observations in the chosen window, including old data points, which may be an undesirable feature. The measure of risk may change abruptly once an old observation is dropped from the window. The choice of the window size is open to debate. In the case of a short window size, the VaR estimates will be very sensitive to accidental outcomes from the recent past. A long window size, on the other hand, has the disadvantage of including past data which might no longer be relevant to the current situation. Hence to be on the safe side, a short window size of 250 and a long window size of 500 should be used.

Secondly, the Monte Carlo Simulation method was used to predict the underlying stocks prices' VaR at 99% and 95% levels of confidence, and the windows sizes were chosen to be 5 000 and 10 000. In this section, the Geometric Brownian Motion (GBM) was used. This means that the stock price follows a random walk and is consistent with the weak form of the efficient market hypothesis (EMH): past price information is already incorporated and the next price movement is conditionally independent of past price movements<sup>22</sup>.

The formula for GBM is found below, where  $S$  is the current stock price,  $\Delta S$  is the change in the stock price,  $r$  is the continuously compounded risk-free rate,  $\sigma$  is the volatility of the stock<sup>23</sup> and  $\Delta t$  is the length of time over which the stock price change occurs. The variable  $\varepsilon$  is a random number generated from a standard normal probability distribution. Recall that the standard normal random variable has a mean of zero, a standard deviation of 1.0 and occurs with a frequency corresponding to that associated with the famous bell-shaped curve.

$$\Delta S = Sr\Delta t + S\sigma\varepsilon\sqrt{\Delta t} \quad (3.3)$$

The first term is a drift and the second term is a shock. For each time period, our model assumed the price would drift up by the expected return. But the drift will be shocked (added or subtracted) by a random shock. Random simulations were then run. To illustrate this, Microsoft Excel has been used to run the simulations<sup>24</sup>.

A standard normal random variable is generated and then, inserted into the right-hand side of the above formula for  $\Delta S$ . This procedure is repeated 5 000 and 10 000 times to build up a probability distribution for the stock price changes. Then VaR is calculated as the appropriate percentile of the probability distribution of the stock price change. The 1-day 99% VaR is the

---

<sup>22</sup>For more details about efficient markets and the Markov, refer to: Brealey (1983) and Cootner (1964).

<sup>23</sup>The volatility of a stock was estimated historically using closing prices from daily data over the most recent 90 days (Hull, 2000).

<sup>24</sup>Excel's Rand() function. The Rand() function produces a uniform random number between 0 and 1, meaning that it generates numbers between 0 and 1 with equal probability. A good approximation for a standard normal variable is obtained by the Excel formula "= Rand() + Rand() + Rand() + Rand() + Rand() + Rand() + Rand() + Rand() + Rand() + Rand() + Rand() + Rand() - 6.0", or simply 12 uniform random numbers minus 6.0.

value of the stock price change for the 50<sup>th</sup> worst outcome when one uses 5000 different sample values of the stock price change. The results of this VaR' quantification together with the actual price changes are shown in Figure 3.3 in appendix A.3. As can be seen in the figures that are summarised in Table 3.2 above, the 1-day 99% VaRs that are obtained using 5 000 and 10 000 simulations are greater than the actual price changes for 97% of the forecasted period (see Table 3.2). Thus, the 1-day 99% VaR using 5 000 and 10 000 simulations is more efficient when Monte Carlo Simulation is used.

Backtesting, performed on historical simulation VaR and the Monte Carlo simulation VaR, involves testing how well the VaR estimates would have performed in the past. In the case of a 1-day 99% VaR, backtesting would involve looking at how often the loss in a day exceeded the 1-day 99% VaR. If this happened on about one percent of the days, one can feel reasonably comfortable with this methodology for calculating VaR. "If it happened on, say 10% of days, then the methodology would definitely be suspect" (Hull, 2000:357). According to the analysis carried out, all VaRs that are obtained via historical simulation were accepted by backtesting. However 16% of the VaRs obtained via the Monte Carlo method were rejected. On the other hand, conditional VaRs ("C-VaRs") significantly reduced the backtesting rejection from the Monte Carlo simulation to 50%. Thus, it is accurate to say that conditional VaR measures losses in "extreme" market conditions, while VaR measures losses in "normal" market conditions.

### 3.6: Conclusions

In this chapter, underlying stocks listed on the JSE were used for the VaR calculations, as shown in Table 3.2. The VaR is calculated in this work using the historical simulation based on 250 and 500 simulations and the Monte Carlo method based on 5 000 and 10 000 simulations. Results show that if the level of confidence is 99%, then the VaR calculated using the historical methodology is greater than the actual price changes in 100% of the forecasted cases in the time period. It was also concluded that although the estimated VaR via

the Monte Carlo simulation approach is greater than the actual price changes in 97% of the cases in the forecasted period, it is not as efficient as that calculated using the historical methodology. However, high option prices imply higher stock volatility, which means a wide possible range of movement in the underlying stock prices. Our results demonstrated a good measure of actual volatility of the underlying stock via 99% historical simulation VaR. Consequently, VaR is assumed to be a good indicator for option prices.

Finally, the conclusion is that Value-at-Risk Volatility, calculated by any method, is a reliable measure of option pricing for whoever is concerned with the actual volatility of the underlying stock, firm managers or individual traders.

University of Cape Town

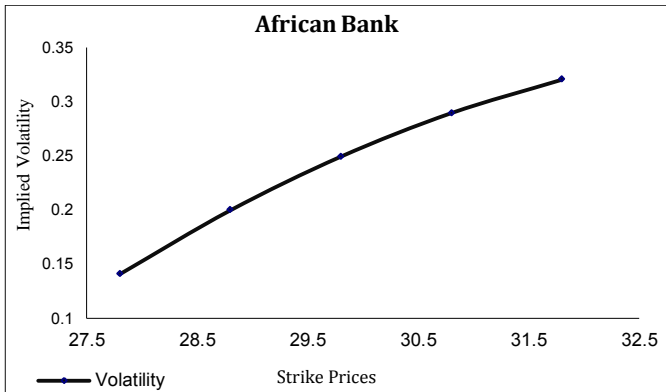
## 3.7: Appendix A

### 3.7.1: A.1 Data Outline

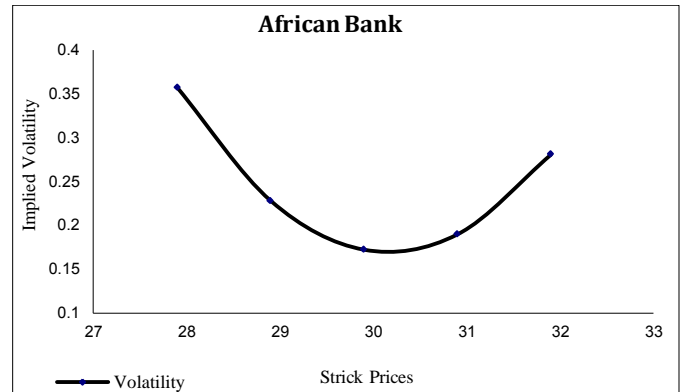
Table 3.1 Data Outline: fourteen stocks listed on JSE Limited and fifty-nine options

1: 14 companies stock listed on JSE							
<b>Underlying Stock</b>	African Bank	BHPBILL	FIRSTRAND	GOLD FIELDS	HARMONY GOLD	IMPALA PLATINUM	MITTAL
<b>JSE Code</b>	ABL	BIL	FSR	GFI	HAR	IMP	MLA
<b>Frequency</b>	200 Days	200 D	200 D	200 D	200 D	200 D	200 D
<b>Start</b>	27/04/2006	25/05/2006	27/03/2006	24/02/2006	12/01/2006	18/05/2006	03/04/2002
<b>End</b>	31/01/2007	28/02/2007	29/12/2006	30/11/2006	18/10/2006	21/02/2007	03/04/2007
<b>Jarque-Bera</b>	2.1	2.9	3.3	4.7	1.8	1.2	4.9
<b>ex-dividend</b>	NO	No	discounted	discounted	discounted	discounted	discounted
<b>Underlying Stock</b>	MTN GROUP	PICKN PAY S	SAPPI	SASOL	Standard Bank	STEINHOFF	TELKOM
<b>JSE Code</b>	MTN	PIK	SAP	SOL	SBK	SHF	TKG
<b>Frequency</b>	200 D	200 D	200 D	200 D	200 D	200 D	200 D
<b>Start</b>	09/02/2006	26/04/2006	02/02/2006	27/04/2006	27/03/2006	16/03/2006	17/04/2006
<b>End</b>	03/04/2007	30/01/2007	8/11/2008	31/01/2007	29/12/2006	20/12/2006	19/01/2007
<b>Jarque-Bera</b>	3.3	2.3	1.6	1.4	3.3	5.4	4.7
<b>ex-dividend</b>	discounted	No	discounted	discounted	discounted	No	No
2: 59 Call and Put options with different style, exercise price and time of expiration, all traded on JSE							
<b>Call/Put warrant</b>	ABLABE	ABLDBA	ABLIBE	ABLNBB	ABLSBK	TKGIBI	TKGNBL
<b>Underlying Stock</b>	ABL	ABL	ABL	ABL	ABL	TKG	TKG
<b>Issuer</b>	ABSA Bank	Deutsche Bank	Investec	NedBank	Standard Bank	Investec	NedBank
<b>Exercise Price</b>	31.00	26.50	33.00	31.00	32.00	160.00	160.00
<b>Expiry Date</b>	6/6/2007	7/6/2007	2/7/2007	23/5/2007	3/7/2007	2/7/2007	24/5/2007
<b>Style</b>	American/call	American/call	American/call	American/call	American/call	American/call	American/call
<b>Conv. Ratio</b>	5	5	8	8	8	35	25
<b>PIKDBA</b>	BILABI	BILDDB	BILIBJ	BILNBL	BILSBB	BILSBC	FSRABG
<b>PIK</b>	BIL	BIL	BIL	BIL	BIL	BIL	FSR
<b>Deutsche Bank</b>	ABSA Bank	Deutsche Bank	Investec	NedBank	Standard Bank	Standard Bank	ABSA Bank
<b>38.00</b>	155.00	145.00	140.00	155.00	140.00	165	21.50
<b>16/8/2007</b>	6/8/2007	26/7/2007	2/7/2007	31/7/2007	3/7/2007	2/8/2007	6/6/2007
<b>American/call</b>	American/call	American/call	American/call	American/call	American/call	American/call	American/call
<b>8</b>	25	20	35	32	35	40	4
<b>FSRSBB</b>	GFIABI	GFIDBM	GFISBN	HARDBI	IMPABI	IMPDBI	SBKSBH
<b>FSR</b>	GFI	GFI	GFI	HAR	IMP	IMP	SBK
<b>Standard Bank</b>	ABSA Bank	Deutsche Bank	Standard Bank	Deutsche Bank	ABSA Bank	Deutsche Bank	Standard Bank
<b>24.00</b>	140.00	140.00	140.00	105.00	210.00	168.75	100.00
<b>7/3/2007</b>	6/6/2007	6/7/2007	5/8/2007	6/7/2007	6/6/2007	6/7/2007	7/03/2007
<b>American/call</b>	American/call	American/call	American/call	American/call	American/call	American/call	American/call
<b>6</b>	45	30	50	40	50	62	30
<b>IMPIBI</b>	MLAABE	SBKDBC	MLADBC	TKGSBC	BILSBC	IMPABJ	MLASBI
<b>IMP</b>	MLA	SBK	MLA	TKG	BIL	IMP	MLA
<b>Investec</b>	ABSA Bank	Deutsche Bank	Deutsche Bank	Standard Bank	Standard Bank	ABSA Bank	Standard Bank
<b>220.00</b>	130.00	90	112	150.00	165	245.00	105
<b>7/2/2007</b>	8/6/2007	6/7/2007	8/16/2007	3/7/2007	8/2/2007	8/6/2007	7/3/2007
<b>American/call</b>	American/call	American/call	American/call	American/call	American/call	American/call	American/call
<b>65</b>	15	14	25	35	40	50	25
<b>IMPIBI</b>	MLAABE	SBKDBC	MLADBC	TKGSBC	BILSBC	IMPABJ	MLASBI
<b>MTNABH</b>	SOLABI	SOLABJ	SOLDBO	SOLIBJ	SOLIBK	SOLNBM	SBKABH
<b>MTN</b>	SOL	SOL	SOL	SOL	SOL	SOL	SBK
<b>ABSA Bank</b>	ABSA Bank	ABSA Bank	Deutsche Bank	Investec	Investec	NedBank	ABSA Bank
<b>80.00</b>	285.00	270.00	270.00	270.00	250.00	275.00	98
<b>6/6/2007</b>	6/6/2007	8/6/2007	6/7/2007	4/5/2007	7/2/2007	7/31/2007	6/6/2007
<b>American/call</b>	American/call	American/call	American/call	American/call	American/call	American/call	American/call
<b>14</b>	59	50	50	60	55	60	14
<b>SHFDBB</b>	GFIDBW	SBKDBU	TKGSBX	SAPIBS	ABLDBP	BILNBT	BILSBP
<b>SHF</b>	GFI	SBK	TKG	SAD	ABL	BIL	BIL
<b>Deutsche Bank</b>	Deutsche Bank	Deutsche Bank	Standard Bank	Investec	Deutsche Bank	NedBank	Standard Bank
<b>24</b>	115.00	78	130.00	95.00	23.50	125.00	140.00
<b>2007/ 6/7</b>	6/7/2007	6/7/2007	8/5/2007	9/4/2007	6/7/2007	4/11/2007	8/2/2007
<b>American/call</b>	European Put	European Put	European Put	European Put	European Put	European Put	European Put
<b>10</b>	30	14	25	20	20	30	27
<b>FSRSBY</b>	IMPDBT	IMPIBT	IMPNBW	IMPSTV	MLADBQ	MLADBR	MLAIBG
<b>FSR</b>	IMP	IMP	IMP	IMP	MLA	MLA	MLA
<b>Standard Bank</b>	Deutsche Bank	Investec	NedBank	Standard Bank	Deutsche Bank	Deutsche Bank	Investec
<b>18.00</b>	182.00	170.00	185.00	170.00	83	98	110
<b>5/8/2007</b>	7/26/2007	7/2/2007	7/31/2007	7/3/2007	7/6/2007	16/8/2007	7/2/2007
<b>European Put</b>	European Put	European Put	European Put	European Put	European Put	European Put	American/call
<b>60</b>	50	50	50	14	25	60	25
<b>SBKABH</b>		SBKDBL		ABLDBP		IMPSTV	
<b>SBK</b>	2007/06/06	SBK	2007/06/07	ABL	2007/06/07	IMP	2007/07/03
<b>ABSA Bank</b>	American Call	Deutsche Bank	American Call	Deutsche Bank	European Put	Standard Bank	American Call
<b>98</b>	14	90	14	23.50	5	200.00	65

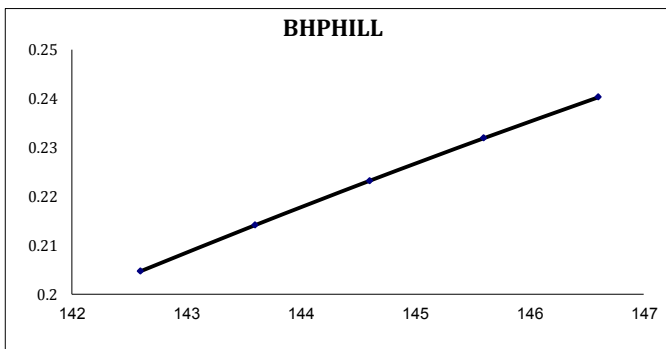
### 3.7.2: A.2 Computing Implied Volatilities



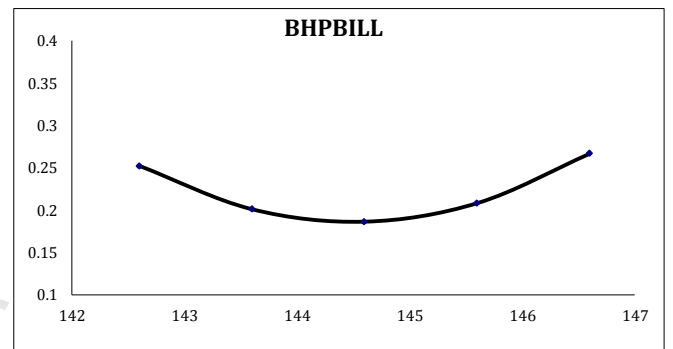
A single option (ABLABE) written on African Bank's stock



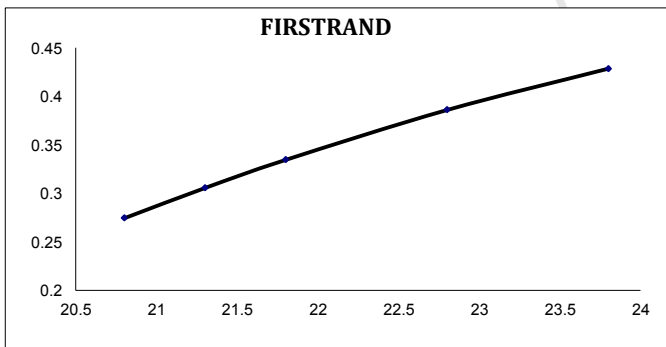
A group of options (ABLABE, ABLDBA, ABLIBE, ABLNBB, ABLSBK and ABLDBP) written on African Bank's stock



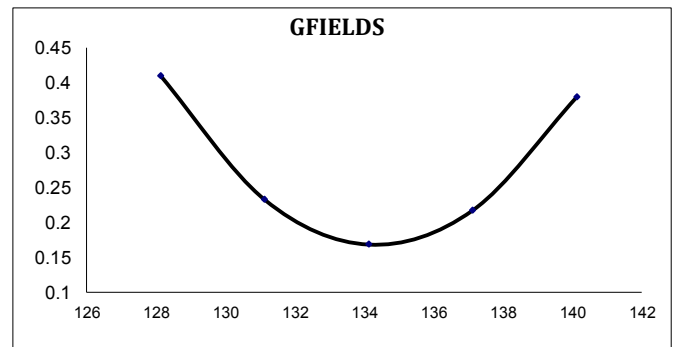
A single option (BILABI) written on BHP BILLION's stock



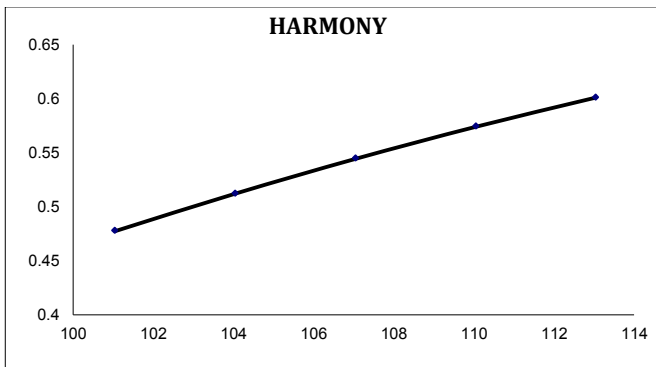
A group of options (BILABI, BILDDBJ, BILIBJ, BILNBL, BILSBB and BILSBC) written on BHP BILLION's stock



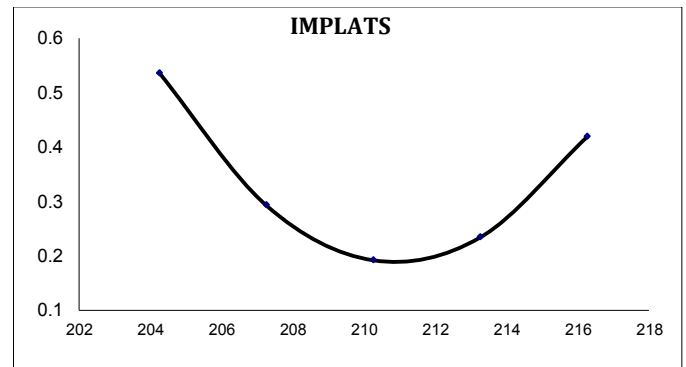
A single option (FSRABG) written on FIRSTRAND's stock.



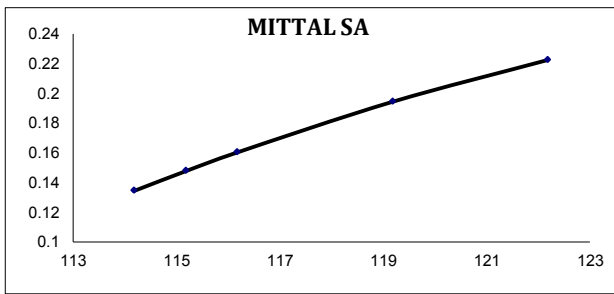
A group of options (GFIABI, GFIDBM and GFISBN) written on Gold Fields stock



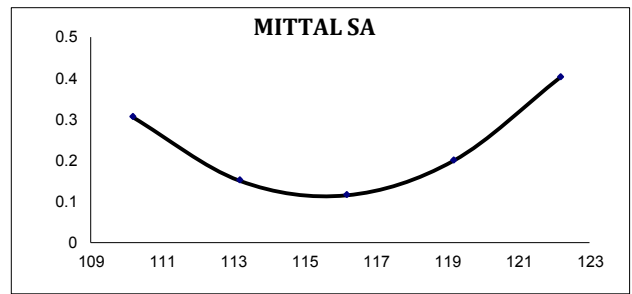
A single option (HARDBI) written on Harmony Gold's stock.



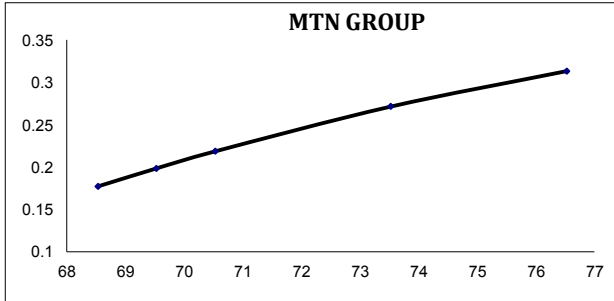
A group of options (IMPNBM, IMPABI, IMPABJ, IMPDBI, IMPIBI, IMPSBD, IMPDBT, IMPIBT, IMPNBW and IMPSBV) written on Impala Platinum's stock



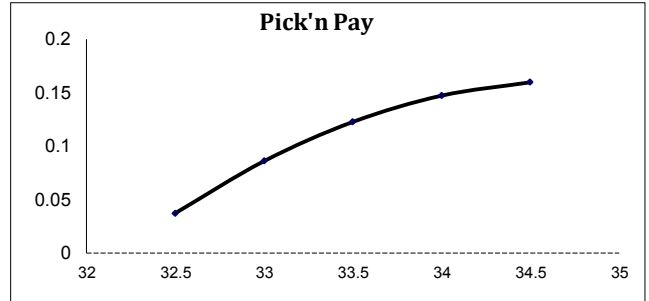
A single option (MLAABE) written on Mittal Steel's stock



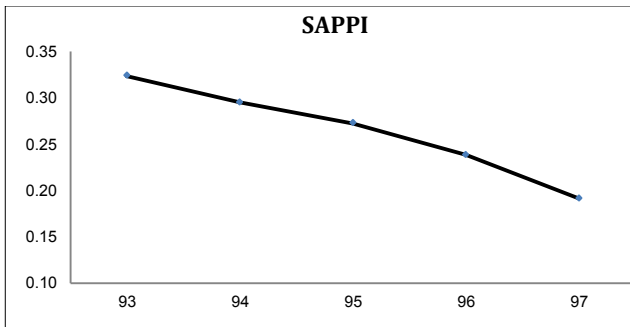
A group of options (MLAABE, MLADBC, MLAIBG, MLASBI, MLADBR and MLADBQ) written on Mittal Steel's stock



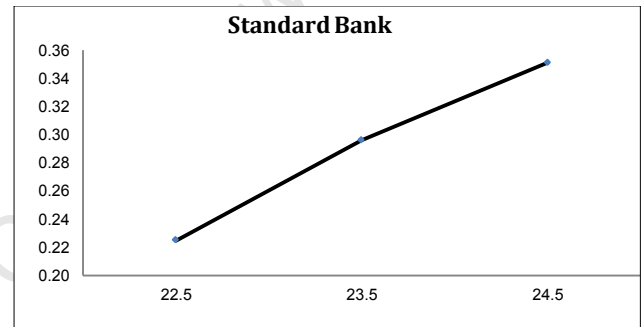
A single option (MTNABH) written on MTN's stock.



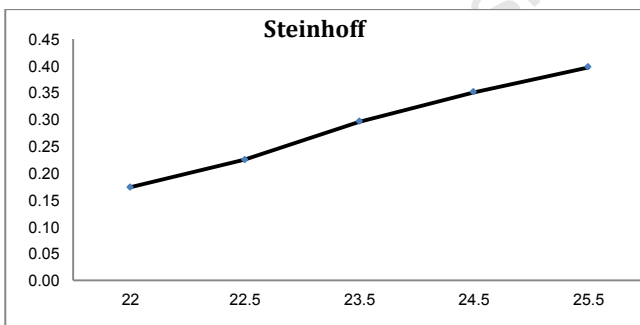
A single option (PIKDBA) written on Pick'n Pay's stock.



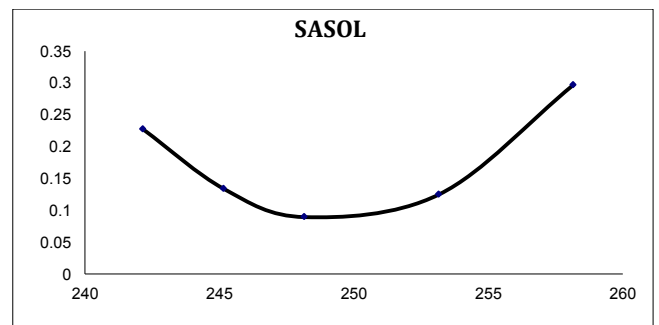
A single Put option (SAPIBS) written on SAPPI's stock.



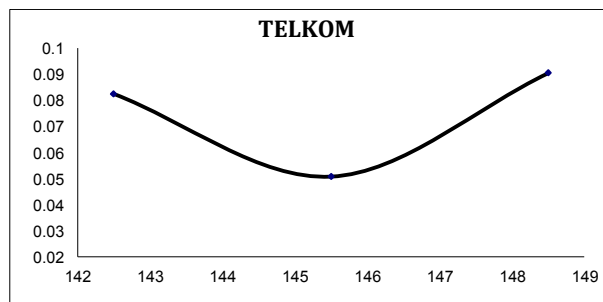
A group of options (SBKSBH, SBKABH and SBKDBL) written on Standard Bank's stock



A single Put option (SHFDDB) written on SAPPI's stock.



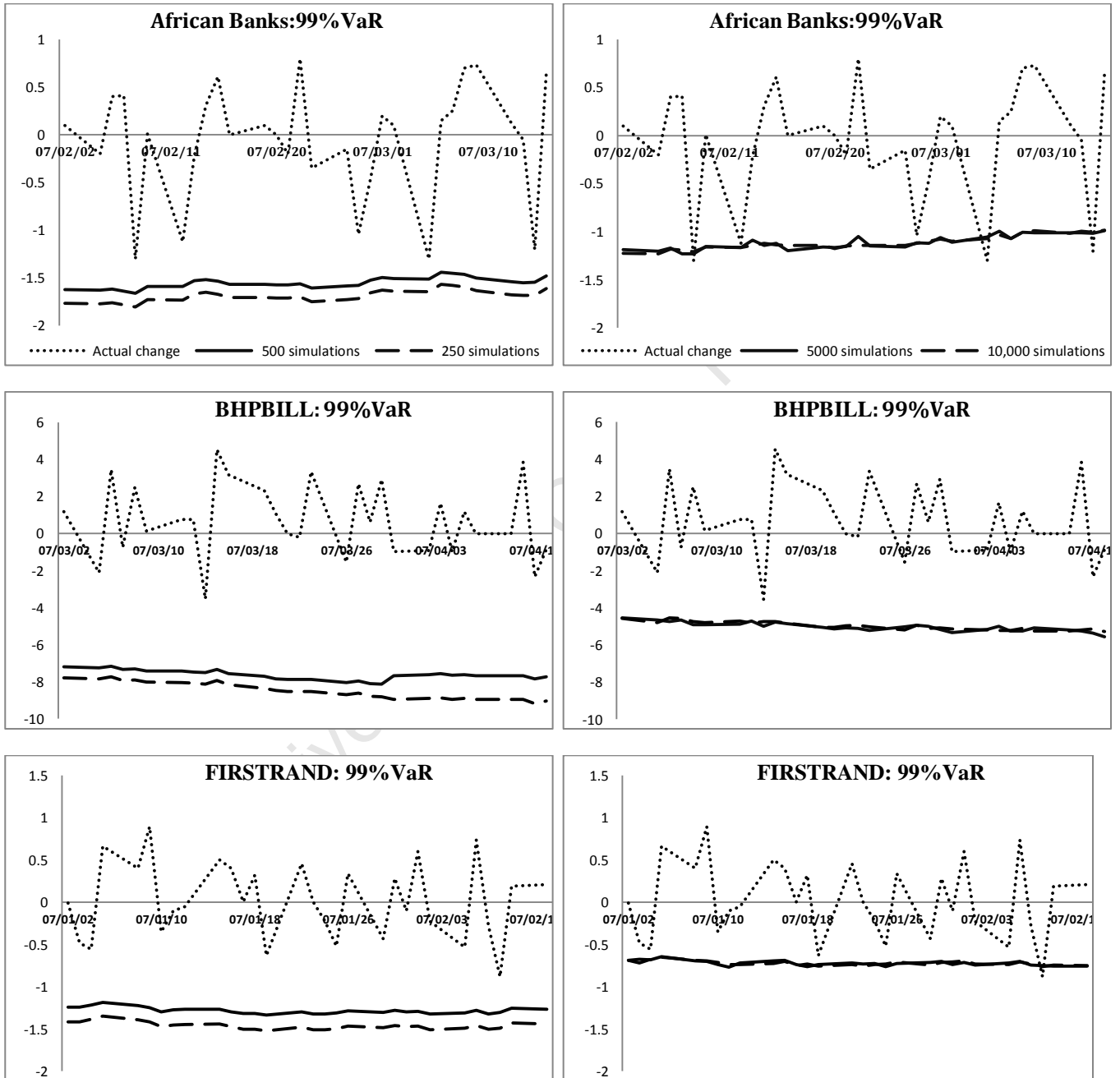
A group of options (SOLABI, SOLABJ, SOLDBO, SOLIBJ, SOLIBK and SOLNBM) written on SASOL's stock

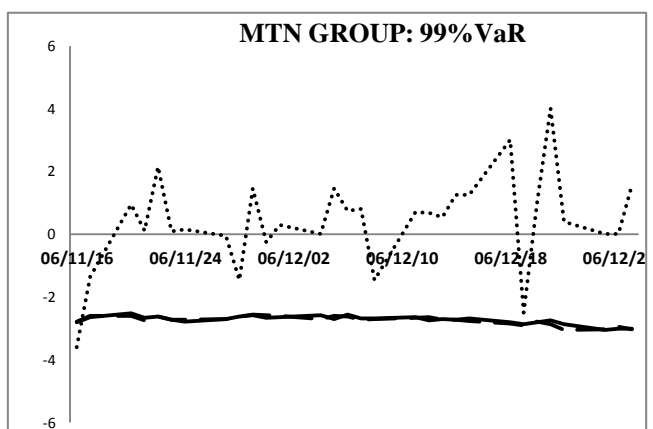
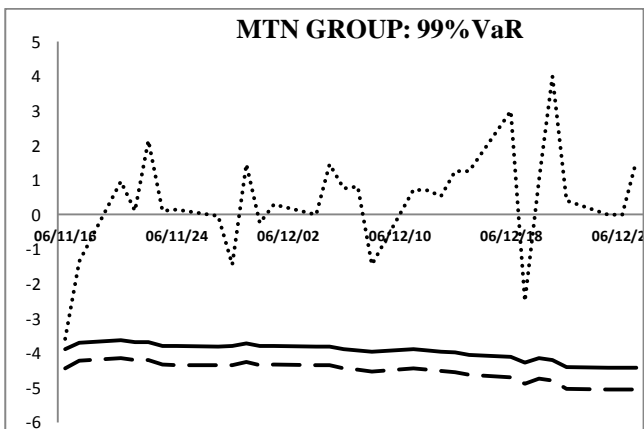
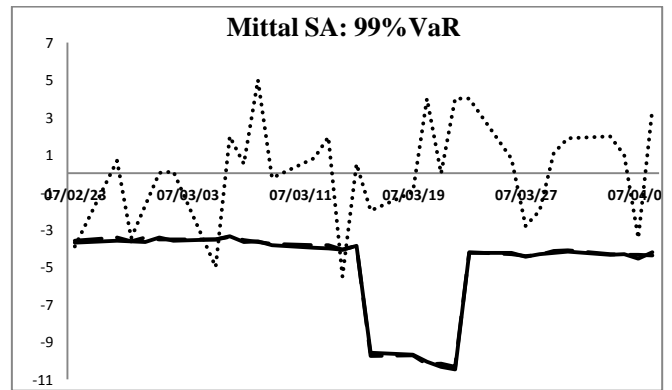
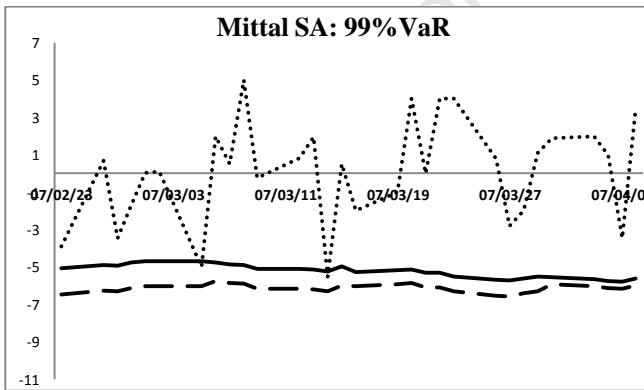
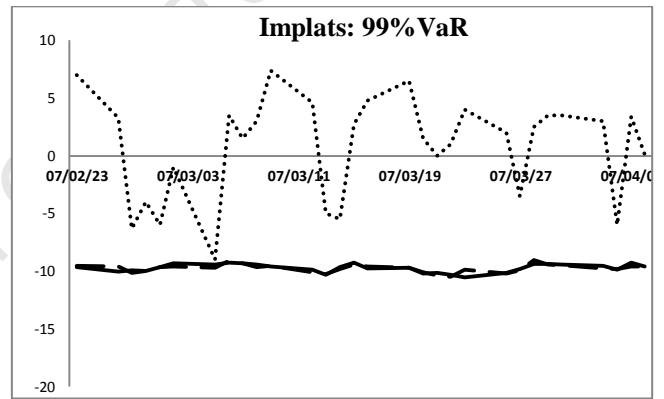
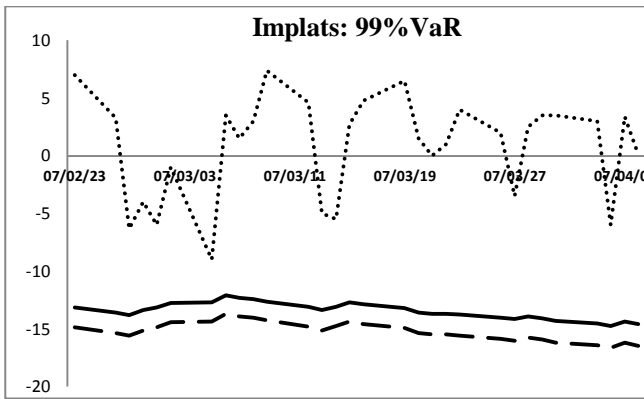
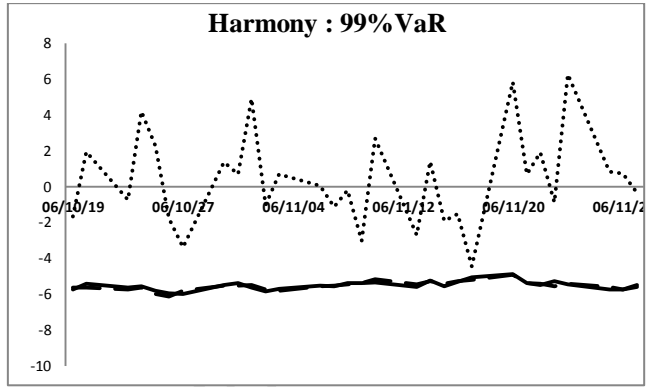
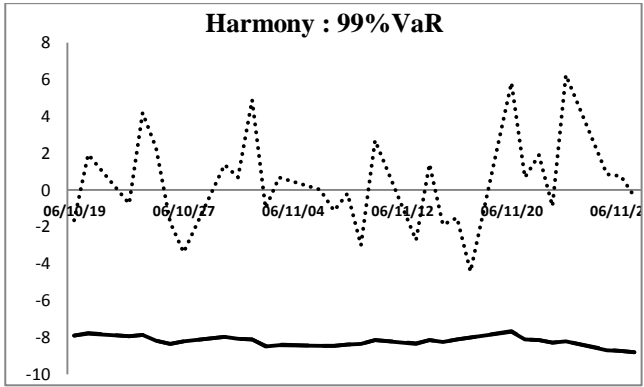
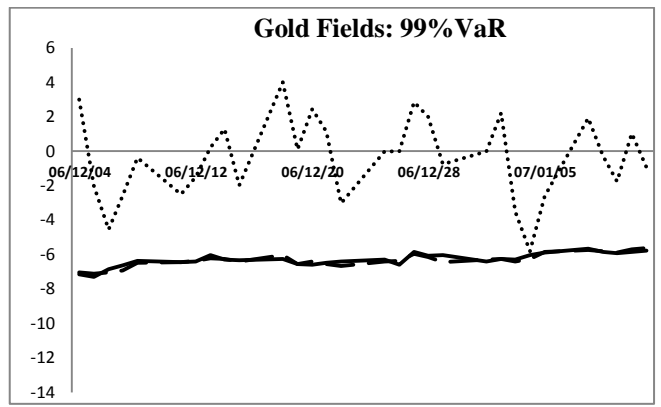
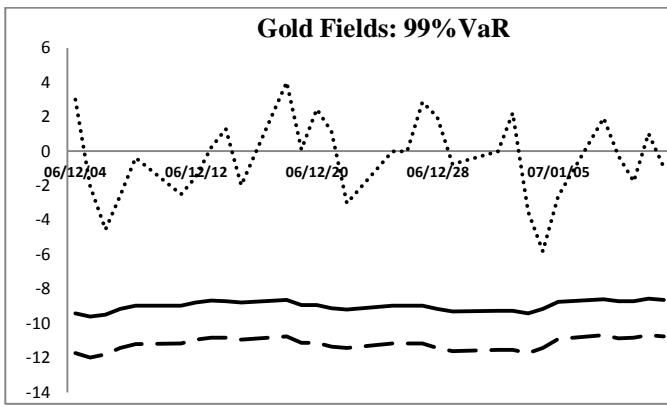


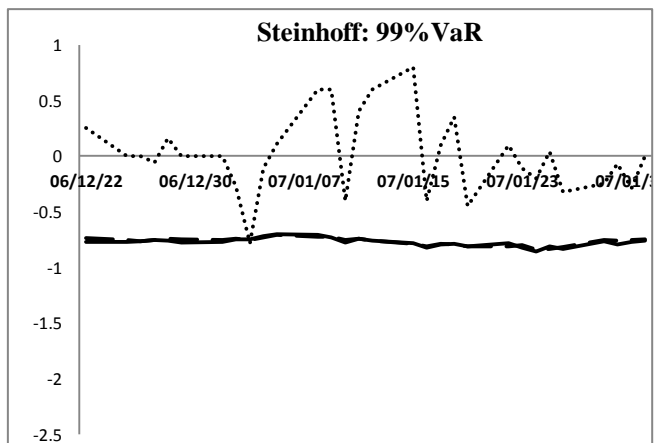
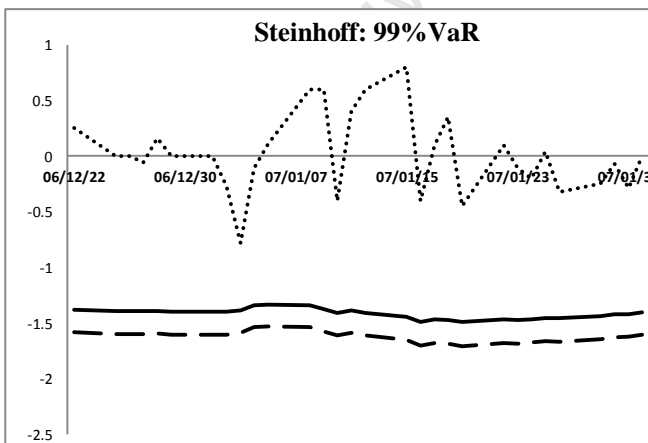
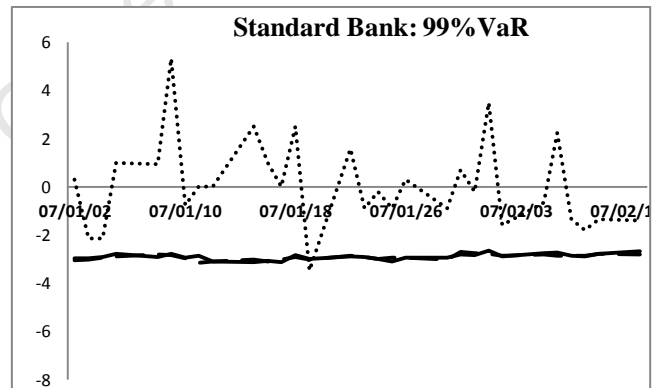
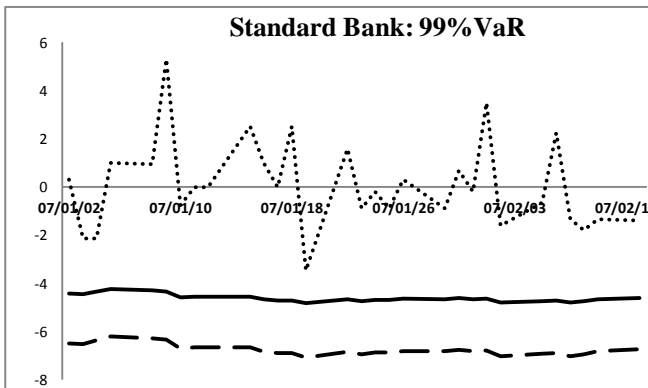
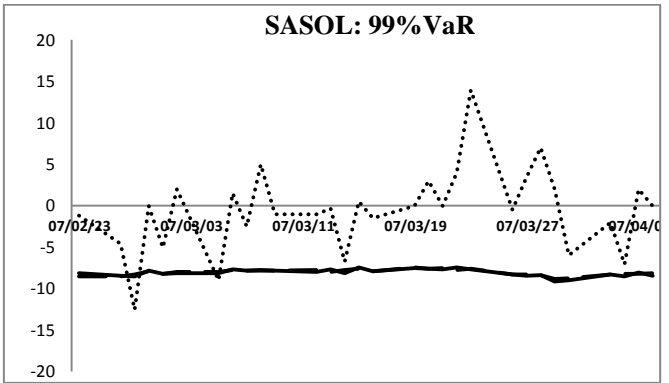
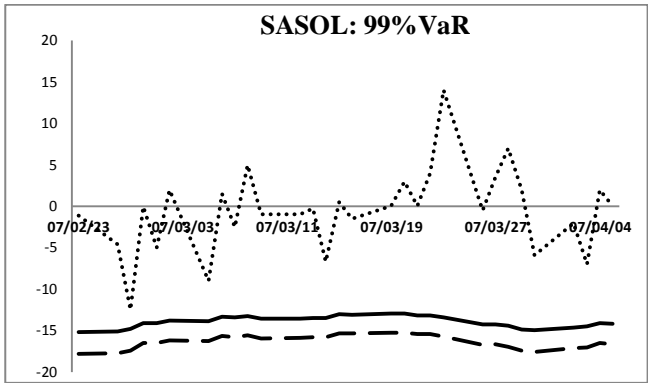
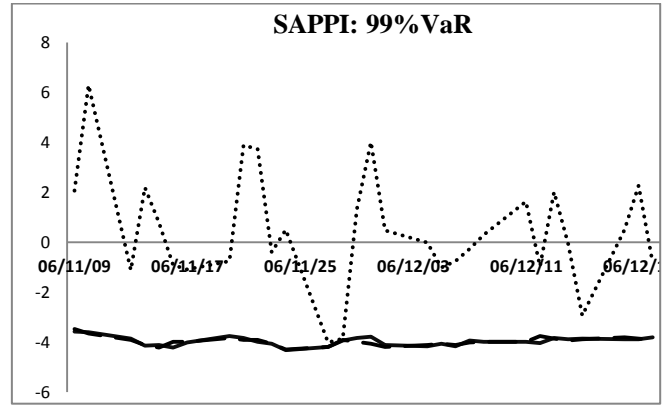
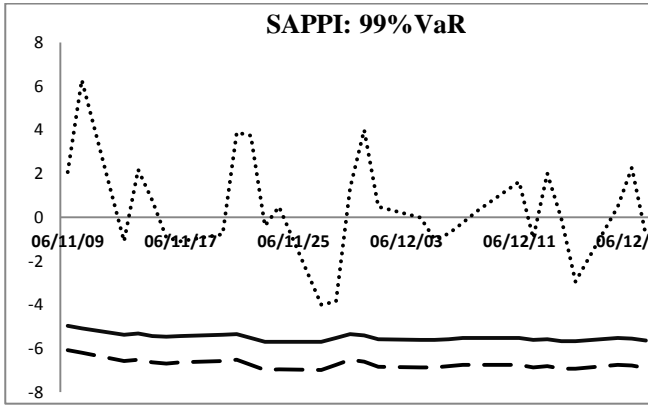
A group of options (TKGIBI, TKG NBL and TKGSBX) written on Telkom's stock.

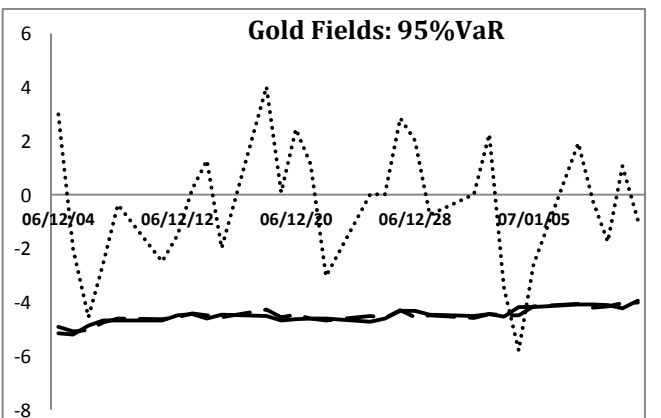
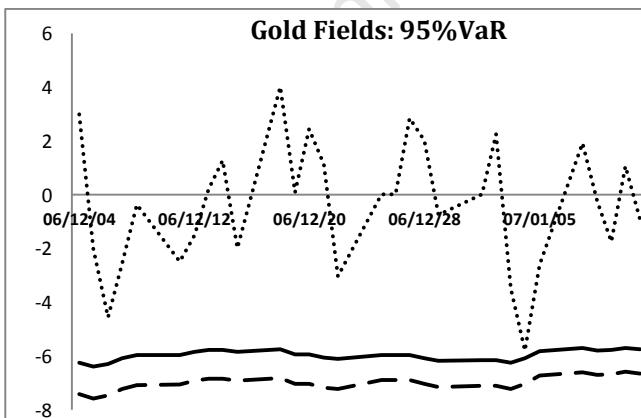
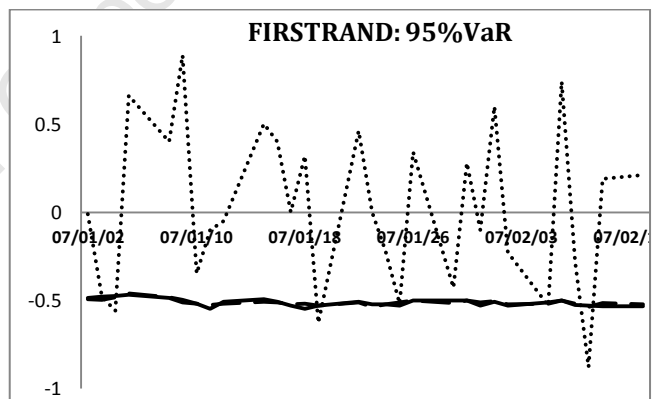
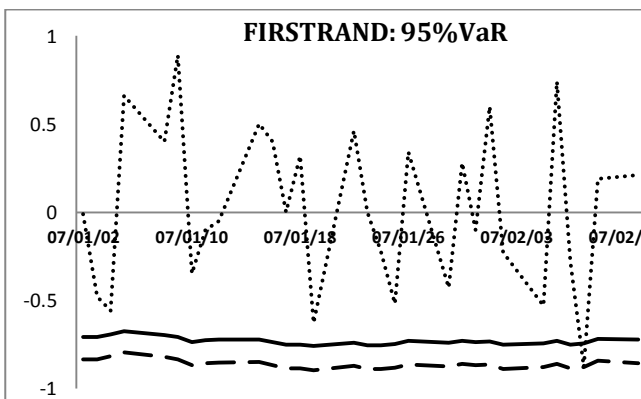
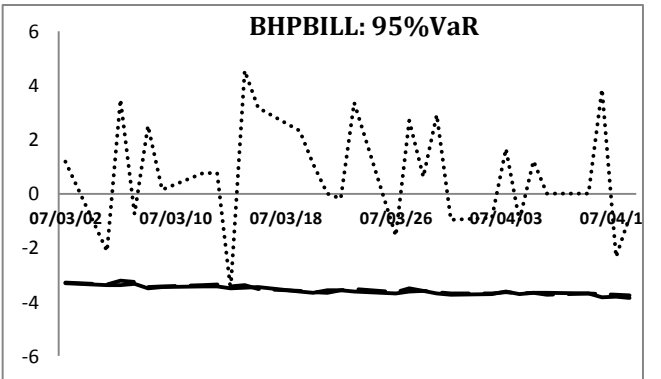
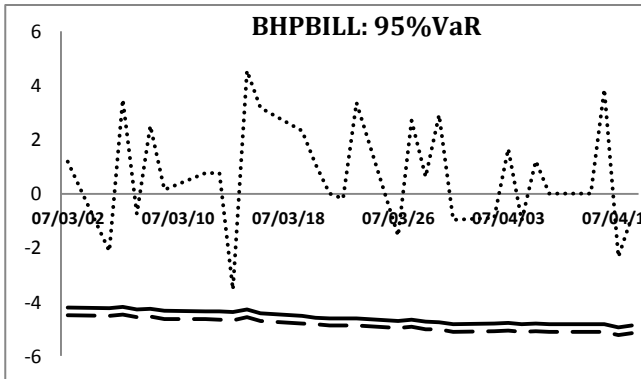
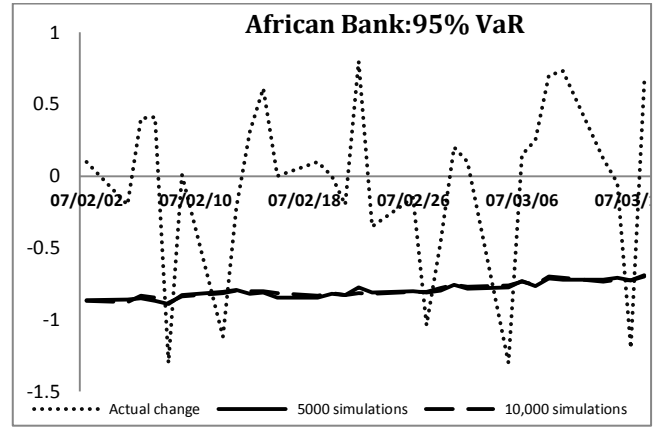
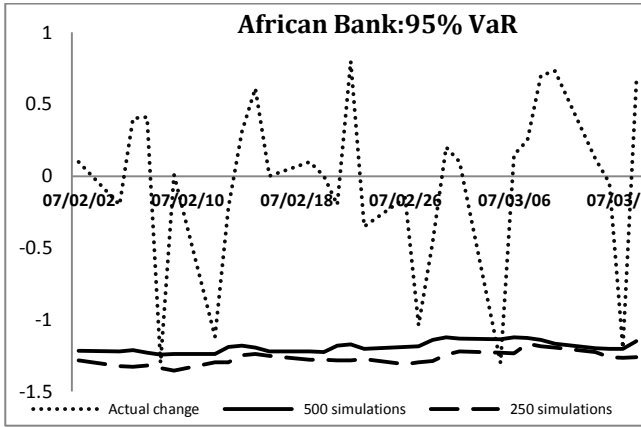
Figure 3.1 reports the slopes and the curves resulted from plotting different strike prices and implied volatilities. Implied volatilities were calculated from reversing the Black-Scholes (1973) option pricing model.

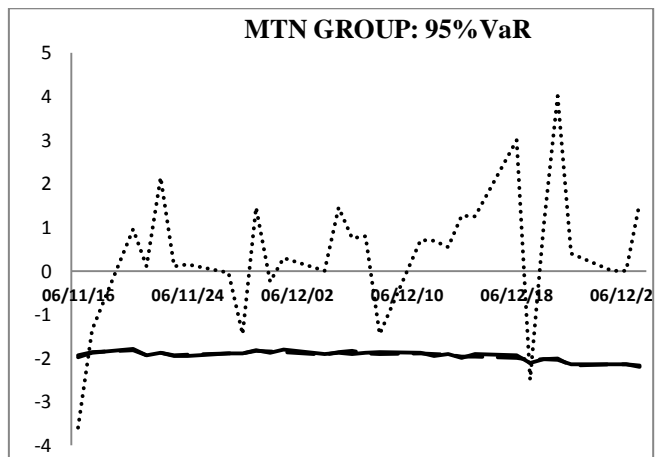
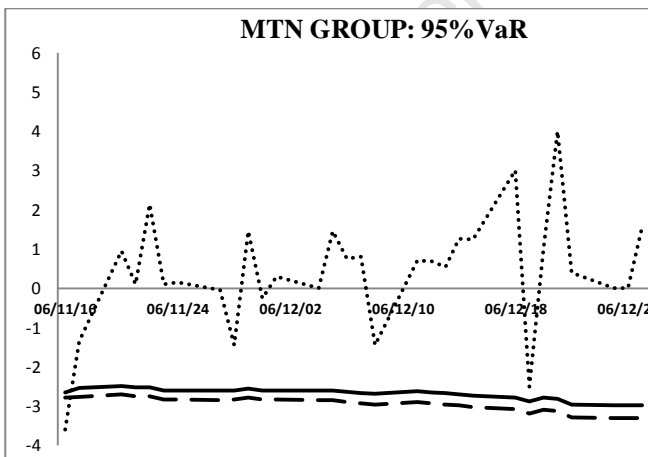
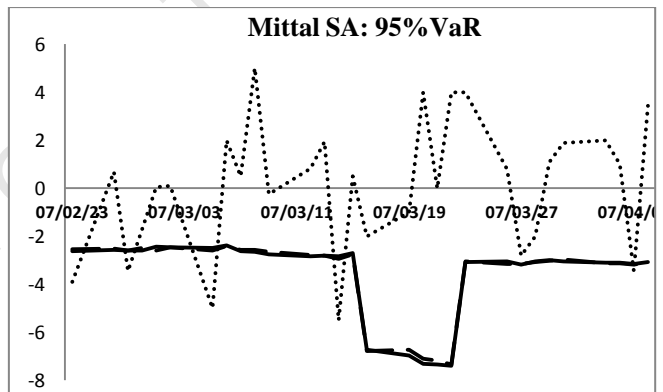
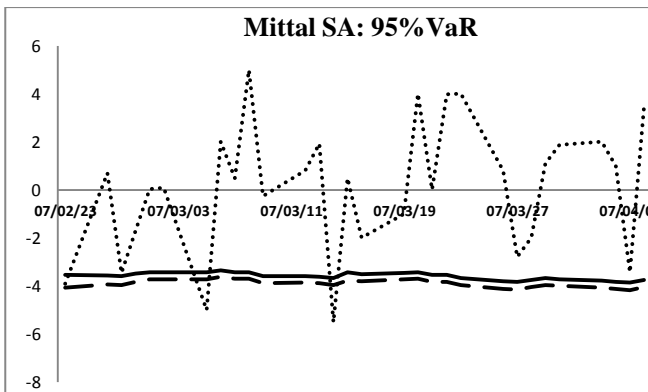
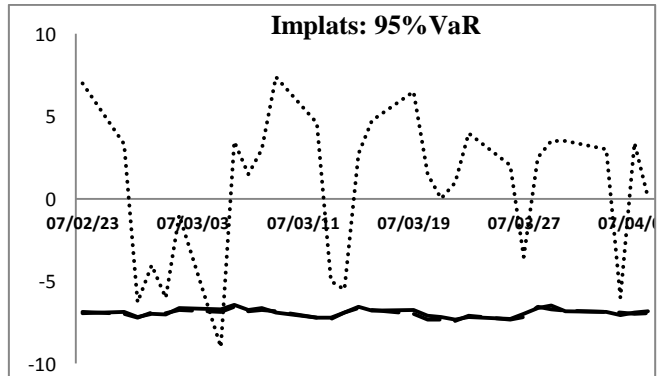
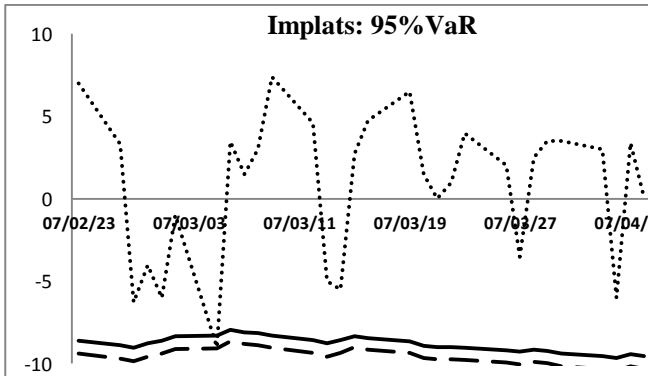
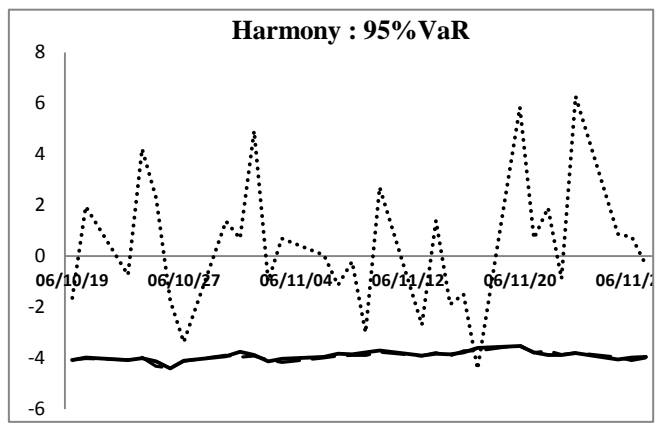
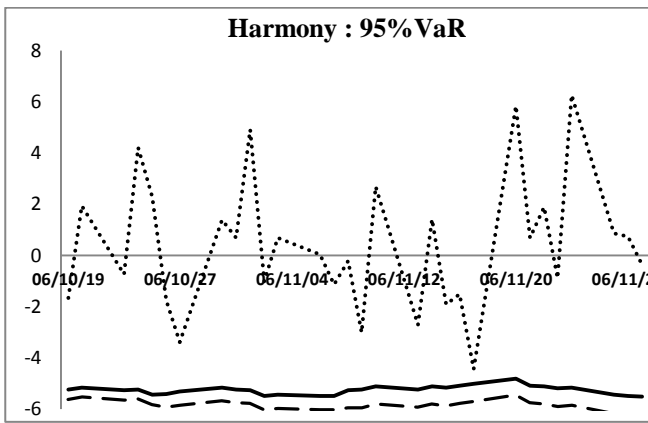
### 3.7.3: A.3 Comparison of Value-at-Risk Methodologies

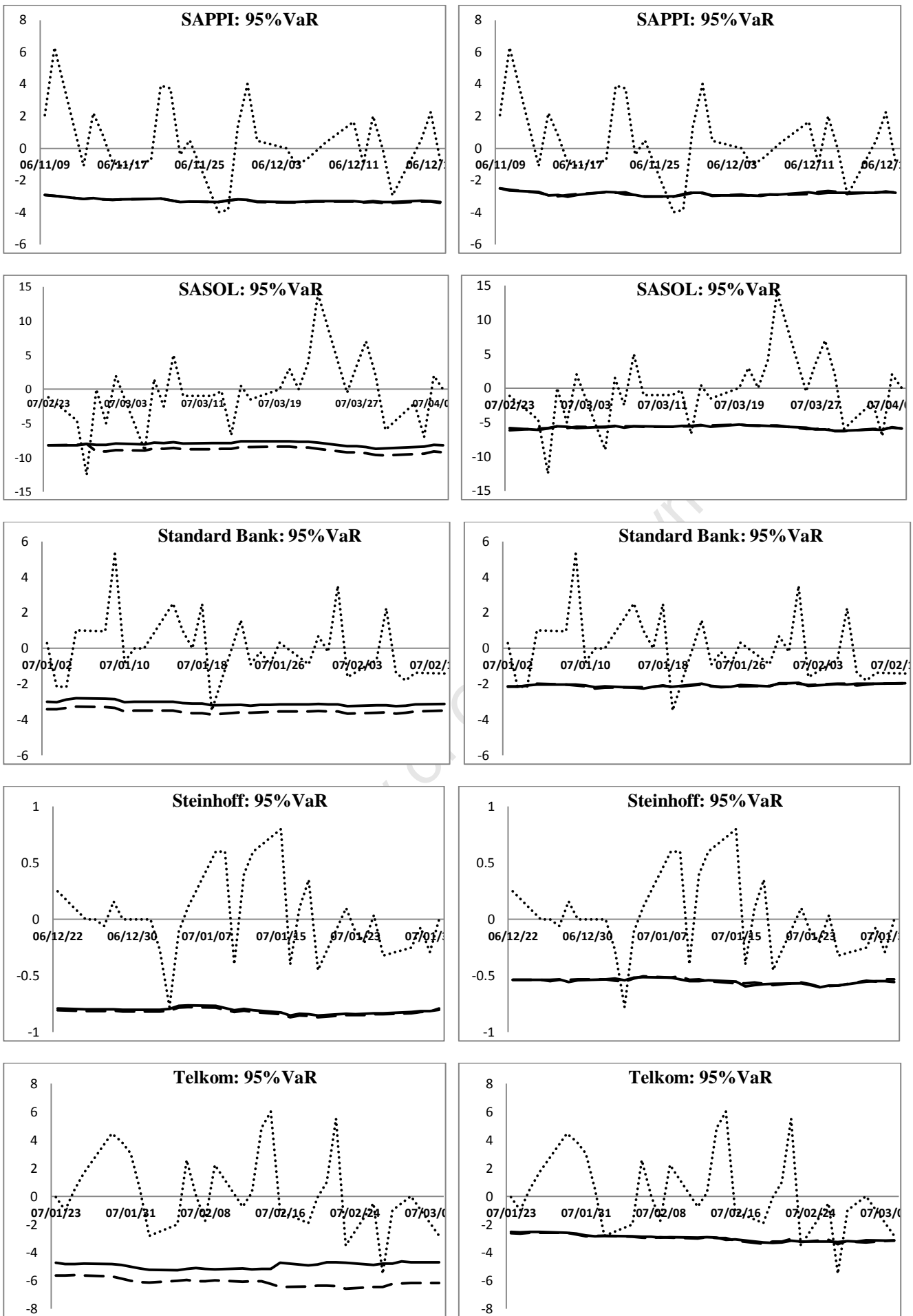












**Figure 3.2** The actual stock price changes and 1 day 95% and 99% VaRs obtained through Historical Simulation method using 500 and 250 simulations.

**Figure 3.3** The actual stock price changes and 1 day 95% and 99% VaRs obtained through Monte Carlo Simulation method using 5,000 and 10,000 simulations.

# Chapter 4

## Market segmentation, relative supply of shares and share price premium: Evidence from the Chinese market

### 4.1: Introduction

Some equity markets, including both developed and emerging ones, allow listed companies to issue different types of stocks. It is common for these stocks, which are issued by the same company, to share the same firm-specific risk, and in most cases also to enjoy the same dividend and voting policies. The only difference between these shares is their restrictions with regard to investors, that is, who can own the stocks.

One typical strategy is to segment investors by their citizenship; that is, a company can issue two types of stocks, one which is available to domestic investors and the other which is otherwise identical, but is only available to foreign investors. Such a segmented issuance strategy has been well researched; examples include the papers by Stulz and Wasserfallen (1995) on Switzerland, Beiley and Jagtiani (1994) on Thailand, Domowitz et al. (1997) on Mexico, Hietala (1989) on Finland, and Bergstorm et al. (1993) on Sweden.

The common phenomenon among these countries is the low price of the shares that are restricted to domestic investors relative to that of the corresponding shares accessible to foreign investors. In China, however, the price differential is the other way around: the shares restricted to domestic investors have a higher price level than the corresponding shares accessible to foreign investors.

In 1990 the Chinese government established two stock exchanges: the Shanghai Stock Exchange (SSE) in Shanghai, and the Shenzhen Stock Exchange (SZSE) in Shenzhen. These two exchanges have expanded rapidly over the past decade and are now both open to foreign investors, with the classes of shares owned by domestic and foreign investors having been separated.

Domestic shares are known as *A* shares, whilst foreign shares are referred to as *B* shares. A third category, *H* shares, comprises foreign shares which can be traded in both the Hong Kong and Shanghai exchanges. However, these shares account for only a very small proportion of the total market.

*A* shares are traded in the Chinese Renminbi<sup>25</sup> currency, whilst *B* shares are traded in foreign currencies (the US dollar in the Shanghai exchange and the Hong Kong dollar in the Shenzhen exchange). At the time of the inception of the market, foreigners could not legally buy *A* shares, and domestic residents could not legally buy either *B* or *H* shares. Apart from the quoted currency and the citizenship of the holders, *A* and *B* shares are otherwise identical in all related rights; however *B* share prices are much lower than those of *A* shares.

In June 2001 the Chinese government opened up the *B* share market to domestic residents and the *A* share market was opened to overseas investors in December 2002. The only difference now between *A* and *B* shares is the quoted and trading currency; nevertheless, *B* share prices remain lower than those of *A* shares. This obviously violates the law of one price.

The significant price difference between domestic *A* shares and foreign *B* shares of the same company is an important issue in the Chinese equity market that calls for a rational explanation. Financial literature has suggested an array of possible explanations, including differences in required returns, liquidity disparity, and relative information available. But do foreign and domestic investors price assets in these markets the same way? In such an

---

<sup>25</sup> Chinese money is called Renminbi (RMB) which means “The People’s Currency”. Yuan is the base unit for RMB - like the Dollar in USD. The RMB and Yuan are the same currency.

environment with more instability and unique institutional constraints, can we still expect the same risk and return relationships predicted by standard CAPM models? These are mainly the questions addressed in this chapter, focusing on the companies listed on the SSE.

However, the establishing of a formal asset pricing model is difficult, if not impossible. Given the controversy around factors that should be included in the model, this chapter will use only the standard Capital Asset Pricing Model (CAPM). This is a one-factor model to directly estimate the *betas* of *A* and *B* shares. The estimated betas should provide insight into the risk and expected return relationship of *A* and *B* shares.

Furthermore, in the Chinese stock markets, the information system is not transparent; very limited information is provided to investors and the general public. Against such a background, it is useful to test whether market efficiency exists in the Chinese stock market. The outcome might explain the differential between *A* and *B* share prices.

The rest of the chapter is organized as follows: section 4.2 will provide an overview of the Chinese equity markets, in which the institutional structure and major market characteristics will be outlined and summarized. Section 4.3 provides a brief literature review of Chinese stock market research concerning the price difference between *A* and *B* shares. Section 4.4 will deal with asset pricing and risk and return relation for *A* and *B* shares. Section 4.5, 4.6 and 4.7 will present the methodology adopted and data used and report on the empirical results of the research. Readers will find the conclusion in the last Section (section 4.8).

## 4.2: Institutional Facts about the Chinese Stock Industry

### 4.2.1: Stock market structure

The Chinese stock market consists of one regulator, two exchanges, one clearing company, and numerous securities exchange companies. The China Securities Regulatory Commission (CSRC) and the State Council Securities Committee (SCSC) were established in 1993. They

consolidated in 1998, and the China Securities Regulatory Commission is now the regulator of the securities industry. SSE and the SZSE were established in December 1990. By the end of 2003, there were 746 *A* shares and 54 *B* shares listed on the SSE and 489 *A* shares and 57 *B* shares listed on SZSE. By the end of 2003, SSE had 2980.5 billion (70.2%) total market capitalization, while the SZSE had Yuan 1265.3 billion (29.8%) total market capitalization. In March 2000, the China Securities Depository and Clearing Company (CSDCC) was established as the central securities clearing company.

As of February 2008, 861 companies were listed on the SSE and the total market capitalization of SSE reached RMB 23,340.9 billion (US\$3,241.8 billion; US\$1 = RMB 6.82) see Table 4.1.

Table 4.1: SSE Trading Summary for 2007 (Shanghai Stock Exchange<sup>26</sup>).

<i>Data updated on 18 February 2008</i>			
	Stock listings	Market value (billion yuan)	Annual turnover value (billion yuan)
<i>A</i> shares	850	26,849.7	30,196.0
<i>B</i> shares	54	134.2	347.4
<b>Total</b>	<b>904</b>	<b>26,983.9</b>	<b>30,543.4</b>

By late November 2009, the Shenzhen Stock Exchange, based in China's fast-growing south-east region, had grown to a total of 801 listed companies and traded 1,126 different listed securities. It is now the second-biggest exchange in China with a current market capitalization of US\$732 billion, up 113% on the 2008 figure, while the Shanghai Stock Exchange has a current market cap of US\$2.43 trillion. According to data from the World Federation of Exchanges<sup>27</sup>, the SZSE is the seventh-largest stock exchange in the Asia-Pacific region by market cap.

#### 4.2.2: Share Structure

To attract foreign investment, the government allows the coexistence of *A* and *B* shares for listed companies. *A* shares were restricted to domestic investors before December 2002 (the

<sup>26</sup> "SSE indices". Shanghai Stock Exchange. 15 November. 2009 < [http://www.sse.com.cn/sseportal/en\\_us/ps/home.shtml](http://www.sse.com.cn/sseportal/en_us/ps/home.shtml) >

<sup>27</sup> World Federation of Exchange. 20 November. 2009 < <http://www.world-exchanges.org/> >

restriction was lifted after December 2002). *A* shares are RMB denominated<sup>28</sup>. *B* shares were restricted to foreign investors before February 2001 (the restriction was lifted after February 2001). They are U.S dollar denominated on the Shanghai Stock Exchange and Hong Kong-dollar denominated on the Shenzhen Exchange.

In terms of the number of shares issued, capital raised, and market capitalization, the *B* share market is much smaller than the *A* share market. Until December 31, 2003, 87 companies were dual listed in *A* and *B* shares. These 87 paired *A* and *B* share companies have similar or even identical business and operating performance. They represent the same voting rights, trade simultaneously on the Shanghai or Shenzhen stock exchanges, and cannot cross-list on the above two exchanges (Bailey, 1994).

It is widely reported in the literature that there is a large *A* to *B* share price premium. The relevant arguments will be presented in more detail in a later section.

With reference to state-owned enterprises (SOEs), 37% and 27% of the shares of the listed companies are held by the state (government) and legal persons (enterprises and institutions), respectively, and are non-tradable. Tradable public shares comprise only 35% of the market. This represents another distinctive feature of the share structure of the Chinese stock market.

### 4.2.3: Investors

Insurance funds and social security funds cannot participate in the stock market and mutual funds are very limited. By 2000, according to the CSRC, individual investors overwhelmingly dominated the *A* share market, holding over 99.5% of the accounts, with less than 0.5% held by institutional investors. In the *B* share market, institutional investors dominated.

---

<sup>28</sup> Besides *A*, *B* shares issued by domestic stock exchanges, there are *H* shares, Red Chip, and *N* shares related to Chinese companies. *H* shares are Chinese companies listed in Hong Kong. Red Chips are Chinese companies incorporated in Hong Kong and listed in Hong Kong. *N* shares are Chinese companies listed on the New York Stock Exchange.

## 4.3: The Puzzle of China's A-Share and B-Share Price Disparities

This section provides a brief literature review of Chinese stock market research that is concerned with the price difference between *A* and *B* shares.

### 4.3.1: The price gap between *A* and *B* shares narrows rapidly

The academic literature is growing on Chinese stock markets, especially on price differences between *A* and *B* shares. For example, Bailey (1994) initiated the research by analyzing eight Chinese stocks from March 1992 to March 1993. He found a substantial discount in *B* share prices relative to the *A* share prices. A follow-up study by Su (1998) investigated 47 stocks from 1993 through 1996 and discovered that the average daily discount on *B* shares relative to *A* shares was about 62.2 percent.

By examining a sample consisting of 68 firms issuing both *A* and *B* share stocks, Chen, Lee, and Rui (2001) also found that the average *B* share discount on the SSE was about 66.2%, while that on the SZSE was about 52.4% from 1992 to 1997. The existence of persistent price differentials between *A* and *B* shares has led to various testable theories. One such theory is the Differential risk theory. Su (1998) presents evidence that the cross-sectional spread between *A* and *B* share returns is correlated with the difference in the risk factors.

Another theory is the Differential liquidity theory. Chen, Lee, and Rui (2001) reported that the price difference is primarily due to illiquid *B* share markets. Their findings also suggest that *B* share prices are more related to market fundamentals, while *A* share prices are more likely to be influenced by non-fundamental factors. Another theory is the Asymmetric information theory was developed by Chakravarty, Sarkar, and Wu (1998) who used a media-coverage variable to reflect foreign investors' language and other barriers and concluded that foreign investors require a discount to hold *B* shares. Another theory is the Differential demand theory of Sun and Tong (2000) argued that since there are only limited investment

alternatives for domestic investors, and the returns on other alternatives were too low to be attractive. The demand for *A* shares is very high compared to that for *B* shares, which are one of the many investment substitutes for foreign investors. So there is a huge *A* to *B* share price premium.

All of the above-mentioned studies were carried out before February 2001, when the Chinese Securities Regulatory Commission allowed Chinese investors to own *B* shares. Karolyi and Li (2003) compared the changes in the *B* share discount relative to the *A* share before and after February 11, 2001. They found that the *B* share discount declined from 75% to 8%, and their findings do not support differential demand or liquidity theories but support differential risk and asymmetric information theories (Tan, 2005).

This chapter will continue with an investigation based on the work of Karolyi and Li (2003) comparing the performance of *A* and *B* shares' before and after permission was given to Chinese investors to own *B* shares; and before and after permission was granted to overseas investors to own *A* shares.

#### 4.3.2: The relationship between *A* and *B* shares prices

Several studies have been undertaken on the relationship between *A* and *B* share prices. These studies focused on the lead-lag or cross-autocorrelation between the prices and returns of *A* and *B* shares (Kim and Shin, 2000, Chui and Kwok, 1998). While these studies provide some insight into the *A* and *B* share pricing relationship, most do not utilize a standard asset-pricing model. Studies that included asset-pricing models and present value models (and modifications of present value models) have been selected here to examine *A* and *B* share-pricing issues.

For example, Fernald and Rodgers (2002) utilized the standard present value model to conjecture that the difference in *A* and *B* share prices of the same company can be attributed

to differences in the discount rates applied by foreign and Chinese investors. In their argument, the expected rate of return  $r$  for Chinese and foreign investors in the model:

$$P_A = \frac{D_t}{r_A - g} = k \frac{E_t}{r_A - g} \quad (4.1a)$$

$$P_B = \frac{D_t}{r_B - g} = k \frac{E_t}{r_B - g} \quad (4.1b)$$

is the key that determines the difference between  $A$  and  $B$  share prices. Where:  $P_A$  is the price of an  $A$  share,  $P_B$  is the price of the corresponding  $B$  share,  $D_t$  is the dividend paid at time  $t$ ,  $r_A$  is the constant expected rate of return of Chinese investors for  $A$  shares. Similarly,  $r_B$  is the expected rate of return of foreign investors for  $B$  shares,  $K$  is the ratio of dividends to earnings  $E_t$ , and  $g$  is the constant growth rate of dividends.

Since  $A$  and  $B$  shares are issued by the same company in China, the earnings are the same for  $A$  and  $B$  shareholders. The above equation shows that:

$$\frac{P_B}{P_A} = \frac{r_A - g}{r_B - g} \quad (4.2)$$

Equation (4.1a, b) implies that  $\frac{E}{P} = \frac{r-g}{k}$  therefore, it follows that:

$$r_A - r_B = k \left[ \frac{E}{P_B} - \frac{E}{P_A} \right] \quad (4.3)$$

However, this argument is not fully convincing. Since we had wide variations in the gap between prices of  $A$  and  $B$  shares, the fixed effect of the difference in  $r$  for Chinese and foreign investors is not sufficient to explain cross company differences in the relative prices

paid by foreign and Chinese investors for the same asset claims<sup>29</sup>. For example, on average, the *A* share prices of “SVA Electron” (SSE’s listed company in our dataset) are about three and a half times more than the *B* share prices. But the difference between the price of *A* and *B* shares of the company “Shanghai Potevio” (also listed on the SSE) in our dataset is as large as seven times. It seems that the price difference of *A* and *B* shares cannot be explained theoretically by a simple present value model.

Starting with a present value model, Fernald and Rodgers concluded their analysis by using a modified version of the *P/E* ratio analysis based on equation (4.3)<sup>30</sup>. They formed a panel of Chinese companies to “identify variables associated with cross-company differences in the relative price paid by foreigners and the earnings-price ratio”. These variables include “a dummy variable for whether the firm exports a high share of its output; the percentage of total shares owned by the state; sales (lagged one period) as a proxy for size; turnover, defined as the average ratio of daily trading volume to shares outstanding; and observed sales growth from 1993-1998” (Fernald and Rodgers, 1999). In their analysis, *betas*<sup>31</sup> were included as one variable to explain the relative prices of *A* and *B* shares. But this study is not a direct test of the CAPM model. In the results shown in their paper, the domestic and foreign *betas* failed to explain the difference in the relative prices of *A* and *B* shares. In addition, the *A* domestic share *beta* and *B* share *beta* yield opposite signs when they are included as independent variables to explain the earnings to price ratio of *A* and *B* shares respectively in their model.

This study makes use of the CAPM one-factor model to explain the asset pricing of *A* and *B* shares. The study differs from that of the Fernald and Rodgers’ present value-model analysis in 1999. Instead of inducing the conjecture from the present value model that the expected rate of return  $r$  is the key to explaining *A* and *B* share prices, this examination will use the

---

<sup>29</sup> If  $r_A$  and  $r_B$  is the only factor that leads to the difference of *A* and *B* share prices, as predicted by the present value model, then the price difference should be same for all the companies that issue both *A* and *B* shares according to equation (3).

<sup>30</sup> In their regression, the dependent variable is *E/P*.

<sup>31</sup> See the following section for the explanation of *betas*.

CAPM one-factor model to directly estimate the *betas* of *A* and *B* shares. The estimated *betas* should provide insight into the risk and the expected return relationship of *A* and *B* shares.

#### 4.3.3: The Information Efficiency of China's Stock Market

Since it is an emerging market that has been established in less than twenty years, China's stock market exhibits many characteristics revealed as inefficient in previous research. It is recapitulated as a "high return, high risk" market by many investors. While fundamental firm-specific factors, that is, the earnings-to-price ratio, dividend yield and liquidity, lack the power to explain stock returns (Wang and Di, 2007).

Market segmentation with evidence of information diffusion between *A* shares and *B* shares is also mentioned in Sjoo and Zhang (2000). Moreover, government intervention, the dominant position taken by individual investors, information asymmetry, and the prohibition of short sales are generally considered as the main characteristics preventing China's stock market from being efficient.

In this case, prices in China's stock market are assumed to be predictable and a constant excess return should be available for investors. For example, Kang, Liu and Ni (2002) document significant abnormal returns from short-horizon contrarian and intermediate-horizon momentum strategies using data from 1993 to 2000. This chapter includes a test to ascertain whether market efficiency exists in the Chinese stock market. The result might explain the price different between *A* and *B* shares.

The present value (PV) model of stock prices has become a fashionable tool in testing for market efficiency. For example, Shiller (1981) referred to the PV model as an "efficient market model" and viewed volatility tests as a method of evaluating market efficiency in a different way from more conventional regression methods.

This study uses the PV model which is known as “first variance bounds” tests by Shiller (1981) and LeRoy and Parter (1981) to test the efficiency of the SSE. This test has sparked much criticism (e.g. Flavin, 1983 and Kleidon, 1986) and further inquiry (Camerer, 1987 and West, 1988a). Under this test of efficient markets, actual stock prices are determined by rational agents as the present discounted value of future dividends. If the model holds, any movements in stock prices are due to new information concerning future dividends and stock prices that reflect all the available information. The validity of the model can be examined by testing the restrictions that are implied by the model. A violation of the restrictions would suggest that prices do not completely reflect all the available information and the efficient market model would then be rejected.

#### 4.4: Asset Pricing: *A* and *B* shares

The difference in price levels of *A* and *B* shares reveals that these two groups of investors actually value the *A* and *B* shares quite differently. Fundamental optimal portfolio theories predict that although *A* and *B* shares represent the same dividend flow (especially if we ignore the exchange rate risk<sup>32</sup>), in equilibrium, *A* and *B* shares need not have the same price levels.

Optimal portfolio theory implies that the price of any one share depends not only upon its own dividend flow but also upon the characteristics of the dividend flows of other assets that individuals may purchase. Since foreign investors and domestic investors have distinct sets of assets from which they can compose their portfolios, optimal portfolio theory provides a basis for accounting for differences in the prices for *A* shares and *B* shares. This explains that just because *A* shares and *B* shares are issued by the same company, and therefore represent the same dividend flow, this condition is not sufficient to guarantee the same equilibrium price levels for these two types of shares. The investment opportunities and market portfolios are

---

<sup>32</sup> Although lately there have been many discussions about the pressure for Chinese RMB to be revalued (mainly to appreciate), the exchange rates between Chinese Yuan and U.S. dollars have been relatively stable in the period of our study. The Chinese government adopts a pegged exchange rate policy, which means that the exchange rate is not determined by international capital markets.

different for Chinese and foreign investors. Hence, *A* and *B* shares will be valued differently by these two segmented groups of investors.

If optimal portfolio theories successfully predict the divergence of *A* and *B* share price levels, can we expect to utilize standard asset pricing theories based on the notion of market portfolios to predict the same risk and return relationships for these two types of shares? This study focuses mainly on examining whether *A* and *B* shares' returns can be explained by systematic risks in China and the US as well as idiosyncratic risks of the listed companies.

## 4.5: Methodology and Tests

This section describes the scientific methods chosen for this analysis. This section starts by describing a simple examination of historical *A* and *B* share prices. After which the section will present a standard CAPM one-factor model, as well as variance bounds tests.

### 4.5.1 Historical analysis Test

This is a simple examination of historical *A* and *B* share prices which confirms that two groups of investors actually value the *A* and *B* shares quite differently. The difference in the price levels of *A* and *B* shares would normally induce arbitrage incentives, but legal restrictions on cross-trading between *A* and *B* shares prevent such activities.

Furthermore, this section investigates the impact of allowing domestic residents to invest in the *B* share stock market, as well as the permission granted to foreigners to invest in the *A* share stock market. A simple examination of historical *A* and *B* share prices is also used for three different time periods: period one, from May 1<sup>st</sup> 1998 until February, 16<sup>th</sup> 2001, just before the lifting of the restrictions on share *B*; period two, from February, 20<sup>th</sup> 2001 until November, 29<sup>th</sup> 2002, just before the lifting of the restrictions on share *A*; and period three,

from December, 2<sup>nd</sup> 2002 until May, 1<sup>st</sup> 2008 in which there were no restrictions at all (see Figure 4.1).

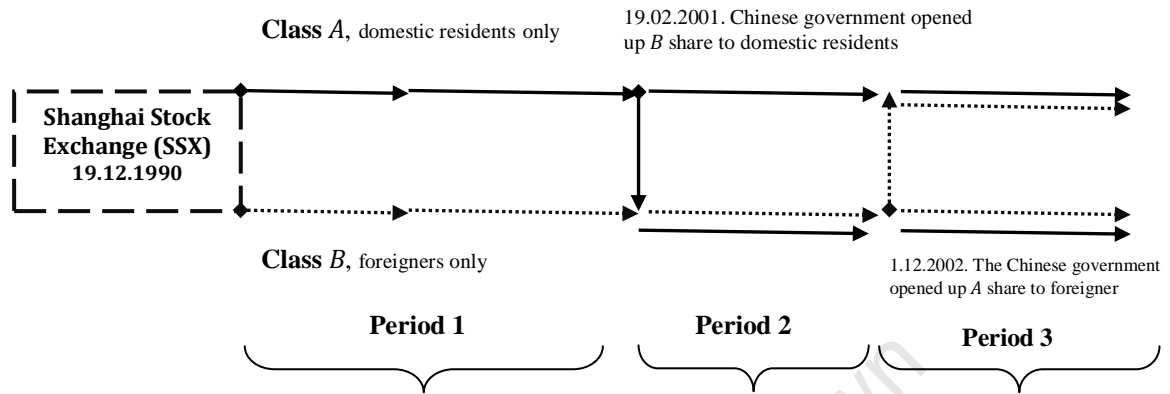


Figure 4.1: Three stages in the removing of restrictions

#### 4.5.2: The Capital-asset pricing model (CAPM)

In my analysis, the CAPM one-factor model is adopted to study the asset pricing issues of A and B shares.

As Cochrane indicated in his book, Asset Pricing (2001), all CAPM factor models are special cases of consumption-based asset pricing models. Suppose we have a basic asset pricing equation:

$$P_t = E \frac{[\alpha U'(C_{t+1})X_{t+1}]}{U'(C_t)} \quad (4.4)$$

Where  $U$  represents individual's utility function,  $C_t$ , and  $C_{t+1}$  represents consumption,  $P_t$  is the current asset price and  $X_{t+1}$  is the payoff of the asset. Define the discount factor  $m$  as:

$$m_{t+1} = \alpha \frac{\mu'(C_{t+1})}{\mu'(C_t)} \quad (4.5)$$

Then we can rewrite the basic asset-pricing equation (4.4) as

$$P_t = E_t[m_{t+1}X_{t+1}] \quad (4.6)$$

Most empirical work testing this equation has shown that the discount factor using consumption data, as specified in (4.5), does not fit the data very well. Obtaining asset price  $P_t$  and asset payoff  $X_{t+1}$  is a fairly easy job. All the empirical tests of asset-pricing models encounter difficulties associated with the task of obtaining good measures of the discount factor  $m_{t+1}$  and linking them to the data.

This has inspired scholars to search for alternative models like CAPM as a proxy to the discount factor. The essence of the CAPM models lies in this equation:

$$m_{t+1} = a + bR_{t+1}^w \quad (4.7)$$

$R_{t+1}^w$  is the wealth portfolio return, which is normally replaced by the return on a broad-based stock portfolio in practice. Equation (4.7) linearises the discount factor specification (4.5) and expresses  $m_{t+1}$  in terms of “factors.”

The key connection between the CAPM single *beta* model and the traditional consumption based asset-pricing model is the assumption that the agents’ utility function takes a quadratic form. This utility form will ensure that the discount factor can be transformed into a linear representation, as specified in 4.7.

I will follow Cochrane’s notations to derive the CAPM model (Cochrane, 2001:155-160).

Assume an investor has a two-period quadratic utility function with no labour income:

$$U(C_t, C_{t+1}) = -\frac{1}{2}(C_t - C^*)^2 - \frac{1}{2}\beta E[(C_{t+1} - C^*)^2] \quad (4.8)$$

With the budget constraint:

$$C_{t+1} = W_{t+1}$$

$$W_{t+1} = R_{t+1}^W(W_t - C_t)$$

$$R_{t+1}^W = \sum_{i=1}^N w_i R_{t+1}^i,$$

$$\sum_{i=1}^N w_i = 1$$

The problem facing each investor is to choose consumption in two periods  $C_t$ ,  $C_{t+1}$  and portfolio weights  $w_i$  to maximize the utility, as specified in (4.8). Each agent starts with the endowment wealth  $W_t$  and lives for two periods.  $R_{t+1}^i$  is asset  $i$ 's return in the second period and,  $R_{t+1}^W$  is the portfolio return to the investor in the second period.  $C^*$  is the peak of the parabolic curve.

A standard first order condition solution will give:

$$m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} = \beta \frac{(C_{t+1} - C^*)}{(C_t - C^*)} \quad (4.9)$$

Substitute the budget constraint to (4.9) yields the following form:

$$m_{t+1} = \beta \frac{R_{t+1}^W(W_t - C_t) - C^*}{C_t - C^*} = \frac{-\beta C^*}{C_t - C^*} + \frac{\beta(W_t - C_t)}{C_t - C^*} R_{t+1}^W \quad (4.10)$$

which can be written as:

$$m_{t+1} = a + bR_{t+1}^W$$

Where  $a_t = \frac{-\beta C^*}{C_t - C^*}$  and  $b_t = \frac{\beta(W_t - C_t)}{C_t - C^*}$

The CAPM model is often stated in equivalent *beta* terms:<sup>33</sup>

$$E(R^i) = \gamma + \beta_{i,R^W}[E(R^W) - \gamma] \quad (4.11)$$

In the above equation,  $R^i$  is an individual asset  $i$ 's return;  $R^W$  is the wealth portfolio return;  $\beta_{i,R^W}$  is the so called *Beta* which is closely related to the covariance of the individual asset's return and the wealth portfolio's return. The one-year Treasury bill rate for the US market and the one-year deposit rate for the Chinese market is a proxy for  $\gamma$ , the risk-free interest rate in my analysis<sup>34</sup>.

Moving  $\gamma$  to the left, equation (4.11) states that an individual asset's excess return is simply this particular asset's *beta* multiplied by the market excess return<sup>35</sup> and that the intercept of this regression should be zero. The *beta* of each asset captures the systematic risk facing the whole market which cannot be avoided through portfolio diversification.

Since the available assets to form portfolios for Chinese and foreign investors are different, the wealth portfolios to Chinese and foreign investors are different too. Therefore, according to equation (4.11),  $A$  shares' excess return should be regressed on the Chinese market excess return. The corresponding  $B$  shares' excess return should be regressed on the US market excess return.<sup>36</sup>

### 4.5.3: Tests of Information Efficiency of China's Stock Market

According to Shiller (1981), the efficient market model can be stated as asserting that the price  $P_t$  of a share or portfolio of shares representing an index equals the mathematical

---

<sup>33</sup> For detailed proof of the equivalence of equation (4.11) and (4.7), see Qi (2004) as well as Cochrane (2001), Chapter 6, "Asset Pricing," 2001. William Sharpe also showed the derivation of equation (4.11) in his paper "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of risk," *The Journal of Finance* 19(3): 425-442.

<sup>34</sup> "One-year deposit rate issued by the People's Bank of China's is a risk-free investment is identical to the 1 year U.S. yield on Treasury Bill".

<sup>35</sup> An asset's excess return is defined as the difference of an asset's return and the risk free interest rate. The market excess return is defined as the difference of the composite stock index return and the risk free interest rate.

<sup>36</sup> In reality, of course not all the foreign investors who hold B shares are American investors. But if the B shares are traded in U.S. dollars and the dividends are also paid in U.S. dollars, the average opportunity cost of investing in Chinese B shares is the average return forgone if they had invested in the U.S. capital market.

expectation, conditional on all information available at the time, of the present value  $P^*$  of actual subsequent dividends accruing to that share.  $P_t^*$  is an unknown at time  $t$  and has to be forecasted. Efficient markets maintain that price is equal to optimal forecast. Different forms of the efficient market model differ in the choice of the discount rate in the present value, but the general efficient market model can be written as:

$$P_t = E_t P_t^* \quad (4.12)$$

where  $E_t$  refers to the mathematical expectation conditional on public information available at time  $t$ . This equation asserts that any surprising movements in the stock market must have at their origin some new information about the fundamental value  $P_t^*$ . It follows therefore from the efficient markets model that

$$P_t^* = P_t + U_t \quad (4.13)$$

where  $U_t$  is a forecast error.

The forecast  $U_t$  has to be uncorrelated to any information variable available at time  $t$ , otherwise the forecast would not be optimal; it would not take into account all the information. Since the price  $P_t$  itself, is the information at time  $t$ .  $P_t$  and  $U_t$  must be uncorrelated to each other. Since the variance of the sum of two uncorrelated variables is the sum of their variances, it follows that the variance of  $P_t^*$  must equal the variance of  $P_t$  plus the variance of  $U_t$ , and since the variance of  $U_t$  cannot be negative, the variance of  $P_t^*$  must be greater than or equal to that of  $P_t$ . Figure 4.1 illustrates these patterns

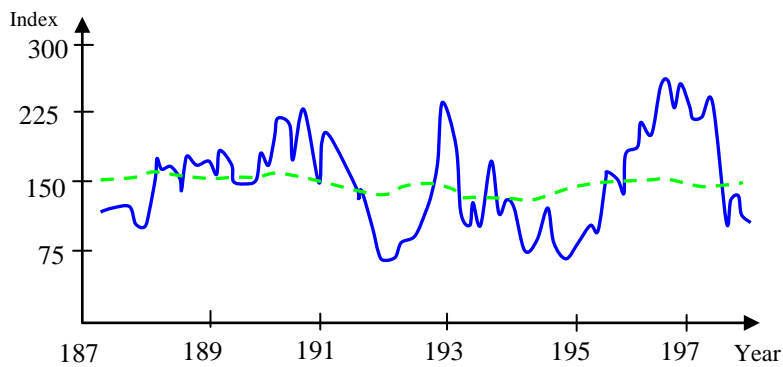


Figure 4.2: Real S&P Composite Stock Price Index (solid line  $p$ ) and ex-post rational price (dotted line  $p^*$ ), 1871-1979, both de-trended by dividing a long-run exponential growth factor. The variable  $p^*$  is the present value of actual subsequent real de-trended dividends, subject to an assumption about the present value in 1979 of dividends thereafter (Shiller, 1981).

Shiller (1981, 2000) and others obtained the time series data on actual dividends and (using some additional assumptions) calculated values for  $P^*$ . Comparing  $P$  and  $P^*$ , they claimed that the data show that in reality  $var(P) > var(P^*)$ . In other words, stock prices exhibit excess volatility (Shiller, 1981). This chapter tests the efficiency of the SSE by applying the first variance bounds test of Shiller (1981).

## 4.6: Data, Variables and Sample Characteristics

The individual assets examined here are the  $A$  shares and  $B$  shares of the companies who issue both classes of shares listed on the SSE. In the datasets used, 33 companies listed on SSE issue both  $A$  and  $B$  shares. Daily stock prices from May 1, 1998 to May 1, 2008, that is 2610 days were obtained from the University of Cape Town DataStream database to examine the correlation between  $A$  and  $B$  share price movements.

Furthermore, the daily individual asset returns were obtained for the same companies for the same period specified above. The dataset also includes a daily composite index of returns on

the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ, and the Shanghai *A* share index returns for the same period<sup>37</sup>.

The model specified by equation (4.11) also requires the risk-free interest rate. As mentioned above, this examination has also included the one-year Treasury bill rate and the one-year Chinese deposit rate as a proxy for the risk-free interest rate for US and China respectively.

The SSE *A* share index minus the one-year Chinese deposit rate is a proxy for the market excess return for Chinese investors, while the NYSE, AMEX and NASDAQ composite index minus the US one-year Treasury bill rate is a proxy for the US market excess return. The regressions are based on equation (4.11)

Annual dividends paid on the Shanghai *A* share index during the interval of January 1992 to June 2008, have been also collected to examine the efficiency of the SSE. The constant discount rate used in the present value calculation is the geometric-average real return on stocks over the full sample. That is a measure of rational investors' required rate of return. The use of a variable discount rate does not change the basic conclusions, as shown by Shiller (2003).

## 4.7: Results and Explanations

### 4.7.1: Results of the Historical Analysis

We can now empirically test the models described in the previous section. The data include 33 companies listed on the SSE issuing both class *A* and *B* shares.

Figure 4.3 in appendix B.1 shows the prices of *A* shares and *B* shares of 33 companies listed. *B* share prices are converted into Chinese Yuan at official exchange rates. It is clear that

---

<sup>37</sup> If the *B* shares are traded in U.S. dollars and the dividends are also paid in U.S. dollars, the average opportunity cost of investing in Chinese *B* shares is the average return forgone if they had invested in the U.S. capital market. However, *A* shares are traded in Chinese currency and the dividends are also paid in Chinese currency on Shanghai *A* share index only.

although there appears to be some degree of co-movement in *A* and *B* shares prices, domestic and foreign investors price these two types of shares differently. The data is daily *A* and *B* share price data from May 1, 1998 to May 1, 2008. After converting *B* shares into Chinese Yuan, further calculation shows that, on average, *A* shares are priced 5.41 times higher than their corresponding *B* shares.

However, after dividing the time period described above into 3 different time periods, as explained in section 4.5.1 and illustrated in Figure 4.1, our calculation shows that, on average, *A* shares are priced 7.93 times higher than their corresponding *B* shares in the first period (from May 1<sup>st</sup> 1998 until February, 16<sup>th</sup> 2001, just before the lifting of the restrictions on share *B*) and 6.1 times higher in the second period (from February, 20<sup>th</sup> 2001 until November, 29<sup>th</sup> 2002, just before the lifting of the restrictions on share *A*) and 3.95 times higher in the third period (from December, 2<sup>nd</sup> 2002 until May, 1<sup>st</sup> 2008 where there are no restrictions at all).

This is the first finding obtained in this chapter, which indicates that volatility transmissions between *A* and *B* share markets accelerate when domestic residents start to invest in *B* shares, and further accelerate when overseas investors start to invest in *A* shares. It is therefore concluded that permitting domestic residents to invest in *B* shares shrinks the gap between *A* and *B* share prices. Further, permitting overseas investors to invest in *A* shares, can shrink the gap even more between *A* and *B* share prices.

This finding was followed up by a comparison of the distribution of daily return variance, skewness, and kurtosis between *A* and *B* share prices in the three periods specified in Figure 4.1. The major findings of this analysis are as follows:

(1) In the first period, the average daily return variance is about 11.37 times higher on the *A* share than on the *B* share. The average kurtosis is also higher on the *A* share than on the *B*

share, while average skewness is higher on *B* than on the *A*. These results are illustrated in Figure 4.4 below.

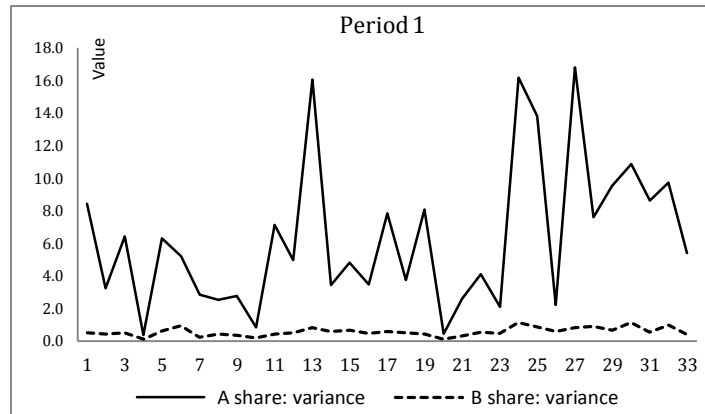


Figure 4.4: Variances of *A* and *B* share in the first period from May 1<sup>st</sup> 1998 to February, 16<sup>th</sup> 2001, just before the lifting of the restrictions on *B* shares

(2) In the second period, the average daily return variance is about 2.5 times higher on the *A* share than on the *B* share, while the average skewness and kurtosis is higher on the *B* than on the *A*, (see figure 4.5).

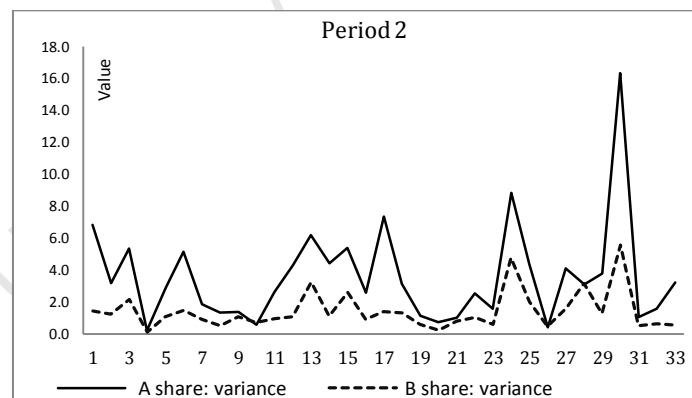


Figure 4.5: Variances of *A* and *B* share in the second period from February, 20<sup>th</sup> 2001 to November, 29<sup>th</sup> 2002, just before the lifting of the restrictions on *A* shares

(3) In the third period, the average daily return variance is about 2.40 times higher on the *A* share than on the *B*, while the average skewness and kurtosis is higher on the *B* than on the *A* share (see Figure 4.6).

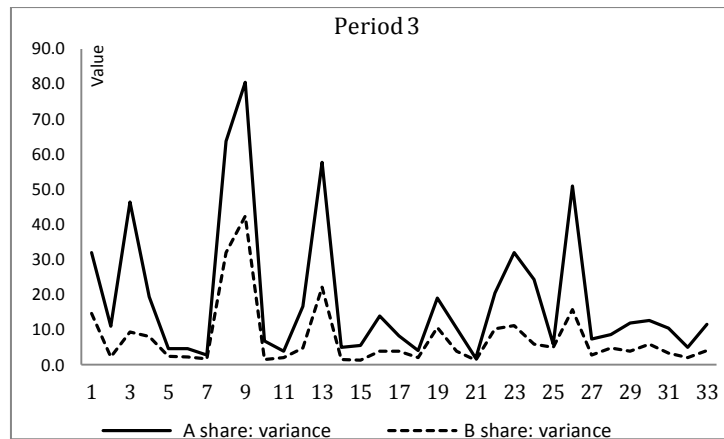


Figure 4.6: Variances of *A* and *B* share in the third period from December, 2<sup>nd</sup> 2002 to May, 1<sup>st</sup> 2008 after which there were no restrictions

The higher variance of *A* share prices means that *A* share prices are more efficient than those of the *B* shares, as these prices incorporate new information faster.

#### 4.7.2 The Risk and Return Relationship for *A* and *B* Shares

The previous empirical finding was not surprising. Fundamental optimal theories predict that although *A* and *B* shares represent the same dividend flow, in equilibrium, *A* and *B* shares need not have the same price levels. Optimal portfolio theory states that the price of any one share depends not only upon its own dividend flow, but also upon the characteristics of dividend flows of other assets that individuals may purchase.

Since foreign investors and domestic investors have distinct sets of assets from which they can compose their portfolios, optimal portfolio theory provides a basis for accounting for differences in the prices of *A* and *B* shares. This explains the fact that although *A* shares and *B* shares are issued by the same company, and therefore represent the same dividend flow, this is not sufficient to guarantee the same equilibrium price levels for the two types of shares. Hence *A* and *B* shares can be valued differently by these two segmented groups of investors.

Table 4.2 in appendix B.2 shows the regression results for the share prices from 33 companies that issue both *A* and *B* shares. For the regressions concerning the Chinese market,

all *A* shares in the dataset confirmed the linear and significant relationship between individual asset's excess returns and the market's excess returns. The estimated beta range is 0.89-1.01.

However, for the regressions related to Chinese *B* shares on the US market's excess returns, all *B* shares in the dataset confirmed the linear and significant relationship between individual asset's excess returns and the market's excess returns. The estimated beta range is 0.86-0.91. The t-test shows that all *B* shares are significantly influenced by the US market.

Thus, price differences between domestic *A* shares and foreign *B* shares for the same company could be explained by the different systematic risks in China and the US. This is valid if all *A* shares in the dataset show a significant relationship between individual asset's excess returns and the Chinese market's excess returns and *B* shares confirm a significant relationship between individual asset's excess returns and the US market's excess returns.

Moreover, the estimated betas of the *B* shares are lower than their counterparts in the *A* shares. This shows that *A* shares issued by Chinese companies present a higher systematic market risk to domestic investors. But on the other hand, the higher systematic risk offers a higher excess return for *A* shares and justifies their higher price in comparison to the *B* share price.

### 4.7.3: Information Efficiency of China's Stock Market

Figure 4.7 below represents our empirical results using the Shiller (1981) model to test the efficient market hypothesis in the Shanghai *A* share index. We find significant evidence against the efficient market hypothesis, as shown in the Figure 4.7 below, although this market has become more efficient at a later stage. These findings are consistent with Shiller's findings (1981), as well as those of Stephen LeRoy and Richard Porter (1981).

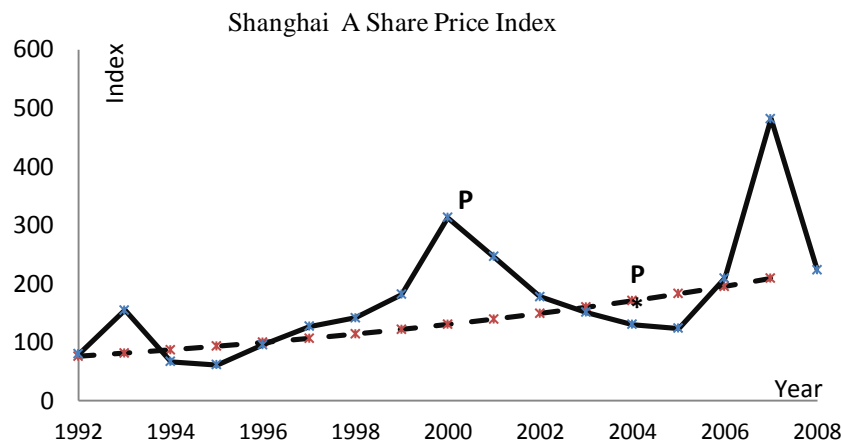


Figure 4.7: Shanghai A share index (solid line p) and ex-post rational price (dotted line p\*), 1992- 2008. The variable p\* is the present value of actual subsequent real de-trended dividends.

## 4.8: Conclusion

This study examined issues of asset pricing as well as risk and return relationships in a special environment in China's stock market. This market has distinctive features such as a large percentage of non-tradable shares and separation of domestic and foreign investors. The research finds that domestic and foreign investors do not price A shares (available for domestic investors) and B shares (available for foreign investors) differently. Moreover, it was discovered that the standard risk and return relationships implied by CAPM models comply with A shares and therefore, domestic investors price asset risk as predicted by CAPM models in China. However, the study also revealed that the standard risk and return relationships implied by CAPM models also comply with B shares and foreign investors price asset risk as predicted by CAPM models in the US. Thus, price differences between domestic A shares and foreign B shares for the same company could be explained by the different systematic risks in China and the US. The investment opportunities and market portfolios are different for Chinese and foreign investors. Hence, A and B shares will be valued differently by these two segmented groups of investors.

The results also suggest that the different pricing can be explained by interventions in the stock market by the Chinese government. Figures 4.3 4.4, 4.5, and 4.6 and the analysis of skewness and kurtosis show sharp jumps and discontinuities, reflecting the high *A* share price influenced by the high volatility of the Chinese stock market. Interestingly, these jumps and discontinuities showed up in the data mostly in the first few years, from (May 1<sup>st</sup> 1998 to February, 16<sup>th</sup> 2001, just before the lifting of the restrictions on share *B*), reflecting the fact that government interventions and policy changes are major sources of volatility in the Chinese stock market, and for the *A* share price in particular, as was discussed in section 4.2.

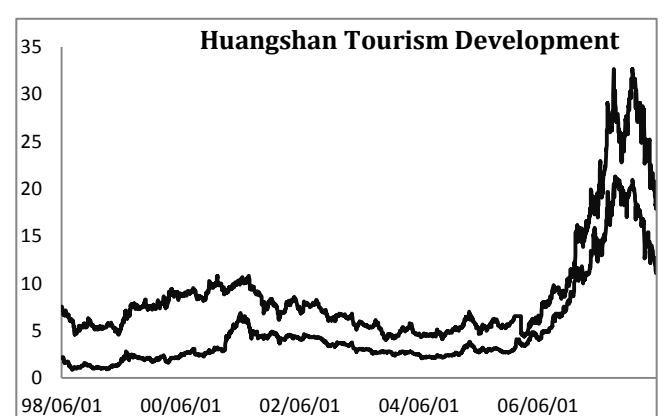
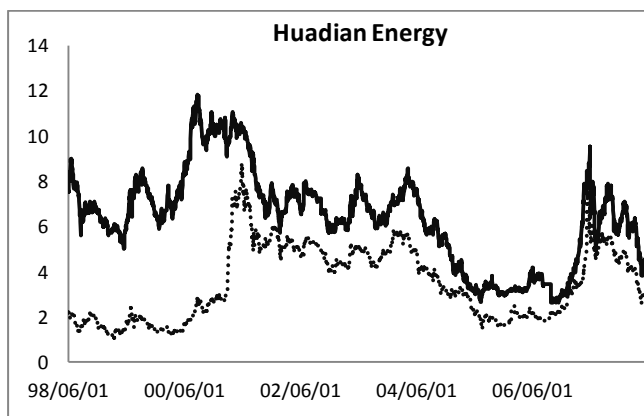
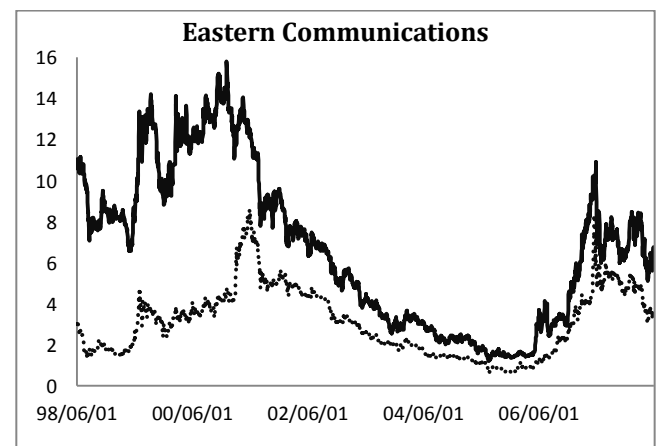
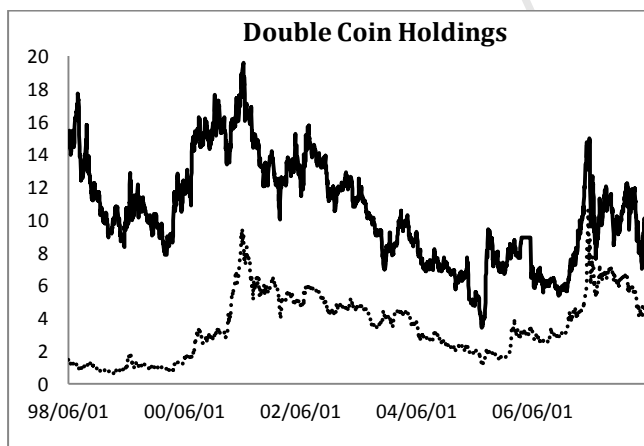
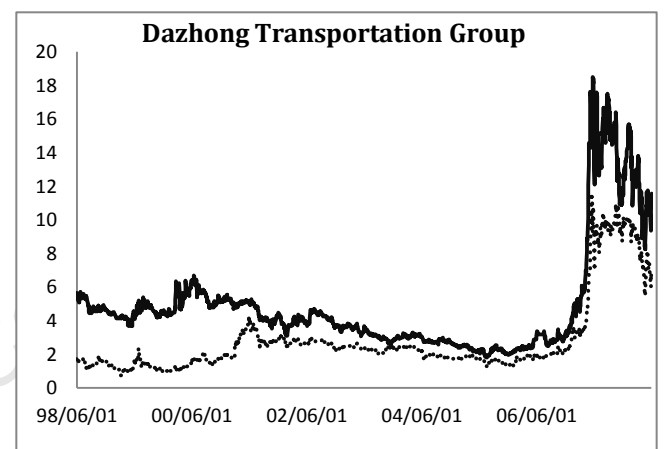
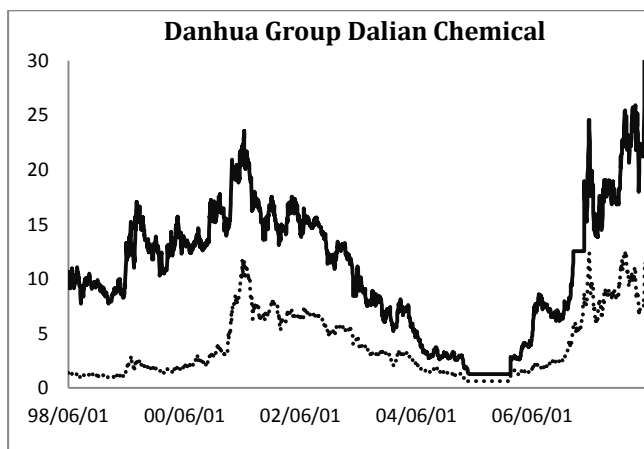
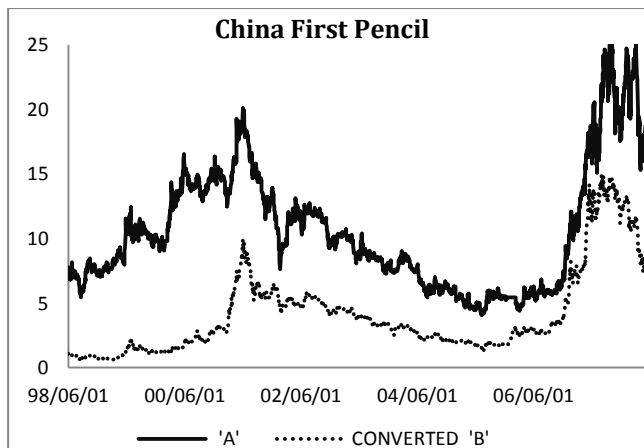
Such government interventions and policy changes preclude foreigners from buying *A* shares, and domestic residents from buying *B* shares. These include changes of interest rate, control of the growing supply of new shares traded in the exchanges, and changes of stock transaction regulations, such as the imposition and removal of daily price change limits. Other political events also disturbed the Chinese stock markets.

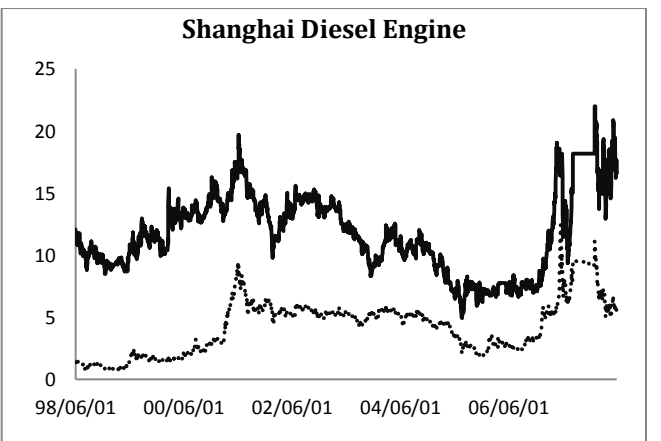
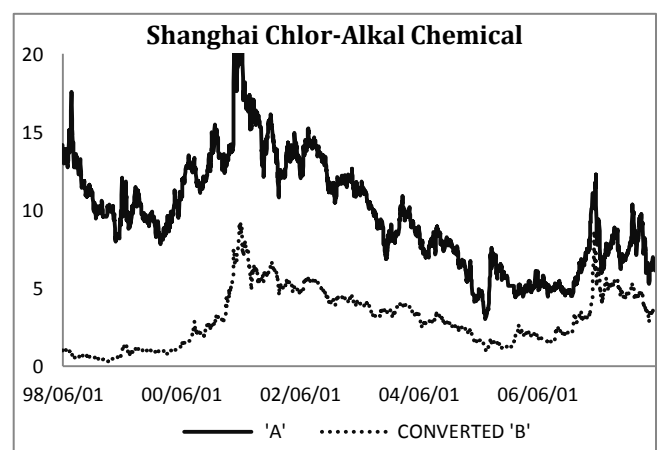
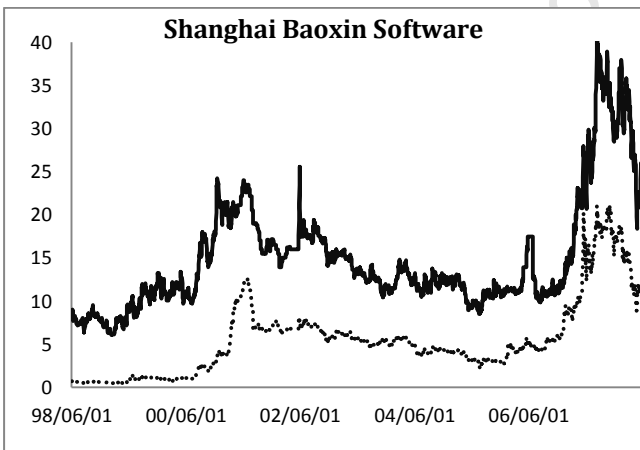
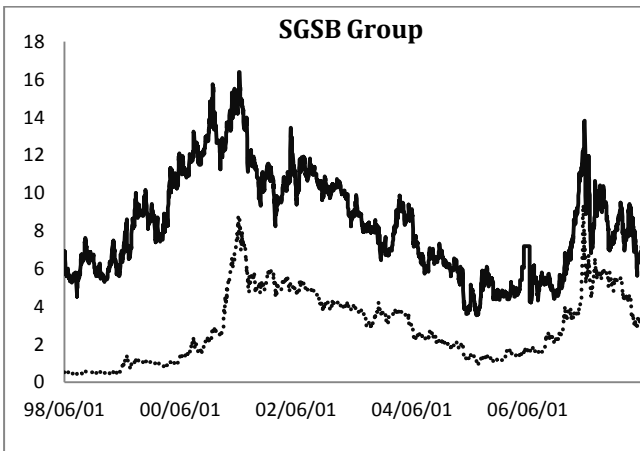
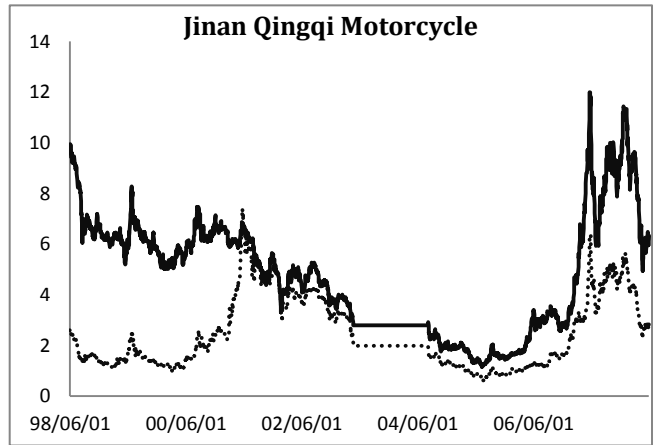
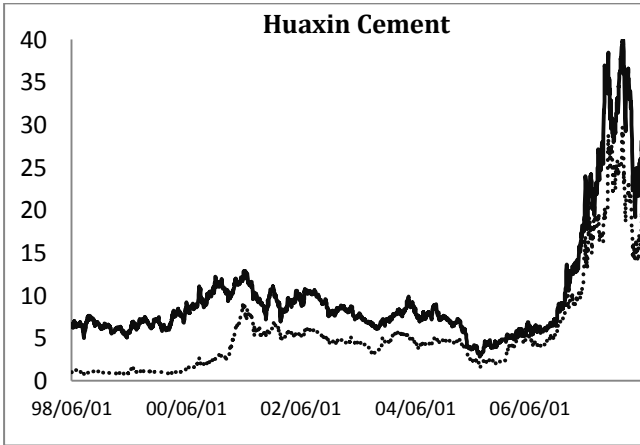
However, we can also see continuous volatility decreases from February, 20<sup>th</sup> 2001 to November, 29<sup>th</sup> 2002, just after the lifting of the restrictions on *B* shares, and from December, 2<sup>nd</sup>, 2002 to May, 1<sup>st</sup> 2008 when there were no restrictions at all, as shown by the variance of the returns. This clearly shows that the Chinese government improved its capital market management by applying more market-oriented methods and becoming less directly involved in the equity markets, leaving them to their own devices.

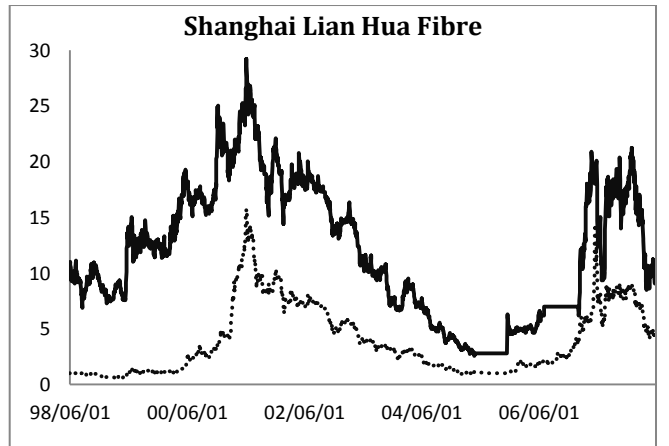
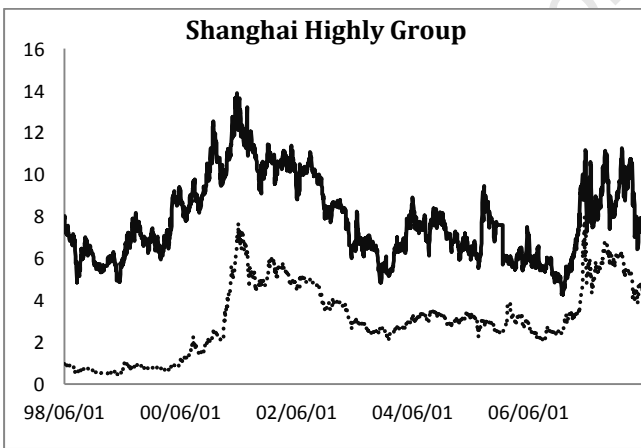
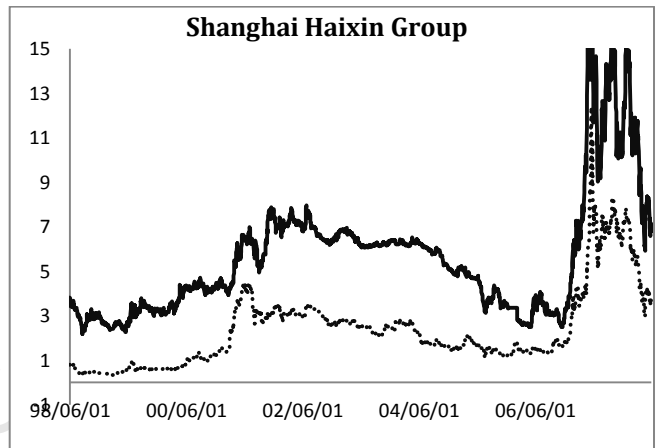
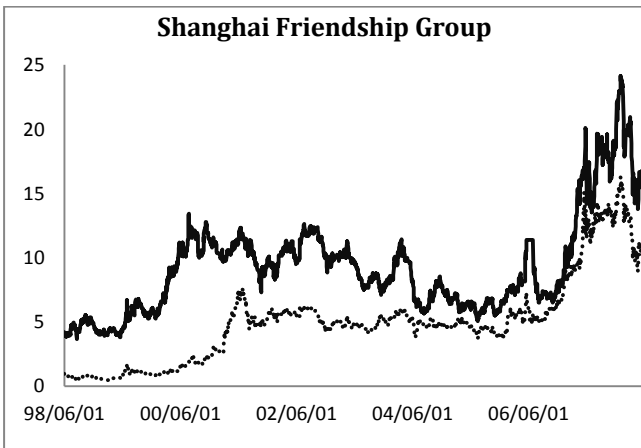
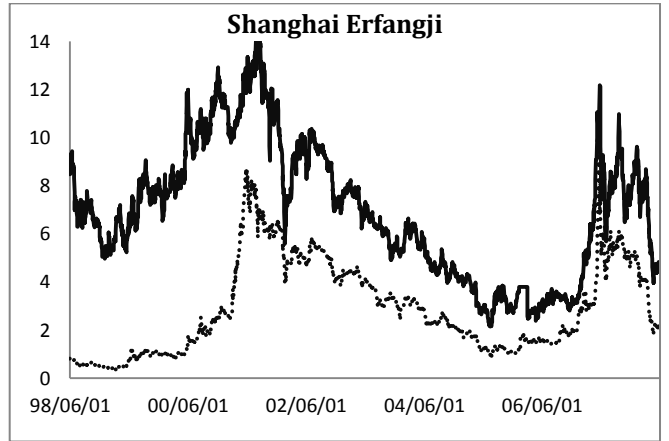
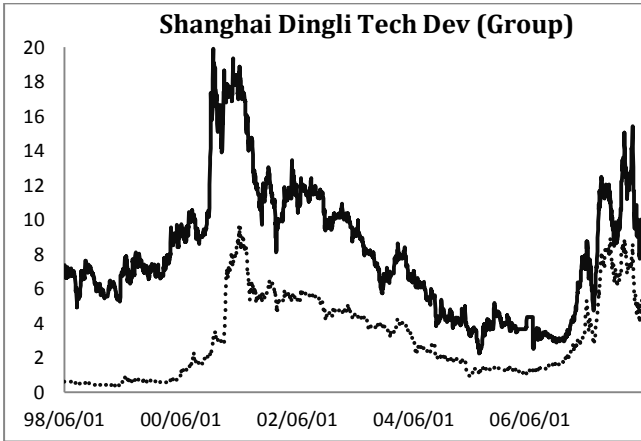
The increase in efficiency over time, indicated by the continuous shrinkage in the gap between daily *A* and *B* share prices in the more recent periods, can be explained by the increasing deregulation and liberalization of the Chinese stock markets, which in the early 1990s were heavily interrupted by unpredictable market interventions by the government. This made these markets persistent rather than neutral. Such non-neutral market persistence allows for profit-making arbitrage opportunities, making these markets unfair. The Chinese markets have become more efficient and neutral in the last few years, and no longer allow abnormal profits, and have therefore become fairer for all traders.

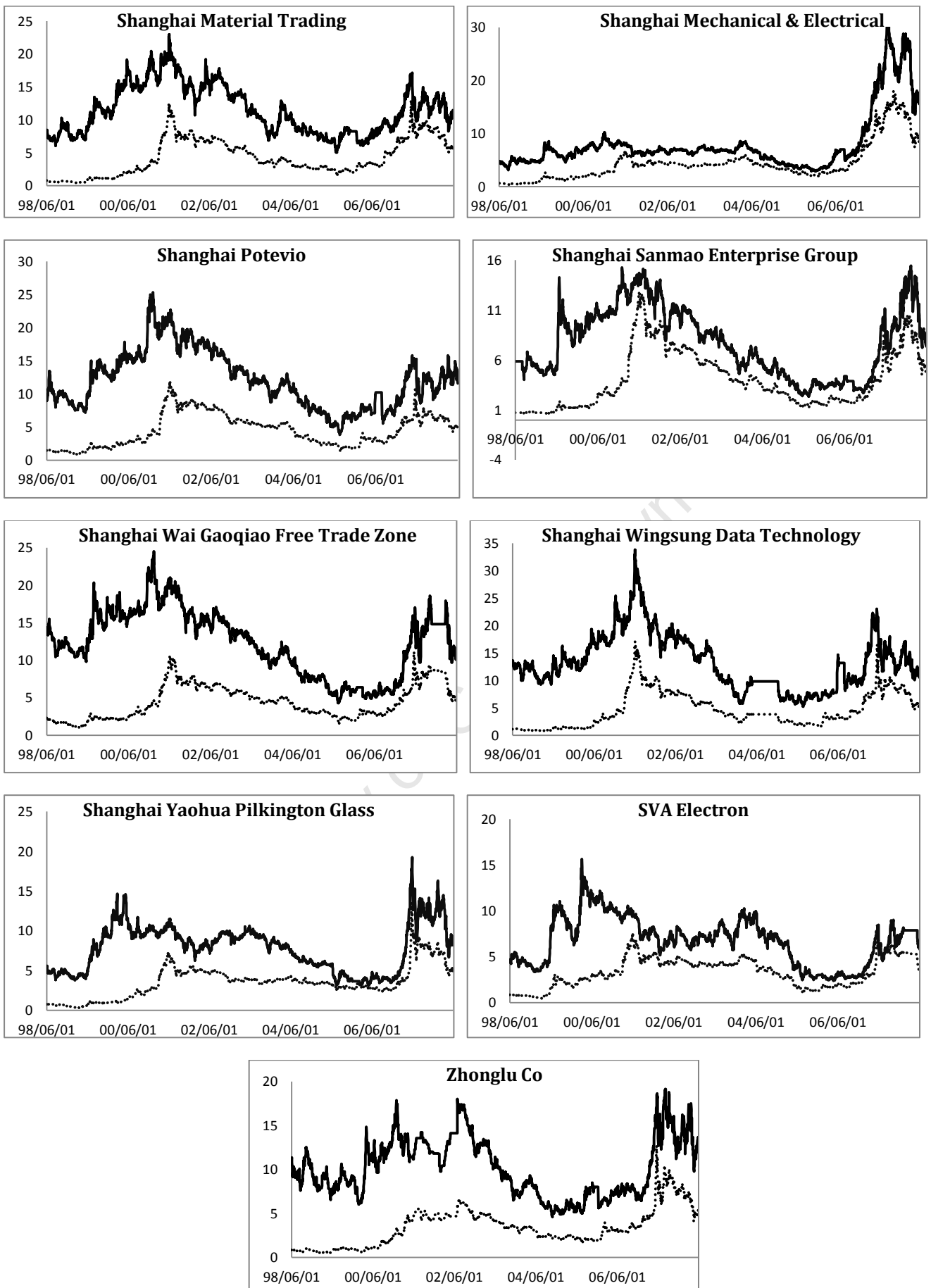
## 4.9: Appendix B

### 4.9.1: B.1 Examination of historical *A* and *B* share prices









**Figure 4.3 Prices of A and B Shares**

*B* share prices, which are denoted in U.S. dollars, are converted into Chinese currency, Yuan, with official exchange rates. The daily data of *A* and *B* shares are obtained from University of Cape Town DataStream. The data for the 33 companies included in this figure cover the period of May 1, 1998 to May 1, 2008.

#### 4.9.2: B.2 Regression Result of A and B shares for CAPM Model

Table 4.2: Regression Result of A and B shares for CAPM. Start date 1998/06/01. End date 2008/06/02. 2610.

		A Shares	B Shares
Company		A-Beta	B-Beta
1	CHINA FIRST PENCIL	<b>0.97</b>	<b>0.88</b>
	t Stat	68.59	51.53
2	CHINA TEX.MCH.	<b>0.91</b>	<b>0.88</b>
	t Stat	57.42	46.55
3	DANHUA CHEMICAL TECH.	<b>0.89</b>	<b>0.88</b>
	t Stat	38.41	44.64
4	DAZHONG TRSP.(GROUP)	<b>0.97</b>	<b>0.90</b>
	t Stat	75.19	54.48
5	DOUBLE COIN HOLDINGS	<b>0.98</b>	<b>0.89</b>
	t Stat	63.8	50.81
6	EASTERN COMMS.	<b>1.00</b>	<b>0.90</b>
	t Stat	65.54	53
7	HUADIAN ENERGY	<b>0.98</b>	<b>0.91</b>
	t Stat	84.25	58.94
8	HUANGSHAN TOURISM DEV.	<b>0.94</b>	<b>0.90</b>
	t Stat	73.83	59.52
9	HUAXIN CEMENT	<b>0.97</b>	<b>0.88</b>
	t Stat	68.62	52.20
10	JINAN QINGQI MCYCLES.	<b>0.92</b>	<b>0.91</b>
	t Stat	69.15	54.39
11	SGSB GROUP	<b>0.98</b>	<b>0.88</b>
	t Stat	58.56	44.55
12	SHAI ATMTN. INSTRUMENTATION	<b>0.95</b>	<b>0.87</b>
	t Stat	58.91	47.39
13	SHAI.BAOSIGHT SOFTWARE	<b>0.91</b>	<b>0.87</b>
	t Stat	56.47	48.24
14	SHAI.CHLOR - ALKALI CHM.	<b>0.98</b>	<b>0.86</b>
	t Stat	62.89	34.11
15	SHANGHAI DAJIANG GROUP STOCK	<b>0.92</b>	<b>0.88</b>
	t Stat	54.51	45.14
16	SHAI.DIESEL ENGINE	<b>0.96</b>	<b>0.89</b>
	t Stat	66.24	53.81
17	SHAI.DINGLI TECH.DEV. (GROUP)	<b>0.93</b>	<b>0.87</b>
	t Stat	55.99	49
18	SHAI.ERFANGJI	<b>1.01</b>	<b>0.88</b>
	t Stat	61.94	45.17
19	SHAI.FRIENDSHIP GROUP INCO	<b>0.95</b>	<b>0.88</b>
	t Stat	72.50	53.76
20	SHAI.HAIXIN GP.	<b>0.95</b>	<b>0.90</b>
	t Stat	68.33	50.81

21	SHAI.HIGHLY	<b>0.97</b>	<b>0.89</b>
	t Stat	65.84	51.27
22	SHAI.JINJIANG INTL. INDL.	<b>0.98</b>	<b>0.89</b>
	t Stat	75.61	53.66
23	SHAI.JIN JIANG INTL. HTLS.DEV	<b>0.99</b>	<b>0.89</b>
	t Stat	76.10	58.04
24	SHAI.JINQIAO EXPT. PROC.ZONE DEV.	<b>1.00</b>	<b>0.89</b>
	t Stat	75.59	57.49
25	SHAI. LIAN HUA FIBRE	<b>0.95</b>	<b>0.88</b>
	t Stat	42.12	50.84
26	SHAI. MRA.TRD	<b>0.98</b>	<b>0.88</b>
	t Stat	65.51	51.70
27	SHAI. MECH. & ELECT. IND.	<b>0.96</b>	<b>0.89</b>
	t Stat	66.98	51.14
28	SHAI.POTEVIO	<b>0.97</b>	<b>0.89</b>
	t Stat	62.12	56.41
29	SHAI.SANMAO ENTER.(GP.)	<b>0.98</b>	<b>0.88</b>
	t Stat	58.64	51.88
30	SHAI. WAI GAOQIAO FREE TRADE	<b>0.98</b>	<b>0.90</b>
	t Stat	69.44	59.30
31	SHAI.WINGSUNG DATA TECH.	<b>0.93</b>	<b>0.89</b>
	t Stat	53.59	53.09
32	SHAI.YAOHUA PILKINGTON GLASS	<b>0.94</b>	<b>0.89</b>
	t Stat	62.07	50.62
33	SVA ELECTRON	<b>0.98</b>	<b>0.90</b>
	t Stat	65.68	54.06
34	ZHONGLU	<b>0.89</b>	<b>0.88</b>
	t Stat	52.31	50.94

# Chapter 5

## Incorporating the Sharpe Ratio and the Market Price of Risk into Asset Allocation

### 5.1: Introduction

Finding reliable and accurate measures to assess and compare the performance of portfolios and stocks has been a subject commanding the attention of the finance literature for a long time. Before the 1960s investors evaluated portfolio performance using the rate of return only; risk was not included in the analysis. The development of portfolio theory and the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965), and Black (1972) provided the foundation for risk-adjusted analysis. Risk, measured by either the standard deviation or beta, has since then been included in the evaluation process.

In the portfolio theory, it is assumed that investors always prefer higher returns to lower returns for a given level of risk; likewise for a given level of return, the assumption is that investors prefer a lower risk to a higher risk. As return and risk are two important quantities in measuring the performance of an investment, it is crucial to consider both return and risk in the selection of assets.

The market price of risk, or the Sharpe Ratio<sup>38</sup> is one of the most popular performance measures. It is defined as the ratio of the expected return to the standard deviation of the

---

<sup>38</sup> The market price of a risk is the alternative term to the Sharpe ratio. To distinguish between them in this chapter, the measurement will be referred to as the “market price of risk” wherever the assumption is made that the stock prices behave in accordance with a Geometric Brownian Motion (GBM). However, this measurement will be termed a “Sharpe ratio” wherever it is not assumed that the stock prices behave in this way.

returns. It captures both return and risk. According to the generalized Sharpe rule, a new asset with a higher Sharpe ratio has a higher probability of being selected (Sharpe, 1966, 1975, 1994; Dowd, 1999, 2000; Hodges, 1998 and Amin and Kat, 2002).

The aim of this chapter is to examine and compare the performance of three methods of measuring the trade-off between the risk and the return of trading stocks in both South Africa and China. This chapter has two primary objectives:

The first objective is to compare the performance of the market price of the risk and the Sharpe ratio for 13 companies listed on the JSE.

In the market price of the risk test employed in achieving this first objective, it was assumed that the stock prices behave according to GBM. Firstly, the constant drift and the volatility is estimated, then the drift term is decomposed into a risk-free rate and the market price of risk-multiplying volatility. The market price of risk is assumed to be constant and independent of time. The Maximum Likelihood Method is adopted to estimate the parameters.

In the Sharpe ratio test, the Maximum Likelihood Estimation Method is adopted to estimate the parameters in the Sharpe ratio equation, developed by Sharpe in 1966. In this way the Sharpe Ratio becomes a forward-looking risk measurement tool instead of a backward-looking risk measurement tool. t-statistics are provided to show the significance of the difference between the market price of risk and the Sharpe ratio.

The Second Objective is to study the risk and return relationships of *A* and *B* shares listed on the Shanghai Stock Exchange (SSE).

The first test proposed to achieve this objective is a test to ascertain whether *A* and *B* shares have a different market price of risk. Both *A* and *B* shares are issued by the same company and are issued with virtually the same voting rights and dividends. The difference that exists may be caused by different risk-free rates or different volatilities, as this chapter will show. In

other words, the test is to determine whether they have the same market price of risk. The procedure in this test is exactly as it is in the first test to achieve the first objective described above. However, the data used for this test are the daily closing prices of *A* and *B* shares of the companies who issue both classes of shares listed on the SSE. The theory behind the analysis is straightforward: since the corresponding *A* and *B* shares are issued by the same company and have identical voting policies and dividend rights, if company-specific fundamentals are accepted as given and the prices of the corresponding *A* shares *B* shares are derived from the fundamentals are assumed, then their market price of risk should be identical since they share the same company-specific risk. If investors view the firm-specific risk as the only risk they bear, then the shares should have the same market price of risk.

On the other hand, if the market price is different this indicates that, although sharing the same firm-specific risk, *A* and *B* shares are considered to be at different market-risk levels and thus are expected to have different excess returns for investors. Furthermore, besides the comparison of the market price of risks for individual *A* and *B* shares, we can also stack all *A* shares or *B* shares' returns and test the average market price of the risks for the two groups. This test is reliable insofar as it relates to the individual results since it uses the average of the individual estimators and thus provides more intuitive results for *A* and *B* shares as a whole.

The second and the final test to achieve the second objective concerns estimating and comparing the Sharpe ratios of *A* shares and *B* shares-based historical-return data.

The rest of the chapter is organized as follows: section 5.2 will provide an overview of the risk and return relationship. Section 5.3 presents the econometric methodology, Section 5.4 and 5.5 describe the data and report on the empirical results of the research. Readers will find the conclusion in the last section (section 5.6).

## 5.2: Risk and Return in the Equity Markets

### 5.2.1: The Risk-Return Relationship

The importance of the relationship between risk and return in the field of finance cannot be overstated. It is the most basic, yet most important theoretical concept of the discipline. Commonly referred to as “no free lunch”, this principle states that in the long term, it is not possible to improve returns without incurring a proportionally larger degree of risk. All asset-pricing models are based on this assumption. For example, Sharpe’s (1964) Capital Asset Pricing Model CAPM introduces the beta parameter, defined as the ratio of the covariance of the return of an asset with that of the market, divided by the variance of the return on the market. Both the numerator and the denominator in the beta calculation are therefore risk measures.

Although asset-pricing models state that only systematic risk should affect returns, several authors have developed models that take idiosyncratic risk into account. Building on the principles of the CAPM, Mao (1971), Levy (1978), Merton (1987), Malkiel and Xu (2002) constructed elaborate models of limited diversification. Four main reasons are given to explain why individuals may choose to hold such “undiversified” or limited, portfolios, namely: transaction costs, taxes, employment compensation, and private information. These extensions of the CAPM result in an additional beta with respect to a market wide measure of idiosyncratic risk.

Kryzanowski and To (1982) compared and reconciled two main contributions to the literature on asset-pricing. Taking into account the imperfect information available, they extracted two testable empirical relationships. The first testable relationship was given by Levy (in 1978) and stated that an individual investor’s portfolio is mean-variance efficient with regard to the number of securities held in the portfolio and that any security included in the portfolio is

linearly related to the intra-portfolio risk of that security. The second testable hypothesis, albeit much less easily tested, was a component of Mao's model (1971).

Mayers (1976) also began with the CAPM framework, but included a non-traded human capital factor. Barberis and Huang (2001) have more recently presented a different perspective by distinguishing between different levels of loss aversion. They concluded that investors exhibit loss aversion to fluctuations of owned individual stocks as opposed to fluctuations of their entire portfolio.

Over the last thirty years an enormous amount of empirical literature has attempted to prove the positive link established by financial theory between expected market returns and the conditional volatility of the aggregate stock market. Pindyck (1984) stated that an increase in volatility during the 1970s led to an increase in the expected risk premium. This was defined as the difference between the market return and the risk-free rate, which in turn led to a decline in stock prices<sup>39</sup>. However, Pindyck (1984) failed to provide a direct test of the relationship between expected returns and volatility. French, Schwert and Stambaugh (1987) set out to test this relationship. Using two separate models, they obtained positive but insignificant results. However, they did obtain a significantly negative relationship between unexpected volatility and excess holding period returns<sup>40</sup>, which they interpreted as supporting the positive relationship between volatility and expected returns<sup>41</sup>.

Turner, Startz and Nelson (1989) developed a model where the market switches between two states: a high-variance state and a low-variance state. Thus, excess return is drawn from two normal densities and the state in each period determines which of the two normal densities will be used for that period. This model structure is heteroskedastic in construction and has a strong time-dependence, which according to the authors improves the forecasting ability for

---

<sup>39</sup> In order to obtain the higher expected risk premium, given a constant cash flow stream generated by the firm, one must be able to buy the stock for less, that is, at a lower stock price.

<sup>40</sup> If the standard deviation increases today, the positive relationship with the risk premium will cause the discount rate for future cash flows to increase, thereby effectively decreasing the present value of these future cash flows and the stock price also.

<sup>41</sup> French, Schwert and Stambaugh (1987) also showed that this large negative relationship is too large to be explained solely by the leverage effect proposed by Black (1979) and Christie (1982).

the conditional variance of the market and, in turn, the risk-return model. By using this two-state model on post World-War II S&P data, Turner, et al. (1989) found a negative correlation between risk and return. Bailie and DeGennaro (1990) also examined the relationship between stock returns and volatility and concluded that the empirical evidence suggests that investors consider another measure of risk to be more important than the variance of portfolio returns in predicting returns.

Campbell and Hentschel (1992) revisited the “volatility feedback” concept first discussed by Pindyck (1984) and French, Schwert and Stambaugh (1987). They developed a complete formal model which accounts for the asymmetric properties of volatility. This is the notion that large negative stock returns are more common than large positive stock returns, and that volatility is typically higher after stock market declines than after stock market increases<sup>42</sup>.

The Campbell and Hentschel (1992) model accounted for the excess kurtosis of stock markets and the persistence of volatility<sup>43</sup>. Campbell and Hentschel concluded that volatility feedback has little effect on returns and contributes little to the unconditional variance of stocks.

Glosten, Jagannathan and Runkle (1993) examined the possibility that the GARCH-M methodology used by most of the previous research was misspecified and led to the inconclusive results found in the literature. They applied three modifications to the methodology; specifically, they allowed seasonal patterns in volatility; they allowed positive and negative innovations to returns with differing impacts; and they allowed nominal interest rates to help predict conditional variances. Their results support the negative correlation between volatility and expected returns found by Fama and French (1977), Campbell (1987), Breen, Glosten and Jagannathan (1989) and Harvey (1991). Whitelaw (1994) also found a negative contemporaneous correlation between risk and return.

---

<sup>42</sup>This was first discussed by Black (1976).

<sup>43</sup>Excess kurtosis refers to the fact that extreme stock price movements are more frequent than is expected if the changes are sampled from a normal distribution.

Because of the risk-return tradeoff, investors must be aware of their personal risk tolerance when choosing investments for their portfolios. Taking on some risk is the price of achieving returns; therefore, if you want to make money, you can't cut out all risk. The goal instead is to find an appropriate balance between risk and return.

### 5.2.2: Composite Risk and Returns

This chapter focuses on the trade-off between return and risk. One way of representing this trade-off is to combine return and risk by taking the expected return and dividing it by the risk measured. This single quantity is called the risk-adjusted performance ratio. There are several risk-adjusted measures, based on different notions of risk. An example of this is the Sharpe Ratio (the focus of this chapter) developed by Sharpe (1966). Sharpe examined the return of thirty four mutual funds in the period 1954-1963. He concluded that the differences in returns were due to the expenses of the mutual funds. He also found that a large proportion of the sample mutual funds failed to outperform the Dow Jones Index.

Treynor (1965) introduced the first formal technique to combine both risk and return in a single performance measure, known as the Treynor Measure. Jensen (1968) developed the Jensen alpha and examined the return of 115 mutual funds in the period 1945-1964 to estimate how much a manager's forecasting ability contributes to a fund's returns. He concluded that funds were on average not able to predict security prices well enough to outperform a buy-the-market-and-hold policy, and also that there was very little evidence that any individual fund was able to do significantly better than expected with mere random chance.

The Sharpe Ratio (SR) takes the mean of the excess return and divides it by the standard deviation of the excess return (e.g. Sharpe, 1970 and Sharpe, 1994). One important implication of using only the first and second moments of the excess returns is that positive

returns and negative returns are treated identically; large positive and negative returns of the same magnitude have the same effect on the risk measure.

Dowd (2000) analysed the Sharpe ratio using the Value at Risk (VaR) approach. He proposed a new generalised rule for risk adjustment and performance evaluation. Also, he proposed to use the VaR rather than the standard deviation. The theoretical methodology helped Dowd to conclude that the generalized Sharpe rule is straightforward to implement and can be easily programmed into software for decision makers to use. He declared in his paper that the Sharpe ratio structure is better than other ratios used to measure return over risk. However, correlations between portfolios can be a problem for the Sharpe ratio.

Asgharian and Hanson (2005) extended the work of MacKinlay and Pastor (2000) on the evaluation of missing risk factors when using the optimal orthogonal portfolio approach. They modified the Sharpe ratio squared that Mackinlay et al. (2000) had used. They used monthly returns for ten US portfolios. In their methodology, they used one exact factor model with an orthogonal portfolio, and two orthogonal models. They also used an out-of-sample Sharpe ratio to compare orthogonal portfolios. They confirmed the general importance of exploiting the relationship between return and risk.

Sharpe (1994) undertook a survey and summary of the different applications of the Sharpe ratio. He re-established this ratio as a measure of the expected return per unit of risk for a zero investment strategy. He pointed out that the Sharpe ratio was designed to measure the return adjusted by risk, and, when properly used, could improve the management of investments.

This study differs from the extant literature in several unique ways. Firstly, this examination makes the Sharpe Ratio a more powerful measurement of the trade-off between risk and return when the Maximum Likelihood Estimation Method is adopted to forecast this measure. Secondly, this study uses a more recent data sample with a daily frequency.

Another example of risk-adjusted measures based on different notions of risk is the market price of risk, which is also the focus of this chapter. The market price of risk is defined as the total return above the risk-free rate per unit of risk. For analytical purposes we can define the risk-free rate as the short-term money rates. The market price of risk captures the degree to which investors require a higher return for bearing the risk associated with an asset.

Under a risk-neutral valuation, which considers the market price of risk, the relevant Brownian motion for a traded asset, security or future contract can be specified as:

$dS = \mu S dt + S \sigma dz$ , which is a natural stochastic process where the  $\mu$  represents natural growth rates, the  $\sigma$  represents the standard deviation of the percentage change of the stochastic variables,  $S$  represents the stock price and  $dz$  represents a Wiener Process of the form  $\varepsilon t$ , where the  $\varepsilon$  values are standard normal deviates (i.e.,  $\varepsilon \sim N(0,1)$ ).

In terms of arbitrage pricing under the Black-Scholes-Merton argument the natural stochastic process and the derivation of the Black-Scholes model are given by:

$dS = r S dt + S \sigma dz$ , where  $r$  is the rate of return on default-free government bonds. Under the arbitrage model only the risk-free rate is relevant and not the natural rate, since it can be hedged.

In a general equilibrium framework the risk neutral valuation is given by:

$dS = (\mu - \lambda \sigma) S dt + S \sigma dz$ , which applies to both traded and non-traded assets, where  $\mu$  is the natural growth rate and  $\lambda \sigma$  is the risk premium. In equilibrium, with arbitrage opportunities, we can now state the following conditions:

$$r = \mu - \lambda \sigma$$

$$\mu = r + \lambda \sigma$$

$$\mu = r + \beta(R_m - r), \text{ where } \beta(R_m - r) \text{ is the risk premium.}$$

An estimation of the above relationship would provide an insight into the consistency of the arbitrage and equilibrium models.

## 5.3: Methodology

### 5.3.1: The Sharpe Ratio

One of the many risk/return trade-off measures to arise from the CAPM theory is the Sharpe Ratio (Sharpe, 1963). The formula for the Sharpe ratio is exceedingly simple:

$$SR_t = \frac{R_t}{\sigma} \quad (5.1)$$

Where,  $R_t = \mu_t - r_f$ ,  $\sigma = \left\{ \frac{\sum_{t=1}^n (\pi_{t-1} - \bar{\pi})^2}{n} \right\}^{\frac{1}{2}}$   $\mu_t = \frac{S_t - S_{t-1}}{S_{t-1}}$  (5.2)

Where  $r_f$  is the rate of return on a risk-free asset,  $S_t$  denotes the price of an asset at time  $t$  and  $\pi_t$  denotes some profit at time  $t$ . The time subscript  $n$  in the formula for the standard deviation is the number of lag values needed to compute the variance for the firm's profits. Thus far this is all standard fare. The decision rule then for investors is to maximize the level of profits (or returns) for a given level of risk. It would be irrational for a profit maximizer who is risk averse to invest in an asset with a higher risk level and a lower level of profits.

The Sharpe ratio is invariant to the  $R - \sigma$  ratio, so that as both the numerator and denominator increase the index remains the same. This invariance is a result of appealing to the mean-variance assumption which underlies the CAPM (for example, Szego, 1980 or Sharpe, 1964). For a model explicitly taking into account the Arrow-Pratt measure of risk aversion in the form of skewness (see Jaro and Na, 2001), this assumption for portfolio decision-making identifies the mean return as the first moment of a possible distribution of investments, and the variance of this return as the risk inherent in the investment.

All the relevant information concerning the portfolio of investments is present in these two moments, the mean and the variance. Since no other moments of the profit/return distribution enter the computation, the Sharpe index is unable to cope with the skewness or the kurtosis in the probability density function for an asset or portfolio's return. The above, though unintended, assumption in the Sharpe index is that when viewed from an activity-analysis perspective the only input necessary to produce a return  $R$  is  $\sigma$ . Obviously, then the inclusive nature of  $\sigma$  in (5.1) requires that  $\sigma$  be an adequate proxy for risk.

From equation (5.1) and (5.2) we can establish the following relationship:

$$S_t - S_{t-1} = S_{t-1}(r + SR\sigma) \quad \text{or} \quad dS = S_{t-1}(r + SR\sigma) \quad (5.3)$$

where  $SR$  is the Sharpe ratio. In order to estimate the parameters  $SR$  and  $\sigma$ , in (5.3) the Maximum Likelihood Estimation Method is proposed. Under these circumstances the Sharpe Ratio will be a forward-looking risk-measurement tool instead of a backward-looking risk-measurement tool. To the best of the researcher's knowledge, no one so far has derived equation (5.3) and obtained the Sharpe ratio by this method.

### 5.3.2: The Sharpe Ratio for Chinese $A$ and $B$ shares

This section compares the performance of the Sharpe ratios of  $A$  and  $B$  shares based on equation 5.1. The Sharpe ratio analyses are based on historical data for both  $A$  and  $B$  shares. The comparison follows the common rule to annualize the Sharpe ratios. This allows for a fair comparison of the different trading strategies even if the time scales of the strategies are not comparable. The aim is to test whether they have the same Sharpe ratios.

However, since  $A$  shares are traded in the domestic currency and  $B$  shares are traded in a foreign currency, the risk-free rate applied to estimate the Sharpe ratios should also be different. For  $A$  shares, the domestic risk-free rate will be applied, and, for  $B$  shares, the corresponding US risk-free rate. Since  $A$ -share investors trade in Chinese currency, they can also invest in domestic risk-free assets. Thus they will face a risk-free rate in Chinese Yuan. Alternatively, since  $B$ -share investors in the Shanghai stock exchange trade in US Dollars, they can instead earn a risk-free rate in US Dollars for the same reason; see Ma (1996) for discussion of market segmentation.

### 5.3.3: The Market Price of Risk

Consider the dynamics of the stock satisfy the following Stochastic Differential Equation (SDE):

$$dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t \quad (5.4)$$

$S_t$  is the stock price,  $\mu(t, S_t)$  and  $\sigma(t, S_t)$  denote the drift and volatility of the stock price process and both are a deterministic function of  $t$  and  $W_t$  is the corresponding Wiener process for the stock<sup>44</sup>.

Generally speaking it is hard to solve the SDE analytically. However, in some cases this can be done if we assume some specific form for  $\mu(t, S_t)$  and  $\sigma(t, S_t)$ . The most widely-used model is based on the assumption that stock prices follow Geometric Brownian Motion (GBM), in that case, the SDE (5.4) can be expressed as

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (5.5)$$

---

<sup>44</sup> a more detailed introduction to the Wiener process can be found in Hull, 2006.

The underlying assumption of GBM is that instead of having constant drift and constant variance, as for the Wiener process, the expected return is constant and the variability of the percentage return in an infinitesimal interval  $dt$  is constant.

Now we consider decomposing the expected return into two parts: the risk-free rate and the market price of risk. Now the dynamics of stock price can be written as follows:

$$dS_t = (r_f + \lambda\sigma)S_t dt + \sigma S_t dW_t \quad (5.6)$$

$r_f$  is the risk-free rate at time  $t$  and  $\lambda$  is the corresponding market price of risk. However the volatility term remains constant, now the parameters need to be estimated are  $\theta = (\lambda, \sigma)$ . The Maximum Likelihood Estimation Method is adopted to estimate these parameters.

#### 5.3.4: The Market Price of Risk for Chinese $A$ and $B$ shares

Consider that a company issues  $A$  and  $B$  shares, as shown in equation 5.5 above, assume that stock prices follow GBM, in this case the dynamics of both shares can be expressed as

$$dS_{At} = \mu_A S_{At} dt + \sigma_A S_{At} dW_{At} \quad (5.7)$$

$$dS_{Bt} = \mu_B S_{Bt} dt + \sigma_B S_{Bt} dW_{Bt} \quad (5.8)$$

$$dW_{At} dW_{Bt} = \rho dt$$

$S_{At}$  and  $S_{Bt}$  are the prices of  $A$  and  $B$  shares respectively;  $\mu_A$ ,  $\mu_B$ ,  $\sigma_A$  and  $\sigma_B$  denote the drift and volatility of stock price processes; and both the drift and volatility terms are constant.

$W_{At}$  and  $W_{Bt}$  are the corresponding Wiener process values for  $A$  and  $B$  shares, while  $\rho$  is the correlation coefficient between them.

Now we consider decomposing the expected return into two parts: the risk-free rate and the market price of risk. Now the dynamics of stock prices can be written as follows:

$$dS_{At} = (r_{f,At} + \lambda_A \sigma_A) S_{At} dt + \sigma_A S_{At} dW_{At} \quad (5.9)$$

$$dS_{Bt} = (r_{f,Bt} + \lambda_B \sigma_B) S_{Bt} dt + \sigma_B S_{Bt} dW_{Bt} \quad (5.10)$$

$r_{f,At}$  and  $r_{f,Bt}$  are the domestic and foreign risk-free rates at time  $t$  and  $\lambda_A$  and  $\lambda_B$  are the corresponding domestic and foreign market prices of risk. We will still adopt the maximum likelihood method to estimate the parameters, as in the previous section. The probability density function is the same as that described in appendix C.1, but we need to substitute the constants  $\mu_A$  and  $\mu_B$  with time-varying drift terms, as in (5.9) and (5.10). However, the volatility term remains constant (Zhu, 2008).

## 5.4: Data Description

The data used in this chapter are from two different markets: the JSE and the SSE.

### 5.4.1. The JSE

In this market, we will compare the Share ratio specified in equation (5.3) with the market price of risk in (5.6) for companies listed on the JSE. Our interest is to test the difference in the two risk measurements, and furthermore to determine whether this is significant.

Recall (5.5) from the model section described above, then the GBM specification assumes the following:

$$\frac{dS}{S} \sim \phi(\mu dt, \sigma \sqrt{dt})$$

By applying Ito's lemma, the logarithm of the price is normally distributed with mean  $(\mu - \frac{\sigma^2}{2})$  and variance rate  $\sigma^2$  (Hull, 2006). Thus,

$$\ln S_T - \ln S_0 \sim \phi \left[ \left( \mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right]$$

It is therefore necessary to test whether the natural logarithms of the stock prices in our data set follow a log-normal distribution. The natural logarithms of the stock prices were therefore tested for normality, using the Jarque-Bera normality test<sup>45</sup>. Only thirteen companies listed on the JSE for 200 daily closing prices satisfied Jarque-Bera normality test as shown in Table 5.1 below.

Table 5.1: Thirteen companies listed on JSE used in our analysis

Company	Observations	Starting Date	Ending Date	Jarque-Bera
AFRICAN BANK	200	2006/04/27	2007/01/31	2.108
BHP BILLITON	200	2006/05/25	2007/02/28	2.927
FIRSTRAND	200	2006/03/27	2006/12/29	3.252
GOLD FIELDS	200	2006/02/24	2006/11/30	4.719
HARMONY GOLD	200	2006/01/12	2006/10/18	1.795
IMPALA PLATINUM	200	2006/05/18	2007/02/21	1.247
MITTAL SA	200	2006/05/18	2007/02/21	4.899
MTN GROUP	200	2006/02/09	2006/11/15	3.287
PICK N PAY STORES	200	2006/04/26	2007/01/30	2.321
SASOL	200	2006/05/18	2007/02/21	1.429
STANDARD BK	200	2006/03/27	2006/12/29	4.056
STEINHOFF INTL	200	2006/03/16	2006/12/20	5.436
TELKOM	200	2006/04/17	2007/01/19	4.699

<sup>45</sup> Refer to footnote number 15 in page 41.

These findings indicate that GBM properties exist in this data set and should therefore, using GBM, produce accurate results. Nevertheless, short term money rate was used as a proxy for determining the risk-free rate.

#### 5.4.2. The SSE

The segmentation of *A* and *B* share markets has been regarded as a stylized fact, and most previous studies used data prior to February 11, 2001, when *B* shares were available only to Foreign investors. Since February 2001, the *B* share market has been conditionally<sup>46</sup> available to domestic investors also. With a more liberal investment environment, it becomes more meaningful to reinvestigate the dynamic relationship between *A* and *B* share markets by incorporating the impact of policy changes and other exogenous shocks.

Firstly, this study investigates whether *A* and *B* shares have the same market price of risk where the GBM is adopted as a benchmark. This is to test the difference in expected returns  $\mu_A - \mu_B$  and the volatilities  $\sigma_A - \sigma_B$ , and furthermore, to ascertain whether they are significant, and whether this difference is caused by the different market prices of risk for *A* and *B* shares.

Secondly, the study examines whether *A* and *B* shares have the same Sharpe ratio based on the historical data. Both investigations are undertaken using companies' issue *A* and *B* shares listed on the SSE.

In February 2001, the Chinese capital market regulator announced a policy to lift restrictions on domestic investors trading in *B* shares. The policy was announced on February 19, 2001 and as a consequence, the price of *B* shares increased dramatically in the week after the

---

<sup>46</sup>Chinese currency RMB is not freely tradable, but foreign currencies are freely tradable among themselves. So, only investors who have access to foreign currency can invest in *B* shares freely.

announcement. The policy change led to a price discount of less than - 40% immediately after its announcement. To exclude this period the analysis would only be applied from 2008<sup>47</sup>.

Firstly, *A* and *B* share prices were collected; the *B* share prices were converted into Chinese Yuan using official exchange rates. Secondly, the natural logarithms of the stock prices were tested for normality using the Jarque-Bera normality test, as suggested by the GBM properties. Only thirteen companies listed on the SSE satisfied Jarque-Bera normality test as shown in Table 5.2 below.

Table 5.2: Fifteen companies listed on the SSE which issue both *A* and *B* shares used in our analysis

Company	Observations	Starting Date	Ending Date	A-Share J-B test	B Share J-B test
Dazhong Transportation Group	60	2008/03/10	2008/05/30	0.255	2.955
Double Coin Holdings	60	2008/03/10	2008/05/30	1.374	5.763
Eastern Communications	50	2008/03/10	2008/05/16	2.979	2.333
Huaxin Cement	60	2008/03/10	2008/05/30	1.586	4.244
Jinan Qingqi Motorcycle	50	2008/03/10	2008/05/16	2.871	5.347
SGSB Group	50	2008/03/24	2008/05/30	4.442	0.660
Shanghai Automation Instrumentation	60	2008/03/10	2008/05/30	4.867	2.958
Shanghai Dajiang Group	60	2008/03/10	2008/05/30	3.834	0.052
Shanghai Dingli Tech Dev (Group)	50	2008/03/24	2008/05/30	1.331	0.914
Shanghai Friendship Group	60	2008/03/10	2008/05/30	1.889	0.055
Shanghai Highly Group	60	2008/03/10	2008/05/30	5.848	0.664
Shanghai Jinjiang Intl Hotels Dev	60	2008/03/10	2008/05/30	2.641	5.999
Shanghai Jinqiao Export Process	60	2008/03/10	2008/05/30	4.995	1.620
Shanghai Yaohua Pilkington Glass	50	2008/03/10	2008/05/16	2.432	5.230
Zhonglu Co	60	2008/03/10	2008/05/30	2.136	3.062

The model specified by the market price of the risk equation in (5.9) and (5.10) also requires the use of a risk-free interest rate. As mentioned above, the 3-month US Treasury bill rate and the 3-months Chinese deposit rate have been used as proxies for the risk-free interest rate for the US and China respectively.

<sup>47</sup> The entire data set was collected from the University of Cape Town DataStream database on November 15, 2008.

## 5.5: Study Results

### 5.5.1: The JSE Results and Analysis

The study tested the null hypothesis for the equality of the Sharpe ratio as specified in equation (5.3) with the market price of risk in (5.6) for several companies listed on the JSE.

Table 5.3 in appendix C.2 presents the estimation results of the Sharpe ratio and the market price of risk, as well as the drift and volatility for each measurement. Table 5.3 illustrates several features of these estimated parameters. Firstly, almost all the drift terms obtained via equation (5.3) of the Sharpe ratio are identical to those obtained via equation (5.6) of the market price of risk, as shown, in Figure 5.1.

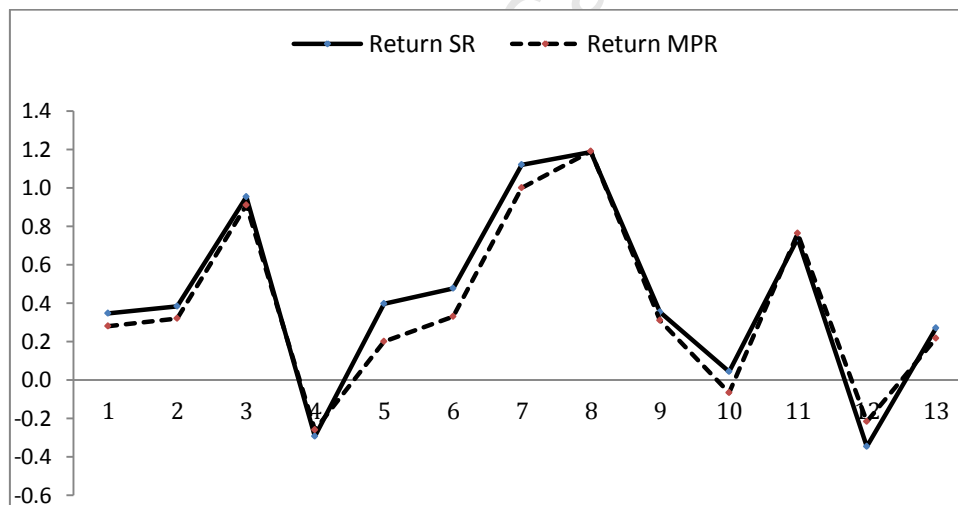


Figure 5.1: Drift terms obtained by Sharpe ratio equation (Solid line) and drift terms obtained by market price of risk equation (dotted line)

Secondly, the annual volatilities obtained in (5.3) are very closely related to those obtained via (5.6) with almost the same degree of co-movement, as shown in Figure 5.2.

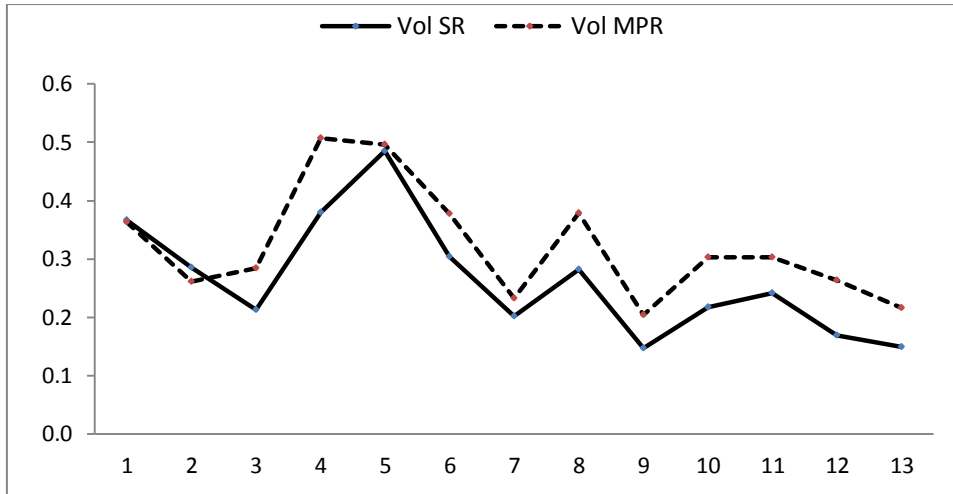


Figure 5.2: Volatility terms obtained by Sharpe ratio equation (Solid line) and volatility terms obtained by market price of risk equation (dotted line)

Thirdly, let us focus on the estimation of the  $SR$  (Sharpe ratio) and  $\lambda$ , the (market price of risk). We have shown that  $\mu_{SR}$ , the drift terms obtained via equation (5.3) of the Sharpe ratio are almost identical to  $\mu_{MPR}$ , the drift terms obtained via equation (5.6) of the market price of risk. Table 5.3 shows that it is also the same for the Sharpe ratio and the market price of risk.

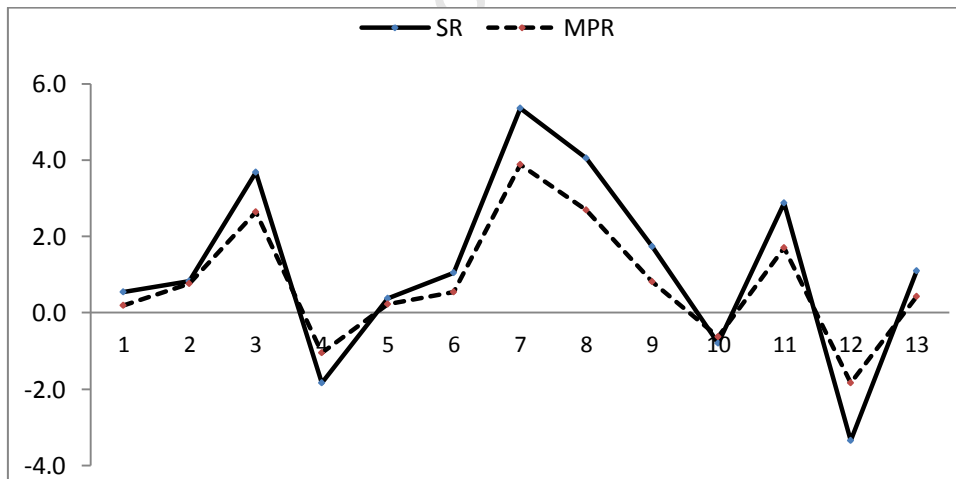


Figure 5.3: Sharpe ratio (Solid line) and market price of risk (dotted line)

Almost all the Sharpe ratio terms are larger than the market price of the risk terms, but the  $t$ -value is not significant for the difference. The results confirm that the Sharpe ratio  $SR$  obtained via (5.3) is as efficient as the market price of risk  $\lambda$  obtained in (5.6). To the best of the researcher's knowledge, this is the first study that has derived equation (5.3) and used the

Maximum Likelihood Estimation Method to estimate the Sharpe ratio  $SR$  from (5.3), and compared these results with the market price of risk  $\lambda$  and obtained this interesting result.

## 5.5.2: The SSE Results and Analysis

### 5.5.2.1: Constant Expected Return and Volatility

Table 5.4 in appendix C.3 presents the estimation results of the drift and the volatility values. Table 5.4 shows several features of these estimated parameters. Firstly 53% of the drift terms of  $B$  shares are larger than those of the corresponding  $A$  shares and 46% are lower. However, the t-values are not significant for these differences, as shown in Figure 5.4.

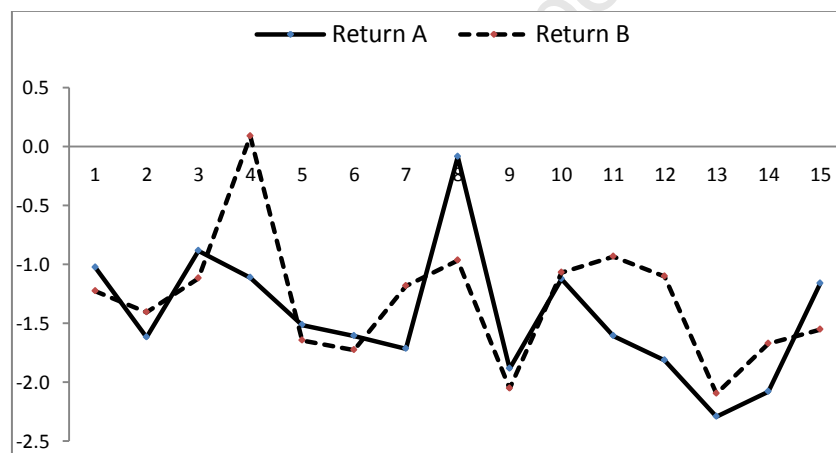


Figure 5.4: The drift terms of  $A$  shares (Solid line) and the corresponding  $B$  shares (dotted line)

From this result it can be established that the expected returns of the  $B$  shares are on average closely related to those of the  $A$  shares.

Secondly, the annual volatility values for all  $A$  and  $B$  shares are higher than those found in more mature markets. For example, Campbell, Lo and Mackinlay (1997) provide the estimated log-run annualised volatility values in the US stock market these were below 0.3.

However, in this study's estimation, the SSE shows much higher volatilities for all the shares. None of the estimates is below 40% and the largest value is above 70%. This kind of high volatility is a feature of an emerging market, as pointed out by many researchers. Taking the short development period of the Chinese stock market into consideration, the high volatility can be taken as a reflection of more fluctuation and speculation in investors' performance.

The interesting thing is that most of the volatility terms of *A* shares are also higher than those of the corresponding *B* shares, as shown in figure 5.5.

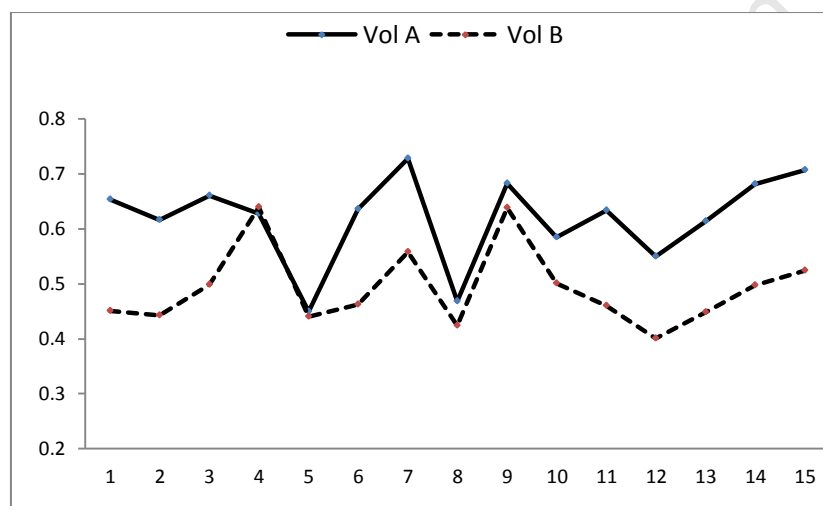


Figure 5.5: The Volatility terms of *A* shares (Solid line) and the corresponding *B* shares (dotted line)

This result seems to be consistent with those found in previous studies. For example, some researchers argue that the *B* share market is less liquid than the *A* share market and thus investors require a liquidity premium in order to compensate for *B* shares. This partly contributes to the *B* share-pricing puzzle. Since *B* shares are less liquid than *A* shares, it is reasonable to assume that the volatility of *B* shares is also less than that of the corresponding *A* shares.

*B* shares have a lower trading volume than corresponding *A* shares. This leads to *B* shares exhibiting a lower volatility than *A* shares. Another factor that can be seen to contribute to this matter is the Chinese government's opening up of the *B* share market to domestic

residents in June 2001 and the *A* share market to overseas investors in December 2002. *A* share prices fluctuated more frequently than *B* share prices around that time. This also increased the volatility of these shares.

The last row of Table 5.4 (in appendix C.3) presents the average differences for drifts and volatilities. *t*-statistics show only the average differences in volatilities as positive and significant. Thus, it is safe to say that as a whole the volatility of *A* shares is higher than that of *B* shares.

### 5.5.2.2: Market Price of Risk with Constant Volatility

Table 5.4 in appendix C.3 presents the estimation results of the market price of risk and volatility terms. It has been shown that *A* and *B* shares have different expected returns  $\mu$ , but those differences are not significant. Table 5.4 shows that this is also the same for the market price of risk, that is, the difference between the market price of risk for the two types of shares  $\lambda_A - \lambda_B$  is positive for most pairs, but the difference is not significant. However, the level is not the same as the difference between the expected returns  $\mu_A - \mu_B$  dealt with in the previous subsection.

The *t*-values are bigger than those of the expected returns. This means that as a whole the market price of risk for *A* shares is higher but not significantly so than that for *B* shares, as shown below in figure 5.6.

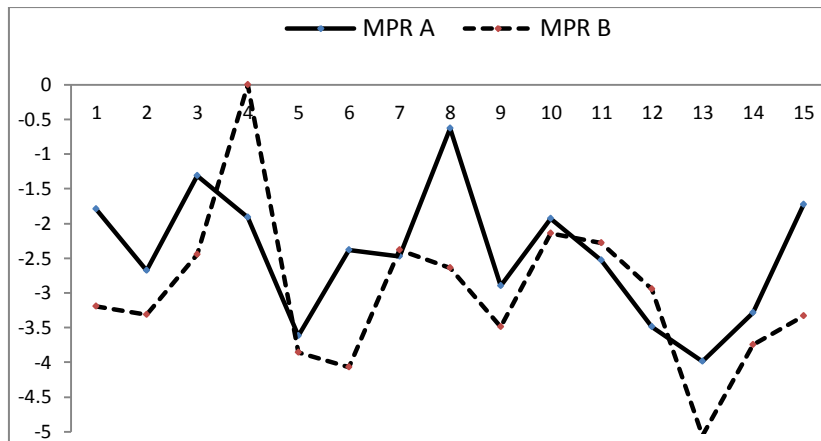


Figure 5.6: The Market price of risk terms of *A* shares (Solid line) and the corresponding *B* shares (dotted line)

This makes sense because both *A* and *B* shares are issued by the same company and have virtually the same voting rights and dividends. Thus, *A* and *B* shares should have the same market price of risk and the same return, as this study has shown. However, the difference in prices is caused by different risk-free rates and different volatilities.

All in all, the empirical results confirm that the price discount is closely related to different volatilities. However, there is no significant difference between the market prices of risk for these twin shares.

### 5.5.2.3: The Sharpe Ratio

Table 5.5 in appendix C.4 presents the estimation results of the Sharpe ratio, as well as the drift and the volatility values using historical data related to the *A* and *B* shares.

Table 5.5 illustrates several features of these estimated parameters. First, all of the drift and 90% of the volatility terms of the *A* shares are larger than those of the corresponding *B* shares. In the last row in Table 5.5, an averaged difference for drifts and volatilities is presented. Both are positive and the t-statistics show that they are significant. Thus, it is safe

to say that both the expected returns and the volatility values for *A* shares are higher than those for *B* shares.

This study has shown that *A* shares have a higher expected return  $\mu$  and volatility  $\sigma$  than corresponding *B* shares. Table 5.5 illustrates the finding that this is also the same for the Sharpe ratio, that is, the difference between the Sharpe ratio  $SR_A - SR_B$  is positive for all pairs, as shown in Figure 5.7, and this difference is significant.

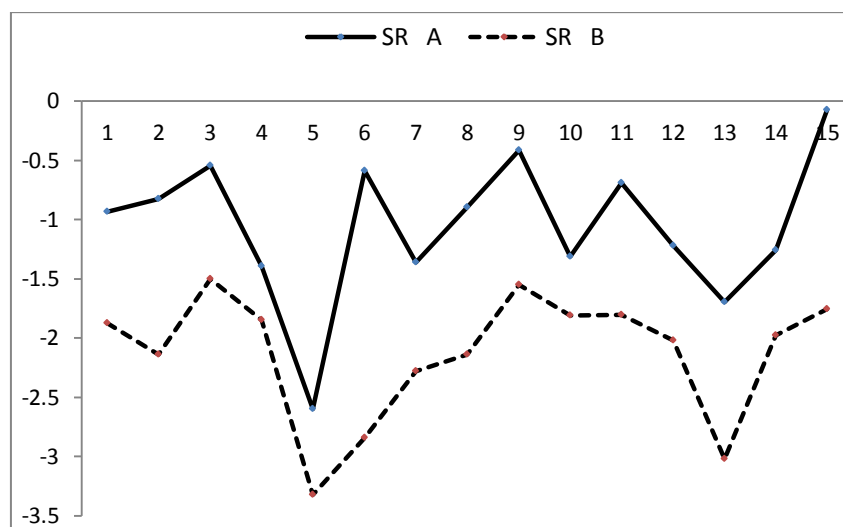


Figure 5.7: The Sharpe ratio terms of *A* shares (Solid line) and the corresponding *B* shares (dotted line)

From the last row, in which the averaged differences are presented, it is clear that the three (return, volatility and Sharpe ratio) are all positive and significant at a level of 10%. This means that as a whole the Sharpe ratio for *A* shares is higher than that for *B* shares.

## 5.6. Conclusion

This chapter first investigated the difference between the market price of risk and the Sharpe ratio for several companies listed on the JSE. In obtaining the market price of risk, the stock price was assumed to behave as GBM, and then the drift term was decomposed into a risk-

free rate and the market price of risk-multiplying volatility. The market price of risk was assumed to be constant and time-independent. The Maximum Likelihood Method was adopted to estimate the market price of risk.

The Maximum Likelihood Estimation Method was adopted to estimate the Sharpe ratio. By this method was the basis from which to compare the two risk measurements, as both of them are forward-looking risk-measurement tools.

The result shows that all Sharpe ratio terms were larger than the market price of risk terms, but the t-value was not significant for the difference, at the level of 1%. From the results we can demonstrate that the Sharpe ratio  $SR$  obtained via (5.3) is as efficient as the market price of risk  $\lambda$ , obtained in (5.6)

Secondly, this chapter investigated the behaviour of the corresponding stock prices in segmented markets: the stock prices of  $A$  and  $B$  shares for domestic and foreign investors. Both the  $A$  and  $B$  shares were issued by the same company, with the same voting rights and the same dividends, yet they are held by different investors and priced differently. The  $B$  shares are priced at a significant discount compared to the corresponding  $A$  shares. The GBM model was adopted to describe the dynamics of the stock prices. Firstly, the price discount can be explained by the higher volatility of the  $A$  shares. Furthermore, the market price of risk for  $A$  shares  $B$  shares is almost identical and the difference between them is not significant at a level of 1% for all companies. It was concluded that the difference in prices is caused by different risk-free rates and different volatilities. However, the historical estimation of the Sharpe ratio for  $A$  shares  $B$  shares suggests that the different price level between  $A$  shares  $B$  shares has resulted in a difference in the Sharpe ratio between them. The return, volatility and Sharpe ratio for  $A$  shares are all significantly larger than those of the corresponding  $B$  shares. This result suggests holding  $A$  shares is less risky than holding  $B$  shares. Thus, the different price levels between  $A$  shares  $B$  shares have resulted in difference in the Sharpe ratio between them.

## 5.7: Appendix C

### 5.7.1: C.1 The probability density function

In the following procedure, we assume that the time to liquidation  $T - t$  is a constant number, our interest is to test the difference in expected returns  $\mu_B - \mu_A$ , and furthermore if it is significant, whether this difference is caused by different market price of risk for  $A$  and  $B$  shares. From (5.7) and (5.8) we know that the pair of log price follows the Bivariate Distribution:

$$\begin{pmatrix} r_{A,t} \\ r_{B,t} \end{pmatrix} \sim N \left( \begin{pmatrix} \mu_A - \frac{1}{2}\sigma_A^2 \Delta t \\ \mu_B - \frac{1}{2}\sigma_B^2 \Delta t \end{pmatrix}, \begin{pmatrix} \sigma_A^2 \Delta t & \rho \sigma_A \sigma_B \Delta t \\ \rho \sigma_A \sigma_B \Delta t & \sigma_B^2 \Delta t \end{pmatrix} \right), r_{A,t} = \log S_{A,t} - \log S_{A,t-\Delta t}, r_{B,t} = \log S_{B,t} - \log S_{B,t-\Delta t}$$

Then the joint pdf for  $r_{A,t}, r_{B,t}$  is

$$f(r_{A,t}, r_{B,t}; \theta) = \frac{1}{2\pi\sigma_A\sigma_B\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)} \left[ \frac{\left(r_{A,t} - \left(\mu_A - \frac{1}{2}\sigma_A^2\right)\Delta t\right)^2}{\sigma_A^2\Delta t} - 2\rho \frac{\left(\mu_A - \left(\mu_A - \frac{1}{2}\sigma_A^2\right)\Delta t\right)\left(r_{B,t} - \left(\mu_B - \frac{1}{2}\sigma_B^2\right)\Delta t\right)}{\sigma_A\sigma_B\Delta t} + \frac{\left(r_{B,t} - \left(\mu_B - \frac{1}{2}\sigma_B^2\right)\Delta t\right)^2}{\sigma_B^2\Delta t} \right]\right\}$$

And  $\theta$  is the parameter vector:

$$\theta = (\mu_A, \mu_B, \sigma_A, \sigma_B, \rho)$$

The conditional log likelihood function of  $r_{A,t}, r_{B,t}$  is therefore:

$$l_t(r_{A,t}, r_{B,t}; \theta) = -\log(2\pi) - \log(\sigma_A) - \log(\sigma_B) - \log(\Delta t) - \frac{1}{2}\log(1-\rho^2) - \frac{1}{2(1-\rho^2)} \left[ \frac{\left(r_{A,t} - \left(\mu_A - \frac{1}{2}\sigma_A^2\right)\Delta t\right)^2}{\sigma_A^2\Delta t} - 2\rho \frac{\left(r_{A,t} - \left(\mu_A - \frac{1}{2}\sigma_A^2\right)\Delta t\right)\left(r_{B,t} - \left(\mu_B - \frac{1}{2}\sigma_B^2\right)\Delta t\right)}{\sigma_A\sigma_B\Delta t} + \frac{\left(r_{B,t} - \left(\mu_B - \frac{1}{2}\sigma_B^2\right)\Delta t\right)^2}{\sigma_B^2\Delta t} \right]$$

The log likelihood function of the whole data series is

$$L(r_{A,1}, r_{B,2}, \dots, r_{A,T}, r_{B,T}; \theta) = \sum_{t=1}^T l_t(r_{A,t}, r_{B,t}; \theta)$$

The maximum likelihood estimator is therefore the choice of parameters  $\theta$  that maximize this equation.

## 5.7.2: C.2 Sharpe ratio and Market price of risk estimation for JSE

Table 5.3: Sharpe ratio and Market price of risk estimation for JSE (totally 13)

	Sharpe Ratio			Market Price of Risk			The Significant Different		
	$\mu_{SR}$	$\sigma_{SR}$	$SR$	$\mu_{MPR}$	$\sigma_{MPR}$	$\lambda$	$\mu_{SR} - \mu_{MPR}$	$\sigma_{SR} - \sigma_{MLH}$	$SR - \lambda$
AFRICAN BANK	0.347	0.366	0.550	0.281	0.364	0.192	0.066	0.003	0.358 (0.114)
BHP BILLITON	0.384	0.285	0.827	0.318	0.261	0.766	0.066	0.024	0.061 (0.017)
FIRSTRAND	0.954	0.213	3.682	0.908	0.284	2.641	0.046	-0.071	1.041 (0.249)
GOLD FIELDS	-0.294	0.380	-1.836	-0.261	0.507	-1.045	-0.033	-0.128	-0.791 (-0.215)
HARMONY GOLD	0.395	0.485	0.380	0.200	0.496	0.227	0.195	-0.011	0.153 (0.045)
IMPALA PLATINUM	0.476	0.304	1.043	0.331	0.377	0.541	0.145	-0.073	0.502 (0.123)
MITTAL SA	1.120	0.202	5.362	1.001	0.233	3.882	0.119	-0.031	1.480 (0.333)
MTN GROUP	1.187	0.282	4.048	1.191	0.379	2.684	-0.004	-0.096	1.364 (0.388)
PICK N PAY STORE	0.354	0.147	1.725	0.309	0.204	0.808	0.045	-0.057	0.917 (0.202)
STANDARD BK	0.041	0.218	-0.798	-0.067	0.303	-0.633	0.108	-0.085	-0.165 (-0.040)
SASOL	0.736	0.241	2.881	0.764	0.303	1.700	-0.029	-0.062	1.181 (0.349)
STEINHOFF	-0.348	0.169	-3.354	-0.217	0.264	-1.842	-0.130	-0.095	-1.512 (-0.318)
TELKOM	0.270	0.150	1.091	0.217	0.216	0.418	0.054	-0.067	0.673 (0.148)
<b>Averaged Difference</b>							<b>0.061</b>	<b>-0.053</b>	<b>0.336</b> <b>(0.091)</b>

$H_0: SR - \lambda = 0$  the value in parentheses are the t-statistics \* Significance level of 10%, \*\* Significance level of 5%, Significance level of 1%,

### 5.7.3: C.3 Market price of risk estimation for SSE

Table 5.4: Market price of risk estimation for fifteen companies listed in SSE (March. 10, 2008 – May. 30, 2008)

	<i>A – share</i>			<i>B – share</i>			<i>The Significant Different</i>		
	$\mu_A$	$\sigma_A$	$\lambda_A$	$\mu_B$	$\sigma_B$	$\lambda_B$	$\mu_A - \mu_B$	$\sigma_A - \sigma_B$	$\lambda_A - \lambda_B$
Dazhong Transportation Group	-1.029	0.654	-1.788	-1.230	0.451	-3.192	0.202 (0.072)	0.202*** (15.799)	1.404 (0.273)
Double Coin Holdings	-1.617	0.616	-2.679	-1.406	0.443	-3.313	-0.211 (-0.084)	0.173*** (16.957)	0.634 (0.136)
Eastern Communications	-0.884	0.660	-1.311	-1.118	0.498	-2.448	0.234 (0.091)	0.162*** (14.644)	1.137 (0.256)
Huaxin Cement	-1.116	0.628	-1.908	0.088	0.639	-0.005	-1.204 (-0.351)	-0.012 (-0.985)	-1.903 (-0.356)
Jinan Qingqi Motorcycle	-1.513	0.449	-3.615	-1.644	0.440	-3.854	0.132 (0.061)	0.009** (2.305)	0.239 (0.049)
SGSB Group	-1.609	0.636	-2.380	-1.728	0.463	-4.069	0.119 (0.043)	0.174*** (14.481)	1.689 (0.341)
Shanghai Automation Instrumentation	-1.718	0.727	-2.472	-1.184	0.559	-2.380	-0.535 (-0.177)	0.169*** (14.992)	-0.093 (-0.020)
Shanghai Dajiang Group	-0.088	0.469	-0.631	-0.966	0.424	-2.642	0.878 (0.409)	0.045*** (6.157)	2.011 (0.422)
Shanghai Dingli Tech Dev (Group)	-1.884	0.682	-2.899	-2.053	0.638	-3.491	0.169 (0.053)	0.044*** (3.762)	0.593 (0.121)
Shanghai Friendship Group	-1.122	0.585	-1.929	-1.068	0.501	-2.147	-0.054 (-0.022)	0.084*** (11.483)	0.217 (0.048)
Shanghai Highly Group	-1.609	0.633	-2.532	-0.932	0.460	-2.285	-0.677 (-0.261)	0.173*** (15.691)	-0.246 (-0.051)
Shanghai Jinjiang Intl Hotels Dev	-1.812	0.550	-3.492	-1.100	0.401	-2.949	-0.711 (-0.311)	0.148*** (21.485)	-0.543 (-0.113)
Shanghai Jinqiao Export Process Zone	-2.293	0.614	-3.984	-2.094	0.449	-5.054	-0.200 (-0.077)	0.165*** (12.425)	1.070 (0.220)
Shanghai Yaohua Pilkington Glass	-2.082	0.682	-3.284	-1.674	0.498	-3.752	-0.408 (-0.135)	0.184*** (15.365)	0.468 (0.092)
Zhonglu Co	-1.164	0.707	-1.725	-1.556	0.524	-3.330	0.392 (0.139)	0.184*** (16.471)	1.605 (0.351)
Averaged Difference							-0.125 (-0.037)	0.127*** (12.069)	0.552 (0.118)

$H_0: \mu_A - \mu_B = 0$ ,  $\sigma_A - \sigma_B = 0$  and  $\lambda_A - \lambda_B = 0$ , the value in parentheses are the t-statistics.

The \* Significance level of 10%, \*\* Significance level of 5%, Significance level of 1%

### 5.7.4: C.4 Sharpe Ratio Estimation for SSE

Table 5.5: Sharpe ratio estimation for fifteen companies listed on SSE (March. 10, 2008 – May. 30, 2008)

	<i>A – share</i>			<i>B – share</i>			<i>The Significant Different</i>		
	$\mu_A$	$\sigma_A$	$SR_A$	$\mu_B$	$\sigma_B$	$SR_B$	$\mu_A - \mu_B$	$\sigma_A - \sigma_B$	$SR_A - SR_B$
Dazhong Transportation Group	-0.602	0.659	-0.935	-0.828	0.457	-1.871	0.226*** (3.126)	0.202*** (15.843)	0.936*** (7.174)
Double Coin Holdings	-0.505	0.622	-0.827	-0.942	0.446	-2.138	0.437*** (7.014)	0.176*** (17.169)	1.312*** (13.220)
Eastern Communications	-0.363	0.665	-0.542	-0.756	0.502	-1.500	0.393*** (5.194)	0.164*** (14.609)	0.958*** (8.004)
Huaxin Cement	-0.862	0.635	-1.390	-1.154	0.644	-1.845	0.292*** (5.437)	-0.009 (-0.782)	0.455*** (5.353)
Jinan Qingqi Motorcycle	-1.162	0.453	-2.598	-1.462	0.443	-3.319	0.300*** (2.722)	0.010** (2.454)	0.722*** (3.073)
SGSB Group	-0.395	0.642	-0.589	-1.308	0.466	-2.839	0.914*** (11.737)	0.176*** (14.630)	2.250*** (17.555)
Shanghai Automation Instrumentation	-1.019	0.734	-1.362	-1.276	0.563	-2.279	0.257* (1.918)	0.170*** (15.145)	0.918*** (4.715)
Shanghai Dajiang Group	-0.402	0.472	-0.896	-0.907	0.427	-2.135	0.505*** (5.830)	0.046*** (6.191)	1.239*** (6.688)
Shanghai Dingli Tech Dev (Group)	-0.297	0.687	-0.414	-1.011	0.644	-1.549	0.714*** (6.826)	0.044*** (3.736)	1.135*** (7.660)
Shanghai Friendship Group	-0.749	0.587	-1.313	-0.905	0.503	-1.810	0.156** (2.406)	0.084*** (11.464)	0.497*** (4.669)
Shanghai Highly Group	-0.447	0.637	-0.691	-0.813	0.463	-1.802	0.366*** (5.025)	0.174*** (15.745)	1.111*** (9.326)
Shanghai Jinjiang Intl Hotels Dev	-0.661	0.552	-1.218	-0.802	0.404	-2.017	0.141** (2.025)	0.149*** (21.424)	0.799*** (6.386)
Shanghai Jinqiao Export Process Zone	-1.045	0.618	-1.696	-1.341	0.453	-3.016	0.296*** (4.244)	0.165*** (12.408)	1.320*** (13.627)
Shanghai Yaohua Pilkington Glass	-0.877	0.688	-1.258	-0.996	0.502	-1.978	0.119 (1.325)	0.186*** (15.458)	0.720*** (5.657)
Zhonglu Co	-0.054	0.711	-0.072	-0.942	0.528	-1.757	0.888*** (9.194)	0.184*** (16.434)	1.685*** (11.277)
Averaged Difference							0.400*** (4.935)	0.128*** (12.129)	1.070*** (8.292)

$H_0: \mu_B - \mu_A = 0, \sigma_B - \sigma_A = 0$  and  $SR_B - SR_A = 0$ , the value in parentheses are the t-statistics.  
The \* Significance level of 10%, \*\* Significance level of 5%, Significance level of 1%

# Chapter 6

## Bubbles: Econometric Analysis and Empirical Evidence

### 6.1: Introduction

#### 6.1.1: The 1990s US Stock Market Bubble

On August 9, 1995, Netscape Communications, a 15-month-old company with no profits and whose primary product was distributed for free, held its initial public offering. The stock, initially valued at \$28 a share, closed the day at  $\$58\frac{1}{4}$ . At that price, the company was valued at \$2.2 billion. Four months later, Netscape stock was trading at around \$170 a share, valuing the company at close to \$6.5 billion.

Approximately five years later, on March 9, 2000, the technology stock that dominated the NASDAQ stock exchange closed for the first time above 5,000. It had risen almost 500% since August of 1995. A little over one month later, the NASDAQ had lost 1,727 points and in the process eviscerated over \$2 trillion in “paper” wealth. An additional \$1.5 trillion would be lost on the NASDAQ over the next year, much of it extracted from dot-com companies<sup>48</sup>. Once considered among the foundation companies of the new economy, Amazon.com lost 77% of its stock market value, Yahoo lost 86%, and Priceline.Com lost a shocking 97%.

What caused the dot-com era? Why were massive amounts of capital suddenly invested in a completely new (and unproven) industry? Why did millions of investors suddenly value small start-up companies more than many industrial stalwarts? Why did hundreds of seasoned,

---

<sup>48</sup> A dot-com company, or simply a dot-com (alternatively rendered dot.com or dot com), is a company that does most of its business on the Internet, usually through a website that uses the popular top-level domain, “.com” (in turn derived from the word “commercial”). “dot com company”. The free dictionary. 27 Feb. 2009 <<http://www.thefreedictionary.com/dot+com+company>>

experienced managers quit comfortable, safe, and lucrative jobs with Fortune 500 companies for the chance at working in a dot-com? And why did all of this suddenly collapse?

Conventional wisdom holds that the answer to these questions is that the dot-com era represents a classic “speculative bubble”<sup>49</sup>. The dot-com era was much like the stock market prior to the crash of 1929, England’s experience with the South Sea Company, and the famous Dutch example of Tulipmania. It was, they allege, a period of mass hysteria in which investors foolishly bid up the price of speculative assets to a ridiculous degree, only to see prices collapse when the supply of additional investors eventually ran out.

### 6.1.2: Measuring Stock Market Bubbles

“Bubble” is not a word specific to the stock market. The initial opinions about so-called price bubbles refer to various kinds of assets, such as foreign exchange, gold, real estate, and stock. Bubbles have been concerned with driving up all these asset prices. Following Blanchard and Watson (1982) who stated that bubbles are more likely to exist in the price of an asset with difficult to understand fundamental values, it is expected that bubbles hardly exist if the fundamental value of an asset is easily identified. With this idea in mind, it is expected that research concerning bubbles is best conducted in stock markets where the fundamental values of stocks are unclear.

When discussing stock market bubbles, three questions are often considered. The first is a question of whether stock market bubbles exist at all and how to verify them. If the answer to the first question is that bubbles do in fact exist, then two valid follow-up questions to ask are: what causes stock market bubbles and how can these causes be measured?

Research concerning rational bubbles has focused on verifying the existence of bubbles, and the research field of behavioural finance enriched quantitative price models with the introduction of categorised trader behaviours. In other words, behavioural finance studies

---

<sup>49</sup>An economic bubble (sometimes referred to as a speculative bubble, a market bubble, a price bubble, a financial bubble, or just a bubble) is “trade in high volumes at prices that are considerably at variance with intrinsic values.

market bubbles by examining the cause of investor behaviour rather than seeking to verify its existence, which is the aim of rational bubbles studies.

These two research areas are closely related. For example, a rational bubble is defined as the outcome of self-fulfilling behaviours, that is, investors buy stocks and drive the price up, with a belief that the price will increase. This scenario includes investor behaviour defined in behavioural theories, such as positive feedback trading and arbitrageurs' anticipatory trading<sup>50</sup>, which are referred to as noise trading or irrationality in behavioural finance.

Two of the three questions pertaining to stock market bubbles described above are topics of concern in rational bubbles studies and the field of behavioural finance. The third question is not covered by these studies and is therefore examined here.

This chapter employs different Value at Risk (VaR) approaches to measure and detect stock market bubbles. VaR was developed in response to the financial disasters of the early 1990s that engulfed Orange County, Barings, Daiwa, and many others. The common lesson of these crises is that billions of dollars can be lost because of poor supervision and management of financial risks. Spurred into action, financial institutions and regulators turned to VaR, an easy-to-understand method for quantifying market risk.

The structure of this chapter is as follows: section 6.2 is devoted to a review of current bubble theories; section 6.3 is an overview of the analysis framework; section 6.4 examines bubble estimations for two countries, US (the NASDAQ) and South Africa (the Johannesburg Stock Exchange JSE). This section describes the methodology, data and results of this study.

Conclusions are presented in section 6.5.

---

<sup>50</sup>Positive feedback trading is the practice of buying securities after prices rise and selling after prices fall (Shleifer, 2000:155)

## 6.2: Asset-Price Bubbles: Theoretical Background

This section reviews the literature related to bubble research from the 1980s when the financial theory of the efficient market hypothesis (EMH) was challenged by studies of the excess volatility of stock prices. The review is also extended to rational bubbles studies and behavioural research, which attempt to formulate a framework for the examination of stock bubbles to explain the real-world phenomena in recent years.

### 6.2.1: Review of Research on Stock Market Bubbles

Bubble research is needed to understand the cause of the volatility of stock prices. In order to find out the reason for excessive price volatility, two schools of research are utilised. One looks for an explanation for stock price volatility from the movement of dividends and discount rates respectively. The other analyses stock price fluctuation as a result of bubbles. The latter defines a stock price as consisting of two basic components: market fundamentals and bubbles.

The bubble corresponding to noise trading and irrationality has been mentioned in the literature on superior asset price movements. However, the bubble phenomena shouldn't be confined to studies of irrational investor behaviour. Since the 1980s, rational bubbles have been viewed as a reason for capricious behaviours in stock markets, while financial economists are vexed about explanations for all financial market behaviours under the EMH. For example, Blanchard and Watson (1982) portrayed rational bubbles as the deviation of the price from its fundamental value. The market fundamental value of an asset as an intractable issue with regard to bubble research and is defined as the present expected discounted value of dividends (Flood and Garber, 1980). Diba and Grossman (1988b) developed a more theoretical definition which stated that "a rational bubble reflects a self-confirming belief that an asset's price depends on a variable (or a combination of variables) that is intrinsically

irrelevant, i.e. not part of market fundamentals or of truly relevant variables in a way that involves parameters that are not part of market fundamentals.”

## 6.2.2: Rational Bubbles: Theoretical Perspective and Empirical Studies

The fundamental price model assumes that stock prices equal the present discounted value of future cash flows (Lucas, 1978). However, this model has been challenged by many theoretical and empirical findings (Shiller, 1981; Leroy and Porter, 1981; Blanchard and Watson, 1982). These challenges to the fundamental price model have often been taken as evidence for the existence of rational bubbles. The rational bubbles theoretical model is based on an expectation formula and is illustrated clearly by West, 1987:

$$P_t = k \cdot E(P_{t+1} + D_{t+1}) \setminus I_t \quad \text{with } k \leq 1 \quad (6.1)$$

Where  $P_t$  is the observed price at  $t$ ,  $E(P_{t+1} + D_{t+1})$  is the expected sum of price and dividends of the next period with the present information  $I_t$  and  $k = \frac{1}{1+j}$ . With the assumption of constant discount rate  $i$ , the equation (6.1) can be resolved recursively forward to get:

$$P_t = \sum_{i=1}^n k^i E D_{t+i} \setminus I_t + k^n E P_{t+n} \setminus I_t \quad (6.2)$$

If the transversality condition  $\lim_{n \rightarrow \infty} k^n E P_{t+n} \setminus I_t = 0$  is achieved, the observed price equals the fundamental value  $P^f$ :

$$P_t^f = \sum_{i=1}^{\infty} k^i E D_{t+i} \setminus I_t \quad (6.3)$$

Similar to solutions documented in the literature before West (1987), such as Blanchard and Watson (1982) and Shiller (1978), the failure of the transversality condition means the observed stock price is not equal to the fundamental value  $P_t^f$ . Thus the price  $P_t$  can be thought of as the sum of the fundamental value  $P_t^f$  and a bubble  $B_t$ :

$$P_t = P_t^f + B_t \quad B_t \geq 0 \quad (6.4)$$

$$B_t = k^{-1} E_t B_{t+1} \quad (6.5)$$

Equation (6.5) implies that an investor who pays for an asset today is expected to be rewarded by an even higher value than the fundamental-expected value of the next period.

Therefore, although investors know rationally that the current market price exceeds the present value of future dividend payments they still invest in the market (Donaldson and Kamstra, 1996). This bubble is considered to be the result of self-fulfilling behaviour which is referred to as a rational or speculative bubble.

Researchers raise two issues regarding the non-negative bubble path in the theoretical assumption above. First, negative bubbles are impossible if  $B_t$  is a negative value today then (6.5) implies that there is a positive probability that at some point  $t + i$ ,  $B_{t+i}$  will be negative enough to make the price negative. Second, if bubbles exist, they must start on the first day, and will not restart after bursts (Diba and Grossman, 1988b). However, these implications derived from (6.5) are obviously inconsistent with evidence in the real world.

The evidence of bubbles is based on the rejection of the transversality condition. The representative investor model pictures an equilibrium price at which the transversality condition is achieved, that is, a competitive agent will always buy undervalued stocks and sell overvalued ones, which adjusts the demand so as to draw the stock price back to the

equilibrium point (fundamental value). However, this theory is little more than an oversimplified conception which pays no attention to the special property of stock markets which is that in these markets fundamental values are uncertain. Fundamental values depend on future dividends which don't appear in the present and cannot be forecasted accurately by statistical modelling techniques.

As many researchers realised, the above theory is fragile due to the naturally weak assumption of equation (6.5). In order to overcome this problem in the initial theory of rational bubbles, some new bubble paths are specified.

Blanchard and Watson (1982) as well as West (1987) illustrated two bubble paths. The first one is called a deterministic bubble:

$$B_t = B_0 + k^{-t} \quad (6.6)$$

Another one with an explosive property is accordingly called a stochastic bubble:

$$\begin{aligned} \text{State 1: } B_{t+1}^S &= \left(\frac{1+r}{\pi}\right) B_t^S && \text{with probability } \pi, && (6.7) \\ \text{State 2: } B_{t+1}^S &= 0 && \text{with probability } 1 - \pi, \end{aligned}$$

Where  $B_t^S$  denotes the stochastic bubbles at period  $t$ . It is easy to verify that stochastic bubbles satisfy equation (6.5). In each period, stochastic bubbles will remain with probability  $\pi$  or crash with probability  $1 - \pi$ . Stock prices including these bubbles diverge from the fundamental price level, as long as they last. However, when stochastic bubbles crash, the stock prices heavily decline to the fundamental price level.

Diba and Grossman (1988b) show that once rational bubbles have crashed, they never restart. Stock prices remain at the fundamental price level after the crash of stochastic bubbles. In other words, stochastic bubbles can never grow again after a complete crash.

Norden and Schaller (1993) generalized the Blanchard and Watson (1982) bubble paths in two ways: first, the probability of collapse is enlarged with the bubble growth; second, the model allows the collapsed bubbles to be above zero (partially collapsed).

Afterwards, Diba and Grossman (1988a) mentioned that bubbles periodically shrink. This periodically collapsed bubble is illustrated by Evan (1991):

$$\begin{aligned}
 B_{t+1} &= (1+r)B_t u_{t+1} && \text{if } B_t \leq \alpha \\
 B_{t+1} &= [\delta + \pi^{-1}(1+r)\theta_{t+1}(B_t - (1+r)^{-1}\delta)]u_{t+1} && \text{if } B_t > \alpha \\
 0 < \delta < (1+r)\alpha && 0 < \pi < 1 && (6.8)
 \end{aligned}$$

$\delta$  and  $\alpha$  are positive parameters, and changes in these can alter the frequency with which bubbles erupt and the average length of time before collapse;  $u_{t+1}$ <sup>51</sup> is an exogenous independently and identically distributed positive random variable with a mean of 1, and  $\theta_{t+1}$ <sup>52</sup> is an exogenous independently and identically distributed Bernoulli process which takes the value 1 with a probability of  $\pi$  and 0 with a probability of  $1 - \pi$ .

The characteristics of bubbles can be adjusted by varying the parameters  $\delta$ ,  $\alpha$  and  $\pi$ . In (6.8), only if  $B_t \leq \alpha$ , bubbles grow at a rate of  $(1+r)$ . As long as  $B_t > \alpha$ , bubbles move into an eruption pattern until collapse. When bubbles collapse, they fall to a mean value of  $\delta$ , that is, bubbles can restart after a collapse.

Periodically collapsing bubbles can be regarded as a combination of stochastic bubbles and log-normally distributed bubbles. Periodically collapsing bubbles are more realistic than stochastic bubbles since they periodically burst. However, as equation (6.8) shows,

---

<sup>51</sup>This follows an exogenous identically and independently log-normal distribution whose mean is 1.

<sup>52</sup> $\theta_{t+1}$  is a random variable similar to stochastic bubbles

periodically collapsing bubbles are much more complicated than deterministic bubbles and stochastic bubbles.

In order to integrate the foregoing descriptions of bubbles, Fukuta (1998) devised a three-state bubble model in an incompletely bursting bubble environment.

**State 1:** the state of large bubbles

$$B_{t+1} = (1 + r) \left( \frac{\omega_1}{\pi_1} \right) B_t \quad \text{With probability } \pi_1$$

**State 2:** the state of small bubbles

$$B_{t+1} = (1 + r) \left( \frac{\omega_2}{\pi_2} \right) B_t \quad \text{With probability } \pi_2$$

**State 3:** the state of incomplete bursts

$$B_{t+1} = (1 + r) \left( \frac{1 - \omega_1 - \omega_2}{1 - \pi_1 - \pi_2} \right) B_t \quad \text{With probability } 1 - \pi_1 - \pi_2 \quad (6.9)$$

Where  $\omega_1$  and  $\omega_2$  are arbitrary with assumption of  $0 < \omega_1 < 1$ ,  $0 < \omega_2 < 1$  and  $0 < 1 - \omega_1 - \omega_2 < 1$  this assumption implies that the bubbles never completely crash.  $\pi_1$ ,  $\pi_2$  and  $1 - \pi_1 - \pi_2$  are the probability of each state and they are strictly positive. We can easily verify that the bubbles defined by equation (6.9) satisfy equation (6.5) and can thus be included in the class of rational bubbles. The condition of  $(1 - \omega_1 - \omega_2)/(1 - \pi_1 - \pi_2) < \omega_2/\pi_2 < \omega_1/\pi_1$  is also assumed. Based on this and some assumptions, we can show that  $B_{t+1}$  in state 1 is larger than  $B_t$   $B_{t+1}$  in state 3 is smaller than  $B_t$  Hence, state 1, state 2 and state 3 can be referred to as a large bubble state, a small bubble state and an incompletely bursting state, respectively.

With various assumptions of  $\omega$  and  $\pi$ , (6.9) can be transformed into the same category of other bubble models described before (Deterministic bubbles, Stochastic bubbles and Periodically bubbles). This work follows Fukuta (1998) in assuming bubbles can have three states; a large bubble state, a small bubble state and an incomplete burst state. Only incomplete burst state  $B_{t+1}$  is smaller than  $B_t$ , thus we can detect this bubble stage using VaR as long as VaR can be interpreted as a worst case scenario which involves an extreme event with a small chance of occurring within a time horizon. This belief leads us to an innovative work on measuring bubbles.

As a parsimonious alternative to the theory of rational bubbles, Froot and Obstfeld (1991) defined an intrinsic bubble which is the function of only dividends. The idea stems from a belief that bubbles are generated from an overreaction regarding dividend news, and the model appears to fit the data in the US stock market during both the 1960s and 1970s. However, at the same time, this model with the newly defined bubble is inconsistent with the conventional description of fundamental values which is defined as inconsistent with the bubble as a concept.

However, overreaction-driven bubbles should in fact generally be free dividends, although they are the result of news about dividends. For example, investors push up the price by buying stocks because they believe the fundamental value of stocks will increase due to the good news about dividends. Since investors are heterogeneous, and no two can ever react identically, different levels of overreaction are not due to dividends but to the heterogeneity of investors' decisions. Therefore, the intrinsic bubble seems to lack a sound theoretical foundation.

Improvements in the bubble assumption have further developed the rational bubble theory. However, one problem remains; it is still quite difficult to mimic a bubble path, since bubbles are influenced by uncertain decisions from heterogeneous investors.

### 6.2.3: Bubbles in Behavioural Studies

While rational bubble theorists struggle to extend the efficient market theory to a more realistic model of rational bubbles, with the accumulation of theoretical challenges and empirical deviations, the substance of EMH has been eroded. Instead, a new set of explanations of empirical regularities, as well as a new set of predictions, has generated behavioural finance, as a study of human fallibility in competitive markets. The behavioural economists contend that “financial markets are not expected to be efficient and the market efficiency only emerges as an extreme special case unlikely to hold under plausible circumstances” (Shleifer, 2000). Shiller (2002) similarly asserted that the efficient market theory is only “a half-truth”. While irrational traders are often depicted as “noise traders” and rational traders are as “arbitrageurs” who cultivate riskless and costless profit in their investment, behavioural finance theorists argue that the strategies adopted by rational investors are not necessarily arbitrages since they are often risky and costly. As a result, mispricing can remain unchallenged (Thaler, 2005).

Shleifer (2000) summarises three areas in which investors deviate from the standard decision making model: attitudes toward risk, non-Bayesian expectation formation, and sensitivity of decision. In addition, Black (1986) indicates that many investors trade on noise rather than information, namely “noise traders” or “unsophisticated traders”. When investors’ beliefs conform to the psychological evidence rather than the economic model, this is referred to as “investor sentiment”.

Two major foundations of behavioural finance theory are “limited arbitrage” and “investor sentiment”, which are in direct contradiction of the principal assumption of EMH – the irrelevance of irrationality. Under EMH, markets are viewed as fully rational, since irrational trading strategies are uncorrelated and offset each other. Rational arbitrageurs, who “simultaneous purchase and sale the same or essentially similar security in two different markets at advantageously different price”, bring the security prices in line with their

fundamental values and squeeze the irrational traders out of the market. As the alternative approach to the study of the financial markets, behavioural finance aims to theoretically and empirically model the real world, in which “arbitrage is risky and therefore limited” and investors form their beliefs by sentiment. Although the initial aim of those models is to display a price forming process with a consideration of investors’ psychological factors, there are some strong resemblances to notions on bubble theories in the models. Four models, namely the noise trader risk model, the model of relative returns of noise traders and arbitrageurs, the model of investor sentiment and the positive feedback model, are reviewed below with the intention of identifying some pioneering ideas about bubbles in the behavioural finance field.

With regard to research on arbitrage, DeLong et al (1990) defined two kinds of risks that arbitrageurs may face. The first is the risk caused by imperfect substitutes of securities, and the second is called “noise trader risk”<sup>53</sup>. The latter is the possibility that mispricing becomes worse due to noise trading. Furthermore, Shleifer (2000) introduces two models which are against the assumption of rational markets. One is a pricing function which describes how noise traders affect the price (for more details refer to Shleifer (2000)).

The central point of the noise trader risk model described by Shleifer (2000) is to identify the impact of noise traders on the stock price, which implies a self-evident extrapolation that the price deviation can be traced to the irrational behaviours of noise traders. In other words, from the standpoint of bubble research, noise trading behaviours contribute to the bubble by keeping arbitrageurs from driving prices back to their fundamental value. This calls for another model which is concerned with the misperception of EMH about the noise trader, that is, it is not always the case that noise traders are weeded out of markets since they can earn a higher return than the arbitrageurs. The view is obtained by analysing the expected difference between noise traders’ and arbitrageurs’ total return (Shleifer, 2000).

---

<sup>53</sup>This is the risk that noise traders’ beliefs become even more extreme before they revert to the mean. “An arbitrageur selling an asset short when bullish noise traders have driven its price up must remember that noise traders might become even more bullish tomorrow, and so must take a position that accounts for the risk of a further price rise when he has to buy back the asset.” Shleifer (2000: 29)

The implication of the behavioural models discussed above for the study of bubbles is clear. First, financial markets are not efficient due to the persistent presence of irrationality. Second, the deviation of prices from fundamental values, the so-called bubble, is the outcome of noise trading.

The model of investor sentiment is devoted to the simulation of belief formation using psychological theories. There are two important psychological phenomena involved: representativeness and conservatism. As a result of representativeness, “people see patterns in truly random sequences” (Shleifer, 2000). The slow updating of models in the face of new evidence is the result of conservatism (Edwards, 1968). The two psychological phenomena are responsible for investors’ overreaction and the underreaction of prices. A model of investor sentiment introduced by Shleifer (2000) illustrates the deviation of the price from its “correct value” as a result of investors’ ignorance of randomly walking earnings. Instead, the price is modelled as an expectation formula, not as a set of random true numbers.

The term “bubble” appeared in behavioural research in the positive feedback trading theory and bubbles are considered to occur in a situation of price soaring without news. Three kinds of investors, namely noise traders, passive investors and arbitrageurs, are identified in the model. Since noise traders are positive feedback traders who buy securities after prices rise and sell after prices fall, they play the role of trend chasing. In contrast, passive investors, who do not play an active role in the business, will purchase investments with the intention of long-term appreciation and limited maintenance. Meanwhile, the stabilising power of arbitrage is challenged, because arbitrageurs amplify positive feedback trading, that is, arbitrageurs who buy more today based on superior information will stimulate buying more tomorrow, and drive prices above fundamental values. The model of positive feedback trading explains the bubble as a result of price-chasing-up behaviours after arbitrageurs’ anticipatory pumping up of the price, for more details see Scheinkman and Xiong (2003), Hong, Scheinkman and Xiong (2006) as well as Yang (2006).

### 6.3: Defining a Bubble: Empirical Framework

Concerning bubble research, two opposite arguments are presented: the notion of the existence of bubbles and the view that there are no bubbles. Theorists arguing against bubbles try to construct a fundamental estimation model which matches the observed price through analyses of historical data. For example, Donaldson and Kamstra (1996) used the neural network technique to generate a satisfactory result. However, their simulated fundamental path still fails to overlap with the observed actual price movement, although their bubble-dismissing result is verified on the basis of the unit root test, their method is questionable.

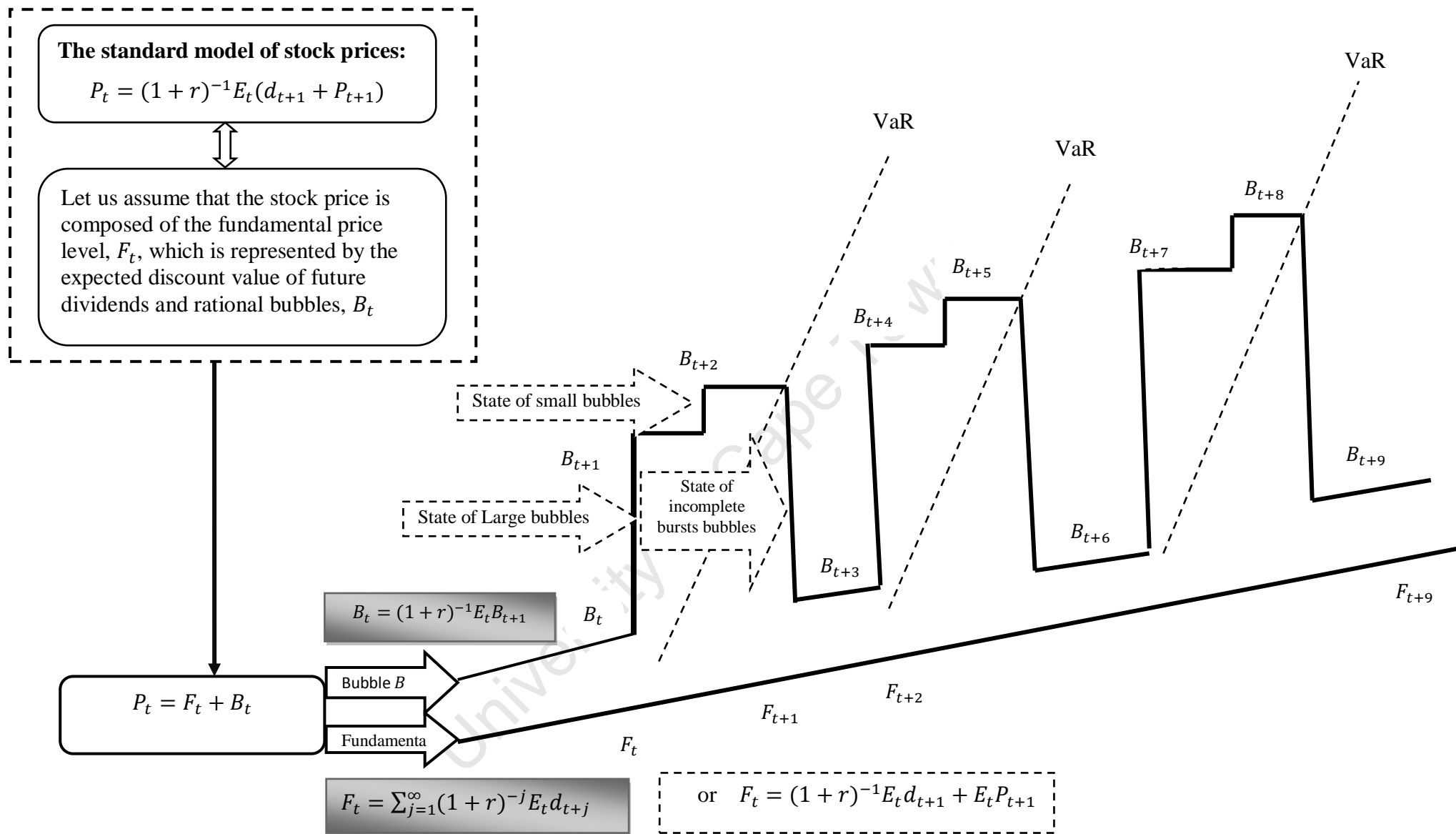
The argument for the existence of bubbles is supported by the belief that a stock price consists of two parts: the fundamental value and the excess value over the fundamental value. However, all existing bubble testing methods enable us to test the existence of bubbles over a time period, but not to estimate bubbles at a particular point in time. This problem imposes a serious discrepancy in studying bubbles and, in particular, in studying what determines bubbles, due to the failure of the method to estimate bubble changes over time. This problem calls for rethinking of current methods in estimating bubbles. Can we really estimate the magnitude of bubbles at a point in time (a crash, for instance)? Bearing this question in mind, this study takes a new approach to investigate bubbles, which is fundamentally different from existing approaches in terms of its theoretical framework and statistical estimation method.

The fundamental notion of our work is that bubbles persist in the stock market. This opinion can be traced to the work of Binswanger (1999), in which persistent bubbles are considered to be sustainable if bubbles move with the development of a real economy. His empirical work in 2000 further verified and highlighted the persistence of bubbles since the early 1980s.

Summers (1986) documented the finding that both theoretical and empirical considerations suggest the existence of continuous and substantial deviations from fundamental values. In fact, many researchers, such as West (1988), Shiller (1984), and Debondt and Thaler (1985), have realised that there is a significant stationary component in a stock price, referred to as a

fad. Furthermore, Lee (1998) and Chung and Lee (1998) empirically identified fads in several stock markets. However, there is no general agreement concerning the distinction between fads and bubbles. For example, following Cochrane (1991), Lee (1998) and Chung and Lee (1998) considered price deviations which slowly return to fundamental values as fads, whereas bubbles are expected to continue until bursts occur. Conversely, Shiller (1988) defined a bubble as a fad if the influence of the fad occurs through prices. Faced with this confusion, Bingswanger (2004) did not distinguish between bubbles and fads. Instead, he interpreted persistent deviations of stock prices from fundamental values as bubbles. This work follows Bingswanger's view that any non-fundamental components in stock prices, except for statistical noises, will be recognised as bubbles which are persistent in a stock market and these assumptions exclude exceptional shocks at a point in time. This belief leads to an innovative method of measuring bubbles.

Our work will follow rational bubbles studies and will stem from two basic opinions: firstly, bubbles persist in stock markets since they result from optimistic beliefs and speculative behaviours which dominate the market always. Accordingly, our work follows Fukuta (1998) in which he presents a class of rational bubbles called "incompletely bursting bubbles". He determined these bubbles have three states: a large bubble state, a small bubble state and an incomplete burst state. Only in the incomplete burst state is  $B_{t+1}$  smaller than  $B_t$  as explained in section 6.2.2 and described in figure 6.1, that is, the expected bubble tomorrow is less than the bubble today. Secondly, the incomplete burst bubble state where only  $B_{t+1}$  is smaller than  $B_t$  can be detected using VaR as long as VaR can be used to estimate the potential loss from adverse price movements. Refer to figure 6.1 for a clearer picture of this.



**Figure 6.1** The figure suggests the asset price consists of two parts: the fundamental value and bubble term. The fundamental value grows at risk free interest rate. The bubble term is always persistent in the price and has only 3 possible states; a large state, a small state and incomplete burst state. The figure also reports how and in which state VaR could detect the bubble.

VaR is a worst case scenario with regard to loss, when an unlikely extreme event occurs within a time horizon under normal market conditions, whereas a stock bubble is an abnormal potential profit which is usually followed by an abnormal loss of a stock of financial instruments under abnormal market conditions. A bubble burst must violate the value of VaR, and therefore, we can use VaR to detect the downside (burst) of the bubble. In this work VaR is used as a benchmark for a bubble. Any violation of this benchmark will be recognised as the result of a bubble burst.

## 6.4: Empirical Method for Testing Bubbles in two Countries

This study concerns the status of market efficiency in the NASDAQ and JSE as a starting point of an investigation of stock market bubbles. The variance bound test presented by Shiller and Leroy and Porter (1981) will be employed for this purpose for each market. The methodology as far as VaR is concerned is presented next.

Consider a single asset portfolio, whose underlying asset follows GBM pricing process

$$\frac{dS}{S} = \mu dt + \sigma dz \quad (6.10)$$

where  $S$  is the price of a stock,  $dS$  is the change of price of the stock,  $\mu$  is the expected return,  $dt$  is a period of time, and  $\sigma dz$  is the white noise for the path of the stock price.

Assuming that the percentage return of the stock price in a short period of time is normally distributed, so

$$\frac{\delta S}{S} \sim \phi(\mu \delta t, \sigma \sqrt{\delta t}) \quad (6.11)$$

where  $\delta S$  is a small time interval,  $\mu \delta t$  is the mean of the normal distribution and  $\sigma\sqrt{\delta t}$  is the standard deviation. By Ito's Lemma, the following lognormal property can be derived for stock price.  $S_T$ , at a future time  $T$ , where  $S_0$  is the stock price at time zero.

$$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right] \quad (6.12)$$

A continuously compounded annual rate of return for stock price is defined as  $\eta$ , and therefore it follows that

$$S_T = S_0 e^{\eta T} \quad (6.13)$$

Therefore,

$$\eta = \frac{1}{T} \ln \frac{S_T}{S_0} \quad (6.14)$$

Alone with equation (6.12), we can see that  $\eta$  has the following normal distribution property with mean as  $\mu - \frac{\sigma^2}{2}$  and standard deviation of  $\frac{\sigma}{\sqrt{T}}$ . That is

$$\eta \sim \phi\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right) \quad (6.15)$$

The volatility of a stock is now defined as a measure of the uncertainty about the returns of stock. Based on equation (6.15), the annual volatility can be seen as the same as the standard deviation  $\frac{\sigma}{\sqrt{T}}$ , where  $T = 1$ , so the annual volatility of the stock is  $\sigma$ . It also can be understood that the daily volatility of the stock would be  $\frac{\sigma}{\sqrt{252}}$ , when using 252 trading days per year.

When we discuss the daily returns of stock, it is reasonable to assume that the expected return would be zero, because in such a short time horizon, the expected return is relatively small. Therefore, based on the normality property in equation (6.15), it can be easily estimated that the distribution of returns from daily volatility. For example, it can be estimated that 99% of

rates of returns will be greater than -2.33 times the daily volatility. If the VaR indicates how much actual amount of money might be lost for a portfolio at 1% of chance in 1 day, the VaR is equal to  $2.33 \times \frac{\sigma}{\sqrt{252}} \times P$ , where the  $P$  is the current value in dollars (Rand) of the portfolio. Thus, the general formula of VaR would be

$$VaR(P, \alpha, \delta t) = P \cdot k \cdot \sigma \sqrt{\delta t} \quad (6.16)$$

Where

$$\Pr(\delta P \geq k \cdot \sigma \sqrt{\delta t}) = \alpha \quad (6.17)$$

Here,  $\alpha$  is the left tail percentage of the distribution,  $\delta P$  is the absolute value of a negative return and  $P, k$  is the number of standard deviations according to  $\alpha$  and  $\Pr(\cdot)$  is the probability based on the distribution. When  $\alpha$  is set as 1% or 5%, then  $k$  would be 2.33 and 1.645 respectively for a normal distribution. In practice,  $\alpha$  needs to be selected as small as possible to catch external events, and 1% has been pretty much a standard setting in the financial sector. One key point is that no matter how small  $\alpha$  is, as long as it is finite, there will always be some probability that extreme events may occur far away from the VaR estimation.

The basic concept of VaR is appealing because it is not difficult to estimate compared to other market risk measure instruments. Most methods use prior data of returns of financial assets to estimate the VaR. However, when using historical data for estimation of current behaviours, there is one key question to be clarified: are the patterns of past analysis sufficient to predict current patterns? In VaR practice, it is necessary to understand the behaviour of distributions for returns of financial assets and be able to make proper assumptions when needed. It needs to be confirmed that such distributions are constant over time. If they are not, is it possible to effectively pattern the change of behaviour?

Existing models need to have a general framework in order to calculate VaR. The first step is to estimate the distribution of portfolio returns. Then it is necessary to select the basic parameters (time horizon, confidence level, time of measurement), and VaR calculation. These models differ with regard to how the estimation is distributed. As mentioned previously, the existing models can be organised into three categories: parametric, nonparametric and semiparametric models. The focus on this chapter will be on the non-parametric model, simply because the non-parametric model includes historical simulation and the Monte Carlo approach, drop any assumption about the distribution shape of the risk factors. Chapter three provides a full description of the Historical simulation approach as well as Monte Carlo simulation testing procedures.

### 6.4.1: Data Description

The published empirical research on the study of bubbles is based on time-series data related to stock prices and dividends (Shiller, 1981; West, 1984; Campbell and Shiller, 1987; Evans, 1991; and McQueen and Thorley, 1994). The stock price of a market is represented by stock price indices, including Standard & Poor's Index, the modified Dow-Jones Index, the Hang Seng Index and the Shanghai Stock Index<sup>54</sup>. Real monthly returns for both equally and value-weighted portfolios of all New York Stock Exchange (NYSE) stocks have also been used in this type of research (McQueen and Thorley, 1994).

Similarly, the main dataset employed by this chapter is from two different stock price indices: the JSE All Share Index and the NASDAQ-100 Index<sup>55</sup>.

---

<sup>54</sup>The observed prices and dividends from Standard & Poor's index are frequently used by researchers, such as Shiller (1981), West (1987), Froot and Obstfeld (1991), and Donaldson and Kamstra (1996). Flood and Hodrick (1986) developed a new empirical analysis using S&P and the modified Dow Jones Index respectively.

<sup>55</sup>The reason of choosing these 2 different markets is to test the capability of my proposed test to detect bubbles in a big market like NASDAQ during the Dot.com Bubble (1995-2000) and its capability to detect bubbles in a relatively smaller market as JSE.

The data employed for efficient market estimation include the JSE All Share Price Index, dividends on the Index and the South African (SA) T-bill 91 days (as a proxy for the risk-free interest rate for SA) from 10.07.1995 to 10.03.2009.

The NASDAQ-100 Price Index, dividends on the Index and the United States (US) T-bill 91 days (as a proxy for the risk-free interest rate for US) from 10.07.1994 to 10.07.2009 will also be employed for testing the efficiency of the NASDAQ-100.

The data employed for VaR calculation via historical simulation include the JSE All Share Price Index from 10.07.1995 to 10.03.2009, as well as the NASDAQ-100 Price Index from 10.07.1994 to 10.07.2009. A 90% and 95% confidence level, 100 time horizon and a one month holding period are specified for VaR parameters.

For obtaining VaR via the Monte Carlo Simulation method, the stock prices (represented by stock price indices) were assumed to follow GBM. The JSE All Share Price index from 10.12.2003 to 10.03.2009 as well as the NASDAQ-100 Price Index from 10.07.1994 to 10.07.2009 was employed. A 90% and 95% confidence level, 5,000 simulations and a one month holding period are specified for VaR parameters (for more details about the Monte Carlo Simulation method when stock prices are assumed to follow GBM refer to equation 3.3 in chapter three)<sup>56</sup>.

## 6.4.2: Empirical Estimation and Results

### 6.4.2.1: Efficiency in the NASDAQ and JSE

In order to test excessive price volatility, Shiller (1981) first employed Standard and Poor's series data that have been used by most of the subsequent researchers. He defined separately

---

<sup>56</sup>All the data was obtained from the Inet-Bridge and DataStream databases in the University of Cape Town Library. The date for the data collection was the 2<sup>nd</sup> of October 2009. However, monthly data were used throughout the analysis and for the purpose of the analysis, a 252 day year was assumed and a continuous risk free rate was used throughout the analysis.

“perfect foresight rational price”  $P^*$ , which is the present discounted value of actual dividends, and its optimal forecast value-actual price  $P$ . He proposed that if markets are efficient, the actual price  $P$ , which is the expected discounted value of future dividends, should have less variance than  $P^*$ , since the expected value of a set of numbers must be more stable than the numbers themselves, from which the variance bound inequality  $V(P) \leq V(P^*)$  is deduced. The results of his statistical tests and the plot analysis give a positive answer to the question of whether stock prices move too much to be justified by changes in dividends.

Meanwhile, LeRoy and Porter (1981) undertook a similar test by incorporating the earnings variable in the model. They reach the same conclusion as Shiller (1981) which is that stock prices are too volatile to be explained by the efficient capital market model. However, this study concerns the status of market efficiency in the NASDAQ and JSE as a starting point for the main investigation of stock market bubbles. The Shiller (1981) test was applied to data from the NASDAQ-100 Index and the JSE All Share Index to test the efficiency of these markets. Interesting results were obtained, as shown in Figures 6.2 and 6.3 below:

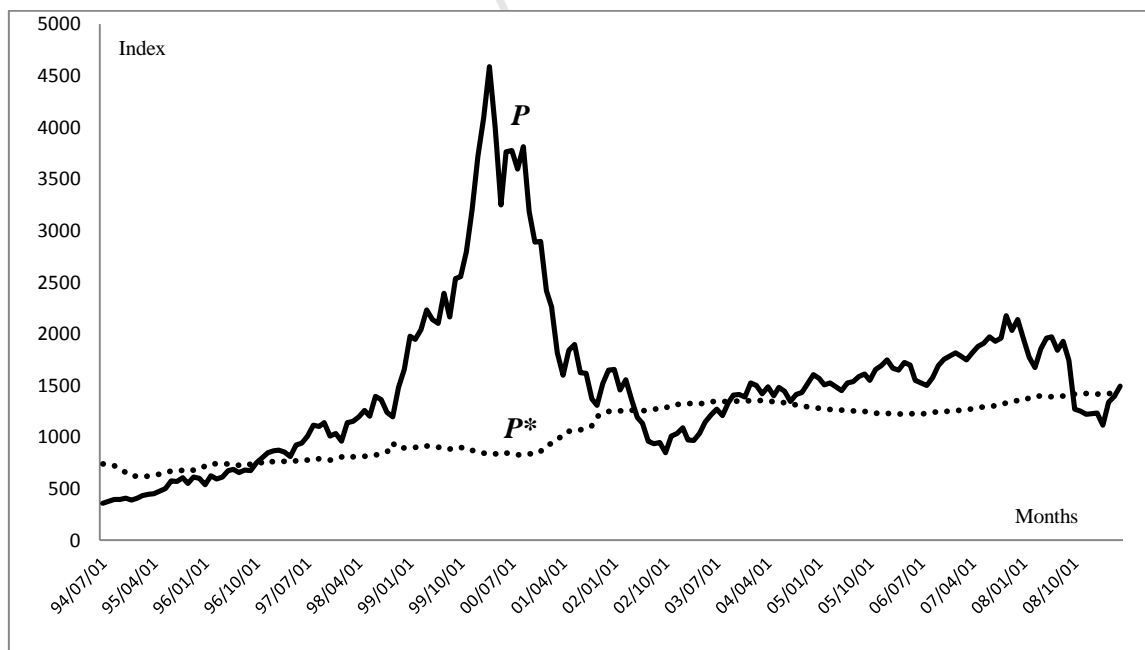


Figure 6.2: The NASDAQ 100 Stock Price Index (solid line  $p$ ) and ex-post rational price (dotted line  $p^*$ ), 10.07.1995-10.03.2009, the variable  $p^*$  is the present value of actual subsequent real detrended dividends.

The findings on the NASDAQ (Figure 6.2) are consistent with Shiller's findings which provide evidence of stock market inefficiency; because the ability to predict prices would indicate that all available information was not already reflected in stock prices. This result showed that the actual stock price movement from 1998 to 2000 is too large to be explained by changes in fundamentals. Therefore, the efficient market hypothesis, the notion that stocks already reflect all available information, may not be correct or stock market bubbles would be likely to exist in this market.

However, the findings on the JSE stock market, as illustrated in Figure 6.3, are also consistent with Shiller's findings which provide evidence of stock market inefficiency, where the actual stock price movement is too large to be explained by changes in the fundamentals from 2006 to 2008, suggesting evidence of bubbles during this period.

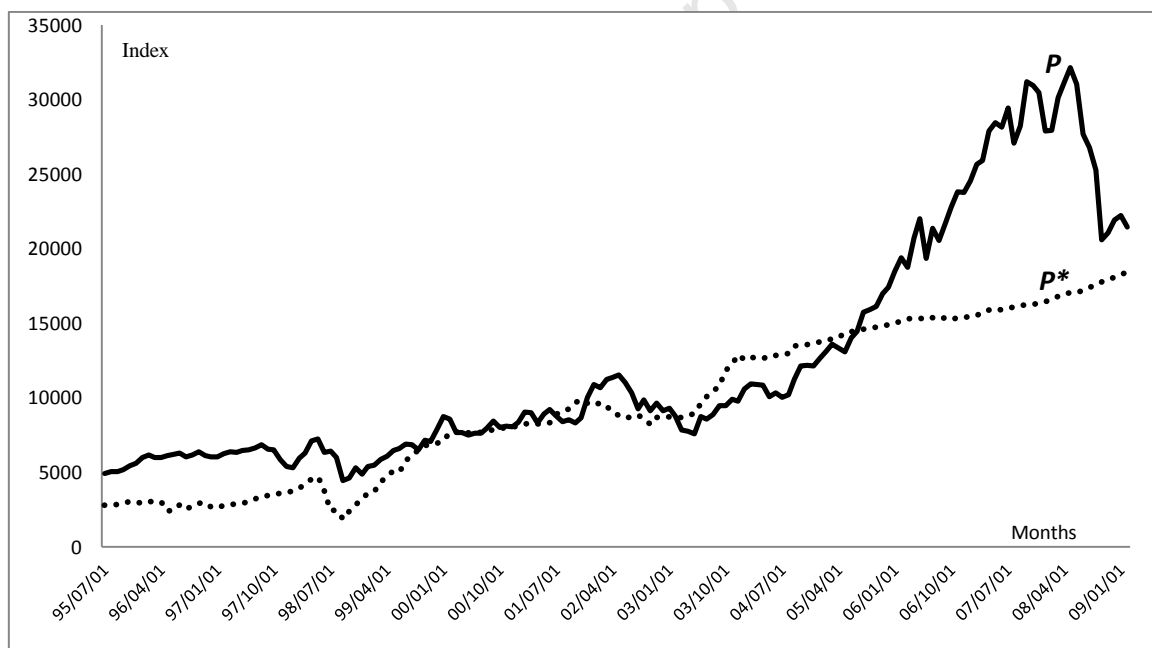


Figure 6.3: The JSE All Stock Price Index (solid line  $p$ ) and ex-post rational price (dotted line  $p^*$ ), 10.07.1994-10.07.2009, the variable  $p^*$  is the present value of actual subsequent real detrended dividends.

Accordingly, this shows, as Blanchard and Watson (1982) indicated, that fundamentals are only part of what determines the price of assets, and there can be rational deviations of the price from its value, which are rational bubbles.

Many assumptions with regard to bubble movements have been documented in the literatures, as discussed in section 6.2. Unfortunately, these efforts do not dramatically improve the authenticity of the theory; in fact they make work around bubble research ambiguous due to their unrealistic assumptions on bubble paths. In particular, in the conventional theory of Blanchard's bubbles<sup>57</sup>, bubbles are assumed to crash completely and never restart thereafter; in other words bubbles are assumed to crash only once, which is apparently an unrealistic assumption. In order to overcome shortcomings in former research, improvements in bubble modelling were made by Fukuta (1998). A new three-state bubble model, namely incompletely bursting bubbles, was designed by him. In this new model, bubble paths are classified into three states: a large bubble state, a small bubble state and an incomplete burst state, which integrates and enhances previous bubble models by exhibiting a more reasonable picture of rational bubbles, in which bubbles are allowed to occur again after incomplete crashes. This work follows Fukuta's view that bubbles persist in the stock market and incomplete crashes follow the bubble state. Next VaR will be employed to detect and measure the incomplete crashes stage in the selected data, as suggested by Fukuta (1998). If the worst loss suggested by VaR was violated by the actual stock price fluctuation, then this will be considered to be the result of incomplete crashes which follow a bubble state. To the best of the researcher's knowledge, no one has previously used VaR as a measurement of stock market bubbles.

#### 6.4.2.2: VaR and Bubble Detection

The VaR analysis results on the NASDAQ-100 Index are reported in figure 6.5 and 6.7 in appendix D.1 and D.2 respectively. The VaR results on the JSE All Share Index are reported in figure 6.9 and 6.11, also in appendix D.1 and D.2 respectively. The VaR return estimates for each method are compared with the realized returns each month. The numbers of violations of the VaR estimates were counted, and the ratio of violations to the length of the

---

<sup>57</sup>Blanchard (1979) and Blanchard Watson (1982).

testing period was compared with the critical value. This was done for several critical values. This is perhaps the simplest possible testing procedure.

Starting with the NASDAQ, the historical simulation VaR were shown to have been violated from May 2000 until early 2001 and in the last quarter of 2008, as figure 6.5 in appendix D.1 illustrates. VaR via Monte Carlo simulation were also violated in the same periods as shown in figure 6.6 in appendix D.2. The violations of VaR obtained via historical simulation were larger than the violations of VaR obtained via Monte Carlo simulation. These results are very much correlated with this study's EMH test on the NASDAQ-100 Index, shown in figure 6.2, which provided evidence of stock market inefficiency and showed that actual stock price movement from 1998 to 2000 is too large to be explained by changes in fundamentals which correspond to the "dot-com bubble" during the period 1998 to March 2000. The VaR results show that VaR was first violated on May 2000 through to early 2001 which corresponds to the dot-com bubble burst on March 10, 2000 and thereafter.

The same story was documented in the 2008 NASDAQ stock market crash. The market was inefficient by deviating from the market fundamental as suggested by EMH and rational bubble theory, and then the market crashed, as our VaR results demonstrated.

The case was very similar with regard to the JSE, where the historical simulation VaR were violated from 2006 until early 2009, as figure 6.9 in appendix D.1 illustrates. VaR via Monte Carlo simulation were also violated during the same period, as shown in figure 6.11 in appendix D.2. The violations of VaR obtained via historical simulation were also larger than the violations of VaR obtained via Monte Carlo simulation. These results show that the JSE All Share Index experienced market bubbles six times followed by six occurrences of incomplete bursting bubbles, as figure 6.9 and 5.11 show (appendix D.1 and D.2 respectively). This result confirms that bubbles persist in the stock market, as we assumed and these bubbles have restarted after all crashes as we also assumed following the view of Fukuta (1998).

Our VaR results show that VaR has successfully detected all crashes occurring in the NASDAQ and JSE markets during our data period. Accordingly, EMH and rational bubbles estimations are good measures to verify the existence of bubble states in general and VaR is a good indicator for the presence of bubbles in the market, where any violation of VaR are considered as bubble crashes.

## 6.5: Conclusion

Ongoing research on stock bubbles can be divided into two fields: the study of rational bubbles and behavioural finance. Rational bubbles studies attempt to verify the existence of bubbles based on diversified assumptions of bubble paths and advanced econometrics techniques. As opposed to EMH, behavioural finance argues that deviations in asset prices are brought about by the presence of traders who are not fully rational (Thaler, 2005). However, neither of these approaches has studied bubbles through detecting them at a certain point of time. This limitation calls for alternative research on bubbles which is the aim of this chapter of our study.

This chapter investigates the presence of rational speculative bubbles in the NASDAQ and JSE stock markets by employing different VaR techniques to detect and measure a stock market bubble. In this study, the Monte Carlo simulation model, as well as the historical simulation method were employed to compute VaR.

Using monthly data, we report the existence of rational speculative bubbles in the NASDAQ-100 stock market during 1998-2000 and a bubble burst in 2000-2001 related to the dot-com bubble and the crisis thereafter. We also report the existence of rational speculative bubbles in the JSE stock market during 2006-2008 and a bubble burst in this stock market in 2009.

## 6.6: Appendix D

### 6.6.1: D.1 Indexes Performances and VaR Historical Simulation Approach

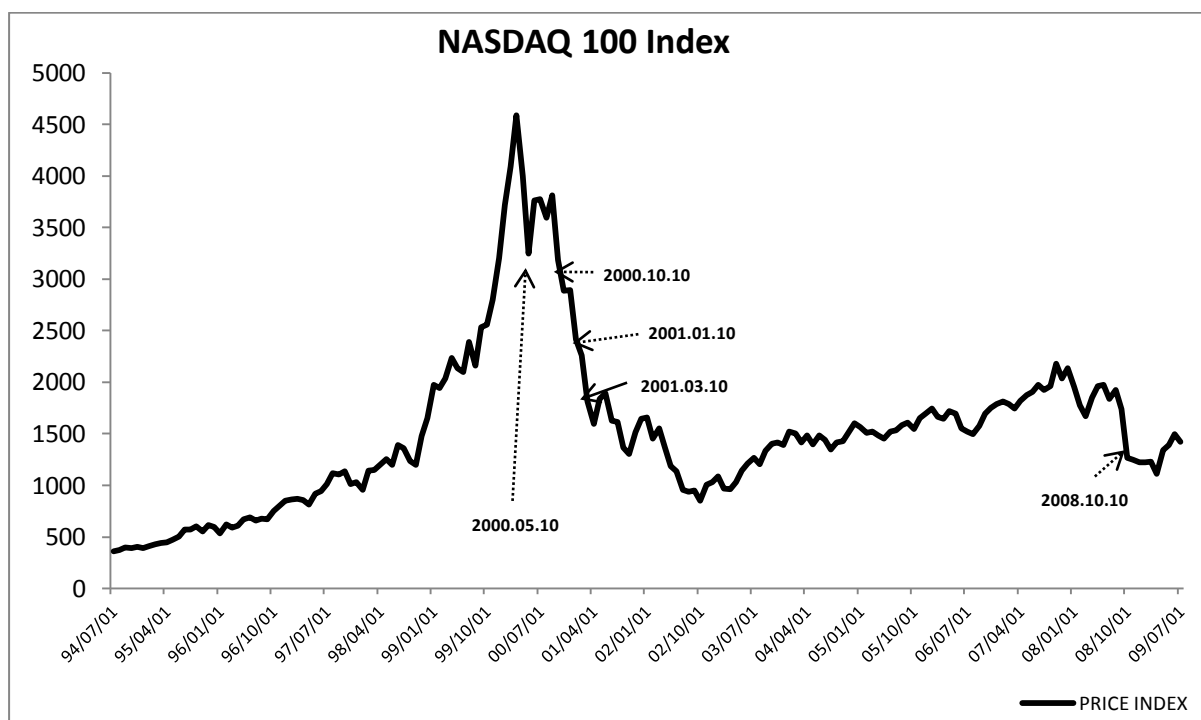


Figure 6.4: A fifteen year graph of the Nasdaq 100 tells the story. From early 1994 to the late summer of two-thousand and nine the tech-heavy index skyrocketed from 1,000 points to 5,000. Investors felt the profit potential for some technology companies was so great they bid up the price of their stocks to hundreds of times earnings.

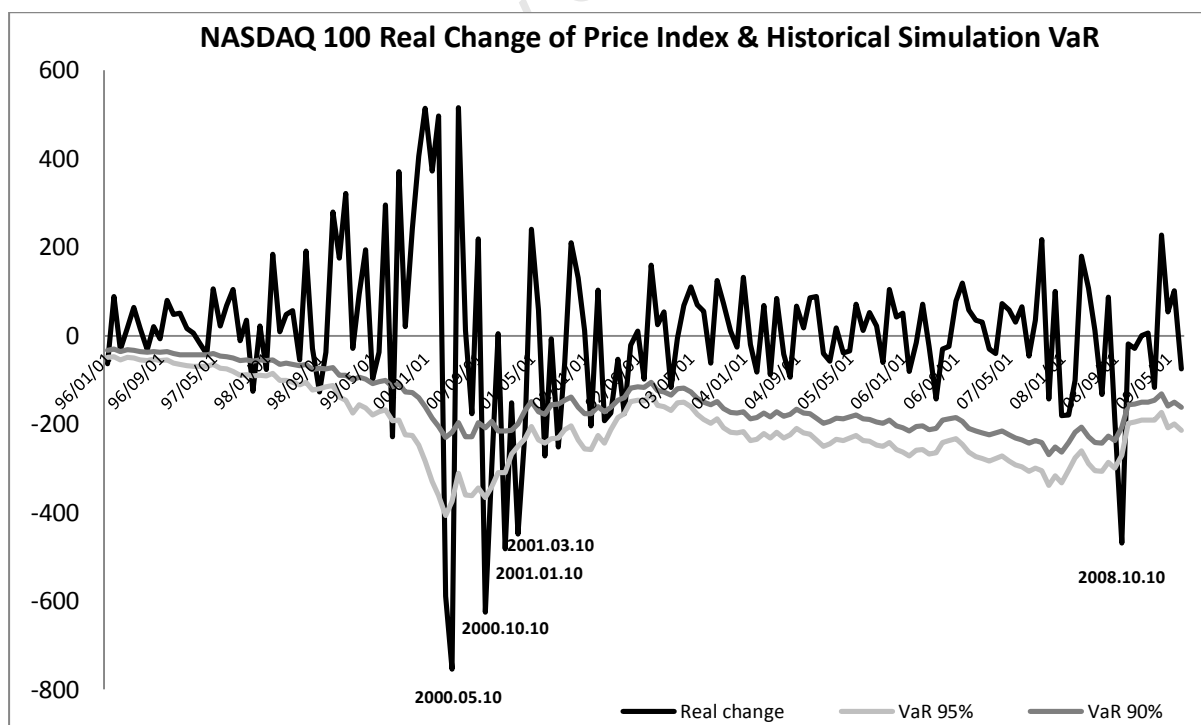


Figure 6.5: Reports a comparison of NASDAQ-100 price index fluctuations with 90% and 95% VaR, obtained via historical simulation method with 100 simulations

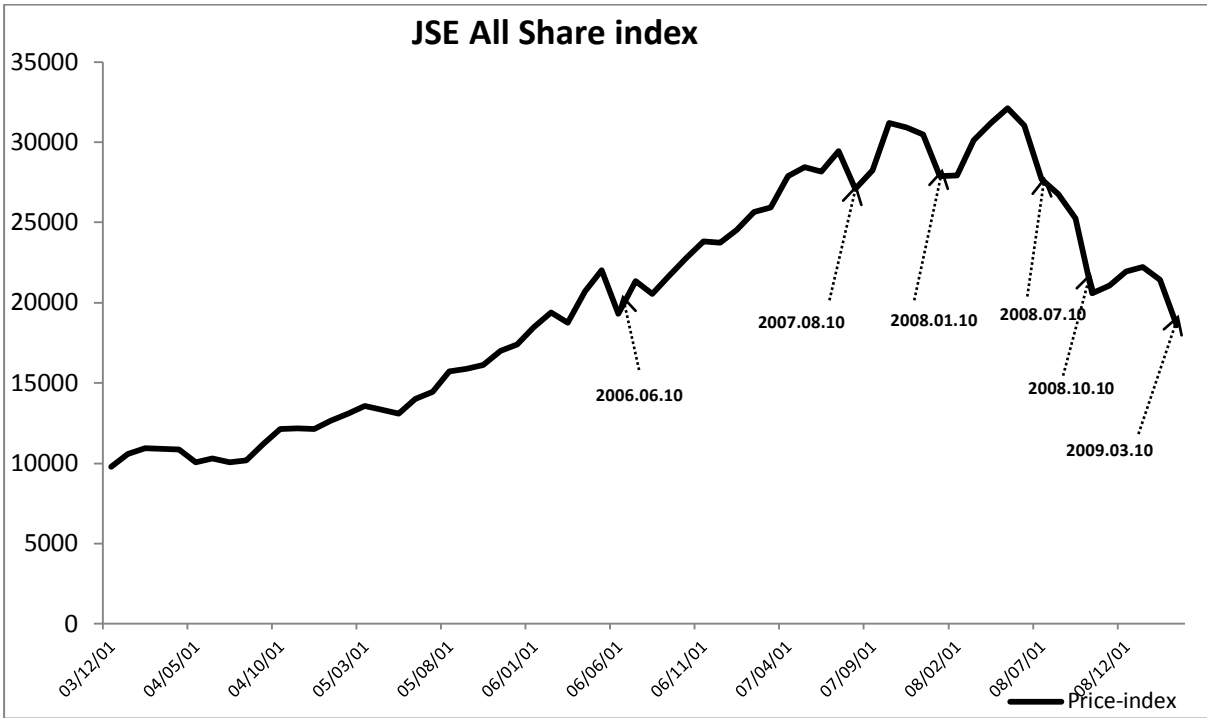


Figure 6.8: A six years graph of JSE All Share Index Performance, from early 2003 to early of two-thousand and nine.

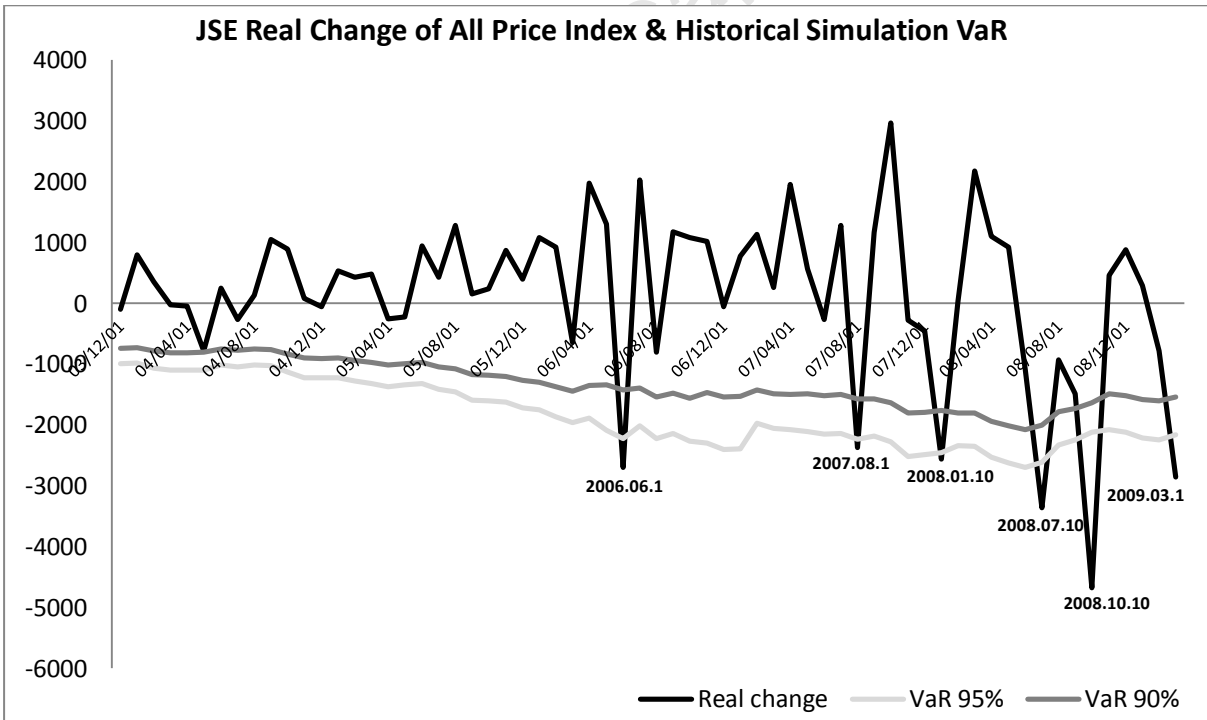


Figure 6.9: JSE All Share price index fluctuations with 90% and 95% VaR obtained through historical simulation method using 100 simulations.

## 6.6.2: D.2 Indexes Performances and VaR Monte Carlo Simulation Approach

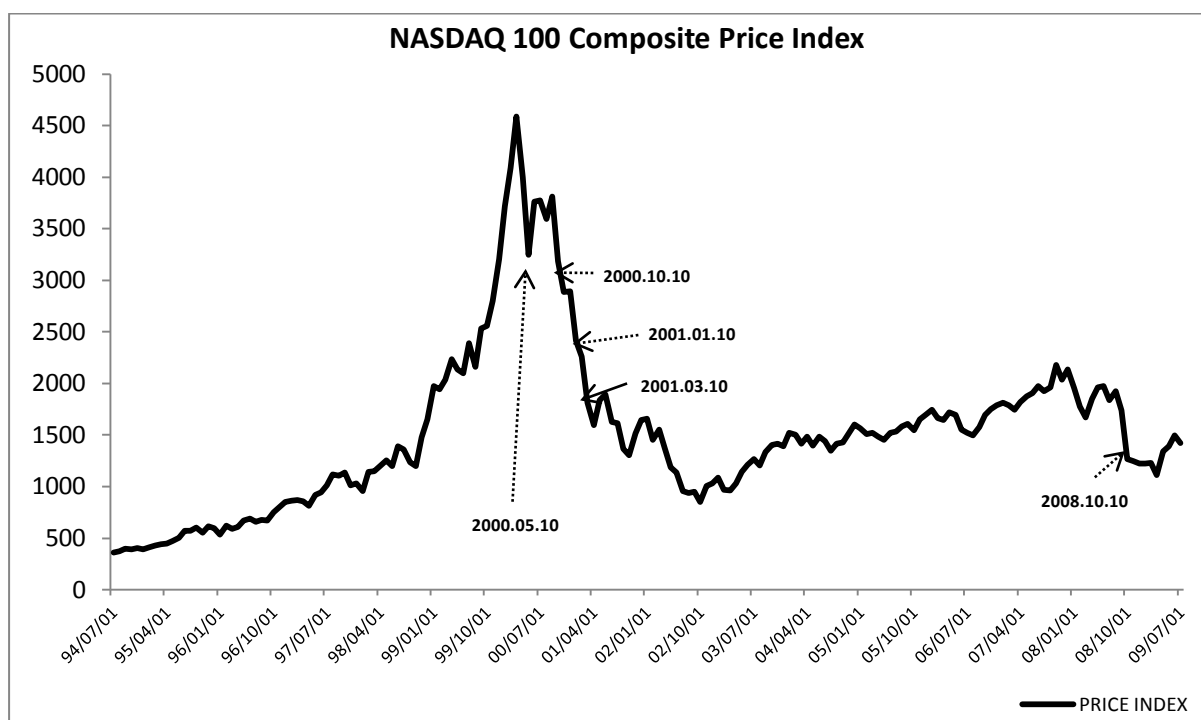


Figure 6.6: A fifteen year graph of the Nasdaq 100 tells the story. From early 1994 to the late summer of two-thousand and nine the tech-heavy index skyrocketed from 1,000 points to 5,000. Investors felt the profit potential for some technology companies was so great they bid up the price of their stocks to hundreds of times earnings.

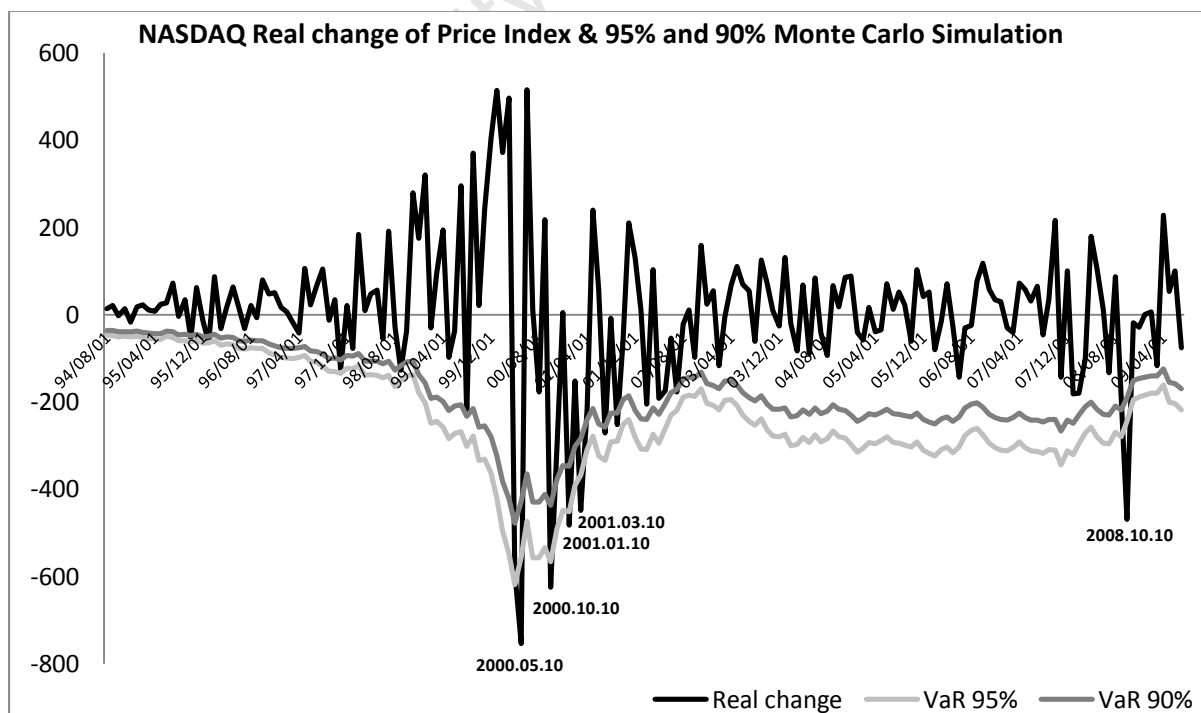


Figure 6.7: Reports a comparison of NASDAQ 100 price index fluctuations with 90% and 95% VaR, obtained via a Monte Carlo simulation method with 5,000 simulations.

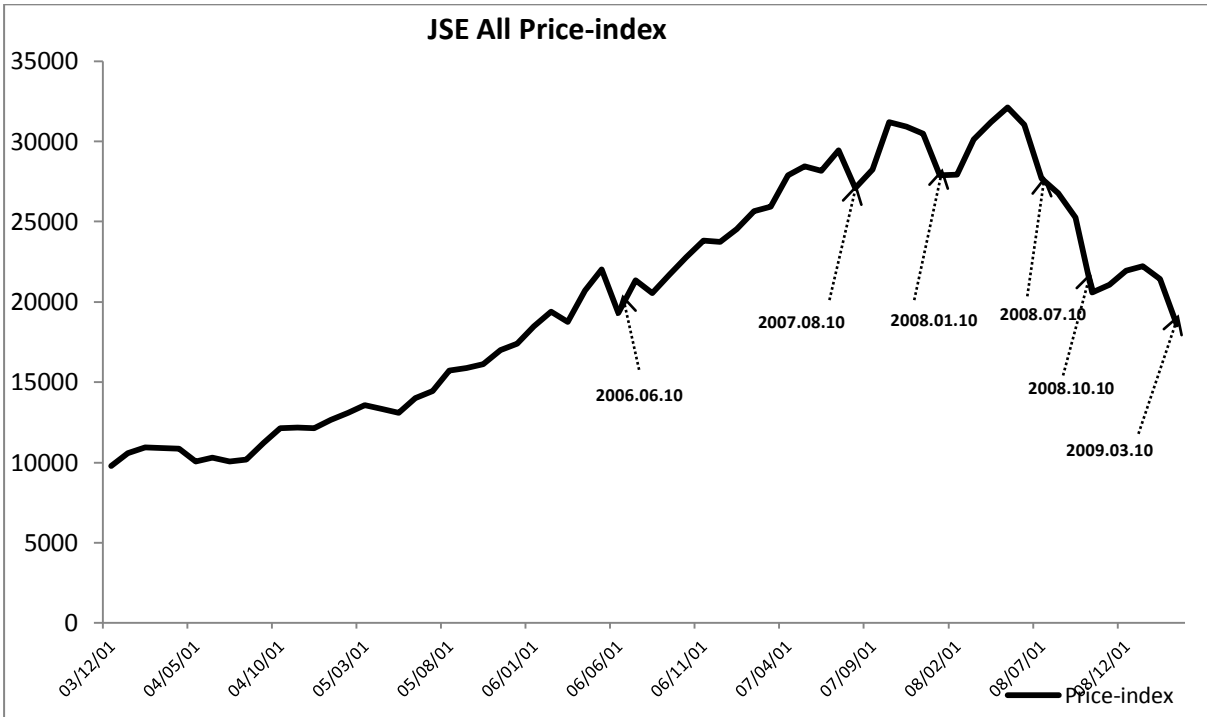


Figure 6.10: Six years graph of JSE All Share Index Performance, from early 2003 to early of two-thousand and nine.

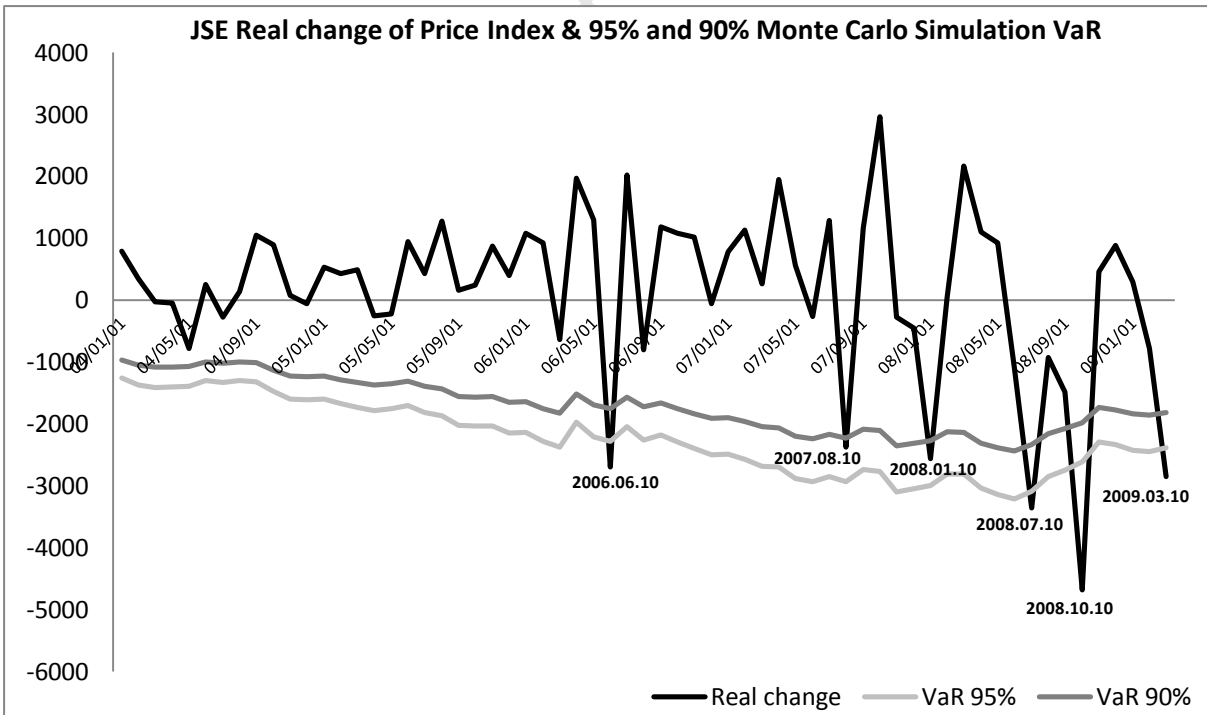


Figure 6.11 Reports a comparison of JSE All Share price fluctuation with 90% and 95% VaR, obtained via a Monte Carlo simulation method with 5,000 simulations.

# Chapter 7

## Conclusions and Future Work

### 7.1: Conclusions

Volatilities play an important role in financial economics and especially in the valuation of various types of options. Volatility is not directly observable in the market. Volatility can only be estimated in the context of a model. Black-Scholes pricing models have been widely used among pricing models. A prediction of the Black-Scholes formula is that all option prices on the same underlying security with the same expiration date but with different exercise prices should have the same implied volatility. The primary focus of the first paper in this thesis was to test the accuracy of the implied volatility derived from the Black-Scholes model. The study also investigates the volatility smile in a South African context.

The empirical findings show that the implied Black-Scholes volatilities vary systematically with strikes, a phenomenon usually referred to as the volatility smile. In the equity market the implied volatilities for options with the same maturity usually decrease as the strikes increase. In other words, the Black-Scholes model underprices deep out-of-the-money put options and overprices deep out-of-the-money call options.

The existence of multiple implied volatilities, regardless of whether they arrange themselves in a smile-like pattern, is somewhat confusing. How can the market show that there is more than one volatility for a stock? Clearly, there is something wrong with the Black-Scholes model. This is that it fails to consider all of the factors that enter into the pricing of an option. It accounts for the stock price, the exercise price, the time to expiration, the dividends, and the risk-free rate, but there must be some factor that it overlooks. The implied volatility here

is more or less a catch-all term, capturing whatever variables are missing, as well as the possibility that the model is improperly specified or blatantly wrong.

In the conclusion of this section, the implied volatility and the existence of the volatility smile are suggested to be the result of using a model that does not capture everything that affects the price of options. Practitioners and academics largely accept the limitations of the model and consider the smile a means of forcing the model to reveal information it is not designed to reveal. The strengths of the Black-Scholes model even with its attendant defects may outweigh the disadvantages of other more complex models. The Black-Scholes model is easily employed to compute the option price by using the no-arbitrage argument or risk-neutral method.

The Value at Risk (VaR) method was proposed to examine underlying stock volatilities and their forecasts. The VaR is calculated in this work using historical simulation based on 250 and 500 simulations and the Monte Carlo method based on 5 000 and 10 000 simulations. Results show that if the level of confidence is 99%, then the VaR calculated using the historical method is greater than the actual price changes in 100% of the forecasted cases in the time period under review. It was also concluded that although the estimated VaR via the Monte Carlo simulation approach is greater than the actual price changes in 97% of the cases in the forecasted period, this method is not as efficient as the historical method. However, high option prices imply higher stock volatility, which means a wide possible range of movement in the underlying stock prices. Our results demonstrated a good measure of actual volatility of the underlying stock via 99% historical simulation VaR. Consequently, VaR is assumed to be a good indicator for option prices.

The conclusion is that Value-at-Risk Volatility, calculated by any method, is a reliable measure of option pricing for concerning the actual volatility of the underlying stock.

A trader ideally would like to have very high profit (return on investment) with very low risk (variability of return). However, there is a fundamental relationship between profit and risk:

the higher the expected profit, the greater the risk and, similarly, the lower the expected profit, the lower the risk. Papers two and three focus on the trade-off between profit and risk, as opposed to profit or risk alone, as in VaR. One way of representing this trade-off is to combine profit and risk by taking the expected profit and dividing it by the risk measure. This single quantity is called the reward-to-risk or risk-adjusted performance. There are several risk-adjusting measures based on different notations of risk. Examples are the beta coefficient (the focus of the second paper), the Sharpe Ratio and the market price of risk (the focus of the third paper).

In paper two we examined issues of asset pricing as well as risk and return relationships in a special environment in China's stock market. This market bears distinctive features such as a large percentage of non-tradeable shares and separation of domestic and foreign investors. We find that domestic and foreign investors do not price *A* shares (available for domestic investors) and *B* shares (available for foreign investors) differently. Moreover, we discover that the standard risk and return relationships implied by CAPM models comply with *A* shares. Therefore, domestic investors price asset risk as predicted by CAPM models in China. *A* shares' excess return was regressed on the Chinese market excess return.

We also discovered that the standard risk and return relationships implied by CAPM models comply with *B* shares. Therefore, foreign investors price asset risk as predicted by CAPM models in US. *B* shares' excess return was regressed on the US market excess return. *B* shares are traded in US dollars and the dividends are also paid in US dollars, and the average opportunity cost of investing in Chinese *B* shares is the average return forgone by investing in the US capital market.

Thus, the price differences between domestic *A* shares and foreign *B* shares for the same company could be explained by the different systematic risks in China and the US. The investment opportunities and assets to form portfolios are different for Chinese and foreign investors. Hence *A* and *B* shares will be valued differently by these two segmented groups of

investors. It is conjectured in this study that the *A* share price premium is determined by the limited alternative investment opportunities available to retail investors in China.

The results also suggest that the different pricing can be explained by interventions in the stock market by the Chinese government. Government interventions and policy changes precluded foreigners from buying *A* shares, and domestic residents from buying *B* shares. Other government interventions include changes of interest rate, control of the growing supply of new shares traded in the exchanges, and changes of stock transaction regulations, such as the imposition and removal of daily price change limits. Political events also disturbed the Chinese stock markets during the period under examination.

The results also show the continuous volatility decreases from February, 20<sup>th</sup> 2001 to November, 29<sup>th</sup> 2002, just after the lifting of the restrictions on *B* shares, and from December, 2<sup>nd</sup>, 2002 to May, 1<sup>st</sup> 2008, when there were no restrictions at all. This clearly shows that the Chinese government improved its capital market management skills by applying more market-oriented methods and thereby becoming less directly involved in the equity markets, leaving them to their own devices.

The increase in efficiency over time, indicated by the continuous shrinkage in the gap between daily *A* and *B* share prices in the more recent periods, can be explained by the sequential deregulation and liberalization of the Chinese stock markets, which in the early 1990s were heavily interrupted by unpredictable market interventions by the government. This made these markets persistent rather than neutral. Such non-neutral market persistence allows for profit-making arbitrage opportunities, making these markets thereby unfair. The Chinese markets have become more efficient and neutral in the last few years, and no longer allow abnormal profits, and have become much fairer for all traders.

Paper three investigated the difference between three methods of measuring the trade-off between the risk and return of trading stocks in South Africa and China. This work was organized into two main parts: the first compared the performance of the Market price of risk

and the Sharpe ratio of trading stocks on the JSE, and the second investigated whether Chinese *A* and *B* shares listed on the SSE have a different market price of risk to explain continued pricing disparities in the two types of shares.

In obtain the market price of risk, the stock price was assumed to behave as GBM, and the drift term was decomposed into a risk-free rate and the market price of risk-multiplying volatility. The Maximum Likelihood Method was adopted to estimate the market price of risk. In the Sharpe ratio test, the Maximum Likelihood Estimation Method was adopted to estimate the parameters in the Sharpe ratio equation developed by Sharpe in 1966. By this method we found a basis from which to compare the two risk measurements, as both of them are forward-looking risk-measurement tools. In South Africa we found the Sharpe ratio is as efficient a measure as the market price of risk. However, with regard to the Chinese shares we found the *A* and *B* share price premium can be explained by the higher volatility of the *A* shares.

Findings were that the Chinese *B* share market is less liquid than the *A* share market and thus investors require a liquidity premium in order to compensate for *B* shares. This partly contributes to the *B* share-pricing discrepancy. Since *B* shares are less liquid than *A* shares, it is reasonable to assume that the volatility of *B* shares is also less than that of corresponding *A* shares. *B* shares have a lower trading volume than corresponding *A* shares and therefore a lower volatility. The market price of risk for *A* shares and *B* shares is almost identical. This makes sense because both *A* and *B* shares are issued by the same company and have virtually the same voting rights and dividends. Thus, *A* and *B* shares should have the same market price of risk and the same return, as this study has shown.

Paper four investigates the presence of rational speculative bubbles in the NASDAQ and JSE stock markets. This study suggests an empirical framework on a possible mechanism to describe asset-price bubbles. The relationship between stock market value based on market fundamentals and the stock market price was analyzed in the context of rational speculative

bubble theory. The theory of speculative bubbles predicts that stock market prices fluctuate around a fundamental value path and price bubbles develop as a series of small persistent steps away from this path. Any sudden movement back to the fundamental path is the bursting of the bubble. This study tests this theory by attempting to capture these characteristics using different VaR techniques.

In this study, a bubble burst is identified as a violation of VaR. Using monthly data, this study reports the existence of rational speculative bubbles in the NASDAQ-100 stock market during 1998-2000 and the bubble burst in 2000-2001 relating to the dot-com bubble and the crisis thereafter. We also report the existence of rational speculative bubbles on the JSE stock market during 2006-2008 and the bubble burst in 2009. Success in estimating the bubble burst could open a new path for academic research in finance. VaR captures extreme events and helps us learn about fat-tailed distributions of risk factors which might be caused by rational bubbles.

Although there are theoretical and empirical problems with regard to the measurement of rational bubbles, there is much to be learnt from studying them. At the very least research into bubbles can demonstrate how it is possible to obtain large and persistent swings in prices with only small effects on expected returns in any one period.

In the extreme case of rational bubbles, price movements are so persistent that they have no effect on expected returns at all. These large swings may not be due only to 'irrational' behaviour, as is commonly implied and therefore merit further study. Part of the reason for the failure to exploit bubbles seems to stem from greed. Investors who believe that assets are over-priced want to generate profits from stock market bubbles, but are hampered by the difficulty of determining when a bubble will burst. The price of overvalued assets may then become even more exorbitant. These overvaluations can stretch over years. This study contends that the impact of psychological behaviour on modern finance theory can no longer be ignored.

## 7.2: Future Study

An extension of this research could include:

- Answering the question: which implied volatility provides the best measure of future volatility? The volatility smile/skew phenomenon makes it unclear which implied volatility provides the best measure of market volatility expectation over the remaining life of the options.
- Further investigation into the unique nature of Chinese stock markets. Chinese listed companies raise foreign capital through issuing domestically listed *B* shares, overseas listed H shares in Hong Kong and ADRs in the US. Though companies view cross-listing as value enhancing, future research could examine how the change in liquidity and volatility, and the cost of trading following cross-listing may adversely affect the quality of the domestic equity.
- Related to the above research, comparing the value at risk VaR of the domestic *A* shares and the foreign *B* shares. This comparison will show which share is more risky to hold.
- Examining whether there was any real economic gain from overinvestment following the US stock price bubble in the late 1990s and investigating why stock return volatility is typically higher after the stock market falls than after it rises (referred as asymmetric volatility)

# Bibliography

- Ahn, C. (1992). Option pricing when jump risk is systematic, *Mathematical Finance* 2:299-308.
- Alexander, C. (2001). *Market Models*, England: John Wiley and Sons
- Alexander, C. and Nogueira, L. (2004). Hedging with Stochastic Local Volatility, *ISMA Centre Discussion Papers in Finance No. DP2004-11*.
- Alexander, G. and Baptista, A. (2004). A comparison of VaR and CVaR constraints on portfolio selection with mean-variance model, *Management Science* 50(9):1261–1273.
- Alexander, G. and Baptista, A. (2002). Economic implications of using a mean-VaR model for portfolio selection: a comparison with mean-variance analysis, *Journal of Economic Dynamics and Control* 26:1159-93.
- Amin, G. and Kat, H. (2002). Generalization of the Sharpe Ratio and the Arbitrage-Free Pricing of Higher Moments, *Alternative Investment Research Centre Working Paper No. 2002-15*.
- Amin, G. and Kat, H. (2002a). Portfolios of Hedge Funds: What Investors Really Invest In, *ISMA Discussion Paper No. 07*.
- Amin, K. and Ng, V. (1993). Option valuation with systematic stochastic volatility, *Journal of Finance* 48:881-910.
- Asgharian, H. and Hansson, B. (2005). Evaluating the Importance of Missing Risk Factors Using the Optimal Orthogonal Portfolio Approach, *Journal of Empirical Finance* 12:556-75.
- Bachelier, L. (1900). Théorie de la Speculation. *Annales Scientifiques de L'École Normale Supérieure 3d ser., 17*, 21-88. Translation in P. Cootner, ed. *The Random Character of Stock Market Prices*. Cambridge: MIT Press, (1964), pp17-79.
- Bailey, W. (1994). Risk and Return on China's New Stock Markets: Some Preliminary Evidence, *Pacific-Basin Finance Journal* 2(2/3):243-260.
- Bailey, W. and Jagtiani J. (1994). Foreign ownership restrictions and stock prices in the Thai capital market, *Journal of Financial Economics* 36(1):57-87.
- Baillie, R. and DeGennaro, R. (1990). Stock Returns and Volatility, *Journal of Financial and Quantitative Analysis* 25:203-214.

Barberis, N. and Huang, M. (2001): Mental accounting, loss aversion, and individual stock returns, *Journal of Finance* 56:1247-1292.

Barone, A. and Whaley, R. (1987). Efficient analytic approximation of American option values, *Journal of Finance* 42:301-20.

Basak, S. (2005). Asset Pricing with Heterogeneous Beliefs, *Journal of Banking and Finance*, 29:2849-2881

Basak, S. and Shapiro, A. (2001). Value-at-risk-based risk management: Optimal policies and asset prices, *The Review of Financial Studies* 14(2):371–405.

Bates, D. (1991). The crash of 87: Was it expected? The evidence from options markets, *Journal of Finance* 46:1009-1044.

Bates, D. (1996). Jumps and stochastic volatility: exchange rate process implicit in Deutsche market option, *Review of financial studies* 9:69-107

Beder, T. (1995). VAR: Seductive but Dangerous, *Financial Analysts Journal* 51(5):12-24.

Bergstrom, C., Rydqvist, K. and Sellin, P. (1993). Asset Pricing with In-and Outflow Constraints: Theory and Empirical Evidence from Sweden, *Journal of Business Finance and Accounting* 20(6):865-879.

Bernstein, P. (1992). *Capital Idea*, New York: The Free Press.

Binswanger M. (1999). *Stock Markets, Speculative Bubbles and Economic Growth*, Aldershot: Edward Elgar.

Binswanger, M. (2004). How Important are Fundamentals? Evidence from a Structural VAR Model for the Stock Markets in the US, Japan and Europe, *Journal of International Financial Markets* 14:185-201.

Black F. (1976). Studies of stock price volatility changes, Proceedings of the 1976 Meeting of Business and Economic Statistics Section, *American Statistical Association* 177-81.

Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing, *Journal of Business* 45(3):444-454.

Black, F. (1975). Fact and Fantasy in the Use of Options, *Financial Analysts Journal* 31(4):36-72.

Black, F. (1976). The pricing of commodity contracts, *Journal of Financial Economics* 3:167-79.

- Black, F. (1979). Business cycles in general equilibrium, *MIT working paper, unpublished*
- Black, F. (1986). Noise, *Journal of Finance* 41:529-43.
- Black, F. and Scholes, M. (1972). The valuation of option contracts in a test of market efficiency, *Journal of finance* 27(3):399-417.
- Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* 81(3):637-59.
- Black, F., Derman, E. and Toy, W. (1990). A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options, *Financial Analysts Journal* 46:33-39.
- Blanchard, O. (1979). Backward and Forward Solutions for Economies with Rational Expectations, *American Economic Review* 69:114-118.
- Blanchard, O. and Watson, M. (1982) .Bubbles, Rational Expectations and Financial Markets. In P. Wachtel, ed, *Crisis in the Economic and Financial Structure*. Lexington Books: Lexington, MA. pp. 295-315.
- Blattberg, R. and Gonedes, N. (1974). A Comparison of Stable and Student Distributions as Statistical Models for Stock Prices, *Journal of Business* 47:244-280.
- Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics* 31:307-327.
- Boness, A. (1964). Elements of a theory of stock-option value, *Journal of Political Economy* 72:163 -175.
- Boudoukh, J., Richardson, M. and Whitelaw, R. (1998). The Best of Both Worlds, *Risk* 11:64-67.
- Boyle, P. (1977). Options: A Monte Carlo Approach, *Journal of Financial Economics* 4:323-338.
- Brealey, R. (1983). *An introduction to risk and return from common stocks*, Cambridge: Cambridge University Press.
- Brealey, R. and Myers, S. (1996). *Principles of Corporate Finance*, 5<sup>th</sup> ed. New York: McGraw-Hill.
- Breeden, D. and Litzenberger, R. (1978). Price of state-contingent claims implicit in option prices, *Journal of Business* 51(4):621-651.
- Breen, W., Glosten, L.R. and Jagannathan, R. (1989). Economic significance of predictable variations in stock index returns, *Journal of Finance* 44:1177-1189.

- Britten, J. and Neuberger, A. (2000). Option prices, implied prices processes, and stochastic volatility, *Journal of Finance* 55(2):839-866.
- Butler, J. and Schachter, B. (1986). Unbiased Estimation of the Black/Scholes Formula, *Journal of Financial Economics* 15:341-357.
- Camerer, C. (1987). Do biases in probability judgement matter in markets? Experimental evidence, *American Economic Review* 77:981-997.
- Campbell, J. (1987). Stock Returns and the Term Structure, *Journal of Financial Economics* 18:373-99.
- Campbell, J. and Hentschel, L. (1992). An asymmetric model of changing volatility in stock returns, *Journal of Financial Economics* 31:281-318
- Campbell, J. and Shiller, R. (1987). Cointegration and Tests of Present Value Models, *Journal of Political Economy* 95(5):1062-1088.
- Campbell, J., Lo, A. and Mackinlay, C. (1997). *The econometrics of financial markets*, New Jersey: Princeton University Press.
- Carr, P. and Madan, D. (1998). Option valuation using the fast Fourier transform, *Journal of Computational Finance*, 2:61-73.
- Castelli, C. (1877). *The Theory of Options in Stocks and Shares*, London: F.C. Mathieson.
- Chakravarty, S., Sarkar, A. and Wu, L. (1998). Information Asymmetry, Market Segmentation and the Pricing of Cross-Listed Shares: Theory and Evidence from Chinese A and B Shares, *Journal of International Financial Markets, Institutions and Money* 8(3-4):325-355.
- Chen, G., Lee, B. and Rui, O. (2001). Foreign Ownership Restrictions and Market Segmentation in China's Stock Markets, *Journal of Financial Research* 24(1):133-155.
- Christie, A. (1982). The stochastic behaviour of common stock variances: values leverage and interest rate effects, *Journal of Financial Economics* 10:15-36.
- Chung, H. and Lee, B. (1998). Fundamental and Nonfundamental Components in Stock Prices of Pacific-Rim Countries, *Pacific-Basin Journal of Finance* 6:321-346.
- Chui, A. and Kwok C. (1998). Cross-Autocorrelation Between A Shares And B Shares In The Chinese Stock Market, *Journal of Financial Research*, 21(3): 333-353.

Cochrane, J. (1991). Volatility Tests and Efficient Markets, *Journal of Monetary Economics* 27:463-485.

Cochrane, J. (2001). *Asset Pricing*, Princeton: Princeton University Press.

Cootner, P. (1964). *The random character of stock market prices*, Cambridge: Cambridge University Press.

Courtadon, G. (1982). A More Accurate Finite Difference Approximation for the Valuation of Options, *Journal of Financial and Quantitative Analysis* 17:697-703.

Cox, J. and Ross, S. (1976). The Valuation of Options for Alternative Stochastic Processes, *Journal of Financial Economics* 3:145-166.

Cox, J., Ross, S. and Rubinstein, M. (1979). Option Pricing: A Simplified Approach, *Journal of Financial Economics* 7:229-263.

Danielsson, J. and de Vries, C. (1997c). Value at Risk and Extreme Returns, London School of Economics, *Financial Markets Group Discussion Paper No. 273*.

Das, S. and Foresi, S. (1996). Exact prices for bond and option prices with systematic jump risk, *Review of derivatives research* 1:7-24.

DeBondt, W. and Thaler, R. (1985). Does the Stock Market Overreact?, *Journal of Finance* 40:793-808.

DeLong, J., Shleifer, A., Summers L., and Waldman, R. (1990). Noise Trader Risk in Financial Markets, *Journal of Political Economy* 98:703-738.

Derman, E. and Kani, I. (1994). Riding on a Smile, *RISK*, 7(2):32-39.

Diba, B. and Grossman, H. (1988b). Explosive Rational Bubbles in Stock Prices?, *American Economic Review* 78(3):520-528.

Domowitz, I., Glen, J. and Madhavan, A. (1997). Market Segmentation and Stock Prices: Evidence from an Emerging Market, *The Journal of Finance* 52(3):1059-1085.

Donaldson, R. and Kamstra, M. (1996). A new dividend forecasting procedure that rejects bubbles in asset prices: The case of 1929's stock crash, *Review of Financial Studies* 9(2):333-383

Dowd, K. (1998). *Beyond Value at Risk: the new science of risk management*, New York: John Wiley & Sons, Inc

- Dowd, K. (1999). A Value at Risk Approach to Risk-Return Analysis, *Journal of Portfolio Management* 25(4):60-67.
- Dowd, K. (1999a). Financial Risk Management, *Financial Analysts Journal* 55:65-71.
- Dowd, K. (2000). Adjusting for Risk: An Improved Sharpe Ratio, *International Review of Economics and Finance* 9:209-222.
- Dowd, K. (2000). Estimating Value-at-Risk: A subjective approach, *The Journal of Risk Finance* 1(4):43-46.
- Duan, J. (1995). The GARCH option pricing model, *Mathematical Finance* 5:13-32.
- Duffie, D. and Pan, J. (1997). An Overview of Value at Risk. *Journal of Derivatives* 4(3):7-49.
- Dumas, B., Fleming, J. and Whaley, R. (1998). Implied volatility functions: empirical tests, *Journal of Finance* 53(6):2059-2106.
- Dupire, B. (1994): Pricing with a Smile, *RISK* 7 (1):18-20.
- Edwards, W. (1968). Conservatism in Human Information Processing. In B.Kleinmuntz, ed, *Formal Representation of Human Judgment* New York: Wiley. pp. 17–52.
- Engle, R. (1982) Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica* 50:987-1007.
- Engle, R. and Manganelli, S. (2001). CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles, *NBER Working Paper No.W7341*.
- Engle, R. and Manganelli, S. (2004). CAViaR: Conditional autoregressive Value at Risk by regression quantile, *Journal of Business and Economic Statistics* 22:367-381.
- Estrella, A. (1996). Taylor, Black and Scholes: Series Approximations and Risk Management Pitfalls. In Board of Governors of the Federal Reserve System ed. *Risk Measurement and Systemic Risk, Proceedings of a Joint Central Bank Research Conference*. Washington, D.C. pp 359-379.
- Evans, G. (1991). Pitfalls in Testing for Explosive Bubbles in Asset Prices, *American Economic Review* 81(4):922-930.
- Fama, E. (1965). The behaviour of stock market prices, *Journal of Business* 38:34-105.
- Fama, E. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance* 25:383-417.

- Fama, E. (1991). Efficient capital markets: II, *Journal of Finance* 96:1575–1617.
- Fama, E. and French, K. (1977). Asset Returns and Inflation, *Journal of Financial Economics* 5:115-46.
- Feinstein, S. (1989). Forecasting stock market volatility using options on index futures, *Economic Review, Federal Reserve Bank of Atlanta* 97:115-154.
- Fernald, J. and Rogers, J. (2002). Puzzles in the Chinese Stock Market, *Review of Economics and Statistics* 84:416-432.
- Figlewski, S. (1989). Options Arbitrage in Imperfect Markets, *the Journal of Finance* 44(5):1289-1311.
- Fisher, R and Tippett, L. (1928). Limiting forms of the frequency distribution of the largest or the smallest member of a sample, *Proceeding of Cambridge Philosophical Society* 24:180-190.
- Flavin, M. (1983). Excess volatility in the financial markets: a reassessment of the empirical evidence, *Journal of Political Economy* 91:929-56.
- Fletcher, R. (1981): *Practical Methods of Optimization*, London: John Wiley and Sons.
- Flood, R. and Garber, P. (1980). Market Fundamentals versus Price-Level Bubbles: The First Tests, *Journal of Political Economy* 88(4):745-770.
- Flood, R. and Hodrick, R. (1986). Asset Price Volatility, Bubbles and Process Switching, *Journal of Finance* 41:831-42.
- French, K., Schwert. G. and Stambaugh. R. (1987). Expected stock returns and volatility, *Journal of Financial Economics* 19:3-29.
- Froot, K. and Obstfeld M. (1991). Intrinsic Bubbles: the Case of Stock Prices, *American economic review* 1189-1215.
- Fukuta, Y. (1998). a Simple Discrete-Time Approximation of Continuous-Time Bubbles, *Journal of Economic Dynamics and Control* 22:937-954.
- Galai, D. (1977). Tests of market efficiency of the Chicago Board Options Exchange, *Journal of business* 50(2):167-197.
- Galai, D. (1983). A Survey of Empirical Tests of Option-Pricing Models. In M. Brenner, ed. *Option Pricing: Theory and Applications*. Lexington Books: Lexington Mass, (1983), pp. 45-80

- Garman, M. and Kohlhagen, W. (1983). Foreign Currency Option Values, *Journal of International Money and Finance* 2:231-237.
- Gencay, R. and Selcuk, F. (2004). Extreme value theory and Value-at-Risk: Relative performance in emerging markets, *International Journal of Forecasting* 20:287-303.
- Geske, R. (1979). A Note on an Analytic Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends, *Journal of Financial Economics* 7: 375-380.
- Geske, R. (1979). The Valuation of Compound Options, *Journal of Financial Economics* 7(1):63-81.
- Glosten, L. Jagannathan R. and Runkle D. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Returns on Stocks, *Journal of Finance* 48:1779-1801.
- Goldman, M., Sosin, H. and Gatto, M. (1979). On contingent claims that insure ex-post optimal stock market timing, *Journal Finance* 34:401-414.
- Grabbe, O. (1983). The Pricing of Call and Put Options on Foreign Exchange, *Journal of International Money and Finance* 2:239-253.
- Gua, W. and Su, T. (2006). Option Put-Call Parity Relations When the Underlying Security Pays Dividends, *International Journal of Business and Economics* 5(3):225-230.
- Halek, M. and Eisenhauer, J. (2001). Demography of Risk Aversion, *Journal of Risk and Insurance* 68:1-24.
- Harvey, C. (1991). The World Price of Covariance Risk, *Journal of Finance* 46:111-157.
- Heston, S. (1993). A Closed-Form Solution for Options with Stochastic Volatility, with Application to Bond and Currency Options, *The Review of Financial Studies* 6:327-343.
- Heston, S. and Nandi, S. (2000). A closed-form GARCH option valuation model, *Review of Financial Studies* 13:585-625.
- Hietala, P. (1989). Asset pricing in partially segmented markets: Evidence from the Finish market, *Journal of Finance* 44(4):697-718.
- Ho, T. and Lee, S-B (1986). Term Structure Movements and Pricing Interest Rate Contingent Claims, *the Journal of Finance* 41:1011-1029.
- Hodges, S. (1998). A Generalization of the Sharpe Ratio and its Applications to Valuation Bounds and Risk Measures, *Financial Options Research Centre, University of Warwick Working Paper No. 88*.

- Hong, H., Scheinkman J. and Xiong, W. (2006). Asset Float and Speculative Bubbles, *Journal of Finance* 3:1073-1117.
- Huisman, R., Koedijk, K. and Pownal, R. (1998). VaR+: Fat Tails in Financial Risk Management, *Journal of Risk* 1:47-61.
- Hull, J. (1997). *Options, Futures, and Other Derivative*, Toronto: Prentice Hall.
- Hull, J. (2000). *Options, Futures and other Derivatives*, Toronto: Prentice Hall.
- Hull, J. (2006). *Options, futures and Other Derivatives*, New Jersey: Prentice-Hall
- Hull, J. and White, A. (1987). The pricing of options on assets with stochastic volatilities, *Journal of Finance* 42(2):281-300.
- Hull, J. and White, A. (1990). Pricing Interest Rate Derivative Securities, *The Review of Financial Studies* 3:573-592.
- Ingersoll, J. (1976). A Theoretical and Empirical Investigation of the Dual Purpose Funds, *Journal of Financial Economics* 3:81-123.
- Ingersoll, J. (1989). *Theory of Financial Decision Making*, Totowa: Rowman & Littlefield.
- Jackwerth, C. and Rubinstein, M. (1996). Recovering probability distributions from option prices, *Journal of Finance* 51(5):1611-1631.
- Jackwerth, J. (1999). Option-Implied Risk-Neutral Distributions and Implied Binomial Trees: A Literature Review, *Journal of Derivatives* 7(2):66-82.
- Jansen, D. and de Vries, C. (1991). On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective, *the Review of Economics and Statistics* 73:18-24.
- Jarque, C. and Bera, A. (1987). A test for normality of observations and regression residuals, *International Statistical Review* 55(2):163-172
- Jarrow, R. and Rosenfeld, E. (1984). Jump risks and the intertemporal capital asset pricing model, *Journal of business* 57:337-351.
- Jarrow, R. and Rudd, A. (1982). Approximate option valuation for arbitrary stochastic processes, *Journal of Financial Economics* 10:347-369.
- Jensen, M. (1968). The Performance of Mutual Funds: 1945-64, *The Journal of Finance* 23(2):389-416.

- Jensen, M. (1978). Some Anomalous Evidence Regarding Market Efficiency, *Journal of Financial Economics* 6(2/3):95-101.
- Johnson, H. and Shanno, D. (1987). Option Pricing when the Variance is Changing, *Journal of Financial and Quantitative Analysis* 22:143-151.
- Jorion, P. (2000). *Value at Risk, The New Benchmark for Managing Financial Risk*, New York: McGraw-Hill.
- Jorion, P. (1988). On Jump Processes in the Foreign Exchange and Stock Markets, *The Review of Financial Studies* 1(4):427-445.
- Jorion, P. (2000): *Value at Risk*, Chicago et al: McGraw-Hill.
- Kang, J., Liu, M. and Ni, S. (2002). Contrarian and momentum strategies in the China stock market: 1993–2000, *Pacific-Basin Finance Journal* 10(3):243-265.
- Karolyi, G. and Li, L. (2003). A resolution of the Chinese discount puzzle, *Ohio State University Working paper* 2003, p. 34.
- Kataoka, S. (1963). A Stochastic Programming Model, *Econometrica* 31:181-196.
- Kim, D. and Kon, S. (1994). Alternative Models for the Conditional Heteroscedasticity of Stock Returns, *Journal of Business* 67(4):563-598.
- Kim, Y. and Shin, J. (2000). Interactions among China-related stocks, *Asian-Pacific Financial Markets* 7(1):97-115.
- Kleidon, A. (1986). Variance bounds tests and stock price valuation models, *Journal of Political Economy* 94:953-1001.
- Kon, S. (1984). Models of Stock Returns: A Comparison, *Journal of Finance* 39(1):147-165.
- Kryzanowski, L. and To, M. C. (1983), General Factor Models and the Structure of Security Returns. *Journal of Finance and Quantitative Analysis* 17 (September):31-32.
- Latane, H. and Rendleman, R. (1976). Standard deviation of stock price ratios implied in option prices, *Journal of Finance* 31(2):369-381.
- Leadbetter, M., Lindgren, G. and Rootzen, H. (1983). *Extremes and Related Properties of Random Sequences and Processes*. New York: Springer-Verlag.
- Lee, B. (1998). Permanent, Temporary and Nonfundamental Components of Stock Prices, *Journal of Finance and Quantitative Analysis* 33:1-32.

- LeRoy, S. and Porter, R. (1981). The Present-Value Relation: Tests Based on Implied Variance Bounds, *Econometrica* 49:555-577.
- Levy, H. (1978). Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in a Portfolio, *American Economic Review* 68:643-658.
- Li, Qi. (2004). *Three Essays on Emerging Capital Markets*. Ph.D. University of Pittsburgh.
- Lin, Tan. (2005). *Empirical Analysis of Chinese Stock Market Behavior: Evidence from Dynamic Correlations, Herding Behavior, and Speed of Adjustment*. Ph.D. Drexel University.
- Lintner, J. (1965). The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics* 47(1):13-37.
- Lucas, R. (1978). Asset prices in an exchange economy, *Econometrica* 46:1429-1445.
- Ma, X. (1996). Capital Controls, Market Segmentation and Stock Prices: Evidence from the Chinese Stock Market, *Pacific-Basin Finance Journal* 4:219-239.
- MacBeth, J. and Merville, L. (1979). An Empirical Examination of the Black-Scholes Call Option Pricing Model, *Journal of Finance* 34(5):1173-1186.
- MacKinlay, A. and Pastor, L. (2000). Asset Pricing Models: Implications for Expected Returns and Portfolio Selection, *Review of Financial Studies* 13:883-916.
- MacMillan, L. (1986). An Analytic Approximation for the American Put Option, *Advances in Futures and Options Research* 1:119-139.
- Madan, D., Carr, P. and Chang, E. (1998). The variance gamma process and option Pricing, *European Finance Review* 2:79-105.
- Malkiel, B. G., and Xu, Y. (2002). Idiosyncratic risk and security returns, *Department of Economics, University of Texas at Dallas Working Paper*, February 2002.
- Manganelli, S. and Engle, R. (2001). Value at risk models in finance, *European Central Bank Working Paper No. 75*.
- Mao, J. C., (1971). Security pricing in an imperfect capital market, *Journal of Financial and Quantitative Analysis* 6:1105-1116.
- Mandelbrot, B. (1963). The Stable Paretian Income Distribution when the Apparent Exponent is Near Two, *International Economic Review* 4(1):111-115.

Mayers, D. (1976). Nonmarketable assets, market segmentation, and the level of asset prices, *Journal of Financial and Quantitative Analysis* 11(1):1-12.

McNeil A. (1998). Calculating Quantile Risk Measures for Financial Time Series Using Extreme Value Theory, *Swiss Federal Technical University ETH Working Paper*, ([www.math.ethz.ch/~embrechts](http://www.math.ethz.ch/~embrechts)).

McQueen, G. and Thorley S. (1994). Bubbles, Stock Returns and Duration Dependence, *Journal of Financial and Quantitative Analysis* 29(3):379-401.

Melino, A. and Turnbull S (1995). Misspecification and the Pricing and Hedging of Long Term Foreign Currency Options, *Journal of International Money and Finance* 14(3):373-393

Melino, A. and Turnbull, S. (1990). Pricing foreign currency options with stochastic volatility, *Journal of econometrics* 45:239-265.

Merton R. (1973). Theory of rational option pricing, *Bell Journal of Economics and Management Science* 4(1):141-183.

Merton R. (1976a). Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics* 3(1/2):125-44.

Merton, R. (1976b). The Impact on Option Pricing Specification Error in the Underlying Stock Price Returns, *Journal of Finance* 31(2):333-350.

Merton, R. (1987). A Simple Model of Capital Market Equilibrium with Incomplete Information, *Journal of Finance* 52:483-510.

Merton, R. (1987). Presidential address: A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42:483-510.

Minnich, M. (1998). A Primer on Value at Risk. In J. Fabozzi, ed. *Perspectives on Interest Rate Risk Management for Money Managers and Traders*, (1998), pp.39-50.

Morgan, J. (1996). *Risk Metrics-Technical Document*, New York: J.P. Morgan.

Nabdi, S. (1998). How important is the correlation between returns and volatility in a stochastic volatility Model? Empirical evidence from pricing and hedging in the S&P 500 Index options market, *Journal of banking and Finance* 22:589-610.

Nagot, I. and Trommsdorff, R. (1999). The tree of knowledge, *Risk* 12(8):99-102.

Naik, V. and Lee, M. (1990). General Equilibrium Pricing of Options on the Market Portfolio with Discontinuous Returns, *The Review of Financial Studies* 3:493-521.

- Ncube, M. and Satchell, S. (1997). The Statistical Properties of the Black-Scholes Option Price, *Mathematical Finance* 7:287-305.
- Norden, S. and Schaller, H. (1993). The predictability of stock market regime: Evidence from the Toronto stock exchange, *Review of Economics and Statistics* 75(3):505-514.
- Pindyck, R. (1984). Risk, inflation, and the stock market, *American Economic Review* 74:334-351.
- Praetz, P. (1972). The Distribution of Share Price Changes, *Journal of Business* 45:57-65.
- Roll, R. (1977). An Analytical Formula for Unprotected American Call Options on Stocks with Known Dividends, *Journal of Financial Economics* 5:251-258.
- Roy, A. (1952). Safety First and the Holding of Assets, *Econometrica* 20(3):43-449.
- Rubinstein, M. (1985). Nonparametric Tests of Alternative Option Pricing Models, *Journal of Finance* 40(2):455-480.
- Rubinstein, M. (1994). Implied binomial trees, *Journal of Finance* 49:771-818
- Samuelson, P. (1965). Rational theory of warrant pricing, *Industrial Management Review* 6:13-31.
- Scheinkman, J. and Xiong, W. (2003). Overconfidence and Speculative Bubbles, *Journal of Political Economy* 111(6):1183-1219.
- Schmanlensee R. and Trippi, R. (1978). Common Stock Volatility Expectations Implied by Option Premia, *Journal of Finance* 33(1):129-147.
- Schwartz, E. (1997). The valuation of warrants: implementing a new approach, *Journal of Financial Economics* 4:79-93.
- Scott, L. (1987). Option Pricing when the Variance changes randomly: Theory, Estimation, and an Application, *Journal Financial and Quantitative Analysis* 22(4):419-438.
- Sharpe, W. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance* 19(3):425-442.
- Sharpe, W. (1966). Mutual fund performance, *Journal of Business* 39(1):119-138.
- Sharpe, W. (1970). *Portfolio theory and capital markets*, New York: McGraw-Hill.
- Sharpe, W. (1975). Adjusting for Risk in Portfolio Performance Measurement, *Journal of Portfolio Management* 1(2):29-34.

- Sharpe, W. (1978). *Investments*, Englewood Cliffs, New Jersey: Prentice-Hall.
- Sharpe, W. (1994). The Sharpe Ratio, *Journal of Portfolio Management* 21(1):49-58.
- Sharpe, W. F., (1966). Mutual Fund Performance, *Journal of Business* 39:119-138.
- Sharpe, W. F., (1975). Adjusting for Risk in Portfolio Performance Measurement, *Journal of Performance Measurement* Winter 29-34.
- Sharpe, W. F., (1975). The Sharpe Ratio, *Journal of portfolio management* 21(1):49-58.
- Sharpe, W.F. (1963). A simplified model for portfolio analysis. *Management Science* 9(2):277-293.
- Sheikh, A. (1991). Transaction Data Tests of S&P 100 Call Option Pricing, *Journal of Financial and Quantitative Analysis* 26(4):459-475.
- Shiller, R. (1978). Rational expectations and the dynamic structure of macroeconomic models: A critical review, *Journal of Monetary Economics* 5:1-44.
- Shiller, R. (1981). Do stock prices move too much to be justified by subsequent movements in dividends?, *American Economic Review* 71(3):421-36.
- Shiller, R. (1984). Stock Prices and Social Dynamics. *Brookings Papers on Economic Activity* 2:457-498.
- Shiller, R. (1988). Fashions, fads and bubbles in financial markets. In: J. Coffee, ed, *Knights, Riders and Targets*, Oxford: Oxford University Press, pp. 56-68.
- Shiller, R. (2000). *Irrational Exuberance*. Princeton: Princeton University Press.
- Shiller, R. (2002). Bubbles, Human Judgment, and Expert Opinion, *Financial Analysis Journal* 58(3):18-26.
- Shiller, R. (2003). From Efficient Markets Theory to Behavioral Finance, *Journal of Economic Perspectives* 17(1):83-104.
- Shimko, D. (1993). Bounds of probability, *RISK* 6(4):33-37.
- Shleifer, A. (2000). *Inefficient Markets: An Introduction to Behavioural Finance*, London: Oxford University Press.
- Sjoo, B. and Zhang, J. (2000). Market Segmentation and Information Diffusion in China's Stock Markets, *Journal of Multinational Financial Management* 10:421-438.

- Smith, C. (1976). Option pricing: a review, *Journal of Financial Economics* 3:3-52
- Sprenkle, C. (1964). Warrant Prices as Indicators of Expectations and Preferences. In P. Cootner, ed. *The random character of stock market prices*. Cambridge: MA: MIT Press, (1964), pp.412-474.
- Stein, E. and Stein, J. (1991). Stock price distributions with stochastic volatility: an analytic approach, *Review of Financial studies* 4(4):727-752.
- Stulz, F. and Wasserfallen W. (1995). Foreign Equity Investment Restrictions, Capital Flight, and Shareholder Wealth Maximization: Theory and Evidences, *Review of Financial Studies* 8(4):1019-1057.
- Su, D. (1998). The Behavior of Chinese Stock Markets. In J.J Choi and J. Doukas, ed. *Emerging Capital Markets*. Westport, CT: Quorum Books, pp 253-273.
- Summers, L. (1986). Does the Stock Market Rationally Reflect Fundamental Values?, *Journal of Finance* 41:591-601.
- Sun, Q. and Tong, W. (2000). The Effect of Market Segmentation on Stock Prices, *Journal of Banking and Finance* 24(12):1875-1902.
- Szegö, G. (1980). *Portfolio Theory with Application to Bank Asset Management*, New York: Academic Press.
- Tan, W. (2005). Culture conflicts in Sino-foreign ventures, *Beijing Review*, 23 June.
- Telser, L. (1956). Safety First and Hedging, *Review of Economic Studies* 23(1):1-16.
- Thaler, R. (2005). *Advances in Behavioural Finance*, Princeton: Princeton University Press.
- Treynor, J. (1965). How to Rate Management of Investment Funds?, *Harvard Business Review* 43(1):63-75.
- Tucker, A. (1992). A Re-examination of Finite- and Infinite-variance Distributions of Daily Stock Returns, *Journal of Business and Economic Statistics* 10(1):73-81.
- Turner, C., Startz, R. and Nelson, C. (1989) A Markov model of heteroskedasticity, risk, and learning in the stock market, *Journal of Financial Economics* 25:3-22.
- Wang, Y. and Di, A. (2007). The cross-sectional relationship between stock returns and domestic and global factors in the Chinese A-share market, *Review of Quantitative Finance and Accounting*, Springer 29(2):181-203.

- West, K. (1984). Speculative Bubbles and Stock Price Volatility, *Princeton University Financial Research Memorandum No. 54*.
- West, K. (1987). A Specification Test for Speculative Bubbles, *The Quarterly Journal of Economics* 102(3):553-80.
- West, K. (1988). Bubbles, Fads, and Stock Price Volatility Tests: a Partial Evaluation, *Journal of Finance* 43:639-55.
- West, K. (1988a). Dividend Innovations and Stock Price Volatility, *Econometrica* 56(1):37-61.
- Whaley, R. (1981). On the Valuation of American Call Options on Stocks with Known Dividends, *Journal of Financial Economics* 9(2):207-211.
- Whitelaw, R. (1994). Time Variation and Covariations in the Expectation and Volatility of Stock Market Return, *Journal of Finance* XLIX (2):515-541.
- Wiggins, J. (1987). Option Values Under Stochastic Volatility: Theory and Empirical Estimates, *Journal of Financial Economics* 19(2):351-372.
- Wiggins, J. (1987). Option Values Under Stochastic Volatility: Theory and Empirical Estimates, *Journal of Financial Economics* 19(2):351-372.
- Yang, Q. (2006). *Stock bubbles: The theory and estimation*. Ph.D. Business School Brunel University.
- Zhu, J. (2008). Testing for Expected Return and Market Price of Risk in Chinese A-B Share Market: A Geometric Brownian Motion and Multivariate GARCH Model Approach, *School of Economics and Management, University of Aarhus Research Paper No. 2008-15*.