

**Teaching Grade 10 Girls concepts of the
Sine Trigonometric Function
using ChatGPT and GeoGebra**

by

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Abstract

This research investigates how ChatGPT, a generative AI tool, and GeoGebra, an interactive mathematical software, can be employed to enhance Grade 10 girls' learning experience and comprehension of sine trigonometric function concepts. The study seeks to address the main research question – “How can the use of ChatGPT and GeoGebra as instructional tools enhance the teaching of sine trigonometric function concepts in a Grade 10 mathematics class?”. The study is grounded in the socio-constructivist approach, which emphasizes active student-centred learning, and positions teachers as facilitators that promote student engagement during teaching and learning. Trigonometry, particularly concepts related to trigonometric functions, presents significant challenges for South African Grade 12 students. Traditional, teacher-centred methods, often based on lectures, may contribute to this limited understanding. A design-based research methodology, utilizing iterative cycles of design, implementation, and refinement was adopted to facilitate the data collection and analyses from the three lessons used. The research sample consisted of 20 Grade 10 girls. Data was gathered through video recordings, worksheets, focus group interviews of students, and teacher interviews. Thematic analysis was conducted to analyze the qualitative data from the data collection instruments. The main findings indicate that GeoGebra enhanced explorative learning and significantly boosted student engagement. ChatGPT was perceived as most effective when it offered step-by-step guidance for problem-solving or methods to achieve specific outcomes. Both students and the teacher expressed a preference for using ChatGPT and GeoGebra in tandem, seamlessly switching between the two rather than using each tool in isolation. The simultaneous integration of ChatGPT and GeoGebra in teaching sine trigonometric function concepts promoted independent exploration and critical thinking among students. This study underscores the potential benefits of using both ChatGPT and GeoGebra software in the classroom for teaching sine trigonometric function concepts. The findings highlight the pedagogical value of employing ChatGPT and GeoGebra in teaching sine trigonometric functions, which can be insightful for teacher educators, curriculum advisors involved in professional development for in-service teachers, and mathematics teachers. A limitation of the study is its reliance on a small sample from a single school. Future research could address this by repeating the study with a larger sample that includes students from diverse educational and cultural backgrounds.

Declaration

I, Malindri Eastes, hereby declare that the work presented in this dissertation titled "Teaching Grade 10 Girls concepts of the Sine Trigonometric Function using ChatGPT and GeoGebra" is my own and has not been submitted previously for any degree or examination at any other institution. All sources used and contributions from other scholars and researchers have been duly acknowledged and referenced in accordance with the requirements.

I also declare that, to the best of my knowledge, the content of this thesis is a true and accurate representation of my research and findings.

Malindri Eastes

5 December 2024

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LIST OF ACRONYMS

AI	Artificial Intelligence
CAST	Cosine-All-Sine-Tangent
DBR	Design-based Research
ICT	Information and Communications Technology
WCED	Western Cape Education Department
LLM	Large Language Model
MMOE	Malaysian Ministry of Education
WiFi	Wireless Fidelity
MATLAB	Matrix Laboratory

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CHAPTER 1: BACKGROUND OF STUDY

1.0 Introduction

The chapter provides an overview of the study. It begins with a discussion of the learning challenges of trigonometric functions from a global and South African perspective, before outlining the statement of the problem, the rationale behind the study, the study's aims and objectives, research questions, significance of the chosen methodology, and ethical considerations of the study. The chapter concludes with an outline of the subsequent chapters to follow.

1.1 Trigonometric functions learning challenges - Global and South African perspective

Trigonometry is a topic taught in the South African Mathematics high school curriculum and is introduced at the 10th-grade high school level. It plays a pivotal role in the world of mathematics, with wide-ranging applications in fields like statistics, economics, architecture, natural sciences and engineering (Weber, 2005). Students, however, encounter significant hurdles in comprehending trigonometry concepts, particularly those associated with trigonometric functions (Ngcobo et al., 2019; Siyepu, 2015).

Student struggles with learning trigonometry seems to be a global problem. For example, Law et al. (2015) found that Malaysian students faced challenges with conceptual understanding of trigonometric functions. In Serbia, Kamber and Takaci (2018) highlighted that among other things, students have difficulties with properties of periodicity of trigonometric functions. The Republic of Indonesia's 2019 National Examination revealed that a very small fraction of students demonstrated proficiency in various trigonometric aspects, including determining trigonometric ratios, graphing trigonometric functions, and solving problems related to trigonometric rules and identities (Bazuri & Arliani, 2022).

The difficulty students face with trigonometry in South Africa is clearly demonstrated by the results of the South African National Senior Certificate Grade 12 Examinations, which are conducted annually. Below are two figures showing the average performance per question in Paper 2 in 2021 (*National Senior Certificate Diagnostic Report, 2021, p. 198*) and in Paper 2 in 2022 (*National Senior Certificate Diagnostic Report, 2022, p. 208*).

Graph 10.6.1 Average performance per question in Paper 2

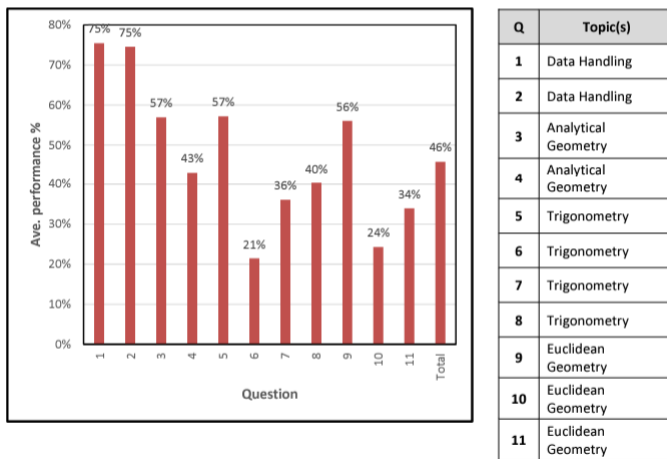


Figure 1: National Senior Certificate Diagnostic Report (2021, p. 198)

A thorough analysis of the statistical data presented in Figure 1 for Paper 2 of the 2021 examination provides valuable insights into student performance on trigonometry-related questions (Questions 5 to 8). Question 5 required students to use reduction formulae to simplify trigonometric expressions, solve trigonometric identities, and apply triangle trigonometric ratios. Question 6 involved simplifying expressions and solving equations using compound and double angle identities and was identified as the lowest-performing question across the entire paper. Question 8, which focused on applying the sine and cosine rules to solve problems involving three-dimensional trigonometric shapes, also revealed performance issues. Additionally, Question 7, which centred on trigonometric functions, was the second lowest-performing question among the trigonometry-related questions and the third lowest-performing question overall, highlighting the significant challenges students faced with trigonometric function concepts in this examination.

Graph 10.6.1 Average performance per question in Paper 2

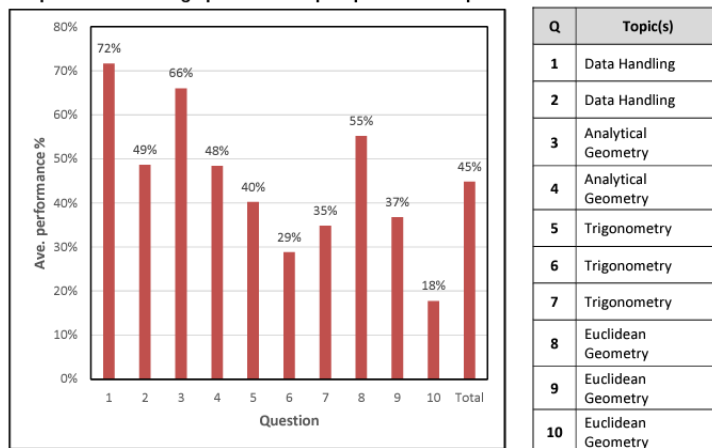


Figure 2: National Senior Certificate Diagnostic Report (2022, p. 208)

Figure 2, which displays the average scores for each question in Paper 2 of the 2022 examination, reveals a similar pattern in student performance on trigonometry-related questions (Questions 5 to 7). Question 5, which required students to use reduction formulae to simplify trigonometric expressions, solve trigonometric identities, and apply triangle trigonometric ratios, was the highest-performing question among the trigonometry-related questions. However, it was still the fourth lowest-performing question across the entire paper. Question 8, which focused on applying the sine and cosine rules to solve problems involving three-dimensional trigonometric shapes, also showed performance issues. Notably, questions related to trigonometric concepts yielded some of the lowest scores on the paper, with Question 6, which centred on trigonometric functions, ranking as the lowest-performing question among the trigonometry-related questions and the second lowest overall.

It is clear from the data above that trigonometry questions, in particular questions related to trigonometric function concepts, pose a challenge to South African Grade 12 students.

1.2 Statement of the Problem

A possible explanation for the limited understanding of trigonometric functions may be linked to traditional teaching methods. These methods often follow a teacher-centred approach, where instructors primarily convey knowledge through lectures or traditional chalk-and-talk sessions. Various studies suggest that these approaches, although efficient in information delivery, may not promote deep learning or engagement and have frequently proven ineffective in helping students establish the crucial interconnections between trigonometric function concepts (Chin, 2013; Demir et al., 2012; Klappa, 2015; Levitt & Grubaugh, 2023).

A method of instruction is needed that emphasizes conceptual comprehension, with the primary goal of ensuring that students effectively grasp trigonometric functions and their interrelationships (Rittle-Johnson & Schneider, 2014). The focus of this study is thus on investigating alternative instructional strategies that can potentially enhance students' understanding of trigonometric function concepts, with the goal of ultimately improving their performance in this area in the future.

1.3 Rationale behind using technology and a socio-constructivist teaching approach

Digital technologies can significantly enhance learning and comprehension of trigonometric function concepts. Studies have shown positive impacts on student achievement using various interactive software technologies (Ross et al., 2011; Urrutia et al., 2019). These tools are used to facilitate exploration, visualization, and dynamic manipulation of trigonometric function concepts, promoting deeper understanding (Kissane & Kemp, 2009; Sacristán et al., 2010). These interactive environments also allow students to transition between algebraic and graphic representations, enhancing their conceptual understanding (Brown, 2015; Nordlander, 2021). However, the effective use of technology in trigonometry education requires careful consideration of instructional sequencing and pedagogical strategies to maximize its potential (Godwin & Sutherland, 2004; Ross et al., 2011; Sacristán et al., 2010).

A socio-constructivist approach to pedagogy emphasizes that educators should shift from being mere conveyors of knowledge to becoming facilitators. This pedagogical approach is noted to promote social interaction, nurture critical thinking abilities and facilitate holistic learning experiences and active participation in exploration and problem-solving activities (Bay et al., 2012; Clark & Dumas, 2015; Le & Nguyen, 2024; Lo & Monge, 2013). Several researchers highlight the value of adopting a socio-constructivist approach to teaching mathematics (Adams, 2006; Bayaga et al., 2019; Gano, 2010, October; Gray, 1995; Kipng'etich et al., 2023).

Condie and Munro (2007) propose that digital technologies promote socio-constructivist learning environments in several ways. They can have a metacognitive impact by emphasising active involvement in learning, stimulating discussions which facilitate experiential learning; assisting with concept explanation and finally motivating students (Condie & Munro, 2007).

For these reasons, I have chosen strategically to integrate technology and employ a socio-constructivist teaching approach in my research study, potentially to enhance student comprehension of concepts specifically related to the sine trigonometric function.

1.4 Aims and Objectives

This study aims to explore how integrating ChatGPT, a generative AI tool, and GeoGebra, an interactive mathematical software, can enhance Grade 10 girls' learning experience and

comprehension of sine trigonometric function concepts. By combining these technologies to address conceptual gaps and engage students, the research offers actionable recommendations for educators to improve instructional outcomes. The study also examines the strengths and limitations of using ChatGPT and GeoGebra both together and individually. It concludes with a set of guidelines to help educators optimize these tools for teaching trigonometric sine function concepts.

Given the acknowledged efficacy of constructivist learning environments in fostering optimal learning outcomes, this study directs its attention towards leveraging ChatGPT and GeoGebra to cultivate such an environment. The primary objective is to harness these tools to facilitate authentic problem-solving, foster critical thinking, encourage collaborative learning, and nurture the cultivation of personalized knowledge construction among Grade 10 students within a selected girls' high school. Through this approach, the aim is ultimately to enhance the understanding and mastery of trigonometric sine function concepts among the students.

1.5 Research Questions of the study

The main research question of this study: How can the use of ChatGPT and GeoGebra as instructional tools enhance the teaching and learning of the sine trigonometric function concepts in a Grade 10 mathematics class?

With the sub-research questions being:

- How can ChatGPT be used as an instructional tool in the teaching and learning of the sine trigonometric function concepts?
- How can GeoGebra be used as an instructional tool in the teaching and learning of the sine trigonometric function concepts?
- How can the combination of ChatGPT and GeoGebra be used as an instructional tool in the teaching and learning of the sine trigonometric function concepts?

1.6 Significance of the Study

The significance of this study is twofold. Firstly, it offers an alternative, student-centred teaching method that can potentially enhance Grade 10 students' comprehension of sine

trigonometric functions. Secondly, by investigating the combination of ChatGPT's natural language capabilities with GeoGebra's dynamic visualizations, the study aims to enhance understanding specifically of the sine trigonometric function in a Grade 10 mathematics class for girls. This research can provide teachers, curriculum advisors, teacher educators, and Education Departments who do professional development of teachers with guidelines on using these technologies together, to achieve optimal learning outcomes.

The uniqueness of this study lies in its focus on ChatGPT, a new educational tool. Few studies have explored the combined use of ChatGPT and GeoGebra in mathematics classrooms, making this research particularly novel and significant.

1.7 Methodology

The research sample consisted of a Grade 10 mathematics class of 20 girls. Data collection involved multiple instruments: video recordings of class observations, worksheets with instructions and questions for students to complete, and interviews with both the teacher and focus groups conducted after each lesson. The qualitative data was analysed using thematic analysis.

The research followed a design-based research (DBR) approach as the aim of this study was to ultimately provide practical guidance for educators seeking to leverage ChatGPT and GeoGebra to enhance students' comprehension of the sine trigonometric function and to foster meaningful connections between the trigonometric concepts.

DBR involves iterative cycles of design, implementation, and refinement, which aim to improve learning outcomes and generate practical knowledge for educators (Reisman, 2008; Zheng, 2015). Utilising DBR allowed me to bridge the gap between theory and practice by potentially creating practical solutions that can be implemented in real educational settings (Fowler et al., 2023). It is particularly valuable in technology-enhanced learning environments, supporting the development and testing of educational innovations like in this study where ChatGPT and GeoGebra were utilised (Ford et al., 2017; Wong et al., 2011).

The methodology of the study is discussed in more detail in Chapter 3.

1.8 Ethical Considerations

I secured ethical clearance from the University of Cape Town Faculty of Humanities Research Ethics Committee and Western Cape Education Department (WCED). I also obtained the necessary permissions to conduct the study at a local private school for girls in Cape Town.

Consent was actively sought from various stakeholders, including the school principal, the maths teacher who conducted the lessons, the students, and parents of the students. In the letters seeking consent from the students, it was explicitly communicated that participation is entirely voluntary, and students had the right to withdraw from the study if they wished to do so. Additionally, to preserve anonymity, participants were identified by codes rather than their actual names.

In compliance with the data management practices of the University of Cape Town, all data collected during this study was securely stored with restricted access to authorized personnel only. Additionally, the study adhered to ethical guidelines by obtaining appropriate permissions for any data sharing or public access, and all shared data was handled in compliance with UCT's policies and ethical standards.

1.9 Outline of Chapters

Chapter 1 provides an overview of the study aimed at integrating ChatGPT and GeoGebra to enhance Grade 10 girls' understanding of sine trigonometric functions. It discusses the challenges in teaching trigonometric functions both globally and in South Africa, justifies the use of technology and a socio-constructivist approach, and outlines the study's aims, research questions, and significance. The chapter also details the methodology, limitations, ethical considerations, and provides an outline of subsequent chapters. The rest of the dissertation is structured as follows: Chapter 2 explores the challenges and strategies for teaching trigonometric functions in South Africa, emphasizing how the integration of ChatGPT and GeoGebra can possibly enhance students' understanding of sine function concepts. Chapter 3 outlines the research methodology, detailing the interpretivist paradigm, research approach, research setting, research sample, and research instruments used. It also explains the Design-Based Research framework, covering the analysis of the practical problem, development of solutions, iterative testing cycles, and the reflection process following data analysis in each cycle. Chapter 4 presents the study's findings, results, and qualitative data analysis, which are

then discussed in Chapter 5 in relation to other research, identifying both similar and contradictory findings in other studies. Chapter 6 concludes by summarising the main findings of the study. Firstly, ChatGPT prompts should be carefully crafted to specify the target audience and request concise, summarized responses. Secondly, users must be mindful of ChatGPT's inconsistency, as it may occasionally provide incorrect or irrelevant answers. Thirdly, the preferred use of ChatGPT involved providing step-by-step instructions, with participants also favoring a sequence where hands-on GeoGebra activities were followed by ChatGPT, enabling students to formulate and validate their conjectures. Fourthly, GeoGebra proved valuable in creating dynamic visualizations, enhancing understanding, contextualizing concepts, and promoting self-exploration. Lastly, key lesson design factors included introducing new material to sustain engagement, avoiding repetitive tasks, and incorporating electronic worksheets to streamline assignments. Chapter 7 addresses the limitations of the study and provides recommendations for further studies. These components collectively inform the ultimate conclusions and recommendations aimed at addressing the research questions.

1.10 Conclusion

South African examination statistics underscore the challenges that high school Grade 10 to Grade 12 students encounter with trigonometric concepts, particularly trigonometric functions (*National Senior Certificate Diagnostic Report, 2022*). In response to these challenges, this study investigates the possibilities of a student-centred approach that fosters a socio-constructivist learning environment using ChatGPT and GeoGebra. A DBR approach was employed to collect qualitative data to develop guidelines for stakeholders to better use these technologies, potentially to enhance students' comprehension of sine trigonometric function concepts. The next chapter offers a review of the challenges and strategies for teaching trigonometric functions in South Africa, emphasizing how the integration of ChatGPT and GeoGebra can possibly enhance students' understanding of sine function concepts.

CHAPTER 2: LITERATURE REVIEW

2.0 Introduction

This chapter examines the theoretical foundations and instructional strategies for teaching the trigonometric sine function, focusing on the use of ChatGPT and GeoGebra. It begins by discussing how trigonometric functions are taught in South Africa, introducing a socio-constructivist approach in mathematics education to create meaningful learning experiences. The chapter then explores how GeoGebra enables interactive, hands-on learning and how ChatGPT can support teaching. It concludes by discussing the combined potential of these tools to deepen students' understanding of the sine function, providing a more comprehensive and impactful learning experience.

2.1 Teaching and learning trigonometric functions – International context

Trigonometry has long been a key component of secondary school mathematics curricula worldwide (Kissane & Kemp, 2009). Looking at the teaching and learning of trigonometric function concepts through a global lens, historically, the process of learning trigonometry involved tedious manual calculations and traditional instructional methods that lacked interactive or dynamic visual aids and did not significantly enhance student understanding (Kissane & Kemp, 2009). As a result, students often struggle with grasping abstract concepts, such as the constancy of trigonometric ratios, and rely heavily on memorization techniques and mechanical application of algorithms (Weber, 2005).

Recent advancements in technology have introduced more effective methods for teaching trigonometry. In Austria, for instance, dynamic applets on the Austrian Maths Online website enable students to explore trigonometric ratios interactively, demonstrating that these ratios remain constant despite changes in triangle size (Embacher & Oberhuemer, 2024). The United Kingdom's NRICH site provides unit circle applets that help students understand circular functions and their applications (NRICH, 2024). In the United States of America, tools like GeoGebra applets and Casio's fx-9860G II graphics calculator allow students to trace unit circles, analyze trigonometric graphs, and connect these concepts to real-world data (Kissane & Kemp, 2009).

While many countries are turning to technology to improve students' understanding of trigonometry, a key challenge remains: most teachers are not yet prepared or comfortable with effectively integrating these tools into their classrooms. In Malaysia, for example, the use of dynamic mathematical software like Geometer's Sketchpad has been officially incorporated into the secondary school mathematics curriculum (Johari et al., 2010). The Malaysian Ministry of Education (MMOE) even secured a nationwide license for the software in 2004, with the expectation that it would benefit both teachers and students. However, its adoption has been limited as highlighted by Boon et al. (2005), who found that many schools have yet to fully utilize Geometer's Sketchpad. Significant barriers, such as teacher readiness and willingness to embrace this technology, continue to hinder its successful implementation.

2.2 Teaching and learning trigonometric functions – The South African context

Educators in South Africa encounter persistent challenges in teaching trigonometry concepts (Mavhunga & Kibirige, 2015). Despite efforts to reform mathematics education, students continue to face difficulties in mastering trigonometric concepts, especially understanding trigonometric functions (*National Senior Certificate Diagnostic Report, 2022*). Consequently, there is a pressing need to critically examine existing literature to identify effective instructional approaches and potential solutions to enhance the teaching and learning of trigonometry functions.

Studies indicate that traditional, authoritarian teaching styles persist in many South African classrooms, with educators struggling to adopt inclusive, inquiry-based practices (Lehesvuori et al., 2017; Mpho, 2018). This trend is attributed to various factors, including an examination-oriented curriculum, large class sizes, and inadequate teacher training (Meier, 2003; Mpho, 2018). Despite curriculum reforms aimed at promoting student-centred approaches, studies reveal that many educators continue to employ traditional teaching methods, indicating a gap between policy and practice (Alexander et al., 2010; Lotz-Sisitka, 2011).

Central to student-centred approaches in the mathematics classroom are the learning theories that inform our teaching philosophy in mathematics education. In South Africa, traditional pedagogical approaches are predominantly used, often relying on behaviourist or cognitivist methods that emphasize rote memorization and procedural learning. (Gandhi & Mukherji, 2021). However, contemporary perspectives advocate for constructivist and socio-cultural

theories, which emphasize active engagement, collaboration, and meaning-making in learning (Smith, 2022). By delving into these theories, this study can discern their relevance and applicability to teaching trigonometric function concepts, as well as explore opportunities for integration and synthesis.

In line with constructivist theories, the integration of technology has shown great potential for improving mathematics education in South Africa. Mokoena's (2021) study highlights how constructivism can be effectively aligned with the use of technologies like GeoGebra and Heymath in the classroom by employing a flexible, inductive, and multi-perspective approach that promotes active, collaborative, and self-discovery learning. In this study Mokoena (2021) provides compelling evidence that integrating ICT tools in South African mathematics classrooms can significantly enhance the teaching and learning of trigonometric functions through visualization, simulation, and hands-on exploration.

The integration of ICT tools within constructivist frameworks offers a promising avenue for addressing the challenges in teaching trigonometric functions in South African classrooms. It is essential to further explore the theoretical underpinnings that inform effective instructional approaches. The next section discusses constructivism and its implications for teaching and learning trigonometric function concepts. This will provide a foundation for examining how constructivist principles can be applied to address the persistent challenges faced by educators in this critical area of mathematics education.

2.3 The theory of constructivism

To address the poor performance in the trigonometric functions section reported in the National Senior Certificate examinations of 2021 and 2022, a potential solution could be to use constructivist learning theory for lesson design.

Constructivism, as emphasized by Piaget's theory, underscores that learning entails a continual process of construction, wherein the student's internal representation of the world, or 'reality' in a broader sense, is consistently evolving. This fundamental notion is referred to as 'constructivism' (Gandhi & Mukherji, 2021).

While there are diverse theoretical viewpoints within constructivism, they all share common characteristics: the active construction of knowledge, the connection of new information with existing knowledge, the role of personal experiences in knowledge creation, and the

involvement of active cognitive processes in learning (Mosese & Ogbonnaya, 2021; Roblyer & Doering, 2014; Smith, 2022; Sukarma, 2013).

Constructivism has various benefits for teaching and learning in mathematics. Firstly it promotes the development of conceptual understanding and problem-solving skills by building on students' existing knowledge (Bhowmik, 2015). Secondly, it leads to more permanent learning and positive attitudes towards mathematics compared to traditional methods (Gyan et al., 2021; Inan, 2013). Thirdly, it encourages student participation, exploration, and creative thinking in mathematics (Gyan et al., 2021). Lastly, specifically focusing on the teaching of trigonometric functions, it helps students transition from 'triangle' to 'circle' trigonometry and support the development of algebraic, geometric, and graphical reasoning (Altman & Kidron, 2016; Serpe & Frassia, 2021).

To ensure effective learning in trigonometric functions, it is essential to shift from traditional 'teacher tells' methods to a more investigative approach (Siyepu, 2013). This approach shifts focus from teacher-centred instruction to student-centred learning, with teachers acting as facilitators (Simon, 1995; Smith, 2022). In this study, the data collection tool was structured to position the teacher as a facilitator with a minimized intervention role. This approach was adopted to encourage an investigative learning atmosphere.

While Piaget and Vygotsky's approaches differ, they are not entirely incompatible. Piagetian theorists must recognize the impact of social context on knowledge construction, just as Vygotskyian theorists must acknowledge the role of individual experience (Smith, 2022).

While constructivism underscores the student's autonomy in knowledge construction, it is essential for teachers not to solely prioritize its theoretical principles. Learning frequently occurs within social contexts like classrooms, where students interact and share ideas. Thus, although students actively construct their understanding, they also draw upon peers and teachers for guidance (Smith, 2022). Similarly, within the scope of this study, the designed data collection tool will additionally emphasize the integration of group work or peer-to-peer discussions. This approach aims to facilitate active construction of understanding among students through social interaction and the articulation of their comprehension.

While constructivism offers no specific teaching model, it informs pedagogical decisions and task design (Simon, 1995). Constructivism remains influential in mathematics education

research and reform efforts providing a framework for developing more effective teaching and learning approaches (Ndlovu, 2013). Within the constructivist learning environment framework, Honebein (1996) delineates seven core pedagogical objectives. These objectives are: providing experiences, real-world contexts; multiple modes of representation (e.g., video, audio, text); diverse perspectives by evaluation of alternative solutions; promoting student-centred learning; encouraging reflection and metacognition. These seven pedagogical objectives form the focal point of this research in devising a lesson design framework.

These objectives provide valuable avenues for promoting social interaction, nurturing critical thinking abilities, and facilitating holistic learning experiences (Bayaga et al., 2019). In conclusion, this study's lesson design, based on constructivist theory, seeks to move away from traditional teacher-centred methods in the mathematics classroom toward a more student-centred approach. Technology can play a pivotal role in supporting and advancing the seven pedagogical objectives of constructivist learning environments discussed earlier.

2.4 Enhancing constructivist learning with technology in trigonometry functions

Technology integration can enhance constructivist learning by providing visually stimulating environments that encourage exploration and knowledge construction (Castillo, 2008; Jiménez-Molotla et al., 2007; Malabar & Pountney, 2002, July). A few studies show that technology can assist in the learning of trigonometric functions. Bornstein (2020), for example, investigated the use of a MATLAB program called TrigReps to teach transformations of trigonometric functions. MATLAB (Matrix Laboratory) is a high-level programming language and environment used primarily for numerical computing, data analysis, and algorithm development (MathWorks, 2024). TrigReps is a MATLAB program that provides multiple representations of trigonometric functions, including graphical, auditory, and dynamic visualizations. The study found that TrigReps helped students understand various transformations, particularly additive transformations and how horizontal transformations can be counterintuitive. Initial results were promising, showing that students could make productive observations and connections. In another study Ross et al. (2011) explored whether technology has a greater impact on student achievement and attitudes when integrated before or after traditional whole-class teaching in the context of learning trigonometric function transformations. The results showed that while students benefited more when technology was used after whole-class teaching in immediate post-tests, there were no significant long-term

differences, suggesting that a well-integrated combination of both approaches is most effective. Furthermore, Bedada and Machaba (2022) found that GeoGebra, a dynamic mathematics software, improves students' ability to make associations between representations of trigonometric functions and to interpret graphs. These studies suggest that technology can make a significant difference in understanding trigonometric function concepts. This study adds to the field by exploring whether combining two technological tools—ChatGPT and GeoGebra—can further enhance comprehension of trigonometric function concepts.

2.5 Challenges in the teaching and learning of trigonometric function concepts.

There are various factors that hinder students' learning and comprehension of the trigonometric function: Trigonometry entails grappling with abstract notions such as angles, ratios, and functions within a geometric framework, rendering visualization and comprehension a daunting task for students (Darussalam, 2015; Hu, 2006). The learning process is further impeded by insufficient emphasis on grasping concepts, over-reliance on rote memorization, and the struggle to bridge procedural and conceptual knowledge (Darussalam, 2015). The introduction of novel terminology, symbols, and mathematical operations exacerbates the complexity of trigonometry learning, as students grapple with the abstractness of algebra, deciphering transformations, and navigating deductive teaching methodologies (Bazuri & Arliani, 2022). Trigonometric concepts understanding necessitates proficiency in algebraic, geometric, and graphical reasoning, often complicating the isolation of discrete concepts and entangling them within a web of interdependence (Bedada & Machaba, 2022). Moreover, the intricate nature of trigonometric functions, characterized by periodic behaviour and nonlinear relationships, poses additional hurdles to students' comprehension (Gerhana et al., 2017, September). Additionally, the geometric essence of trigonometry demands abstract spatial reasoning abilities, essential for conceptualizing relationships within triangles and coordinate systems (Kamber & Takaci, 2018). Deciphering trigonometric concepts symbolically also presents a challenge, as students grapple with interpreting and manipulating mathematical notation (Kamber & Takaci, 2018).

Teaching trigonometric functions also poses significant difficulties for educators. Ramorola (2013) investigated the challenges South African senior secondary school teachers face in effectively integrating technology into teaching and learning. Through in-depth interviews, observations, and document reviews, the study identified several key issues: the absence of a

technology policy, insufficient equipment, a lack of qualified teachers in technology integration, and ongoing technical and maintenance problems, which further exacerbate these difficulties. Despite efforts to introduce digital technology, many teachers feel inadequate and unprepared, with older educators finding it particularly daunting (Msila, 2015). The uptake of technology remains low, and when technology is used, it often serves as a substitute for traditional methods without fundamentally changing teaching practices (Graham et al., 2021; Padayachee, 2017). This limited and often superficial use of technology fails to address the deeper pedagogical challenges inherent in teaching complex mathematical concepts like trigonometric functions.

2.6 Teaching and learning of trigonometric sine function concepts in this study

The focus of this study is specifically on the teaching and learning of sine trigonometric function concepts.

To counter the above challenges, the teaching of trigonometric sine function concepts can be significantly enhanced through using a few strategies that have been proven useful through research. The ratio method, defining trigonometric functions as ratios of sides in right-angled triangles, has been found more useful than the unit circle method for introductory trigonometry (Kendal & Stacey, 1996, June). Rosjanuardi and Jupri (2020), however, argues that a combined approach to teaching trigonometric functions, using both the right-angle triangle and unit circle contexts, is more effective as it helps students to better grasp the connection between these two fundamental concepts, which they often struggle to differentiate. Consequently, for the purposes of this study, a combination approach of the above methods was used in the design of the lesson plans.

Studies also highlight the affordances of employing a constructivist approach to teaching trigonometric sine function concepts (Mesa et al., 2023; Serpe & Frassia, 2021). For example, Mesa et al. (2023) conducted a study using a constructivist approach, combined with realistic mathematics education and GeoGebra, to teach trigonometric sine functions to tenth-grade students at San Vicente Educational Institution. By focusing on real-world problem-solving and applying the concept of mathematization levels, the study explored the transition from informal to formal knowledge. The results showed that students, through engaging activities, developed a deeper understanding of periodic phenomena within realistic contexts. In another

study Serpe and Frassia (2021) investigated how to improve the teaching and understanding of trigonometric functions using technological tools. The authors designed a teaching approach that integrates problem-solving and dynamic geometry software (GeoGebra) to help students grasp key trigonometric concepts. The study was grounded in the instrumental and semiotic mediation theories, emphasizing the role of artefacts in learning. A classroom activity was created where students used a clinometer to measure a tree's height, fostering connections between geometry and real-world applications. The findings suggest that integrating technology into teaching can enhance students' conceptual understanding and problem-solving skills in trigonometry. Adopting a constructivist approach to teach sine trigonometric function concepts fosters a shift from memorization towards understanding reasoning behind solutions (O'Meara et al., 2017). These student-centred strategies have been shown to promote conceptual mastery (Holtman et al., 2018). Creating constructivist environments can promote collaborative, participatory learning, benefiting fields like trigonometry that require knowledge application in different domains (Gandhi & Mukherji, 2021). Consequently, for the purposes of this study, a constructivist approach was used in the design of the lesson plans.

Technology-driven exploration, integrating both inductive and deductive reasoning, can significantly aid students in understanding the trigonometric sine function (Daher, 2020). In Daher's (2020) study, three high-achieving 11th-grade students utilized both types of reasoning, facilitated by GeoGebra, to explore and grasp new concepts related to the sine function within the unit circle and coordinate system. Data collection involved video recording the students and their computer screens during three 45-minute lessons. GeoGebra played a dual role in the learning process: as a static visual mediator for deductive reasoning and a dynamic one for inductive reasoning. The combination of student interactions, technology, and teacher guidance helped foster the students' understanding of key mathematical concepts, such as sameness, encapsulation, and reification. Similarly, the present study aims to enhance the comprehension of sine trigonometric function concepts using technology, but it differs by employing two tools, ChatGPT and GeoGebra, instead of just one.

2.7 Why use ChatGPT and GeoGebra to teach the trigonometric sine function?

Technologies are highlighted as tools for exploration, collaboration, and individual expression (Calvert, 2001), aligning with socio-constructivist principles from Piaget and Vygotsky by promoting both individual and social learning.

In socio-constructivist classroom environments, where learning is built through social interaction and collaborative problem-solving, ChatGPT offers valuable support as a conversational tool with human-like text processing abilities. ChatGPT is an AI-powered large language model (LLM) developed by OpenAI that uses artificial intelligence as a machine learning model to comprehend and generate human language text. It can provide instant feedback, answer questions, and generate explanations, making it a versatile tool for various applications (ChatGPT, 2023).

In a study by Zhao et al. (2023), the aim was to explore how ChatGPT could enhance student engagement and academic outcomes in China. Their study evaluated factors such as students' autonomy in learning, motivation for problem-solving, critical thinking, and overall academic performance. Based on a comprehensive literature review examining students' attitudes and learning outcomes when using ChatGPT, their study found that ChatGPT significantly improves students' learning engagement. Engaged students are more likely to take an active role in problem-solving, inquiry, and constructing new knowledge, all key principles of socio-constructivism.

Patero (2023) conducted a mixed-methods study, combining quantitative and qualitative approaches to examine ChatGPT's impact on mathematics education. Participants included secondary school students (grades 9-12) and math educators with traditional teaching experience and an interest in technology-based pedagogy. The quantitative phase used pre- and post-intervention surveys to assess students' attitudes, self-efficacy, engagement, and performance on assessments and problem-solving tasks. In the qualitative phase, interviews and focus groups with students and educators explored their experiences and perceptions of ChatGPT in learning. The study found that ChatGPT significantly improved students' learning outcomes, including higher scores on assessments, quizzes, and problem-solving tasks, indicating its potential to enhance comprehension of mathematical concepts. In socio-constructivist classrooms, where feedback and reflection are vital, ChatGPT's real-time responsiveness can be an asset to reaching learning outcomes.

Truong (2023) explored the integration of ChatGPT in high school Mathematics in Vietnam, focusing on enhancing students' learning experiences and outcomes. The paper discussed the benefits and challenges of incorporating ChatGPT into the Vietnamese Mathematics curriculum. One of the key findings was that ChatGPT's 24/7 accessibility allows students to

engage with educational content anytime, accommodating different learning schedules. This flexibility supports the socio-constructivist principle of self-directed, active learning.

According to Serhan and Welcome (2024), ChatGPT's real-time assistance and conversational interface fosters a student-centred learning environment, making learning more interactive and enjoyable. Their study investigated student perceptions of ChatGPT's role in learning calculus. They surveyed 64 undergraduate students enrolled in a calculus course, using a 5-point Likert-scale questionnaire and two open-ended questions. The results revealed that most students favored ChatGPT, noting its positive impact on their learning, participation, and classroom engagement.

Wardat et al. (2023) also examined perspectives on using ChatGPT to teach mathematics. Through a qualitative case study, they analyzed interviews and user experiences. The findings showed that both teachers and students recognized ChatGPT for its increasing mathematical proficiency and its ability to simplify complex geometry concepts, making them more accessible. This research supports my study, which aims to use ChatGPT in a mathematics classroom to make challenging concepts, specifically sine trigonometric functions concepts, more accessible and to improve students' understanding of these concepts.

ChatGPT also has some limitations. The second stage of Wardat et al.'s (2023) study, examined user experiences across three educational scenarios, which highlighted issues such as ChatGPT's limited understanding of geometry and its inability to correct misconceptions effectively. It was found that the tool's accuracy is influenced by the complexity of tasks, input data, and instructions provided. Despite these challenges, ChatGPT is expected to improve in solving more complex mathematical problems (Wardat et al., 2023). Other limitations of using ChatGPT in mathematics education were highlighted in a study done by Frieder et al. (2024) who assessed the mathematical capabilities of two versions of ChatGPT (from January 2023) and GPT-4 by testing them on both publicly available and specially designed datasets. The aim was to evaluate their performance in advanced mathematics, particularly at the graduate level. The findings revealed that while ChatGPT and GPT-4 can be useful for tasks like querying facts and serving as mathematical search engines, they fall short in handling more complex, graduate-level mathematical problems. This study aims to investigate both the advantages and challenges of utilizing ChatGPT in a student-centred learning environment to teach sine

trigonometric concepts. Additionally, the study will identify potential limitations of ChatGPT and attempt to address these through iterative lesson designs.

GeoGebra is a dynamic mathematics software that enables hands-on exploration of geometry, algebra, and calculus through interactive visualization, supporting Piaget's concept of discovery-based learning (GeoGebra, 2024). It engages students actively, encouraging individual exploration and visualization through animations and simulations, which aligns well with constructivist learning principles (Smith, 2022). Studies highlight GeoGebra's ability to enhance creative thinking, support learning of mathematical periodicity (like the trigonometric sine function), and improve student attitudes and performance in mathematics (Arbain & Shukor, 2015; Mokoena, 2021). Research, including work by Trung (2014), shows that students using GeoGebra outperform those receiving traditional instruction, with greater engagement and understanding.

Additional studies (Bedada & Machaba, 2022; Mosese & Ogbonnaya, 2021) confirm GeoGebra's effectiveness in teaching trigonometry, emphasizing improvements in understanding trigonometric graphs and shifting learning from a teacher-centred to a student-centred approach. Bedada and Machaba's (2022) study particularly underscores GeoGebra's role in enhancing learning through cooperative activities, demonstrating the tool's effectiveness within a constructivist framework, which my study also aims to replicate to foster a richer understanding of the sine function.

Swaniker (2023) advocates that, integrating diverse knowledge areas, such as combining different technologies, can assist in solving complex problems and can enhance interdisciplinary learning. This is precisely why this study aims to use ChatGPT alongside GeoGebra to teach sine trigonometric function concepts to girls, aiming to foster a deeper understanding and potentially to encourage the development of critical thinking and problem-solving skills.

The combination of ChatGPT and GeoGebra in this study aims to support both individual and collaborative learning, enhancing engagement and understanding of trigonometric function concepts in a constructivist classroom. This dual approach leverages each tool's strengths to provide a comprehensive learning experience focused on active, student-centred education.

2.8 Enhancing learning through combined use of ChatGPT and GeoGebra.

By using GeoGebra for visual and interactive exploration and ChatGPT for detailed explanations and problem-solving support, this study aims to address the challenges of abstract trigonometric concepts by investigating how these two technologies can be utilized to potentially enhance comprehension.

GeoGebra enhances student engagement and achievement by catering to diverse learning styles, making it particularly effective for the VARK (visual, auditory, reading/writing, and kinaesthetic) framework which focuses on catering for different students learning styles (Bhatti et al., 2017; Wassie & Zergaw, 2019). The VARK model categorizes students based on their sensory preferences, helping educators tailor their teaching methods to better accommodate different learning needs, thereby improving educational outcomes (Fleming & Mills, 1992). It is rare for a single mode to be used or to be sufficient, which is why there are two-part, three-part, and four-part VARK preferences. Individuals who do not have a dominant mode, with one preference significantly higher than the others, are considered multimodal (Cadigan, 2024). ChatGPT could potentially augment the VARK model by providing interactive and adaptive support tailored to visual, auditory, reading/writing, and kinesthetic learning preferences. This will be explored in my study.

This multimodal approach aligns with the findings of O'Halloran (2015) and Reisa (2010), which highlight the advantages of integrating diverse tools to support different learning styles and improve mathematical understanding. Multimodal approaches enhance mathematics learning by integrating symbolic language, visual aids, and natural language (Joutsenlahti & Kulju, 2017; Morales et al., 2003). These approaches enable students to make connections through various resources, such as speech, calculator inputs, and written work, and involve complex interactions of perceptual and cognitive experiences (Ferrara, 2013). Additionally, multimodal strategies not only reinforce the understanding of mathematical concepts but also aid in evaluating learning outcomes (Joutsenlahti & Kulju, 2017).

2.9 Conclusion

Chapter 2 explored constructivism as a theoretical framework for teaching trigonometric sine function concepts, highlighting the need for effective strategies to address learning challenges. It discussed how ChatGPT and GeoGebra can enhance understanding through interactive and

inquiry-based learning. The combined use of these technologies shows promise in improving student engagement and conceptual grasp. The chapter concluded with an overview of Design-Based Research (DBR) methodology, emphasizing its iterative, collaborative, and practical nature as a framework for implementing and evaluating these technological interventions in the classroom. In preparation for the next chapter, the focus will shift to detailing the research methodology employed in this study.

CHAPTER 3: RESEARCH METHODOLOGY.

3.0 Introduction

This chapter outlines the methodology employed to address the primary research question: How does the use of ChatGPT and GeoGebra software enhance Grade 10 girls' comprehension of the trigonometric sine function? The chapter begins with a discussion of the research paradigm. This is followed by a detailed explanation and justification of the research approach and design. Additionally, the chapter describes the research setting, population, and sample, along with the data collection and analysis methods used. Finally, issues of trustworthiness and ethical considerations are addressed.

3.1 Interpretivism Research Paradigm

Willis (2007) describes a paradigm as a comprehensive belief system or worldview that directs research and practice within a field. Paradigms facilitate rigorous research methodologies that aid educators in refining their practices (Taylor & Medina, 2011).

The philosophical foundation of this study, which investigates Grade 10 girls' comprehension of the sine trigonometric function in a private school setting, is based on the interpretive paradigm within a DBR framework. The interpretivist paradigm emphasizes understanding the subjective meanings and experiences of individuals within their social contexts, focusing on the complexity of human behaviour and interactions (Schwandt, 1994). This paradigm is supported by scholars such as Baxter and Jack (2008), Golafshani (2003), and Adom et al. (2016), who assert that qualitative research often aligns with constructivist or interpretive perspectives.

DBR aligns with the interpretive paradigm due to several key characteristics inherent in both approaches: Firstly, DBR involves continuous interaction between researchers and participants to iteratively design, implement, and refine educational interventions. This aligns with the interpretive paradigm's emphasis on understanding the meaning and context of participants' experiences through interaction and engagement (Anderson & Shattuck, 2012; Collective, 2003). Secondly, the interpretive paradigm prioritizes understanding the context-specific nuances and the subjective experiences of participants, which DBR also seeks to address by

developing interventions that are sensitive to specific educational settings and the unique needs of participants (Barab & Squire, 2016; Wang & Hannafin, 2005). Thirdly, both DBR and the interpretive paradigm utilize a variety of data collection methods to capture the complexity of educational interventions and the multifaceted experiences of participants (Collins et al., 2016; Golafshani, 2003). Lastly, both DBR and the interpretive paradigm are rooted in constructivist theories, which posit that knowledge is constructed through social interactions and engagement with the environment. Vygotsky's (1978) theory of social constructivism, which underpins both DBR and interpretive research, suggests that learning occurs through social interaction and the co-construction of knowledge, which DBR operationalizes through collaborative design processes (Reeves, 2006).

For investigating the teaching of Grade 10 girls the concepts of the sine trigonometric function using ChatGPT and GeoGebra, I have selected interpretive phenomenology as the preferred research methodology. This methodology, influenced by Heidegger's philosophy, acknowledges multiple realities and emphasizes the co-construction of knowledge (Koch, 1995; Mackey, 2005). It is well-suited to exploring how students experience and interpret sine trigonometric function concepts within a technologically mediated learning environment.

3.2 Research Approach

This study employs a qualitative research approach, leveraging in-depth methods such as interviews and observations to explore complex phenomena in natural settings (Creswell, 2015; Patton, 2014). Qualitative research is particularly effective for capturing the depth and complexity of educational experiences and provides a rich, descriptive account of participants' perspectives (Merriam & Tisdell, 2015). This approach, chosen over quantitative methods, allows for nuanced understanding and adaptation to different contexts (Bogdan & Biklen, 1997; Falk, 2007). Data collection strategies included focus group and teacher interviews, observations and worksheets which provided triangulation to enhance reliability and validity (Cohen et al., 2007; Gill et al., 2008; Patton, 2014). Triangulation ensured a comprehensive view of how ChatGPT and GeoGebra impacted students' understanding of the sine function, while iterative validation with participants and rigorous data management upholds the study's credibility (Barrett & Twycross, 2018; Bogdan & Biklen, 1997; DiCicco-Bloom & Crabtree, 2006). Integrating these strategies within a DBR framework addressed the qualitative research limitations, providing reliable and applicable insights into educational practice.

3.3 Research Setting

A research setting is perceived as the site of participants (a natural setting) where the researcher goes to conduct research (Lee, 2006). The current study was conducted at a private all-girls school in the southern suburbs of Cape Town, South Africa. The school, which serves middle- and upper-class families, offers education from grades 8 to 12. The research focused on a Grade 10 class (of only girls), chosen due to logistical timetable limitations and my own teaching commitments. Concepts of the sine trigonometric function are introduced in the Mathematics curriculum in Grade 10 which is the reason this topic was chosen. The girls at the school are streamed according to their mathematical ability, and the class used in this study was the top-streamed class, consisting of students with exceptional mathematical skills. I selected this school for the following reasons: the school is intent on incorporating more AI into their lesson plans and policies, necessitating guidelines for future reference and exploration of potential benefits; the researcher had easy access to the school as she teaches there herself; the availability of advanced educational resources, including WiFi connectivity across the campus and all classrooms, and the fact that all students have their own devices to use in class; and the potential academic benefits for the participants involved in the study, including a better comprehension of the concepts on the sine trigonometric function.

3.4 Research Sample

Qualitative research focuses on participants selected for their specific characteristics and knowledge relevant to the study, emphasizing exploration within their natural context rather than broad generalization (Lodico et al., 2010). Purposeful sampling, particularly homogeneous sampling, was used in this study to select Grade 10 students with high mathematical ability from a private all-girls school in Cape Town. This approach, chosen for its ability to provide in-depth insights into the research objectives (Benoot et al., 2016; Patton, 2002), allowed for a detailed investigation of how ChatGPT and GeoGebra enhance understanding of sine trigonometric concepts. Consent was obtained from 20 students and their parents, ensuring privacy and confidentiality throughout the study (Arifin, 2018).

3.5 Research Instruments

In this study, various research instruments were used to collect data on the use of ChatGPT and GeoGebra for teaching the sine trigonometric function. These instruments included an observation guide, worksheets, and interviews.

3.5.1 Data Collection

3.5.1.1 Observation guide

Observations were conducted during all three lessons with the same group of 20 participants. The focus was on how ChatGPT and GeoGebra were used to teach the sine trigonometric function, particularly with respect to the seven constructivist pedagogical principles. Field notes and video recordings were used to document classroom interactions and to analyze aspects such as student motivation, engagement, and reliance on the teacher.

3.5.1.2 Worksheets

Worksheets were used in all three lessons to guide students through the learning activities and contained the content of the lessons. The students were instructed to work in pairs most of the time and in Lesson two they were instructed to work in groups of four for an activity. These worksheets monitored comprehension of the sine function and evaluated the effectiveness of the pedagogical strategies implemented. They facilitated students' exploration of the sine graph using ChatGPT and GeoGebra, aligned with Grade 10 content. The first two lessons built on prior knowledge of sine ratios and basic trigonometry, while the third lesson introduced new content.

3.5.1.3 Interviews

Focus-group interviews were conducted with four randomly selected voluntary participants from the group of 20. These interviews aimed to gather additional insights into the use of ChatGPT and GeoGebra. Teacher interviews were conducted to gather participant opinions and recommendations. An educational expert, the teacher, was also consulted to refine the lesson design for improved learning outcomes to support the iterative design followed by the selected DBR framework. All interviews were audio-recorded, transcribed verbatim and analyzed to support or clarify findings from other data sources.

3.5.2 Data Analysis

3.5.2.1 Worksheets

Data from the worksheets was analyzed to assess students' comprehension of sine trigonometric function concepts and their responses to questions about how they perceived the use of ChatGPT and GeoGebra in supporting pedagogical strategies. The analysis also examined the extent to which ChatGPT and GeoGebra contributed to or hindered students' understanding of sine trigonometric concepts.

3.5.2.2 Interviews

Transcribed interview data were analyzed to gain further insights into participants' experiences on the use of ChatGPT and GeoGebra to teach sine trigonometric concepts. This analysis aimed to support or clarify findings from observations and worksheets and to refine lesson design based on feedback from teachers and the educational expert.

3.5.2.2 Observational guide through video recordings

I used video recordings of the classroom as my observational guide to capture authentic interactions, student engagement, and instructional dynamics in real time. Video allows for a more accurate and detailed review of classroom behaviours and responses, ensuring that no significant moments are missed. I only incorporated my observations from the recordings when they were directly applicable to the focus of my research, ensuring that the findings remain relevant and aligned with the study's objectives.

3.6 Design Based Research

In DBR, the process involves iterative phases where researchers work closely with participants to design, implement, and refine educational interventions, with the aim of understanding and improving their use in real-world settings (Anderson & Shattuck, 2012; Wang & Hannafin, 2005). Figure 3 below illustrates the four stages of DBR. In Stage 1, the problem is identified and analyzed. In Stage 2, solutions to the problem are developed, which are then tested through iterative cycles in Stage 3. Finally, all the data is analyzed and reflected upon in Stage 4.

DBR's focus on generalizability means that the theoretical insights and practical interventions developed through the research can be applied across different settings, although success in new contexts is not guaranteed (McKenney & Reeves, 2018). These design principles, however, offer valuable guidance for addressing educational challenges in ways that are

tailored to specific contexts, emphasizing the broader potential impact of the study's outcomes (Plomp, 2013).

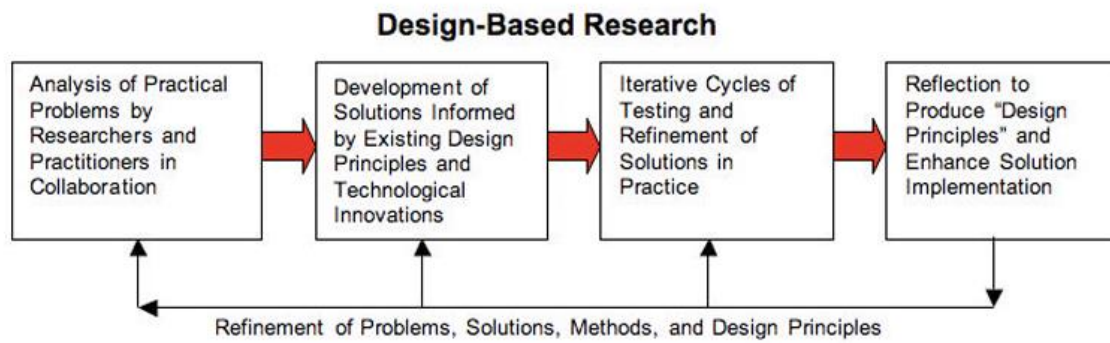


Figure 3: Predictive and design-based research approaches in educational technology research. Adapted from: Amiel and Reeves (2008, p. 34)

Utilising a design-based research approach, the study allows for iterative development and refinement of instructional interventions, considering the diverse needs and preferences of students for instruction models. In the following section, each phase of the DBR will be defined in the context of this study.

3.6.1 Phase 1: Analysis of Practical Problem

The identified problem to be investigated in this study is the problem statement discussed earlier which elaborates on the poor comprehension of trigonometric functions among students in South Africa and worldwide. Research delved into the successful teaching strategies employed by other researchers in addressing similar problems. This research endeavor was geared towards enhancing my own understanding of the issue and served as the foundation for the initial instructional intervention. This intervention was guided by a set of globally recognised socio-constructivist design principles mentioned above, and its perceived effectiveness and impact on students' grasp of concepts surrounding the sine trigonometric function was evaluated through the utilisation of ChatGPT and GeoGebra. The goal was to formulate a comprehensive set of design principles that can be readily employed by other educators looking to utilise ChatGPT and GeoGebra for teaching trigonometric functions.

3.6.2 Phase 2: Development of Solutions

In the context of constructivist learning environments, Honebein (1996) identifies seven key pedagogical objectives that will be the primary focus of this research in the development of a lesson design framework: Firstly, a constructivist mathematics design should provide

experiences that engage students in the process of constructing knowledge, allowing them to understand how they learn. Secondly, the lesson should encourage exposure to and appreciation for diverse perspectives, involving the assessing and analysis of alternative solutions. Thirdly, learning should be situated in real-world contexts, emphasising authentic tasks. Fourthly, the lesson should promote student ownership and give students a say in the learning process, fostering student-centred learning. Fifthly, the lesson should integrate learning with social experiences, emphasising collaboration. Sixth, it should encompass multiple modes of representation, including video, audio, and text. Lastly, the lesson should cultivate an awareness of the knowledge construction process, encouraging reflection and metacognition. These objectives provide valuable avenues for promoting social interaction, nurturing critical thinking abilities, and facilitating holistic learning experiences (Bayaga et al., 2019).

The seven key pedagogical objectives in constructivist learning environments mentioned above were the global design principles used to design all three intervention lessons for this study and will be referred to for the data analysis and reflection to focus on effectiveness, practicality, and level of focus on each of these objectives in each lesson design. Lessons 1, 2 and 3 were meticulously designed using these principles as a framework, implementing them in the following ways:

3.6.2.1 Providing Experiences.

The use of GeoGebra significantly enhances the understanding of trigonometric functions by fostering active exploration, and offering greater benefits than traditional teaching methods (Bedada & Machaba, 2022). Mosese and Ogbonnaya (2021) found that students who used GeoGebra to input equations and generate trigonometric graphs gained valuable time for exploration and graph interpretation. A study done by Clements (2000) aligns with this view that immediate feedback from ICT tools promotes curiosity and exploratory learning. According to Weber (2005), using GeoGebra supports the development of a deep understanding of trigonometric functions by connecting new knowledge with existing knowledge and experiences and enables students to remember, deduce, and explain sine trigonometric function properties effectively.

Considering the points discussed above, all the lessons (interventions) utilised GeoGebra to offer students a visual learning experience. The students were given opportunities to

experiment and interact with GeoGebra, allowing them to explore mathematical concepts through hands-on activities. This aligns with Honebein's principle of providing rich experiences that foster deep understanding (Honebein, 1996). In Lesson 1, two key activities, Task 3 and Task 4 were designed to facilitate an understanding of sine ratios through interactive exploration using GeoGebra (see Appendix A on page 116). In Lesson 2, Task 2 was designed to facilitate an understanding of the sine ratio in different quadrants in the Cartesian plane (see Appendix B on page 121). In Lesson 3, two key activities, Task 2 and 4 were designed to facilitate exploration of the sine graph and how the shape links with the ratio in different quadrants (see Appendix C on page 130).

3.6.2.2 Real-world contexts

Darussalam (2015) and Mutodi and Ngirande (2014) assert that practical, real-world applications of mathematical concepts, including trigonometry, significantly enhance students' understanding. Wong et al. (2012) add that such applications not only foster critical thinking in students but also deepen educators' comprehension. In this study, GeoGebra was used to model real-world problems, while ChatGPT generated real-world scenarios related to sine trigonometric functions, demonstrating the value of applying mathematical concepts in real-world contexts.

The lessons involved investigating the sine ratio in various triangle scenarios, culminating in solving a real-world problem. In all three lessons students could see in real-time as changes were made to certain parameters which affected the sine ratio's answer or the shape of the graph. In Lesson 2 Task 4 (Appendix B on page 121) ChatGPT was prompted to generate real-world context questions to students to test their knowledge on the sine function in different quadrants. Students then critically analysed and attempted to answer or solve the questions generated by ChatGPT.

3.6.2.3 Multiple modes of representation (e.g., video, audio, text).

Students should be exposed to information in various forms, such as text, visuals, and interactive experiences, to promote a deeper understanding and accommodate different learning styles (Sankey & Gardiner, 2010, January). In all three lessons, in the DBR process of this study, ChatGPT provided text and GeoGebra provided a visual and an interactive experience. Students were also provided with a worksheet to facilitate the learning activity and to document and demonstrate their understanding.

In all three lessons, GeoGebra was used for visualization, movement, and simulation, while ChatGPT provided textual definitions and explanations. Utilizing multiple modes of representation as mentioned in the previous chapter, caters to diverse learning styles and enhances conceptual understanding (Mayer, 2002).

3.6.2.4 Diverse perspectives by evaluation of alternative solutions.

Both ChatGPT and GeoGebra were utilized to offer different perspectives on the same mathematical concept. This approach aligns with the constructivist view that learning is enhanced when students are exposed to multiple viewpoints (Jonassen, 1994). In Lesson 1, ChatGPT was used in Task 2 to explain the sine ratio in words and define the sine ratio to understand and use this in the questions to follow (see Appendix A on page 116). In Lesson 2, ChatGPT was used in Tasks 3 and 4 to firstly read in prompt to explain why the sign of the sine ratio changes in different quadrants. In Task 4, another prompt was given to ask ChatGPT to generate questions/examples and answers to these for students to do and analyse to test their knowledge on the given concept (see Appendix B on page 121). In Lesson 3, ChatGPT was used in Task 1 where ChatGPT was prompted to give detailed steps to reach a learning outcome independently (see Appendix C on page 130).

Changes in how ChatGPT was used in these lessons were made based on findings from previous iterations, reflecting the principles of design-based research. Each lesson was refined using insights gained from student performance and feedback, allowing for continuous adjustments to instructional strategies.

3.6.2.5 Promoting student-centred learning

Lovitt and Clarke (2011) emphasised that effective mathematics teaching should involve active student participation in the learning process. Utilisation of student-centred teaching strategies can enhance the trigonometric function teaching effectiveness thus teachers need to be encouraged to adopt these kinds of approaches over teacher-centred ones (Spangenberg, 2021).

The lessons were designed to be student-centred, with the teacher acting as a facilitator. Additionally students were involved in the learning design through focus group interviews and worksheets, where they provided recommendations for the use of ChatGPT and GeoGebra in subsequent lessons. This approach fosters a sense of ownership and active engagement in the learning process (Hannafin & Land, 1997).

3.6.2.6 Collaboration

The intervention lessons aimed to foster collaborative learning through peer interactions, resource use, and teacher support. Drawing on insights from Spangenberg (2021), I aimed to enhance students' mathematical skills and confidence by encouraging them to explain concepts to each other. By facilitating collaboration with both peers and the teacher, who acted as a coach, I sought to foster the social and intellectual negotiation essential for constructing mental models (Jonassen, 1994). DeJarnette (2014) suggested that pairing students for problem-solving, particularly with technology, could increase participation and enhance argument formulation. To support this, I integrated ChatGPT and GeoGebra to encourage peer discussions and cooperative problem-solving. The study balanced individual and group work, using digital tools to enhance learning outcomes, with students working in pairs and groups throughout the lessons. This collaborative approach encouraged peer learning and critical thinking (Vygotsky & Cole, 1978).

3.6.2.7 Encouraging reflection and metacognition.

The lessons provided gradual scaffolding to support students' progression from basic to advanced understanding, reducing teacher and worksheet support as students gained independence. This approach, like Blackett and Tall (1991), required students to independently connect triangle diagrams and numerical relationships, a form of reasoning not always familiar to high school and college students (Weber, 2005).

The lessons also encouraged metacognitive reflection, helping students develop self-regulation and problem-solving skills, and articulate and justify mathematical procedures (Weber, 2005). At the conclusion of each lesson, students were asked to solve a problem independently without the aid of ChatGPT and GeoGebra. This activity served as a means for reflection and metacognition, allowing students to assess their understanding and think about their own learning processes (Schraw & Moshman, 1995). See Appendix A Task 5, Appendix B Task 4, and Appendix C Tasks 1 and 5.

The iterative cycles of design helped refine certain strategies and the use of technology, but all three lessons were consistently structured around integrating as many of the seven key pedagogical objectives as possible. Each lesson was carefully crafted to exemplify a strong constructivist learning environment, fostering deep, meaningful, and student-centred learning experiences. These pedagogical objectives provided valuable avenues for promoting social

interaction, nurturing critical thinking abilities, and facilitating holistic learning experiences (Bayaga et al., 2019).

3.6.3 Phase 3: Iterative Cycles of Testing

The iterative cycles of testing in this DBR study were three lessons which were meticulously crafted with a steadfast commitment to the seven-core constructivist pedagogical objectives. The aim was to adeptly employ ChatGPT and GeoGebra to fulfil each objective within every lesson. Subsequently, an in-depth analysis was conducted to discern the most effective level of utilisation and focus and practical combinations to yield the most effective outcome with the main aim being better comprehension of the sine trigonometric function and its concepts. Below, I discuss the three sine trigonometric function lessons which constituted part of the research instruments.

3.6.3.1 Lesson 1

Investigating similar triangles and their ratios, right-angled triangles, ratios of sides, and trig ratios to relate this to the definition of the sine ratio. ChatGPT was used to give a definition or explanation of the sine ratio and GeoGebra was used to visualise the changes to the lengths of sides of the right-angled triangles and how this affects the sine ratio.

3.6.3.2 Lesson 2

Investigating right-angled triangles in different quadrants in the CAST diagram and changing the right-angled triangles acute angle measurement to note the relevant changes to the sine ratio in different quadrants. ChatGPT was used to generate questions to test a student's understanding of the sine ratio in different quadrants. Students were asked to first attempt to solve these questions in pairs and then to ask ChatGPT to generate the correct answers to the questions posed. Students then had to analyse ChatGPT's answers to see if they or ChatGPT made mistakes in their reasoning. GeoGebra was used to visualise how the changes of the triangles in each quadrant changes the answer of the sine ratio.

3.6.3.3 Lesson 3

This lesson focused on how to draw the sine trigonometric graph and how certain transformations to the sine trigonometric function change the graph. ChatGPT was utilised to give students steps to follow in drawing a sine graph. Students had to follow these steps working in pairs to plot the sine graph. GeoGebra was then utilised to show how the sine ratio of a triangle drawn in different quadrants in the cartesian plane effects and produces the sine graph.

These 3 lessons of the study were conducted, implementing the refined learning activities in the classroom. Throughout, I continued to collect data and refine the design based on ongoing feedback and observations.

3.6.3.4 Data Collection Process

The data collection focused on students' comprehension, engagement, autonomy, and intrinsic motivation, as well as the implementation of seven constructivist pedagogical teaching strategies.

Data collection took place over seven weeks, aligning with qualitative research principles that emphasize active participation. This approach is consistent with constructivist theories that stress students' active engagement in knowledge construction (Mather, 2003). To strengthen the validity and reliability of the study's findings, data was collected using multiple methods, including class observations, worksheets, focus-group interviews, and teacher interviews.

DBR promoted a continuous feedback loop between theory and practice. The findings then informed and refined the conceptual framework, with adjustments or additions made based on the empirical evidence after each iteration.

3.6.4 Phase 4: Data Analysis and Reflection

The collected data were analyzed to identify patterns, trends, and insights related to the impact of technology on student comprehension and learning experiences. This analysis was framed within the seven pedagogical teaching principles.

The data used in this study included a range of textual data which consisted of transcriptions of teacher interviews and focus-group interviews and visual data like a video recording of the lessons which, according to Brooks et al. (2018), are all consistent with qualitative data.

The data was analysed to look for common themes, known as thematic analysis. Brooks et al. (2018) advocates that thematic analysis is one of the most common ways in which to do the analysis of qualitative data and it involves examining and recording patterns within the data relating to the research questions in this study.

The thematic analysis followed five systematic steps mentioned by Cho and Lee (2014). Firstly, interviews and observations were recorded in audio format and subsequently transcribed. Secondly, the data from transcripts, observation notes, and document analyses was carefully

reviewed to gain a comprehensive understanding. Individual words and phrases were coded inductively, and their frequency was documented. Thirdly, these codes were organized into sub-categories based on their units of meaning. Fourthly, the sub-categories were grouped into predefined categories through axial coding, which is a type of qualitative coding which analyses the properties of elements and discovers their interrelationships (Onwuegbuzie et al., 2016).

Lastly, once the coding process was complete, the data was thoroughly examined to identify patterns and draw conclusions in response to the sub-research questions. A cross-case analysis was conducted to ascertain commonalities among the observed elements in teachers' practices (Cho & Lee, 2014). The data analysis was structured into three phases: Phase 1 encompassed the analysis of observations, Phase 2 focused on examining worksheets, and Phase 3 involved the analysis of the focus-group and teacher interviews. These phases culminated in a summary for each lesson, presenting elements related to the research questions from each data collection instrument.

Based on the findings and reflections from the initial implementation, the learning activities and technology integration strategies were refined and redesigned to address challenges or issues that emerged during the initial phases of Lessons 1 and 2.

Each lesson in the study was documented, including design decisions, implementation details, data collection, and analysis. Reports were prepared alongside publications to communicate the study's findings, implications, and contributions to the field of mathematics education.

There are various criticisms levelled at qualitative analysis; for example, issues relating validity, reliability and credibility (Brooks et al., 2018). In this study, I addressed these through a range of methods including triangulation of data. By combining different types of data it strengthened the analyses and addressed criticisms of single-method approaches. The interviews and focus groups may yield socially acceptable but not genuine responses thus the video observation mitigated this issue by providing additional context and insights (Brooks et al., 2018) and transparency of approach.

The validity of qualitative data, or its trustworthiness in reflecting reality, differs from traditional scientific validity, which relies on replication. Due to the context-specific nature of qualitative research, replication is not feasible. Instead, I had to focus on the context of the

interviews and their own roles, reflect on and disclose any assumptions or biases, including their theoretical perspectives and relevant past experiences (Brooks et al., 2018).

The themes resulting from this form of analysis illuminated participants' meanings, actions and social contexts relating to the phenomena under consideration.

3.6.4.1 Reflexive Statement on Researcher Positioning

In researching Grade 10 girls' understanding of the sine function through ChatGPT and GeoGebra, I reflected on my position and potential biases to ensure the qualitative data's credibility. Grounded in constructivist theory, I used triangulation involving multiple data sources (e.g., student interviews, classroom observations, teacher feedback, student worksheets) and also used member validation by seeking feedback from participating students through using a questionnaire within the worksheets and through the interview questions to accurately represent participant experiences and reinforce research validity.

3.7 Conclusion

This methodology chapter detailed a design-based approach to integrating ChatGPT and GeoGebra for teaching the sine function, emphasizing iterative lesson refinement through student feedback. Guided by constructivist principles, it ensured pedagogical goals were met. The next chapter will analyze the results, examining student comprehension, engagement, and preference in the context of teaching and learning sine trigonometric function concepts. The next chapter will also provide insights into the instructional strategies employed and the overall contributions and or limitations of ChatGPT and GeoGebra to the learning process.

CHAPTER 4: FINDINGS

4.0 Introduction

This chapter presents the findings from three intervention lessons investigating how ChatGPT and GeoGebra can be employed as instructional tools to enhance the teaching and learning of sine trigonometric function concepts in a Grade 10 mathematics class. The detailed lesson designs and findings of each iterative cycle are organized and discussed in the sections that follow.

4.1 Lesson 1 Design

Lesson 1 was designed to introduce the concept of the sine ratio and its application in calculating the sides of a triangle. Although the students had prior knowledge of trigonometric ratios and their use in finding sides and angles within triangles, this lesson aimed to deepen their understanding through a structured, student-centred, technology-enhanced approach.

The lesson began with the teacher welcoming the class and providing clear instructions. Students were reminded to follow the worksheet instructions carefully, answer all questions honestly, and provide thoughtful feedback on the lesson activities. The 20 students were then divided into pairs and worked collaboratively to complete the worksheet, which took approximately 45 minutes. Afterwards, the worksheets were collected, the video recording was stopped, and a group of students was selected for a post-lesson focus group interview.

Students were given five tasks to complete in total. Task 1 tested students' prior knowledge of the definition of the sine of an angle, serving as a baseline for their understanding at the beginning of the lesson. In Task 2, ChatGPT was introduced to provide a definition and explanation of the sine of an angle. Students were given a simple prompt: "Ask ChatGPT what sine of an angle means" and were encouraged to record ChatGPT's response. The purpose of using ChatGPT at this stage was to reinforce the concept, even though students already had a foundational understanding. Their feedback on the usefulness of ChatGPT in clarifying the sine ratio provided valuable insight into whether it contributed to their comprehension.

In Task 3 and Task 4, GeoGebra activities provided a hands-on opportunity for students to see the real-time application of the definition provided by ChatGPT and explore why the sine of an angle remains constant across different triangles.

In Task 3 (see Appendix A on page 116) students used GeoGebra to explore the sine ratio for various angles within a right-angled triangle by moving a triangle vertex B in the GeoGebra interface. They calculated and recorded the sine ratio (the ratio of the opposite side length to the hypotenuse) by positioning point B in three different locations. An example of a student pair answersheet is shown below:

Task 3:
 Click on the link below:
<https://www.geogebra.org/m/dPqv8ACE#material/kcuema3n>
 When you open the GeoGebra file you will see a triangle with angles size of 55°.
 Using the given triangle and **without moving point B**, give the answers to the following using the given sine side ratios $\frac{\text{opposite}}{\text{hypotenuse}}$. Round your answers to 2 decimal places.

$$\sin 55^\circ = \frac{5,51}{6,72} = 0,82$$

Now move point B to any position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin 55^\circ = \frac{6,35}{7,75} = 0,82$$

Now move point B to another position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin 55^\circ = \frac{6,02}{7,35} = 0,82$$

Figure 4: Example of student answer for Task 3 Lesson 1 using GeoGebra with a constant angle only moving vertex B in the plane

All student pairs successfully understood the task and obtained consistent ratio values rounded to two decimal places.

In the same task they adjusted the angle measurement using a slider and repeated the exercise, again achieving identical sine ratio values across all transformations of the point B. An example of a student pair answer sheet is shown below:

Now change the angle size to any angle of your choice by dragging the black dot to the right. Give the values of sin(of your chosen angle). Round your answers to 2 decimal places.

$$\sin 34 = \frac{2,93}{5,24} = 0,56$$

Don't change your angle size but move point B to another position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin 34 = \frac{4,01}{7,16} = 0,56$$

Don't change your angle size but once again move point B to another position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin 34 = \frac{1,68}{3} = 0,56$$

Figure 5: Example of student answer for Task 3 Lesson 1 using GeoGebra by changing the angle measurement with a slider and moving vertex B in the plane

In a second GeoGebra activity (Task 4), students manipulated the vertices of a right-angled triangle by dragging the white dots and observed the sine ratio displayed as a fraction in the activity. Students were instructed to note how the ratio behaved as they moved the triangle's vertices while keeping the angles constant, which helped solidify their comprehension of the sine ratio. An example of a student pair answer sheet is shown below.

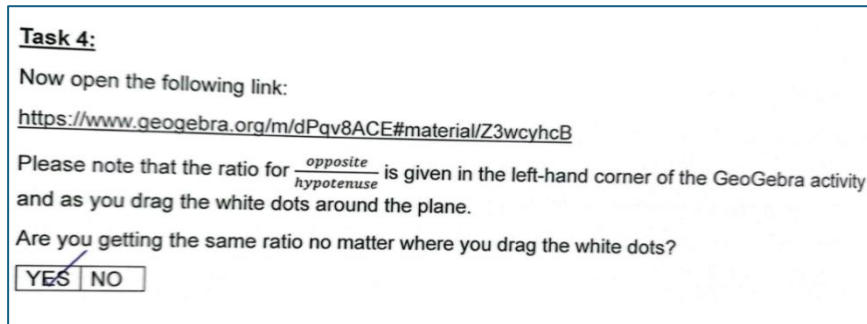


Figure 6: Example of student response for Task 4 Lesson 1

Once again, all pairs understood the task and concluded that the sine ratio remained constant, regardless of the positions of the vertices. Finally, after completing both activities on ChatGPT and GeoGebra, the students' understanding was assessed in Task 5. Students were tasked with calculating the sine ratio for various angles within a triangle. (See Appendix A on page 116). All students successfully answered the questions and were able to apply the knowledge they had gained. Below is an example of one student pair's responses to the given question:

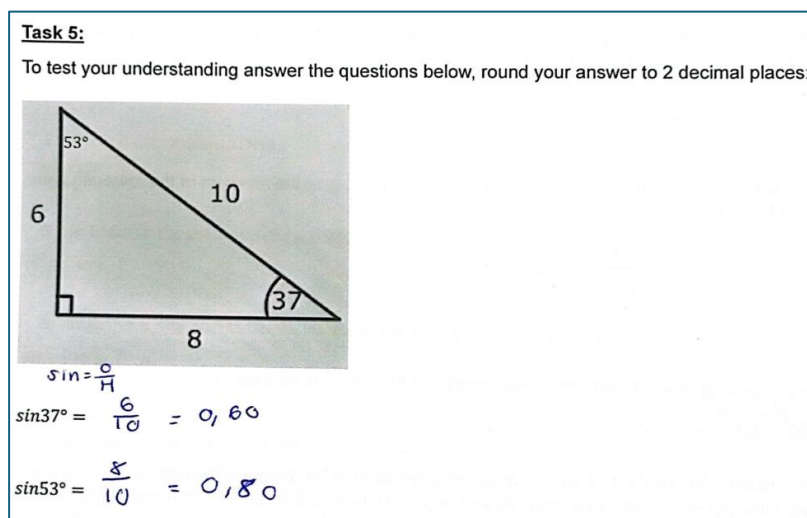


Figure 7: Example of student answer for Task 5 Lesson 1 where their understanding of the concept was tested.

The lesson concluded with a set of reflective questions, allowing students to provide feedback on their learning experience during Lesson 1.

4.1.1 ChatGPT as an Instructional Tool in Lesson 1

When looking at how ChatGPT can be used as an instructional tool in the teaching and learning of sine trigonometric function concepts, the thematic data analysis from Lesson 1 revealed four distinct themes, reflecting various aspects of ChatGPT's instructional use within the lesson design. The four themes identified were: complexity of ChatGPT's responses, ChatGPT's responses lack conciseness, need for ChatGPT prompt refinement and lack of engagement with ChatGPT. Evidence supporting these themes is discussed below

4.1.1.1 Complexity of ChatGPT's Responses

The theme "Complexity of ChatGPT Responses" became evident through various student responses from the questions answered on the worksheet, from the responses from the focus group interview, as well as feedback from the teacher, illustrating the difficulty in understanding ChatGPT's explanations of the sine trigonometric ratio due to the overly complex and abstract nature of its answers.

In the worksheets, students were asked: "Was there any part of ChatGPT's response that lacked correctness, detail, or that you struggled to understand?" Student Pair 3 responded, "We didn't understand what the second sentence meant, but upon further questioning of the Chatbot we reached our understanding." Student Pair 1 shared, "The response didn't lack correctness; however, we struggled to understand the big words." Similarly, Student Pair 2 remarked, "No, but it's quite wordy; it could be simpler," while Student Pair 4 added, "ChatGPT tried to explain...which was a bit confusing to understand." Lastly, Student Pair 5 succinctly stated, "Too complicated."

Further complexity was highlighted in the focus group interviews when students were asked whether there were any parts of ChatGPT's response that they didn't understand. Student 1 noted, "...but if you read it, the length and the ratio and grammar is just not understandable." Student 2 commented, "I felt that ChatGPT just liked using fancy words which just overcomplicated things." Student 3 added, "...the way in which ChatGPT explained this was really confusing."

When asked in the focus group, "Did you find using ChatGPT helpful?" Several students voiced similar concerns. Student 1 stated, "...and with the math it was very much overly complicated." Student 2 said, "If I had not learned the content before, I would be like it's just random terms to me...can't quite visualize it." Student 3 reflected, "ChatGPT's answer was a bit difficult to visualize, but then it was also explaining stuff about why sine is always between -1 and 1 —the way in which ChatGPT explained this was really confusing." Student 4 added, "And if I hadn't learned it previously, I wouldn't have understood it at all...it was almost grammatically confusing."

The teacher, when asked in an interview whether the ChatGPT and GeoGebra activities enhanced students' understanding of the sine trig ratio, responded, "ChatGPT on its own was a bit more abstract and harder to understand."

These direct quotes underscore the challenges students and the teacher experienced with ChatGPT, especially its tendency to overcomplicate explanations by using advanced vocabulary and abstract phrasing. This often made it difficult for students to grasp the sine ratio concept, particularly when they lacked prior knowledge, highlighting the complexities of using ChatGPT as a stand-alone educational tool.

4.1.1.2 ChatGPT's responses lack conciseness

The theme "ChatGPT's responses lack conciseness" emerged clearly from student feedback, revealing that while some students found ChatGPT's responses complex, others struggled with the length of the explanations.

In response to the worksheet question, "Was there any part of ChatGPT's response that lacked correctness, detail, or that you struggled to understand?" Student Pair 5 pointed out that the responses were, "Too complicated and not concise enough."

During the focus group interview, when asked, "Did you find using ChatGPT helpful?" Student 4 noted, "And we had to ask ChatGPT to shorten the answers," suggesting that while the content may have been helpful, the responses were too long for effective learning. This issue of lengthy explanations was reinforced when students were asked, "Were there any parts of ChatGPT's response that you didn't understand?" Student 1 responded, "...but if you read it, the length and the ratio and grammar is just not understandable," highlighting how the length of responses affected comprehension.

Finally, when asked, "Was there any part of the activity that you found challenging?" Student 1 expressed frustration with the length of ChatGPT's explanations, stating, "we were trying to get the answer that ChatGPT gave us, as short and sweet as possible, because it gave quite long answers..."

The worksheets recorded several examples of student prompts aimed at shortening ChatGPT's answers. These prompts included requests such as: "What does the sine of an angle mean? (summarise and make it 2 sentences)," "Shortly summarise what the sine of an angle means," and "What is the sine of an angle?" followed by a follow-up prompt: "Make it shorter, please." I had not anticipated that students would modify the prompts to better suit their needs.

These direct quotes and evidence of prompts used in the worksheet emphasizes a recurring issue with ChatGPT's responses, that is, while they were often accurate, many students found them overly detailed, making it necessary to request shorter answers to better aid their understanding. This lack of conciseness impacted the usability of ChatGPT's explanations, particularly when students were looking for straightforward and digestible information.

4.1.1.3 Need for Prompt Refinement

The theme "Need for Prompt Refinement" emerged from both teacher and student feedback, highlighting the importance of crafting specific and refined prompts when using ChatGPT to enhance the learning experience.

When asked, "How did you find the activity where they used ChatGPT, did you pick up on anything that you found interesting and do you think it had value?" the teacher responded that, "the girls adapted to changing their prompts...and kept asking more questions to understand certain words," indicating that students quickly recognized the necessity of adjusting their prompts to receive clearer and more relevant explanations from ChatGPT. This adaptability was a key element in improving their learning experience.

In the focus group interview, students were asked, "Did you find using ChatGPT helpful?" Student 4 pointed out that they had to refine their questions, saying, "And we had to ask ChatGPT to shorten the answers," suggesting that the initial prompts were producing lengthy responses.

When asked, "Was there any part of the activity that you found challenging?" Student 1 explained, "...so we asked ChatGPT to summarise for a shorter version of the answer," further

reinforcing that prompt refinement was essential in managing the complexity of ChatGPT's responses. This challenge revealed the students' need to adjust their prompts to receive answers that were better suited to their level of understanding.

Finally, in response to the question, "Do you think using the two technologies together can be helpful without having been taught the concepts beforehand?" Student 2 remarked, "I feel like for ChatGPT we did need to come up with like more specific prompts, just so that the answers that it gives us is like more suitable for our level of understanding." Student 1 added, "I think that ChatGPT can be useful if you have a clear prompt; if you haven't been taught it before, then even that prompt, you wouldn't know what to take out and what to actually take as valuable information. So you will need to know what to type into it to get the right or suitable answer." These responses highlight that without prior knowledge of the subject, students may struggle to generate effective prompts, underscoring the need for prompt clarity to maximize the educational benefits of ChatGPT.

Overall, this evidence demonstrates that while ChatGPT can be a valuable resource, its utility heavily depends on the ability to refine and specify prompts to align with the students' level of understanding.

4.1.1.4 Lack of engagement with ChatGPT

The theme "Lack of engagement with ChatGPT" emerged from student feedback and observational notes, revealing challenges in maintaining student interest and motivation during the activity.

During the focus group interview, students were asked, "Did you find using ChatGPT helpful?" Student 2 expressed her disinterest, stating, "Well, I didn't find it overly helpful. First because we learnt this already in class, it was like I already knew what ChatGPT was talking about." This sentiment was echoed by Student 3, who noted, "...and I feel like I understood the first part of the answer that it gave, which is basically the definition of what sine is, but also because we've done it in class." These comments suggest that the prior knowledge students had gained in class diminished the perceived value of using ChatGPT, leading to a lack of engagement.

Student 4 also highlighted this disengagement, remarking, "I definitely felt like it wasn't engaging as well." This feeling of disconnection is significant, as it indicates that the use of ChatGPT did not sufficiently stimulate interest or curiosity in the topic.

Observational notes from the video recordings provided further evidence of this lack of engagement. Although students appeared interested at the beginning of the task, their enthusiasm waned during Task 2. Notably, two pairs of students displayed visible frustration when asked to write down the entire response generated by ChatGPT. This reaction suggests that the activity may have felt burdensome or tedious, further contributing to a decline in engagement.

Overall, the evidence indicates that while ChatGPT has the potential to serve as a valuable educational tool, it can be undermined by a lack of novelty and engagement. Students' existing knowledge diminished the perceived value of the tool, and the structured tasks did not capture their interest sufficiently, resulting in a less interactive learning experience.

4.1.2 GeoGebra as an Instructional Tool in Lesson 1

When looking at how GeoGebra can be used as an instructional tool in the teaching and learning of sine trigonometric function concepts the thematic data analysis from Lesson 1 identified three distinct themes, highlighting various aspects of GeoGebra's instructional use: GeoGebra visualizes concepts, GeoGebra enhances understanding, and GeoGebra promotes exploration. Evidence supporting these codes is detailed below.

4.1.2.1 Visualizing Concepts

The theme of GeoGebra's ability to visualize concepts emerged strongly during the focus group discussions, highlighting its role in enhancing student comprehension of mathematical concepts, particularly the sine ratio within triangles. When asked in the focus group interview, "Did you find using GeoGebra helpful?" Student 1 responded, "GeoGebra was very helpful in my opinion... I got to visualize," emphasizing the tool's contribution to providing clear visual representations of the mathematical principles they were learning. Similarly, Student 3 echoed this sentiment by stating, "GeoGebra was more helpful in visualising it," reinforcing the idea that the visual aspects of GeoGebra significantly aided their understanding.

Student 4 further elaborated, indicating that GeoGebra allowed them "to see how it actually worked in like a situation," which suggests that the tool facilitated a connection between abstract concepts and real-world applications. This practical visualization enabled students to "apply it to the visual type of sense," which is crucial for grasping complex mathematical ideas.

The teacher's insights also supported the theme of visualization as a key strength of GeoGebra. When asked, "How did you find the activity where they used GeoGebra? Do you think it had value?" she remarked, "GeoGebra was great in showing them visually what was meant with the concept." She emphasized that it "worked well to help them see the concept," suggesting that the visual representation of the sine ratio within a triangle helped clarify the material for the students.

Another question posed to the teacher was, "Which activity did you find the most effective for comprehension, the activity using ChatGPT or the activity in GeoGebra or a combination of the two?" In response, she concluded that "GeoGebra was more effective than ChatGPT, especially for a concept that is so visual." This assessment underscores the importance of visual learning tools in enhancing students' understanding of concepts that benefit from graphical representation.

In summary, the evidence from both student responses and teacher observations strongly indicates that GeoGebra is an invaluable tool for visualizing mathematical concepts. Its ability to provide a visual context not only aids comprehension, but also enhances student engagement, making it a critical component of effective lesson design.

4.1.2.2 Enhancing Understanding through Contextualization

The theme of how GeoGebra enhanced understanding and comprehension emerged prominently from the focus group interviews and teacher feedback. When students were asked, "Did you find using GeoGebra helpful?" Student 1 remarked, "I actually got to expand what I knew," highlighting how the tool allowed them to build on their existing knowledge and deepen their understanding of mathematical concepts. Student 3 reinforced this perspective by stating, "GeoGebra was more helpful in understanding what actually is going on because you're putting it in context." This suggests that the contextualization provided by GeoGebra played a significant role in helping students grasp the underlying principles of the sine function.

The teacher's observations further supported this theme. When asked about the use of GeoGebra, she stated, "GeoGebra definitely helped to enhance the understanding." This indicates that the visual representation of mathematical concepts facilitated clarity and comprehension, making the material more accessible to students.

Additionally, the worksheet responses indicated that 80% of student pairs favored GeoGebra for comprehension. This statistic underscores the widespread acknowledgment among students of the tool's positive impact on their understanding of the subject matter.

Overall, the combined evidence from student feedback, teacher observations, worksheet analysis, and video recordings of the class highlights that GeoGebra significantly deepened students' understanding of the sine ratio within triangles. Its ability to visually contextualize concepts not only helped students grasp complex ideas but also fostered greater engagement with the material. This was further supported by the video footage, which showed students actively interacting with GeoGebra, enthusiastically moving points and sliders to independently draw conclusions based on their observations.

4.1.2.3 Fostering Exploration

The theme of GeoGebra promoting exploration emerged strongly from both student and teacher responses during the focus group interviews. When students were asked, “Did you find using GeoGebra helpful?” Student 2 noted, “...we were fiddling around and doing our own thing, and then like seeing the answers you could come to our own conclusion about what is happening when we use GeoGebra.” This highlights the sense of autonomy and discovery that GeoGebra facilitated, allowing students to engage with the material in a hands-on manner and draw their own conclusions based on their interactions.

Student 4 echoed this sentiment, stating, “Because we were able to, like, play with it... But at the same time we also get to experiment more.” This indicates that the exploratory nature of GeoGebra not only encouraged playfulness, but also provided opportunities for experimentation, fostering a deeper understanding of the sine ratio concepts through active involvement.

The teacher’s insights further reinforced the theme of exploration. When asked about the value of the GeoGebra activity, she responded, “...they could pull the triangles around themselves and change certain parameters...” This emphasizes the interactive aspect of GeoGebra, enabling students to manipulate visual elements and observe the effects of their changes in real time.

Together, these responses illustrate how GeoGebra serves as a powerful tool for exploration, allowing students to engage actively with mathematical concepts. By providing opportunities for manipulation and experimentation, GeoGebra encourages a deeper level of inquiry and discovery, ultimately enhancing students' learning experiences.

4.1.3 Reflection and Design Adjustments for Lesson 2

The reflection on Lesson 1 revealed several insights that informed the design of Lesson 2 in this study. One of the major successes of Lesson 1 was the use of GeoGebra to reinforce concepts through exploration and self-discovery. According to the worksheet responses, when asked which technology they preferred, 80% of the student pairs favored GeoGebra for engagement and comprehension. The second success of the lesson was the combinational use of ChatGPT and GeoGebra, as highlighted by the teacher's response, when asked if this approach enhanced students' understanding of the sine trigonometric ratio. The teacher affirmed that "the combination works better" and emphasized that "if GeoGebra was used in isolation, they would not have the understanding." This was further supported by 70% of the student pairs who, in their worksheet responses, indicated that using both ChatGPT and GeoGebra together deepened their understanding of the sine of an angle within a right-angled triangle.

However, certain aspects of Lesson 1 did not work as intended. Using ChatGPT to define the sine of an angle proved challenging. The prompt provided to ChatGPT was too broad and the answers given by ChatGPT were too lengthy, resulting in responses that students found overly complex and long. The prompt should have been more detailed and should have specified that the definition was intended for students with no prior experience in trigonometry, aiming for a less complicated response. Feedback from the worksheet supported this, with Student Pair 1 recommending that prompts to ChatGPT be "more specific and less broad," and Student Pair 3 suggesting that the prompts include guidance on the intended age group for the response. To address this, the prompt for Lesson 2 was revised to provide students with more detailed instructions, aiming to elicit clearer, more focused responses to enhance their understanding.

Additionally, the way ChatGPT was used in Lesson 1 could be improved. During the focus group interview, Student 1 proposed that ChatGPT be used to generate questions for students to answer, allowing them to test their understanding of the concept. Student 2 suggested that students input their answers into ChatGPT so the tool could analyze their responses and highlight any mistakes. These adjustments were incorporated into the design of Lesson 2. However, rather than having students input their own answers, they were be tasked with analyzing responses generated by ChatGPT.

Evidence of student disengagement during Lesson 1 was also noted in their worksheet responses, where one student pair suggested the need for "more challenging tasks of work we haven't covered in class yet." This feedback highlights differing opinions on the optimal timing for using these two technologies in the learning process. Student 2 suggested that the combination of ChatGPT and GeoGebra should not be used for an initial introduction to the sine ratio, while Student 3 felt that the activities could build on a foundational lesson covering the sine concept. For Lesson 2, this recommendation could not be fully implemented, as the content covered in Lesson 2 built on concepts introduced in Lesson 1.

Specifically, Lesson 2 focused on how the sine ratio changed across different quadrants of the Cartesian plane, material the students were already familiar with. As a result, disengagement persisted, particularly when determining the optimal points for introducing or reinforcing these concepts using ChatGPT and GeoGebra. Further exploration was needed to identify the most effective timing for integrating these technologies.

Another challenge was the sequence in which the technologies were used. In Lesson 1, ChatGPT was used first to explain the sine function, followed by GeoGebra for visual exploration. The teacher suggested restructuring this sequence for Lesson 2, recommending that the lesson should start with GeoGebra to introduce the diagrams, followed by ChatGPT for a detailed explanation, and concluding with further exploration in GeoGebra. This restructuring was applied in Lesson 2; however, due to the nature of the concept being explored, an additional GeoGebra activity was neither necessary nor feasible, as the concept was addressed within a single exploration. As a result, the sequence of technology use in Lesson 2 will be adjusted, starting with GeoGebra and transitioning to a ChatGPT activity. The alternating use of technologies was not implemented, as the lesson content did not accommodate such a structure.

In response to these suggestions, Lesson 2 was restructured to address the challenges observed in Lesson 1. ChatGPT was used to generate questions, encouraging students to compare their answers with ChatGPT's responses, which aimed to foster deeper understanding and critical thinking. The prompts used for ChatGPT were refined to aim to ensure that responses are appropriate for the students' level and needs. GeoGebra was used to support visualization, exploration, and comprehension, as this was highly successful in Lesson 1. Additionally, a new sequence was implemented where students started with GeoGebra to visualize the concept, then move to ChatGPT for detailed explanations. This approach, along with clearer and more

detailed prompts to ChatGPT, aimed to better engage students and enhance their learning experience in Lesson 2.

4.2 Lesson 2 Design

Lesson 2 was designed to introduce the concept of the sine ratio and its behaviour in different quadrants of the Cartesian plane. Although students had prior knowledge of trigonometric ratios and their application in determining angles and sides within the Cartesian plane, the lesson aimed to deepen their understanding through a structured, student-centred approach, enhanced by technology.

The lesson began with the teacher welcoming the class and reminding students to carefully follow the worksheet instructions, answer all questions honestly, and providing thoughtful feedback on the lesson design. The teacher's instructions did not change from Lesson 1 to Lesson 2. Eighteen students were then divided into pairs to collaboratively complete Tasks 1 to 3. For Task 4, they worked in groups of four to encourage more discussion, idea-sharing, and collaborative problem-solving. The lesson took approximately one hour. After the completion of the tasks, worksheets were collected, the video recording was stopped, and a focus group interview was conducted with selected students.

Task 1 assessed students' prior knowledge of the sine ratio's behaviour in different quadrants of the Cartesian plane. Seven student pairs expressed confidence in identifying in which quadrant the sine ratio is positive or negative and understood why. However, two student pairs indicated that they knew in which quadrants the sine ratio changes but did not fully grasp the underlying reasoning. Their responses are shown below in Figure 8 and Figure 9.

Task 1:

Do you know in what quadrants of the Cartesian plane an acute-angled triangle's sine ratio will give a positive answer? (with one of its vertices on the origin)

YES NO

Do you understand why the sine ratio of such a triangle might be positive or negative in different quadrants?

YES NO

Figure 8: Example of student answer for Task 1 Lesson 2 showing lack of understanding prior to the ChatGPT and GeoGebra activities

Task 1:

Do you know in what quadrants of the Cartesian plane an acute-angled triangle's sine ratio will give a positive answer? (with one of its vertices on the origin)

YES NO

Do you understand why the sine ratio of such a triangle might be positive or negative in different quadrants?

YES NO

Figure 9: Example of student answer for Task 1 Lesson 2 showing lack of knowledge and lack of understanding prior to the ChatGPT and GeoGebra activities

In response to a recommendation from Lesson 1, Task 2 began with an activity using GeoGebra. Students were asked to open a provided link, observe the sine ratio for 36 degrees displayed in the tool, round the answer, and then verify the result using their calculators. This exercise helped them see that both GeoGebra and their calculators provided consistent values. The next step involved calculating the angle when given a sine ratio, reinforcing how to use the calculator to determine either the angle or the ratio in a triangle, depending on the information provided. Students then moved point P around the Cartesian plane, observing the sine ratio's sign and the angle's classification (acute, obtuse, reflex) in each quadrant. After completing the activity, both previously uncertain pairs reported a clearer understanding of why the sine ratio changes across quadrants. Their understanding was further demonstrated through written explanations which are shown below in Figure 9 and Figure 10

e) After doing this activity in GeoGebra do you have a better understanding of why the sine ratio is positive or negative in certain quadrants?

YES NO

Explain your understanding in words:

The sine ratio is positive because both the hypoteneus and opposite side is positive in the first and second quadrant.

Figure 10: Example of a student's explanation in Task 2 Lesson 2 on why the quadrants influence the sign of the sine ratio, giving a vaguely correct explanation.

e) After doing this activity in GeoGebra do you have a better understanding of why the sine ratio is positive or negative in certain quadrants?

YES NO

Explain your understanding in words:

Between 0° and 180° (1st and 2nd quadrant) the ratio will be positive. But above 180° (3rd and 4th quadrant) the ratio will be negative. This because the coordinates of the points on either the opposite or hypotenuse lines will have a negative x or y value.

Figure 11: Example of a student’s explanation in Task 2 Lesson 2 on why the quadrants influence the sign of the sine ratio, showing a slight misconception with referring to the x-values.

Task 3 required students to engage with ChatGPT by asking a specific prompt provided on the worksheet. As recommended after Lesson 1, the prompt was designed to be more detailed, ensuring a comprehensive response.

The prompt given was: "Explain why the sine ratios of an angle in different quadrants of the Cartesian plane have different signs. Consider the geometric interpretation of the sine function in relation to the lengths of the sides of a right triangle, as well as how the signs of these lengths vary across quadrants. Use examples and diagrams, if necessary, to illustrate your explanation."

Students recorded ChatGPT’s answers, one student pair’s answer from ChatGPT is shown below in Figure 12.

We added this

Task 3:

Use the following prompt to ask ChatGPT:

Briefly

A "Explain why the sine ratios of an angle in different quadrants of the Cartesian plane have different signs. Consider the geometric interpretation of the sine function in relation to the lengths of the sides of a right triangle, as well as how the signs of these lengths vary across quadrants. Use examples and diagrams, if necessary, to illustrate your explanation.

ChatGPT's ANSWER:

In a Cartesian plane, each quadrant has its own unique characteristics regarding the signs of coordinates. The signs of coordinates determine the signs of lengths of sides in a right triangle, which in turn affect the sine ratio of an angle.

1. Quadrant I: Both x and y coordinates are positive. Therefore, all sides of the right triangle are positive ~~are~~ resulting in a positive sine ratio.
2. Quadrant II: The x coordinate is negative but the y coordinate is positive thus only the opposite side (y coordinate) is positive, leading to a positive sine ratio.
3. Quadrant III: ~~The x coordinate is~~ Both x and y coordinates are negative & consequently both the opposite and hypotenuse sides are negative, resulting in a negative sine ratio.
4. ~~Quadrant~~ Quadrant IV: The x coordinate is positive but the y coordinate is negative, hence only the opposite side (y coordinate) is negative, causing a negative sine ratio.

In summary the signs of the sine ratios change across quadrants because the signs of the length of sides in a right triangle vary based on the signs of the coordinates in each quadrant.

Figure 12: Example of one student pair's ChatGPT response given after prompt was used for Task 3 Lesson 2

The student pairs then compared their understanding of the sine ratio from the ChatGPT activity in Task 3 with what they learned through the GeoGebra activity in Task 2. They were also asked to evaluate the clarity of ChatGPT's explanations. In Task 4, pairs merged into groups of four and used another detailed prompt to interact with ChatGPT.

The prompt used was: "Imagine you're teaching a friend about the CAST diagram, which helps determine the signs of the trigonometric sine function in each quadrant of the Cartesian plane. Using the CAST diagram as a reference, create questions that test their understanding of whether the trigonometric sine function is positive or negative in different quadrants. Consider asking about angle locations, function signs, and real-life applications. Be sure to include a variety of scenarios to challenge your friend's comprehension."

One student pair's recorded ChatGPT response to the prompt above is shown below in Figure 13.

Task 4:

Work in groups of 4 for this Task.

Every group of 2 needs to fill in the worksheet below.

Use ChatGPT and ask:
 Imagine you're teaching a friend about the CAST diagram, which helps determine the signs of the trigonometric sine function in each quadrant of the Cartesian plane. Using the CAST diagram as a reference, create questions that test their understanding of whether the trigonometric sine function is positive or negative in different quadrants. Consider asking about angle locations, function signs, and real-life applications. Be sure to include a variety of scenarios to challenge your friend's comprehension. *Make it concise.*

Write ChatGPT's answer below:

1) Quadrant indication:

- Question: In which quadrant(s) is the sine function positive?

- Answer: Quadrants I & II

2) Negative Sine angle:

- Question: If the sine of an angle is negative, in which quadrant could the angle be located?

- Answer: Quadrants III & IV

3) Quadrant IV sine:

- Question: For an angle in quadrant IV, what is the sign of the sine function?

- Answer: Negative

4) Sign change across quadrants:

- Question: How does the sign of the sine function change as an angle moves from quadrant II to quadrant III?

- Answer: It becomes negative.

5) Real-life Application:

- Question: Can you think of a real-life scenario where understanding the signs of trigonometric functions is important?

- Answer: Navigation, where it helps determine direction relative to a reference point.

Figure 13: Example of a student pair's ChatGPT response on the prompt given in Task 4 Lesson 2

The groups analyzed the questions generated by ChatGPT, attempted to answer them, and critiqued the quality of the questions. Afterwards, they requested that ChatGPT provide answers to its own questions and analyzed these responses for any mistakes or missing information.

Each group was then tasked with creating a real-life challenge question related to the sine ratio in different quadrants, intended to test the understanding of other groups. However, due to time constraints, this part of the activity was not completed. Students were asked to reflect on Task 4, specifically on their experience with ChatGPT, and whether the activity promoted discussion, critical thinking, and learning.

The lesson concluded with students providing feedback on which activity, ChatGPT or GeoGebra, they preferred for learning the sine ratio in different quadrants, as well as which activity they found more challenging and engaging in terms of critical thinking. They also commented on which activity contributed most to their understanding of the concept.

4.2.1 ChatGPT as an Instructional Tool in Lesson 2

When examining the use of ChatGPT as an instructional tool for teaching sine trigonometric function concepts, the thematic analysis from Lesson 1 revealed four key themes, which were also evident in the data from Lesson 2. These recurring themes included: complexity of ChatGPT's responses, ChatGPT's responses lack conciseness, a need for ChatGPT prompt refinement and lack of engagement with ChatGPT. Additionally, a new theme emerged in the analysis of Lesson 2, highlighting mistakes in ChatGPT's responses. The evidence supporting these themes, along with a comparison to the outcomes from Lesson 2, are discussed below.

4.2.1.1 Complexity of ChatGPT's Responses

The theme "Complexity of ChatGPT's Responses" emerged from various student responses to the worksheet questions, feedback from the focus group interview, and insights from the teacher. This highlights the challenge students faced in comprehending ChatGPT's explanations of the sine trigonometric ratio, largely due to the overly complex and abstract nature of its answers.

In Task 3, ChatGPT was used to explain why the sine ratio is negative in different quadrants, using a more detailed prompt than that employed in Lesson 1. Of the eight student pairs who completed the worksheet, four indicated that ChatGPT's response was unclear, leaving them unable to grasp the underlying rationale. During the focus group interview, when students were asked, "How did your experience using ChatGPT this time, where it provided questions instead of just definitions, compare to the first lesson?" two students shared that they found ChatGPT's explanations sufficient. Student 2 remarked, "I liked when we could ask ChatGPT the first question to explain why the sine ratio of triangles in different quadrants have different signs. Then it gave us a really nice description on it and it worked really well with what we saw on GeoGebra." Student 3 added, "...it explained to me how the y values are positive when it's above the X axis and why it's negative when it's below, which influences the sign of the sine

ratio of the triangles, which I'd never learned about before. So that really helped me." However, Student 4 noted that she found the explanations very wordy and confusing, indicating mixed feelings regarding the complexity of ChatGPT's explanations in Task 3.

In Task 4, ChatGPT was tasked with generating questions to test the students' comprehension of the lesson's concepts. During the focus group interview, students expressed their opinions about ChatGPT's questions. Student 1 stated, "Sometimes ChatGPT overcomplicates it," while Student 3 noted, "...simple question but they worded it quite weirdly..." and added, "A lot of the questions were really confusing." When the teacher was asked about the extent to which ChatGPT's explanations and guidance aided students' understanding of trigonometric sine functions when used independently, the teacher responded, "ChatGPT was harder to follow..." Overall, compared to Lesson 1, the responses indicate that ChatGPT's explanations and responses remain too complex for many students, making comprehension difficult.

4.2.1.2 ChatGPT's responses lack conciseness

The recurring theme: "ChatGPT's responses that lack conciseness" emerged from various student responses to the worksheet questions, feedback from the focus group interview and insights from the teacher interview. In Lesson 2, ChatGPT's responses in both Task 3 and Task 4 tended to be lengthy. During the focus group interview, when asked, "How did your experience using ChatGPT this time, where it provided questions instead of just definitions, compare to the first lesson? Student 5 pointed out that ChatGPT often over-explained concepts, leading to excessive writing and overly complex sentences. Similarly, Student 2 noted the lengthy nature of ChatGPT's initial responses. The teacher corroborated these observations, indicating that students found ChatGPT's answers to be excessively long. Evidence of this issue is also reflected in the worksheet, where student pairs reported modifying the prompt to elicit shorter responses. Despite using a more detailed prompt compared to Lesson 1, the answers provided by ChatGPT remained too long and wordy.

4.2.1.3 Need for Prompt Refinement

The recurring theme of "Need for Prompt Refinement" emerged from both teacher and student feedback, underscoring that while the prompts used in Lesson 2 were more detailed than those in Lesson 1, they still require further refinement. When asked about the effectiveness of integrating GeoGebra and ChatGPT in her students' learning process, the teacher noted,

"Maybe if the prompts were more specific to get concise answers out, it would have helped more."

In the focus group interview, students provided recommendations to improve the use of ChatGPT for generating questions and answers in future lessons. Student 5 suggested, "When it comes to asking ChatGPT for questions, maybe like a certain amount because we got an excess number of questions." Student 3 echoed this sentiment, stating, "Maybe if they were more precise in how many questions you ask...".

Additionally, it was observed that prompts should be structured to encourage shorter responses, using terms such as "summarize" or "briefly." Student 2 remarked, "The summary prompt I found really helpful because it explained to me... At first ChatGPT gave a really long answer, and then we asked for it to be summarized." The teacher also noted that some students modified their prompts to include "summarize the previous answer." Student 5 concluded, "If ChatGPT gave a concise or summarized version, that would be quite interesting."

In summary, for the prompts to be more effective in supporting student learning, they should not only be detailed like in Lesson 1, but also explicitly instruct ChatGPT to provide summarized or concise answers tailored to the student's level of understanding.

4.2.1.4 Lack of engagement with ChatGPT

The theme of "Lack of engagement with ChatGPT" emerged from student feedback and observational notes, revealing significant challenges in maintaining student interest and motivation during Lesson 2 ChatGPT activities (Task 3 and Task 4). Unlike Lesson 1, where it was evident from the data analysis that engagement issues were predominantly attributed to the familiarity of the material, leading students to perceive the activity as monotonous, Lesson 2 presented a different set of engagement concerns. When the teacher was asked to evaluate student engagement while utilizing ChatGPT compared to traditional instructional methods, she indicated, "They lost confidence in the answers given... they did not engage well for the duration of the lesson." Although students commenced the lesson with enthusiasm, their interest diminished as the activity progressed. The teacher further suggested that "if it was shorter, they probably would have engaged better," underscoring the importance of concise content in lesson design in sustaining student attention.

In the focus group interview, students expressed concerns about the overwhelming amount of writing involved in the ChatGPT activity which caused a lack of engagement. As Student 5 explained, “I felt like it was a lot of writing... it was a bit overwhelming.” And Student 4 agreed by commenting: “I agree with not writing a very long paragraph because that took a very long time, and that was where I got a bit bored” When asked to suggest improvements for Lesson 3, students proposed two key solutions to address this issue. Firstly, Student 4 recommended using electronic worksheets, saying, “I think electronic worksheets will be better as writing the long paragraph took a very long time, and that was where I got a bit bored.” Student 5 agreed, adding, “Definitely an electronic copy... then we can copy and paste for the ChatGPT answers.” This suggests that switching to digital formats could streamline tasks and reduce fatigue. The second solution involved refining the ChatGPT prompt. Student 2 observed that initially, ChatGPT provided lengthy answers, but requesting ChatGPT for a summary yielded clearer, more concise explanations: “At first ChatGPT gave a really long answer, and then we asked for it to be summarized, and it gave us a really nice explanation, and it worked really well.” Additionally, Student 5 proposed limiting the number of questions generated to prevent information overload: “When it comes to asking ChatGPT for questions, maybe ask it to only give a certain amount because we got an excess number of questions...”

These insights suggest that the extensive responses generated by ChatGPT in Lesson 2 contributed to a decline in student engagement, indicating that more concise prompts or having the students answer the worksheet electronically may enhance motivation and focus on the next lesson design. This stands in contrast to the challenges observed in Lesson 1, where disengagement stemmed from a lack of novelty rather than the lengthy responses from ChatGPT.

4.2.1.5 ChatGPT’s incorrect responses

The theme "ChatGPT's Incorrect Responses" emerged as a key issue in the analysis, highlighting concerns regarding the accuracy of the AI-generated answers. Both the teacher and students identified instances where ChatGPT provided incorrect information, underscoring the need for careful consideration of its use in an educational setting. The teacher noted inaccuracies in ChatGPT’s questions and answers generated, and several students echoed this sentiment. For example, Student 2 noted that “some of ChatGPT’s answers to its generated questions were inaccurate.” Similarly, Student 3 observed that certain questions generated by ChatGPT were irrelevant or not applicable.

A specific example of this occurred when ChatGPT posed the question, “Can you provide an example of an angle in the second quadrant where the sine function is negative?”, a question that is fundamentally flawed, as sine is positive in the second quadrant. Additionally, when asked what happens to the sign of the sine function in the second quadrant compared to the first, ChatGPT incorrectly stated that it becomes negative. These inaccuracies were recorded by one group of students, as shown in Figure 14 and Figure 15, where they documented both the faulty questions and incorrect answers provided by ChatGPT.

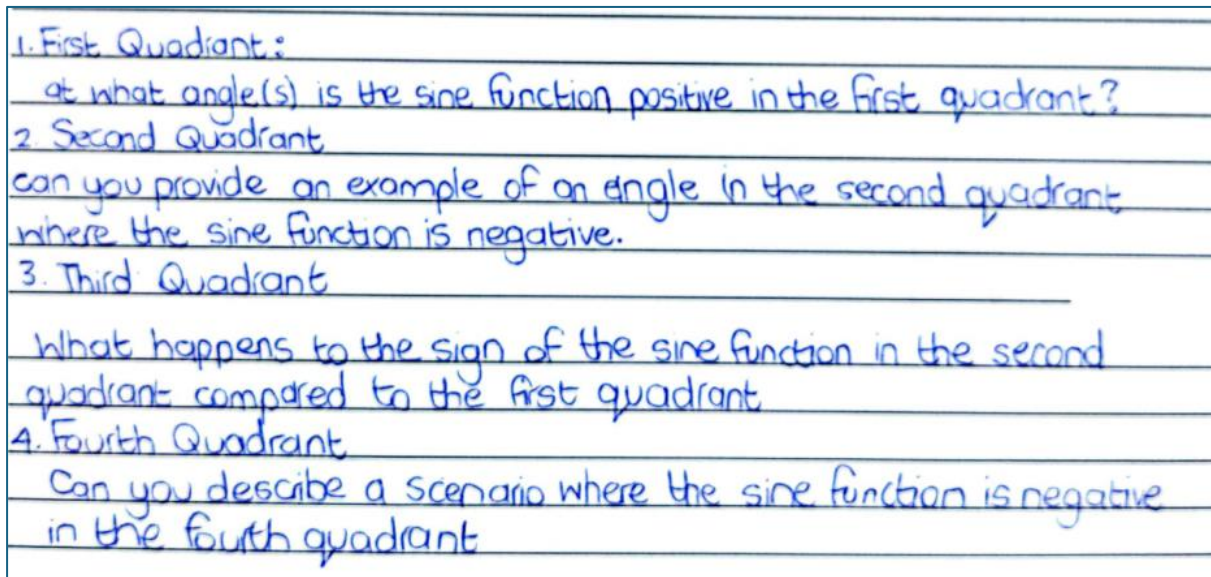


Figure 14: Example of a student pair’s ChatGPT questions generated for Task 4 Lesson 2 where the second question generated was irrelevant and impossible to answer.

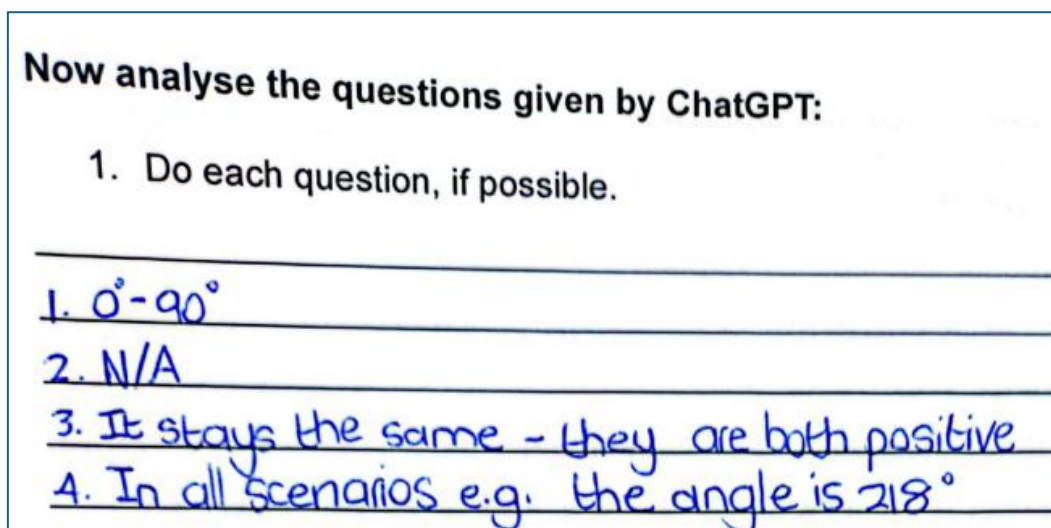


Figure 15: The student pair’s answers to the questions generated by ChatGPT in Figure 14 showing that the second question generated was irrelevant and impossible to answer.

Despite these issues, there was evidence that engaging with ChatGPT's incorrect responses prompted critical thinking among students. One student pair noted in their worksheet that the task was "interesting as you had to engage critically with ChatGPT's answers," and reported that it enhanced their understanding of sine, despite the complexity of some responses. However, they also pointed out that this kind of engagement might not be accessible to students without prior knowledge of the topic, as the errors could be more confusing than beneficial for those new to trigonometry.

This suggests that while ChatGPT can introduce errors, these mistakes can be leveraged as learning opportunities, fostering deeper engagement and understanding, provided the students have sufficient foundational knowledge to critically assess and correct the inaccuracies.

4.2.2 GeoGebra as an Instructional Tool in Lesson 2

When looking at how GeoGebra can be used as an instructional tool in the teaching and learning of sine trigonometric function concepts the thematic data analysis from Lesson 2 identified three recurring themes, highlighting various aspects of GeoGebra's instructional use: GeoGebra visualizes concepts, GeoGebra enhances understanding, and GeoGebra promotes exploration. Evidence supporting these codes is detailed below.

4.2.2.1 Visualizing Concepts

The theme of "GeoGebra Visualizes Concepts" became apparent in both Lesson 1 and Lesson 2, with feedback from students and the teacher highlighting how GeoGebra helps make mathematical ideas visual and in return more accessible. In Lesson 1, GeoGebra was particularly valued for its ability to present a visual representation of abstract mathematical concepts, making it easier for students to grasp challenging topics. Both student responses and teacher observations suggested that the visual aspect of GeoGebra played a key role in boosting comprehension.

In Lesson 2, this trend continued, with students expressing a preference for using GeoGebra to visualize the mathematical concepts. The following question was asked to the students during the focus group interview: "Starting with GeoGebra in this lesson versus the first lesson where we started with ChatGPT, what was your experience of this activity?"

In response, Student 1 stated, "I like GeoGebra more as it is visual," while Student 2 added, "I liked doing the GeoGebra first because it's nicer to be able to visualize and see what's

happening." Similarly, Student 3 noted, "I liked doing the GeoGebra first, because then you could see what the triangle looks like in all the quadrants and the size of the angles." These comments underline how GeoGebra's visual aids helped students better understand geometric relationships and maintain their focus throughout the lesson.

The teacher was also asked to reflect on the use of GeoGebra with the question: "How would you rate the overall experience of your students with the GeoGebra and ChatGPT tools for learning sine trigonometric function concepts?" In response, the teacher confirmed, "Overall, I think they found GeoGebra easier to follow as it was visual." This echoes the student feedback, emphasizing how the visual nature of GeoGebra simplified complex mathematical ideas and contributed to student engagement.

Comparing this to Lesson 1, where GeoGebra was also appreciated for its visual benefits, Lesson 2 also highlighted the advantage of starting with visual representations. This suggests that, for a topic like the sine trigonometric function, using a visual tool like GeoGebra in the learning process allows students to build a clearer understanding from the outset through visualization of the concepts.

4.2.2.2 Enhancing understanding through contextualization

The theme "GeoGebra Enhancing Understanding through Contextualization" emerged clearly from the analysis of Lesson 2, with both student feedback and worksheet responses showing how GeoGebra supported a deeper comprehension of trigonometric concepts. During the focus group interview, students were asked: "Starting with GeoGebra in this lesson versus the first lesson where we started with ChatGPT, what was your experience of this activity?"

In response, several students emphasized that beginning with GeoGebra made the material more accessible. Student 4 noted, "I also liked using GeoGebra first because I find it easier to understand," while Student 5 remarked, "I also found that it's a lot simpler and the methods are easier to understand." This indicates that using GeoGebra first provided a straightforward introduction to the content, easing the transition to more abstract concepts covered later in the lesson.

When asked another question in the focus group: "Did the combination work best or would you still rather just prefer to work with ChatGPT or just with GeoGebra?", Student 5 responded, "...you get a basic understanding from using GeoGebra and it helps you when you're learning

a new concept." This sentiment reflects the broader consensus that GeoGebra laid a strong conceptual foundation, supporting the development of new ideas in a more tangible and approachable way.

In a parallel discussion with the teacher, the question was posed: "Did the combination of ChatGPT and GeoGebra complement each other effectively in your students' learning process? How did their integration enhance understanding compared to using them individually?" The teacher responded, "For this lesson, I almost think that GeoGebra would have been better on its own." This suggests that while GeoGebra significantly contributed to understanding, ChatGPT's involvement may have detracted from the clarity or simplicity that GeoGebra alone provided.

The worksheet analysis further supported this theme. In Task 1 of the GeoGebra activity, students were tasked with investigating the sign of the sine ratio in different quadrants. All student pairs successfully followed the instructions and answered the question correctly, demonstrating their ability to apply what they observed using GeoGebra. Following Task 2, students were asked: "After doing this activity in GeoGebra, do you have a better understanding of why the sine ratio is positive or negative in certain quadrants?" The responses varied in depth but generally indicated that the visual representation provided by GeoGebra enhanced their understanding.

For example, Student pair 1 demonstrated a clear understanding, correctly identifying that sine ratios are positive in quadrants 1 and 2 and negative in quadrants 3 and 4. Student pair 2 offered a more detailed explanation, connecting the sign of the sine ratio to the positive or negative values of the opposite side and hypotenuse in each quadrant. Student pair 3 also grasped the concept but focused primarily on the negative values of the opposite side in the lower quadrants. Other student pairs, such as student pair 6, exhibited partial understanding, recognizing the relationship between the quadrant and sine ratio but struggling to articulate the reasoning behind it.

In the conclusion section of the worksheet, students were asked two key questions: "Which activity did you learn from the most, ChatGPT, GeoGebra, or the combination of the two technologies?" and "Which activity gave you the best understanding of the concept of different signs of a sine ratio in different quadrants?"

Notably, 5 out of 8 student pairs indicated that they learned the most from GeoGebra, and that it provided them with the clearest understanding of the concept of sine ratios in different

quadrants. This feedback aligns with earlier observations that students found GeoGebra to be a helpful tool for grasping trigonometric concepts, particularly the signs of sine in different quadrants.

In contrast, Lesson 1 similarly highlighted GeoGebra's role in enhancing understanding, but the conclusions drawn were somewhat broader. Feedback from Lesson 1, encompassing both student comments and teacher observations, indicated that GeoGebra consistently promoted comprehension of the sine ratio in triangles by offering visual context to complex ideas. However, while the focus of Lesson 1 was on GeoGebra as a useful tool to generally aid comprehension, Lesson 2 revealed a more nuanced picture, where students not only found GeoGebra beneficial but also preferred it as the starting point for learning, suggesting that its role in laying foundational knowledge was more critical than its supplementary role alongside ChatGPT.

While both lessons highlighted GeoGebra's value in improving understanding, Lesson 2 demonstrated its strength as a tool for initial learning, especially when compared to ChatGPT. The student responses and teacher feedback in Lesson 2 underscored that the visual and interactive nature of GeoGebra allowed for clearer conceptualization and understanding of trigonometric sine function concepts.

4.2.2.3 Fostering Exploration

The theme of "GeoGebra Promotes Exploration" emerged strongly from both Lesson 1 and Lesson 2, emphasizing how the tool fosters active student engagement and a deeper understanding of mathematical concepts through exploration and manipulation.

In Lesson 2, when the teacher was asked, "How helpful was GeoGebra in visualizing and exploring sine trigonometric function concepts for your students on its own?", the response highlighted the exploratory nature of the tool: "I think it was very helpful as they could see how the ratio changed to become negative if they moved the triangle into different quadrants. They engaged with it and understood the concept well."

This feedback illustrates how GeoGebra allowed students to directly interact with and manipulate the trigonometric functions, facilitating a hands-on approach to learning. By moving the triangle into different quadrants and observing the corresponding changes in the sine ratio, students were able to visualize the impact of these transformations, enhancing their

conceptual understanding of trigonometry. The interactive experience encouraged them to explore on their own and arrive at a clearer grasp of sine ratio behaviour across quadrants.

Across both lessons, GeoGebra emerged as a vital tool for promoting exploration. It provided students with the freedom to interact with the mathematical concepts at their own pace, test different ideas, and draw conclusions based on their visual and hands-on experiences. This exploratory nature allowed for deeper conceptual engagement, encouraging students to take ownership of their learning and enhancing their overall understanding of trigonometry.

4.2.3 Reflection and Design Adjustments for Lesson 3

The reflection on Lesson 2 revealed several key insights that shaped the design of Lesson 3 in this study. One of the major successes in both Lesson 1 and Lesson 2 was the use of GeoGebra to reinforce mathematical concepts through exploration and self-discovery, which significantly enhanced students' understanding as seen in the evidence stated above.

Additionally, the evidence demonstrated the preference from students and the teacher to introduce and use GeoGebra first to establish a solid conceptual foundation before using ChatGPT for further clarification. Several students expressed a preference for this sequence of activities during the focus group interview, with Students 2, 3, and 4 commenting, "I liked doing the GeoGebra first." This feedback indicates that beginning with visual, hands-on learning effectively prepared them for the subsequent engagement with ChatGPT. In the design approach for Lesson 3, ChatGPT was employed first in Task 1 to deliver step-by-step instructions and a clear method for graphing a sine function using the calculator. Following this, Task 2 utilized GeoGebra exclusively, allowing students to engage hands-on with the concepts explained in Task 1. In Tasks 3 and 4, as recommended, GeoGebra was used first for students to explore and formulate individual conjectures and conclusions. Subsequently, ChatGPT was integrated into the same tasks to verify these conjectures and provide additional context and detail regarding the "what" and "why" of the concepts being studied.

A further aspect of Lesson 2's design was the emphasis on group discussion and critical thinking, achieved using ChatGPT to generate questions and answers for students to analyze in Task 4. All student pairs indicated in their worksheet responses that this task effectively promoted discussion and deeper thinking. In Lesson 3, the revised sequence of using ChatGPT and GeoGebra further aimed to encourage these skills by incorporating tasks where students applied what they had learned, fostering both discussion and critical thinking.

Some aspects of the lesson design that proved less successful were related to how ChatGPT was used in Lesson 2, an issue that also emerged in Lesson 1. In contrast to Lesson 1, where ChatGPT primarily provided definitions, in Lesson 2, it was tasked with not only generating a definition but also to generate both questions and answers for student analysis. Unfortunately, some of ChatGPT's responses and generated questions were inaccurate or not applicable, leading to student confusion. When asked whether they preferred ChatGPT's definitions and question generation, GeoGebra's interactive explanations, or a combination of both for understanding the signs of the sine ratio in different quadrants, five out of nine pairs indicated a preference for GeoGebra alone, while four pairs favored a combination. This feedback underscores the importance of a more intentional integration of ChatGPT into lesson design. In Lesson 3, ChatGPT was utilized differently from previous lessons, focusing on delivering clear, step-by-step instructions and precise methods. It also provided detailed explanations of the "why" and "how" behind what students observed in the GeoGebra activities, helping them independently draw a sine function graph using the calculator.

In contrast to the observations from Lesson 1, the integration of ChatGPT and GeoGebra in Lesson 2 did not yield the desired outcomes, as reflected in the teacher's feedback. The teacher remarked, "For this lesson, I almost think that GeoGebra would have been better on its own. I am not convinced that ChatGPT helped in the understanding." Only two out of eight student pairs indicated in their worksheet responses that the integration of both technologies provided the best understanding of the different signs of the sine ratio in various quadrants. This indicates that the approach to utilizing these two technologies together requires further refinement to enhance their collaborative impact on student learning. Student feedback from Lesson 2 indicated a preference for using the two technologies simultaneously, allowing them to move between GeoGebra and ChatGPT as needed. On the worksheet responses, Student Pair 3 suggested, "Use both GeoGebra and ChatGPT simultaneously, jumping between the two." When asked how the integration of the two tools could be improved for Lesson 3, the teacher reiterated a point made after Lesson 1, stating, "It might be good to start with GeoGebra, move to ChatGPT, and then return to GeoGebra again. This could help students better understand the text answers in ChatGPT, as they can revisit the functions after reading the responses." To enhance the integration of the two technologies in Lesson 3, adjustments were made aiming to foster greater cohesion between ChatGPT and GeoGebra. In Lesson 3, ChatGPT was employed in Task 1 to provide clear, step-by-step instructions and precise methods for students to independently draw a sine function graph using the calculator. In Task 2, GeoGebra was used

to explore these concepts in real-time, building on what students learned in Task 1. In Tasks 3 and 4, students first investigated graph transformations using GeoGebra, drawing their own conclusions before consulting ChatGPT for clarification on the "why" and "how" of each transformation. This integrated approach sought to create a more cohesive learning experience by incorporating the recommendation to alternate between the technologies within the lesson design, ultimately enhancing students' overall understanding.

An additional element of Lesson 2 that required adjustments was the prompt used for ChatGPT, which, like Lesson 1, was too broad and resulted in overly lengthy and complex responses. This made it difficult for students to fully engage with the material. The prompt should have been more focused, specifying concise answers intended for students with no prior experience in trigonometry. The lack of clarity in the prompt led to confusion and frustration among the students. During the focus group interview, Student 5 observed, "When it comes to asking ChatGPT for questions, maybe limit the number because we got an excess amount." Similarly, Student Pair 1 noted in their worksheet responses, "Use ChatGPT when confused, but don't put in long, wordy prompts, or else you'll get a long, confusing answer." Student Pair 5 suggested, "Give more explicit instructions to ChatGPT and make sure it's summarized; otherwise, it's too dense. Ask ChatGPT to use simpler language." The teacher also emphasized this, stating, "The students found the ChatGPT answers very lengthy, and it was exhausting to write everything down. Some students changed their prompts to say 'summarize the previous answer,' which helped, but it was still too long." This feedback highlights that the downfall was not the use of ChatGPT itself, but rather how the prompt was framed. To rectify this in Lesson 3, the prompt will be revised to include more specific instructions, asking ChatGPT to provide concise, age-appropriate responses for 13-year-old students with no prior knowledge of trigonometry. The revised prompt will also include a request for summarized answers, ensuring clearer, more manageable responses that align with students' cognitive levels, ultimately aiming to enhance their understanding.

In Lesson 2, another key challenge identified was the lack of student engagement, which stemmed from two main factors. The first issue was that students had to manually write down all ChatGPT's responses, resulting in a tedious and drawn-out experience. During the focus group interview, Student 3 suggested, "...have a little less writing so that we don't get bored writing out ChatGPT's answers." Student 4 added, "I agree with not writing a very long paragraph because that took a very long time, and that was where I got a bit bored. Also, maybe having a break in between lessons would help, because otherwise our brains get really tired."

Student 5 echoed this sentiment: "It felt like there was a lot of ChatGPT in this thing. It was a lot of writing, and with the questions and answers, you'd be confused, and it felt overwhelming with too much at once. Splitting it up could work." The teacher also noted that the lesson felt long and suggested that shorter lessons might improve engagement. Furthermore, she observed that writing down ChatGPT's lengthy answers was exhausting for the students. She recommended allowing students to type out or summarize the answers in their own words, as some students had adjusted their prompts to ask ChatGPT for summaries, which helped but was still a lengthy process.

Another factor contributing to disengagement was the level of challenge in the tasks. Some students, as observed in both Lesson 1 and Lesson 2, felt that the material was too familiar and not challenging enough. In Lesson 2, Student Pair 2 commented, "It's nice to use GeoGebra for visual thinking, but it could be nicer to make the questions, when using GeoGebra, more difficult." The class observation video also revealed moments of disengagement, as students who had completed writing ChatGPT's responses often became distracted, engaging in unrelated conversations while waiting for others.

To address these issues, Lesson 3 was designed with several adjustments to increase engagement and reduce the writing burden. An electronic copy of the worksheet was provided, allowing students to copy and paste ChatGPT's answers rather than manually writing them out. Additionally, hard copies with graph grids were distributed for students to plot graphs themselves, promoting hands-on learning. Another key adjustment made was the introduction of new content in Lesson 3, focusing on how to draw and generate the graph of the sine function, concepts that are new to the class. This shift to unfamiliar material was intended to keep students engaged by challenging them with tasks they have not yet encountered in class. Through these changes, the aim is to enhance student involvement and create a more cohesive integration of the two technologies.

A further difficulty in the design of Lesson 2 was Task 4, where students were asked to create their own real-life scenario questions to test other groups' understanding of the sine ratio in different quadrants. Many students found this task particularly difficult. In the focus group interview, Student 2 commented, "We found this question quite hard to answer because we couldn't think of any real-life application situations, so we didn't really know how to do this one." Similarly, Student 3 mentioned, "...two of them were wrong, and the others were a bit confusing or irrelevant. So, when we had to come up with our own, we weren't really sure what

to say." While some students managed to find guidance by prompting ChatGPT for ideas, it was still challenging for most. Student 4 noted, "ChatGPT gave us a nice suggestion, but it was a bit wordy, so we just cut out some of the words." The teacher also observed this difficulty, commenting, "They really struggled to do this... couldn't come up with examples." In response to this feedback, Lesson 3 no longer included the task of generating a real-life scenario question.

In response to the challenges identified in Lesson 2, Lesson 3 was restructured with several key adjustments. The sequencing of technology use was revised to enhance cohesion and achieve a more seamless integration of both tools. To reduce the writing burden on students, an electronic worksheet was provided, allowing them to copy and paste ChatGPT's responses instead of manually writing them out. Additionally, hard copies with graph grids were distributed to facilitate hands-on graph plotting, further promoting active engagement in the learning process. Moreover, Lesson 3 introduced new content focused on how to draw and generate a sine function graph, challenging students with tasks they had not yet encountered, thereby aiming to maintain their engagement. The prompts given to ChatGPT were also revised to ensure concise, age-appropriate responses suitable for 13-year-old students, with a specific request for summarized answers to enhance clarity and manageability. This adjustment was intended to align the responses with students' cognitive levels.

4.3 Lesson 3 Design

Lesson 3 was designed to teach the concept of how a sine trigonometric function is drawn and how and why the graph of the sine function is shaped as it does. Students had no prior knowledge of trigonometric ratios and their application in graphing the sine trigonometric function. The lesson aimed to deepen their understanding through a structured, student-centred approach, enhanced by technology.

The lesson began with the teacher welcoming the class and reminding students to carefully follow the worksheet instructions, answer all questions honestly, and providing thoughtful feedback on the lesson design. The teacher's instructions did not change from Lesson 2 to Lesson 3. Sixteen students were then divided into pairs to collaboratively complete Tasks 1 to 5. The lesson took approximately 45 minutes. After the completion of the tasks, worksheets were collected, the video recording was stopped, and a focus group interview was conducted

with selected students. The students were given the worksheet to complete electronically on their devices.

Task 1 began with a ChatGPT activity in which students were provided with a specific prompt to guide them in using ChatGPT to obtain clear, step-by-step instructions for drawing the sine function using a calculator. The prompt read: “What are the steps to follow when drawing the function $y = \sin x$ from zero to 360 degrees using a calculator, plotting a point every 30 degrees? Summarize the steps in simple language that a 13-year-old student can understand. Provide the angles in degrees.” Students were instructed to input this prompt into ChatGPT and copy its response into the designated space on their worksheet.

Following this, each student pair was given a printed grid where they applied ChatGPT’s instructions to physically draw the sine function themselves. An example of this can be seen in Figure 16, which displays Student Pair 1’s drawn sine function.

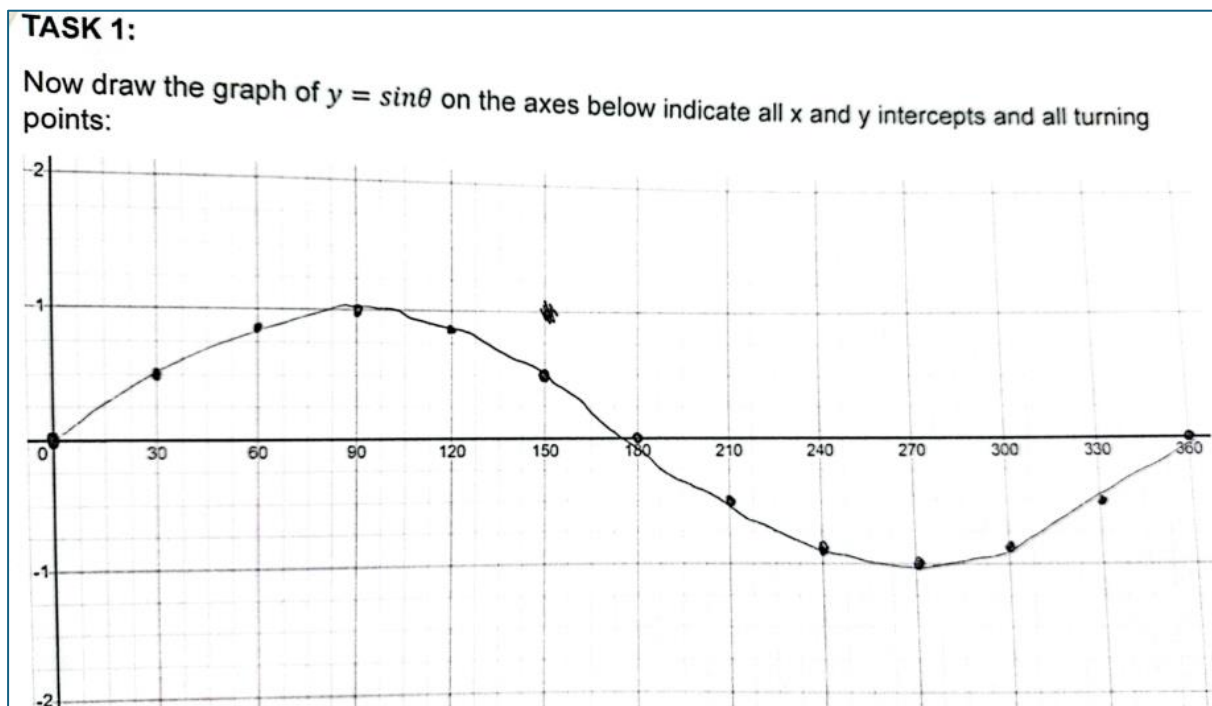


Figure 16: Sine trigonometric function graph drawn by Student pair 1 for Task 1 Lesson 3

After completing the graph, students were tasked with answering four questions based on the sine function they had drawn. Figure 17 below shows the responses from Student Pair 1.

Looking at the drawn graph above, answer the following questions:

What is the maximum value of the graph (how high does the graph go? 1)

What is the minimum value of the graph (how low does the graph go? -1)

Give the coordinates of the turning points? (1;90) and (-1;270)

Give the coordinates of the x-intercepts? (0;0) (180;0) (360;0)

Figure 17: Responses by Student pair 1 for Task 1 Lesson 3 about reading values from their drawn graph

In Task 2, students were directed to click on a provided link that led to a pre-configured GeoGebra activity. Upon accessing the activity, they were informed that the radius was set to 1 unit, and the initial angle measurement was given as 30 degrees, as illustrated in Figure 18 below.

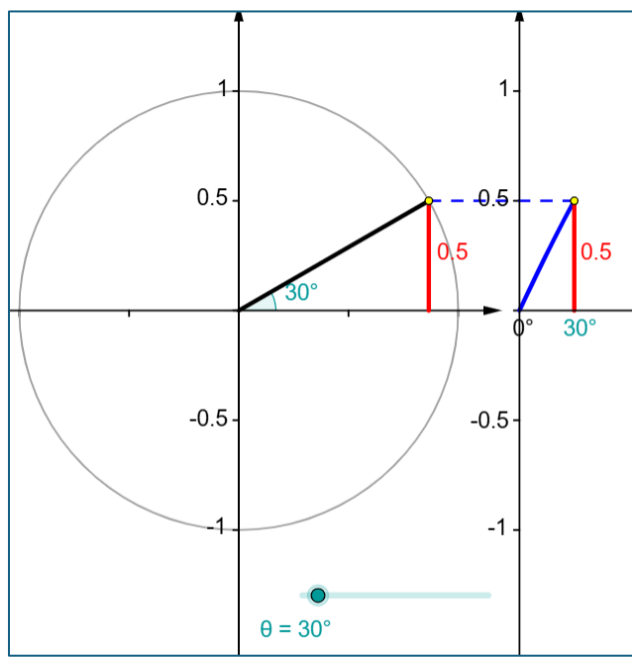


Figure 18: GeoGebra activity Task 2

Students were instructed to focus on the vertical length of the triangle (represented by the red line) and note its measurement. They were then asked to calculate the sine ratio for 30 degrees and record their answer as a decimal. Next, students were directed to observe the graph on the right-hand side and recognize that the y-value plotted corresponded to the sine ratio they had just calculated. Following this, the instructions guided students to drag the slider to increase the angle measurement, repeating the process for two angle measurements in each quadrant of the Cartesian plane. An example of this angle investigation can be seen in Figure 19 below.

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 200° .

The radius (black line) is 1 unit long, and the angle at the moment is 200°

What is the vertical length (red line) of the triangle when the angle is 200° ? 0.35

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

0.35

How does this answer link to where the graph is on the axes drawn on the right at this point?

it is the same

Figure 19: Part of GeoGebra activity Task 2

After completing these repetitive questions, all yielding the same outcome, students reinforced their understanding of how the sine graph is plotted and how the y-value of the graph corresponds to the sine ratio as it changes from one quadrant to the next. An example of the completed graph generated by GeoGebra is shown in Figure 20 below.

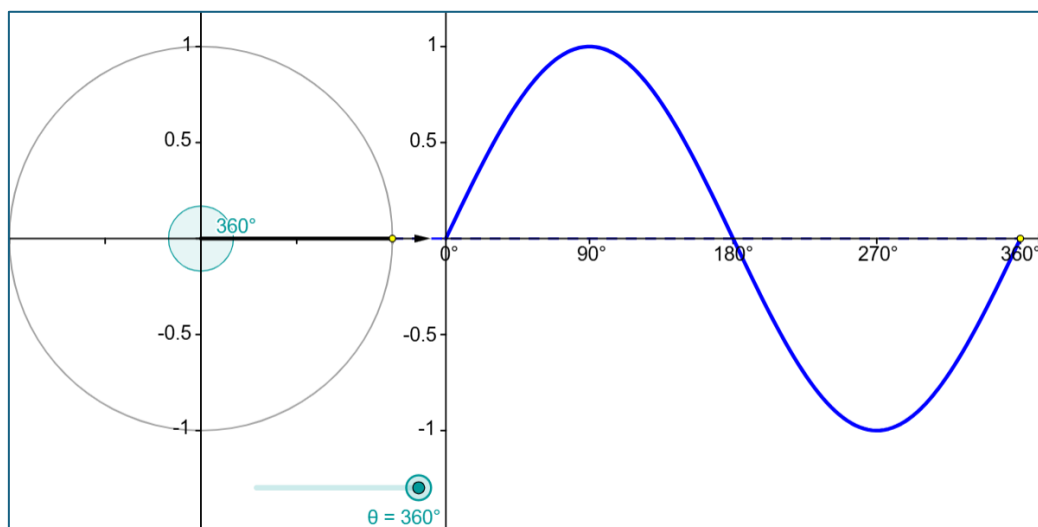


Figure 20: Sine Graph drawn by GeoGebra in Task 2

Students were then instructed to review the completed graph generated in GeoGebra and respond to a set of questions like those posed after Task 1. Refer to Figure 21 for Student Pair 1's responses to these questions. By repeating the same questions from Task 1, the goal was to reinforce the idea that both the graph created using ChatGPT's step-by-step instructions and

the interactive GeoGebra activity produced identical results. This aimed to help emphasize the consistency between the two methods in generating the sine graph.

Looking at the drawn graph in GeoGebra on the right, answer the following questions:

What is the maximum value of the graph (how high does the graph go? _____ 1 _____)

What is the minimum value of the graph (how low does the graph go? _____ -1 _____)

Give the coordinates of the turning points? _____ (1;90) and (-1;270) _____

Give the coordinates of the x-intercepts? _____ (0;0) (180;0) (360;0) _____

Figure 21: Student pair 1’s responses after the GeoGebra activity in Task 2 on reading coordinates of special points from the graph.

In Task 3, students were provided with another GeoGebra activity, which opened a pre-configured GeoGebra activity. They were instructed to observe that the graph displayed represented the equation $y = a \cdot \sin(x)$ where $a = 1$. Students were then asked to determine whether this graph was like the one they had drawn in the previous tasks, assuming π equals 180 degrees. All students responded "yes," recognising that the graph was identical to those produced in Task 1 and Task 2. The sine graph from Task 3 is shown in Figure 22 below.

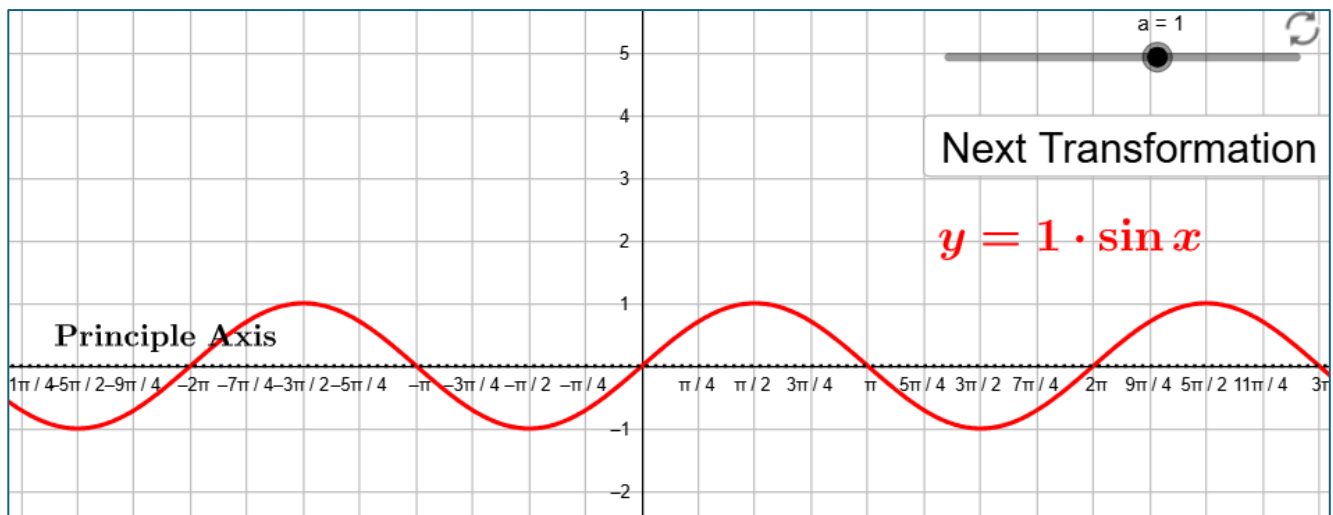


Figure 22: Sine Graph given in GeoGebra activity in Task 3 (see link: <https://www.geogebra.org/m/QhZdJGKy>)

Students were then instructed to adjust the slider to set $a = 2$ and later to $a = 3$, observing how these changes affected the graph. As this was their first encounter with the concept, students did not yet possess the appropriate vocabulary to fully describe the transformations

they were witnessing. Nonetheless, all student pairs successfully noted that the graph "flipped" and that the "peaks" and "troughs" of the graph shifted in response to the changes in a .

After noting and commenting on these changes, the students were instructed to open ChatGPT once more and copy and paste the following prompt: "Explain what happens to the sine graph when the function is multiplied by a constant value. Use simple language suitable for 13-year-old students who are encountering this specific transformation for the first time." They were then directed to paste ChatGPT's response in the designated space on the electronic worksheet. See one such ChatGPT response from Student pair 1's worksheet in Figure 23 below.

Copy and Paste ChatGPT's answer below:

Sure! Let's imagine you have the function $y = \sin(x)$, which gives you the values of the sine graph. Now, if you multiply this function by a constant value, let's say 2, the new function becomes $y = 2 \cdot \sin(x)$.

Here's what happens:

When you multiply the sine values by 2, it's like stretching the graph vertically. Imagine you have a rubber band representing the sine curve. If you pull the rubber band upwards, making it longer, the curve will stretch out, making the highs higher and the lows lower.

So, multiplying the sine function by 2 makes the highs of the sine curve twice as high as they were before, and the lows become twice as low. But the shape of the curve stays the same; it's just stretched or compressed vertically.

Figure 23: Transformation explanation by ChatGPT in Task 3

Following this, students were asked to compare ChatGPT's explanation with their own descriptions of the observed transformation. Additionally, they were prompted to reflect on whether ChatGPT's explanation helped them better understand the transformation. All student pairs noted that while ChatGPT's answer closely aligned with their own, it provided more detail, enhancing their understanding of why the graph behaves as it does by offering a thorough explanation of the transformation.

In Task 4, the students engaged with the same GeoGebra activity and followed the identical instructions from Task 3, but this time they were directed to click on the option labeled "NEXT TRANSFORMATION." They observed that the equation of the sine function had changed to $y = 1 \cdot \sin(x) + d$. The new graph transformation created by GeoGebra is depicted in Figure 24 below.

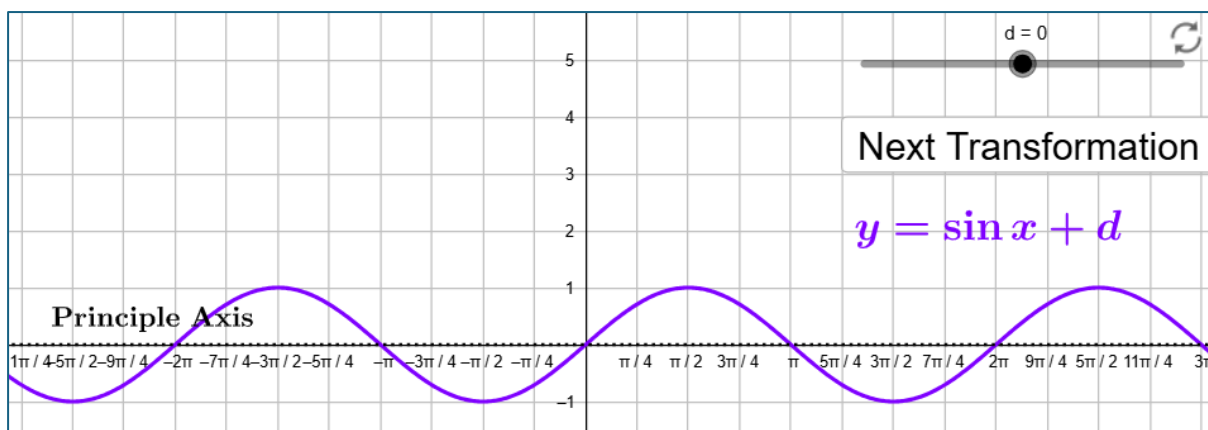


Figure 24: GeoGebra graph transformation activity in Task 4 3 (see link: <https://www.geogebra.org/m/QhZdJGKy>)

Next, the students were instructed to drag the slider representing the variable d to a value of 2 and to comment on the changes they observed in the graph. They were then asked to compare their observations to the explanation provided by ChatGPT using the prompt, "Explain what happens to the sine graph if you add or subtract the function with a certain constant value." A clear ChatGPT response regarding this transformation is shown in Figure 25 below.

Adding or subtracting a constant value to the sine function moves the entire graph up or down vertically.

Imagine you have a picture on a wall. If you lift the picture up, it moves higher on the wall. If you push it down, it moves lower on the wall. Adding or subtracting a constant to the sine function does the same thing to its graph.

If you add a positive constant, the graph moves up. It's like lifting the picture higher on the wall.

Figure 25: ChatGPT's transformation explanation in Task 4

All student pairs noted that ChatGPT's explanation of the transformation was more accurate and detailed but generally aligned with their own observations. They found that ChatGPT's response made the concept more understandable and significantly improved their comprehension of the transformation being investigated.

In the final task of this lesson design, students' understanding of the concepts learned through the activities was assessed. They were provided with two hard copy grids on which to graph two trigonometric functions: $y = 2\sin x + 1$ and $y = \cos x$. Six of the eight student pairs successfully drew the sine graph with the two transformations, while the same number of pairs

accurately represented the cosine graph. Students were also instructed that they could use ChatGPT for guidance if needed.

The lesson design concluded with a reflective section in which student pairs were asked to share their experiences of the lesson and their use of ChatGPT and GeoGebra throughout the tasks.

4.3.1 ChatGPT as an Instructional Tool in Lesson 3

An analysis of the use of ChatGPT as an instructional tool for teaching sine trigonometric function concepts in Lesson 3 revealed two new themes that were distinct from those identified in the data analyses of Lessons 1 and 2. The primary themes that emerged regarding ChatGPT's role as an instructional tool in Lesson 3 were: Clear Instructions and Explanations, and Improved Comprehension.

In Lesson 3, ChatGPT was implemented following the adjustments recommended from Lesson 2. It was first utilized as a stand-alone activity in Task 1, where it provided step-by-step guidance on drawing the sine function using a calculator. Subsequently, in Tasks 3 and 4, ChatGPT was integrated with GeoGebra to explain the transformations occurring in the sine function. The prompts for these tasks were significantly different from those used in Lessons 1 and 2, as they specifically directed ChatGPT to deliver explanations suitable for a 13-year-old with no prior knowledge and requested a summary of the answer. Additionally, students had the option to use ChatGPT voluntarily in Task 5 for extra support.

4.3.1.1 Clear instructions and explanations

The theme of "Clear Instructions and Explanations" regarding the use of ChatGPT in Lesson 3 emerged prominently through various sources of evidence gathered during the lesson activities and subsequent discussions with students and the teacher.

After completing Task 1, students participated in a focus group interview, where they were asked two primary questions: "How helpful was the first activity where you used ChatGPT by itself to give the steps to draw the sine graph?" and "Why do you think it was effective or not?" Student 1 noted, "I found it effective as it gave a very clear explanation. Especially because it was for like a 13-year-old. So it was a very simple explanation of it and it gave a really good understanding of how to do it." Similarly, Student 2 remarked, "We found the first task very nice because the steps were clear and it was very detailed. ChatGPT told you, which buttons to press." Student 3 echoed this sentiment, adding, "I also found it very effective, and the method

was very clear and simple and easy to follow.” However, Student 2 pointed out a potential issue, stating, “In our second step, ChatGPT instructed us to put the angle in the calculator before you press the sine button, and then it would cause an error. But obviously, we knew to put it afterwards.” Student 4 also pointed out this limitation, noting that “ChatGPT asked us to switch to a different mode on the calculator and find something, but that option wasn’t available.” This feedback suggests that the prompt should have specified the calculator model being used in class to ensure students could follow all steps without confusion.

The teacher observed that while the instructions were simple enough for most students to plot the points, they did not fully grasp the underlying concepts of the graph’s shape yet at this point, stating, “It gave simple enough instructions to follow. Most girls could plot the points and join them, but did not really understand what they were doing and why the graphs have that shape.” Additionally, worksheet responses confirmed that the instructions were clear, as all eight student pairs successfully drew the sine graph without errors. When asked in the worksheet, “Which task was easier?” five out of eight student pairs indicated Task 1. For example, Student Pair 1 stated, “Task 1, it was easier to find the coordinates for the graph,” and Student Pair 2 noted, “In Task 1, it was easier to have to draw the graph ourselves...”

In Tasks 3 and 4, ChatGPT was utilized alongside GeoGebra, enabling students to visualize transformations in real-time while receiving clarifying explanations from ChatGPT after forming their own conclusions from the GeoGebra activity. In a subsequent focus group interview, students shared their experiences with this combined approach. They remarked on the clarity and simplicity of ChatGPT's explanations, with Student 1 stating, “When you asked ChatGPT very simply to explain it, it gave a very easy to understand and simple explanation, which was incredibly useful in understanding.” Student 2 added, “When we used ChatGPT, it explained to us why it happened.” Analysis of the worksheet responses revealed that student pairs 1, 3, 5, and 8 found ChatGPT to be highly beneficial, appreciating its clear and straightforward guidance in both tasks. For instance, Student Pair 5 noted after using ChatGPT in Task 3 that “it was detailed and easy to understand English,” and after Task 4, they commented, “It explained it in more detail.”

Ultimately, the feedback from students in Lesson 3 supports the emergence of the theme "Clear Instructions and Explanations." ChatGPT provided clear guidance and explanations in Lesson 3 that enhanced students’ understanding of sine trigonometric functions, underscoring the importance of clear and detailed prompts to be provided to ChatGPT in the learning process.

4.3.1.1 ChatGPT enhanced understanding

The theme of "ChatGPT-enhanced understanding" emerged strongly in Lesson 3, supported by multiple sources of evidence gathered from lesson activities and follow-up discussions with both students and the teacher.

During the focus group interview, when asked about ChatGPT's role in the lesson, both Student 2 and Student 5 emphasized how it facilitated interactive learning, allowing them to ask follow-up questions that deepened their comprehension. For example, Student 2 remarked, "You could ask it another question, which then provided greater understanding," underscoring the value of ChatGPT's interactive features in enhancing their learning experience.

During Tasks 3 and 4, students first used GeoGebra to explore sine graph transformations and make their own conclusions, before turning to ChatGPT to further enhance and verify their conjectures. An analysis of their worksheets revealed that seven out of eight student pairs reported improved understanding after using ChatGPT in this way. In the focus group interview after asking students to comment on their experience of Task 3 and Task 4 the students emphasized the value of using GeoGebra first to form their own conclusions, then deepening their understanding with ChatGPT. For instance, as Student 1 noted, "I found these tasks very useful because you could see the changes in GeoGebra when you adjusted the values, and when you asked ChatGPT to explain why, it provided a simple and easy-to-understand explanation, which was incredibly useful." Student 2 described the experience as immersive, and said, "It was really cool to see what was happening in GeoGebra, and we could make our own conclusions. And then ChatGPT explained why our conclusions were correct" Student 5 echoed this, appreciating the opportunity to think independently before consulting ChatGPT. Student 6 highlighted the benefit of hypothesizing about transformations before asking ChatGPT, noting that ChatGPT's role went beyond simply providing answers, it prompted critical thinking about the mathematical concepts, leading to a deeper level of comprehension. This illustrates the importance of allowing students to engage with the material before consulting ChatGPT, reinforcing their understanding of the concepts.

The data analysis revealed that the use of clear and detailed prompts significantly enhanced students' comprehension of concepts when engaging with ChatGPT. During the focus group discussion where students discussed their experiences with the use of ChatGPT in Lesson 3 compared to Lesson 1 and 2, Student 1 emphasized the impact of specificity in the prompts,

stating, “By making the prompts more specific to make it simpler for us, it was incredibly useful in terms of fixing our understanding of the topics.” This highlights how precise and well-crafted prompts not only facilitated clarity but also actively contributed to correcting misunderstandings, ultimately enhancing the overall learning experience.

The evidence illustrates that employing clear and detailed prompts is crucial in optimizing student comprehension when utilizing ChatGPT, ultimately leading to a more effective learning process. Furthermore, ChatGPT enhances understanding by allowing students to ask follow-up questions, thereby deepening their comprehension. Its value is further amplified when used after GeoGebra, as it enables students to first form their own conjectures and then verify them against ChatGPT’s responses.

4.3.2 GeoGebra as an Instructional Tool in Lesson 3

In exploring the role of GeoGebra as an instructional tool for teaching sine trigonometric function concepts, the thematic analysis from Lesson 3 revealed three recurring themes that underscore its instructional value: GeoGebra Visualizes Concepts, GeoGebra Enhances Understanding through contextualization, and GeoGebra Promotes Exploration. Additionally, a new theme emerged from participants' critical feedback regarding the Repetitive GeoGebra Activities. The evidence supporting these themes is presented below.

4.3.2.1 Visualizing Concepts

The theme of "GeoGebra Visualizes Concepts" became apparent in all three lessons, with feedback from students and the teacher highlighting how GeoGebra helps make mathematical ideas visual and in return more accessible. In Lesson 1 and Lesson 2, GeoGebra was particularly valued for its ability to present a visual representation of abstract mathematical concepts. Throughout Lesson 3, students consistently again expressed a preference for using GeoGebra to provide clear, visual representations of the content.

In the focus group interview, students highlighted how GeoGebra helped them experience the material visually. When asked about their experience with Task 2 using GeoGebra by itself, Student 1 remarked, "it provided a visual medium to see it," and Student 2 added, "it gave a nice visual." These comments emphasize the role of GeoGebra in transforming abstract concepts into tangible, visual forms.

Similarly, when students were asked what their experiences were while doing Tasks 3 and 4, Student 3 noted that "GeoGebra helped to visualize it and it helped us to show how the graph moves up and down, "illustrating how GeoGebra allowed them to track transformations." When asked which task they preferred, Student 4 responded, "I really enjoyed task 4, just being able to visualize it," further reinforcing the positive impact of visual learning on student engagement.

The value of GeoGebra for new concept introduction was also echoed in the worksheet response analysis. Student Pair 5 suggested, "Use GeoGebra for new concepts to visualize the work," underlining its importance as a tool for conceptual clarity. Additionally, when asked whether using GeoGebra in the mathematics classroom is valuable and enhances learning, Student Pair 2 affirmed, "Yes, we think it can help us better visualize what we are learning and calculating." This sentiment was echoed in the teacher interview, where, when asked about her opinion on using ChatGPT and GeoGebra in a mathematics lesson, she expressed a preference for GeoGebra, stating, "it is visual and interactive."

Overall, these findings suggest that GeoGebra plays a critical role in enhancing students' learning experience by providing a visual context for abstract mathematical concepts, making it easier for them to grasp and apply what they are learning.

4.3.2.2 Enhancing understanding through contextualization

The theme "GeoGebra Enhancing Understanding through Contextualization" emerged clearly from the analysis of all three lessons. Feedback from both students and the teacher, along with worksheet responses, highlighted how GeoGebra supported a deeper comprehension of trigonometric concepts by providing contextualized learning experiences.

During the focus group interview, students were asked, "Which task out of everything that you did today did you find most effective? And why?" Student 1 responded, "I found Task 4 incredibly effective because GeoGebra paired with ChatGPT provided a visual understanding and explanation...which then provided greater understanding." This response underscores how GeoGebra facilitated contextualized learning, enabling students to visualize and clarify their understanding of the material.

When asked about their experience with Task 2 using GeoGebra only, Student 4 noted, "GeoGebra is quite useful because it does help you understand what's happening. And while it

is repetitive, it engraves it in your head.” This indicates that the repetitive visual engagement with GeoGebra helped solidify students’ comprehension of the concepts.

Further reinforcing this theme, the worksheet response analysis revealed that when asked for recommendations on using ChatGPT and GeoGebra in a lesson, Student Pair 5 commented, “Use GeoGebra for new concepts to better understand...the work.” This recommendation highlights the importance of GeoGebra in introducing new concepts through contextual visualization and interaction.

In the teacher interview, when asked, “Do you think that using ChatGPT with GeoGebra in a mathematics lesson can be useful for reaching learning outcomes? The teacher stated, “I prefer GeoGebra as it can be so valuable to use in an investigative way, leading to deeper understanding.” This perspective reinforces the idea that GeoGebra not only aids in visualization but also fosters an investigative approach to learning, allowing students to engage more deeply with mathematical concepts.

Overall, the evidence from both student and teacher feedback supports the theme that GeoGebra enhances understanding through contextualization, enabling students to visualize, interact with, and engage in their learning processes more effectively.

4.3.2.3 Fostering Self-exploration

The theme of "GeoGebra Promotes Exploration" emerged prominently from both Lesson 1 and Lesson 2, highlighting how the tool encourages active student engagement and enhances understanding of mathematical concepts through exploration and manipulation. However, in Lesson 3, this theme evolved to emphasize the promotion of self-exploration facilitated by GeoGebra.

The theme of "GeoGebra Promotes Self-Exploration" was particularly evident in the feedback gathered from students and the teacher regarding the use of GeoGebra in the classroom. When asked if they believed that using GeoGebra enhances learning in mathematics, Student Pair 2 stated in their worksheet response, “Yes, we think it can help us as it allows individual learning without pressuring teachers.” This response highlights how GeoGebra facilitates personalized learning experiences, empowering students to explore concepts at their own pace without feeling overwhelmed by direct teacher oversight.

The teacher also emphasized the value of GeoGebra, commenting on its visual and interactive nature: “I prefer GeoGebra as it is so visual and interactive...it can be so valuable to use in an investigative way, leading to deeper understanding.” This perspective underlines the potential of GeoGebra to encourage investigative learning, allowing students to engage actively with mathematical concepts through self-exploration.

During the focus group interview, students reflected on their experiences with the third and fourth activities, where they used both ChatGPT and GeoGebra simultaneously to investigate graph transformations. Student 1 remarked, “I actually found that activity very useful because you saw what happened in GeoGebra when you changed the values...” This indicates that visual exploration enhanced their understanding.

Student 2 added, “I thought it was really cool to see what was happening, and then before we used ChatGPT, we made our own conclusions....” This shows that students valued the opportunity to form their own hypotheses before validating their understanding with ChatGPT, promoting a deeper level of self-exploration.

Student 5 emphasized the importance of the initial self-exploration process, stating, “I think it was nice that we were asked to explore first before we could ask ChatGPT. The fact that we had to answer why we thought the graph was changing allowed us to compare our ideas with ChatGPT’s response afterward.” This approach of encouraging students to first articulate their reasoning fosters a sense of ownership over their learning, enhancing their exploratory experience.

Overall, the evidence from both student and teacher feedback strongly supports the theme that GeoGebra promotes self-exploration, enabling students to engage with mathematical concepts more deeply and independently.

4.3.2.4 Repetitive GeoGebra activities

The theme of "Repetitive GeoGebra Activities" emerged distinctly during Lesson 3, revealing students' mixed feelings about the use of repetitive tasks in the learning process. When asked about their experience with the second activity, which involved using GeoGebra to draw the sine graph, Student 1 remarked, “It was useful because it again provided a visual medium to see it, but I think because we had already drawn the sine graph in Task 1, it was quite repetitive.”

This response indicates that while the visual aspect was beneficial, the familiarity of the task diminished its effectiveness due to repetition.

Student 3 echoed this sentiment, stating, “It was nice to see it the first few times, but when we did it over and over again, we already knew how it would look.” This highlights that the initial engagement with the visual representation of the graph became less stimulating as the activity progressed. Student 2 added, “Yeah, it gave a nice visual, but it was repetitive, and you could always predict what the next step was because we had already done it with ChatGPT in Task 1.” This demonstrates a sense of predictability that can stifle exploration and engagement, as students felt they were simply going through the motions.

Despite the criticism of repetitiveness, some students acknowledged the potential benefits of such activities. Student 4 noted, “GeoGebra is quite useful because it does help you understand what's happening. And whilst it is repetitive, it does engrave it in your head.” This suggests that repetition can reinforce learning by solidifying concepts in students' minds, even if it initially feels tedious.

Student 5 reflected on the initial confusion, stating, “I also thought that it was a bit repetitive, but at the beginning of the task, we didn't really know what it was leading to. We didn't realize that we would be drawing the same graph as in the previous question. If we had known that beforehand, it would have been easier to understand.” This implies that providing context for repetitive activities could enhance their educational value and reduce potential frustration.

Overall, the theme of "Repetitive GeoGebra Activities" reflects the nuanced experiences of students as they navigate the balance between reinforcement and engagement. While the visual medium of GeoGebra is recognized for its educational value, the repetitiveness of tasks can lead to predictability and disengagement unless thoughtfully structured with clear objectives.

4.3.3 Reflection on the Design and Instructional Adjustments

The reflection on Lesson 3 uncovered several important insights into the lesson design. One of the major successes in all three lessons was the use of GeoGebra to reinforce mathematical concepts through exploration and self-discovery, which significantly enhanced students' understanding as seen in the evidence stated above.

The participants showed a clear preference for the combined use of ChatGPT and GeoGebra over using either tool in isolation. The students valued this combinational approach and

highlighted in the focus group interview that the two technologies complemented each other. For example, Student 6 commented that, “I do think that using the combination of the two could be effective in classrooms.” Student 3 echoed this sentiment by saying, "I think ChatGPT and GeoGebra go very well hand in hand," while Student 4 commented that the approach was "incredibly helpful." The teacher also supported this approach, stating, "Using both ChatGPT and GeoGebra was more effective."

Not only did participants prefer using both tools together, but they also appreciated the specific sequencing of their use. The simultaneous integration of GeoGebra and ChatGPT in Tasks 3 and 4 of Lesson 3 stood out as particularly successful. Starting with GeoGebra to build a foundational understanding and then using ChatGPT to clarify and deepen that knowledge. The worksheet data further supports these insights, with four out of eight student pairs agreeing that the combination of both tools was most beneficial, while the other pairs found ChatGPT alone more helpful. Student 1 observed that “GeoGebra's visual tools vividly illustrated graph transformations, while ChatGPT complemented this with clear and accessible explanations”. Student 4 reinforced this by stating, “Using it together at the same time was incredibly helpful”. This sequencing allowed for reflective learning, as Student 6 remarked, “We had to answer why we thought the graph was changing... then afterwards we saw ChatGPT's answer.” Student pair 3 found the comparison between their own interpretations and ChatGPT's feedback especially valuable, stating it “was better than the other lessons, as it allowed for reflective learning.”

The teacher also emphasized that this approach encouraged critical thinking, noting, “Students had to evaluate their findings with ChatGPT's explanations, which clarified confusion and engaged them in critical thinking.” Student 6 summed it up by saying, “...you could use these tools to help us with critical thinking and to help us first see if we can understand the concept by ourselves before we go to a teacher.” This feedback highlights not only the success of using both ChatGPT and GeoGebra together but also the importance of sequencing.

In contrast to Lessons 1 and 2, where the prompts were either too vague or overly broad, a key improvement in the design of Lesson 3 was the use of more detailed and precise prompts. These clearer instructions ensured that students received more relevant and helpful responses, significantly enhancing their understanding. Notably, no responses from ChatGPT were reported as incorrect or confusing in this lesson.

Another key improvement in the lesson design of Lesson 3 compared to Lesson 2 was that it felt less lengthy and tedious for students. The electronic worksheet allowed them to type or summarize answers in their own words, eliminating the need to write out all of ChatGPT's prompts and responses. Instead, they could easily copy and paste, creating a smoother and more efficient flow throughout the lesson.

A notable improvement in Lesson 3 compared to Lessons 1 and 2 was the introduction of new content, which significantly boosted student interest and engagement. As Student 6 remarked, "The fact that this is new work was what made the difference in today's lesson." Student 5 echoed this sentiment, stating, "The difference between already knowing the work and learning something new... you also mentally engage more." Other students shared similar views, with Student 4 noting that, unlike previous lessons where they already knew the material, "this time it actually provided great understanding." Student 3 added, "We were learning new work, and we were more engaged in activities... whereas in the previous ones, it just felt like an extra task." This fresh material clearly contributed to a more dynamic and stimulating learning experience.

One aspect of the lesson design that proved less successful was the repetitive use of GeoGebra, which risked diminishing student interest and engagement. The lesson structure should aim to avoid overuse of similar activities to prevent the process from becoming too predictable and tedious. When students know exactly what to expect, the learning experience can feel prolonged and less stimulating, ultimately impacting their overall engagement.

4.4 Conclusion

The analysis of Lessons 1, 2, and 3 provided valuable insights into the impact of integrating ChatGPT and GeoGebra on student learning and engagement in teaching trigonometric sine function concepts. The study found that ChatGPT and GeoGebra can enhance the teaching and learning of sine trigonometric functions when used thoughtfully. Early lessons with vague prompts caused confusion, highlighting the need for clear, concise, and tailored prompts. Critical analysis of ChatGPT's responses helped students develop analytical skills and encouraged peer collaboration. Lesson 3 showed that combining ChatGPT's step-by-step guidance and explanations with GeoGebra activities allowed students to explore, validate, and deepen their understanding of concepts. GeoGebra improved students' engagement and comprehension by offering real-time visualizations and connecting sine functions to practical applications like wave modeling. Independent exploration with GeoGebra fostered self-directed learning, while investigative activities encouraged critical thinking and collaboration. A carefully planned lesson design was crucial for success. Engagement improved when new content was

introduced, while repetitive tasks led to frustration. Using electronic worksheets made learning more efficient by reducing the burden of traditional assignments.

Overall, combining ChatGPT and GeoGebra within well-planned lessons created a productive, interactive learning environment that enhanced student understanding and engagement. The connections and differences between these findings and other studies are further explored in Chapter 5.

CHAPTER 5: DISCUSSION

5.0 Overview

This study explored how ChatGPT and GeoGebra can be employed as instructional tools to enhance the teaching and learning of sine trigonometric functions in a Grade 10 mathematics class. The analysis of Lessons 1, 2, and 3 underscored both the advantages of integrating these technologies into lesson design and the challenges faced during implementation. This section addresses the research questions, exploring how ChatGPT and GeoGebra were employed both individually and together in this study, as well as in other research. The similarities and differences in the findings are discussed below.

5.1 How to employ ChatGPT in Teaching Sine Trigonometric Functions

The findings of the study showed that the design of prompts plays a critical role in determining how effectively students engage with ChatGPT's content. Vague or open-ended prompts, as used in the current study, often led to overly complex responses that overwhelmed students, particularly those at earlier stages of learning. This aligns with Mondal et al. (2024), who emphasize that ambiguous or overly broad prompts tend to produce verbose outputs, making it necessary to specify response length or use key terms like "concise" and "summarized" to avoid wordy responses.

Leung (2024), also highlights the value of different types of prompts, proposing that closed-question prompts (prompts that often have a limited range of responses) are effective for grasping basic concepts, while open-ended prompts (prompts that encourage the respondent to explore and elaborate on ideas), encourage higher-order thinking. Open-ended prompts were integrated into all the lessons in this study. These prompts encouraged ChatGPT to provide detailed explanations, aiming to enhance students' understanding of trigonometric function concepts. Additionally, the study employed prompts that connected theory with practice, such as in Lesson 2, Task 4, where ChatGPT was prompted to generate real-world-related questions. Students then analyzed both the questions and ChatGPT's responses. This approach aligns with Leung (2024), who emphasizes that well-crafted prompts should foster critical thinking and support the application of knowledge in real-world contexts.

However, the present study also noted limitations, particularly when using ChatGPT's responses without adequate scaffolding. Students without prior knowledge struggled to generate effective prompts or interpret ChatGPT content, as also noted by Shoufan (2023), who found that learners without foundational knowledge have difficulty using ChatGPT effectively. This underscores the importance of educator involvement in creating prompts and guiding students through ChatGPT's responses.

Moreover, the use of ChatGPT for collaborative learning in this study, resonates with Heimans et al. (2023), who emphasize that ChatGPT can support group discussions and problem-solving when used with careful prompt design. This aligns with the approach taken in this study, where students were asked to analyze ChatGPT's questions and answers, promoting deeper engagement and critical thinking. Additionally, the study found that combining ChatGPT with tools like GeoGebra allows students to form their own conjectures before verifying them with ChatGPT's responses, enhancing their learning experience. This reflects findings from Remoto (2023), who noted that AI models like ChatGPT can effectively support problem-solving and the understanding of mathematical concepts when integrated into interactive learning environments.

This study's findings closely align with existing research on the importance of prompt engineering in educational settings. While prompt design significantly influences the effectiveness of ChatGPT's content, both this study and prior research highlight the necessity of educator involvement and scaffolded learning to maximize the potential of ChatGPT in fostering critical thinking, collaboration, and deeper understanding.

5.2 How to employ GeoGebra in Teaching Sine Trigonometric Functions

The findings of this study showed strong support for GeoGebra's ability to make mathematical concepts visual. GeoGebra's dynamic visualization capabilities, allowed students to visualize geometric shapes and perform mathematical calculations simultaneously. This aligns with a study done by Sur (2020) which highlighted how visualization capabilities were crucial in helping students grasp complex topics such as definite integrals. GeoGebra's ability to represent both 2D and 3D geometric shapes has been shown to improve students' visual representation skills, as evidenced by Azizah et al. (2021), who reported significant post-test score improvements following the use of GeoGebra. Dockendorff and Solar (2018), similarly,

emphasized the importance of visualization as a core mathematical competence, noting that incorporating GeoGebra into classrooms has led to shifts in teaching practices and improved mathematical processes.

GeoGebra's diverse features not only support the visualization of mathematical concepts but also make learning more exploratory by enabling students to observe connections between analytical and visual representations, as was identified in this study. Students were able to manipulate variables in real-time and observe the effects on sine graphs and trigonometric relationships, promoting independent learning. This finding is in line with Garber et al. (2010), who highlighted the value of GeoGebra's hands-on manipulation of graphs, formulas, and shapes in fostering a deeper exploration of algebraic and geometric concepts. Additionally, Fahlberg-Stojanovska and Stojanovski (2009) noted that GeoGebra's freedom for exploration is particularly valuable in addressing declining mathematical and logical thinking skills among young people, a concern that this study also addressed by promoting self-guided learning through GeoGebra's interactive features.

This study also found that students' motivation to learn mathematics increased when using GeoGebra, echoing Yerizon et al. (2021), who highlighted GeoGebra's role in enhancing student engagement with mathematical material through interactive visualizations.

The present study also highlighted how GeoGebra facilitated students' understanding of trigonometric concepts, such as finding angles, calculating sine ratios, and generalizing trigonometric graph behaviours. This was consistent with other studies that found GeoGebra's dynamic features allowed students to intuitively grasp derivatives by visualizing transformations of secant lines into tangent lines, as noted by Yerizon et al. (2021). Furthermore, studies by Ridha et al. (2020) and Hayati and Ulya (2022) demonstrated that GeoGebra-assisted learning approaches significantly improved students' understanding of calculus, geometry, and other mathematical concepts when compared to conventional methods. In a study done by Velichova (2011) it was advocated that simply drawing mathematical objects and figures is insufficient for building a deep understanding of basic mathematical concepts. Instead, dynamic and creative activities are essential for developing both conceptual understanding and technological content knowledge. This aligns with the broader idea of this study that mathematical understanding involves the ability to navigate between various representations of concepts when illustrated dynamically. GeoGebra serves as a didactic tool, facilitating these processes in a user-friendly and intuitive way.

This study's findings align with a broader body of research that underscores GeoGebra's role in enhancing mathematical understanding through dynamic visualization, real-time manipulation, and self-exploration. GeoGebra consistently outperforms traditional teaching methods by making abstract concepts more tangible and fostering greater student engagement and comprehension.

5.3 Other lesson design factors using ChatGPT and GeoGebra in combination

This study identified three key factors that affected student engagement during the lesson design, while using ChatGPT and GeoGebra. The first factor is related to prior knowledge. Students who engaged with the technologies on topics they were already familiar with showed lower levels of engagement. This finding aligns with studies on exploratory and investigative teaching approaches, which highlight that prior knowledge plays a significant role in student engagement. For instance, students with low prior knowledge benefit from feedback during exploratory activities (Fyfe et al., 2012), while those with higher knowledge tend to perform better without feedback. This mirrors the current study's observation that new, unfamiliar material significantly improves engagement, as seen in Lesson 3, reinforcing the importance of timing exploration before instruction to maximize engagement and learning.

The second factor contributing to disengagement was the repetitive nature of certain tasks, particularly when students perceived them as predictable or redundant, as observed in the findings from Lesson 3. This contrasts with studies that emphasize the benefits of repetition in learning, such as improved conceptual understanding and retention (Lomibao & Ombay, 2017). However, the design of repetitive tasks is crucial, studies indicate that repetitive tasks without provided solutions encourage students to develop their own solutions, fostering deeper learning (Norqvist et al., 2023). While repetition can aid intellectual development (Millousheva-Boykina, 2021), this study found that when tasks were too repetitive and predictable, they became disengaging, which has also been observed in other research on exploratory mathematics (Wilkie et al., 2023).

The third factor was the lack of engagement when students were required to manually record prompts and responses from ChatGPT, which they found tedious. This study found that using electronic worksheets significantly enhanced engagement, as it made tasks less time-consuming and more interactive. This finding is supported by other research, which shows that

electronic worksheets enhance cognitive learning outcomes and engagement (Aida & Wulandari, 2024), provide greater flexibility (Macmillan, 2004), and can make learning more game-like, thus increasing motivation (Sumanik et al., 2023). Additionally, electronic worksheets foster student autonomy and can facilitate problem-based learning approaches, making them a valuable tool in modern educational settings (Faizah, 2021).

This study also explored the sequential use of GeoGebra and ChatGPT, finding that alternating between the two technologies, using GeoGebra first for exploration and ChatGPT for explanations, elicited the best feedback. While no other studies have specifically examined the combined use of these two technologies, research supports the notion that self-exploration before instruction improves learning outcomes in both physics (Bego et al., 2022) and mathematics (Decaro & Rittle-Johnson, 2012). Exploration before instruction allows students to experiment with different strategies, leading to better conceptual understanding and preparation for formal learning (Fyfe & Rittle-Johnson, 2012). This aligns with the current study's findings, suggesting that alternating technologies and providing opportunities for self-exploration can enhance engagement and learning outcomes.

5.4 Conclusion

In summary, this chapter discussed how ChatGPT and GeoGebra were integrated into the teaching of sine trigonometric functions and reflected on lesson design, prompt structuring, and technology sequencing on student engagement and learning. The findings were mostly supported by several studies; however, some studies countered the observed student disengagement with repetitive tasks, suggesting that when designed properly, repetition can enhance retention and understanding. The findings of this study showed that an effective strategy for supporting the teaching and learning of trigonometric function concepts is educator involvement in crafting detailed prompts. These prompts should include instructions for ChatGPT to provide concise responses tailored to the recipient's age or knowledge level. Additionally, the study found that guiding students through ChatGPT's responses within the lesson design is essential for achieving the desired learning outcomes. The sequential use of GeoGebra for exploration followed by ChatGPT for explanation emerged as another strategy for maximizing learning outcomes, further aligning with research advocating for exploration before formal instruction. The next chapter will conclude the study by summarising key insights, addressing the research implications.

CHAPTER 6: CONCLUSION

This chapter concludes the study by summarizing the key insights and findings, addressing each sub-research question, and exploring potential implications for future research.

Key insights on using ChatGPT to teach sine trigonometric function concepts

The first sub-research question focused on how ChatGPT can be employed as an instructional tool for teaching sine trigonometric function concepts. The findings highlighted several key insights.

Firstly, the broad and vague prompts can lead to confusion. In Lesson 1, where ChatGPT provided definitions and explanations, and in Lesson 2, where it generated questions and answers for students to analyze, broad and vague prompts led to overly complex, lengthy, or irrelevant responses. This frustrated students by failing to build on their prior knowledge or provide meaningful value. The findings showed that these challenges can be addressed by designing prompts that are specific to students' knowledge levels or age groups and by explicitly requesting concise or summarized responses. When using ChatGPT to generate questions and answers for analysis, prompts should also specify the number of examples to prevent cognitive overload. Implementing these strategies can improve the clarity, relevance, and conciseness of ChatGPT's responses, enhancing its use as a teaching tool.

Secondly, it is recommended to incorporate predefined prompts into the lesson design when using ChatGPT, as the findings showed that students may struggle to generate effective prompts without prior knowledge of the subject. This approach can better support their learning process.

Thirdly, it is important to exercise caution when using ChatGPT to generate questions and answers, as some outputs may contain inaccuracies or irrelevant content. The study indicated that it is crucial for students to critically analyze these responses to identify any misconceptions or errors. This analytical process was found to enhance students' critical thinking skills and promote meaningful discussion and collaboration among peers.

Fourthly, the lesson design should explicitly encourage students to use ChatGPT at any point during the lesson when they feel the need for clarification or wish to explore additional questions. Allowing students to ask follow-up questions can significantly enhance their

understanding, as it enables them to deepen their comprehension of the material at any time during the lesson.

Lastly, the sequence in which the two technologies are integrated into the lesson is important. The findings revealed that many participants preferred the way ChatGPT was utilized in Lesson 3 compared to its implementation in other lessons. Lesson 3 demonstrated that ChatGPT's value was significantly enhanced when it provided students with step-by-step instructions, followed by a hands-on investigation of the same concept using GeoGebra. The students also appreciated the opportunity to engage with ChatGPT again later in the lesson, after each GeoGebra activity. This approach allowed students to first formulate their own conjectures based on their observations and then validate them against ChatGPT's responses. By interacting with ChatGPT after drawing initial conclusions and making conjectures with GeoGebra, students can gain deeper insights into the reasoning behind their observations, which can enrich their overall learning experience.

Key insights on using GeoGebra to teach sine trigonometric function concepts

The second research question focused on how GeoGebra should be employed in lesson design for teaching sine trigonometric function concepts. The findings highlighted several key insights regarding the preferred methods of using GeoGebra.

Firstly, using GeoGebra to create dynamic visual representations of sine functions, allows students to manipulate parameters and observe the resulting changes in real-time. This approach significantly enhanced students' visualization skills and mathematical thinking, leading to improved engagement and a deeper understanding of complex concepts.

Secondly, using GeoGebra to present mathematical concepts within real-world contexts, enabled students to relate sine function concepts to practical applications, such as modelling waves. This contextualization enhanced students' comprehension and demonstrated the relevance of sine functions in various fields.

Thirdly, independent exploration promotes self-directed learning. The lesson design incorporated tasks that encouraged students to explore sine trigonometric function concepts independently using GeoGebra. By allowing students to experiment with different parameters and investigate their effects, the tool promoted self-directed learning and fostered a sense of ownership over their educational journey.

Lastly, GeoGebra can facilitate exploratory and investigative learning. In this study, students engaged in hands-on activities where they formulated hypotheses about sine trigonometric function concepts and used ChatGPT to test their conjectures. This interactive approach encouraged critical thinking and collaboration among peers, as students discussed their findings and insights.

Key lesson design considerations for integrating ChatGPT and GeoGebra to teach sine trigonometric function concepts

The findings of the study also highlighted several important lesson design factors for employing ChatGPT and GeoGebra in combination to teach sine trigonometric function concepts.

The first lesson design factor that needs to be addressed is student engagement due to prior knowledge. This research study indicated that student engagement significantly improved in Lesson 3, where new content was introduced. This change contrasted sharply with earlier lessons, where students' familiarity with the concepts led to boredom and frustration. To maintain engagement, lesson designs should include a variety of new and challenging material that encourages exploration and interest. This can be achieved by integrating concepts that build on prior knowledge while introducing novel elements to stimulate curiosity.

The second lesson design factor to take note of is to avoid repetitive tasks. The study revealed that tedious writing tasks in Lesson 1 and repetitive GeoGebra activities in Lesson 3 contributed to student disengagement. Students expressed that the predictability of similar tasks diminished their excitement for discovery. Therefore, lesson designs should aim to vary tasks and challenges to maintain a sense of novelty.

The third lesson design factor that was highlighted in the findings of this study was to provide electronic worksheets. Incorporating electronic worksheets into the lesson design can significantly reduce the tediousness associated with traditional paper-based tasks. These worksheets can streamline the process of capturing responses, allowing students to focus more on their learning rather than the mechanics of writing. By making it easier to complete assignments, electronic worksheets can enhance student engagement and promote a more efficient learning experience.

The integration of ChatGPT and GeoGebra in teaching sine trigonometric function concepts has demonstrated promising results in enhancing students' understanding and engagement. This

study underscores the significance of effective prompt engineering, the benefits of visualization through dynamic geometry software like GeoGebra, and the necessity for meticulously planned lesson designs to achieve optimal learning outcomes while fostering a socio-constructivist learning environment throughout the process.

CHAPTER 7: RECOMMENDATIONS FOR FUTURE RESEARCH AND LIMITATIONS

This study, which explored the use of ChatGPT and GeoGebra in teaching Grade 10 girls the concepts of the sine trigonometric function, presents several recommendations for future research and identifies key limitations. This chapter outlines these recommendations alongside the study's limitations, offering insights to inform and enhance the design of future studies.

7.1 Recommendation for future studies

The findings suggest that future studies should explore the applicability of these tools across a broader range of mathematical topics and educational contexts, as the current focus was limited to sine trigonometric function concepts.

Additionally, while the study demonstrated improved comprehension among the girls, as evidenced by their ability to solve trigonometry questions and sketch fluid curves, it also raises concerns about resource dependency. Implementing such technologies effectively in under-resourced schools presents challenges, making it essential to consider researching how these tools can be adapted or supplemented in less affluent educational settings.

Future studies could also address the limitation of this study's small sample size, as it involved only one class of 20 girls. Repeating the study with a larger, more diverse sample would provide broader insights and enhance the generalizability of the findings.

7.2 Limitations

This study's focus on a small, well-resourced sample of high-achieving Grade 10 girls and a singular mathematical topic—the sine trigonometric function—raises concerns regarding the generalizability of its findings. Such a narrow scope may not capture the diverse experiences of students in other educational contexts, particularly those in under-resourced environments or without access to similar technological tools. Additionally, the exclusive focus on girls in a top-set class may not reflect outcomes for boys, students in co-educational settings, or those with varying mathematical abilities, especially those who struggle with the subject. Consequently, the results may not apply broadly across the diverse South African school context, where many institutions operate with limited resources and technological infrastructure.

Future research should consider a wider array of educational contexts, encompassing schools with varying resource levels, different demographic profiles, and a broader range of mathematical topics. Such an approach would help determine whether the advantages and disadvantages of the technologies examined in this study are transferable to diverse learning environments and student populations.

Addressing these recommendations will support the development of technology-enhanced learning strategies in mathematics education that are relevant and effective across different demographics and settings, promoting more inclusive and equitable educational practices.

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Appendix A

LESSON 1 Worksheet: The Sine Ratio

Lesson outcome: Defining the trigonometric ratio of $\sin \theta$

- You can work in groups of two for this activity.

Task 1:

Using your knowledge on trigonometrical ratios from term one.

Tick yes or no to the questions below:

1. The sine of an angle represents an acute angle measurement in a right-angled triangle.

YES	NO
-----	----

2. The sine of an angle represents a ratio of 2 specific sides in any triangle.

YES	NO
-----	----

3. The sine of an angle represents a ratio of 2 specific sides in any right-angled triangle.

YES	NO
-----	----

4. The sine of an angle represents the ratio of the adjacent side over the hypotenuse side in any right-angled triangle.

YES	NO
-----	----

5. The sine of an angle represents the ratio of the opposite side over the hypotenuse side in any right-angled triangle.

YES	NO
-----	----

Task 2:

Open ChatGPT by clicking on the link: <https://chat.openai.com/>

Ask ChatGPT what the sine of an angle means? And write down ChatGPT's answer below.

1. Do your answers in Task 1 correspond to ChatGPT's answer above?

YES	NO
-----	----

2. Do you understand ChatGPT's answer?

YES	NO
-----	----

3. Is ChatGPT's answer clear enough?

YES	NO
-----	----

4. Was there any part to ChatGPT's answer that you did not understand?

YES	NO
-----	----

5. Was there any part of ChatGPT's response that lacked correctness, detail or that you struggled to understand? please elaborate if needed.

6. Would you say your understanding of the sine ratio is more comprehensive after this activity?

YES	NO
-----	----

Task 3:

Click on the link below:

<https://www.geogebra.org/m/dPqv8ACE#material/kcuema3n>

When you open the GeoGebra file you will see a triangle with angles size of 55° .

Using the given triangle and **without moving point B**, give the answers to the following using the given sine side ratios $\frac{\textit{opposite}}{\textit{hypotenuse}}$. Round your answers to 2 decimal places.

$$\sin 55^\circ =$$

Now move point B to any position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin 55^\circ =$$

Now move point B to another position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin 55^\circ =$$

Are you getting more or less the same answer for all of the above cases?

YES	NO
-----	----

Now change the angle size to any angle of your choice by dragging the black dot to the right. Give the values of \sin (of your chosen angle). Round your answers to 2 decimal places.

$$\sin \underline{\hspace{2cm}} =$$

Don't change your angle size but move point B to another position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin \underline{\hspace{2cm}} =$$

Don't change your angle size but once again move point B to another position of your choice, give the answers of the following again. Round your answers to 2 decimal places.

$$\sin \underline{\hspace{2cm}} =$$

Are you getting more or less the same answer for all the above cases?

YES NO

Task 4:

Now open the following link:

<https://www.geogebra.org/m/dPqv8ACE#material/Z3wcyhcB>

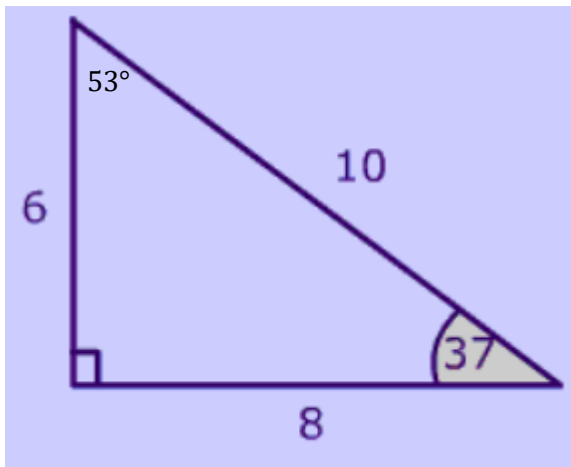
Please note that the ratio for $\frac{\textit{opposite}}{\textit{hypotenuse}}$ is given in the left-hand corner of the GeoGebra activity and as you drag the white dots around the plane.

Are you getting the same ratio no matter where you drag the white dots?

YES NO

Task 5:

To test your understanding answer the questions below, round your answer to 2 decimal places:



$\sin 37^\circ =$

$\sin 53^\circ =$

CONCLUSION

- 1) After doing Task 3 and Task 4, do you have a better understanding of the sine ratio?

YES	NO
-----	----

- 2) If you only did the GeoGebra activity (Task 3 and Task 4) without having done the activity on ChatGPT (Task 2), do you think you would have understood clearly what the sine ratio means?

YES	NO
-----	----

- 3) Do you think that the combination of the use of ChatGPT and GeoGebra assisted in a deeper understanding of the sine function?

YES	NO
-----	----

- 4) Do you feel that you were engaged in critical thinking or problem solving during any of the tasks above.

YES	NO
-----	----

- 5) Which task did you find the most engaging or fun?

- 6) Which task did you find the most understandable?

- 7) Which task did you find the most challenging?

- 8) Do you have any recommendations as to how ChatGPT can be used differently in the next lesson to improve the lesson engagement and/or critical thinking and/or problem solving?

Appendix B

LESSON 2 Worksheet: Sine ratio in different Quadrants

- You can work in groups of two for this activity.
- Answer all the questions by ticking the relevant box
- Follow all instructions carefully.
- Time duration: 1 hour and 30 minutes

Task 1:

Do you know in what quadrants of the Cartesian plane an acute-angled triangle's sine ratio will give a positive answer? (with one of its vertices on the origin)

YES	NO
-----	----

Do you understand why the sine ratio of such a triangle might be positive or negative in different quadrants?

YES	NO
-----	----

Task 2:

Open the following link:

<https://www.geogebra.org/m/k8SWu6Dv>

Click on the *sinθ* box and the **Show values** box

Without moving any points, what does the given ratio $\frac{0.588}{1}$ represent? Tick the right box

$\frac{\textit{opposite}}{\textit{adjacent}}$	$\frac{\textit{opposite}}{\textit{hypotenuse}}$	$\frac{\textit{adjacent}}{\textit{hypotenuse}}$

Give the ratio value of $\sin 36^\circ$ given in GeoGebra to two decimal place:

Now use your calculator and type in $\sin 36^\circ =$

Is your calculator answer the same as the given ratio's answer of $\frac{0.588}{1}$ in GeoGebra?

YES	NO
-----	----

Use your calculator and calculate $\sin^{-1}\left(\frac{0.588}{1}\right)$

(This is the inverse of the sine ratio which will give you the acute angle measurement, to do this press the shift button on the calculator then $\sin\frac{0.588}{1} =$)

ANSWER: _____

Is this answer and the angle measurement given on GeoGebra the same?

YES	NO
-----	----

a) Now move the point P to any other positions between 0° and 90° in the 1st quadrant.

As you move point P between 0° and 90° in the first quadrant, are your different sine ratio answers positive or negative in the first quadrant?

positive	Negative

The angle measurement from the 0° to somewhere between 0° and 90° in this quadrant is:

Acute	Obtuse	Reflex between 180° and 270°	Reflex between 270° and 360°

b) Now move the point P to any other positions between 90° and 180° in the 2nd quadrant.

Using the appropriate sides of the new triangles formed, are your different sine ratio answers positive or negative in the 2nd quadrant?

Positive	Negative

The angle measurement from the 0° point to somewhere between 90° and 180° in this quadrant is:

Acute	Obtuse	Reflex between 180° and 270°	Reflex between 270° and 360°

c) Now move the point P to any other positions between 180° and 270° in the 3rd quadrant.

Using the appropriate sides of the new triangles formed, are your different sine ratio answers positive or negative in the 3rd quadrant?

positive	Negative

The angle measurement from the 0° point to somewhere between 180° and 270° in this quadrant is:

Acute	Obtuse	Reflex between 180° and 270°	Reflex between 270° and 360°

d) Now move the point P between 270° and 360° in the 4th quadrant.

Using the appropriate sides of the new triangles formed, are your different sine ratio's answers positive or negative in the 4th quadrant?

positive	Negative

The angle measurement from the 0° point to somewhere between 270° and 360° in this quadrant is:

Acute	Obtuse	Reflex between 180° and 270°	Reflex between 270° and 360°

e) After doing this activity in GeoGebra do you have a better understanding of why the sine ratio is positive or negative in certain quadrants?

YES	NO
-----	----

Explain your understanding in words:

Task 3:

Use the following prompt to ask ChatGPT:

"Explain why the sine ratios of an angle in different quadrants of the Cartesian plane have different signs. Consider the geometric interpretation of the sine function in relation to the lengths of the sides of a right triangle, as well as how the signs of these lengths vary across quadrants. Use examples and diagrams, if necessary, to illustrate your explanation.

ChatGPT's ANSWER:

After reading this answer from ChatGPT answer the following questions:

In which of the following quadrants will the sine ratio will be positive?

Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4

In which of the quadrants will the sine ratio will be negative?

Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4

Reading this explanation from ChatGPT do you now, compared to your understanding after doing the GeoGebra activity, understand why the sine ratio might be positive or negative in different quadrants better?

YES	NO
-----	----

Does the answer from ChatGPT explain clearly why the sine ratio might be positive or negative in different quadrants?

YES	NO
-----	----

Which task would you prefer using to explain clearly why the sine ratio might be positive or negative in different quadrants?

Task 2	Task 3
--------	--------

4. Taking all of the above questions and answers given by ChatGPT into account, design a better real-life application question to pose to the other groups to test their understanding of the sine ratio, angle measurements and/or side length's of any right-angled triangle in different quadrants in the Cartesian Plane?

Did you find Task 4 interesting and engaging?

YES	NO
-----	----

Did Task 4 teach you anything new?

YES	NO
-----	----

Explain:

Did Task 4 promote group discussions?

YES	NO
-----	----

Did Task 4 promote critical thinking?

YES	NO
-----	----

Conclusion

Would you rather use the ChatGPT explanation, GeoGebra's interactive explanation or a combination of both explanations when finding the signs of the sine ratio in different quadrants?

ChatGPT	GeoGebra	Combination of both

Explain:

Which activity did you find the most challenging?

ChatGPT	GeoGebra	Combination of both

Which activity did you find you were engaged in critical thinking the most?

ChatGPT	GeoGebra	Combination of both

Which activity did you learn the most from?

ChatGPT	GeoGebra	Combination of both

Which activity gave you the best understanding the concept of different signs of a sine ratio in different quadrants?

ChatGPT	GeoGebra	Combination of both

Do you have any recommendations how we can use ChatGPT and/or GeoGebra differently in our next lesson to maximise learning and critical thinking?

Appendix C

LESSON 3 Worksheet: Trigonometric Sine Function

- You can work in groups of two for this activity.

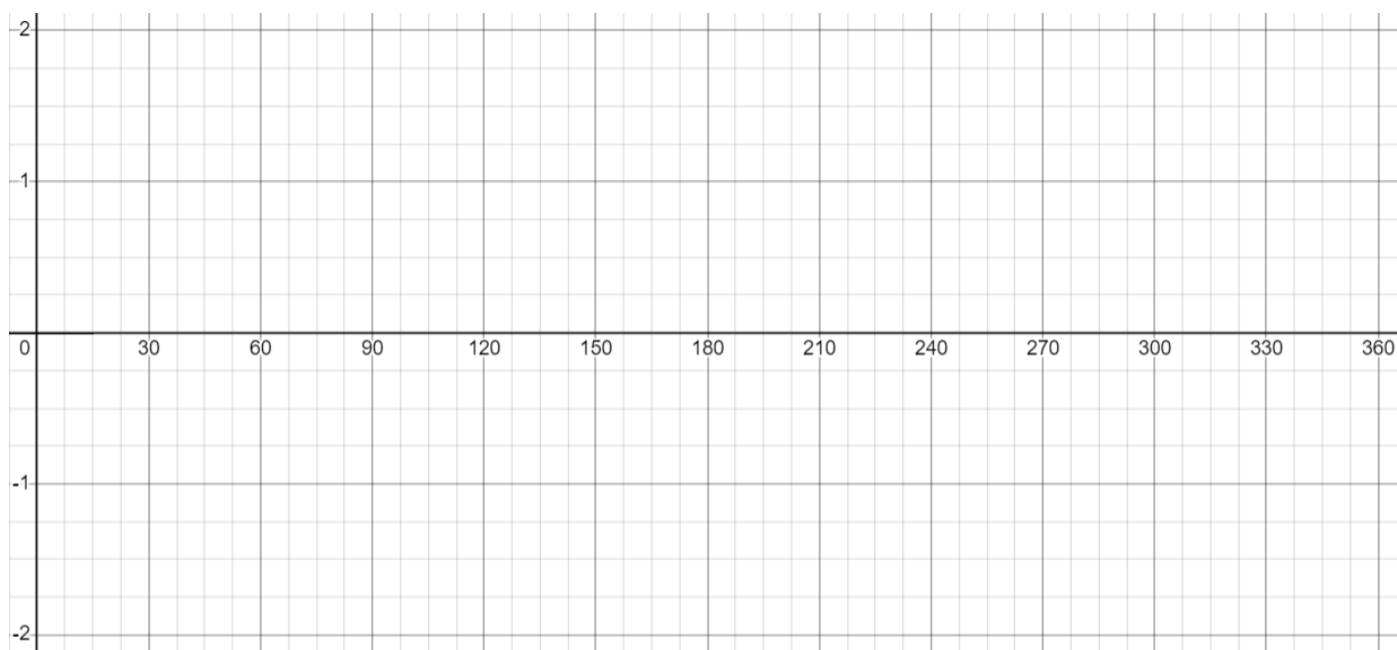
Task 1:

Copy and paste the following question into ChatGPT:

“What are the steps to follow when drawing the function $y = \sin x$ from zero to 360 degrees by using a calculator and giving a point on the graph every 30 degrees. Summarise the steps and use very simple language that a student of 13 years old will understand. Give the angles in degrees.”.

Copy and paste ChatGPT’s answer below:

Now using the steps, can you draw the $y = \sin x$ function below.



Looking at the drawn graph above, answer the following questions:

What is the maximum value of the graph (how high does the graph go)? _____

What is the minimum value of the graph (how low does the graph go)? _____

Give the coordinates of the turning points? _____

Give the coordinates of the x-intercepts? _____

Task 2:

Open the link below:

<https://www.geogebra.org/m/S2gMrkbD>

The radius (black line) is 1 unit long, and the angle at the moment is 30°

What is the vertical length (red line) of the triangle when the angle is 30° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right, to the right until the angle is close to 60° .

The radius (black line) is 1 unit long, and the angle at the moment is 60°

What is the vertical length (red line) of the triangle when the angle is 60° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 90° .

The radius (black line) is 1 unit long, and the angle at the moment is 90°

What is the vertical length (red line) of the triangle when the angle is 90° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 150° .

The radius (black line) is 1 unit long, and the angle at the moment is 150°

What is the vertical length (red line) of the triangle when the angle is 150° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 180° .

The radius (black line) is 1 unit long, and the angle at the moment is 180°

What is the vertical length (red line) of the triangle when the angle is 180° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 200° .

The radius (black line) is 1 unit long, and the angle at the moment is 200°

What is the vertical length (red line) of the triangle when the angle is 200° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 270° .

The radius (black line) is 1 unit long, and the angle at the moment is 270°

What is the vertical length (red line) of the triangle when the angle is 270° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 290° .

The radius (black line) is 1 unit long, and the angle at the moment is 290°

What is the vertical length (red line) of the triangle when the angle is 290° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Slowly drag the green sliding button at the bottom right to the right, until the angle is close to 360° .

The radius (black line) is 1 unit long, and the angle at the moment is 360°

What is the vertical length (red line) of the triangle when the angle is 360° ? _____

At this point, take the opposite side length's y value (red line), and the radius length as the hypotenuse length of the triangle, and calculate the sine ratio of the angle at this point? Write your answer as a fraction and then as a decimal answer:

How does this answer link to where the graph is on the axes drawn on the right at this point?

Looking at the drawn graph in GeoGebra on the right, answer the following questions:

What is the maximum value of the graph (how high does the graph go)? _____

What is the minimum value of the graph (how low does the graph go)? _____

Give the coordinates of the turning points? _____

Give the coordinates of the x-intercepts? _____

Task 3:

Open the following link:

<https://www.geogebra.org/m/QhZdJGKy>

Notice that the graph given now is $y = a \cdot \sin x$.

Does this graph look like the graph drawn in the previous GeoGebra activity and to the graph you drew on hard copy provided if $\pi = 180^\circ$

Answer YES or NO

Drag a to the right such that $a = 2$ and then to the left such that $a = -3$

What do you notice happened to the graph:

Copy and paste the following question into ChatGPT:

“Explain what happens to the sin graph if you multiply the function with a certain constant value. Use a simple explanation that students of 13 years old will understand if this is the first time they are investigating this specific transformation”

Copy and Paste ChatGPT’s answer below:

Was your answer above and ChatGPT’s explanation the same? If not, how did it differ?

After reading ChatGPT’s answer do you understand the transformation better?

Answer Yes and No, Give a reason for you answer

Task 4:

Use the same link as task 3 but now click on the words: **NEXT TRANSFORMATION** until you get to the variable d.

Notice that the graph given now is **$y = 1.\sin x + d$**

Drag d to the left such that a = 2.

What do you notice happened to the graph:

Copy and paste the following question into ChatGPT:

“Explain what happens to the sin graph if you add or subtract the function with a certain constant value”

Was your answer above and ChatGPT’s explanation the same? If not, how did it differ?

After reading ChatGPT’s answer do you understand the transformation better?

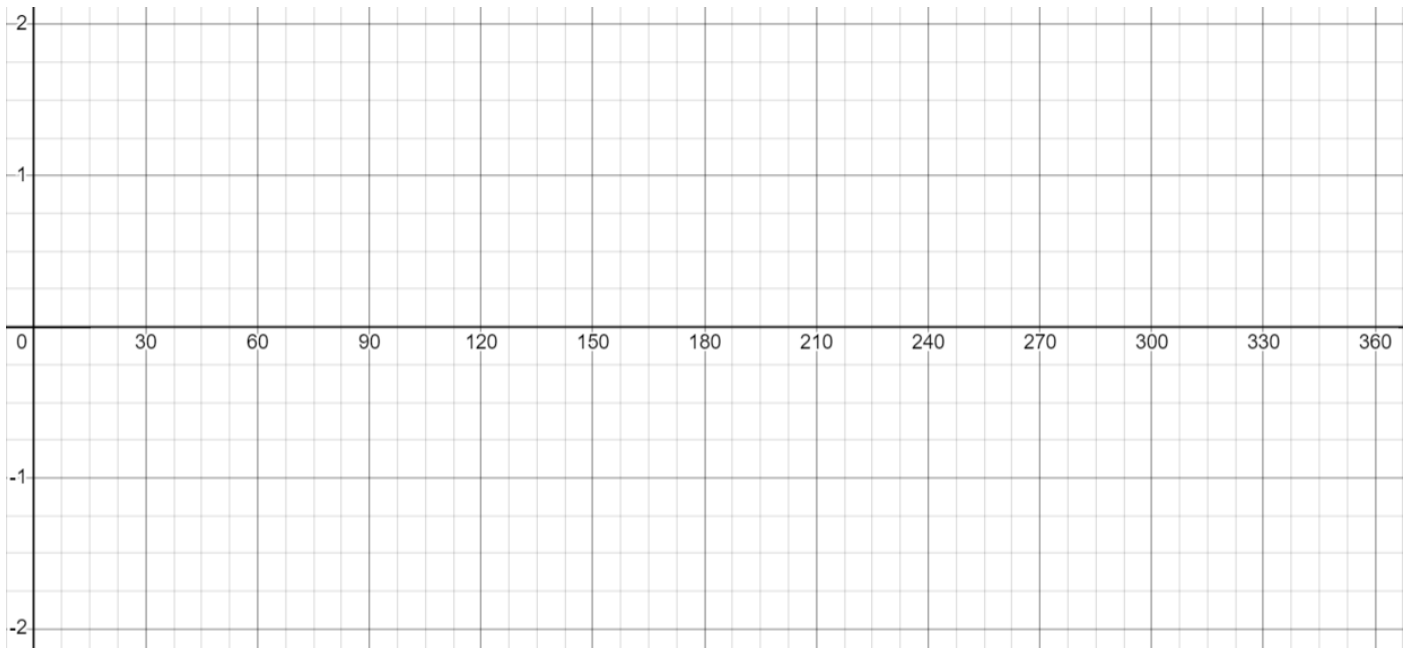
Answer Yes and No. Give a reason for you answer.

Task 5

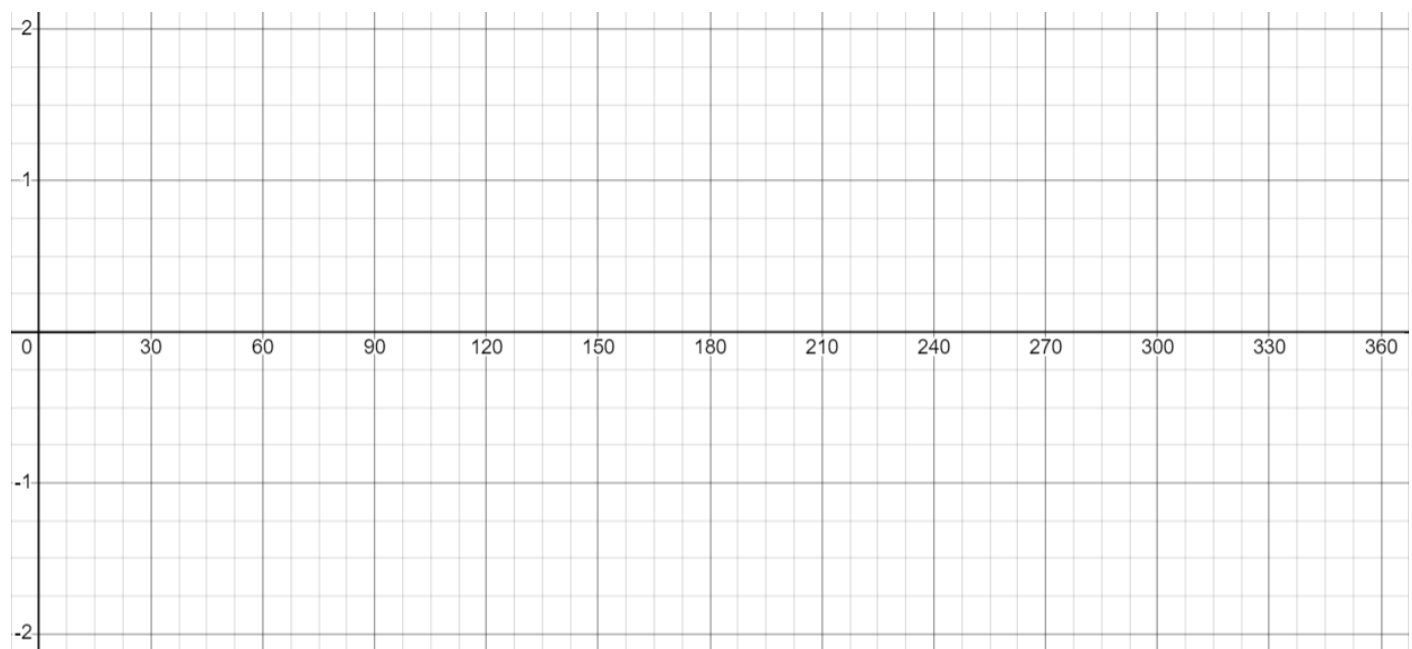
After doing the activities above can you draw the following graph on the hard copy axes provided:

$$y = 2\sin x + 1$$

If you need help you can ask ChatGPT to help, please just copy and paste your prompts used and ChatGPT's responses below:



Now draw the graph of $y = \cos\theta$ on the axes below indicate all x and y intercepts and all turning points:



CONCLUSION:

a) Which of the 5 tasks did you find easier:

Task 1	Task 2	Task 3	Task 4	Task 5

Explain:

a) Which of the 5 tasks did you find the most challenging:

Task 1	Task 2	Task 3	Task 4	Task 5

Explain:

b) If you had to learn how to draw the sine graph, would you choose to use the steps that ChatGPT gave, the steps given in the activity in GeoGebra or a combination of the two activities?

ChatGPT	GeoGebra	Combination of both

c) Which activities was most effective in engaging you in critical thinking?

ChatGPT	GeoGebra	Combination of both	Neither

d) Which activities was most effective for understanding?

ChatGPT	GeoGebra	Combination of both	Neither

e) Did you learn anything new in this lesson? Answer YES or NO and explain your answer if YES

f) Was there anything in this lesson that worked better than the previous lessons?

g) Was there anything in this lesson, compared to the previous lessons, that didn't work?

h) In which lesson did you find the combinational use of ChatGPT and GeoGebra most effective for learning:

Lesson 1 (what is sine ratio)	Lesson 2 (sine in different quadrants)	Lesson 3 (sine graph)	NONE

i) Would you rather use ChatGPT for:

Give definitions, explaining concepts in mathematics	Giving examples to test mathematical understanding	Giving Steps to a certain outcome.	Not in any of these ways

Give a reason for you answer above:

j) Any recommendations that you can give to other teachers to use ChatGPT and GeoGebra effectively to teach concepts of Trigonometric sine function?

k) Do you think that using ChatGPT in the Mathematics classroom is valuable and can it enhance the learning in the classroom?

l) Do you think that using GeoGebra in the Mathematics classroom is valuable and can it enhance the learning in the classroom?

m) Do you think that using ChatGPT and GeoGebra in combination in the Mathematics classroom is valuable and can it enhance the learning in the classroom?