Investigation of factor rotation routines in principal component analysis of stock returns

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

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Abstract

This paper investigates rotation routines that will produce uncorrelated rotated principal components for a dataset of stock returns, in an attempt to identify the macroeconomic factors that best explain the variability among risk-adjusted stock returns on the Johannesburg Stock Exchange. An alternative to the more traditional rotation approaches is used, which creates subsets of principal components with similar variances that are rotated in turn. It is found that only one of the three normalisation constraints examined can retain uncorrelated principal components after rotation. The results also show that when subspaces of components are rotated that have close eigenvalues, the different rotation criteria used to rotate principal components will produce similar results. After rotating the suitable subsets using varimax rotation, it is found that the first rotated component can be explained by the African Industrials sector, the second rotated component is related to the African Consumer Services sector while the third rotated component shows a significant relationship to the African Finance factor.

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1 Introduction

The prediction of stock price behaviour has perhaps been one of the most studied topics in finance. One widely applied theory is the arbitrage pricing theory developed by Ross in 1976. It states that there exists a linear relationship between stock returns and a number of common risk factors (Ross, 1976). Since then, a large number of studies have produced evidence of the existence of a relationship between stock returns and various different macroeconomic and fundamental variables. Studies on the US markets as well as the South African market have collectively found numerous factors that could help explain stock return volatility (Fama (1981), DeBondt and Thaler (1985), Van Rensburg (2000)). However, it is not desirable to include all these risk factors in the arbitrage pricing theory model, since the model should be as simple and accurate as possible. It has thus been the objective of a number of studies to reduce the number of factors to only those that explain the most of the stock return volatility. Various methods have been employed in an attempt to identify a smaller number of the most significant factors. One of the more popular methods is principal component analysis.

Principal component analysis reduces the dimensionality of a dataset by replacing the original correlated variables by a smaller number of uncorrelated principal components that account for most of the variability in the dataset (Jolliffe, 1989). The new smaller set of uncorrelated variables becomes much easier to understand than the original large correlated dataset (Dunteman, 1989). When interpretation of the extracted principal components is difficult, they are often rotated in an attempt to ease interpretation. However, there are a number of drawbacks associated with rotating principal components. The main drawback being that the rotated principal components appear to be no longer uncorrelated (Jolliffe, 1989).

This paper attempts to find a rotation routine that will produce uncorrelated rotated principal components. These are found for a dataset of stock returns in an attempt to identify the macroeconomic factors that best explain the variability among risk-adjusted stock returns on the Johannesburg Stock Exchange.

The paper proceeds as follows: Section 2 looks at previous work done on the arbitrage pricing theory, while Section 3 provides a background into principal component analysis. Component rotation and its main drawbacks are discussed in Section 4. This is followed by Section 5, which describes the data and methodology used in this paper. The results are then presented and analysed in Section 6 and conclusions are drawn in Section 7.

2 The arbitrage pricing theory

According to the arbitrage pricing theory, there exists a linear relationship between stock returns and a number of common risk factors (Ross, 1976). However, the theory neither specifies how many factors nor what type of risk factors should be included in this linear model. This has resulted in many studies attempting to identify what the appropriate number of factors is and what these factors should be.

In trying to identify the possible risk factors to include in the model, a large number of papers have produced evidence of the existence of a relationship between stock returns and certain macroeconomic and fundamental variables. Studies conducted on the US stock market returns have found a number of macroeconomic variables to be able to explain stock returns. Fama (1981) finds stock price behaviour to be influenced by inflation; while Chen, Roll and Ross's (1986) results show that industrial production and twists in the yield curve are able to explain expected stock returns. Other studies conducted on the US market have found relationships between stock returns and other variables such as past returns (DeBondt and Thaler, 1985) and earnings-to-price ratios (Basu, 1983) over the periods of their studies.

Similar results are seen in studies on the South African market. When examining the Johannesburg Securities Exchange (JSE) over the period 1985 to 1995, van Rensburg (2000) finds the rate on long-term bonds, gold price in Rands and the level of gold and foreign exchange reserves, to represent risk factors within the arbitrage pricing theory framework. In van Rensburg's (2002) more famous paper, he uses principal component analysis to show that the JSE Financial and Industrial Index and the Resources Index are the best observable proxies to use in a two-factor arbitrage pricing theory model. Later, van Rensburg and Robertson (2003) provide results that indicate the ability of price-to-NAV, price-to-earnings, size, divided yield and cashflow-to-price to explain stock returns on the JSE.

These are just a few of the many factors found in the literature that display a relationship with stock returns. However, the aim of the arbitrage pricing theory is to obtain a model, which is as simple and accurate as possible. Thus, it is not desirable to include all these factors in the model. Many studies have been conducted which attempt to find a smaller number of factors that account for most of the stock return volatility. One commonly used approach to identify these factors is principal component analysis.

3 Principal component analysis

Principal component analysis is a method, which reduces the dimensionality of a dataset from its original p variables to a smaller number of m variables, called principal components (Jolliffe, 2002). These principal components are uncorrelated and retain the variance of the original p variables.

The first principal component is a linear combination of the original variables $x_1, x_2, \dots, x_p$ such that

$$z_1 = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1p}x_p$$

where the weight vector $\left(a_{11}, a_{12}, \dots, a_{1p}\right)$ is chosen to maximise the variance of z_1 subject to the constraint:

$$\sum_{i=1}^{p} {a_{1i}}^2 = 1$$

(Dunteman, 1989). The second principal component is another linear combination of the original variables. Here, the weight vector (also known as loadings) $\left(a_{21}, a_{22}, \dots, a_{2p}\right)$ maximises the variance of

$$z_2 = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2p}x_p$$

subject to two constraints,

$$\sum_{i=1}^{p} {a_{2i}}^2 = 1$$

and z_2 must be uncorrelated with z_1 (Dunteman, 1989). This procedure is continued until p principal components have been found.

These principal components can be expressed in matrix form as

$$\boldsymbol{Z} = \boldsymbol{XA}$$

where $\boldsymbol{Z}$ is the vector of p principal component scores, $\boldsymbol{A}$ is the $p \times p$ matrix of weight vectors, $\boldsymbol{X}$ is the column vector of the p original variables and $\boldsymbol{A}'\boldsymbol{A} = \boldsymbol{I}$ where $\boldsymbol{I}$ is the identity matrix (Dunteman, 1989).

The i^{th} principal component's variance is the i^{th} largest eigenvalue (or latent root) of the variables' covariance matrix, denoted by l_i. It indicates how much of the variability in the dataset the principal component explains. The associated eigenvector is then the principal component's weight vector, which enables the interpretation of the principal component (Dunteman, 1989). The latent root of successive principal components decreases, thus only the first m (smaller than p) principal components are often retained to account for most of the variation among the original variables (Dunteman, 1989).

In this way the original variables can be reduced to a much smaller number of uncorrelated principal components, which explain as much of the variability in the dataset as possible. The next step in the process is therefore to find the optimal number m.

3.1 The number of components to retain

The aim of principal component analysis is to reduce the number of variables in a dataset, while at the same time retaining as much information as possible (Jolliffe, 2002). However, there is no set theory stating how many principal components should be included in order to adequately account for the total volatility in the dataset. A number of studies have focused on this problem and have suggested numerous different rules to determine how many principal components are required to describe a reasonable amount of the variability of the dataset, while also reducing its dimensionality. A few of these are detailed below.

One approach starts with deciding what percentage of the total variance the principal components are desired to contribute (Jolliffe, 2002). The smallest amount of components required to reach this percentage of variation is then the appropriate number.

Another widely adopted approach is the Kaiser's rule (Kaiser, 1960). As Jolliffe (2002) explains, this rule stipulates that principal components with latent roots of more than one should be included, while those below should not. However, Dunteman (1989) argues that this rule could exclude principal components, which may be important despite being small, since they can be useful in describing the structure of the data. Fifield, Power and Sinclair (2002) thus develop a different rule. They include components with eigenvalues above as well as slightly below one, while also retaining enough principal components to account for at least 80 per cent of the variance in the dataset.

The scree test proposed by Cattell (1966) is also widely used in studies. Here the latent root of each component is plotted on a graph to create a latent root curve from the largest latent root to the smallest (Mitchum, 1993). A point k is then found where the curve is steep to the left and flat to the right of k

(Dunteman, 1989). This point is the cut-off value, resulting in k principal components being retained. The drawback of this test is that it is very subjective and often such lines do not exist for a dataset (Dunteman, 1989).

Another more objective test, as pointed out by Mitchum (1993), is the Guttman test. This approach calculates the average latent root and includes all components with latent roots above this value. Each of the components chosen should thus account for more than the average amount of variability (Mitchum, 1993).

These are a few of the many rules and approaches developed and applied to find the most appropriate number of components. Since there is no single test that is optimal for all datasets, a somewhat subjective choice needs to be made as to which rule to apply to a dataset. The amount of components that are required will always vary and depends on the objectives and requirements of the principal component analysis (Jolliffe, 2002). However, since all the rules are arbitrary, Dunteman (1989) warns that one should always apply them with caution.

3.2 Normalisations

In the derivation of principal component analysis above, the i^{th} principal component is defined as $\boldsymbol{Z}_i = \boldsymbol{X}\boldsymbol{A}_i$ where $\boldsymbol{A}_i$is the weight vector and $\boldsymbol{X}$ the vector of original variables (Jackson, 1991). To specify $\boldsymbol{Z}_i$ uniquely the normalisation constraint

$$\boldsymbol{A}_i'\boldsymbol{A}_i = 1 \tag{1}$$

is imposed so that the weight vectors $\boldsymbol{A}_i$ have unit length and are orthogonal. This results in uncorrelated principal components whose variances are their latent roots (Jackson, 1991). However, the normalisation is only required in order to prevent the weights from becoming very large when maximising the variance of the principal component. The nature of the component will remain unchanged when the weight vector is multiplied by an arbitrary constant (Jackson, 1991).

Therefore, there are a wide variety of alternative normalisation constraints that can be employed to obtain a unique result (Jackson, 1991).

A different normalisation that has also found widespread use and is used by most computer packages, scales the weight vectors to their latent roots such that

$$\boldsymbol{A}_i'\boldsymbol{A}_i = l_i \tag{2}$$

(Jackson, 1991). This normalisation will still produce uncorrelated principal components but their variances are now equal to the squares of their latent roots (Jackson, 1991). Another popular normalisation is

$$\boldsymbol{A}_i'\boldsymbol{A}_i = l_i^{-1} \tag{3}$$

where the weight vectors are scaled to the reciprocal of their latent roots (Jackson, 1991). These principal components are uncorrelated with variances equal to unity (Jackson, 1991). Any of these normalisations can be used in a principal component analysis and there is still no uniformity of normalisation in the literature (Jackson, 1991).

4 Component rotation

Since by construction principal components are a linear combination of all the original variables, interpreting these components may become difficult (Jolliffe, 2002). However, principal components are only useful if they can be easily interpreted. Various methods have thus been developed to make the interpretation of principal components easier. Possibly the most widely used method is the rotation of the principal components. Its sole objective is to make it as simple as possible to interpret the rotated components (Jolliffe, 2002). This is achieved by relaxing the requirement that the components must account for the maximum variance, in order to obtain components that better resemble the original variables.

Jolliffe (2002) argues that it is less important to interpret the m principal components separately and more important to interpret the m-dimensional space defined by the m components. Graphically, the aim of rotating the axes in this m-

dimensional space about the origin is to redistribute the variance among the principal components in an attempt to make the interpretation of the axes significantly easier (Jolliffe, 2002).

The goal of rotation is to achieve a so-called simple structure. As Richman (1986) explains, Thurstone (1947) was the first to suggest that five criteria are required to achieve simple structure. Firstly, there should be at least one zero loading on a component for every variable (Richman, 1986). Secondly, every component should have as many zero loadings as the number of components or more (Richman, 1986). Also, for every pair of components, variables should have only significant loadings on one of the two components and not the other (Richman, 1986). In addition, if there are more than four components then the pair of components should both have a large proportion of zero loadings (Richman, 1986. Lastly, pairs of components should only have a small amount of complex variables (Richman, 1986).

Since these criteria are very strict, many academics use a more relaxed definition of a simple structure. Most rotation criteria attempt to achieve simplicity by either moving the principal component loadings towards zero or their maximum absolute value (Jolliffe, 2002). The rotated components with large absolute values are then seen as important, while those close to zero are considered unimportant. Loadings with values between these two extremes are avoided as best as possible, so as to make interpretation easier.

4.1 Types of rotation

Many different rotation algorithms have been developed over the last few decades that attempt to obtain simple structure. Different rotation criteria will either simplify the rows or the columns of the rotated loadings matrix in an attempt to obtain simple structure (Dunteman, 1989). They can be sorted into two main categories, namely orthogonal rotation and oblique rotation. Orthogonal rotation assumes the components are uncorrelated and results in the rotated axes still being orthogonal to each other (Jolliffe, 2002). The main orthogonal methods are varimax and quartimax. When simple structure can be obtained using orthogonal rotation, many academics agree that varimax rotation is the most efficient (Kline, 1994). Oblique rotation on the other hand assumes the components are correlated and does not require the rotated axes to be orthogonal (Jolliffe, 2002). These oblique rotations include quartimin, direct oblimin, maxplane, oblimax and promax (Richman, 1986). These are just some of the more widely used rotation methods. Again, the most appropriate method to use will vary according to what the definition and objective of simple structure is (Jackson, 1991).

The simplicity of orthogonal rotation and its neat results is one of the main advantages of using orthogonal rotation and enables easy interpretation of the rotated principal components (Rummel, 1970). Oblique rotation on the other hand is very flexible, allowing the simple structure to be improved better than orthogonal rotation can. However, two main drawbacks of oblique rotation are that the rotated components become difficult to interpret and more importantly, are no longer uncorrelated (Dunteman, 1989). Therefore, the rest of this paper will only consider orthogonal rotation, since the aim is to find uncorrelated principal components.

4.1.1 Orthogonal rotation

Most orthogonal rotation methods attempt to obtain simple structure by maximising the criterion Q, defined as:

$$Q = \sum_{j=1}^{m} \left\{ \sum_{i=1}^{p} {b_{ij}}^4 - \frac{c}{p} \left(\sum_{i=1}^{p} {b_{ij}}^2 \right)^2 \right\} \tag{4}$$

where b_{ij} is the new rotated loading and c is an arbitrary constant that depends on which rotation method is chosen (Jackson, 1991). The orthogonal rotation methods aim to carry the original loadings matrix $\boldsymbol{A}$ into a new loadings matrix $\boldsymbol{B}$**,** for which Q is maximised, by finding an orthogonal transformation matrix $\boldsymbol{T}$ (Harman, 1976). The orthogonal rotation of any variable x_i, in the plane of principal components r and s for an angle φ, will carry the a loadings into new rotated loadings b by post-multiplying the matrix of the pair of columns r and s of matrix $\boldsymbol{A}$ by the transformation matrix

$$\boldsymbol{T_{rs}} = \begin{pmatrix} \cos \varphi_{rs} & -\sin \varphi_{rs} \\ \sin \varphi_{rs} & \cos \varphi_{rs} \end{pmatrix}$$

(Harman, 1976).

The new loadings are therefore given by the equations:

$$b_{ir} = a_{ir}\, cos\, \varphi_{rs} + a_{is}\, sin\, \varphi_{rs}$$

$$b_{is} = -a_{ir}\, sin\, \varphi_{rs} + a_{is}\, cos\, \varphi_{rs}.$$

The aim is to determine the angle of rotation φ_{rs} for each pair of principal components, which will maximise Q (Harman, 1976). The transformation to the rotated matrix $\boldsymbol{B}$ is then achieved after rotating all combinations of components. The rotated loadings matrix is then the product of the transformation of every combination of pairs of principal components such that

$$\boldsymbol{B} = \boldsymbol{A}\boldsymbol{T}_{12}\boldsymbol{T}_{13} \dots \boldsymbol{T_{rs}} \dots \boldsymbol{T}_{(m-1)m}$$

where $r = 1,2, \dots, (m-1)$ and $s = r+1, r+2, \dots, m$ (Harman, 1976). The components are rotated two at a time and the cycle is complete after $\frac{m(m-1)}{2}$ pairings of components and Q calculated after each cycle (Harman, 1976). Cycles of transformations are repeated until Q is at a maximum (no longer increases) (Harman, 1976).

Quartimax rotation sets $c = 0$ and aims to maximise the sum of the fourth powers of the loadings. It thus maximizes the sum of squares of the rotated matrix by row (Jackson, 1991). The maximum is achieved when every variable has a loading of one on a component and zero on all other components. The function therefore increases high loadings and decreases middle loadings for each variable.

For this rotation criterion the angle φ, which will maximise Q for any rotation $\boldsymbol{T_{rs}}$ is found to be

$$\tan 4\varphi = \frac{2\sum_{j=1}^{p}(2a_{jr}a_{js})({a_{jr}}^2 - {a_{js}}^2)}{\sum_{j=1}^{p}\left[\left({a_{jr}}^2 - {a_{js}}^2\right)^2 - \left(2a_{jr}a_{js}\right)^2\right]} \qquad (5)$$

(Harman, 1976). Harman (1976) shows that Q has the period $\frac{\pi}{2}$ after rotation, therefore the angle of rotation φ will as well. He also notes that while equation (5) will yield a critical value of φ it may not necessarily be a maximum but instead a minimum or stationary value. Obtaining a maximum value depends on the sign of the denominator and numerator in (5). Harman (1976) shows in which quadrant φ must lie in order to obtain a maximum for Q for different signs of the numerator and denominator. This is used to determine in which quadrant φ will lie in order to obtain a maximum for Q.

Varimax rotation sets $c = 1$ and was developed by Kaiser (1958). It maximises the sum of squares across the columns of the rotated matrix (Jackson, 1991). This criterion is a modification of the quartimax criterion and attempts to simplify the columns rather than the rows of the loadings matrix. It thus simplifies the principal components instead of the variables. However, in addition this criterion normalizes the rows of the loadings matrix before rotation. The vectors of variables are rescaled to have unit length and individual b_{ij} are replaced with $\frac{b_{ij}}{h_i}$ in equation (4) where $h_j = \sum_{i=1}^{m} a_{ji}$ for $j = 1,2,..,p$ (Harman, 1976). The rotation is then carried out after which the vectors are brought back to their original length by multiplying each rotated loading by its appropriate h_j again (Harman, 1976).

Kaiser (1958) finds that the angle that maximises Q is given by the equation:

$$\tan 4\varphi = \frac{2\sum_{j=1}^{p} u_j v_j - \frac{2}{n}\left(\sum_{j=1}^{p} u_j\right)\left(\sum_{j=1}^{p} v_j\right)}{\sum_{j=1}^{p}\left({u_j}^2 - v_j^2\right) - \frac{1}{n}\left(\left(\sum_{j=1}^{p} u_j\right)^2 - \left(\sum_{j=1}^{p} v_j\right)^2\right)} \qquad (6)$$

where $u_j = \left(\frac{a_{jr}}{h_j}\right)^2 - \left(\frac{a_{js}}{h_j}\right)^2$ and $v_j = 2\left(\frac{a_{jr}}{h_j}\right)\left(\frac{a_{js}}{h_j}\right)$ (Harman, 1976).

As with quartimax rotation, the angle of rotation φ will have period $\frac{\pi}{2}$ and Harman's (1976) findings are used to determine in which quadrant φ will lie in order to obtain a maximum for Q.

4.2 Drawbacks of rotation

Despite the main advantage of making the principal components easier to interpret, rotating principal components also has a number of disadvantages. Firstly, choosing which rotation criteria to use can be difficult due to the large number of possible rotations listed above (Jolliffe, 2002). However, when examining orthogonal rotation methods, Jolliffe (2002) remarks that there is often little difference in results after rotation.

Another drawback Jolliffe (1989) recognises, is the influence of the number of components chosen on the resultant rotated components. When increasing the number of principal components from m to n, the original m components remain unchanged. Yet, after rotating the components, the n rotated components may be very different to the rotated m components. Therefore, as Jolliffe (2002) shows, the rotated components may be greatly affected by the number m of components chosen. The choice of subspace may thus be seen as more important than the choice of rotation (Jolliffe, 2002).

Additionally, while rotation still retains the maximum variance attainable, it redistributes this variance more evenly among the components than before the rotation (Jolliffe, 1989). Therefore, Jolliffe (1989) indicates that it may result in the loss of information on the most dominant sources of variance.

Lastly, Jolliffe (1995) observes that imposing different normalisation constraints on the loadings, results in different solutions after rotation. He algebraically derives the properties of the principal components after orthogonally rotating them under the three different normalisation criteria described in Section 3.2 (see Appendix A for the theoretical proof). In particular, he investigates the effect on the orthogonality of the loadings and the uncorrelatedness of the components after orthogonal rotation. He shows that despite both these properties being present before rotation, only one of the properties can hold after rotation.

The choice of normalisation will determine which of the two properties will be retained. Under normalisation (1) the orthogonally rotated loadings remain orthogonal; however, the rotated components are no longer uncorrelated. When using the popular normalisation (2), the loadings are not orthogonal anymore and the components become correlated after orthogonal rotation. Uncorrelated principal components can only be retained after orthogonal rotation if normalisation (3) is employed. However, the loadings will not remain orthogonal after rotation using this normalisation.

Jolliffe (1995) therefore recommends not using the most common form of normalisation, equation (2), since neither of the two properties is retained after rotation. Instead, he suggests employing normalisation (1) when interpretability and orthogonality of the loadings is important, while normalisation (3) should be used when uncorrelatedness is a concern. Jolliffe's (1995) observation highlights how many academics are incorrect in stating that orthogonal rotation will retain the two properties of orthogonal loadings and uncorrelated components. They fail to specify that the choice of normalisation will determine which of these properties are retained after orthogonal rotation. It is important that enough consideration is given to the choice of normalisation used and its effect on the rotated loadings' and component's properties.

4.3 Alternative approaches

Due to the discovery of the above drawbacks, academics have attempted to remove these by finding alternatives to the standard rotation techniques. Hawkins (1973) and Jeffers (1981) for example suggest rotating all p principal components instead of only the first m components chosen. This would prevent the number of principal components chosen to influence the results. However, it may result in the loss of information on the most dominant sources of variance, since the variance is redistributed more evenly among many more components.

A different solution is proposed by Jolliffe (1989). He suggests creating subsets of the principal components and then rotating the components in each subset separately. These subsets should be chosen such that its principal components have nearly equal variances, while they are also well separated from those of other subsets Jolliffe (1989). As Jolliffe (1989) explains, there are a number of advantages to this method. First, since the variability within each subset is relatively evenly distributed among the components there should be no loss of information. The choice of normalisation constraint will also have a much smaller effect on the rotated components, since the latent roots for all the principal components in a subset will be almost the same (Jolliffe, 1989). In addition, there is no longer a need to choose the number of principal components to include (Jolliffe, 1989).

Thus, the only remaining arbitrary choice to be made is into which subsets the components should be divided. This is achieved by determining how close two consecutive eigenvalues are (and therefore how stable the components are). The components that display eigenvalues in close proximity to each other are then grouped together to create well-defined subspaces (Jolliffe, 1989).

As with the choice of how many components to retain, a number of academics have suggested tests to determine how close two eigenvalues are. Some papers suggest using the percentage of difference between two consecutive eigenvalues $\left(\frac{l_i - l_{i+1}}{l_i}\right)$ as a rule for determining closeness (Jolliffe, 1989). However, Jolliffe (1989) proposes that examining the absolute difference between the components' eigenvalues $(l_i - l_{i+1})$ may be a simpler rule. Nevertheless, for both of these rules a choice still needs to be made as to what cut-off value is used to determine the closeness of the eigenvalues.

An alternative and more ad hoc rule to finding well-defined subspaces is developed by North et al. (1982). This rule states that the difference between two consecutive eigenvalues should be greater than one or two times the estimate of their respective standard errors. These estimates are defined as

$$\delta l \sim l_i \left(\frac{2}{n}\right)^{1/2}$$

where n is the sample size (North et al., 1982). This test only requires an arbitrary choice to be made regarding which multiple (between one and two) of the estimates is used.

Though a decision still has to be made regarding the rule and criteria used to divide components into well-defined subsets, Jolliffe (1989) argues that it is often more arbitrary to choose the number of components to retain in order to account for most of the variation.

The alternative approach of rotating subspaces of principal components rather than the first few principal components appears to address all the above mentioned drawbacks associated with traditional rotation. Nevertheless, Jolliffe (1989) acknowledges that in practice it may still be more useful in some circumstances to rotate the first few components despite their different eigenvalues. Thus, every dataset still needs to be analysed individually in order to determine which rotation technique is optimal.

5 Stock market application

The objective of this paper is to find uncorrelated macroeconomic factors that explain the risk-adjusted returns of stocks on the JSE. Therefore, two separate sets of data are analysed, namely, stock return series and macroeconomic factor series. Two different approaches can be used to identify the most significant macroeconomic or fundamental factors using principal component analysis.

The first approach is to conduct principal component analysis on the set of all macroeconomic factors in order to reduce the dimensionality of the dataset to a much small number of principal components. These principal components are then used in a regression analysis to explain the excess stock returns according to the arbitrage pricing theory (e.g. Fifield, Power and Sinclair (2002), Rao and Radjeswari (2000)).

The other method conducts the principal component analysis on the set of risk-adjusted stock returns rather than the macroeconomic factors (e.g. Curto, Pinto and Fernandes (2006)). Using regression analysis it is then determined which factors best explain the returns of the extracted principal components. An advantage of this method is that the analyst does not need to decide which macroeconomic factors influence the stock returns before conducting the analysis. Instead, any macroeconomic factor can be compared to the extracted principal components of the stock returns. This second approach is implemented in this paper and applied as follows.

The monthly total returns[1] of a selection of JSE stocks as well as the JSE All Share Index (ALSI) are obtained from Bloomberg for the period 30 June 2003 to 28 February 2013. This time period and frequency was used, as it was most easily available for the index series. Since thin trading of small market capitalisation stocks can distort the returns, only the top 100 market capitalisation stocks on the exchange as at 7 February 2013 are considered. Of these stocks, 16 stocks are only listed on the exchange after 30 June 2008 and therefore are excluded from the sample. This results in a total sample of 84 stocks over 116 months. Table 1 below lists the stocks along with their market sectors.

Before obtaining the principal components of these stock returns, the dataset is risk-adjusted, de-trended and standardised. The logarithm returns are calculated from the stock price series (in order to make the dataset stationary). Each of these return series are then regressed on the logarithm returns of the ALSI in order to estimate the beta of each stock over the full period. The risk-adjusted excess returns, x_i, are then calculated for each stock and standardised so that each variable x_i has mean zero and standard deviation one. The data is standardised to prevent the variable with the largest volatility dominating the first principal component extracted.

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[1] Corporate events such a dividends and stock splits are accounted for.

Table 1: List of JSE shares

Ticker	Share Name	Sector Name
ABL	African Bank Investments Ltd	Financial Services
ACL	Arcelor Mittal South Africa Ltd	Industrial Metals & Mining
ACP	Acucap Properties Ltd	Real Estate Investment & Services
AEG	Aveng Ltd	Construction & Materials
AFE	AECI Ltd	Chemicals
AFX	African Oxygen Ltd	Chemicals
AGL	Anglo American PLC	Mining
AMS	Anglo Platinum Ltd	Mining
ANG	Anglogold Ashanti Ltd	Mining
APN	Aspen Pharmacare Holdings Ltd	Pharmaceuticals & Biotechnology
ARI	African Rainbow Minerals Ltd	Mining
ASA	Absa Group Ltd	Banks
ASR	Assore Ltd	Mining
AVI	AngloVaal Industries Ltd	Food Producers
BAT	Brait Se	Financial Services
BAW	Barloworld Ltd	General Industrials
BIL	BHP Billiton PLC	Mining
BVT	Bidvest Group Ltd	Support Services
CFR	Compagnie Financiere Richemont SA	Personal Goods
CLS	Clicks Group Ltd	Food & Drug Retailers
CML	Coronation Fund Managers Ltd	Financial Services
CPI	Capitec Bank Holdings Ltd	Banks
CPL	Capital Property Fund	Real Estate Investment Trusts
DST	Distell Group Ltd	Beverages
DSY	Discovery Holdings Ltd	Life Insurance
DTC	Datatec Ltd	Software & Computer Services
EXX	Exxaro Resources Ltd	Mining
FPT	Fountainhead Property Trust	Real Estate Investment Trusts
FSR	FirstrandLtd	Banks
GFI	Gold Fields Ltd	Mining
GND	Grindrod Ltd	Industrial Transportation
GRT	Growthpoint Properties Ltd	Real Estate Investment & Services
HAR	Harmony Gold Mining Company Ltd	Mining
HCI	Hosken Consolidated Investments Ltd	Financial Services
HYP	Hyprop Investments Ltd	Real Estate Investment & Services
ILV	Illovo Sugar Ltd	Food Producers
IMP	Impala Platinum Holdings Ltd	Mining
INL	Investec Ltd	Financial Services
INP	Investec PLC	Financial Services
IPL	Imperial Holdings Ltd	Industrial Transportation
JDG	JD Group Ltd	General Retailers
LBH	Liberty Holdings Ltd	Life Insurance
LON	Lonmin PLC	Mining
MDC	Medi-Clinic International Ltd	Health Care Equipment & Services
MMI	MMI Holdings Ltd	Life Insurance
MPC	Mr Price Group Ltd	General Retailers
MSM	Massmart Holdings Ltd	General Retailers
MTN	MTN Group Ltd	Mobile Telecommunications
MUR	Murray & Roberts Holdings Ltd	Construction & Materials
NED	Nedbank Group Ltd	Banks
NHM	Northam Platinum Ltd	Mining
NPK	Nampak Ltd	General Industrials
NPN	Naspers Ltd	Media
NTC	Netcare Ltd	Health Care Equipment & Services
OCE	Oceana Group Ltd	Food Producers
OML	Old Mutual PLC	Life Insurance
OMN	Omnia Holdings Ltd	Chemicals
PIK	Pick N Pay Stores Ltd	Food & Drug Retailers
PPC	Pretoria Portland Cement Company Ltd	Construction & Materials
PSG	PSG Group Ltd	Financial Services
PWK	Pick n Pay Holdings Ltd	Food & Drug Retailers
RDF	Redefine properties Ltd	Real Estate Investment & Services
REM	Remgro Ltd	General Industrials
RES	Resilient Prop Inc Fund Ltd	Real Estate Investment & Services
RLO	Reunert Ltd	Electronic & Electrical Equipment
RMH	RMB Holdings Ltd	Banks
SAB	SABMiller PLC	Beverages
SAP	Sappi Ltd	Forestry & Paper
SBK	Standard Bank Group Ltd	Banks
SHF	Steinhoff International Holdings Ltd	Household Goods & Home Construction
SHP	Shoprite Holdings Ltd	Food & Drug Retailers
SLM	Sanlam Ltd	Life Insurance
SNT	Santam Ltd	Nonlife Insurance
SOL	Sasol Ltd	Oil & Gas Producers
SUI	Sun International Ltd	Travel & Leisure
TBS	Tiger Brands Ltd	Food Producers
TFG	The Foshini Group Ltd	General Retailers
TKG	Telkom SA Ltd	Fixed Line Telecommunications
TON	Tongaat Hulett Ltd	Food Producers
TRE	Trencor Ltd	Industrial Transportation
TRU	Truworths International Ltd	General Retailers
TSH	Tsogo Sun Holdings Ltd	Travel & Leisure
WBO	Wilson Bayly Holmes-Ovcon Ltd	Construction & Materials
WHL	Woolworths Holdings Ltd	General Retailers

Next, the principal components are found. The 84×84 correlation matrix is calculated in order to find the eigenvalues and associated eigenvectors. The extracted eigenvalues (or loadings) are then used to calculate the scores of each principal component.

Lastly, the factors are examined. The monthly closing prices of a selection of macroeconomic factors are obtained from Bloomberg, iNet Bridge, the JSE and the Bureau of Economic Research (Stellenbosch) for the same period. A total of 160 macroeconomic factors are included in the sample and are shown in Table 2 below. This set of factors is a collection of global and domestic macroeconomic factors, which are often considered as indicators of stock market movements. These include exchange rates, commodity prices (e.g. Gold, Platinum, Brent Crude) in different currencies, local interest rates and Consumer Price Indices as well as indices and market sectors in South Africa, Africa and emerging markets. For some of the factors, lagged series were also included in the dataset in an attempt to identify leading indicators.

The logarithm returns are calculated and standardised for each of these factors over the period given. These are then regressed on the scores of each of the principal components. The R-squared obtained is examined to assess the fit of each factor to the components. If the fit is not good, it may be necessary to rotate the components. The new scores are then calculated after rotating the loadings and each R-squared is recalculated to assess the fit.

While there are a number of computer programmes and packages available that can perform principal component analysis as well as rotation, most can be very restrictive and do not allow for different rotations or normalisations to be applied. Consequently, the following results were generated in Excel by coding the above methodology in VBA.

Table 2: List of factors and descriptions

Factor	Description
USDZAR	USD/ZAR exchange rate
EURUSD	EUR/USD exchange rate
EURZAR	EUR/ZAR exchange rate
Brent USD	Brent Crude price in USD
Brent EUR	Brent Crude price in EUR
Brent ZAR	Brent Crude price in ZAR
Alu USD	Aluminium price in USD
Alu EUR	Aluminium price in EUR
Alu ZAR	Aluminium price in ZAR
Plat USD	Platinum price in USD
Plat EUR	Platinum price in EUR
Plat ZAR	Platinum price in ZAR
Cop USD	Copper price in USD
Cop EUR	Copper price in EUR
Cop ZAR	Copper price in ZAR
Gold USD	Gold price in USD
Gold EUR	Gold price in EUR
Gold ZAR	Gold price in ZAR
Com USD	Goldman Sachs Commodity price in USD
Com EUR	Goldman Sachs Commodity price in EUR
Com ZAR	Goldman Sachs Commodity price in ZAR
CPI m	CPI MoM
3 month	3 month JIBAR
6 month	6 month JIBAR
1 year	1 year zero rate
3 year	3 year zero rate
5 year	5 year zero rate
10 year	10 year zero rate
15 year	15 year zero rate
JALSH RSI	ALSI Relative Strength Index
Small-Big	Small less big market capitalisation stocks
Value-Growth	Value less growth stocks
ALBI	All Bond Index
PMI m	PMI MoM
LEAD. INDI m	Composite business cycle lead indicators MoM
OIL PRICE m	Oil price MoM
PETROL PRICE m	Petrol price MoM
RETAIL TRADE m	Retail trade MoM
BUILDING m	Growth in new buildings completed MoM
VEH. SALES m	Vehicle sales MoM
PPI m	PPI MoM
CREDIT m	Credit extended by all monetary institutions MoM
Carhart's 4th	Carhart's 4th Market Moment
AFR TECHNOL	African technology sector
AFR FINAN	African financial sector
AFR TELECOM	African telecommunication sector
AFR CON SRV	African consumer services sector
AFR HEALTH	African health sector
AFR CON GDS	African consumer goods sector
AFR INDUST	African industrials sector
AFR BAS MAT	African basic materials sector
AFR RES	African resources sector
Sec IND MET	JSE industrial metals sector
Sec MINING	JSE mining sector
Sec CONSTR	JSE construction and mater sector
Sec GEN IND	JSE general industrials sector
Sec FOOD PR	JSE Food and drug retailers sector
Sec GEN RET	JSE general retailers sector
Sec BANKS	JSE banks sector
Sec GEN FIN	JSE general financial sector
EMBI	Emerging Market Bond Index
MSCI-EM	MSCI Emerging Markets Index
ALSI	JSE All Share Index
USDZAR (L1)	USD/ZAR exchange rate (1 month lag)
EURUSD (L1)	EUR/USD exchange rate (1 month lag)
EURZAR (L1)	EUR/ZAR exchange rate (1 month lag)
USDZAR (L2)	USD/ZAR exchange rate (2 month lag)
EURUSD (L2)	EUR/USD exchange rate (2 month lag)
EURZAR (L2)	EUR/ZAR exchange rate (2 month lag)
USDZAR (L3)	USD/ZAR exchange rate (3 month lag)
EURUSD (L3)	EUR/USD exchange rate (3 month lag)
EURZAR (L3)	EUR/ZAR exchange rate (3 month lag)
CPI m (L1)	CPI MoM (1 month lag)
CPI m (L2)	CPI MoM (2 month lag)
CPI m (L3)	CPI MoM (3 month lag)
3 Month (L1)	3 month JIBAR (1 month lag)
3 Month (L2)	3 month JIBAR (2 month lag)
3 Month (L3)	3 month JIBAR (3 month lag)
PMI m (L1)	PMI MoM (1 month lag)
LEAD. INDI m (L1)	Composite business cycle lead indicators MoM (1 month lag)
OIL PRICE (L1)	Oil price MoM (1 month lag)
PETROL PRICE m (L1)	Petrol price MoM (1 month lag)
RETAIL TRADE m (L1)	Retail trade MoM (1 month lag)
BUILDING m (L1)	Growth in new buildings completed MoM (1 month lag)
VEH. SALES m (L1)	Vehicle sales MoM (1 month lag)
PPI m (L1)	PPI MoM (1 month lag)
CREDIT m (L1)	Credit extended by all monetary institutions MoM (1 month lag)
PMI m (L2)	PMI MoM (2 month lag)
LEAD. INDI m (L2)	Composite business cycle lead indicators MoM (2 month lag)
OIL PRICE (L2)	Oil price MoM (2 month lag)
PETROL PRICE m (L2)	Petrol price MoM (2 month lag)
RETAIL TRADE m (L2)	Retail trade MoM (2 month lag)
BUILDING m (L2)	Growth in new buildings completed MoM (2 month lag)
VEH. SALES m (L2)	Vehicle sales MoM (2 month lag)
PPI m (L2)	PPI MoM (2 month lag)
CREDIT m (L2)	Credit extended by all monetary institutions MoM (2 month lag)
PMI m (L3)	PMI MoM (3 month lag)
LEAD. INDI m (L3)	Composite business cycle lead indicators MoM (3 month lag)
OIL PRICE (L3)	Oil price MoM (3 month lag)
PETROL PRICE m (L3)	Petrol price MoM (3 month lag)
RETAIL TRADE m (L3)	Retail trade MoM (3 month lag)
BUILDING m (L3)	Growth in new buildings completed MoM (3 month lag)
VEH. SALES m (L3)	Vehicle sales MoM (3 month lag)
PPI m (L3)	PPI MoM (3 month lag)
CREDIT m (L3)	Credit extended by all monetary institutions MoM (3 month lag)
RETAIL TRADE y	Retail trade YoY
BUILDING y	Growth in new buildings completed YoY
VEH. SALES y	Vehicle sales YoY
CPI y	CPI YoY
PPI y	PPI YoY
CREDIT y	Credit extended by all monetary institutions YoY
RETAIL TRADE y (L1)	Retail trade YoY (1 month lag)
BUILDING y (L1)	Growth in new buildings completed YoY (1 month lag)
VEH. SALES y (L1)	Vehicle sales YoY (1 month lag)
CPI y (L1)	CPI YoY (1 month lag)
PPI y (L1)	PPI YoY (1 month lag)
CREDIT y (L1)	Credit extended by all monetary institutions YoY (1 month lag)
RETAIL TRADE y (L2)	Retail trade YoY (2 month lag)
BUILDING y (L2)	Growth in new buildings completed YoY (2 month lag)
VEH. SALES y (L2)	Vehicle sales YoY (2 month lag)
CPI y (L2)	CPI YoY (2 month lag)
PPI y (L2)	PPI YoY (2 month lag)
CREDIT y (L2)	Credit extended by all monetary institutions YoY (2 month lag)
RETAIL TRADE y (L3)	Retail trade YoY (3 month lag)
BUILDING y (L3)	Growth in new buildings completed YoY (3 month lag)
VEH. SALES y (L3)	Vehicle sales YoY (3 month lag)
CPI y (L3)	CPI YoY (3 month lag)
PPI y (L3)	PPI YoY (3 month lag)
CREDIT y (L3)	Credit extended by all monetary institutions YoY (3 month lag)
EMBI (L1)	Emerging Market Bond Index (1 month lag)
MSCI-EM (L1)	MSCI Emerging Markets Index (1 month lag)
EMBI (L2)	Emerging Market Bond Index (2 month lag)
MSCI-EM (L2)	MSCI Emerging Markets Index (2 month lag)
EMBI (L3)	Emerging Market Bond Index (3 month lag)
MSCI-EM (L3)	MSCI Emerging Markets Index (3 month lag)
S&P 500	Standard & Poor's 500

6 Results

After calculating and analysing all the estimates, a number of interesting results are observed.

6.1 Principal component analysis

After obtaining the eigenvectors of the correlation matrix, the corresponding eigenvalues are ordered from largest to smallest as can be seen in Table 3 below. From the table it is clear that the eigenvalue of the first principal component extracted is significantly larger than those of the remaining components. While the largest eigenvalue accounts for 20.5 per cent of variation among the stocks, the second largest only explains 6.2 per cent of the total variation.

The relationships of the factors to the extracted principal components are shown in Table 4 below. The table shows the three factors that display the largest R-squared for each extracted principal component. The first principal component displays a R-squared of 76 per cent for both the JSE General Retailers sector and the African consumer services sector as well as a R-squared of 72 per cent for the African Resources sector. The second principal component shows a significantly smaller R-squared of 29 per cent to the JSE construction sector, while the remaining principal components do not show any significant relationships with any factors.

Due to the insignificant relationships obtained for all but the first principal component, there is strong motivation to rotate these principal components in an attempt to better capture the variability among the stocks. However, before rotating the principal components, a number of decisions need to be made. The appropriate normalisation as well as rotation methodology needs to be chosen. In addition, it needs to be decided how many components to retain or how subsets will be divided. These aspects will be examined in more detail below.

Table 3: Eigenvalues of the extracted principal components along with the percentage of total variation explained individually and cumulatively

Principal component	Eigenvalue	Percentage of total variation	
		Individual	Cumulative
1	17.26	20.5%	20.5%
2	5.17	6.2%	26.7%
3	4.16	4.9%	31.6%
4	3.45	4.1%	35.8%
5	2.93	3.5%	39.2%
6	2.68	3.2%	42.4%
7	2.45	2.9%	45.4%
8	2.29	2.7%	48.1%
9	2.23	2.7%	50.7%
10	2.03	2.4%	53.1%
11	1.98	2.4%	55.5%
12	1.87	2.2%	57.7%
13	1.67	2.0%	59.7%
14	1.56	1.9%	61.6%
15	1.48	1.8%	63.3%
16	1.45	1.7%	65.1%
17	1.39	1.7%	66.7%
18	1.33	1.6%	68.3%
19	1.28	1.5%	69.8%
20	1.16	1.4%	71.2%
21	1.13	1.3%	72.6%
22	1.09	1.3%	73.9%
23	1.06	1.3%	75.1%
24	1.05	1.2%	76.4%
25	1.03	1.2%	77.6%
26	0.98	1.2%	78.8%
27	0.93	1.1%	79.9%
28	0.90	1.1%	80.9%
29	0.85	1.0%	81.9%
30	0.81	1.0%	82.9%
31	0.74	0.9%	83.8%
32	0.72	0.9%	84.6%
33	0.71	0.8%	85.5%
34	0.68	0.8%	86.3%
35	0.65	0.8%	87.1%
36	0.63	0.8%	87.8%
37	0.61	0.7%	88.5%
38	0.58	0.7%	89.2%
39	0.52	0.6%	89.9%
40	0.50	0.6%	90.5%
41	0.49	0.6%	91.0%
42	0.46	0.5%	91.6%
43	0.44	0.5%	92.1%
44	0.43	0.5%	92.6%
45	0.41	0.5%	93.1%
46	0.38	0.4%	93.6%
47	0.36	0.4%	94.0%
48	0.34	0.4%	94.4%
49	0.32	0.4%	94.8%
50	0.31	0.4%	95.1%
51	0.29	0.3%	95.5%
52	0.28	0.3%	95.8%
53	0.26	0.3%	96.1%
54	0.24	0.3%	96.4%
55	0.24	0.3%	96.7%
56	0.23	0.3%	96.9%
57	0.22	0.3%	97.2%
58	0.21	0.2%	97.4%
59	0.20	0.2%	97.7%
60	0.19	0.2%	97.9%
61	0.17	0.2%	98.1%
62	0.17	0.2%	98.3%
63	0.15	0.2%	98.5%
64	0.14	0.2%	98.7%
65	0.13	0.2%	98.8%
66	0.12	0.1%	99.0%
67	0.11	0.1%	99.1%
68	0.10	0.1%	99.2%
69	0.09	0.1%	99.3%
70	0.08	0.1%	99.4%
71	0.07	0.1%	99.5%
72	0.07	0.1%	99.6%
73	0.06	0.1%	99.6%
74	0.05	0.1%	99.7%
75	0.05	0.1%	99.8%
76	0.04	0.0%	99.8%
77	0.03	0.0%	99.8%
78	0.03	0.0%	99.9%
79	0.03	0.0%	99.9%
80	0.02	0.0%	99.9%
81	0.02	0.0%	100.0%
82	0.02	0.0%	100.0%
83	0.01	0.0%	100.0%
84	0.00	0.0%	100.0%

Table 4: The three factors that display the largest R-squared for each principal component extracted

Principal component	Factor 1	R-squared	Factor 2	R-squared	Factor 3	R-squared
1	Sec GEN RET	76%	AFR CON SRV	76%	AFR RES	72%
2	Sec CONSTR	29%	EMBI (L1)	28%	MSCI-EM (L1)	24%
3	AFR FINAN	17%	Sec BANKS	16%	EMBI	13%
4	Gold EUR	24%	Gold ZAR	20%	Gold USD	16%
5	AFR TELECOM	13%	Sec BANKS	11%	Plat ZAR	8%
6	AFR CON GDS	21%	Sec CONSTR	11%	USDZAR (L3)	7%
7	AFR CON GDS	6%	Sec IND MET	6%	Sec GEN FIN	4%
8	Sec BANKS	8%	EMBI (L1)	8%	AFR BAS MAT	4%
9	RETAIL TRADE y	6%	Sec FOOD PR	5%	AFR CON GDS	5%
10	AFR HEALTH	5%	CPI m (L2)	5%	VEH. SALES m	4%
11	PETROL PRICE m (L2)	15%	OIL PRICE (L3)	9%	PETROL PRICE m (L1)	6%
12	PMI m	5%	Plat ZAR	4%	Plat EUR	4%
13	AFR TELECOM	18%	EURZAR (L3)	6%	USDZAR (L3)	5%
14	PMI m	9%	Sec GEN IND	7%	USDZAR (L2)	5%
15	EMBI (L2)	10%	MSCI-EM (L1)	6%	Brent USD	6%
16	AFR TELECOM	4%	RETAIL TRADE m	4%	Plat EUR	3%
17	AFR CON GDS	5%	EURZAR	4%	Sec MINING	3%
18	Brent EUR	6%	Brent ZAR	5%	6 month	4%
19	MSCI-EM (L2)	4%	VEH. SALES m (L2)	3%	CPI m (L3)	3%
20	AFR TECHNOL	9%	AFR TELECOM	4%	Alu EUR	4%
21	PETROL PRICE m (L2)	5%	PETROL PRICE m (L1)	5%	BUILDING y (L1)	4%
22	CPI m (L3)	7%	AFR TECHNOL	6%	3 Month (L1)	5%
23	CREDIT m (L2)	7%	PETROL PRICE m (L2)	6%	RETAIL TRADE m (L3)	5%
24	CPI m (L3)	7%	PMI m (L3)	5%	Sec IND MET	4%
25	Carhart's 4th	6%	MSCI-EM (L2)	4%	PMI m (L1)	4%
26	CREDIT m	5%	BUILDING m (L3)	4%	Brent EUR	3%
27	CREDIT m (L2)	7%	Alu EUR	7%	3 Month (L2)	7%
28	Brent USD	6%	EURZAR (L2)	5%	Com USD	5%
29	Alu EUR	7%	EURZAR (L1)	6%	Alu USD	6%
30	PPI m	8%	Com ZAR	4%	VEH. SALES m (L2)	4%
31	CPI y (L2)	6%	CPI y (L1)	6%	CPI y	5%
32	BUILDING y (L2)	7%	BUILDING y (L1)	4%	PETROL PRICE m (L1)	4%
33	EURUSD (L3)	10%	Sec IND MET	7%	BUILDING y (L3)	4%
34	Sec FOOD PR	7%	Value-Growth	6%	VEH. SALES m	5%
35	AFR TECHNOL	6%	PMI m (L3)	5%	3 Month (L2)	4%
36	CREDIT y (L3)	4%	LEAD. INDI m (L2)	4%	VEH. SALES m	4%
37	PETROL PRICE m (L2)	8%	OIL PRICE (L3)	7%	VEH. SALES m (L1)	4%
38	JALSH RSI	6%	RETAIL TRADE m (L3)	3%	BUILDING y	2%
39	VEH. SALES m (L2)	5%	CPI m (L1)	4%	Carhart's 4th	3%
40	3 Month (L2)	4%	CREDIT m	3%	USDZAR	3%
41	BUILDING y (L3)	9%	BUILDING y	8%	BUILDING m (L2)	6%
42	CPI m (L3)	7%	OIL PRICE m	5%	Cop ZAR	5%
43	CPI m (L1)	6%	PPI m (L1)	3%	CREDIT m (L1)	3%
44	PMI m	4%	RETAIL TRADE m (L3)	3%	USDZAR (L1)	3%
45	Com EUR	5%	AFR BAS MAT	4%	PPI m (L3)	4%
46	VEH. SALES y (L3)	4%	PMI m (L2)	3%	LEAD. INDI m (L2)	3%
47	BUILDING m (L2)	7%	CPI m	5%	EURZAR (L3)	4%
48	CREDIT m (L1)	4%	BUILDING y (L2)	4%	PPI m (L1)	4%
49	3 month	7%	VEH. SALES m (L3)	6%	6 month	6%
50	3 Month (L1)	8%	Carhart's 4th	7%	3 Month (L2)	5%
51	PMI m (L2)	8%	3 Month (L3)	5%	6 month	5%
52	EMBI	6%	USDZAR (L1)	5%	MSCI-EM (L2)	5%
53	BUILDING m (L3)	5%	15 year	3%	Brent ZAR	2%
54	AFR CON GDS	4%	AFR BAS MAT	4%	CPI m (L3)	3%
55	EURZAR (L3)	3%	EURZAR (L2)	3%	Cop ZAR	3%
56	PMI m (L2)	5%	Gold EUR	4%	MSCI-EM (L3)	3%
57	USDZAR (L1)	3%	LEAD. INDI m (L1)	3%	MSCI-EM (L1)	3%
58	RETAIL TRADE m (L3)	4%	3 Month (L3)	3%	CREDIT y (L1)	2%
59	EURUSD	4%	BUILDING m (L2)	4%	BUILDING m (L1)	4%
60	EURZAR (L2)	8%	USDZAR (L2)	6%	OIL PRICE (L2)	4%
61	LEAD. INDI m (L3)	7%	3 Month (L3)	6%	LEAD. INDI m (L2)	6%
62	VEH. SALES m	7%	PMI m	6%	3 year	5%
63	BUILDING m (L1)	4%	RETAIL TRADE m (L1)	3%	Cop ZAR	2%
64	MSCI-EM (L1)	4%	OIL PRICE (L2)	3%	CPI m (L2)	2%
65	Carhart's 4th	7%	VEH. SALES y (L2)	6%	RETAIL TRADE y (L3)	5%
66	CREDIT m (L1)	5%	PMI m (L1)	4%	Cop EUR	3%
67	VEH. SALES m (L3)	4%	EURZAR	3%	PPI y (L2)	3%
68	VEH. SALES m (L3)	6%	BUILDING m (L3)	3%	USDZAR (L2)	3%
69	BUILDING y (L2)	3%	CREDIT y (L1)	3%	EURZAR (L3)	3%
70	Sec GEN FIN	4%	PPI m (L2)	3%	Alu USD	3%
71	LEAD. INDI m (L1)	7%	PMI m (L2)	7%	VEH. SALES y (L3)	5%
72	LEAD. INDI m (L2)	7%	MSCI-EM (L2)	5%	LEAD. INDI m (L1)	5%
73	RETAIL TRADE m (L2)	5%	PETROL PRICE m	4%	VEH. SALES y (L2)	4%
74	CPI m (L2)	7%	CPI y	5%	CPI y (L2)	5%
75	PPI m (L2)	6%	PMI m (L1)	6%	CREDIT m (L2)	5%
76	CREDIT m	8%	PPI m	5%	CPI m (L1)	3%
77	VEH. SALES m (L2)	7%	LEAD. INDI m (L1)	5%	RETAIL TRADE m	5%
78	USDZAR (L3)	4%	EURUSD (L3)	4%	MSCI-EM (L3)	3%
79	PETROL PRICE m (L3)	4%	PPI y (L3)	3%	RETAIL TRADE m (L2)	3%
80	RETAIL TRADE m (L1)	10%	PETROL PRICE m (L1)	4%	CREDIT m (L1)	4%
81	PMI m (L3)	5%	OIL PRICE (L1)	5%	PETROL PRICE m	3%
82	PETROL PRICE m	3%	15 year	3%	Alu ZAR	3%
83	LEAD. INDI m (L1)	4%	OIL PRICE m	4%	OIL PRICE (L1)	3%
84	LEAD. INDI m	4%	Cop EUR	4%	PPI y	4%

6.2 Normalisation constraints

The effect of the choice of normalisation criteria on the components' properties is examined when rotating the components. Orthogonal rotations are conducted under the three different normalisation constraints mentioned in Section 3.2 above. In each case the correlations among the components are examined.

It is found that under both normalisations (1) and (2) the rotated components display correlations among each other. However, in all rotations performed, normalisation (3) produced an identity matrix as a correlation matrix after rotation. To illustrate this, the correlation matrices obtained when rotating the first three principal components under the three different normalisations are shown in Table 5 below. These were rotated using the varimax rotation criteria. The table shows that under normalisation (1) the three rotated principal components display a correlation of between 35 per cent and 55 per cent.

Table 5: Correlations of the rotated principal components under the three different normalisation criteria

Correlation matrix under normalisation criteria (1)

	PC1	PC2	PC3
PC1	1	0.42	0.55
PC2	0.42	1	0.35
PC3	0.55	0.35	1

Correlation matrix under normalisation criteria (2)

	PC1	PC2	PC3
PC1	1	0.74	0.84
PC2	0.74	1	0.66
PC3	0.84	0.66	1

Correlation matrix under normalisation criteria (3)

	PC1	PC2	PC3
PC1	1	0	0
PC2	0	1	0
PC3	0	0	1

The results after using normalisation (2) are even larger, with correlations between 66 per cent and 84 per cent. Only when normalisation (3) is used, the rotated principal components display zero correlation to one another. Similar results are observed when using quartimax rotation.

These findings are in line with Jolliffe’s (1995) theory that the application of the more popular normalisation criteria (2) will result in correlated components, while normalisation (3) will produce uncorrelated rotated principal components. Since this paper aims at obtaining uncorrelated rotated principal components, the optimal normalisation to use when orthogonally rotating the components is normalisation (3).

6.3 The number of components to retain

To determine how many components should be retained for rotation, the different tests mentioned in Section 3.1 are examined. The scree test plot is shown in Figure 1 below. The analysis of the test is subjective. Nevertheless, the plot does not show any significant change in slope. The only conclusion that can be drawn from the plot is that the first component clearly accounts for a larger amount of variation than the remaining components.

The more objective Kaiser’s rule is considered next. From Table 3 above it can be seen that the first 25 components have an eigenvalue above 1 and account for 78 per cent of the variation. Therefore, using Kaiser’s rule would result in 25 components being rotated. Any other rule (Fifield, Power and Sinclair, 2002; Jolliffe, 1972), such as requiring 80 per cent or more of the variation to be explained, will result in more than 25 principal components being retained. However, the first principal component is significantly larger and well separated from the remaining principal components, which are not all well separated. Therefore, rotating the first 25 principal components may result in the loss of the dominant source of variation found in the first principal component.

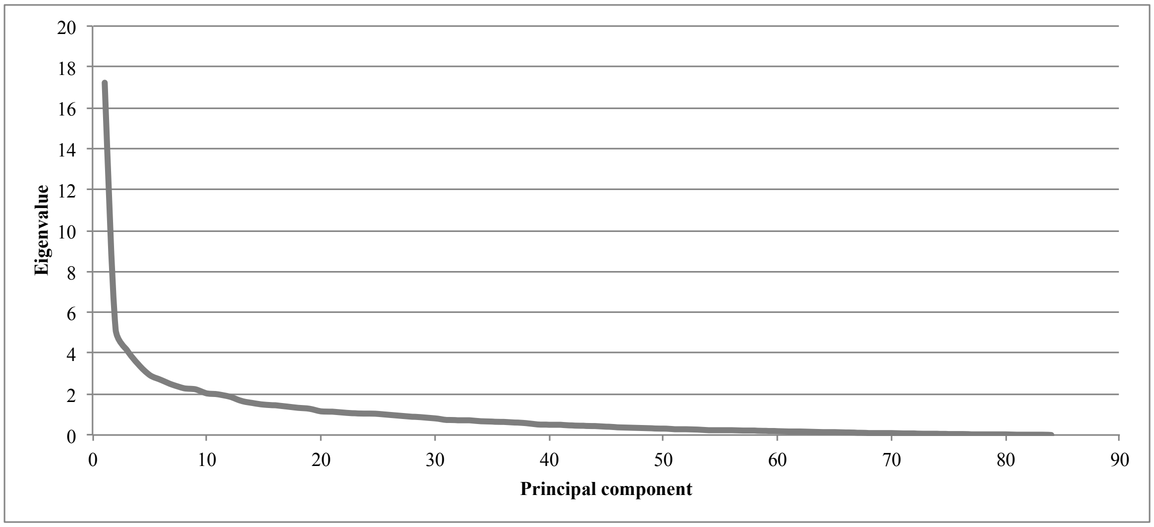

Figure 1: Plot of the eigenvalues of each principal component extracted

The resultant eigenvalues obtained after rotating the first 25 principal components using normalisation constraint (1) under varimax rotation are shown in Table 6 below. These findings clearly confirm the above notion.

As can be seen, while the rotated components still account for the same amount of variance in total, the first eigenvalue now only accounts for 7 per cent of the variation after rotation compared to the original 21 per cent. Additionally, after rotation the highest R-squared displayed for the first principal component is only 40.1 per cent. Thus, the initial dominant source of information found in this dataset is lost when rotating the first few principal components. Similar results are obtained when using normalisation (2) and (3). Therefore, the traditional rules and approaches used to decide how many components to retain and rotate are not optimal for this dataset.

Table 6: Eigenvalues of the rotated principal components

Principal component	Eigenvalue	Percentage of total variation	
		Individual	Cumulative
1	5.87	7.0%	7.0%
2	5.63	6.7%	13.7%
3	4.14	4.9%	18.6%
4	3.51	4.2%	22.8%
5	3.34	4.0%	26.8%
6	3.26	3.9%	30.7%
7	3.04	3.6%	34.3%
8	2.80	3.3%	37.6%
9	2.37	2.8%	40.4%
10	2.30	2.7%	43.2%
11	2.28	2.7%	45.9%
12	2.20	2.6%	48.5%
13	2.19	2.6%	51.1%
14	2.15	2.6%	53.7%
15	2.09	2.5%	56.2%
16	2.08	2.5%	58.6%
17	2.06	2.5%	61.1%
18	2.01	2.4%	63.5%
19	1.99	2.4%	65.8%
20	1.77	2.1%	67.9%
21	1.73	2.1%	70.0%
22	1.69	2.0%	72.0%
23	1.69	2.0%	74.0%
24	1.50	1.8%	75.8%
25	1.48	1.8%	77.6%
26	0.98	1.2%	78.8%
27	0.93	1.1%	79.9%
28	0.90	1.1%	80.9%
29	0.85	1.0%	81.9%
30	0.81	1.0%	82.9%
31	0.74	0.9%	83.8%
32	0.72	0.9%	84.6%
33	0.71	0.8%	85.5%
34	0.68	0.8%	86.3%
35	0.65	0.8%	87.1%
36	0.63	0.8%	87.8%
37	0.61	0.7%	88.5%
38	0.58	0.7%	89.2%
39	0.52	0.6%	89.9%
40	0.50	0.6%	90.5%
41	0.49	0.6%	91.0%
42	0.46	0.5%	91.6%
43	0.44	0.5%	92.1%
44	0.43	0.5%	92.6%
45	0.41	0.5%	93.1%
46	0.38	0.4%	93.6%
47	0.36	0.4%	94.0%
48	0.34	0.4%	94.4%
49	0.32	0.4%	94.8%
50	0.31	0.4%	95.1%
51	0.29	0.3%	95.5%
52	0.28	0.3%	95.8%
53	0.26	0.3%	96.1%
54	0.24	0.3%	96.4%
55	0.24	0.3%	96.7%
56	0.23	0.3%	96.9%
57	0.22	0.3%	97.2%
58	0.21	0.2%	97.4%
59	0.20	0.2%	97.7%
60	0.19	0.2%	97.9%
61	0.17	0.2%	98.1%
62	0.17	0.2%	98.3%
63	0.15	0.2%	98.5%
64	0.14	0.2%	98.7%
65	0.13	0.2%	98.8%
66	0.12	0.1%	99.0%
67	0.11	0.1%	99.1%
68	0.10	0.1%	99.2%
69	0.09	0.1%	99.3%
70	0.08	0.1%	99.4%
71	0.07	0.1%	99.5%
72	0.07	0.1%	99.6%
73	0.06	0.1%	99.6%
74	0.05	0.1%	99.7%
75	0.05	0.1%	99.8%
76	0.04	0.0%	99.8%
77	0.03	0.0%	99.8%
78	0.03	0.0%	99.9%
79	0.03	0.0%	99.9%
80	0.02	0.0%	99.9%
81	0.02	0.0%	100.0%
82	0.02	0.0%	100.0%
83	0.01	0.0%	100.0%
84	0.00	0.0%	100.0%

6.4 Rotation of subspaces

Next, the division of subspaces is considered. The tests detailed in Section 4.3 are conducted in an attempt to find well-defined subspaces that can be rotated in turn. The eigenvalues are examined and divided into appropriate groups according to the proximity to the other eigenvalues. This proximity is determined as follows.

First, the components are divided into subspaces according to their percentage differences. An arbitrary cut-off value of 9 per cent is used to determine the subspaces. Any two components that are less then 9 per cent apart are considered close and part of a subset. The resultant subsets are shown in Table 7 below.

Table 7: Subspaces created when using the percentage difference rule

Subspace	Principal components
1	5 - 12
2	13 -19
3	20 - 30
4	31 - 38
5	39 - 84

The table shows that using this test results in the first four principal components being considered well separated and do thus not form part of a subspace. According to Jolliffe’s (1989) theory, these four principal components would not need to be rotated. However, from Table 4 above it is clear that components two, three and four do not show any significant relationship to any factors. It is desirable to rotate these in order to find a better fit. Therefore, another subset is added to the above subsets, which consists of the more stable principal components two, three and four.

These subspaces are rotated in turn and the resultant relationships between the rotated components and the factors are shown in Table 8. The results found after quartimax rotation are very similar to those shown for varimax rotation. Since the first principal component was not rotated, it's eigenvalue and loadings do not change. Therefore its R-squared values remain the same. The highest R-squared found for the remaining principal components appear to have decreased after rotating the specified subspaces. None of the principal components have been rotated such that a better relationship to a factor is obtained. Therefore, rotation of these subspaces does not result in an optimal solution and an alternative test has to be considered.

The estimation rule derived by North et al. (1982) is examined next to determine the stability of principal components. This test results in fewer and larger subspaces than the previous test. The subspaces created after using 1.5 times the estimates are shown in Table 9 (the same subspaces are created when using 2 time the estimate). As can be seen in the table, this test only considers the first principal component to be well separated from the remaining components. All other components are divided into subspaces that can be rotated.

Table 8: Regression analysis results after rotating the first set of subspaces

Principal component	Factor 1	R-squared
1	Sec GEN RET	76%
2	Sec CONSTR	25%
3	3 year	18%
4	Gold EUR	14%
5	AFR TELECOM	15%
6	Sec BANKS	19%
7	3 Month (L3)	6%
8	Sec FOOD PR	4%
9	AFR CON GDS	21%
10	VEH. SALES m (L2)	6%
11	Sec IND MET	10%
12	OIL PRICE (L3)	8%
13	AFR TELECOM	19%
14	Brent ZAR	7%
15	Brent EUR	10%
16	EURUSD	5%
17	EURUSD (L2)	6%
18	PMI m	7%
19	MSCI-EM (L3)	5%
20	AFR TECHNOL	14%
21	CPI m (L3)	7%
22	PETROL PRICE m (L3)	10%
23	PPI m (L3)	5%
24	CREDIT m (L1)	10%
25	Alu ZAR	7%
26	Sec IND MET	8%
27	RETAIL TRADE m (L3)	6%
28	Alu ZAR	9%
29	PPI m (L2)	6%
30	BUILDING m (L3)	8%
31	RETAIL TRADE y (L2)	6%
32	MSCI-EM (L2)	5%
33	Sec IND MET	10%
34	BUILDING y (L3)	8%
35	VEH. SALES m	7%
36	AFR TECHNOL	7%
37	Sec IND MET	4%
38	JALSH RSI	6%
39	LEAD. INDI m (L1)	5%
40	RETAIL TRADE m	6%
41	BUILDING m (L1)	8%
42	Carhart's 4th	4%
43	OIL PRICE (L2)	4%
44	VEH. SALES m (L1)	3%
45	VEH. SALES y (L3)	5%
46	LEAD. INDI m (L2)	5%
47	JALSH RSI	5%
48	6 month	6%
49	PETROL PRICE m (L3)	7%
50	3 Month (L1)	10%
51	OIL PRICE (L2)	8%
52	VEH. SALES m	6%
53	3 Month (L3)	4%
54	PMI m (L3)	3%
55	BUILDING y	5%
56	PMI m (L1)	7%
57	3 Month (L3)	3%
58	CREDIT m	7%
59	OIL PRICE m	3%
60	PMI m (L3)	7%
61	Cop EUR	4%
62	EURZAR (L3)	4%
63	USDZAR (L2)	6%
64	PETROL PRICE m (L1)	4%
65	6 month	6%
66	3 Month (L1)	4%
67	PPI m (L3)	7%
68	BUILDING y (L1)	5%
69	Alu EUR	7%
70	LEAD. INDI m (L2)	8%
71	3 Month (L3)	4%
72	USDZAR (L2)	6%
73	VEH. SALES m (L3)	5%
74	VEH. SALES m (L2)	5%
75	PMI m (L1)	5%
76	PPI m (L2)	12%
77	MSCI-EM (L2)	6%
78	RETAIL TRADE m	4%
79	CPI y	6%
80	PPI m	6%
81	BUILDING m	6%
82	EURUSD (L2)	6%
83	OIL PRICE m	6%
84	MSCI-EM (L2)	4%

Table 9: Subspaces created when using the North et al. (1982) test

Subspace	Principal components
1	2 - 75
2	76 - 80
3	81 - 84

The results after rotating the above subspaces are shown in Table 10 below. Due to the first subspace being very large, the total variance of the principal components is distributed more evenly among them. Once again, the interpretability of the remaining principal components does not appear to have materially improved after rotating the specified subspaces. Only the rotated components 11 and 16 show a significant relationship to the JSE Industrial Metals sector and African Telecommunication sector respectively. However, both of these components only account for 1 per cent of the total variation in the dataset. The components explaining a more significant amount of the variation are not explained by any of the factors. Similar conclusions are drawn after using the quartimax criteria to rotate these subspaces. Therefore, this division of subspaces also does not enable a better interpretation of the principal components after rotation.

An alternative division of subspaces will need to be found. Consider again the first subset that was rotated (components 2 – 4, 5 – 12, 13 – 19, 20 – 30, 31-38, 39 – 84). The rotation of the first subset (components two, three and four) did not improve the interpretability of the components. Therefore, the first component will now be added to this first subspace, while the remaining subsets remain the same. The results after rotating these subsets using the varimax method are obtained and analysed.

Table 10: Regression analysis results after rotating the subspaces: 2 – 75, 76 – 80, 81 – 84

Principal component	Factor 1	R-squared
1	Sec GEN RET	76%
2	PMI m (L1)	8%
3	Sec BANKS	23%
4	CPI m (L1)	7%
5	Alu ZAR	5%
6	OIL PRICE (L2)	4%
7	AFR CON GDS	18%
8	VEH. SALES y (L1)	10%
9	PPI y (L3)	5%
10	EURUSD (L3)	6%
11	Sec IND MET	52%
12	PETROL PRICE m (L3)	4%
13	CPI m (L3)	5%
14	PMI m (L2)	8%
15	BUILDING y (L1)	11%
16	AFR TELECOM	57%
17	Gold EUR	11%
18	AFR HEALTH	14%
19	AFR TECHNOL	37%
20	Sec GEN IND	10%
21	BUILDING y (L3)	7%
22	EMBI (L1)	24%
23	VEH. SALES y (L2)	10%
24	PPI m	8%
25	CPI y	7%
26	BUILDING m (L3)	6%
27	PPI m (L2)	9%
28	PETROL PRICE m	6%
29	EURZAR (L2)	7%
30	VEH. SALES y (L3)	4%
31	3 month	3%
32	3 Month (L3)	7%
33	3 month	4%
34	Sec FOOD PR	16%
35	BUILDING m (L1)	10%
36	Sec CONSTR	5%
37	Carhart's 4th	12%
38	EURZAR (L2)	6%
39	AFR HEALTH	12%
40	AFR HEALTH	5%
41	PPI y	5%
42	BUILDING y (L1)	5%
43	EMBI (L3)	7%
44	OIL PRICE (L1)	4%
45	AFR CON GDS	15%
46	Cop EUR	5%
47	3 month	6%
48	Alu EUR	9%
49	EMBI	9%
50	EURZAR (L3)	7%
51	CPI y	8%
52	PETROL PRICE m (L2)	7%
53	Sec BANKS	7%
54	EURUSD	4%
55	EURZAR (L3)	5%
56	CREDIT m (L1)	6%
57	PMI m (L3)	8%
58	Alu EUR	7%
59	AFR CON GDS	9%
60	USDZAR (L1)	10%
61	Gold ZAR	21%
62	MSCI-EM (L3)	4%
63	CPI m	8%
64	Sec CONSTR	9%
65	Sec CONSTR	13%
66	CREDIT m (L2)	6%
67	PPI m (L1)	4%
68	BUILDING y (L3)	8%
69	Cop ZAR	7%
70	EMBI	6%
71	Gold EUR	5%
72	3 Month (L2)	6%
73	Sec GEN RET	5%
74	MSCI-EM (L2)	6%
75	PPI m (L2)	8%
76	CREDIT m	6%
77	EURUSD (L2)	5%
78	LEAD. INDI m (L1)	5%
79	RETAIL TRADE m (L3)	5%
80	RETAIL TRADE m (L1)	8%
81	PETROL PRICE m	6%
82	OIL PRICE (L1)	5%
83	OIL PRICE m	3%
84	3 Month (L3)	4%

Table 11: Regression analysis results after rotating (varimax) the subspaces:

1 - 4, 5 - 12, 13 - 19, 20 - 30, 31 - 38, 39 - 84

Principal component	Factor 1	R-squared
1	AFR INDUST	51%
2	AFR CON SRV	46%
3	AFR FINAN	45%
4	Gold ZAR	21%
5	AFR TELECOM	15%
6	Sec BANKS	19%
7	3 Month (L3)	6%
8	Sec FOOD PR	4%
9	AFR CON GDS	21%
10	VEH. SALES m (L2)	6%
11	Sec IND MET	10%
12	OIL PRICE (L3)	8%
13	AFR TELECOM	19%
14	Brent ZAR	7%
15	Brent EUR	10%
16	EURUSD	5%
17	EURUSD (L2)	6%
18	PMI m	7%
19	MSCI-EM (L3)	5%
20	AFR TECHNOL	14%
21	CPI m (L3)	7%
22	PETROL PRICE m (L3)	10%
23	PPI m (L3)	5%
24	CREDIT m (L1)	10%
25	Alu ZAR	7%
26	Sec IND MET	8%
27	RETAIL TRADE m (L3)	6%
28	Alu ZAR	9%
29	PPI m (L2)	6%
30	BUILDING m (L3)	8%
31	RETAIL TRADE y (L2)	6%
32	MSCI-EM (L2)	5%
33	Sec IND MET	10%
34	BUILDING y (L3)	8%
35	VEH. SALES m	7%
36	AFR TECHNOL	7%
37	Sec IND MET	4%
38	JALSH RSI	6%
39	LEAD. INDI m (L1)	5%
40	RETAIL TRADE m	6%
41	BUILDING m (L1)	8%
42	Carhart's 4th	4%
43	OIL PRICE (L2)	4%
44	VEH. SALES m (L1)	3%
45	VEH. SALES y (L3)	5%
46	LEAD. INDI m (L2)	5%
47	JALSH RSI	5%
48	6 month	6%
49	PETROL PRICE m (L3)	7%
50	3 Month (L1)	10%
51	OIL PRICE (L2)	8%
52	VEH. SALES m	6%
53	3 Month (L3)	4%
54	PMI m (L3)	3%
55	BUILDING y	5%
56	PMI m (L1)	7%
57	3 Month (L3)	3%
58	CREDIT m	7%
59	OIL PRICE m	3%
60	PMI m (L3)	7%
61	Cop EUR	4%
62	EURZAR (L3)	4%
63	USDZAR (L2)	6%
64	PETROL PRICE m (L1)	4%
65	6 month	6%
66	3 Month (L1)	4%
67	PPI m (L3)	7%
68	BUILDING y (L1)	5%
69	Alu EUR	7%
70	LEAD. INDI m (L2)	8%
71	3 Month (L3)	4%
72	USDZAR (L2)	6%
73	VEH. SALES m (L3)	5%
74	VEH. SALES m (L2)	5%
75	PMI m (L1)	5%
76	PPI m (L2)	12%
77	MSCI-EM (L2)	6%
78	RETAIL TRADE m	4%
79	CPI y	6%
80	PPI m	6%
81	BUILDING m	6%
82	EURUSD (L2)	6%
83	OIL PRICE m	6%
84	MSCI-EM (L2)	4%

This division of subspaces appears to produce better results. Although the first principal component has lost a large amount of its variation, it still accounts for 9 per cent of the total variation. The second and third rotated principal components now account for a larger amount of variation, namely 9 per cent and 8 per cent respectively. Each of the rotated components' associated R-squared values are displayed in Table 11 below. While the first components displays a lower R-squared after rotation (51 per cent), this is still a relatively significant relationship to the African Industrials sector. By distributing the variance of this first component more evenly among components in the subspace, components two and three appear to be more related to the factors than previously. Component two now displays a 46 per cent R-squared for the African Consumer Services sector, while the third component shows a relationship to the African Finance factor (with an R-squared of 45 per cent). While these relationships are not highly significant, they can be considered large enough. The remaining rotated principal components do not show any significant relationships with any factors.

While for most of the previous subspace rotations the varimax criteria and quartimax criteria have produced very similar results, this is not so for the last group of subsets. After rotating the first four components using quartimax rotation, the variance explained by the first principal component does not change significantly and its relation to the General Retail sector remains significant at 75 per cent. However, since this rotation does not redistribute much of the dominant source of variance, the remaining components explain almost the same amount of total variation. As a result, components two and three do not display any significant R-squared values after quartimax rotation. Similar results are observed when rotating the first 25 components.

Therefore, it appears that the results obtained after rotating components under different rotation algorithms will not differ greatly when subsets of components are rotated that have close eigenvalues. However, the choice of rotation criteria may impact the components very differently when a subspace of well-separated components is rotated. From the results it appears that the varimax rotation criteria redistributed most of the variance among the rotated components more evenly. On the other hand, quartimax rotation does not result in a large redistribution of total variance. Thus, the varimax rotation is more effective for this dataset, since the first principal component extracted accounts for a large amount of variation in comparison to the remaining components. This variance has to be redistributed among a few components to obtain rotated components, which are significant as well as easier to interpret.

Therefore, this analysis shows that the African Industrials, Consumer Services and Financial sectors can explain the three main factors driving the JSE stock returns over the period. These findings are different to those of van Rensburg (2002), who finds that the Financial and Industrial Index and the Resources Index best explain the stock returns on the JSE. However, in his study van Rensburg (2002) rotates the principal components using an oblique promax rotation. Thus, the factors he finds are likely to be correlated, unlike the factors extracted in this study. In addition, van Rensburg (2002) looks at the returns over the period from 1993 to 2000, while this paper analyses the returns over the period 2003 till 2013. These differences could be the reason for the differing results obtained. It also highlights how the South African stock market has changed over the past decade.

The JSE's largest sectors by market capitalisation are the Resources, Consumer Services, Financials and Industrials. Historically (mainly over the period van Rensburg (2002) analysed), the resources sector and financial sector have dominated the stock market. Despite the resource sector still constituting a large part of the market, in more recent years this sector has underperformed in contrast to other sectors. While the financial sector has produced good returns, the industrial sector and consumer services sector in particular have gained a large amount of market capitalisation and these shares have shown significant growth in recent years. The results of this analysis confirm this by indicating that most of the volatility over the period from 2003 to 2013 can be explained by the industrials, consumer services and financial sectors.

7 Conclusion

This paper examines rotation routines that will produce uncorrelated rotated principal components for a dataset of stock returns, in an attempt to identify the macroeconomic factors that best explain the variability among risk-adjusted stock returns on the Johannesburg Stock Exchange.

Principal component analysis is first conducted on the risk-adjusted stock return series. The extracted principal components are then regressed onto the macroeconomic factor series. The initial principal components extracted do not display significant relationships to the factors and interpreting these principal components is difficult. Thus, orthogonal rotation is employed in an attempt to make the principal components easier to interpret. It is found that for this dataset, the more traditional methods of deciding how many principal components to retain and rotate do not improve the interpretability of the rotated components. In addition, the choice of normalisation constraint applied to the data influences the correlation among the principal components. Only one of the three normalisation constraints examined can retain the uncorrelatedness property after rotation. This normalisation constraint is used for the rest of the paper.

An alternative rotation approach is used in this paper. Subsets of principal components with similar variances are created and rotated in turn. This method appears to produce the best results for the dataset. The results also show that the choice of rotation criteria used to rotate principal components will affect the resultant rotated components if the eigenvalues are well-separated. However, when subspaces of components are rotated that have close eigenvalues, the rotation criteria will produce similar results.

After rotating the suitable subsets using varimax rotation, it is found that the first component can be explained by the African Industrials sector, the second component is related to the African Consumer Services sector while the third component shows a significant relationship to the African Finance factor.

8 Further research

There is one main improvement that can be made in future analysis. Among the PCA literature there is agreement that the number of observations to included in a sample must be more than the number of variables being analysed. However, there appears to be no consensus as to what the optimal number of observations should be. According to Curto, Pinto and Fernandes (2006) a general rule is to have a number of observations that is at least five times the number of variables. The sample in this study does not comply with this rule due to the large number of stocks being analysed. Using a larger set of observed stock returns (i.e. using weekly or daily rather than monthly returns) may improve the accuracy of the results obtained from this analysis.

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Appendix A

The properties of rotated principal components – a theoretical proof

The following proof is an expansion of Jolliffe's (1989) original proof.

Let $\boldsymbol{A}$ be the $p \times m$ matrix where column i is the vector of loadings $(a_{i1}, a_{i2}, \dots, a_{ip})$ for the i th principal component. The matrix of principal component scores $\boldsymbol{Z}$ is therefore defined as $\boldsymbol{Z} = \boldsymbol{XA}$, where $\boldsymbol{X}$ is the original dataset. Lastly, define the matrix $\boldsymbol{L}^2$ to have diagonal elements equal to the eigenvalues of $\boldsymbol{X'X}$.

The principal components obtained from performing this initial principal component analysis will have the following properties:

1. The loadings are orthogonal, since
$$\boldsymbol{A'A} = \boldsymbol{I}$$
where $\boldsymbol{I}$ is the identity matrix
2. The principal components are uncorrelated, since
$$\begin{aligned}\boldsymbol{Z'Z} &= \boldsymbol{A'}\mathrm{X'XA}\\ &= \boldsymbol{A'AL}^2\boldsymbol{A'A}\\ &= \boldsymbol{L}^2\end{aligned}$$
since $\boldsymbol{A}$ is orthogonal.

These principal components are then rotated using an orthogonal rotation technique. The new matrix of rotated loadings $\boldsymbol{B}$ is found such that $\boldsymbol{B} = \boldsymbol{AT}$ where $\boldsymbol{T}$ is the orthogonal transformation matrix. The rotated matrix of principal component scores therefore becomes

$$\begin{aligned}\boldsymbol{Z}^R &= \boldsymbol{XB}\\ &= \boldsymbol{XAT}\\ &= \boldsymbol{ZT}\end{aligned}$$

The properties of the rotated principal components are derived using three different normalisations. The first normalisation (1) is $\boldsymbol{A}_i'\boldsymbol{A}_i = \boldsymbol{1}$ or $\boldsymbol{A}'\boldsymbol{A} = \boldsymbol{I}$. The properties obtained after orthogonal rotation are as follows:

1. The rotated loadings are still orthogonal, since

$$\begin{aligned} \boldsymbol{B}'\boldsymbol{B} &= \boldsymbol{T}'\boldsymbol{A}'\boldsymbol{A}\boldsymbol{T} \\ &= \boldsymbol{T}'\boldsymbol{T} && \text{since } \boldsymbol{A}'\boldsymbol{A} = \boldsymbol{I} \\ &= \boldsymbol{I} && \text{since } \boldsymbol{T} \text{ is orthogonal} \end{aligned}$$

2. The rotated components are no longer uncorrelated, since

$$\begin{aligned} \boldsymbol{Z}'^R\boldsymbol{Z}^R &= \boldsymbol{T}'\boldsymbol{Z}'\boldsymbol{Z}\boldsymbol{T} \\ &= \boldsymbol{T}'\boldsymbol{A}'\boldsymbol{X}'\boldsymbol{X}\boldsymbol{A}\boldsymbol{T} \\ &= \boldsymbol{T}'\boldsymbol{A}'\boldsymbol{A}\boldsymbol{L}^2\boldsymbol{A}'\boldsymbol{A}\boldsymbol{T} \\ &= \boldsymbol{T}'\boldsymbol{L}^2\boldsymbol{T} \end{aligned}$$

   which is not diagonal.

Next, the properties after rotating the principal components using normalisation (2) are examined. This normalisation specifies that $\boldsymbol{A}_i'\boldsymbol{A}_i = l_i$ or $\widetilde{\boldsymbol{A}} = \boldsymbol{A}\boldsymbol{L}$ and results in the following:

1. The loadings are not orthogonal after rotation, since

$$\begin{aligned} \boldsymbol{B}'\boldsymbol{B} &= \boldsymbol{T}'\widetilde{\boldsymbol{A}}'\widetilde{\boldsymbol{A}}\boldsymbol{T} \\ &= \boldsymbol{T}'\boldsymbol{L}'\boldsymbol{A}'\boldsymbol{A}\boldsymbol{L}\boldsymbol{T} \\ &= \boldsymbol{T}'\boldsymbol{L}^{\boldsymbol{2}}\boldsymbol{T} \end{aligned}$$

which is not diagonal.

2. The rotated components are correlated, since

$$\begin{aligned} \mathbf{Z}'^R\mathbf{Z}^R &= \mathbf{T}'\mathbf{Z}'\mathbf{Z}\mathbf{T} \\ &= \boldsymbol{T}'\widetilde{\boldsymbol{A}}'\boldsymbol{X}'\boldsymbol{X}\widetilde{\boldsymbol{A}}\boldsymbol{T} \\ &= \boldsymbol{T}'\widetilde{\boldsymbol{A}}'\boldsymbol{A}\boldsymbol{L}^{\boldsymbol{2}}\boldsymbol{A}'\widetilde{\boldsymbol{A}}\boldsymbol{T} \end{aligned}$$

$$= \boldsymbol{T'L'A'AL^{2}A'ALT}$$
$$= \mathbf{T'L^{4}T}$$

which is not diagonal.

Lastly, Jolliffe (1989) shows that the normalisation required in order to retain the property of uncorrelated components after rotation is normalisation (3). It divides each column of $\boldsymbol{A}$ by the square root of its eigenvalue such that $\boldsymbol{A}_i'\boldsymbol{A}_i = l_i^{-1}$ or $\tilde{\tilde{\boldsymbol{A}}} = \boldsymbol{AL^{-1}}$. This finding is derived as follows:

1. The loadings are not orthogonal after rotation, since

$$\boldsymbol{B'B} = \boldsymbol{T'}\tilde{\tilde{\boldsymbol{A}}}'\tilde{\tilde{\boldsymbol{A}}}\boldsymbol{T}$$
$$= \boldsymbol{T'L^{-1'}A'AL^{-1}T}$$
$$= \boldsymbol{T'L^{-2}T}$$

which is not diagonal.

2. However, the rotated components remain uncorrelated , since

$$\mathbf{Z}'^{R}\mathbf{Z}^{R} = \mathbf{T'Z'ZT}$$
$$= \boldsymbol{T'}\tilde{\tilde{\mathrm{A}}}'\boldsymbol{X'X}\tilde{\tilde{\mathrm{A}}}\boldsymbol{T}$$
$$= \boldsymbol{T'}\tilde{\tilde{\mathrm{A}}}'\boldsymbol{AL^{2}A'}\tilde{\tilde{\mathrm{A}}}\boldsymbol{T}$$
$$= \boldsymbol{T'L^{-1'}A'AL^{2}A'AL^{-1}T}$$
$$= \mathbf{T'T}$$
$$= \mathbf{I}$$