Investigation of the Impact of Demand Elasticity and System Constraints on Electricity Market Using Extended Cournot Approach

by

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Abstract

Since late 1990s, privatization and deregulation in the South African Electricity supply industry (ESI) have gradually been underway. Due to the issues which arose in the international markets, such as the case of UK liberalization, the level of market power was underestimated although the capacity divestments have been substantial since the early 1990s. Therefore, the deregulation process is now at the stage of intensive research and planning.

There are a number of models that are used to investigate the competition in the electricity market. They include Bertrand model, Cournot model, Supply function equilibrium model, conjectural variations model, etc. These models differ in their modeling methodology and assumptions. As the quantity that is produced by independent power producers is a key market indicator, the classical Cournot competition under game theory has been often used in modeling the case with the existence of transmission constraints to identify strategic behaviors of the market participants. However, the classical Cournot model focuses on finding the Nash equilibrium as a solution. In this case, there is a high possibility of mismatch between the supply and demand of power. No power producer intends to move away from the Nash equilibrium. The participation of demand side and the demand elasticity of power demand are underestimated.

In today's electricity supply industry, demand side participation is considered an important factor that can influence the market performance and output effectively. Demand elasticity shows the sensitivity of demand side to the market price, and thus can provide potential adjustment of demand in the market.

The purpose of this research is to study the impact of demand elasticity on power producers' market competition output. An analytical model, called "Extended Cournot model" is developed in this thesis based on the classical Cournot model. Through the integration with conjectural variation model, in which power producers consider both the generation and price level, the extended Cournot model can analyze electricity market results under the conditions of different constraints.

In the classical Nash Cournot model, capacity withdrawal exists in most cases especially when transmission constraint occurs. In contrast, the newly developed analytical model ensures that demand is always satisfied at all time. Demand elasticity is incorporated directly into the market results calculation instead of using the market clearing price. This approach enables the load demand to directly obtain the market results by tuning its demand elasticity. The intention is to show that demand side should be more encouraged to participate in the market competition. In the classical economic dispatch, the load demand is highly inelastic. From the load curve, there is only a change of physical volume of demand. The demand responsiveness, which is represented by demand elasticity, has been understated.

In this thesis, the hypothesis is that demand elasticity and system constraint have critical influence on the power producers' competition results in terms of market clearing price, individual output and profit. Load demand can make use of demand elasticity to affect its final payment to the market. Such ability is expected to be limited in the case where system constraints, i.e. generation limits and transmission limits, exist. For simplicity, a small network and number of power producers are used in this thesis to investigate the effectiveness of the Extended Cournot model. However, this model can be applied to more complex networks with different market environments.

In order to compare the different impact of load demand's influence on market competition, the market results are calculated based on two cases, i.e. change of physical load demand with constant demand elasticity and change of demand elasticity with constant physical load demand. In each case, three scenarios as described below were modeled:

1) without generation or transmission constraints,
2) with generation constraints, and
3) with generation and transmission constraints.

Therefore, in total six case studies were conducted to analyze the combined impact of demand elasticity and system constraints on power producers' market results together with load payment. The optimization problem is solved based on the profit maximization of all power producers.

The market indicators used for the investigation include:

- Market clearing price
- Power producers’ output
- Power producers’ profit
- Load payment

To investigate the impact of the change in demand elasticity on the marginal change of price and power producers’ quantity, additional two indicators are considered:

- Derivative of market clearing price over demand elasticity
- Derivative of individual production over demand elasticity

In the case of change in total demand, the results show that change in total quantity does not affect market price, in the scenario without constraints. In contrast, market clearing price is determined by the total quantity in the traditional Cournot model. However, the price will change when generation and transmission constraints reach a certain extent. This is mainly due to the fact that the supply of the most economical power producer’s capability to supply the load demand is diminished due to the above constraints. Any additional demand will be allocated to the less economical power producer with high marginal cost. For the same reason, power producers’ profits and load payment will increase when constraints are introduced.

To validate the above hypothesis, simulations were performed in this thesis. The analytical model is validated by comparing its results to the results obtained using an industrial electricity market simulation tool, known as “Plexos”.

Plexos has the options to include demand elasticity to model Cournot competition. Plexos also allows to introduction generation and transmission limits in its input date session. Both the analytical model and Plexos simulation use the Nash Cournot competition approach. The simulation results show that the analytical model and Plexos software package are in agreement.

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For some of the simulated scenarios, such as the one with both generation and transmission constraints, there are variations in the results between the analytical model and Plexos output. The discrepancies in the results are mainly caused by the variations of input data assumptions in the modeling of load demand.

It is shown that the assumptions on marginal costs have significant influence on the power producers' production pattern in different scenarios.

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Notation

Roman symbols

a	Intercept of the demand function
d_i	Percentage of Q_i goes through line L_{jk}
b	Slope of the demand function
C_i	Marginal cost of GenCo$_i$
$\overline{C}$	Average marginal cost
n	Number of GenCos in the market
P	Market clearing price
Q_i	Individual output of GenCo$_i$
Q	Total production output
$Q_i^{\max}$	GenCo$_i$'s maximum generation capacity
T_{jk}	Transmission capacity of line L_{jk}

Greek symbols

α	The multiplier determined by the relationship between $\sum_{i=1}^{n} d_i Q_i$ and T_{jk}.
π_i	Individual profit of GenCo$_i$ for $i = 1,2,3 \ldots n$
ε	Demand elasticity
μ_i	The multiplier determined by the relationship between Q_i and $Q_i^{\max}$,

Abbreviations

DG	Distribution generation
ECM	Extended Cournot model
GenCo	Generation company

HHI	Herfindahl-Hirschman Index
ISO	Independent system operator
KKT	Karush-Kuhn-Tucker
LMP	Locational marginal price
MPE	Mathematical program with equilibrium
NEM	National electricity market
PIPA	Penalty Interior Point Algorithm
RSM	Response surface method
RTOs	Regional transmission organizations

Chapter 1

Introduction

1.1 Competition in the electricity market

The electricity supply industry has been undergoing continuous deregulation over the last twenty years. During this restructuring process, the exercise of market power has been found in a number of electricity markets [1]. Market power is usually defined as a market supplier that owns the ability to drive the price above the competitive level, or control output for a significant period of time. The exercise of market power creates inefficiencies in both resource production and allocation. The most common effects of market power include less production in the market which results in lower consumption, and higher price levels compared to marginal cost. One of the main threats of market power is diminishing consumer surplus which transfers social benefits from the consumers to the suppliers [1].

Many methods have been developed to assess market power, such as Lerners index and the Herfindahl-Hirschman Index (HHI index) [1, 2]. More recent research studies have raised the concern of lack of market dynamics in the above indices. It has been found in many studies that generators are more likely to exercise market power within the presence of potential network constraints [3, 4].

According to the market mechanism, the electricity markets have been defined by two main models: the PoolCo model and the Bilateral model [1, 3]. The PoolCo model is more centralized, in which the independent system operator determines the economics of dispatch and maintains system security. The Bilateral model is a decentralized mechanism in which the market determines the economic dispatch and electricity price. A hybrid model, which is a combination of the PoolCo and Bilateral models, has also been suggested as a solution to complement the central dispatch with bilateral contracts [1].

It is found that generators have more potential to behave strategically in the PoolCo model compared to the Bilateral model. The strategic behavior can be modeled through imperfect competition using game theory. Previous research has shown that generators can influence the transmission constraints and market price with their generation output decisions [2, 3, 4, 5].

1.2 Game theoretical approach in modeling electricity competition

The basic strategies for generators to play in a market include price, quantity, and supply function. Under the regulations, any explicit price increase by generators will be very sensitive and might be penalized by the system operators or regulators. On the other hand, changing the generation supply curve is more implicit and less likely to be detected. The most usual theory used to analyze price and quantity gaming and competition in electricity markets is game theory [6].

In game theory, a pricing game is usually defined as a Bertrand game, and a quantity game is defined as a Cournot game. Players in the Bertrand game will usually raise their bids above their marginal costs. For the classic Bertrand model, there are a number of assumptions as follows [7]:

1) Product homogeneity and identical unit costs.
2) No capacity constraints.
3) Generation Companies (GenCos) choose price simultaneously.
4) Competitors have the incentive to undercut each other’s prices vigorously, which results in a competitive outcome.

However, when network constraints exist, Generation Companies may not choose to price at a competitive level. Therefore, it is believed that the nature of the above assumptions may affect market equilibrium significantly. In the case of high demand together with the existence of significant constraints, Cournot competition is often used to predict power producers' behavior [7].

The classic Cournot model of competition enables the power producers to compete against each other with quantity strategies. The assumptions under classic Cournot competition are as follows [7]:

1) Homogeneous products and identical unit production costs.
2) GenCos bid once in the market and choose quantities non-cooperatively and simultaneously.
3) A price will be determined that equates demand and supply.

The first assumption is the same as that of the Bertrand game. The implication of the second assumption is that power producers choose their production level based on their own interest, and such choice is independent of the choices of other power producers. This is why it is known as a non-cooperative game. The choice each power producer makes is also based on the anticipation of other power producers' quantities.

The Cournot model utilizes two functions, the profit function and demand function. The profit function includes the market price, total and individual production, and individual marginal cost. When demand function is introduced, the first order of profit maximization will yield the results of individual quantities. Once quantities are chosen, market price will be determined. When there is only one unique equilibrium, a Nash-Cournot equilibrium exists.

Although the outcome of the classic Cournot competition model is often questionable as the supply side ultimately chooses the price level, Cournot equilibrium is still favored in investigating imperfect competition in various industries, including the electricity market. The withholding of output by the power producers in order to raise the market price is a typical feature in Cournot competition.

Nash-Cournot equilibrium is a well-known strategy adopted by market players during market competition [8]. This thesis focuses on the quantity game which is the foundation of Nash-Cournot equilibrium. The reason for choosing the Nash-Cournot competition model is that it focuses on quantity strategies which will influence market price.

One of the main features of Cournot competition is its ability to incorporate quantity as part of price in a demand function. In the demand function, price is a function of quantity demanded in the market. The function describes an inverse relationship between price and quantity. This creates opportunities for the generators to withhold capacity in order to raise the market price. Depending on the level of elasticity of demand, the reduction in generation will result in a relatively greater increase in price and the profit of the individual generator will increase.

Both demand function and elasticity of demand in Cournot competition have important implications in electricity market modeling. The Cournot model is found to be applicable for interpreting the electricity market features and has been widely used to represent generator behavior such as the matrix approach that is used in [9] to solve a three-player game. However, it is argued that the presence of capacity constraints should not be the necessary motivation for Cournot modeling. This means that, in most studies that are using Cournot competition to analyze generator behaviors, capacity constraint has not been considered as a constraint in the models.

More realistically, there will be arbitragers, or traders, involved in the electricity market trading process. More attention should be paid to this as the Cournot competition includes conjectural variation, in which generators decide the generation and price level on the assumption that arbitragers will not change their purchases. The arbitragers are treated exogenously in the Cournot Conjectural variation model [10].

In the classical Nash Cournot model, capacity withdrawal exists in most cases, especially when transmission constraint occurs. On the contrary, the analytical model ensures that demand is always satisfied at all times. Demand elasticity is incorporated directly into the market results calculation instead of using the market clearing price. This approach enables the load demand to directly obtain the market results by tuning its demand elasticity. The intention is to show that there should be more encouragement of demand side participation in market competition. In the classical economic dispatch, the load demand is highly inelastic. From the load curve, there is only change in physical volume of demand. The demand responsiveness, which is represented by demand elasticity, has been understated.

1.3 Hypothesis

The research hypothesis is that demand elasticity and system constraints will have a critical influence on the power producers' competition results in terms of market clearing price, individual output and profit. Load demand can utilize demand elasticity to affect its final payment to the market. Such ability is expected to be limited in the case where system constraints, i.e. generation limit and transmission limit, exist. Extended Cournot model can be used for analyzing the impact of demand elasticity and system constraints on electricity market outputs.

1.4 Contributions of this thesis

From the literature review, it can be concluded that the impact of different types of constraints on GenCo market strategies has not been sufficiently addressed. In addition, the impact of demand elasticity has not been fully investigated.

The purpose of this thesis is to develop an Extended Cournot model (ECM) based on conjectural variations models. One of the objectives of this thesis is to use elasticity in the ECM with different constraints, such as generation and transmission constraints. The main advantage of ECM is the ability to interpret the relationship between elasticity of demand and other key market indicators, for example, market price, individual generation, and individual profit. With the introduction of constraints, the different impact of elasticity on the market can be shown [61], [62], [63], [64], and [65].

The main contributions of this thesis are summarized as follows.

- The Extended Cournot model developed in this thesis considers more factors affecting the outcomes of market competition, such as demand elasticity, generation capacity constraint, transmission constraint as compared to the classical Cournot model. As a result, the importance of demand responsiveness has been highlighted and investigated. Significant impact of demand elasticity on the market outputs has been identified.

- This thesis has provided a tool for quantifying the impact of the demand side to the market outputs, which includes market clearing price, individual production and profit, and load payment. In the case of change in demand elasticity, the proportional change of market price and individual production against demand elasticity has been quantified. The results show how demand side participation can effectively influence the market outputs. It is also found that, when the generation and transmission constraints are implemented, the impact of demand elasticity on the market outputs will be affected.

- It has been found that demand elasticity has much higher impact on the market clearing price at its lower bound session. This indicates that the demand side can effectively reduce the market clearing price by becoming more responsive.

- The Extended Cournot model proposed in this thesis can be used to analyze imperfect competition on a scenario-to-scenario basis, given specific assumptions. In a scenario without any system constraints, this model can be used to predict realistic market results. Therefore, unlike the classical cournot model, the Extended Cournot model can be applied to situations with and without system constraints.

1.5 The contents of this thesis

This thesis is organized as follows:

Chapter 2 gives a comprehensive literature review on the issues of electricity market competition, game theory, Nash Cournot competition, network congestion, and demand elasticity.

Chapter 3 presents the derivation of the Extended Cournot model which is integrated with elasticity of demand. It includes two cases, namely, change of total demand, and change of demand elasticity. System constraints have been investigated separately as generation and transmission capacity constraints.

Chapter 4 analyses the calculation results with two cases: 1) change in total demand and 2) change in demand elasticity. For each case, three scenarios were investigated, i.e., i) without constraint, ii) with generation capacity constrain, iii) with generation and transmission constraints,. The purpose is to show the different impact demand elasticity has on market results with and without system constraints during market competition.

Chapter 5 compares the simulation results from a market simulator, Plexos, with the results from Chapter 4. Plexos has the option to include demand elasticity when modeling market competition. Therefore, it is a practical to compare the results from the Extended Cournot model with the ones from Plxos.

Chapter 6 gives the conclusions and discusses the recommendations for future potential research.

Chapter 2

Literature review

This literature review has covered a number of market-related issues in the electricity supply industry. First, market competition, which includes pricing and transmission congestion analysis, is reviewed as these marketing elements are the foundation of the research hypothesis. Second, Game Theory and Nash Cournot competition and their application to the electricity market are discussed in order to compare them with the Extended Cournot model proposed in this thesis. Third, demand responsiveness that has been considered as an important market variable, is discussed.

2.1 Market competition related issues

The analysis of decision-making process is becoming more important to the oligopolistic electricity market competition. Simple learning rules have been implemented in dynamic models for strategic bidding [11]. In [12], the authors define near-equilibrium as a case where generators maximize their profits and consumers maximize their economic utilities. They try to analyze the equilibrium in a competitive market with single time period using locational marginal price (LMP). It is suggested that the market near-equilibrium is an efficient approach for market monitoring on the generators' profits. In [13], the author emphasizes the benefits of LMP compared to the actual market. It uses the Ontario market as a case study to prove that LMP can significantly improve both system reliability and market performance.

Reference [14] analyzes changes in LMPs with respect to operational parameters, i.e., demands, generator cost parameters, and voltage bounds. It is believed that the changes in LMPs as parameters vary, providing insight into the functioning and behavior of the electric energy system. This information on sensitivity might help producers and consumers to establish their respective bidding strategies, and the regulator to assess the degree of competitiveness of the electricity market. Simple analytical expressions are developed in [14] to compute LMP sensitivities with respect to changes in demands throughout an electric power network.

In [15], a novel LMP policy for distribution system with significant penetration of distribution generation (DG) in a competitive electricity market is presented. A new LMP method is based on remunerating DG units for their participation in reduced levels of energy losses in distribution systems brought about by participation of all DG units in meeting demand. Since the LMP prediction methods used in [15] contain error, uncertainty modeling is implemented for modeling the effect of error in prediction on the LMP calculation.

In [16], the authors investigate the marginal costs and prices in the electricity industry. The intention is to identify whether the consumers will be charged at the price levels that are indicative of the costs of supply. It is argued that appropriate market prices can send signals about both short run and long run costs. There is agreement with the opinion that a price that is higher than marginal cost indicates the exercise of market power. This can be understood as the recovery of long term capital costs.

Reference [17] assesses the benefits and costs of the formation of regional transmission organizations (RTOs) by evaluating the marginal loss pricing scheme. It is argued that, for generators and consumers to receive appropriate short and long term signals in the transmission network, the loss component must be priced using marginal cost methods. A case study on the transmission losses in New York is used to illustrate the importance of marginal cost of transmission losses that will affect the generator’s bidding behaviors.

In [18], the author assesses the market based price differentials in zonal and LMP market design. He tries to use LMP to reflect the actual costs of delivering energy by using an accurate network model to price in both congestion and losses with the market design simulations at the California Independent System Operator. The major finding is that depending on the location of congestion, the simulations show that LMP will not create sustained adverse impact on market prices. Differences in LMPs are a result of increased transparency of prices that show the value of energy throughout the grid.

A one snap-shot Nash equilibrium is presented in [19] to analyze the market competition with transmission constraints. In this model the authors demonstrate how to analyze the impact of transmission capacity on competition. The model is applied to the western U.S. electricity market. The results show that transmission constraints enhance the exercise of market power.

Reference [20] focuses on congestion management and risk management issues in a practical case in Europe. A coordinated congestion mechanism has been developed by the authors in order to achieve higher level of market competition, maximum interconnection usage, and decreased risk for market participants.

An analysis for the impact of network constraints on the market equilibrium is presented in [21]. The authors solved a two-level optimization problem in the model they designed. Firstly, the independent system operator (ISO) dispatches generation and sets transmission price through solving optimal power flow. Secondly, the individual generators use Nash-Supply function equilibrium strategy to submit their generation bids after taking into account the ISO’s decision. It is found that the generators have the intention to bid for generation close to the constraint boundary. This resulted in multi-Nash equilibriums, which means that generators may change their bidding behaviors.

In [22], supplier equilibrium strategy is analyzed by considering transmission constraints. The Cournot model is used to simulate the market competition with a 2-bus and a 3-bus network. The results show that, when there is potential transmission constraint, there might be multiple equilibriums in the Cournot competition. Another finding is that the transmission capacity must be far higher than the actual power flow on the specific line in order to obtain the same equilibrium without transmission constraint.

In [23] and [24], bidding strategies from the supply side under possible network constraint have been investigated to assess the level of competition and the market performance. The authors found that strategic behavior may mitigate the seriousness of congestion, while congestion offers additional opportunities for gaming to the producers.

To characterize LMPs that appear in the payment cost objective function, the authors of [25] established and embedded Karush-Kuhn-Tucker (KKT) conditions of economic dispatch as constraints. They investigated the payment cost minimization problem with transmission capacity constraints as formulated. A regularization method was used to satisfy constraint qualifications. Their numerical results show that payment cost minimization leads to consumer payment savings as compared to bid cost minimization for the same set of supply bids.

References [11] to [18] have focused on oligopolistic competition, and market pricing mechanisms, but they did not investigate further on the transmission constraints or the involvement of demand side. References [19] to [25] have investigated the impact of transmission constraints on a competitive electricity market. The constraints presented are focused on transmission side. However generation capacity constraint should also be investigated.

2.2 Cournot modeling in electricity market competition

A study on the market power of generators in the electricity market with transmission constraints is done in [26]. It is argued that many authors do not obtain the same results, nor do they indicate their assumptions in the Cournot models. A simplified duopoly model with one transmission line is presented in the paper. A transmission limit is applied to the model and a linear demand function is defined. With the involvement of the System Operator, it is concluded that there is no direct relation between the use of transmission capacity and the transmission price outcome.

A two-settlement equilibrium in competitive electricity markets is formulated in [27]. In this model, each generation GenCo solves a Mathematical Program with Equilibrium Constraints (MPE). The model includes forward and spot markets. It implements a two-solution approach and applies these solutions to a 6–node and 8-line model. Two approaches, iterative Response Surface Method (RSM) and iterative Penalty Interior Point algorithm (PIPA), are compared. The authors expect that the generators' market power is encouraged by raising the forward price to increase profits.

Other supplier side modeling mechanism, such as the supply function equilibrium model, are also used to compare the market equilibrium with the one from Cournot competition model in the case of transmission constraint [28, 29]. It was found that strategic generators were able to capture all the congestion rental of the constrained line. They recommend that market rules be designed to restrict the bidding flexibility and eventually reduce the equilibrium prices.

In [30], Cournot equilibrium is investigated in a simple network. The network consists of three buses, and each bus has one generator and one load. The study analyzes the Cournot competition with the existence of transmission constraints. The results from the model show that generators can significantly affect the line flows and market price through strategic behaviors. When transmission constraints existed, the market outcome became much more uncertain.

A demonstration on the importance of active participation of transmission rights owners in a competitive market to achieve an efficient outcome is addressed in [31]. It is argued that, even without market concentration, congestion, and passive transmission rights will cause implicit collusion among generators. As a result, the market price will be higher than the marginal cost. The price deviation can also result in short and long term inefficiency.

In [32], the authors modeled Nash equilibrium through relaxation procedure and applied it to an objective function, known as the Nikaido-Isoda function. The Nikaido-Isoda function is defined as a transformation of an equilibrium problem into an optimization problem. The relaxation algorithm is used to calculate Nash equilibrium under constraint. The Nash equilibrium is applied to the IEEE 30-bus system. The emphasis is on the importance of price-demand elasticity. As elasticity changes, generators will also change their ways in order to collude.

The strategic behaviors of the generators that may result in transmission congestion is discussed in [33]. The author calculates Nash-Cournot equilibrium in a critically congested 6–bus network. The finding is that Nash-Cournot equilibrium does exist in the case of transmission congestion.

Both [34] and [35] used a simplified three generation power system model to investigate oligopolistic competition in the electricity market. The authors in [34] conducted a set of experiments on the oligopolistic markets with three generating companies, and their results show that market competition will converge into the results between perfect competition equilibrium and Nash Cournot equilibrium. In [35], a simple numerical example with three generation companies is demonstrated to show the basic idea of the authors' proposed method. They focused on the analytical study of the equilibrium of N-company generation market. They obtained Nash equilibrium in a uniform price auction in the spot market.

In [36], an analysis is conducted on the potential market power in a restructured New Jersey electricity market. The calculation of Nash-Cournot equilibrium is done with different demand levels during peak and off-peak periods. A price responsive demand curve is also included. It is found that there is a threshold level of demand at which the Cournot competition will result in higher market price if the actual demand is higher than the threshold level. When the demand level is lower than the threshold level, the effect of a Nash Cournot game on the price is relatively much smaller. A rapid price increase is identified when there is a potential transmission constraint between New Jersey and the rest of the PJM market.

Two Cournot models of imperfect competition among generators, one with arbitrage (Bilateral model) and the other with arbitrage (PoolCo model), are formulated in [37] and [38]. A congestion pricing scheme for transmission is concluded. The importance of including the transmission network, while modeling the Cournot competition and the choice of transmission pricing scheme, is addressed.

A new algorithm is used in [39] to calculate the Cournot equilibrium without the presence of transmission congestion. The Cournot game is transformed into a three level decision-making process with economic signal exchange. The advantages and disadvantages between two popular methods (i.e., Cournot Equilibrium and Supply Function Equilibrium (SFE)) are compared to model competition. The model is tested by using a market comprising the generating units of the IEEE 3-area RTS9.

References [40] and [41] have compared Cournot competition with Bertrand competition. In [40], the Australia National Electricity Market (NEM) was utilized as an example test system to validate the authors' method. The comparison between Cournot model and other models, such as the Bertrand model, shows that the Cournot model has a more reasonable perfomance in the NEM. The comparison between Bertrand and Cournot competition in [41] shows that Bertrand competition can also yield Cournot outcomes depending on both the strategic variables and the context in which those variables are employed.

Capacity withhold by generation companies has been investigated under Nash Cournot competition in [42] and [43]. The simultaneous move and sequential-move games in applied mathematics are used to model the interactions of the generation companies, transmission network and market operator. The results from the simulations in the six-bus Garver's example system and the IEEE 14-bus system show that market power can be successfully modeled in the transmission augmentation algorithm [42]. The authors in [43] considered physical withholding of capacity in the energy market and the HHI index is utilized to measure market concentration. They found that market design is crucial to determine the possible existence of capacity withhold and market power.

Other papers, such as [44], [45] and [46] focused on electricity markets that are cleared by merit order using pure strategy Nash equilibrium. They analyzed how market power is influenced by the number and size of the competing generation companies.

There are other studies done on Cournot competition, such as [47], [48], [49], [50], [51] and [52]. Reference [47] modeled Cournot prices with generator availability and demand uncertainty, but with no consideration of transmission congestion. Reference [48] used an agent-based test bed, AMES (a market software package), to study the power market operations subject to generation constraints, transmission constraints, and strategic behaviors. Reference [49] investigated Nash Cournot equilibrium in power markets in both PoolCo and Bilateral models. The authors found that Cournot competition among producers yields the same outcomes for both market designs. A competitive co-evolutionary algorithm is used to model agents' interactions in the market by finding the Nash Cournot Equilibrium [50]. A Game theoretical approach is also compared with other decision making methods, such as cost benefit analysis [51]. A direct computation is developed to calculate Nash Equilibrium of electricity markets using the supply function model [52].

A large number of simulations have been done in [26] to [52] on the Cournot competition with the presence of transmission constraints. However, they did not specify the important impact that demand elasticity could have on market competitions.

2.3 Demand elasticity

The responsiveness of customers to price changes is characterized by their "price elasticity of demand" [53]. Demand responsiveness plays a vital role in increasing efficiency and reducing price volatility in the electricity markets. The authors of [53] investigated demand-bid price sensitivity and supply-offer price caps on LMPs. They modeled a restructured wholesale power market operating through time subject to transmission line constraints, generation capacity constraints and strategic trader behavior

The authors of [54] modeled demand response programs in the power markets. They developed an extended responsive load economic model based on price elasticity and customer benefit function. In [55], the authors investigated the influence of price responsive demand shifting bidding on congestion and LMPs in pool-based day-ahead electricity markets. They compared the price responsive demand and conventional price responsive and price taking bids. They found that the conventional price taking bids encourages the GenCos to exercise market power and may lead to uncontrolled LMPs and system congestion.

Authors of [56] found that increment in the demand elasticity provides the expected positive results in market performance in terms of the Lerner index and the reduced congestion in the network. Similarly, the results from the simulations in [57] which attempt to quantify the effect of demand response show that market clearing price tends to reduce with increasing level of demand shifting.

However, the study conducted in [58] shows that, although demand response provides an important contribution to the electricity market, its contribution has limited impact on reliability of supply side resource and the capacity market. There is also a study that focuses on the analysis of a competitive power market with constant elasticity function, using Cournot game to determine the market equilibrium and price level [59]. The behavior of customers under different demand response programs is modeled considering incentive and penalty mechanisms [60].

In [53] to [60], demand elasticity, sometimes also called demand response, is introduced in the discussion of power market. But the impact of change in the physical quantity demanded and demand elasticity has not yet been clarified.

The following issues need further investigations:

- The system constraints can be considered in two different areas, i.e. generation constraints and transmission constraints. Most of the studies that mention constraint are only focused on transmission constraint. The generation capacity itself also implies a constraint to the electricity market and network. Especially when the marginal costs are different among generators, the generation capacity constraint does significantly affect the outcome of Cournot equilibrium. The issue of generation capacity constraint has not yet been identified fully in the literature.

- The price elasticity of demand has been mentioned in several papers, but none of them has included it as an input variable into their mathematical models. The elasticity of demand is a key factor to help include dynamics into the calculations of Cournot competition. The effect of change of demand elasticity on the individual generation of each generator and their profits needs to be investigated

- The price derivative against elasticity has also not been addressed. One of the key indicators in measuring generators' strategic behaviors is the market price level. As the elasticity of demand changes, market price will also change as consumers may change their demand with the response to the change in price.

- The individual quantity derivate against elasticity has not been investigated. The individual quantity refers to the generation of each generator, which is determined by the Cournot equilibrium. As the elasticity of demand is included, the total quantity demanded by the customer may change, and this will result in a change in the corresponding total generation. Therefore, the share of the total generation among the generators will be different.

Chapter 3

Model derivations

3.1 Classic Cournot model and Nash equilibrium in game theory

In the classic model of Cournot competition, players compete with each other through their quantity bids. This competition model applies to the game comprising two or more players. Within Cournot competition, the most popular solution concept is called Nash equilibrium. Nash equilibrium exists when each player has chosen a strategy and no player can benefit by changing his strategy while the other players keep theirs unchanged. There are two important assumptions for Cournot competition. Firstly, one player decides his strategy or decision on quantity bid, taking into account the others' quantity decisions. This means that the player's payoff or profit depends not on his own quantity but also the quantities of other players. Another assumption is that players decide their quantity bids simultaneously. The simultaneous aspect implies that each player chooses his quantity without knowing the choices of others [6], [31], [34], [35], [36], [40].

A two-stage game between expected profit maximizing GenCos has shown an unique equilibrium in the Cournot competition. In the first stage, GenCos decide their production simultaneously and independently. In the second stage, GenCos choose price simultaneously and independently and the demand is allocated among GenCos at the market clearing price level. If the demand function is concave, the Cournot outcome is the unique equilibrium outcome. The result shows an important finding that quantity competition is a choice of scale with GenCos competing by means of their cost functions. The weight of each GenCo's own cost over average GenCos' sum costs determines the quantity produced by each GenCo [41].

However, the classical Cournot model initially does not consider the constraints, such as generation capacity and transmission network constraints. It seeks the equilibrium based on the supply side, for example, the marginal costs of the GenCos have significant impact on the individual output. There is a high possibility that demand may not be 100% satisfied. Although demand function, with demand intercept and slope, is used for calculation, the demand elasticity has not been specified.

In this Chapter, an Extended Cournot model is developed. It includes the consideration of elasticity of demand as an important input to the model itself. In this thesis, system constraints are modeled as generation and transmission constraints. Demand elasticity is included which allows load to express its willingness to reduce consumption when price changes. Therefore, the investigation is focused on the impact of generation constraints, transmission constraints and demand elasticity on GenCo' market strategies. Table 3.1 represents a comparison between the Cournot classical model and the Cournot extended model. In the Cournot extended model, demand elasticity becomes a very critical input variable. With the analysis of price derivate and generation quantity derivative, it is possible to demonstrate the impact of marginal change in elasticity of demand on the market price and quantity produced by each generator.

Table 3.1

Comparison between two different Cournot models

	Classical Cournot model	**Extended Cournot model**
Input variables	• Marginal cost • Total demand	• Marginal cost • Total demand • Demand elasticity • Generation constraints • Transmission constraints
Output variables	• Generation • Market clearing price	• Generation • Market clearing price • Price derivative • Generation quantity derivative

3.2 Derivation of Extended Cournot model

Cournot competition has been used extensively in duopoly competition, with two players, and oligopoly competition, with multiple players, to investigate how GenCos are maximizing their profits by using quantity as a strategy.

The maximization problem of individual profit in the case of three GenCos is derived as follows:

$$\text{Max}\ (\pi_i) \qquad (3.1)$$

where,

$\pi_i \quad i = 1,2,3 \ldots n$	-	individual profit of GenCo_i

As each GenCo normally acts in its own self-interest, the profit maximization function of a company in an oligopoly competition with three GenCos can be described as follows:

$$\pi_i = P * Q_i - C_i * Q_i \qquad (3.2)$$

where,

π_i	-	individual profit of GenCo_i
P	-	market clearing price
Q_i	-	individual output of GenCo_i
Q	-	total production output
C_i	-	marginal cost of GenCo_i

In the competition of a Cournot model, the Nash Cournot equilibrium is the solution of the game. However, the assumption is that each GenCo chooses its own quantity level assuming that the quantities of other GenCo are fixed [8]. As each GenCo maximizes its own profit against its quantity, Nash Cournot equilibrium only requires the first order (F.O.) partial derivative of Equation (3.2) to be equal to 0 (i.e., $\frac{\partial \pi_i}{\partial Q_i} = 0$) to ensure the profit maximization. Setting the partial derivative of Equation (3.2) to equal to zero gives the following:

$$\frac{\partial \pi_i}{\partial Q_i} = \partial P * Q_i + P * \partial Q_i - \partial C_i * Q_i - C_i * \partial Q_i = \frac{\partial P}{\partial Q} * Q_i + P - C_i = 0 \qquad (3.3)$$

In Equation (3.3), $\frac{\partial P}{\partial Q}$ is the price derivative. It shows the marginal change in market price over the marginal change in quantity produced by all GenCo.

From Equation (3.3), the following is obtained

$$Q_i = \frac{C_i - P}{\frac{\partial P}{\partial Q}} \qquad (3.4)$$

The relationship between market clearing price and total quantity, i.e. $\frac{\partial P}{\partial Q}$, is normally considered as negative, which means that when price increases, the total quantity demanded at the load is going to decrease. Therefore, Equation (3.4) can be simplified to the following,

$$Q_i = P - C_i \geq 0 \qquad (3.5)$$

Equation (3.5) implies that a GenCo$_i$'s quantity change has an inverse relationship with its marginal cost. This means that a GenCo with higher cost will produce less output. This describes the rule of quantity distribution among the GenCos in the quantity game. When a GenCo's costs rise, its production will decrease. This also indicates a potential relationship between GenCo*i*'s costs and the output of the other

GenCos. For example, the increase in GenCo*i*'s costs may result in an increase in the production of the other GenCos.

In a given demand function, the market clearing price can be given as:

$$P = a + bQ = a + b\sum_{i=1}^{n} Q_i \quad (3.6)$$

where,

P	-	market clearing price
a	-	intercept of the demand function
b	-	slope of the demand function
Q_i	-	individual quantity production of GenCo$_i$

3.3 Cournot Extended model with no constraints

The objective function in the case with no constraints is Equation (3.1). After taking the first derivative, Equation (3.7) is the condition for profit maximization.

Elasticity of demand is defined as follows:

$$\varepsilon = \frac{\%\Delta Q}{\%\Delta P} = \frac{\frac{\partial Q}{Q}}{\frac{\partial P}{P}} = \frac{\partial Q}{\partial P} * \frac{P}{Q} = \frac{P}{\frac{\partial P}{\partial Q} Q} \quad (3.7)$$

where,

ε	-	Demand elasticity,
$\%\Delta Q$	-	Percentage change in quantity demanded by consumer
$\%\Delta P$	-	Percentage change in market price
$\frac{\partial P}{\partial Q}$	-	Price derivative, usually, it can also be noted as P'.

From Equation (3.7), the following is obtained

$$\frac{\partial P}{\partial Q} = \frac{P}{Q\varepsilon} \quad (3.8)$$

To introduce elasticity of demand into Equation (3.3), $\frac{\partial P}{\partial Q}$ in Equation (3.3) is replaced by $\frac{P}{Q\varepsilon}$ of equation (3.8). The following Equation (3.9) is then obtained,

$$Q_i\,(\frac{P}{Q\varepsilon}) + P - C_i = 0 \quad (3.9)$$

From Equation (3.9), the following Equation (3.10) is obtained

$$Q_i = (C_i - P)\frac{Q\varepsilon}{P} \quad (3.10)$$

As the total demand equals the total supply, which is the sum of all individual production, we have

$$Q = \sum_{i=1}^{n} Q_i \quad (3.11)$$

where

n - the number of GenCos in the market.

Q_i in Equation (3.11) is replaced by $(C_i - P)\frac{Q\varepsilon}{P}$ of Equation (3.10), and the following is obtained,

$$Q = \sum_{i=1}^{n}[(C_i - P)\frac{Q\varepsilon}{P}] = \sum_{i=1}^{n} C_i\,\frac{Q\varepsilon}{P} - n\varepsilon Q \quad (3.12)$$

Using Equation (3.12), the following Equation (3.13) is obtained:

$$P = \frac{\varepsilon}{(n\varepsilon + 1)} \sum_{i=1}^{n} C_i \tag{3.13}$$

Using Equation (3.10), and by replacing P with $\frac{\varepsilon}{(n\varepsilon + 1)} \sum_{i=1}^{n} C_i$ as shown in Equation (3.13), the following Equation (3.14) is obtained

$$Q_i = \left(\frac{C_i (n\varepsilon + 1) - \varepsilon \sum_{i=1}^{n} C_i}{\sum_{i=1}^{n} C_i}\right) Q \tag{3.14}$$

The above equations are based on the assumptions that demand elasticity ε, total demand Q, and marginal cost of all generation companies C_i are known, and, the market price P and individual output Q_i are unknown.

According to Equation (3.13), the market price is determined by the marginal cost, elasticity of demand, and the number of GenCos. It can be seen that market price is independent of the total quantity demanded by the load.

This is another key difference between the Extended Cournot model and the classical Cournot model. In the classical Cournot model, the market price is highly dependent on the total quantity that is expressed in a demand function. Whereas in the Extended Cournot model, market price is not depending on the total quantity.

In Equation (3.14), GenCo's individual quantity is determined by marginal cost, elasticity of demand, number of GenCos and total demand.

Note that the sign of demand elasticity is negative, which represents the inverse relationship between market price and quantity. For example, comparing the levels of demand elasticity between -0.75 and -0.5, it is stated that the elasticity of demand is higher at -0.75 than the one at -0.5 in absolute value. The price elasticity of -1.0 means that a 1% increase in price, will result in a 1% decrease in quantity.

The partial derivative of market price with respect to demand elasticity is given by Equation (3.15):

$$\frac{\partial P}{\partial \varepsilon} = \frac{\sum_{i=1}^{n} C_i}{(n\varepsilon + 1)^2} \tag{3.15}$$

The partial derivative of GenCo$_i$' quantity with respect to demand elasticity is given by Equation (3.16):

$$\frac{\partial Q_i}{\partial \varepsilon} = \frac{C_i Q}{\overline{C}} - Q \tag{3.16}$$

where

$$\overline{C} = \frac{\sum_{i=1}^{n} C_i}{n}$$ is the average marginal cost

Rearranging Equation (3.16) gives:

$$\frac{\partial Q_i}{\partial \varepsilon} = \frac{C_i Q}{\overline{C}} - Q = \left(\frac{C_i}{\overline{C}} - 1 \right) Q \tag{3.17}$$

Once elasticity of demand is given, the quantity of power generated has an inverse relationship with the marginal cost. It means that the lower the marginal cost of the *i*th GenCo, the higher the quantity produced. It looks similar to the classical economic dispatch, however Equation (3.17) does not take into account the weight of individual capacity Q_i in the measure of average marginal cost $\overline{C}$. In principle, the maximum capacity of the GenCos can be different. In this thesis, for simplicity purpose, the maximum capacities of the GenCos are set to be all equal.

Equation (3.17) also shows that output increases when the cost of the *i*th GenCo is lower than the average cost, and decreases when the cost of the *i*th GenCo is higher than the average cost. It remains constant (derivative equals to zero) when the cost of the *i*th GenCo is equal to the average cost, i.e. $C_i = \overline{C}$.

3.4 Derive Extended Cournot model with constraints under Kuhn-Tucker conditions

The Kuhn-Tucker theorem is used to solve the profit maximization problem of the players in the Extended Cournot model setting with constraints. In the Extended Cournot model, partial derivative is taken for individual profit against individual quantity production. This is not an optimization of the sum of total profits.

According to the Kuhn-Tucker theorem, the non-linear programming problem can be described as follows [65]:

$$\max f(x) \qquad \text{subject to} \quad g_i(x) \geq 0 \qquad i = 1, \ldots, n \tag{3.18}$$

where f(x) is the objective function and g_i(x) is the constraint function.

The assumption is that all functions are differentiable.
There are three conditions to hold:

1) x^* is feasible

then, there exist multipliers $\lambda_i \geq 0, i = 1,\ldots,n$, such that

2) $\lambda_i g_i(x^*) = 0 \quad i = 1,\ldots,n$, and

3) $\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x^*) = 0$

where $\nabla f(x^*)$ is maximization of the objective function

The Lagrange function can be shown to be [65]:

$$L(x,\lambda) = f(x) + \lambda_i g_i(x) = f(x) + \sum_{i=1}^{n} \lambda_i g_i(x) \tag{3.19}$$

Two multipliers, μ and α, are introduced for the generation capacity and transmission capacity constraints, in sections 3.4.1 and 3.4.2, respectively.

3.4.1 Generation capacity constraint

μ_i is assumed to be the multiplier determined by the relationship between Q_i and $Q_i^{\max}$, then, the following is obtained

$$\mu_i (Q_i^{\max} - Q_i) = 0, \; \mu_i \geq 0 \tag{3.20}$$

where,

$Q_i^{\max}$ - GenCo$_i$'s maximum generation capacity

Referring to the Kuhn–Tucker theorem, the term $\mu_i (Q_i^{\max} - Q_i)$ should be zero which means that both factors can be zero. For Equation (3.20), this means that if a generator is fully dispatched, i.e. $Q_i^{\max} = Q_i$, then the multiplier μ_i can be zero or non-zero in this case. But if a generator is not fully dispatched, i.e., $Q_i^{\max} \rangle Q_i$, μ_i equals to zero for Equation (3.20) to hold.

This means that μ_i can be considered as a penalty imposed on the GenCos when their real outputs are equal to the maximum. In this way, the penalty will be accounted for when calculating the marginal cost of a GenCo. For example, if GenCo$_1$ is producing at maximum capacity, its marginal cost becomes $C_1 + \mu_1$. This makes GenCo$_1$ more expensive and not as economical as before. The same principle applies to other GenCos.

The maximization problem is the profit maximization of each generator, which is described as follows:

Max π_i (3.21)

subject to $Q_i^{\max} - Q_i \geq 0$

$\mu_i (Q_i^{\max} - Q_i) = 0,\ \mu_i \geq 0$

Therefore, the Lagrange function can be rewritten as follows:

$$L(\pi_i, \mu_i) = \pi_i (Q_i) + \mu_i g(Q_i) = (P * Q_i - C_i * Q_i) + \mu_i (Q_i^{\max} - Q_i) \tag{3.22}$$

s.t. $Q_i^{\max} - Q_i \geq 0$

$\mu_i (Q_i^{\max} - Q_i) = 0$

The following is defined,

$$\mu = (\mu_1, \ldots, \mu_i)^t$$
$$(Q_i^{\max} - Q_i) = [(Q_1^{\max} - Q_1), \ldots, (Q_i^{\max} - Q_i)]^t \tag{3.23}$$

Based on equation (3.22), the individual profit function can be rewritten to the following,

$$\pi_i = (P * Q_i - C_i * Q_i) + \mu_i (Q_i^{\max} - Q_i) \tag{3.24}$$

By taking the first order derivative of equation (3.24.) with respect to Q_i, the following Equation (3.25) is obtained:

$$\frac{\partial \pi_i}{\partial Q_i} = \frac{\partial P}{\partial Q} Q_i + P - C_i - \mu_i = 0 \qquad (3.25)$$

with the same derivation process in section (3.3), the following is obtained,

$$\mu_i = \max\left\{ 0, \frac{(\overline{Q}_i + \varepsilon Q)(\sum_{i=1}^{n} C_i + \mu)}{(n\varepsilon + 1)Q} - C_i \right\} \qquad (3.26)$$

$$P = \frac{\varepsilon}{(n\varepsilon + 1)} (\sum_{i=1}^{n} C_i + \mu) \qquad (3.27)$$

$$Q_i = \frac{(C_i + \mu_i)(n\varepsilon + 1)Q}{\sum_{i=1}^{n} C_i + \mu} - \varepsilon Q \qquad (3.28)$$

The above equations are based on the assumptions that demand elasticity ε, total demand Q, and marginal cost of all generation companies C_i and the sum of individual generation capacity multiplier μ are known, and, the market price P, individual output Q_i and individual generation capacity multiplier μ_i are unknown.

According to Equation (3.27), the market price is depending on μ. In Equation (3.28), Q_i is determined by μ_i and μ.

To investigate the impact of the marginal change of price and quantity with respect to the change of elasticity of demand, derivatives on Equations (3.27) and (3.28) were taken and the following was obtained:

$$\frac{\partial P}{\partial \varepsilon} = \frac{\sum_{i=1}^{n} C_i + \mu}{(n\varepsilon + 1)^2} \tag{3.29}$$

$$\frac{\partial Q_i}{\partial \varepsilon} = \frac{(C_i + \mu_i)Q}{\overline{C} + \overline{\mu}} - Q \tag{3.30}$$

where $\overline{\mu}$ is the average value of μ_i and is defined as follows,

$$\overline{\mu} = \frac{\sum_{i=1}^{n} \mu_i}{n} = \frac{\mu}{n} \tag{3.31}$$

and $\overline{C}$ as defined previously.

Equations (3.30) is rearranged as:

$$\frac{\partial Q_i}{\partial \varepsilon} = \frac{(C_i + \mu_i)Q}{\overline{C} + \overline{\mu}} - Q = \left(\frac{C_i + \mu_i}{\overline{C} + \overline{\mu}} - 1 \right) Q \tag{3.32}$$

3.4.2 Generation and transmission capacity constraints

After the introduction of generation capacity limit as a constraint, another constraint is imposed on the transmission line between bus *j* and bus *k*. The idea is to create a potential transmission constraint for transmitting power from the most economical GenCo to the load. It is necessary to note that the transmission limit between buses *j* and *k* can also be modeled with constraint.

The maximization problem is still the profit maximization of each generator as described in Equation (3.21). However it is subject to the following:

subject to $Q_i^{\max} - Q_i \geq 0$

$$\mu_i(Q_i^{\max} - Q_i) = 0, \ \mu_i \geq 0$$

$$T_{jk} - \sum_{i=1}^{n} d_i Q_i \geq 0$$

where,

d_i - percentage of Q_i goes through line L_{jk}

T_{jk} - transmission capacity of line L_{jk}

The possible transmission constraint is assumed on line L_{jk}, and this constraint applies to every GenCo's profit function.

Therefore, the general expression of the lagrangian functions is as follows:

$$\begin{aligned} L(\pi_i, \mu_i, \alpha) &= \sum_{i=1}^{n} \pi_i(Q_i) + \mu_i g(Q_i) + \alpha(T_{jk} - \sum_{i=1}^{n} a_i Q_i) \\ &= \sum_{i=1}^{n} (P * Q_i - C_i * Q_i) + \mu_i(\overline{Q_i} - Q_i) + \alpha(T_{jk} - \sum_{i=1}^{n} a_i Q_i) \end{aligned} \tag{3.33}$$

For the transmission capacity constraint, α is assumed as the multiplier determined by the relationship between $\sum_{i=1}^{n} a_i Q_i$ and T_{jk}.

After taking first derivative, the following is obtained:

$$\frac{\partial P}{\partial Q} Q_i + P - C_i - \mu_i - \alpha \sum_{i=1}^{n} d_i Q_i = 0 \tag{3.34}$$

with the same derivation process in section (3.3), the following is obtained,

$$P = \frac{\varepsilon}{(n\varepsilon + 1)} (\sum_{i=1}^{n} (C_i + \mu_i) + \alpha \sum_{i=1}^{n} a_i Q_i) \tag{3.35}$$

$$Q_i = \left[\frac{(C_i + \mu_i + a_i \alpha)(n\varepsilon + 1)}{\sum_{i=1}^{n} (C_i + \mu_i) + \alpha \sum_{i=1}^{n} a_i Q_i} - \varepsilon \right] Q \tag{3.36}$$

$$\mu_1 = \max \left\{ 0, \frac{(Q_i^{\max} + \varepsilon Q)(\sum_{i=1}^{n} (C_i + \mu_i) + \alpha \sum_{i=1}^{n} a_i)}{(n\varepsilon + 1)Q} - C_i - a_i \alpha \right\} \tag{3.37}$$

Again, when first derivate is taken on Equations (3.35) and (3.36) in respect to demand elasticity, the following is obtained:

$$\frac{\partial P}{\partial \varepsilon} = \frac{(\sum_{i=1}^{n} (C_i + \mu_i) + \alpha \sum_{i=1}^{n} a_i)}{(n\varepsilon + 1)^2} \tag{3.38}$$

$$\frac{\partial Q_i}{\partial \varepsilon} = \left[\frac{n(C_i + \mu_i + a_i \alpha)}{\sum_{i=1}^{n} (C_i + \mu_i) + \alpha \sum_{i=1}^{n} a_i} - 1 \right] Q \tag{3.39}$$

Based on the above equations, simulations have been conducted using the Extended Cournot model will be discussed in chapters 4 and 5.

Chapter 4

Simulation results of analytical model

In order to illustrate the hypothesis more clearly and easily, a simplified three-bus benchmark network as shown in Figure 4.1 is used. It should be mentioned that in a few papers [30], [31], [33] a similar simplified power system network was used to model Nash Cournot competition in the presence of transmission constraints. In principle, the extended Cournot competition can be applied to a more complicated power system network.

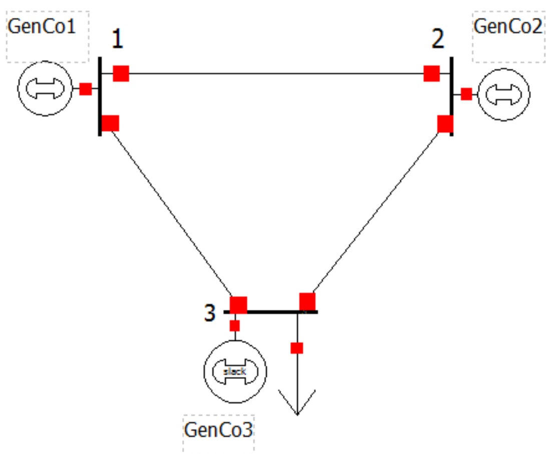


Fig. 4.1 Simplified three bus benchmark DC network

As shown in Figure 4.1, there is one GenCo on each node, and a load is placed on node 3 where the most expensive generator is located. It is important to note that the position of load in the network is critical to the market competition output. The reason to place the load on bus 3 is to distance it from the more economical generators, which are generators 1 and 2. The load position is important in the case of imposing transmission constraints, in which the load may not be able to acquire as much power as it wishes from the more economical generators.

The simulations performed in this thesis covered market operations under the following two cases:

- Case 1: Change in total demand
- Case 2: Change in elasticity of demand

In each case, three different scenarios are presented based on the level of constraints that were introduced, which include:

- Scenario 1: No constraints
- Scenario 2: Generation capacity constraints
- Scenario 3: Transmission and generation capacity constraints

To present the market competition results, the focus is on the following output variables for analysis

- GenCos' individual output
- GenCos' individual profits
- Market clearing price
- Load payment

4.1 Case formulation

For these simulations, the following assumptions were made:

- The change of quantity demanded does not affect the market clearing price in a Nash-Cournot game as discussed earlier, given that demand elasticity is constant. This is different from the traditional understanding that the change of demand will cause an increase in price due to the decrease in the supply side, which is generally an economic dispatch approach.

- With the introduction of system constraints, the impact of demand elasticity on market competition will be more limited. The demand side is usually expected to utilize its demand elasticity to bid in order to reduce the market clearing price and the load payment. The existence of systems constraints, especially the transmission constraints can significantly offset the effect of demand elasticity.

4.1.1 Marginal cost C*i*

The marginal costs (C_i) in Rand (R) terms of GenCos are assumed as follows:

$C_1 = R100$

$C_2 = R150$

$C_3 = R200$

It is possible to change the difference interval between the GenCos' costs as they may significantly affect the allocation of demand among GenCos. However this is not the focus of this thesis and can therefore be investigated in future research.

4.1.2 Generation capacity

The maximum outputs of the GenCo are denoted as O.

In scenario 1, with no constraints, the maximum generation capacities of three GenCos are:

$$O_1^{\max} = O_2^{\max} = O_3^{\max} = 1000MW ,$$

In scenarios 2 and 3, the maximum generation capacities of three GenCos are:

$$O_1^{\max} = O_2^{\max} = O_3^{\max} = 250MW$$

It is assumed that due to the existence of generation capacity constraints for all GenCos, the impact of demand elasticity on market competition will decrease. The multiplier μ_i can be considered as a penalty added to the marginal cost when a specific GenCo is fully dispatched. This will cause an increase in market clearing price when one or more GenCos are supplying at full capacity.

4.1.3 Transmission capacity

In scenario 3, with generation and transmission constraints, for example as shown in Figure 4.2, a network constraint with a maximum transmission capacity of 225 MW is applied to line L_{13}, which is the transmission line between bus 1 and bus 3. The maximum transmission capacities for line L_{12} and L_{23} are set to be at 500MW.

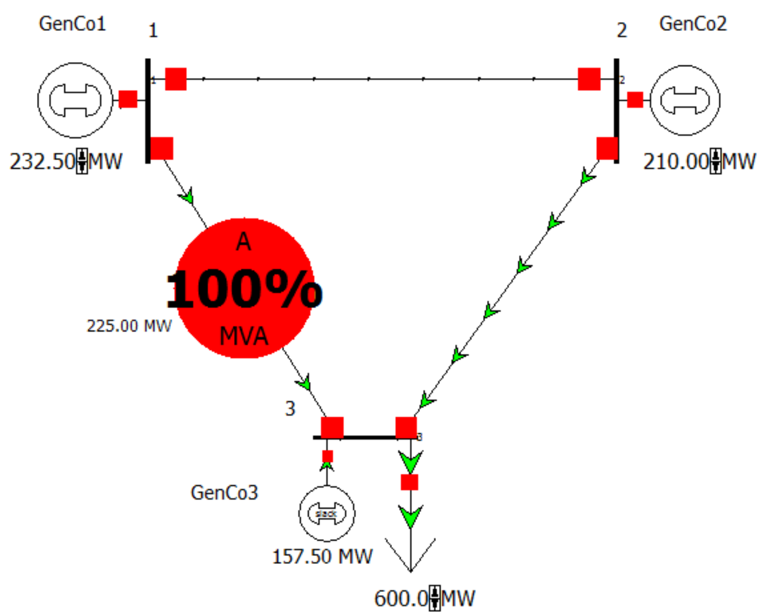


Fig. 4.2 An example of 3-bus network with transmission network constraints with total demand of 600 MW

In general, if the percentages of Q_1 and Q_2 goes through line L_{13} increase, which are d_1 and d_2, respectively, the flow of power from bus 1 to bus 3 via line L_{13} will increase. This will affect the proportion of power to be delivered from $GenCo_1$ to the load.

The value of d_i, which is the percentage of Q_i goes through line L_{jk}, can be changed for further research calculation and interpretation.

In the calculations in this thesis, it is assumed that all three lines have the same impedance to ensure that the power flow satisfies Kirchhoff Law. This is also due to the consideration of a simplified network.

4.1.4 Demand elasticity

To define within which area elasticity can vary, based on Equations (3.13), the condition for the price to be positive is that,

$$n\varepsilon + 1 < 0,$$

Since $n\varepsilon + 1$ is the denominator in Equation (3.13), it should be less than, but not equal to, zero.

In this study, the number of GenCos is three, therefore the value of n equals to 3.

Then, based on the above equation, $\varepsilon < -\frac{1}{3}$

In Equation (3.14), the condition for the individual quantity Q_i to be positive is that,

$$\frac{C_i(n\varepsilon + 1)}{\sum_{i=1}^{n} C_i} - \varepsilon \geq 0$$

For n=3, $\sum_{i=1}^{n} C_i = R100/MW + R150/MW + R200/MW = R450/MW$

Therefore, without considering Q, $\varepsilon \leq \frac{C_i}{3 * C_i - 450}$

To meet the above condition, C_i is replaced with the marginal cost of $GenCo_1$, $GenCo_2$ and $GenCo_3$, respectively. Then ε is obtained as follows:

$$\varepsilon \geq -\frac{4}{3}$$

Therefore, the area of elasticity is found as follows:

$$-\frac{4}{3} \leq \varepsilon < -\frac{1}{3}$$

In the calculation, it is found that the market results for value of demand elasticity within the area of -0.3334 to -0.4334 become extremely volatile. There is large change in the output variables, such as market clearing price, individual quantities, against the demand elasticity. This is due to the fact that the impact of demand elasticity, at the lower bound of its value, is significantly high on the above mentioned variable. Therefore, the results with the area of demand elasticity between -0.4334 to -1.3334 are considered in this thesis.

It is necessary to pay attention to the negative sign: -1.3334 which is lower (more negative) than -0.4334, but economically speaking, elasticity is higher (in absolute value) at -1.334.

4.2 Case Discussion

4.2.1 Change of total demand with constant demand elasticity

The assumptions in this section are as follows:

- Demand elasticity stays constant, with -0.6667 (there is no particular reason for choosing the demand elasticity at -0.6667, it can be any value between -0.4334 to -1.3334)

- Marginal cost stays constant at R100/MW, R150/MW and R200/MW, respectively
- Total demand varies from 100 MW to 800MW with 100 MW interval

The change of total demand can be explained by Figure 4.3. The total demand curves in Figure 4.3 have the same interception on the vertical axis, which represents the highest market clearing price. The horizontal axis represents the total demand. When the demand elasticity is constant, a horizontal dashed line shows that price stays the same at P* at different total demand. The elasticity of demand is the same at point A, B, C and D.

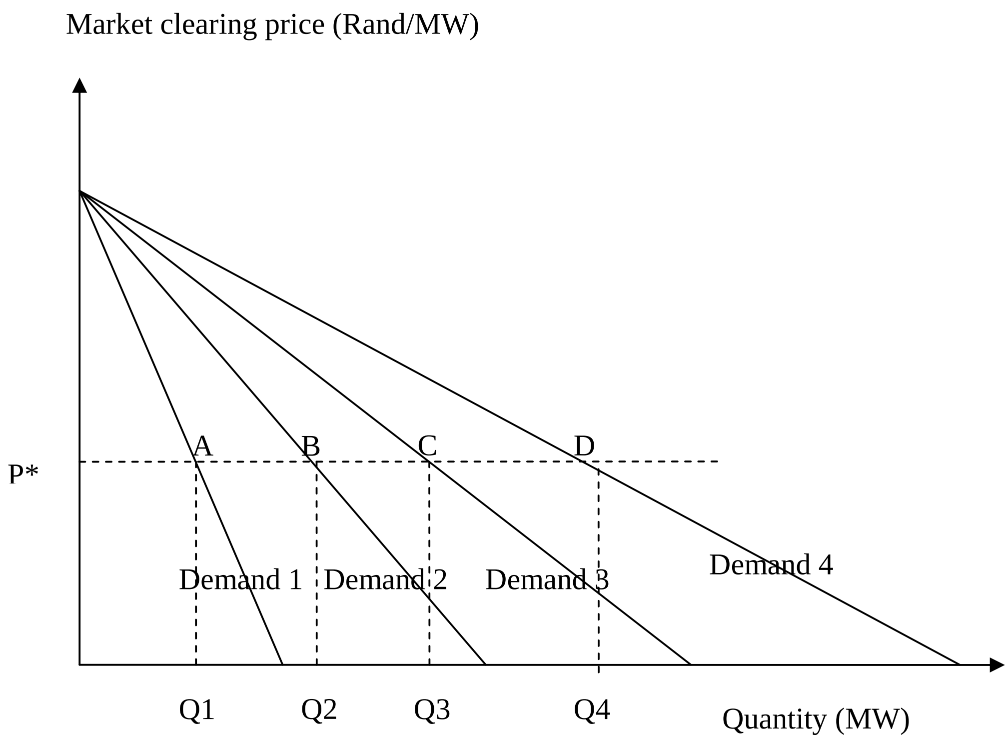


Fig. 4.3 Change of total demand with constant demand elasticity

4.2.2 Change of demand elasticity with constant total demand

The assumptions in this section are as follows:

- Demand elasticity varies from -0.4334 to -1.3334
- Marginal cost stays constant with R100/MW, R150/MW and R200/MW, respectively
- Total demand stays constant at 600 MW

The change of demand elasticity can be described in Figure 4.4. The demand elasticity at point E, F, G and H are different from each other. The vertical dashed line represents the same quantity demanded at Q*. As demand elasticity changes with constant total demand, different market clearing prices are obtained.

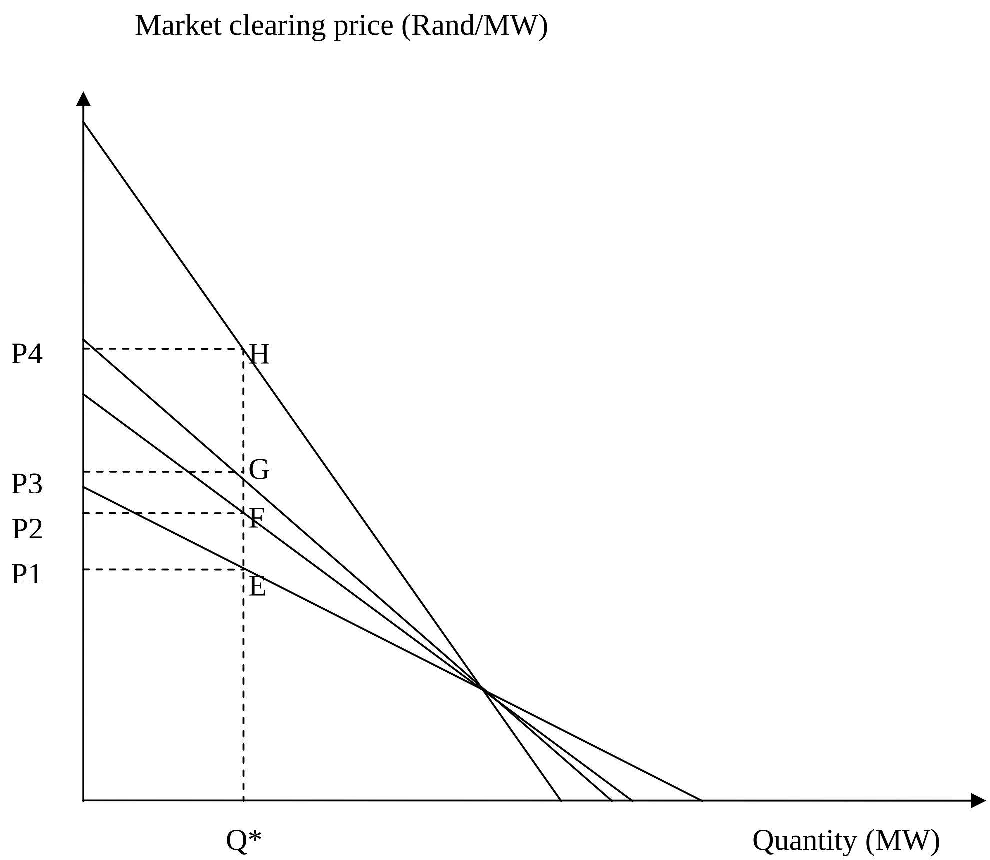


Fig. 4.4 Change of demand elasticity with fixed total quantity demand and constant marginal cost

4.3 Case 1 - Change of total demand with constant demand elasticity

4.3.1 Scenario 1 - with no system constraints

Figure 4.5 shows the value of the market clearing price over the change of total demand. It is found that the market price does not change as total demand increases. According to the conventional electricity supply industry, as demand increases, price will change. For example, price at peak demand will be higher than at off-peak demand.

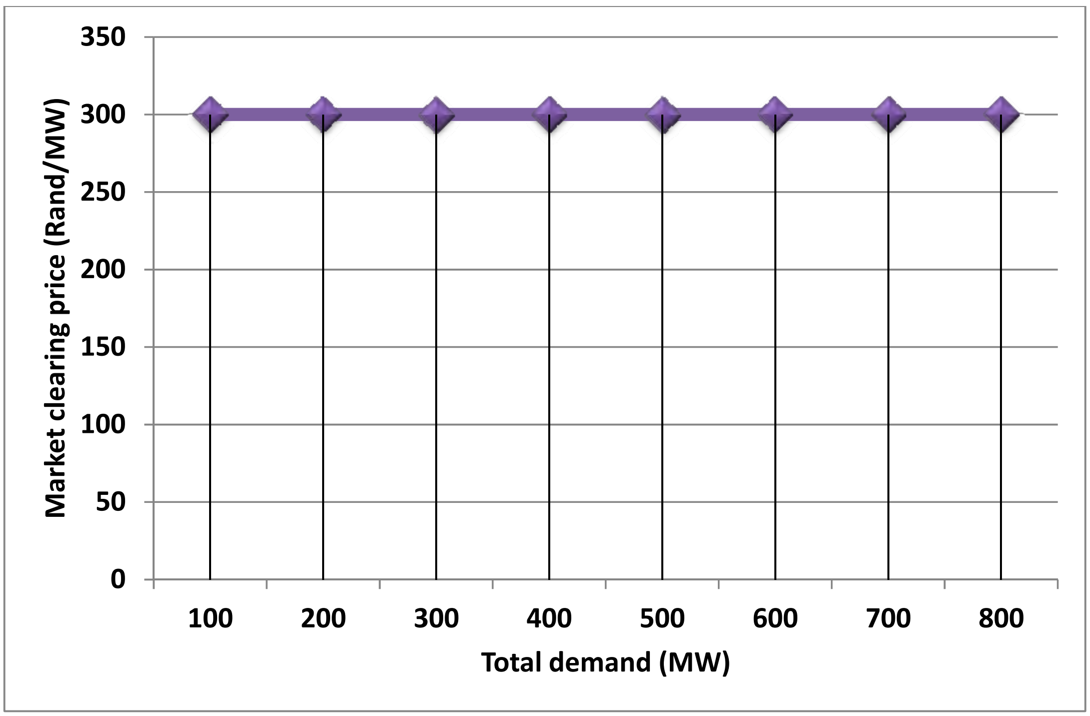


Fig. 4.5 Market clearing price with 100MW change interval of total demand, where demand elasticity stays constant at -0.6667

However, in the case of Extended Cournot model competition, the market clearing price is not determined by the total demand. As shown in Equation (3.13), market clearing price is determined by the demand elasticity and the sum of the marginal cost of all GenCos. In the assumption, the marginal costs of GenCos are constant. The results match the expectation that the market clearing price does not change if total demand changes.

Figure 4.6 shows the level of GenCos' individual outputs when total demand changes. All three GenCos' individual outputs increase as total demand increases. The additional demand is allocated among GenCos according to their marginal cost. Each GenCo's output is determined by the weight of its marginal cost over the sum of all the GenCos' marginal costs.

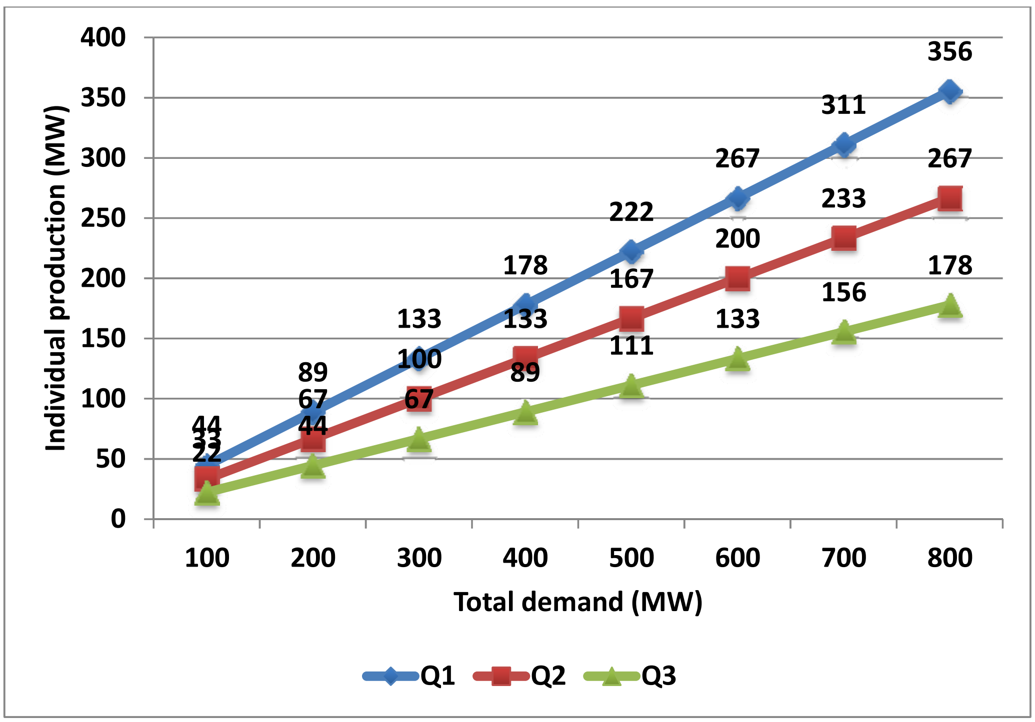


Fig. 4.6 Individual outputs with change of total demand, where demand elasticity stays constant at -0.6667

The results show that the lower the marginal cost of a GenCo, the more output will be required from this GenCo. Based on the assumption, $GenCo_1$ is the most economical generator with the lowest marginal cost. The output of $GenCo_1$ is the highest. Conversely, $GenCo_3$ is the most expensive generator with the highest marginal cost, and its output is the lowest. Although the quantity allocation in Cournot competition is determined partly by the marginal costs of GenCos, it is fundamentally different from the conventional economic dispatch. The conventional cost-based dispatching mechanism determines the output of generators based on their marginal costs. The most economical generator will be utilized as much as possible to satisfy the load before the more expensive generator is accepted.

The Cournot competition is a quantity game, or a quantity competition. The equilibrium is based on the profit maximization of each individual GenCo. At the equilibrium point, the output levels of all GenCos are optimal. Even the more expensive GenCo will be dispatched before the less expensive GenCo is fully dispatched. The change of output levels in Figure 4.6 shows that any additional demand will be satisfied with the same pattern, or proportion, among the GenCos.

Figure 4.7 shows the GenCos' individual profits against the change of total demand. It can be seen from Figure 4.7 that individual output, Q_i, increases as total demand, Q, increases. Therefore, individual profits increase as total demand increases. The proportional increase of each GenCo's profit has a similar pattern to the example with an increase in quantity.

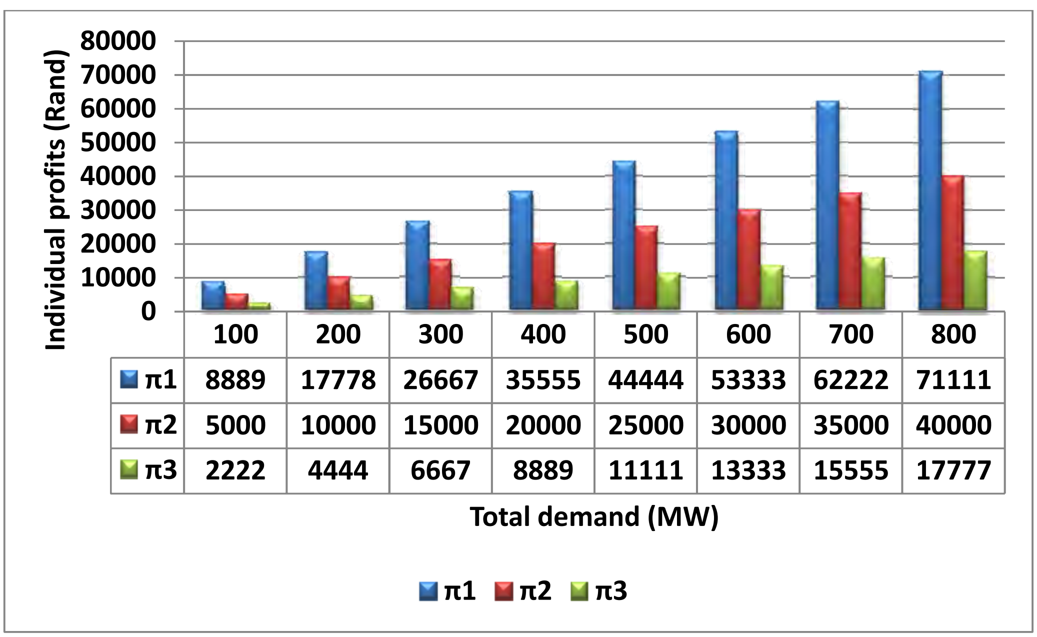


Fig. 4.7 Individual profits with change of total demand, where demand elasticity stays constant at -0.6667

Figure 4.8 shows the change of load payments as total demand changes and demand elasticity stays constant (-0.6667). As load payment is equal to the market clearing price multiplied by the total quantity demanded, the load payment increases as total demand increases.

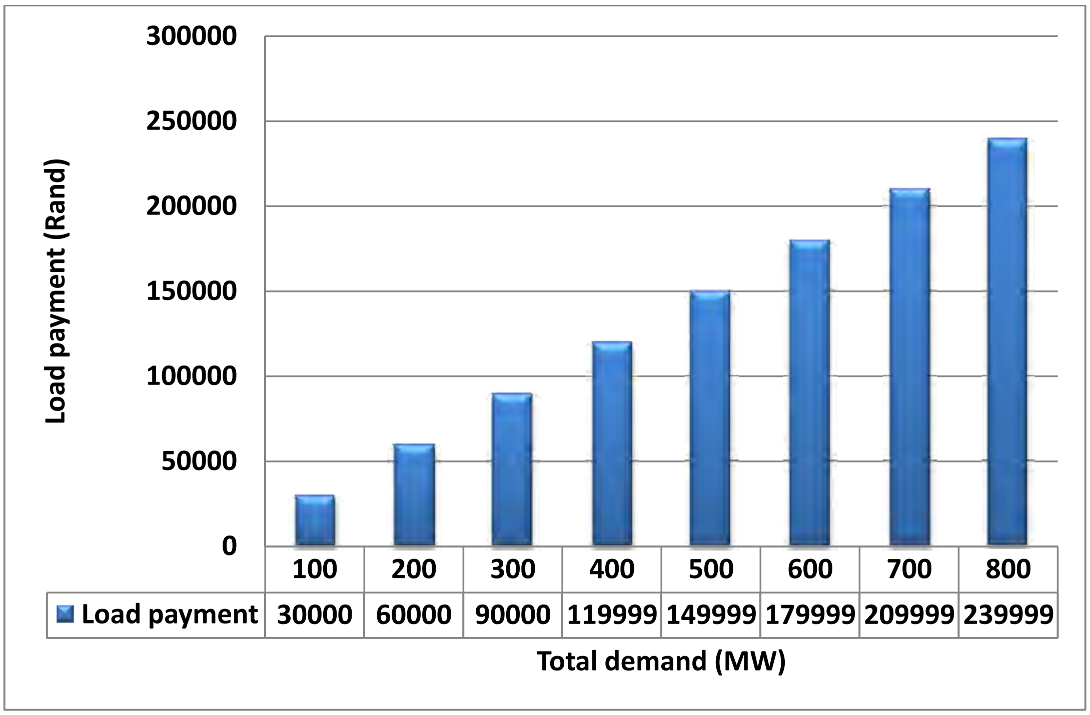


Fig. 4.8 Load payments with change of total demand, where demand elasticity stays constant at -0.6667

4.3.2 Scenario 2: with generation capacity constraint

With generation capacity constraint, the market clearing price increases as total demand increases. When total demand was low, the allocation of output among GenCos was the same as the one without capacity constraint, as μ_i equals to zero.

Figure 4.9 shows the change of market clearing pricing when total generation reaches its limit. As shown in Figure 4.9, when total demand increases, the most economical GenCo will be dispatched fully, i.e. $\overline{Q_1} = Q_1$ and μ_i will be non-zero. The quantity allocation among the three GenCos for a total demand of 600 MW is 250 MW, 207 MW and 143 MW, respectively. The market clearing price reaches P=R311 at this demand level. The increase of market clearing price becomes non-linear at an accelerating rate. This means that, when the level of total demand becomes close to the total generation capacity limits of all three GenCos, the proportional increase of the market clearing price is higher than the increase of total demand.

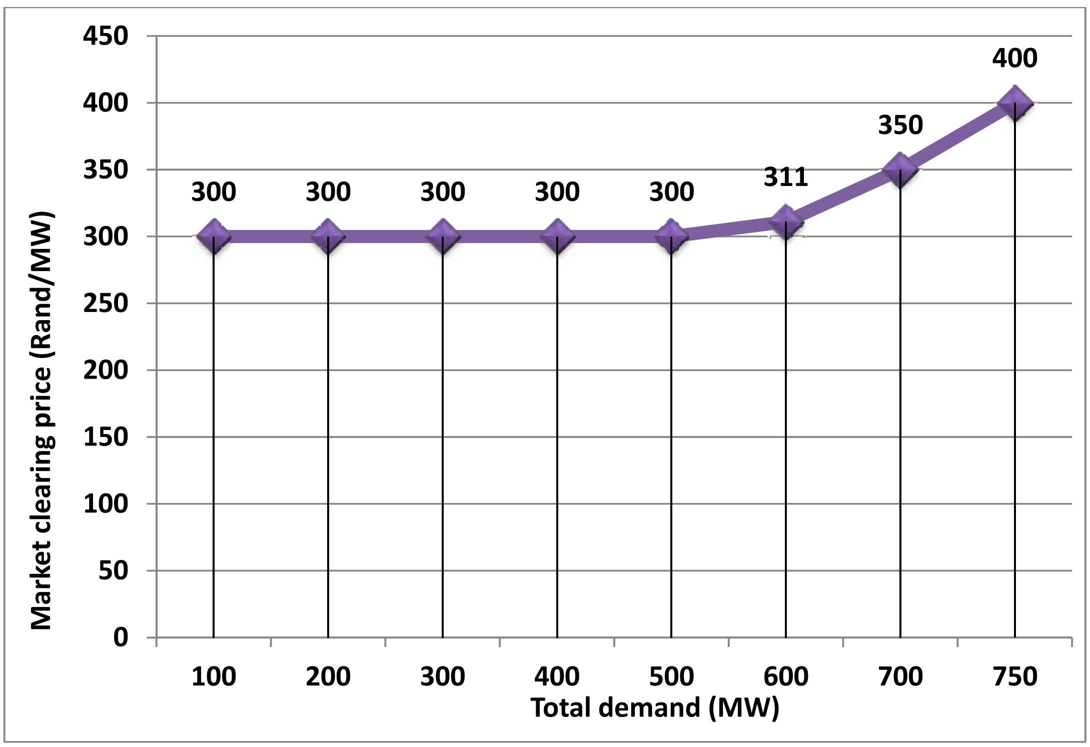


Fig. 4.9 Market clearing price with constant demand elasticity under generation constraint

Figure 4.10 presents the change of individual production given the generation capacity constraint. As shown in Figure 4.10, before reaching the maximum generation capacity, all GenCos' output levels are the same as the ones without generation capacity constraints. When the capacity limit of a specific GenCo is reached, its output will remain constant. When $GenCo_1$ is fully dispatched, $GenCo_2$ and $GenCO_3$ both increase their outputs to meet the additional demand. For example, after the total demand reaches 600 MW, $GenCo_1$ is fully dispatched; $GenCo_2$ and $GenCo_3$ are producing 207 MW and 143 MW, respectively. The proportional increase of all GenCos has changed due to the generation capacity constraint.

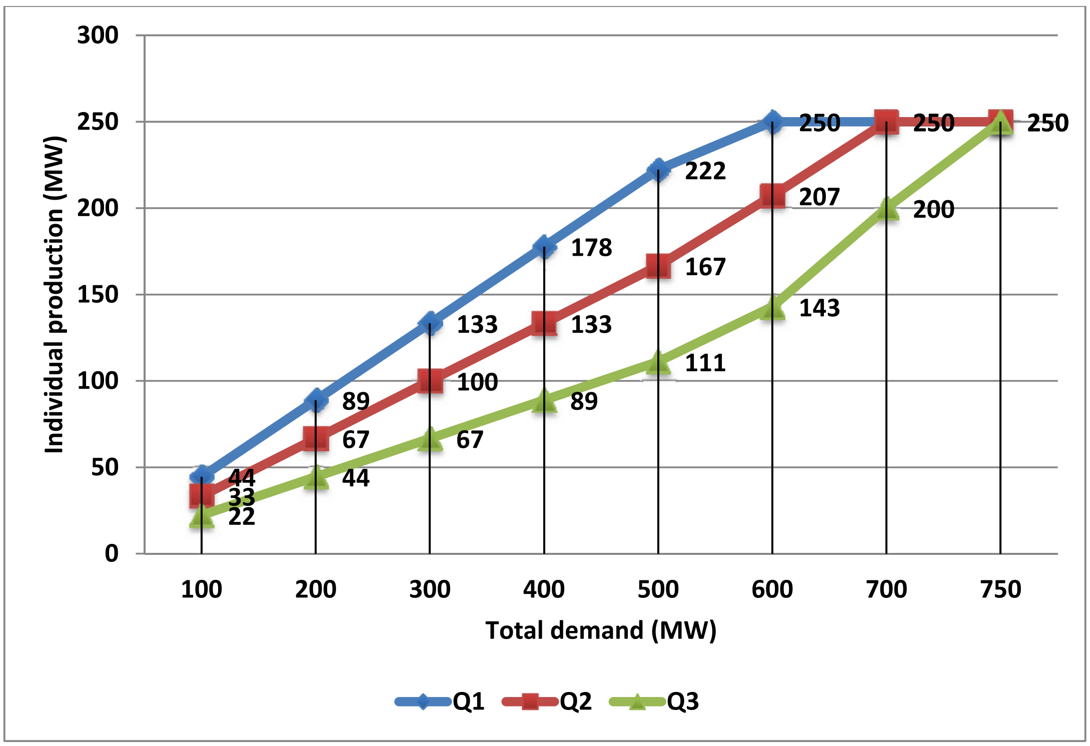


Fig. 4.10 Individual production with constant demand elasticity under generation constraint

The individual profit change against total demand is shown in Figure 4.11. The individual profit is determined by both market clearing price and individual production. When both variables increase, the GenCos' profit will increase at an accelarating rate. This applies to $GenCo_2$ and $GenCo_3$ in the case when $GenCo_1$ is fully dispatched, as the market clearing price will only increase when one of the GenCo's generation capacity limit is reached. In this case, $GenCo_1$'s profit will only increase as the market clearing price increases as its production reaches maximum. But the profits of $GenCo_2$ and $GenCo_3$ will increase dramatically as both their production and market clearing price are increasing.

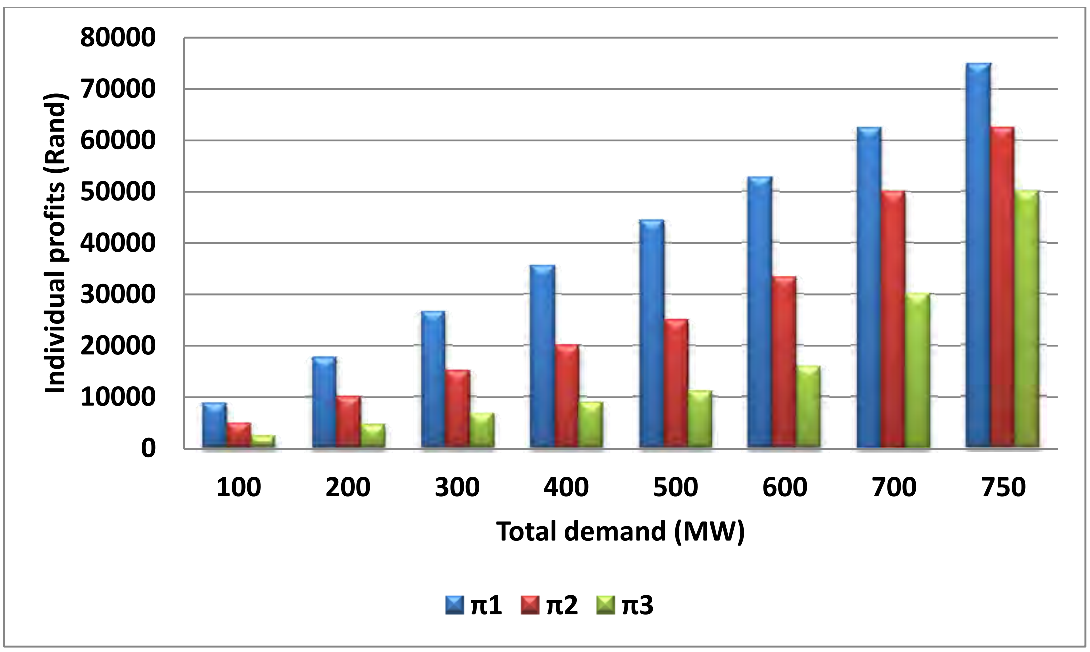


Fig. 4.11 Individual profits with constant demand elasticity under generation constraint

As shown in Figure 4.12, the change pattern of load payment is also affected by the generation capacity constraint. Load payment depends on total quantity demanded and the market clearing price. This indicates that load demand is bearing the cost of system constraint.

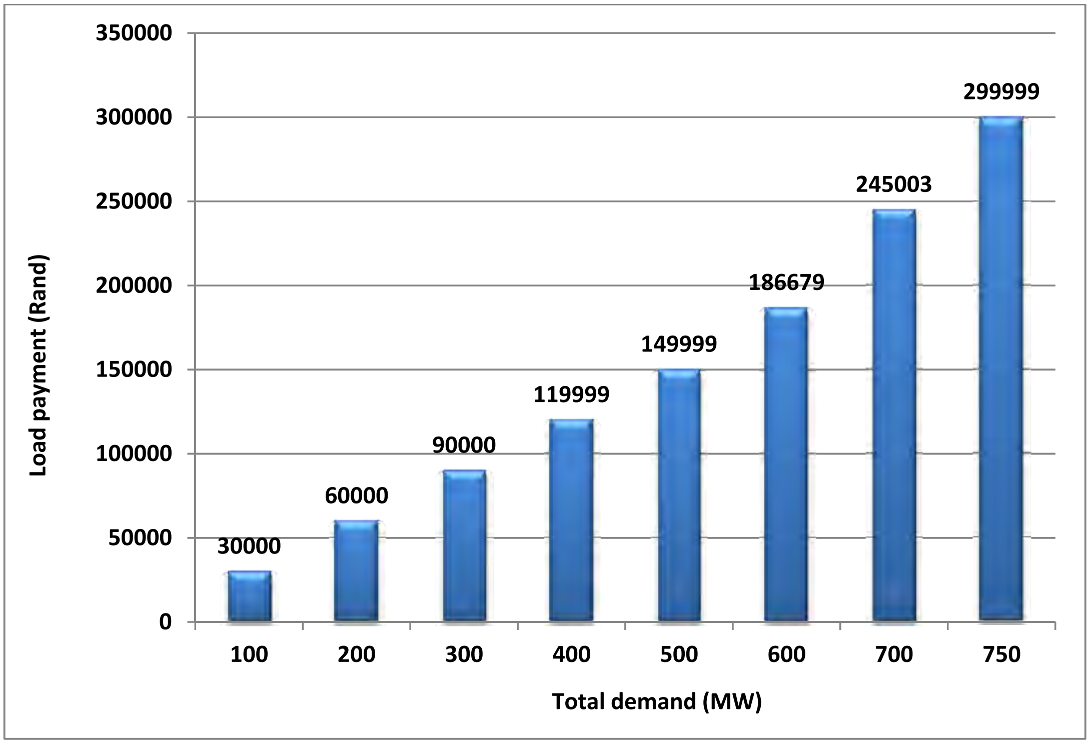


Fig. 4.12 Load payment with constant demand elasticity under generation constraint

4.3.3 Scenario 3: with generation and transmission capacity constraints

Figure 4.13 presents the market clearing price with both generation and transmission constraints. By comparing with the price level in scenarios 1 and 2, the level of market clearing price is the highest among three scenarios. From the simulations, the total demand can only increase up to 735 MW under both constraints. This means that, with the existence of transmission constraints, $GenCo_1$ and/or $GenCo_2$ may not be able to be fully dispatched. This has also pushed the market clearing price higher as there is no marginal unit that can be supplied by the more economical GenCos, that is, $GenCo_1$ and $GenCo_2$, to the load at bus 3.

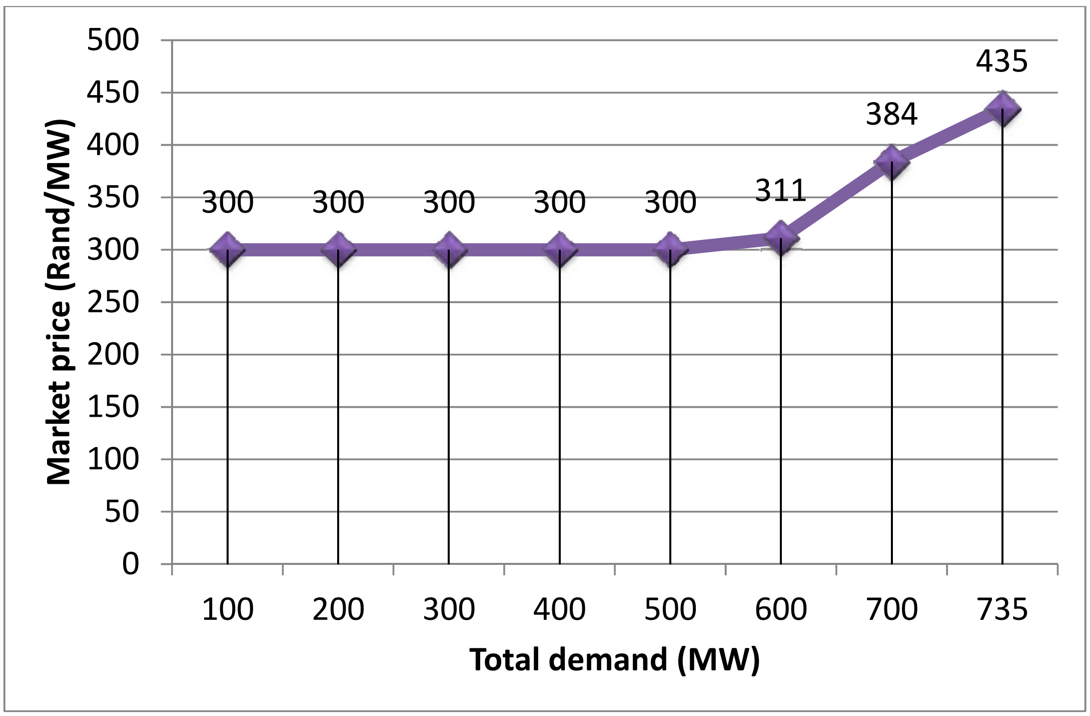
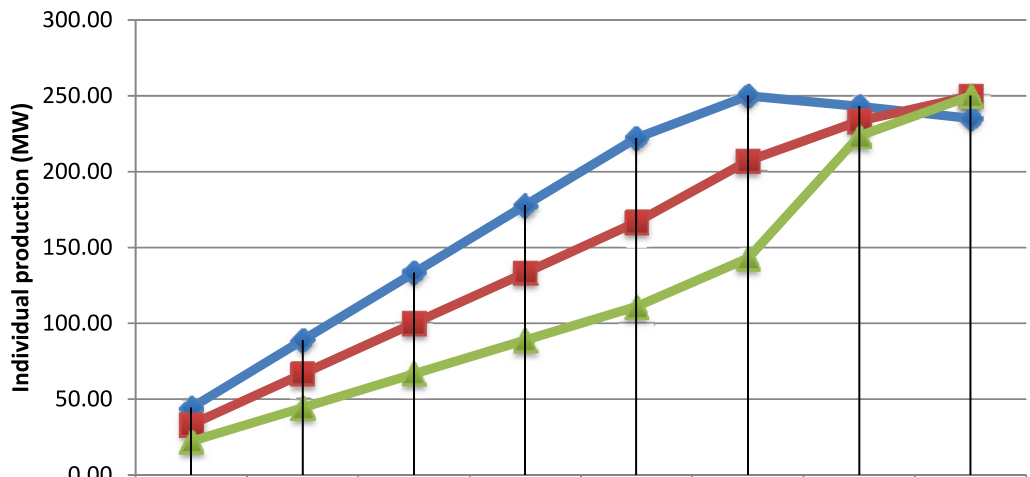


Fig. 4.13 Market clearing price with constant demand elasticity under generation and transmission constraints

It is interesting to see that the individual production of $GenCo_1$ first increases to its maximum capacity and then decreases as shown in Figure 4.14. $GenCo_2$ and $GenCo_3$ both increase consistently until they reach their generation capacity limits. This pattern has clearly shown the impact of transmission capacity constraint on the market results, i.e. the production of $GenCo_1$ is forced to decrease as total demand increases in order to avoid reaching the maximum transmision rate of line $L_{1\text{-}3}$.

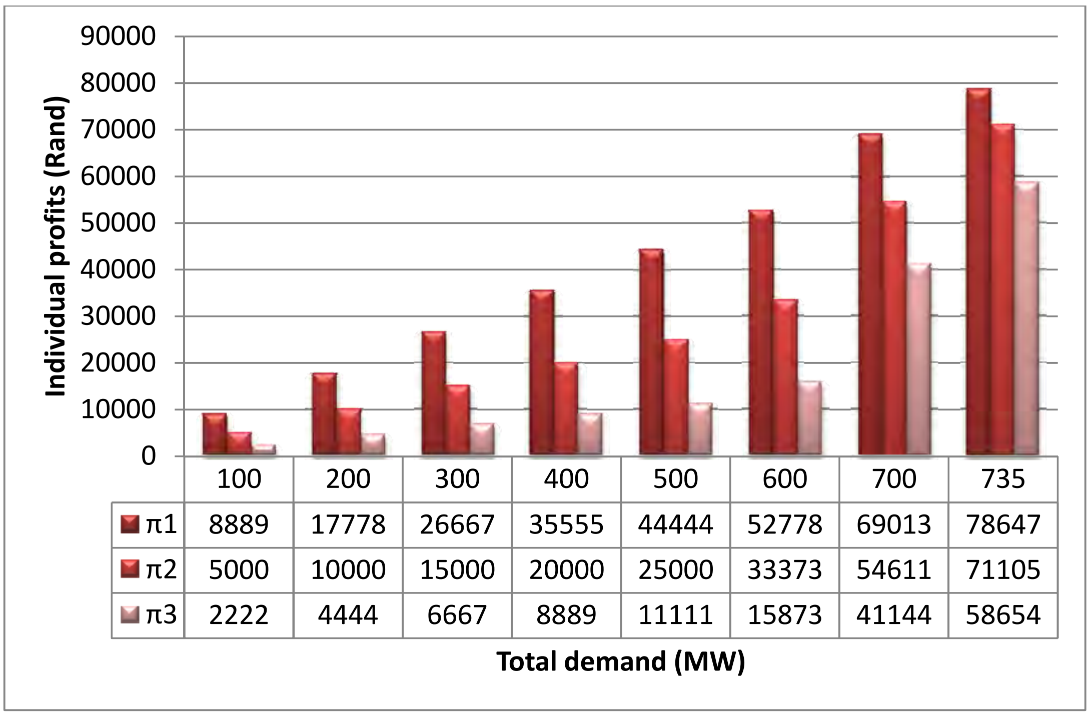


	100	200	300	400	500	600	700	735
Q1	44.40	88.90	133.30	177.80	222.20	250.00	243.00	235.10
Q2	33.30	66.70	100.00	133.30	166.70	207.10	233.38	249.90
Q3	22.20	44.40	66.70	88.90	111.10	142.90	223.61	250.00

Total demand (MW)

Fig. 4.14 Individual production with constant demand elasticity under generation and transmission constraints

Figure 4.15 shows the individual profits under generation and transmission constraints. The individual profits of three GenCos are the same as scenario 2 until transmission constraint starts to take effect. In Figure 4.15, although GenCo1 has less production due to the transmission constraint, its profit is still higher than in the other two cases. This is due to the higher increase in market clearing price compared to a lower decline in production. GenCo2 and GenCo3 have also benefited from the existence of transmission constraints by supplying more production with a higher market clearing price.



	100	200	300	400	500	600	700	735
π1	8889	17778	26667	35555	44444	52778	69013	78647
π2	5000	10000	15000	20000	25000	33373	54611	71105
π3	2222	4444	6667	8889	11111	15873	41144	58654

Fig. 4.15 Individual profits with constant demand elasticity under generation and transmission constraints

Load payment under generation and transmission constraint is shown in Figure 4.16. Load payment is higher than when there are no constrains and when there is generation constraint only. The main contribution towards the higher load payment is the increase in the market clearing price. This indicates that load demand is disadvantaged by overpaying for the same capacity it requires from the network.

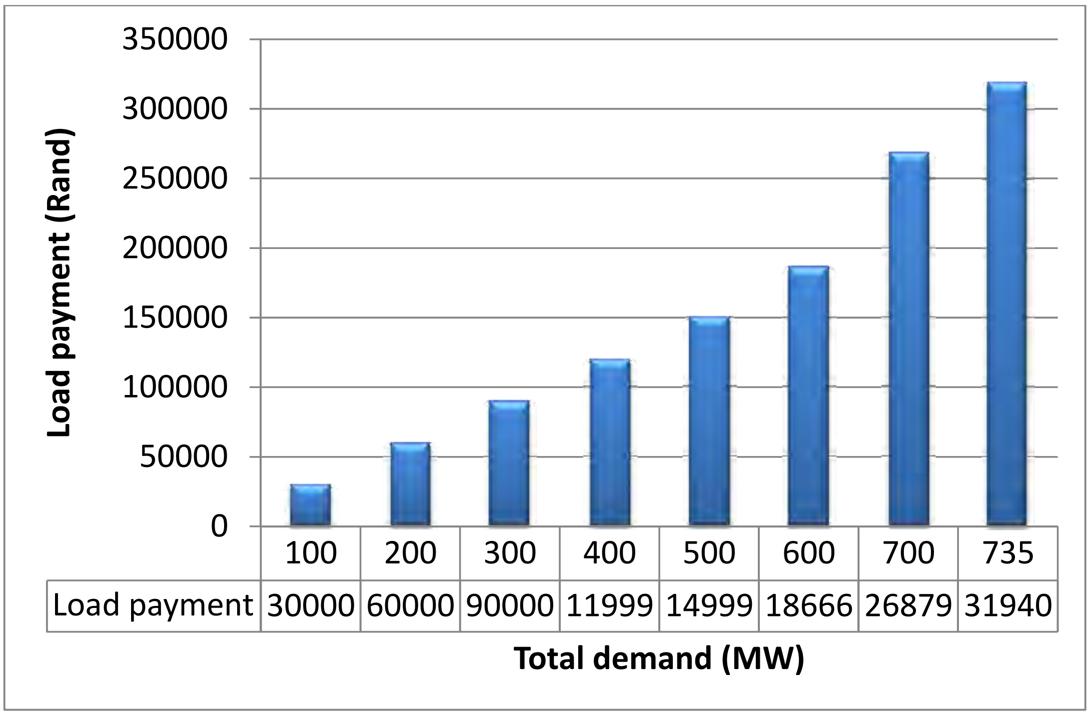


Fig. 4.16 Load payment with constant demand elasticity under generation and transmission constraints

4.3.4 Summary of case 1

The comparison of market results from all scenarios under case 1 is presented in this section. Since the maximum total demand varies from no constraint (800MW) to the one with generation capacity constraint (750MW) and transmission capacity constraint (R735MW), the total demand range that is used for comparison in this section is from 100MW to 700MW.

Table 4.1 shows the individual production in case 1. For $GenCo_{1,}$ production reaches maximum at 250MW when total demand increases to 600MW in scenario 2 and scenario 3. $GenCo_1$'s production decreases when total demand reaches 700MW in scenario 3 in order to avoid transmission congestion. A similar situation applies to $GenCo_2$, when total demand increases to 700MW, its production reaches its maximum capacity at 250MW in scenario 2, but also needs to reduce to 233MW in scenario 3 in order to avoid the transmission congestion on line L_{13}. Although $GenCo_3$ has the highest marginal cost, it takes advantage of its position of being at the same bus as the load demand. When total demand increases to 700MW, $GenCo_3$'s production reaches 224MW in scenario 3, compared to 156MW and 200MW in scenarios 1 and 2, respectively.

Table 4.1

Comparison of individual production in case 1

Total Demand (MW)	**Q_1 (MW)**			**Q_2 (MW)**			**Q_3 (MW)**		
	Scenario 1	**Scenario 2**	**Scenario 3**	**Scenario 1**	**Scenario 2**	**Scenario 3**	**Scenario 1**	**Scenario 2**	**Scenario 3**
100	44	44	44	33	33	33	22	22	22
200	89	89	89	67	67	67	44	44	44
300	133	133	133	100	100	100	67	67	67
400	178	178	178	133	133	133	89	89	89
500	222	222	222	167	167	167	111	111	111
600	267	250	250	200	207	207	133	143	143
700	311	250	243	233	250	233	156	200	224

Table 4.2 shows the comparison of market clearing price from all three scenarios in case 1. In scenario 1, without constraints, market clearing price stays unchanged. With the introduction of generation and transmission capacity constraints in scenarios 2 and 3, market clearing price increases once the generation and transmission capacity limits are reached. When total demand increases to 700MW, market clearing price in scenario 3 is R384/MW, higher than R300/MW in scenario 1 and R350/MW in scenario 2, respectively.

Table 4.2

Comparison of market clearing price in case 1

Total Demand (MW)	Scenario 1 (Rand/MW)	Scenario 2 (Rand/MW)	Scenario 3 (Rand/MW)
100	300	300	300
200	300	300	300
300	300	300	300
400	300	300	300
500	300	300	300
600	300	311	311
700	300	350	384

Table 4.3 shows the comparison of individual profits in case 1. As individual profit is determined by the change of market clearing price and individual production, with the increase of market clearing price in scenarios 2 and 3, individual profits tend to increase. This is especially so when demand increases to 700MW, as all three GenCos reach highest profits in scenario 3 due to the presence of generation and transmission constraints.

Table 4.3

Comparison of individual profits in case 1

Total Demand (MW)	π_1 (Rand)			π_2 (Rand)			π_3 (Rand)		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
100	8889	8889	8889	5000	5000	5000	2222	2222	2222
200	17778	17778	17778	10000	10000	10000	4444	4444	4444
300	26667	26667	26667	15000	15000	15000	6667	6667	6667
400	35555	35555	35555	20000	20000	20000	8889	8889	8889
500	44444	44444	44444	25000	25000	25000	11111	11111	11111
600	53333	52783	52778	30000	33380	33373	13333	15878	15873
700	62222	62501	69013	35000	50001	54611	15555	30002	41144

Table 4.4 presents the comparison of load payment in case 1. With the same level of total demand, after the introduction of generation and transmission constraints, load payment becomes higher. When total demand increases to 700MW, load payment reaches the highest level of R268,799 in scenario 3.

Table 4.4

Comparison of load payment in case 1

Total Demand (MW)	Scenario 1 (Rand)	Scenario 2 (Rand)	Scenario 3 (Rand)
100	30000	30000	30000
200	60000	60000	60000
300	90000	90000	90000
400	119999	119999	119999
500	149999	149999	149999
600	179999	186679	186667
700	209999	245003	268799

4.4 Case 2 - Change of demand elasticity with constant total demand

4.4.1 Scenario 1: with no capacity constraints

The market clearing price changes as elasticity changes as shown in Figure 4.17.

It can be seen from Figure 4.17 that the market clearing price is extremely high when demand elasticity is low. The price was R750, 150/MWh at the extreme lower bound of elasticity. The price level varies against demand elasticity, between the values of -0.4334 and -1.3334, as shown on the horizontal axis. This pattern explains the reason why only the value of demand elasticity between -0.4334 to -1.3334 is chosen for simulation. As mention above, in this thesis, the area of demand elasticity is calculated between -0.4334 to -1.3334. When the number of GenCos and the marginal costs of GenCos change, the area of demand elasticity will change accordingly.

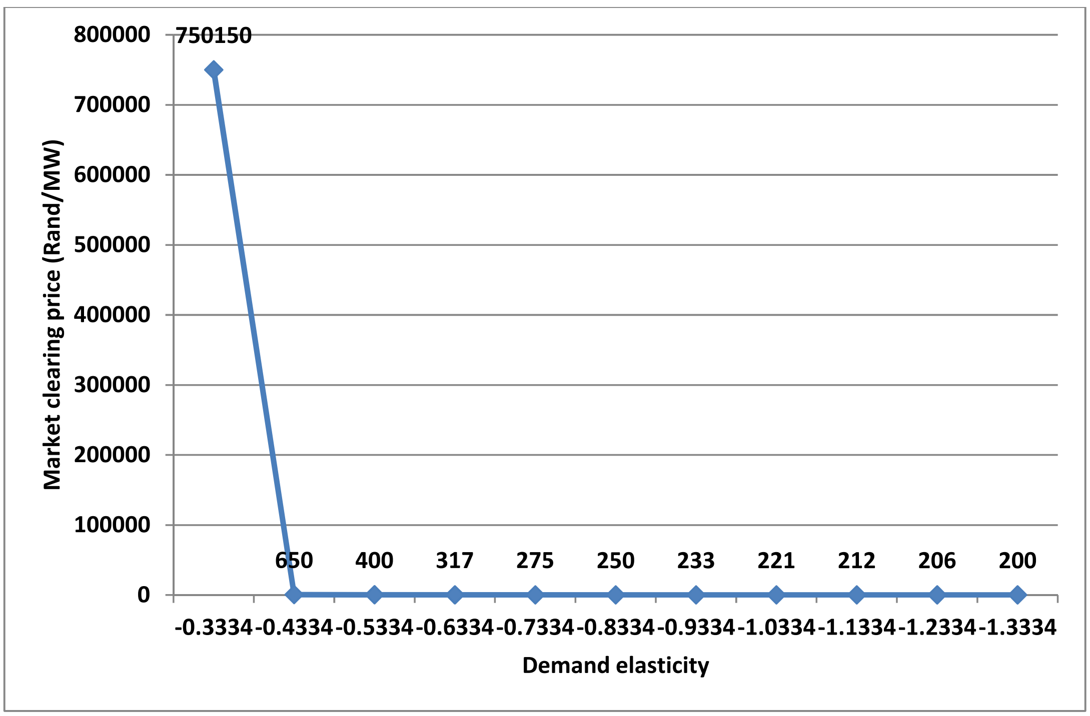


Fig. 4.17 Market clearing price with the change of demand elasticity, where total demand stays constant at 600MW

Figure 4.18 shows a clearer change of market price after zooming area of demand elasticity between -0.4334 to -1.3334, which cannot be seen clearly in Figure 4.17. This applies to all subsequent analysis in this thesis.

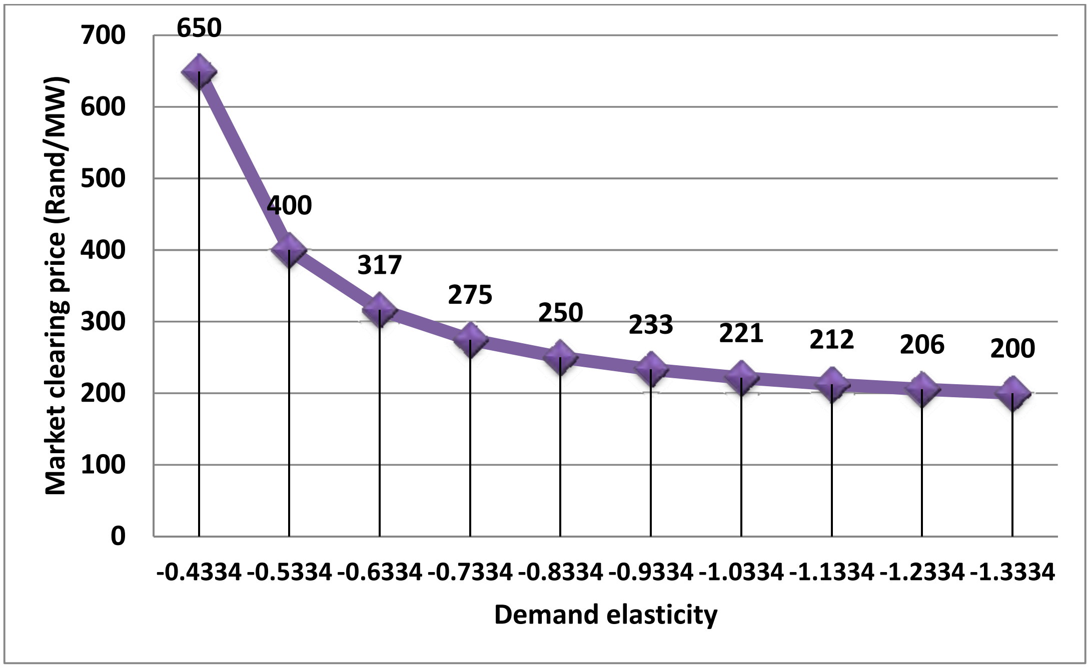


Fig. 4.18 Market price without extreme lower bound of elasticity, where total demand stays constant at 600MW

In Figure 4.18, the pattern of price is similar to the one in figure 4.17. With the enlarged view, it can be seen that market clearing price approaches R200/MW at demand elasticity of -1.3334. It means that, as the demand becomes more responsive, market price drops. This indicates that the demand side can effectively reduce the market clearing price by becoming more responsive.

Figure 4.19 shows the individual production of GenCos against the change of demand elasticity. As shown in Figure 4.19, the individual output increases for $GenCo_1$, it stays the same for $GenCo_2$ and decreases for $GenCo_3$. The reason for this type of change is that, when demand is more responsive to price, the most economical generator will be dispatched more fully first. In contrast, the more expensive a generator is, the less it will be dispatched. Marginal cost plays an important role in the determination of the output change pattern. The marginal cost of $GEenCo_2$ is the mean of the marginal costs of $GenCo_1$ and $GenCo_3$. Therefore, its output level does not change. If the marginal cost of $GenCo_2$ is close to that of $GenCo_1$, its output will increase. On the other hand, if the marginal cost of $GenCo_2$ is close to the one of $GenCo_3$, its output will decrease.

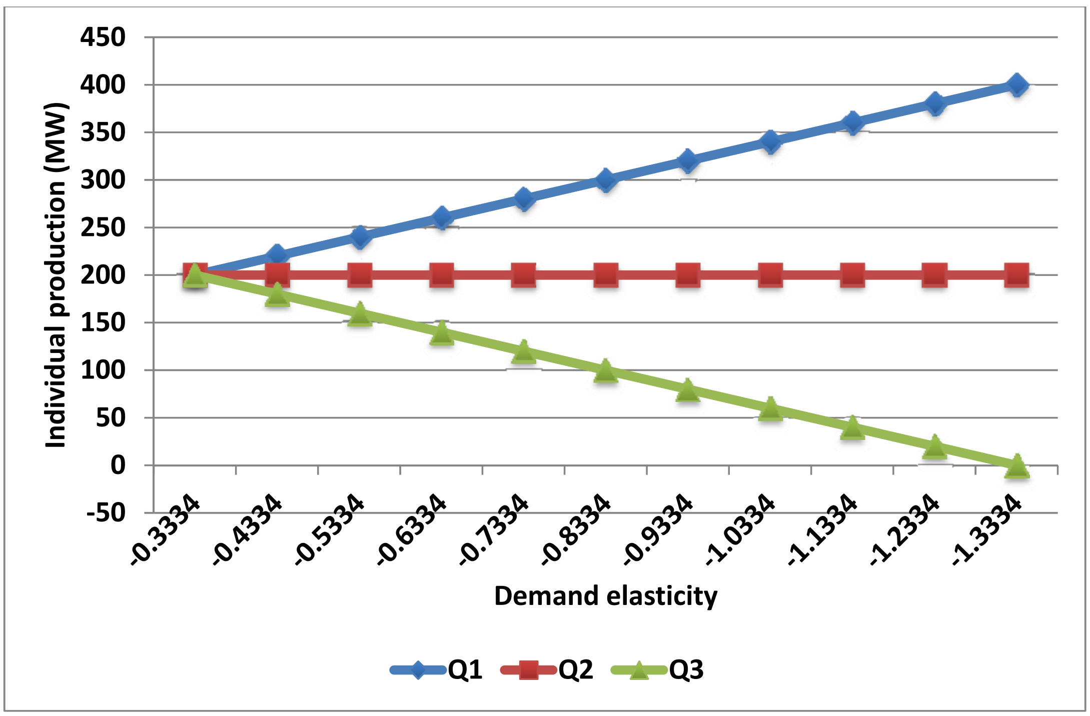


Fig. 4.19 Individual outputs with change of demand elasticity, where total demand stays constant at 600MW

In Figure 4.20, when demand becomes more responsive to price, the more expensive GenCo will not be dispatched as fully as before. When demand elasticity is close to -1.3334 for example, $GenCo_3$ is not dispatched, and its profit becomes zero. The profits of $GenCo_1$ and $GEenCo_2$ decrease due to the decrease in market price.

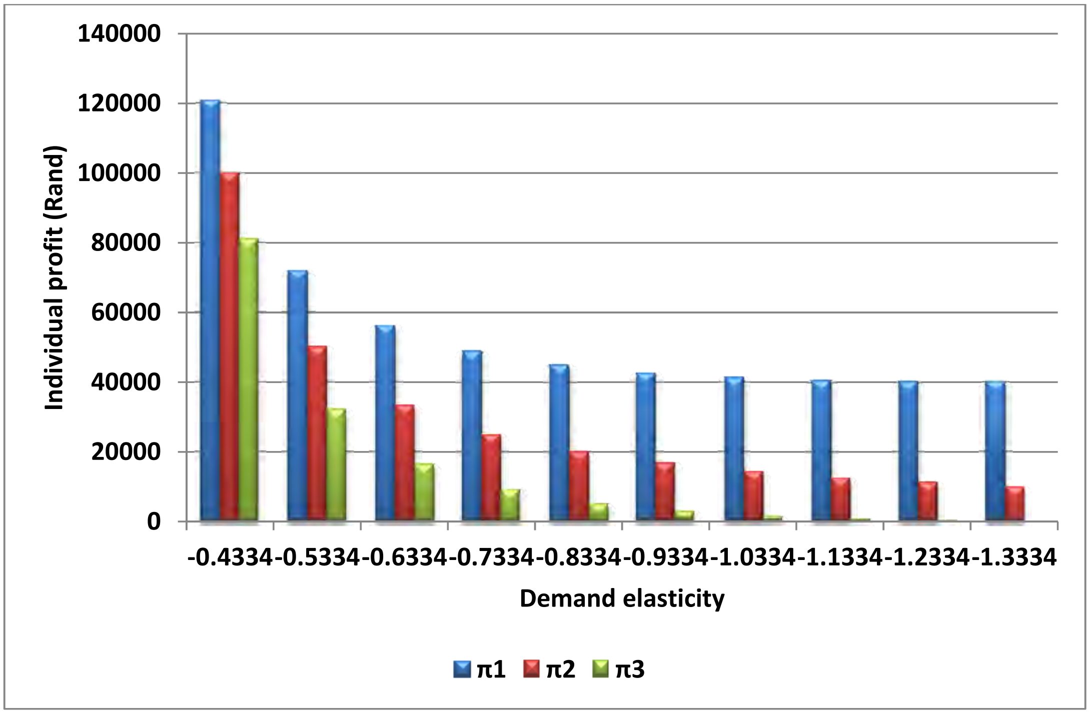


Fig. 4.20 Individual profits with change of demand elasticity without lower bound of elasticity, where total demand stays constant at 600MW

Figure 4.21 shows that load can effectively change its payment by changing its elasticity. It means that load payment can be reduced excessively if the demand is more responsive to market price. When the load is more elastic, it pays lower market price per unit for more quantity demanded.

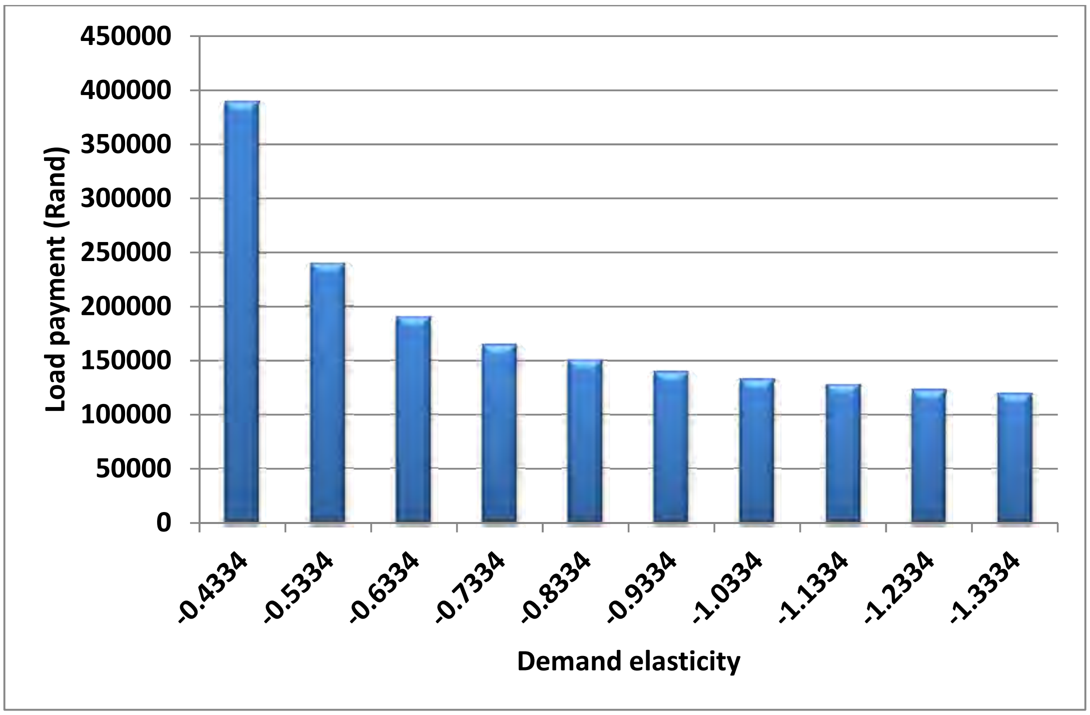


Fig 4.21 Change of load payment with change of demand elasticity without lower bound of elasticity, where total demand stays constant at 600MW

Figure 4.22 shows the marginal change of market clearing price over demand elasticity. As mentioned above, the absolute value of demand elasticity determines its level of impact. As demand becomes more elastic, for example changes from -0.5334 to -0.6334, its marginal impact on market clearing price decreases until it is equal to the marginal cost of the most expensive GenCo. It can be seen that demand elasticity has much higher impact on the market clearing price at its lower bound session.

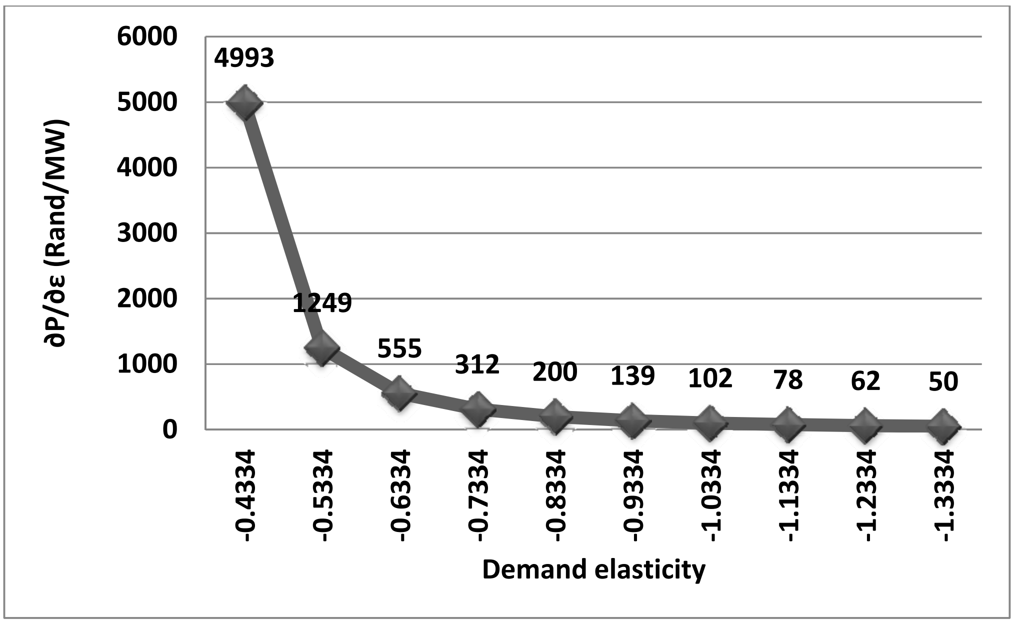


Fig. 4.22 Derivative of market clearing price over demand elasticity

In Figure 4.23, the shape of the derivatives of individual quantitiy output over demand elasticity for three GenCos are presented. As the three GenCos have the same difference interval of their marginal costs, the marginal change of GenCo2' output, $\frac{\partial Q_2}{\partial \varepsilon}$, is not affected by the change of demand elasticity as its marginal cost equals to the average cost. Therefore, its derivative equals to zero. The outputs of $GenCo_1$ and $GenCo_3$ have constant and inverse pattern.

It is important to note that as the value of demand elasticity is negative, the marginal change of the indivual production over demand elasticity should be intepreted inversely. It means that the maginal change of demand elasticity with 1 unit will cause 200 MW increase of Q_1, zero change of Q_2 and a 200 MW decrease of Q_3. The change of demand elasticity is 0.1 in each interval, and has 10 intervals in total. Therefore, the changes in individual production of Q1, Q2 and Q3, are +20, 0 and -20, respectively. Figrue 4.23 also shows that the maginal impact of demand elasticity on individual production is constant at different level of demand elasticity.

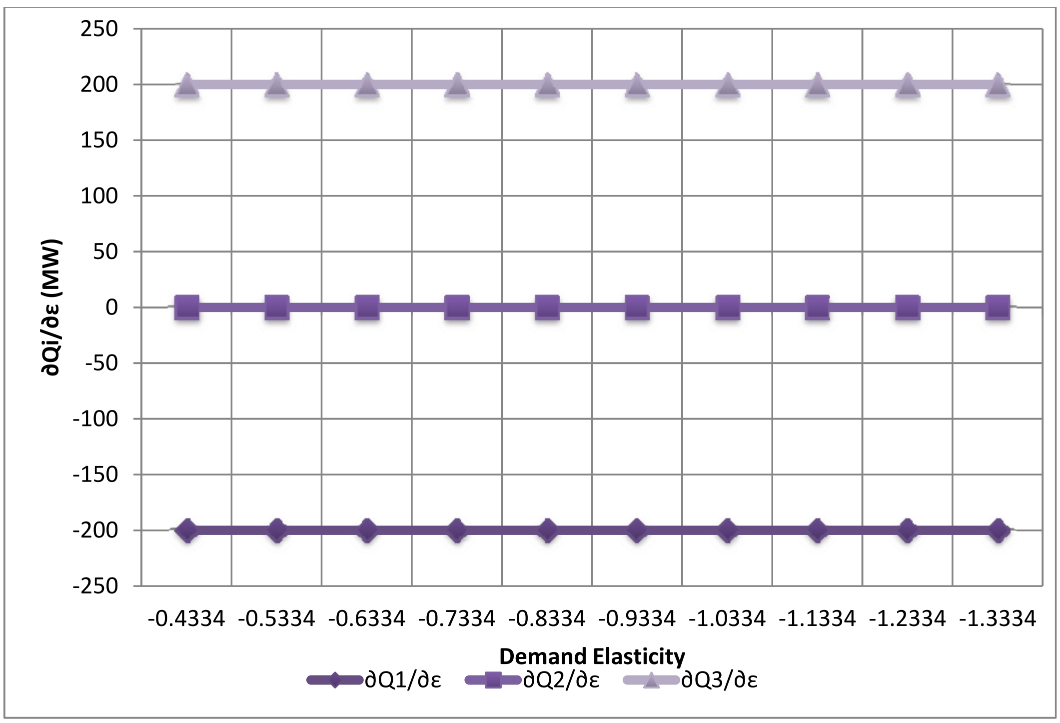


Fig. 4.23 Derivative of individual quantity output over demand elasticity

4.4.2 Scenario 2: with generation capacity constraint

Fig. 4.24 presents the individual production with change of demand elasticity under generation constraint. The pattern of individual production of all GenCos changes after the demand elasticity reaches -0.6334 as shown in Figure 4.24. As $GenCo_1$'s production stops increasing, the additional load demand is allocated between $GenCo_2$ and $GenCo_3$. When demand elasticity reaches its maximum of -1.3334, $GenCo_1$ and $GenCo_2$ both reach their maximum capacity. The rest of the load demand is supplied by $GenCo_3$. This is different from the case in which system constraint does not exist. The continuous supply from the most expensive generation company, i.e. $GenCo_3$, is the main contribution to the high market clearing price. The higher price signal should be captured as an alert of a supply shortage, especially of the lower cost generation supply.

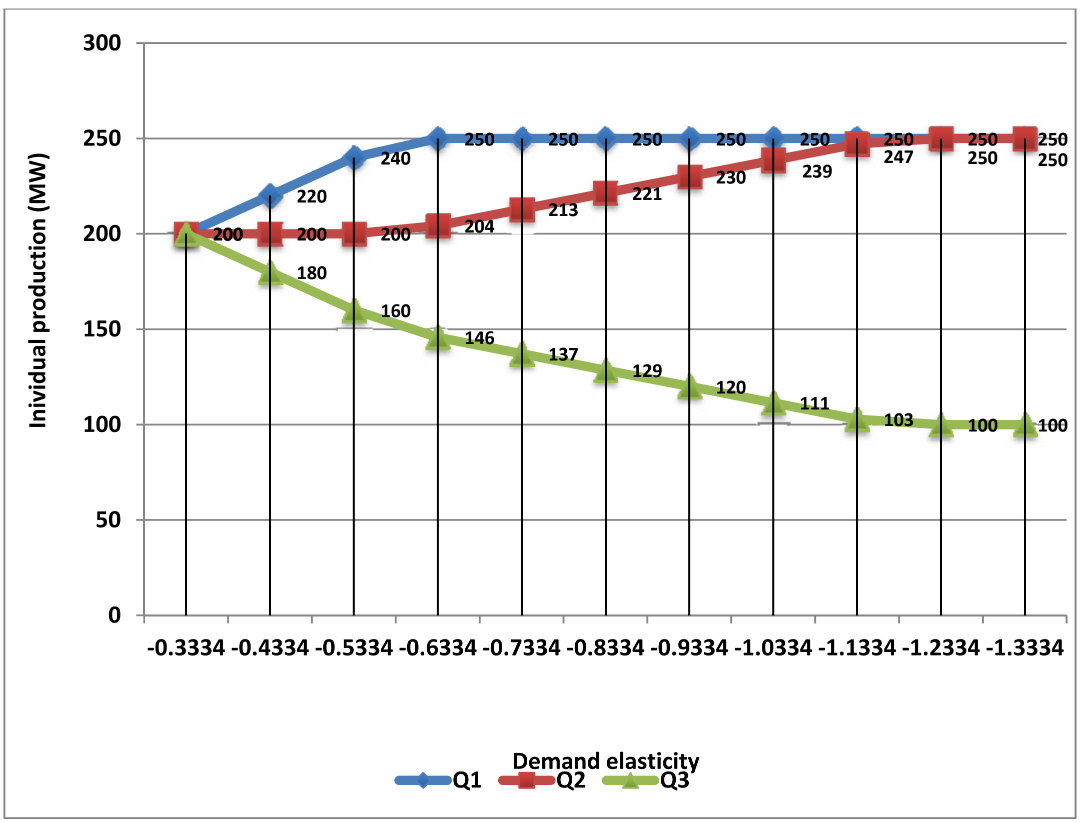


Fig. 4.24 Individual production with change of demand elasticity under generation constraint

The market clearing price is decreasing as the demand becomes more responsive. However, the impact of demand elasticity on market clearing price has been limited due to the generation capacity constraint. As shown in Figure 4.25, due to the effect of the multiplier of the relationship between the GenCo's actual generation quantity and maximum generation capacity, mu, market clearing price is relatively higher after the demand elasticity reaches -0.6334, compared to the one in scenario 1. When demand elasticity is within its upper bound, the effect of the multiplier reaches its maximum. Even when the demand elasticity has reached its highest level, at -1.3334, the market clearing price does not go down to R200/MW as does $GenCo_3$'s marginal cost because of the existence of the multiplier. Therefore, the market clearing price stays higher than in scenario 1.

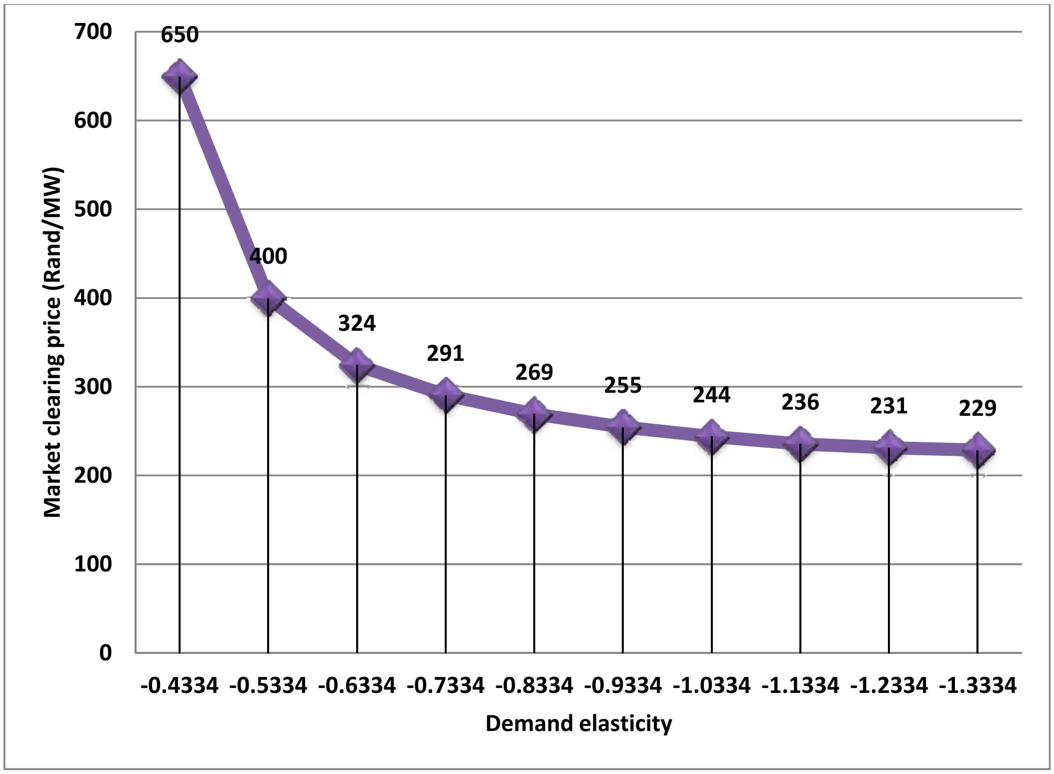


Fig. 4.25 Market clearing price with change of demand elasticity under generation constraint

Figure 4.26 shows the individual profit with change of demand elasticity under generation constraint. The individual profits of all GenCos are decreasing during the increase of demand elasticity in absolute value as shown in Figure 4.26. $GenCo_3$ is still obtaining profit even when the demand elasticity reaches its maximum. $GenCo_1$ obtains less profit under generation constraint, after demand elasticity reaches -0.6334, compared to scenario 1. The sum of individual profits with generation capacity constraint is higher than those without constraint. This is mainly due to the higher market clearing price and the continuous supply from the more expensive generation, i.e. GenCo3. In general, $GenCo_1$ is worse off with the generation capacity constraint due to the fact that its dispatched generation declines. $GenCo_2$ and $GenCo_3$ are better off with the generation capacity constraint. This can be explained by the fact that after reaching its generation capacity limit, $GenCo_1$ keeps supplying the same amount of power at its low price. Therefore, its profit decreases more quickly than in the case where there is no constraint.

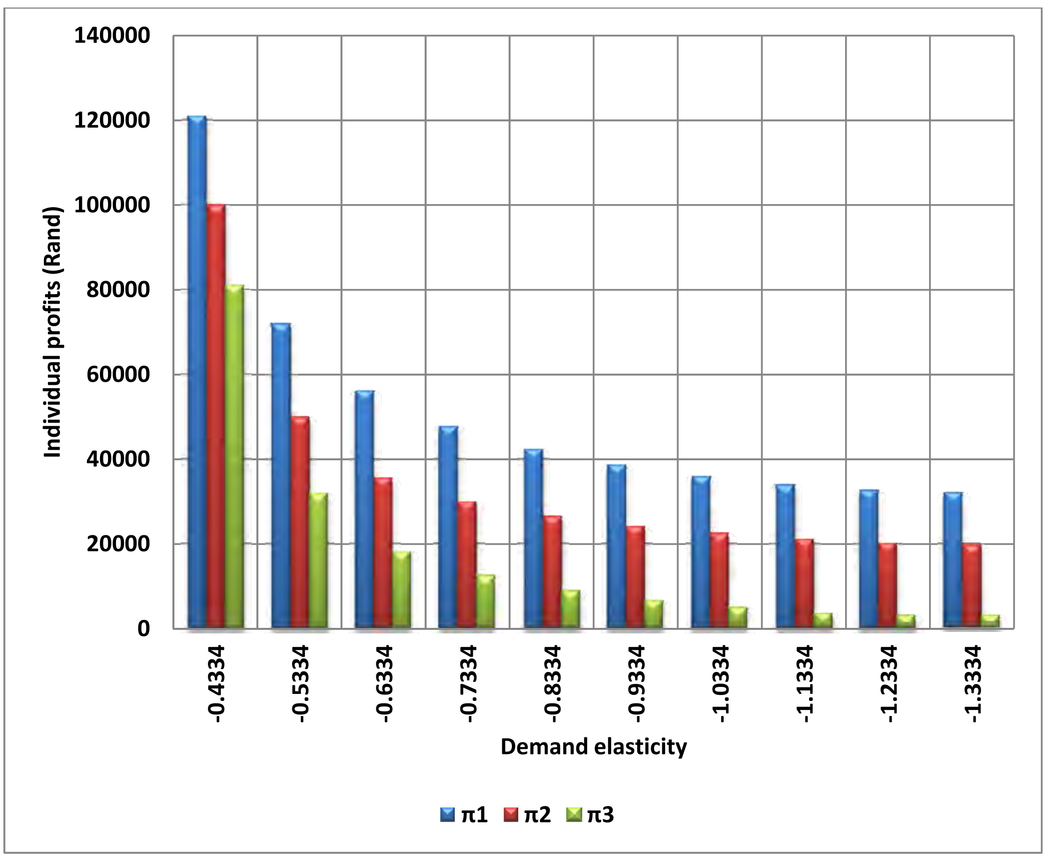


Fig. 4.26 Individual profit with change of demand elasticity under generation constraint

Figure 27 shows the change of load payment against the change of demand elasticity. With the increase of demand elasticity, as shown in Figure 4.27, load payment decreases similarly to the scenario without constraint. However, the decreasing rate of load payment is less than in the case without generation capacity constraint. Since the total quantity demand remains unchanged at 600MW, as demand elasticity increases, the load payment is heavily dependent on the market clearing price. The market clearing price with generation capacity constraint includes the consideration of μ_i. Therefore, the decrease of market price in the case with generation capacity constraint is not as much as the one without constraint. This has resulted in higher load payment in the case with generation capacity constraint.

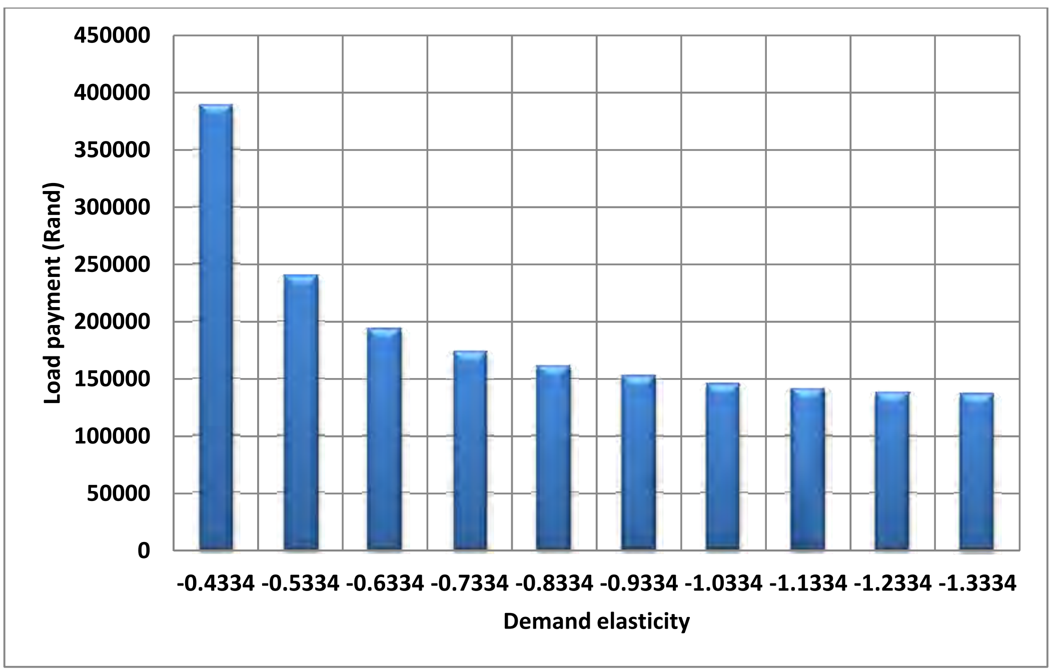


Fig. 4.27 Load payment with change of demand elasticity under generation constraint

The pattern of the derivative of market clearing price over demand elasticity is presented in Figure 4.28. In the case of generation capacity constraint, the derivative of market clearing price over demand elasticity is not only determined by the sum of marginal cost and the value of demand elasticity, but also on the value of mu. It is important to note that, although the value of the derivative is higher than the one in the case without constraint, it does not necessary mean that demand elasticity has caused a higher change in the market clearing price in this case. This is mainly due to the penalty effect on the generation constraint from the supply side.

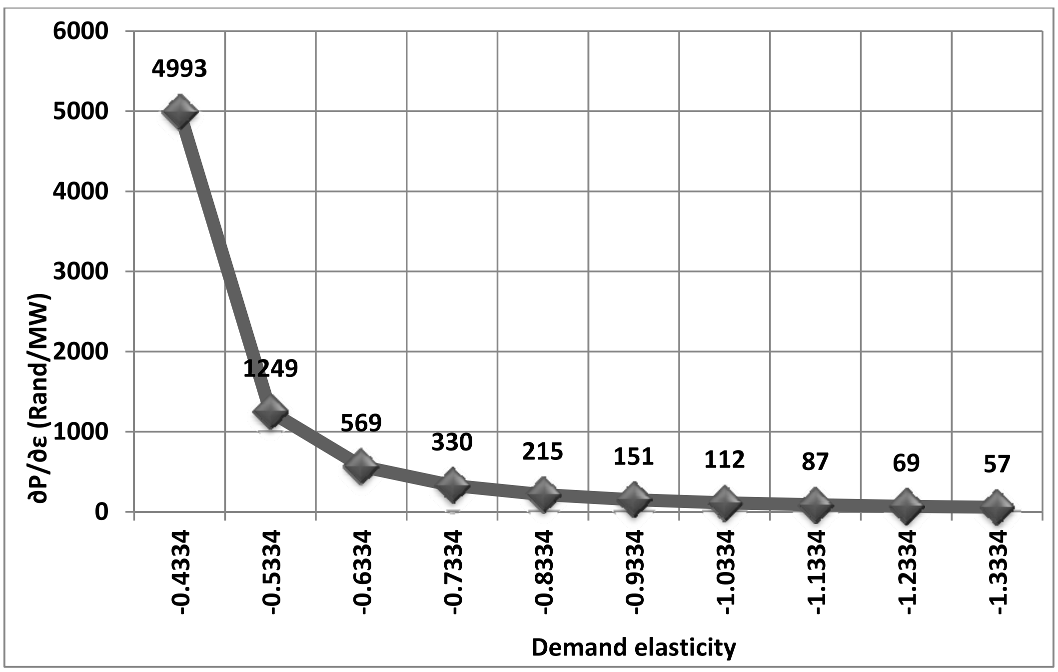


Fig. 4.28 Derivative of market clearing price over demand elasticity

In Figure 4.29, the derivative of individual production over demand elasticity under generation capacity constraint is presented. The value of derivatives without generation capacity constraint is constant for all GenCos, but the values become non linear after the value of μ becomes non zero in the case with generation capacity constraint. As demand elasticity increases further, the impact of demand elasticity becomes weaker on the increase in $GenCo_1$'s production. Over a certain point, $GenCo_2$'s marginal production also starts to increase. The decreasing rate of $GenCo_3$'s marginal production over demand elasticity is less than in the case without constraint.

Different from scenario 1, the derivative of individual production over demand elasticity under generation capacity constraint is no longer constant after the level of demand elasticity increases over -0.5334.

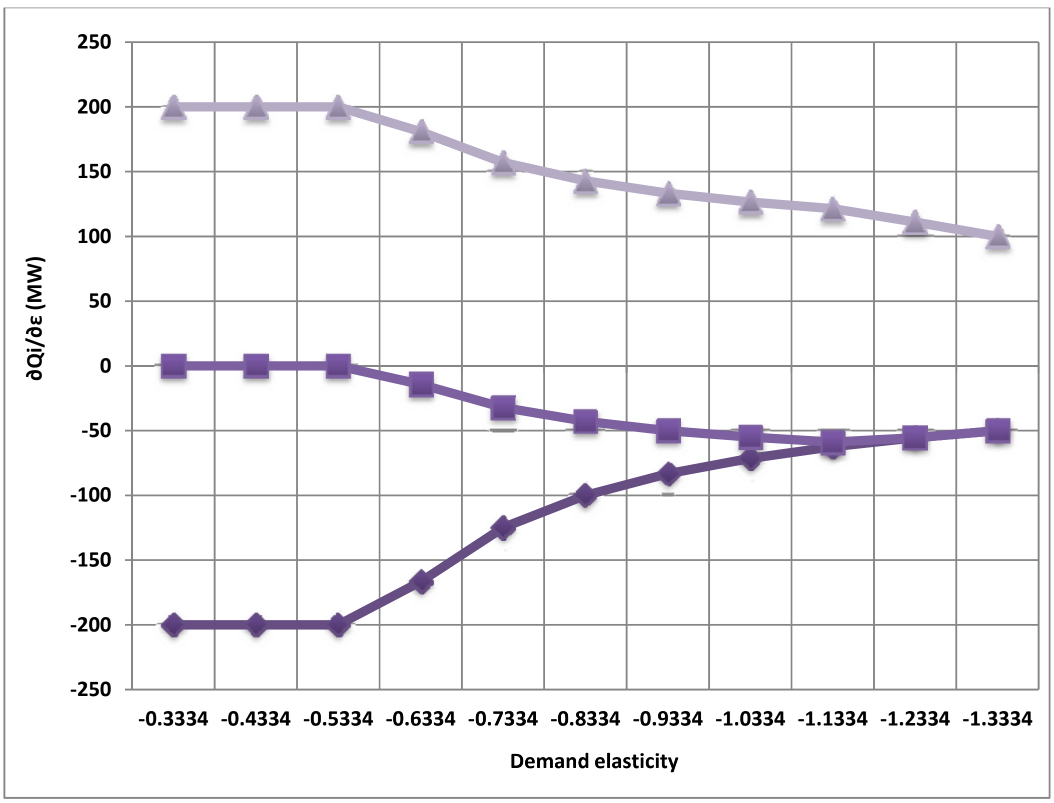


Fig. 4.29 Derivative of individual production over demand elasticity

4.4.3 Scenario 3: with generation and transmission capacity constraint

Figure 4.30 shows individual production with change of demand elasticity under generation and transmission constraints. Individual production of all GenCos changes in the beginning and remains almost the same when demand elasticity is beyond -0.8334. This means that demand elasticity has no further impact on individual productions at and above a certain point. It is believed that there is no room for the more economical GenCos, (i.e. $GenCo_1$ and $GenCo_2$) to supply any additional MW to the load due to the transmission constraint. For a quantity game, this will be the optimal solution among the GenCos.

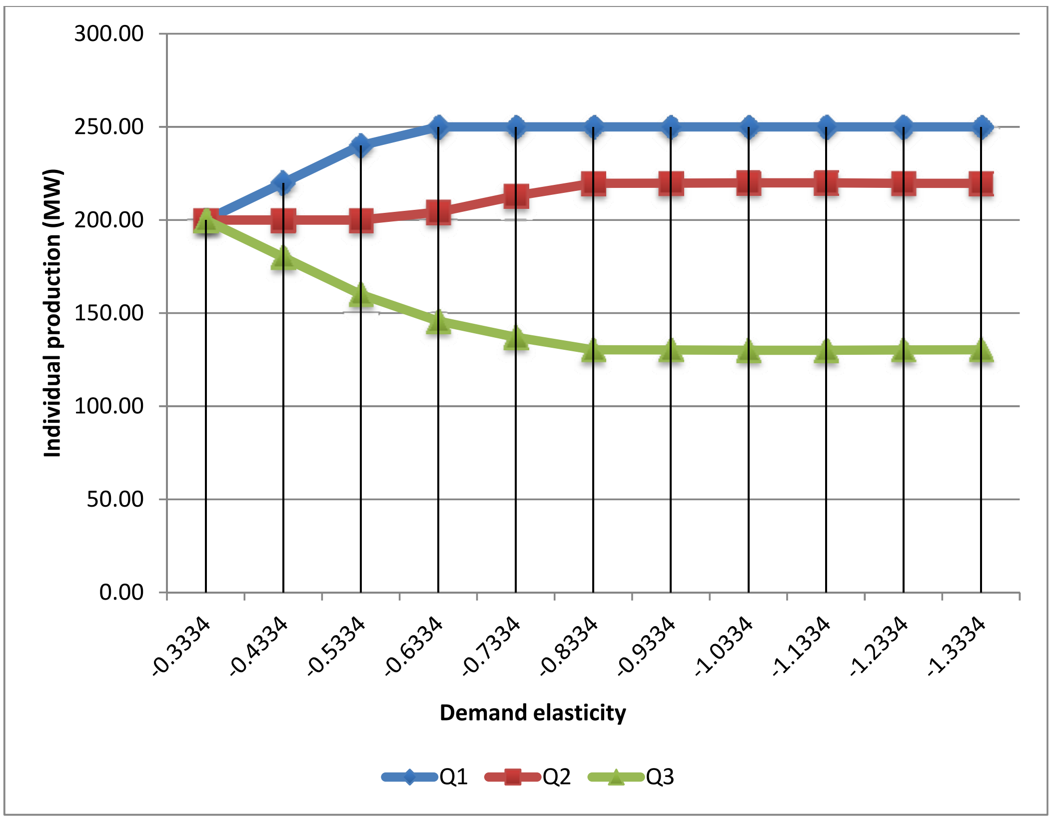


Fig. 4.30 Individual production with change of demand elasticity under generation and transmission constraints

Figure 4.31 presents the market clearing price change against demand elasticity under generation and transmission constraints. As shown in Figure 4.31, although market clearing price decreases similarly to the decreases in scenarios 1 and 2, the price level generally sits above those in cases 1 and 2. When transmission constraint exists, the more economical GenCo, such as $GenCo_2$, may not be fully dispatched. The more expensive GenCo, such as $GenCo_3$, will be dispatched more due to its position on the same bus as the load. This suggests that system constraints can effectively reduce the impact of demand elasticity on the market output.

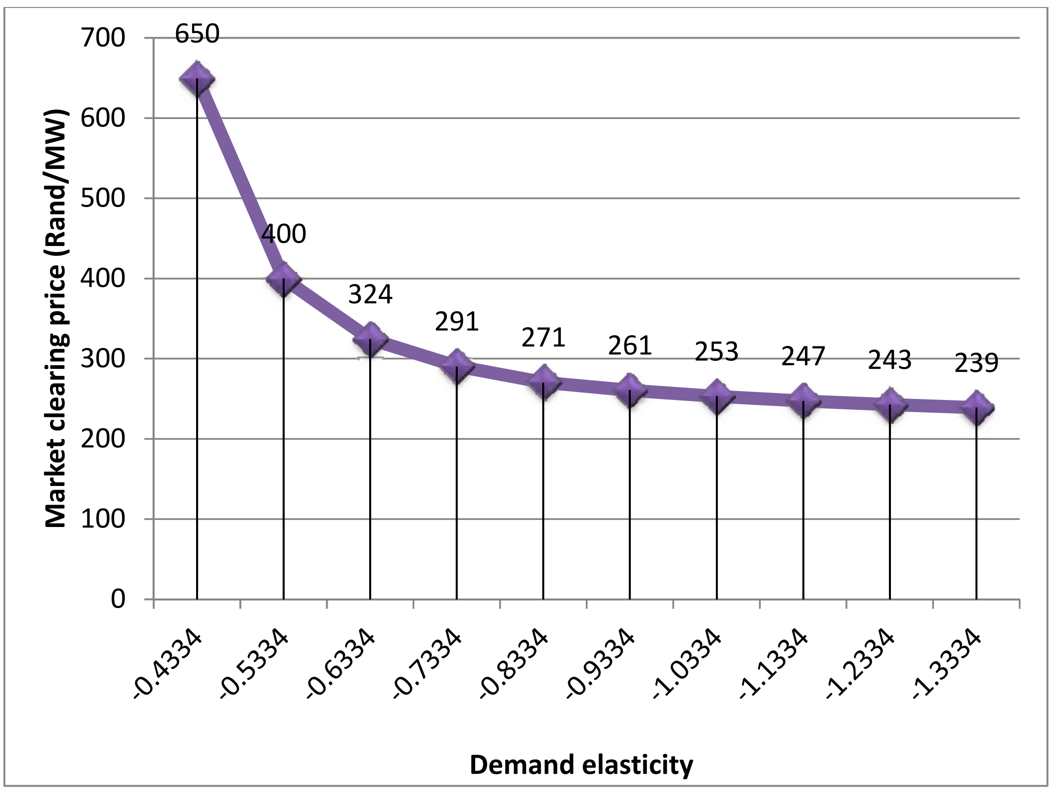


Fig. 4.31 Marke clearing price with change of demand elasticity under generation and transmission constraints

In Figure 4.32, all GenCos' individual profits against demand elasticity are presented. When demand elasticity reaches and beyond -0.8334, the individual profits are different for all GenCos from those in the other two scenarios. For $GenCo_1$, its production does change, with a higher market clearing price received, its profit becomes higher. $GenCo_2$ faces higher price and lower production, and as the former is greater than the latter, its production also increases. The generation company which benefits the most is $GenCo_{3,}$ almost doubling its profit compared to the scenario 2 while the transmission constraint takes effect on the network. This is mainly due to the higher market price and the higher individual production of $GenCo_3$ under both generation and transmission constraints.

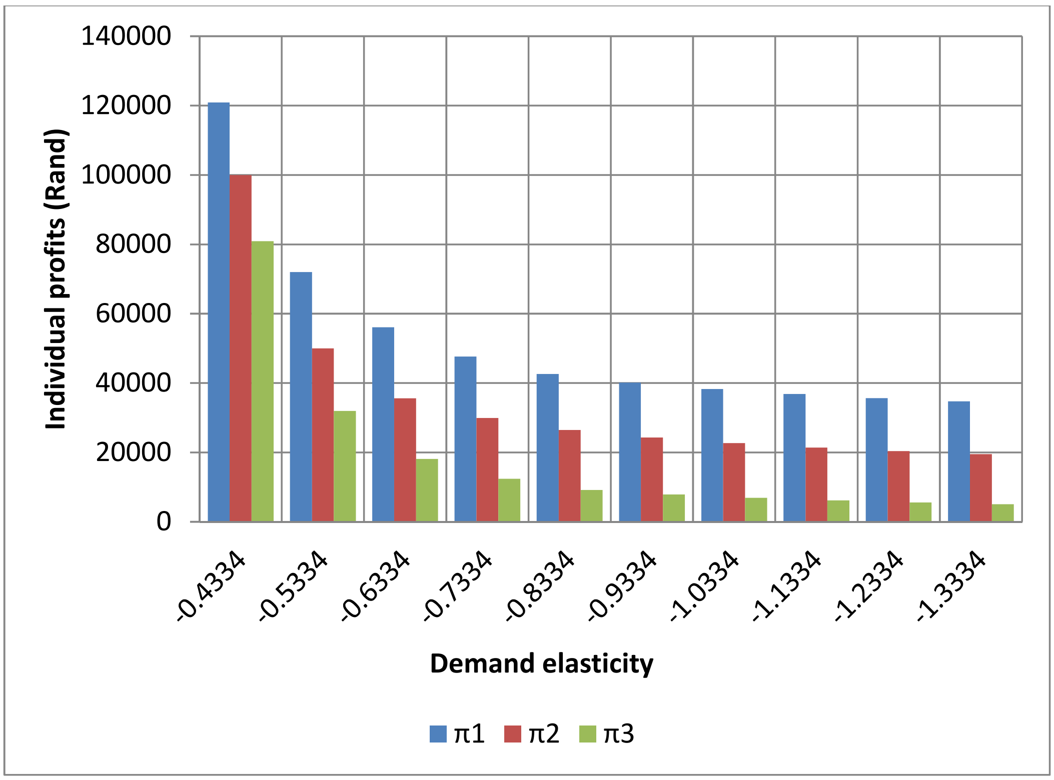


Fig. 4.32 Individual profit with change of demand elasticity under generation and transmission constraints

In Figure 4.33, load payment with generation and transmission constraint is presented. With both generation and transmission capacity constraints, load payment is the highest out of all three scenarios. The higher market clearing price is the main driving force behind the increase in load payment. It causes the reduction in customer surplus in the market due to the penalty charge on transmission constraint.

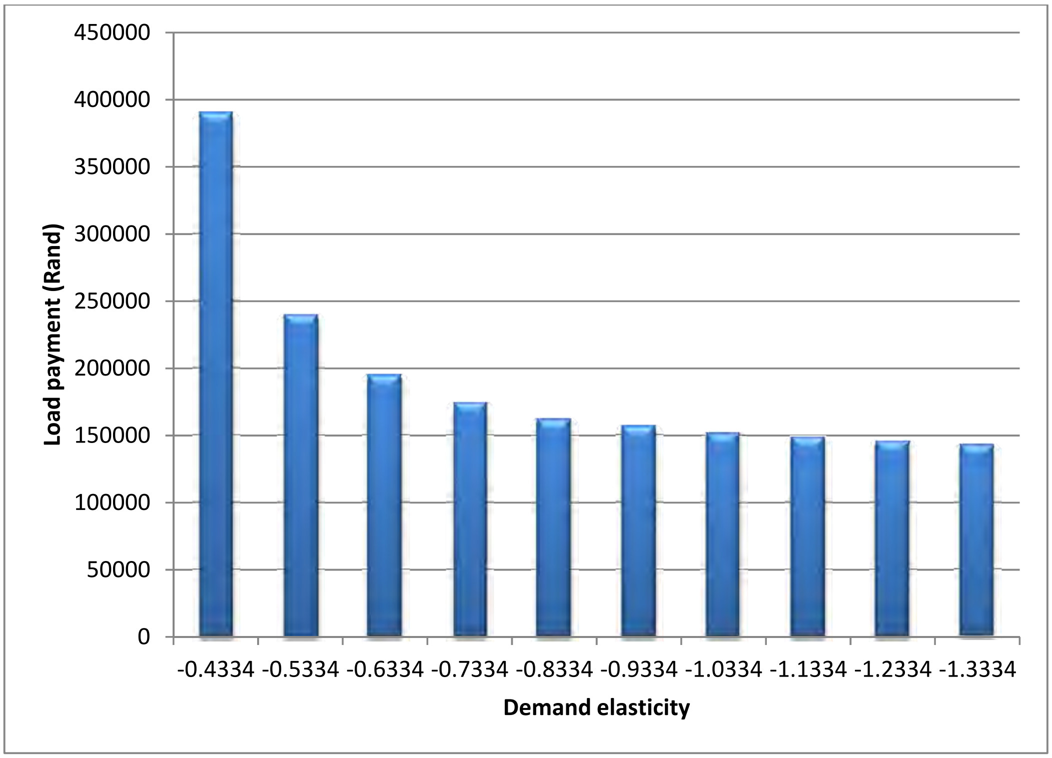


Fig. 4.33 Load payment with change of demand elasticity under generation and transmission constraints

Figure 4.34 shows the derivative of market clearing price over demand elasticity under the presence of both generation and transmission constraints. The value of the derivative is higher than in scenarios 1 and 2. This is due to the penalty effect on the generation and transmission constraints from the supply side and the network.

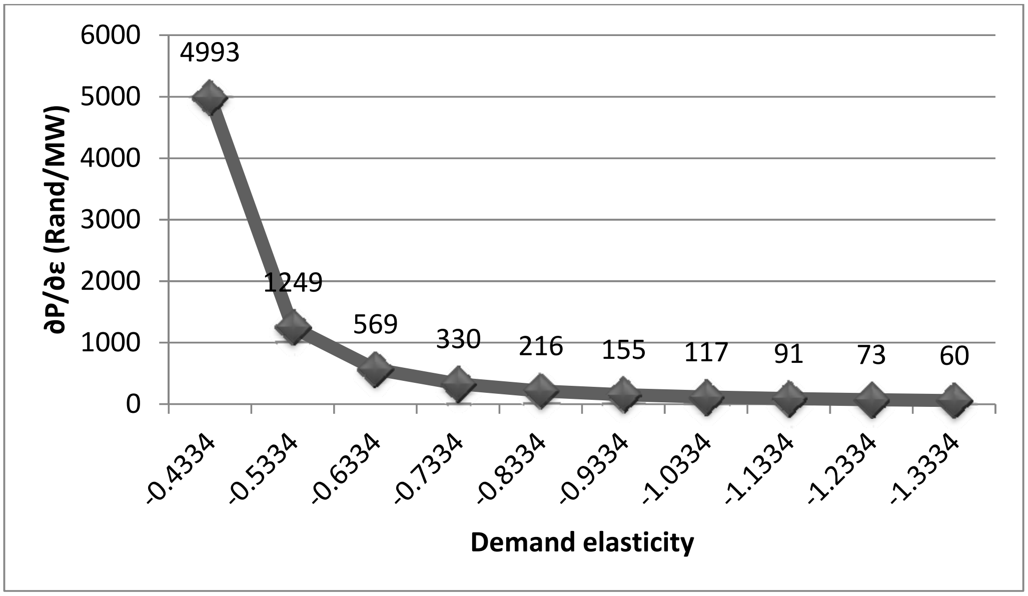


Fig. 4.34 Derivative of market clearing price over demand elasticity under generation and transmission constraints

Figure 4.35 shows the derivative of individual production over demand elasticity in scenario 3 with generation and transmission constraints. As shown in Figure 4.35, the value of the derivative for $GenCo_1$ is almost no different to the one in scenario 2. This can be explained by the fact that its production is almost identical in scenario 2 and scenario 3. Due to lower production in scenario 3 for $GenCo_2$, compared to scenario 2, the derivative shows less increase in its production. The opposite result applies to $GenCo_3$ with its higher increase in individual production resulting in higher value in the derivative of production over demand elasticity.

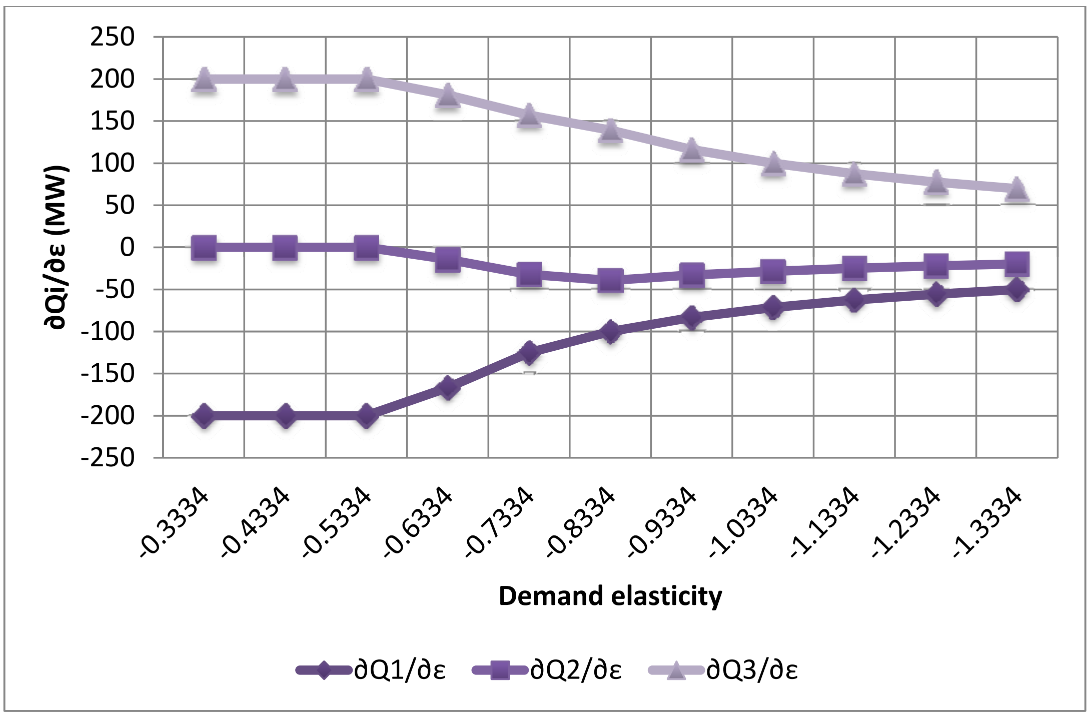


Fig. 4.35 Derivative of individual production over demand elasticity under generation and transmission constraints

4.4.4 Summary of case 2

Table 4.5 shows the comparison of market clearing price from all three scenarios in case 2. According to Table 4.5, market clearing price is highest in scenario 3 compared to the other two scenarios. The diminishing effect of demand elasticity on market clearing price can be identified.

Table 4.5

Comparison of market clearing price in case 2

Demand elasticity	**Scenario 1 (Rand/MW)**	**Scenario 2 (Rand/MW)**	**Scenario 3 (Rand/MW)**
-0.4334	650	650	650
-0.5334	400	400	400
-0.6334	317	324	324
-0.7334	275	291	291
-0.8334	250	269	271
-0.9334	233	255	261
-1.0334	221	244	253
-1.1334	212	236	247
-1.2334	206	231	243
-1.3334	200	229	239

Table 4.6 presents the comparison of individual production in case 2. In Table 4.6, in scenario 2 with generation capacity constraint, when $GenCo_1$ and $GenCo_2$ reach their maximum generation capacity, $GenCo_3$ will be dispatched in order to meet the demand. Since $GenCo_3$'s marginal cost is the highest, market clearing price will increase as the additional unit being dispatched determines the price level. In scenario 3, with generation and transmission constraints, $GenCo_1$ is fully dispatched. However, $GenCo_2$ is not fully dispatched due the transmission capacity limit. Therefore, $GenCo_3$ will be dispatched to meet the demand.

Table 4.6

Comparison of individual production in case 2

Demand elasticity	Q_1 (MW)			Q_2 (MW)			Q_3 (MW)		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
-0.4334	220	220	220	200	200	200	180	180	180
-0.5334	240	240	240	200	200	200	160	160	160
-0.6334	260	250	250	200	204	204	140	146	146
-0.7334	280	250	250	200	213	213	120	137	137
-0.8334	300	250	250	200	221	220	100	129	130
-0.9334	320	250	250	200	230	220	80	120	130
-1.0334	340	250	250	200	239	220	60	111	130
-1.1334	360	250	250	200	247	220	40	103	130
-1.2334	380	250	250	200	250	220	20	100	130
-1.3334	400	250	250	200	250	220	0	100	130

Table 4.7 presents the comparison of individual profits in case 2. Although individual profits of all GenCos tend to decrease in all scenarios, with the existence of more constraints, the decreasing rate of individual profits becomes lower and lower. $GenCo_3$ benefits from the existence of constraints. The general increase of individual profit for $GenCo_3$, from scenarios 1 to 2 and 3, indicates that the effect of demand elasticity on individual profit is more limited with the existence of constraints.

Table 4.7

Comparison of individual profits in case 2

Demand elasticity	π_1 (Rand)			π_2 (Rand)			π_3 (Rand)		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
-0.4334	120934	120934	120934	99933	99933	99933	80934	80934	80934
-0.5334	71984	71984	71984	49983	49983	49983	31984	31984	31984
-0.6334	56327	56092	56092	33326	35623	35623	16327	18122	18122
-0.7334	48997	47637	47637	24996	29918	29918	8997	12418	12418
-0.8334	44998	42302	42626	19997	26395	26472	4998	8897	9188
-0.9334	42665	38633	40149	16665	24042	24308	2665	6543	7891
-1.0334	41285	35952	38273	14284	22380	22673	1285	4881	6906
-1.1334	40500	33908	36824	12499	21163	21399	500	3664	6151
-1.2334	40111	32811	35680	11110	20311	20375	111	3124	5564
-1.3334	40000	32143	34729	9999	19643	19534	0	2858	5071

Table 4.8 presents the comparison of load payment in case 2. It can be seen that load payment in scenario 3 is the highest of the three scenarios. As total demand stays constant, load payment will increase as market clearing price increases. This indicates that the existence of both generation and transmission constraints has significantly affected the demand side. The increase of load payment due to the constraints should send a clear signal of the need for long term planning on generation and transmission capacity expansion.

Table 4.8

Comparison of load payment in case 2

Demand elasticity	**Scenario 1 (Rand)**	**Scenario 2 (Rand)**	**Scenario 3 (Rand)**
-0.4334	389800	389800	389800
-0.5334	239950	239950	239950
-0.6334	189978	194622	194622
-0.7334	164988	174329	174329
-0.8334	149992	161525	162301
-0.9334	139994	152718	156358
-1.0334	132853	146286	151855
-1.1334	127497	141380	148378
-1.2334	123331	138747	145632
-1.3334	119998	137144	143350

Table 4.9 shows the comparison of market clearing price over demand elasticity in case 2. It can be seen that demand elasticity has more impact on the market clearing price at its lower bound session. With the penalty effect on the constraints, from the supply side and the network, the value of the derivative of market clearing price over demand elasticity increases.

Table 4.9

Comparison of derivative of market clearing price over demand elasticity in case 2

Demand elasticity	Scenario 1 (Rand/MW)	Scenario 2 (Rand/MW)	Scenario 3 (Rand/MW)
-0.4334	4993	4993	4993
-0.5334	1249	1249	1249
-0.6334	555	569	569
-0.7334	312	330	330
-0.8334	200	215	216
-0.9334	139	151	155
-1.0334	102	112	117
-1.1334	78	87	91
-1.2334	62	69	73
-1.3334	50	57	60

Table 4.10 shows the comparison of the derivative of individual production over demand elasticity in case 2. As the value of demand elasticity is negative, the marginal change of the individual production over demand elasticity is interpreted inversely. It can be seen that, with the increase of demand elasticity, the increase of individual production from $GenCo_1$ and $GenCo_2$ is more limited in the presence of generation and transmission constraints. On the other hand, the influence of demand elasticity on the reduction of $GenCo_3$'s production is diminished when more constraints are introduced.

Table 4.10

Comparison of derivative of individual production over demand elasticity in case 2

Demand elasticity	$\partial Q_1/\partial \varepsilon$ (MW)			$\partial Q_2/\partial \varepsilon$ (MW)			$\partial Q_3/\partial \varepsilon$ (MW)		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
-0.4334	-200	-200	-200	0	0	0	200	200	200
-0.5334	-200	-200	-200	0	0	0	200	200	200
-0.6334	-200	-200	-200	0	0	0	200	200	200
-0.7334	-200	-167	-167	0	-14	-14	200	181	181
-0.8334	-200	-125	-125	0	-32	-32	200	157	157
-0.9334	-200	-100	-100	0	-43	-39	200	143	139
-1.0334	-200	-83	-83	0	-50	-33	200	133	116
-1.1334	-200	-71	-71	0	-55	-28	200	127	100
-1.2334	-200	-63	-62	0	-59	-25	200	121	87
-1.3334	-200	-56	-56	0	-56	-22	200	111	77

Chapter 5

Plexos simulation results and comparison with analytical model

Simulation results obtained with Plexos software package are presented in this chapter. Results from Plexos and the analytical model are also compared.

5.1 Brief description of Plexos

Plexos is a power market simulator with solution methods based on linear, mixed integer and quadratic optimization. It is developed by Energy Exemplar.

The Cournot Models that have been adopted in Plexos are based on the following assumptions [66]:

- Producers behave as Cournot players, that is, each individual producer chooses its output to maximize its profit given its rivals' outputs are fixed;
- The transmission system is represented as a linearized DC network on which power flows are consistent with Kirchhoff's laws;
- Transmission pricing is based on locational marginal pricing; and
- Both producers and arbitragers make output and pricing decisions under the assumption that their decisions do not affect transmission constraints and the fees that the Independent System Operator ISO charges for transmission services.

Figure 5.1 shows the Cournot models used in Plexos with PoolCo and Bilateral models. In the PoolCo model, all participants bid supply and demand to the ISO. In the bilateral model, trading activities are conducted between electricity generators and consumers. There are two situations under the bilateral model, i.e., with and without arbitragers.

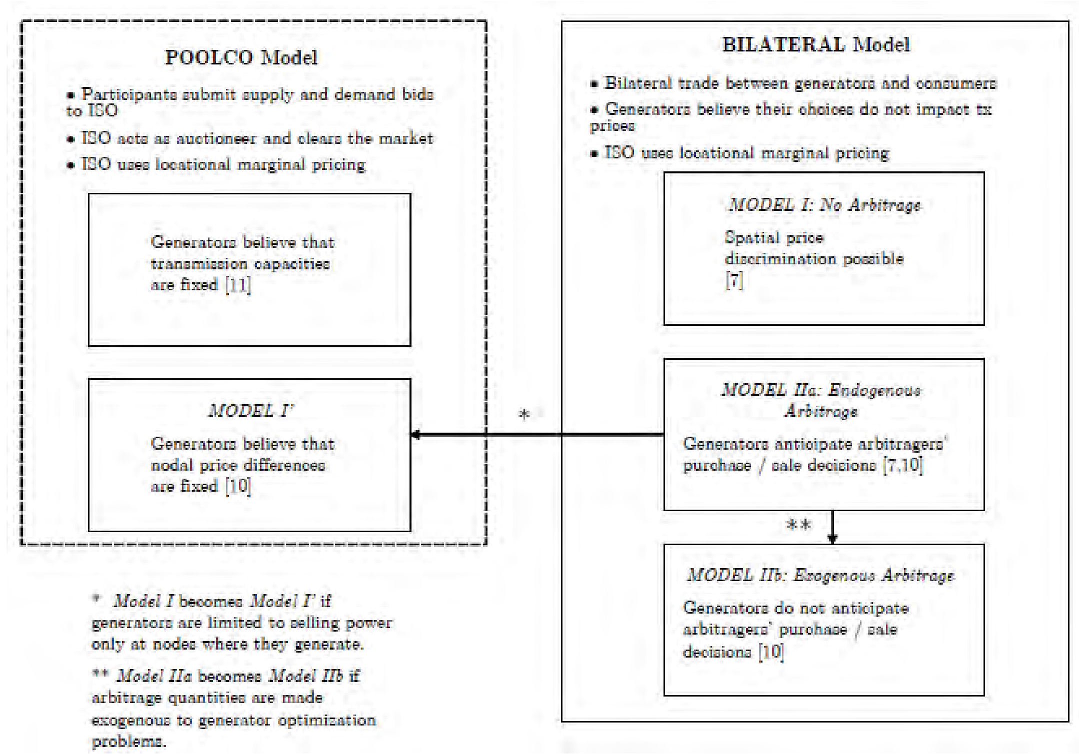


Fig. 5.1 Illustration of Cournot Models in Plexos with PoolCo and Bilateral models [66]

According to reference [66], the Cournot models implemented in Plexos simulate imperfect competition among generators that produce and sell electricity in a bilateral model. Reference [66] also demonstrated that if perfect competition exists among arbitragers in a bilateral model, then the PoolCo model and the bilateral model yield the same equilibrium prices under either perfect competition or Cournot competition.

While implementing the Cournot competition models in Plexos, three common aspects are considered, which includes the following [66]:

1) **<u>Transmission</u>**

   The electricity network is modelled as a finite set of nodes and a finite set of directed arcs. In general, constraints on transmission are represented by a finite number of linear inequalities. For the purposes of the models, the constraints are limited to i) constraints on the transmission capacities, and to ii) constraints on flow balance. Usually, the models use Kirchhoff's laws and

Power Distribution Factors to implement a linearized DC network flow model. The reactive power flow is not taking into consideration.

2) **Consumers**

Consumers at node *i* consume q_i MW such that the price at node *i* (p_i) is related linearly to consumption by the nodal demand function. The use of a linear demand function allows for the use of a Linear Complementarity or Quadratic Problem formulation in the model.

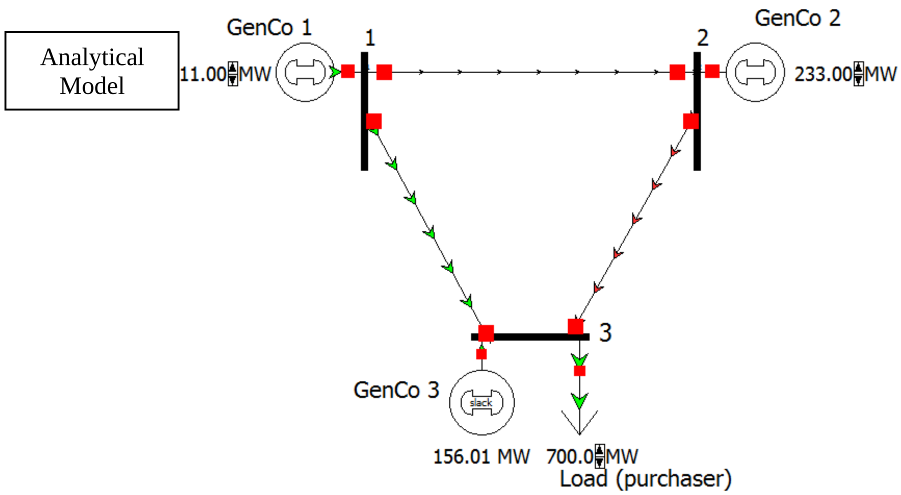
3) Producers

A producer maximizes its profit, i.e. revenues less costs, under an assumption of imperfect competition. The imperfect competition is modeled as Cournot behavior with respect to sales in the power market. A critical assumption of the models is that producers do not anticipate the impact of their output decisions on transmission congestion and prices. This assumption, therefore, eliminates the situations in which producers manipulate transmission in a strategic manner to their benefit. The disadvantage of the assumption is that it does not allow for the possibility that, in reality, producers recognize the impact of their decisions on transmission prices. The advantage, however, is that the models are solvable for realistically large networks.

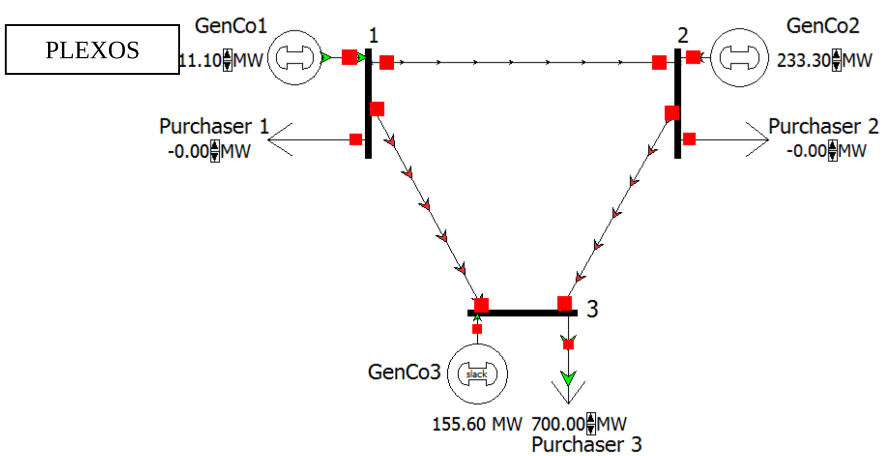
Although Plexos provides the platform for modeling the Cournot competition, there is a fundamental requirement for the demand side during the simulation, i.e. there must be at least one load demand on each node of the generation company. It means that, instead of having one load on bus 3 only, there are also two other loads situating at bus 1 and bus 2.

However, as shown in Figure 5.2, in order to be consistent with the analytical model, the demand slope and intercept of load on bus 1 and 2 are set below the accepted market clearing price. This enables Plexos to simulate Cournot competition. It means that although there are physically three loads situating on three buses, respectively, load 1 and 2 are not going to require any MWs from the system as the market price will be above the maximum price level that they are willing to pay.

Figure 5.2 shows an example of the different system settings between the analytical model and Plexos, when the total demand equals to 700MW with no constraint.



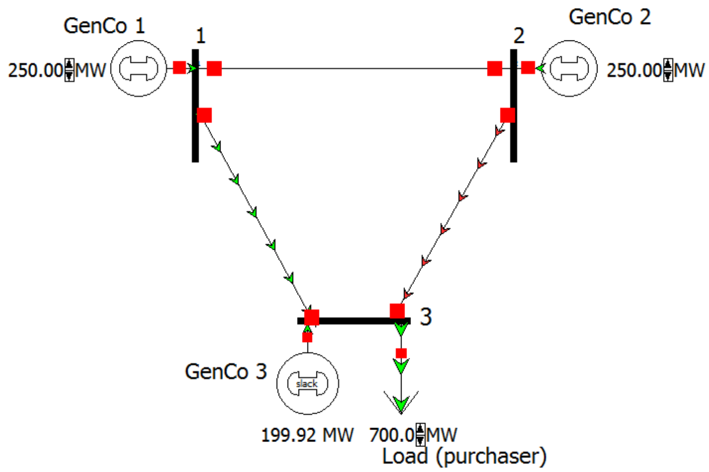
(a) Results from Analytical model calculation in case 1scenario 1 with total demand at 700MW



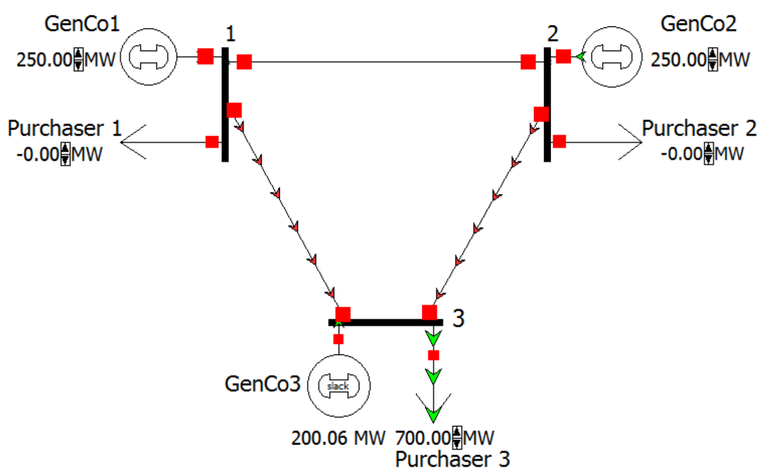
(b) Results from Plexos simulation in case 1 scenario 1 with total demand at 700MW

Fig. 5.2 Comparison of network configuration between analytical model and Plexos

For scenario 2, a generation capacity limit of 250MW is imposed to all GenCos. Figure 5.3 shows an example of market results under generation capacity constraint between the analytical model and Plexos. The results between the analytical model and Plexos seem to match.



(a) Results from Analytical model calculation in case 1scenario 2 with total demand at 700MW



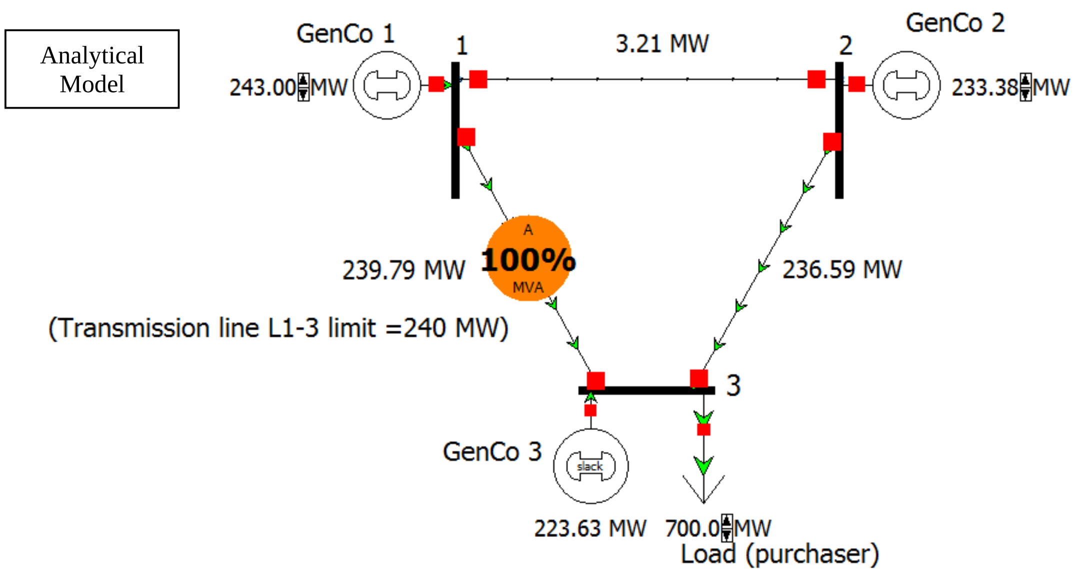
(b) Results from Plexos simulation in case 1 scenario 2 with total demand at 700MW

Fig. 5.3 Results from analytical model and Plexos with generation capacity constraint

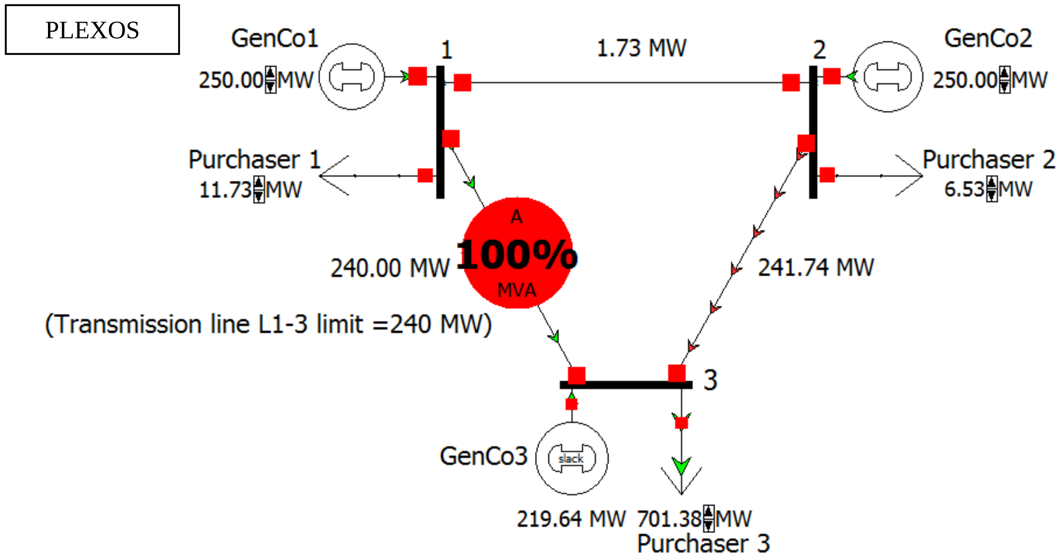
For scenario 3, with the inclusion of transmission capacity constraint, when the quantity demanded changes, the results from the analytical model and Plexos are similar in most cases. The deviation is below the 10% region. As mentioned earlier, Plexos requires load on each bus that has a generation company.

Figures 5.4 (a) and (b) illustrate an example of the comparison of case 1 scenario 3 with a transmission capacity constraint of 240MW on line L_{13}.

When total demand reaches 700MW, the analytical model shows that the generation allocation among the three GenCos, GenGo1, GenGo2 and GenGo3 are 243MW, 233MW and 224MW, respectively (see Fig, 5.4-(a)). The results in Figure 5.4-(b) show that the production allocation is 250MW, 250MW and 219.64MW respectively. If purchaser3 alone is considered as the only total demand in the network, its demanded quantity of 701,38MW equals to the total production of 719.64 less the demand by Purchaser_1 (11.73MW) and Purchaser_2 (6.53MW) on bus 1 and 2, respectively.



(a) Results from Analytical model calculation in case 1scenario 3 with total demand at 700MW



(b) Results from Plexos simulation in case 1 scenario 3 with total demand at 700MW

Fig. 5.4 Results from analytical model and Plexos with transmission capacity constraint of 240MW on line L_{13}

5.2 Comparison of Case 1 Scenario 1 (change of total demand with no constraint)

For case 1, the results on change of total demand from 100MW to 800MW show that there is an almost 100 percent match between the analytical model and the Plexos simulations. As shown in Table 5.1, the market clearing price remains unchanged at the level of 300 unit price. There is very little difference in individual profits and load payment at the demand of 700MW and 800MW.

Table 5.1

Comparison of case 1 scenario 1 with change of Q

Analytical model hand calculation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
100	44	33	22	8889	5000	2222	30000	300
200	89	67	44	17778	10000	4444	60000	300
300	133	100	67	26667	15000	6667	90000	300
400	178	133	89	35555	20000	8889	119999	300
500	222	167	111	44444	25000	11111	149999	300
600	267	200	133	53333	30000	13333	179999	300
700	311	233	156	62222	35000	15555	209999	300
800	356	267	178	71111	40000	17777	239999	300
Plexos simulation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
100	44.4	33.3	22.2	8889	5000	2222	30000	300
200	88.9	66.7	44.4	17778	10000	4444	60000	300
300	133.3	100.0	66.7	26667	15000	6667	90000	300
400	177.8	133.3	88.9	35556	20000	8889	120000	300
500	222.2	166.7	111.1	44444	25000	11111	150000	300
600	266.7	200.0	133.3	53333	30000	13333	180000	300
700	311.1	233.3	155.5	62218	34998	15555	209986	300
800	355.6	266.7	177.8	71111	40000	17778	240000	300

Table 5.1 shows that the results from analytical model and Plexos match perfectly in case 1 scenario1. This proves that this part of the analytical models can be validated by the simulation results from Plexos.

5.3 Comparison of Case 1 Scenario 2 (change of total demand with generation capacity constraint)

Similar to the results in case 1 scenario 1, the comparison of scenario 2 with a change in total demand indicates a fair match between the analytical model and Plexos simulations as shown in Table 5.2,. The slight difference on individual profit and load payment between the two sets of results is very small (i.e., less than 1% variation) and can be overlooked. The total demand reaches the highest amount of 750MW due to the generation capacity constraint. Market clearing price remains identical between the two sets of results.

Table 5.2

Comparison of case 1 scenario 2 with change of Q

Analytical model hand calculation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
100	44	33	22	8889	5000	2222	30000	300
200	89	67	44	17778	10000	4444	60000	300
300	133	100	67	26667	15000	6667	90000	300
400	178	133	89	35555	20000	8889	119999	300
500	222	167	111	44444	25000	11111	149999	300
600	250	207	143	52783	33380	15878	186679	311
700	250	250	200	62501	50001	30002	245003	350
750	250	250	250	75000	62500	50000	299999	400
Plexos simulation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
100	44	33	22	8889	5000	2222	30000	300
200	89	67	44	17778	10000	4444	60000	300
300	133	100	67	26667	15000	6667	90000	300
400	178	133	89	35556	20000	8889	120000	300
500	222	167	111	44444	25000	11111	150000	300
600	250	207	143	52777	33372	15872	186691	311
700	250	250	200	62500	50000	30000	245000	350
750	250	250	250	75005	62505	50000	300012	400

5.4 Comparison of Case 1 Scenario 3 (change of total demand with generation and transmission capacity constraint)

While comparing the results in the case with generation and transmission capacity constraints when total demand increases, the main difference appears in individual quantity, individual profit, load payment and market clearing price. The largest variation is found when total demand is beyond 700MW. For example, as shown in

Table 5.3, the net variation of $GenCo_1$'s individual production and individual profit, load payment and market clearing price, at total demand of 700MW, are 3%, 4%, 1% and 8%, respectively.

Table 5.3

Comparison of case 1 scenario 3 with change of Q

Analytical model hand calculation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
100	44	33	22	8889	5000	2222	30000	300
200	89	67	44	17778	10000	4444	60000	300
300	133	100	67	26667	15000	6667	90000	300
400	178	133	89	35555	20000	8889	119999	300
500	222	167	111	44444	25000	11111	149999	300
600	250	207	143	52778	33373	15873	186667	311
700	243	233	224	69013	54611	41144	268799	384
735	235	250	250	78647	71105	58654	319405	435
Plexos simulation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
100	44	33	22	8889	5000	2222	30000	300
200	89	67	44	17778	10000	4444	60000	300
300	133	100	67	26667	15000	6667	90000	300
400	178	133	89	35556	20000	8889	120000	300
500	222	167	111	44444	25000	11111	150000	300
600	250	207	143	52777	33372	15872	186691	311
700	250	250	220	63759	54470	39694	267029	355
735	250	250	250	75490	67814	60137	322665	402

5.5 Comparison of Case 2 Scenario 1 (change of demand elasticity with no constraint)

With the change of demand elasticity, the results from the analytical model and Plexos simulations also show a good match. There is no significant difference between the two sets of results. The only notable difference in individual profits, load payment and market clearing price, at the demand elasticity level of -1.2334, is less than 2%. The comparison of the results is shown in Table 5.4.

Table 5.4 shows that the results from analytical model and Plexos match perfectly. This proves that analytical models can be validated by the simulations results from Plexos.

Table 5.4

Comparison of case 2 scenario 1 with change of e

Analytical model hand calculation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
-0.4334	220	200	180	120934	99933	80934	389800	650
-0.5334	240	200	160	71984	49983	31984	239950	400
-0.6334	260	200	140	56327	33326	16327	189978	317
-0.7334	280	200	120	48997	24996	8997	164988	275
-0.8334	300	200	100	44998	19997	4998	149992	250
-0.9334	320	200	80	42665	16665	2665	139994	233
-1.0334	340	200	60	41285	14284	1285	132853	221
-1.1334	360	200	40	40500	12499	500	127497	212
-1.2334	380	200	20	40111	11110	111	123331	206
-1.3334	400	200	0	40000	9999	0	119998	200
Plexos simulation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
-0.4334	220	200	180	121000	100000	81000	390000	650
-0.5334	240	200	160	72000	50000	32000	240000	400
-0.6334	260	200	140	56377	33367	16357	190082	317
-0.7334	280	200	120	49000	25000	9000	165000	275
-0.8334	300	200	100	45000	20000	5000	150000	250
-0.9334	320	200	80	42613	16633	2653	139903	233
-1.0334	340	200	60	41339	14316	1294	133256	221
-1.1334	360	200	40	40500	12500	500	127727	212
-1.2334	380	200	20	40066	11088	109	122920	205
-1.3334	400	200	0	39998	9999	0	119999	200

5.6 Comparison of Case 2 Scenario 2 (change of demand elasticity with generation capacity constraint)

With the change in demand elasticity, the results of the comparison between the analytical model and the Plexos simulation are also well matched . The only minor difference is in individual profits and load payment while demand elasticity increases.

As shown in Table 5.5, the individual production and market clearing price remain identical between the results of the two sets during the change of demand elasticity.

Table 5.5

Comparison of case 2 scenario 2 with change of e

Analytical model hand calculation results								
Total demand (MW)	GenCo$_1$ production Q_1 (MW)	GenCo$_2$ production Q_2 (MW)	GenCo$_3$ production Q_3 (MW)	GenCo$_1$ profit π_1 (MW)	GenCo$_2$ profit π_2 (MW)	GenCo$_3$ profit π_3 (MW)	Load payment	Market clearing price
-0.4334	220	200	180	120934	99933	80934	389800	650
-0.5334	240	200	160	71984	49983	31984	239950	400
-0.6334	250	204	146	56092	35623	18122	194622	324
-0.7334	250	213	137	47637	29918	12418	174329	291
-0.8334	250	221	129	42302	26395	8897	161525	269
-0.9334	250	230	120	38633	24042	6543	152718	255
-1.0334	250	239	111	35952	22380	4881	146286	244
-1.1334	250	247	103	33908	21163	3664	141380	236
-1.2334	250	250	100	32811	20311	3124	138747	231
-1.3334	250	250	100	32143	19643	2858	137144	229
Plexos simulation results								
Total demand (MW)	GenCo$_1$ production Q_1 (MW)	GenCo$_2$ production Q_2 (MW)	GenCo$_3$ production Q_3 (MW)	GenCo$_1$ profit π_1 (MW)	GenCo$_2$ profit π_2 (MW)	GenCo$_3$ profit π_3 (MW)	Load payment	Market clearing price
-0.4334	220	200	180	121000	100000	81000	390000	650
-0.5334	240	200	160	72000	50000	32000	240000	400
-0.6334	250	204	146	56085	35612	18115	194676	324
-0.7334	250	213	137	47634	29911	12413	174169	291
-0.8334	250	221	129	42303	26396	8898	161592	269
-0.9334	250	230	120	38631	24038	6541	152524	255
-1.0334	250	239	112	35964	22403	4891	146310	244
-1.1334	250	247	103	33906	21159	3663	141188	236
-1.2334	250	250	100	32806	20306	3120	138827	231
-1.3334	250	250	100	32144	19644	2858	136922	229

In case 2 scenario 2, the results between the analytical model and Plexos show a good match.

5.7 Comparison of case 2 scenario 3 with generation and transmission capacity constraints

With the change in demand elasticity, the difference between the two sets of results appears to increase when demand elasticity is beyond -0.8334. The largest variation in individual production is 14% for $GenCo_2$ at demand elasticity of -1.2334 and -1.3334. $GenCo_3$'s individual profit has the highest variation at 14% when demand elasticity reaches -1.2334. Load payment has 1% variation with demand elasticity at the values between -0.9334 to -1.33334. Market clearing price also varies between two to five percent when demand elasticity increase from -0.9334 to -1.3334.

Table 5.6

Comparison of case 2 scenario 3 with change of e

Analytical model hand calculation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
-0.4334	220	200	180	120934	99933	80934	389800	650
-0.5334	240	200	160	71984	49983	31984	239950	400
-0.6334	250	204	146	56092	35623	18122	194622	324
-0.7334	250	213	137	47637	29918	12418	174329	291
-0.8334	250	220	130	42626	26472	9188	162301	271
-0.9334	250	220	130	40149	24308	7891	156358	261
-1.0334	250	220	130	38273	22673	6906	151855	253
-1.1334	250	220	130	36824	21399	6151	148378	247
-1.2334	250	220	130	35680	20375	5564	145632	243
-1.3334	250	220	130	34729	19534	5071	143350	239
Plexos simulation results								
Total demand (MW)	$GenCo_1$ production Q_1 (MW)	$GenCo_2$ production Q_2 (MW)	$GenCo_3$ production Q_3 (MW)	$GenCo_1$ profit π_1 (MW)	$GenCo_2$ profit π_2 (MW)	$GenCo_3$ profit π_3 (MW)	Load payment	Market clearing price
-0.4334	220	200	180	121000	100000	81000	390000	650
-0.5334	240	200	160	72000	50000	32000	240000	400
-0.6334	250	204	146	56085	35612	18115	194676	324
-0.7334	250	213	137	47634	29911	12413	174169	291
-0.8334	250	222	129	42487	26595	9045	161895	270
-0.9334	250	230	127	38887	24721	7469	155529	256
-1.0334	250	239	125	36038	23301	6346	151019	244
-1.1334	250	248	123	33871	22281	5459	147483	235
-1.2334	250	250	121	32961	21447	4813	144431	232
-1.3334	250	250	121	32181	20637	4410	142450	229

Chapter 6 Conclusions and recommendations

6.1 Conclusions

In this thesis, an Extended Cournot model has been developed. It is used to investigate the impact of demand elasticity on key market outputs during electricity market competition.

The classical Cournot model focuses more on the supply side, which in most cases will lead to a mismatch between the supply and demand in the electricity market as the power suppliers will use the demand function to determine their outputs on the assumption that all suppliers behave in the same way simultaneously. Demand elasticity is proven to play a significant role in the system management especially during peak time, and its relationship with other market factors needs to be investigated further. The Extended Cournot model which includes demand elasticity and generation and transmission constraints has been proposed. This gives the load side more ability to influence the market results. By changing the demand elasticity, load can have certain influence on the market price, GenCos' outputs and profits and load payment.

As transmission constraint is not a necessary market condition to modeling Cournot competition, the unbundling of generation and transmission constraints can provide more scope for the Extended Cournot model to conduct investigations in different environments. The section with generation constraint has shown important findings on the evidence that, when demand elasticity is introduced and stays constant, market price is only going to change when the generation capacity limit of certain power producers reaches its maximum. This can in fact provide a signal to the system operator to assess the balance between electricity supply and demand. Necessary generation capacity expansion can then become an issue for planning and the market price can assist by providing an indication of the level of planning needed. However, when the planning is not feasible in the short term, demand response will help to adjust the load demand.

In the classical Cournot model, power suppliers will ultimately determine the market price via choosing their production level. In certain circumstances, if a load demand already exists, there is a high possibility that the load demand will not be hundred percent satisfied. The results from the simulations show that increased demand elasticity will reduce load payment which reinforces the fact that demand should participate actively in the market in order to prevent abuse of market power.

While the results on one hand show significant impact of demand elasticity on the market outputs, on the other hand, they also show that demand elasticity has a limited impact on the market outputs as the level of influence diminishes with the increase in demand elasticity. This is especially true when both generation and transmission constraints are implemented in the models.

One of the major assumptions in the model is that the marginal cost is constant and known. Therefore, the results depend on the level of each GenCo's marginal cost compared to the average cost.

Simulation results from Plexos are compared with the ones from the developed Extended Cournot model. The preliminary results show agreement with the simulation results obtained using Plexos. There are some advantages and disadvantages to using both models. Plexos has the advantage that its model can be applied to larger networks. However, since it is based on the classic Cournot model, it cannot adequately handle transmission and generation constraints. The Extended Cournot model has the advantage of introducing demand elasticity in a simpler way by changing the input data, compared to the more complicated way in Plexos which needs to calculate the demand slopes for each bus first based on the total demand curve. During the data input process, loads need to be imposed on all buses although their values are set to be equal to zero. Another advantage of the Extended Cournot model is that it can adequately handle network constraints.

6.2 Recommendations

The work in progress extends application of the Extended Cournot model to the following areas:

1) Different level of marginal cost of the GenCos.

One of the main assumptions of the Extended Cournot model is that the marginal cost of all power producers is fixed. The proportional difference between the marginal costs of the three power producers is the same. Therefore, it is recommended that the impact of a variable marginal cost on the market outputs be investigated. This may also provide the opportunity to use both quantity and price to identify the behaviors of power producers in any potential exercise of market power.

2) Different level of generation capacities of the GenCos

The assumption of the Extended Cournot model in the scenario with generation capacity constraint, is that the generation capacities of the three power producers are the same and constant. In future work, generation capacity can also become variable. As locational marginal pricing is adopted in this thesis, in a larger network it is possible to consider overall power supply in certain regions instead of each individual power producer's supply.

3) Different possibilities of transmission capacities and the network

The transmission constraint introduced in scenario 3 is to place a transmission limit on line $L_{1\text{-}3}$. The reason for this is to limit access to the load to the most economical power producer. In a lager network, this may not necessarily be the case. Load demand may have less access to any power producers that have lower bids/marginal cost. The choice of a simplified 3-bus network is that such a network has been used in certain research work for demonstration purposes.

However, the Extended Cournot model can be applied to a larger and more complex network.

Another application could be that the demand side involvement contribute to analysis in the electricity market such as smart grid as the information about the behaviors of both suppliers and consumers needs to be gathered and considered in order to improve the efficiency, reliability, economics, and sustainability of the production and distribution of electricity.

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Journal papers

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Appendices

Appendix 1. Example of Plexos Excution Log file – Case 1 Scenario 1: Change of total demand (Q=100) with constant demand elasticity (e= -0.6667) and without constraint

```
Started Project NC (Model "NC-Q=100") -->
- Using Metric Units
Primary Compilation Started:
----------------------------------------------------------------------------
Element Records
----------------------------------------------------------------------------
Objects....................................................................4
Memberships...............................................................107
Properties.................................................................58
Scenario "Q=100"............................................................3
----------------------------------------------------------------------------
Total.....................................................................172
----------------------------------------------------------------------------
Primary Compilation Completed. Time: 1.83 sec.
Model Initialization Started -->
- Random Number Seed = 560619933 (enter this number to repeat this sequence).
Requesting licenses:
- PLEXOS 4.914 R2
- PLEXOS Base Product
- MOSEK Base Product
- MOSEK Parallel
- MOSEK Dual Simplex
- Machine Name: MACHINESLAB11
- Physical Memory: 1,022.73 MB
- CPU Type: Intel(R) Pentium(R) 4 CPU 3.00GHz
- CPU Count: 2
- Maximum Threads: 2
- MOSEK 5.0.0.128
Licenses received.
Planning Horizon Set up Started
Set up 1 Day(s). Time: 0.17 sec.
Secondary Compilation Started:
Secondary Compilation Completed. Time: 0.34 sec.
In-Memory Cache Setup Started
In-Memory Cache Setup. Time: 2.63 sec.:
- 0 reallocations
- 10000.00 kB
- 0.00 % efficiency
<-- Model Initialized. Time: 7.44 sec.
Preschedule -->
Preschedule step 1 of 1: Wednesday, January 01, 2014 - Wednesday, January 01, 2014
```

-->
<-- Preschedule step 1 of 1. Time: 0:00:00

Preschedule Completed. Time: 0:00:00

Setup.. 51.61 %
Solution....................................... 0.00 %
Other.. 48.39 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

ST Schedule -->

OPF Element Count

Single Slack Bus.."1"
Nodes (busbars)...3
Nodes in Service...3
Load Points..0
Generation Injection Points..3
Total Injection Points...3
Lines (branches)...3
Lines in Service...3
AC Lines...3
Flow Limits >= 0 kV..3
Flows Modeled >= 0 kV..3
DC Lines...0
Phase Shifters...0
Phase Shifters in Service..0
Generators...3
Generators in Service..3
Unique AC Paths..3
Modeled AC Paths...3
Required PTDF..9
Finished 3 Projections. Stored 6 PTDFs out of 9 (Compressed 33.33%). Time: 0.00 sec.
Started ST Schedule Step 1 of 24: Jan 1, 2014 12:00:00 AM - Jan 1, 2014 12:59:00 AM
Task Size:
- Columns (variables): 26
- Rows (constraints): 16
- Non-zeros (coefficients): 54
- Memory (MB estimated): 32.66
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 1 of 24. Time: 1.06 sec.
Started ST Schedule Step 2 of 24: Jan 1, 2014 1:00:00 AM - Jan 1, 2014 1:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 2 of 24. Time: 0.00 sec.
Started ST Schedule Step 3 of 24: Jan 1, 2014 2:00:00 AM - Jan 1, 2014 2:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 3 of 24. Time: 0.02 sec.
Started ST Schedule Step 4 of 24: Jan 1, 2014 3:00:00 AM - Jan 1, 2014 3:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 4 of 24. Time: 0.00 sec.
Started ST Schedule Step 5 of 24: Jan 1, 2014 4:00:00 AM - Jan 1, 2014 4:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 5 of 24. Time: 0.02 sec.
Started ST Schedule Step 6 of 24: Jan 1, 2014 5:00:00 AM - Jan 1, 2014 5:59:00 AM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 6 of 24. Time: 0.00 sec.
Started ST Schedule Step 7 of 24: Jan 1, 2014 6:00:00 AM - Jan 1, 2014 6:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 7 of 24. Time: 0.02 sec.
Started ST Schedule Step 8 of 24: Jan 1, 2014 7:00:00 AM - Jan 1, 2014 7:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 8 of 24. Time: 0.02 sec.
Started ST Schedule Step 9 of 24: Jan 1, 2014 8:00:00 AM - Jan 1, 2014 8:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 9 of 24. Time: 0.00 sec.
Started ST Schedule Step 10 of 24: Jan 1, 2014 9:00:00 AM - Jan 1, 2014 9:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Model (NC-Q=100) Log

Completed ST Schedule Step 10 of 24. Time: 0.02 sec.
Started ST Schedule Step 11 of 24: Jan 1, 2014 10:00:00 AM - Jan 1, 2014 10:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 11 of 24. Time: 0.02 sec.
Started ST Schedule Step 12 of 24: Jan 1, 2014 11:00:00 AM - Jan 1, 2014 11:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 12 of 24. Time: 0.02 sec.
Started ST Schedule Step 13 of 24: Jan 1, 2014 12:00:00 PM - Jan 1, 2014 12:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 13 of 24. Time: 0.03 sec.
Started ST Schedule Step 14 of 24: Jan 1, 2014 1:00:00 PM - Jan 1, 2014 1:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 14 of 24. Time: 0.00 sec.
Started ST Schedule Step 15 of 24: Jan 1, 2014 2:00:00 PM - Jan 1, 2014 2:59:00 PM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 15 of 24. Time: 0.00 sec.
Started ST Schedule Step 16 of 24: Jan 1, 2014 3:00:00 PM - Jan 1, 2014 3:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 16 of 24. Time: 0.02 sec.
Started ST Schedule Step 17 of 24: Jan 1, 2014 4:00:00 PM - Jan 1, 2014 4:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 17 of 24. Time: 0.02 sec.
Started ST Schedule Step 18 of 24: Jan 1, 2014 5:00:00 PM - Jan 1, 2014 5:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 18 of 24. Time: 0.00 sec.
Started ST Schedule Step 19 of 24: Jan 1, 2014 6:00:00 PM - Jan 1, 2014 6:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 19 of 24. Time: 0.00 sec.
Started ST Schedule Step 20 of 24: Jan 1, 2014 7:00:00 PM - Jan 1, 2014 7:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 20 of 24. Time: 0.02 sec.
Started ST Schedule Step 21 of 24: Jan 1, 2014 8:00:00 PM - Jan 1, 2014 8:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 21 of 24. Time: 0.00 sec.
Started ST Schedule Step 22 of 24: Jan 1, 2014 9:00:00 PM - Jan 1, 2014 9:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 22 of 24. Time: 0.02 sec.
Started ST Schedule Step 23 of 24: Jan 1, 2014 10:00:00 PM - Jan 1, 2014 10:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 23 of 24. Time: 0.00 sec.
Started ST Schedule Step 24 of 24: Jan 1, 2014 11:00:00 PM - Jan 1, 2014 11:59:00 PM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 100.00 100.00 0.00 13.89 30.00 0.00 $300.00

Completed ST Schedule Step 24 of 24. Time: 0.03 sec.
Regional Summary:

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 2,400.00 2,400.00 0.00 333.33 720.00 0.00 $300.00

ST Schedule Completed. Time: 0:00:02

Setup... 22.49 %
MOSEK:.. 0.92 %
Solution.. 13.49 %
Other... 63.09 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

<-- ST Schedule
<-- Project NC (Model "NC-Q=100") Completed. Time: 0:00:12

Appendix 2. Example of Plexos Excution Log file – Case 1 scenario 2: Change of total demand (Q=750) with constant demand elasticity (e= -0.6667) and with generation capacity constraint

Model (NC-Q=750) Log
Started Project NC (Model "NC-Q=750") -->
- Using Metric Units
Primary Compilation Started:

Element Records

Objects..4
Memberships..107
Properties...54
Scenario "Q=750"...6

Total...171

Primary Compilation Completed. Time: 1.80 sec.
Model Initialization Started -->
- Random Number Seed = 770296122 (enter this number to repeat this sequence).
Requesting licenses:
- PLEXOS 4.914 R2
- PLEXOS Base Product
- MOSEK Base Product
- MOSEK Parallel
- MOSEK Dual Simplex
- Machine Name: MACHINESLAB11
- Physical Memory: 1,022.73 MB
- CPU Type: Intel(R) Pentium(R) 4 CPU 3.00GHz
- CPU Count: 2
- Maximum Threads: 2
- MOSEK 5.0.0.128
Licenses received.
Planning Horizon Set up Started
Secondary Compilation Started:
Secondary Compilation Completed. Time: 0.30 sec.
In-Memory Cache Setup Started
In-Memory Cache Setup. Time: 2.61 sec.:
- 0 reallocations
- 10000.00 kB
- 0.00 % efficiency
<-- Model Initialized. Time: 7.13 sec.
Preschedule -->
Preschedule step 1 of 1: Wednesday, January 01, 2014 - Wednesday, January 01, 2014
-->
<-- Preschedule step 1 of 1. Time: 0:00:00

Preschedule Completed. Time: 0:00:00

Setup.. 100.00 %
Solution....................................... 0.00 %
Other.. 0.00 %
Memory:
100000 kB
Reallocations................................ 0
Efficiency.................................. 0.00 %

ST Schedule -->

OPF Element Count

Single Slack Bus.."1"
Nodes (busbars)...3
Nodes in Service..3
Load Points..0
Generation Injection Points...3
Total Injection Points..3
Lines (branches)..3
Lines in Service..3
AC Lines..3
Flow Limits >= 0 kV...3
Flows Modeled >= 0 kV...3
DC Lines..0
Phase Shifters..0
Phase Shifters in Service...0
Generators..3
Generators in Service...3
Unique AC Paths...3
Modeled AC Paths..3
Required PTDF...9
Finished 3 Projections. Stored 6 PTDFs out of 9 (Compressed 33.33%). Time: 0.00 sec.
Started ST Schedule Step 1 of 24: Jan 1, 2014 12:00:00 AM - Jan 1, 2014 12:59:00 AM
Task Size:
- Columns (variables): 26
- Rows (constraints): 16
- Non-zeros (coefficients): 54
- Memory (MB estimated): 32.66
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 1 of 24. Time: 0.53 sec.
Started ST Schedule Step 2 of 24: Jan 1, 2014 1:00:00 AM - Jan 1, 2014 1:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 2 of 24. Time: 0.00 sec.
Started ST Schedule Step 3 of 24: Jan 1, 2014 2:00:00 AM - Jan 1, 2014 2:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 3 of 24. Time: 0.00 sec.
Started ST Schedule Step 4 of 24: Jan 1, 2014 3:00:00 AM - Jan 1, 2014 3:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 4 of 24. Time: 0.02 sec.
Started ST Schedule Step 5 of 24: Jan 1, 2014 4:00:00 AM - Jan 1, 2014 4:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 5 of 24. Time: 0.00 sec.
Started ST Schedule Step 6 of 24: Jan 1, 2014 5:00:00 AM - Jan 1, 2014 5:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 6 of 24. Time: 0.02 sec.
Started ST Schedule Step 7 of 24: Jan 1, 2014 6:00:00 AM - Jan 1, 2014 6:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 7 of 24. Time: 0.00 sec.
Started ST Schedule Step 8 of 24: Jan 1, 2014 7:00:00 AM - Jan 1, 2014 7:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 8 of 24. Time: 0.02 sec.
Started ST Schedule Step 9 of 24: Jan 1, 2014 8:00:00 AM - Jan 1, 2014 8:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 9 of 24. Time: 0.00 sec.
Started ST Schedule Step 10 of 24: Jan 1, 2014 9:00:00 AM - Jan 1, 2014 9:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 10 of 24. Time: 0.03 sec.
Started ST Schedule Step 11 of 24: Jan 1, 2014 10:00:00 AM - Jan 1, 2014 10:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 11 of 24. Time: 0.00 sec.
Started ST Schedule Step 12 of 24: Jan 1, 2014 11:00:00 AM - Jan 1, 2014 11:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 12 of 24. Time: 0.02 sec.
Started ST Schedule Step 13 of 24: Jan 1, 2014 12:00:00 PM - Jan 1, 2014 12:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 13 of 24. Time: 0.00 sec.
Started ST Schedule Step 14 of 24: Jan 1, 2014 1:00:00 PM - Jan 1, 2014 1:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 14 of 24. Time: 0.02 sec.
Started ST Schedule Step 15 of 24: Jan 1, 2014 2:00:00 PM - Jan 1, 2014 2:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 15 of 24. Time: 0.00 sec.
Started ST Schedule Step 16 of 24: Jan 1, 2014 3:00:00 PM - Jan 1, 2014 3:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 16 of 24. Time: 0.03 sec.
Started ST Schedule Step 17 of 24: Jan 1, 2014 4:00:00 PM - Jan 1, 2014 4:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 17 of 24. Time: 0.02 sec.
Started ST Schedule Step 18 of 24: Jan 1, 2014 5:00:00 PM - Jan 1, 2014 5:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 18 of 24. Time: 0.02 sec.
Started ST Schedule Step 19 of 24: Jan 1, 2014 6:00:00 PM - Jan 1, 2014 6:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 19 of 24. Time: 0.00 sec.
Started ST Schedule Step 20 of 24: Jan 1, 2014 7:00:00 PM - Jan 1, 2014 7:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 20 of 24. Time: 0.00 sec.
Started ST Schedule Step 21 of 24: Jan 1, 2014 8:00:00 PM - Jan 1, 2014 8:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 21 of 24. Time: 0.02 sec.
Started ST Schedule Step 22 of 24: Jan 1, 2014 9:00:00 PM - Jan 1, 2014 9:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 22 of 24. Time: 0.02 sec.
Started ST Schedule Step 23 of 24: Jan 1, 2014 10:00:00 PM - Jan 1, 2014 10:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 23 of 24. Time: 0.00 sec.
Started ST Schedule Step 24 of 24: Jan 1, 2014 11:00:00 PM - Jan 1, 2014 11:59:00 PM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 749.97 749.97 0.00 112.49 300.01 0.00 $400.02

Completed ST Schedule Step 24 of 24. Time: 0.02 sec.
Regional Summary:

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(GWh) (GWh) (GWh) ($000's) ($000's) (MWh)

R 18.00 18.00 0.00 2,699.88 7,200.12 0.00 $400.02

ST Schedule Completed. Time: 0:00:01

Setup... 0.00 %
MOSEK:.. 6.85 %
Solution....................................... 1.80 %
Other... 91.35 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

Solution Tables Indexed. Time: 0.86 sec.
<-- ST Schedule
<-- Project NC (Model "NC-Q=750") Completed. Time: 0:00:12

Appendix 3. Example of Plexos Excution Log file – Case 1 scenaior 3: change of total demand (Q=700) with constant demand elasticity (e= -0.6667) and with generation and transmission capacity constraints

Started Project NC (Model "NC-Q=700") -->
- Using Metric Units
Primary Compilation Started:

Element Records

Objects..4
Memberships..107
Properties...55
Scenario "Q=700"...6

Total...172

Primary Compilation Completed. Time: 2.58 sec.
Model Initialization Started -->
- Random Number Seed = 1092358493 (enter this number to repeat this sequence).
Requesting licenses:
- PLEXOS 4.910 R2
- PLEXOS Base Product
- MOSEK Base Product
- MOSEK Parallel
- MOSEK Dual Simplex
- Machine Name: JACKY
- Physical Memory: 1.50 GB
- CPU Type: Intel(R) Pentium(R) 4 CPU 3.00GHz
- CPU Count: 2
- Maximum Threads: 2
- MOSEK 5.0.0.79
Licenses received.
Planning Horizon Set up Started
Secondary Compilation Started:
Secondary Compilation Completed. Time: 0.36 sec.
In-Memory Cache Setup Started
In-Memory Cache Setup. Time: 3.05 sec.:
- 0 reallocations
- 10000.00 kB
- 0.00 % efficiency
<-- Model Initialized. Time: 8.56 sec.
Preschedule -->
Preschedule step 1 of 1: Sunday, January 01, 2006 - Sunday, January 01, 2006 -->
<-- Preschedule step 1 of 1. Time: 0:00:00
Preschedule Completed. Time: 0:00:00

Setup... 86.24 %
Solution.. 0.00 %
Other... 13.76 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

ST Schedule -->

OPF Element Count

Single Slack Bus.."1"
Nodes (busbars)...3
Nodes in Service..3
Load Points...0
Generation Injection Points...3
Total Injection Points..3
Lines (branches)..3
Lines in Service..3
AC Lines..3
Flow Limits >= 0 kV...3
Flows Modeled >= 0 kV...3
DC Lines..0
Phase Shifters..0
Phase Shifters in Service...0
Generators..3
Generators in Service...3
Unique AC Paths...3
Modeled AC Paths..3
Required PTDF...9
Finished 3 Projections. Stored 6 PTDFs out of 9 (Compressed 33.33%). Time: 0.00 sec.
Started ST Schedule Step 1 of 24: Jan 1, 2006 12:00:00 AM - Jan 1, 2006 12:59:00 AM
Task Size:
- Columns (variables): 26
- Rows (constraints): 16
- Non-zeros (coefficients): 54
- Memory (MB estimated): 32.66

Class Name Flow (MW) Limit (MW) Loading

Line "1-3"(1) 2.460703e+002 2.400000e+002 102.53 %

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 1 of 24. Time: 0.64 sec.
Started ST Schedule Step 2 of 24: Jan 1, 2006 1:00:00 AM - Jan 1, 2006 1:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 2 of 24. Time: 0.02 sec.
Started ST Schedule Step 3 of 24: Jan 1, 2006 2:00:00 AM - Jan 1, 2006 2:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 3 of 24. Time: 0.00 sec.
Started ST Schedule Step 4 of 24: Jan 1, 2006 3:00:00 AM - Jan 1, 2006 3:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 4 of 24. Time: 0.02 sec.
Started ST Schedule Step 5 of 24: Jan 1, 2006 4:00:00 AM - Jan 1, 2006 4:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 5 of 24. Time: 0.02 sec.

Started ST Schedule Step 6 of 24: Jan 1, 2006 5:00:00 AM - Jan 1, 2006 5:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 6 of 24. Time: 0.02 sec.
Started ST Schedule Step 7 of 24: Jan 1, 2006 6:00:00 AM - Jan 1, 2006 6:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 7 of 24. Time: 0.02 sec.
Started ST Schedule Step 8 of 24: Jan 1, 2006 7:00:00 AM - Jan 1, 2006 7:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 8 of 24. Time: 0.02 sec.
Started ST Schedule Step 9 of 24: Jan 1, 2006 8:00:00 AM - Jan 1, 2006 8:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 9 of 24. Time: 0.02 sec.
Started ST Schedule Step 10 of 24: Jan 1, 2006 9:00:00 AM - Jan 1, 2006 9:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 10 of 24. Time: 0.02 sec.
Started ST Schedule Step 11 of 24: Jan 1, 2006 10:00:00 AM - Jan 1, 2006 10:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 11 of 24. Time: 0.00 sec.
Started ST Schedule Step 12 of 24: Jan 1, 2006 11:00:00 AM - Jan 1, 2006 11:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 12 of 24. Time: 0.00 sec.
Started ST Schedule Step 13 of 24: Jan 1, 2006 12:00:00 PM - Jan 1, 2006 12:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 13 of 24. Time: 0.02 sec.
Started ST Schedule Step 14 of 24: Jan 1, 2006 1:00:00 PM - Jan 1, 2006 1:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 14 of 24. Time: 0.00 sec.

Started ST Schedule Step 15 of 24: Jan 1, 2006 2:00:00 PM - Jan 1, 2006 2:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 15 of 24. Time: 0.02 sec.
Started ST Schedule Step 16 of 24: Jan 1, 2006 3:00:00 PM - Jan 1, 2006 3:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 16 of 24. Time: 0.03 sec.
Started ST Schedule Step 17 of 24: Jan 1, 2006 4:00:00 PM - Jan 1, 2006 4:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 17 of 24. Time: 0.00 sec.
Started ST Schedule Step 18 of 24: Jan 1, 2006 5:00:00 PM - Jan 1, 2006 5:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 18 of 24. Time: 0.02 sec.
Started ST Schedule Step 19 of 24: Jan 1, 2006 6:00:00 PM - Jan 1, 2006 6:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 19 of 24. Time: 0.02 sec.
Started ST Schedule Step 20 of 24: Jan 1, 2006 7:00:00 PM - Jan 1, 2006 7:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 20 of 24. Time: 0.02 sec.
Started ST Schedule Step 21 of 24: Jan 1, 2006 8:00:00 PM - Jan 1, 2006 8:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 21 of 24. Time: 0.02 sec.
Started ST Schedule Step 22 of 24: Jan 1, 2006 9:00:00 PM - Jan 1, 2006 9:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 22 of 24. Time: 0.01 sec.
Started ST Schedule Step 23 of 24: Jan 1, 2006 10:00:00 PM - Jan 1, 2006 10:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 23 of 24. Time: 0.00 sec.

Started ST Schedule Step 24 of 24: Jan 1, 2006 11:00:00 PM - Jan 1, 2006 11:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 719.64 719.64 0.00 106.43 255.50 0.00 $355.03

Completed ST Schedule Step 24 of 24. Time: 0.00 sec.
Regional Summary:

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(GWh) (GWh) (GWh) ($000's) ($000's) (MWh)

R 17.27 17.27 0.00 2,554.28 6,131.95 0.00 $355.03

ST Schedule Completed. Time: 0:00:01

Setup... 3.05 %
MOSEK:.. 9.12 %
Solution.. 1.55 %
Other... 86.32 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

<-- ST Schedule
<-- Project NC (Model "NC-Q=700") Completed. Time: 0:00:13

Appendix 4. Example of Plexos Excution Log file – Case 2 scenario 1: change of demand elasticity (e= -0.4334) with constant total demand – (Q=600) and without constraint

Started Project NC-Case1-change of elasticity (Model "e=-0.4334") -->
- Using Metric Units
Primary Compilation Started:

Element Records

Objects..4
Memberships..119
Properties...58
Scenario "e=-0.4334"..6

Total..187

Primary Compilation Completed. Time: 1.86 sec.
Model Initialization Started -->
- Random Number Seed = 566868342 (enter this number to repeat this sequence).
Requesting licenses:
- PLEXOS 4.914 R2
- PLEXOS Base Product
- MOSEK Base Product
- MOSEK Parallel
- MOSEK Dual Simplex
- Machine Name: MACHINESLAB11
- Physical Memory: 1,022.73 MB
- CPU Type: Intel(R) Pentium(R) 4 CPU 3.00GHz
- CPU Count: 2
- Maximum Threads: 2
- MOSEK 5.0.0.128
Licenses received.
Planning Horizon Set up Started
Set up 1 Day(s). Time: 0.20 sec.
Secondary Compilation Started:
Secondary Compilation Completed. Time: 0.33 sec.
In-Memory Cache Setup Started
In-Memory Cache Setup. Time: 2.61 sec.:
- 0 reallocations
- 10000.00 kB
- 0.00 % efficiency
<-- Model Initialized. Time: 7.52 sec.
Preschedule -->
Preschedule step 1 of 1: Wednesday, January 01, 2014 - Wednesday, January 01, 2014
-->
<-- Preschedule step 1 of 1. Time: 0:00:00

Preschedule Completed. Time: 0:00:00

Setup.. 34.04 %
Solution....................................... 0.00 %
Other.. 65.96 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency................................... 0.00 %

ST Schedule -->

OPF Element Count

Single Slack Bus.."1"
Nodes (busbars)...3
Nodes in Service..3
Load Points...0
Generation Injection Points..3
Total Injection Points...3
Lines (branches)..3
Lines in Service..3
AC Lines..3
Flow Limits >= 0 kV..3
Flows Modeled >= 0 kV..3
DC Lines..0
Phase Shifters..0
Phase Shifters in Service...0
Generators..3
Generators in Service..3
Unique AC Paths...3
Modeled AC Paths..3
Required PTDF...9
Finished 3 Projections. Stored 6 PTDFs out of 9 (Compressed 33.33%). Time: 0.00 sec.
Started ST Schedule Step 1 of 24: Jan 1, 2014 12:00:00 AM - Jan 1, 2014 12:59:00 AM
Task Size:
- Columns (variables): 26
- Rows (constraints): 16
- Non-zeros (coefficients): 54
- Memory (MB estimated): 32.66
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 1 of 24. Time: 0.98 sec.
Started ST Schedule Step 2 of 24: Jan 1, 2014 1:00:00 AM - Jan 1, 2014 1:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 2 of 24. Time: 0.05 sec.
Started ST Schedule Step 3 of 24: Jan 1, 2014 2:00:00 AM - Jan 1, 2014 2:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 3 of 24. Time: 0.02 sec.
Started ST Schedule Step 4 of 24: Jan 1, 2014 3:00:00 AM - Jan 1, 2014 3:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 4 of 24. Time: 0.00 sec.
Started ST Schedule Step 5 of 24: Jan 1, 2014 4:00:00 AM - Jan 1, 2014 4:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 5 of 24. Time: 0.02 sec.
Started ST Schedule Step 6 of 24: Jan 1, 2014 5:00:00 AM - Jan 1, 2014 5:59:00 AM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 6 of 24. Time: 0.00 sec.
Started ST Schedule Step 7 of 24: Jan 1, 2014 6:00:00 AM - Jan 1, 2014 6:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 7 of 24. Time: 0.00 sec.
Started ST Schedule Step 8 of 24: Jan 1, 2014 7:00:00 AM - Jan 1, 2014 7:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 8 of 24. Time: 0.02 sec.
Started ST Schedule Step 9 of 24: Jan 1, 2014 8:00:00 AM - Jan 1, 2014 8:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 9 of 24. Time: 0.02 sec.
Started ST Schedule Step 10 of 24: Jan 1, 2014 9:00:00 AM - Jan 1, 2014 9:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 10 of 24. Time: 0.00 sec.
Started ST Schedule Step 11 of 24: Jan 1, 2014 10:00:00 AM - Jan 1, 2014 10:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 11 of 24. Time: 0.02 sec.
Started ST Schedule Step 12 of 24: Jan 1, 2014 11:00:00 AM - Jan 1, 2014 11:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 12 of 24. Time: 0.00 sec.
Started ST Schedule Step 13 of 24: Jan 1, 2014 12:00:00 PM - Jan 1, 2014 12:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 13 of 24. Time: 0.02 sec.
Started ST Schedule Step 14 of 24: Jan 1, 2014 1:00:00 PM - Jan 1, 2014 1:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 14 of 24. Time: 0.02 sec.
Started ST Schedule Step 15 of 24: Jan 1, 2014 2:00:00 PM - Jan 1, 2014 2:59:00 PM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 15 of 24. Time: 0.01 sec.
Started ST Schedule Step 16 of 24: Jan 1, 2014 3:00:00 PM - Jan 1, 2014 3:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 16 of 24. Time: 0.03 sec.

Started ST Schedule Step 17 of 24: Jan 1, 2014 4:00:00 PM - Jan 1, 2014 4:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 17 of 24. Time: 0.00 sec.
Started ST Schedule Step 18 of 24: Jan 1, 2014 5:00:00 PM - Jan 1, 2014 5:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 18 of 24. Time: 0.00 sec.
Started ST Schedule Step 19 of 24: Jan 1, 2014 6:00:00 PM - Jan 1, 2014 6:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 19 of 24. Time: 0.03 sec.
Started ST Schedule Step 20 of 24: Jan 1, 2014 7:00:00 PM - Jan 1, 2014 7:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 20 of 24. Time: 0.00 sec.
Started ST Schedule Step 21 of 24: Jan 1, 2014 8:00:00 PM - Jan 1, 2014 8:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 21 of 24. Time: 0.00 sec.
Started ST Schedule Step 22 of 24: Jan 1, 2014 9:00:00 PM - Jan 1, 2014 9:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 22 of 24. Time: 0.02 sec.
Started ST Schedule Step 23 of 24: Jan 1, 2014 10:00:00 PM - Jan 1, 2014 10:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 23 of 24. Time: 0.00 sec.
Started ST Schedule Step 24 of 24: Jan 1, 2014 11:00:00 PM - Jan 1, 2014 11:59:00 PM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.00 600.00 0.00 88.00 390.00 0.00 $650.00

Completed ST Schedule Step 24 of 24. Time: 0.03 sec.
Regional Summary:

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(GWh) (GWh) (GWh) ($000's) ($000's) (MWh)

R 14.40 14.40 0.00 2,112.00 9,360.00 0.00 $650.00

ST Schedule Completed. Time: 0:00:02

Setup... 21.28 %
MOSEK:.. 1.90 %
Solution.. 14.88 %
Other... 61.94 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

<-- ST Schedule
<-- Project NC-Case1-change of elasticity (Model "e=-0.4334") Completed. Time: 0:00:12

Appendix 5. Example of Plexos Excution Log file – Case 2 scenario 2: change of demand elasticity (e= -1.334) with constant total demand – (Q=600) and with generation capacity constraint

Started Project NC (Model "e=-1.3334") -->
- Using Metric Units

Primary Compilation Started:

Element Records

Objects..4
Memberships..119
Properties...58
Scenario "e=-1.3334"...6

Total..187

Primary Compilation Completed. Time: 1.73 sec.
Model Initialization Started -->
- Random Number Seed = 1625828854 (enter this number to repeat this sequence).

Requesting licenses:
- PLEXOS 4.914 R2
- PLEXOS Base Product
- MOSEK Base Product
- MOSEK Parallel
- MOSEK Dual Simplex
- Machine Name: MACHINESLAB11
- Physical Memory: 1,022.73 MB
- CPU Type: Intel(R) Pentium(R) 4 CPU 3.00GHz
- CPU Count: 2
- Maximum Threads: 2
- MOSEK 5.0.0.128

Licenses received.
Planning Horizon Set up Started
Secondary Compilation Started:
Secondary Compilation Completed. Time: 0.30 sec.
In-Memory Cache Setup Started
In-Memory Cache Setup. Time: 2.55 sec.:
- 0 reallocations
- 10000.00 kB
- 0.00 % efficiency

<-- Model Initialized. Time: 6.91 sec.
Preschedule -->
Preschedule step 1 of 1: Wednesday, January 01, 2014 - Wednesday, January 01, 2014 -->
<-- Preschedule step 1 of 1. Time: 0:00:00
Preschedule Completed. Time: 0:00:00

```
----------------------------------------------------------------------------
Setup.......................................... 100.00 %
Solution....................................... 0.00 %
Other.......................................... 0.00 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %
----------------------------------------------------------------------------
ST Schedule -->
----------------------------------------------------------------------------
OPF Element Count
----------------------------------------------------------------------------
Single Slack Bus........................................................"1"
Nodes (busbars)...........................................................3
Nodes in Service..........................................................3
Load Points................................................................0
Generation Injection Points................................................3
Total Injection Points.....................................................3
Lines (branches)...........................................................3
Lines in Service..........................................................3
AC Lines...................................................................3
Flow Limits >= 0 kV........................................................3
Flows Modeled >= 0 kV......................................................3
DC Lines...................................................................0
Phase Shifters.............................................................0
Phase Shifters in Service..................................................0
Generators.................................................................3
Generators in Service......................................................3
Unique AC Paths............................................................3
Modeled AC Paths...........................................................3
Required PTDF..............................................................9
Finished 3 Projections. Stored 6 PTDFs out of 9 (Compressed 33.33%). Time: 0.00 sec.
Started ST Schedule Step 1 of 24: Jan 1, 2014 12:00:00 AM - Jan 1, 2014 12:59:00 AM
Task Size:
- Columns (variables): 26
- Rows (constraints): 16
- Non-zeros (coefficients): 54
- Memory (MB estimated): 32.66
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)
------------------------------------------------------------------------------------------------
R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58
------------------------------------------------------------------------------------------------

Completed ST Schedule Step 1 of 24. Time: 0.47 sec.
Started ST Schedule Step 2 of 24: Jan 1, 2014 1:00:00 AM - Jan 1, 2014 1:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)
------------------------------------------------------------------------------------------------
R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58
```

Completed ST Schedule Step 2 of 24. Time: 0.00 sec.
Started ST Schedule Step 3 of 24: Jan 1, 2014 2:00:00 AM - Jan 1, 2014 2:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 3 of 24. Time: 0.02 sec.
Started ST Schedule Step 4 of 24: Jan 1, 2014 3:00:00 AM - Jan 1, 2014 3:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 4 of 24. Time: 0.00 sec.
Started ST Schedule Step 5 of 24: Jan 1, 2014 4:00:00 AM - Jan 1, 2014 4:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 5 of 24. Time: 0.02 sec.
Started ST Schedule Step 6 of 24: Jan 1, 2014 5:00:00 AM - Jan 1, 2014 5:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 6 of 24. Time: 0.02 sec.
Started ST Schedule Step 7 of 24: Jan 1, 2014 6:00:00 AM - Jan 1, 2014 6:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 7 of 24. Time: 0.00 sec.
Started ST Schedule Step 8 of 24: Jan 1, 2014 7:00:00 AM - Jan 1, 2014 7:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 8 of 24. Time: 0.03 sec.
Started ST Schedule Step 9 of 24: Jan 1, 2014 8:00:00 AM - Jan 1, 2014 8:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 9 of 24. Time: 0.00 sec.

Started ST Schedule Step 10 of 24: Jan 1, 2014 9:00:00 AM - Jan 1, 2014 9:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 10 of 24. Time: 0.01 sec.
Started ST Schedule Step 11 of 24: Jan 1, 2014 10:00:00 AM - Jan 1, 2014 10:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 11 of 24. Time: 0.00 sec.
Started ST Schedule Step 12 of 24: Jan 1, 2014 11:00:00 AM - Jan 1, 2014 11:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 12 of 24. Time: 0.02 sec.
Started ST Schedule Step 13 of 24: Jan 1, 2014 12:00:00 PM - Jan 1, 2014 12:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 13 of 24. Time: 0.00 sec.
Started ST Schedule Step 14 of 24: Jan 1, 2014 1:00:00 PM - Jan 1, 2014 1:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 14 of 24. Time: 0.02 sec.
Started ST Schedule Step 15 of 24: Jan 1, 2014 2:00:00 PM - Jan 1, 2014 2:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 15 of 24. Time: 0.00 sec.
Started ST Schedule Step 16 of 24: Jan 1, 2014 3:00:00 PM - Jan 1, 2014 3:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 16 of 24. Time: 0.02 sec.
Started ST Schedule Step 17 of 24: Jan 1, 2014 4:00:00 PM - Jan 1, 2014 4:59:00 PM

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 17 of 24. Time: 0.00 sec.
Started ST Schedule Step 18 of 24: Jan 1, 2014 5:00:00 PM - Jan 1, 2014 5:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 18 of 24. Time: 0.02 sec.
Started ST Schedule Step 19 of 24: Jan 1, 2014 6:00:00 PM - Jan 1, 2014 6:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 19 of 24. Time: 0.00 sec.
Started ST Schedule Step 20 of 24: Jan 1, 2014 7:00:00 PM - Jan 1, 2014 7:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 20 of 24. Time: 0.00 sec.
Started ST Schedule Step 21 of 24: Jan 1, 2014 8:00:00 PM - Jan 1, 2014 8:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 21 of 24. Time: 0.02 sec.
Started ST Schedule Step 22 of 24: Jan 1, 2014 9:00:00 PM - Jan 1, 2014 9:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 22 of 24. Time: 0.00 sec.
Started ST Schedule Step 23 of 24: Jan 1, 2014 10:00:00 PM - Jan 1, 2014 10:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 23 of 24. Time: 0.03 sec.
Started ST Schedule Step 24 of 24: Jan 1, 2014 11:00:00 PM - Jan 1, 2014 11:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 600.02 600.02 0.00 82.50 137.15 0.00 $228.58

Completed ST Schedule Step 24 of 24. Time: 0.00 sec.
Regional Summary:

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(GWh) (GWh) (GWh) ($000's) ($000's) (MWh)

R 14.40 14.40 0.00 1,980.08 3,291.58 0.00 $228.58

ST Schedule Completed. Time: 0:00:01

Setup... 0.00 %
MOSEK:.. 5.68 %
Solution.. 3.62 %
Other... 90.70 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

Solution Tables Indexed. Time: 0.97 sec.
<-- ST Schedule
<-- Project NC (Model "e=-1.3334") Completed. Time: 0:00:11

Appendix 6. Example of Plexos Excution Log file – Case 3 scenario 3: change of demand elasticity (e= -1.2334) with constant total demand (Q=600MW) and with generation and transmission capacity constraints

Started Project NC (Model "e=-1.2334") -->
- Using Metric Units
Primary Compilation Started:

Element Records

Objects..4
Memberships...119
Properties...58
Scenario "e=-1.2334"..6

Total...187

Primary Compilation Completed. Time: 1.78 sec.
Model Initialization Started -->
- Random Number Seed = 1166189730 (enter this number to repeat this sequence).
Requesting licenses:
- PLEXOS 4.914 R2
- PLEXOS Base Product
- MOSEK Base Product
- MOSEK Parallel
- MOSEK Dual Simplex
- Machine Name: MACHINESLAB11
- Physical Memory: 1,022.73 MB
- CPU Type: Intel(R) Pentium(R) 4 CPU 3.00GHz
- CPU Count: 2
- Maximum Threads: 2
- MOSEK 5.0.0.128
Licenses received.
Planning Horizon Set up Started
Secondary Compilation Started:
Secondary Compilation Completed. Time: 0.30 sec.
In-Memory Cache Setup Started
In-Memory Cache Setup. Time: 2.50 sec.:
- 0 reallocations
- 10000.00 kB
- 0.00 % efficiency
<-- Model Initialized. Time: 6.87 sec.
Preschedule -->
Preschedule step 1 of 1: Wednesday, January 01, 2014 - Wednesday, January 01, 2014
-->
<-- Preschedule step 1 of 1. Time: 0:00:00
Preschedule Completed. Time: 0:00:00

Setup... 100.00 %
Solution....................................... 0.00 %
Other... 0.00 %
Memory:
100000 kB
Reallocations.................................. 0
Efficiency..................................... 0.00 %

ST Schedule -->

OPF Element Count

Single Slack Bus.."1"
Nodes (busbars)...3
Nodes in Service..3
Load Points...0
Generation Injection Points...3
Total Injection Points..3
Lines (branches)..3
Lines in Service..3
AC Lines..3
Flow Limits >= 0 kV...3
Flows Modeled >= 0 kV...3
DC Lines..0
Phase Shifters..0
Phase Shifters in Service...0
Generators..3
Generators in Service..3
Unique AC Paths...3
Modeled AC Paths..3
Required PTDF...9
Finished 3 Projections. Stored 6 PTDFs out of 9 (Compressed 33.33%). Time: 0.00 sec.
Started ST Schedule Step 1 of 24: Jan 1, 2014 12:00:00 AM - Jan 1, 2014 12:59:00 AM
Task Size:
- Columns (variables): 26
- Rows (constraints): 16
- Non-zeros (coefficients): 54
- Memory (MB estimated): 32.66

Class Name Flow (MW) Limit (MW) Loading

Line "1-3"(1) 2.446748e+002 2.400000e+002 101.95 %

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 1 of 24. Time: 0.45 sec.

Started ST Schedule Step 2 of 24: Jan 1, 2014 1:00:00 AM - Jan 1, 2014 1:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 2 of 24. Time: 0.00 sec.
Started ST Schedule Step 3 of 24: Jan 1, 2014 2:00:00 AM - Jan 1, 2014 2:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 3 of 24. Time: 0.00 sec.
Started ST Schedule Step 4 of 24: Jan 1, 2014 3:00:00 AM - Jan 1, 2014 3:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 4 of 24. Time: 0.00 sec.
Started ST Schedule Step 5 of 24: Jan 1, 2014 4:00:00 AM - Jan 1, 2014 4:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 5 of 24. Time: 0.00 sec.
Started ST Schedule Step 6 of 24: Jan 1, 2014 5:00:00 AM - Jan 1, 2014 5:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 6 of 24. Time: 0.02 sec.
Started ST Schedule Step 7 of 24: Jan 1, 2014 6:00:00 AM - Jan 1, 2014 6:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 7 of 24. Time: 0.02 sec.
Started ST Schedule Step 8 of 24: Jan 1, 2014 7:00:00 AM - Jan 1, 2014 7:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 8 of 24. Time: 0.03 sec.
Started ST Schedule Step 9 of 24: Jan 1, 2014 8:00:00 AM - Jan 1, 2014 8:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 9 of 24. Time: 0.00 sec.
Started ST Schedule Step 10 of 24: Jan 1, 2014 9:00:00 AM - Jan 1, 2014 9:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 10 of 24. Time: 0.02 sec.
Started ST Schedule Step 11 of 24: Jan 1, 2014 10:00:00 AM - Jan 1, 2014 10:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 11 of 24. Time: 0.00 sec.
Started ST Schedule Step 12 of 24: Jan 1, 2014 11:00:00 AM - Jan 1, 2014 11:59:00 AM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 12 of 24. Time: 0.03 sec.
Started ST Schedule Step 13 of 24: Jan 1, 2014 12:00:00 PM - Jan 1, 2014 12:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 13 of 24. Time: 0.00 sec.
Started ST Schedule Step 14 of 24: Jan 1, 2014 1:00:00 PM - Jan 1, 2014 1:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 14 of 24. Time: 0.02 sec.
Started ST Schedule Step 15 of 24: Jan 1, 2014 2:00:00 PM - Jan 1, 2014 2:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 15 of 24. Time: 0.00 sec.
Started ST Schedule Step 16 of 24: Jan 1, 2014 3:00:00 PM - Jan 1, 2014 3:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 16 of 24. Time: 0.03 sec.
Started ST Schedule Step 17 of 24: Jan 1, 2014 4:00:00 PM - Jan 1, 2014 4:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 17 of 24. Time: 0.02 sec.
Started ST Schedule Step 18 of 24: Jan 1, 2014 5:00:00 PM - Jan 1, 2014 5:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 18 of 24. Time: 0.00 sec.
Started ST Schedule Step 19 of 24: Jan 1, 2014 6:00:00 PM - Jan 1, 2014 6:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price

(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 19 of 24. Time: 0.02 sec.
Started ST Schedule Step 20 of 24: Jan 1, 2014 7:00:00 PM - Jan 1, 2014 7:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 20 of 24. Time: 0.00 sec.
Started ST Schedule Step 21 of 24: Jan 1, 2014 8:00:00 PM - Jan 1, 2014 8:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 21 of 24. Time: 0.02 sec.
Started ST Schedule Step 22 of 24: Jan 1, 2014 9:00:00 PM - Jan 1, 2014 9:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 22 of 24. Time: 0.00 sec.
Started ST Schedule Step 23 of 24: Jan 1, 2014 10:00:00 PM - Jan 1, 2014 10:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 23 of 24. Time: 0.02 sec.
Started ST Schedule Step 24 of 24: Jan 1, 2014 11:00:00 PM - Jan 1, 2014 11:59:00 PM
Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(MWh) (MWh) (MWh) ($000's) ($000's) (MWh)

R 621.13 621.13 0.00 86.73 144.01 0.00 $231.85

Completed ST Schedule Step 24 of 24. Time: 0.02 sec.
Regional Summary:

Region/Area Demand Generation Net I/C Gen. Cost Load Cost USE Price
(GWh) (GWh) (GWh) ($000's) ($000's) (MWh)

R 14.91 14.91 0.00 2,081.44 3,456.17 0.00 $231.85

ST Schedule Completed. Time: 0:00:01

Setup... 0.00 %
MOSEK:.. 9.72 %
Solution.. 5.78 %
Other... 84.50 %
Memory:
100000 kB
Reallocations................................. 0
Efficiency.................................... 0.00 %

<-- ST Schedule
<-- Project NC (Model "e=-1.2334") Completed. Time: 0:00:10

Appendix 7. Values of μ_i and α in case 1

Total Demand (MW)	Scenario 1			Scenario 2			Scenario 3			
	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	α
100	-	-	-	0	0	0	0	0	0	0
200	-	-	-	0	0	0	0	0	0	0
300	-	-	-	0	0	0	0	0	0	0
400	-	-	-	0	0	0	0	0	0	0
500	-	-	-	0	0	0	0	0	0	0
600	-	-	-	16.7	0	0	16.7	0	0	0
700	-	-	-	62.5	12.5	0	0	0	0	126

Appendix 8. Values of μ_i and α in case 2

Demand elasticity	Scenario 1			Scenario 2			Scenario 3			
	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	α
-0.4334	-	-	-	0	0	0	0	0	0	0
-0.5334	-	-	-	0	0	0	0	0	0	0
-0.6334	-	-	-	11	0	0	11	0	0	0
-0.7334	-	-	-	25.5	0	0	25.5	0	0	0
-0.8334	-	-	-	34.6	0	0	31.9	0	0	5
-0.9334	-	-	-	40.9	0	0	27.6	0	0	25
-1.0334	-	-	-	45.5	0	0	24.4	0	0	40
-1.1334	-	-	-	49	0	0	21.7	0	0	52
-1.2334	-	-	-	53.1	3.1	0	19.4	0	0	62
-1.3334	-	-	-	57.1	7.1	0	17.6	0	0	70