UNIVERSITY OF CAPE TOWN

A CONTINGENT CLAIMS ANALYSIS OF THE PRICING OF RIGHTS ISSUES WITH DISCONTINUOUS DIFFUSION PROCESSES

A dissertation submitted in partial satisfaction of the requirements for the Degree of Master of Commerce (by coursework and dissertation).

by

Russel John Botha

Cape Town

December 1998

ACKNOWLEDGEMENTS AND DECLARATION

I, Russel John Botha, do hereby declare that this research report entitled :

A Contingent Claims analysis of the Pricing of Rights Issues with Discontinuous Diffusion Processes

is my own unaided work save to the extent stated in the acknowledgement. I certify that this work has not been submitted as a dissertation at any other university.

I would like to express my sincere gratitude to the following persons for their support in the compilation of this research report :

- Professor Derek Botha of UCT for valuable guidance and support throughout;
- Francois du Plessis of HSBC Simpson McKie for assistance in the initial stages of the thesis and data collection;
- Michael Davidow of the Johannesburg Stock Exchange for current market statistics;
- Professor Gewers of the Stellenbosch Graduate School of Business for the final data set obtained;
- Praveen Kumar of Lehman Brothers (New York) for practical advice;
- Professor Mike Page of the UCT Graduate School of Business for technical assistance.

Russel John Botha

ABSTRACT

This research proposed to identify the most accurate method of pricing rights using option pricing models, including the Black Scholes model, the Cox constant elasticity of variance model and the Merton jump diffusion model, and to determine the set of input parameters that lead to the most optimal results. The empirical results indicated that on average all of the models are able to estimate the actual rights trading prices relatively well. Some models performed better than others and these findings were consistent with the original reasonings. The market was shown to not account for the effect of dilution. The best model prices were obtained when calculating volatility over a one year historical period that included the actual rights trading period. The hypothesis regarding trading volume showed that there is a significant impact of trading volume on the estimation of accurate option prices. The filter rule of rejecting rights prices below 10 cents and 100 cents also improved the results thus showing a bias for lower priced rights to be incorrectly valued and possibly some inefficiency in this sector of the market.

BRIEF CONTENTS

DETAILED CONTENTS

CHAPTER 1 - INTRODUCTION

1.1 INTRODUCTION TO CONTINGENT CLAIMS ANALYSIS

Contingent claims analysis involves the valuation of claims whose payoffs are contingent on the value of one or more underlying assets. The most well known application of this type of analysis is in the field of option pricing, the topic of this research.

Whilst options have existed for centuries, it was the opening of the Chicago Board Options Exchange in 1973 that revolutionized the trading of options. Before this, options were relatively obscure financial instruments. In the same year that the exchange opened, Fisher Black and Myron Scholes developed a model capable of accurately pricing European put and call options. Their model, commonly known as the Black Scholes model, was to the field of finance what Newton's laws were to physics. This is primarily due to the model being of such a general nature that it can be used to not only value share options but also have applications to the valuation of real investment projects, calculating the cost of capital and valuing other corporate securities (Brigham and Gapenski, 1985). The path-breaking article by Black and Scholes (1973) has formed the basis for many subsequent academic studies on option pricing and there has hence been a wealth of research in this field.

1.2 DEFINITION OF AN OPTION

The standard option pricing terminology used in this paper requires an introduction as it has become so accepted in this field that it is often used without explanation. A standard option is a financial contract which gives its holder the right to buy or sell the underlying asset at a pre-specified price (striking price) within a pre-specified time (expiration date). Options to purchase securities which are written by individuals are termed call options, options to sell securities which are written by individuals are termed put options. Options can be further subclassified into vanilla verses exotic options. If the option may be exercised prior to the expiration date then it is termed an American option. If the option may only be exercised on the expiration date then it is termed a European option. Options granted by corporations are termed warrants. If the strike price of a call is lower (higher) than the spot price of the underlying asset it is called an in the money call option or ITM (out of the money call option or OTM). If the strike price of the call is equal to the spot price of the underlying asset it is called an at the money option. This paper will only concentrate on vanilla European call options due to their similarity to rights.

1.3 RAISING CAPITAL IN THE MARKET AND THE LINK TO OPTION PRICING

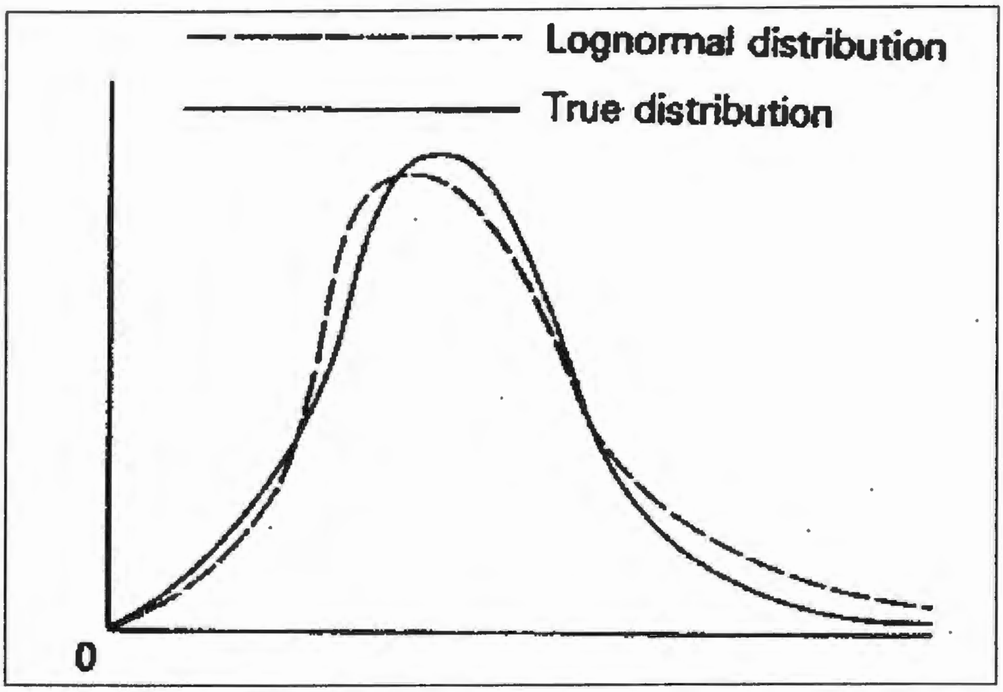
Firms require additional finance for strategic financial investments to provide growth, investment in capital expenditure or for the development of new products. This finance can be obtained inter alia from ordinary shareholders' equity, external debt, preference shares and derivative securities such as options, warrants and convertible bonds. Ordinary shareholders' equity is the dominant source of finance (Flynn and Weil, 1991) and a rights issue is one of the primary methods of issuing ordinary equity (Brealey and Myers, 1991), especially in South Africa where a company's articles of association are required to give the current shareholders first right to an equity issue (Fenwick, 1994).

In a rights issue each shareholder is issued an option to buy a specified number of new shares at a specified price, termed the subscription price, within a specified time, after which the rights expire. A rational shareholder will only subscribe to the rights offering if the subscription price is below the market price of the stock on the offer's expiration date. Thus the right itself must have some value - to be able to buy a share of stock for less than its market value suggests that the right must have a value approximately equal to the difference between the market price of the stock and the subscription price (Diamond, 1994). Smith (1976) hypothesized that the above characteristics make a right similar in nature to a European call option and the value of a right should thus be able to be calculated using the Black Scholes model.

1.4 ANALYSIS OF PRICING BIASES

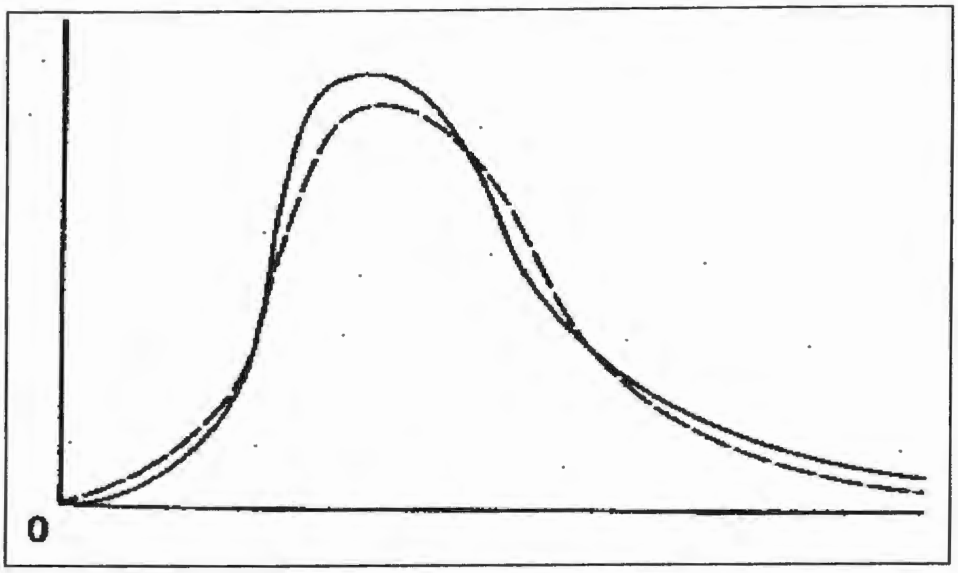
The original Black Scholes model is, however, based on a number of restrictive assumptions. The aim of this research is to price rights while relaxing some of these assumptions : (1) of a lognormally distributed stock price, and (2) of a constant variance of the underlying stock price process. There is a significant amount of empirical research that has shown the above assumption of lognormality of stock prices and stationary variance processes to be inaccurate (Mandelbrot, 1963 and Goldenberg, 1991). The price dynamics of the underlying instruments are critical in solving for equilibrium option prices (Hull, 1991) and if the terminal stock price distribution does not accord with the Black Scholes assumptions then the model prices will deviate from the market prices in systematic ways.

There are three primary departures from lognormality that are observed in the market-place where the terminal stock price distribution can differ from a lognormal distribution and yet still have the same mean and standard deviation for the stock price return (Hull, 1991). These will be illustrated next as they explain the justification for the use of the alternative models employed in this study.



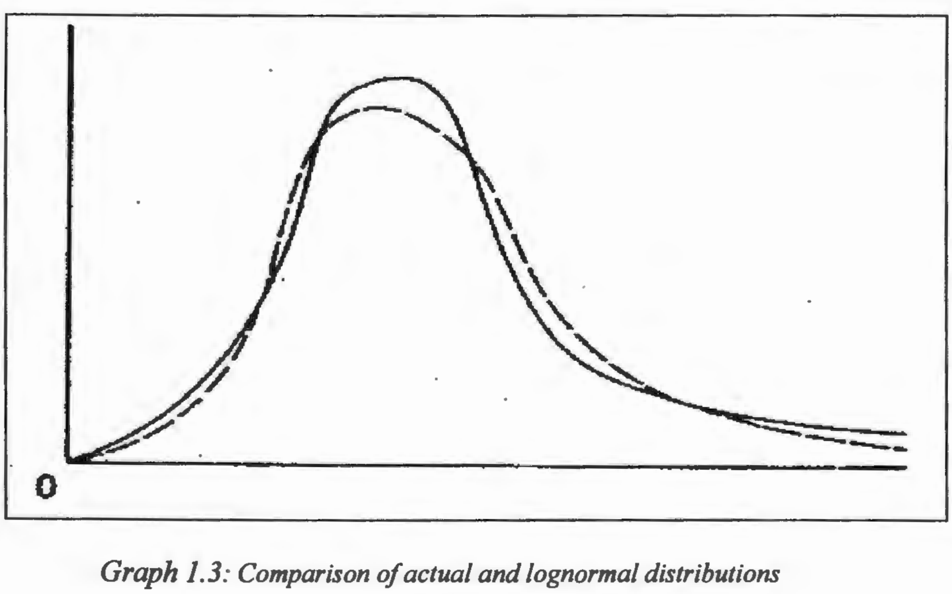
Graph 1.1 : Comparison of actual and lognormal distributions

In Graph 1.1 the true distribution has a fatter left tail and a thinner right tail. The result of this distribution is that the Black Scholes model will overprice out of the money calls and in the money puts. Conversely it will underprice out of the money puts and in the money calls. The reasoning behind this is that when the stock price increases, its instantaneous volatility decreases which in turn implies that very high stock prices are less likely to occur than under geometric Brownian motion. Conversely, when the stock price decreases, the instantaneous volatility will increase and thus very low stock prices are more likely to occur than under geometric Brownian motion. This distribution is typically generated by a stock diffusion process where the stock price and volatility are negatively correlated. This paper will apply the Cox (1976) Constant Elasticity of Variance model with a negative correlation between the stock price and instantaneous volatility to compensate for such a distribution.



Graph 1.2: Comparison of actual and lognormal distributions

In Graph 1.2 the right tail is fatter than that of a lognormal distribution and the left tail is thinner. The result of this is that the Black Scholes model will overprice out of the money puts and in the money calls. Conversely it will underprice in the money puts and out of the money calls. This distribution is typically generated by a stock diffusion process where the stock price and volatility are positively correlated. This phenomenon has no empirical or theoretical grounding and will thus not be examined.



Graph 1.3: Comparison of actual and lognormal distributions

In Graph 1.3 both tails are seen to be fatter than the lognormal distribution. In this scenario the Black Scholes model underprices out of the money and in the money calls and puts. The most common explanation for such a process is that the stock price process has a jump component which would give rise to the fatter tails. Merton (1976) proposed a jump diffusion model where the stock price process has jumps (either positive or negative) superimposed upon a geometric Brownian motion continuous process. This paper will employ the Merton (1976) Jump Diffusion model to compensate for such a distribution.

1.5 MOTIVATION FOR THE RESEARCH

An understanding of the pricing of rights issues is crucial in understanding the South African equity market. This is primarily due to the fact that rights issues are so important in raising equity capital in South Africa. However, previous studies using the Black Scholes model to price rights and gilts showed it to be an in-efficient estimator of actual option prices (Law, 1992; Fenwick, 1994) although the results were somewhat inconsistent. It is thus necessary to ascertain whether there is an option pricing model that is able to accurately calculate the prices of rights in the South African market.

At present, except for warrants, all options in South Africa are traded on an over the counter basis. The warrants market has only recently begun to develop and the liquidity of the market is consequently fairly low, thus not lending itself favorably to empirical testing. The gilt market, on the other hand, has been in existence for some time and has been the subject of at least two empirical studies with fairly accurate results. It was thus decided to focus on the market for rights issues as it is an unresearched area and particularly interesting in that while most research in option pricing has focused on exchange traded options, a right is dissimilar in that it is more like a real option and it is thus important to determine empirically whether the theoretical link between a right and a European call option in fact holds.

1.6 THE RESEARCH PROBLEM

This research proposes to identify the most accurate method of pricing rights (nil paid letters) using option pricing models, including the Black Scholes model, the Cox constant elasticity of variance model and the Merton jump diffusion model, previously applied to the valuation of other financial assets, and to determine the set of input parameters that lead to the most optimal results.

1.7 RESEARCH METHODOLOGY

The methodology for testing excess returns in other studies on option pricing (Macbeth and Merville, 1980) and warrant pricing (Kremer and Roenfeldt, 1993) has been followed. The methodology involves testing the differences between the actual rights prices observed and those calculated using the various option pricing models being tested. All rights issues during the period 1 Jan 1992 to 30 April 1998 by companies listed on the Johannesburg Stock Exchange will be examined, where there are no confounding events at the time of the rights issue. Three models will be tested against the daily nil paid letters prices. The basic model against which the other models will be compared is the Black Scholes (1973) model. The Cox (1976) constant elasticity of variance model will be employed to account for the hypothesized negative relationship between share price and instantaneous volatility. The

Merton (1976) jump diffusion model will be used to account for the possibility of a discontinuous stock diffusion process.

1.8 CONTRIBUTION TO KNOWLEDGE

The international research on the valuation of rights issues using option pricing models is fairly limited, with the papers generally only employing the Black Scholes model. In contrast, the constant elasticity of variance model and jump diffusion model have been employed in studies of stock options, foreign currency options and warrants, with impressive results. They have not been used before to price rights issues and there does appear to be a solid grounding for their use therein considering the structure of volatility processes and share price diffusion processes. South African research on option pricing has been very limited, generally to a few applications of the Black Scholes model to pricing warrants and gilts, with conflicting results primarily due to thin trading.

This study is thus the first of its kind to employ advanced option pricing models to the field of rights issues. It is also the first study that addresses the issue of which volatility estimator results in a parameter that prices options most accurately. Furthermore the research concentrates on pricing options in the presence of non-normality of returns which in an illiquid market like ours is a very real phenomenon and not taken into account in any of the previous research in this field. This research specifically adjusts the models for the thin trading phenomenon. The research is concentrated on short maturity option pricing and will show whether the assumptions underlying the Black Scholes model are so significant and unrealistic as to force market participants to use other option pricing models to be able to accurately price derivative instruments. The results are directly extendible to longer maturity options and as such could form the basis for further research into options of a longer term e.g. the warrants market and LEAPS.

The research results will also have significant practical applications. The most efficient model will be able to be employed by companies in evaluating the cost of underwriting a rights offer via the put-call parity principle (Marsh, 1980). It will necessarily also enable market participants to trade profitably in the nil paid letters market and identify any arbitrage opportunities (and thus perhaps lead to a more efficient market). The applications of the results are numerous and limited only to the creative capacity of the user.

1.9 STRUCTURE OF THE RESEARCH THESIS

Chapter 2 traces the history of options and their pricing. Chapter 3 is the foundation chapter for the thesis in that it develops the mathematics of the various stock price diffusion processes that are hypothesized to underlie the option pricing models employed in the later chapters. Chapter 4 details the formulation of the Black Scholes model including the alternative Cox-Ross-Rubinstein binomial lattice approach and its convergence to the Black Scholes model in continuous time.

Chapter 5 addresses the issue of share price volatility. The various hypothesized volatility processes are explained and their implication on option pricing highlighted. This leads to Chapter 6 on the Cox Constant Elasticity of Variance option pricing model. Chapter 7 describes the various share price diffusion processes that have been put forward in the literature and Chapter 8 develops the Merton (1976) Jump Diffusion model to account therefore.

Chapter 9 explains the theory behind warrants and rights issues in order to highlight the relationship between rights pricing, warrant pricing and option pricing. The nature of the thesis is such that it has a joint hypothesis problem in that any test of the models employed is necessarily also a test of the efficiency of the rights issue market and market efficiency is accordingly addressed in chapter 10.

Chapter 11 details the research methodology employed including a detailed analysis of the sample data. The results of the tests performed on the various models and across the different input parameters employed are illustrated in Chapter 12. Chapter 13 draws the conclusion on the research results.

CHAPTER 2 - THE HISTORY OF OPTIONS AND OPTION PRICING

2.1 INTRODUCTION

This chapter will trace the early beginnings of options trading both internationally and in South Africa. The current status of the South African options market will also be discussed. As an introduction to the later chapters that discuss some of the option pricing models in detail, the latter half of this chapter will briefly describe the early empirical research into option pricing that preceded the Black Scholes model.

2.2 THE ORIGINS OF OPTIONS TRADING

2.2.1 THE INTERNATIONAL ARENA

Many regard the formation of the Chicago Board Options Exchange in 1973 as being the origin of options trading but the origin of options contracts can be traced much farther back in time to the ancient Greeks (Gastineau, 1979). The earliest recorded story relating to the concept of options was due to Aristotle. Aristotle related the story of a wise man named Thales. Thales was a philosopher with the gift of being able to read the stars. He was trapped in poverty and often challenged as to why, with all his great expertise, he was unable to become rich. One winter Thales predicted a great harvest by the winter stars. With the little money that he had, he gave deposits for the use of all the olive presses in the surrounds. The deposits can be likened to paying a premium for an option, the underlying instrument being the rental rate for the presses (Thompson, 1995). When the good harvest came and everyone desperately needed the presses, Thales let them out at outrageous rates and made a handsome profit.

Early options usage was not limited to the ancient Greeks however. Both the Phoenicians and the Romans granted options on cargoes transported by their ships. The first extensive use of options after the Middle Ages occurred during the Dutch tulip bulb mania of the early seventeenth century (Hull, 1993). Dealers in tulips would sell bulbs for future delivery based on call options granted to them by growers. The growers in turn, could guarantee a minimum price for their tulips by purchasing puts from the dealers. Options were thus very useful to the growers in guaranteeing a price for future delivery. Unfortunately this market was unregulated and undisciplined. When the market for tulips broke in 1637, many put writers were destroyed. With the losers either unable or unwilling to pay, the Provincial Council at the Hague was summoned to resolve the problem. In the end, writers were never required to perform on their contracts. Ironically, the Dutch continued to use options, despite the tulip debacle. Puts, calls and straddles were traded in Amsterdam on the shares of the Dutch West India Company only a few years later.

Organized trading in puts and calls on securities in London began late in the seventeenth century. Considerable opposition developed (Barnard's Act of 1733) but was not effective in stopping the trade. Trading in puts and calls continued until the financial crisis of 1931 (Gastineau, 1979). Options were also banned for a period from W.W.II to the late 1950's.

In the United States, option trading began in the late eighteenth century. The options trading arena had some rather questionable practices but these were eliminated in the 1930's with the imposition of securities legislation, including the Securities and Exchange Commission and National Association of Securities Dealers. The most profound impact on options trading came with the establishment of the Chicago Board Options Exchange in 1973, which today is the largest options exchange in the world.

2.2.2 THE SOUTH AFRICAN BEGINNINGS

According to Payne (1980) options were first traded in South Africa soon after the establishment of the Johannesburg Stock Exchange (hereafter referred to as the JSE) in 1887 although no records exist from that era to substantiate this contention. Following the establishment of the JSE, rules and regulations were adapted from those used on the London Stock Exchange. The combined influence of the British and Dutch in South Africa together with the development of the mining industry, led to the development of options trading in this country. Initially, European non-transferable options were created (Mathew, 1990). This adoption process is not a typical world wide trend, as the procedure has generally been to float a futures market first.

Prior to WWII options were written inter alia by the mining houses and this continued for approximately ten years after the war. They were not written on shares of their own stock but rather on the shares of companies further down the company structure i.e. subsidiaries. It was only in the late 1940's that stockbroking firms began to deal in options to any great extent, when two members of the JSE, M.Johnson and R.Anderson, began to write options. In the 1960's fixed interest stocks were issued with an early redemption (put) option for either the issuer or the holder. These customized, over the counter options were the only type of option on fixed interest securities in South Africa until the early 1980's (Mathew, 1990).

During the early 1980's negotiable certificates of deposit and Krugerrand options were introduced as formal derivatives. Neither of these instruments lasted vary long due to insufficient volatility and funds. In July 1984 three banks opened up Reuters screens offering call and put options on gilts. However, until 1987 the options market was not standardized and customized settlement dates, strike prices and declaration dates prevailed. In 1987 the first standardized option contract was developed - the Eskom 168 gilt option. This was followed by the introduction in Parliament in April 1989 of the draft Financial Markets Control Act for governance of all non-equities related markets in South Africa

In 1992 the Johannesburg Stock Exchange launched the Traded Options Market (TOM). This was, however, a total failure due to its high costs and administrative problems and not even the institutions could be enticed to invest (Markos, 1995). The JSE is currently investigating resurrecting TOM and the South African Futures Exchange has already been granted permission to list individual equity futures and options by the Registrar of Financial Markets. Time will tell whether an active options market can be successfully developed in South Africa. Presently, a number of warrants are traded on the JSE but the volume is thin and there are only approximately eleven listed warrants which trade regularly These appear to be the only exchange traded option-like instruments available to investors at present.

2.3 THE ORIGINS OF OPTION PRICING

With the trading in options occurring, there was a necessity to find a true value for these contracts between a willing buyer and a willing writer. The development of these models appears to have begun long after the origins of trading in the instruments. Prior to the Black-

Scholes (1973) option pricing model, various incomplete models of call option pricing were developed. It is important to summarize the development of these models as it highlights the major contribution made by Black and Scholes (1973) - that of risk neutral valuation whereby the call option could be valued by creating and continuously rebalancing a risk-free hedge. It is also important to highlight the great contributions made by the early researchers in option pricing, lest their work be forgotten.

In 1900 Bachelier put forward the first option pricing model using the assumption that stock prices follow an arithmetic Brownian motion process. Thus price changes are posited to be independent, identically distributed random variables that follow the process given by :

$$prob[S_T \le S^* | S_t = S] = F(S^* - S; T)$$

where S represents the current stock price, T the terminal date, t the current date and S^* the terminal stock price. This describes arithmetic Brownian motion but it does not restrict stock prices to positive values and the critical assumption of limited liability is thus broken. Arithmetic Brownian motion allows for a constant drift, whereas the currently popular model of geometric Brownian motion allows for a constant drift rate. Bachelier (1900) incorrectly deduced that the above formula implies that the density function thereof must be the normal distribution. This is incorrect as it must be further assumed that the variance is finite before the normal distribution can be utilized and Bachelier's (1900) function would be satisfied by any member of the stable Paretian distributions. Bachelier (1900) also assumed that the mean expected price change per unit time is zero which is clearly not in agreement with daily observations of stock market prices. He arrived at the following integral :

$$C = E(C^*) = \int_X^\infty (S^* - X).N'(S^*)dS^*$$

to which the solution for the call price is :

$$C = S.N\left(\frac{S-X}{\sigma.\sqrt{T}}\right) - X.N\left(\frac{S-X}{\sigma.\sqrt{T}}\right) + \sigma.\sqrt{T}.N'\left(\frac{X-S}{\sigma.\sqrt{T}}\right)$$

It is evident that as the time to maturity is increased, the call price increases without bound thus violating Merton's (1973) restriction that the call price is restricted to the share price (see Appendix 6).

Sprenkle (1964) partly overcame the assumptions of arithmetic Brownian motion and zero expected return by assuming that stock prices are lognormally distributed and additionally allowed for drift in the random walk. Under this model the expected value of the call on expiration is set to :

$$E(C^*) = \int_X^\infty (S^* - X).L'(S^*)dS^*$$

The solution is :

$$E(C^*) = e^{\rho.T}.S.N\left(\frac{\ln(S/X) + [\rho + (\sigma^2/2)].T}{\sigma.\sqrt{T}}\right) - X.N\left(\frac{\ln(S/X) + [\rho - (\sigma^2/2)].T}{\sigma.\sqrt{T}}\right)$$

The formula is then modified for risk :

$$E(C^*) = e^{\rho.T}.S.N\left(\frac{\ln(S/X) + [\rho + (\sigma^2/2)].T}{\sigma.\sqrt{T}}\right) - (1-k).X.N\left(\frac{\ln(S/X) + [\rho - (\sigma^2/2)].T}{\sigma.\sqrt{T}}\right)$$

The formula is flawed however, as it requires the assumption that interest rates are zero and thus ignores the concept of the time value of money.

The next improvement was made by Boness (1964), who furthered Sprenkle's (1964) model by allowing for the time value of money, but failed to accept the notion of different levels of risk for the stock and its associated call option. The expected terminal value of the option was calculated as :

$$E(C^*) = [E(S^*|S^* > X) - E(X|S^* > X)].prob(S^* > X)$$

Knowing that :

$$E(S^*|S^* > X) = \int_X^{\infty} S^* L'(S^*)dS^* \Big/ \int_X^{\infty} L'(S^*)dS^*$$
$$E(X|S^* > X) = X$$
$$prob(S^* > X) = \int_X^{\infty} (S^* - X).L'(S^*)dS^*$$

and to allow for the time value of money the terminal call price is discounted back to the present yielding the option price as :

$$C = e^{-\rho.T}.\int_X^{\infty} (S^* - X).L'(S^*)dS^*$$

This integral is solved to yield the call price as :

$$C = S.N\left(\frac{\ln(S/X) + [\rho + (\sigma^2/2)].T}{\sigma.\sqrt{T}}\right)$$
$$- e^{-\rho.T}.X.N\left(\frac{\ln(S/X) + [\rho - (\sigma^2/2)].T}{\sigma.\sqrt{T}}\right)$$

This formula is flawed however, in that one must estimate ρ, the expected rate of growth of the stock price.

Samuelson (1965) derived the model that ultimately led to the development of the Black Scholes model by assuming that stock prices follow geometric Brownian motion with positive drift, ρ, thus allowing for positive interest rates and risk premiums :

$$E(S^*/S) = e^{\rho.T}$$

If the option price also grows at the rate k, then :

$$E(C^*/C) = e^{k.T}$$

With the assumption of a lognormal terminal stock price distribution, the value of the option is then :

$$C = e^{-k.t} E(C^*)$$
$$= e^{-k.t} \int_X^{\infty} (S^* - X).L'(S^*)dS^*$$

This differential equation is solved for the call price :

$$C = e^{(\rho - k).T}.S.N\left(\frac{\ln(S/X) + [\rho + (\sigma^2/2)].T}{\sigma\sqrt{T}}\right) - e^{-k.T}.X.N\left(\frac{\ln(S/X) + [\rho - (\sigma^2/2)].T}{\sigma\sqrt{T}}\right)$$

As argued by Black and Scholes (1973), the procedure of assuming k to be a constant is inappropriate as a base for the theory of option pricing under capital market equilibrium. The culmination of these theories is the now famous Black Scholes (1973) model which will be examined in Chapter 4.

CHAPTER 3 - THE DYNAMICS OF SHARE PRICE DIFFUSIONS IN CONTINUOUS TIME

3.1 INTRODUCTION

Merton (1990) developed the basic theorems required for continuous time analysis using probability theory and calculus. The mathematics of continuous time models is specialized due to the fact that while the sample paths for stochastic variables generated by diffusion processes are continuous everywhere, they are differentiable nowhere (in the usual sense) and thus a more general type of differential equation is required to express the dynamics of such processes. The development of this model will be summarized together will all the necessary economic assumptions required to formulate the model, as it is on this model of stock price dynamics that the derivative pricing models in this paper are based. The solutions for the stock price diffusion processes derived below will be used in the individual chapters relating to each of the different option pricing models employed, to derive the option pricing model concerned.

3.2 DEFINITIONS

The following definitions are required for notational purposes :

h - the minimum amount of time between successive transactions

$X(t)$ - the price of a security at time t

n - the number of trading intervals between time 0 and T

$n.h$ - the amount of time between two periods

The change in price of a share from $t = 0$ to $T \equiv n.h$ is simply the sum of the changes in share price over all the individual trading periods in the time interval :

$$X(T) - X(0) = \sum_{k=1}^{n} [X(k.h) - X((k-1).h)]$$

by substituting k for $k.h$ this can be simplified to :

$$X(T) - X(0) = \sum_{1}^{n} [X(k) - X(k-1)] \qquad \{1\}$$

Define $\varepsilon(k)$ as the difference between the actual share price change ($X(k) - X(k-1)$) and the expected share price change $E(X(k) - X(k-1))$ anticipated at time $(k-1)$:

$$\varepsilon(k) \equiv X(k) - X(k-1) - E_{k-1}\{X(k) - X(k-1)\} \qquad \{2\}$$

Since ε (k) is defined as the unanticipated price change of the security, conditional on being at time $k-1$, it must hold that $E_{k-1}\{\varepsilon(k)\} = 0$. Three assumptions are then necessary to restrict the price movements in continuous time to reasonable values (Merton, 1990).

1. For each finite interval $[0, T]$ there will still be some variance of the future share price change (even as $h \rightarrow dt$). Define $A_1 > 0$, then symbolically this can be represented as :

$$Var(S_n) \equiv E_0\{[\sum_{1}^{n} \varepsilon(k)]^2\} \geq A_1$$

2. The variance does not become unbounded. Define $A_2 < \infty$, then :

$$Var(S_n) \equiv E_0\{[\sum_{1}^{n} \varepsilon(k)]^2\} \leq A_2$$

3. There is significant price uncertainty in all trading periods and it is not only concentrated in a few of the periods. Define $V(k) \equiv E_0\{\varepsilon^2(k)\}$ as the expected variance of returns of the security between time *k-1* and time *k* based on information available as at time zero. The maximum variance is $V \equiv \max_k V(k)$ so that :

$V(k)/V \geq A_3$ where $0 < A_3 \leq 1$

These three assumptions lead to the proposition that the variance, $V(k)$, is asymptotically proportional to h i.e. $V(k) \sim h$. Let $\varepsilon(k)$ take on any one of $j = 1,\ldots,m$ distinct values denoted $\varepsilon_j(k)$. The conditional probability :

$$p_j(k) = prob[\varepsilon(k) = \varepsilon_j \mid info.\ available\ as\ at\ t = 0]$$

implies that :

$$\sum_{j=1}^{m} p_j(k).\varepsilon_j^2(k) = O(h)$$

and thus $p_j(k).\varepsilon_j^2(k) = O(h)$ and $p_j(k).\varepsilon_j^2(k) \neq o(h)$ so that it is asymptotically proportional to h ($p_j.\varepsilon_j^2 \sim h$). Merton (1990) then posited that there exist numbers q_j, r_j such that $p_j \sim h^{q_j}$ and $\varepsilon_j \sim h^{r_j}$. Hence the values taken must satisfy :

$$q_j + 2.r_j = 1$$

Thus for large values of r_j the outcome will be of smaller magnitude than for outcomes with small values of r_j, but at the same time, the rule must hold, so that for the large r_j the p_j will be small and vice versa. Thus outcomes of large magnitude have smaller probability than outcomes of smaller magnitude. These values thus determine the asymptotic distributional properties of the stock price process.

The possible continuous-sample-path processes can be apportioned into three groupings based on the above assumption. Type I and II outcomes cover the scenario where $0 < r_j \leq 0.5$ and type III outcomes are where $r_j = 0$. The distributional characteristics of type I and II outcomes are the same so they will be discussed together and then contrasted to type III outcomes.

3.3 TYPE I AND II OUTCOMES

Define the conditional expected dollar return per unit time on a security as :

$$\propto_k \equiv E_{k-1}\{X(k) - X(k-1)\} / h \qquad \{3\}$$

The expected return during any period is assumed to be bounded and thus $|\propto_k| \leq \infty$, thus ensuring that returns do not become infinite as the time interval is decreased. The dollar return on a security can thus be expressed by combining {2} and {3}:

$$X(k) - X(k-1) \equiv \propto_k .h + \varepsilon(k) \qquad \{4\}$$

If the price diffusion process is either type I or II then the continuous time sample path will be continuous. However, this path is almost nowhere differentiable as $[X(k) - X(k-1)] / h \sim 1 / h^{0.5}$, which diverges as $h \to 0$. Looking at the moment properties of the stock price changes reveals that all moments higher than the second are asymptotically insignificant compared with the first two. From assumptions one and three the second moment is :

$$E_{k-1}\left[(X(k) - X(k-1))^2\right] = \sigma_k^2 .h + o(h)$$

Moments of higher $(N > 2)$ order are :

$$E_{k-1}\left[|\varepsilon(k)|^N\right] = o(h)$$

$$E_{k-1}\left[(X(k)-X(k-1))^N\right] = E_{k-1}\left[|\varepsilon(k)|^N\right] + o(h^{N/2})$$

Based on the above, Merton (1990) derived the stochastic difference equation for type I and II outcomes as :

$$X(k)-X(k-1) = \propto_k .h + \sigma_k .u(k).h^{1/2} \qquad \{5\}$$

where $u(k) = \dfrac{\varepsilon(k)}{(\sigma_k^2 .h)^{1/2}}$ and by construction has the following properties :

$$\begin{aligned} &E_{k-1}[u(k)] = 0 \\ &E_{k-1}[u^2(k)] = 1 \\ &E_{k-1}[|u(k)|^N] = O(1) \quad for\, N > 2 \end{aligned}$$

Let $F(t) = f(X,t)$ *if* $X(t) = X$ where f is a C^2 function with bounded third partial derivatives. The Taylor series expansion is :

$$\begin{aligned} f(X_j,k) = {} & f(X,k-1) + f_1(X,k-1).(\alpha_k h + \sigma_k u_j h^{1/2}) + f_2(X,k-1).h \\ & + \frac{1}{2}.f_{11}(X,k-1).(\alpha_k h + \sigma_k u_j h^{1/2})^2 + R_j \end{aligned}$$

where $|R_j| = O(h^{3/2}) = o(h)$. Thus :

$$\begin{aligned} F(k)-F(k-1) = {} & \left[f_1(X(k-1),k-1).\alpha_k + f_2(X(k-1),k-1) + \frac{1}{2}.f_{11}(X,k-1)(\sigma_k^2 u_j)\right].h \\ & + f_1(X,k-1).(\sigma_k u_j h^{1/2}) + o(h) \end{aligned}$$

Taking expectations and defining $\mu_k \equiv E_{k-1}[F(k)-F(k-1)]/h$:

$$\begin{aligned} F(k)-F(k-1) = {} & \mu_k .h + \frac{1}{2}.f_{11}[X(k-1),k-1].\sigma_k^2.(u^2(k)-1).h \\ & + f_1[X(k-1),k-1].\sigma_k .u(k).h^{1/2} + o(h) \end{aligned}$$

Merton (1991) showed the conditional moments to be :

$$E_{k-1}[(F(k)-F(k-1))^2] = \left[f_1[X(k-1),k-1].\sigma_k\right]^2.h + o(h)$$
$$E_{k-1}[(F(k)-F(k-1))^N] = O(h^{N/2}) = o(h) \quad for\, N > 2$$

It is thus evident that the order relation for the conditional moments of $F(k)-F(k-1)$ is the same as for the conditional moments of $X(k)-X(k-1)$ and indeed their co-movements are perfectly correlated so that in continuous time they will be perfectly correlated. Over longer time intervals the cumulative error of approximation goes to zero as the trading interval becomes infinitesimally small and thus in the limit of continuous time, with probability one :

$$F(T)-F(0) = \sum_1^n [F(k)-F(k-1)]$$
$$= \sum_1^n \mu_k h + \sum_1^n f_1[(X(k-1),k-1].\sigma_k.u(k).h^{1/2}$$

and using the limiting arguments for Riemann integration :

$$F(T)-F(0) = \int_0^T \mu(t)dt + \int_0^T f_1[X(t),t].\sigma(t).u(t).(dt)^{1/2}$$

and the corresponding stochastic differential equation (hereafter denoted SDE) is :

$$dF(t) = \mu(t)dt + f_1[X(t),t].\sigma(t).u(t).(dt)^{1/2} \qquad \{6\}$$

The corresponding SDE for X itself can be written :

$$dX(t) = \alpha(t)dt + \sigma(t).u(t).(dt)^{1/2} \qquad \{7\}$$

Merton's (1990) final assumption is that the stock process is Markov. Defining $p(x,t) \equiv prob[X(T) = X | X(t) = x] \quad t < T$, then provided p is a well-behaved function of x and t, it will satisfy the above SDE for $F(t)$. p is a probability and thus its expected change is zero so :

$$0 = \frac{1}{2}\sigma^2(x,t).p_{11}(x,t) + \alpha(x,t).p_1(x,t) + p_2(x,t)$$

which is a linear partial differential equation (Kolmogorov backward equation). Define $Z(t)$ to be a random variable similar to the previously defined change in stock price $X(k)-X(k-1)=\alpha_k.h+\sigma_k.u(k).h^{1/2}$, except that $\alpha_k=0,\sigma_k=1$. Then, over time :

$$Z(T)-Z(0)=\sum_1^n[Z(k)-Z(k-1)]$$

$$=T^{1/2}.\left(\frac{\sum_1^n u(k)}{n^{1/2}}\right)$$

but the $u(k)$ are independent and identically distributed with zero mean and unit variance so by the Central Limit Theorem (Feller, 1966) in the limit of continuous trading $Z(T)-Z(0)$ will be normally distributed with zero mean and variance T. Thus the solution to the above equation for $p(x,t)$ with $\alpha=0,\sigma^2=1$ is :

$$p(x,t)=\frac{e^{-\frac{1}{2}.\frac{(X-x)^2}{(T-t)}}}{(2.\pi.(T-t))^{1/2}}$$

which is a normal density function. Thus $dZ(t)=u(t).(dt)^{1/2}$ and where the $u(t)$ are independent identically distributed. This is a Wiener or Brownian motion process. Rewriting $dX(t)$ then :

$$dX(t)=\alpha[X(t),t].dt+\sigma[X(t),t].dZ(t) \qquad \{8\}$$

which is an Ito process. Ito's lemma states that if $f(X,t)$ takes the form above then the time dependent random variable $F\equiv f$ has stochastic differential :

$$dF=f_1(X,t).dX+f_2(X,t).dt+\frac{1}{2}.f_{11}(X,t).(dX)^2 \qquad \{9\}$$

following the rules $(dZ)^2=dt, DZ.dt=0,(dt)^2=0$. Thus if the above set of assumptions holds and unanticipated security price changes have only type I and II outcomes, then in continuous trading models of that structure, security price dynamics can always be described by an Ito process. Equation {8} will be used as the stock price model for Chapter 4.

3.4 TYPE III OUTCOMES

This describes paths where the outcomes for $\varepsilon(k)$ can be type I, II or III. Virtually all the observations will be type I outcomes and while the type III outcomes with their probabilities proportional to h are the rarest of the admissible outcomes, the magnitude of these outcomes is the largest. Merton (1990) stated that if for k = 1,...,n at least one possible outcome for $\varepsilon(k)$ is a type III outcome, then the continuous time sample path for the price of the security will no longer be continuous. Like type I and II outcomes the first and second unconditional moments of $X(k)-X(k-1)$ are asymptotically proportional to h. However, unlike those processes, the Nth unconditional absolute moments $2<N<\infty$ are asymptotically proportional to h as well :

$$E_0\{|\varepsilon(k)|^N\} = \sum_1^m p_j.|\varepsilon_j|^N$$

$$= O\left(\sum_1^m h^{(N-2).r_j+1}\right)$$

$$= O(h) \qquad \text{for } N>2$$

(because $r_j = 0$ for all type III outcomes). Thus all the absolute moments of $X(k)-X(k-1)$ are of the same order of magnitude and cannot be neglected in the limit of continuous trading. However it can be shown that the contribution of the type I outcomes to the moments higher than the second is asymptotically insignificant.

Let $F(t)=f(X,t)$ if $X(t)=X$ where f is a function with bounded third partial derivatives. For a given outcome $y(k)=y$:

$$f[X(k-1)+\alpha_k.h+y,k] = f[X(k-1)+y,k-1]+f_1[X(k-1)+y,k-1].\alpha_k.h+ f_2[X(k-1)+y,k-1].h+o(h)$$

Similarly, for a given outcome $u(k) = u$:

$$f[X(k-1)+\alpha_k.h+u.h^{1/2},k] = f[X(k-1),k-1]+f_1[X(k-1),k-1].(\alpha_k.h+u.h^{1/2})+$$
$$f_2[X(k-1),k-1].h+\frac{1}{2}f_{11}[X(k-1),k-1].u^2.h+o(h)$$

Following from the conditional expectation probabilities set up previously :

$$E_{k-1}\{F(k)-F(k-1)\} = \lambda(k).h.E_{k-1}^{y}\{F(k)-F(k-1)\}+(1-\lambda(k).h).E_{k-1}^{u}\{F(k)-F(k-1)\}$$

Substituting the previous expressions into the above :

$$E_{k-1}\{F(k)-F(k-1)\} = (\frac{1}{2}.f_{11}[X(k-1),k-1]\sigma_u^2+f_1[X(k-1),k-1].(\alpha_k-\lambda.\bar{y})$$
$$+f_2[X(k-1),k-1]+\lambda.E_{k-1}^{y}\{f[X(k-1)+y(k),k-1]$$
$$-f[X(k-1),k-1]\}).h+o(h)$$

Define the conditional expected change in F per unit time as $\mu_k \equiv E_{k-1}\{F(k)-F(k-1)\}/h$, dividing the above expression by h and taking the limit as h approaches zero, the instantaneous conditional expected change in F per unit time is :

$$\mu(t) = \frac{1}{2}.f_{11}[X(t),t]\sigma_u^2(t)+f_1[X(t),t].(\alpha(t)-\lambda(t).\bar{y}(t))+f_2[X(t),t]+\lambda.E_t^{y}\{f[X(t)+y(t),t]$$
$$-f[X(t),t]\}$$

The higher conditional moments for the change in F can be written :

$$E_{k-1}\{[F(k)-F(k-1)]^2\} = (\lambda.E_{k-1}^{y}\{f[X(k-1)+y(k),k-1]-f[X(k-1),k-1]\}^2$$
$$+f_1^2[X(k-1),k-1]\sigma_u^2).h+o(h)$$

and for $N>2$:

$$E_{k-1}\{[F(k)-F(k-1)]^N\} = \lambda.E_{k-1}^{y}\{f[X(k-1)+y(k),k-1]-f[X(k-1),k-1]\}^N+o(h)$$

The results are as for the moments of $X(k)-X(k-1)$ as all the moments of $F(k)-F(k-1)$ are of the same order of magnitude and only the type III outcomes contribute significantly to the moments higher than the second.

Merton (1990) made the assumption that the stock process is Markov. Then $\lambda(k) = \lambda[X(k-1), k-1]; \alpha_k = \alpha_k[X(k-1), k-1]; \sigma_u^2(k) = \sigma_u^2[X(k-1), k-1]$ and the conditional density function for $y(k)$ can be written $g[y(k); X(k-1), k-1]$. As before let $p(x,t)$ denote the conditional probability density for $X(T) = X$ at time T, conditional on $X(t) = x$. Because p is a probability density its expected change is zero - substituting that $\mu(t) = 0$, p must satisfy :

$$0 = \frac{1}{2}\sigma_u^2 p_{11}(x,t) + (\alpha - \lambda.\bar{y})p_1(x,t) + p_2(x,t) + \lambda.\int p(x+y,t)g(Y;x,t)dy$$

It can thus be shown that the asymptotic distribution for $X(t)$ is identical to that of a stochastic process driven by a linear superposition of a continuous-sample-path diffusion process and a Poisson-directed process. Let $Q(t+h) - Q(t)$ be a Poisson-distributed random variable with characteristic parameter $\lambda[X(t),t].h$, in the limit as h approaches dt :

$$\begin{aligned} dQ(t) &= 0 \quad \text{with probability} \quad 1 - \lambda[X(t),t]dt + o(dt) \\ &= 1 \quad \text{with probability} \quad \lambda[X(t),t]dt + o(dt) \\ &= N \quad \text{with probability} \quad o(dt), N \geq 2 \end{aligned}$$

Hence the continuous trading dynamics for $X(t)$ are represented by the SDE :

$$dX(t) = \{\alpha[X(t),t] - \lambda[X(t),t].\bar{y}(t)\}.dt + \sigma[X(t),t].dZ(t) + y(t).dQ(t) \quad \ldots \{10\}$$

where α is the instantaneous expected change in X per unit time, σ^2 is the instantaneous variance of the change of X conditional on the change being a type I outcome; λ is the probability per unit time that the change in X is a type III outcome and $y(t)$ is the random variable outcome for the change in X conditional on the change being a type III outcome.

Define the following random variables :

- $dX_1(t) = y(t)dQ(t)$ which is a Poisson directed process
- $dX_2(t) = \alpha' dt + \sigma' dZ$ where dZ is a Wiener process so this is a diffusion process with a continuous sample path.

The SDE for F is then :

$$dF(t) = \{\frac{1}{2}\sigma^2 f_{11}[X(t),t] + (\alpha - \lambda.\bar{y})f_1[X(t),t] + f_2[X(t),t]\}dt + \sigma.f_1[X(t),t].dZ(t)$$
$$+ \{f[X(t)+y(t),t] - f[X(t),t]\}.dQ(t)$$

thus if the dynamics of *X(t)* can be described by a superposition of diffusion and Poisson-directed processes, then the dynamics of well behaved functions of *X(t)* can be described in the same way (the transformation rule corresponds to Ito's lemma for pure diffusion processes).

In type III processes then, the diffusion process component describes the frequent local changes in prices and is sufficient in structures where the magnitudes of the state variables cannot change radically in a short period of time (Merton, 1990). The Poisson directed process is used to capture those rare events when the state variables have non-local changes and security prices "jump". This combined "normal" and "jump" process for stock price dynamics will be investigated further in Chapter 8 on the Merton Jump Diffusion option pricing model which specifically accounts for the possibility of a discontinuous stock price path (explained in Chapter 7).

CHAPTER 4 - THE BLACK SCHOLES MODEL

4.1 INTRODUCTION

The recent rapid development of option pricing theory and the application of this theory to practice can be traced to the path-breaking paper by Fischer Black and Myron Scholes (1973). In their paper, Black and Scholes provided the first explicit general equilibrium solution to the option pricing problem for simple European puts and calls. The solution was then extended to provide a basis for the general analysis of contingent claim assets. Most practitioners adopted the Black Scholes model as the premier model for pricing and hedging options (Brenner and Subrahmanyam, 1994) and although there are many extensions of the model, the original version is still by far the most widely used by professionals as a benchmark for pricing such instruments (Zhang, 1997). To some degree the Black Scholes model has facilitated option trading and aided the growth of the financial derivatives market as a whole.

The development of the model and the mathematics behind the model will be discussed in this chapter. Due to the arbitrage argument underlying the theory of option pricing, this will be discussed first and then used in the proof of the Black Scholes model. The alternate discrete time model - the Cox-Ross-Rubinstein binomial model - will also be investigated to gain a deeper understanding of the Black Scholes model. A comparative statics analysis of the Black Scholes model will be performed to examine the impact that each of the input parameters has on the model outputs.

4.2 ARBITRAGE PRICING AND THE BLACK SCHOLES MODEL

The field of option pricing is distinct in that it relies on the concept of arbitrage as opposed to the more common procedure of calculating expected values of payoffs, as used in capital budgeting and net present value calculations. An example will emphasize the notion of using the arbitrage argument and show why the expected value approach leads to incorrect option prices. Assume that a share is currently trading at R100 (P_0), can either rise to R125 (P_{T_1}) with probability 0.50, or fall to R80 (P_{T_2}) with probability 0.50, one year hence and there exists an option on the share expiring in one period with strike price equal to R95 (*K*). The one year riskfree rate is 10%. If one owns the call option, the possible terminal payoffs can be represented as follows :

	P_{T_1}	P_{T_2}
TERMINAL STOCK PRICE (P_T)	80	125
VALUE OF CALL OPTION (*C*)	0	30

Next assume that the investor has purchased three calls and not just one, the payoff structure becomes :

	P_{T_1}	P_{T_2}
TERMINAL STOCK PRICE (P_T)	80	125
VALUE OF CALL OPTIONS (*C*)	**0**	**90**

The exact same payoff structure could be obtained if the investor had purchased two shares at the current price and partly financed this purchase by borrowing the present value of R80 for each share :

	P_{T_1}	P_{T_2}
TERMINAL STOCK PRICE (P_T)	80	125
VALUE OF TWO SHARES	160	250
VALUE OF DEBT	(160)	(160)
VALUE OF PORTFOLIO	**0**	**90**

If the two payoffs are the same then, to prevent riskless arbitrage, it must be that their setup costs are the same and hence the cost of purchasing the three call options must equal the cost of purchasing the two shares and borrowing :

$$3.C = 2.P_0 - 2.PV(80)$$
$$\therefore 3.C = 2.(100) - 2.(\frac{80}{1.1})$$
$$\therefore C = R\ 18.18$$

Thus, in terms of arbitrage, the value of the call must be R 18.18. Had one valued the call using its expected value the result would have been (incorrectly) :

$$3.C = \frac{(0.5 \times 0) + (0.5 \times 90)}{1.10}$$
$$\therefore C = R\ 13.64$$

The option could still have been valued in terms of expected values, however, had the investor used the equivalent martingale measure of 60.6% (Shimko, 1992). It should be noted though, that not all options can be valued in terms of the arbitrage principle.

4.3 BLACK SCHOLES MODEL ASSUMPTIONS

In deriving their now famous model of valuing call options, Black and Scholes (1973) made certain assumptions which they called the "ideal conditions" in the market for the stock and the option :

1. The short term interest rate is known and is constant through time.
2. The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite time interval is lognormal. The variance rate of the return of the stock is constant.

3. The stock pays no dividends or other distributions.
4. The option is European.
5. There are no transaction costs in buying or selling the stock or option.
6. It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short term interest rate.
7. There are no penalties for short selling. A seller who does not own the security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

Black and Scholes (1973) also indirectly assumed that :

8. The standard form of the Sharpe-Lintner-Mossin Capital Asset Pricing Model holds for intertemporal trading and trading takes place continuously in time.
9. There are no dividends or exercise price changes over the life of the contract.

It is under this set of restrictions then, that it will be possible to create a hedged position consisting of a long position in the stock and short position in the option, the value of which will not depend on the price of the stock but instead will depend only on time and variables that are taken to be know constants. It is the intuition of this riskless hedge that leads to the Black-Scholes formula.

4.4 DERIVATION OF THE MODEL

Black and Scholes (1973) made use of the Samuelson (1965) application to warrant pricing of the Bachelier-Einstein-Dynkin derivation of the Fokker-Planck equation to express the expected return on the option in terms of the option price function and its partial derivatives (due to the distributional assumptions and because the option price is a function of the common stock price). From the equilibrium condition on the option yield, such a partial differential equation is derived and its corresponding solution for a European call option solved.

If a variable follows an Ito process :

$$dx = a(x,t).dt + b(x,t).dz \qquad ...\{12\}$$

where dz is a Wiener process and the variable x has a drift rate of a and a variance rate of b^2, which may both be functions of the price x and time t, Ito's lemma shows that a function G, of x and t, follows the process :

$$dG = (\frac{dG}{dx}.a + \frac{dG}{dt} + \frac{1}{2}.\frac{d^2G}{dx^2}.b^2).dt + \frac{dG}{dx}.b.dz$$

where dz is the same Wiener process as in the above equation (see Appendix 3 for proof). Thus G also follows an Ito process. Hull (1993) shows that it has a drift rate of :

$$\frac{dG}{dx}.a + \frac{dG}{dt} + \frac{1}{2}.\frac{d^2G}{dx^2}.b^2$$

and a variance rate of :

$$(\frac{dG}{dx})^2.b^2$$

Using the stochastic model developed for stock price behaviour for Type I and II outcomes from Chapter 3 for a continuous sample path (equation {8}) and assuming a constant drift and variance rate, Ito's lemma shows that a function f, of S and t is :

$$df = (\frac{df}{dS}.\mu.S + \frac{df}{dt} + \frac{1}{2}.\frac{d^2f}{dS^2}.\sigma^2.S^2).dt + \frac{df}{dS}.\sigma.S.dz \qquad ...\{13\}$$

where, as previously stated, in terms of Ito's lemma, the dz terms are the same. Black and Scholes (1973) posited that a portfolio be set up, consisting of the stock and call option, which will be riskless. Since the call price is a function of the stock price and the remaining time to maturity, the changes in the call price can be expressed as a function of the changes in the stock price and the remaining time to maturity. Thus choose a portfolio of the stock and derivative security that will eliminate the Wiener process, which is the sole source of uncertainty. Black and Scholes (1973) showed that the appropriate portfolio to eliminate the Wiener process is :

-1 : derivative security

$+\frac{df}{dS}$: shares

The value of this portfolio is defined as :

$$\Pi = -f + \frac{df}{dS}.S$$

and the change in the value of the portfolio is thus :

$$\Delta\Pi = -\Delta f + \frac{df}{dS}.\Delta S \qquad ...\{14\}$$

Substituting from the stock price behaviour model and {13}:

$$\Delta\Pi = -[(\frac{df}{dS}.\mu.S + \frac{df}{dt} + \frac{1}{2}.\frac{d^2f}{dS^2}.\sigma^2.S^2).\Delta t + \frac{df}{dS}.\sigma.S.\Delta z] + \frac{df}{dS}.[\mu.S.dt + \sigma.S.dz]$$

$$= -\frac{df}{dS}.\mu.S.dt - \frac{df}{dt}.dt - \frac{1}{2}.\frac{d^2f}{dS^2}.\sigma^2.S^2.dt - \frac{df}{dS}.\sigma.S.\Delta z + \frac{df}{dS}.\mu.S.dt + \frac{df}{dS}.\sigma.S.dz$$

$$= (-\frac{df}{dt} - \frac{1}{2}.\frac{d^2f}{dS^2}.\sigma^2.S^2).dt \qquad ...\{15\}$$

Thus the change in the value of the portfolio has no stochastic element dz and if the terminal value is known with certainty it is, by definition, riskless. It is this most remarkable insight that the portfolio can be made into a riskless hedge at any point in time by choosing an appropriate mixture of the stock and calls such that any increase (decrease) in the portfolio due to a profit (loss) on the stock is set off against a decrease (increase) in the portfolio due to a loss (profit) on the call that results in the Black and Scholes (1973) formulation. It necessarily follows from the capital asset pricing model (see assumption 7) that any riskfree investment must earn the riskfree rate to prevent arbitrage. Thus the return on the portfolio must be :

$$\Delta\Pi = r.\Pi.\Delta t$$

Substituting for the two terms gives :

$$(-\frac{df}{dt} - \frac{1}{2}.\frac{d^2 f}{dS^2}.\sigma^2.S^2).dt = r.(f - \frac{df}{ds}.S).dt$$

$$\therefore -\frac{df}{dt}.dt - \frac{1}{2}.\frac{d^2 f}{dS^2}.\sigma^2.S^2.dt = r.f.dt - r.\frac{df}{ds}.S.dt$$

$$\therefore r.f.dt = \frac{df}{dt}.dt + \frac{1}{2}.\frac{d^2 f}{dS^2}.\sigma^2.S^2.dt + r.\frac{df}{ds}.S.dt$$

$$\therefore r.f = \frac{df}{dt} + r.S.\frac{df}{ds} + \frac{1}{2}.\sigma^2.S^2.\frac{d^2 f}{dS^2}$$

which is the Black Scholes (1973) formula. This formula has many solutions depending on the boundary conditions used for the value of the underlying variable. For a European call option the terminal boundary condition used is :

$$f = \max(S - X, 0) \text{ when } t = T$$

The solution under this boundary condition was shown by Black and Scholes (1973) to be the solution to the heat transfer equation in physics, which was first solved by Churchill (1963) and will be outlined next.

4.5 SOLUTION TO THE BLACK SCHOLES PARTIAL DIFFERENTIAL EQUATION

The heat or diffusion equation models the diffusion of heat in one space dimension where *u(x,t)* represents the temperature in a long, thin, uniform bar of material whose sides are perfectly insulated so that its temperature varies only with distance x along the bar and with time t:

$$\frac{du}{dt} = \frac{d^2u}{dx^2} \quad \text{for } -\infty < x < \infty$$

with

$$u(x,0) = u_0(x)$$

where

1. $u_0(x)$ is sufficiently well behaved
2. $\lim_{|x|\to\infty} u_0(x).e^{-a.x^2} = 0$ for any $a > 0$
3. $\lim_{|x|\to\infty} u_0(x,t).e^{-a.x^2} = 0$ for any $a > 0, t > 0$

This is termed the initial value problem (Wilmot et al, 1997). The solution to the diffusion equation is:

$$u_\delta(x,t) = \frac{1}{2.\sqrt{\pi.t}}.e^{-x^2/(4.t)}$$

This models the evolution of a unit amount of heat initially concentrated into a single point and is called the fundamental solution of the diffusion equation. The explicit solution of the initial value problem is :

$$u(x,t) = \frac{1}{2\sqrt{\pi.t}} \int_{-\infty}^{\infty} u_0(s).e^{-(x-s)^2/(4.t)}ds$$

with initial data :

$$u(x,0) = \frac{1}{2\sqrt{\pi.t}} \int_{-\infty}^{\infty} u_0(s).\delta(s-x)ds = u_0(x)$$

where δ $(\cdot)$ is the Dirac delta function. This solution is then used to solve the Black Scholes partial differential equation. This equation can be transformed by setting :

$$S = E.e^x, \; t = T - t/(\frac{1}{2}\sigma^2), \; C = E.v(x,t)$$

into the equation :

$$\frac{dv}{dt} = \frac{d^2v}{dx^2} + (k-1).\frac{dv}{dx} - k.v$$

with $k = r / (\frac{1}{2}\sigma^2)$. This changes the initial condition to :

$$v(x,0) = \max(e^x - 1, 0)$$

This equation is then transformed into the heat diffusion equation by setting :

$$v = e^{\alpha.x + \beta.t}.u(x,t)$$

Setting $\alpha = -\frac{1}{2}(k-1),\ \beta = -\frac{1}{4}(k+1)^2$ gives the equation :

$$v = e^{-\frac{1}{2}(k-1).x - \frac{1}{4}(k+1)^2.t}.u(x,t)$$

where :

$$\frac{du}{dt} = \frac{d^2u}{dx^2}$$

with :

$$u(x,0) = u_0(x) = \max(e^{\frac{1}{2}(k+1).x} - e^{\frac{1}{2}(k-1).x}, 0)$$

The solution is thus as for the diffusion equation with the different initial condition given above. Evaluating the integral requires the change of variable, $x' = (s - x) / \sqrt{2.t}$ so that :

$$u(x,t) = \frac{1}{\sqrt{2.\pi}} \int_{-\infty}^{\infty} u_0(x'\sqrt{2.t} + x).e^{-\frac{1}{2}x'^2} dx'$$

$$= \frac{1}{\sqrt{2.\pi}} \int_{-x/\sqrt{2.t}}^{\infty} e^{\frac{1}{2}(k+1)(x + x'\sqrt{2.t})}.e^{-\frac{1}{2}x'^2} dx' - \frac{1}{\sqrt{2.\pi}} \int_{-x/\sqrt{2.t}}^{\infty} e^{\frac{1}{2}(k-1)(x + x'\sqrt{2.t})}.e^{-\frac{1}{2}x'^2} dx'$$

Evaluating the integrals by completing the square in the exponent to get the standard integral results in :

$$u(x,t) = e^{\frac{1}{2}(k+1)x + \frac{1}{4}(k+1)^2 t}.N(d_1) - e^{\frac{1}{2}(k-1)x + \frac{1}{4}(k-1)^2 t}.N(d_2)$$

where :

$$d_1 = \frac{x}{\sqrt{2.t}} + \frac{1}{2}(k+1)\sqrt{2.t}$$

$$d_2 = \frac{x}{\sqrt{2.t}} + \frac{1}{2}(k-1)\sqrt{2.t}$$

Substituting $x = \log(S/E), t = \frac{1}{2}\sigma^2(T-t), C = E.v(x,t)$ gives the solution to the Black Scholes partial differential equation :

$$C = S.N(d_1) - X.e^{-r(T-t)}.N(d_2)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)(T-t)}{\sigma.\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/X) + (r - \sigma^2/2)(T-t)}{\sigma.\sqrt{T-t}}$$

The price of the call option is thus dependent on five variables :

1. the stock price,
2. the variance rate on the stock price,
3. the exercise price of the option,
4. the time to maturity of the option,
5. the risk-free interest rate.

Note that the option price does not depend on the expected return on the common stock (thus the term risk-neutral valuation), risk preferences of investors or the aggregate supply of assets. The only argument of the solution that is not directly observable is the variance rate of the stock but this can be approximated using a sequence of past prices (Smith, 1976).

4.6 THE BINOMIAL LATTICE OPTION PRICING MODEL

The models in the Black-Scholes and Merton articles are mathematically complex so William Sharpe devised a simpler method of arriving at the same result. Option pricing models may be simplified into two distinct groupings depending on their assumption of the underlying asset price movement :

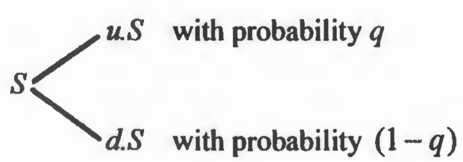
- continuous models, and
- discrete models

The most popular discrete model is the binomial model (Zhang, 1997). The binomial model was originally developed by William Sharpe to price standard options. It was then extended to the trinomial tree model in which the underlying asset's price is assumed to follow three different paths in each following period.

Since the late 1970's, the lattice and tree based method have been widely used in pricing essentially all kinds of derivative products, especially path dependent and other complicated products such as interest rate derivatives (Black and Derman and Toy, 1991).

4.7 THE BINOMIAL OPTION PRICING FORMULA

The stock is assumed to either move up or down as follows :



The interest rate, denoted *r*, is assumed constant and positive with $d < r < u$. Let C denote the value of the call :

$$C \begin{cases} C_u = \max[0, u.S - K] & \text{with probability } q \\ C_d = \max[0, d.S - K] & \text{with probability } (1-q) \end{cases}$$

Form a portfolio of Δ shares of the stock and B in riskless bonds. This will cost $S\Delta + B$ and at the end of the period be worth :

$$C \begin{cases} u.S.\Delta + r.B & \text{with probability } q \\ d.S.\Delta + r.B & \text{with probability } (1-q) \end{cases}$$

Choose Δ, B in such a way as to replicate the end of period values for the call i.e.

$$u.S.\Delta + r.B = C_u$$

$$d.S.\Delta + r.B = C_d$$

The solution to these simultaneous equations is :

$$\Delta = \frac{C_u - C_d}{(u-d).S}$$

$$B = \frac{u.C_d - d.C_u}{(u-d).r}$$

There is thus an equivalent portfolio to the call option. If there are to be no riskless arbitrage opportunities then the current value of the call cannot be less than the current value of the equivalent portfolio. Thus :

$$\begin{aligned} C &= S.\Delta + B \\ &= S.\frac{C_u - C_d}{(u-d).S} + \frac{u.C_d - d.C_u}{(u-d).r} \\ &= \frac{C_u - C_d}{(u-d)} + \frac{u.C_d - d.C_u}{(u-d).r} \\ &= \left[\left(\frac{r-d}{u-d}\right).C_u + \left(\frac{u-r}{u-d}\right).C_d\right] / r \end{aligned}$$

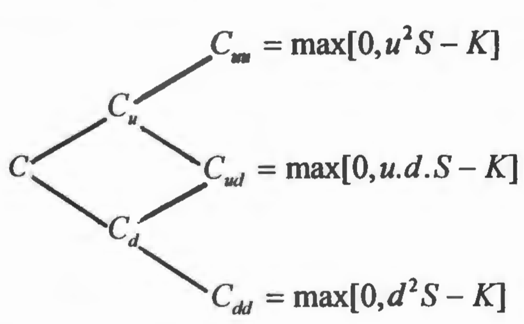
This can be simplified by defining $p \equiv (r-d)/(u-d)$ then $(1-p) \equiv (u-r)/(u-d)$ and then the call price can be written as :

$$C = [p.C_u + (1-p).C_d]/r$$

There are four critical points that arise from this formula :

- the probability q does not appear - thus even if different investors have different subjective probabilities of upward and downward movements in the stock, they could still agree on the value of the call option;
- the call value does not depend on investors attitudes toward risk - the only assumption made in the formula is that investors prefer more wealth to less - the same formula would be obtained if investors were risk averse or risk lovers (this confirms the technique of arbitrage valuation and the possibility of not having to take account of investor risk preferences as required in the more complex models based on stochastic volatility);
- the only random variable on which the call depends is the stock price itself;
- the $p \equiv (r-d)/(u-d)$ is always greater than zero and less than unity so that is has the properties of a probability, hence the call can be valued as the present value of its expectation in a risk neutral world.

Extended to a three period binomial model the structure is such :



so that :

$$C_u = [p.C_{uu} + (1-p).C_{ud}]/r$$

$$C_d = [p.C_{du} + (1-p).C_{dd}]/r$$

and again :

$$C = [p.C_u + (1-p).C_d]/r$$

which can be re-expressed as :

$$C = [p^2.C_{uu} + 2.p.(1-p).C_{ud} + (1-p)^2.C_{dd}]/r^2$$
$$= (p^2.\max[0,u^2.S-K] + 2.p.(1-p).\max[0,u.d.S-K] + (1-p)^2.\max[0,d^2.S-K])/r^2$$

There is thus a recursive procedure to solve for the value of the call with any number of periods to go, starting at the expiration date and calculating expected values under the measure p and discounting back, because the derivative process is a p martingale, so that :

$$C = \left\{\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right).p^j.(1-p)^{n-j}.\max[0,u^j.d^{n-j}.S-K]\right\}/r^n$$

This can be simplified by letting a be the minimum number of upward moves the stock must make over the next n periods for the call to finish in the money. Thus it will be the smallest non-negative integer satisfying :

$$u^a.d^{n-a}.S > K$$

The values of $j < a$ can be ignored as they sum to zero and the summation is hence made over all $j \geq a$ so that :

$$C = \left\{\sum_{j=a}^{n}\left(\frac{n!}{j!(n-j)!}\right).p^j.(1-p)^{n-j}.[u^j.d^{n-j}.S-K]\right\}/r^n$$

By breaking the above into two terms the call price can be re-expressed :

$$C = S.\left\{\sum_{j=a}^{n}\left(\frac{n!}{j!(n-j)!}\right).p^j.(1-p)^{n-j}.(\frac{u^j.d^{n-j}}{r^n})\right\} - \frac{K}{r^n}\left\{\sum_{j=a}^{n}\left(\frac{n!}{j!(n-j)!}\right).p^j.(1-p)^{n-j}\right\}$$

Letting $p' = (u/r).p$ and $(1-p') = (d/r)(1-p)$ the two expression can be interpreted as the complementary binomial distribution function as follows :

$$C = S.\Phi[a;n,p'] - \frac{K}{r^n}.\Phi[a;n,p]$$
$$\begin{aligned} &where \\ &p \equiv (r-d)/(u-d) \\ &p' \equiv (u/r).p \\ &a \equiv \text{the smallest non-negative integer greater than } \ln(K/S.d^n)/\ln(u/d) \end{aligned}$$

Because it is unrealistic that the stock price only take on one of two values over say a day or week of trading, the formula is adapted to the limit where the trading interval becomes infinitesimally small. This formula converges to the Black Scholes formula in continuous time and thus :

$$\Phi[a;n,p'] \rightarrow N(x) \quad and \quad \Phi[a;n,p] \rightarrow N(x - \sigma.\sqrt{t})$$

so that :

$$C = S.N(x) - \frac{K}{r^t}.N(x - \sigma.\sqrt{t})$$
$$where\ x = \frac{\log(S/K.r^{-t})}{\sigma.\sqrt{t}} + 0.5\sigma.\sqrt{t}$$

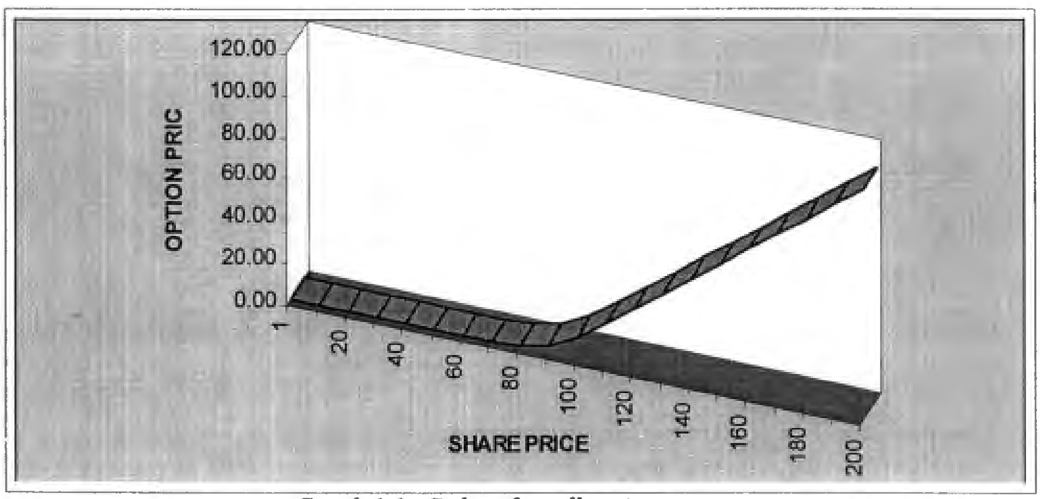
This proof is included in Appendix 7 and shows that the binomial model indeed leads to the same solution as the Black Scholes model.

4.8 A COMPARATIVE STATICS ANALYSIS OF THE BLACK SCHOLES PARAMETERS

As shown above, the Black Scholes model is a function of only five variables : the stock price, the exercise price, the time to maturity, the risk free interest rate and the instantaneous variance rate on the stock price. The influence that each of these has on the option price will be examined next. The analysis will only examine the impact of a change in the parameters on call options as this research is based on rights issues which can be likened to short term call options (as opposed to put options).

4.8.1 VARIATIONS IN STOCK PRICE (DELTA Δ)

The delta measures how fast the option's price changes with respect to the price of the underlying asset. The mathematical interpretation is that it is the first order partial derivative of the option price with respect to the underlying asset price, while economists interpret it as the sensitivity of the option price toward the price of the underlying asset.



Graph 1.1 : Delta of a call option

With a log-normal distribution of stock prices, the expected terminal price is a positive function of the current price and thus an increase in the stock price increases the expected payoff to the option. The formula for the delta of a call option is :

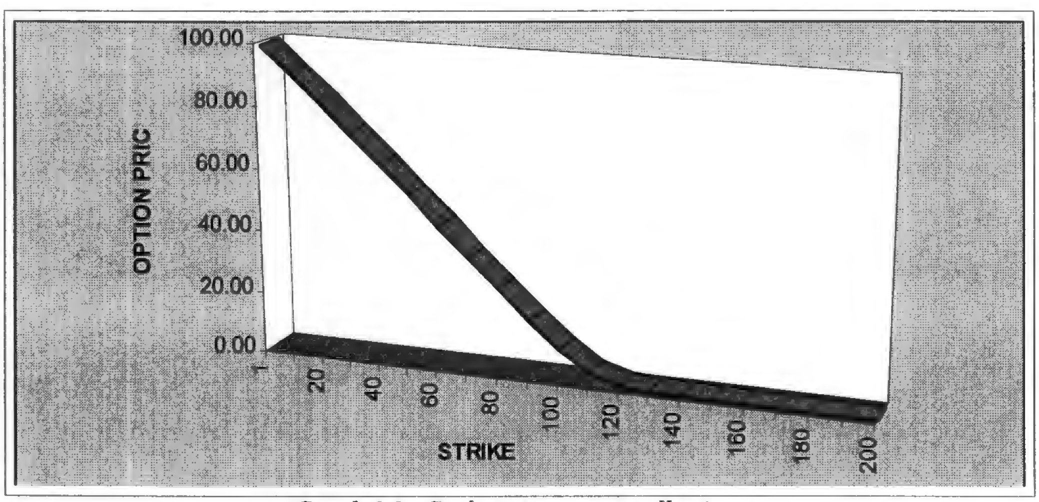
$$\frac{dC}{dS} = N\left(\frac{\ln(S/X) + [r + (\sigma^2/2)].T}{\sigma\sqrt{T}}\right) > 0$$

Deep out of the money options have deltas close to zero, implying that these option prices change little with the underlying asset prices. Deep in the money options have deltas close to one, implying that these option prices change about the same amount as the underlying asset prices.

4.8.2 VARIATIONS IN EXERCISE PRICE

As the exercise price rises the call price falls. This simply reflects the reduced probability of the call ending up in the money at maturity. The formula relating the exercise price to the call price is :

$$\frac{dC}{dX} = -e^{-r.T}.N\left(\frac{\ln(S/X)+[r-(\sigma^2/2)].T}{\sigma\sqrt{T}}\right) < 0$$

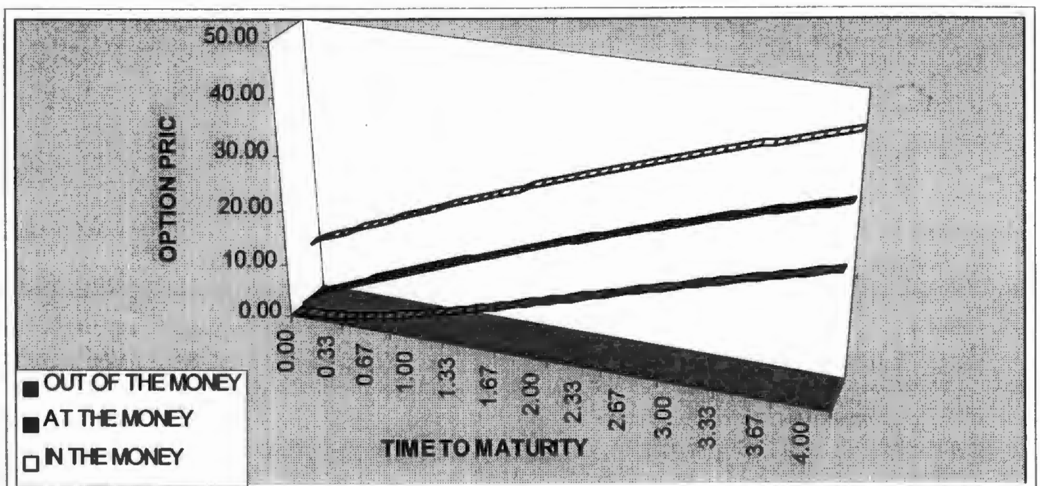


Graph 1.2 : Strike price verses call price

The concept of the volatility smile which occurs when one graphs the implied standard deviation against the option price for out of, at and in the money calls is related to the above graph. This phenomenon is re-addressed in Chapter 5 on the topic of volatility.

4.8.3 VARIATIONS IN TIME TO EXPIRATION (THETA Θ)

An option's theta is also referred to as the time decay of the option. Theta measures the sensitivity of the option price with respect to the time to maturity. An option's value is made up of its intrinsic value as well as its time value. As the time to expiration increases the call price increases due to the fact that the present value of the exercise price is lower the longer the time to maturity and there is always more possibility for the price of the underlying asset to change whenever there is more time to expiration.



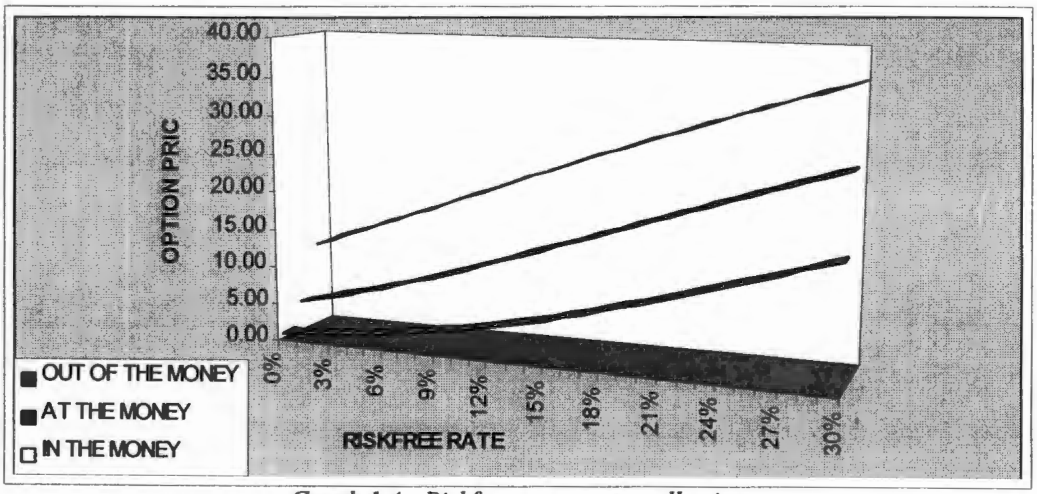
Graph 1.3 : Time to maturity verses call price

Thus an option always has positive theta. The formula for the theta of an option is :

$$\frac{dC}{dT} = X.e^{-r.T}.\left[\frac{\sigma}{2\sqrt{T}}.N\left(\frac{\ln(S/X)+[r-(\sigma^2/2)].T}{\sigma\sqrt{T}}\right)\right]$$
$$+r.N\left(\frac{\ln(S/X)+[r-(\sigma^2/2)].T}{\sigma\sqrt{T}}\right) > 0$$

4.8.4 VARIATIONS IN THE RISKFREE RATE (RHO *rho*)

An option's rho measures the sensitivity of the option's value with respect to the fluctuation of the riskfree rate. The level of the riskfree rate represents the opportunity cost of holding options (Zhang, 1997). Thus the higher the interest rate, the higher the opportunity cost of holding the call option and hence the higher the price of the call option.



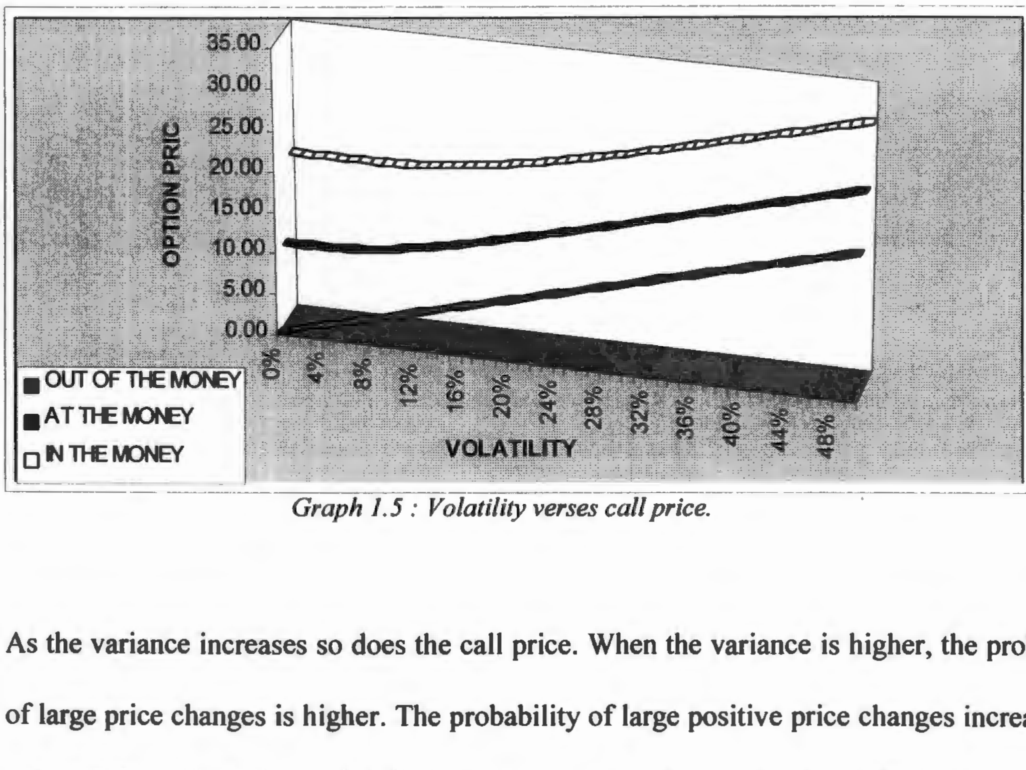
Graph 1.4 : Riskfree rate verses call price

As the riskless rate of interest rises the call price rises and this follows from the explanation for the time to expiration due to the fact that the present value of the exercise price falls when the riskless rate rises. The formula for the rho of an option is :

$$\frac{dC}{dr} = T.X.e^{-r.T}.N\left(\frac{\ln(S/X)+[r-(\sigma^2/2)].T}{\sigma\sqrt{T}}\right) > 0$$

4.8.5 VARIATION IN THE VARIANCE RATE(VEGA Λ)

Vega measures the sensitivity of the option price with respect to its volatility. It is thus the first order partial derivative of the option price with respect to the volatility of the underlying asset. Zhang (1997) likened volatility to options as wind is to kites. Kites cannot fly without wind, and they tend to crash if there is too much wind. Similarly, options would not exist without volatility and they cannot trade smoothly if there is too much volatility. If there is not enough noise in the market then prices of the underlying assets will remain relatively stable and there is little need for options on such assets.



Graph 1.5 : Volatility verses call price.

As the variance increases so does the call price. When the variance is higher, the probability of large price changes is higher. The probability of large positive price changes increases the value of the call but the probability of negative price changes has less effect as the terminal option price is bounded by zero. The formula for the vega of an option is :

$$\frac{dC}{d\sigma^2} = X.e^{-r.T}.N'\left(\frac{\ln(S/X) + [r-(\sigma^2/2)].T}{\sigma\sqrt{T}}\right).\frac{\sqrt{T}}{2.\sigma} > 0$$

CHAPTER 5 - SHARE PRICE VOLATILITY

5.1 INTRODUCTION TO VOLATILITY

The significant determinant and sole uncertain parameter in determining option value in the traditional Black Scholes (1973) framework is the volatility of the underlying instrument. The reliability and accuracy in estimating volatility are thus key to success in options trading, portfolio management and speculation. This was noted by Bookstaber (1981) : "*The investor who can overcome the difficulties in estimating the future volatility of the underlying security and get the best estimate will also get the greatest profits*" and later emphasized by Christie (1982). Since a rights issue is a form of call option, the correct estimation of volatility is the key factor in correct pricing of the rights via some option pricing model. A detailed investigation of volatility processes is thus required.

This chapter will outline the various volatility processes that are proposed to exist and introduce the new option pricing models that compensate for these differing volatility structures. The methods that have been put forward in the literature for the calculation of volatility will be examined. The chapter will conclude with the impact of non-stationary volatility on option pricing.

5.2 RELATIONSHIP OF VOLATILITY TO SHARE PRICE

Various theories have been put forward regarding the structure of volatility. As previously mentioned, the volatility of the underlying asset price is one of the primary determinants of option prices and hedge ratios. An option pricing model that does not properly capture the evolution of the volatility process can thus give rise to option prices that do not accurately estimate the prices observed in the market and consequently deteriorate an investor's ability to hedge risk.

In trying to address these biases, researchers have attempted to relax the traditional Black Scholes (1973) assumption of a constant variance of the underlying stock price process. There does not appear to be any consensus on how to accurately model the underlying volatility process however. If volatility is not deterministic it could be modeled as geometric Brownian motion (similar to the stock price process) :

$$dV = \alpha.Vdt + \sigma.VdW$$

or as an Ohrnstein-Uhlenbeck mean reverting process (similar to interest rates) :

$$dV = k(\mu - V)dt + \sigma.V^{Y}dW$$

Various approaches have been used but the two most popular are that volatility either evolves as a separate process in itself or, alternatively is driven by the same factors that drive the asset returns (Nandi, 1998).

If volatility does evolve as a separate process it is assumed that the innovations governing the evolution of the volatility process are different from the innovations governing the asset returns process. The two may, however, be correlated to some degree. The models that cover the above volatility process are termed stochastic volatility models. These models have been researched by Johnson and Shanno (1987), Hull and White (1987) and Wiggins (1987) amongst others. Unfortunately the models developed for the stochastic volatility process generally require the numerical solution of a two dimensional parabolic partial differential equation. The computation required is thus rather demanding and inefficient and, furthermore, some of the models require unrealistic assumptions regarding zero correlation between the asset return and the volatility process. This zero correlation phenomenon will later be shown to not accord with empirically observed volatility behaviour but for now can be regarded as being in direct conflict with the volatility smile observed in the market-place which stems from a negative skewness in the risk neutral distribution of asset returns. Nandi (1998) confirmed that this negative skewness, in the absence of a jump diffusion process in the underlying asset, can only be achieved through a negative correlation between returns and volatility.

If the innovations driving the volatility process and the asset returns process are the same, then the volatility process can be modeled via a generalized autoregressive conditional heteroskedacity process (GARCH). These models have been studied by Amin and Ng (1993), Engle and Mustafa (1992) amongst others.

An alternative, however, is the deterministic model that relates the general observation that the instantaneous stock volatility and stock price are negatively correlated. This observation was tested by Christie (1982), Beckers (1980), Black (1976), Macbeth and Merville (1980), Schmalensee and Trippi (1978) and Thorpe (1976). The claim of a negative relationship can be traced as far back as Burton Crane's work in 1964. In the most extensive study of the price-variance relationship, Christie (1982) tested the market folklore contention that there is a variance/stock price relation - the usual claim being that of a negative one so that when the stock price increases the variance rate declines. Christie (1982) explained this contention via the assumption of a Modigliani Miller world with a constant interest rate, no dividends and a single class of riskless debt and constant firm volatility. It can then be shown that :

$$\sigma_{S,t} = \sigma_V (1 + LR_t)$$

where σ denotes standard deviation of rate of return, $LR = D / S(V)$ is the market financial leverage ratio, t denotes time and V, S, D represent the market values of the firm, equity and debt respectively. In this scenario the volatility of equity, σ_S, is a positive increasing function of financial leverage. The elasticity, θ_S, of the equity volatility with respect to the stock price can be calculated as :

$$\theta_S = (\frac{d\sigma_S}{\sigma_S}) / (\frac{dS}{S}) = -[LR / (LR + 1)] \qquad -1 \leq \theta_S \leq 0$$

It can thus be seen that the elasticity of equity volatility with respect to the stock price in this simple model is negative and there is thus a negative relation between volatility and the value of equity. To test this contention empirically, Christie (1982) ran the following regression for a sample of over 379 firms :

$$\ln(\hat{\sigma}_{S,t} / \hat{\sigma}_{S,t-1}) = \beta_0 + \theta_S.[\ln(S_t / S_{t-1})] + u_t$$

The cross-sectional mean elasticity, θ_S, was found to be -0.23 but varied slightly across firms. This thus gives some credence to the market folklore. Christie (1982) then attempted to strengthen this finding using a maximum likelihood estimator (hereafter MLE) of θ_S that is not dependent on any regression equation, to test the contention that one source of variation in the volatility of equity is changes in financial leverage. Models tested included the consol model which is part of the class of risky debt models that treat the common stock of levered firms as an option on the value of the firm, that can be exercised by the bondholders :

$$\sigma_{S,t} = \beta_0 + \beta_0.QR_t + w_t$$

where QR is a surrogate for the leverage ratio calculated as the face value of debt divided by the market value of equity (thus leading to slightly biased results as the face value of debt will necessarily differ to the current market value). The behaviour of the consol model was found to confirm the previous findings of a negative relation between stock price level and volatility (note that it does differ somewhat from the previous model in that the elasticity does not decline monotonically but this is not relevant to this study as it still displays the negative relationship and this study is not concerned with the detailed formulation of this relationship).

Christie (1982) summarised his findings as being that equity volatility is an increasing function of financial leverage, but increasing at a decreasing rate, and the relationship between σ_S and financial leverage is sufficient to induce a negative elasticity between σ_S and the value of equity. Using an MLE, the mean elasticity was found to be -0.20 while the regression resulted in a value of -0.24. Christie (1982) concluded that : "*... the postulated relationship exists and is in substantial part attributable to financial leverage ... volatility is an increasing function of financial leverage and this relation can cause the elasticity of volatility with respect to value of equity to be negative under a broad range of circumstances*." This was confirmed more recently by Hull and White (1996).

Beckers (1980) attempted to justify the inverse relationship by arguing that a simple economic mechanism might cause an inverse relationship between the stock price and the variance of its return. He argued that if a firm's stock price falls, the market value of its equity tends to fall more rapidly than the market value of its debt, causing the debt equity ratio to rise and hence the riskiness of the stock to increase. If a firm has no debt there will be a similar effect due to the fixed costs which must be met irrespective of income levels - a decrease in income will decrease the value of the firm and simultaneously increase its riskiness. Thus both operating and financial leverage arguments can be used to explain the inverse relationship between variance and stock price observed in empirical examinations. Furthermore, Black (1976) noted that the cause and effect may be inverted, so that an economic downturn might lead to an increase in stock price volatility and thus declining stock prices.

While Christie's (1982) results show that θ_S is usually a negative random variable that is a function of financial leverage, it is convenient for both analytical and practical reasons to consider models in which θ_S is assumed to be constant. The constant elasticity of variance diffusion processes (Cox and Ross, 1976) embodies this statistical representation :

$$dS = \mu.Sdt + \lambda.S^{\theta+1}dZ$$

or in terms of rates of return :

$$dS / S = \mu dt + \lambda.S^{\theta}dZ$$

which thus implies that the rate of return on the equity over some small time interval *dt* is normally distributed with mean $\mu.dt$ and volatility $\lambda.S^{\theta}\sqrt{dt}$. The value of the characteristic parameter can be estimated by least squares techniques or alternatively via maximum likelihood estimators. A regression of the form :

$$\ln(\hat{\sigma}_{S,t}) = \ln\lambda + \theta.\ln S_t + u_t$$

can be run over the data sample (based on the fact that $\sigma_S = \lambda.S^{\theta}$) where *u* is distributed as the square root of a Gamma variable adjusted to have zero mean. This procedure may, however, lead to inaccurate results as the usual *t*-test statistics rely on the error term being normally distributed and it has been shown to have a non-Guassian distribution. The alternative maximum likelihood estimator for a sample of size *T* is :

$$L = -T/2\ln(2\pi) - T/2\ln(\lambda^2) - \theta.\sum \ln S - \frac{1}{2}\lambda^{-2}\sum r_S^2.S^{-2.\theta}$$

Empirical testing using the above formulas revealed that the characteristic parameter of the constant elasticity of variance process is in the region of -0.20, with the MLE estimator and regression estimator generally yielding similar results (Beckers, 1980; Christie, 1982; Randolph, 1991).

The above findings of a negative relation between asset returns and volatility thus appear to be well founded in the literature and cannot be ignored when formulating a model to price options in the market-place due to the significant impact that the variance rate has on the price of the option through its vega (see Chapter 4).

5.3 THE VOLATILITY SMILE

This negative price-variance phenomenon is related to the volatility smile observed in market prices of traded options. The way in which implied volatility of a stock varies with strike price for options of a fixed expiration is termed the volatility smile. The volatility smile relates to the real-world measure of the deviation of market option prices from Black-Scholes theory (Chriss, 1997) and is one of the two components of an equity market's particular implied volatility structure, the other being the term structure of volatility. Examining graphs of the volatility smile of the S&P 500 index it is evident that for out of the money calls, as strike price increases, implied volatility decreases and the reverse is true for in the money calls (Chriss, 1997). Cox (1996) noted that while this is a complex phenomenon, most would agree that the negative correlation between stock price changes and volatility changes is a primary ingredient.

The volatility smile has also been researched in a number of recent studies by Derman and Kani (1994), Dupire (1994) and Rubinstein (1994). These studies generally relied on the inverse relationship between stock price changes and volatility changes thus lending further credence to the above hypothesis.

5.4 ESTIMATING STOCK VOLATILITY

To estimate the volatility parameter, it is most often the case that daily prices or index data from a certain number of historical days are used (Zhang, 1997). The daily prices (called level prices) are then converted into daily gross returns and the logarithm of these returns calculated. This thus results in a daily standard deviation which can be annualized. There are however, various other ways to estimate the volatility parameter depending on the particular market circumstances. The various estimation procedures most commonly used in empirical research will be outlined here in order to obtain an understanding of the possible biases caused by using the traditional method as well as obtaining insight into better approximation methods that could be used.

Suppose a point particle undergoes a one-dimensional continuous random walk with a diffusion constant *D*. Parkinson (1980) showed that the probability of finding the particle in the interval *(x,x+dx)* at time *t*, if it started at point x_0 at time *t=0* is :

$$\frac{dx}{\sqrt{2.\pi.D.t}}.e^{-\frac{(x-x_0)^2}{2.D.t}}$$

and by comparison with the normal distribution one can see that *D* is the variance of the displacement $x-x_0$ after a unit time interval. This suggests the naive estimator for *D* to be :

$$D_x = \frac{1}{n-1}\sum_{i=1}^{n}(d_i-\bar{d})^2$$

with

$$\bar{d} = \frac{1}{n}\sum_{m=1}^{n} d_m$$

$$d_i = x(i)-x(i-1) \qquad i=1,\ldots,n$$

and this is termed the *traditional* method.

Instead of using the closing prices the measure D_l could represent the difference between the high and low prices during each time interval. This variance formulation, termed the *extreme value* method, is expressed as :

$$D_l = \frac{0.361}{n}\sum_{i=1}^{n} l_i^2$$

Parkinson (1980) calculated the efficiency of these ratios as follows :

$$E[(D_x - D)^2] = \left[\frac{E(x^4)}{E(x^2)} - 1\right].\frac{D^2}{N_x} = \frac{2.D^2}{N_x}$$

$$E[(D_l - D)^2] = \left[\frac{E(l^4)}{E(l^2)} - 1\right].\frac{D^2}{N_l} = \frac{0.41.D^2}{N_l}$$

where N is the number of observations. It is thus evident that to obtain the same variance using the two methods requires $N_x \approx 5.N_l$ thus showing that the extreme value method is far superior to the traditional method. The importance of high efficiency is that estimates of improved confidence may be constructed from the available data and this has the corollary that researchers can adopt the tactic of purposely restricting data usage to combat unforeseen nonstationarities (the most recent data set presumably has more predictive content in the presence of unknown nonstationarities). Additionally, by including opening and closing prices in the formula the efficiency ratio increases from 5.2 to 6.2. This shows the benefit of using the opening and closing prices in addition to the daily high and low prices in a volatility estimator. The basis for using high and low prices is that they require continuous monitoring in order to establish their values whereas the opening and closing prices are mere snapshots of the process. Based on its superior efficiency ratio, the extreme value method should be preferred to the traditional method in calculating the volatility for a stock price series.

Garman and Klass (1980) assumed that security prices can be modeled by a diffusion process of the form :

$$P(t) = \phi(B(t))$$

where P is the security price, t is time, ϕ is a monotonic time-independent transformation and $B(t)$ is the diffusion process with the differential representation :

$$dB = \sigma.dZ$$

where dZ is the standard Gauss-Wiener process and σ is an unknown constant to be estimated. The model accounts for paths that are not everywhere observable (thus accounting for market closures and discrete transactions) by assuming that the continuous Brownian motion is followed during periods between transactions as well as periods during which the market is closed. They focused their paper on estimators of the form $D(u,d,c)$ where the u, d, c terms represent the daily high, low and stock price respectively. Note that the close of trade interval is set equal to zero so today's opening price is then equal to yesterday's closing price. Any minimum variance estimator should inherit the invariance properties of the joint density of (u,d,c). Two such invariance properties are as follows :

$$g(u,d,c;\sigma^2) = g(-d,-u,-c;\sigma^2)$$
$$g(u,d,c;\sigma^2) = g(u-c,d-c,-c;\sigma^2)$$

The first condition represents price symmetry and the second condition represents time symmetry. They thus sought decision rules which satisfied the above criteria. The scale invariance property is also required to hold and the scale invariance decision rule can be stated as :

$$g(\lambda.u,\lambda.d,\lambda.c) = \lambda.^2\, g(u,d,c) \qquad \lambda > 0$$

The resultant model developed is :

$$D(u,d,c) = a_{200}u^2 + a_{020}d^2 + a_{002}c^2 + a_{110}ud + a_{101}uc + a_{011}dc$$

Finding the minimum variance measure results in the following minimum variance estimator which is quadratic in u, d and c under the assumption of no drift :

$$\sigma_{GK}^2 = 0.511(u-d)^2 - 0.019[c.(u+d) - 2.u.d] - 0.383c^2$$

and the calculated efficiency ratio is in the region of 7.4 thus showing further improvement on the aforementioned estimators.

Rogers et al (1994) formulated a model for the case of non-zero drift in the underlying stock's diffusion process. The usual assumption that the price of an asset is generated by the process :

$$ds(t) = \alpha.s(t)dt + \sigma.s(t)dW(t)$$

was used. Their formula requires the following definitions : let *h(t)*, *l(t)* and *s(t)* be the high, low and closing prices from day *t* respectively. Divide these values by the opening price *o(t)* and define :

$$H(t) = \ln(h(t) / o(t))$$
$$S(t) = \ln(s(t) / o(t))$$
$$L(t) = \ln(l(t) / o(t))$$

thus making these estimates independent across days since they are functions of non-overlapping increments of Brownian motion. The naive estimator of the variance in a driftless world $(\alpha = 0)$ is then the same as the previously defined Parkinson estimator, while a mean adjusted unbiased alternative for the case of drift is :

$$\hat{\sigma}_1^2 = \frac{1}{(n-1)}\sum_{t=1}^{n}[S(t)]^2 - \frac{1}{n.(n-1)}[\ln(S(n)) - \ln(S(0))]^2$$

The Rogers et al (1994) (hereafter referred to as RS) estimator that is unbiased irrespective of the drift parameter is :

$$\sigma_{RS}^2 = H(H-S) + L(L-S)$$

This formula is a member of the unbiased quadratic class of Garman and Klass (hereafter referred to as GK) so $\hat{\sigma}^2_{GK}$ will outperform it in the case of zero drift. However as the drift increases $\hat{\sigma}^2_{GK}$ goes badly astray while $\hat{\sigma}^2_{RS}$ remains close to the true value (Rogers et al, 1994). However, even the above formulas (except for the naive estimators) are biased due to their use of the daily high and low values. This is due to the fact that the observed highs and lows from the market are less in absolute magnitude than the highs and lows of the idealized continuous process and that the time period over which it is estimated is shortened to the span of exchange trading hours excluding closing hours. These technical limitations are posited to produce a downward bias.

Testing by Rogers et al (1994) led to the conclusion that the adjusted estimators are always larger than their unadjusted counterparts - this indicates the substantial effect of transactions on volatility and in general they found that most estimators are significantly correlated with the number of transactions. Their conclusion is that if the drift is constant, the adjusted GK estimator appears to be preferred on the basis of both bias and efficiency. As the drift is allowed to vary, the naive estimators and GK perform badly and the RS estimator outperforms all the others.

In conclusion, it is well known that the variance of the aggregate stock returns changes over time and thus the presence of heteroskedacity prevents estimation of the variance over a long time period. If the volatility of stock returns is not constant then recent data will be preferred to predict future volatility. Empirical investigations might thus purposely restrict data usage to combat unforeseen non-stationarities and thus the procedures above will perform well as they give high efficiency without using too long a time series and seem able to cope with certain types of non-stationarity.

5.5 IMPLICATIONS OF NON-CONSTANT VOLATILITY ON OPTION PRICING

If volatility is a stochastic process, arbitrage based arguments are no longer sufficient to explain option prices and more complicated preference-based models are required (Brockman and Chodhury, 1997). This can be explained as follows. If stock prices follow a diffusion process while bond prices and stock return volatility follow a deterministic process then there is but one source of uncertainty in the option valuation model - a Wiener process (dW) which can be extinguished by maintaining a dynamic hedge of a specific number of stocks (long) against a specific number of calls (short) - the delta neutral hedging technique proposed by Black and Scholes (1973). This delta neutral hedging technique results in a partial differential equation with no random components :

$$\frac{dc}{dt} = -r.c - r.S.\frac{dc}{dS} - \frac{1}{2}.\frac{d^2c}{dS^2}\sigma^2 S^2$$

and the solution thereto is the Black Scholes option pricing model. If however, one introduces a stochastic volatility term into the process, so that volatility itself follows the process :

$$dV = u.Vdt + \xi.VdZ$$

a riskless hedge can no longer be formed and the Black Scholes formula breaks down due to there now being an additional Wiener process dZ. Deriving a new formula requires solving the new partial differential equation :

$$\frac{df}{dt} + \frac{1}{2}\left[\sigma^2 S^2 \frac{d^2 f}{dS^2} + 2.\rho.\sigma^3.\xi.S.\frac{d^2 f}{dS.dV} + \xi^2.V^2 \frac{d^2 f}{dV^2}\right] - r.f = -r.s.\frac{df}{dS} - \mu.\sigma^2.\frac{df}{dV}$$

Without making unrealistic assumptions about investor preferences or requiring enormous computational time, the solution to the above equation has proven elusive as the models are complex and do not lead to closed form solutions without extra assumptions (Brockman and Chodhury, 1997).

Brockman and Chodhury (1997) applied chaos techniques to an implied volatility series in order to distinguish a deterministic from a stochastic series. This is done by testing for randomness via plotting the data series in *n*-dimensional space and observing any resulting clustering. They found correlation dimensions of 0.9280, 0.9944, 1.0415, 1.0796, 1.1118 and 1.1339 for embedding dimensions of 5, 10, 15, 20, 25 and 30 respectively. This result implies that volatility is described by a low correlation dimension and thus contains a great deal of structure and the possibility of modeling the structure thus appears possible. Further testing using the Brock residual test rejected the hypothesis of deterministic chaos and thus by default accepted the notion of stochastic implied volatility. This would imply that the Black Scholes model and its deterministic offspring are misspecified with respect to volatility.

If a stock and its volatility are instantaneously uncorrelated then the price of a European option on the stock is equal to the Black Scholes price integrated over the probability distribution of the average variance rate (Hull and White, 1996). Provided that the underlying stock, S, follows a process of the form :

$$dS = \mu.S.dt + \sqrt{V}.S.dW$$

with the usual notation for the Wiener process and the drift rate, the call price can be expressed as :

$$c = \int C(\overline{V}).g(\overline{V})d\overline{V}$$

where $\overline{V}$ is the average variance rate during the lifetime of the derivative, g is the probability distribution of V in a risk neutral setting and $C(V)$ is the price of the derivative assuming that the variance rate is constant and equal to V. Hull and White (1996) noted that the shape of the function $C(V)$ depends on the terms of the derivative. If the option is at the money, the function is concave for all values of V. Thus a stochastic variance rate always has the effect of reducing the option price when the price of the underlying and its variance rate are uncorrelated. However, when the option is significantly in the money or out of the money, the function *C(V)* is predominantly convex and a stochastic variance rate tends to increase the price of the option. Hull and White (1987) produced a solution in series form for the case where the stock price is instantaneously uncorrelated with the volatility. Their option price is found to be lower than the Black Scholes price when the option is close to being at the money and higher when it is deep in or deep out of the money. They assumed that the stock price, S, and volatility, V, conform to the following stochastic processes :

$$dS = \phi.S.dt + \sigma.S.dW$$
$$dV = \mu.Vdt + \xi.VdZ$$

Their series solution is :

$$f(S\sigma^2) = C(\sigma^2) + \frac{1}{2}.\frac{S\sqrt{T-t}.N'(d_1)(d_1d_2-1)}{4\sigma^3}.\left[\frac{2\sigma^4(e^k-k-1)}{k^2}-\sigma^4\right]$$
$$+\frac{1}{6}.\frac{S\sqrt{T-t}.N'(d_1)[(d_1d_2-3)(d_1d_2-1)-(d_1^2+d_2^2)]}{8\sigma^5}$$
$$\times\sigma^6.\left[\frac{e^{3.k}-(9+18.k).e^k+(8+24.k+18.k^2+6.k^3}{3.k^3}\right]+\ldots$$

However, when the correlation between the price of the underlying stock and its variance rate is non-zero the above results do not hold.

If the Black Scholes model is indeed misspecified, then it is necessary to examine the magnitude of the errors that result when using a constant volatility assumption. If the resultant errors are insignificant, then traders would not have to concern themselves with the vastly more complex models that are required if one is to account for a stochastic volatility process. Boyle and Ananthanarayanan (1977) noted that finance theory indicates that the true value of the variance of the return on the underlying stock should be used to compute option prices. However, they observed that in practice what is usually done is to use an estimate of the variance based on a sample of historical stock prices. It is then clear that the sampling error associated with calculating this variance gives rise to error in the option price. The option pricing formula is non-linear in the variance, so an unbiased estimate of the variance will not give rise to an unbiased estimate of the option price. Methods must then be sought to reduce this estimation risk to obtain the most accurate option prices possible. The most obvious alternative is to increase the sample size. Using a longer series of historical returns however, may not be optimal as there is evidence to suggest that the variance is non-stationary (Praetz, 1972), so extending the observation period may in fact make matters worse. The alternative is to increase the frequency of data sampling. This leads to its own set of statistical problems as well as the fact that generally only daily data are available, thus limiting the possible sample size.

Boyle and Ananthanarayanan (1977) analyzed the nature and extent of the induced estimation risk. To explore the dimensions of the estimation risk, a sample of size n was used to obtain an unbiased estimate s^2 of σ^2. They then compared the distribution of option prices $W(s^2,\bullet)$ to the true option prices $W(\sigma^2,\bullet)$. They found that the sign of this bias cannot be unambiguously determined because, although W is an increasing function of s^2, it is neither a strictly convex nor a strictly concave function of s^2. As the sample of size n is increased the bias decreases as expected. The bias is found to be quite small even when n = 15 and as n becomes larger (near 50) the bias becomes negligible.

As a rough measure of stochastic volatility it was decided to calculate the volatility for a sample of the shares included in this research over a number of years. If there are substantial changes in volatility from year to year then the Black Scholes assumption of constant volatility is inappropriate. The results are displayed in the following table.

	E-R-P-M	GENCOR	ISCOR	LONRHO	RAINBOW	UNISPIN	USKO
1993	9.94%	6.86%	4.56%	4.30%	7.40%	16.67%	11.55%
1994	5.01%	4.14%	3.82%	4.02%	4.77%	9.05%	11.20%
1995	5.09%	2.26%	2.11%	2.46%	5.18%	8.06%	6.16%
1996	7.01%	3.85%	4.01%	3.25%	10.05%	8.49%	2.93%

Table 5.1 Annual volatility calculations

The results vary for the sample with some companies showing fairly stable annual variances and others having highly erratic variances, perhaps indicating that the Black Scholes assumption of constant variance is indeed misspecified. Further analysis of the effect of non-stationery variance in the South African market is thus required before a model for the variance of stock returns can be developed.

5.6 CONCLUSION

In conclusion it thus appears that there is ample empirical as well as theoretical evidence that stock prices and volatility levels are negatively correlated. It has been shown that the impact of this correlation on the Black Scholes model is significant and a model that is able to account for this relationship is thus sought. The stochastic volatility models introduced above generally do not lead to closed form solutions and are also reliant on additional assumptions which do not always reflect reality. Fisher Black, in 1976, noted from his own empirical work that there appeared to be a negative relationship between stock price and volatility as evidenced by the observed volatility smile (perhaps smirk would be more accurate). This prompted him to request John Cox to derive an option pricing model along the original Black

Scholes lines that was able to incorporate this negative correlation. This research resulted in the Cox Constant Elasticity of Variance Process which leads to a closed form solution that is testable under two simplifications. This model will be discussed in Chapter 6 and employed in this study to specifically account for the now commonly accepted negative relationship between stock price and underlying volatility.

CHAPTER 6 - CONSTANT ELASTICITY OF VARIANCE OPTION PRICING MODELS

6.1 INTRODUCTION

One of the basic assumptions of the Black-Scholes option pricing model is that the stochastic process for the stock and dividends depends at most on the stock price and the remaining time to maturity. This translates into the assumption that the underlying stock price process must have constant volatility. As shown in Chapter 5, empirical studies of common stock returns reveal that this assumption has little justification. The constant volatility assumption can, however, be relaxed in favor of a deterministic function of time process and the essence of the derivation will still remain valid (Merton, 1973). Thus if the stock price follows a diffusion process while bond prices and stock volatility follow a deterministic process, then there is but one source of uncertainty in the option valuation model and this can be hedged perfectly to equate to the differential equation with no random components.

The constant elasticity of variance model (CEV model) developed by John Cox (1975) incorporates the above regularities. It specifically allows the instantaneous volatility of the stock to be negatively correlated with the level of the stock price. Volatility is thus not assumed to be constant and is instead related to the underlying stock price itself. The volatility is, however, still a deterministic function and the difficulties relating to stochastic volatility processes can thus be ignored. Thus one of the basic assumptions of the Black

Scholes model, that of constant variance of the underlying stock, can be relaxed and an alternative closed form solution developed.

6.2 APPLICABILITY TO THE PRICING OF RIGHTS

Much empirical research has been performed on the reaction of share *prices* to the announcement of rights issues. A much less thoroughly researched area, however, is the impact of the rights issue announcement on the underlying share price *volatility*. If it can be shown that the volatility of the underlying share is significantly different in the pre-rights announcement period to the post-rights announcement period then, using the traditional method of calculating the volatility for the Black Scholes model based on a historical estimate will lead to incorrect model prices.

Chan (1997) investigated this issue in an attempt to explain the conflicting results of studies on the valuation of underwriting agreements using put option pricing models. If share price volatility in the pre-announcement period were found to be significantly different from that in the rights trading period, the option pricing models using the pre-announcement period volatility as their volatility estimate would incorrectly model the returns to underwriters. Using a Wilcoxon signed rank test, Chan (1997) found that there was a statistically significant increase in volatility in the post-announcement period. The difference is so significant as to swing the return to underwriters from positive to negative when employing a put pricing model based on the volatility estimate including the rights trading period and that excluding the period respectively.

The implication of the above finding is that volatility is not constant when viewed across the pre- and post-announcement periods of a rights issue. Based on the findings of Chan (1997) it appears that the volatility increases and this may in turn lead to a decrease in the share price. It is accordingly critical to obtain an accurate estimate of the volatility in the rights trading period when attempting to value the rights using an option pricing model because using the volatility in the pre-announcement period will result in inaccurate model prices. The CEV model specifically accounts for this factor, allowing the volatility to change as the share price changes from the pre- to the post-announcement period.

6.3 DEVELOPMENT OF THE MODEL

The Black Scholes model is a member of the family of constant elasticity of variance option pricing models. While the Black Scholes model assumes that the instantaneous variance is constant through time (Black and Scholes, 1975), the other members of the family allow the volatility to change with the stock price (Cox, 1976). John Cox originally developed the constant elasticity of variance model in an unpublished Stanford working paper entitled "*Notes on Option Pricing 1 : Constant Elasticity of Variance Diffusions*". The paper was in response to a request from Fischer Black to develop a general model that would include an inverse dependence between stock prices and volatility. This led to the option pricing solution for processes called constant elasticity of variance diffusions - processes where the instantaneous variance of the stock price, *S*, is given by $\sigma^2 S^\beta$ with $0 \leq \beta < 2$.

The stochastic differential equation for a constant elasticity of variance diffusion process follows from the geometric Brownian motion process and is (see Chapter 5) :

$$dS = \mu.Sdt + \delta.S^{\beta/2}dZ$$

so that the standard deviation of return (volatility) at any instant of time is :

$$\sigma_t = \delta . S_t^{(\beta-2)/2}$$

where the elasticity factor β takes on the values $0 \leq \beta < 2$. Thus the instantaneous variance of the price change is equal to $\frac{\sigma^2}{S^{2-\beta}}$ and hence is a direct inverse function of the stock price. The traditional Black Scholes framework, which corresponds to the lognormal model, is the limiting case when $\beta = 2$ and thus presumes that the variance rate is not a function of the stock price itself, and is constant through time (Black and Scholes, 1973).

If the stock price follows a diffusion process, its evolution will be completely specified by its instantaneous mean (drift) coefficient $\mu(S,t)$, its instantaneous variance (diffusion) coefficient $\sigma^2(S,t)$ and its behaviour at accessible boundaries. Any security, such as an option, whose value can be written as a twice-differentiable function of the stock price and time will then also follow a diffusion process that is a transformation of the basic process (Chriss, 1997). Cox (1996) calculated that the transformed process will have the following distributional properties :

- instantaneous mean equal to $\frac{1}{2}\sigma^2(S,t).P_{SS} + \mu(S,t).P_S + P_t$ and,
- instantaneous variance equal to $(P_S)^2\sigma^2(S,t)$

where subscripts denote partial derivatives.

Using the other standard assumptions inherent in the formulation of the Black Scholes model and with dividend payments set equal to $b(S,t)$, the following partial differential equation is obtained :

$$\frac{1}{2}\sigma^2(S,t).P_{SS} + (r.S - b(S,t)).P_S - r.P + P_t = 0$$

Cox and Ross (1976) showed that if a perfectly hedged portfolio of the stock and option can be constructed, the value of a European option will be the expectation of its terminal value in a risk neutral world discounted to the current time period. Considering only proportional dividend policies $b(S,t) = a.S$, the instantaneous mean of the stock price in a risk neutral world becomes $(r-a).S$ (Wilmott et al, 1997). The valuation of the option then becomes a procedure of finding the distribution of a variable that has instantaneous mean $(r-a).S$ and instantaneous variance $\sigma^2 S^\beta$ and an absorbing barrier at zero. The transition probability for this process was solved by Feller (1951) and is :

$$f(S_T,T;S_t,t) = (2-\beta).k^{\frac{1}{2-\beta}}.(x.z^{1-2\beta})^{\frac{1}{2}(\frac{1}{2-\beta})}.I_{\frac{1}{2-\beta}}(2(x.z)^{\frac{1}{2}})$$

with

$$k = \frac{2.(r-a)}{\sigma^2(2-\beta).(e^{(r-a)(2-\beta)(T-t)}-1)}$$
$$x = k.S_t^{2-\beta}.e^{(r-a)(2-\beta)(T-t)}$$
$$z = k.S_T^{2-\beta}$$

where $I_q(.)$ is the modified Bessel function of the first kind of order q. Cox (1996) showed that using the above solution and taking the expectation of $\max(S_T - E,0)$ and discounting to the present time t, yields the option valuation formula for a call with exercise price E :

$$C(S_t,t) = S_t.e^{-a(T-t)}.\sum_{n=0}^{\infty}\frac{e^{-x}.x^n.G\left(n+1,\frac{1}{2-\beta}.k.E^{2-\beta}\right)}{\Gamma(n+1)}$$
$$-E.e^{-r(T-t)}.\sum_{n=0}^{\infty}\frac{e^{-x}.x^{n+\frac{1}{2-\beta}}.G\left(n+1,k.E^{2-\beta}\right)}{\Gamma(n+1+\frac{1}{2-\beta})}$$

where G is the standard complementary gamma distribution function.

Cox (1996) formulated the above solution for values of $\beta < 2$. For the case where $\beta > 2$ the analysis is not identical and the final formula differs due to the different boundary behaviour

(Emanuel and MacBeth, 1982). This can be explained as follows. The value of a binding commitment to pay E for one share at time T is :

$$S_t e^{-\alpha t} - E.e^{-rt}$$

which is the current stock price adjusted by the attenuation caused by dividends over time, less the present value of the exercise price. An option to pay E for one share at time T is necessarily worth more than a binding commitment. The difference will be the present value of the money saved when the option is not exercised. This difference is worth (Emanuel and MacBeth, 1982) :

$$\int_0^E f(S_T,T;S_t,t).e^{-rt}.(E-S_T)dS_T$$

and the total value of the option is the sum of the above :

$$C = S_t e^{-\alpha t} - E.e^{-rt} + \int_0^E f(S_T,T;S_t,t).e^{-rt}.(E-S_T)dS_T$$

Emanuel and MacBeth (1982) formulated the solution to this equation as (with the term θ replacing β):

$$C = S_t.e^{-\alpha t}.(1-.\sum_{n=0}^{\infty}\frac{e^{-x}.x^{n+\frac{1}{\theta-2}}.G\left(k.E^{2-\theta}\middle|n+1\right)}{\Gamma(n+1+\frac{1}{\theta-2})})$$

$$-E.e^{-rt}.(1-\sum_{n=0}^{\infty}\frac{e^{-x}.x^{n}.G\left(k.E^{2-\theta}\middle|n+1+\frac{1}{\theta-2}\right)}{\Gamma(n+1)}$$

For the case where $\beta < 2$ the value of the call option is :

$$C = S.N_1 - E.e^{-rt}.N_2$$

$$N_1 = \sum_{n=0}^{\infty} g(S^*|n+1).G(E^*|n+p)$$

$$N_2 = \sum_{n=0}^{\infty} g(S^*|n+p).G(E^*|n+1)$$

For the case where $\beta > 2$ the value of the call option is :

$$C = S.N_1 - E.e^{-r.t}.N_2$$

$$N_1 = 1 - \sum_{n=0}^{\infty} g(S^*|n+p).G(E^*|n+1)$$

$$N_2 = 1 - \sum_{n=0}^{\infty} g(S^*|n+1).G(E^*|n+p)$$

With the terminology as follows :

$$S^* = [\frac{2.r.e^{r.t.(2-\theta)}}{\delta^2(2-\theta).(.e^{r.t.(2-\theta)}-1)}].S^{2-\theta}$$

$$E^* = [\frac{2.r}{\delta^2(2-\theta).(.e^{r.t.(2-\theta)}-1)}].E^{2-\theta}$$

$$g(x|m) = \frac{e^{-x}.x^{m-1}}{\Gamma(m)}$$

$$G(x|m) = \int_x^{\infty} g(y|m)dy$$

$$p = 1 + \frac{1}{|2-\theta|}$$

6.4 INTERPRETATION OF THE CEV FORMULA

The name of the model is derived from the specification that the elasticity of the variance of the return (the percentage change in variance for a one percent change in the stock price) equals $(2.\beta - 2)$ which for the case where the parameter is below unity, is a negative constant :

$$\frac{dVar[\Delta S_t / S_t]}{dS_t}.\frac{S_t}{Var[\Delta S_t / S_t]} = \frac{(2.\beta-2).\sigma^2.S_t^{2.\beta-3}.S_t}{\sigma^2.S_t^{2.\beta-2}} = (2.\beta-2) < 0$$

Interpreting the solution to the SDE will provide the intuition behind the use of the formula in this paper :

$$C(S_t,t;r,\sigma,T,K,\beta) = S.\sum_{n=0}^{\infty} g(\lambda.S_t^{-\phi},n+1).G\left(\lambda.(K.e^{-r.t})^{-\phi},n+1-\frac{1}{\phi}\right)$$
$$- K.e^{-r.t}\sum_{n=0}^{\infty} g(\lambda.S_t^{-\phi},n+1-\frac{1}{\phi}).G\left(\lambda.(K.e^{-r.t})^{-\phi},n+1\right)$$

The first summation to infinity corresponds to the normal distribution in the Black-Scholes model and is the expected stock price at expiration given that it exceeds the exercise price $E[S_T|S_T > K].prob(S_T > K)$. The second summation is the probability of ending up in the money $prob(S_T > K)$. From the original variance assumption that $Var[\Delta S_t / S_t] = \sigma^2(S_t,t).\Delta t$, this implies that $Var[\Delta S_t / S_t] = \sigma^2 S_t^{2.\beta-2}.\Delta t$. For the case where $\beta < 1$ as the share price increases the variance will decline and conversely as the share price decreases the variance will increase :

$$\frac{dVar[\Delta S_t / S_t]}{dS_t} = (2.\beta - 2).\sigma^2.S_t^{2.\beta-3}.\Delta t < 0$$

One of the justifications for this effect is given by Jarrow and Rudd (1983) as being the change in the firm's financial leverage. As the stock price declines, the market value of the firm's liabilities will also fall because of an increased perception of bankruptcy. Of the two, the decrease in the equity will be larger, they argue, and this produces an increase in the firm's debt to equity ratio. This increase in financial leverage causes an increase in the equity's risk leading to a rise in stock volatility (investigated in Chapter 5).

6.5 TWO SPECIAL CASES OF THE CEV MODEL

Two special cases of the model have exact solutions derived for them - the square root model and the absolute model. Due to the complexity of the final solution derived by Cox (1976), it is fairly difficult to use the model for empirical testing and it requires substantial computational time and facilities. The two special cases allow the model to be tested empirically.

6.5.1 SQUARE ROOT MODEL

For the square root model $\beta = \frac{1}{2}$ and the instantaneous standard deviation of the stock becomes :

$$\sigma(S_t, t) = \frac{\sigma}{\sqrt{S_t}}$$

and hence the reason for the name the square root model. The elasticity of variance is exactly minus unity and hence a ten percent decrease in the stock price is accompanied by a ten percent increase in the variance of the stock process. The original model can be simplified when inserting the square root value and Cox (1976) simplified it as follows :

$$C = S.N(q(4)) - K.e^{-r.t}.N(q(0))$$

where for $w = 0$ *or* $w = 4$

$$q(w) = \frac{1 + h.(h-1).\left(\frac{w+2.y}{(w+y)^2}\right) - h.(h-1).(2-h).(1-3.h).\left(\frac{(w+2.y)^2}{2.(w+y)^4}\right) - \left(\frac{z}{w+y}\right)^h}{\left[2.h^2.\left(\frac{w+2.y}{(w+y)^2}\right).\left(1-(1-h).(1-3.h).\left(\frac{w+2.y}{(w+y)^2}\right)\right)\right]^{\frac{1}{2}}}$$

$$h(w) = 1 - 2.(w+y).(w+3.y)(w+2.y)^{-2}/3$$

$$y = 4.r.S/(\sigma^2.(1-e^{-r.t}))$$

$$z = 4.r.K/(\sigma^2.(e^{r.t}-1))$$

and with N representing the cumulative standard normal distribution as in the Black-Scholes formula.

6.5.2 ABSOLUTE MODEL

For the absolute model $\beta = 0$ and the instantaneous standard deviation of the stock becomes :

$$\sigma(S_t, t) = \frac{\sigma}{S_t}$$

This process would be descriptive of a stock whose price changes have constant variance. Cox and Ross (1976) showed that this formula simplifies to :

$$C = (S - K.e^{-r.t}).N(h_1) + (S + K.e^{-r.t}).N(h_2) + v(N'(h_1) - N'(h_2))$$
$$where$$
$$v = \sigma.\sqrt{((1 - e^{-2.r.t})/2.r)}$$
$$h_1 = (S - K.e^{-r.t})/v$$
$$h_2 = (-S - K.e^{-r.t})/v$$

with N representing the cumulative standard normal distribution as in the Black-Scholes formula and N' representing the density function of the standard normal random variable.

6.6 APPLICABILITY OF THE MODEL

Emanuel and MacBeth (1982) attributed the superiority of the constant elasticity of variance model over the Black Scholes model primarily to two factors :

- the model corrects the mispricing of out of the money and in the money options;
- the model predicts changes in volatility.

The value of the characteristic parameter determines the difference between the Black Scholes model prices and the CEV model prices. For values of $\beta < 2$ the volatility, σ_t, increases as the stock price decreases. For values of $\beta > 2$ the volatility σ_t decreases as the stock price decreases. For values of $\beta < 2$ the Black Scholes model price with the correct volatility will be greater than the market price for out of the money options and less than the market price for in the money options. For values of $\beta > 2$ the Black Scholes model price with the correct volatility will be less than the market price for out of the money options and greater than the market price for in the money options.

It should be noted that Emanuel and MacBeth (1982) excluded options with less than ninety days to expiration from their sample positing that as expiration approaches, option prices become rather insensitive to volatility. This will be investigated in the current study, due to the fact that rights issues generally trade for a period substantially less than ninety days.

6.7 ESTIMATING THE CHARACTERISTIC PARAMETER AND RESULTS OF EMPIRICAL TESTING

The constant elasticity of variance model requires the estimation of the characteristic parameter for the stock price process. This procedure has proven problematic in empirical studies of the model as it is not directly observable in the market and will thus be investigated here .

The instantaneous standard deviation of the percentage price change is given as $\sigma.S_t^{(\beta-2)/2}$. Beckers (1980) showed that this can be restated as :

$$\ln\left(stdv \frac{S_{t+dt}}{S_t} \right) = \ln\sigma + \frac{(\beta - 2)}{2} . \ln S_t$$

To estimate the parameters a regression of the form :

$$\ln\left(stdv \frac{S_{t+dt}}{S_t} \right) = a + b . \ln S_t + w_t$$

could be used to determine the value of β. However Beckers (1980) noted that there are several problems in implementing this procedure using daily returns, including the fact that the regression equation is expressed over a finite period of time while the relationship above is continuous. There is also the problem of calculating $stdv \frac{S_{t+1}}{S_t}$ since only one return observation is available on any given day. Beckers (1980) proved that $\left| \ln \frac{S_{t+1}}{S_t} \right|$ can be used as a surrogate if the stock price process is lognormally distributed. For a normally distributed $N(\mu, \sigma^2)$ random variable :

$$E(|x|) = \frac{1}{\sqrt{2\pi.\sigma^2}} \int_{-\infty}^{\infty} |x| . e^{-\frac{1}{2} \cdot \frac{(x-\mu)^2}{\sigma^2}}$$

letting $y = \dfrac{x - \mu}{\sigma}$:

$$E(|x|) = \frac{1}{\sqrt{2\pi.\sigma^2}} \int_{-\infty}^{\infty} |\sigma y + \mu| . e^{-\frac{1}{2} \cdot y^2}$$

$$= \sigma . \sqrt{2/\pi} \int_{\mu/\sigma}^{\infty} y . e^{-\frac{1}{2} \cdot y^2} dy + \mu - \mu . \sqrt{2/\pi} \int_{-\infty}^{-\mu/\sigma} e^{-\frac{1}{2} \cdot y^2} dy$$

Because μ is negligibly small in comparison to σ and close to zero for daily data :

$$E(|x|) = \sigma . \sqrt{2/\pi}$$

Beckers (1980) accordingly ran the regression :

$$\ln\left| \ln \frac{S_{t+1}}{S_t} \right| = a + b \ln S_t + w_t$$

against a sample of stocks and found that the characteristic parameter was significantly negative thus supporting the hypothesis of an inverse relationship. For his sample of 47 stocks, Beckers found that 38 of them had a significant inverse relationship between stock price and variance. He was also able to reject the notion that one parameter was the same for all stocks as a Chow test showed that the parameter was not constant across all stocks. When examining whether this was as a result of increased financial leverage he found that an increase in leverage does significantly affect the common stock risk although there were a number of other contributing factors to.

Beckers (1980) claimed that it is generally true that for in the money and at the money options the model price increases as the characteristic exponent decreases whereas exactly the opposite is true for out of the money options. Thus model prices for the square root and absolute models will be higher than the Black Scholes model prices for at the money and in the money options and this behaviour is consistent across stock price levels. The difference between the models was noted to become more apparent as the time to maturity and volatility increased. Beckers (1980) noted that in contrast to the above, prices for out of the money options seemed to decrease as one moved further away from the lognormal diffusion process.

There is an alternative approach that can be used to estimate the characteristic parameter. The implied volatility of an at the money option can be used to estimate the parameters of the CEV model because it is a reasonable approximation of the true volatility on any day regardless of the value of β (Black, 1976). Thus for any value of β, the following equation can be used with the current stock price and the implied volatility of an at the money option to find the corresponding value of δ :

$$\sigma_t = \delta . S_t^{(\beta-2)/2}$$

The findings of the Emanuel and MacBeth (1982) study revealed that β is not constant and also that there is positive covariability of these values across stocks. It is furthermore interesting to note that Emanuel and Macbeth (1982) found that for 1976 the estimated values of β were less than two, thus predicting that during this period volatility would increase as the stock price declined. Their results showed that these predictions were valid. However, for the year of 1978, estimated values of β rose from the neighbourhood of two and then declined back to two later in the year thus predicting that volatility would increase as the stock price increased. Their results for 1978 were thus that the Black Scholes model outperformed the CEV model due to its constant variance assumption and the incorrect specification of the β parameter in the CEV model.

Emanuel and MacBeth (1982) concluded that volatility is indeed negatively correlated with the stock price. This is the relationship that the CEV model predicts with a β parameter less than two. However when β is estimated as greater than two, the volatility is predicted to vary in proportion to the stock price and the CEV model predictions of option prices are worse than the Black Scholes model. Thus they concluded that the CEV model works best when the β is less than two. They did however caution that the CEV model is unable to fully explain volatility changes and as such is only a partial predictor of volatility.

MacBeth and Merville (1980) used the Cox CEV diffusion process to model the heteroskedacity in returns to common stocks. As a justification for its use they posited that theoretically, firms may internally change their common stock return distribution through technological innovations and/or mergers and acquisitions. MacBeth and Merville (1980) also posited another explanation for dynamic variances from multiperiod consumption-investment theory : if in each period aggregate consumer-investors plan their consumption and investment over multiple future periods then the variances for securities may change over time as new information arises and new individuals (preferences) bid for risky assets in the capital markets.

MacBeth and Merville (1980) confirmed the previous findings that β is in general less than two and that the CEV model with β less than two fits the data better than the Black Scholes model and thus with precise estimates of β and δ the CEV model should yield even better results. They do, however, warn that an overriding consideration should be whether the extra effort in estimating the β and δ parameters is justified by the improved results relative to the Black Scholes model.

6.8 CONCLUSION

Based on the results of the extensive empirical testing that has been performed using the constant elasticity of variance model, it appears that the model with a characteristic parameter β less than two will result in more accurate option prices than the Black Scholes model. The model is thus able to relax one of the Black Scholes assumptions - that of a constant variance. The model should thus be most appropriate to the valuation of rights where it has been shown that the variance is not stationary across the pre- and post-announcement periods (Chan, 1997).

CHAPTER 7 - STOCK DIFFUSION DISTRIBUTIONS

7.1 INTRODUCTION

This chapter will investigate the distributions of share price changes that are hypothesized to exist. It will be shown that the currently widely accepted assumption of a normal distribution for the log of share price changes does not accord with the empirically observed share price behaviour. The impact of this on the Black Scholes model will be demonstrated and the theoretical basis for an alternative model, the Merton jump diffusion model, formulated. This alternative model will be examined in Chapter 8.

7.2 THE SHARE PRICE DIFFUSION DEBATE AND EVIDENCE

There has long been a tradition among financial economists that prices in speculative markets behave like random walks. This random walk theory was first postulated by Eugene Fama and is based on two main assumptions :

- price changes are independent random variables;
- the changes conform to some probability distribution.

Prior to the work of Benoit Mandelbrot (1963) the usual assumption was that the distribution of price changes in a speculative series was approximately Guassian. This was supported theoretically by Bachelier (1900) and Osborne (1959). The Bachelier (1900) model states that if $Z(t)$ is the price of a security at time *t*, then it is assumed that successive price differences of the form $Z(t+T)-Z(t)$ are independent, Guassian, random variables with zero mean and variance proportional to the differencing interval T. This process has subsequently come to be known as Brownian motion. It is generally represented by the differential equation :

$$dS = \mu.dt + \sigma.dW \qquad \text{for arithmetic Brownian motion}$$

$$dS = \mu.S.dt + \sigma.S.dW \qquad \text{for geometric Brownian motion}$$

where S is the share price and W a Wiener process. If the price changes from transaction to transaction are independent, identically distributed random variables with finite variance, and if transactions are fairly uniformally spaced through time, then one can invoke the central limit theorem to show that price changes across differencing intervals such as a day, week or month will be normally distributed since they are simple sums of the changes from transaction to transaction (Shimko, 1992). This was supported empirically by Kendall and Moore (1977) who found security prices to be approximately normally distributed. It is on this basis of the lognormality of future share price returns that the Black Scholes model is reliant.

Kendall and Moore (1977) did however note that the extreme tails of the empirical distributions were higher than those of the normal distribution. Mandelbrot (1963) and Fama (1965) confirmed this finding and thus concluded that the Guassian distribution does not account for the movement in stock prices because the empirical distributions of price changes are usually too "peaked" to be relative samples from Guassian distributions. There are far too many outliers to fit the normal distribution and a property termed *leptokurtosis* is observed. In a subsequent study Turner and Wiegel (1990) found that the stock market's probability of a three sigma event is roughly twice that of the Guassian random numbers and that the $T^{1/2}$ rule commonly used for scaling of standard deviation over longer periods is incorrect : monthly and quarterly volatility were higher than they should be compared to annual volatility and conversely, daily volatility was found to be lower than it should be. Regarding the Johannesburg Stock Exchange, Bowie (1994) examined the residuals of the JSE Actuaries Index for various return periods and his tests revealed a high degree of kurtosis and thus non-normality. Bowie (1994) also made a comparison between the kurtosis and skewness coefficients of the JSE and the NYSE. The JSE was found to exhibit a much higher degree of non-normality than the NYSE and led to the conclusion that : "*This finding makes it all the more important to incorporate non-normality into any analysis of the financial statistics of the JSE* ".

7.3 THE NECESSITY FOR A NEW THEORY

The leptokurtosis phenomenon introduced above has now been widely accepted in the financial research arena and a modification to capital markets analysis is warranted to explain the fat tails of the distribution.

Econometric analysis assumes that if there are no outside influences then a system is at rest (in equilibrium). By perturbing the system, exogenous factors take it away from equilibrium. The system then reacts to the perturbation by reverting to equilibrium in a linear manner. The linear paradigm assumes that investors react in a linear fashion to information - they react to information as it is received and not in a cumulative fashion to a series of events. This is built into the rational investor concept because past information has already been discounted into security prices. Thus the linear paradigm implies that security returns should have approximately normal distributions. The system reacts immediately because it wishes to be in equilibrium and abhors being out of balance. However, Peters (1991) argued that if we look at nature, and thus at a free market economy, we observe evolving structures which abhor equilibrium. Equilibrium in a system implies the system's death. Peters (1991) conjectured that a likely model for the capital markets is a non-linear, dynamic system. These characteristics arise only when a system is far from equilibrium and Peters (1991) proposed that they describe the market we know from the empirical evidence

To explain the leptokurtosis it is easiest to assume that information shows up in infrequent clumps rather that in a smooth and continuous fashion. The market reaction to these clumps of information then, results in the fat tails. The theory thus holds that because the distribution of price information is leptokurtotic the distribution of price changes is also leptokurtotic. Peters (1991) expanded this theory by disagreeing with the rational investor concept whereby information is assimilated in a linear fashion. He postulated that it may be the reaction to information which occurs in clumps. If investors ignore information until trends are well in place and then react in a cumulative fashion to all the information previously ignored, this to would create the fat tailed distribution. This implies that investors react to information in a non-linear fashion. Once the level of information passes a critical level, people will react to all the information that they have ignored up to that point. This however implies that the present is influenced by the past - a clear violation of the efficient markets hypothesis where it is held that information is received and reacted to by changing the price, to reflect the new information (See Chapter 10).

7.4 THE STABLE PARETIAN APPROACH

Mandelbrot (1963) formulated a radically new approach to the theory of random walks in speculative prices, called the stable Paretian hypothesis, to model the leptokurtotic distribution.

This theory relies on two basic assumptions :

- the variances of the distributions behave as if they are infinite;
- the distribution conforms best to the non-Guassian members of a family of distributions called stable Paretian.

The logarithm of the characteristic function for the stable Paretian family of distributions is :

$$\log f(t) = \log \int_{-\infty}^{\infty} e^{(i.u.t)} dP(\tilde{u} < u) = i.\delta.t - \gamma.|t|^{a}.[1 + i.\beta.(\frac{t}{|t|})\tan(a.\pi/2)]$$

Each of the parameters of the function will be discussed below in order to gain an understanding of the characteristic function and its relation to the normal distribution :

- a is the characteristic exponent and is the most important for comparing to the Guassian distribution. It determines the height, or total probability, contained in the tails of the distribution and takes its value in the interval $0 < a \leq 2$. When :
    - $a = 2$ the distribution is the normal distribution (the logarithm of the characteristic function of the normal distribution is $\log f(t) = i.u.t - \frac{\sigma^2}{2}.t^2$ which is a stable Paretian distribution with $a = 2, u = \delta, \gamma = \sigma^2 / 2$),
    - $0 < a < 2$ the extreme tails of the distribution are higher than those of the normal distribution with the total probability in the tails increasing as a approaches zero,
    - $a = 2$ the variance is finite, for any other value of a the variance is infinite,
    - $a > 1$ the mean is finite, for any other value of a the mean is undefined;
- δ is the location parameter and if $a > 1$ then δ is equal to the expected value of the distribution;

- β is an index of skewness which is in the interval $-1 \leq \beta \leq 1$. When :
  1. β =0 the distribution is symmetric,
  2. β >0 the distribution is skewed right (and the skewness increases as β approaches 1),
  3. β <0 the distribution is skewed left (and the skewness increases as β approaches -1);
- γ is the scale parameter.

Mandelbrot (1963) postulated that the distribution of a speculative price series follows a stable Paretian function with $1 < a < 2$ so that the distributions have finite means but infinite variances as opposed to the normal distribution with $a = 2$.

The stable paretian distribution has different properties to the normal distribution. The three most important properties are :

1. the asymptotically Paretian nature of the extreme tail areas;
2. invariance under addition (stability);
3. the fact that these distributions are the only possible limiting distributions for sums of independent, identically distributed, random variables.

By definition, the distribution of sums of independent, identically distributed, stable Paretian random variables is itself stable Paretian and can be expressed as follows :

$$\log f(t) = i.(n.\delta).t - (n.\gamma).|t|^{a}.[1 + i.\beta.(\frac{t}{|t|})\tan(a.\pi/2)]$$

where n is the number of variables in the sum and $\log f(t)$ is the logarithm of the characteristic function of the individual summands. This expression is the same as the expression for $\log f(t)$ except that the location parameter δ and the scale parameter γ are multiplied by n. So the distribution of the sum is, except for the origin and scale, exactly the same as the distribution for the individual terms. Stability thus means that the parameters a, β remain constant under addition. This does however assume that the individual variables are independent and identically distributed i.e. all have the same four parameters a, β, δ, γ. Fama (1965) showed however, that stability still holds when the location and scale parameters are not the same for each individual variable in the sum. Because the change in price over a long period can be regarded as the sum of changes in price over a number of short periods, if the individual changes are independent, identically distributed, stable Paretian variables then daily, weekly and monthly changes will follow stable Paretian distributions of the same form except for origin and scale.

A most important corollary property of stable Paretian distributions is that they are the only possible limiting distributions for sums of independent, identically distributed, random variables. If such variables have finite variance then the distribution for their sum will be the normal distribution. If such variables have infinite variance then it will be a stable Paretian distribution.

Fama (1963) postulated that the price changes in a speculative series can be regarded as the influx of new information into the market and the re-evaluation of existing information. In the simplest case the price changes implied by individual bits of information may themselves follow stable Paretian distributions and then, if the effects of individual bits of information combine in a simple additive fashion, the price changes from transaction to transaction will also be stable Paretian and furthermore will have the same parameters a, β. Even if the price changes implied by individual bits of information are not stable (but still asymptotically Paretian) the Pareto-Doebline-Gnedenko condition will be satisfied and if there are many bits of information involved in a transaction then price changes between transactions will be stable Paretian. The Pareto-Doebline-Gnedenko condition states that in order for the limiting sums to be stable Paretian with characteristic exponent a $(0 < a < 2)$ it is necessary and sufficient that :

$$\frac{F(-u)}{1-F(u)} \rightarrow \frac{C_1}{C_2} \qquad \text{as } u \rightarrow \infty$$

and for every constant $k > 0$:

$$\frac{1-F(u)+F(-u)}{1-F(k.u)+F(-k.u)} \rightarrow k^a \qquad \text{as } u \rightarrow \infty$$

where F is the cumulative distribution function of the random variable $\tilde{u}$ and C_1, C_2 are constants. Any variable that is asymptotically Paretian (whether or not stable) will satisfy these conditions.

Thus, as long as the effects of individual bits of information combine in a way which makes the price changes from transaction to transaction asymptotically Paretian with exponent a, then according to the Pareto-Doebline-Gnedenko condition the price changes for longer differencing intervals will be stable Paretian with the same value of a.

7.5 THE SUPPORTING EMPIRICAL EVIDENCE

Peters (1990) conjectured that there is a finite long memory process underlying most systems - the length of the memory depending on the composition of the non-linear dynamic system that produces the fractal time series. A statistic capable of measuring this is the R/S statistic which is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Give a sample of returns $X_1, X_2, X_3, \ldots, X_n$ for n periods and sample mean $\overline{X}_n$, the classical rescaled range is :

$$R/S_n = \frac{1}{S_n} \cdot \left[\max \sum_{j=1}^{k} (X_j - \overline{X}_n) - \min \sum_{j=1}^{k} (X_j - \overline{X}_n) \right]$$

where S_n is the standard deviation estimator. Hurst found these observations to be well represented by the relation :

$$R/S_n = a.n^H$$

where H is the Hurst exponent. Using a logarithmic transformation, a regression of the form :

$$Log(R/S_n) = Log(a) + H.(Log(n))$$

can be run to calculate the Hurst exponent. The H values can be summarized as follows :

- $H = 0.5$ implies a random walk confirming the efficient markets hypothesis. Yesterday's events do not impact today and today's events do not impact tomorrow. The events are uncorrelated. Old news has already been absorbed and discounted by the market.
- $H > 0.5$ implies that today's events do impact tomorrow. Information received today continues to be discounted by the market after it has been received. This is not simply serial correlation where the impact of information quickly decays either. It is a longer memory function - the information can impact the future for very long periods and it goes across all time scales - all six months periods influence all following six month periods ... The impact does decay with time but at a slower rate than short term dependence. The cycle length is the decorrelation time of the series.

High H values show less noise, more persistence, and clearer trends than low H values and thus imply less risk because there is less noise in the data.

R/S analysis on the S&P500 for monthly data over the period January 1950 to July 1988 revealed that a long memory process is at work for less than approximately 48 months

(Peters, 1991). After this point the graph begins to follow the random walk line of $H = 0.5$. The peak in the graph occurs at $N = 4$ years with $H = 0.78$ which is thus the Hurst exponent estimate for the S&P 500. This high value for H shows that the stock market is fractal and not a random walk. Thus the independence assumption particularly regarding long memory effects is seriously flawed - market returns are persistent time series with an underlying fractal probability distribution and they follow a biased random walk. The market exhibits trend reinforcing behaviour not mean reverting behaviour. Because the system is persistent it has cycles and trends with an average cycle of 48 months. Peters (1990) also noted that stocks in the same industry exhibited similar H values and similar cycle lengths. Industries with high levels of technology innovation tended to have high H values with short cycle lengths and conversely utilities tended to have low H values but longer cycle lengths.

Nawrocki (1995) attempted to clear the contradictory results obtained by Peters (1991) who found long term persistent dependence with finite non-periodic cycles in stock market indices and Lo (1991) who could not find such cycles using the modified rescaled range (R/S) formula :

$$S_n(q) = \left\{ \frac{1}{n} \sum_{j=1}^{n} (X_j - \overline{X}_n)^2 + \frac{2}{n} \sum_{j=1}^{q} w_j(q) \left[\sum_{i=j+1}^{n} (X_i - \overline{X}_n).(X_{i-j} - \overline{X}_n) \right] \right\}^{0.5}$$

which adjusts for autocorrelation. Nawrocki (1995) ran tests on the CRSP index and the S&P500 from 1926 to 1992. His results showed that there is a local maximum in the Hurst exponent around 50 months which ties in closely to the 4 year period observed by Peters (1991). Using the Lo modified statistic further cycles of 5 and 10 years were found indicating that there is persistent finite memory in stock prices.

7.6 THE LARGE PRICE CHANGE PHENOMENON EXPLAINED

In the Guassian case any occurrence of a large value of $L(t,1) = \ln Z(t+1) - \ln Z(t)$ should be traceable to a rare conjunction of large changes in all or most of the periods. In the stable Paretian case one should, on the contrary, expect large changes in $L(t,1) = \ln Z(t+1) - \ln Z(t)$ to be traceable to one or a small number of the contributing periods - this is closer to the empirically observed behaviour of the capital markets.

Thus if the price increase over a long period of time happens ex-post to have been unusually large then in a stable Paretian market one would expect to find that this change was mostly performed during a few periods of exceptionally high activity i.e. the majority of the contributing daily changes are distributed on a fairly symmetric curve while a few especially high values fall well outside this curve.

7.7 IMPLICATIONS OF THE STABLE PARETIAN HYPOTHESIS ON THE BLACK SCHOLES MODEL

Peters (1990) drew the following implications for the stock market. Investors are postulated to value securities within a range of prices. This range is determined partly by fundamental information using fundamental analysis. The second component of the price range is what investors feel other investors would be willing to pay - this sentiment component is valued using technical analysis and sets a range around the fair value. If the fundamentals are good, the price will rise toward a fair value. As other investors see the trend confirming their positive outlook on the security they will begin to buy it as well. Yesterday's activities influence today's, the market retains a memory of its recent trend. The bias will change when the price hits the upper range of its fair value. At that point the bias will shift.

The hypothesis implies that there are a larger number of abrupt changes in the economic variables that determine equilibrium prices in speculative markets than would be the case under a Guassian hypothesis. The fact that there are a larger number of abrupt changes means that such a market is inherently more risky for the investor than a Guassian market - in a stable Paretian market investors cannot usually protect themselves from large losses by means of such devices as stop-loss orders. This is due to the fact that in a market that is stable Paretian with $a < 2$, a large price change across a long time interval will more than likely be the result of very large price changes that took place during smaller subintervals i.e. if the price level is going to fall very much, the total decline will probably be accomplished very rapidly.

This fractal nature of the market thus contradicts the efficient markets hypothesis and all the quantitative models that are derived from it, including the Black Scholes model, which depend on the normal distribution and finite variance.

In the normal distribution pricing is considered to be continuous. This assumption of continuous pricing made the Black Scholes derivation possible - an investor could synthetically replicate an option like a call by continuously rebalancing between the risky asset and cash. However, in a fractal distribution as explained above, large changes occur through a small number of large changes. Large price changes can be discontinuous and abrupt. When the share price diffusion process is no longer continuous, a new stochastic differential equation is required and the Black Scholes model is no longer valid. In response thereto Merton (1976) posited that this new diffusion process can be modeled by a combined normal distribution, which models the "normal" price diffusion process, together with an exponential distribution, which models the random discontinuous price "shocks". This model will be investigated in the following chapter.

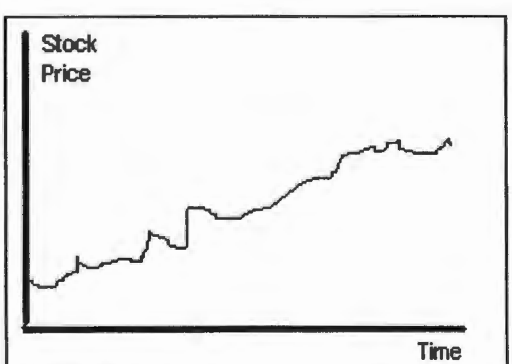
CHAPTER 8 - JUMP DIFFUSION PRICING MODELS

8.1 INTRODUCTION

This chapter will investigate the jump diffusion concept and relate it to the stable Paretian distributions discussed in the previous chapter as a likely model for share prices. The Merton (1976) jump diffusion model will be introduced and its derivation highlighted. Due to the complexity of estimating the model parameters the procedures used in other studies that employed the Merton model will be examined.

8.2 THE JUMP DIFFUSION CONCEPT

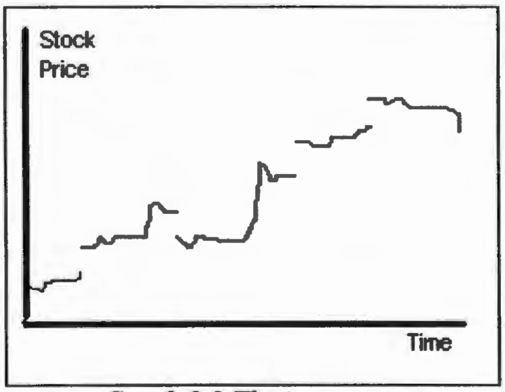
The validity of the Black-Scholes option pricing formula is dependent on the capability of investors to follow a dynamic portfolio strategy in the underlying stock, that replicates the payoff structure to the option (see Chapter 4). The critical assumption required for such a strategy to be feasible is that the underlying stock-return dynamics can be described by a stochastic process with a continuous sample path.



Graph 8.1 The continuous time diffusion process

The Black-Scholes solution is no longer valid if the stock price dynamics cannot be represented by a stochastic process with a continuous sample path. An alternative stock price process proposed by Merton (1976), is a jump stochastic process allowing for a positive probability of a stock price change of extraordinary magnitude, no matter how small the time interval. The Merton jump diffusion model specifically incorporates a discontinuous stock price distribution into an adjusted option pricing model that thus overcomes the continuous sample path assumption required by the Black Scholes model. This agrees with the empirical studies of stock price series which revealed leptokurtosis (see Chapter 7). The modified total stock price change is posited to be made up of two components :

1. the "normal" vibrations in price which produce a marginal change in the stock price;
2. the "abnormal" vibrations in price which produce more than a marginal change in the stock price.



Graph 8.2 The jump process

Graph 8.2 clearly indicates the discontinuous nature of the stock price distribution. Practically, such large jumps represent a worst case failure of Black Scholes while theoretically, such large jumps are related to the stable Paretian hypothesis of Mandelbrot (1965). This jump process is related to the Type III stock price path as explained in Chapter 3. These large jumps inflict maximum losses on a hedge constructed on the fiction of pure diffusion and the losses on the Black Scholes hedge are thus unbounded (Jones, 1984).

8.3 DERIVATION OF THE MERTON JUMP DIFFUSION MODEL

The derivation of the model will be divided into first describing and formulating the process followed by the underlying stock and then, secondly, applying this process to the formulation of an option on the stock. The proof follows that of Merton (1991).

8.3.1 THE STOCK PRICE DYNAMICS

The total stock price change is posited to be made up of two components :

1. The "normal" vibrations in price due to e.g. temporary imbalance between supply and demand or changes in the economic outlook. These produce a marginal change in the stock price. This component is modeled by a standard geometric Brownian motion with a constant variance per unit time and has a continuous sample path. The natural prototype process is a Wiener process.
2. The "abnormal" vibrations in price due to e.g. firm specific events. These produce more than a marginal change in the stock price. Such important information arrives only at discrete points in time and is modeled by a jump process. The natural prototype process is a Poisson driven process.

The Poisson distributed event is the arrival of an important piece of information about the stock and the arrivals are assumed to be independent identically distributed (hereafter IID). The probabilities of events are as follows :

P(the event does not occur in the interval $(t,t+h)$) $= 1-\lambda.h+o(h)$

P(the event does occur in the interval $(t,t+h)$) $= \lambda.h+o(h)$

P(the event occurs more than once in the interval $(t,t+h)$) $= o(h)$

where λ is the mean number of arrivals per unit time. Given that a Poisson event occurs, there is a drawing from a distribution $(Y \geq 0)$ to determine the impact this information has on the stock price so that if a Poisson event occurs then $S(t+h) = S(t).Y$. The stochastic differential equation for these mixed processes can be written :

$$\frac{dS}{S} = (\alpha - \lambda.k)dt + \sigma dZ + dq$$

with α the instantaneous expected return on the stock, σ^2 the instantaneous variance of the return conditional on no Poisson event occurring, *dZ* the standard Gauss-Wiener process, *q(t)* the independent Poisson process and $k = \varepsilon(Y-1)$ where *Y-1* is the random percentage change in the stock price if the Poisson event occurs. So the $\sigma.dZ$ describes the unanticipated return due to "normal" price vibrations whereas *dq* describes the portion due to "abnormal" price variations. The possible outcomes each period are then :

$$\frac{dS}{S} = (\alpha - \lambda.k)dt + \sigma dZ \qquad \text{if the Poisson event does not occur}$$

$$\frac{dS}{S} = (\alpha - \lambda.k)dt + \sigma dZ + (Y-1) \qquad \text{if the Poisson event does occur}$$

Thus the path is continuous most of the time, with finite jumps of differing signs and amplitudes occurring at discrete points in time. The partial differential equation can be solved to provide the following solution :

$$\frac{S(t)}{S} = e^{\left[\left(\alpha - \frac{\sigma^2}{2} - \lambda.k\right)t + \sigma.Z(t)\right].Y(n)}$$

which is the process followed by the underlying stock price if it assumed to follow a mix of a continuous and jump diffusion process.

8.3.2 THE OPTION PRICE DYNAMICS

Let $W(t) = F(S,t)$ then :

$$\frac{dW}{W} = (\alpha_w - \lambda.k_w)dt + \sigma_w dZ + dq_w$$

with parallel symbolism from the stock price model. Using Ito's lemma for the continuous part and an analogous lemma for the jump part, Merton (1991) showed that :

$$\alpha_W = \frac{0.5\sigma^2.S^2.F_{SS}(S,t) + (\alpha - \lambda.k).S.F_S(S,t) + F_t + \lambda.\varepsilon\{F(SY,t) - F(S,t)\}}{F(S,t)}$$

$$\sigma_W = \frac{F_S(S,t).\sigma.S}{F(S,t)}$$

Note that the Poisson event for the option price occurs only if the Poisson event for the stock price occurs.

Consider a portfolio strategy that holds the stock, option and riskless asset in proportions w_1, w_2, w_3. Merton (1976) showed that the return dynamics of the portfolio are :

$$\frac{dP}{P} = (\alpha_P - \lambda.k_P)dt + \sigma_P dZ + dq_P$$

with parallel symbolism from the stock price model. Thus :

$$\alpha_P = w_1(\alpha - r) + w_2(\alpha_W - r) + r$$

$$\sigma_P = w_1.\sigma + w_2.\sigma_w$$

$$Y_P - 1 = w_1(Y - 1) + \frac{w_2[F(SY,t) - F(S,t)]}{F(S,t)}$$

Inspection of these equations reveals that there is not a set of portfolio weights (w_1, w_2) that will eliminate the jump risk. The Black-Scholes hedge will thus not be riskless. Let P^* denote the value of the portfolio if the Black-Scholes hedge is followed, then :

$$\frac{dP^*}{P^*} = (\alpha_P^* - \lambda.k_P^*)dt + dq_P^*$$

The return on the portfolio is thus a pure jump process as the continuous parts of the stock and option price movements have been hedged out :

$$\frac{dP^*}{P^*} = (\alpha_P^* - \lambda.k_P^*)dt \qquad \text{if the Poisson event does not occur}$$

$$\frac{dP^*}{P^*} = (\alpha_P^* - \lambda.k_P^*)dt + (Y_P^* - 1) \qquad \text{if the Poisson event does occur}$$

Thus, most of the time the return on the portfolio will be predictable, but on average once every $1/\lambda$ units of time the portfolio's value will take an unexpected jump equal to: .

$$Y_P^* - 1 = \frac{w_2^*[F(SY,t) - F(S,t) - F_S(S,t)(SY - S)]}{F(S,t)}$$

By the convexity of the option price $F(SY,t) - F(S,t) - F_S(S,t)(SY - S)$ is positive for all Y. Hence if w_2^* is positive then the unanticipated return on the hedged portfolio will always be positive and if w_2^* is negative then the unanticipated return will be negative. Merton (1976) showed that an investor following the Black-Scholes hedge who is :

- long the stock and short the option ($w_2^* < 0$) will earn more than the expected return on the hedge but in the rare occasions when the stock price jumps he will suffer a comparatively large loss;
- short the stock and long the option ($w_2^* > 0$) will earn less than the expected return on the hedge but in the rare occasions when the stock price jumps he will earn a comparatively large profit.

In practice then, in "quiet" periods when little company specific information is arriving, writers of options will make what appear to be positive excess returns and buyers will lose.

However, in the relatively infrequent "active" periods the writers will suffer large losses and the buyers will profit.

Let $g(S,t)$ be the equilibrium instantaneous expected rate of return on the option when the current stock price is S and the option expires at some time t in the future, then :

$$0 = 0.5\sigma^2.S^2.F_{SS} + (\alpha - \lambda.k).S.F_S - F_t - g(S,t).F + \lambda.\varepsilon\{F(SY,t) - F(S,t)\}$$

subject to boundary conditions

$$F(0,t) = 0$$
$$F(S,0) = \max(0, S - E)$$

The jump component represents non-systematic risk in the CAPM framework and will thus be uncorrelated with the market. The P^* portfolio must thus have a zero beta as the only source of uncertainty is the non-systematic jump component of the stock. If the CAPM holds, then the expected return on all zero beta securities must equal the risk free rate and thus $\alpha^*_P = r$ but this implies that :

$$\alpha^*_P - r = 0 = w^*_1(\alpha - r) + w^*_2(\alpha_W - r)$$

and substitution leads to :

$$\frac{\alpha - r}{\sigma} = \frac{\alpha_W - r}{\sigma_W}$$

and thus F must satisfy :

$$0 = 0.5\sigma^2.S^2.F_{SS} + (r - \lambda.k).S.F_S - F_t - r.F + \lambda.\varepsilon\{F(SY,t) - F(S,t)\}$$

and is thus independent of $\alpha, g(S,t)$. A complete closed form solution cannot be formulated without a further specification of the distribution for Y. Merton (1976) defined the random variable X_n to have the same distribution as the product of n independently and identically distributed random variables, each identically distributed to the random variable Y and with

$X_0 = 1$. With ε_n defined as the expectation operator over the distribution X_n, the solution for the option price can be written :

$$F(S,t) = \sum_{n=0}^{\infty} \frac{e^{-\lambda t}(\lambda.t)^n}{n!}.\varepsilon_n\{W[SX_n e^{(-\lambda k t)},t;E,\sigma^2,r]\}$$

The formula has two special cases where the above solution can be simplified. The first is the case where there is positive probability of immediate ruin (Samuelson, 1972), but this is unrealistic for the purposes of this study. The second special case is where the random variable *Y* has a lognormal distribution. The equilibrium call option value for this special case is :

$$F(\mu,\sigma^2,\lambda,\delta^2;S,K,r,t) = \sum_{n=0}^{\infty} \frac{e^{-Et}(E.t)^n}{n!}.W((\sigma^2 + n.\delta^2 / t);S,K,$$
$$(r + n(\mu + \frac{1}{2}\delta^2)/t - \lambda(e^{\mu+\frac{1}{2}\delta^2} - 1)),t)$$

where

$$E = \lambda.e^{\mu+\frac{1}{2}\delta^2}$$

Merton (1976) then assumed that $E(Y) = 1$ so that $\mu = -\frac{\delta^2}{2}$ thus simplifying the formula to :

$$\boxed{F(\sigma^2,\lambda,\delta^2;S,K,r,t) = \sum_{n=0}^{\infty} \frac{e^{-\lambda t}(\lambda.t)^n}{n!}.W((\sigma^2 + n.\delta^2 / t);S,K,r,t)}$$

8.4 DIVERSIFIABILITY OF THE JUMP RISK

If the jump component of an individual security's return does represent non-systematic risk, it should be eliminated within a well diversified portfolio of individual securities. The Merton (1976) formula is reliant on the assumption that securities satisfy the CAPM framework and that the jump component of a security's return is uncorrelated with the market.

To investigate the validity of this assumption, Kim et al (1994) conducted research to determine whether the jumps found in stock prices are driven by a firm specific factor (thus diversifiable) or driven by a systematic market-wide risk factor (hence non-diversifiable). Only if they are diversifiable will the Merton (1976) jump diffusion model be valid. The SDE for the Kim et al (1994) stock process if a jump occurs (with probability $\lambda.dt$) is :

$$\frac{dS_t}{S_t} = y.dt + \sigma.dW_t + (Y - 1)$$

and if a jump does not occur (with probability 1- $\lambda.dt$) it is :

$$\frac{dS_t}{S_t} = y.dt + \sigma.dW_t$$

Using a sample of the MMI and its twenty component stocks Kim et al (1994) rejected the hypothesis that $\lambda = 0$ for the MMI at the 1% level in certain cases thus implying that the MMI still contained a significant jump component. Because the MMI is an index of twenty stocks one would expect that were the jump component diversifiable it would not be evident in the MMI as a sample of size twenty should be sufficient to diversify away a majority of this jump risk. Thus the jumps in stock price may represent systematic or non-diversifiable risk. The tests did however show different results for different time periods and the hypothesis could not be rejected in all cases. Thus the hypothesis necessary for the Merton jump diffusion model to hold, that the jump risk is diversifiable, cannot be conclusively rejected and more research is required to determine the validity of this assumption. For the purposes of this study, with no conclusive evidence to the contrary, an assumption will be made that the jump risk is diversifiable.

8.5 MODIFICATIONS OF THE MERTON MODEL

Oldfield et al (1977) proposed a model for common stock returns which is composed of a calendar time diffusion process and a jump process where the magnitudes of the jump may be autocorrelated. Considering the two general classes of continuous time stochastic processes - diffusion processes (continuous with probability one) and jump processes (discontinuous with probability one), they derived a model which is a combination of these :

$$\frac{dP}{P} = \alpha.dt + \beta.dW + z.d\pi$$

where P is the share price, dW a Wiener process, z the percent change in share price resulting from a jump and $d\pi$ the jump process (equal to one when a jump occurs else equal to zero). This can be written as :

$$\ln[P(t+s)/P(t)] = (\alpha - \beta^2/2).s + \beta.\sqrt{s}.W + \sum_{i=1}^{N} \log Z(i)$$

where the first two terms are due to the diffusion process and the last term due to the jump process. Oldfield et al (1977) calculated the conditional mean and variance as :

$$E\{\ln P(t+s)/P(t)]|N\} = (\alpha - \beta^2/2).s + N.\mu$$
$$Var\{\ln P(t+s)/P(t)]|N\} = \beta^2 s + N.\sigma^2 + 2.\sigma^2.\sum_{j=1}^{N-1}(N-j).\rho_j$$

This can then be compared to the Merton (1976) model where the corresponding conditional mean and variance are :

$$E\{\ln P(t+s)/P(t)]|N\} = (\alpha - \beta^2/2).s + N.\mu$$
$$Var\{\ln P(t+s)/P(t)]|N\} = \beta^2 s + N.\sigma^2$$

This highlights the difference due to the autocorrelation term as Merton (1976) assumed $\rho_j = 0$ for all j and the time between jumps to be exponentially distributed, while the

Oldfield et al (1977) model assumes the time to be gamma distributed. Applying their model to transaction data, Oldfield et al (1977) concluded that the geometric Brownian motion process does not alone describe the sample data very well and instead stock returns appear to follow an autoregressive jump process.

More recently, Jones (1984) developed a model similar to the Merton (1976) jump diffusion model except that he was able to eliminate the actual probabilities of jumps from the valuation formula. For this case the Merton (1976) model must be adjusted for the jump amplitude term to be $\ln Y \sim N(\mu, \sigma^2)$ and let $\sigma^2 \to \infty$. The option valuation formula then becomes :

$$C = S[1 - e^{-\theta.t} N(-b_1)] - E.e^{-n.t}.N(b_2)$$
$$with$$
$$b_1 = [\ln(S/E) + (n - \theta + \frac{1}{2}\sigma^2).t]/(\sigma.\sqrt{t})$$
$$b_2 = b_1 - \sigma.\sqrt{t}$$

where n and θ are related to the probability of a jump and the expected jump amplitude respectively.

These values are only ascertainable from two other options in the hedge and thus not determinable for our purposes as this involves using options to hedge against other options. This is not possible in the case of nil paid letters as there is but one such option on the underlying share and thus no such hedge can be constructed. Thus the Jones (1984) riskless hedge of the stock, three options and a risk free asset is not constructable when attempting to value nil paid letters.

8.6 ESTIMATING THE MODEL PARAMETERS

The use of the jump diffusion model requires various additional parameters to be calculated. In addition to the usual parameters of the Black Scholes model, the formula requires the following parameters :

- the mean jump amplitude;
- the mean jump variance;
- the diffusion mean.

Methods of solving for these parameters will be investigated.

The original paper on jump diffusion processes which attempted to calculate the characteristic component parameters was by Press (1967). The Press (1967) model is able to incorporate the high kurtosis observed in empirical data via decomposing the log returns into a continuous diffusion part and a discontinuous jump part (assumed to be distributed according to a Poisson process) much like the Merton (1976) model. Press (1967) constrained the instantaneous expected return on the security to be zero and the density is thus given by :

$$p(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} . \phi(x; n.\mu, \sigma^2 + n.\delta^2)$$

with

$$\phi(x; \mu, v^2) = \frac{1}{\sqrt{2\pi . v^2}} . e^{-\frac{(x-\mu)^2}{2.v^2}}$$

with the cumulants as follows :

$$K_1^P = \lambda . \mu$$
$$K_2^P = \sigma^2 + \lambda . (\mu^2 + \delta^2)$$
$$K_3^P = \lambda . \mu . (\mu^2 + 3.\delta^2)$$
$$K_4^P = \lambda . (\mu^4 + 6\mu^2 . \delta^2 + 3.\delta^4)$$

However, restricting the mean of the diffusion process to be zero is highly unrealistic and not in accordance with empirical observations.

Beckers (1981) attempted to solve the difficulties associated with estimating the parameters of the Poisson mixture of lognormal distributions stock price model put forward by Press (1967).

Beckers (1981) showed that the stock price returns are governed by the following differential equation :

$$\frac{dS}{S} = \alpha.dt + \sigma.dZ + dq$$

where dq is a Poisson process and λ is the mean number of arrivals of new information per unit time. The jump size (Y) is assumed to be itself a random variable with distribution $\ln Y \sim N(\mu, \delta^2)$. Then :

$$\ln\frac{S(T)}{S(t)} \sim \sum_{n=0}^{\infty} \frac{e^{-\lambda t}(\lambda.t)^n}{n!}.N(\alpha.\tau + n.\mu, \sigma^2.\tau + n\delta^2)$$
$$with$$
$$\tau = T - t$$

so that the log return is stationary over time and is described as a Poisson mixture of normal distributions. The kurtosis can be written (Kendall and Stuart, 1977) :

$$\frac{K_4}{K_2^2} = \frac{\lambda.\tau.(3\delta^4 + 6\mu^4\delta^2 + \mu^4)}{(\sigma^2\tau + \lambda.\tau.\delta^2 + \lambda.\tau.\mu^2)^2}$$

and can be seen to always be positive and thus describe a leptokurtotic distribution which appears to better describe the empirical stock return data. Due to the infinite summation term Beckers (1981) used a variant of the method of moments, the cumulant matching method, in order to estimate the parameters of the model. The method relies upon the theoretical relationship between the population cumulants, which are unknown and thus substituted by the sample cumulants, and the parameters of the distribution. The cumulants can be summarized as follows :

- 1st cumulant - the mean,
- 2nd cumulant - the variance,
- 3rd cumulant - the skewness,
- 4th cumulant - the kurtosis.

Inspection of Becker's (1981) formula reveals that the variance estimates, σ^2 and δ^2, do not always have positive sign. This is because there is no reason why, even in large samples :

$$\overline{K}_2 > \frac{5.\overline{K}_4^2}{3.\overline{K}_6}$$

The procedure is only found to give satisfactory results for those stocks with high kurtosis. To overcome this problem, Beckers (1981) made the additional assumption that the probability of a jump be the same for all the stocks under consideration. An average probability estimate is obtained by pooling the data using the following procedure. Let :

$$\frac{K_4}{K_2^2} = \alpha_1$$
$$\frac{K_6}{K_3^2} = \alpha_2$$

assuming that $\frac{\delta^2}{\sigma^2} = n$ is constant across stocks :

$$\alpha_1 = \frac{3.n^2.\lambda}{(1+\lambda.n)^2}$$
$$\alpha_2 = \frac{15.n^3.\lambda}{(1+\lambda.n)^3}$$

The above system of two equations in two unkowns is solved for $\hat{\lambda}, \hat{n}$ using the sample averages above as inputs. The following set of equations needs to be solved for each stock :

$$K_1 = \alpha$$
$$K_2 = \sigma^2 + \lambda.\delta^2$$
$$K_4 = 3\delta^4\lambda$$

The undesirability of this procedure is that λ is restricted to be the same for all stocks however, Beckers (1981) conjectured that this could for a number of stocks correspond to the probability of significant new information reaching the market.

Using an alternative method to solve for the jump diffusion parameters, Ball and Torous (1985) investigated the maximum likelihood estimation of the parameters. The mean logarithmic jump size was set to zero. Assuming a sample of size m of daily returns $X = (x_1, x_2, \ldots, x_m)$ and letting $Y = (\lambda, \sigma^2, \delta^2, \alpha)$, the logarithm of the corresponding likelihood function is :

$$\ln L(X;Y) = \sum_{i=1}^{m} \ln b(x_i;Y)$$

Necessary conditions for the existence of a maximum likelihood estimator are provided by :

$$\frac{d \ln L(X;Y)}{dY_i} = 0 \qquad i = 1,2,3,4$$

whereas corresponding sufficient conditions require :

$$\frac{d^2 \ln L(X;Y)}{dY_i dY_j} < 0 \qquad i,j = 1,2,3,4$$

There is, however, an infinite sum in the Merton formula and Ball and Torous (1985) truncated this at *N*, denoting the resultant approximation error as *B(N)*. They showed that :

$$0 \le B(N) \le \sum_{n=N+1}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} . \frac{1}{\sqrt{2.\pi.\sigma^2}}$$

and using a Taylor series expansion for the exponential function and integrating by parts *N* times sequentially it can be shown that :

$$\sum_{n=N+1}^{\infty} \frac{e^{-\lambda} \lambda^{n}}{n!} = e^{-\lambda} \int_{s=0}^{\lambda} \frac{(\lambda - s)^{N}}{N!} . e^{s} ds$$

so

$$B(N) \leq \frac{1}{\sqrt{2.\pi.\sigma^{2}}} \int_{s=0}^{\lambda} \frac{(\lambda - s)^{N}}{N!} ds \qquad \text{for } 0 \leq s \leq \lambda$$

Completing the integration leads to the following bound for the truncation error :

$$B(N) \leq \frac{1}{\sqrt{2.\pi.\sigma^{2}}} . \frac{\lambda^{N+1}}{(N+1)!}$$

While the truncation error is necessarily a function of the parameter values, Ball and Torous (1985) found that for plausible values of the parameters, truncation at $N = 10$ provided double precision computer accuracy. They then used a multidimensional Newton Raphson procedure to estimate the parameters. The truncation limit of ten will be used in this study as well and the effect of this truncation is shown in Appendix 2 - due to the short time to maturity a truncation limit of ten is highly accurate (at least to the fourth decimal). It is only when the period to maturity is close to one year or more that one observes errors occurring by limiting the series at ten. With an average period of thirty days to maturity for a rights issue this is thus not a concern.

8.7 A COMPARISON TO THE BLACK SCHOLES MODEL

Jarrow and Rudd (1982) examined the impact of the underlying distribution (summarized by its moments) on the option price using a generalized Edgeworth series expansion which involves approximating a given probability distribution *f(s)* with an alternative distribution *a(s)*. Thus using *f(s)* as the true distribution of the stock price at maturity, the expected value at maturity of an option on that stock can be obtained by using an Edgeworth series expansion to give an approximate value of the option at expiration in terms of the approximating distribution *a(s)*. The expression for the approximate option price is (Jarrow and Rudd, 1982) :

$$C(F) = C(A) + e^{-rt}\frac{K_2(F) - K_2(A)}{2!}.a(K) - e^{-rt}\frac{K_3(F) - K_3(A)}{3!}.\frac{da(k)}{dS_t}$$
$$+ e^{-rt}\frac{(K_4(F) - K_4(A)) + 3.(K_2(F) - K_2(A))^2}{4!}.\frac{d^2a(k)}{dS_t^2} + E(K)$$

where *C(A)* represents the standard Black Scholes value. The expression thus gives three adjustment terms to the Black Scholes valuation formula which will bring its value closer to the true option value (up to the error term *E(K)*). The adjustment terms operate as follows :

- the first term corrects for differing variance and if the true distribution has larger variance than the approximating lognormal then this term will be positive. The magnitude of the adjustment depends on whether the option is in or out of the money and how deep in or out of the money it is;
- the second term adjusts for differing skewness and will be at a maximum when the option is deep in or deep out of the money;
- the third term reflects the differing kurtosis and will be positive for deep in and out of the money options and negative otherwise.

Jarrow and Rudd (1981) analyzed the error term when using the above approximation to estimate option prices when the underlying stock price follows a jump diffusion process. An interesting result from the theoretical model is that due to the jump component being ever present, the model will tend to overprice deep in the money and deep out of the money options when compared to the Black Scholes model. This is because while a deep out of the money option assuming a continuous sample path for the underlying stock, has near no possibility of expiring in the money, with the jump component present, the possibility of expiring in the money is increased and hence makes the option more valuable. The presence of the possibility of a large finite jump thus causes the above model to place more value on the option than the traditional Black Scholes model. The reverse explanation applies to deep in the money options and the model prices are also above the Black Scholes model prices. Merton (1991) suggested that this may go some way in explaining what practitioners often claim to observe in the market for option prices - deep in the money and deep out of the money options tend to be underpriced by the Black Scholes model. More relevant to this study is the fact that for short maturity options this difference will be magnified (Merton, 1991) and the Merton model should thus provide better option prices for short maturity options than the Black Scholes model.

Applying MLE estimated jump diffusion parameters to the prices of options on 30 NYSE traded stocks, Ball and Torous (1985) found that the mean percentage difference between the Black Scholes and Merton model prices was positive for in the money and out of the money call options and negative for at the money call options. The magnitude of the bias was found to be small. It was however noted that the bias for out of the money calls tends to increase with decreasing term to expiration. Significant discrepancies are only likely to develop if the underlying common stock return process is predominated by large jumps which occur infrequently.

The majority of empirical research employing the Merton jump diffusion model has been performed on the pricing of warrants and is accordingly discussed under that topic in Chapter 9.

8.8 CONCLUSION

The Merton jump diffusion model incorporates a discontinuous stock price distribution into an adjusted option pricing model. This model thus relaxes the Black Scholes assumption of a continuous stock price path and is related to the stable paretian hypothesis discussed in Chapter 7. For short maturity options the impact of a discontinuous sample path will be magnified (Merton, 1991) and the Merton model should thus provide better option prices for short maturity options than the Black Scholes model. Due to rights having a maturity of not more than thirty days, the Merton model should lead to better rights prices than the traditional Black Scholes model.

CHAPTER 9 - WARRANT PRICING AND RIGHTS ISSUES

9.1 INTRODUCTION

This chapter will examine the relationship between warrants and rights issued by a firm. The aim is to link the pricing of options and warrants and then examine the valuation models employed in previous research on valuing warrants and attempt to use these models as a basis for the valuation of rights. The approach will be to outline the theory of warrant pricing and then link this to a possible theory for rights pricing. The sparse empirical testing on warrant pricing will also be examined.

9.2 THE RELATIONSHIP BETWEEN WARRANTS AND OPTIONS

The major difference between warrants and options is that warrants are issued by the corporation whose stock is to be purchased whereas options are contracts between investors. Warrant exercise thus directly affects the corporation, unlike option exercise. When a warrant is exercised new shares of stock are issued and the cash payment that is made increases the assets of the issuing firm. Options, however, are created by individuals, and when exercised, already existing shares must be delivered. Thus the cash payment does not go to the corporation whose stock is delivered but to the party who wrote the option instead. It is because of these differences that the Black Scholes model cannot be used directly to value warrants (Ingersoll, 1991) but with some adjustments is able to give fairly accurate results.

Indeed, it should be noted that the Black and Scholes (1973) paper was in fact motivated by prior research on warrant pricing.

9.3 THE RELATIONSHIP BETWEEN RIGHTS AND WARRANTS

Most articles of incorporation give shareholders pre-emptive rights to new-issue stock subscriptions. This provision reflects the common-law practice of reserving to current owners viz. the stockholders, rights associated with their property (the common stock), including a potentially valuable opportunity to acquire new stock. Most pre-emptive rights are presented to current stockholders as the "right" to acquire new shares in some proportion to their present shareholdings at a preferential price. Smith (1977) claimed that the rights are an option issued by the firm to purchase new shares and this was confirmed by Brealey (1971) who noted that rights are equivalent to short term options to buy the stock at the offer price. The right states the relevant terms of the option specifying the number of rights required to purchase a share, the subscription (exercise) price and the expiration date (see Appendix 1 for details of rights issues).

It is thus evident that a rights issue is very similar to issuing call options on the firm, or in fact, more similar to short term warrants due to them also being issued by the firm rather than individuals. Ingersoll (1991) posited that the method of valuing these rights using warrant pricing models should then, theoretically, lead to fairly accurate results. Commenting on the applicability of the original Black Scholes assumptions, Marsh (1980) noted that while they are not strictly met in the case of rights issues they do appear to be reasonable : the underwriting option is European; due to the short duration the interest rate will effectively be known and constant; since the aggregate supply of options is zero in a rights issue it appears reasonable to assume that the variance is constant and in most instances dividends are not paid during the underwriting period.

9.4 THE THEORY OF WARRANT PRICING

Galai and Schneller (1978) derived the value of warrants in their early study which was the first conclusive study on the relationship between call options and warrants. Assumptions required are that the investment policy of the firm is given and unaffected by its financial decisions and that markets are perfect. They demonstrated that the issue of warrants does not affect the wealth of the current security holders of the firm but it may increase the current market value of the firm's future cash flows which is then offset by the added liability of its security holders in such a way as to leave their wealth unchanged.

Assuming a pure equity firm with n shares that is expected to be liquidated at time t_1 and denoting its assets' stochastic liquidation value by X_1, the value of a share at t_1 must be :

$$P_1^* = \frac{X_1}{n}$$

If the firm now issues warrants with a striking price equal to k and with the ratio of the number of warrants to the number of shares q, and relying on the assumption that the proceeds from selling the warrants will be immediately distributed to the current shareholders in the form of dividends then, if the warrants are exercised, the gross value of the firm at time t_1 will exceed X_1 by the amount of the proceeds from exercising the warrants, which equals $n.p.q$. This event will only take place if the warrants are exercised. The warrants will be exercised only if the value of a converted warrant as a share :

$$P_1 = \frac{(X_1 + n.p.q)}{n.(1+q)}$$

is greater than the price for its conversion, k. The condition for exercising the warrant is thus (Galai and Schneller, 1978) :

$$P_1 = \frac{(X_1 + n.p.q)}{n.(1+q)} = \frac{P_1^* + q.k}{1+q} > k$$

which is equivalent to $P_1^* > k$ when the terms are simplified.

Galai and Schneller (1978) then compared the return on a call option written on the firm which does not issue warrants to the return on the warrant. The call option will be exercised at t_1 if $P_1^* > k$. By comparing the return on the call option to the return on the warrant Galai and Schneller (1978) showed that they are perfectly correlated and that the warrant return is merely a proportion of the call return for all values of X_1. This is because :

- if $P_1^* < k$, neither the call nor the warrant will be exercised and they thus both have nil value;
- if $P_1^* > k$ the value of the warrant is $\frac{P_1^* + q.k}{1+q} - k = \frac{P_1^* - k}{1+q}$ and the value of the call is $P_1^* - k$.

The conclusion is that the return on the warrant is always equal to $1/(1+q)$ of the return on the call. In perfect capital markets, if two assets' yields are perfectly correlated then their prices should be proportional. Thus to eliminate any arbitrage possibility it must be that :

$$W = \frac{C}{(1+q)}$$

where W is the price of the warrant and C is the price of an identical call option on the same underlying share. This relationship of the price of the warrant to the price of the call explicitly accounts for the dilution effect.

Ingersoll (1991) followed the analysis of Galai and Schneller (1978) in developing the dilution adjustment required for the valuation of warrants. An important finding of his, however, is that the warrants should be priced as a contingent claim on the firm as a whole and not merely on the stock. This means that instead of using the share price as an input into the Black Scholes model, the total value of the firm is substituted. This requires the calculation of the volatility of the firm value and not merely of the stock value, as is the usual case. Schultz and Trautmann (1994) noted that for this case the variance of the rate of return on these assets is neither observable nor inferable when the outstanding warrants are not traded in an informationally efficient market and this is why warrants are often valued as otherwise identical call options on the firm's common stock. Assuming the value of the firm, *V*, follows a constant variance diffusion process during the lifetime of the outstanding warrants and that dividends are not paid during this period, Schultz and Trautmann (1994) developed the following warrant pricing model for a contingent claim on the value of the firm as a whole :

$$W = \frac{1}{1+q}\left(\frac{V}{N}N(d_1) - K.e^{-r.T}.N(d_2)\right)$$

where

$$d_1 = \frac{\ln(V / N.K) + (r + \sigma_V^2 / 2).T}{\sigma_V \sqrt{T}}$$

$$d_2 = d_1 - \sigma_V.\sqrt{T}$$

While the formula is theoretically correct, the chief obstacle to its use is that neither the true value of the firm, *V*, nor its instantaneous volatility, σ_V, can be observed. Schultz and Trautmann (1994) were thus forced to make various additional assumptions in using the above formula but application to a sample of traded warrants revealed that the results were no more accurate than the traditional model with the contingent claim assumed to be on the equity of the firm rather than on the firm as a whole. Thus the dilution adjustment proposed by Galai and Schneller (1978) applied to the standard Black Scholes model should provide fairly accurate results for warrant prices.

9.5 THE THEORY OF RIGHTS PRICING

While this is not a study of rights issues per se, it is important to investigate the behaviour of share prices around the rights trading period as this will impact the option pricing models employed and the underlying share volatility (see Chapter 5). Various theories have been put forward on what the share price implications of a rights issue should be, together with the reasons therefore. The major theories on expected price reaction to the announcement of equity issues can be summarised as follows :

- The capital structure hypothesis where, in terms of the Modigliani Miller theorem (Modigliani and Miller, 1963), a new equity issue will be unfavorably received as it lowers the leverage ratio. However if the leverage ratio is currently so high as to have a negative impact on firm value, an equity issue should have a positive price effect.
- The information theories are due to information asymmetry between management and investors whereby the equity issue may be seen as a negative signal regarding the future cash flows of the company or that the shares are overpriced (Mikkelson and Partch, 1986). If management themselves have large equity holdings the impact may not be so severe.
- The application of funds theory states that the intended use to which the funds will be put affects the price reaction and so expansionary motives may have a less negative impact (Mikkelson and Partch, 1986).
- The price pressure hypothesis states that an increase in the supply of shares causes a decline in the firm's stock price due to a downward sloping demand curve for shares

(Asquith and Mullins, 1985). This thus relies on the assumption that a firm's shares are unique and there are no close substitutes.

- The transaction cost hypothesis whereby investors must be compensated for the transaction costs they bear in adjusting their portfolios to absorb the new shares (Barclay and Litzenberger, 1987).
- The tax advantage of debt hypothesis relates the unanticipated decline in leverage due to the additional equity raised to a share price decline due to the advantages of debt financing foregone.
- The redistribution hypothesis whereby the decrease in financial leverage results in a transfer of wealth from the stockholders to the bondholders and thus a share price decline (Smith and Warner, 1979).

In a sample of 696 rights issues Brealey (1971) found that in the months leading up to the issue the stocks showed unusual price gains whereas during the actual month of issue the stocks fell by an average of 0.3 % and thereafter there was little price movement in either direction. Lambrechts and Mostert (1980) conducted a study to determine the effect of rights issues on the underlying share prices but they could not find any statistically significant evidence that the announcement of a rights issue has a favorable impact on the price of the underlying share. In a later study, Youds et al (1993) showed a definite increasing price trend in the twenty days before the announcement and a decreasing price trend after the announcement, with a significant price decline around day ten after the announcement which is consistent with the ex-rights date.

The above theories display the lack of consensus as to just what the underlying causes are for the observed price changes in an equity issue. The empirical evidence is even more puzzling.

9.6 EMPIRICAL EVIDENCE ON WARRANT PRICING

As is the case with option pricing, the majority of studies have been performed in the United States on data from the New York Stock Exchange. There is however some literature on the South African warrant market and this will be summarized to.

9.6.1 INTERNATIONAL STUDIES

Kremer and Roenfeldt (1992) posited that the pricing of warrants constitutes a natural application for option pricing models due to the numerous similarities between warrants and call options. They applied the Merton (1976) jump diffusion model to the pricing of warrants. While the Merton (1976) model is theoretically superior to the Black Scholes model, it suffers from the additional complexity and difficulty in parameter estimation for describing the intertemporal movements in the underlying stock price (see Chapter 5). Merton's jump diffusion option pricing model :

$$C^{*} = \sum_{n=0}^{\infty} \frac{e^{-\lambda t}(\lambda . t)^{n}}{n!} . C_{n}$$

was used in its standard form with the Galai and Schneller (1978) formula for dilution :

$$W = \frac{C}{(1+q)}$$

where q is the number of new shares created by immediate block warrant exercise. To apply the Black Scholes model, estimates must be made of the underlying stock's volatility. A general approach in option pricing is to use implied volatilities estimated from an at the money option from other similar maturity options. It is unlikely however, that a company has more than one warrant outstanding and this approach is thus not feasible, especially considering that the warrants are generally issued way out of the money to be of value to investors and such options result in biased implied standard deviations (Rubinstein, 1985). Thus in warrant valuation, volatilities must be estimated from historical stock returns data. Kremer and Roenfeldt (1992) were thus forced to apply the Beckers (1981) cumulant matching procedure to a sample of 75 traded warrants and found the following :

- both the Black Scholes and Jump Diffusion models, unadjusted for dividends, overpriced warrants;
- both the Black Scholes and Jump Diffusion models, adjusted for dividends, underpriced warrants.

Both models were found to perform poorly for short maturity warrants, although the deviations for the Jump Diffusion model were smaller. Overall it was shown that the Jump Diffusion model outperformed the Black Scholes model.

Using an alternative testing mechanism Lauterbach and Schultz (1990) used a dilution adjusted version of the Black Scholes model to estimate implied equity standard deviations (hereafter ISDs). The following regression was run :

$$ISD_t = \alpha_0 + \alpha_1 \left(\frac{S - \sum_i D_i . e^{-r.t_i} - e^{-r.T} x}{e^{-r.T} x} \right)_{t-1} + \alpha_2 . r_{t-1} + \varepsilon_t$$

for each warrant to determine the ISD and its dependence on stock prices and interest rates. The results showed that the ISDs of warrants are inversely related to the value of the underlying equity. This inverse relation between variance and stock prices may thus be useful in pricing warrants. Exploring alternative explanations for the observed bias, Lauterbach and Schultz (1990) found that alternatives to the Black Scholes model that allow for early exercise, stochastic equity variance or stochastic interest rates do not improve the warrant prices. Models that allow for an inverse relationship between equity value and equity volatility such as the constant elasticity of variance model may be more promising. They thus applied the square root CEV model. Their findings showed that it is a consistently more accurate predictor of market prices than the Black Scholes model. It was also shown that the superiority over the Black Scholes model became more pronounced as time passed from the ISD estimation and thus the automatic adjustment of variances in the CEV model reduces the need for the type of ad-hoc adjustments frequently made in practice.

Various studies have also been performed on other types of warrants. Noreen and Wolfson (1981) studied executive warrants by assuming that stock prices follow a CEV diffusion process and then tested the two cases where the characteristic parameter is equal to one and two. Their findings showed that there are no important differences between the two models with the estimated warrant prices almost always within 2% of each other and never more than 5% apart. Rogalski and Seward (1991) studied the market for foreign currency exchange warrants (each FCEW entitles the holder to receive from the issuing company the cash value in US dollars of the right to purchase a fixed amount of US dollars at a fixed exchange rate). Their warrant valuation procedures proved that fairly accurate results could be obtained by modifying the Black Scholes model sufficiently.

9.6.2 SOUTH AFRICAN STUDIES

The South African study performed on warrant pricing was by Le Plastrier and Thomas (1984). The results showed that the Black Scholes model was unable to accurately calculate warrant prices and the Galai and Schneller (1978) dilution adjustment in fact led to even less accurate results. Based on this study and no other empirical evidence to the contrary, it appears that the South African warrant market is highly inefficient. It should however be noted that this study was performed several years ago and the primary problem was thus one of low liquidity. Were the study repeated and applied to the current warrant market, the results might be far superior.

9.7 CONCLUSION

It thus appears that, based on international evidence, the Black Scholes model can be used with a dilution adjustment as proposed by Galai and Schneller (1978) to price warrants and thus, theoretically, rights. The constant elasticity of variance model with a negative relationship between share price and volatility and the Merton (1976) Jump Diffusion model should provide even better results. Both of these models will be tested in this research.

CHAPTER 10 - MARKET EFFICIENCY

10.1 INTRODUCTION

Any test of the efficiency of the options market is necessarily a combined test of the pricing model being used to test the market and market efficiency itself. The test is thus unable to distinguish between the two hypotheses and a joint hypothesis problem exists (Galai, 1977). In this study, the accuracy of various option pricing models and their extensions, is tested against the prices of nil paid letters. By the nature of the tests performed, the efficiency of the equity rights market and the validity of the various models for the valuation of the rights themselves are tested simultaneously (Fenwick, 1996). It is accordingly necessary to determine if the rights market is efficient in order to be able to draw conclusions on the validity of the pricing models tested. If the rights market can be accepted as being efficient then the results of the option pricing model tests can be accepted as tests of the models themselves and the joint hypothesis problem ignored.

This chapter will outline the theory of both stock market and options market efficiency testing. The empirical evidence on options market efficiency will then be investigated.

10.2 THE THEORY OF MARKET EFFICIENCY

In the most comprehensive study of market efficiency to date, Fama (1970) defined three categories of market efficiency :

- weak form, where current share prices fully reflect all historical information,
- semi-strong, form where share prices reflect all generally available public information,
- strong form, where share prices reflect all public and private information.

In a sequel to his earlier paper Fama (1991) refined his definition of an efficient market as one where security prices fully reflect all available information. This is given as the strong version, a precondition to which is that the costs of getting prices to reflect information are always zero. Economically this is unrealistic and thus a weaker condition is given that prices reflect information to the point where the marginal benefits of acting on information do not exceed the marginal costs (Jensen, 1978).

There have been numerous tests on market efficiency but they are well summarised in Weston and Brigham (1993) who showed that empirical work on event studies indicates that, on average, stock prices adjust rapidly to information about investment decisions, dividend changes, capital structure changes and the like, thus lending belief that prices adjust efficiently to firm-specific information. As regards private information however, it has been conclusively shown that corporate insiders have private information which leads to abnormal returns (Jaffe, 1974). Regarding the predictability of stock returns based on past returns and other variables, the majority of studies have failed to support the efficient markets hypothesis. Evidence on the predictability of returns from other variables shows that returns for short and long horizons are predictable from dividend yields, price earnings ratios and default spreads of low over high grade bond yields (Campbell and Shiller, 1988; Fama and French, 1989). Term spreads and the level of short rates are also able to predict future returns to a one year time horizon (Chen, 1991).

The South African market has its own specific characteristics including, amongst others the problem of thin trading and concentrated control structures. Any emerging market such as ours is also likely to be less efficient than a large and sophisticated market such as the New York Stock Exchange.

The empirical evidence on weak form efficiency of the JSE has been mixed : Jammine and Hawkins (1974) and Hadassin (1976) showed that daily returns applied to serial correlation and runs tests showed share price behaviour that is inconsistent with the random walk hypothesis; the profitability of these inconsistencies was shown by Brummer and Jacobs (1981) to be too small to be exploited. Regarding tests of trading rules, Firer et al (1987) conducted a study where a portfolio of a high beta asset (held during bull markets) and low beta asset (held during bear markets) was constructed. With the inclusion of trading costs the results were shown to support weak form efficiency. Bhana (1989) investigated share price reaction to favorable and unfavorable company-specific news and found evidence consistent with the overreaction hypothesis for short-term negative events (but only weak evidence for positive events). Further testing by Page and Way (1992) on the overreaction hypothesis confirmed the presence thereof and provided conclusive evidence of long run weak form inefficiency.

Semi-strong form testing has concentrated on the impact of FIFO/LIFO changes (Knight and Affleck-Graves, 1983), changes to dividend policy (Knight and Affleck-Graves, 1987; Bhana, 1991) and share splits (Cross and Firer, 1986; Biger and Page, 1992) amongst others. The conclusions reached have been mixed with some studies directly contradicting the results of a similar earlier study.

Strong form tests on issues such as unit trust performance (Gilbertson and Vermaak, 1982; Knight and Firer, 1989) have generally shown evidence of strong form inefficiency of the JSE. Bhana (1987) showed that insiders have access to and trade on information during takeovers and in a further study Bhana (1991) showed that insiders were able to profit from dividend policy changes thus confirming that the JSE is strong form inefficient.

Keane (1989) posited that there are degrees to which a market may be efficient as opposed to a clear-cut efficient verses inefficient conclusion. In his framework, the debate is whether the market pricing mechanism is sufficiently unreliable to enable ordinary investors to use publicly available information to identify significantly mispriced securities. Following in this vein, Bhana (1995) described an operationally efficient market as one that provides superior returns for those whose expertise and efforts sustain its efficiency which is thus efficient to the vast majority of investors but inefficient to the few experts. A moderately inefficient market includes such factors as the weekend effect, small firm effect and the PE effect. An operationally inefficient market contains inefficiencies which are exploitable not only by the expert but also by the average reasonably informed investor (as well as the ability of the expert to transmit information to the average investor due to delayed market reaction to new information). After reviewing the previous market efficiency studies of the JSE, Bhana (1995) concluded that there can be little dispute that the JSE is a highly efficient information processor with respect to highly traded shares and as such is operationally efficient. His findings can be summarized as follows :

- the market is a rapid and relatively reliable interpreter of fundamental economic data;
- with the exception of insider trading, superior investment performance appears to be confined to a relatively small number of highly skilled information processors;
- it is futile for investors to look beyond security prices on the basis of their personal interpretation of publicly available information.

It thus appears that one can take comfort that the JSE is to some extent weak and semi-strong form efficient but definitely not strong form efficient. The implications for this study are that the trading prices of the underlying shares can be assumed to be efficient and should thus provide a reliable basis on which to test the various option pricing models employed in this study.

10.3 THE THEORY OF OPTIONS MARKET EFFICIENCY TESTS

Black and Scholes (1973) determined that the value of an option contract under their set of specific assumptions is only a function of the stock price and time remaining to maturity. There is thus a unique option value such that the stock and option are always in equilibrium. If the price of the option were different from that calculated using the model, there would be a portfolio combination of a long (short) position in the stock together with a short (long) position in the option that would be virtually riskless in the short term and would give a return in excess of the riskfree rate (with the necessary hedge adjustments being made over the life of the option contract). While not necessarily profitable in the short run, the hedge requires that the option revert to its equilibrium value eventually, as its terminal value will be the greater of zero and the difference between the stock and strike prices. Since in equilibrium a riskless investment should not yield more than the riskfree rate (in terms of the CAPM), the option must be priced such that market traders could not establish this hedge and earn a certain profit - the market efficiency argument. It is on this argument that the majority of empirical testing on the efficiency of the stock options market has been based. This appears logical as the basis for the Black Scholes (1973) model and most of the extensions thereof, is the arbitrage argument.

10.4 THE INTERNATIONAL EVIDENCE

Black and Scholes (1975) tested the efficiency of the options market by establishing whether significant profits could have been made by buying undervalued options and selling overvalued options at the prevailing market prices. At the writing date of the call, the Black Scholes model was used to value the contract. If the market price of the call option was greater than the model price, the contract was determined to be overvalued. If the market price was below the model price, the option contract was determined to be undervalued. Black and Scholes (1975) then set up four different portfolios :

1. all call contracts were purchased at the market prices,
2. all call contracts were purchased at the model prices,
3. undervalued calls were purchased and overvalued calls were sold all at the model prices (a test of the accuracy of the model),
4. undervalued calls were purchased and overvalued calls were sold all at the market prices (a test of market efficiency).

Empirical results showed that the returns on the first two portfolios were insignificantly different from zero. The third portfolio yielded significantly negative returns while the fourth portfolio yielded significantly positive returns, thus questioning the efficiency of the options market.

They also noted that options on high variance securities tended to be priced too low while options on low variance securities were valued too high by the market. The model thus overestimates the value of options on high variance securities and underestimates the value of low variance securities. The market appears to do the converse. There thus appeared to be profitable trading strategies in the market. It is however, necessary to take transactions costs into account as it is possible that they may be so large as to distinguish any profitable trading opportunities. When the effect of transaction costs was taken into account, the above profit opportunities were found to disappear. Thus Black and Scholes (1975) concluded that the options market is efficient in its pricing of options.

The most comprehensive study of the efficiency of the Chicago Board Options Exchange (CBOE) was undertaken by Galai who conducted a detailed investigation comparing the Black Scholes model prices to the market prices. Galai (1977) split his tests into ex post tests which would thus indicate the existence of contemporaneous deviations from equilibrium and ex ante tests which can be used to test market efficiency. In the ex post tests the original position in the option is determined (by reference to whether the option is overvalued or undervalued) and maintained until the expiration date (with the necessary adjustments required to keep the hedge delta neutral). The first test did not yield any significant profits but the second portfolio showed that the model is able to identify overvalued and undervalued options fairly well and substantial profits are available in the market. When changing the period of systematic adjustment to two days it was found that the return fell, but Galai (1977) noted that this was not a general result as the exact direction of the change in systematic risk and in realized returns cannot be determined in advance without complete knowledge of all hedged positions and changes in stock and option prices. French and Martin (1988) confirmed the contention that the daily hedge adjustment justifies an expected riskless return for correctly valued options.

The test for market efficiency then, involves ascertaining whether abnormal profit opportunities existed in the market - if such opportunities are found and they persist over time one can argue that the market was inefficient over that period. Performing a similar test to that in the ex post testing and with a one day lag Galai (1977) found returns to be below the ex post returns and by changing the holding period to two days, they dropped significantly. There is thus evidence that profit opportunities did exist, however, this is before the effect of transaction costs (Galai, 1977). Looking at the impact of transactions costs, Galai (1977) found that by imposing a 1% transaction cost on trades the profit was reduced to nearly nil, thus confirming the efficiency of the CBOE.

Macbeth and Merville (1979) tested the Black Scholes model by solving for the implied volatility to determine whether there are different values of volatility on the same stock and whether this can be explained in some systematic way. They found that the implied variance appears to change from day to day i.e. non-stationary, and that the implied variance rates decline as the exercise price increases. This phenomenon is termed the volatility smile and is discussed in detail in Chapter 5. According to the Black-Scholes model, whether an option is in or out of the money, the implied variance should be the same - the mere act of changing the strike price should have no effect on the volatility of the underlying stock as volatility is related to the stock and not the exercise price. However, their results showed that implied variances are different depending on whether the stock is in, near or out of the money. There was also a tendency for in the money options with a short time to expiration to have larger implied standard deviations than options with the same exercise price but a longer time to expiration. In contrast however, out of the money options with a short time to expiration were found to have smaller implied standard deviations than options with the same exercise price but a longer time to expiration.

These findings are consistent with Black's (1975) statement. Thus given that the partial derivative of the call price with respect to the volatility is greater than zero, the Black Scholes model must yield call prices which exceed observed market prices for options out of the money and call prices which are below market prices for options that are in the money (because the ISDs decline as the strike price increases).

The results of the Macbeth and Merville (1978) study can be summarized as follows :

- the Black Scholes model prices are on average less (greater) than the market prices for in the money (out of the money) options;
- with the exception of out of the money options with less that ninety days to expiration, the extent to which the Black Scholes model underprices (overprices) an in the money (out of the money) option increases with the extent to which the option is in the money (out of the money) and decreases as the time to expiration decreases;
- Black Scholes model prices with less that ninety days to expiration are on average greater than market prices but there is no consistent relationship between the extent to which these options are overpriced and the degree to which they are out of the money or the time to expiration.

Sterk (1982) attempted to explain the conflicting results of previous option pricing market efficiency research :

- Black's (1975) findings that the model underprices (overprices) deep out of the money (in the money) call options, and
- Merton (1973) and MacBeth and Merville's (1979) findings of exactly the opposite in their studies - the model overprices (underprices) deep out of the money (in the money) call options,

by means of a dividend adjusted Black Scholes model. Sterk (1982) used the Roll Geske Whaley model which specifically accounts for dividends and the possibility of early exercise. Testing a sample of CBOE options and specifically accounting for the degree to which they were in or out of the money, it was found that the median deviations for the out of the money options were positive for low dividend stocks but became progressively more negative as the dividend increased. For in the money options the reverse was found to apply - the model underpriced (overpriced) out of the money options when the dividend was small (large). The model overpriced (underpriced) in the money options when the dividend was small (large). This behaviour could thus explain the seemingly contradictory results of the previous research findings - if the research sample was based on stocks with a low average dividend the conclusion would be that the model underprices (overprices) out of the money (in the money) options. However if the reverse was true - the sample consisted on average of high dividend stocks - just the opposite conclusion would be reached. The magnitude of the dividend on the underlying stock appears to influence the over or underpricing of the model :

- out of the money options on stocks with low dividends are underpriced by the models,
- in the money options on stocks with low dividends are overpriced by the models,

and this dividend induced behaviour reverses as the dividend increases.

An alternative approach to testing the efficiency of the options market is to make use of the relationship between put and call options on the same stock (Hull, 1992). The put-call parity relationship exists because the put, call and underlying stock form an interrelated securities complex in which any of the three instruments can be combined in such a manner to yield the profit and loss opportunities of the third instrument. Due to the conversion possibilities, theoretical put call parity models can be developed to determine a put (call) price given a corresponding call (put) price and other relevant information. Should the actual put or call price deviate substantially from its parity price, then there would be an arbitrage opportunity for a riskless profit to be made. Klemkosky and Resnick (1979) set up a sample of such portfolios and tested whether there were in fact arbitrage opportunities in the market place. Their findings were consistent with put-call parity and thus supported the notion of market efficiency in the options market.

10.5 THE SOUTH AFRICAN EVIDENCE

Le Plastier et al (1986) investigated the valuation of warrant and gilt option prices in the SA market. Only warrants on AMIC, East Daggafontein, ERPM and Western Deep were included in the sample due to the thinness of trading of the other warrants. The warrant prices were calculated using a dividend-adjusted Black Scholes model. Due to the warrants being of a long term nature and the fact that they are issued out of the money and thus volatility will impact the option price heavily, they calculated the volatility based on a historical period of 26 weeks and volatility for the gilts on a 13 week period. The findings for warrant valuation were rather surprising in that only the Western Deep Level's warrant price difference was found to not be statistically different from zero. Additionally, when the values were recalculated with a dilution adjustment it was found that this has only a relatively small effect and for three of the four warrants actually weakened the result. They attributed this to the fact that the number of warrants is usually small in relation to the number of shares. It should be noted that warrants are thinly traded in the market place and it may thus be possible that the market is in fact valuing them incorrectly.

The results for the valuation of gilts were more impressive and this can be attributed to the much larger volume of trading in options on gilts. A paired difference t-test showed that there was no statistical difference between the actual and calculated option price even at the 20% significance level. This is however not surprising as the market makers, the banks, use the same model to determine their bid and ask prices in the market (Le Plastrier and Thomas, 1984).

Marsh (1986) studied the accuracy of the Black Scholes model in calculating prices of nil paid letters. His first t-test rejected the hypothesis of the accuracy of the Black Scholes model with a t-statistic of -3.4. Postulating that the poor estimate of volatility was the cause of such mispricing, Marsh (1986) replaced the historical volatility used with the actual volatility observed over the trading period. Surprisingly, the t-statistic worsened to -6.96. The model appeared to be overvaluing the nil paid letters. It is thus possible that the volatility was not the main reason for the inaccurate results. Marsh (1986) was thus forced to conclude that the Black Scholes model was not able to price nil paid letters accurately.

Fenwick (1996) investigated the efficiency of the South African rights issue market using a dividend adjusted Black Scholes model. While realizing that the tests are unable to distinguish between the efficiency of the market and the validity of the model, his results indicated that a dilution adjusted Black Scholes model, on average, predicted the actual prices fairly well. It should be noted that the opposite conclusion was reached by Law (1994) who additionally found that the dilution adjustment caused the results to be worse. Fenwick (1996) did however note that the degree of liquidity improved the efficiency of the pricing of the rights issues market.

10.6 CONCLUSION

The above results are somewhat discouraging for the researcher interested in the valuation of options on equities. While the valuation of options on gilts appears to give fairly accurate results, the valuation of warrants and nil paid letters shows that the South African market is not very efficient in valuing these instruments. The only comfort than can be obtained is from the Fenwick (1996) study which concluded that the South African rights issues market is efficient. These results are thus fairly inconclusive and this factor will be accounted for when testing the various models.

CHAPTER 11 - RESEARCH METHODOLOGY

11.1 OUTLINE TO METHODOLOGY

The axial center around which this research effort turns will be motivated here and thereafter formulated into testable statements of hypotheses. The assumptions and limitations of the study will be stated, followed by the research methodology. The data sample will then be analyzed with the necessary descriptive statistics.

11.2 THE RESEARCH PROBLEM

This research proposes to identify the most accurate method of pricing nil paid letters using option pricing models, including the Black Scholes model, the Cox Constant Elasticity of Variance model and the Merton Jump Diffusion model, previously applied to the valuation of other financial assets, and to determine the set of input parameters that lead to optimal results.

11.3 THE SUB-PROBLEMS

11.3.1 THE FIRST SUBPROBLEM

The first subproblem is to identify which measure of volatility results in the most accurate model prices.

11.3.2 THE SECOND SUBPROBLEM

The second subproblem is to adjust the model prices for the effects of nil trading volume to identify the impact of nil trading volume on model pricing accuracy.

11.3.3 THE THIRD SUBPROBLEM

The third subproblem is to identify whether the absolute magnitude of the nil paid letter prices has an impact on the accuracy of the model prices.

11.3.4 THE FOURTH SUBPROBLEM

The fourth subproblem is to identify whether the market takes cognisance of the effect of dilution when pricing nil paid letters.

11.4 THE HYPOTHESIS STATEMENTS

11.4.1 MAIN HYPOTHESIS STATEMENTS

NULL HYPOTHESIS

H_0 The Black Scholes model, Cox Square Root Constant Elasticity of Variance model, Cox Absolute Constant Elasticity of Variance model and Merton Jump Diffusion model will on average correctly value nil paid letters.

ALTERNATE HYPOTHESIS

H_1 The above models will on average not be able to correctly value nil paid letters.

11.4.2 SUB-HYPOTHESIS STATEMENTS

H_2 Calculating volatility based on different historical periods will not affect the accuracy of the model prices.

H_3 Trading volume will not impact on the accuracy of the model prices.

H_4 The absolute magnitude of the nil paid letters price will not affect the accuracy of the model prices.

H_5 Adjustment of the models for dilution will not affect the accuracy of the model prices.

11.5 ASSUMPTIONS AND LIMITATIONS OF THE STUDY

The following are the basic assumptions of the study which are consistent with the Black and Scholes (1975) assumptions as well as those generally used in other empirical studies on option pricing (Hull, 1991; Macbeth and Merville, 1980) :

1. The short term interest rate is known and is constant through time.
2. The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite time interval is lognormal. The variance rate of the return of the stock is constant.

3. The stock pays no dividends or other distributions.
4. The option is European.
5. There are no transaction costs in buying or selling the stock or option.
6. It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short term interest rate.
7. There are no penalties to short selling. A seller who does not own the security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

Additionally, the Merton jump diffusion model relies on the CAPM. Some of the assumptions are relaxed when the different pricing models are used. The other limitations of the study relate to the data used. The possibility of miscoded security prices has been minimized by running a Visual Basic routine programmed to detect closing stock and rights prices which differ by more than 10% of the average of the day's high and low closing price. Any outliers were investigated and if they appeared to be errors the stock was excluded from the final sample studied. The only possible errors noted were a few stocks for which the daily low price had not been captured. These stocks were eliminated from the sample as the above reasonability test could not be performed on them.

11.6 THE NEED FOR THIS STUDY

There has been a lack of empirical research in South Africa on the topic of derivative pricing in general and option pricing in particular. The few studies that have examined option pricing on the shares of companies have only employed the Black Scholes model and the conclusions reached were generally in direct conflict to similar international studies (see Chapter 9). It is thus necessary to ascertain whether there is an option pricing model that is able to accurately predict the prices of nil paid letters in the South African market and to attempt to identify the reason why previous studies' results contradicted the international empirical evidence.

11.7 BASIC RESEARCH METHODOLOGY

The methodology of testing for excess returns in other studies on option pricing (Macbeth and Merville, 1980) and warrant pricing (Kremer and Roenfeldt, 1993) has been followed here with some additional modifications to account for the specifics of the South African securities market such as thin trading. The methodology involves testing the observed differences between the actual rights prices and those calculated using the various option pricing models employed. Due to the large sample size of 196 rights issues, parametric tests were employed to test for statistical significance.

11.8 SAMPLE SELECTION

All rights issues by companies listed on the Johannesburg Stock Exchange during the period 1 Jan 1992 to 30 April 1998 will be examined subject to the following filter rule :

- only rights issues of that company's shares;
- only companies without confounding events (including mergers and acquisitions);
- only companies for which a complete data set is available.

The rights were selected from the Monthly JSE Bulletins from 1992 through 1998. This resulted in a sample of 196 rights issues in the final sample. The sample is summarised in Appendix 4.

Daily closing, high and low share prices and nil paid letter (hereafter NPL) prices together with their respective trading volumes were obtained from the University of Stellenbosch Business School. The 90 day BA rate was obtained from the same source and used as the riskfree rate. Dilution ratios and rights issue size statistics were obtained from the Monthly JSE Bulletins. Incomplete data sets were investigated further with the JSE.

11.9 SAMPLE DESCRIPTIVE STATISTICS

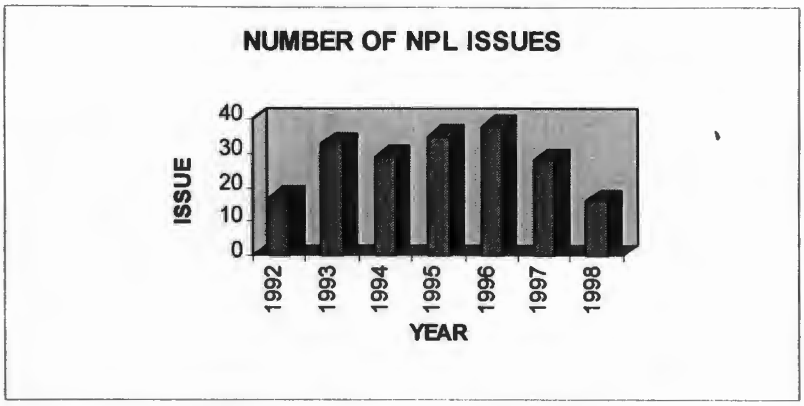
The data sample will be examined via the use of descriptive statistics to ascertain the nature and characteristics of the underlying data employed in this study.

11.9.1 VOLUME OF RIGHTS ISSUES

The NPL issues were spread fairly evenly across the sample of years. 1992 and 1998 issues are lower than the average as 1998 was only investigated to April and 1992 data required 12 months of trading data to pass the filter rule for which the earlier portions of the year failed the test as the data was only available from July 1991. This is illustrated in Table 11.1

1992	1993	1994	1995	1996	1997	1998
17	33	29	35	38	28	16

Table 11.1 NPL Issues by Year



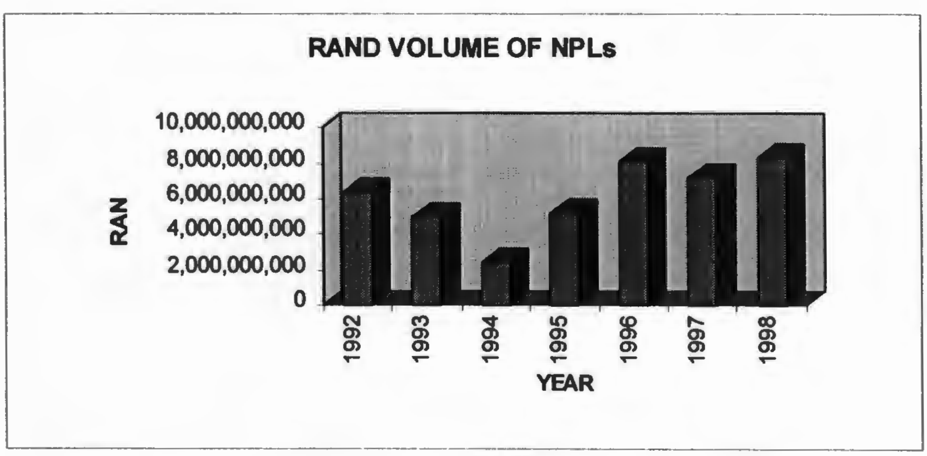
Graph 11.1 Number of NPL Issues by Year

It is evident from Graph 11.2 that the average Rand size of issues has increased from R374 million in 1992 to R517 million in 1998. The 1998 data are however distorted by two very large rights issues, by Rand Merchant Bank Holdings and Momentum Life, which were both abnormally large when compared to the average rights issue, thus distorting the statistics slightly.

1992	1993	1994	1995	1996	1997	1998
6,362,080,026	4,933,405,611	2,416,995,985	5,200,570,837	8,011,256,486	7,171,707,684	8,272,529,925

Table 11.2 NPL Rand Values by Year

This is graphed below.



Graph 11.2 Rand Values of NPLs by Year

11.9.2 STOCK PRICE ANALYSIS

The stock price and return statistics were investigated to ascertain their distributional parameters. In option pricing one of the fundamental assumptions of the Black Scholes model is that the terminal stock price distribution is lognormal (Black and Scholes, 1973). Statistics of the sample of stock data used show, as expected and consistent with the other empirical work on stock price and return distributions (Taylor, 1986), that there is a high degree of leptokurtosis in the returns. This is the familiar fat tailed distribution which deviates from the usual assumption of lognormality (see Chapter 1). Departures from normality can be tested in a variety of ways including :

1. fitting a normal curve to the data and then using a chi squared test for goodness of fit,
2. calculating certain functions of the moments of the data and testing for significance of their departure from the expected values for a normal population,
3. calculating Geary's ratio.

This research will use the second method as it is the preferred method in the other studies that have tested stock price returns for normality (Ozen, 1977). The two functions that will be investigated are the sample skewness and kurtosis parameters. It should be noted that all of the above methods assume that the underlying data are continuous. They may still be employed however, if the data are discrete but unimodal and equidistant, and therefore may be approximated by a continuous distribution.

11.9.2.1 SAMPLE SKEWNESS ANALYSIS

The moment ratios of the logarithmic stock return distributions have been calculated as using the following formulas :

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$m_r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^r}{n}$$

where :

n	is the total number of stock return observations
x_i	is the i-th observation
$\bar{x}$	is the mean of the observations
m_r	is the rth sample moment about the mean

The Pearson co-efficient of skewness is then calculated as :

$$\frac{m_3}{m_2^{3/2}}$$

In a normal population this co-efficient takes on the value of zero. Departure from this value indicates skewness in the distribution. A value less than zero indicates negative skewness and a value greater than zero indicates positive skewness. The following table indicates the values obtained for the sample of shares under investigation :

COUNT	196
AVERAGE	0.46
MAX	8.37
MIN	(6.30)
MEDIAN	0.41

Table 11.3 Sample skewness statistics

A Pearson statistic of 0.46 is sufficiently close to zero to prevent the rejection of an assumption of normality of returns. This statistic compares favorably to other South African stock return distributions research where it was generally also not far from zero (Ozen, 1977 and Bowie, 1994). An interesting finding in these other studies is that log price relatives appear to be positively skew in a uptrend and negatively skew in a downtrend. The fact that the current sample extends over a period of seven years would thus eliminate such biases.

11.9.2.2 SAMPLE KURTOSIS ANALYSIS

The sample moments are calculated as shown above and then used as inputs into the kurtosis formula :

$$\frac{m_4}{m_2^2}$$

In a normal population this parameter takes on the value of three. Departures from this value indicate kurtosis, with values less than three indicating platykurtosis and values greater than three indicating leptokurtosis. The greater the absolute difference from three, the greater the departure from normality.

COUNT	196
AVERAGE	16.83
MAX	96.37
MIN	0.43
MEDIAN	13.36

Table 11.4 Sample kurtosis statistics

The mean kurtosis statistic of 16.83 in Table 11.4 indicates a clear departure from a normal distribution. This departure in daily log price relatives is consistent with the findings of Mandelbrot (1963) and Fama (1965). It is also consistent with South African research on stock returns distributions as both Ozen (1977) and Bowie (1994) found significant departures from normality for JSE stock returns using kurtosis analysis. Bowie (1994) investigated the issue further and found that thin trading severely impacted the kurtosis of returns with the most thinly traded stocks generally having high correlation co-efficients with high kurtosis statistics. Ozen (1977) also postulated that the more closely held shares are likely to reveal more leptokurtosis than widely held shares due to their trading volumes generally being lower. The South African market is characterized by such close holdings via the mechanisms of pyramid structures which may thus exacerbate the non-normality of returns. Either way, the non-normality of returns is a very evident phenomenon as indicated by the high degree of leptokurtosis and an option pricing model such as the Merton jump diffusion model that relaxes the assumption of the normality of the terminal stock price distribution, should be far more accurate in pricing options on equities in the South African market.

11.9.3 VOLATILITY ANALYSIS

Volatility estimation is absolutely crucial for the Black Scholes model. Volatility was calculated based on historical log returns, adjusted for the effects of dividends and stock splits calculated on a daily basis. Rogalski (1978) noted that the market price of a warrant regularly lies between prices obtained from the Black Scholes model employing :

- an estimate of historical volatility, and
- the actual volatility that occurs from the observation date to the expiry date.

Thus it seems that investors' expectations are based upon past volatility adjusted by correct anticipation of the direction of future volatility changes. Kremer and Roenfeldt (1992) incorporated this observation by employing two different volatility estimates for their model inputs :

1. over the 181 trading days before the observation date;
2. over a period of 90 trading days before the observation date to 90 trading days after the observation date.

The first estimate thus being based purely on historical data and the second incorporating an adjustment for future volatility changes. In a similar vein, for the purposes of this research volatility was calculated over four different periods :

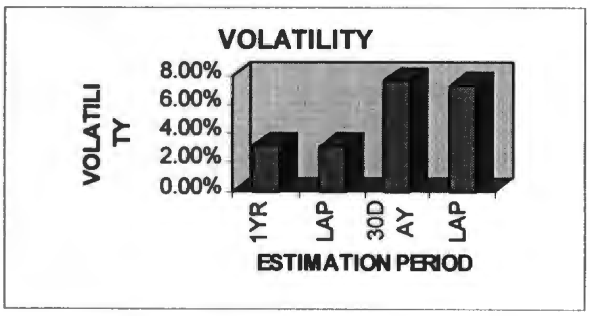
1. The 250 trading day period preceding the rights issue (termed "1 YR");
2. The 250 trading day period preceding the rights issue and including the rights trading period (termed "1 YR LAP");
3. The 30 day period preceding the rights issue (termed "30 DAY");
4. The 30 day period preceding the rights issue and including the rights trading period (termed "30 DAY LAP").

Method 3 is the traditionally favored. It was felt that it may be necessary, however, to use a longer period to get a better estimate of future volatility assuming that the past year's volatility is reflective of future volatility and thus the inclusion of method 1. Methods 2 and 4 explicitly account for the price volatility over the actual trading period. This is presumed to result in a better prediction of the actual NPL price as Black and Scholes (1975) mention that using the actual volatility over the life of the call should give more accurate model prices when using the Black Scholes model. The annualized volatility for each of these periods is calculated for the entire sample in Table 11.5.

	1YR	LAP	30DAY	LAP
COUNT	196	196	196	196
AVERAGE	3.14%	3.13%	7.69%	7.38%
MAX	20.48%	19.23%	31.75%	44.84%
MIN	0.06%	0.11%	0.00%	0.52%
MEDIAN	2.37%	2.37%	6.39%	5.63%
MODE	5.65%	6.09%	0.00%	24.97%
TTEST		22.20%	0.10%	0.07%
TSTAT		1.23	3.34	3.44

Table 11.5 Volatility t-test statistics

It is evident that the 1 year volatility estimates are much lower than either the 30 day or 30 day lap period annualized. It appears that there is a significant increase in volatility in the pre-rights trading period compared to the 1 year historical period as evidenced by the two sample matched pairs t-statistics of 3.34 and 3.44 at the 95% confidence level, and this continues into the trading period. This could be attributed to the increased trading volume during the rights trading period and is consistent with the findings of Chan (1997). These volatilities are graphed below.



Graph 11.3 Different Volatility Estimates

Using the pre-announcement volatility as an estimate of the volatility input for the Black Scholes model would thus lead to mispricing of the NPLs. This issue will be investigated when comparing model prices incorporating the 30 day interval volatility measure and those calculated based on the 250 day interval volatility measure.

11.9.4 TRADING PERIOD ANALYSIS

The number of days for which the rights issues traded is summarized in Table 11.6. There is a fair amount of variation and not all rights traded for exactly the same period.

COUNT	196
AVERAGE	17
MAX	23
MIN	7
MEDIAN	18
MODE	18

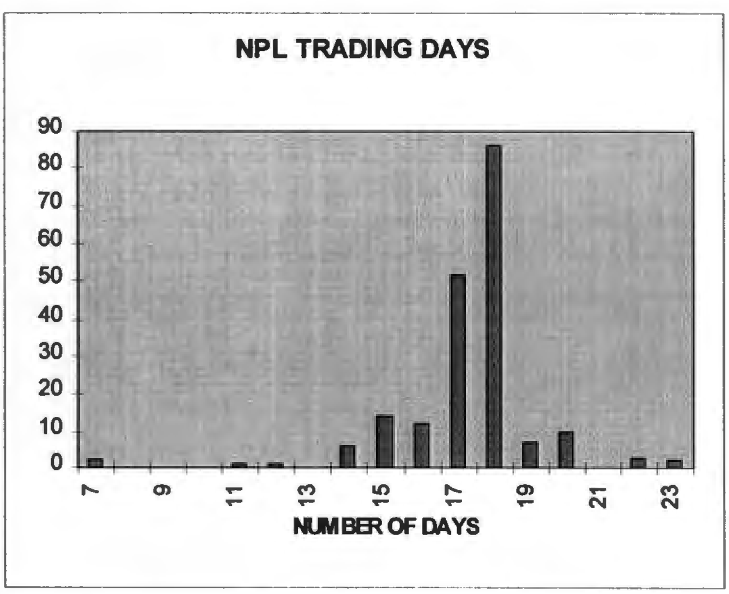
Table 11.6 NPL Trading Days Statistics

The average trading period is 17 days with one NPL that traded for a full 23 days and one as little as 7 days. The sample should thus provide reasonable results with an average trading period long enough to be tested thoroughly.

INTERVAL	COUNT
6 to 10	2
11 to 15	22
16 to 20	167
21 to 25	5

Table 11.7 Interval of NPL Trading Days

The distribution of trading days is thus concentrated in the 16 to 20 days category as expected and this is displayed in Graph 11.4.



Graph 11.4 Distribution of Number of Days the NPL Traded

11.10 INPUT PARAMETER ESTIMATION

This section will describe the issues encountered when calculating the various models' input parameters. Each of the models requires a set of input parameters based on the underlying process for the stock price. The use of the Black Scholes formula requires the input of five variables :

1. stock price
2. exercise price
3. time to maturity
4. interest rate
5. stock volatility

The first four parameters are relatively easy to obtain. The last parameter, the stock volatility, is the only input that is not directly observable and thus requires further analysis.

The Cox constant elasticity of variance model requires an additional parameter - the characteristic parameter of the diffusion process. In this study however, two simplified versions of the model are employed where the characteristic parameter takes on the values of zero or unity and estimation thereof is thus not required (see Chapter 6).

The Merton jump diffusion model requires an additional three parameters to be calculated. These are the :

6. diffusion process variance
7. jump process variance
8. jump process mean

Prior empirical research has shown these parameters to be particularly complex to calculate (see Chapter 8). The calculation of the above parameters is outlined below.

11.10.1 UNDERLYING SHARE PRICE

The underlying share price was calculated as the closing price for that particular share on the day concerned. Closing prices were compared to the high and low for the day to check for reasonableness in case of any recording errors. Where shares changed name these were investigated via the use of McGregors (1997, 1995, 1994) and the original shares traced for prices. Where the original share could not be obtained, thus limiting the number of share price observations to less than one year, the share was excluded from the final sample.

One issue confronted was when the share paid a dividend or there was a stock split. The adjustment processed is in terms of Cox and Rubenstein (1991). For share splits, when calculating the log return for the day, the numerator is adjusted by the split ratio and thereafter calculated as normal to remove what would appear to be a large price gain or loss for the day. For stock dividends, the dividend paid is added to the ex-dividend share price to similarly avoid an erroneous one day return.

11.10.2 EXERCISE PRICE

Exercise prices were extracted from the Monthly JSE Bulletins. No NPLs were noted with changing exercise prices so this factor could be ignored. Where shareholders were offered a choice between ordinary shares and some other security e.g. debentures or preference shares, the rights issue was treated as if it were for the ordinary shares only. While not theoretically correct, the few cases where this occurred were recalculated based on the exercise price being the present value of the other security concerned (appropriately discounted at the riskfree rate) and no significant differences were noted.

Exercise prices were evaluated for reasonableness and, where considered incorrect, were investigated further via press correspondence.

11.10.3 RISK FREE RATE

The interest rate used should measure the risk-free borrowing and lending rate over the period of the option (Bookstaber, 1990). Borrowing and lending rates typically differ and an average of the two is thus usually used. Bookstaber (1990) recommended using the Certificate of Deposit rate while Cox and Rubenstein (1990) recommended using the Treasury bill rate. It is also necessary to obtain a rate that matches the duration of the option contract. It was thus decided to use the Bankers Acceptance rate (hereafter BA rate) as the risk-free rate. This is a fairly riskless security and is also short term and covers the roughly one month period for which NPL issues trade. While the rate does not match the maturity of the option precisely, the theta of the option with such a short maturity is fairly insensitive to the interest rate used (see Chapter 4).

The BA rates were calculated from the first day that the particular NPL began to trade. It was initially decided to use the BA rate on each day for which a valuation was performed, seeing as model prices are calculated for each day, but due to the short time to maturity and extensive computation required to change the interest rate each day this was not undertaken and the original BA rate existing on the first day of trading was used for each calculation for that particular share. Due to the short time to maturity this had an insignificant effect which was confirmed by testing a sample of NPL models with the actual interest rate for each day against the model using the original day one interest rate. The rates were converted to annualized continuously compounded rates due to the Black Scholes model being a continuous trading model and thus requiring continuously compounded interest rates. This was done using the standard compounding formula (Zhang, 1997) :

$$FACTOR = \lim_{n \to \infty}\left(1 + \frac{r}{n}\right)^{t.n} = \left[\lim_{n \to \infty}\left(1 + \frac{r}{n}\right)^{n/r}\right]^{r.t} = e^{r.t}$$

where FACTOR is the continuous compounding factor. The adjustment to compound rates was found to result in insignificant differences in the model prices but was employed as it is theoretically the correct rate to use.

11.10.4 TIME TO MATURITY

The time to maturity is calculated as the calendar time outstanding between the day on which a valuation is performed and the expiry date of the offer. The expiry dates were extracted from the Monthly JSE Bulletins. The calendar days outstanding were calculated using a Visual Basic macro and based on a year of 365 trading days as suggested by Cox and Rubenstein (1985).

Due to the debate over whether a 250 day trading year or a 365 day calendar year should be used, the calculations were performed over a 365 day calendar year and a 250 day trading year but no significant differences were noted, primarily due to the theta of the option having such a small effect when the time to maturity is so short (see Chapter 4).

11.10.5 VOLATILITY

The volatility is the only input of the Black Scholes model which requires subjective calculation and is uncertain. Numerous researchers have disagreed as to how the volatility is to be calculated and there are thus a host of methods that can be used (see Chapter 5). Due to the majority of current empirical studies as well as practitioners using the standard deviation of past returns (Cox and Rubinstein, 1985), it was decided to employ this method.

There appears to be much debate as to over which period the volatility is to be calculated. One ought not to go too far back in time as the volatility thus calculated may differ from what it is expected to be in the near future over the NPL trading period. One ought also not to choose too short a period as it may not be reflective of the underlying share's actual volatility. A balance must thus be sought between capturing the structure of the underlying share's volatility but also adjusting this for any changes expected over the life of the option. Black and Scholes (1975) mention that using the volatility over the actual life of the option leads to better results. It has also been noted in tests of implied standard deviations that practitioners appear to use measures of historical volatility but updated for expected future changes therein.

Volatility was calculated over four different intervals so that the model outputs using each of these volatility estimates could be compared. The four different intervals are as follows, together with the reasons for their selection :

1. Volatility was calculated over the one year period preceding the rights issue. This measure should capture the long term volatility of the underlying share and is a long enough period to encapsulate the underlying process which the volatility follows.

2. Volatility was calculated over the one year period preceding the rights issue and including the actual rights issue period. This measure should better reflect the actual volatility as it includes the actual volatility over the rights trading period.
3. Volatility was calculated over the thirty trading days prior to the rights issue. This measure reflects the most recent past of the underlying share and may thus be better reflective of future volatility. It also ignores the effects of share price movements before the thirty day period which may no longer be relevant if the share does not follow a deterministic volatility process.
4. Volatility was calculated over the thirty trading days prior to the rights issue and including the actual rights issue period. The measure improves on the above interval as it specifically accounts for the volatility over the actual rights trading period.

The variances were calculated using the Cox and Rubenstein (1991) formula :

$$\sigma^2 = \frac{1}{n-1} . \sum_{k=1}^{n} (\ln(R_k - \mu)^2$$

where n is the number of days returns used to calculate the variance (based on the above four categories), R_k is the natural log return for that particular day calculated as :

$$\ln(R_k) = \frac{\ln S_2}{\ln S_1}$$

where S_2, S_1 are the share prices on the final and previous day respectively. The Parkinson (1977) as well as the Garman and Klass (1980) methods were employed for a sample of the shares (see Chapter 5). The variance estimates were insignificantly different however and owing to the low vega of the option (see Chapter 4) when so close to maturity and the fact that these estimators require considerably more computation time, the traditional method was employed instead.

11.10.6 MERTON JUMP DIFFUSION PARAMETERS

The sample moments for the stock are required in order to calculate the parameters of the jump diffusion process. To compute the jump diffusion model three additional parameters thus require estimation :

- λ the mean number of Poisson jumps per unit time;
- δ^2 the variance of the Poisson jump size;
- y^2 the diffusion process variance in the absence of Poisson jumps.

Beckers (1981) developed the cumulant matching procedure for this purpose. Stock returns over the estimation period are used to calculate the first six sample moments :

$$m_s = \frac{1}{T}\sum_{t=1}^{T}[\Delta Z(t)]^s$$

where ΔZ is the change in the natural log of the security price during time t and T is the number of days in the estimation period. Using the sample moments above, Kendall and Stuart (1977) showed that the sample cumulants can be calculated as :

$$\begin{aligned}
K_1 &= m_1 \\
K_2 &= m_2 - m_1^2 \\
K_4 &= m_4 - 4.m_3.m_1 - 3.m_2^2 + 12.m_2.m_1^2 - 6.m_1^4 \\
K_6 &= m_6 - 6.m_5.m_1 - 15.m_4.m_2 + 30.m_4.m_1^2 - 10.m_3^2 + 120.m_3.m_2.m_1 \\
&\quad - 120.m_3.m_1^3 + 30.m_2^3 - 270.m_2^2.m_1^2 + 360.m_2.m_1^4 - 120.m_1^6
\end{aligned}$$

Beckers (1981) then calculated the parameter estimates as :

$$\lambda = \frac{25.K_4^3}{3.K_6^2}$$

$$y^2 = K_2 - \frac{5.K_4^2}{3.K_6}$$

$$\delta^2 = \frac{K_6}{5.K_4}$$

This procedure was performed for all the stocks in the sample. Unfortunately the same problem experienced by Beckers (1981) was encountered in this study - some of the variance statistics calculated using the above procedure were negative. In general this occurs if the stock does not exhibit high kurtosis. Thus a modified method is required, as the option pricing models will not accept negative variance estimates. An additional assumption that the probability of a jump be the same for all the stocks under consideration was thus made which is in accordance with the methodology employed in other studies using the Merton jump diffusion model (Beckers, 1981 and Kremer and Roenfeldt, 1992). An average probability estimate is obtained by pooling the data using the following procedure. Let :

$$\frac{K_4}{K_2^2} = \alpha_1$$
$$\frac{K_6}{K_3^2} = \alpha_2$$

Assuming that $\frac{\delta^2}{\sigma^2} = n$ is constant across stocks :

$$\alpha_1 = \frac{3.n^2.\lambda}{(1+\lambda.n)^2}$$
$$\alpha_2 = \frac{15.n^3.\lambda}{(1+\lambda.n)^3}$$

The above system of two equations in two unkowns is solved for $\hat{\lambda}$, $\hat{n}$ using the sample averages above as inputs. The following system of equations then needs to be solved for each stock :

$$K_1 = \alpha$$
$$K_2 = \sigma^2 + \lambda.\delta^2$$
$$K_4 = 3\delta^4\lambda$$

The pooled estimates were calculated based on calendar years as it was felt that to calculate an overall average across all the years of the study would be incorrect as the mean is unlikely to be stationery and this procedure thus reduces the bias somewhat. The jump diffusion parameters calculated for each stock are listed in Appendix 5.

11.11 VALUATION MODELS EMPLOYED

The hypothesis statement consists of finding which model best estimates the actual trading prices of NPLs observed. Three main models were employed in this study with one model being employed under two different assumptions :

1. Black Scholes (1973) model;
2. Cox Constant Elasticity of Variance model (1975) with its two sub-categories being :
   2.1. Square root model,
   2.2. Absolute model;
3. Merton Jump Diffusion model (1975).

11.11.1 BLACK SCHOLES MODEL

The traditional Black Scholes model is used as formulated by Black and Scholes (1973). This model will be used as the basic model from which to compare the results of the other models. The model is used as a base as it is the most simplistic of the three in that it has the most restrictive assumptions which the other models attempt to relax. The model is as follows :

$$C = S.N(d_1) - X.e^{-r(T-t)}.N(d_2)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)(T-t)}{\sigma.\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/X) + (r - \sigma^2/2)(T-t)}{\sigma.\sqrt{T-t}}$$

The terminology used and the model itself are explained in Chapter 4. The Visual Basic routine is included in Appendix 8.

11.11.2 CONSTANT ELASTICITY OF VARIANCE MODEL

This model is employed to account for the inverse relationship between a stock's price and its instantaneous variance rate as put forward by Christie (1978). This model thus relaxes the assumption of constant variance required for the Black Scholes model. The share is presumed to have a constant elasticity of variance. The model has a complex solution which is not easy to implement in practice. Cox (1975) simplified the model for two special cases of its parameters which enable it to be tested empirically.

11.11.2.1 SQUARE ROOT PROCESS

The equation for this process is :

$$C = S.N(q(4)) - K.e^{-r.t}.N(q(0))$$

where for $w = 0$ *or* $w = 4$

$$q(w) = \frac{1 + h.(h-1).\left(\frac{w+2.y}{(w+y)^2}\right) - h.(h-1).(2-h).(1-3.h).\left(\frac{(w+2.y)^2}{2.(w+y)^4}\right) - \left(\frac{z}{w+y}\right)^h}{\left[2.h^2.\left(\frac{w+2.y}{(w+y)^2}\right).\left(1-(1-h).(1-3.h).\left(\frac{w+2.y}{(w+y)^2}\right)\right)\right]^{1/2}}$$

$$h(w) = 1 - 2.(w+y).(w+3.y)(w+2.y)^{-2} / 3$$

$$y = 4.r.S / (\sigma^2.(1-e^{-r.t}))$$

$$z = 4.r.K / (\sigma^2.(e^{r.t} - 1))$$

and with N representing the cumulative standard normal distribution as in the Black-Scholes formula. The terminology and model are described in Chapter 6. The Visual Basic routine is included in Appendix 8.

11.11.2.2 ABSOLUTE PROCESS

The equation for this process is :

$$C = (S - K.e^{-rt}).N(h_1) + (S + K.e^{-rt}).N(h_2) + v(N'(h_1) - N'(h_2))$$

$$\begin{aligned} &where \\ &v = \sigma.\sqrt{((1-e^{-2rt})/2.r)} \\ &h_1 = (S - K.e^{-rt})/v \\ &h_2 = (-S - K.e^{-rt})/v \end{aligned}$$

and with N representing the cumulative standard normal distribution as in the Black-Scholes formula and N' representing the density function of the standard normal random variable. The remaining terms are as for the Black Scholes formula described above. The Visual Basic routine is included in Appendix 8.

11.11.3 MERTON JUMP DIFFUSION MODEL

The Merton (1975) model takes account of the fact that the underlying stock process is not continuous. This thus relaxes the assumption of a continuous process for the underlying stock returns required for the Black Scholes model. The model is applied because it appears to more realistically represent the path of stock price movements empirically observed and should thus lead to better option price estimates. The model, assuming the jump process to be lognormally distributed is :

$$F(\sigma^2,\lambda,\delta^2;S,K,r,t)=\sum_{n=0}^{\infty}\frac{e^{-\lambda t}(\lambda.t)^n}{n!}.W((\sigma^2+n.\delta^2/t);S,K,r,t)$$

where W represents the Black Scholes model outlined above. The above formula has an infinite sum which does not lend itself to easy testing. Following the findings outlined in Chapter 8, the summation will be truncated at ten which should provide accurate results. The effect of truncation is summarized in Appendix 2. The individual terms of the model are described in detail in Chapter 8 on the Merton model. The Visual Basic routine is included in Appendix 8.

11.12 STATISTICAL TESTING PROCEDURE

For each of the models used and then across each of the different filter rules, the percentage difference between the model price and the actual trading price of the nil paid letter, will be calculated as follows :

$$\%\ \text{Difference} = \frac{\text{Actual Price - Model Price}}{\text{Actual Price}} \times 100$$

If the particular model accurately estimates the market price of the nil paid letter, then the percentage difference will be close to zero. An average actual nil paid letter price and model price is then calculated based on the number of days that the rights existed and the above formula applied thereto.

Due to the large sample size, it can be assumed that the differences will be normally distributed (Taha, 1987) about the mean and that the mean is zero (Marsh, 1986). T-statistics were then calculated for each model. These were based on a two tail test at the 95% confidence level. Comparison of the t-statistics will then show which model is able to most accurately calculate nil paid letters prices.

CHAPTER 12 - RESULTS OF TESTING

12.1 RESULTS OF PRIMARY HYPOTHESIS TEST

The primary null hypothesis that there is on average no difference between the mean actual prices and the prices calculated by the individual models, was tested using an *F*-test at the 5 % significance level. The null hypothesis was accepted for the unadjusted models per the following table :

MSB	24,502	TEST STAT	0.16
MSW	153,058	F STAT	1.85

Table 12.1 F-test for equality of means

This result is as expected for it was proposed that the models should all be able to accurately price nil paid letters and the above result confirms this contention. This result is in accordance with the international studies that have been performed on the pricing of rights issues, except that this study has additionally shown that other models to, are able to price the rights accurately. The fact that the Black Scholes model prices were accepted by the *F*-test is thus in accordance with the South African research by Fenwick (1994) but in direct conflict with the results of Law (1992) where the Black Scholes model was rejected as being able to calculate nil paid letters prices accurately.

The above results are comforting in that one can now accept the models as being able to accurately price nil paid letters and conduct further investigation into which of the models is the most accurate and which set of input parameters lead to optimal prices. The approach for the remainder of this chapter will be to examine the results of each model separately to gain a deeper understanding of the model and the applicability of its underlying assumptions and then investigate the relationship of the various input and adjustment factors that affect the results. First the unadjusted models will be tested. This will be followed by a comparison of the results across the various volatility estimation intervals. Thereafter the effect of adjusting for dilution will be examined. Thin trading adjustments and absolute value of price adjustments will be examined last whereafter the results will be summarised.

The terminology used relating to the various models tested requires some explanation. The following abbreviations will be used in the graphs and tables of this chapter :

BLACK SCHOLES	The traditional Black Scholes model (see Chapter 4).
SQUARE ROOT CEV	The CEV model with the square root special case for the characteristic parameter (see Chapter 6).
ABSOLUTE CEV	The CEV model with the absolute special case for the characteristic parameter (see Chapter 6).
JUMP DIFFUSION (ACTUAL)	The Merton jump diffusion model with the actual mean jump parameter calculated for each individual stock (see Chapter 8).
JUMP DIFFUSION (AVERAGE)	The Merton jump diffusion model using a pooled mean jump diffusion parameter for the stocks (see Chapter 8).

When comparing the different models it should be noted that the Merton jump diffusion model results are only calculated for the one year period of volatility estimation and not the thirty day period. To calculate the parameters of the jump diffusion model one needs a fairly lengthy historical period to capture the underlying diffusion process that is followed by the stock. Beckers (1978) suggested that a period of at least 180 days be used. The 30 day period was investigated but the results were meaningless, as expected, and have thus been excluded.

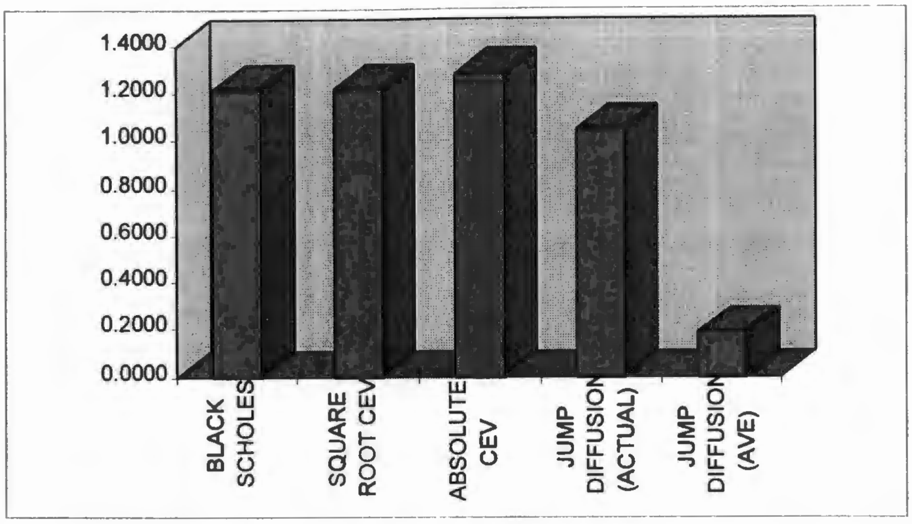
12.2 TESTING OF UNADJUSTED MODELS

The unadjusted models do not account for such factors as thin trading, dilution or low rights prices. They are thus the basic original models formulated, against which the adjusted models will be compared in the later sections. This section will begin with a comparison between the models based on volatility being calculated over a one year historical period as this appears to be the favoured interval employed in the majority of other empirical studies on option pricing and will thus highlight which model is the most accurate in pricing the rights using the most commonly accepted methodology for calculating the input parameters.

MODEL	1 YEAR PROB	T
BLACK SCHOLES	22.43%	1.2189
SQUARE ROOT CEV	22.49%	1.2173
ABSOLUTE CEV	20.09%	1.2832
JUMP DIFFUSION (ACTUAL)	30.16%	1.0443
JUMP DIFFUSION (AVE)	85.66%	0.1809

Table 12.2 T-Test results for unadjusted models

Table 12.2 summarises the results of two tailed matched pairs t-tests performed between each model and the actual nil paid letters prices observed. By the nature of the test, the lower the t-value, the closer are the model prices to the actual prices. Accordingly, the model that exhibits the lowest t-statistic will be the most accurate. For ease of comparison the individual t-values have been graphed in Graph 12.1.

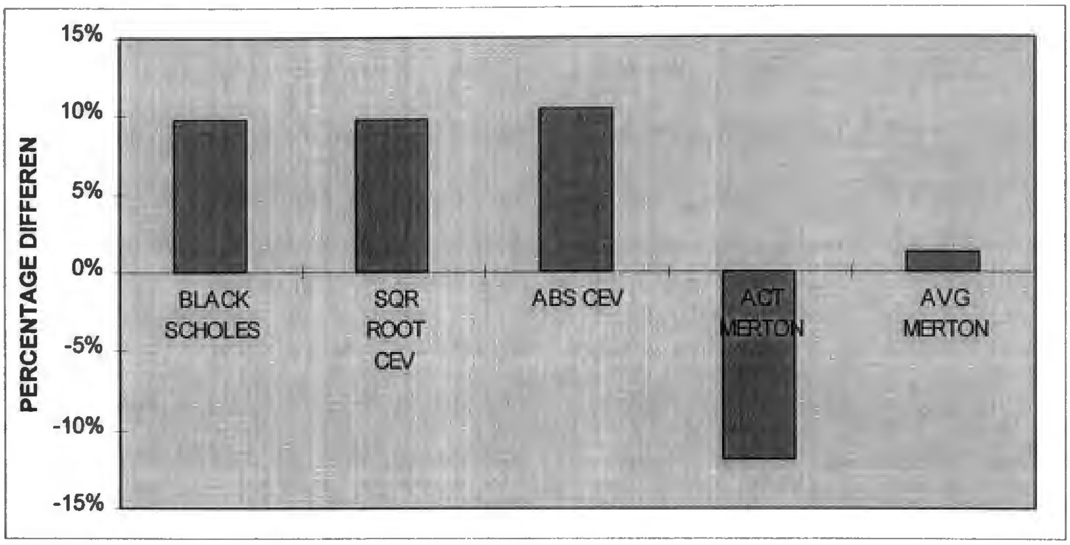


Graph 12.1 All Models Unadjusted for any Effects

The above results indicate that the Merton jump diffusion model calculated using an average mean jump parameter is superior to any of the other models. The t-statistic is well below that of any other model and additionally it will be shown later that this result holds across all the adjustments tested including dilution, nil trading etc.

It is interesting to note that the Merton model calculated using an averaging technique produces superior results to the same model with the actual input parameters and this will be discussed later. The absolute CEV model appears to be the most inaccurate of those tested. This may indicate that the underlying process does not follow such a distribution with characteristic CEV parameter of zero.

It is important to investigate the sign of the price differences from the actual prices to ascertain whether there is an under or overpricing phenomenon. This is displayed in Graph 12.2.



Graph 12.2 Percentage price differences from actual

It is evident that all of the models overprice the rights except for the Merton model with the inputs calculated as the actual diffusion parameters which appears to underprice the rights. This result could be due to a number of factors but is most likely to be caused by an overestimation of the underlying volatility as the phenomenon is consistent across each of the three models that make use of the standard historical volatility estimate.

12.3 TESTING THE DIFFERENT VOLATILITY ESTIMATION INTERVALS

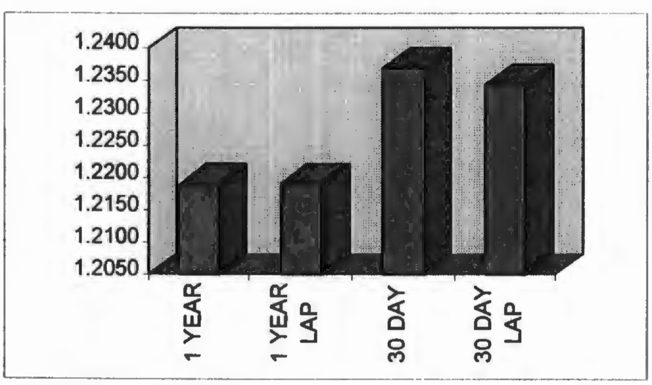
The above results are only based on calculating volatility across a one year historical period and the results thus need comparison across the different volatility intervals to see if they hold consistently and if there is one volatility calculation period which is superior to the others. The t-tests across the various volatility estimation intervals are summarised below in Table 12.3.

MODEL	1 YEAR PROB	1 YEAR T	1 YEAR LAP PROB	1 YEAR LAP T	30 DAY PROB	30 DAY T	30 DAY LAP PROB	30 DAY LAP T
BLACK SCHOLES	22.43%	1.2189	22.43%	1.2190	21.76%	1.2370	21.85%	1.2344
SQUARE ROOT CEV	22.49%	1.2173	22.49%	1.2173	22.49%	1.2174	22.49%	1.2174
ABSOLUTE CEV	20.09%	1.2832	20.09%	1.2832	20.09%	1.2832	20.09%	1.2832
JUMP DIFFUSION (ACTUAL)	30.16%	1.0443	30.16%	1.0443				
JUMP DIFFUSION (AVE)	85.66%	0.1809	85.66%	0.1809				

Table 12.3 T-Test results for unadjusted models

The table appears to show a trend that the one year historical period is favoured over the thirty day period. To test this contention the results for each model will be examined individually and these results then consolidated into a conclusion.

12.3.1 BLACK SCHOLES MODEL



Graph 12.3 Black Scholes Model by Variance Calculation

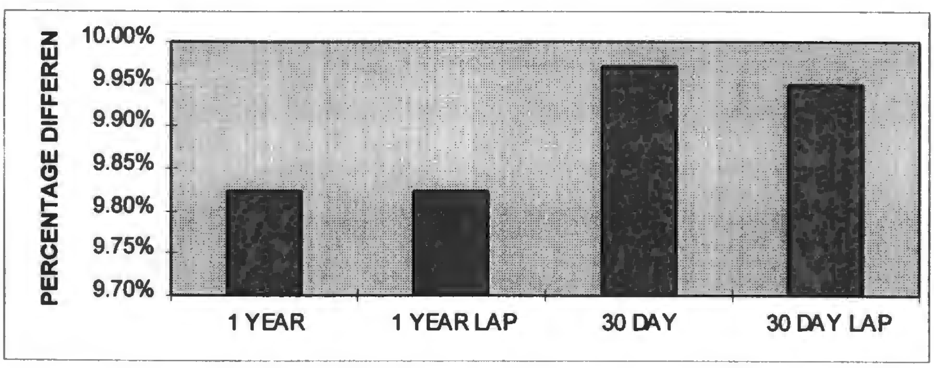
Graph 12.3 displays the t-test statistics of the Black Scholes model calculated for each of the volatility intervals. The "lap" periods refer to calculating volatility over the period including the actual rights trading period. The results are consistent with Fisher Black's statement that the Black Scholes model will produce best results when the volatility is calculated over a period that includes the actual trading period of the option. For both the thirty day interval (although only marginal) and the one year interval it is evident that the calculating volatility based on including the lap period produces superior results to excluding the lap period. The fact that this result holds across both the time intervals chosen gives clear support to the hypothesis.

The most accurate model prices are obtained when using a historical period of one year, as opposed to thirty days, to calculate the underlying asset's volatility. This appears logical as most empirical research has shown that volatility is non-stationary and may in fact be mean reverting following a stochastic process such as :

$$dV = k(\mu - V)dt + \sigma.V^{Y}dW$$

If this contention is accurate, then one needs to calculate the volatility over a period that is able to encapsulate the above process and it is considered that thirty days is too short to achieve this.

Looking at the effect on over or underpricing in Graph 12.4:

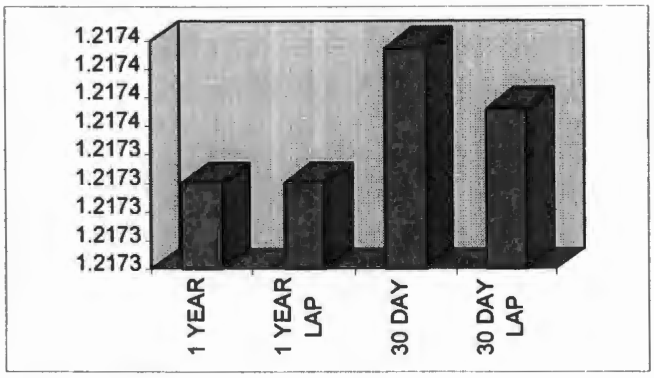


Graph 12.4 Percentage difference in Black Scholes model prices

it is evident that each of the estimates still lead to an overpricing of the rights.

12.3.2 SQUARE ROOT CONSTANT ELASTICITY OF VARIANCE MODEL

The results for the CEV models are consistent with the above findings for the Black Scholes model and are graphed below.

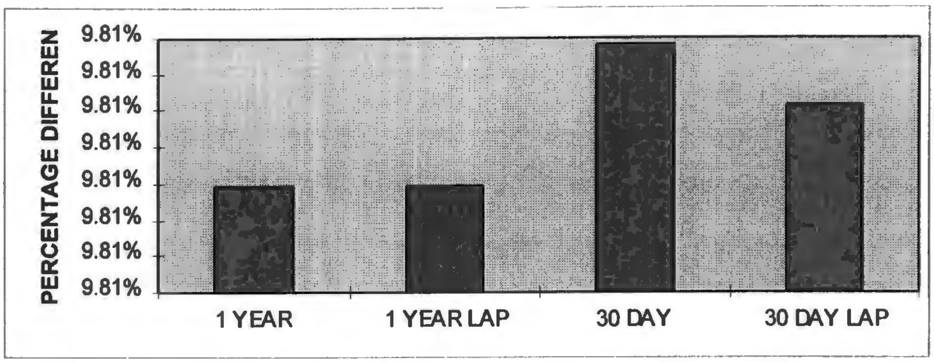


Graph 12.5 Square Root CEV Model by Variance Calculation

Overall the t-statistics are lower than for the Black Scholes model thus indicating the superiority of this model (although only marginally so). Consistent with the findings for the Black Scholes model, the lap period provides better model prices than the interval that excludes the lap period. This result is consistent across the one year and thirty day periods thus showing its validity.

The model prices calculated using a one year volatility interval are once again superior to those employing a thirty day interval. In a CEV process this appears even more relevant than the Black Scholes result as the CEV stochastic process requires an accurate formulation of the volatility structure so that the hypothesized negative relationship between stock price and volatility can be modeled correctly.

Looking at the effect on over or underpricing in the graph below :

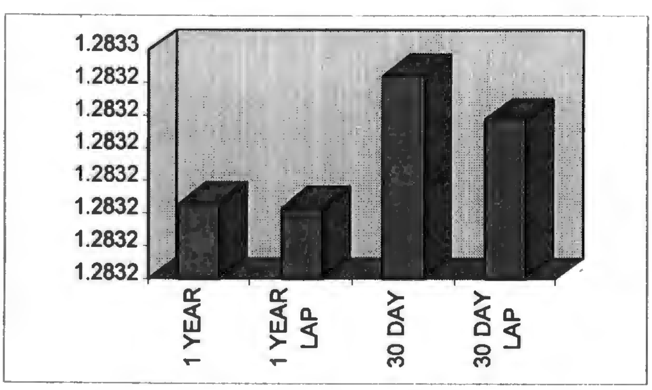


Graph 12.6 Percentage difference in Square Root CEV model prices

it is evident that each of the estimates still lead to an overpricing of the rights.

12.3.3 ABSOLUTE CONSTANT ELASTICITY OF VARIANCE MODEL

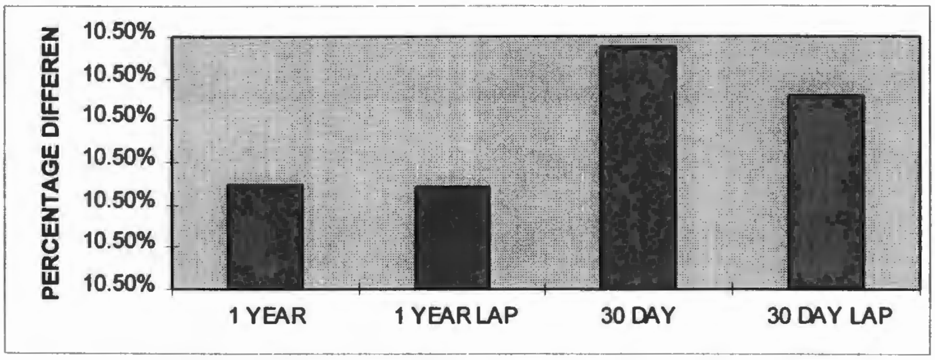
The Absolute CEV results are worse than those of any of the other models. This model does not appear able to capture the process followed by the underlying stock and option. This is consistent with expectations as volatility is unlikely to follow such a process and the test was employed to confirm this finding.



Graph 12.7 Absolute CEV Model by Variance Calculation

Consistent with the findings for the Black Scholes and square root CEV models, the lap period provides better model prices than the interval that excludes the lap period. This result is consistent across the one year and thirty day periods as well.

The model prices calculated using a one year volatility interval are once again superior to those employing a thirty day interval. Looking at the effect on over or underpricing in the graph below :



Graph 12.8 Percentage difference in Merton model prices

it is evident that each of the estimates still lead to an overpricing of the rights.

12.3.4 VOLATILITY SUMMARY

The finding that using a one year interval to calculate volatility provides more accurate results than a thirty day period is consistent across all the models tested and thus leads to acceptance of the hypothesis that a one year period is favoured over the shorter thirty day period. This appears logical as one would expect that a longer period is required to capture the correct volatility process using the traditional method for calculating volatility.

The inclusion of the lap period appears to provide even better model prices and this finding is consistent across both the different models employed and the two different volatility estimation periods. Thus the hypothesis that calculating volatility over the period that includes the actual option trading period will provide better results than when the trading period is excluded, is accepted. This is as expected as the models should provide the most accurate prices when they employ the actual volatility over the trading period as opposed to an estimate of what it will be based on past volatility (Black, 1976).

12.4 ADJUSTING FOR DILUTION

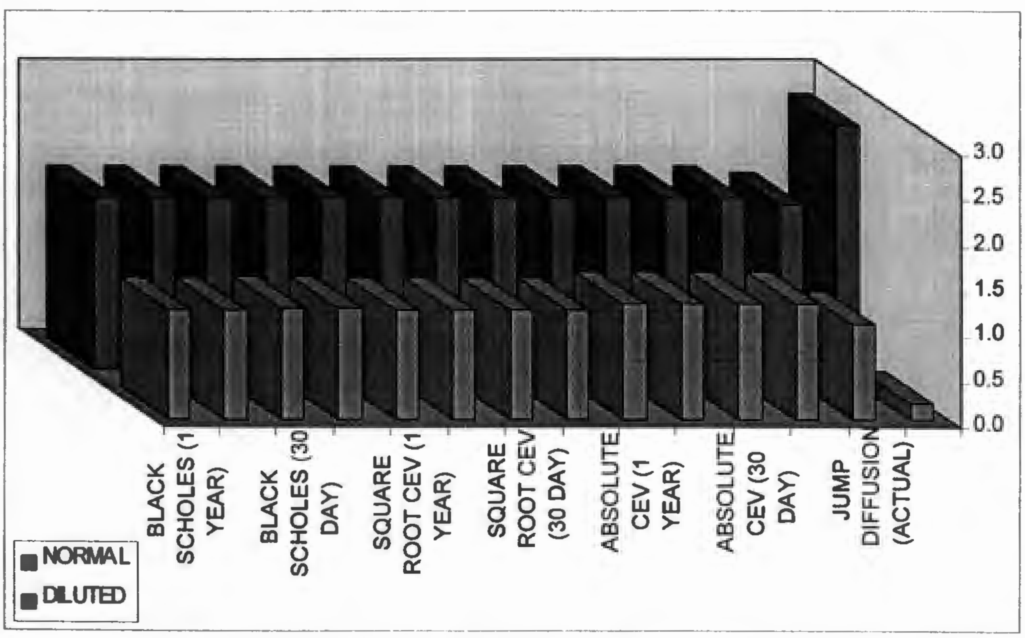
12.4.1 TESTING OF DILUTION ADJUSTED MODELS

When the prices are adjusted for dilution using the Galai dilution adjustment, the model prices become significantly worse for all of the models tested. Table 12.3 is reproduced below with the t-statistics for the dilution adjusted models (in italics) included below each model's unadjusted statistics.

MODEL	1 YEAR		1 YEAR LAP		30 DAY		30 DAY LAP	
	PROB	T	PROB	T	PROB	T	PROB	T
BLACK SCHOLES	22.43%	1.2189	22.43%	1.2190	21.76%	1.2370	21.85%	1.2344
DILUTION ADJUSTED	*5.86%*	*1.9030*	*5.86%*	*1.9029*	*5.96%*	*1.8950*	*5.97%*	*1.8943*
SQUARE ROOT CEV	22.49%	1.2173	22.49%	1.2173	22.49%	1.2174	22.49%	1.2174
DILUTION ADJUSTED	*5.85%*	*1.9039*	*5.85%*	*1.9039*	*5.85%*	*1.9038*	*5.85%*	*1.9038*
ABSOLUTE CEV	20.09%	1.2832	20.09%	1.2832	20.09%	1.2832	20.09%	1.2832
DILUTION ADJUSTED	*5.85%*	*1.9039*	*5.85%*	*1.9039*	*5.85%*	*1.9038*	*5.85%*	*1.9038*
JUMP DIFFUSION (ACTUAL)	30.16%	1.0443	30.16%	1.0443				
DILUTION ADJUSTED	*7.31%*	*1.8342*	*7.31%*	*1.8342*				
JUMP DIFFUSION (AVE)	85.66%	0.1809	85.66%	0.1809				
DILUTION ADJUSTED	*0.73%*	*2.7167*	*0.73%*	*2.7167*				

Table 12.4 T-Test results for unadjusted models verses dilution adjusted models

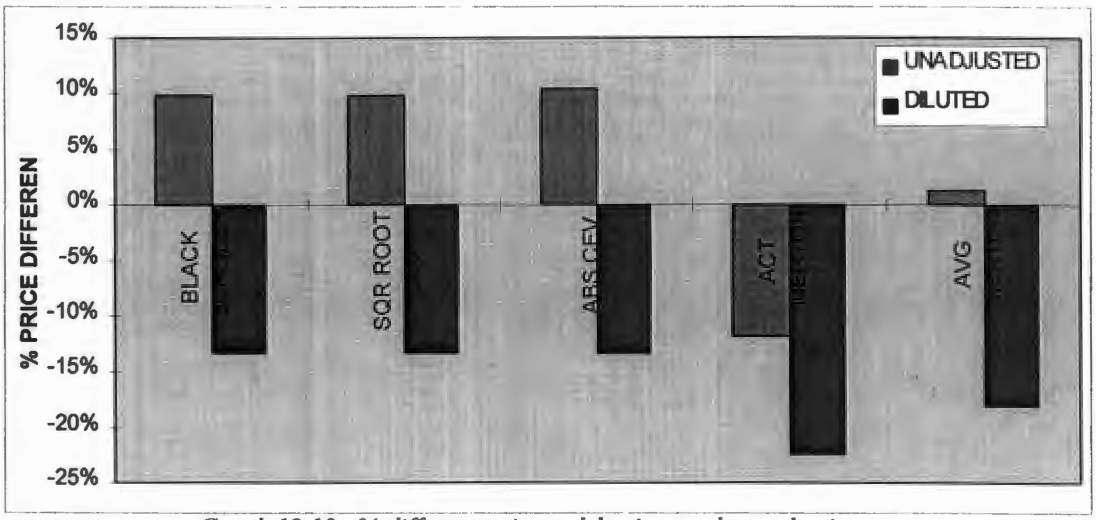
For each model tested and for each of the volatility estimation intervals, the t-statistics are larger for the dilution adjusted models than their unadjusted counterparts. This thus indicates a worsening of model pricing. The fact that this result is consistent for each model tested and for each of the volatility estimation intervals, indicates that the result is conclusive. This is consistent with Kremer and Roenfeldt's (1992) finding that because the number of warrants outstanding is so small compared to the total number of shares in issue, the dilution effect is ignored by the market. This is also consistent with the findings of Law (1994) in the South African rights issue market. The consistent deterioration of model prices by accounting for the effect of dilution is illustrated in Graph 12.9 where the larger t-statistics for the dilution adjusted models are evident.



Graph 12.9 Unadjusted verses dilution adjusted model prices (1 year volatility period)

There is thus not a single case where the dilution adjustment improves the accuracy of the model prices. The market does not appear to account for the dilution effect and may thus be considered inefficient in this regard.

It is interesting to note what effect the dilution adjustment has on the model prices. The following graph of the percentage difference between the model price and the actual price for the unadjusted and then the dilution adjusted models indicates that the dilution adjustment has the effect of inducing an underpricing of the nil paid letters.



Graph 12.10 % differences in model prices and actual prices

Thus while all of the models bar the Merton model incorporating the actual diffusion parameters, tend to overprice before the dilution adjustment, with the dilution adjustment they all underprice the NPLs. The effect of the dilution adjustment is thus to swing the mispricing in the opposite direction and to a greater magnitude evidenced by the increased t-statistics.

12.4.2 CONCLUSION ON DILUTION ADJUSTMENT

In summary, the dilution adjustment does not appear to warrant inclusion in pricing models of rights issues. This is evidenced by each model providing worse results when the dilution adjustment is applied.

12.5 TRADING VOLUME ADJUSTMENTS

12.5.1 TESTING OF MODELS ADJUSTED FOR NIL VOLUME

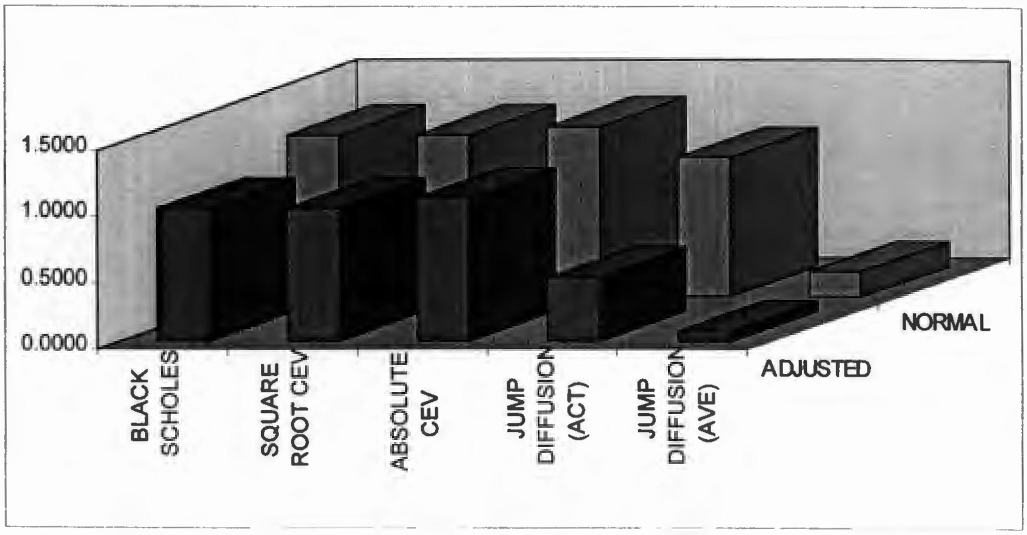
The pricing tests were repeated for each day of each rights issue where only both the right and the underlying share traded. This should theoretically lead to better results as it eliminates the problem of non-simultaneity of rights and stock price quotations to some degree. This happens due to the stock not trading for the day and the closing price being the same as the previous day's closing while the right may trade and thus be out of synch with the underlying stock price. This phenomenon is often encountered in event studies (Brown and Warner, 1985) where non-synchronous trading results in serial correlation of returns. The following table shows the calculated t-statistics for model prices on days where both the underlying share and nil paid letter trade.

MODEL	1 YEAR		1 YEAR LAP		30 DAY		30 DAY LAP	
	PROB	T	PROB	T	PROB	T	PROB	T
BLACK SCHOLES	31.36%	1.0105	31.34%	1.0108	30.56%	1.0274	30.58%	1.0269
DILUTION ADJUSTED	*3.62%*	*2.1111*	*3.62%*	*2.1107*	*3.68%*	*2.1034*	*3.71%*	*2.1011*
SQUARE ROOT CEV	31.46%	1.0084	31.46%	1.0084	31.45%	1.0086	31.45%	1.0085
DILUTION ADJUSTED	*3.61%*	*2.1122*	*3.61%*	*2.1122*	*3.61%*	*2.1121*	*3.61%*	*2.1121*
ABSOLUTE CEV	28.19%	1.0791	28.19%	1.0791	28.19%	1.0792	28.19%	1.0792
DILUTION ADJUSTED	*3.61%*	*2.1122*	*3.61%*	*2.1122*	*3.61%*	*2.1122*	*3.61%*	*2.1122*
JUMP DIFFUSION (ACTUAL)	63.33%	0.4802	63.33%	0.4802				
DILUTION ADJUSTED	*15.67%*	*1.4397*	*15.67%*	*1.4397*				
JUMP DIFFUSION (AVE)	92.91%	0.0891	92.91%	0.0891				
DILUTION ADJUSTED	*0.29%*	*3.0213*	*0.29%*	*3.0213*				

Table 12.5 T-statistics of trading volume adjusted models

The model prices for all of the models improve considerably when thin trading is accounted for, evidenced by the consistently lower t-test statistics for the each of the models as compared to the "unadjusted models" t-statistics in Table 12.3. This is illustrated in Graph

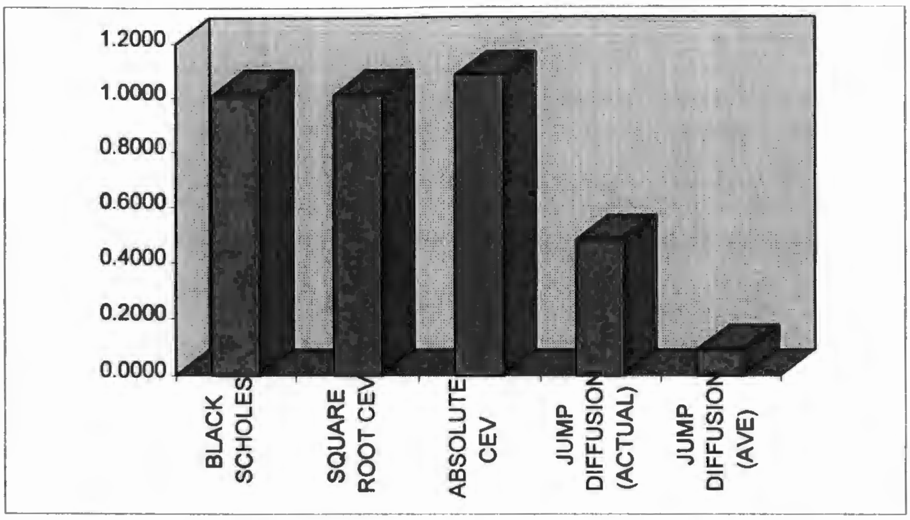
12.11 which compares the t-statistics for the models without any adjustment for thin trading to the models adjusted for thin trading. Note that only the results for the one year volatility interval have been graphed as the findings are consistent across all the different volatility estimation intervals.



Graph 12.11 T-statistics of unadjusted verses trading volume adjusted model prices

The jump diffusion average model thus provides the best results of all the models once again and this is highly significant and clearly an excellent predictor of actual rights trading prices in the market place.

It is evident that the results bear the same relationships observed in the previous section which is comforting in that it reinforces the previous findings. The t-test statistics are graphed below for ease of comparison (note that only the one year interval has been graphed as the results in the above table are consistent across all the interval periods).



Graph 12.12 All Models Adjusted for Nil Volume Trades

Once again thus superiority of the jump diffusion model is displayed. The finding that the jump diffusion model with the averaging parameters as opposed to the actual parameters is the better model is valid again. The results are consistent with the previous sections where the jump diffusion model was shown to be the most accurate pricer of nil paid letters, followed by the square root CEV model and then the Black Scholes model. The absolute CEV model is consistently the worst of the alternatives.

The one year period once again is shown to provide the best estimate of historical volatility. The lap period does not appear to produce better estimates of volatility in the one year interval but does provide better results for the thirty day interval. This result appears to conflict somewhat with the previous findings of the lap period consistently providing better volatility estimates than the non-lap period. However the lap period is still superior in the thirty day interval.

Consistent with the previous findings, adjustment for dilution provided worse results in all cases. The dilution adjustment does thus not appear to be accounted for by the market.

12.5.2 CONCLUSION ON ADJUSTMENT FOR TRADING VOLUME

The adjustment for trading volume, whereby the rights are only priced on the days where both the underlying share and the right trade, provides more accurate model prices for each of the models tested. This thus indicates that the non-simultaneity of rights and share price trades is a serious concern for empirical testing of rights.

12.6 ADJUSTMENT FOR THE ABSOLUTE VALUE OF RIGHTS PRICES

Some researchers exclude the low value option prices as part of their filter rule when conducting empirical research on option pricing models and option market efficiency (Kremer and Roenfeldt, 1992). This is generally so as to exclude the biases caused by options that are way out of the money and yet still trade for a notional price. In the sample used in this study it was noted that most of the way out of the money rights still traded for a price, generally equal to one or two cents. This is mainly due to the market recording system which does not allow the right to trade for nil value as it must have some value inherent in it. It was thus decided to adjust the analysis via another filter rule that excluded rights prices below two levels :

1. 10 cents
2. 100 cents

The first category will eliminate the bias caused by rights that should be trading for close to zero cents but due to currency definitions are unable to trade for a fraction of a cent and will thus cause differences between the actual and model prices. The second filter is aimed at only including those rights of high value. These rights are generally of the higher Rand value shares which appear to often be the more "blue chip" shares which are actively watched by traders and analysts. It was felt that this category would have a far more liquid market and thus a more efficient market in both the underlying share and the actual rights themselves.

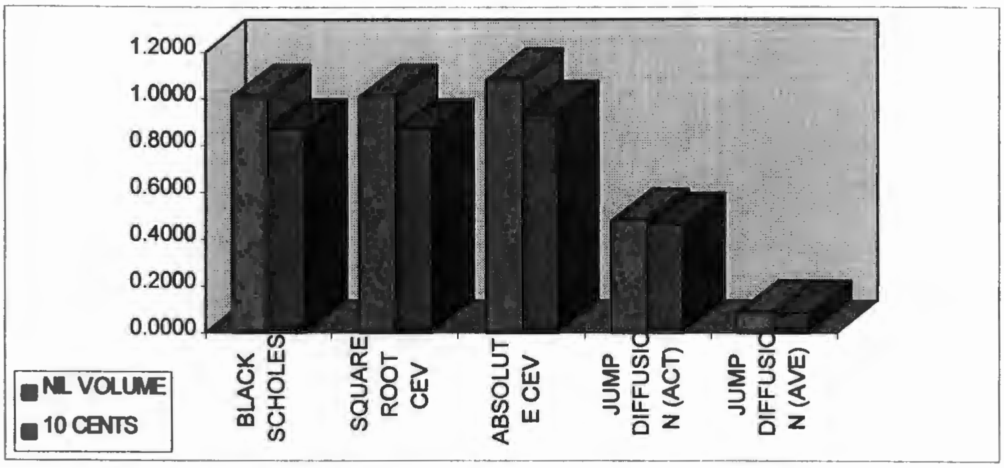
12.6.1 FILTER RULE REJECTING RIGHTS BELOW 10 CENTS

Table 12.6 summarises the t-test statistics for the case where the nil trading adjustment and 10 cent filter rule adjustments are included for the models.

	1 YEAR		1 YEAR LAP		30 DAY		30 DAY LAP	
MODEL	PROB	T	PROB	T	PROB	T	PROB	T
BLACK SCHOLES	38.84%	0.8653	38.81%	0.8658	38.75%	0.8669	38.50%	0.8715
DILUTION ADJUSTED	*2.46%*	*2.2732*	*2.46%*	*2.2728*	*2.47%*	*2.2718*	*2.50%*	*2.2672*
SQUARE ROOT CEV	38.86%	0.8648	38.86%	0.8648	38.86%	0.8648	38.86%	0.8649
DILUTION ADJUSTED	*2.46%*	*2.2736*	*2.46%*	*2.2736*	*2.46%*	*2.2736*	*2.46%*	*2.2736*
ABSOLUTE CEV	35.04%	0.9371	35.04%	0.9371	35.04%	0.9371	35.04%	0.9371
DILUTION ADJUSTED	*2.46%*	*2.2736*	*2.46%*	*2.2736*	*2.46%*	*2.2736*	*2.46%*	*2.2736*
JUMP DIFFUSION (ACTUAL	64.64%	0.4633	64.64%	0.4633				
DILUTION ADJUSTED	*15.27%*	*1.4660*	*15.27%*	*1.4660*				
JUMP DIFFUSION (AVE)	92.97%	0.0884	92.97%	0.0884				
DILUTION ADJUSTED	*0.18%*	*3.1845*	*0.18%*	*3.1845*				

Table 12.6 T-statistics for 10 cent absolute value adjusted models

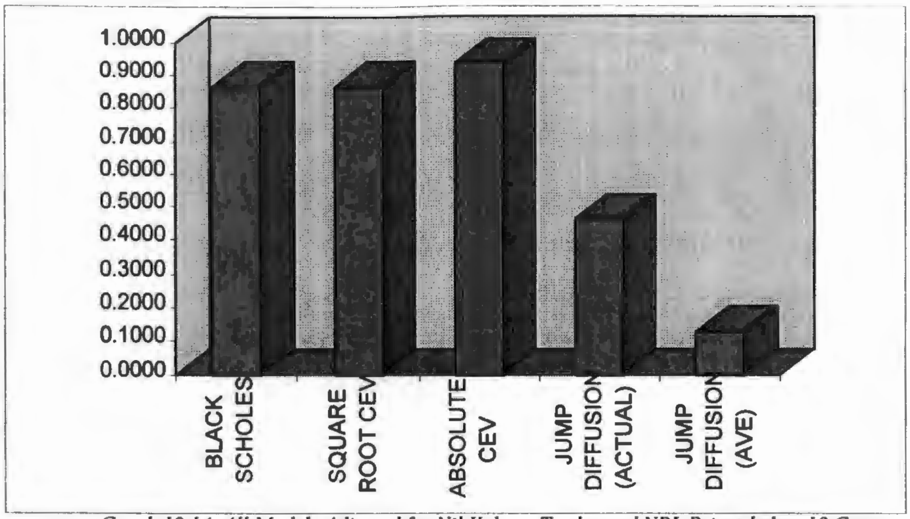
The improvement in model prices can be seen in the following graph which compares the t-statistics of the nil trading adjusted models to the nil trading adjusted models with the 10 cent filter rule.



Graph 12.13 Comparison of nil volume and 10 cent adjusted model t-statistics

The Black Scholes model prices improve as do all the other model prices. The 10 cent filter rule appears however to only marginally improve the results as the t-statistics are not significantly lower for any of the models. This does lend some support however to the contention that the market for the lower priced rights is less efficient than for the higher priced rights as each of the models are better able to price the rights when those trading for less than 10 cents are excluded from the sample.

These results are consistent with both of the previous sections. The square root CEV model outperforms the Black Scholes model but the absolute CEV model is once again the worst performer. The jump diffusion actual is outperformed by the jump diffusion average model and they are both significantly better than the other models used. This can be seen in the following graph.



Graph 12.14 All Models Adjusted for Nil Volume Trades and NPL Prices below 10 Cents

As expected from the previous results, the dilution adjustment adds no improvement to the model prices.

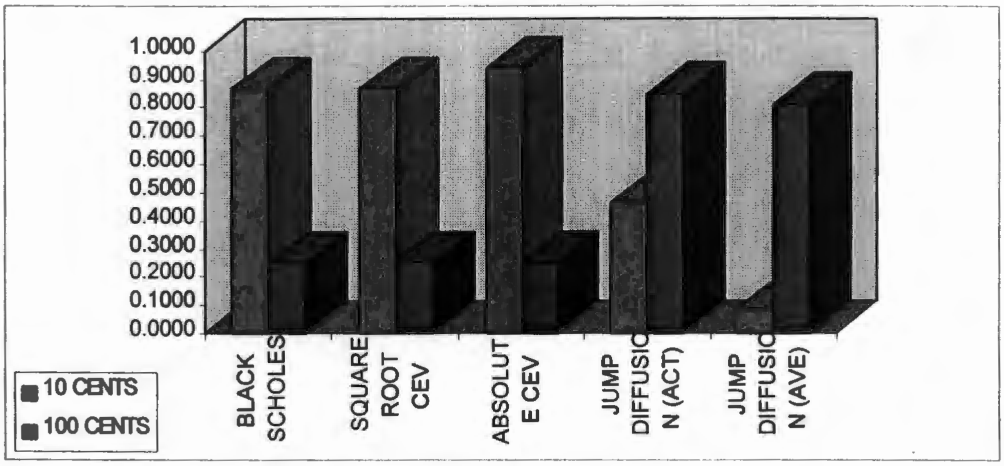
12.6.2 FILTER RULE REJECTING RIGHTS BELOW 100 CENTS

This test was performed to see if the results are consistent with the sample that excluded all prices below 10 cents and if the higher priced rights are indeed more efficiently valued by the market. The following table summarises the t-statistics when testing the 100 cent filter rule.

	1 YEAR		1 YEAR LAP		30 DAY		30 DAY LAP	
MODEL	PROB	T	PROB	T	PROB	T	PROB	T
BLACK SCHOLES	80.29%	0.2506	80.29%	0.2506	80.22%	0.2515	80.04%	0.2539
DILUTION ADJUSTED	*1.87%*	*2.4120*	*1.87%*	*2.4120*	*1.87%*	*2.4113*	*1.89%*	*2.4087*
SQUARE ROOT CEV	80.30%	0.2504	80.30%	0.2504	80.30%	0.2504	80.30%	0.2504
DILUTION ADJUSTED	*1.87%*	*2.4122*	*1.87%*	*2.4122*	*1.87%*	*2.4122*	*1.87%*	*2.4122*
ABSOLUTE CEV	80.30%	0.2504	80.30%	0.2504	80.30%	0.2504	80.30%	0.2504
DILUTION ADJUSTED	*1.87%*	*2.4122*	*1.87%*	*2.4122*	*1.87%*	*2.4122*	*1.87%*	*2.4122*
JUMP DIFFUSION (ACTUAL	41.04%	0.8468	41.04%	0.8468				
DILUTION ADJUSTED	*13.10%*	*1.5975*	*13.10%*	*1.5975*				
JUMP DIFFUSION (AVE)	42.19%	0.8084	42.19%	0.8084				
DILUTION ADJUSTED	*0.10%*	*3.4651*	*0.10%*	*3.4651*				

Table 12.7 T-statistics for 100 cent absolute value adjusted models

The results here are slightly inconsistent with the results obtained when only considering rights prices above ten cents and are evidenced graphically below.



Graph 12.15 T-statistics compared between absolute value filter rule

Both the Black Scholes and CEV models perform better, while the jump diffusion models perform considerably worse than before and are actually outperformed by the other two models. This is in direct contrast with all of the previous findings.

Further analysis revealed that the results are due to the sample sizes having become so small as to be reduced below ten. This is because very few rights traded at a price greater than 100 cents. The statistical results thus cannot be relied upon without using non-parametric tests which would then not allow comparison to the other testing results and they have accordingly been ignored for the remainder of the study.

As an aside what is evident however, is that the Black Scholes and CEV model prices are even better when the 100 cent filter rule is applied as against when the 10 cent filter rule is applied. The contention that the higher priced rights are priced more efficiently in the market thus appears to have some basis.

The dilution effect is once again negative and consistently provides worse results.

12.6.3 CONCLUSION ON ADJUSTING FOR ABSOLUTE VALUE OF RIGHTS PRICES

The models appeared to give more accurate prices when adjusted for the absolute value of the rights prices. Filtering out the lower priced rights showed better results for each of the models tested and across each of the volatility estimation intervals. The conclusion is difficult to formulate as it could either be due to the market not pricing the lower priced rights efficiently (perhaps due to their low value) or else that the models are unable to price these rights as accurately as the higher priced rights. This will be addressed in the concluding chapter.

CHAPTER 13 - CONCLUSION

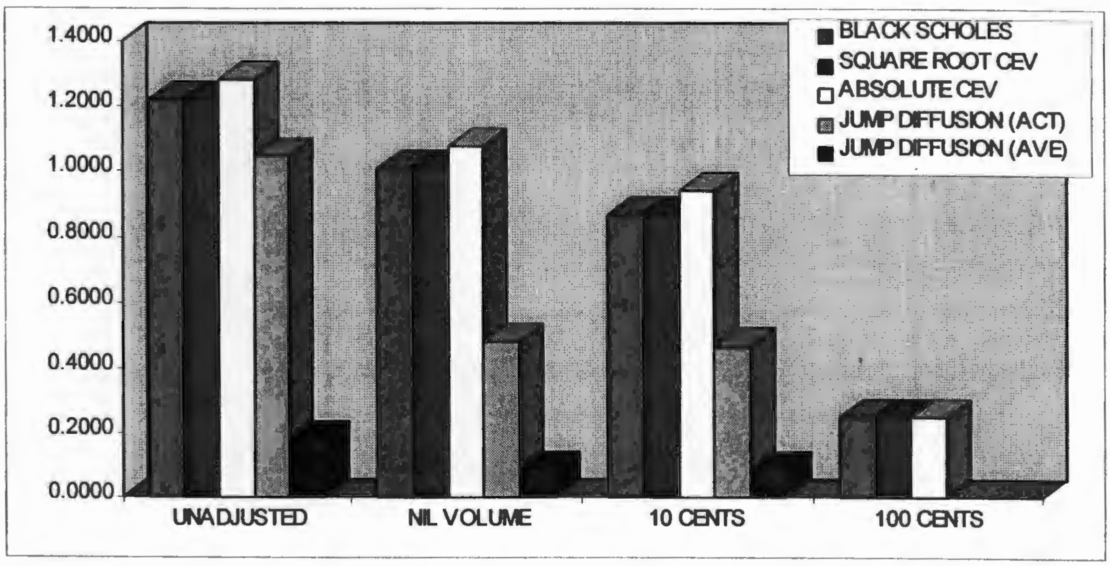
13.1 THE RESEARCH PROBLEM

This research proposed to identify the most accurate method of pricing nil paid letters using option pricing models, including the Black Scholes model, the Cox constant elasticity of variance model and the Merton jump diffusion model and to determine the set of input parameters that lead to the most optimal results.

13.2 THE FINDINGS

The main hypothesis of this study relates to evaluating whether there is one model that consistently outperforms some or all of the other models and thus leads to better estimates of the actual rights trading prices observed in the marketplace. If such a model exists then it must have some superior theoretical or practical foundation not captured by the other models. The Black Scholes model has some very restrictive assumptions which have theoretically and empirically been shown to be unreflective of real market conditions. The other models used in this paper have relaxed some of the more important assumptions in the hope that this will reveal which of them may cause the Black Scholes model to misprice actual rights values.

The empirical results indicated that on average all of the models are able to estimate the actual rights trading prices relatively well. Some models did however perform better than others. The market was shown to not account for the effect of dilution. The best model prices were obtained when calculating volatility over a one year historical period that included the actual rights trading period. The hypothesis regarding trading volume showed that there is a significant impact of nil trading volume on the estimation of accurate rights prices. The filter rule of rejecting rights prices below 10 cents and 100 cents also improved the results thus showing a bias for the lower priced rights to be incorrectly valued and possibly some inefficiency in this sector of the market. These results are best illustrated graphically.



Graph 13.1 All Models Graphed for Comparison across the different filter rules

The jump diffusion model results can be seen to consistently outperform the other models no matter what the adjustments are to the model inputs. The extent of the superior ability of the jump diffusion model to estimate the actual rights prices is evident from the stark difference between the t-values shown in Graph 13.1 with the jump diffusion average considerably below the other models' t-values. Within the two versions of the jump diffusion model, the model that calculates an annual jump mean based on a pooling method appears to outperform the model that calculates the jump mean individually for each rights issue. This result is useful for practical purposes as it allows an annual average to be calculated and thus results in one less parameter to be estimated for the model each time it is used.

In conclusion, the hypothesis :

H_0 The Black Scholes model, Cox Square Root Constant Elasticity of Variance model, Cox Absolute Constant Elasticity of Variance model and Merton Jump Diffusion model will on average correctly value nil paid letters.

is accepted at the 95% confidence level and the individual models can be ranked as follows based on their t-statistics :

1. Jump diffusion model with pooled jump mean
2. Jump diffusion model with actual jump mean
3. Square root CEV model
4. Black Scholes model
5. Absolute CEV model

The results are thus as expected with the models that relax the Black Scholes assumptions of a constant variance and a continuous stock price processes outperforming the traditional Black Scholes model.

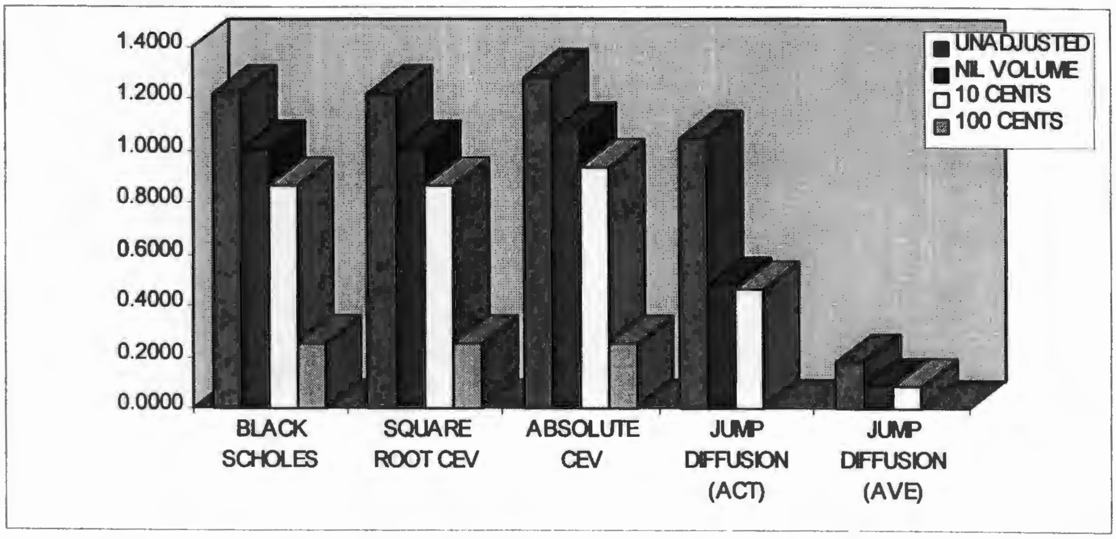
The sub-hypothesis tests related to searching for the optimal set of input parameters for each of the models. The results showed that each of the adjustments that were made improved the accuracy of the model prices and this improvement was consistent for each of the models tested :

1. Calculating volatility over a longer interval.
2. Calculating volatility over a period including the actual rights trading period.

3. Thin trading adjustments.

4. Filtering out the lower priced rights.

Only the dilution adjustment was shown to not improve the accuracy of the models. This is evidenced by the lower average t-statistics in Graph 13.2 as each of the adjustments is incorporated into the models - as one scans the graph for each model from left to right there is a clear decrease in the t-statistic and this is consistent for each of the models.



Graph 13.2 All Models Graphed Across Adjustments

Each of the sub-hypotheses stated below were thus able to be rejected :

H_2 Calculating volatility based on different historical periods will not affect the accuracy of the model prices.

H_3 Trading volume will not impact on the accuracy of the model prices.

H_4 The absolute magnitude of the nil paid letters price will not affect the accuracy of the model prices.

H_5 Adjustment of the models for dilution will not affect the accuracy of the model prices.

Thus a model that incorporates each of the above adjustments, excluding a dilution effect, should provide the most reliable rights prices.

13.3 OVERALL SUMMARY OF FINDINGS

It must be borne in mind that the above tests are unable to distinguish between the validity of the model being tested and the notion of market efficiency. Due to four different models having been tested and the results being consistent across the individual models some comfort on the validity of the models can be placed however. It appears that the Merton Jump Diffusion Model outperformed the other models across all categories of volatility, trading volume and rights prices. This was followed by the square root Constant Elasticity of Variance Model and then the Black Scholes Model. The models that relax the basic assumptions of the Black Scholes model thus appear to do so fairly well and thus give results more accurate and reflective of the actual prices observed in the market.

13.4 THE PROPOSED RIGHTS PRICING MODEL

These results show that a new model capable of incorporating a negative stock price/variance relationship together with an assumption of discontinuous sample paths for the underlying stock diffusion process will provide the most accurate rights prices. While no such model is know to currently exist, it is postulated that the following stochastic differential equation would incorporate the necessary factors into the stock price process :

$$dS = (\alpha - \lambda.k).Sdt + \sigma.S^{\beta/2}dZ + SdQ$$

with the usual terminology for the two Wiener processes. The model is thus a combination of the constant elasticity of variance process together with a jump process. Together with the necessary boundary conditions and the assumption that the jump risk is diversifiable, the above stochastic differential equation should be able to be solved for the corresponding call price. Further research into the solution of the above model may thus yield an option pricing model able to provide highly accurate results for the pricing of rights issues. This however remains a topic for future research.

CHAPTER 14 - BIBLIOGRAPHY

Bachelier L. "Theorie de la Speculation" *Annales de l'Ecole Normale Superiere*, Vol. 17, 1900, p.21-86. English translation by A.J. Boness in The Random Character of Stock Market Prices edited by P.Cootner,1967, p.17-78, Cambridge, Massachusetts, MIT Press

Ball C. & Torous W. "A Simplified Jump Process for Common Stock Returns" *Journal of Financial and Quantitative Analysis*, Vol.18, March, 1983, p.53-66.

Ball C. & Torous W. "The Maximum Likelihood Estimation of Security Price Volatility : Theory, Evidence and Application to Option Pricing" *Journal of Business*, Vol. 57, No. 1, 1984, p. 97-112

Ball C. & Torous W. "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing" *The Journal of Finance*, Vol. 90, No. 1, March, 1985, p.155-173

Ball C. & Roma A. "Stochastic Volatility Option Pricing" *Journal of Financial and Quantitative Analysis*, Vol. 29, No. 4, December, 1994, p. 589-612

Bates D. "The Crash of '87 : Was It Expected ? The Evidence from the Options Markets" *The Journal of Finance*, Vol. 106, No. 3, July, 1991, p.1009-1044

Baxter M. & Rennie A. Financial Calculus, Cambridge University Press, 1998

Beckers S. "The Constant Elasticity of Variance Model and Its Implications for Option Pricing" *Journal of Finance*, Vol. 35, June, 1980, p.661-673

Beckers S. "A Note on Estimating the Parameters of a Diffusion-Jump Model of Stock Returns" *Journal of Financial and Quantitative Analysis*, Vol. 36, March, 1981, p.127-140.

Beckers S. "Standard Deviations Implied in Option Prices as Predictors of Future Stock Price Variability" *Journal of Banking and Finance*, Vol. 5, Sep, 1981, p.363-382

Beckers S. "Variances of Security Price Returns Based on High, Low, and Closing Prices" *Journal of Business*, Vol. 56, January, 1983, 97-112.

Bhana N. "A Review of the Efficiency of the Johannesburg Stock Exchange" *De Ratione*, Vol. 8, No. 2, Summer 1994/1995, p. 79-98

Bick A. "Comments on the Valuation of Derivative Assets," *Journal of Financial Economics*, Vol. 10,November, 1982, p. 331-345.

Black F. "Fact and Fantasy in the Use of Options" *Financial Analysts Journal,* Jul-Aug, 1975, p. 36-72

Black F. "Studies of Stock Price Volatility Changes," *Proceedings of the 1976 Meeting of the American Statistical Association, Business and Economic Statistics Section*, August, 1976, p.177-181.

Black F. & Scholes M. "The Valuation of Option Contracts and a Test of Market Efficiency" *Journal of Finance*, Vol. 27, May, 1972, p.399-418

Black F. & Scholes M. "The Pricing of Options and Corporate Liabilities" *Journal of Political Economy*, Vol. 81, May-Jun, 1973, p. 637-659

Blattberg R. and Gonedes N. "A Comparison of the Stable and Student Distribution of Statistical Models for Stock Prices" *Journal of Business*, Vol. 47, April, 1974, p. 244-280

Bloch E. Inside Investment Banking, Dow Jones-Irwin, Second Edition, 1989

Boness J. "Elements of a Theory of Stock-Option Value" *Journal of Political Economy*, Vol. 72, April, 1964, p.163-175

Bookstaber R. "Observed Option Mispricing and the Nonsimultaneity of Stock and Option Quotations" *Journal of Business*, Vol. 54, Jan, 1981, p.141-155

Bowie D. "Thin Trading, Non-normality and the Estimation of Systematic Risk on Small Stock Markets" Unpublished Ph.D. Thesis submitted to the University of Cape Town, 1994

Boyle P. and Ananthanarayanan A. "The Impact of Variance Estimation in Option Valuation Models" *Journal of Financial Economics*, Vol. 5, December, 1977, p. 375-388

Boyle P. & Emanual D. "Discreetly Adjusted Option Hedges" *Journal of Financial Economics*, Vol. 8, September, 1980, p. 255-282

Bradfield D. & Ardington C. “A Note on the Riskiness of Long Term Investment on the JSE” *Journal for Studies in Economics and Econometrics*, Vol. 21, No. 3, 1997, p.67-77

Brealey R. Security Prices in a Competitive Market : More About Risk and Return from Common Stocks, MIT Press, 1971, Cambridge, Massachusetts

Brealey R. & Myers S. Principles of Corporate Finance, Fifth Edition, 1996

Brennen M. “The Pricing of Contingent Claims in Discrete Time Models” *Journal of Finance*, Vol. 34, March, 1979, p.53-68

Brennan M. & Schwartz E. “Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion” *Journal of Finance*, Vol. 32, December, 1977, 1699-1716.

Brennen M. & Schwartz E. “Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A Synthesis” *Journal of Financial and Quantitative Analysis*, Vol. 17, December, 1982, p.697-703

Brenner M. & Subrahmanyam M. “A Simple Approach to Option Valuation and Hedging in the Black Scholes Model” *Financial Analysts Journal*, March/April, 1994, p.25-28

Brigham E. & Gapenski L. Intermediate Financial Management, Harcourt Brace & Company, 4th Edition, 1993

Brite R.L. Business Statistics, Addison Wesley Publishing Company, 1980

Brockman P. & Chowdhury M. "Deterministic verses Stochastic Volatility : Implications for Option Pricing Models" *Applied Financial Economics*, 1997, Vol. 7, p. 499-505

Brown S. & Warner J. "Using Daily Stock Returns" *Journal of Financial Economics*, Vol. 14, 1985, p.3-31

Bullard D. "Options Trading in the Gilt Market" *The Investment Analysts Journal*, Winter, November, 1987, p.40-45

Chan H. "The Effect of Volatility Estimates in the Valuation of Underwritten Rights Issues" *Applied Financial Economics*, Vol. 7, 1997, p.473-480

Chen A. "A Model of Warrant Pricing in a Dynamic Market" *Journal of Finance*, Vol. 25, December, 1970, p.1041-1060

Chicago Board Options Exchange Manual, 1996

Chiras D. & Manaster S. "The Information Content of Option Prices and a Test of Market Efficiency" *Journal of Financial Economics*, Vol. 6, June-September, 1978, p. 213-234

Chriss N. Black-Scholes and Beyond : Interactive Option Pricing Toolkit - A Guide to In-Depth Option Pricing Models, McGraw Hill, 1997

Chriss N. Black-Scholes and Beyond : Option Pricing Models, McGraw Hill, 1997

Christie A. "The Stochastic Behavior of Common Stock Variances," *Journal of Financial Economics*, Vol. 10, December, 1982, p. 407-432

Cornell B. "Using the Option Pricing Model to Measure the Uncertainty Producing Effect of Major Announcements," *Financial Management*, Vol. 7, Spring, 1978, p. 54-59

Cox J. "The Constant Elasticity of Variance Option Pricing Model" *The Journal of Portfolio Management*, Special Issue 1996, p.15-17

Cox J. & Ross S. "A survey of some new results in financial option pricing theory" *Journal of Finance*, Volume 31, May, 1976, p. 383-402

Cox J. & Ross S. "Valuation of options for alternative stochastic processes" *Journal of Financial Economics*, Vol. 3, Nos 1 and 2, 1976, p. 145-166

Cox J. & Ross S. & Rubinstein M. "Option pricing : a simplified approach" *Journal of Financial Economics*, No. 3, 1979, p. 229-263.

Diamond W. "Rights Issues. The Impact on Share Price Around the Time of the Announcement of Rights Issues" A Technical Report presented to the Graduate School of Business University of Cape Town, 1994

Emanuel D. "A Theoretical Model for Valuing Preferred Stock" *Journal of Finance*, Vol. 38, September, 1983, p. 1133-1155

Emanuel D. "Warrant Valuation and Exercise Strategy," *Journal of Financial Economics*, Vol. 12, August, 1983, p. 211-236.

Emanuel D.C. and MacBeth J.D. "Further Results on the Constant Elasticity of Variance Call Option Pricing Models" *Journal of Financial and Quantitative Analysis*, Vol. 17, November, 1982, p. 533-554.

Epps T.W. "Security Price Changes and Transaction Volumes: Theory and Evidence," *American Economic Review*, September, 1975, p. 586-597.

Fama E. "Mandelbrot and the Stable Paretian Hypothesis." *Journal of Business*, Vol. 36, October, 1963, p. 420-429

Fama E. "The Behavior of Stock Market Prices," *Journal of Business*, Vol. 38, January, 1965, p. 34-105

Fama E. "Efficient Capital Markets II" *The Journal of Finance*, Vol. XLVI, No. 5, December 1991, p.1575-1611

Fama E. Foundations of Finance : Portfolio Decisions and Securities Prices, Basic Books, 1977

Fenwick G. "The Pricing of South African Rights Issues Using the Black Scholes, Implications for Market Efficiency" A Technical Report presented to the Graduate School of Business University of Cape Town, 1996

Finnerty J.E. "The CBOE and Market Efficiency," *Journal of Financial and Quantitative Analysis,* Vol. 13, March, 1978, p. 29-38

French D. & Martin L. "The Measurement of Option Mispricing" *Journal of Banking and Finance*, Vol. 12, 1988, p.537-550

Fuller R.J. "Factors Which Influence Listed Call Option Prices" *Review of Business and Economic Research,* Vol. 13, Winter, 1977-78, p. 21-34

Galai D. "Tests of Market Efficiency of the Chicago Board Options Exchange" *The Journal of Business*, Vol. 50, 1977, p.167-197

Galai D. "A Convexity Test for Traded Options" *Quarterly Review of Economics and Business,* Vol. 19, Summer, 1979, p. 83-90

Galai D. "A Survey of Empirical Tests of Option Pricing Models" in *Option Pricing*, ed. Menachem Brenner, pp. 45-80. Lexington, Mass.: D.C. Heath, 1983.

Galai D. "Empirical Tests of Boundary Conditions for CBOE Options" *Journal of Financial Economics,* Vol. 6, June-September, 1978, p. 187-211

Galai D. "Pricing of Optional Bonds" *Journal of banking and Finance*, Vol. 7, September, 1983, p. 323 - 337.

Galai D. "The Components of the Return from Hedging Options Against Stock" *Journal of Business,* Vol. 56, January, 1983, p. 45-54.

Galai D. & Schneller M. "Pricing Warrants and the Value of the Firm" *Journal of Finance*, Vol. 33, December, 1978, p. 1333-1342.

Galai D. & Masulis R. "The Option Pricing Model and the Risk Factor of Stock" *Journal of Financial Economics*, Vol. 3, January-March, 1976, p. 53-81.

Garman M. & Klass M. "On the Estimation of Security Price Volatilities from Historical Data" *Journal of Business*, Vol. 53, January, 1980, p. 67-78.

Gastineau G.L. 1979. *Stock options manual*. New York: McGraw-Hill.

Geske R. "Pricing of Options with Stochastic Dividend Yield" *Journal of Finance*, Vol. 33, May, 1978, p.617-625

Geske R. "The Valuation of Compound Options" *Journal of Financial Economics*, Vol. 7, March, 1979, p.63-81

Geske R. "The Valuation of Corporate Liabilities as Compound Options," *Journal of Financial and Quantitative Analysis*, Vol. 12, November, 1977, p. 541-552.

Geske R. & Roll R. "On Valuing American Call Options with the Black Scholes European Formula" *Journal of Finance*, Vol. 31, June, 1984, p.443-455

Geske R. and Johnson H. "The Valuation of Corporate Liabilities as Compound Options: A Correction," *Journal of Financial and Quantitative Analysis*, Vol. 19, June, 1984, p. 231-232.

Gleit A. "Valuation of General Contingent Claims : Existence, Uniqueness and Comparison of Solutions" *Journal of Financial Economics*, Vol. 6, Mar, 1978, p.71-88

Goldenberg D.H. "A Unified Method for Pricing Options on Diffusion Processes" *Journal of Financial Economics*, Vol. 29, 1991, p.3-34

Goldenberg D.H. & Schmidt R.J. "On Estimating the Expected Rate of Return in the Diffusion Price Models with Application to Estimating the Expected Return on the Market" *Journal of Financial and Quantitative Analysis*, Vol. 31, No. 4, Dec, 1996, p.605-631

Goldman M. & Sosin H. & Gatto M. "Path Dependent Options: Buy at the Low, the Sell at the High," *Journal of Finance*, Vol. 34, December, 1979, p. 1111-1128.

Goodman R. Introduction to Stochastic Models, Benjamin Cummings , 1988

Granger C. & Morgenstern O. Predictability of Stock Market Prices, Heath Lexington Books , 1970

Hilliard J.E. & Reis J. "Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot" *Journal of Financial and Quantitative Analysis*, Vol. 33, No. 1, March, 1998, p. 72-86

Hsu D and Miller R. and Wichern D. "On the Stable Paretian Behavior of Stock Market Prices," *Journal of the American Statistical Association*, Vol. 69, March, 1974, p. 108-113.

Hull J. & White A. "The Pricing of Options on Assets with Stochastic Volatilities" *The Journal of Finance*, Vol. 42, June, 1987, p. 98-109

Hull J. & White A. "An Analysis of the Bias in Option Pricing Caused by a Stochastic Volatility"*Advances in Futures and Options Research*, Vol. 3, 1988, p.29-61

Hull J. & White A. Hull-White on Deriatives, Risk Publications, 1996

Ingersoll J.E., Jr. "An Examination of Corporate Call Policies on Convertible Securities" *Journal of Finance*, Vol. 32, May, 1977, p. 463-478.

Ingersoll J.E., Jr., "A Contingent-Claims Valuation of Convertible Securities," *Journal of Financial Economics*, Vol. 4, May, 1977, p. 289-322.

Ingersoll Jr. J. Theory of Financial Decision Making, Rowman and Littlefield, 1983

Jarrow R. & Rudd A. "Approximate Option Valuation for Arbitrary Stochastic Processes" *Journal of Financial Economics*, Vol. 10, November, 1982, p.347-369

Jarrow R. & Rudd A. Option Pricing, Dow Jones-Irwin, 1983

Jarrow R.and Rudd A. "Tests of an Approximate Option-Valuation Formula," in *Option Pricing*, ed. Menachem Brenner, p. 81-100. Lexington, Mass.: D.C. Heath, 1983.

Jones E. "Option Arbitrage and Strategy with Large Price Changes" *Journal of Financial Economics*, Vol.13, 1984, p.91-113

Kalay A. and Subrahmanyam M.G. "The Ex-Dividend Day Behavior of Option Prices" *Journal of Business*, Vol. 57, January, 1984, p.113-128.

Kapp J. "The Effects of a Rights Issue Announcement on the Daily Trading Volume of Shares", A Technical Report presented to the Graduate School of Business University of Cape Town, 1996

Kassouf S.T. "An Econometric Model for Option Price with Implications for Investors' Expectations and Audacity" *Econometrica*, Vol. 37, October, 1969, p. 685-694.

Katz R.C. "The Profitability of Put and Call Option Writing," *Industrial Management Review*, Fall, 1963, p. 55-69.

Kendall Sir M. & Stuart A. The Advanced Theory of Statistics, Charles Griffin & Company Limited, Volume 1, 4th Edition, 1977

Kim M. & Oh Y. & Brooks R. "Are Jumps in Stock Returns Diversifiable ? Evidence and Implications for Option Pricing" *Journal of Financial and Quantitative Analysis*, Vol. 29, No. 4, December 1994, p. 609-631

Klemkosky R.C. & Resnick B.G. "Put-Call Parity and Market Efficiency" *Journal of Finance*, Vol. 34, December, 1979, p.1141-1155.

Klemkosky R.C. and Resnick B.G. "An Ex Ante Analysis of Put-Call Parity" *Journal of Financial Economics*, Vol. 8, December, 1980, p. 363-378.

Klerck W.G. & du Toit G.S. "An Investigation into the Return Distributions of Ordinary Industrial Shares on the JSE" *The Investment Analysts Journal*, May, 1986, p.19-27

Kremer J. & Roenfeldt R. "Warrant Pricing : Jump-Diffusion verses Black-Scholes" *Journal of Financial and Quantitative Analysis*, Vol. 28, No. 2, June 1992, p.255-272

Lambrechts I. & Mostert F. "An Analysis of the Behaviour of Market Prices During Rights Issues" *The Investment Analysts Journal*, 1980, No. 15, p. 25-33

Latane, H., and Rendleman, R. "Standard deviations of stock price ratios implied in option prices" *Journal of Finance*, Vol. 31, May, 1976, p.369-381.

Lauterbach B. & Schultz P. "Pricing Warrants : An Empirical Study of the Black Scholes Model and its Alternatives" *The Journal of Finance*, Vol. XLV, No. 4, September 1990, p.1181-1209

Law Y. "A Reliability Study of Volatility Measures in Predicting Market Prices on Equity Rights Issues" A Technical Report presented to the Graduate School of Business University of Cape Town, 1994

Le Palstrier M. & Thomas W. & Affleck-Graves J. "The Valuation of Call Options on Gilts and Warrants in South Africa" *The Investments Analysts Journal*, May, 1986, p.23-29

Le Plastrier M.R. & Thomas W.W. "The Application of the Black-Scholes Equation to the Valuation of Call Options on Gilts and Warrants in South Africa" A Technical Report presented to the Graduate School of Business University of Cape Town, 1984

Leabo D.A. and Rogalski R.J. "Warrant Price Movements and the Efficient Market Model" *Journal of Finance,* Vol. 30, March, 1975, p. 163-177.

Lux T. "The Stable Paretian Hypothesis and the Frequency of Large Returns : An Examination of Major German Stocks" *Applied Financial Economics*, Vol. 6, 1996, p.463-475

MacBeth J.D. & Merville L.J. "An Empirical Examination of the Black-Scholes Call Option Pricing Model," *Journal of Finance*, Vol. 34, December, 1979, p. 1173-1186.

MacBeth J.D. & Merville L.J. "Tests of the Black-Scholes and Cox Call Option Valuation Models," *Journal of Finance,* Vol. 35, May, 1980, p. 285-300.

Manaster S. and Rendleman, Jr. R.J. "Option Prices as Predicators of Equilibrium Stock Prices" *Journal of Finance*, Vol. 37, September, 1982, p. 1043-1058.

Mandelbrot B. "The Variation of Certain Speculative Prices" *Journal of Business*, Vol. 36, October, 1963, p. 394-419.

Mandelbrot B. and Taylor H. "On the Distribution of Stock Price Differences," *Operations Research*, Vol. 15, November, 1967, p. 1057-1062.

Marsh R.A. "An Anlysis of the Applicability of the Black Scholes Equation to Nil Paid Letters and the Analysis of Trading Strategies for Gilt Options" A Technical Report presented to the Graduate School of Business University of Cape Town, 1986

Marsh P. "Valuation of underwriting agreements for UK rights issues" *Journal of Finance*, Vol. 35, No. 3, 1980, p. 693-716.

Merton R. & Scholes M. & Gladstein M. "The Returns and Risks of Alternative Call-Option Portfolio Investment Strategies," *Journal of Business*, Vol. 51, April, 1978, p. 183-242.

Merton R. "The Relationship Between Put and Call Option Prices" *Journal of Finance*, Vol. 28, March, 1973, p.183-184

Merton R. "Theory of rational option pricing" *Bell Journal of Economics and Management Science*, Vol. 4, Spring, 1973, p. 141-183.

Merton R. "Option pricing when underlying stock returns are discontinuous" *Journal of Financial Economics*, Vol. 3, March, 1976, p. 125-144.

Merton R. "The impact on option pricing of specification error in the underlying stock price returns" *Journal of Finance*, Vol. 31, May, 1976, p. 333-350.

Merton R. " On the pricing of contingent claims and the Modigliani-Miller theorem." *Journal of Financial Economics*, Vol. 10, November, 1977

Merton R. Continuous-Time Finance, Basil Blackwell, Cambridge, Massachusetts, 1990

Merville L.J. & Pieptea D.R. "Stock-Price Volatility, Mean-Reverting Diffusion and Noise" *Journal of Financial Economics*, Vol. 24, 1989, p. 193-214

Nandi S. "How Important is the Correlation Between Returns and Volatility in a Stochastic Volatility Model ? Empirical Evidence from Pricing and Hedging in the S&P 500 Index Options Market" *Journal of Banking and Finance*, Vol. 22, 1998, p.589-610

Nawrocki D. "R/S Analysis and Long Term Dependence in Stock Market Indices" *Managerial Finance*, Vol. 21, No. 7, 1995, p.78-92

Ncube M. & Satchell S. "A Bias Adjusted Black and Scholes Option Pricing Model" *Applied Financial Economics*, Vo. 5, 1995, p.51-60

Noreen E. and Wolfson M. "Equilibrium Warrant Pricing Models and Accounting for Executive Stock Options" *Journal of Accounting Research*, Vol. 19, Autumn, 1981, p. 384-398.

Oldfield G.S. Jr. and Rogalski R.J. and Jarrow R.A. "An Autoregressive Jump Process for Common Stock Return," *Journal of Financial Economics*, Vol. 5, December, 1977, p. 389-418.

Osborne M.F.M. "Brownian Motion in the Stock Market," *Operations Research*, Vol. 7, March-April, 1959, p. 145-173.

Ozen M.F. "An Investigation of the Normal Hypothesis of Log-Price Relatives in Homogenous Time Segments" Unpublished MBA Research Report submitted to the University of the Witwatersrand, 1977

Parkinson M. "Option Pricing : The American Put" *Journal of Business*, Vol. 50, Jan, 1977, p.21-36

Parkinson M. "The Extreme Value Method for Estimating the Variance on the Rate of Return," *Journal of Business*, Vol. 53, January, 1980, p. 61-65.

Payne A. "Options Trading" *The Investment Analysts Journal*, October, 1980, p.23-27

Peters E.E. Chaos and Order in the Capital Markets, John Wiley & Sons Inc., 1993

Praetz P.D. "The Distribution of Share Price Changes," *Journal of Business*, Vol 45, January 1972, p. 49-55.

Press S. J. "A Compound Events Model for Security Prices," *Journal of Business*, Vol. 40, July, 1967, p. 317-335.

Rendelman R. & Barter B. "Two State Option Pricing" *Journal of Finance*, Vol. 34, Dec, 1979, p.1093-1110

Rogalski R.J. "Variances and Option Prices in Theory and Practice," *Journal of Portfolio Management*, Vol. 4, Winter, 1978, p. 43-51.

Rogers L., Satchell S. & Yoon Y. "Estimating the Volatility of Stock Prices : A Comparions of Methods that use High and Low Prices" *Applied Financial Economics*, 1994, Vol. 4, p.241-247

Rush D.F. and Melicher R.W. "An Empirical Examination of Factors Which Influence Warrant Prices," *Journal of Finance*, Vol. 29, December, 1974, p. 1449-1466.

Samuelson P. "A Theory of Rational Warrant Pricing" *Industrial Management Review*, Vol. 6, Spring 1967, p.13-31

Samuelson P. & Merton R. "A Complete Model of Warrant Pricing that Maximises Utility" *Industrial Management Review*, Vol. 10, Winter 1969, p.17-46

Schmalensee R. & Trippi R. "Common Stock Volatility Expectations Implied by Option Premia," *Journal of Finance*, Vol. 33, March, 1978, p. 129-147.

Schulz G. & Trautmann S. "Robustness of Option-Like Warrant Valuation" *Journal of Banking and Finance*, Vol. 18, 1994, p. 841-859

Schwartz E.S. "The Valuation of Warrants: Implementing a New Approach," *Journal of Financial Economics*, Vol. 4, January, 1977, p. 79-93.

Shimko D. Finance in Continuous Time, Kolb Publishing Company, 1992

Smidt S. "A New Look at the Random Walk Hypothesis" *Journal of Financial and Quantitative Analysis*, Vol. 3, September, 1968, p. 235-261.

Smith C. "Alternative Methods for Raising Capital : Rights Verses Underwritten Offerings" *Journal of Financial Economics*, Vol.5, 1977, p.273-307

Smith C. Jr "Option Pricing : A Review" *Journal of Financial Economics*, Vol. 3, Jan-Mar, 1976, p.3-51

Sterk W.E. "Comparative Performance of the Black-Scholes and Roll-Geske-Whaley Option Pricing Models," *Journal of Financial and Quantitative Analysis*, Vol. 18, September, 1983, p. 345-354.

Sterk W.E. "Tests of Two Models for Valuing Call Options on stocks with Dividends" *Journal of Finance*, Vol. 37, December, 1982, p. 1229-1238.

Review in The Economist : The Mathematics of Financial Markets : A Survey of the Frontiers of Finance, *The Economist*, October, 9, 1993, Vol. 329, p.5-20

Thompson A. "Greek Lesson :What Thales the Milesian perfected is Baring fruit Today" *Finance Week*, May 25-31, 1995, p.22

Thompson A. & Ward M. "The Johannesburg Stock Exchange as an Efficient Market : A Review" *Journal of Studies in Economics and Econometrics*, Vol. 19, No. 3, 1995, p. 33-61

Trennepohl G.L. & Dukes W.P "CBOE Options and Stock Volatility" *Review of Business and Economic Research*, Vol. 14, Spring, 1979, p. 49-60.

Trippi R. "A Test of Option Market Efficiency Using a Random-Walk Valuation Model" *Journal of Economics and Business*, Vol. 29, Winter, 1977, p. 93-98.

Ozen M.F. "An Investigation of the Normal Hypothesis for Log-Price Relatives in Homogenous Time Segments" A Research Report presented to the Faculty of Business Administration, University of the Witwatersrand, 1977

Van Horne J.C. "Warrant Valuation in Relation to Volatility and Opportunity Costs" *Industrial Management Review*, Vol. 10, Spring, 1969, p. 19-32.

Whaley R. "On the Valuation of American Call Options on Stocks with Known Dividends" *Journal of Financial Economics*, Vol. 9, June, 1981, p.207-212

Whaley R. "Valuation of American Call Options on Dividend Paying Stocks: Empirical Tests" *Journal of Financial Economics*, Vol. 10, March, 1982, p. 29-58.

Wilmott P. & Howison S. & Dewynne J. The Mathematics of Financial Derivatives, Cambridge University Press, 1997

Youds D. & Firer C. & Ward M. "The Impact of Rights Issue Announcements on Share Prices" *Journal of Studies in Economics and Econometrics*, Vol. 17, No. 3, 1993, p.21-39

Zhang P.G. Exotic Options : A Guide to Second Generation Options, World Scientific Publishing Co., 1997

APPENDIX 1 - THE JSE LISTINGS REQUIREMENTS FOR RIGHTS OFFERS

The following sections, relevant to rights issues, have been extracted from thé Johannesburg Stock Exchange Rules.

Description

5.42 A rights offer is an offer to existing holders of securities to subscribe for or purchase further securities in proportion to their holdings made by means of the issue of a renounceable letter or other negotiable document which may be traded (as "nil paid" rights) for a period before payment for the securities is due.

Specific Requirements

5.43 Letters of application, allocation or acceptance are to be issued for the rights offer and must be renounceable. The Committee may in exceptional circumstances waive with requirement.

Underwriting

5.44 A rights offer need not be underwritten. However, if it is not underwritten, the offer must be conditional upon the minimum subscription being received that will fulfill the purpose of the rights offer. In these circumstances, the renounceable letters to be issued shall contain a statement, in a bold typeface, that the offer is conditional upon the minimum subscription being received and that in the event of the minimum subscription not being received, any purchaser of the letter will have no claim against the JSE, the listed company (including its directors, officers and advisers) or the seller of the letter. The second press announcement referred to in paragraph 5.57 shall contain a similar statement.

5.45 If the offer is underwritten, the underwriter must satisfy the Committee that it can meet its commitments.

5.46 Any underwriting commission paid to a shareholder of the company should not be above the current market rate payable to independent underwriters.

5.47 In a rights offer which includes the right to apply for excess securities, the right to apply for excess securities must be transferable upon renunciation of a letter of allocation.

5.48 In respect of applications for excess securities, the pool of excess securities should be allocated equitably, taking cognisance of the number of securities held by the shareholder (including those taken up as a result of the rights offer) and the excess securities applied for by such shareholder and may be used to round holdings up to multiples of 100 securities.

<u>Ratio for rights offers</u>

5.49 The ratio should not give rise to fractions of securities that have more than six decimal places.

General

5.50 Rights offers priced at above the ruling price require the approval of the Committee if it could increase the number of shares held by a shareholder and its associates in that class to more than 50%.

5.51 Unless circumstances are such as to warrant a concession being granted by the Committee, the Committee requires the letters of allocation to be listed.

5.52 In respect of the letter of allocation, only Form A (Form of Renunciation) requires the signature of the renouncee. Form B (Registration Application Form) and Form C (Application for split forms) are not be required to be signed.

Documents to be submitted to Committee

5.53 The documents detailed in paragraph 16.15 should be submitted to the Committee at the relevant times as specified within the timetable set out in paragraph 5.57 below.

Documents to be published

5.54 Press announcements should be published giving the following information, respectively:

a) the last date for shareholders to register to participate in the rights offer;

b) the terms of the rights offer;

c) the salient dates relating to the rights offer;

d) the results of the rights offer.

5.55 In addition a circular or pre-listing statement should be sent to shareholders.

5.56 The press announcements, rights offer circular or pre-listing statement should comply with the requirements of paragraphs 11.13 to 11.18 and should be issued according to the timetable set out below.

Timetable

5.57 The following timetable is applicable to a listed company making a rights offer:

Day	Event
Friday (D+0)	Latest date for the first press announcement giving the last day for registration for the rights.
Monday (D+10)	Second press announcement giving the terms of the rights offer including the statement referred to in paragraph 5.44.
Wednesday (D+12)	Third press announcement giving the salient dates for the rights offer. (All documentation described in paragraph 16.15 must have been submitted to an approved by the Committee).
Friday (D+14)	Last day to register for the rights offer.
Monday (D+17)	Letters of allocation listed. Securities listed ex rights.
Wednesday (D+19)	Last day for receipt of postal registrations.
Friday (D+21)	Circular and / or pre-listing statement and letters of allocation posted to shareholders registered for

	the rights offer.
Wednesday (D+40)	Last day for dealing in letters of allocation.
Thursday (D+41)	Last day for splitting letters of allocation (14h30). Securities that are the subject of the rights offer listed (if granted) if the rights offer is fully underwritten.
Friday (D+42)	Offer closes (14h30) (earliest date).
Wednesday (D+47)	Last day for postal acceptances of the rights offer.
Friday (D+49)	Fourth press announcement giving the results of the rights offer. The securities that are the subject of the rights offer listed (if granted) if the rights offer is not fully underwritten.

DOCUMENTS TO BE SUBMITTED BY LISTED COMPANIES

Rights

16.5 The following information is required to be submitted to and approved by the Committee before listing can be granted :

a) the circular or pre-listing statement;

b) the information with respect to any underwriting described in paragraph 16.10 g;

c) the application for listing complying with Schedule 2;

d) the provisional allotment letter;

e) copies of any exchange control (see paragraph 16.27) approvals required;

f) copies of any experts' consents (see paragraph 7.F.5) appearing in the circular or pre-listing statement; and

g) the appropriate documentation and listing fee as per Section 17.

APPLICATION FOR THE LISTING OF SECURITIES RESULTING FROM RIGHTS OFFERS

2.1 The application for the listing of securities resulting from a rights offer should include:

a) description and number of renounceable letters for which a listing is applied for, and the relevant dates;

b) description and number of securities for which a listing is applied, and the relevant dates;

c) brief description of the offer;

d) date on which renounceable letters and the circular or pre-listing statement will be posted;

e) date on which certificates will be issued;

f) last day for splitting and that the renounceable letters will be split as often as required;

g) date on which the offer closes;

h) the authorised and issued share capital of the applicant prior to the issue of the rights or claw-back securities;

i) the issued capital after the issue of the rights securities; and

j) in addition to the above information the following undertakings must be given:

   i) all renounceable letters dispatched by the applicant to registered shareholders will be sent by registered / certified mail and by airmail wherever this is possible; and

   ii) all acceptances for the offer sent by post by the beneficial holders will be accepted by the applicant provided the envelope bears the postmark of a day on or before the closing of the offer and provided such acceptances are received within 3 business days of the closing of the offer.

2.2 The application must be signed by the secretary and a director of the applicant and the sponsoring broker.

2.3 The application must be accompanied by a resolution of the directors of the applicant authorising the application for listing together with the relevant listing fee.

APPENDIX 2 CONVERGENCE OF THE JUMP DIFFUSION MODEL

INTRODUCTION

This appendix will illustrate that truncating the infinite series in the Merton Jump Diffusion model at 10 leads to accurate results at least to the fourth decimal place. Two examples will be tested to show the effect of different truncation limits on the option prices :

(1) Example 1 is an option that has 30 days remaining to maturity and is thus similar in nature to a right.
(2) Example 2 is an option that has one year remaining to maturity and is thus similar in nature to a warrant.

Each of these options will be tested for different strike prices thus allowing comparison of results for options that are in, at and out of the money which may thus affect the truncation. Interest rates of both 10% and 15% will be used in case there is an effect. The Merton model parameters chosen are based on those for the sample used in this study. The truncation will be done at 1, 2, 5, 10, 50 and 100 iterations. This will thus indicate whether truncation at a limit of 10 leads to materially different prices from a truncation at 100.

EXAMPLE 1 PARAMETERS

SHARE PRICE	100
STRIKE PRICE	100
RISKFREE RATE	15%
TIME TO EXPIRY	0.0833
JUMP MEAN	0.0243
DIFFUSION VARIANCE	0.0004
JUMP VARIANCE	0.0882

1.1 TESTING WITH RISKFREE RATE OF 10%

	TRUNCATION LIMIT					
STRIKE	1	2	5	10	50	100
80	21.0002	21.0002	21.0002	21.0002	21.0002	21.0002
90	11.1310	11.1311	11.1311	11.1311	11.1311	11.1311
100	1.2678	1.2678	1.2678	1.2678	1.2678	1.2678
110	0.0171	0.0171	0.0171	0.0171	0.0171	0.0171
120	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115

1.2 TESTING WITH RISKFREE RATE OF 15%

	TRUNCATION LIMIT					
STRIKE	1	2	5	10	50	100
80	21.0002	21.0002	21.0002	21.0002	21.0002	21.0002
90	11.1310	11.1311	11.1311	11.1311	11.1311	11.1311
100	1.2678	1.2678	1.2678	1.2678	1.2678	1.2678
110	0.0171	0.0171	0.0171	0.0171	0.0171	0.0171
120	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115

<u>EXAMPLE 2 PARAMETERS</u>

SHARE PRICE	100
STRIKE PRICE	100
RISKFREE RATE	15%
TIME TO EXPIRY	1.0000
JUMP MEAN	0.0243
DIFFUSION VARIANCE	0.0004
JUMP VARIANCE	0.0882

2.1 TESTING WITH RISKFREE RATE OF 15%

	TRUNCATION LIMIT					
STRIKE	1	2	5	10	50	100
80	31.1635	31.1735	31.1736	31.1736	31.1736	31.1736
90	22.5966	22.6049	22.6050	22.6050	22.6050	22.6050
100	14.0532	14.0599	14.0600	14.0600	14.0600	14.0600
110	5.5370	5.5424	5.5425	5.5425	5.5425	5.5425
120	0.2923	0.2967	0.2968	0.2968	0.2968	0.2968

CONCLUSION ON TRUNCATION LIMIT

From the above testing it is evident that a truncation limit of 10 provides highly accurate results (at least to the fourth decimal place). Even for the option with one year to maturity, truncation at 10 provides fairly accurate prices.

A limit of 10 will thus be used when employing the Merton Jump diffusion model in the empirical testing.

APPENDIX 3 - DERIVATION OF ITO'S LEMMA

The following simplified proof of Ito's Lemma follows the argument supplied by Hull (1991) as it is fairly easy to understand and does not get overly involved in the complex mathematics behind the proof. If Δx is a small change in x and ΔG the resulting change in G then the Taylor series expansion is :

$$\Delta G = \frac{dG}{dx}.\Delta x + \frac{1}{2}.\frac{d^2G}{dx^2}.\Delta x^2 + \frac{1}{6}.\frac{d^3G}{dx^3}.\Delta x^3 + ...$$

For a continuous and differentiable function of two variables (x, y) the analogous Taylor series expansion is :

$$\Delta G = \frac{dG}{dx}.\Delta x + \frac{dG}{dy}.\Delta y + \frac{1}{2}.\frac{d^2G}{dx^2}.\Delta x^2 + \frac{1}{2}.\frac{d^2G}{dy^2}.\Delta y^2 + \frac{d^2G}{dx.dy}.\Delta x.\Delta y + ...$$

For a variable that follows a generalised Wiener process $dx = a(x,t).dt + b(x,t).dz$ the Taylor series expansion is :

$$\Delta G = \frac{dG}{dx}.\Delta x + \frac{dG}{dt}.\Delta t + \frac{1}{2}.\frac{d^2G}{dx^2}.\Delta x^2 + \frac{1}{2}.\frac{d^2G}{dt^2}.\Delta t^2 + \frac{d^2G}{dx.dt}.\Delta x.\Delta t + ...$$

all terms of magnitude Δt^2 can be ignored thus leaving :

$$\Delta G = \frac{dG}{dx}.\Delta x + \frac{dG}{dt}.\Delta t + \frac{1}{2}.\frac{d^2G}{dx^2}.\Delta x^2 + \frac{d^2G}{dx.dt}.\Delta x.\Delta t$$

The third term $\frac{1}{2}.\frac{d^2G}{dx^2}.\Delta x^2$ then needs to be evaluated. Knowing that the discrete version of the generalised Wiener process is :

$$\Delta x = a(x,t).\Delta t + b(x,t).\varepsilon.\sqrt{\Delta t}$$

and without arguments this is :

$$\Delta x = a.\Delta t + b.\varepsilon.\sqrt{\Delta t}$$

it can be shown that :

$$(\Delta x)^2 = a^2.\Delta t^2 + 2.a.b.\varepsilon.\Delta t^{3/2} + b^2.\varepsilon^2.\Delta t$$

$$= b^2.\varepsilon^2.\Delta t + \textit{terms of higher} \text{ order in } \Delta t$$

$$\approx b^2.\Delta t$$

The variance of a standardised normal distribution is 1. This implies $E(\varepsilon^2) - [E(\varepsilon)]^2 = 1$. But $E(\varepsilon) = 0$ so it follows that $E(\varepsilon^2) = 1$. The expected value of $\varepsilon^2.\Delta t$ is thus Δt. The variance of $\varepsilon^2.\Delta t$ is of the order Δt^2 (which can thus be ignored) and thus $\varepsilon^2.\Delta t$ becomes non-stochastic and equal to its expected value of Δt as Δt tends to zero. The term thus becomes $b^2.dt$ as Δt tends to zero so that $\frac{1}{2}.\frac{d^2G}{dx^2}.\Delta x^2 \approx \frac{1}{2}.\frac{d^2G}{dx^2}.b^2.\Delta t$

Evaluating the last term $\frac{d^2G}{dx.dt}.\Delta x.\Delta t$ shows that :

$$\Delta x = a.\Delta t + b.\varepsilon.\sqrt{\Delta t}$$

$$\therefore \Delta x.\Delta t = (a.\Delta t + b.\varepsilon.\sqrt{\Delta t}).\Delta t$$

$$\therefore \Delta x.\Delta t = a.\Delta t^2 + b.\varepsilon.\Delta t^{3/2}$$

thus $\frac{d^2G}{dx.dt}.\Delta x.\Delta t$ involvs terms of a higher order than Δt which can be ignored. Thus Ito's Lemma is proven :

$$\Delta G = \frac{dG}{dx}.\Delta x + \frac{dG}{dt}.\Delta t + \frac{1}{2}.\frac{d^2G}{dx^2}.\Delta x^2 + \frac{d^2G}{dx.dt}.\Delta x.\Delta t$$
$$\approx \frac{dG}{dx}.\Delta x + \frac{dG}{dt}.\Delta t + \frac{1}{2}.\frac{d^2G}{dx^2}.b^2.\Delta t$$
$$\approx \frac{dG}{dx}.dx + \frac{dG}{dt}.dt + \frac{1}{2}.\frac{d^2G}{dx^2}.b^2.dt$$

APPENDIX 4 - DATA SAMPLE

YEAR	COMPANY NAME	MONTH	NUMBER OF SHARES	STRIKE PRICE (CENTS)	TOTAL RAISED (RANDS)
1992	CROWN	Aug	12,197,328	90	10,977,595
1992	EGOLI	Jul	10,587,002	35	3,705,451
1992	FEDSURE	Apr	5,967,500	1000	59,675,000
1992	FIRSTBK	Mar	10,913,052	5000	545,652,600
1992	GENBEHEER	Feb	127,196,060	900	1,144,764,540
1992	GENCOR	Feb	199,949,913	1000	1,999,499,130
1992	INHOLD	Apr	2,000,000	1750	35,000,000
1992	MANRO	Dec	23,265,306	50	11,632,653
1992	METKOR	Jun	11,939,858	220	26,267,688
1992	MJM	Dec	11,759,244	35	4,115,735
1992	PEPGRO	Jul	11,289,696	1200	135,476,352
1992	PEPKOR	Jul	23,850,443	1100	262,354,873
1992	PLATE-GL	Apr	8,236,193	4300	354,156,299
1992	PRIMA	Jun	44,370,000	63	27,953,100
1992	SASOL	Mar	56,382,400	1850	1,043,074,400
1992	SBIC	Dec	10,797,016	6000	647,820,960
1992	USKO	Jun	199,814,600	25	49,953,650
1993	ABS	Dec	24,772,900	120	29,727,480
1993	BERGERS	Jun	8,479,800	200	16,959,600
1993	BERTRAD	Jun	17,974,000	100	17,974,000
1993	BIDCORP	Jul	392,792	3650	14,336,908
1993	BIDVEST	Jul	278,053	7000	19,463,710
1993	BOLAND	Mar	12,106,188	840	101,691,979
1993	CLINICS	Jul	73,260,000	540	395,604,000
1993	DALYS	Apr	1,400,000	4100	57,400,000
1993	DIDATA	Mar	5,660,156	850	48,111,326
1993	EERSLNG	Jul	7,745,756	80	6,196,605
1993	E-R-P-M	Dec	110,602,800	500	553,014,000
1993	F-S-GROUP	Apr	113,102,304	100	113,102,304
1993	F-S-I	Apr	24,370,205	1000	243,702,050
1993	HIGATE	Mar	9,267,971	680	63,022,203
1993	HOLDAIN	Feb	4,857,176	4000	194,287,040
1993	HYPROP	Sep	10,101,398	630	63,638,807
1993	INHOLD	Jan	1,100,000	2050	22,550,000
1993	KAROS	Oct	30,692,960	130	39,900,848
1993	KLIPTON	Dec	5,109,350	120	6,131,220
1993	LONRHO	Jan	101,476,821	634	643,363,045
1993	MACMED	Nov	21,138,057	55	11,625,931
1993	MEDHOLD	Apr	5,692,500	50	2,846,250

1993	NORTHAM	Jan	24,192,000	1475	356,832,000
1993	PICAPLI	Aug	25,932,721	150	38,899,082
1993	POWTECH	Feb	15,744,914	400	62,979,656
1993	RHOVAN	Mar	175,950,230	20	35,190,046
1993	SA-DRUG	May	9,150,965	2200	201,321,230
1993	SUNCRUSH	Apr	271,258	40000	108,503,200
1993	TARGET	Nov	43,703,572	300	131,110,716
1993	TARGET	Nov	43,703,572	300	131,110,716
1993	TEMPORA	Apr	13,654,766	1800	245,785,788
1993	W-&-A	Apr	373,505,030	175	653,633,803
1993	WAICOR	Apr	433,414,383	70	303,390,068
1994	AMREL	Sep	20,725,794	750	155,443,455
1994	APEX	Dec	87,956,449	100	87,956,449
1994	AUTOPGE	Mar	8,383,556	290	24,312,312
1994	BASREAD	May	22,000,000	105	23,100,000
1994	CITYHLD	Sep	4,000,000	40	1,600,000
1994	ETTINGTN	Oct	10,834,455	400	43,337,820
1994	GROWPNT	Mar	8,600,575	320	27,521,840
1994	INHOLD	Jun	3,900,000	4500	175,500,000
1994	INVSTEC	Jun	5,653,389	5400	305,283,006
1994	JDGROUP	Dec	33,223,742	540	179,408,207
1994	JOEL	Apr	97,980,267	290	284,142,774
1994	KH-PROPS	Oct	10,120,972	520	52,629,054
1994	LASER	Nov	8,000,000	450	36,000,000
1994	LONSUGR	Feb	676,692	5000	33,834,600
1994	MAINPRO	Dec	68,065,789	185	125,921,710
1994	MASCON	Sep	2,708,306	150	4,062,459
1994	MASHOLD	Sep	4,684,928	150	7,027,392
1994	MASHOLD	Sep	2,708,306	150	4,062,459
1994	MORKELS	Nov	20,932,500	200	41,865,000
1994	NEI-AFR	May	6,703,127	450	30,164,072
1994	NEIHOLD	May	7,350,992	225	16,539,732
1994	NSA	Jul	21,480,000	600	128,880,000
1994	OZZ	Mar	5,077,360	900	45,696,240
1994	PROFURN	Nov	77,268,643	25	19,317,161
1994	SECHOLD	Apr	40,087,418	310	124,270,996
1994	SENTRCHEM	Mar	34,643,947	850	294,473,550
1994	SHOREDITS	Nov	9,155,350	210	19,226,235
1994	SILOAK	Jun	27,740,250	185	51,319,463
1994	UMDONI	Apr	39,000,000	190	74,100,000
1995	BEARMAN	Mar	947,636	2600	24,638,536
1995	BOLAND	Jul	7,667,252	5000	383,362,600
1995	COROHLD	Dec	3,559,432	2500	88,985,800
1995	DBN-DEEP	Nov	2,640,000	3000	79,200,000
1995	ESTVAAL	Nov	75,744,000	550	416,592,000
1995	FALVEST	Feb	798,868	1413	11,288,005

1995	HARWILL	May	2,150,000	225	4,837,500
1995	ISCOR	Mar	344,531,002	380	1,309,217,808
1995	JASCO	Dec	7,921,575	285	22,576,489
1995	KETTER	Feb	11,929,018	60	7,157,411
1995	MARLIN	Sep	117,845,532	51	60,101,221
1995	MCPHAIL	Feb	7,554,011	325	24,550,536
1995	MERHLD	Mar	2,634,638	360	9,484,697
1995	NORTHAM	Feb	20,448,000	2500	511,200,000
1995	NRB	Mar	4,154,015	375	15,577,556
1995	NSAINV	Aug	27,323,885	700	191,267,195
1995	NUWORLD	Nov	2,702,913	800	21,623,304
1995	ORYX	Nov	78,858,028	150	118,287,042
1995	PUBHOLD	Nov	1,623,828	270	4,384,336
1995	PUBLICO	Nov	2,233,635	270	6,030,815
1995	RICHWAY	Nov	16,961,588	410	69,542,511
1995	SAGEGRP	Mar	10,680,477	1150	122,825,486
1995	SCHAMIN	Nov	11,870,000	200	23,740,000
1995	SMART	Oct	7,356,898	675	49,659,062
1995	SPESCOM	Dec	11,139,088	275	30,632,492
1995	STOCKS	Jun	12,873,840	600	77,243,040
1995	STRAND	Feb	15,027,990	110	16,530,789
1995	SUPALEK	Aug	19,350,000	140	27,090,000
1995	TELJOY	Jun	25,516,480	400	102,065,920
1995	TEMPORA	Sep	10,716,074	2100	225,037,554
1995	UNISPIN	Sep	240,500,000	50	120,250,000
1995	W-&-A	Jun	159,557,647	500	797,788,235
1995	WALHOLD	Sep	8,079,972	800	64,639,776
1995	WALTONS	Sep	15,468,453	800	123,747,624
1995	WESWITS	Oct	13,830,000	285	39,415,500
1996	AFLIFE	Mar	14,632,838	865	126,574,049
1996	AKJ	Jun	4,950,000	300	14,850,000
1996	AMIC	Oct	8,196,340	14500	1,188,469,300
1996	BATECOR	Nov	16,460,048	190	31,274,091
1996	BLYVOOR	Apr	6,599,993	600	39,599,958
1996	BUILDMX	Oct	16,800,000	200	33,600,000
1996	CHOICE	Dec	13,634,236	750	102,256,770
1996	CONCOR	May	1,939,394	2375	46,060,608
1996	CULLINAN	Dec	29,156,338	150	43,734,507
1996	DATATEC	Jun	3,288,811	750	24,666,083
1996	ENGEN	Aug	19,190,838	3150	604,511,397
1996	GLODINA	May	10,144,645	145	14,709,735
1996	HARVEST	Apr	4,590,000	4000	183,600,000
1996	HLH	Sep	121,777,255	620	755,018,981
1996	HUNTCOR	Sep	46,364,406	1240	574,918,634
1996	IMPERIAL	Mar	23,783,797	4230	1,006,054,613
1996	JOEL	Apr	164,606,848	25	41,151,712

1996	LIBSIL	Oct	56,396,747	1350	761,356,085
1996	LOGTEK	Jun	1,288,404	590	7,601,584
1996	MACMED	May	12,858,025	160	20,572,840
1996	MANCARE	May	5,789,418	210	12,157,778
1996	MASCON	Feb	2,924,970	220	6,434,934
1996	MASHOLD	Feb	5,059,722	220	11,131,388
1996	MATH-ASH	Mar	6,073,800	200	12,147,600
1996	MORKELS	Aug	8,405,361	500	42,026,805
1996	MOTOLNK	Aug	15,011,392	300	45,034,176
1996	NRB	Aug	28,801,170	420	120,964,914
1996	NSAINV	Oct	20,083,057	1400	281,162,798
1996	OMEGA	Jul	4,829,958	425	20,527,322
1996	PAG	Jun	8,477,304	250	21,193,260
1996	PIONEER	Mar	23,393,163	255	59,652,566
1996	PREM-GRP	Aug	84,380,476	540	455,654,570
1996	PRESTAS	Dec	8,612,821	105	9,043,462
1996	PRIMA	Jun	33,277,500	55	18,302,625
1996	RA-HOLD	Nov	89,531,168	450	402,890,256
1996	RAINBOW	Sep	884,400,000	85	751,740,000
1996	SMG HOLDINGS	Aug	119,141,086	100	119,141,086
1996	SOWITS	Dec	10,500,000	14	1,470,000
1997	ADVTECH	Nov	52,454,291	200	104,908,582
1997	AUCKLND-N	Aug	17,901,291	400	71,605,164
1997	CAPITAL	Mar	22,984,470	2450	563,119,515
1997	CARSON	Jun	2,154,227	1200	25,850,724
1997	CHILLRS	Jul	10,750,000	100	10,750,000
1997	CLINICS	Nov	223,677,713	333	744,846,784
1997	CONTRAV	Oct	15,000,000	100	15,000,000
1997	COROHLD	Aug	17,656,836	7375	1,302,191,655
1997	CORWIL	Mar	929,345	350	3,252,708
1997	CRENDEL	Jan	283,365,444	18	51,005,780
1997	GEN-OPTIC	Mar	2,166,622	410	8,883,150
1997	ILCO	Mar	35,713,881	350	124,998,584
1997	INTRUST	May	25,645,356	780	200,033,777
1997	INVICTA	Oct	19,111,574	570	108,935,972
1997	KALGOLD	Mar	27,585,780	290	79,998,762
1997	KING	Nov	24,371,154	165	40,212,404
1997	KUDU	Oct	72,996,910	85	62,047,374
1997	MARLIN	Feb	143,534,478	85	122,004,306
1997	NAIL	Sep	318,911,432	315	1,004,571,011
1997	NEWPORT	May	7,242,665	400	28,970,660
1997	OMEGA	Dec	19,697,922	550	108,338,571
1997	PLASTAL	May	5,132,119	500	25,660,595
1997	PUBLICO	Nov	3,700,000	390	14,430,000
1997	ROADCOR	Jul	23,530,408	190	44,707,775
1997	SAFLIFE	May	112,500,859	2000	2,250,017,180

1997	SUB-N	Aug	18,137,492	49	8,887,371
1997	UNISPIN	Dec	204,685,000	20	40,937,000
1997	VESTCOR	Jul	15,835,090	35	5,542,282
1998	AIDA	Jan	3,000,000	85	2,550,000
1998	CITIZEN	Mar	7,278,111	1750	127,366,943
1998	CITYHLD	Feb	37,015,991	22	8,143,518
1998	DELTA	Mar	1,890,983	1650	31,201,220
1998	DON	Jan	252,415,974	25	63,103,994
1998	KOLOSUS	Jan	500,400,000	50	250,200,000
1998	LASER	Feb	6,265,512	480	30,074,458
1998	METJE-&-Z	Mar	1,005,420	620	6,233,604
1998	MIH	Mar	28,358,818	1250	354,485,225
1998	MOMENTUM	Apr	572,696,739	900	5,154,270,651
1998	MONEX	Mar	36,653,881	900	329,884,929
1998	NUWORLD	Mar	3,084,621	2400	74,030,904
1998	OPUS	Apr	4,745,585	460	21,829,691
1998	RMBH	Apr	112,225,933	1500	1,683,388,995
1998	SHOCRAF	Apr	32,500,000	400	130,000,000
1998	SPICER	Feb	30,346,288	19	5,765,795

APPENDIX 5 - JUMP DIFFUSION PARAMETERS

The following table lists the parameters calculated for the Merton Jump Diffusion Model using the Beckers cumulant matching procedure. Firstly, the actual paramaters are calculated under the headings "Actual". Scanning the variance column it is evident that some of the variance estimates are negative. As explained in Chapter 8, an adjusted method was thus used whereby the jump mean was calculated on a "pooled" basis for each year and the other parameters derived from the set of simultaneous equations. This is evident from the pooled jump mean column which displays the constant jump mean for each year.

YEAR	COMPANY NAME	ACTUAL JUMP MEAN	ACTUAL DIFFUSION VARIANCE	ACTUAL JUMP VARIANCE	POOLED JUMP MEAN	DERIVED DIFFUSION VARIANCE	DERIVED JUMP VARIANCE
1992	CROWN	0.0793	0.0002	0.0192	0.0196	0.0010	0.0387
1992	EGOLI	0.5652	-0.0007	0.0051	0.0196	0.0017	0.0276
1992	FEDSURE	0.0705	0.0001	0.0031	0.0196	0.0002	0.0058
1992	FIRSTBK	1.3130	0.0000	0.0001	0.0196	0.0001	0.0008
1992	GENBEHEER	34.5542	0.0033	-0.0001	0.0196	0.0005	0.0034
1992	GENCOR	0.8009	0.0001	0.0004	0.0196	0.0004	0.0026
1992	INHOLD	0.9843	-0.0001	0.0003	0.0196	0.0001	0.0023
1992	MANRO	0.0289	-0.0003	0.0886	0.0196	0.0001	0.1076
1992	METKOR	0.3187	-0.0001	0.0020	0.0196	0.0004	0.0080
1992	MJM	0.0799	-0.0001	0.0032	0.0196	0.0000	0.0064
1992	PEPGRO	0.0364	-0.0074	0.4720	0.0196	0.0028	0.6434
1992	PEPKOR	0.0741	0.0000	0.0047	0.0196	0.0002	0.0092
1992	PLATE-GL	0.4680	-0.0001	0.0006	0.0196	0.0001	0.0027
1992	PRIMA	15.8689	-0.0020	0.0002	0.0196	0.0004	0.0044
1992	SASOL	4.6480	0.0008	-0.0001	0.0196	0.0003	0.0016
1992	SBIC	0.1934	0.0000	0.0008	0.0196	0.0001	0.0026
1992	USKO	0.5943	-0.0046	0.0169	0.0196	0.0037	0.0931
1993	ABS	0.1779	-0.0001	0.0049	0.0159	0.0006	0.0163
1993	BERGERS	0.1563	-0.0012	0.0232	0.0159	0.0013	0.0727
1993	BERTRAD	0.0326	-0.0006	0.0667	0.0159	0.0001	0.0956
1993	BIDCORP	0.0378	0.0001	0.0103	0.0159	0.0002	0.0159
1993	BIDVEST	0.2899	-0.0002	0.0017	0.0159	0.0002	0.0071
1993	BOLAND	1.2313	-0.0002	0.0002	0.0159	0.0001	0.0022
1993	CLINICS	0.1663	-0.0001	0.0040	0.0159	0.0004	0.0129
1993	DALYS	0.3443	-0.0001	0.0006	0.0159	0.0001	0.0028
1993	DIDATA	0.7899	-0.0002	0.0009	0.0159	0.0004	0.0063
1993	EERSLNG	0.0293	-0.0005	0.0707	0.0159	0.0001	0.0960
1993	E-R-P-M	0.0726	0.0000	0.0177	0.0159	0.0007	0.0378
1993	F-S-GROUP	0.0402	0.0000	0.0357	0.0159	0.0005	0.0568

1993 F-S-I	0.0288	-0.0051	0.4793	0.0159	0.0016	0.6456
1993 HIGATE	0.1997	0.0000	0.0005	0.0159	0.0000	0.0016
1993 HOLDAIN	0.4211	0.0000	0.0004	0.0159	0.0001	0.0020
1993 HYPROP	0.2052	0.0000	0.0007	0.0159	0.0001	0.0025
1993 INHOLD	0.1637	0.0000	0.0012	0.0159	0.0001	0.0037
1993 KAROS	0.1974	0.0000	0.0048	0.0159	0.0007	0.0171
1993 KLIPTON	0.4156	-0.0004	0.0024	0.0159	0.0004	0.0121
1993 LONRHO	0.1701	0.0006	0.0084	0.0159	0.0016	0.0275
1993 MACMED	0.1068	-0.0004	0.0241	0.0159	0.0012	0.0623
1993 MEDHOLD	0.5427	-0.0016	0.0066	0.0159	0.0014	0.0387
1993 NORTHAM	0.2959	-0.0002	0.0020	0.0159	0.0003	0.0086
1993 PICAPLI	0.0688	0.0000	0.0084	0.0159	0.0003	0.0174
1993 POWTECH	0.3152	0.0000	0.0011	0.0159	0.0003	0.0049
1993 RHOVAN	0.3348	-0.0011	0.0154	0.0159	0.0029	0.0707
1993 SA-DRUG	0.0272	-0.0052	0.4801	0.0159	0.0021	0.6275
1993 SUNCRUSH	0.0904	0.0000	0.0020	0.0159	0.0001	0.0048
1993 TARGET	0.0504	-0.0015	0.1712	0.0159	0.0023	0.3047
1993 TARGET	0.0504	-0.0015	0.1712	0.0159	0.0023	0.3047
1993 TEMPORA	7.9992	-0.0006	0.0001	0.0159	0.0002	0.0024
1993 W-&-A	0.2335	-0.0001	0.0073	0.0159	0.0012	0.0279
1993 WAICOR	0.3037	-0.0003	0.0073	0.0159	0.0014	0.0320
1994 AMREL	0.2695	-0.0002	0.0029	0.0139	0.0004	0.0129
1994 APEX	0.0197	-0.0003	0.0722	0.0139	0.0000	0.0858
1994 AUTOPGE	0.2692	-0.0001	0.0040	0.0139	0.0007	0.0176
1994 BASREAD	1.5927	-0.0025	0.0028	0.0139	0.0016	0.0304
1994 CITYHLD	0.0957	-0.0017	0.0480	0.0139	0.0012	0.1259
1994 ETTINGTN	0.1776	0.0000	0.0024	0.0139	0.0003	0.0087
1994 GROWPNT	0.1049	-0.0003	0.0102	0.0139	0.0004	0.0279
1994 INHOLD	0.2561	-0.0001	0.0015	0.0139	0.0002	0.0066
1994 INVSTEC	0.2902	0.0000	0.0011	0.0139	0.0002	0.0050
1994 JDGROUP	0.0745	-0.0001	0.0071	0.0139	0.0002	0.0165
1994 JOEL	0.2423	0.0046	-0.0038	0.0139	0.0035	0.0159
1994 KH-PROPS	0.1362	0.0000	0.0007	0.0139	0.0001	0.0022
1994 LASER	0.2269	-0.0001	0.0031	0.0139	0.0005	0.0124
1994 LONSUGR	0.0680	-0.0002	0.0083	0.0139	0.0001	0.0184
1994 MAINPRO	0.0189	-0.0013	0.1767	0.0139	0.0008	0.2058
1994 MASCON	0.0216	-0.0076	0.9986	0.0139	0.0033	1.2436
1994 MASHOLD	0.0215	-0.0068	0.9996	0.0139	0.0026	1.2416
1994 MASHOLD	0.1721	-0.0009	0.0196	0.0139	0.0015	0.0688
1994 MORKELS	0.1429	-0.0001	0.0137	0.0139	0.0013	0.0440
1994 NEI-AFR	0.1633	0.0000	0.0032	0.0139	0.0003	0.0111
1994 NEIHOLD	0.0813	-0.0001	0.0071	0.0139	0.0002	0.0171
1994 NSA	0.0392	-0.0003	0.1932	0.0139	0.0028	0.3242
1994 OZZ	0.2195	0.0000	0.0011	0.0139	0.0002	0.0042
1994 PROFURN	0.2539	-0.0002	0.0170	0.0139	0.0031	0.0727
1994 SECHOLD	0.1884	-0.0006	0.0173	0.0139	0.0017	0.0636

1994 SENTRCHEM	9.9096	0.0009	-0.0001	0.0139	0.0002	0.0019
1994 SHOREDITS	0.1520	-0.0002	0.0086	0.0139	0.0007	0.0285
1994 SILOAK	0.0810	0.0000	0.0129	0.0139	0.0007	0.0310
1994 UMDONI	0.0490	0.0001	0.0032	0.0139	0.0001	0.0061
1995 BEARMAN	0.1620	0.0000	0.0015	0.0355	0.0001	0.0032
1995 BOLAND	0.6373	-0.0001	0.0007	0.0355	0.0002	0.0032
1995 COROHLD	0.3682	0.0000	0.0008	0.0355	0.0002	0.0025
1995 DBN-DEEP	0.0935	0.0002	0.0088	0.0355	0.0005	0.0143
1995 ESTVAAL	6.2901	-0.0020	0.0005	0.0355	0.0008	0.0066
1995 FALVEST	0.4004	-0.0014	0.0074	0.0355	0.0007	0.0248
1995 HARWILL	0.1069	-0.0001	0.0059	0.0355	0.0001	0.0102
1995 ISCOR	26.1956	-0.0013	0.0001	0.0355	0.0003	0.0018
1995 JASCO	0.0734	0.0002	0.0059	0.0355	0.0003	0.0084
1995 KETTER	4.6210	-0.0027	0.0009	0.0355	0.0012	0.0106
1995 MARLIN	0.1551	0.0014	0.0152	0.0355	0.0026	0.0318
1995 MCPHAIL	0.3429	-0.0003	0.0033	0.0355	0.0005	0.0104
1995 MERHLD	0.2620	-0.0001	0.0018	0.0355	0.0002	0.0048
1995 NORTHAM	0.3941	0.0001	0.0036	0.0355	0.0011	0.0119
1995 NRB	0.0922	-0.0001	0.0098	0.0355	0.0002	0.0158
1995 NSAINV	0.0320	-0.0025	0.2201	0.0355	0.0029	0.2089
1995 NUWORLD	0.1657	0.0000	0.0016	0.0355	0.0001	0.0035
1995 ORYX	0.0538	0.0011	0.0176	0.0355	0.0013	0.0216
1995 PUBHOLD	0.0468	-0.0002	0.0208	0.0355	0.0000	0.0238
1995 PUBLICO	0.0724	-0.0001	0.0143	0.0355	0.0002	0.0204
1995 RICHWAY	0.1719	0.0000	0.0005	0.0355	0.0000	0.0011
1995 SAGEGRP	0.3657	-0.0001	0.0010	0.0355	0.0001	0.0033
1995 SCHAMIN	0.1020	0.0001	0.0066	0.0355	0.0004	0.0111
1995 SMART	0.1216	0.0000	0.0006	0.0355	0.0000	0.0010
1995 SPESCOM	0.0859	0.0006	0.0078	0.0355	0.0009	0.0121
1995 STOCKS	3.5067	-0.0013	0.0007	0.0355	0.0009	0.0067
1995 STRAND	0.0377	-0.0141	1.4916	0.0355	0.0124	1.5364
1995 SUPALEK	0.1766	0.0004	0.0111	0.0355	0.0015	0.0247
1995 TELJOY	0.4344	0.0000	0.0018	0.0355	0.0005	0.0063
1995 TEMPORA	0.6174	0.0000	0.0002	0.0355	0.0001	0.0010
1995 UNISPIN	0.5367	-0.0003	0.0054	0.0355	0.0019	0.0209
1995 W-&-A	0.4204	0.0000	0.0044	0.0355	0.0013	0.0153
1995 WALHOLD	0.3871	-0.0001	0.0008	0.0355	0.0001	0.0025
1995 WALTONS	0.8571	-0.0002	0.0005	0.0355	0.0002	0.0026
1995 WESWITS	31.4383	-0.0035	0.0001	0.0355	0.0009	0.0042
1996 AFLIFE	0.7267	-0.0001	0.0005	0.0140	0.0002	0.0037
1996 AKJ	0.3221	-0.0003	0.0034	0.0140	0.0005	0.0162
1996 AMIC	2.9270	-0.0002	0.0002	0.0140	0.0002	0.0023
1996 BATECOR	0.0776	0.0002	0.0049	0.0140	0.0004	0.0116
1996 BLYVOOR	4.0470	-0.0020	0.0008	0.0140	0.0010	0.0134
1996 BUILDMX	0.0666	0.0005	-0.0008	0.0140	0.0004	0.0018
1996 CHOICE	0.0537	0.0003	0.0064	0.0140	0.0005	0.0125

1996 CONCOR	0.2183	0.0000	0.0021	0.0140	0.0003	0.0083
1996 CULLINAN	6.2741	-0.0033	0.0007	0.0140	0.0010	0.0151
1996 DATATEC	0.4351	-0.0002	0.0021	0.0140	0.0006	0.0115
1996 ENGEN	0.1014	0.0001	0.0030	0.0140	0.0003	0.0081
1996 GLODINA	0.1956	0.0002	0.0034	0.0140	0.0007	0.0127
1996 HARVEST	0.3087	0.0000	0.0003	0.0140	0.0001	0.0015
1996 HLH	0.6106	-0.0002	0.0011	0.0140	0.0004	0.0076
1996 HUNTCOR	0.0546	0.0001	0.0069	0.0140	0.0003	0.0136
1996 IMPERIAL	0.2203	0.0000	0.0007	0.0140	0.0001	0.0029
1996 JOEL	0.0192	0.0021	-0.0047	0.0140	0.0019	0.0055
1996 LIBSIL	0.1598	0.0003	0.0015	0.0140	0.0004	0.0050
1996 LOGTEK	0.0983	0.0001	0.0029	0.0140	0.0003	0.0077
1996 MACMED	1.0199	-0.0005	0.0018	0.0140	0.0011	0.0152
1996 MANCARE	0.0830	-0.0003	0.0177	0.0140	0.0005	0.0431
1996 MASCON	0.2703	-0.0003	0.0025	0.0140	0.0003	0.0111
1996 MASHOLD	0.2178	-0.0001	0.0024	0.0140	0.0003	0.0096
1996 MATH-ASH	0.0745	-0.0015	0.0651	0.0140	0.0013	0.1502
1996 MORKELS	0.1822	0.0000	0.0017	0.0140	0.0002	0.0060
1996 MOTOLNK	26.8034	0.0075	-0.0002	0.0140	0.0010	0.0103
1996 NRB	0.0469	-0.0002	0.0544	0.0140	0.0009	0.0996
1996 NSAINV	2.3556	-0.0004	0.0003	0.0140	0.0003	0.0037
1996 OMEGA	0.0242	-0.0078	0.9862	0.0140	0.0021	1.2985
1996 PAG	0.3255	-0.0011	0.0154	0.0140	0.0029	0.0742
1996 PIONEER	0.2109	0.0000	0.0007	0.0140	0.0001	0.0027
1996 PREM-GRP	4.5986	-0.0004	0.0002	0.0140	0.0003	0.0029
1996 PRESTAS	6.0007	-0.0034	0.0008	0.0140	0.0014	0.0174
1996 PRIMA	0.2964	-0.0001	0.0011	0.0140	0.0002	0.0051
1996 RA-HOLD	2.3814	-0.0012	0.0008	0.0140	0.0007	0.0110
1996 RAINBOW	0.9361	-0.0004	0.0019	0.0140	0.0011	0.0154
1996 SMG HOLDING	1.2043	-0.0008	0.0014	0.0140	0.0007	0.0128
1996 SOWITS	0.4240	-0.0001	0.0098	0.0140	0.0033	0.0537
1997 ADVTECH	53.5000	0.0169	-0.0003	0.0513	0.0015	0.0090
1997 AUCKLND-N	0.0237	-0.0031	0.5018	0.0513	0.0087	0.3406
1997 CAPITAL	0.4220	-0.0002	0.0012	0.0513	0.0002	0.0033
1997 CARSON	0.1447	0.0008	0.0100	0.0513	0.0013	0.0168
1997 CHILLRS	0.3085	0.0022	-0.0020	0.0513	0.0014	0.0049
1997 CLINICS	0.1413	0.0002	0.0033	0.0513	0.0004	0.0054
1997 CONTRAV	0.0576	0.0004	0.0216	0.0513	0.0005	0.0229
1997 COROHLD	0.0243	0.0001	0.0252	0.0513	0.0002	0.0173
1997 CORWIL	0.1319	-0.0005	0.0121	0.0513	0.0001	0.0194
1997 CRENDEL	0.0573	-0.0293	1.7186	0.0513	0.0240	1.8157
1997 GEN-OPTIC	0.0424	-0.0002	0.0368	0.0513	0.0003	0.0334
1997 ILCO	15.0500	-0.0088	0.0007	0.0513	0.0016	0.0127
1997 INTRUST	1.2437	-0.0001	0.0003	0.0513	0.0002	0.0016
1997 INVICTA	0.9625	-0.0002	0.0012	0.0513	0.0007	0.0054
1997 KALGOLD	0.2480	0.0008	0.0101	0.0513	0.0021	0.0223

1997 KING	1.7701	0.0014	-0.0003	0.0513	0.0007	0.0020
1997 KUDU	0.1175	0.0004	0.0073	0.0513	0.0007	0.0110
1997 MARLIN	0.1941	0.0013	-0.0013	0.0513	0.0009	0.0025
1997 NAIL	1.3593	-0.0002	0.0007	0.0513	0.0006	0.0037
1997 NEWPORT	0.1976	0.0000	0.0016	0.0513	0.0001	0.0030
1997 OMEGA	0.2208	0.0002	0.0033	0.0513	0.0006	0.0069
1997 PLASTAL	0.3630	-0.0004	0.0058	0.0513	0.0009	0.0153
1997 PUBLICO	0.9881	-0.0009	0.0028	0.0513	0.0012	0.0123
1997 ROADCOR	0.0584	0.0002	0.0115	0.0513	0.0002	0.0123
1997 SAFLIFE	0.1023	0.0004	0.0043	0.0513	0.0005	0.0060
1997 SUB-N	2.3292	-0.0098	0.0083	0.0513	0.0067	0.0560
1997 UNISPIN	0.5560	0.0012	0.0026	0.0513	0.0022	0.0084
1997 VESTCOR	0.1668	-0.0020	0.0955	0.0513	0.0051	0.1722
1998 AIDA	0.1986	-0.0005	0.0212	0.0176	0.0025	0.0710
1998 CITIZEN	0.3195	0.0002	0.0055	0.0176	0.0016	0.0236
1998 CITYHLD	1.0516	0.0002	0.0047	0.0176	0.0045	0.0360
1998 DELTA	0.1410	0.0001	0.0033	0.0176	0.0003	0.0092
1998 DON	0.2590	0.0016	0.0085	0.0176	0.0033	0.0327
1998 KOLOSUS	6.2820	-0.0041	0.0010	0.0176	0.0020	0.0193
1998 LASER	0.1100	0.0003	0.0105	0.0176	0.0010	0.0261
1998 METJE-&-Z	0.0908	-0.0001	0.0052	0.0176	0.0002	0.0118
1998 MIH	0.2515	0.0001	0.0020	0.0176	0.0005	0.0076
1998 MOMENTUM	0.0292	-0.0121	1.0072	0.0176	0.0056	1.2959
1998 MONEX	46.0902	-0.0101	0.0003	0.0176	0.0014	0.0131
1998 NUWORLD	0.6143	-0.0001	0.0007	0.0176	0.0002	0.0040
1998 OPUS	0.3090	0.0002	0.0063	0.0176	0.0016	0.0264
1998 RMBH	0.0292	-0.0122	1.0288	0.0176	0.0055	1.3241
1998 SHOCRAF	0.0807	-0.0009	0.1303	0.0176	0.0047	0.2788
1998 SPICER	13.0866	0.0161	-0.0009	0.0176	0.0041	0.0241

APPENDIX 6 - RESTRICTIONS ON RATIONAL OPTION PRICING

Following the work of Merton (1973), certain restrictions can be derived on option pricing formulas. These restrictions can then be used to determine the value of the option from the stochastic differential equation derived as the solution to the equation is entirely dependent on the boundary conditions entered for the derivative security. Merton (1973) made no assumptions about the process generating the stock price over time, the restrictions he derived depend only on dominance arguments. In equilibrium no dominant or dominated asset can exist. If a dominant security did exist, everyone would buy that security thus bidding up its price until the dominance disappeared and vice versa. The results of the dominance arguments can thus be used as a general consistency criteria against which subsequent models may be conveniently measured.

Denote $F(S,t;E)$ as the value of an American warrant and $f(S,t;E)$ as the price of a European warrant with share price S, time t to expiration and exercise price E. Then from the definition of a warrant and using the limited liability concept :

$$F(S,t;E) \geq 0 \qquad \text{and} \qquad f(S,t;E) \geq 0$$

and at expiration both contracts must satisfy :

$$F(S,0;E) = f(S,0;E) = \max(0, S - E)$$

From the conditions of arbitrage it follows that (not necessarily for a European warrant) :

$$F(S,t;E) \geq \max(0, S - E)$$

ASSUMPTION 1

An option must be priced such that it is neither a dominant or dominated security so that :

$$F(S,t_2;E) \geq F(S,t_1;E) \quad \text{if } t_2 > t_1$$

and that :

$$F(S,t;E) \geq f(S,t;E)$$

Furthermore two identical warrants except for their exercise prices must satisfy :

$$F(S,t;E_2) \leq F(S,t;E_1)$$

$$f(S,t;E_2) \leq f(S,t;E_1) \quad \text{if } \mathrm{E}_2 > E_1$$

A common stock is equivalent to a perpetual warrant with zero exercise price so :

$$S = F(S,\infty,0) \geq F(S,t;E)$$

and the warrant must be worthless if the stock is to :

$$F(0,t;E) = f(0,t;E) = 0$$

THEOREM 1

Let $P(t)$ represent the present value of a riskless loan which pays one dollar t years from now (so $1 = P(0) > P(t_1) > P(t_2) > \ldots$). If there is a European warrant where no payouts (dividends) are made to the common stock over the life of the warrant then :

$$f(S,t;E) \geq \max[0, S - E.P(t)]$$

THEOREM 2

If the conditions for Theorem 1 hold then an American warrant will never be exercised prior to expiration and hence has the same value as a European warrant. Thus early exercise must be due to unfavorable changes in the exercise price or lack of protection against dividends. It is common to refer to $\max(0, S - E)$ as the intrinsic value of the warrant and to state that the warrant must always sell for at least its intrinsic value. Merton (1973) showed that this value should actually be $\max(0, S - E.P(t))$ to hold.

THEOREM 3

If the conditions for Theorem 1 hold then the value of a perpetual warrant must equal the value of the common stock. Theorem 1 demonstrates that the warrant price must be a function of $P(t)$. The effect of a change in P is similar to a change in the exercise price which is negative - so the warrant price is an increasing function of the interest rate (the higher the interest rate, the lower the PV of the exercise price; the lower the exercise price, the higher the value of the option).

THEOREM 4

If $F(S,t;E)$ is a rationally determined warrant price, then F is a convex function of its exercise price E.

THEOREM 5

If $f(S,t;E)$ is a rationally determined European warrant price, then :

$$-P(t).(E_2 - E_1) \le f(S,t;E_2) - f(S,t;E_1) \le 0 \quad \text{for} \quad E_1 < E_2$$

and if f is a differentiable function of its exercise price :

$$-P(t) \le \partial f(S,t;E) / \partial E \le 0$$

THEOREM 6

Let $Q(t)$ be the price of a common stock at time t and $F_Q(Q,t;E_Q)$ the price of a warrant to purchase one share of stock on or before a given time t years in the future when the current price of the shares is Q. If k is a positive constant such that $Q(t)=k.S(t)$ and $E_Q = k.E$ then

$$F_Q(Q,t;E_Q) \equiv k.F(S,t;E)$$

Thus the rational warrant pricing function is homogeneous of degree one in S and E with respect to scale.

THEOREM 7

The rationally determined warrant price is a non-decreasing function of the riskiness of its associated common stock. Thus the more uncertain one is about the outcomes on the common stock, the more valuable is the warrant.

THEOREM 8

If the distribution of returns per dollar invested in the common stock is independent of the level of the stock price, then $F(S,t;E)$ is homogeneous of degree one in the stock price per share and the exercise price.

THEOREM 9

If the distribution of returns per dollar invested in the common stock is independent of the level of the stock price, then $F(S,t;E)$ is a convex function of the stock price.

THEOREM 10

If Assumption 1 holds and the borrowing and lending rates are equal then :

$$g(S,t;E) = f(S,t;E) - S + E.P(t)$$

APPENDIX 7 PROOF OF CONVERGENCE OF THE BINOMIAL MODEL AND BLACK SCHOLES MODEL

This Appendix proves that the binomial formula converges to the Black Scholes formula in continuous time where the binomial fomula is :

$$C = S.\Phi[a;n,p'] - \frac{K}{r^n}.\Phi[a;n,p]$$

The proof follows that of Cox, Ross and Rubenstein (1979). As explained in Chapter 6, what needs to be proven is that in continuous time :

$$\Phi[a;n,p'] \rightarrow N(x) \quad and \quad \Phi[a;n,p] \rightarrow N(x - \sigma.\sqrt{t})$$

Looking at $\Phi[a;n,p]$ this is the probability that the sum of n random variables each taking the value 1 with probability p and 0 with probability (1 - p) will be greater than or equal to a. The random value of this sum has mean $n.p$ and standard deviation $\sqrt{n.p.(1-p)}$ and thus :

$$1 - \Phi[a;n,p] = prob[j \le a-1] = prob\left[\frac{j - n.p}{\sqrt{n.p.(1-p)}} \le \frac{a-1-n.p}{\sqrt{n.p.(1-p)}}\right]$$

The share price return can be written :

$$\log(S^* / S) = j.\log(u/d) + n.\log d$$

so that :

$$\hat{\mu_P} = p.\log(u/d) + \log d$$
$$\hat{\sigma_P^2} = p.(1-p).[\log(u/d)]^2$$

Then :

$$\frac{j-n.p}{\sqrt{n.p.(1-p)}}=\frac{\log(S^{*}/S)-\hat{\mu_P}.n}{\hat{\sigma_P}.\sqrt{n}}$$

Using the definition of a in the binomial formula :

$$a-1=\log(K/S.d^{n})/\log(u/d)-\varepsilon=[\log(K/S)-n.\log d]/\log(u/d)-\varepsilon$$

where ε is a number between zero and one. Thus :

$$\frac{a-1-n.p}{\sqrt{n.p.(1-p)}}=\frac{\log(K/S)-\hat{\mu_P}.n-\varepsilon.\log(u/d)}{\hat{\sigma_P}.\sqrt{n}}$$

Combining the above results in the complementary binomial distribution :

$$1-\Phi[a;n,p]=prob\left[\frac{\log(S^{*}/S)-\hat{\mu_P}.n}{\hat{\sigma_P}.\sqrt{n}}\leq\frac{\log(K/S)-\hat{\mu_P}.n-\varepsilon.\log(u/d)}{\hat{\sigma_P}.\sqrt{n}}\right]$$

The central limit theorem can then be applied contingent on the higher order moments being insignificant as n approaches infinity, so that :

$$\frac{q.\left|\log u-\hat{\mu_P}\right|^{3}+(1-q).\left|\log d-\hat{\mu_P}\right|^{3}}{\hat{\sigma}_P^{3}.\sqrt{n}}=\frac{(1-p)^{2}+p^{2}}{\sqrt{n.p.(1-p)}}\rightarrow 0\quad as\quad n\rightarrow\infty$$

Then :

$$p\rightarrow 0.5+0.5\left(\frac{\log r-0.5\sigma^{2}}{\sigma}\right).\sqrt{\frac{t}{n}}$$

and thus the initial condition holds. As n approaches infinity :

$$\hat{\mu_P}.n\rightarrow(\log r-0.5\sigma^{2}).t$$
$$\hat{\sigma_P}.\sqrt{n}\rightarrow\sigma.\sqrt{t}$$
$$\log(u/d)\rightarrow 0$$

For the application to the central limit theorem :

$$\frac{\log(K/S) - \hat{\mu_P}.n - \varepsilon.\log(u/d)}{\hat{\sigma_P}.\sqrt{n}} \rightarrow z \equiv \frac{\log(K/S) - \log(r - 0.5\sigma^2).t}{\sigma.\sqrt{t}}$$

and thus :

$$1 - \Phi[a;n,p] \rightarrow N(z) = N\left[\frac{\log(K.r^{-t}/S) - \hat{\mu_P}.n}{\sigma.\sqrt{t}} + 0.5\sigma^2.\sqrt{t}\right]$$

By the symetry of the normal distribution :

$$\Phi[a;n,p] \rightarrow N(-z) = N\left[\frac{\log(S/K.r^{-t}) - \hat{\mu_P}.n}{\sigma.\sqrt{t}} - 0.5\sigma^2.\sqrt{t}\right] = N(x - \sigma.\sqrt{t})$$

A similar argument holds for $\Phi[a;n,p']$ which thus proves that in the limit of continuous time the binomial option pricing model converges to the Black-Scholes formula.

APPENDIX 8 - VISUAL BASIC ROUTINES WRITTEN

6.1 BLACK SCHOLES MODEL ROUTINE

```
Function BS (SHARE, EXERCISE, RISKFREE, VOLATILITY, TOEXPIRE)

  D = (Log(SHARE / EXERCISE) + (RISKFREE + (VOLATILITY ^ 2) / 2) * TOEXPIRE)
      /(VOLATILITY * Sqr(TOEXPIRE))
  DD = D - (VOLATILITY * Sqr(TOEXPIRE))
  BS = (SHARE * Application.NormSDist(D)) - (EXERCISE * Exp(RISKFREE * (-
       TOEXPIRE)) * Application.NormSDist(DD))

End Function
```

6.2 CONSTANT ELASTICITY OF VARIANCE MODEL

6.2.1 SQUARE ROOT MODEL

```
Function CEV(STOCK, STRIKE, RISKFREE, VOLATILITY, EXPIRY)

Rem Define the variables required
   Y = (4 * RISKFREE * STOCK) / ((VOLATILITY ^ 2) * (1 - Exp(-RISKFREE *
      EXPIRY)))
   Z = (4 * RISKFREE * STRIKE) / ((VOLATILITY ^ 2) * (Exp(RISKFREE * EXPIRY) -
      1))

Rem This handles the case where w = 4
      H_4 = 1 - (2 * (4 + Y) * (4 + (3 * Y)) * ((4 + (2 * Y)) ^ (-2))) / 3
      TERM1 = H_4 * (H_4 - 1) * (4 + (2 * Y)) / ((4 + Y) ^ 2)
      TERM2 = H_4 * (H_4 - 1) * (2 - H_4) * (1 - (3 * H_4))
      TERM3 = ((4 + (2 * Y)) ^ 2) / (2 * (4 + Y) ^ 4)
```

```
    TERM4 = (Z / (4 + Y)) ^ H_4
    Q_4_NUMER = 1 + TERM1 - TERM2 * TERM3 - TERM4
    TERM5 = 2 * (H_4 ^ 2)
    TERM6 = (4 + (2 * Y)) / ((4 + Y) ^ 2)
    TERM7 = (1 - H_4) * (1 - (3 * H_4))
    TERM8 = (4 + (2 * Y)) / ((4 + Y) ^ 2)
    TERM9 = 1 - TERM7 * TERM8
    Q_4_DENOM = (TERM5 * TERM6 * TERM9) ^ (1 / 2)
    Q4 = Q_4_NUMER / Q_4_DENOM

Rem This handles the case where w = 0
   H_0 = 1 - (2 * (0 + Y) * (0 + (3 * Y)) * ((0 + (2 * Y)) ^ (-2))) / 3
   TERM11 = H_0 * (H_0 - 1) * (0 + (2 * Y)) / ((0 + Y) ^ 2)
   TERM12 = H_0 * (H_0 - 1) * (2 - H_0) * (1 - (3 * H_0))
   TERM13 = ((0 + (2 * Y)) ^ 2) / (2 * (0 + Y) ^ 4)
   TERM14 = (Z / (0 + Y)) ^ H_0
   Q_0_NUMER = 1 + TERM11 - TERM12 * TERM13 - TERM14
   TERM15 = 2 * (H_0 ^ 2)
   TERM16 = (0 + (2 * Y)) / ((0 + Y) ^ 2)
   TERM17 = (1 - H_0) * (1 - (3 * H_0))
   TERM18 = (0 + (2 * Y)) / ((0 + Y) ^ 2)
   TERM19 = 1 - TERM17 * TERM18
   Q_0_DENOM = (TERM5 * TERM6 * TERM9) ^ (1 / 2)
   Q0 = Q_0_NUMER / Q_0_DENOM

Rem This calculates the CEV formula value

  CEV = (STOCK * Application.NormSDist(Q4)) - (STRIKE * Exp(-RISKFREE *
        EXPIRY)) * Application.NormSDist(Q0)

End Function
```

6.2.2 ABSOLUTE MODEL

```
Function ABSCEV(STOCK, STRIKE, RISKFREE, VOLATILITY, EXPIRY)

Rem Define the variables
   V = VOLATILITY * Sqr((1 - Exp(-2 * RISKFREE * EXPIRY)) / (2 * RISKFREE))
   H_1 = (STOCK - (STRIKE * Exp(-RISKFREE * EXPIRY))) / VOLATILITY
   H_2 = (-STOCK - (STRIKE * Exp(-RISKFREE * EXPIRY))) / VOLATILITY

Rem Calculate the formula values
   TERM1 = STOCK - (STRIKE * Exp(-RISKFREE * EXPIRY))
   TERM2 = STOCK + (STRIKE * Exp(-RISKFREE * EXPIRY))
   TERM3 = (1 / Sqr(2 * Application.Pi())) * Exp((-1 / 2) * ((H_1) ^ 2))
   TERM4 = (1 / Sqr(2 * Application.Pi())) * Exp((-1 / 2) * ((H_2) ^ 2))

Rem Calculate the Absolute CEV formula
   ABSCEV = TERM1 * Application.NormSDist(H_1) + TERM2 *
              Application.NormSDist(H_2)  + VOLATILITY * (TERM3 - TERM4)

End Function
```

6.3 MERTON JUMP DIFFUSION MODEL

```
Function MERTON(STOCK, EXERCISE, EXPIRATION, RISKFREE, MEAN, VARJUMP,
        VARNORM)

Dim I As Integer

For I = 0 To 10

   VOLATILITY = VARNORM + (I * VARJUMP / EXPIRATION)
   D = (Log(STOCK / EXERCISE) + (RISKFREE + (VOLATILITY) / 2) * EXPIRATION) /
     (Sqr(VOLATILITY) * Sqr(EXPIRATION))
```

```
DD = D - (Sqr(VOLATILITY) * Sqr(EXPIRATION))
POISS = (Exp(-MEAN * EXPIRATION)) * ((MEAN * EXPIRATION) ^ I) /
    (Application.Fact(I))

TEMP = TEMP + POISS * ((STOCK * Application.NormSDist(D)) - (EXERCISE *
    Exp(RISKFREE * (-EXPIRATION)) * Application.NormSDist(DD)))

Next I

MERTON = TEMP

End Function
```