

The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.

Pricing and Hedging Credit-risky Derivatives using Corporate Bonds.

Submitted by

Stephen Ngonidzashe Murefu

Supervised by

Professor Haim Abraham

in fulfilment of the requirements for the degree of **Master of Commerce** (*Financial
Economics*)

**University of Cape Town
School of Economics
December 2003**

ABSTRACT

The benefits of being a bondholder are well appreciated and documented in the world of investments. However, most of these holdings are in the risk free (no chances of defaulting) government bonds (Treasuries). It follows then that by investing in the riskier bonds (corporate bonds); the investor should reap more benefits (higher returns). The argument lies in the trade off, yield and risk, higher yield results in higher credit risk (the probability of default is higher). The answer is to invest in corporate bonds and simultaneously find ways to minimise the credit risk associated with those purchases. Credit derivatives (options in particular for this paper) are financial instruments that can aid in the management of credit risk by insuring against adverse movements in the credit quality of the borrower. That is, if the borrower defaults, the bondholder will incur loss on the bond investment but the losses can be offset by gains in the credit derivative. The credit risk can be fully offset on condition that the credit derivative is priced and hedged properly.

This paper looks at the use of the Hull & White Model and the Jarrow & Turnbull Model to price and hedge credit risky options using corporate bonds and their comparable Treasury bonds. The models are taken from their papers, "*The Price of Default*", (1992) and "*Pricing Derivatives on Financial Securities Subject to Credit Risk*", (1995), respectively. Their models develop procedures to estimate the expected loss of default on a derivative using the price of risky debt issued by the counterparty in the derivative contract.

The Jarrow & Turnbull Model is taken from their paper that uses the 'Foreign Currency Analogy' of Jarrow and Turnbull (1991). It decomposes the dollar payoff from the risky security into a certain payoff and a "spot exchange rate" to price both the credit risk from the underlying asset and the credit risk of the writer of the derivative security. It is a discrete time model with two variables of interest, the one-period default-free rate of interest and the default event where both variables follow a binomial process. The model can be used to value the three classes of credit risky derivatives as defined in the section on credit derivatives.

The Hull and White model is both a discrete and continuous time model that looks at the impact of credit risk on class 2 & 3 derivatives as defined later in the section on credit derivatives. Ideally, the paper will explore the use of these models to manage credit risk of corporate bonds from Emerging markets, such as South Africa, and Brazil. For this paper, due to reasons mentioned later, US bonds, specifically the US Treasuries and US corporate bonds are used instead of Treasuries and corporate bonds from either South Africa, or Brazil.

Two different options are used for all the calculations, we explore the use of a warrant (special case of an option) issued by a Triple-A firm and secondly use of options on bonds (both risky and risk free bonds). The former is explored as an alternative to using a triple-A option because it proved difficult to obtain these triple-A options in the market. This is as a result of the fact that most options are over-the-counter issues (contracts between private parties) and thus, getting the data is difficult. Options on bonds, the most suitable credit derivatives to use with these models are also over-the-counter instruments. However, this is overcome by pricing options on the available bonds using the Black-Scholes model or precisely the Black Model (see Hull: 2000). The project also explores the use of both coupon and zero-coupon bonds in the pricing models, on condition that the risky debt is used relative to its particular benchmark Treasury bond.

Section 1 gives an overview of bonds, specifically looking at the definition of bonds, the bond markets and the risks associated with bonds. Then Section 2 looks at the two models, the Jarrow & Turnbull model and the Hull & White model, their application in pricing (credit risk management tool) credit risky options using corporate bonds. Before discussing the two models above, a brief look at the derivatives and their history, as well as examples of credit derivatives are given. Lastly, Section 3 concludes this paper with a comparison of the two models' results achieved when using US corporate bonds and US Treasury bonds as model inputs.

ACKNOWLEDGEMENTS

I would like to express my gratitude to *Professor Haim Abraham* of the UCT School of Economics, whose belief in my capabilities went beyond the call of duty.

I am also grateful to my wife, *Nyaradzo* for pushing me to see this project through and for consistently believing in my potential even those times that I myself failed to see it.

You are indeed a good thing from above!

To my family, *mai Katiyo, baba namai Kaseke* thank you so much for everything. You are the greatest.

To my friends, thank you all for your support and prayers. Be blessed!!!

To *Johan Du Preez, Ron Mangani, HSBC crew (Jonathan and Nick)* thank you so much for your input, suggestions and above all your help with my data collection.

Thank you Lord!

TABLE OF CONTENTS

1. OVERVIEW OF BONDS	4
1.1 INTRODUCTION	4
1.2 DEFINITION	4
1.2.1 ISSUER	5
1.2.2 TERM TO MATURITY	6
1.2.3 THE PRINCIPAL	7
1.2.4 THE COUPON	7
1.3 THE WORLD BOND MARKET	8
1.3.1 THE US BOND MARKET	9
1.3.2 EMERGING MARKETS	14
1.3.3 THE BRAZILIAN MARKET	14
1.3.4 THE SOUTH AFRICAN BOND MARKET	15
1.4 RISKS ASSOCIATED WITH INVESTING IN BONDS	18
1.4.1 MARKET/INTEREST RISK	18
1.4.2 INFLATION RISK	19
1.4.3 REINVESTMENT RISK	19
1.4.4 CURRENCY RISK	20
1.4.5 CALL RISK	20
1.4.6 LIQUIDITY RISK	21
1.4.7 VOLATILITY RISK	21
1.4.8 DEFAULT/CREDIT RISK	21
1.4.9 PRICE OF RISK USING RISK PREMIUMS	23
2. RISK MANAGEMENT	25
2.1 INTRODUCTION	25
2.2 DERIVATIVES AND CREDIT DERIVATIVES	26
2.2.1 EXAMPLES OF CREDIT DERIVATIVES	27
2.2.2 CLASSIFICATION OF CREDIT DERIVATIVES	29
2.3 THE BASIC MODEL REQUIREMENTS	30
2.4 THE JARROW & TURNBULL MODEL	31
2.4.1 THE FOREX ANALOGY	33
2.4.2 RESULTS: JARROW & TURNBULL MODEL	41
2.5 THE HULL & WHITE MODEL	48
2.5.1 RESULTS: HULL & WHITE MODEL	54
3. COMPARISON OF THE TWO MODELS AND CONCLUSION	60
3.1 INTRODUCTION	60
3.2 RESULTS	60
3.3 VOLATILITY	63
3.3.1 ESTIMATING VOLATILITY USING HISTORICAL DATA	63
3.3.2 GARCH (1, 1) MODEL	64
3.4 CONCLUSION	68

3.4.1 BOND INPUTS SUMMARY	70
REFERENCES	72
APPENDIX A.1	74
MOODY'S INVESTORS SERVICE	74
APPENDIX A.2	75
MOODY'S INVESTORS SERVICE, STANDARD & POOR'S, FITCH IBCA, DUFF & PHELPS	75
APPENDIX A.3	76
REUTERS CORPORATE SPREADS FOR BANKS	76
REUTERS CORPORATE SPREADS FOR INDUSTRIALS	76
REUTERS CORPORATE SPREADS FOR TRANSPORTATION	77
REUTERS CORPORATE SPREADS FOR UTILITIES	77
REUTERS CORPORATE SPREADS FOR FINANCIALS	78
REUTERS CORPORATE SPREADS FOR 30-YEAR MATURITIES	78
APPENDIX A.4	83
ZERO COUPON BONDS	83
APPENDIX B.1	85
CORPORATE BOND YIELD SPREADS	85
APPENDIX B.2	85
YIELD SPREAD GRAPHS	85
APPENDIX B.3	85
COMBINED SPREADS	85
APPENDIX B.4	86
DOLLAR PRICE OF RISK	86
APPENDIX GARCH (1,1) COUPONS	86

VOLATILITY CALCULATIONS	86
APPENDIX GARCH (1,1) ZEROS	86
VOLATILITY CALCULATIONS	86
APPENDIX HW.1-8	86
EXPECTED BAA1 OPTION PRICES USING THE HW MODEL	86
APPENDIX JT.1-8	86
EXPECTED BAA1 OPTION PRICES USING THE JT MODEL	86

LIST OF FIGURES

Figure 1: The US corporate bond issuance between 1980 and 1999	12
Figure 2: The US corporate bonds outstanding between 1980 and 1999	13
Figure 3: The BESA bond sectors in nominal value as at October 2002	16
Figure 4: The corporate spreads for corporate bond issues of different credit ratings	22
Figure 5: The default-free zero-coupon bond price process for the two-period economy	35
Figure 6: The payoff ratio process for XYZ debt in the two-period economy	36
Figure 7: The XYZ zero-coupon bond price process for the two-period economy in XYZs	37
Figure 8: The XYZ zero-coupon bond price process for the two-period economy in dollar	38
Figure JT.1: The expected Baa1 option prices using zero-coupon bonds, a triple-A warrant and the JT model	42
Figure JT.2: The expected Baa1 option prices using coupon bonds, a triple-A warrant and the JT model	43
Figure JT.3: The expected Baa1 option prices using zero-coupon bonds, bond options using the 30-year Treasury bond yield and the JT model	44
Figure JT.4: The expected Baa1 option prices using coupon bonds, bond options using the 30-year Treasury bond yield and the JT model	45
Figure JT.5: The expected Baa1 option prices using zero-coupon bonds, bond options using the 6-month Treasury bill rate and the JT model	46
Figure JT.6: The expected Baa1 option prices using coupon bonds, bond options using the 6-month Treasury bill rate and the JT model	46
Figure JT.7: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 10-year Treasury note yield, and the JT model	47
Figure JT.8: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 6-month Treasury bill rate and the JT model	47
Figure HW.1: The expected Baa1 option prices using zero-coupon bonds, a triple-A warrant and the HW model	55

Figure HW.2: The expected Baa1 option prices using coupon bonds, a triple-A warrant and the HW model	56
Figure HW.3: The expected Baa1 option prices using zero-coupon bonds, bond options using the 30-year Treasury bond yield and the HW model	56
Figure HW.4: The expected Baa1 option prices using coupon bonds, bond options using the 30-year Treasury bond yield and the HW model	57
Figure HW.5: The expected Baa1 option prices using zero-coupon bonds, bond options using the 6-month Treasury bill rate and the HW model	57
Figure HW.6: The expected Baa1 option prices using coupon bonds, bond options using the 6-month Treasury bill rate and the HW model	58
Figure HW.7: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 10-year Treasury note yield, and the HW model	58
Figure HW.8: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 6-month Treasury bill rate and the HW model	59
Figure R.1: Comparison of the expected Baa1 option prices obtained using the JT model vs. the HW model, with zero coupon bonds, or coupon bonds, and the triple-A warrant as inputs	60
Figure R.2: Comparison of the expected Baa1 option prices using the JT model vs. the HW model, with zero coupon bonds, the bond options from Black model using the 30-year Treasury bond yield as the risk free rate	61
Figure R.3: Comparison of the expected Baa1 option prices using the JT model vs. the HW model, with zero coupon bonds, the bond options from Black model using the 6-month Treasury bill rate as the risk free rate	61
Figure R.4: Comparison of the expected Baa1 option prices using the JT model vs. the HW model, with 10-year coupon bonds, the bond options from Black model using the 30-year Treasury bond yield and the 6-month Treasury bill rate as the risk free rate	62
Figure G.1a zeros: The daily volatilities of the HW model and the JT model prices using zero coupon Treasury bond and zero coupon corporate bond	66
Figure G.1b zeros: The daily volatilities of the HW model and the JT model prices using zero coupon Treasury bond and zero coupon corporate bond	66
Figure G.1a coupons: The daily volatilities of the HW model and the JT model prices using coupon Treasury bond and coupon corporate bond	67
Figure G.1b coupons: The daily volatilities of the HW model and the JT model prices using coupon Treasury bond and coupon corporate bond	67

LIST OF TABLES

Table 1: The BESA bond sectors in nominal value as at October 2002	16
Table 2: The statistical summary for coupon bonds for both their normal prices and their log prices	70
Table 3: The statistical summary for zero-coupon bonds for both their normal prices and their log prices	71

1. OVERVIEW OF BONDS

1.1 Introduction

This section begins with the definition of bonds and a further expansion of the components that are enveloped in the definition. It is followed by a look at the world bond markets, their development and characteristics. The bond market discussion is then divided into a discussion on the developed and emerging market bond markets. The United States of America (referred to as the US from henceforth) is used to represent the developed markets as well as the standard for the structure and constitution of bond markets in general. Brazil together with South Africa (SA) is used as representatives of the emerging bond markets. A brief look at Brazil's corporate bond's credit relationship with the developed market (US) corporate debt and the reasons for such a relationship is discussed. This is followed by a brief look at the SA bond market, specifically, the SA bond exchange (BESA) and its notable historical events, and a discussion on the different sectors that make up the SA bond market.

1.2 Definition

A bond is a debt instrument requiring the *issuer* (also known as the debtor or borrower) to repay to the *bondholder* (lender or investor) the amount borrowed (*the principal*) plus *interest* (coupon payment) over a specified period of time (*term to maturity*). At the end of the period (at *maturity*) the borrower repays the full initial amount borrowed (Fabozzi: 2000). Myers (1984) suggested that firms prefer retained earnings (available liquid assets) as their main source of funds for investments. Next in order of preference is debt, and last comes external equity (issuing new shares). This is mainly because unlike issuing new equity it does not result in a shareholding dilution for existing shareholders and secondly, the ownership composition of the firm does not change. In other words, issuing new shares results in change of ownership structure whereas issuing a bond does not result in ownership changes. As a bondholder then, one is concerned about getting the promised periodic interest payments and

the principal amount lent to the debt-issuing firm. Bonds are suitable for both investors looking for capital gains and income growth.

A bond provides three sources of income/cash flow to an investor over the time it is held:

1. The contractual periodic interest payments when the counterparty (borrower) honours its promise or their contractual obligations according to the indenture or bond covenants.

2. Interest gained from reinvestment of the periodic interest payments. As one receives the periodic interest payments one invests the amount at the prevailing market rates although when pricing bonds it is assumed that these payments are reinvested at the yield-to-maturity.

3. Capital gains resulting from the disposal of the security whenever market interest rates fall. A basic fixed trading rule in the market for fixed income securities is that the interest rates and the security prices always move in opposite directions. When interest rates rise, prices fall, and when interest rates drop, prices will therefore rise (capital gains received from selling the bond).

All the important facts dealing with the rights of the holder and the obligations of the issuer are contained in the “*indenture*” agreement (*Contract note*), the legal document that spells out its terms and conditions. The agreement details the face value of the bond, the repayment schedule (of the coupon, and the principal amounts), the frequency of payment, the description of any property to be pledged as collateral, the steps that will be taken by the bondholder in the event of default, and callable features that may be present. A brief description of each of the components to the indenture agreement follows.

1.2.1 Issuer

One of the most important characteristics of a bond is the nature of its issuer. The three largest issuers of debt are the government and its agencies, municipal governments, and corporations (both domestic and foreign). They issue sovereign bonds (Treasuries), municipal bonds, and corporate bonds (also simply known as corporates), respectively. Within each of these classes of issuers, however, one can find additional and significant differences. These could be divisions defined according to their abilities to satisfy their

contractual obligations to the investors or lenders. Domestic corporations, for example, include regulated utilities as well as unregulated manufacturers.

1.2.2 Term to maturity

This is the number of years over which the issuer has promised to meet the conditions of the obligation as contained in the bond's indenture. The maturity of a bond is the date that the debt ceases to exist, at which the issuer will redeem the bond by paying the principal. In practise, the term to maturity of a bond is simply referred to as its term or maturity.

Technically, maturity denotes the date the bond will be redeemed, and the term to maturity denotes the number of years until that date.

Bonds can be classified into three categories as a result of their term to maturity: *Short-term* bonds have a maturity of one to five years, *medium-term* bonds have a maturity of between five and twelve years, and finally *long-term* bonds have a maturity of more than twelve years. Usually, the maturity of a corporate bond is between 10 and 30 years, the shorter maturities are more characteristic of banking and financial issues, and utilities are more likely to employ the longer maturities. Government bonds range in life from 1 to 20 or more years (though technically, treasury issues of 1 to 10 years are known as Notes), but the number of bonds with maturities exceeding 10 years is relatively small (Michael D Joehnk: 1983).

The maturity of a bond is a very important feature mainly because it indicates the time period over which the bondholder can expect to receive the coupon payments and the number of years before the principal amount is paid in full. Secondly, the yield received on a bond (its annual rate of return) depends on its term to maturity. Thirdly, the volatility (the price fluctuations) of a bond's price is dependent on its maturity too, specifically the longer the maturity the greater the price volatility resulting from market yield changes. Finally, bonds with long terms may be safer than debts with shorter maturities. The long term bond issues have a higher likelihood of finding favourable conditions for retirement, which usually occurs through refinancing, than the obligors whose bonds have shorter life span.

1.2.3 The principal

As previously mentioned, the principal is the amount that the issuer borrows and agrees to repay the bondholder (lender) either at maturity or at those times when the bond is called or retired according to sinking fund provisions. It is also the basis on which the coupon rests; the coupon is the product of the principal and the coupon rate. It is also known as the redemption value, maturity value, par value, or *face value* as it will be referred to for the rest of this paper.

1.2.4 The coupon

A coupon is the annual amount of the interest payment made to the bondholders during the life of the bond. The *coupon rate* multiplied by the face value gives the monetary value amount of the coupon where the coupon rate or nominal rate is the interest rate that the issuer agrees to pay each year. This is also the yearly sum of the periodic amounts paid per year. The periodic payments can be annually, semi-annually, or quarterly.¹ The coupons can either be fixed or floating rates, most bonds are still the traditional fixed rate securities. Floating rates on the other hand, are bonds that have variable interest rates that are adjusted periodically according to an index tied to short-term Treasury bills or money markets or LIBOR rate. While such bonds offer protection against increases in interest rates, their yields are typically lower than those of fixed-rate securities with the same maturity.

Most bonds are “*bearer bonds*” whose investors clip coupons and send them to the obligor for interest payments. Nobody's name is on the bond or the coupon; therefore, they are referred to as coupon bonds. The coupons are submitted twice a year and the authorized bank pays the interest. For instance, a twenty-year \$1,000 bond paying 8% interest would have 40 coupons for \$40 each. Bearer bonds can be used like cash and are highly negotiable. There

¹ Ashland Inc issued a corporate bond (ASH7.97) on 27th of February 1995 with a maturity of 10 years paying coupon semi-annually at a coupon rate of 7.97% and had a face value of US\$100.00. Thus the coupon was \$7.97 per year and 7.97/2, \$3.99 semi-annually. See appendix B.1

are still many in circulation, however, the Tax Reform Act of 1982 ended the issuance of bearer bonds in the US.

Some bonds are “*registered bonds*”, and their owners receive the payment automatically at the appropriate time. There exists a cross between a coupon bond and registered bond known as a *partially registered bond*. These bonds come registered to a particular investor; however, it has coupons attached, which the bondholder has to send in for payment. *Zero coupon bonds* can also be issued, that is bond issues without any interest payments over the life of the bond except the repayment of the face value at maturity. The interest is indirectly embedded in the issue because the bond is always issued at a discount and redeemed for the full; face value at maturity ensuring that the lender gets compensated for lending and foregoing interest income in alternative investments.

In Appendix A.4, an article by The Federal Reserve Bank of New York on zeros and their history in the US Bond market is shown. It defines the zeros similarly to the definition provided above and looks at their birth and the initial perceptions of these new instruments by the bond investors, the public and the Federal authorities.

1.3 The world bond market

The bond market has experienced major expansion of bond markets especially in many emerging markets (the East Asian and Latin American countries such as Singapore, Mexico, and Brazil to name only a few). The bond markets are rapidly expanding world wide, but in many countries the growth is strongly biased towards government issued or government-backed bonds. Trading volumes of these government issued bonds are very large mainly because of their attributes; default free (risk free), high liquidity, less monitoring. Whereas the corporate bonds seldom offer high liquidity and less monitoring because of the risk associated with them and to some extent the volumes they trade at.

Bond markets world wide are built on:

- the number of issuers with long-term financing needs;
- investors with a need to invest in interest bearing securities. (A weakness in Africa and other emerging markets is that domestic savings are poorly mobilised and foreign investment needs to be vigorously encouraged to sustain growth).
- intermediaries that bring together prospective investors and issuers; and
- infrastructure that provides a secure, efficient and transparent structure.

We take a look at the US bond market as a developed market followed by a look at Brazil and South Africa has the representatives of the emerging markets' bond markets.

1.3.1 The US bond market

United States is one of the very few countries with a flourishing and very liquid corporate bond market and an even larger over-the-counter market. It is evidently the largest bond market in the world. Most of the corporations use hybrid debt issues (issues with embedded derivatives such as convertible bonds) as a way to manage the risks involved rather than issuing straight debt and then buying/selling the necessary derivatives. Smithson and Chew (1992) argued that hybrid debt offered corporate treasurers an efficient means of managing a variety of financial and operating risks, risks that in many cases could not be managed if the firm issued straight debt and then purchased derivatives.

Fabozzi (2000) summarized the different sectors that constitute the US bond market, and consequently other markets as follows:

The *treasury sector* includes securities issued by the U.S government and therefore, they have the full backing of the U.S government. It is argued that there is no probability of default at all by the government, considering that it always has the option to print more money as a last resort to service its contractual obligations. And some in that respect will argue that such actions will bring up the question of the Reserve bank's independence from the Government policies and authority. It has several types of issues, Treasury bills, notes, and bonds.

Treasury Bills (T-bills) have maturities of 3 months and 6 months. They are auctioned once every week and once every month, 1 year T-bills are auctioned. These are a direct short-term obligation of the U.S. government. T-bills do not pay interest they are purchased at a discount, for example one might buy a \$10,000 three-month T-bill for \$9,700. The investor would then receive \$10,000 when the T-bill reached maturity in 3 months. T-bills are the only Treasury security issued at a discount. They are also the only Treasury security issued without a stated interest rate. The interest rate is determined at auction. T-bills are also offered in book entry form only, that is, the investor does not receive a certificate. T-bills are also highly liquid. *U.S. Treasury Notes* (T-notes) are direct obligations of the U.S. government. These notes have maturities from one year to ten years. T-notes pay interest on a semi-annual basis and they always expire at par value. The different length notes are auctioned at different periods throughout the year. *U.S. Treasury Bonds* (T-bonds) are also direct obligations of the U.S. government. They pay interest on a semi-annual basis. These have long-term maturities of 10 years to 30 years. The 30-year T-bonds are callable beginning 5 years prior to maturity.

The *agency sector* is the smallest sector in the U.S. It includes securities issued by federally related institutions and government backed enterprises. The U.S government does not directly issue them but they are, however, considered as 'moral obligations' of the U.S government. These include the likes of Federal Home Loan Banks (FHLB), Federal Home Loan Mortgage Corporation, Federal National Mortgage Association (Fannie Mae), and the latter sister agent, the Government National Mortgage Association (GNMA's or Ginnie Mae).

The *municipal sector* is where the state and local governments raise their funds by issuing debt securities. They issue mainly IOUs. The *mortgage sector* is where the securities are backed by mortgage loans. These are loans borrowed by individuals in order to purchase residential property or an entity to purchase commercial property.

The *corporate sector* includes securities issued by both non-U.S and U.S corporations in the United States (mainly dollar denominated) namely; bonds, medium-term notes, structured notes, and commercial paper. The dollar denominated bonds issued by Non-US corporations

is commonly known as *Yankee Bonds*. This sector is further sub-divided into the *investment grade* and *non-investment grade* sectors as defined by the rating agencies. The rating agencies include Standard & Poor's Co, Moody's Investors Service, etc. Using the Standard and Poor's (S&P) classification, securities rated BBB or above is regarded as *investment grade*; lower rated bonds (non-investment grade) are more speculative and sometimes given the derogatory, and somewhat unjustified, name of *junk bonds* (see Jorion and Khoury: 1996).²

Some corporate bonds are issued with property (such as land, buildings, machinery, or other equipment) as collateral against the loan, just as you might offer collateral to a bank in exchange for a personal loan. These bonds are known as *Secured bonds*. When the issuing firm defaults on its obligations, or becomes insolvent the bondholders of these secured loans will claim any proceeds from the property sales. They have first claim over the proceeds ahead of other bondholders and shareholders (both ordinary and preferred shareholders). Bonds issued without collateral (*Unsecured bonds*), are called debentures. The value of a debenture is guaranteed by the good faith of the issuing corporation and the capacity of its earnings to repay interest and principal. If issued by a strong corporation the debenture can be a highly secure investment. In the event of liquidation, the holders of debenture bonds are placed ahead of ordinary and preferred shareholders but behind the holders of secured bonds. If you buy a secured bond, you will "pay" for the extra safety by receiving a lower interest rate or pay a higher price than you would have on a comparable unsecured bond.

The US corporate bond market is the most liquid in the world, with daily trading volumes estimated at \$10 billion. Issuance for 1999 was an estimated \$677.0 billion. The total market value of outstanding corporate bonds in the United States at the end of 1999 was approximately \$3.0 trillion. Buying and selling of corporate bonds is done on the New York Stock Exchange (NYSE), where major corporations' debt issues are quoted and traded daily. Surprisingly, more corporate bonds than stocks are listed on the NYSE. The diagram below shows the growth of corporate bonds issued in the US from 1980 to 1999 in billions of dollars.

² For rating agencies' definitions of the different class ratings refer to Appendix A.1.

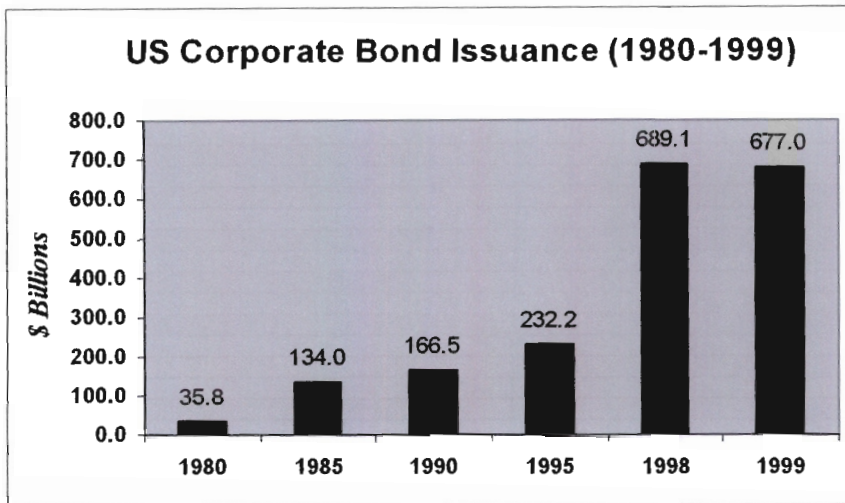


Figure 1: The US corporate bond issuance between 1980 and 1999.

Includes all non-convertible debt and medium-term-note issues, but excludes all federal and agency debt.

Source: Thomson Financial Securities Data (2000), (See http://www.slk.com/bond/ig_corp.html).

The rest of the corporate bonds are traded on the “over-the-counter (OTC)” market, which has no central location. The market is made up of bond dealers and brokers around the country who trade in these corporate bonds and many other types of debt securities. The OTC market is much bigger than the exchange market because most bond transactions, and even those involving listed issues, take place in this market. Investors in corporate bonds include large financial institutions, such as pension funds, endowments, mutual funds, insurance companies and banks as well as individuals

Below is a diagrammatic representation of the corporate bonds outstanding over the years from 1980 to 1999 in US bond market. It shows a growth similar to the growth in corporate bond issuance shown earlier.

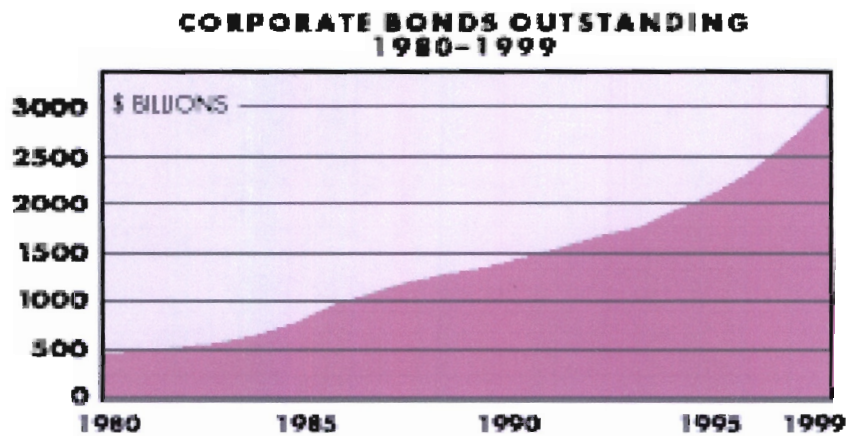


Figure 2: The US corporate bonds outstanding between 1980 and 1990

Source: The Bond Market Association estimates; Federal Reserve System (2000). (See http://www.slk.com/bond/ig_corp/big.html).

It will be plausible to ask why an investor would be compelled to invest in corporate bonds considering their risk, when there are readily available risk-free Treasury issues. The following attributes listed below are the main motivations behind such compellation:

1. Attractive yields. Corporate bonds offer higher yields than comparable-maturity government bonds or Certificate of Deposits (CDs) to compensate investors for taking on that extra risk over and above the risk-free rate. Consequently this high-yield potential is unfortunately accompanied by higher risks.

2. Dependable income flow. Bonds provide steady income while preserving the investor's principal amount.

3. Safety. They are evaluated and assigned a rating based on credit history and ability to repay obligations, the higher the rating, and the safer the investment.

4. Diversity. Corporate bonds provide the opportunity to choose from a variety of sectors, structures and credit-quality characteristics to meet ones investment objectives something that the government bonds cannot offer. The variety is a result of the different types of corporate issuers that are available; utilities, transportation, industrial, financial services and conglomerates. As mentioned earlier they may be foreign firms including foreign governments as well.

5. Marketability. As mentioned earlier, the corporate bond market is the largest bond sector therefore offers high liquidity. Thus if an investor must sell a bond before maturity, the investor can easily and quickly do so (Spear, Leeds & Kellogg: 2000).

1.3.2 Emerging markets

Emerging markets have almost non-existent corporate bond exchanges though the OTC market is considerably large by all means. Thus accessing valuable and relevant information is a daunting task for one to effectively price and manage the risk exposures of the bond issues. Fortunately there are a few exceptions to this norm that have almost flourishing bond markets largely because they are well supported by the developed markets. For example Brazil, one of the emerging markets, is well supported by the US market. Most of the Brazilian issues are listed on the US exchanges and are dollar denominated. A look at a Brazilian firm with a particular rating and a corresponding bond issued by a US corporation will help to show the credit relationship of the emerging market corporate bond with the developed bond market debt issues. One of the major factors that contribute to the poor credit quality of emerging market issues is the risk inherent to the country in question. Political and currency risk are the main risks that determine a country's corporate credit quality. An article written by Marijke Zewuster (2002) for ABN AMRO that is summarised and discussed below attempts to contextualise emerging bond markets and the factors mentioned above which govern them.

1.3.3 The Brazilian market

During the period from 2000 to 2002, the Brazilian economy was able to withstand the economic crisis that hit Argentina, another emerging market. This was attributed to Brazil's then stable political climate. The firm actions carried out by the monetary authorities assured the stability and the extensive multilateral and bilateral support established by the government with established economies including US. The monetary authorities were able to transform the once extremely closed and inefficient economy into a more open and more market-oriented economy. Moreover, over this two-year period the government finally started to sort out the public finances, and despite disappointing economic growth it easily achieved the primary budget surpluses (i.e. excluding interest payments) agreed with the IMF. Even then, Brazil was not fully shielded from the global economic downturn experienced in 2001. Economic growth declined from 4.2% in 2000 to 1.5% in 2001, while

owing to the Real's depreciation inflation rose from 6.0% in 2000 to 7.7% in December 2001. The lower growth and more favourable exchange rate did not prevent an increase in the current-account deficit over the same period from 4.1% of GDP to 4.6%. Lower commodity prices and a reduction in world trade volume were the main reasons for a disappointing trade performance. A striking feature was that despite the unfavourable conditions for emerging markets, the inflow of foreign direct investment remained considerable and virtually covered the current-account deficit.

After the 2001 downturn, early in 2002 the first signs of recovery became visible. The outlook for the export sector was encouraging, despite the loss of the Argentine market, and the trade balance was set to improve considerably. On the financial markets the fears of infection by the Argentine crisis had waned significantly. The interest-rate differentials between Brazilian long-term foreign debt papers and US paper with a similar maturity climbed beyond 1,000 basis points in 2001, but since then they had fallen back again to around 700 early 2002. The Real, which depreciated steadily until October 2001, had since recovered appreciably. This is all more remarkable given that Brazil was to hold presidential and legislative elections later in 2002, which would have been a source of political risk (change of economic policies) from an investors' point of view. Fortunately, the government's presidential candidate, José Serra, who was then the minister of health, was widely expected to coast to victory, and this meant a continuation of the successful economic policies was guaranteed. In conclusion, the relationship between emerging markets and the developed markets greatly depends on the level of political stability of the emerging market and its effect on the economic policies pursued.

1.3.4 The South African bond market

Like any other emerging market, South Africa has a bond market, BESA (Bond Exchange of South Africa) where both the primary and the secondary market is predominantly government issued securities (RSA bonds-Republic of SA bonds). Ninety percent (90%) of BESA turnover is in RSA bonds of which 34% is spot trades, 66% in repurchase agreements ("Repo") and 0.25% is options exercised. Repos have had an important effect on the liquidity for the BESA listed securities. Securities dealers use repos to facilitate long or short

positions. Approximately 22% of BESA trade is concluded with non-residents and another 5% is traded offshore and settled through the South African settlement system (Allen Jones: 2002). The RSA bond issues include the well-known R150, R153, and R157 treasury bonds as well as government backed issues such as Telkom bond (TK01). The more common R153 is regarded, as the benchmark for all comparisons; in the last four years there has been an expressed intention to adopt the R157 as the next benchmark bond. It also has a few municipal issues such as the Umgeni Water Project (UG55). Corporate bonds are more popular on the Over-the-counter (OTC) market as much as the government bonds are on the bond exchange in terms of trading. Allen Jones (2002) summarised the proportion of BESA held by each of the above mentioned bond issues according to their prospective sectors based on the sectors nominal values in issue as at October 2002. These nominal values are summarised below and graphically represented in Figure 3 below.

Table 1: The BESA bond sectors in nominal value as at October 2002.

Central Government	R 330 656m
Municipal Bonds	R 131m
Parastatals / Utility Bonds	R 38 015m
Water Authorities	R 17 519m
Banking Sector	R 27 356m
Corporate Sector	R 14 705m
Securitisation	R 10 315m
Total in issue (Nominal)	R438 700m

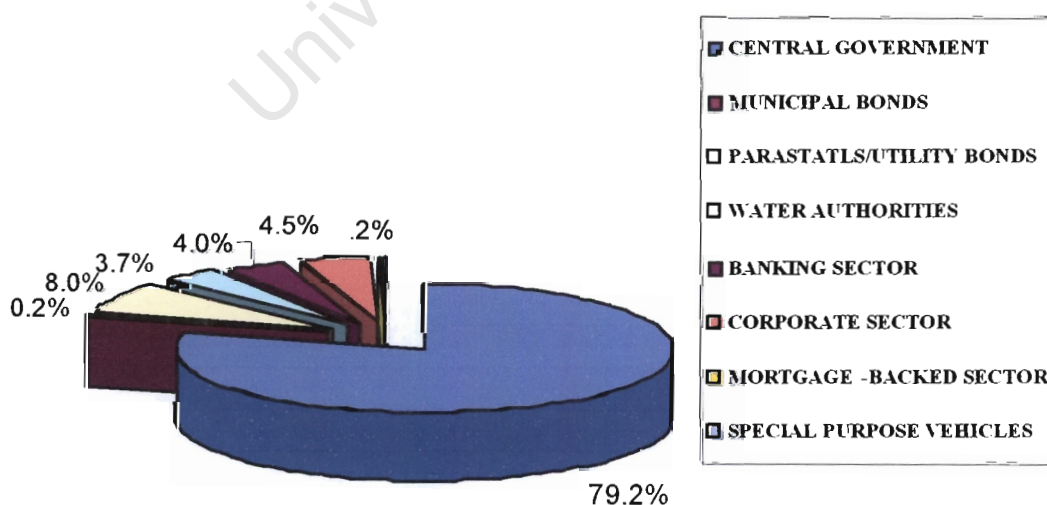


Figure 3: The BESA bond sectors in nominal value as at October 2002.

A brief summary of notable events in the history of BESA follows (Allen Jones: 2002),

- **1987** – Stals/Jacobs inquiry into financial markets recommended that either the participants or the Central Bank regulate the fragmented bond markets. The participants choose self-regulation and the Bond Market Association (“BMA”) was formed.
- **Prior to 1989** - Institutions were subject to prescribed investments; therefore interest rates were kept at an artificial level. There was an active money market and an open market monetary policy; however, the bond market was fragmented and illiquid. In the later part of the 1980’s Eskom started a market in its own bonds and the E168 bond became the benchmark (E168’s yield was lower than the RSA bond yield). This led Transnet and Telkom to start making markets in their own bonds.
- **1990** - The National Treasury consolidates a number of smaller issues to create the R150 and R153 bonds.
- **1991** - The South African Reserve Bank commences market making in government bonds. The R150 bond replaces the E168 bond as the benchmark.
- **1992** - The first corporate bond listed on BMA, issued by SA Breweries Limited.
- **1996** - The BMA is formally licensed and becomes the Bond Exchange of South Africa (“BESA”).
- **1997** - BESA moves to *t+3* rolling settlement and achieves full compliance with G30 “Recommendations for Clearing and Settlement”, the first exchange in Africa to do so. The first Collateralised Debt Obligation listed (INCA BOND).
- **1998** - National Treasury appoints 12 Primary Dealers to make a market in seven government bonds. The open outcry-trading floor closed as floor trading activity dwindles to less than 10% of total turnover.
- **2000** - Members book all trades on new Bond Automated Trading System (“BATS”). BESA implements the Total Return Index (“TRI”) with the All Bond Index (“ALBI”) comprising 20 different bonds selected for their size and liquidity. Two sub-sections of the ALBI are the Government Bond Index (“GOVI”) and the Other Bond Index (“OTHI”). 80% of bonds listed are dematerialised. The first CPI-linked bond issued by National Treasury listed.

- **2001** - Corporate Bond market starts to take off. BESA lists first mortgage-backed securitisation. The National Treasury introduces strip programme for: R150, R153, R157, R186 and R194. The national Treasury implements 'Buy Back' programmes and switches.
- **2002** – BESA lists first receivable and credit swap synthetic securitisations and Index-linked contract. BESA issues new listing disclosure requirements and rules. The BESA members approve the BESA restructuring proposal.

BESA has never had any liquidation default and no claim has been made on the Guarantee Fund in its history. BESA has never closed its market during market disruptions such as the October 1998 Russian and Asian problem or even the unprecedented September 11 tragedy. In February 2000 Standard and Poor's raised its foreign currency issuer credit rating on South Africa from double-B, that is, BB (See Appendix A.2 for definitions) to triple-B and its local currency issuer ratings to A from BBB. It also upgraded the ratings on South Africa's senior unsecured foreign and local currency debt to BBB and single-A-minus, respectively as a result of credible economic policy framework and sound economic fundamentals in the country.

In summary, South African corporate bond investors face the same challenges faced by all the other emerging markets. Specifically, the corporate bond investors have to trade on an informal corporate bond market.

1.4 Risks associated with investing in bonds

1.4.1 Market/ Interest risk

After the bond is issued the higher the market interest rates the lower the bond value/price, and vice-versa. The directional movement of interest rates results in an opposite directional change in the price of bonds posing a risk of capital loss to an investor if interest rates increase. This is called market/interest risk. Heath, Jarrow, and Morton (1992) model provides a process for interest rate risk modelling and risk management. The initial market interest rate level, determines the magnitude and the direction of the movement or the change

in the bond price. The lower the coupon and the longer the term to maturity the higher the interest risk will be when market rates increase (the larger the capital loss).

The inverse relationship between bonds and interest rates - that is, the fact that bonds are worth less when interest rates rise can be easily explained: when interest rates rise, new issues come to market with higher yields than older securities, making those older ones worth less. Hence, their prices go down. When interest rates decline, new bond issues come to market with lower yields than older securities, making those older, higher-yielding ones worth more. Hence, their prices go up. As a result, if you have to sell your bond before maturity, it may be worth more or less than amount you paid for it.

1.4.2 Inflation risk

Various economic forces affect the level and direction of interest rates in the economy. Interest rates typically climb when the economy is growing, and fall during economic downturns. Similarly, rising inflation leads to rising interest rates (although at some point, higher rates themselves become contributors to higher inflation), and moderating inflation leads to lower interest rates. Inflation is one of the most influential forces on interest rates. For all but floating rate bonds (unless if the coupon is inflation-indexed) an investor is exposed to inflation risk because the interest rate the issuer promises to make is fixed over the issue's life.

1.4.3 Reinvestment risk

As a bondholder, one is entitled to coupon payments. These interim cash flows will have to be invested at the prevailing market rate. Thus, the bondholder runs the risk of reinvesting these cash flow amounts at interest rates lower than those offered by the bond itself. In bond pricing, it is assumed that the coupon payments received by the bondholder are reinvested at the bond's yield to maturity and not the prevailing rates. In the real world this assumption is clearly flawed, the yield to maturity rate of the bond will always differ from the spot interest rates when the payments are received. Thus the received coupons will be invested at a rate

higher or lower than the bond's yield to maturity thereby presenting the holder with reinvestment risk. The impact of this risk on the bondholder depends primarily on the difference between the bond's yield to maturity and prevailing rate as well as the size of the coupon (the larger the coupon the greater the resulting reinvestment risk).

1.4.4 Currency risk

Currency risk occurs when one holds an issue whose cash flows are denominated in a foreign currency. It is also commonly known as *exchange rate risk*.

1.4.5 Call risk

If the bond's indenture contains a "call" provision, the issuer retains the right to retire (that is, redeem) the debt, fully or partially, before the scheduled maturity date. For the issuer, the chief benefit of such a feature is that it permits the issuer to replace outstanding debt with a lower-interest-cost new issue. A call feature creates uncertainty as to whether the bond will remain outstanding until its maturity date. Investors risk losing a bond paying a higher rate of interest when rates have declined and issuers decide to call in their bonds. When a bond is called, the investor must usually reinvest in securities with lower yields. Calls also tend to limit the appreciation in a bond's price that could be expected when interest rates start to slip. Because a call feature puts the investor at a disadvantage, callable bonds carry higher yields than noncallable bonds, but higher yield alone is often not enough to induce investors to buy them. As further encouragement, the issuer often sets the call price (the price investors must be paid if their bonds are called) higher than the principal (face) value of the issue. The difference between the call price and principal is the call premium.

Generally, bondholders do have some protection against calls. An example would be a bond that has a 10-year final maturity, not callable for the first two years. This means the investor is protected from a call for two years, after which time the issuer has the right to call the bonds. They can also demand sinking fund provisions with their issue. A sinking fund is money taken from a corporation's earnings that is used to redeem bonds periodically, before

maturity as specified in the indenture. If a bond issue has a sinking-fund provision, a certain portion of the issue must be retired each year.

One investor benefit of a sinking fund is that it lowers the risk of default by reducing the amount of the corporation's outstanding debt over time. Another is that the fund provides price support to the issue, particularly in a period of rising interest rates. However, the disadvantage - which usually weighs more heavily on investors' minds, especially in a falling-rate environment, is that bondholders may receive a sinking-fund call at a price (often par) that may be lower than the current market price of the bonds.

1.4.6 Liquidity risk

The ease with which one can sell an issue at or near its value is referred to as liquidity or *marketability risk*. The wider the spread between bid and ask price of an issue the more the liquidity risk and the less liquid the security is, and likewise, the narrower the spread the more liquid the issue is and the less the liquidity risk is. Treasury bonds have less liquidity risk than corporate bonds because there are more marketable (default free) and their market has depth.

1.4.7 Volatility risk

Volatility risk is the risk that a change in volatility of interest rates will affect the price of a bond adversely.

1.4.8 Default/Credit risk

Finally, default/credit risk is the risk that an issuer will be unable to make the contractual principal and interest payments (the risk of default by the issuer). The extent of the exposure to credit risk depends on the issuer and its credit rating or standing as by credit rating companies mentioned earlier.³ Credit risk of bond issues can be deduced from the *Credit*

³ See Appendix A.1 for an example of credit rating definitions. Appendix A.2 shows the different symbols used by different rating agencies for the different credit classes.

Spread, the yield spread between the corporate bond yield and its comparable Treasury bond (benchmark bond) yield. The government yield is the also known as the Treasury rate when the government borrows in its own currency. The spread profile can then define the different credit ratings for the different bonds as shown in the figure below. In the event of default the insufficient recovery of full debt is known as *Recovery Risk*. This is the market value for the residual assets of a firm in default.

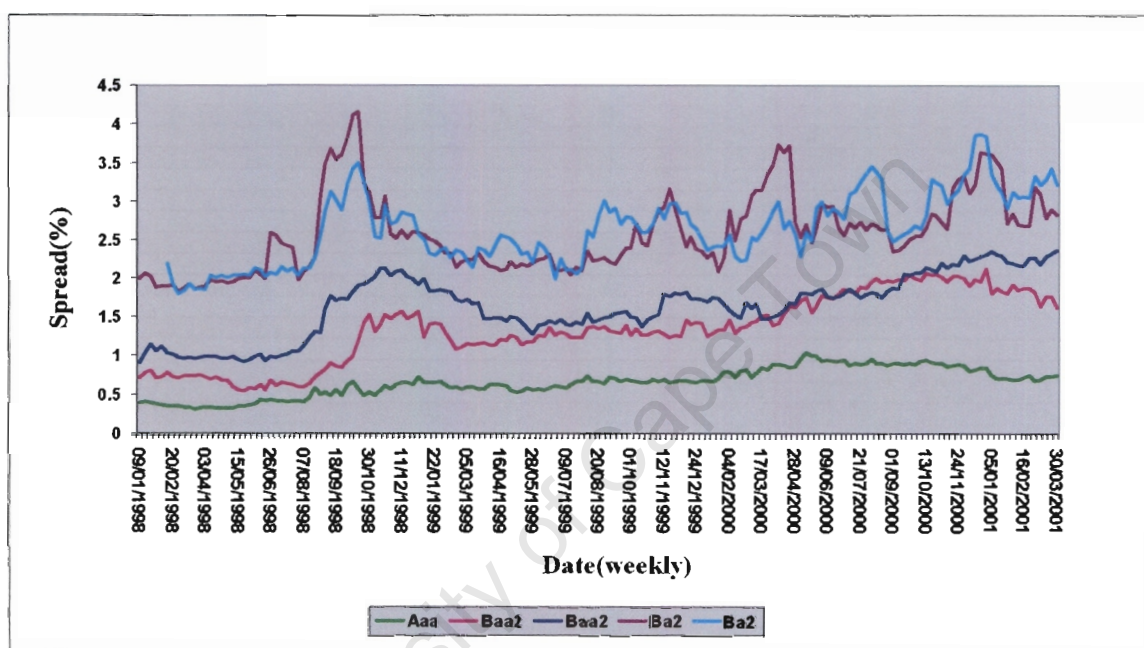


Figure 4: The corporate spreads for corporate bond issues of different credit ratings. The graph is obtained from Appendix B.3, which is a result of combining the graphs shown in Appendix B.2. See also Appendix B.1 for the data (the corporate bond yields for different credit ratings and their corresponding Treasury bond yields). The graph shows that the higher the credit rating (Aaa) the lower the yield spread and likewise, the lower the credit rating (Ba2) the higher the yield spread.

In most instances, default risk and recovery risk are deemed to be one form of risk. In terms of definition, the two are separable because the former deals with the likelihood of the default event and not the loss incurred where the latter deals with the loss incurred only after the default event has occurred already. For this paper however, the two will be deemed as one such that the credit spreads used represent the difference in the probability of default and the size of recovery once in default between a risky corporate bond and a risk-free Treasury bond. Consequently, in this paper all other bond issues are expected to get at least that risk

free rate plus an extra rate above it to compensate for the credit risk. Put differently, all other bond issues will trade at lower prices (at a discount to the price of the risk free government bond). This excess return required by investors to take on risky bonds is called the bond's *Risk Premium*.⁴ Common sense tells us that the riskier the bond the greater the risk premium required, or the lower the price investors will be willing to buy the bond at. Thus, the risk premium required from an Aaa rated bond will be lower than that required on a Caa rated bond because the former is regarded to be less risky compared to the latter.⁵

1.4.9 Price of risk using risk premiums

Taking six corporations with bond issues in the United States that are classified in the four credit ratings shown and explained in Appendix A.1, the dollar price of risk was calculated. Assuming that U.S Treasury bond (T6.5) was the universal benchmark for these six corporate bond issues. Each bond issue's indenture was assumed to have the same bond features as the Treasury bond. That is, the maturity date, the settlement date, the coupon rate, the number of payments in a year (frequency), the principal amount (redemption), and the basis are the same for both the corporate bond and the benchmark Treasury bond. This leaves the yields as the only different variable between the two bond issues implying that it represented the difference in credit quality between them. The bond prices with the above features are then calculated using the formula,

4 For example, on the 22nd of May 1998 ASH7.97 was trading at 6.261 % yield and T6 government bond was trading at 5.722% yield. Hence the Ash7.97 was offering a risk premium of 0.539% (6.261- 5.722).

5 As an example, on the 15th of May 1998 KFW 7.5 an Aaa rated bond was offering 6.095% yield and its benchmark was offering 5.748%. Therefore offering a risk premium of 0.347% (6.095- 5.753). On the same day, FGH4.5 a Caa rated bond was offering 7.483% and its benchmark was offering 5.748% resulting in a risk premium of 1.735%.

Bond price = Pv (settlement, maturity, rate, yield, redemption, frequency, basis)

$$Bp = Pv (S, M, r, Y, R, F, B)^6$$

The right hand side of the equation represents the price per \$100 face value of the bond that pays periodic interest. In other words, Bp is present value function of $S, M, r, Y, R, F,$ and B . As such, the difference between the KFW bond price and T6.5 bond price is the dollar value of the credit risk embedded in KFW corporate bond issue. Generally, taking P as the price of a corporate bond at time t , P_g as the price of the relevant Treasury bond at time t and the credit risk price P_{risk} is

$$P_{risk} = P_g - P$$

Also taking Y as the yield on a corporate bond at time t , Y^* as the yield on the relevant treasury at time t , and the yield spread as Y_{spread}

$$Y_{spread} = Y - Y^*$$

In our previous example our yield spread is 5.304 minus 4.55, 0.754%. An example on calculating the dollar price of risk for a corporate bond using the above formula is shown in Appendix B.4.

⁶ KFW corporate bond at 30/03/2001 with a yield of 5.304% and T6.5 with a yield of 4.55% and both with the first settlement date 21/04/1995, maturity 21/04/2005, rate 0.075, redemption \$100, frequency 2, and basis of zero had prices of \$116.71 and \$123.29, respectively.

2. RISK MANAGEMENT

2.1 Introduction

Having obtained the price of credit risk for the corporate bonds held, the paper now seeks to explore the options available to an investor to manage or control the credit risk exposure faced. This process of managing the risk exposure is known as *Risk Management*. It is a very complex process that requires not only high level of technical knowledge, but also a great understanding of the dynamics involved with the financial markets and their instruments. Fortunately, a breed of financial instruments sprung up in the late 1990s that made the risk management process less complex. These instruments are known as *Derivatives*. Risk management involving derivatives is mainly centred on the pricing and hedging of the derivatives used to control the risk exposure faced.

This section starts off with a brief history on derivatives and their application in risk management processes. It is followed by examples of credit derivatives that an investor can engage in for the purpose of hedging their risky asset holdings. Then a brief look at the classification of credit derivatives is given, as the pricing and hedging process that can be followed depends heavily on the credit derivative's classification. Examples of such attempted processes are also discussed in this subsection. The basic model that has been followed in the credit risk management process is summarised before the two chosen models are explored. The two models namely, the Jarrow & Turnbull model (JT model) and the Hull and White (HW model) are then discussed. For each model, a result summary of the pricing of credit derivatives (options & warrants) using the model and the corporate bond data available is given.

2.2 Derivatives and Credit derivatives

Derivatives are securities that can be generally defined as financial securities valued in reference to more basic underlying assets. The underlying assets are usually prices of traded assets, for example a stock option is a derivative depended on a particular stock/share price. In recent years, derivatives have become increasingly important in the world of finance (Hull: 2000). Derivatives grew in at least three dimensions. First, Futures and Options emerged as the building blocks for second and third generation derivatives that span complex hybrid, contingent, and path-dependent risks. They are the basic form of derivatives and the most common ones in practice. Second, new applications expanded use of derivatives beyond the specific management of price and event risk to the strategic management of portfolio risk, capital, balance sheet growth, shareholder value, and overall business performance. Finally, derivatives extended beyond the common underlying assets (interest rates, currencies, commodities, and equities) to new underlying risks including catastrophe, pollution, electricity, inflation, and credit (JP Morgan: 2000).

Until recently, credit remained the major component of business risk for which no tailored risk management products existed. Credit risk management for the bond manager meant a strategy of portfolio diversification backed by line limits, with an occasional sale of positions in the secondary market. Users relied heavily on purchasing insurance, letters of credit, and guarantees and so on. These strategies proved inefficient, mainly because they failed to separate the management of credit risk from the asset with which that risk is associated. For example, consider a corporate bond, which represents a bundle of risk as explained earlier on, also including duration, convexity, callability, and credit risk. If the only way to adjust credit risk is to buy or sell that bond, and consequently affect positioning across the entire bundle of risks, then surely the strategy available is inefficient.

The introduction of fixed income derivatives facilitated the management of risks such as duration, convexity, and callability independently of bond positions. Credit derivatives then completed the process by allowing independent management of credit risk. Credit derivatives are bilateral contracts that isolate specific aspects of credit risk from an underlying

instrument and transfer that risk between two parties. That is, they separate the ownership and management of credit risk from other aspects of ownership of financial assets. They can help investors and corporations manage the credit risk of their investments by insuring against adverse movements in credit quality of the borrower.

If a borrower defaults, the investor (bondholder) will suffer losses on the investment but the losses can be fully or partially offset by the gains from the credit derivative. The reference entity, whose credit risk is being transferred, needs neither be a party to nor aware of a credit derivative transaction. They are the first mechanism via which short sales of credit instruments can be executed with any reasonable liquidity. Where it is impossible to short-sell a bond/loan by synthetically purchasing credit protection using a credit derivative one can achieve the economics of a short position. Credit derivatives, except when in structured notes, are off-balance sheet instruments.

2.2.1 Examples of credit derivatives

1. Credit default swap. This is a contract where company X (protection buyer) has the right to sell a bond issued by company Y for its face value to company Z (protection seller) in the event that there is default on the bond. In return, company X makes periodic payments to company Z. In other words, it is an agreement in which a periodic fixed-rate payment, or up front fee, is exchanged for the promise of some specified payment(s) to be made only if a particular, predetermined credit event occurs. A credit event is defined as a default or a credit downgrade, where a default could include bankruptcy, insolvency, or failure to make payments within a predetermined amount of time. A lowering of the credit by public rating agencies below a certain pre-specified level would constitute a downgrade.

2. Total return swap. The return from one bond or a group of bonds is swapped for the return of another. Alternatively, it is a swap agreement in which the total return of a bank loan(s) or credit-sensitive security(s) is exchanged for some other cash flow, usually tied to LIBOR or some other loan(s) or credit-sensitive security(s). A total return swap can be considered a synthetic loan or security because while no principal amounts are exchanged and no physical change of ownership occurs, the swap allows participants to “effectively go long or short the

underlying. The total return swap is distinctly different from a credit swap in that it exchanges the total economic performance of a specified asset for another fixed cash flow. The payments between the parties are based upon changes in the market valuation of a specific credit instrument, irrespective of whether a credit event has occurred.

Another example would be a *forward contract* on a corporate bond (*A swap* where the cash flows from a corporate bond are paid and cash flow from a treasury instrument is received) and a *futures contract* on the spread between a corporate bond yield and the Treasury bond yield.

3. Credit spread option. This is an option on the spread between yields earned on two assets. The option provides a payoff whenever the spread exceeds some level (the strike spread), meaning that the payoff of the option is linked to underlying credit spread or credit-sensitive asset price. That is, for example, an investor holding a corporate bond could buy an option that pays off whenever the corporate bond yield exceeds the Treasury bond yield by say 400 basis points (4%). Typically the options are structured to knock-out (expire worthless) upon a default so that the economics of the instruments separate spread risk and default risk.

4. A credit risk option (CRO). A CRO is an option where the writer agrees to compensate the buyer for a pre-determined and agreed fall or rise in the credit rating of a particular corporation. They are also known as *Credit Rating Options*. That is, they are financial instruments with payouts contingent upon a rating event (downgrade or upgrade). As explained earlier the fall in the credit rating (downgrade) is the lowering of the credit by public rating agencies. This could be for example the fall of a firm's credit rating from Baa (Investment grade) to Caa considered as a speculative rating. Note that the fall in credit quality subsequently results in a larger spread for the particular firm's debt issues. The pricing models for CROs must be of class that incorporate transitions between various ratings. Pricing models such as Das and Tufano (1996) create a risk neutral transition probability matrix between different class ratings that can be directly used to price a host of these CROs.

2.2.2 Classification of credit derivatives

Credit derivatives can be classified on the basis of how their values are affected by the level of credit risk.

1. The derivatives are issued by default-free counterparties on credit-risky underlying assets. For example, option on a corporate bond written by an Aaa company.
2. The credit derivatives are issued by credit-risky counterparties on default-free underlying assets whose values are not affected by the event of default by the writer. For example, if a Baa firm writes an option on a Treasury bond or index fund, the option would fall in this class of derivatives.
3. Lastly, there are derivatives that have both levels of default risk where the derivative is issued on a credit-risky underlying asset by a credit-risky counterparty. In the options market, options that have both levels of credit risk are known as vulnerable options.

Clearly, there is a great need to develop credit risk management tools that can manage both levels of credit risks simultaneously. Developing derivative pricing models that effectively incorporate the impact of both types of credit risks in their pricing process is one such effective way of managing these risks. There have been several attempts by people to develop models to that effect and many have fallen short of providing a practical and observable model that captures both risks. For example, Hull and White (1991) addresses a model that prices one type of the credit risk (vulnerable options' pricing) only whilst Litterman and Iben (1991) looks at the risk inherent in the underlying asset instead. This is mainly a result of the differences in the assumptions and set ups used in developing the models. For example, differing assumptions were made on the determination of the payoff ratio in default (others considered this as exogenously given while others took it as an implicitly determined value).

Other models ignored the existence of credit risk and instead priced them as default free interest rate options (see Ho and Singer:1984). This however, ignores the need for an arbitrage-free set up and also does ignore the existence of credit spreads (due to difference in credit quality) between Treasury default free bonds and the risky corporate bonds. Another problem inherent with some of these models is that they require valuation of non-tradable assets as part of the underlying asset pool as a model input.

2.3 The basic model requirements

When analysing the effect of credit risk on the value of any financial instrument including derivatives the process can be divided into two stages.

1. Estimating the expected loss (amount) in the event of a default. This greatly depends on the type of contract, the credit exposure and the chance of recovery in the event of default.
2. Calculating the probability of default on the security.

For example, analysing the impact of credit risk on the value of a loan,

- The expected loss in the event of a default is the principal plus any accrued interest less any expected recovery (deduced from the past).
- The probability of default occurring can be determined using historical default rates provided by rating agencies.

It is however, not that simple with derivatives where the exposure only occurs when the value of the contract is positive to the particular investor and negative to the counterparty. For example, an interest rate swap at inception has a zero value and there is no exposure to the counterparties but when interest rates move resulting in a positive value for the investor exposure occurs. The exposure on a derivative is considered as a payoff from an option on the derivative with a strike price of zero. It is also difficult to determine the relationship between the event of default and the credit exposure for derivative contracts. As previously suggested it is very easy to determine this relationship for loan contracts where the exposure is always close to principal amount, unlike derivatives where it is variable. The expected loss

on the derivative is relatively large when probability of default is high and the exposure is large, and it will be relatively small when the probability is low and the exposure is large.

2.4 The Jarrow & Turnbull Model

The above-mentioned model was presented by Jarrow and Turnbull (1995) and adapted for this paper as discussed in the following section.

As the norm, we assumed the existence of a frictionless, competitive and continuous trading economy where the investment horizon or trading horizon is $[0, \tau]$. A frictionless trading market is assumed to have no transaction costs and trading barriers, a dream that the world will never get to realise. A competitive market from an economic perspective is the market where all trading participants are pure price takers; their individual trading does not affect the equilibrium price. Continuous trading market as the name suggests is a market where participants can continuously trade the portfolio components in order to adjust their portfolios accordingly.

These assumptions made are somewhat far fetched for some markets. Transaction costs do exist, for example brokerage fees when an investor buys any financial security through the registered traders (Brokerage firm). Monopolies as well as oligopolies also do exist together within competitive markets. In the financial markets however, there is room to believe that there are very competitive markets where arbitrage opportunities exist far in between. If an arbitrage opportunity exists, the investors (traders) will all pounce on that profiteering chance forcing prices to adjust and consequently erasing the arbitrage opportunity.

Continuous markets do exist but the extent or degree of continuity very much depends on the liquidity of the traded assets. As an example one could argue that the Treasury bond market is a more continuous market compared to the corporate bond market because the Treasuries are more liquid than corporate bonds. The continuity is also greatly impeded by the fact that it takes time to execute a trade, the settlement date is usually plus or minus two days after the trading order date.

So taking the set of trading dates as discrete $\{0, 1, 2, \dots, \tau\}$ or as continuous $[0, \tau]$, we assumed that there were three types of issuers whose issues were currently trading on the bond market. That is, the US government trading a default-free zero coupon Treasury bond (TB), a firm trading specific Moody's rating (Baa1) risky zero coupon corporate bond XYZ, and a second firm trading a triple-A zero coupon corporate bond. These three zero coupon bonds were obtained by bootstrapping existing coupon bonds. The bonds used were specifically:

- a) the US Treasury maturing 15 May 2011 trading as US TREASURY (CATS) ZERO 15 May 2011.
- b) the risky zero coupon bond XYZ was obtained by bootstrapping a Baa1 Moody's rating General Motors Corporation 7.75% coupon bond maturing March 2036.
- c) the triple-A corporate bond used was trading as INTL.BK.RECON & DEV (IRBD) ZERO 15 August 2003.

We assumed that both the US Treasury bond and the triple-A corporate bond were the relevant benchmark bonds for the risky bond for modelling purposes. The argument in support of this assumption was the fact that options issued by and on triple-A firms are considered risk free according to the credit ratings definitions therefore the corresponding bond issued by the same firm would be deemed risk free.

Letting $p_0(t, T)$ denote the time t dollar value of the default free zero coupon bond that paid a definite dollar (\$) at time $T \geq t$, for simplicity in our observations and analysis we assumed positive bond prices $p_0(t, T) > 0$, and default free $p_0(t, t) \equiv 1$. Probability of default for these zero coupon bonds was therefore zero, that is, there was no chance of the Treasury bond defaulting because it had the full backing and the credit quality of the US government. On the same note, we let $v_1(t, T)$ be the time t value of the XYZ zero-coupon bond promising a dollar (\$) at date $T \geq t$. Again we assumed the prices were positive at all times for simplicity i.e. $v_1(t, T) > 0$. Note that our XYZ zero-coupon bond is a corporate bond that carries with it credit risk among other risks whose magnitude is dependent on the issuer's (General Motors Corporation) credit rating. It therefore promises to pay a dollar unlike the default free bond that pays a sure dollar at maturity.

By using the default-free term structure, a money market account was developed. It invested a dollar in the shortest maturity (one-period maturity) bond, and then subsequently rolled

over into the next one period bond. In other words, a dollar invested today $t = 0$ in a one period bond gains the specific interest from this bond and this future value of the investment at $t = 1$ is invested in the next one period bond until maturity at $t = 2$, and so on until the end of the trading horizon, τ . Let $B(t)$ represent the time t value of this money market investment.

2.4.1 The Forex analogy

The basic insight of the approach is to show that pricing options on risky debt is the same exercise as pricing options on foreign currencies. This argument follows that, because pricing the latter is well documented and understood if the similarities of the two pricing problems is established, pricing the former will consequently be easily solved. To demonstrate the ‘Forex analogy’, suppose that the XYZ zero coupon bond payoffs in Rand (R) and not in dollars (\$). The introduction of the rand brings into play an exchange rate, e_1 between the rand and the dollar (i.e. price of Rand in dollars (\$/R)).

$$e_1(t) \equiv v_1(t, t)$$

This is the time t dollar value of one promised Rand delivered at time t . In Rand, XYZ bond is of course default free. However, at maturity an XYZ promised dollar might not be worth an actual dollar. This occurs if XYZ firm is in default and $e_1(t)$ is less than 1. It follows then that if XYZ firm is solvent $e_1(t)$ is unity (one), and each promised Rand is actually worth a dollar. Let $p_1(t, T)$ be the time t price in Rand of an XYZ promised dollar delivered at time T for sure (Term structure of XYZ debt in Rand).

From the above, the dollar value of XYZ debt can be written as:

$$v_1(t, T) = p_1(t, T) e_1(t)$$

The left hand side is the dollar value of a Rand (foreign currency) denominated bond. While the right hand side is the Rand value of a rand denominated bond multiplied by the spot exchange rate. Pricing and hedging options on credit risky XYZ debt is therefore analogous

to pricing and hedging options on a rand denominated bond in dollars (The Forex Analogy). Here the spot exchange rate acts a payoff ratio when XYZ is in default.

A two period economy is used to illustrate the foreign currency analogy as applied to credit risky options. The two-period economy is in discrete-time state and can easily be generalized to the continuous-time setting. The discrete-time setting results in trading dates $t \in \{0, 1, 2\}$. The default-free bond price process is assumed to be solely dependent on the spot interest rate. Thus, the current ($t = 0$) one period spot interest rate is

$$r(0) = 1/p_0(0, 1)$$

In the “up-state,” the one period spot rate is

$$r(1)_u \equiv 1/p_0(1, 2)_u$$

And in the “down-state”

$$r(1)_d \equiv 1/p_0(1, 2)_d$$

Probability of state u occurring is π_0 and the probability of state d occurring is $1 - \pi_0$, assuming that $p_0(1, 2)_u < p_0(1, 2)_d$

Let $B(t)$ be the value of the Money Market Account at time t after investing a dollar at $t = 0$. Note that, because we know the prevailing one period rate from the default-free zero-coupon bond term structure $B(t+1)$ is known at time t .

$$\Rightarrow B(0) \equiv 1, (t = 0)$$

$$\text{After one period } B(1) \equiv B(0)r(0) \quad \Rightarrow \quad B(1) \equiv r(0) \text{ i.e. at trading date } t = 1$$

$$\text{At } t = 2 \text{ in the upstate } B(2)_u \equiv r(0)r(1)_u, \text{ and in the down state } B(2)_d \equiv r(0)r(1)_d.$$

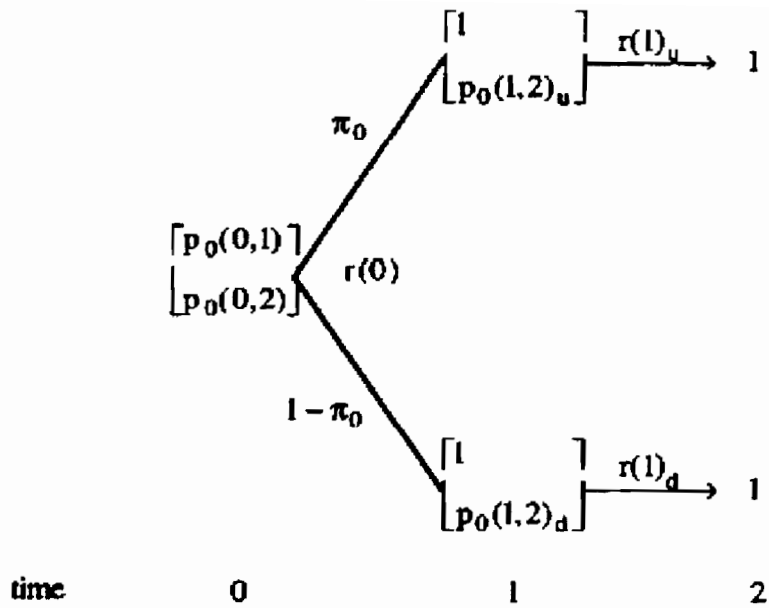


Figure 5: The default-free zero-coupon bond price process for the two-period economy.

The figure shows the evolution of the spot interest rate process and the zero-coupon bond prices over the period 0, 1, and 2. $p_0(t, T)$ denotes the time t dollar value of the default free zero-coupon bond paying a sure dollar at time T .

Harrison and Pliska, (1981) show that the no-arbitrage market condition is the existence of pseudo-probabilities π_0 , $\lambda\mu_0$ and $\lambda\mu_1$ such that $p_0(t, 1)/B(t)$, $p_0(t, 2)/B(t)$, $v_1(t, 1)/B(t)$, $v_1(t, 2)/B(t)$ are martingales. Market completeness is equivalent to the uniqueness of these probabilities and implies that any contingent claim against the securities can be synthetically constructed through trading in the primary securities. The probability π_0 is determined in the default free bond market as mentioned above.

From figure 5 above,

$$p_0(0, 2) = [\pi_0 p_0(1, 2)_u + (1 - \pi_0) p_0(1, 2)_d] / r(0)$$

Today's zero-coupon bond price is its next period's value discounted using π_0 . That is, the current bond price equals the discounted expected future value.

Therefore π_0 , rearranging the above is given by

$$\pi_0 = [p_0(1, 2)_d - r(0)p_0(0, 2)] / [p_0(1, 2)_d - p_0(1, 2)_u]$$

Thus, π_0 exists, is unique, and satisfies $0 < \pi_0 < 1$ if and only if

$$p_0(1, 2)_u < r(0)p_0(0, 2) < p_0(1, 2)_d$$

Alternatively, the short-term zero-coupon bonds must not dominate long-term zero-coupon bonds, and that it earns more return in state d than in state u . Taking the zero coupon bond's default/no default state at time t , as b (default) and n (no default).

Now for a credit risky zero-coupon bond at maturity, two states are possible i.e. default occurs, and default does not occur. If default occurs, the payoff is less than the face value of the bond. And if default does not occur, the payoff is exactly the same as the face value of the bond at maturity. Assuming that the payoff per unit of face value is δ , in terms of the foreign currency analogy, the spot exchange rate at time 0 is unity, $e(0) = 1$ and at times 1 and 2 $e(t)$ takes on the values as shown in Figure 6.

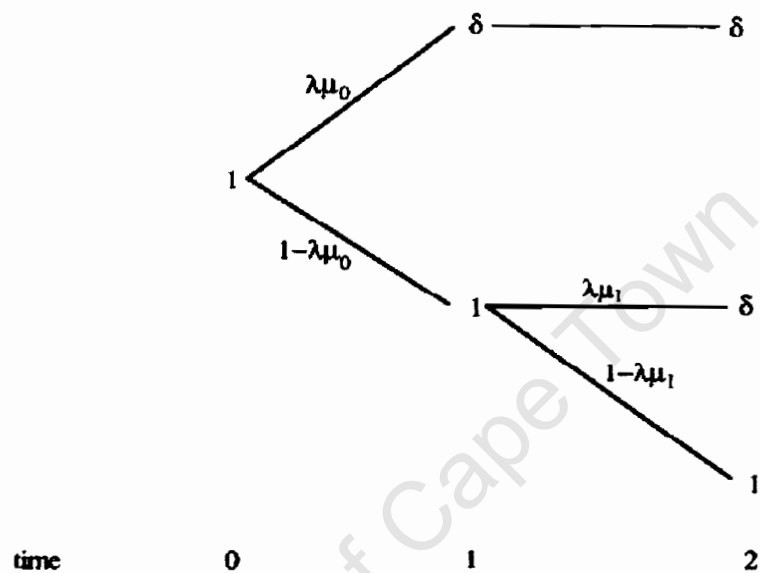


Figure 6: The payoff ratio process for XYZ debt in the two-period economy. The figure shows the binomial process followed by the payoff ratio for XYZ debt over the three periods.

At time 1, with probability $\lambda\mu_0$, default occurs and the payoff is δ of the face value. The payoff remains the same at time 2 because the XYZ will still be in default. If however, default does not occur at time 1 ($1 - \lambda\mu_0$) the probability of default occurring at time 2 is given by $\lambda\mu_1$.

Figure 7 below shows the stochastic movement of the XYZ bond denominated in Rand, it is similar to Figure 5 except that there are more states involved: the combinations of the up/down states and default/no default states.

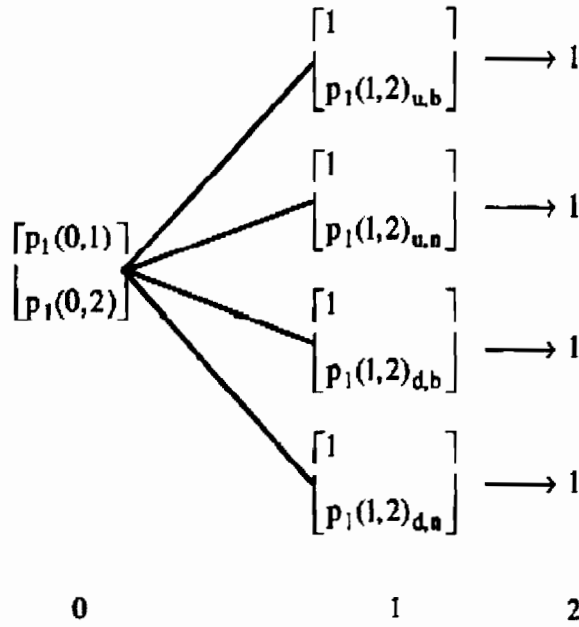


Figure 7: The XYZ zero-coupon bond price process for the two-period economy in XYZs. The figure presents the XYZ zero-coupon bond prices in XYZs (currency) in the two-period economy.

Figure 8 below shows the XYZ bonds in dollars, a product of Figures 6 and 7, assuming that the spot interest rate process (Figure 5) and the default process (Figure 6) are independent under the pseudo-probabilities. The product results in the states combinations given by

$$v_1(1, 2)_{u, b} = \delta p_1(1, 2)_{u, b} = \delta / r(1)_u \quad (\text{Up, default state})$$

$$v_1(1, 2)_{u, n} = p_1(1, 2)_{u, n} = [\lambda \mu_1 \delta + (1 - \lambda \mu_1)] / r(1)_u \quad (\text{Up, No default states})$$

$$v_1(1, 2)_{d, b} = \delta p_1(1, 2)_{d, b} = \delta / r(1)_d \quad (\text{Down, default state})$$

$$v_1(1, 2)_{d, n} = p_1(1, 2)_{d, n} = [\lambda \mu_1 \delta + (1 - \lambda \mu_1)] / r(1)_d \quad (\text{Down, No default states})$$

Giving $\lambda \mu_1$ as

$$\lambda \mu_1 = [1 - p_1(1, 2)_{u, n} r(1)_u] / [1 - \delta] = [1 - p_1(1, 2)_{d, n} r(1)_d] / [1 - \delta]$$

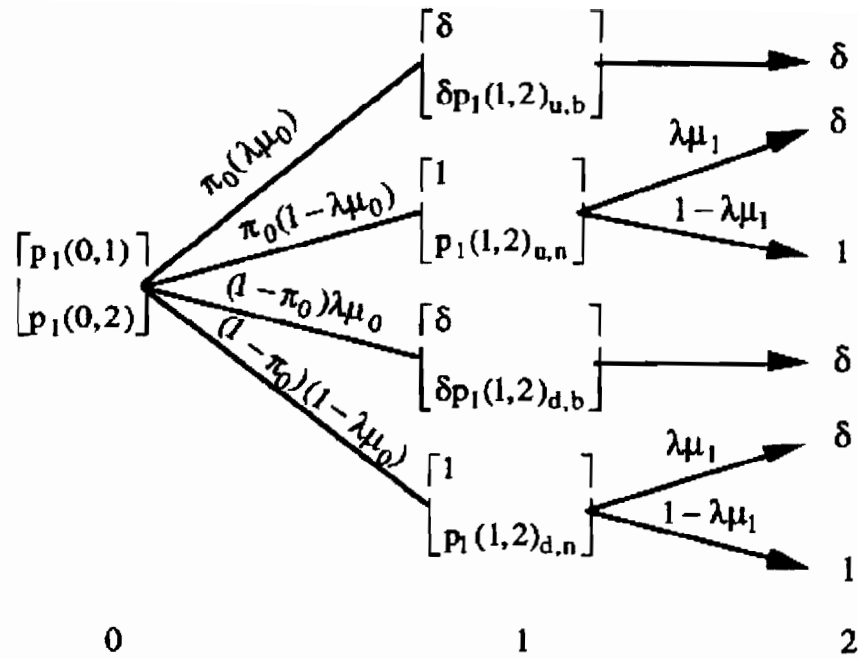


Figure 8: The XYZ zero-coupon bond price process for the two-period economy in dollars. The figure shows the binomial process followed by the XYZ zero-coupon bond prices in dollars over the three periods. It is a product of the payoff ratio process and the XYZ zero-coupon bond price (in XYZs) process over the period.

Which exists, is unique, and satisfies $0 < \lambda\mu_1 < 1$ if and only if

1. $p_I(1,2)_{u,b} = 1/r(1)_u$
2. $p_I(1,2)_{d,b} = 1/r(1)_d$
3. $\delta/r(1)_u < p_I(1,2)_{u,n} < 1/r(1)_u$
4. $\delta/r(1)_d < p_I(1,2)_{d,n} < 1/r(1)_d$
5. $r(1)_u p_I(1,2)_{u,n} = r(1)_d p_I(1,2)_{d,n}$

Where conditions,

(1) and (2) show that in the state of default (b), the default-free bonds in dollars are equal to the XYZ bonds in Rand because the only uncertainty left is that of the default-free spot interest rates, (3) and (4) state that given no default at time 1, the dollar value for the XYZ bond must be less than the dollar value of a claim paying one dollar for sure and greater than a claim paying δ dollars for sure. (5) Guarantees the independence of the $\lambda\mu_1$ from the spot interest rate process.

(2)

Also from figure 8 we obtain,

$$6. \quad v_1(0, 1) = p_1(0, 1) = [\lambda\mu_0\delta + (1 - \lambda\mu_0)] / r(0)$$

$$7. \quad v_1(0, 2) = p_1(0, 2) = [\pi_0(\lambda\mu_0) \delta p_1(1, 2)_{u, b} + \pi_0(1 - \lambda\mu_0) \delta p_1(1, 2)_{u, n} + (1 - \pi_0)\lambda\mu_0 \delta p_1(1, 2)_{d, b} + (1 - \pi_0)(1 - \lambda\mu_0)p_1(1, 2)_{d, n}] / r(0)$$

Again, the current prices are period one discounted expected values. Substituting conditions for π_0 and $\lambda\mu_1$ into (7) results in conditions: the dollar value of XYZ bond maturing at period one must be worth less than receiving a sure dollar and greater than receiving δ dollars for sure. XYZ bond maturing at period 2 must be worth more than receiving δ dollars for sure and less than receiving $r(1)_{d, n} p_1(1, 2)_{d, n}$ dollars for certain. Also, the default process is independent of the default-free spot interest rate process.

Now let $C(t)$ be the time t price of a European call option on the two-period XYZ zero-coupon bond. Its value at expiration is

$$C(1) = \max [v_1(1, 2) - X, 0]$$

where X is the exercise price at time 1. Using risk-neutral procedures the option's current value can be calculated as the discounted expectation of its time 1 payoff.

$$C(0) = (1 - \lambda\mu_0)[\pi_0 C(1)_{u, n} + (1 - \pi_0) C(1)_{d, n}] / r(0) + (\lambda\mu_0)[\pi_0 C(1)_{u, b} + (1 - \pi_0) C(1)_{d, b}] / r(0)$$

That is, it's the sum of the discounted expectation of its payoff at expiration in both the default and no-default states. Figure 7 shows that the XYZ bond process has four branches (combination of the up/down and default/ no-default states), and consequently, to hedge an option three assets plus the money market are needed. That is, α shares of the two-period XYZ zero coupon bond, β shares of the one period XYZ zero coupon bond, γ shares of the two-period default-free zero coupon bond, and lastly ϵ shares of the money market investment to give

$$\alpha v_1(1, 2)_{u, b} + \beta v_1(1, 1)_{u, b} + \gamma p_0(1, 2)_{u, b} + \epsilon r(0) = C(1)_{u, b}$$

$$\alpha v_1(1, 2)_{u, n} + \beta v_1(1, 1)_{u, n} + \gamma p_0(1, 2)_{u, n} + \epsilon r(0) = C(1)_{u, n}$$

$$\alpha v_1(1,2)_{d,b} + \beta v_1(1,1)_{d,b} + \gamma p_0(1,2)_{d,b} + \varepsilon r(0) = C(1)_{d,b}$$

$$\alpha v_1(1,2)_{d,n} + \beta v_1(1,1)_{d,n} + \gamma p_0(1,2)_{d,n} + \varepsilon r(0) = C(1)_{d,n}$$

Maintaining the market completeness and no arbitrage conditions the call option's price is given

$$C(0) = \alpha v_1(0,2) + \beta \alpha v_1(0,1) + \gamma p_0(0,2) + \varepsilon$$

This valuation process is used to price credit options by looking at the risk where the underlying asset (XYZ bond) may default. This can be extended or modified to include pricing vulnerable options (the option writer defaulting).

Assuming that like XYZ zero-coupon bond issuer, the writer ABC has zero-coupon bonds issued against its own assets with value $v_2(t, T)$ for $T \geq t$ and that they follow the same price process as the XYZ bonds. They have unique pseudo-probabilities that result in the relative prices of all these bonds to be martingales. The default process for the payoff ratio, $e_2(t)$ is independent of the default-free spot interest rate process and independent of the payoff ratio of XYZ.

Assuming that ABC instead of XYZ writes the previous call option at maturity, the cash flow to the buyer represents a promise by ABC to make payment $C(1)$

$$e_2(1)C(1)$$

Thus,

$$\begin{aligned} C_2(0) &= E_0(e_2(1)C(1)/B(1)) \\ &= E_0(e_2(1)C(0)) \end{aligned}$$

The price of the call option written by ABC is the price of a call option written by a default free (AAA company) writer discounted by the expected payoff from ABC.

$$C_2(0) = [v_2(0,1)/p_0(0,1)]C(0)$$

The XYZ dollar bond price divided by the default free bond price equals the expected payoff to the credit risky bond at maturity. Therefore, a vulnerable option is therefore always less valuable than a non-vulnerable option $\{C_2(0) < C(0)\}$ because $[v_2(0,1)/p_0(0,1)] < 1$.

In general,

$$C_v(t) = [v_1(t, T)/p_0(t, T)]C(t)$$

where $C_v(t)$ is the time t value of a option written by a firm with risky debt or on a credit risky firm by a riskless writer ($v_1(t, T)$), and $C(t)$ is the time t value of the same option written by an otherwise riskless writer or on a risk free debt.

2.4.2 Results: Jarrow & Turnbull model

The results of using the JT model presented below are broken into three different parts according to the bond inputs and the option inputs used. From the payoff result shown by the 'forex analogy' process we expect to observe expected Baa1 option prices that are lower than their corresponding risk free option prices. That is, to attract investors into buying a risky option you would expect the option to sell at a lower price compared to a risk free option. On that same note, we therefore expect the expected Baa1 option obtained using the Treasury bond to have a higher price than the expected Baa1 option obtained using the triple-A corporate bond.

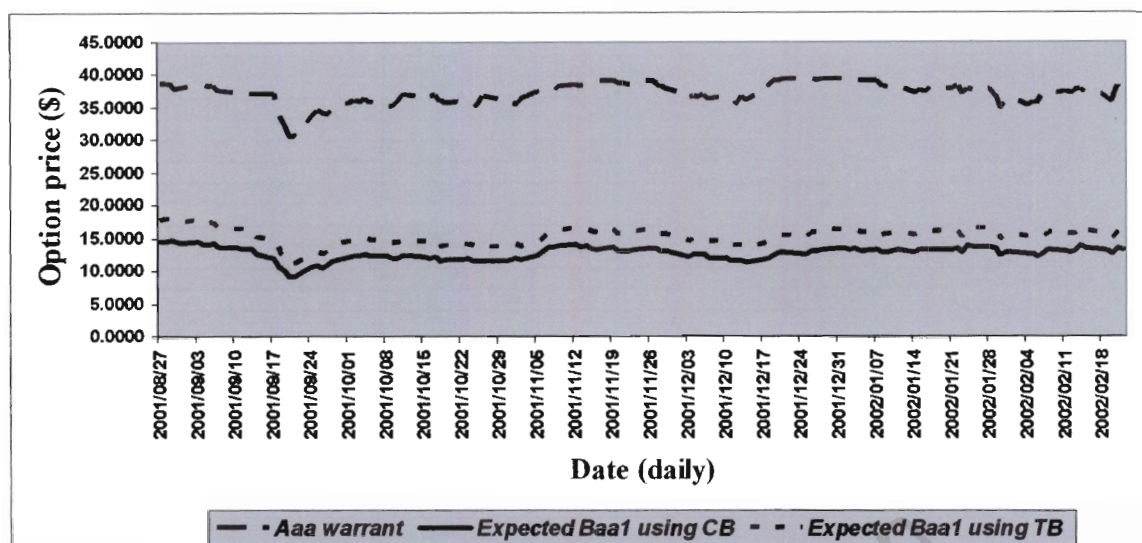


Figure JT.1: The expected Baa1 option prices using zero-coupon bonds, a triple-A warrant and the JT model. The figure shows the expected Baa1 option prices obtained when using a triple-A warrant as the risk free option, using the Treasury zero coupon bond as well as the triple-A zero coupon corporate bond as the risk free debt over six months.

A warrant issued by a triple-A firm (triple-A warrant) is used to represent a triple-A option (option written by an otherwise less risky writer or on a risk free debt). In this scenario no corresponding risky warrant was available to validate or compare with the expected risky warrant prices calculated using the JT model. This is as a result of the less liquid nature of warrants issued by risky firms and therefore very few active issues can be found in the market. As shown in the figure above, the expected Baa1 option prices resulting from the JT model using both the Treasury bond and the triple-A bond are lower than the corresponding prices for the risk free option used. For full details see Appendix JT. 1

An interesting result was obtained when we used coupon bonds as inputs instead of zero-coupon bonds with the same triple-A warrant. The expected Baa1 option prices using the Treasury bond were higher than the prices for the expected Baa1 option obtained using the triple-A coupon corporate bond. This was expected but they were slightly higher than the risk free option prices as well, contradicting the payoff system suggested by the forex analogy.

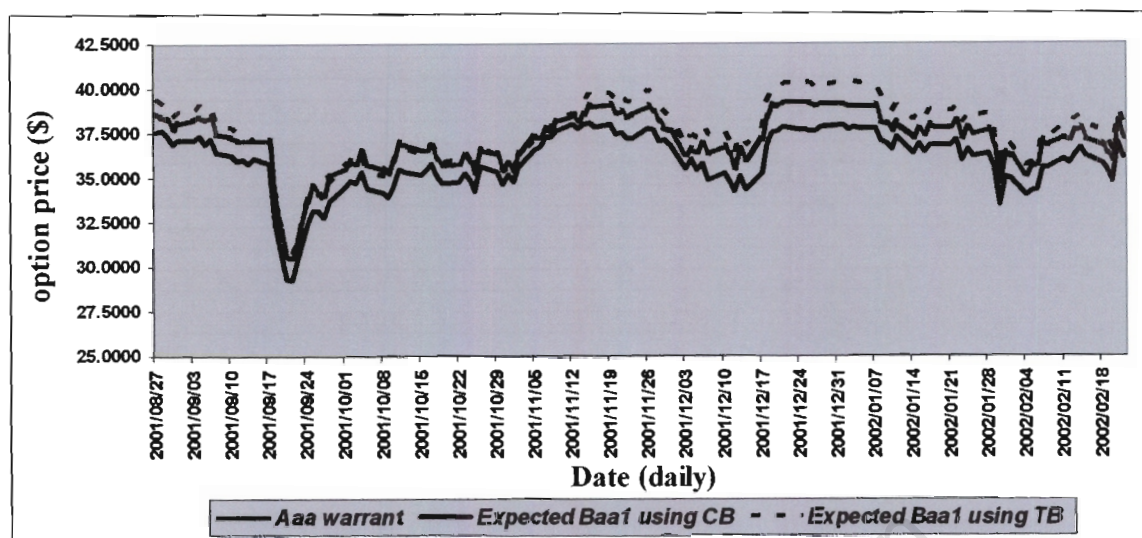


Figure JT.2: The expected Baa1 option prices using coupon bonds, a triple-A warrant and the JT model.

Obtaining more warrants proved difficult just as much as trying to get the options on bonds (bond options) from the market, even for the most liquid market, the US option market. The latter proved to be impossible mainly because most if not all bond options are traded over-the-counter and therefore there is no ready and well-updated database to extract the prices from. Unfortunately, these bond options are the most ideal options used with the JT model because the bond option used would be the one ranking equally with the bond issue used. To overcome that hurdle, this paper looked at calculating these bond options using the bond issues that were readily obtained from the bond market and using option pricing models.

To calculate the bond options, Black-Scholes model's special case model the *Black Model* was used (Black: 1976). Of note is that the volatility used in the Black model is the implied volatility calculated using historical bond prices, where in this paper the historical prices used are then used as the current bond prices for the JT model. The bond option calculated is a six month option on the US Treasury bond, triple-A corporate bond and finally on a Baa1 corporate bond for both zero coupon and coupon bonds. In other words, the implied volatility is in actual fact the volatility of the bonds over the six-month period. For the bond volatilities and the Black model option prices see pages before each corresponding appendix (Appendix JT.3-6).

The six month bond option on Treasury bond calculated using the Black model had a price distribution which was higher than for the expected Baa1 option from the JT model. However, as shown in the figure below the option prices did move or fluctuate correspondingly over the six months. This is a result that was closely shared when we used the six month bond option on the triple-A zero-coupon corporate bond calculated using the Black model and the resultant expected Baa1 option using the JT model as shown in figure JT.4.

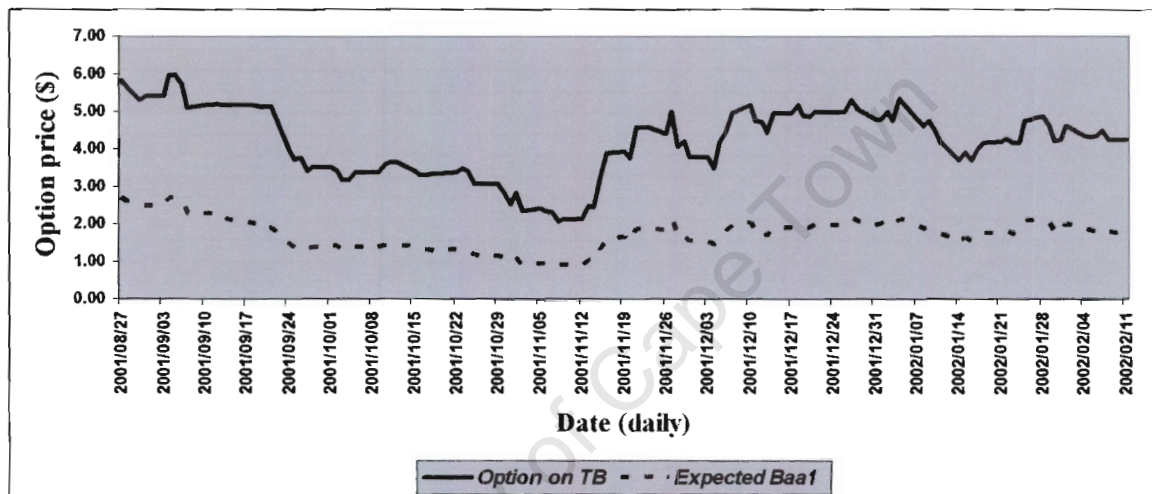


Figure JT.3: The expected Baa1 option prices using zero-coupon bonds, bond options using the 30-year Treasury bond yield and the JT model. This shows the expected Baa1 option prices obtained from the JT model using as inputs the six month bond options calculated using the Black model together with zero coupon bonds for the US Treasury bond and the Baa1 corporate bond. To calculate the six-month bond options using the Black model, the 30-year Treasury bond yield was used as the risk free rate in determining the discount rate.

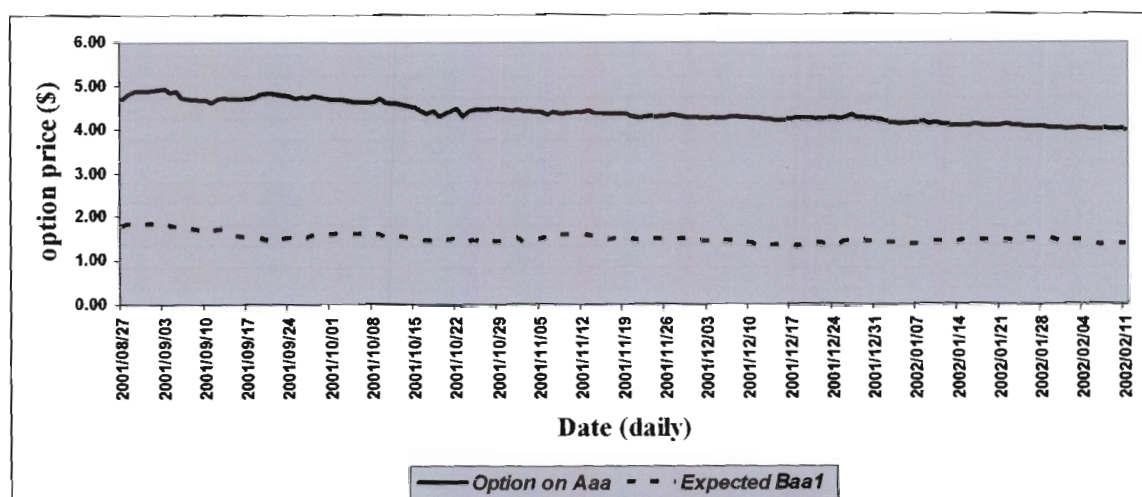


Figure JT.4: The expected Baa1 option prices using coupon bonds, bond options using the 30-year Treasury bond yield and the JT model.

For the above two figures when calculating the bond option prices using the Black model, the discounting rate used is the sum of the 30-year US Treasury bond yield and the average risk premium for a 30-year triple-A corporate bond, or for a 30-year Baa1 bond. The risk premium was the average risk premium (basis points above the Treasury bond) for 30-year corporate bonds issued under banks, industrials, transportation, utilities, and financials in the US corporate sector.⁷ We then looked at using the same bond inputs as for the above results but changing the discount rate used in the Black model calculations. It is the sum of the average risk premium of the corporate bonds and the 6-month Treasury bill rate and not the 30-year US Treasury bond yield. This is arguably the better risk free rate to use because it represents the same time horizon as the bond option being calculated. It is also argued that the short term Treasury debt is more risk free than the longer dated debt issues; hence it is preferred as the risk free rate in asset valuations.

Figure JT.5 that has the same zero-coupon bond inputs as for figure JT.3 which shows the same results as figure JT.3. The movement and changes in the option prices over the six month period are identical. Similarly, figure JT.6 has corresponding results as figure JT.4

⁷ For example, a 30-year Baa1 bond has the following risk premiums for the above mentioned sectors respectively, 177,168,218,136,202 basis points. This gives an average risk premium of 180 basis points (1.8%) above the Treasury yield (5.17%). The discount rate used in the Black Model for a 4-month option on a Baa1 30 year bond is therefore the sum of the last two figures, 5.17% and 1.8% that is 6.98%.

whose only difference lies in the discount rate used in calculating the bond options using the Black model. Despite observing these same results in both scenarios the use of the Treasury bill yield as the risk free rate in calculating the discount rate to be used in the Black model is recommended.

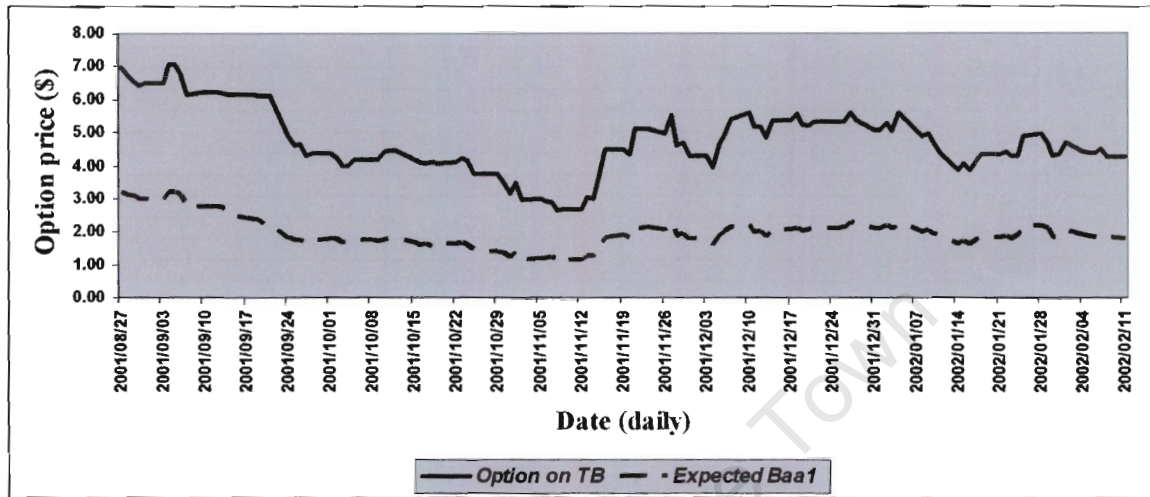


Figure JT.5: The expected Baa1 option prices using zero-coupon bonds, bond options using the 6-month Treasury bill rate and the JT model. The Black model bond options are calculated by using the 6-month Treasury bill rate as the risk free rate in determining the discount rate to be used.

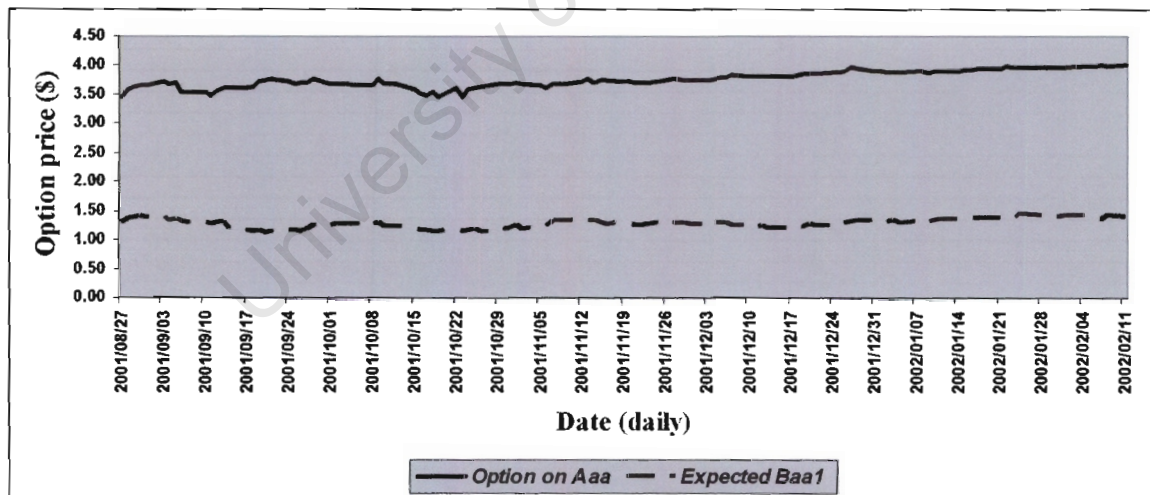


Figure JT.6: The expected Baa1 option prices using coupon bonds, bond options using the 6-month Treasury bill rate and the JT model.

Finally, we looked at the JT model using a Treasury note (T-Note) that was previously defined as a Treasury bond with a maturity of ten years or less and a corresponding coupon corporate bond. The bonds used were both 10-year maturity issues. A 6-month bond option

was calculated using the Black model and used as part of the inputs into the JT model. The 6-month bond options were calculated using the same implied volatility concept as before and the discount rate used was the sum of the 10-year Treasury note yield and the average risk premium for a 10 year Baa1 corporate bond. See Appendix JT.7 for full details.

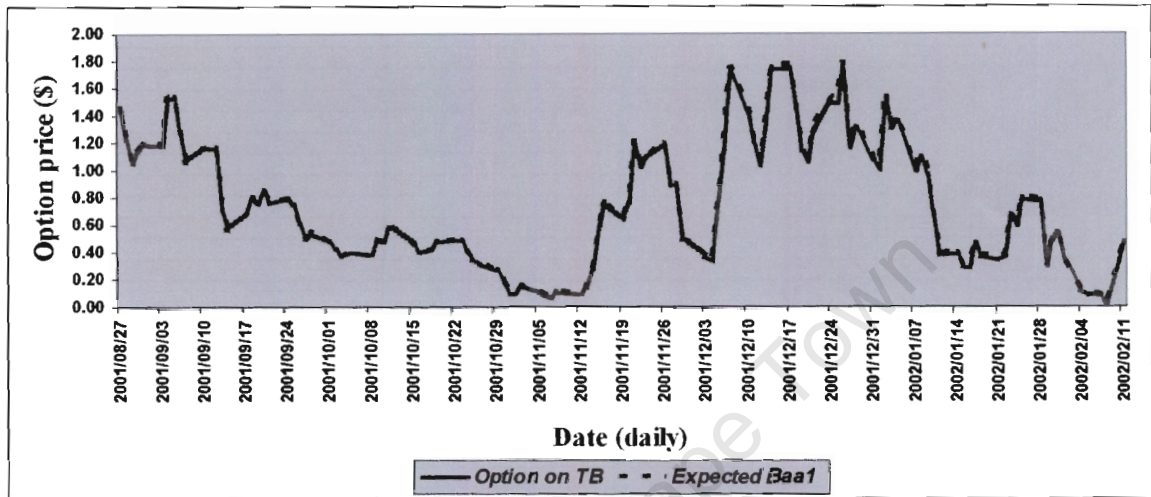


Figure JT.7: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 10-year Treasury note yield, and the JT model.

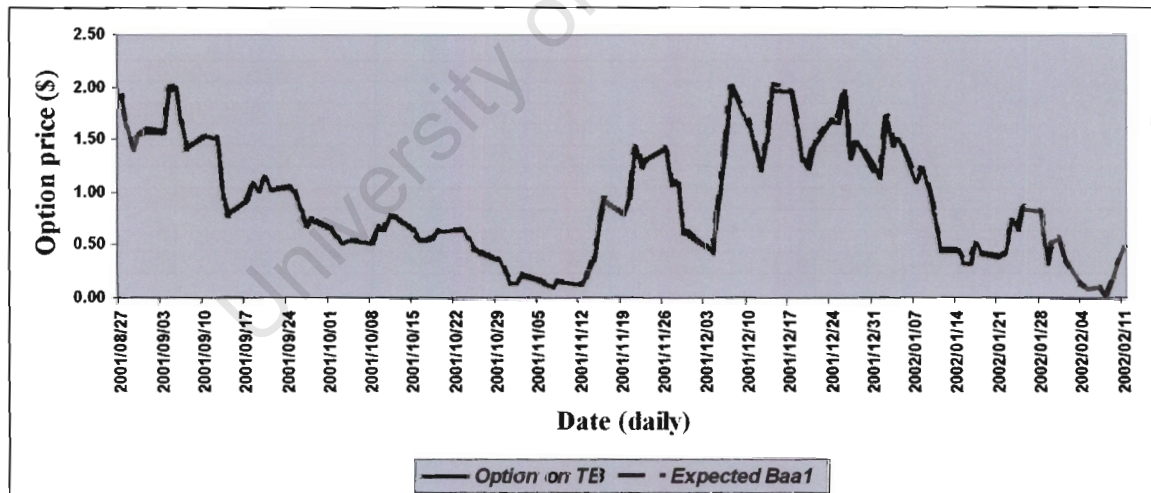


Figure JT.8: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 6-month Treasury bill rate and the JT model.

In both instances, that is, when we used the 10-year Treasury note yield or the 6-month Treasury bill rate, the results were the same. There was no difference between the risk free option (option on TB) and the risky option (the expected Baa1 option from the JT model).

This indifference meant that the payoff of the Baa1 corporate bond used in the model was close to 1, or the bond itself was close to being risk free issue. This is a result that is contrary to what one expects given the definition in terms of risk according to Moody's ratings for a Baa1 corporate bond.

2.5 The Hull & White Model

The Hull & White model discussed in this section was presented by Hull & White (1992). It is a model used to price derivatives on financial securities subject to credit risk. Simply put, they define how to price derivatives on risky underlying assets taking account of their yield spreads or default probabilities. It aims to answer the question whether estimated yields on corporate bonds (those that exist and those still to be issued) can be used to provide credit-adjusted quotes for derivatives. With any derivative, losses can only be incurred when the value of the derivative is positive to the financial institution and negative to the counterparty. This portrays an option-like function of the no default value. That is, $\max(V, 0)$, where V is the derivative's no-default value to the financial institution. For contracts such as long options, V is always positive and thus the exposure equals to V , unlike swaps and forward contracts.

A bond XYZ is defined as ranking equally with an option ABC if the recovery made in the event of a default as a proportion of the exposure is the same as the option. Assuming that two groups of variables affect the value of a derivative security when default is possible: that is,

- the variables affecting its value in a no-default world
- the variables affecting the occurrence of defaults by the counterparty and the proportional recovery made in the event of a default.

A simplifying assumption that makes practical use of the model feasible is that the first set of variables is independent of the second. The question of how realistic this assumption arises. The answer to it will be that the value of the contract under consideration has a negligible bearing on the ability of the counterparty to meet its liabilities as they become due. This

means that contract is a very small part of the counterparty's portfolio of assets and liabilities, or that the contract's risk is entirely hedged by the counterparty.

The values that actually cause defaults are also assumed to be uncorrelated with the variables affecting the no-default value of the derivative security. In other words, the variables such as the market interest rate term structure the value of the underlying asset that affect the value of say a call option were independent from those variables that affect the value of the same option looking from the probability of default of the counterparty. Such variables include the likes of total assets and liabilities of the counterparty firm and their impact on default probability and the recovery made on securities like options issued by the same firm.

From a pricing point of view this assumption will be difficult to accept given that pricing of any security involves taking into consideration all the available variables that affect the security in both the no-default world and where probability of default exists. However, as argued before if we take the security in question to be part of a large pool of securities that are held by the firm the impact of this particular security on the firm's portfolio becomes very small and consequently its impact on the firm's probability of default is also very small and can therefore be ignored. Thus we can separate the variables that affect the option or derivative from those that affects the firm as whole and its default probabilities. As a side note, it would be of great importance to see the validity of such an assumption probably by trying to isolate the individual variables that affect the option/derivative and those that affect the firm as whole and quantify their impact on the value of those two.

Suppose first that the option is European and matures in time T . Define F as the value of the option taking account of the possibility of defaults and F^* as the value of a similar default-free option (we assumed that both a European option on a triple-A corporate bond or on a Treasury bond is the default free).

The relationship between F and F^* is:

$$F = F^* \exp. \{- (Y - Y^*) T\} \dots \dots \dots HW$$

where Y is the yield on a zero coupon bond issued by the counterparty ranking equally with the option in the event of a default and, Y^* is the yield on a similar default-free bond (both continuously compounded).

As an example of the application of this formula, consider a two-year OTC option with a default-free value of \$3. Suppose that it is estimated that the two-year zero coupon bond issued by the option writer and ranking equally with the option in the event of a default would yield 150 basis points (1.5%) over the Treasuries. Default risk has the effect of reducing the option value by about 3% to \$2.911 ($3e^{-(0.015)2}$).

Before presenting the model's results, the different assumptions are highlighted and discussed in detail below.

1. Bond prices issued by the counterparty and ranking equally with the option are readily available. From experience, bond traders are able to model the price structures of non-existent issues using the particular firm's financial information and issues issued by other firms that rank equally with the firm concerned. Therefore the problem of non-existent bond issues for the counterparty can be easily solved. However, the task at hand is not easy considering that these simulated zero coupon prices should be for issues that pay off the same proportion, as do options issued by the same firm. Using bond issues issued by other firms that rank equally with the firm whose bond prices are being modelled, one can assume that if two firms have the same rating then their bond issues will have the same rating ignoring the differences between the two firms profile. Thus the bond prices can be modelled from similar credit rated firms.

Intuitively, there has to be an adjustment to the bond issue or the option so that their duration or term to maturity as well as their underlying assets has the same credit value for them to rank equally and thus pay the proportion when in default. For instance, bond issues have longer term to maturities compared to options (call/put) and their underlying assets may be different from the bond's underlying. The latter's underlying assets are usually the residual assets in the event of default or there can be secured assets (secured bonds) whereas the option issued by the firm or a firm with similar credit rating may have a different firm as the

underlying asset. Assuming that the bonds are secured debts and that warrants are used as the call options ranking equally as the bond issues the above problem can somewhat be resolved. Warrants are discussed in more detail on page 53.

2. There are Treasury default-free zero coupon bonds and a specific credit rated firm's risky zero coupon corporate bonds currently trading in the market. As previously mentioned, for on-the-run Treasury bonds corresponding STRIPS (Separate Trading of Registered Interest and Principal of Securities) can easily be found in the market. If they don't exist one can easily strip the existing on-the-run Treasury to form a portfolio of zeros consisting of the Treasury's periodic coupon payments and the face value of the issue.

With the periodic coupon payments taken as principal amounts the periodic yield can be calculated. Consider a 10-year semi-annual paying Treasury, the resultant yields will be for periods: six months (period 1), one year (period 2), one and half years (period 3)10 years (period 20). This is further simplified by the fact that there exist a large market of Treasury bonds with very high liquidity profile and thus readily available bond prices. Unfortunately, the same cannot be said of corporate bonds let alone zero coupon corporate bonds due to their nature as explained previously.

It is important that the Treasury zeros and the corporate zeros (referred to as zeros henceforth) have the same profile in terms of duration, term to maturity, and face value amount. These are vital requirements as they affect not only the prices of the zeros but their yield to maturity. Consequently, the accuracy of the yield spread between the two instruments is affected. An accurate yield spread will represent the precise difference in the credit worthiness of the Treasury zero and the corporate zero and the payoff of holding the risky corporate zero over the Treasury zero.

What is interesting, as depicted in Appendix A.3 is the difference that exists for zeros issued by issuers in different market sectors. According to the graphs in Appendix A.3, the corporate spreads for Utilities, for example, are different to the spreads for the Banking sector yet the bond issues have the same credit rating and maturity. Another interesting observation is that for a corporate zero with a particular maturity, the corporate spreads for the different sectors diverge as we move from the investment grade issues (Aaa) towards the

speculative issues (Caa). In other words, for a particular credit rating the corresponding corporate spreads are different from one sector to another. This is probably so because the different sector issues use different Treasury benchmarks. Therefore one needs to be careful on the benchmark used.

3. *A triple-A European option is assumed to be a default free option.* Considering that in the market a triple-A option is the highest credit rated option that exists this assumption should have no impact on the model. After all, the calculated/ expected prices for the risky option from the model may or in fact, will not exist. Therefore holding the triple-A options, which are readily available, and the risky bond will create a portfolio that mimics the payoff of the expected risky option.

In brief, the triple-A option is the ideal risk-free option and furthermore if it has a risk-free asset as the underlying it becomes a risk free vulnerable option. The risk free underlying asset could be an interest rate index, a Treasury security etc. What are the chances of getting the above ideal option whose maturity, duration matches that of the risky and risk free zeros the other inputs in the model? The market seems to suggest that the chances of getting such an ideal option are rare, if at all there is ever a chance. One way to overcome that would be to use warrants in place of normal European options. They are readily available compared to normal European options and thus will be easier to find and use. Ignoring the difference in maturity and duration between the zeros and the warrants they can be used as inputs for the models above using time-series data.

Warrants are special call options differing from the ordinary option in terms of their embedded relationship between writer and the underlying stock. A warrant is a call option written by a firm with the firm's own common stock as the underlying asset. This allows the assumption that the credit rating of the warrant will be the same as the credit rating of the issuing firm. Hence, they can be valued as vulnerable options. They are often issued as attachments to notes, bonds or preferred stocks, or in conjunction with common stock offerings.

There are two main warrant features that clearly distinguish them from normal traded options,

- they have a longer life, five to ten years at issue compared to traded call options that have anything from one month to two years,
- on their exercise the writing firm must issue new shares, resulting in a holdings dilution for existing shareholders and cash inflow to the firm.

Unlike, with ordinary call options there is no cash inflow or shareholding dilution to the firm because the contract is between investors with the underlying asset/firm not taking part in the contract. Dilution occurs if for example, the initial shares outstanding was 50 and upon a warrant's exercise the writing firm issued 15 new shares, new outstanding shares would then be 65. In this case, initially the existing shareholders had $1/50$ claim on the firm, now after issuing new shares they own only $1/65$ of the revised company shareholdings.

When warrants are issued as attachments to other security issues they act as 'sweeteners', they provide the issuing firm with a cheaper source of capital or in other cases they solve a problem that normal debt/equity issue alone cannot solve. They are separated and traded on their own as soon as the security to which they will attach to is issued. They are more liquid compared to call options; because call options are short term investors consider them as 'speculative securities' thus they opt to buy more of the long-term issues.

As warrants are long term call options, they can be valued using the Black-Scholes model allowing for a few alterations to the inputs. Dividends are deducted, that is, the usual S in the model is replaced by $S \exp(-\delta t)$. The expected average long-term volatility is used to compensate the long warrant life. As previously mentioned, warrant exercise results in share dilution, that is, more shares are issued and the share price falls. Galai and Schneller (1978) showed that the resultant fall in share price when a warrant is exercised should be taken into account when the warrant is valued. In short, Schulz and Trautmann (1990) showed that valuing a warrant as a long-term call option, based upon the share price (modified for dividends) and totally ignoring dilution when using Black-Scholes model is not totally inaccurate. The degree of error is only large if the warrant is both close to maturity (two

years or less) and not yet in-the-money. Not surprisingly, practitioners currently value warrants that way.

To tie in with the HW model assumption that the derivative used/calculated always has positive value, warrants never have negative values. Therefore their exposure is always the no-default value at any time during the warrant's trading life. A warrant is always worth at least, the value of the common stock minus the warrant's exercise price, its intrinsic value. At expiration, the warrant is either equal to its intrinsic value or zero (when it has no positive value to the holder, no exercise occurs).

4. The default-free bond price process is assumed to solely depend on the spot interest rate.

There is a negative or inverse relationship between bond prices and interest rates. That is, bond prices decrease as market interest rates rise and vice-versa. Therefore the only variable that causes a change in price and the direction of the change is the spot rate (the current or prevailing short-term interest rate). The magnitude and direction of the change depends heavily on the size of interest rate movement, that is, a movement of 100 (1%) basis points will result in a different magnitude in the bond price change compared to a movement of 300 (3%) basis points.

2.5.1 Results: Hull & White model

The results of using the HW model presented below are also broken into different parts according to their bond and option inputs as explained earlier for the JT model. For full details of the results represented by the figures below refer to Appendix HW1-8 correspondingly.

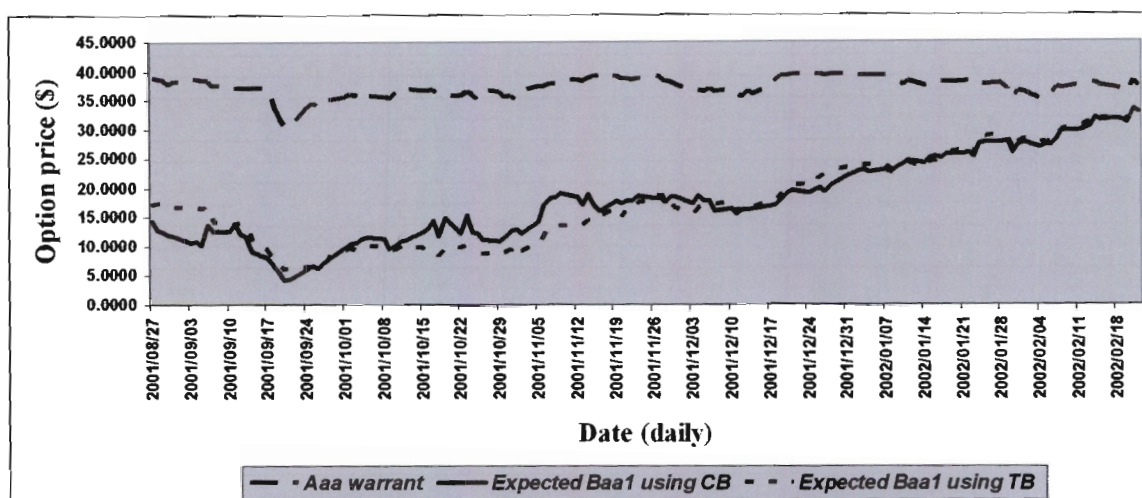


Figure HW.1: The expected Baa1 option prices using zero-coupon bonds, a triple-A warrant and the HW model. This figure shows the expected Baa1 option prices obtained when using a triple-A warrant as the risk free option, using the Treasury zero coupon bond as well as the triple-A zero coupon corporate bond as the risk free debt over six months.

As observed on the JT model results for the same bond and option inputs, the triple-A warrant prices are higher than the expected Baa1 option prices obtained from the HW model using both the triple-A corporate bond and the Treasury bond. The relationship between the two expected Baa1 options prices is mixed, with the expected Baa1 option prices using the triple-A bond being higher than the expected Baa1 option prices using the Treasury bond. Suggesting that at those times the triple-A corporate bond was less risk compared to the Treasury bond.

For the coupon bond inputs and the triple-A warrant the results confirmed the relationship between the triple-A warrant prices and the expected Baa1 option prices. Of note was that the two expected Baa1 option prices were the same suggesting that the two coupon bonds used had the same credit worthiness as shown below.

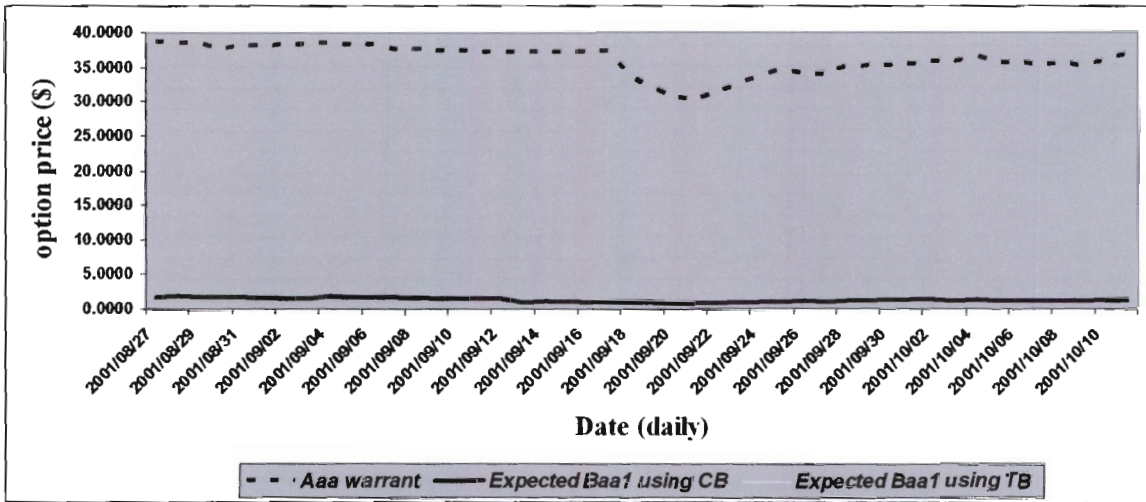


Figure HW.2: The expected Baa1 option prices using coupon bonds, a triple-A warrant and the HW model.

The expected Baa1 option prices using zero-coupon Treasury bond and zero-coupon Baa1 corporate bond as well as using the 30-year Treasury bond as the risk free rate in the Black model were lower than the corresponding risk free option prices. The two prices converged at the end of the six-month term because the exercise value of the two options was the same at expiration.

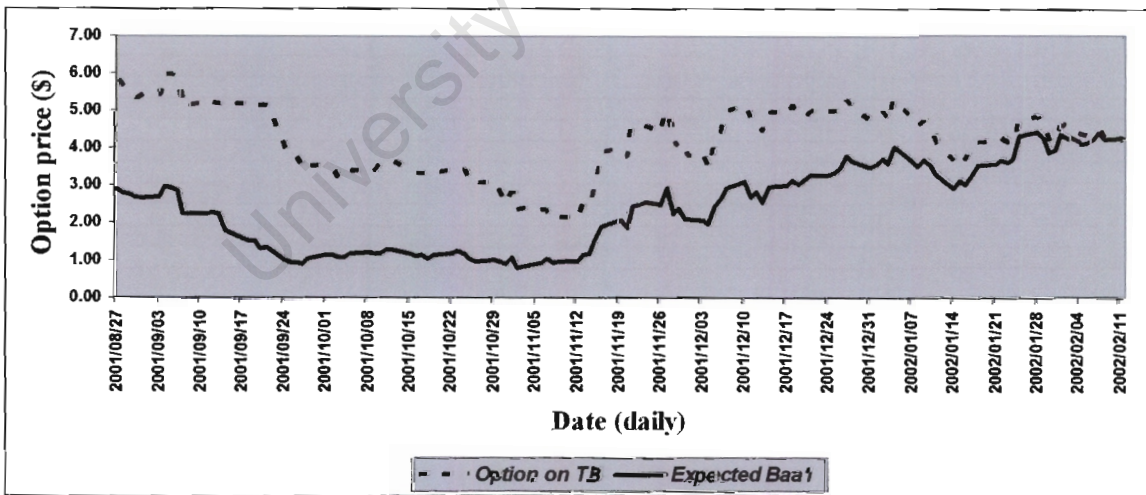


Figure HW.3: The expected Baa1 option prices using zero-coupon bonds, bond options using the 30-year Treasury bond yield and the HW model. The figure shows the expected Baa1 option prices obtained from the HW model using as inputs the six month bond options calculated using the Black model together with zero coupon bonds for the US Treasury bond and the Baa1 corporate bond. The Black model used the 30-year Treasury bond yield as the risk free rate in determining the discount rate used to calculate the six month bond options.

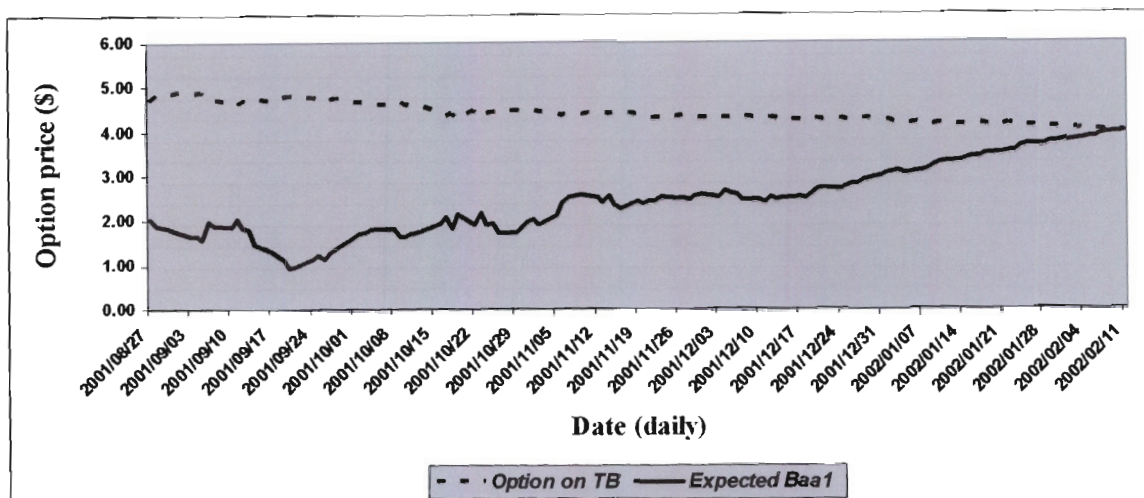


Figure HW.4: The expected Baa1 option prices using coupon bonds, bond options using the 30-year Treasury bond yield and the HW model. This shows similar results to figure HW.3, with a convergence in the two option prices at maturity. The Black model used the 30-year Treasury bond yield as the risk free rate in determining the discount rate used to calculate the six month bond options.

Figure HW.5 and HW.6 below are derived as the above two figures respectively, except that the discount rate used in the Black model is the sum of the average risk premium of the corporate bonds and the 6 month Treasury bill rate. As in the case of the JT model, the results obtained are exactly the same as those obtained when the Black model used the 30-year Treasury bond yield as the risk free rate. This is shown below.

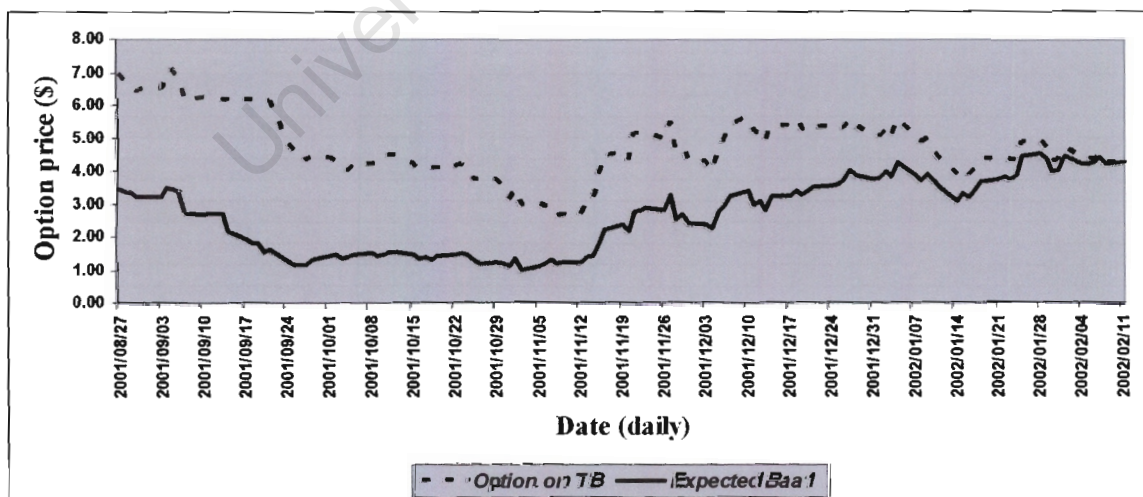


Figure HW.5: The expected Baa1 option prices using zero-coupon bonds, bond options using the 6-month Treasury bill rate and the HW model. The Black model bond options are calculated by using the 6-month Treasury bill rate as the risk free rate in determining the discount rate to be used.

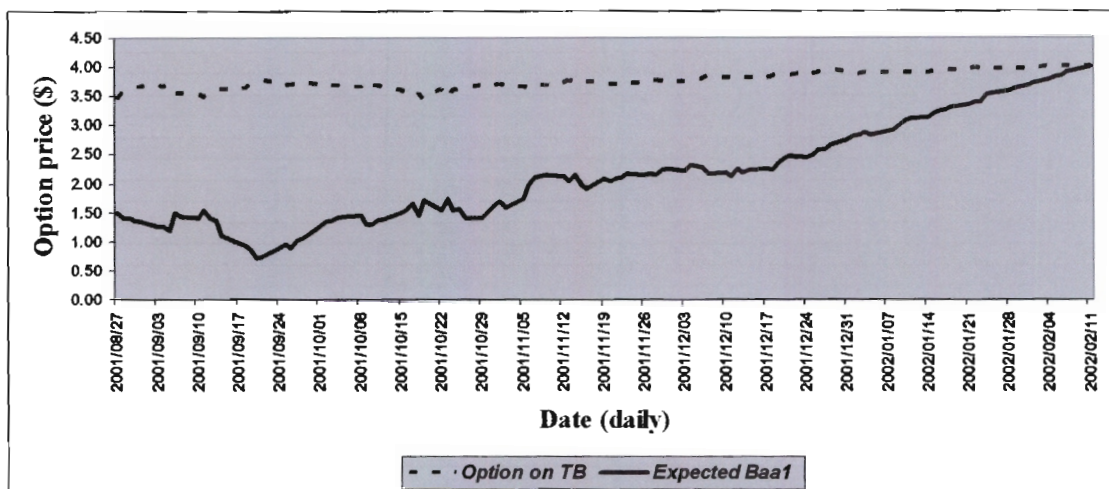


Figure HW.6: The expected Baa1 option prices using coupon bonds, bond options using the 6-month Treasury bill rate and the HW model. The Black model bond options are calculated by using the 6-month Treasury bill rate as the risk free rate in determining the discount rate to be used.

Likewise, when we used the 10-year Treasury note yield or the 6-month Treasury bill rate, the results were the same for the HW model as for the JT model. There was no significant difference between the risk free option and the risky option prices. This indifference means that the payoff of the Baa1 corporate bond used in the model was close to 1, or the bond itself was close to being a risk free issue as shown in the two figures below.

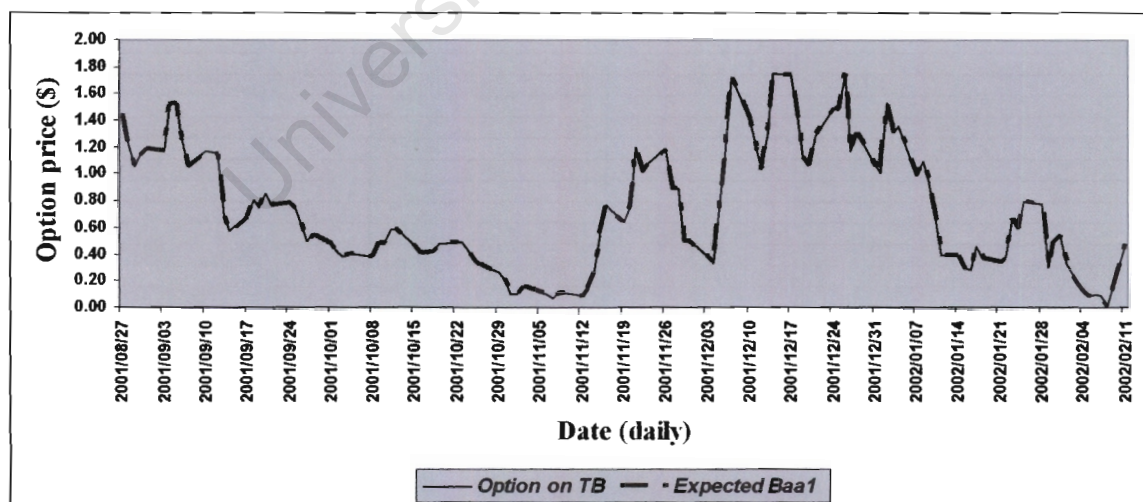


Figure HW.7: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 10-year Treasury note yield, and the HW model. The expected Baa1 option prices observed are equal to the risk free option (option TB calculated using the Black model).

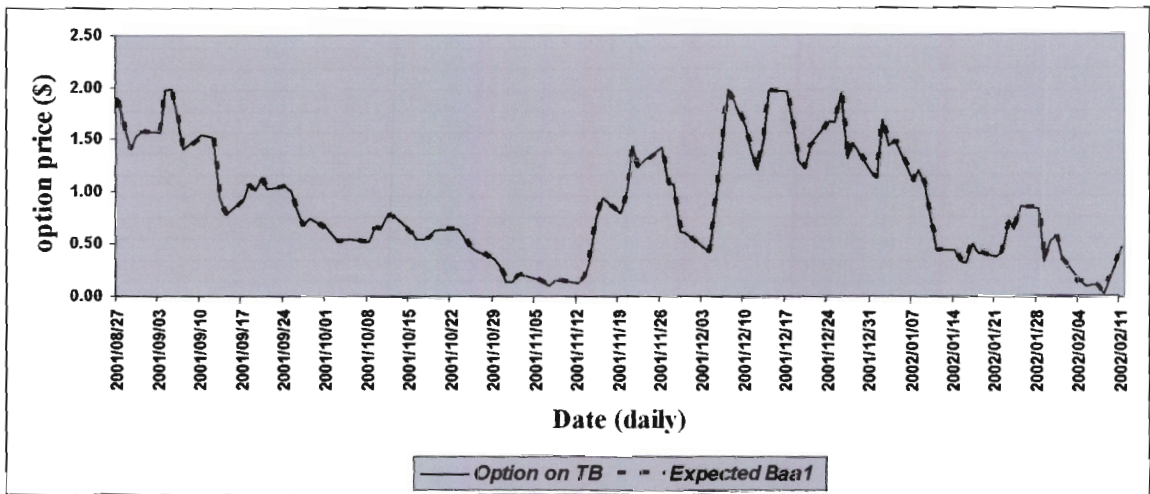


Figure HW.8: The expected Baa1 option prices using 10-year coupon bonds, bond options using the 6-month Treasury bill rate and the HW model. As in figure HW.7 the two option prices observed are equal suggesting that the Treasury bond and the Baa1 bond have the same credit worthiness.

University of Cape Town

3. COMPARISON OF THE TWO MODELS AND CONCLUSION

3.1 Introduction

The individual model results discussed above are combined to facilitate a comparison between the Jarrow & Turnbull and the Hull & White model results when used with the same bond inputs and option inputs. The results are shown diagrammatically with a brief summary on the inputs used to obtain the results. This is followed by an analysis on the model results. The analysis model used for the comparison is the volatility analysis. The volatility analysis is carried out by constructing a portfolio consisting of the “expected options” together with the actual warrants and in the other case, the Black model expected bond option prices. The section ends with a conclusion deduced from the model results and a summary of the whole paper.

3.2 Results

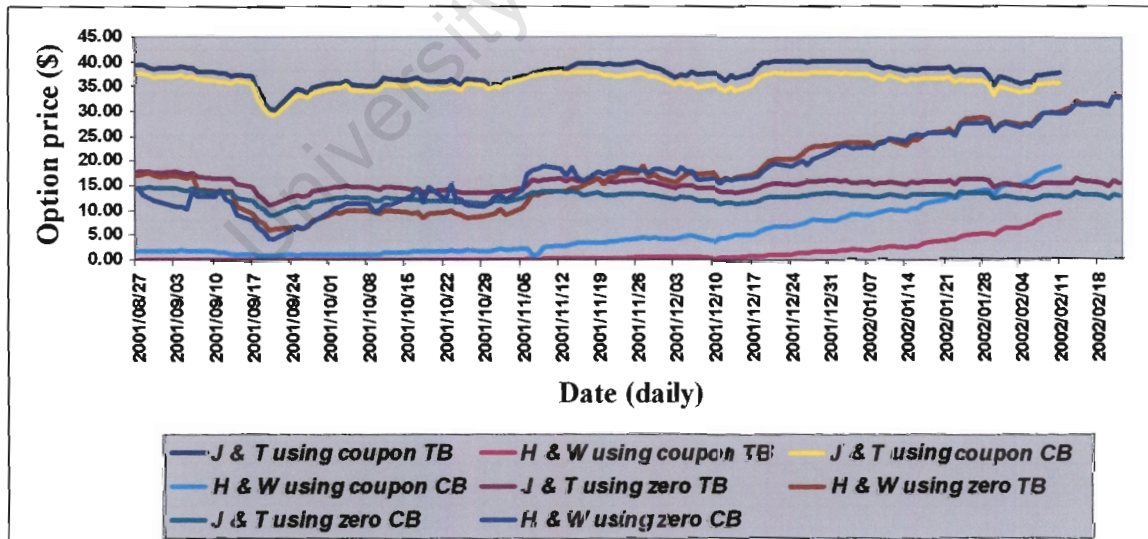


Figure R.1: Comparison of the expected Baa1 option prices obtained using the JT model vs. the HW model, with zero coupon bonds, or coupon bonds, and the triple-A warrant as inputs. For all the different combination of inputs the two models do not produce any expected Baa1 option prices that are equal.

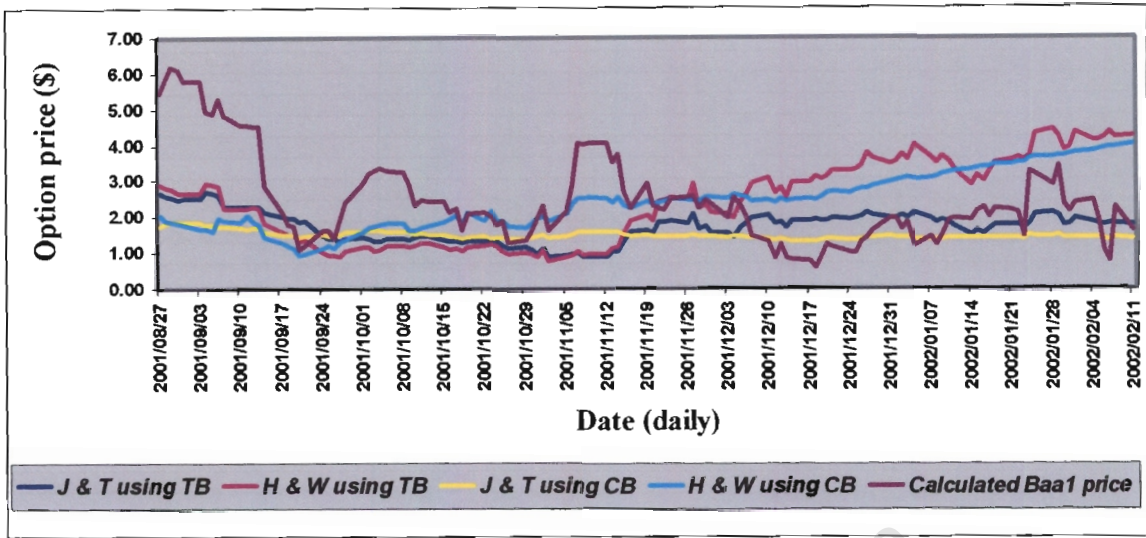


Figure R.2: Comparison of the expected Baa1 option prices using the JT model vs. the HW model, with zero coupon bonds, the bond options from Black model using the 30-year Treasury bond yield as the risk free rate. The calculated Baa1 price is from the Black model with the 30-year Treasury bond yield as the risk free rate. The two models do not produce the same expected Baa1 option prices when using both the Treasury bond and the triple-A corporate bond.

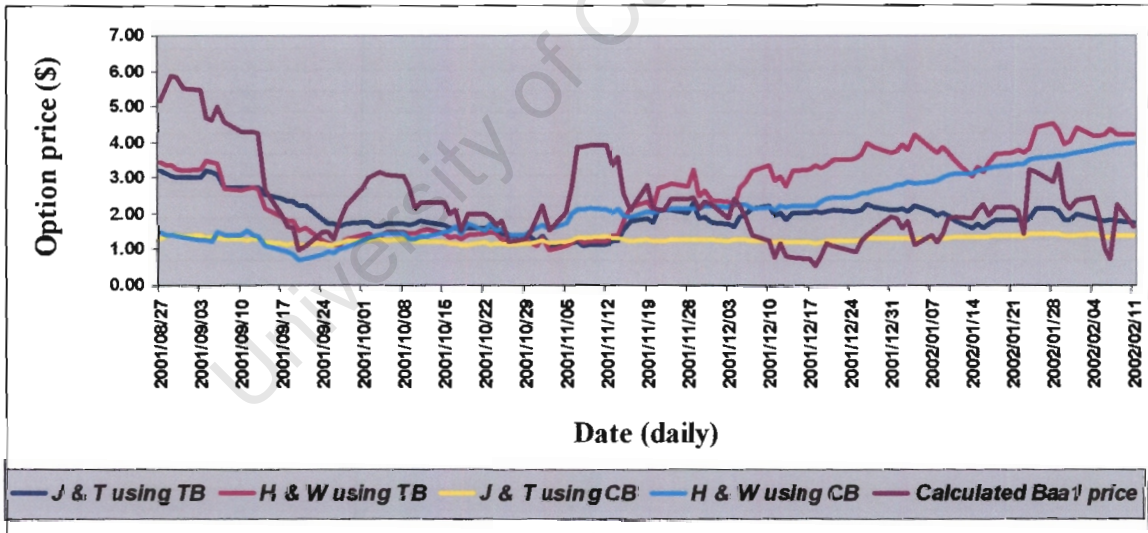


Figure R.3: Comparison of the expected Baa1 option prices using the JT model vs. the HW model, with zero coupon bonds, the bond options from Black model using the 6-month Treasury bill rate as the risk free rate. The calculated Baa1 price is from the Black model with the 6-month Treasury bill rate as the risk free rate. The expected Baa1 option prices are different for the two models.

All three figures show clearly that the JT model and the HW model produce expected Baa1 options that are significantly different. Indeed this is a concerning result as one would expect

them to be the same or at most slightly different given that the same inputs are used in either model. The probable reason for such big differences between the two models' results lies in the degree to which each model is sensitive to the adjustments and Black model calculations carried out on the inputs.

In spite of the above results, different results were observed when the 10 year coupon bond inputs were used. The risk free option (option on the 10-year Treasury bond) was calculated from the Black model using the 10-year Treasury bond yield or the 6-month Treasury bill rate as the risk free rate. The results shown in figure below show that the expected Baa1 option prices for both the JT model and the HW model were considerably the same but significantly different from the calculated Baa1 using the Black model.

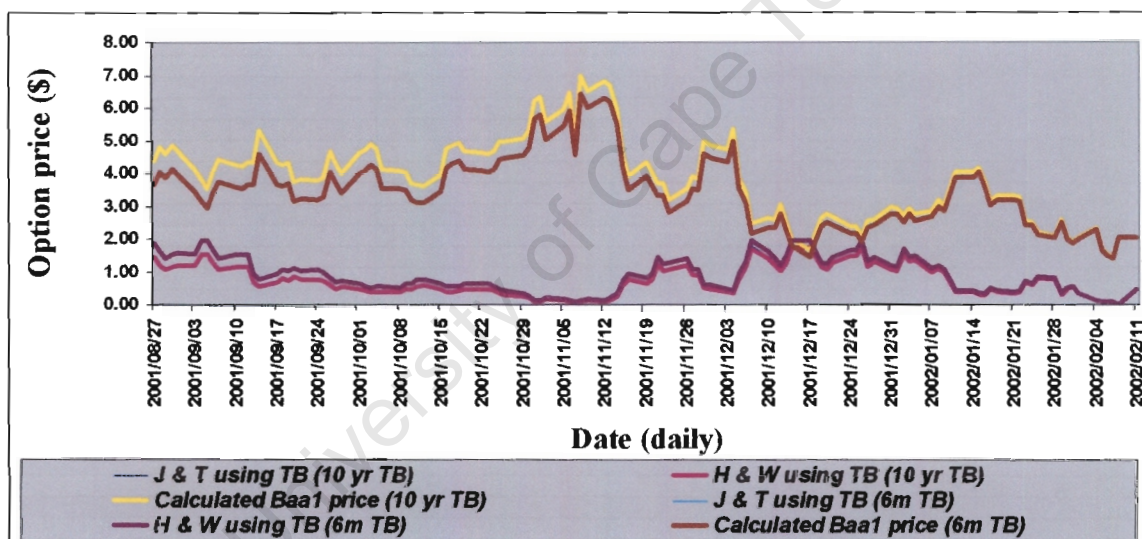


Figure R.4: Comparison of the expected Baa1 option prices using the JT model vs. the HW model, with 10-year coupon bonds, the bond options from Black model using the 30-year Treasury bond yield and the 6-month Treasury bill rate as the risk free rate. The calculated Baa1 price is also from the Black model with the 30-year Treasury bond yield and the 6-month Treasury bill rate as the risk free rate.

For in-depth comparison of the above results we explored the use of volatility analysis. This analysis was carried out to see whether there was any meaningful relationship in the price movement (price change) in the calculated and expected Baa1 options. Any such meaningful relationship would help ascertain that one could still price and hedge their security holdings using the two models (the JT model and the HW model).

3.3 Volatility

Volatility, σ , is the measure of the uncertainty of the return realised on an asset. It can also be defined as the standard deviation of the asset's return in one year when the return is continuously compounded. Note that it is also the standard deviation of the natural logarithm of the asset's price at the end of one year. Using historical price data one can produce estimates of current levels of volatilities and correlations, as well as forecasts of the future prices. The volatilities are modelled using the Value-at-risk, VaR, which is defined as the minimum expected loss with a 1% confidence level for a given time horizon (usually 1 or 10 days) calculation process (Hull 2000).

3.3.1 Estimating volatility using historical data

Observing the asset's price daily define:

- $n + 1$ as the number of observations
- S_i as the asset price at the end of i th interval ($i = 0, 1, 2, \dots, n$), where the intervals of time in our case is daily closing prices.
- τ as the length of time interval in years

Let,

$$u_i = \ln (S_i / S_{i-1}) \quad (1)$$

for $i = 1, 2, 3, \dots, n$.

That is u_i is the continuously compounded return in the i th interval. Using s , the standard deviation of the u_i 's which is given by $\sigma / (\tau)^{0.5}$ it follows that given s and τ , σ , the volatility per annum of the asset can be easily estimated. Empirical research shows that only trading days are used in volatility calculations (Fama: 1965 and French: 1980). That is, non-trading days are ignored and thus a year has 252 days (trading days). In words, the volatility σ per annum is the product of the volatility per trading day s and the square root of the number of trading days in year.

For daily volatilities, let σ_n be the volatility of a market variable (e.g option price, bond price) on day n , estimated at $n-1$, and let σ_n^2 be the variance rate on day n .

It follows that,

$$\sigma_n^2 = (1/m - 1)(\sum (u_{n-i} - \bar{u})^2) \quad (2)$$

\bar{u} is the average of the u_i 's and $(\sum (u_{n-i} - \bar{u})^2)$ is the sum of all deviation squared of m observed daily returns from their mean.

As mentioned earlier, to analyse our model predicted option prices versus the actual option market prices we aim to look at the relationship of their volatilities over time. Assuming we hold a hypothetical portfolio consisting of an option whose prices are model-determined and another option whose prices are the actual market prices (Black-Scholes prices) one can use the volatility modelling used for VaR calculations. Hence the above equation is adjusted to accommodate VaR calculation,

$$\sigma_n^2 = (1/m)(\sum u_{n-i}^2) \quad (3)$$

u_i is now the proportionate change in the asset price, the mean of u_i 's is assumed to be equal to zero and $m - 1$ is replaced by m and all u_i^2 's are given equal weightings (Hull: 2000).

3.3.2 GARCH (1, 1) Model

$$\sigma_n^2 = \gamma V + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (4)$$

σ_n^2 is calculated from a long run average variance rate, V , with a weighting γ , u_{n-1}^2 , with weighting α , and σ_{n-1}^2 , with a weighting β . The sum of the weightings is equal to one. In words, σ_n^2 is determined by the recent u^2 's and the most recent variance rate estimate.

Now let ω be equal to the long run average variance rate, V , weighted by γ .

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (5)$$

To calculate or to estimate the parameters, ω , α , and β the *Maximum Likelihood Method* is used. It estimates the parameter values that maximises the likelihood of the data occurring. Letting $v_i = \sigma_i^2$ be the variance rate for day i , estimating the parameters iteratively requires maximising the expression

$$\sum [-\ln (v_i) - (u_i^2 / v_i)] \quad (5)$$

where i , ranges from 1,2,3,....., m . This is known as the *Likelihood measure*.

Using Solver in Microsoft Excel to estimate the optimal parameters in the GARCH (1, 1) by maximising the above expression the following constraints/conditions for the parameters are used. The long run variance rate, V , is equal to the sample variance (*Variance Targeting*).

From variance targeting, $\omega = \gamma V = V(1 - \alpha - \beta)$

$$\gamma + \alpha + \beta = 1 \text{ or } (V/\omega) + \alpha + \beta = 1$$

$$\alpha + \beta \leq 1$$

$$\alpha \leq 1 - \beta, \beta \leq 1 - \alpha$$

$$\gamma, \alpha, \beta \geq 0$$

$$\gamma, \alpha, \beta \leq 1$$

Please note that only parameters α, β are estimated using the above constraints. Using the values of $v_i = \sigma_i^2$, variance rate for day i , resulting from the optimal parameters the daily volatility is computed as the square root of the variance rate for that particular day.

In both Figure G.1 zeros and Figure G.1 coupons, the volatilities of the expected Baa1 option prices using both the JT and HW model as well as using the Treasury bond and the triple-A corporate bond as the reference bonds and a triple-A warrant as the risk free option are shown below. For full details of Figure G.1 zeros see Appendix GARCH (1,1) zeros that shows the volatility calculations for the expected Baa1 options prices obtained using HW model and the triple-A zero coupon corporate bond as the risk free bond. Likewise, for the full details pertaining to Figure G.1 coupons see Appendix GARCH (1,1) coupons that show the volatility calculations for the expected Baa1 option prices using the JT model and using the triple-A coupon corporate bond as the reference bond.

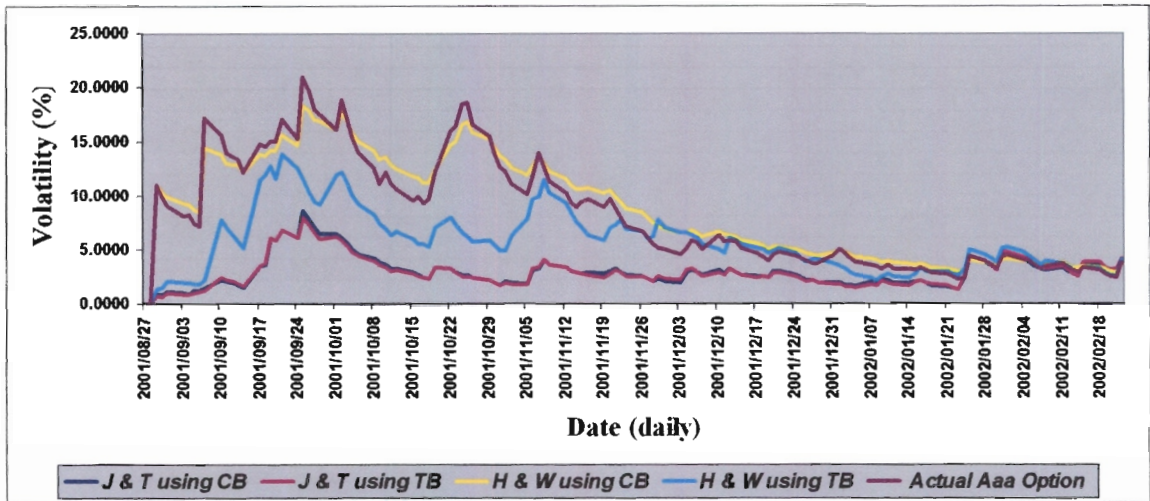


Figure G.1a zeros: The daily volatilities of the HW model and the JT model prices using zero coupon Treasury bond and zero coupon corporate bond. The figure is a Line Chart that displays the volatility trend over time. The figure shows that the peaks and troughs of the daily volatilities occur at the same time periods but differ in magnitude for the two models.

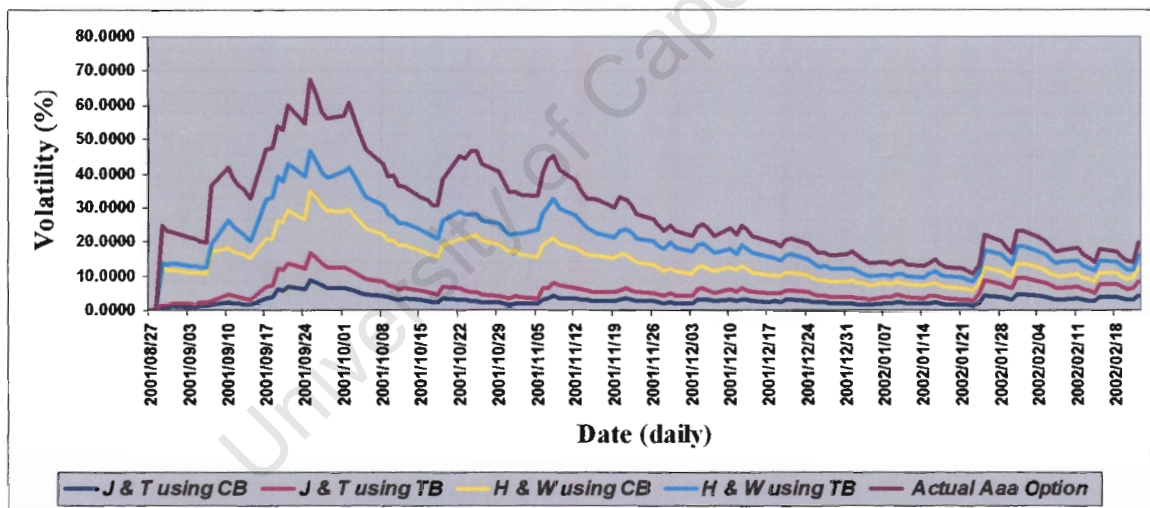


Figure G.1b zeros: The daily volatilities of the HW model and the JT model prices using zero coupon Treasury bond and zero coupon corporate bond. The figure above is a Stacked Line Chart that is displaying the volatility trend of the contribution of each daily volatility value over time.

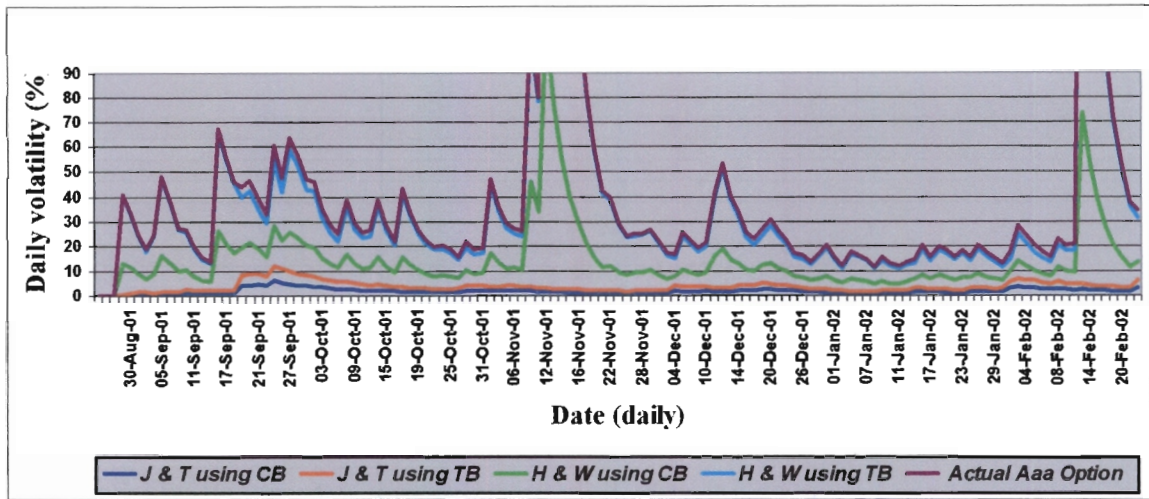


Figure G.1a coupons: The daily volatilities of the HW model and the JT model prices using coupon Treasury bond and coupon corporate bond. This figure is a Line Chart that displays the volatility trend over time.

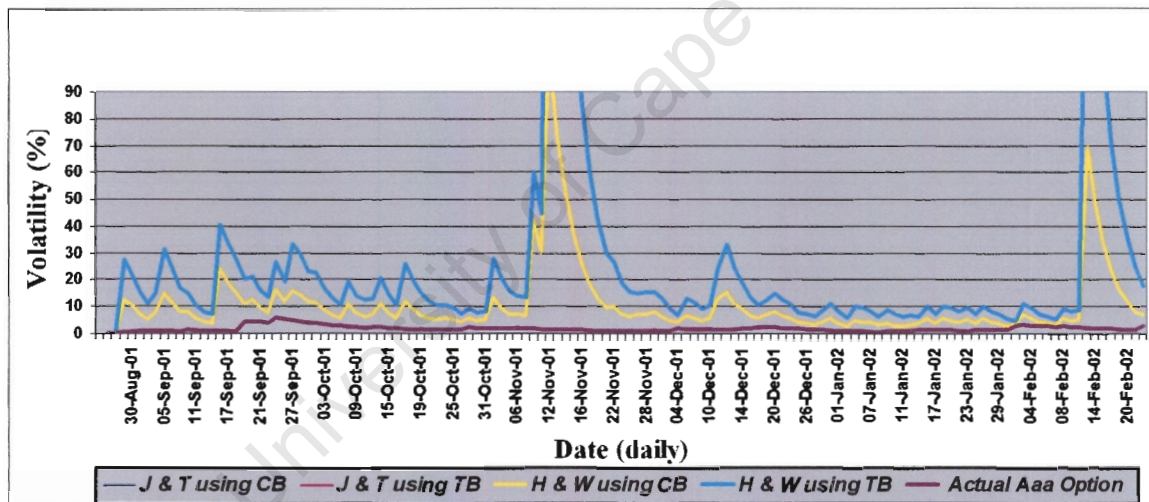


Figure G.1b coupons: The daily volatilities of the HW model and the JT model prices using coupon Treasury bond and coupon corporate bond. This figure is a Line Chart that displays the volatility trend over time.

In Figure G.1 coupons and Figure G.1 zeros, all the volatilities of the model prices have their peaks and troughs exactly like the triple-A option with the difference only appearing in the magnitude of those troughs and peaks. This should be expected considering that the models are deterministic and therefore greatly dependent on the reference risk free option used. The peaks and troughs are as a result of the payoff ratio of the risky debt to the risk free bond

used. For each of the above figures, the corresponding Appendix for their GARCH model volatilities is presented with the answer report, sensitivity report and a limits report. These reports are just a summary of the parameters and their characteristics used in the iterative process to calculate or model the daily volatilities.

3.4 Conclusion

From the results shown above, clearly the two models failed to price the expected risky option price given the bond inputs and the risk free option or warrant. This is greatly attributed to the characteristics or nature of these vital inputs used. A brief statistical look at the bond inputs used confirmed that the predictive failure of the two models was indeed as a result of the bond inputs' nature (See table 2 & 3 below).

In Table 2 & 3 the summary shows that the bond prices and yields used did not follow a normal distribution series, mainly because the skewness and kurtosis needed for a normal distribution were not achieved by any of these bond inputs.⁸ The bond options calculated using the Black model requires the bond input prices to be normally distributed and also the two models JT and HW assume normal distribution in the bond prices and the option prices used and obtained. The reason being that if you use normally distributed variables the resultant product will also assume normal distribution.

In other words, because of the non-normality in the bond inputs used, the bond option prices calculated using the Black model would be inaccurate. Consequently, explaining the mixed results observed in the expected Baa1 option prices from the JT and the HW model.

⁸ Normal distribution series requires that the prices exhibit a skewness of 0 and a kurtosis of 3. The former meaning that the distribution is symmetrical about the mean and the latter means the distribution has a proportionate peak and the values are not concentrated about the mean.

Henceforth, it was concluded that for the JT and HW models to work, or accurately predict or price the risky options the following conditions must be met.

These inputs must

- be normally distributed
- have corresponding maturities as well as the other bond features such as the coupon size.
- be of issues in the same sector, as shown by the Reuters tables that the risk premium required for bond issues of the same rating but from different sectors are different.
- on-the-run Treasury issues must be used because they provide the most liquidity (they are highly traded); therefore they are the best risk free bonds to use.
- as previously mentioned, it is difficult to get such perfectly corresponding bond issues and options to use in the models in order to predict a risky option's price. Therefore it will be necessary and important to investigate methods that can be used in practise to adjust the imperfectly corresponding inputs so as to be able to use the resultants inputs in the models.

University of Cape Town

3.4.1 Bond inputs summary

A statistical summary was done for the normal bond price and yield, and their corresponding log values. Same results are observed for the log of the prices and yields as in the normal case.

Table 2: The statistical summary for coupon bonds for both their normal prices and their log prices
Descriptive Statistics (Normal market prices)

	BAA1	BAA1Y	TB	TBY
Mean	102.63170	6.37103	101.38250	4.81611
Median	102.35000	6.40900	100.81250	4.88600
Max	105.88000	6.70600	106.21880	5.21300
Min	100.29000	5.92600	98.46875	4.18200
SD	1.10200	0.15821	1.87293	0.24879
Skewness	0.97374	-0.98171	0.66114	-0.61931
Kurtosis	3.78258	3.72438	2.69332	2.64118
Jarque-Bera Prob	22.20895 0.00002	22.08137 0.00002	9.28910 0.00961	8.38385 0.01512
Sum	12418.44000	770.89500	12267.28000	582.74900
Sum sq Dev	145.72890	3.00346	420.94540	7.42753
Observations	121	121	121	121

Descriptive Statistics (Log prices)

	Ln BAA1	LN BAA1Y	Ln TB	Ln TBY
Mean	4.63109	1.85145	4.61873	1.57061
Median	4.62840	1.85770	4.61326	1.58637
Max	4.66231	1.90300	4.66550	1.65116
Min	4.60807	1.77935	4.58974	1.43079
SD	0.01068	0.02516	0.01837	0.05263
Skewness	0.94447	-1.04778	0.62688	-0.71996
Kurtosis	3.72570	3.85484	2.63646	2.81639
Jarque-Bera Prob	20.64425 0.00003	25.82410 0.00000	8.59131 0.01363	10.62315 0.00493
Sum	560.36190	224.02550	558.86660	190.04380
Sum sq Dev	0.01370	0.07594	0.04049	0.33241
Observations	121	121	121	121

Table 3: The statistical summary for zero-coupon bonds for both their normal prices and their log prices

<u>Descriptive Statistics (Normal market prices)</u>						
	AAA	AAAY	BAA1	BAA1Y	TB	TBY
Mean	98.48131	2.11399	33.61028	3.20719	82.12702	2.08773
Median	98.43800	2.10810	33.62900	3.19790	82.29600	2.07230
Max	99.00300	2.44250	37.51700	3.59270	84.36200	2.36090
Min	97.90400	1.92080	29.30100	2.85910	79.65600	1.79580
SD	0.29492	0.09401	1.79044	0.15780	1.05585	0.12816
Skewness	0.16068	0.86118	0.00265	-0.01004	-0.31530	-0.13176
Kurtosis	1.80579	4.38713	2.47119	2.57745	2.90074	2.64783
Jarque-Bera	7.71082	24.65691	1.41001	0.90221	2.05447	0.97543
Prob	0.02117	0.00000	0.49411	0.63692	0.35800	0.97543
Sum	11916.24000	255.79310	4066.84400	388.06990	9937.37000	252.61490
Sum sq Dev	10.43704	1.06055	384.68060	2.98824	133.77910	1.97101
Observations	121	121	121	121	121	121
<u>Descriptive Statistics (Log prices)</u>						
	Ln AAA	Ln AAAY	Ln BAA1	Ln BAA1Y	Ln TB	Ln TBY
Mean	4.58986	0.74762	3.51342	1.16419	4.40819	0.73419
Median	4.58943	0.74579	3.51539	1.16249	4.41032	0.72866
Max	4.59515	0.89302	3.62479	1.27890	4.43512	0.85904
Min	4.58399	0.65274	3.37762	1.05051	4.37772	0.58545
SD	0.00299	0.04377	0.05342	0.04934	0.01289	0.06189
Skewness	0.15719	0.69488	-0.11563	-0.12609	-0.34980	-0.28447
Kurtosis	1.80569	3.95930	2.50653	2.59078	2.90942	2.75090
Jarque-Bera	7.68965	14.37737	1.49737	1.16487	2.50903	1.94484
Prob	0.02139	0.00076	0.47299	0.55854	0.28522	0.37817
Sum	555.37330	90.46187	425.12380	140.86700	533.39040	88.83668
Sum sq Dev	0.00108	0.22987	0.34243	0.29214	0.01992	0.45966
Observations	121.00000	121	121	121	121	121

REFERENCES

Allen Jones November 2002, Developing African Bond Markets, and what should be done.

Black, F., 1976, The pricing of Commodity contracts. *Journal of Financial Economics*, 3, 167-79

Fama, E.E., 1965, The behaviour of Stock Market Prices. *Journal of Business*, 38, 34-105

Francis, J. C., Frost, J. A, and Whittaker, J. G., 1999, *The handbook of Credit derivatives*. M^cGraw-Hill.

French, K.R., 1980, Stock Returns and the Weekend Effect. *Journal of Financial Economics*, 8, 55-69

Galai, D. and Schneller, M., 1978, Pricing warrants and the value of the firm. *Journal of Finance*, 33, 1333-42

Hull, J. C. and White, A., 1995, The impact of Default Risk on the Prices of Options and other Derivative Securities. *Journal of Banking and Finance*, 19, 299-322

Hull, J. C. and White, A., September 1992, The Price of Default. *Risk*. 101-103

Hull, J. C., 2000. *Options, Futures, and Other Derivatives* (4th edition). Prentice-Hall International, Inc.

Jarrow, R. A. and Turnbull, S. M., 1991, The Foreign Currency Analogy.

Jarrow, R. A. and Turnbull, S. M., 1995, Pricing Derivatives on Financial Securities Subject to Credit Risk. *Journal of Finance*, 50 (1), 53-85

Jarrow, R.A (Editor), 1995, OVER THE RAINBOW: Developments in Exotic Options and Complex Swaps. Risk Publications.

Jarrow, R.A and Turnbull, S.M, 1999, DERIVATIVES SECURITIES: The Complete Investor's Guide. Thomson Learning Centre.

Lauterbach, B. and Schultz, P., 1990, Pricing warrants: An empirical study of the Black-Scholes model and its alternatives. Journal of Finance, 45 (4), 1181-1209

Moore T. W, 2001. Real Options and Option-embedded Securities. John Wiley and Sons

Schulz, G. and Trautmann, S, 1990, Valuation of Warrants: Theory and empirical tests for warrants written on German stocks. European Finance Association

Tennican, M. L. 1975 Convertible Debentures and Related Securities. Harvard University

Zewuster, M, April 2002 ABN AMRO Economics Department: Brazil: resilient enough to withstand infection by Argentina.

Appendix A.1 is extracted from <http://www.wrhambrecht.com/ind/auctions/ratings.html>, June 2003.

Appendix A.3 the spread tables are extracted from <http://www.bondsonline.com/asp/corp/spreadBank.html>, June 2003.

Appendix A.4 is an extract from <http://www.bondsonline.com/asp/bondpage/zero.html>, The Federal Reserve Bank of New York, June 2003.

Appendix A.1

Moody's Investors Service

Issuer ratings

Foreign currency: Moody's foreign currency ratings are opinions of the ability of entities to honour senior unsecured financial obligations and contracts denominated in foreign currency.

Domestic currency: Moody's ratings are opinions of the ability of entities to honour senior unsecured financial obligations and contracts denominated in their domestic currency.

Derivative Product Companies: Issuer ratings that are assigned to derivative product companies are opinions of the financial capacity of an obligor to honour its senior obligations under financial contracts, give the appropriate documentation and authorisations.

Four class ratings as per Moody's Investors Services are explained and examples of firms for each class are also given in table below.

Aaa. The counterparties offer exceptional financial security and have the lowest degree of risk. While the financial strength of these entities may change, such changes as can be visualized are most unlikely to impair the entities' strong position (e.g. KFW International Finance and General Electric).

Baa. The counterparties offer adequate financial security. However, certain protective elements may be lacking or may be characteristically unreliable over any great period of time (e.g. Diamond Shamrock and Ashland Inc).

Ba. The counterparties offer questionable financial security. Often the ability of these entities to meet their contractual obligations may be uncertain and thereby not well safeguarded in the future (e.g. American Standard and Hollinger Pub).

Caa. The counterparties offer very poor financial security. Such counterparties may be in default, or there may be present elements of danger with regard to financial capacity (e.g. Amazon.com Inc and Friede Goldman).

Appendix A.2

Moody's Investors Service, Standard & Poor's, Fitch IBCA, Duff & Phelps

Credit ratings	Moody's	Standard and Poor's	Fitch IBCA, Duff & Phelps
<i>Investment Grade</i>			
Highest quality	Aaa	AAA	AAA
High quality (very strong)	Aa	AA	AA
Upper medium grade (strong)	A	A	A
Medium grade	Baa	BBB	BBB
<i>Speculative bonds</i>			
Lower medium grade (Somewhat speculative)	Ba	BB	BB
Low grade (Speculative)	B	B	B
Poor quality (may default)	Caa	CCC	CCC
Most speculative	Ca	CC	CC
No interest being paid or Bankruptcy petition filed	C	C	C
In default	C	D	D

Appendix A.3

Reuters corporate spreads for Banks

Ratings	Maturity						
	1yr	2yr	3yr	5yr	7yr	10yr	30yr
Aaa/AAA	16	19	30	33	48	60	80
Aa1/AA+	25	34	35	42	58	71	91
Aa2/AA	27	40	42	46	61	73	94
Aa3/AA-	29	43	44	51	65	77	103
A1/A+	53	59	63	66	81	95	118
A2/A	56	62	65	68	82	97	122
A3/A-	60	65	68	72	86	100	123
Baa1/BBB+	73	84	92	97	129	151	177
Baa2/BBB	76	92	100	104	134	157	182
Baa3/BBB-	83	97	105	108	139	164	187
Ba1/BB+	565	575	585	595	615	635	655
Ba2/BB	575	585	595	605	625	645	665
Ba3/BB-	585	595	605	615	635	655	675
B1/B+	735	745	755	785	825	865	915
B2/B	745	755	765	795	835	875	925
B3/B-	755	765	775	808	845	885	935
Caa/CCC	1155	1165	1175	1200	1230	1290	1240

Reuters corporate spreads for Industrials

Ratings	Maturity						
	1yr	2yr	3yr	5yr	7yr	10yr	30yr
Aaa/AAA	10	15	17	20	30	37	52
Aa1/AA+	15	20	26	30	40	47	60
Aa2/AA	20	30	32	35	48	57	65
Aa3/AA-	25	35	37	43	58	67	72
A1/A+	35	45	51	62	76	85	99
A2/A	45	55	65	78	90	105	122
A3/A-	55	70	81	88	109	120	143
Baa1/BBB+	77	92	106	116	129	147	168
Baa2/BBB	97	112	120	132	145	160	183
Baa3/BBB-	117	125	133	145	157	172	215
Ba1/BB+	375	400	355	350	325	300	325
Ba2/BB	400	525	530	470	425	400	375
Ba3/BB-	475	575	580	525	475	450	525
B1/B+	575	650	725	575	525	475	575
B2/B	600	675	725	600	550	550	850
B3/B-	825	925	880	870	825	900	925
Caa/CCC	1650	1725	1680	1530	1425	1500	1650

Reuters corporate spreads for Transportation

Ratings	Maturity						
	1yr	2yr	3yr	5yr	7yr	10yr	30yr
Aaa/AAA	43	53	62	71	85	93	103
Aa1/AA+	47	58	67	81	95	103	118
Aa2/AA	53	63	72	86	110	113	128
Aa3/AA-	58	68	77	96	115	128	143
A1/A+	63	78	87	106	130	143	163
A2/A	68	88	97	121	140	163	183
A3/A-	78	93	107	131	150	178	203
Baa1/BBB+	98	113	127	146	170	193	218
Baa2/BBB	113	128	142	161	185	213	238
Baa3/BBB-	133	148	162	181	205	228	253
Ba1/BB+	650	600	550	500	450	475	425
Ba2/BB	1000	950	900	700	650	750	600
Ba3/BB-	1300	1250	1150	1000	900	900	1000
B1/B+	1500	1400	1300	1200	1100	1200	1250
B2/B	1600	1500	1400	1350	1250	1300	1400
B3/B-	1700	1600	1500	1400	1300	1350	1500
Caa/CCC	2300	2100	1900	1800	1600	1500	1600

Reuters corporate spreads for Utilities

Ratings	Maturity						
	1yr	2yr	3yr	5yr	7yr	10yr	30yr
Aaa/AAA	16	20	20	22	33	39	61
Aa1/AA+	26	29	34	47	47	46	77
Aa2/AA	30	32	36	50	53	64	82
Aa3/AA-	33	38	45	67	63	75	103
A1/A+	36	46	49	72	73	83	101
A2/A	48	59	63	77	77	93	103
A3/A-	53	67	64	89	87	94	131
Baa1/BBB+	62	82	84	106	105	112	136
Baa2/BBB	75	93	98	113	116	137	153
Baa3/BBB-	81	123	107	121	126	146	161
Ba1/BB+	955	997	1010	883	665	695	720
Ba2/BB	965	990	1015	885	710	730	780
Ba3/BB-	975	1000	1020	905	740	785	830
B1/B+	1085	1125	1255	1160	985	1045	1120
B2/B	1215	1240	1305	1210	1020	1145	1170
B3/B-	1290	1330	1410	1310	1170	1245	1320
Caa/CCC	1380	1450	1565	1510	1295	1320	1420

Reuters corporate spreads for Financials

Ratings	Maturity						
	1yr	2yr	3yr	5yr	7yr	10yr	30yr
Aaa/AAA	15	20	23	32	52	70	85
Aa1/AA+	44	45	48	58	78	91	108
Aa2/AA	46	48	50	64	81	94	116
Aa3/AA-	48	52	54	72	85	97	117
A1/A+	85	112	115	120	126	146	162
A2/A	100	120	122	123	132	151	167
A3/A-	105	124	126	126	137	156	170
Baa1/BBB+	118	137	138	147	165	184	202
Baa2/BBB	123	141	146	159	176	190	212
Baa3/BBB-	128	146	151	167	187	210	222
Ba1/BB+	577	587	597	625	645	670	720
Ba2/BB	587	597	607	635	655	680	730
Ba3/BB-	597	607	617	645	665	690	740
B1/B+	740	760	770	795	830	855	905
B2/B	750	770	780	805	850	955	1055
B3/B-	760	780	790	815	880	1105	1205
Caa/CCC	1105	1125	1140	1190	1240	1295	1355

Reuters corporate spreads for 30-year maturities

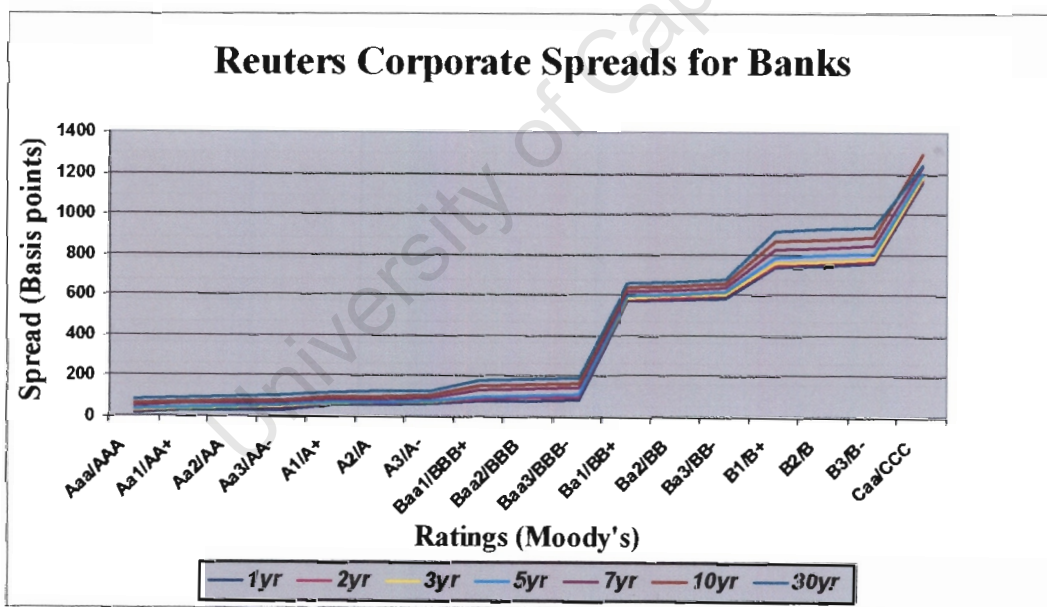
Ratings	30yr Maturity				
	Banks	Industrials	Transportation	Utilities	Financials
Aaa/AAA	80	52	103	61	85
Aa1/AA+	91	60	118	77	108
Aa2/AA	94	65	128	82	116
Aa3/AA-	103	72	143	103	117
A1/A+	118	99	163	101	162
A2/A	122	122	183	103	167
A3/A-	123	143	203	131	170
Baa1/BBB+	177	168	218	136	202
Baa2/BBB	182	183	238	153	212
Baa3/BBB-	187	215	253	161	222
Ba1/BB+	655	325	425	720	720
Ba2/BB	665	375	600	780	730
Ba3/BB-	675	525	1000	830	740
B1/B+	915	575	1250	1120	905
B2/B	925	850	1400	1170	1055
B3/B-	935	925	1500	1320	1205
Caa/CCC	1240	1650	1600	1420	1355

Note: Reuters evaluator spreads for bullet bonds.

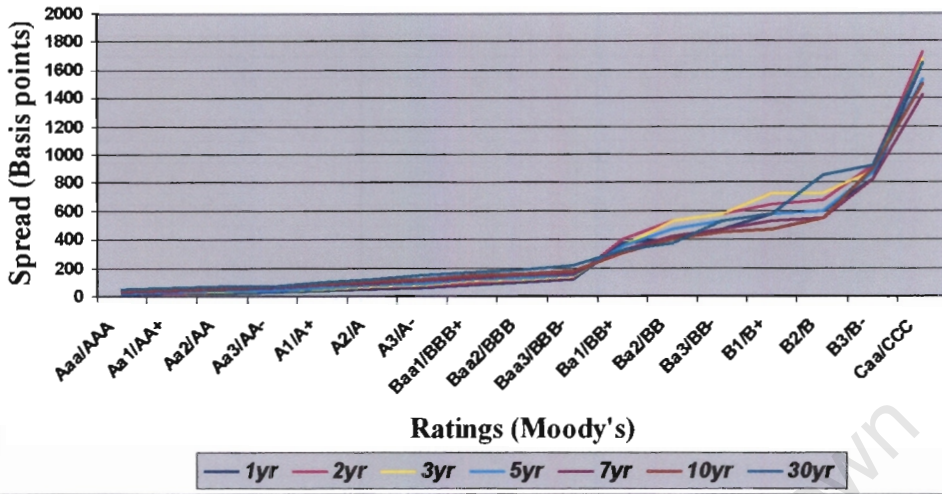
Source: <http://www.bondsonline.com/asp/corp/spreadbank.html>, June 2003.

The first five tables above show the credit spread in basis points for an issue of particular credit rating, with a specific term to maturity and issued in a specific sector. For example, the credit spread for a two year Baa1/BBB+ corporate debt issued in the financial sector is 137 bp (basis points) and for a Thirty year Baa1/BBB+ corporate debt issued in the same sector has 202 bp of credit spread. The last table above was compiled by combining the thirty-year corporate bonds issues from the five sectors (Banks, Utilities, Transportation, Industrials, and Financials) represented above of different credit ratings to give the corporate spreads for thirty-year maturities.

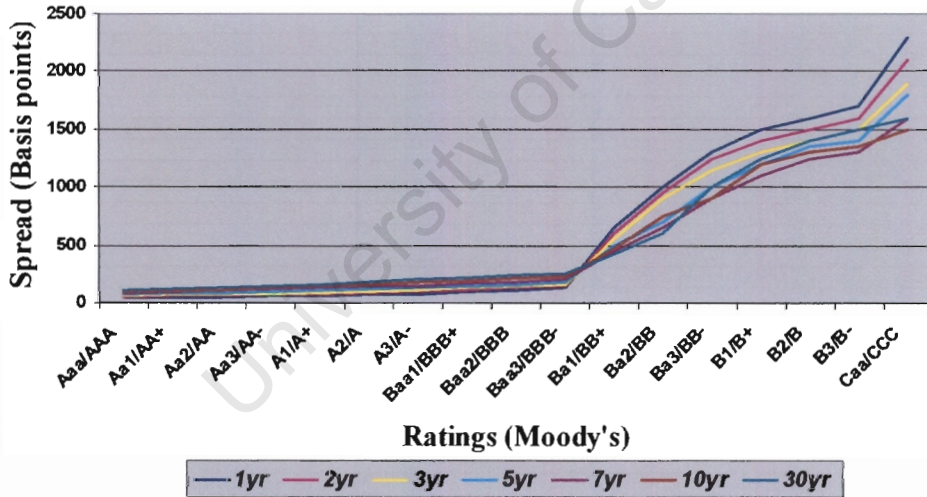
For each of the above tables, a graph is shown below. The graph depicts the corporate spread against credit ratings for specific bond maturities, as shown below. The last graph represents the corporate spread against the credit ratings for thirty-year corporate bonds from the five different sectors.



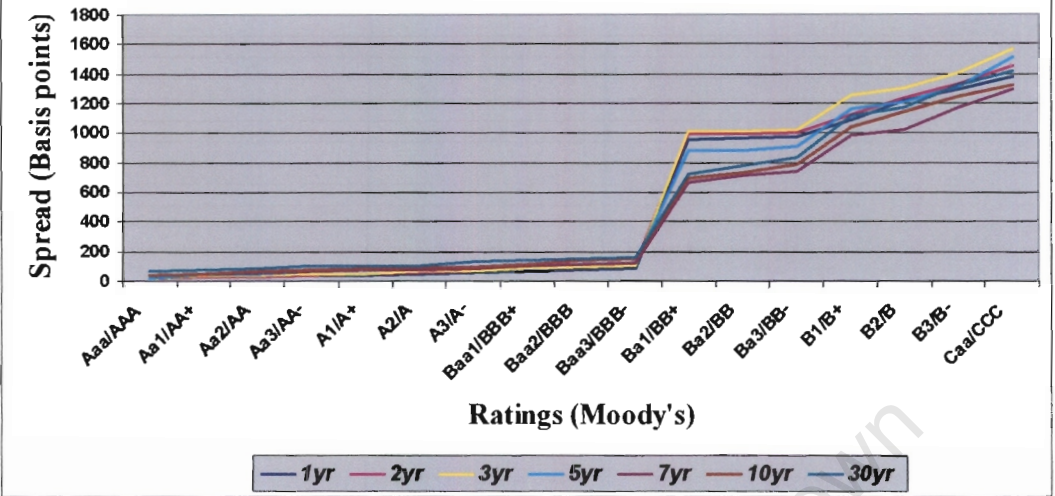
Reuters Corporate Spreads for Industrials



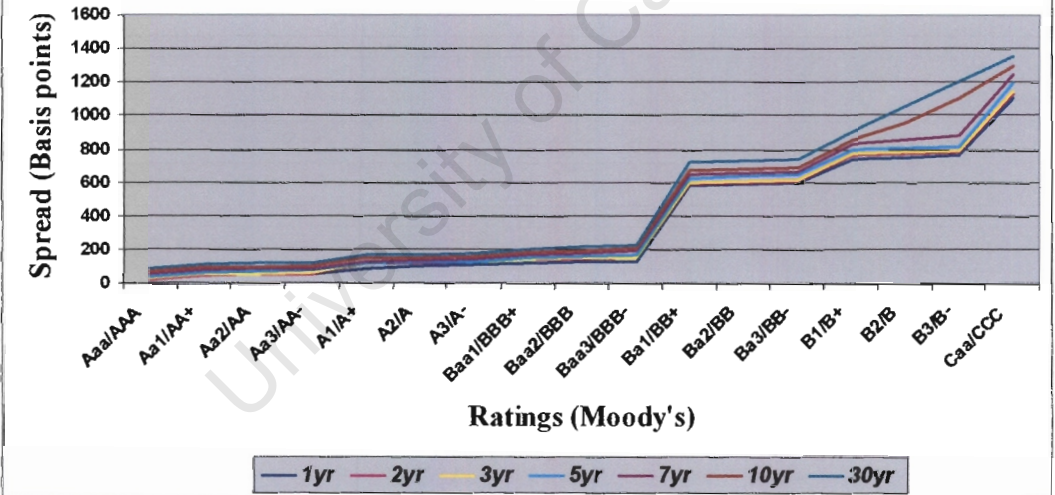
Reuters Corporate Spreads for Transportation



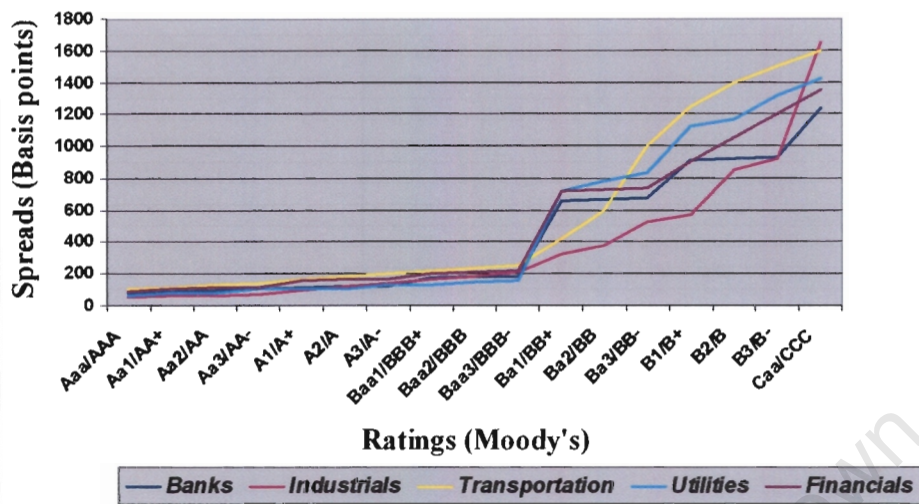
Reuters Corporate Spread for Utilities



Reuters Corporate Spreads for Financials



Reuters Corporate Spreads for 30-year Maturities



University of Cape Town

Appendix A.4

Zero coupon bonds

Below is an expansion on the previous discussion on Zero coupon bonds:

- Zero coupon bonds result from the separation of coupons from the body of a security.
- Zeros sell at deep discounts from face value.
- The difference between the purchase price of the zero and its face value when redeemed is the investor's return.
- Zeros can be purchased from private brokers and dealers, but not from the Federal Reserve or any government agency.

Creating Zeros by Coupon Stripping

Coupon stripping is the act of detaching the interest payment coupons from a note or bond and treating the coupons and the body as separate securities. Each coupon, or interest payment, entitles its owner to a specified cash return on a specific date; the body of the security calls for repayment of the principal amount at maturity. The body of the stripped securities and the separate coupons are known as "zero coupons" or "zeros" because there are no periodic interest payments on each instrument. After stripping, the body and coupons are sold at a deep discount from their face values. An STRIP owner benefits only from the difference between the purchase price and the payment received upon sale or at maturity. For example, a 20-year bond with a face value of \$20,000 and a 10% interest rate could be stripped into its principal and its 40-semi-annual interest payments. The result would be 41 separate zero coupon instruments, each with its own maturity date. The principal would be worth \$20,000 upon maturity, and each interest coupon \$1,000, or one-half the annual interest of 10% on \$20,000. Each of the 41 securities, now possessing a distinct ID number, could be traded separately until its maturity date at prices determined by the market.

Proliferation of Treasury STRIPS

Some Treasury securities were traded in the secondary market without one or more of their interest coupons in the late 1970s. Stripped securities offered investors a financial instrument that had abundant supply, no default risk, and low incidence of being "called," or paid off,

before their maturity date. However, their popularity raised fears within the Treasury Department that zeros would result in a sizable loss of tax revenues. Detached coupons and the body of the security were sold at deep discounts, \$0.05 or \$0.10 on the dollar. After purchase, an investor claimed a capital loss on the difference between the sale price of the security and its face value, thus reducing the investor's overall tax liability. The Tax Equity and Fiscal Responsibility Act (TEFRA) of 1982 adjusted the tax treatment of stripped securities to reduce their tax advantage. The Treasury Department then withdrew its objections to coupon stripping, prompting several securities dealers to create new products incorporating receipts for stripped debt securities. TEFRA also required the Treasury to begin issuing all of its securities in book-entry (electronic) form only from January 1983. This provision eliminated Treasury issues of bearer notes and bonds with coupons attached. Physical stripping would no longer be possible.

In response, bond dealers began to market receipts that evidenced ownership of Treasury zeros held by a custodian. The first of these "receipt products" were named Treasury Investment Growth Receipts, or TIGRS. Similar products appeared in 1984, such as Certificates of Accrual on Treasury Securities (CATS) and Treasury Receipts (TRs). However, most of these securities were not exchangeable with other stripped securities, and thus lacked the liquidity customers had come to expect from "zero" instruments.

In February 1985, the Treasury took a more active role by introducing its own coupon-stripping program called STRIPS, an acronym for Separate Trading of Registered Interest and Principal of Securities. The STRIPS program was intended primarily to reduce the cost of financing the public debt "by facilitating competitive private market initiatives."

Under the STRIPS program, U.S. government issues with maturities of ten years or more became eligible for transfer over Fedwire. The process involves wiring Treasury notes and bonds to the Federal Reserve Bank of New York and receiving separated components in return. This practice also reduced the legal and insurance costs customarily associated with the process of stripping a security. In May 1987, the Treasury began to allow the reconstitution of stripped securities.

Part of a Balanced Portfolio

Stripped securities can be purchased only from private dealers and brokers. Although the Federal Reserve provides services to the zero coupon market, it does not actually sell these securities for the Treasury. Financial services companies decide when and what portion of an eligible security are stripped and sold. Because their increase in value is taxable yearly as it accrues, zeros have become most popular for investments on which taxes can be deferred, such as individual retirement accounts and pension plans, or for nontaxable accounts. However, their known cash value at specific future dates enables savers and investors to tailor their use to a wide range of portfolio objectives.

Source: Federal Reserve Bank of New York (June 2003).

Appendix B.1

Corporate bond yield spreads

The bond yield spread of four of the six corporate bonds issued by the six chosen US companies that were used to explain the different credit ratings in Appendix A.1 are shown here in Appendix B.1.

The four companies tabulated have ratings: Aaa, Ba2, and Baa2.

See Appendix B.1 on disk.

Appendix B.2

Yield spread graphs

The bond yield spreads tabulated above in Appendix B.1 as well as from the other two corporate bonds issued by the US firms not shown in Appendix B.1 but explained in Appendix A.1 are diagrammatically shown in Appendix B.2. See Appendix B.2 on disk.

Appendix B.3

Combined spreads

The graphs in Appendix B.2 are combined to show the relationship of the yield spreads for corporate bond issues that have different credit ratings over time. See Appendix B.3 on disk.

Appendix B.4

Dollar price of risk

Appendix B.4 gives an example of the calculation of the dollar price of risk for a corporate bond as explained in section 1.4.1 above. See Appendix B.4 on disk.

Appendix GARCH (1,1) coupons

Volatility calculations

Appendix GARCH (1,1) coupons shows the volatility calculations for the expected Baa1 option prices using the JT model and using the triple-A coupon corporate bond as the reference bond. See Appendix GARCH (1,1) coupons on disk.

Appendix GARCH (1,1) zeros

Volatility calculations

Appendix GARCH (1,1) zeros shows the volatility calculations for the expected Baa1 options prices obtained using HW model and the triple-A zero coupon corporate bond as the risk free bond. See Appendix GARCH (1,1) zeros on disk.

For each of the above two Appendices, each appendix includes the answer report, sensitivity report and a limits report. These reports are just a summary of the parameters and their characteristics used in the iterative process to calculate or model the daily volatilities. See Hull (2000) for the constraints and characteristics of the parameters used.

Appendix HW.1-8

Expected Baa1 option prices using the HW model

See disk attached for Appendix HW.1-8

Appendix JT.1-8

Expected Baa1 option prices using the JT model

See disk attached for Appendix JT.1-8 together with the volatility calculations as well as the Black model calculations used.