Title: The Optimal Valuation of Black Economic Empowerment Transactions in South Africa

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Abstract

There is uncertainty about how best to determine the final payoff at maturity of various BEE transactions entered into by a number of South African companies. This thesis attempts to develop a method which will price these transactions accurately based on a sample of transactions entered into between 2004 and 2011. The primary valuation methods which will be used are Real Options Valuations, Risk–Neutral Valuations, Implied Trinomial Tree Valuations and valuations performed using the Black-Scholes-Merton Model. Various amendments to each of these methods are introduced in order to correct for pricing biases inherent in each valuation model.

The results presented in this thesis suggest that it is possible to price any BEE transaction provided that the underlying contractual terms are understood.

List of Abbreviations

ABC	-	ABC Recruitment (Proprietary) Limited
BBBEE	-	Broad-Based Black Economic Empowerment
BEE	-	Black Economic Empowerment
BLM	-	Binomial Lattice Model
BS	-	Black Scholes
BSM	-	Black Scholes Merton Model
BSS	-	Bathusi Staffing Services (Proprietary) Limited
COG	-	Capital Outstanding Group (Proprietary) Limited
CRR	-	Cox - Ross – Rubinstein
DCF	-	Discounted Cash Flow
DEPS	-	Diluted Earnings Per Share
D&K	-	Derman and Kani
EPS	-	Earnings Per Share
FAS	-	Financial Accounting Standard
FASB	-	Financial Accounting Standard Board
FCFE	-	Free Cash Flow to Equity
FCFF	-	Free Cash Flow to the Firm
HEPS	-	Headline Earnings Per Share
JB	-	Jarque –Bera
JR	-	Jarrow – Rudd
JSE	-	Johannesburg Stock Exchange
IAS	-	International Accounting Standards
IFRS	-	International Financial Reporting Standards
MS	-	Main Street
NAV	-	Net Asset Value Per Share
NPV	-	Net Present Value
RFR	-	Risk Free Rate
RNV	-	Risk Neutral Valuation
ROI	-	Return on Investment
SCE&C	-	Sanyati Civil Engineering and Contruction
SENS	-	Stock Exchange News South Africa
SPV	-	Special Purpose Vehicles

TEC - Top Empowerment Companies

TNAV - Total Net Asset Value Per Share

WAC - Weighted Average Cost of Capital

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1. Introduction

The South African government used Black Economic Empowerment (BEE) as a policy to attempt to rectify the effects of welfare inequalities in the South African society. Little scientific analysis took place during the BEE implementation period. Corporate South Africa depicted BEE in a depoliticized and technical manner, with the key challenge being to determine the fair value of the various BEE transactions entered into by the different companies.

A key proposition for determining the fair value of these BEE transactions has been to treat a given BEE transaction as a call option, and then to use the standard market capitalization as a yardstick to determine the fair value. In certain instances the Black-Scholes, Binomial and Monte-Carlo simulation methods are used to determine the fair value.

Researchers in the field of financial economics have found that any given investment strategy encompasses a portfolio of carefully selected investment projects which the company deems feasible and that option pricing may be used as an analytical tool to evaluate such projects. The Black-Scholes model has certain limitations such as the underestimation of extreme moves and the assumption of instant costless trading. The accuracy of the results produced by the Black-Scholes model is marred by the model's failure to take into account different options such as expansion, abandonment and deferral opportunities embedded in the transactions. An approach called "Real Options Analysis" has been developed in an attempt to deal with the problems encountered when using traditional option pricing techniques. In South Africa, Real Option Valuation techniques have been used in the construction and telecommunication industries and in determining the value of game lodge concessions.

1.1 The Economic Problem Facing South Africa

Researchers in the field of financial economics have found that using the market capitalization at the inception of the transaction as a yardstick misinterprets the economic value of a transaction. Thus, option pricing models are often used as a basis for determining the fair value of BEE transactions. Owing to the subjectivity involved in doing these calculations, mispricing in the fair value is common.

1.2 Objectives of this Investigation

The main objectives of this investigation are as follows:

1) to use empirical data to determine the fair value of BEE transactions at maturity.
2) to apply the principles of real options pricing, because it is forward looking, to determine the value of assets exchange in a BEE deal. The outcome will be compared to the actual values at each transaction's maturity date.

Similar procedures will be performed for each transaction using the Black-Scholes Model, Binomial and Trinomial Trees, the Monte Carlo Simulation methods and the Binomial Lattice Model. Comparisons of results produced by each method will highlight which method produced the most accurate estimate of the final payoff at maturity.

2. Literature Review

When valuing BEE transactions, initiators often use the principles of "International Financial Reporting Standard (IFRS) 2 Share Based Payments" and the beneficiaries apply the principles of "International Accounting Standard (IAS) 39 Financial Instruments", when determining the fair value of the transaction (See Van der Merwe (2009)). In option pricing terms, the fair value is equivalent to the payoff at maturity.

Different models can be applied to determine the option value. The results produced by each model vary and this can be attributed to the factors inherent in the valuation equation. The different models have been examined below. All methods will be used for empirical analysis. For expansions of the equations, refer to *Appendix B*.

2.1 Valuation Models

2.1.1 Black-Scholes Option Pricing Model

The Black-Scholes (BS) option pricing model is arguably the most famous valuation model. Black and Scholes (1973) became the pioneers in the theory of option pricing, when they presented their analysis, based on the work of Bachelier (1900), to the option trading world.

The valuation formula is given by:

$$w(x,t) = xN(d_1) - ce^{r(t-t^*)}N(d_2)$$

where:

$$d_1 = \frac{\ln\frac{x}{c} + \left(r + \frac{1}{2}v^2\right)(t^* - t)}{v\sqrt{t^* - t}}$$

$$d_2 = \frac{\ln\frac{x}{c} + \left(r - \frac{1}{2}v^2\right)(t^* - t)}{v\sqrt{t^* - t}}$$

The function $N(x)$therefore represents the cumulative probability distribution function for a standardized normal distribution.

and:

$w_1(x,t)$= the value of an option

x = the stock price

c = the exercise price

t^* = the maturity date

t = the current date

v^2 = the variance rate of the return on the stock

r = the short-term interest rate.

2.1.2 Binomial and Trinomial Trees

The Binomial model is a discrete option valuation model which can be used to value derivatives. A valuation method for BEE contracts is not readily available, which makes this model suitable to value transactions as this method is commonly used to value derivatives. The Binomial model was created by Cox, Ross and Rubinstein (1979). Their model is a departure from the Black-Scholes option-pricing model, which is also based on stochastic processes. A primary assumption is that as the time-step becomes smaller the model approaches the lognormal assumption for stock prices which underlies the Black-Scholes model. It is important to note that when valuing options with this model, use is made of both the no-arbitrage arguments and a principle known as Risk-Neutral Valuation.

The Binomial model states:

$$f = e^{-rT}[pf_u + (1-p)f_d]$$

Where:

$$p = \frac{e^{rT} - d}{u-d}$$

Where:

r = the risk-free interest rate

T = the time period between the different nodes

d = the percentage decrease in a stock's downward movement

u = the percentage increase in a stock's upward movement

f_u = the payoff from the stock if the stock price moves up

f_d = the payoff from the stock if the price moves down

p = the risk–neutral probability

2.1.2.1 Risk - Neutral Valuation

The Risk - Neutral Valuation principle is a method of pricing options whether in the simple Binomial model or when using the Black-Scholes-Merton differential equation. The equation does not include any variable which may be affected by the risk preferences of investors. Risk-Neutral Valuation may be compared to replication as both may produce the same option price. The following procedures must be performed when applying the principles of Risk-Neutral Valuation in the binomial model:

(1) Compute the probabilities p and $1-p$ of the states u and d that make the expected return on the stock equal to the risk-free rate *r.*
(2) Compute the expected payoff from the option at maturity under the probabilities p and $1-p$.
(3) Discount the expected payoffs back to the current period using the risk-free rate (RFR), r.

The final result is expected to be exactly equal to the arbitrage-free price of an option that is obtained by replication.

There are several variations to Risk-Neutral Valuation. Hull (2009) suggests that in a risk-neutral world all individuals are indifferent to risks, thus investors will require no compensation for such risks. The expected return on securities valued will therefore be the risk-free rate. When pricing options using this approach, the expected return ($\hat{E}(S_t)$) in a risk-neutral world is represented by the equation:

$$\hat{E}(S_t) = S_0 e^{rT}$$

The mean return on a given tree is defined by the following equation:

$$e^{(r-q)\Delta t} = pu + (1-p)d$$

where:

q = the dividend yield

Δt = the change in time

Hull (2009) introduces variations to the initially proposed techniques which can be used to determine the final payoff from exercising the option.

(A) Control Variate Technique

This method is used to improve the accuracy of the pricing of an American option. In this thesis, it is assumed that BEE transactions are European in nature. This method is included for comparability. This technique entails using the same binomial tree to calculate both the value of an American option, f_A, and the corresponding European option, f_E. The option value produced by the BS model is denoted by f_{BS}. The error produced by the tree in the pricing of a European option is assumed to be equal to that given by the tree in the pricing of the American option. The estimated American option price is given by:

$$f_A + f_{BS} - F_E$$

(B) Setting *p* = 0.5

This method removes the restrictions of the Cox, Ross and Rubinstein approach where $u = 1/d$ and instead sets p = 0.5 as this is the only scenario where the equation will hold true. This method allows trees to be built for options on stocks, indices, foreign exchange and futures. An advantage of this method is that fixing the probabilities to 0.5 means that the impact of the value of σ and the number of time-steps can be disregarded. This method can be applied to a BEE transaction, as the transactions are options on stocks. The upwards and downwards movement are represented by:

$$u = e^{(r-q-\sigma^2/2)\Delta t+\sigma\sqrt{\Delta t}}$$

$$d = e^{(r-q-\sigma^2/2)\Delta t-\sigma\sqrt{\Delta t}}$$

Where:

q = the dividend yield for the period. This is assumed to be 0 in order to be consistent with the rest of the work.

2.1.2.2 Trinomial Trees

Trinomial trees are used as the alternative to binomial trees for pricing options. Trinomial trees are lattice based computational models and will be included for the sake of comparability to the Binomial Lattice method. The general form of any tree incorporates the following variables, p_u, p_m and p_d. These represent the probabilities of an upward, middle and downward movement at each node, and Δt represents the length of the time-step. The work of Hull (2009) and Derman, Kani and Chriss (1996) will be used to perform the valuations for this exercise.

Derman et al (1996) show that in option markets, where there is a significant or persistent volatility smile, implied tree models may be used to price exotic options consistently. They build implied trinomial tree models for the volatility smile, as trinomial trees inherently have more parameters than binomial trees. When implied trees are implemented discretely to allow volatility to vary from node to node, then the tree is made more "flexible" so that the market prices of all standard options can be matched.

Derman et al (1996) use various methods to construct the constant volatility trinomial tree which can serve as initial state spaces for implied trees. All their applied methods converge to the same theory, i.e. the constant-volatility Black-Scholes theory in continuous limit. Three equivalent methods are used for building constant volatility trinomial trees: (a) combining two steps of a Cox, Ross and Rubinstein (CRR) binomial tree; (b) combining two steps of a Jarrow-Rudd (JR) binomial tree; and (c) an equal-probability tree. In all instances the variables remain defined as:

S_u = the stock price reached from an upward move.

S_m = the stock price at the beginning of the time step.

S_d = the stock price reached from a downward move.

p = the probability for an upward move

q = the probability for a downward move

r = the risk-free interest rate

σ = the constant volatility for the period

The Trinomial Tree Valuation methods are represented by the following:

(a) Combining two steps of a CRR binomial tree

$S_u = Se^{\sigma\sqrt{2\Delta t}}$

$S_m = S$

$S_d = Se^{-\sigma\sqrt{2\Delta t}}$

(b) Combining two steps of a JR binomial tree

$$S_u = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{2\Delta t}}$$

$$S_m = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t}$$

$$S_d = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t - \sigma\sqrt{2\Delta t}}$$

(c) Equal-probability tree

$$S_u = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{3\Delta t/2}}$$

$$S_m = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t}$$

$$S_u = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{3\Delta t/2}}$$

d) Incorporating forward rates

Derman et al (1996) and Hull (2009) propose different methods for incorporating the effect of forward rates on the option value. Derman et al (1996) state that in all instances the transition probabilities for any node must lie between 0 and 1, otherwise the implied tree allows riskless arbitrage which is inconsistent with the rational option prices. Hull (2009) introduces two additional methods which will be used to price the different options. The first method is an alternative to the binomial tree, and makes use of the concept of upward and downward movements, but an additional inclusion is the probability of a middle movement.

The equation consists of the following:

$$e^{-r\Delta t}(p_u f_u + p_m f_m + p_d f_d)$$

Where:

f_u = the value of the option at an up node

f_m = the value of the option at a middle node

f_d = the value of the option at a down node

Hull (2009) also explains how the principles of Risk-Neutral Valuation can be extended to handle the valuation of real options. The concept of mean reversion introduced previously during the Risk-Neutral Valuation forms the basis of this valuation. The General Tree-Building Procedure is based on the Hull-White model. An extension of the previous method is an equation for the risk-neutral process followed by a given commodity price S defined by the following:

$$d \ln S = [\theta(t) - a \ln S]dt + \sigma\, dz$$

The above term is based on the principles of Brownian motion, thus the variable ln$S$ is assumed to follow a similar process to variable X, except for a time-dependent drift. The tree is calculated based on the 1-year, 2-year and 3-year futures prices calculated using the prevailing spot price. The variables for a and σ are defined by the following:

α = 0.1

σ = 0.2

The different techniques proposed above incorporate different variables and are expected to produce different results. The results are expected to mimic the fair values.

2.1.3 Monte Carlo Simulations

Many BEE transaction valuations are based on the Monte Carlo simulation valuation methods. These methods are a departure from the traditional Black-Scholes and Binomial option pricing models. Monte Carlo methods are often the primary tool used in nonparametric option valuation methods and may be used to sample the random outcomes for a stochastic process.

The work of Hull (2009) will be used primarily as the basis of formulating a unique simulation which can be applied to BEE transactions. In this study the Monte Carlo method was used to determine the strike price, K, which will be used in all valuation techniques. The derivation of the simulation is explored in the methodology section.

2.1.4 Binomial Lattice Model

BEE transactions fall within the provisions of IFRS 2 and may therefore be valued as employee stock options. Hull and White (2004) examine the valuation of employee stock options. Their work is based on the argument against expensing employee stock options owing to the difficulty of determining the fair value at the time the options are granted. Their approach is based on that suggested by the American Financial Accounting Standards Board's (FASB) Statement of Financial Accounting Standards No.123, i.e. FAS123r Stock Option Accounting (See Hull et al. (2004)).

Hull et al. (2004) suggest that the traditional way of accounting for stock options in the United States in the past has been to use the intrinsic value based method. This method was based on the Accounting Principles Board Opinion No.25, issued in 1972 (Hull et al. (2004)). This method requires the compensation cost of an employee stock option be assumed to be the excess, if any, of the market price of the stock, over the exercise price on the date that the option is granted. They found, however, that in the most common situation, in which options are granted with an exercise price equal to the current market price, the intrinsic-value based method calculates the compensation cost as zero. As a result of this, the FASB issued a statement in October 1995, FAS123r, which encouraged companies to adopt a fair-value based method of accounting for stock options, instead of the intrinsic value based method. Appendix B of FAS123 suggests the following three-step valuation procedure:

(1) Estimate the expected life of the option
(2) Use the Black and Scholes (1973) model or the Cox, Ross and Rubinstein (1979) binomial tree to value the option, with the expected life as the time to maturity.
(3) Adjust the value to allow for the possibility of the employee leaving the company during the vesting period.

Radke (2007) introduces a suggested improvement to the proposed methods. In this method all companies will be required to conform to the fair value methods suggested by Hull et al. (2004) above. Radke (2007) introduces two methods which incorporate the three-step valuation approach introduced by Hull et al. (2004) to value options. She proposes that the Black – Scholes – Merton and Binomial Lattice Valuations may be used as follows:

(i) Black-Scholes-Merton (BSM) Model

(a) Use the simple well established BSM to model a single outcome where a stock appreciates, according to a defined volatility, each year for the option's life (expected term).

(b) Upon exercise, the model produces a profit which is present-valued, using the risk-free rate of return, to determine the value of the option today.

(ii) Binomial Lattice Model

This model takes into consideration that in each period the stock price can go either up or down to produce a range of potential future stock prices (and thus multiple valuations). The outcomes form a tree with the grant being the root of the tree. This model allows for early exercise, unlike the BSM model which assumes that the grantees will hold their options to the end of the expected term. This cash value is present-valued at the point of exercise to reflect the time-value of money using the risk-free rate of return. In order for an outcome to be considered, the value must be greater than zero. The average of the discounted ending values produces a single number which represents the calculated option values.

Hull et al. (2004) also introduce an amendment to the binomial tree which allows the tree to take into account employee exit rates.

It is defined by:

$$f_{N,j}=\max(S_{N'j}-K,0)$$

From the review of the BEE disclosure in various Annual Reports, it was found that, in some instances, the employee exit rates were not significant; thus the newly-proposed method will not be taken into consideration when performing the initial calculations.

2.1.5 Real Options

Embedded in many BEE contracts are option-like features which can be valued using the traditional Net Present Value (NPV) methods, but this requires the availability of historical information. Real option valuations were developed to correct these shortcomings of the traditional NPV calculations as the Real options valuation technique is forward-looking. This technique will therefore form a core part of our analysis.

Damodaran (2010) expands on the theories introduced by Latimore (2001). Damodaran (2010) extends the argument that traditional discounted cash-flow models do not always capture the value of the options embedded in corporate actions. The value-enhancing properties of these embedded options are thus often not taken into account even though they have the ability to determine whether investments and acquisitions that would not be justifiable are in fact value-enhancing. Damodaran's (2010) theories used to value embedded options are based on the mechanics driving the valuation of a call and a put option, and use the principles of the Binomial Model and the Black-Scholes Model to price the various options.

A crucial contribution to the theory of option pricing from this thesis is the impact of exercise of the option on the value of the underlying asset. This aspect is not taken into account in the Black-Scholes model, which is based upon the assumption that exercising an option does not affect the value of an underlying asset.

Damodaran (2010) highlights that many investments and acquisitions contain hidden options, which may be value-enhancing. In investment analysis and capital budgeting, 3 options may be embedded in a given project:

(i) The option to delay a project – especially when a firm has exclusive rights to a project.

(ii) The option to expand a project to cover new products or markets sometime in the future.

(iii) The option to abandon a project if cash flows do not measure up to expectations.

The impact of the above may be interpreted as follows:

(i) The Option to Delay a Project

In an environment in which a project can only be taken by one firm, because of legal restrictions and barriers to entry to competitors, the changes in the project's value over time give it the characteristics of a call option.

The net present value of a project is given by:

$$\mathbf{NPV} = \mathbf{V} - \mathbf{X}$$

Where:

X = the initial upfront investment

V = the present value of expected cash inflows computed at the present moment

Using this method allows a given firm to take up or abandon a project as it highlights the financial implications of investing in a project. In instances where a firm elects not to take up the project, it incurs no additional cash flows, though it will lose what it originally invested in the project.

(ii) The option to expand a project to cover new products or markets sometime in the future.

(Also called Multi–Stage Projects / Investments)

Damodaran (2010) finds that this occurs when firms have the option to enter the business in stages. Doing so reduces the potential upside benefits, but also protects the firms against downside risk, by allowing them to gauge demand and decide whether to go on to the next stage.

A standard project can thus be seen as a series of options to expand, with each option being dependent on the previous one.

(iii) The Option to Abandon a Project

This is the final embedded option and it should be considered when the cash flows do not measure up to expectations. Damodaran found that those projects with inadequate cash flows typically only work for multi-stage projects and require probability inputs at each stage of the project. Option pricing provides a general way of estimating and building the value of abandonment into the value of an option.

The payoff from owning an abandonment option is defined by the following:

If $V > L$ the payoff is zero

If $V \leq L$, the payoff is $L - V$

The option to abandon takes on the characteristics of a put option, which are similar to those of previous options.

The option is always treated as a put option where the value of the option is given by:

$$p = Ke^{-rT}N(-d_2) - S_0\,N(-d_1)$$

Damodaran (2010) found that the Black-Scholes model can be applied to value equity as an option. A firm is valued by estimating cash flows over a long time-horizon and discounting the cash flows back at a discount rate that reflects the riskiness of the cash flows. The value of equity is obtained by subtracting the value of debt from the firm value. He found that discounted cash-flow models understate the value of equity in firms with high financial leverage and negative operating income, since they do not reflect the option that equity investors have to liquidate the firm's assets.

The payoff to equity investors on liquidation can thus be defined as :

If $V > D$ then the payoff = $V - D$

If $V \leq D$ then the payoff = 0

Equity can be viewed as a call option to the firm, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercise price) is paid off.

The payoff can thus be defined as:

Payoff on exercise = $S - K$ if $S > K$

=0 if $S \leq K$

Each of techniques explored above will be evaluated in this paper to determine the fair value of the options embedded in the BEE transactions. The fair value will be compared to the actual outcomes at maturity of the transactions to determine if they produce a result which could accurately predict the actual payoff from the transactions.

3. Methodology

Real Option analysis is based primarily on determining the value of the initial investment and determining the present value of expected cash flows. These values are then used as the values of S and K in the BSM equation. Different sources will therefore be used to determine the variables underlying each technique. The annual financial statements and financial information contained in publicly issued circulars will be used in all instances to find all the necessary information required to perform the Real Options Valuation.

The success of the Binomial and Trinomial Tree Valuations as well as the Binomial Lattice Valuation is largely dependent on the change in volatility of the stock price over the contractual term. All variables needed to satisfy their equations will thus be derived from the stock price behaviour.

3.1 Empirical Tests

The Jarque-Bera test was applied to all the companies included in the initial sample (refer to *Appendix B*) for which there was data available. The Jarque-Bera test is a goodness-of-fit test used to determine whether the sample data have skewness and kurtosis which match those of a normal distribution.

The test statistic JB is defined by the following equation:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

In instances where the share prices of a company mimic a normal distribution, the JB statistic asymptotically has a chi-squared distribution with two degrees of freedom. Hence, the statistic may be used to test the hypothesis that the data are from a normal distribution. The null hypothesis may be defined as a joint hypothesis of the skewness, of which the excess kurtosis is zero. In instances where the excess kurtosis is 3, it is equivalent to an expected excess kurtosis of 0. Where there is any deviation from this principle, the JB statistic is increased.

One of the fundamental assumptions underlying the theory of option pricing, which has been incorporated in the Black-Scholes option pricing formula, requires that the natural logarithms of the spot price of the underlying stock should be normally distributed, and hence that they follow a normal distribution. The natural logarithms of the underlying stocks were thus tested for normality using the Jarque-Bera normality test. For each company included in the sample group, the tests were performed given a level of significance of 0.05. Therefore, $\chi^2 0.95$ = 5.99 from $\chi^2(2)$. In instances where JB > 5.99, then the Null hypothesis of asymptotic normality was rejected. But where JB ≤ 5.99, normality could not be rejected.

The JB test was applied to a sample of 49 companies. Only the following companies' data produced results where the JB statistic was ≤ 5.99:

1) Adcorp Holdings Limited (Adcorp)
2) Kumba Resources (Kumba)
3) Primeserv Group Limited (Primeserv)
4) Santam Limited (Santam)
5) Sanyati Holdings Limited (Sanyati)
6) Standard Bank of South Africa Limited (Standard Bank)

To determine the fair value associated with an Empowerment Transaction, the different valuation techniques were applied to the data pertaining to the six companies where normality could not be rejected.

The change in share price across the contractual term was based on the Monte Carlo Simulation techniques presented in Hull (2009). The primary equation is given by:

$$\Delta S = S r_c \Delta t + S\sigma\varepsilon\sqrt{\Delta t}$$

To determine S, it was observed in what month the BEE deal was announced for a given company and performed a simulation using the listed share price for every trade day in that specific month. The value for r_c is based on the risk-free rate of a 3-month treasury-bill for that specific month.

The volatility is derived primarily from the model of stock price behaviour used by Black, Scholes and Merton (1973). This model assumes that the percentage changes in the stock price in a short period of time are normally distributed.

To determine the probability distribution of the continuously compounded rate of return earned on a stock between the time interval [0 and T], reliance was placed on the lognormal property of stock prices.

Using the guidance given in Hull (2009), for the purpose of this study the volatility of the stock price is defined as the standard deviation of the return provided by a stock in the first year in instances where the return is expressed using continuous compounding. Hull (2009) estimates the volatility of a stock price empirically using historical stock prices.

The volatility can thus be estimated as:

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}}$$

Where:

τ = length of time-intervals in years

s = historic standard deviation

The standard error of the estimate can is given by $\hat{\sigma}/\sqrt{2n}$.

Hull (2009) suggests that more data increases the accuracy of any calculation. Hull (2009) proposes a method of using closing prices from daily returns over the most recent 90 and 180 days. This was used as a basis for determining the volatility in this study. It was assumed that there are 252 trading days per year, thus τ = 1/252.

As ε is a random number generated from a standard normal probability distribution, it was imperative to identify the number of random numbers required for each trade-day share price tested. The trade day coincides with the announcement date of the transaction.

In this study, the Monte Carlo simulation was based on the principles explained in Hull (2009). The number of weeks between the announcement date and the transaction implementation date were determined. Using the number of weeks as a basis, the number of random numbers required to represent ε were determined. The path for each stock price over the period was thus determined by sampling repeatedly for ε from ϕ (0,1) and then substituting into the equation below:

$$\Delta S = S r_c \Delta t + S\sigma\varepsilon\sqrt{\Delta t}$$

As the time interval is based on weeks, Δt is equivalent to 1 week which is 0.0192. For each week, the change in stock price is calculated. This is added to the initial stock price at the beginning of the period, S, to produce a new stock price at the commencement of the following week. Because the process being simulated is Markovian, the samples for ϵ are independent of each other. The final stock price, adjusted for the final change in stock price over the period, becomes the exercise price, K, in all valuation models.

For purposes of performing all valuations, it was assumed that the dividend yield is zero, as dividends are used in most instances to repay the debt taken out to finance the transactions.

3.2 Sample Selection and Data

The theories introduced previously will be tested on an independently selected sample of BEE transactions in order to determine their validity. The sample was constructed by locating all companies included in the Economic Empowerment Rating Agency's (Empowerdex) list of most empowered companies for the year ended 31 December 2011, as well as all mining companies included in the Empowerdex publication "A summary of the BEE Transactions in the Mining Sector 2004 – 2005". From the companies included in the 'Most Empowered Companies' list for the year 2011, only the top 43 companies, based on the ratings obtained, were included in the final sample. All the mining companies included in the publication relating to the mining sector were included in the final sample. There were only six mining companies, yet the total value of these transactions amounted to R43 billion. (refer to *Appendix A* for the complete list of companies included in the final sample together with a brief description of the BEE transaction.)

Subsequently, the following information was gathered for each company included in the sample for the fiscal years 2004 – 2011:

1) Data pertaining to each company's BEE transaction was collected from two sources. The first source was articles freely available containing details of the specific BEE transaction entered into by the company such as:
    i. the expected transaction date
    ii. the transaction market value
    iii. the expected period over which the transaction should be finalized
    iv. the total number of shares to be issued
    v. details relating to the BEE partners
2) In instances where no information was freely available, the Circular to Shareholders for the respective company was used, or the full notice of announcement released on the JSE's Stock Exchange News (SENS).

Share data for each of the companies included in the sample was collected from DataStream. The data was obtained for the period 1 January 2004 – 30 June 2012.

4. Real Options

4.1Practical Application of the Real Options Method

For the six companies included in the final sample, different option pricing techniques will be used to determine the final implication of the transaction. Depending on the characteristics of the transaction, the values may be determined by valuing the different components as call options, abandonment options or expansion options. This will be consistent with the approach suggested by Damodaran (2010). In all instances, unless indicated otherwise, the strike price, K, will be the value calculated using the Monte Carlo simulation method introduced in the previous section.

The following techniques will be applied to the different companies:

i) Adcorp Holdings Limited

Adcorp Holdings Limited entered into a BEE transaction to facilitate the acquisition of a 25% + 1 vote-direct ownership interest in the total issued share capital of Adcorp by the BEE parties. In addition, the company acquired 100% of the issued share capital of Capital Outstanding Group (Proprietary) Limited (COG) for a total purchase consideration of R238 million. The announcement took place on 12 April 2007 and was implemented on 7 May 2007. The call option value was calculated as R23.89.

When the cash consideration received was compared to the BS call value it was found that the cash received exceeded the value significantly. A similar comparison was done between the market value on transaction date and the perceived value calculated by the company on transaction date. In addition the market value based on the trading share price of R38.15 on transaction date, significantly exceeded the option value, based on the BS model. The company estimated the transaction to be valued at R94,800,000. This was found to be significantly below the calculated BS call value.

The second part of the transaction relates to the acquisition of 100% share capital in COG. The purchase consideration of R238,000,000 was settled by an issue of 218,750 ordinary shares at a price of R 32.00 per share and a private placement of 4,000,000 ordinary shares at an issue price not less than the 30day volume-weighted average price of Adcorp at transaction date. When the total transaction value based on the BS value is compared to the actual cash consideration received, the cash proceeds exceed the BS value.

The results indicate that the transaction value could potentially be overstated. Adcorp enjoyed high financial gains from entering into this transaction as the cash flows which they received from the initial placement and the subsequent private placement significantly exceeded the amounts paid to acquire the shares initially.

ii) Santam

Santam's BEE initiative was implemented by way of an arrangement in terms of which BEE SPV, a special purpose vehicle, acquired an aggregate of 10% of Santam's shares following the settlement of the repurchase offer and the Sanlam offer, for a consideration of R82 per share.

All shares held by BEE SPV company would be "locked in" to the SPV until 28 February 2015. The valuation of the call option per share will be performed for a period of 3, 4 and 5 years. This coincides with the group's policy, which states that the vesting period is five years and lapses after the sixth year. All share options may be exercised from the third year and should be fully exercised after the sixth year. The valuation was also performed for a period of 8 years to take into account the fact that all shares will be "locked in" for a period of eight years.

The inclusion of the early exercise dates could potentially indicate that these options are American in nature. For purposes of this study, the transactions will be treated as European in nature. This was done for the sake of comparability.

The announcement of the transaction took place on 26 March 2007 and was implemented on 21 May 2007. The call option values were calculated as R21.93, R31.35, R32.43 and R45.90 for the 3, 4, 5 and 8 year period.

The transaction was funded primarily by way of subscription for 7-year cumulative preference shares, to the value of R450 million. Sanlam Life provided up to R600 million in bridging for the outstanding balance. Comparing the total BS call values calculated for each of the respective years to the total cash consideration received from entering into this transaction, it was found that the cash received significantly exceeded the transaction value in all instances. This indicates that in all instances, the payoff from exercising the option is favourable to the holder of the option.

The actual share price achieved on the three-to-five year vesting dates, 21 May 2010, 2011 and 2012, was compared to the acquisition price of R82. The gain in share price fell within a range of 23.30% to 103.05%.

On acquisition date, the cash consideration of R82.00 per Santam share represented a discount of 10.20% to the weighted average price of R93.98 per share. The average dilution as a result of achieving the 10% BEE ownership was 2.8%. On the option exercise dates, the average increase in value exceeded this amount significantly. This indicates that when structuring the transaction, it was correctly predicted that there will be an increase in the future share price. The effects of offering the shares at a discount therefore resulted in positive future financial effects.

iii) Sanyati Holdings Limited

Sanyati Holdings Limited entered into a transaction whereby 48,000,000 of all ordinary shares would be issued to a BEE consortium. An additional 10,000,000 ordinary shares would be issued to the Sanyati 2007 Acquisitions Share Incentive Scheme. The share options are treated as share based payments and had a "lock in" period of 12 months after the date of issue. The company's policy is to exercise all options after a period of 3 years, when they vest. This vesting period is similar to Santam's transaction, thus the transaction will also be treated as European in nature for the sake of comparability.

The values of the call options per share for the different years were calculated as R1.23 and R1.55 for years 3 and 4 to 5 respectively.

When the total value of the transaction based on the BS values is compared to the cash considerations received from both the initial transaction and the specific issue to the Sanyati 2007 Acquisition Share Incentive Scheme, it was found that the cash received exceeded the transaction value in both instances.

When the BS call value per share is compared to the actual share price on the vesting dates, which has been determined as 16 May 2010, 2011 and 2012, it was found that the call value exceeded the share price in all instances. This is attributed to the large decline in share price following the transaction conclusion.

The unaudited interim financial statements for the period ended 31 August 2006 were reviewed in order to determine the impact of the issue to the BEE consortium and the Sanyati 2007 Acquisitions Share Inventive Scheme. It was found that the Earnings Per Share (EPS), Headlines Earnings Per Share (HEPS) and Diluted Earnings Per Share (DEPS) values increased by 2.27% after the issue to the BEE consortium. When the effects of the issue to the Sanyati 2007 Share Incentive Scheme were taken into account, it was found that the EPS, HEPS and the DEPS figures declined by (6.11)%. This is an interesting observation, especially considering the significant excess cash received by the Holding company for entering into this transaction.

iv) Assore

Assore was included by virtue of the fact that Standard Bank was a significant player in their BEE transaction. Information pertaining to specific empowerment transactions within the Standard Bank Group could not be obtained, therefore this approach was followed. The transactions were not subjected to the log-normality test mentioned earlier as these were already applied to Standard Bank. The different components of Assore's transaction will thus be assessed individually.

A) First Empowerment Transaction

In February 2006 the Assore Group entered into an empowerment transaction whereby 15.02% of the issued ordinary share capital would be acquired by two black economic empowerment (BEE) entities, namely Shanduka Resources, a subsidiary of Shanduka Group (Proprietary) Limited (Shanduka) and the Bokamoso Trust. The Bokamoso Trust is a broad-based trust and was founded for the benefit of the communities in and around Assore's areas of operation. The trust holds its effective equity interest in Assore through MS350. Shanduka Resources acquired a 11.76%,equity interest and the Bokamoso Trust purchased a 3.26%, equity interest.

The deal was announced on 10 November 2005 and was concluded in February 2006. The call value per share was calculated as R7.17. When the total transaction value based on the BS call value was compared to the total market value based on the share price of R25.40 on 10 November 2005, it was found that the market value significantly exceeded the calculated value.

In order to participate in the First Empowerment Transaction, MS 350 required funding to acquire its direct interest. The funding was provided as follows:

(i) The Standard Bank of South Africa Limited provided preference share funding to MS 350 amounting to R53,600,000. This was achieved by means of subscription for "A" class preference shares. The total shares held are 381.

(ii) Assore provided preference share funding to MS 350 amounting to R25,000,000, by subscribing to "B" class preference shares. Assore still holds the 25 "B" class preference shares.

When the BS call value was compared to the total funding received, it was found that the funding received exceeded the fair value significantly. This indicates that there were financial gains for the Holding company.

B) Second Empowerment Transaction

Assore entered into a second empowerment transaction in which an additional 2,723,653 shares, comprising 9.88% of the issued ordinary share capital at the date, were acquired by an entity in which Assore and the Bokamoso Trust have a 49% and 51% interest respectively. A specific issue of 349,747 treasury shares took place, resulting in the Trust achieving control of 14.28% of the issued share capital after the transaction date.

The announcement took place on 25 June 2008 and was implemented on 27 November 2009. The call option value was determined as R124.48. When the total BS call value was compared to the market value on implementation date, which was based on a share price of R215.00, it was found that the market value significantly exceeded the transaction value.

In order to participate in the Second Empowerment Transaction, MS350 procured a refinancing of existing preference-share funding to Standard Bank and Assore respectively, in terms of which MS350 will create certain “C” class preference shares and “D” class preference shares. Assore would subscribe for a given number of “C” preference shares at an aggregate cash subscription price of R65,000,000. MS350 will utilize the aggregate subscription price of R65,000,000 to redeem the existing “A” Preference shares and the “B” Preference shares in the issued capital of MS350.

When the BS call value was compared to the total funding received from the “C” Preference Share Funding, it was found that the fair value significantly exceeded the cash consideration received. This is contrary to the previous transactions where the cash consideration received significantly exceeded the call option value. This could indicate that Assore Limited identified potential gains elsewhere in the transaction.

From the review of the Circular dated 2 December 2009, the approximate value of the total shares included in the transaction above, based on the 30-day volume-weighted average share price on the JSE as at 27 November 2009 of R668.32 per share, was approximately R2,054.01 million. This figure significantly exceeds the total BS call option value, the total market value and the total funding received. The potential appreciation in share price as a result of entering into this transaction can be attributed to the increased returns for the Holding company.

C) Third Empowerment Transaction

This transaction is being implemented in two phases. During the first phase, sale shares from MS343, a wholly-owned subsidiary of Shanduka Resources, was acquired by MS904, a special purpose vehicle incorporated to facilitate the Third Empowerment Transaction, owned 51% by the

Ficker Road Trust and 49% by the Assore Employee Trust. The total purchase price was R2,683,705,350.

In order to acquire the Sale Shares, MS904 was capitalized by way of a loan facility in the amount of R2.73 billion, advanced to MS904 by Standard Bank in terms of the Facility Agreement.

Phase two of the transaction involves the introduction of a long-term vendor-financing structure by Assore in respect of MS904 and its holding of Ordinary Shares. Phase two will be facilitated by Standard Bank subscribing for the Assore Preference Shares in the authorized but unissued share capital of Assore, to the value of R2.85 billion. Assore will capitalize MS904 by subscribing for the MS904 Preference Shares in the authorized but unissued share capital of MS904, to the value of R2.85 billion. This will provide MS904 with the aggregate funding amounting to R2.85 billion required to discharge its obligations to Standard Bank, in terms of the Facility Agreement.

Assore will redeem the Assore Preference Shares over a maximum period of 5 years, with the option to redeem the whole or any portion of the unredeemed amount at any time in advance of its scheduled redemptions.

The deal was announced on 9 December 2011 and was implemented on 13 February 2012. The calculated BS call value was determined as R94.18. The market value of this transaction, based on the share price of R246.22 as at 9 December 2011, significantly exceeded the fair value.

To acquire the Sale Shares from MS343 a loan facility of R2.73 billion was capitalized. To facilitate phase two of the Third Empowerment transaction, Standard Bank subscribed for Preference shares to the value of R2.85 billion. The total BS call option value was compared to the cash consideration received, and it was found that the total cash significantly exceeded the fair value of the transaction. This indicates that the option should be exercised as there is a highly favourable payoff.

v) Primeserv

Primeserv Group Limited announced on 2 December 2004 that, subject to shareholder approval and certain other conditions precedent, an agreement has been reached between Primeserv and Bathusi Staffing Services (Proprietary) Limited (BSS), whereby BSS will acquire from Primeserv ABC Recruitment (Proprietary) Limited (ABC), a wholly owned subsidiary of Primeserv Group Limited, the business and related assets and liabilities of the Secunda branch of ABC. This will constitute the ABC disposal. The transaction would be effective from 1 July 2004.

This disposal may be viewed as an abandonment option. The approach of Damodaran (2006) was followed, whereby the option to abandon a project is used when a project's cash flows does not measure up to the company's expectations. The Primeserv transaction will thus be valued as a put option.

The value of the put option, which represents the value of abandonment, was determined as R1,408,980. This amount was added to the value of the underlying asset, S, of R3,612,000, yielding a total net present value with the abandonment option of R5,020,980, V. The abandonment value, L, of R6,000,000 exceeds the net present value with the abandonment option by R979,020. This represents the payoff from owning this option. As the value of abandonment is higher, it indicates that this transaction was favourable for the Holding company.

The benefits of the transaction have been that it resulted in a capital surplus of R4.2 million for Primeserv. In addition, Bathusi Staffing Services (Proprietary) Limited, which had previously been accounted a subsidiary, became an associate with effect from 29 January 2005. Only earnings from associate and the balance of the investment in associate are thus reflected in the Group's annual results.

The group publication stating the pro-forma effects of the ABC Disposal on audited results as at 30 June 2004 were reviewed and it was found that the ABC disposal increased the EPS and HEPS by 43.10% and 11.80% respectively. In addition, the Net asset value per share (NAV) and the Tangible net asset value per share (TNAV) increased by 9.70% and 8.70% respectively. This indicates that the group enjoyed positive results following the disposal of this division.

vi) Kumba Resources

Kumba Iron Ore

Kumba's empowerment transaction resulted in Kumba being split into two companies namely Kumba Iron Ore and Newco. The components underlying the transaction are as follows:

- Kumba transferred 80% of its iron ore assets housed in Sishen Iron Ore company to Kumba Iron Ore. Newco retained a 20% stake in the entity.
- Newco acquired Eyesizwe Coal in a bid to enhance its coal asset portfolio.
- Newco acquired Anglo America's heavy mineral operation, Namakwa Sands.
- Newco acquired a 26% stake in Black Mountain zinc and Gamsberg zinc.
- Kumba acquired 48.5% of Ticor Australia.

The announcement of the transaction took place on 13 October 2005 and the entities were listed on 20 November 2005. As the above transactions entail the acquisition of different projects various options will be valued using the method of Damodaran (2006). The various projects can be viewed as options to expand. The option payoff is only positive if the present value of the expected cash flows at that point in time exceeds the cost to expiration.

The method was applied to each of the components of the transaction mentioned above. The Sishen Iron Ore, Eyesizwe and Ticor transactions relate to Kumba Iron ore, thus the information relating to Kumba will be used to determine the underlying components necessary to determine the option values. Namakwa Sands, Black Mountain and Gamsberg zinc are all divisions of Anglo American plc, thus the latter company's information will be used to determine the underlying option values.

Kumba Iron Ore's and Anglo American plc's published annual financial statements for the year ended 31 December 2005 were the primary source used to perform the valuation. The total option value was determined as R 1,089,780.

At the transaction announcement, Kumba estimated the transaction value at R14 billion. The call value, although positive, indicates that the transaction could possibly have been overvalued.

4.2 Comparison of the Computed Option Values to Actual Results

After 1995, there was a growth in merger activity aimed at increasing the participation of formerly disadvantaged individuals in the South African economy. This was encouraged by the introduction of the Black Economic Empowerment legislation by the new South African government. The trend in these transactions has been increased activity for the period 1995 to 2001. This decreased substantially from 2002 to 2004, owing to the markets' uncertainty surrounding the BEE deals. An increase was however experienced again from 2004 to 2007, with the mining sector making a considerable contribution to these deals. All of the companies in this study's sample entered into their transactions in or before 2007. A number of conditions specified in the contracts have been fulfilled to date, thus comparisons of the actual performance of the shares to the value of the call options calculated in the results section can be made.

A typical deal is structured in such a way that a Special Purpose Vehicle (SPV) acquires a number of shares in the Holding company. The purchase price is either financed by the Holding company via vendor funding and or by external financiers. The shareholder or beneficiary of the SPV will thus be obliged to contribute a nominal amount to the SPV. Before the option matures, interest will accrue on the funding at either a fixed rate or a floating rate. All dividends received by the SPV on the shares held will be used to service and repay the funding. Adcorp Holdings and Santam Limited implemented the latter to their advantage.

The results for all the transactions have been summarized below:

Real Options	Adcorp Holdings Limited	Assore Phase 1	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Total option value	401 955 060	494 244	161 460 543	1 089 780 000	1 253 000 000	470 183 222	-8 640 000
			2 072 546 840				
Actual	468 179 888		2 300 000 000	1 081 500 000	1 260 000 000	470 000 000	-8 908 800
Difference:	66 224 828		227 453 160	-8 280 000	7 000 000	-183 222	-268 800
Deviation as a percentage of actual:	14%		10%	-1%	1%	0%	3%
Difference when compared to McGreggor BFA:							-0.56%

Table 1 Comparison of results produced using "Real Options" Pricing to actual results at maturity.

(i) Santam

Santam's empowerment transaction, which saw 10% of the short-term insurer's shares sold to a range of previously disadvantaged individuals, resulted in an improvement of their broad-based black economic empowerment (BBBEE) scorecard from a level 6 contribution to a level 3 contribution. At the time of announcing the transaction in 2007, the value of the transaction was estimated at R915 million.

The publicly issued value of the scheme, according to the Annual Report for the year ending 31 December 2010, was approximately R470 million. During 2010, the dividend income received by the structure in which the scheme was housed was also sufficient to service the senior debt facility fully and to make a proportional payment towards servicing the mezzanine debt.

When the estimated value of R 470 million is compared to the BS option value calculated for year 4, the deviation is only 0.03%. The BS value is thus an accurate estimation of the actual. This highlights the fact that the initial published value of R915 million at transaction date was significantly overstated.

(ii) Adcorp

Adcorp has a similar success story. During the current year, Adcorp received the accolade, for the second consecutive year, of being the most empowered entity on the JSE in the Financial Mail's survey of Top Empowerment Companies (TEC). The group improved on their empowerment score of

81.69% in the previous year to produce 88.71%, a feat which made them the only level 2 BEE contributor listed on the JSE.

In 2007, Adcorp's BEE partners, including a consortium, Sandile Zungu, made an exit. After a strategic rethink Adcorp replaced the Zungu consortium with a R512 million transaction which transferred 15% of the Group's shares to a consortium made up of Women Investment Portfolio Holdings and Simeka Group.

When the share option value calculated previously is compared to the market value as at 28 February 2012, being Adcorp Holdings' financial year end, the deviation is 14.16%. This too presents a fairly accurate prediction of the actual payoff. This is mainly attributable to the market achieving a higher return, with a trading price of R27.83, which is greater than the calculated option value of R23.89. It should however be noted that the share price of R30.43 used on transaction date to determine the total value of R512 million is greater than both prices at expiry of the options.

The total issue price of 2.5 cents per share is significantly below the BS option value. This indicates that all beneficiaries who entered into the transaction are likely to receive major payoffs as the shares were acquired at a significant discount. The group issued the shares at such a discount after having to rethink the group's BEE strategy following the failure of a number of BEE transactions.

(iii) Primeserv

Primeserv also created an associate, but they achieved this by disposing of one of their operations. The group disposed of the business operations of its Secunda Branch, together with the assets and liabilities related thereto, to BSS for a purchase consideration of R6,000,000. A clause was included whereby the purchase price may be increased to an amount equal to six times the excess of the average profits, before tax and interest, on the purchase consideration loan for the years 2005, 2006 and 2007 over an amount of R1,058,500, but limited to a maximum amount of R10,000,000.

The transaction left Bathusi Staff Services (Proprietary) Limited (BSS) with a deficit in total shareholders' funds. As of 2011 the "Net Shareholders' Funds" had failed to recover following the transaction, with a net deficit of R3,174,000 being recorded.

The Group's Associate specializes in the provision of outsourced staff to the petrochemical, engineering and construction industries, as well as to the mining and allied industries. Following the conclusion of the transaction, the result of the division ceased to be consolidated as a subsidiary and is now treated as an associate. The only balances reflected in the Group's Annual Financial Statements are therefore the balances related to Earnings from Associate and Investment in

Associate. Following the disposal, the division delivered an unstable performance, largely attributable to delays in planned shutdowns and restructuring at certain key customers. The review of the Chief Executive Officer's Annual report for the eighteen months ended 31 December 2006 disclosed that the contribution to the Group results from the Bathusi operation is largely reflected in cost recoveries and the interest-received line. In 2006, the group received an amount of R1,4 million in interest which they regarded as an operational recovery, rather than interest earned on surplus funds.

When the earnings for the group from the period 2005 to 2011, including interest received, is compared to the put option payoff calculated above, the deviation is only 0.01%. This indicates that valuing this as an abandonment option produced very accurate results.

In 2011, the Group advanced cash, in the form of an interest-bearing loan, to the division as the division was making losses. This increased the value of the investment in associate significantly, following the reduction in value from the period 2005 to 2009 owing to repayments to the Holding company. The figure increased significantly in 2011 as a result of a cash advance received from the Holding company. This was done as the division delivered a loss, which was caused by the loss of a significant customer as part of a tender process.

The 2006 Chief Executive Officer's report mentions that while re-evaluating the division's practices and processes it became evident that too much emphasis was placed on developing capacity and capability and on service excellence. However, not enough emphasis was placed on driving sales processes and leveraging market opportunities. Special mention was in this report of the fact that the division had advanced its BEE strategy and managed to receive a strong level-four contributor empowerment rating during the review period that year. The Group voiced their continued commitment to ongoing BEE transformation activities, which remained a key strategic objective. This could therefore serve as an explanation for the somewhat unexpected cash advance in 2011 to the Bathusi operation as the operation neared the repayment of the initial loan.

(iv) Sanyati

Sanyati entered into a transaction similar to that of Primeserv, but the outcome was different owing to a crash in the share price. After Sanyati Holdings Limited's BEE transaction, the company was rated as the twenty-eighth most empowered company on the JSE in a survey conducted and published by the Financial Mail in 2011. The Group voiced their commitment to continued achievement of genuine and sustainable improvements in the required categories of the Construction Charter scorecard.

Comparison of the calculated option values for a period of 5 years revealed that the share price had declined sharply since the implementation of the transaction. In addition, the deficit figures reported by the group for the period 31 May 2009 to 31 May 2012 ranged between R13,976,000 and R39,000. As a result, Sanyati Holdings engaged in a Business Rescue Plan in May 2012, in a bid to recover the ailing share price. The options associated with the BEE deal are all out of the money, these options hold no value.

On 10 July 2012, Sanyati Holdings Limited issued a press release stating that the company would cease all business rescue proceedings for the Sanyati Civil Engineering and Contruction (SCE&C) and would proceed with the liquidation of this unit. This decision indicated that the company as a whole would most likely liquidate as Sanyati was largely dependent on the trading, income and profitability of the SCE&C unit.

Previously attempts were made to value the BEE transaction using the Monte Carlo and Black-Scholes methods. These however produced results which did not depict reality, as the company's share price declined sharply after the announcement of the BEE deal. Since the company faced possible liquidation, the principles of Damodaran (2006), where equity is valued as an option, can be applied to determine the payoffs of the option.

Whilst the valuations performed were similar to those for Kumba and Primeserv, the inputs used were extracted primarily from the annual financial statements. Owing to the unexpected declines in share price over the period, methods used previously to determine various financial rates were deemed inappropriate. All financial data was thus extracted from McGreggor BFA (as at 20 August 2012) to finalize the computations. The final call option value was estimated as R(53,792,570). It can therefore be concluded that the firm will have no value once liquidated as the liquidation value of the firm does not exceed the value of outstanding debt. The option thus has a payoff of zero.

The value of one share at liquidation date was calculated as R(0.18). On 20 August 2012, McGreggor BFA's report indicated that for the year ended 28 February 2011 the return on equity was (18.58)%. Comparison of this to the result above indicates that, for every R1 invested in Sanyati, the shareholder can expect a return of R(0.18), which may be equated to (18)%. This closely resembles the amount obtained from McGreggor BFA.

Valuing the option using the real options technique therefore produces a more accurate value than using the Monte Carlo and Black-Scholes combination.

(v) Assore

An analysis of the BEE ownership transactions by Empowerdex revealed that the majority were made to secure between 10%-25% and 25%-50% ownership. This was done primarily to secure the Mining Charter scorecard requirements of 15% by 2009 and 26% by 2012. Kumba's and Assore's transactions fall into this category as similar ownership percentages were achieved by these transactions. Both transactions produced favourable results when compared with actual performance to date.

In June 2011, Cyril Ramaphosa's Shanduka Group disposed of its 11.8% black economic stake in the Assore Group. The interest was acquired through a BEE transaction in which it purchased 5.7% from Old Mutual and received another 6.06% in the form of newly issued shares.

In terms of the initial agreement, Shanduka was not permitted to sell those shares before the "lock-in" period of 2014, without permission from Assore. The early disposal was however negotiated between Shanduka and Assore because it produced benefits for both parties. After the sale, Assore announced their "third empowerment transaction", in terms of which the 11.8% stake will be sold to a special purpose vehicle, called MS904, which is owned by two independent empowerment trusts set up by Assore. The aim of the transaction is to ensure that the shares will ultimately be owned and controlled by and for the benefit of BBBEE (broad-based black economic empowerment) groupings who will eventually become long-term shareholders in Assore.

A unique facet of the Assore BEE deal is that it consists of three phases. In order to determine the payoff from the sale of shares, the principles of Damodaran (2006), introduced previously, will be applied to value multi-stage projects or investments. The gain in value from embarking on the multi-stage investments will be weighed off against the cost to determine the final payoff.

When Shanduka acquired the stake in 2005, they acquired 3,29 million shares at the then market price of R127 a share. The deal was estimated to have a value of R418 million. Shanduka, however, did not pay the stated nominal price of R127, as the 6.06% interest acquired was issued at a price of R85. The company announcement relating to the share issue did not specify the amount paid for the shares acquired from Old Mutual. The initial transaction was funded solely through preference shares provided by Standard Bank and facilitated by Assore. Hence, Shanduka did not put up any of their own funds to acquire the shares.

In September 2010, following the sharp increase in the price of an Assore share as a result of the iron ore boom, all Assore's shares were split five for one. The 11.8% stake owned by Shanduka thus consisted of 16,46 million shares. Assore's spokesperson, Jacques de Brie, released a statement on 28 June 2011, stating that Shanduka's profit on the deal amounted to more than R2 billion. The sum of the payoffs calculated for each stage of the transaction (Phase I and II) gives a result that is sufficiently close to the actual profit realized on the transaction, as the deviation is 9.88%. The profit is well above the R2 billion released by Assore.

In 2005, when Shanduka Group entered into the transaction, the value of the transaction was estimated at R418 million. However, when the value of the transaction is estimated using the BS option valuation the total value is only R23,606,040. This could indicate that the value of the transaction was overstated at inception.

(vi) Kumba

Similarly, Kumba's deal also resulted in a favourable position for their shareholders. Exxaro was formed in November 2006 following the unbundling of Kumba Resources. The benefits from entering into the BEE transaction will therefore be measured based on the results of Exxaro. In December 2011, Exxaro Resources Limited announced that beneficiaries of the Exxaro Employee Empowerment Participation Scheme, which was established after the unbundling transaction, were set to benefit significantly from pay-outs as the scheme had reached maturity at the end of the five-year capital appreciation period.

The shares generated approximately R1 billion for distribution to the beneficiaries. In previous years, beneficiaries had already benefited from the robust growth in Exxaro's share price, which had trebled since November 2006. Beneficiaries received a total of R81.5 million in dividends.

When these values are compared to the option values calculated previously, they return a deviation of only 0.77% of the actual value. In 2006, when the deal was announced, the total value was estimated at R14 billion. The above calculation, however, indicates that the transaction was significantly overvalued at inception. This is similar to all the previous transactions where the market value at announcement date was used to determine the value of a transaction. The market value is possibly not the best method to use when establishing the total deal value.

5. One – Step Binomial Model

5.1 Evaluation of the Suitability of the One – Step Binomial Model

When performing the simple one-step Binomial calculation, the time period was taken as 12 months before maturity of the option. In addition, in order to be consistent with the techniques used in Hull (2009) , the calculation was also performed for a period of 3 months before maturity. The valuation date is consistent with that applied when valuing the options using the Monte Carlo and Black-Scholes models.

The maximum and minimum stock prices over a given period represent the highest and lowest stock prices reached during the 12 months and 3 months for which the calculation is performed. The current stock price is taken as the actual stock price 12 months or 3 months before valuation or expiry date. The payoffs and the probabilities were then calculated accordingly. Refer to the table below for results obtained for each period:

One-Step Binomial Model						
	Adcorp Holdings Limited	Assore Limited	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
12 Months						
Deviation as a percentage of actual:	91%	92%	-147%	100%	66%	127%
3 Months						
Deviation as a percentage of actual:	97%	95%	-63%	100%	92%	100%

Table 2 Comparison of the results produced by the One-Step Binomial Model for periods of 3 and 12 Months.

When the computations were performed, the model produced payoffs ranging between 0.28% and 33.87% for the combined transactions. Using this approach however significantly misprices all transactions and is not suited to the current purpose.

The following trends emerged when the calculations were performed:

(a) None of the calculations for either the 3-month or 12-month time period produced a result which approximated the actual payoff for the transactions.

(b) In all instances a period of 3 months, prior to expiry or valuation date, produced lower results than those for the 12-month period.

(c) The results produced by each period for all mining companies was 64% lower for the 3-month period when compared to the 12-month period. The results for the other companies varied between 0% and 85.86%.

(d) The result for the 3-month period for Sanyati Limited correctly predicts that the option will have no value once it is expires or reaches valuation date.

(e) Primeserv was the only transaction where no exchange or issue of shares took place, yet they recorded a positive payoff, even if it was the lowest payoff. This could possibly indicate that the market sees value in entering into a BEE transaction where share issues or exchanges take place; hence the company is rewarded.

As the results above do not provide an accurate reflection of reality, alternative techniques need to be applied. Dubofsky and Miller (2007) state that when the Binomial Option Pricing Model is used in instances where the time period is longer, and if the upwards and downwards movements are consistent with generating a lognormal distribution for the stock price, then the Binomial Option Pricing Model should converge to the Black-Scholes Option Pricing Model. They suggest using a Multi-Period Binomial Option Pricing Model, which, owing to its extreme flexibility, can be used to value American options (which may be exercised early) and most, if not all, exotic options. This model will be explored in the next section in the form of the Binomial Lattice Model.

6. Binomial Lattice Model

6.1 Application of the Binomial Lattice Model

The Binomial Lattice Model (BLM) is included as the BEE share options contain characteristics of regular stock options. To determine the fair value of these options, Radke's (2007) techniques, which are largely based on the approach suggested by FAS123r, Stock Option Accounting, will be used. Refer to the table below for results:

The Binomial Lattice Model						
	Adcorp Holdings Limited	**Assore**	**Kumba**	**Primeserv Group Limited**	**Santam**	**Sanyati Holdings Limited**
Binomial Lattice Model						
Computed Value:	405 262 432	2 111 967 500	1 691 400 329	67 747 333	420 460 272	-77 280 000
Actual:	468 179 888	2 300 000 000	1 081 500 000	1 260 000	470 000 000	-8 918 400
Difference:	**62 917 456**	**188 032 500**	**-609 900 329**	**-66 487 333**	**49 539 728**	**68 361 600**
Deviation as a percentage of actual:	**13%**	**8%**	**-56%**	**-5277%**	**11%**	**-767%**

Table 3 Results produced by the application of the Binomial Lattice Model.

The BLM produced good results for Adcorp Holdings Limited and Assore Limited, where the computed results represented 87% and 88% of the actual payoffs respectively. For the rest of the companies, the deviations were significant.

Radke (2007) recognizes that because of the complexity and the large number of outcomes produced when making use of the BLM, a software implementation will be required by a given company wishing to perform a computation, in order to compute the option value correctly. The results above indicate that the current technique, used to compute the option values for Kumba Iron Ore, Primeserv Group Limited, Santam and Sanyati Holdings Limited, could possibly not be the best suited as the deviations were significant. A variation to the technique above was devised and used to determine an alternative method of option valuation. It is based on the following:

a) Obtain the actual share prices for the four companies during the contractual term. (This is obtainable as most of the options have reached maturity.)
b) For all the final outcomes produced on the final nodes of the tree, assess whether the value is above or below the range of actual share prices reached during the contractual term.
c) Eliminate all outcomes which do not fall in the range.
d) Compute a new average using the remaining outcomes from "c" above.

Applying the above produced the following results for shares with significant deviations:

(i) Kumba Iron Ore

Although the proposed amendments produced improved results, the deviations remained significant. It was found that the best possible option value was produced when only the option values for two of the share prices in the range, R149.99 and R150.68, which are close to the actual value at maturity date, were taken into account. The final payoff produced a deviation of 56% when compared to the actual.

(ii) Primeserv Group Limited

The proposed amendments produced an increased deviation from the initial value. This indicates that the appreciation in share price was not significant enough to give rise to large payoff. Again, taking into account the fact that no share issue took place in this BEE transaction, this suggests that the payoff is attributable to some other factor not related to the share prices. This technique can therefore not be used to determine the fair value of this transaction.

(iii) Santam

The proposed amendments did not produce the desired results for Santam. Santam's option valuation included two share prices, R158.25 and R152.24, which fell outside the range of R63.50 – R135.80 for share prices during the contractual term. The BEE transaction is however structured to allow for exercise one year after valuation date. The range of share prices up to 30 June 2012 were reviewed and noted that the maximum share price reached was R178.38 on 19 June 2012. The highest outcomes included in the initial range which exceeded the maximum share price reached were thus excluded from the calculation. The new valuation produced a result that is a 29% deviation from the actual, which is an improvement from the initial deviation of (58)%.

The payoffs from R158.25 and R152.24 are close to the actual payoff. These figures were used only to calculate an option and the deviation dropped to 11%.

(iv) Sanyati Holdings Limited

The results for Sanyati were computed by removing the share price outcomes which fell outside the share price range of R0.19 - R4.04 for the period in question. Although the results produced a deviation of (767)%, the option value correctly predicts that the outcome from exercising the option would be negative at the time of expiration, as the options will be out of the money. The previous calculation failed to predict this as it indicated that the payoff would be positive. This is therefore a better technique to apply when determining the nature of the final option payoff.

Hull et al. (2004) and Radke (2007) correctly point out that the BSM model may also be used to determine the value of options values using this model were calculated and compared it to the actual results achieved.

The BSM produced good results for Adcorp, Assore and Santam. The deviations for Kumba, Primeserv and Sanyati were significant, however. Interestingly, the results for Kumba and Sanyati closely resemble the results from the BLM, once the initial proposed model is adjusted. Sanyati produced a positive option value, similar to the results of the initial BLM. This indicates that this model was not suitable for determining the option value.

Radke (2007) suggests that the ideal model is dependent on the companies' circumstances. She points out certain characteristics which would indicate what model would be best suited to the different companies. She suggests the following:

(a) BSM model is most suited to companies with:

- Limited Option Activity
- A stable Stock that Increases in a Predicted Pattern
- Stock Option expense which is Immaterial or Insignificant
- Low Option Holder Turnover
- Little History of Early Exercise

(b) BLM model is most suited to companies with:

- Significant Option Activity
- A high level of Volatility
- A stock Price that has periods of declining value as well as periods of increasing value
- Stock Option Expense which is significant
- High Option Holder Turnover
- Significant History of early Exercise

Results were obtained using both methods, it is easy to compare the results and then to determine which method is most suited to the different companies after considering the factors indicated previously.

The results were as follows:

Black Scholes Merton Model vs the Binomial Lattice Model						
	Adcorp Holdings Limited	Assore Phase	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
BSM as percentage of actual	14%	10%	-64%	99%	21%	934%
Binomial Lattice Model as a percentage of actual	13%	8%	-56%	95%	11%	-767%
Difference:	1%	2%	-8%	4%	11%	1700%

Table 4 Comparison of results produced by the BSM and Binomial Lattice Model.

In all instances, the BLM produced better option values than the BSM model. The differences for Assore and Adcorp were insignificant as the values were close. For Santam, the BLM outperformed the BSM model significantly. The same can be said of Primeserv Holdings, but the calculated option values for Primeserv were not significant, in a similar way to the results from the simple One-Step Binomial Model. This means that the contribution of the BEE transaction to share appreciation has been insignificant. Although both models produced results that did not reflect the actual results for Kumba and Sanyati Holdings Limited, the BLM produced a better result in both instances. It is interesting to note that the payoff from the Sanyati Holdings option using the BLM of R(77,280,000) represents the FCFE of R(73,019,000) calculated in Section 4 - Real Options (see above), when assessing the value of equity in a distressed firm. In both instances Kumba's payoff was not accurately calculated, but the BLM produced a lower misstatement.

On applying the characteristics identified by Radke (2007) to the companies above, it was found that only Adcorp and Assore had significant option activity during the period. In fact, they were actively pursuing additional BEE transactions. Apart from Sanyati Holdings Limited, which experienced steady declines in share prices during the contract period, the companies experienced share prices that had periods of declining value as well as of increasing value. In South Africa all BEE share option expenses are considered significant. IFRS 2 requires all expenses to be recognized in the entity which received the BEE credentials, irrespective of whose equity instruments are issued. The expense should also be recognized in the year in which the service conditions are satisfied. In addition, an expense should be corrected for any non-market-related vesting conditions. This can therefore be considered not very relevant when deciding which model to use.

All companies had high levels of volatility, but also had limited option holder turnover and little history of early exercise. For most companies the criteria for choosing the BLM (four of six) has been met. Only three of the five criteria required for use of the BSM model have been met; this suggests that the BLM is the model more suited to value the six transactions. This assumption is also supported by the results above.

7. Risk-Neutral Valuation

7.1 Risk-Neutral Valuation

The underlying principle of Risk-Neutral Valuation (RNV) is based on the method of Binomial Trees. The valuation will be performed using the risk-free rate (RFR) as the discount rate. A similar valuation will be performed using the90-day volatility as the discount rate. An additional approach will be included which entails using both the RFR and volatility as the discount rates, after incorporating the changes in volatility over the contractual term. The different volatilities applied to the different periods during the contractual term are similar to the values used in the BLM. The original 90-day historic volatility is used as the discount factor in order to prevent distortions caused by fluctuations in volatility during the contractual term. The results for both the RFR as a discount factor and volatility as a discount factor will be combined in this section.

It is expected the option values for Primeserv to approximate the actual values as it has been established, that based on the result of the BS and BLM methods, that Primeserv is an anomaly. Results for Primeserv will however be displayed for the sake of completeness. Negative option payoff values will be included for Sanyati as the company had a negative return on equity.

All results will be based on the percentage deviation of the computed value from the actual payoff.

7.1.1 Risk-Neutral Valuation Based on Hull (2009)

The results from all the applied methods were summarized and reviewed and produced a variety of outcomes.

Instances where the RFR was used as the discount rate produced the best results for three of the seven scenarios under examination. The amended methods of allowing for early exercise and computing an average option value based on the next two share prices outside the price range produced the best option values for Adcorp, Primeserv and Sanyati when incorporating negative values. Using the 90-day historic volatility as a discount rate produced two of the most accurate option values. These values, for Kumba and Sanyati, were determined based on the approach suggested originally by Hull (2009). When the changes in volatility over the contractual period were taken into account, and then followed by using the RFR as the discount factor, this produced the best option value for Assore. Using the original 90-day volatility under this section produced good results for two scenarios, Kumba and Santam.

It is interesting to note that using the average method produced four of the best overall results and in two instances the discount rate was the RFR. Early exercise using the RFR as the discount factor produced one best overall result, whilst using the volatility as a discount factor produced the remaining two best overall results. Incorporating the change in volatility during the contractual period proved to be significant as three of the most accurate results were produced when this was done.

Kumba and Sanyati produced the best results when the volatility was used as a discount factor. It is necessary to note that these transactions were previously valued using the real options approach as their contractual terms differ from those of the others. Kumba was treated as an "Expansion Option" and Sanyati was valued using the "Equity in a Distressed Firm" approach. Although none of the proposed methods could produce a result which mimicked the real option value, all the trees incorporating negative option values correctly predicted that in instances where the share price declined below R2.10, the option payoff will be negative. Assore, which was previously treated as a multi-stage expansion option, produced a deviation of (10%). Although this was not the most accurate result, the mispricing is insignificant. This could possibly indicate that volatility can be used as a discount factor to value real options exclusively.

In no instance did the original method proposed by Hull (2009) produce an accurate option value, but amendments of the method produced superior results. The claims made by Hull (2009) and Sundaram et al. (2011) that the option value always approximates the actual value using Risk-Neutral Valuation are thus questionable.

7.1.2 Additional Procedures Using Risk-Neutral Valuation

7.1.2.1 Control Variate Technique

The proposed equation by Hull (2009) was amended and did not determine both the value of the American option and European option using the same tree, as it has been observed that early exercise can severely misprice any given option value. The value of the European option was calculated using the same tree, option values from early exercise in instances where max$(S_T - K)$ is reached before maturity were removed.

The value of the American option is based on the initial values calculated using the method proposed by Hull (2009) and will thus correspond to the unadjusted risk-neutral values used in the previous section.

The tests were performed as before where the RFR and 90-day historical volatility are used as the discount rate. The effects of changes in volatility over the contractual term are also taken into account.

Instances where the historic volatility was used as the discount rate produced three of the best overall results. The three companies, Adcorp, Assore and Santam, all have 'vanilla' transactions, that is, they have no complicated contractual terms. When the RFR was used as the discount rate, it produced the remaining three best overall results. The mispricing on these transactions was however significant.

Although incorporating the changes in volatility over the period produced some of the best overall results, the mispricing on all of the transactions when the RFR or volatility is used as the discount factor proved to be significant. Only Assore produced an acceptable result with a deviation of only 9% from the actual when the volatility is used as the discount rate. It can therefore be conclude that incorporating changes in volatility over the contractual term does not significantly impact the option value under this method.

The Control Variate Technique involves using the pricing tree to calculate the difference between the European and American price. From the results above it can be can concluded that this is only achieved when the RFR is used as the discount rate and the transactions do not contain any real options elements.

7.1.2.2 Valuing the Options as European Options

Upon performing the Control Variate Technique, it can be noted that in certain instances where the option value is estimated as a European option, the final option value approximates the actual payoff. The value of the European option was determined based on similar principles to those described above. Four of the best overall results were produced when the RFR was used as the discount rate after incorporating the changes in volatility over the contractual term. The mispricing on the Adcorp and Sanyati option values, the latter value being determined when the volatility figure is used as the discount rate, is however significant.

Instances where the historic volatility was used as the discount rate produced good results. The pricing on the Kumba option produced the best result in comparison with other models, with the mispricing being only 11%. Sanyati also produced their best result using this method, but the option value was significantly mispriced. Both of these transactions contain aspects of real options in their contracts. This could indicate that using the volatility as the discount rate when dealing with real options is the best alternative.

The final best overall result was produced when the RFR was used as the discount rate. This was achieved on the Primeserv share, but the mispricing proved to be significant.

7.1.2.3 Setting $p = 0.5$

When the RFR was used as the discount factor, it produced four of the six best overall results. Using the volatility as a discount rate produced the remaining two best overall results, those for Kumba and Sanyati. As mentioned earlier, these companies contain elements of real options in their contracts. When changes in volatility are taken into consideration and the RFR is used as the discount factor, two of the most accurate results are produced.

On the whole, apart from the Kumba and Santam option values where the deviations were minimal, this method misprices the options.

7.2 Summary of All Results

In order to determine the best way to perform Risk-Neutral Valuation, the most accurate results, using each of the techniques explored above, will be compared to determine which model is superior. The results are as follows:

Summary of Results (based on deviations)	Adcorp Holdings Limited	Assore	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
(i) Risk - Neutral Valuation	3%	-6%	-22%	-51%	6%	396%	14%
(ii) Control Variate Technique	5%	3%	-95%	-1236%	14%	428%	476%
(iii)European Options	35%	23%	11%	24%	-17%	154%	127%
(iv)Alternative procedures	41%	32%	-21%	-202%	13%	229%	267%
Best Results	**3%**	**3%**	**11%**	**24%**	**6%**	**154%**	**14%**

Table 5 Summary of results produced by the different alternatives of Risk – Neutral Valuation.

The Risk - Neutral Valuation method produced three of the best overall results, each falling within the following categories:

1) Adcorp – Calculating the average option value with the RFR used as the discount factor.

2) Santam – Calculating the average option value with the volatility used as the discount factor. The changes in volatility over the contractual term should be taken into account.
3) Sanyati (with negative values) – Considering early exercise with the RFR used as the discount factor.

The Control Variate Technique produced one of the best results, that for Assore. This was achieved when the 90-day historical volatility was used as the discount factor.

Performing Risk-Neutral Valuation as a European option, whilst using the 90-day historical volatility as the discount factor, produced three of the best overall results. These were achieved for Kumba, Primeserv and Sanyati.

From the results above it can be noted that the risk-neutral methods proposed by Hull (2009) and Sundaram et al. (2011) did not produce the expected results, but rather that modifications of their suggested approaches produced more accurate results. Using the volatility as a discount rate produced the best results for contracts which contain elements of real options, such as Assore, Kumba and Sanyati. In other instances, the volatility significantly misprices the final option value.

The following propositions can therefore be made:

(1) In instances where transactions are 'vanilla' and are issued at a significant discount, using the RFR as the discount factor will produce the most accurate results when taking into account the average option value. When transactions are issued at a minimal discount, the volatility should be used as the appropriate discount rate.
(2) In instances when a transaction can be classified as "Equity in a distressed firm", using the RFR as the discount factor will produce the best option value when early exercise is taken into consideration.
(3) In instances where transactions can be classified as expansion or abandonment options, treating them as European options using Hull (2009) and using the 90-day volatility as a discount rate will produce the most accurate option value.
(4) When a transaction can be classified as a multi-stage expansion option then using the Control Variate Technique with the 90-day volatility as the discount rate will produce the most accurate option value.

Hull's (2009) claims that his risk-neutral approach would produce prices which are correct in all worlds could therefore not be justified. Sundaram et al (2011) used a similar approach to Hull, but they claimed that the final result will be precisely the arbitrage-free price of the option that is obtained by replication. This was not the case in this instance, thus neither claim could be justified.

7.3 Comparison of All Results Obtained Using the Different Models

Three different methods were applied in an attempt to obtain a model which prices the options most accurately. The best results produced by each of the three models are presented below. They are measure-based on the percentage deviation from the actual.

	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Binomial Lattice Valuation	13%	8%	-56%	95%	11%	-767%
Black Scholes Merton Model	14%	11%	-64%	-1085%	21%	934%
Risk Neutral Valuation	3%	3%	11%	24%	6%	14%
Best Results	**3%**	**3%**	**11%**	**24%**	**6%**	**14%**

Table 6 Comparison of results produced by different pricing methods.

In all instances Risk-Neutral Valuation produced the best results. This model is therefore considered the most accurate in comparison to previously applied methods when attempting to determine the final option payoff.

8. Trinomial Trees

8.1 Trinomial Trees as an Alternative Valuation Method

Trinomial Trees are used as an alternative method to Binomial Trees. Trinomial Trees will therefore also be covered, in addition to the Binomial Lattice Model and Risk-Neutral Valuation.

When performing the valuation using the six Trinomial Tree techniques presented in the literature review, the change in time is always taken as one year. In instances where the option's contractual term takes into account a fraction of a year, this is adjusted for in the final year when performing the discounting. In all calculations, the exercise price will be the value of K, calculated using the Monte Carlo method discussed in Section 4 - Real Options. This is consistent with the other techniques applied previously.

Separate calculations are not performed for European options exclusively, as the final option values resemble the value of an American option exercised only at maturity. It was established that early exercise options gave rise to lower results in most instances. The only instance where this needs to be considered is when the impact of forward rates is taken into account. For each of the techniques, the valuations will be performed using the RFR as the discount rate, after which the volatility will also be used as the discount rate. For some of the methods, the impact of the change in volatility over the contractual term will also be taken into account to determine if incorporating the change in volatility has any impact on the final option value.

The BSM, BLM and Risk-Neutral Valuation methods did not produce accurate results for Primeserv and Sanyati, owing to the failure of the models to capture the unique facets of these two transactions. these transactions will therefore be valued using Trinomial trees.

The basis of measurement is the percentage deviation from the actual of the final option value produced by the tree.

8.1.1 CRR Binomial Tree

This method produced insignificant mispricing on Adcorp, Assore and Santam, but other methods have previously produced better results.

8.1.2 JR Binomial Tree

This method produced the best results for Adcorp and Assore, producing virtually no deviations, but mispriced the other options. The best results for both companies were produced when the volatility was taken as the discount factor. There is a similarity between these two companies in that both transactions were initiated at a significant discount with minimal cash flows being exchanged to acquire the shares. This will be assessed to determine if a trend exists between volatility, the contractual terms and the share price.

8.1.3 Equal-Probability Tree

The most accurate results produced by this method occurred when the volatility was used as the discount factor. Adcorp and Assore produced results with close to 0% deviations in instances where the volatility was used as the discount rate. Similarly to the results produced by the JR tree, this supports the suggestion that there could be a link between the volatility and contractual features incorporated in Adcorp's and Assore's transactions. Santam also produced good results when the changes in volatility over the contractual term were taken into account, even though the RFR was used as the discount factor.

8.1.4 Hull (2009)

The most significant result from this exercise is the valuation of Kumba's share option which produced the most accurate results compared to the other methods. This was obtained when the volatility was used as the discount rate. This agrees with the finding previously introduced in Section 7 - Risk-Neutral Valuation, that in instances where the BEE transaction contains elements of an expansion option, the volatility should be used as the discount rate when determining the final option value.

Adcorp and Assore also produced good results, but only when the change in volatility over the contractual term was taken into account. This indicates that incorporating the volatility significantly impacts the option price. The final option value was obtained by taking the average option value. This result can therefore not be considered as a significant final outcome at expiry as other trees produced a result which resembled the actual.

8.1.5 Incorporating the Effects of Forward Rates

Formulas introduced in the initial section were therefore used to determine the impact on the option value of incorporating forward rates. For each transaction, the forward price one year from the initial inception of the transaction was calculated.

After this, the following equations were applied to determine into what category the probability of an upward or downward movement would fall:

1) $S_{i+1} < F_i < S_{i+2}$

Or:

2) $S_i < F_i < S_{i+1}$

For this calculation the change in volatility over the contractual term is ignored as it would entail calculating new probabilities for each contractual year. This was done because it was noted previously that the effect of incorporating the change in volatility does not always impact the final option value significantly. In addition, the effect of forward rates will be assessed when determining the effect of the volatility smile on the "Implied Trinomial Tree".

The option value used to perform the discounted option price at each node, apart from at the final node, will be the value of treating the option as American. This is done as negative payoffs will leave the option holder with no choice but not to exercise the option. It was noted that this value is often lower than the value produced by the continuing of $e^{-r\Delta t}(p_u f_u + p_m f_m + p_d f_d)$, and that in most instances this was higher than the American option value of $(S - K)$, and resulted in significant mispricing.

A condition of using the method above is that negative option values will need to be taken into account, as this thesis opts not to ignore negative probabilities and not to replace them with positive call option values as suggested by the authors. In instances where the option value is negative, the option value which will be used to perform the valuation would be:

The lower of the negative values produced by:

(a) $e^{-r\Delta t}(p_u f_u + p_m f_m + p_d f_d)$, And

(b) $(S-K)$,

This is done as it is assumed that the option holder will be left with no choice but not to exercise the option and instead to hold it to maturity.

In instances where the American option value is negative and the value of continuing is positive, the value of continuing defined by (a) above will be used to price the option.

Sanyati's value can be obtained from a given node prior to maturity for all the option valuation methods. At inception, the shareholder was not aware that the return on the share would be negative, as this could not be predicted at the time. For the sake of comparability with previous methods, this transaction is valued only at maturity and the early exercise option is ignored.

A. Option 1

Apart from the Sanyati transaction, all share prices increased from initial inception. all the Share prices at year 2 is expected, following an upwards movement from inception, to satisfy equation 1 above. the suggested equations were used to determine the value of p_i and d_i.

None of the results produced by this method were found to be favourable. The Equal-Probability Tree produces the lowest mispricing. An interesting observation is that all models produced results within the same range for Sanyati.

B. Alternative to Option 1

Owing to the severe mispricing as a result of applying the constraint $S_{i+1} < F_i < S_{i+2}$, this thesis attempt to use the other constraint, $S_i < F_i < S_{i+1}$, ignoring that the forward rate will exceed the stock price one year after inception, to determine if it will produce a better result.

Removing the initial constraint produced worse results than not doing so; it can therefore be inferred that the substitution does not have a positive impact in most instances. Instead, the results suggest that the Equal-Probability method is superior when performing those option valuations which incorporate forward rates.

The most significant result produced by this method is for Primeserv where the deviation is 0%. Apart from the real options approach, this is the only method which managed to produce a result of this nature. This could indicate that in instances where the forward rates are taken into consideration when pricing a transaction which involved no exchange of share capital, the final payoff will approximate the actual payoff when applying the JR Binomial tree.

B. Option 2

Apart from those for Primeserv, the methods suggested under option 1 failed to produce accurate results. On reviewing the calculations, it was noted that this is attributed to the current equations used to calculate the values of the probability of an upwards or downwards movement, which in most instances attribute the greatest probability to a downward move. Incorporating the second constraint mispriced the options severely because of the inclusion of negative probabilities. The authors suggest deleting all negative amounts and then replacing them with positive call option values. For this calculation, this work kept the negative amounts when determining the call option values.

However, owing to the severe mispricing given by the standard approaches described above this thesis will add the following amendments:

(a) Calculate the probabilities using the original techniques proposed by Derman, Kani and Chriss (1996).
(b) Using the values obtained by “a” above, attribute the largest probability to an upward move, the second largest to a middle move and the lowest to a downward move.

The approach is suggested as it is known that all shares, apart from Sanyati, increased in value since the inception of the BEE transaction. The upward move should thus receive the highest probability. The calculations will be determined using the original constraint of $S_{i+1} < F_i < S_{i+2}$. This was found to resemble the final option values best.

For the sake of completeness and comparability the calculations will also be performed for $S_i < F_i < S_{i+1}$ as this takes into account the likelihood of a large downwards move.

C.

(i) Maximum probabilities when $S_{i+1} < F_i < S_{i+2}$.

This method produces good option values for Adcorp, Assore and Santam, where the mispricing was less than 5% from the actual. An important consideration is the fact that all these transactions are ‘vanilla’ in nature. Kumba, Primeserv and Sanyati’s results remained significantly mispriced.

(ii) Maximum probabilities when $S_i < F_i < S_{i+1}$.

A summary of overall results produced by this method is presented below:

	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Si < F(i) < S(i+1)						
CRR Binomial Tree	34%	-12%	-207%	-563%	24%	1315%
Equal-Probability Tree	49%	-15%	-87%	-713%	20%	1236%
Hull (2009)	2%	-35%	-311%	-861%	6%	1672%
JR binomial tree	32%	-38%	-199%	-553%	-1%	1506%
Best Results:	**2%**	**-12%**	**-87%**	**-553%**	**-1%**	**1236%**

Table 7 Results obtained from the Incorporating of forward rates.

The Equal-Probability Tree method produced the largest number of results which were the most accurate in their category. The mispricing on some of the options was however very significant. The most accurate option values, when using the Equal-Probability Tree were produced by transactions which are 'vanilla' in nature, that is, the Adcorp, Assore and Santam transactions. No method produced accurate results for the Sanyati transaction, with this option producing the highest mispricing.

8.1.5.1 Combined Summary of All Four Methods

When the best results from each of the four options explored above is taken into account, the option values are as follows:

	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Original methods:						
S(i+1) < F(i) < S(i+2)	97%	91%	88%	61%	91%	115%
Si < F(i) < S(i+1)	112%	121%	163%	0%	29%	-278%
Alternative methods:						
S(i+1) < F(i) < S(i+2)	0%	-5%	-98%	-711%	-2%	1236%
Si < F(i) < S(i+1)	2%	-12%	-87%	-553%	-1%	1236%
	0%	**-5%**	**-87%**	**0%**	**-1%**	**1236%**

Table 8 Comparison of overall results produced by the different forward rates methods.

The Alternative methods produced more accurate results than the Original methods in most instances, with those calculations which kept the original constraint producing the most accurate results. The transactions which were most accurately priced were those for 'vanilla' BEE transactions such as Adcorp, Assore and Santam. Primeserv produced the best results when the original method was used. No method could produce accurate results for the Kumba and Sanyati transactions. The original method which kept the original constraint produced a lower mispricing than when the alternative procedures are used.

An important trend which emerged was that the Equal-Probability Tree method consistently produced the most accurate option values. The JR binomial tree also produced fairly good results. Using the methods of Hull (2009) and the CRR Binomial tree did not produce good results in any instance.

8.1.6 Real Option Analysis Using Mean Reversion

Hull (2009) also uses the concept of forward rates, introduced previously, to construct Trinomial trees. His method however differs from the approach used by Derman et al (1996). Using the principles of Hull (2009), it was noted previously that in instances where both the RFR and the volatility are used as a discount rate, different option values are produced. The same approach will be used here. As the contractual term in all instances will be limited to 3 years, because the forward rates are calculated for a 3-year period, the change in volatility over the contractual term will not be taken into account.

See below for a summary of combined results:

Real Options - Mean Reversion	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
Risk-free rate is discount factor	-11%	-43%	-333%	-20848%	-86%	1140%	1308%
Volatility is discount factor	12%	17%	-195%	-9054%	-27%	330%	367%
Best Results	-11%	17%	-195%	-9054%	-27%	330%	367%

Table 9 Results produced by the Mean Reversion method.

When the results of the two methods are compared, it becomes evident that using the volatility as the discount rate produced the best results. Another observation is that this method produced the lowest mispricing for Adcorp, Assore and Santam, the three 'vanilla' transactions. The mispricing on Primeserv, Kumba and Sanyati remained significant. An interesting observation is that the result for Kumba is significantly mispriced. This is inconsistent with the previous methods where using the volatility as the discount factor produced good results for Kumba.

The results from this method are however not better than the option values produced by previous methods. This could be attributed to the shorter contractual term, limits imposed on the variables a and σ and fixed values allocated to these variables. In addition, uniform probabilities are applied to various points on each node. This introduces an additional constraint.

8.1.7 Summary of results from the application of all methods

The results from all techniques are as follows:

Summary of Results (based on deviations)	Adcorp Holdings Limited	Assore	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
(i) CRR Binomial Tree	-9%	-9%	-165%	-782%	10%	799%	490%
(ii) JR Binomial Tree	0%	-1%	-245%	-694%	-40%	587%	79%
(iii)Equal-probability Tree	0%	2%	-211%	-821%	0%	467%	483%
(iv)Hull (2009)	-19%	-37%	-7%	-127%	3%	152%	124%
(v)Incorporating the effect of forward rates	0%	-5%	-87%	0%	-1%	1236%	-
(vi)Real Options - Mean Reversion	-11%	17%	-195%	-9054%	-27%	330%	367%
Best Results	**0%**	**-1%**	**-7%**	**0%**	**-1%**	**152%**	**79%**

Table 10 Comparison of results produced by the different Trinomial Tree methods.

From the above, it is evident that this method produces the best results for Adcorp, Assore and Santam compared to previous methods, with the deviations in mispricing being close to 0%. The JR Binomial Tree and Hull (2009) produced the best results. Apart from Kumba's share option value, Hull's (2009) results, produced results which are significantly mispriced. For each of the companies the most accurate option values were produced using the following methods:

1) **Adcorp:**
    a. JR Binomial method (incorporating the change in volatility with volatility as the discount rate.)
    b. Equal-Probability Tree (incorporating the change in volatility with volatility as the discount rate.)
    c. Incorporating the effect of forward rates (alternative methods using the constraint $S_{i+1} < F_i < S_{i+2}$)

2) **Assore:**
    a. JR Binomial method (using the 90-day volatility as the discount rate.)

3) Kumba:

a. Hull (2009) (using the 90-day volatility as the discount rate.)

b. Hull (2009) (incorporating the change in volatility over the period (using the 90-day volatility as the discount rate.))

4) Primeserv:

a. Incorporating the effect of forward rates (original methods using the constraint $S_i < F_i < S_{i+1}$)

5) Santam:

a. Incorporating the effect of forward rates (alternative methods using the constraint $S_i < F_i < S_{i+1}$)

b. Equal-Probability Tree (incorporating the change in volatility with the RFR as the discount rate.)

6) Sanyati:

a. Hull (2009) (incorporating the change in volatility over the period (using the RFR as the discount rate.)

b. JR Binomial Method (using the RFR as the discount rate.)

c. JR Binomial Method (incorporating the change in volatility over the period, sing the 90-day volatility as the discount rate.)

The JR method produced the most accurate results when the change in volatility over the contractual term was taken into account and the volatility was used as the discount rate. This method proved to be the best-fit method for transactions containing properties of real options as the Assore and Sanyati transactions were valued most accurately when using the method. For expansion options, Hull (2009) produced the best results. No other transaction was priced accurately using this method. It can be inferred that Hull (2009) is the best method to apply when valuing expansion options. The Equal-Probability Tree proved to be the most suitable method for 'vanilla' transactions. In most instances the change in volatility over the contractual terms must be taken into account. It was found that when the transactions were issued at a significant discount, the RFR had to be used as the discount rate. When minimal discounts took place, the volatility had to be used as the discount rate.

It becomes evident that incorporating the effects of volatility in the form of changes over the contractual term, or using the 90-day historical volatility as the discount rate, does have an impact on the final option values calculated. This concept will therefore be explored in the next section to determine if any relationship exists between the two variables.

For Sanyati's transaction, none of the methods could price this option accurately. However, when one determines the option value to be simply $S_0 - K$, at the initial node, the method returns an option value of R0.05. This however requires that the option should be treated as American and should be exercised at inception. When the Minutes issued by the board of directors of the meeting held on 25 July 2012 were reviewed to determine the future of Sanyati, it was noted that it was announced that all creditors would receive 7 cents in the Rand post-liquidation. This figure is significant as it resembles the value of R0.05 which was predicted to be the option payoff at inception. What can be confirmed is that in certain instances where early exercise is considered, all trees, for all applied methods, will have a value which approximates the final outcome at expiry when taking into consideration the final share price of R0.20.

When the option valuations were performed, it was noted that some option values, valued at inception, closely approximate the share price on transaction date. This will be explored later to determine if a possible relationship exists.

8.2 Impact of Volatility on Call Option Values

Notably from the results of option valuations performed, it has been established that the volatility has an impact on the final option value. The valuations for Adcorp, Assore and Santam produced virtually no mispricing when the volatility was taken into account. The 90-day historical volatility together with the values of the different volatilities calculated across the years were therefore obtained. It was found that no definite relationships emerged. Apart from Sanyati's 90-day historical volatility, all transactions had similar volatilities. Upon the review of the change in volatility across the contractual term, no distinct trends emerged.

This thesis therefore concludes that the excellent results achieved by the pricing of the option must be attributable to factors inherent in the contractual terms. The one aspect that all transactions have in common is that in instances where shares were issued, it was done at a discount, resulting in significant dilutions. What distinguishes Adcorp and Assore from the rest is that minimal cash flows took place to finance the transactions. The Adcorp shares were issued for 2.5 cents a share, while, although a price was agreed at inception of the contract for Assore, no cash flows took place between the company and the beneficiaries to act as financing for the shares. The Assore shares were financed solely by the bridging finance facility provided by Standard Bank.

Santam's shares were acquired at a price of R82 per share, with bridging finance being received from various sources. This sets it apart from the Adcorp and Assore transactions. However, when one reviews the contracts, one notices that the contractual terms in each of these transactions is 10 years or more. When the transactions are priced using the traditional Black-Scholes Pricing methods, they return values which resembles the actual. This thesis therefore conclude that where minimal cash exchange takes place to acquire the options, the longer the contractual term, the more accurate the option price will be when applying the various Trinomial methods.

Kumba's transaction is different from the rest as it involves a combination of different components. With the method of Damodaran (2002) described in the real options section, it was found that this transaction may be treated as an expansion option. Applying the principles of Risk-Neutral Valuation, it became evident that in instances where the volatility is used as the discount rate for expansion options, an accurate option value is returned. This also applies to Trinomial Valuations, as Hull (2009) accurately priced this option.

Sanyati remains an anomaly, as none of the methods applied could produce an accurate option value.

8.3 Relationship between the Option Value and the Share Price

It was noted that in certain instances, the final share option value, when working backwards in the tree, produces values which are close to the share price at inception. The results for each company, using each of the six techniques used to perform the trinomial valuation were compared. Each method will be assessed to determine if any links exist.

The resulting table is color-coded as follows:

(i) **Green** – This represents the best overall option value after taking all methods into account.

(ii) **Yellow** – This represents the best option value for the given valuation method.

(iii) **Blue** – This represents an option value which, although not the best value, still closely approximates the best value obtained.

Use is made of the following terms:

(i) "in–category" - defined as the specific valuation method

(ii) "in–segment" - defined as the discount rate category

See below for the various methods and their corresponding results.

I. CRR Binomial Tree

CRR Binomial Tree	Adcorp Holdings Limited	Assore 1	Assore 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
(1) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	16.88	10.47	102.31	45.43	0.12	35.60	1.51	0.86
Percentage of share price:	**46%**	**41%**	**47%**	**41%**	**58%**	**32%**	**57%**	**32%**
(2) 90 day historic volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	30.66	25.30	147.72	84.98	0.18	63.14	2.42	2.42
Percentage of share price:	**83%**	**99%**	**68%**	**77%**	**90%**	**57%**	**91%**	**91%**
(3) Incorporating change in volatility over the period								
(i) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	17.45	11.63	103.11	61.49	0.14	48.79	1.30	0.73
Percentage of share price:	**47%**	**45%**	**48%**	**56%**	**72%**	**44%**	**49%**	**27%**
(i) Volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	33.33	36.15	221.85	190.68	1.11	154.56	2.15	2.11
Percentage of share price:	**90%**	**141%**	**102%**	**173%**	**555%**	**140%**	**81%**	**79%**

Table 11 Comparison of the final option value and the share price using the CRR Binomial Tree.

The following trends emerged from the results presented above:

Trend 1: Where volatility is used as the discount factor, the more closely the option value represents the share price, the more accurate the option value.

Trend 2: Where RFR is used as the discount factor, the less closely the option value represents the share price, the more accurate the option value.

II. The JR Binomial Tree

JR Binomial Tree (2009)	Adcorp Holdings Limited	Assore 1	Assore 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
(1) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	16.97	11.62	140.39	59.13	0.15	60.51	0.91	-0.04
Percentage of share price:	**46%**	**45%**	**65%**	**54%**	**77%**	**55%**	**34%**	**-1%**
(2) 90 day historic volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	27.96	24.75	136.44	83.04	0.15	63.25	1.96	2.40
Percentage of share price:	**76%**	**97%**	**63%**	**75%**	**76%**	**57%**	**74%**	**90%**
(3) Incorporating change in volatility over the period								
(i) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	18.28	10.97	101.67	43.18	0.11	35.80	1.25	0.69
Percentage of share price:	**49%**	**43%**	**47%**	**39%**	**55%**	**32%**	**47%**	**26%**
(i) Volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	27.74	19.98	146.07	61.53	0.16	55.85	2.23	2.36
Percentage of share price:	**75%**	**78%**	**67%**	**56%**	**78%**	**51%**	**84%**	**89%**

Table 12 Comparison of the final option value and the share price using the JR Binomial Tree.

It was found that the results produced by this method strengthen the assumptions made when using the CRR Binomial Tree to value options. The following trends also emerged:

Trend 3: In instances where the percentage of share option value is a large percentage of the share price, the option value is more accurate when the discount rate is the RFR.

Trend 4: For all 'vanilla' transactions, the options are most accurately priced when the share option value is a high percentage of the share price and where either the volatility or the RFR is used as the discount rate.

III. Equal-Probability Tree

Equal - probability Tree	Adcorp Holdings Limited	Assore 1	Assore 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
(1) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	16.95	10.34	98.78	48.30	0.12	53.23	0.68	0.71
Percentage of share price:	**46%**	**40%**	**46%**	**44%**	**61%**	**48%**	**26%**	**27%**
(2) 90 day historic volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	27.53	20.12	132.76	71.33	0.16	79.18	1.17	2.20
Percentage of share price:	**74%**	**79%**	**61%**	**65%**	**82%**	**72%**	**44%**	**83%**
(3) Incorporating change in volatility over the period								
(i) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	22.49	15.73	111.77	53.43	0.13	39.95	1.84	1.23
Percentage of share price:	**61%**	**61%**	**52%**	**49%**	**66%**	**36%**	**69%**	**46%**
(i) Volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	31.23	23.63	154.30	72.53	0.18	59.41	2.77	2.90
Percentage of share price:	**84%**	**92%**	**71%**	**66%**	**89%**	**54%**	**104%**	**109%**

Table 13 Comparison of the final option value and the share price using the Equal–Probability Tree.

No additional trends emerged from the application of this method. The results however strengthened the assumptions of Trends 1 and 2.

IV. Hull (2009)

Hull (2009)	Adcorp Holdings Limited	Assore 1	Assore 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
(1) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	13.90	9.40	68.94	31.88	0.08	18.27	3.17	2.80
Percentage of share price:	**38%**	**37%**	**32%**	**29%**	**41%**	**17%**	**119%**	**105%**
(2) 90 day historic volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	6.10	2.30	45.70	15.95	0.03	12.39	0.10	0.05
Percentage of share price:	**16%**	**9%**	**21%**	**14%**	**15%**	**11%**	**4%**	**2%**
(3) Incorporating change in volatility over the period								
(i) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	15.19	13.16	80.49	65.65	0.63	38.51	3.65	3.36
Percentage of share price:	**41%**	**51%**	**37%**	**60%**	**313%**	**35%**	**137%**	**127%**
(i) Volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	6.77	3.51	41.07	18.28	0.04	22.19	0.28	0.23
Percentage of share price:	**18%**	**14%**	**19%**	**17%**	**19%**	**20%**	**11%**	**8%**

Table 14 Comparison of the final option values and the share price using Hull (2009).

This method produced results which contributed to the assumptions of Trend 2. An additional trend arose:

Trend 5: In instances where the transaction comprises an expansion option, using the volatility as the discount rate will produce the best option values.

V. Incorporating the effects of forward rates

For this method, only the RFR was used as the discount factor. The effect of using the volatility as a discount rate is ignored as it produced worse results than when using the RFR.

(a) Option 1

Incorporating the effects of forward contracts	Adcorp Holdings Limited	Assore Phase 1	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
S(i+1) < F(i) < S(i+2)							
(1)CRR Binomial Tree							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	30.95	22.88	195.38	74.37	2.77	73.94	2.24
Percentage of share price:	**84%**	**89%**	**90%**	**68%**	**1385%**	**67%**	**84%**
(2)Equal-Probability Tree							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	19.26	5.14	147.25	46.75	0.11	35.15	2.11
Percentage of share price:	**52%**	**20%**	**68%**	**43%**	**54%**	**32%**	**79%**
(3)Hull (2009)							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	27.73	12.72	169.47	70.51	0.16	40.80	4.18
Percentage of share price:	**75%**	**50%**	**78%**	**64%**	**79%**	**37%**	**157%**
(4)JR binomial tree							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	32.19	8.23	162.88	34.01	0.13	40.45	2.64
Percentage of share price:	**87%**	**32%**	**75%**	**31%**	**65%**	**37%**	**99%**

Table 15 Comparison of the final option values and the share price using Option 1.

A distinguishing factor is that the best results were obtained for each of the 'vanilla' transactions, which were Adcorp, Assore and Santam's transactions, when using this method. This gives rise to a new trend:

Trend 6: When a transaction contains elements which make it a 'vanilla' transaction, and a significant consideration is paid to acquire the shares, then the lower the option value, the more accurately the option will be priced when the RFR is used as the discount factor.

When insignificant cash flows take place, the volatility should be used as the discount factor. The option value will also return a higher percentage of the share price.

Hull's (2009) real option analysis section treats abandonment options as the inverse of expansion options. It has therefore been established that the expansion options are best valued using the volatility as the discount rate. Owing to the relationship between the two, it can be conclude that when pricing a BEE transaction containing elements of an abandonment option, and where there is an exchange of shares, it is best to value the transaction using volatility as the discount rate.

(b) Option 2

	Adcorp Holdings Limited	Assore Phase 1	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
S(i) < F(i) < S(i+1)							
(1)CRR Binomial Tree							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	18.35	7.12	155.12	52.69	0.09	30.09	2.26
Percentage of share price:	**50%**	**28%**	**71%**	**48%**	**44%**	**27%**	**85%**
(2)Equal-Probability Tree							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	14.10	4.82	159.30	32.04	0.09	31.94	1.24
Percentage of share price:	**38%**	**19%**	**73%**	**29%**	**43%**	**29%**	**47%**
(3)Hull (2009)							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	27.37	11.82	185.96	70.46	0.13	37.23	2.92
Percentage of share price:	**74%**	**46%**	**86%**	**64%**	**64%**	**34%**	**110%**
(4)JR binomial tree							
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66
Option Value	19.05	30.54	186.22	51.29	0.11	40.22	2.61
Percentage of share price:	**51%**	**119%**	**86%**	**47%**	**54%**	**36%**	**98%**

Table 16 Comparison of final option values and share prices using Option 2.

The alternative method produces the best overall result for Santam, when the JR method is used. The percentage option value as a percentage of the share price of 36% is the lowest in the category. This confirms **Trend 6**.

VI. Real Options – Mean Reversion

Real Options - Mean Reversion	Adcorp Holdings Limited	Assore 1	Assore 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
(1) Risk free Rate - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	30.95	22.88	195.38	74.37	2.77	73.94	2.24	1.93
Percentage of share price:	**84%**	**89%**	**90%**	**68%**	**1385%**	**67%**	**84%**	**73%**
(2) 90 day historic volatility - Discount Factor								
Share price at inception	37.00	25.60	217.00	110.00	0.20	110.57	2.66	2.66
Option Value	24.46	12.78	113.68	50.60	1.21	50.64	0.50	0.43
Percentage of share price:	**66%**	**50%**	**52%**	**46%**	**606%**	**46%**	**19%**	**16%**

Table 17 Comparison of final option values and share prices using Real Options – Mean Reversion.

This method does not produce any of the best overall results. The only comparison which can be made is in category comparisons.

Adcorp produces the best result when the RFR is used as the discount rate, and the percentage option value as a percentage of the share price is significantly higher than the other transactions. This is consistent with **Trend 6**.

The rest of the transactions produced the best results in the category when the volatility was used as the discount rate. This result is expected, because all six transactions were valued as real options containing characteristics of expansion options. This is consistent with **Trend 5**.

None of the transactions produced best overall results because of the contractual terms' constraint of 3 years, and also because of fixed probabilities and constant values imposed by this method. In addition, only Kumba contains elements of expansion options, from which it can be expected that the pricing to be most accurate using this method. This was however not the case, as Kumba produced the worst results. This method can therefore not be accepted as an appropriate method to value real options.

8.4 Summary of Results

From the above, it becomes evident that no single method is superior. Instead, they have all contributed to the confirmation the following relationships between the share price, option price, volatility and discount rates:

a. **Trend 1:** Where volatility is used as the discount factor, the more closely the option value represents the share price, the more accurate the option value.
b. **Trend 2:** Where the RFR is used as the discount factor, the less closely the option value represents the share price, the more accurate the option value.
c. **Trend 3:** In instances where the percentage of share option value is a large percentage of the share price, the option value is more accurate when the discount rate is the RFR.
d. **Trend 4:** For all 'vanilla' transactions, the options are most accurately priced when the share option value is a high percentage of the share price and where either the volatility or the RFR is used as the discount rate.
e. **Trend 5:** In instances where the transaction includes an expansion option, using the volatility as the discount rate will produce the best option values.
f. **Trend 6:** When a transaction contains elements which make it a 'vanilla' transaction, and a significant consideration is paid to acquire the shares, the lower the option value, the more accurately the option will be priced when the RFR is used as the discount factor.
When insignificant cash flows take place, the volatility should be used as the discount factor. The option value will also return a higher percentage of the share price.

8.5 Comparison of the Results Obtained Using the BSM, BLM, Risk - Neutral Valuation and the Trinomial Tree

Four different methods have been applied in an attempt to obtain a model which prices the options the most accurately. The best results produced by each of the three models are presented below. They are measure-based on the percentage deviation from the actual.

Summary of Results produced by all methods	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Binomial Lattice Valuation	13%	8%	-56%	95%	11%	-767%
Black Scholes Merton Model	14%	11%	-64%	-1085%	21%	934%
Risk Neutral Valuation	3%	3%	11%	24%	6%	14%
Trinomial Trees	0%	-1%	-7%	0%	-1%	79%
Best Results	**0%**	**-1%**	**-7%**	**0%**	**-1%**	**79%**

Table 18 Comparison of results produced using different valuation models.

The trinomial tree methods produced the best results for five of the six transactions. The only transaction which could not be priced accurately was Sanyati's transaction. The success of the trinomial tree methods is attributable to these five transactions containing properties which make the techniques of real options pricing the more appropriate method to price the transactions. Sanyati's share price declined sharply after the transaction date, which impacted the terms of the transaction. A 14% deviation from the actual was produced, but this occurred when early exercise was permitted. In no other instance was early exercise permitted. The trinomial method produced trees where negative option values exist which could resemble the actual if the early exercise option is taken. For the sake of comparability, all options were priced at maturity using the trinomial method. The result of 79% for Sanyati can be compared to the results produced by Risk-Neutral Valuation following the removal of the early exercise option, where the range of the deviation is between 127% and 476%. This means that the trinomial method is more suited to pricing the Sanyati Transaction. As the trinomial method produced five of the six best results, it is deemed to be the most appropriate option pricing model.

9. Additional Procedures

9.1 Improvement to Valuation Techniques

A number of papers have been published suggesting improvements to the techniques which have been used in this work, so as to improve the final option values produced by various models. The most relevant works will be reviewed being, those of Rubinstein (1994) and Leippold and Wiener (2004). The latter is important as Hull's (2009) Mean Reversion method is based on a similar concept and did not produce satisfactory results previously.

Rubinstein (1994) develops a new method for inferring risk-neutral probabilities from simultaneously observed prices of European options. His work is based on the amended Longstaff's Method and Shimko's methods.

It was found however that one could not apply Rubinstein's (1994) suggested method to this work, as different values for associated call options, with different strike prices but similar time to expiration, are not freely available in the South African market. In addition, the values which are currently published by the different companies were found to be not representational of the actual option value.

Leippold and Wiener (2004) followed a different approach to Rubinstein (1994). They developed a model for the implementation and calibration of a one-factor short rate model. The suggested approach was reviewed and found no inherent properties which would make it superior to the "General Tree-Building Procedures" suggested in Hull (2009), which incorporates the method of the Hull-White model applied by Leippold et al. (2004). When the results based on the forward induction method of Hull and White (1994) were compared to the other results obtained by the application of different methods, it was found that other methods produced far superior results.

9.1.1 Improvements Based on Results Obtained

In the case of the Risk-Neutral and Trinomial Tree valuations, it was noted that in instances where a transaction contains properties which make it a real option, the volatility, σ can be used instead of the risk free rate, r, as the discount factor. The Black-Scholes and Binomial Lattice methods were performed without evaluating this aspect. These methods will be revisited to determine if this phenomenon also impacts the results when applied to them.

I. Black-Scholes Pricing Method

The traditional Black –Scholes formulas were amended to the following:

$$c = S_0\,N(d_1) - Ke^{-rT}N\,(d_2)$$

Where:

$$d_1 = \frac{\ln(S_0/K) + (\sigma + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (\sigma - \sigma^2/2)T}{\sigma\sqrt{T}}$$

The expression $N(d_2)$ represents the probability that the option will be exercised in a risk-neutral world, so that $KN\,(d_2)$ is the strike price times the probability that the strike price will be paid. $S_0\,N\,(d_1)e^{rT}$ is the expected value in a risk-neutral world of a variable that is equal to S_T if $S_T > K$ and zero otherwise.

(i) Volatility as the discount rate

When the proposed amendments to the original formula, which incorporated the effects of volatility are applied, none of the values approximate any of the actual payoffs. This could indicate that the volatility has not been used correctly.

(ii) Examining $\boldsymbol{KN\,(d_2)}$

$KN\,(d_2)$ is the strike price times the probability that the strike price will be paid. In this paper, the payoff is driven largely by the value of K, calculated using the Monte Carlo simulation method. The volatility was used as the discount rate and examined the values produced by $KN\,(d_2)$ only, as $N\,(d_2)$ represents the payoff in a risk-neutral world. The results are as follows:

Black Scholes Calculation - *(K)*							
	Adcorp Holdings Limited	Assore Phase 1	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
BSM (volatility as discount rate)							
Value per option	1.15	5.90	38.96	18.36	0.01	45.39	0.14
Total number of shares	16 822 849	3 290 000	16 460 000	63 017 896	95 231 000	11 820 643	48 000 000
Total option value	**19 346 276.35**	**19 411 000.00**	**641 281 600.00**	**1 157 008 570.56**	**1 222 766.04**	**536 538 985.77**	**6 720 000.00**
			660 692 600.00				
Actual	**468 179 888.00**		**2 300 000 000.00**	**1 081 500 000.00**	**1 260 000.00**	**470 000 000.00**	**-8 918 400.00**
Difference:	**-448 833 611.65**		**-1 658 718 400.00**	**75 508 570.56**	**-37 233.96**	**66 538 985.77**	**15 638 400.00**
Deviation as a percentage of actual:	**-96%**		**-72%**	**7%**	**-3%**	**14%**	**-175%**

Table 19 Results produced when examining the terms $KN\ (d_2)$ in isolation.

All of the options are significantly mispriced except the ones for Kumba, Primeserv and Santam. Kumba's result matches the best result produced for this option using the Trinomial tree method. A different strike price, however, was used to achieve this result. The Kumba Iron Ore share only recorded a listed value on the JSE on 20 November 2006, a year after the implementation of the transaction. Using the initial strike price of R110.39, the mispricing becomes 32%. When this amended strike is applied to the method previously used, it was noted that it distorts values obtained previously. It is therefore only suited to this method. The result however, confirms Trend 5 which states:

Trend 5: In instances where the transaction includes an expansion option, using the volatility as the discount rate will produce the best option values.

Although Santam's result is good and somewhat unexpected, no significant inferences can be made, as prior methods have produced better results.

Primeserv's result of 3% is slightly worse than the result of 1% produced using the Trinomial tree method. It is however the best result produced by the Black-Scholes method. This transaction is the only transaction where there has been no issue of shares. It can therefore be assumed:

Trend 7: In instances where there has been no issue of shares, or where shares have been issued at a minimal discount, using the volatility as the discount rate for the term $KN(d_2)$ will produce a result which approximates the actual payoff at maturity.

(iii) Summary of best results

A summary of the best possible results produced by the Black-Scholes pricing method, incorporating the above, is as follows:

Black Scholes Merton Model						
	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Black Scholes Merton Model (original)	14%	11%	-64%	99%	21%	934%
Black Scholes Merton Model (volatility only)	29%	27%	433%	1336%	33%	-1465%
Black Scholes Merton Model (K)	-96%	-72%	7%	-3%	14%	-175%
Best Results	**14%**	**11%**	**7%**	**-3%**	**14%**	**-175%**

Table 20 Results produced by the different variations to the traditional BSM model.

II. Binomial Lattice

The effects of the impact of volatility on the payoff were incorporated by substituting σ in the place of r in the term e^{-rT}, which is used to discount the payoff in Radke's (2007) technique. The change in volatility across the contractual term is already taken into account as this is the basis used to calculate the maximum and minimum share prices reached at the end of each year of the contractual term. The results are as follows:

Binomial Lattice Valuation							
	Adcorp Holdings Limited	Assore Phase 1	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Binomial Lattice (Volatility as discount rate)							
Value per option	6.85	4.51	71.87	32.89	0.01	38.98	-0.88
Total number of shares	16 822 849	3 290 000	16 460 000	63 017 896	95 231 000	11 820 643	48 000 000
Total option value	**115 236 515.65**	**14 837 900.00**	**1 182 980 200.00**	**2 072 658 599.44**	**947 309.39**	**460 768 664.14**	**-42 240 000.00**
			1 197 818 100.00				
Actual	**468 179 888.00**		**2 300 000 000.00**	**1 081 500 000.00**	**1 260 000.00**	**470 000 000.00**	**-8 918 400.00**
Difference:	**352 943 372.35**		**1 117 019 800.00**	**-991 158 599.44**	**312 690.61**	**9 231 335.86**	**33 321 600.00**
Deviation as a percentage of actual:	**75%**		**49%**	**-92%**	**25%**	**2%**	**-374%**

Table 21 Results produced using the Binomial Lattice Valuation model.

Only Santam produced a good result. The other options were significantly mispriced. In the Black-Scholes section, Santam also produced good results. This could indicate another trend:

Trend 8: In instances where the share is issued at a minimal discount, using the Black-Scholes and Binomial Lattice Valuation model will produce an accurate option value when the historical volatility is used as the discount rate.

When the alternative procedures are applied to Radke's (2007) approach to the Binomial Lattice, it gives the following amended results:

(i) Kumba Iron Ore

The result returned a value which was a 21% deviation from the actual. This is an improvement on the result produced previously. As the mispricing is not significant, it supports the finding that volatility is the discount rate most suitable for expansion options.

(ii) Sanyati Limited

Application of this method to Sanyati's transaction produced a payoff that is a 33% deviation from the actual. Although this result is a substantial improvement on prior results, the mispricing remains significant. When this approach was applied to the remaining companies, the results worsened, and therefore they were discarded.

(iii) Summary of best results

A summary of the best results produced by the Binomial Lattice method is as follows:

Binomial Lattice Valuation						
	Adcorp Holdings Limited	**Assore Phase 2**	**Kumba**	**Primeserv Group Limited**	**Santam**	**Sanyati Holdings Limited**
Binomial Lattice Valuation	13%	8%	-56%	95%	11%	-767%
Binomial Lattice Valuation (Volatility as discount rate)	75%	49%	-21%	25%	2%	33%
Best Results	**13%**	**8%**	**-21%**	**25%**	**2%**	**33%**

Table 22 Results produced by the different variations to the Binomial Lattice Valuation model.

This method produced results in which there are still significant deviations from the actual, despite significant modifications to the original pricing method. It can therefore be accepted that this method is not suitable for pricing options. The method was however valuable in this study as the valuations methods which incorporate the change in volatility over the contractual term were based on the principles underlying this method. By using this approach, it was also discovered that limiting the valuation to the maximum share price reached during the option's contractual term produces better results in certain instances.

III. Mean Reversion

This is Hull's (2009) proposed method of valuing real options and forms a major part of Leippold's (2004) work. Ironically, it produced the worst results in comparison to the rest of the Trinomial tree methods. Previously it was noted that the following limitations are inherent in this model:

(i) Fixed term limited to 3 years

(ii) Consistent probabilities apply at different nodes

(iii) Constant volatility and a fixed variable for the constant.

In rare instances will one obtain forward prices exceeding a three year term. This thesis accepts this limitation. To deal with the limitation of constant probabilities, the variable for a was substituted, calculated previously in Section 7 (Risk-Neutral Valuation), a variable of 2, as suggested by Hull (2009) for j in the proposed equations. This was done in order to determine a specific probability which could be applied to each node.

It was found that the probabilities produced values which significantly mispriced the options. This approach was therefore rejected. Hull (2009) proposes using $a = 0.1$ and $\sigma = 0.20$. The equations were amended to take the actual historical volatility into account for each of the transactions. As this did not produce accurate results, it was revisited the model and made use of the proposed variables.

The results from the using the risk-free rate and the volatility as the discount rate are as follows:

Real Options - Mean Reversion							
	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
Risk-free rate is discount factor	-1%	-41%	-291%	-14109%	-86%	1058%	1308%
Volatility is discount factor	10%	-5%	-217%	-10406%	-31%	757%	1308%
Best Results:	**-1%**	**-5%**	**-217%**	**-10406%**	**-31%**	**757%**	**1308%**

Table 23 Results produced by the Real Options – Mean Reversion Valuation model when different discount rates.

The only transactions which are not significantly mispriced are the transactions for Adcorp and Assore. Although these results are good, other valuation methods have produced better results for these companies.

A comparison of the combined results of Mean Reversion using the historical volatility and a volatility of 20% yields the following:

Real Options - Mean Reversion							
	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited	Sanyati Holdings Limited (with negative values)
Risk-free rate is discount factor	-11%	-43%	-333%	-20848%	-86%	1140%	1308%
Volatility is discount factor	12%	17%	-195%	-9054%	-27%	330%	367%
Real Options - Mean Reversion (20%)							
Risk-free rate is discount factor	-1%	-41%	-291%	-14109%	-89%	1058%	1308%
Volatility is discount factor	10%	-5%	-217%	-10406%	-31%	757%	1308%
Best Results:	-1%	-5%	-195%	-9054%	-27%	330%	367%

Table 24 Results produced by the different variations to the Real Options – Mean Reversion Valuation model.

The overall results are not superior to those produced by valuation methods discussed previously. One can thus conclude that although this is the suggested method for valuing real options, it is not the most appropriate method.

IV. Application of the Black-Scholes model to the Sanyati transaction.

Sanyati Holdings Limited entered into a transaction whereby 48,000,000 of all ordinary shares would be issued to a BEE consortium. An additional 10,000,000 ordinary shares will be issued to the Sanyati 2007 Acquisitions Share Incentive Scheme. The share options are treated as share-based payments and have a "lock in" period of 12 months after the date of issue. The company's policy is to exercise all options after a period of 3 years, when they vest. The company opted to treat these options as call options and the value of each option calculated using the BS pricing model in Section 4 - Real Options was determined as R1. 54.

After inception, the share price of the transaction began to decline consistently across the maturity term. This thesis proposes that instead being treated as call options, the options should be treated as put options, as these options are valuable when the share price declines sharply.

It is proposed that the following amendments to the approach suggested by Black and Scholes (1973):

(a) The value for d_1 will remain (-d_1). Instead of determining the value from the distribution table for $N(x)$ when$x \leq 0$, the value should be read from the table for $N(x)$ when$x \geq 0$. This is done because it is assumed that x represents the share price. In no given instance will one invest in a share with a negative price.

When pricing the original call options the inputs are as follows:

S_0 = 2.67; K = 2.61; r = 7.64%; t = 5; and σ = 57.91%.

The value for d_1 and d_2 were determined as:

d_1 = 0.9639 thus $N(0.9639)$ = 0.8315

d_2 = -0.3349 thus $N(-0.3349)$ = 0.3707

The above is used to price a put option using the amendment previously introduced. The value of the put options is therefore defined by:

$p = Ke^{-rT}N(-d_2) - S_0\,N(-d_1)$

= $2.61^{-0.0764\,(5)}N(0.9082) - 2.67\;(0.5160)$

$= 0.2296$

Where:

d_1 = $1 - 0.9639 = 0.0361$ thus $N(0.0361)$ = 0.5160

d_2 = $1 - (0.3349) = 1.3349$ thus $N(1.3349)$ = 0.9082

The value of the call option does not approximate the total loss on the transaction of R(1.89) at the date of liquidation. Instead it approximates the actual share price, five years after the inception date, of R0.23.

It can therefore be assumed that in instances where put options are used to price transactions with declining share prices, which were originally treated as call options, one can estimate the final share price at maturity.

Cox and Rubinstein (1985) explain the meaning of the terms $N(d_1)$ and$N(d_2)$. $N(d_1)$ is defined as the factor by which the present value of the contingent receipt of stock exceeds the stock price. Therefore, the expected value, computed using the risk-adjusted probabilities, of receiving the stock at expiration of the option, contingent upon the option finishing in the money, is $N(d_1)$ multiplied by the current stock price and the riskless compounding factor. $N(d_2)$is defined as the risk adjusted probability of exercise.

Nielsen (1992) states that in instances where the exercise is completely random and unrelated to the stock price, the present value of the contingent receipt of the stock would be the current stock price multiplied by $N(d_2)$. This assumption is applied to value the put option, as it is assume that because the share price declined the decision to exercise would be based on an independent factor.

The value of a put option will thus be:

$p = Ke^{-rT}N(-d_1) - \ S_0\, N(-d_2)$

= $2.61^{-0.0764\ (5)}N(0.5160) - \ 2.67\ (0.9082)$

$= (1.5057)$

Where:

d_1 = $1 - 0.9639 = 0.0361$ thus $N(0.0361)$ = 0.5160

d_2 = $1 - (0.3349) = 1.3349$ thus $N(1.3349)$ = 0.9082

When the final value of R(1.5057) is compared to the actual loss on each share at maturity of R(1.89), the deviation is 19.48%. This is the first instance where one could find a value using the BS model which could approximate the loss at maturity.

9.1.2 Summary of Overall Results

The different approaches to the transactions were compared in order to determine the best method, that is, that which values BEE transactions the most accurately. The result produced by each model is displayed below. The yellow represents the best overall result.

Summary of Results produced by all methods	Adcorp Holdings Limited	Assore Phase 2	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Real Options Method	14%	10%	-1%	1%	0%	3%
Binomial Lattice Valuation	13%	8%	-21%	25%	2%	33%
Black Scholes Merton Model	14%	11%	7%	-3%	14%	-175%
Risk Neutral Valuation	3%	3%	11%	24%	6%	14%
Trinomial Trees	0%	-1%	-7%	0%	-1%	79%
Best Results	**0%**	**-1%**	**-7%**	**0%**	**-1%**	**33%**

Table 25 Overall summary of results produced using the different valuation methods.

It can be seen that the Real Options Valuation and the Trinomial Tree Method produced the highest number instances where the results were the most accurate reflection of the actual option payoff. The Real Options method produced more results which were close to the actual option payoff, but this method suffers from a drawback, as it is very laborious. In addition, the approach involving determining the option value based on the principles of $FCFE$ is not well known, and is not the norm in practice. However, the advantage of this method is that it takes into account factors not related to share issues, such as the effect of the disposal of significant divisions and the impact of liquidations on the share price. This is the reason that this method could produce very accurate results for Sanyati. All the other methods failed to produce a result which reflected the actual payoff as they were not engineered to take specific aspects into consideration. The One-Step Binomial Tree produced a deviation of 0% for Sanyati. This result, however, is ignored as the principle behind the one-step model states that in instances where the option value is negative, one should allocate a value of 0% to the option as the option is out of the money. As a result of this one cannot say that this valuation method is superior.

The Trinomial Trees produced good results as well. This method is based on principles which can easily be applied on a day to day basis. One can therefore consider this to be the most appropriate method of valuing options.

10. Application of Net Present Values to obtain final option values

10.1 Using Option Pricing Methods to Value Flexibility

Option pricing methods have been developed to value projects as they are deemed superior to the traditional Direct Cash Flow (DCF) approaches. This is because they take into account the value of flexibility when determining the investment value. In certain instances the investment may not be made immediately. Instead, it could be advisable to defer the project to the future. Option pricing methods will allow for this flexibility factor, whilst the DCF method assumes that the project ignores this aspect.

To perform the valuation using real options, use will be made of the techniques proposed by Copeland, Koller and Murrin (2000). The following are proposed when performing the real options valuation:

$$NPV = \max_{t=0}\left[\frac{Expected\ cash\ flows}{Cost\ of\ Capital}, 0\right]$$

$$Option\ value = Expected\ \left[\max_{t=0}\left(\frac{Expected\ cash\ flows}{Cost\ of\ Capital}, 0\right)\right]$$

The NPV is the maximum, decided at present, of the expected discounted cash flows or zero, while the option value is the expected value of the maximums, decided when the information becomes publicly available, of the discounted cash flows. The authors propose that the value of a project using option pricing will always exceed the value of a project using NPV.

For all calculations it is assumed that the NPV will be the amount paid at inception of all BEE transactions to acquire the shares or the investment. The expected cash flows have been obtained using the Free Cash Flow valuation techniques proposed by Copeland et al (2000) and Damodaran (2000). The sum of the two will give the total undiscounted return on the transaction. Refer to *Appendix F* for full disclosure of the various techniques.

The source of all information used to perform the calculation is the Annual Report for the given company for the specific year ended. The group results were used at all times. In all instances, the impact of deferred tax assets and liabilities has been ignored as the effect is already taken into account when determining the taxation figure in the Income Statement or the Statement of Comprehensive Income. The impact of the Foreign Currency Cash flows was only considered when there was a specific adjustment in the Cash Flow Statement or Notes to the Income Statement or the Statement of Comprehensive Income. Current liabilities always exclude the current portion of long-term interest-bearing debts. The effect of bank-borrowings is always netted off against cash and cash equivalents.

When estimating the Cost of Capital (WAC), the Gordon Growth method was used to estimate the Cost of Equity. In instances where it produced an impossible value, the growth component was always estimated using the dividend yield (DY), net income, or by estimating the growth in annual dividends declared over the option period. Refer to *Appendix C* for the valuation technique.

In all instances, the value determined to be the final option value was compared to:

a) The share price at acquisition
b) The share price at maturity date
c) The strike price(K), which was determined using the Monte Carlo method.
d) The actual option payoff per share at maturity date

In all instances, in addition to the WAC, the risk-free rate (RFR) and the historical volatility (volatility) at inception were used as discount factors. This was done to be consistent with previous valuation methods.

For the sake of completeness, the non-discounted sum of the NPV plus the cash flows over the given period was compared to the actual total option payout at maturity. A good result would constitute a deviation below 20%. The results will be given in the next section.

10.2. Results using Copeland, Koller and Murrin

Applying the valuation technique to the method proposed by Copeland et al. (2000) gives the following results:

Adcorp

To acquire the Adcorp shares, subscribers only paid 2.5 cents per share. This represents a significant discount on the prevailing share price. In addition to this value, the actual share price on the acquisition date is used to determine what the NPV would have been, had the shares not been issued at a discount.

a) Alternative 1 (2.5 cents per share)

Using volatility as the discount rate, the final value gives an accurate estimation of what the actual share price at inception and maturity date will be, as the deviation here was the least significant. The estimation of the strike price and the final option payoff per share is also the most accurate when this is taken into account. The deviation of the strike price was the lowest, meaning that this method best estimated this value. The WAC and RFR produced poor results in all instances.

b) Alternative 2 (R37.00 per share)

The RFR and the WAC gave the most accurate estimates in this instance. No good results were produced when taking volatility into account. The RFR produced good results for all four categories measured, whilst the WAC could only accurately estimate the share price at maturity and the final option value per share. The RFR however produced superior results in these categories with deviations of only 4%.

When the results are compared to those of "Alternative 1", where the volatility performed well, it was found that the current approach produced better results. This could indicate that in instances where shares are issued at a significant discount, using the volatility as the discount rate will produce accurate estimates of all four variables. In instances where no discounts or minimal discounts are part of the contract, using the RFR as the discount factor will be the most appropriate method to estimate the share price at acquisition or maturity, the strike price and the final option value per share.

Assore Limited

A unique facet of the Assore valuation is that the share subscription consisted of three empowerment transactions. In addition, post-acquisition, the shares were split one for five. Previously, multi-stage valuations were performed to take into consideration the effects of phase 1 and phase 2. Phase 2, however, produced negative NPVs, thus only one valuation was performed for the transaction as a whole. No cash transfer took place to acquire the shares as it was funded by bridging finance provided by Standard Bank. The NPV is thus assumed to be zero. Two calculations were performed to accommodate the total final number of 16,460,000 shares issued after the share split, and the total before the split of 7,280,837.

a) Alternative 1 (16,460,000 shares)

The best estimates were produced by this method when the share price at acquisition was estimated and the strike price whilst using the volatility as the discount rate. The WAC and RFR produced the best estimates of the final option payoff per share at maturity. The deviation was greater than 20% for all results; the alternative method will therefore be reviewed in order to determine if it produces better results.

b) Alternative 2 (7,280,837 shares)

The RFR produced accurate estimations of the share price at inception, maturity and the strike price, whilst the WAC produced good results for the share price at inception, strike price and the option payoff. Other than for the share price at maturity, the WAC produced the lowest deviations.

It can therefore be assumed that when the NPVs are applied to multi-stage expansions, and if all changes in the share capital post-inception are ignored, then using the WAC and the RFR will produce accurate estimates of the share price at inception and maturity, strike price and the option payoff.

Kumba Iron Ore Limited

This transaction is described as an expansion transaction, as many components were acquired in this BEE transaction. In most instances, the deviations were significant, apart from the estimations of the strike price where the WAC and RFR produced minimal misstatements, with the WAC producing the best result. The sum of R14.185 billion for the Free Cash Flows across the contractual term is significant as this figure closely resembles the disclosed transaction value of R14 billion at the inception of the transaction.

Primeserv

This transaction mimics an abandonment option as a division was sold to the BEE partner. No share issue took place.

In this instance using the WAC as the discount rate produced the most minimal deviations when estimating the share price at acquisition and the strike price. The RFR produced good results for similar categories. In no instance did the volatility produce good results, but it managed to produce a value of R0.04. This closely resembles the actual payoff of R0.01.

Santam Limited

Santam's shares were acquired at a discount of 10% of the share price. Using all 3 discount factors produced good estimates for the share price at maturity and the strike price. Using the volatility as the discount rate produced the best results when estimating the share price at maturity whilst using the RFR as a discount rate best estimated the strike price. None of the methods produced accurate estimates of the final option payoff.

Sanyati Holdings Limited

This transaction is different owing to the company liquidating before maturity of the transaction. In order to determine the WAC, the information relating to the cost of equity and all returns required by the shareholders was obtained from McGreggors BFA. In no instance did any method prove superior. What can however be confirmed is that the negative computed values indicate that the transaction is not viable and should not be entered into because of the negative results produced under all categories. What can be confirm is that the computed values best represent the total loss experienced when one adds the final loss of 18.56 cents per share to the share price at acquisition date of R2.67.

B. Trends from the valuation

The following observations have emerged from the valuations performed previously:

Adcorp and Santam are two 'vanilla' transactions. In the instance of Adcorp, where the shares were acquired for almost no cost, the volatility gave the best estimate of the share price at acquisition and maturity as well as the strike price and the final option payoff. When a consideration is paid, the RFR produces good estimates for all categories, whilst the WAC accurately estimates the share price at maturity and the final option payoff.

In the case of Santam, using the volatility as the discount rate produced accurate estimates of the share price at maturity only, whilst the RFR could accurately estimate the strike price.

The following can therefore be assumed:

Trend 1: In instances where the transaction is a 'vanilla' transaction and the shares are acquired at a significant discount, using the volatility as a discount rate will give an accurate estimate of the share price at inception and maturity, strike price and the final option payoff.

Trend 2: In instances where a consideration which represents the share price at inception date is paid, using the RFR as a discount rate can accurately estimate the share price at acquisition and maturity, the strike price and the final option payoff. The WAC can only estimate the option value at maturity and the share price at maturity.

Trend 3: In instances where the transaction is a 'vanilla' transaction and the shares are issued at a minimal discount, using the volatility as a discount rate will give a good estimate of the share price at maturity whilst the RFR will accurately estimate the strike price.

Trend 4: Where the transaction contains properties which make it a multi-stage expansion option, using the WAC and RFR will allow the accurate estimation of the strike price, with the WAC producing the most accurate value.

Trend 5: Where the transaction contains facets which make it an expansion option, using the RFR will accurately estimate the share price at inception and maturity as well as the strike price. The WAC will produce good estimates of the share price at inception, the strike price and the option payoff per share.

Trend 6: In instances where the transaction is an abandonment option, using the WAC and RFR will accurately estimate the share price at acquisition and the strike price. The WAC will produce the better results.

Trend 7: When performing the valuation for a firm in financial distress, none of the discount rates will produce an accurate estimate. Instead, negative final values will be produced which indicates that the investment decision is not appropriate. When the cost per share at acquisition and the final value are added, the result will give a good approximation of the total loss per share.

10.3 Results using Damodaran

Applying the valuation technique to the method proposed by Damodaran (2002) gives the following results:

Adcorp

a) Alternative 1 (2.5 cents per share)

This result is a vast contrast to that of Copeland (2000), as the volatility produced no good result. The WAC and RFR produced good estimates of the share price at acquisition, whilst the WAC produced a good estimate of the strike price.

b) Alternative 2 (R37.00 cents per share)

In no instance did this method produce good results for any of the proposed categories. It can therefore be assumed that this valuation method is not suited to this transaction.

Assore Limited

a) Alternative 1 (16 460 000 shares)

This method produced results which were a total contrast to the previous results. No method produced good results except when the volatility is used as the discount rate to estimate the option value per share. Using the approach of Copeland et al (2000) also failed to estimate the final option value per share accurately. It can therefore be assumed that in instances where the transaction is a multi-stage expansion option, then using the volatility as the discount rate will produce an accurate estimate of the final option value.

b) Alternative 2 (7,280,837 shares)

Unlike previous methods, this method produced no accurate results.

Kumba

For Kumba, both the WAC and RFR could accurately estimate what the final share price at maturity is expected to be. In instances where volatility is used as the discount rate, it produces an accurate estimate of the strike price.

Primeserv

Using the volatility as a discount rate previously produced no good results. In this method, it produced the only good result, as it produced the best estimates of the strike price and the share price at acquisition. No accurate prediction of the final payoff per share could be made.

Santam

In this instance, none of the 3 discount factors could accurately predict the final option payoff per share. When volatility was used as the discount rate, the share price at inception could accurately be estimated, as well as the strike price. The WAC produced accurate estimates of the share price at maturity and the strike price, whilst the RFR could only produce good results for estimating the share price at maturity.

Sanyati Holdings Limited

The method when applied to this company gives results which are consistent with the previous results, as in no instance was the final payoff accurately predicted. The computed values however give a good estimate of the difference between the share price at maturity date and the acquisition price at inception date. This represents the actual loss resulting from entering into this transaction.

B. Trends from the valuation

The results produced by this method contradicted the trends previously identified. This is largely owing to the application of different techniques when determining the Free Cash Flow amount. From the results obtained, one can assume the following:

Trend 1: In instances where the transaction can be considered a 'vanilla' transaction and the shares are acquired at a significant discount, using the volatility will accurately estimate the option value per share at maturity.

In instances where the WAC and RFR are used, the RFR will accurately predict the share price at acquisition whilst the WAC will accurately predict the share price at acquisition and the strike price.

In instances where the shares are acquired at no discount, no good estimates will be produced.

Trend 2: In instances where the shares are acquired at a discount, using the volatility will accurately predict the share price at inception and the strike price. The WAC will accurately predict the share price at maturity and the strike price whilst the RFR will accurately estimate the share price at maturity.

Trend 3: In instances where the transaction is a multi-stage expansion option, using the volatility as the discount rate will accurately predict the final option payoff per share.

Trend 4: In instances where the transaction is an expansion option, using the volatility as the discount rate will accurately predict the strike price.

Trend 5: In instances where the transaction is an abandonment option, using the volatility as the discount rate will accurately predict the strike price and the share price at acquisition.

Trend 6: In instances where the transaction is for a firm facing financial distress, all 3 discount factors will produce negative values. The total loss for each transaction will serve as an estimate of the value of the share price at maturity date less the cost of acquiring the shares.

10.3.1 Results using Damodaran (Alternative 1)

In addition to the approach mentioned above, Damodaran proposes two additional methods which can be applied to determine the FCFF (refer to *Appendix C* for details). These approaches failed to produce accurate results for any of the transactions. The only number which can be considered to be of any significance is the amount R0.01, which arose for Primeserv when the volatility was used as the discount rate. This number approximates the final option payoff per share.

B. Trends from the valuation

In no instance did this approach produce good results for any of the companies. It is therefore doubtful whether this method is appropriate for the purpose of determining the value of a given company. The primary shortcoming of this method is that the value of the company is determined based on the following year's free cash flow only. The methods of Copeland et.al (2000) produced better results as they take the entire cash flows over the option term into account.

10.3.2 Results using Damodaran (Alternative 2)

This method produced results similar to Alternative one and only produced good results for the Primeserv transaction. This method produced good results, using all three discount factors, when estimating the share price at acquisition and the strike price. The WAC produced the best results in both instances.

The results for Sanyati approximated the actual loss per share at maturity. This is consistent with all the previous methods.

B. Trends from the valuation

From the results given above one can assume the following:

Trend 1: In instances where the transaction constitutes an abandonment option and where there has been no issue of shares, using the WAC, RFR and volatility as discount factors will accurately estimate the share price at acquisition and the strike price.

Trend 2: When valuing a firm in financial distress, all three discount factors indicate that the payoff from the investment will be negative. The payoff will approximate the loss per share, that is, the share price at valuation date less initial cost of the share.

10.4. Comparison of the Non-discounted Option Value to the Actual Option Payoff at Maturity

The final values produced by each method were compared to the actual value to determine if any relationships exist. The results are as follows:

NPV Plus FCF Comparison To Actual							
	Adcorp (a)	Adcorp (b)	Assore (a)	Kumba	Primeserv	Santam	Sanyati
A) Copeland							
Sum of NPV + FCFF	1 401 019 000	759 647 311	2 017 588 000	14 185 000 000	27 246 000	1 679 000 000	-41 036 000
Actual Option Value:	468 179 888	468 179 888	2 300 000 000	1 081 500 000	1 260 000	470 000 000	-8 918 400
Difference:	932 839 112	291 467 423	-282 412 000	13 103 500 000	25 986 000	1 209 000 000	-32 117 600
Deviation:	**199%**	**62%**	**-12%**	**1212%**	**2062%**	**257%**	**360%**
B) Damodaran							
Sum of NPV + FCFF	955 831 760	314 460 071	11 026 363 190	53 505 980 000	96 348 440	2 482 520 000	3 575 850
Actual Option Value:	468 179 888	468 179 888	2 300 000 000	1 081 500 000	1 260 000	470 000 000	-8 918 400
Difference:	487 651 872	-153 719 817	8 726 363 190	52 424 480 000	95 088 440	2 012 520 000	12 494 250
Deviation:	**104%**	**-33%**	**379%**	**4847%**	**7547%**	**428%**	**-140%**

Table 26 Comparison of results produced using Copeland and Damodaran valuation methods.

In this comparison, all transactions had significant deviations apart from the Assore transaction using the approach of Copeland (2000), where the deviation was only 12%. The valuation of R14,185,000,000 for Kumba approximates the actual published transaction value at inception. One can therefore assume the following:

Trend 1: In instances where a transaction constitutes a multi-stage expansion option, the sums of Free Cash Flows over the transaction period plus the NPV will be equivalent to the actual payoff at the end of the period.

Trend 2: In instances where a transaction constitutes an expansion option, the sums of Free Cash Flows over the transaction period plus the NPV will be equivalent to the strike price at inception.

10.5 Comparison of the Different Valuation Models

Different approaches produced different trends. This indicates that the valuation method and the discount factor have a substantial impact on the final value produced. The results produced for each of the identified categories by the different models were compared to identify which model produced the best results.

From the review of all combined results, it becomes evident that the method applied by Copeland et al. (2000) produced the lowest deviations in comparison with other models, except in the cases of Adcorp and Sanyati when the NPV is significantly discounted. In these instances Damodaran (2002) produced more accurate results. It was also noted that in instances where the transaction is accompanied by heavy capital expansion projects, acquisitions and non-cash transactions, using Copeland et al. (2000) will produce the most accurate results as the model has been designed to take these factors into account. When the results were summarized the proposition by Copeland et al. (2000) which states that the Cost of Capital is the most appropriate discount factor were taken into account. The results by category were as follows:

a) Share Price at Acquisition

In this instance, using the WAC as the discount rate produced the most accurate estimates, with Damodaran's approach producing two of the three best results. In one instance, the volatility produced one accurate result. This thesis therefore examined the successful estimates closely to identify whether they contain properties which could influence the valuations.

The three successful companies were Adcorp (2.5 cents a share), Assore (16,460,000 shares) and Primeserv. The companies belong to different categories as one is a 'vanilla' option, whilst the others are expansion and abandonment options respectively. The one property that they have in common is that at the inception of the transaction, minimal or no cash transfers took place to acquire the shares. One can therefore assume the following:

Trend 1: In instances where minimal cash is invested to acquire shares, using the WAC as the discount rate will produce a result which is close to the share price at acquisition.

The volatility produced a good result for Santam only. One can thus assume the following:

Trend 2: In a 'vanilla' transaction where the shares are issued at a minimal discount, using the volatility as the discount rate will produce a result which mimics the share price at transaction date.

b) Share price at maturity

Only four companies produced results which could estimate the share price at maturity: Adcorp (share price R37.00), Assore (7,280,837 shares), Primeserv and Sanyati. Primeserv produced this result using Damodaran (2002) Alternative 2, where the WAC was used as a discount rate. One can thus assume the following:

Trend 3: When a transaction constitutes an abandonment option and the free cash flow takes into account capital expenditure, and when it is adjusted for a growth factor which only considers next year's cash flows, then using the WAC as a discount rate will produce an accurate estimate of the share price at maturity.

Adcorp (share price R37.00) and Assore produced good results when using Copeland (2000) and when the RFR was used as the discount factor. One can thus assume the following:

Trend 4: When a transaction is a 'vanilla' transaction and the shares are acquired at a minimal discount, using the RFR as a discount factor will produce an accurate estimate of the share price at maturity.

The same will hold true for a multi-stage expansion option.

Santam produced an accurate estimate of the share price at maturity, but in this instance volatility was used as the discount rate. One can therefore assume the following:

Trend 5: When a transaction is a 'vanilla' transaction and the shares are issued at a minimal discount, using the volatility as a discount factor will produce a result which mimics the share price at maturity.

c) Strike Price

Adcorp, Assore, Kumba and Santam were the only companies for which the strike price could be accurately predicted. For Assore and Kumba these estimates were produced when the WAC was used as a discount factor. Assore's result was produced using the approach of Copeland (2000), while Kumba's was achieved using Damodaran's (2000) approach. One can thus assume the following:

Trend 6: When the transaction is a multi-stage expansion or 'vanilla' expansion option, using the WAC as the discount rate will produce an accurate estimate of the strike price.

Trend 7: In instances where the transaction constitutes a 'vanilla' transaction, and where the shares have been issued at a minimal discount, using the RFR will return the share strike price.

Trend 8: When the transaction is a 'vanilla' transaction and the shares have been issued at a price which closely approximates the actual, using the volatility as the discount rate will return the strike price.

d) Option value per share

Only for two companies could the methods accurately estimate the final option payoff. This is ironic as estimating the final option value is the fundamental aim of this exercise. It is also interesting that in both instances the estimates were obtained when the RFR and the volatility were used as the discount factors. Adcorp (2.5 cents per share) produced an accurate result when the RFR was applied to Copeland's (2000) methodology. Assore achieved one using Damodaran's (2002) approach, which used volatility as the discount rate. Kumba previously had an accurate estimate when the undiscounted final value is compared to the actual payoff. It is therefore surprising that an accurate estimate could not be obtained for it in this instance. One can therefore assume the following:

Trend 9: Where the transaction is a 'vanilla' transaction and the shares are acquired at a significant discount, using the RFR as a discount factor will accurately estimate the final option payoff.

Trend 10: Where the transaction is a multi-stage expansion option, using the volatility as the discount rate will return an accurate estimate of the final option payoff.

It was noted from a review of the results that the application of different methods gave rise to different trends. The results for Sanyati were similar in all sections, which leads us to assume that these assumptions hold true in that case. Apart from the generalizations produced in the last section, one can say that in instances where certain techniques are applied, the identified trends above should emerge.

11. Implied Trinomial Trees of the Volatility Smile

The Black-Scholes model suffers from various limitations, one being the inability to produce an accurate option value when used in its originally-intended form. Traders therefore adjusted the formula to allow the volatility, used in the equation to price an option, to depend on the strike price and the time to maturity of the option. This phenomenon can be represented graphically as the implied volatility. The implied volatility of an option can therefore be explained as the relationship between a volatility smile and the risk-neutral probability distribution being assumed for a future price. Typically the market-implied volatilities of share index options often show that the “volatility smile” decreases with the strike level and increases with the time to maturity.

Derman, Kani and Chriss (1996) propose that their suggested CRR Trinomial Tree, together with the constant local volatility assumption in Black-Scholes theory, will eliminate the volatility smile. Previously the CRR Trinomial Tree produced results which were not very accurate. This thesis will therefore use the theorem on which the CRR Trinomial Tree is based to construct Implied Trinomial Trees in order to determine if the results are desirable. In the CRR method, the upward (S_u), middle (S) and downward movements (S_d) are presented by the following:

$$S_u = Se^{\sigma\sqrt{2\Delta t}}$$

$$S_d = Se^{-\sigma\sqrt{2\Delta t}}$$

Where:

σ and t represent the 90-day historic volatility and the time to maturity respectively. In all instances, S represents the initial share price and remains constant for the middle nodes throughout the contractual term.

The transitional probabilities are represented by:

$$p_i = \frac{e^{r\Delta t}C\left(S_{i+1,t_{n+1}}\right) - \sum_{j=i+1}^{2n}\lambda_j(F_j - S_{i+1})}{\lambda_i(S_{i+2} - S_{i+1})}$$

$$q_i = \frac{F_i - p_i(S_{i+2} - S_{i+1}) - S_{i+1}}{S_i - S_{i+1}}$$

$$m = 1 - p_i - q_i$$

Where:

$r =$ the known riskless forward interest rate between level n and $n+1$

s_i = the known share price at the node (n, i)

F_i = the known forward price at level $(n+1)$ of the share price s_i at level n

S_i = the known share price at node ($+1, i$) and also the strike for options expiring at level $(n+1)$

λ_i = the known Arrow-Debreu price at node (n, i)

C = the known Call option value

The known share price, S_i, is the same price used in the original CRR Trinomial Tree valuations. It represents the price at announcement date. As the known forward prices could not be obtained, call option values or the Arrow–Debreu prices as these are not readily available. the Black–Scholes call option values, calculated in Section 6 – Binomial Lattice Model, were therefore used to represent the known call option value. The known forward price was based on the following:

$$F_o = S_0 e^{(r-\delta)\Delta t}$$

Where:

S_0 = the known share price at inception;

δ = the dividend yield;

r and t remain defined as before.

For the sake of consistency, it was assumed that the dividend yield remains zero, as before. It was noted that the changes in share prices for all transactions were significant during the contractual terms. A unique forward rate on each anniversary date, using the prevailing share price and risk-free rate on that date was therefore calculated.

As the Arrow-Debreu prices were not readily available, the Derman and Kani (D&K) algorithm was used to determine them. In the D&K algorithm, the Arrow–Debreu price, $\lambda_{n,i}$, is the price of an option that pays one unit payoff in one and only one state i at nth level, and otherwise pays 0. The Arrow-Debreu price can be obtained by the formula $\lambda_{1,1} = 1$.

Where:

$$\lambda_{n+1,i+1} = e^{-r\Delta t}\{\lambda_{n,i}p_{n,i} + \lambda_{n,i+1}(1 - p_{n,i+1})\}, \quad 2 \leq i$$

and

$$p_{n,i} = \frac{F_{n,i} - s_{n+1,i}}{s_{n+1,i+1} - s_{n+1,i}}$$

All variables remain defined as before. The values for $s_{n+1,i}$ and $s_{n+1,i+1}$ were obtained from the initial CRR Trinomial Trees produced in Section 8 (Trinomial Tree Valuations).

All known variables were substituted in equations p_i and q_i to obtain the risk-neutral transitional probabilities from node (n, i) to node $(n + 1, \ i + 2)$ and from node (n, i) to $(n + 1, \ i)$. The resulting probabilities were greater than one in all instances. The middle probability was a large negative number in all instances. In order to avoid arbitrage, the transitional probability, $p_{n,i}$, at any given node should lie between 0 and 1, as the following must hold true at all times:

$$F_{n,i} < s_{n+1,i+1} < F_{n,i+1}$$

The principle of Risk-Neutral valuations was violated as the final option values using these amounts were significantly mispriced. This method was thus abandoned. Derman et al. (1996) suggest that known put option prices should be substituted in the place of the known call option price to determine the transition probabilities from all nodes below (and including) the center node $(n + 1, n)$ at time t_n. This exercise was not performed as the results would most likely be the same as the initial results.

An amendment to the initial suggested approach was introduced and allocated the values of $p_{n,i}$ and $\lambda_{n+1,i+1}$ to represent the risk-neutral transition probabilities for each node for an upward and downward movement in the given year. The middle movement is therefore represented by :

$$1 - p_{n,i} - \lambda_{n+1,i+1}$$

This thesis abandoned Derman and Kani's suggestion of calculating an unique combination for each node. Using the newly proposed method, two sets of Arrow-Debreu prices were calculated. In Method A, the forward rate was based merely on the share price at announcement date. The second method, Method B, is based on the unique forward rates for each year which were calculated using the prevailing share price and risk-free rate on the contract's anniversary date. These probabilities were used to perform the Implied Trinomial Tree valuations.

11.1 Three Branch Tree

In general, it has been noted that a share will either move up, down or remain constant. There is therefore only a one in three chance for each of these possibilities after a year. With all valuation methods, it was noted that during each stage (year) the trees produce many possible share prices. Different probabilities are therefore allocated to the different share prices calculated at the different nodes. This has been viewed this as “noise” and therefore introduced a new valuation technique, where the Trinomial Tree consists of three branches only, which grow through the contractual term. It can be viewed as:

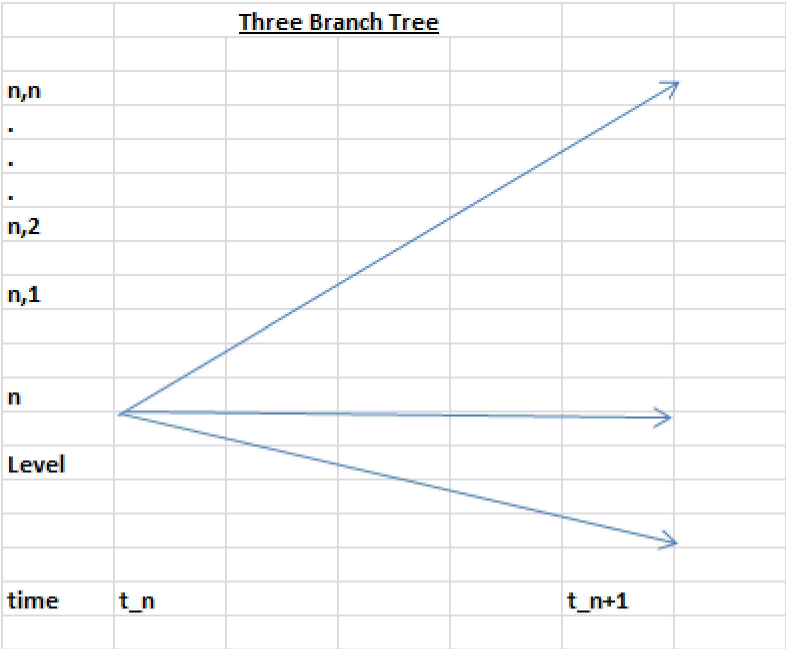

Figure 1 The Three Branch Tree

The transitional probabilities will therefore be the Arrow-Debreu prices calculated previously. This idea has been introduced from Derman’s and Kani’s statement that the Arrow-Debreu price can be applied to one and only one state. Each year therefore becomes a state when using this method, as each year will only have three possible prices.

11.2 Methodology:

For each transaction this thesis will perform procedures similar to Trinomial Trees using the following:

a) Perform the CRR Implied Trinomial Tree calculation using the original formulas and the risk-free rate as the discount rate.

b) Perform the CRR Implied Trinomial Tree calculation using the original formulas and the 90-day historic volatility as the discount rate.

c) Perform the CRR Implied Trinomial Tree calculation, amending the original formulas to incorporate the changes in volatility which took place during the contractual term. The different volatilities for each year were calculated previously when performing the Trinomial Tree calculations. The risk-free rate and the volatility at inception were used as the discount rates respectively for every given year. This is consistent with the approach followed before.

For each method introduced previously different variations to the risk-neutral transitional probabilities to see if they produce different results. The following variations were introduced:

(1) VALUATIONS PERFORMED USING METHOD A

In this method the Arrow–Debreu price is based on the values produced by the application of the forward rate equation.

(2) VALUATIONS PERFORMED USING METHOD B

In this method the Arrow–Debreu price is based on the values which are determined by substituting the actual share price and risk-free rate prevailing for a given year in the standard forward rate formula.

(3) VALUATIONS PERFORMED USING CONSTANT PROBABILITIES

Instead of using the unique Arrow-Debreu combination for a given year to determine the option values during that period, the combination for year 1 is used across the period. This is consistent with other Trinomial pricing methods reviewed in Section 8.

(To determine which Arrow-Debreu combination to use, valuations were performed using methods A and B above. The method which produced option values which were closer to the actual was deemed more appropriate. The remaining variations were then based on this method. In most instances Method A always produced the more accurate value.)

(4) VALUATIONS PERFORMED USING THE RISK-FREE RATE TO DETERMINE THE SHARE PRICE FROM AN UPWARD AND DOWNWARD MOVEMENT.

Barle and Cakici (1998) proposed an improvement of the D&K algorithm. They suggests taking the riskless interest rate into account when determining the share price of the central nodes. If $(n+1)$ is odd then $Se^{rn\Delta t}$. In instances where it is even, different methods are applied to determine the share price. Their method only adjusts the central node. From the Arrow–Debreu probabilities produced previously, it was noted that the fewest probabilities are always allocated to the middle share price. In most instances it was below 10%. The risk-free rate (RFR) was therefore substituted in the place of the volatility, to determine the upward and downward movement of each share price. Instead the following was used:

$$S_u = Se^{r\sqrt{2\Delta t}}$$

$$S_d = Se^{-r\sqrt{2\Delta t}}$$

All variables remain defined as before.

(5) VALUATIONS PERFORMED USING AMENDED PROBABILITIES

In this technique, it is assumed that the share price will always increase. The transition probability of a downward movement will therefore always receive the lowest Arrow-Debreu price. The downward movement probability was therefore allocated to a middle movement.

The transition probability of a middle movement will therefore receive the Arrow-Debreu price allocated to the downward movement. In all instances the upward movement remains untouched as this represents the "true" probability of an upward movement.

(6) VALUATIONS PERFORMED USING PROBABILITIES MIMICKING THE ACTUAL SHARE PRICE MOVEMENT

In this technique, it is assumed that the transitional probabilities must always mimic the share price movement. The most relevant probabilities are the p_i and q_i, as these are the drivers of the final option value. The probability allocated to the middle movement therefore remains constant.

In years where the share price increased, p_i received the highest probability. In years where the share price decreased, the downward movement received the highest probability.

The results of the testing will be given in the next section

11.3 RESULTS

The results have been summarized in terms of the percentage deviation produced by each method. The colours on the table represent the following:

Yellow – Best result for a given share.

Green – Result closely approximates best result with deviations < 20%.

Blue – Result closely approximates the share price at inception of the transaction with deviations < 5%.

Orange – Results closely approximates the strike price with deviations < 5%.

Red - Results closely approximates the final share price with deviations < 5%.

I.ADCORP

Adcorp - Implied CRR Trinomial Trees (Overall Summary of Results)	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	Probabilities using method - B	RFR used to determine Share Price	Amended Probabilities	Probabilities mimic movement in Share Price
ORIGINAL PROPOSED METHOD						
I. Using the Risk-free rate as the discount factor	79%	-37%	98%	96%	-12%	-8%
II. Using the 90 day Historic Volatility as the discount factor	94%	63%	100%	99%	70%	71%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	78%	-52%	98%	96%	-29%	-19%
b)Using the 90 day Historic Volatility as the discount factor	94%	60%	100%	99%	66%	68%
3 BRANCH METHOD:						
I. Using the Risk-free rate as the discount factor	-26%	-406%	39%	90%	-29%	-633%
II. Using the 90 day Historic Volatility as the discount factor	67%	-35%	84%	97%	66%	-95%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-25%	-403%	40%	90%	-29%	-628%
b)Using the 90 day Historic Volatility as the discount factor	67%	-34%	84%	97%	66%	-94%

Table 27 Adcorp – Summary of results produced by the Implied CRR Trinomial Trees.

This transaction is unique as the shares were acquired for a significant discount on the share price. In order to test the various valuation techniques introduced previously, one hypothesize the following:

Hypothesis 1:

When shares are acquired at a significant discount, allowing the probabilities to mimic the share price movement will provide an accurate estimate of the final option value, as the amended probabilities create a risk-neutral environment.

When Hypothesis 1 is tested , it is found that the most accurate result was produced when using the original method and when the probabilities mimicked the change in share price over the period. Good results were also produced when the changes in volatility over the contractual term were taken into account and when the original method, using the amended probabilities, was applied. One can thus accept Hypothesis 1 as true.

To test other relationships additional hypotheses are introduced:

Hypothesis 2:

When shares are acquired at a significant discount, allocating the highest probabilities to the upward and middle transition probabilities will return the initial share price.

Hypothesis 3:

When the shares are acquired at a significant discount and the Three Branch Method is applied, using the original method and the RFR as the discount rate, the result will approximate the strike price.

Hypothesis 4:

When shares are acquired at a discount and the Three Branch Method is applied, using the volatility as the discount rate will return the initial share price.

The results confirmed Hypothesis 2 as true, as the originally-proposed method produced two instances where the final option value returned an approximation of the initial share price. This occurred when the original method was used and the probabilities were kept constant, and when the changes in volatility over the contractual term were taken into account and the RFR was used as the discount factor. The latter produced the better result. In both instances the highest probabilities were allocated to an upward and middle movement.

When test for hypotheses 3 and 4 were performed, it is found that the Three Branch Method produced results which closely approximate the initial share price and the strike price. When the initial method was applied and the RFR was used as the discount rate, it returned values which approximated the strike price. When changes in volatility over the contractual term were taken into account and the RFR was used as the discount rate, it also returned the strike when using the original method and the amended probabilities method. The original method produced the best result in both instances.

The Three Branch method also produced results which closely resemble the initial share price, when the RFR is used as the discount factor and the probabilities are constant. The same holds true when the original method which incorporates the change in volatility, and which uses the volatility as a discount rate. Using the volatility as a discount rate produced the best results. These results therefore indicate that one can accept the hypotheses 3 and 4 as true.

II. ASSORE

Assore - Implied CRR Trinomial Trees (Overall Summary of Results)	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	Probabilities using method - B	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD						
I. Using the Risk-free rate as the discount factor	74%	70%	92%	87%	21%	15%
II. Using the 90 day Historic Volatility as the discount factor	85%	84%	100%	92%	55%	53%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	68%	63%	89%	87%	1%	-16%
b)Using the 90 day Historic Volatility as the discount factor	82%	81%	100%	92%	44%	36%
3 BRANCH METHOD:						
I. Using the Risk-free rate as the discount factor	0%	-119%	-80%	73%	-17%	-311%
II. Using the 90 day Historic Volatility as the discount factor	49%	23%	34%	85%	39%	-58%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-65%	-247%	-186%	74%	-82%	-558%
b)Using the 90 day Historic Volatility as the discount factor	15%	-25%	-7%	82%	5%	-160%

Table 28 Assore – Summary of results produced by the Implied CRR Trinomial Trees.

This transaction is different from the other transactions as it is an expansion transaction which consists of multiple stages, being stages I, II and III. An alternative hypothesis was therefore devised which takes this aspect into account:

Hypothesis 5:

When the Three Branch Method is applied and the change in volatility over the contractual term is taken into account, and when the volatility is used as the discount rate, the answer will approximate the final option value.

Good results are produced when the RFR is used as the discount rate using the amended probabilities method and the movement in share price methods. Although the original approach produced good results, the Three Branch Method produced the best results, with a 0% deviation, when the RFR was used as the discount rate. For the Three Branch method to be applied correctly, changes in the quantity of shares during the period must be ignored and only the final number of shares outstanding at maturity, 16,460,000, need be taken into account. Previously, the option value from phase 1 was obtained, based on the initial shares issue of only 3,290,000.

It is also interesting to observe that the Three Branch Method produces good results when the volatility is used as the discount rate after taking the changes in volatility over the contractual term into account. Good results are produced by the original method, Method B and the amended probabilities method. One can therefore assume the following:

The Three Branch Method also produced a good estimate of the initial price and the strike price for phase 2 of the transaction. This occurred when the movement in the share price was taken into account and the volatility was used as the discount rate, which produced the strike price. When the change in volatility was taken into account and the RFR was used as the discount rate, it produced a good estimate of the initial share price. Not enough instances were noted which produced the initial share price and the strike price, thus no definite assumptions could be made.

The results however indicate that one can accept Hypothesis 4 as true.

III. KUMBA

Kumba - Implied CRR Trinomial Trees (Overall Summary of Results)	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	Probabilities using method - B	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD						
I. Using the Risk-free rate as the discount factor	-14%	-522%	-1490%	81%	-414%	-658%
II. Using the 90 day Historic Volatility as the discount factor	64%	-96%	-401%	93%	-62%	-139%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-119%	-1316%	-3660%	81%	-978%	-1642%
b)Using the 90 day Historic Volatility as the discount factor	29%	-346%	-1085%	81%	-240%	-139%
3 BRANCH METHOD:						
I. Using the Risk-free rate as the discount factor	-476%	-1855%	-1701%	45%	-476%	-2451%
II. Using the 90 day Historic Volatility as the discount factor	-81%	-516%	-467%	83%	-81%	-704%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-1275%	-4569%	-4199%	45%	-1275%	-2451%
b)Using the 90 day Historic Volatility as the discount factor	-333%	-1371%	-1255%	83%	-333%	-1819%

Table 29 Kumba – Summary of results produced by the Implied CRR Trinomial Trees.

The Kumba transaction is different from the rest as it is an expansion option comprised of many different components. The main difference between this transaction and Assore's transaction is that the latter one occurred in multiple stages. To test this transaction, one can hypothesized the following:

Hypothesis 6:

When a transaction is a normal expansion option, then using the original Method A will produce the most accurate results.

Only one method could produce an accurate value, which was the original method where the RFR is used as the discount rate. In two instances the final option value returned approximated the strike price. This occurred when constant probabilities were used, but the RFR was used as the discount rate. Using the original method and the volatility after taking into consideration the changes in volatility over the contractual term was the second method. As the results in this column are much dispersed, no definite conclusions can be made.

The results therefore indicate that one can accept Hypothesis 6 as true.

IV. PRIMESERV

Primeserv - Implied CRR Trinomial Trees (Overall Summary of Results)	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	Probabilities using method - B	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD						
I. Using the Risk-free rate as the discount factor	73%	-1814%	-227%	94%	-718%	-1631%
II. Using the 90 day Historic Volatility as the discount factor	97%	-146%	58%	99%	-5%	-122%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	1%	-20526%	-1327%	94%	-3034%	-12230%
b)Using the 90 day Historic Volatility as the discount factor	87%	-2551%	-83%	99%	-303%	-1485%
3 BRANCH METHOD:						
I. Using the Risk-free rate as the discount factor	-1252%	-15404%	-1598%	41%	-1268%	-25551%
II. Using the 90 day Historic Volatility as the discount factor	-74%	-1893%	-118%	92%	-76%	-3197%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-42947%	-494058%	-53977%	41%	-42963%	-817488%
b)Using the 90 day Historic Volatility as the discount factor	-5433%	-63412%	-6850%	92%	-5435%	-104981%

Table 30 Primeserv – Summary of results produced by the Implied CRR Trinomial Trees.

Previously, difficulties were encountered with valuing the Primeserv share as no actual issue of shares took place. Because of this one can expect adjustments to the volatility which reflects the changes over the contractual term to produce an accurate estimate of the final option value, as this is the only aspect which does not remain constant. One can therefore hypothesize the following:

Hypothesis 7:

When the change in volatility over the contractual term is taken into account and the RFR is used as the discount rate, then the answer will approximate the final option value.

This method provided an instance where the deviation in valuation was only 1%. This was obtained when the changes in volatility over the contractual term were taken into account and the RFR was used as the discount rate. This is an expected result, because the CRR Trinomial tree is an implementation of the process:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t$$

where Z_t is the standard Wiener process and μ and σ are constants. Derman and Kani state that their model was engineered to construct a discrete approximation of the model above, on the basis of observed option prices, yielding the variable volatility $\sigma\ (S_t, t)$.

The Primeserv share is thus in its undisturbed state; one can expect, based on the model, that the option value will be largely impacted by changes in volatility over the contractual term. Hypothesis 7 is thus confirmed as true. To test additional relationships which the results produced for this transaction, one can hypothesized the following:

Hypothesis 8:

When the volatility is used as the discount rate and the Amended probabilities method is applied, then the final value will approximate the final option value.

Hypothesis 9:

When no share issue takes place and the transitional probabilities mimic the movement in share price, using the volatility as the discount rate will return the final share price.

Hypothesis 10:

When no share issue takes place and the original Three Branch Method using the RFR as the discount factor is used, the final option value will approximate the strike price. The same will hold true when the change in volatility over the contractual term is taken into account and the RFR is used as the discount rate.

The results indicated that different methods produced results which on the whole resembled significant numbers. Although the deviations were significant, a number of methods produced option values between R0.00 and R0.05. In rand terms, these would constitute a close approximation of the final option value. These results were produced primarily when the volatility was used as a discount rate and the original method plus the variations were applied. When the changes in volatility over the contractual term, using the volatility as a discount rate for Method A and B, was used, then the RFR and the Amended probabilities all produced values between R0.00 to R0.05. These results confirm Hypothesis 8.

The Three Branch Method also produced good estimations of the final option payoff. The results were produced when the RFR was used as the discount factor using the original method and when the change in volatility over the term using the RFR as the discount factor was applied to the original method. The results, however, are not as consistent as the volatility results above.

Another significant finding is that in instances where the change in volatility over the contractual term is taken into account and the volatility is used as the discount rate, the final option value approximates the final share price at maturity. This also occurs when the Three Branch Method is used and the volatility is used as the discount rate on the initial method. Both results were produced when the probabilities reflected the movement in share price. This confirms Hypothesis 9.

The results for Primeserv also produced instances where the final option value approximated the strike price. This occurred when the change in volatility over the contractual term was taken into account and the RFR was used as the discount factor. Using the Three Branch Method and the RFR as the discount rate to the original and amended probabilities method also produced values which approximated the strike price. The results indicate that one can accept Hypothesis 10 as true.

V. SANTAM

Santam - Implied CRR Trinomial Trees (Overall Summary of Results)	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	Probabilities using method - B	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD						
I. Using the Risk-free rate as the discount factor	42%	8%	89%	79%	-69%	30%
II. Using the 90 day Historic Volatility as the discount factor	65%	45%	93%	88%	13%	58%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-25%	-104%	77%	79%	-241%	-52%
b)Using the 90 day Historic Volatility as the discount factor	25%	-23%	88%	86%	-105%	8%
3 BRANCH METHOD:						
I. Using the Risk-free rate as the discount factor	-61%	-138%	-45%	58%	-64%	-164%
II. Using the 90 day Historic Volatility as the discount factor	3%	-43%	13%	75%	1%	-59%
III. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-317%	-519%	-278%	58%	-321%	-588%
b)Using the 90 day Historic Volatility as the discount factor	-151%	-272%	-127%	75%	-153%	-313%

Table 31 Santam – Summary of results produced by the Implied CRR Trinomial Trees.

This table represents a ‘vanilla’ transaction where the shares were acquired at minimal discount. This transaction is similar to Adcorp’s, with the main difference being that the shares were issued at a minimal discount. The best results for Adcorp were achieved when the RFR and volatility using the originally-proposed method was applied. In section 8 (Trinomial Trees), it was discovered that when the volatility is used as the discount rate, accurate option values may be produced as this removes any dilution effects. Using this as a basis one can formulate the following hypothesis to test the transaction:

Hypothesis 11:

When the transaction is a 'vanilla' transaction and the shares are issued at a minimal discount, using the volatility as the discount rate will return the final option value. In instances where the change in volatility over the contractual term is taken into account and the volatility is used as the discount rate, the final value will approximate the actual share price.

Using the original approach only produced good estimates of the actual option value in three instances, two of which were when the volatility was used as the discount rate. In two instances the option value returned the value which would have been obtained had 15 million shares been issued in the BEE transaction with a value close to R 31.22. This occurred when the original method, in which the share price movement using the RFR as discount rate, and the change in volatility over the period using Method A with the volatility as a discount rate, was applied.

The most prominent feature of this method is the application of the Three Branch Tree. Using the volatility as a discount rate produced good results in two instances and the best result in one instance, all when using the volatility as a discount rate applied to the original method. Another significant finding is that when the change in volatility over the contractual term is taken into account, the final payoff represents the actual share price at inception. This occurs when the Original method A and the Amended probabilities method are used.

One can therefore accept the hypothesis as true.

VI. SANYATI

The Sanyati transaction is the only transaction where the company was liquidated before the option reached maturity. Methods A and B, with and without negative final payoffs, will therefore be considered because of the large decline in share price over the contractual term. The original valuation method was designed to value a call option, where the share price is assumed to be increasing steadily over the contractual term. One can therefore expect none of the methods to value the option accurately as the market followed a different trend. To account for this, it is proposed that the results would resemble other figures which are consider significant. To test the proposed amendment, one can hypothesize the following:

Hypothesis 12:

When the transaction includes a share price that is rapidly declining, and when using the RFR as the discount rate, the payoff will approximate the share price at acquisition. When the volatility is used as a discount rate the final payoff will approximate the final share price.

Hypothesis 13:

When the share price is declining sharply, using the proposed method which takes the change in volatility over the contractual term into account, and where the greatest probability is always allocated to the downward movement and the least to the upward movement, will produce a value which approximates the share price at maturity.

a) Sanyati – Method A (without negative payoffs)

Sanyati - Implied CRR Trinomial Trees (Overall Summary of Results - Without negative pay-offs)	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD					
I. Using the Risk-free rate as the discount factor	147%	66%	104%	1186%	147%
II. Using the 90 day Historic Volatility as the discount factor	103%	229%	100%	177%	103%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	140%	2008%	104%	1042%	140%
b)Using the 90 day Historic Volatility as the discount factor	103%	236%	100%	167%	103%
3 BRANCH METHOD:					
I. Using the Risk-free rate as the discount factor	1577%	22563%	120%	1591%	1577%
II. Using the 90 day Historic Volatility as the discount factor	205%	1699%	102%	206%	205%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	2275%	33195%	120%	2290%	2275%
b)Using the 90 day Historic Volatility as the discount factor	255%	1699%	102%	206%	205%

Table 32 Sanyati – Summary of results produced by the Implied CRR Trinomial Trees (Method A).

No significant results were produced using this method. What is noticeable here is that in many instances the final payoff is close to zero. This was expected, as the CRR Trinomial tree can accurately predict instances where the option will finish out of the money.

Another point to highlight is that when the change in volatility over the contractual term is taken into account, the final option value is R 0.07. This figure is significant, as at liquidation each creditor received R 0.07 for every R 1.

When the Three Branch Method is used to perform valuations, using the RFR as the discount rate returns a value which approximates the actual share price, when using the Method A, Amended probabilities and movement in share price methods. When the volatility is used as the discount factors, the same method produces values which approximate the final option value. These results are of significance as they confirm Hypotheses 12 and 13.

b) Sanyati – Method A (With negative payoffs)

Sanyati - Implied CRR Trinomial Trees (Overall Summary of Results - With negative pay-offs)	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD					
I. Using the Risk-free rate as the discount factor	-632%	1494%	-140%	1099%	-632%
II. Using the 90 day Historic Volatility as the discount factor	48%	199%	83%	171%	48%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-627%	1606%	-63%	965%	-627%
b)Using the 90 day Historic Volatility as the discount factor	48%	207%	83%	162%	48%
3 BRANCH METHOD:					
I. Using the Risk-free rate as the discount factor	745%	22053%	-247%	1631%	745%
II. Using the 90 day Historic Volatility as the discount factor	146%	1663%	62%	201%	146%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	1439%	32682%	-222%	2217%	1439%
b)Using the 90 day Historic Volatility as the discount factor	195%	2420%	62%	251%	195%

Table 33 Sanyati – Summary of results produced by the Implied CRR Trinomial Trees (Method A) with negative payoffs.

No significant findings were produced using this method. What can be confirmed is that in instances where the original method is used, and the RFR is used as the discount rate, the final option value is negative, indicating that the option has no value. In two instances, using the volatility as the discount rate produced the share price at maturity, whilst using the RFR as the discount rate returned the strike price. One can assume that this is the inverse of the previous assumption.

c) Sanyati – Method B (Without negative payoffs)

Sanyati - Implied CRR Trinomial Trees (Overall Summary of Results - Without negative pay-offs)					
	Probabilities using method - B	Probabilities using method - B (Constant probabilities)	RFR used to determine Share Price	Amended Probabilities	Probabilities mimic movement in Share Price
ORIGINAL PROPOSED METHOD					
I. Using the Risk-free rate as the discount factor	100%	1137%	100%	447%	100%
II. Using the 90 day Historic Volatility as the discount factor	100%	174%	100%	125%	100%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	100%	1142%	100%	365%	100%
b)Using the 90 day Historic Volatility as the discount factor	100%	174%	100%	119%	100%
3 BRANCH METHOD:					
I. Using the Risk-free rate as the discount factor	175%	19163%	102%	190%	175%
II. Using the 90 day Historic Volatility as the discount factor	105%	1457%	100%	106%	105%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	209%	28186%	102%	224%	209%
b)Using the 90 day Historic Volatility as the discount factor	108%	2100%	100%	109%	108%

Table 34 Sanyati – Summary of results produced by the Implied CRR Trinomial Trees (Method B).

This method did not produce an option value which approximated the actual. What it did reflect was a number of zero payoffs. This indicates that the option is out of the money. When the Three Branch method is applied and the change in volatility over the term is taken into account and the RFR is used as the discount rate, the final option value represents the actual share price. This was achieved when using Method B and the movement in share price method.

d) Sanyati – Method B (With negative payoffs)

Sanyati - Implied CRR Trinomial Trees (Overall Summary of Results - With negative pay-offs)	Probabilities using method - B	Probabilities using method - B (Constant probabilities)	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD					
I. Using the Risk-free rate as the discount factor	-778%	622%	-204%	312%	-778%
II. Using the 90 day Historic Volatility as the discount factor	38%	137%	78%	115%	38%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-774%	650%	-204%	245%	-774%
b)Using the 90 day Historic Volatility as the discount factor	38%	139%	55%	110%	38%
3 BRANCH METHOD:					
I. Using the Risk-free rate as the discount factor	-678%	18603%	-248%	116%	-678%
II. Using the 90 day Historic Volatility as the discount factor	45%	1417%	59%	101%	45%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-648%	27622%	-248%	150%	-648%
b)Using the 90 day Historic Volatility as the discount factor	47%	2059%	58%	104%	47%

Table 35 Sanyati – Summary of results produced by the Implied CRR Trinomial Trees (Method B) with negative payoffs.

Using the original method B and the movement in share price method produces a value of R(1.63). This value is significant as the actual loss on entering into the transaction, based on an acquisition price of R2.10 per share, is R(1.89). This is a close approximation of the actual loss on the transaction. Similar results are produced when the change in volatility over the period is taken into account and the RFR is used as the discount rate. The option value is R(1.62) in those instances. When the volatility is used as the discount rate and the probabilities are kept constant the option value is R0.07. This represents the final amount payable to each creditor for every R1 owed at liquidation date.

There were no additional significant findings, except that when the volatility is used as the discount rate using Method B, it produces a deviation of 38%, with an option value returned of R(0.10). This is the lowest value produced by any of the trees.

Method B produced better results than Method A for Sanyati, as Method B could capture the actual movement in the share when calculating the transition probabilities, as the forward rates were based on actual.

The results do not indicate that additional hypotheses are necessary. Instead they indicate that one can accept the hypotheses as true.

11.4 CONCLUSION

In this section assumptions are introduced which are extensions of the assumptions introduced in the Trinomial trees section. The best results produced with this method are as follows:

Adcorp – Using the original method A with the RFR as the discount rate and with the probabilities reflecting the movement in the share price.

Assore– Using the Three Branch Method A with the RFR as the discount rate.

Kumba – Using Method A with the RFR as the discount rate.

Primeserv – Using Method A after incorporating the change in volatility over the contractual term, and using the RFR as the discount rate.

Santam – Using the Three Branch method with amended probabilities and the RFR as the discount rate.

Sanyati – Using Method B after incorporating the change in volatility over the contractual term, and using the volatility as the discount rate.

From the above, it can be noted that the Three Branch Methods may be used to estimate the strike price accurately and also the share price at inception. This indicates that there may be value in this method.

The following table shows the comparison of the results produced using this method are to those of other methods:

Summary of Results produced by all methods	Adcorp Holdings Limited	Assore	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Real Options Method	14%	10%	-1%	1%	0%	3%
Binomial Lattice Valuation	13%	8%	-21%	25%	2%	33%
Black Scholes Merton Model	14%	11%	7%	-3%	14%	-175%
Risk Neutral Valuation	3%	3%	11%	24%	6%	14%
Trinomial Trees	0%	-1%	-7%	0%	-1%	79%
Implied Trinomial Trees	-8%	0%	-14%	1%	1%	38%
Best Results	**0%**	**0%**	**-7%**	**0%**	**-1%**	**14%**

Table 36 Summary of results produced by the different valuation methods.

Ignoring the results produced by the "Real Options" method, it is found that in two of the six instances the Implied Trinomial tree method produced the best results. Sanyati's result may be included above the Binomial Lattice Valuation result because for the first time there is an option value, produced by a tree, which approximates the actual of R(0.18). The model produced values of R(0.12) and R(0.18). Although the results for Adcorp and Kumba were not the best results produced, they are of a higher quality than those for other methods, as the results were produced by a method in its purest form, where no significant adjustments were necessary.

It would appear that this method works because the strike price K remains constant and does not change as suggested by the "volatility smile". The change in volatility is not deemed to be a significant factor. It can be seen that the only instance where the change in volatility must be taken into account is where no share issue takes place, as in the case of Primeserv, or where the share is issued at a minimal discount, as in the case of Santam. The share is left to follow an independent drift, where the only factor which may affect the share price and the final option payoff is volatility. Changes in the strike price should not be considered significant. Using the Implied Trinomial tree method captures this aspect. It may therefore be the most appropriate valuation method.

12. Application of limitations to the "Implied Volatility Tree" produced for Sanyati

In the previous section it is noted that Sanyati produced the highest deviation from the actual, with a value of 38%. This difference is considered to be too high, as other transactions produced deviations below 20%. In some instances deviations close to 0% were obtained. Additional procedures were therefore performed with the aim of re-examining this result.

This thesis draws on the idea introduced previously using the BS model, which deals with instances where there are large declines in the share price post-transaction date. This idea suggests placing a limit on the share price and then taking a position in the opposite direction. This will produce an estimate of the final loss at expiry. This will be applied to the Sanyati share by taking into consideration only the final payoffs produced by the tree for share prices below R2.67. This implies that a payoff of zero will be applied at maturity to shares where the share price, computed by the tree, is above R2.67. The application of this method would mean that only the option values produced by the bottom half of the tree will be considered.

In previous attempts to value the Sanyati share, it is noted that, in instances where negative payoffs are taken into consideration, the final option value was a better estimate of the actual. For this valuation, therefore, one would only consider the "Implied Volatility Trees" produced using "Methods A and B" where the negative payoffs were taken into account. To test this method, one can hypothesize the following:

Hypothesis 1:

When the share price declines significantly, using the RFR as the discount rate for trees where only option values in the bottom half are taken into account will produce an accurate estimate of the loss per share at maturity.

A summary of overall results is presented below:

i) ***Method A***

Sanyati - Implied CRR Trinomial Trees *(Share Price Limited to R2.67)*	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD					
I. Using the Risk-free rate as the discount factor	-679%	-323%	-143%	20%	-679%
II. Using the 90 day Historic Volatility as the discount factor	45%	70%	83%	94%	45%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-666%	-301%	-143%	30%	-666%
b)Using the 90 day Historic Volatility as the discount factor	45%	71%	83%	95%	45%
3 BRANCH METHOD:					
I. Using the Risk-free rate as the discount factor	-730%	-409%	-240%	44%	-730%
II. Using the 90 day Historic Volatility as the discount factor	41%	64%	60%	96%	41%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-735%	-412%	-240%	44%	-735%
b)Using the 90 day Historic Volatility as the discount factor	-735%	64%	-240%	44%	-735%

Table 37 Results from Implied CRR Trinomial Trees with share price limited to R2.67 (Method A).

The best result produced by this method is a deviation of 20%. This is produced when the highest probability is allocated to the upwards and middle movement in the share price. This is contrary to the results produced previously, which suggested that in instances where unexpected declines are faced in the share price, the probabilities should mimic the movement in the market value. It is for this reason that "Method B" was found to be best suited to value this transaction.

Interestingly, in instances where the RFR was used as the discount factor, the final value approximated an amount greater than 75% of the actual loss of R(1.89) at maturity on the share (denoted by the purple regions). These results indicate that one can accept Hypothesis 1 as true.

ii) Method B

Similar procedures to the trees produced using "Method B" were applied. As Method A could not produce an accurate estimate of the final option value, an alternative hypothesis more suited to the testing was devised:

Hypothesis 2:

When the share price declines significantly, then using the “Method B – Amended Probabilities” approach with the RFR as the discount rate, for trees where only option values in the bottom half are taken into account, will produce an accurate estimate of the final options value at maturity, provided that the change in volatility over the contractual term is taken into account.

The results are presented below:

Sanyati - Implied CRR Trinomial Trees *(Share Price Limited to R2.67)*	Probabilities using method - B	Probabilities using method - B (Constant probabilities)	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
ORIGINAL PROPOSED METHOD					
I. Using the Risk-free rate as the discount factor	-778%	-413%	-204%	-26%	-778%
II. Using the 90 day Historic Volatility as the discount factor	37%	63%	78%	91%	37%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-775%	-391%	-204%	-11%	-775%
b)Using the 90 day Historic Volatility as the discount factor	38%	65%	78%	92%	38%
3 BRANCH METHOD:					
I. Using the Risk-free rate as the discount factor	-751%	-459%	-249%	43%	-751%
II. Using the 90 day Historic Volatility as the discount factor	39%	60%	59%	96%	39%
III. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-756%	-462%	-249%	42%	-756%
b)Using the 90 day Historic Volatility as the discount factor	39%	60%	58%	96%	39%

Table 38 Results from Implied CRR Trinomial Trees with share price limited to R 2.67 (Method B).

Results similar to those of “Method A” were produced, with good representations of the final loss at maturity. This method produced the best result to date for an estimate of the final option value of (18.68) cents. This was produced using the “Amended Probabilities” approach, similar to Method A when using the RFR as the discount rate. This value is more accurate, owing to the procedure taking into consideration the change in volatility over the contractual term.

Although the value of 11% is accurate, it is produced in a scenario which defies the proposition of the probabilities in any trinomial tree being representational of the movement in the market, as the highest probabilities are allocated to the upwards and middle nodes. This is similar to results produced using “Method A”. This cannot be the optimal solution, as it is known that the share price declined over the option’s contractual term. This scenario suggests that the share price increased steadily across the contractual term. This thesis will thus have to search for a solution which takes this aspect into account. All results indicate that the alternative hypothesis is true.

13. Use of Implied Trinomial Tree to Create a Risk-Neutral Environment

13.1 Amendments to existing Implied Trinomial Tree Theory

In the previous section it was established that, when using the results of Sanyati, a good estimate of the final option payoff at maturity may be produced when the "Amended Probabilities" method is applied to share prices which decline unexpectedly after transaction inception date. One would therefore like to produce a scenario where the use of the Arrow–Debreu probabilities takes this into account and which matches the behaviour of the share during the contractual term.

It has been established that Primeserv is the optimal share, whose model is based on a discrete model which captures all the elements of a standard Wiener process. It has also been established that in instances where the change in volatility is taken into account and the RFR is used as the discount rate, the final value will represent the actual payoff at maturity. The results also indicated that in instances where the change in volatility is taken into consideration and the volatility is used as the discount rate, the final values will produce the best estimates, which fall within a close range of the actual payoff. In addition, estimates of the final share price at inception and maturity and of the strike price will be produced by the different methods.

It has been noted that in the instance of Santam, when the change in volatility over the contractual term is taken into account and the volatility is used as the discount rate, the final value will approximate the actual payoff. Instances where the final payoff approximated the share price at inception were also presented.

In order to produce the best estimates of the payoff at maturity, it was attempted to build surrogate "Implied Trinomial Trees" for all the transactions, based on the principles contained within Primeserv's transaction, as these represent the risk - neutral state. In instances where transactions were impacted by discounts, was taken into consideration the situation of Santam, as a minimal discount was given in this instance, and it has been noted that use of the volatility as the discount rate removes the dilution effect.

The following procedures were performed:

(1) "Implied Trinomial Trees" which take into account the change in volatility across the contractual term were constructed. Two trees were produced using the RFR and volatility as the discount rate.

(2) The pattern in the share price movement across the contractual term was also examined. It was assumed that entering into a BEE transaction which involves an exchange of share capital impacts the "independent path" that a share must follow, where "independent path" is defined as the route on which the share is positioned on transaction date. It was assumed that this would take place in the first year of the contractual term. After one year, the share would have found a way to deal with the impact of entering into the transaction, and would thus find another independent path which leads to the final payoff. It was assumed that one of the underlying principles of the Wiener process should be that "shocks" to the share price over the contractual term are expected and are gradually adjusted for by the share taking one of an infinite number of "detour routes", which leads to one and only one independent predetermined final outcome. The effects of "jumps" post-transaction date are therefore irrelevant as they will be adjusted for automatically by the market.

(3) From Santam's transaction, it was noted that use of the volatility adjusts for the impact of transactions entered into at a discount. The pattern of the movement in share price over the contractual terms for all transactions was therefore examined. It was noted that, other than for Primeserv's share, there is an upwards surge in the share price after the transaction implementation date, which is then followed by a period of steady decline. Attempts were made to adjust for the effect of this "upwards surge". Where the RFR is used as the discount rate in the Implied Trinomial Tree, the volatility is used as the discount rate in the first year. The RFR will remain the discount rate for all the remaining years. In instances where the volatility is used as the discount rate, the RFR is used as the discount rate in the first year. The volatility will remain the discount rate for all the remaining years.

The procedures above would be applied to the trees produced by the Three Branch Method as well, as it has been noted that accurate results are produced when using this method.

The results will be presented as follows:

(1) Primeserv and Sanyati will be presented first as they form the basis of this argument.

(2) Adcorp, Assore and Kumba will follow. These transactions all have unique facets which will be discussed separately.

(3) Sanyati using "Methods A and B" will be presented last. For this calculation, the results produced using the limits set on the share price of "R2.67" were taken into account as these results were a better approximation of the actual.

In all instances, the results produced by the Implied Trinomial Trees when using the original method, where the RFR and the 90-day historic volatility were used as the discount rate were tested and it was noted that the mispricing proved to be too severe. These results were therefore ignored when performing this exercise.

In the results which follow, the key remains as defined before:

Yellow – Best result for a given share

Green – Result closely approximates best result with deviations < 20% observed.

Blue – Result closely approximates the share price at inception of the transaction with deviations < 5%.

Orange – Results closely approximates the strike price with deviations < 5%.

Red - Results closely approximates the final share price with deviations < 5%.

I. PRIMESERV AND SANTAM

Implied CRR Trinomial Trees *(Combination of discount rates)*	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	Probabilities using method - B	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
Primeserv						
ORIGINAL PROPOSED METHOD						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	25%	-15543%	-983%	96%	-2277%	-9252%
b)Using the 90 day Historic Volatility as the discount factor	83%	-3395%	-142%	99%	-431%	-1989%
3 BRANCH METHOD:						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-32548%	-374686%	-40914%	55%	-32560%	-619987%
b)Using the 90 day Historic Volatility as the discount factor	-7195%	-83641%	-9064%	90%	-7197%	-138450%
Santam						
ORIGINAL PROPOSED METHOD						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-10%	-80%	80%	79%	-201%	-34%
b)Using the 90 day Historic Volatility as the discount factor	15%	-39%	84%	86%	-133%	-4%
3 BRANCH METHOD:						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-267%	-445%	-233%	63%	-271%	-505%
b)Using the 90 day Historic Volatility as the discount factor	-185%	-323%	-158%	71%	-187%	-369%

Table 39 Results of Risk–Neutral Implied CRR Trinomial Trees for Primeserv and Santam.

The Primeserv and Santam transactions are closely related because in one instance no shares were issued and in the other shares were issued at a minimal discount. The dilution effects are thus minimal in both instances. To test these transactions, one can therefore hypothesize the following:

Hypothesis 1:

When shares are issued at a discount or premium, the market takes this into account and will adjust to remove the impact of the discount or premium. In instances where no shares are issued, the market operates in its perfect undisturbed state. This state represents the Risk-Neutral state.

The results for Primeserv are as expected, as none of the methods could produce a result which approximated the actual. The best estimate was a result of 25% produced when using the RFR as the discount rate. The mispricing on the share is the most severe. This is largely attributable to changing the environment from a perfect state where the share can follow its path independently and then introducing factors which are not taken into consideration. By this, it is meant that the environment which has been created could only be balanced if an actual share issue had taken place, as this is the missing variable. This confirms that here the environment is operating in the risk-neutral state.

Santam's share produced results which were expected. Where the original method is used and both the RFR and volatility are used as the discount rates, the results approximate the actual. It is good to see that these results are produced when the probabilities use the "Original" method and when the probabilities "Mimic the movement in the share price". The best result for this method is produced when the volatility is used as the discount rate. This supports the initial assumptions made in Section 11(Implied Trinomial Trees) which suggested that using the volatility as the discount rate for this particular transaction will produce the best results.

The Three Branch Tree produced results which are contrary to the results produced in the initial application. Previously, using the volatility as the discount rate produced good estimates of the share price at inception. The same methods now produced good estimates of the strike price.

The results produced when using "Method B" are new but support the theory that the application of inverse methods will produce two different results. When the RFR is used as the discount rate, it produces a good estimate of the final share price. When the volatility is used as the discount rate it produces a good estimate of the share price at inception.

The results indicate that one can accept Hypothesis 1 as true.

II. ADCORP, ASSORE AND KUMBA

Implied CRR Trinomial Trees *(Combination of discount rates)*	Probabilities using method - A	Probabilities using method - A (Constant probabilities)	Probabilities using method - B	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
Adcorp Holdings Limited						
ORIGINAL PROPOSED METHOD						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	83%	-16%	99%	97%	1%	8%
b)Using the 90 day Historic Volatility as the discount factor	92%	47%	99%	99%	55%	59%
3 BRANCH METHOD:						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-26%	-406%	39%	90%	-29%	-633%
b)Using the 90 day Historic Volatility as the discount factor	67%	-35%	84%	97%	66%	-95%
Assore						
ORIGINAL PROPOSED METHOD						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	73%	53%	43%	89%	8%	-22%
b)Using the 90 day Historic Volatility as the discount factor	78%	71%	82%	91%	29%	14%
3 BRANCH METHOD:						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-35%	-172%	-126%	78%	-50%	-422%
b)Using the 90 day Historic Volatility as the discount factor	-2%	-55%	-33%	82%	-15%	-221%
Kumba						
ORIGINAL PROPOSED METHOD						
I. Using the Risk-free rate as the discount factor	9%	-395%	-1165%	85%	-309%	-503%
II. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-74%	-1027%	-2891%	85%	-757%	-1286%
b)Using the 90 day Historic Volatility as the discount factor	11%	-461%	-1389%	93%	-327%	-200%
3 BRANCH METHOD:						
I. Incorporating the changes in volatility over the contractual term						
a) Using the Risk-free rate as the disount rate	-994%	-3614%	-3320%	56%	-994%	-4746%
b)Using the 90 day Historic Volatility as the discount factor	-445%	-1749%	-1603%	78%	-445%	-2313%

Table 40 Results of Risk–Neutral Implied CRR Trinomial Trees for Adcorp, Assore and Kumba.

These transactions each contain unique facets and different methods were applied previously to determine the optimal option value. To test the Adcorp transaction, one can formulate the following hypothesis based on previous results:

Hypothesis 2:

Where shares are issued at a significant discount, then using the RFR as the discount rate, combined with the "Amended probabilities" method will produce the final option value.

The original method failed to produce good results for Adcorp initially. With modification, it produced the best result for the Implied Trinomial trees with a deviation of only 1%. This result was produced using the RFR as the discount rate and the "Amended Probabilities" method. Here one expects the technique of applying the highest probabilities to the upward and middle movement to produce the best result. In Section 11 (Implied Trinomial Trees), this method also produced the best result. There is a likely reason for this result. At inception, the shares were issued at a significant discount, as the price of a share was only 1 cent, while the listed price was R37.00. In order for the share to reach a predetermined payoff at maturity, it will choose a path where the maximum probabilities will be allocated to the upward and middle movements. This is done so as to ensure that there is a strong upwards surge in the share in order to drive the price back to its originally intended path. One can therefore suggest that this is part of the mechanics of the market.

This theory is supported by good results which are produced when the "Probabilities mimic the movement in share price" method is used.

This method also produced good estimates of the share price at acquisition. This is similar to the estimates produced with other methods (see above). A good estimate of the strike price was also produced by this method. From both the previous findings and these later ones, one can accept Hypothesis 2 as true.

Assore remains special, as it is the only transaction where use of the Three Branch Tree can produce a good estimate of the final payoff. This result is achieved when the volatility is used as the discount rate. In Sections 7 (Risk-Neutral Valuation), 8 (Trinomial Trees) and 11 (Implied Trinomial Trees), it is established that where a transaction is an expansion option, using the volatility as the discount rate is the best method to price the option. Although the Assore transaction is a multi-stage expansion option, one can assume that the theory can be applied in this instance too. To test the transaction, one can therefore hypothesize the following:

Hypothesis 3:

When the transaction is a multi-stage expansion option, using the volatility as the discount rate will produce the final option value.

It has been noted that the Three Branch Tree can also be used to estimate the final share price at maturity. Using the Original Method produces good estimates of the actual option value when both the RFR and volatility are used as the discount rate. One can therefore accept Hypothesis 3 as true.

The results for Kumba remained as before. In no instance was the Three Branch Tree able to produce a price which represented the actual. One can therefore draw on the results obtained in section 11 to devise a hypothesis to test the transaction:

Hypothesis 4:

Where a transaction is an expansion option, using the original method with the RFR as the discount rate will produce the best option value.

When the original method was used and the volatility was used as the discount rate, the final value produced a payoff which approximated the actual. This confirms trend 3 above which suggests that for expansion options, the volatility is the optimal discount rate.

In Section 11 – Implied Trinomial Trees the best result for the Kumba transaction was produced using the original method with the RFR as the discount rate. As this was the only instance in which a best result was produced, an exception was made and included the results from applying the concepts to this transaction. The deviation produced using this method was better than all results produced and was only 9%. The results therefore indicate that one can accept Hypothesis 4 as true.

III. SANYATI

Implied CRR Trinomial Trees *(Combination of discount rates)*	Probabilities using method - (A/B)	Probabilities using method - (A/B) (Constant probabilities)	RFR used to determine Share Price	Amended Probabilities	Probabilities mimick movement in Share Price
Sanyati - Method A					
ORIGINAL PROPOSED METHOD					
I. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-363%	-142%	-47%	58%	-363%
b)Using the 90 day Historic Volatility as the discount factor	10%	53%	71%	92%	10%
3 BRANCH METHOD:					
I. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-405%	-209%	-106%	66%	-405%
b)Using the 90 day Historic Volatility as the discount factor	2%	40%	60%	93%	2%
Sanyati - Method B					
ORIGINAL PROPOSED METHOD					
I. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-429%	-197%	-84%	33%	-429%
b)Using the 90 day Historic Volatility as the discount factor	38%	65%	78%	92%	38%
3 BRANCH METHOD:					
I. Incorporating the changes in volatility over the contractual term					
a) Using the Risk-free rate as the disount rate	-417%	-240%	-111%	65%	-417%
b)Using the 90 day Historic Volatility as the discount factor	-1%	34%	30%	93%	-1%

Table 41 Results of Risk–Neutral Implied CRR Trinomial Trees for Sanyati (Method A and B).

After the results in the previous section had been produced, it has been discovered that only taking into consideration the bottom half of the tree produced better results than previous methods. In an attempt to produce better results from the test performed, one can hypothesized the following:

Hypothesis 5:

When the share price declines rapidly post-transaction date, then using volatility as the discount rate will produce the most accurate estimate of the final option value.

The best result produced previously for Sanyati was a deviation of 14%. It has been established that the condition under which this result was produced is not optimal as it assumed that share prices would be increasing. This paper therefore devised the current method in an attempt to find a better solution. The results above are not surprising, as all the good results were produced in instances where the initial methods A and B and the "Probabilities mimic the movement in the share price" method were used. It is interesting to note that in no instance did using the RFR as the discount rate produce any good results. Using method A produced good results when both the original method and the Three Branch tree were used to perform the valuations. The best overall result for this method was produced using method B and the volatility as the discount rate, as the deviation was only 1%.

One can conclude from the results above that the previous good results produced by Sanyati were fortuitous and that this method is the most suitable valuation method for firms in financial distress. The results also indicate that one can accept Hypothesis 5.

13.2 CONCLUSION

A comparison of the results produced by the two approaches is as follows:

Comparison of Results produced by all methods						
	Adcorp Holdings Limited	**Assore**	**Kumba**	**Primeserv Group Limited**	**Santam**	**Sanyati Holdings Limited**
Implied Trinomial Trees (Original)	-8%	0%	-14%	1%	1%	38%
Implied Trinomial Trees (Amended)	1%	-2%	9%	25%	-4%	-1%
Best Results	**1%**	**0%**	**9%**	**1%**	**-1%**	**-1%**

Table 42 Comparison of results produced by the Original CRR Implied Trinomial Trees and the Amended Trees.

The results are distributed evenly between the two approaches. The original method was most suited to Assore, Primeserv and Santam with minimal differences between Assore and Santam. The amended approach was most suitable for Adcorp, Kumba and Sanyati.

The worst result produced by this method is for Kumba, which suggests that there is an unknown aspect inherent in the contract which still leads to mispricing. When the share price movement was investigated, it was noted that the decline post-announcement date was experienced for a period of two years. the calculation was therefore performed, taking this aspect into account, and obtained deviations of only 0.55%. This was achieved when taking the changes in volatility into account and using the volatility as the discount rate. An amendment to the initial method was introduced, whereby the RFR was only used for a period of two years after inception as the discount rate, as the company took two years post-announcement to list on the JSE again. Also, in the two adjustment years, when the share price exceeded the maximum value of R500 reached during the contractual term, the volatility was used as the discount rate. This was done because as noted in Section 6 - Binomial Lattice Valuations, setting limits on the range of the share price produces more accurate estimates. The opposite occurred where the RFR was used as the discount rate. Because the results are ambiguous, one cannot come to a firm conclusion on this method. What can be confirmed is that applying combination of discount rates will produce an answer which is closer to the actual than 9%.

A comparison to the results produced by the other methods can be summarized as follows:

Summary of Results produced by all methods	Adcorp Holdings Limited	Assore	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Real Options Method	14%	10%	-1%	1%	0%	3%
Binomial Lattice Valuation	13%	8%	-21%	25%	2%	33%
Black Scholes Merton Model	14%	11%	7%	-3%	14%	-175%
Risk Neutral Valuation	3%	3%	11%	24%	6%	14%
Trinomial Trees	0%	-1%	-7%	0%	-1%	79%
Implied Trinomial Trees	1%	0%	1%	1%	-1%	-1%
Best Results	**0%**	**0%**	**1%**	**0%**	**-1%**	**-1%**

Table 43 Summary of results produced by the different valuation methods.

When one compares the results produced by this method to those for other methods it is noted that the best results are split between the Trinomial Trees and Implied Trinomial Trees. This is to be expected, as the two methods are based on similar underlying principles. The Implied Trinomial Tree method is superior as it produced a variable which no other method could produce, that of the Sanyati option. An additional factor which sets the "Implied Trinomial Tree" method apart from the rest is that the values were obtained in a risk–neutral world.

14. Creating a Risk-Neutral Environment using the Black-Scholes Model

Previously Implied Trinomial trees were used to create a risk-neutral environment by examining the patterns in the share price movement across the contractual terms. It was discovered that in most instances that, in the year that the transaction is implemented, there is a peak in the share price caused by an unexpected increase, which is followed by a decline in the share price in the year of inception. This however only holds true in instances where a share issue has taken place. In the previous section, in order to remove the impact of these peaks on the final share value, it was adjusted the discount rates used during the given period. This was possible because Implied Trinomial trees are discrete valuation models and can easily be adjusted to incorporate this. The Black-Scholes (BS) Model is different, as it is a continuous valuation method. The model is designed to fix all the variables at inception, thus the option values produced by this model will not capture any extreme changes in any input variable which may occur during the contractual term. It has been have assumed that entering into a BEE transaction is responsible for the peaks which usually occur in the year of inception. At valuation date, the BS value will therefore be determined without taking the impact of these peaks into account, as the model currently ignores this aspect.

In an attempt to introduce an element of flexibility in the value of the final option payoff produced by the model, an attempt will be made to determine the portion of the value of the final option payoff attributable to these peaks. The following procedures will therefore be performed:

1. Each share price pattern was examined to determine the date of commencement and ending of these peaks. The method introduced in Section 3 was revisited. As before, the share price S was taken at the prevailing share price at commencement of the peak date. The strike price K was determined as before, using Monte Carlo simulation. The number of random variables required, ε, was based on the number of weeks that the peak prevailed. The risk-free rate was based on the 3-month treasury-bill for that month. The volatility, σ, was derived from the model of stock price behaviour used by Black, Scholes and Merton. All traded share prices during the period identified as the peak were taken into consideration when determining the volatility. The time period, t, is taken as the time interval for which the peak prevails. This value is presented in years. These variables were then inserted in the standard BS model to produce the portion of the value of a call or put attributable to the peak.

2. It was assumed that the only disruption to the standard Wiener process is the BEE share issue. After a peak, one can assume that the share will return to the initial Wiener process which follows the risk-neutral path. All other “jumps” are caused by events independent from the BEE transaction. One can therefore assume that it may be possible to estimate the payoff at maturity after the peak using adjusted values for r, σ and t. The value for S will therefore become the share price on the last day of the peak. The value for K will be the value calculated for the process described above. Monte Carlo simulation is a stochastic process used for sampling random outcomes. The value of K will therefore be the value calculated during the peaks. In order to remove the effect of the sharp change in volatility during the peaks, the volatility was estimated using the same method as before, but only for the period between announcement and the commencement of the peak date. This was done because it is believed that the share price was undisturbed during this period and that this is a true reflection of the volatility during that year. To determine whether the volatility impacts the final option value significantly, a similar calculation was performed using the prevailing volatility during the peak period. These two variables were then used in the BS calculation for the remaining post-peak period.
3. In Section 9, it was confirmed that the terms, $KN\ (d_2)$, represent the payoff in a risk-neutral world. In certain instances, where σ is substituted for r, the final option value approximates the actual option value at maturity. Similar procedures to Section 9 were performed here. In this instance, however, both volatility variables described in “2” above were used to determine if they significantly impacts the final option value.
4. To be consistent with the previous methods applied, σ was substituted in the place of r in the original BS formula to determine how it would affect the final option value.

A table for the results follows. In all instances the key remains as defined before:

Yellow – Best result for a given share

Green – Result closely approximates the best result with deviations of less than 20% observed.

All results are presented in terms of percentage deviation from the actual.

I. ADCORP

Adcorp - BSM Valuations				
Summary of Results:				
	Value During Peak	**Value After Peak**	**Value After Peak with Peak Volatility**	**Volatility Discount Rate**
Adjusted Values	2%	-25%	-13%	-6%
Examining K N(d2)	25%	71%	73%	7%

Table 44 Adcorp – Results produced by the proposed amendments to the original BSM model.

Adcorp's shares were issued at a significant discount and were acquired for 1 cent per share. Because the guidelines of the BS model are strictly defined, this fact is irrelevant as the share price prevailing on the announcement date forms the basis of the valuation. This is important, as it implies that with any given share issue, the amount at which the shares are purchased remains insignificant and does not affect the final option payoff at maturity. Thus, the most important drivers are the share price at transaction date and the strike price. One can also assume that, because the transaction was entered into at a significant discount, the return during the peak should be sufficient to remove any dilution effects. One can therefore hypothesize the following:

Hypothesis 1:

Where transactions are entered into at a significant discount, the value of the call option determined during the peak should be added to the initial BS call value, which has been estimated without taking the peak into account, to determine the final option value.

When one determines the call value during the peak of R 3.46 and adds it to the initial BS calculated value of R 23.89, the final answer of R 27.35 was only a 2 % deviation from the actual value of R 27.83. This result is important as it confirms that, because the BS model is a continuous valuation model based on rigid inputs, it fails to capture any unexpected increases in value which may occur during the contractual term. In instances where attempts were made to value the option for the remaining term after the peak, significant deviations were produced. This is attributed to the decline in the share price which took place post-inception date. This aspect is not captured by the valuation model and the final BS option value will reflect a value based on a higher share price. This supports the assumption made for the One-Step Binomial valuations, which suggested that in instances where the valuation term is reduced, poor results will be produced by any valuation model. However, when the volatility during the peak is used during the remaining term valuation, it produces a better result. The contrary is suggested here. Substituting the volatility in place of the risk-free rate (RFR) and examining the term, which is then added to the initial BS value, also produced good estimates for this method This result supports the assumption that using the volatility as the discount rate removes any dilution effects in the share price caused by acquiring the shares at a minimal discount price.

One can therefore accept Hypothesis 1 as true.

II. ASSORE

Assore - BSM Valuations				
Summary of Results:				
	Value During Peak	Value After Peak	Value After Peak with Peak Volatility	Volatility Discount Rate
Adjusted Values	-2%	0%	5%	-10%
Examining K N(d2)	-32%	-1%	4%	-64%

Table 45 Assore – Results produced by the proposed amendments to the original BSM model.

Assore's transaction can be classified as a multi-stage expansion option, as the transaction was initiated in multiple stages. This transaction is different, because the first stage was initiated in 2005, when the share price was a dismal R 25.40. Since that date, there has been a rapid upsurge in the share price, which only declined on 27 February 2009. After this date there was an increase in the share price again. Before the inception of this transaction, it was publicly disclosed that a rapid increase in the share price was expected. One can therefore not take the entire period from 2005 to 2009 as a peak caused by the BEE transaction, as this will lead to mispricing. Stage 1 and 2 of the transaction took place during the period of tremendous upsurge. There was no definite decline in the share price after the inception of stage 1. However, Stage 2's inception date was on 15 June 2008. This means that it falls within the "expected" peak period. It can therefore be assumed that, because the initial BS valuation was done in stages and does not consist of only one amount, it would be necessary to include the apportioned effect of the second transaction as it took place before the peak end date. To test the transaction, one therefore hypothesized the following:

Hypothesis 2:

When a transaction comprises a multi-stage expansion option, then obtaining the BS call value after the peak for the remaining contractual term will produce an accurate estimate of the final option value.

The computed values were added to the initially-calculated combined BS option value of R 131.66. Very good results were produced using this method. The best result occurred, as expected, where the valuations were performed post-peak. This is a substantial contrast to Adcorp's results. It confirms the assumption that the increase in the share price post-peak follows the standard Brownian motion and that the increase in share price was not entirely attributable to the BEE transaction. It also confirms that the share returns to the risk-neutral path post-peak. It is worth noting that only two of the eight valuation methods applied failed to produce an accurate estimate of the actual payoff. However, using the volatility as the discount rate produced a value which was less than a 5% deviation from the final share price.

One can therefore accept Hypothesis 2 as true.

III. KUMBA

Kumba - BSM Valuations					
Summary of Results:					
	Value During Peak	Value After Peak	Value After Peak with Peak Volatility	Volatility Discount Rate (During Peak)	Volatility Discount Rate (After Peak)
Adjusted Values	-357%	-160%	-280%	-470%	-552%
Adjusted Values in Isolation	-193%	4%	-116%	-306%	-388%
Examining K N(d2)	-258%	-262%	-236%	-165%	-161%
Examining K N(d2) in Isolation	-94%	-98%	-72%	-1%	3%

Table 46 Kumba – Results produced by the proposed amendments to the original BSM model.

The Kumba transaction is an expansion option. Adding the computed values to the original BS values produced severe mispricing. Alternative procedures were therefore performed where one examines the option values in isolation. The following hypothesis formed the basis of the testing:

Hypothesis 3:

Where a transaction is an expansion option, using the BS model to estimate the call value for the remaining term after the peak and using the volatility measured during the peak as the discount rate will produce an accurate estimate of the final option value.

As expected, accurate prediction of the payoff after the peak with a deviation of only 4% could be obtained. This supports Adcorp's and Assore's results, for which it was established that after the peak, the share returns to its risk-neutral path. In Section 9, it was noted that when using the volatility as a discount rate, an accurate prediction of the option payoff may be obtained, provided that the only term taken into account is $K\ N(d_2)$. It was applied differently in this instance, as the only term taken into account was for periods post-peak, where the volatility was used as the discount rate. Two computations were performed. In one, the value of S and K remain as defined during the peak. This was done as the results showed that the payoff during the peak does not have a significant impact on the final option value. The values for t and σ therefore remained as they were in the peak period with only the value for r being adjusted. The second calculation was performed where the values for S and K reflected the variables computed for the post-peak period. The only volatility used was the volatility computed during the peak. It was only after these adjustments that accurate results could be obtained, with the best result of a deviation of 1% being produced. This finding confirms the initial assumption that using the volatility as a discount rate when examining $K\ N(d_2)$ is the most appropriate for expansion options. One can therefore accept Hypothesis 3 as true.

IV. PRIMESERV

Primeserv - BSM Valuations					
Summary of Results:					
	Value During Peak	Value After Peak	Value After Peak with Peak Volatility	Volatility Discount Rate (During Peak)	Volatility Discount Rate (After Peak)
Adjusted Values	-563%	-267%	-822%	-481%	-911%
Examining K N(d2)	1243%	-498%	-598%	-4151%	-911%

Table 47 Primeserv – Results produced by the proposed amendments to the original BSM model.

In previous BS valuations, it was noted that in instances where there has been no issue of shares, or where shares have been issued at a minimal discount, using the volatility as the discount rate for the term $K\ N(d_2)$ will produce a result which approximates the actual payoff at maturity. This forms the basis of the hypothesis:

Hypothesis 4:

Where there has been no share issue, examining the term $K\ N(d_2)$ in isolation will produce an accurate estimate of the final option value at maturity.

Here the Primeserv share produced a deviation of (3)%. It is assumed that this is the risk-neutral state for this share and that in no other circumstances will any other method produce a more accurate result. This assumption is confirmed by the results above, where it is evident that no valuation method could produce an accurate estimate of Primeserv's payoff. It is assumed that this is attributable to the share price pattern not reflecting the peak post-announcement of the transaction.

When the valuations were performed, it was noted that later there was a sharp decline in the share price pattern in 2009. In an attempt to adjust for this, no accurate results could be produced, which suggests that this phenomenon is not related to the BEE transaction. This is important as it supports the assumption that independent events which occur post-inception date have no impact on the final option value at maturity. Hypothesis 4 can therefore be accepted as true.

V. SANTAM

Santam - BSM Valuations						
Summary of Results:						
	Value During Peak	Value After Peak	Value After Peak with Peak Volatility	Volatility Discount Rate (During Peak)	Volatility Discount Rate (After Peak)	Amendment
Adjusted Values	-12%	18%	-15%	-50%	14%	-28%
Examining K N(d2)	-87%	7%	-28%	-131%	4%	-79%
Examining K N(d2) in Isolation	-9%	86%	51%	-52%	83%	0%

Table 48 Santam – Results produced by the proposed amendments to the original BSM model.

Santam's transaction is a 'vanilla' transaction entered into at a minimal discount. Similar procedures to that of Adcorp's transaction will be performed, as Adcorp also represents a 'vanilla' transaction, but one which occurred at a significant discount. To test the transaction, one will hypothesize the following:

Hypothesis 5:

When a transaction has been entered into at a minimal discount, and where the values for S and K remain as estimated during the peak, and the volatility remains as estimated before the peak, then examining the term $K\ N(d_2)$ in isolation will produce an accurate estimate of the final option value.

When the computed option values are added to the original BS option values the deviations are minimal. In Section 9, as with Kumba's transaction, it has been noted that, in certain instances, using volatility as a discount rate can produce accurate option payoff estimates, when examining the term $N(d_2)$. When this is applied to Santam, it was noted that good results were produced when examining the period after the peak. This is an expected result as it has been established that using volatility as a discount rate removes the effect of dilutions when shares are issued at a minimal discount. This suggests that the risk-neutral environment has not been altered significantly and, as in all previous transactions that the share returns to the risk-neutral path post-peak.

In Section 9, both Kumba's and Santam's transactions could be valued accurately when the volatility was used as the discount rate. A valuation method similar to that which produced the best results for Kumba was introduced, where the value of S and K remain as defined during the peak over the remainder of the contractual term, and the volatility is used as the discount rate. This also produced the best result for Santam with a deviation of 0%. The volatility therefore removes the effect of the discount and restores the risk-neutral environment. This confirms the assumption that term $K\ N(d_2)$ represents the risk-neutral state and that Hypothesis 5 is true.

VI. SANYATI

Sanyati - BSM Valuations			
Summary of Results:			
	Value During Peak	Value After Peak	Value After Peak with Peak Volatility
Adjusted Values	150%	-892%	-1031%
Examining K N(d2)	-43%	-318%	-640%

Table 49 Sanyati – Results produced by the proposed amendments to the original BSM model (Peak 1).

Previously, it was assumed that, because the share price declined sharply after the inception date, the transaction must be treated as a put option. In order to treat it as a put option, the effect of the peak on the total transaction value was determined. The total call value calculated during the peak amounted to R1.63. This exceeded the call value, calculated at inception, of R 1.54. The peak was ignored altogether, owing to the later occurrence, and to value the option only as a put option post-peak. Despite this amendment, it was noted that none of the methods could accurately price the option. The values produced were of significance, however, even though they were mispriced. The option value computed after the peak is of significance as the value of R(1.84) closely resembled the actual loss on the transaction of R(1.89). The value of R(2.10), computed when using the volatility as a discount rate, represents the total acquisition price of R2.10 lost by each shareholder who entered into the transaction.

Previously, this thesis suggested interchanging the variables for $N(d_1)$ and $N(d_2)$ when valuing a put option where the exercise is completely random and unrelated to the stock price. This thesis also suggested limiting the parameters to $N(x) \geq x$. In this particular instance it proved to be irrelevant.

The share pattern was re-examined to determine if additional anomalies are evident and it was discovered a second less-defined peak, which occurs slightly after the end of the initial peak. Similar calculations were therefore performed as before in an attempt to find a better solution. The results from this approach are listed below:

Sanyati - BSM Valuations				
Summary of Results:				
	Value During Peak	**Value After Peak**	**Value After Peak with Peak Volatility**	**Volatility Discount Rate**
Adjusted Values	-633%	-957%	-606%	-966%
Examining K N(d2)	-351%	-564%	-151%	-665%

Table 50 Sanyati – Results produced by the proposed amendments to the original BSM model (Peak 2).

The value of the call option produced during the second peak amounted to R0.18. Although it is lower than the initial call value of R1.54, it more closely resembles the share price of R0.21 at liquidation date. Another interesting figure which appeared was a value of R0.23, which represents the final share price at maturity and which is obtained when measuring the return after the peak using the volatility estimated before the peak. Procedures similar to the initial procedure were therefore performed. No method could produce an accurate estimate of the payoff at maturity. The results ranged between R(0.47) and R(1.98). Some of the values, however, represented good estimates of the loss at maturity.

As no method could produce an accurate estimate of the final option payoff, an alternative method was devised, in which it was decided to return to the initial contractual terms and valued the option as a call option. This thesis suggest that the final payoff at maturity should be the initial total call value adjusted for the effect of peaks during the period, as it has been confirmed that this has an effect on the final option value. This would be of great assistance here, as the values calculated previously exceeded the actual payoff at maturity significantly. To test the amended approach, one can hypothesize the following:

Hypothesis 6:

When there is a sudden unexpected decline in the share price, then deducting the call value during the peak from the initial BS call value will produce an accurate estimate of the final option value.

The adjusted effects are as follows:

Sanyati - Value as a Call Option		
	Peak 1	Peak 2
Original Call Value	1.53	1.53
Less:		
Value of Call Peak 1	1.63	1.53
Value of Call Peak 2	-	0.18
Net:	-0.10	-0.1742
As Percentage of (18.56)% :		**94%**

Table 51 Sanyati – Results produced by the proposed amendments to the original BSM model (Peak 1 and 2).

For peak 1, the net effect is a negative value. This is close to the actual negative payoff. When an adjustment is made for the effect of peak 2, the effect of peak 1 is limited to the original call value. This is done because it is know that the BS model is based on the cumulative probability distribution function. Incorporating the total effect will thus over-estimate the loss and will introduce an element of mispricing. When the results using this method are compared to the actual, the deviation was 6%. In Rand terms the deviation is only R 0.00116, which is very insignificant. This is the best result produced by the BS method. The precision could be further improved if one had access to the cause of peak 2 and the exact period in question. This result confirms that the second peak was not caused by an independent event but rather an event directly related to this transaction and thus it significantly impacted the final option value. The overall result confirms that the hypothesis is true.

(ii) Summary of Results

A comparison of the amended results to the initial best results as given in Section 9 is presented below:

Black Scholes Merton Model	Adcorp Holdings Limited	Assore	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Original Values	14%	11%	7%	-3%	14%	-175%
Adjusted Values	2%	0%	-1%	-267%	0%	6%
Best Results	**2%**	**0%**	**-1%**	**-3%**	**0%**	**6%**

Table 52 Comparison of the results produced by the original BSM model and the amended BSM.

From the above, it becomes evident that this approach produced better results than the initial estimated values. This is largely because the current approach takes into consideration the different changes which take place across the contractual term. An element of flexibility has therefore been introduced in the to the BS model. The results are significant, because they support the suggestion, introduced in the Implied Trinomial tree section, that the peaks are caused by the BEE share issues and hence peaks have an impact on the final option values.

The following table shows the results compared to the results produced by the other valuation methods:

Summary of Results produced by all methods	Adcorp Holdings Limited	Assore	Kumba	Primeserv Group Limited	Santam	Sanyati Holdings Limited
Real Options Method	14%	10%	-1%	1%	0%	3%
Binomial Lattice Valuation	13%	8%	-21%	25%	2%	33%
Black Scholes Merton Model	2%	0%	-1%	-3%	0%	6%
Risk Neutral Valuation	3%	3%	11%	24%	6%	14%
Trinomial Trees	0%	-1%	-7%	0%	-1%	79%
Implied Trinomial Trees	1%	0%	1%	1%	-1%	-1%
Best Results	**0%**	**0%**	**1%**	**0%**	**0%**	**-1%**

Table 53 Summary of results produced by the different valuation methods.

The amended BS values produced three of the six best results for the transactions, with the remaining best results being produced by the Trinomial and Implied Trinomial methods. This is significant because it confirms the assumption that binomial and trinomial trees, as the step becomes smaller, lead to the lognormal assumption for stock prices that underlies the Black-Scholes model.

15. Conclusion

In this thesis different valuation methods were explored, in an attempt to derive a method which will best price any option at maturity. The application of Net Present Value methods was examined in order to obtain the final option values at maturity.

It was found that the Real Options Valuation provides an accurate estimate for each transaction, owing to the various categories available under which an option may be priced. This technique suggests that, when determining the value of each company, the underlying contractual terms should be examined as this is valuable when pricing a company. Although this method proved to be a successful pricing method, it suffers from the drawback that various sources have to be reviewed to gather the necessary information required in the pricing formulas. This proved to be a laborious task.

The Binomial Lattice Valuation method provided the worst results. All attempts to adjust the existing method in order to provide improved results were unsuccessful. However, it is from this method that it was noted that in order to price any option successfully, using the change in volatility over the contractual term and limiting the prices in the pricing model to the maximum share price reached during the contractual term would provide a better estimate of the final option value.

The Risk-Neutral valuation method also provided good results. The major contribution from this method, however, was the discovery that the volatility may be used as a discount rate. This was found to be particularly successful when used as the discount rate for pricing expansion options.

The Trinomial Tree Valuations produced good results for all but on transaction. Using this valuation method it was found that there are important relations between the final option values produced by any tree and the share price, strike price and final share price at maturity, as the final values closely resembled the latter values in many instances. As this method could not provide an accurate result for a company in financial distress, alternative methods were devised in order to price these companies accurately. The Implied CRR Trinomial Trees were therefore examined and the Black-Scholes-Merton Model in order to find a value which approximates the actual.

This paper presents amendments to the Black-Scholes-Merton and the Implied Trinomial Trees models. The major contribution of this paper is the discovery of peaks in the share price in the year of announcement of the transaction, which are generally attributed to the company entering into the given transaction. It was found that the inclusion of the effect of the call option value calculated during these peaks on the final call option value provides the best results when pricing any option. It is primarily for this reason that the Implied CRR Trinomial Tree Methods and the Black-Scholes-Merton Models interchangeably provided the best results, as these methods could be easily amended to take the effect of the peaks into consideration.

When the various Net Present Value Methods were examined to determine if they provided suitable pricing method, various trends emerged. The results suggested that, instead of always using the suggested weighted average cost of capital as the discount rate for Net Present Value calculations, the risk-free rate and the volatility should also be used. Another important finding was that, depending on the method applied, the final value would estimate the share price at inception and maturity, the strike price, or the final payoff at maturity.

The results therefore suggest that any given valuation method will provide an accurate estimate of the final option value at maturity. When choosing which valuation method to use, the most important aspect to consider is the necessity of understanding the underlying contractual terms of the transaction.

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All share price information was extracted from McGregor BFA.

Appendices

Appendix A

Rank 2011	Company	Details of transactions	Year
1	Standard Bank of South Africa Ltd	JSE listed iron-ore, manganese, chrome and ferroalloy producer Assore announced that it would buy its lack partner Shanduka's 11.8% stake in the company for R2.7 billion and pass the benefit to its BBBEE shareholders. Standard Bank was actively involved in the structuring of the BEE deal.	2011
2	Sekunjalo Investments Ltd	Saab SA is the designated BEE partner. Sekunjalo Investments Ltd secured 25% of Saab SA's voting rights and a 5% economic interest with an option, linked to Saab SA's performance over the next five years to extend the economic interest to 25%.	2007
3	Nedbank Ltd	The issue of new ordinary shares to black controlled entities beneficially owned by black employees, clients and distributors, communities and Black Business Partners	2005
4	Kelly Group Ltd	The Group had its origin in the Kelly business, which dates back to 1969. It was acquired by Brait private Equity (BPE) in April 2001. Prior to its listing in 2007, BPE owned 47% of the group, BEE investor Safika owned 25% and the remainder of the shares were owned by management.	2007
5	Hosken Consolidated Investments Ltd	HCI acquired 25.1% of Clover.	2005
6	Adcorp Holdings Ltd	Women Investments Portfolio Holdings Limited ("WIPHOLD") and Simeka Group (Pty) Limited identified as BEE partners	2007
7	Old Mutual plc	The transaction will result in a direct black ownership of 11% of the company's shares. 6.14% will be allocated to staff and management, 3.5% to Black Business Partners and 1% to Empowerment Trusts.	2005
8	Pretoria Portland Cement Ltd	The proposed sale of 75% of Afripak Limited to a management and empowerment consortium to establish a credible Black Economic Empowerment force in the industry	2003
9	TongaatHulett Ltd	The sal of a 25% stake in TongaatHulett and a 15% stake in Hulamin to BEE investors achored by Ayavuna Women's Investments, Sangena Investments, Imbewu Consortium and Makana Investment Consortium.	2007
10	Kagiso Media Ltd	Kagiso Media and MSG Afrika Media acquired a 50.1% stake in Clear Channel Merafe.	2007
11	Group Five Ltd	The introduction of a new BEE partner iLima consortium. The transaction will achieve 26.1% direct black ownership in the company following the issue of shares to iLima consirtium, Mvelaphanda Group Ltd and broad based employees.	2005
12	Aveng Group Ltd	The introduction of BEE partners ("BEE Consortium") into Avengs African operations namely Aveng (Africa) and Trident Steel, Kagiso Tiso Holdings (Pty) Ltd , the merged entity of Tiso Group (Pty) Ltd ("Tiso") and Kagiso Trust Investments (Pty) Ltd. The BEE Consortium and has established a seven year partnership with Aveng.	2004
13	Sun International Ltd	The proposed extension of the R60 million financial guarantee previously provided by Sun International to Depfin, for certain obligations of Sun International's business partner, Dinokana.	2010
14	Wilson Bayly Holmes - Ovcon Ltd	The details of their charter will ensure that 30% of the company is in black ownership by 2013.	2006
15	Brimstone Investment Corporation Ltd	Brimstine investment acquired a 6% stake in Galaxy Gold, resulting in Galaxy Gold exceeding the Mining Charter's 26% equity target ahead of the April 2014 deadline.	2011
16	Metropolitan Holdings Ltd	Over the last 5 years Kagiso Trust and KTI acquired a meaningful stake in the company's equity via phase I and II of the BEE deal. Phase III was introduced to increase the equity stake.	2009
17	Oceana Group Ltd	Oceana, Ocfish Holding Company Limited (Ocfish), Tiger Brands Limited (Tiger Brands) and Real Africa Holdings Limited(RAH) have entered into an agreement which will result in : (i)Disposal by RAH of its entire shareholding in Ocfish to Tiger Brands (ii)Offer by Tiger Brands to otherOchfish minority shareholders to acquire their shares. (iii)The subsequnet unbundling of approximately 46% interest in Ocean to Ocfish shareholders (iv)Sale by Tiger Brands of Oceana share to a BEE consortium comprising a black employee share trust and a strategic partner. (v) The new issue of Oceana ordinary shares to the black employee share trust. (vi) Oceana intends to enter into an agreement with Brimstone Investment Corporation Limited as strategic BEE partner whereby Brimstone will acquire 10% of Oceana shares.	2006
18	Barloworld Ltd	The company announed a deal that will achiev eblack ownership of 28.99%. Strategic business partners will hold 5.88% of the shares, 2.39% will ba allocated to employees, 0.78% to educational trusts and 0.95% to community service groups.	2008
19	Optimum Coal Holdings Limited	Glencore International plc expressed interest to acquire a controlling interest in Optimum Coal Holdings Limited.	2011
19	Netcare Ltd	Netcare concluded a BEE transaction transferring 160 million shares to the Health Partners for Life (HPFL) Trusts for a range of beneficiaries	2005
21	Tiger Brands Ltd	The company sold 4% of their shares to staff (including staff of subisiaries being Tiger Food Brands, Adcock Ingram, Adcock Ingram HealthCare, Adcock Ingram Critical Care and Enterprise Foods.)	2005
22	Primeserve Group Ltd	The commercialisation of the operations of Bathusi Staffing Services (Pty) Ltd and the introduction of Black Economic Shareholders into the Group operating subsidiaries providing empowered status.	2004
23	Bidvest Group Ltd	Bidvest concluded a BEE ownership transaction with Dinatla, facilitating the acquisition of the company by Dinatala in a transaction valued at R2.1 billion at transaction date.	2003
24	Cadiz Holdings Ltd	Makana Financial Services (Proprietary) Limited will enhance its strategic investment in Cadiz through the acquisition of a further 5% of the issued share capital.	2010
25	Allied Electronics Corporation Ltd	Thebe Investement Corporation (Pty) Ltd and Identity Capital Partners (Pty) Ltd acquired a combined 25% plus one share equity shareholding in the company.	2010
26	African Oxygen Ltd	The company sold 69% shareholding in Afrox Healthcare Ltd to Mvelaphanda Strategic Investments.	2003
27	AdaptIT Holdings Ltd	The acquisition of a majority shareholding in ITS Holdings by way of an exchange of R 16 million for a 51% shareholding.	Outstanding
28	Sanyati Holdings Ltd	The purchase of 9,722,222 ordinary shares in Sanyati Holdings Limited by Growie Holdings Limited.	2008
29	Beige Holdings Ltd	Thebe Investments Holdings (Pty) Ltd acquired 563 294 600 shares in Beige Holdings Ltd increasing their shareholding to 33.45%.	2008
30	Firstrand Ltd	The acquisition of 6.5% of FirstRand by the FirstRand Empowerment Trust, a trust specifically created for the benefit of the four participating broad-based BEE groups namely Kagiso Trust , the Mineworkers Investment Trust, WDB Trust and and the FirstRand Empowerment Foundation. Secondly the acquisition of 3.5% of FirstRand, through various trusts, by the Group's black South African employees and black non-executive directors.	2005

Appendix A (continued)

31	Gijima Holdings (Pty) Ltd	Their transaction led to the formation of GijimaAst, resulting in a 32% direct black ownership. Transaction was based on the independently assessed underlying value of Gijima's ICT subsidiaries and AST Group. The unique features of the transaction in the ICT sector entails that it has no funding structures, preferential shares or special voting rights.	2005
32	Workforce Holdings Ltd	The company listed on the JSE for the first time on the 21 November 2006. The company has a 26% BEE shareholding in the form of Vunani Capital Holdings (Pty) Ltd and Pha Phama Africa Employee Trust. The listing will enable BEE shareholders to cement their shareholding in Workforce and offer equity participation by management and staff while enabling Workforce to attract and retain key human resources.	2006 / 2007
33	Paracon Holdings Ltd	Paracon and Adcorp Holdings Limited entered into an agreement whereby all the share capital in Paracon will be acquired by Adcorp.	2011
34	City Lodge Hotels Ltd	City Lodge Hotels entered into a BEE transaction involving its employees, students from the University of Johannesburg's school of tourism and hospitality and a black controlled investment company called Vuwa Investments.	2008
35	Jasco Electronics Holdings Ltd	Afrocentric acquired 34,9% of telecom's group Jasco, which, in turn acquired 34% of M-Tec.	2008
36	Santam Ltd	The sale of 10% of issued shares to Santam staff, broad based community trusts and strategic business partners.	2007
37	Glenrand MIB Ltd	The introduction of two new BEE partners, Ayavuna Women's Investments and Mateneku Investments, which increases the BEE shareholding in the group to 27.4%.	2009
38	Avusa Ltd	Avusa acquired the entire share capitals of Universal Print Group (Pty) Ltd, Hirt & Carter (Pty) Ltd from Communications (Pty) Ltd as well as claims that UHC may have against UPG and H&C. As part of the settlement of the acquisition price 20,555,555 shares will be issued in Avusa Ltd.	2010
39	Growthpoint Properties Ltd	A BEE consortium was established to acquire 100 million Growthpoint shares and debentures or linked units (approximately 14% of the total Growthpoint linked units in issue)	2005
40	MTN Group Ltd	The transaction, called MTN Zakhele, states that 4% stake of the company will be sold to black investors.	2010
41	Mutual & Federal Insurance Company Ltd	The transaction will result in a direct black ownership of 11% of the company's shares. 6.14% will be allocated to staff and management, 3.5% to Black Business Partners and 1% to Empowerment Trusts.	2005
42	Allied Technologies Ltd	The company entered into a transaction which involves the transfer of 25% plus 1 share in Altech's wholly owned subsidiaries, Altech Netstar and Altech Netstar Fleet Solutions.	2009
43	Absa Group Ltd	Absa shareholders approved the allocation of 73,152,300, redeemable cumulative option-holding Absa preference shares to Batho Bonke Capital Limited. Each redeemable preference share carries the option to acquire one Absa ordinary share	2004
Mining Related Transactions			
1	Kumba Resources	Kumba Resources (Anglo American and IDC) sold mining assets excluding coal to a BEE Consortium (Eyesizwe Mining, Eyabantu Capital, Tiso Group and South African Women.)	2005
2	Impala Platinum	Royal Bafokeng Nation acquired 13.4% of the shares of Impala Platinum.	2006
3	Northam Platinum	Mvelaphanda Resources sold the Booysendal Platinum project to Northam Platinum for shares in Northam. (The transaction resulted in Mvelaphanda acquiring and owning 63% of Northam Platinum.)	2007
4	AngloPlatinum	Royal Bafokeng entered into a 50:50 joint venture agreement with Anglo Platinum on Rasimone Platinum.	2008
5	Sasol	R 7.8 billion of the R 26 billion broad BBBE transaction was issued to the public with hopes of attracting between 100,000 and 200,000 new black shareholders	2008
6	Goldfields Ltd	Mvelaphanda Resources Ltd acquired 15% of Gold Fields Ltd, after the option intruments granted in 2004, reached vesting date.	2009

Table 54 Table showing total number of companies included in the initial sample including details of the transaction and the year of announcement.

Appendix B

1) Black-Scholes option pricing model

$$w(x,t) = xN(d_1) - ce^{r(t-t^*)}N(d_2)$$

where:

$$d_1 = \frac{\ln\frac{x}{c} + (r + \frac{1}{2}v^2)(t^* - t)}{v\sqrt{t^* - t}}$$

$$d_2 = \frac{\ln\frac{x}{c} + (r - \frac{1}{2}v^2)(t^* - t)}{v\sqrt{t^* - t}}$$

and:

$w_1(x,t)$ = the value of an option

x = the stock price

c = the exercise price

t^* = the maturity date

t = the current date

v^2 = the variance rate of the return on the stock

r = the short-term interest rate.

2) Binomial Method

$$f = e^{-rT}[pf_u + (1-p)f_d]$$

Where:

$$p = \frac{e^{rT} - d}{u - d}$$

Where:

r = the risk-free interest rate

T = time period between the different nodes

d = percentage decrease in a stock's down movement

u = percentage increase in a stock's up movement

f_u = the payoff from the stock if the stock price moves up

f_d = the payoff from the stock if the price moves down

p = the probability of a stock moving up by a certain percentage and a certain probability of the stock moving down by a certain percentage

3) Risk-Neutral Valuation

$$e^{(r-q)\Delta t} = pu + (1-p)d$$

Where:

$p = \dfrac{a-d}{u-d}$

$u = e^{\sigma\sqrt{t}}$

$d = e^{-\sigma\sqrt{t}}$

$a = e^{(r-q)\Delta t}$

(A) Control Variate Technique

$f_A + f_{BS} - F_E$

Where:

f_{BS} = the Black – Scholes price of a European option. This will be equivalent to the value calculated under the real options section.

(B) Setting *p* = 0.5

$u = e^{(r-q-\sigma^2/2)\Delta t + \sigma\sqrt{\Delta t}}$

$d = e^{(r-q-\sigma^2/2)\Delta t - \sigma\sqrt{\Delta t}}$

Where:

q = the dividend yield for the period. This is assumed to be 0 in order to be consistent with the rest of the work.

4) Trinomial Trees

(A) Combining two steps of a CRR binomial tree

$S_u = Se^{\sigma\sqrt{2\Delta t}}$

$S_m = S$

$S_d = Se^{-\sigma\sqrt{2\Delta t}}$

$$p = \left[\frac{e^{r\Delta t/2} - e^{-\sigma\sqrt{\Delta t/2}}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\Delta t/2}}}\right]^2$$

$$q = \left[\frac{e^{\sigma\sqrt{\Delta t/2}} - e^{r\Delta t/2}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\Delta t/2}}}\right]^2$$

Where:

S_u = Stock price reached from an up move.

S_m = Stock price at the beginning of the time step.

S_d = Stock price reached from a down move

p = The probability from an up move

q = The probability from a down move

r = risk-free interest rate

σ = The constant volatility for the period

(B) Combining two steps of a JR binomial tree

$$S_u = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{2\Delta t}}$$

$$S_m = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t}$$

$$S_d = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t - \sigma\sqrt{2\Delta t}}$$

(C) Equal-Probability Tree

$$S_u = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{3\Delta t/2}}$$

$$S_m = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t}$$

$$S_u = Se^{\left(r-\frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{3\Delta t/2}}$$

(D) Incorporating forward rates

If $S_{i+1} < F_i < S_{i+2}$

then:

$$p_i = \frac{1}{2}\left[\frac{F_i - S_{i+1}}{S_{i+2} - S_{i+1}} + \frac{F_i - S_i}{S_{i+2} - S_i}\right]$$

And:

$$q_i = \frac{1}{2}\left[\frac{S_{i+2} - F_i}{S_{i+2} - S_i}\right]$$

And if $S_i < F_i < S_{i+1}$ then:

$$p_i = \frac{1}{2}\left[\frac{F_i - S_i}{S_{i+2} - S_i}\right]$$

And

$$q_i = \frac{1}{2}\left[\frac{S_{i+2} - F_i}{S_{i+2} - S_i} + \frac{S_{i+1} - F_i}{S_{i+1} - S_i}\right]$$

Where:

F_i = The forward rate defined by: $F_A = Se^{(r-\delta)\Delta t}$

Where:

δ = the dividend yield for the period. This is assumed to be zero in order to be consistent with the rest of the work. In both instances the middle probability is equal to $1 - p_i - q_i$.

(E) Hull 2009

$u = ^{\sigma\sqrt{3\Delta t}}$

$d = \frac{1}{u}$

$p_d = -\sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - q - \frac{\sigma^2}{2}\right) + \frac{1}{6}$

$p_m = \frac{2}{3}$

$p_u = \sqrt{\frac{\Delta t}{12\sigma^2}\left(r - q - \frac{\sigma^2}{2}\right) + \frac{1}{6}}$

The variables contained in each equation remain defined as before.

(F) General Tree-Building Procedures using Mean Reversion

(i) First Stage

The Hull-White model for the instantaneous short rate r is defined by :

$dr = [\theta(t) - ar]dt + \sigma\, dz$

The time step on the tree is constant and equal to Δt. The Δt rate, R, is assumed to follow the same process as r, and is defined by the following:

$dR = [\theta(t) - aR]dt + \sigma\, dz$

This appears to be reasonable as the limit as Δt tends to zero. To construct a tree for this model entails constructing a variable R^*, which is initially zero and follows the following process:

$dR^* = -aR^* dt + \sigma\, dz$

The process is assumed to be symmetrical about $R^* = 0$.The variable $R^*(t + \Delta t) - R^*(t)$ is normally distributed. If terms of higher order than Δt are ignored, the expected value of

$R^*(t+\Delta t) - R^*(t)$ is defined by $-aR^*(t)\Delta t$ and the variance of $R^*(t+\Delta t) - R^*(t)$ is defined by $\sigma^2\Delta t$. The spacing between interest rates on each tree, ΔR, is defined as:

$$\Delta R = \sigma\sqrt{3\Delta t}$$

The probabilities of the highest, middle and lowest branches on the trinomial tree are defined by the following equations:

$$p_u = \frac{1}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 - a\, j\, \Delta t)$$

$$p_m = \frac{2}{3} - a^2 j^2 \Delta t^2$$

$$p_d = \frac{1}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 + a\, j\, \Delta t)$$

Where:

a = a constant

j = a positive or negative integer.

(ii) Second Stage

The second stage when constructing the tree is to convert the tree for R^* into a tree for R. This is achieved by displacing all the nodes on the R^* tree so that the initial term structure of the interest rates is exactly matched. It is defined by the following equation:

$$\alpha(t) = R\,(t) - R^*(t)$$

Where:

$\alpha_i = (i\Delta t)$, the value of R at the time $i\ \Delta t$ on the R - tree minus the corresponding value of R^* at the time $i\ \Delta t$ on the r^* - tree.

6. Binomial Lattice Model

The equations to calculate the above are given by:

$$f_{N,j=\max(S_{N'j} - K,0}$$

When $0 \le i \le N - 1$

If $i\delta t > v$ and $S_{i,j} \geq KM$

Then:

$f_{i,j} = S_{i,j} - K$;

If $i\delta t > v$ and $S_{i,j} < KM$

Then:

$f_{i,j} = (1 - e\delta t)e^{-r\delta t}\left[pf_{i+1,j+1} + (1-p)f_{i+1,j}\right] + e\delta t max\left(S_{i,j} - K, 0\right)$;

If $i\delta t < v$,

Then:

$f_{i,j} = (1 - e\delta t)e^{-r\delta t}[pf_{i+1,j+1} + (1-p)f_{i+1,j}]$

Where:

e =employee exit rate

M =early exercise multiple. M is always equal to the average ratio of the stock price to the strike price when employees have made voluntarily early exercise decisions in the past and these decisions were not made immediately after the end of the vesting period.

$e\delta t$ =probability that the option will be forfeited.

δt =time period

N =time steps of length δt

$S_{i,j}$ =stock price at jth node of the tree at time $i\delta t$

$f_{i,j}$ =the value of the option at this node

f_{00} =the value of the option.

K =the strike price of the option

v =the time when the vesting period ends

r =the risk-free rate

p =the probability of an up movement in the binomial tree.

7.Real Options

(A) The Option to Delay a Project

The net present value of a project is given by:

$\mathbf{NPV = V - X}$

Where:

V = the present value of expected cash inflows computed today.

X = the initial up-front investment

If:

$V > X$ Take the project: Project has positive net present value

$V < X$ Do not take the project: Project has negative net present value

The project is valued by obtaining the present value of a call option using the Black-Scholes parameters of:

$$w(x,t) = xN(d_1) - ce^{r(t-t^*)}N(d_2)$$

i. **Value of the underlying asset:** The present value of expected cash flows from initiating the project now, not including the up-front investment, which is obtained by doing a standard capital budgeting analysis.

ii. **Variance in the value of the asset**: This can be estimated in three ways:
   - (i) If similar projects have been introduced in the past, the variance in these cash flows from those projects can be used as an estimate.
   - (ii)Probabilities can be assigned to various market scenarios, cash flows estimated under each scenario and the variance estimated across present values. Alternatively, the probability distributions can be estimated for each of the inputs into the project analysis (i) the size of the market (ii) the market share and the profit margin (iii) simulations used to estimate the variance in the present values that emerge.
   - (iii) The variance in the firm value of firms involved in the same business.

(iii) **Exercise price On Option:** The cost of making the investment. Costs remain constant; any uncertainty associated with the product is reflected in the present value of cash flows on the product.

(iv) **Expiration of the Option:** The option expires when the rights to the project lapse. Investments made after the project rights expire are assumed to deliver a net present value of zero as competition drives returns down the required rate

(v) **The Riskless Rate:** The rate which corresponds to the expiration of the option.

(vi) **Cost of Delay (Dividend Yield):** Occurs once the net present value turns positive. The cost of delay is written as :

$$\text{Annual cost of delay} = \frac{1}{n}$$

(B) The option to expand a project to cover new products or markets sometime in the future.

(Also called Multi – Stage Projects / Investments)

(i) Value of Abandonment

The payoff from owning abandonment option if given by the following:

If $V > L$, the payoff is zero

If $V \leq L$, the payoff is $L - V$.

Where:

V – the remaining value on a project if it continues to the end of its life

L – the liquidation or abandonment value for the same project at the same point in time.

(ii) The payoff to equity investors on liquidation

The payoff to equity investors, on liquidation can thus be defined as :

If $V > D$ then the payoff = $V - D$
If $V \leq D$ then the payoff = 0

Where:
V = Liquidation Value of the firm
D = face Value of outstanding debt and other external claims

Equity can be viewed as a call option to the firm, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercise price) paid off.

The payoff can thus be defined as:

Payoff on exercise = $S - K$ if $S > K$

= 0 if $S \leq K$

Where:

S = value of the firm

K = face value of outstanding debt

The remaining inputs to the Black –Scholes model are obtained as follows:

S (Value of the Firm): Obtained in two ways, either by the market value of outstanding debt or equity cumulated, assuming that all debt and equity are traded to obtain firm value.

Secondly: the market values of assets of the firm are estimated either by discounting expected cash flows at the weighted average cost of capital or by using prices from a market that exists for these assets.

***σ* (Variance in Firm value):** The variance in the firm value can be obtained directly if both stocks and bonds in the firm trade in the market place. The variance is given by:

$\sigma^2_{\text{firm}} = w_e^2\sigma_e^2 + w_d^2\sigma_d^2 + 2\, w_e w_d \rho_{ed}\sigma_e\sigma_d$

Where:

ρ_{ed} = the correlation between the stock and the bond prices

σ_e^2 = the variance in the stock price

σ_d^2 = the variance in the bond price

w_e = the market-value weight of equity

w_d = the market value weight of debt

***t* (maturity of the debt)** =Both the coupon payments and the maturity of the bonds are taken into account to estimate the duration of each debt issue and to calculate a face value weighted average of the durations of the different issues. The value-weighted duration is then used as a measure of the time to expiration of the option.

8. Jarque-Bera Test statistic

The test statisticJB is defined by the following equation:

$$JB = \frac{n}{6}\,(S^2 + \frac{1}{4}(K-3)^2$$

Where:

n = the number of observations (or degrees of freedom)

$S = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2)^{3/2}}$

$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2)^2}$

And where:

$\hat{\mu}_3$and$\hat{\mu}_4$ = the estimates of the third and fourth central moments

$\bar{x}$ = the sample mean

$\hat{\sigma}^2$ = the estimate of the second central moment, i.e. the variance

9. Monte Carlo simulation Techniques

9.1 Primary Valuation Technique

$$\Delta S = S r_c \Delta t + S\sigma\varepsilon\sqrt{\Delta t}$$

Where:

S = the current stock price

ΔS = the change in the stock price

r_c = the continuously compounded risk-free rate

σ = the volatility of the stock

Δt = the length of the time interval over which the stock price change is occurs.

ε = the random number generated from a standard normal probability distribution

9.2 Volatility

$$\ln\frac{S_T}{S_0} \sim \emptyset\ [\ (\mu - \frac{\sigma^2}{2})T, \sigma^2\ T\ ]$$

Where:

S_T = the stock price at a future time T

S_0 = the stock price at time 0

μ= the expected return on stock per year

σ = volatility of the stock price per year

Appendix C

(A) Copeland (2002)

Copeland (2002)	
Free Cash Flow calculation	**Amount**
Net income from continuing operations	XXX
Add: Depreciation	XXX
Gross Cash Flow	**XXX**
(Increase)/decrease in working capital	XXX
Less: Capital Expenditure	(XXX)
(Incease)/decrease in other assets, net of liabilities	XXX
Foreign currency translation effect	XXX
Gross investment	**XXX**
Free Cashflow before goodwill	XXX
Investment in goodwill (and adjustments)	XXX
Free cash flow	**XXX**
Cash flow from non-operating investments	XXX
After-tax interest income	XXX
Increase/(decrease) in excess marketable securities	(XXX)
Cash Flow available to investors	**XXX**

Table 55 Table displaying valuation technique applied for Copeland (2002).

B) Damodaran (2002)

Damodaran (2002)	
Free Cash Flow calculation	**Amount**
EBIT (1-t)	XXX
Add: Depreciation	XXX
Gross Cash Flow	**XXX**
(Increase)/decrease in working capital	(XXX)
Less: Capital Expenditure	(XXX)
Free cash flow	**XXX**
Cash flow from non-operating investments	XXX
After-tax interest income	XXX
Increase/(decrease) in excess marketable securities	(XXX)
Cash Flow available to investors	**XXX**

Table 56 Table displaying valuation technique applied for Damodaran (2002).

C) Damodaran (2002) Alternative 1

Step 1

Estimate the return on capital by using the following:

$$Return\ on\ capital = \frac{EBIT(1-t)}{(Book\ value\ of\ debt + Book\ value\ of\ equity)}$$

where:

$EBIT$ = Earnings before interest and tax

t = prevailing income tax rate for the given year

Step 2

Estimate the reinvestment rate:

$$Reinvestment\ rate = g/ROC$$

where:

g = growth rate for the given year

ROC = the return on capital

Step 3

Estimate the firm's expected free cash flow to the firm as follows:

(1) Expected cash flow = $EBIT(1-t)next\ year$
(2) Less: Determined expected reinvestment rate = $EBIT(1-t)(Reinvestment\ rate)$

The result of the sum above gives rise to the free cash flow to the firm.

Step 4

Estimate the firm's final valuation by performing the following:

Value of a Firm	
	Amount
Value of operating assets	XXX
Add: value of cash and marketable securities:	XXX
Less: Value of debt	(XXX)
Value of Equity	XXX

Table 57 Table displaying valuation technique applied for Damodaran (2002) Alternative 1.

where:

Value of firm = $\frac{FCFF_1}{(WACC - g_n)}$

and:

$FCFF_1$ = Expected FCFF next year

$WACC$ = Weighted average cost of capital

g_n = Growth rate in the FCFF forever

In certain instances the growth factor exceeded the cost WACC; hence the proposed amendment was used being:

Value of firm = $\frac{FCFF_1}{(1 + g_n)}$

In the above equation the value of cash and marketable securities is always net of the bank-overdraft or short - term liabilities balances.

D) Damodaran (2002) Alternative 2

Similar steps are followed apart from the calculation of the expected free cash flow in Step 3. The value is determined as follows:

Free Cash Flow to the Firm	
	Amount
EBIT(1-t)	XXX
Add: Depreciation	XXX
Less: Capital expenditure	(XXX)
Adjusted for (increases)/decreases in working capital	
Value of Equity	**XXX**

Table 58 Table displaying valuation technique applied for Damodaran (2002) Alternative 2.