MCOM DISSERTATION

PORTFOLIO DIVERSIFICATION UTILISING ROLLING ECONOMIC DRAWDOWN CONSTRAINTS AND RISK FACTOR ANALYSIS

By

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ABSTRACT

This study investigates a new asset allocation technique termed Factor Adjusted Rolling Economic Drawdown (FAREDD), whereby resources are allocated to different assets by way of integrating Principle Component Analysis (PCA) with existing Rolling Economic Drawdown Methods (REDD). The primary purpose of this model is to create a portfolio with low drawdown levels, that can withstand turbulent market periods thus protecting portfolio value through providing stronger diversification benefits while still seeking to maximise risk adjusted and overall return. This will have strong implications for investors as it could provide an additional method and tool to be considered during the asset allocation decision stage if they have a strong drawdown aversion.

The concept of FAREDD is developed in this study within a South African context and compares this method with several traditional allocation methods including mean-variance optimised models, risk parity as well as traditional rolling economic drawdown models. So far, at the point of writing this study, the author has been unable to find any previous studies documenting this type of application of PCA to REDD.

In addition to this, all previous studies that has investigated rolling economic drawdown has been conducted exclusively on the United States of America. The literature finds that REDD provides a viable and superior alternative to traditional asset allocation in the long run. Thus, as part of this study, a second objective is to investigate whether REDD models provide sufficient protection and superior returns in a developing economy with a significantly lower number of available liquid assets and higher volatility due to increased political, economic and business risk, when compared to alternative more traditional allocation techniques.

The key findings of this study are that the FAREDD model does outperform the traditional REDD model that it is compared to for the period and it also meets the objective of providing low drawdowns and volatility while achieving strong risk-adjusted returns. However, the model does not provide the strongest drawdown protection of all portfolios tested. The FAREDD model is surpassed by the minimum-variance portfolio in this regard but from a risk adjusted basis and an overall return perspective it far outperforms the minimum-variance portfolio. Therefore, the performance of the FAREDD model is mixed and its optimality would need to be assessed relative to an investor's risk appetite and risk-return trade-off.

In addition to this, the paper finds that the performance of traditional REDD models in the South African context are mixed when compared to traditional asset allocation techniques thereby indicating

that REDD models may not be superior in the South African market place at all times. However, they can provide relevant and potential asset allocation alternatives for mangers to consider.

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CHAPTER 1: INTRODUCTION

The concept of diversification and its importance is one that has been well documented and is highly respected amongst all types of investors. However, as Meccui (2010) points out, there is no broadly accepted and specific methodology to calculate or quantify diversification, yet most investors will agree that they consider a portfolio diverse if it is not heavily exposed to individual shocks. Idzorek and Kowara (2013) indicate that since the ground-breaking publication of the mean-variance portfolio (MVP), which is the basis of modern portfolio theory, by Markowitz (1952), the primary building blocks of the optimal portfolio has been largely centred around different asset classes. These classes come with the inherent belief that different asset classes alone can provide strong diversification benefits. This is not unfounded, nor is it without merit, as in a constant and relatively stable market there have been numerous, documented, results confirming that often different asset classes have low or negative correlations (Greenspan, 2008). However, as can be seen in large downside events – such as the financial crisis, the Russian Ruble crisis, the dot.com bubble and the Asian debt crisis just to name a few – assets across asset classes exhibit the phenomenon of contagion which can lead to large if not irrecoverable capital losses. This is primarily due to the fact that the correlations and statistical models governing our portfolio allocation on the basis of diversification and mean-variance optimisation are based on datasets that do not adequately, if at all, represent the correlations and volatilities of assets during large market contractions (Chua, Kritzman & Page, 2009).

Investors have created a variety of measures in order to account for this in some way and incorporate these measures into their portfolio selection and management strategy. One such measure is drawdown - which can be most commonly be defined as a percentage loss of current wealth from a previous all-time high (Yang & Zhong, 2013). The drawdown of an asset or portfolio illustrates and provides a form of measurement for a portfolio's downside risk and performance. This in turn - if one considers the definition of diversification provided by Meccui (2010) - can be related implicitly to the strength of the portfolio's diversification. One would expect a well-diversified portfolio to have lower drawdown levels. For this reason, drawdown is a measure for risk control while optimising a portfolio and as such, is an area of research that has grown in importance since the publication of modern portfolio theory (Xie, Xu & Yu, 2014).

The implications of a large and/or prolonged drawdown can be devastating for both an investor and their fund manager. This statement is supported by the findings that fund managers clients are unlikely to tolerate a drawdown of more than 50%, fund managers themselves may shut down funds with drawdowns exceeding 20% and that an account is most likely to be closed by a client if a fund is in drawdown – even a minor one – for more than 2 years (Chekhlov, Uryasev & Zabarankin, 2005). This has led to a variety of different drawdown optimisation frameworks aimed at creating an optimal portfolio while also trying to minimise drawdown over market cycles. One such framework which has gained increasing attention over the past decade, especially in light of the dot.com bubble and the financial crisis, is economic drawdown (EDD) developed by Grossman and Zhou (1993). This has subsequently formed the basis of a new and improved model namely Rolling Economic Drawdown (REDD) developed by Yang and Zhong (2013) which has demonstrated promise as a strong potential asset allocation model available to portfolio managers (Xie, Xu & Yu, 2014).

However, the REDD model still bases its assumptions of diversification on traditional diversification methods and thus looks towards the correlations between assets that exist under normal market conditions when allocating resources to assets. This is not always realistic as asset class correlations tend to be asymmetric – highly correlated in down-markets and uncorrelated in up-markets (Chua, Kritzman & Page, 2009). This can result in the model failing to achieve its fundamental purpose during downturns of limiting drawdown and protecting the portfolio.

Therefore, the main aims and objectives of this study are twofold. Firstly, this study will look to develop a factor adjusted REDD model (FAREDD) with stronger diversification benefits. It will place an emphasis on the levels of diversification during turbulent periods, while also aiming to provide strong returns. Thereby, moving the model further towards optimality with a focus on reducing the maximum drawdown experienced by a portfolio while also optimising the risk adjusted return achieved by a portfolio as measured by the Sharpe Ratio. This will be investigated by developing a novel approach to the current framework by adapting it to incorporate multiple uncorrelated risk factors using principle component analysis. As a part of this analysis the paper will evaluate different rebalancing periods to determine the optimal rebalancing period for the model. The optimal rebalancing period for the FAREDD model will then be evaluated.

Secondly, the study will investigate whether the various economic drawdown models that have been developed as well as the FAREDD model provide improved performance in an emerging market where markets have fewer liquid tradeable assets and more volatility due to increased exposure to global and local shocks. Therefore, the models will be investigated in a South African context using South African factors and data.

Thus, the hypotheses proposed in this study are that:

1) Portfolio performance in terms controlling drawdown (through improved diversification) while achieving the optimal long-term growth rate will be improved by using the FAREDD model over other economic drawdown orientated models.
2) The various economic drawdown models that have been developed will provide improved performance in a South African context when compared to other traditional asset allocation models available to investors

This section has provided a thorough introduction to the various concepts that forms the basis of this study. Further to this section, in Chapter 2 we discuss the current literature on drawdown models as well as literature on factor analysis.

CHAPTER 2: LITERATURE REVIEW

This section will review the theory, models, applications and existing studies surrounding the three main components that make up the FAREDD model. The section provides additional background to economic drawdown models as well as risk factors as portfolio building blocks compared to asset classes and then provide a motivation for their respective inclusions to develop a FAREDD model.

2.1: ECONOMIC DRAWDOWN MODELS

INTRODUCTION TO ECONOMIC DRAWDOWN

Economic drawdown can be most easily defined as the percentage loss of current economic wealth from an all-time economic maximum (Yang & Zhong, 2013). This model was pioneered by Grossman and Zhou (1993) where they approached mean-variance optimisation of a portfolio by incorporating expected utility theory and defined portfolio optimality as maximising the long-term growth rate in the power law wealth utility function. They then incorporated the constraint of an investor who is not able to lose more than a fixed percentage of his/her maximum wealth. This linkage of mean-variance optimisation and drawdown using utility theory allows for the development of resource allocation based on the direct evaluation of asset drawdowns thus allowing economic drawdown to be a measure of performance as well as the primary risk metric variable in the portfolio optimisation problem.

This section will cover the various economic and rolling economic drawdown models developed including their mathematical framework and findings.

2.1.1: ECONOMIC DRAWDOWN FRAMEWORK

Mathematically, economic drawdown is represented by $f(t) = 1 - \frac{Wt}{EM(t)}$, where $W(t)$ represents the current wealth of the portfolio and $EM(t)$ is some economic maximum that was achieved between inception and time *t* and is calculated as follows:

$$EM(t) = \underset{0 \le s \le t}{\mathrm{Max}} \left\{ \left(1 + r_f\right)^{t-s} W(s) \right\}, \qquad (1)$$

where $W(s)$ is the optimal wealth level that could have been achieved before a drawdown. The equation accounts for the economic decay of a portfolio at the risk-free rate r_f that would impact the economic maximum had an investor shifted all their resources from risky asset(s) to the risk-free asset(s) at time *s* just before the drawdown occurred (Grossman & Zhou, 1993).

The model assumes a frictionless economy with one risky asset and one risk-free asset – similar to a Black Scholes economy - and continuous rebalancing under random walk return dynamics (Grossman & Zhou, 1993). It calculates the portion to be allocated to the risky asset as follows:

$$x(t) = \left(\frac{\frac{\lambda}{\sigma}+\frac{1}{2}}{1-\delta\cdot\gamma}\right)\cdot\left(\frac{\delta-EDD(t)}{(1-EDD(t))}\right), \tag{2}$$

Where σ represents the long term standard deviation of the risky asset. λ is the long term expected Sharpe ratio, δ is the fixed drawdown constraint chosen by an investor and γ is the is the complement of risk aversion relevant to the investor.

This model was extended to incorporate multiple risky assets by Cvitanic and Karatzas (1994). However, they used a discrete time interval rebalancing frequency and linear leverage constraints (Cvitanic & Karatzas, 1994). This resulted in lost portfolio optimality due to the discrete time period rebalancing frequency as the model would use the current portfolio value as a base for the allocation calculation but the actual adjustment in allocation only occurs after a finite time delay (Klass & Nowicki, 2005).

Yang and Zong (2013) further exhibited that the model presented by Grossman and Zhou (1993) above is not optimal as a fixed drawdown look-back period from current levels to portfolio inception can result in over and underweighting of risky assets in the incorrect market conditions (for example overweighting a risky asset while the market is falling or underweighting a risky asset while the market is rising). This is especially evident when a market cycle has a sub-cycle that is longer than the discrete rebalancing frequency - leading to sub-optimal allocations and resulting in lower long-term growth (Yang & Zhong, 2013).

Further to this, Yang and Zhong (2013) argued that the EDD model as developed by Grossman and Zhou (1993) with the long term fixed window look-back represents an idealistic mental accounting by investors due to the fact that the model retrospectively evaluates how much better off an investor would have been had they exited the risky asset at some previous perfect time in history. Yet we know that investors have inception and memory differences as they may enter or be forced to exit at different times and under varying conditions due to lock in periods, asset liquidity, fund size etc. leading to ineffective asset allocations for investors with varying requirements and investment horizons (Yang & Zhong, 2013).

In addition to this, it can be seen from equation (1) that the EM gets larger as time increases due to r_f – effectively accounting for the economic decay of the portfolio over time. Practically, this could lead to lost performance due to slower movement into the risky asset, than would be desired, during the beginning of a market recovery due to the higher economic maximum anchor level (Yang & Zhong, 2013). Therefore, due to its design the model doesn't account for current market conditions which may influence asset allocation and could lead to improved absolute performance in the long term. This is evidenced by the loss in optimality and poor performance of the model during the recovery from the 2008-2009 financial crisis (Yang & Zhong, 2013).

Despite these limitations the EDD method produces strong results with Grossman & Zhou (1993) showing that the portfolios performance is extremely close to that of the MVP but has the added benefit of having reduced drawdowns.

2.1.2: ROLLING ECONOMIC DRAWDOWN (REDD)

To address the shortcomings that exist in Grossman and Zhou's model, Yang and Zhong (2013) modified the definition of an EM in order to improve the forward-looking market timing of the model. They incorporated a constant rolling time period window of length *H* – in place of the fixed time period window – when calculating an EM to act as an anchor and thus enable the model to take into account a drawdown reference lower than the EM reference as defined by Grossman and Zhou (1993). This essentially enabled the model to allocate more resources to the risky asset at the beginning of a market recovery than would otherwise have been allowed (Yang & Zhong, 2013). They argue that due to mean reversion and the positive long run average return of asset markets historically indicate that there is a higher probability that the risky asset market will revert upwards in the current period of length *H* if it has been falling in the previous period of length *H* (Yang & Zhong, 2013). They called this constant rolling time variable the rolling economic max (REM) which is defined as follows:

$$REM(t,H) = \max_{t-H \le s \le t} \left\{ \left(1 + r_f\right)^{t-s} W(s) \right\}, \tag{3}$$

Thus, the concept of rolling economic drawdown (REDD) is defined as:

$$REDD(t,H) = 1 - \frac{W_t}{REM(t,H)}, \tag{4}$$

It can be seen from the above equation that there is no difference between the REM and the EM when the returns of the risky asset moves back to the portfolio high within the time period *H* (Yang & Zhong, 2013). Thus it can be inferred that in order for the REDD method to provide an additional performance benefit - by allowing for better market timing during a recovery period – the time period *H* will need to be shorter than the decline period length of the market cycle (Yang & Zhong, 2013). This creates a highly subjective component for the model and one needs to balance the risks of choosing a short length for *H* (which could result in an overweighting of risky assets should the market continue to decline) with the additional benefit to be had in the long run, by a faster entry into the risky asset if the market begins to recover. Thus, Yang and Zhong (2013) argue that *H* should relate to fundamental economic factors and for this reason they proved that *H* = one year is the most optimal time span.

The second alteration to the EDD model that Yang and Zhong (2013) make is to suggest that $\delta = \lambda$ as with portfolio drawdown strategies one is not concerned with the risk aversion of each individual investor but rather with that of the fund manager who, typically, has a lower risk aversion than that of one associated with a typical household investor. A fund manager is also directly concerned with and effected by drawdown levels as this level is one of his primary performance metrics and thus one can infer that his drawdown tolerance will be somewhat if not exactly similar to his risk aversion (Yang & Zhong, 2013). To emphasise this point, one can reiterate that fund managers' clients are unlikely tolerate a drawdown of more than 50%, fund managers themselves may shut down funds with drawdowns exceeding 20% and an account is most likely to be closed by a client if a fund is in drawdown for more than 2 years (Chekhlov, Uryasev & Zabarankin, 2005).

With these two adjustments Yang and Zhong (2013) proposed an economic drawdown controlled strategy with dynamic allocations between a risk free asset and risky asset(s) looking to maximise the long term growth rate of the portfolio with a constraint of $REDD \leq \delta$(Yang & Zhong, 2013). As such the asset allocation to a single risky asset with a short sale constraint is as follows:

$$X(t) = \operatorname{Max}\left\{0, \left(\frac{\lambda/\sigma + 1/2}{1-\delta^2}\right) \cdot \left[\frac{\delta - REDD(t,H)}{1 - REDD(t,H)}\right]\right\}, \tag{5}$$

Thus, the allocation to the risk-free asset (*Y*) at time *t* can be calculated as $Y(t) = 1 - X(t)$. The model was also extended to two risky assets with the asset allocations, with short sale constraints, calculated as follows:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{1}{1-\rho^2} \begin{bmatrix} \left(\lambda_1 + (1/2)\sigma_1 - \rho \cdot (\lambda_2 + (1/2)\sigma_2)\right) \big/ \sigma_1 \\ \left(\lambda_2 + (1/2)\sigma_2 - \rho \cdot (\lambda_1 + (1/2)\sigma_1)\right) \big/ \sigma_2 \end{bmatrix} \cdot Max\left[0, \frac{1}{(1-\delta^2)} \cdot \left(\frac{\delta - REDD(t,H)}{1 - REDD(t,H)}\right)\right]. \tag{6}$$

Where ρ represents the risky assets long term return correlation and will evidently affect the position size in the asset allocation process. In their research and back testing Yang and Zhong (2013) factored in the long-term Sharpe ratio, σ and ρ by calculating the values from the whole sample set.

Yang and Zhong (2013) compared their REDD model and Grossman and Zhou's (1993) EDD model - as discussed in the previous subsection - and proved that over time their REDD model consistently and significantly outperformed the EDD model on a risk-adjusted return, annualised return, traditional drawdown and EDD/REDD basis. They showed that their model was indeed superior to the EDD model.

This result has been supported by findings from Xie, Xu and Yu (2014) who compared the EDD and REDD models to their REDDp strategy – see subsection 2.1.4.

Yang and Zhong (2013) performed an empirical study of their REDD model containing two risk factors and compared its performance to a variety of different portfolio construction models including traditional 60/40, Risk Parity and Mean Variance portfolios – an extract of the results comparing the portfolios can be seen below in table 1. In addition, they compared different drawdown constraint levels and determined that all portfolios other than the constraint of $\delta = 1/3$ met their required target. They further found that any portfolio of $\delta \geq 1/4$ failed to control drawdown consistently. Their final investigation and observation evaluated the best model rebalancing frequency taking into account transaction costs, with the monthly frequency emerging superior (Yang & Zhong, 2013).

As can be seen from the table below, the REDD portfolios provided actual annualised returns in line or slightly higher than the traditional 60/40 model, the actual indices themselves (i.e. the S&P500 total return index (SPTR) and the Barclays Capital 20+ years US treasuries index (TLT)) and the risk parity portfolio (RPP). Annualised volatility measurements were also very much in line with the 15% REDD model outperforming all other models while the levered RPP performed the worst. On terms of risk-adjusted return Sharpe ratios there is no outright winner amongst the minimum variance portfolio (MVP), RPP and REDD portfolios as they all have similar and higher Sharpe ratios than the 60/40 portfolios.

It is important to note that direct comparison across the various portfolios is tricky due to the differing target outcomes of each method. In addition, the fact that for the 20% and 25% REDD target models the average exposure, as well as maximum exposure was greater than 100% indicates the ability to leverage the portfolios. This is not compatible with a long only 60/40 portfolio or an un-levered RPP. Thus the 25% REDD portfolio should be compared to the 60% levered RPP and the 15% REDD (or at a stretch the 20% REDD portfolio) should be compared to the more traditional portfolio methods.

On this basis, the 25% REDD model significantly outperforms the levered RPP in terms of drawdown (both on a rolling economic drawdown and the traditional drawdown basis) while also having a slightly higher annualised return and Sharpe ratio. Although, the RPP portfolio has lower volatility the 15% REDD model outperforms significantly in terms of drawdowns, both in terms of a REDD and traditional drawdown comparison with the 15% REDD model having a max traditional drawdown 15% lower than the 60/40 portfolios and a REDD 13% less than the 60/40 portfolios. Its closest competitor, in terms of performance, was the MVP yet the 15% REDD portfolio outperformed across the max drawdown categories as well as the on a risk-adjusted return, annualised return and volatility basis. Yang and Zhong (2013) attribute these differences to the market timing of the drawdown control mechanism in the REDD model that effectively moves resources between risky assets and risk-free assets more efficiently.

What stands out regarding the results of the REDD model is the fact that during the 2008-2009 financial crisis, significant drawdowns were still experienced with all models reaching their limits and in the case of the drawdown limit of 1/3, the limit was significantly breached with the maximum drawdown being 38% (Yang & Zhong, 2013). This differential could be explained by sudden increases in asset correlations and the pace at which prices fell during the crisis, although these reasons were not explicitly examined by Yang & Zhong (2013).

Table 1: Performance statistics of two risky asset portfolios between January 1991 and June 2011

	SPTR	TLT	60/40 Portfolio (No Rebalance)	60/40 Portfolio (Monthly Rebalance)	MVP	RPP	60% Levered RPP	15% REDD target	20% REDD target	25% REDD target
Annualised return (%)	9.22	8.15	8.82	9.26	8.97	9.07	12.16	9.03	10.91	12.85
Annualised Standard Deviation (%)	14.89	10.73	10.47	9.64	8.41	8.51	13.58	8.32	11.29	14.16
Max REDD (%)	46.76	21.51	27.14	26.68	15.67	16.25	26.04	13.06	17.65	22.42
Avg. REDD (%)	6.18	4.79	4.11	3.52	2.92	2.87	4.63	3.38	4.59	5.88
Max Drawdown (%)	50.95	21.40	30.30	28.63	15.65	17.22	28.16	14.89	20.45	26.07
Sharpe Ratio	0.384	0.433	0.508	0.598	0.650	0.655	0.638	0.655	0.656	0.646
Average Total Exposure (%)	100	100	100	100	100	100	160	93.99	127.84	163.80
Max Total Exposure (%)	100	100	100	100	100	100	160	125.47	170.35	218.04

***Source:* Yang and Zhong (2013)**

These results, while not always being easily comparable, and not without their concerns, still indicate that the REDD model provides significant performance advantages when looking to protect portfolio wealth while maximising an investors long term growth. It also provides an alternative method to approaching portfolio construction that should, at the very least, be considered by managers, depending on their mandate and investment requirements. Further to this, even though the model improved the drawdowns experienced by portfolios during crisis periods the REDD models' performances were still impacted by the effect of increased correlations during the crash. This indicates that diversification benefits may have been reduced during those periods.

2.1.3: ROLLING ECONOMIC DRAWDOWN OF RISKY ASSET PRICES (REDDp)

Xie, Xu and Yu (2014) noted that the way Yang and Zhong (2013) factored in the long-term Sharpe ratio, σ and ρ by calculating the values from the whole sample set was not possible in a practical setting. Investors would not have access to future information and would thus need to estimate the long term expected Sharpe ratio, which is subjective and unreliable (Xie, Xu & Yu, 2014). For this reason, combined with the fact that future returns are most affected by the most recent information available, a more practical adjustment would be to utilise a rolling λ, σ and ρ (Xie, Xu & Yu, 2014). This allows for current information and conditions to be of greater impetus in the model and will allow the rolling variables to match the rolling drawdown adjustment, which in turn will allow for a more accurate asset allocation on a shorter term rolling basis as conditions evolve which, as proven by Xie, Xu and Yu (2014) will lead to improved performance over the longer term.

In addition to this Xie, Xu and Yu (2014) made another practical observation regarding the REDD method. They noted that for any investor who is entering the market or looking to adjust their portfolio, the investor does not consider the entire portfolios wealth level when making asset allocation decisions, but rather evaluates the drawdown of that risky asset itself and its specific impact on the whole portfolio (Xie, Xu & Yu, 2014).

In order to address this consideration in the model, they proposed a rolling economic drawdown on risky asset prices that incorporated a rolling sharpe ratio of time period *h* (Xie, Xu & Yu, 2014). This model was named Rolling Economic Drawdown of Risky Asset Prices Strategy (REDDp) which looks back at each risky asset's prices for rolling period of length *H* with REMp being described as follows where *P(s)* is the maximum price of the risky asset:

$$REMp\ (t,H) = \max_{t-H \le s \le t} \left\{ \left(1+r_f\right)^{t-s} P(s) \right\}, \tag{7}$$

Similarly, the rolling economic drawdown of asset prices is then defined by:

$$REDDp\ (t,H) = 1 - \frac{P_t}{REMp\ (t,H)}, \tag{8}$$

When calculating the asset allocation equation Yu, Xie and Xu (2014) substituted the long term Sharpe ratio and volatility for each risky asset with the rolling Sharpe ratio (*RS*) and volatility measured over the same rolling time window [*t-h, t*]. Thus, the asset allocation formula with short sale constraints is written as follows:

$$X(t) = Max\left\{0, \frac{1}{(1-\delta^2)} \cdot \left(\frac{RS(t,h)}{\sigma(t,h)} + \frac{1}{2}\right) \cdot \frac{(\delta - REDDp(t,H))}{(1-REDDp(t,H))}\right\}, \tag{9}$$

Therefore, it follows that when considering multiple risky assets the formula becomes:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{1}{1-\rho^2} \begin{bmatrix} \left(RS_1(t,h) + (1/2)\sigma_1(t,h) - \rho \cdot \left(RS_2(t,h) + (1/2)\sigma_2(t,h)\right)\right) \Big/ \sigma_1(t,h) \\ \left(RS_2(t,h) + (1/2)\sigma_2(t,h) - \rho \cdot \left(RS_1(t,h) + (1/2)\sigma_1(t,h)\right)\right) \Big/ \sigma_2(t,h) \end{bmatrix} \cdot Max\left[0, \frac{1}{(1-\delta^2)} \cdot \left(\frac{\delta - REDD(t,H)}{1-REDD(t,H)}\right)\right], \tag{10}$$

The benefits of using a REDDp strategy over and above the two other models (namely EDD and REDD) goes beyond simply improving performance over the long term, although that is the ultimate goal. Firstly, it allows the investor to differentiate between two or more seemingly similar risky assets. For example, if the investor is evaluating a number of risky assets with equal long-term Sharpe ratios and equal volatility, the EDD and REDD strategy would allocate resources to these assets equally (Xie, Xu & Yu, 2014). However, logically the investor should favour the assets that have the lower drawdown as this provides additional benefit in itself and thus under a REDDp strategy resources are allocated more efficiently (Xie, Xu & Yu, 2014).

Secondly, the REDDp strategy improves portfolio flexibility (Xie, Xu & Yu, 2014). The drawdown constraint in the REDD strategy looks solely at the portfolio drawdown levels and thus has the same impact on all risky assets and may lead to investors moving out of the incorrect assets at the incorrect times (Xie, Xu & Yu, 2014). For example, consider a portfolio that is equally weighted between a variety of risky assets. It could well happen that during a downturn the majority of risky assets in the portfolio are losing value and thus the portfolio is experiencing a drawdown overall, yet there may well be one risky asset that is appreciating in value. The REDD portfolio, as it stands, would move resources out of all the risky assets based on the fact that the portfolio as a whole is in drawdown. However, one would then lose any potential gain to be had from the single risky asset that is appreciating. In addition to this, due to the fact that the one risky asset is appreciating, the total portfolio drawdown would be offset somewhat resulting in too slow of a movement out of the assets performing poorly. This is not optimal.

In contrast, the REDDp strategy considers the drawdown for each individual risky asset and thus evaluates the viability of each individual asset that makes up or could make up the overall portfolio allowing the investor to move out of the least attractive risky assets and into the more attractive risky assets on a continuous basis (Xie, Xu & Yu, 2014). This is more in line with the practical applications of active management where investors seek to generate superior returns by actively selecting assets and undertaking strategies that will outperform.

In their paper Xie, Xu and Yu (2014) demonstrate and prove that the REDDp strategy performs the same function as the REDD and EDD strategy in controlling drawdown risks, yet it generates superior returns to either of the other two strategies by allocating resources based on the REDDp of each individual asset while incorporating a rolling Sharpe ratio thereby making it a more practical and sophisticated model.

Table 2: Performance statistics of two risky asset portfolios between March 1992 and December 2014 with $\delta = 1/3$

	SPTR	DJUBS	REDD strategy	REDDp Strategy
Annualised return (%)	7.13	1.50	4.80	7.64
Annualised Standard Deviation (%)	14.65	14.96	5.30	6.77
Max REDD (%)	47.90	54.79	19.93	7.96
Avg. REDD (%)	6.51	9.16	2.59	1.99
Max Drawdown (%)	52.56	54.52	21.27	8.13
Sharpe Ratio	0.2891	-0.0929	0.3602	0.7016

***Source: Xie, Xu and Yu* (2013)**

The results of the comparison done by Xie, Xu and Yu (2014) can be seen above in table 2 where they compared the REDDp strategy to the REDD strategy as well as the index returns with the two risky assets being the S&P500 total returns index (SPTR) and the Dow Jones UBS-commodity total return Index (DJUBS). What they do differently to Yang and Zhong (2013) in order to make their results more comparable is to normalise their allocation weights. This ensures that their capital exposure is no more than 100% regardless of drawdown limit. Xie, Xu and Yu (2014) do not compare their portfolios to other potential market portfolios such as the MVP, RPP or 60/40 portfolios, this makes it difficult to ascertain whether, under the new time period and drawdown constraints, these two models still provide improved performance over more traditional portfolio construction methods.

As can be seen from table 2; over the period investigated the REDDp – with its forward looking rolling Sharpe ratio and individual asset pricing focus – significantly outperforms the REDD strategy and provides improved performance across every variable. This provides investors with the significant ability to control drawdown and protect investor wealth while also generating superior returns in the long run. It thus allows investors to withstand sudden market shocks and provides for a strong alternative method for portfolio construction, at least in theory, that should be tested and investigated further.

In addition to this, it is evident from its construction that the REDDp is based on the same underlying assumptions as Yang and Zhong (2013) in the sense that they base allocations on traditional asset classes. Thus, the model is still exposed to sudden increases in asset correlations during market crisis periods. Thereby resulting in lost diversification benefits, especially when the model is potentially expanded to more than two risky assets. What's more, is that when the model is expanded to multiple risky assets the correlation calculations and thereby the weighting calculations become much more computationally challenging as correlations between numerous factors need to be considered.

For these reasons, this study will look to extend the work done by Xie, Xu and Yu (2014) and will use their REDDp strategy – and by default the work of Yang and Zhong (2013) as well as Grossman and Zhou (1993) - as the basis for the development of the FAREDD strategy. The details to which are discussed further in the method section of the paper (section 3).

This study also seeks to contribute to the existing literature on Economic and Rolling Economic Drawdown models by investigating their potential applications in a developing emerging market economy where volatility and illiquidity is higher as all existing literature is focused on developing the model in the setting of a developed market.

2.2: RISK FACTORS

2.2.1: INTRODUCTION TO RISK FACTORS – A BRIEF HISTORY

The use of asset classes as the primary diversification tool - and by default the primary building blocks of a portfolio - has been undisputed up until recently. Due to the phenomenon of contagion and highly correlated assets, there has been a rapid increase in the amount of research performed on a new set of potential building blocks - risk factors.

The term "risk factors" is a rather broad and non-specific term that can be interpreted in a variety of ways. For the purposes of this study we will use the definition provided by Podkaminer (2013) that risk factors are the smallest systematic units that explain an asset's return and risk characteristics. With this definition in mind, the potential benefit of a risk based resource allocation approach becomes apparent – the investor is theoretically able to select exposure to specific factors and create a portfolio with exposures to factors that are uncorrelated, thus increasing diversification benefits (Podkaminer, 2013).

Risk factors have their roots in mean-variance optimisation, as initially developed by Markowitz (1952), and as investors began to use the methodology increasingly to develop their portfolios, the process was broken down into two distinct steps; the asset allocation (beta / market timing) decision where resources are allocated to various asset classes on the basis of exposing the portfolio to systematic risk and the alpha decision where specific individual securities within the asset class are selected in order to provide the best performance (Idzorek & Kowara, 2013). The problem is that the alpha decision often involves having to evaluate thousands of individual securities and thus, in order to optimise these, one needs to estimate the future returns and the correlations between all these securities – which is practically impossible. This resulted in the development of multifactor models in order to identify a reasonable number of common factors to explain the returns of each individual security (Idzorek & Kowara, 2013).

The Sharpe-Lintner-Mossin-Treynor Capital Asset Pricing model was developed in the 1960's and assumed that the market is the primary risk factor effecting returns and each individual asset had some correlation to the returns of the market as a whole (Idzorek & Kowara, 2013). This basic concept was further extended by Ross (1976), who proposed that multiple risk factors, other than just simply the market contribute to asset returns known as arbitrage pricing theory (APT), although he did not indicate exactly what those factors may be. Fama and French (1992) took the research into multiple risk factors, further using a fundamental factor based model and identified size and value as factors that explained returns not previously explained by the market risk factor. They used this to develop a 3-factor model including, size, value and market risk to explain asset returns and volatility. In 2013, they further proposed a 5-factor model that added the additional two factors of profitability and an investment factor - companies that have high asset growth yet below average returns (Fama & French, 2013). They did however receive criticism for ignoring momentum as a factor even though Jegadeesh and Titman (1993) had proven that it was applicable as early as 1993.

As pointed out by Idzorek and Kowara (2013), it is important to note that the development of these models as well as the identification of these initial factors was to explain the market and its anomalies rather than to develop ways in which to build a portfolio in such a way where exposure to specific factors was the primary goal. This was first proposed and developed by Grinold and Kahn (2000). However, since a number of relatively strong market shocks in the recent past, there has been increased interest in developing portfolios based first and foremost on these risk factors and then building up multi-asset portfolios rather than starting with specific assets and their related returns and then evaluating their risks after the fact. Although, even if an investor chooses to build a portfolio based on asset classes, having a risk factor analysis and methodology available to further their understanding of the risk factor exposures in their portfolio could go a long way to improve their performance.

2.2.2: EVIDENCE OF ASSET CLASS CORRELATIONS

Since the primary motivation behind using risk factors as the factor allocation model is because traditional asset allocation models do not create portfolios with strong diversification characteristics when it is truly needed due to asset classes being more correlated than expected. It is prudent to investigate the literature on asset class correlations both within a single market as well as correlation interdependence between markets.

As noted by Idzorek and Kowara (2013), asset class correlations within portfolios can be high as many assets returns and volatility within different asset classes are or can be influenced by the same factors. This statement is supported by a wide combination of empirical evidence. In a comprehensive paper Chua, Kritzman and Page (2009) investigate the asymmetry present in asset correlations and the subsequent challenge it presents to portfolio construction. They used a conditional correlation model that estimates asset correlations from a normal distribution of asset returns and then compared these to conditional correlations obtained via empirical evidence to ascertain that asset correlations are asymmetrical across a wide range of asset pairs and that many correlations increase during turbulent periods while decreasing during steady bull market periods (Chua, Kritzman & Page, 2009). They note that this is in contrast to a typical investor's desires (who relies on equity returns to drive growth) who would seek to introduce assets into the portfolio that move in tandem with stock markets when they are bullish and decouple from the stock market in times of market crisis – thus creating diversification benefits (Chua, Kritzman & Page, 2009). This decoupling has been shown to take place in certain instances, for example, Gulko (2002) found that equity shares and treasury bonds tend to decouple during market crashes, mainly due to the fact that as risky assets sell off and uncertainty in the market spikes, investors flock to the safety of Government obligations because - in theory at least - they are close to risk free. However, it must be noted that Gulko (2002) only investigated correlations during market crashes between 1987 and 1999 and as Statman and Scheid (2008) demonstrate; asset correlations are higher now in the last decade than it was previously - before the early 2000's.

In support of Statman and Scheid (2008) findings - a more recent paper than Gulko (2002) found that historically only very few asset classes offer desirable downside diversification in turbulent times – namely government credit, convertible arbitrage and in some instances mortgage backed securities (MBS) and high yield bonds (due to reserve banks lowing interest rates leading to bond prices increasing dramatically) (Chua, Kritzman & Page, 2009). However, Chua, Kritzman and Page (2009) identify that all of the favourable asset classes failed to diversify each other during a number of more recent of large financial crises due to massive risk sell-offs caused by a complete loss of confidence in the financial system as a whole. A case in point is the recent subprime mortgage crisis in 2008 to 2009. This finding is supported by a number of proponents of behavioural finance who argue that due to investors being subject to irrational biases such as overconfidence, overreaction, loss of risk aversion and herding, this can result in the high correlations that can be seen between assets in crisis periods as all investors panic together (Lo, 2004).

It is also relevant to note that, with large technological advancements and the phenomenon of globalisation taking place, markets and asset classes around the world have become more integrated and thus more correlated. This is supported by a large amount of literature dedicated to investigating asset allocations between international markets. Unlike the mixed results found between asset classes within one domestic market, empirical research concludes convincingly that exposure to different country's assets across a variety of asset classes offer significantly less diversification in bear markets and during market crashes than in bull markets (Chua, Kritzman & Page, 2009). Ferreira and Gama (2004) investigated global industry returns and found the same asymmetric phenomenon to occur. Ang, Chen and Xing (2002) found the same result for individual stock returns between international equity markets and Capiello, Engle and Sheppard (2006) found the same asymmetry between international bond markets. The asymmetry and loss of diversification benefits discussed is also present in the international hedge fund space, as proven by Van Royen (2002). This increase in correlation between international market asset classes is depicted in Figure1 below where it can be seen that correlation coefficients between the S&P500 and the EAFE index have increased dramatically over both 5 and ten year periods and also shows the volatility of correlations over time - especially around market crashes.

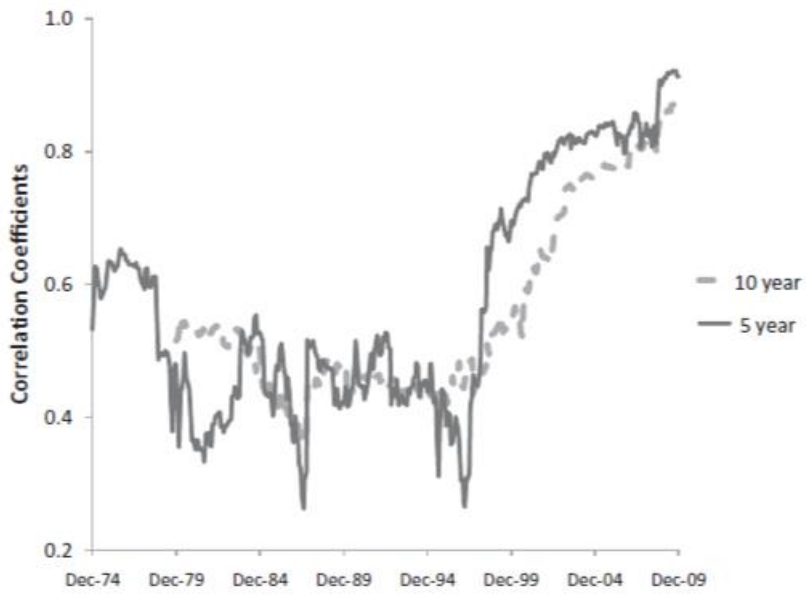

Figure 1: Correlations between S&P500 and EAFE measured over rolling five and ten-year periods

Source: **Marston (2011)**

In a South African context, a working paper published by Duncan and Kabundi (2011) – the first study of its kind – studied asset class volatility interactions and spill overs, found that in times of economic and financial distress both locally and internationally, led to significant increases in the volatility and correlations between asset classes. They studied the daily volatility between various assets over a period of almost 14 years from October 1996 to June 2010 and identified that equities are the most important source of volatility spill overs to other asset classes except in certain circumstances (Duncan & Kabundi, 2011). This Indicates that shocks to the equity market will cause ripple effects into the bonds, cash, property and commodity markets and thus increase these assets correlations to each other. In addition to this, their study also finds that in terms of volatility spill overs between asset classes, the South African marketplace has considerably higher connections between asset classes than more developed economies such as the US (Duncan & Kabundi, 2011). This is not totally unexpected due to South Africa being a small developing economy with fewer assets. These economies tend to have higher exposure to political, economic and environmental shocks due to increased global integration (Saunders & Walter, 2002).

Although diversification as a part of portfolio construction is a crucial element to get right, it is by no means a simple or trivial task. Simple portfolio allocation models that rely on traditional historical and evidently unstable asset class correlations can often result in dangerous capital exposures during crisis periods and market downturns.

2.2.3: RISK FACTOR PRACTICAL CONSIDERATIONS & IDENTIFICATION MODELS

As previously mentioned, factor based allocation provides an investor with a new and unique way of developing a portfolio with exposure to specific risk factors. These factors are uncorrelated and each is individually rewarded by the market for their level of risk, which in turn will create a portfolio with stronger diversification characteristics such as; lower volatility, smaller drawdowns and better Sharpe ratios (Idzorek & Kowara, 2013). Therefore, although the actual individual assets making up the factors may change, the factors themselves remain constant throughout certain time periods and are relatively uncorrelated thereby combating the main weakness caused by the asymmetry experienced in traditional asset class allocations. However, the significance of the impact that each risk factor has on the returns and risk levels of assets may vary over time (Zorn, 2013).

Another advantage to using risk factors is that while there is a relatively small pool of different asset classes – probably around 15 to 20 at the most – to which one can allocate resources, there can be a large number of different factors that may explain parts of returns (Idzorek & Kowara, 2013). This creates increased opportunity for investors to create a more robust and diverse model specific for their individual needs.

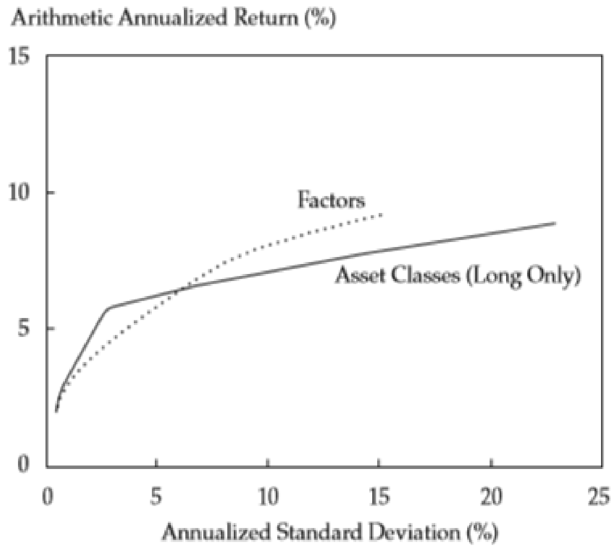

Figure 2: Asset class vs. Classic Risk Factors
Jan 1979 – Dec 2011
Source: Idzorek & Kowara (2013)

Proponents for factor allocation make the argument that, because of these benefits and because it focuses on the effect of changes in factors at a microscopic level of risk and return, essentially making it a bottom up approach, it is inherently superior (Kaya, Lee and Wan, 2012). However, Idzorek and Kowara (2013) argue that its perceived benefits do not make it inherently superior and that the empirical investigations that have been used to prove such a superiority have not compared like to like.

Much of the literature comparing asset class allocations base their empirical observations on observing how a factor allocated model performs relative to a long only asset class based model (Idzorek & Kowara, 2013). This is not ideal.

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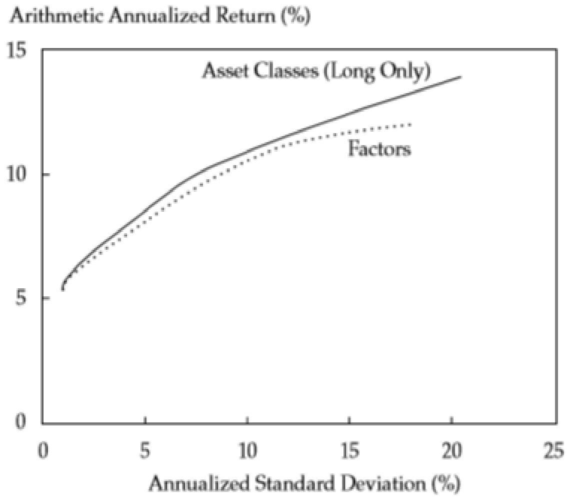

Figure 3: Asset class vs. Classic Risk Factors
Jan 2002 – Dec 2011
***Source*: Idzorek & Kowara (2013)**

Podkaminer (2013) states that to obtain exposure to a particular risk factor premia involves going long and short on a particular trait inherent in asset classes. For example, if an investor wants to expose themselves to the value factor to take advantage of this, they would need to be long value stocks and short growth stocks as you are essentially betting that value will outperform growth. Overweighting value stocks in the portfolio is simply providing a factor tilt on the portfolio and is not the equivalent of gaining exposure to some factor premia (Podkaminer, 2013).

This is one of the major practical stumbling blocks with the risk factor allocation model, it is impossible for all investors to hold the same portfolio as going long on value would require everyone in the world to short growth stocks (Idzorek & Kowara, 2013). Idzorek an Kowara (2013) went on to illustrate that even if this stumbling block is ignored and like comparisons are not done, the performance of risk factor models compared to asset allocation models is highly period dependent with different selections yielding different results, as can be seen when comparing figures 2 and 3. This shows that the superiority of risk factors is not clearly cut by consistent performance. They do, however, concede that research does support the fact that risk factors tend to outperform during market crisis periods (Idzorek & Kowara, 2013). Thus this limitation does not undermine the potential advantages of using risk factors as a resource allocation tool in certain circumstances where diversification is beneficial such as during economic regime changes (Bender et al., 2010).

The other major hurdle for risk factors in a practical sense is that it is often very difficult to find investible, liquid proxies to mimic a risk factor (Zorn, 2013). As Idzorek and Kowara (2013) indicate, this is mainly due to the fact that investible assets are often exposed to multiple risk factors and thus do not fit perfectly into mutually exclusive categories. This makes gaining meaningful exposure to factors rather difficult. However, it should be mentioned that as Roll (2013) points out, passive investment vehicles such as EFT's are making it increasingly easy to gain access to multiple indices and allow for new and innovative ways to classify assets, which has allowed for easier investability in proxies that would then allow for the subsequent creation of synthetic factor exposures at a much lower cost than traditional asset allocation models would allow.

In addition to this the final major obstacle that factor models face at this stage is what Ceria, Saxena and Stubbs (2012) dub *"factor alignment problems"*. They argue that risk factor models that are used to forecast expected returns are inherently forward looking and are judged on their ability to accurately forecast returns while factor models that are used to forecast risk have a different mandate to have both a forward looking element but also rely heavily on historical data to explain the cross-sectional variance of the returns process and are thus judged on their ability to capture systematic risk factors and the correlations that exists between these factors, before aiming to reduce the exposures between correlated factors (Ceria, Saxena &Stubbs, 2011). This combined with the fact that investment managers and portfolios have various constraints that are often incompatible with factor based models, results in a misalignment between the factors selected to forecast expected return and the factors that would be selected to forecast risk (Ceria, Saxena & Stubbs, 2011).

The perfect example of constraints that may present issues to risk factor models is evident in the rules governing South African pension funds where resource allocation is governed by limits linked to specific asset classes. Thus, it is completely feasible that these particular asset classes exhibit significant exposure to similar risk factors yet, a portfolio built on risk factor methodologies may look to invest heavily in one asset class and could still significantly reduce overall risk factor exposure and improve diversification yet, will be prohibited from doing so.

Ceria, Saxena and Stubbs (2011) went on to provide evidence of these misalignments and show that when combined with a portfolio optimisation tool they go on to create portfolios that significantly underestimate and over-expose the investor to systematic risk despite resources being allocated between low or uncorrelated factors. This is one of the motivating factors for utilising factor analysis in the rolling economic drawdown methodology for portfolio construction, due to the fact that the drawdown optimisation process would help to reduce the exposure to systematic risk as market conditions change, thereby reducing the potential downfalls of this excess exposure while capitalising on the gains to be had.

In addition to the considerations to be had regarding the benefits and challenges of using factors seen above, the literature on risk factor identification has examined three different models in which to attempt to identify and measure factors. Primarily (1) a macroeconomic factor model, (2) a statistical factor model and (3) a fundamental factor model (Connor & Korajczyk, 2010).

The first model, the macroeconomic model, as the name suggests is based on economic theory. Cheb, Roll and Ross (1986) were the first to develop a macroeconomic factor model using observable macroeconomic events, compiling them into a time series and using this to estimate factor betas via ordinary least squares regressions of each assets returns against the series of factors. Chen, Roll and

Ross (1986) argue that common factors must be tied in some way to the causes of market shocks and should thus impact either cash flows or the risk-adjusted discount rate. They find that some relevant factors in the US market are inflation, interest rates and other business cycle related factors (Chen, Roll & Ross, 1986). However, this approach has a noticeable weakness in the frequency of economic information (such as interest rate decisions) being low, thus resulting in a weak empirical fit for the data while also causing the model to exhibit a lack of robustness in certain instances (Shanken & Weinstein, 2006).

The statistical model on the other hand is highly robust from a theoretical perspective and ensures that factors are uncorrelated. It is a purely statistical approach to risk factor analysis and does not depend on any economic theory as the resulting factors are not tied to any external data source (Zorn, 2013). The most common method uses the covariance of asset returns to extract factors from the returns' variance-covariance matrix in a process referred to as principle component analysis (PCA). PCA uses eigenvalue decomposition to convert a set of time series observations that have correlations into a set of linearly uncorrelated variables, thus reducing the dimensionality of the original data while retaining as much of the variation in data as possible (Jolliffe, 1986). However, like the economic model it too has its drawbacks. These include the fact that factors that influence returns may vary over time periods and therefore should be continuously recalculated. In addition, statistical models do not allow for a direct link to a specific factor as there is no way to identify what a specific factor represents in economic theory (Connor & Korajczyk, 2010).

The final method is the fundamental factor model. This is essentially a characteristic based model that was developed by Fama and French (1993), is an extension of the CAPM and determines potential factors that are related to returns in two main steps. In the first they sort all individual assets into groups using some fundamental characteristic (such as market value in equities) and then in the second step, they conduct a time series regression on the returns of the individual groups in order to identify factor betas for that fundamental characteristic (Zorn, 2013). However, the main argument against this method is that none of these potential factors are guaranteed to be related to risk in any way and thus, they cannot always be considered risk factors as defined by Idzorek and Kowara (2013).

Each model has its benefits and limitations and for this reason there seems to be no inherently superior approach to identifying risk factors. The model that one would choose and the factors that would be desired depend entirely on the investor's preferences and portfolio requirements. For the purposes of this model we will disregard the macroeconomic factor model due to the poor empirical fit of the economic model caused by the infrequency of economic data. We will ignore the fundamental model due to the lack of a causal link to risk as this study is centred around creating improved diversification within portfolios. The statistical PCA approach provides the best opportunity to create a theoretically robust FAREDD model that has minimal correlations between factors which in turn will help investigate its potential to improve diversification and protect portfolios during market crashes.

2.2.4: EMPIRICAL EVIDENCE

The literature on statistical factor analysis in developed markets is well documented with researchers analysing the various types of factors that exist, as well as the models that have been developed. Many of these studies focus on fundamental and economic factors. Due to the fact that we have already indicated that this study will focus on statistical factors, investigating factors based on fundamental and economic principles – as well as their applicable models - is beyond the scope of this study. Therefore, we shall focus solely on the statistical factor findings.

Authors have attempted to use PCA methodologies as well as PCA factors in new and unique ways in order to develop more robust portfolio selection methods as well as improve how we understand and measure diversification. Caputo and Partovi (2004) were one of the first to propose an asset allocation mechanism using uncorrelated principle portfolios which can be realised through long and short positions. This led to the development of one of the main applications for statistical PCA risk factors in recent times, the enhancement of risk parity portfolios with numerous studies being done on their performance when compared to traditional portfolio construction methods and asset class based risk parity portfolios, with results indicating that they tend to outperform on a risk-adjusted basis but not always on absolute performance (Lohre, Opfer & Orszag, 2013).

Further to this, Meucci (2009) used PCA factors to develop a conditional principle components portfolio that takes into account any linear constraint when constructing portfolios. This model used principle components to present an alternative approach to risk budgeting and risk parity portfolios by introducing an Effective Number of Bets which measure the true contributions of uncorrelated factors rather than the marginal contribution to a portfolio that is provided when using correlated factors, thereby extracting the main drivers of the asset's variance (Meucci, 2009). Meucci (2009) along with d-fine (2011), prove that a conditional PCA model can provide a practical and effective method to define and identify levels of portfolio diversification relevant to risk factors and can be a useful tool in portfolio construction. These findings are further evidenced by Frahm and Wiechers (2011) who prove that the process of mean-variance optimisation across asset classes does not provide for portfolios with reduced diversifiable risk when evaluating the portfolios based on statistical correlations. They go on to show that using PCA factors to identify the level to which a portfolio is diversified can help improve performance (Frahm & Wiechers, 2011). This provides some additional evidence that PCA can be used to improve diversification measures within portfolios and does provide additional explanatory power during portfolio construction.

Due to the fact that PCA does not tie factors directly to some economic or fundamental base, researchers have been able to show that these factors exist. However, they cannot with perfect certainty tie them to any one explanatory variable, such as GDP growth (economic) or size (fundamental) for example. The most recent studies conducted by Bhansali (2011) compared nine different asset classes in the US market and identified that 4 to 5 PCA factors explained close to 100% of the returns of the assets over a period of 50 years. In addition, Bhansali (2011) provided evidence that risk factor based models utilising PCA, for example risk parity models utilising PCA methodologies, provided significant reductions in asset correlations - PCA factors exhibited correlations of approximately 1,6%-2% while asset classes exhibited correlations ranging between 30% and 51% - through the period of March 1994 to December 2009. This had the increased benefit of also assisting in the control of portfolio drawdowns over the period by allowing managers to rebalance their portfolios as the portfolio exposures changed, or by allowing them to hedge against those exposures through the use of options thereby, significantly limiting the impact of potential market crises (Bhansali, 2011). Subsequently, in a slightly more recent study, Bhansali et al. (2012) argue that of the five most important PCA factors only two that are linked to growth and inflation are responsible for 68% of the total return of nine different asset classes. This indicates that although there are theoretically endless possibilities of risk factors that could affect asset's returns they are mainly influenced by similar factors, thereby reducing the asset allocation decision significantly as one can get exposure to similar risks no matter what asset class was chosen.

In a South African context, it has been noted that due to South Africa's smaller, more illiquid emerging market economy, South African assets often experience higher levels of volatility during market shocks compared to developed market economies (Duncan & Kabundi, 2014). This has led to a number of studies on potential statistical factors effecting returns with a variety of results. However, the literature investigating statistical risk factors in a South African marketplace is significantly smaller than that of developed economies.

The first study on record conducted by Page (1986) applied factor analysis and identified that the most significant risk factors were highly correlated to the mining and industrial sector respectively. This is due to the fact that the mining sector was historically the largest sector of the economy with mining firms dominating the JSE by way of market capitalisation due to the fact that they formed the initial backbone of the South African Marketplace. This led Venter, Bradfield and Bowie (1992) to suggest that an appropriate major share index related to the various sectors should be used in the CAPM model rather than the overall index for JSE shares. This was due to the size of the mining sector and thus the JSE index was exposed more heavily to this area, which in turn meant that the all share index did not adequately explain the returns of firms that did not operate in these major areas (Venter, Bradfield & Bowie, 1992).

In 1997 a study conducted by Van Rensburg and Slaney (1997) applied a similar factor analysis, as was used by Page in 1986, and found significant evidence supporting the use of three factors; a gold/mining factor, an industrial factor and a non-precious metal factor.

However, in 1998 the Johannesburg Stock Exchange (JSE) initiated a process to revise index classifications. This resulted in the shuffling of various firms into different index classifications making the previous factors identified outdated. Van Rensburg (2002) re-conducted the factor analysis with the new classifications and identified that a two-factor model was more appropriate than the three-factor model found in 1997 to explain asset returns. The new factors were identified as the financial-industrials index and the resources index – where the resources index was a broad index that included mining as well as other related sectors (Van Rensburg, 2002).

However, since 2002 the JSE reorganised its sectors again in 2006 ,and the subsequent literature on relevant statistical factors seems rather thin. When writing this study, we have been unable to identify any updated and available studies that investigate the presence of various factors using a PCA statistical analysis approach. In addition to this, since the late 1990's and early 2000's there has been a clear and definitive shift in the weightings of the various sectors on a market cap index with the mining/resources sector dropping significantly in size when compared to its earlier size of close to 50% in the 1990's (JSE, 2017). This has the added implication that previous factors may no longer be relevant or accurate. Therefore, for the purposes of this study we will recalculate asset correlations across the 10 available industry classifications which will result in potential explanatory factors that will then allow for the FAREDD model to be implemented. This is further discussed in Section 3.

2.2.5: MOTIVATION FOR USE IN THE FAREDD MODEL

The main motivation for including risk factors as calculated using PCA as opposed to asset classes is due to the evidence obtained proving that asset class correlations are often more correlated than expected. So far, all drawdown models developed have looked to essentially improve a portfolios market timing when moving between risky and risk free assets. However, they have - so far - not considered the possibility of constructing and rebalancing the portfolio based on uncorrelated risky assets which, as we have seen above can significantly improve diversification and risk-adjusted performance within a portfolio which, in turn can provide increased benefits to an investor.

Therefore this study seeks to extend the work done one economic drawdown models and apply the model to uncorrelated risk factors in order further improve portfolio protection and diversification during a drawdown event – especially in a volatile and more illiquid emerging market economy.

2.3: COMPARISON PORTFOLIOS

In order to effectively and fairly compare the performance of the FAREDD model to other techniques that could be used by investors, we need to select optimisation methods with similar characteristics, most specifically the ability to provide an alternative to portfolio managers with the goal of reducing potential portfolio wealth loss by limiting exposure to risk while attempting to maximise return, i.e. maximising the sharpe ratio– especially during market crashes and downward market conditions. Therefore, for the purposes of this study, the performance of the developed FAREDD model will be compared to a risk parity portfolio as well as two mean-variance optimised portfolios with different objectives. In addition to this, the model will also be compared to the traditional REDDp model as developed by Yu,Xie and Xu (2014) as discussed earlier in this study in section 2.1.4. The first mean-variance objective will be to minimise variance and the second which looks to maximise the Sharpe ratio accordingly.

These four alternative portfolios , other than the REDDp strategy which has already been discussed in this literature review, and the theory underlying them, will be briefly discussed below in order to provide some theoretical background for these portfolios.

2.3.1: RISK PARITY

The unlevered risk parity portfolio has gained acceptance and popularity, mainly from passive fund managers, as an alternative approach to asset allocation (Qian, 2011). The concept of Risk parity is centred around allocating weights to the asset classes in such a way that allows each asset class to contribute an equal weight to the overall portfolio. The theory behind this is that capital diversification should be diversified on a basis of risk as opposed to a more traditional but less accurate measure of a 60/40 approach for example. Quin (2011) goes on to prove that although the risk parity portfolio performs worse than the market portfolio or a 60/40 portfolio on a total return basis, it outperforms consistently on a risk adjusted basis as evidenced by a consistently superior Sharpe ratio which Quin (2011) believes directly indicates improved diversification.

Some arguments against risk parity have been made whereby it is argued that the belief that a better Sharpe ratio is automatically indicative of a portfolio being superior is incorrect, as this measurement basis of supposed superiority is dependent on an investors perspectives and goals (Asness et al., 2012).

For example, a portfolio that has a superior Sharpe ratio but a lower total return than another portfolio will not be preferred by an investor whose sole goal is to maximise absolute investment return.

Chaves *et al* (2010) found that while a Risk Parity Strategy is able to achieve a higher Sharpe Ratio, it is unable to consistently outperform all alternative portfolios and thus, is extremely dependent on the time period selected. However, it does provide for a portfolio with low standard deviation and risk over all time periods.

2.3.2: MEAN-VARIANCE OPTIMAL PORTFOLIOS

Mean-Variance optimisation (MVO) first came to the fore when it was proposed by Markowitz (1952, 1956) - and is the basis of Modern Portfolio theory - when he postulated that any rational investor would aim to maximise expected return on their portfolio for a given level of risk. This created an optimal portfolio, also referred to as a tangency or market portfolio, which is the tangency point on the efficient frontier and represents the greatest utility that an investor can receive given the investor's constraints and the risk-free rate that is available in the market (Greig, 2016).

The mean variance framework, however, is not without its criticisms. Maillard, Roncalli and Teiletche (2009) argue that the optimal portfolio is over concentrated in a small area of the investible universe while also being extremely sensitive to input parameters. Asness *et al* (2012) also identify a downfall to the MVO portfolio by indicating that, from a risk perspective the market portfolio is not well diversified, as equities are historically more volatile than other asset classes meaning that the market portfolio is mainly an equities weighted portfolio as the main area of volatility in the market and thus the most variation in the performance of the portfolio is explained by equities and thus offers less diversification than one would like across asset classes. Despite these drawbacks, many investors adopt strategies derived from MVO due to the fact that MVO is easy to calculate and compute and is perceived to be a robust tried and tested method (Maillard, Roncalli & Teiletche, 2009). The two preferred strategies tend to be the equal-weighted portfolio and the minimum variance portfolio (Greig, 2016). One is also able to use the mean-variance optimisation framework to solve for the tangency portfolio which has the highest Sharpe ratio and thus evidently provides the best return per unit of risk (Markowitz, 1956).

Therefore, the tangency portfolio, along with the minimum variance portfolio, will be used as comparatives as they provide to realistic alternatives to construct portfolios that are readily available to asset managers, while trying to achieve a similar goal of reducing risk and downside moves while the maximum Sharpe ratio portfolio also aims to achieve the highest level of return available for each specific level of risk.

CHAPTER 3: DATA AND METHOD

The analysis done, and the results presented in this study are based on stock, bond and property daily total return price data, denominated in South African Rand for the period of 03 January 2006 – 31 August 2017 from the *Bloomberg* online database. The risk-free rate used in the models is derived from the 3 Month JIBAR rate. All strategies for tests 2 and 3 were rebalanced on a monthly basis to match the optimum rebalancing period for the FAREDD model – as discussed in Section 4 of this study. All strategies had an initial training period of 12 months, simplified to 252 trading days' worth of data, meaning that this data is not used in calculating the portfolios performance but is used as a baseline to calculate initial weights and the like – especially in the FAREDD model. The models do not allow for leverage - in the sense of being able to borrow at the risk-free rate - but do allow for short sales, which is effectively another form of leverage available (with the exception of the risk parity portfolio which equally allocates risk and thus cannot practically allow for short sales). The comparison portfolios have also been provided with box constraints of -1 to 1 in order to allow the portfolios to be comparable to the FAREDD model by allowing them the flexibility to go long and short across various asset classes as necessary. The reason why the portfolios have been allowed to short sell is due to the nature of the PCA analysis and the fact that any constraints on the maximum exposure to a specific asset or a constraint regarding short sales would render the objective of obtaining uncorrelated principle components useless, as the resulting principle components would then no longer be uncorrelated which defeats the purpose of the model.

Therefore, all comparison strategies have been selected to allow short sales (with the exception of the risk parity portfolio). While this consideration does reduce the practicality of the strategy as an asset selection method for fund managers who have strict short sale and fully invested constraints (such as pension and mutual funds), the main purpose of this study is to investigate whether focusing on a FAREDD strategy has the potential to outperform similar portfolios and protect investors from downside risk. Therefore, although the practical implications of parts of the model is important to note, they ultimately do not affect the objectives and subsequent findings of this study. In addition to this the strategy still holds potential weight for less conservative and restricted funds - such as hedge funds - allowing the opportunity for such these fund managers to utilise this strategy as a viable option when considering their portfolio construction process.

The table below provides some descriptive statistics of the 13 indices used throughout the various tests in this study. Note that the Sharpe Ratio has been calculated using a risk-free rate equal to the average 3 month JIBAR rate over the period of 7.24%. During the Financial crisis period test, as the sub test incorporated into Test 2 to evaluate the performance of the portfolios during a time of market crisis characterised by high volatility and drawdowns the JIBAR rate used is 9.91%.

Table 3: Descriptive Statistics of the 13 Indices used

Index	Annualised Return (%)	Annualised Standard Deviation (%)	Annualised Sharpe Ratio	Maximum Drawdown (%)	Maximum REDD (%)
GOVI	7.054	8.250	-0.028	13.58%	18.85%
OTHI	7.700	10.81	0.0558	19.26%	23.55%
JBIND	6.055	30.361	-0.0391	64.64%	66.04%
JGIND	12.269	18.091	0.2779	50.12%	51.56%
SASOL (Oil & Gas)	8.536	33.762	0.0384	55.07%	58.17%
JCCGD	21.072	19.990	0.6919	27.93%	33.93%
JNCCD	17.104	20.164	0.4892	52.94%	53.43%
JCSEV	23.709	21.644	0.7608	41.02%	60.77%
JNCSV	11.930	30.992	0.1339	49.78%	52.62%
JITECH	17.445	22.842	0.4467	63.75%	67.21%
JBANKS	13.565	27.455	0.2303	48.25%	52.67%
JLFEA	13.106	25.029	0.2343	47.32%	48.07%
JSPAPY	16.350	15.251	0.6505	41.49%	39.02%

From the table above one can see that the lowest Sharpe ratio is the GOVI bond index which was close to 0 for the period under review. This is not unexpected as government bonds are expected to be risk free and thus provide a return in-line with the risk free rate. JCSEV (Consumer Services sector) exhibited the best Sharpe Ratio while all equity indices exhibited moderate to high levels of volatility, this is indicative of the equities market in an emerging market with political uncertainty. The JBANKS (Banking) index had the 4th Lowest Sharpe ratio due to its high volatility. This is not surprising as the Rand is one of the most volatile currencies in the world which has a knock-on effect to the banking sector which is a sector that is also highly exposed to political uncertainty. The Property index performed well compared to other listed equity indices and provided the lowest standard deviation while achieving strong growth of close to 16.5% on an annualised basis after bonds resulting in it obtaining the 3rd highest Sharpe ratio.

What is critical to note from the data is that all of the indices exhibited high levels of drawdown and REDD with all but the GOVI and OTHI indices easily breaching the limit of 20% that investors will tolerate with many getting close to or breaching the 50% mark (Chekhlov, Uryasev & Zabarankin, 2005). As can be seen in Figure4 below, almost all of the largest REDD experienced by the different indices occur during the financial crisis period of 2009 and 2010, which is not unexpected. However, an interesting observation can be seen that a second large set of REDD's occurred in certain sectors in South Africa during 2015, 2016 and 2017. This is also not completely unexpected due to South Africa's political landscape and the economic headwinds that have been experienced over this period.

Therefore, since the main objective of this model is to provide greater downside protection to investors, this study will focus on the overall performance of the model relative to comparable strategies as well as the performance of the model during a specific volatile event, namely the 2009 Financial crisis period and will briefly touch on the models performance over the 2015 – 2017 period.

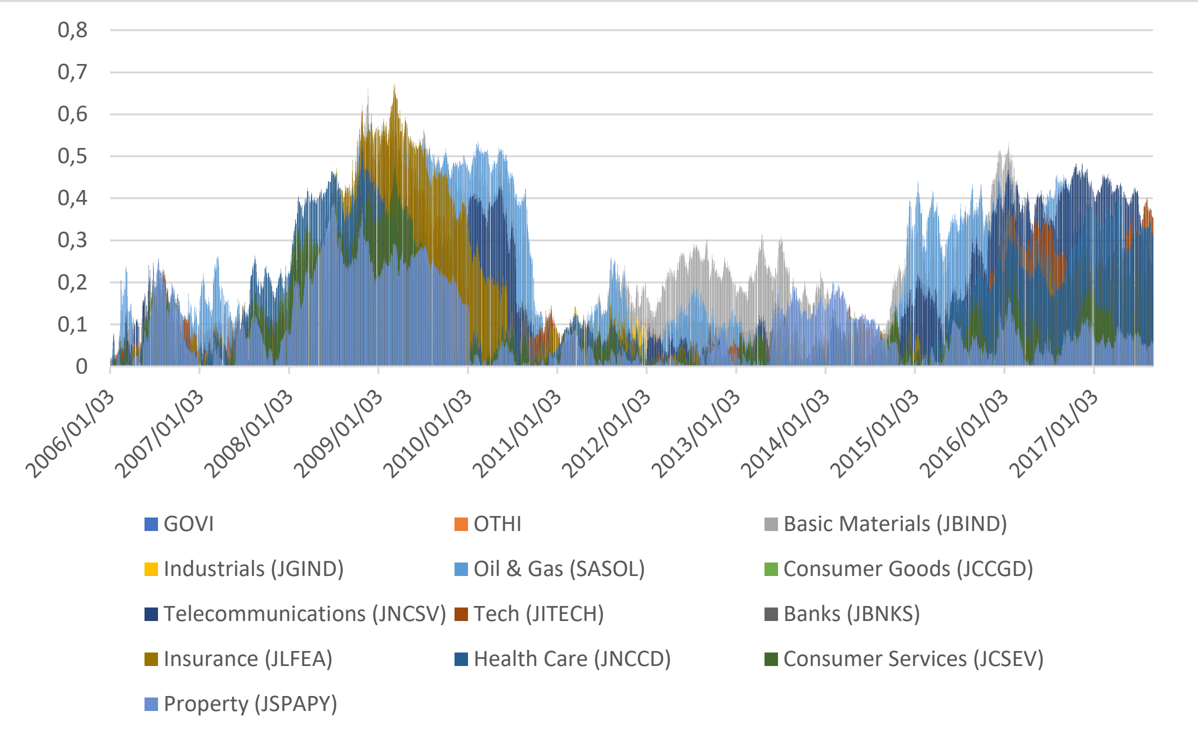

Figure 4: REDD values for the various Indices over the period of January 2006 to August 2017

3.1 : REDD STRATEGIES

This study consists of two main tests.

The first test looks to determine the optimal rebalancing period for the FAREDD model and will evaluate the performance of the fund over a daily, monthly, quarterly and yearly rebalance period in order to investigate what the optimal rebalance period is for the strategy in the South African marketplace.

The second test will investigate whether the FAREDD will outperform other selected models including drawdown orientated models, risk parity and mean-variance optimised models. This will be evaluated based on thirteen different asset classes namely the JSE equity sector indices (JBIND, JGIND, SASOL, JCCGD, JNCSV, JITECH, JBNKS, JLFEA, JNCCD AND JCSEV), the government bond index (GOVI), the other bond index (OTHI) and the South African Property index (JSPAPY). These indices are all market cap weighted and th place the most weight on larger, liquid corporations, thereby reflecting realistic investment possibilities. The portfolio with the best performance will then be measured against other potential strategies available to investors – namely a risk parity portfolio, and a mean-variance optimisation strategy and the REDDp strategy (see sections 3.1.2, 3.1.3 and 3.1.4 respectively).

The criteria that will be used to evaluate performance throughout the tests must take into account the objective of what we are trying to achieve using the model – namely improved diversification that leads to lower drawdowns and lower loss of capital during market falls but strong upside potential during bull market runs and market recoveries, while limiting the portfolios exposure to risk. Thus, the main criteria that will be used includes annualised standard deviation, annualised returns, maximum drawdown, maximum REDD, Sharpe ratios and an ending multiple that evaluates the increase in portfolio value over the period.

3.1.1: FAREDD MODEL

In developing the FAREDD strategy, the strategy selection methodology was built on the REDDp strategy as developed by Xie, Xu and Yu (2014) and discussed in Section 2.1.4 of this study. The FAREDD strategy differs on the actual underlying asset's to be weighted and subsequently included in the model.

Instead of calculating the various weightings of the assets to be included using each individual asset class, the FAREDD model first calculates the daily returns from the individual assets (in this case the different indices), it then performs a principle component analysis on the correlation matrix of various asset class returns[1] over a rolling period of window *H* which is equivalent to the window period used for the REDD calculations. The length of the window was set at *H*= 1 year (252 trading days). This was kept in-line with previous findings by Yang and Zhong (2013) that a lookback period of 1 year provides the best balance between short term gain to be had by "rolling" into risky assets more quickly during potential market recovery, and the downside risks of a second declining period of length *H,* as markets tend not to fall dramatically for longer than 12-month periods. This finding is supported by reasoning stemmed from a Bayesian point of view that, markets are more likely to rebound to a rising price

[1] The program *R Studio* will be used with its inbuilt *prcomp()* function to perform the PCA analysis.

course if they had been falling in a previous period of length *H,* yet if one has too short a look-back period, it can result in even further losses if markets continue to fall, thus *H* must be long enough to reduce this risk but be short enough to allow for sufficient benefit to be achieved from rolling into risky assets sooner. For the purposes of this study a lookback period of 1 year is assumed to provide the best balance between the rewards and risks associated with it as previously found by Yang and Zhong (2013). This relationship is assumed to hold for the South African marketplace and it goes beyond the scope of this study to investigate the optimum lookback period for South Africa.

Once the square principle component matrix has been obtained with its various factor weights, the FAREDD model then calculates the principle component (PC) returns by utilising the relationship between the eigenvectors and the returns of the asset classes over the period. This relationship is defined in Meucci (2010) and is calculated as follows:

$$R_{pc} \equiv E^{-1} R_N \, , \tag{11}$$

Where $E \equiv (e_1, \dots, e_N)$ represents the eigenvectors of the PCA matrix which defines the set of *N* uncorrelated PC "portfolios" whose returns can be calculated by taking the inverse of eigenvectors multiplied by the returns for each asset. Thus, the eigenvectors essentially act as "weights" of the asset returns that will be incorporated into the PC returns as seen in equation 11 above with each principle component being decreasingly responsible for the returns in the market (Meucci, 2010). Note that if the eigenvector weights are negative, the portfolio is effectively short selling that asset. The cash that would have then been generated by that short sale has been assumed to be invested JIBAR which is the risk free rate for the purposes of this study.

Once the uncorrelated principle components were compiled, the weights of the PC's making up the FAREDD portfolio can be calculated using a simplified version of the REDDp formula as described in Equation 10 in Section 2 of this study. This simplification is possible due to the fact that the PC's are uncorrelated which means the ρ can be set to 0. This results in the weighting of the PC's in the FAREDD model being able to be calculated as follows, where *N* is each principle component to be included in the FAREDD model:

$$[X_N] = \left[\left(RS_1(t,h) + \left({}^1\!/_2 \right) \sigma_1(t,h) \right) \Big/ \sigma_1(t,h) \right] \cdot Max\left[0, \frac{1}{(1-\delta^2)} \cdot \left(\frac{\delta - REDD(t,H)}{1 - REDD(t,H)} \right) \right], \tag{12}$$

For the purposes of this study the drawdown limit (δ) was set at 20% in line with previous findings by Chekhlov, Uryasev and Zabarankin (2005), that managers themselves may shut down funds with drawdowns exceeding 20%. Thus, for the purposes of this study we have assumed that the maximum drawdown an investor is willing to withstand is 20%. This, however, is an assumption and the investigation as to the maximum size of the drawdown limit for South African investors is beyond the scope of this study.

Once the weights have been calculated, the model was tested for the best rebalancing time length, namely daily, monthly, quarterly and yearly in order to calculate the optimum performance of the portfolio and to investigate which rebalancing period yield the best results for this strategy in the South African marketplace. The most practical and optimal portfolio as a result of this test, was the FAREDD model that was used to compare it’s results relative to that of the Risk Parity and Mean Variance optimised portfolio. The results of which are discussed in Chapter 4.

The full calculation algorithm for the FAREDD model is further explained in Appendix A of this study.

3.1.2: RISK PARITY

The risk parity strategy is focused around increasing diversification and minimising risk to investors. This is brought about by the model allocating resources to asset classes so that they all contribute equally to the overall portfolio risk level, thereby attempting to achieve a superior Sharpe ratio when compared to other strategies (Grieg, 2016). Thus, this strategy is a fully invested one whereby the *ex post* risk contributions of all the various asset classes are equal.

Therefore, this model provides a reasonable alternative to the FAREDD model as they both have similar objectives to maximise risk adjusted return while not exposing the investor to large levels of risk or potential wealth loss. The one downside to this model as a comparable portfolio is, that due to its very nature of achieving equal risk contribution of each asset class to the overall risk of the portfolio, it does not allow for short sales.

The *Portfolio Analytics* package in R will be used in order to calculate and evaluate this model. The package uses the *DEoptim* solver and for the purposes of this study we add Risk Budget objective and run the optimisation using a training period of 12 months (252 days) with monthly rebalancing so that the weights sum to 1 at each rebalance date.

3.1.3: MEAN VARIANCE OPTIMISATION (MVO)

As previously mentioned, this study will use both a minimum-variance optimised MVO model as well as a maximum Sharpe Ratio optimised MVO model. The reasons for selecting these models are discussed below. However, both models will be calculated using the *Portfolio Analytics* package in R and, since mean-variance optimisation is a quadratic problem, the solver used will be used for the purposes of this study will be *ROI.* Both models will be rebalanced monthly and have a training period of 12 months (252 trading days) of data.

Minimum Variance

This portfolio lies on the leftmost side of the efficient frontier and has a suboptimal Sharpe ratio. However, it is a well known and studied portfolio which has the ability to represent the chosen portfolio of the most risk averse investor and thus looks to reduce risk as much as possible. Thus, it provides a sound and practical alternative to the FAREDD model and will be constructed by setting the portfolio objective to minimise risk.

Maximum Sharpe Ratio

This portfolio will be constructed by adding the *MaxSR* objective to the mean variance optimisation model in R's *Portfolio Analytics* package. Due to the fact that the model has allowed for short sales and have no box constraints, as the results will show there are occasionally large positions taken in any one industry of up to 50% of the portfolio residing in one index. In addition, there are often significant short positions taken which allow the portfolio to fund larger positions in other indices. These considerations could result in many issues for fund managers with regards to their mandates and risk profiles. Therefore, like the FAREDD model, this model may only be a possible strategy for managers and investors with little to no restrictions on short sales and maximum position sizes. However, the portfolio provides a strong theoretical alternative that looks to maximise the Sharpe ratio of a portfolio. In other words, the portfolio will seek to maximise the risk adjusted return, however there is less emphasis on protecting the portfolio against sharp declines in value. Thus, this portfolio will be used to compare the performance of the FAREDD model on a risk adjusted return basis.

3.1.4: REDD of RISKY ASSESTS PRICING STRATEGY

As discussed in the literature review of this study (Section 2), the REDDp strategy has been developed from previous REDD and EDD models and is the best performing economic drawdown model at the point of writing this study. The model as developed by Yu, Xie and Xu (2014) was designed in order to control the drawdown of a portfolio and prevent losses while also seeking to boost risk adjusted return performance. It is also the basis for the FAREDD strategy and thus the FAREDD effectively seeks to improve on the REDDp strategy. For this reason, they inherently share the same objectives and look to provide strong downside protection to the investor while looking to maximise return. In addition to this, the model provides a practical strategy to asset managers and investors and is thus the closest potential comparison to the FAREDD model.

In addition to this, for the purposes of this study, we will only evaluate the REDDp strategy rather than include all other REDD strategies, due to the fact that as discussed in the literature review – section 2 of this study – the REDDp strategy has been proven to be more efficient and provide stronger performance than all other REDD strategies, thereby making its performance the benchmark for the FAREDD to be compared to as the other models would provide inferior performance making them redundant.

At the time of conducting this study there is no built-in optimisation function that can be used in R to run this model and therefore the model is built manually following the below formula as developed by Yu, Xie and Xu (2014) which allows for short sales.

The weight allocation of each asset to the portfolio for multiple risky assets with no short sale constraint is calculated as follows:

$$X(t) = [\sigma(t,h)^{-1}\mu(t,h)]^T \sigma(t,h)^{-1} \frac{1}{(1-\delta^2)} (\frac{\delta - REDD(t,H)}{1-REDD(t,H)}), \tag{13}$$

Where $\sigma(t,h)$ is the volatility matrix ($\Omega(t,h) = \sigma(t,h)\sigma(t,h)^T$; $\Omega(t,h)$ is the covariance matrix in [*t-h,t*]. $\mu(t,h)$ is the return drift vector such that $\mu(t,h) = R_i(t,h) - r + \sigma_i{}^2(t,h)/2$ (Yu, Xie & Xu, 2014).

The above strategies are then all compared to each other to evaluate the performance and effectiveness of the various models, both overall as well as during the financial crisis period. They will be compared based on their level of returns, standard deviation, Sharpe Ratio, drawdown and REDD. For the purposes of this study, a drawdown limit of 20% will be considered the maximum drawdown an investor is willing to withstand.

The portfolios will also be compared to the FTSE/ALSI as a benchmark during the period, in order to provide the reader and investors considering the various portfolios with a passive benchmark for the period that will allow them to determine whether the portfolios constructed provide a better trade-off than simply investing in the overall market and only exposing themselves to market risk

CHAPTER 4: RESULTS

4.1: TEST 1 – CALCULATING THE OPTIMAL REBALANCE PERIOD

The table below shows the performance of the FAREDD model over different rebalancing period dates.

Table 4: Results of the FAREDD model under different rebalancing timelines for the period January 2007 to August 2017

	Daily	Monthly	Quarterly	Yearly
Annualised Return	20.48%	13.67%	19.50%	8.93%
Annualised σ	59.25%	27.18%	30.57%	19.39%
Sharpe Ratio	0.22	0.23	0.40	0.09
Max Short Exposure	1.89x	1.02x	3.10x	5.38x
Average Short Exposure	0.79x	0.69x	0.98x	0.54x
Max Drawdown	-32.50%	-19.69%	-21.38%	-9.19%
Max REDD	-34.10%	-25.53%	-28.32%	-18.25%
Ending Multiple	6.44x	3.60x	5.94x	2.79x

It is important to view the results presented in Table 4 as well as the results in the remaining tests below with the main purpose of the model in mind which is to reduce drawdown exposure, increase portfolio diversification while also seeking to provide strong risk adjusted returns thereby protecting an investor during downturns.

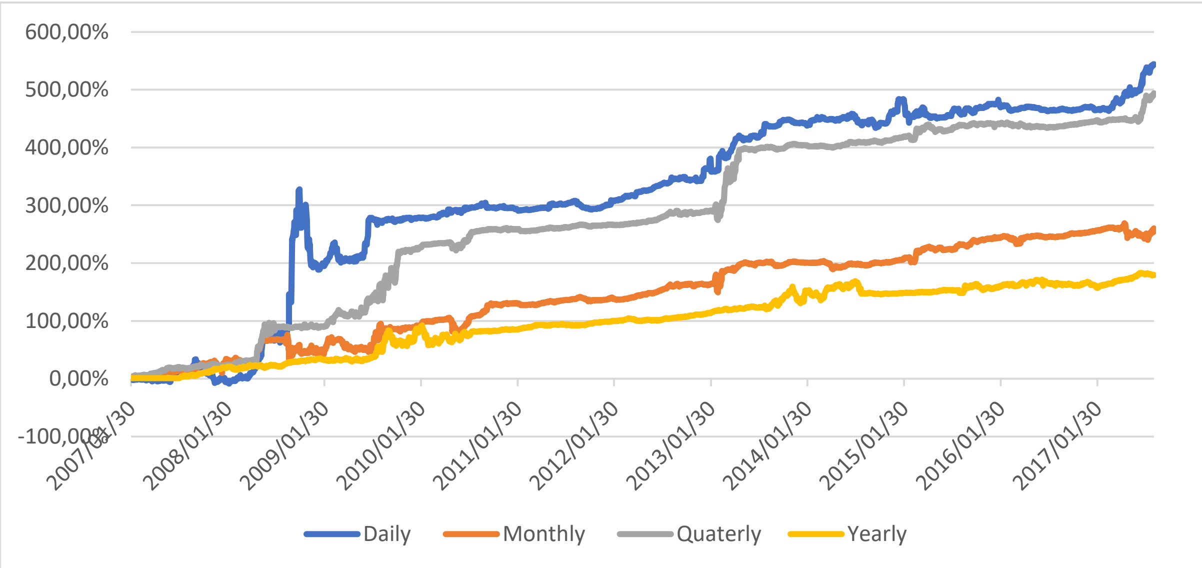

Figure 5: Cumulative returns of the FAREDD model for different rebalancing periods for the period January 2007 to August 2017

An important observation is that all the models take major short positions over the test period which has practical implications and considerations for investors looking to use this model as an allocation technique. Both the quarterly and yearly rebalance periods hit maximum short positions that were 3.10x and 5.38x, with average short positions of 0.98x and 0.54x, greater than the size of the fund respectively which has strong practical implications as the model is assuming that one could short large values of shares at any one time not to mention the added risk into the portfolio as these short positions would not be hedged in any way and would thus be considered to be highly risky to any rational investor.

From an optimum rebalancing point of view as evidenced in the table, the quarterly rebalance period outperforms all other FAREDD models across all the performance criteria bar the maximum drawdown experienced, max REDD and the annualised standard deviation where the monthly and yearly rebalancing period experienced more favourable outcomes.

The yearly rebalancing period achieved significantly low drawdown levels with a maximum drawdown of 9%. However, the portfolio also achieved the lowest annualised return over the period. As can be further evidenced in Figure5 which depicts the cumulative return of all the portfolios, the yearly portfolio had the lowest compound return followed by the monthly then quarterly and then daily. This subsequently resulted in the portfolio having the lowest Sharpe Ratio of 0.09. This is followed by the daily return period Sharpe ratio of 0.22.

The daily return period while having the strongest cumulative returns and an annual return of 20.48% had a large annual standard deviation value of almost 59.20% and a maximum drawdown of 32.50% resulting in a Sharpe Ratio of 0.22. In addition, the portfolio had a REDD of almost 34.10%. This does not achieve the objectives of the model in terms of protecting the investor against drawdown as the model breaches the 20% limit set for the purposes of this study. In addition to this, a major practical implication of rebalancing daily is that it would result in high transaction costs which would erode returns and would be highly computationally challenging.

Therefore, it can be seen that both the daily and yearly rebalancing period perform the worst in terms of risk adjusted returns and the daily period performs the worst when evaluating the drawdown levels. The daily period experiences considerably high drawdown levels and the yearly rebalance period has a low Sharpe ratio when compared to monthly and quarterly rebalance periods. This is caused by both lower annualised returns and higher standard deviations with large amounts of volatility present in the daily model. This indicates that the daily rebalancing is too short of a time frame and thus the principle component portfolios if recalculated daily is not necessarily indicative of the conditions going forward to the next day, which results in portfolios that are not adequately uncorrelated in the next time period and can lead to reduced performance and sudden losses in value, essentially the daily rebalance period seems to place too much emphasis on more current information that may be random and not indicative of the true market conditions. Conversely, it seems that the yearly rebalance period is too long and thus while it provides the best results in terms of drawdown it has a poor risk adjusted return and a low annual return while also having a relatively high REDD of 18% when compared to its slow annual growth rate. This seems to indicate that a year-long rebalance period does not accurately take into account current information in the market and is thus slow to adjust and react to changes in market conditions and additional information becoming available.

The quarterly period seems more attractive from a risk-adjusted basis as measured by a Sharpe ratio of 0.40 when compared to a monthly rebalance period’s Sharpe Ratio of 0.23, which are still lower than desired. The large exposure to short sales as well as the drawdown experienced by the quarterly rebalancing FAREDD is significant as one of the main purposes of the model was to limit risk to investors and prevent large sudden decreases in value in order to better protect one’s portfolio. The large exposure to short selling of 3x the portfolio value introduces additional risk into the portfolio that may not be represented in the annual standard deviation measure. In addition to this, such large short sale exposure creates a number of practical consideration issues which need to be taken into account when considering this model as a practical alternative. The model also experienced a maximum drawdown of 21% and this breached the drawdown limit set by the model as well as the practical limit of 20% as found by Chekhlov, Uryasev and Zabarankin (2005). In addition to this, the quarterly rebalance period experiences a slightly larger REDD of 28% when compared to the monthly period of 25%. Therefore, overall, the quarterly and monthly models both have significant pros and cons and ultimately it would come down to the risk appetite of the investor as to which period to choose. However, for the purposes of this study we must consider the practical considerations of the large short sales as well as larger average short sale level of the quarterly period when compared to the monthly in addition to the higher annual standard deviation, REDD and drawdown experienced by the quarterly model. With these factors in mind the quarterly rebalance period is not seen as the optimal rebalance period for the purposes of this study.

Therefore, in the tests going forward, the rebalance period will be monthly as opposed to quarterly, yearly or daily in order to stay in-line with the objectives of this study. However, it can be noted that from a practical perspective an investor who is willing to tolerate a higher drawdown and level of risk in order to achieve a greater level of return may consider the quarterly rebalancing period as optimal to their requirements. This essentially represents the risk-return trade off that any investor faces.

4.2: TEST 2 – COMPARATIVE PERFORMANCE

The table below depicts a summary of the descriptive statistics for the various strategies under consideration.

Table 5: Results of test 2 evaluated over the period January 2007 to August 2017

	Minimum Variance (MVP)	Maximum Sharpe Ratio	Risk Parity	FAREDD Model (monthly)	REDDp
Annualised Return	8.94%	15.95%	9.37%	13.67%	12.71%
Annualised Standard Deviation	10.91%	35.89%	16.66%	27.18%	29.45%
Sharpe Ratio (rf = 7.2409)	0.15	0.25	0.13	0.23	0.19
Maximum Drawdown	-9.481%	-36.48%	-26.67%	-19.69%	-27.50%
Max REDD	-17.68%	-43.17%	-34.18%	-25.54%	-36.30%
Ending Multiple	2.35x	4.51x	2.45x	3.60x	3.31x

As can be seen in table 5 the performance of the various portfolios is mixed with the FAREDD model outperforming the other portfolios, with the exception of the maximum Sharpe Ratio portfolio, in terms of risk adjusted return. It also presented the second lowest drawdown and REDD percentage as well as the second highest annual return over the period.

Test 2 will be evaluated as two sub tests, with the first evaluation looking at how the rolling economic drawdown models compare to traditional models and the second evaluation looking at how the FAREDD model compares to the REDDp strategy.

4.2.1 :THE PERFORMANCE OF THE REDDP & FAREDD MODELS COMPARED TO TRADITIONAL MODELS

Both the FAREDD model as well as the REDDp model provide strong results relative to the other models over the period. This is further evidenced in Figure6 below where it can be seen that the cumulative returns for the FAREDD model are only lower than the optimised Maximum Sharpe ratio portfolio followed by the REDDp model and then by the risk parity and MVO models respectively which are significantly lower.

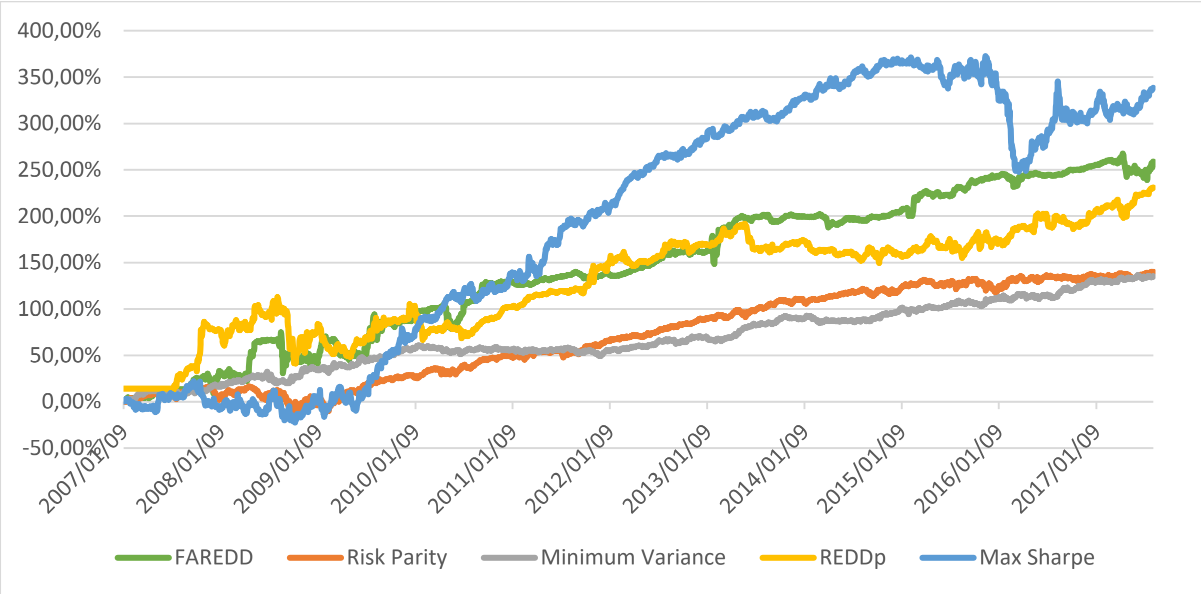

Figure 6: Cumulative returns of the comparable portfolios for the period January 2007 to August 2017

From an absolute performance perspective, one can see from table 5 as well as from Figure6 that the FAREDD model outperforms the other models when considering the objectives of this study. It returned the best risk adjusted return over the period, while also preventing the maximum drawdown experienced by the model from breaching 20% of the portfolio value, thereby achieving the objectives of the paper to minimise loss and still achieve optimal returns. The only other portfolio to achieve this was the MVP portfolio which provided for the lowest drawdown, however performed poorly when it came to annualised returns and risk adjusted returns.

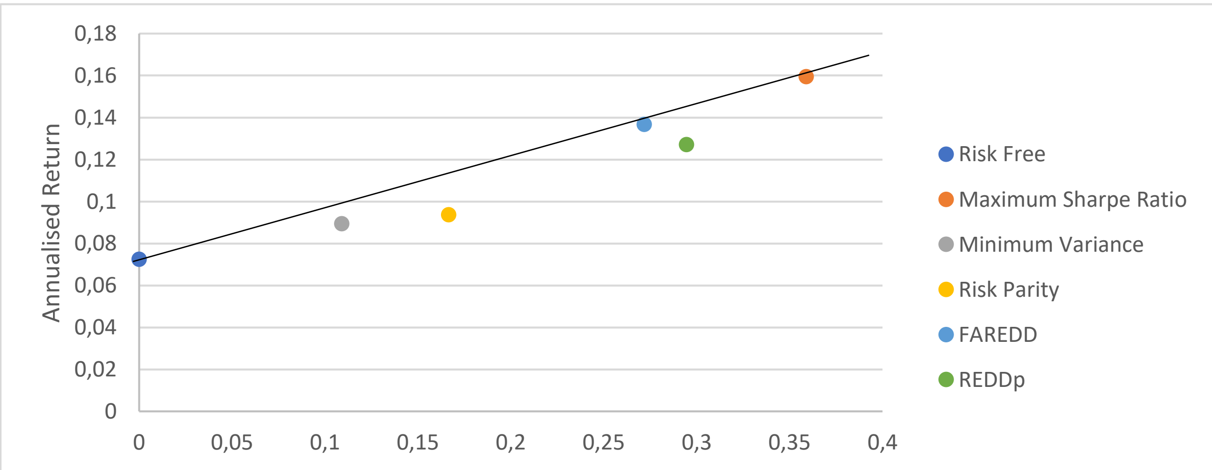

Figure 7: Annualised returns and standard deviations for the period

It is also immediately evident in Figure6 that the maximum Sharpe ratio portfolio experienced negative returns and large volatility during the 2009 crisis period as well as towards the end of 2015 and through the beginning of 2016, possibly as a result of the Nenegate scandal in South Africa whereby the country experienced political, economic and exchange rate turbulence as a result of ratings downgrades, poor economic growth and political uncertainty. Interestingly, the other portfolios did not experience the same level of decrease and volatility, they seem to be more effected by the 2009 crisis which may be as a result of the fact that the 2009 crisis was significantly longer than the 2016 shock in South Africa which was simply a quick and sudden market shock with a faster recovery. This may indicate that the that the maximum Sharpe Ratio portfolio is the riskier portfolio with greater volatility while the other portfolios, which had more of a focus on risk and controlling drawdown, were less affected, which may indicate that due to their focus on either drawdown/risk and/or return may be better protected and diversified during those periods.

Figure 7 above shows the efficient frontier and plots the annualised standard deviations and returns of the 6 tested strategies. The maximum Sharpe ratio portfolio acts as the tangency portfolio. As can be seen from the figure. The MVP and Risk Parity portfolios provide similar returns for differing levels of risk over the period, but neither are close to the capital allocation line. The FAREDD model achieves the closest Risk and Return combination that comes closest to meeting the capital allocation line. However, it has larger levels of standard deviation attached to it than the minimum variance or risk parity portfolio. Similarly, the REDDp model provides a higher level of standard deviation yet achieves a lower annualised return, thus the REDDp model is inferior to the FAREDD model over the entire period, which is clearly depicted in the efficient frontier.

From a risk perspective, as defined by standard deviation, the performance of the FAREDD model and REDDp model was overshadowed by the MVP and Risk Parity portfolios which produced a standard deviation of 10.91% and 16.66% respectively, however these low values are to be expected as both models are designed to reduce the standard deviation of a portfolio. In addition to this, the high standard deviations experienced by the FAREDD and REDDp model may be as a result of the model being allowed to take short positions on assets over the course of the period thereby effectively introducing additional variation into the models, while the risk parity portfolio did not take any short positions due to its equal weighting nature while the MVP model is optimised with the objective of minimising standard deviation

Now, if one was to go back to the definition of a diversified portfolio as defined by Meucci (2009), whereby a portfolio would be considered to be diversified not by its level of standard deviation but rather by whether it prevents a large and sudden decrease in portfolio value, only the MVP portfolio followed by the FAREDD would be successful here as all other portfolios experience a drawdown of greater than 20%. With all except the MVP portfolio breaching 20% REDD drawdown.

Figure 8 below depicts the REDD of the various models over the test period and it is clear that the Max Sharpe ratio performs the worst in this regard, followed by the REDDp and Risk parity portfolio.

In the graph in Figure6, as well as in Figure8, below it is evident that there were high levels of volatility during the 2008 -2010 periods over the course of the financial crisis where the maximum Sharpe ratio as well as risk parity portfolio fell into negative territory before making a recovery. Figure 7 depicts the REDD's of the portfolios over the test period. What is most evident here is that during the financial crisis portfolios experienced significant losses in value. Thus, considering that the purpose of this model is to prevent portfolio losses during times of uncertainty and crisis this study will evaluate the performance of the portfolios over this time period from January 2007 to January 2010 as a test period during times of crisis. Figures 9 to 13 depict the return and drawdown of each model over the financial crisis period.

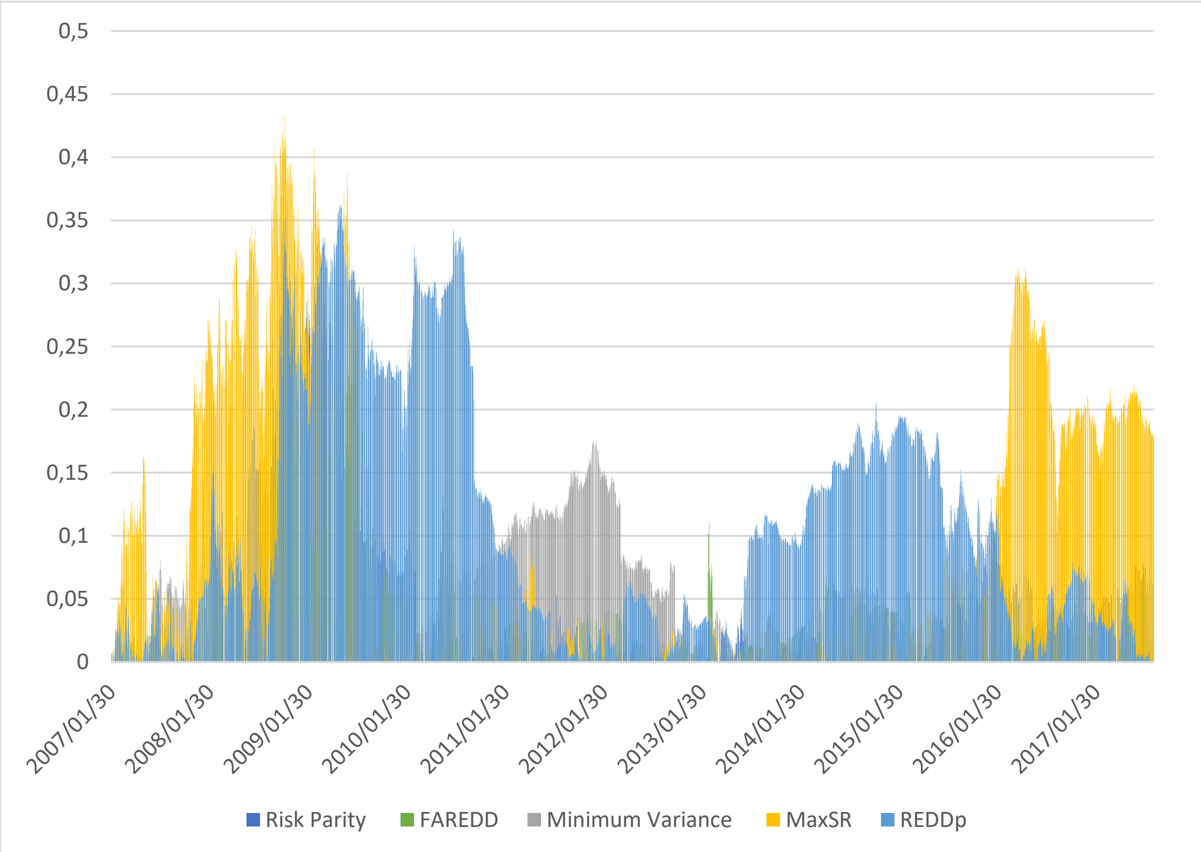

Figure 8: REDD over the period January 2007 to August 2017

As can be seen in figures 9 to 13 below, the models that perform optimally in terms of managing both drawdown and targeting the best risk adjusted returns over the crisis period are again the FAREDD model and the MVP model with max REDD levels of 25% and 12% respectively during the period. In addition, the FAREDD model while being more volatile during the period (with an annualised standard deviation of 37.13% compared to 13.16%), achieved a cumulative return of 76% while the MVP achieved 57%. The REDDp model on the other hand had larger REDD's over the period and greater volatility of 40.34% yet achieved a cumulative return of 97% over the period. The worst performing portfolio in terms of achieving the dual objectives set in this study is the Risk Parity portfolio which delivered cumulative returns of 28% and experienced REDD's of near 35% for a large portion of the period with an annualised standard deviation of 13.93%. The Sharpe ratios for the portfolios as can be seen in table 6 below, show that the MVP portfolio provides the best risk-adjusted return over the period followed by the REDDp method and the FAREDD method while the Risk Parity portfolio fails during this period and provides negative risk adjusted returns.

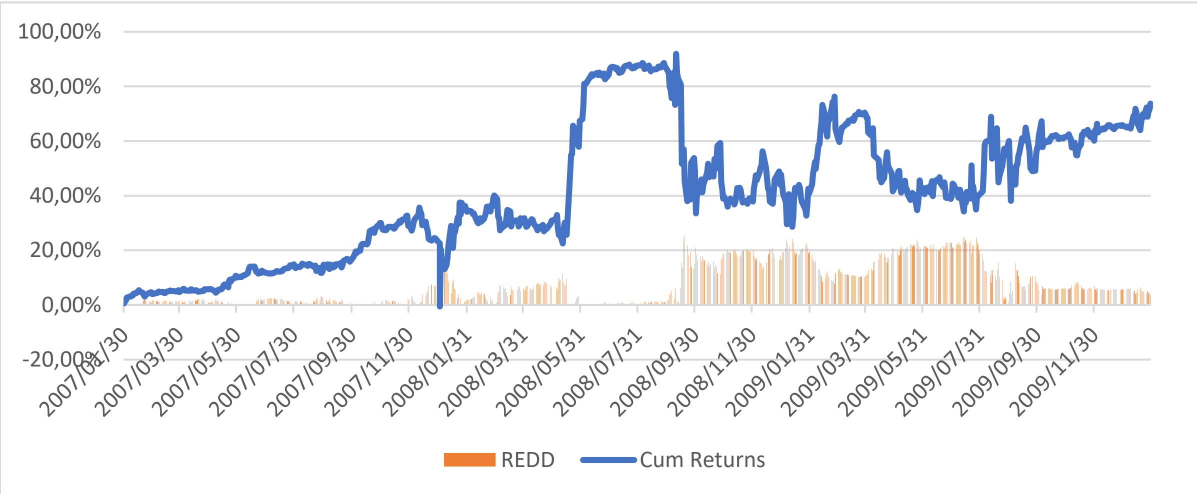

Figure 9: FAREDD Performance over the period Jan 2007 to Dec 2009

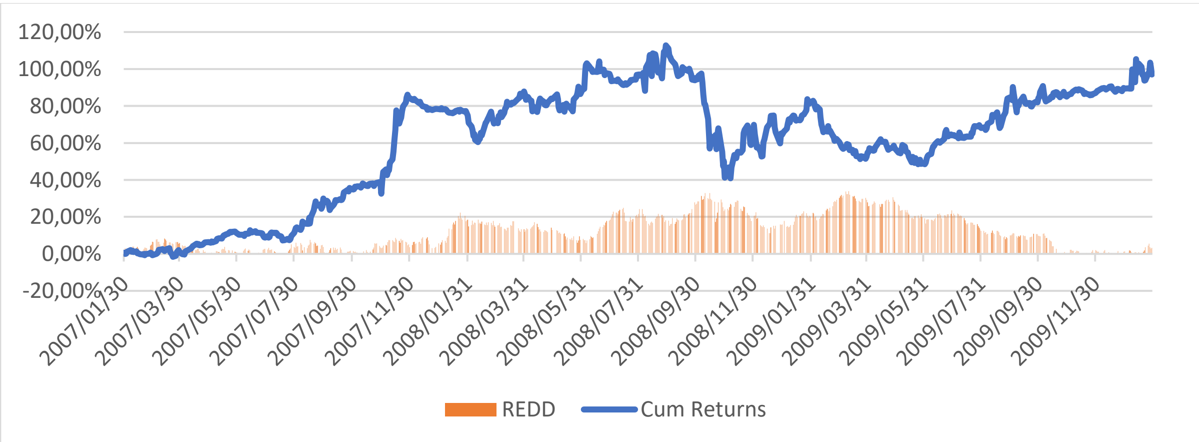

Figure 10: REDDp Performance over the period Jan 2007 to Dec 2009

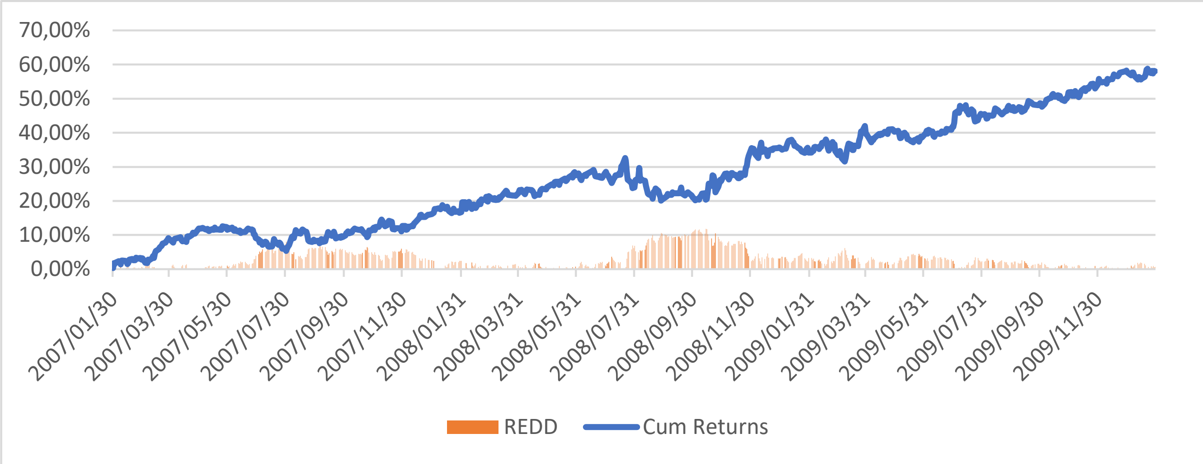

Figure 11: Minimum Variance Performance over the period Jan 2007 to Dec 2009

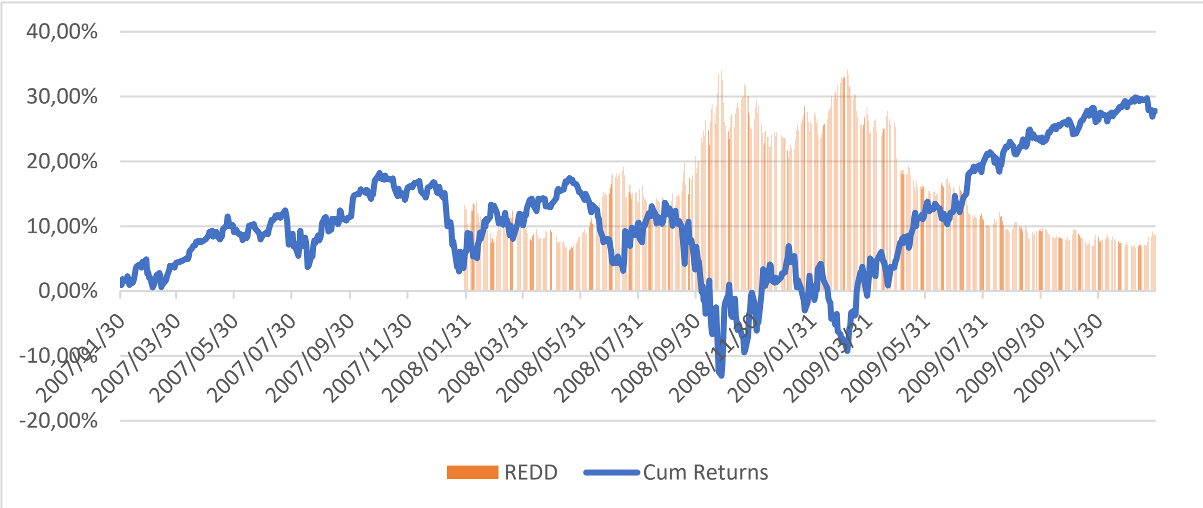

Figure 12: Risk Parity Performance over the period Jan 2007 to Dec 2009

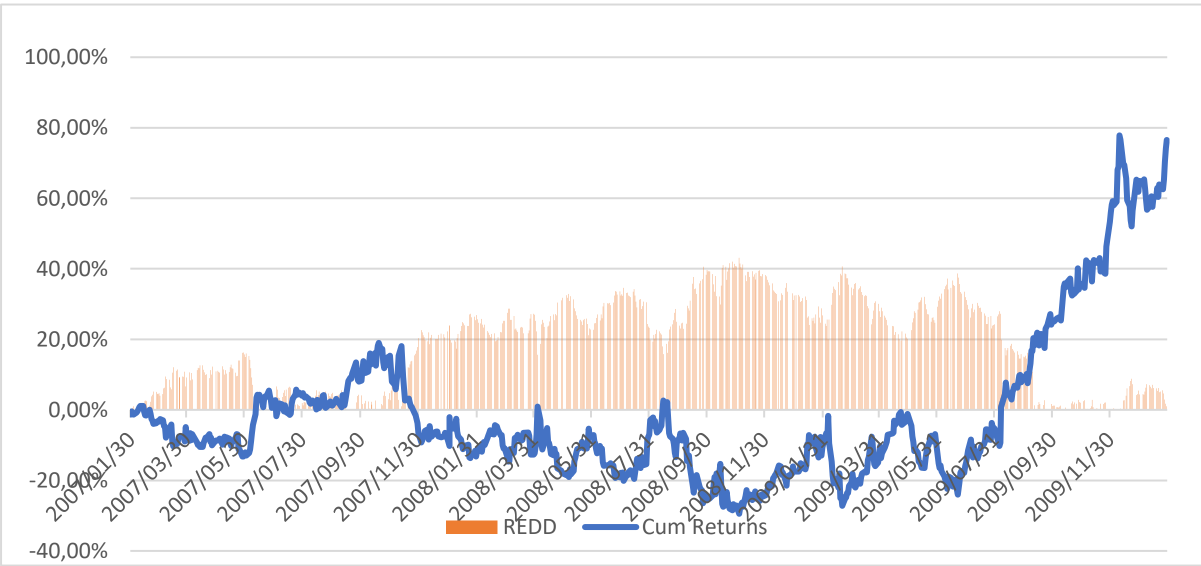

Figure 13: Maximum Sharpe Ratio Performance over the period Jan 2007 to Dec 2009

In addition, the Max Sharpe ratio portfolio provides a Sharpe Ratio of 0.32 over the period due to large standard deviations of 35.89% indicating that it, as well as the FAREDD model, experienced the most volatility while it achieved a cumulative return of close to 80% and experienced REDD's of over 40% during the period. This indicates that the Max Sharpe ratio is the most volatile with the least diversification during the period. Thus, the maximum Sharpe Ratio portfolio does not achieve the objectives of this study and is therefore not considered optimal for the purposes of this study.

However, it must be reiterated that investors with different risk-reward trade-offs or different objectives may consider other models more optimal than the FAREDD or the MVP as these portfolios may better achieve their main objective.

Table 6: Sharpe Ratios of the portfolios during Jan 2007 to Dec 2009

	FAREDD	REDDp	MVP	Risk Parity	Max Sharpe Ratio
Sharpe Ratio	0.31	0.40	0.54	-0.05	0.32

Therefore, it can be seen that during a market crisis the FAREDD model as well as the REDD model seem to be inferior to the MVP in terms of protecting the portfolio from losses while also providing strong risk adjusted returns over a crisis / downturn period. Conversely, the models provide the most optimal performance when considering the cumulative return of all models tested. Therefore, there is no conclusive outperformance of any one portfolio over the crisis period. However, if one is solely interested in maximining risk adjusted return while also limiting drawdown which are two of the main considerations of this study the MVP seems to be the most optimal followed by the FAREDD and the REDD model.

Surprisingly, the Risk Parity did not perform optimally during the financial crisis when compared to the other portfolios, and still experienced significant drawdown. This may be as a result of the way in which it is comprised and having to be invested equally in the various assets at all times resulting in it having been exposed to all industries while they all experienced contagion thereby preventing it from reducing the risk of the portfolio during this period while conversely all other models were able to short assets during this time and also reduce their fully invested exposure.

When one considers the overall period of the test, from January 2007 to August 2017, the MVP portfolio falls back and underperforms from a return and risk-adjusted return point of view as by reducing variance it is naturally reducing the exposure to risk and thus limiting the return of the portfolio. However, the benefit of doing so results in significantly less annualised volatility for the period. The FAREDD model and REDDp model provide stronger risk-adjusted returns as well as annualised returns in the long run and as such outperform on a return basis while also providing the best downside protection after the MVP model. The Maximum Sharpe ratio portfolio is by far the most volatile and from a drawdown, REDD and thus from a portfolio protection point of view is inferior to all other models when considering the objectives of this study.

4.2.2 : THE PERFORMANCE OF THE FAREDD MODEL COMPARED TO THE REDDp MODEL

When comparing the performance of the FAREDD model to the REDDp model one must keep in mind the objective of the FAREDD model which was to improve upon the REDDp model and provide for stronger diversification benefits as measured by both standard deviation and the ability of the model to withstand sudden drawdowns while also outperforming in the long run.

Therefore, with this in mind, it can be seen from Figure9 and 10 as well as Table 6 that the FAREDD is not as optimal as the REDDp model during a crisis period in terms of risk adjusted returns – with a Sharpe Ratio of 0.31 as compared to the 0.40 for the REDDp model. This suggests that there is not necessarily any additional benefit of utilising principle components to outperform from a risk-adjusted perspective during sudden market downturns. However, the FAREDD model does provide lower REDD drawdowns than the REDDp model, has a lower volatility over the period as well as a lower overall maximum drawdown – as depicted in table 5 above. All of these factors indicate that it protects a portfolios value more effectively while also providing a slightly more stable model in terms of volatility. Therefore, if one returns to Meccui's understanding of diversification it can be seen that the FAREDD model provides slightly better diversification benefits during both the market crisis as well over the entire period as a result of its principle component approach and their uncorrelated nature.

In addition to this, the FAREDD model provides stronger performance over the entire test period with significant outperformance of the REDDp model in terms of risk-adjusted returns, annualised return, drawdown, standard deviation and REDD. This indicates that over the longer term the FAREDD model is able to provide a less volatile performance and is able to achieve more sustainable returns over time.

4.2.3: COMPARING THE PORTFOLIOS TO A BENCHMARK

As previously discussed in chapter 3, the portfolios are compared to a benchmark in order to provide a reference point to readers and investors as to how the portfolios compare to a passive portfolio.

The FTSE/ALSI index for the period experienced a Sharpe ratio for the period of just 0.072 while returning an annualised return of 8.31% and an annualised standard deviation of 14.88%. In addition to this, the index experienced a maximum drawdown of 41.45% and a max REDD of 46.71% that occurred during the financial crisis. This is further depicted in Figure13 below.

Further to this, it can be seen that the portfolio experienced significant REDD's during the financial crisis period before growing rapidly between July 2009 and December of 2014. For the years following 2014, the ALSI performance remained flat before picking up slightly towards June 2017. This is in contrast to the consistent upward trend as experienced by the REDDp and the FAREDD model as can

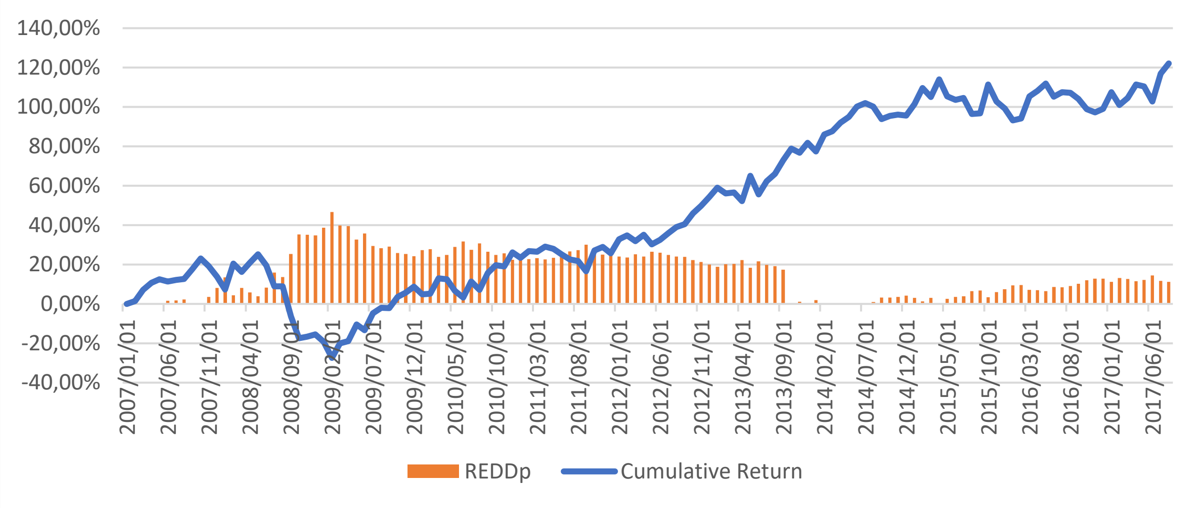

Figure 14: FTSE/ALSI Performance over the period of January 2007 to August 2017

be depicted in Figure6 above where it can be seen that during 2014 to 2017 the portfolios continued to make positive returns due to the portfolios' ability to evaluate each sector's performance and REDD levels and allocate more resources toward positive performing industries, while also being able to short sell assets. Thereby allowing the portfolios to perform positively in sideways markets. In contrast to this the Risk Parity, Maximum Sharpe Ratio and Minimum Variance portfolios as depicted in Figure6 tended to follow a similar trend as the ALSI index, whereby they flattened out and provided very little additional return during the period of 2014 to 2017, with the maximum Sharpe ratio actually losing value.

In addition to this, when one compares the FTSE/ALSI performance in Figure14 over the period of January 2007 to December of 2009 to the performance of the portfolios in figures 9 to 13 one can see that all except the maximum Sharpe Ratio portfolio experienced lower REDD's during the period and all achieved better cumulative returns during the period. On a risk-adjusted basis the ALSI index provided a Sharpe Ratio of -0.21 and provided a total cumulative return of 8.72% over the entire 2007 - 2009 period. Therefore, the ALSI was also the worst performing portfolio on a risk adjusted basis over the course of the financial crisis period.

CHAPTER 5: CONCLUSION, STUDY LIMITATIONS AND AREAS OF FURTHER RESEARCH

5.1 CONCLUISION

The concept of asset allocation and asset allocation strategies has been a critically important element to both fund managers, academics and investors as individuals look for and investigate new ways in which to improve their performance and protect their portfolios. This has been most relevant now than ever before, especially since the 2009 financial crisis. This has led to a variety of different strategies being created and academics challenging or looking to improve upon these frameworks.

This study falls in line with this discussion in two main forms, firstly by attempting to improve upon the existing REDDp framework by developing the FAREDD model while also investigating how these two models perform in a South African context when compared to more traditional portfolios namely the minimum variance, maximum Sharpe Ratio and Risk Parity portfolios. Further to this the paper focused on preventing drawdown and protecting a portfolio during turbulent times. Therefore, the portfolio investigated how the various portfolios performed both over the entire forecast period as well as specifically over the financial crisis period.

As can be seen in section 4.2.2 of this study, the FAREDD model outperforms the REDDp model in the long term, providing the highest Sharpe ratio behind that of the Maximum Sharpe ratio portfolio. In addition to this, during the financial crisis period the FAREDD model experiences lower drawdowns and REDD's than the REDDp model as well as all other models tested bar the minimum variance portfolio. Thus, when considering the purposes of this study whereby the objectives of the model were to reduce risk and protect a portfolio more effectively during a downturn while still achieving high long-term returns and strong risk adjusted returns, one can see that the FAREDD model was able to achieve these objectives more efficiently than the REDD model.

In addition to this, the paper evaluated the optimal rebalancing period for the developed FAREDD model and although the monthly rebalance period provided the lower Sharpe Ratio of 0.23 when compared to the Quarterly rebalanced portfolios Sharpe ratio of 0.40. The Quarterly period had a number of practical restrictions and also experienced greater drawdowns and REDD's which exceeded the 20% drawdown limit set by the portfolio which resulted in the finding that the optimal period for the South African market was monthly.

Finally, the paper investigated whether existing drawdown models, the REDDp model as well as the FAREDD model, would provide improved performance when compared to the traditional asset allocation models. The results of this question were mixed as the REDD portfolio did provide a better Sharpe ratio than all other portfolios bar the FAREDD portfolio and the Maximum Sharpe Ratio portfolio. However, the maximum drawdowns the REDD portfolio experienced and the max REDD's experienced by the portfolio were much higher than expected with the drawdown breaching the 20% limit easily and reaching close to 28%, resulting in the REDDp portfolio experiencing larger drawdowns and REDD's than the MVP, FAREDD and Risk Parity portfolios. However, during the financial crisis period the REDDp portfolio provided the second strongest Sharpe Ratio behind the mean-variance portfolio while providing the best cumulative return of all of the portfolios achieving close to 100% return over the period. However, its REDD levels and maximum drawdown during the period was surpassed by both the MVP and FAREDD model over the course of the period. Therefore, if one was to consider the performance of the REDD model in terms of standard deviation and drawdown it outperformed 2 of the 3 traditional portfolios during both the financial crisis and over the entire test period. However, if one considered risk adjusted return, cumulative return and the rate at which the portfolio recovered from downturns as the main elements of performance metrics, the portfolio would be considered as one of the better performing. This is the same for when one considers the performance of the FAREDD model whereby over the long term and across all metrics being considered across returns, downside risk measures and volatility it could be seen to perform the best at meeting the requirements of this study to maximise returns while protecting the portfolio. However, if one was to solely consider downside metrics, i.e.: standard deviation and drawdowns the FAREDD model would be inferior to the minimum variance portfolio.

Therefore, as with most asset management decisions the strategy selected needs to be aligned with the investors risk appetite and requirements. Therefore, there is no conclusive evidence to indicate that the FAREDD or the REDD model outperforms all other models over all time periods. This means that the model that would be considered most optimal will be highly dependent on an investors risk profile and risk-return trade-off that they are willing to make.

With that being said, the REDDp and FAREDD strategies have been proven to provide competitive performance to traditional models and can thus be considered as relevant alternative asset allocation methodologies with the FAREDD model proving to be superior over the REDDp portfolio over the long term.

5.2: LIMITATIONS

There are several limitations to the model that need to be addressed. The most critical of which to mention is the fact that the model, by its very nature is retrospective. It relies on historical information in order to calculate asset weights and exposures. This in turn means that a fall would have already taken place and there is a delay before the model moves resources out of the risky assets into a risk free. This therefore means that, this model would not be optimal in a highly volatile market with sharp drops and recoveries – for example the crypto currency market – where large shifts can happen very quickly within a day as the model would miss a large portfolio these sudden changes.

The second important limitation of the model that one must be aware of is that it makes two major assumptions; firstly that one is able to short sell large quantities of any of the assets at any one time while also assuming a liquid market. What is meant by this is that the model assumes that during a sudden market downturn or fall, one is able to find a willing buyer for the assets in the portfolio when moving resources out of the risky asset and into the risk free asset. For large portfolios and for portfolios that hold assets that become bad – for example portfolios that held collateralised debt instruments in 2009 – it is sometimes a case that one cannot find buyers for their assets. Meaning that in these cases, even with the model, the portfolio may experience large losses in wealth and significant drawdown levels.

Finally, this model does not factor in the effect of brokerage or transaction fees which would reduce the excess returns and post fee Sharpe ratios of the various portfolios.

5.3: AREAS OF FURTHER STUDY

There are several areas of further study that one could use the FAREDD or REDDp model as a base. Firstly, one could look at adjusting the model to incorporate semi-deviation rather than standard deviation in the allocation formula, this would result in the model focusing more on downside deviation rather than penalising both upside and downside movements and may help make the portfolio more resistant to downturn movements.

In addition to this, another area that the model could be extended to is utilising the model by using investible ETF's as the various asset classes in order to increase diversification and the ease of investment while also having the implication of reducing transaction costs.

It is important to note that due to the nature of short sales being a crucial element of the FAREDD model, the comparable REDDp model was developed to allow for short sales. The performance of a traditional REDDp model that is long only in a South African marketplace or another developing economy is still an area in which research could be undertaken.

In addition to this, this study used the Sharpe ratio as the main metric to evaluate risk-adjusted return, however one could develop a new metric, that is testable, that studies the performance of the portfolio by viewing risk as drawdown rather than standard deviation and thus evaluates return compared to drawdown of a portfolio.

APPENDIX:

A: Step by Step Process Explanation of the FAREDD model

Step 1: Calculate the Rolling Economic Drawdown (REDD) by using formulas 7 and 8 of this study for each asset over the entire time period in excel. A lookback period of 252 trading days as a rolling window and the average risk-free rate for that rolling period was used.

Step 2: Import the asset Total Returns Prices Data (TRT) into R Studio for the entire period as well as the REDD data as calculated in Step 1 for each individual asset

Step 3: Calculate the TRT percentage returns and convert along with the REDD data into a time series in R

Step 4: Perform a PCA analysis on the asset class returns for the test period of 252 days

Step 5: Extract the eigenvalues from the PCA analysis that will act as factor loadings

Step 6: Using the formula as developed by Meucci (2010) – equation 11 of this study – what the returns would have been for each principle component for each day in the 252day test period. For short positions taken in each PC, it was assumed that the short positions would be invested at the risk-free rate for the period.

Step 7: Calculate the cumulative returns, standard deviation, resulting sharpe ratio and REDD for each principle component for the test period of 252 days

Step 8: Using this information calculate the initial portfolio weights by using the REDDp asset weights allocation equation as defined in equation 10 of this study

Step 9: Calculate the weight allocated to the risk-free asset by subtracting the weights of each asset calculated in Step 8 from 1. Inserted a leverage limit so that if the amount allocated to the risk free rate was less than 0 then the allocations would be proportionally adjusted to net out to 0. This was done by adding all the weights and dividing each weight by the cumulative total to proportionally adjust each PC's allocation. These weights then form the FAREDD portfolio.

Step 10: Create a loop that will move by a value of (x) where x represents the period of time, in days, one wishes to rebalance the portfolio

> Step 10.1: Calculate the daily returns data for the portfolio for the period of (x) days, using the asset allocation weights as found in step 8 and 9 and store these returns

Step 10.2: Repeat steps 3 to 9 having moved the rolling 252 day period forward by (x) days until the loop stops at the end of the period

Step 11: Save and export the resultant data into excel to analyse further

B: Code Used in R

```
##################INSTALLING ADDITIONAL PACKAGES#################
install.packages("xlsx")
library(xts)
library(xlsx)
library(PerformanceAnalytics)
library(PortfolioAnalytics)
library(DEoptim)
library(ROI)
library(ROI.plugin.glpk)
library(ROI.plugin.quadprog)
library(synchrony)
library(quantmod)
library(lattice)
library(factoextra)
library(rio)
library(data.table)
library(tawny)
library(quadprog)

###################IMPORTING DRAWDOWN DATA AND FORMULAS ########
attach()
data=(REDDP_H_1year_)
attach(RiskFree)
attach(Data)
attach(Tot_R)

#################INITIAL VALUES ###################
```

```
x=2662

y=2914

a=1

b=252

c=a+252

DD=0.25

i=data[x:y,]

j=Data[x:y,]

k=RiskFree[x:y,]

s=Shorting_Asset_Returns_Daily[x:y,]


###########PCA ANALYSIS FOR DIFFERENT LENGTH PERIODS########


###INITIAL FAREDD PORTFOLIO###


#Calculating PC weights and PC portfolio returns#


prices=Tot_R[x:y,]

stocks<-xts(prices[,-1],order.by=as.POSIXct(prices$Date)) #converting prices to a time-series

SR<-CalculateReturns(stocks,method = c("discrete"))

SR[is.na(SR)]<-0

SAR<-xts(s[,-1],order.by=as.POSIXct(s$Date)) #Converting daily short sale returns to a time-series

SAR[is.na(SAR)]<-0

SRM<-t(SR) #Transposing Returns into a matrix

rf<-colMeans(k,na.rm=FALSE,dims=1) #Calculating the average risk free rate for the period


pca<-prcomp(SR) #Calculating Principle Components on drawdowns


fviz_eig(pca,xlab="Principle Components") #ScreePlot of eigenvalues

Eig<-get_eigenvalue(pca) #eigen values and variance explained for each PCA
```

```
evec<-pca$rotatio[] #Extracting loadings per PC

inv.evec<-solve(evec) #Inverse of the eigenvectors

inv.evecP<-ifelse(inv.evec>0,inv.evec,0) #Only positive factor loadings

inv.evecN<-ifelse(inv.evec<0,abs(inv.evec),0) #Only negative factor loadings


PC.Return.Pos<-inv.evecP%*%SRM

PC.Return.Neg<-inv.evecN%*%t(SAR)

PC.Return<-PC.Return.Pos+PC.Return.Neg

PC.Return<-t(PC.Return)

Cum_PC.Return<-colSums(PC.Return)


Eigenvalue<-Eig$eigenvalue #Correlates to variance of PC's


StdDev<-StdDev.annualized(PC.Return)


SR.PC<-(Cum_PC.Return-(rf))/(StdDev) #Calculating the rolling sharpe ratio for the period


#PC Portfolio REDD's#


PC.REDDp_Pos<-t(inv.evecP%*%t(i)) #Calculating the drawdowns if factors are not short

PC.REDDp_Pos<-PC.REDDp_Pos[a:a,] #Selecting drawdowns as rebalance takes place

PC.REDDp_Pos<-as.matrix(PC.REDDp_Pos)


stocks_Beg<-stocks[a:a,] #Stock Prices at begining of the period

stocks_Beg<-as.matrix(stocks_Beg)

stocks_End<-stocks[(a+b),] #Stock Prices at rebalance day

stocks_End<-as.matrix(stocks_End)


stocks_DD<-(1-(stocks_Beg*(1+rf)^(b/252))/stocks_End) #Calculating Drawdowns if was short the
entire period

stocks_DD<-ifelse(stocks_DD<0,0,stocks_DD) #Eliminate neg drawdown values
```

```
stocks_DD<-as.matrix(stocks_DD)

PC.REDDp_Neg<-inv.evecN%*%t(stocks_DD)
PC.REDDp<-PC.REDDp_Pos+PC.REDDp_Neg

#PC Portfolio Allocations#

denom<-(1-PC.REDDp)
numer<-(DD-PC.REDDp)
f1<-1/(1-DD^(2))

DFc<-f1*(numer/denom)#Calculating part 2 of allocation equation
DFc<-ifelse(DFc>0,DFc,0)

RS.Adj<-(SR.PC/StdDev)+(1/2)#Calculating part 1 of allocation equation

PCAllocation<-RS.Adj*t(DFc)
rfAllocation<-(1-rowSums(PCAllocation))
if(rfAllocation < 0){
  Adjustweights<-(PCAllocation/rowSums(PCAllocation))
  PCAllocation<-Adjustweights*1
  rfAllocation<-1-rowSums(PCAllocation)
} else {
  rfAllocation
} # Leverage constraint of 1x of equity value

AssetAllocation=PCAllocation%*%t(evec)



###Portfolio Performance - REBALANCING as per b###
b=21 #Change as required for daily monthly weekly and yearly
```

```
z<-0
return.port=NULL
rfAllocations=NULL
Dates=NULL
AssetWeights=NULL
f=b-1

while(x>0){
 x=x-b
 y=y-b
 z<-1+z
 i=data[x:y,]
 j=Data[x:y,]
 k=RiskFree[x:y,]
 s=Shorting_Asset_Returns_Daily[x:y,]

 prices=Tot_R[x:y,]
 stocks<-xts(prices[,-1],order.by=as.POSIXct(prices$Date)) #converting prices to a time-series
 SR<-CalculateReturns(stocks,method = c("discrete"))
 SR[is.na(SR)]<-0
 SAR<-xts(s[,-1],order.by=as.POSIXct(s$Date)) #Converting daily short sale returns to a time-series
 SAR[is.na(SAR)]<-0
 SRM<-t(SR) #Transposing Returns into a matrix
 rfOld<-rf

 PC.Return.Pos2<-inv.evecP%*%SRM
 PC.Return.Neg2<-inv.evecN%*%t(SAR)
 PC.Return2<-PC.Return.Pos2+PC.Return.Neg2
 Performance_PC2<-PCAllocation%*%PC.Return2
 Performance_rf2<-rfAllocation*((1+rfOld)^(1/252)-1)
 Performance_Port2<-Performance_PC2+Performance_rf2
```

```
Performance_Port2<-t(Performance_Port2)
returns<-Performance_Port2[(253-f):253,]
Date<-prices$Date
Date<-Date[1]
return.port=append(return.port,returns)
rfAllocations=append(rfAllocations,rfAllocation)
Dates=append(Dates,Date)
AssetAllocation=PCAllocation%*%t(evec)
AssetWeights=append(AssetWeights,rbind(AssetAllocation))

print(AssetWeights)

pca<-prcomp(SR) #Calculating Principle Components on stock prices

evec<-pca$rotatio[] #Extracting loadings per PC
inv.evec<-solve(evec) #Inverse of the eigenvectors
inv.evecP<-ifelse(inv.evec>0,inv.evec,0) #Only positive factor loadings
inv.evecN<-ifelse(inv.evec<0,abs(inv.evec),0) #Only negative factor loadings

rf<-colMeans(k,na.rm=FALSE,dims=1) #Calculating the average risk free rate for the period

PC.Return.Pos<-inv.evecP%*%SRM
PC.Return.Neg<-inv.evecN%*%t(SAR)
PC.Return<-PC.Return.Pos+PC.Return.Neg
PC.Return<-t(PC.Return)
Cum_PC.Return<-colSums(PC.Return)

StdDev<-StdDev.annualized(PC.Return)

SR.PC<-(Cum_PC.Return-(rf))/(StdDev) #Calculating the rolling sharpe ratio for the period
```

```
#PC Portfolio REDD's#

PC.REDDp_Pos<-t(inv.evecP%*%t(i)) #Calculating the drawdowns if factors are not short
PC.REDDp_Pos<-PC.REDDp_Pos[a:a,] #Selecting drawdowns as rebalance takes place
PC.REDDp_Pos<-as.matrix(PC.REDDp_Pos)

stocks_Beg<-stocks[a:a,] #Stock Prices at begining of the period
stocks_Beg<-as.matrix(stocks_Beg)
stocks_End<-stocks[(a+b),] #Stock Prices at rebalance day
stocks_End<-as.matrix(stocks_End)

stocks_DD<-(1-(stocks_Beg*(1+rf)^(b/252))/stocks_End) #Calculating Drawdowns if was short the
entire period
stocks_DD<-ifelse(stocks_DD<0,0,stocks_DD) #Eliminate neg drawdown values
stocks_DD<-as.matrix(stocks_DD)

PC.REDDp_Neg<-inv.evecN%*%t(stocks_DD)

PC.REDDp<-PC.REDDp_Pos+PC.REDDp_Neg

#PC Portfolio Allocations#

denom<-(1-PC.REDDp)
numer<-(DD-PC.REDDp)
f1<-1/(1-DD^(2))

DFc<-f1*(numer/denom)#Calculating part 2 of allocation equation
DFc<-ifelse(DFc>0,DFc,0)

RS.Adj<-(SR.PC/StdDev)+(1/2)#Calculating part 1 of allocation equation
```

```
  PCAllocation<-RS.Adj*t(DFc)
  rfAllocation<-(1-rowSums(PCAllocation))
  if(rfAllocation < 0){
    Adjustweights<-(PCAllocation/rowSums(PCAllocation))
    PCAllocation<-Adjustweights*1
    rfAllocation<-1-rowSums(PCAllocation)
  } else {
    rfAllocation
  }
}

pc=data.frame(Dates,return.port,rfAllocations)
AssetW=data.frame(Dates,AssetWeights)
write.xlsx(pc,file="Model.Returns.xlsx")
write.xlsx(AssetW,file="ModelWeights.xlsx")

StdDev.annualized(return.port)
Return.annualized(return.port)
################## Compareable Portfolios #####################
################REDDp - No Short sale constraint###############
#####Initial Risky Asset allocation###########
x=2662
y=2914
a=1
b=252
c=a+252
DD=0.20
v=1
p=252
i=data[x:y,]
j=Data[x:y,]
```

```
k=RiskFree[x:y,]
s=Shorting_Asset_Returns_Daily[x:y,]

prices=Tot_R[x:y,]
stocks<-xts(prices[,-1],order.by=as.POSIXct(prices$Date)) #converting prices to a time-series
SR<-CalculateReturns(stocks,method = c("discrete"))
SR[is.na(SR)]<-0
SAR<-xts(s[,-1],order.by=as.POSIXct(s$Date)) #Converting daily short sale returns to a time-series
SAR[is.na(SAR)]<-0
SRM<-t(SR) #Transposing Returns into a matrix
rf<-colMeans(k,na.rm=FALSE,dims=1) #Calculating the average risk free rate for the period

StdDv<-StdDev.annualized(t(SRM))
VolM<-var(t(SRM))
CovM<-cov(t(SRM))
CumR<-rowSums(SRM)# Cumulative return
CumR<-data.matrix(CumR)
F1<-CumR-rf
F2<-t((StdDv^2)/2)
RDV<-F1-F2 # Return Drift vector for the period

P1<-((1/VolM)%*%RDV)^P
P1<-(1/(VolM))%*%P1
P2<-1/(1-(DD^2))
P3<-(DD-data[x:x,])/(1-data[x:x,])
P3<-as.matrix(P3)
Y<-P2%*%P3
Y<-c(Y)
Y<-diag(Y)

REDPpWeights<-Y%*%P1
```

```
REDDpWeights<-(REDPpWeights/sum(REDPpWeights))
rfAllocation<-(1-rowSums(REDDpWeights))
if(rfAllocation < 0){
  Adjustweights<-(REDDpWeights/rowSums(REDDpWeights))
  REDDpWeights<-Adjustweights*1
  rfAllocation<-1-rowSums(REDDpWeights)
} else {
  rfAllocation
}
###########REDDp Simulation #####################
b=21 #Change as required for daily monthly weekly and yearly rebalance
z<-0
p=
REDDP.Returns=NULL
rfAllocations=NULL
Dates=NULL
f=b-1

while(x>0){
  x=x-b
  y=y-b
  p=p+b
  z<-1+z
  i=data[x:y,]
  j=Data[x:y,]
  k=RiskFree[x:y,]
  s=Shorting_Asset_Returns_Daily[x:y,]

  prices=Tot_R[x:y,]
  stocks<-xts(prices[,-1],order.by=as.POSIXct(prices$Date)) #converting prices to a time-series
  SR<-CalculateReturns(stocks,method = c("discrete"))
```

```
SR[is.na(SR)]<-0
SAR<-xts(s[,-1],order.by=as.POSIXct(s$Date)) #Converting daily short sale returns to a time-series
SAR[is.na(SAR)]<-0
SRM<-t(SR) #Transposing Returns into a matrix
rf<-colMeans(k,na.rm=FALSE,dims=1) #Calculating the average risk free rate for the period

REDDpReturns<-t(SRM)%*%REDDpWeights
REDDpReturns<-REDDpReturns[(253-f):253,]
Date<-prices$Date
Date<-Date[1]
REDDP.Returns=append(REDDP.Returns,REDDpReturns)
rfAllocations=append(rfAllocations,rfAllocation)
Dates=append(Dates,Date)

StdDv<-StdDev.annualized(t(SRM))
VolM<-var(t(SRM))
CovM<-cov(t(SRM))
CumR<-rowSums(SRM)# Cumulative return
CumR<-data.matrix(CumR)
F1<-CumR-rf
F2<-t((StdDv^2)/2)
RDV<-F1-F2 # Return Drift vector for the period

P1<-((1/VolM)%*%RDV)^p
P1<-(1/(VolM))%*%P1
P2<-1/(1-(DD^2))
P3<-(DD-data[x:x,])/(1-data[x:x,])
P3<-as.matrix(P3)
Y<-P2%*%P3

Y<-c(Y)
```

```
  Y<-diag(Y)

  REDPpWeights<-Y%*%P1
  REDDpWeights<-(REDPpWeights/sum(REDPpWeights))
  rfAllocation<-(1-rowSums(REDDpWeights))
  if(rfAllocation < 0){
   Adjustweights<-(REDDpWeights/rowSums(REDDpWeights))
   REDDpWeights<-Adjustweights*1
   rfAllocation<-1-rowSums(REDDpWeights)
  } else {
   rfAllocation
  }
}
pc=data.frame(Dates,REDDP.Returns)
write.xlsx(pc,file="REDDp.xlsx")
###################### MVP, RISK PARITY, MAX-SHARPE ######################
Prices2=Tot_R
St<-xts(Prices2[,-1],order.by=as.POSIXct(Prices2$Date)) #converting prices to a time-series
Rtn<-CalculateReturns(St,method = c("discrete"))
Rtn[is.na(Rtn)]<-0

#initial portfolio#

funds<-colnames(Rtn)
head(funds)
port1<-portfolio.spec(assets=funds)
port1

port1=add.constraint(portfolio=port1, type ="leverage", min_sum=0.99,max_sum=1.01)
port1=add.constraint(portfolio=port1,type="box",min=-1,max=1)
por1=add.constraint(portfolio=port1,type="leverage_exposure", leverage=NULL)
```

```
### Min variance ###

port1=add.objective(portfolio=port1,type="risk",name="StdDev", arguments=list(p=0.95),
min_conentration=TRUE)

min_risk = optimize.portfolio.rebalancing(R=Rtn,portfolio=port1,
                              rebalance_on ="months",
                              optimize_methos="ROI",
                              training_period=252)

print.default(min_risk)

min_risk

weights_min_risk=extractWeights(min_risk)
min_risk_ret=Return.rebalancing(Rtn,weights=weights_min_risk,rebalance_on = "months")

charts.PerformanceSummary(R=min_risk_ret,main = "Min Risk Performance")
chart.TimeSeries(R=min_risk_ret)
chart.Weights(min_risk,main="Actual Weights")

write.xlsx(min_risk_ret,file="MVPModel.Returns.xlsx")

StdDev.annualized(min_risk_ret)
Return.annualized(min_risk_ret)

### Max Sharpe Ratio ###

port2<-portfolio.spec(assets=funds)
```

```
port2=add.constraint(portfolio=port2, type ="leverage", min_sum=0.99,max_sum=1.01)
port2=add.constraint(portfolio=port2,type="box",min=-1,max=1)
port2=add.objective(portfolio=port2,type="return",name="mean", maxSR=TRUE)
port2=add.objective(portfolio=port2,type="risk",name="StdDev")

max_SR=optimize.portfolio.rebalancing(R=Rtn, portfolio=port2,
                    rebalance_on = "months",
                    training_period = 252,
                    optimize_method = "ROI")

print.default(max_SR)

weights_max_SR=extractWeights(max_SR)
maxSR_ret=Return.rebalancing(Rtn,weights=weights_max_SR,rebalance_on = "months")
chart.Weights(max_SR,main="Actual Weights")


StdDev.annualized(maxSR_ret)
Return.annualized(maxSR_ret)


write.xlsx(maxSR_ret,file="Max_SR_Model.Returns4.xlsx")

### Risk Parity ###

port3<-portfolio.spec(assets=funds)
port3=add.constraint(portfolio=port3, type ="leverage", min_sum=0.99,max_sum=1.01)
port3=add.constraint(portfolio=port3,type="box",min=0,max=1)
por3=add.constraint(portfolio=port3,type="leverage_exposure", leverage=NULL)
```

```
port3=add.objective(portfolio=port3,type="risk_budget",name="StdDev", arguments=list(p=0.95),
min_conentration=TRUE)

port3=add.objective(portfolio=port3,type="risk_budget",name="StdDev",
arguments=list(p=0.95),min_concentration=TRUE)

RiskParity=optimize.portfolio.rebalancing(R=Rtn, portfolio=port3,
                              rebalance_on = "months",
                              optimize_method ="DEoptim",
                              training_period=252,trace=TRUE,
                              itermax=999,
                              seachsize=4000)

print.default(RiskParity)

weights_RiskParity=extractWeights(RiskParity)
RiskParity_Return=Return.rebalancing(Rtn,weights=weights_RiskParity,rebalance_on = "months")

write.xlsx(RiskParity_Return,file="RiskParity_Model.Returns.xlsx")
chart.Weights(RiskParity,main="Actual Weights")

extractStats(RiskParity)

StdDev.annualized(RiskParity_Return)
Return.annualized(RiskParity_Return)
```

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