# ESTIMATING VALUE AT RISK AND EXPECTED SHORTFALL: A KALMAN FILTER APPROACH

by

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# ABSTRACT

Calculating Value-at-Risk (VaR) to estimate the maximum loss a portfolio may incur at a given confidence level and over a specified time has undergone several adaptations, iterations, and additions since its inception in 1994. In 2013, the Basel Committee on Banking Supervision (BCBS) replaced VaR with Expected Shortfall (ES), or Conditional VaR (CVaR), as the new primary measure for banking institutions to forecast market risk and hence allocate the relevant amount of regulatory market risk capital. ES measures the probability weighted losses beyond VaR, so VaR remains a crucial step in its computation and retains its significance in estimating market risk and associated measures.

A Kalman filter is used for the first time to estimate both VaR (and ES) to provide an alternative technique to existing industry methods. Modelling the volatility of asset returns as a stochastic process, the Kalman filter uses Bayesian statistics to forecast unobservable data by identifying underlying patterns required to predict future values. Back-testing results (in which the number of times VaR or ES forecasted too low a value to cover the following day's market loss is compared with the prescribed confidence level) indicate that the Kalman filter is a reliable and robust contender in the volatility framework milieu, outperforming GARCH, EWMA and equally weighted measures of volatility in both volatile and calm market conditions.



**Keywords:** Kalman filter, Value-at-Risk, Expected Shortfall

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# 1. INTRODUCTION

Risk which arises from the activities of any given institution has historically required the use of metrics such as VaR and ES (or CVaR) to effectively identify and evaluate the potential losses a portfolio may face over a given time horizon and at a given confidence level. VaR defines the *minimum* expected loss for a given portfolio under normal market conditions, resulting in a *frequency* measure for losses beyond a certain confidence interval. Using a hypothetical daily portfolio VaR of $1 million at a 99% confidence level, there is a 99% chance that the portfolio will not exceed $1 million in losses for the specified day. Calculating the ES for the same portfolio requires a few additional steps to quantify the magnitude of losses beyond the VaR threshold. Put simply, it provides an average of the losses that exceed the loss level, at a prescribed confidence, determined by VaR (Acerbi & Tasche, 2002).

The application of accurate VaR estimates, particularly in banking institutions and investment firms, is of considerable importance. Capital allocation for market risk may be misaligned if the underlying risk is not adequately estimated, jeopardising the stability of institutions that rely on these metrics. Several approaches, both non-parametric and parametric, have been established to better understand and manage the varying forms of risk (credit, operational, liquidity, and market) a firm may face (Manganelli & Engle, 2001). Economic reforms are often witnessed in a country, and markets can be susceptible to internal and external shocks, such as currency movements, credit rating changes, inflation, and shifts in risk premiums. Higher volatility during turbulent periods can result in financial returns having more distorted distributions than normal, making it difficult to assess VaR using standard methods (Miletic & Miletic, 2015). Krause and Tse (2016) highlight how recent empirical findings corroborate the earlier theoretical claims in existing literature that risk management leads to increased firm value and returns, while simultaneously decreasing return and cash flow volatility.

Although VaR has gained significant popularity in modern finance, it is plagued by some limitations and assumptions. ES has emerged as a preferred risk measurement tool in certain scenarios, offering advantages over (but still dependent upon) its progenitor. In 2013, the BCBS replaced VaR with ES as the new primary measure for banking institutions to forecast market risk (BCBS, 2013). The estimation of VaR and ES can be accomplished using various methods, each varying in popularity and complexity. In this essay, a selection of these popular methods implemented in modern financial markets will be referred to and matched against the Kalman filter (Kalman, 1960), an algorithm providing estimates of unknown variables and unobservable parameters through dynamic system estimation. While originally applied primarily in the field of engineering, the Kalman filter has more recently found applications in finance and economics. It has exhibited competence in estimating various factors, such as inflation expectations, commodity futures prices, and hedge ratios for interest rate contracts (Arnold, Bertus & Godbey, 2008). The objective is for the Kalman approach to serve as an alternative, and potentially more effective, method for financial analysts to quantify market risk.

The literature governing the application of the Kalman filter to financial risk is relatively scarce as the approach is still reasonably novel. This work is one of the first to provide robust, extensive results of comprehensive back-testing. Most similar to this study was that of Berardi, Corradin, and Sommacampagna (2002), who used the Kalman filter to calculate VaR by estimating portfolio $\beta$s, treating the $\beta$ parameter as if it were unobservable and followed a first order autoregressive process. Berardi et al (2002) determined a sequence of values for $\beta_{i,t}$ and these were used to calculate the portfolio VaR. More recently, Das (2019) assessed modified adaptive Kalman filters to gauge their effectiveness in market risk $\beta$ and VaR estimation. The study revealed that the filters demonstrated comparable performance to the selected benchmark, even when considering the adaptive noise covariance assumptions.

In this research, volatilities of asset returns are modelled as a stochastic process and the Kalman filter – using Bayesian statistics to forecast unobservable data by identifying underlying patterns required to predict future values – then forecasts these. Back-testing estimates have provided encouraging results, indicating that the Kalman filter is a reliable and robust contender in the volatility framework milieu, outperforming varying measures of volatility in both volatile and calm market conditions.

The remainder of this essay proceeds as follows: Section 2 reviews the literature surrounding existing VaR and ES estimation methods, the Kalman filter, and its application in financial risk management. Section 3 sets out the underlying data used in the essay and provides a summary of the relevant mathematics used to estimate VaR and ES as well as a detailed description of the workings of the Kalman filter. Section 4 presents and discusses the results of the subsequent analysis while Section 5 provides recommendations for further research and concludes.

## 2. LITERATURE REVIEW

Precise quantification of overall risk for a given institution or portfolio exposed to several systematic and non-systematic influences can be challenging. The coverage of risk detection in financial markets has historically focused on broad statistical concepts in standard deviation (or variance). Since its introduction by J.P. Morgan for the first edition of its RiskMetrics programme in 1994 (J.P. Morgan, 1996), VaR has gone through several adaptations, iterations, and additions in its rise to prominence as a suitable risk management tool. The tool's effectiveness has been previously validated by its ability to consolidate various components of market risk within a firm into a single quantitative measure. This attribute received substantial endorsement from industry and regulatory bodies, particularly in the late 1990s when the methodologies associated with the tool first became widely accessible (Marshall & Siegel 1997).

Early contributions to VaR can be attributed to Markowitz (1952) and Roy (1952), who both emphasised the incorporation of covariances among risk factors to reflect diversification and hedging effects. However, due to limited processing power during subsequent decades, VaR remained primarily a theoretical concept. It was only when financial institutions began adopting VaR as a routine tool for assessing market risk and establishing risk limits that the Markowitz (1952) methodology gained widespread usage. In the late 1980s, J.P. Morgan developed RiskMetrics, a system capable of modelling numerous risk factors and employing various VaR metrics (J.P. Morgan, 1996). Prior to the introduction of VaR, commercial banks typically assessed risk on a departmental basis rather than evaluating the overall exposure of the entire company (Chen, 2014).

Despite its widespread adoption in industry, VaR has been subject to scrutiny since its inception due to the identification of certain limitations and flaws. Artzner, Delbaen, Eber, and Heath (1999) and Acerbi and Tasche (2002) have previously questioned the viability of VaR, citing its lack of coherence as a risk measure. The primary argument was that there will consistently be a probabilistic chance of an extreme event taking place which falls a significant distance away from the estimate that VaR produces. This implied that VaR should not be relied on as a sole risk management tool.

In addition to subadditivity, three requirements for a coherent risk measure were defined by Artzner et al. (1999) as homogeneity, monotonicity, and translation invariance. *Homogeneity* implies a proportional level of risk relative to size; for instance, if the number of future contracts is doubled or trebled, the margin requirement doubles or triples as well. This is primarily due to factors such as the risk of liquidation in larger asset positions, where an increase in size corresponds to a commensurate rise in the level of risk. *Monotonicity* dictates that if a given portfolio (X) has a future value that exceeds another portfolio's (Y) future value, then a 'monotonic' risk measure will be lower for portfolio X than for Y, indicating that portfolio Y is riskier. *Translation invariance* refers to the proportionate decrease in risk as a certain quantity of cash, or a risk-free asset, is added to a portfolio.

VaR generally does *not* meet the *subadditivity* requirement, which states that a combination of the given risks for hypothetical assets A and B ultimately will not lead to an aggregate risk that is higher than the total of the individual risks (Daníelsson, Jorgensen, Samorodnitsky, Sarma, & de Vries, 2012). This is not the case with VaR, ultimately discouraging diversification. Practical implications of VaR not meeting the subadditivity requirement have been highlighted in the management of credit portfolio risk, for example. Credit instruments frequently feature 'fat' tails in their return distributions and generally exhibit asymmetric return characteristics related to default risk – there may be a higher concentration of credit risk due to VaR's inability to meet the subadditivity requirement (Albanese, 1997).

The limitations of VaR were tolerated, despite being acknowledged, until certain events such as the credit crisis between 2007-2008 highlighted the understatement of potential losses prior to the crash. As a result, these shortcomings could no longer be overlooked, leading to the introduction of ES in the "Fundamental Review of the Trading Book" overhaul by the BCBS (BCBS, 2013). The ability of ES to evaluate tail risk and its proven subadditivity are noted as reasons for its emergence as a preferred risk measurement tool (in certain scenarios) over VaR. For any given portfolio, ES measures the probability weighted losses *beyond* VaR (Taylor, 2019). Then, by definition, VaR remains a crucial step in its computation and retains its significance in estimating market risk and associated measures. When focusing on methods with asymmetrical risk profiles, such as writing option contracts, ES efficiently captures the lowest possibility of suffering larger losses than what is expected. VaR, on the other hand, erroneously lowers risk estimations since it overestimates the size of prospective losses for such methods (Jorion, 2007).

Despite its limitations, VaR is often preferred over sub-additive risk measures such as ES by both industry and regulators in the banking sector due to its practical benefits, which include smaller data requirements, ease of backtesting, and, in some cases, ease of calculation (Daníelsson et al., 2012). Orhan and Köksal (2012) contend that despite research highlighting the lack of sub-additivity and convexity in VaR, the measure is still the most effective way to quantify risk. More recently, the emergence of complex financial derivatives and associated volatilities has called for the development of an indicator capable of handling the highly unpredictable nature of these regularly changing products (Adamko, Spuchľáková & Valášková, 2015).

There is no singular approach that banking institutions are encouraged to adopt for estimating VaR, primarily because research has not identified an optimal method that outperforms others on a consistent basis for conducting such estimates. The BCBS does not prescribe any particular method and favour each of three mentioned approaches (variance-covariance matrices, historical simulations, or Monte Carlo simulations) equally. According to the Basel II Capital Accord, banking institutions may use any model, so long as each model implemented can capture all the material risks faced by the company (BCBS, 1996).

These approaches, along with others that have been developed more recently, differ in terms of their computational and modelling complexity. This has led to trade-offs between methods that may offer optimal performance but require more resources to implement. Among the most popular methods to estimate VaR are the historical, Variance-Covariance (VCV), and Generalised AutoRegressive Conditional Heteroskedasticity (GARCH) approaches. Within the VCV framework, two common methods for estimation include the Equally Weighted (VCV EW) and Exponentially Weighted Moving Average (VCV EWMA) approaches. Monte Carlo simulation, a well-recognised method, was not included in this study as it does not replicate the actual distribution of risk factors for a portfolio or index. The use of Monte Carlo simulation primarily relates to portfolios characterised by a significant concentration of derivatives, wherein the absence of pre-existing historical data necessitates the generation and simulation of historical prices. For instance, when considering an Over the Counter (OTC) derivative contract executed between two counterparties, the absence of a recorded price history necessitates the simulation of plausible historical price scenarios.

Effective measurement of the performance of the Kalman filter in estimating VaR must consider the existing methodologies, as well as their associated assumptions and logical flaws.

### 2.1 Historical Estimation

Historical estimation represents the simplest method of chosen approaches, assuming prices of assets behave in a similar manner to what has been witnessed in the past (Sharma, 2012). Current weights are applied to a time-series of historical asset returns, focusing on reconstructing the history of a hypothetical portfolio based on its current position (Adamko et al., 2015). The simplicity of the historical approach, its ability to easily incorporate stress scenarios, the logical time horizon measurement period (based on length of holding time), and the omission of standard deviation or correlation requirements (stemming from an empirical loss distribution rather than an imposed one) have established the method as a compelling choice in industry. However, several key limitations are evident – older returns which are potentially irrelevant to the context of the current market are weighted the same as recent returns, and there is an increased requirement for historical data coupled with an inability to isolate short term data in contrast with other methods. Underlying changes in implicit volatility can also take longer periods of time to be realised with this approach (Adamko et al., 2015).

To address the limitations of the historical approach, Žiković and Filer (2009) compared the effectiveness of VaR and ES models using a hybrid historical simulation. This analysis spanned the period before and after the 2008 financial crisis, encompassing both developed and emerging markets. The hybrid model employed a combination of nonparametric bootstrapping and parametric GARCH volatility forecasting. Through backtesting, it was determined that the hybrid approach offered equivalent protection to extreme value (EV) models, but with significantly lower capital reserve requirements. The hybrid approach was found to yield the smallest error statistics for ES, particularly in developed markets.

**2.2 VCV EW Estimation**

The general VCV method operates under the assumption that the risk factors influencing the portfolio's value follow a multivariate normal distribution. As a result, the fluctuations in the value of a linear portfolio follow a normal distribution (de Raaji & Raunig, 1999). This implies that the VaR output is a multiple of the standard deviation, and is given by:

$$VaR = -\alpha \sqrt{w' \sum w}$$

where $\alpha$ is a scaling factor representing a given confidence interval (usually 1.65 at 95% or 2.33 at 99%), $w$ and $w'$ denote a vector of absolute portfolio weights and its transpose respectively, and $\sum$ is a variance-covariance matrix (J.P. Morgan, 1996).

Compared to the historical approach and the previously mentioned Monte Carlo simulation, the VCV method offers a distinct advantage in allowing for the prediction of volatilities in financial returns (J.P. Morgan, 1996). Moreover, the method is straightforward to implement and has demonstrated satisfactory precision and accuracy, requiring fewer data compared to the historical approach.

To apply the VCV method, an approximation of the covariance matrix of the risk factors is required. In most cases, the variances (and covariances) are computed based on the daily historical time series of returns for the corresponding risk factors, employing equally weighted moving averages:

$$\sigma_{ijT}^2 = \sum_{t=T-n}^{T-1} \frac{r_{it} r_{jt}}{n}$$

where $\sigma_{ijT}^2$ is the variance (or covariance) at time $T$, $n$ is the number of observations, and $r_{it} r_{jt}$ are the corresponding risk factor returns.

### 2.3 VCV EWMA Estimation

VCV EWMA differs from VCV EW in that current values are weighted more heavily than past values. J.P. Morgan (1996) define the EWMA estimator in its recursive form by:

$$\sigma_{ij/t}^2 = \lambda\, \sigma_{ij/t-1}^2 + (1-\lambda) r_{it-1} r_{jt-1}$$

where $\lambda$ measures the declining weighting scheme of observations, $\sigma_{ijT}^2$ is the variance (or co-variance) at time $t-1$, and $r_{it-1} r_{jt-1}$ are previous day's returns. This weighting tilt allows for a faster reaction to market crashes or significant changes in a given economy.

For the stock prices of a well-known multinational firm, Galdi and Pereira (2007) investigated the effectiveness of VaR estimation techniques for VCV EWMA, GARCH, and stochastic volatility (SV) across a sampled 1 500-observation window. Relative to more ‘sophisticated’ methods in GARCH and SV, VSV EWMA did *not* produce inferior violation test results. Additionally, the model required less computational effort to implement.

Although the VCV methods (VCV EW and VCV EWMA) have straightforward implementation, nonlinear financial instruments such as derivatives containing non-normal distributions of profit or loss are problematic for VCV calculations (Best, 2000). If the underlying risk factors are not normally distributed, finding their associated distribution can be challenging.

### 2.4 GARCH Estimation

Like VCV EWMA, the GARCH approach is non-linear, yet differs through its accounting of asset volatility reverting to a long-term mean (Poon & Granger, 2003). The magnitude of standard deviation, a key component of VaR, is effectively tracked by GARCH in comparison to other models. The GARCH formulation, derived as a generalisation of the autoregressive conditional heteroscedasticity model (ARCH) was proposed by Bollerslev (1986) and is as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{u} \alpha_i \alpha_{t-i}^2 + \sum_{j=1}^{v} \beta_j \sigma_{t-j}^2$$

where the sum of $\alpha_i$ and $\beta_i$ determines how persistent 'shocks', or unexpected deviations, to volatility will be. A significant shock to volatility in a previous period (i.e., yesterday) raises the likelihood of a shock to volatility today, which is a helpful representation for the clustering in volatility that is typically seen in series of returns (Tsay, 2010). A limitation of this model is that it does not distinguish between the effects of positive and negative shocks, which frequently have different effects (Restrepo, 2012).

Similar to VCV EWMA, GARCH models imply serial correlations in the returns of financial assets. More recent results are favoured over earlier ones and as a result, both models estimate volatility based on the most recent return data (Best, 2000). So and Yu (2006) applied seven variations of the GARCH model to four foreign exchange rates and 12 market indices to evaluate the VaR levels at varying confidence intervals. The findings suggest that both fractionally integrated and stationary GARCH models are more effective than RiskMetrics in predicting VaR at a 1% level. Patten, Ziegel, and Chen (2019) looked to address the challenge of "elicitability" in ES estimation by employing a joint modelling approach that incorporates both VaR and ES. By applying this approach to daily returns on four international equity indices, it was observed that the joint model outperformed GARCH models in terms of forecasting accuracy.

**2.5 Kalman Filter Estimation**

The Kalman filter, proposed by Kalman (1960) and with origins in autonomous navigation processes and trajectory tracking, has more recently been applied to financial markets. In mathematical finance, the issue of estimating unobserved latent variables from observable market data regularly occurs. Calibrated to solve similar issues in engineering and econometrics, the Kalman filter is employed in applications for data smoothing as well as the construction of time series models for variable forecasting (Date & Ponomareva, 2010).

Berardi et al. (2002) established that VaR estimation using a Kalman methodology was both feasible and suitable. In a different manner to what will be presented in this essay, the authors employed a first-order autoregressive process to estimate portfolio $\beta$s in their approach. The sequence of values for $\beta_{i,t}$ could be estimated and the final $\beta_{i,T}$ was then calculated to derive the VaR for the portfolio. A portfolio comprising ten stocks traded on the Nasdaq stock market was examined. Initially, an equal percentage of investment was assumed for each asset, followed by a random simulation of 5 000 portfolio compositions. In both scenarios, the portfolio composition remained unchanged over time. Backtesting analysis revealed that the Kalman filter-based approach exhibited sensitivity to changes in market volatility, yielding notable and significant results.

Previous studies, such as the work conducted by Bernales, Beuermann, and Cortazar (2014), have demonstrated the effectiveness of the Kalman filter in calculating market risk measures. The Kalman methodology was applied to a thinly traded fixed income portfolio to assess its ability to provide appropriate risk measures for a market where the portfolio is traded infrequently. The methodology employed a three-stage process. Firstly, the Kalman filter was used to extract a complete price dataset, even in situations where there were only a few price observations available, allowing for prices to be estimated on days with limited price information. In the second stage, market risk measures, specifically VaR, were estimated using the complete price dataset obtained from the first stage. Finally, a back-test was conducted to verify the reliability of the Kalman approach in estimating the price model. The empirical evidence presented suggests that the Kalman filter approach provided reliable measures of VaR for securities that are traded infrequently. The Kalman approach also outperformed the conventional method of simply replicating the last traded price in calculating the chosen risk measure (Bernales, Beuermann, and Cortazar, 2014).

Date and Bustreo (2015) proposed the Kalman filter as a method to measure VaR for sovereign debt portfolios by simulating bond prices with a two-factor short rate model. The Kalman approach only required a simulation of a vector of two random variables for one-step ahead forecasts, resulting in computational "cheapness" in comparison to principal components analysis which utilises more than two principal components. The results indicated an arguably more transparent and accurate reflection of market conditions associated with highly liquid government securities.

Fundamental Sharpe ratios were estimated by Gatfaoui (2016) using a Kalman filter approach. The Sharpe ratio is a measure of risk-adjusted performance for a portfolio or individual security, defined as follows:

$$SR_a = \frac{E[R_a - R_b]}{\sigma_a}$$

where $R_a$ is the asset return, $R_b$ is the riskless asset return, and $\sigma_a$ is the asset's excess return standard deviation. Contrary to risk measures of *loss* in VaR and ES, this research focused on assessing the accuracy of risk-adjusted *performance* estimation using the Kalman filter. To account for the time variation, idiosyncratic risk, and market trend bias, the Sharpe ratios were adjusted into filtered Sharpe ratios (FSRs). The FSRs were designed to isolate the fundamental component of the Sharpe ratio on a time series basis. By applying the Kalman filter methodology, the time-varying FSRs were captured, thereby excluding any previous biases inherent in the metric. Thereafter, a comparative analysis of various modeling techniques was performed, including GARCH and Monte Carlo simulations. Equally weighted portfolios were constructed, incorporating the highest performing equities identified by each measure. The FSR portfolio, when compared to the comparable portfolios, demonstrated reduced VaR forecasts and higher expectations of gains. This research showcases the capabilities of the Kalman filter in extracting fundamental Sharpe ratios, which are free from bias and serve as pure performance indicators, distinct from the traditional Sharpe ratios.

Thomson and van Vuuren (2018) decomposed the time series of hedge fund returns into market timing and stock selection factors using the Kalman filter. Representing the first application of Kalman in this manner, the model was used to determine whether statistically significant abnormal profits are truly generated by hedge fund managers, in accordance with popular belief. Through an extension of the capital asset pricing model (CAPM) equation, stock selection, market timing, and market exposure components may be separated from hedge fund results. The authors conclude that one could use the Kalman filter, which employs Bayesian variance reduction, to get the parameters required for this enhanced CAPM. The paper found that top-performing hedge funds obtained the majority of their $\alpha$ from consistent stock selection and somewhat from market timing. These funds also showed less fluctuation in return. The worst-performing funds had variable market timing $\alpha$ and greater volatility, implying that attempts to time the market frequently cause volatility and reduce long-term returns.

Das (2019) employed an adaptive Kalman filter approach to effectively track and estimate the market risk $\beta$ and VaR in the Indian market. This approach did not rely on assuming the noise covariance (i.e., uncertainties). The adaptive Kalman filter demonstrated similar performance to an ordinary filter, reinforcing previous observations that sector $\beta$ estimates are dynamic and not constant in nature. Das (2019) presents recent findings that highlight the efficacy of utilising the mathematical principles of Kalman in accurately estimating VaR.

Van Rooyen and van Vuuren (2022) explored asset allocations using the Kalman filter, estimating $\alpha$ and $\beta$ parameters as they appear in the CAPM to forecast asset returns. Two approaches in Tactical Asset Allocation (TAA) and Strategic Asset Allocation (SAA) were examined in the paper. To forecast asset returns, TAA uses quantitative methods, notably the CAPM framework and estimations using the Kalman filter. By dynamically altering asset class weights based on the anticipated returns, this strategy seeks to enhance portfolio performance and risk characteristics. The results indicated that, when compared to a ‘static’ SAA allocation, the TAA strategy, which makes use of the Kalman filter and dynamic asset allocation, can improve portfolio performance and risk characteristics.

In the research explored by Claver, Dave, and Che (2023), the Kalman filter was employed to develop a dynamic system for predicting the price movements of a single equity. By simulating the equity's movement using the Kalman filter, future price levels were aimed to be forecasted with greater accuracy relative to more traditional approaches. Through the conditions of Kalman, it was discovered that the filtering method proved to be effective based on multiple error metrics.

# 3. DATA AND METHODOLOGY

## 3.1 Data

The data comprise daily returns from Bloomberg on the Dow Jones Industrial Average (DJIA), a price-weighted index of 30 publicly traded companies that are widely recognised, covering the period from January 2017 to April 2023. The selected timeframe spans roughly one business cycle (about seven years), encompassing the pre-Covid-19 period of relative financial stability and the subsequent Covid-19 period of turbulence. Other significant global events having recent influence on financial markets have been included, such as the Russian invasion of Ukraine, which began in February 2022 and continues at the time of writing (June 2023) with no resolution pending. The first year (2017) is employed to allow for a full year of daily returns and standard deviations to train and calibrate the Kalman model to estimate parameters required for the remaining (out of sample) period.

Daily returns are used to create a daily estimation of VaR, as recommended by the Bank for International Settlements (BIS), for back testing analyses focused on assessing the differences in a VaR model output and the selected portfolio value on an ex-post basis (BIS, 2019). Using a 99% confidence interval, the analysis measures whether daily losses beyond VaR are experienced 1% of the time - in accordance with the Basel II Capital Accord requirements. Rather than extracting data on individual equities, this approach used data on the prices of a *single* popular index (the DJIA). An index is frequently recalculated, and its composition varies constantly over time, which eliminates inactive equites and reduces the likelihood of survivorship bias in equity selection. To conform with its goals as an index, the DJIA also changes and redistributes weights accordingly.

Estimates were computed for the historical, VCV EW, VCV EWMA, GARCH, and Kalman filter methods.

### 3.2 Kalman Filter

The Kalman filter is a Bayesian updating method designed to optimise the accuracy of estimating unknown parameter values (Koch, 2006). This filter deals with the broader issue of estimating the state $[x \in \Re^n]$ of a discrete, time-controlled process that follows a linear stochastic difference equation as follows:

$$x_t = \boldsymbol{F}x_{t-1} + \boldsymbol{B}u_{t-1} + w_{t-1} \tag{1}$$

with a measurement $[z \in \Re^n]$:

$$z_t = \boldsymbol{H}x_t + v_t \tag{2}$$

where $\boldsymbol{F}$ denotes the state transition matrix responsible for transitioning between states, $\boldsymbol{B}$ represents the control matrix that maps control variables to state variables, and $\boldsymbol{H}$ represents the measurement matrix responsible for mapping measurements onto the state.

The random variables $w$ and $v$ denote process white noise and measurement white noise, respectively. It is assumed that these variables are independent of each other, meaning there is no correlation between them. Both $w$ and $v$ are assumed to follow normal probability distributions:

$$w(\cdot) \sim N(0, \boldsymbol{Q})$$
$$v(\cdot) \sim N(0, \boldsymbol{R}).$$

In practical applications, the covariance matrices $Q$ and $R$, which represent the process noise and measurement noise respectively, may vary at each time step. However, in this context, they are assumed to remain constant, as stated by Koch (2006). These constant values were estimated using maximum likelihood methods.

The state transition matrix $\boldsymbol{F}$, with dimensions $2 \times 1$ in this case, connects the state at the previous time step $t-1$ to the current state at step $t$, assuming the absence of any driving function or process noise. On the other hand, the control matrix $B$, with dimensions $2 \times 2$, establishes the relationship between the optional control input $u \in \Re^l$ and the state $x$. Additionally, the $2 \times 1$ matrix $H$ in the measurement describes the relationship between the state and the measurement $z_k$. Although in practice, $F$ and $H$ may vary with each time step, in this scenario, both matrices are assumed to remain constant.

The intended procedure for the mechanical process is as follows:

**PREDICT**

| | | |
|---|---|---|
| Project state 1 time step ahead | $\hat{\boldsymbol{x}}_{t\|t-1} = \boldsymbol{F}_t \hat{\boldsymbol{x}}_{t-1\|t-1} + \boldsymbol{B}_t \boldsymbol{u}_t$ | (3) |
| Project error covariance 1 step ahead | $\boldsymbol{P}_{t\|t-1} = \boldsymbol{F}_t \boldsymbol{P}_{t-1\|t-1} \boldsymbol{F}_t^T + \boldsymbol{Q}_t$ | (4) |

**UPDATE**

| | | |
|---|---|---|
| Compute Kalman gain | $\boldsymbol{K}_t = \boldsymbol{P}_{t\|t-1} \boldsymbol{H}_t^T \left( \boldsymbol{H}_t \boldsymbol{P}_{t\|t-1} \boldsymbol{H}_t^T + \boldsymbol{R}_t \right)^{-1}$ | (5) |
| Update estimate with measurement $\boldsymbol{y_t}$ | $\hat{\boldsymbol{x}}_{t\|t} = \hat{\boldsymbol{x}}_{t\|t-1} + \boldsymbol{K}_t \left( \boldsymbol{y}_t - \boldsymbol{H}_t \hat{x}_{t\|t-1} \right)$ | (6) |
| Update error covariance | $\boldsymbol{P}_{t\|t} = (\boldsymbol{I} - \boldsymbol{K}_t \boldsymbol{H}_t) \boldsymbol{P}_{t\|t-1}$ | (7) |

where $\hat{x}$ represents the estimated state, $F$ denotes the state transition matrix responsible for transitioning between states, $u$ represents the control variables, $B$ represents the control matrix that maps control variables to state variables, $P$ represents the state variance matrix, $Q$ represents the process variance matrix that captures errors caused by the process, $y$ represents the measurement variables, $H$ represents the measurement matrix responsible for mapping measurements onto the state, $K$ represents the Kalman gain, and $R$ represents the measurement variance matrix that accounts for errors originating from measurements.

Subscripts represent:

| | |
|---|---|
| $t\|t$: | current time |
| $t-1\|t-1$: | previous time, and |
| $t\|t-1$: | intermediate steps. |

The observation equation is the VaR, which can be expressed as follows:

$$VaR_{CI}^{1d}(t) = \mu(t) + \sigma(t) \cdot N^{-1}(CI) + \epsilon(t) \qquad \epsilon(t) \sim N(0, \sigma_\epsilon^2) \tag{8}$$

where $\mu$ represents the average of daily returns over the previous period, $\sigma$ denotes the daily standard deviation of the portfolio or security return, and $\epsilon$ represents a noise term. The noise term $\epsilon$ is assumed to be independently and identically distributed (i.i.d.) with a normal distribution $\sim N(0, \sigma_\epsilon^2)$, where $0, \sigma_\epsilon^2$ represents the variance of $\epsilon$.

The specific form of the transition equation depends on the stochastic process assumed for the time-varying $\alpha$s and $\beta$s. It can be modelled using either an autoregressive, mean-reverting (AR(1)) model or a random walk process. Research has shown that the random walk model provides a more robust characterisation of time-varying $\beta$s (Denrell, 2004). On the other hand, AR(1) forms of the transition equation may encounter convergence issues, which can indicate misspecification of the transition equation, particularly for certain return series (Faff et al., 2000).

The random walk model (RWM) utilised in this essay assumes that both $\alpha$ and $\beta$ follow a random walk process. In other words, the current market exposure is considered a normally distributed random variable, with its mean being the exposure of the previous period. The un-correlated system noises, including the evolution of $\alpha$ and $\beta$, are also assumed to be normally distributed.

The state variables $x(t) \in \Re^2$ are the time-varying coefficients:

$$x(t) = \begin{bmatrix} \mu(t) \\ \sigma(t) \end{bmatrix}$$

at each time $t$. Both are assumed to follow the random walk model. The state equation is:

$$\begin{bmatrix} \mu(t+1) \\ \sigma(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mu(t) \\ \sigma(t) \end{bmatrix} + \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \tag{9}$$

where

$$\begin{bmatrix} \gamma \\ \delta \end{bmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_\gamma^2 & 0 \\ 0 & \sigma_\delta^2 \end{bmatrix}\right)$$

and the measurement equation is:

$$VaR_{CI}^{1d}(t) = \begin{bmatrix} 1 & CI(t) \end{bmatrix} \begin{bmatrix} \mu(t) \\ \sigma(t) \end{bmatrix} + \epsilon(t) \tag{10}$$

### 3.3 Expected shortfall

The ES at a selected quantile, $q$, denoted as $ES_q$, is computed as the probability-weighted average of values in the tail below q, such that:

$$ES_q = E\left(L \middle| L < VaR_q\right)$$

For a normal distribution, $$ES_q = \frac{f\left(VaR_q\right)}{q}$$

where $$f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

In other words, the probability density of the normal distribution is used to calculate $ES_q$, where $\sigma_t$ is the volatility. The function $f(x)$ refers to the probability density function of the normal distribution $N(0, \sigma^2)$, and it is assumed that $\mu = 0$.

To calculate $ES_q$ for any volatility, $\sigma$, and at any significance level, $q$, the function below must be integrated:

$$\begin{aligned} ES_q &= \int_{-\infty}^{q} x \cdot f(x) dx \\ &= \int_{-\infty}^{q} \frac{x}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{x^2}{2\sigma^2}\right) dx. \end{aligned}$$

Let

$$\chi = \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \text{then} \quad d\chi = -\frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \quad \text{so} \quad -\sigma^2 d\chi = x \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

Substituting

$$ES_q = -\frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{q} d\chi \qquad = -\frac{\sigma}{\sqrt{2\pi}} \cdot \exp\left(-\frac{x^2}{2\sigma^2}\right)\Bigg|_{-\infty}^{q} \qquad = -\frac{\sigma}{\sqrt{2\pi}} \cdot \exp\left(-\frac{q^2}{2\sigma^2}\right)$$

The observation that the distribution of asset price returns has fatter tails compared to a normal distribution has led to the introduction of the student's-t distribution. This distribution is employed to more accurately model the excessive kurtosis observed in asset returns (Fama, 1965; Bekaert, Erb, Harvey & Viskanta, 1998). When calculating the ES for a portfolio using the student's-t distribution, the integration process follows a similar procedure as with the normal distribution.

$$ES_q = \int_{-\infty}^{q} t \cdot f(t) dt$$

In this case, $f(t)$ is the probability density function of the $t$-distribution, which is (for $\mu = 0$ and standard deviation, $\sigma$):

$$f(t) = \frac{\Gamma(\nu + 1)}{\sqrt{\nu\pi} \cdot \Gamma\left(\frac{\nu}{2}\right) \cdot \sigma} \left(1 + \frac{t^2}{\sigma^2 \nu}\right)^{-\left(\frac{\nu+1}{2}\right)}$$

where $\nu$ counts the degrees of freedom, calculated using:

$$k = \frac{6}{\nu - 4} + 3,$$

and where $k$ is the kurtosis of the data (Rozga & Arnerić, 2009)

For $\nu$ even: $$\frac{\Gamma(\nu+1)}{\sqrt{\nu\pi} \cdot \Gamma\left(\frac{\nu}{2}\right)} = \frac{(\nu-1)\cdot(\nu-3)\cdots 5\cdot 3}{2\sqrt{\nu}(\nu-2)\cdot(\nu-4)\cdots 4\cdot 2}$$

and for $\nu$ odd: $$\frac{\Gamma(\nu+1)}{\sqrt{\nu\pi} \cdot \Gamma\left(\frac{\nu}{2}\right)} = \frac{(\nu-1)\cdot(\nu-3)\cdots 4\cdot 2}{\pi\sqrt{\nu}(\nu-2)\cdot(\nu-4)\cdots 5\cdot 3}$$

To calculate $ES_q$ for any volatility, $\sigma$, any number of degrees of freedom, $\nu$, and any significance level, $q$, the integral below must be determined:

$$ES_q = \int_{-\infty}^{q} t \cdot \frac{\Gamma(\nu+1)}{\sigma\sqrt{\nu\pi} \cdot \Gamma\left(\frac{\nu}{2}\right)} \cdot \left(1 + \frac{t^2}{\sigma^2\nu}\right)^{-\left(\frac{\nu+1}{2}\right)} dt.$$

Let $\Theta = \frac{\Gamma(\nu+1)}{\sqrt{\nu\pi} \cdot \Gamma\left(\frac{\nu}{2}\right)}$ and $\chi = 1 + \frac{t^2}{\sigma^2\nu}$ then $d\chi = \frac{2t}{\sigma^2\nu} dt$ so $\frac{\sigma^2\nu}{2} d\chi = tdt.$

Substituting

$$ES_q = \frac{\Theta\sigma\nu}{2}\int_{-\infty}^{q} \chi^{-\left(\frac{\nu+1}{2}\right)} d\chi \quad = \frac{\Theta\sigma\nu}{1-\nu} \cdot \chi^{\frac{1-\nu}{2}}\Big|_{-\infty}^{q} \quad = \frac{\Gamma(\nu+1)}{\Gamma\left(\frac{\nu}{2}\right)} \cdot \left(\frac{\sigma}{1-\nu}\right) \cdot \sqrt{\frac{\nu}{\pi}} \cdot \left(1 + \frac{q^2}{\sigma^2\nu}\right)^{\frac{1-\nu}{2}}$$

## 4. RESULTS AND DISCUSSION

Figures 1 and 2 illustrate the rebased index prices and daily returns for the DJIA over the period from Jan-17 to Apr-23 (prices rebased in Jan-17 to 100) respectively. Figure 1 displays elevated volatility between Jan-20 and Jul-20, which can largely be attributed to the influence of the Covid-19 pandemic on global financial markets. This period can be identified as an evident 'extreme' event, wherein risk measures such as VaR and ES hold significant potential in mitigating substantial losses for given institutions and banks. The daily returns of the DJIA have predominantly fluctuated within a range of 10% (comprising a 5% positive return and a 5% negative return) both preceding and following the aforementioned period of pronounced volatility. Therefore, the focus will be on the effectiveness of each measure in relation to Kalman filter over this period.


**Figure 1.** Times series of the DJIA, rebased to 100 in Jan-17.

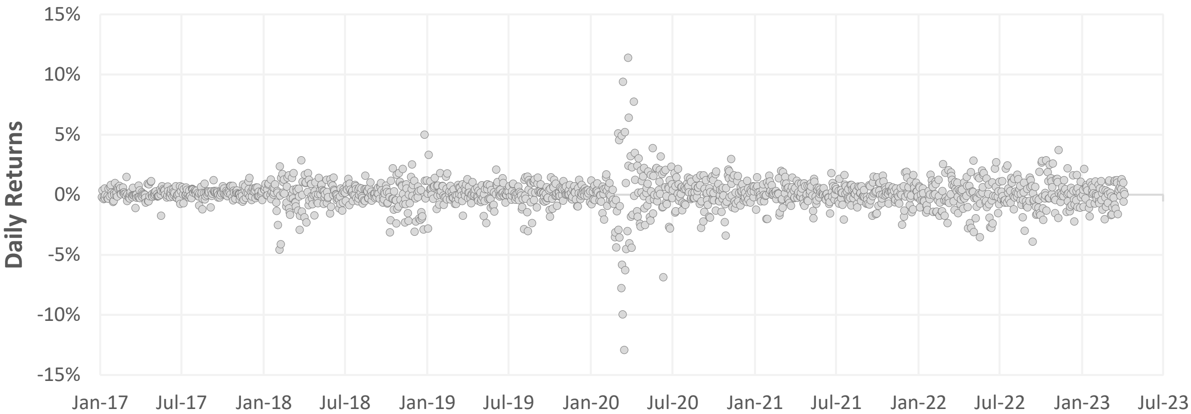



**Figure 2.** Daily return series of the DJIA.

The different methods selected to estimate the VaR and ES for the DJIA over the period were compared against the Kalman estimation approach. The results of these estimations are presented in Figures 3-4 and Figures 5-6, respectively. Each figure plots the lines for the different VaR and ES estimation approaches at the $99^{th}$ percentile. Each point on the graph represents a *negative* daily return on the DJIA over the period, as VaR and ES are only concerned with downside risk. A return that falls below a line indicates that the estimation technique was unsuccessful in forecasting and capturing market risk – i.e., the method was 'wrong'.

Throughout the entire period, the trajectory of historical VaR closely aligns with that of Kalman VaR in Figure 3. Given that both approaches encompass all the historical data from the given sample, it is unsurprising that this observation holds true. The Kalman approach takes an additional step by employing variance reduction techniques to mitigate noise and produce instantaneous measures (Thomson & van Vuuren, 2018).

The difference between the two approaches emerges during the highly volatile period, where the Kalman filter estimates on average approximately one percentage point more negative values than the historical VaR (-8% compared with -7%, on average). During this period, the disparity between the two estimations does not seem to offer distinct advantages in terms of capturing greater downside risk. Both approaches respond to the market downturn; however, they fall short in capturing the most severe losses that transpired, especially during the specific period of Mar-20. The historical approach performs equally as well as the Kalman approach during this isolated period, while exhibiting a slightly less sensitive estimation level. The more significant variation in terms of accuracy becomes apparent in the post-Apr-21 period, where the Kalman VaR demonstrates a greater ability to capture a few additional negative returns compared to the historical approach.

The VCV EW approach exhibits a comparatively lower level of sensitivity to the 'extreme' event, with VaR estimates hovering around -5% as opposed to the Kalman approach at -8%, which captures a slightly greater number of returns in that region. Similar to the historical VaR, the VCV EW VaR fails to capture as many negative returns as the Kalman approach, both in the post-Apr-21 period and the pre-Jan-20 period.

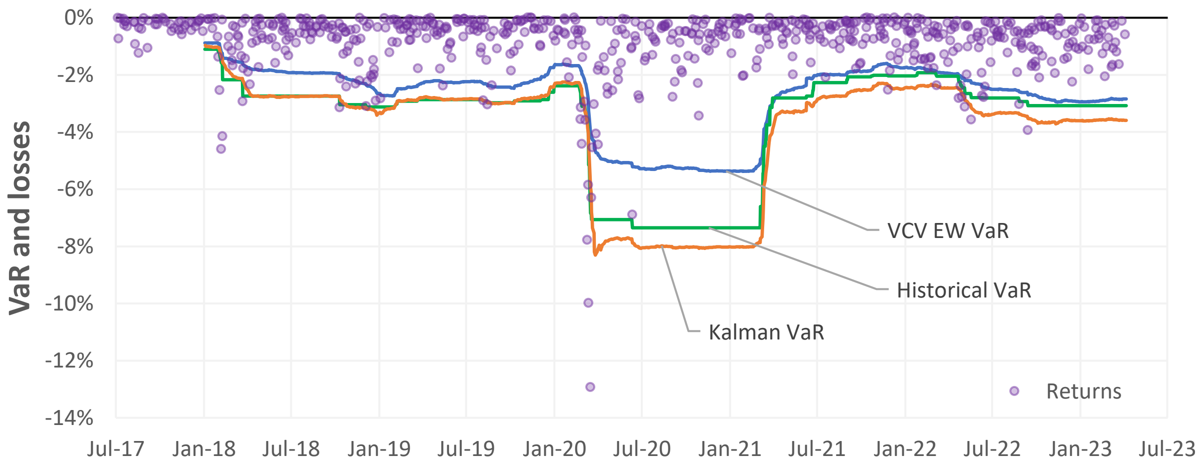



**Figure 3.** Comparison of VaR estimation approaches using historical and VCV EW techniques with the Kalman filter method.

Contrary to the historical and VCV EW approaches, the VCV EWMA and GARCH techniques exhibit a pronounced increase in VaR estimation sensitivity in response to fluctuations in returns. This effect is prominent during the period of high impact caused by the Covid-19 pandemic, wherein both methods demonstrate a more pronounced reaction compared to the Kalman approach. The simultaneous spikes, evident with GARCH (moving more than -18% in one instance), indicate a heightened sensitivity to extreme events compared to the Kalman estimation. While this sensitivity can be advantageous in highly volatile circumstances, it noticeably leads to inadequate forecasts and captures of market risk, as evidenced by the respective timeframes before and after the most volatile Covid-19 market period where the methods were less successful in capturing returns.

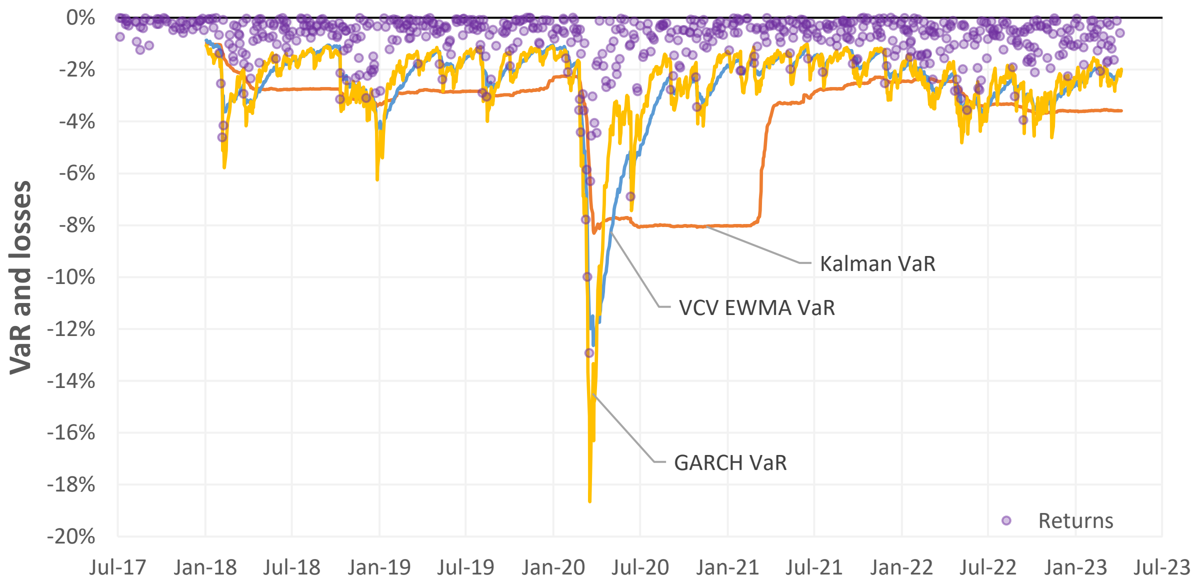



**Figure 4.** Comparison of VaR estimation approaches using VCV EWMA and GARCH techniques with the Kalman filter method.

Comparing Figure 3, which illustrates the Kalman filter alongside historical VaR and VCV EW VaR, it can be observed that the positioning of historical ES has moved downwards in relation to Kalman ES in Figure 5. historical ES is computed by averaging VaR values, meaning that the average only changes when the VaR value changes. Consequently, a new set of returns is averaged until the VaR changes again. The VCV EW approach demonstrates a relatively lower sensitivity to extreme events, with VaR estimates ranging from -6% to -7% compared to the -9% to -10% range of the Kalman and historical approaches. All approaches in Figure 5 provide more downside coverage in general when compared to their associated VaR measures, which is expected considering the scale of ES predictions. Both the Kalman and historical ES approaches exhibit a similar trajectory, posing challenges in immediately discerning which method outperformed the other over the given period. At a later stage in the results, the percentage of unsuccessful (and if reversed, successful) VaR forecasts by each method is provided.

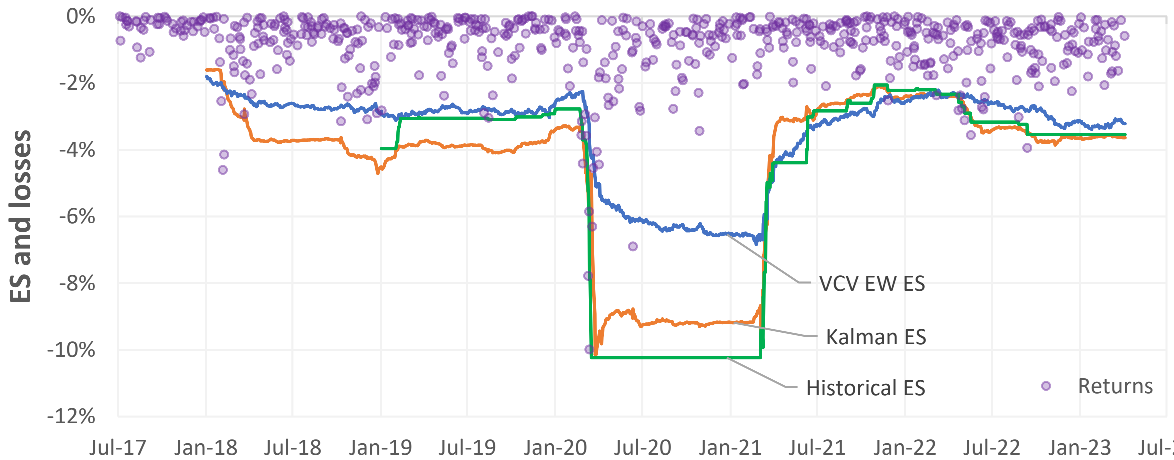



**Figure 5.** Comparison of ES estimation approaches using historical and VCV EW techniques with the Kalman filter method

Figure 6 presents a comparison of Kalman ES, GARCH ES, and VCV EWMA ES, which exhibits a similar pattern to their respective VaR counterparts in Figure 4, albeit on a larger scale. Once again, both the VCV EWMA and GARCH techniques demonstrate a more noticeable increase in sensitivity when estimating ES in response to returns fluctuations. However, in this case, all ES approaches have captured a greater number of returns before and after the volatile Covid-19 period.

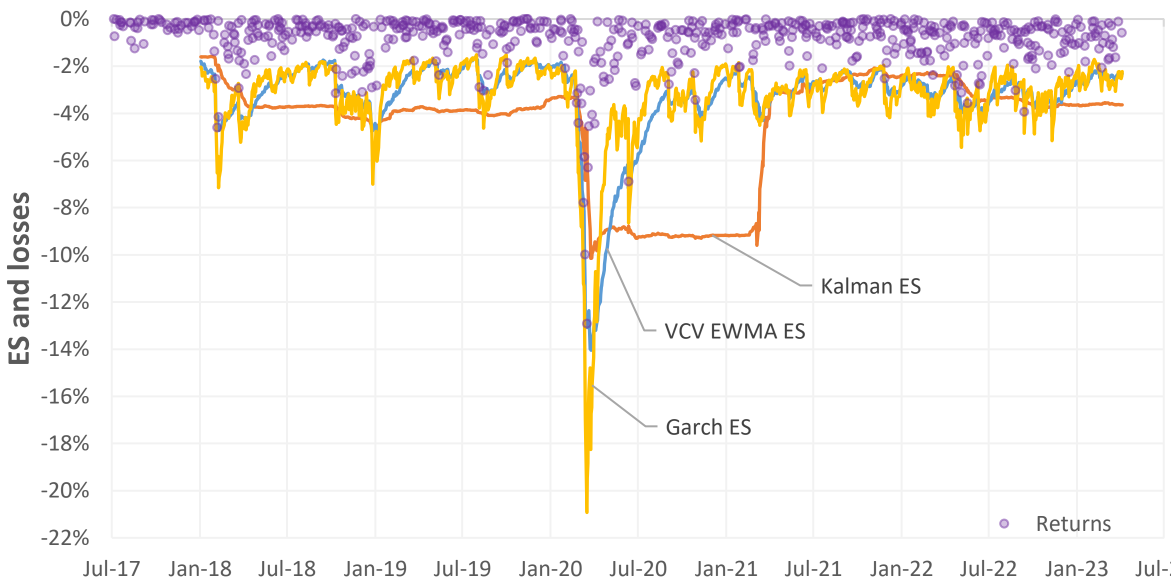
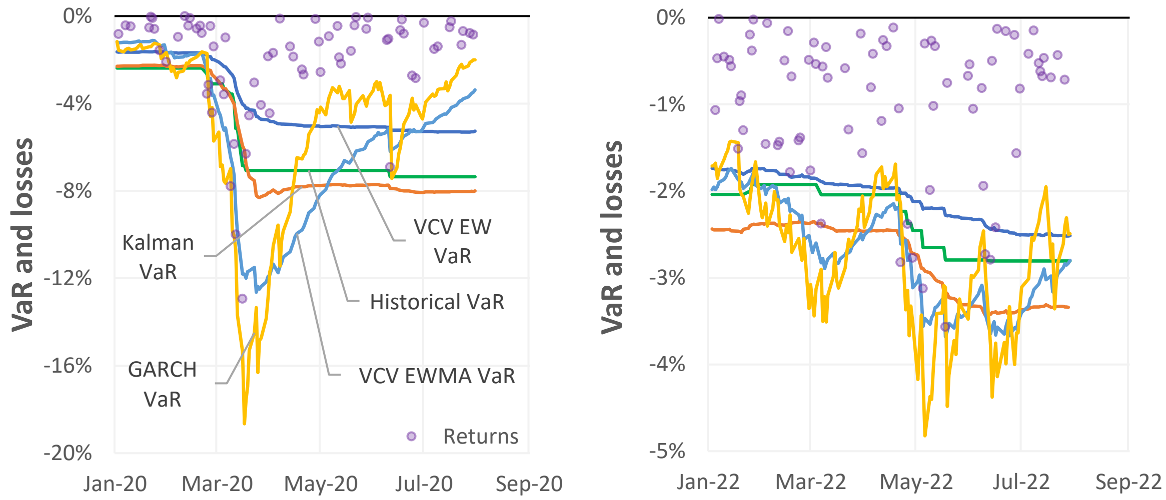



**Figure 6.** Comparison of ES estimation approaches using VCV EWMA and GARCH techniques with the Kalman filter method.

Figures 7 and 8 provide detailed representations of specific time periods within the sampled data, highlighting periods of high volatility and comparatively lower volatility, respectively. The sensitivities of the VCV EWMA and GARCH methods are more apparent, while the Kalman approach typically falls in between these two methods as well as the historical and VCV EW methods. The Kalman approach reacts more quickly than the standard VCV and historical approaches because the current set up calibrates the filter's response based off a standard deviation as a measure of volatility rather than an exponentially weighted technique. Using the latter as a volatility measure calibrates the filter's response such that it responds even more quickly to market turbulence than the EWMA and GARCH techniques. This characteristic provides the Kalman approach with a distinct advantage, particularly in volatile periods.




**Figures 7 & 8.** Select period comparison (comparably high and low volatilities) of popular VaR estimation techniques in relation to the Kalman method.

Figure 9 provides a summary of all VaR estimation approaches used over the entire period in relation to the Kalman estimation method.

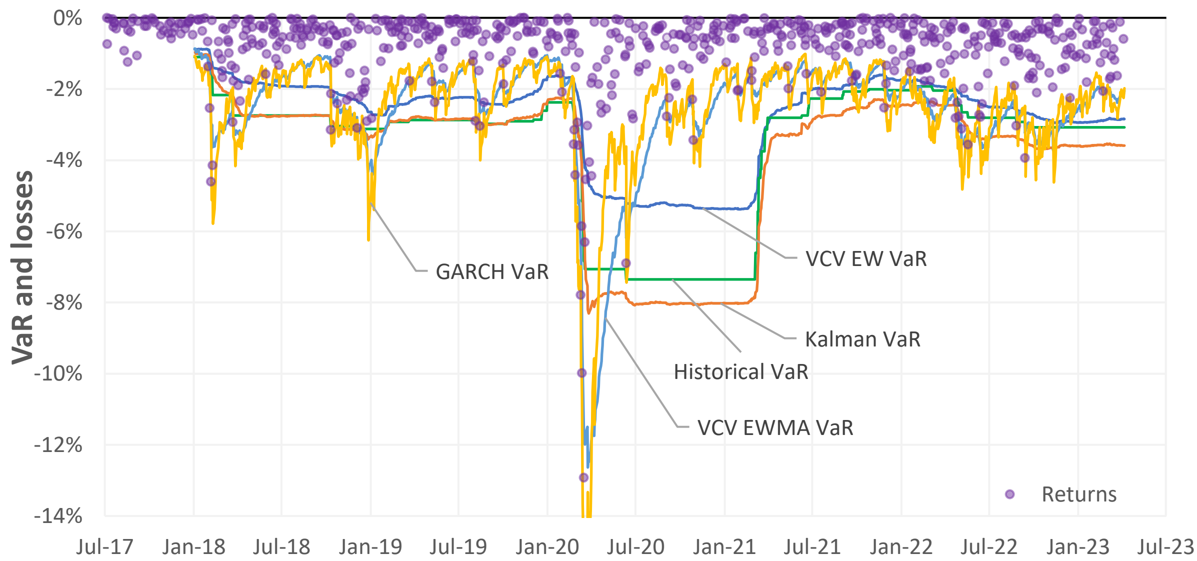



**Figure 9.** Efficiency comparison of popular VaR estimation techniques in relation to the Kalman method.

Figure 10 provides a summary of all ES estimation approaches used over the entire period in relation to the Kalman estimation method.

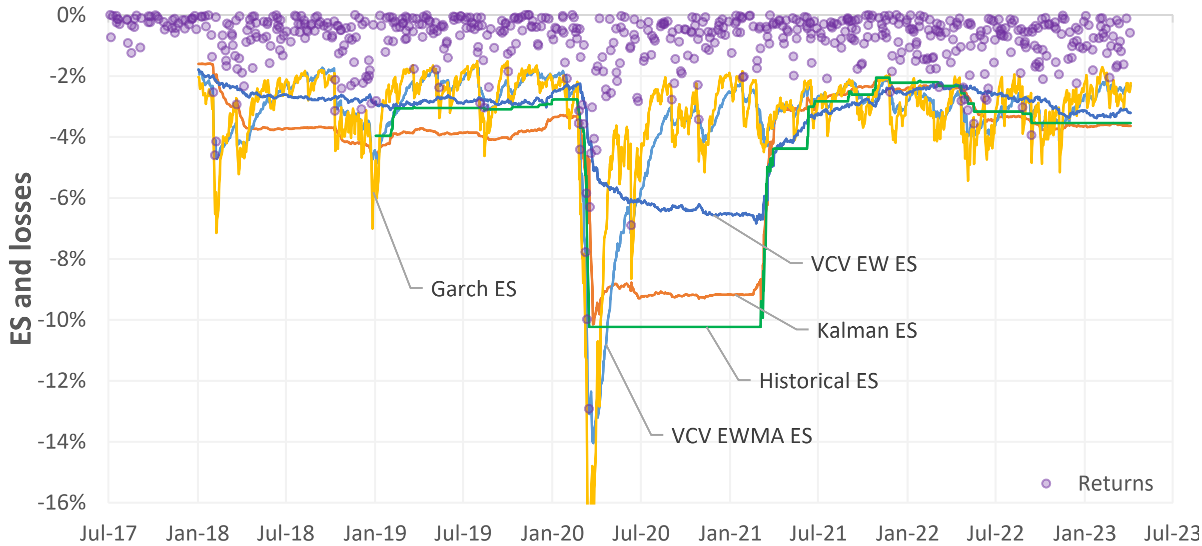



**Figure 10.** Efficiency comparison of popular ES estimation techniques in relation to the Kalman method.

Figure 11 presents a cumulative count of VaR exceptions for each method. To determine these exceptions, the VaR forecast from the previous day is compared with the current day's return using the time series of VaR outputs and DJIA daily returns. If yesterday's forecast, which represents the amount set aside based on what VaR estimates, is smaller than the current day's return, it is recorded as an exception.

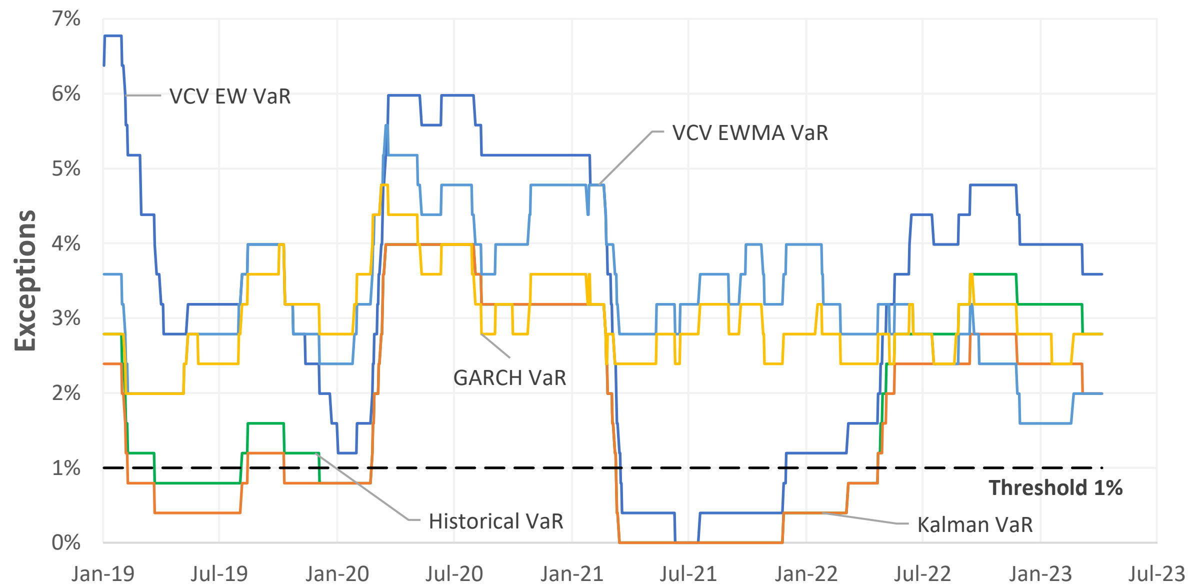



**Figure 11.** Cumulative count of VaR exceptions in relation to the Kalman method.

In instances where the line of a given model crosses above the horizontal 1% line over time, it signifies that the number of exceptions has surpassed the anticipated level at that specific moment. This occurrence is an unexpected event, considering that the computed VaR is designed to capture losses with a 99% confidence level. Ideally, a model should accurately achieve this, but during periods of significant volatility, models fail to adjust rapidly enough, leading to a significant increase in the number of exceptions. Models demonstrating greater accuracy are therefore those with fewer instances of exceeding the 1% threshold. In other words, the higher the number of exceptions beyond 1%, the poorer the model's forecasting performance. Throughout the selected timeframe, the Kalman VaR and historical VaR methods exhibit superior performance, with the Kalman approach outperforming the historical method during the Feb-19 to Aug-19 period. The VCV EW VaR, although briefly dipping below the 1% threshold between Mar-21 to Nov-21, demonstrates significant volatility before and after this period of relative stability.

Figure 12 provides an overview of the percentage of unsuccessful (and if reversed, successful) VaR forecasts by each method. The Kalman filter estimation outperformed all approaches, followed closely by historical VaR. Following the most consistent approaches, VCV EWMA VaR and VCV EW VaR display the least consistent performance over the analysed period.

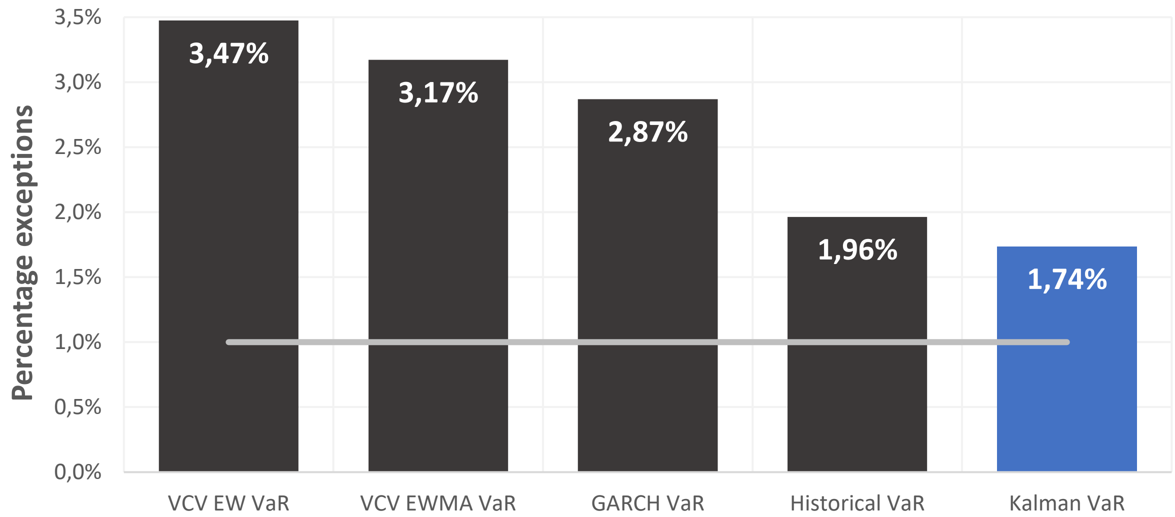



**Figure 12.** Comparison of VaR estimation inaccuracies relative to the Kalman method.

Figure 13 provides an overview of the percentage of unsuccessful (and if reversed, successful) ES forecasts by each method. In line with the VaR forecasts, the estimation of ES using the Kalman filter exhibited a relatively low forecasting error over the analysed period - this time it was matched by the GARCH ES estimation. Among all the approaches considered, the historical ES method exhibited the fewest inaccuracies, albeit by a small margin compared to the Kalman filter. This finding is promising nonetheless and highlights the robust performance of the Kalman approach in estimating both VaR and ES measures. Notably, the Kalman approach stands out for its ability to achieve such accuracy while imposing minimal limiting assumptions as discussed in Section 3.2 in relation to the other approaches, further enhancing its appeal and practicality in risk estimation. It is also worth reiterating, as supported by existing literature, that ES is considered a superior alternative to VaR for effective risk management. All methods generated fewer unsuccessful forecasts compared to their respective VaR approaches shown in Figure 12.

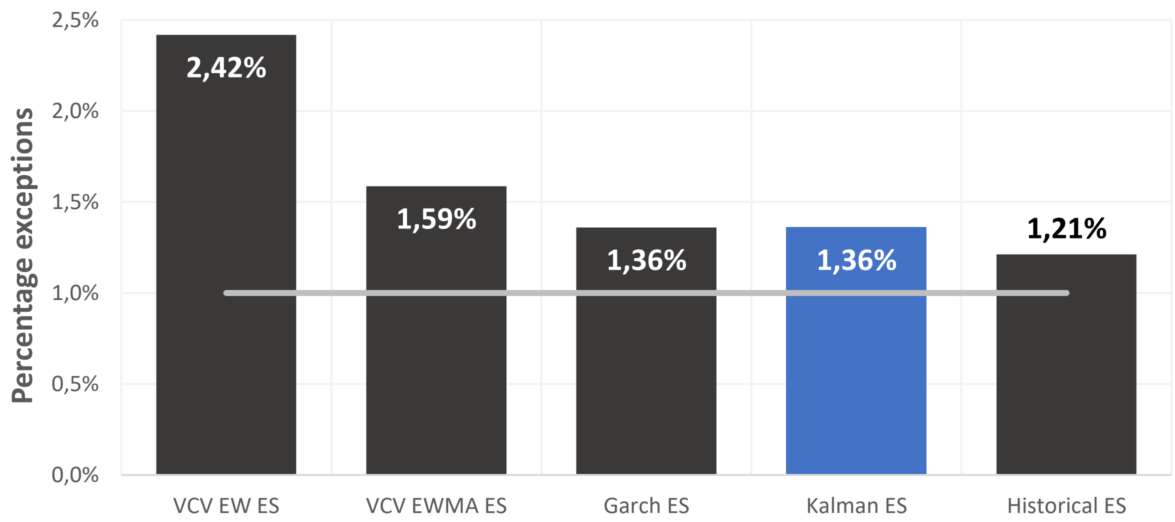



**Figure 13.** Comparison of ES estimation inaccuracies relative to the Kalman method.

## 5. CONCLUSIONS AND SUGGESTIONS

The empirical analysis presented in this essay explored a novel approach to assess VaR and ES risk measures using the Kalman filter. In comparison to well-established and commonly used approaches, the Kalman approach demonstrated superior performance exclusively in terms of VaR. It also achieved the same level of performance as GARCH for the ES back-test, spanning a period characterised by both high and low phases of volatility and distinct market shocks.

The results observed provide evidence that the Kalman estimation method is responsive to changes in market volatility, corroborating existing literature that highlights the model's capacity to enhance the precision of estimating unknown parameter values (Koch, 2006). This indicates that the Kalman filter is a reliable and robust contender in the volatility framework milieu. Moreover, this study contributes to the expanding body of literature (Albanese, 1997; Artzner et al., 1999; Acerbi & Tasche, 2002; Jorion, 2007; Daníelsson et al., 2012) that highlights the superiority of ES over VaR in effectively measuring risk. This superiority is evidenced by the discrepancy in estimation inaccuracies observed between the two methodologies over the sampled multi-year index period.

To better understand the effectiveness of the Kalman filter approach in different market environments, further research could extend this methodology to various financial markets. Each market is exposed to unique risks stemming from factors such as currency fluctuations, political landscapes, and market compositions. Application of the Kalman filter in risk management can also be extended to other popular measures, an example of which being extreme value theory (EVT). In addition to traditional volatility estimation approaches in GARCH and VCV EWMA, for example, the evaluation of the Kalman filter approach can be extended to more advanced machine learning techniques. Models comprised of deep learning, support vector machines, random forests, and many others have gained prominence in recent years. Assessing the performance of the Kalman filter against these modern methodologies would provide a comprehensive comparison and highlight the relative strengths and weaknesses of each approach in volatility estimation and risk management.

Policy implications include an updating of regulatory guidelines and regulations related to risk management in financial institutions which encourage the use of more sophisticated risk measurement models, such as the Kalman filter approach, to enhance accuracy and reliability. In addition, an increased focus on the validation and testing of risk models within financial institutions may be warranted. Regulators should require institutions to demonstrate the effectiveness of their risk models, and internal validation processes could become more rigorous. Regulatory reporting requirements may also need to be adjusted to incorporate more sophisticated risk metrics.

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