MATHEMATICAL MODELLING STUDIES

OF HUMAN LOCOMOTION.

by

CECIL HERSHLER.

The University of Cape Town

Thesis presented in partial fulfillment of the requirements for the degree of Master of Science in the Department of Physics of the University of Cape Town. South Africa.

June 1973.

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The University of Cape Town

TO

MY FATHER

ABRAHAM DOV HERSHLER

ACKNOWLEDGEMENTS.

The author's sincere thanks go to Dr.Morris Milner, Head of the Department of Bio-engineering and Medical Physics, for his encouragement and supervision during the entire period. His warmth and personality were to a large measure responsible for the initiation and fulfillment of this thesis.

The very generous assistance derived from the grant made by the Harry Crossley Foundation to the Human Locomotor Engineering Research and Development Programme in the Department of Bio-engineering and Medical Physics, is gratefully acknowledged.

Professor D.H.Jacobson and Dr.C.K.Chow are thanked for their personal assistance in certain aspects of this work.

Thanks are due to the Staff of the Princess Alice Orthopaedic Hospital; Groote Schuur Hospital and Computer Centre of the University of Cape Town for their co-operation and for the use of their facilities.
Special mention must be made of the clinical photographer of Groote Schuur Hospital, Mr.C.E.Wilberforce, for his invaluable photographic assistance.
Further thanks are also due to the author's colleagues in the department and to the secretaries, Mrs.A.Mathew and Mrs.L.Whitcher.

Special thanks to the author's mother, Mrs.Bella Hershler, for typing the drafts and the final version of the thesis and to the author's wife, Ruth, for her unfaltering support and encouragement throughout.

SUMMARY.

MATHEMATICAL MODELLING STUDIES OF HUMAN LOCOMOTION.

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0.1 Introduction.

This thesis was almost wholly conducted in the confines of the Department of Bio-engineering and Medical Physics, University of Cape Town and the Bio-engineering Unit, Princess Alice Orthopaedic Hospital, Cape Town.
It is an interdisciplinary thesis in that an effort has been made to bridge and thus unify the different scientific disciplines of Physics, Biology, Electrical Engineering and Mathematics.
Thus the author hopes the work would appeal to as wide an audience as possible.
The thesis is directed generally towards a deeper understanding of normal and pathological biped locomotion.
It involves theoretical considerations in respect of model building, model testing and practical work concerned with the collection of pertinent data.

0.2 Motivation.

Two important practical goals of locomotion study are the better design of prostheses for disabled persons and the possibility of programmed electro-stimulation of paralyzed muscle. The ultimate aim is to duplicate the functional performace of their normal human counterparts as closely as possible.
C.K.Chow and D.H.Jacobson (1971) have made a theoretical study of normal human locomotion. On page 242 of their paper they write:-

> "The present study is primarily concerned with the various mechanical moments that actuate the locomotion system and the resulting gait patterns it produces.
> By knowing the moment histories, we can then concentrate on the neuralmuscular transformation (Electromyographic activity (E.M.G.) and the like) for the electrical patterns of activity and eventually the neural action".

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0.3 Thesis.

This thesis tested their model by utilizing it using locally derived data on a normal subject for a range of walking speeds.
One pathological situation encountered is the surgical "freezing" of the ankle joint where damage might have been wrought by an accident.
The use of this model was extended to the study of this case also using data acquired locally.
The theoretical gait patterns were compared with their experimental counterparts.
An attempt was made to extend their study by correlating the phasic properties of the knee moment profiles with the corresponding electromyographic patterns obtained experimentally.
The results obtained showed a definite correlation between the predicted optimal knee moment profiles and the muscle activities of certain groups of muscles (Quadriceps during stance, Gastrocnemius during deploy and Tibialis Anterior during swing).
Certain discrepancies between experiment and theory which occurred during stance and deploy were explained by the need for experimental ground reaction profiles.
Theoretical results obtained for the work done externally by the muscles confirmed, to a certain extent, the experimental findings that energy expenditure was a minimum for certain combinations of speed and step length.

0.4 References.

Chow, C.K. and Jacobson, D.H. (1971): "Studies of Human Locomotion via Optimal Programming". Mathematical Biosciences 10: 239-306.

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CHAPTER 1.

STRUCTURE OF THESIS

1.1 Introduction.

The basic aims of research in the field of Bio-engineering have been neatly summed up by Milner (1968) who said, referring to his work, as follows:- "It is considered that the study presented typifies research efforts in the relatively newly-emergent field of Bio-engineering, which, amongst its aims, endeavours to understand the design of living systems in order to gain design information to possibly make better engineering systems.
Another facet is the development of diagnostic apparatus and instrumentation for clinical use, following from the understanding of the biological mechanism.
The essential aim of the work is to illuminate the manner in which mathematical modelling, aided by suitable computational facilities, assists in the scientific exploration of some of the mechanisms of nerve function".

Another important by-product, becoming more apparent with time, is the opening of channels of communication with their inherent feedback between engineers, physicists and chemists on the one hand and clinical diagnosticians, physiologists and anatomists on the other.

It is for this very reason that a large amount of background material, taken mainly from the literature and well-known text-books, has been considered necessary and included in the earlier chapters.

1.2 Structure of the thesis.

Chapter 2 presents a historical survey of human locomotion studies.
Chapter 3 reviews the anatomical aspects of the lower part of the body from the hip to the toes of the foot.
Chapter 4 discusses the various physiological factors in human locomotion. The lengthier section on the mechanical aspects of isotonic contraction in striated muscle shows that, up until the present time, Hill's force-velocity relationship (1938) is still considered to be of sufficient accuracy to justify its inclusion in the theoretical model of Chow and Jacobson (1971).

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Chapter 5 is a general review of experimental techniques used in human locomotion.

Chapter 6 introduces the mathematical model of Chow and Jacobson (1971).

The development of the dynamic equations and kinematic constraints for normal locomotion and their subsequent simplification has been reproduced in appendix 3.

The optimal programming approach is introduced and the important derivation (with all its approximations) of the energy performance criterion is shown in appendix 4.

In addition to a study of normal locomotion at various speeds, a study has been made of a certain type of pathological gait. The particular type of pathological gait under consideration here is the one caused by an arthrodesed ankle. There is no rotation of the foot about the ankle. The alterations to the various mathematical equations and constraints, introduced by the choice of this particular gait, are outlined.

Chapter 7 shows how the problem is formulated in an optimization context and the method of solution is outlined. Also described is the computer programming and listings of the various programs are shown in appendix 6.

Chapter 8 deals in detail with the experimental side of this thesis.

Chapter 9 compares the numerically simulated results with the actual experimental results taken from the subjects concerned.

The results obtained show a definite correlation between the predicted optimal knee moment profile and the muscle activities of certain groups of muscles (Quadriceps during stance, Gastrocnemius during deploy and Tibialis Anterior during swing).

Certain discrepancies between experiment and theory which occur during stance and deploy are explained by the need for experimental ground reaction profiles.

A number of additional ideas are introduced as a possible way of increasing the accuracy of future work in this area.

1.3 <u>References;</u>

Chow,C.K. and Jacobson D.H (1971): "Studies of Human Locomotion via Optimal Programming". Mathematical Biosciences <u>10</u>: 239-306.

Hill, A.V.(1938): "The Heat of Shortening and the Dynamic Constants of Muscle". Proc.Roy.Soc.(Lon.) Ser B 126: 136.

Milner,M (1968):"Models of Nerve Excitation and Propagation with special reference to Multifibre Peripheral Nerve". Ph.D.Thesis, University of Witwatersrand, South Africa.

CHAPTER 2.

HISTORICAL SURVEY OF HUMAN LOCOMOTION.

2.1 Notes:

It is generally considered (Encyclopaedia Brittanica (1952)) that Aristotle was the earliest person in our history to comment meaningfully on animal gait.
According to Bernstein, Popova and Mogilansky (1934), however, it was Leonardo da Vinci who was the first to record scientific observations concerning human locomotion.

J.A.Borelli (1743) published a physico-mechanical dissertation on muscle but it was the Weber brothers (1836) who published the first basic papers on the biomechanics of gait. Their work, however, was more qualitative than quantitative.

Marey (1885) was probably the first to contribute to gait analysis using the technique of photography. Marey dressed his human subject in a tightly fitted black suit with white stripes on the side. This method was modified by Braune and Fischer (1895) who used luminous tubes instead of the white stripes. This enabled them to study the biomechanical factors of gait in greater detail.

Anatomical deduction is historically the oldest and most widely employed method of determining the function of a muscle, to ascertain its origin and insertion, and to deduce what motion would occur should the muscle undergo contraction. This procedure had obvious limitations and has led to many erroneous conclusions. Palpation, which reached its highest development in the hands of Scherb (1927) consists in the exploration of the individual muscle with the examining fingers while the subject is walking on a treadmill.
The alternate hardening and softening of the muscle in relationship to the position of the extremity permits some evaluation of its function. The accuracy of such a method is open to question, and the results cannot be utilized for quantitative data.

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The analytic method has not been utilized extensively, but has attained some degree of accuracy in the hands of Liberson (1936) and Elftman (1939). This technique is purely a mathematical one and consists in the theoretical calculation of the forces producing motion, based upon the centres of gravity and the acceleration of parts involved. The forces thus calculated are attributed to the muscles acting in the general direction of the calculated force. This may lead to very accurate results, but it is not purely objective.

Recently this field has been extended (Inman (1966)) to the study of potential and kinetic energy time patterns during locomotion. This is usually done by analysing data obtained from strain gauges at the point of ground support under the feet, and from accelerometers near the centre of gravity of the body.

Electromyography, which is the amplification and recording of the action potentials of muscles as a method of studying their activities, is a relatively new tool. Its main advantage is its ability to reveal the precise phase relationship of muscle action.

In 1947, a lengthy and concentrated study of human locomotion was conducted by the University of California Biomechanics Department using electromyographic data. Gray and Basmajian (1968), using electromyography, have studied the activity of involved muscles during walking in order to elucidate the functions of the different muscle groups with a view to the appropiate design and development of prosthetic devices.

The above research was primarily experimental in nature largely due to the complexity of the process. In 1971, however, there appeared a critical study at the theoretical level instituted by Chow and Jacobson.
First, it is proposed that walking obeys a certain "principle of optimality". This means simply that when a person walks he minimizes a certain quantity which in most cases will be

energy or the mechanical work done by the muscles externally. Based on external characteristics of muscles and certain assumptions regarding normal locomotion, a simple quadratic type of performance index is obtained. This performance criterion is shown to be related to the mechanical work done during walking. Next, at the dynamic level, modern control theory is used to derive the optimal moment profiles about the hip and the knee which actuate the locomotor elements to sythesize the observed patterns of the gait. The walking cycle is divided into three distinct stages of phasic activity. Each stage involves dynamic equations of motion and kinematic constraints which reflect the particular nature of the phasic activity. Invoking the necessary conditions of optimal control theory, a multipoint boundary-value problem is then employed for an iterative numerical solution on a digital computer.

2.2 References.

Bernstein,N.,Popova, and Mogilansky,(1934): Technique of the Study of Movements. Moscow,Gossidat.

Borelli,J.A.,(1743): De Motu Animalium,Dissertationibus physiomechanicis de motu musculorum, et de effervescentia, et fermentatione. Hagae Comitum. P.Gosse.

Braune,C.W., and Fischer,O.(1895): Der Gang des Menschen. I-Teil: Versuche am unbelasteten Menschen, Leipzig.

Chow,C.K., Jacobson,D.H.(1971):"Studies of Human Locomotion via Optimal Programming" Mathematical Biosciences 10:239-306.

Elftman,H.(1939):"Forces and Energy Changes in the Leg During Walking; The Function of Muscles in Locomotion" Amer.J.Physiol 125:339-366.

Encyclopaedia Britannica,(1952): On the Gait of Animals, in Great Books of the Western World: Works of Aristotle, Vol.II,9:243-254, Chicago.

Gray,E.G.,Basmajian,J.V(1968):"Electromyography and Cinematography of Leg and Foot ("Normal" and Flat) during Walking". The Anatomical Record, 161, No.I,1-16.

Inman,V.T.,(1966):"Human Locomotion". Canadian Medical Association Journal; 94:1047-1054.

Liberson,W.T.,(1936):"Une Nouvelle Application du Quartz Piézoélectrique: Piézoélectrographie de la Marche et des Mouvements Volontairs". Paris, Le Travail Humain ;4:1-7.

Marey,E.J.,(1885):"Locomotion de l'homme; Images stereo-scopiques des trajectoires que decrit dans l'espace un point du tronc pendant la marche, la course et les autres allures. Compt Rend Acad Sc Paris; 100,.

Scherb,R.,(1927): "Mittielungen zur Myo Kinesiographie". Ztschr f Orthop Chir; 49,.

University of California Final Report No.12 (1947)."Report on Fundamental Studies of Human Locomotion relating to the design of artificial limbs".

Weber,W., and Weber,E.,(1836): Mechanik der Menschlichen Gehwerkzenge. Göttingen,Dieterich.

CHAPTER 3.

ANATOMICAL ASPECTS.

3.1 Introduction.

In an effort to keep this thesis as complete as possible it has been decided to include this fairly detailed description of the skeletal system and musculature of the lower half of the body. Most of the content has been taken from Basmajian's text (1970) but detail has been sacrificed for ease in perusal by one not conversant with anatomy. A summary of the functions of the major muscle groups of the leg during locomotion is included for convenience at the end of the chapter.

An important aspect of the mathematical modelling used in this thesis is the decomposition of the body dynamics into two parts. One part describes the lower extremity motion with its consequent generation of gait patterns and the second part pertains to the motion of the upper trunk (including head and arms). As this thesis deals only with the former part, those interested in a study of the torso motion are referred to a second paper published by Chow and Jacobson (1972).

3.2 Anatomical Aspects of Human Locomotion.

The skeletal system that is being considered during the process of human locomotion consists of:-

A. The hip bone
B. The femur or thigh bone
C. The tibia and fibula
D. The patella or knee-cap
E. The bones of the foot

A. The Hip Bone. (Fig.3.1)

The acetabulum is a socket for the reception of the spherical head of the femur or thigh bone and it faces lateralwards, downwards, and slightly forward.

B. The Femur or Thigh Bone. (Fig.3.2)

The femur or thigh bone is the longest bone in the body

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being about 65 cms in length. Its ends are very specialized for the hip and knee joints but the shaft is relatively simple. The shaft slopes downwards and medially while the spherical articular head fits accurately in the acetabulum of the hip bone. As the knee is approached, the cylindrical shaft becomes expanded sideways and a large, smooth, triangular area exists which forms the upper part of the region at the back of the knee.

C. The Tibia. (Fig. 3.3)

The strong and massive bone on the medial side of the leg is the tibia. It alone receives weight transmitted to it by the femur and it conveys that weight to the foot.

The salient features of the tibia may readily be made out since the bone lies just deep to the skin where, as the shin, it is palpable from knee to ankle.
The shaft of the tibia tapers from the expanded upper end until it approaches the ankle when it again expands; the lower end, however, is less expanded than the upper.

The lateral bone of the leg, the fibula, is like a long thin stick; its shaft is twisted and is moulded by the origins and dispositions of the numerous muscles that arise from and are in contact with it.

The functions of the fibula are threefold
(1) to give origin to muscles
(2) to act as a pulley for tendons passing behind it at the ankle
(3) to act as a lateral 'splint' for the ankle joint.
Without the fibula the whole security of that joint is lost.

D. The Patella. (Fig. 3.4)

The patella or knee-cap is a bone developed in a tendon which is triangular in outline with its apex down. It is thick and more or less flat.
Since it lies just deep to the skin its size and shape are readily made out. It is said to enhance the power of the already powerful extensor muscle of the knee, in whose tendon it is developed, by increasing the leverage for that muscle.

E. <u>The Bones of the Foot</u>.(Fig. 3.5)

The human foot, in contrast to the hand, has sacrificed all other functions to concentrate on the duties of weight-bearing and <u>locomotion</u>.

The anterior half includes all the metatarsals and phalanges; the posterior half includes all seven tarsal bones.

The talus and calcaneum are the largest of the tarsal bones and are concerned with receiving the weight of the body above. The talus rests on the calcaneum or 'heel-bone' and occupies the socket of the ankle joint.
On the upper surface of the talus i.e. at the ankle joint, occur only the simple hinge movements of flexion and extension. The calcaneum, on the other hand, accommodates itself to the irregularities of the ground with which it is in contact and it does so without in any way disturbing the superimposed talus.

The posterior third of the calcaneum projects behind the ankle joint as the heel. The heel is a lever whereby the powerful calf muscles can extend the ankle and raise the body on tip-toe.

Viewed from above, the bones of the foot may be divided longitudinally into two segments (1) calcaneum, cuboid, metatarsals 4 and 5, (2) talus, navicular, three cuneiforms and metatarsals 1,2 and 3. There is some functional significance to this arrangement.
The lateral segment of the foot consists of a low longitudinal arch supported chiefly by ligaments. The medial segment consists of a high longitudinal arch supported chiefly by ligaments during standing but also by muscles during locomotion.

In walking, the weight is first borne by the heel and then is spread forwards along the lateral edge of the foot quickly to the metatarsal heads. The 'push-off' thrust is concentrated at the head of metatarsal 1, accounting for the large diameter of this bone.

3.3 The Muscular System.

The muscular system pertinent to the process of human locomotion consists of the muscles of the lower limb:

A. The Muscles of the hip and thigh
B. The Muscles of the leg.

The lower limb is the limb of stability, its movements are limited, coarse and often stereotyped. The paramount duty of its muscles is locomotion. The most powerful muscles lie alternatively at the back of the hip, at the front of the thigh, and at the back of the leg. The existence in the thigh of so many 'two-joint' muscles is explained by the requirements of the act of walking. Hip flexed and knee extended is the position of the limb when the advanced foot is about to be placed on the ground. Hip extended and knee flexed is the position of the 'hind' limb when the foot leaves the ground. If, by passing over both joints, certain muscles can perform the two required movements simultaneously, they achieve an economy of effort.

A. The Muscles of Hip and Thigh.

These can be subdivided into

(i) Muscles crossing the front of the hip joint.
(ii) The three gluteal muscles (and the Tensor Fascia Latae)
(iii) The six lateral rotators.
(iv) Muscles of the front of the thigh
(v) Muscles of the medial side of the thigh
(vi) Muscles of the back of the thigh

(i) Muscles crossing the front of the hip joint (Fig.3.6)

Psoas Major and Iliacus.
These two muscles come together, fuse and cross the front of the hip joint close to the head of the femur. Because they share a common tendon of insertion and have a common action, they are often regarded as one - the Iliopsoas. Both parts of the Iliopsoas are concerned with the flexion of the hip joint.

(ii) The three gluteal muscles are the Gluteus Maximus, Gluteus Medius, Gluteus Minimus.
The Gluteus Maximus is an extensor of the hip joint but it is used only when the joint has to be extended with power. It is also a powerful lateral rotator of the extended thigh.

The tensor fascia latae is inserted with the Gluteus Maximus into the iliotibial tract of the fascia lata. It is a flexor and medial rotator of the hip joint as well as an extensor of the knee joint. Its important use is to brace the knee so that, in walking, the knee joint can take the weight without 'buckling' while the other foct is off the ground and body is swinging forward.

The Gluteus Medius and Gluteus Minimus except for their origin and insertion onto the femur are identical (for all practical purposes) in action and use. Their functions are to abduct (turn outwards) the thigh and the anterior parts of the muscle rotate the thigh medialwards.

(iii) The six lateral rotators originate from the back of the hip bone, pass behind the hip joint and all except one crowd to be inserted into the medial surface of the greater trochanter. All six are lateral rotators of the hip and probably do little else.

(iv) The muscles of the front of the thigh. (Fig.3.7)
These are the three Vasti and the Rectus Femoris which are grouped together under the name Quadriceps Femoris.
In addition there is the long obliquely-running Sartorius. The Sartorius originates from the Ilium and is inserted on the upper part of the tibia. The Sartorius is not a powerful muscle and does not seem to play an important role in walking.

The Rectus Femcris arises by means of two tendons from a region just above the acetabulum.
It runs down the front of the thigh and is inserted by means of a tendon on the upper border of the patella. The Rectus Femoris is the only member of the Quadriceps group that crosses the hip joint and that can, therefore, flex the hip and/or extend the knee by which means the limb is advanced in walking.

The Peronei evert the foot. The Peroneus Longus is a plantarflexor of the transverse tarsal joint.

(iii) <u>Superficial Plantar-flexors</u>. (Fig. 3.11, 3.12)

These are the Gastrocnemius and Soleus. Relatively unimportant is the associated Plantaris. The Gastrocnemius crosses two joints and its two bulging medial and lateral heads arise from the back of the femur just above the condyles. These two heads unite to form a single muscle. The Soleus arises from the back of the tibia and fibula and lies beneath the Gastrocnemius. The Plantaris shares its origin with the lateral head of the Gastrocnemius. All three muscles share a single tendon of insertion (Tendo Achillis) which is inserted into the back of the calcaneus at the back of the ankle joint.

The fibres of the Gastrocnemius are so short that the muscle can either flex the knee or plantar-flex the ankle but never both together. The Soleus plantar-flexes the ankle when the knee is flexed. Both muscles are active during the 'take-off' phase of the foot in walking and running.

(iv) <u>Deep Plantar-flexors</u>.

These consist of three muscles; the Flexor Hallucis Longus, Flexor Digitorum Longus and Tibialis Posterior.

The Flexor Hallucis Longus arise from the lower two-thirds of the fibula posterior and is inserted finally onto the distal phalanx of the great toe. It flexes the great toe and assists in plantarflexion of the ankle.

The Flexor Digitorum Longus arises from the posterior surface of the tibia and is inserted finally by means of four tendons to the four smaller toes. Flexion of the small toes is its only confirmed important action.

The Tibialis Posterior is the deepest muscle of the leg. It arises from the inner surface of the osseofibrous pocket (a compartment between the tibia and fibula) and is inserted finally into the tuberosity of the navicular. It inverts the foot and plantarflexes the transverse tarsal joint.

The Muscles of the Foot

These occur in four distinct layers in the sole of the foot and other than help to maintain the arches of the foot during locomotion the functions and actions of these muscles do not appear to be important in human locomotion.

3.4 Summary.

The action of the calf group may be summarized as follows:
1. Resists ankle flexion after mid-stance and therefore creates a new centre of rotation for the weight bearing extremity at the ball of the foot.

2. Actively extends the ankle creating an upward and forward ground which is utilized to propel first the entire body mass and then the swing-through extremity.

3. Flexes the knee of the weight bearing extremity,controlling the vertical height of the body and preparing the extremity for an efficient swing-through position.

The action of the quadriceps group can be summarized as follows:
1. Deceleration of the downward movement of the centre of gravity by permitting controlled flexion of the knee to occur.

2. By extension of the knee with the foot fixed, acceleration of the body in upward direction is accomplished.

3. Oscillatory Action: At certain speeds the backward oscillation of the extremity at the hip-joint is retarded and the movement reversed resulting in forward acceleration of the lower extremity for swing-through activity.

The activity of the hamstring group may be summarized as follows:
1. Decelerates the swing-through extremity, reverses its directions of movements and plants the heel to the floor.

2. Acts to extend the hip and cppose gravitational and inertial moments of flexion of the pelvis and torso around the hip.

3. Under certain circumstances assists in shortening of the swing-through extremity by flexion of the knee.

The action of the pre-tibial group may be summarized as follows:

The major activity of the pre-tibial group occurs during the period the toe is approaching the ground and the foot is becoming plantar flexed in relation to the tibia. Therefore, the motion which is normally associated with activity of this muscle, dorsi-flexion of the foot, not only does not occur but the converse, plantar-flexion, takes place. The muscles are both lengthened, despite their contraction, as they resist the force of body acting down the tibia about the fulcrum at the heel. Thus the tendency of the forepart of the foot to slap to the ground is controlled while decelerating the conditions wherein muscles are being lengthened against their normal motion of contraction. Work is therefore being done upon the muscles rather than by the muscle upon the skeletal system.

The secondary and much smaller peak of contraction at toe-off tries to dorsi-flex the foot toward the right angle. Plantar-flexion associated with calf group propulsion has just occurred. Thus maximum clearance is afforded to the toe during the swing-through period by this lifting action. In addition the entire extremity is somewhat shortened in total length and the ankle position is returned to optimum angle for the next major contraction after heel strike.

3.5 References.

Basmajian, J.V. (1970): Primary Anatomy. Sixth Edition The Williams & Wilkins Company Baltimore. 180-206.

Chow, C.K., Jacobson, D.H. (1972): "Further Studies of Human Locomotion: Postural Stability and Control". Mathematical Biosciences. 15: 93-108.

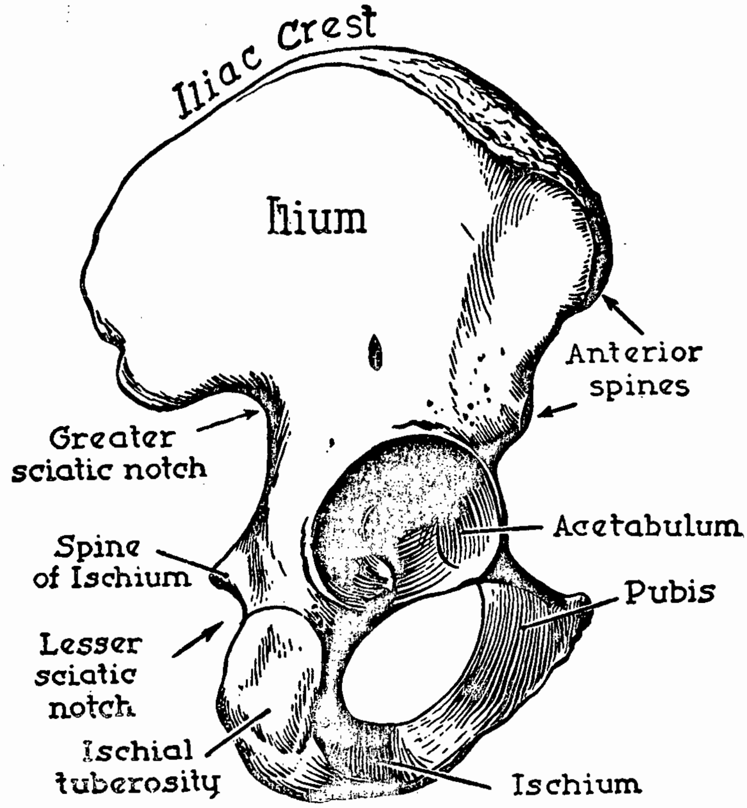

Fig. 3.1 Outer Aspect of Right Hip Bone.

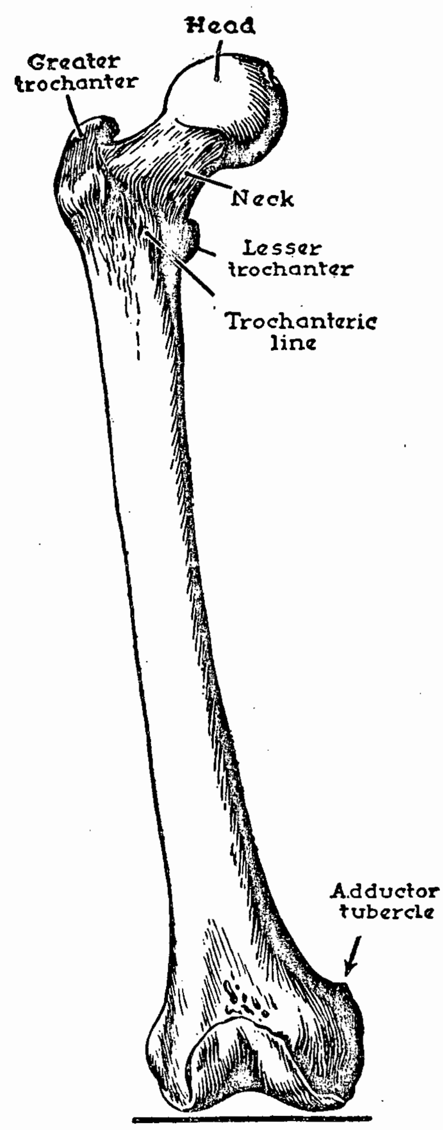

Fig. 3.2 Right Femur From in Front.

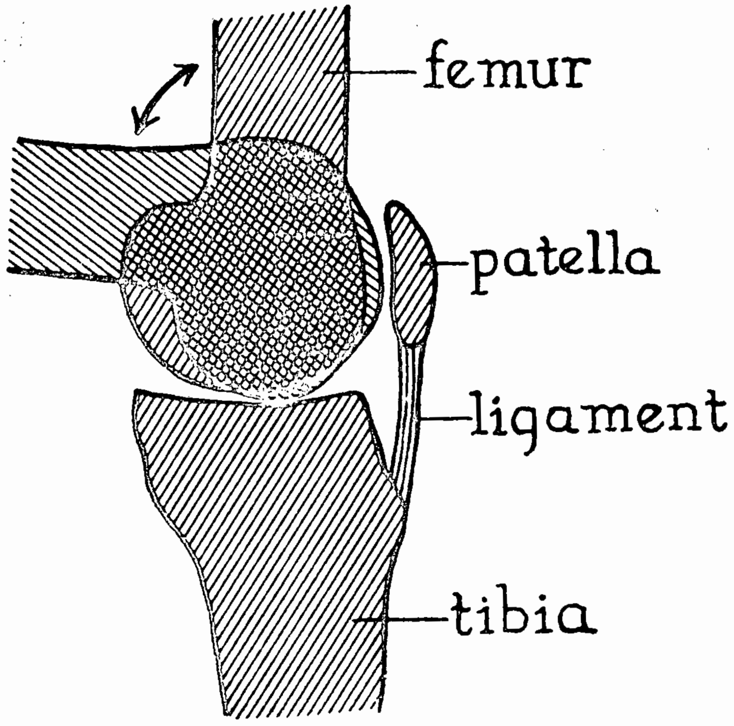

Fig. 3.4 Position of Patella during Flexion and Extension of Knee.

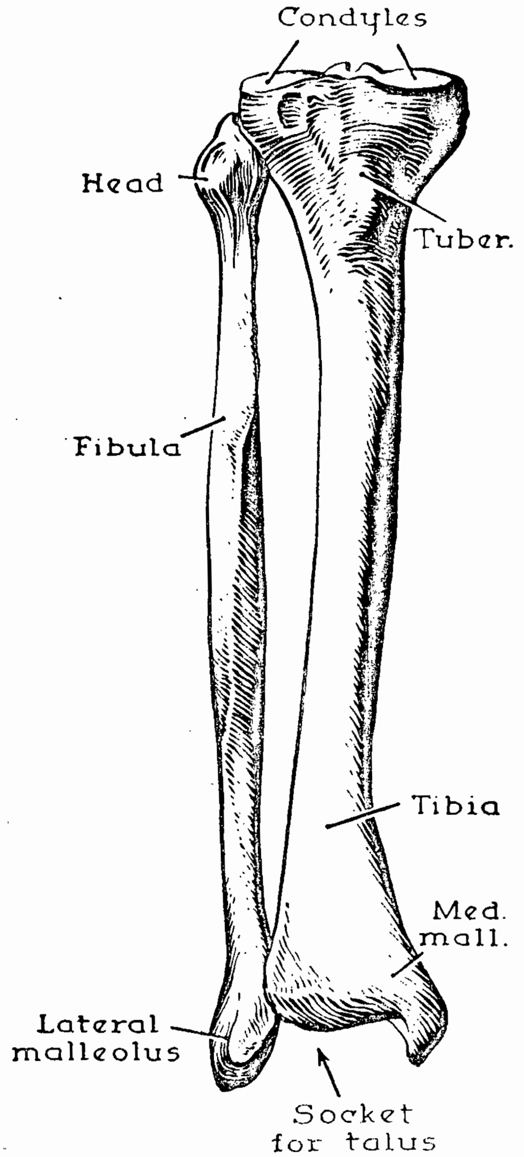

Fig. 3.3 Right Tibia and Fibula from in Front.

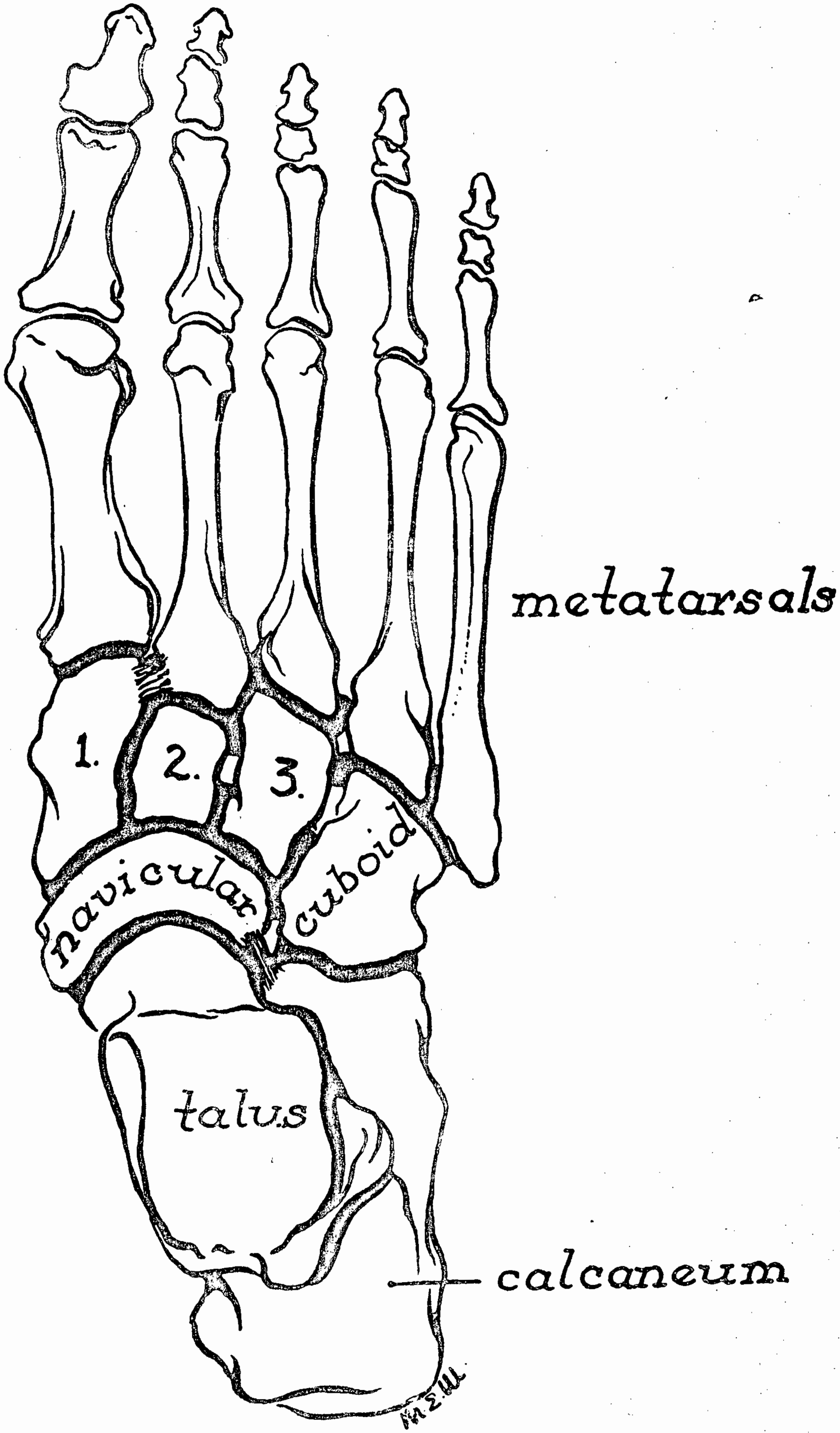

Fig. 3.5 Bones of Right Foot Viewed from Above
1,2,3 = cuneiforms.

1. Tensor Fasciae Latae.
2. Ilio-Tibial Band.
3. Vastus Lateralis.
4. Sartorius.
5. Iliacus.
6. Psoas.
7. Pectineus.
8. Adductor Longus.
9. Gracilis.
10. Adductor Magnus.
11. Rectus Femoris.
12. Vastus Medialis.
13. Tendon of Rectus.
14. Patella.

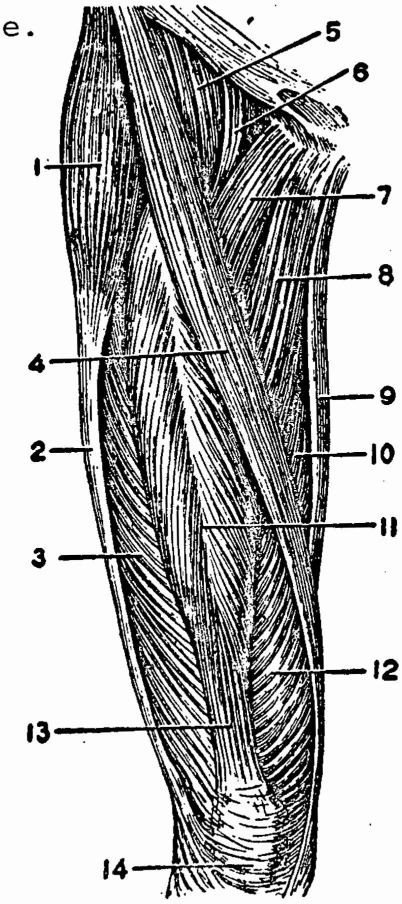

Fig. 3.6 Dissection of Muscles of Front of Right Thigh.

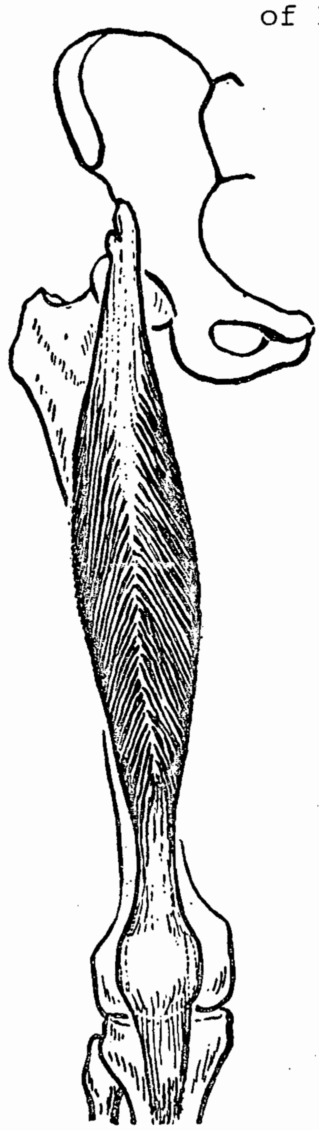

Fig. 3.7 Right Rectus Femoris.

1. Obturator Internus.
2. Gluteus Maximus.
3. Quadratus Femoris.
4. Adductor Magnus.
5. Gracilis.
6. Short Head of Biceps.
7. Long Head of Biceps.
8. Origin of Biceps and Semitendinosus.
9. Origin of Semi-membranosus.
10. Insertion of Semi-membranosus.

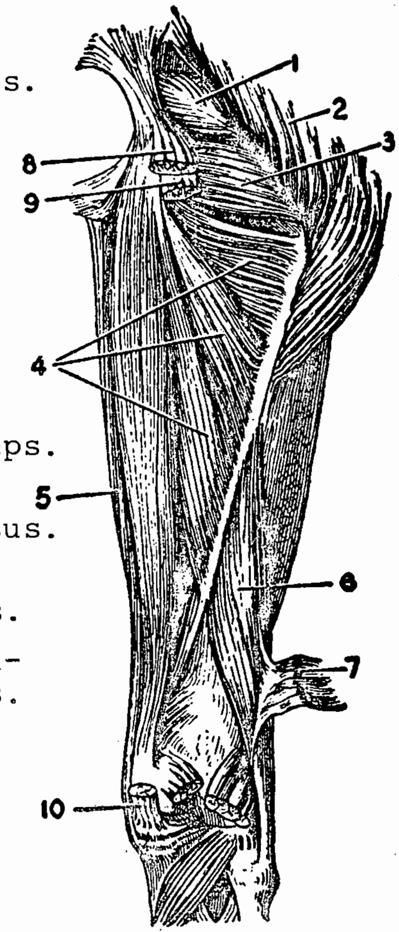

Fig. 3.9 Deep Dissection of Muscles of Back of Right Thigh.

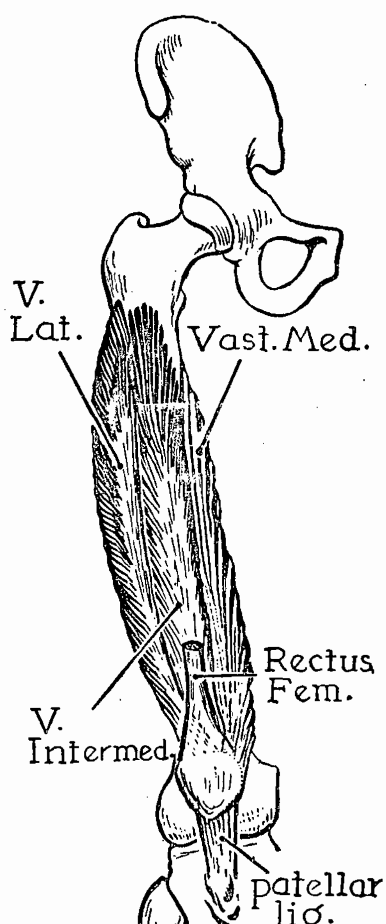

Fig. 3.8 Vasti from in Front with Rectus Femoris Removed.

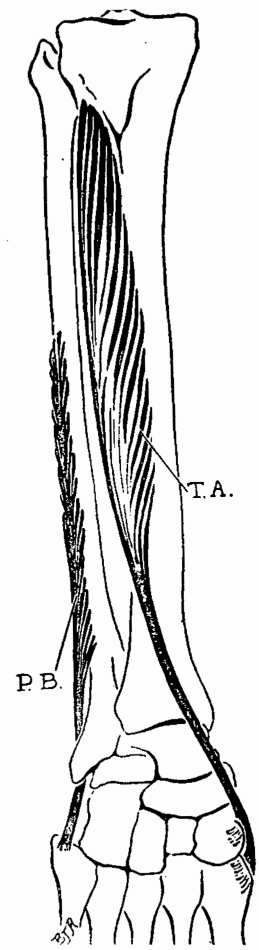

Fig.3.10 Right Tibialis Anterior and Peroneus Brevis from in Front.

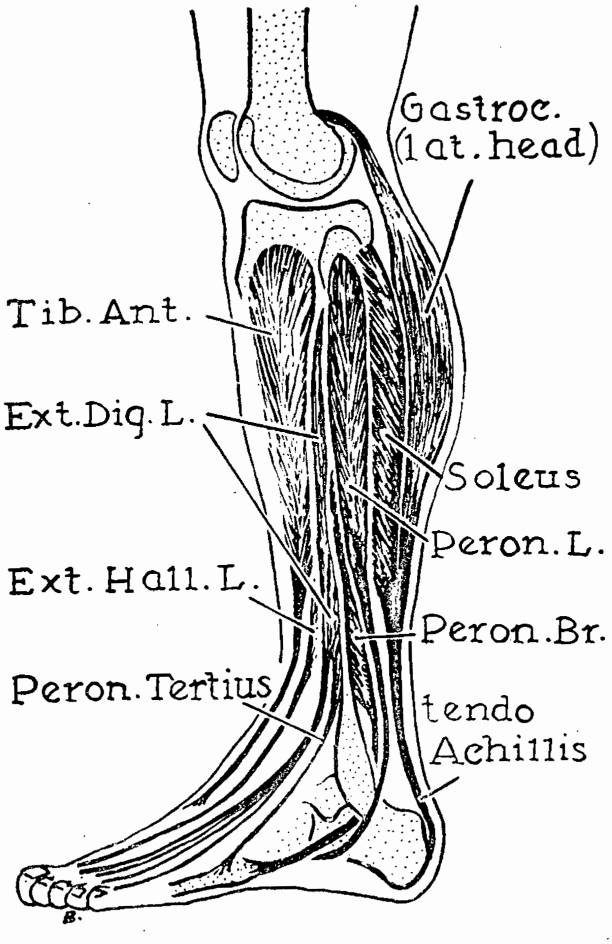

Fig. 3.11 Muscles of Left Leg from Lateral side.

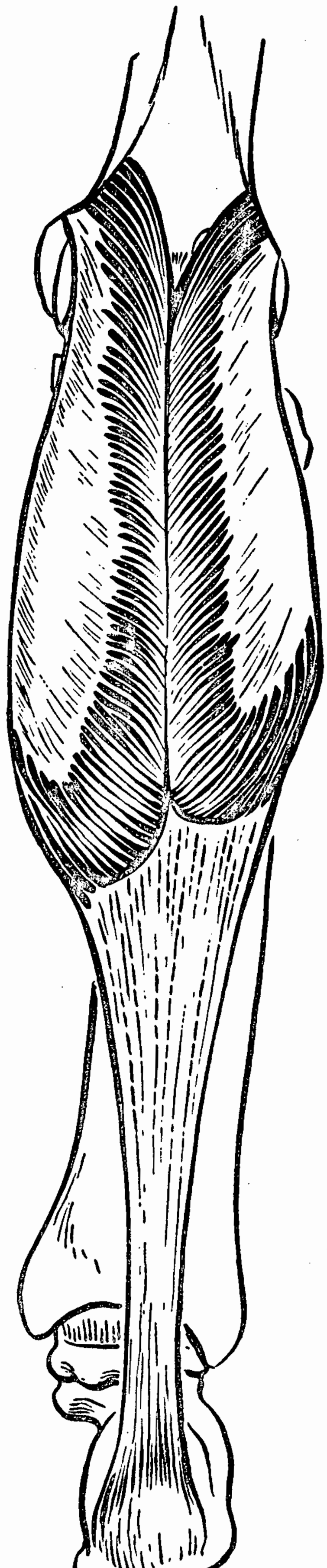

Fig. 3.12 Right Gastrocnemius and Tendo Calcaneus from Behind.

CHAPTER 4.

PHYSIOLOGICAL FACTORS IN HUMAN LOCOMOTION.

4.1 Introduction.

Human beings move by contracting their muscles. Hence muscular contraction is one of the key processes of life. Muscular contraction initiates a reversible chain of events involving structure and associated electrical, chemical, thermal and mechanical changes.

A brief discussion below on the neuromuscular connection, muscular structure and associated electrical, chemical and thermal attributes of muscle precedes a lengthier review on the mechanical properties of muscle. In the latter the emphasis is on those semi-empirical models which are considered to give the correct mechanical changes which occur during muscular contraction.

4.2 The Nervous System.

Locomotion activity usually involves decision and supervision by the 'higher centres' in the brain. The brain, in controlling muscles, directs to the neural organizations directly connected with the muscles (viz. the spinal segments) command volleys of impulses.
Under the action of these command impulses, and impulse signals arriving at the spinal segment from the proprioceptors of the muscle itself, a purposeful contraction of the muscle occurs, and a corresponding purposeful motion of the skeletal bones.

Many investigations, (Bernstein (1947),Hammond (1956)), have been carried out on the control mechanism of muscular activity. By control mechanism will be understood the law of generation of these command impulses (i.e. under which conditions they arise and where they are delivered) and methods for their generation.

Aizerman (1968) has studied a mechanism conventially called simple search mechanism (S.S.M) This mechanism controls muscles in conditions where it is required to maintain precisely a certain found position or velocity (in certain cases a frequency) of a part of the skeleton or joint angles e.g. the mechanism takes active part in the maintenance of the vertical position.

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Aizerman expressed the hypothesis that the S.S.M. was also used to control time variation of the lengths of muscles in periodically repeating movements like habitual walking.

An experiment (see appendix 1) in the Dept. of Bio-engineering and Medical Physics, U.C.T. was designed to test this hypothesis. In this experiment processed electromyographic activity of the flexor and extensor muscles about the knee is correlated with the absolute angular velocity about the knee. The subjects walked comfortably repeating approximately the same step-period and step-length with the speed approximately constant. It was found that a peak of muscle activity occurred just before heel-strike in both the extensor (quadriceps) and flexor (hamstring) as the absolute value of the angular velocity ($\frac{d}{dt}\theta_{knee}$), increasing, passed through a certain threshold (or band of threshold values).

This confirmed experimentally the theoretical prediction of Aizerman. Namely, that the S.S.M. mechanism could serve not only to maintain minimum 'unpleasantness' but also to control time variation of the lengths of muscles (in particular in joint movements) for example in the simplest, periodically repeating movements (breathing, habitual walking etc.,).

4.3 Muscle Structure. (Fig.4.1)

A skeletal muscle consists of long cylindrical muscle fibres which vary in length from 1 to 40 mm., and in diameter from 10 to 100 μ. Each muscle fibre is composed of smaller units called myofibrils, which are in the sarcoplasm. Myofibrils range from 1 to 2 μ in diameter. Each muscle fibre is enclosed by a structureless membrane - the sarcolemma.

Individual muscle fibres have a striated appearance. The banding of the myofibrils comprises alternate dark (A band) and light (I band) areas. In the centre of each A band is a less dense area called the H zone. Each I band is bisected by a narrow dark line called the Z-line. The unit which encompasses all the material between two Z-lines is called the sarcomere. Consider the myofibrils to be arranged in parallel fashion.

4.4 Electrical Attributes (Fig.4.2)

The electrical properties of skeletal muscle parallel those properties present in nerve fibres. Around each muscle fibre is an electrically polarized membrane the inside of which is 90 mv negative with respect to the outside. During muscular activity, the membrane is temporarily depolarized and the muscle contracts. By this means the activity of the muscles is controlled by the nervous system. An impulse travelling down a motor nerve is transmitted to the muscle membrane whereupon a wave of depolarization (action potential) sweeps down the muscle fibre causing the muscular twitch. The significant aspect of the muscle action potential is that the electrical response always occurs before, and is completed by , the time the mechanical response begins.

Fig.4.2 illustrates the form of the action potential.

4.5 Thermal Changes.

During the events resulting from excitation and contraction there occur within the muscle fibres a number of chemical reactions resulting in liberation of energy. Associated with this energy liberation is the production of heat.

Inactive muscle liberates a resting heat. Activated muscle liberates heat in excess of the resting base level. In the case of a single twitch, this heat is called activation heat whereas in the case of tetanic contraction (summation of twitches) it is called maintenance heat.
A muscle which contracts and shortens under a constant load is said to contract isotonically.
A muscle which contracts but is held at its two ends in such a way that it does not shorten is said to contract isometrically. Such a contracting muscle operates during the maintenance of posture and equilibrium and in holding an object.

Activation heat is seen whether the muscle contracts isotonically or isometrically and represents energy expenditure for maintenance of the active state. If, however, the muscle shortens under a load then more heat in the form of shortening heat is liberated. Shortening heat is independent of the work done by the muscle and also of the speed of muscle shortening and is dependent only on the amount of shortening.

However the total energy (heat and work) expended by the contracting muscle must increase with load. When muscle contraction is over, relaxation begins, and as the load is lowered there is a further liberation of heat in the form of relaxation heat. This appears as the result of the muscle being stretched by the load.

4.6 Chemical Changes. (Fig. 4.3 and 4.4)

The chemical reactions which provide the energy for contraction must therefore be controlled not only by the change in the length of the muscle, but also by the tension placed on the muscle during the change. From the chemical point of view, the contractile structure of muscle consists of mainly three proteins myosin, actin and tropomyosin. It seems that the myosin filaments mainly reside in the A band while the actin filaments reside in the I band.

The exact mechanism involved in the activity of the contractile elements when the cell membrane becomes excited has not been completely elucidated. The currently accepted theory is the interdigitation or sliding of the myofilaments relative to each other. The myosin filaments are about 100 A^o thick and 450 A^o apart and extend through the A band. The actin filaments are half as thick and extend from a Z line to the H line. The actin filaments are arranged hexagonally and each actin filament is surrounded by three myosin filaments in the A band. When a muscle changes its length the two sets of filaments slide past each other. (Fig. 4.3).

Huxley (1958) believes a 'catch' mechanism occurs in which side extensions (cross-bridges) move the myofilaments past one another by a rachetlike process. The energy of contraction comes from adenosine triphosphate (ATP) containing energy-rich phosphate binds. (Fig. 4.4).
P.T.Hadingham(1972) considers Huxley's sliding-filament model and finds an expression for the sliding shear σ between the two contractile proteins actin and myosin as a product of the activity $A(t)$ and the amount of overlap Δ. Activity being the number of cross-bridges activated per unit length of overlap x force generated per cross-bridge i.e.

$$\sigma = A(t) \times \Delta .$$

Some comments are also made concerning the physical mechanism of this shear.

4.7 Mechanical Properties.

All striated muscle when stimulated will contract and develop tension. Under any given situation a muscle contracting tetanically exerts its maximum tension. However, the amount of tension that is developed depends upon a number of factors, one of which is the length of the muscle at the time of stimulation. Thus if a muscle is set at rest length, the tension it develops during tetanus is greater than that developed with longer or lesser length than the rest length.

A Muscle Model (Fig.4.5)

The most commonly used model for skeletal muscle is a combination of a series elastic component (S E C) a contractile component (CC) and a parallel elastic component (PEC), each of which contributes to the total force at the muscle insertion in the bone. (Pringle (1960), Sandow (1961)). This is illustrated in Fig.4.5. The empirical formulation of this model for whole tetanized muscle is

$$P = b.P_o - a\left(V + \frac{d}{dt}f(P)\right) . \left(V + \frac{df(P)}{dt} + b\right)$$

where

P = External muscle force.

P_o = Tetanic isometric muscle force.

V = Contraction velocity

a, b = Empirical Hill's constants

$f(P)$ = stress - strain curve of the SEC (Hill (1939). Wilkie (1956))

The basic micro-structure of skeletal muscle as well as the muscular contraction have been described above in sections 4.3 and 4.6 respectively.

This contraction mechanism is represented by the CC in the model.

Sandow (1961) showed that the SEC accounts for less than 20% of the total muscle resting length. It can thus be neglected in any length calculations. Also, Wilkie (1950) showed it to have negligible influence in velocity calculations. It has thus been decided to omit a SEC in the subsequent discussion.

Also the PEC may be neglected in an active muscle and only becomes important in passive muscle stretched beyond 30% of its resting length. The experimental work presented in this thesis was mostly done at speeds where the leg angles did not reach more than 25 or 30 degrees. Hence it was concluded (Galiana (1968)) that the muscles concerned are never stretched beyond this 30% limit and that the PEC can also be neglected in the muscle model.

We thus describe the model by a contractile component only.

<u>Force-Velocity Relationship</u> (Fig.4.6,4.7)

If one plots isotonic force against initial velocity of shortening, a curve of characteristic shape is obtained. The shape of the force-velocity-curve is more or less the same for muscles from many different types of animals(Fig.4.6) Many algebraic equations can be fitted to the empirical force-velocity curve. Among the more well-known ones are:

(i) $(P+a)\,(V+b) = b.(Po+a)$ Hill (1938) (4.1)

P = force V = velocity

Po = isometric tension

a,b constants

Note that the correction due to the SEC has been neglected.

(ii) $P = Po\,e^{-aV} - kV$ Fenn & Marsh (1935) (4.2)

a,k constants.

(iii) $P = Po\;e^{-\,pV/bPo} - Va/b$ Polissar (1952) (4.3)

a,b constants as in Hill's eqn.

Other mathematical relations that have been attempted are:

(iv) $P = Po(L) + Pi\;(L)$ (Carlson 1957) (4.4)

$Po(L)$ is the isometric tetanic length tension curve and $Pi(L)$ is some viscous-like function.

(v) $r\dot{P} - P = KL - rf\dot{L}$ Brown (1959) (4.5)

r,f,K are not constants but depend on P,L and $\dot{L}$ in a nonlinear fashion.

Hill's equation can be interpreted in the following way: The rate of extra energy liberation (in excess of isometric) $(P + a)v$ is a rather exact linear function of the load P, increasing as P diminishes, being zero when $P = Po$ in an isometric contraction and having its greatest value for zero load. Thus

$$(P + a)\ (V + b) = b\ (Po + a) \qquad (4.6)$$

represents a rectangular hyperbola (Fig.4.6) with asymptotes at $P = -a$, $V = -b$ where

a = apparent resistance to shortening.
P = load
V = velocity of shortening
Po = isometric tension
b = constant defining the absolute rate of energy liberation and can also be written as

$$(P + a)\ V = b(Po - P) \qquad (4.7)$$

We can also re-write above expression in the form

$$(P + a)\ V = \frac{b(Po + a)\ V}{V + b} \qquad (4.8)$$

i.e. as speed of shortening increases, the total rate of energy liberation rises rapidly at first and then more slowly which shows directly that amount of chemical change per unit amount of shortening is not a constant.

It is thus clear that an advantage of Hill's equation over the others is its mathematical simplicity as well as simplicity in interpretation. Another advantage is its symmetry which becomes clear if we normalize it by introducing the variables

$\nu = P/Po$ & $\mu = V/Vm$ in the expression:

$$(P + a)\ (V + b) = b(Po + a) = a(b + Vm) \qquad (4.9)$$

where Vm = velocity of contraction when load is zero.

$\frac{a}{Po} = \frac{b}{Vm} = n$ Then the functional form of P(V) is identical to that of V(P), a situation reminiscent of the phenomenological relations of non-equilibrium thermo-dynamics.

S.R.Caplan (1972) has shown that skeletal muscle may in principle belong to the class of simple autonomic (self-regulated) linear energy converters for which the Hill force-velocity relation is the general output relation and as such contains only a single adjustable parameter - the degree of coupling. The tighter the coupling, the greater the curvature of the Hill hyperbolae. A characteristic of the Hill equation is that at a high degree of coupling the power output remains essentially constant over a considerable range of load resistance which is roughly the case in muscle as was pointed out by Fenn & Marsh (1935).

We should note, however, that in Hill's equation

$$(P + a) V = b (Po - P) \tag{4.10}$$

Po appears as a constant and this equation applies only for small length changes in the region near the flat top of the tension - length curve. (Fig. 4.7).
Hill's equation, however, can be modified to apply at other lengths by arranging that Po should vary with muscle length L according to the tension - length curve F2(L)

$$(P + a) V = (F2(L) - P) \tag{4.11}$$

This equation then describes the full range of shortening of the tetanized muscle with fair accuracy (Abbott & Wilkie, (1953)).
Hill's equation can be modified still further by taking into account the decline in activity following a single stimulus by writing.

$$Po (L,t) = \frac{F2(L).F4(t)}{Po} \tag{4.12}$$

Where Po is the original Po of Hill's equation i.e. the tetanic tension at body-length.
The equation $(P + a)V = b(\frac{F2(L).F4(t)}{Po} - P)$ can be tested by inserting experimentally determined values for Po,a,b and for the tension-length and active-state curves; then integrating to find how L changes with t for different values of P i.e. one predicts the shapes of isotonic twitches against various loads (Ritchie & Wilkie (1955)).
This modified Hill's equation could tentatively be regarded as the general equation describing the mechanical properties of muscle.

Some equation other than Hill's, may be found whose form is more closely related to the physical chemistry of contraction, but it is clear that any theory which accounts both for the shape of the force-velocity curve and for the occurrence of shortening heat must at the same time be consistent with the known energy turn-over: Shortening heat and work are proportional to (isometric tension corresponding to instantaneous length) - (tension on contractile element) (Abbott & Wilkie (1953)).
This is the very criticism that could be levelled at Polissar's theory (iii) which is very comprehensive as regards mechanical and thermal data and could probably accommodate the known chemical events, but it takes no account of muscle structure and does not fit with some important facts about heat production.

From observations of tension versus change in muscle length it was apparent that non-linearities are involved in the muscle under study (e.g. the constant hysteresis loop in the phase relationship between length and tension regardless of frequency are indicators of the non-linearities)(See Fig.4.7)
Brown,A.C.(1959) proposed a model to account for the non-linearities: $r\dot{P} - P = KL - rf\dot{L}$ where r,f and K are not constants but are dependent on P,L and $\dot{L}$ in some cases in a non-linear fashion.
However despite its generality and inclusion of non-linearities there was no analytical expression given to r,f and K and they had to be generated and plotted graphically for each muscle.

Hill stated:- "The real question as between the various equations is which is the most useful? Helping to make a complicated mixture of observed facts more digestible, pointing out where to look for new ones and providing a 'take off' for the next jump into the unknown".

Caplan (1972) argues that Hill's equation satisfies this above criterion of usefulness and that its probable significance in the description of muscle behaviour has been overlooked.

Caplan concerns himself with a regulator which responds uniquely to the load. The input to the working element of the muscle being the input of chemical energy controlled by the regulator.

Modification of Hill's Equation. (Fig.4.8)

If equation 4.1 is now modified as shown below, it can also describe the behaviour of the CC of submaximally stimulated muscle (Bigland and Lippold (1954), Milsum (1966)).

$$P(L,v,Z) = ZPo(L)\frac{(1 - \frac{V}{Vm})}{(1 + \frac{V}{nVm})} \qquad (4.13)$$

or

$$P=ZPo(1 - \frac{V}{Vm})(1 - \frac{V}{nVm} + \frac{V^2}{2!n^2 Vm^2} - \ldots) \qquad (4.14)$$

Vm = maximum velocity of contraction (at P=o)

$n = \frac{a}{Po} = \frac{b}{Vm}$

Z = neural stimulation level (O<Z<1)

P = muscle force at that level of stimulation

Po, the isometric muscle force at maximal (100%) stimulation, is proportional to the muscle cross-sectional area and a function of the muscle length.

Again because of the low walking speeds and leg angles, we assume

V<Vm and Po constant

Under these conditions, equation (4.14) reduces to

$$P=ZPo.(1 - \frac{V(1 + \frac{1}{n})}{Vm}) \qquad (4.15)$$

which is the formula used for contracting muscle by Chow & Jacobson (1971) in their derivation of an energy performance criterion.

However equation (4.15) only holds when the activated muscle is contracting, that is shortening, under load. When an activated muscle is stretched under a load greater than Po,

4.8 REFERENCES.

Abbott, B.C. & Wilkie,D.R.(1953):"The relationship between velocity of shortening and the tension-length curve of skeletal muscle". J.of Physiol 120: 214-223.

Aizerman, M.A and Andreeva,E.A.(1968): "Simple Search Mechanism for Control of Skeletal Muscles" in Automation and Remote Control. Issue N3 pg 452.

Bernstein,N.A.(1947): "The Structure of Motion (In Russian) Meditsina.

Bigland,B. & Lippold,O.C.J.(1954):"The Relation Between Force, Velocity and Integrated Electrical Activity in Human Muscles". J.Physiol; 123: 214-224.

Brown,A.C.(1959):"Analysis of the myotatic reflex". Ph.D. dissertation, University of Washington.

Caplan,S.R(1972):"Autonomic Energy Conversion.
I. The Input Relation: Phenomenological & Mechanistic Considerations.
II. An Approach to the Energetics of Muscular Contraction".
Biophysical J.8 : 1146-1166.
1167-1193.

Carlson,F.D.(1957): "Kinematic studies on mechanical properties of muscle" in Tissue Elasticity J.W.Remington,Ed.Washington,D.C. Amer.Physiol Soc. pp 55-72.

Fenn & Marsh. (1935):"Muscular Force at Different Speeds of Shortening". J.of Physiol. (Lon) 85 : 277- 297.

Galiana,H.L.(1968): "Modelling the Human Leg in Walking". M.Sc.thesis Dept of Electrical Engineering. Mc.Gill University Montreal. Canada.

Hadingham,P.T.(1972):Research report No.6 "A new muscle model: Preliminary thoughts". Dept. of Bio-engineering and Medical Physics. University of Cape Town.

Hammond,P.H. and Merton,R.A.(1956):"Nervous gradation of muscular contraction". Brit.Med.Bulletin 12 : No.3.

Hill, A.V: "Trails and Trials" in Physiology. Ed.Arnold (London) pg 68.

Hill,A.V.(1938): "The Heat of Shortening and the Dynamic Constants of Muscle". Proc.Roy.Soc.(Lon)Ser B 126: 136-195.

Hill,A.V.(1939): "The Dynamic Constants of Human Muscle". Proc.Roy.Soc.(London),Ser R,Vol.128: 263-274.

Huxley, H.E (1958): "The Contraction of Muscle" Sci.Amer. 199: 67-82.

Katz,B.(1939): "The relation Between Force and Speed in Muscular Contraction", J.Physiol., 96: 45.

Polissar (1952): "Physical Chemistry of Contractile Process in Muscle". Parts I,II Amer.J.of Physiol.168: 766-792.

Pringle,J.W.S.(1960): "Models of Muscle" in Models and Analogues in Biology, Symp. of Soc. for Exp.Biol.,14 Cambridge University Press.

Ritchie,J.M.& Wilkie,D.R.(1955): "The dynamics of muscular contraction". J.of Physiol. 130 : 488.

Sandow,A.(1961): "Energetics of Muscular Contraction", in Biophysics of Physiological and Pharmacological Actions,Amer.Assoc. for the Advancement of Science, pub.69,ed.A.M.Shanes.

Stark,L.,Houk,J,,Willis,P.A.,Elkind,J.,(1962).Jan.: "The Dynamic Characteristics of a Muscle Model Used in Digital-Computer Simulation of an Agonist-Antagonist Muscle System in Man". MIT QPR 64, Res.Lab. of Electronics.

Wilkie,D.R.(1950):"The Relation Between Force and Velocity in Human Muscle". J.Physiol., 110: 249-280.

Wilkie,D.R.(1956): "The Mechanical Properties of Muscle". Brit. Med. Bull. 12: 177.

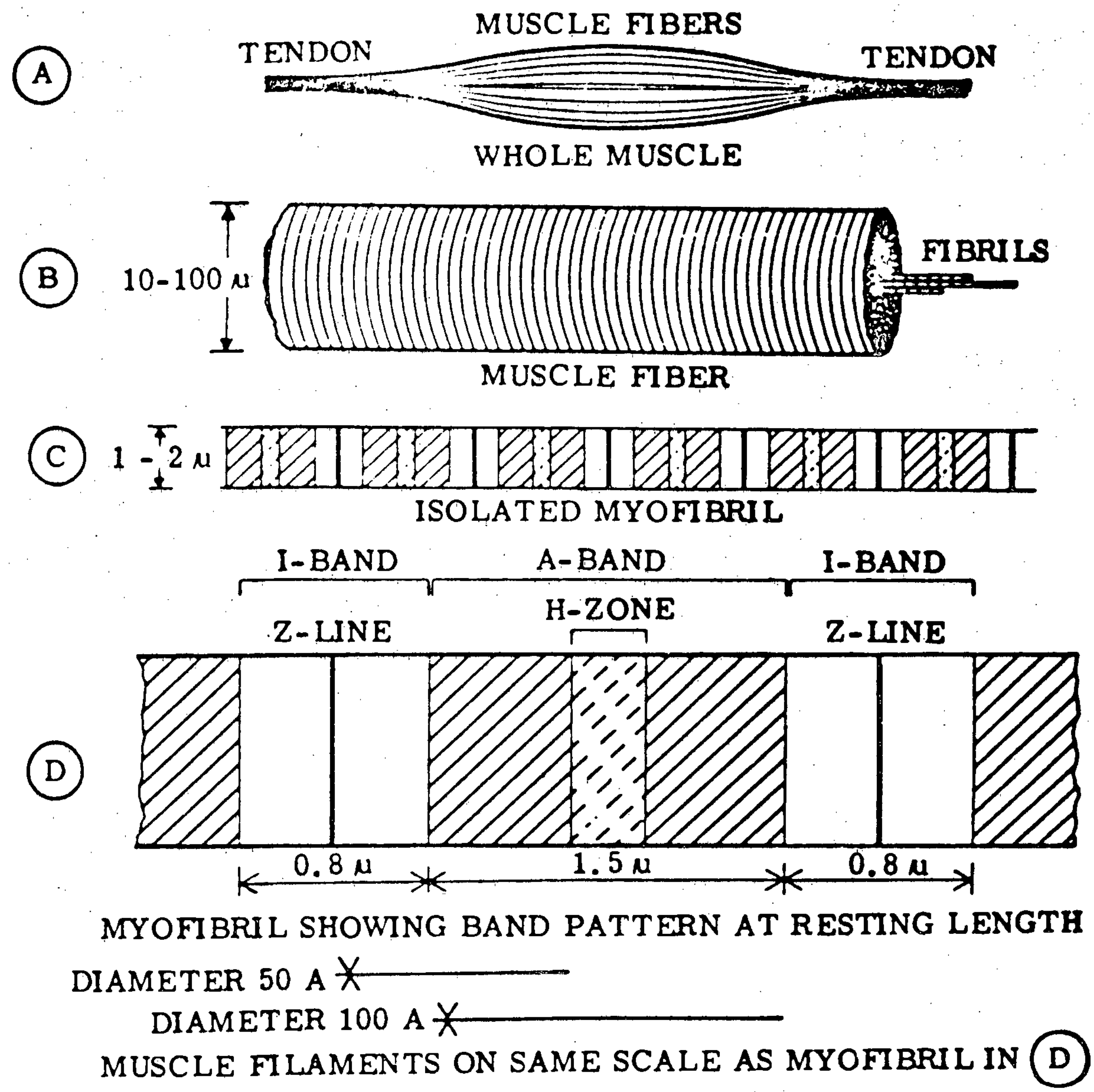

Fig. 4.1 Structure of Muscle.

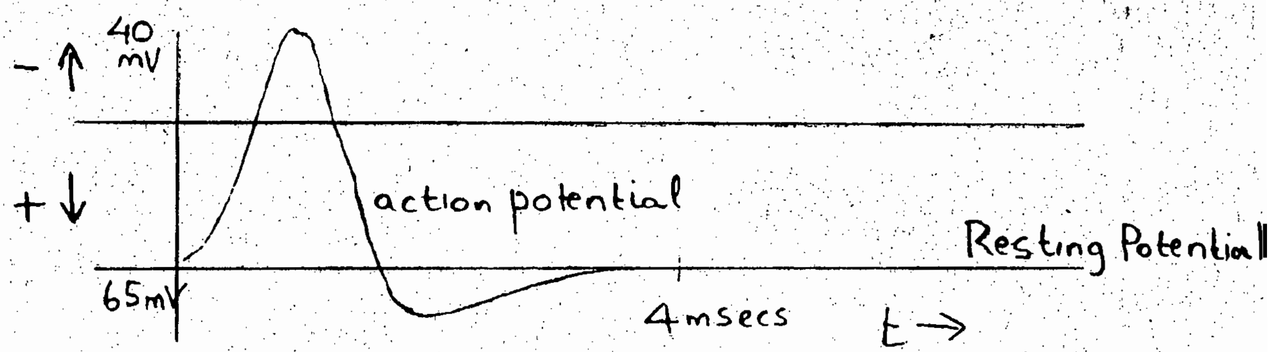

Fig. 4.2 The Action Potential.

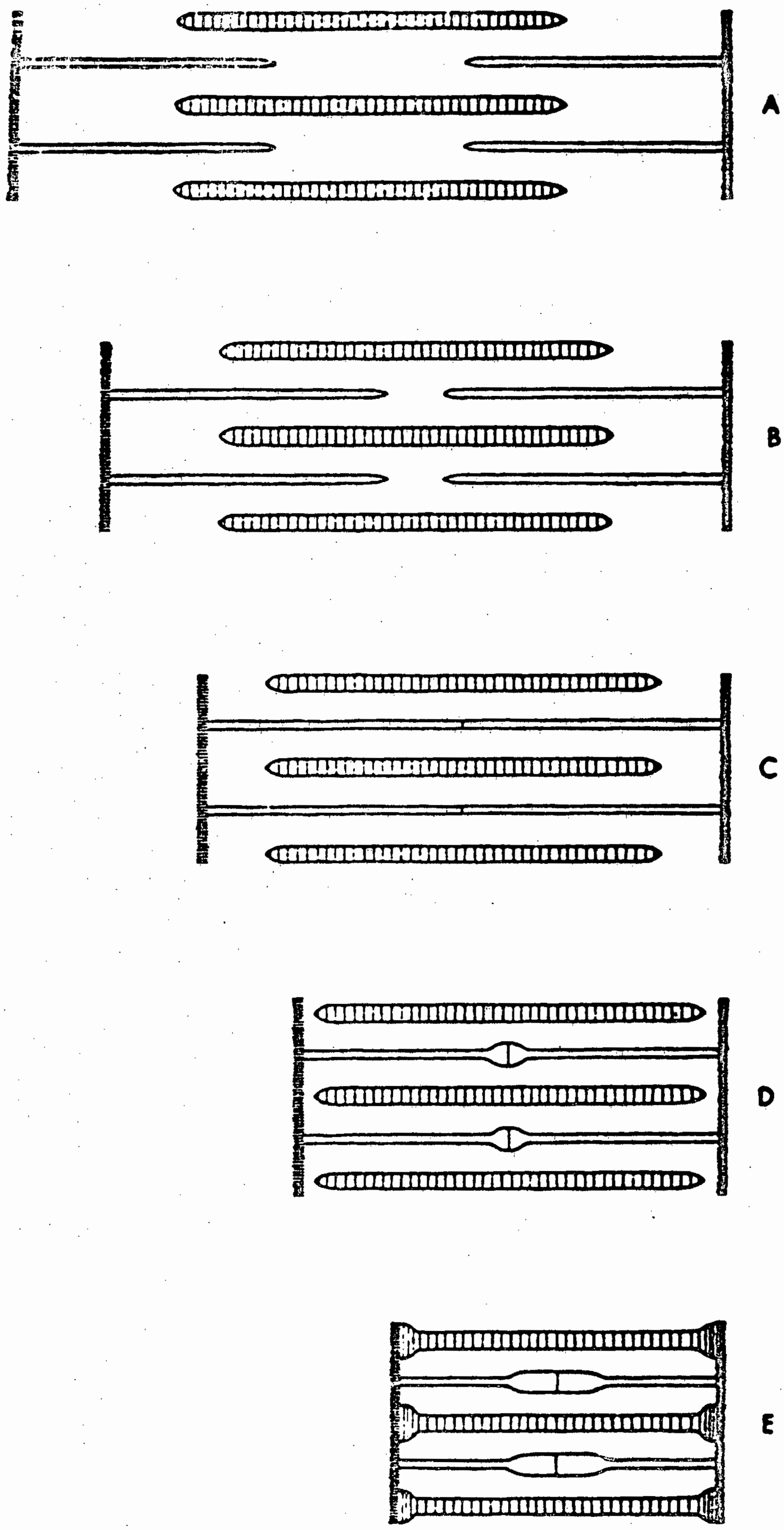

Fig. 4.3 Change in Length of the Muscle changes the Arrangement of the Filaments.

In A the muscle is stretched; in B it is at its resting length; in C,D,and E it is contracted. In C the thin filaments meet; in D and E they crumple up. (From Huxley (1958)).

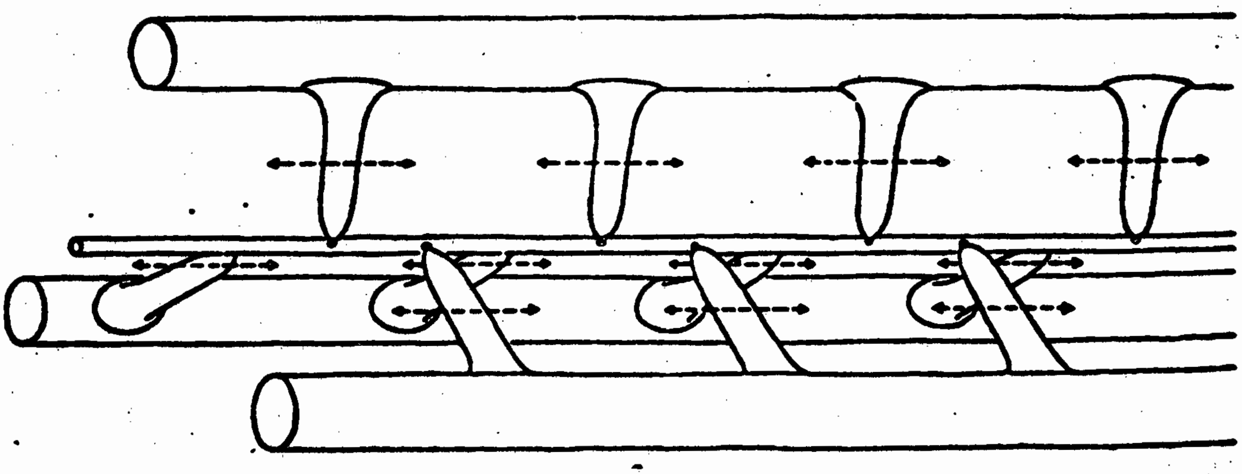

Fig. 4.4 Arrangement of cross-bridges suggests that they enable the thick filaments to pull the thin filaments by a kind of ratchet action. In this schematic drawing one thin filament lies among three thick ones. Each bridge is a part of a thick filament, but it is able to hook onto a thin filament at an active site (dot). Presumably the bridges are able to bend back and forth (arrows). A single bridge might thus hook onto an active site, pull the thin filament a short distance, then release it and hook onto the next active site. (From Huxley (1958)).

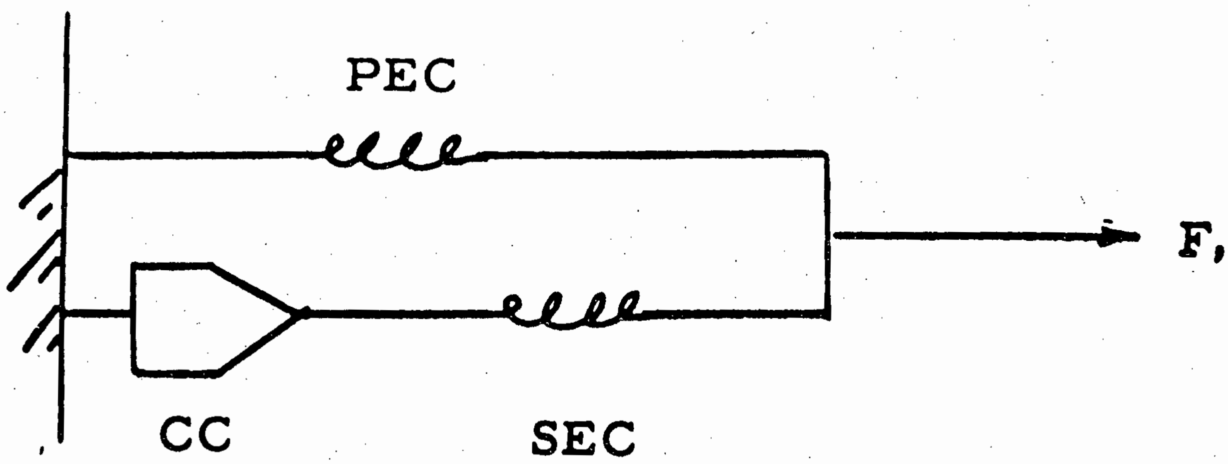

Fig. 4.5   A Muscle Model.

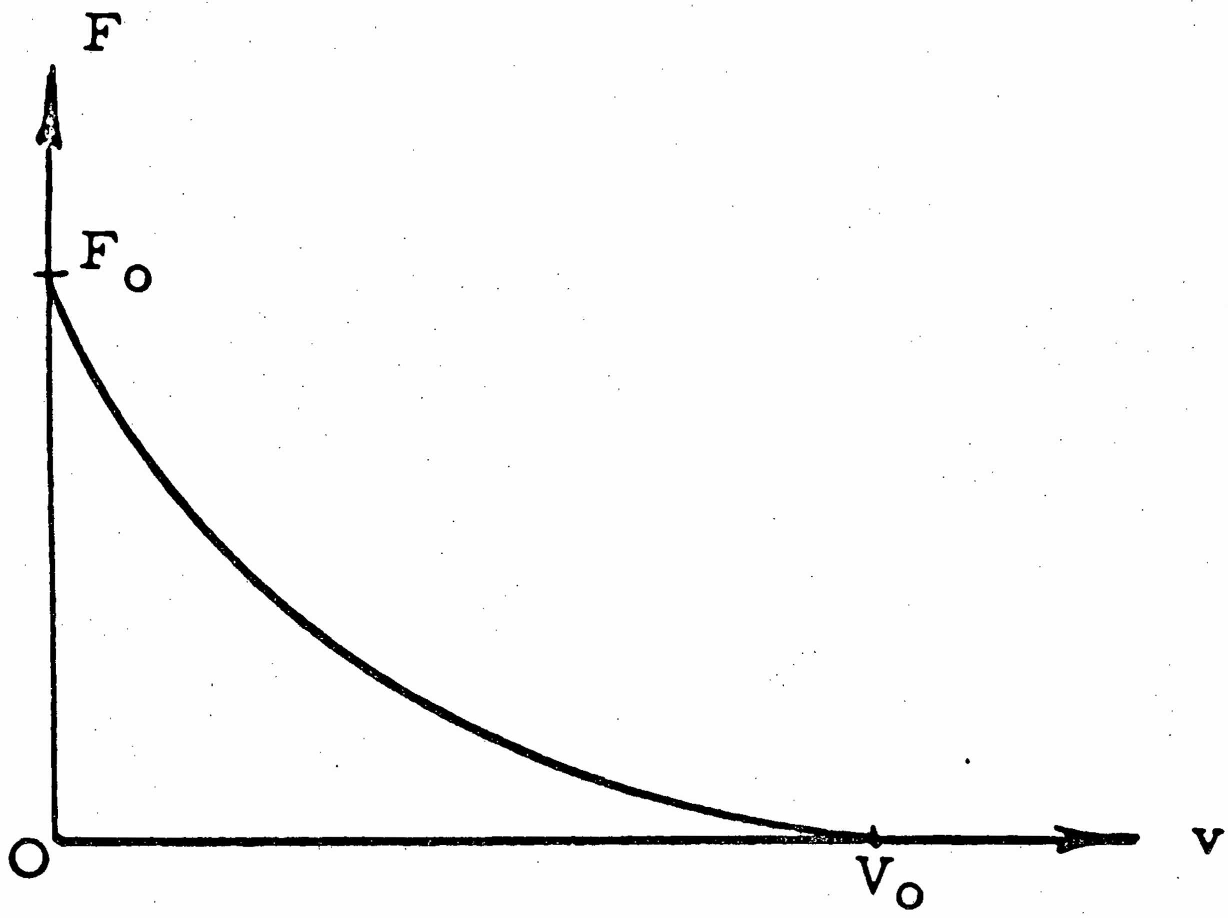

Fig. 4.6 Force-Velocity Curve for Skeletal Muscle.

P = F = Force

V = Velocity

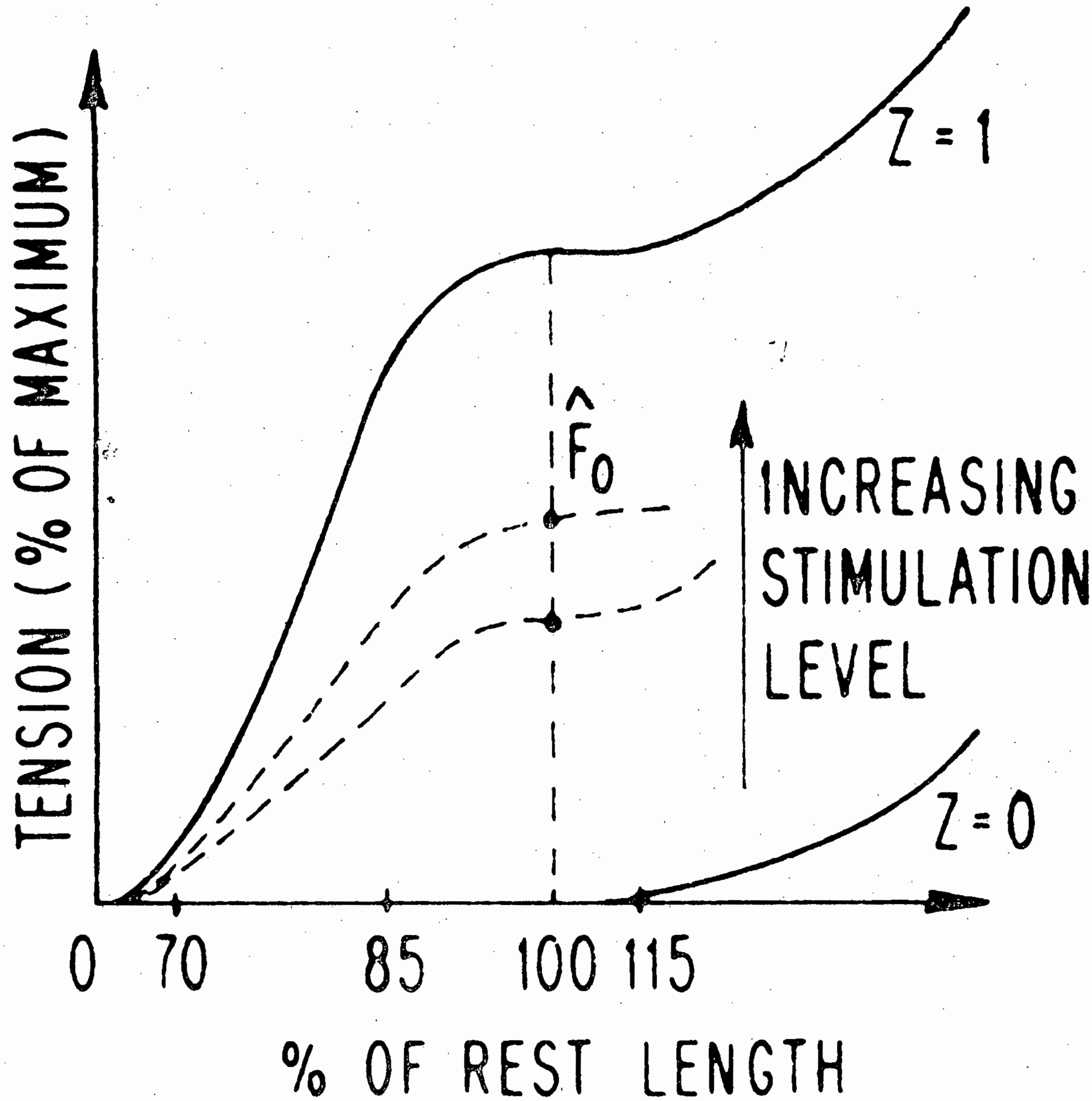

Fig. 4.7 Tension - Length Curve for Skeletal Muscle.

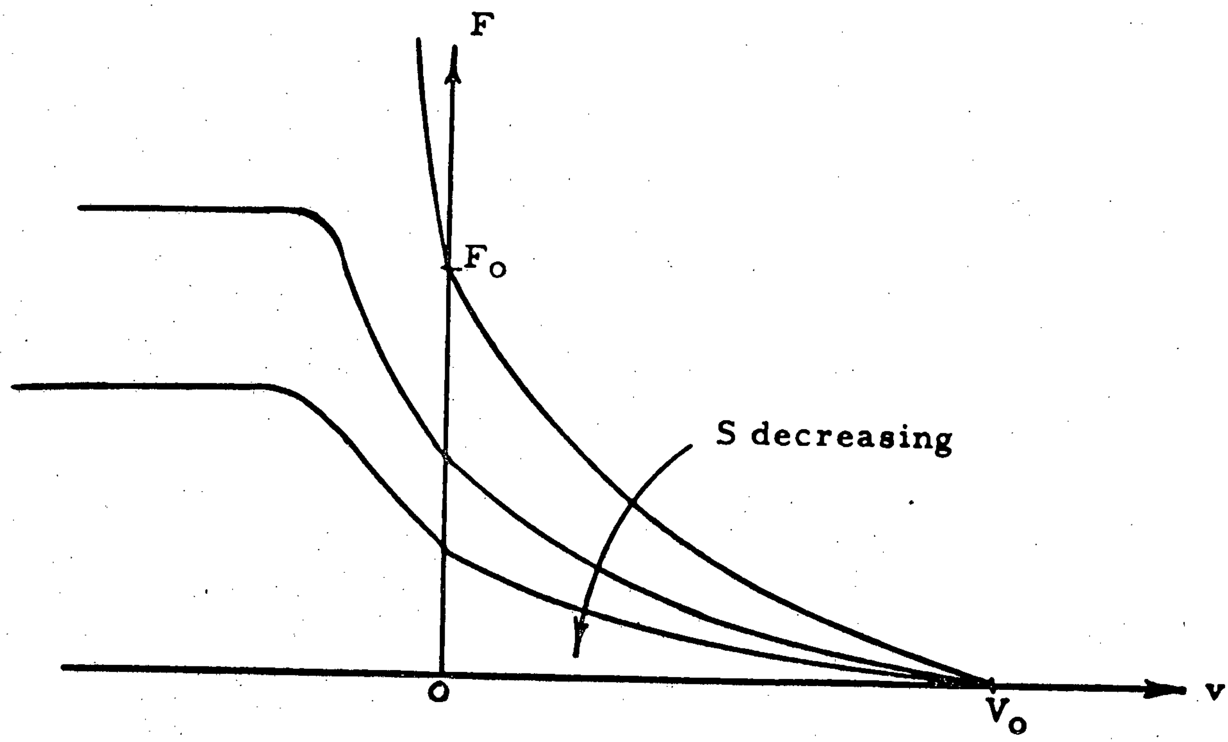

Fig. 4.8 Force - Velocity Curves.

Muscle is stretched under a load greater than Fo.
Po=Fo= Isometric muscle force at maximal stimulation.

CHAPTER 5.

A BRIEF REVIEW OF EXPERIMENTAL TECHNIQUES IN THE STUDY OF HUMAN LOCOMOTION

5.1 Photographic Techniques. (Fig. 5.1)

These techniques involve the study of trajectories in space and time.

Examples of such trajectories are:

(a) Displacement (vertical and horizontal) of Hip, Knee and Ankle versus Time.

(b) Displacement (vertical and horizontal) of Hip, Knee and Ankle versus Distance.

(c) Variation in the angle between Thigh and Shank versus Time.

(d) Angle about the Hip versus Angle about the Knee.

Photography, either still or movie usually provides the most useful tool for the above studies.
The subject is usually constrained to walk along a walkway.
The most useful example is the glass walkway.
This type of walkway enables studies of the displacements of points on the body to be made in the three planes of space simultaneously. A mirror placed below the transparent walkway and inclined at 45^{o} enables records of the side and bottom view of the subject to be made with one camera. An additional synchronously operated camera placed at the end of the walkway adds the view in the third plane.

A type of photography becoming increasingly popular in this study is interrupted - light - photography or strobe - photography. Here consecutive phases of action is exposed upon a single film. This demonstrates sequential actions of a particular motion and provides information for the determination of forces acting upon the limb(s) concerned. Reflector markers are placed on joints of limbs and joining the exposed points on the developed film with straight lines leads to a 'stick-diagram'. (Fig.5.1)

An alternative to the above technique is the use of electro-goniometers. This type of device incorporates a potentiometer and enables one to measure the joint angle

By feeding the output voltage through a differentiating circuit one can obtain $\frac{d\theta}{dt}$ and further information can be analysed (appendix 1).

Other walkways used are those made of a conducting material. Both shoes worn by the subject are fitted with contacts (foot-switches) at the heel and toe and a foot-switch pattern can be obtained indicating the time-sequence of ground contact of the extremities.
Alternatives are pressure-sensitive flooring or changing capacitance between the subjects foot and the floor (Goldkamp 1968).

5.2 Accelerographic Study of Gait.

Acceleration is one of the most important parameters of gait. Force is equal to mass x acceleration. Hence a vectorial representation of acceleration permits the calculation of the force vector. This method has an advantage over the use of force plates, since it permits an analysis not only of stance but of the swing phase.

Liberson (1936) used an accelerometer which incorporated a piezoelectric quartz crystal. The principle involved is the sensitiveness of the quartz crystal to the pressure exerted along a certain axis. This sensitiveness is revealed by a development of an electric potential proportional to the pressure along another axis perpendicular to the one subjected to the pressure. Recently strain gauges have been used instead of the quartz crystal. In the strain gauge accelerometer, variable pressure produces changes in the resistance of the strain gauge.
The technique of using strain gauges was further modified by the use of paired accelerometers connected in opposition. This permits the recording of angular acceleration Lieberson (1962).

Another technique is the use of a tachograph to record the instantaneous forward velocity of the trunk during human locomotion (In Contini et al (1964) "Human Gait Characteristics").

5.3 Techniques associated with the study of the forces involved in human locomotion.

The forces in operation are muscular, gravitational and floor reactions.

Electromyography.

The amplification and recording of the action potentials of muscles as a method of studying their activities is a relatively new tool. While the apparatus required and the technique are complex, this method has many advantages.

The ideal method for the study of the action of muscles in the human body should give the following information:
(a) The exact phase of activity of the muscle being studied in relationship to the movement of the particular skeletal part should be demonstrated. This phase relationship should include the exact time the muscle begins activity, and the exact time at which such activity is terminated. It should also indicate the varying degree of activity during its period of action.

(b) The study should also indicate the magnitude of the force being developed in the muscle at any instant so that, together with its direction as determined by anatomical studies, vector diagrams can be drawn.

Electromyography fulfills the first of these requirements of the ideal method for the study of muscle activity. It reveals the precise phase relationship of muscle action. However, it gives only suggestive or qualitative information concerning the forces generated withing the muscle. This failure is inherent in the physiological characteristics of the muscle, because the electrical changes are not directly proportional to the contraction processes.

5.4 Principles Underlying Electromyography.

(Milner (1968) University of California Report No.12(1947)).

The muscles of the body are accumulations of individual muscle fibres. These fibres constitute the ultimate functional unit and possess common physiological characteristics.

One of these characteristics is the so called 'all or none' response. This consists of the following series of events.
If the muscle fibre is adequately stimulated, it will undergo a contraction which is maximal under the circumstances. A short period then follows in which the muscle recovers from its contraction. During the early phase of activity, it will not respond to further stimulation; this interval is known as the 'refractory period'. Subsequently it will respond to an additional stimulus and again undergo its maximal contraction. After stimulation and just preceding its contraction, the covering of the muscle fibre changes its permeability and the surface of the fibre becomes negatively charged as compared to surrounding tissues. This difference in potential can be picked up from single fibres or larger aggregates by electrodes and amplified and recorded. The recording of these 'action potentials' from muscle constitutes electromyography (Fig 4.2)

The electrical changes occurring on the surface of the muscle fibre are not identical with the contraction processes, but may be looked upon as epiphenomena. The action potential remains constant each time the muscle fibre is activated, but the force exerted by the contraction of this fibre is dependent upon other factors inherent in the dynamics of the muscle. Therefore the recording of action potentials gives no direct information as to the tension developed in the fibres during its contraction, but it does indicate the precise moment at which muscle activity is initiated or suppressed.
Because of the 'all or none' response of single muscle fibres, smooth motion is achieved by collecting many fibres together and activating them asynchronously. Graduations in the total tension of the muscle are obtained by varying the number of motor fibres acting at a particular time. The greater the number of individual fibres contracting, the greater the total tension will be exerted by the muscle at its attachments. Therefore, to a certain extent, the action potentials do parallel the forces developed by the muscle insofar as greater total electrical potentials indicate a greater number of active individual motor units.

The methods of sampling muscles are numerous and the techniques employed depend upon the aim and interest of the investigator. If single motor units are being studied, small concentric needle electrodes may be employed. If the action of an entire muscle is to be studied, then the problem becomes one of statistical sampling to achieve a satisfactory representation which will indicate the average activity of all fibres. In the present investigation skin electrodes were employed and placed as near the centre of the muscle belly as was anatomically practically. The field of the electrodes was at right angles to the long axis of the muscle fibres; the electrodes were separated to include as much of the muscle substance as possible without running the risk of picking up stray potentials from adjacent groups.

Electromyography as a tool to study active muscle function has been utilized by Basmajian (1962) Inman (1951) Joseph (1960) and others. Electrodes placed over the four main muscle groups of the lower limbs (Quadriceps, Tibialis Anterior, Gastrocnemius, Hamstrings) pick up muscular activity and long trailing lengths of recording leads connect the subject and the amplifying equipment.

5.5 Telemetry.

Another approach is the use of telemetry (Ziskind and Milner (1972)).

The signal sub-carriers are amplitude-modulated and these are combined and transmitted by frequency modulation. A commercially available FM receiver is utilized in conjuction with the appropriate filters for recovery of the desired signals. Range is approximately 30 metres in an open space. If desired, other physiological variables can easily be accommodated in place of the e.m.g. signals.

The portion of the system carried by the subject weighs 500 gms(approx.) and is not unduly bulky. Flexibility of use and portability are two essential features. For example, subjects can be studied with very little constraint imposed upon them.

5.6 Force Plate Studies.

Force plate studies have been conducted (Inman (1966)) using strain gauges which record the magnitudes of the ground re-actions, bending moments and horizontal shears of the foot. Combining such data with simultaneously recorded stick diagrams, fairly complete descriptions of the external forces acting on the limbs and the moments acting at the joints are obtained.

5.7 Computer analysis of electro-myographic data.

Instead of the electro-myographic signals being ultimately fed into an ultra-violet pen-recorder, they can be fed into a 14 - channel analogue tape recorder. The analogue tape is then incorporated in an analogue-to-digital converter and the data is ultimately put on to an IBM tape in the correct format so as to be able to be read in the nearest computer. (Hershler,McConnell and Milner (1972)). A computer program has been devised to analyse this data. Appendix 2 contains a listing of the program as well as a description of its potential.

5.8 REFERENCES.

Basmajian,J.V.(1962): Muscles Alive. Their functions revealed by electromyography, Baltimore, Williams & Wilkie Co.

Goldkamp,O.(1968): "Telemetric recording of gait phase activity in muscle". Arch.Phys.Med. & Rehab. June : 349-352.

Hershler,C. and McConnell, V. and Milner,M.(1972): Processing of Human Locomotion Data.
(i) Analogue-to-digital Conversion of Data using a Varian Mini-Computer.
(ii) A program for reading the magnetic tape output. Laboratory Technical Report: Dept.of Bio-engineering and Medical Physics,U.C.T.

Inman,V.T.(1966): "Human Locomotion" Can. Med.Assoc. J. 94: 1047-1054.

Inman,V.T.(Nov.1951):"Relation of Human electromyograms to muscular tension". Advisory Committee on Artificial Limbs, National Research Council, Northwestern University Prosthetic Research Centre, Series 2, Issue 18.

Joseph,J.(1960): Man's Posture: Electro-myographic studies, Springfield,Ill.,Charles C.Thomas.

Liberson,W.T.(1936): Le Travail Human 4: 1-7.

Liberson,W.T.(1962): Arch.Phys.Med.& Rehab.43: 547-551.

Milner,M.(1968): "Models of Nerve Excitation and Propogation with special reference to Multifibre Peripheral Nerve". Ph.D. Thesis, University of Witwatersrand, South Africa.

University of California (1947):"Fundamental studies of human locomotion and other information relating to the design of artificial limbs". Berkeley, California.

Ziskind,A & Milner,M.(1972):"A Biotelemetry System and some applications specifically concerning electromyography in gait". Dept.of Bio-engineering and Medical Physics, University of Cape Town (unpublished).

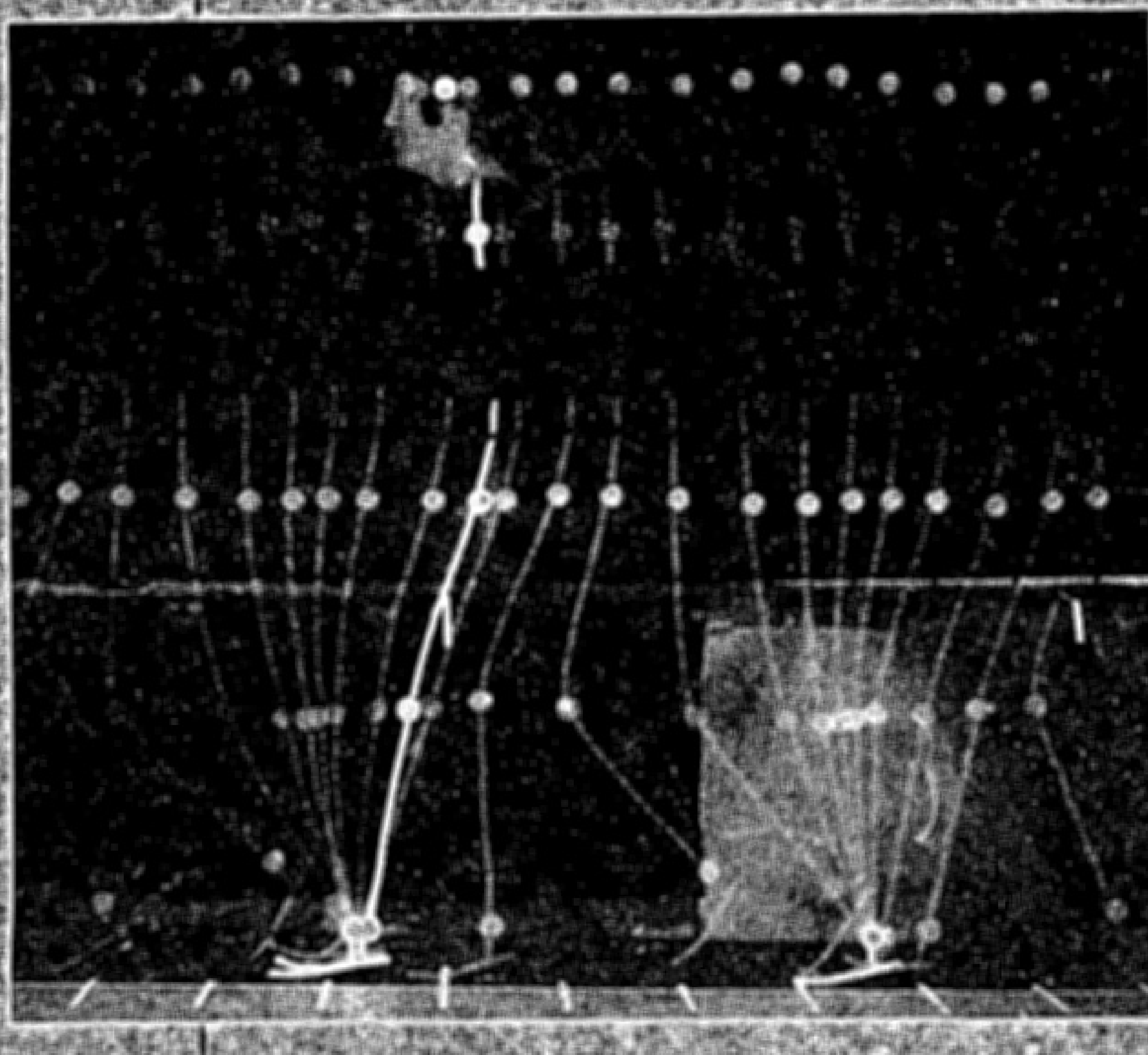

Fig. 3.17. A "Stick - Diagram".

Subject C.H.
Walking speed = 0.4 metres/second.

CHAPTER 6

MATHEMATICAL MODELLING OF HUMAN LOCOMOTION.

6.1 The Mathematical Model

In the preceding chapters various aspects of human locomotion have been reviewed. It is advantageous now to examine more closely the model of Chow and Jacobson(1971).
In order to use optimal programming, locomotion has to be transformed into an optimization problem. To do this it was proposed that locomotion obeyed a principle of optimality i.e. that one minimizes the mechanical work done by the muscles of the lower extremity during walking.
Such a hypothesis has not been confirmed conclusively by experiment but certain results do support it.
M.Milner et al (1970), for example, have shown that EMG activity is a minimum when the human subject is allowed to walk at a pace frequency of his own choice for a given speed of walking.

The dynamic equations and kinematic constraints for the system can now be developed using simple physical laws and gait information. A reproduction of part of the development of the system dynamics and auxiliary constraints taken from the paper by Chow and Jacobson is shown in appendix 3. Appendix 4 contains a summary of their very important derivation of an energy performance criterion.

Since we now have a mathematical model of the loco-motor system including kinematic and dynamic constraints, a performance criterion for the purpose of minimization and initial, terminal and ' in-flight' boundary conditions, we are in a position to formulate the problem explicitly. We then use modern control theory for numerical solution of the optimization problem. This leads to specification of the various moment quantities and the simulation of the gait patterns on the basis of minimum mechanical energy expenditure

6.2 Modifications introduced : Normal subject.

For the purposes of this investigation, the form of the dynamic equations and kinematic constraints derived for the three phasic portions of the walk-cycle (stance,deploy and swing) has been kept intact.

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The only modifications introduced have been the substitution of constants in the equations which more accurately reflect the physical characteristics of the subject.

Table I contains a list of these parameters. The starting values for the angular displacements have been obtained from inspection of photographic records of the subject (Chapter 8). The same source gives the lengths in time of the swing, deploy and stance portions of the walk-cycle.

A crucial part of the theory is the need for a prescribed vertical and horizontal hip trajectory in time. The experimental trajectories for subject C.H. are shown in figures 6.1 and 6.2. Mathematical functions were fitted to these curves.

For the vertical hip trajectory in figure 6.1, a Fourier Series Mathematical Package of the Univac 1106 was used. A brief description of this package can be found in appendix 5. It turned out that in most of the cases studied the second harmonic dominated. This then resulted in a mathematical function for the vertical hip trajectory of the form $A\,COS(\frac{4\pi T}{To})$ or $B\,SIN(\frac{4\pi T}{To})$

where A,B are constants and

To = period of the walk-cycle.

Because of its linearity throughout the walk-cycle, the best straight line fit to the data in figure 6.2 gave the horizontal hip trajectory.

In the stance portion prescribed vertical and horizontal ankle trajectories in time are also needed. These are again obtained by mathematically fitting functions to the experimental curves. These curves with the points pertaining to stance clearly marked are shown in figures 6.3 and 6.4.

Because of their obvious linearity during stance, a first-order polynomial fit was sufficient. Since the subject is normal, the motion of only one of the legs is studied, and it is assumed that the motion of the other leg and hence the resultant gait patterns are the same except for a possible phase difference.

6.3 Modifications introduced: Pathological Gait.

The particular type of pathological gait under consideration here is the one caused by an arthrodesed ankle. There is no rotation of the foot about the ankle joint. The same set of dynamic equations derived in appendix 3, is used except for substitution of more physically realistic constants (Table II) and owing to the fact there is no rotation of the foot about the ankle joint, the ankle moments ($M\alpha$ and $M\alpha'$) during stance and deploy are taken to be zero.
As described in section 6.2 the prescribed hip and ankle trajectories are obtained by experimental curve fitting and are shown in figures 6.5 to 6.8.

6.4 Kinematic Constraints for the Arthrodesed Ankle.

The effect of a "fixed"foot is to make the stance portion after heelstrike and the deploy portion merge together kinematically (appendix 3).
The kinematic constraints in these two periods were taken to be identical.
With the hip motion prescribed, the angles X1 and X2 are no longer independent but subjected to an equality constraint relationship.
Hence we have

$$S_1^d(x;t) = S_1^{rs}(x;t) = g(t)+L_1t_2 + L_2t_4-E_o-p(t)=o$$

and

$$S_2^d(x;t) = S_2^{rs}(x;t) = V_o(t+t_o) + L_1t_1-L_2t_3-q(t)=o$$

where $S^d \triangleq$ deploy constraint

$S^{rs} \triangleq$ stance constraint

$g(t)$ = prescribed vertical hip trajectory

$p(t)$ = prescribed vertical ankle trajectory

$Vo(t+to)$ = prescribed horizontal hip trajectory

$q(t)$ = prescribed horizontal ankle trajectory

L_1, L_2 are the lengths of the thigh and shank resp.

t_1 = sin (X1-XZ)

t_2 = cos (X1-XZ)

t_3 = sin (X2-X1+XZ)

t_4 = cos (X2-X1+XZ)

XZ = constant

Eo = average height of hip above ground

During swing motion the kinematic constraint is the same as for normal walking.
The performance criterion outlined in appendix 4 for normal walking has been assumed to apply equally well to the particular gait under discussion here.
Since motion of left leg was now different to the motion of the right leg (over and above a phase difference) the study was repeated for both legs.
We now possess a constrained mechanical model and a performance criterion which will form the basis of our optimization problem.
We are now in a position to formulate the problem and to evolve a method of solution.

6.5 References.

Chow, C.K. and Jacobson, D.H. (1971): "Studies of Human Locomotion via Optimal Programming". Mathematical Biosciences 10; 239-306.

Milner, M. and Quanbury, A.O, and Basmajian, J.V. (1970). Progress report No.4 - Analysis of muscle activities at various speeds and pace periods from EMG's with indwelling electrodes, LTR - CS - 33, NRC, Ottawa, Canada.

TABLE I.

Subject C.H.

Parameter.	Value.
Body weight W	154 lb
Mass A_0	4.812 slugs
Mass of thigh M_1	0.462 slugs
Mass of shank M_2	0.308 slugs
Moment of inertia of thigh I_1	0.1249 slugs-ft^2
Moment of inertia of shank I_2	0.0826 slugs-ft^2
Link length of thigh L_1	1.2196 ft.
Link length of shank L_2	1.2029 ft.
Distance from hip joint to centre of gravity of thigh α_1	0.521 ft.
Distance from knee joint to centre of gravity of shank α_2	0.518 ft.
Length of foot (Deploy)	0.60 ft.
(Swing)	0.83 ft.

Composite Parameters in Simulation.

$$A_1 = I_1 + m_1\alpha_1^2 + m_2 L_1^2 = 0.8951 \text{ slug-ft}^2$$

$$A_2 = I_2 + m_2\alpha_2^2 = 0.1663 \text{ slug-ft}^2$$

$$C_1 = m_1\alpha_1 + m_2 L_1 = 0.6163 \text{ slug-ft}^2$$

$$C_2 = m_2\alpha_2 = 0.1596 \text{ slug-ft}^2$$

$$C_3 = C_2 \cdot L_1 = 0.1946 \text{ slug-ft}^2$$

Note: The various parameters have been expressed using units in the F.P.S. system to agree with the usage in the original program. The results, however, have been converted into the M.K.S. system in line with official requirements.

TABLE II.

Subject P.B.

Parameter.	Value.
Body weight W	160 lb
Mass A_0	4.974 slugs
Mass of thigh M_1	0.477 slugs
Mass of shank M_2	0.318 slugs
Moment of inertia of thigh I_1	0.1286 slugs-ft^2
Moment of inertia of shank I_2	0.0764 slugs-ft^2
Link length of thigh L_1	1.3940 ft
Link length of shank L_2	1.1316 ft
Distance from hip joint to centre of gravity of thigh α_1	0.6039 ft
Distance from knee joint to centre of gravity of shank α_2	0.490 ft
Length of foot (Deploy)	0.64 ft
(Swing)	0.82 ft

Composite Parameters in Simulation.

$A_1 = I_1 + m_1\alpha_1^2 + m_2L_1^2 = 0.8757$ slug-ft^2

$A_2 = I_2 + m_2\alpha_2^2 = 0.1528$ slug-ft^2

$C_1 = m_1\alpha_1 + m_2L_1 = 0.7320$ slug-ft^2

$C_2 = m_2\alpha_2 = 0.1559$ slug-ft^2

$C_3 = C_2 \cdot L_1 = 0.2173$ slug-ft^2

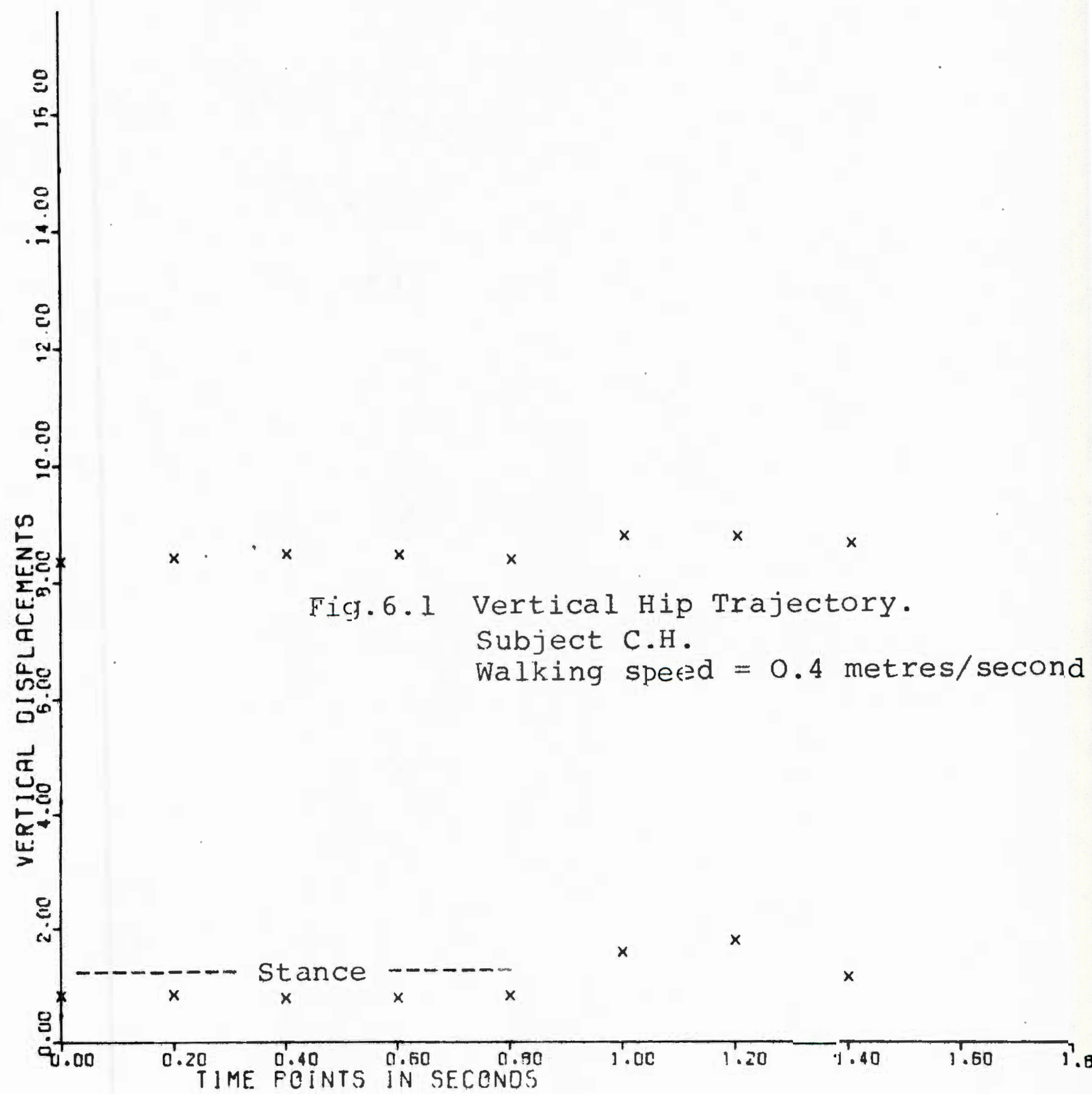

Fig.6.1 Vertical Hip Trajectory.
Subject C.H.
Walking speed = 0.4 metres/second

Fig. 6.3 Vertical Ankle Trajectory.

Subject C.H.
Walking speed = 0.4 metres/second.

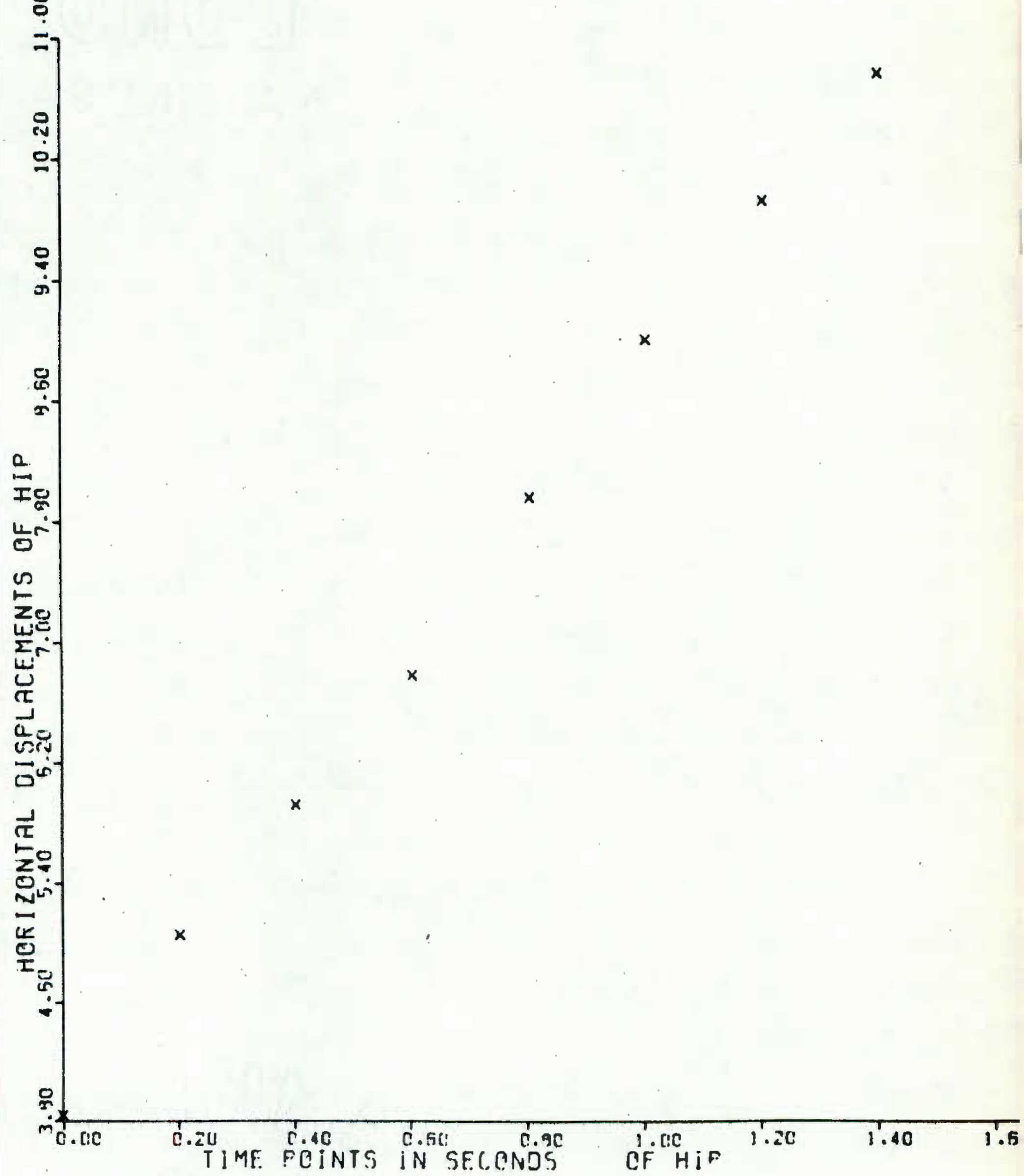

Fig. 6.2 Horizontal Hip Trajectory.

Subject C.H.
Walking speed = 0.4 metres/second.

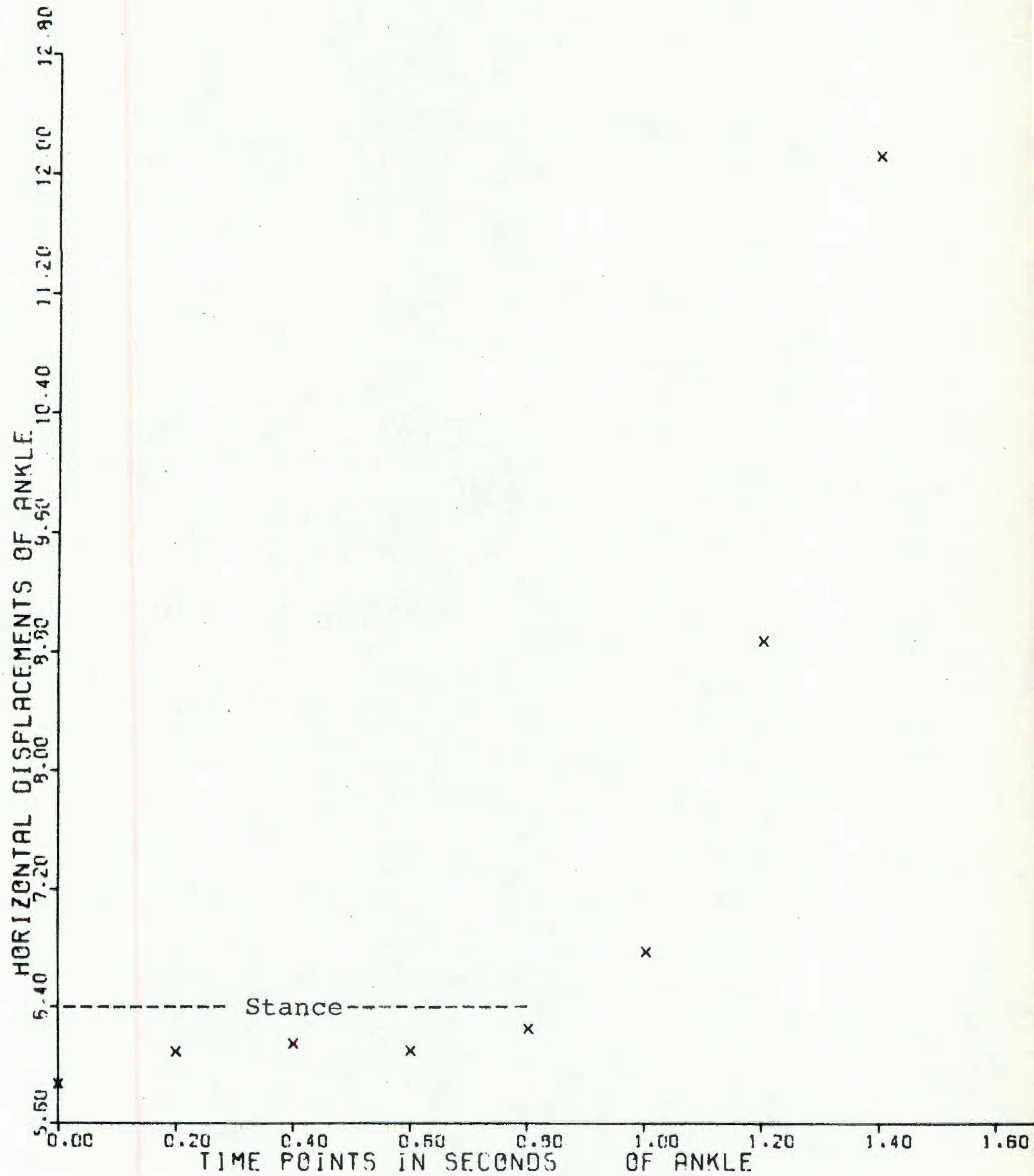

Fig. 6.4 Horizontal Ankle Trajectory.

Subject C.H.
Walking speed = 0.4 metres/second.

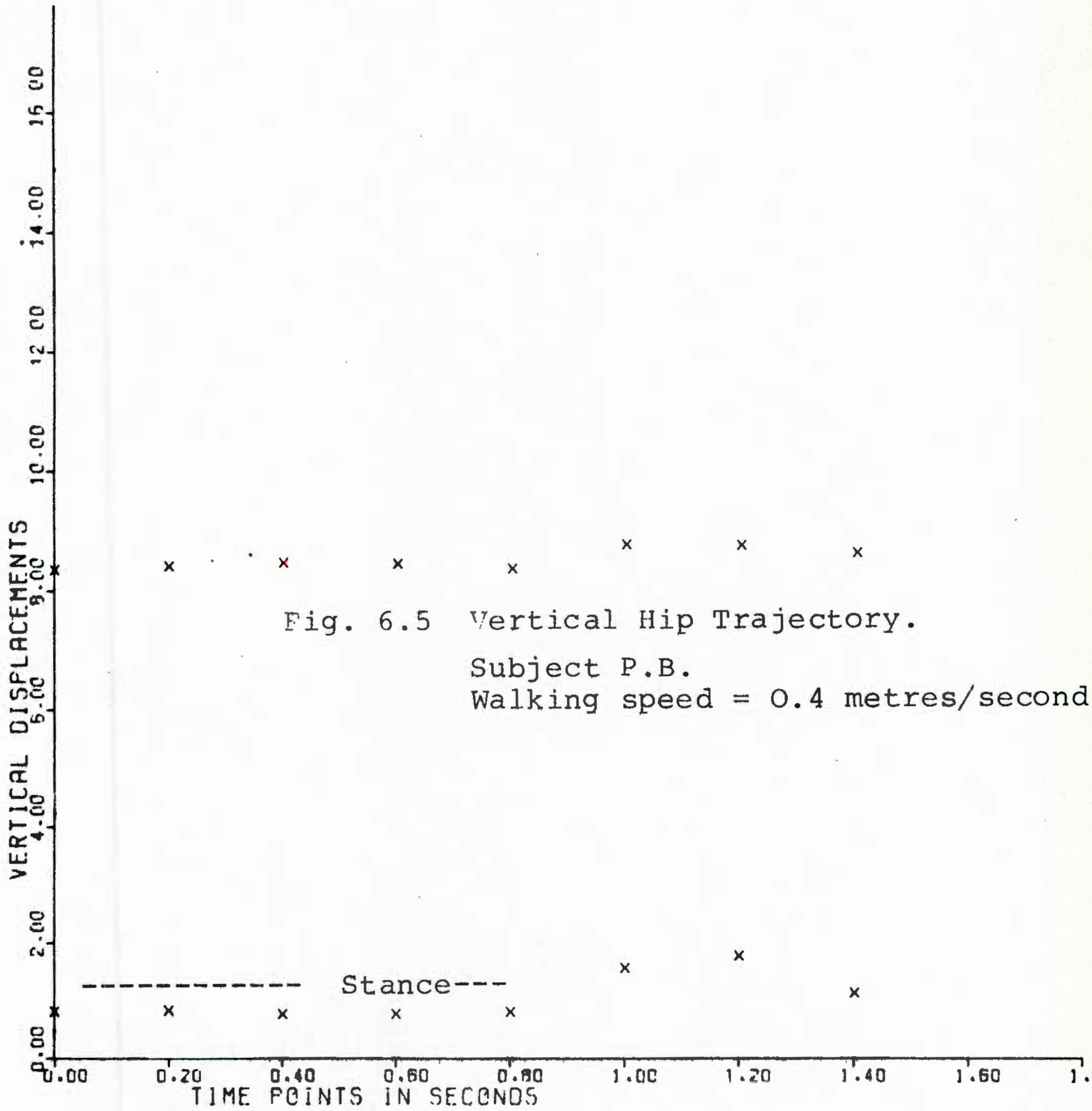

Fig. 6.5 Vertical Hip Trajectory.

Subject P.B.
Walking speed = 0.4 metres/second

Fig.6.7 Vertical Ankle Trajectory.

Subject P.B.
Walking speed = 0.4 metres/Second

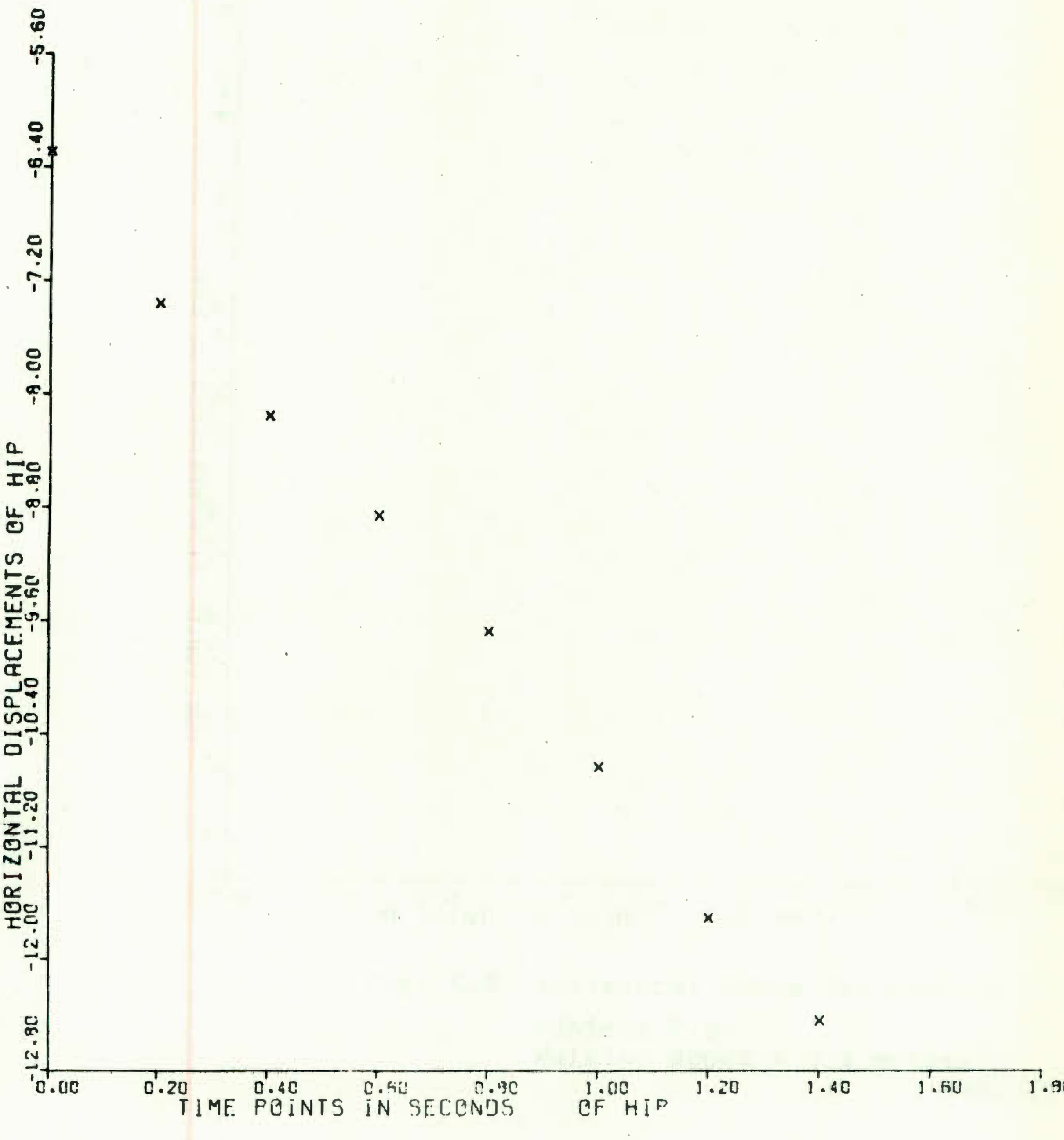

Fig. 6.6 Horizontal Hip Trajectory.

Subject P.B.

Walking Speed = 0.4 metres/second.

CHAPTER 7

OPTIMIZATION.

7.1 Formulation of Problem and Method of Solution.

In appendix 6, the formulation of the problem is shown and its modifications after the necessary conditions of optimalit have been invoked. As is mentioned there, the basis is the conversion of the original constrained optimization problem into a sequential unconstrained problem. In the sequential unconstrained problem, the original performance criterion is modified for the stance, deploy and swing portions.

Chow and Jacobson (1971) use numerical optimization only in deploy and swing, while an algebraic method is used to solve the simultaneous equations in stance.
For the purposes of this thesis, however, it was considered necessary to minimize the performance criterion in stance as well as in deploy and swing, in order to obtain the optimal hip and knee moment profiles.
The prescribed hip and ankle trajectories, which determine the motion of the thigh and shank during stance, are obtained by fitting mathematical functions to the experimenta curves.
This introduces a certain amount of error (as often there is insufficient experimental points) which can be minimized by optimization. The error is usually sufficiently large to forbid an algebraic method for solution in this portion.

7.2 The Computer Program.

The walking-cycle is divided into three portions - stance, deploy and swing for separate solution.
The listing shown in appendix 6 is for solution in the swing portion. Deploy and stance phase solutions are very similar except for modification of equation expressions.

Program Features:

(a) Written in Fortran IV, single precision.

(b) Run on the Univac 1106

(c) Contains a numerical optimization package in the form of a first-order algorithm which is the standard gradient method with fixed step size EPS1 ($\varepsilon 1$)

(d) Contains a fourth-order Runge-Kutta method with fixed integration step size for the integration of the state and adjoint equations.

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(e) For each phasic portion, the computation is done in series with increasing positive values of the weighing factor SIGMA (σ).

(f) In addition, during the swing calculation, computation is done for decreasing positive values of GAMMA (γ).

7.3 References.

Chow,C.K. and Jacobson, D.H.(1971): "Studies of Human Locomotion via Optimal Programming".
Mathematical Biosciences 10: 239-306.

CHAPTER 8.

EXPERIMENTAL THESIS WORK

8.1 Introduction.

In Chapter 5 a brief but general review of experimental techniques in human locomotion was presented. For the purposes of this thesis, the two experimental techniques concentrated on were:

(a) Interrupted light or strobe-photography and

(b) Electromyography.

8.2 Strobe-Photography

The aim of this study is to obtain the trajectories traced out in space and time of selected points on the subject's body. The points selected were the head (between eye and ear) shoulder, hip, knee, ankle and toe. The use of a camera with a Polaroid back and a stroboscopic flash unit in a darkened room enabled this study to be carried out.

(a) The Walkway. (Fig.8.1)

The subject walked a distance of approximately 8 metres along a straight line. The perpendicular distance from straight line to the location of the camera is about 3,5 metres. The strobe light source is mounted directly below the camera. A suitable black back-ground and white markers located at fixed intervals along the walkpath enabled scaled measurements to be made from the film records.

(b) The Camera (Fig.8.2)

The camera used was a Polaroid 350 Automatic Land Camera with the magic eye exposure meter blacked out. This yielded an 8,5 x 10, 5 cm print with type 107 pack film rated at 3000 ASA.

(c) The Stroboscope.

A stroboscope flash system with an adjustable flashing rate was constructed. The circuit diagram is given in Fig.I. of the paper of Milner,Wilberforce and Brennan(1972). Fixed rates available are 5,10,15,20 and 25 flashes/sec.

(d) Subject Preparation.

As shown in Fig.8.3, the subject wears a black cat suit with long sleeves. Reflectors (REG Circular Bicycle reflectors 4 cm diameter) are fitted by means of Velcro bands and adhesive tape over the points whose trajectories it is desired to follow. Strips of white pressure sensitive scotch tape 1 cm wide are adhered to the subject and connect reflector to reflector.
A velocity meter (with illuminated face) (Fig.8.4) is placed in line with the subject's eyes to enable him to adjust his walk to different speeds.
An example of a typical photograph (enlarged) showing the type of results obtained is shown in Fig.8.5.

(e) Data Processing.

The co-ordinates (x,y) of each point are read off using an electronic digitizer (9107A Hewlett Packard Digitizer) and these co-ordinates are fed as input into a computer program.
An explanation and listing of this program is given in appendix 7.
The program calculates the angles about the hip, knee and ankle during one complete step and also offers a number of plot options.

8.3 Electromyography.

The subject walks naturally along a metal walkway facing a velocity-meter (Fig.8.6) so that various speeds of walking can be obtained. Both shoes worn are fitted with contacts (foot-switches) at the heel and toe (Fig.8.7) and a foot-switch pattern can then be obtained indicating the time sequence of ground contact of the extremities of the underside of the foot.

An area of skin covering the main bulk of the quadriceps muscle is shaved and cleansed with alcohol and three self-adhesive electrodes * filled with electrolyte paste

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* The electrodes are obtained from Becton-Dickinson Dispos-el disposable electrode kits.
Address: Ecton, Dickinson & Co.,Rutherford, New Jersey 07070, U.S.A.

(one electrode was the earth) were placed adjoining each other on the skin. Two similar electrodes were placed in a like manner over the main bulk of the hamstring muscle, tibialis anterior muscle and gastrocnemius muscle (Fig.8.8). The raw electromyographic (emg) signal passes from the muscle via the electrodes to an amplifier (gain 100) and after a further stage of amplification (gain 10) is then recorded in its analogue form on a 14-channel magnetic tape recorder. Six of these channels are taken up by the four emg signals and the two foot-switch patterns. (Fig.8.9). The tape recorder can now be played back to pass the raw-signal through a rectifier and averager so that the final emg pattern recorded on an Ultraviolet (u-V) pen-recorder is an averaged unidirectional emg envelope.

An alternative available is to pass the averaged emg signal in its analogue form through an analogue-to-digital conversion system (Hershler,McConnell and Milner (1972)) with the digital output recorded on a 9 - channel IBM magnetic tape. This tape can then be read by the UNIVAC 1106 computer and by means of a computer program (appendix 2) analysis of the various electromyographic signals can be attempted.

8.4 REFERENCES.

Hershler,C.,and McConnell, V.A., and Milner,M.(1972): "Processing of Human Locomotion Data".
(i) Analogue - to - digital Conversion of Data using a Varian Mini-Computer.
(ii) A program for reading the magnetic tape output. Laboratory Technical Report: Dept.of Bio-engineering and Medical Physics, U.C.T.

Milner,M.and Wilberforce,C.B.A.and Brennan,P.K (1972) "Stroboscopic Polaroid Photography In Clinical Studies of Human Locomotion".
Technical Note: Dept. of Bio-engineering and Medical Physics and Department of Clinical Photography, Groote Schuur Hospital/U.C.T. S.A.Med.Jour. (In the press).

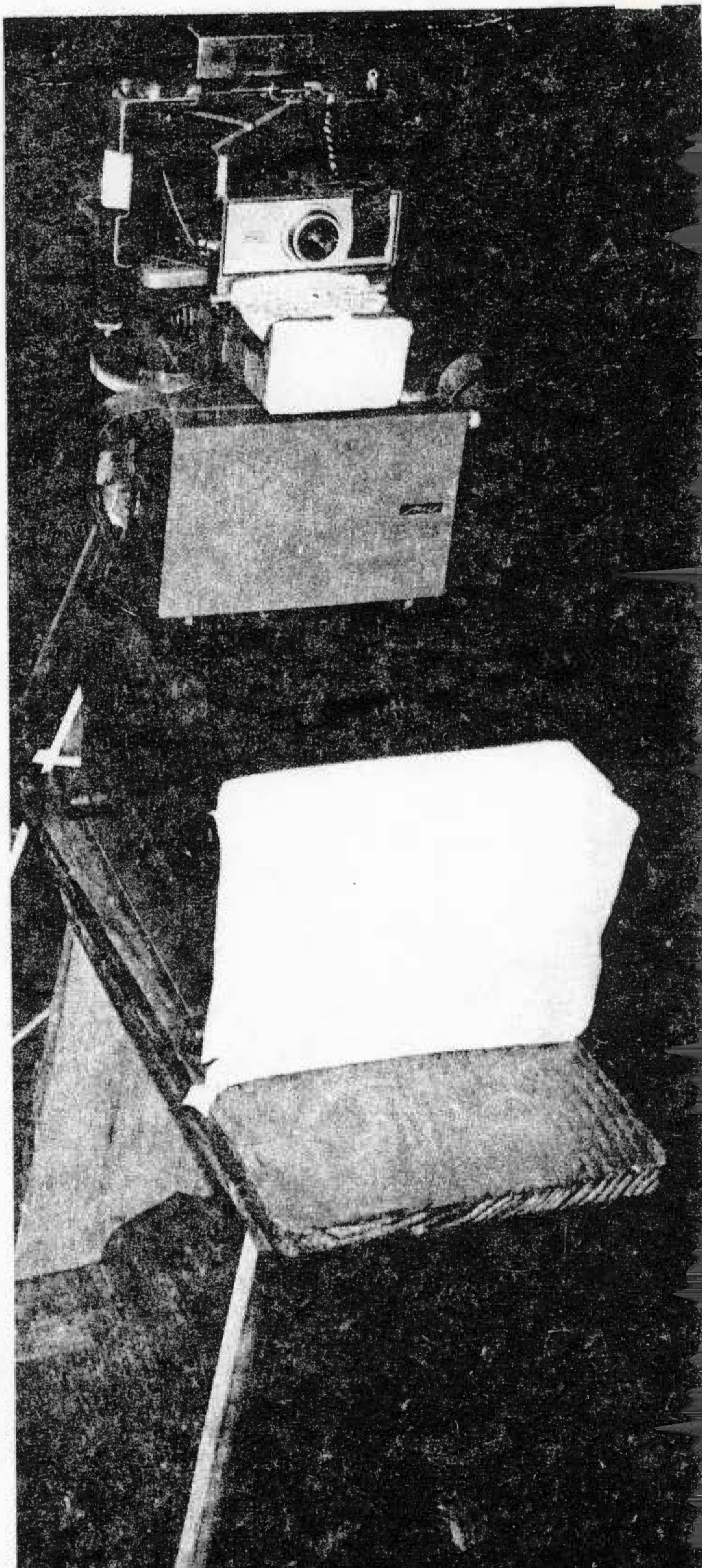

Fig. 8.2 The Camera.

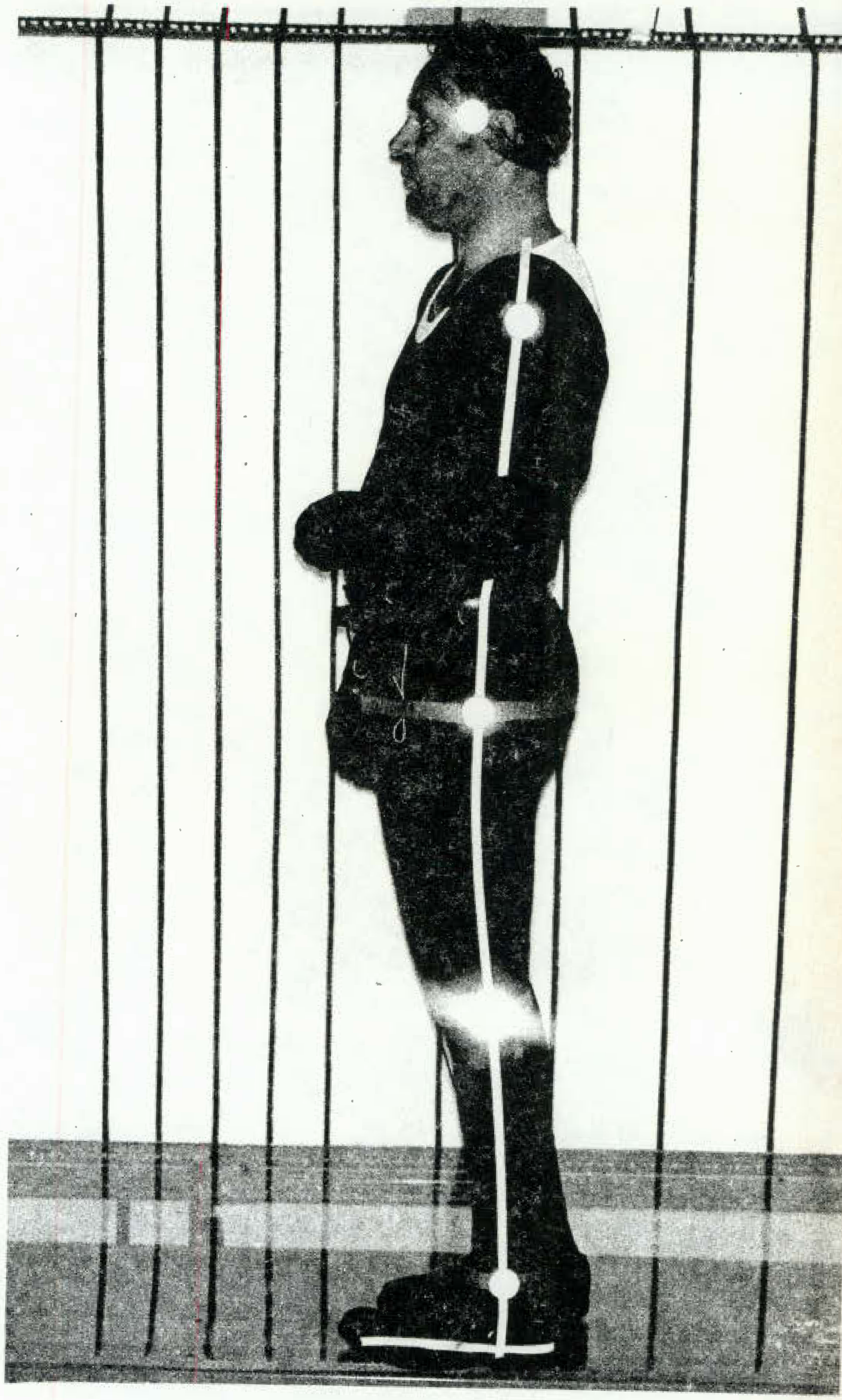

Fig. 8.3 Subject fitted out with black cat suit and reflectors.

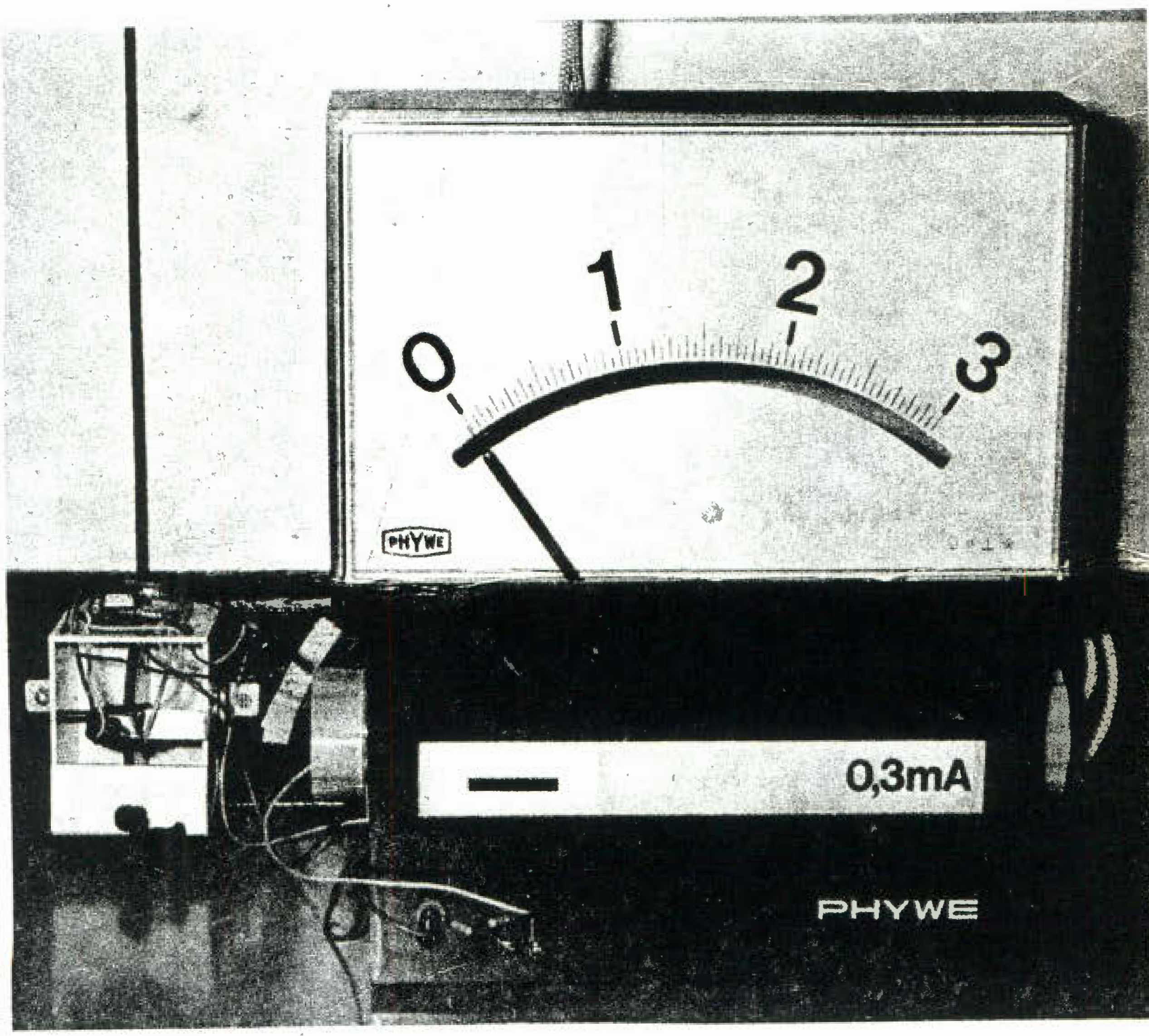

Fig. 8.4 The Velocity - Meter.

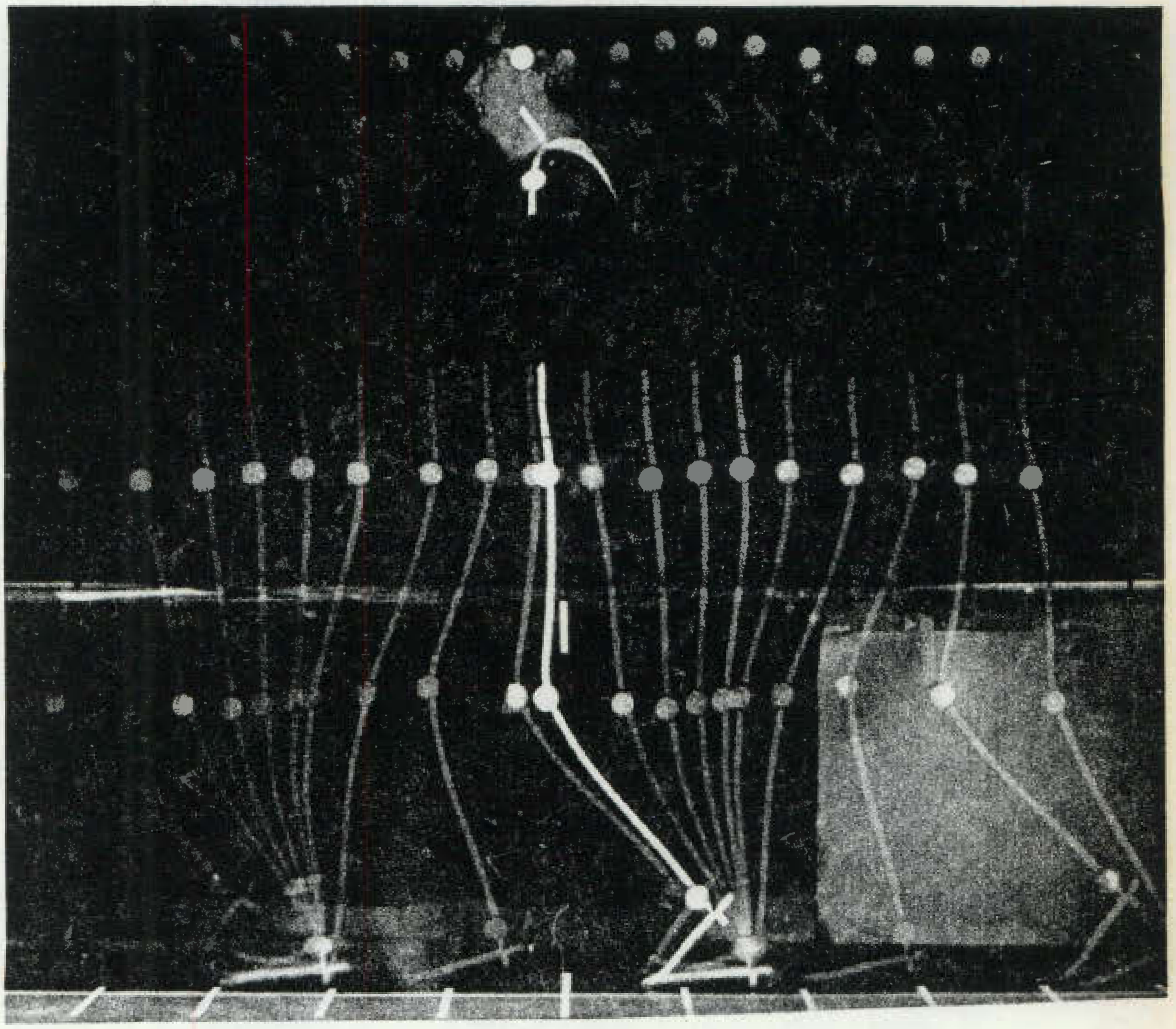

Fig. 8.5 Strobe Photograph.
Subject P.B.
Walking speed = 0.4 metres/second.

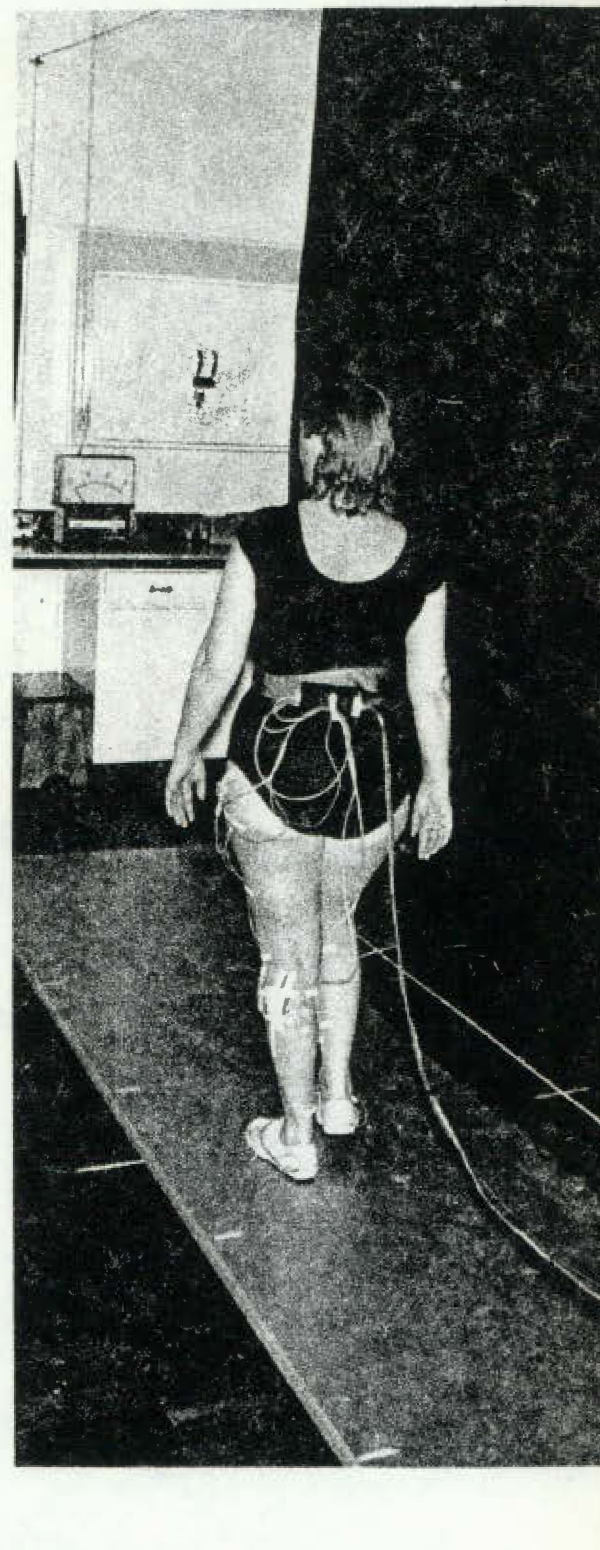

Fig. 8.6 Subject walking along metal walkway facing velocity-meter.
Long trailing leads connect subject to recording equipment.

Fig. 8.7 Shoes fitted with contacts at heel and toe.

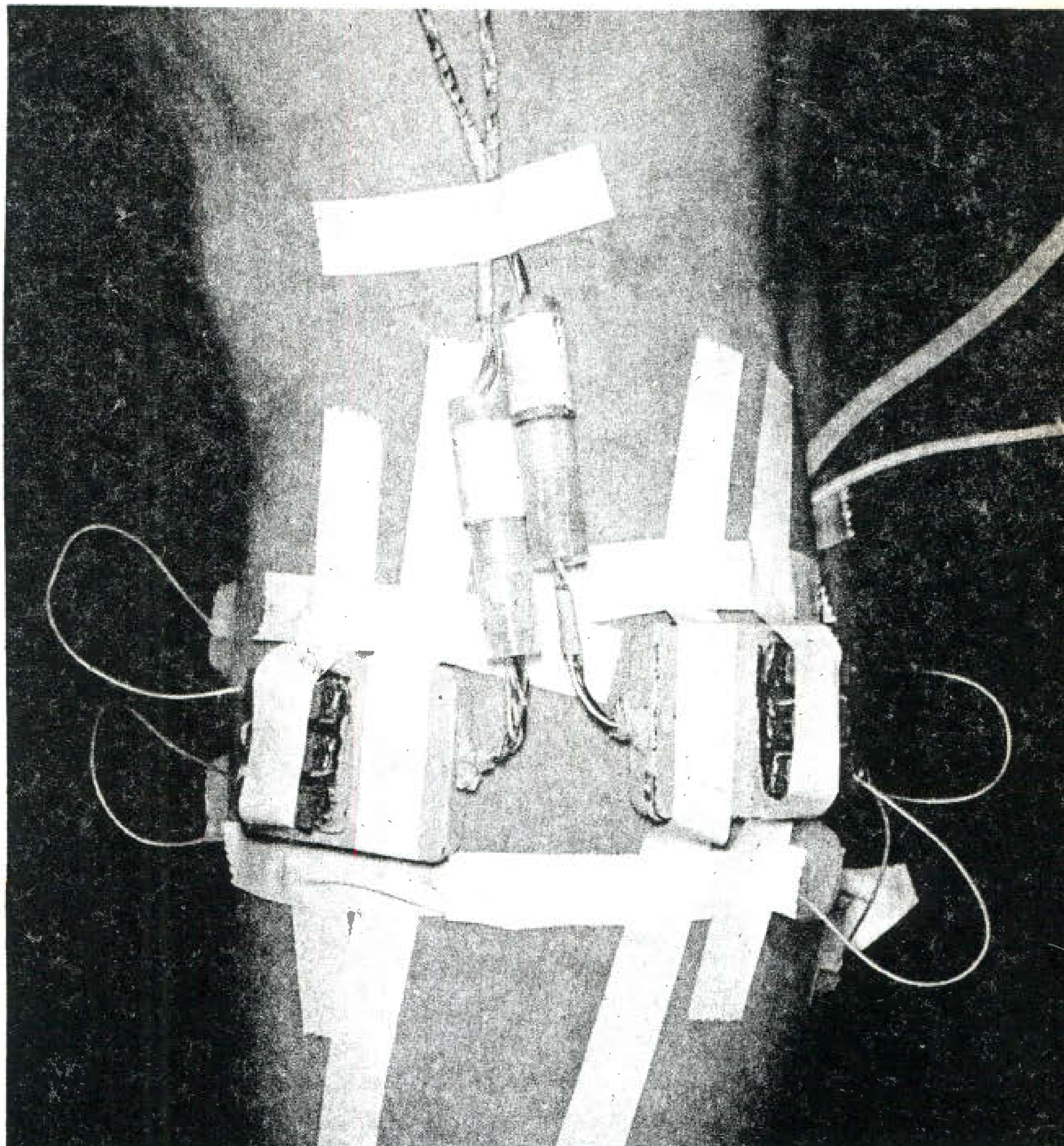

Fig. 8.8 Surface electrodes and amplifiers placed over the main bulk of the Tibialis Anterior and Gastrocnemius muscles.

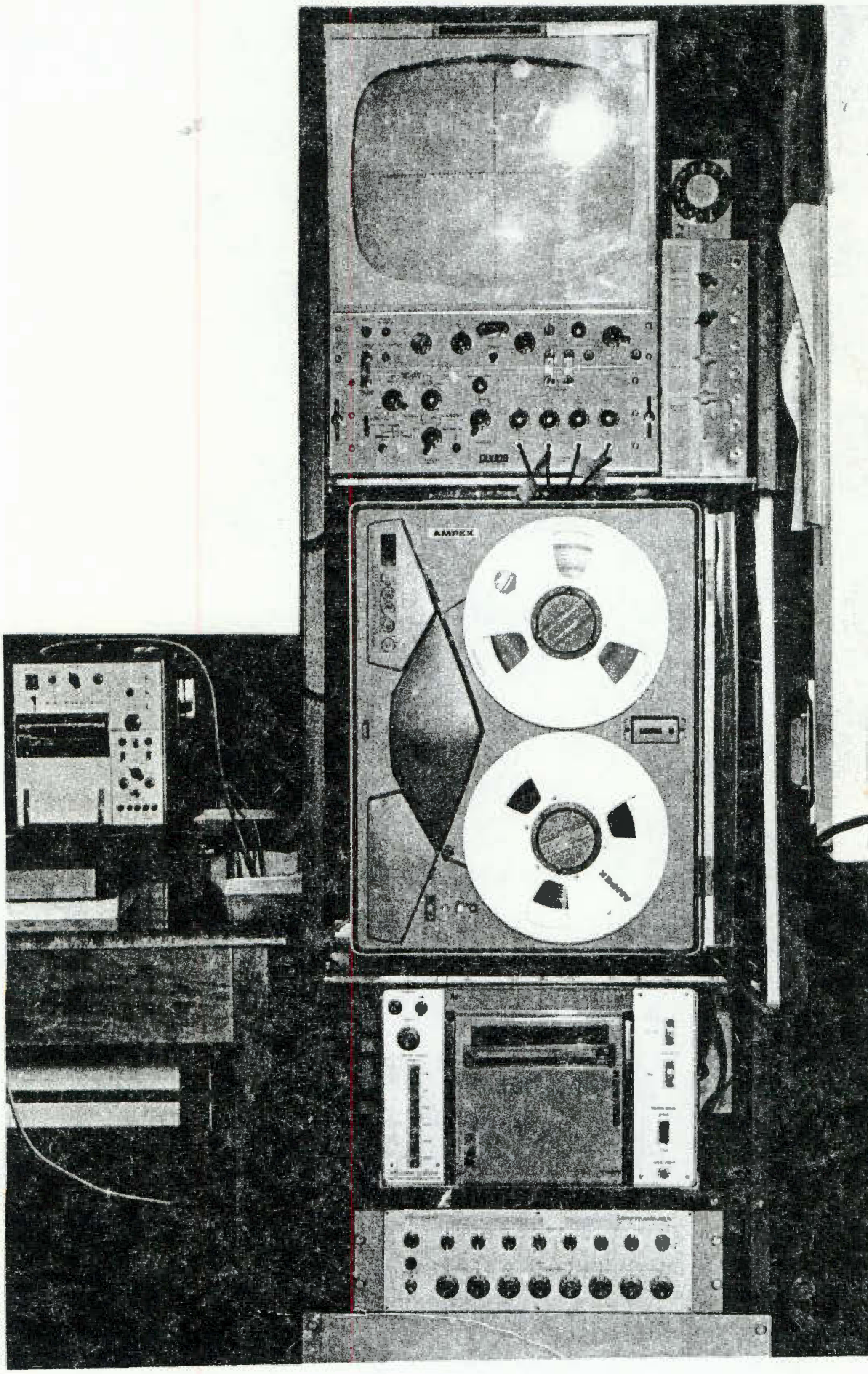

Fig. 8.9 Recording Equipment.
(Oscilloscope, Tape recorder and Ultra-violet pen recorder).

CHAPTER 9

DISCUSSION OF RESULTS.

9.1 Introduction

It is hoped that the preceding chapters have given the reader a feeling for both the experimental and theoretical aspects of this work.

It would be pertinent to review at this point the extent and limitations of the study. For the normal locomotion study, one normal typical subject C.H. was chosen and the study was repeated three times at walking-speeds 0.4 m/s, 1.6 m/s and 2.1 m/s. The second speed being his average relaxed walking speed ("comfy cadence").
The quantity of computer time needed to achieve the final optimal results was prohibitively large. It was felt, however, that enough results had been obtained to test the Chow and Jacobson model as well as being a good first step towards a more comprehensive study. In such a study at least six different normal subjects could be chosen to walk at a number of different speeds.
For the pathological gait study, subject P.B. walked at a speed of 0.4 m/s and a separate study was initiated on both his right and left legs.
In any further study, force plate data on each subject would be essential in order to make the results more meaningful. This point has been expounded upon more fully in section 9.4.

A discussion of the results now follows where the latter has been divided into two sections. The first comprises the experimental results while the second contains the theoretically predicted results.

9.2 Experimental Results.

Subject: C.H. Normal Locomotion.

Figures 9.1, 9.2 and 9.3 are photographic copies of the electromyographic and footswitch patterns for the above subject walking at three different speeds.

It must be stated at the very outset, that any deductions concerning the phasic properties of the electromyographic

patterns are approximate.
Chapter 5 section 5.3 describes how electromyography, although only able to give qualitative information concerning the forces generated within the muscle, is able to reveal the precise phase relationship of muscle action. The limitation in this experimental work is the use of the surface electrodes (Section 5.4).
The error arises not so much in <u>when</u> a peak in the muscular activity occurs but rather in <u>what</u> particular muscle or muscle group is being monitored.

As an illustration of this ambiguity , consider the surface electrode placed over the major muscle mass of the front of the thigh. A careful perusal of Chapter 3 section 3.3A(iv shows that the electrical signal picked up by the electrode is the collective effort of the muscle group called the Quadriceps Femoris. Here one presumes that an additional muscle, called the Sartorius, since it is considered not to play an important role in walking, does not contribute significantly to the electrical signal.
Even neglecting the effect of the Sartorius, it would be impossible to know which member of the Quadriceps dominates in the muscular activity associated with walking. At best, the electromyographic pattern associated with the Quadriceps is the collective result of the muscle activities of the Rectus Femoris and the three Vasti.
The same argument could be used for the Hamstrings,Tibialis Anterior and Gastrocnemius. (Chapter 3 section 3.3).

This suggests a modification to the experimental set-up for future work in this area. With the aid of an anatomist, needle-electrodes could be inserted into a particular muscle. This would help to localize the electric signal due to the muscle action more effectively and it would reduce any relative motion between electrode and muscle when the subject is walking. As these are the very limitations associated with surface electrodes, this would help to eliminate interference from adjoining muscles.

The footswitch pattern is now divided into three parts giving the intervals of time for stance, deploy and swing.

Inspection of the concurrent muscle activities of the Gastrocnemius, Tibialis Anterior, Hamstrings and Quadriceps now gives the positions of peaks (if they exist) as a percentage of the particular phasic interval.

Subject: P.B. Right Arthrodesed Ankle.

Figs. 9.4 and 9.5 are photographic copies of the electromyographic and footswitch patterns of the right and left legs of the above subject walking at one particular speed.

9.3 Theoretical Results.

Subject: C.H. Normal Locomotion.

Stance.

During stance, the Quadriceps group is active during extension of the knee with the foot fixed (knee-locking). It would be pertinent, therefore, to compare the variation of peak position in the theoretically predicted absolute knee moment as the walking speed changes with the variation of peak position in the experimental muscle activity of the Quadriceps as the walking speed changes (Fig.9.6). Fig.9.7 shows the Knee Moment during Stance vs Time for a particular speed of walking.

Deploy.

The Gastrocnemius is active during the deploy portion hence peak positions in its muscle activity during deploy can be compared with peak positions in the absolute Knee Moment as the walking speed changes (Fig.9.8).

Fig. 9.9 shows how the Knee Moment during Deploy varies with time for a particular speed of walking.

Swing.

The Tibialis Anterior is used in walking to bend the foot up as the advancing limb swings forward and hence again its muscle activity during swing can be compared with the absolute Knee Moment during swing as the walking speed changes (Fig.9.10). Fig.9.11 shows the Knee Moment during Swing vs Time for a particular walking speed.

Subject:P.B. Right Arthrodesed Ankle.

Predicted peak positions in the Absolute Knee Moment for the right leg and left leg are shown for the one walking speed and compared with the peak positions of the Quadriceps, Gastrocnemius and Tibialis Anterior during stance,deploy and swing respectively. This is shown in Figs.9.12,9.13 and 9.14.

9.4 Discussion of Results.

The performance criterion which has been discussed in appendix 4 has been shown to be related to the mechanical work done externally by the muscles of the leg during walking. From the computed results, numerical values can be obtained for the work done during one complete step for the three different walking speeds.
For the three speeds, (0.4:1.6:2.1) the work done by the muscles externally was found to be in the following proportion

2,67 : 1,00 : 1,27

Such a theoretical result serves to confirm to a certain extent some of the experimental findings of Atzler and Herbst (Elftman (1966)). They showed that energy expenditure was a minimum for certain combinations of speed and step length.

For normal locomotion, the general shape of the theoretical curve for the predicted peak positions of the various muscle groups outlined above as the walking speed changes agrees satisfactorily with the experimental curve.

There is at times disagreement in the prediction of the actual numbers concerned. An explanation for this discrepancy and an important point to remember for future work in this field is the horizontal and vertical ground reaction profiles.

These profiles are part of the prescribed input for the theoretical optimization which leads ultimately to the predicted hip and knee moments.

For the work done in this thesis theoretical ground reaction profiles (Inman (1966)) were utilized. A far more satisfactory arrangement would have been to use a force plate. (See Chapter 5 section 5.6).
The particular subjects studied could be then caused to walk upon the force plate and experimental ground reaction profiles obtained for each walking speed.

It is the contention of the author that inclusion of a force plate in future work will lead to better agreement between the actual numbers.

Significant is the fact that during swing (when force plate data is not needed) agreement between the actual numbers as well as the shape of the curve is greatly enhanced.

The above discussion applies equally well to the results of the pathological gait study of subject P.B. Here the force plate study would have been of especial importance.

One of the limitations encountered in this type of work was the enormous amount of computer time used before a useful result could be obtained. It would be possibly advantageous in the future to experiment with a number of different numerical algorithms in the optimization package and to choose ultimately the one which saves the most computer time.

Once the above modifications have been introduced it could be possible to test some of the assumptions made in the theoretical model.

For example, in appendix 4, an expression for the incremental change in the neural stimulation level Z has been derived using various assumptions viz.

$$\Delta Z \simeq \frac{Fs}{Fo(L)} (1 + b) \frac{Vs}{Vm} \propto Vs, \quad \frac{Vs}{Vm} << 1$$

One could now alter the above expression in a suitable manner and start the whole optimization process once again to obtain theoretical predictions which could be compared with experimentalresults. The increased accuracy of the prediction could now indicate a possible better expression for ΔZ and one now has a handle with which to study the nervous system.

9.5 Conclusions.

The results obtained show a definite correlation between the predicted optimal knee moment profile and the muscle activities of certain groups of muscles (Quadriceps during Stance, Gastrocnemius during deploy and Tibialis Anterior during swing).

The curves of peak positions versus walking speed have the same shape. Certain discrepancies between experimental numbers and theoretical predictions occur during stance and deploy and are explained by the need for experimental ground reaction profiles obtained from force plate data.

Experimental results during swing as well as the theoretical predictions agree both in the shape of the curve as well as numerical precision. This underlines the need for force plate data on each subject studied.

A number of additional ideas are introduced as a possible way of increasing the accuracy of future work in this area.

(i) The use of needle electrodes instead of surface electrodes. These can be inserted into a particular muscle with the aid of an anatomist and will lead to a clearer understanding of the electromyographic patterns.

(ii) If a force plate is not available, an alternative would be to change arbitrarily the ground reaction profile and to see what effect this has on the hip and knee moment outputs.

(iii) By repeating the optimization using different values for r1 and r2 (see appendix 4). This results in a less smoothed-out version of the hip and knee moment profiles. More peaks appear and a more comprehensive comparison with the electromyographic data may be achieved..

(iv) Assumptions like $\frac{de}{ds} \simeq 1$ (appendix 4) could be re-checked in the light of more recent experimental knowledge.

(v) Inital hip and knee moment profiles which are used as inputs could be varied and the optimization procedure repeated.

If in each case the same result for the final hip and knee moment profiles is obtained then one could feel reasonably sure that one had reached a local minimum.

(vi) The possible inclusion of a better numerical algorithm for integration of the differential equations. This could save computer time and costs.

(vii) The "semi-empirical" method of altering certain assumptions and repeating the optimization to see whether this results in an increase in accuracy of prediction. This could lead to one gaining more of an insight into the complexities of the neuromuscular transformation.

9.6 References.

Elftman H.(1966): Biomechanics of muscle J.Bone and Joint Surgery 48A No.2

Inman V.T. (1966): Human Locomotion Can.Med. Assoc. J.94 : 1047-1054

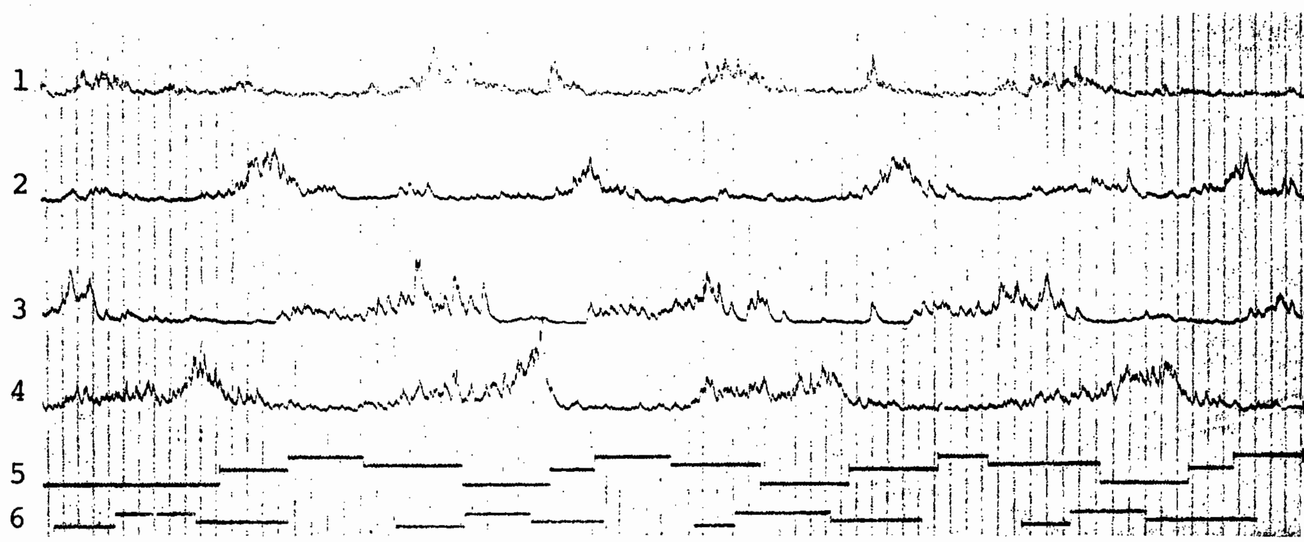

Fig.9.1 Electromyographic (E.M.G.) and Footswitch Patterns.

Subject C.H.

Walking Speed 0.4 metres/second.

Key

1. Quadriceps E.M.G. pattern.
2. Hamstring E.M.G. pattern.
3. Tibialis Anterior E.M.G. pattern.
4. Gastrocnemius E.M.G. pattern.
5. Left Footswitch pattern.
6. Right Footswitch pattern.

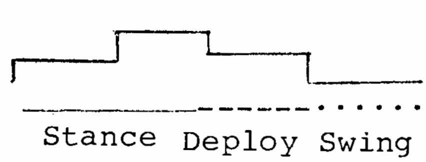

Fig.9.2 Electromyographic (E.M.G.) and Footswitch Patterns.

Subject C.H.

Walking speed 1.6 metres/second.

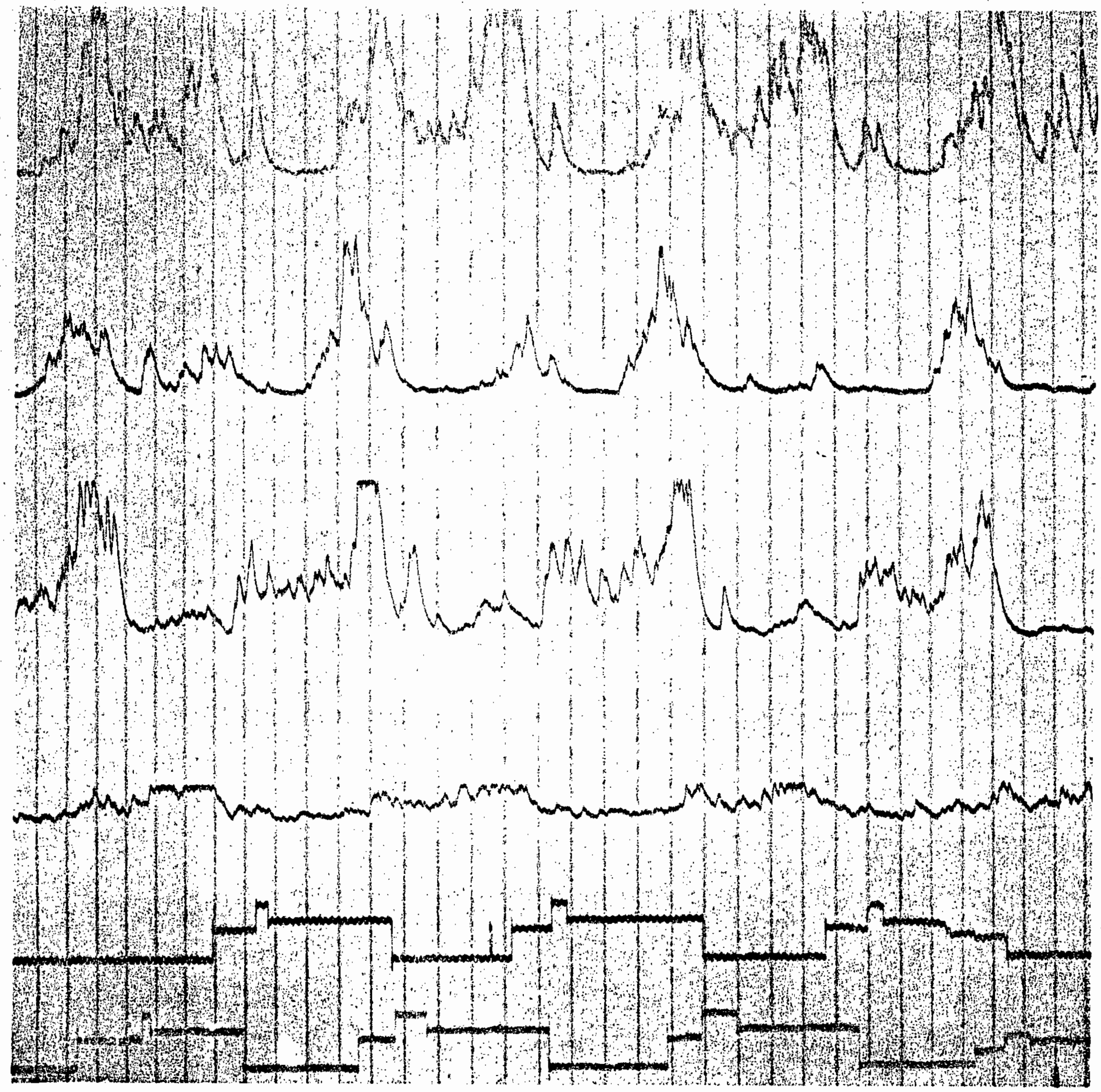

Fig. 9.3 Electromyographic (E.M.G.) and Footswitch Patterns.

Subject C.H.

Walking speed 2.1 metres/second.

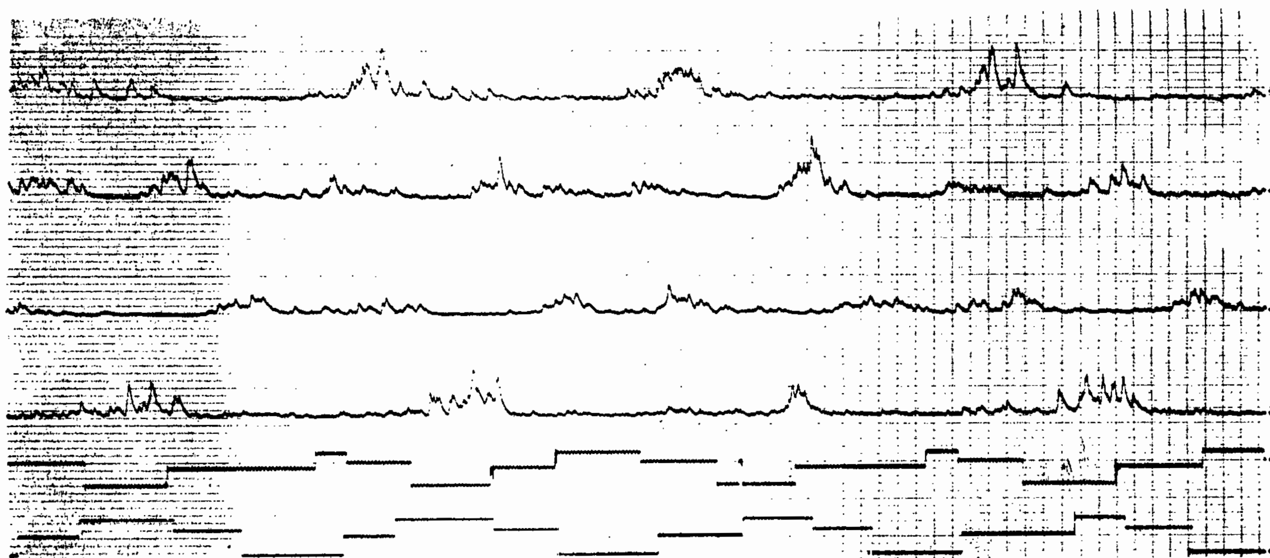

Fig. 9.4 Electromyographic (E.M.G.) and Footswitch Patterns.

Subject P.B. (Right Leg)

Walking speed 0.4 metres/second.

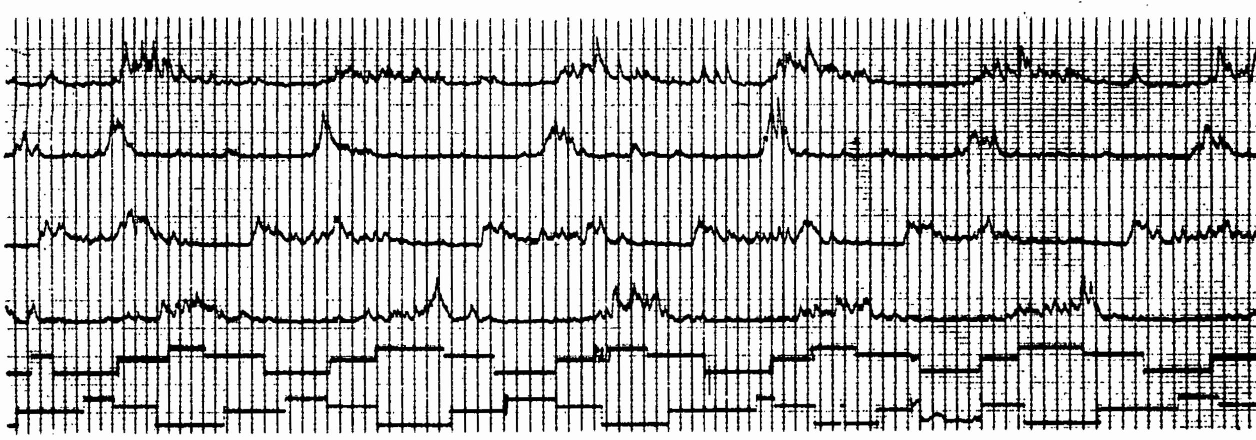

Fig. 9.5 Electromyographic (E.M.G.) and Footswitch Patterns.

Subject P.B. (Left Leg)

Walking speed 0.4 metres/second.

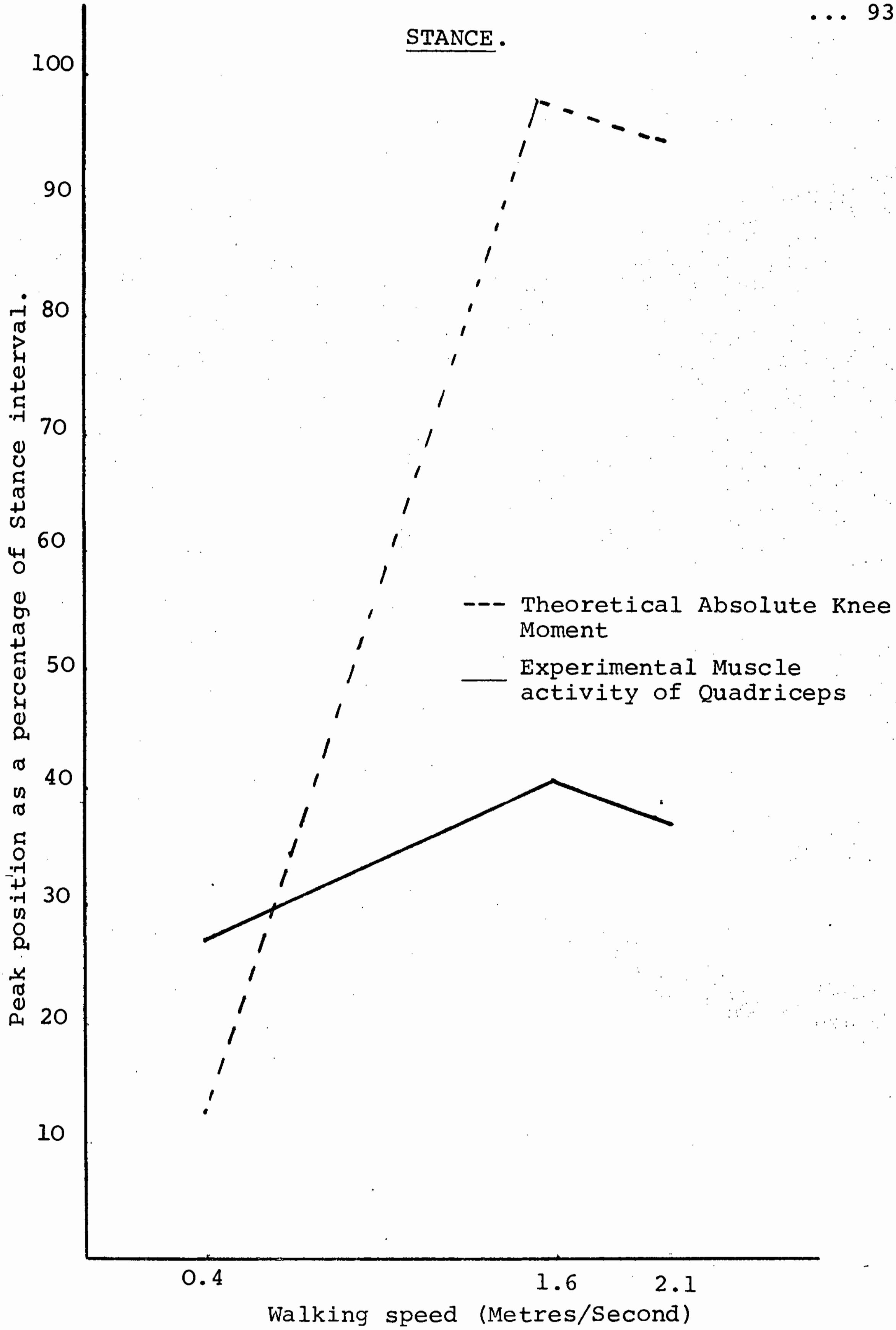

Fig.9.6 PEAK POSITION VS. WALKING SPEED.

SUBJECT C.H.

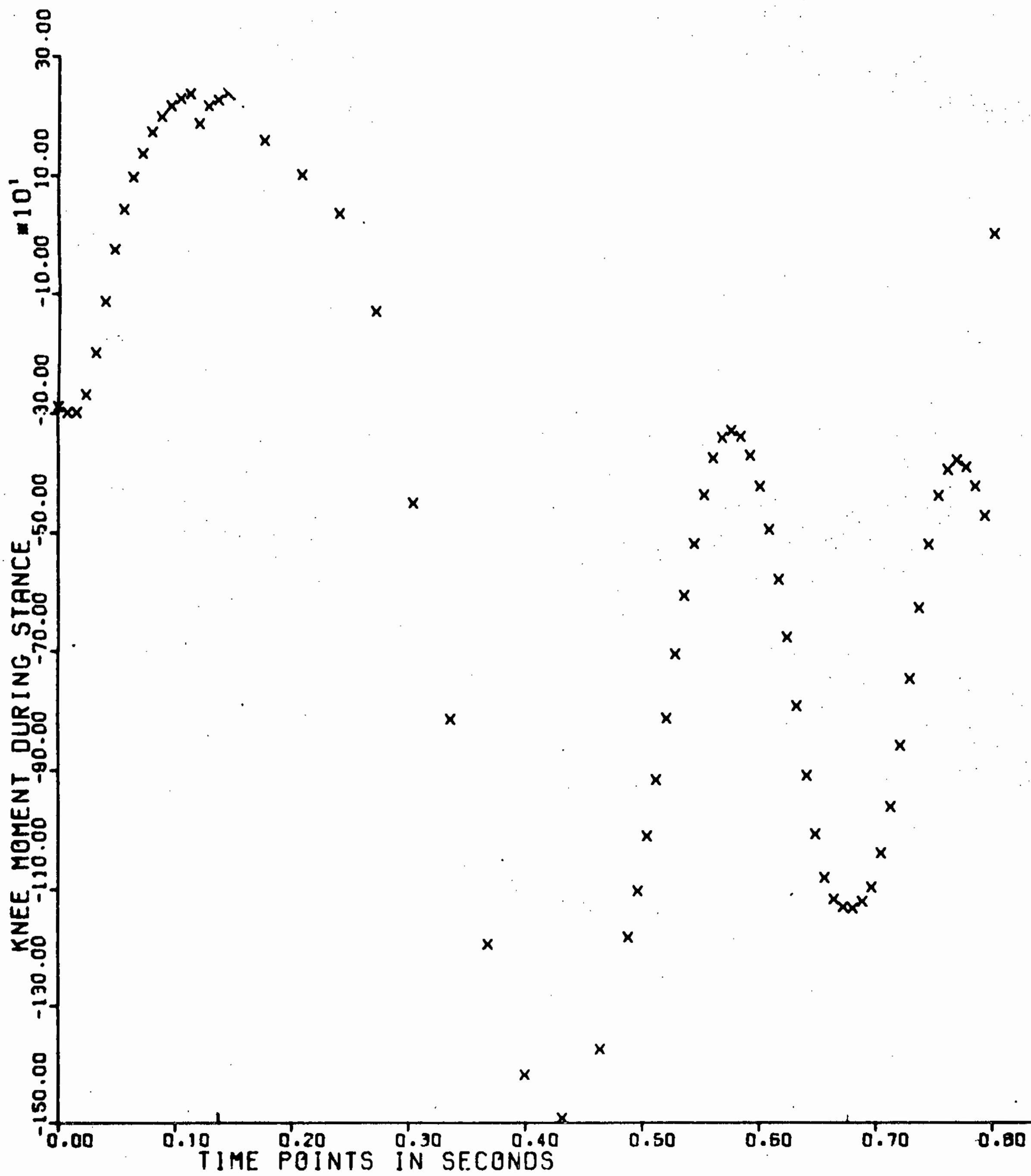

Fig.9.7 Knee Moment (Stance) vs Time.

Subject C.H.

Walking speed 0.4 metres/second

DEPLOY

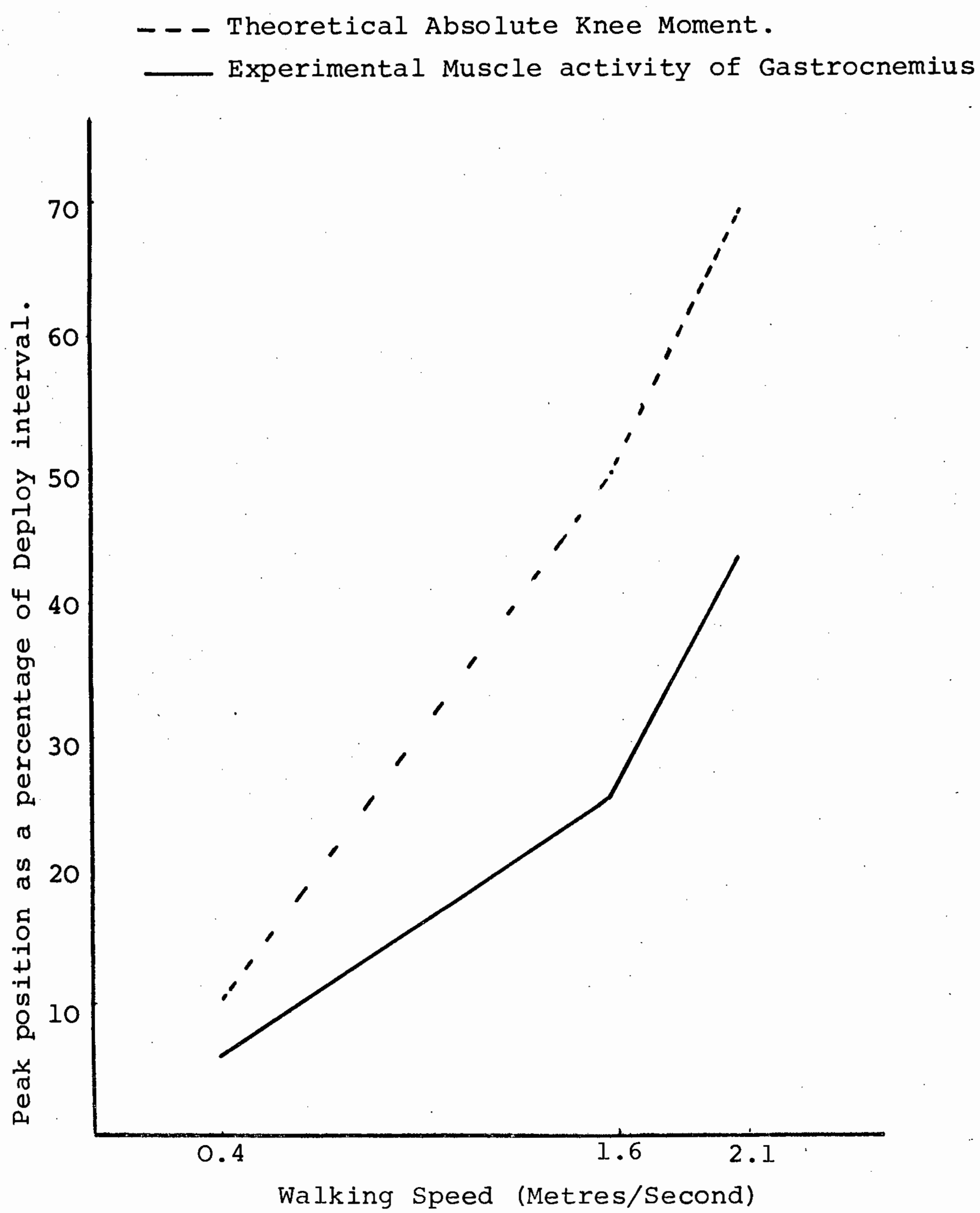

Fig. 9.8 PEAK POSITION VS.WALKING SPEED.
SUBJECT C.H.

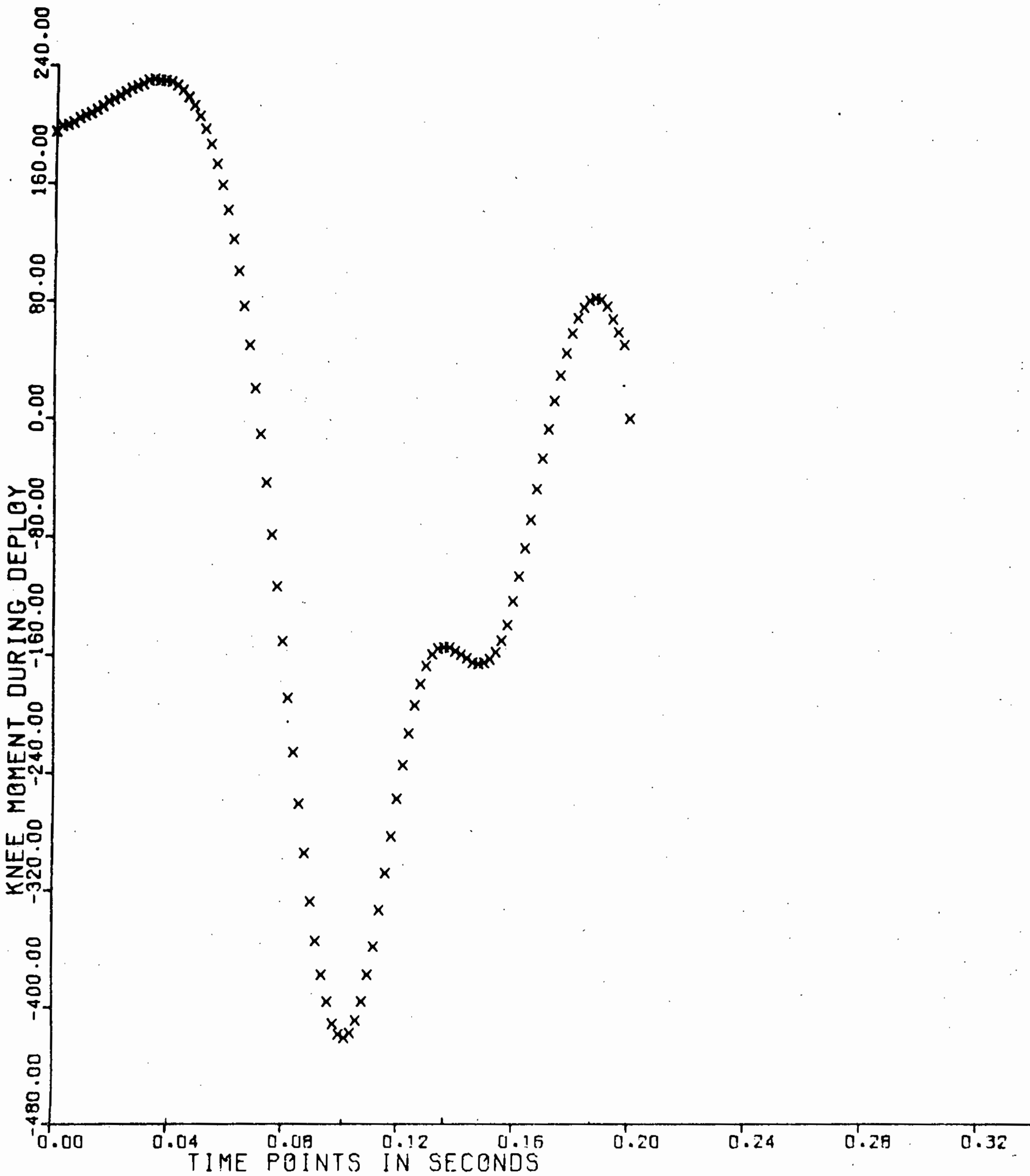

Fig.9.9 Knee Moment (Deploy) vs Time.

Subject C.H.

Walking speed 1.6 metres/second

SWING

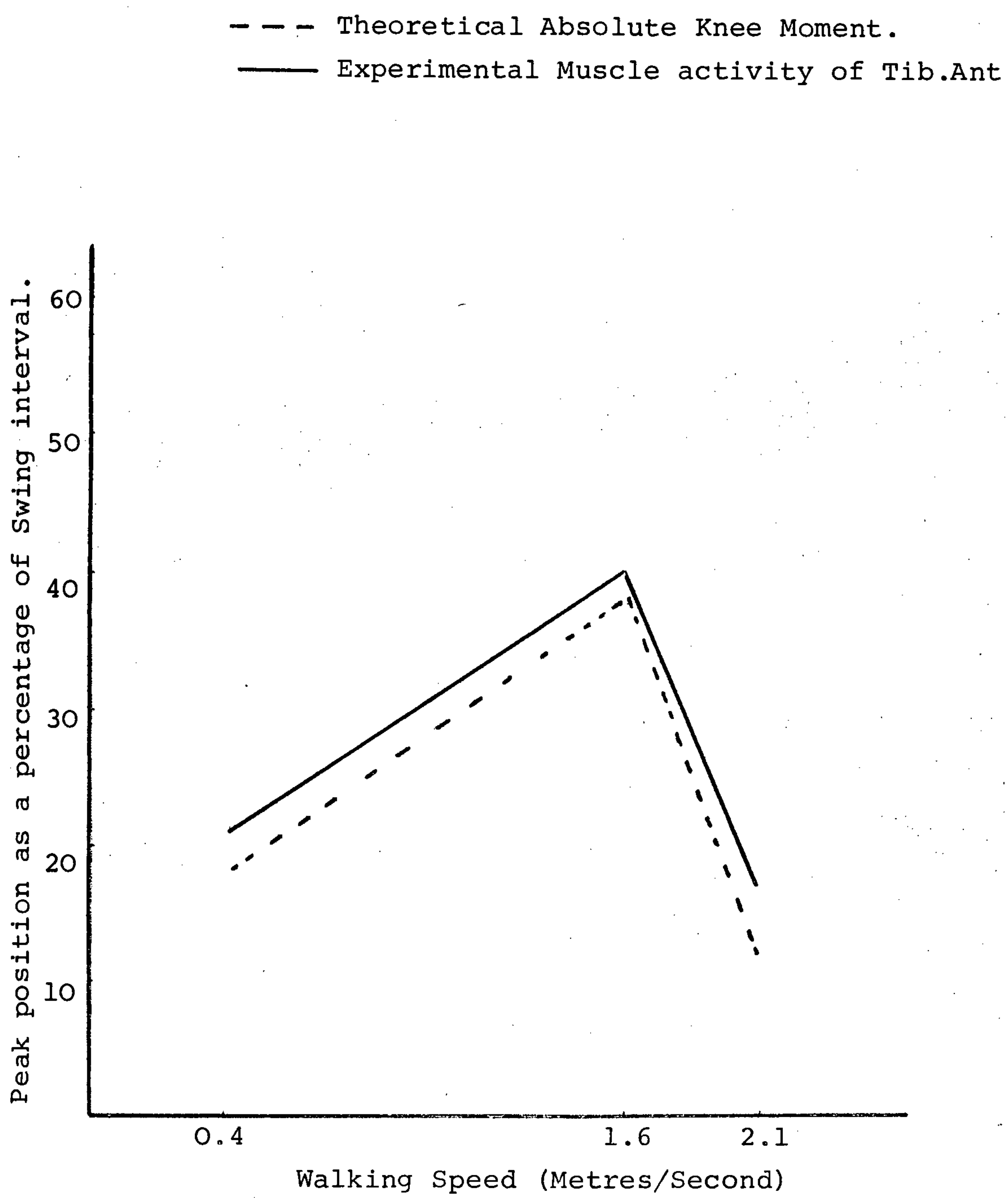

Fig. 9.10 PEAK POSITION VS. WALKING SPEED.
SUBJECT C.H.

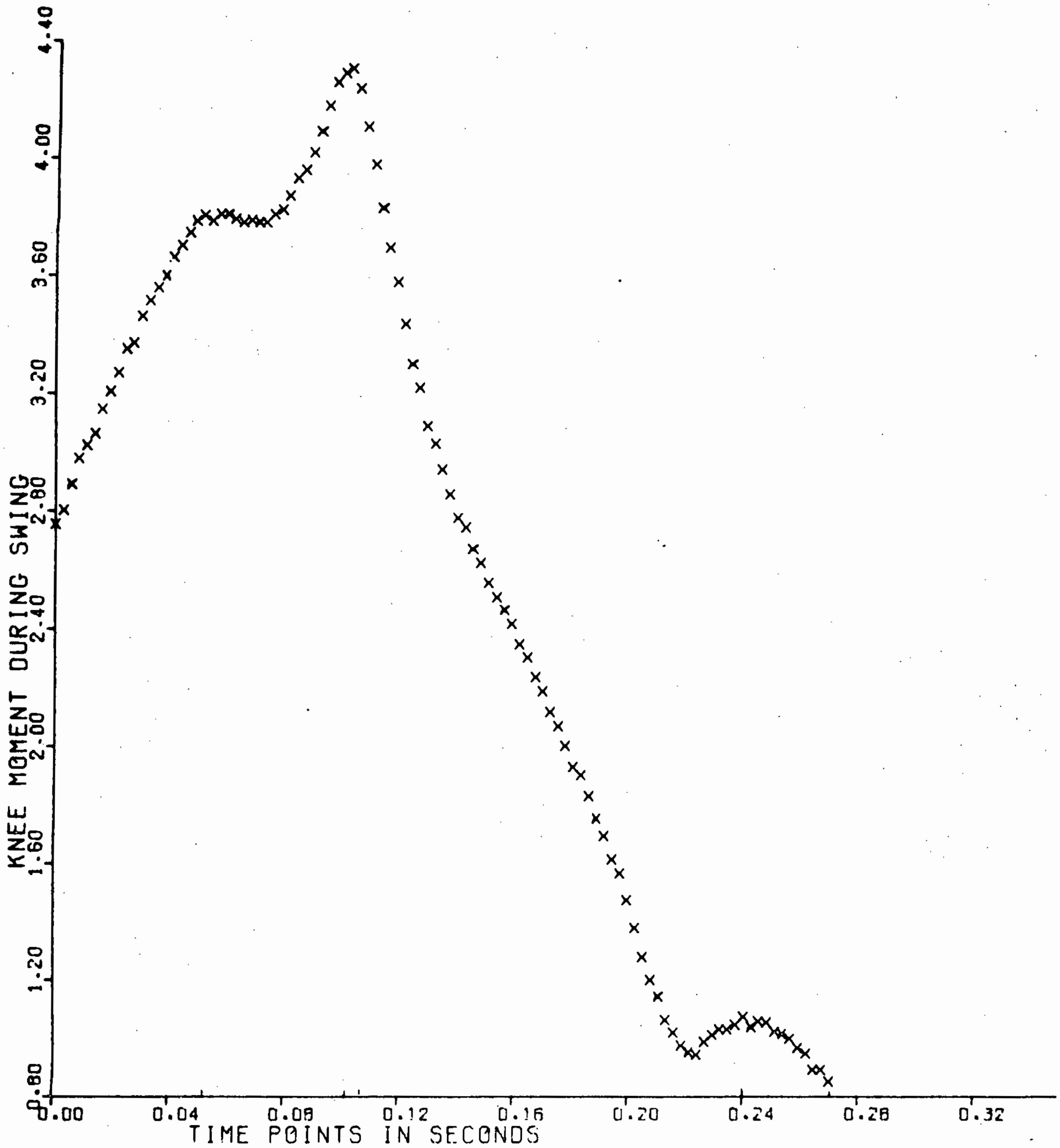

Fig.9.11 Knee Moment (Swing) vs Time.

Subject C.H.

Walking speed 1.6 metres/second

STANCE.

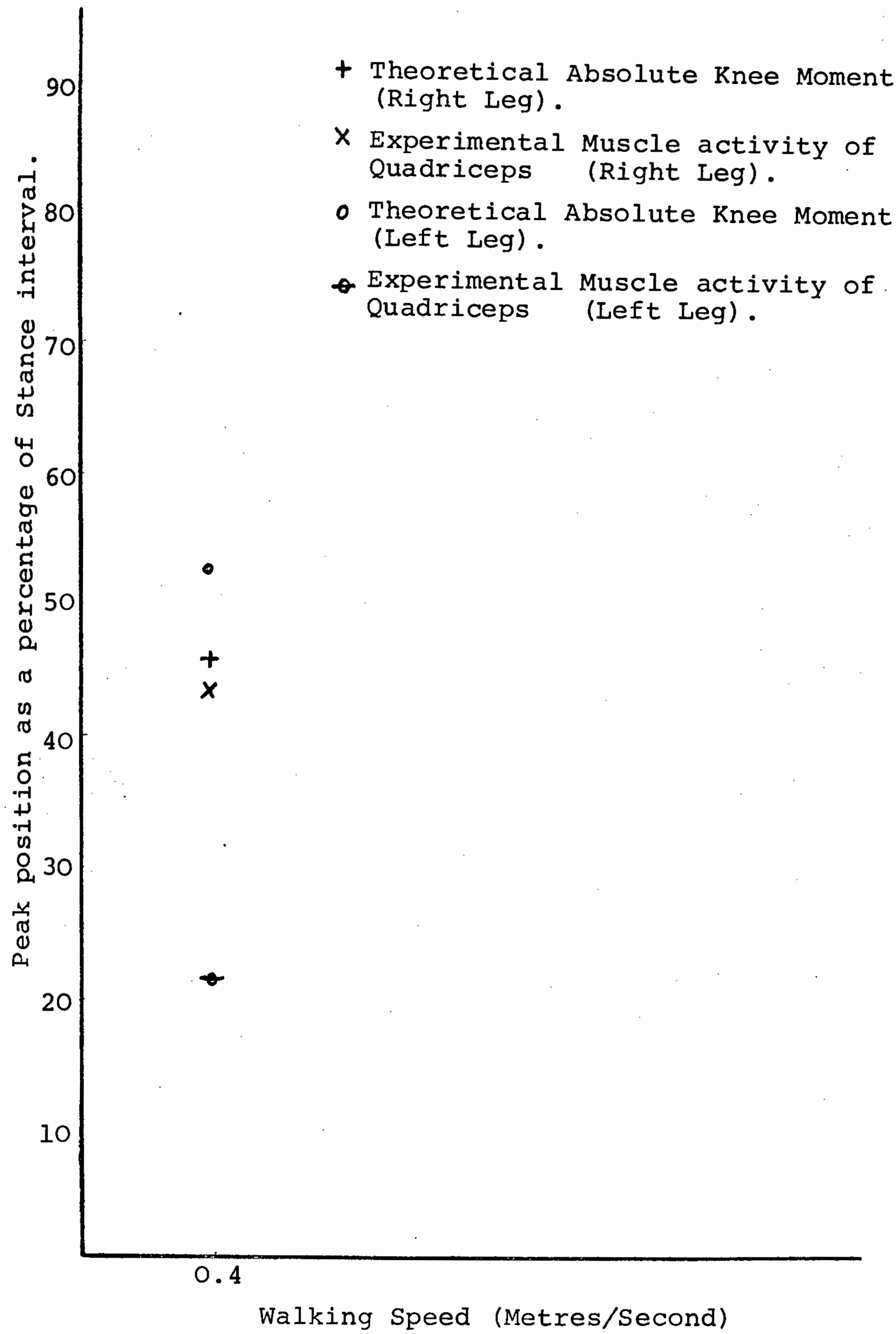

Fig. 9.12 PEAK POSITIONS FOR RIGHT AND LEFT LEGS AT ONE WALKING SPEED.

SUBJECT P.B.

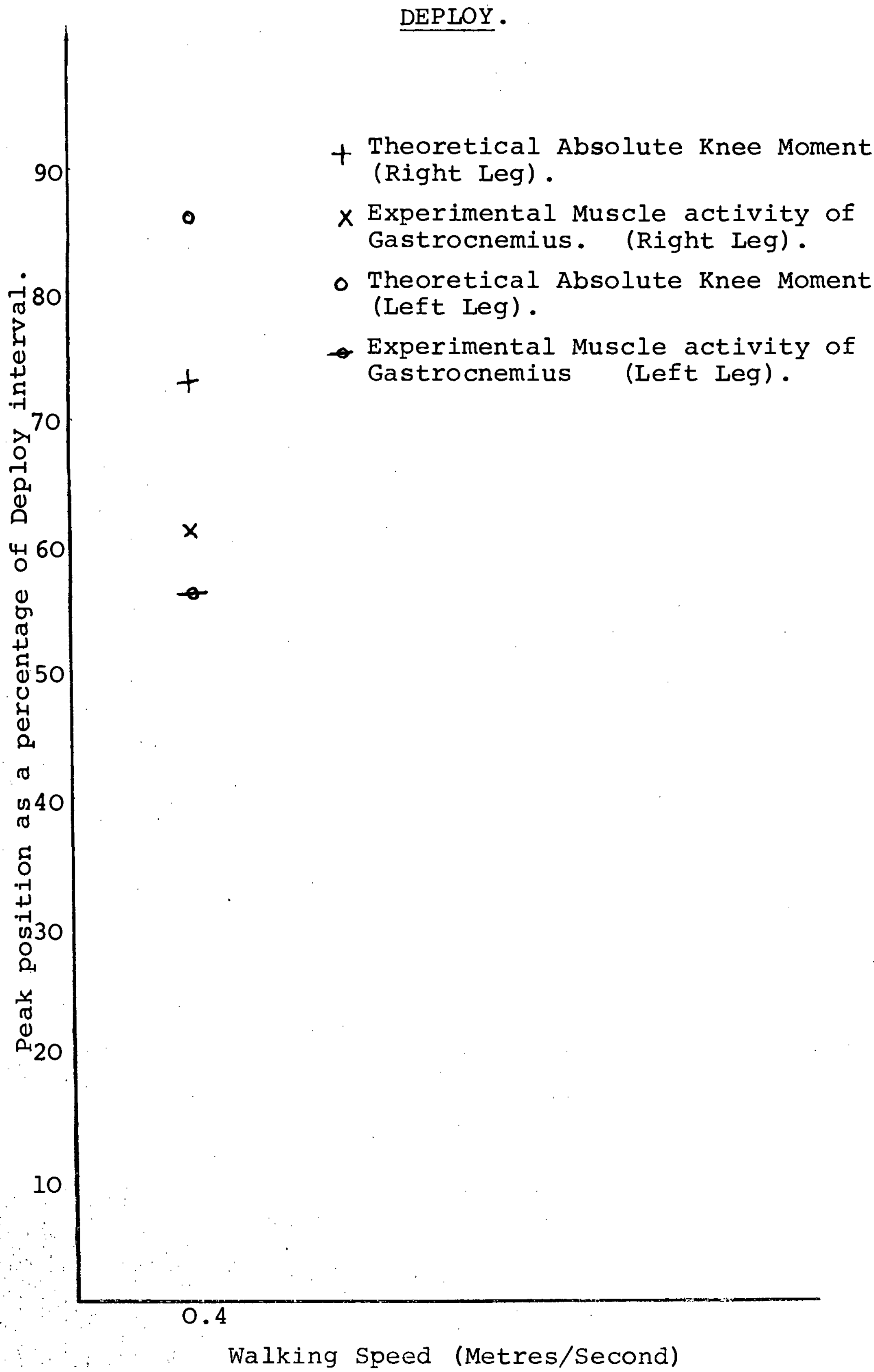

Fig. 9.13 PEAK POSITIONS FOR RIGHT AND LEFT LEGS AT ONE WALKING SPEED.

SUBJECT P.B.

SWING.

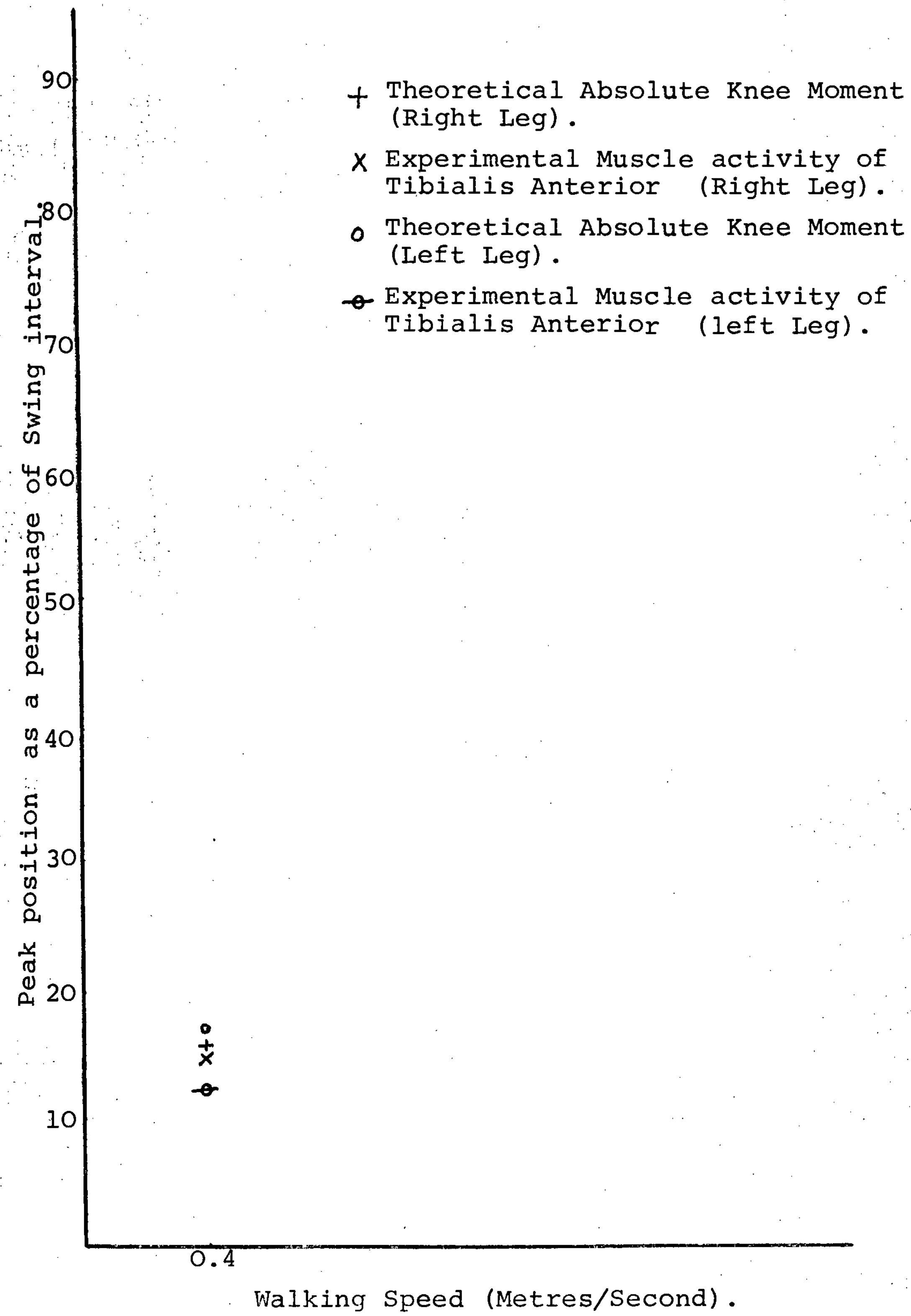

Fig. 9.14 PEAK POSITIONS FOR RIGHT AND LEFT LEGS AT ONE WALKING SPEED.

SUBJECT P.B.

APPENDIX 1.

A NOTE ON THE SIMPLE SEARCH MECHANISM (SSM) AS RELATED TO HUMAN LOCOMOTION

Introduction

The Simple Search Mechanism (SSM) has been suggested as the mechanism which controls muscles in conditions where it is required to maintain precisely a certain found position or velocity (in certain cases frequency) of a part of the skeleton or joint angles. For example, SSM controls muscles in conditions of maintenance of muscle tension or magnitudes of joint angles for which some stimulus is minimal.

The hypothesis was expressed by Aizerman and Andreeva (1968) that the SSM was also used to control time variation of the lengths of muscles in periodically repeating movements like habitual walking. The experiment reported here was designed to test this hypothesis. In this experiment the subjects walked in "comfy cadence" i.e. each subject walked comfortably repeating approximately the same step-period and probably the same step length. The speed was approximately constant. The results of this experiment can be compared with other experiments connected with the operation of SSM. Briefly, in this experiment processed electromyographic (emg) activity of the flexor and extensor about the knee is correlated with the absolute angular velocity about the knee.

It was found that a peak of muscular activity occurring just before heel-strike starts in both the extensor (quadriceps) and flexor (ham string) as the absolute value of the angular velocity, increasing, passes through a certain threshold (or band of threshold values). The theoretical basis for this study is excellently summarized in the paper by Aizerman (1968). After Chernov (1968) it will be assumed that the absolute angular velocity between the shank and the femur is the "unpleasantness function" for the aspect of "comfy cadence" under surveillance here.

Note:

The subject in this experiment could have been constrained to walk an exactly equal repeating step-length but it was felt that this was not true habitual walking whereas walking in "comfy cadence" was more natural and an approximately constant stepping period was soon obtained. See Table A1.1.

Experiment

To measure the knee-joint angular velocity simultaneously with emg's of flexor and extensor muscles about the knee while the subject is walking naturally in "comfy cadence". Comparison of these curves will confirm or deny the hypothesis of Aizerman & Andreeva (Aizerman(1968)).

Method

The potentiometer is mechanically driven in response to the angular variation between two perspex shafts. The voltage proportional to the angle is fed to a differentiating RC circuit (see Fig. A1.1) The subject walks naturally along a metal walkway so that an approximately constant stepping period is soon obtained (see Table A1.1). Both shoes worn are fitted with contacts (foot-switches) at the heel and toe and a foot-switch pattern can then be obtained indicating the time sequence of ground contact of the extremities. An area of skin covering the main bulk of the quadriceps muscle was shaved and cleansed with alcohol and three self-adhesive electrodes* filled with electrolyte paste (one electrode was the earth) were placed adjoining each other on the skin. Two similar electrodes were placed over the main bulk of the hamstring muscle. The raw emg signal passes from the muscle via the electrodes to an amplifier (gain 100) and then through a rectifier and averager so that the final emg pattern recorded on an Ultraviolet (U-V) recorder is an averaged emg envelope. The signal representing the joint angle θ_{knee} and the output signal from the differentiating RC circuit pass to additional channels of the U-V recorder giving θ_{knee} and $\frac{d\theta_{knee}}{dt}$.
The experiment was repeated on four male subjects (C.H.,F.R., P.H.,and A.Z.) with the position of the electrodes kept as far as possible the same in each case. Results were recorded directly for each subject.(For C.H. and F.R. the power leads in the potentiometer circuit were reversed.) Fig. A1.2 to A1.5 are records of three (right-foot) steps of each subject.

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* The electrodes are obtained from Becton-Dickinson Dispos-el disposable electrode kits.
Address: Becton, Dickinson and Co.,
Rutherford, New Jersey, 07070, USA.

Key to Interpretation of Output.

From top to bottom

Channel (1) Quadriceps emg. Channel (2) Hamstring emg.

Channel (3) θ_{knee} Channel (4) $\frac{d}{dt}\theta_{knee}$

Channel (5) Left - Footswitch pattern.

Channel (6) Right - Footswitch pattern.

The distance between two vertical lines on paper output, is 0.5 cm and represents 0.1 seconds.

No scale is given for θ_{knee}, $\frac{d}{dt}\theta_{knee}$ due to the fact that only a correlation between the slope of $\frac{d}{dt}\theta_{knee}$ and the emg patterns is required.

Results (see attached Figures A1.2 - A1.5)

The e.m.g. amplifier gains used were constant in each run.

Conclusions.

From inspection of the various outputs of the 4 different subjects one can say

(i) From heel-strike to heel-strike of the same right-foot.

θ_{knee}: decreases - increases - decreases - increase
(heel-strike)- (heel & toe)- (toe only) - (toe-off & swing)

i.e. there occur two peaks in the θ_{knee} trajectory during one complete step where θ_{knee} is the angle between the shank and femur. The slight oscillation that occurs in the θ_{knee} trajectory during the heel and toe phase of the step appears to be due to a combination of knee-locking and outward-bunching of the quadriceps muscle which caused the perspex shafts to move slightly. The relative contributions are estimated at 50% each.

(ii) A peak of muscle activity starts in both the extensor (quadriceps) and flexor (hamstring) as the absolute value of the angular velocity ($\frac{d}{dt}\theta_{knee}$), increasing, passes through a certain threshold.

In general, there will be a band of threshold values so that when the absolute value of the knee angular velocity passes through this band it coincides with the start of a peak in the extensor and flexor e.m.g.'s.

(iii) In <u>some</u> cases, during one complete step, the initial peak is followed by a second one(usually smaller in magnitude). In these cases, however, the correlation between the absolute value of the angular velocity (increasing) passing through the threshold band and the start of the second peak is not good.

It is considered that the second peak occurs randomly. The hypothesis, therefore, that the SSM mechanism controls the time variation of lengths of muscles in habitual walking seems to be confirmed experimentally if one considers only the first peak of muscular activity in the e.m.g.'s of the flexor and extensor muscles occurring just before heelstrike.

References

(1) Aizermann, M.A. and Andreeva, E.A.(1968): "Simple Search Mechanism for control of Skeletal Muscles". Automation and remote control pg.452. (Translated from Automatika i Telemekhanika No.3 pp. 103 - 118 March, 1968).

(2) Chernov. V.I. (1968): "Control of one muscle and a pair of muscle-antagonists under accurate search conditions". Automation and Remote Control No.7, (in Russian).
Chernov uses the absolute angular velocity of the wrist joint as his "unpleasantness function".

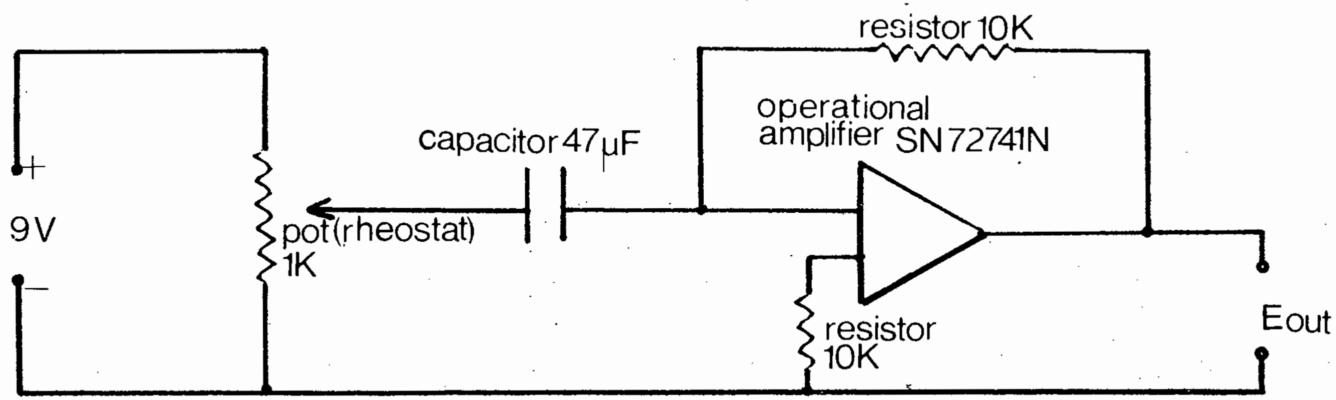
TABLE A1.1.

SUBJECT	PERIODS OF SUCCESSIVE STEPS (SECONDS)	AVERAGE PERIOD OF STEP(SECONDS)	STANDARD DEVIATION (SECONDS)
P.H.	1.30 1.28 1.28	1.28	± 0.02
F.R.	1.34 1.28 1.32	1.31	± 0.04
A.Z.	1.46 1.46	1.46	± 0.0
C.H.	1.38 1.30 1.32	1.33	± 0.06

The above Table indicates that when a subject walks naturally in "comfy cadence" an approximately constant steplength-period is soon obtained.

FIGURE A1.1

ELECTRICAL CIRCUIT OF POTENTIOMETER



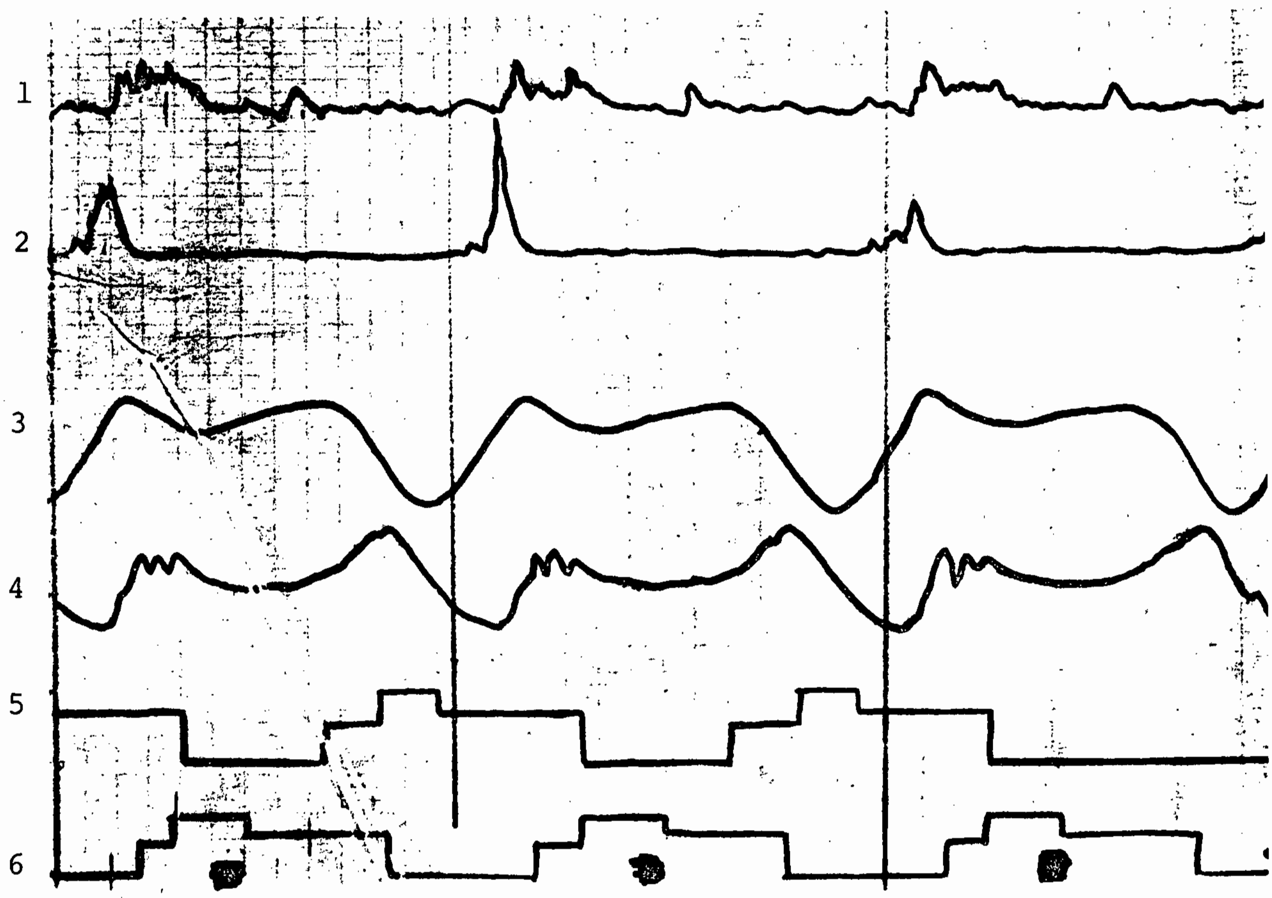


Fig. A1.2 Output for S.S.M.Experiment.
Subject C.H.

Key to interpretation of output is on page 104.

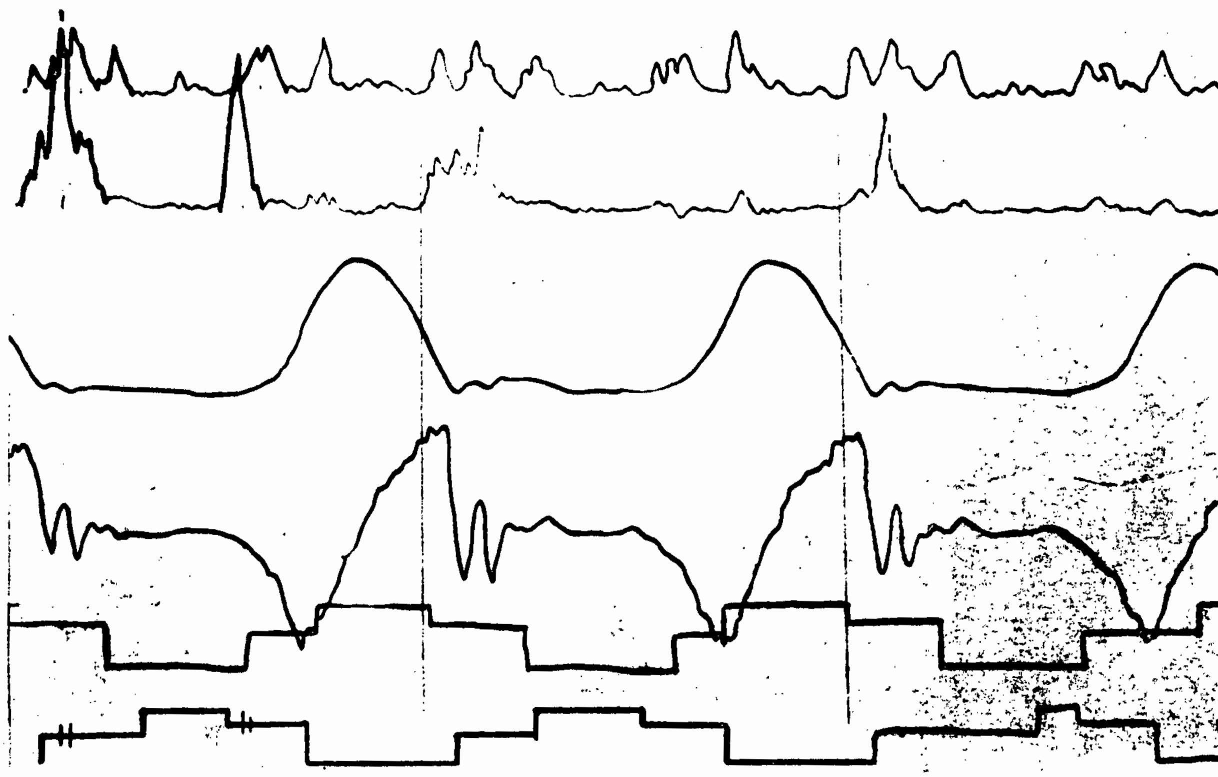


Fig. A1.3 Output for S.S.M. Experiment.
Subject A.Z.

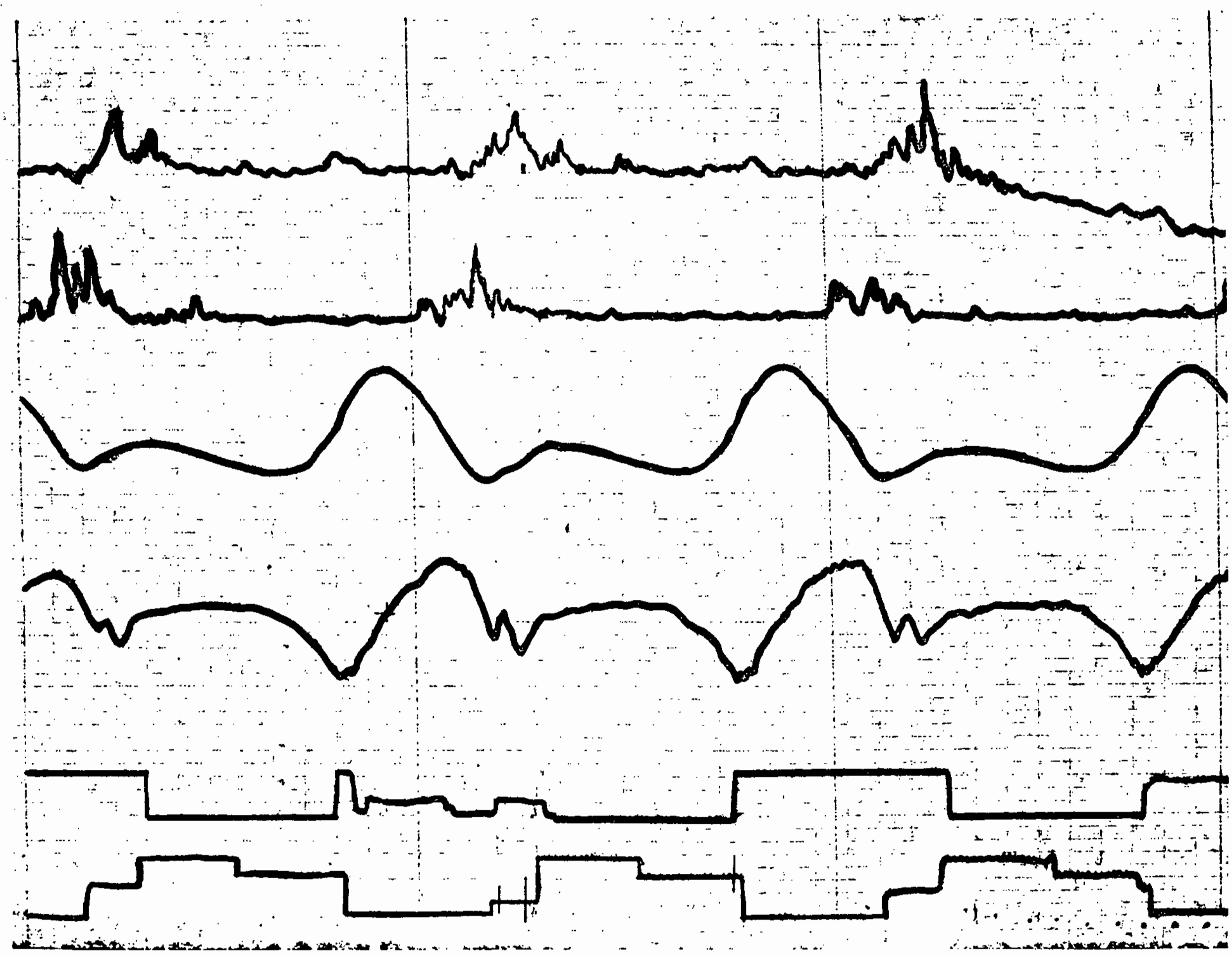


Fig. A1.4 Output for S.S.M. Experiment.

Subject P.H.

Fig. A1.5 Output for S.S.M. Experiment.

Subject F.R.

APPENDIX 2

A Computer Program for the Analysis of Electromyographic Data.

This program originally was designed for the analysis of electromyographic (EMG) data collected from the major muscle groups of the leg as a subject walks along a metal walkway, but it can equally be used (with a slight modification) for the analysis of EMG data taken off any part of the human anatomy.

Input.

The program requires as input up to a maximum of four EMG channels and two footswitch (F.S.) channels. These are fed in from a digital magnetic tape as six arrays of numbers.

Aim of Program.

The principle aim is to correlate an average EMG signal (of a particular muscle or group of muscles) with the F.S pattern so as to give an idea of the phasic properties of the muscle activity.

Taking the time from heel-strike to heel-strike of the same foot as one complete period of a double-step, the peak position, peak-width, peak-width at half-height and peak-width at quarter height can be determined as a percentage of this period.

These parameters are tabulated (ultimately they will be plotted graphically) to show their variation as step-length or walking-speed changes.

The phasic properties of the different muscles (or muscle groups) can be compared.

Output.

Plots of the four averaged EMG signals and the two F.S. patterns can be obtained.

The standard deviation and standard error of the mean is obtained in the y-direction at a discrete number of points of the EMG signal and in the x-direction at the beginning and end of a peak. Information concerning the peak position, peak-width etc. as a percentage of heel-strike to heel-strike period is presented.

Listing of Program.

... 114/

```
@FOR,IS RUTH,MAIN,RUTH,MAIN
      REAL LF
      DIMENSION A(50),C(50),E(50),F(50),O(50),H(10),R(50),G(10),P(50),
     1INDEX(4),SUM(6),ZB(50,10),TIM(50),B(50),          RF(50),LF(50),
     2Q(50),D(50),ARRAY(50,10),ARREY(50,10),ORRAY(50,10),
     3ORREY(50,10),DRRAY(50,10),DRREY(50,10),AZB(50,10),BZB(50),EMG1(50)
     4,ERRAY(50,10),ERREY(50,10),T(50),U(50),URRAY(50,10),IBUF( 5000)
     5,EMG2(50),EMG3(50),EMG4(50),O2(50),D3(50),D4(50),Q2(50),Q3(50),
     6Q4(50),U2(50),U3(50),U4(50),T2(50),T3(50),T4(50)
      COMMON INDEX,NUM,THRE,NSTIM,N,II,JJ
      READ(8,100) THRE, NUM
  100 FORMAT(F6.3,I3)
      DO 50 K=1,NUM
   50 AZB(K,1) = 0.
      LL=0
      TT = 0
      MM = 1
      KK=1
      WRITE (5,400)
  400 FORMAT (40X,37H ANALYSIS OF FOOTSWITCH AND EMG DATA //)
      WRITE (5,401)
  401 FORMAT (20X,69H (DATA ANALYSED IN THE FOLLOWING SEQUENCE LF DATA,R
     1F DATA,EMG1------)///)
      CALL PLOTS(IBUF, 5000,6)
      CALL PLOT(0,0,0.5,-3)
   70 READ(8,101) (RF(I),I=1,NUM)
      READ(8,101)(LF(I),I=1,NUM)
      READ(8,101)(EMG1(I),I=1,NUM)
      READ(8,101)(EMG2(I),I=1,NUM)
      READ(8,101)(EMG3(I),I=1,NUM)
      READ(8,101)(EMG4(I),I=1,NUM)
  101 FORMAT()
      WRITE(12) (RF(I),I=1,NUM)
      WRITE(12)(  LF(I),I=1,NUM)
      WRITE(12)(EMG1(I),I=1,NUM)
      WRITE(12)(EMG2(I),I=1,NUM)
      WRITE(12)(EMG3(I),I=1,NUM)
      WRITE(12)(EMG4(I),I=1,NUM)
      IF(RF(1)-1.)30,31,30
   30 GO TO 70
C     TO RECALL LF DATA FROM STORE TO WORKING MEMORY
   31 ICOUNT=1
      N=2
C     N EQUALS THE NUMBER OF STEPS TO BE AVE /2
      DO 305 J=1,N
   13 CALL SETADR(12,ICOUNT)
      READ (12)(LF(I),I=1,NUM)
   20 DO 464 I=1,NUM
      A(I) = LF(I)
  464 CONTINUE
      ICOUNT = ICOUNT+6
```

```
      CALL SETADR(12,ICOUNT)
      READ (12) (LF(I),I=1,NUM)
   19 DO 465 I=1,NUM
      C(I) = LF(I)
  465 CONTINUE
C     SUBROUTINE TO LINE UP STARTS OF FOOT SWITCH DATA
      CALL LINUP( A)
      CALL LINUP( C)
C     SUBROUTINE TO FIND POSITIONS OF JUMPS ALONG X-AXIS
   55 CALL POSJU(A,H,R)
      CALL POSJU(C,G,P)
      WRITE (5,402)
  402 FORMAT (10X,115H XDIFF1          XDIFF1-THRE          XDIFF2          XD
     1IFF2-THRE          XDIFF3          XDIFF3-THRE          SUM(XDIFF-THRE)/
     2)
      SUM(1) = 0.
      L=1
  116 DO 16 M = 1,3
      L = L+1
C     CALCULATES DIFF BETWEEN JUMPS ON X AXIS
       DIFF =  ABS( R(M) -  P(M))
C     COMPARISON OF DIFF VALUES WITH THRESHOLD VALUE
      ZDIFF = DIFF - THRE
      IF(ZDIFF)1,1,2
    1 ZDIFF = 1.
      GO TO 120
    2 ZDIFF = 0.
  120 IF(M-2)10,11,12
   10 WRITE (5,112) DIFF,ZDIFF
  112 FORMAT (13X,F3.1,14X,F3.1)
      GO TO 17
   11 WRITE (5,113) DIFF,ZDIFF
  113 FORMAT (46X,F3.1,14X,F3.1)
      GO TO 17
   12 WRITE (5,115) DIFF,ZDIFF
  115 FORMAT (79X,F3.1,14X,F3.1)
   17 SUM(L) = SUM(L-1) + ZDIFF
   16 CONTINUE
      WRITE (5,800) SUM(L)
  800 FORMAT (116X,F6.3)
      CALL PREP(A,E,TT)
      CALL PREP (C,F,TT)
      DO 40 K = 1,NUM
      ARRAY(K,J) = E(K)
      ARREY(K,J) = F(K)
      ORRAY(K,J) = A(K)
   40 ORREY(K,J) = C(K)
C     CRITERION FOR AVERAGING
      IF(SUM(L)-0.)27,303,303
  303 DO 700 K=1,NUM
  700 ZB(K,J) = (ORRAY(K,J)+ORREY(K,J))/2.0
  305 ICOUNT=ICOUNT+6
  888 DO 42 K=1,NUM
      NN = N +1
      DO 43 J = 2,NN
      AZB(K,J) = AZB(K,J-1) + ZB(K,J-1)
      AZB(K,J-1) = AZB(K,J)
   43 CONTINUE
   42 CONTINUE
      DO 5 K=1,NUM
    5 BZB(K) = AZB(K,N)/N
      WRITE (5,901)
      WRITE(5,701) (BZB(K),K=1,NUM)
      WRITE (5,900)
  455 FORMAT(2X,17H FIRST STEP DATA,I3/)
  456 FORMAT(2X,18H SECOND STEP DATA,I3/)
```

```
  901 FORMAT (2X,15H AVERAGED DATA /)
  900 FORMAT(/)
  155 FORMAT ()
  701 FORMAT()
      CALL PREP (BZB,O,TT)
      MN = NSTIM-1
      DO 51 L = 1,MN
   51 URRAY(L ,MM) = O(L)
      MM = MM+1
      CALL STERR(ARRAY,ARREY,O)
      IF(TT-1.)752,850,850
  850 CALL STERY (ORRAY,ORREY,BZB)
  752 IF(TT-1.)777,778,778
C     PLOTTING ROUTINE FOR AVR EMG PATTERN
  778 CALL PLOT(10.0,0.0,-3)
      DO 600 J=1,NUM
      TIM(J) = J
  600 B(J) =  BZB(J)
      CALL SCALE(TIM (1)   ,9.0,25 ,1)
      CALL SCALE(   B(1)   ,9.0,25 ,1)
      CALL AXIS(0.0,0.0,27H TIME POINTS OF WALK CYCLE ,-27,9.0,0.0,TIM(2
     16),TIM(27))
      CALL AXIS(0.0,0.0,19H EMG VOLTAGE LEVELS,+19,9.0,90.0,B(26),B(27))
      CALL LINE(TIM,B,25,1,-1,4)
      GO TO 765
C     PLOTTING ROUTINE FOR AVR. FOOTSWITCH PATTERN
  777 DO 500 J=1,NUM
      TIM(J) = J
  500 B(J) =  BZB(J)
      CALL SCALE(TIM (1)   ,9.0,25 ,1)
      CALL SCALE(   B(1)   ,9.0,25 ,1)
      CALL AXIS(0.0,0.0,27H TIME POINTS OF WALK CYCLE ,-27,9.0,0.0,TIM(2
     16),TIM(27))
      CALL AXIS(0.0,0.0,19HFOOT VOLTAGE LEVELS,+19,9.0,90.0,B(26),B(27))
      CALL LINE(TIM,B,25,1,-1,4)
      CALL SYMBOL(0.5,9.6,0.21,24H MMC2  C.HERSHLER BIOENG, 0,24)
  765 CONTINUE
      IF(LL-1)23,33,27
   23 ICOUNT=0
      SUM(L) = 0.
C     TO RECALL RF AND EMG1-4 FROM STORAGE TO WORKING MEMORY
      DO 406 J=1,N
    9 CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
      DO 466 I=1,NUM
      A(I) = RF(I)
  466 CONTINUE
      ICOUNT = ICOUNT+2
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I), I=1,NUM)
      DO 467 I=1,NUM
      EMG1(I) = RF(I)
      D(I) = EMG1(I)
  467 CONTINUE
      ICOUNT=ICOUNT+1
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
      DO 1000 I=1,NUM
      D2(I)=RF(I)
 1000 CONTINUE
      ICOUNT=ICOUNT+1
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
      DO 1001 I=1,NUM
      D3(I) = RF(I)
 1001 CONTINUE
```

```
      ICOUNT=ICOUNT+1
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
      DO 1002 I=1,NUM
      D4(I) = RF(I)
 1002 CONTINUE
      ICOUNT=ICOUNT+1
  611 CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
  610 DO 468 I = 1,NUM
      C(I) = RF(I)
  468 CONTINUE
      ICOUNT = ICOUNT+2
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I), I=1,NUM)
    8 DO 469 I=1,NUM
      EMG1(I) = RF(I)
      Q(I) = EMG1(I)
  469 CONTINUE
      ICOUNT=ICOUNT+1
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
      DO 1003 I=1,NUM
      Q2(I) = RF(I)
 1003 CONTINUE
      ICOUNT=ICOUNT+1
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
      DO 1004 I=1,NUM
      Q3(I) = RF(I)
 1004 CONTINUE
      ICOUNT=ICOUNT+1
      CALL SETADR(12,ICOUNT)
      READ(12) (RF(I),I=1,NUM)
      DO 1005 I=1,NUM
      Q4(I) = RF(I)
 1005 CONTINUE
      CALL LINOP(A,D,D2,D3,D4)
      CALL LINOP(C,Q,Q2,Q3,Q4)
      IF(J-N)6,4,6
    4 LL=LL+1
    6 CALL POSJU(A,H,R)
      CALL POSJU(C,G,P)
      WRITE(5,402)
      SUM(1) = 0.
      L=1
  117 DO 299 M=1,3
      L = L+1
C     CALCULATES DIFF BETWEEN JUMPS ON X AXIS
       DIFF =  ABS( R(M) -  P(M))
C     COMPARISON OF DIFF VALUES WITH THRESHOLD VALUE
      ZDIFF = DIFF - THRE
      IF(ZDIFF)53,53,22
   53 ZDIFF=1.
      GO TO 121
   22 ZDIFF=0.
  121 IF(M-2)73,71,72
   73 WRITE(5,112)DIFF,ZDIFF
      GO TO 177
   71 WRITE(5,113)DIFF,ZDIFF
      GO TO 177
   72 WRITE(5,115)DIFF,ZDIFF
  177 SUM(L)=SUM(L-1) +ZDIFF
  299 CONTINUE
      WRITE (5,800) SUM(L)
      CALL PREP(A,E,TT)
```

```
      CALL PREP (C,F,TT)
      CALL PREP (D,U,TT)
      CALL PREP(D2,U2,TT)
      CALL PREP(D3,U3,TT)
      CALL PREP(D4,U4,TT)
      CALL PREP (Q,T,TT)
      CALL PREP(Q2,T2,TT)
      CALL PREP(Q3,T3,TT)
      CALL PREP(Q4,T4,TT)
      DO 48 K=1,NUM
      ARRAY(K,J) = E(K)
      ARREY(K,J) = F(K)
      ORRAY(K,J) = A(K)
      ORREY(K,J) = C(K)
      ERRAY(K,J) = U(K)
      ERREY (K,J) = T (K)
      DRRAY(K,J) = D(K)
   48 DRREY(K,J) = Q(K)
C     CRITERION FOR AVERAGING
      IF(SUM(L)-0.)27,308,308
  308 DO 708 K=1,NUM
  708 ZB(K,J) = (ORRAY(K,J) + ORREY(K,J))/2.0
  406 ICOUNT=ICOUNT+1
      DO 933 K = 1,NUM
  933 AZB(K,1) = 0.
      CALL PLOT(10.0,0.0,-3)
      GO TO 888
   33 DO 505 J=1,N
      DO 470 I = 1,NUM
      ORRAY(I,J) = DRRAY(I,J)
      ORREY(I,J) = DRREY(I,J)
      ZB(I,J) = (ORRAY(I,J) + ORREY(I,J))/2.0
      ARRAY (I,J) = ERRAY (I,J)
      ARREY (I,J) = ERREY (I,J)
  470 CONTINUE
  505 CONTINUE
      LL=LL+1
      TT = TT+1
      DO 934 K = 1,NUM
  934 AZB(K,1) = 0.
      GO TO 888
   27 CALL RAG(MM,URRAY,BZB)
      IF(LL-5) 15,99,15
   15 IF(KK-2) 1100,1101,1102
 1100 DO 407 J=1,N
      DO 49 K=1,NUM
      ERRAY(K,J)=U2(K)
      ERREY(K,J)=T2(K)
      DRRAY(K,J)=D2(K)
   49 DRREY(K,J)=Q2(K)
  407 CONTINUE
      KK=KK+1
      GO TO 33
 1101 DO 408 J=1,N
      DO 102 K=1,NUM
      ERRAY(K,J) = U3(K)
      ERREY(K,J) = T3(K)
      DRRAY(K,J) = D3(K)
  102 DRREY(K,J) = Q3(K)
  408 CONTINUE
      KK=KK+1
      GO TO 33
 1102 DO 409 J=1,N
      DO 103 K=1,NUM
      ERRAY(K,J) = U4(K)
      ERREY(K,J) = T4(K)
```

```
      DRRAY(K,J) = D4(K)
  103 DRREY(K,J) = Q4(K)
  409 CONTINUE
      GO TO 33
   99 CALL PLOT(12.0,0.0,999)
      CALL EXIT
      END
@FOR,IS RUTH.LINUP,RUTH.LINUP
      SUBROUTINE  LINUP( X)
C     TO RECORD X-AXIS POSITION OF JUMP
      DIMENSION INDEX(4),X(50)
      COMMON INDEX,NUM,THRE,NSTIM,N,II,JJ
      I=2
C     CHECKING ARRAY FOR FIRST JUMP
    9 IF(X(I)-X(I-1))1,10,1
   10 I=I+1
      GO TO 9
C     RE-ALIGNING ARRAY WITH FIRST JUMP AS POSITION 1
    1 DO 20 J=1,NUM
   20 X(J) = X(I+J-1)
      RETURN
      END
@FOR,IS RUTH.POSJU,RUTH.POSJU
      SUBROUTINE POSJU(X,Y,Z)
      DIMENSION INDEX(4),X(50),Y(50),Z(50)
      COMMON INDEX,NUM,THRE,NSTIM,N,II,JJ
      INDEX(1 ) = X(1)
      K=1
      M = 2
  151 DO 30 JUMP = 1,3
    6 K=K+1
      IF (X(K)-X(K-1))2,6,2
    2 Y(JUMP) = X(K)
C     CALCULATION OF INDEX IN ORDER TO COMPARE WITH STD INDEX/OTHR INDICES
      INDEX(1 ) = INDEX(1 ) + Y(JUMP)**M
      M=M+1
   30 Z(JUMP) = K
      IF (INDEX(1 )- 150)4,5,4
    5 WRITE(5,116)
  116 FORMAT(51X,'150')
    4 WRITE(5,117)INDEX(1 )
  117 FORMAT (51X,I3)
      RETURN
      END
@FOR,IS RUTH.PREP,RUTH.PREP
      SUBROUTINE PREP(X,Y,TT)
      DIMENSION INDEX(4),X(50),Y(50)
      COMMON INDEX,NUM,THRE,NSTIM,N,II,JJ
      NSTIM = 1
      K = 1
C     TO FIND POSITION ON X AXIS WHERE EXCITATION STARTS
    7 DO 1 I=K,NUM
      IF(I-NUM)9,8,9
    9 IF(X(I))2,1,2
    1 CONTINUE
    2 Y(NSTIM) = I-1
      II=Y(NSTIM)
    3 FORMAT (2X,F6.3)
      NSTIM = NSTIM+1
C     TO FIND POSITION ON X AXIS WHERE EXITATION ENDS
      DO 4 J=I,NUM
      IF (J-NUM)10,8,10
   10 IF (X(J))4,5,4
    4 CONTINUE
    5 Y(NSTIM)=J
      JJ=Y(NSTIM)
```

```
    6 FORMAT(2X,F6.3)
      NSTIM = NSTIM+1
      IF (TT-1.) 20,21,21
C     TO FIND POSITION OF SECOND START/JUMP ON AXIS
   20 DO033 L=J,NUM
      IF(L-NUM)22,8,22
   22 IF(X(L))24,33,24
   33 CONTINUE
   24 Y(NSTIM) = L-1
      NSTIM = NSTIM + 1
      K=L
      GO TO 7
   21 K=J
      GO TO 7
    8 RETURN
      END
@FOR,IS RUTH.STERR,RUTH.STERR
      SUBROUTINE STERR(ARRAY,ARREY,Z)
      DIMENSION INDEX(4),SIGMA(50,10),STGMA(50,10),Z(50),ARRAY(50,10),
     1ARREY(50,10)
      COMMON INDEX,NUM,THRE,NSTIM,N,II,JJ
      L = NSTIM-1
      DO 3 I = 1,L
    3 SIGMA(I,1) = 0.
C     TO CALC. X AXIS STD ERRORS OF THE MEAN
      WRITE (5,420)
  420 FORMAT (2X,34H ERROR STATISTICS OF AVERAGED DATA/)
      WRITE (5,421)
  421 FORMAT(2X,59H (X-AXIS TIMES ARE SUCCESSIVE BEGINNINGS AND ENDS OF
     1PEAKS)/)
      WRITE (5,422)
  422 FORMAT(2X,35H (Y-AXIS TIMES ARE EACH TIME POINT)/)
      WRITE (5,22)
   22 FORMAT (10X,81H AVERAGE CHOSEN TIMES          STANDARD DEVIATION
     1    STANDARD ERROR OF THE MEAN/)
      WRITE (5,23)
   23 FORMAT (2X,8H X-AXIS /)
C     L = NUMBER OF POINTS UPON WHICH STATISTICS IS DONE FOR X-ERROR ANALYSIS
C     ARRAY KEEPS VALUES OF POINTS FOR 1ST STEPS OF PAIR
C     ARREY KEEPS VALUES OF POINTS FOR 2ND STEPS OF PAIR
      DO 20 I = 1,L
      NN = N+1
      DO 2 M= 2,NN
      SIGMA(I,M) = SIGMA(I,M-1) + (ARRAY(I,M-1) - Z(I))**2 +
     1(ARREY(I,M-1) -Z(I))**2
    2 CONTINUE
      SIGMA(I,NN)= SIGMA(I,NN)/(2*N)
  101 FORMAT ()
      SIGMA(I,NN)= SQRT(SIGMA(I,NN))
      STGMA(I,NN) = SIGMA(I,NN)/SQRT(2*N)
      WRITE(5,21) Z(I),SIGMA(I,NN),STGMA(I,NN)
   21 FORMAT (18X,F6.3,22X,F6.3,25X,F6.3)
   20 CONTINUE
      RETURN
      END
@FOR,IS RUTH.LINOP,RUTH.LINOP
      SUBROUTINE LINOP(X,Y,Z,W,V)
      DIMENSION INDEX(4),X(50),Y(50),Z(50),W(50),V(50)
      COMMON INDEX,NUM,THRE,NSTIM,N,II,JJ
      I=2
C     TO CHECK RF ARRAY FOR 1ST JUMP
    9 IF (X(I) -X(I-1))1,10,1
   10 I=I+1
      GO TO 9
C     REALIGNING RF ARRAY AND EMG ARRAY WITH 1ST RF JUMP POSN
    1 DO 20 J=1,NUM
```

```
      IF(YPEAK(I)-PEAKY)2,11,10
   10 PEAKY=YPEAK(I)
      PEAKX=XPEAK(I)
      GO TO 2
   11 PEAKY=YPEAK(I)
      PEAKX=(PEAKX+XPEAK(I))/2
    2 CONTINUE
      HALFP=PEAKY/2
      WRITE (5,18) PEAKY,HALFP,PEAKX
   18 FORMAT(2X,13H PEAK HEIGHT =,F6.3,19H HALF PEAK HEIGHT =,F6.3,
     118H X-VALUE OF PEAK =,F6.3)
      QRT=PEAKY/4
      WRITE(5,1009)QRT
 1009 FORMAT(2X,22H QUARTER PEAK HEIGHT =, F6.3)
C     QRT IS THE QUARTER PEAK HEIGHT
      PEAK=PEAKX*100/LP
      WRITE(5,16) PEAK
   16 FORMAT(2X,35H PEAK POSN-% OF HEEL TO HEEL TIME =, I3)
C     HALFP IS THE HALF PEAK HEIGHT
      DO 88 L=I1,JJ
      IF(X(L)-HALFP) 88,99,999
   99 HALF1 = L
      GO TO 45
  999 X1 = L
      X2 = L-1
      Y1 = X(L)
      Y2 = X(L-1)
      HALF1 = (HALFP - Y1 + ((Y1-Y2)/(X1-X2))*X1)/((Y1-Y2)/(X1-X2))
 9432 FORMAT(2X, 15HX1X2Y1Y2HALF = ,5F12.5)
      GO TO 45
   88 CONTINUE
   45 DO 77 M=JJ,I1,-1
      IF(X(M)-HALFP)77,66,777
   66 HALF2 = M
      GO TO 44
  777 X1=M
      X2=M+1
      Y1=X(M)
      Y2=X(M+1)
      HALF2 = (HALFP - Y1 + ((Y1-Y2)/(X1-X2))*X1)/((Y1-Y2)/(X1-X2))
      WRITE(5,9432) X1,X2,Y1,Y2,HALF2
      GO TO 44
   77 CONTINUE
   44 DIFF=HALF2-HALF1
      PERFS=DIFF*100/LP
      WRITE(5,19) PERFS
   19 FORMAT(2X,41H HALF PEAK WIDTH-% OF HEEL TO HEEL TIME =, I3)
      DO 1000 L=I1,JJ
      IF(X(L)-QRT)1000,1002,1003
 1002 QRT1=L
      GO TO 1004
 1003 X1=L
      X2 = L-1
      Y1 = X(L)
      Y2 = X(L-1)
      QRT1 = (QRT-Y1+((Y1-Y2)/(X1-X2))*X1)/((Y1-Y2)/(X1-X2))
      GO TO 1004
 1000 CONTINUE
 1004 DO 1001 M=JJ,I1,-1
      IF(X(M)-QRT)1001,1006,1007
 1006 QRT2=M
      GO TO 1008
 1007 X1=M
      X2=M+1
      Y1=X(M)
      Y2=X(M+1)
      QRT2 = (QRT-Y1+((Y1-Y2)/(X1-X2))*X1)/((Y1-Y2)/(X1-X2))
      GO TO 1008
 1001 CONTINUE
 1008 DIFF=QRT2-QRT1
      PERFS=DIFF*100/LP
      WRITE(5,1005) PERFS
 1005 FORMAT(2X,42H QRT PEAK WIDTH - % OF HEEL TO HEEL TIME =, I3)
      RETURN
      END
```

The total kinetic energy of the link system is

$$T = T\phi + T\gamma + T_{x_1} + T_{x_2} + T_\omega$$

After manipulation, rearranging, and regrouping terms, the total kinetic energy can be concisely expressed as

$$\begin{aligned} T = {} & \tfrac{1}{2}A_0(\dot{h}^2+\dot{v}^2) + \tfrac{1}{2}A_1\dot{\phi}^2 + \tfrac{1}{2}A_2(\dot{\gamma}-\dot{\phi})^2 + \tfrac{1}{2}A_1\dot{x}_1^2 + \tfrac{1}{2}A_2(\dot{x}_2-\dot{x}_1)^2 \\ & + \tfrac{1}{2}A_\omega\dot{\omega}^2 + C_1\dot{x}_1(\dot{h}t'_2 + \dot{v}t'_1) + C_2(\dot{x}_2-\dot{x}_1)(-\dot{h}t'_4+\dot{v}t'_3) \\ & - C_3\dot{x}_1(\dot{x}_2-\dot{x}_1)t'_6 + C_1\dot{\phi}(\dot{h}t_2+\dot{v}t_1) + C_2(\dot{\gamma}-\dot{\phi})(-\dot{h}t_4+\dot{v}t_3) \\ & - C_3\dot{\phi}(\dot{\gamma}-\dot{\phi})t_6 + C_5\dot{\omega}(\dot{h}\cos\omega-\dot{v}\sin\omega) \end{aligned}$$

where

$A_0 = 2(m_1 + m_2) + m_\omega$	$C_1 = m_1\alpha_1 + m_2L_1$,
$A_1 = I_1 + m_1\alpha_1^2 + m_2L_1^2$,	$C_2 = m_2\alpha_2$,
$A_2 = I_2 + m_2\alpha_2^2$,	$C_3 = m_2\alpha_2L_1 = C_2L_1$,
$A_\omega = I_\omega + m_\omega\alpha_\omega^2$,	$C_5 = m_\omega\alpha_\omega$,
$t_1 = \sin(\phi-\phi_0)$,	$t'_1 = \sin(x_1-\phi_0)$,
$t_2 = \cos(\phi-\phi_0)$,	$t'_2 = \cos(x_1-\phi_0)$,
$t_3 = \sin(\gamma-\phi+\phi_0)$,	$t'_3 = \sin(x_2-x_1+\phi_0)$,
$t_4 = \cos(\gamma-\phi+\phi_0)$,	$t'_4 = \cos(x_2-x_1+\phi_0)$,
$t_5 = \sin(\gamma)$,	$t'_5 = \sin(x_2)$,
$t_6 = \cos(\gamma)$,	$t'_6 = \cos(x_2)$.

Similarly, the total potential energy of the system is

$$\frac{V}{g} = (A_0v - C_1t_2 - C_2t_4 - C_1t'_2 - C_2t'_4 + C_5\cos\omega).$$

The equations of motion are obtained by substituting the T and V expressions into

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_i}\right] - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = M_i$$

where q_i represents the angular variables and M_i the effective moment for the appropriate link. The result is a system of five nonlinear,coupled second-order differential equations. Implicitly the system of equations can be expressed as

$$F(\phi,\dot{\phi},\ddot{\phi};\gamma,\dot{\gamma},\ddot{\gamma};x_1,\dot{x}_1,\ddot{x}_1;x_2,\dot{x}_2,\ddot{x}_2;\omega,\dot{\omega},\ddot{\omega};h,v) = M_\phi$$

$$F_\gamma(\phi,\dot{\phi},\ddot{\phi};\ldots;h,v) = M_\gamma,$$

$$F_1(\phi,\dot{\phi},\ddot{\phi};\ldots;h,v) = M_1,$$

$$F_2(\phi,\dot{\phi},\ddot{\phi};\ldots;h,v) = M_2,$$

$$F_\omega(\omega,\dot{\omega},\ddot{\omega};h,v) = M_\omega.$$

Explicitly we have

(1) leg in stance portion:

$$A_0(\ddot{v}.\dot{v}_\phi + \ddot{h}.\dot{h}_\phi) + A_1\ddot{\phi} - A_2(\ddot{\gamma}-\ddot{\phi}) + C_1(\ddot{h}t_2+\ddot{v}t_1)$$
$$+C_1\ddot{\phi}(\dot{h}_\phi t_2+\dot{v}_\phi t_1) + C_1\dot{\phi}^2(-\dot{h}_\phi t_1+\dot{v}_\phi t_2) - C_2(-\ddot{h}t_4+\ddot{v}t_3)$$
$$+C_2(\ddot{\gamma}-\ddot{\phi})(-\dot{h}_\phi t_4+\dot{v}_\phi t_3) + C_2(\dot{\gamma}-\dot{\phi})^2(\dot{h}_\phi t_3+\dot{v}_\phi t_4)$$
$$-C_3t_6(\ddot{\gamma}-2\ddot{\phi}) + C_3t_5\dot{\gamma}(\dot{\gamma}-2\dot{\phi}) + C_1\ddot{x}_1(\dot{h}_\phi t_2'+\dot{v}_\phi t_1')$$
$$+C_1\dot{x}_1^2(-\dot{h}_\phi t_1'+\dot{v}_\phi t_2') + C_2(\ddot{x}_2-\ddot{x}_1)(-\dot{h}_\phi t_4'+\dot{v}_\phi t_3')$$
$$+C_2(\dot{x}_2-\dot{x}_1)^2(\dot{h}_\phi t_3'+\dot{v}_\phi t_4') + C_5\ddot{\omega}(\dot{h}_\phi \cos\omega-\dot{v}_\phi \sin\omega)$$
$$-C_5\dot{\omega}^2(\dot{h}_\phi \sin\omega+\dot{v}_\phi \cos\omega) + \left[A_0\frac{\partial \bar{v}}{\partial\phi} + C_1t_1 - C_2t_3\right]g = M_\phi,$$

$$A_0(\ddot{v}.\dot{v}_\gamma+\ddot{h}\,\dot{h}_\gamma) + A_2(\ddot{\gamma}-\ddot{\phi}) + C_1\ddot{\phi}(\dot{h}_\gamma t_2+\dot{v}_\gamma t_1)$$
$$+C_1\dot{\phi}^2\ (-\dot{h}_\gamma t_1+\dot{v}_\gamma t_2) - C_2(-\ddot{h}t_4+\ddot{v}t_3)$$
$$+C_2(\ddot{\gamma}-\ddot{\phi})(-\dot{h}_\gamma t_4+\dot{v}_\gamma t_3) + C_2(\dot{\gamma}-\dot{\phi})^2\ (\dot{h}_\gamma t_3+\dot{v}_\gamma t_4)$$
$$-C_3t_6\ddot{\phi} + C_3t_5\dot{\phi}^2 + C_1\ddot{x}_1(\dot{h}_\gamma t_2'+\dot{v}_\gamma t_1') + C_1\dot{x}_1^2(-\dot{h}_\gamma t_1'+\dot{v}_\gamma t_2')$$
$$+C_2(\ddot{x}_2-\ddot{x}_1)(-\dot{h}_\gamma t_4'+\dot{v}_\gamma t_3') + C_2(\dot{x}_2-\dot{x}_1)^2(\dot{h}_\gamma t_3'+\dot{v}_\gamma t_4')$$
$$+\left[A_0\frac{\partial v}{\partial\gamma} + C_2t_3\right]g = M_\gamma,$$

where

$$\dot{h}=\frac{d}{dt}h;\qquad \ddot{h}=\frac{d^2}{dt^2}h;\qquad \dot{h}_\phi=\frac{\partial}{\partial\phi}h$$

and similar expressions hold for $\dot{v}_\phi, \dot{v}, \ddot{v}$, and so on. The effective moments M_ϕ and M_γ are given by

$$M_\phi = u_1 + M_\alpha + Y(L_1t_1-L_2t_3) + X(L_1t_2+L_2t_4),$$

$$M_\gamma = u_2 - M_\alpha + YL_2t_3 - XL_2t_4,$$

where u_1= moment generated by muscle action about the hip joint, u_2= moment generated by muscle action about the knee joint, M_α= ankle moment, Y = vertical component of the internal reaction force at the ankle joint, and X = horizontal internal reaction at the ankle.

Similar expressions can be obtained for the leg in deploy and swing portions.
It turns out that the equations in the stance and deploy portions are identical except for the interchange of variables: x_1 and x_2 for ϕ and γ.

To give further simplification, prescribed time functions are obtained for h and v the horizontal and vertical hip trajectories and all partial derivatives such as $h\dot{\phi}$ and $v\dot{\phi}$ are taken to be zero. Because of the similarity in the equations, a common notation is adopted. Let x_1 and x_2 stand for the thigh and shank angles respectively (i.e., for ϕ and γ in stance; x_1 and x_2 in deploy and swing). Define also $x_3=\dot{x}_1, x_4=\dot{x}_2$; then we obtain the equations in first-order canonical form

$$\dot{x}(t) = f(x,\tilde{u};t)$$

where

$$x \triangleq (x_1, x_2, x_3, x_4)^T;$$

$$\dot{x}_1 = x_3,$$

$$\dot{x}_2 = x_4,$$

$$\dot{x}_3 = \frac{1}{\delta}\{(\tilde{R}_3+\tilde{u}_1)A_2+t_7(\tilde{R}_4+\tilde{u}_2)\},$$

$$\dot{x}_4 = \frac{1}{\delta}\{t_7(\tilde{R}_3+\tilde{u}_1) + t_8(\tilde{R}_4+\tilde{u}_2)\};$$

$$\tilde{R}_3 = -C_3t_5x_4(x_4-2x_3) - (C_1t_1-C_2t_3)(\ddot{\tilde{g}}+g)$$

$$\tilde{R}_4 = -C_3t_5x_3^2-C_3t_6(\ddot{\tilde{g}}+g);$$

$\tilde{u}_1 = u_1 + M_\alpha + Y(L_1t_1-L_2t_3) + X(L_1t_2+L_2t_4)$ in stance,
$\tilde{u}_1 = u_1 + M'_\alpha + Y'(L_1t_1-L_2t_3) + X'(L_1t_2+L_2t_4)$ in deploy,
$\tilde{u}_1 = u_1$ in swing,
$\tilde{u}_2 = u_2 - M_\alpha + YL_2t_3 - XL_2t_4$ in stance,
$\tilde{u}_2 = u_2 - M'_\alpha + Y'L_2t_3 - X'L_2t_4$ in deploy,
$\tilde{u}_2 = u_2$ in swing,
$\delta = A_2t_8 - t_7^2 \triangleq$ determinant of the dynamic coupling matrix;
$t_8 = A_1 + 2C_3t_6,\quad t_7 = A_2 + C_3t_6,\quad t_9 = t_8 - t_7.$

Kinematics

In the deploy portion, the ankle of the foot describes a circular arc about the ball of the foot. With the hip motion prescribed, the angles x_1 and x_2 are no longer independent but subjected to an equality constraint relationship. Suppose that deploy starts at time $\tilde{t}_1$ and ends at $\tilde{t}_2$.

Vertical and horizontal displacements (Fig.A3.2) are summed and after some mathematical manipulation the desired kinematic constraint is obtained.

$$S^d(x;t) = L_1^2 + L_2^2 - d^2 + (e_1-\tilde{g})^2 + (e_2 - v_0 t)^2 + 2L_1L_2t_6$$
$$-2(e_1-\tilde{g})(L_1t_2 + L_2t_4) - 2(e_2-v_0t)(L_1t_1 - L_2t_3) = 0.$$

where

$$e_1 = \tilde{g}(\tilde{t}_1) + (L_1t_2 + L_2t_4)_{\tilde{t}_1} + d\sin\alpha(\tilde{t}_1).$$

and

$$e_2 = v_0\tilde{t}_1 + (L_1t_1 - L_2t_3)_{\tilde{t}_1} + d\cos\alpha(\tilde{t}_1).$$

At time $\tilde{t}_2$, the angle δ between the shank and foot approaches a limiting maximum value δ_{max}. During swing motion, $\delta = \delta_{max}$ (i.e. foot locking) while the toes of the foot are kept above ground. From Fig.A3.2 we thus have the inequality constraint

$$S^s(x;t) = \tilde{g}(t) + L_1t_2 + L_2t_4 + d\cos\beta(t) - e_0 \leq 0,$$
$$\beta = \pi - \delta_{max} - \phi_0 + x_1(t) - x_2(t)$$

for $\tilde{t}_2 \leqslant t \leqslant \tilde{t}_3$, $\tilde{t}_3$ being the end of swing.
In addition to state variable inequality constraint, the angle $x_2(t)$ has to be positive ($x_2>0$) during the swing. However, the multilinkage mechanical model does not take into account such a restriction. This aspect of model limitation does not appear to have been noticed in previous investigations. A useful device to alleviate the difficulty is to adjoin a "soft" penalty to the performance criterion J_0 under optimization (J_0 has yet to be defined); that is define

$$J_s \triangleq J_0 + \gamma_s \int_{t_2}^{t_3} \frac{dt}{x_2(t)}$$

for a sequence of $\gamma_s: \gamma_0 > \gamma_1 > \gamma_2 \cdots > \gamma \to 0_+$.

In the stance portion, the foot is more or less stationary except for the brief moment following heel strike. The dynamic behaviour is difficult to describe because a rigid link cannot describe the behaviour of a "round" foot. Fig.A3.3 illustrates the rollover situation where the weight-bearing leg rotates about its ankle (O), the primary rotation center.

The pressure center where the ground reaction forces are acting is gradually transferred from the heel toward the toe. The effective kinematic constraints in this period are that the ankle (x_A, y_A) coordinates follow a prescribed trajectory $(q(t), p(t))$. In a rough analysis, $q(t)$ and $p(t)$ can be taken as constant. Thus we have the kinematic constraints in stance

$$S_1^{rs}(x;t) = \tilde{g}(t) + L_1 t_2 + L_2 t_4 - e_0 + p(t) = 0,$$

$$S_2^{rs}(x;t) = v_0(t+t_0) + L_1 t_1 - L_2 t_3 - q(t) = 0.$$

References

Chow, C.K. and Jacobson, D.H. (1971): "Studies of Human Locomotion via Optimal Programming". Mathematical Biosciences 10: 239-306.

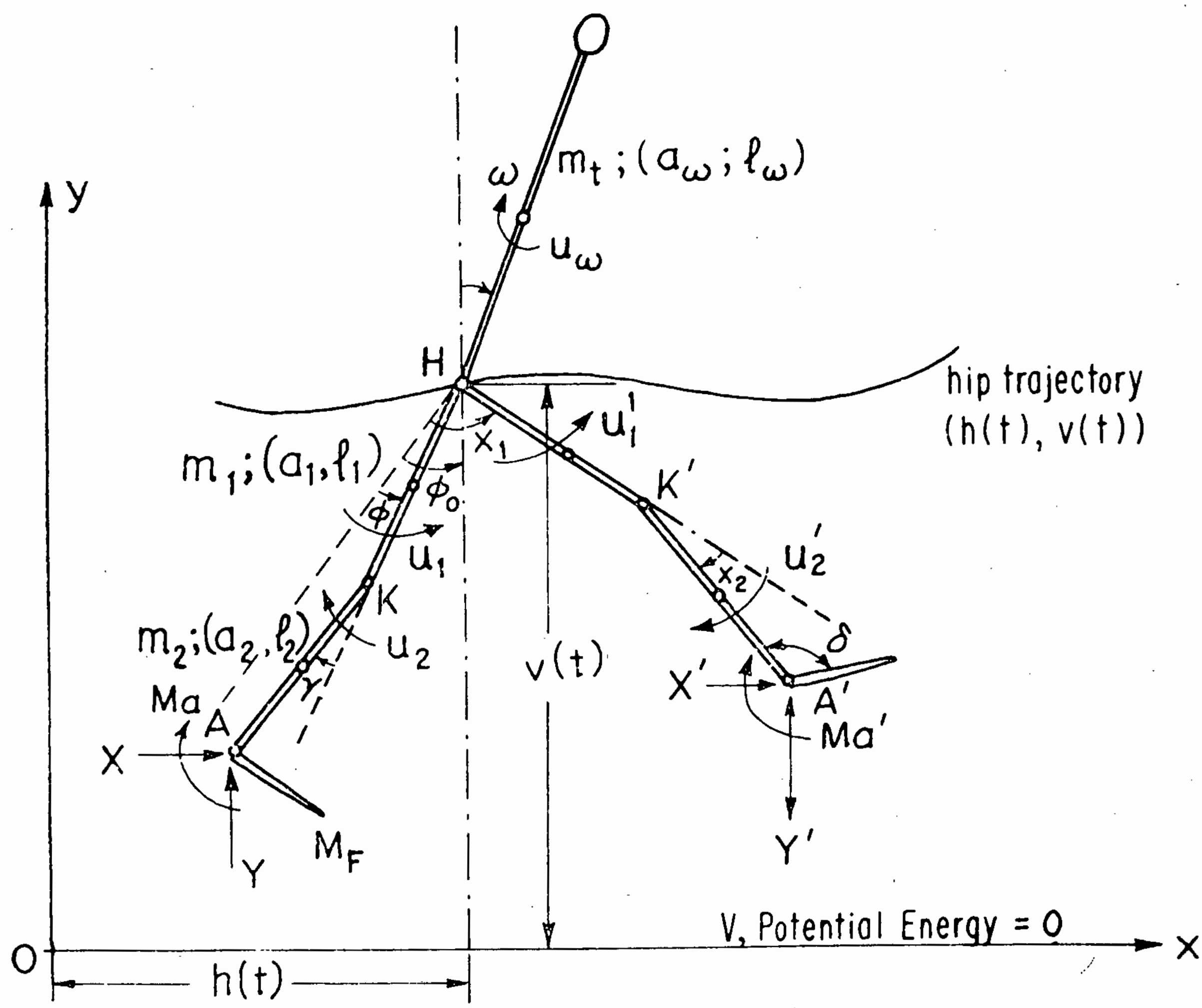


Fig.A3.1 Coordinate System for the Human Locomotor System Model.

α_1= distance of center of gravity of the thigh segment from hip joint (H)

α_2= distance of center of gravity of the shank segment from knee joint (K)

ℓ_1= length of the thigh segment

ℓ_2= length of the shank segment

$m_i \triangleq$ mass of the i-th segment (i=1 thigh; i=2 shank; i=ω HAT section)

Y,X= vertical and horizontal reactions at the ankle

Mα= ankle moment

u_i= effective moment for the i-th link.

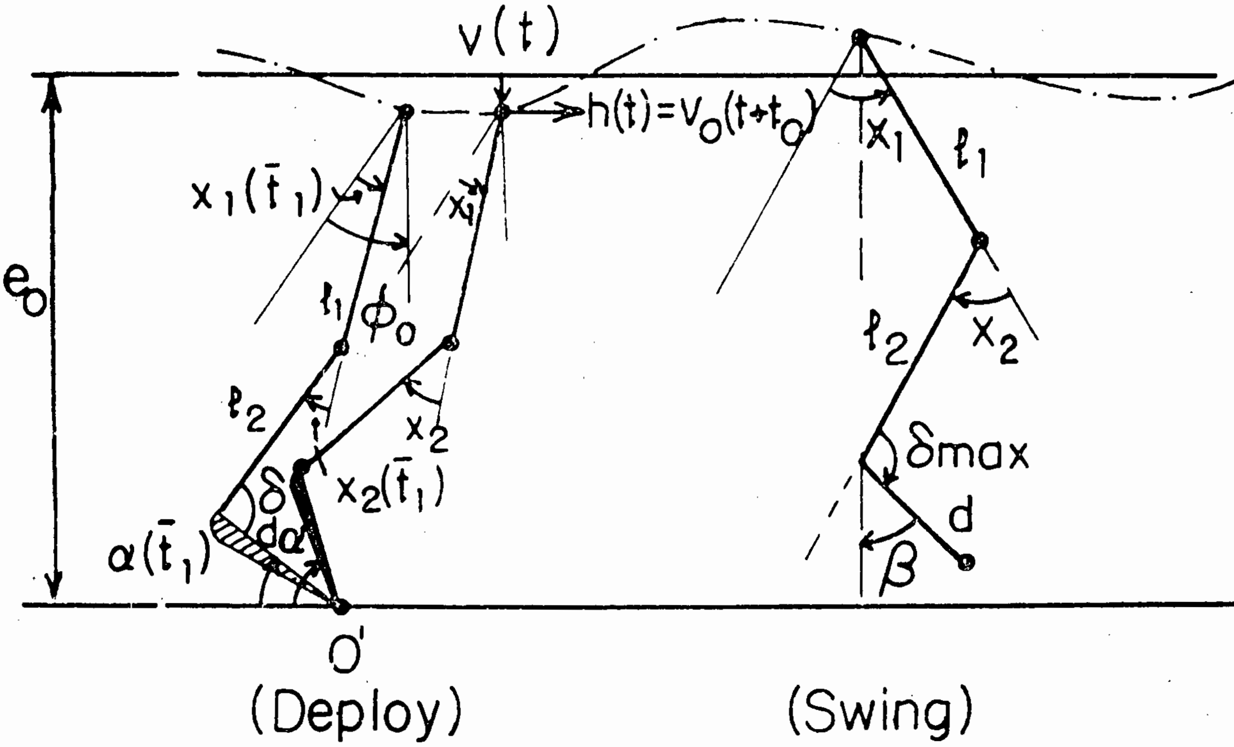


Fig. A3.2 Kinematic constraints in Deploy and Swing Portions.

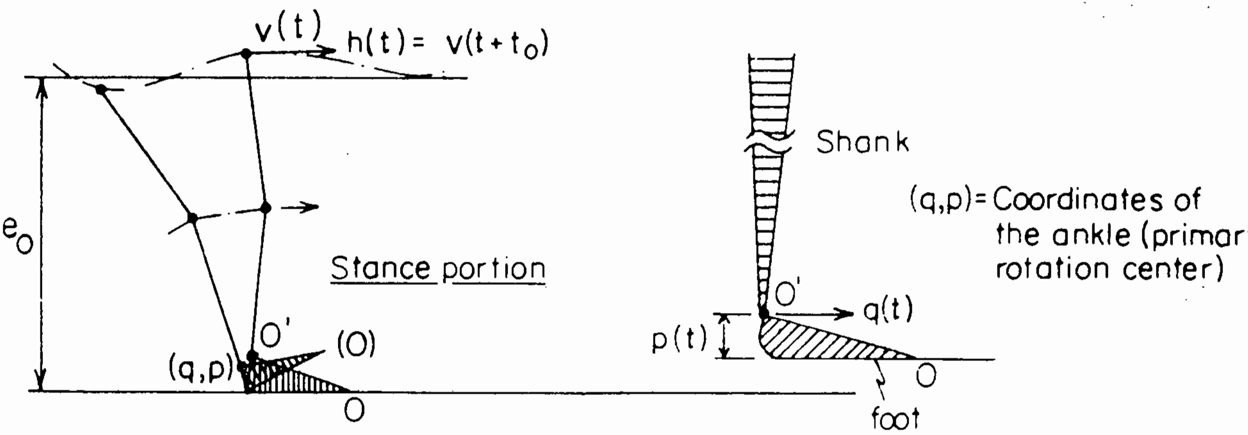


Fig.A3.3 Kinematic Constraints in Stance Portion.

APPENDIX 4

The Energy Performance Criterion.

Introduction

The following summary of the derivation of an energy performance criterion in human locomotion can be studied in more detail on pages 263-271 of the paper by Chow & Jacobson (1971).

Assumptions

The mechanical work done (by or on) a muscle, which is the quantity of interest for the derivation of an energy performance criterion, is evaluated by taking the time integral of the product of the force generated by the muscle in causing rotation and its velocity of shortening.

The neural-viscoelastic model for the muscle is based on the work of Bigland and Lippold (1954). This model has previously been discussed in Chapter 4, section 4.7. It rests essentially on the following functional form for the force generated by a muscle

$$F(L,v,Z) = ZF_0(L)\frac{1 - (v/v_m)}{1 + (v/bv_m)} \qquad \text{(A4.1)}$$

where

$F_0(L)$ = maximum isometric force at muscle length L

Z = neural stimulation level ($0 \leqslant Z \leqslant 1$)

v_m = maximum velocity for the isotonic case (F=0)

b = constant (typical values between 0.25 and 0.3)

The derivation is confined to a one-joint agonist/antagonist muscle system. Since the human locomotor system is equipped with an abundance of two-joint muscles, it can be shown (Steindler (1935), Eberhart (1968)) that two-joint muscles can be functionally decomposed into the one-joint variety.

Muscle in Shortening.

From A4.1 the "shortening" force is

$$F_s = ZF_0(L)\frac{1 - (v_s/v_m)}{1 + (v_s/bv_m)}$$

For normal locomotion activity, F_s is not significantly different from the static component $\hat{F}_0 = Z_0F_0(L_0)$, so that it can be satisfactorily represented as

$$\delta F_s = \hat{F}_s - \hat{F}_0$$

$$= \left(\frac{\partial F_s}{\partial Z}\right)_{Z_0} . (Z-Z_0) + \left(\frac{\partial F_s}{\partial L}\right)_{L_0} . (L-L_0) + \left(\frac{\partial F}{\partial v_s}\right)_{v_0} (v-\phi_0)$$

and

$$\left(\frac{\partial F_s}{\partial Z}\right)_{Z_0} = F_0(L_0), \quad \left(\frac{\partial F}{\partial v_s}\right)_{v_0} = -Z_0 F_0 (L_0) \frac{1 + (1/b)}{v_m}$$

$$\left(\frac{\partial F}{\partial L}\right)_{L_0} \simeq 0 \quad \text{(small)}$$

That the linear terms constitute the main contribution is supported by experimental results obtained in normal walking.

The next crucial step is to obtain a relationship between stimulation increment $\Delta Z = Z - Z_0$ and velocity v_s. Bigland and Lippold showed that for approximately constant F_s, v_s and ΔZ are linearly related for small velocities.

$$\Delta Z \simeq \frac{F_s}{F_0(L)} (1+b) \frac{v_s}{v_m} \propto v_s \quad \text{for} \quad \frac{v_s}{v_m} \ll 1.$$

<u>Normal Range of Locomotion Activity</u>.

From a study of the characteristic surface of a skeletal muscle it can be shown that the above linear relationship between ΔZ and v_s still holds but with a slightly larger slope dependent on the maximum value of F_s. Thus we have an operating relationship of the form

$$\Delta Z = k_s((\hat{F}_s)_{max}; \hat{F}_0) v_s$$

Hence

$$\delta F_s = (\hat{F}_0 k_s - B_{m_s}) v_s$$

Since $(F_s - F_0) > 0$ for a shortening muscle, then $(F_0 k_s - B_{m_s}) > 0$. The work done by a muscle in shortening is then

$$W_s = \int_{t_0}^{t_f} \frac{(F_s - F_0)^2}{(F_0 k_s - B_{m_s})} dt$$

$$= \int_{t_0}^{t_f} \frac{u_s^2(t)\,dt}{(\hat{F}_0 k_s - B_{m_s}) . d_s^2(t)} = \frac{1}{2} \int_{t_0}^{t_f} r_s(t) u_s^2(t)\,dt$$

where $u_s(t)$ is the moment generated by the shortening muscle and $d_s(t)$ the moment arm.

$$\tfrac{1}{2} r_s(t) \triangleq \frac{1}{(F_0 k_s - B_{m_s}) d_s^2}$$

Muscle in lengthening

In a similar manner an expression can be obtained for the work done on the muscle in lengthening viz.

$$W_e = -\int_{t_0}^{t_f} \frac{(F_e - \hat{F}_0)^2}{(B_{m_e} - \hat{F}_0 k_e} dt$$

$$= -\tfrac{1}{2} \int_{t_0}^{t_f} \frac{(2) \cdot u_e^2(t) dt}{(B_{m_e} - \hat{F}_0 k_e) \cdot d_e^2(t)}$$

$$= -\tfrac{1}{2} \int_{t_0}^{t_f} r_e (t) u_e^2(t) dt$$

where

$$r_e(t) = \frac{2}{(B_{m_e} - \hat{F}_0 k_e) d_e^2(t)}$$

and $d_e(t) \triangleq$ moment arm for the lengthening muscle ($\approx d_s(t)$).

Total Mechanical Work.

The total work done is the sum of lengthening and shortening contributions

$$W = W_e + W_s$$

$$= \tfrac{1}{2} \int_{t_0}^{t_f} (r_s u_s^2 - r_e u_e^2) dt.$$

The net moment acting on the limbs is given by

$$u = \frac{F_s - F_e}{d_s}$$

$$= \frac{(F_s - \hat{F}_0) - (F_e - \hat{F}_0)}{d_s}$$

$$= u_s - \left(\frac{d_e}{d_s}\right) u_e$$

$$\approx u_s - u_e \text{ for } \frac{d_e}{d_s} \approx 1.$$

Now,

$$u^2 = (u_s - u_e)^2 = u_s^2 + u_e^2 - 2u_s u_e < 2(u_s^2 + u_e^2).$$

But,

$$u_s^2 - u_e^2 < u_s^2 + u_e^2;$$

therefore,

$$u^2 = \frac{k}{2}(u_s^2 - u_e^2), \quad k = \text{constant factor},$$

$$W = \frac{1}{2}\int_{t_0}^{t_f} (r_s u_s^2 - r_e u_e^2)\,dt$$

$$\cong \text{constant} \cdot \int_{t_0}^{t_f} u^2(t)\,dt.$$

Hence, in the normal range of activity, the sum total of mechanical energy expenditure by the muscle-activating system is proportional to the integral of the square of the net moment.

References.

Bigland,B.and Lippold,O.C.J.(1954): "The relation between force, velocity and integrated electrical activity in human muscle." J.Physiol. 123: 214.

Chow,C.K.and Jacobson,D.H.(1971):"Studies of Human Locomotion via Optimal Programming". Mathematical Biosciences 10:239-306.

Steindler,A.(1935): Mechanics of Normal and Pathological Locomotion,Charles C.Thomas, Publisher,Springfield,Ill.,

Wilson,P.D. and Klopsteg,P.E.(1968): Human Limbs and Their Substitutes,McGraw-Hill, New York.

Univac 1106 Subroutine to find the Coefficients of the Fourier Series on a Discrete Range.

The large scale systems Math-Pack UP - 7542 Rev.1. Programmer's Reference Manual contains a subroutine

DFSRIE (NP,NH,Y,A,B)

This subroutine determines the coefficients of a Fourier series given discrete data points where

NP is the number of data points in the Y array. NP must be greater than or equal to 2NH+1.

NH is the number of harmonics to be used in the Fourier series.

Y is a one-dimensional array of NP locations containing the value of the function $F(t)$ at NP points.

A is a one-dimensional array of (NH+1) locations containing the computed coefficients A_n of the Fourier series.

B is a one-dimensional array of (NH+1) locations containing the computed coefficients Bn of the Fourier series.

Mathematical Method

A Fourier series, or harmonic analysis, is represented by the equation

$$F(t) = \frac{Ao}{2} + \sum_{p=1}^{NH} \left(A_p \cos \left(\frac{2\pi t}{T}\right) p + B_p \sin \left(\frac{2\pi t}{T}\right) p \right)$$

Where T = period

Hence once the A_o, A_p and B_p are obtained as output from the subroutine one is in a position to see which harmonics dominate.

The Necessary Conditions of Optimality.

Introduction.

The following brief summary of the necessary conditions of optimality has been taken from the paper of Chow & Jacobson (1971). A more detailed description can be found in the book of Bryson and Ho (1968).

Conditions (necessary) of Optimality.

Solution for the deploy and swing portions are shown. Solution for stance is not shown as it was considered to be similar in form to that of deploy.

The technique is to convert the original constrained optimization problem into a sequential unconstrained problem. Let $S(x;t)$ denote the state constraint (equality or inequality) in either portion. Define an extra state variable $x_5(t)$ such that it satisfies the differential equation

$$\dot{x}_5(t) = \mathrm{sgn}(S)\cdot S^2(x;t),$$

$$\mathrm{sgn}(S) \triangleq \begin{cases} 1 & \text{if } S=0, \\ 0 & \text{if } S<0, \end{cases}$$

for $0\leqslant t\leqslant t_1 ; t_1\leqslant t\leqslant t_2$. The original state vector satisfies, of course, the dynamic equation

$$\dot{x}(t)=f(x,u;t); \qquad x=(x_1,x_2,x_3,x_4)^T.$$

With the initial condition set to zero, that is, $x_5(t_1)= 0$ for swing, $x_5(0)=0$ for deploy, respectively, then

$$\min x_5(t\ ) = \int_{(0);(t_1)}^{(t_1);(t_2)} \mathrm{sgn}(S)S^2(x;\eta)d\eta = 0$$

$$\Rightarrow \mathrm{sgn}(S)\cdot S^2(x;\eta) = 0 \qquad \eta\varepsilon\ 0,t_1 \quad \text{or } \eta\varepsilon\ t_1,t_2\ .$$

The state constraint is thus satisfied, A mathematical proof of the fore-going penalty function technique is given by Lele and Jacobson (1969). As far as the initial and terminal conditions are concerned, a quadratic penalty function is used. With these, the necessary conditions of optimality can be separately derived for the deploy and swing portions.

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In deploy, we have the modified performance criterion for the sequential unconstrained problem, that is,

$$\min_{u} \hat{J}=\sigma_K\left\{x_5(t_1) +\psi_1^2(x(t_1);t_1)+\psi_2^2(x(t_1);t_1)+(x_3(t_1)-d_3)^2 + (x_4(t_1)-d_4)^2\right\}+\frac{1}{2}\int_0^{t_1}(r_1u_1^2+r_2u_2^2)dt$$

Specifically, we solve the above for a monotonically increasing sequence $0<\sigma_1<\sigma_2<\cdots<\sigma_K$ and subject to the equations

$$\dot{x}(t)=f(x,u;t), \qquad x(0)=x_0,$$
$$\dot{x}_5(t)=(S^d(x;t))^2, \quad x_5(0)=0.$$

The signum function $sgn(S^d)=1$ because the state equality constraint is effective throughout the interval. Define a variational Hamiltonian $H^d(x,u,\lambda,\sigma_k;t)$ as

$$H^d \triangleq \tfrac{1}{2}(r_1u_1^2+r_2u_2^2) +\sum_{j=1}^{4}\lambda_j f_j(x,u;t) +\lambda_5(t)(S^d(x;t))^2$$

The λ's are the influence functions or the adjoint variables of the optimization problem. The extremal condition is

$$\frac{\partial H^d}{\partial u_i} = 0 \Rightarrow r_iu_i+\sum_{j=1}^{4}\lambda_j\frac{\partial f_j}{\partial u_i}= 0 \quad \text{for } i=1,2.$$

Therefore,

$$r_1u_1+\frac{1}{\delta}(A_2\lambda_3+t_7\lambda_4)=0, \qquad r_2u_2+\frac{1}{\delta}(t_7\lambda_3+t_8\lambda_4)=0 \qquad \text{(A)}$$

Note. $\delta^2H^d/\delta u_i^2=r_i>0$ for $r_i>0;i=1,2$. The influence equations and their terminal conditions at t_1 are

$$-\dot{\lambda}_i= \frac{\delta H^d}{\delta x_i}$$

$$=\sum_{j=1}^{4}\lambda_j\frac{\delta f_j}{\delta x_i} + 2\lambda_5\cdot S^d(x;t)\frac{\delta S^d}{\delta x_i}, \qquad i=1,\ldots,4,$$

$$-\dot{\lambda}_5= 0,$$

$$\lambda_1(t_1)= 2\sigma_k\left(\psi_1\frac{\delta\psi_1}{\delta x_1}+\psi_2\frac{\delta\psi_2}{\delta x_1}\right),$$

$$\lambda_2(t_1)= 2\sigma_k\left(\psi_1\frac{\delta\psi_1}{\delta x_2}+\psi_2\frac{\delta\psi_2}{\delta x_2}\right),$$

$\lambda_3(t_1) = 2\sigma_k(x_3(t_1)-d_3)$,

$\lambda_4(t_1) = 2\sigma_k(x_4(t_1)-d_4)$,

$\lambda_5(t_1) = \sigma_k$

Explicitly, expressions for the λ equations and $\delta S^d/\delta x_i$ are given as follows.

$$-\dot{\lambda}_1 = \frac{1}{\delta}\left(A_2 R_{31} + t_7 R_{41}\right)\lambda_3 + \left(t_7 R_{31} + t_8 R_{41}\right)\lambda_4 + 2\lambda_5 S^d \frac{\delta S^d}{\delta x_1}$$

$$-\dot{\lambda}_2 = \frac{\lambda_3}{\delta}\left\{A_2 R_{32} - C_3 t_5 (R_4+u_2) + t_7 \cdot R_{42} - \frac{\delta_2}{\delta}\left[A_2(R_3+u_1) + t_7(R_4+u_2)\right]\right\}$$
$$+\frac{\lambda_4}{\delta}\cdot\left\{t_7 R_{32} - C_3 t_5 (R_3+u_1) + t_8 R_{42} - 2C_3 t_5 (R_4+u_2)\right.$$
$$\left.+\frac{\delta_2}{\delta}\cdot\left[t_7(R_3 \pm u_1) + t_8 \cdot (R_4+u_2)\right]\right\} + 2\lambda_5 S^d \frac{\delta S^d}{\delta x_2},$$

$$-\dot{\lambda}_3 = \lambda_1 + \frac{1}{\delta}\left[(A_2\lambda_3 + t_7\lambda_4)\cdot R_{33} + (t_7\lambda_3 + t_8\lambda_4)\cdot R_{43}\right],$$

$$-\dot{\lambda}_4 = \lambda_2 + \frac{R_{34}}{\delta}(A_2\lambda_3 + t_7\lambda_4),$$

$$-\dot{\lambda}_5 = 0,$$

$$R_3 = -C_3 t_5 x_4(x_4-2x_3) - (C_1 t_1 - C_2 t_3)(g+\ddot{\tilde{g}}),$$

$$R_4 = -C_3 t_5 x_3^2 - C_3 t_6 (g+\ddot{\tilde{g}})$$

$$R_{31} = -(C_1 t_2 + C_2 t_4) t_{10}, \qquad t_{10} \triangleq g+\ddot{\tilde{g}},$$

$$R_{41} = C_2 t_4 t_{10} + l_1 t_2 Y - l_1 t_1 X,$$

$$R_{32} = -C_3 t_6 x_4 (x_4-2x_3) + C_2 t_4 t_{10},$$

$$R_{42} = -C_3 t_6 x_3^2 - C_2 t_4 t_{10},$$

$$R_{33} = 2C_3 t_5 x_4, \qquad R_{43} = -2C_3 t_5 x_3, \qquad R_{34} = -2C_3 t_5 (x_4-x_3),$$

$$S^d = S_1^2 + S_2^2 - d^2,$$

$$S_1 = e_1 - l_1 t_2 - l_2 t_4 - \tilde{g}(t), \quad S_2 = e_2 - l_1 t_1 + l_2 t_3 - v_0 t,$$

$$\frac{\delta S^d}{\delta x_1} = 2S_1(l_1 t_1 - l_2 t_3) - 2S_2(l_1 t_2 + l_2 t_4),$$

$$\frac{\delta S^d}{\delta x_2} = 2l_2(S_1 t_3 + S_2 t_4),$$

$$\delta = A_2 t_8 - t_7^2, \qquad \delta_2 \triangleq \frac{\delta S}{\delta x_2} = 2C_3^2 t_5 t_6 .$$

Similarly, in the swing portion, we have the augmented performance criterion

$$\min_u \hat{J}_s = \sigma_k \left\{ x_5(t_2) + \sum_{i=1}^{4} (x_i(t_2) - \tilde{d}_i)^2 \right\} + \gamma_k \int_{t_1}^{t_2} \frac{dt}{x_2(t)} + \frac{1}{2}\int_{t_1}^{t_2} (r_1 u_1^2 + r_2 u_2^2)\,dt$$

for $0<\sigma_1< \ldots \sigma_k \to \infty$ and $\gamma_1>\gamma_2> \ldots >\gamma_k \to 0_+$. The dynamic equations are now

$\dot{x}(t)=f(x,u;t)$, with $x(t_1)$ determined from $\psi_1=0, \psi_2=0$ at terminal deploy, $\dot{x}_5(t)=\mathrm{sgn}(S^S)(S^S(x;t))^2$, $x_5(t_1)=0$.

The presence of the "soft" penalty is to ensure that $x_2(t)>0$ during the entire swing portion. This requires that the initial and all iterative trajectories be feasible. Similar to the deploy portion, define a variational Hamiltonian as

$$H^S(x,u,\lambda;\sigma_k,\gamma_k;t) \triangleq \tfrac{1}{2}(r_1 u_1^2 + r_2 u_2^2) + \frac{\gamma}{x_2(t)} + \sum_{j=1}^{4} \lambda_j f_j(x,u;t) + \lambda_5(t)\,\mathrm{sgn}(S^S)(S^S(x;t))^2 .$$

Again, the extremal condition implies

$$(H^S)_u = 0 \Rightarrow r_i u_i + \sum_{j=1}^{4} \lambda_j \frac{\partial f_j}{\partial u_i} = 0, \quad i=1,2.$$

The equation set (A) is thus obtained. The influence equations and their appropriate boundary conditions are

$$-\dot{\lambda}_i = \sum_{j=1}^{4} \lambda_j \frac{\partial f_j}{\partial x_i} + 2\lambda_5\,\mathrm{sgn}(S^S).\,S^S \frac{\partial S^S}{\partial x_i} - \frac{\gamma_k \cdot \delta(2,i)}{x_2^2(t)},$$

$$-\dot{\lambda}_5 = 0,$$

$$\lambda_i(t_2) = 2\sigma_k (x_i(t_2) - \tilde{d}_i),$$

$$\lambda_5(t_2) = \sigma_k,$$

for $i=1,\ldots,4$ and $\delta(2,i)=1$ if $i=2$ and 0 otherwise.

The expressions for the λ equations are similar to those for the deploy portion except for the substitution of inequality for equality constraint expressions.

Expressions related to the constraint are

$$\frac{\partial S^s}{\partial x_1} = - l_1 t_1 + l_2 t_3 - d \sin\beta(t),$$

$$\frac{\partial S^s}{\partial x_2} = - l_2 t_3 + d \sin\beta(t),$$

$$\frac{\partial S^s}{\partial x_3} = 0, \qquad \frac{\partial S^s}{\partial x_4} = 0.$$

For both the deploy and swing portions, we thus have a two-point boundary value problem (TPBVP)-for the respective unconstrained optimization problems.

References.

Bryson,A.E. and Ho.Y.C.(1968): Applied Optional Control: Optimization, Estimation and Control,Blaisdell Publishing Company,Waltham,Mass.,

Chow,C.K. and Jacobson,D.H.(1971): "Studies of Human Locomotion via Optimal Programming". Mathematical Biosciences 10: 239-306.

Lele,M.M and Jacobson,D.H,(1969):"A proof of the convergence of the Kelley-Bryson penalty function technique for state-constrained control problems". J.Math.Anal.Appl.26: 163-170.

```
C-----MAIN PROGRAM
   55 REAL L1,L2,MFOOT
C-----BLOCK 0 FOR TRANSFER OF PARAMETERS
C-----BLOCK 1 FOR TRANSFER OF TRAJECTORIES DATA
      COMMON/BLK1/UIPL1(2,101),NCONTR,GAMMA
      COMMON/BLK2/X(5,101),XO(5),ALAMB(5,101),ALAMBF(5)
      COMMON/BLK3/NSTATE,NSTEP,DELTA
      COMMON/BLK4/GIPL1(2,101)
      COMMON/BLK5/UI(2,101),SI(2,101)
      COMMON/BLK6/XNOM(5),XFINAL(5),NOPT,ICOUNT,SIGMA
      COMMON/DATA1/L1,L2,AO,A1,A2,C1,C2,C3,G
      COMMON/DATA2/E1,E2,XZ,ALPHAO
      COMMON/DATA3/R1,R2
      COMMON/DATA4/PI,PPI,VO,TF,TC
      COMMON/DATA5/OMEGA,EPSI
      COMMON/DATA6/DD,MFOOT,BETA
      COMMON/DATA7/E3,E4
      COMMON/REACT/FVP1(101),FHP1(101),DYN1(2,101)
C-----VALUES OF COMMON PARAMETERS
      IR = 8
      IW = 5
      INTEGER TOLER3
      G = 32.17
      TF = 1.40
      PI = 3.14159265
      PPI = 4.0 * PI/TF
      L1 = 1.3940
      L2 = 1.1316
      DFOOT = 0.82
      DD = DFOOT + DFOOT
      XZ = 1.7*PI/18.0
      GG = (10.185)/(2.54*12.0)
      VO = -4.346667
      TC = (-6.345583)/(-4.346667)
      AAD = 16.759
      E1 = (AAD /2.0)*GG
      E2 = -VO*TC
      E3 = 0.0
      E4 = 0.0
      ALPHAO = 0.51959188
      R1 = 1.
      R2 = 1.
      OMEGA = 2.
      AO = 160.0/G
      A1 = 0.8757
      A2 = 0.1528
      C1 = 0.7320
      C2 = 0.1559
      C3 = 0.2173
      MFOOT = AO*0.015
C-----READ EXTERNAL DATA AND PARAMETERS------
      READ(13,10) NSTATE,NCONTR,NSTEP,DELTA,NOPT
      READ(13,12) (XO(I),I=1,NSTATE)
      DO 86 K = 1,101
      READ(16,89)FVP1(K)
   89 FORMAT (E15.6)
   86 CONTINUE
      DO 96 J = 1,101
      READ(16,89)FHP1(J)
   96 CONTINUE
      REWIND 16
      DO 9007 J = 1,101
      FVP1(J) = FVP1(J)*160.0/130.0
      FHP1(J) = FHP1(J)*160.0/130.0
 9007 CONTINUE
      DO 4 I= 1,2
      DO 40 M = 1,101
```

```
      DO 48 M = 1,101
      READ (12,30)  UIPL1(1,M)
   30 FORMAT (E15.6)
   48 CONTINUE
    4 CONTINUE
      REWIND 12
   99 CONTINUE
      READ(13,18) TOLER1,TOLER2,TOLER3
      READ(13,20) IPRINT
      READ(13,22) SIGMA,EPS1
      READ(13,77)BETA
      READ(13,8053) GAMMA
      REWIND 13
   20 FORMAT(I1)
 8053 FORMAT(E12.5)
   18 FORMAT(E12.5,F10.7,I4)
   22 FORMAT(2E12.5)
   12 FORMAT(E12.5,5X,E12.5,5X,E12.5,5X,E12.5,5X,E12.5)
   77 FORMAT(F5.2)
   10 FORMAT(I2,3X,I2,3X,I4,1X,F10.7,I1)
      IF (SIGMA.GT. 0.005 ) GO TO 58
      CALL GRAD(TOLER1,TOLER2,IPRINT,TOLER3)
      IF (SIGMA.GT. 0.005 ) GO TO 58
      GO TO 55
   58 END
 BFOR,IS HER,GRAD,HER,GRAD
      SUBROUTINE GRAD(TOLER1,TOLER2,IPRINT,TOLER3)
      DIMENSION SIPL1(2,101),GI(2,101)
      IW = 5
      REAL L1,L2
      INTEGER TOLER3
C----- BLOCK 0
      COMMON/DATA1/L1,L2,AU,A1,A2,C1,C2,C3,G
      COMMON/DATA2/E1,E2,XZ,ALPHA0
      COMMON/DATA3/R1,R2
      COMMON/DATA4/PI,PPI,V0,TF,TC
      COMMON/DATA5/OMEGA,EPS1
      COMMON/DATA6/DD, FOOT,BETA
      COMMON/REACT/FVP1(101),FHP1(101),DYN1(2,101)
C-----BLOCK 1
      COMMON/BLK1/UIPL1(2,101),NCONTR,GAMMA
      COMMON/BLK2/X(5,101),X0(5),ALAMB(5,101),ALAMBF(5)
      COMMON/BLK3/NSTATE,NSTEP,DELTA
      COMMON/BLK4/GIPL1(2,101)
      COMMON/BLK5/UI(2,101),SI(2,101)
      COMMON/BLK6/XNOM(5),XFINAL(5),NOPT,ICOUNT,SIGMA
C-----FUNCTION SUBPROGRAMS FOR CALCULATION
      EXTERNAL AHSQ
      EXTERNAL COST
      INTEGER FILE1
      FILE1 = 12
C *** PRINTOUT OF THE TOLERANCES FOR THE PARTICULAR RUN
      WRITE(IW,201)
      WRITE(IW,202) TOLER1
      WRITE(IW,204) TOLER3
  201 FORMAT(/////,10X,24H TOLERANCES FOR THIS RUN)
  202 FORMAT(//,28H TOLERANCE ON HSUBUSQ WAS =,E12.5)
  204 FORMAT(//,29H TOLERANCE ON ITERATIONS WAS,I4)
C  FORMATION OF NOMINAL TRAJECTORY
C  INTEGRATION OF THE STATE EQUATIONS
   90 ICOUNT = 0
   91 CALL DIFFEQ(X,X0,NSTATE,NSTEP,DELTA,0)
C  INTEGRATION OF THE ADJOINT EQUATIONS
      CALL DIFFEQ(ALAMB,ALAMBF,NSTATE,NSTEP,DELTA,1)
C EVALUATION OF THE GRADIENT
      CALL AHSUBU(X,ALAMB,UIPL1,GIPL1,NSTATE,NCONTR,NSTEP,DELTA)
```

```
C  CHECKING THE VALUE OF HSUBU SQUARED OVER THE INTERVAL TO TO TF
      CALL SIMPIN(AHSQ,DELTA,NSTEP,GIPLIS)
C  PRINTOUT OF THE NOMINAL TRAJECTORY
      IF (IPRINT - 1) 101,200,101
  200 WRITE(IW,30) NSTEP
      DO 210 K = 1,NSTATE
      WRITE(IW,32) K
      WRITE(IW,34) (X(K,L),L=1,NSTEP)
  210 CONTINUE
      WRITE (IW,42)
      DO 215 K = 1,NSTATE
      WRITE(IW,44)K
  215 WRITE(IW,34) (ALAMB(K,L),L = 1,NSTEP)
      WRITE(IW, 36)
      DO 220 K = 1,NCONTR
      WRITE(IW,38) K
      WRITE(IW,34) (UIPL1(K,L),L = 1,NSTEP)
  220 CONTINUE
  101 RATIO1 = GIPLIS
      WRITE(IW,16) EPS1
   16 FORMAT('0','SEARCH STEP SIZE FOR SIMPLE GRADIENT=',1PE15.8)
      WRITE(IW,40) GIPLIS
      DO 228 K = 1,NCONTR
      DO 226 L = 1,NSTEP
  226 UI(K,L) = UIPL1(K,L)
  228 CONTINUE
      F = 0.
      CALL COSTF(0.,F)
      WRITE(IW,46) F
      DO 225 K = 1,NCONTR
      DO 224 L = 1,NSTEP
  224 SIPL1(K,L) = -GIPL1(K,L)
  225 CONTINUE
      IF (IPRINT - 1)111,112,111
  112 WRITE(IW,229)
  229 FORMAT(///,6H GIPL1,///)
      DO 232 K = 1,NCONTR
      WRITE(IW,230) (GIPL1(K,L),L=1,NSTEP)
  230 FORMAT(5(E16.6,4X)/)
  232 CONTINUE
  111 WRITE(IW,238) SIGMA
  238 FORMAT(//,8H SIGMA =,E10.1,///)
C  SETTING THE ITERATION COUNTER
C----- BLOCK III
C  SETS I = I + 1
  300 DO 310 K = 1,NCONTR
      DO 305 L = 1,NSTEP
      UI(K,L) = UIPL1(K,L)
      GI(K,L) = GIPL1(K,L)
  305 SI(K,L) = SIPL1(K,L)
  310 CONTINUE
      GISQ = GIPLIS
      FI = F
C-----BLOCK IV GRADIENT ALGOTHIRM ITERATION WITH STEP SIZE EPSI
      ICOUNT = ICOUNT + 1
      RATIO2 = RATIO1
      F = 0.
      CALL COSTF(EPS1,F)
C  SWEEPING THE ADJOINT AND THE STATE EQUATIONS
  610 CALL DIFFEQ(ALAMB,ALAMBF,NSTATE,NSTEP,DELTA,1)
      CALL AHSUBU(X,ALAMB,UIPL1,GIPL1,NSTATE,NCONTR,NSTEP,DELTA)
      CALL SIMPIN(AHSQ,DELTA,NSTEP,GIPLIS)
      DO 430 K = 1,NCONTR
      DO 425 L = 1,NSTEP
      BETAI = 0.0
  425 SIPL1(K,L) = -GIPL1(K,L) + BETAI*SI(K,L)
```

```
  430 CONTINUE
C   PRINTOUT OF THE ITERATION
      IF (IPRINT - 1) 402,401,402
  401 WRITE(IW,48) NSTEP,ICOUNT
      DO 440 K = 1,NSTATE
      WRITE (IW,32) K
      WRITE(IW,34) (X(K,L) , L = 1,NSTEP)
  440 CONTINUE
      DO 445 K = 1,NCONTR
      WRITE(IW,38) K
      WRITE (IW,34) (UIPLI(K,L) ,L=1,NSTEP)
  445 CONTINUE
      DO 450 K = 1,NSTATE
      WRITE(IW,44) K
  450 WRITE(IW,34) (ALAMB(K,L),L=1,NSTEP)
      WRITE(IW,452)
  452 FORMAT(///,6H GIPLI,///)
      DO 456 K = 1,NCONTR
      WRITE(IW,454) (GIPLI(K,L),L=1,NSTEP)
  454 FORMAT(5(E16.6,4X)/)
  456 CONTINUE
      WRITE(IW,40) GIPLIS
      WRITE(IW,46) F
      WRITE(IW,238) SIGMA
C*----BLOCK V CHECKING FOR CONVERGENCE
  402 RATIO1 = GIPLIS
      RR = RATIO1/RATIO2
      WRITE(IW,40) GIPLIS
      IF (RR.GT.10.0) WRITE(IW,17)
   17 FORMAT('0','GRADIENT BLOWS UP'//)
      DO 1001 I = 1,2
      DO 1010 M= 1,101
      WRITE (FILE1,144) UIPLI(I,M)
  144 FORMAT (E15.6)
 1010 CONTINUE
 1001 CONTINUE
      ENDFILE FILE1
      REWIND FILE1
      WRITE(13,10)     NSTATE,NCONTR,NSTEP,DELTA,NOPT
   10 FORMAT(I2,3X,I2,3X,I4,1X,F10.7,I1)
      WRITE(13,82)     (XO(I),I=1,NSTATE)
   82 FORMAT(E12.5,5X,E12.5,5X,E12.5,5X,E12.5,5X,E12.5)
      WRITE(13,18)     TOLER1,TOLER2,TOLER3
   18 FORMAT(E12.5,F10.7,I4)
      WRITE(13,20)     IPRINT
   20 FORMAT(I1)
      WRITE(13,22)     SIGMA,EPS1
      WRITE(13,77)BETA
      WRITE(13,8053) GAMMA
 8053 FORMAT(E12.5)
      ENDFILE13
      REWIND 13
      IF (IPRINT - 1) 640,403,640
  403 WRITE(IW,16) EPS1
  640 IF (ICOUNT-TOLER3) 490,980,980
  490 IF (GIPLIS-TOLER1) 495,495,300
  495 SIGMA = OMEGA*SIGMA
      EPS1= OMEGA*EPS1
      WRITE(13,10)     NSTATE,NCONTR,NSTEP,DELTA,NOPT
      XO(1) = 2.9288
      XO(2) = 9.5936
      XO(3) = 119.46
      XO(4) = 268.76
      XO(5) = 0.0000000
      WRITE(13,82)     (XO(I),I=1,NSTATE)
      WRITE(13,18)     TOLER1,TOLER2,TOLER3
```

```
      WRITE(13,20)    IPRINT
      WRITE(13,22)    SIGMA,EPSI
   22 FORMAT(2E12.5)
      WRITE(13,77)BETA
      WRITE(13,8053) GAMMA
      ENDFILE13
      REWIND13
      DO 8500 I = 1,2
      DO 8400 M = 1,101
      WRITE (14,8300) DYN1(I,M)
 8300 FORMAT(E15.6)
 8400 CONTINUE
 8500 CONTINUE
      END FILE 14
      REWIND 14
      WRITE(IW,12) SIGMA
C-----BLOCKVI PRINT OUT THE LAST ITERATION AND PUNCH CONTROL TRAJECTORY
C-----FOR LATER CALCULATIONS
      WRITE(IW,52)
      DO 2001 I = 1,2
      DO 2010 M= 1,101
      WRITE (FILE1,144) UIPL1(I,M)
 2010 CONTINUE
 2001 CONTINUE
      ENDFILE FILE1
      REWIND FILE1
      WRITE(IW,46) F
      WRITE(IW,40) GIPLIS
      WRITE(IW,76)BETA
   76 FORMAT(2X, 8H BETA = ,F5.2)
      WRITE(IW,72) GAMMA
   72 FORMAT(2X,8H GAMMA =,F10.5)
      DO 6032 I = 1,4
      DO 6053 M= 1,101
      WRITE(15,6083) X(I,M)
 6083 FORMAT(E15.6)
 6053 CONTINUE
 6032 CONTINUE
      ENDFILE15
      REWIND 15
      WRITE(IW,7030) (X(I,101),I=1,5)
 7030 FORMAT(1X,18H X(I,101),I=1,5 = ,5(E10.5,2X))
      GO TO 566
  980 WRITE(IW,50) ICOUNT
      WRITE(IW,46) F
      WRITE(IW,40) GIPLIS
      DO 3001 I = 1,2
      DO 3010 M= 1,101
      WRITE (FILE1,144) UIPL1(I,M)
 3010 CONTINUE
 3001 CONTINUE
      DO 982 K = 1,NCONTR
          WRITE(IW,14) (UIPL1(K,L),L=1,NSTEP)
  982 CONTINUE
      ENDFILE FILE1
      REWIND FILE1
  566 GO TO 999
C   FORMAT STATEMENTS
   77 FORMAT(F5.2)
   12 FORMAT(2X,14H NEW SIGMA = ,E10.3)
   14 FORMAT(4(E15.6,5X))
   30 FORMAT(    19HINOMINAL TRAJECTORY,5X,15H NO. OF STEPS=,I4///)
   32 FORMAT(10X,2H X,I2//)
   34 FORMAT(5(E16.6,4X)//)
   36 FORMAT(/////,16H NOMINAL CONTROL ///)
   38 FORMAT(10X,2H U,I2//)
```

```
   40 FORMAT (/////,34H VALUE OF HSUBUSQ OVER TO TO TF =,E16.6//)
   42 FORMAT (///,9H LAMBDA'S,///)
   44 FORMAT(10X,7H LAMBDA,I2//)
   46 FORMAT (///,16H COST FUNCTION =,E16.6///)
   48 FORMAT(16H1 NO. OF STEPS =,I4,5X,12H ITERATION =,I2//)
   50 FORMAT (31H1 METHOD FAILED TO CONVERGE IN ,I4,12H ITERATIONS,//)
   52 FORMAT(45H1 CONVERGENCE ATTAINED. PRINTOUT OF OPTIMAL,/////)
   54 FORMAT(19H OPTIMAL TRAJECTORY,10X,15H NO. OF STEPS =,I4////)
  999 RETURN
      END
@FOR,IS HER.DIFFEQ,HER.DIFFEQ
      SUBROUTINE DIFFEQ(Y,YO,NSTATE,NSTEP,DELTA,ICHK)
C     RUNGE-KUTTA SCHEME OF THE FOURTH ORDER EMPLOYED FOR THE FORWARD SWEEP
C     AND ALSO FO R THE BACKWARD SWEEP
      REAL L1,L2,MFOOT
      DIMENSION Y(5,101),YO(5),YDOT(5),FK1(5),Y1(5),FK2(5),FK4(5),
     1FK3(5)
      COMMON/BLK1/UIPL1(2,101),NCONTR,GAMMA
      COMMON/BLK5/UI(2,101),SI(2,101)
      COMMON/BLK6/XNOM(5),XFINAL(5),NOPT,ICOUNT,SIGMA
      COMMON/DATA1/L1,L2,A0,A1,A2,C1,C2,C3,G
      COMMON/DATA2/E1,E2,XZ,ALPHA0
      COMMON/DATA4/PI,PPI,V0,TF,TC
      COMMON/DATA6/DD,MFOOT,BETA
      COMMON/DATA7/E3,E4
      IF (ICHK) 10,10,100
   10 DO 12 K = 1,NSTATE
      Y1(K) = YO(K)
   12 CONTINUE
      NCSTR = 0
      L = 1
      DO 14 K = 1,NSTATE
   14 Y(K,L) = YO(K)
      DO 90 L = 2,NSTEP
      T = L - 2
      T = T * DELTA
      LM1 = L - 1
   16 DELT2 = DELTA/2.
      CALL AHX(Y1,YDOT,T,LM1,NSTATE,ICHK,NCSTR)
      DO 20 K = 1,NSTATE
      FK1(K) = DELTA * YDOT(K)
      Y1(K) = Y(K,LM1) + FK1(K)/2.
   20 CONTINUE
      TO = T+DELT2
      CALL AHX(Y1,YDOT,TO,LM1,NSTATE,ICHK,NCSTR)
      DO 30 K = 1,NSTATE
      FK2(K) = DELTA*YDOT(K)
      Y1(K) = Y(K,LM1) + FK2(K) /2
   30 CONTINUE
      CALL AHX(Y1,YDOT,TO,LM1,NSTATE,ICHK,NCSTR)
      DO 40 K = 1,NSTATE
      FK3(K) = DELTA * YDOT(K)
      Y1(K) = Y(K,LM1) + FK3(K)
   40 CONTINUE
      T3 = T + DELTA
      CALL AHX(Y1,YDOT,T3,LM1,NSTATE,ICHK,NCSTR)
      DO 45 K = 1,NSTATE
      FK4(K) = DELTA * YDOT(K)
   45 CONTINUE
      DO 50 K = 1,NSTATE
      Y(K,L) = Y(K,LM1) + (FK1(K)+FK4(K)+2.*(FK2(K)+FK3(K)))/6.
      Y1(K) = Y(K,L)
   50 CONTINUE
   90 CONTINUE
      IF (NOPT.EQ.1 ) GO TO 92
      RETURN
```

```
   92 DO 94 K = 1,NSTATE
      XFINAL(K) = Y(K,NSTEP)
   94 CONTINUE
      RETURN
  100 MI = NSTEP + 1
      NCSTR = 0
      IF (NOPT.EQ.1) GO TO 104
      GO TO 101
  104 SIGMAA = 2. * SIGMA
      PX1 = XFINAL (1) - XZ
      PX2 = XFINAL(2) - PX1
      CT1 = SIN(PX1)
      CT2 = COS(PX1)
      CT3 = SIN(PX2)
      CT4 = COS(PX2)
      GG = (10.185)/(2.54*12.0)
      AA2 = 0.0
      WTF = 4*PI*5/TF
      HTF = AA2*COS(WTF)
      CS1 = E1-L1*CT2-L2*CT4-HTF
      CS2 = E2-L1*CT1+L2*CT3-0.2*V0
      C = 2.0*(CS1*(L1*CT1-L2*CT3)-CS2*(L1*CT2+L2*CT4))
      D = 2.0*L2*(CS1*CT3+CS2*CT4)
      G1 = 0.3482
      G2 = 0.5669
      Y0(1) = SIGMAA*(XFINAL(1) - G1)
      Y0(2) = SIGMAA*(XFINAL(2) - G2)
      Y0(3) = SIGMAA*(XFINAL(3) - E3)
      Y0(4) = SIGMAA*(XFINAL(4) - E4)
      Y0(5) = SIGMA
  106 CONTINUE
  101 DO 102 K = 1,NSTATE
  102 Y1(K) = Y0(K)
      DO 105 K = 1,NSTATE
  105 Y(K,NSTEP) = Y0(K)
      DO 190 L = 2,NSTEP
      T = L-2
      T = T * DELTA
      M = MI-L
      MP1 = M + 1
      CALL AHX(Y1,YDOT,T,MP1,NSTATE,ICHK,NCSTR)
      DO 114 K = 1,NSTATE
      FK1(K) = DELTA*YDOT(K)
      Y1(K) = Y(K,MP1) - FK1(K)/2.
  114 CONTINUE
      TO = T+0.5*DELTA
      CALL AHX(Y1,YDOT,TO,MP1,NSTATE,ICHK,NCSTR)
      DO 116 K = 1,NSTATE
      FK2(K) = DELTA*YDOT(K)
      Y1(K) = Y(K,MP1) - FK2(K)/2.
  116 CONTINUE
      CALL AHX(Y1,YDOT,TO,MP1,NSTATE,ICHK,NCSTR)
      DO 117 K=1,NSTATE
      FK3(K) = DELTA*YDOT(K)
      Y1(K) = Y(K,MP1) - FK3(K)
  117 CONTINUE
      T3 = T + DELTA
      CALL AHX(Y1,YDOT,T3,MP1,NSTATE,ICHK,NCSTR)
      DO 118 K = 1,NSTATE
  118 FK4(K) = DELTA*YDOT(K)
      DO 120 K = 1,NSTATE
      Y(K,M) = Y(K,MP1) - (FK1(K)+FK4(K)+2.*(FK2(K)+FK3(K)))/6.
      Y1(K) = Y(K,M)
  120 CONTINUE
  190 CONTINUE
      RETURN
```

```
@FOR,IS HER.AHX,HER.AHX
      SUBROUTINE AHX(P1,PDOT,T,M,NSTATE,ICHK,NCSTR)
C     COMPUTES THE RIGHTHAND-SIDE FOR BOTH THE ADJOINT AND THE STATE
C     DIFFERENTIAL EQUATIONS
      DIMENSION P1(NSTATE),PDOT(NSTATE),COEFQ(3),COEFP(3)
      REAL L1,L2,MFOOT
      COMMON/BLK1/UIPL1(2,101),NCONTR,GAMMA
      COMMON/BLK2/X(5,101),XO(5),ALAMB(5,101),ALAMBF(5)
      COMMON/BLK6/XNOM(5),XFINAL(5),NOPT,ICOUNT,SIGMA
      COMMON/DATA1/L1,L2,A0,A1,A2,C1,C2,C3,G
      COMMON/DATA2/E1,E2,XZ,ALPHAO
      COMMON/DATA4/PI,PPI,VO,TF,TC
      COMMON/DATA6/DD,MFOOT,BETA
      COMMON/REACT/FVP1(101),FHP1(101),DYN1(2,101)
      IF (ICHK) 10,10,100
C-----RIGHT HAND SIDE EXPRESSIONS FOR THE DYNAMIC EQUATIONS
   10 MP1 = M + 1
      U1 = UIPL1(1,M)
      U2 = UIPL1(2,M)
      Y3 = P1(3)
      Y4 = P1(4)
      X1 = P1(1) - XZ
      X2 = P1(2) - P1(1)+XZ
      X3 = P1(2)
      T1 = SIN(X1)
      T2 = COS(X1)
   51 T3 = SIN(X2)
      T4 = COS(X2)
   52 T5 = SIN(X3)
      T6 = COS(X3)
      C2T3 = C2 * T3
      C2T4 = C2*T4
      C3T5 = C3 * T5
      C3T6 = C3 * T6
      T7 = A2 + C3T6
      T8 = A1 + 2.0 * C3T6
      T9 = T8-T7
      GG = (10.185)/(2.54*12.0)
      AA2 = 0.0
      W = 4*PI*T/TF
      H = AA2*COS(W)
      DDH = -PPI*PPI*H
      T10 = DDH+G
      DELTA = A2*T8-T7*T7
      S2 = E2-VO*T-L1*T1+L2*T3
      S1 = E1 - L1*T2 - L2*T4 - H
      DFOOT = 0.82
      DELMAX = (110.0)*(3.142)/(180.0)
      ETA = PI - DELMAX - XZ + X1 - X2
      SS = H + L1*T2 + L2*T4 + DFOOT*COS(ETA) - E1
      COEFP(1) = 0.744200
      COEFP(2) = -0.027
      COEFQ(1) = -7.934200
      COEFQ(2) = -0.178
      Q = COEFQ(1) + COEFQ(2)*T
      Q    = Q*GG
      P = COEFP(1) + COEFP(2)*T
      HH      = L2*T4 + L1*T2 + P + H -E1
      AA      = L1*T1 - L2*T3 -Q + VO*(T+TC)
      B = SS
C-----*-*******************-----------
      FVP = FVP1(M)
      FHP = FHP1(M)
      ANKLE = FVP*S2+FHP*S1
      DYN1(1,M) = +MFOOT*(L1*T1-L2*T3)*G+ANKLE-(L1*T1-L2*T3)*FVP-(L1*T2
```

```
     1+L2*T4)*FHP
      DYN1(1,M) = 0.0
      DYN1(2,M) = +MFOOT*L2*T3*G-ANKLE-L2*T3*FVP+L2*T4*FHP
      DYN1(2,M) = 0.0
      R3 = -C3T5*Y4*(Y4-2.0*Y3)-(C1*T1-C2T3)*T10
      R4 = -C3T5*Y3*Y3-C3T6*T10
      PDOT (1) = P1(3)
      PDOT (2) = P1(4)
      PDOT (3) = (A2*(R3+U1)+T7*(R4+U2))/DELTA
      PDOT(4) = (T7*(R3+U1)+T8*(R4+U2))/DELTA
      PDOT(5) = B*B
      RETURN
C-----RIGHT HAND SIDE EXPRESSIONS FOR THE LAMBDA EQUATIONS (ADJOINT EQUATIONS
  400 NN1 = M - 1
      V1 = UIPL1(1,M)
      V2 = UIPL1(2,M)
      Y1 = X(1,M)
      Y2 = X(2,M)
      Y3 = X(3,M)
      Y4 = X(4,M)
      Y5 = X(5,M)
      PX1 = Y1-XZ
      PX2 = Y2 - PX1
      PX3 = Y2
      T1 = SIN(PX1)
      T2 = COS(PX1)
      T3 = SIN(PX2)
      T4 = COS(PX2)
      T5 = SIN(PX3)
      T6 = COS(PX3)
      C2T3 = C2 * T3
      C2T4 = C2 * T4
      C3T5 = C3 * T5
      C3T6 = C3 * T6
      T7 = A2 + C3T6
      T8 = A1+2.0 * C3T6
      T9 = T8 - T7
      GG = (10.185)/(2.54*12.0)
      AA2 = 0.0
      W = 4*PI*T/TF
      H = AA2*COS(W)
      DDH = -PPI*PPI*H
      T10 = DDH + G
      DELTA = A2 * T8 - T7 * T7
      DET2 = 2.0 *C3*C3 *T5*T6
      CT = 0.2-T
      S2 = E2-L1*T1+L2*T3-V0*CT
      S1 = E1-L1*T2-L2*T4-H
      DFOOT = 0.82
      DELMAX = (110.0)*(3.142)/(180.0)
      ETA = PI - DELMAX - XZ + X1 - X2
      SS = H + L1*T2 + L2*T4 + DFOOT*COS(ETA) - E1
      COEFQ(2) = -0.178
      COEFQ(1) = -7.934200
      COEFP(2) = -0.027
      COEFP(1) = 0.744200
      Q = COEFQ(1) + COEFQ(2)*T
      Q   = Q*GG
      P = COEFP(1) + COEFP(2)*T
      HH     = L2*T4 + L1*T2 + P + H -E1
      AA     = L1*T1 - L2*T3 -Q + V0*(T+TC)
      BX2 = +L2*T3 + DFOOT*SIN(ETA)
      BX1 = -L1*T1 + L2*T3 - DFOOT*SIN(ETA)
      B = SS
      FVP = FVP1(M)
      FHP = FHP1(M)
```

```
      R3=-C3T5*Y4*(Y4-2.0*Y3)-(C1*T1-C2T3)*T10
      R31 = -(C1*T2+C2T4)*T10+BETA*(-MFOOT*(L1*T2+L2*T4)*G )
      R32 = -C3T6*Y4*(Y4-2.0*Y3)+C2T4*T10+BETA*(+MFOOT*L2*T4*G)
      R33 = 2.0*C3T5*Y4
      R34 = -2.0*C3T5*(Y4-Y3)
      R4 = -C3T5*Y3*Y3-C2T3*T10
      R41 = C2T4*T10+BETA*(+MFOOT*L2*T4*G+L1*T2*FVP-L1*T1*FHP)
      R42 = -C3T6*Y3*Y3-C2T4*T10+BETA*(-MFOOT*L2*T4*G)
      R43 = -2.0*C3T5*Y3
      PDOT(1) = (P1(3)*(A2*R31+T7*R41)+P1(4)*(T7*R31+T8*R41))/DELTA+2.0
     1*B*BX1*P1(5)
      PDOT(2) = (P1(3)/DELTA)*(A2*R32-C3T5*(R4+V2)+T7*R42-(DET2/DELTA)*
     1 (A2*(R3+V1)+T7*(R4+V2)))+(P1(4)/DELTA)*(T7*R32-C3T5*(R3+V1)+T8*
     2R42-2.0*C3T5*(R4+V2)+(DET2/DELTA)*(T7*(R3+V1)+T8*(R4+V2)))+2.0*
     3B*BX2*P1(5)   - GAMMA/(X2)**2
      PDOT(3) = P1(1)+(((A2*P1(3)+T7*P1(4))*R33+(T7*P1(3)+T8*P1(4))*
     1R43)/DELTA)
      PDOT(4) = P1(2)+(R34/DELTA)*(A2*P1(3)+T7*P1(4))
      PDOT(5) = 0.0
      RETURN
      END
@FOR,IS HER.COSTF,HER.COSTF
      SUBROUTINE COSTF(X,F)
      REAL MFOOT
      COMMON/BLK1/UIPL1(2,101),NCONTR,GAMMA
      COMMON/BLK2/Y(5,101),Y0(5),ALAMB(5,101),ALAMBF(5)
      COMMON/BLK3/NSTATE,NSTEP,DELTA
      COMMON/BLK5/UI(2,101),SI(2,101)
      COMMON/BLK6/XNOM(75),XFINAL(5),NOPT,ICOUNT,SIGMA
      COMMON/DATA1/L1,L2,A0,A1,A2,C1,C2,C3,G
      COMMON/DATA2/E1,E2,XZ,ALPHA0
      COMMON/DATA4/PI,PPI,V0,TF,TC
      COMMON/DATA6/DD,MFOOT,BETA
      COMMON/DATA7/E3,E4
      EXTERNAL COST
      DO 11 K = 1,NCONTR
      DO 5 L = 1,NSTEP
    5 UIPL1(K,L) = UI(K,L)+X*SI(K,L)
   11 CONTINUE
      CALL DIFFEQ(Y,Y0,NSTATE,NSTEP,DELTA,0)
      CALL SIMPIN(COST,DELTA,NSTEP,F)
      PX1 = Y(1,NSTEP)-XZ
      PX2 = Y(2,NSTEP)-PX1
      CT1 = SIN(PX1)
      CT2 = COS(PX1)
      CT3 = SIN(PX2)
      CT4 = COS(PX2)
      GG = (10.185)/(2.54*12.0)
      AA2 = 0.0
      WTF = 4*PI*5/TF
      HTF = AA2*COS(WTF)
      CS1 = E1-L1*CT2-L2*CT4-HTF
      CS2 = E2-L1*CT1+L2*CT3-0.2*V0
      G2 = 0.5669
      G1 = 0.3482
      SUM = SIGMA*(Y(5,NSTEP)+(Y(1,NSTEP)-G1 )**2+(Y(2,NSTEP)-G2  )**2
     1+(Y(3,NSTEP)-E3)**2+(Y(4,NSTEP)-E4)**2)
      F = F + SUM
      RETURN
      END
@FOR,IS HER.AHSUBU,HER.AHSUBU
      SUBROUTINE AHSUBU(Y,P,Q,R,NSTATE,NCONTR,NSTEP,DELTA)
      REAL L1,L2
      DIMENSION Y(5,101),P(5,101),Q(2,101),R(2,101)
      COMMON/DATA1/L1,L2,A0,A1,A2,C1,C2,C3,G
```

```
      COMMON/DATA3/R1,R2
      DO 10 L = 1,NSTEP
      Y2 = Y(2,L)
      T7 = A2+C3*COS(Y2)
      T8 = A1+2.0*C3*COS(Y2)
      DOM = A2*T8-T7*T7
      R(2,L) = R2*Q(2,L)+(T7*P(3,L)+T8*P(4,L))/DOM
      R(1,L) = R1*Q(1,L)+(A2*P(3,L)+T7*P(4,L))/DOM
   10 CONTINUE
      RETURN
      END
@FOR,IS HER.COST,HER.COST
      FUNCTION COST(M)
      COMMON/BLK1/UIPL1(2,101),NCONTR,GAMMA
      COMMON/BLK2/X(5,101),X0(5),ALAMB(5,101),ALAMBF(5)
      COMMON/DATA3/R1,R2
      COST = 0.5*(R1*UIPL1(1,M)*UIPL1(1,M)+R2*UIPL1(2,M)*UIPL1(2,M))
     1 + GAMMA/X(2,M)
      RETURN
      END
@FOR,IS HER.AHSQ,HER.AHSQ
      FUNCTION AHSQ(M)
      COMMON/BLK4/R(2,101)
      AHSQ = (R(1,M)**2)+(R(2,M)**2)
      RETURN
      END
@FOR,IS HER.SIMPIN,HER.SIMPIN
      SUBROUTINE SIMPIN(Y,DELTA,NSTEP,F)
      DIMENSION      Z(101)
      HT = 0.3333333*DELTA
      L1 = 1
      L2 = 2
      L3 = 3
      L4 = 4
      L5 = 5
      L6 = 6
      SUM1 = Y(L2) + Y(L2)
      SUM1 = SUM1 + SUM1
      SUM1 = HT*(Y(L1) + SUM1+Y(L3))
      AUX1 = Y(L4) + Y(L4)
      AUX1 = AUX1 + AUX1
      AUX1 = SUM1 + HT*(Y(L3)+AUX1+  Y(L5))
      AUX2=HT*(Y(L1)+3.875*(Y(L2)+Y(L5))+2.625*(Y(L3)+Y(L4))+Y(L6))
      SUM2 = Y(L5) + Y(L5)
      SUM2 = SUM2 + SUM2
      SUM2 = AUX2-HT*(Y(L4)+SUM2+Y(L6))
      Z(L1) = 0.
      AUX = Y(L3) + Y(L3)
      AUX = AUX + AUX
      Z(L2) = SUM2-HT*(Y(L2)+AUX+Y(L4))
      Z(L3) = SUM1
      Z(L4) = SUM2
      DO 4 I = 7,NSTEP,2
      SUM1 = AUX1
      SUM2 = AUX2
      AUX1 = Y(I-1)+Y(I-1)
      AUX1 = AUX1 + AUX1
      AUX1 = SUM1 + HT*(Y(I-2)+ AUX1+Y(I))
      Z(I-2) = SUM1
      IF (I-NSTEP) 3,6,6
    3 AUX2 = Y(I) + Y(I)
      AUX2 = AUX2 + AUX2
      AUX2 = SUM2 + HT*(Y(I-1) + AUX2+Y(I+1))
    4 Z(I-1) = SUM2
    6 Z(NSTEP-1) = SUM2
      Z(NSTEP) = AUX1
      F = Z(NSTEP)
      RETURN
      END
@MAP,IS HER.PROG,HER.PROG
  IN  HER.MAIN
  IN  HER.GRAD
  IN  HER.DIFFEQ
  IN  HER.AHX
  IN  HER.AHSUBU
  IN  HER.COSTF
  IN  HER.COST
  IN  HER.AHSQ
  IN  HER.SIMPIN
@XQT HER.PROG
@FIN
```

APPENDIX 7

The Strobe Program.

Introduction.

This program has as its immediate aim the extraction of relevant data from the strobe photograph and the further analysis of this data.

Input.

Input required for the initiation of the program are the (x,y) co-ordinates of the head (between eye and ear) shoulder, hip, knee, ankle and toe. These are read off the photograph by means of a Hewlett-Packard digitizer (9107A) and they are thereafter manually punched in the correct format onto cards.

In addition to the (x,y) co-ordinates, 3 constants must be specified

(i) JS = Total number of strobe flashes used in the input (This constant is determined by inspection of one complete period (heel-strike to heel-strike) on the photograph).

(ii) NUM = Total number of points used.

(iii) FREQ = Number of flashes per second.

The Program.

The program scans the photographic data giving

(i) Hip angle (Flexion(+), neutral, extension(-))

(ii) Knee angle (Flexion(+), neutral, hyperextension(-))

(iii) Ankle angle (Dorsiflexion(-), Plantarflexion(+))

These angles are then tabulated. (See Fig. A7.1)

There are now a number of plot options open to the user:

... 154/

```
      ANGL3(J) = ANGLE(3,J)
      ANGL5(J) = ANGLE(5,J)
      XX(J) = X(3,J)
      ZZ(J) = X(5,J)
      WRITE(5,6033)  X(3,J),X(5,J)
 6033 FORMAT(2X,2F16.6)
  600 CONTINUE
      DO 3000 K = 1,JS
      ANG33(K) = ABS(ANGL3(K))*(9.0)/(45.0)
      ANG49(K) = ABS(ANGL4(K))*(9.0)/(72.0)
   29 FORMAT(2X,5F10.3)
 3000 CONTINUE
      DO 800 J = 1,JS
      TIM(J) = (J-1)/FREQ
      WRITE(5,6036) TIM(J)
 6036 FORMAT(2X,F16.6)
  800 CONTINUE
      CALL PLOTS(IBUF,25000,6)
      IF (IA-1)250,101,250
  101 CALL PLOT(0.0,0.5,-3)
      CALL SCALE(X(1,1),9.0,NUM,1)
      CALL SCALE(Y(1,1),9.0,NUM,1)
      CALL AXIS(0.0,0.0,46HHORIZONTAL DISPLACEMENTS OF HIP,KNEE ETC.
     1 ,-46,9.0,0.0,X(1,JS+1) ,X(2,JS+1))
      CALL AXIS(0.0,0.0,46H  VERTICAL DISPLACEMENTS OF HIP,KNEE ETC.
     1 ,+46,9.0,90.0,Y(1,JS+1),Y(2,JS+1))
      CALL LINE(X,Y,NUM,1,-1,4)
      CALL SYMBOL(0.5,9.6,0.21,24H MPC3  C.HERSHLER BIOENG, 0,24)
      CALL SYMBOL(0.5,9.2,0.14,24HSUBJECT   P.B. DATA 79A, 0,24)
      DO 400 J = 1,JS
      DO 399 I = 1,6
      TIME(I,J) = (J-1)/FREQ
  399 CONTINUE
  400 CONTINUE
  201 FORMAT()
  200 CONTINUE
  250 IF(IB-1)202,809,202
  809 CALL PLOT(11.0,0.5,-3)
      CALL SCALE(TIME(1,1),9.0,NUM,1)
      CALL SCALE(Y(1,1),9.0,NUM,1)
      CALL AXIS(0.0,0.0,47HTIME POINTS IN SECONDS    OF HIP,KNEE ETC.
     1  ,-47,9.0,0.0,TIME(1,JS+1),TIME(2,JS+1))
      CALL AXIS(0.0,0.0,46H  VERTICAL DISPLACEMENTS OF HIP,KNEE ETC.
     1 ,+46,9.0,90.0,Y(1,JS+1),Y(2,JS+1))
      CALL LINE(TIME,Y,NUM,1,-1,4)
  202 IF(IC-1)605,203,605
  203 CALL PLOT(11.0,0.5,-3)
      CALL SCALE(TIME(1,1),9.0,NUM,1)
      CALL SCALE(X(1,1),9.0,NUM,1)
      CALL AXIS(0.0,0.0,47HTIME POINTS IN SECONDS    OF HIP,KNEE ETC.
     1  ,-47,9.0,0.0,TIME(1,JS+1),TIME(2,JS+1))
      CALL AXIS(0.0,0.0,46HHORIZONTAL DISPLACEMENTS OF HIP,KNEE ETC.
     1 ,+46,9.0,90.0,X(1,JS+1),X(2,JS+1))
      CALL LINE(TIME,X,NUM,1,-1,4)
  605 IF(ID-1)505,601,505
  601 CALL PLOT(11.0,0.5,-3)
      CALL SCALE(ANGL3(1)  ,9.0, JS,1)
      CALL SCALE(ANGL4(1)  ,9.0, JS,1)
      CALL AXIS(0.0,0.0,46H  HIP ANGLE IN DEGREES
     1 ,-46,9.0, 0.0, ANGL3(JS+1),ANGL3(JS+2))
      CALL AXIS(0.0,0.0,47HKNEE ANGLE IN DEGREES
     1 ,+47,9.0,90.0,ANGL4(JS+1),ANGL4(JS+2))
      CALL LINE(ANGL3,ANGL4,JS,1,-1,4)
  505 IF(IE-1)612,501,612
  501 CALL PLOT(11.0,0.5,-3)
      CALL SCALE(TIM (1)  ,9.0, JS,1)
```

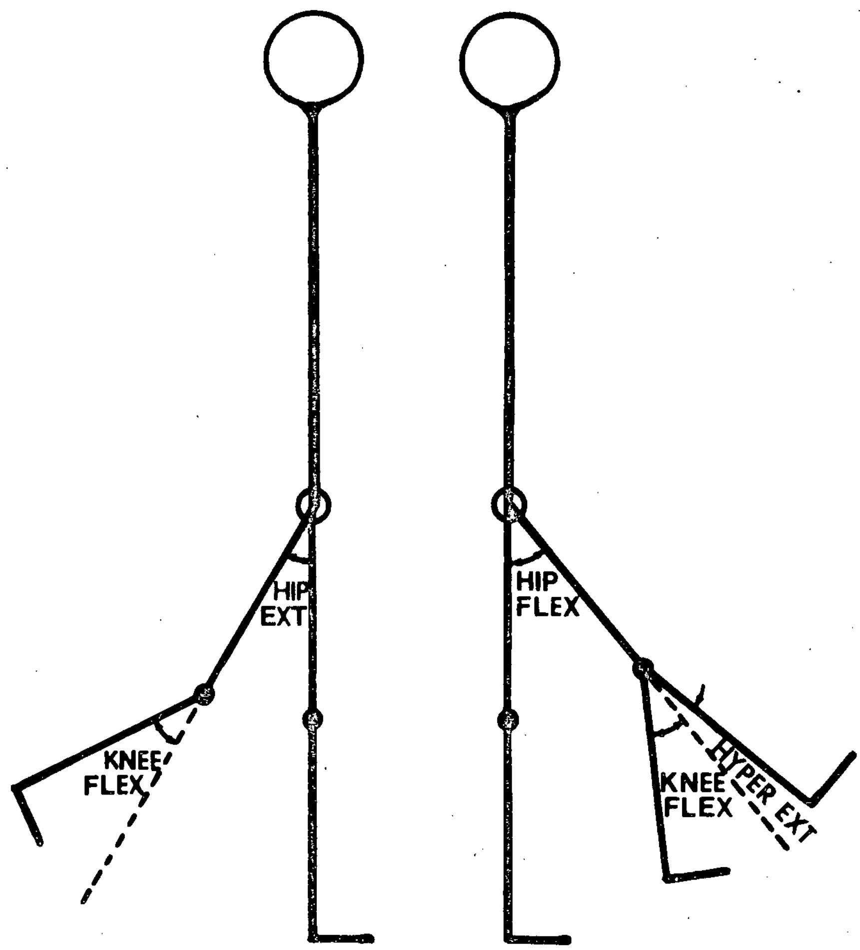

Fig.A7.1 DIRECTION OF FLEXION AND EXTENSION