A STUDY OF NUCLEON UNSTABLE STATES PRODUCED IN DIRECT NUCLEAR REACTIONS

by

Kurt Miklós Hartmann

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In Memory of

my brother, Péter

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Abstract

We study excited states of a nucleus that are unstable against nucleon decay. An expression is derived for the wave function of such an excited nucleus, using the continuum shell-model theory. The bound and resonant states of the shell-model Hamiltonian are treated on an equal footing. The resonant part of this wave function is substituted into the DWBA transition amplitude for stripping reactions. Simple expressions are obtained for the energy spectrum and angular distribution of the uncaptured nucleon(s). Comparison with the corresponding elastic scattering cross section is made and the extraction of spectroscopic information is discussed. Angular distributions and excitation functions for a number of projectile-target systems are calculated. We also suggest nucleon induced inelastic scattering as an alternative direct reaction mechanism with which these excited states may be studied.

Contents

1. Introduction

In recent years there has been some interest in the study of nucleon unstable excited states of nuclei which have been formed during direct nuclear reactions. Because these direct reactions affect only a small number of internal degrees of freedom of the colliding systems, they are ideally suited for studying the structure of such simple excitations. For instance in one-nucleon transfer reactions, the majority of nucleons remain passive during the interactions, i.e. occupy the same configuration in the initial and final nucleus. These passive nucleons form an inert core to which particles can be added. The validity of this picture is measured, in an actual transfer reaction, by the spectroscopic factor. The value of this spectroscopic factor is to be compared with the value calculated from an appropriate nuclear model.

One such nuclear model is the shell model. Since we are interested in excited states that lie above the nucleon threshold energy, an attempt must be made to treat both bound and continuum states consistently within the framework of the shell model [1-3]. A problem in these theories is the treatment of the single particle resonances of the shell-model Hamiltonian. A resonance is in many ways similar to a bound state, the main difference being that the resonance state has a finite lifetime. If the potential well would be somewhat deeper the resonance would be transferred into a bound state. Indeed in the simplest versions of the shell model and the R-matrix theories, the wave function of the resonance is

approximated by a linear superposition of bound states. More elaborate theories [4-7] take into account the effects of the coupling of the bound states to the continuum. In the work of Mahaux and Saruis [5] the wave function of a resonance is obtained, inside the nucleus as a linear superposition of shell model bound and resonant states (evaluated at the real energy of the resonance) with complex configuration-mixing coefficients (square roots of the spectroscopic factors). It is also shown that these resonant states are populated with a Breit-Wigner probability distribution.

Several authors [8-13] have advocated the use of complex energy eigenstates (Gamow states) as the most natural definition of a potential resonance state. We use these Gamow states together with the bound states and non-resonant continuum states of the shell model Hamiltonian to construct, within the frame-work of the continuum shell-model theory, expressions similar to those in ref.[5] for the wave function at resonance.

As has been mentioned above, one-nucleon stripping reactions populating particle unstable states may be used to study the various theoretical descriptions of the resonating system. In the experiments the decayed particle is usually not observed, and background contributions to the cross-secion are subtracted away during data analysis. In this way a cross-section very similar to the one in stripping to bound states is measured. The theoretical expression for the differential cross-section is usually evaluated in distorted-wave Born approximation (DWBA) [14,15]. The evaluation of the resulting radial integrals causes various convergence problems depending on the description used for the resonating

system. If the scattering radial wave function, evaluated at the (real) energy of the resonance, is used, the convergence of the integrals is slow $\left(\sim \frac{1}{r}\right)$. To speed up the convergence, the "convergence factor method" [14] or the "contour integration method" [16] is usually employed. These two methods may also be used when the resonant state is described by a Gamow function [10,13,17]. For this latter case, we shall give a physical interpretation of the convergence procedure.

In stripping to bound states, the normalization of the single-particle angular distribution to the experimental angular distribution, yields the spectroscopic factor [18]. There has been some controversy as to what exactly this normalization procedure measures in stripping to unbound states. Vincent and Fortune [16] and Baur and Trautmann [19] conclude that it measures the factor by which the single particle width must be multiplied in order to give the actual width of the resonance. Coker and Hoffmann [13] on the other hand take it to measure the spectroscopic factor.

Since these particle unstable states are above nucleon threshold, they may also be populated via the appropriate elastic scattering experiment. A comparison between stripping and elastic scattering to unstable states has been made by Fuchs et al. [20,21]. The observed similarities and differences have been explained by Lipperheide and Möhring [22-24], by Barz et al. [25] and by Baur and Trautmann [19] . Lipperheide and Möhring use stripping reactions to unbound states to study the off-the-energy-shell behaviour of the elastic transition amplitude. The intension of their work is thus somehwat different to ours.

Other kinds of direct reactions can also be used to investigate nucleon unstable states. For instance a high energy nucleon may knock out an inner shell nucleon, leaving the residual nucleus in a highly excited state. This nucleus may deexcite by emitting a further nucleon [26-30]. Another example is of a high energy projectile nucleon which may excite a single target nucleon into a low-lying resonating state. The results of this inelastic scattering to unstable states may be compared with stripping and elastic scattering to the same states. Experiments with this intension have, to the author's knowledge, not be performed.

In chapter 2 we derive expressions for the resonance behaviour of the eigenstate of, firstly, the shell-model Hamiltonian and then the full Hamiltonian. In chapter 3 simple expressions for the energy spectrum and angular distribution of the uncaptured outgoing nucleon(s) are derived in the DWBA. Angular distributions and energy spectra are calculated for a number of projectile target systems. We suggest experiments and derive relevant formulae for a study of inelastic scattering leading to unstable states in chapter 4. Chapter 5 contains a summary and a brief discussion of the results. In Appendix A we recall some of the properties of Gamow states while in Appendix B we give the orthogonality and completeness relations resulting from including the Gamow states with the bound and real energy scattering states. In Appendix C we discuss the problem of evaluating the background phase shift and wave function. Appendix D is devoted to the derivation of the scattering matrix for the full Hamiltonian and a sum rule for the widths. Details of the angular distribution and excitation function calculations are given in Appendix E.

2. The Wave Function at Resonance

In this chapter we study, in the frame of the continuum shell-model theory, the behaviour of the many-particle scattering wave function in the vicinity of a resonance. In the definition of the various quantities and in the derivation we have kept as close as possible to the notation used in Mahaux and Weidenmüller's book [3].

In section 2.1 the Hamiltonian of the many-particle system is decomposed into an unperturbed Hamiltonian and a perturbing potential. The eigenfunctions of the unperturbed Hamiltonian are given in section 2.2. These consist of eigenfunctions in which all the nucleons of the system are in bound orbitals and eigenfunctions in which one nucleon is in the continuum. The resonance behaviour of the latter set of eigenfunctions is discussed in section 2.3. An expression for the wave function of the total Hamiltonian is obtained in section 2.4 in terms of the eigenstates of the unperturbed Hamiltonian and the perturbing potential. Certain features of the result are also discussed in this section. We conclude, in section 2.5, with a comparison with other work of a similar nature.

2.1 The Decomposition of the Hamiltonian

A system of A nucleons is characterized by a Hamiltonian $H_A \equiv H(1,\ldots,A)$. The numbers $1,\ldots,A$ give the space, spin and isospin coordinates of the A nucleons. We write H_A as a

sum of two terms

$$H_A = H_{o,A} + V_A \quad . \tag{2.1.1}$$

The "unperturbed" Hamiltonian $H_{o,A}$ is further split into two parts

$$H_{o,A} = H_{A-1} + h_o(A) \quad . \tag{2.1.2}$$

The first part is the exact (to be qualified in subsection 2.2.a below) Hamiltonian of the (A-1) nucleon system. The second part consists of the kinetic energy operator t(A) and the shell-model potential $v_o(A)$ of the Ath nucleon,

$$h_o(A) = t(A) + v_o(A) \quad . \tag{2.1.3}$$

The perturbation V_A is the difference between the exact and the shell-model potentials acting between nucleon A and the other A-1 nucleons

$$V_A = \sum_{i=1}^{A-1} v(i,A) - v_o(A) \quad . \tag{2.1.4}$$

In order to solve the eigenvalue problem for the (A-1) nucleon system, H_{A-1} is also decomposed into a shell-model Hamiltonian $H_{o,A-1}$ and a residual interaction V_{A-1}

$$H_{A-1} = H_{o,A-1} + V_{A-1} \quad , \tag{2.1.5}$$

with

$$H_{o,A-1} = \sum_{i=1}^{A-1} \left[t(i) + v_o(i) \right] \quad , \tag{2.1.6}$$

and

$$V_{A-1} = \sum_{i<j=1}^{A-1} v(i,j) - \sum_{i=1}^{A-1} v_o(i) \quad . \tag{2.1.7}$$

The above decomposition of the Hamiltonian H_A is slightly different from the one in ref. [3], in that the residual interaction between the A-1 nucleons is included in $H_{o,A}$ both when the Ath nucleon is in a bound or scattering state. This choice is justified since we have in mind the study of single-particle states produced by, for instance, dropping a nucelon onto a target system of (A-1) nucleons in a stripping experiment.

In the following section we construct the eigenstates of the Hamiltonian $H_{o,A}$.

2.2 The Eigenfuctions of $H_{o,A}$

Since a basic restriction of the continuum shell-model theory is that only one nucleon (the Ath nucleon) is allowed to be in a scattering state, the states of the (A-1) nucleon system must contain only those configurations in which all (A-1) nucleons are in bound orbits. We construct in turn the eigenstates of H_{A-1} , $h_o(A)$ and $H_{o,A}$.

2.2.a Eigenstates of H_{A-1}

Because the Hamiltonian $H_{o,A-1}$ of eqn. (2.1.6) is a sum of single particle operators, its eigenfunction can easily be found. In order to ensure that all these eigenfunctions correspond to bound states, the shell-model potentials $v_o(i)$ may be made infinitely deep. In principle this gives rise to an infinite number of bound states, and in a particular calculation, a finite number of these must be chosen. Alternatively, if the potentials $v_o(i)$ are taken to be of finite depth, only those configurations in which all (A-1) nucleons are in bound orbitals are to be retained. There will be a finite number of such configurations. Then, if w_i are the eigenstates of $H_{o,A-1}$, we have

$$H_{o,A-1}\, w_i = \varepsilon_i\, w_i \quad , \qquad \langle w_i | w_j \rangle = \delta_{ij} \quad . \tag{2.2.1}$$

Each function w_i is an antisymmetrized linear combination of products of single-particle wave functions, coupled to a definite total spin I with projection μ.

The eigenstates of H_{A-1} with spin I and projection μ are expanded into a set of shell-model states

$$\varphi_{t,A-1} = \sum_i O_{ti}\, w_i \quad , \tag{2.2.2}$$

$$H_{A-1}\, \varphi_{t,A-1} = \varepsilon_t\, \varphi_{t,A-1} \quad . \tag{2.2.3}$$

The subscript t stands for the set of quantum numbers $\{I\mu\alpha\}$

where α denotes any further quantum numbers necessary to specify $\varphi_{t,A-1}$ completely. The eigenvalues ε_t are obtained by solving the determinant

$$\det\left[(\varepsilon_i - \varepsilon_t)\,\delta_{ij} + V_{ij}\right] = 0 \quad , \tag{2.2.4}$$

where

$$V_{ij} = \langle w_i \,|\, V_{A-1} \,|\, w_j \rangle \quad . \tag{2.2.5}$$

The coefficients O_{ti} are then determined from the set of coupled equations

$$\sum_j \left[(\varepsilon_i - \varepsilon_t)\,\delta_{ij} + V_{ij}\right] O_{ti} = 0 \quad . \tag{2.2.6}$$

2.2.b Eigenstates of $h_o(A)$

The wave function for the Ath nucleon satisfies the equation

$$h_o(A)\,\psi(\underline{r}) = \left[t(A) + v_o(A)\right]\psi(\underline{r}) = \varepsilon\,\psi(\underline{r}) \quad . \tag{2.2.7}$$

This wave function may be written in the form

$$\psi(\underline{r}) = \sum_{\ell j m} c_{\ell j m}\, \mathcal{Y}^{m}_{\ell j}(\hat{r})\, \frac{1}{r}\, u_{\ell j}(r,k) \quad , \tag{2.2.8}$$

where we have denoted the eigenfunctions of ℓ^2, j^2 and j_z by

$$\mathcal{Y}^m_{\ell j}(\hat{r}) = i^\ell \sum_\lambda (\ell \tfrac{1}{2} \lambda\ m-\lambda | j m)\ Y^\lambda_\ell(\theta,\phi)\ \Theta^{m-\lambda}_{\frac{1}{2}} \quad , \qquad (2.2.9)$$

and the radial wave function and spin function by $u_{\ell j}(r,k)$ and $\Theta^{m-\lambda}_{\frac{1}{2}}$ respectively.

If the energy eigenvalue ε is negative, then the Ath nucleon is in a bound state and its radial wave function is denoted by $w^b_{\ell j}$. These functions are chosen to be real and are normalized according to

$$\int_0^\infty dr\ w^b_{\ell j}(r, k^{\ell j}_m)\ w^b_{\ell j}(r, k^{\ell j}_n) = \delta_{mn} \quad . \qquad (2.2.10)$$

The complex quantities $k^{\ell j}_n$, are the wave numbers corresponding to the bound state energies $\varepsilon^{\ell j}_n$

$$\varepsilon^{\ell j}_n = \frac{\hbar^2}{2m} (k^{\ell j}_n)^2 \quad , \quad (\mathrm{Im}\, k^{\ell j}_n > 0) \quad , \qquad (2.2.11)$$

where m is the reduced mass of the nucleon.

For positive energies, the Ath nucleon is in a scattering state and the radial wave function is denoted by $u_{\ell j}$. These are made to satisfy the normalization condition

$$\int_0^\infty dr\ u_{\ell j}(r,k)\ u_{\ell j}(r,k') = \delta(\varepsilon-\varepsilon') \quad , \qquad (2.2.12)$$

with

$$\varepsilon = \frac{\hbar^2}{2m} k^2 \quad . \qquad (2.2.13)$$

From the fact that $u_{\ell j}$ is real and normalized according to eqn. (2.2.12) we have

$$u_{\ell j}(r,k) = \frac{i}{\hbar}\left(\frac{m}{2\pi k}\right)^{1/2}\left[e^{-i\delta_{\ell j}(k)} I_{\ell j}(r,k) - e^{i\delta_{\ell j}(k)} O_{\ell j}(r,k)\right] , \qquad (2.2.14)$$

where $I_{\ell j}$ and $O_{\ell j}$ correspond asymptotically to incoming and outgoing waves and $\delta_{\ell j}(k)$ is the potential scattering phase shift.

For later purposes we also introduce the functions

$$u^{(\pm)}_{\ell j}(r,k) = e^{\pm i\delta_{\ell j}(k)} u_{\ell j}(r,k) \quad . \qquad (2.2.15)$$

2.2.c Eigenstates of $H_{o,A}$

We now construct the eigenstates of $H_{o,A}$ using the eigenstates of H_{A-1} and of $h_o(A)$. Firstly we define the channel surface function:

$$\varphi_c = \frac{1}{r}\sum_m (j\ I\ m\mu | JM)\ \mathcal{Y}^m_{\ell j}(\hat{r})\ \varphi_{t,A-1} \quad . \qquad (2.2.16)$$

The index c stands for the set of quantum numbers $\{\ell j I J M \alpha\}$. When the Ath nucleon is in a bound single particle orbital, the eigenstate of $H_{o,A}$ with total angular momentum J and projection M is denoted by Φ_n :

$$\Phi_n = \mathcal{A}_A \left\{ \omega^b_{\ell j}(r, k_n^{\ell j}) \varphi_c \right\} \quad . \tag{2.2.17}$$

Here, $\mathcal{A}_A$ denotes the antisymmetirzation with respect to the target nucleons and the Ath nucleon. The states Φ_n are sometimes referred to as bound states embedded in the continuum (BSEC) or quasi-bound states.

When the Ath nucleon is in a scattering state, we denote by χ^c_E the eigenstates of $H_{o,A}$ with total angular momentum J and projection M:

$$\chi^c_E = \mathcal{A}_A \left\{ u_{\ell j}(r,k) \varphi_c \right\} \quad . \tag{2.2.18}$$

The unperturbed Hamiltonian $H_{o,A}$ is defined by its matrix elements

$$\langle \Phi_m | H_{o,A} | \Phi_n \rangle = E_n \delta_{nm} \quad , \tag{2.2.19}$$

$$\langle \Phi_n | H_{o,A} | \chi^c_E \rangle = 0 \quad , \tag{2.2.20}$$

$$\langle \chi^{c'}_{E'} | H_{o,A} | \chi^c_E \rangle = E \delta_{cc'} \delta(E-E') \quad . \tag{2.2.21}$$

We also have

$$E_n = \varepsilon_t + \frac{\hbar^2}{2m} (k_n^{\ell j})^2 \quad , \tag{2.2.22}$$

$$E = \varepsilon_t + \frac{\hbar^2}{2m} k^2 \quad . \tag{2.2.23}$$

We introduce the notation

$$\chi_E^{c(\pm)} = \mathcal{A}_A \left\{ u_{\ell j}^{(\pm)}(r,k)\, \varphi_c \right\} \quad . \tag{2.2.24}$$

2.3 The Behaviour of $u_{\ell j}^{(\pm)}$ and $\chi_E^{c(\pm)}$ Near a Single Particle Resonance

The potential scattering phase shift $\delta_{\ell j}(k)$ appearing in eqn. (2.2.14) may display an increase by π over a narrow energy range. This is called a single particle resonance (SPR). In the neighbourhood of such a SPR, the scattering radial wave function $u_{\ell j}$ increases drastically as a function of r within the range of the nuclear potential. Inside the potential well $u_{\ell j}$ resembles the bound state radial wave function $w_{\ell j}^b$. In this section we aim to separate out the resonance behaviour of $u_{\ell j}$ from its behaviour far from resonance. Consider the function $u_{\ell j}^{(+)}(r,\mathcal{k})$ in the complex k-plane.† Using eqn. (2.2.14) we have

$$u_{\ell j}^{(+)}(r,\mathcal{k}) = \frac{i}{\mathcal{k}} \left(\frac{m}{2\pi\mathcal{k}}\right)^{1/2} \left[I_{\ell j}(r,\mathcal{k}) - e^{2i\delta_{\ell j}(\mathcal{k})} O_{\ell j}(r,\mathcal{k}) \right] \quad . \tag{2.3.1}$$

A SPR of the Hamiltonian $h_o(A)$ may be associated with a pole

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†) script letters $\mathcal{k}$ and $\mathcal{E}$ denote complex wave numbers and energies while k and ε are reserved for the corresponding real qualities.

of the scattering function

$$S_{\ell j}(k) = e^{2i\delta_{\ell j}(k)} \quad , \tag{2.3.2}$$

in the complex k-plane. We will be interested in poles at $k = k_n^{\ell j}$ with $\mathrm{Re}\,k_n^{\ell j} \neq 0$, $\mathrm{Im}\,k_n^{\ell j} < 0$ and $|\mathrm{Re}\,k_n^{\ell j}| > |\mathrm{Im}\,k_n^{\ell j}|$.

As a function of complex energy, $\mathcal{E}$, the scattering function defined by eqn. (2.3.2) has the approximate form

$$S_{\ell j}(\mathcal{E}) \simeq e^{2i\xi_n^{\ell j}}\left[1 - \frac{i\Gamma_n^{\ell j}}{\mathcal{E} - \mathcal{E}_n^{\ell j} + \frac{i}{2}\Gamma_n^{\ell j}}\right] \quad , \tag{2.3.3}$$

where

$$\mathcal{E}_n^{\ell j} - \tfrac{i}{2}\Gamma_n^{\ell j} = \frac{\hbar^2}{2m}\left(k_n^{\ell j}\right)^2 \quad , \tag{2.3.4}$$

and $\xi_n^{\ell j}$ is the background (non-resonant) phase shift. Near a pole of $u_{\ell j}^{(+)}$ we can write, using eqns (2.3.1) - (2.3.3),

$$u_{\ell j}^{(+)}(r,k) = u_{\ell j}^{(+)\,NR}(r,k) - e^{i\xi_n^{\ell j}} \frac{\left(\frac{1}{2\pi}\Gamma_n^{\ell j}\right)^{1/2}}{\mathcal{E} - \mathcal{E}_n^{\ell j} + \frac{i}{2}\Gamma_n^{\ell j}} \omega_{\ell j}^{res}(r, k_n^{\ell j}) \quad . \tag{2.3.5}$$

In this equation, the first term on the RHS gives the non-resonant part of the radial wave function. This term is regular at the pole $\left(\mathcal{E}_n^{\ell j} - \frac{i}{2}\Gamma_n^{\ell j}\right)$, and is given by

$$u_{\ell j}^{(+)\,NR}(r,k) = \frac{i}{\hbar}\left(\frac{m}{2\pi k}\right)^{1/2}\left[I_{\ell j}(r,k) - e^{2i\xi_n^{\ell j}(k)}\, O_{\ell j}(r,k)\right] . \qquad (2.3.6)$$

The numerator of the second term is the residue of $u_{\ell j}^{(+)}$ at the complex pole. We have used the Mahaux-Weidenmüller definition [3] of the wave function at resonance (Gamow function)

$$\omega_{\ell j}^{res}(r, k_n^{\ell j}) = \left(\frac{m\Gamma_n^{\ell j}}{k^2 k_n^{\ell j}}\right)^{1/2} e^{i\xi_n^{\ell j}}\, O_{\ell j}(r, k_n^{\ell j}) , \qquad (2.3.7)$$

(see Appendix A for further details).

For real values of the energy ε (or the wave number k) near the complex resonance energy (or wave number), the scattering radial wave function, eqn. (2.3.5), takes the form

$$u_{\ell j}^{(+)}(r,k) = u_{\ell j}^{(+)\,NR}(r,k) - e^{i\xi_n^{\ell j}} \frac{\left(\frac{1}{2\pi}\Gamma_n^{\ell j}\right)^{1/2}}{\varepsilon - \varepsilon_n^{\ell j} + \frac{i}{2}\Gamma_n^{\ell j}}\, \omega_{\ell j}^{res}(r, k_n^{\ell j}) . \qquad (2.3.8)$$

We have thus isolated the violent energy dependence of $u_{\ell j}^{(+)}$ near a SPR. Further, inside the potential well the function $u_{\ell j}^{(+)NR}$ is negligibly small, provided the angular momentum barrier in the neighbourhood of the resonance energy is high. Outside the well, the function $w_{\ell j}^{res}$, as it stands, has no physical significance. It has however been shown by Berggren [9], Romo [12] and Gyarmati and Vertse [17] how the Gamow function may be normalized. We try to attach a physical

significance to these regularization procedures in Appendix A. If the residue of $u_{\ell j}^{(+)}$ at a resonance is evaluated not at the complex wave number, $k_n^{\ell j}$, but at its corresponding real part, the RHS of eqn. (2.3.8) is defined for all r. This form for the resonance wave function has been used by Baur and Trautmann [19]. Inside the potential well the Gamow function is similar whether evaluated at the complex wave number or at the corresponding real wave number [10] .

In principle, for a particular orbital angular momentum ℓ, there are several resonances corresponding to increasing principal quantum number n. Thus in eqn. (2.3.8), the resonance term should also contain a sum over these resonances. However, in practice, these resonances are well separated in energy and, apart from the lowest lying of these, they are broad ($|\mathrm{Re}\, k_n^{\ell j}| \lesssim |\mathrm{Im}\, k_n^{\ell j}|$). In the following, therefore, we retain only the narrow resonance in a particular channel.

We give a procedure for determining the background wave function, $u_{\ell j}^{NR}$, in Appendix C, while in Appendix B we discuss some of the orthonormality and completeness relations arising from the form of eqn. (2.3.8).

The form of $\chi_E^{c(+)}$ (eqn. (2.2.23)) near a SPR is easily constructed. Using eqns. (2.2.3) and (2.3.8) we obtain

$$\chi_E^{c(+)} = \chi_E^{c(+)\,NR} - e^{i\xi_c} \frac{\left(\frac{1}{2\pi}\Gamma_c\right)^{1/2}}{E - E_c + \frac{i}{2}\Gamma_c} W_c \quad . \tag{2.3.9}$$

We have replaced the indices n, ℓ and j in the quantities $\Gamma_n^{\ell j}$ and $\xi_n^{\ell j}$ in eqn. (2.3.8) by the single subscript c, used

the relation

$$E_c = \varepsilon_t + \varepsilon_n^{\ell j} \quad , \tag{2.3.10}$$

and introduced the many-body Gamow state

$$W_c = \mathcal{A}_A \left\{ \omega_{\ell j}^{res}(r, k_n^{\ell j}) \, \varphi_c \right\} \tag{2.3.11}$$

The wave function $\chi_E^{c(-)}$, satisfying incoming boundary conditions is given by

$$\chi_E^{c(-)} = \chi_E^{c(-)NR} - e^{-i\xi_c} \frac{(\frac{1}{2\pi}\Gamma_c)^{1/2}}{E - E_c - \frac{i}{2}\Gamma_c} \tilde{W}_c \quad , \tag{2.3.12}$$

with

$$\tilde{W}_c = \mathcal{A}_A \left\{ \tilde{\omega}_{\ell j}^{res}(r, \tilde{k}_n^{\ell j}) \, \varphi_c \right\} \quad , \tag{2.3.13}$$

(see Appendix A).

2.4 The Eigenstates of H_A

Suppose the eigenstates of the Hamiltonian H_A defined in eqn. (2.1.1), in which the A nucleons are coupled to total angular momentum J and component M, are denoted by ψ_E^c . In the shell-model approach to nuclear reactions these states are written as linear superpositions of the eigenstates of $H_{o,A}$:

$$\Psi_E^c = \sum_{n=1}^{M} |\Phi_n\rangle\langle\Phi_n|\Psi_E^c\rangle + \sum_{c'=1}^{\Lambda} \int_{\varepsilon_{c'}}^{\infty} dE' \, |\chi_{E'}^{c'}\rangle\langle\chi_{E'}^{c'}|\Psi_E^c\rangle \quad . \tag{2.4.1}$$

We assume here that there are M bound states and Λ channels of $H_{o,A}$ which couple to J and M. The energy $\varepsilon_{c'}$ denotes the threshold energy in channel c' .

The wave function ψ_E^c with outgoing boundary conditions satisfies the Lippmann-Schwinger equation

$$\psi_E^{c(+)} = \chi_E^{c(+)} + \sum_{n=1}^{M} \frac{|\Phi_n\rangle\langle\Phi_n|}{E - E_n} V_A \psi_E^{c(+)} + \sum_{c'=1}^{\Lambda} \int_{\varepsilon_{c'}}^{\infty} dE' \, \frac{|\chi_{E'}^{c'}\rangle\langle\chi_{E'}^{c'}|}{E^+ - E'} V_A \psi_E^{c(+)} \quad . \tag{2.4.2}$$

The explicit solution of $\psi_E^{c(+)}$ may be obtained provided the Born expression of the resolvent of the kernel

$$\sum_{c'=1}^{\Lambda} \int_{\varepsilon_{c'}}^{\infty} dE' \, \frac{|\chi_{E'}^{c'}\rangle\langle\chi_{E'}^{c'}|}{E^+ - E'} V_A \quad , \tag{2.4.3}$$

converges. This will not be the case if there are SPR in the eigenstates $\chi_{E'}^{c'}$ of the shell-model Hamiltonian $H_{o,A}$. Suppose there are SPR in the channels denoted by $c_o'(= 1, \ldots, \Lambda_o)$. In these channels we use a separable approximation for the Green's function

$$\sum_{c_o'=1}^{\Lambda_o} \int_{\varepsilon_{c_o'}}^{\infty} dE' \, \frac{|\chi_{E'}^{c_o'}\rangle\langle\chi_{E'}^{c_o'}|}{E^+ - E'} \quad , \tag{2.4.4}$$

which was derived by Romo [12] (see also end of Appendix A). This uses the many-body Gamow states $W_{c_o'}$ (see eqn. (2.3.11)) with complex energies $E_{c_o'} - \frac{i}{2}\Gamma_{c_o'}$. We have for eqn. (2.4.3)

$$\sum_{c'=1}^{\Lambda} \int_{\varepsilon_{c'}}^{\infty} dE' \frac{|\chi_{E'}^{c'}\rangle\langle\chi_{E'}^{c'}|}{E^+ - E'} V_A = \sum_{c_o'=1}^{\Lambda_o} \frac{|W_{c_o'}\rangle\langle\widetilde{W}_{c_o'}|}{E - E_{c_o'} + \frac{i}{2}\Gamma_{c_o'}} V_A + K^{(+)} V_A \quad , \tag{2.4.5}$$

where

$$K^{(+)} = \sum_{c'=1}^{\Lambda} \int_{\varepsilon_{c'}}^{\infty} dE' \frac{|\chi_{E'}^{c'}\rangle\langle\chi_{E'}^{c'}|}{E^+ - E'} - \sum_{c_o'=1}^{\Lambda_o} \frac{|W_{c_o'}\rangle\langle\widetilde{W}_{c_o'}|}{E - E_{c_o'} + \frac{i}{2}\Gamma_{c_o'}} \quad . \tag{2.4.6}$$

The kernel $K^{(+)}V_A$ no longer contains SPR and may be expanded in a Born series.

If the channel c in eqn. (2.4.2) is one of the channels c_o, we also employ eqn. (2.3.9) to separate out the resonance behaviour in $\chi_E^{c(+)}$:

$$\chi_E^{c(+)} = \chi_E^{c(+)NR} - e^{i\xi_c} \frac{\left(\frac{1}{2\pi}\Gamma_c\right)^{1/2}}{E - E_c + \frac{i}{2}\Gamma_c} W_c \, \delta_{cc_o} \quad . \tag{2.4.7}$$

If there are no SPR in the channels, we have from eqns. (2.3.2) and (2.3.3) that the phase shifts δ_c and ξ_c are identical and

$$\chi_E^{c(+)} \equiv \chi_E^{c(+)NR} \quad . \tag{2.4.8}$$

Substituting eqns. (2.4.5) and (2.4.7) into the Lippmann-Schwinger equation we obtain

$$\psi_E^{c(+)} = \chi_E^{c(+)} + \left\{ \sum_{\alpha=1}^{M+\Lambda_o} \frac{|\theta_\alpha\rangle\langle\tilde{\theta}_\alpha|}{E-E_\alpha} + K^{(+)} \right\} V_A \psi_E^{c(+)} - \sum_{\alpha=1}^{M+\Lambda_o} e^{i\xi_\alpha} \frac{\left(\frac{1}{2\pi}\Gamma_\alpha\right)^{1/2}}{E-E_\alpha} \theta_\alpha \delta_{\alpha c_o} \delta_{c_o c} \quad . \tag{2.4.9}$$

We have introduced the notation

$$\theta_\alpha = \begin{cases} \Phi_n & \\ W_{c_o'} & \end{cases}, \qquad E_\alpha = \begin{cases} E_n & \alpha \leq M \\ E_{c_o'} - \frac{i}{2}\Gamma_{c_o'} & \alpha > M \end{cases} . \tag{2.4.10}$$

The factor $\delta_{\alpha c_o}\delta_{c_o c}$ ensures that the last term appears only if there is a SPR of the shell-model Hamiltonian in the channel $c = c_o$.

On defining the quantity $a_{\alpha c}^{(+)}$ by

$$a_{\alpha c}^{(+)} = \frac{\langle\tilde{\theta}_\alpha | V_A | \psi_E^{c(+)}\rangle}{E-E_\alpha} - e^{i\xi_\alpha} \frac{\left(\frac{1}{2\pi}\Gamma_\alpha\right)^{1/2}}{E-E_\alpha} \delta_{\alpha c_o}\delta_{c_o c} \quad , \tag{2.4.11}$$

eqn. (2.4.9) can be written as

$$\psi_E^{c(+)} = \chi_E^{c(+)NR} + \sum_{\alpha=1}^{M+\Lambda_o} \theta_\alpha a_{\alpha c}^{(+)} + K^{(+)} V_A \psi_E^{c(+)} \quad . \tag{2.4.12}$$

Rearranging the above equation,

$$\Psi_E^{c(+)} = (1 - K^{(+)} V_A)^{-1} \left\{ \chi_E^{c(+)NR} + \sum_{\alpha=1}^{M+\Lambda_0} \theta_\alpha a_{\alpha c}^{(+)} \right\} . \qquad (2.4.13)$$

Substituting eqn. (2.4.13) into eqn. (2.4.11) gives

$$a_{\alpha' c}^{(+)} = \sum_{\alpha=1}^{M+\Lambda_0} (\underset{\sim}{C}^{-1})_{\alpha'\alpha} \left\{ \langle \tilde{\theta}_\alpha | V_A (1 - K^{(+)} V_A)^{-1} | \chi_E^{c(+)NR} \rangle \right.$$

$$\left. - e^{i\xi_\alpha} \left(\frac{1}{2\pi}\Gamma_\alpha\right)^{1/2} \delta_{\alpha c_o} \delta_{c_o c} \right\} , \qquad (2.4.14)$$

with

$$C_{\alpha'\alpha} = (E - E_{\alpha'}) \delta_{\alpha'\alpha} - \langle \tilde{\theta}_{\alpha'} | V_A (1 - K^{(+)} V_A)^{-1} | \theta_\alpha \rangle . \qquad (2.4.15)$$

Combining eqn. (2.4.14) and (2.4.12) we have

$$\Psi_E^{c(+)} = (1 - K^{(+)} V_A)^{-1} \chi_E^{c(+)NR} + \sum_{\alpha,\alpha'} (1 - K^{(+)} V_A)^{-1} | \theta_\alpha \rangle (\underset{\sim}{C}^{-1})_{\alpha\alpha'} \qquad (2.4.16)$$

$$\left\{ \langle \tilde{\theta}_{\alpha'} | V_A (1 - K^{(+)} V_A)^{-1} | \chi_E^{c(+)NR} \rangle - e^{i\xi_{\alpha'}} \left(\frac{1}{2\pi}\Gamma_{\alpha'}\right)^{1/2} \delta_{\alpha' c_o} \delta_{c_o c} \right\} .$$

The quantity $(1 - K^{(+)} V_A)^{-1}$ is expanded in a Born series:

$$(1 - K^{(+)} V_A)^{-1} = 1 + K^{(+)} V_A + (K^{(+)} V_A)^2 + \ldots , \qquad (2.4.17)$$

which is substituted into eqn. (2.4.16). Neglecting channel-channel coupling terms ($\langle \chi_E^{c'NR} | V_A | \chi_E^{cNR} \rangle = 0$) we obtain

$$\Psi_E^{c(+)} = \chi_E^{c(+)NR} + \sum_{\alpha,\alpha'} (1 + K^{(+)} V_A) | \theta_\alpha \rangle (\underset{\sim}{C}^{-1})_{\alpha\alpha'}$$

$$\left\{ \langle \tilde{\theta}_{\alpha'} | V_A | \chi_E^{c(+)NR} \rangle - e^{i\xi_{\alpha'}} \left(\frac{1}{2\pi}\Gamma_{\alpha'}\right)^{1/2} \delta_{\alpha' c_o} \delta_{c_o c} \right\} , \qquad (2.4.18)$$

with

$$C_{\alpha\alpha'} = (E - E_\alpha)\delta_{\alpha\alpha'} - \langle \tilde{\theta}_\alpha | V_A | \theta_{\alpha'} \rangle - \langle \tilde{\theta}_\alpha | V_A K^{(+)} V_A | \theta_{\alpha'} \rangle \quad . \tag{2.4.19}$$

Suppose $O_{t\alpha}$ are the elements of the complex orthogonal matrix $\underline{O}$ which diagonalises the matrix $\underline{C}$, i.e.

$$\sum_{\alpha\alpha'} O_{t\alpha}\, C_{\alpha\alpha'}\, (\underline{O}^{-1})_{\alpha' t'} = (E - E_t + \tfrac{i}{2}\Gamma_t)\,\delta_{tt'} \quad . \tag{2.4.20}$$

It is understood that the quantities $O_{t\alpha}$, E_t and Γ_t vary smoothly with the energy E. Introducing the definition

$$\varphi_{t,A} = \sum_{\alpha=1}^{M+\Lambda_o} O_{t\alpha}\, \theta_\alpha \quad , \tag{2.4.21}$$

eqn. (2.4.18) becomes

$$\psi_E^{c(+)} = \chi_E^{c(+)NR} - e^{i\xi_c} \sum_t (1 + K^{(+)} V_A)\varphi_{t,A}\, \frac{G_{tc}}{E - E_t + \frac{i}{2}\Gamma_t} \quad , \tag{2.4.22}$$

with

$$G_{tc} = O_{tc_o} (\tfrac{1}{2\pi}\Gamma_{c_o})^{1/2} \delta_{c_o c} - \langle \tilde{\varphi}_{t,A} | V_A | \chi_E^{c\,NR} \rangle \quad . \tag{2.4.23}$$

The complete wave function for the nucleus A is obtained by summing over all open channels c that can couple to JM:

$$\psi_{E,A}^{J,M(+)} = \sum_{c(open)} \psi_E^{c(+)} \quad . \tag{2.4.24}$$

Eqn. (2.4.22) is a generalization of eqn. (2.3.9) to the many channel case. As a function of energy it contains a smoothly varying background term and a Breit-Wigner shaped resonance term. The quantity $(1 + K^{(+)}V_A)\varphi_{t,A}$ describes the state into which the incident wave function χ_E^{cNR} decays. This state consists of the function $\varphi_{t,A}$ which is a linear superposition of the bound and Gamow states of the shell-model Hamiltonian. Due to the residual interaction V_A, there is a leakage into all the channels $\chi_E^{c'NR}$ ($c' = 1,\ldots,\Lambda$) described by the term $K^{(+)}V_A\varphi_{t,A}$. Both the bound and the Gamow states have large amplitudes inside the nucleus whereas, for large angular momentum barriers, $\chi_E^{c'NR}$ are zero inside the nucleus.

The decay into the state $\varphi_{t,A}$ is described by (neglecting channel-channel coupling) the quantity G_{tc}. If a SPR of the shell-model Hamiltonian is present in the channel $c = c_o$, then decay from that channel is caused either by the residual interaction V_A, or by the natural width of the Gamow state W_{c_o}. Because this latter width is multiplied by the admixture coefficient O_{tc_o}, it need not be larger than the width due to V_A.

The decay amplitude G_{tc} is in general a complex quantity because of the presence of the Gamow states in $\tilde{\varphi}_{t,A}$ and the admixture coefficients $O_{t\alpha}$. However the imaginary component of G_{tc} will be small for narrow resonances. The finite range of V_A ensures that only the real parts of the Gamow states contribute to the amplitude $\langle\tilde{\varphi}_{t,A}|V_A|\chi_E^{c,NR}\rangle$. In the case of narrow resonances, a real matrix $\underline{O}$ will almost diagonalize the matrix $\underline{C}$ with elements given by eqn. (2.4.19).

Because of the presence of the Gamow states in $\varphi_{t,A}$,

eqn. (2.4.22) is defined only inside the nucleus. An expression for $\psi_E^{c(+)}$ valid outside the nucleus may be obtained by evaluating the Gamow states at the real energy of the SPR (see sec (2.3)). The asymptotic form of the $\psi_E^{c(+)}$ will be required in Appendix D where we construct the S-matrix.

We now mention a number of limiting cases that eqn. (2.4.22) reduces to:

(i) When the physical states $\varphi_{t,A}$ are well separated in energy, only one term in the summation over t will contribute at a particular energy. Further, if we are interested only in the resonance contribution to $\psi_E^{c(+)}$, the background term may be omitted. Then

$$\psi_E^{c(+)} \simeq -e^{i\xi_c}(1+K^{(+)}V_A)\varphi_{t,A}\frac{G_{tc}}{E-E_t+\frac{i}{2}\Gamma_t} \quad . \qquad (2.4.25)$$

(ii) When the (A-1) nucleon system is a doubly closed nucleus, the residual interaction V_A will be negligible. In this case the physical state $\varphi_{t,A}$ will be identical to the shell-model state W_{c_o}. We have further that

$$O_{t\alpha} = \delta_{t\alpha} \qquad (\alpha = c_o) \quad ,$$

$$E_t = E_{c_o} \quad , \qquad \Gamma_t = \Gamma_{c_o} \quad , \qquad (2.4.26)$$

and eqn. (2.4.22) reads

$$\psi_E^{c_o(+)} = \chi_E^{c_o(+)NR} - e^{i\xi_{c_o}}\frac{\left(\frac{1}{2\pi}\Gamma_{c_o}\right)^{1/2}}{E-E_{c_o}+\frac{i}{2}\Gamma_{c_o}}W_{c_o} \quad , \qquad (2.4.27)$$

which is identical to eqn. (2.3.9).

(iii) When the depth of the shell-model potential $v_o(A)$ tends to infinity, we recover the bound state wave function for the A nucleon system. Under this condition, the threshold energies $\varepsilon_c \to \infty$, $(c = 1,\ldots,\Lambda)$, the SPR become bound states ($\Gamma_{c_o} \to o$, $c_o = 1,..,\Lambda_o$) and the violent energy behaviour in the quantity

$$\frac{G_{tc}}{E - E_t + \frac{i}{2}\Gamma_t} \quad , \tag{2.4.28}$$

is restricted to a narrow energy region around E_t. Eqn. (2.4.23) becomes

$$\psi_E^{c(+)} \to \phi_{t,A} = \sum_n O_{tn}\, \Phi_n \quad , \tag{2.4.29}$$

and the coefficients O_{tn} are real.

2.5 Comparison with Other Work

In this section we review, very briefly, work by other authors that is relavant to eqn. (2.4.22).

The resonance behaviour of the continuum function $\chi_E^{c(+)}$ near an isolated SPR has been made explicit in a way similar to ours in the work of Bang and Zimányi [10], Coker and Hoffmann [13] and Baur and Trautmann [19]. The main advantage of the present formulation is that the normalization of the Gamow state (eqn. (2.3.7)) follows naturally from the normalization of $u_{\ell j}$ adopted in eqn. (2.2.14), and from the form of the scattering function near resonance (eqn. (2.3.3)). The

treatments of the SPR in refs. [10,13,19] and others will be dealt with in section 3.4 when discussing the various theories of stripping to resonant states.

Returning to the many channel case (eqn. (2.4.22)), the other versions also use the method of the continuum shell-model theory. However they differ with the present work and with each other in the way in which the SPR of the shell-model Hamiltonian are included with the bound and non-resonant continuum states. We therefore deal only with this aspect of these papers.

(i) Levin [15] does not make explicit the resonant behaviour of the continuum function $\chi_E^{c(+)}$. He does assume that the Hamiltonian H_A has been somehow diagonalized in the space of the continuum functions. The energy dependence of $\psi_E^{c(+)}$ will therefore be explicitly displayed only for those decaying states which do not have a large overlap with the SPR of the shell-model Hamiltonian.

(ii) Huby [6] ensures that no SPR occur in the eigenstates of $H_{o,A}$ by including the potential responsible for the SPR in the perturbation. Nevertheless the effects of the SPR are exactly determined by writing the main part of the perturbing potential in a separable form.

(iii) Mahaux and Saruis [5] use the Wong-Shakin prescription for the SPR [32] . The resonances are projected out of the continuum states χ_E^c and added to the bound states Φ_n . The model Hamiltonian $H_{o,A}$ is modified so as to have as eigenstates the new (non-resonant) continuum states and (extended set of) bound states. The perturb-

ing potential V_A is correspondingly altered.

In contrast to the methods in (ii) and (iii) we do not alter the shell-model Hamiltonian when including explicitly the effects of the SPR. Our treatment is in many ways similar to that of Unger [33] . We do not however review his work because his theory is rather different from ours.

3. Stripping to Unstable States

We compare some of the features of particle unstable states, discussed in the previous chapter, with experiment. The unstable states are formed during single-nucleon transfer reactions. The angular distribution of the outgoing uncaptured nucleon (or nucleons) is analysed using the distorted-wave Born approximation (DWBA). Since these decaying states lie above the nucleon threshold energy, they may also be populated via the appropriate elastic scattering experiment. Certain features emerge when the stripping and elastic scattering experiments are compared. These features are also accounted for.

A survey of some relevant experiments is given in section 3.1. General formulae for the DWBA transition amplitude and cross-section are recalled in section 3.2. A number of simplifications are made to these formulae. In the following section the extraction of spectroscopic information and the connection with total elastic scattering is discussed. We end this section with an application of the formulae to a simple example. In section 3.4 the present approach is compared with certain of the numerous other treatments of stripping to unstable states. Finally, in section 3.5, we present numerical results for a number of typical stripping reactions leading to nucleon unstable states.

3.1 A Brief Survey of Some Experiments

Many experiments have been performed which study stripping to resonant states [20,21,34-45] . The widths of these states range from tens of keV up to about 0.5 MeV and their energies occur up to several MeV above the single particle threshold. These reactions should be describable in terms of a direct reaction model because of the similarity of the angular distributions to those for stripping to bound states. Most of these angular distributions are forward peaked, and the peak cross-sections are of the same order of magnitude as those for stripping to the ground and low-lying states. Thus in most cases it should be possible to assign ℓ-values and spectroscopic factors in analogy with stripping to bound states. We should like to mention in particular the following angular distribution measurements for which calculations will be performed in section 3.5.

(i) $^{16}O(d,p)^{17}O$ populating the 5.08 MeV, $d^3/_2$ SPR in ^{17}O [42,43]. The angular distributions are forward peaked and very similar in shape and magnitude to the angular distributions of the $d^5/_2$ ground state.

(ii) $^{15}N(d,n)^{16}O$ leading to unbound negative parity states of ^{16}O [45]. Spectroscopic factors extracted from a fit to the angular distributions may be compared with theoretical calculations which treat these states as 1p-1h states.

(iii) $^{92}Mo(^3He,d)^{93}Tc$ reaction, forming a number of unstable states in ^{93}Tc [41]. These data have been analysed in a way similar to our method.

A very instructive comparison has been performed between stripping reactions and elastic scattering reactions populating the same unstable states. The energy distribution of the proton in the reaction $^{15}N(d,p)^{16}N$ has been measured by Fuchs et al. [20,21]. The proton spectra display patterns very similar to the energy dependence of the corresponding $n-^{15}N$ total elestic cross-section, including interference phenomena between resonances and continuum. These authors also find that the ratio of intensity in (d,p) to that in (n,n) strongly increases with the angular momentum transfer ℓ.

3.2 The Transition Amplitude and the Cross-section

We study the transition amplitude and the cross-section for the stripping reaction

$$d + (A-1) \rightarrow y + A^* \rightarrow y + x + (A-1) \quad , \qquad (3.2.1)$$

where d is the projectile; (A-1) represents the target consisting of (A-1) nucleons; A^* is the excited nucleus formed by the transfer of a nucleon x from d to (A-1) and y is the other nucleon (or nucleons) making up the projectile.

The transition amplitude for the reaction (3.2.1) may be written in the distorted wave approximation as [46,47] :

$$T(d,y)x \simeq \lim_{\alpha \to 0} \langle \chi^{(-)}_{yA}(\underset{\sim}{k}_y)\, \psi^{(-)}_{E,A} \,|\, e^{-\alpha r} (V_{xy} + Q V_{yA-1} P) \,|\, \varphi_d \varphi_{A-1} \chi^{(+)}_{dA-1}(\underset{\sim}{k}_d) \rangle \quad . \qquad (3.2.2)$$

The transfer of n from the deuteron to the target is described by the interaction V_{xy} , which leaves the target unaffected, plus the interaction $QV_{yA-1}P$, which excites the target nucleus. The projection operators P and Q project respectively onto the ground and excited states of the target nucleus (A-1).

The distorted waves $\chi^{(+)}_{dA-1}$ and $\chi^{(-)}_{yA}$ describe the elastic scattering of d by (A-1) and y by A respectively. The function φ_d represents the internal motion of the projectile and φ_{A-1} the ground state of the target. The target plus stripped nucleon system is described by $\psi^{(-)}_{E,A}$. This wave function may be written as a sum over all spin states J :

$$\psi^{(-)}_{E,A} = \sum_J \psi^{J(-)}_{E,A} \quad , \tag{3.2.3}$$

where $\psi^{J(-)}_{E,A}$ is given by eqn. (2.4.23) of the previous chapter. Eqn. (3.2.3) may therefore be written as a sum of two terms:

$$\psi^{(-)}_{E,A} = \sum_{J,c} \chi^{(-)NR}_{c,E} - \sum_{J,t,c} e^{-i\xi_c} \frac{G^*_{tc}}{E - E_t - \frac{i}{2}\Gamma_t} \left[1 + K^{(-)} V_A\right] \tilde{\varphi}_{t,A} \quad . \tag{3.2.4}$$

The transition amplitude in eqn. (3.2.2) may also be written as a sum of two terms: the first term describes the non-resonant breakup of the projectile while the second term gives the resonant contribution.

The convergence factor $e^{-\alpha r}$ in the transition amplitude ensures that the radial integrals converge. In the case of stripping to a bound state, this factor satisfies the condition imposed on a convergence factor and may thus be omitted.

Because of the normalization adopted in eqn. (2.2.14), the cross-section for the reaction (3.2.1) reads

$$\frac{d^2\sigma}{d\hat{k}_y dE} = \frac{m_d\, m_y}{(2\pi\hbar^2)^2}\,\frac{k_y}{k_d}\,\frac{1}{(2J_{A-1}+1)(2s_d+1)}\sum\left|T(d,y)x\right|^2 , \qquad (3.2.5)$$

where m_d and m_y are the reduced masses of d and y, $T(d,y)x$ is the matrix element of eqn. (3.2.2) and the summation is over the magnetic quantum numbers.

There are two basic conditions that must be fulfilled if the above formulae are to apply:

(i) In the non-resonant breakup part of the transition-amplitude, we have written the wave function for x and y in product form. This will be valid provided there are no final state interactions between x and y. A sufficient condition for this is that the relative x-y energy be large. Since we are primarily interested in low lying resonant states, this condition should be well met.

(ii) The second condition affects the resonant part of the transition amplitude, but is not unrelated to the first condition. The unstable nucleus, A, must be long enough lived that the nucleon(s) y does not affect its subsequent decay. As mentioned by Levin [15] this puts an upper limit on the width of the unstable states that can be studied within this framework.

The form of eqn. (3.2.4) suggests that violent interference effects may arise in the cross-section, eqn. (3.2.5). Such

interference effects may occur in two different ways:

(i) The background term in eqn. (3.2.4) becomes comparable in magnitude with a resonance term, within the range of the nuclear forces. This is likely to occur in the case of a SPR with low orbital angular momentum, lying fairly far above the particle threshold, i.e. when the single particle width, Γ_{c_o} , is large.

(ii) There are overlapping resonances in the energy region of interest. Conditions will be especially favourable when one of the resonances is a SPR (large width) while the others are much narrower compound nucleus resonances.

Baur and Trautmann [19] and Lipperheide and Möhring [23,24] have invoked interference between breakup and resonance in the same channel, while Barz et al. [25] used interference between different resonances to account for the observed interference patterns [20,21] .

The general cross-section formula given by eqns. (3.2.2)-(3.2.5) is rather complicated. However great simplifications occur if certain approximations are made. These approximations are well met in most experimental situations of interest. They are

(a) Only one term in the summation over J in eqn. (3.2.3) is important at a particular energy.

(b) The resonances with a particular J are all well separated in energy.

(c) Background terms may be neglected.

(d) Only the zero order term in V_A is retained in eqn. (3.2.4).

The above four approximations lead to the following form for the wave function describing the state A

$$\psi^{(-)}_{E,A} = -\sum_{c(open)} e^{-i\xi_c} \frac{G^*_{tc}}{E - E_t - \frac{i}{2}\Gamma_t} \tilde{\varphi}_{t,A} . \qquad (3.2.6)$$

(e) In the functions W_{co} and Φ_n occurring in $\varphi_{t,A}$ and G_{tc} of eqn. (3.2.6), the wave function of the transferred nucleon x, is not antisymmetrized with the wave functions of the target nucleons. These quantities are then given by (see eqns. (2.2.17) and (2.3.11))

$$\Phi_n = \omega^{b}_{\ell j}(r, k_n^{\ell j})\, \varphi_c \quad , \qquad (3.2.7)$$

$$W_{co} = \omega^{res}_{\ell j}(r, k_n^{\ell j})\, \varphi_c \quad . \qquad (3.2.8)$$

(f) The target nucleus is assumed to remain in its ground state during the transfer process. The transfer is then caused only by the interaction V_{xy} .

(g) The function $\varphi_{t,A}$ is approximated by (see eqns. (2.4.21) and (2.4.10))

$$\varphi_{t,A} \simeq O_{t\alpha}\, \theta_\alpha = O_{\ell j}\, \omega^{\{b}_{res\}}_{\ell j}\, \varphi_{A-1} \quad . \qquad (3.2.9)$$

The notation $w^{\{b}_{\ell j}$ denotes that the radial wave function of the nucleon x is either a bound state or a Gamow state wave function. The function φ_{A-1} ($\equiv \varphi_c$) contains the ground state wave function of the target.

(h) Finally, V_{xy} is approximated by a zero-range force and spin-orbit coupling is neglected.

We may then use the result of Levin [15] to evaluate the cross-section, eqn. (3.2.5). In the notation of Satchler [48] we have

$$\frac{d^2\sigma}{d\hat{k}_y dE} = \frac{\sum_{c(open)} |G_{tc}|^2}{(E-E_t)^2 + \frac{1}{4}\Gamma_t^2} \; |O_{lj}|^2 \left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} \quad . \tag{3.2.10}$$

The single particle cross-section $\left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.}$ is defined by

$$\left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} = \frac{m_d m_y}{(2\pi\hbar^2)^2} \frac{k_y}{k_d} \left(\frac{2J_A+1}{2J_{A-1}+1}\right) \frac{D_o^2}{(2s_x+1)} \sum_{\lambda} |\beta_{l\lambda}(\underset{\sim}{k}_y, \underset{\sim}{k}_d)|^2 \quad , \tag{3.2.11}$$

where

$$\beta_{l\lambda}(\underset{\sim}{k}_y, \underset{\sim}{k}_d) = \frac{i^{-l}}{\sqrt{2l+1}} \lim_{\alpha\to 0} \int d^3r \; e^{-\alpha r} \; \chi^{(-)*}_{yA}(\underset{\sim}{k}_y, \underset{\sim}{r}') \; \frac{\omega^{\{b_{res}\}}_{lj}(r, k^{lj}_{res})}{r} \; Y^*_{l\lambda}(\hat{r}) \; \chi^{(+)}_{dA-1}(\underset{\sim}{k}_d, \underset{\sim}{r}) \quad , \tag{3.2.12}$$

$$|\underset{\sim}{r}'| = \frac{M_{A-1}}{M_A} |\underset{\sim}{r}| \quad , \tag{3.2.13}$$

and the integral over the projectile coordinates is denoted by D_o.

The "convergence problem" arising when w^{res}_{lj} occurs in eqn. (3.2.12) is discussed by several authors. Of relevance

to the present formulation are the works of Bang and Zimányi [10] and Coker and Hoffmann [13] (see also appendix A).

In the following section we derive an expression for the angular distribution, $d\sigma/d\hat{k}_y$, and discuss the problem of extracting structure information. We also relate eqn. (3.2.10) to the total elastic cross-section.

3.3 Structure Information from Stripping to Unstable States and Relation to Elastic Scattering

3.3.a Angular Distributions and Extraction of Spectroscopic Factor

The angular distribution of y may be obtained from eqn. (3.2.10) by integrating over the energy of the resonance state in the system A. This integral may be performed explicitly if the quantities Γ_t and G_{tc} vary slowly with the energy. Using the result

$$\int_{-\infty}^{\infty} dE \, \frac{1}{(E-E_t)^2 + \frac{1}{4}\Gamma_t^2} = \frac{2\pi}{\Gamma_t} \quad , \tag{3.3.1}$$

the angular distribution is

$$\frac{d\sigma}{d\hat{k}_y} = \frac{2\pi \sum_{c(\text{open})} |G_{tc}|^2}{\Gamma_t} \, |O_{lj}|^2 \left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} \quad . \tag{3.3.2}$$

We observe that

$$\frac{d\sigma}{d\hat{k}_y} = |O_{kj}|^2 \left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} \quad , \tag{3.3.3}$$

as is the case in the theory of stripping to bound states [18], since the following sum rule is shown to be valid for isolated resonances in Appendix D.

$$\Gamma_t = 2\pi \sum_{c(open)} |G_{tc}|^2 \quad , \tag{3.3.4}$$

The quantity G_{tc} is given by eqn. (2.4.23) viz.

$$G_{tc} = O_{tc_o}\left(\frac{1}{2\pi}\Gamma_{c_o}\right)^{1/2}\delta_{c_oc} - \sum_{\alpha=1}^{M+\Lambda_o} O_{t\alpha}\langle\theta_\alpha|V_A|\chi_{E_t}^{c,NR}\rangle \quad . \tag{3.3.5}$$

Eqn. (3.3.4) is a generalization of the sum rule for isolated resonances derived in ref. [3]. The quantity G_{tc} contains not only the decay amplitudes of the states Φ_n but also those of the SPR. These SPR have a "natural" decay width caused by the unperturbed Hamiltonian and a width due to the perturbation V_A. The natural width can only occur in the channel c_o containing the SPR, while the width due to V_A may be detected in all channels c which can couple to the channel c_o.

In general, the natural width is larger than the width due to V_A. However this does not imply that the first term in eqn. (3.3.5) is the dominant one. The dominant term on the RHS of eqn. (3.3.5) is determined by the product of the admixture coefficient and the decay amplitude. If the admixture coefficient O_{tc_o} and the width Γ_{c_o} are of such a magnitude that the first term in eqn. (3.3.5) dominates the total decay amplitude

G_{tc} , then eqn. (3.3.4) yields

$$|O_{tc_o}|^2 \simeq \Gamma_t / \Gamma_{c_o} \quad . \tag{3.3.6}$$

It may also happen that there are no SPR in any of the channels c into which $\phi_{t,A}$ is allowed to decay, or that the width and admixture of a state Φ_n is such that eqns. (3.3.5) and (3.3.4) may be approximated to give

$$|O_{tn}|^2 \simeq \Gamma_t / \gamma_n \quad , \tag{3.3.7}$$

where

$$\gamma_n = 2\pi \sum_{c(open)} |\langle \bar{\Phi}_n | V_A | \chi_{E_t}^{c,NR} \rangle|^2 \quad . \tag{3.3.8}$$

Eqns. (3.3.6) and (.3.3.7) give the spectroscopic factor as the ratio of the actual and the single particle widths. A relation between the spectroscopic factor and the widths has also been discussed by Huby [6] and by Bunakov et al. [49,25].

It has been suggested [49,25] that the spectroscopic factor for the unstable state may be measured in two ways. It is given as the ratio of the experimental and single particle cross-sections (eqn. (3.2.3). In obtaining the single particle cross-section the physical state of the unstable nucleus is replaced by the shell model state with the largest admixture coefficient. Alternatively eqn. (3.3.6) or eqn. (3.3.7) may be used, provided the width Γ_t is measured. These two equations are obtained by calculating the products of the admixture

coefficients and the corresponding decay amplitudes and seeing which product gives the largest contribution to the RHS of eqn. (3.3.5). The above results suggest that the ratio of experimental-to-single-particle-cross-section procedure gives spectroscopic information about the dominant mode of formation, while the ratio-of-widths procedure measures the dominant mode of decay of the unstable nucleus. Only if these two modes are the same, have we two ways at our disposal for determining the spectroscopic factor.

Eqn. (3.3.3) for the angular distribution is a generalization of the expression for stripping to bound states. The bound states radial wave function is normalized according to eqn. (2.2.10), while the Gamow state radial wave function normalization is given by eqn. (2.3.7).

When the target nucleus is allowed to become excited during the transfer process, the largest admixture to the physical state $\phi_{t,A}$, may come from a shell-model configuration Φ_n whose energy is above the single nucleon threshold even though all the nucleons are in bound orbitals. The cross-section for this "rearrangement stripping" is expected to be small [46]. Further, since these states are fairly high in the continuum, many channels will be available for decay, so that the width due to these states should be large. Then the assumption that the nucleon y does not interfere with the decay of the nucleus A may no longer be fulfilled.

3.3.b Similarity Between Deuteron Stripping and Total Elastic Cross-section

The cross-section given by eqn. (3.2.10) may be written as a product of the total elastic scattering cross-section of x on (A-1) and a "stripping enhancement factor".

Using the sum rule, eqn. (3.3.4), eqn. (3.2.10) reads

$$\frac{d^2\sigma}{d\hat{k}_y\, dE} = \frac{\frac{1}{2\pi}\Gamma_t}{(E-E_t)^2 + \frac{1}{4}\Gamma_t^2} \, |O_{lj}|^2 \left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} \quad . \tag{3.3.9}$$

Near resonance, the total elastic cross-section of the nucleon x by the nucleus (A-1) is

$$\sigma_{el} = \frac{\pi}{k_x^2} \frac{(2J_A+1)}{(2J_{A-1}+1)(2s_x+1)} \frac{|O_{lj}|^2 \Gamma_t^2}{(E-E_t)^2 + \frac{1}{4}\Gamma_t^2} \quad . \tag{3.3.10}$$

Hence eqn. (3.3.9) may be written as

$$\frac{d^2\sigma}{d\hat{k}_y\, dE} = \sigma_{el} \; F \quad , \tag{3.3.11}$$

where the "stripping enhancement factor" is defined by

$$F = \frac{(2s_x+1)(2J_{A-1}+1)}{(2J_A+1)} \frac{k_x^2}{2\pi^2 \Gamma_t} \left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} \quad . \tag{3.3.12}$$

It will be noticed that the stripping enhancement factor is inversely proportional to the width Γ_t . Because of barrier

penetration effects, these widths increase with increasing angular momentum ℓ of the transferred nucleon x. Thus the magnitude of the stripping cross-section increases relative to the total elastic cross-section with increasing ℓ.

Eqns. (3.3.10) and (3.3.9) show that the total (x,x) cross-section and the corresponding portion of the y spectra from (d,y)x have a very similar resonance structure. Eqns. (3.3.12) and (3.3.11) demonstrate that the ratio of intensity in (d,y)x to that in (x,x) increases with the angular momentum transfer. Both these effects have been observed in a comparison of the reactions $^{15}N(n,n)^{15}N$ and $^{15}N(d,p)^{16}N^*$ [20,21]. A similar interpretation of these effects has been given in refs. [19] and [25] .

3.3.c Physical Interpretation of Results

We now give an interpretation of the formulae derived above. We consider the reaction

$$^{15}N(d,n)^{16}O^* \quad , \tag{3.3.13}$$

leading to unstable states of ^{16}O. It is assumed that the target ground state can be described as a hole state in the $1p^1/_2$ shell. We also include in the discussion an excited state of ^{15}N which can be described as a hole state in the $1p^3/_2$ shell. The transferred proton can drop into the $1d^5/_2$, $2s^1/_2$ and $1d^3/_2$ single particle levels. The energies of the hole and the particle states relative to the ground state of

^{16}O may be obtained from the experimentally observed ground and excited states of ^{15}N and ^{17}F. The 1p-1h shell-model states of ^{16}O are shown in fig. 3.3.1. Only the proton configurations which can couple to $J^{\pi} = 1^{-}$ are indicated. In a structure calculation the residual interaction between these bound, resonant and continuum states is switched on and

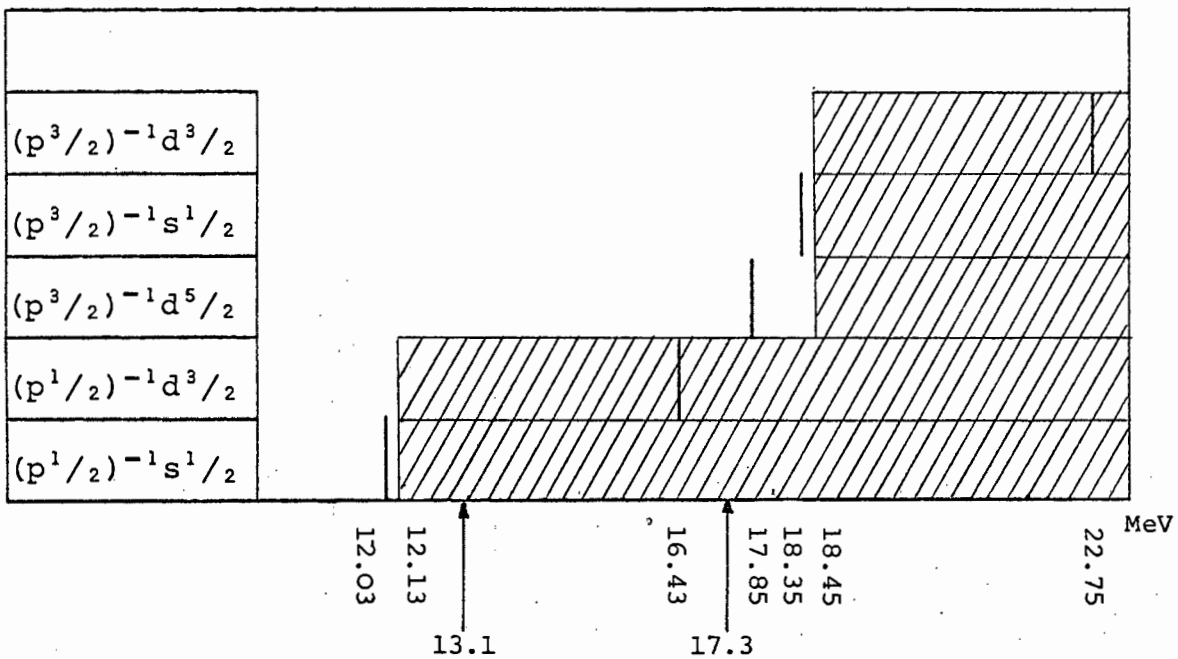

Fig.3.3.1. The 1p-1h shell-model proton states of ^{16}O which can couple to $J^{\pi} = 1^{-}$. The energy plotted is the excitation energy in ^{16}O. The bound and resonant states are indicated by vertical lines, while the continuum states are indicated by the shaded area. Arrows show some of the physical states of ^{16}O.

a matrix with elements given by eqn. (2.4.19) is diagonalized. The resulting energies should correspond to some of the

observed excited states of ^{16}O. One such experimental state φ_t of ^{16}O occurs at 13.1 MeV. It is interpreted [31] as existing predominantly in the

$$\Phi_1 = (p\,1/2)^{-1}\; s\,1/2 \quad , \tag{3.3.14}$$

configuration. Therefore in the single particle cross-section $(d\sigma/d\hat{k}_y)_{s.p.}$ of eqn. (3.2.11) the bound state wave function $w^b_{o\frac{1}{2}}$ appears, and is evaluated at 0.1 MeV binding energy. The residual interaction causes the energy shift to 13.1 MeV and the decay. The state φ_t can decay into two channels with configurations given by

$$\chi^1_E = (p\,1/2)^{-1}\; s\,1/2 \quad , \qquad \chi^2_E = (p\,1/2)^{-1}\; d\,3/2 \quad . \tag{3.3.15}$$

There is a SPR of width 0.9 MeV at 16.43 MeV in the channel χ^2_E . The various quantities which enter in the expression for G_{tc} (eqn. (2.4.23)) have been calculated by Beres and Macdonald [3.27]

$$2\pi\,|\langle \varphi_t\,|\,V_A\,|\,\chi^1_{13.1}\rangle|^2 = 0.122 \text{ MeV} \quad ,$$

$$2\pi\,|\langle \varphi_t\,|\,V_A\,|\,\chi^2_{13.1}\rangle|^2 = 0.0 \text{ MeV} \quad ,$$

$$(O_{t2})^2\,\Gamma_2 = (0.53\times10^{-5})\times 0.9 = 0.477\times10^{-5} \text{ MeV} \quad .$$

Hence the decay is essentially into the channel χ^1_E .

The authors also give the admixture of Φ_1 in φ_t as

$$S_{t,1} = 0.996 \quad . \tag{3.3.16}$$

Hence the decay of Φ_1 into the channel χ_E^1 is

$$2\pi |\langle \Phi_1 | V_A | \chi'_{13.1} \rangle|^2 = 0.123 \quad MeV \quad .$$

Experimentally, the 13.1 MeV resonance is observed to have a width [50]

$$\Gamma_t = 0.11 \; MeV \quad .$$

Hence relation (3.3.2) for the spectroscopic factor gives

$$S_{t,1} = 0.90 \quad ,$$

in fair agreement with eqn. (3.3.16).

In the above we have used the "effective-binding-energy prescription" (EBEP) [51] in obtaining spectroscopic information from stripping reactions. The eigenenergy of $w_{\ell j}^{\{b_{res}^{s}\}}$ is determined from the binding energy of the shell-model single-particle level. More frequently employed is the "separation-energy prescription" (SEP) [52] . The eigenenergy of $w_{\ell j}^{\{b_{res}^{s}\}}$ is determined from the Q-value of the reaction. In bound state stripping analyses the spectroscopic information tends to be more reliable from the former method. These two methods are compared for unbound stripping in section 3.5. One consequence of using the SEP is that all decaying states can be treated as SPR. A survey of the literature reveals that this is indeed the most common method of description.

3.4 Comparison with Other Work

Many different theoretical methods have been developed for describing stripping to unstable states [10,13-16,19, 22-25,49,53-61] . Below we mention only those which are of relevance to the present work [10,13,19,16,54,60,61] . In nearly all these formulations, the simplest possible form of the wave function at resonance has been taken. In our formulation this is given by eqn. (2.3.9).

The Gamow function is used for the resonant state by Coker and Hoffmann [13] and by Bang and Zimányi [10] . The former authors write (in effect) for the resonant state

$$u^{(+)}_{\ell j} \simeq - e^{i\xi^{\ell j}_n} \frac{\left(\frac{1}{2\pi}\Gamma^{\ell j}_n\right)^{1/2}}{E - E^{\ell j}_n + \frac{i}{2}\Gamma^{\ell j}_n} \omega^{res}_{\ell j}(r, k^{\ell j}_n)_{C-H} , \qquad (3.4.3)$$

and replace in the bound state stripping cross-section, the bound state form factor by the quantity $w^{res}_{\ell jC-H}$. The spectroscopic factor is obtained by renormalizing this single particle angular distribution to the experimentally observed one. Our more general formulation confirms the conclusions of this more intuitive approach.

Bang and Zimányi [10] use the following form for the resonant state

$$u^{(+)}_{\ell j} \simeq - e^{2i\xi^{\ell j}_n} \frac{\frac{1}{2}\Gamma^{\ell j}_n}{E - E^{\ell j}_n + \frac{i}{2}\Gamma^{\ell j}_n} \omega^{res}_{\ell j}(r, k^{\ell j}_n)_{B-Z} . \qquad (3.4.2)$$

They adopt the Berggren normalization for the Gamow state [9] which is shown in Appendix A to correspond to our normalization up to zero order in $(\mathrm{Im}k_n^{\ell j}/\mathrm{Re}k_n^{\ell j})$. Eqn. (3.4.2) differs from eqns. (3.4.1) and (2.3.8) by the factor $\left(\frac{\pi}{2}\,\Gamma_n^{\ell j}\right)^{\frac{1}{2}}$. Their angular distribution formula differs from ours and from that of ref. [13] by the corresponding factor.

Baur and Trautmann [19] do not use wave functions corresponding to a complex energy. They take the wave function at resonance to be of the form

$$u_{\ell j}^{(+)} = -\frac{1}{\kappa}\left(\frac{m}{2\pi\, k_n^{\ell j}}\right)^{\frac{1}{2}} \frac{\Gamma_n^{\ell j}}{E-E_n^{\ell j}+\frac{i}{2}\Gamma_n^{\ell j}}\, e^{2i\xi_n^{\ell j}}\, O_{\ell j}(r, k_n^{\ell j}) \quad . \tag{3.4.3}$$

The corresponding cross-section becomes, after evaluating the energy integral,

$$\frac{d\sigma}{d\hat{k}_y} = \left(\frac{m\,\Gamma_n^{\ell j}}{\kappa^2\, k_n^{\ell j}}\right)\left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} \quad , \tag{3.4.4}$$

where the single particle cross-section is given by eqn. (3.2.11) with

$$\beta_{\ell\lambda}(\underline{k}_y, \underline{k}_d) = \frac{i^{-\ell}}{\sqrt{2\ell+1}} \lim_{\alpha\to 0} \int d^3r\, e^{-\alpha r}$$

$$\chi_{yA}^{(-)*}(\underline{k}_y, \underline{r}')\left[\frac{O_{\ell j}(r, k_n^{\ell j})}{r} Y_{\ell\lambda}^{*}(\hat{r})\right] \chi_{dA-1}^{(+)}(\underline{k}_d, \underline{r}) \quad . \tag{3.4.5}$$

They then allow $\Gamma_n^{\ell j}$ to become a free parameter, and normalize the cross-section of eqn. (3.4.4) to the experimental value

$$\left(\frac{d\sigma}{d\hat{k}_y}\right)_{exp} = \left(\frac{m\Gamma_t}{\hbar^2 k_n^{\ell j}}\right)\left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.} \quad . \tag{3.4.6}$$

The quantity $(m\Gamma_t/\hbar^2 k_n^{\ell j})$ corresponds to the quantity $N_{\ell j}^2 S_{\ell j}$ occurring in stripping to bound states formulae, $N_{\ell j}$ being the normalization of the bound state wave function and $S_{\ell j}$ the spectroscopic factor. In the case of stripping to unbound states (see eqn. (2.3.7))

$$N_{\ell j}^2 = \frac{m\Gamma_n^{\ell j}}{\hbar^2 k_n^{\ell j}} \quad , \tag{3.4.7}$$

and therefore

$$S_{\ell j} = \Gamma_t / \Gamma_n^{\ell j} \quad . \tag{3.4.8}$$

This is the same result as in eqn. (3.3.6). However in this formulation it is not obvious that the quantity Γ_t measures the actual width of the resonant state.

The approach of Vincent and Fortune [16] is in many ways similar to that of Baur and Trautmann's. However, asymptotically, the resonating part of the radial wave function, $u_{\ell j}^{(+)}$, is written as a sum of incoming and outgoing waves. We believe that this is incorrect. In section 2.3, we have shown that the violent energy behaviour of $u_{\ell j}^{(+)}$ occurs, asymptotically, only in the outgoing wave. The <u>complete</u> radial wave function $u_{\ell j}^{(+)}$, does of course behave, for large r, as a superposition of incoming and outgoing waves.

Other methods, which make use of the properties of the wave function near resonance are given in refs. [54],[60] and

[61]. The radial wave function function of a particular ℓ is made to resonate at the observed energy of the resonance. This wave function is then modified by cutting it off smoothly in the region of the first node outside the nucleus. It is then normalized to unity and substituted as $w^{res}_{\ell j}$ into the DWBA transition matrix element. This method differs from ours in the normalization procedure adopted for $w^{res}_{\ell j}$.

Levin [15] deals mainly with stripping to doorway states of the shell-model Hamiltonian. In such states all nucleons are in bound orbitals although the energy of the system is above the single nucleon threshold. However such doorway states correspond to fairly complicated many-particle resonances of the compound nucleus. Indeed in the example of subsection 3.3.c, the doorway states $(p^3/_2)^{-1}\ d^5/_2$ and $(p^3/_2)^{-1}\ s^1/_2$, can only be populated by exciting the target during the transfer process. As we have argued at the end of subsection 3.3.9, the cross-section for populating such states should be small.

3.5 Calculations

3.5.a Angular distributions

In this subsection we compare the angular distribution predicted by eqn. (3.3.3) with the results of experiments for a variety of projectile-target systems. In particular we investigate the following:

(i) The validity of an approximation to the Gamow function based on physical grounds.

(ii) The extraction of spectroscopic information using the "effective-binding-energy" and the "separation-energy" prescriptions (see end of section 3.3).

Calculations using the Gamow state, $w_{\ell j}^{res}(r,k_n^{\ell j})$, evaluated at complex $k_n^{\ell j}$ and with purely outgoing asymptotics, have been performed by Coker and Hoffmann [13] . We compare our angular distribution resulting from an approximate evaluation of $w_{\ell j}^{res}$ with their results. The depth of the central potential is adjusted until the potential phase shift, $\delta_{\ell j}$, equals $\pi/2$ at the required energy $E_n^{\ell j}$. The width, $\Gamma_n^{\ell j}$, is taken from a phase shift analysis to be the distance between the energies such that

$$\sin^2\left[\delta_{\ell j}(E)\right] = \frac{1}{2} \qquad (3.5.1)$$

The radial wave function $w_{\ell j}^{res}$ is evaluated at the (real) energy $E_n^{\ell j}$. Provided the resonance width is small, the form of $w_{\ell j}^{res}$ whether evaluated at $E_n^{\ell j}$ or at $E_n^{\ell j} - \frac{i}{2}\Gamma_n^{\ell j}$ is the same inside the potential well [10] . Also, with this approximation the function $w_{\ell j}^{res}$ is real. It has been demonstrated by Mahaux and Weidenmüller [3] that $w_{\ell j}^{res}$, corresponding to a complex energy, is almost real within the range of the nuclear potential provided the resonance is narrow.

For a narrow resonance, the resonating part of the scattering radial wave function has a large amplitude inside the nucleus, this amplitude being greatly reduced outside the nucleus.

We use this fact to set the wave function equal to zero after the first node outside the nuclear radius. A similar procedure has been used in refs. [54,60,61] . However we do

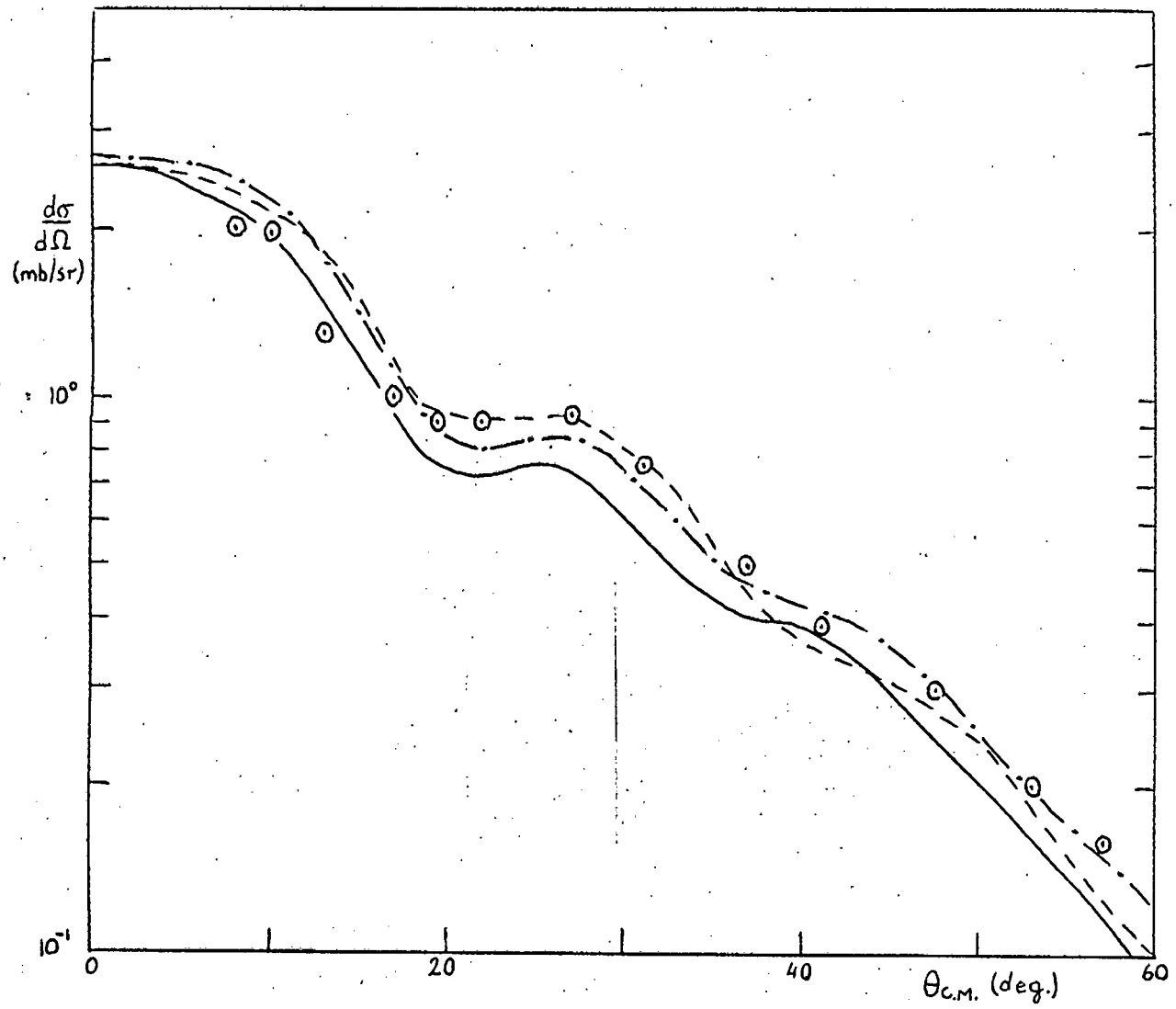

Fig. 3.5.1. Data (points) for the ^{92}Mo(^{3}He,d)^{93}Tc reaction leading to the unbound state at 8.41 MeV [41]. together with DWBA calculations using various descriptions for the resonating state. Dashed curve is from ref. [13] . Our approximation to the Gamow state is shown by the solid line, while the result of using a slightly bound state is indicated by the dashed-dot curve.

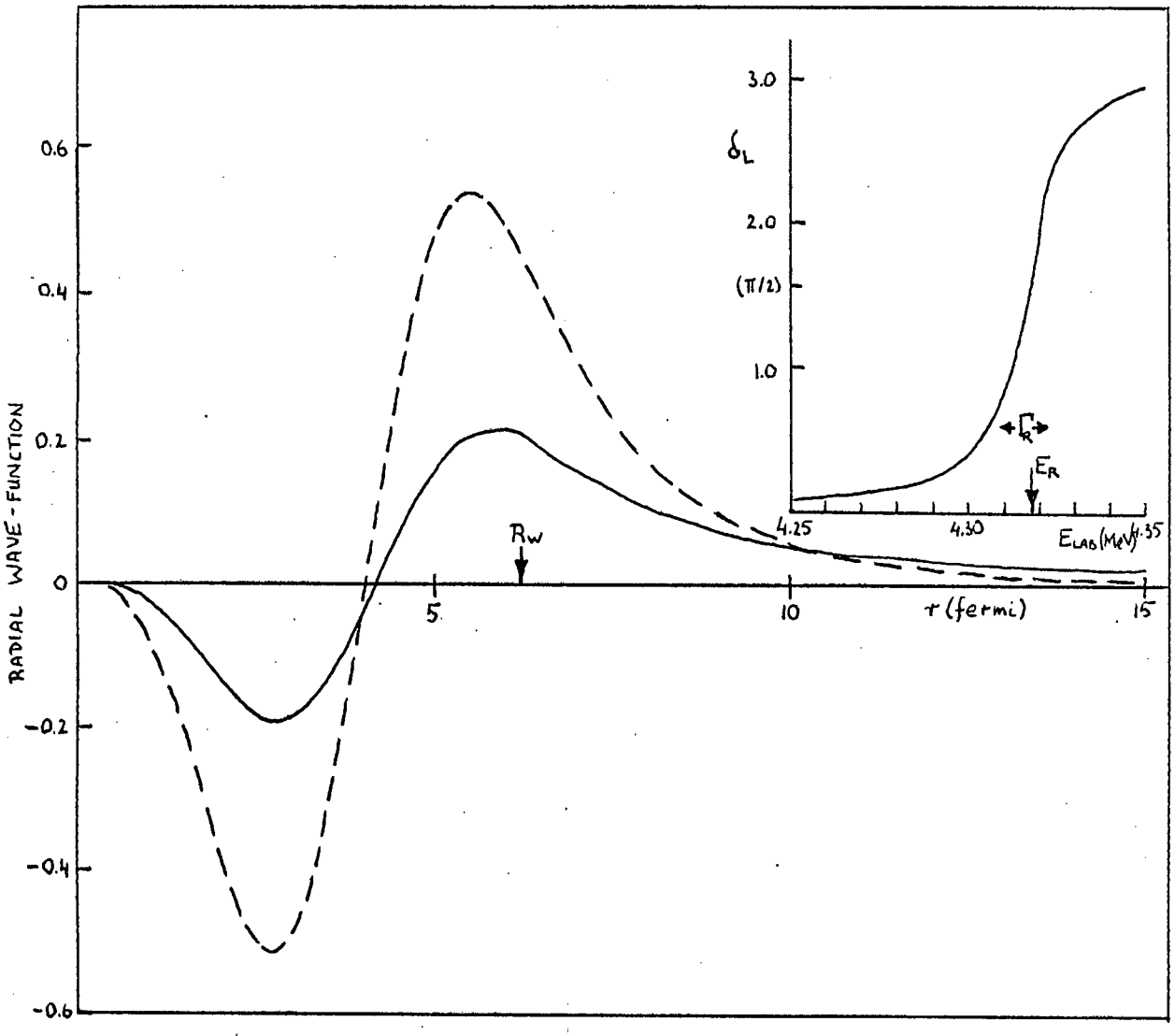

Fig. 3.5.2. The radial dependence of the bound (dashed) and resonant (solid) single-particle wave functions for the $2d^5/_2$ state of ^{92}Mo + p at 4.317 MeV (lab). The quantity R_W indicates the radius of the absorptive potential. Also shown is the phase-shift near the SPR.

TABLE 3.5.2 Potential parameters used in the analysis of $^{16}O(d,p)^{17}O$

Channel	V_o	W_o	r_{ov}	r_{ow}	a_{ov}	a_{ow}	r_{oc}
$^{16}O + d$	85.29	12.75^{v}	1.25	0.958	0.606	1.578	1.25
$^{17}O + p$	59.99	5.729^{v}	1.25	1.517	0.501	0.494	1.25
$^{16}O + n$	$40.9^{\dagger}$	-	1.325	-	0.5	-	-

†Gives a resonance energy of 0.99 MeV (lab) and s.p. width of 0.115 MeV (c.m.)

v volume absorption

resulting from the bound and the Gamow state descriptions of the SPR are normalized to have a spectroscopic factor of unity. The difference in magnitude between these latter two angular distributions is again attributed to the difference in magnitude between the respective radial wave functions describing the SPR (see fig. 3.5.4). The radial integrals are sensitive to the form of the wave functions, $w^{b}_{\ell j}$ and $w^{res}_{\ell j}$, for $r \gtrsim 3.9$ fermi. The phase shift analysis gives a single-particle width of 0.115 MeV compared with 0.09 MeV obtained from neutron scattering experiments.

In order to simplify their calculation, Vincent and Fortune have factored the energy dependence out of the form factor. We have indicated in section 3.4 that their method of doing this seems to be conceptually incorrect. The fact

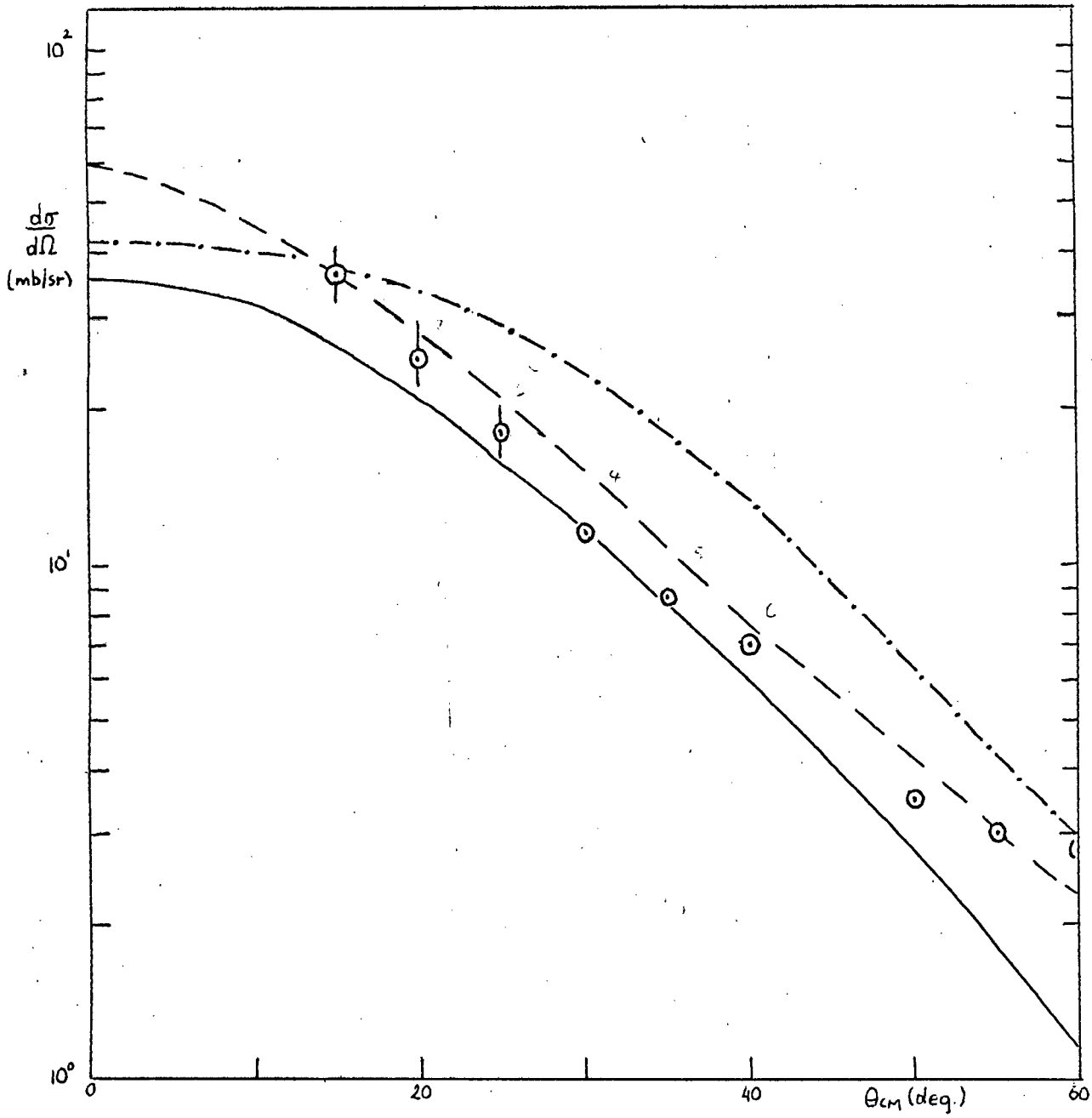

Fig. 3.5.3. Data (points) for the $^{16}O(d,p)^{17}O$ reaction leading to the unbound state at 0.99 MeV [42,43], together with DWBA calculations using various descriptions for the resonating state. Dashed curve is from ref. [16]. Our approximation to the Gamow state is shown by the solid line, while the result of using a slightly bound state is indicated by the dashed-dot curve.

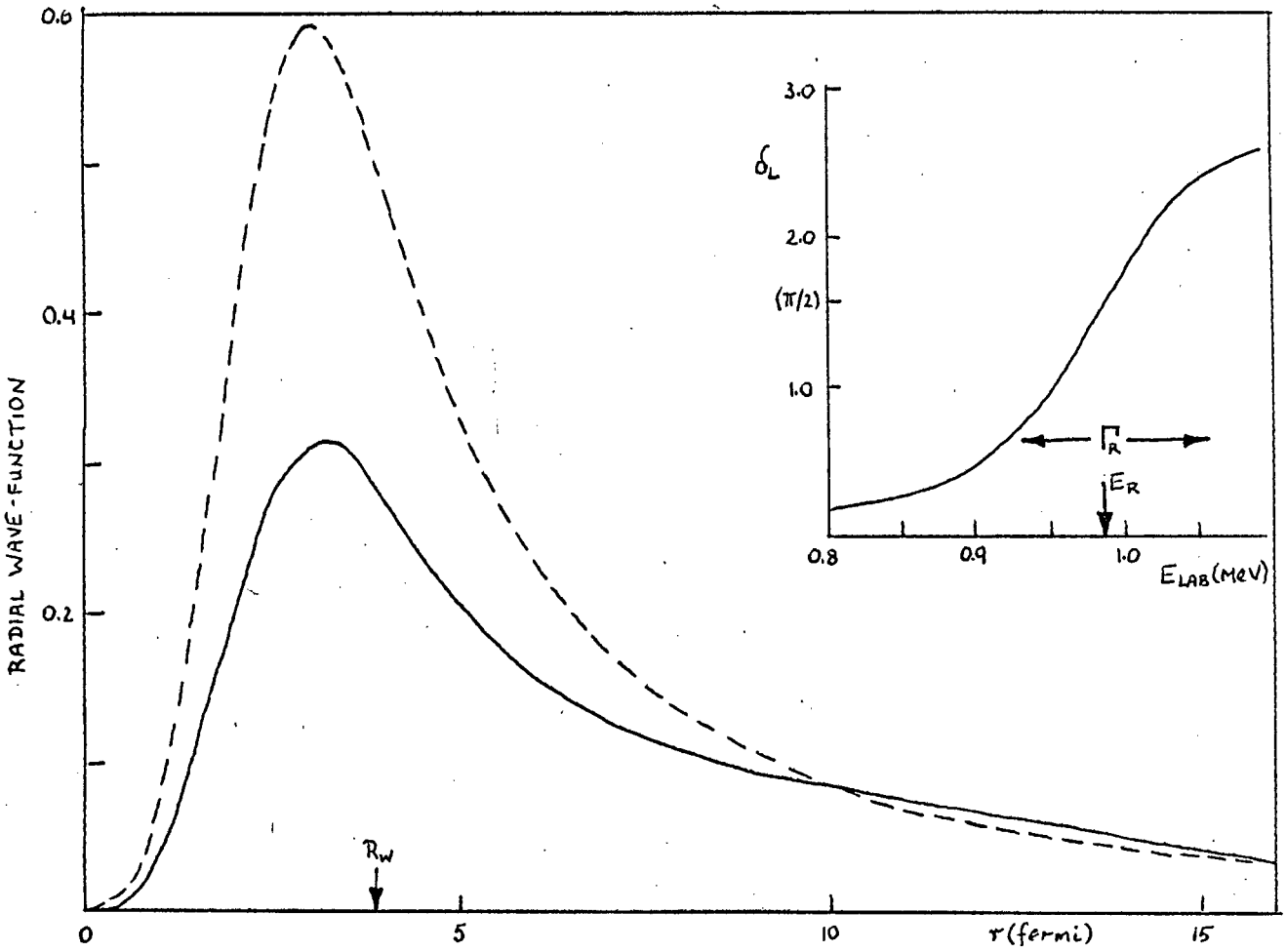

Fig. 3.5.4. The bound (dashed) and resonant (solid) single-particle wave functions for the $d^3/_2$ state of ^{16}O + n at 0.99 MeV (lab). The absorptive potential radius is denoted by R_W. Also shown is the behaviour of the phase shift near the SPR.

that they find it necessary to multiply their results by the factor 0.78 may be due to this.

The results of the foregoing calculations indicate that the main features of the Gamow state are contained in the simplified form we have adopted for it.

Bohne et al. [45] have investigated the reaction $^{15}N(d,n)^{16}O$

leading to unstable states of ^{16}O. Their analysis of the negative parity states gave spectroscopic factors that were much lower than the ones obtained from shell-model calculations. A possible source of discrepancy may be due to the way in which they obtain the form factors for the resonating single particle states.

We analyse the angular distributions obtained from populating the 12.967 MeV, 13.129 MeV and 13.358 MeV states of ^{16}O [45]in the following two ways:

(i) The form factors are obtained by evaluating the radial wave functions for the transferred proton using the binding energies of the shell model single particle levels (effective binding energy prescription, EBEP). The dominant shell-model configurations for all these states is thought to be $(p^1/_2)^{-1}d^5/_2$ with a binding energy of 0.6 MeV.

(ii) The form factors were obtained by evaluating the radial wave functions for the transferred proton at the observed energies of the resonances (separation energy prescription, SEP). The Gamow states are evaluated using the approximations mentioned on pages 49-50.

The form factors resulting from these two methods of calculation are compared in fig. 3.5.8 for the 15.129 MeV level. Angular distributions for stripping to the three unbound states are shown in figs. 3.5.5 - 3.5.7. The optical model parameters [45] are listed in table 3.5.3, while the potential parameters for the ^{15}N + p system are given in table 3.5.4 together with the energy position and width of the $d^5/_2$ level.

TABLE 3.5.3. Optical potential parameters used in the analysis of $^{15}N(d,n)^{16}O$ [45]

Channel	V_o	W_o	r_{ov}	r_{ow}	a_{ov}	a_{ow}	r_{oc}
$^{15}N + d$	72.0	8.0^{v}	1.4	1.35	0.7	0.8	1.3
$^{16}O + n$	$49.3-0.33E_n$	23.0^{s}	1.25	1.25	0.65	0.7	1.3

v volume absorption , s surface absorption

TABLE 3.5.4. Potential parameters for the system $^{15}N + p$

Level (MeV)	EBEP or SEP	$E_{d^5/2}$ (MeV)	$\Gamma_{d^5/2}$ (MeV)	V_o	r_{ov}	a_{ov}	r_{oc}
12.967	EBEP	-0.60	-	58.89	1.25	0.65	1.3
	SEP	0.892	0.0110	55.75			
13.129	EBEP	-0.60	-	58.89	1.25	0.65	1.3
	SEP	1.069	0.0094	55.40			
13.258	EBEP	-0.60	-	58.89	1.25	0.65	1.3
	SEP	1.205	0.0128	54.90			

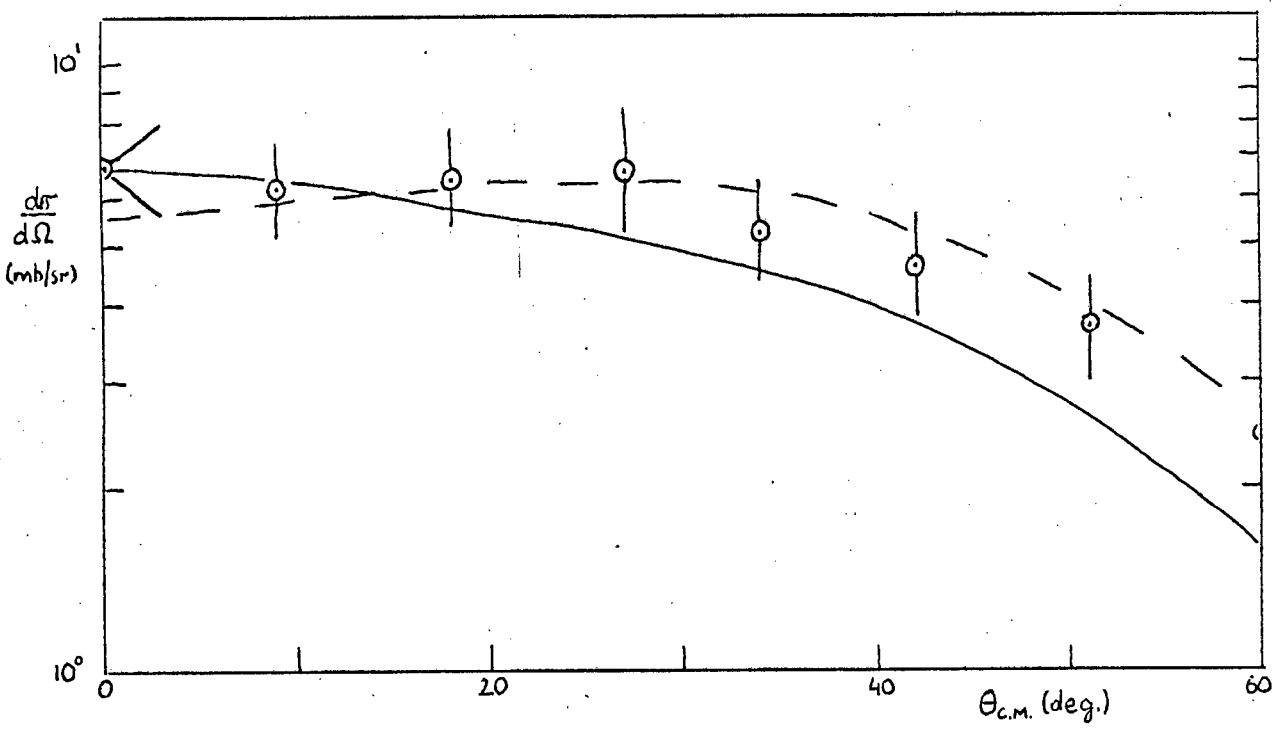

Fig. 3.5.5. Data (points) for the $^{15}N(d,n)^{16}O$ reaction leading to the $J^{\pi}_{x} = 2^{-}$, E_{x} = 12.967 MeV state in ^{16}O, together with DWBA calculations using the EBEP (dashed line) and the SEP (solid line) for the resonating state.

For all three levels in ^{16}O, the spectroscopic information deduced from the SEP and the EBEP is the same (unity) and agrees with the predictions of shell-model calculations. The shape of the angular distributions at forward angles is better reproduced by the SEP. The data are expected to be most accurate at forward angles. The radial integrals are sensitive to the form factors from the nuclear surface outwards. In the EBEP the behaviour of the form factors in this region are incorrect.

We have also attempted to calculate the angular distribution for the E_{x} = 13.093 MeV level in ^{16}O whose configuration is $(p^{1}/_{2})^{-1}s^{1}/_{2}$. However the ℓ = o phase shift analysis gave a

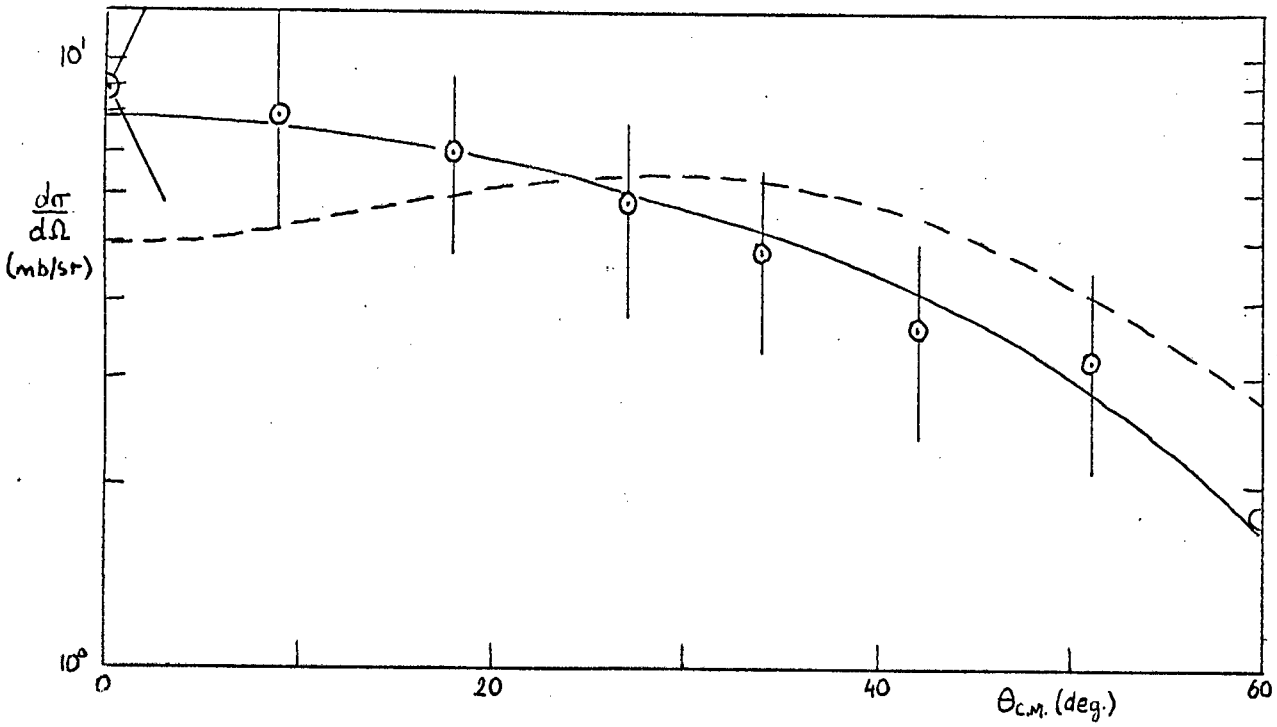

Fig. 3.5.6. Same as in fig. 3.5.5 except for the $J_x^\pi = 3^-$, E_x = 13.129 MeV state.

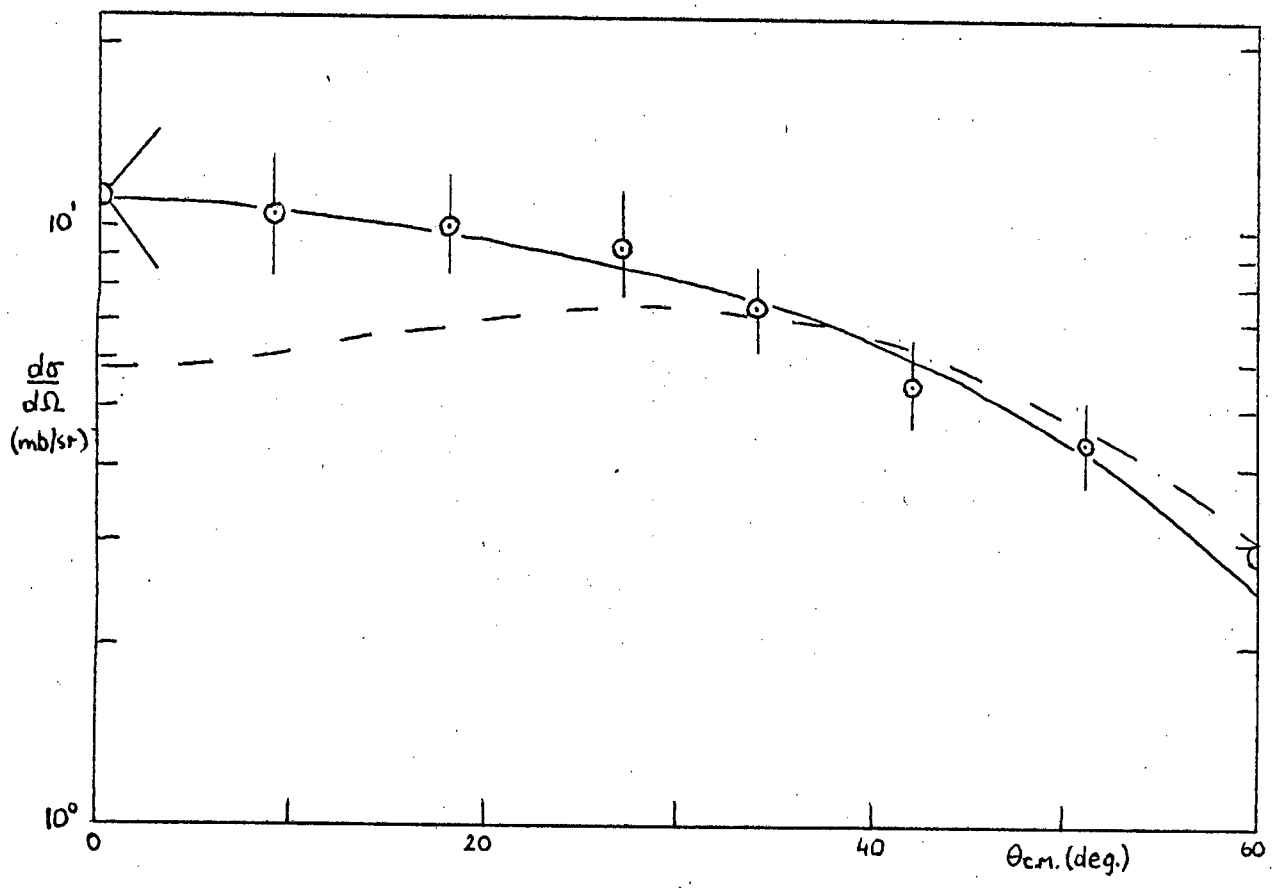

Fig. 3.5.7. Same as in fig. 3.5.5 except for the $J_x^\pi = 3^-$, E_x = 13.258 MeV state.

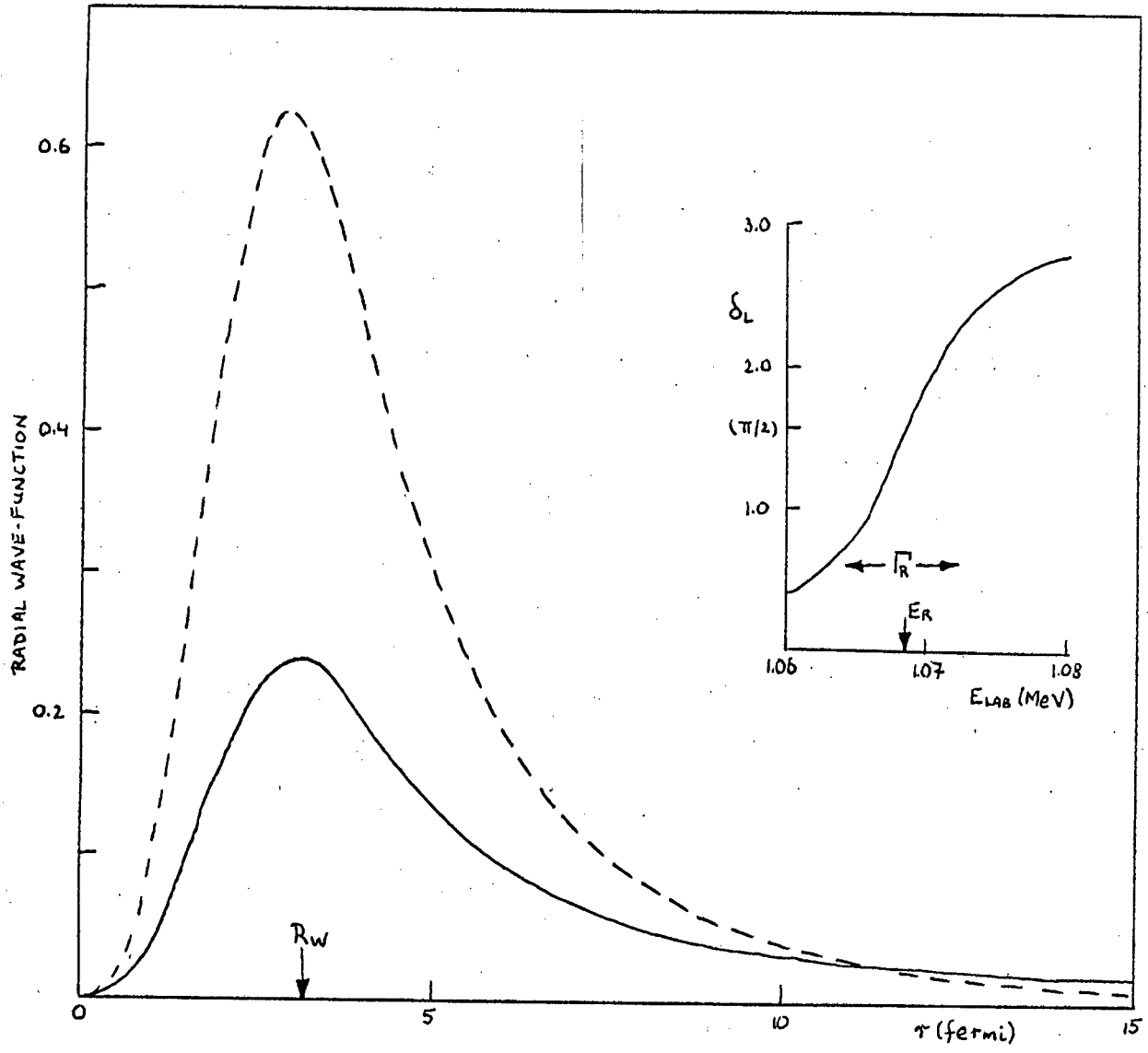

Fig. 3.5.8. Bound (E_B = 0.6 MeV) (dashed) and resonant (E_R = 1.069 MeV) (solid) single particle wave functions for the $1d^5/_2$ proton in ^{16}O ($J^\pi_x = 3^-$, E_x = 13.129 MeV). The absorptive potential radius is denoted by R_w. Also shown is the behaviour of the phase shift near the SPR.

broad width at the resonance energy in contrast with the narrow experimental [50] and theoretical [31] widths. Our approximate method for evaluating the Gamow state for s-waves is not expected to be accurate because the wave function has equally large amplitudes inside and outside the nucleus.

3.5.b Energy Spectra

A simplified expression for the excitation function may be derived as follows. We assume that the target nucleus remains in its ground state during the transfer process. Then the sum over J in eqn. (3.2.3) is replaced by the sum over ℓ and j for the transferred nucleon. Levin has shown [15] that different ℓ,j contribute incoherently to the cross-section. Therefore, in order to investigate the resonance behaviour of the energy spectrum, we need only retain the channel (ℓ,j) in which the transferred nucleon resonates. Then the following expression for the energy spectrum, which includes implicitly the effects of the background and the resonance in the channel (ℓ,j) may be derived (cf. eqn.(3.2.10))

$$\frac{d^2\sigma}{d\hat{k}_y dE} = \frac{m_d m_y}{(2\pi\hbar^2)^2}\,\frac{k_y}{k_d}\left(\frac{2J_A+1}{2J_{A-1}+1}\right)\frac{D_0^2}{(2s_x+1)}\sum_{\lambda}\left|\beta_{\ell\lambda}(\underset{\sim}{k}_y,\underset{\sim}{k}_d)\right|^2 \quad , \tag{3.5.2}$$

where

$$\beta_{\ell\lambda}(\underset{\sim}{k}_y,\underset{\sim}{k}_d) = \frac{i^{-\ell}}{\sqrt{2\ell+1}}\lim_{\alpha\to 0}\int d^3r\, e^{-\alpha r}$$

$$\chi^{(-)*}_{yA}(\underset{\sim}{k}_y,\underset{\sim}{r}')\,\frac{u^{(-)*}_{\ell j}(r,k_x)}{r}\,Y^*_{\ell\lambda}(\hat{r})\,\chi^{(+)}_{dA-1}(\underset{\sim}{k}_d,\underset{\sim}{r}) \quad , \tag{3.5.3}$$

and $u_{\ell j}^{(-)}$ is given by eqns. (2.2.14) and (2.2.15).

We have used eqn. (3.5.2) to calculate the excitation function for the reaction $^{15}N(d,n)^{16}O$ in the neighbourhood of the 13.129 MeV resonance (fig. (3.5.9)). The radial integrals in eqn. (3.5.3) have been evaluated out to 30 fermi. Beyond this distance the integrand of the radial integrals will oscillate. Thus the major contribution to the cross-section will come from the nuclear interior. Similar results are obtained for the other two resonances at 12.967 MeV and 13.258 MeV. Because these resonances are well below the $\ell = 2$ angular momentum barrier the widths are narrow ($\Gamma_R/E_R \simeq 10^{-2}$) and the background wave function is small inside the nucleus. Hence no interference patterns are seen. The shape of the resonance as a function of scattering angle is unaltered. This is also a consequence of the fact that the background wave function is small inside the nucleus. Equation (3.2.10) shows that, on neglecting background terms, the dependence of the energy spectrum on the scattering angle and the energy factors.

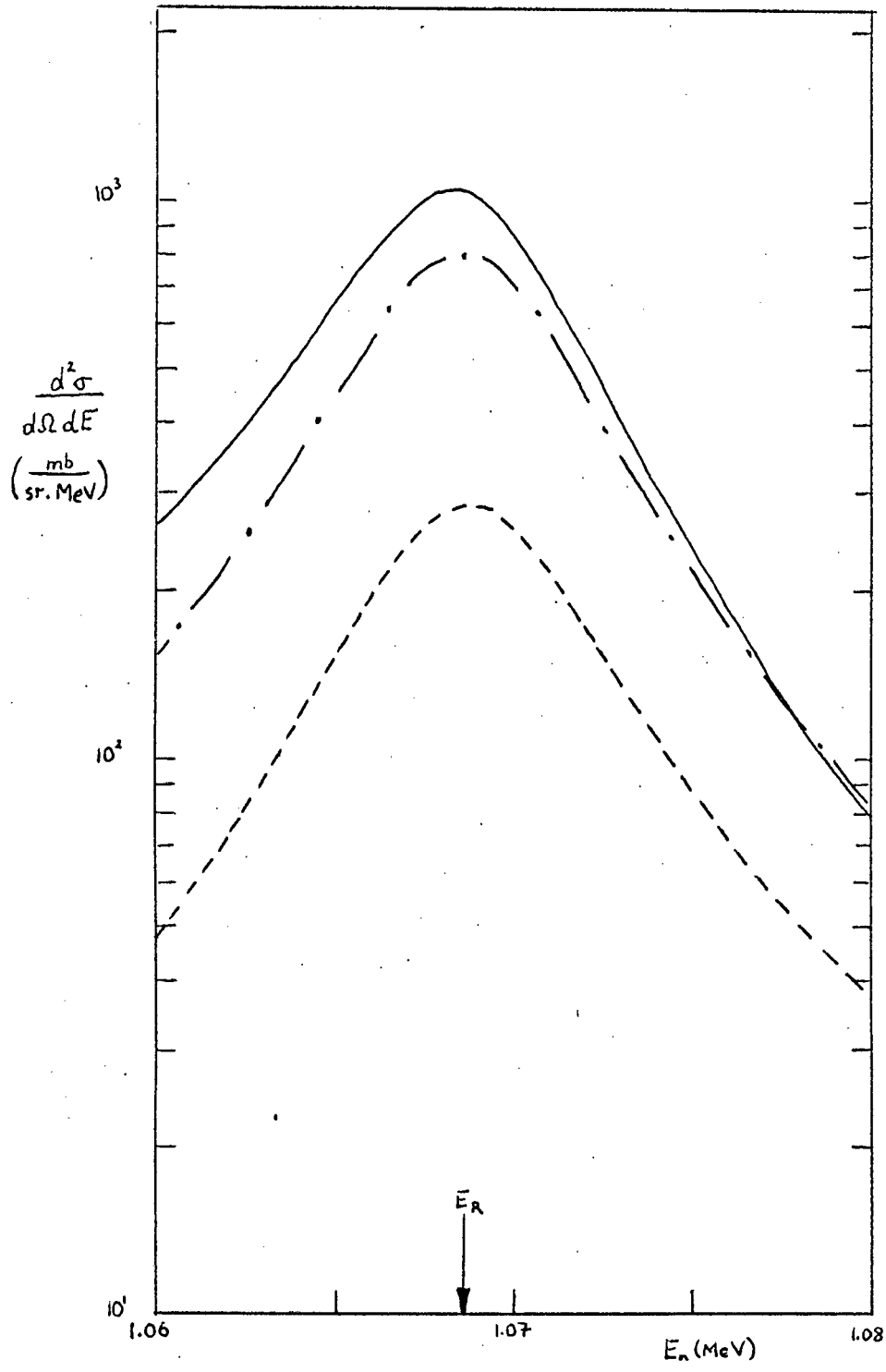

Fig. 3.5.9. Theoretical excitation functions for the reaction $^{15}N(d,n)^{16}O$ populating the 13.139 MeV level in ^{16}O. The angular momentum transfer is $\ell = 2$. The centre-of-mass angles are 20° (full line), 40° (dashed-dot line) and 60° (dashed line). The position of the resonance energy is denoted by E_R.

4. Inelastic Scattering to Unstable States

In this chapter we suggest an alternative direct reaction process with which particle unstable nuclei may be studied. Nucleon induced inelastic scattering at low energies excites predominantly collective modes of excitation. At higher energies the probability that the projectile nucleon interacts strongly with just one target nucleon increases. In this way the same nucleon unstable states may be populated as with stripping reactions. A comparison between these two quite different direct reaction mechanisms can be made.

In section 4.1 we indicate in some detail the kind of inelastic experiments required to produce particle unstable states. We also mention quasi-free scattering experiment results where unstable hole-states are produced. Formulae for the transition amplitude and the cross-section are derived in the following section. These results are compared with those of stripping reactions leading to unstable states. The wave function for a hole-state is discussed in section 4.3.

4.1 The Experimental Situation

An inelastic reaction in which a particle unstable nucleus is formed may be written symbolically as

$$y + A \rightarrow y' + A^* \rightarrow y' + x + (A-1) \quad . \qquad (4.1.1)$$

In order that the projectile nucleon y have a large probability to interact with just one nucleon, x, of the target nucleus A, the energy of y must be high. This is in contrast with the stripping reaction, where relatively low energy projectiles are sufficient to populate unstable states. An advantage of using reaction (4.1.1) is that "inner nucleons" may be excited. In this way fairly high lying unstable states of A may be studied. For instance, in the reaction $^{16}O(p,p')^{16}O^*$, a $p^3/_2$ proton may be excited, thus forming excited states in ^{16}O with predominantly $(p^3/_2)^{-1}s^1/_2$ or $(p^3/_2)^{-1}d^5/_2$ configurations (see fig. (3.3.1)). Such states are expected not to be strongly populated in the corresponding $^{15}N(d,n)$ reaction because the target must be excited during the transfer process. An example of an inelastic process populating an essentially single particle resonance state is $^{17}O(p,p)^{17}O^*$, leading to the $d^3/_2$ state in ^{17}O. The low energy particle, x, in reaction (4.1.1) need not be observed. The corresponding particle is usually not observed in stripping experiments to unbound states.

Experiments with the above intention have, to the author's knowledge, not been performed. There seems to be only one instance in the literature where the reaction mechanism of eqn. (4.1.1) is invoked to explain the experimental data [62]. It is suggested that in the reaction $^{16}O(p,2p)^{15}N$ at an incident proton energy of 45 MeV and with final proton energies of about 16 MeV [63] , a collective state in ^{16}O at about 29 MeV excitation energy is populated. Since this state is well above the proton threshold of about 12 MeV, the ^{16}O deexcites by emitting a proton.

Much more numerous are quasi-free (p,2p) experiments with incident proton energies above 200 MeV. Such reactions have been reviewed recently by Tyren et al. [64] , Jackson [65] and by Jacob and Maris [66]. Experiments deal almost exclusively with situations where the two protons in the exit channel have equal (high) energies. Of special interest to the present work are situations where a deeplying proton is dislodged by the incident proton. In this way the residual nucleus may be left in an excited state (hole state) whose energy is greater than the particle threshold energy. This nucleus will therefore deexcite by emitting a further nucleon [26-30,67,68] .

An example of this is the reaction $^{12}C(p,2p)$ studied by Tyrén et al. [64] with 460 MeV protons in a coplanar symmetric geometry. The broad peak of width 10 MeV in the separation energy spectrum at 36 MeV is attributed to the knockout of a $1s^1/_2$ proton from ^{12}C. The highly excited ^{11}B nucleus decays rapidly by emitting a further low energy nucleon [26-30].

4.2 Theory of Inelastic Scattering to Low Lying Unstable States

4.2.a Formulae for Transition Amplitude and Cross-Section

The distorted wave transition amplitude for the reaction (4.1.1) in which a projectile nucleon y excites a particle unstable state of a target nucleus A is

$$T(y,y')x \simeq \lim_{\alpha\to 0} \langle \chi^{(-)}_{y'A}(\underset{\sim}{k}'_y)\, \psi^{(-)}_{E,A} \,|\, e^{-\alpha r} V_{yA} \,|\, \phi_A \chi^{(+)}_{yA}(\underset{\sim}{k}_y)\rangle . \quad (4.2.1)$$

The distorted waves $\chi^{(+)}_{yA}$ and $\chi^{(-)}_{y'A}$ describe the elastic scattering of y by A in the entrance and exit channels respectively. The function φ_A is the ground state wave function of the target, while $\psi^{(-)}_{E,A}$ denotes the ((A-1) + x) nucleon system. This latter function is given by eqns. (3.2.3) and (3.2.4). The excitation is caused by the interaction V_{yA}, which is the difference between the exact and optical model potentials acting between the projectile and target. The convergence factor $e^{-\alpha r}$ ensures that all radial integrals converge.

The cross-section for the reaction is

$$\frac{d^2\sigma}{d\hat{k}_{y'}dE} = \frac{(m_y)^2}{(2\pi\hbar^2)^2}\,\frac{k_{y'}}{k_y}\,\frac{1}{(2J_A+1)(2s_y+1)}\sum|T(y,y')x|^2 \quad . \qquad (4.2.2)$$

Eqn. (4.2.1) for the transition amplitude will be valid provided the energy difference between x and y' is large and provided the unstable state is long enough lived for y' not to interfere with its decay.

As in the case of stripping to decaying states, interference effects may occur in the above cross-section. The conditions for this to happen are either that the background term in $\psi^{(-)}_{E,A}$, describing the non-resonant knockout of x from A, is comparable in magnitude with the resonance term or that there are several overlapping resonances in the energy region E.

In order to simplify further discussion we make the following approximations in eqn. (4.2.1):

(a) The projectile y interacts with just one target nucleon x. Then the potential V_{yA} may be replaced by V_{yx} .

(b) The interaction V_{yx} is of zero range and of strength V_o . At sufficiently high projectile energies, the struck nucleon may be regarded as essentially free, so that the two body interaction, V_{xy} , may be summed to all orders and replaced by the free two-body t-matrix. If it is assumed that this t-matrix is a function of momentum transfer q only, then in the subsequent equations V_o is to be replaced by the t-matrix t(q).

(c) Spin orbit coupling is neglected.

(d) It is physically meaningful to write the target ground state wave function ϕ_A as a product of a wave function for the nucleon x in the shell model orbit with angular momenta ℓ_o, j_o and a wave function for the other (A-1) nucleons

$$\varphi_A = O_{t_o n_o}\, \omega^{b}_{\ell_o j_o}(r, k_{n_o}^{\ell_o j_o})\, \varphi_{A-1} \quad . \tag{4.2.3}$$

(e) The same assumptions are made about the wave function for the unstable nucleus, $\psi^{(-)}_{E,A}$ as in section 3.2 of chapter 3.

Eqn. (4.2.2) may then be written as

$$\frac{d^2\sigma}{d\hat{k}_{y'}\, dE} = \sum_{c(\text{open})} \frac{|G_{tc}|^2}{(E-E_t)^2 + \frac{1}{4}\Gamma_t^2}\; |O_{\ell j}|^2 \left(\frac{d\sigma}{d\hat{k}_{y'}}\right)_{S.P.} \quad . \tag{4.2.4}$$

The single-particle cross-section $(d\sigma/d\hat{k}_{y'})_{s.p.}$ is defined by

$$\left(\frac{d\sigma}{d\hat{k}_{y'}}\right)_{s.p.} = \frac{(m_y)^2}{(2\pi\hbar^2)^2}\,\frac{k_{y'}}{k_y}\left(\frac{2J_A^*+1}{2J_A+1}\right)\,\frac{V_0^2\,|O_{t_0n_0}|^2}{(2s_y+1)} \sum_{\lambda} |\beta_{\ell\lambda}(\underset{\sim}{k}_{y'}, \underset{\sim}{k}_y)|^2 \quad , \tag{4.2.5}$$

where

$$\beta_{\ell\lambda}(\underset{\sim}{k}_{y'}, \underset{\sim}{k}_y) = \frac{i^{-\ell}}{\sqrt{2\ell+1}} \lim_{\alpha\to 0} \int d^3r\; e^{-\alpha r}\, \chi^{(-)*}_{y'A}(\underset{\sim}{k}_{y'}, \underset{\sim}{r}) \left[\frac{\omega^{\{b\}}_{\{res\}\ell_j}(r, k_n^{\ell_j})}{r} Y^*_{\ell\lambda}(\hat{r})\right]\left[\frac{\omega^{b}_{\ell_0 j_0}(r, k_{n_0}^{\ell_0 j_0})}{r} Y_{\ell_0\lambda_0}(\hat{r})\right] \chi^{(+)}_{yA}(\underset{\sim}{k}_y, \underset{\sim}{r}') \quad , \tag{4.2.6}$$

and

$$|\underset{\sim}{r}'| = \frac{M_{A-1}}{M_A}\,|\underset{\sim}{r}| \quad . \tag{4.2.7}$$

Provided the quantities G_{tc} and Γ_t are independent of energy, the angular distribution of y' may be written as

$$\frac{d\sigma}{d\hat{k}_{y'}} = \frac{2\pi \sum_{c(open)} |G_{tc}|^2}{\Gamma_t}\,|O_{\ell j}|^2 \left(\frac{d\sigma}{d\hat{k}_{y'}}\right)_{s.p.} \tag{4.2.8}$$

The above results are very similar to the corresponding results for stripping to unstable states. The quantity D_0^2 appearing in eqn. (3.2.11) is replaced by the quantity $V_0^2|O_{t_0n_0}|^2$ in eqn. (4.2.5). The spectroscopic factor

$|O_{t_o n_o}|^2$ gives the probability that the nucleon x in the physical ground state, t_o, of the target is in a shell model state n_o. It may be obtained either from a structure calculation or from a study of direct knockout reations in the energy region where no unstable states occur [65].

The integrand in eqn. (4.2.6) converges without the convergence factor when the Gamow function $w_{\ell j}^{res}$ is used, provided the binding energy of the nucleon x in the ground state of the target is greater than the resonance width of x in the SPR state. This condition is expected to be satisfied in most cases of interest.

4.2.b Comparison with Results for Stripping Reactions

Eqn. (4.2.8) shows that the ratio

$$\left(\frac{d\sigma}{d\hat{k}_{y'}}\right) \Big/ \left(\frac{d\sigma}{d\hat{k}_{y'}}\right)_{s.p.} = |O_{\ell j}|^2 \quad , \tag{4.2.9}$$

as in stripping reaction leading to unstable states since (see Appendix D)

$$\Gamma_t = 2\pi \sum_{c(open)} |G_{tc}|^2 \quad . \tag{4.2.10}$$

As a further parallel with the stripping reaction, eqn. (4.2.4) demonstrates that the energy spectrum may also be written as a product of the total elastic scattering cross-section of the nucleon x by the nucleus (A-1) and an "inelastic scattering enhancement factor". This enhancement factor is

also proportional to the inverse of the width Γ_t. Therefore it is expected that the ratio of intensity in (y,y')x to that in (x,x) should increase with angular momentum transfer. Further, at the resonance energy, E_t, the ratio of intensity in (d,y)x to that in (y,y')x should be roughly equal to

$$\left(\frac{d\sigma}{d\hat{k}_y}\right)_{s.p.}[\text{eqn}(3.2.11)] \Big/ \left(\frac{d\sigma}{d\hat{k}_{y'}}\right)_{s.p.}[\text{eqn}(4.2.5)] \quad . \tag{4.2.11}$$

In this respect it would be interesting to compare the energy spectra of neutrons and protons from $^{15}N(d,n)^{16}O^*$ and $^{16}O(p,p)^{16}O^*$.

4.3 The Wave Function for a Hole State

We return to the $^{12}C(p,2p)$ reaction [64] mentioned in section 4.1 and indicate how the highly excited ^{11}B nucleus may be described in our formalism. Since it is assumed that a $1s^1/_2$ proton has been knocked out of ^{12}C it is natural to describe the physical state φ_t of ^{11}B as being essentially composed of the BSEC, Φ_n, with a proton hole in the $1s^1/_2$ orbital. This state may decay into channels c in which there are two holes in the $1p^3/_2$ orbital and one nucleon in the continuum. The unstable ^{11}B state may be written as

$$\psi_E^{J(+)} \simeq \sum_{c(\text{open})} e^{i\xi_c} \frac{\left(\frac{1}{2\pi}\gamma_{nc}\right)^{1/2}}{E - E_t + \frac{i}{2}\Gamma_t} \Phi_n \quad , \tag{4.3.1}$$

where

$$E_t = E_n + \langle \bar{\Phi}_n | V_A | \bar{\Phi}_n \rangle + \sum_c \mathcal{P} \int dE' \frac{\langle \bar{\Phi}_n | V_A | \chi_{E'}^c \rangle^2}{E - E'} \quad ,$$

$$\Gamma_t = 2\pi \sum_c \langle \bar{\Phi}_n | V_A | \chi_E^c \rangle^2 = \sum_c \gamma_{nc} \quad . \qquad (4.3.2)$$

Eqns. (4.3.1) and (4.3.2) lead to the autoionization process discussed by Pittel and Austern [30] . Lipperheide et al. [29] obtain a similar result except that they include several Φ_n in the description of ^{11}B.

We have neglected the background term in eqn. (4.3.1) for $\psi_E^{J(+)}$. Its inclusion would describe the simultaneous knockout of a fast proton and a slower nucleon from the $1p^3/_2$ shell. Thus this term describes the "double-excitation process" of Pittel and Austern [30].

As an alternative to the above description, we mention the work of Herscovitz et al. [67,68] , Shanta [69] and Berggren and Ohlén [70] . The cross-section for the (p,2p) process is proportional to the square of the overlap integral, which may be written symbolically as

$$\langle \tilde{\varphi}_{A-1} | \varphi_A \rangle \quad , \qquad (4.3.3)$$

where φ_A is the initial target state and $\tilde{\varphi}_{A-1}$ is the final unstable state. This overlap integral is approximated by a single nucleon shell model wave function which describes the motion of the knocked out nucleon in the target nucleus before the reaction has taken place. In order to account for the decaying nature of the final state, the single particle wave function is evaluated at a pole of the S-function in the

complex momentum plane. Such a pole is situated in the upper half k-plane to ensure that the wave function decreases exponentially for large r. This is also the asymptotic behaviour of eqn. (4.3.1).

5. Summary and Conclusions

We have studied within the framework of the continuum shell-model theory nuclear excitations that are unstable against nucleon emission. An expression has been derived for the wave function of a nucleus in the neighbourhood of such an excited state. As a function of energy this wave function consists of a smoothly varying background term and a Breit-Wigner shaped resonance term. As a function of displacement, the background term is, for a sufficiently high angular momentum barrier, zero inside the nucleus. On the other hand, the resonance term, which consists of a linear superposition of shell-model bound and resonant (Gamow) states, has a large amplitude inside the nucleus. The decay of the state is caused by the natural widths of the shell-model resonant states and by the residual interaction (difference between the exact and the shell-model Hamiltonians).

The validity of this description is investigated by an analysis of nucleon transfer reactions populating unstable states. In the DWBA expression for the transition amplitude, the usual bound state wave function describing the target-plus-captured-nucleon system is replaced by the resonating part of the wave function for this system. Simple expressions are obtained for the energy spectrum and the angular distribution of the uncaptured, outgoing nucleon (or nucleons). In these expressions the form factor is given by the radial wave function for the captured nucleon. Because of absorption in the entrance and exit channels, the transition amplitude is sensitive

to the form of this wave function only in the region outside the nuclear interior. The "stripping to bound states" method for extracting spectroscopic factors from the angular distribution is shown to apply also to stripping reactions leading to unstable states. The energy spectrum is related to the total elastic cross-section populating the same unstable state. Angular distributions have been calculated for a variety of projectile and target systems. The Gamow state has been evaluated in an approximation which has been shown to be valid for narrow resonances. The inclusion of spin-orbit coupling, finite range and nonlocality effects should affect the angular distributions only slightly. More important from our point of view is the approximation of the form factor by the radial single-particle wave function. This is correct only for stripping by a closed shell target nucleus. In general it is necessary to determine the form factor by calculating the overlap between the target ground state wave function and the target-plus-stripped-nucleon wave function from nuclear structure theories.

We have also suggested another direct reaction mechanism - nucleon induced inelastic scattering at high energies ($\gtrsim$ 100 MeV) - with which these unstable states may be studied. Similarities between this reaction, the stripping reaction and elastic scattering, all populating the same particle unstable states, are pointed out.

Appendix A : Properties of the Gamow Function

We recall here the form of the wave function of a single particle resonance, and discuss its normalization.

It is shown by Romo [12] and by Mahaux and Weidenmüller [3] that the residue of the Green function $g_{\ell j}^{(+)}(r,r';k)$ factorizes in r and r' in the vicinity of a SPR (see also end of this Appendix):

$$g_{\ell j}^{(+)}(r,r';k) \simeq \frac{\omega_{\ell j}^{res}(r,k_n^{\ell j})\,\omega_{\ell j}^{res}(r',k_n^{\ell j})}{\varepsilon - \varepsilon_n^{\ell j} + \frac{i}{2}\Gamma_n^{\ell j}} \quad . \tag{A.1}$$

In this equation $k_n^{\ell j}$ is the position of the resonance in the complex k-plane

$$k_n^{\ell j} = k_1 - i k_2 \quad , \qquad (k_1 > k_2 > 0) \quad , \tag{A.2}$$

and $\varepsilon_n^{\ell j} - \frac{i}{2}\Gamma_n^{\ell j}$ is the corresponding complex energy.

The function $w_{\ell j}^{res}$ defined by

$$\omega_{\ell j}^{res}(r,k_n^{\ell j}) = \left(\frac{m\Gamma_n^{\ell j}}{\hbar^2 k_n^{\ell j}}\right)^{1/2} e^{i\xi_n^{\ell j}}\, O_{\ell j}(r,k_n^{\ell j}) \quad , \tag{A.3}$$

is a solution of the one-body Schrödinger equation with complex energy $\varepsilon_n^{\ell j} - \frac{i}{2}\Gamma_n^{\ell j}$ and with purely outgoing boundary conditions. It is called a Gamow function. The function $\tilde{w}_{\ell j}(r,\tilde{k}_n^{\ell j})$, satisfies the same Schrödinger equation with purely in-going asymptotics provided $\tilde{k}_n^{\ell j} = -k_n^{\ell j *}$ and $\tilde{w}_{\ell j}^{res}(r,\tilde{k}_n^{\ell j}) = w_{\ell j}^{res}(r,k_n^{\ell j})^*$ [9] :

$$\tilde{\omega}_{\ell j}^{res}(r,\tilde{k}_n^{\ell j}) = \left(\frac{m\Gamma_n^{\ell j}}{\hbar^2 \tilde{k}_n^{\ell j}}\right)^{1/2} e^{-i\xi_n^{\ell j}} I_{\ell j}(r, k_n^{\ell j *}) \quad . \tag{A.4}$$

To zeroth order in (k_2/k_1) the quantity $[m\Gamma_n^{\ell j}/(\hbar^2 k_n^{\ell j})]^{1/2}$ reduces to $\sqrt{2k_2}$ - the normalization of Berggren [9].

Because of its asympototic form, the Gamow function cannot be normalized in the usual way. Berggren [9] shows that the Gamow function describing resonances with ($\mathrm{Im}k_n^{\ell j} < \mathrm{Re}k_n^{\ell j}$) may be ortho-normalized in the following sense

$$\lim_{\alpha\to 0} \int_0^\infty dr\, e^{-\alpha r^2} \omega_{\ell j}^{res}(r,k_n^{\ell j}) \omega_{\ell j}^{res}(r,k_m^{\ell j}) = \delta_{nm} \quad . \tag{A.5}$$

To prove this he shows that

$$\lim_{\alpha\to 0} \int_0^\infty dr\, e^{-\alpha r^2} e^{2ik_n^{\ell j} r} = \frac{1}{2ik_n^{\ell j}} \quad , \tag{A.6}$$

and

$$\lim_{\alpha\to 0} \int_0^\infty dr\, e^{-\alpha r^2} r\, e^{2ik_n^{\ell j} r} = \left(\frac{1}{2ik_n^{\ell j}}\right)^2 \quad . \tag{A.7}$$

To regularize the Gamow function, Romo [12] evaluates the integral for poles situated at $k_n'^{\ell j}$ and $k_m'^{\ell j}$ in the <u>upper half</u> k-plane and then performs an analytic continuation to the resonance poles $k_n^{\ell j}$ and $k_m^{\ell j}$ in the <u>lower half</u> k-plane:

$$\lim_{\substack{k_n'^{\ell j}\to k_n^{\ell j} \\ k_m'^{\ell j}\to k_m^{\ell j}}} \int_0^\infty dr\, \omega_{\ell j}^{res}(r,k_n'^{\ell j}) \omega_{\ell j}^{res}(r,k_m'^{\ell j}) = \delta_{nm} \quad . \tag{A.8}$$

Gyarmati and Vertse [17] demonstrate that the regularization procedures of (A.5) and (A.8) are identical by performing the integrals (A.6) and (A.7) in the complex r-plane.

We should like to attach a physical meaning to these regularization procedures. The divergence of the Gamow state is only apparent because we are treating an inherently time-dependent problem in a time-independent formalism [71]. It was shown by Rosenfeld [72] that in a time-dependent treatment, the wave function at resonance increases exponentially with r up to the distance travelled by the wave packet in a time t. For r-values larger than this distance, the wave function is zero. (In most situations matrix elements which contain a Gamow function have an r-cutoff in the form of a nuclear potential and/or a bound state wave function.) More exactly, the time dependent asymptotic form of the Gamow function is proportional to

$$\exp\left\{ i\left[k_1 r - (\mathcal{E}_n^{\ell j}/\hbar)t\right] + k_2 r - (\Gamma_n^{\ell j}/2\hbar)t \right\} \qquad (A.9)$$

For resonances with $k_1 > k_2 > 0$, Rosenfeld gives the following relation between the time t and the distance r

$$t = c\left(\frac{m}{\hbar}\right)\left(\frac{r}{k_1 - k_2}\right) \quad , \qquad (A.10)$$

where c is a constant greater than unity. Using eqn. (A.10) and the relation $\Gamma_n^{\ell j} = (2\hbar k_1 k_2)/m$, the real part of the exponent in eqn. (A.9) becomes

$$\bar{k}_2 r = \left(k_2 - c\,\frac{k_1 k_2}{k_1 - k_2}\right) r \quad , \qquad (A.11)$$

which is less than zero for all (asymptotic) r. Thus the inclusion of the time development of the Gamow state is effectively to ensure that the complex pole always occurs on the upper half k-plane. Relation (A.6) may be obtained by replacing k_n^{lj} in the integrand by $\bar{k}_n^{lj}$

$$\bar{k}_n^{lj} = k_1 - i\bar{k}_2 \qquad (k_1>0, \bar{k}_2<0) \; . \tag{A.12}$$

The convergence factor, $e^{-\alpha r^2}$, may be omitted, and we have

$$\int_0^\infty dr\, e^{2i\bar{k}_n^{lj} r} = \left(2i\bar{k}_n^{lj}\right)^{-1} \; . \; . \tag{A.13}$$

We now take the "time-independent limit", $c \to o$, and obtain the desired result. Eqn. (A.8) may be evaluated similarly.

We may also give the convergence factor $e^{-\alpha r}$ appearing in eqns. (3.2.12) and (4.2.6) a physical meaning by associating α with the quantity $(ck_1k_2)/(k_1-k_2)$ of eqn. (A.11). The convergence factor method is really useful only when the radial integral can be evaluated in closed form. This is the case in nucleon transfer between heavy ions when the incident energies are below the Coulomb barrier [73,19] and when the strong absorption model is applicable [74-76,57] . However, so far no experimental results of heavy ion transfer reactions leading to unbound states have been reported. The contour integration method seems to be most appropriate when the radial integral is solved numerically [16] (see however the method of Coker and Hoffmann [13] . When the Gamow state is not used to describe the SPR (e.g. [16]), the convergence procedure cannot be given a physical interpretation.

Finally, we should like to show, using eqn. (2.3.8), that near a narrow SPR, the Green's function

$$g^{(+)}_{\ell j}(r,r';k) = \lim_{\eta\to 0}\int d\varepsilon' \frac{|u^{(+)}_{\ell j}(r,k')\rangle\langle u^{(+)}_{\ell j}(r',k')|}{\varepsilon - \varepsilon' + i\eta} \quad , \tag{A.14}$$

may be approximately written in a separable form inside the nucleus. For a narrow resonance $u^{(+)}_{\ell j}$ takes on the approximate form

$$u^{(+)}_{\ell j}(r,k) \simeq e^{i\xi^{\ell j}_n} \frac{\left(\frac{1}{2\pi}\Gamma^{\ell j}_n\right)^{1/2}}{\varepsilon - \varepsilon^{\ell j}_n + \frac{i}{2}\Gamma^{\ell j}_n}\ \omega^{res}_{\ell j}(r,k^{\ell j}_n) \quad , \tag{A.15}$$

inside the nucleus. Using eqn. (A.15) the Green's function, eqn. (A.14), reads

$$g^{(+)}_{\ell j}(r,r';k) \simeq \lim_{\eta\to 0}\ \left(\frac{1}{2\pi}\Gamma^{\ell j}_n\right)\int d\varepsilon' \frac{\omega^{res}_{\ell j}(r,k^{\ell j}_n)\,\omega^{res}_{\ell j}(r',k^{\ell j}_n)}{\left(\varepsilon' - \varepsilon^{\ell j}_n + \frac{i}{2}\Gamma^{\ell j}_n\right)(\varepsilon - \varepsilon' + i\eta)\left(\varepsilon' - \varepsilon^{\ell j}_n - \frac{i}{2}\Gamma^{\ell j}_n\right)} \tag{A.16}$$

We evaluate the integral in the upper half ε'-plane using the Cauchy integral formula. The poles occur at $\varepsilon' = \varepsilon + i\eta$ and at $\varepsilon' = \varepsilon^{\ell j}_n + \frac{i}{2}\Gamma^{\ell j}_n$, and we have

$$g^{(+)}_{\ell j}(r,r';k) \simeq 2\pi i \left(\frac{1}{2\pi}\Gamma_n^{\ell j}\right)\left\{ - \frac{\omega^{res}_{\ell j}(r,k_n^{\ell j})\,\omega^{res}_{\ell j}(r',k_n^{\ell j})}{(\varepsilon-\varepsilon_n^{\ell j}+\frac{i}{2}\Gamma_n^{\ell j})(\varepsilon-\varepsilon_n^{\ell j}-\frac{i}{2}\Gamma_n^{\ell j})} \right.$$

$$\left. + \frac{\omega^{res}_{\ell j}(r,k_n^{\ell j})\,\omega^{res}_{\ell j}(r',k_n^{\ell j})}{i\Gamma_n^{\ell j}(\varepsilon-\varepsilon_n^{\ell j}-\frac{i}{2}\Gamma_n^{\ell j})} \right\} ,$$

$$= \frac{\omega^{res}_{\ell j}(r,k_n^{\ell j})\,\omega^{res}_{\ell j}(r',k_n^{\ell j})}{\varepsilon-\varepsilon_n^{\ell j}+\frac{i}{2}\Gamma_n^{\ell j}} \quad . \qquad \text{(A.17)}$$

This is the result given by Romo [12] and by Mahaux and Weidenmüller [3].

Appendix B : Completeness and Orthonormality Relations

Completeness and orthonormality relations resulting from the inclusion of Gamow states with the bound and real energy scattering states have been derived by Berggren [9] and Romo [12] . We discuss these relations using eqn. (2.3.8) for the real energy scattering states. The discussion is restricted to one channel (ℓ,j),and we shall drop these two suffixes. It is understood that all radial integrals containing Gamow states are evaluated using one of the methods mentioned in Appendix A. Eqn. (2.3.8) reads, on including resonances with increasing principal quantum number n,

$$u^{(+)}(r,k) \simeq u^{(+)NR}(r,k) - \sum_n e^{i\xi_n} \frac{\left(\frac{1}{2\pi}\Gamma_n\right)^{1/2}}{\varepsilon-\varepsilon_n+\frac{i}{2}\Gamma_n} \omega^{res}(r,k_n) \quad . \tag{B.1}$$

The completeness relation for the bound and real energy scattering state radial wave functions is

$$\sum_m \omega^b(r,k_m)\omega^b(r',k_m) + \int_0^\infty d\varepsilon\, u^{(+)}(r,k)\left[u^{(+)}(r',k)\right]^* = \delta(r-r') \quad . \tag{B.2}$$

In the above equation, we substitute eqn. (B.1) for $u^{(+)}$. If the resonances of $u^{(+)}$ are well separated in energy, we obtain for the completeness relation (B.2),

$$\sum_m \omega^b(r,k_m)\omega^b(r',k_m) + \sum_n \omega^{res}(r,k_n)\omega^{res}(r',k_n)$$

$$+ \int_0^\infty d\mathcal{E}\; u^{(+)NR}(r,k)\left[u^{(+)NR}(r',k)\right]^*$$

$$- \sum_n \left(\frac{1}{2\pi}\Gamma_n\right)^{1/2}\left\{\omega^{res}(r,k_n)\int_0^\infty d\mathcal{E}\;\frac{u^{NR}(r',k)^*}{\mathcal{E}-\mathcal{E}_n+\frac{i}{2}\Gamma_n}\right. \qquad (B.3)$$

$$\left. + \omega^{res}(r',k_n)\int_0^\infty d\mathcal{E}\;\frac{u^{NR}(r,k)}{\mathcal{E}-\mathcal{E}_n-\frac{i}{2}\Gamma_n}\right\} = \delta(r-r')\ .$$

The energy integrals in the last term on the left hand side of eqn. (B.3) may be evaluated approximately. We replace $u^{NR}(r,k)$ by its value at the resonance energy viz. $u^{NR}(r,k_n)$. Then we have for the last term

$$-i\sum_n \left(\frac{\Pi}{2}\Gamma_n\right)^{1/2}\left\{\omega^{res}(r',k_n)u^{NR}(r,k_n) - \omega^{res}(r,k_n)u^{NR}(r',k_n)^*\right\}\ . \qquad (B.4)$$

If in eqn. (B.1), the sum over n contains only the narrow resonances, $u^{NR}(r,k_n)$ will be negligibly small inside the nucleus. Hence for r and r' inside the nucleus, the completeness relation becomes

$$\sum_{p=m,n} \omega^{\{b\atop res\}}(r,k_p)\,\omega^{\{b\atop res\}}(r',k_p) + \int_0^\infty d\mathcal{E}\; u^{(+)NR}(r,k)\left[u^{(+)NR}(r',k)\right]^* = \delta(r-r')\ . \qquad (B.5)$$

This result is similar to Berggren's [9]. Here, however, the energy integral is along the real ε-axis. Contributions to this integral come from upward of a sufficiently high energy in order that $u^{NR}(r,k)$ and $u^{NR}(r',k)$ be non-vanishing inside the nucleus. When studying low-lying states, the contribution from virtual excitations high into the continuum is expected to be small. Therefore the low lying physical states consist

essentially of the states $w^{\{b\}}_{res}$.

The wave functions $w^{b}(r,k_m)$, $w^{res}(r,k_n)$ and $u(r,k)$ are all eigenstates of the same Hamiltonian. Romo shows [12] that the following orthonormality relations hold among these functions

$$\langle \tilde{w}^{\{b\}}_{res}(r,\tilde{k}_p) | w^{\{b\}}_{res}(r,k_{p'}) \rangle = \delta_{pp'} \quad , (p,p' = m,n) \; , \tag{B.6}$$

$$\langle \tilde{w}^{\{b\}}_{res}(r,\tilde{k}_p) | u(r,k) \rangle = 0 \quad , \tag{B.7}$$

$$\langle u(r,k') | u(r,k) \rangle = \delta(\varepsilon' - \varepsilon) \quad . \tag{B.8}$$

The non-resonant radial wave function $u^{NR}(r,k)$ is not an eigenstate of the above mentioned Hamiltonian. We may use eqns. (B.6) - (B.8) and eqn. (B.1) for $u^{(+)}(r,k)$ to derive the following relations

$$\langle w^{b}(r,k_m) | u^{NR}(r,k) \rangle = 0 \quad , \tag{B.9}$$

$$\langle \tilde{w}^{res}(r,\tilde{k}_n) | u^{NR}(r,k) \rangle = \frac{(\frac{1}{2\pi}\Gamma_n)^{1/2}}{\varepsilon - \varepsilon_n + \frac{i}{2}\Gamma_n} \quad , \tag{B.10}$$

$$\langle u^{(+)}(r,k') | u^{(+)NR}(r,k) \rangle = \delta(\varepsilon' - \varepsilon) \quad , \tag{B.11}$$

$$\langle u^{(+)NR}(r,k') | u^{(+)NR}(r,k) \rangle = \delta(\varepsilon' - \varepsilon) + \sum_n \frac{\frac{1}{2\pi}\Gamma_n}{(\varepsilon - \varepsilon_n + \frac{i}{2}\Gamma_n)(\varepsilon' - \varepsilon_n - \frac{i}{2}\Gamma_n)} \quad . \tag{B.12}$$

Eqns. (B.9) - (B.12) reduce to eqns. (B.7) and (B.8) far from resonance ($u^{NR} \equiv u$). In the limit when the SPR becomes a bound state ($\varepsilon_n < 0$, $\Gamma_n = 0$), eqn. (B.10) reduces to eqn. (B.9) and eqn. (B.12) reduces to eqn. (B.8).

Appendix C : Determination of the Background Term in eqn. (2.3.8)

In the application of the wave function at resonance to a study of direct reactions, we did not require to know the background phase shift, $\xi_n^{\ell j}$, and the background wave function, $u_{\ell j}^{(+)NR}$, appearing in eqns. (2.3.7) and (2.3.8). In a study of angular distributions we were only interested in the contribution of the resonance, and the phase factor, $e^{i\xi_n^{\ell j}}$, appearing in the definition of the Gamow function went out when the modulus of the transition amplitude was taken. When we investigated the interference effects in the energy spectrum we used the function $u_{\ell j}^{(+)}$, and did not separate out the background and resonance contributions. However if we are to perform the structure calculation implied in section 2.4 (i.e. diagonalize the matrix $\underline{C}$ in eqn. (2.4.19)) we need to know both $\xi_n^{\ell j}$ and $u_{\ell j}^{(+)NR}$.

The Gamow function, $w_{\ell j}^{res}$, may be determined up to the phase factor $e^{i\xi_n^{\ell j}}$ by the method of Coker and Hoffmann [13]. They solve the equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + v_{eff}(r)\right] \omega_{\ell j}^{res}(r, k_n^{\ell j}) = \left(\varepsilon_n^{\ell j} - \tfrac{i}{2}\Gamma_n^{\ell j}\right) \omega_{\ell j}^{res}(r, k_n^{\ell j}) \quad , \qquad (C.1)$$

where the effective potential is given by

$$v_{eff}(r) = \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2} + v_{COUL}(r) + v_{CENT}(r) + v_{S.O.}(r)\,\mathcal{K}_{\ell j} \quad , \qquad (C.2)$$

$$\mathcal{K}_{\ell j} = \begin{cases} -(\ell+1) \quad , & \text{for } j = \ell - \tfrac{1}{2} \quad , \\ +\ell \quad , & \text{for } j = \ell + \tfrac{1}{2} \quad . \end{cases} \qquad (C.3)$$

The full scattering radial wave function $u_{\ell j}(r,k)$ satisfied the equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + v_{\ell\text{eff}}(r)\right] u_{\ell j}(r,k) = \mathcal{E}\, u_{\ell j}(r,k) \qquad (C.4)$$

In eqn. (C.4), we replace $u_{\ell j}(r,k)$ by

$$u^{NR}_{\ell j}(r,k) - \frac{\left(\frac{1}{2\pi}\Gamma_n^{\ell j}\right)^{1/2}}{\mathcal{E} - \mathcal{E}_n^{\ell j} + \frac{i}{2}\Gamma_n^{\ell j}}\, \omega^{RS}_{\ell j}(r, k_n^{\ell j}) \qquad (C.5)$$

Using eqns. (C.1), (C.4) and (C.5), we obtain

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + v_{\ell\text{eff}}(r)\right] u^{NR}_{\ell j}(r,k) = \mathcal{E}\, u^{NR}_{\ell j}(r,k) - \left(\frac{1}{2\pi}\Gamma_n^{\ell j}\right)^{1/2} \omega^{RS}_{\ell j}(r, k_n^{\ell j}) \qquad (C.6)$$

The most general regular solution of eqn. (C.6) may be written as

$$u^{NR}_{\ell j}(r,k) = A(k)\, u_{\ell j}(r,k) + \left(\frac{1}{2\pi}\Gamma_n^{\ell j}\right)^{1/2} \int_0^\infty dr'\, g^{(+)}_{\ell j}(r,r';k)\, \omega^{RS}_{\ell j}(r', k_n^{\ell j}) \,, \qquad (C.7)$$

where the Green function $g_{\ell j}(r,r';k)$ is a solution of the equation

$$\left[\mathcal{E} + \frac{\hbar^2}{2m}\frac{d^2}{dr^2} - v_{\ell\text{eff}}(r)\right] g_{\ell j}(r,r';k) = \delta(r-r') \qquad (C.8)$$

Indeed, if we use the separable approximation for $g^{(+)}_{\ell j}$ valid near $k \simeq k_n^{\ell j}$ and the normalization property of the Gamow

function, eqn. (C.7) reduces to eqn. (2.3.8) with $A(k) = e^{i[\delta_{\ell j}(k) - \xi_n^{\ell j}]}$.

A practical method for obtaining $\xi_n^{\ell j}$ and $u_{\ell j}^{NR}$ is the following. The solution of eqn. (C.1) gives $w_{\ell j}^{res}$ (up to the phase factor $e^{i\xi_n^{\ell j}}$), $\varepsilon_n^{\ell j}$ and $\Gamma_n^{\ell j}$ while the solution of eqn. (C.4) gives $u_{\ell j}(r,k)$. By performing a phase shift analysis we obtain the phase shift $\delta_{\ell j}(\varepsilon_n^{\ell j})$ at the energy $\varepsilon_n^{\ell j}$. Then the difference

$$\delta_{\ell j}(\varepsilon_n^{\ell j}) - \pi/2 \quad , \tag{C.9}$$

gives the background phase shift $\xi_n^{\ell j}$. From eqn. (2.3.8) we have

$$u_{\ell j}^{NR}(r,k) = e^{i[\delta_{\ell j}(k) - \xi_n^{\ell j}]} u_{\ell j}(r,k) + \frac{(\frac{1}{2\pi}\Gamma_n^{\ell j})^{1/2}}{\varepsilon - \varepsilon_n^{\ell j} + \frac{i}{2}\Gamma_n^{\ell j}} w_{\ell j}^{res}(r, k_n^{\ell j}) \tag{C.10}$$

Using eqn. (2.2.20) we obtain χ_E^{cNR} as

$$\chi_E^{cNR} = \mathcal{A}_A \{ u_{\ell j}^{NR}(r,k) \varphi_c \} \quad . \tag{C.11}$$

The above procedure gives χ_E^{cNR} inside the nucleus. In a structure calculation, because the background wave function χ_E^{cNR} appears only in matrix elements of V_A , this is the only region of r-space that is required.

Appenxix D : Determination of the Scattering Matrix from eqn (2.4.22)

We may obtain an expression for the scattering matrix by looking at the asymptotic form of eqn. (2.4.22). As we have noted in section 2.4, in order for the wave function, $\psi_E^{\bar{c}(+)}$ to have a meaning for large distances, the Gamow states in $\varphi_{t,A}$ must be evaluated at the real energies of the SPR. From eqns. (C.11), (2.3.6) and (2.2.15) we find

$$\lim_{r\to\infty} \chi_E^{c(+)NR} = \lim_{r\to\infty} \left\{ \varphi_c(1,\ldots,A) \frac{i}{k} \left(\frac{m}{2\pi k}\right)^{1/2} \exp(i\xi_c) A^{-1/2} \right. \times \left. \left[\exp(-i\xi_c) I_{\ell j}(r,k) - \exp(i\xi_c) O_{\ell j}(r,k) \right] \right\} . \tag{D.1}$$

In an open channel c', the asymptotic behaviour of the Green function

$$K_{c'}^{(+)} = \int dE' \frac{|\chi_{E'}^{c'NR}\rangle\langle\chi_{E'}^{c'NR}|}{E^+ - E'} , \tag{D.2}$$

is [3]

$$\lim_{r\to\infty} K_{c'}^{(+)} = 2\pi i \lim_{r\to\infty} \left\{ \varphi_{c'}(1,\ldots,A) \frac{i}{k'} \left(\frac{m}{2\pi k'}\right)^{1/2} A^{-1/2} \right. \times \left. O_{\ell'j'}(r,k') \exp(i\xi_{c'}) \chi_{E'}^{c'}(1',\ldots,A') \right\} . \tag{D.3}$$

If there is a SPR in the channel c' (= c'_o), the asymptotic contribution from the function $\varphi_{t,A}$ is, using eqns. (2.4.21), (2.3.11) and (2.3.7),

$$\lim_{r\to\infty} \varphi_{t,A} = 2\pi i \lim_{r\to\infty} \left\{ \varphi_{c_0'}(1,\ldots,A) \frac{i}{k} \left(\frac{m}{2\pi\hbar^2}\right)^{1/2} A^{-1/2} \right.$$

$$\left. \times \left[-\left(\frac{1}{2\pi}\Gamma_{c_0'}\right)^{1/2} \exp(i\xi_{c_0'}) O_{\ell' j'}(r,k') \right] \right\} \quad . \tag{D.4}$$

In eqn. (D.4) we have replaced the complex wave number $k_{n'}^{\ell' j'}$ by the real wave number in its neighbourhood.

Substituting eqns. (D.1), (D.3) and (D.4) into eqn. (2.4.22) we obtain for an isolated resonance, in an open channel c' and for $r \to \infty$,

$$\psi_E^{c(+)} \sim \varphi_{c'}(1,\ldots,A) \frac{i}{k} \left(\frac{m}{2\pi\hbar^2}\right)^{1/2} A^{-1/2}$$

$$\times \left[\delta_{cc'} I_{\ell' j'}(r,k') - S_{cc'} O_{\ell' j'}(r,k') \right] \quad , \tag{D.5}$$

where

$$S_{c'c} = \exp(i\xi_c + i\xi_{c'}) \left[\delta_{c'c} - 2i\pi \frac{G_{tc'} G_{tc}}{E - E_t + \frac{i}{2}\Gamma_t} \right] \quad , \tag{D.6}$$

and

$$G_{tc} = O_{tc_0} \left(\frac{1}{2\pi}\Gamma_{c_0}\right)^{1/2} \delta_{cc_0} - \langle \varphi_{t,A} | V_A | \chi_E^{c\,NR} \rangle \quad . \tag{D.7}$$

Since in obtaining the quantities $O_{t\alpha}$ ($\alpha = 1,\ldots, M, \ldots, \Lambda_o$) and $\langle \varphi_{t,A} | V_A | \chi_E^{cNR} \rangle$, the Gamow functions are needed only inside the nucleus, eqn. (D.7) is almost identical with eqn. (2.4.23).

Equation (D.6) is consistent with the unitarity of the scattering matrix provided the amplitudes G_{tc} are real and satisfy

$$\Gamma_t = 2\pi \sum_{c(\text{open})} G_{tc}^2 \quad . \tag{D.8}$$

This is the sum rule that must be satisfied if the ratio of the experimental and the single-particle cross-sections are to measure the spectroscopic factor in stripping and inelastic scattering to decaying states (eqns. (3.3.4) and (4.2.10)).

Finally, we note that eqn. (D.6) reduces, in the single particle limit, to the form

$$S_{cc} = e^{2i\xi_c}\left[1 - i\frac{\Gamma_c}{E-E_c+\frac{i}{2}\Gamma_c}\right] \quad , \tag{D.9}$$

(c.f. eqn. (2.3.3)).

Appendix E : Details of the Calculation

We evaluate the quantity in eqn. (3.2.12) viz.

$$\beta_{\ell\lambda}(\underline{k}_y, \underline{k}_d) = \frac{i^{-\ell}}{\sqrt{2\ell+1}} \lim_{\alpha \to 0} \int d^3r \, e^{-\alpha r} \chi^{(-)*}_{yA}(\underline{k}_y, \underline{r}') \frac{\omega^{\{b_{res}\}}_{\ell j}(r, k_n^{ij})}{r} Y^*_{\ell\lambda}(\hat{r}) \chi^{(+)}_{dA-1}(\underline{k}_d, \underline{r}) \quad . \tag{E.1}$$

Choosing the coordinate system such that $\hat{k}_d$ defines the z-axis and $\hat{k}_d \times \hat{k}_y$ the y-axis, we expand $\chi^{(+)}_{dA-1}$ and $\chi^{(-)*}_{yA}$ in partial waves

$$\chi^{(+)}_{dA-1}(\underline{k}_d, \underline{r}) = \frac{\sqrt{4\pi}}{k_d r} \sum_{\ell'} \sqrt{2\ell'+1} \; i^{\ell'} u_{\ell'}(k_d, r) Y_{\ell' 0}(\hat{r}) \quad , \tag{E.2}$$

$$\chi^{(-)*}_{yA}(\underline{k}_y, \underline{r}') = \frac{4\pi}{k_y r'} \sum_{\ell''\lambda''} i^{-\ell''} u_{\ell''}(k_y, r') Y^*_{\ell''\lambda''}(\hat{r}) \, Y_{\ell''\lambda''}(\theta, 0) \quad . \tag{E.3}$$

In eqns. (E.2) and (E.3), u_ℓ, and $u_{\ell''}$ are the distorted radial wave functions. Asymptotically they behave as

$$u_\ell(k, r) \simeq \frac{i}{2} e^{i\sigma_\ell} \left[I_\ell(k, r) - S_\ell(k) O_\ell(k, r) \right] \quad , \tag{E.4}$$

where σ_ℓ is the Coulomb phase shift, S_ℓ is the S-function for elastic scattering in a complex potential and O_ℓ and I_ℓ are the Coulomb-distorted outgoing and incoming solutions.

Substitution of eqns. (E.2) and (E.3) into eqn. (E.1) and integration over the angle $\hat{r}$ yields

$$\beta_{\ell\lambda}(\underset{\sim}{k}_\gamma, \underset{\sim}{k}_\alpha) = \frac{4\pi}{k_\alpha k_\gamma} \frac{M_A}{M_{A-1}} \sum_{\ell', \ell''} i^{\ell'-\ell''-\ell} \sqrt{2\ell''+1}\, (\ell''0\ell 0 | \ell' 0)$$

$$(\ell''\lambda\, \ell\, -\lambda | \ell' 0)\, Y_{\ell''-\lambda}(\theta, 0)\, \mathcal{R}^{\ell}_{\ell''\ell'}(k_\gamma, k_\alpha) \quad , \tag{E.5}$$

where

$$Y_{\ell''-\lambda}(\theta, 0) = \left[\frac{2\ell''+1}{4\pi} \frac{(\ell''-|\lambda|)!}{(\ell''+|\lambda|)!}\right]^{1/2} P_{\ell'',-\lambda}(\cos\theta) \quad , \tag{E.6}$$

and

$$\mathcal{R}^{\ell}_{\ell''\ell'}(k_\gamma, k_\alpha) = \lim_{\alpha\to 0} \int_0^\infty dr\, e^{-\alpha r}\, u_{\ell''}(k_\gamma, r) \left\{\frac{\omega^{\{b\}}_{\ell j}(r, k_n^{\ell j})}{r}\right\} u_{\ell'}(k_\alpha, r) \quad . \tag{E.7}$$

We have used the programs developed by Dr. S.M. Perez to evaluate the following quantities:

(i) The Clebsch-Gordan coefficients $(\ell_1\lambda_1\ell_2\lambda_2 | \ell_3\lambda_3)$

(ii) The associated Legendre functions $P_{\ell,\lambda}(\cos\theta)$

(iii) The distorted radial wave functions $u_\ell(k,r)$

(iv) The bound state radial wave functions $w^b_\ell(r, {}^{\ell}_{n})$

(v) The scattering state radial wave function $w^{res}_\ell(r, k^\ell_n)$, for real potential and giving a phase shift of $\frac{\pi}{2}$ at the wave number k^ℓ_n .

When the function $w^b_{\ell j}$ is present in eqn. (E.7), the convergence factor is not needed. In our approximated evaluation of the Gamow state, $w^{res}_{\ell j}$, the convergence factor is again not needed. The integration in eqn. (E.7) is

performed using the Simpson 3-point formula. The integration is stopped at r = R such that

$$\omega^{b}_{\ell j}(r \geq R, k_n^{\ell j}) < 10^{-3} \quad ,$$

or at r = R such that

$$\omega^{res}_{\ell j}(R, k_n^{\ell j}) = 0 \quad , \qquad R > R_{NUCLEAR} \, .$$

The summation over ℓ' and ℓ'' in eqn. (E.5) is stopped after the moduli of the elastic scattering functions $S_{\ell'}$ and $S_{\ell''}$ have become unity.

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