

The Strange-Quark Mass From QCD Sum Rules: An Update

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Three different ways of determining the strange quark mass using QCD sum rules are reviewed. First, from a QCD sum rule determination of the up and down quark masses, together with the current algebra ratio $m_s/(m_u + m_d)$. Second and third, from QCD sum rules involving the strangeness changing vector and axial-vector current divergences, respectively. Present results are encompassed in the value: $m_s(1 \text{ GeV}) = 150 \pm 70 \text{ MeV}$. It is argued that until *direct*, and precision measurements of the relevant spectral functions become available, the above error is not likely to be reduced.

It is probably unnecessary, in such a workshop, to emphasize the importance of knowing the value of the strange quark mass with reasonable accuracy. Most of kaon-physics, plus CP violation, depend strongly on m_s . Also, sophisticated modern multiloop calculations in weak hadronic physics will not achieve their full potential of improving theoretical precision, unless the uncertainty on the value of m_s is brought under control. A couple of years ago, it seemed that this was the case, as progress was made in understanding and removing quark mass singularities from light-quark correlators [1]-[2]. This allowed for a better control of the theoretical (QCD) side of the sum rules in e.g. the $\Delta S = 1$ scalar channel, where experimental data on the $K - \pi$ phase shifts allows, in principle, to *reconstruct* the hadronic spectral function. The word *reconstruct* is crucial, since it is no substitute for a direct and precision measurement of the spectral function, as I shall argue here. In fact, a recent re-analysis [3] of these QCD sum rules, using the same $K - \pi$ phase shift primary data, but a different reconstruction of the spectral function, has made this evident.

I start with the determination of m_s from the current algebra ratio [4]

$$\frac{m_s}{m_u + m_d} = 12.6 \pm 0.5, \quad (1)$$

together with a QCD sum rule determination of $(m_u + m_d)$. The latter was discussed some years ago [5] in the framework of QCD Finite Energy Sum Rules (FESR) in the pseudoscalar channel, at the two-loop level in perturbative QCD, and including non-perturbative condensates up to dimension-six, with the result

$$(m_u + m_d)(1 \text{ GeV}) = 15.5 \pm 2.0 \text{ MeV}. \quad (2)$$

This result, together with Eq.(1), implies

$$m_s(1 \text{ GeV}) = 195 \pm 28 \text{ MeV}. \quad (3)$$

A recent re-analysis [6] of the same QCD sum rules, but including the next (3-loop) order in perturbation theory, quotes

$$(m_u + m_d)(1 \text{ GeV}) = 12.0 \pm 2.5 \text{ MeV}, \quad (4)$$

which, using Eq.(1), leads to

$$m_s(1 \text{ GeV}) = 151 \pm 32 \text{ MeV}. \quad (5)$$

The problem here is that essentially the same raw data for resonance masses and widths, plus the same threshold normalization from chiral perturbation theory, has been used in [5] and [6]. The main difference is in the reconstruction of the spectral function from the resonance data; with a more ornamental functional form being adopted in [6]. In fact, the difference between the results of both analyses cannot be accounted

for by the inclusion (or not) of the 3-loop perturbative QCD contribution, which amounts to a reduction of the 2-loop result, Eq.(2), of only a few percent. The difference lies in the functional form of the hadronic parametrization of the same set of raw resonance data. This exposes a type of *systematic* uncertainty of the QCD sum rule method, and will cease to be an uncertainty only after the pseudoscalar hadronic spectral function is measured *directly* and accurately (e.g. from tau-lepton decays). In the meantime, both results for m_s , Eqs. (3) and (5), are equally acceptable, and taken together provide a measure of underlying systematic uncertainties.

Next, I turn to the second alternative, which is based on the correlator of the strangeness-changing vector current divergences

$$\psi(q^2) = i \int d^4x e^{iqx} \langle 0 | T(\partial^\mu V_\mu(x) \partial^\nu V_\nu^\dagger(0)) | 0 \rangle, \quad (6)$$

where $V_\mu(x) =: \bar{s}(x)\gamma_\mu u(x) :$, and $\partial^\mu V_\mu(x) = m_s : \bar{s}(x)iu(x) :$, and the up and down quark masses have been neglected. To summarize the QCD sum rule technique [7] for determining m_s , one first makes use of Wilson's Operator Product Expansion (OPE) of current correlators at short distances, modified so as to include non-perturbative effects (parametrized by quark and gluon vacuum condensates). This provides a QCD, or theoretical expression of the two-point function which involves the quark mass to be determined, the QCD scale Λ_{QCD} , and the vacuum condensates (whose values are determined independently). Next, invoking analyticity and QCD-hadron duality, the QCD correlator can be equated to a weighted integral of the hadronic spectral function. Depending on the choice of weight, one obtains e.g. FESR, Laplace and Hilbert transform sum rules, Gaussian sum rules, etc.. The upper limit of the hadronic integral, the so called continuum (or asymptotic freedom) threshold, is a free parameter signalling the end of the resonance region and the beginning of the perturbative QCD domain. Ideally, predictions should be stable against reasonable changes in this parameter. If the hadronic spectral function

is known experimentally, then the quark mass is extracted with an uncertainty which depends on the experimental errors of the hadronic data, the value of the continuum threshold, and the uncertainties in Λ_{QCD} and the vacuum condensates. If the hadronic spectral function is not known directly from experiment, it can be reconstructed from information on resonance masses and widths in the chosen channel. This procedure introduces two major uncertainties, viz. the value of an overall normalization, and the question of a potential background (either constructive or destructive). The choice of a resonance functional form (e.g. Breit-Wigner), while contributing to the uncertainty, is not as important. The overall normalization can be constrained considerably by using chiral-perturbation theory information at threshold. This idea was first proposed in [8], and is now widely adopted in most applications of QCD sum rules. However, there still remains the underlying assumption that chiral symmetry is realized in the orthodox fashion. If this were not the case, as advocated e.g. in [9], then the overall normalization would have to be modified accordingly. The potential existence of a background, interfering with the resonances, should be a source of more serious concern. Depending on its size and sign, it could modify considerably the result for the quark mass, as will be discussed shortly. On the theoretical side, the OPE entails a factorization of short distance effects (absorbed in the Wilson expansion coefficients), and long distance phenomena (represented by the vacuum condensates). The presence of quark mass singularities of the form $\ln(m_q^2/Q^2)$ in the correlator Eq.(6) spoils this factorization. Early determinations of m_s [10] were limited in precision because of this. A first attempt to deal with this problem was made in [11], and a more complete treatment of mass singularities is discussed in [1]-[2].

In the specific case of the correlator (6), the availability of experimental data on $K - \pi$ phase shifts [12] allows, in principle, for a reconstruction of the hadronic spectral function, from threshold up to $s \simeq 7 \text{ GeV}^2$. In both [1] and [2], the functional form chosen for this reconstruction consisted in a superposition of two Breit-Wigner res-

onances, corresponding to the K_0^* (1430) and the K_0^* (1950) [13], normalized at threshold according to conventional chiral-symmetry. It was argued in [1] and [2] that the non-resonant background implicit in this threshold normalization was important to achieve a good fit to the $K - \pi$ phase shifts. In [1] and [2], the correlator (6) was calculated in perturbative QCD at the 3-loop level, with mass corrections up to the quartic order, and including non-perturbative quark and gluon vacuum condensates up to dimension four (the $d = 6$ condensates can be safely neglected [2]). Using Laplace transform sum rules, the results for the strange quark mass thus obtained were

$$\begin{aligned} m_s(1 \text{ GeV}) &= 171 \pm 15 \text{ MeV} \quad ([1]) \\ &= 178 \pm 18 \text{ MeV} \quad ([2]) \end{aligned} \quad (7)$$

The errors reflect uncertainties in the experimental data, in the value of Λ_{QCD} ($\Lambda_{\text{QCD}} \simeq 200 - 500 \text{ MeV}$), in the continuum threshold s_0 ($s_0 \simeq 6 - 7 \text{ GeV}^2$), and in the values of the vacuum condensates.

As mentioned before, an example of a potential systematic error affecting these results would be the presence of a background, beyond the one implicit in the chiral-symmetry normalization of the hadronic spectral function at threshold. Obviously, this is not included in (7). A reanalysis of this QCD sum rule determination of m_s [3] has uncovered this uncertainty. In fact, it is claimed in [3] that by using the Omnes representation to relate the spectral function to the $K - \pi$ phase shifts, it is necessary to include a background interacting destructively with the resonances. As a result, the hadronic spectral function is considerably smaller than that used in [1]-[2]. This in turn implies smaller values of m_s , viz.

$$m_s(1 \text{ GeV}) = 140 \pm 20 \text{ MeV} . \quad (8)$$

Still another source of systematic uncertainty, this time of a theoretical nature, has been unveiled in [14]. This has to do with the fact that the QCD expression of the correlator (6) is of the form

$$\psi(Q^2) \propto m_s^2(Q^2) \left(1 + a_1 \left(\frac{\alpha_s(Q^2)}{\pi} \right) + \right.$$

$$\begin{aligned} & a_2 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 + a_3 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3 + \\ & b_1 m_s^2(Q^2) \left(1 + c_1 \frac{\alpha_s(Q^2)}{\pi} + \dots \right) + \\ & \left. b_2 m_s^4(Q^2) \left(1 + c_2 \frac{\alpha_s(Q^2)}{\pi} + \dots \right) + \dots \right) \quad (9) \end{aligned}$$

Knowing both $m_s(Q^2)$ and $\alpha_s(Q^2)$ to a given order, say 3-loop, the question is: *to expand or not to expand* in the inverse logarithms of Q^2 appearing in (9) ?. A similar question arises after Laplace transforming the correlator, i.e. *to expand or not to expand* in inverse logarithms of the Laplace variable M_L^2 ?. It has been argued in [14] that it makes more sense to make full use of the perturbative expansions of the quark mass and coupling (known to 4-loop order), and hence not to expand them in (9). Numerically, it turns out that the non-expanded expression is far more stable than the truncated one, when moving from one order in perturbation theory to the next. In particular, as shown in [14], logarithmic truncation can lead to sizable overestimates of radiative corrections. This in turn implies an underestimate of the quark mass. In fact, after using untruncated expressions, together with the same hadronic spectral function parametrization as in [1]-[2], the authors of [14] find

$$m_s(1 \text{ GeV}) = 203 \pm 20 \text{ MeV} , \quad (10)$$

to be compared with the results (7) obtained from truncated expressions. Until the question of truncation (or not) becomes satisfactorily settled, one has to take the value (10) together with (7), and include (8) as well

Finally, I wish to present preliminary results [15] of Laplace transform QCD sum rules in the strange-pseudoscalar channel, i.e. for the correlator

$$\begin{aligned} \psi_5(q^2) &= i \int d^4 x e^{iqx} \\ &< 0 | T(\partial^\mu A_\mu(x) \partial^\nu A_\nu^\dagger(0)) | 0 > , \end{aligned} \quad (11)$$

where $A_\mu(x) =: \bar{s}(x) \gamma_\mu \gamma_5 u(x) :$, and $\partial^\mu A_\mu(x) = m_s : \bar{s}(x) i \gamma_5 u(x) :$. The QCD expression of this two-point function is trivially obtained from that of (6). The hadronic expression, though, is

quite different. There is, at present, preliminary information from tau-decays [16] in a kinematical range restricted by the tau-mass. We have reconstructed the spectral function, including in addition to the kaon-pole its radial excitations $K(1460)$ and $K(1830)$, normalized at threshold according to conventional chiral symmetry. In addition, we have incorporated the resonant sub-channels $\rho(770) - K$ and $K^*(892) - \pi$, which are of numerical importance (particularly the latter). We find

$$m_s(1\text{ GeV}) = 165 \pm 35\text{ MeV}, \quad (12)$$

which agrees nicely with results from the scalar channel.

The various QCD sum rule results discussed here can be summarized in the value

$$m_s(1\text{ GeV}) = 150 \pm 70\text{ MeV}. \quad (13)$$

The given error, although rather large, still does not include the possibility of an unconventional chiral symmetry normalization of the hadronic spectral functions [9]. Unfortunately, one can not do better at the moment. The good news are that logarithmic mass singularities can be dealt with satisfactorily [1]-[2], and that the 4-loop ($\mathcal{O}(\alpha_s^3)$) perturbative corrections are small. This reduces considerably the theoretical uncertainties of the method. However, some still remain, largely in the issue of the expansion (or not) in inverse logarithms, when taking products of (running) quark masses and the coupling constant. They affect $m_s(1\text{ GeV})$ by up to 20 - 25 %. The major source of error, though, lies in the hadronic sector. Direct and accurate measurements of the relevant spectral functions appear as the only hope to reduce the uncertainty in the quark masses.

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