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Performance Evaluation of Detection Algorithms for MIMO OFDM Systems

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requirement for the degree of Master of Science in Engineering

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Author's Declaration

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Abstract

The introduction of new applications and the internet have led to demand in speed, capacity and reliability. Next generation communication systems are challenged to offer high data rates with better quality of service. Current systems have got limitations and some of them stem from the physical layer performance.

Performance of wireless communication systems is limited by the fading in the radio channels. In order to exploit channel characteristics like multipath fading, Multi Input Multi Output (MIMO) technique was proposed to improve signal quality. Air interface has evolved from Single Input Single Output (SISO) in 1st generation network to MIMO which is currently being used in Wireless Local Area Network (WLAN). Improvement of signal quality and tolerance to noise also increased the system coverage and capacity. Radio channels introduce severe Inter-symbol Interference (ISI) in MIMO systems and this would require complex equalization. MIMO would be used in broadband systems that exhibit frequency selective fading. Orthogonal Frequency Division Multiplexing (OFDM) turns frequency selective channels into parallel flat fading channels hence coping with ISI. This has lead to intensive research in MIMO and OFDM systems because they offer great improvements when combined thus they have been proposed by different parties (Nortel, Ericsson and Motorola) as the basis of Next Generation Network (NGN) air interface.

Introduction of MIMO OFDM as the base air interface method for NGN will face a number of challenges from hostile channel conditions to interference from other users. This would result in an increase of detection complexity required for mobile systems. Complex detection will reduce the battery life of mobile devices because of the many calculations that have to be done to decode the signal. Very powerful detection algorithms exist but they introduce high detection complexity. Next Generation Networks will employ different MIMO systems, but this research will consider spatially multiplexed MIMO which is used to improve the data rate and network capacity. In NGN different

multi access modulation schemes will be used for uplink and downlink but they both have OFDM as the basic building block.

In this work performance of MIMO OFDM is investigated in different channels models and detection algorithms. A low complexity detection scheme is proposed in this research to improve performance of MIMO OFDM. The proposed detection scheme is investigated for different channel characteristics. Realistic channels conditions are introduced to evaluate the performance of the proposed detection scheme.

We analyze weaknesses of existing linear detectors and the enhancements that can be done to improve their performance in different channel conditions. Performance of the detectors is evaluated by comparison of Bit Error Rate (BER) and Symbol Error Rate (SER) against signal to noise ratio (SNR). This thesis proposes a detector which shows a higher complexity than linear detectors but less than Maximum Likelihood Detector (MLD). The proposed detector shows significant BER improvement in all channel conditions. For better performance evaluation this work also investigates performance of MIMO OFDM detectors in realistic channels like Kronecker and Weichselberger channel models.

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Acronyms and abbreviations

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
CP	Cyclic Prefix
CDMA	Code-Division Multiple Access
DL	Down Link
FDD	Frequency Division Duplex
FDM	Frequency Division Multiplexing
FEC	Forward Error Correction
FFT	Fast Fourier Transform
D-BLAST	Diagonal Bell Laboratories Layered Space Time
H-ARQ	Hybrid- Automatic Repeat Request
IFFT	Inverse Fast Fourier Transform
ISI	Inter-symbol Interference
LTE	Long Term Evolution
MISO	Multiple-Input Single-Output
MIMO	Multiple-Input Multiple-Output
MLD	Maximum Likelihood Detector
MMSE	Minimum Mean Square Error
M_T	Number of transmit antennas

N_R	Number of receive antennas
NGN	Next Generation Networks
OFDM	Orthogonal Frequency Division Multiplexing
PIC	Parallel Interference Cancellation
OSIC	Ordered Successive Interference Cancellation
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
PSD	Power Spectral Density
QAM	Quadrature Amplitude Multiplexing
QoS	Quality of Service
QPSK	Quadrature Phase Shift Keying
SC-FDMA	Single Carrier Frequency Division Multiple Access
SER	Symbol Error Rate
SFBC	Space Frequency Block Coding
SIC	Successive Interference Cancellation
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SM	Spatial Multiplexing
SNR	Signal to Noise Ratio
STBC	Space Time Block Coding
STTC	Space Time Trellis Coding

SVD	Single Value Decomposition
T-BLAST	Turbo Bell Laboratories Layered Space Time
TDD	Time Division Duplex
UL	Uplink
V-BLAST	Vertical BLAST
WLAN	Wireless Local Area Networks
WiMAX	Worldwide Interoperability for Microwave Access
ZF	Zero Forcing

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Chapter 1

1. Introduction

One of the challenges faced by future wireless communication systems is to provide high data rates at high quality of service (QoS). Combined with the fact that spectrum is a scarce resource and propagation conditions are hostile this requires radical increase in spectral efficiency and link reliability [1]. The system requirements can be met by the combination of two powerful technologies in the physical layer design: multi input and multi output (MIMO) antennas and orthogonal frequency division multiplexing (OFDM) modulation [2]. These two are considered to be the key enabling technologies that will help Long Term Evolution (LTE) and next generation networks exceed the current system performance [3]. Use of multiple antennas can offer significant increase in data throughput, spectral efficiency and link range without additional bandwidth or transmit power [4]. Link reliability and diversity are improved but this also increases frequency selective fading in the channels [5]. The primary advantage of OFDM is its ability to cope with severe channel conditions like frequency selective fading due to multipath propagation, attenuation of high frequency in long copper wire and narrowband interference [4].

Spatial multiplexing has been recognized as the MIMO technique to increase transmission capacity. In spatial multiplexing, Maximum Likelihood Decoder (MLD) is the best performing detection algorithm [6] but its complexity increases with the number of transmit antennas and constellation size. Therefore current research is trying to find well performing less complex detection algorithms [7] for different channel models and conditions.

Simulated performance of MIMO is strongly influenced by the choice of the channel model. Modeling of the radio channel is essential for system design and performance evaluation [8]. Inaccurate channel model selected for MIMO-OFDM can lead to suboptimal transmissions and incorrectly assessed performance of the algorithms [8]. Channel conditions like spatial correlation have been noticed to substantially impair the performance of MIMO wireless communication systems. Most of the research in MIMO OFDM performance has been concentrated on using independent and identically distributed Rayleigh channel model which will also be considered along with more realistic channel models with

spatial correlation. The overall goal of this work is to evaluate and improve the performance of spatial multiplexing (SM) MIMO OFDM system.

1.1 Problem Definition

It is clear that the combination of MIMO and OFDM will greatly improve the performance of the NGN but we should note that some of the individual problems of these techniques will be carried over to the new system. In MIMO systems as the number of antennas increases the capacity improves linearly but complexity of decoding the transmitted data grows.

Maximum Likelihood Detector complexity increases exponentially with the number of antennas and when modulation schemes with higher order constellation are used. Based on the MLD different suboptimal detection algorithms like Sphere Decoding (SD) have been proposed but they still offer higher complexity compared to linear detectors. Linear detectors are characterized by less complexity but they cannot be employed in practical systems because they have high BER. It has been shown in [9] that the performance can further be improved by the use of iterative decoding. Complexity of these detectors for spatially multiplexed systems is greatly reduced in comparison to MLD and SD.

Iterative decoding performs interference cancellation but when symbol cancellation is used the order in which the components are detected becomes important to the overall performance of the system [9]. Linear detector performance can be improved by applying it with interference cancellation. In interference cancellation other symbols are treated as interference to the symbol of interest. Two well discussed methods exist that are successive and parallel interference cancellation [27,70]. In parallel method, interference cancellation is performed in parallel to all the symbols transmitted whilst in successive, cancellation is performed to one symbol at a time. Even though successive has a longer delay than parallel method it has been shown to provide a better performance in multiuser systems [12]. On the other hand parallel interference cancellation is less complex than successive method. Parallel interference cancellation that has only been applied to multiuser systems will be applied to spatially multiplexed signals. A combination of linear detector and interference cancellation methods which has less complexity than MLD method [49] is investigated as a better solution for our system model.

A less complex interference cancellation method is introduced and compared to successive interference. In this thesis we propose a detection scheme which will be compared to the already existing detectors. For a better estimation of the detector performance different realistic channels models are used and the best detection method is recommended for the NGN systems.

1.2 Thesis Objectives

The main objective of this thesis is to carry out the performance evaluation of SM MIMO OFDM systems in different channel models. This work investigates the performance with different detection algorithms. The effect of detection algorithm on the overall system performance is brought into question. BER and output noise are used to compare the performance of detection algorithms.

The interference from other signals in spatially multiplexed systems is considered to be the same multi access interference, thus interference cancellation methods are used. The interference cancellation methods introduced are successive interference cancellation and parallel interference cancellation. The interference cancellation methods extended the linear detection algorithms to improve the system performance. The performance of these cancellation methods is investigated on different channel conditions. After the evaluation of the SM MIMO OFDM linear detectors the best performing suboptimal scheme is recommended for the next generation networks air interface.

1.3 Methodology

The system performance will be investigated through simulation in Matlab. First the SM MIMO OFDM system is implemented in Matlab and Monte-carlo simulations are carried out. Linear detectors are employed and tested in different channel conditions. Second, depending on their performance the best performing detectors are recommended for implementation with interference cancellation method.

The two interference cancellation methods used are serial and parallel cancellation. A new detection scheme is introduced and compared to cancellation methods. Lastly, the detectors are compared in realistic channel models and the system performance is evaluated. Recommendations for the system parameters and detection algorithms are made.

1.4 Scope and Limitations

This thesis investigates SM MIMO OFDM systems, evaluating the effect the detection algorithm has on the overall performance. This research also presents a brief discussion of the different types of MIMO technologies. This work will only discuss SM MIMO OFDM to detail as the basic technology for future networks air interface. This investigation begins with linear detector implementation extending to interference cancellation methods to improve system performance. The performance of the detectors will also be evaluated based their output noise power.

1.5 Report Structure

This thesis is organized in follows:

Chapter 2 Presents a background literature review of the two main technologies MIMO and OFDM that will be considered in this thesis. It also discusses proposed air interfaces for future networks highlighting the target performance expectations of the system under investigation.

Chapter 3 Provides an extensive discussion of different channel conditions. Ill-conditioning of channel matrix and its effect on performance are highlighted. A brief discussion is given for uncorrelated and correlated channels which are also referred as realistic channel. Kronecker and Weichselberger channel model correlation and ill-conditioning characteristics are discussed.

Chapter 4 Gives a brief description of SM MIMO and OFDM systems mathematical representation. Linear detectors are presented and their theoretical performance is discussed. Successive interference cancellation based detectors performance is evaluated through simulations and analytical calculation for output noise.

Chapter 5 describes the system model and the parameters used for the simulation. Investigate the performance of the linear detectors and interference cancellation methods including a hybrid canceller in different channel conditions. A detection scheme is proposed that uses the best performing linear detector.

Chapter 6 Investigates the antenna configuration for MIMO OFDM system. The performance of the introduced detector scheme is evaluated in comparison with the existing detectors. The bit error rate and output noise are used to evaluate the performance of the detectors. The best performing detectors are then tested with different realistic channel models.

Chapter 7 Concludes the thesis based on the simulation results and highlights contributions of this work. Thereafter recommendations for future research in this area are presented.

Appendix A: Simulation code in Matlab noise output calculation

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Chapter 2

2. Literature Review

This chapter presents a literature review of the work on MIMO-OFDM. The review starts with a discussion of the general expectations of next generation wireless communication networks highlighting the main technologies to be applied in air interface. It then addresses the advances in MIMO-OFDM research including their current application. Lastly, it gives a general overview on the MIMO and OFDM technologies. These two technologies are promising but this chapter highlights their weaknesses, detection complexity in MIMO and effect of correlation on system performance. In this work diversity schemes are discussed highlighting the features of MIMO making it suitable for next generation systems.

2.1 Background Information

The recent technological advances in wireless communication have been driven by emerging high data rate multimedia services. Advances have been made by the introduction of current 3G networks but in future they would not be complying with the demand. Recently we have seen proliferation of multimedia application such as video conferencing, network gaming and high quality video streaming. In order to cater for the changes, different bodies in wireless communication have taken paths to develop more efficient next generation networks (4G). The third generation partnership project (3GPP) has started the development and testing of Long Term Evolution (LTE) of 3G [59]. The other paths to 4G involve the Wireless Local Area Network (WLAN) and Wireless Metropolitan Area Network (WMAN) standards such as IEEE802.11n and IEEE802.16m (WIMAX). One of the technical challenges of next generation networks physical layer is to provide high spectral efficiency ($\approx 10 \text{ bit/Hz/s}$) and to handle severe frequency selectivity due to use of large bandwidth [61]. It is shown in [67] that the number of mobile broadband users in the world increases every year, thus the network capacity should increase too. The objectives of 4G are to offer connection anytime, anywhere and to anyone. In [66] the data rates targets were set to 100Mb/s for high mobility systems and up to

1Gbps for low mobility systems. Figure 2-1 shows the approximate data rate against mobility of wireless network [60].

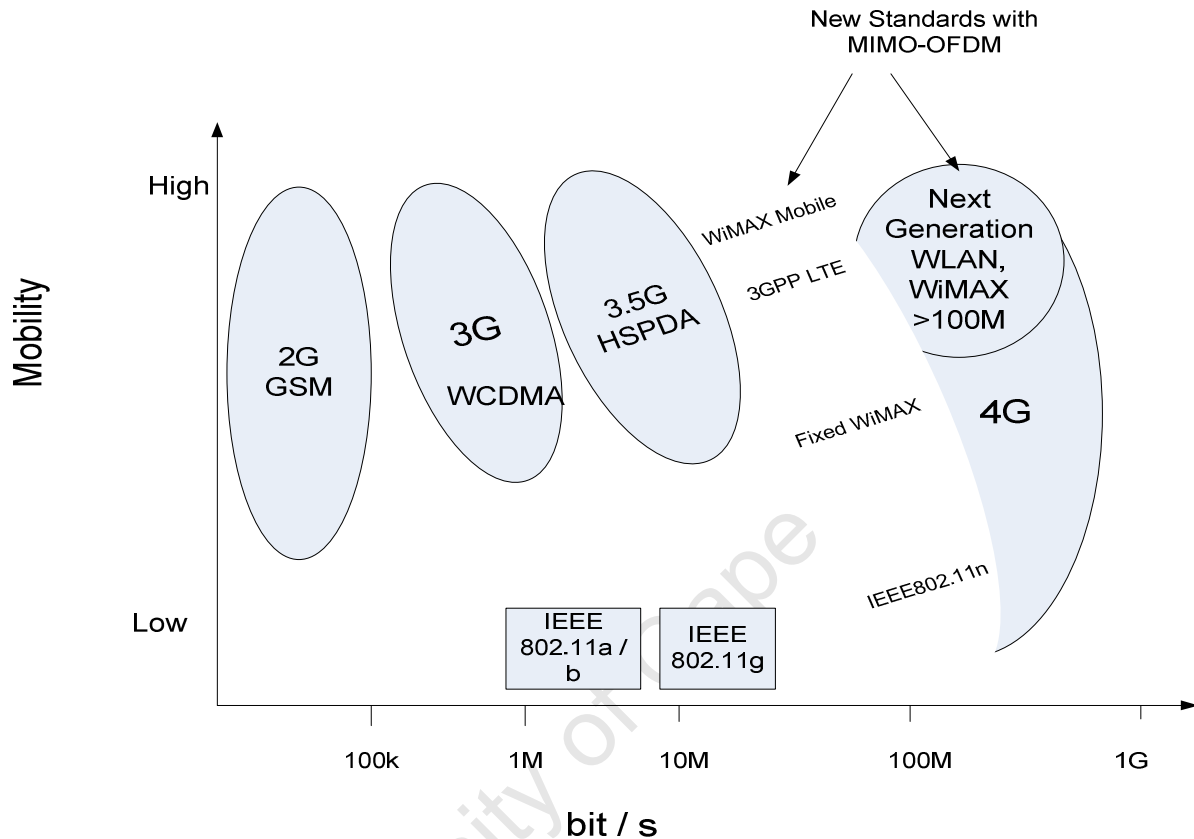


Figure 2-1: Mobility versus bit rate for communication networks.

Mobile wireless systems performance is lower than that of stationary wireless networks, thus the key for better systems is held in the air interface. Different air interface technologies are being discussed for next generation systems but all the contributing parties agree that the basis for the systems will be made of two main technologies MIMO and OFDM. Multiple Input Multiple Output (MIMO) according to [11] can offer increased spectral efficiency through spatial multiplexing gain and improved link reliability due to antenna diversity. Large scale MIMO systems deployment has been seen recently in WLANs and in WMANs. Spatially multiplexed MIMO systems called diagonal Bell Layered Space Time (D-BLAST) were first introduced by Foschini in [62]. Later Vertical Bell Layered Space Time (V-BLAST) was introduced by Bell labs in [16] as a more computationally efficient method than D-BLAST. This led to increase in research in MIMO systems and a simple two

transmitter two receiver (2×2) system Space Time Block Coding (STBC) method was proposed in [18] by Siavash M. Alamouti and the STBC architectures is further generalized to M_T transmitters and N_R receivers in [19].

Traditionally multiple antennas were used at the receiver side to offer interference cancellation and achieve diversity and array gain through coherent combining [1]. The use of multiple antennas on both sides has been shown [1, 11, 16, 18, and 62] to offer additional gain called spatial multiplexing gain that improves the spectral efficiency. MIMO performance greatly degrades in the channels that exhibit frequency selective fading that causes Inter-Symbol Interference (ISI). OFDM modulation turns the frequency selective channel into a set of parallel flat fading channels and hence it was attractive solution for coping with ISI. OFDM was initially introduced in the 60s by Chang when he published work on multichannel transmission [64]. OFDM could not maintain orthogonal subchannels, this was solved by the pioneering work of Peled and Ruiz [76] in 1980 who introduced the cyclic prefix. The wide area deployment of OFDM only came into the picture around the 90s when it was used in Digital Audio Broadcast (DAB) and Digital Video Broadcast (DVB) standards [1]. MIMO OFDM will be introduced by standards currently still under definition which include IEEE802.11n and IEEE802.16e/m. Future data communication cellular network, LTE which is still under testing will be using MIMO OFDM as the basic technology.

According to [1] OFDM use eliminates the ISI but the computational complexity of MIMO-OFDM spatial multiplexing receivers is high. MIMO-OFDM system is currently been tested in labs [12, 57, 58] and a number of design constrains have been highlighted. Antenna induced spatial channel correlation significantly reduces diversity and multiplexing gains. The gains in MIMO are reduced significantly when complex fades of more than 0.7 are present [12]. The performance of spatially multiplexed MIMO is dependent on the independence of channel paths but Line of Sight (LOS) introduces dependence in channel matrix. Large capacity gains are possible in MIMO when statistical distribution of condition number is low. LOS usually creates high condition numbers in channel matrix. Condition number is the ratio of maximum and minimum eigenvalues of MIMO channel matrix [12, 51]. In [63] David Gesbert and Jabran Akhar have investigated the effect of ill-conditioning in MIMO system proposing the use of constellation multiplexing.

Detection complexity of MIMO systems increases with the number of receivers [74, 48]. Maximum likelihood detector has the best performance but the implementation complexity is high. In spatially multiplexed MIMO, V-BLAST detection was proposed in [15] and it has low complexity and better performance than linear detectors. V-BLAST detection is also called successive interference cancellation which was previously used in multiuser Code Division Multi Access (CDMA) system. The performance of Successive Interference Cancellation (SIC) can be improved by optimal ordering as shown in [15, 16] which minimizes error propagation effect. In case of ill-conditioned channel matrix the performance of MIMO detectors is degraded, thus in this thesis we are going to look for a better performing less complex detector.

In literature authors have reported two transmitter two receiver (2×2) real time space time coding MIMO testbed reaching data rates of 30Mbps/s [54] and three transmitter three receiver (3×3) MIMO testbed reaching data rate of 281.25Mbps/s [53]. From 1999 to 2004 as shown by the Table [1-1] there has been a increase in the practical implementation of MIMO testbed that use OFDM as a modulation scheme. Bell laboratories testbed (8×12) spatially multiplexed MIMO system achieved data rate of 777.6Kbits/s. University of Bristol implemented a (4×6) MIMO-OFDM tested with QPSK modulation that offered 96Mbps/s [41].

Table 1-1: Practical MIMO OFDM testbed system performance comparison

Name	MIMO Configuration	Data Rate	Frequency/ Bandwidth	Spectral efficiency	Modulation
Georgia Tech testbed#1[53]	3x3	281.25Mbps	2.435 GHz/19.5 MHz	14.4 bits/Hz/s	64-QAM / OFDM
Georgia Tech testbed #2 [54] (real time mode)	2x2 (space time coding)	30Mbps	2.435 GHz/19.5 MHz	14.8 bits/Hz/s	64-QAM / OFDM
Bell Laboratory testbed [44]	8x12	777.6Mbps	1.9 GHz/30kHz	25.92 bits/Hz/s	16-QAM
Iospan wireless testbed [55]	2x3	13.6Mbps	2.5-2.6 GHz/2 MHz	6.8 bits/Hz/s	64-QAM / OFDM
University of Bristol testbed [55]	4x6	96Mbps	5 GHz/12 MHz	8 bits/Hz/s	QPSK / OFDM
Motorola testbed [56]	2x2	180Mbps	3.65 GHz/20 MHz	9 bits/Hz/s	64-QAM/OFDM
BYU testbed [57] (real time mode)	4x4	2Mbps	2.45 GHz/250 KHz	8 bits/Hz/s	QPSK
University of Michigan [52]	4x4	525Mbps		19.2bits/Hz/s	OFDM

Spatially multiplexed MIMO-OFDM systems with high configurations offer high throughput but the interest has been shifted to low number of transmitter and receivers systems (3×3) and (4×4) . This is because they require less complex channel estimations methods and detection algorithms. Georgia Tech testbed with three transmitters and three receivers reached 281.25Mbps/s with 64-QAM/OFDM modulation scheme. In [52] a four transmitter four receiver (4×4) wireless prototype managed to reach 525Mbps with OFDM modulation. In this work we investigate the effect of detection algorithms on the overall system performance. Simulations are going to be done to determine the performance of the detectors but for better performance evaluations realistic MIMO channel are used to test detectors

2.2 Diversity Schemes

Wireless channel suffers attenuation due to destructive superposition of multipaths in the propagation media and interference from other users. This makes it difficult for the receiver to reliably determine the transmitted signal. Multipath fading occurs when the transmitted signal reaches the receiver at a variety of paths with different angles, delays or frequency due to scattering of electromagnetic waves. The received signal power fluctuates in space (due to angle spread) and due to time (Doppler spread) apart from random superposition of multipath components [17]. Fading causes the reduction in detection reliability and the system is further constrained by the limitations of power and scarce frequency bandwidth.

Diversity schemes are used to improve the receiver's capability of determining the transmitted signal [12]. Diversity involves sending of less attenuated signal replicas to the receiver, so that it can reliably determine the transmitted signal [11]. In diversity the information reaches the receiver through statistically independent channels. Diversity can be classified into the following groups [12]:

- a. Temporal diversity the signals are transmitted in different time slots and the received signals are uncorrelated. For sufficient decorrelation the temporal distance must be at least $1/2v_{\max}$

where v_{\max} is the maximum Doppler frequency. It can be realized in different ways:

- i. Repetition coding is the simplest form where the signal is repeated several times with the repetition intervals being long enough to achieve correlation. Diversity is

achieved, but it is also bandwidth inefficient. Spectral efficiency decreases with a factor equal to number of repetitions.

- ii. Automatic repeat request (ARQ). Here the receiver sends a message to transmitter to indicate whether it has received data with sufficient quality. If not so the transmission is repeated after wait period that achieves decorrelation. It performs better than repetition method but the downside is that ARQ requires feedback channel.
 - iii. Combination of interleaving and coding. This can be viewed as a more advanced repetition coding which uses forward error correction and interleaving. Different symbols of a codeword are transmitted in different times which increase probability of at least receiving a signal with good SNR.
- b. Frequency diversity. Equal signals are transmitted in different frequencies. These frequencies are spaced apart by more than the coherence bandwidth of the channel, so their fading is approximately independent. The probability is low that the signal is in deep fade at both frequencies simultaneously.
- c. Polarization diversity uses antennas with different polarization for both transmit to receiver. Transmitter sends signal with same polarization but channel effects lead to depolarization. The fading of signals with different polarizations is statistically independent and dual polarized antenna can be used at the receiver side thus providing diversity. Polarization can provide diversity up to 6: three components of the E-field and three components of the H-field can all be exploited [13]. Propagation characteristics as well as practical consideration limit full exploitation of the diversity order.
- d. Angle diversity. The signals are transmitted at different angles. A fading dip is created when signals from different directions interfere destructively. Co-located antennas with different patterns see the received signal differently.

- e. Spatial diversity employs antenna diversity. It uses two or more antennas to improve the signal quality. The performance is influenced by correlation of the signals between the antenna elements. Time interleaving results in large delays for the system when channel is slowly varying therefore spatial diversity performs better than time diversity. Antenna diversity is a practical, effective technique for reducing multipath effect. Multiple antennas can be used at transmitter side offering transmit diversity in MISO or at the receiver offering receive diversity in SIMO or a combination of both in MIMO

2.3 MIMO systems

MIMO systems have attracted much attention because of high spectrum efficiency [13, 14] since they offer significant increases in data throughput or link range without additional bandwidth or transmit power. Single antenna system exploit, time and frequency dimensions only but the leverages of MIMO are realized by exploiting spatial dimension. Figure 2-1 shows the block diagram of a three transmitter three receiver (3 × 3) system.

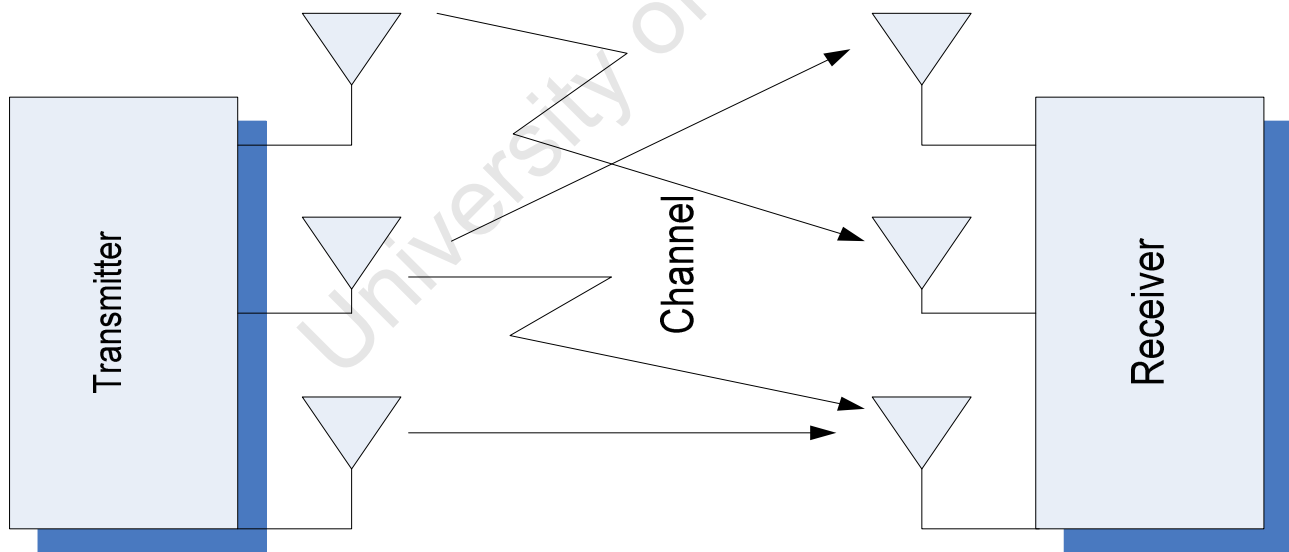


Figure 2-2: MIMO system block diagram

MIMO systems can be categorized into two groups; diversity coding and spatial multiplexing

- i. Diversity coding - Signals are coded in space time or frequency before transmission. Depend signals are transmitted at different times and used to decode the original signals. Diversity coding systems like STBC have got high diversity gains compared to any other systems.
- ii. Spatial multiplexing - If individual transmit and receive antenna paths fade independently, the channel matrix is well conditioned, this creates multiple parallel spatial channels. Transmitting the data through the spatial channels will increase the data rate. This effect is called spatial multiplexing and this can be exploited in BLAST receiver family.

2.3.1 Benefits of MIMO

Multiple antennas can improve system performance in two aspects which are reliability and supporting higher data rates. MIMO systems can provide significant performance gains through array gain, spatial diversity gain, spatial multiplexing gain and interference reduction [17]. These gains are described below

- i. Array gain: this can also be called power gain; it results from coherent detection of the transmitted signal at a receiver through spatial processing or spatial pre processing at transmitter antennas. It is called power gain because it improves the signal SNR.
- ii. Interference reduction and avoidance: interference occurs when multiple signal from multiple users are transmitted sharing time and frequency domains. Exploiting the spatial domain will help reduce the interference between users by increasing the separation. Array gain increases the noise tolerance hence improving interference signal to noise plus interference ratio (SINR). This also helps increase the system range and coverage.
- iii. Diversity Gain: this provides the transmitter with multiple copies of the signal in space, time or frequency domain increasing the reliability of decoding the transmitted signal. All the transmitted signals following different paths experience different fading, therefore one of them will be decoded correctly. A system with M_T transmitters and N_R receivers will offer $M_T N_R$ independent fading paths hence the spatial diversity gain is $M_T N_R$.

- iv. Multiplexing Gain: this is achieved by spatially multiplexing independent data streams through multiple transmitters. Spatial multiplexing helps increase the data rates within operating bandwidth and the gain increases system capacity. The number of data streams that be supported by a MIMO channel equals the minimum number of transmit antennas and number of receive antennas that is $\min\{M_T, N_R\}$.

Maximizing one of the gains will not necessarily maximize the other. The discussed MIMO systems will offer different gains. Orthogonal designs in STBC, Space Frequency Block Coding (SFBC) and Space Time Trellis Coding (STTC) will maximize diversity gain but will also provide low spatial multiplexing gain. On the other hand the BLAST family will provide spatial multiplexing gain. Space Time layered architectures offer increase in capacity promising a linear growth with the size of the antenna array under some circumstances. Advanced MIMO spatial multiplexing techniques (2 or 4) × (2 or 4) downlink and uplink supported and also multi- user MIMO also supported

2.3.2 Spatial Multiplexing Systems

In 1996 Foschini proposed a diagonal layered architecture called D-BLAST. From this stemmed the other systems like V-BLAST and Turbo BLAST [15]. In this section we discuss all the BLAST family members highlighting their problems and also noting the reasons why V-BLAST has been the most successful for practical implementation.

Turbo BLAST

Turbo BLAST [15] was proposed in [16] by Sellathurai and Haykin. It is based on Turbo principle and it was later generalized to Threaded Space Time Architecture (TST) in [17] by ElGamal and Hammons.

Encoder: The information bits are demultiplexed and the obtained substreams are independently encoded with FEC codes. Diagonal interleaver is used to interleave the substreams before they are mapped to symbols which are then transmitted. Each symbol can have bits from more than one stream therefore the symbol error spreads the bit error across streams.

Decoder: Decoding the interweaved streams is very expensive computationally, being exponential in the number of substreams, constellation and block sizes. The suboptimal algorithm is based on the

interpretation of the Turbo encoder as a group of block codes where “outer code” is connected with an “inner coder” through parallel interleavers. The receivers therefore has two decoders, inner decoder is meant to cope with inter symbol interference and the outer decoder aims to correct symbol errors that occurred during transmission. After required iterations the soft decisions from the decoders are sent to hard limiters.

D-BLAST

It was proposed by Foschini and it reached capacities close to Shannon limit [15]. It consists of M transmitters and N receivers. Information bit streams from source are demultiplexed into substreams which are then coded separately into symbols. These symbols are then dispersed “diagonally” across antennas and time. As shown in figure [2-2] at an instant each antenna transmits a different layer from the other. D-BLAST cannot reach Shannon capacity because time and space is wasted before the transmission of new symbol. This is experienced when the number of symbols in layer is more than the number of antennas. Symbols are spread across the antennas, therefore this system captures transmit diversity.

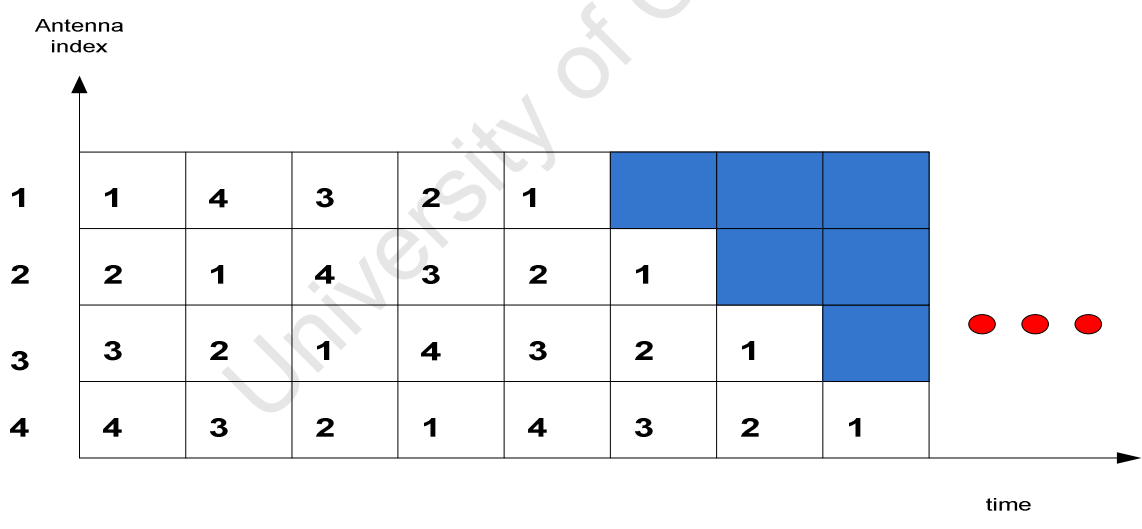


Figure 2-2. Numbers in boxes represent the layer that can transmit at that antenna and the filled blocks show space time wastage

Decoder: Symbols are decoded one layer at a time with the first symbol of a layer granted to be corrected since it was transmitted alone. The next symbol is decoded but the interference also increases as we progress. Substream associated with layer is decoded after all the symbols in layer

have been detected. The decoded layer is subtracted to expose the next layer but it should be decoded correctly to reduce error propagation in the system. A simplified version of D BLAST was developed that tries to reduce the computational complexity but doing so the transmit diversity is lost [15]

V-BLAST

V-BLAST has an extra ordinarily bandwidth efficient approach with spectral efficiency ranges from 20 to 40 bps/Hz whilst traditional systems have ranges from 1 to 5 bps/Hz for mobile cellular [16].

Encoder: The user's information data is split into multiple substreams which are transmitted simultaneously from the multiple antennas. Generally the system consists of M transmitting antennas and N receiving antennas such that $N_R \geq M_T$. Each substream at the transmitter is mapped to a symbol by the same constellation as the other $M - 1$ transmitters. The laying is horizontal thus all symbols from the same stream are transmitted through same antenna

Decoder: Signals are decoded immediately at the receiver using maximum likelihood (ML). The ML decoder solves;

$$\hat{s} = \underset{s}{\operatorname{argmin}} \left\| y - \sqrt{\frac{E_s}{N_T}} Hs \right\|_F^2 \quad (2.1)$$

where s is the transmitted signal, H is the channel matrix, y is the received signal. Several detectors like zero forcing and minimum mean square error can be used for detection of the transmitted signal. Linear detectors have got less complexity compared but ML detector has better performance. Successive interference cancellation when the other symbols are treated as interference to the symbol of interest.

Comparison of Space Time Coding and Spatial Multiplexing

As shown in table 2-2 in [33] Alexis Dowhuszko summarizes the difference in performance of the two MIMO schemes.

Table 2-2. Space Time Block Coding and Spatial Multiplexing comparison

MIMO Scheme	STBC	Spatial Multiplexing
Data rate	U → High S → Low	S → High U → Low
Diversity gain	S	U
Spatial correlated channel	S	S → Low U → High
Channel estimation	Insensitive	Insensitive → Low Sensitive → High
LOS	S	U
Antenna configuration	S → 2Tx	S → Tx ≥ Rx
S: Suitable U: Unsuitable		

Advanced MIMO spatial multiplexing techniques (2 or 4) × (2 or 4) downlink and uplink supported in LTE and also Multi user MIMO also supported [26].

2.4 OFDM

Wireless communication systems are affected by frequency selective fading degrading system performance. OFDM is used to increase robustness against frequency selective or narrowband interference. OFDM is a technique used to broadcast media like European terrestrial digital television (DVB-T) and Digital Audio broadcasting (DAB) [1].

2.4.1 OFDM Advantages

OFDM can be considered as a modulation or multiplexing scheme [20]. Modulation is the mapping of the information on a waveform with varying phase or frequency or amplitude or any combination in order to carry information. Multiplexing is the method of combining signals to share medium like bandwidth. OFDM is a multicarrier technique which divides the available channel into subchannels of low rate. OFDM is similar to frequency division multiplexing (FDM), some authors argue it's a type of FDM [21] but OFDM saves bandwidth compared to FDM. In FDM the frequency channel is divided into M non overlapping subchannels. These subchannels are modulated separately whilst in OFDM the subcarriers overlap thus saving bandwidth. The subchannels in OFDM are orthogonal and close to each other compared to FDM. An illustrative comparison of OFDM and FDM is shown in figure 2-3

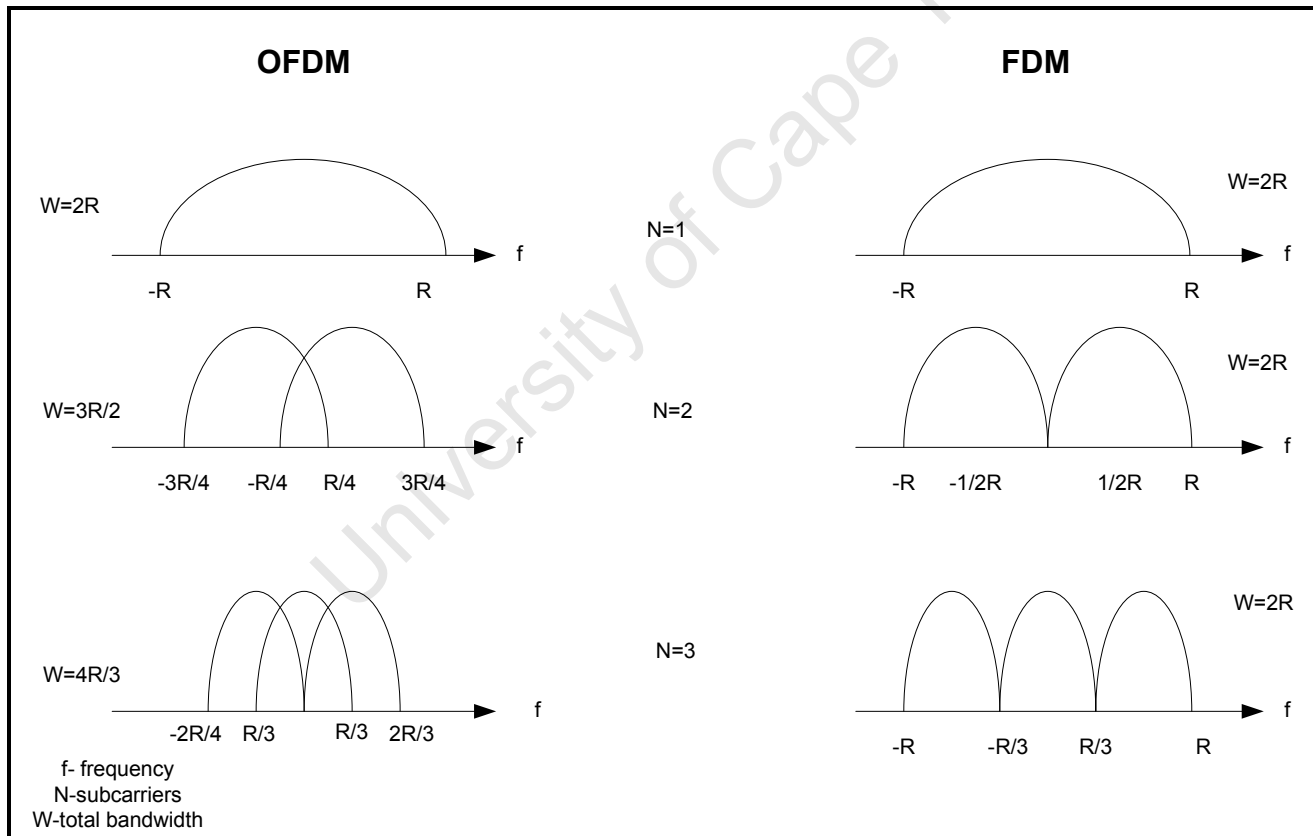


Figure 2-3. OFDM versus FDM technique

FDM signal occupies $2R$ bandwidth whilst OFDM occupies $(4/3)R$ which saves 33% of the bandwidth. The main advantages of OFDM signal is its robustness against frequency selective fading.

Figure 2-4 shows how frequency selective fading affects a signal transmitted in single and multiple channels.

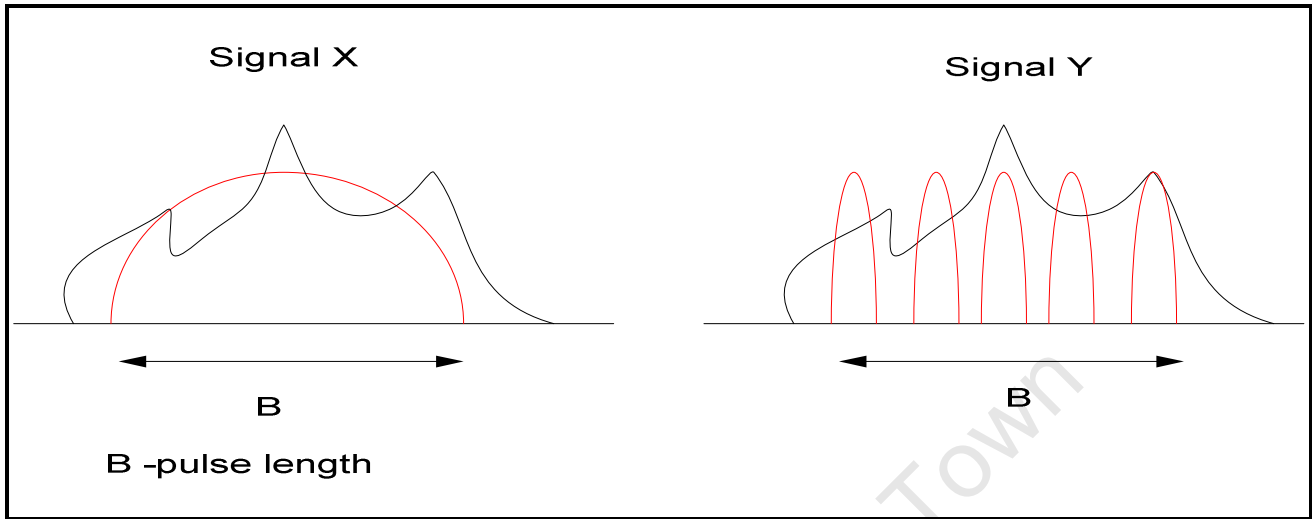


Figure 2-4. Frequency selective fading on OFDM

In Figure 2-4 in signal X the data is transmitted over one subchannel and this happens in conventional FDM. When affected by frequency selective fading the information subchannel is distorted. Signal Y shows the case data is transmitted over five subchannels each occupying B/N and in the presence of frequency selective fading not all subchannels are affected. Some of the information is not distorted in signal Y and this is utilized in OFDM transmission

For the orthogonality between the OFDM signal to be maintained the following must be true:

1. transmitter and receiver should be perfectly synchronized
2. the components of transmitter and receiver should be of high quality
3. there should be no multipath channel [22]

Multipath occurs when signals reach a receiver through more than one path. This phenomenon is caused by reflection and refraction along the path from natural and man-made objects. The received multiple signals differ in path delays and phases. Multipath channel causes errors on received signal because a dispersive channel causes intersymbol interference (ISI) and orthogonality between carriers lost thus causing inter carrier interference. The delay spreads can cause ISI when adjacent data symbols overlap. In the channel the symbols experience different path delays on different propagation

paths thus causing ISI. In high data transmission systems the symbols have short duration ($T_s < \tau$) the effect of ISI increases receiver complexity. Cyclic prefix (CP) was introduced by Peled and Ruiz to overcome the multipath effect. Cyclic prefix is the copy of the last part of the OFDM symbol that is added to the beginning of the transmitted signal as shown in figure 2-5.

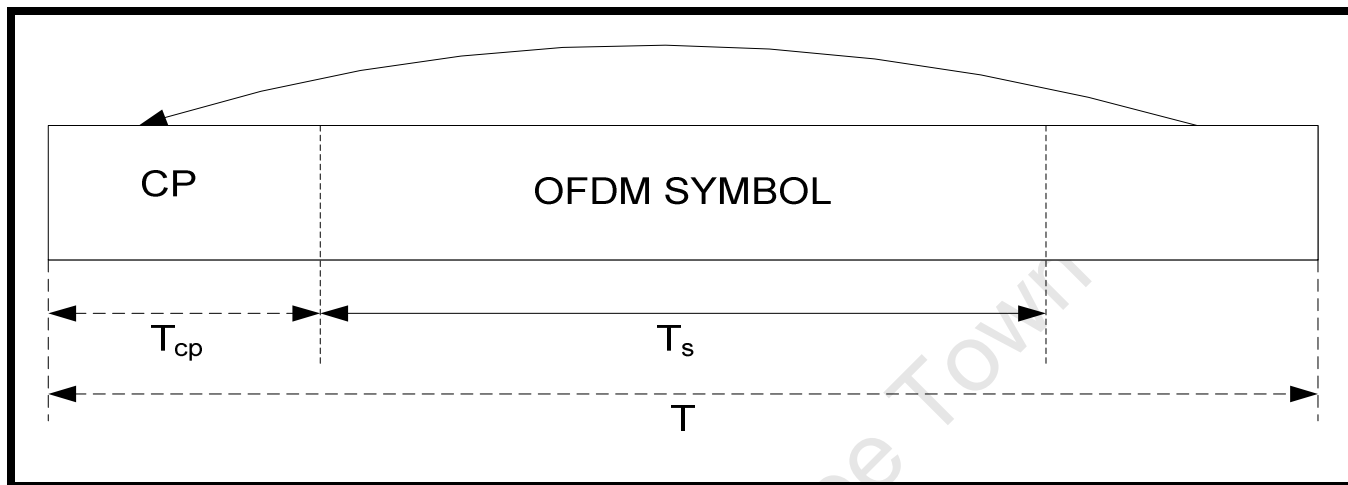


Figure 2-5. OFDM symbol with cyclic prefix

where T_{cp} is the length of CP , T_s is the length OFDM symbol and T is length of transmitted symbol

$$T = T_{cp} + T_s \tag{2.2}$$

The cyclic prefix should be longer than the significant part of the impulse response experienced by the signal in the channel. Cyclic prefix acts as a guard between two consecutive OFDM symbols thus preventing ISI. It also converts the linear convolution [24]. It should not be made longer than necessary because it causes SNR loss and efficiency loss. The SNR loss is given by the formula below

$$SNR_{loss} = -10 \log_{10} \left(1 - \frac{T_{cp}}{T} \right) \tag{2.3}$$

The required SNR for BER values is increased compared to normal performance and this referred as SNR loss. The rate of symbols transmitted per Hz of bandwidth is also reduced to $R(1 - T_{cp})$ where R is the baud rate [24]. Data rate is reduced because we are transmitting redundant information .

Diversity schemes are going to be used in next generation air interface to improve estimate of transmitted symbols. Antenna diversity is employed both at the transmitter and receiver in MIMO systems. In this work we are going to employ SM MIMO systems which provide multiplexing gain. Performance of MIMO is affected in frequency selective fading and OFDM has been shown to increase robustness against frequency selective fading. In OFDM multipath causes delay in OFDM symbols making them overlap and this is causes intersymbol interference. Cyclic prefix is used to remove effects of ISI but it reduces data rate because CP is redundant data added to transmission. In the next chapter we will discuss the channel model that is going to be used for simulation of the NGN air interface.

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Chapter 3

3. MIMO Channel Models

This chapter gives a review of MIMO channels models. Firstly it discusses ill-conditioning a channel condition that reduces capacity and increase BER in MIMO systems. The chapter also gives a brief description of correlation in realistic channel models. Secondly the chapter goes on further to study two main groups of MIMO channel model that is simulation channels models and realistic channels. It motivates the reader why realistic channel like Kronecker and Weichselberger should be considered more than simulation channels.

3.1.1 Ill-conditioning

David Gesbert and Jabran Akhar [63] proposed the use of constellation multiplexing whereby higher order QAM created from distinct M-QAM streams is used but it should have rate equivalent to sum of original signals rate. It has been shown in [50] that ill conditioning can be observed in a channel matrix when line of sight (LOS) exists for transmitted signals. It is predicted in [50] that if the LOS is dominating, the system capacity falls to SISO system level.

Spatial multiplexing MIMO systems rely on the linear independence between the channel responses corresponding to each transmit receive antenna pair. They suffer greatly from ill conditioning in MIMO channel matrix brought about by fading correlation and Rice components. The numerical condition of a matrix can be measured by the condition number. The 2-norm of the condition number in [51] is defined as the ratio of the largest eigenvalue to the smallest eigenvalue.

$$\gamma = \frac{\lambda_{\max}}{\lambda_{\min}} \quad (3.1)$$

$1 \leq \gamma \leq \infty$ where a value of one shows well conditioned matrix and large value indicates ill-conditioning.

Procedure for generating ill conditioned matrix

Singular Value Decomposition (SVD) method is employed to change a conditioned matrix to ill-

conditioned. SVD is used to decompose an $N_R \times M_T$ into a product of three matrices:

$$[A] = [U] [S] [V^H] \quad (3.2)$$

where A is a $n \times m$ matrix

U is an $n \times n$ unitary matrix

S is a $n \times m$ matrix has singular values and is diagonal

V is an $m \times m$ unitary matrix

As defined earlier the condition number is the ratio of the largest eigenvalue to the smallest. Firstly SVD is used to decompose the matrix. Secondly the smallest eigenvalue in S is divided by factor greater than 1. The $S^H S$ matrix is normalized for the new matrix. The new channel matrix is calculated and the conditioning number is recalculated. Figure [3-1] shows the change in conditioning number.

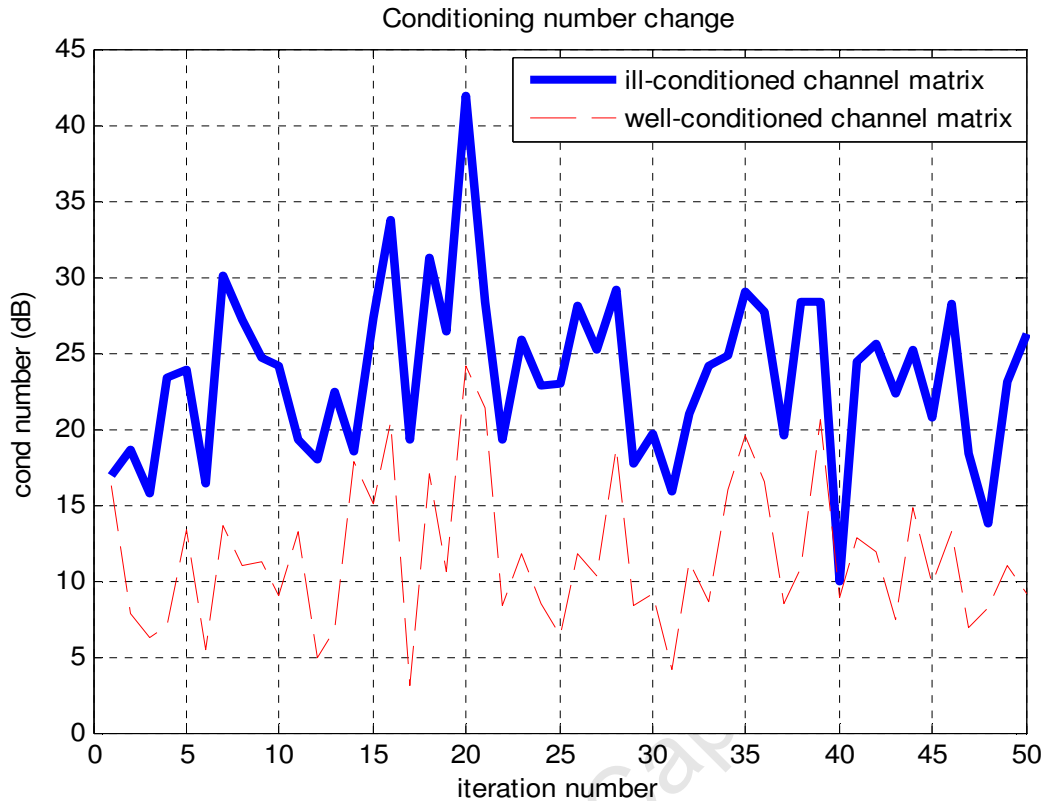


Figure 3-1. Introduction of Ill-conditioning in MIMO channel

The ill conditioning effect will be shown on the BER performance of MIMO OFDM. Linear detectors performance is greatly reduced in the presence of ill conditioning in the channel matrix. Ill-conditioning induces vector dependence in a matrix therefore it becomes almost non-invertible. Regular matrix inversion methods used by linear detector cannot invert ill-conditioned matrix correctly except for MLD which does not invert matrix but its complex.

3.1.2 Correlation

The performance of spatial multiplexed data is dependent on the independence of channels. It has been shown in [9] that the performance of MIMO systems is reduced by correlation. The effects of correlation on spatial multiplexed information will be investigated by the use of realistic channel models that will be presented in this work.

3.2. Simulation Channel models

3.2.1 Additive White Gaussian channel

Noise is an inherent problem in wireless communication and it is introduced as thermal noise to the transmitted signal. Thermal noise is introduced in wireless systems by electric components, antennas and amplifiers because of particle motion in materials at temperatures above absolute temperatures ($0K / -273^{\circ}C$). Therefore noise exists even when fading, multipath and interference are present. Additive White Gaussian Noise (AWGN) channel model only adds noise to the signal passing through it. In this model the amplitude response is flat and the phase response is linear for all frequencies. This means the signal passes through the channel without amplitude and phase change [35]. The received signal is,

$$r(t) = x(t) + n(t), \quad (3.3)$$

where $r(t)$ is the received signal at time t , $x(t)$ is the transmitted signal and $n(t)$ is the white Gaussian noise. Noise is represented as narrowband after passing through a bandpass filter [67]. The noise expression is;

$$n(t) = x_1(t) \cos(\omega_0 t) + y_1(t) \sin(\omega_0 t) \quad (3.4)$$

where $x_1(t)$ and $y_1(t)$ are uncorrelated stochastic processes which represent random in-phase and quadrature components of noise respectively. $\omega_0(t)$ is phase

Noise in this model has a flat Power Spectral Density (PSD) for all frequencies. The noise PSD is

$$N(f) = N_0(f) \quad -\infty < f < \infty \quad (3.5)$$

This channel model does not exist but channels are assumed to be AWGN when the signal bandwidth is smaller than channel bandwidth. No channel has infinite bandwidth but in LOS radio channels like fixed satellite links in good weather behave like AWGN.

Rayleigh channel

Channel fading can be categorized in two groups large scale fading and small scale fading. Large scale represents average signal power attenuation or path loss due to motion over large areas. The transmitted signal is affected by terrain as it pass to the receiver and large scale fading provides a chance to model or estimate path loss as function of distance. Small scale fading refers to dramatic changes in signal amplitude and phase as a result of small changes. Small scale fading manifests in the following two mechanisms:

1. time spreading of the signal
2. time variant behavior of the channel response

In small scale fading when there is no light of sight the received signal is described by Rayleigh probability density function (p.d.f) whereas when there is a dominant line of sight it is defined by a Rician pdf.

Small fading can be called Rayleigh fading if multiple reflective paths exist and there is no line of sight component. A signal transmitted in the channel is affected by fading component as shown by equation

$$r(t) = \alpha x(t) + n(t) \quad (3.6)$$

where α is fading coefficient introduced by channel and it is statistically described by a Rayleigh p.d.f. A random variable $x = x_1 + jx_2$ where x_1 and x_2 are independent and identically distributed (i.i.d) with variance σ_x^2 has Rayleigh distribution. Rayleigh distribution is obtained by summing independent field components and its probability distribution [36].

$$P(r) = \frac{r}{\sigma^2} \exp \left[-r^2 / (2\sigma^2) \right] \quad , \quad r \geq 0 \quad (3.7)$$

where the envelope $r(t)$ of complex signal $E(t)$ is given as

$$r(t) = \sqrt{[Re E(t)]^2 + [Im E(t)]^2} \quad (3.8)$$

where δ^2 is the mean power and $r^2(t)/2$ is the short term signal power. Figure 3-2 shows the probability density function of Rayleigh distributions

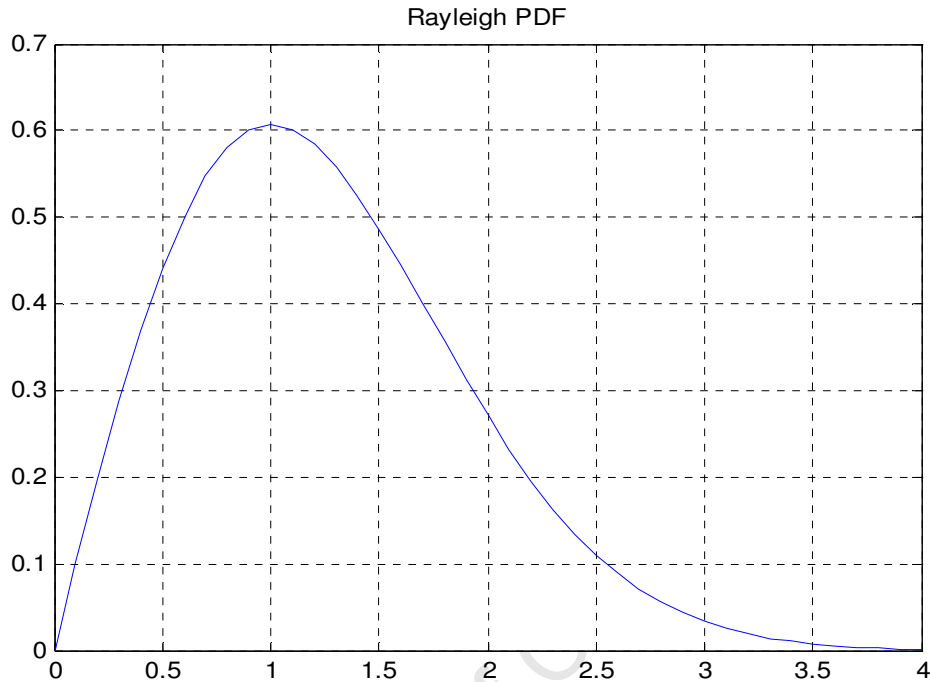


Figure 3-2. Rayleigh distribution probability density function

Rician channel

This is a channel that has a dominant line of sight signal and several multipath fading components. The envelope or amplitude of the received signal follows Rice probability density function

$$pdf_r = \frac{r}{\delta^2} \exp\left[-\frac{r^2 + A^2}{2\delta^2}\right] \cdot I_0\left(\frac{rA}{\delta^2}\right) \quad , \quad 0 \leq r \leq \infty \quad (3.9)$$

where $I_0(x)$ is the modified Bessel function of the first kind zero order and A is the signal amplitude. The mean square of the rice distributed random variable r is:

$$\overline{r^2} = 2\delta^2 + A^2 \quad (3.10)$$

The ratio of the power in the LOS component to the power in the diffuse component $A^2/(2\sigma^2)$ is called the rice factor K [12].

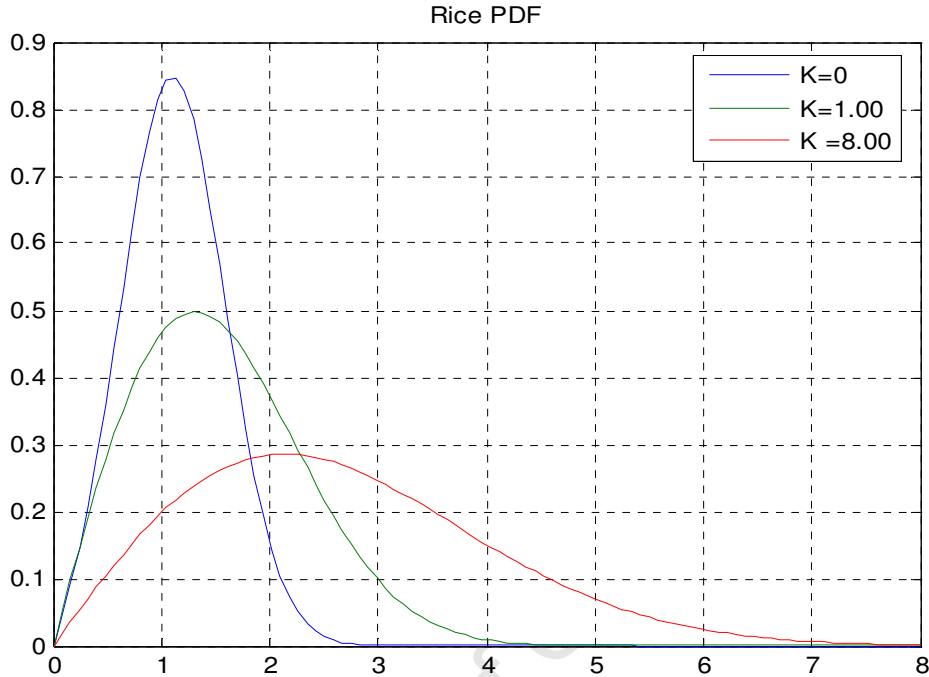


Figure 3-3. Rician probability density function with varying factor

As shown by figure [3-3] , if $K \rightarrow 0$ the distribution becomes Rayleigh distribution while large values of K approximates a Gaussian distribution with mean value A

3.3 Realistic Channels

The MIMO channel capacity grows linearly with the number of antennas if the fading between each antenna pair is independent and identically Rayleigh distributed. In a realistic environment there is spatial correlation between subchannels because of antenna configuration and the position of scatters between the antennas [28]. Correlation at either transmit or receive side will cause a reduction in system capacity, therefore stochastic MIMO radio channels models have been developed to account for this spatial correlation in a realistic way [29]. Correlation not only reduces channel capacity but also degrades signal processing performance [30].

Many works have been done to quantify the fading correlation effect on receive diversity ($M_T = 1, N_R \geq 1$) schemes and different approaches have been used but two of them will be discussed below. One approach is to record a large number of typical channel realizations through field measurements or through ray simulation. Another method is to use the scatter model that captures reasonable scattering of the wireless environment. In this work we will consider two analytical models based on correlation:

- i. Kronecker model
- ii. Weichselberger model

Kronecker Channel model

Kronecker model assumes that the correlation at the transmitter and receiver can be separated. Kronecker channel is a correlated fading channel that has Rayleigh fading parameters in the channel. The underlying validation of this approach is to assume that only the immediate surroundings of the antenna array impose the correlation between array elements and have no impact on the correlation observed between elements of the array at the other end of the link which is a reasonable assumption for indoor environments [31]. A correlated Rayleigh fading channel $\tilde{H} \in \mathbb{C}^{M_T \times N_R}$ according to [29] can be generated according to

$$\text{vec}(\tilde{H}) = R_H^{1/2} \text{vec}(G) \quad (3.11)$$

where $\text{vec}(\cdot)$ is a vector operator which stacks the columns of a matrix into one column, $(\cdot)^{1/2}$ denotes any square root matrix fulfilling

$$R_H^{1/2} \cdot (R_H^{1/2})^H = R_H \text{ and}$$

$$R_H = E\{\text{vec}(H)\text{vec}(H)^H\}$$

R_H is a $(M_T \cdot N_R) \times (M_T \cdot N_R)$ channel correlation matrix containing the complex correlation between elements of the $(M_T \cdot N_R)$ channel matrix H . $(\cdot)^H$ is the complex conjugate transpose and $E\{\cdot\}$ is the

expectation. G is a stochastic (M_T, N_R) matrix with independently identically distributed (i.i.d) complex Gaussian elements with variance one and zero mean. Under the assumption that the transmitter and receiver correlation can be separated the matrix can be given by Kronecker product of transmitter correlation matrix R_{Tx} and receiver correlation matrix R_{Rx} .

$$R_H = \frac{1}{\text{tr}\{R_{Rx}\}} R_{Tx} \otimes R_{Rx} \quad (3.12)$$

where $R_{Tx} = E\{(H^H H)^T\}$, $R_{Rx} = E\{HH^H\}$, \otimes denotes the Kronecker product using the identity $[B^T \otimes A] \text{vec}(D) = \text{vec}(ADB)$ simplifies to the Kronecker model

$$\tilde{H} = \frac{1}{\sqrt{\text{tr}\{R_{Rx}\}}} R_{Rx}^{1/2} G (R_{Rx}^{1/2})^T \quad (3.13)$$

Kronecker model simplifies the analytical treatment, simulation of MIMO systems and it allows independent optimization of the Tx and Rx, hence it has become popular [29]. The correlation between adjacent transmit antennas is given by [32]

$$E_{h_i} \{ | h_{i,j} h_{i,j+1}^* | \} = \rho_t, \quad j = \{1 \dots M_t - 1\}, \rho_t \in \mathbb{R}, 0 \leq \rho_t \leq 1 \quad (3.14)$$

where $h_{i,j}$ represents the channel impulse response coupling between transmitter j and receiver antenna i . Transmit antenna correlation coefficients are independent from receive antenna whose correlation coefficients are given by

$$E_{h_i} \{ | h_{i,j} h_{i,j+1}^* | \} = \rho_r, \quad j = \{1 \dots N_r - 1\} \rho_r \in \mathbb{R}, 0 \leq \rho_r \leq 1 \quad (3.15)$$

The full transmit and receive correlation matrices obtained from [32] are given below. The channel correlation matrices for R_t and R_r is derived from a single spatial correlation model proposed in [68].

The correlation is dependent on the distance separating the antennas on transmitter and receiver each side resulting in the Toeplitz structure of correlation matrices.

$$R_t = R^T = \begin{Bmatrix} 1 & \rho_t & \rho_t^2 & \dots & \rho_t^{M_t-1} \\ \rho_t & 1 & \rho_t & \dots & \rho_t^{M_t-2} \\ \rho_t^2 & \rho_t & 1 & \dots & \rho_t^{M_t-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_t^{M_t-1} & \rho_t^{M_t-2} & \rho_t^{M_t-3} & \dots & 1 \end{Bmatrix} \quad (3.16)$$

$$R_r = R^T = \begin{Bmatrix} 1 & \rho_r & \rho_r^2 & \dots & \rho_r^{N_r-1} \\ \rho_r & 1 & \rho_r & \dots & \rho_r^{N_r-2} \\ \rho_r^2 & \rho_r & 1 & \dots & \rho_r^{N_r-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_r^{N_r-1} & \rho_r^{N_r-2} & \rho_r^{N_r-3} & \dots & 1 \end{Bmatrix} \quad (3.17)$$

with real valued coefficients.

From [28] the general stochastic MIMO channel matrix can be generated as

$$H = R_{H,RR}^{\frac{1}{2}} G R_{H,TX}^{\frac{1}{2}} \quad (3.18)$$

where G is a stochastic $N_T \times N_R$ matrix with i.i.d complex Gaussian elements with variance one and zero mean. $R_{H,TX}$ is $N_T \times N_T$ and $R_{H,RR}$ is $N_R \times N_R$.

In [73] the correlation values are measured for a 4×4 MIMO system. The experiment was done at a test bed at 5.8GHz (ISM of IEEE802.11a) with signal bandwidth of 20MHz. It is shown that there is significant drop in capacity for MIMO systems at $\rho_t = \rho_r > 0.4$. The error probability of the employed model is

$$\Psi(R, \check{R}) = \frac{\|R - \check{R}\|_F}{\|R\|_F} \quad (3.19)$$

where $\|\cdot\|_F$ is the frobenius norm, R is the correlation Kronecker product of the transmitter and receiver correlation $R = R_{TX} \otimes R_{RX}$. The biggest error of 28% is obtained when the correlation matrix

$\tilde{\mathbf{R}}$ is equal to identity matrix. Table [3-1] shows the collected correlation values for 2×2 MIMO subchannels.

Table 3-1. Practical measured correlation values for spatial correlated Kronecker model

	Maximum	Mean ν	Minimum
ρ_r	0.2913	0.1649	0.0408
ρ_t	0.2085	0.1060	0.0670

The above correlation values are tested for the 2×2 MIMO Kronecker channel because they depict a realistic indoor office environment.

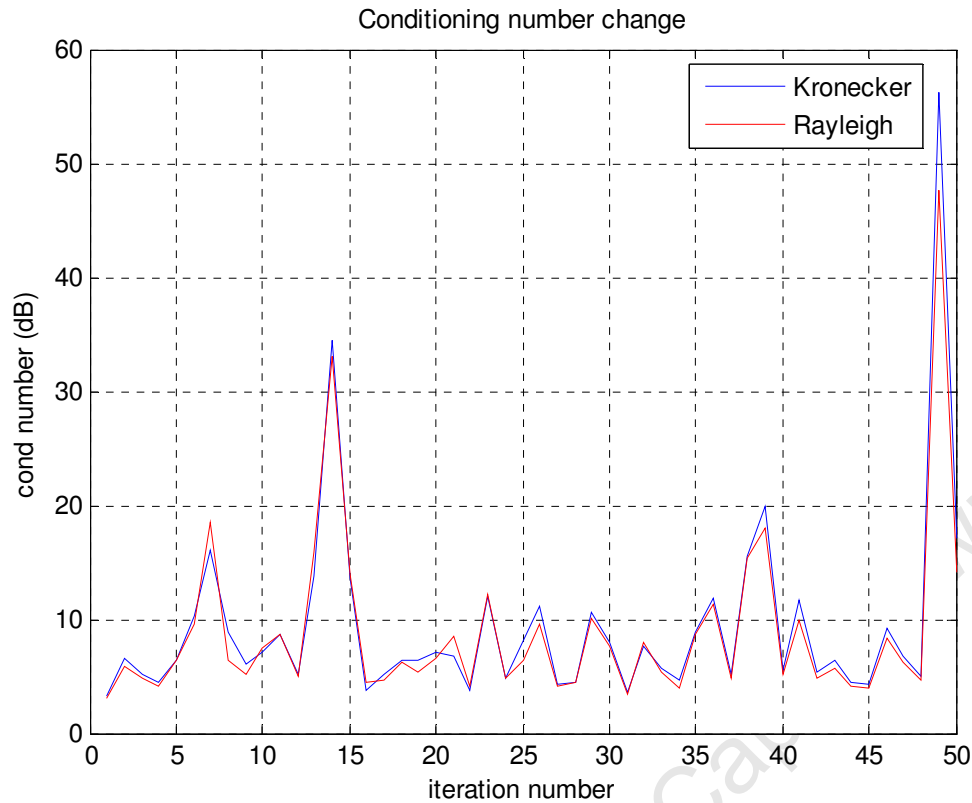


Figure [3-4].Comparison of Kronecker and Rayleigh channel model conditioning numbers

As shown in figure 3-4 the conditioning number of Kronecker conditioning number is higher than Rayleigh fading channel. The sum of Kronecker conditioning number minus Rayleigh conditioning number is 26.5665 for 50 iterations. This shows that the realistic channel has ill conditioned channel matrices higher than the Rayleigh channel.

Weichselberger Channel Model

Measurements of the channel conditions at outdoor scenarios agree with the Kronecker model assumptions but indoor data seem to deviate more [29] . Kai te. al [31] has shown that the Kronecker model can correctly predict the channel capacity for system with less than three transmit and receive antennas. As a consequence a more general model was developed by Weichselberger et al [37].This model is less restrictive than the Kronecker model on the correlation between the transmitter and receiver antenna. It models the correlation between the transmitter and receiver jointly by allowing

coupling between their eigenbases. This was achieved by generalizing the MISO eigenmodes to MIMO. The eigenmode decomposition is unique in that eigenmodes are orthogonal; this results in smallest possible number of modes that fade independently. MIMO eigenmodes are matrices therefore, we excite them by vector modes which are similar to eigenmodes [38]. Weichselberger imposed a condition or structure to these modes in the form of eigenvalue decomposition of the R_{Rx} and T_{Tx} correlation matrices [39] as

$$R_{Rx} = U_{Rx} \Lambda_{Rx} U_{Rx}^H \quad (3.20)$$

$$R_{Tx} = U_{Tx} \Lambda_{Tx} U_{Tx}^H \quad (3.21)$$

The model definition is defined as

$$H_{\text{Weichsel}} = U_{Rx} (\bar{\Omega}_{\text{Weichsel}} \odot G) U_{Tx}^T \quad (3.22)$$

where eigenvalues Λ_H have no constraint except they are either zero or positive, U_{Rx} , U_{Tx} are eigenvector matrices given by eigenvector decomposition of the correlation matrix the receive (transmit) elements respectively and \odot is element-wise product, $\bar{\Omega}_{\text{Weichsel}}$ element-wise square root of power coupling matrix Ω_{Weichsel} . The elements $\Omega_{\text{Weichsel}, ij}$ are calculated as average power coupling of between the i th transmit and j th receive eigenmode.

$$\Omega = E\{(U_{Rx}^H H U_{Tx}^*) \odot (U_{Rx}^T H^* U_{Tx})\} \quad (3.23)$$

The Weichselberger equation captures the channel correlation of the environment in conjunction with the statistics of the transmitted signal. Weichselberger model analysis has higher complexity than Kronecker and this has reduced its acceptance in a MIMO system.

In figure 3-5 it shows the conditioning number change for Weichselberger channel model.

Weichselberger channel model is an ill conditioned channel matrix is much higher than Rayleigh fading channel. The sum of the difference in conditioning number is 1.2546e+003 after 50 iterations.

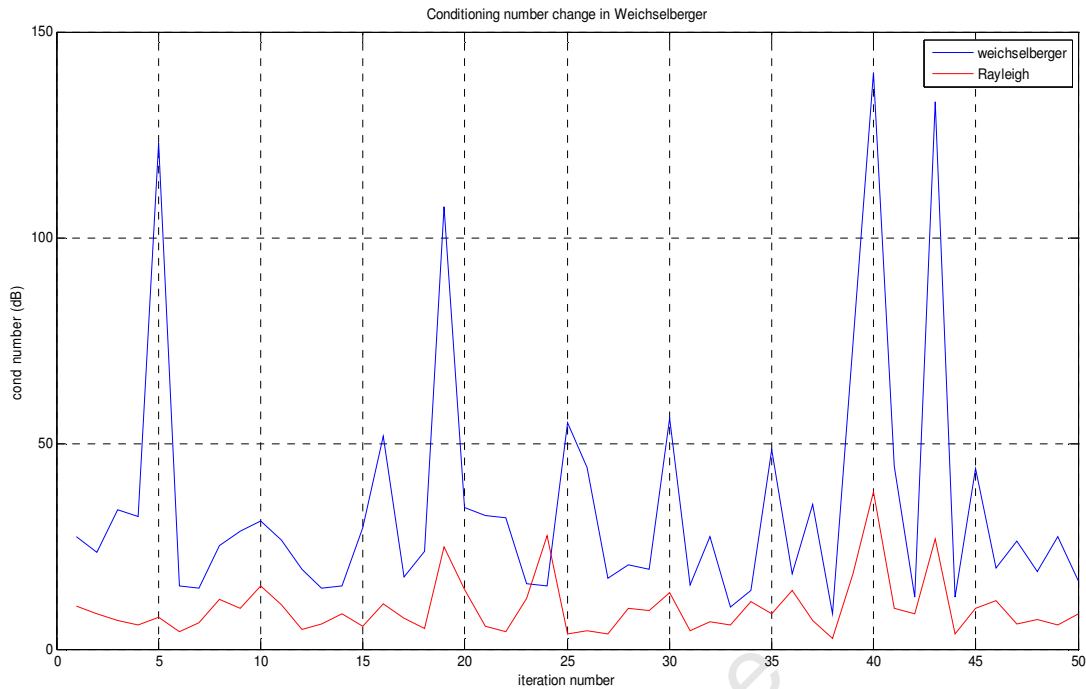


Figure 3-5. Comparison of Weichselberger and Rayleigh channel model conditioning numbers

The Weichselberger channel model, as seen above has shown to have high conditioning number thus it is another efficient way of introducing ill conditioning in the channel. Kronecker channel model has high conditioning number than Rayleigh fading channel. Realistic channels combine both correlation and ill conditioning in the channel matrix. Weichselberger shows represents a more realistic channel but the model more complex compared to Rayleigh, Rician and Kronecker. In the later chapters the Weichselberger and Kronecker will be used to test the performance of proposed detectors.

Chapter 4.

4. OFDM and MIMO System

This chapter discusses the two main technologies that are going to be the driving force for next generation air interface. The chapter considers OFDM and SM-MIMO technologies individually. Linear detectors and SIC are discussed in this chapter investigating their effect on output noise.

4.1 OFDM System

OFDM is a special form of multicarrier modulation where a single data stream is transmitted over a number of low rate subcarriers [25]. The block diagram of OFDM system model is shown in figure 4-1.

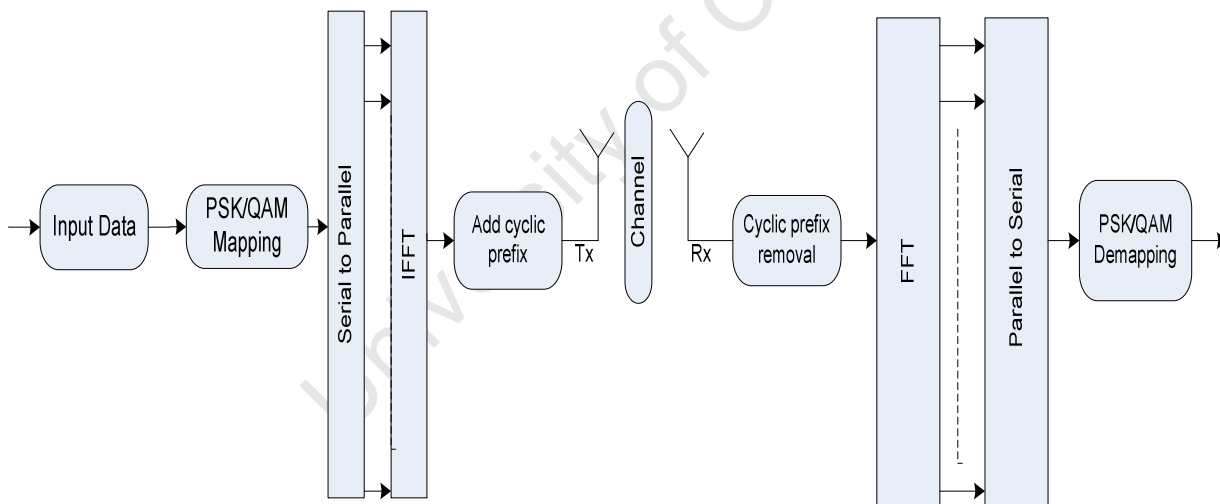


Figure 4-1. OFDM system model

Serial source symbols from PSK/QAM mapper are passed through a Serial to Parallel (S-P) converter. The output of the S-P is the vector, $\mathbf{x}_m = (x_{0,m} \ x_{1,m} \ \dots \ x_{N_c-1,m})^T$, where N_c is the number of

subchannels. IDFT is performed on each data symbol block and a cyclic prefix of length N_{cp} is added. IDFT converts the data signal to a time spectrum and this ensures that the subchannels are orthogonal. The resulting complex baseband discrete time signal of the m^{th} subchannel symbol of the OFDM shown above is represented as,

$$s_m = f(x) = \begin{cases} \frac{1}{N_c} \sum_{k=0}^{N_c-1} x_{k,m} e^{j2\pi k(n-N_c)/N_c} & \text{if } n \in [0, N_c + N_{cp} - 1] \\ 0 & \text{otherwise,} \end{cases} \quad (4.1)$$

where n is the discrete time index. The time domain representation of the subchannel symbol, $s(n)$, is given by,

$$s(n) = \sum_{m=0}^{\infty} (s_m(n - m(N_c + N_{cp}))). \quad (4.2)$$

The received signal, $r(n)$, in (equation 4.3) is the sum of the linear convolution of the discrete channel impulse response $h(n)$ and additive white Gaussian noise $n(n)$. We assume that channel fading is constant during one OFDM symbol, thus it is referred to as a quasi-static fading channel. The transmitter and receiver are assumed to be perfectly synchronized therefore no synchronization error is considered for the calculation,

$$r(n) = \sum_{\eta=0}^{N_{cp}} h(\eta) s(n - \eta) + n(n). \quad (4.3)$$

The incoming sequence, $r(n)$, is split into blocks and the cyclic prefix of each block is removed. The resultant vector is $r_m = (r(z_m + 1) \dots \dots r(z_m + N_c - 1))^T$, with $z_m = (N_c + N_{cp}) + N_{cp}$.

The received data symbol, $y_{k,m}$, is obtained by performing IDFT on received vector r_m . The $y_{k,m}$ is given by,

$$y_{k,m} = \sum_{n=0}^{N_c-1} r(z_m + n) e^{-j2\pi kn/N_c}. \quad (4.4)$$

For a more compact notation, a matrix equivalent is used. For a single OFDM symbol, it equals

$$y_m = H \circ x_m + n_m = \text{DIAG}(H) \cdot x_m + n_m, \quad (4.5)$$

where \circ denotes Hadamard operator, which is simply the element wise product; $\text{DIAG}(H)$ is the vector with diagonal elements of H ; and,

$$y_m = (y_{0,m} \ y_{1,m} \ \dots \ \dots \ y_{N_c-1,m})^T$$

$$n_m = (n_{0,m} \ n_{1,m} \ \dots \ \dots \ n_{N_c-1,m})^T$$

$$H = (h_0 \ h_1 \ \dots \ \dots \ h_{N_c-1})^T$$

From [24] the m^{th} OFDM symbol in matrix notation is given as:

$$Y_m = H \circ X_m + N_m = \text{DIAG}(H) \cdot X + N, \quad (4.6)$$

where: $Y = (y_0 \ y_1 \ \dots \ \dots \ y_{N_c-1})$; $X = (x_0 \ x_1 \ \dots \ \dots \ x_{N_c-1})$ and

$N = (n_0 \ n_1 \ \dots \ \dots \ n_{N_c-1})$. The complete time signal $s(n)$, is given by the combination of all OFDM symbols that were transmitted.

4.2 V-BLAST Architecture

Multipath , which can be viewed as impairment by other systems, is exploited by the V-BLAST method. Spatial multiplexing is applied in LTE and WiMAX to increase the system capacity. The diversity gain for V-BLAST is only $N_R - M_T + 1$. There are two remedies available:

- (i) channel dependent ordered decoding at receiver and,
- (ii) allocation of rates and powers across the transmitter antennas.

However, even with these remedies, the diversity is upper bounded by N_R . It has been proved that the application of ordering to V-BLAST cannot improve diversity gain of the V-BLAST scheme although the ordering yields $10 \log_{10} M_T$ dB coding gain for zero forcing V-BLAST in high SNR [27]. At symbol time t , the transmitted signal vector is $s^t = [s_1^t, s_2^t, \dots, s_M^t]^T$ as shown by the transmitter in figure 4-2.

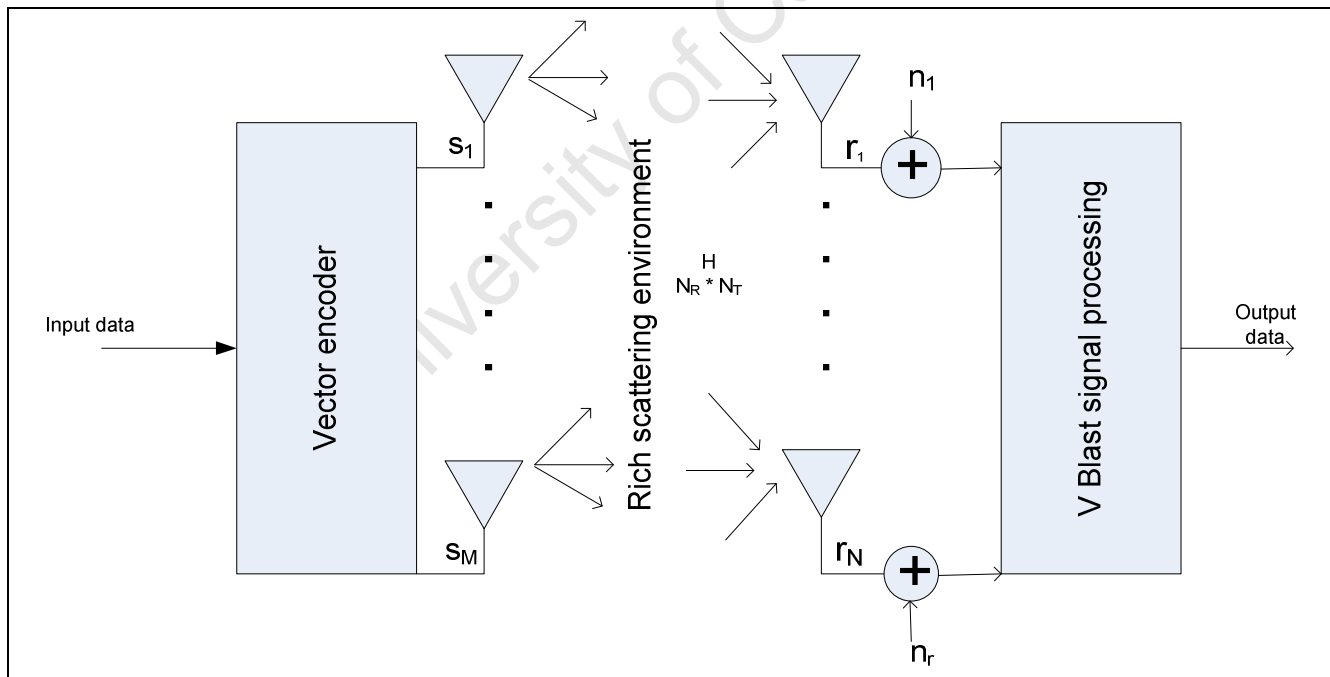


Figure 4-2. V-BLAST system model

At each time sequence $\{s^t\}_j, (j = 1, 2 \dots \dots, M)$, the time sequence is referred to as a layer [16]. Vector r^t (components in equation 4.7 below) is received at the receiver and it is a superposition of the transmitted signal s^t corrupted by additive white Gaussian noise n_j^t . The noise is i.i.d. complex Gaussian with zero mean and unit variance. The r^t is defined as,

$$r_l^t = \sum_{j=1}^M h_{lj}^t s_j^t + n_j^t, \tag{4.7}$$

where j is the transmit antenna and l is the receive antenna, h_{lj}^t is the channel gain parameter and the channel is assumed to be quasi-static flat fading. The above equations can be represented in matrix notation by [16],

$$r = Hs + n, \tag{4.8}$$

where $H = \begin{bmatrix} h_{11} & \dots & h_{1N} \\ \vdots & \ddots & \vdots \\ h_{M1} & \dots & h_{MN} \end{bmatrix}$, $r = \begin{bmatrix} r_1 \\ \vdots \\ r_M \end{bmatrix}$, $s = \begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix}$ and $n = \begin{bmatrix} n_1 \\ \vdots \\ n_M \end{bmatrix}$.

4.3 Detection

In this section, SM MIMO detectors will be divided into two main groups, namely linear detectors and interference cancellers. A brief discussion of the maximum likelihood detector is given, highlighting its high complexity. Lastly an analytical comparison of the noise power of the ZF detector and successive interference canceller is compared. First though, we investigate the relationship between output noise power for different square configurations for SM MIMO.

4.3.1 Zero Forcing

The received signal in MIMO detection for generalized MIMO OFDM systems is given by,

$$r_1 = Hs + n, \tag{4.9}$$

where $\mathbf{s} = (s_1, s_2, \dots, s_{M_T})$ represent the transmitted signal, \mathbf{n} is the Gaussian white noise with zero mean and variance σ^2 equal to one. The inverse of the channel matrix \mathbf{H} , \mathbf{G}_{ZF} , is given as,

$$\mathbf{G}_{ZF} = \mathbf{H}^\dagger.$$

The output signal is given by,

$$\mathbf{y}_1 = \mathbf{s} + \mathbf{H}^\dagger \mathbf{n}, \quad (4.10)$$

where \mathbf{H}^\dagger is the inverse of the channel matrix of dimension $N_R \times M_T$ given $N_R = M_T$. \mathbf{H} is of full column rank (column rank is the maximum number of independent columns in a matrix). The ZF detector decouples the channel matrix into M_T parallel scalar channels, but noise is enhanced. Equations (4.11) and (4.12) show input and output signal to noise ratio (SNR) respectively [44, 79]. These equations will be used to calculate the noise enhancement effect of detectors.

$$SNR_{in} = \frac{E\{\|\mathbf{H}\mathbf{s}\|^2\}}{E\{\|\mathbf{n}\|^2\}} \quad (4.11)$$

$$SNR_{out} = \frac{E\{\|\mathbf{H}\mathbf{s}\|^2\}}{E\{\|\mathbf{n}\|^2\}} \quad (4.12)$$

4.3.2 Minimum Mean Square Error

The noise enhancement effect of the ZF detector can be reduced by using the MMSE detector, $\mathbf{G}_{MMSE} = (\mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}^H$, which is the \mathbf{G}_{MMSE} minimizing the mean square error $E\{\|\mathbf{G}_{MMSE} \mathbf{r}_1 - \mathbf{s}\|^2\} = 0$ [74, 75], where \mathbf{s} is the transmitted symbol. Let the obtained output signal be,

$$\mathbf{y}_{MMSE} = (\mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}^H \mathbf{r}_1, \quad (4.13)$$

where \mathbf{H}^H is the Hermitian transpose of \mathbf{H} , σ^2 is the noise variance and \mathbf{I} is the identity matrix. As $\sigma^2 \rightarrow 0$, $\sigma^2 \mathbf{I}$ matrix approaches zero matrix therefore \mathbf{G}_{MMSE} function tends to $(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$ and

according to [45] the channel matrix \mathbf{H} 's pseudo inverse is given by $(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$, which is equal to \mathbf{G}_{ZF} . Therefore the decoded signal is:

$$\mathbf{y}_{ZF} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{r} = \mathbf{s} + \mathbf{G}_{ZF}\mathbf{n}. \quad (4.14)$$

(4×4) system will be shown in chapter 6 to offer low BER and moderate receiver complexity. In our simulations we applied (4×4) system because of its advantages. In [52,57] it was shown the to have a high data rate of 2Mbps with QPSK and 525Mbps with OFDM.

As shown above ZF detector can be derived from MMSE but MMSE performs better because it takes into account the noise variance of system. It has been shown in research that MMSE detector has superior performance as proved through simulation of system model in figure 4-3

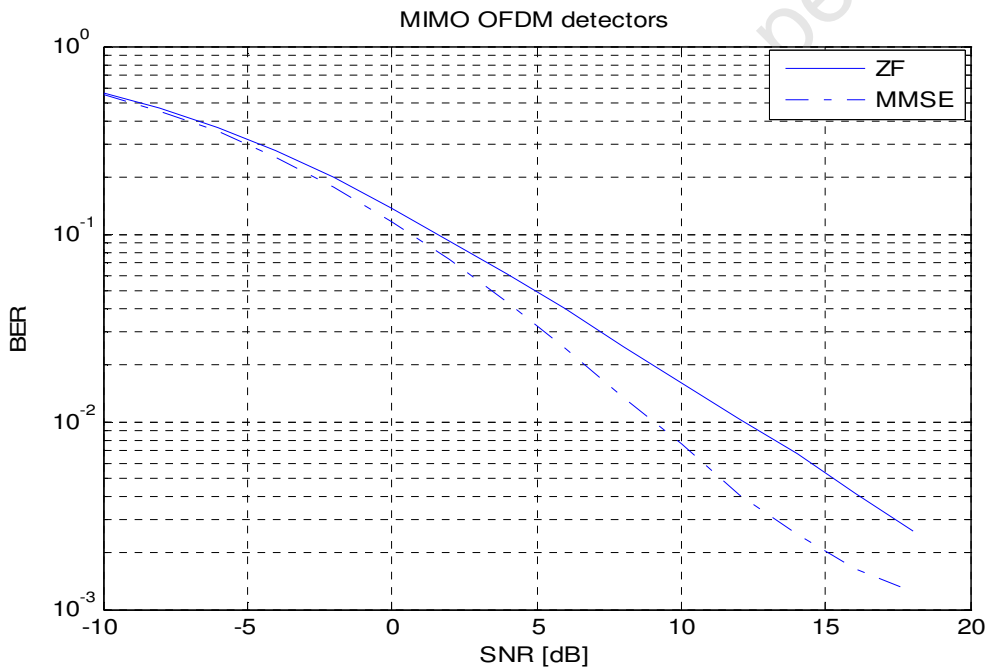


Figure 4-3. Comparison of Zero forcing detector and Minimum Mean Square Error detector

As shown by Figure 4-3 above, the MMSE detectors performs better than the ZF detector. At low SNR, which is at -10dB, the two detectors perform the same. The MMSE detector at -10dB has BER

of 0.0912 and ZF has BER of 0.0912. However, at higher SNR 15dB, the performance of the detectors differs, with MMSE having a lower BER compared to ZF.

4.3.3 Successive Interference cancellation

This detection algorithm was discussed in [16, 44] as a basic interference cancellation method that is based on either linear detector, that is either ZF or MMSE. The ZF-SIC detection method first decodes the first layer then, it cancels the decoded signal from the received signal. The process is repeated for the remaining signal. The performance of V-BLAST can be improved by ordering as shown in [44]. The strongest layer is decoded first and cancelled from received signal to reduce interference. The received signal in MIMO detection for generalized MIMO OFDM system is given by,

$$r_1 = Hs + n, \tag{4.15}$$

where $s = (s_1, s_2, \dots, s_{M_T})$ and n the noise.

In this model the other signals are considered as interference to the symbol of interest. The signal post detection SNR used to determine the order of detection for the k_i^{th} is given by [44],

$$\rho_{k_i} = \frac{\langle |s_{k_i}|^2 \rangle}{\sigma^2 \|w_{k_i}\|^2} \tag{4.16}$$

where w_{k_i} is orthogonal to $(H)_{k_i}$, it also satisfies the condition $w_{k_i}^T (H)_{k_i} = \begin{cases} 0 & j \geq i \\ 1 & j = i \end{cases}$, $(H)_{k_i}$ is the k_i^{th} column of H .

Transmitted signals use the same constellation; therefore the signal with the smallest ρ_{k_i} will dominate the error performance of the system. As shown by [44], choosing the best ρ_{k_i} at each stage in the detection process leads to global optimal ordering.

ZF SIC detection is a recursive algorithm which includes the determination of optimal ordering as follows [16]:

Initialization:

$$i \leftarrow \mathbf{1} \quad (4.17a)$$

$$G_i = H^\dagger \quad (4.17b)$$

$$k_i = \text{argmin} \|(G_i)\|^2 \quad (4.17c)$$

Recursion :

$$w_{k_i} = (G_i)_{k_i} \quad (4.17d)$$

$$y_{k_i} = w_{k_i}^T r_i \quad (4.17e)$$

$$\hat{s}_{k_i} = Q(y_{k_i}) \quad (4.17f)$$

$$r_{i+1} = r_i - \hat{s}_{k_i} (H)_{k_i} \quad (4.17g)$$

$$G_{i+1} = H_{\bar{k}_i}^\dagger \quad (4.17h)$$

$$k_{i+1} = \text{argmin} \|(G_{i+1})_j\|^2 \quad (4.17i), \quad j \text{ is not in the subset of } \{k_1, \dots, k_i\}$$

$$i \leftarrow \mathbf{1} + 1 \quad (4.17j)$$

where $(G_i)_j$ is the j^{th} row of G_i , $H_{\bar{k}_i}^\dagger$ denotes the matrix obtained by zeroing the columns k_1, k_2, \dots, k_i of H and $()^\dagger$ is the Moore-Penrose pseudo-inverse operator [45]. (4.17c and i) determine the optimal ordering. (4.17d to f) computes respectively the ZF nulling vector, the decision statistic and estimated component of \mathbf{s} , (4.17g) performs the cancellation of the detected component from the received vector and (4.17h) computes the new pseudo-inverse for the next iteration.

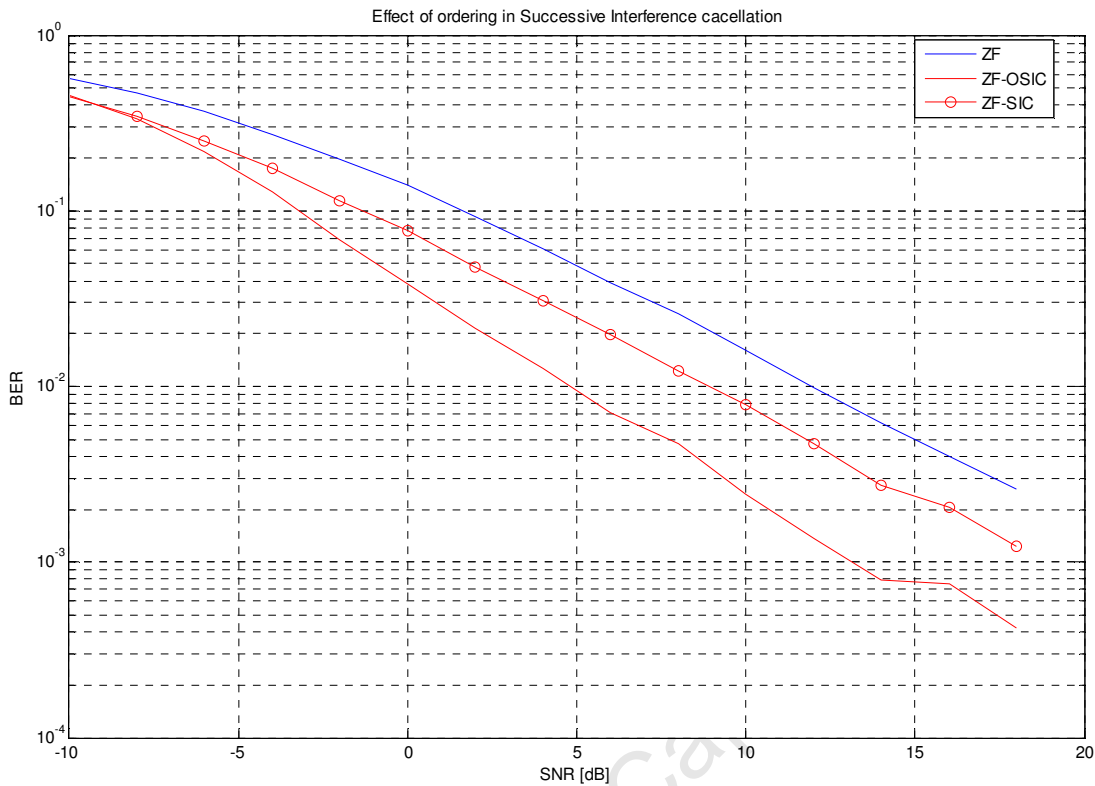


Figure 4-4. Ordering effects on Interference Cancellation performance

System model simulation shown in Figure 4-4 above, it is shown that successive interference cancellation improves the performance of the ZF detector and the MMSE detector. Successive interference cancellation initially decodes the symbol then subtracts its effects from the received signal, thus increasing the diversity order of the next stage and the reliability of estimating the second symbol. If the previous decision was correct the diversity of the next stage of SIC is $(N_R - M_T + i)$ where $i = 1, \dots, M_T$, for example in (4×4) matrix situation, the diversity for second stage ($i = 2$) is 2. In the interference cancellation shown in [16], the post SNR of the remaining symbol changes after cancellation of the decoded signal. In a linear detector, all the symbols are detected with the same diversity with all interference from the other sent symbols. Ordering as shown above improves the system performance by 3dB, that is to 8dB. The MMSE-OSIC weighting matrix, \mathbf{G}_i , is chosen by the MMSE rule as in [15] and the recursive algorithm is given below:

Initialization:

$$i \leftarrow 1 \quad (4.18a)$$

$$G_i = (H^H H + \sigma^2 I)^{-1} H^H \quad (4.18b)$$

$$k_i = \operatorname{argmin} \| (G_i) \|^2 \quad (4.18c)$$

Recursion:

$$w_{k_i} = (G_i)_{k_i} \quad (4.18d)$$

$$y_{k_i} = w_{k_i}^T r_i \quad (4.18e)$$

$$\hat{s}_{k_i} = Q(y_{k_i}) \quad (4.18f)$$

$$r_{i+1} = r_i - \hat{s}_{k_i} (H)_{k_i} \quad (4.18g)$$

$$G_{i+1} = (H_{k_i}^H H_{k_i} + \sigma^2 I)^{-1} H_{k_i}^H \quad (4.18h)$$

$$k_{i+1} = \operatorname{argmin} \| (G_{i+1})_j \|^2 \quad (4.18i) \quad , \quad \text{where } j \text{ is not in the subset of } \{k_1 \dots \dots k_i\}.$$

$$i \leftarrow i + 1 \quad (4.18j)$$

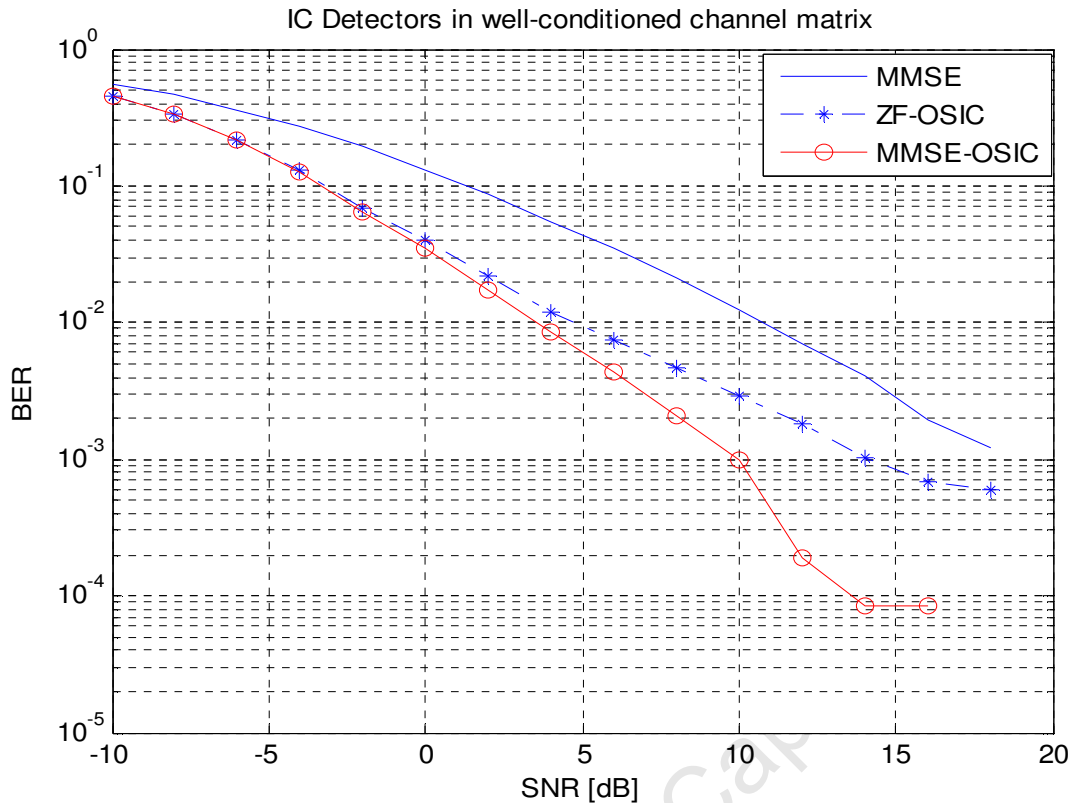


Figure 4-5. Comparison of MMSE and Interference Cancellation methods

Interference cancellation method employs linear detectors for detecting the transmitted signal therefore the overall performance is determined by detector used. The MMSE detector performs better than ZF detector in a Rayleigh fading channel therefore the performance of MMSE-OSIC is expected to be better than ZF-OSIC. As shown by Figure 4-5 the performance of the two interference cancellation detectors is almost the same for signal to noise ratio below 5dB the same characteristics are shared by Figure 4-1 comparing the linear detector. One thing to note is that the successive interference cancellation greatly improves performance of the linear detectors as shown by the BER performance of MMSE detector and MMSE-OSIC detector above.

V-BLAST SIC detection, assuming without error propagation, converts the channel into parallel paths with increasing diversity at each successive stage. In reality error propagation exists and error performance is dominated by the weakest first stream decoded, therefore increased diversity does not help.

A short description of the Maximum likelihood (ML) detector will be given but it will not be considered in this chapter. ML detector performs vector decoding and it is an optimal decoder. It chooses a vector \hat{s} that solves $\hat{s} = \underset{s}{\operatorname{argmin}} \|r - Hs\|_F^2$, see [42], where the optimization is performed through a search of all candidate vector symbol s . The computational complexity grows exponentially with the number of transmit antennas M_T ; and it might be too high for moderate systems and big constellation sizes. Fast algorithms that employ ML have been proposed, for example sphere decoding techniques, but they still have higher computational complexity [48].

4.3.4 Output Noise Analysis

This section investigates the general formula for detector effect on input noise. The calculations can be done and in fact was done in Matlab and the code can be seen in Appendix A.

Zero Forcing Detectors.

Noise calculation for (2×2) MIMO OFDM block is represented by,

$$y = h * x + n \quad \text{where} \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} * x_1 + h_{12} * x_2 + n_1 \\ h_{21} * x_1 + h_{22} * x_2 + n_2 \end{pmatrix}.$$

The output of the detector is,

$$\hat{x} = x + \operatorname{inv}(h) * n.$$

The output noise coefficient determined is then,

$$\frac{1}{B} \begin{pmatrix} h_{22} * n_1 - h_{12} * n_2 \\ -h_{21} * n_1 + h_{11} * n_2 \end{pmatrix} \quad \text{where } B = h_{11} * h_{22} - h_{12} * h_{21}$$

The norm of the output noise vector is used to compare the coefficients of the input and noise values

$$\frac{1}{B^2} (h_{22}^2 * n_1^2 + 2 * h_{12} * h_{21} * n_1 * n_2 - h_{21}^2 * n_2^2 + h_{21}^2 * n_1^2 + 2 * h_{21} * h_{11} * n_1 * n_2 - h_{11}^2 * n_2^2)$$

Randomly generated fading coefficients shown below are used to estimate the effect of ZF detector on input noise. $h_{11} = 0.8147$, $h_{12} = 0.1270$, $h_{21} = 0.9058$, $h_{22} = 0.9134$

The norm for the output noise is calculated to be **5.8985** which more than twice the input noise. The above calculation shows what has been proved in the literature; that the ZF detector increases the magnitude of the input noise. In our second calculation we are going to show the reduction of output noise value. As discussed earlier the ZF-SIC detector utilizes a ZF detector but has the advantage of cancelling the interference effect on the successive symbols.

The noise coefficients of the first decoded symbols were found to be $\frac{1}{B} (h_{21} * n_1 - h_{11} * n_2)$, where $B = (h_{11} * h_{22} - h_{12} * h_{21})$.

After interference cancellation of the decoded signal x_1 , zero forcing will be used to decode the remaining symbol x_2 . From Appendix A, the (2×2) SIC code shows the noise coefficients to be,

$$\frac{1}{B_{t2}} (\text{conj}(h_{12}) * n_1 + \text{conj}(h_{22}) * n_2)$$

$$\text{where } B_{t2} = (\text{conj}(h_{12}) * h_{12} + \text{conj}(h_{22}) * h_{22}) * \text{conj}(h_{12})$$

The noise coefficients for the (3×3) MIMO with ZF SIC detector has, as noise coefficients for the first decoded signal, the following:

$$\frac{1}{B_{t21}} [(h_{22} * h_{33} - h_{23} * h_{32}) * n_1 + (-h_{12} * h_{33} + h_{13} * h_{32}) * n_2 + (h_{12} * h_{23} - h_{13} * h_{22}) * n_3].$$

where,

$$B = (h_{11} * h_{22} * h_{33} - h_{11} * h_{23} * h_{32} - h_{21} * h_{12} * h_{33} + h_{21} * h_{13} * h_{32} + h_{31} * h_{12} * h_{23} - h_{31} * h_{13} * h_{22})$$

Second decoded signal noise coefficients:

$$\frac{1}{B_{t22}} (\text{abs}(h_{12}^2 * h_{23}^2) + \text{abs}(h_{12}^2 * h_{33}^2) - \text{conj}(h_{22}) * h_{23} * \text{conj}(h_{13}) * h_{12} - \text{conj}(h_{32}) * h_{33} * \text{conj}(h_{13}) * h_{12}) / h_{12} * (n_1) - (-\text{abs}(h_{22}^2 * h_{13}^2) - \text{abs}(h_{22}^2 * h_{33}^2) + \text{conj}(h_{12}) * \text{conj}(h_{23}) * h_{22} * h_{13} + \text{conj}(h_{23}) * \text{conj}(h_{32}) * h_{33} * h_{22}) / h_{22} * (n_2) + (\text{abs}(h_{32}^2 * h_{13}^2) + \text{abs}(h_{32}^2 * h_{23}^2) - \text{conj}(h_{12}) * \text{conj}(h_{33}) * h_{32} * h_{13} - \text{conj}(h_{33}) * \text{conj}(h_{22}) * h_{23} * h_{32}) / h_{32} * (n_3)$$

$$\begin{aligned}
B_{t32} = & (\text{conj}(h_{12}) * h_{12} * \text{conj}(h_{23}) * h_{23} + \text{conj}(h_{12}) * h_{12} * \text{conj}(h_{33}) * h_{33} + \text{conj}(h_{22}) * h_{22} * \text{conj}(h_{13}) \\
& * h_{13} + \text{conj}(h_{22}) * h_{22} * \text{conj}(h_{33}) * h_{33} + \text{conj}(h_{32}) * h_{32} * \text{conj}(h_{13}) * h_{13} + \text{conj}(h_{32}) \\
& * h_{32} * \text{conj}(h_{23}) * h_{23} - \text{conj}(h_{12}) * h_{13} * \text{conj}(h_{23}) * h_{22} - \text{conj}(h_{12}) * h_{13} * \text{conj}(h_{33}) \\
& * h_{32} - \text{conj}(h_{22}) * h_{23} * \text{conj}(h_{13}) * h_{12} - \text{conj}(h_{22}) * h_{23} * \text{conj}(h_{33}) * h_{32} - \text{conj}(h_{32}) \\
& * h_{33} * \text{conj}(h_{13}) * h_{12} - \text{conj}(h_{32}) * h_{33} * \text{conj}(h_{23}) * h_{22})
\end{aligned}$$

The noise coefficients for the third decoded symbol:

$$\frac{1}{B_3} (\text{conj}(h_{13}) * n_1 + \text{conj}(h_{23}) * n_2 + \text{conj}(h_{33}) * n_3)$$

$$\text{where } B_3 = (\text{conj}(h_{13}) * h_{13} + \text{conj}(h_{23}) * h_{23} + \text{conj}(h_{33}) * h_{33})$$

Noise coefficients for (4×4) the last decoded signal:

$$\frac{1}{B_4} (\text{conj}(h_{14}) * n_1 + \text{conj}(h_{24}) * n_2 + \text{conj}(h_{34}) * n_3 + \text{conj}(h_{44}) * n_4)$$

$$\text{where } B_4 = (\text{conj}(h_{14}) * h_{14}) + (\text{conj}(h_{24}) * h_{24}) + (\text{conj}(h_{34}) * h_{34}) + (\text{conj}(h_{44}) * h_{44})$$

Looking at the last decoded signal the numerator coefficients for the noise for (2×2) , (3×3) and (4×4) are always derived from the last column in a uniform way. The denominator of the coefficients is always divided by the sum of absolute values squared of the diagonal entries. This can be used to predict the noise coefficients of (5×5) as shown below

$$\frac{1}{B_5} (\text{conj}(h_{15}) * n_1 + \text{conj}(h_{25}) * n_2 + \text{conj}(h_{35}) * n_3 + \text{conj}(h_{45}) * n_4 + \text{conj}(h_{55}) * n_5)$$

where

$$B_5 = (\text{conj}(h_{15}) * h_{15}) + (\text{conj}(h_{25}) * h_{25}) + (\text{conj}(h_{35}) * h_{35}) + (\text{conj}(h_{45}) * h_{45}) + (\text{conj}(h_{55}) * h_{55})$$

The above calculations have been proved by simulations in MatLab as shown in Appendix A.

Table [4-1] from [69] gives a summary of the diversity order for the above discussed detectors. ZF detector has the least diversity order of the all detectors whilst the ML detector has the highest order of M_R .

Table 4-1: Detectors diversity order

Receiver	Diversity Order
ZF	$N_R - M_T + 1$
MMSE	$\approx N_R - M_T + 1$
SUC	$\approx N_R - M_T + 1$
OSUC	$> N_R - M_T + 1, < N_R$
ML	N_R

As shown by the Table [4-1] above ordering increases the diversity order of successive cancellation receiver. Successive cancellation receiver employs successive interference cancellation for detecting symbols. Table [4-2] gives a short summary of the discussed detectors.

Table 4-2: Detector performance

Receiver	Complexity	Performance
ZF	Low	Poor
MMSE	Low (higher than ZF)	Good
SUC	High (higher than ZF)	Good (better than MMSE)
ML	Highest	Best

Successive cancellation receivers have higher complexity than MMSE detector. SUC detector performance is between MMSE and ML detector.

Chapter 5

5. SM MIMO OFDM detector in ill-Conditioned Channel Model

SM MIMO OFDM system model is discussed in this chapter with emphasis on detection algorithms. As evident from Chapter 4, interference cancellation improves the performance of linear detectors. MMSE with interference cancellation has a lower BER compared to MMSE. MMSE SIC also has a higher diversity order than MMSE but with higher complexity. This chapter investigates the performance of different interference cancellation methods in Rayleigh fading channel. The detectors are further tested in ill-conditioned channel model and showing how the performance of detector is greatly reduced. We propose a method to improve the detector BER in the presence of ill-conditioned channel matrix.

5.1. System Model

Spatial multiplexed OFDM system has been proposed by different researchers as a solution to increase the capacity of communication networks [40-42]. In this thesis, a discrete baseband spatially multiplexed MIMO OFDM model is considered with $N_R \geq M_T$ antennas.

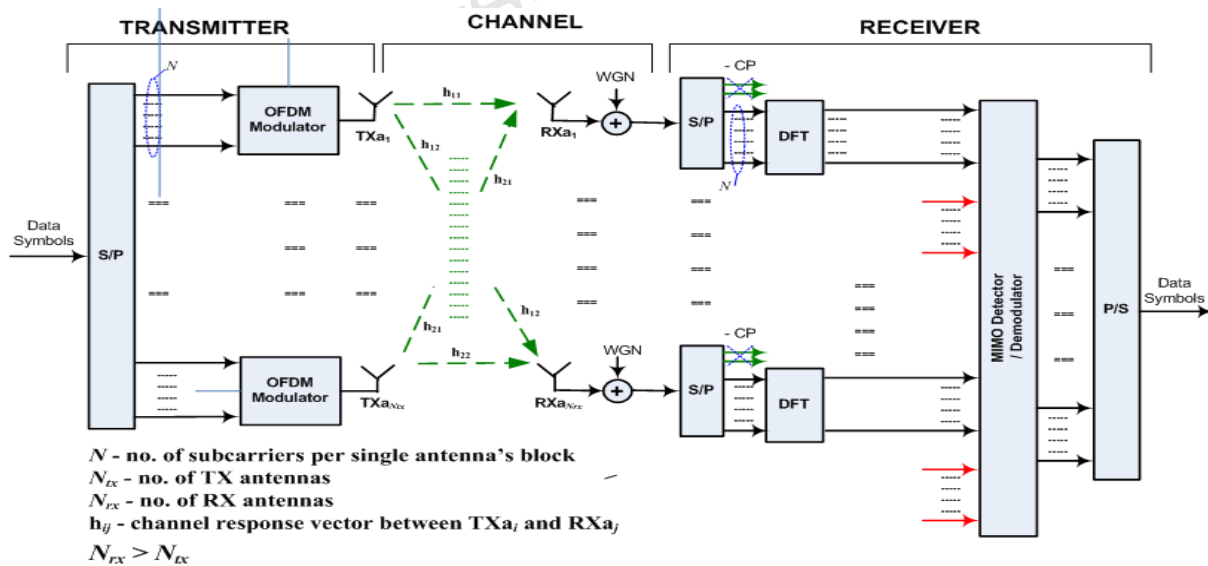


Figure 5-1. MIMO-OFDM system

At the transmitter, input bits are divided into M_T parallel streams which are mapped to PSK or QAM creating N complex valued symbols $x_{n,m}, n \in [0, N - 1]$. The N complex symbols form a block which is loaded to N orthogonal subcarriers. The channel is considered to be slow fading and it remains constant during the transmission of each block. The block is then IDFT transformed and a cyclic prefix is added to the block, the prefix should be sufficiently long enough such that $N_{cp} \geq L$ to suppress any possible ISI. If the N is a power of 2, then IFFT can be utilized for the OFDM modulator.

The Channel Impulse Response (CIR) is described by the vector $h_m(j, t) = [h_{l,m}(j, t)]_{L \times 1}$, where $l \in [0, L - 1]$, $t \in [0, M_s - 1]$ and $j \in [0, N_R - 1]$ [43]. $h_m(j, t)$ is linked to a quasi-static approximation of a wide sense stationary uncorrelated scattering (K path) response model [43], the channel response model is given by,

$$g(t, j, i) = \sum_{m=0}^{\infty} \sum_{k=0}^{K-1} a_{k,m}(j, i) \delta[t - mT - \tau_k(j, i)]. \quad (5.1)$$

According to the formula,

$$h_m(j, t) = \sum_k (j, t) a_m(j, t), \quad (5.2)$$

where k is the path index, τ_k is the path delay, T is the block duration including CP and path gains $a_m(j, t) = [a_m(j, t)]_{K \times 1}$, $k \in [0, K - 1]$ represent a zero mean complex Gaussian random variable vector [43]. Signal $y_{m,n}$ received at the j^{th} antenna within the m^{th} block is,

$$y_{n,m}(j) = \sum_{i=0}^{M_T-1} x_{n,m}(i) h_{n,m}(j, t) + w_{n,m}(j), \quad (5.3)$$

where $h_{n,m}(j, i)$ is the Channel Frequency Response (CFR) corresponding to the (i, j) th spatial layer, and $w_{n,m}(j)$ is the Gaussian noise. The received signal cyclic prefix is removed and it is transformed by DFT before the decoding of transmitted blocks. Detection is done block by block. According to [42] the spatial multiplexing for MIMO OFDM can be represented simpler in matrices as,

$$y[k] = \sqrt{\frac{E_s}{M_T}} H[k]s(k) + n(k), \quad k = 0, 1, 2, \dots, N-1 \quad (5.4)$$

where $s[k]$ is the signal vector with M_T data symbols launched over the k^{th} tone. NM_T scalar symbols are transmitted over each one OFDM symbol with M_T symbols transmitted over each tone. $y(k)$ is a vector of size N_R and $H[k]$ is a $N_R \times M_T$ matrix.

5.2 Parallel Interference Cancellation

In this thesis we propose the use of parallel interference cancellation method that has been applied to multiuser systems in [70]. In successive interference cancelation, other users are treated as interference to the signal/user of interest. Signals are decoded one at a time starting with the most reliable when ordering is applied. The PIC method is a suboptimal detector and compared to other MUD methods it offers low computational complexity [49].

$$r_1 = Hs + n \quad (5.5)$$

We describe the PIC method; it consists of M_T identical branches and in the first iteration all the signals are decoded using a linear decoder. The first stage makes hard or soft decisions for all users:

Initialization

Stage 1

$$\hat{s} = H^\dagger r_1, \quad (5.6a)$$

In this stage all the transmitted signals are detected

Recursive

Stage 2

$$\hat{\mathbf{s}} = Q(\hat{\mathbf{s}}).$$

Hard decision of the estimated symbols from zero forcing is taken.

Stage 3

The interfering signals are retransmitted over the channels and subtracted from the total received signal:

$$\hat{\mathbf{s}}_i = \mathbf{r}_i - \sum_{\substack{j=0 \\ j \neq i}}^{M_T-1} \hat{\mathbf{s}}_j \mathbf{H}_j \quad (5.6b)$$

Stage 4

$$\mathbf{G}_i = \mathbf{H}_i^{\dagger},$$

where \mathbf{H}_i^{\dagger} denotes the matrix obtained by zeroing columns k_1, k_2, \dots, k_i of \mathbf{H} except column i and \mathbf{O}^{\dagger} is the Moore Penrose pseudo-inverse.

Stage 5

$$\hat{\mathbf{s}} = \mathbf{G}_i \hat{\mathbf{s}}_i$$

Stage 5 shows the final estimate of the transmitted signal. Stages 2 to 5 are the recursive methods. Every symbol is treated as the last symbol of SIC as shown in stage 5 when decoding the final estimate.

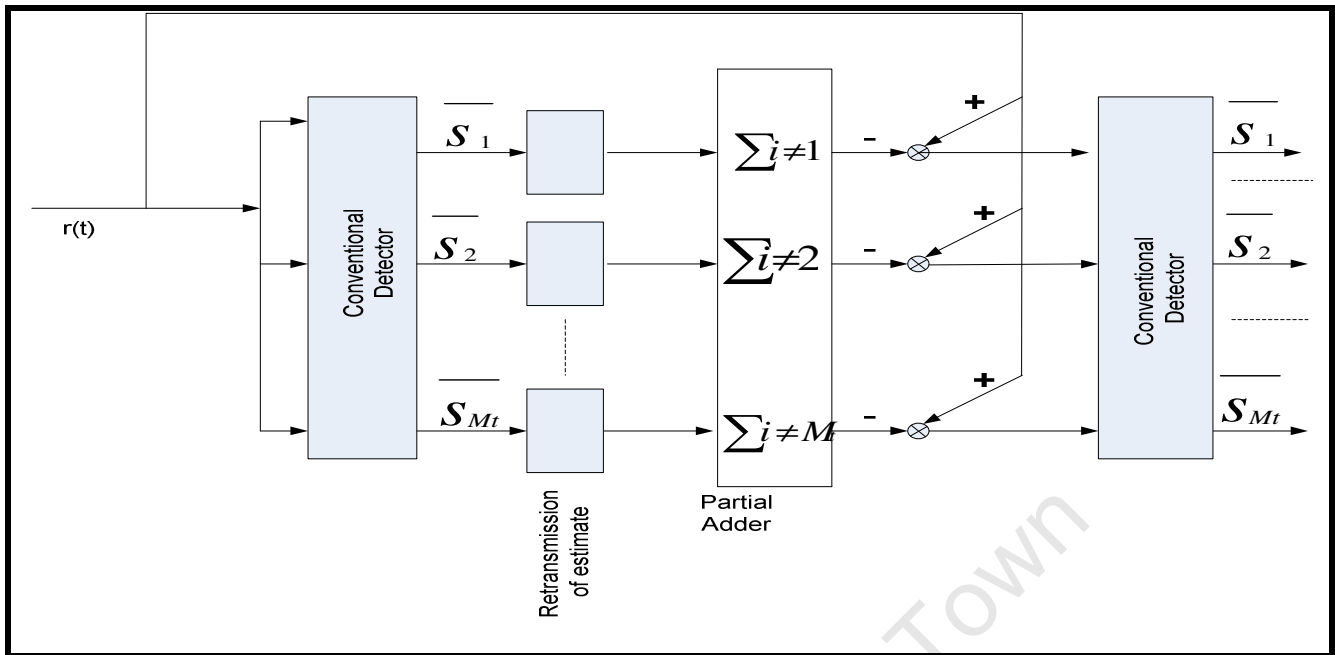


Figure 5-2a. Last Order Successive Interference Cancellation in MIMO OFDM

The method is recursive and one symbol is detected every time cancelling out the other symbols, thus it has been called last order successive interference cancellation. The number of iterations is dependent on the rate of convergence of the estimate this is when the estimated value is the same as previous estimate or it can be set to reduce number of iterations. This process is repeated until the decoded signals no longer change from iteration to iteration. Figure 5-2b shows the SIC (on the left) and PIC detector. As shown in the diagram below SIC is a recursive process for removing interference whilst on the other hand PIC has less number of iterations.

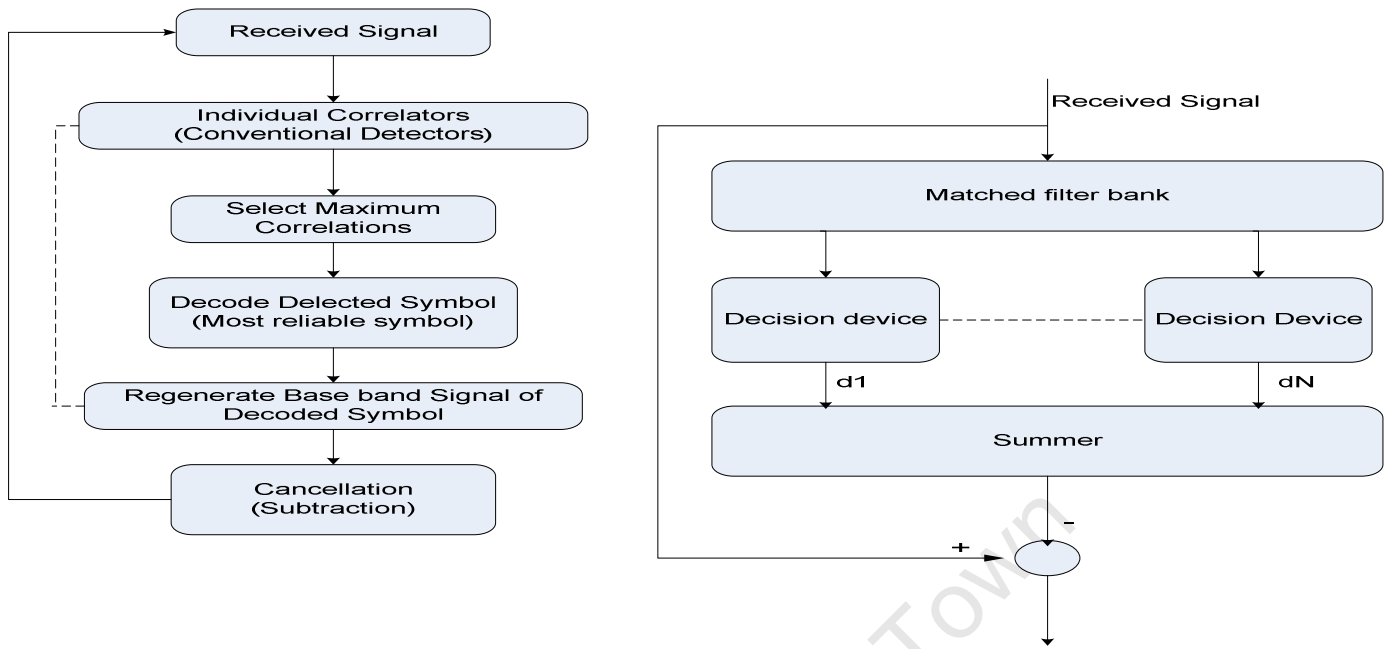


Figure 5-2b: Comparison of SIC and PIC

Simulation of the system model with parallel interference cancellation is investigated and the results are shown in Figure 5-3

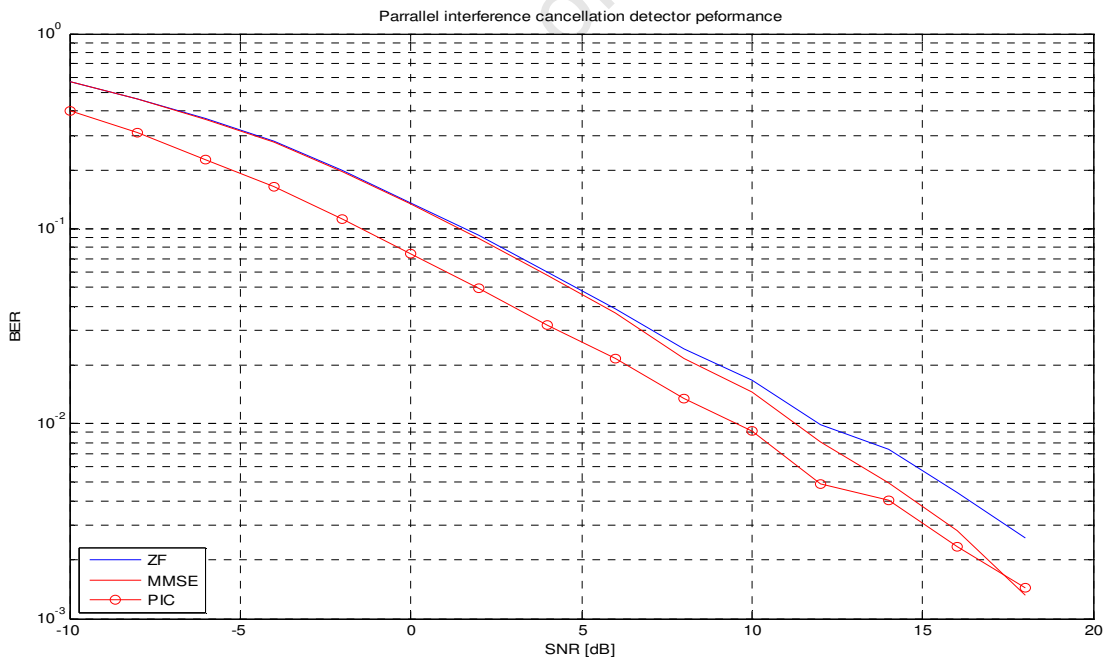


Figure 5-3. PIC detector performance

Parallel interference cancellation performance in multiuser CDMA systems has been proved [71] to perform better than linear detectors. As shown in Figure 5-3, at 8dB, the PIC detector has BER of 0.013, whilst the second best detector is MMSE with bit error rate is 0.021 and ZF has the highest bit error rate of 0.024. The performance of the PIC depends on the estimation of the interference from the other transmitted symbols thus PIC using MMSE will perform better than PIC employing ZF detector. (All figure graphs were simulated for this project)

Error propagation is also a significant problem for parallel interference cancellation. One solution would be to divide the signals into several groups based on SINR. For a decision of class m , we use the feedback from classes 1 up to m ; the more reliable decisions are used to cancel out the interference in class m [12]. Only the last stage uses hard decisions.

In this work we combine the PIC and SIC detector to create a Hybrid Interference Canceller (HIC) detector. In a practical implementation of multiuser systems, PIC been shown to suffer from “ping pong” effect whereby BER in even stages converges to one value. SIC is not affected by such an effect but has considerable processing delay [12]. The HIC detector combines the advantages of the two detectors. HIC detector offers a tradeoff between the complexity and performance: for M_T transmitted symbols from different transmitters, the strongest K symbols are decoded after ordering by SIC and $M_T > K$. K symbols are successively decoded and subtracted from the received signal. The remaining $(M_T - K)$ symbols are decoded through PIC method. HIC detector hardware complexity is higher than SIC but less than PIC [78]. Figure 5.4 shows the performance of ZF, ZF-SIC, Hybrid Detector and PIC detection algorithms with our system model simulated in Matlab software.

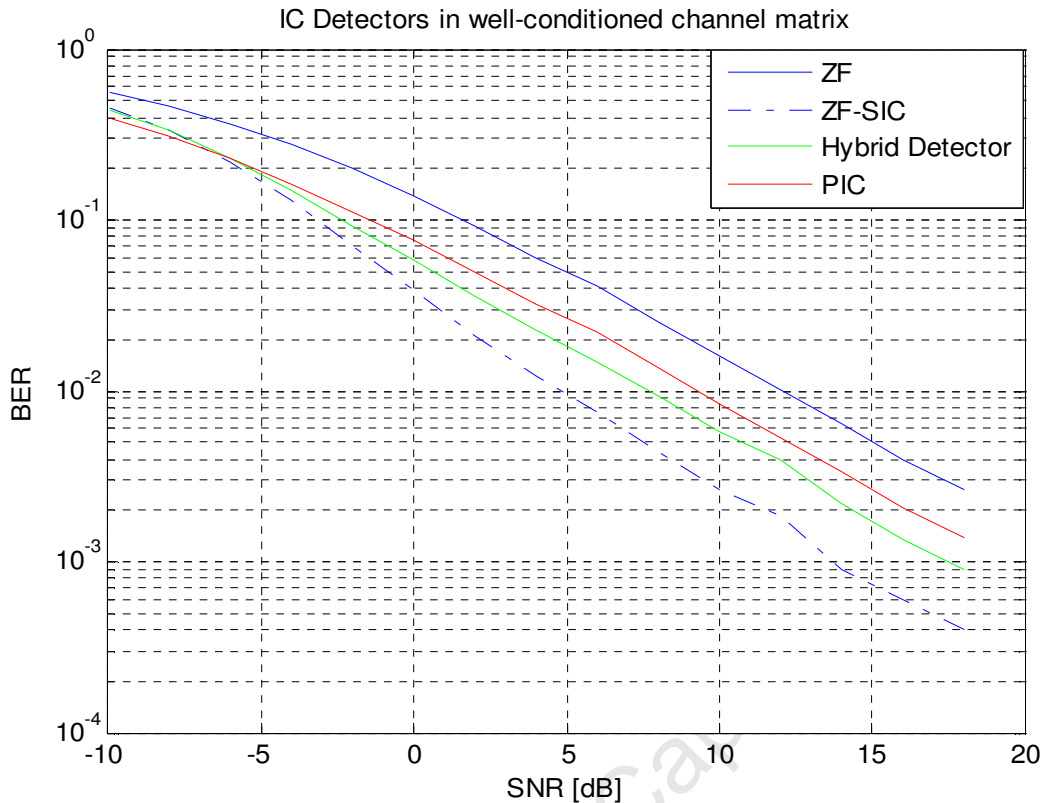


Figure 5-4. Inference cancellation detectors in well conditioned Rayleigh channel

Figure 5-4 shows that the BER performance of HIC is inferior to SIC but it offers an advantage in practical implementations were HIC offers less processing time. Instead of cancelling all the symbols in series or parallel, they are cancelled partially in series and parallel [70]. The performance of detectors above can further be improved by use of a MMSE linear detector before cancellation. MMSE estimation of the inverse channel matrix is affected by the estimation of the noise variance. Considering a hybrid interference cancellation detector for a (4×4) system, first two symbols are decoded by SIC then the remaining two by PIC.

5.3 Proposed solution

Either fading correlation or Rician factor introduces ill conditioning in the channel matrix. Ill-conditioning reduces system diversity of the channel matrix. Spatial multiplexing system performance is dependent on the number of independent channel in a system for transmission. Ill-conditioning induces linear dependency on the channel matrix eigenvectors thus reducing eigenvector independence. Reduction in diversity reduces the channel capacity and it increases the system BER of a detector as shown in figure 5-5.

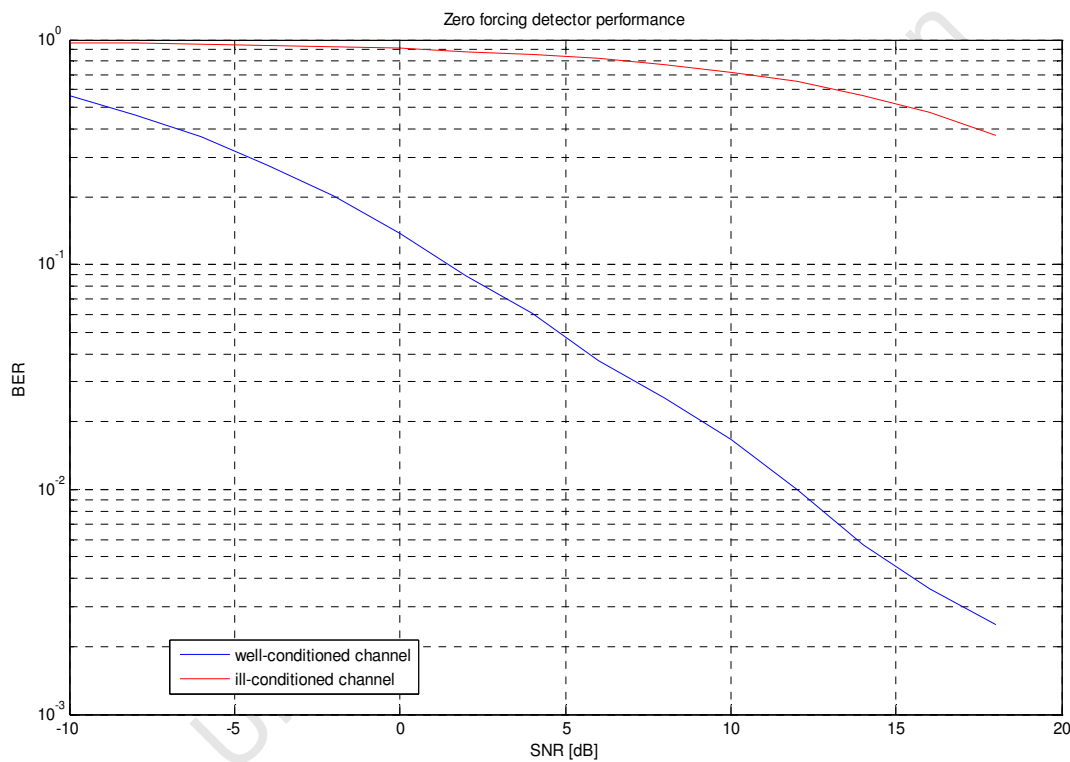


Figure 5-5. Zero forcing detector performance in ill and well conditioned channel

BER of a ZF detector in an ill conditioned channel is high compared to a well conditioned channel. As shown by the figure above the detector crashes at low SNR (2dB) the detector in ill conditioned channel has BER of 0.8866 whilst in well conditioned channel BER is 0.0896. An increase in SNR does not improve the performance of the detector at high SNR (20dB) the detector in ill-conditioned

channel has BER of 0.3759 whilst in well conditioned the BER is 0.0025. An ill-conditioned matrix has got inverse but its an almost singular matrix. Thus an accurate estimate of the channel matrix inverse is required. This can be done by the MMSE detector which takes noise variance into consideration when determining the inverse of channel matrix but it also crashes as shown in the figure 5-6

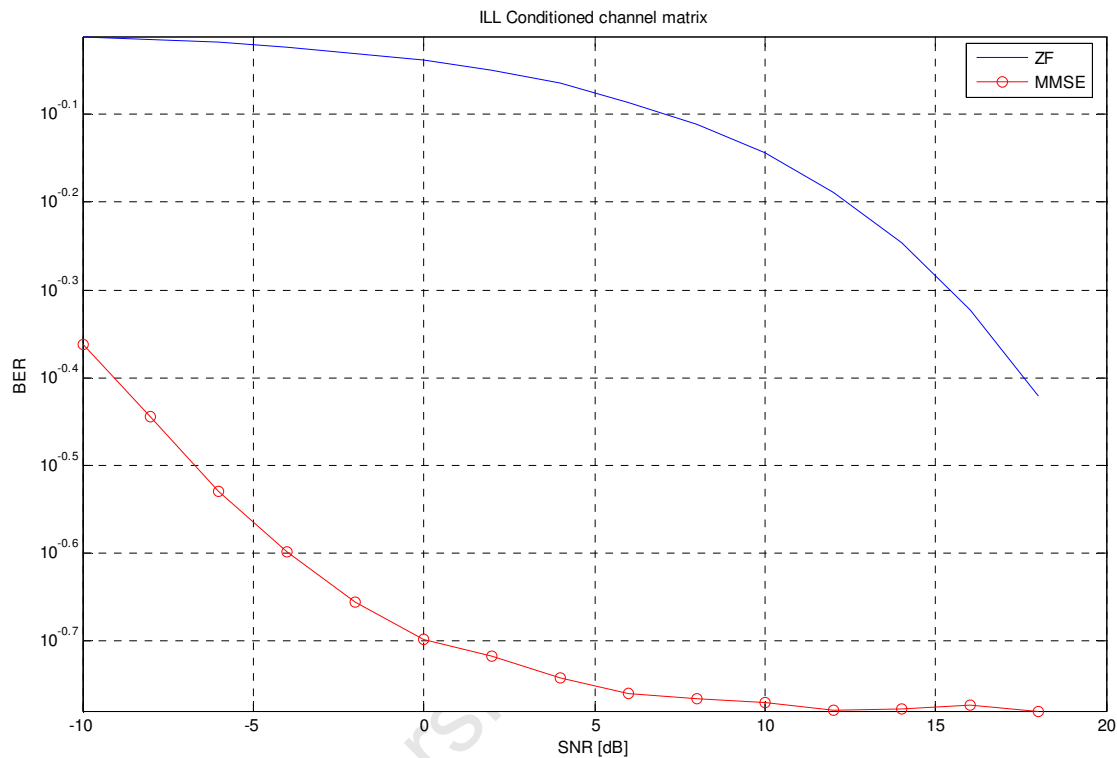


Figure 5-6. Comparison of linear detector in ill-conditioned channel

In Figure 5-6 simulation of our system model with MMSE detector is shown to be a better detector but its BER is still high considering that 100 000bits were transmitted but the best error it could provide was 0.1659 at high SNR (20dB). On the other hand the MMSE is better than ZF whose BER at high SNR (20dB) is 0.3790. System model simulation shows an expected of the interference cancellation methods employing linear detector as shown by figure 5-7.

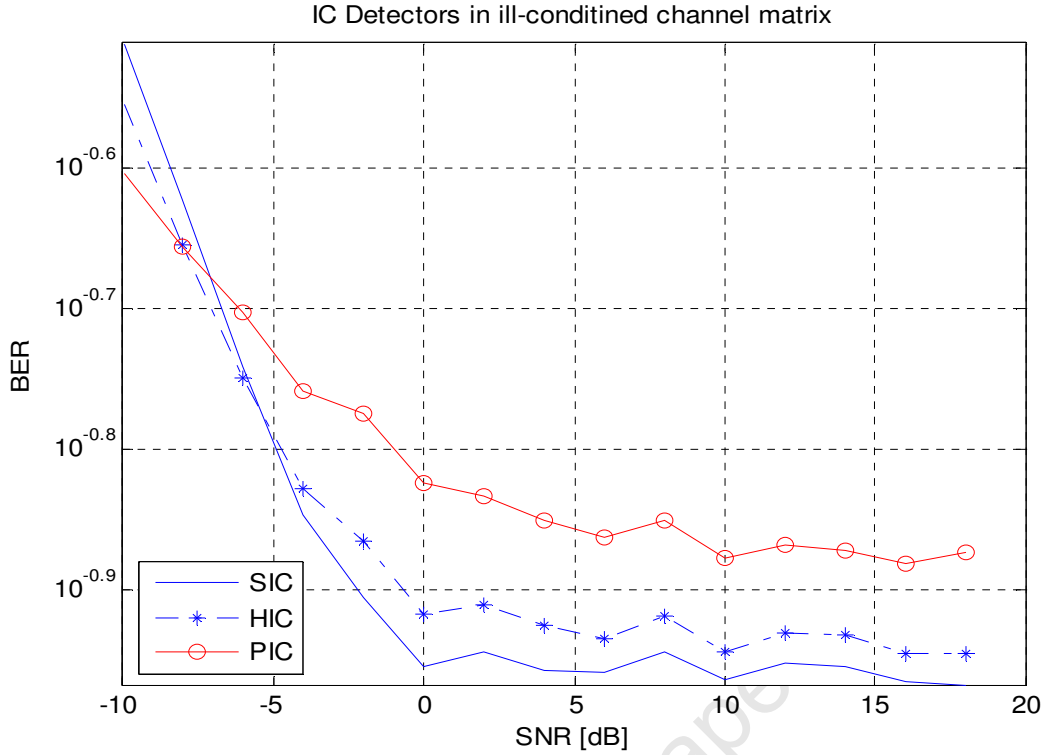


Figure 5-7. MMSE based detectors in ill conditioned Rayleigh channel

In figure 5-7 the performance of all interference cancellation detectors is greatly reduced in the presence of ill-conditioning in the channel. SIC has the best performance between the three detectors but with a very high BER values. MMSE detector can be improved by biasing the detector through the noise variance estimate. This can be done through regularization methods which can improve the conditioning by modifying the covariance matrix [72]. In radar systems and image processing ill-conditioning is introduced into a system through introduction of noise and Tikhonov regularization has been used to solve the problem. In this work we are going to consider Tikhonov regularization to improve our covariance estimate because it is the most popular method [34]. Regularization is used to solve ill posed problems of linear equations for example,

$$Ax = b, \quad A \in \mathbb{C}^{m \times n}, \quad x \in \mathbb{C}^n, \quad b \in \mathbb{C}^m. \quad (5.7)$$

Tikhonov regulation for least squares amounts to solving the problem.

$$\min \|Ax - b\|_2^2 + \alpha \|Lx\|_2^2 \quad (5.8)$$

In Tikhonov regulation an approximate solution of eqn. (5.8) is given by

$$x_{\alpha} = (A^T A + \alpha I)^{-1} A^T b \quad (5.9)$$

where $\alpha > 0$ is the regularization parameter, A^T is the adjoint operator of A and I is an identity matrix. The regularization parameter α associated with solution of x_{α} is determined by Morozov's discrepancy principle. In [47] this is done as follows

choose $\alpha_0 = 0 < d < 1$

set $j = 0$ and solve (5.9) for x_{α} corresponding to α_0

while $\|Ax_{\alpha_j} - f_{\delta}\| > \delta$

where $\|Ax_{\alpha_j} - f_{\delta}\| = \delta$, f_{δ} is the noisy data

$j = j + 1$

$\alpha_j = d\alpha_{j-1}$

Compute x_{α_j} from (5.9)

end

$\alpha_{max} = \alpha_{j-1}, \alpha_{min} = \alpha_{j-1}$

While not $\|Ax_{\alpha_j} - f_{\delta}\| = \delta$

$\alpha = (\alpha_{max} + \alpha_{min})/2$

Compute x_{α} from (5.9)

If $\|Ax_{\alpha_j} - f_{\delta}\| > \delta$ then $\alpha_{max} = \alpha$ else $\alpha_{min} = \alpha$

end

The above method used to determine the tikhonov parameter is iterative hence it will increase the system complexity; therefore using fixed parameter will reduce complexity. In [46], the Constrained Minimum Output Energy (CMOE) receiver using Tikhonov regularization was proposed which replaces the matrix \mathbf{A} with $\mathbf{A} + \nu \mathbf{I}_{MN}$ as expressed by

$$\tilde{\mathbf{w}} = (\tilde{\mathbf{A}} + \nu \mathbf{I}_{MN})^{-1} \tilde{\mathbf{h}} \quad (5.10)$$

where $\nu = \alpha \times \text{trace}(\tilde{\mathbf{A}})$ is a positive parameter and $\tilde{\mathbf{A}} = (\mathbf{H}^H \mathbf{H} + \delta^2 \mathbf{I})$. In [65] the system is simulated at fixed tikhonov parameters for system with perfect power control when $P_i = 0$ (where P_i is the interfering power relative to the desired transmission). The least squares is run on noise variance where $\alpha = 0.0001$. This value is used to calculate tikhonov parameter in the next chapter of results. Constant parameter reduces the number of iterations required to calculate the best estimate of the matrix inverse in a detector.

Chapter 6

6. System Results

The previous chapter showed that ill-conditioning negatively affects the BER of detection algorithms. Chapter 3 showed that realistic channel have got ill-conditioned channel matrix. To get a better picture of our detector performance we tested them in realistic channel (Kronecker and Weichselberger). We conclude recommending the best performing detector to be used. All the graphs in this chapter were simulated using our system model in Matlab. The assumptions for simulations are:

- The receiver and transmitter are stationary
- BPSK and QAM modulation are used
- No error coding is employed before transmission
- Peak to Power Ratio is not considered in OFDM systems
- 256 subchannels for OFDM transmission

It has been shown in literature that the capacity of space time codes increase linearly with the number of antennas. In figure 6-1 we investigate the relationship between square antenna configurations and BER.

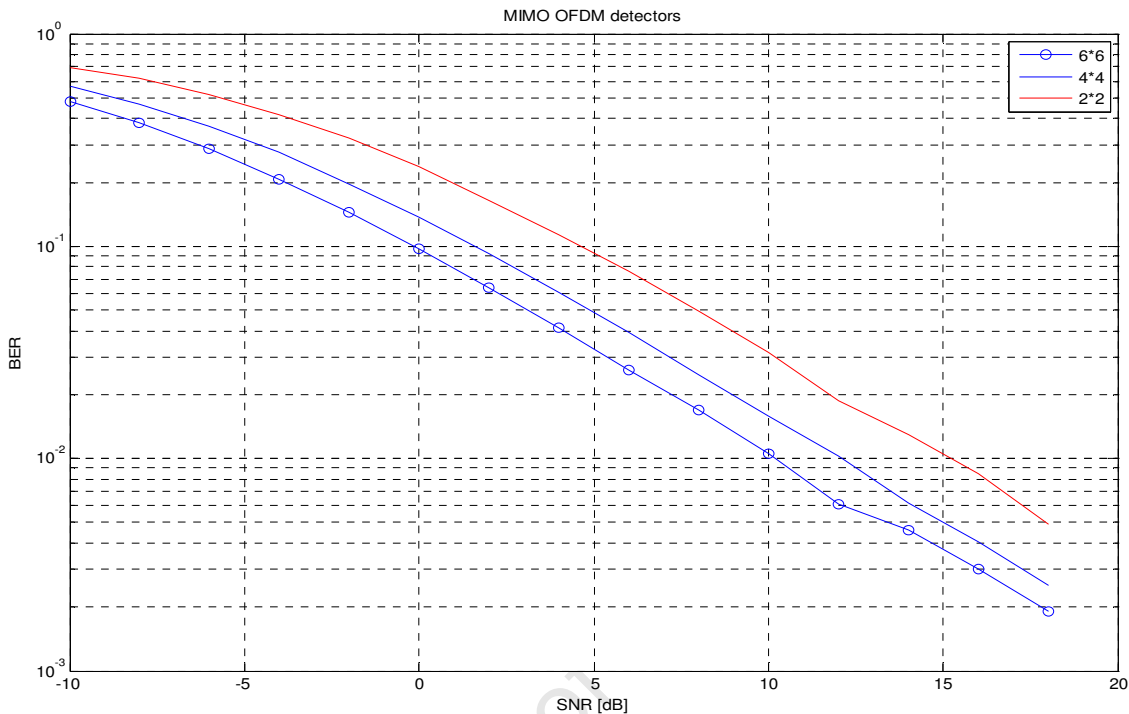


Figure 6-1. BER performance MIMO-OFDM square antenna configuration over Rayleigh fading MIMO channel.

6x6 MIMO performs better than the other square systems. This can be proved by BER at 10dB and high SNR (15dB) as shown in table [6-1]

Table 6-1. High and Low SNR comparison of systems

MIMO ($N_R \times M_T$)	2x2	4x4	6x6
BER (5db)	0.0930	0.0487	0.0343
BER (15dB)	0.0106	0.0053	0.0035

Higher square matrix offer higher spatial multiplexing gain therefore they have better performance. As shown by the table [6.1] above at low signal to noise ratio (6×6) MIMO has BER of **0.0343** whilst (4×4) has a BER of 0.0487 and (2×2) has the highest BER of 0.0930. The performance of the three systems is almost an asymptotic from low SNR (5dB) to high SNR (15dB). The performance spatially multiplexed systems is limited by the number of receivers as show by figure 6-2.

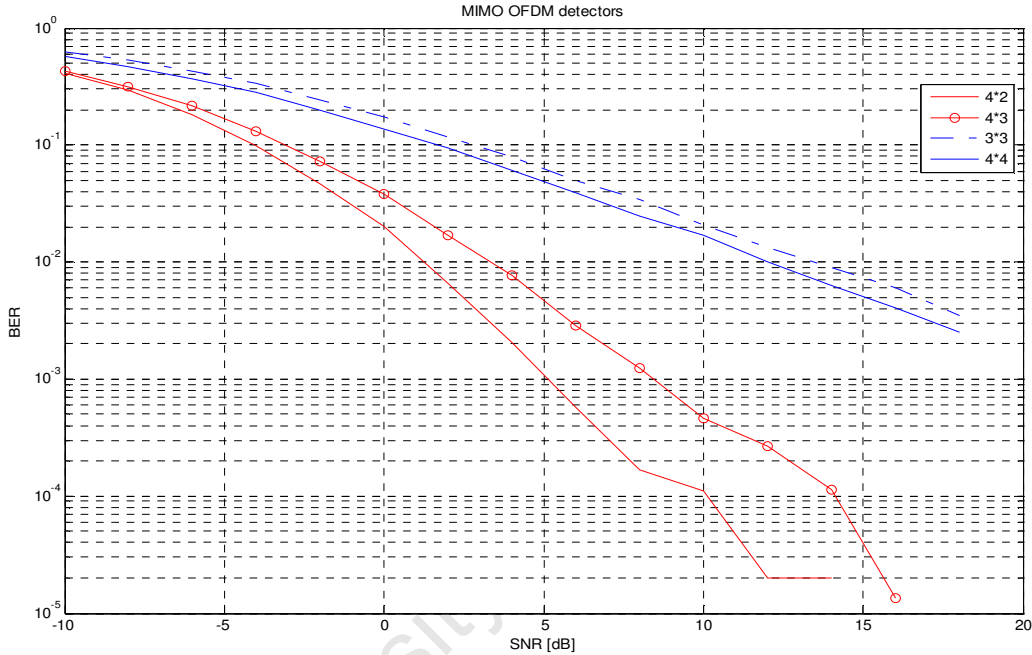


Figure 6-2. Comparison of square and rectangle configurations with ZF detector

In the above diagram the antenna configuration is ($N_R \times M_T$) therefore (4×2) represents two transmitters and four receivers. As discussed earlier 4×4 performs better than 3×3 systems but 4×3 system performs much better. In 4×3 the number of unknown transmitted is less than the number of equations used by receiver to estimate them. More equations give a better estimation of the unknown. More receivers mean more spatial paths therefore better estimation of the transmitted signals. 4×2 has the highest diversity order of 3 ($4-3+1$). This noise effect is greatly reduced as shown by the example given below

Example

$$H = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \\ 6 & 7 & 3 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, n = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$$

$$y = Hx + n$$

At decoding

$$\hat{x} = x + H^{-1}n$$

where H^{-1} is the inverse of the channel matrix

the factor $H^{-1}n$ determines the estimate of soft decoded signal for the above 3×3 MIMO system the noise in decoded signal is

$$\hat{n} = \begin{bmatrix} -0.6585 & 0.1220 & 0.1951 \\ 0.5854 & -0.2195 & 0.0488 \\ -0.0488 & 0.2683 & -0.1707 \end{bmatrix} \times \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$$
$$= \begin{bmatrix} -0.0683 \\ 0.0829 \\ 0.0098 \end{bmatrix}$$

The Frobenius norm equation below is used to measure the size of the elements in the matrix and norm for the new noise is 0.1079

$$\text{Frobenius norm} = \sqrt{\text{sum}(\text{diag}(B' \times B))}$$

For a 4×4 system with channel matrix

$$H = \begin{bmatrix} 1 & 3 & 2 & 5 \\ 4 & 5 & 6 & 4 \\ 6 & 7 & 2 & 3 \end{bmatrix} \text{ therefore the inverse is } H^{-1} = \begin{bmatrix} -0.1127 & 0.0122 & 0.1039 \\ 0.0188 & -0.0508 & 0.1057 \\ -0.1109 & 0.2485 & -0.1380 \\ 0.2557 & -0.0713 & -0.0290 \end{bmatrix} \text{ with the same}$$

transmitted signal and noise as above, $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $n = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$ respectively. The factor $H^{-1}n$ for the new

noise effect is

$$\hat{n} = H^{-1}n = \begin{bmatrix} 0.0007 \\ 0.0147 \\ -0.0001 \\ 0.0311 \end{bmatrix} \text{ and the Frobenius norm is } 0.0344. \text{ Comparing the power of the noise}$$

coefficients it can be shown that the noise for 4×4 has less noise compared to 2×2 thus it had better performance.

Table 6-2. Comparison of rectangle and square configuration at low SNR

MIMO ($N_R \times M_T$)	4×2	4×3	3×3	4×4
BER (0db)	0.0196	0.0362	0.1738	0.1378
BER (5dB)	0.0013	0.0052	0.0647	0.0489

4×2 performs better than 4×3 because it offers better diversity, this can be seen from table 6-2 above. Table [6-2] above shows that BER of the different MIMO configurations at high and low SNR. In figure 6-2 with a constant number of transmitters and variable number of receivers it can be shown that BER performance improves as receivers increase.

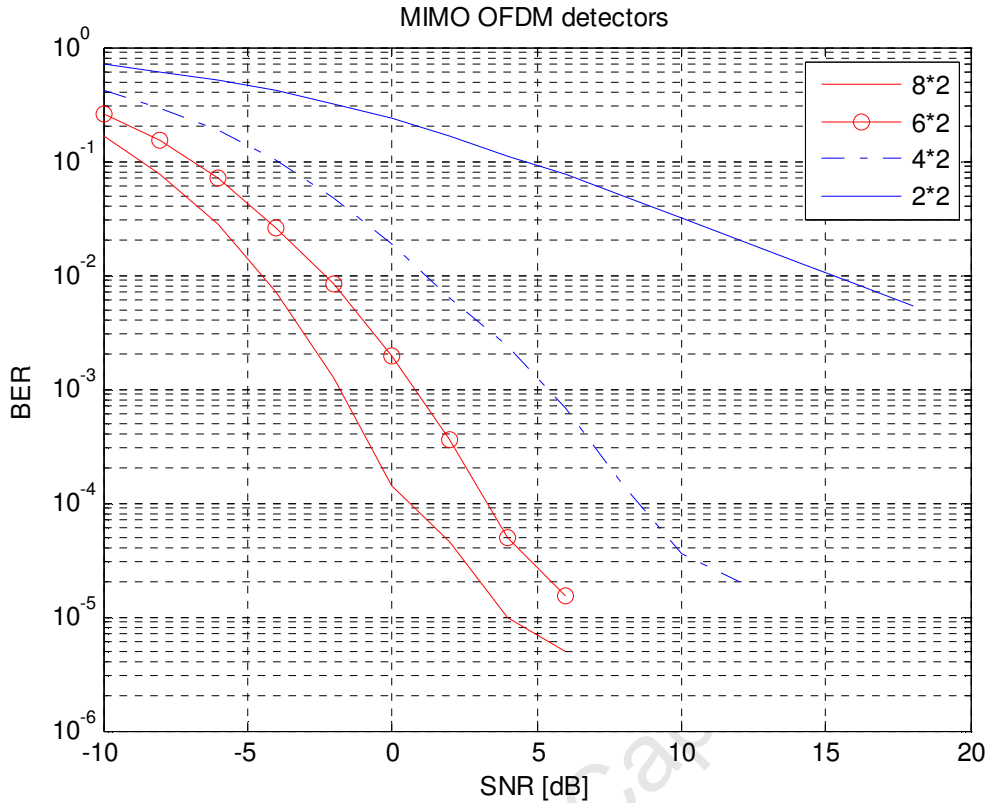


Figure 6-3. BER performance MIMO-OFDM with varying number of receive antennas

8×2 systems performs better than all of the other configuration in comparison because of the high diversity gain ($N_R - M_T + 1 = 7$). 6×2 which has the second high diversity offers the second best performance followed by 4×2 then lastly 2×2 system. At low SNR (8×2) BER is 2.05×10^{-4} which is lower than 6×2 BER of 0.0017 but at high SNR they both have low BER. As shown in [52] by the table [1-1], 4×4 practical implementation can offer good performance and high throughput. 4×4 antenna configuration has been implemented on testbed compared to 6×2 and 8×2 because it requires less complex channel estimation methods.

Modified MMSE detectors

In this section we investigate MMSE based detectors that have been modified using fixed parameter tikhonov parameter for regularization. The modified detectors are also referred as tikhonov MMSE detector. As shown in figure 6-4 the performance of tikhonov MMSE based detector is slightly better than MMSE detector.

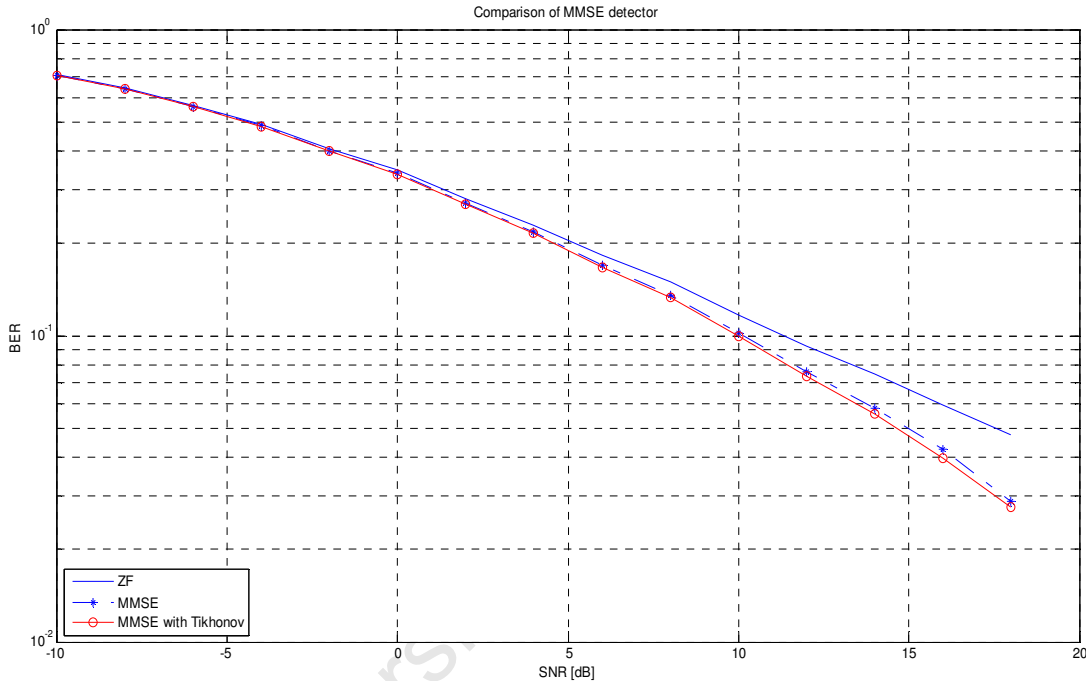


Figure 6-4. Comparison of MMSE and modified MMSE in Rayleigh fading channel (well conditioned channel)

The performance of modified MMSE and MMSE detector is the same in Rayleigh fading channel. Tikhonov parameter has no effect on the estimate of channel matrix inverse in the case of well conditioned channel matrix. A bias of the noise variance parameter in MMSE will not give modified detector a great advantage than the usual detector where no ill conditioning exists as shown by figure 6-4. At low SNR (-6dB) the BER is 0.9170 for modified MMSE whilst the usual MMSE has BER of 0.9225 and ZF detector 0.9412. The performance of the detectors is almost the same for low SNR but the modified MMSE has best performance. Modified MMSE performance improves as the SNR

increase but ZF detector bit error rate is high. In the case of ill conditioned channel the modified MMSE detector performs better than MMSE detector as shown in figure [6-5].

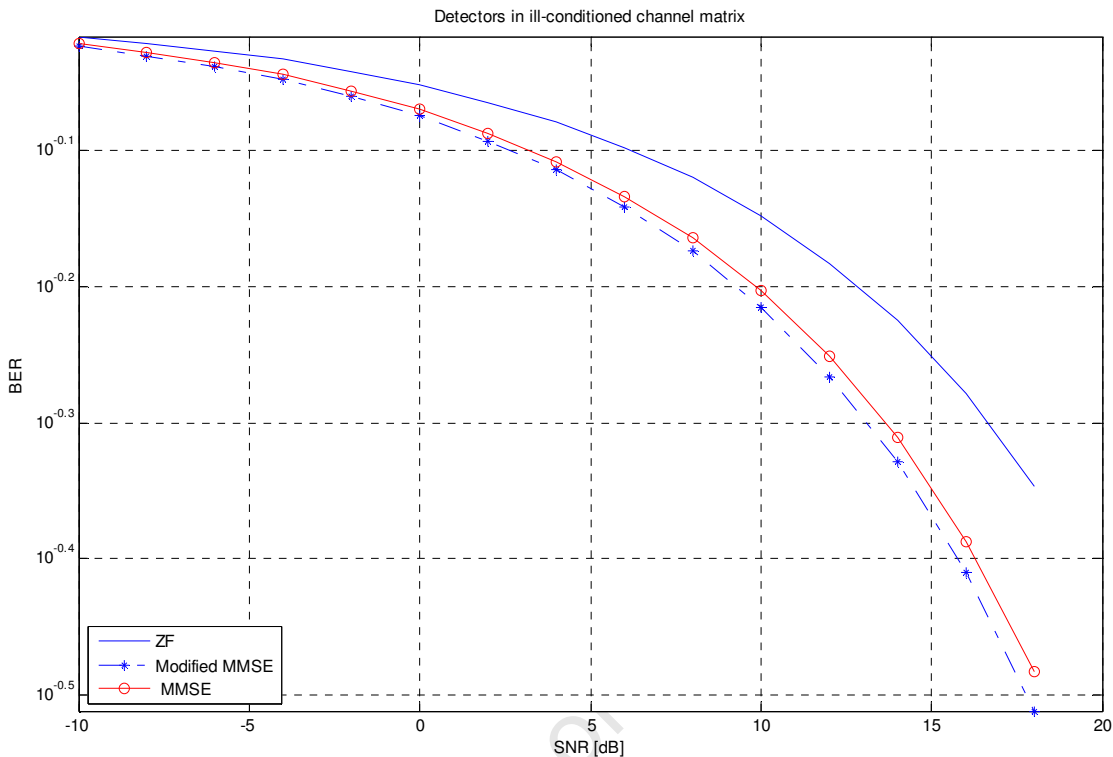


Figure 6-5. Comparison of MMSE and modified MMSE in ill conditioned channel

Ill-conditioning in the channel matrix decreases the system diversity, eigenvalue dependence thus increasing the BER. Modified MMSE has better performance because of a more accurate estimate of the channel matrix inverse. One thousand bits were transmitted and the least BER would be 10^{-5} but the lowest BER is 0.3082. The use of Tikhonov regularization improves the performance of an interference cancellation detector as shown by figure 6-6

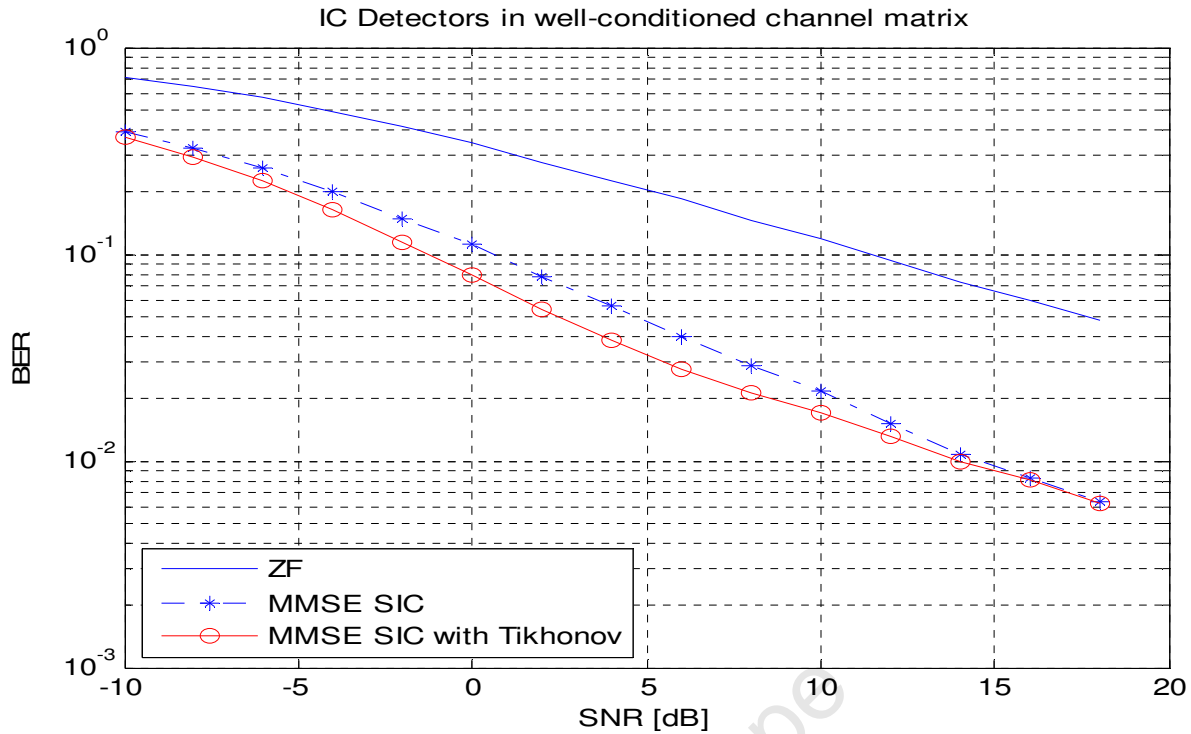


Figure 6-6. Comparison of SIC detector and modified SIC in Rayleigh fading channel

At low SNR the MMSE SIC with tikhonov regularization has a better performance because of the better estimate of inverse matrix. The BER converge high SNR (14dB) because the noise effect is reduced thus regularization parameter effect is also reduced as shown by above figure. This same effect of modified detector converging at high SNR can also be noticed in PIC detector as shown in figure 6-7.

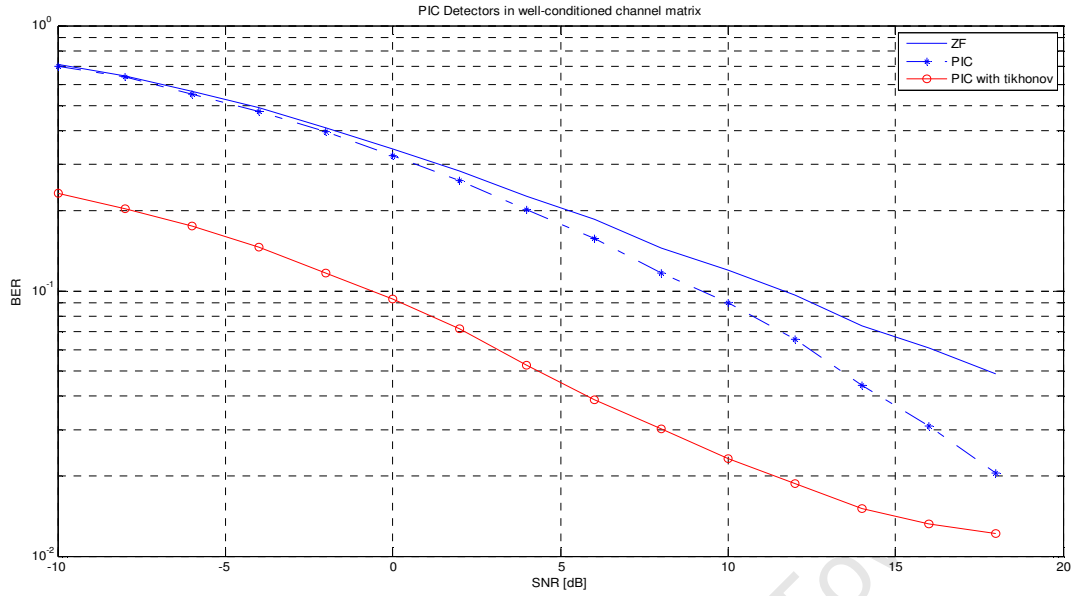


Figure 6-7. Comparison of PIC and modified PIC in ill-conditioned channel

PIC values in high SNR (15dB) show characteristics of going to converge to a single value at very high SNR. In a channel which is ill-conditioned the SIC tikhonov detector offer better performance than Hybrid detector with tikhonov.

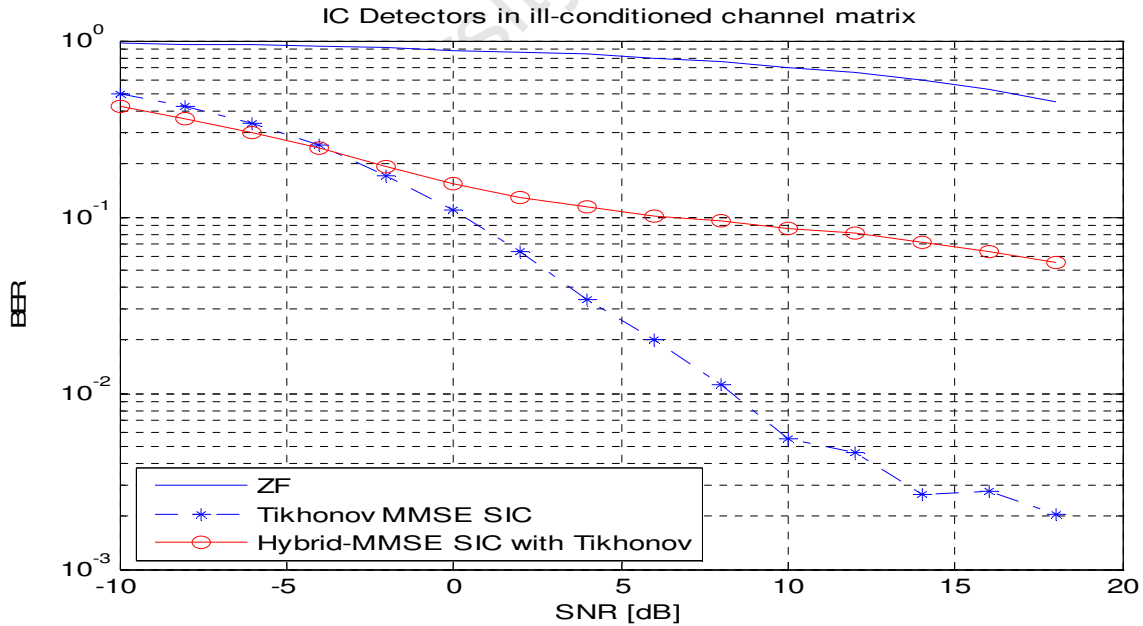


FIGURE 6-8. Comparison OF modified SIC and Hybrid detector in ill -conditioned channel

As shown earlier in figure 6-8 the performance of SIC is better than Hybrid detector which incorporate both SIC and PIC. Thus for highly ill conditioned channel Modified SIC with tikhonov would offer the best performance as shown by the detectors in Weichselberger channel model. A realistic channel would give better estimate of the detector performance therefore in this work the two best detectors are compared in Kronecker channel in figure 6-9.

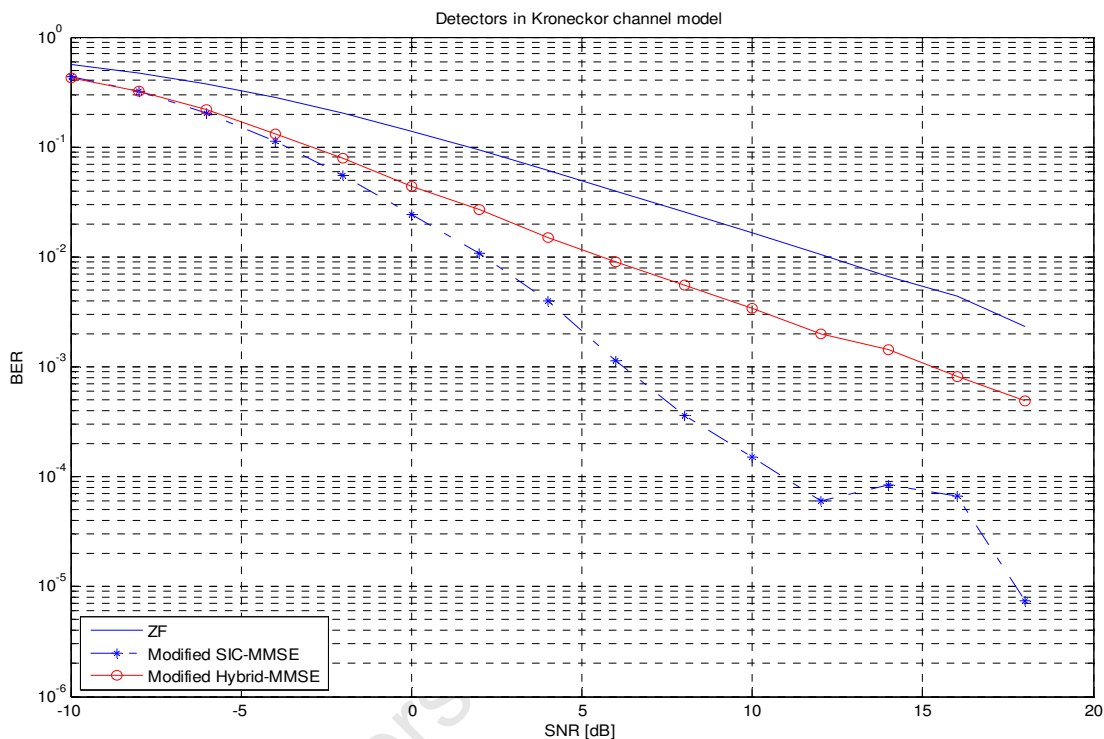


Figure 6-9. Modified Hybrid and SIC detector in Weichselberger channel model.

Kronecker channel model has got higher conditioning number than Rayleigh fading channel. Modified SIC detector is the same with modified Hybrid in low SNR (-6dB) this is because the OSIC performance is depends on ordering symbols post SNR before detection. In slightly conditioned channel matrix ordering does not have advantage compared to hybrid detector. As discussed earlier Weichselberger channel has been shown to be a better estimate of the channel for (4×4) and higher order diversity systems than Kronecker. As shown in figure 3-5 it also displays more ill-conditioning

effect than the Kronecker channel. Figure [6-10] shows the performance of the detectors in Weichselberger channel model

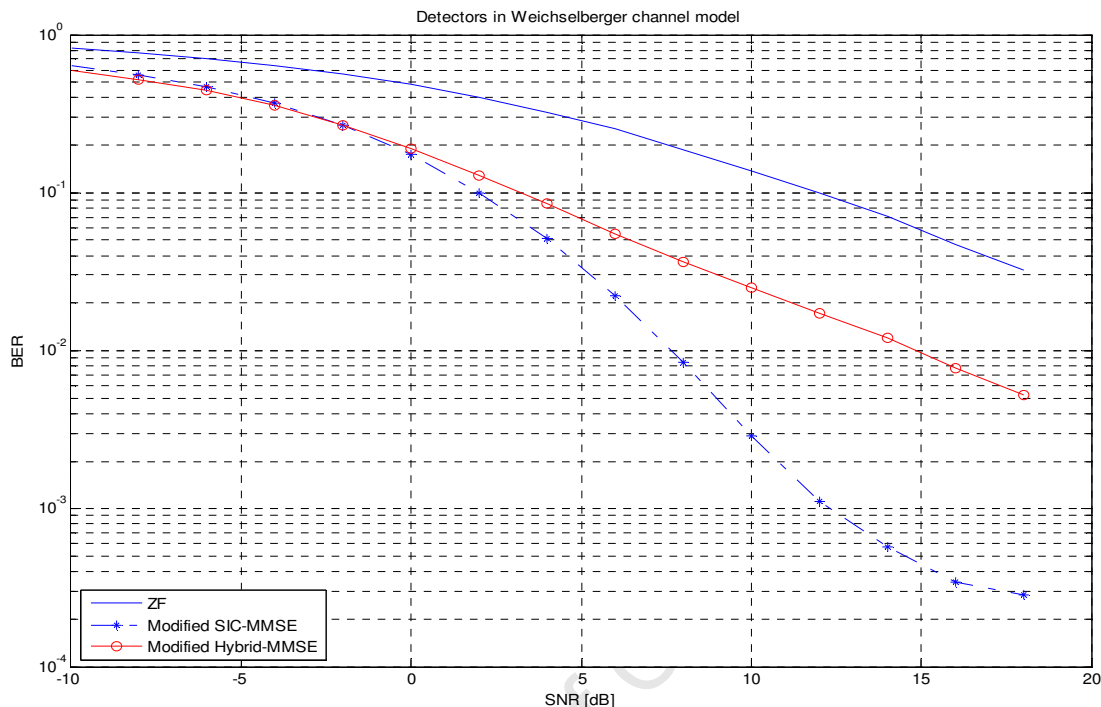


Figure 6-10. Modified Hybrid and SIC detector in Weichselberger channel model.

At low SNR (-10dB) modified Hybrid detector performs then the modified SIC detector. The performance of modified SIC detector only improves the modified Hybrid at SNR of (-4dB). This same characteristic is displayed by PIC compared with SIC detector in a well conditioned channel. SIC detector performance is dependent on the post SNR ordering of the transmitted signals but at very low SNR the values are very low thus successive interference cancellation has no effect because all the transmitted symbols have low SNR. On the other hand PIC detector performance is not dependent on ordering of symbols therefore it performs better than OSIC. We recommended the use of Modified OSIC MMSE detector in MIMO OFDM based systems. This detector has been shown to provide the best BER performance in a real channel models that display correlation and ill conditioning.

Chapter 7

7. Conclusion and Recommendations

This thesis evaluates the performance of MIMO OFDM detectors. The investigated detectors are (1) linear detector (ZF and MMSE), (2) successive interference cancellation (SIC). It is shown the SIC is the same with linear detector with interference cancellation. Analytical calculation for the noise effect of the three detection methods was done to estimate the noise. SIC is shown to be the best performing detector in flat fading Rayleigh channel.

Parallel interference cancellation method is introduced and its performance is investigated in MIMO OFDM systems. SIC has a better BER than PIC but longer processing time this leads us to propose a Hybrid detector that combines the SIC and PIC detector. HIC detector has for a (4×4) system first groups the received the signals based on SNR. PIC detector is used to detect the two signals with highest SNR and SIC is employed to decode the remaining signals. HIC detector switches between two detectors with a set threshold. We compared interference cancellation based detector performance in ill-conditioned Rayleigh fading channel matrix. SIC detector is the best performing but with a high BER compared to normal values in well conditioned channel. It also be noted that interference cancellation detectors use linear detector.

In this work it is shown that MMSE linear detector has better performance than ZF detector therefore Interference cancellation detectors considered are MMSE based detectors. In the presence of ill conditioning the performance of all the detectors is greatly reduced. Ill conditioning has been shown to be reduce systems diversity therefore reducing capacity and BER performance of detectors. Regularization methods have been employed to solve ill posed problems in radar and image processing systems.

Tikhonov regularization method was applied to MMSE detector to improve their performance in the presence of ill conditioning. Performance of MMSE based detectors was investigated in well conditioned channel and ill conditioned channel. Two realistic practical channels were used to

determine the performance of the best modified detectors. The two realistic channels considered for in this thesis are Kronecker and Weichselberger channel models. Modified HIC detector offers a tradeoff between receiver complexity and processing time but Modified OSIC MMSE is the best performing in terms of BER. After considering all the simulated results we recommend the use of Modified OSIC MMSE detector because of its low BER in realistic channels. Modified HIC detector can only be used where processing time of detector is very important for system detection.

Suggestions for future work

In this thesis the noise effect was estimated for the last decoded signal and this can be used in future to cancel out the effect of noise in the detector. Tikhonov regularization might be the most popular but other regularization methods like Total variation regularization should be used. Regularization methods are used to solve ill-posed problems but the solution can be over smoothed. In [77] a local regularization method is proposed that decomposes the regularization into local and global parts in which smoothing is applied in different domains.

Forward error corrections methods are employed in communication to improve the system capacity. System capacity for MIMO OFDM system can be improved by use of low complexity FEC methods like Low density parity codes. The performance of coded OFDM system with MIMO should be investigated with the new modified detector. A combination of STBC and V-Blast would offer a system with high spatial multiplexing gain and diversity gain. MIMO from the combined STBC and V-Blast with coded OFDM would give a system good gain trade-offs and system capacity

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Appendix

Appendix A- Symbolic SIC Noise Coefficient Calculations Code

```
% (2*2) MIMO
syms h11 h22 h12 h21 x1 x2 n1 n2 %defining symbols
x= sym('[x1;x2]');
n= sym('[n1;n2]');
h= sym ('[h11 h12 ; h21 h22]'); %initializing vectors

y = h*x+n; %the normal model

H=inv(h)

%%%%%%%%%%>> B = (h11*h22-h12*h21); %the multiplying factor
hhx1 = H(2,:); %remove first columns of H
%%%%%%%%%%simplify (hhx1*y)
%%%%%%%%%%simplify (B*hhx1*y)
yy1 = y- x1*h(:,1); %blah
hh1 = h(:,2)
hhh1 = (inv(hh1'*hh1))*hh1'
Bt1 =(conj(h12)*h12+conj(h22)*h22)*conj(h12);
simplify (Bt1*hhh1*yy1)
Bt2 =(conj(h12)*h12+conj(h22)*h22);
simplify (Bt2*hhh1*yy1)
at 10dB
noise =
    0.0912
    0.0796
%%%%%%%%%%
conj(h12)^2*(h12*x2+n1)+conj(h12)*conj(h22)*(h22*x2+n2) %determined last coefficients
```

```

% (3*3) MIMO
>> yy2 = yy1 - x2 * hh1(:,1) %Assuming the 1st and 2nd symbols the received after cancelling the two
yy2 =
    h13*x3+n1
    h23*x3+n2
    h33*x3+n3
hhh2 = (hh2'*hh2)*hh2'
>> yy2 = x3*h(:,3)+n
yy2 =
    h13*x3+n1
    h23*x3+n2
    h33*x3+n3
>> hh1 = h(:,3);
>> hhh2 = inv(hh1.'*hh1)*hh1.' % pseudo inverse of the channel matrix multiplying the 3rd symbol
(last symbol)
    hhh2 =
[ 1/(h13^2+h23^2+h33^2)*h13, 1/(h13^2+h23^2+h33^2)*h23, 1/(h13^2+h23^2+h33^2)*h33]
>> B = (h13^2+h23^2+h33^2) %factor used to simplify the pseudo inverse of hh1
B =
h13^2+h23^2+h33^2
>> B*hhh2
ans =
[ h13, h23, h33]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
>> B*hhh2*yy2
ans =
h13*(h13*x3+n1)+h23*(h23*x3+n2)+h33*(h33*x3+n3) %shows the decoded symbol of the 3rd symbol ,it
shows the coefficients of noise

```

```

% (4*4) MIMO
>> yy3 = yy2 - x3*hh2(:,1) %the received symbol after cancelling the first three symbols
correctly

yy3 =

    h14*x4+n1

    h24*x4+n2

    h34*x4+n3

    h44*x4+n4

>> inv (hh3'*hh3)

ans =

1/(conj(h14)*h14+conj(h24)*h24+conj(h34)*h34+conj(h44)*h44)

>> Bt3 = (conj(h14)*h14+conj(h24)*h24+conj(h34)*h34+conj(h44)*h44) %factor used to simplify the
inverse of matrix

Bt3 =

conj(h14)*h14+conj(h24)*h24+conj(h34)*h34+conj(h44)*h44

>> hhh3=(inv (hh3'*hh3))*hh3'

hhh3 =

[ 1/(conj(h14)*h14+conj(h24)*h24+conj(h34)*h34+conj(h44)*h44)*conj(h14),
  1/(conj(h14)*h14+conj(h24)*h24+conj(h34)*h34+conj(h44)*h44)*conj(h24),
  1/(conj(h14)*h14+conj(h24)*h24+conj(h34)*h34+conj(h44)*h44)*conj(h34),
  1/(conj(h14)*h14+conj(h24)*h24+conj(h34)*h34+conj(h44)*h44)*conj(h44)]

>> hhh3=Bt3 * (inv (hh3'*hh3))*hh3'

hhh3 =

[ conj(h14), conj(h24), conj(h34), conj(h44)]

>> hhh3*yy3

ans =

conj(h14)*(h14*x4+n1)+conj(h24)*(h24*x4+n2)+conj(h34)*(h34*x4+n3)+conj(h44)*(h44*x4+n4)

>> simplify (hhh3*yy3) %the simplified decoded symbol before demodulation

ans =

conj(h14)*h14*x4+conj(h14)*n1+conj(h24)*h24*x4+conj(h24)*n2+conj(h34)*h34*x4+conj(h34)*n3+conj(
h44)*h44*x4+conj(h44)*n4

```

```

% (5*5) MIMO
yy5 = hs.*x+n

yy5 =

h15*x1+n1
h25*x2+n2
h35*x3+n3
h45*x4+n4
h55*x5+n5

hhs = inv(hs.'*hs)*hs.'

hhs =

[ 1/(h15^2+h25^2+h35^2+h45^2+h55^2)*h15, 1/(h15^2+h25^2+h35^2+h45^2+h55^2)*h25,
1/(h15^2+h25^2+h35^2+h45^2+h55^2)*h35, 1/(h15^2+h25^2+h35^2+h45^2+h55^2)*h45,
1/(h15^2+h25^2+h35^2+h45^2+h55^2)*h55]

>> B = (h15^2+h25^2+h35^2+h45^2+h55^2);

B*hhs*yy5 %shows the simplified equations of the noise

ans =

h15*(h15*x1+n1)+h25*(h25*x2+n2)+h35*(h35*x3+n3)+h45*(h45*x4+n4)+h55*(h55*x5+n5)

```