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# A modern econometric approach to estimating theoretically consistent demand systems of retail brands

February 15

2011



ABSTRACT

This paper conducts an empirical investigation of the cake mix and frosting demand system using daily data on sales figures, prices and marketing mix variables. SUR maximum likelihood is used to estimate static and dynamic (error correction) linear almost ideal demand systems. Results compare static and dynamic model estimates.

## 1. Introduction

It is claimed that many types of marketing strategies (advertising, promotions, etc.) are capable of shaping consumer preferences. These strategies, that may be brand orientated or generic, may affect the allocation of expenditures between categories of goods, or between the respective brands within those categories. The role of marketing techniques therefore, is to shift consumer preferences towards the products or services of a particular firm.
In recent years, economists have researched the supposed effects of marketing techniques upon the inter-product distribution of demand, often through the estimation of *advertising-augmented* versions of well-known demand systems. As a consequence of the recent burgeoning literature on unit roots and cointegration in time series it has become increasingly apparent that there is a need for the refinement of *estimation* and testing procedures. Cointegration analysis has been developed as a response to the problems associated with non-stationary time series. Here models are specified to include both levels and first differences of the data, and as a result the estimation procedure retains both types of information. This approach uses the variables' levels to estimate the long-run, cointegrating relationships, whilst the variables' differences capture short-run dynamic adjustments that describe how demand moves towards the long-run equilibrium. This process of adjustment would be influenced by various inertia factors that arise from the habitual nature of consumption, adjustment costs, imperfect information and uncertainty. In the periods before these adjustments are completed, consumers will be 'out of equilibrium' and their short-run responses to changes in prices, advertising and income differ significantly from their long-run effects. Similarly, optimal consumer behaviour may differ in the respective long and short-run periods.

The aim of this paper is to derive a systematic framework for modeling the demand for brands using sales and marketing data, while taking the time series properties of the data into consideration. To properly develop this framework, this paper employs the dynamic error-correction specification of Karagiannis and Velentzas (2000). This approach can be interpreted as one that exploits the well-known connection between cointegrated time series and their error-correction representation. The model will be used to test how variables in the marketing mix affect the allocation of demand between two retail store categories, namely; cake mix and frosting (icing sugar). The sales data is collected from scanners at Shoprite and Checkers stores in the Western Cape area.

The structure of the paper is as follows. In the next section, a review of the literature used in writing this paper is presented. Section 3 details the advertising-augmented demand systems. Section 4 contains a description of the data employed and the results of unit root tests. The estimation methods and results are described in section 5, while section 6 contains some final comments and conclusions.

## 2. Literature review

Economists have long debated the inconsistencies between data and theory in applied demand analysis. A paper by Kenzaenkamp and Barten (1995) offers an answer to the question originally raised by Laitinen (1978) indicating three possible sources for the apparent inconsistency. First, the poor quality of price and quantity data (typically this is aggregated), second, a functional misspecification of the demand system, third, the use of inappropriate econometric techniques for estimating. Several efforts (i. e. Ng, 1995; Lewbel, 1996; Ashce, 1996; Balcombe and Davis, 1996; Attfield, 1997; Karagiannis and Velentzas, 1997) have estimated demand systems with modern time series techniques claiming that when price and quantity data have appropriate time-series properties, it is more likely that neither homogeneity nor symmetry is rejected.

The Almost Ideal Demand System (AIDS) model has emerged as the most popular functional form in empirical demand analysis[1]. Its popularity is due to its many desirable attributes: (a) it is an arbitrary first order approximation to any demand system, (b) it satisfies the axioms of choice, (c) it aggregates over consumers, and (d) it is easy to estimate. The estimated coefficients are easy to interpret and it has been extensively used in empirical work.

Traditionally this model has been estimated in a static framework, thereby ignoring the dynamic properties of the data. Early dynamic AIDS models allow for flexible substitution patterns and heterogeneity in consumer tastes. Other dynamic generalizations are well documented and examples include Blanciforti and Green (1983) and Alessie and Kapteyn (1991). Karaginnis and Velentzas (2000) outlined the potential use of error correction models (ECM) in an AIDS. They found that if the time-series properties of the data revealed an existence of a cointegration relationship between the dependent and independent variables, an ECM could be estimated in an iterative seemingly unrelated regression (ISUR) procedure.



[1] See section 3, for further details on the AIDS framework.

# 3. Model specification

## 3.1 Static model

For this study I use the most popular demand framework; the Deaton and Muellbauer (1980) Almost Ideal Demand System (AIDS) specification. This specification models the demand for a good on the price of that good, the price of other goods in the system and the total expenditure across all goods in the system (Duffy, 1995 and 2001). The basic form of the *i*th demand equation in the AIDS is:

$$w_{it} = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \beta_i \ln\left(\frac{M_t}{P_t}\right) \qquad (1)$$

i = 1, 2,..., n; t = 1, 2,... T

n denotes the number of brands in the system, T is the sample size.

The left-hand side of the equation indicates the market share of the *i*th brand in period t, $w_{it}$. This is the proportion of the total per capita expenditure on all n brands in period t that is allocated to the *i*th brand, or $w_{it} = p_{it}q_{it}/M_t$ .

With $p_{it}$ and $q_{it}$ as the price and quantity sold of the *i*th brand at time t, and $M_t$ is the total expenditure on the system of brands given by the following equation; $M = \sum_{i=1}^{n} p_{it}q_{it}$

The right-hand side of the equation models $w_{it}$ on the total expenditure across all brands by consumers as well as the prices of each of the brands in the system. Let j represent the subscript of each of the n brands in the system and $p_{jt}$ is the price of the *j*th brand at time t. The effect that each of the prices of the brands has on $w_{it}$ is introduced by this term;

$$\sum_{j=1}^{n} \gamma_{ij} \ln p_{jt}$$

Where $\gamma_{ij}$ represents the change in the *i*th brand's market share with respect to the change in the *j*th brand's price with real expenditures held constant.

Similarly the effect that the total real expenditure by consumers has on $w_{it}$ is introduced by the following term;

$$\beta_i \ln\left(\frac{M_t}{P_t}\right)$$

Where $P_t$ is a general index of prices in the system which is approximated here by the Stone (1953) form;

$$lnP_t = \sum_{j=1}^{n} w_{jt}\, lnp_{jt}$$

The $B_i$ coefficient represent the change in the *i*th brand's market share with respect to a change in real expenditures with prices held constant.

Since the AIDS equation constitutes the long-run demand equations, the time subscript can be omitted and it is important to recognize that the coefficients represent the long-run effects of the explanatory variables on the budget shares. Thus, this specification does not assume that consumer responses to price and total expenditure are uniform over time. To comply with the theoretical properties of consumer theory the following restrictions are imposed on the parameters in the AIDS model;

- *Adding up restriction; which allows the budget share to sum to unity*

$$\sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{i=1}^{n} \beta_i = 0, \quad \sum_{i=1}^{n} \gamma_{ij} = 0$$

- *Homogeneity; based on the assumption that a proportional change in all prices and expenditure does not affect the quantities purchased. In other words the consumer does not exhibit the money illusion*

$$\sum_{j=1}^{n} \gamma_{ij} = 0$$

- *Symmetry; represents consistency of consumer choices*

$$\gamma_{ij} = \gamma_{ji}$$

In his 2003 paper Duffy explains since prices are not the sole marketing mechanic used by retailers or manufacturers, the AIDS model can be augmented to include additional marketing variables such as advertising and promotional discounts. Duffy specifies the augmented AIDS model as follows;

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \sum_{j=1}^{n} \theta_{ij} \ln a_j + \sum_{j=1}^{n} \vartheta_{ij} \ln d_j + \beta_i \ln\left(\frac{M}{P}\right)$$

Where $a_j$ is a vector of the expenditures spent on each of the brand's advertising, $d_j$ is a vector of the size of the promotional discounts being offered on each of the brands, $\theta_{ij}$and $\vartheta_{ij}$ represent the matrix of own and cross effects of the advertising expenditure and size of promotional discount respectively.

Since the two categories are complements it is not unreasonable to expect that changes in the demand for one category would have an effect on the demand on the other. Cake Mix and Frosting are usually bought in pairs, consequently the sales of one category are affected by sales of the other. To account for the complementary nature of the brands I have included another intercept-shifting term into the model; complementary prevalence ($c_t$). This is the total quantity complementary goods sold at time t. This variable accounts for any changes in the demand for the complementary category. The marketing and complementary augmented AIDS model is specified as follows;

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \sum_{j=1}^{n} \theta_{ij} \ln a_j + \sum_{j=1}^{n} \vartheta_{ij} \ln d_j + \sigma_i \ln c_t + \beta_i \ln\left(\frac{M}{P}\right) \qquad (2)$$

Where $\sigma_i$ is the parameter to be estimated for sales of complementary goods.

The demand elasticities are calculated as functions of the estimated parameters, and have standard implications. The augmented AIDS model in equation 2 can be used to test whether marketing techniques have the power to redistribute the allocation of expenditure by consumers across brands. It may also be used also to measure the strength of these effects, both in absolute terms and in comparison with the responsiveness of demand to changes in other variables such as prices and income. These comparisons are best made using (long-run) elasticities which, according to Green and Alston (1990), can be calculated as follows:

*Income (expenditure) elasticity;* $\eta_{ij} = \frac{\beta_i + w_i}{w_i}$

*Compensated price elasticity;* $\delta_{ij} = \frac{\gamma_{ij} + w_i w_j}{w_i} - \varphi_{ij}$

Where $\varphi_{ij}$ = 1 for i = j and 0 otherwise, and

*Marketing mix elasticity;* $\emptyset_{ij} = \frac{\theta_{ij}}{w_i}, \emptyset_{ij} = \frac{\vartheta_{ij}}{w_i}$

*Complementary elasticity;* $\phi_{ij} = \frac{\sigma_i}{w_i}$

This long-run model implicitly assumes that the consumers' behavior is always in '*equilibrium*', i.e. that there is no difference between consumers' short and long run behavior. However habit persistence, imperfect information, adjustment costs, incorrect expectations and misinterpreted real price changes often prevent consumers from adjusting their expenditure instantly to price and income changes (Anderson and Blundell, 1983). Therefore, until full adjustment takes place consumers would transact at a point that is '*out of equilibrium.*' This is one of the reasons why most static AIDS models have been found to be unsatisfactory (Duffy, 2003), since it ignores the dynamic properties of the data. Furthermore, when such a framework is applied to data that is known to be non-stationary, traditional statistics become unreliable and the results are often spurious. This is the argument for augmenting the long-run equilibrium relationship with a short-run adjustment mechanism.

In a stationary time-series the mean, variances, and autocovariances are independent of time and are said to be integrated of order zero, denoted as I(0). Typically a non-stationary time-series can be made stationary by differencing. A series which needs to be differenced b times in order to yield it stationary is said to be integrated of order b, denoted as I(b). As a general principle, a linear combination of I(b) variables will also be I(b). However, where a linear combination of I(b) variables is I(b-d), d>0, these variables are said to be cointegrated, denoted as CI(b,d). Such a cointegrating relationship implies that even though the individual time-series are non-stationary, the relationship would not be spurious. Hence, this group of variables would exhibit some sort of long-run equilibrium relationship that may be represented by a cointegrating vector (CV). Any deviations from the equilibrium will tend to be (partially) corrected for in the following periods. One can derive an error-correction model (ECM) that describes this behavior, if there is a cointegrating long-run relationship (Engle and Granger, 1987).

The augmented Dickey-Fuller (ADF) statistics can be employed to test the variables concerned for unit roots (stationarity). To ensure that the error terms in the Dickey-Fuller test equation are white noise, a number of lags of the dependent variable are included in the test equation. In this study the lag length was determined by the Schwartz Information Criterion (SIC).

Once the orders of integration of the variables have been identified and one or more of the time series are found to be I(1)[2], the Engle and Granger (1987) two-stage approach can be used to test for a cointegration relationship among the variables in the models. Therefore, if each $w_i$ is integrated of the same order as one or more of the explanatory variables, then a cointegrating relationship would exist if the linear combination of all variables is stationary. Under such conditions, each market share could be written in ECM form. Applications of the ECM-AIDS can be seen in the studies of demand for food and meat products (Balcombe and Davis, 1996; Attfield, 1997l Karagiannis et al. 2000; Karagiannis and Mergos, 2002).

In equation 2 the long-run impact of explanatory variables on the expenditure on the respective brands is described. The error-correction specification would then allow for a flexible pattern of non-uniform consumer responses to advertising over time, allowing for short-run dynamic adjustments towards long-run equilibrium. The ECM of the AIDS used in this article follows that of Karagiannis and Mergos (2002), which is described as;

$$\Delta w_{it} = \delta_i \Delta w_{t-1} + \sum_{j=1}^{n} \gamma_{ij} \Delta \ln p_{jt} + \sum_{j=1}^{n} \theta_{ij} \Delta\ln a_j + \sum_{j=1}^{n} \vartheta_{ij} \Delta\ln d_j + \sigma_i \Delta\ln c_t + \beta_i \Delta\ln\left(\frac{M}{P}\right) + \lambda_i \mu_{it-1} + \mu_t \qquad (3)$$

Where $\mu_{it-1}$ is the ECM term that measures the feedback effects and is estimated from the corresponding cointegrating equation.

This representation allows for market shares to be changed as a result of a previous shock, which is captured by the error correction term, $\lambda_i \mu_{it-1}$. This represents the deviation of the actual market shares in the previous period, $w_{t-1}$, from the values that were desired on the basis of the information available then, $w_{t-1}^*$ (where asterisk denotes a desired value). Consumers in the current period attempt to change $w_t$ from its value in the previous period, $w_{t-1}$, with the aim of closing some of the gap that may have existed between $w_{t-1}$ and $w_{t-1}^*$. These adjustments move market shares in the direction of their desired values, eventually establishing a long-run equilibrium.

This implies certain types of consumer behavior over time. It implies that once consumers have reached their long-run equilibrium values where the allocation of expenditure across products is optimal, they will have in each period

[2] It would also be possible for two of the time series to be I(2) if there was a third series were I(1). In such a case the I(2) series may be cointegrated, which would imply that collectively they are I(1). These series could then combine with the remaining I(1) series to ensure that the system is stationary.

henceforth a '*baseline*' plan, which may be used to allocate their budgeted expenditure across brands in the future periods, so that $\Delta w_t = 0$. In other words, once a pattern of expenditure has been chosen on the basis of previous experience and information about the prices, advertising, promotions and incomes, consumers do not revise that pattern *without good reason*. This baseline pattern can be modified as consumers become aware of new information that has become available since the previous period for example on prices, income, advertising or even health risks.

## 4. Data, variables and time series properties

### 4.1 Data

All the data series used in this study span the period of July 1st 2009 till June 30th 2010. There are two data sets of interest, i.e. the sales data collected from scanner data and marketing data collected from retailers. The data is concerned with two types of fast-moving consumer goods, namely cake mix and frosting (icing sugar). The data pertains to two different stores (one Shoprite and one Checkers) from the Western Cape area. It is sampled at a daily frequency.

To simplify the estimation, I restrict the analysis to the most common product size in each category (i.e. 1 kilogram cake mix and 500 gram icing sugar). This requires the assumption that consumer X only buys 1 size of cake mix and 1 size of frosting and switches between brands, rather than switching to a different size of the product within the same brand. I further simplify this analysis by restricting the number of brands in each category. In the cake mix category I take the top five brands, while in the frosting category I take the top four brands. These top brands make up roughly 90% of sales in each category. The year amounts to 355 days, once all the days where one or more of the stores were closed (i.e. Christmas) have been removed.

By using daily data the prices reflected are not averages but the actual selling prices of goods. Hence there is an accurate record of the quantity of each item bought and the exact price paid for it, which is useful as if avoids aggregation bias.

This database is then combined with a second, which contains marketing information for each brand. This database monitors the expenditure spent on advertising in print or TV for any given week by the retailer. This database allows us to test if advertising expenditures can reallocate sales between brands. Unfortunately these advertising expenditures are in weekly values making it difficult to accurately measure their effectiveness. Consider; if a commercial airs on the last day of the week, its effect will probably only take place in the following week's sales. The database also indicates if the brand was on promotion that day (selling at a discount). This database presents variables which allow us to measure the effects that marketing techniques have on demand for brands.

Finally, I control for seasonal variation in the sales series. To account for possible seasonality in the sales series, I include 4 seasonal dummies in the model, each covering four consecutive months. If necessary, I also include dummies to capture special holidays (Easter and Christmas) and lagged values of these dummies to deal with the dynamic effects of these events.

## 4.2 Operationalization of variables

In summary, the variables in the ECM-AIDS model include:

1. Quantity sold; $q_{cit}$

This is the quantity of brand i in category c sold at time t. For example, 5 units of *Moires* brand of *Cake Mix* category sold at day 11.[3]

2. Store price; $p_{it}$

Price being charged by the store for brand i at time t.

3. Real price index; $Pindex_{it}$

The store price does not always accurately reflect the real price of the brand, as it may be selling on promotion for that day. On promotion days the brand sells for less than it would on non-promotion days. The price the brand sells for on non-promotion days is considered, by the consumer, to be the real price of the brand (as opposed to discounted price). The real price index at time t is measured as the average of the preceding three non-promotion prices being charged for that brand, and is a measure of the real price being charged for the brand. Pindex is a proxy for the store price offered and is used in calculating all values dependant on the price in equation 2 and 3 (i.e. $M_t$; the total expenditure on the system of brands, P; a general index of prices in the system). A similar measure of real price was used in Duffy (2004).

4. Expenditure; $M_{it}$

Total expenditure across all n brands in the category at time t. Calculated as;

$$M = \sum_{i=1}^{n} Pindex_{it} q_{it}$$

5. Brand market share; $w_{it}$

Ratio of consumers' expenditure at current prices on the $i^{th}$ brand to total consumer's expenditure across all brands in the category at time t. Calculated as;

$$w_{it} = Pindex_{it} q_{it} / M_t$$

[3] For the remained of the variables I will drop the category subscript, but please keep in mind that the variables differ between the two categories.

6. Size of promotional discount; $d_{it}$

A measure of the size of the promotion of brand *i* at time t. It is measured as the size of the discount relative to the real price of the good. This is a natural measure for the size of a promotion and it allows for a comparison across brands and categories. A promotional discount is typically also accompanied by an in-store display notifying the customers of the discount. Calculated as;

$$d_{cit} = \frac{Pindex_{it} - P_{it}}{Pindex_{it}}$$

7. Advertising expenditure; $a_{it}$

The amount of expenditure spent by the retailer on advertising for that brand throughout the week of time t.

8. Complementary sales; $c_{it}$

Sales of all goods of the complementary category at time t; for instance all sales of all brands of Cake Mix at time t. Calculated as;

$$c_{it} = \sum_{j=1}^{n*} Pindex_{jt} q_{jt}$$

Where n* is the number of brands in the complementary category and j is a subscript denoting each of the brands in that category.

### 4.3 Issue of endogeneity

An issue of substantial importance is whether the sales of Cake Mix and Frosting are exogenously or endogenously determined. Since these categories are complements their sales could be endogenous as the sales of either category simultaneously influences the other. Similar arguments have been made for the endogeneity between sales and the promotion of brands. A common method for retailers to set their advertising budgets is as proportion of previous period's sales. This could be represented as; $a_{it} = \beta(q_{i,t-1})$. From this it would appear that both advertising and sales of brands should be considered endogenous, as each one simultaneously influences the other.

By setting up the data in the form of a VAR, ECM or VECM, all variables are endogenous, and each variable is specified to be determined by its own lagged values and the lagged values of all other variables in the system. This avoids any simultaneity bias brought about by endogeneity, and is one of the benefits of using a multi-equation estimation technique.

### 4.4 Testing order of integration

It is common practice to test for stationarity and orders of integration in time series data before attempting to estimate long-run, cointegrating relationships.

For studies that employ daily data, the integration is often found to be I(1). As mentioned previously one expects most time-series to be I(1). Where a time-series was found to be non-stationary, the first difference of that series was tested for stationarity, to establish whether the original series was I(1). The ADF tests are sensitive to the number of augmented terms included in the test equation and the “best” test equation was chosen on the basis of the SIC. For the ADF test a trend was not included in the test equation.

The ADF statistics are shown in Tables 4.4A and 4.4B. These suggest that all the variables used in equation 2 are I(1). Given that the time-series under scrutiny are generally non-stationary, the implication of these results is that an approach that does not test for cointegration might yield spurious results. Thus, in order for these results to be economically meaningful, at least one cointegrating relationship must be found. This is the focus of the following section.

### 4.5 Testing for cointegration

Having established that the series are predominantly I(1) (first differencing needed) I continue to test for cointegration between the variables of equation 2 following the Johansen Procedure. In establishing the number of CVs intercepts and trends were included in the tests, which, given a graphical representation of the data, seems appropriate in this context (Patterson, 2000: 627).

The first step was to determine the appropriate lag length. Patterson (2000: 649) explains that the SIC performs well in the context of simultaneously estimating the lag length and the cointegrating rank of a set of equations, and based on this information criterion the optimal lag length was found to be equal to one.

In establishing the number of CV’s of an equation one can use either the maximal eigenvalue or the trace statistic approach. The procedure of establishing the number of CV’s becomes more complicated when dealing with a system of equations as the relationships between variables may differ between equations. For instance there may be a CV between Brand A sales and Brand A advertising expenditure but not between Brand B sales and Brand B advertising expenditure, while at the same time there may be a CV between the advertising of Brand A and Brand B sales. Since the system of demand equations can be estimated in an iterative seemingly unrelated regression (ISUR) procedure we could conduct maximum eigenvalue and trace tests on each of the equations separately. This will give us an idea of the number of CV’s in each of the nine demand equations[4]. These tests are conducted for the demand equations specified in

[4] 5 brands of Cake mix and 4 brands of Frosting amount to 9 demand equations in total

static (equation 2) and dynamic (equation 3) formulation. The results are reported in table 4.5A and 4.5B
The results indicate that the trace and maximal eigenvalue statistics clearly established the presence of two CVs in all nine equations in the static model, and as many as four CV's in the dynamic model.

## 5. Empirical results

### 5.1 Estimating the ECM

The cointegrating relationship in equation 2 can be modeled using the ECM-AIDS described in equation 3. Since the sum of all expenditure shares in the AIDS model is equal to unity, the residuals variance-covariance matrix is singular. The usual solution is to delete an equation from the system and estimate the remaining equations and then calculate the parameters in the deleted equation in accordance with the adding-up restrictions. In this case I arbitrarily drop the equations for the brands with the lowest market share in each category (Brand E and D for Cake Mix and Frosting respectively). First I estimate the unrestricted static AIDS model using equation 2. I add the deterministic components in the form of seasonal dummies and a linear time trend in the model. Estimation is carried out implementing the maximum likelihood (ML) routines for seemingly unrelated regression (SUR) which adjusts for cross-equation contemporaneous correlation. The process of iteration ensures that the estimates obtained approach asymptotically those of maximum likelihood method.

I impose the homogeneity and symmetry conditions separately and then combine them to estimate the restricted models. The likelihood ratios estimated from the unrestricted and restricted models are presented in Tables 5.1A and 5.1B. Results indicate that both the homogeneity and symmetry conditions are satisfied by the static model. Models which impose both the homogeneity and symmetry conditions are unambiguously the most restricted models consistent with data. The restricted price models, which sets all the price coefficients to zero, is accepted at the 5% significance levels. Implying that the inter-product allocation of demand is definitely affected by relative prices both in the short and long-run.

However the restricted advertising and discount models, which set all the advertising expenditure and promotion size coefficients to zero, have ambiguous results. There is little evidence that the promotion variables can reallocate demand in the static (long-run) model, while there is evidence that they can reallocate demand in the dynamic (short-run) model. These results are not all together clear on how whether or how advertising and discounts affect consumer demand for brands. It was decided to give these variables the *benefit of the doubt* and retain them in the analysis for the rest of the study.

The Engle and Granger two-step approach is employed for estimating cointegrating regressions. The residuals from these regressions are obtained and incorporated into equation 3, and the unrestricted ECM-AIDS is estimated using MLE of SUR procedure. Tables 5.1.2A and 5.1.2B contain long-run and short-run estimates of parameters for price, expenditure and complementary sales for the homogeneity and symmetry-constrained version of equation 3. The elasticity estimates calculated from these tables are reported in tables 5.1.3A and 5.1.3B. The long-run and short-run parameter estimates for the promotion variables are displayed in tables 5.1.4A and 5.1.4B and the elasticity estimates calculated from these are displayed in tables 5.1.5A and 5.1.5B.

## 5.2 Elasticity Analysis Results

### 5.2.1 *Price elasticities*

#### *5.2.1.1 Static AIDS (long-run, equilibrium) elasticities*

The estimated long-run own-price elasticities for both categories are completely plausible. The upper halves of Tables 5.1.3A and 5.1.3B show the cross-price elasticities are negative as predicted by demand theory and most are significantly different from zero. These results show that the managers of these brands can, ceteris paribus, discount average price in response to a fall in production costs and expect a revenue increase. On the other hand, they are unable to pass cost increases onto the consumer and thus raise average price without incurring a revenue decrease. These estimates, together with significant cross-price elasticity estimates, confirm the important role that prices play in the determination of the inter-product distribution of demand over the long-run. The long-run results for Cake Mix show that Brand E (with an own price elasticity of -1.31) is the only elastic good, meaning that the percentage change in quantity demanded is *greater than* the percentage change in price. While Brand D is the only elastic good in the Frosting category.

Although many of the cross-price elasticities are positive only a few are significant at the 5% level, meaning there is little evidence to support any price substitution between the brands for either category. However the elasticity estimates indicate support for price substitution between Brands A and C as well as Brands A and D of the Cake Mix category. Similarly, there is support for price substitution between Brands A and B of the Frosting category. The statistical significance here suggests that, on average, price is a particularly effective tactical marketing mechanic in the competitive activity between these brands over the long-run.

#### 5.2.1.2 *Dynamic AIDS (short-run) elasticities*

All of the own-price elasticities of demand in the ECM-AIDS are shown in the lower halves of Tables 5.1.3A and 5.1.3B. Many are significantly different from zero and correctly signed, suggesting that any deviations of brand allocation from the long-run equilibrium allocation are accounted in the dynamic AIDS model. This indicates that habit persistence plays an important role in the brand consumption decision making process for these two categories. In other words, the previous distribution of brand expenditure influences the current decision on brand choice. The coefficients of the ECM are all statistically significant at 5 % level and correctly signed,

The own-price elasticity of demand for the Cake Mix category ranges from -0.43 to -0.8, suggesting that in the short-run each of the brands' consumption have similar sensitivities to their own price, i.e. the effect of a price change is almost equal for all brands. All the brands (with the exception of Brand D of cake mix) are inelastic goods, meaning that the percentage change in quantity demanded is *less than* the percentage change in price.

There is evidence to suggest that, in the Cake Mix category, Brands B and E show substitution properties. A one percent increase in the price of Brand E leads to a 1.55% increase in the demand for B. However the reverse is not true, making it a weak substitute. This implies that Brand B must take the price decisions of E into account when setting its own short-run policy. Similarly evidence suggests that Brands A and B as well as Brands C and D in the Frosting category are price substitutes. This is revealed by the significant cross-price elasticities of demand between the two pairs of Brands.

The overall conclusion for short-run pricing is that Brand B, from Cake Mix, is the most effective with significant steal from E. However Brands B and E (Cake Mix) as well as Brands C and D (Frosting) cannot generate a sustainable competitive advantage over each other through pricing strategies. These elasticity estimates only suggest it is possible to obtain short-term market gains by reducing prices. There are no significant cross-price elasticities for these brands in the static model. The implication is that the market shares of these brands are only temporarily affected its pricing strategies of their substitutes. On the other hand Brands A and B (Frosting) have significant cross elasticities in both the dynamic and static models. In this case the short term pricing strategies can translate into long-term market shares. This insight clearly highlights the benefit of a model capable of separating long-run movements in the underlying brand share level form the short-run dynamics.

### 5.2.2 *Expenditure elasticities*

#### *5.2.2.1 Static AIDS elasticities*

The effect of a change in total expenditure is twofold. Consider the example of an increase in total income of the consumers; firstly, this leads to a proportional increase in total consumption of all goods (homogeneity principle), and secondly, this increased expenditure reallocates demand across all brands in each category. Most of the expenditure elasticity estimates are significantly different from 0. Here the estimated expenditure elasticities for Cake Mix Brands A and B as well as Frosting Brand E are 1.2, 1.15 and 2.74 respectively categorizing them as *luxuries*. These brands are the most sensitive to changes to total expenditures, meaning they are the biggest gainers (losers) of the competing brands when consumers increase (decrease) their expenditures.

#### *5.2.2.2 Dynamic AIDS elasticities*

The expenditure elasticities estimated by the Static and Dynamic model are quite different from each other. There are many more *luxury* brands estimated by this model, all but Brand E (Cake Mix) and Brand C (Frosting). Brand E (Cake Mix) and Brand D (Frosting) change signs and are now negative, meaning the demand for these brands increases as expenditure decreases. Although neither of these brands are significant at the 5% level, they are fairly significant.

### 5.2.3 *Complementary sales elasticities*

#### *5.2.3.1 Static AIDS elasticities*

The long-run estimates of complementary sales elasticities show that Brands A and B (Cake Mix) as well as Brands A and B (Frosting) are significant and positive. Hence the demand for Brands A and B of Cake Mix are influenced by the total sales of all Frosting brands, while Brands A and B of Frosting are similarly influenced by the total sales of all Cake Mix brands. The effects of a change in complementary sales are similar to that described above for a change in expenditure. Increased sales of Cake Mix would presumably increase total sales of the Frosting category and reallocate demand across the different brands of Frosting. Brands A and B (either category) are the only market share gainers (losers) when the sales of complements increase (decrease). The elasticity estimates for these brands are positive and inelastic, meaning an increase in sales of complements results in a less than proportional increase in demand for Brands A and B.

#### *5.2.3.2 Dynamic AIDS elasticities*

The short-run complementary sales elasticity estimates are similar to long-run estimates. Brand A of both categories have significant and positive elasticities, which are less than 1. However Brand B of either category is now not significant. Thus changes in complementary sales can result in short-run changes in demand for Brand B and generate long-run, permanent changes in demand for Brands A and B. This suggests that complementary sales induce short-run market share movements of Brand A which eventually translate into long-run changes in market share allocation.

### *5.2.3 Promotional elasticities*

#### *5.2.3.1 Static AIDS elasticities*

These are shown in the upper halves of Tables 5.1.5A and 5.1.5B. All diagonal own-effects are positive as promotional activity boosts demand.

Brands A and B (Cake Mix) have significant estimates. Suggesting that retailer advertising of each of these brands can generate long-term, permanent shifts in market share. However there is no evidence showing that advertising has permanent cross effects. Note that although the model shows that advertising of brands can increase their market share it is not capable of identifying where the market share comes from, i.e. which brands lose market share due to other Brands' advertising.

Only Brand A (Frosting) has a significant estimate. Suggesting that retailer discounts this brand can generate long-term, permanent shifts in market share. However there is no evidence showing that discounting has permanent cross effects.

The significant elasticities mentioned are all inelastic, i.e. a change in either advertising expenditure or promotion size would result in a less than proportional change in demand.

#### *5.2.3.2 Dynamic AIDS elasticities*

These are shown in the lower halves of Tables 5.1.5A and 5.1.5B. Again all diagonal own-effects are positive as expected, and all significant estimates are inelastic.

Brands A, B and C (Cake Mix) have significant own-advertising. All the off-diagonal terms for advertising expenditure are insignificant in the frosting category. However there is some evidence of advertising expenditure leading to steal between Brands C and E of the Cake Mix. Either one of these brands are capable of stealing short-run market share from each other by increasing advertising expenditure. Compared with the price elasticity results, it is clear that both these Brands compete more on the basis of advertising expenditure in the short-run.

Interestingly, an increase in the advertising expenditure of Brand C tends to raise category awareness to the extent that funds are diverted into additional purchases of D. Positive cross-effects are possible since promotional activity can help to raise awareness of all brands in the category.

Brands A, B and C (Cake Mix and Frosting) have significant own-discount size elasticities. Increasing the size of promotions on these Brands will reallocate short-term market shares in their favor. When Brand A (Cake Mix) increase the size of its promotional discounts its steals from Brand B, similarly Brand C steals from Brand A. Here Brand A needs to be aware of the discount strategy of Brand C, while the reverse is not true.

Brand C's (Frosting) promotional discounts are successful in drawing share from A implying the former is a weak substitute for the latter. This contrasts with the competitive price interaction seen earlier where Brand C's price strategies steal from D's market share. This implies that, in the short-run, Brands A and D need to be aware of C's discount and pricing strategies.

The cross-elasticities of Brands A and B as well as C and D of Frosting implying these are substitute brands. Note that these are the same substitutes as were shown by the dynamic price elasticity estimates. This is understandable, as a price change and a discount can have very similar effects. The only difference between the two is that a discount is accompanied with an in-store display and is typically bigger than day to day price changes.

Again there is a clear distinction between the short and long-run substitution patterns, highlighting promotional policies that enjoy short-term competitive success but cannot contribute to long-term sustainable movements in own or competitor market share. Promotion strategies seem to be more effective at creating short-term advantages than long-term sustainable advantages. This makes sense as discounts at time t encourage consumers to buy the brand at time t but not necessarily again at time t+1 when the brand is selling at wholesale. Also advertising has been shown to be more effective at creating short term awareness of brands rather than creating a competitive advantage over the long-run.

## 6. Conclusion

In this paper I propose an Almost Ideal Demand System – Error Correction model to explain the differences in short-run and long-run effects of price and promotions on sales. The model is applied to daily sales of different cake and frosting brands. Alone, the AIDS model allows for estimating Long-run demand equations; how brands' market shares are affected by variables over the long-run. Augmenting the AIDS model with ECM we can estimate out-of-equilibrium (i.e. short-run) results, transforming the static AIDS model into a dynamic demand model.

Most of the long and short-run results are consistent with consumer behavior. For instance by increasing total expenditure I find that certain brands' market shares increase at the cost of others, revealing luxury brands. Significant own-price elasticities are positive and cross-price elasticities are negative; revealing that decreasing the price increases sales of brand at the cost of market share to other brands and vice versa for increasing the price. These results confirm the important role that prices play in the determination of inter-product distribution of demand over both the short and long-run. The results for promotional effects are less convincing than price effects as there are fewer significant findings; however significant results are still consistent with what one might expect. The results indicate some steal in the short-run, for instance increasing the advertisement expenditure of either Brand C or E of cake mix steals short-run market share from the other; that in the short-run there are brands that compete more on basis of advertising expenditure than prices. The results show a clear distinction between short and long-run substitution patterns. Promotional strategies enjoy short term competitive success but do not contribute to long-term sustainable movements in their own or competitive market share. This highlights how advertising and short term promotional activities are more effective at creating short term awareness and interest than creating long term competitive advantage. The statistical significance suggests that, on average, price is a particularly effective tactical marketing mechanic in the competitive activity between certain brands over the long-run.

An additional point worth mentioning is that cake mix and frosting are complementary goods. There is a strong significant relationship between sales of either good over the long-run. I also show that sales of certain brands of each good have relationships with each other. The model permits measuring the direction of the effect, for instance Brands A and B of Frosting are influenced by the total sales of all Cake Mix brands.

Although the highlight of this paper are the price effects there is good news for advertisers and shows that it is possible for brands to persuade consumers to purchase their products through combative advertising in short-run. This paper only scratches the surface of promotional effects, further studies might include the effects of different media; i.e. the effectiveness of TV advertising as opposed to all other types. This can be taken a step further by measuring the effects of media across different types of consumers (some of which might be more susceptible to different advertising). Further research would do well to include these advertising augmentations to separate the promotional effects.

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## *Appendix*

Table 4.4A: Augmented Dickey-Fuller tests for Cake Mix

| Variable | Levels | 1st diff | Conclusion |
|---|---|---|---|
| $q_{it}$ | -1.23 (2) | -3.06* (1) | I(1) |
| $c_{frosting,t}$ | -1.41 (2) | -4.12 (1) | I(1) |
| $p_{it}$ | -2.15 (2) | -6.52 (2) | I(1) |
| $w_{it}$ | -2.21 (1) | 3.65 (1) | I(1) |
| $a_{it}$ | -2.46 (1) | -3.72 (1) | I(1) |
| $d_{it}$ | 1.1 (1) | 3.82 (0) | I(1) |
| $M_{it}$ | -1.13 (1) | -4.2 (1) | I(1) |
| $P_{it}$ | -0.67 (1) | -3.78 (1) | I(1) |

*Number of lags in parenthesis

Table 4.4B: Augmented Dickey-Fuller tests for Frosting

| Variable | Levels | 1st diff | Conclusion |
|---|---|---|---|
| $q_{it}$ | -1.26 (2) | -3.21* (1) | I(1) |
| $c_{Cake\ Mix,t}$ | -1.62 (2) | -5.01 (1) | I(1) |
| $p_{it}$ | -2.54 (2) | -7.12 (2) | I(1) |
| $w_{it}$ | -2.66 (1) | 4.34 (1) | I(1) |
| $a_{it}$ | -2.81 (1) | -4.02 (1) | I(1) |
| $d_{it}$ | 1.61 (1) | 4.22 (0) | I(1) |
| $M_{it}$ | -1.16 (1) | -4.57 (1) | I(1) |
| $P_{it}$ | -0.61 (1) | -3.53 (1) | I(1) |

*Number of lags in parenthesis



Table 4.5A: Static Model for Cake Mix

| Null hypothesis | Alt. hypothesis | Test statistic | | | | | 95% critical value |
|---|---|---|---|---|---|---|---|
| | | Eigen Value test | | | | | |
| | | A | B | C | D | E | |
| r = 0 | r = 1 | 82.16 | 72.59 | 69.77 | 91.26 | 72.40 | 49.02 |
| r ≤ 1 | r = 2 | 75.52 | 39.77 | 36.76 | 56.03 | 68.52 | 34.99 |
| r ≤ 2 | r = 3 | 22.86 | 14.77 | 9.93 | 17.88 | 26.02 | 29.50 |
| r ≤ 3 | r = 4 | 3.69 | 4.39 | 2.35 | 2.86 | 8.14 | 12.98 |
| r ≤ 4 | r = 5 | 0.33 | 0.42 | 0.29 | 0.31 | 0.78 | 7.02 |
| | | Trace Statistic test | | | | | |
| | | A | B | C | D | E | |
| r = 0 | r = 1 | 132.18 | 133.36 | 116.91 | 111.89 | 133.60 | 79.23 |
| r ≤ 1 | r = 2 | 72.30 | 69.53 | 75.92 | 97.70 | 108.68 | 56.40 |
| r ≤ 2 | r = 3 | 32.15 | 28.91 | 22.77 | 34.06 | 29.44 | 41.26 |
| r ≤ 3 | r = 4 | 13.98 | 4.70 | 3.33 | 18.42 | 4.89 | 24.23 |
| r ≤ 4 | r = 5 | 3.86 | 0.99 | 0.27 | 2.09 | 0.48 | 14.34 |

| Table 4.5A: Dynamic Model for Cake Mix | | | | | | | |
|---|---|---|---|---|---|---|---|
| Null hypothesis | Alt. hypothesis | Test statistic | | | | | 95% critical value |
| Eigen Value test | | | | | | | |
| | | A | B | C | D | E | |
| r = 0 | r = 1 | 89.93 | 83.00 | 107.18 | 109.27 | 104.13 | 60.26 |
| r ≤ 1 | r = 2 | 65.64 | 66.82 | 97.65 | 57.82 | 92.05 | 36.20 |
| r ≤ 2 | r = 3 | 36.16 | 35.14 | 40.55 | 33.70 | 74.24 | 32.30 |
| r ≤ 3 | r = 4 | 6.79 | 14.01 | 27.13 | 5.09 | 17.47 | 15.43 |
| r ≤ 4 | r = 5 | 1.18 | 1.36 | 5.23 | 3.16 | 8.92 | 9.46 |
| Trace Statistic test | | | | | | | |
| | | A | B | C | D | E | |
| r = 0 | r = 1 | 151.40 | 110.97 | 121.14 | 174.80 | 114.77 | 92.76 |
| r ≤ 1 | r = 2 | 89.74 | 91.20 | 91.08 | 145.59 | 80.34 | 72.39 |
| r ≤ 2 | r = 3 | 31.65 | 27.43 | 38.55 | 61.14 | 66.86 | 51.15 |
| r ≤ 3 | r = 4 | 4.97 | 16.89 | 27.30 | 9.07 | 19.11 | 35.72 |
| r ≤ 4 | r = 5 | 0.51 | 2.44 | 6.54 | 0.75 | 1.50 | 17.73 |

| Table 4.5B: Static Model for Frosting | | | | | | |
|---|---|---|---|---|---|---|
| Null hypothesis | Alt. hypothesis | Test statistic | | | | 95% critical value |
| Eigen Value test | | | | | | |
| | | A | B | C | D | |
| r = 0 | r = 1 | 96.97 | 76.63 | 78.86 | 94.92 | 56.39 |
| r ≤ 1 | r = 2 | 50.11 | 73.84 | 51.20 | 88.61 | 41.35 |
| r ≤ 2 | r = 3 | 15.70 | 26.73 | 16.83 | 34.40 | 33.82 |
| r ≤ 3 | r = 4 | 3.24 | 8.53 | 2.50 | 6.39 | 15.13 |
| r ≤ 4 | r = 5 | 0.27 | 2.53 | 0.21 | 0.52 | 7.86 |
| Trace Statistic test | | | | | | |
| | | A | B | C | D | |
| r = 0 | r = 1 | 132.74 | 149.58 | 123.92 | 133.43 | 88.62 |
| r ≤ 1 | r = 2 | 79.82 | 76.05 | 72.55 | 68.76 | 64.63 |
| r ≤ 2 | r = 3 | 25.82 | 20.90 | 22.14 | 20.11 | 48.33 |
| r ≤ 3 | r = 4 | 10.22 | 3.71 | 4.67 | 4.36 | 26.26 |
| r ≤ 4 | r = 5 | 1.37 | 0.76 | 0.64 | 0.36 | 15.95 |

Table 4.5B: Dynamic Model for Frosting

| Null hypothesis | Alt. hypothesis | Test statistic | | | | 95% critical value |
|---|---|---|---|---|---|---|
| Eigen Value test | | | | | | |
| | | A | B | C | D | |
| r = 0 | r = 1 | 96.23 | 98.20 | 115.41 | 109.84 | 70.27 |
| r ≤ 1 | r = 2 | 66.68 | 76.27 | 76.04 | 66.19 | 41.25 |
| r ≤ 2 | r = 3 | 23.91 | 29.95 | 27.34 | 17.74 | 35.49 |
| r ≤ 3 | r = 4 | 6.95 | 4.61 | 6.41 | 2.69 | 16.39 |
| r ≤ 4 | r = 5 | 0.62 | 1.61 | 2.99 | 0.27 | 9.89 |
| Trace Statistic test | | | | | | |
| | | A | B | C | D | |
| r = 0 | r = 1 | 167.75 | 115.33 | 127.83 | 205.96 | 92.97 |
| r ≤ 1 | r = 2 | 91.56 | 58.59 | 64.11 | 152.38 | 80.60 |
| r ≤ 2 | r = 3 | 34.42 | 17.47 | 35.23 | 61.44 | 51.42 |
| r ≤ 3 | r = 4 | 5.75 | 5.58 | 12.16 | 45.77 | 42.12 |
| r ≤ 4 | r = 5 | 0.73 | 0.98 | 1.38 | 3.97 | 20.64 |

Table 5.1A: Constraints Likelihood Ratio Tests for Cake mix

| H1 | H0 | np1 | np0 | df | Calculated Chi2 | p-Value |
|---|---|---|---|---|---|---|
| | | Static | | | | |
| Unrestricted | Homogeneity | 60 | 56 | 4 | 5.807 | 0.214 |
| Unrestricted | Symmetry | 60 | 54 | 6 | 22.46 | 0.001 |
| Unrestricted | Homogeneity and Symmetry | 60 | 50 | 10 | 20.87 | 0.022 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Price responses | 50 | 44 | 6 | 13.27 | 0.039 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Advertising responses | 50 | 30 | 20 | 29.87 | 0.072 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Discount responses | 118 | 98 | 20 | 27.45 | 0.123 |
| | | Dynamic | | | | |
| Unrestricted | Homogeneity | 128 | 124 | 4 | 4.388 | 0.356 |
| Unrestricted | Symmetry | 128 | 122 | 6 | 14.56 | 0.024 |
| Unrestricted | Homogeneity and Symmetry | 128 | 118 | 10 | 24.24 | 0.007 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Price responses | 118 | 112 | 6 | 13.13 | 0.041 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Advertising responses | 118 | 98 | 20 | 32.32 | 0.032 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Discount responses | 118 | 98 | 20 | 33.59 | 0.029 |

H0 and H1 denote the null and alternative hypotheses, respectively and np0 and np1 indicate the number of freely estimated parameters under the null and alternative hypotheses, respectively. Degrees of freedom are equal to np1 - np0

Table 5.1B: Constraints Likelihood Ratio Tests for Frosting

| H1 | H0 | np1 | np0 | df | Calculated Chi2 | p-Value |
|---|---|---|---|---|---|---|
| | | Static | | | | |
| Unrestricted | Homogeneity | 36 | 33 | 3 | 6.997 | 0.072 |
| Unrestricted | Symmetry | 36 | 33 | 3 | 8.203 | 0.042 |
| Unrestricted | Homogeneity and Symmetry | 36 | 30 | 6 | 20.79 | 0.002 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Price responses | 30 | 27 | 3 | 9.022 | 0.029 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Advertising responses | 30 | 18 | 12 | 18.9 | 0.091 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Discount responses | 118 | 98 | 20 | 16.71 | 0.672 |
| | | Dynamic | | | | |
| Unrestricted | Homogeneity | 104 | 101 | 3 | 5.123 | 0.163 |
| Unrestricted | Symmetry | 104 | 101 | 3 | 10.95 | 0.012 |
| Unrestricted | Homogeneity and Symmetry | 104 | 98 | 6 | 14.67 | 0.023 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Price responses | 98 | 95 | 3 | 11.57 | 0.009 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Advertising responses | 98 | 86 | 12 | 22.85 | 0.029 |
| Homogeneity and Symmetry | Homogeneity and Symmetry and Restricted Promotion responses | 118 | 98 | 20 | 40 | 0.005 |

H0 and H1 denote the null and alternative hypotheses, respectively and np0 and np1 indicate the number of freely estimated parameters under the null and alternative hypotheses, respectively. Degrees of freedom are equal to np1 - np0

Table 5.1.2A: Estimates of the preference parameters for prices, expenditure and complementary sales of Cake Mix

| | Prices | | | | | Expenditure | Complementary sales |
|---|---|---|---|---|---|---|---|
| | $Pindex_A$ | $Pindex_B$ | $Pindex_C$ | $Pindex_D$ | $Pindex_E$ | $M$ | $c_{Frosting}$ |
| | | | Static Model | | | | |
| A | -0.05 | -0.10 | 0.12 | 0.15 | 0.03 | 0.06 | 0.10 |
| B | - | 0.11 | 0.05 | 0.15 | -0.01 | 0.04 | 0.23 |
| C | - | - | 0.07 | 0.03 | 0.07 | -0.02 | -0.10 |
| D | - | - | - | 0.03 | -0.06 | -0.06 | -0.12 |
| E | - | - | - | - | -0.04 | -0.02 | -0.11 |
| | | | Dynamic Model | | | | |
| A | -0.01 | -0.06 | 0.10 | -0.06 | -0.10 | 0.03 | 0.10 |
| B | - | 0.02 | 0.09 | -0.06 | 0.11 | 0.05 | 0.27 |
| C | - | - | 0.06 | 0.05 | -0.03 | 0.00 | -0.11 |
| D | - | - | - | 0.07 | 0.00 | 0.02 | -0.15 |
| E | - | - | - | - | 0.01 | -0.10 | -0.12 |

Homogeneity and symmetry imposed. The coefficients for Brand E were derived from the adding up constraints.

Table 5.1.2B: Estimates of the preference parameters for prices, expenditure and complementary sales of Frosting

| | Prices | | | | Expenditure | Complementary sales |
|---|---|---|---|---|---|---|
| | $Pindex_A$ | $Pindex_B$ | $Pindex_C$ | $Pindex_D$ | $M$ | $c_{Cake\ Mix}$ |
| | | | Static Model | | | |
| A | 0.0175 | -0.042 | 0.182 | 0.1505 | -0.1015 | 0.20 |
| B | - | -0.031 | -0.0124 | 0.2077 | -0.1302 | 0.24 |
| C | - | - | 0.026 | 0.01 | -0.012 | -0.21 |
| D | - | - | - | -0.3682 | 0.2437 | -0.23 |
| | | | Dynamic Model | | | |
| A | 0.035 | 0.056 | 0.049 | 0.0525 | 0.2345 | 0.26 |
| B | - | 0.1364 | 0.1271 | 0.0837 | 0.3007 | 0.19 |
| C | - | - | -0.028 | -0.014 | -0.066 | -0.23 |
| D | - | - | - | -0.1222 | -0.4692 | -0.22 |

Homogeneity and symmetry imposed. The coefficients for Brand D were derived from the adding up constraints.

Table 5.1.3A: Estimates of the elasticities of demand with respect to prices, expenditure and complementary sales of Cake Mix

| | Prices | | | | | Expenditure | Complementary sales |
|---|---|---|---|---|---|---|---|
| | $Pindex_A$ | $Pindex_B$ | $Pindex_C$ | $Pindex_D$ | $Pindex_E$ | $M$ | $c_{Frosting}$ |
| | | | | Static Model | | | |
| A | **-0.83 (2.97)** | -0.02 (1.23) | **0.52 (1.76)** | **0.62 (1.81)** | 0.17 (1.06) | **1.2 (1.89)** | **0.32 (1.67)** |
| B | -0.02 (1.51) | **-0.33 (2.53)** | 0.33 (1.14) | 0.68 (1.35) | 0.07 (1.81) | **1.15 (2.47)** | **0.78 (1.82)** |
| C | **1.11 (1.75)** | 0.64 (1.32) | **-0.38 (2.26)** | 0.34 (1.37) | 0.58 (1.51) | 0.84 (2.38) | -0.64 (1.15) |
| D | **1.32 (1.76)** | 1.31 (1.42) | 0.34 (1.09) | **-0.66 (2.04)** | -0.29 (0.99) | 0.58 (1.21) | -0.82 (0.34) |
| E | 0.6 (1.16) | 0.23 (1.17) | 0.97 (1) | -0.48 (1.19) | **-1.31 (1.05)** | 0.77 (2.78) | -1.22 (0.92) |
| | | | | Dynamic Model | | | |
| A | **-0.7 (2.42)** | 0.09 (1.21) | 0.45 (1.59) | -0.03 (1.47) | -0.23 (0.96) | 1.09 (1.8) | 0.33 (2.1) |
| B | 0.1 (1.83) | **-0.63 (2.5)** | 0.47 (0.97) | -0.05 (1.48) | 0.48 (0.64) | 1.17 (2.12) | 0.94 (1.24) |
| C | 0.96 (1.81) | 0.91 (1.5) | **-0.43 (1.65)** | 0.45 (1.33) | -0.08 (1.68) | 1.03 (2.55) | -0.73 (0.13) |
| D | -0.06 (0.93) | -0.1 (0.9) | 0.45 (1.43) | **-0.38 (2.18)** | 0.12 (1.12) | 1.12 (2.28) | -1.01 (1.31) |
| E | -0.82 (1.64) | **1.55 (1.87)** | -0.13 (1.36) | 0.2 (1.06) | **-0.8 (0.94)** | -0.12 (1.94) | -1.30 (0.81) |

Derived from preference parameters estimated with homogeneity and symmetry imposed; see the table 5.1.2A.

The t-ratios are given in parenthesis.

Table 5.1.3B: Estimates of the elasticities of demand with respect to prices, expenditure and complementary sales of Frosting

| | Prices | | | | Expenditure | Complementary sales |
|---|---|---|---|---|---|---|
| | $Pindex_A$ | $Pindex_B$ | $Pindex_C$ | $Pindex_D$ | $M$ | $c_{Cake\ Mix}$ |
| | | | | Static Model | | |
| A | **-0.6 (1.85)** | **0.19 (1.83)** | 0.72 (1.19) | 0.57 (0.89) | 0.71 (2.94) | **0.57 (1.67)** |
| B | **0.21 (1.88)** | **-0.79 (2.23)** | 0.16 (1.27) | 0.81 (1.19) | 0.58 (2.69) | **0.77 (1.87)** |
| C | 1.26 (1.33) | 0.25 (1.21) | **-0.67 (3.13)** | 0.19 (1.13) | 0.94 (2.03) | -1.05 (1.21) |
| D | 1.43 (1.13) | 0.5 (1.41) | 0.27 (0.82) | **-3.49 (1.37)** | **2.74 (1.42)** | -1.63 (1.02) |
| | | | | Dynamic Model | | |
| A | **-0.55 (1.98)** | **0.47 (1.83)** | 0.34 (1.9) | 0.29 (0.98) | 1.67 (2.42) | 0.74 (1.82) |
| B | **0.53 (1.88)** | **-0.25 (2.64)** | 0.61 (1.06) | 0.41 (1.29) | 1.97 (2.35) | 0.61 (0.62) |
| C | 0.6 (1.12) | 0.47 (1.42) | **-0.94 (2.62)** | **0.07 (1.85)** | 0.67 (1.67) | -1.15 (0.14) |
| D | 0.73 (1.27) | 0.11 (1.9) | **0.1 (1.83)** | **-1.73 (0.74)** | -2.35 (1.37) | -1.56 (0.64) |

Derived from preference parameters estimated with homogeneity and symmetry imposed; see the table 5.1.2B.

The t-ratios are given in parenthesis.

| Table 5.1.4A: Estimates of the preference parameters for advertising expenditure and discounts of Cake Mix | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Advertising expenditures | | | | | Discounts | | | | |
| | $a_A$ | $a_B$ | $a_C$ | $a_D$ | $a_E$ | $d_A$ | $d_B$ | $d_C$ | $d_D$ | $d_E$ |
| Static Model | | | | | | | | | | |
| A | **0.0544** | -0.048 | -0.0064 | -0.0352 | -0.0608 | **0.0928** | 0.0544 | -0.0032 | -0.0608 | -0.0128 |
| B | -0.0435 | **0.0435** | -0.0435 | 0.0406 | -0.0348 | -0.0145 | **0.0377** | -0.0551 | 0.0551 | -0.0493 |
| C | -0.015 | -0.0195 | **0.018** | 0.0135 | -0.003 | -0.0075 | -0.0165 | **0.0165** | -0.015 | -0.0255 |
| D | -0.0135 | -0.003 | -0.0075 | **0.0225** | -0.0225 | -0.024 | -0.0165 | -0.003 | **0.015** | -0.0255 |
| E | 0.0176 | 0.027 | 0.0394 | -0.0414 | **0.1211** | -0.0468 | -0.0591 | 0.0448 | 0.0057 | **0.1131** |
| Dynamic Model | | | | | | | | | | |
| A | **0.0928** | 0.0288 | -0.032 | -0.016 | -0.0384 | **0.0416** | -0.0448 | 0.016 | 0.0224 | -0.0096 |
| B | -0.0232 | **0.0232** | -0.0261 | 0.0464 | -0.0348 | 0.0377 | **0.0116** | -0.0435 | -0.0203 | -0.029 |
| C | 0.0165 | -0.0255 | **0.0315** | 0.03 | -0.0225 | -0.021 | 0.021 | **0.036** | -0.0105 | -0.0135 |
| D | -0.0045 | -0.024 | -0.0285 | **0.0345** | 0.0225 | -0.018 | -0.024 | -0.006 | **0.0225** | -0.0105 |
| E | -0.0816 | -0.0025 | 0.0551 | -0.0949 | **0.0732** | -0.0403 | 0.0362 | -0.0025 | -0.0141 | **0.0626** |

Homogeneity and symmetry imposed. The coefficients for Brand E were derived from the adding up constraints.

| Table 5.1.4B: Estimates of the preference parameters for advertising expenditure and discounts of Frosting | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Advertising expenditures | | | | Discounts | | | |
| | $a_A$ | $a_B$ | $a_C$ | $a_D$ | $d_A$ | $d_B$ | $d_C$ | $d_D$ |
| Static Model | | | | | | | | |
| A | **0.0735** | 0.0385 | 0.0105 | 0.0525 | **0.077** | -0.049 | -0.0385 | 0.014 |
| B | 0.0217 | **0.0248** | -0.031 | -0.0217 | -0.062 | **0.0806** | 0.062 | -0.0465 |
| C | -0.024 | -0.014 | **0.03** | -0.02 | -0.018 | -0.02 | **0.026** | 0.006 |
| D | -0.0712 | -0.0493 | -0.0095 | **-0.0108** | 0.003 | -0.0116 | -0.0495 | **0.0265** |
| Dynamic Model | | | | | | | | |
| A | **0.105** | -0.042 | 0.0595 | 0.0595 | **0.063** | 0.042 | -0.0105 | 0.0245 |
| B | -0.0248 | **0.0186** | -0.0124 | 0.0093 | -0.0062 | **0.0837** | 0.0062 | -0.0496 |
| C | -0.024 | 0.012 | **0.052** | 0.002 | -0.004 | 0.022 | **0.016** | -0.008 |
| D | -0.0562 | 0.0114 | -0.0991 | **-0.0708** | -0.0528 | -0.1477 | -0.0117 | **0.0331** |

Homogeneity and symmetry imposed. The coefficients for Brand D were derived from the adding up constraints.

| Table 5.1.5A: Estimates of the elasticities of demand with respect to advertising expenditure and discounts of Cake Mix | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Advertising expenditures | | | | | Discounts | | | | |
| | $a_A$ | $a_B$ | $a_C$ | $a_D$ | $a_E$ | $d_A$ | $d_B$ | $d_C$ | $d_D$ | $d_E$ |
| Static Model | | | | | | | | | | |
| A | **0.17 (1.85)** | -0.15 (1.06) | -0.02 (0.83) | -0.11 (0.72) | -0.19 (0.92) | **0.29 (0.17)** | 0.17 (1.22) | -0.01 (1.2) | -0.19 (1.23) | -0.04 (0.96) |
| B | -0.15 (1.15) | **0.15 (2.21)** | -0.15 (1.1) | 0.14 (1.32) | -0.12 (0.9) | -0.05 (1.28) | **0.13 (1.09)** | -0.19 (0.83) | 0.19 (1.65) | -0.17 (1.27) |
| C | -0.1 (1.27) | -0.13 (1.31) | **0.12 (1.37)** | 0.09 (1.36) | -0.02 (1.3) | -0.05 (1.21) | -0.11 (1.07) | **0.11 (0.75)** | -0.1 (1.11) | -0.17 (0.99) |
| D | -0.09 (1.07) | -0.02 (1.39) | -0.05 (0.8) | **0.15 (0.81)** | -0.15 (1.28) | -0.16 (1.09) | -0.11 (1.29) | -0.02 (1.12) | **0.1 (1.27)** | -0.17 (1.24) |
| E | 0.2 (1.26) | 0.3 (0.97) | 0.44 (1.33) | -0.46 (1.13) | **1.35 (0.68)** | -0.52 (0.83) | -0.66 (1.07) | 0.5 (1.17) | 0.06 (1.29) | **1.26 (1.05)** |
| Dynamic Model | | | | | | | | | | |
| A | **0.29 (2.04)** | 0.09 (1.3) | -0.1 (0.97) | -0.05 (1.44) | -0.12 (1.03) | **0.13 (2.26)** | **-0.14 (1.59)** | 0.05 (1.17) | 0.07 (1.28) | -0.03 (1.21) |
| B | -0.08 (1.27) | **0.08 (2.85)** | -0.09 (0.91) | 0.16 (0.94) | -0.12 (0.9) | 0.13 (1.52) | **0.04 (2.27)** | -0.15 (1.03) | -0.07 (1.12) | -0.1 (1.09) |
| C | 0.11 (1.22) | -0.17 (1.65) | **0.21 (2.85)** | **0.2 (1.82)** | -0.15 (1.01) | **-0.14 (1.72)** | 0.14 (1.22) | **0.24 (2.97)** | -0.07 (1.5) | -0.09 (1.19) |
| D | -0.03 (1.24) | -0.16 (0.92) | -0.19 (0.84) | **0.23 (0.81)** | **0.15 (1.65)** | -0.12 (1.54) | -0.16 (0.98) | -0.04 (0.99) | **0.15 (0.63)** | -0.07 (1.32) |
| E | -0.91 (1.27) | -0.03 (0.86) | **0.61 (1.66)** | -1.05 (0.82) | **0.81 (1.32)** | -0.45 (1.57) | 0.4 (1.38) | -0.03 (1.05) | -0.16 (1.13) | **0.7 (1.06)** |

Derived from preference parameters estimated with homogeneity and symmetry imposed; see the table 5.1.4A.

The t-ratios are given in parenthesis.

| Table 5.1.5B: Estimates of the elasticities of demand with respect to advertising expenditure and discounts of Frosting | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Advertising expenditures | | | | Discounts | | | |
| | $a_A$ | $a_B$ | $a_C$ | $a_D$ | $d_A$ | $d_B$ | $d_C$ | $d_D$ |
| Static Model | | | | | | | | |
| A | **0.21 (1.19)** | 0.11 (1.13) | 0.03 (1.21) | 0.15 (1.01) | **0.22 (2.01)** | -0.14 (1.07) | -0.11 (1.22) | 0.04 (1.03) |
| B | 0.07 (1.15) | **0.08 (0.98)** | -0.1 (1) | -0.07 (1.17) | -0.2 (1.23) | **0.26 (1.18)** | 0.2 (0.9) | -0.15 (1.21) |
| C | -0.12 (1.3) | -0.07 (0.91) | **0.15 (1.22)** | -0.1 (1.15) | -0.09 (1.41) | -0.1 (0.88) | **0.13 (1.33)** | 0.03 (1.04) |
| D | -0.51 (1.03) | -0.35 (1.1) | -0.07 (1.17) | **-0.08 (1.27)** | 0.02 (1.18) | -0.08 (1.27) | -0.35 (1.31) | **0.19 (1.18)** |
| Dynamic Model | | | | | | | | |
| A | **0.3 (1.07)** | -0.12 (1.1) | 0.17 (1.26) | 0.17 (1.11) | **0.18 (2.73)** | 0.12 (0.97) | -0.03 (1.68) | 0.07 (1.64) |
| B | -0.08 (0.83) | **0.06 (1.06)** | -0.04 (1.64) | 0.03 (1.01) | **-0.02 (1.58)** | **0.27 (1.94)** | 0.02 (1.62) | -0.16 (1.64) |
| C | -0.12 (1.1) | 0.06 (1.45) | **0.26 (1.33)** | 0.01 (1.16) | **-0.02 (1.66)** | 0.11 (0.97) | **0.08 (1.69)** | **-0.04 (1.62)** |
| D | -0.4 (1.35) | 0.08 (1.19) | -0.71 (0.88) | **-0.51 (1.35)** | -0.38 (1.63) | -1.06 (0.96) | **-0.08 (1.53)** | **0.24 (1.44)** |

Derived from preference parameters estimated with homogeneity and symmetry imposed; see the table 5.1.4B.

The t-ratios are given in parenthesis.

Table 5.1.6A: Estimates of the error correction term of Cake Mix

| | Product | | | | |
|---|---|---|---|---|---|
| | A | B | C | D | E |
| | | | Static Model | | |
| A | **-0.96 (1.92)** | -0.18 (2.03) | 0.11 (1.3) | -0.08 (1.71) | -0.18 (2.06) |
| B | 0.06 (1.52) | **-0.39 (3.08)** | -0.02 (2.08) | -0.17 (1.76) | 0.1 (2.02) |
| C | -0.12 (1.43) | -0.02 (1.34) | **-0.36 (2.6)** | -0.12 (1.32) | -0.11 (1.94) |
| D | -0.18 (1.59) | 0.19 (1.32) | -0.09 (1.32) | **-0.39 (2.97)** | -0.03 (2.07) |
| E | -0.18 (1.29) | -0.16 (1.21) | -0.16 (1.82) | -0.16 (1.49) | **-0.57 (2.23)** |
| | | | Dynamic Model | | |
| A | **-0.63 (3.25)** | -0.12 (1.57) | 0.1 (2.03) | 0.02 (1.71) | -0.14 (1.9) |
| B | 0.17 (1.24) | **-0.61 (3.93)** | -0.07 (1.53) | -0.17 (1.28) | 0.11 (1.96) |
| C | -0.2 (2.08) | -0.03 (1.89) | **-0.98 (2.32)** | -0.07 (1.29) | -0.14 (2.05) |
| D | 0.12 (1.74) | 0.02 (1.32) | -0.17 (1.74) | **-0.88 (1.83)** | -0.19 (1.82) |
| E | 0.1 (1.38) | 0.06 (1.9) | 0.15 (1.93) | 0.11 (1.68) | **-0.32 (2.47)** |

Table 5.1.6B: Estimates of the error correction term of Frosting

| | Product | | | |
|---|---|---|---|---|
| | A | B | C | D |
| | | Static Model | | |
| A | **-0.44 (3.41)** | -0.12 (1.64) | -0.06 (1.4) | -0.12 (1.25) |
| B | 0.12 (1.33) | **-0.81 (1.98)** | 0.18 (1.62) | -0.12 (1.75) |
| C | 0.2 (2.09) | -0.11 (2) | **-0.99 (3.08)** | -0.18 (1.59) |
| D | -0.18 (2.02) | 0.2 (1.85) | -0.12 (1.26) | **-0.64 (2.77)** |
| | | Dynamic Model | | |
| A | **-1.08 (3.83)** | -0.05 (1.66) | -0.01 (1.7) | -0.19 (1.92) |
| B | -0.11 (1.78) | **-0.62 (3.34)** | -0.18 (1.85) | -0.06 (1.58) |
| C | -0.18 (1.61) | -0.08 (1.36) | **-0.84 (2.85)** | -0.18 (2.08) |
| D | -0.12 (1.5) | -0.06 (1.93) | -0.15 (1.58) | **-0.56 (3.92)** |