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REDISTRIBUTION OF STRANGENESS BETWEEN QUARK-GLUON PLASMA AND HADRONIC GAS

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ABSTRACT

In a baryon-rich hadronic gas s-quarks, unlike antistrange quarks, can be found in baryonic degrees of freedom. This abundance asymmetry induces an associated asymmetry in the otherwise symmetric quark-gluon plasma section of the fireball volume. The magnitude of this effect is established as a function of thermodynamic variables and experimental consequences are explored.

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In a short-lived isolated quark-gluon plasma there is an equal abundance of strange (s) and antistrange ($\bar{s}$) quarks arising dominantly from gluon-gluon interactions[1]. Owing to this symmetry the chemical potential μ_s of strangeness vanishes. The purpose of this study is to explore systematically deviations from this simple picture occurring when the plasma is in close contact with domains of hadronic gas, arising in particular in the hadronisation process of the plasma. In the baryonic hadronic gas the contributions of hyperonic degrees of freedom to the balancing of strangeness induces a nonvanishing strangeness chemical potential. This clash between the unequal value of the strangeness chemical potentials in the two phases has been recorded earlier[2]. The present investigation is undertaken in view of recent claims that re-equilibration of strangeness between the hadronic gas and the quark-gluon plasma is capable of significantly depleting the abundance of $\bar{s}$-quarks in plasma[3]. If this assumption were generally true it could partially negate the proposal to use strange antibaryons as a signature of the quark-gluon plasma[4]. For further details on strangeness in hot matter the reader may consult the recent review[5].

We assume that the space region of the quark-gluon plasma is surrounded by a comparable volume domain of hadronic gas and that the s, $\bar{s}$-quarks can penetrate the phase boundary. This process of re-equilibration erases any discontinuity in the strangeness chemical potential μ_s, establishing its common value at the phase boundary. In consequence and depending on the thermodynamic conditions of the temperature T and the baryochemical potential $\mu_b \simeq 3\mu_q$ an enrichment or depletion of $\bar{s}$-quarks in the plasma domain may result along with the opposite effect, i.e. depletion, resp. enrichment of $\bar{s}$ abundance in the hadronic gas phase. As we will see, the magnitude of this effect will also depend on the relative size of the volume V_p/V_h occupied by plasma ('p') and the participating hadronic gas ('h'). We expect that the plasma phase is only formed in the hottest domains of colliding nuclear matter, and that V_p will decrease with time due to progressive hadronisation. Hence we shall concentrate our attention on the domain $0 < V_p/V_h < 1$. Anyway, for $V_p > V_h$ the large strangeness abundance in the plasma dominates over the abundance in the hadronic gas and the re-equilibration effects will be negligible.

The temperature range of interest to us is 120 MeV < T < 180 MeV corresponding to the various current hadronisation models of a baryon-rich plasma.

Consider the partition function of strange particles in the Boltzmann approximation

$$\ln Z_s = Z^{(1)}_{p,s} + Z^{(1)}_{h,s} \quad , \tag{1}$$

where 's' reminds us that eq.(1) refers only to the strange fraction of all particles and 'p' and 'h' have the same meaning as above. The upper index '(1)' stands for 'one particle' partition function; we have in the plasma

$$Z^{(1)}_{p,s} = (\lambda_s + \lambda_s^{-1}) \frac{V_p T^3}{2\pi^2} g_s \; W(m_s/T) \quad , \tag{2a}$$

$$W(x) = x^2 K_2(x) \quad , \tag{2b}$$

with the statistical degeneracy $g_s = 6$ (3 colours, 2 spins). We have introduced here the fugacity of strangeness

$$\lambda_s = e^{\mu_s/T} \quad , \tag{3a}$$

and similarly of light quarks (see below):

$$\lambda_q = e^{\mu_q/T} \quad . \tag{3b}$$

The s-quark mass is assumed here to be $m_s \approx 160$ MeV. The Boltzmann approximation for strange quarks in a plasma remains valid at $m_s/T \sim O(1)$, provided that $\lambda_s \sim O(1)$ which, as we shall see, will be the case in our study.

In the hadronic gas we must count all important hadronic carriers of s, $\bar{s}$-quarks. We have

$$Z^{(1)}_{h,s} = \frac{V_h T^3}{2\pi^2} [(\lambda_s \lambda_q^{-1} + \lambda_s^{-1}\lambda_q) F_K + (\lambda_s \lambda_q^2 + \lambda_s^{-1}\lambda_q^{-2}) F_Y + (\lambda_s^2 \lambda_q + \lambda_s^{-2}\lambda_q^{-1}) F_\Xi + (\lambda_s^3 + \lambda_s^{-3}) F_\Omega] . \tag{4a}$$

where the kaon (K), hyperon (Y), cascade (Ξ) and omega (Ω) degrees of freedom in the hadronic gas are listed successively. In detail:

$$F_K = \sum_j g_{K_j} \; W(m_{K_j}/T) \quad ; \quad K_j = K,\ K^*,\ K_2^*,\ \ldots \tag{4b}$$

$$F_Y = \sum_j g_{Y_j} \; W(m_{Y_j}/T) \quad ; \quad Y_j = \Lambda,\ \Sigma,\ \Sigma(1385),\ \ldots \tag{4c}$$

$$F_\Xi = \sum_j g_{\Xi_j} \; W(m_{\Xi_j}/T) \quad ; \quad \Xi_j = \Xi,\ \Xi(1530),\ \ldots \tag{4d}$$

$$F_\Omega = g_\Omega \; W(m_\Omega/T) \quad . \tag{4e}$$

All particles listed explicitly will be included in the calculations below, unless excluded specifically. In Fig.1 we show the relative magnitude of the phase space of these degrees of freedom as a function of the temperature. At high T ($\sim$ 180 MeV) we see that the K, Y, Ξ degrees of freedom are of decreasing magnitude separated by a factor $\sim$5 from each other, while Ω is about a factor $\sim$10 smaller than cascades. At the 'small' temperature, T $\sim$ 120 MeV, the kaonic degrees of freedom are most numerous (subject to modification by $\lambda_q \neq 1$). We conclude that it is not always safe to neglect the presence of multiply-strange baryons, especially when $\lambda_q \gtrsim 5$.

We now further assume that the number of s and $\bar{s}$-quarks is equal within the total volume of the fireball comprising V_p and V_h. This implies that we implicitly assume that the loss of strangeness due to pre-equilibrium emission either does not matter, or is rather symmetric between s and $\bar{s}$-quarks. The latter point is not true as the $\bar{s}q(K^o,K^+)$ mesons are the most penetrating strangeness carrying hadrons and therefore the $\bar{s}$-loss indeed will be greater. However, calculations of the absolute rate along the lines of Ref.(6), believed to overestimate pre-equilibrium emission, indicate that it is safe to neglect the magnitude of the effect. In other words, pre-hadronisation kaon emission which populates the high p_t tails of the spectra remains a small fraction of the total kaon abundance. Hence we can neglect this small resulting s-$\bar{s}$ asymmetry in the fireball.

The condition on Z_s, (eq.(1)) that the total strangeness vanish takes the form

$$0 = \langle s\rangle - \langle\bar{s}\rangle = \lambda_s \frac{\partial}{\partial\lambda_s} \ln Z_s \quad . \qquad (5)$$

We find, using eqs.(2,4) in eq.(1), that the requirement (5) is equivalent to:

$$(\lambda_s^{-1} - \lambda_s)\ g_s\ W(m_s/T)\ \frac{V_p}{V_h} =$$

$$= (\lambda_s\lambda_q^{-1} - \lambda_s^{-1}\lambda_q)\ F_K + (\lambda_s\lambda_q^2 - \lambda_s^{-1}\lambda_q^2)\ F_Y + \qquad (6)$$

$$+ 2\ (\lambda_s^2\lambda_q - \lambda_s^{-2}\lambda_q^{-1})\ F_{\Xi} + 3\ (\lambda_s^3 + \lambda_s^{-3})\ F_{\Omega} .$$

This is an implicit equation for λ_s (or $\mu_s = T \ln \lambda_s$) given a certain value of the thermodynamic variables V_p/V_h, T, λ_q (or $\mu_q = T \ln \lambda_q$).

The interesting quantity related to λ_s is the ratio R of strange to antistrange quarks in the plasma:

$$R := \left.\frac{\langle s\rangle}{\langle\bar{s}\rangle}\right|_{plasma} = \lambda_s^2 \quad . \qquad (7)$$

Replacing λ_s in eq.(6) by $R^{1/2}$ we obtain an (implicit) relation between R and the named thermodynamic variables. Explicitly, one easily finds a relation between V_p/V_h and R as displayed in Figures 2,3. In Fig.2a-d the ratio R is shown for four selected values of T: 180(a), 160(b), 140(c), and 120(d) MeV as a function of $V_p/V_h < 1$. In all figures $\lambda_q = 1.5$; e, e^2, 12 has been used as a parameter; in (d) in addition $\lambda_q = e^3$, $e^{3.5}$ is shown. We see that with increasing V_p/V_h all curves tend to unity - clearly the large strangeness abundance in the plasma dominates the re-equilibration condition in that limit. As the plasma volume becomes very small, say one tenth of the total fireball volume, significant deviation of R can result, especially at 'small' temperatures. But as is clearly visible, for sufficiently large μ_q in comparison to the temperature T we obtain R < 1. In Fig.2a (T = 180 MeV) we note a slight enhancement of s over $\bar{s}$ in the plasma (R > 1) for $\mu_q < T$, while a notable enhancement of $\bar{s}$ over s in the plasma results for $\mu_q > T$ which persists for all values of $V_p \lesssim V_h$. We thus learn that in a baryon-rich plasma with $\mu_q \gtrsim m_N/3$

$\sim$ 330 MeV, hadronising at a temperature of about 180 MeV, as stipulated by the current state-of-the-art lattice gauge theory calculations[7], $\bar{s}$-quarks will be enhanced in the plasma phase. s-quarks find a refugium in the hadronic gas phase due to the large hyperon phase space. Another (dynamical) way to look at this result is: s-quarks freeze out from the plasma more easily and rapidly as they can form kaons and hyperons, while $\bar{s}$-quarks freeze out more slowly since the (anti-) hyperon degree of freedom is not available. Consequently their density in the plasma increases. Indeed, their chemical potential μ

$$\mu_s = T \ln \lambda_s = \frac{1}{2} T \ln R \tag{8}$$

can assume significant negative values for $R = 1/2 : \mu_s \sim -0.3T \sim -60$ MeV (at $V_p/V_h \sim 1$, $T = 180$ MeV, $\mu_q \sim 400$ MeV). Note that μ_s must be compared with the value of $m_s \sim 160$ MeV, on which scale 60 MeV is a significant, non-negligible quantity. In Fig.3 we see how the qualitative behaviour of $R(V_p/V_h)$ changes as the hadronisation temperature decreases from 180 to 120 MeV, always assuming $\mu_q = 2T$. There is a clear tendency at fixed μ_q/T to increase R as T decreases; however, should T decrease, μ_q/T will increase and then we will still find $R < 1$, c.c. Figs.(2b,c,d).

To gain better insight into the behaviour of the boundary defined by $R = 1$ we consider eq.(6) neglecting the contributions of the multiply-strange hadrons Ξ and Ω. Solving eq.(6) for λ_s we find

$$R = \lambda_s^2 = \frac{F_s + \lambda_q F_K + \lambda_q^{-2} F_Y}{F_s + \lambda_q^{-1} F_K + \lambda_q^2 F_Y} \tag{9a}$$

where

$$F_s = \frac{V_p}{V_h} g_s W(m_s/T) \quad . \tag{9b}$$

At fixed V_p/V_h and T (i.e. fixed F_s) we see that the condition $R < 1$ (i.e. $\langle s \rangle < \langle \bar{s} \rangle$ in the plasma) corresponds to the requirement

$$\lambda_q F_K + \lambda_q^{-2} F_Y < \lambda_q^{-1} F_K + \lambda_q^2 F_Y \tag{10a}$$

(which indeed corresponds to the complementary statement $\langle \bar{s} \rangle < \langle s \rangle$ in the hadronic gas phase). The contrary requirement $R \gtrsim 1$ of course implies the opposite condition in eq.(10) with '<' being replaced by '>'. Neglecting in eq.(10) the antihyperon term $\lambda_q^{-2} F_Y$ in comparison to $\lambda_q F_K$ kaon [$\bar{s}q$ (K^0, K^+)] and similarly on the right hand side the kaon term $\lambda_q^{-1} F_K$ [$s\bar{q}$($\bar{K}^0, K^-$)] in comparison to the hyperon term $\lambda_q^2 F_Y$ we find that eq.(10a) is equivalent to

$$\lambda_q > F_K/F_Y \quad . \tag{10b}$$

Recalling from Fig.1 that this signifies $\lambda_q >> 1$, all the approximations are indeed justified including neglect of F_{Ξ} as compared with $\lambda_q F_Y$ when deriving eq.(9). We can repeat these arguments for the opposite condition R > 1. The opposite of eq.(10a) then also requires $F_K/F_Y > \sqrt{3}^{\frac{3}{2}} \sim 2.6$ which as we see in Fig.1 is satisfied in our range of temperatures. Although then the opposite of eq.(10b) may in some instances be slightly inconsistent with the neglect of multiply-strange hadrons, the boundary $\lambda_q = F_K/F_Y$ describes qualitatively the division between R > 1 and R < 1. In Fig.4 the associated baryon chemical potential

$$\mu_b = 3\ T\ \ln F_K/F_Y \tag{11}$$

is shown. This boundary shown in Fig.4 highlights the fact that the question of R > 1 or R < 1 at hadronisation depends on where in the μ_b, T region of Fig.4 the phase diagram is going to have its boundary. Matters will be further complicated by significant temperature gradients and decoupling of the hadronic gas from the plasma in case of rapid expansion of the fireball. It is of great importance to record that the boundary shown in Fig.4 depends only on well known phase space factors of the hadronic gas and hence it is very much model-independent.

The important physics point of this paper is: unless by a very unlikely coincidence the boundary shown in Fig.4 is also a phase boundary between gas and plasma, strangeness will flow in a specific direction through the boundary between the hadronic gas and the plasma enriching further the respective s and/or $\bar{s}$ concentrations, as expressed by the finite value of the strangeness chemical potential μ_s (eq.(8)) for $R \neq 1$. This will doubtlessly further facilitate the formation of multiply-strange hadrons and possibly other exotic strange-quark clusters, even beyond the extraordinary levels predicted earlier[5]. The abundance of strange antibaryons, proposed as a signature of the quark-gluon plasma, will be affected in the following way: should the baryon-rich plasma hadronise at high temperature (e.g. $\mu_q > 1.5$ T at T $\sim$ 180 MeV), then s-quarks would hadronise faster than $\bar{s}$ and the clustering of $\bar{s}$-quarks to multiply-strange objects would be enhanced towards the latter stage of the plasma lifetime, possibly enhancing the total strange antibaryon yield. Should hadronisation occur at 'low' temperature T $\sim$ 140 MeV, however, then it is possible that a fraction of all $\bar{s}$-quarks from the plasma will hadronise, preferentially into $\bar{s}q$-kaons, depleting somewhat the anticipated plasma signature. Which of the two scenarios prevails can be determined by comparing Ω to $\bar{\Omega}$ abundance. Indeed,

$$\Omega/\bar{\Omega} \sim \lambda_s^6 = R^3 \quad . \tag{12}$$

Without the strangeness re-equilibration we expect equal abundances of these rare hadrons. Abundance asymmetry is directly related in eq.(12) to a nonvanishing λ_s, with $\bar{\Omega}$ being favoured over Ω by $\lambda_s < 1$, i.e. $R < 1$. We can turn this argument around: if $\Omega > \bar{\Omega}$ is observed, then this can be taken as evidence of hadronisation in the lower left triangle of the (μ_b,T) domain shown in Fig.4. Should even $\Omega >> \bar{\Omega}$ be observed, then the scenario of 'cold' hadronisation apparently employed in the related work[3] would be justified. However, the conventional-wisdom hadronisation pictures of relativistic nuclear collisions with $T \gtrsim 180$ MeV and $\mu_q > T$ has $R < 1$ and hence we are led to expect $\bar{\Omega} > \Omega$.

It has been shown that the difference of the structure of the hadronic gas phase and the quark-gluon plasma with respect to strangeness facilitates the establishment of normally a minor asymmetry between s and $\bar{s}$-quarks in these coexistent phases.

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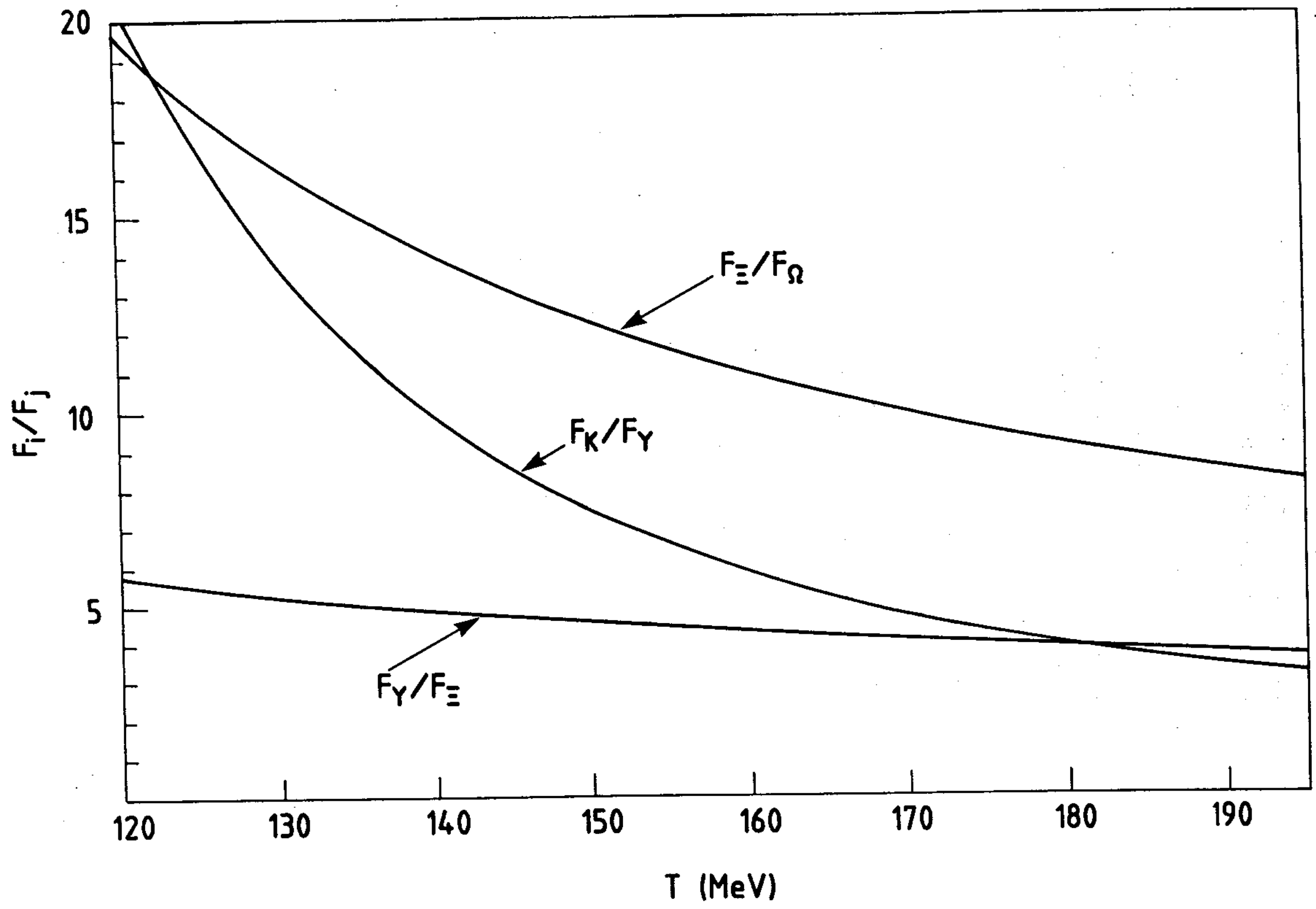
Figure Captions

Fig.1 : Phase-space ratio of kaons and hyperons (F_K/F_Y), hyperons and cascades (F_Y/F_Ξ), cascades and omega (F_Ξ/F_Ω) as functions of temperature T

Fig.2 : Ratio R of strange to antistrange quarks in the plasma as a function of the ratio of plasma volume to hadronic-gas volume for selected values of λ_q (i.e. μ_q) for T = 180 MeV(a), T = 160 MeV(b), T = 140 MeV(c) and T = 120 MeV(d)

Fig.3 : Same as Fig.2 at μ_q = 2T

Fig.4 : Separation of the μ_b-T plane according to s-abundance dominance in the two phases



20
15
10
5
F_i/F_j
120
130
140
150
160
170
180
190
T (MeV)
F_Ξ/F_Ω
F_K/F_Y
F_Y/F_Ξ

FIGURE 1

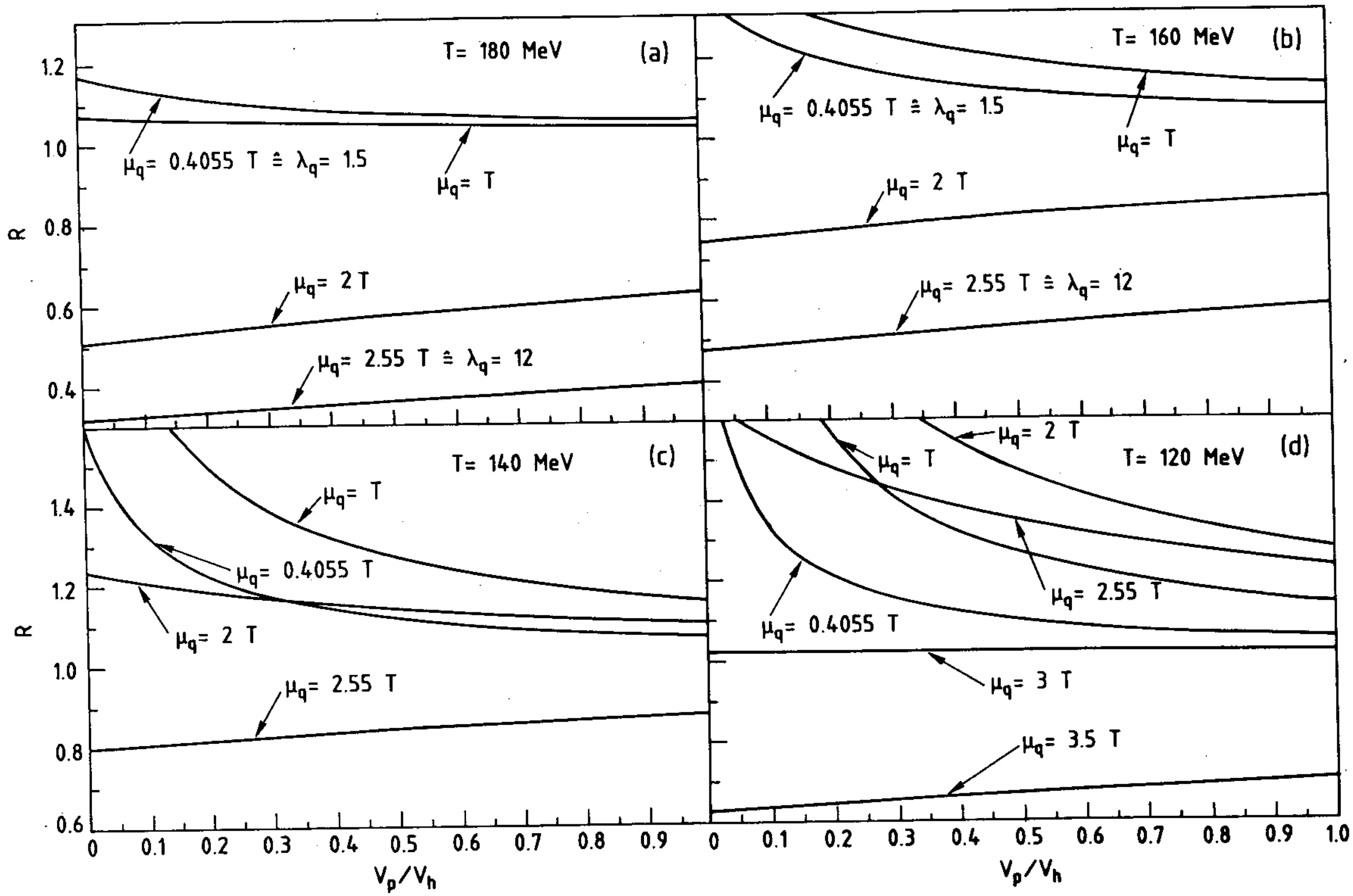
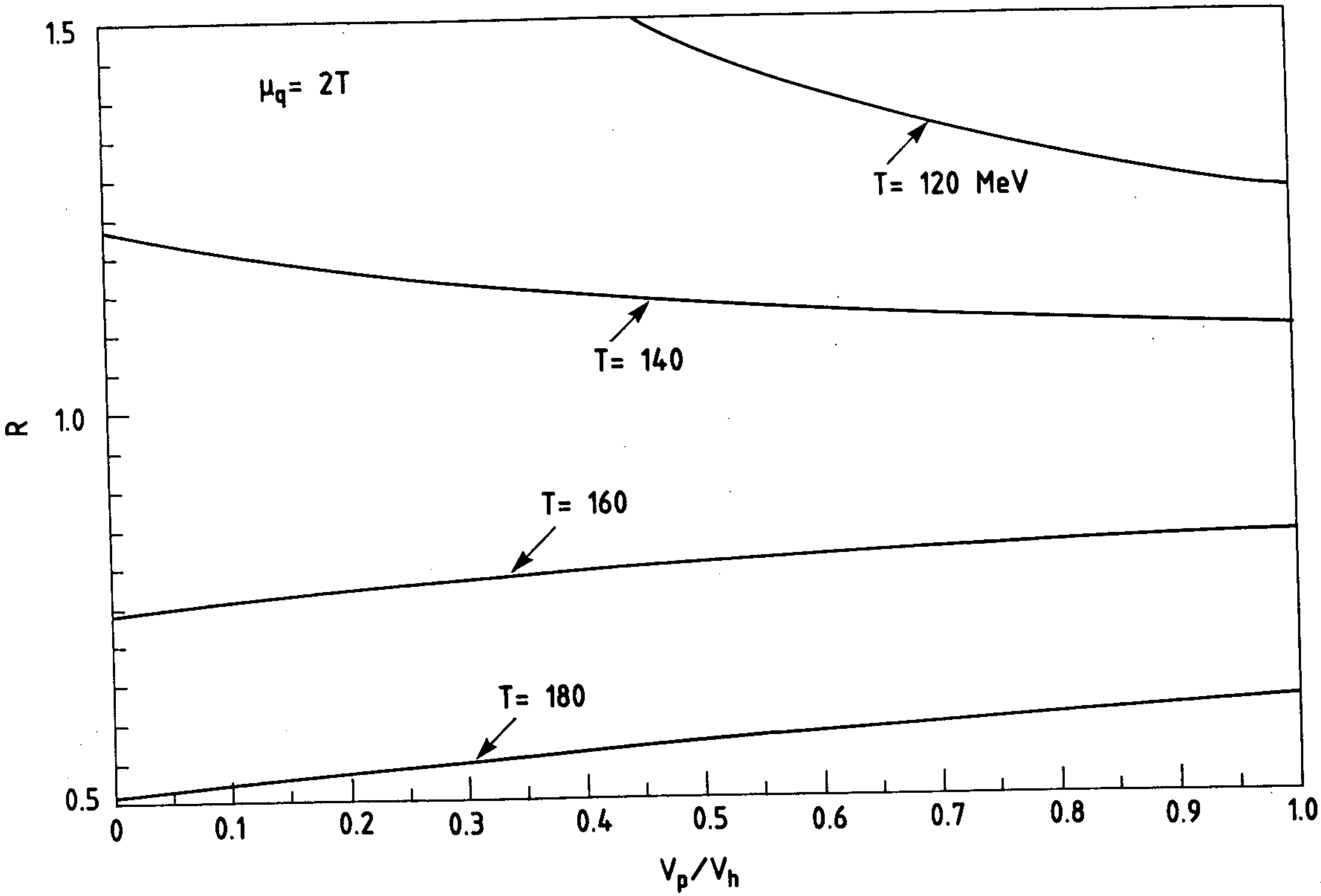

T= 180 MeV
(a)
μq= 0.4055 T ≙ λq= 1.5
μq= T
μq= 2 T
μq= 2.55 T ≙ λq= 12
T= 160 MeV
(b)
μq= 2 T
T= 140 MeV
(c)
μq= 0.4055 T
μq= 2.55 T
T= 120 MeV
(d)
μq= 3 T
μq= 3.5 T
R
Vp/Vh

FIGURE 2



1.5
1.0
0.5
R
μq= 2T
T= 120 MeV
T= 140
T= 160
T= 180
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Vp/Vh

FIGURE 3

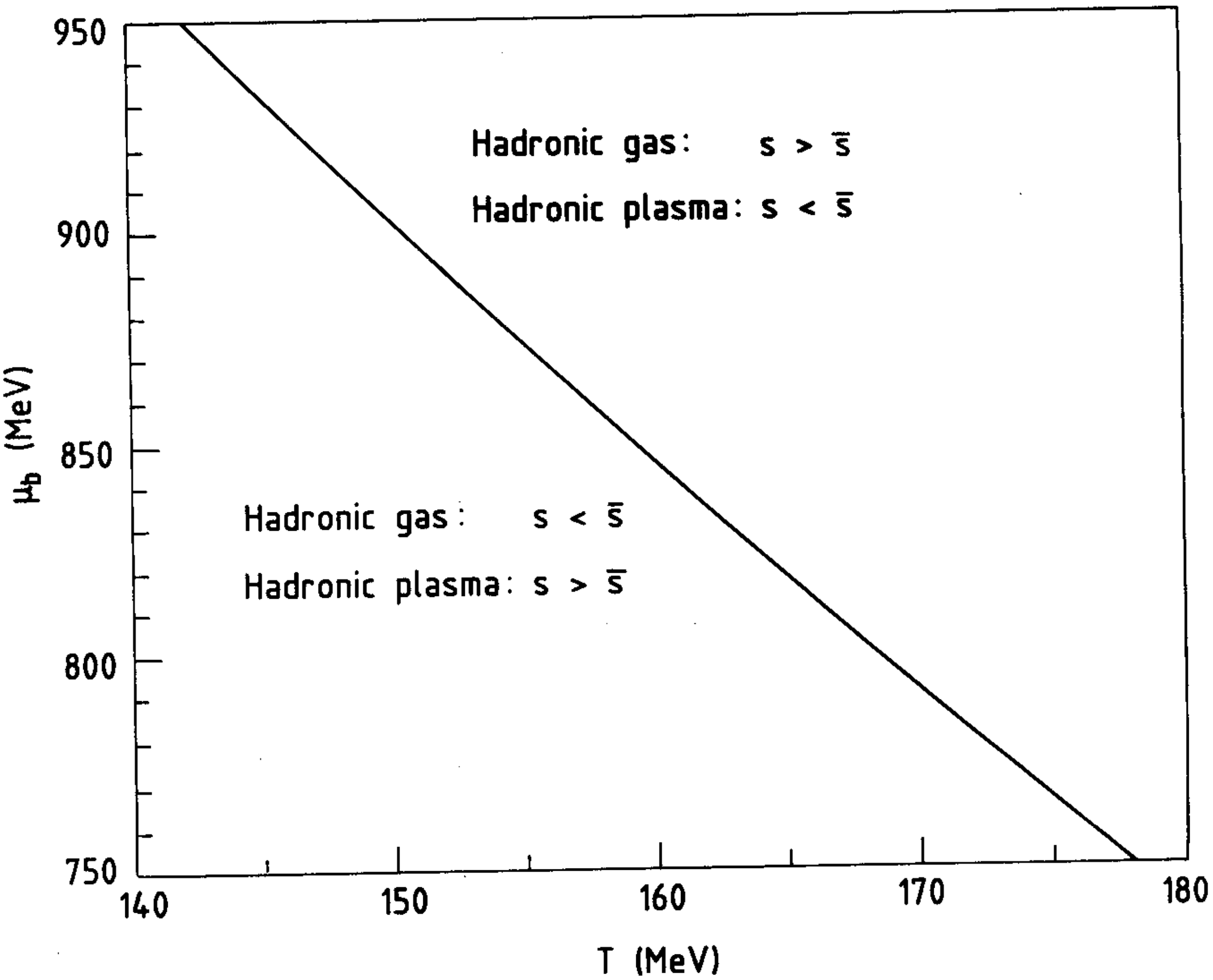

Hadronic gas: s > $\bar{s}$
Hadronic plasma: s < $\bar{s}$
Hadronic gas: s < $\bar{s}$
Hadronic plasma: s > $\bar{s}$
950
900
850
800
750
μ_b (MeV)
140
150
160
170
180
T (MeV)

FIGURE 4