ABSTRACT GEOMETRY IN THE MOTETS OF DUNSTABLE AND DUFAY

VOL. 1

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Abstract Geometry in the Motets of Dunstable and Dufay

This dissertation is submitted in partial fulfilment of
the requirements for the degree of Master of Music
at the University of Cape Town.

Abbreviations

JAMS - Journal of the American Musicological Society

Mus.Brit. - Musica Britannica

MQ - Musical Quarterly

NOHM - New Oxford History of Music

PRMA - Proceedings of the Royal Musical Association

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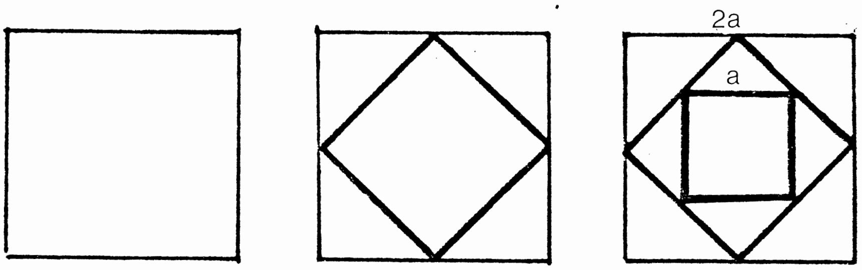
Introduction

This dissertation has been inspired by architectural studies showing the extent to which the medieval architect was steeped in geometrical and mathematical thinking. Researchers such as Lesser, von Simson and Wittkower [1] have shown that, from its overall proportion to its most intimate detail of ornamentation, the Gothic cathedral is derived from the sacred lore of numbers and its concommitent geometric procedures, notably quadrature and triangulation. With a single basic dimension, the Gothic architect developed all the measurements of his groundplan and elevation by geometrical means, using the square and the triangle as his module. Dimensions are related, for example, by squares of which the area diminishes or increases in geometric proportion:

Fig.1. Geometric construction of the 'just measure.'



The measurements obtained in this way were considered to be according to true measure and the application of this method of proportioning could yield an infinite variety of forms. In the Gothic canopy support at fig.2, for example, there is no detail that cannot be related to another by means of quadrature. Every geometric shape within the groundplan consists of a combination of squares and the dimensions of all the squares used are derived

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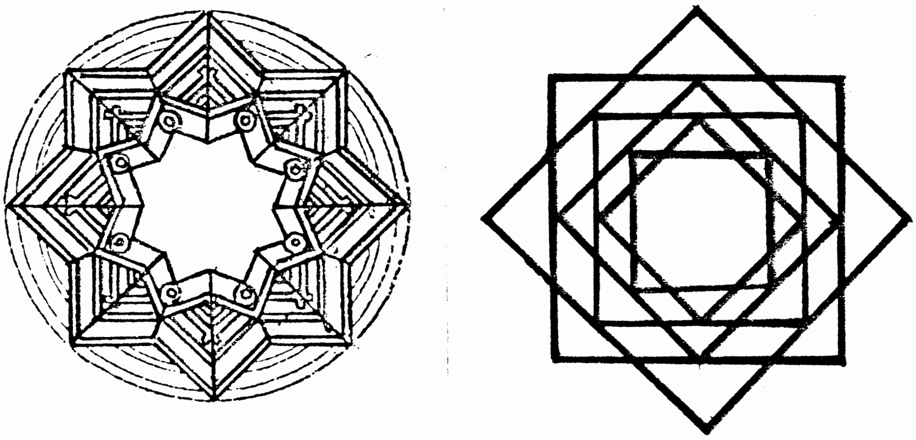
[1] G. Lesser, Gothic Cathedrals and Sacred Geometry London, 1957.
O. von Simson, The Gothic Cathedral, London and New York, 1956.
R. Wittkower, Architectural principles in the Age of Humanism, London 1952, rev.edn.1962.

from the largest one purely by geometric means:

Fig.2.The geometric construction of the groundplan of a Gothic canopy support.



[2]

Clearly these procedures of variation are in essence architectural and do not seem to have any direct bearing on the processes involved in musical composition.However, von Simson,Lesser and Wittkower so often link Gothic and Renaissance architecture to medieval and Renaissance music, that one wonders whether the music itself may not have been influenced by mathematical and geometrical principles.

2.O. von Simson, The Gothic Cathedral,p.15. I have not been able to trace the source of this illustration.

In support of this hypothesis one can make the following points:

1. Philosophical works of the Middle Ages stress the importance of interdisciplinary thinking and many examples do this with explicit reference to music as one of the seven liberal arts. In the Pythagoreo-platonic tradition of the Middle Ages,arts and sciences were related by means of the theory of numbers conceived by Pythagoras. This theory of numbers had geometric implications which, when transposed to architecture, led to an art in which proportional relationships,conceived in spatial terms, played a dominant role. As musical time was also conceived proportionally during the Gothic period, it is quite possible that it, too, was conceived in spatial terms.

2. The whole of medieval thought is in essence symbolic. More than in any other age, philosophers emphasised analogy and allegory. This mental habit, elevated to a philosophical principle in scholasticism, suffused the whole of life and must therefore have had its influence on architecture and music. In such an environment an analogy between architecture and music is not only a possibility, but a likely occurrence.

3. It is furthermore unlikely that such an analogy would only exist on a philosophical level and therefore remain speculative,because Gothic Christianity emphasised the synthesis of life and action. The Gothic cathedrals, for example, were conceived as a reflection of the glory of God on earth and the details of construction were subordinated to philosophical ideas that would reveal this glory. Could these ideas not be reflected in a similar way in the music that was used in these cathedrals?

4. Finally, there are, to my knowledge, five modern scholars who have revealed evidence of geometrical and mathematical proportioning in music of the Gothic period.[3]

In view of these four points it would be necessary, for further historical research on the subject, to uncover the broader frame of reference that underlies the isolated and, as yet, unrelated examples of geometric procedures in music. The question of the use of geometric and mathematical procedures as an important and pervading aspect of musical composition in the Gothic era can only be answered by a detailed analysis of the music, an undertaking that is too vast for this present study. In this dissertation I will therefore limit myself to a study of the isorhythmic motets of Dunstable and Dufay, in order to establish the relevance of proportional and geometric analysis, as applied to these works.

Chapter 1 will be devoted to a detailed study of the four points mentioned above and to a critique of the methods so far used by scholars in their analyses of the proportional and geometric structures of individual works. Included in this chapter is an analysis of five Organa by Léonin.

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[3] N.Powell, "Fibonacci and the Gold Mean: Rabbits, Rumbas and Rondeaux." Journal of Music Theory, 23(1979):227-273.

M.van Crevel, see the introductions in Jacobus Obrecht, Opera Omnia, Vol.1, fasc.6 & 7, ed.M.van Crevel(Amsterdam,1959-1964).

B.Trowell, "Proportion in the Music of Dunstable" Proceedings of the Royal Musical Association, 105(1978-79):100-141.

M.Sandresky, "The Golden Section in three Byzantine Motets of Dufay." Journal of Music Theory, 25(1981):291-306.

C.Warren, "Brunelleschi's Dome and Dufay's Motet." The Musical Quarterly, 59(1973):92-105.

These will establish which geometric procedures, if any, were used during the early stages of the Gothic era. Chapter 2 will serve as a guideline to Volume 2. It consists of analyses of five pieces from the Ars Antiqua and Ars Nova,the isorhythmic and non-isorhythmic motets of Dunstable and the isorhythmic motets of Dufay. The first five examples will serve to illustrate the existence of a continuing tradition of geometric procedures throughout the Gothic period[4]and will form the basis for a more detailed study of proportions in the works of Dufay and Dunstable. In chapter 3 the evidence brought to light in the analyses will be put in a historical perspective. Here the geometric and mathematical procedures used will first be viewed as an isolated aspect of musical composition,then in its relation to'tonality'[5],and finally as a phenomenon with extra-musical and symbolic implications.

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4. On the basis of these few examples one can of course do no more than establish the existence of such a tradition throughout the Gothic period and not whether it was a widespread practice.

5. The term 'tonality' will be used in its broader sense.

Chapter I

Origins of Geometric thinking in the Gothic Period

The historical evidence that leads one to suspect the existence of proportional and mathematical thinking in music is mainly derived from studies in architecture. A feasible starting point for this investigation, therefore, is to establish the extent to which these studies can help us to uncover the degree of influence architecture had on music.

O. von Simson[6] traces the origins of Gothic architecture to the cathedral schools of places such as Chartres and Paris. In his view, the whole development derives from two main sources, namely, Plato's Timaeus and the writings of St. Augustine(notably his treatise De Musica). These sources, discussed and re-interpreted by medieval philosophers connected with the cathedral schools[7],formed the basis on which architects of the Gothic period built the aesthetic ideals which governed the construction of their cathedrals. Whether these cathedral schools ever produced a system of aesthetics that exerted a direct influence on the new architectural style that emerged after 1150 cannot be answered with certainty. However, it can be surmised,on the basis of circumstantial evidence, in the knowledge displayed by architects such as Villard de Honnencourt[8]. This knowledge, von Simson claims,could only have been acquired in cathedral schools laying a particular emphasis on metaphysical speculation of the Pythagoreo-platonic tradition. The main source of this tradition was Plato's Timaeus.

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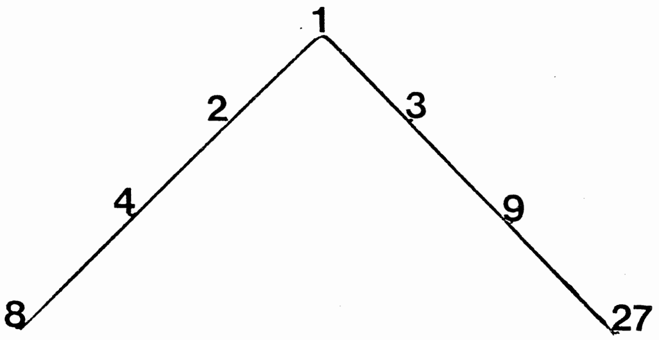
6. O. von Simson, The Gothic Cathedral.
7. Notably Thierry de Chartres, William of Conches and Abelard.
8. Villard de Honnencourt: French architect of the first half of the 13th century, whose architectural drawings - a fragment of 33 folios with about 325 drawings - give a clear insight in the working processes of medieval architecture.

The philosophy expressed in the Timaeus is based on the idea that the World-soul must contain all fundamental principles within itself, which implies that the nature of the World-soul consists of numbers, since these were considered the fundamentals of things. This led to speculation on the mathematical division of the World-soul into its component parts. This division, according to the Timaeus, is as follows:

> First he (the Demiurge) took one portion from the whole, and next a portion double of this; the third half as much again as the second, and three times the first; the fourth double the second; the fifth, three times the third; the sixth, eight times the first; and the seventh twenty-seven times the first.[9]

This produces the series 1,2,3,4,9,8,27. The Demiurge made this division by means of two geometric proportions of four terms each: 1,2,4,8 and 1,3,9,27 (i.e. an even and an odd-numbered series). Since Crantor, the first Timaeus commentator, the Platonic series of seven numbers has been represented in the shape of Lambda:



[10]

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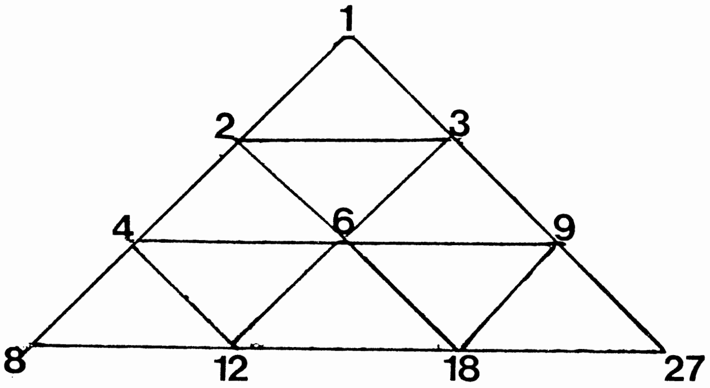
[9] Plato. Timaeus, trans. and ann. F Cornford, London, 1937, p.66.
[10] Plato, p.67.

This arrangement is called the greater or double tetraktys. The series can also be extended by multiplication (x6):



In this manner a new numerical sequence is produced:[11] 1,2,3,4,6,9,8,12,18,27. This series contains the smaller tetraktys (1,2,3,4),[12] the Babylonian Golden proportion (6,9,8,12)[13] and 18 and 27.[14] Calcidius[15] maintains that these numbers express the division effected by the Demiurge in accordance with the harmonic, arithmetic and geometric proportions. In contrast to the smaller tetraktys, the ten numbers in this series contain all three Pythagoreo-platonic proportions:

Geometric proportion:

1 : 2 : 4 = 2 : 4 : 8 = 3 : 6 : 12
1 : 3 : 9 = 2 : 6 :18 = 3 : 9 : 27
4 : 6 : 9 = 8 :12 :18 =12 :18 : 27

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[11] M. van Crevel, Introduction, Obrecht: Opera Omnia, fasc 7, p.LXV.

[12] & [13]. For the connection between the tetraktys and the Babylonian Golden proportion cf. s.v. "Pythagorean Hammers" Harvard Dictionary of Music, London, 2nd.ed. 1970.

[14] 27 = 3^3 = a solid, and is therefore the limit of the series.

[15] Calcidius (late 4th century): first Latin translator of the Timaeus. see M. van Crevel, Introduction, fasc 7, p.LVIII.

Arithmetic proportion: (Inversion of harmonic)

1 : 2 : 3 = 2 : 4 : 6 = 3 : 6 : 9
4 : 8 :12 = 6 :12 :18 = 9 :18 :27
2 : 3 : 4 = 4 : 6 : 8 = 6 : 9 :12

Harmonic proportion: (Inversion of arithmetic)

$$\frac{1}{3} : \frac{1}{2} : \frac{1}{1} =$$

2 : 3 : 6 = 4 : 6 :12 = 6 : 9 :18

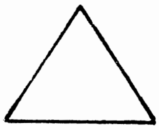
$$\frac{1}{4} : \frac{1}{3} : \frac{1}{2} =$$

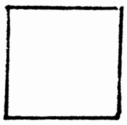
3 : 4 : 6 = 6 : 8 :12 = 9 :12 :18 [16]

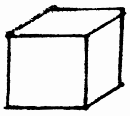
The numbers of the greater tetraktys are linked to geometric figures:

1 = · = point

2 = ——— = line

3 =  = surface

4 ; 9 =  = surface (2^2 & 3^2)

8 ; 27 =  = solid (2^3 & 3^3) [17]

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16. M. van Crevel, *Introduction*, fasc.7,p. LXVI.
17. This explains why, in the enumeration of the 7 Platonic numbers,the cubic number 8 always follows the square number 9.

Von Simson maintains that in the cathedral schools of Chartres and Paris the Timaeus was reinterpreted in a way which put it in agreement with the book of Genesis. Both these works were believed to be in agreement with each other in what they revealed about the creation of the universe and about the creator. The theology of the cathedral schools emphasised mathematics and geometry. For example, Thierry de Chartres[18] sought, with the help of geometry and arithmetic, an explanation of the mystery of the Trinity. According to him, the equilateral triangle expressed the equality of the three persons of the Trinity, and the square the relation between God the Father and God the Son (God the Son is a unity begotten by a unity, as the square results from the multiplication of a number by itself). Another teacher at the School of Chartres, William of Conches[19], and Peter Abelard of the Notre-Dame School in Paris,[20] identified the Platonic World-soul with the Holy Ghost in its creative and ordering effect upon the universe, and they conceived of this effect as musical consonance. In the Timaeus Plato's description of the division of the World-soul according to the ratios of the smaller tetraktys is related to the musical consonances (their ratios being 4:3, 3:2, 2:1, 4:1), and forms the basis of all aspects of creation. Thus, the composition of the world is brought about by fixing quantities in perfect geometrical proportions of squares (1:2:4:8) or cubes (1:3:9:27) which can be continuously related to simple musical ratios or their multiples.

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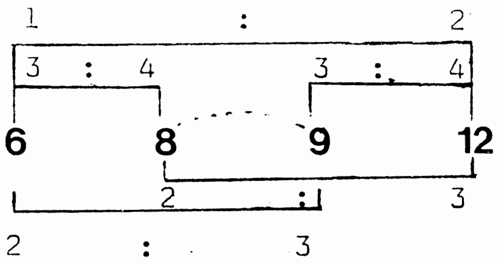
[18] Thierry de Chartres,(d.c.1150)taught at Paris and Chartres. He attempted to harmonise Scripture with Platonic doctrine in his commentaries on Genesis.

[19] William of Conches,(d.1154)also a teacher at Paris and Chartres, wrote glosses on Plato's Timaeus and Boethius' De Consolatione.

[20] Peter Abelard,(1079-1142).

It expresses itself in a universe in which all things are linked in perfect proportion, unified within themselves and free from any internal disharmony.

Medieval philosophers saw these aspects of the Timaeus mainly through the eyes of Augustine and Boethius.[21] Their writings on music clearly show a Pythagoreo-platonic bias. In Boethius' De Musica[22] the Pythagoreo-platonic division of the musical scale is given an extensive exposition. Here the division of the scale is related to the three proportions already discussed. By applying and combining the ratios within these proportions, the intervals of the Greek musical scale[23] are given mathematical justification, for the geometric proportion results in the octave 6:12 (1:2) and the ratios of the harmonic and arithmetic proportions determine the intervals of the fourth, the fifth and the tone:



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In view of the importance of this whole idea, as can be seen from the preceding discussion, one would expect it to influence

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21. see "Plato" Encyclopedia Britannica, Chicago, 1973.

22. Boethius (d.c.525) De Musica expounds the Pythagoreo-platonic theory of music expressed by Nicomachus and Ptolemy.

23. see s.v. "Pythagorean scale." Harvard Dictionary of Music.

24. see s.v. "Pythagorean hammers." Harvard Dictionary of Music.

the arts of the Gothic period in more than just a cursory way.

It is possible, therefore, that the music of Léonin, which was specifically designed for use in the cathedral of Notre-Dame in Paris, may have been influenced by the Pythagoreo-platonic tradition of the cathedral schools. This raises the question as to whether this influence can be shown in the actual form of Léonin's Organa. While most research in proportioning methods in music has been restricted to the late Gothic period and the early Renaissance, the flowering of neo-platonic ideals in philosophy can be traced to the beginning of the Gothic period and since these ideals found their application in architecture as early as 1150, the same aesthetic ideals may have been applied to music as early as this date.

Waite[25] has shown that the beginning of the Gothic period coincides with a return to strict rhythmic measurement, as seen in the Organa of Léonin and Pérotin. The system of rhythmic organisation that evolved was based on the repetition of patterns or rhythmic modes arranged into phrases or ordines. These are established through the interpolation of rests which control the length of phrases and therefore control the organisation of the piece as a whole. In striking parallel with von Simson, who traces the origins of organising principles in Gothic architecture to Augustine's De Musica, Waite traces the origins of modal rhythm also to De Musica.[26]

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25. W.G. Waite, The Rhythm of twelfth-century polyphony, New Haven, 1954.

26. Augustine, 'On Music, De Musica', ed. and trans. R.Taliaferro, The Fathers of the Church, a New Translation: Writings of St. Augustine, ii (New York, 1947) p.153 - 379.

Augustine's treatise presents the theory of modal rhythm in terms of Metrics, which, as part of Grammar, is a discipline belonging to the Trivium and therefore not often linked to the more mathematical arts of the Quadrivium. However, the treatise also points out that Metrics is only a preparatory study for the arts of Arithmetic, Geometry, Music and Astronomy. This correlation of Metrics to the arts of the Quadrivium, seems to imply a Pythagoreo-platonic foundation for the art of measured rhythm as applied to music. Statements in treatises of the thirteenth century echo this opinion.

In the first chapter of Odington's De Speculatione, for example, Metrics is established as the foundation of the whole discipline of music precisely because of its relation to number:

> Since the present treatise concerns music, and music consists of number related to sound, I believe that first of all the ars metrica must be explained, which consists of number per se, without which no noble object can be discussed. [27]

A similar statement is made by Johannes de Garlandia, when he says that "the subject of music is the joining of rests and notes in a necessary, properly observed manner. The predicate is the lawful art of adjusting this same music in suitable proportions by observing diligently all its modes, and for this the ars metrica supplies the philosophical part."[28]

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27. W.G.Waite, p.24 & E. Coussemaker, Scriptorum de musica, I, 183b.

28. W.G.Waite, p.23 & E. Coussemaker, Scriptorum de musica, I, 158a.

While these statements clearly echo the Pythagoreo-platonic tradition, they do not explain the link between music and number in actual practice. It could be that the connection is only superficial and that these writers are only paying tribute to an old and half-understood tradition. In book VI of De Musica, for example, Augustine bases all rhythm on two temporal units, short and long, the latter having twice the length of the former. These quantities, in various combinations produce the different poetic feet (e.g. trochee and iamb), but are always divisible into the same proportional parts(2:1). This,in itself does point to a connection with the Pythagoreo-platonic theory of numbers, but this application of it seems almost inconsequential to an understanding of the form of Organa.

If the Pythagoreo-platonic tradition had any tangible influence on music written in modal rhythm(which Waite has shown is derived from the poetic feet in Augustine's De Musica), this would presumably be reflected in more than the vague connection described above. The connection would probably be similar to the one that von Simson found in architecture and would involve either arithmetic relationships or their expression in geometric terms.

The following is an analysis of metrical lengths in five Organa by Léonin. The transcription used in this analysis can be found in Waite's doctoral thesis[29]. While this is an authoratative edition, there are still many points that are controversial as regards the notation of rhythm

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[29] W.G. Waite, Rhythm of twelfth-century.

in the Notre-Dame school.[30] The interpretation in this dissertation is therefore subject to reinterpretation. In the five diagrams that follow, the numbers within the block marked A indicate the number of poetic feet in each phrase. In the music, end of phrases are demarcated by rests. In Waite's transcription, these rests either carry the value of a quaver or a crochet. In the diagrams a quaver rest is indicated by a comma(,) and a crochet rest by a stroke(/). Occasionally there will be found two numbers underneath each other in the block marked A. These indicate the places where, according to Waite, there is an irregular phrase(e.g. a phrase consisting of 7 poetic feet instead of the more usual 6). These phraselengths, which in themselves are already a peculiar phenomenon, invariably create lopsided metrical structures in pieces which will be seen to be surprisingly strictly organised. They may therefore be the result of misinterpretation on the part of modern scholars.

The numbers underneath, in the block marked B, are an attempt to integrate the seemingly endless succession of phraselengths into a coherent system of proportions.

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[30] I believe that if the use of geometric proportions can be proven to be a general practice in the _Magnus Liber_ a detailed study, which would take this into account, might eventually help to clarify some controversial points of interpretation. For conflicting views of interpretation cf. H. Tischler, "A propos the notation of the Parisian Organa" _JAMS_ 14(1961): 1-8.

In the block marked C, an indication of these unsuspected relationships that reveal themselves is given, by translating the numerical relationships into algebraic relationships.

Example 1. **O1 Judea et iherusalem ℣ Constantes**

A	10 , 8 , 6	4/4,4	4, 8	16 / 8	8, 4	6, 6	4/10 ,4, 6	6 / $\overset{7}{6}$
B	24	12	12	24	12	12	24	12
C	2a	a	a	2a	a	a	2a	a

A	6,4, 4,6/4	18 8 2,4,12 /4, 4	18 8 4,6,8 ,2,6	4,4, 4,4, 4,8	4,6/4	4,6/4,4,4, 6
B	24	26	26	28	14	28
C	2a	c	c	2b	b	2b

A	4/4, 12 /	10
B	20	10
C	2d	d

The pattern 24 : 12 : 12 : 24 : 12 : 12 : 24 : 12 : 24 : 26 : 26 : 28 : 14 : 28 (2a : a : a : 2a : a a : 2a : a : 2a : c : c : 2b : b : 2b) reveals two ratios in quadrature, separated by a central part (c : c) which can be further subdivided into sections of 18 : 8 : 18 : 8. The final 30 creates the proportion 20 : 10 (2d : d) which can also be related to quadrature.

Example 2. O36. Vir iste. ℣ Pro eo ut.

A	6/6,6,4,8	4,6/4/4/6/4,2/4/4/4/14/4	$\overbrace{10,\overset{7}{6},10}^{26}/8/\overbrace{8,8,10}^{26}$
B	30	60	60
C	x	2x	2x

A	10/10/6/6,6,12,14/12/6/8/8/4/6/4	$\overset{7}{6}$/6/4/8/8/8/8/8	8/8/8/$\overset{5}{4}$
B	112	56	28
C	4Y	2Y	Y

The pattern 30 : 60 : 60 : 112 : 56 : 28
1 : 2 : 2 / 4 : 2 : 1

reveals the geometric proportion, which can be associated with the architectural principle of quadrature. It would also be useful to note the radial symmetry which occurs in the third subsection (26:8:26), since this is a formal feature that can be found in other works of the Gothic period.

Example 3.

M22. Alleluya

A	6,4,4/4,4,4/4,4.4,4	6//6/2,2,4/4,4,4,4,4.8/2,8,4,4,4,10/4	4,4,4,8/6/8/4,4,4/4,4,8,2,4/
B	42	84	168
C	x	2x	4x

A	10,4,4,4,6/4,6/6/6,4,2/6,$\overset{5}{4}$/6/6/6,4,4,4,4	10/6/4/8,6/8/
B	124	
C	2y	

A	6,2,4,4,6,4,4,4,6,6/12/$\overset{5}{4}$
B	62
C	y

The pattern 42 : 84 : 168 (x : 2x : 4x), can be seen to overlap with 124 : 62 (2y : y). In both cases this suggests the geometric proportion associated with quadrature.

Example 4.

M15 Alleluya

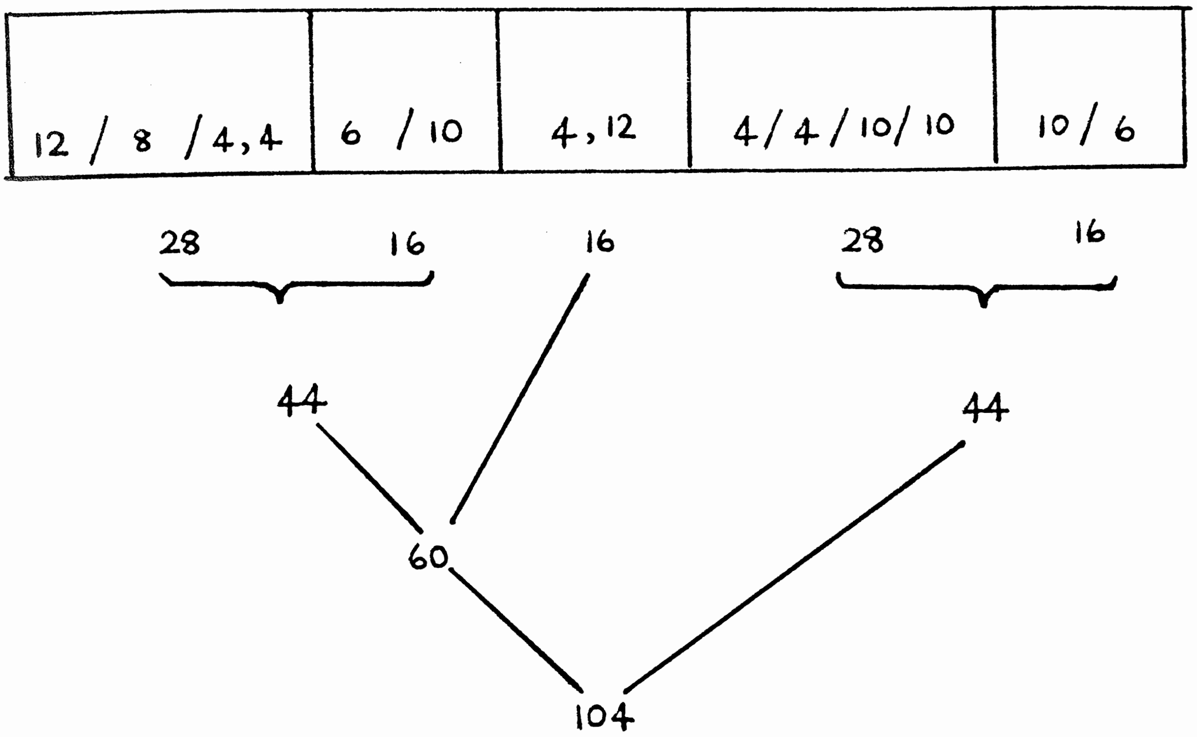
A	8	12/6/6,6/6,$\overset{5}{4}$/18 // 6 / 8	18 18 6,4,8/6/8/4
B		72	36
C		2x	x

A	4/4/4,6,6/3,4,4,6/6	4,4,4,6,8	6/6/4,4,4,4/6/4/2/6/6	18 18 6/8/4/4,8,6
B	52	26	52	36
C	2y	y	2y	x

Excluding the first 8, the pattern is as follows:
72 : 36 : 52 : 26 : 52 :36 (2x : x : 2y : y : 2y : x)
which combines two ratios in quadrature and reveals a radial symmetry.

Example 5.

Gloria Patri



The additive process shown here belongs to the geometric procedure known as the golden section, which will be discussed at a later stage in this chapter. Also involved, is a radial symmetry (44 : 16 : 44). *

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*In all five examples, end of sections correspond to a crochet rest (/). They do not coincide with musically significant subdivisions, and this method of structural organisation in the Organa of Léonin must therefore be viewed independently from the actual musical structure. While this is not the place for a detailed study of the Organa of Léonin, I would like to point out that at least two thirds of the Magnus Liber shows some evidence of similar geometric structures (see footnote 30).

This method of analysis shows that, at least in some of Léonin's pieces, there is a concern with architectural proportioning. The first four examples show, either singly or in combination, the geometric proportion, which in geometry is inextricably linked to quadrature. Where the proportion does not apply, for example in the third section of O 36, Vir Iste (Example **2**) and in Gloria Patri (Example **5**) there is either a symmetry involved, or, as with example **5,** the proportions can be related to the geometric construction known as the golden section. As will be discussed later, radial symmetry reveals itself as a consistent formal pattern during the Gothic period and the golden proportion has been discovered in works of Machaut and Dufay.

The most consistently used pattern is the geometric proportion, which, as shown on p.1 and 2, is inherent to quadrature. As it is the very principle that lies at the basis of most proportioning in Gothic architecture, we have found a tangible connection between the methods employed in architecture and music. This reinforces the possibility of cross-fertilization and interdisciplinary thinking taking place among the arts.

According to Willmann,[31] medieval writers often echoed Vitruvius[32] in that they found a real connection in all branches of learning. These had a reciprocal action upon

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31. O.Willmann, s.v. "The seven liberal Arts" The Catholic Encyclopedia, New York,1945.

32. Vitruvius: Roman architect whose treatise De Architectura (27 B.C.) was the chief authority of early Renaissance architects and was known during the Gothic period.

each other, for, in the opinion of medieval writers, universal science is composed of the special sciences in the same way as a body is composed of its members[33] and the same fundamental features and relations can therefore be seen in all branches of knowledge. These relations could only be expressed in tangible form by means of Arithmetic. Thus, for example, Arithmetic, which is concerned with the relationships of numbers, can be used to reveal relationships in Geometry (although Geometry also involves so-called incommensurable or irrational numbers that cannot be reduced to simple numerical proportions). Numerical proportions derived from and expressing geometric forms can in their turn be applied to the laws of motion revealed in Metrics, which in their turn can be translated into musical terms. The application of geometric form to Metrics and Music naturally implies a spatial conception of time and motion which can be revealed by an analysis of the numerological properties of poetry and music. It is now known that numerology was widely used by ancient Latin authors and that it is common to most Medieval and Renaissance poets.[34] The structural and symbolic function of numerical relationships in the large-scale form of Medieval and Renaissance poems depends on a whole system of numerology derived from the tenets of Pythagoras. These relationships, found in both small-scale and large-scale poems, point to the conclusion that at least some poetry was thought of proportionally and therefore with spatial (i.e. architectural) implications.

33.see P. Vitruvius, De Architectura, I, 1, 12: "Those, therefore, who from tender years receive instruction in the various forms of learning, recognize the same stamp on all the arts, and an intercourse between all studies, and so they more readily comprehend them all." (trans. M.H. Morgan, New York, 1914.)

34.See for example A. Fowler, Triumphal forms, Cambridge, 1970.

Powell,[35] for instance, has demonstrated that both the poetic and the musical structures of the fourteenth century Rondeau are based on the proportions of the golden section, which again implies a geometric derivation pointing to the Pythagoreo-platonic tradition. The above demonstrates clearly that interdisciplinary thinking was not only restricted to the arts of the Quadrivium and the Trivium, but that it had very real implications for the applied arts.

Analogies of this kind reveal something about the general pattern of thought during the Gothic period. It implies, for example, that symbolic thought was of primary importance. Indeed, if such a minute application of the Pythagoreo-platonic theory of numbers did exist, it is possible that the predilection for symbolism was so fundamental to the Medieval mind that their was no need to make it explicit. An example of this mode of thinking can be seen in scholasticism. In a disputatio the scholasticus expounded an argument by means of two propositions, which were called the premises. These two premises were supposed to contain an idea common to both, from which would necessarily result a third proposition called the conclusion.[36] This process, which is in essence syllogistic, was the necessary prerequisite for philosophical argumentation and represented a form or logic to which one strictly adhered. This way of thinking quite naturally lead to symbolism.

Huizinga[37] considers symbolism as a "sort of short-circuit of thought"[38] in which the relation between things is not

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[35] N.Powell,"Fibonacci and the Gold Mean:Rabbits,Rumbas,and Rondeaux." Journal of Music Theory,p.253-258.

[36] s.v. "Scholasticism",Encyclopedia Britanicca,Chicago 1973

[37] J.Huizinga,The Waning of the Middle Ages,Harmondsworth, repr.1979.

[38] J.Huizinga,p.159.

found in its causal connections, but in a comparison of its attributes and qualities. In this way, the seemingly unrelated can be brought together and make tangible the abstract idea that underlies them.Panofsky[39] shows that even the scholastic method of argumentation can be compared to the ordering principles used in architecture and music. He maintains that, unlike Aristotle and Plato,the scholastics felt "compelled to make the orderliness and logic of their thought palpably explicit."[40] In a scholastic treatise the reader is led step by step from one proposition to another.

> The whole is divided into partes which could be divided into smaller partes; the partes into membra, quaestiones or distinctiones, and these into articuli. Within the articuli,the discussion proceeds according to a dialectical scheme involving further subdivision, and almost every concept is split up into two or more meanings (intendi potest, dupliciter, tripliciter etc.)according to its varying relations to others.[41]

Panofsky argues that, because of the educational monopoly of scholasticism, this passion for clarification imparted itself to all people engaged in cultural pursuits. By becoming a mental habit, it must have influenced the artist as well.

He further states that the detailed articulation of subject matter in scholastic thought finds its parallel in music, articulated through an exact and systematic division of

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39.E.Panofsky,Gothic Architecture and Scholasticism,New York, 1957.

40.E.Panofsky,p.34.

41.E.Panofsky,p.33 - 34.

time, and in art and architecture, as revealed in an exact and systematic division of space. He sees this purely as the product of a common mental habit and seems to rule out the possibility of a more direct and concrete cross-fertilization of the arts under the influence of a commonly held philosophy. It is however unlikely that artists, musicians and architects who seem to share the influence of the same ideas and possibly the influence of the same cathedral schools, would not be aware of the similarities that exist in such concrete and tangible aspects of their arts as methods of proportioning.

It is at the same time difficult to prove any direct link between musicians, artists, architects and philosophers of the Gothic era. For that our factual knowledge is too limited. The influence of the Cathedral schools on early Gothic architects is a likely possibility, but it cannot be demonstrated with certainty. With composers of the fourteenth and fifteenth century our task becomes a little easier. Philippe de Vitry, for example, can be placed in a definite intellectual milieu[42], Dunstable's knowledge of astronomy is an established fact[43], and in the case of Dufay we have a specific example of the composer's collaboration with the architect Brunelleschi.[44]

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[42] see E.H.Sanders,"The early motets of Philippe de Vitry." Journal of the American Musicological Society (1975):24-45.

[43] His epitaphs refer to him as 'astrorum conscius ille' and 'Michalus alter novus et Ptolomeus' and three astronomical treatises survive which he copied or owned.See Trowell,"Proportion."p.140.

[44] C.Warren,"Brunelleschi's Dome...".

This kind of information, however, remains general and we are mostly left to conjecture from specific musical examples as to whether they reveal structures which could be interpreted as the result of a relationship between music, architecture and the Pythagoreo-platonic theory of numbers.

The question as to whether geometric and arithmetic methods of proportioning have any relevance to the actual music of the Gothic period has elicited some controversy among scholars. Van Crevel's introductory notes to his edition of Obrecht's Missa Maria Zart and Missa Sub tuum presidium must be the first attempt at explaining the proportions of music by means of number-symbolism.[45] In these introductions van Crevel alludes to a whole gamut of symbolic and numerical interpretations: he suggests hidden meanings derived from Cabbalistic and Gnostic Gematria, refers to neo-platonic interpretations, and, using the researches of von Simson and Wittkower as an example, points to the possibility of a geometric construction. At one point he goes even as far as to draw a parallel with Chinese alchemy. Van Crevel emphasises the highly speculative nature of his remarks, but is convinced that number-symbolism was not an exceptional feature of composition in the fifteenth century.

His speculations were mostly received with reserve. Trowell refers to his "intriguing but at times dotty introductions"[46] and emphasises that the history of symbolism which will ensure the validity and accuracy of certain interpretations still has to be written.

45.M.van Crevel,see the introductions in Jacobus Obrecht, Opera Omnia,I,fasc.6 & 7.

46.B.Trowell,"Proportion",p.100.

Lenaerts[47] dismisses the whole idea on the grounds that what van Crevel interprets as symbolically significant proportions could simply be the incidental result of isorhythmic technique. In his opinion the so-called proportions are brought about incidentally when talea are lengthened or shortened by means of mensuration signs. He also questions van Crevel's interpretation of the historical evidence the latter offers in support of his argument. [48]

While these interpretations are valid to a certain extent and raise some interesting points concerning the methods to be used in analysis, they cannot dispose of the possibility that works of the late Gothic period were filled with symbolic references. Most convincing in this respect are the researches of Trowell, Powell, Sandresky and Warren previously mentioned. These writers approach the problem from diverse points of view and in the process uncover aspects of the problem which should be viewed in conjunction.

Powell is concerned with the use of the Fibonacci series and the golden section in music of the fourteenth and fifteenth century. He aims at revealing certain explicit relationships among disciplines of the Quadrivium by analysing the mathematical structure of the Kyrie of Cornelius Heyns' Missa Pour quelque paine, Machaut's 6th motet, Dufay's motet Ecclesia Militantis and the general form of the Rondeau. These works reveal a specific irregularity in proportion and phrase-structure which, he claims,

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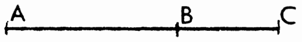
[47] R.Lenaerts,"Musical Structure and Performance Practice in Masses of Josquin and Obrecht." Josquin des Prez,ed.E.Lowinsky,Oxford University Press,1976,p.621 : "Van Crevel's alleged discovery of this symmetrical form tempts him to search for 'secret mathematical meanings'.He overlooks the fact that the presence of an isorhythmic cantus firmus rules out any result other than a mathematically equal number of breves,or an

can be explained by means of a Fibonacci series, which, in turn, can be linked to the geometric construction known as the golden section.[49]

In support of his argument he refers to Pisano's Il liber abbaci, Nicomachus' Introduction to arithmetic, Boethius' De Institutione arithmetica. If these three sources cannot be shown to influence musical composition there still remains the possibility that a principle is operating "that could be unconscious (or subconscious) on the part of the composer."[50]

To explain Powell's argument more fully, I will first demonstrate the means by which a golden section can be constructed geometrically, then explain its relation to the Fibonacci series and finally show how Powell explains its application to music.

When a line is divided into two sections in such a way that the smallest part is in relation to the larger as the larger is in relation to the whole, it is called a golden section (A is to B as B is to C : ).
A golden section can be effected by the following geometric procedure:

a. A line is divided into two equal halves (AB and BC).

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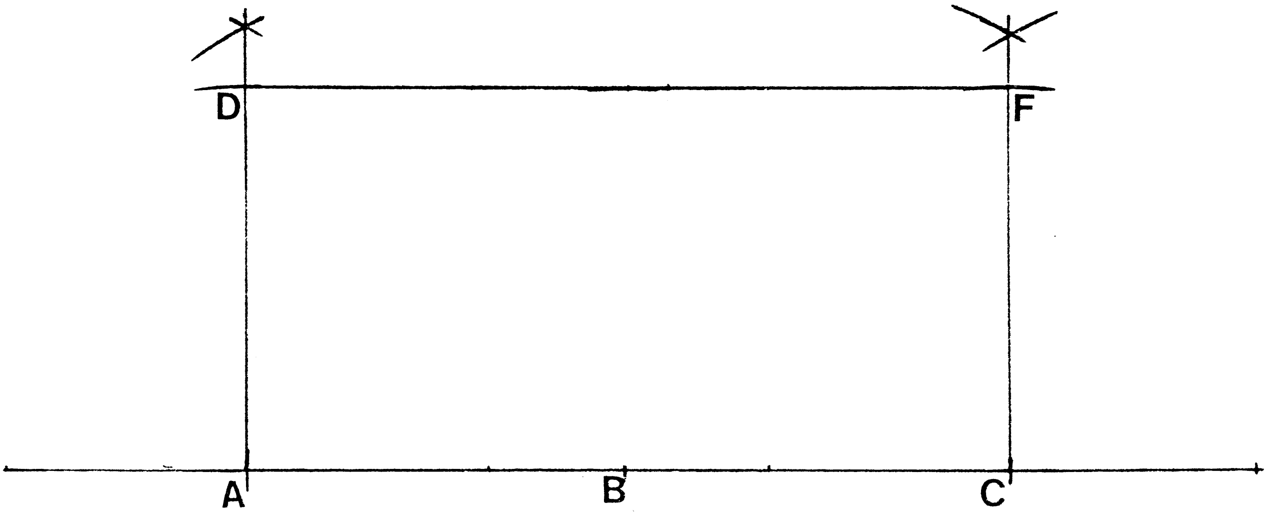
exact multiple or quotient of it."

48. Lenaerts, Symposium, "Problems in editing the music of Josquin des Prez." Josquin des Prez, p.733.

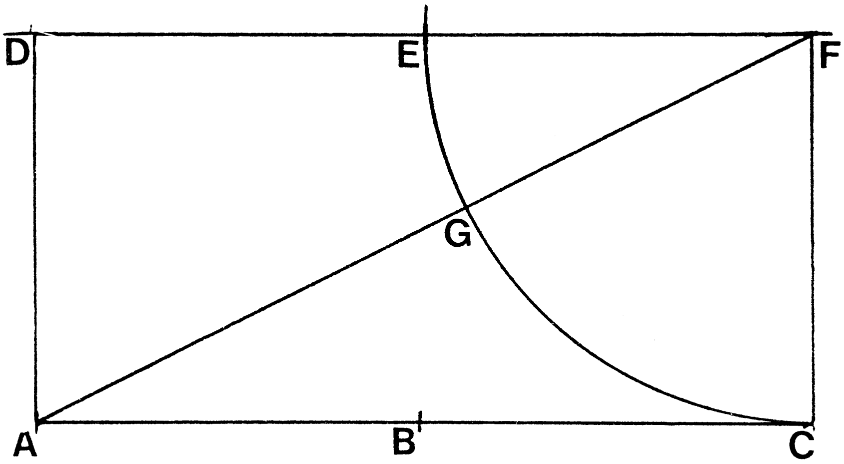
49. Powell, "Fibonacci."

50. N.Powell, "Fibonacci", p.239.

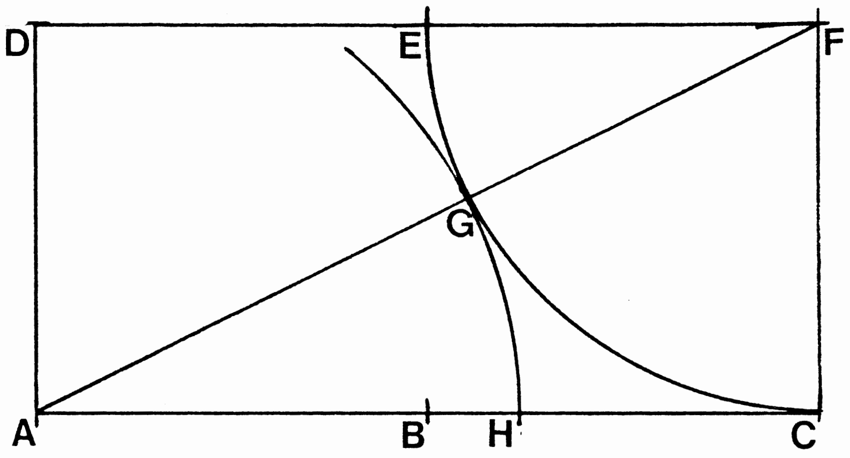
b. Perpendicular lines AD and CF are erected with AD = CF = AB = BC. Then D and F are joined and the result is a rectangle with AC : CF in the ratio of 2 : 1.



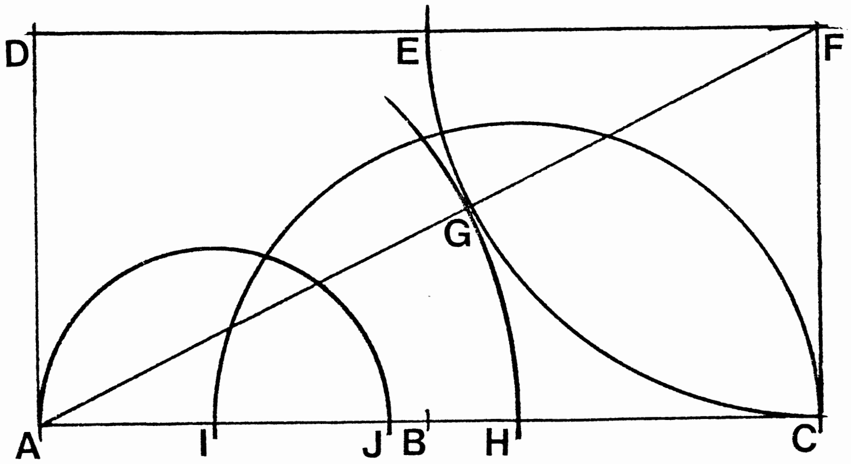
c. The hypotenuse AF is drawn, producing the right-angled triangle ACF. Then AF is divided so that FG = FE = FC.



d. The length AG is transferred to AC, producing AH. HC : AH is now in the same proportion as AH : AC.



e. If section CH is doubled to produce point I and section AI is doubled to produce point J, we have the following interlocking golden sequence: JH : IJ as IJ : IH, AI : IH as IH : AH and HC : AH as AH : AC.



Any ratio derived through further geometric subdivision or extension of line AC will be part of this golden sequence. The result is a series of identical ratios, known as the golden ratio, expressed by the Greek letter phi (ϕ) and equal to 1.61803398875... As this number is irrational, it is impossible to give accurate arithmetic expression to this golden ratio, but it can be approximated, for example in the ratio 2 : 3,which equals 1.666 or 5 : 8 , which equals 1.625.

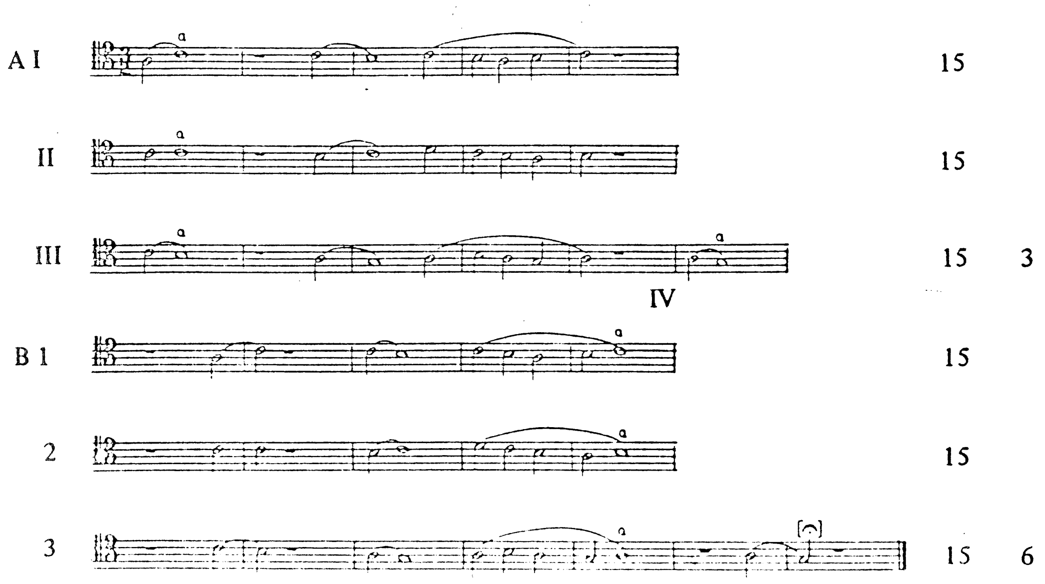
The same applies for a series of interlocking ratios. While the golden sequence, which has the distinction of being the only in which every ratio in the sequence is equal to every other, cannot be expressed arithmetically, there are other sequences approximate to it and which are governed by the same additive process. For the purpose of this study these series will be classified under the general term'additive series',although they may have distinctive names such as the Fibonacci series (1,2,3,5,8,13...), the Lucas series (1,3,4,7,12,19...) and the Evangelist series (2,5,7,12,19...).

Powell maintains that it is these sequences that can be found in music. A good example can be seen in his analysis of Machaut's motet no.6. In his opinion, the structure of this motet, as analysed by Ludwig and Schrade, appears lopsided and he suggests an alternative interpretation which can be correlated to the golden section.

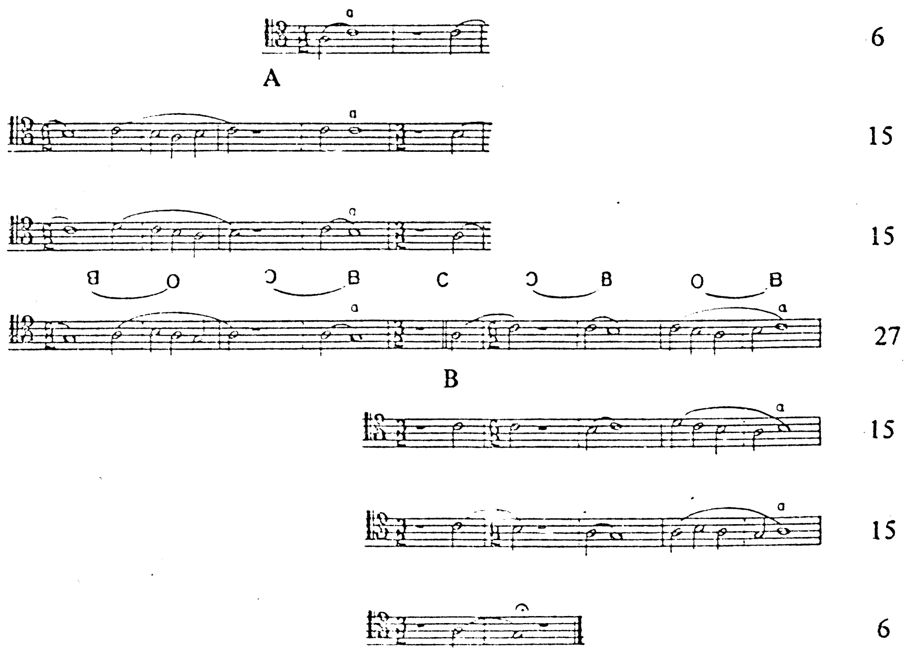
First he illustrates this point by comparing the structure of the tenor as given by Ludwig and Schrade with the one suggested to him by Gombosi:

Fig.3. Guillaume de Machaut, Motet no.6 (tenor)

a. After Ludwig and Schrade (cf. *HAM*, No. 44)



b. Gombosi-Powell Analysis



51.

*Notes marked with an *a* are "altered" breves in the original notation.

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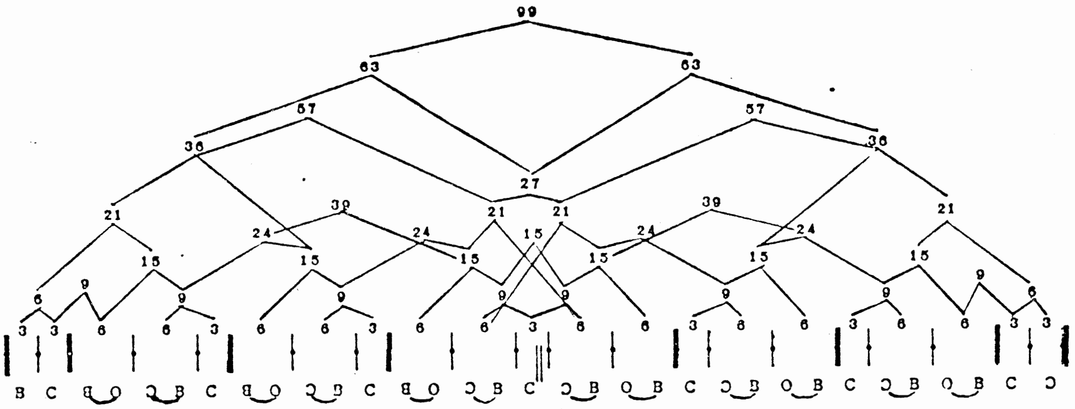
51. N.Powell, p.244 - 245.

Then he points out that the new arrangement produces a symmetrical structure:

6 : 15 : 15 : 27 : 15 : 15 : 6

Next he correlates this symmetrical structure to the additive series known as the Fibonacci, the Lucas and the Evangelist series, either in their simplest form or in multiples, as shown at the bottom of the figure:

Fig.4.Fibonacci Hierarchy in the tenor of Machaut,Motet no. 6. (after Powell)



n	1	2	3	4	5	6	7	8
$3F_n$	3	3	6	9	15	24	39	63
$3E_n$	6	15	21	36	57			
$9L_n$	9	27	36	63	99			

[52]

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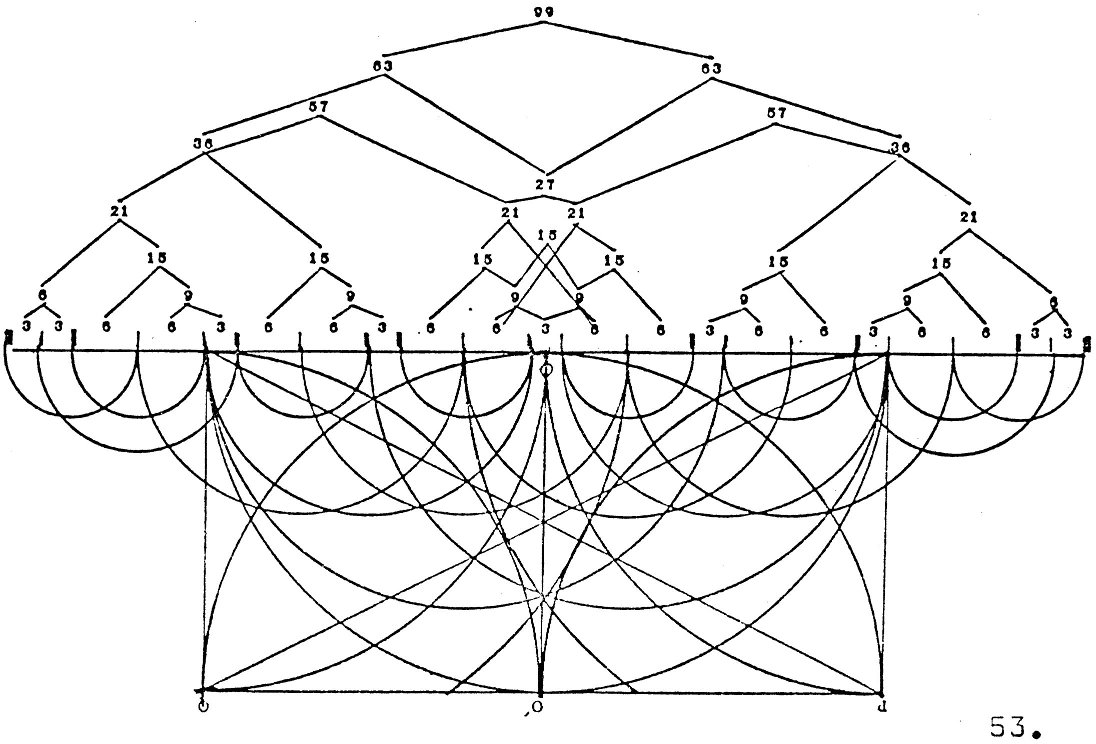
[52] Powell.p.247.

The tenor, with its additive series, is then shown to be related to the golden proportion and its geometric construction:

Fig.5. Machaut, Motet no.6: Fibonacci Hierarchy and Geometric construction compared. (after Powell)



53.

Powell's method and very convincing examples leave no doubt about the tangible correlation of music to disciplines of the Quadrivium. It begs the question whether geometric methods of proportioning were only applied to works with an asymmetrical structure, or whether there were other geometric procedures involved in cases that cannot be correlated to the golden sequence. To this the previous analyses of Organa by Léonin already

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53. Powell, p.251.

furnish a partial answer. These Organa definitely demonstrate a further application of geometric methods of proportioning and reveal that the idea of proportioning had taken root very early in the Gothic period. A prerequisite for further investigation of this idea, is that research must be conducted with this historical perspective in mind. One would then be able to trace the idea from its historical prototype to the more complex geometric expressions which would presumably emerge during the fourteenth and fifteenth centuries.

Powell's approach is not the only one that has been tried. In Sandresky's opinion, proportional analysis can be related closely to the musical material itself. To illustrate this, Sandresky has analysed three motets by Dufay, namely Vasilissa,ergo gaude, Balsamus et munda and Lamentatio Sanctae Matris Constantinopolitanae.[54] In her opinion,the form of Vasilissa is governed by the talea of the isorhythmic tenor and by the length of the prelude. The prelude consists of 8 longs, Talea 1 is 13 longs and Talea 2 is 13 longs. The result is a golden proportion, belonging to the additive series known as the Fibonacci series. But Sandresky goes a step further and divides talea 2 into two sections with the ratio 5 : 8. This further division is done on the grounds that there are three C's at longa 5 of the second talea. This subdivision is then used as a point of departure by which she elicits a further proportion. A symmetrical relationship results between the two outer parts 8 : 13 : 5 : 8 and by pitting the sum of the outer numbers against the sum of the two inner numbers the Pythagorean ratio 16 : 18 is derived. This ratio 16 : 18 = 8 : 9 equals a tone and Sandresky maintains that this is significant,

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54. Sandresky,"The Golden Section".

since the first two intervals of the vocal prelude consist of a tone plus a tone. In this way she attempts to correlate the melodic material to the numerical proportions of the tenor. Whether such a correlation is justifiable will depend on further analytical evidence which, unfortunately, is not yet forthcoming.

In her analysis of Balsamus, Sandresky takes the step of separating the upper and lower voices. By looking for significant landmarks such as imitation and coloration, she discovers two fragments in golden proportion. She determines the three divisions of the golden section by pointing to the cadential feeling with imitation in the upper voices at breves 13 and 21, and to the end of coloration, which occurs at breve 34. However, there are two main inconsistencies in this argument. First of all, there is a lack of logic in using different criteria for marking off the golden section,i.e. imitation with cadence for two points and the end of coloration for the third (never mind all the other points of coloration). Secondly, there are other points at which a cadence with imitation occur (e.g. at breves 4 and 43) which are not mentioned in her analysis. This inconsistency casts doubt on her interpretation. It appears as if she were looking for some golden proportion and would have settled for any landmark in the piece to prove a theory.

In her view, coloration not only contributes to the division of the upper voices into a golden proportion, but also produces the Pythagorean ratio 8 : 16 : 24 : 32 (1 : 2 : 3 : 4). Sandresky maintains that this coloration coincides with triad leaps in the upper voices, as shown in this table:

Table 1. Upper voice pattern in Balsamus et munda

(after Sandresky)

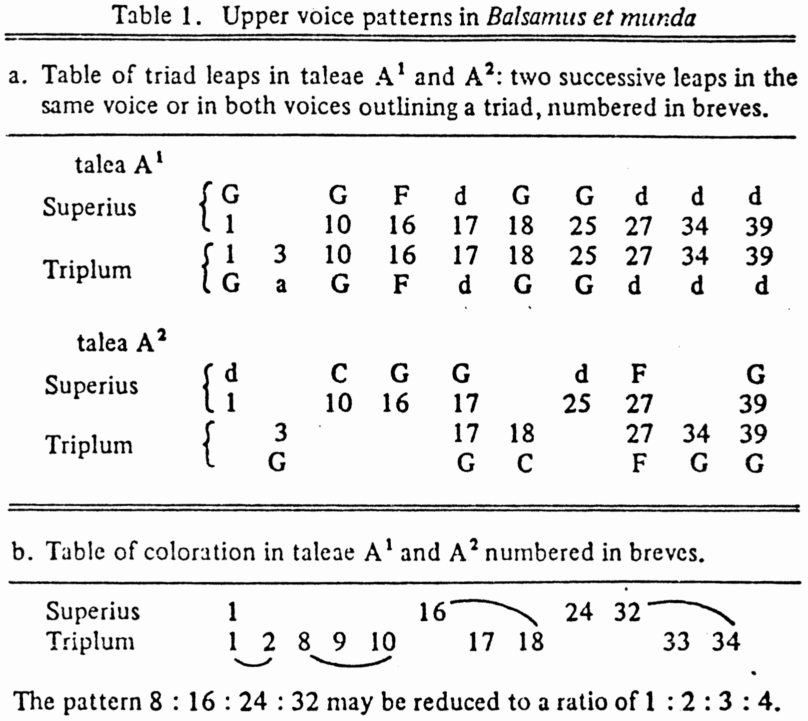

Table 1. Upper voice patterns in *Balsamus et munda*

a. Table of triad leaps in taleae A^1 and A^2: two successive leaps in the same voice or in both voices outlining a triad, numbered in breves.

talea A^1										
Superius	G		G	F	d	G	G	d	d	d
	1		10	16	17	18	25	27	34	39
Triplum	1	3	10	16	17	18	25	27	34	39
	G	a	G	F	d	G	G	d	d	d
talea A^2										
Superius	d		C	G	G		d	F		G
	1		10	16	17		25	27		39
Triplum		3			17	18		27	34	39
		G			G	C		F	G	G

b. Table of coloration in taleae A^1 and A^2 numbered in breves.

Superius	1					16			24	32		
Triplum	1	2	8	9	10		17	18			33	34

The pattern 8 : 16 : 24 : 32 may be reduced to a ratio of 1 : 2 : 3 : 4.

55.

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55. Sandresky,p.302.

The coincidences she finds between the use of triads and coloration are not convincing. For example, there is a triad at breve 10, i.e. at the end point of coloration, but the Pythagorean ratio pattern starts at breve 8, which has nothing to do with the triad at breve 10. The same applies for the coloration at breve 24, which cannot be correlated to the triad leap, which only occurs at breve 25. Neither does the coloration at breve 32 really coincide with the triad leap at breve 34. There are also a number of triads which stand on their own, without coloration. If there is really an intentional use of coloration to express Pythagorean ratios, these are not dependent on thematic material. In this instance the Pythagorean ratios coincide purely with the beginnings of certain colorations.

Sandresky's analysis of the tenor of *Balsamus* is accurate, but incomplete. She points out that the piece continuously produces sections in the ratio of 2 : 1.

	Talea A^1	:	Talea A^2	:	Talea B^1 & B^2
in semi-breves	78	:	78	:	26 : 13 : 26 : 13
					2 : 1 : 2 : 1
		2		:	1

[56]

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[56] see Sandresky, p. 299.

In doing this she ignores the additional 3 semibreves introduced by Dufay at the end of the piece to complete the final cadence. These destroy the carefully constructed symmetries in the piece. For this Sandresky offers no explanation, although in her analysis of the *Lamentatio* it is precisely this addition that becomes the crucial key to her analysis. This inconsistency makes one wonder whether there is not an alternative answer which would explain the details of form more fully.

To this end, if one measures the tenor line of *Balsamus* in breves it yields the following proportions:

Talea A^1	:	Talea A^2	:	Talea B^1 & B^2	:	Cadence
26	:	26	:	26	:	1

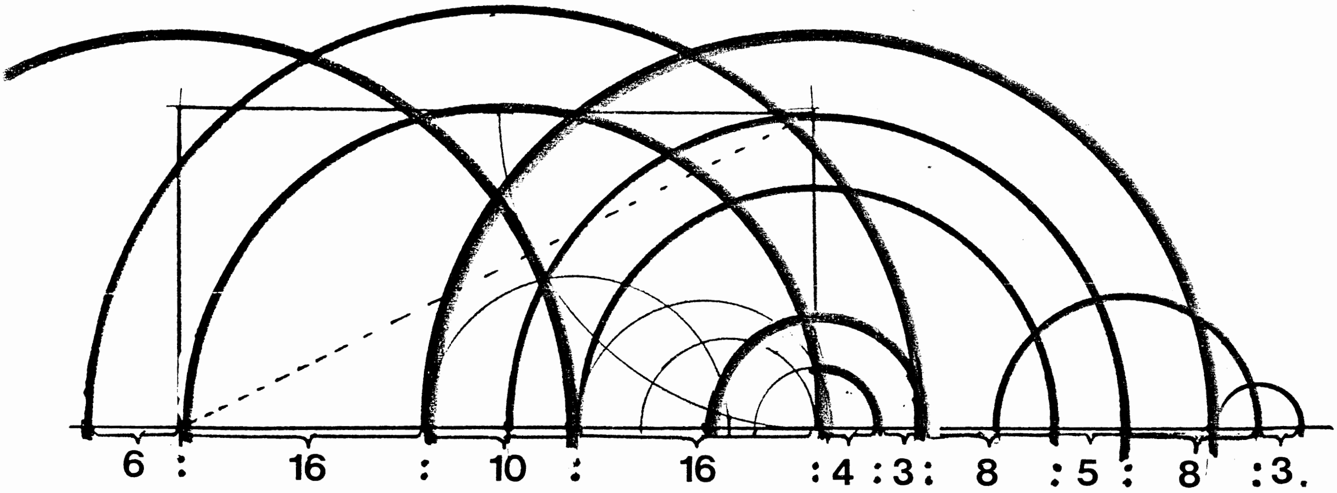
A further subdivision can be obtained by measuring each phrase within the tenor, that is, up to every perfect longa rest:

Talea A^1	:	Talea A^2	:	Talea B^1 & B^2	:	Cadence
6:16:4	:	6:16:4	:	3:8:2:3:8:2	:	1
6:16:	10	:16:4	:	3:8: 5 :8:2	:	1

The result is a series of proportions belonging to the Fibonacci series.: Talea A^1 and A^2 consist of numbers belonging to the Fibonacci series x2 (4,6,10,16,26) and Talea B^1 and B^2 consist of numbers belonging to the basic Fibonacci series (2,3,5,8,13,26).

Out of the tenor one can construct two golden proportions, flanked by numbers belonging to the Fibonacci series and which can be derived geometrically from the construction of the golden section. The following illustrates this by combining the proportional analysis of the tenor with the corresponding geometric construction of the golden section:

Fig.6.Geometric construction of the tenor of Balsamus et munda



In spite of the inconsistencies of Sandresky's analysis, it presents many possibilities and conjectures which need further consideration. Firstly, by considering the upper voices for analysis she has opened up possibilities of investigation not touched upon by Powell, who only looks at tenors. Secondly, by looking at the relation between melodic material and proportion, she unwittingly poses the question of what proportioning methods can reveal about a piece's melodic and tonal structure.[57] Thirdly, Sandresky attempts a comparison of methods used by Dufay at different stages of his life. She suggests that the intricate relationship of material and the overriding importance of proportions derived from the golden section in Vasilissa is more characteristic of the Gothic period, whereas the simpler proportions of Balsamus belong to the early Renaissance aesthetic emerging in Italy and particularly in Florence. The evidence that the golden section does not only appear in the upper voices as a structure superimposed on the simple proportions of the tenor (as is suggested by Sandresky), but actually determines the tenor, is a partial contradiction of her interpretation. Her statement is also marred by the unwarranted assumption that there are no simple proportions in earlier music. For example, the previous analysis shows that the music of Léonin is full of commensurable ratios. While one can safely surmise a different attitude to proportioning in the Renaissance, more examples will be needed before one can state with some certainty in which way this differs from the methods applied during the Gothic period.

Trowell's article on proportion in the music of Dunstable attempts to correlate the musical and mensural structure to the simple proportions of the neo-platonic tradition.[58] In his opinion the music of Dunstable is built on mathematical and proportional structures that the ear can discern and delight in. In its simplest form this can be

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57. An aspect which is discussed in chapter 3.

58. B. Trowell, "Proportion...", PRMA, 105(1979):100-141.

seen in his isorhythmic motets, which either reveal the proportion 9 : 6 : 3 (i.e. the arithmetic proportion) or 6 : 4 : 3 (i.e. the harmonic proportion). Trowell finds four exceptions out of thirteen to this pattern, namely Specialis Virgo, which is built on the proportion 4 : 4 : 3 : 3 : 3, Veni Sancte Spiritus, with the inexplicable proportion 93 : 93 : 58, and two textless motets[59], with the ratios 1 : 1 : 1 : 1 and 12 : 9 : 8 : 6 respectively.

In his analysis of free-style motets, which are not as clearly subdivided, Trowell has adopted the procedure of "throwing in any avenue of approach that yields an apparently significant result."[60] Sometimes the proportions are determined by a change of metre or texture, sometimes by the division inherent in the original plainsong used, and at other times by the actual phrasing of the tenor or discant. In this way Trowell has been able to give a reasonably simple interpretation for all the proportions found in the motets.

Trowell is also concerned with the symbolic meaning of the numbers and proportions found in the music. These symbolisms range from mathematically significant numbers (e.g. a piece that yields a triangular number, or the first 5 terms of the greater tetraktys) to Gnostic Gematria (e.g. the section of a piece may yield a total of 444 semibreves, which equals 888 minims - a Gnostic symbol for Christ and the sum of the letters 'Iesous' in Greek Gematria).

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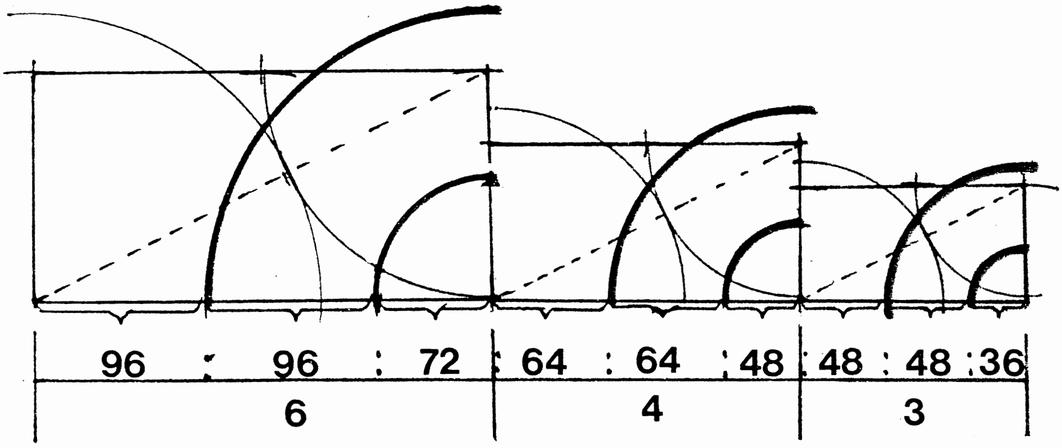
[59] Nos. 34 & 66 in Musica Britannica, ed. Bukofzer, 1953.

[60] B.Trowell, p.106.

As will be seen in chapter 2, Trowell's analyses can be substantiated and amplified when one considers the proportional interrelationships within individual voices as well. Such an analysis, which is based on the principle of measuring phraselengths, rather than sections, reveals subtle structural connections, which can be fitted into the framework of simple proportions discovered by Trowell. Some of the music can also be analysed on the lines adopted by Powell in his analyses of music by Dufay and Machaut. For example, the realised version of the tenor of the isorhythmic motet Albanus roseo rutilat may, as Trowell points out, produce the harmonic proportion 6 : 4 : 3, but a more detailed analysis of the tenor line also reveals a geometric construction,based on the golden section, inside this simple structure:

Fig.7.Geometric construction of the tenor of Albanus roseo rutilat



The divergent methods applied by Trowell and Powell raise the question whether composers thought mainly in mathematical terms or in geometric terms, or, as Sandresky implies, initially in geometric terms and, with the beginnings of the Renaissance, in mathematical terms.

The question of geometric construction or mathematical progression brings us back to the crux of this discussion, namely the link that may exist between music and architecture during the period. Warren's article on Dufay's motet <u>Nuper rosarum flores</u> affords us with a specific example of a collaboration between an architect and a musician.[61] Warren shows that this particular motet was the result of a deliberate attempt on the part of Dufay to create a sounding model of Brunelleschi's architecture for the <u>Duomo</u> in Florence.

The proportions of the cross of the thirteenth century cathedral to which Brunelleschi's dome was added,were derived by means of quadrature. This incommensurable quadrature series was reduced by Brunelleschi to a set of ratios that accord with the Pythagorean musical proportions, by rounding off the series to the nearest whole numbers, thus producing the proportion 6 : 4 : 2 : 3. It is precisely this proportion that Dufay used in the overall planning of his motet written for the consecration of this cathedral. The correspondence is not only restricted to the overall proportion, but can also be seen in the fact that the measurements involved in the construction of the cathedral, when calculated in braccia, correspond

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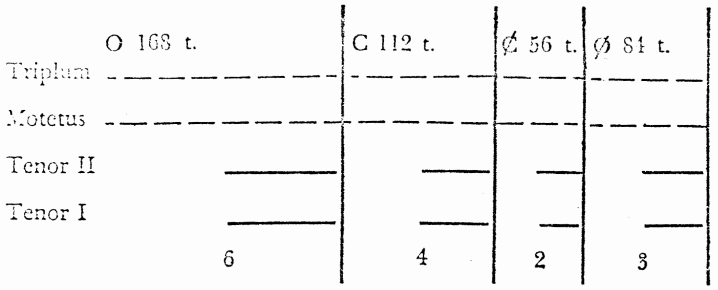
[61] C.Warren, "Brunelleschi's Dome...", <u>MQ</u>, 59 (1973):92-105.

to the total number of breves within each section of the motet.

> Not only do the mensurations in its four sections have a proportional relationship of 6:4:2:3, but the number of tactus (breves) in each of the sections is the same as the number of braccia contained in the modular scheme based on twenty-eight braccia squares: one hundred sixty-eight in the nave, one hundred twelve in the transept, fifty-six in the apse, and eighty-four in the dome of Brunelleschi. What is more, there are twenty-eight breves in each of the two-voice and four-voice subsections of the motet, which means that the "module" of the motet is the same as that of the cathedral:



[62]

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[62] C.Warren, p.96.

Judging from the detailed correspondences between _Nuper rosarum flores_ and the cathedral, one must either attribute a remarkable flight of fancy on the part of Dufay, or one may infer the possibility of a traditional affiliation between architecture and music, which Dufay handled, in this instance, quite literally. With this statement I am not suggesting that one will necessarily find other examples of architectural allegory in music, but it adds to the conjecture that the aesthetic ideals underlying both music and architecture of the period could have the same origin and would therefore evoke parallel traditions and constructing principles.

Chapter II

An analysis of motets by Dunstable and Dufay

This chapter serves as a guideline to the diagrams contained in volume 2 of this dissertation. Their purpose is to demonstrate the proportions, symmetries and other temporal relationships in the structure of the motets of Dunstable and Dufay. They are preceded by selected examples from the thirteenth and fourteenth centuries, in order to show the continuity of the tradition (the approximate date is indicated in the table on p.iii of vol.2).

Unless otherwise stated, the numbers in the diagrams indicate the number of semibreves within each phrase. At times, phrases have been coupled, and the numbers may therefore show the number of semibreves in two adjacent phrases. This has been done in cases where individual phrases did not appear to have a numerical significance within the structure of the whole, or were too short to be considered as distinctive, separate units.

All voices have been treated individually, except where the division of tenor and contratenor seem to correspond. The bottom line indicates the proportional relationships between sections. Brackets are used to show when phrases appear to form a unit ⏟ .Lines \ / indicate numbers related by means of an additive series. Arcs ⌒ show the existence of symmetries and square brackets └→ ←┘ show where the addition of flanking phrases produce a numerically significant result. It will be shown that the addition of flanking proportions reveal numerical relationships which have hitherto gone undetected .

Section A.

a. Two anonymous motets.

Excelsus in numine/Benedictus Dominus/Tenor.[1]

This late thirteenth century double motet, as analysed by Marrocco and Sandon, illustrates the technique of voice exchange (stimmtausch):

> Except at the beginning and end and in the middle of the piece, the triplum and motetus exchange their melodies at regular four-bar intervals, producing the structure:

Triplum		A	B	C	D		E	F	G	H	
Motetus	Introduction	B	A	D	C	middle	F	E	H	G	ending
(Bars)	7	4	4	4	4	3	4	4	4	4	4

[2]

However, if one disregards this musical structure and measures phraselengths of individual voices the following proportional structure results:

Tenor.

The tenor can be divided into the proportion 28 : 32 : 32 (7:8:8) which expresses the golden section based on the additive series 7,8,15, 23. The following is a geometric construction of this tenor:

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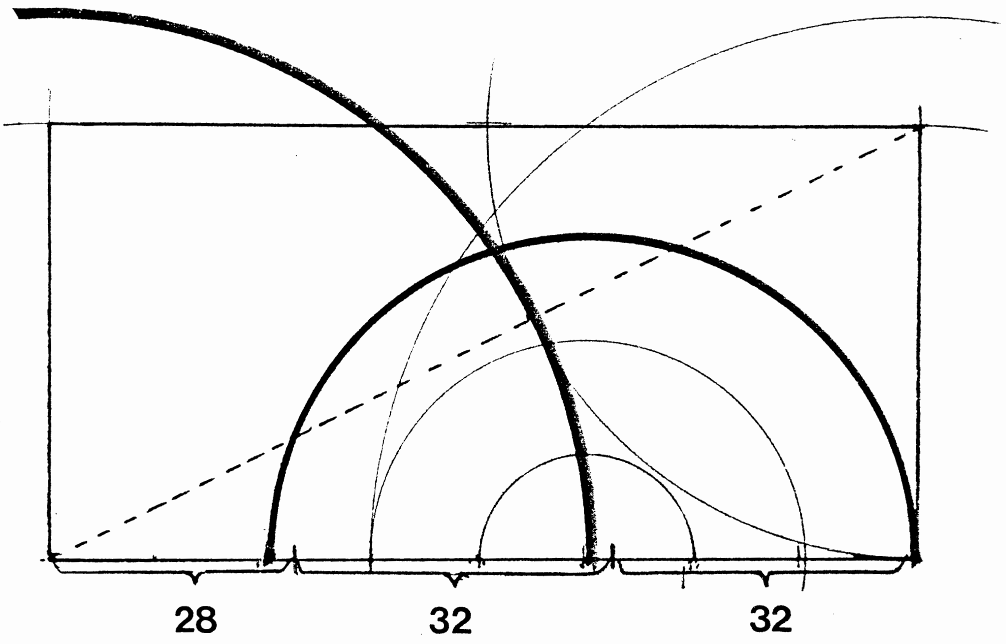
[1] Published in The Oxford Anthology of Music, Medieval Music, ed.W.Marrocco and N.Sandon, Oxford University Press, 1977.p.105.

[2] Oxford Anthology, p.106.

Fig.8.Geometric construction of the tenor of Excelcus in numine



Motetus.

Corroboration of the analysis of the tenor comes when it is found that,although the motetus consists of different phraselengths, it also reveals the proportion 32 : 60. The latter part (60) can be subdivided into the proportion 24 : 12 : 24 (2 : 1 : 2),which is a radial symmetry.

Triplum.

The triplum reveals the radial symmetry 40 : 12 : 40.

O livor / Inter amenitatis / Revertenti.[3]

Tenor.

The isorhythmic tenor consists of eight equal phrases of 12 breves.

Motetus.

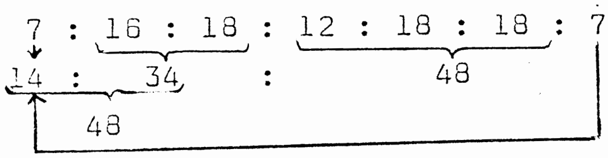
The motetus displays the proportion 9 : 18 : 24 : 15 : 30. This can be expressed in algebraic terms as a : 2a : 2c : b : 2b , where a = 9 ,b = 15 and c relates to the 12 breves continuously found in the tenor.
If this interpretation is accepted, it would mean that all the proportions in this voice can be related to quadrature.

Triplum.

The proportion of the triplum is :

7 : 16 : 18 : 12 : 18 : 18 : 7

Here the 12 : 18 : 18 (2 : 3 : 3) of the second part expresses a golden proportion based on the Fibonacci series (1,2,3,5,8),while the whole structure displays the additive series 14 . 34 , 48 , 82. The latter series can be seen when one adds the flanking proportions (7 and 7)to each other.



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3 . Anonymous,"O livor"Denkmäler der Tonkunst in Österreich, Jahrg.XL-Band 76,Trienter Codices VI,ed.R.von Ficker,Graz 1960.

Section A.

b. Three motets by Philippe de Vitry.

Floret/ Florens/ Neuma.[4]

Tenor.

The isorhythmic tenor can be subdivided into four equal sections revealing the proportion 36 : 72 (1 : 2).

Motetus.

The motetus does not adhere to the tenor's strict division and its overall proportion can be expressed as 126 : 63 : 90 : 180. In algebraic terms this can be expressed as 2a : a : b : 2b. The last 180 semibreves result from adding the proportions 63 : 63 : 54 (7 : 7 : 6) which expresses a golden section based on the additive series 6,7,13,20,33.

Triplum.

Here again there is no strict adherence to the tenor's subdivisions and the overall proportion 171 : 117 : 171 (19 : 13 : 19) expresses a golden section based on the additive series 6,13,19,32,51 and a radial symmetry. In both cases the 171 semibreves are the product of adding a phrase of 90 semibreves to one of 81 semibreves (10 + 9). In the case of the second 171, the 81 and 90 are produced by adding flanking phrases.

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[4] .Philippe de Vitry,"Floret/Florens/Neuma" edited by E.Sanders,"The early Motets of Philippe de Vitry." Journal of the American Musicological Society,1975,p.37-42.

Hugo princeps.[5]

Tenor.

The isorhythmic tenor consists of nine equal phrases of 5 breves.

Motetus.

Provided that one adds the flanking phrases (6 and 4)to each other, the motetus reveals the proportion

$$\begin{tikzcd}[column sep=small] 6 \arrow[d] & : 10 : 10 : 10 : 5 : & 4 \arrow[dll] \\ 10 & & \end{tikzcd}$$

which in algebraic terms can be expressed as

$$2a : 2a : 2a : 2a : a$$

and can possibly be related to quadrature.

Triplum.

The triplum reveals the radial symmetry

$$30 : 74 : 30 \ (5 : 14 : 5)$$

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[5] Philippe de Vitry,"Hugo princeps",DTÖ,Jahrg.XL - Band 76, p.4.

Vos qui admiramini. [6]

Tenor.

The isorhythmic tenor divides into equal segments of 15 for the first section and 9 for the second section. The addition of the final cadence (6), can be added to the previous 9 to produce a 15 bar phrase.

Motetus.

The motetus consists of two golden sections:

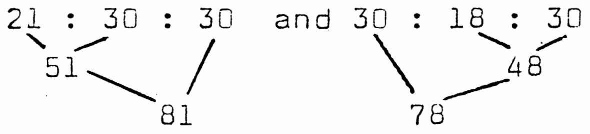
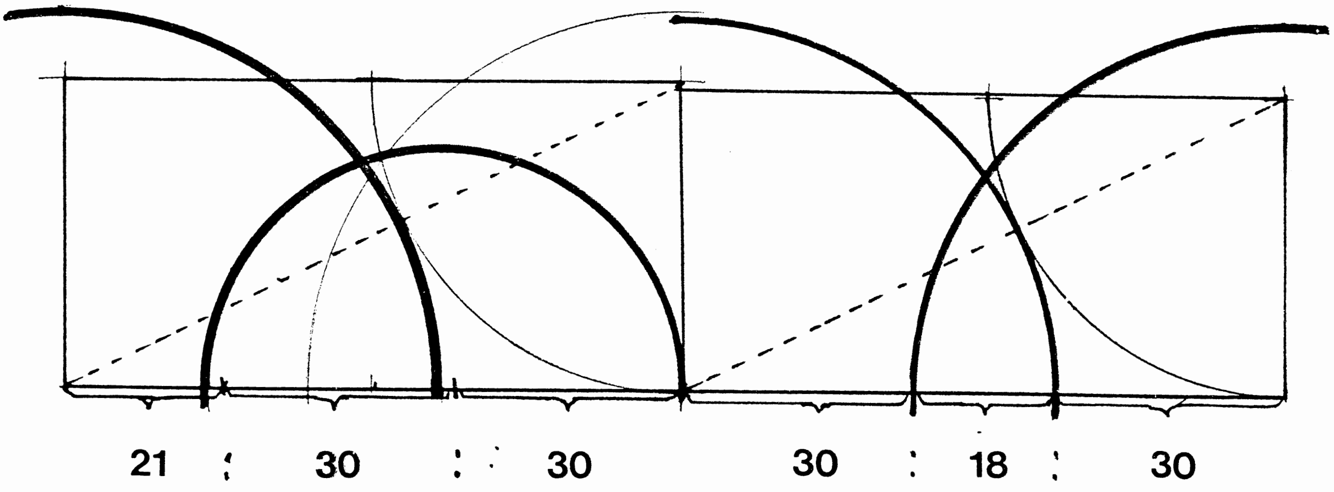


Fig. 9 .Geometric construction of the motetus of Vos qui admiramini.



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[6] . Philippe de Vitry,"Vos qui admiramini.",Oxford Anthology, p.120 - 126.

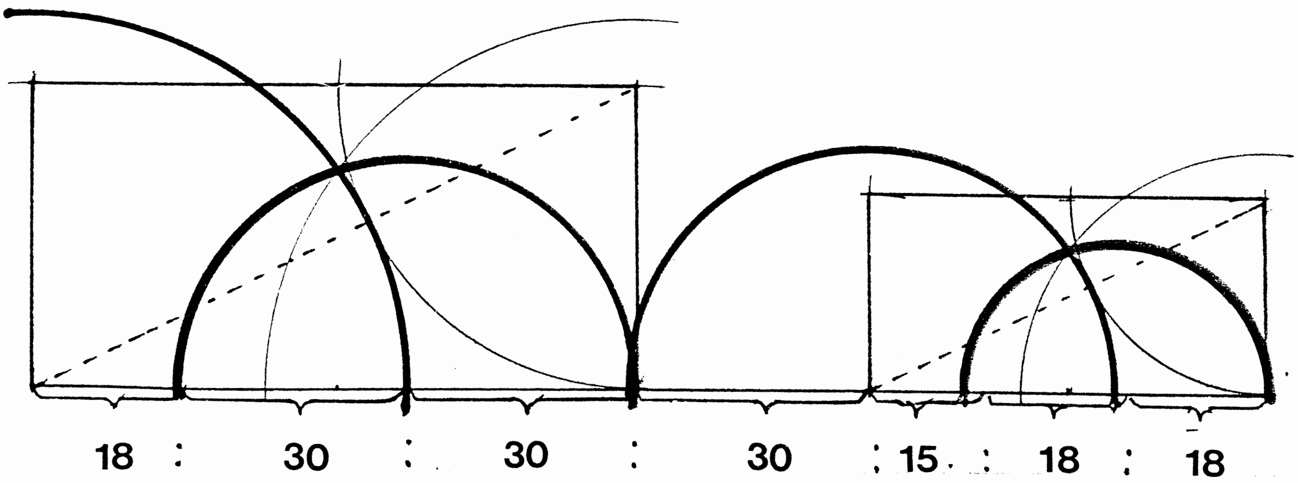
Triplum.

The golden section of the motetus also appears in the triplum, but here the two proportions are inverted:

18 : 30 : 30 and 30 : 51
48
78

In the latter half, the 51 is obtained by means of another golden section, namely 15 : 18 : 18.
33
51

Fig.10.Geometric construction of the triplum of Vos qui admiramini.



Section B.

The Isorhythmic Motets of Dunstable[7]

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[7] John Dunstable: Complete Works (Musica Britannica, viii), ed. M.F. Bukofzer (London, 1953).

Albanus roseo rutilat.[8]

Tenor.

The isorhythmic tenor reveals the proportion :

Section A. 96 : 96 : 72 (4 : 4 : 3)
Section B. 64 : 64 : 48 (4 : 4 : 3)
Section C. 48 : 48 : 36 (4 : 4 : 3)

This is a golden section belonging to the additive series known as the Lucas series (1,3,4,7,11). The overall proportion 264 : 176 : 132,in its simplest form,displays the harmonic proportion 6 : 4 : 3.

Motetus.

The motetus has the same proportion as the tenor.In section A the last phrase of 15 semibreves must be added to the first two,to produce the proportion 72 : 96 : 96 (3 : 4 : 4).In section B the last phrase of 20 semibreves must be added to the first one to produce the proportion 48 : 64 : 64 (3 : 4 : 4).Section C displays the proportion 36 : 48 : 48(3 : 4 : 4).

Triplum.

Section A reveals a radial symmetry in its proportion 66 : 36 : 60 : 36 : 66. It can also be related to the golden section expressed in the additive series 7,10, 17,27,44 (60,102,162,264).

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[8] Mus.Brit.,no.23,p.58. Also see p.45 of this dissertation.

Ave Regina celorum, ave decus. [9]

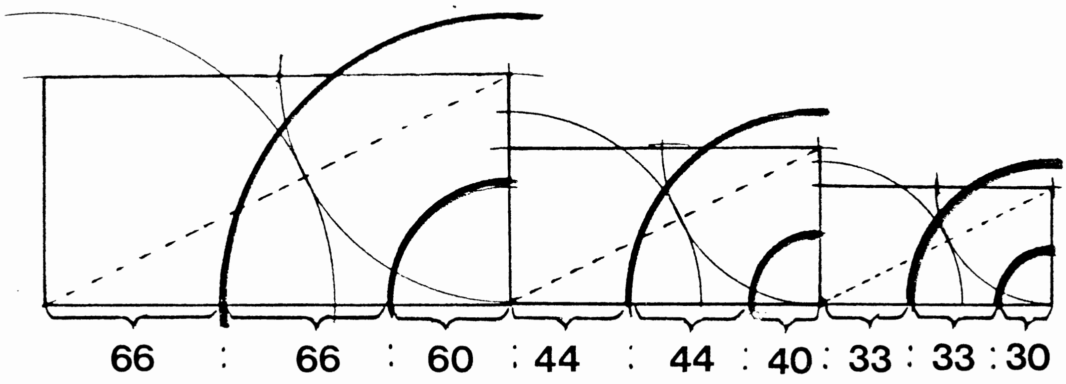
Tenor.

The isorhythmic tenor reveals the proportion:

Section A. 66 : 66 : 60 (11 : 11 : 10)
Section B. 44 : 44 : 40 (11 : 11 : 10)
Section C. 33 : 33 : 30 (11 : 11 : 10)

This is a golden proportion belonging to the additive series 10, 11, 21, 32, 53.

Fig.11.Geometric construction of the tenor of Ave Regina celorum.



The overall proportion 192 : 128 : 96 displays the harmonic proportion 6 : 4 : 3.

Motetus.

Provided one adopts the procedure of adding the two flanking phrases of section A, a radial symmetry results (57 : 39 : 57), flanked by phrases which together equal the central one.

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[9] Mus.Brit.,no.24,p.62.

If we follow the same principle in section B, this results in the proportion 8 : 56 : 8, with flanking phrases equal to the central one (56).

In section C the adding of flanking phrases produces two equal sections (48 : 48).

Triplum.

Pursuing the same principle of combining flanking phrases in each section, as was followed in the motetus the following radial symmetries result:

Section A. 45 : 51 : 45 (+51)=15 : 17 : 15 (+17)
Section B. 34 : 30 : 34 (+30)=17 : 15 : 17 (+15)
Section C. 33 : 15 : 33 (+33)=11 : 5 : 11 (+11)

Christe sanctorum.[10]

Tenor.

The isorhythmic tenor reveals the following equal phrases:

Section A. 54 : 54 : 54 : 54
Section B. 36 : 36 : 36 : 36
Section C. 18 : 18 : 18 : 18

The overall proportion 216 : 144 : 72 displays the arithmetic proportion 9 : 6 : 3.

Motetus.

Phraselengths of the motetus can be reduced to the following proportions:

→63←
Section A. 33 : 45 : 63 : 45 : 30
Section B. 72 : 72
Section C. 21 : 36 : 15
→36←

Section A displays a radial symmetry, while its flanking phrases produce a phrase equal in length to the central one.

Section B consists of two sections of equal length, and, provided one adds the two flanking phrases of section C to each other, two phrases of 36 semibreves result.

Triplum.

Pursuing the principle of adding flanking proportions in each section, the following proportions result:

Section A. 57 : 51 : 57 (+51)
Section B. 38 : 34 : 38 (+34)
Section C. 36 : 36

---

[10] Mus. Brit., no. 25, p 64.

Dies dignus decorari.[11]

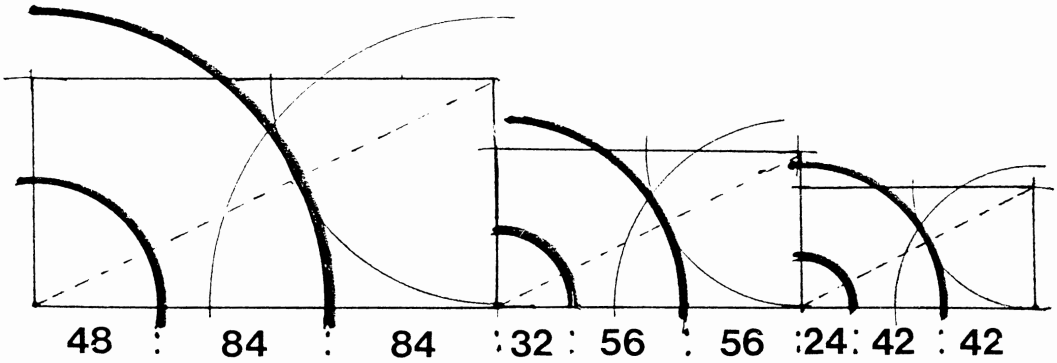
Tenor.

Counted in semibreves, the tenor reveals the following proportion:

Section A. 48 : 84 : 84
Section B. 32 : 56 : 56
Section C. 24 : 42 : 42

In its simplest form this can, in each case, be reduced to the proportion 4 : 7 : 7. This division of the tenor reveals a golden proportion belonging to the additive series 1,3,4,7,11 (i.e. the Lucas series).
The geometric construction of the golden proportion in these three sections is shown in fig. 12:

Fig.12.Geometric construction of the tenor of Dies dignus decorari.



When the semibreves in sections A, B and C are added, they produce the proportion 216 : 144 : 108, which, in its simplest form, produces the harmonic proportion 6 : 4 : 3.

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[11] Mus Brit.,no.26.p.67.

Motetus.

Phraselengths of the motetus can be reduced to the following proportions:

Section A. 30 : 67 : 33 : 67 : 3
Section B. 36 : 24 : 40 : 24 : 4
Section C. 15 : 24 : 30 : 24 : 15

All three sections reveal a radial symmetry. In sections A and B the remaining flanking phrases (30 and 3 in section A and 36 and 4 in section B) produce a phrase equal to the central one when added together. No consistent simple proportion reveals itself and it is therefore unlikely that Dunstable intended any (40 : 24 = 5 : 3 and 24 : 30 = 4 : 5, but 67 : 33 is irreducible).

Triplum.

Pursuing the same principle of combining flanking phrases in each section, as was followed in the motetus, all phrases can be added to produce equal numerical proportions of 36 semibreves each. Section A has six, section B has four and section C has three phrases of 36 semibreves, which seems to solidify the concept of the harmonic proportion as revealed in the isorhythmic tenor.

Gaude felix Anna.[12]

Tenor.

The isorhythmic tenor, which is stated twice in each section, can be divided in the following proportion:

Section A. 99 : 63 : 99 : 63
Section B. 66 : 42 : 66 : 42
Section C. 33 : 21 : 33 : 21

This division can be related to the ratio 11 : 7 from the Lucas series. When the semibreves in sections A, B and C are added, they produce the proportion 324 : 216 : 108, which in its simplest form is the arithmetic proportion 9 : 6 : 3.

Motetus.

Provided one adopts the procedure of adding the two flanking phrases of section A (c.f. Dies dignus decorari) the following proportion results:

$$45 : \underbrace{66 : 30}_{96} : 66 : \underbrace{66 : 30}_{96} : 21$$

$$\xrightarrow{\hspace{6em}} 66 \xleftarrow{\hspace{6em}}$$

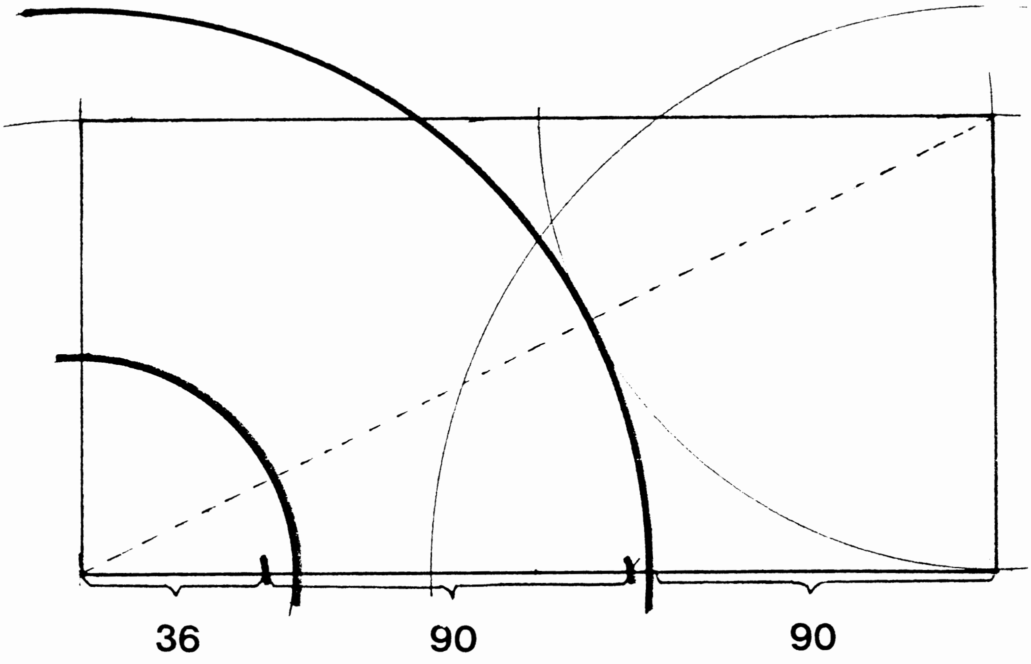
This proportion at the same time reveals a radial symmetry 96 : 66 : 96 (flanked by phrases which correspond to the central one) and an additive series 30, 66, 96, 162, 258. In this case the proportion cannot be related to simple additive series such as the Evangelist or the Lucas sequence. The lower numbers of the sequence, such as 30 : 66 (i.e. 2,2) only very roughly approximate the actual golden ratio (1,618), but the higher numbers produce a more accurate approximation (e.g. 66 : 96 = 1,5).

---

[12] Mus.Brit.,no.27,p.70.

Section B produces the proportion 36 : 90 : 90, which can also be related to an additive series (36, 90, 126, 216) approximating a golden section. Here the same principle applies as the one in section A, with the higher numbers being more accurate in their approximation of the golden ratio.

Fig.13 .Geometric construction of section B of Gaude felix Anna.



Section C yields the proportion 12 : 36 : 18 : 36 : 6. Again a radial symmetry is involved, with the flanking phrases adding up to a phrase that equals the central one. In this case the proportion can be reduced to 2 : 1 : 2 (+1). If one adds the central phrase to the two flanking phrases three equal phrases of 36 semibreves result.

Triplum.

The divisions in section A produce the proportion 30 : 93 : 69 : 93 : 36 : 3. Again, this can be related to an additive series and radial symmetry (provided one adds the flanking phrases to produce a phrase that equals the central one of 69 semibreves).

Section B, with the proportion 21 : 78 : 30 : 78 : 9, is similar to section A in that it displays radial symmetry and, when the 78 is broken down to its components 30 : 48, an additive series is revealed (18, 30, 48, 78, 126).

The division for section C is a conjecture, chosen on the ground that phraselengths of 30 semibreves recur in section B. It produces a radial symmetry 24 : 30 : 24 (4 : 5 : 4),flanked by phrases which add up to the central 30 semibreves.

Gaude Virgo salutata.[13]

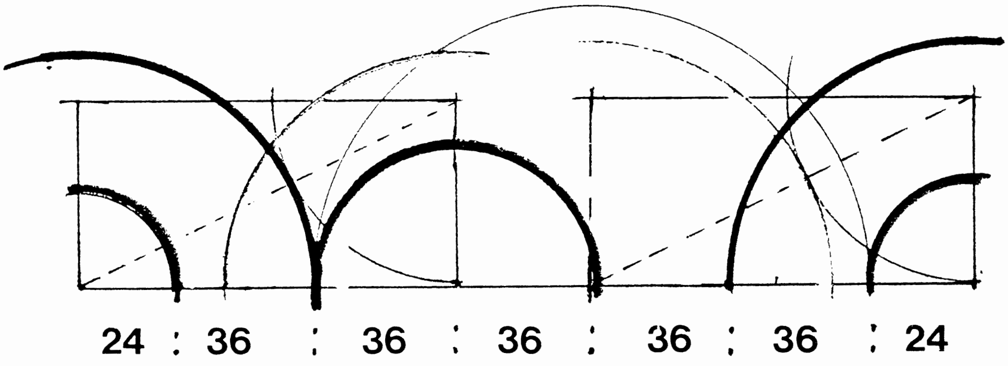
Tenor.

The subdivisions of the isorhythmic tenor are as follows:

Section A. 24 : 36 : 36 : 36 : 36 : 36 : 24
Section B. 16 : 24 : 24 : 24 : 24 : 24 : 16
Section C. 12 : 18 : 18 : 18 : 18 : 18 : 12

In each case, this reveals a central phrase flanked by phrases in golden proportion, which,in its simplest form,produces numbers belonging to the additive series 1, 4, 5, 9, 14.
Radial symmetry is also involved

Fig.14.Geometric construction of the tenor of
Gaude Virgo salutata.



The overall proportion 228 : 152 : 114 is the harmonic proportion 6 : 4 : 3.

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[13] Mus.Brit.,no.28,p.74.

Contratenor.

The three sections yield the following ratio:

Section A. 84 : 144
Section B. 56 : 96
Section C. 42 : 72

Each of these is reducible to the ratio 7 : 12, which can be related to the additive series known as the Evangelist series (2, 5, 7, 12).

In section C, however, the addition of the final cadence (6 semibreves) which stands outside the isorhythmic structure, will produce a radial symmetry 42 : 36 : 42 (7 : 6 : 7).

Motetus.

The phraselengths in section A display the division 51 : 72 : 60 : 45. If one adds the flanking phrases (51 and 45) the proportion 96 : 72 : 60 (8 : 6 : 5) results. This in itself does not seem very significant, but it does show a relationship to the proportions of section A in the tenor.

Section B reveals a bi-lateral symmetry 28 : 48 : 48 : 28. In its simplest form it produces the ratios 7 : 12 : 12 : 7. The fact that the division 7 : 12 occurs in this voice, though now in half values, reinforces the conjecture that the division 7 : 12 in the contratenor and the triplum could have been intentional.

Section C (again with the additional cadential 6 semibreves) produces another radial symmetry 36 : 12 : 36, flanked by two phrases which, when added, equal 36 semibreves.

Triplum.

Section A produces the ratio 84 : 144(provided one adds the last 3 semibreves to the first section),section B the ratio 56 : 96 and section C (excluding the final cadence) the ratio 42 : 72.In all three cases this can be reduced to the ratio 7 : 12,which would therefore link this voice to the contratenor and section B of the triplum.

In section C the addition of the final cadence which stands outside the 7 : 12 ratio,creates a symmetrical pattern identical to that of section C of the contratenor (42 : 36 : 42).

Preco preheminencie.[14]

Tenor.

Each section displays the ratio 1 : 1:

Section A.	135	:	135
Section B.	90	:	90
Section C.	45	:	45

In each section the isorhythm displays a radial symmetry, and the overall proportion 270 : 180 : 90 is the arithmetic proportion 9 : 6 : 3.(3:2:1).

Contratenor.

Provided one adds the flanking proportions in section A and C,all three sections produce equal phraselengths of 90 semibreves(3,2 and 1,matching the arithmetic proportion in the tenor).

Motetus.

Provided one adds the flanking proportions in section A, all three sections produce equal phraselengths of 90 semibreves(again matching the arithmetic proportion).
Section C consists of a central phrase of 45 semibreves, flanked by phrases that add up to 45.

Triplum.

Section A reveals the proportion 45 : 90 : 45 : 90 (1 : 2 : 1 : 2). Section B reveals a radial symmetry 66 : 24 : 66, with the outer phrases added to produce a phrase that equals the central one of 24 semibreves. Section C consists of a central phrase of 45 semibreves,flanked by phrases that add up to 45 semibreves(cf.motetus ,section C).

It can be seen that the entire motet displays the arithmetic proportion in a tangible way.

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[14] Mus.Brit.,no.29,p.78.

Salve scema sanctitatis.[15]

Tenor.

With the flanking phrases added in each section, the tenor produces the following proportion:

Section A. 54 : 108 : 54 : 108
Section B. 36 : 72 : 36 : 72
Section C. 18 : 36 : 18 : 36

In its simplest form each can be reduced to the proportion 1 : 2 : 1 : 2, and the overall proportion 324 : 216 : 108 is the arithmetic proportion 9 : 6 : 3.

Contratenor.

The important point to note here is that, in spite of the fact that the phraselengths involved are different from those of the tenor, the same proportions prevail (54 : 108, 36 : 72, 18 : 36).

Motetus.

All three sections reveal a radial symmetry with flanking sections adding up to a section that equals the central phrase in length.

Section A. 99 : 63 : 99 (+ 63)
Section B. 48 : 60 : 48 (+ 60)
Section C. 15 : 36 : 15 (+ 36)

There is no consistent simple proportion involved.

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[15] Mus.Brit.,no.30,p.81.

Triplum.

Section A reveals a radial symmetry with flanking sections adding up to a phrase that equals the central one: 66 : 96 : 66 (+ 96).

Section B consists of equal phraselengths with flanking sections adding up to another phrase of the same length: 54 : 54 : 54 (+ 54).

Section C reveals radial symmetry with flanking phrases adding up to the central phrase: 21 : 33 : 21 (+ 33).

Again no consistent proportion is revealed.

Specialis Virgo.[16]

Tenor.

The tenor can be divided into two sections. The first section with the proportion 15 : 24 : 24 (5 : 8 : 8) reveals a golden section related to the additive series known as the Fibonacci series 1,2,3,5,8,13 (15,24,39,63,102). The second section is divided into three equal phrases of 18 semibreves each.

Motetus.

The proportion of the duplum 51 : 51 : 30 : 51 : 51 produces a radial symmetry.

Triplum.

The central part of the triplum reveals a radial symmetry (48 : 15 : 6 : 24 : 6 : 15 : 48 or 69 : 24 : 69) while the remaining phrases flanking the symmetry also add up to 69.

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[16] Mus.Brit.,no.31,p.86.

Veni Sancte Spiritus - Veni Creator Spiritus.[17]

Tenor.

The proportions are as follows:

Section A. 27 : 81 : 54 : 81 : 27(3:9:6:9:3)
Section B. 18 : 54 : 36 : 54 : 18(3:9:6:9:3)
Section C. 9 : 27 : 18 : 27 : 9(3:9:6:9:3)

In each case there is a radial symmetry and the overall proportion 270 : 180 : 90 (i.e.the arithmetic proportion 9 : 6 : 3),is maintained internally within each section.

Contratenor.

Section A reveals the proportion 51 :84 : 51 : 84, a proportional relationship which is echoed in section A of the triplum ,as 84 : 51 : 84 : 51. Section B reveals the proportion 42 : 48 : 42 : 48 (7 : 8 : 7 : 8) which is echoed in section B of the motetus. Section C,with the proportion 18 : 27 : 18 : 27 (6 : 9 : 6 : 9)is related to section C of the tenor.

Motetus.

Provided one adds the flanking proportions as shown in section A,all phraselengths can be combined to produce sections of 54 semibreves. In section B the outer phraselengths can be combined to produce the proportion 42 : 48 : 42 : 48 (7 : 8 : 7 : 8) already found in the contratenor. The same principle can be applied to section C,to produce the proportion 21 : 24 : 21 : 24 (also 7 : 8 : 7 : 8).

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[17] Mus.Brit.,no.32,p.88.

Triplum.

In section A two interpretations are possible.Phrases can be coupled to divide the section into the proportion 84 : 51 : 84 : 51,in which case it can be related to section A of the contratenor,or they can be coupled in such a way as to produce the proportion 72 :63 : 72 : 63 (8 : 7 : 8 : 7) in which case it can be related to section B of the contratenor and motetus and section C of the motetus and triplum.

Section B follows the same principle as section A of the motetus. Provided one adds flanking proportions as shown in the diagram,sections of 30 semibreves each result.

Section C reveals the proportion 24 : 21 : 24 : 21 (8 : 7 : 8 : 7).Its relation to other voices has already been mentioned.

Veni Sancte Spiritus (3 voices).[18]

This motet is unique in that it does not display a simple solution.

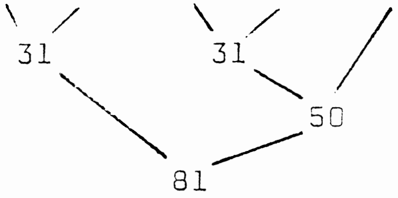
Tenor.

The proportions of the tenor can possibly be interpreted as follows:

Section A. 36 : 57
Section B. 36 : 57
Section C. 24 : 8 : 24 (i.e. 56)

These proportions are problematic in that they cannot be coordinated into a pattern of simple ratios, as can be done with all other isorhythmic motets of Dunstable. However, these proportions very nearly correspond to those of the additive series known as the Evangelist (2, 5, 7, 12, 19, 31, 50, 81). Provided one adds a semibreve to section C, the proportions revealed in the tenor are as follows:

36 : 57 : 36 : 57 : 57 (56) or, in its simplest form:
12 : 19 : 12 : 19 : 19



It is difficult to establish whether such a solution really reflects Dunstable's intention, since it appears to be an isolated case. Within the context of the historically valid solutions which have been discussed in chapter 1, it seems to be the one that is most likely.

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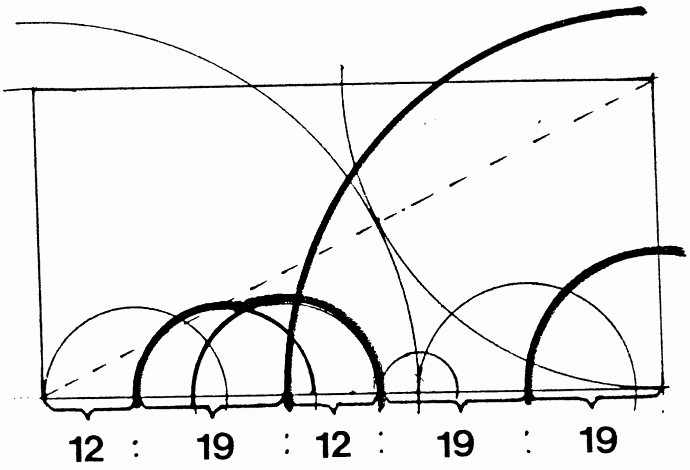
[18] Mus.Brit.,no.33,p.92.

Fig.15.Geometric construction of the tenor of Veni Sancte Spiritus.



Section C provides a possible reason why Dunstable did not make the last section correspond exactly to the Evangelist number 19 (56). Here the proportion 24 : 8 : 24 produces a radial symmetry which would have been disrupted by the addition of another semibreve.

Motetus.

Section A reveals the proportion 57 : 36. The fact that this section so openly divides into the 19 : 12 proportion of the Evangelist series, reinforces the conjecture that the same division could apply to sections A and B of the tenor.

Section B divides into the proportion 66 : 27. This division can also be found in section B of the tenor (as indicated above the line in the diagram), but there it is in inverse proportion 27 : 66.
Section C, with its proportion 12 : 22 : 22 (6 : 11 : 11), can possibly be interpreted as a golden section belonging to the additive series 1, 5, 6, 11, 17.

Triplum.
Both section A and B reveal a proportion of 30 : 63.
Section C with its proportion of 22 : 34, is also related inversely to section C of the contratenor (34 : 22).

Nesciens Mater Virgo.[19]

Tenor.

The tenor can be divided into equal portions of 24 semibreves.

Motetus.

The two sections of the motetus reveal the proportion 30 : 18 (5 : 3). In section A this is produced by adding the first two phrases(9 and 21) and in section B (9 : 30 : 9) by adding the flanking proportions.Section B also reveals radial symmetry.

Triplum.

The proportion of section A,32 : 16 equals 2 : 1.The proportion of section B, 27 : 21 (9 : 7) can be related to section A of the motetus,where it is possible to add the flanking phrases 9 and 18 to produce the same proportion.

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[19] Mus.Brit.,no.34,p.94.

Section C.

Non-isorhythmic motets of Dunstable

Alma redemptoris Mater.[20]

Trowell's proportional analysis[21] is corroborated by further subdivisions within the individual voices of the piece.

The overall proportion is $171 : \underbrace{90 : 81}_{171}$ $(19 : \underbrace{10 : 9}_{19})$.

In section A of the triplum, 171 is clearly divided into the ratio 90 : 81 and it is echoed in the motetus, where the first 15 semibreves must be added to the last segment, to produce the same ratio 81 : 90. Section A of the tenor produces the ratio 120 : 51, which occurs in another Marian piece, *Salve Regina misericordie* . It is therefore possible that this ratio had a symbolic connotation.

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[20] Mus.Brit.,no.40,p.106.

[21] B.Trowell,"Proportion..."p.108.

Beata Dei Genitrix.[22]

Tenor.

The tenor yields the proportion 144 : 72 : 144 : 72, which, in its simplest form, can be reduced to 2 : 1 : 2 : 1, and suggests the geometric proportion.

Motetus.

Section A reveals the proportion 96 : 48 (2 : 1).
If one treats section B and C as one entity it is possible to link phrases in the motètus in such a way that three equal phrases result (96 : 96 : 96).
The beginning of the last 96 semibreves is clearly indicated by the full longa rest.[23]

Triplum.

Section A and B reveal a 1:1 ratio which is found by adding the flanking phrases 144 and 36 (= 180) and placing them with the central portion(180). In this case, the central 180 can be divided into three sections of 60 semibreves (1 : 1 : 1), with one of them being the result of adding flanking phrases. Trowell points out that the number 360, as a symbol of perfection, is often associated with the Virgin.[24] It would seem as if the number carries this symbolic meaning in this piece.
Section B shows an important point of rest[25] which further divides the triplum in the proportion 144 : 120 : 96 : 72, (6 : 5 : 4 : 3) which, as Trowell points out , is the arithmetic proportion.

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[22] Mus.Brit., no.41, p.108.

[23] see bar 93 in Bukofzer edition, p.109.

[24] B.Trowell, "Proportion...", p.120.

[25] see bar 78 in Bukofzer edition, p.109.

Beata Mater.[26]

Trowell offers two possible explanations for the proportions in Beata Mater, depending on the time-signature of the last section.[27]The first one, in which the last section is in imperfect time, yields a proportion of 16 : 11 : 14 for the three sections; the second one, in which the last section is in diminished imperfect time, yields a proportion of 16 : 11 : 7. If sections B and C are added to each other, the first interpretation yields the proportion 16 : 25 ($4^2 : 5^2$) and the second interpretation yields the proportion 16 : 18 (8 : 9), which is the ratio producing the musical consonance of a tone. Of the two interpretations, only the second one, using diminished imperfect time for the last section, yields further significant proportions, as will be seen in the analysis of the contratenor and the motetus.

Contratenor.

Here, section A can be divided into two sections displaying the proportion 66 : 33 (2 : 1). When this section is viewed in conjunction with section B, the resulting proportion 66 : 33 : 66 (2 : 1 : 2) displays a radial symmetry.

Section B displays the proportion 24 : 42 (4 : 7) , which in conjunction with section C (also 42) reveals the golden section which can be correlated to the additive series known as the Lucas series (1,3,4,7.)

Motetus.

If the first two phrases are added to each other, they can be pitted against the last two phrases to produce an overall ratio of 42 : 120 : 42 (7 : 20 : 7), which reveals a radial symmetry. As in the contratenor, section B and C, with the proportion 24 : 42 : 42 (4 : 7 : 7) reveal a golden proportion belonging to the Lucas series.

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[26] Mus.Brit.,no.42,p.110.

[27] B.Trowell, "Proportion",p.109.

Salve Regina misericordie.[28]

Trowell[29] has discovered a number of symbolic relationships in this piece. The most important numerological symbolisms are contained in the recurrence of the number 222(giving a combined total of 444 semibreves or 888 minims)which is the sum of the letters 'Iesous' in Greek Gematria;in the number 152,the numerological symbol for 'Maria'in Gematria; 153 which is the number of the miraculous draught of fishes in John Chapter 21 verse 2;and 33,which represents Christ's years on earth.

Trowell deduced these significant symbolic numbers by adding to one another sections that are in some way related. However,there are other proportional relationships which reveal themselves only when voices are analysed individually. Even the though the following subdivisions may seem arbitrary,there are musical justifications in that each subdivision is demarcated by a significant rest.

In section A of the tenor the proportion 81 : 141 is seen, a proportion which also occurs in the triplum,between sections C + D and E + F (141 : 81) The duplum section A reveals the proportion 156 : 66,a proportion that is also seen between sections Ca and Cb + D + E + F (66 : 156). Another proportion is revealed when one adds the flanking proportions 24 and 27 in section Aa of the triplum (51 : 120).This proportion can be correlated to the proportion appearing between sections Ga and Gb + H,which also reveals the proportion 51 : 120. Although these are not simple proportions,their recurrence could hardly have been a matter of chance.Also intentional may have been the proportion 46 : 92 : 184 (1 : 2 : 4) revealed between sections B : D + E : F + G + H.This would give proportional justification to section B,which is otherwise unrelated.

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[28] Mus,Brit.,no.46,p.115.

[29] B.Trowell,"Proportion." p.117 - 119.

While most of these proportions cannot be reduced to simple proportions,there is no reason to believe that Dunstable was only concerned with straightforward numerical relationships.Other such proportions occur in a number of pieces analysed in this chapter.

Sancta Maria, non est tibi similis.[30]

In Trowell's analysis the tenor is divided in the following way:[31]

$$11 : 4 : 4 : 4 : 11 : 11 : 11 : 4$$

This division, which, in Trowell's opinion, suggests certain symbolic meanings, overlooks one important resting-point which occurs in the middle of the piece.[32] If one considers this rest in the subdivision, the following proportion results:

$$11 : 4 : 4 : \underbrace{4 : 7} : 4 : 11 : 11 : 4$$

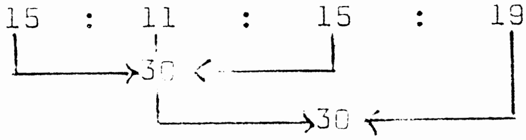
This not only divides the piece into two equal halves, but also reveals symmetrical structures within the two halves:

$$11 : 4 : 4 : 11 \quad \text{and} \quad 4 : 11 : 11 : 4$$

Another possible interpretation is to correlate phrases so that the additive series 11 : 19 : 30 results:

$$11 : \underbrace{4 : 4 : 11}_{19} : \underbrace{4 : 11 : 11 : 4}_{30}$$

This division is of significance because it also reveals itself in the contratenor and the motetus. Although the phraselengths of the contratenor and the motetus differ from those of the tenor, they can both be correlated in such a way as to produce two sections of 30 breves. In the contratenor the 30 : 30 proportion is obtained by means of interlocking proportions:



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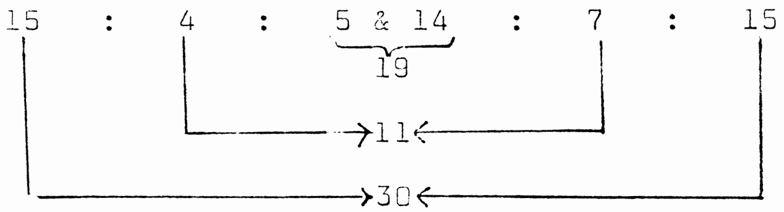
[30] Mus.Brit., no.48, p.121.

[31] Trowell, "Proportion.", p.119 - 120.

[32] see bar 30 in Bukofzer edition, p.122.

In the motetus the same proportion is the result of adding flanking proportions:



Sancta Maria succurre miseris.[33]

Tenor.

If one considers the lower voice of the duo (section B) to be part of the motetus, and not of the tenor, the tenor then reveals in sections A and C the proportion 120 : 30 : 24 (20 : 5 : 4). The 5 : 4 ratio in section C recurs in other voices.

Motetus.

The proportion in this voice, 135 : 54 : 54 (5 : 2 : 2) is a re-arrangement of that of the triplum. The first section (135) can be divided into the ratio 69 : 66, which, as will be seen, is echoed in the triplum.

Triplum.

Here the proportion 54 : 135 : 54 (2: 5 : 2) produces radial symmetry. The middle section 135 displays the ratio 66 : 69.

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[33] Mus.Brit.,no.49,p.122.

Speciosa facta es.[34]

If one adds the flanking phrases of the motetus to each other, the proportion 14 : 21 : 21 (2 : 3 : 3) reveals itself. These numbers belong to the Fibonacci series (1,2,3,5,8), and imply the golden section.

Corroboration of this interpretation comes when the proportion 21 : 35 can be shown in both the tenor and the triplum, although the phraselengths involved in each case are quite different. In the tenor the two flanking phrases 14 and 7 equal 21 semibreves and can be pitted against the three central phrases (4 + 29 + 2 = 35). In the triplum the same procedure, of adding the outer phrases to each other, produces 35 (30 + 5), while the central phrases yield the number 21.

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34. Mus. Brit.,no.50, p. 124.

Section D.

The isorhythmic motets of Dufay.[35]

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[35] Guillelmi Dufay:Opera Omnia (Corpus Mensurabilis Musicae 1,1), ed.Heinrich Besseler(Rome 1966).

Vasilissa, ergo gaude.[36]

Tenor.

The proportion of the tenor, 36 : 45 : 36 , 35 : 45 : 36 (excluding the added cadence) reveals a radial symmetry and can be reduced to the simple proportion 4 : 5 : 4 , 4 : 5 : 4.

Contratenor.

The contratenor uses the same proportion as the tenor, but with its phrases rearranged:

36 : 36 : 45 , 36 : 36 : 45 (4 : 4 : 5 , 4 : 4 : 5)

Motetus.

When one adds the flanking phrases to each other, Section A of the motetus reveals the proportion 24 : 48 (1 : 2).

Section B and C both have a rest at the central point, thus producing the symmetrical pattern 57 : 3 : 57 , 57 : 3 : 57.

Triplum.

Section A of the triplum echoes section A of the motetus, with the proportion 24 : 48 (1 : 2) again produced by adding the two flanking outer phrases to each other.

Sections B and C both consist of three phrases of 39 semibreves, with two of them forming the central part and the third one being the result of adding the flanking phrases to each other.

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[36] Corpus Mensurabilis,no.7,p.21.

All the phrases of sections B and C form part of the Fibonacci series (1,2,3,5,8,13) and can be reduced to the proportion:

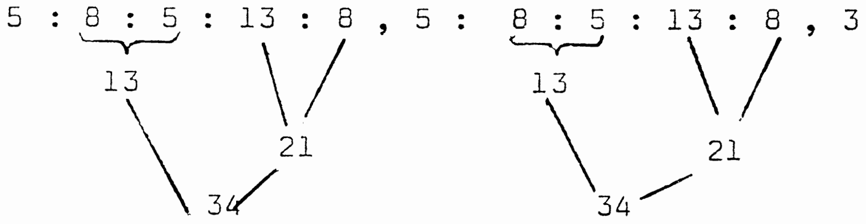
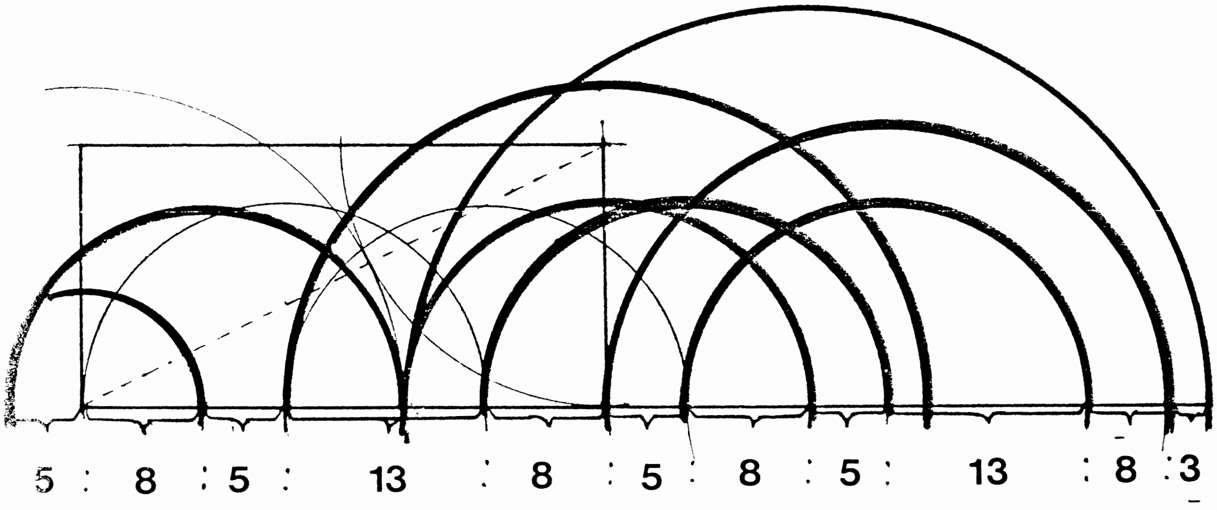


Fig.16.Geometric construction of sections B and C of the triplum of Vasilissa



Overall proportion:

As pointed out by Sandresky,[37] the overall proportion 8 : 13 : 13 is a golden proportion.

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[37] 37. M.Sandresky,"The Golden Section..."p.292 - 293.

O sancte Sebastiane.[38]

Tenor.

In all sections phrases stand in the proportion 54 : 27, which, in its simplest form,equals 2 : 1(excluding the final cadence).

Contratenor.

Sections B, C and D reveal the proportion 36 : 45, which can be reduced to the simple proportion 4 : 5.
Section E consists of three equal phrases (27 : 27 : 27).

Motetus.

Section A reveals the proportion 12 : 24 : 24 : 24 (1 : 2 : 2 : 2).
Sections B,C, D and E all produce the proportion 27 : 54(1:2) already found in the tenor.

Triplum.

Section A displays the proportion 24 : 24 : 12 : 24 (2 : 2 : 1 : 2). (cf. motetus)
Sections B, C and D present a problem, in that the phrases do not reveal an overall logic(42 : 21 : 18). The first two phrases, however, stand in the proportion 2 : 1.
Section E consists of three equal phrases 27 : 27 : 27.

Overall proportion.

The overall proportion is problematic in that it cannot be reduced to a simple ratio:

84 : 81 : 81 : 81 : 81 + 9

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[38] Corpus Mensurabilis,no.8,p.24.

O gemma,lux et speculum.[39]

Tenor.

The proportion in sections A and B, 45 : 54 , 45 : 54 can be reduced to the simple proportion 5 : 6 , 5 : 6 . Section C reveals the proportion 6 : 24 : 12 : 24 : 6, which is symmetrical.

$$\underbrace{6 : 24 : 12 : 24 : 6}_{\longrightarrow 12 \longleftarrow}$$

Contratenor.

The same proportions as those of the tenor occur. In sections A and B the proportion 45 : 54 (5 : 6) can be produced by adding the flanking phrases to each other. In section C the radial symmetry is as follows:

$$\underbrace{\underbrace{6 : 6} : \underbrace{12 : 24 : 12}_{\longrightarrow 24 \longleftarrow} : \underbrace{6 : 6}}_{\longrightarrow 24 \longleftarrow}$$

The result is three equal phrases of 24 semibreves.

Motetus.

In section A the proportion 27 : 45 can be reduced to 3 : 5, which would relate this section to section B and C. One can also add flanking phrases to each other to produce the proportion 48 : 24 (2 : 1), which would relate this section to section A of the triplum.

In section B, which contains a fragment in diminished imperfect time at bar 41 - 49, the proportional relationship 27 : 21 : 27 : 24 results in two interlocking proportions corresponding to those of the contratenor and tenor, namely

$$\begin{array}{c} \longrightarrow 54 \longleftarrow \\ 27 : 21 : 27 : 24 \\ \longrightarrow 45 \longleftarrow \end{array}$$

Section C is the same as section B.

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[39] Corpus Mensurabilis,no.9,p.29.

Section D reveals two phrases of 36 semibreves each when one adds the two flanking phrases on each side of the central phrase:

$$\underbrace{2 : 14} : 36 : \underbrace{14 : 6}$$

$$\longrightarrow 36 \longleftarrow$$

All proportions in the motetus relate to the Fibonacci series (1,2,3,5,8):

A B C D

3 : 5 / 3 : 5 : 3 / 3 : 5 : 3 / 8

Triplum.

Sections A, B and C all reveal a 2 : 1 proportion.
In section A, provided that one adds the flanking phrases 15 and 33, 48 : 24 (2 : 1) is revealed (cf. motetus section A)
In sections B and C there is the proportion 33 : 66 (1 : 2).

Section D corresponds to the same section in the motetus, in that it consists of two equal phrases (36 : 36), which are produced by adding flanking proportions.

Apostolo,glorioso.[40]

Tenor.

In both sections B and C the proportion 54 : 36 (3 : 2) is revealed when one adds the flanking phrases to each other.

Sections D and E consist of two equal phrases 30 : 30.

The final cadence is not involved in any proportion, except, as will be noted,in the triplum and contratenor II.

Contratenor I.

Sections B and C, with the proportion 54 : 36 , 54 : 36 (3 : 2 , 3 : 2) echo the proportion of the tenor sections.

Sections D and E are divided into two equal phrases (15 : 15 : 15 : 15),imitating the tenor 30 : 30.

Contratenor II.

Provided one adds the flanking phrases to each other the proportion 60 : 30 results(2 : 1) in both sections B and C.

Sections D and E can be divided into the proportion 18 : 12 , 18 : 12 (3 : 2 , 3 : 2).

Section A (21 semibreves) and the final cadence (9 semibreves) equal 30 semibreves, which fits into the overall scheme:

21 : 60 : 30 : 60 : 30 : 30 : 30 : 30 : 9

→30←

Motetus.

Provided one adds flanking proportions of sections B and C, these two sections reveal the 30 : 60 (1 : 2) proportion already found in contratenor II.

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[40] Corpus Mensurabilis,no.10,p.33.

Sections D and E are equal 30 : 30.
Section A (30 semibreves) fits into the overall scheme:

30 : 60 : 30 : 60 : 30 : 30 : 30 (excluding the final cadence)

Triplum.

Sections B and C reveal the proportion 54 : 36 (3 : 2) already noted in the tenor and contratenor I.
Sections D and E reveal equal phrases 30 :30. If one adds the final cadence to section E, the new proportion (39) can be pitted against section A, which is also 39 semibreves, producing the proportion:

39 : 60 : 30 : 60 : 30 : 30 : 39

Rite majorem.[41]

All four sections of the tenor and contratenor and sections A and B in the motetus are in some way related to the arithmetical proportion 2 : 3 : 4. In all cases the final cadence (9 semibreves) expresses the basic unit for the proportions in these sections.

Tenor.

Sections A and B: 2 : 3 : 4 : 2 (18 : 27 : 36 : 18)
Sections C and D: 2 : 3 : 3 : 2 (18 : 27 : 27 : 18)

Contratenor.

Sections A and B: 2 : 3 : 2 : 4 (18 : 27 : 18 : 36)
Sections C and D: 2.: 3 : 2 : 3 (18 : 27 : 18 : 27)

Motetus.

Section A and B: 4 : 3 : 4 (36 : 27 : 36)

The remaining sections reveal the following proportions:

Motetus.

Sections C and D reveal the proportion 60 : 30 when one adds the flanking phrases to each other.

Triplum.

Sections A and B, with the proportion 7 : 13 : 13, can be related to the golden section expressed by the additive series 6,7,13 ,20.
Sections C and D reveal the symmetrical proportion 4 : 2 : 4 (36 : 18 : 36).

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41. Corpus Mensurabilis, no.11, p.38.

Ecclesiae militantis.[42]

As shown by Powell,[43] this motet relies on two interlocking additive series to express the golden proportion. An analysis in which the voices are viewed independently reveals that these interlocking series are tangibly expressed in the mensuration signs of the individual voices. This reinforces the conjecture that the use of the golden proportion was a conscious process on the part of Dufay.

Tenor I and II.

Here the Fibonacci series (1,2,3,5,8) is used in all sections, the number of breves involved being dependent on the mensural sign employed:

Ͼ	18 : 27 : 27	¢	9 : 13,5 : 13,5	C	12 : 18 : 18
	2 : 3 : 3		2 : 3 : 3		2 : 3 : 3
¢	6 : 9 : 9	O	18 : 27 : 27	Φ	9 : 13,5 : 13,5
	2 : 3 : 3		2 : 3 : 3		2 : 3 : 3

With the addition of the introductory duo, the overall proportion revealed is

72 : 72 : 36 : 72 : 72 : 36
2 : 2 : 1 : 2 : 2 : 1

Contratenor:

In the contratenor the golden proportion can again be correlated to the mensuration signs:

Φ 108 : ¢ 72 : Φ 108
3 : 2 : 3

Each section can in its turn be divided into the proportion 2 : 1:

Φ 108 = 72 : 36 ¢ 72 = 48 : 24 Φ 108 = 72 : 36

Motetus and Triplum.

In both the motetus and the triplum the second half expresses the proportion ¢ 72 : ¢ 72 : ¢ 45 (the final cadence included).
This reveals the golden proportion 8 : 8 : 5, which is based on the Fibonacci series.

In the motetus, section C can be further subdivided into the proportion 36 : 36 (1 : 1), provided one adds the flanking phrases to each other.

In the triplum, section C can be divided into two equal halves (36 : 36) and section D reveals a radial symmetry 27 : 18 : 27 (3 : 2 : 3), which relates this section to the tenor.

The division in section B of the motetus (30 : 42), is the same as the division in section A of the triplum (42 : 30).

In spite of the fact that the motetus and the triplum sections A and B involve different phraselengths, both reveal the proportion 42 : 63 : 75. In the motetus this is produced by adding flanking phrases and in the triplum by adding adjacent phrases. Of further interest is the radial symmetry revealed in the overall proportion of the triplum 117 : 135 : 117 and the 1 : 3 proportion in the second part of the triplum (63 : 189).

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42. Corpus Mensurabilis, no.12, p.46.
43. N.Powell, "Fibonacci...", p.258-266.

Balsamus et munda.[44]

Tenor and contratenor. (breves)

As already mentioned on p.41 , the tenor and contratenor are both governed by the Fibonacci series and its multiple (X2):

Tenor 6 : 16 : 4 / 6 : 16 : 4 / 3 : 8 : 2 / 3 : 8 : 2 / 1

Contratenor 16 : 6 : 4 /16 : 6 : 4/ 8 : 3 : 2/ 8 : 3 : 2 / 1

Triplum and motetus.

Sections A and B reveal the proportion 45 : 72 (5 : 8),which is produced by adding adjacent phrases in the motetus and flanking phrases in the triplum.

In sections C and D one has to add flanking phrases to each other to produce equal phrases of 42 semibreves.

The proportion 8 : 5 appears in all the sections of the piece, except sections C and D of the motetus and triplum.

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[44] Corpus Mensurabilis,no.13,p.54.

Supremum est mortalibus bonum.[45]

Tenor.

Sections B,C,D,E,F and G of the tenor divide into two very unequal phrases which are proportionally related as 4 : 1. However, a curious fact emerges if one adds the longer phraselengths of these sections together, (resulting in a total of 360 semibreves),and the shorter phrases(resulting in a total of 90 semibreves): 360 : 90 also reveals the proportion 4 : 1.

The remaining section (H), reveals the proportion 36 : 66 : 66, which is the golden proportion based on the additive series 6, 11, 17, 28. However, section H does not fit any kind of overall proportion, and in order to produce this overall proportion, its internal proportion should be 36 : 72 : 72 (1 : 2 : 2). It is possible that a scribal error exists in bar 108. One of the important aims of a study into proportions is to maybe detect scribal errors of this nature. In this case, the production of 36 : 72 : 72 (180) would bring the overall proportion to 270 : 180 : 180 (3 : 2 : 2). This alteration would also make the proportions of the piece symbolically appropriate,since the work is a supplication for peace.[46]

Motetus.

Sections B and C reveal the proportion 30 : 60 (1 : 2). In section B this is the result of adding adjacent proportions and in section C it is the result of adding flanking phrases. Section D reveals the proportion 8 : 4 : 3, which, as will be seen, also occurs in the

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[45] Corpus Mensurabilis,no.14,p.59.

[46] According to Trowell,the number 360 was considered perfect. see footnote no.24 of this chapter.

triplum.

Section E reveals the proportion 20 : 40 (1 : 2), sections F and G the proportion 32 : 28 (8 : 7) and 28 : 32 (7 : 8) respectively. Section H either expresses the golden proportion 36 : 66 : 66 or, with the alteration, the proportion 36 : 72 : 72 (1 : 2 : 2).

Triplum.

Section A has two phrases of equal length (30 : 30), sections B, C, D, E, F and G express the proportion 32 : 16 : 12 (8 : 4 : 3) already encountered in section D of the motetus, or, if one adds the 16 and 12 (4 and 3), it produces the proportion 8 : 7 already encountered in sections F and G. Section H is the same as section H of the tenor and motetus.

Salve flos.[47]

Tenor and Contratenor.

The proportions in these two voices are as follows:

Section A. 114 : 96 : 96
Section B. 72 : 48 : 48
Section C. 96 : 64 : 64
Section D. 48 : 32 : 32

All these proportions can be reduced to the simple proportion 3 : 2 : 2.

The overall proportion 336 : 168 : 224 : 112 equals 6 : 3 : 4 : 2. The last two sections can be added to each other to produce a symmetrical overall proportion:

336 : 168 : 336 (2 : 1 : 2)

Motetus.

Section A can be divided into two halves, each having the proportion 72 : 48 : 48 (3 : 2 : 2).
Section B reveals the proportion 54 : 30 when one adds flanking phrases to each other. In its simplest form this can be expressed as 9 : 5.
Sections C and D divide into equal phrases:

Section C. 56 : 56 : 56 : 56
Section D. 28 : 28 : 28 : 28

Triplum.

Section A divides into two equal halves, each having the proportion 90 : 60 (3 : 2).

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[47] Corpus Mensurabilis, no.15, p.64.

Section B displays the same proportions as section C of the motetus (54 : 30 : 54 : 30).
Section C expresses the proportion 72 : 40 : 72 : 40, provided one adds the flanking phrases in each half.
In its simplest form, this again produces the ratio 9 : 5.
In section D one can add interlocking phrases to produce the proportion 48 : 64 (3 : 4), which links this voice to the 3 : 2 : 2 proportion of the tenor.

Due to the mensuration signs all four voices of this motet reveal the simple overall proportions 2 : 1, and, if one adds sections C and D, a radial symmetry 2 : 1 : 2 is revealed.

Nuper rosarum flores.[48]

Tenor.

The proportions of the tenor are as follows:

Section A. 84 : 12:24:24:24 (84:84)
Section B. 56 : 8:16:16:16 (56:56)
Section C. 28 : 4: 8: 8: 8 (28:28)
Section D. 42 : 6:12:12:12 (42:42)

Motetus.

The duos and tuttis in each section have to be considered as separate entities for any simple proportional relationships to emerge.
In section A the duo reveals the proportion 18 : 24 : 18 : 24 (3 : 4 : 3 : 4). The tutti can be divided into two halves, the first one displaying the proportion 30 : 15 (2 : 1) and the second half the proportion 24 : 15 (8 : 5).

In section B the duo displays the proportion 16 : 12 : 16 : 12 (4 : 3 : 4 : 3) and the tutti the proportion 24 : 32 (also 3 : 4).

Section C 's tutti displays the proportion 18 : 12 (4 : 3). The duo,on the other hand, cannot be subdivided. It is to be noted that in this section the cadence indicating the end of the duo finishes a bar earlier than in the other sections (bar 125).

In section D the duo cannot be subdivided and the tutti reveals the proportion 24 : 18 (4 : 3).

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[48] Corpus Mensurabilis,no.16,p.70.

Triplum.

Section A's duo has the same proportion as the corresponding section of the motetus, namely 18 : 24 : 18 : 24 (3 : 4 : 3 : 4) The tutti also relates to the corresponding section of the motetus, but here the two proportions are inverted :

15 : 30 (1 : 2) and 15 : 24 (5 : 8)

In section B the duo is the same as section B of the motetus and the tutti reveals the 3 : 4 proportion already found in the corresponding section in the motetus, although it is now produced by means of adding flanking phrases.

Section C is identical to the corresponding section of the duplum.

Section D inverts the proportion of the motetus section D: 18 : 24 : 42 (3 : 4 : 7).

Overall proportion.

The overall proportion 168 : 112 : 56 : 84 can be reduced to 6 : 4 : 2 : 3.

Magnanimae gentis.[49]

Tenor.

Including the prelude to the isorhythmic tenor of section A, the proportion 48 : 36 : 48 : 60 : 60 (4 : 3 : 4 : 5 : 5) is shown. Section A without the prelude and sections B, C and D all reveal the simple proportion 3 : 4 : 5 : 5:

Section A. 36 : 48 : 60 : 60
Section B. 24 : 32 : 40 : 40
Section C. 12 : 16 : 20 : 20
Section D. 18 : 24 : 30 : 30

The addition of the final cadence (18 semibreves) in section D reveals the proportion 48 : 24 : 48 (2 : 1 : 2) which is produced by means of interlocking phrases:

$$\begin{tikzcd}
18 \arrow[dr] & : 24 : & 30 \arrow[dl] & : 48 \\
 & 48 & &
\end{tikzcd}$$

Motetus.

Section A, with the introductory prelude, reveals the proportion 84 : 198 (1 : 2). Sections B and C, together, produce the proportion 68 : 136 (1 : 2) and section D with the final cadence results in the proportion 24 : 48 : 48 (1 : 2 : 2). (cf. section D of the tenor)

Triplum.

Section A, including the prelude, reveals the proportion 72 : 108 : 72 (2 : 3 : 2). Sections B and C together produce the proportion 136 : 68 (2 : 1). (compare sections B and C of the motetus.)
When one adds the flanking proportions of section D, two phrases of equal length result : 54 : 54.

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[49] Corpus Mensurabilis, no. 17, p. 76.

Overall proportion.

The proportion of the isorhythmic sections is:

204 : 136 : 68 : 102 (6 : 4 : 2 : 3)

Fulgens iubar.[50]

Tenor I and II.

Sections A, B and D can all be divided into equal phrases of 72 semibreves and section C can be divided into two phrases of 48 semibreves.

Section A. 72 : 72
Section B. 72 : 72
Section C. 24 : 24 : 24 : 24 (48 : 48)
Section D. 36 : 36 : 36 : 36 (72 : 72)

Motetus.

Sections A and B both reveal the proportion 51 : 51 : 42 (17 : 17 : 14), which can be related to the golden section based on the additive series 14 , 17 ; 31 ; 48.
Section C and D both reveal the proportion 5 : 3 : 5 : 3 :

Section C. 30 : 18 : 30 : 18
Section D. 45 : 27 : 45 : 27

The additional 18 semibreves forming the final cadence, fall within the scheme of the final section,45 : 27 : 45 : 27 : 18 being equal to 5 : 3 : 5 : 3 : 2.

Triplum.

Sections A and B both display the proportion 45 : 18 : 45 : 36 (5 : 2 : 5 : 4).
Section C can either be interpreted as 30 : 18 : 30 : 18 (5 : 3 : 5 : 3), which would link it to section C of the motetus,or as 36 : 12 : 36 : 12 (3 : 1 : 3 : 1),which

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[50] Corpus Mensurabilis,no.18,p.80.

would link it to section D of the triplum.
Section D, with the proportion 54 : 18 : 54 : 18 can be reduced to the simple proportion 3 : 1 : 3 : 1, with the final cadence (18 semibreves) forming the basic unit of the proportion.

Overall proportion.

The overall proportion 144 : 144 : 96 : 144 results in the simple proportion 3 : 3 : 2 : 3, or, if one adds sections B and C, a radial symmetry 3 : 5 : 3.

Moribus et genere Christo.[51]

Tenor.

All sections can be divided into equal phraselengths:

Sections A and B. 72 : 72 : 72 : 72
Sections C and D. 48 : 48 : 48 : 48
Sections E and F. 36 : 36 : 36 : 36

Motetus.

Sections A and B reveal the proportion 108 : 108 : 72 (3 : 3 : 2).
Sections C and D present a problem, in that the phrases do not produce a simple proportion, a golden proportion or a radial symmetry: 52 : 52 : 30 : 48 (26 : 26 : 15 : 24)
Sections E and F display the proportion 36 : 72 : 36 (1 : 2 : 1).

Triplum.

Provided one adds the flanking proportions to each other, sections A and B reveal the proportion 180 : 108, which can be reduced to the simple proportion 5 : 3 (cf. motetus).
Sections C and D, with the proportion 56 : 28 : 56 : 52 (84 : 108), can be reduced to the simple proportion 7 : 9.
Sections E and F, like the corresponding sections of the motetus, reveal the proportion 36 : 72 : 36 (1 : 2 : 1).
The addition of the final cadence (18) changes the proportion of the last section to 36 : 72 : 36 : 18 = 2 : 4 : 2 : 1 = 6 : 3 = 2 : 1.

Overall proportion.

Excluding the final cadence sections A & B : C & D : E & F stand in the proportion 72 : 48 : 36 (6 : 4 : 3), which is the harmonic proportion. This is the only motet of Dufay which reveals this pattern, so frequent in Dunstable's motets.

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[51] Corpus Mensurabilis, no.19, p.88.

Chapter III

Proportioning Methods in their Historical Perspective

It is now possible to consider these analyses in their historical context. The period we are dealing with is one of transition, alternately called the waning of the Middle Ages and the beginning of the Renaissance. All the contradictions of a period of transition, its archaic traditions and its newly formed ideals, are often expressed in its musical forms. It is hoped, therefore, that a comparison of the methods of proportioning used in these motets will reveal some of these contradictions.

To this end, this chapter will deal with three different aspects of proportioning. In the first part, proportioning methods will be viewed purely on their own merit. In as far as is possible, the motets will be compared in chronological order and the result of this investigation will be correlated to some of Wittkower's observations concerning proportioning methods as they are found in art and architecture. The second aspect that will be discussed is the possible link that exists between proportioning methods and 'tonality'. This section will start with a summary of five scholars' views on 'tonality'as it applies to this period in general; their observations will then be illustrated by means of examples from the motets, and will be followed by a comparison of tonal and proportional procedures in the music of Dunstable and Dufay. The third part of this chapter will be devoted to the link existing between proportioning methods and symbolism. As this section deals more specifically with the intellectual and emotional overtones of ideas, as they are reflected in the use of proportioning methods, the first part will deal with the expressive means at the disposal of the composer in a general historical context,in order to elucidate the difference in the attitudes of the Gothic and the Renaissance composer. This will then be amplified by two examples from the motets.

Wittkower [1] claims that two different classes of proportions, both derived from the Pythagoreo-platonic world of ideas, were used during the history of European art and architecture. In his opinion, the Middle Ages favoured the Pythagoreo-platonic geometry, while the Renaissance preferred the numerical, that is, the arithmetical side of the tradition. Wittkower's explanation is as follows:

> The arithmetic proportions epitomized in the ratios of the Greek musical scale consist of integral numbers or simple fractions; i.e. they consist of commensurable ratios. By contrast, many of the geometrical proportions cannot be expressed by integral numbers or simple fractions: i.e. they are incommensurable or irrational. In the equilateral triangle, for instance, the height (i.e. the perpendicular) is incommensurable to the length of the sides and can only be expressed by the square root of three. The hypotenuse of the right-angled isosceles triangle, i.e. the diagonal of the square, is related to the shorter sides as $1 : \sqrt{2}$ and the length of a side of the larger to the length of a side of the smaller square in the construction of "the just measure" is related as $1 : \frac{\sqrt{2}}{2}$. The construction of the pentagon implies the cutting of a straight line into "extreme and mean ratio". What Euclid (VI,30) calls "to cut a line in extreme and mean ratio" is nowadays called the Golden Section,

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[1] R.Wittkower, "Systems of Proportion." Architects' Year Book 5,(London 1953):9 - 18.

> in which the smaller part is related to the larger as the larger to the whole, and this proportion is, of course, incommensurable. [2]

Wittkower maintains that these irrational proportions would have confronted the Renaissance artist with a perplexing dilemma, for the Renaissance attitude to proportioning was determined by a new organic approach to nature which involved the empirical procedure of measuring and was aimed at demonstrating that everything was related to everything else by number. Thus commensurability becomes the nodal point of Renaissance aesthetics. To illustrate his argument, Wittkower points out that Leonardo's comprehensive studies of proportion use exclusively numerical proportions. As in this illustration, he measured and compared the proportions of one part of the body to another and expressed these relationships in small ratios, such as 1 : 2 and 1 : 3.

<u>Fig.17.Leonardo da Vinci:Study in Human Proportion. Showing the ratios 1:3:1:2:1:2</u>

[3]

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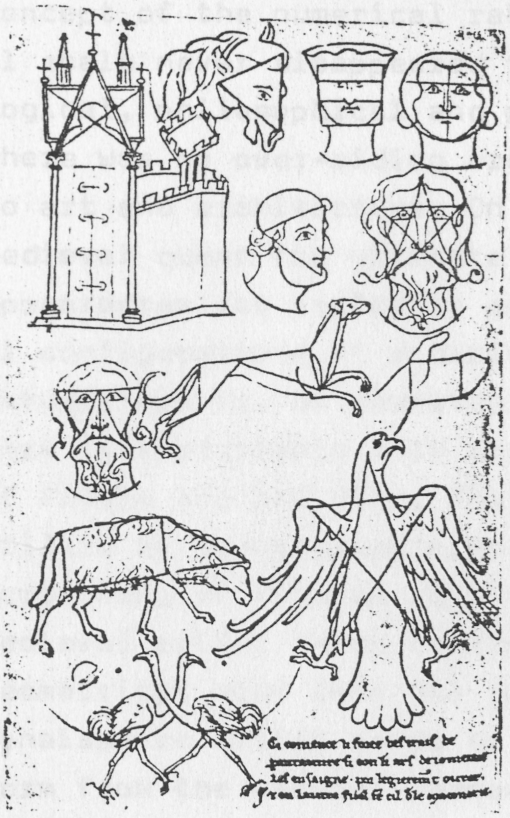
2. R.Wittkower,p.16.
3. R.Wittkower,p.16.

By contrast, the 13th century architect Villard de Honnencourt, in his sketchbook, draws figures the proportions of which were determined by Pythagorean geometrical figures such as the equilateral triangle.

Fig.18.Villard de Honnencourt: Page from Notebook,c.1235.



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4. R.Wittkower,p.16.

Wittkower sums up this difference with the following words:

> While to the organic, metrical Renaissance view of the world rational measure was the sine qua non, for the logical, predominantly Aristotelian medieval approach to the world the problem of metrical measure hardly arose. And although the Pythagoreo-platonic concept of the numerical ratios of the musical scale never disappeared from medieval theological, philosophical and aesthetic thought, there was no over-riding urge to apply them to art and architecture. On the contrary: the medieval quest for ultimate truth behind appearances was perfectly answered by geometrical configurations of a decisively fundamental nature;that is, by geometrical forms which were irreconcilable with the organic structure of figure and building. The contrast between Villard de Honnencourt's and Leonardo's proportioning of figures is a typical one: the medieval artist tends to project a pre-established geometrical norm into his imagery, while the Renaissance artist tends to extract a metrical norm from the natural phenomena that surround him. 5

5.R.Wittkower,p.17.

Since this pattern of development exists in art and architecture, it is indeed pertinent to investigate whether it can be found in music. Incommensurable terms, such as those found in geometry, cannot be expressed in music, but in the use of series approximating the golden proportion and in the use of simple proportions suggesting the use of quadrature, the musician has proportions which in one way or another relate to these incommensurable patterns. It is therefore possible to classify proportioning methods in music according to the degree of geometrical or arithmetical application involved.[6]

Such a classification is attempted in the following diagrams:

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6. In some cases a decision had to be made, whether a proportion, such as 2 : 1, reflects a simple numerical ratio, or a geometrical ratio related to quadrature. Wittkower points out that the same ratio may be used in a metrical and a geometrical context. In the following diagrams, the proportion 2:1, is seen as the result of quadrature when it occurs in various unrelated ratios within one single section of a piece (for example, in _O livor_). Where the ratio appears as an integrated principle within a section or a whole piece, this is considered as the result of metrical thinking (for example, in _O sancte Sebastiane_). Proportions suggesting a golden section are considered geometrical, because of their close connection to the geometrical construction found in chapter 1. Radial symmetries are essentially visual, and are therefore also considered to be geometrical.
see also Wittkower, p.17.

Diagram 1.

Music that employs purely geometric means of proportioning.

a. Quadrature and radial symmetry.	
Leonin:	O 1 Judea et iherusalem O36 Vir iste M15 Alleluya M22 Alleluya
b. Golden proportion and radial symmetry.	
Leonin:	Gloria Patri
c. Golden proportion, quadrature and radial symmetry.	
Anonymous:	Excelsus in numine O livor/Inter amenitatis/Revertenti
De Vitry:	Floret/florens/Neuma Hugo Princeps
d. Golden proportion.	
De Vitry:	Vos qui admiramini

Diagram 2.

Music that uses both geometric and arithmetic methods of proportioning.

DUNSTABLE	DUFAY
Albanus roseo rutilat	O, gemma lux
Ave Regina	Ecclesiae militantis
Dies dignus	Balsamus et munda
Gaude felix Anna	Vasilissa ergo gaude
Gaude Virgo	
Specialis Virgo	
Veni Sancte Spiritus	
Beata Mater	
Speciosa facta es	

Music in which there is a frequent use of simple proportions.

DUNSTABLE	DUFAY
Preco preheminencie	Supremum est
Salve scema	Fulgens iubar
Nesciens Mater	Moribus et genere
Christe sanctorum	O, Sancte Sebastiane
Alma redemptoris	

Music making use of a few simple proportions throughout.

DUNSTABLE	DUFAY
Veni Sancte-Veni Creator	Apostolo glorioso
Beata Dei Genitrix	Rite majorem
Sancta Maria, non est	Salve flos
Sancta Maria succurre	Nuper rosarum
	Magnanimae gentis

From these diagrams it becomes clear that, at least in these works, there is a definite move from a geometric to an arithmetical way of thinking during this transition period. All the early examples (i.e. before the fifteenth century) are exclusively geometrical. With Dunstable and Dufay, on the other hand, there is evidence of both geometric and mathematical ways of thinking. Dunstable, the astronomer versed in the arts of the Quadrivium, makes more frequent application of geometric methods of proportioning than Dufay, who seems to favour simple proportions. The proportions in Dunstable's music are almost made spatial through the frequent use of radial symmetry and the adding of flanking proportions. Dufay, on the other hand, seldom uses flanking proportions.

The use of simple proportions often goes hand in hand with a change in harmonic style, as can be seen in Dunstable's pan-consonant motet *Sancta Maria, non est tibi* and Dufay's *Nuper rosarum flores.* It follows that a more accurate picture can be obtained when one takes these tonal and stylistic features into consideration and see the extent to which changes in style are reflected in changes in proportioning methods. The next section of this chapter is such an attempt.

<u>Tonality during the transition period.</u>

A summary of the views of five modern scholars.

The term tonality, as applied to the music of Dunstable and Dufay, is at the centre of the same historical problem. There has been much discussion of this transition in its relation to tonality, and the concommitant changes in harmonic and melodic styles. However, none take into consideration the relation of tonality to the proportional methods that are in evidence in the music. It seems that proportional methods and tonality should be viewed in conjunction, because both are at the centre of formal preoccupations facing the composer, whether the process is conscious or subconscious. It is most likely that Dunstable and Dufay, for example, set out by selecting a plainchant tenor. This tenor was then cast into an isorhythmic structure with proportional implications, around which the composer could structure his polyphonic complex. It is probable that the composer's selection of proportions would be influenced by the tonal potentialities of the tenor, or that certain tonal potentialities would arise from the proportional structure imposed upon the flexible plainchant tenor. In the actual composition of a work, therefore, the proportional patterns will influence the tonal patterns inherent in the piece. Because the proportions are determined by rests and therefore establish the lengths of phrases, these must necessarily regulate the occurence of cadential patterns, since cadences will naturally coincide with the end of phrases. It follows that the tonal language and the use of proportional methods are inextricably linked.

What follows is a summary and discussion of certain scholars' theories that discuss tonality as an organising principle and form-builder in 15th century music. The summaries of arguments are selective and by no means complete. They have been restricted to those aspects that are directly related to this specific study. After this more general discussion, there follows an application in which the essential aspects of their theories are applied to examples from the motets of Dunstable and Dufay.

For Lowinsky[7] the transition period from Middle Ages to Renaissance is governed by a move from modality to tonality. In his view tonality is set off from modality by a strong sense of direction. This tonal direction is produced by the use of the tonic, reserved for points of departure and arrival, by the use of sequences, through marked and regular distinctions between weak and strong beats, and the use of light and springy upbeats. These, he says, contribute as much to the effect of tonality as does the change of harmonic vocabulary. Lowinsky maintains that the development of tonal thinking in polyphonic art is an evolution from Dunstable, Dufay, the frottola and the villancico, and moves to Josquin, the French chanson, the Italian canzonet and balletto. This development can be seen from two angles:

1. A chronological development from about 1450 to 1550, which he sees as an era of floating tonality, through an era of triadic atonality (±1550 - 1600), to a consolidation of tonal procedures (1575 onwards).
2. A synchronistic development in which "modality", "tonality" and "atonality" coexist and interact - modality

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7. E.Lowinsky, <u>Tonality and Atonality in Sixteenth-Century Music,</u> Berkerley,1962.

subsisting in traditional religious practice, tonality going hand in hand with secularisation and atonality with individualism and the revival of Greek ideals.

In this definition of the historical development, a great emphasis is placed on the function of the cadence as the most important building-block of tonality. These cadences were extended and amplified by means of the floating tonality and triadic atonality of the early English and the Netherlands school, experimentations which, he maintains, were indispensible for further elaboration of the tonal system. In the context of his theory, Dunstable and Dufay move in a territory between the old and the new, still bound to counterpoint and modality, but influenced by the creative impetus of the new harmonic language which, in his opinion, came from Italy.[8].

If one correlates Lowinsky's view with the fact that phrasing in the music of Dunstable and Dufay depended upon proportions, one would expect an evolutionary pattern to take place in the music, in which proportional phrasing would become increasingly linked to cadential patterns. This would hinge more and more upon points of departure and arrival. If one accepts his view, then one would expect that in medieval polyphony the voices would lead more or less independent lives, with phrases that diverge and vary in length, and proceed purely on their own volition and are defined by their independent proportional patterns. In the later works of Dufay, the unifica-

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[8] An opinion that is not universally accepted. Bukofzer, for example, maintains that it was the Fauxbourdon that was the most important catalyst for change. (NOHM, III, 165-213).

tion of voices through cadencing would presumably influence the structuring of phrases and therefore alter and influence the proportional permutations that are possible.

It is by the very nature of Lowinsky's investigation that he does not detail modal counterpoint with as much care as he does tonality. In modal polyphony of the Gothic period, there exists an interaction between voices quite distinct from that existing within a tonal framework.

Salzer[9] attempts a definition of this question. He uses the term "modal/contrapuntal texture" to define the tonal implications of medieval polyphony in general. In his view, "modal/contrapuntal texture" implies contrapuntal writing without harmonic (i.e. triadic) interference or influence. Salzer avoids calling it linear counterpoint, because this term obscures the fact that linear voices may imply or express a common sonority. In the following example all voices, which at first sight seem to be independent, lie within the basic compass of an 8th and imply this underlying interval in its individual voices:

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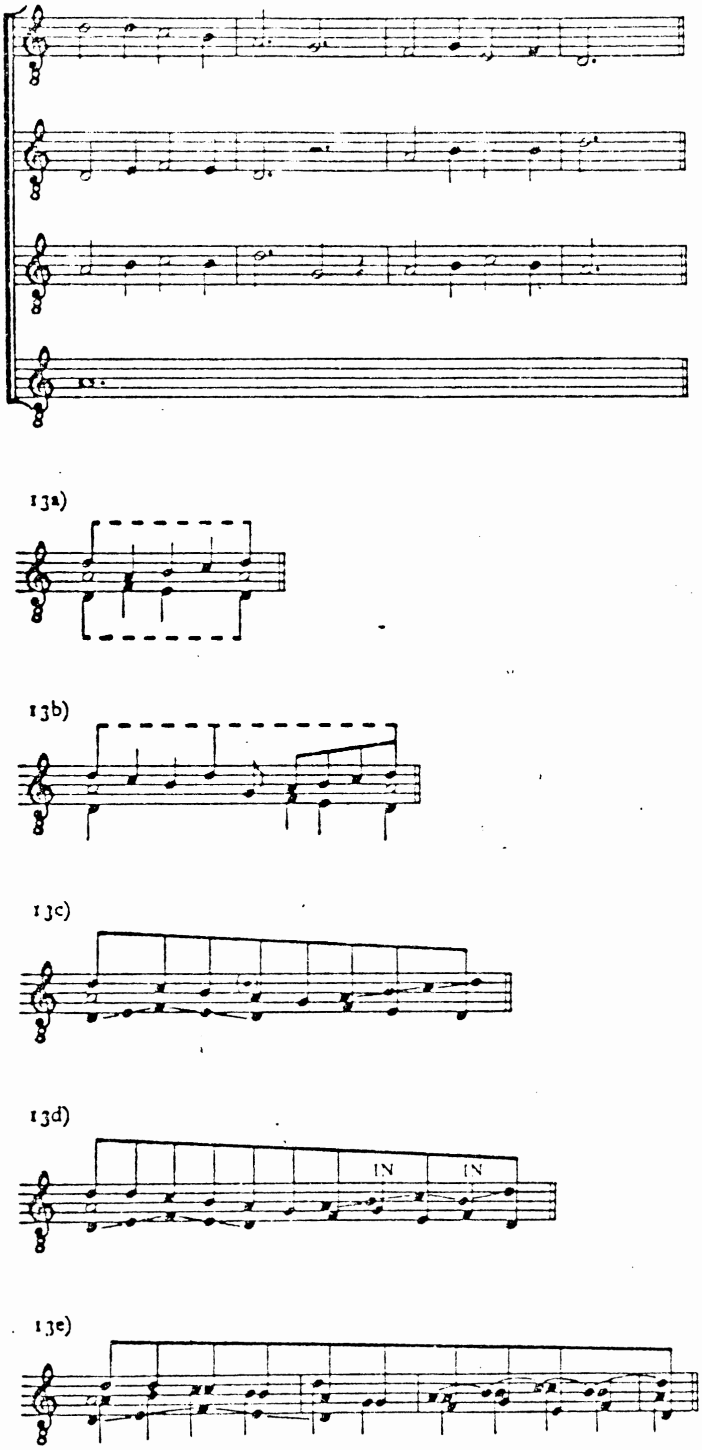
[9] F.Salzer,"Tonality in Medieval polyphony." The Music Forum I , Columbia University Press., 1967.

Example 6. Analysis of a fragment by Pérotin.
(after Salzer)



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10. F.Salzer, "Tonality...",p.79.

Salzer states that during the Middle Ages the underlying intervals expressed were the perfect consonances, that is,the fifth, the fourth, the octave and, in the case of three-voice textures, combinations of these intervals. Within the narrow scope of these intervals, voices frequently cross and even produce dissonances, yet express a common purpose. Gradually through the 14th and 15th centuries, however, a new usage of tonal language came into being, in which triads emerged and in which associations and relationships between triads were found to exist, finally resulting in harmonic progressions based on I - V - I. This, he maintains, gave contrapuntal voice-leading a different orientation and new perspective.

To go one step further than Salzer, this new perspective in the Renaissance could be defined by a complementary word: "harmonic/contrapuntal texture"[11]. In "harmonic/contrapuntal texture", which emerged at the beginning of the Renaissance, there are clear distinctions between the horizontal and vertical aspects of music. The function of a specific note is determined by its context to an even greater degree in "harmonic/contrapuntal texture" than in "modal/contrapuntal texture". In "harmonic/contrapuntal texture", for example, C as the root of a triad, will have a different function from C as the 7th of a dominant seventh, and C as a passing note will have a different function from C as a note of departure. It follows, therefore, that in "harmonic/contrapuntal texture" there are hierarchies among sonorities and voice-leading principles that help to direct the motion

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[11] The term is used in a general sense s.v. "Texture" in the Dictionary of 20th Century Music (London 1974):741-752.

of the music, which do not occur in "modal/contrapuntal texture", and these have formal implications that affect the rhythmic unfolding of melodies.

Music of the fifteenth century will naturally reveal characteristics of both "modal/contrapuntal" and "harmonic/ contrapuntal textures". If these two textures apply to medieval and Renaissance music respectively, they must interact during the period of transition and must influence the structures and length of phrases in different ways. By implication, proportioning will also be influenced by this gradual transition.

Further avenues and implications are explored by Gülke.[12] For him, the move from the Middle Ages to the Renaissance is governed by a purposeful inner logic in which all forms interact in the quest for new stylistic laws.

Among the smaller forms, the Rondeau with its dance-like and song-like elements brought about a simplification of style and melodic line. In this process the sounding together of notes became important, producing an emphasis on the vertical which, in turn, led to functional tonality. At the same time, the poetical metre of the Rondeau cramped the freedom of melodic motion. In the fauxbourdon canon this antimony of old poetic rules and new sounds was resolved. Here the tenor and superius form a duet and the tenor and contratenor another duet. The tenor acts as fundamentum relationis and this allows a greater freedom to the polyphonic lines. The question that remained was how to unite melodic lines.

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[12] P.Gülke,s.v."Niederländische Musik." Musik in Geschichte und Gegenwart(Bärenreiter-Kassel Verlag,1961).

For this to take place in works based on a cantus firmus, the double function given to the tenor had to fall away. It could not be both the carrier of the fundamentum relationis and the cantus firmus. In the isorhythmic motet,the cantus firmus had already lost much of its individuality by being bound to strict talea. Melodic interest therefore shifted to the two upper voices, which, under the influence of the Italian caccia,became imitative. Thus the way was freed for a greater harmonic interaction between tenor and contratenor. Since in this way the cantus firmus lost its structural importance, the motet assimilated the chanson-style. The unity of the whole was endangered by the increasing melodic intensity of the upper voices.

Gülke maintains that by 1430 the problem was solved. Now it was the contratenor that acted as fundamentum relationis so that the double function of the tenor could fall away. The tenor could then be used as a starting point for motivic development and the voices could become more and more homogeneous.

In the mass,the new style was extended and elaborated. There too, the fauxbourdon was influential as the common denominator that could simplify the motley of elements from conductus, motet and chanson. It was also in the mass that the imitative technique of the caccia found more general application. Thus, by 1440, all musical forms were written in the same unified style. The change that had been effected had drawn on feeling for melody as well as a new sensitivity for harmonic sounds.

In the process of change, as expounded by Gülke, the motet takes a central position. The dwindling importance of the tenor, the coupling of voices and the eventual integration of polyphonic parts, must have been reflected in the proportional relationships of the music. Since the changes in style and texture were the result of the new tonal processes at work at the beginning of the fifteenth century, those changes in the interaction and relationship of voices will affect the tonal and proportional characteristics in conjunction. For the purpose of this dissertation, the most important aspect of Gülke's approach is his detailed clarification of the interrelationships between voices at steps in the process of change. He implies that,with the move from a structure in which voices are primarily dependent on the tenor, to one in which voices are functionally dependent on each other, there may have been a gradual movement from an intervallic to a harmonic concept of tonality.

A similar view is expressed by Fox.[13] His analysis of secular songs of the fifteenth century reveals the increasing application of a tertian principle to the music. He maintains that in the age of Binchois and Dufay the superius and the tenor form a satisfactory duet, the tenor and the contratenor form a duet that is harmonically correct, but the superius and the contratenor cannot be performed as a duet, because there are frequent bold dissonances, notably perfect fourths.[14] In the later works of Busnois, on the other hand, any pair of voices will form a satisfactory duet in non-quartal style. This style seems to have been favoured from 1450 to 1520. While some composers were particularly fond of it (e.g. Busnois, Henry VIII, Morton and Hofheimer), no composer seems to have written all his songs in this

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[13] C.Fox,"Non-quartal Harmony in the Renaissance." The Musical Quarterly,XXXI (1945):33 - 53.

[14] Fox points out that theorists writing c.1500 classify this interval as a dissonance.

style. Fox points out that it may be regarded as an"ideal", in the sense that the absence of dissonances such as the fourth in this style, could represent a limit which may or may not be reached.

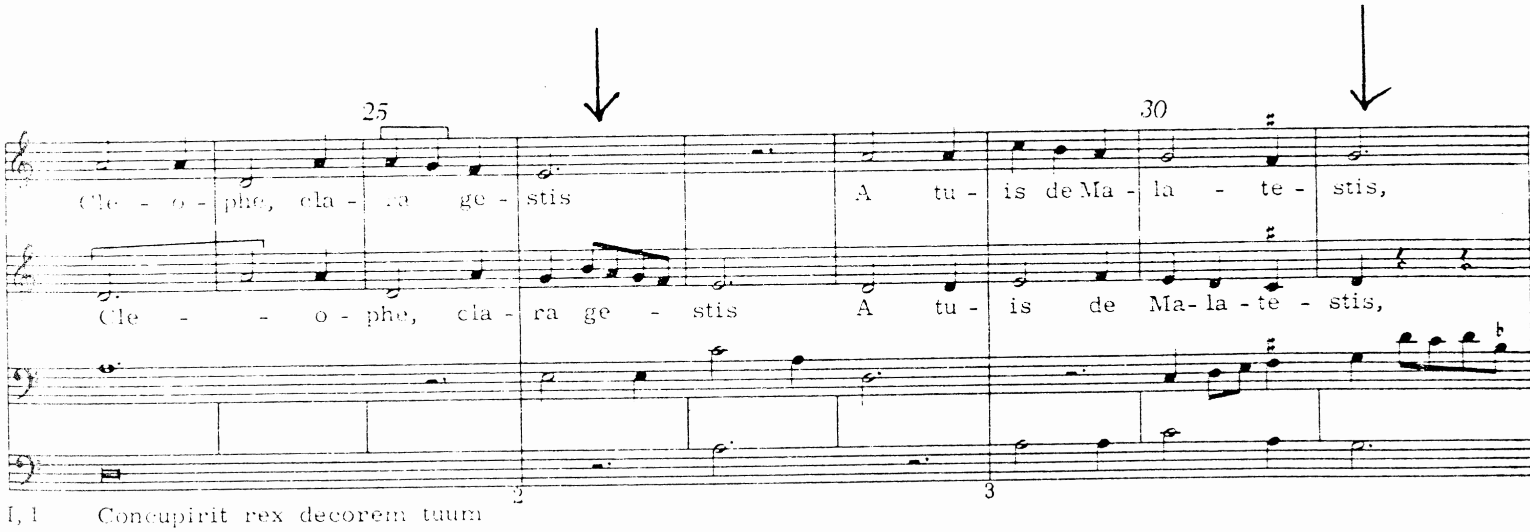
For the transition to be completed the altus had to be added to these non-quartal three-voice structures. The addition of the altus brought about a transformation in the harmonic structure of the music: without the altus, the remaining trios (for superius, tenor and bassus) are either in non-quartal style or practically non-quartal; when the altus is added, the fourth appears. Gradually the altus effected a change. While in the 1520's the altus was still an exceptional voice, from about 1540 the three upper voices came to be treated in the same way as regards the use of fourths. By this time, the only rule in regard to fourths was that they may not be used if they involved the lowest voice. Fox maintains that at this point the intervallic conception of harmony had been forgotten and that the transition from Middle Ages to Renaissance was now complete.

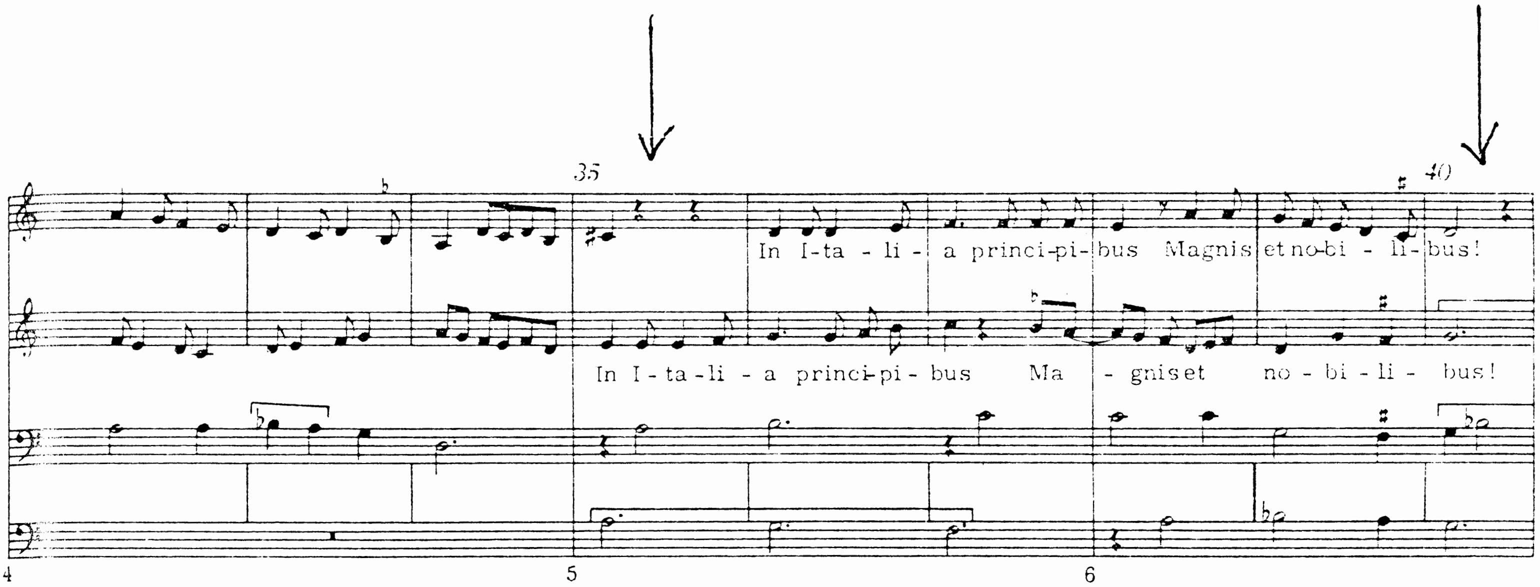
The application of these theories to the motets of Dunstable and Dufay.

The change of style, as it is discussed by these scholars, revolves around one central problem, namely the extent to which the voices are dependent upon each other and interact tonally. Essential to tonality is the degree of coherence achieved in a texture that is contrapuntally orientated. The contrapuntal aspect of the problem is most clearly defined by Gülke , although his delineation of the development can be most fruitfully amplified by taking into consideration the tonal and harmonic aspects as they are defined by Lowinsky, Fox, Salzer and Vinton.

According to Gülke, the first problem that had to be overcome in the transition of style, was the cramping of melodic lines that is in evidence in the Rondeau. Applying Gülke's theory to the motets of Dunstable and Dufay, the piece that best illustrates this problem is Vasilissa, ergo gaude. Although it has a certain amount of flexibility, it is essentially sectional in character. As illustrated by this example, the flow of the piece continuously comes to a standstill:

Example 7. Vasilissa,ergo gaude,bars 23-40.





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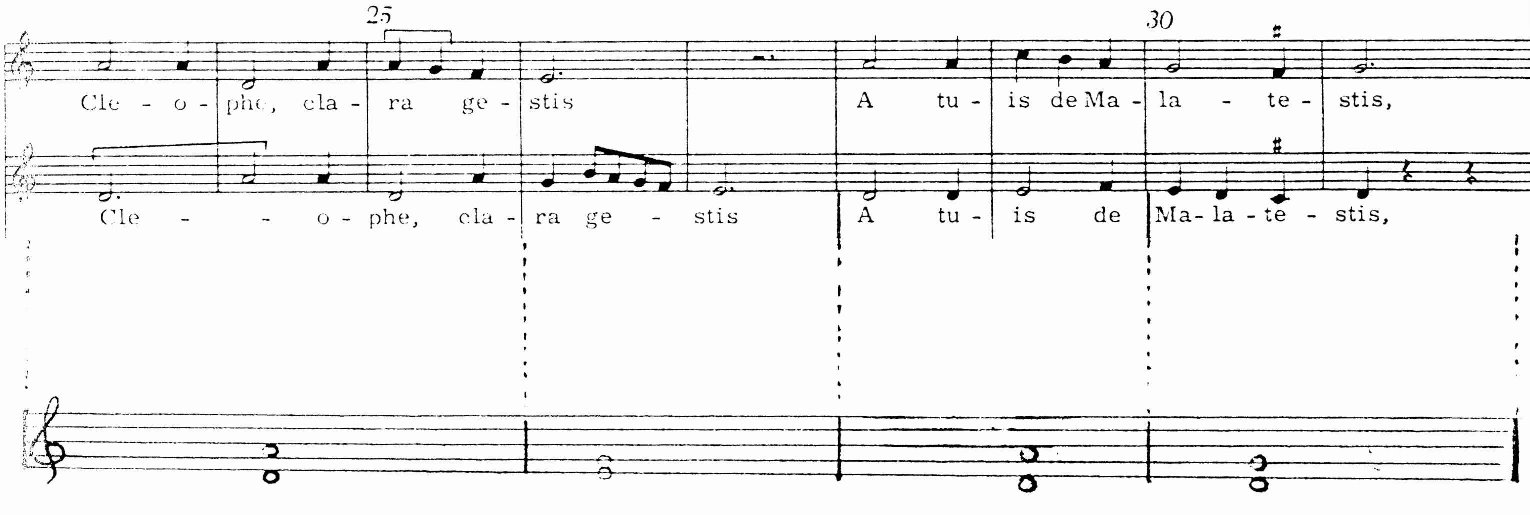
15. Corpus Mensurabilis, no.7, p.21 & 22, bars 23-35.

Here, cadences correspond to the end of phrases in the poetry and this implies a musical structure that is governed by poetic rules, rather than tonal structure. The frequent cadences can also be seen as the result of a musical style under the influence of a "modal/contrapuntal texture", in which the length of a phrase is limited by the narrow range of the interval that governs it:

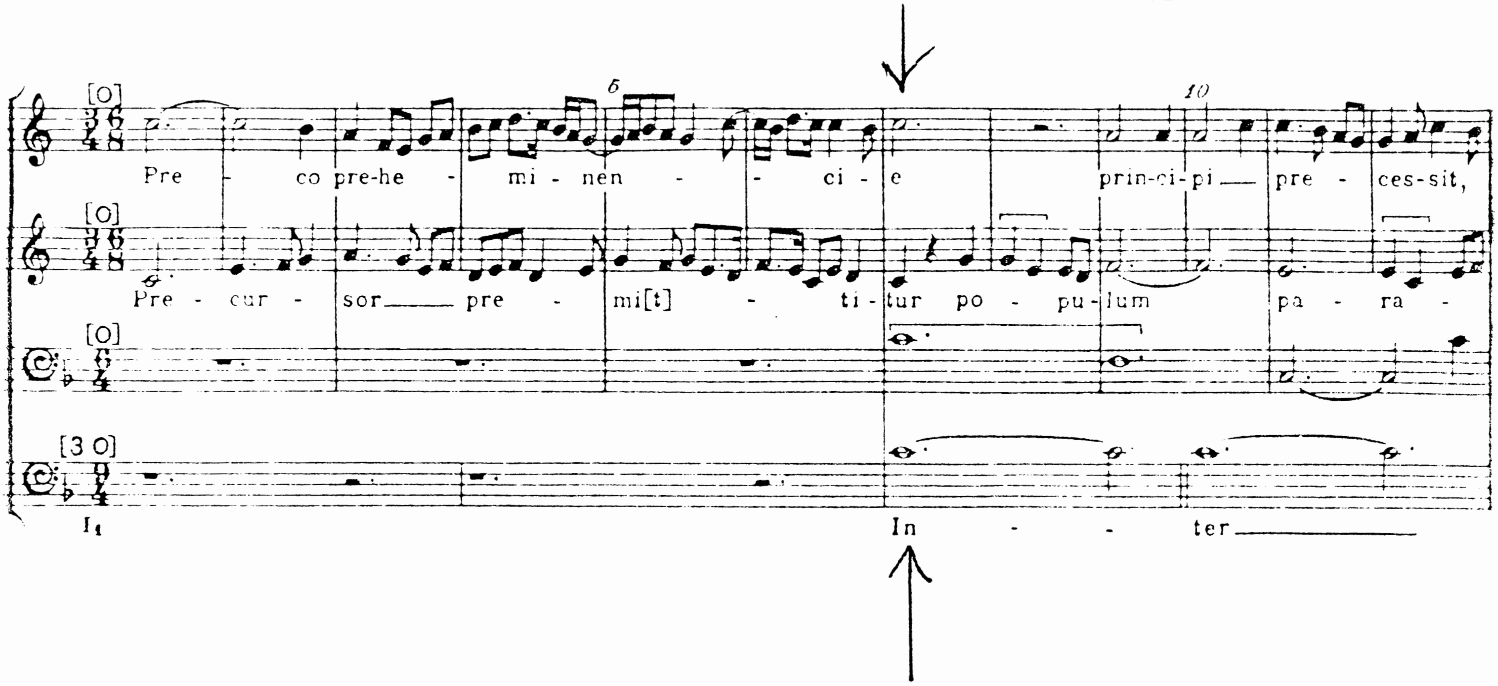
Example 8. Vassilisa, ergo gaude, motetus and triplum, bars 23 - 31, illustrating the narrow voice range.



Certain solutions to this problem can already be seen in Vasilissa. For example, there is already some stratification within the voice structure, tenor and contratenor forming one pair, and motetus and triplum another. This produces a structure in which a certain amount of flexibility exists, in that the duet of tenor and contratenor can move independently from the upper duet of motetus and triplum (see example 7).

This aspect is more fully illustrated in Dunstable's <u>Preco preheminencie</u>, where there are overlapping phrase structures, with one pair of voices starting where the other pair of voices cadences:

<u>Example 9.Preco preheminencie,bars 1 - 12.</u>



[16]

It can also be seen in the fact that while the aurally most recognisable cadences correspond with the proportions of the tenor, there are other, less clearly delineated cadences which occur at end of phrases in either the triplum or the motetus, thus creating a flexible structure.

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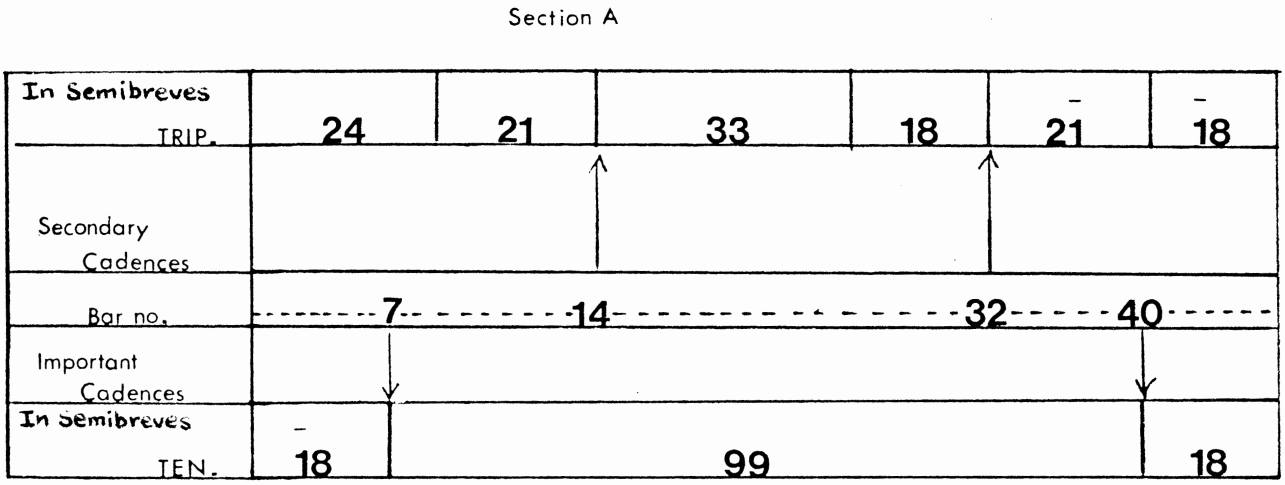
[16] Mus.Brit.,no.29,p.78,bars 1-12.

This is illustrated in the following diagram showing where cadences occur in the first section of Preco preheminencie:

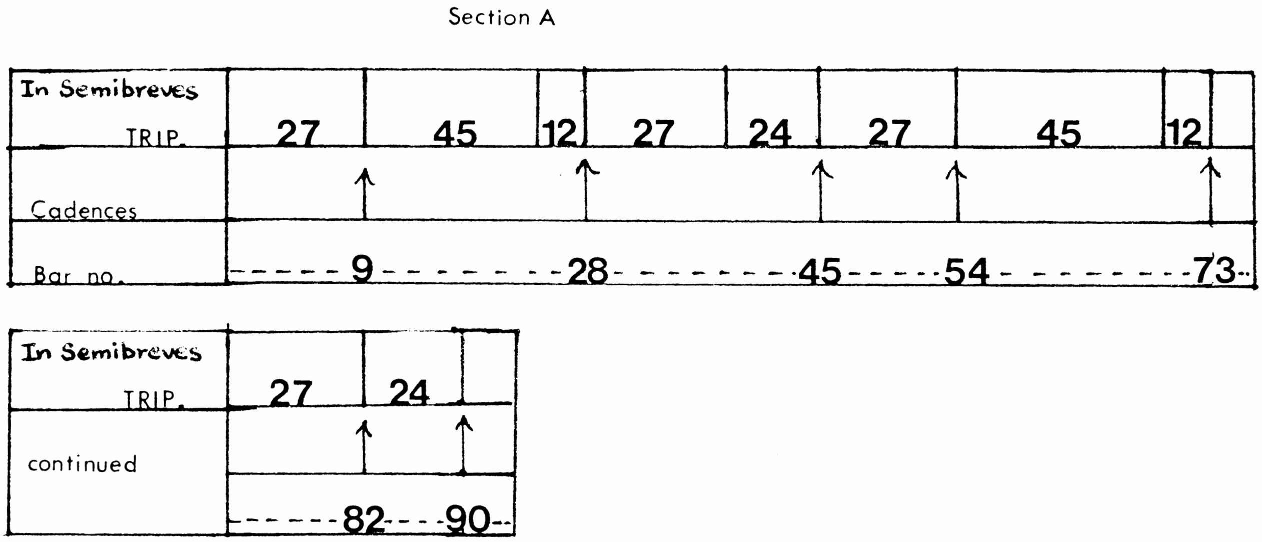
Diagram 3. A comparison of cadential structures and proportions in the first section of Preco preheminencie.



Coupling of voices implies the intervallic conception of harmony which has been noted by Fox. It also makes imitation possible between two voices, thus introducing a means by which voices can be integrated, and yet remain flexible.

With the introduction of a more "harmonic/contrapuntal texture", as can be seen in the pan-consonant style of motets such as *Maria, non est tibi* by Dunstable, the interdependence of coupled voices becomes less important . Here harmonic integration makes the intervallic conception of harmony less necessary. This, as Gülke suggests, led to a further polarisation of voices, and to music in which the melody takes on a dominant role. Dunstable's *Veni Sancte Spiritus - Veni Creator* is an example of this development. Here the most important cadential structures and proportions do not depend on the tenor, but on the triplum:

Diagram 4. A comparison of cadential structures and the proportions of the triplum in Veni Sancte Spiritus.



17

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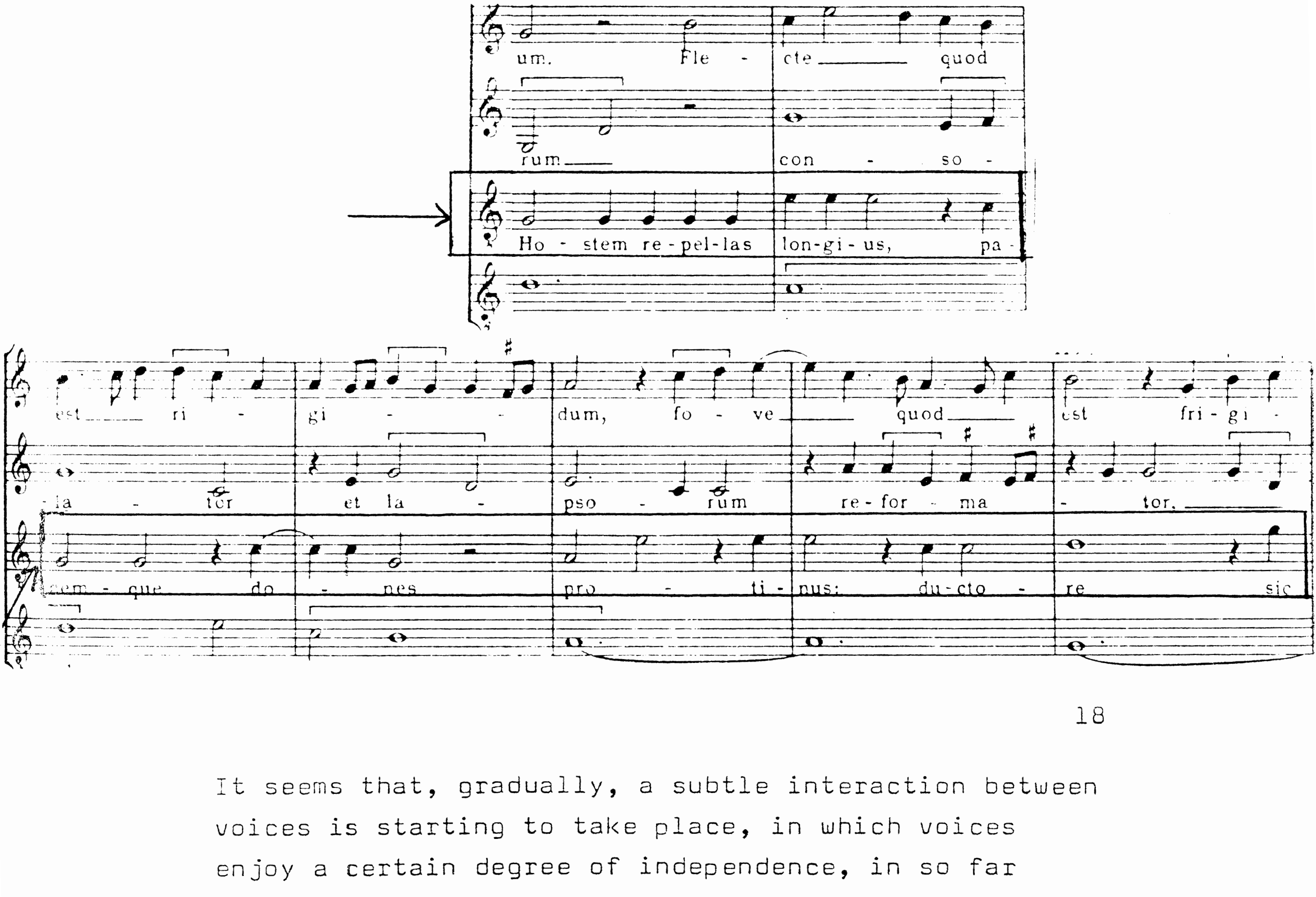
17. Mus.Brit.,no.32,p.88.

An effect of this polarisation is that the three lower voices become more homogeneous. Within this "harmonic/contrapuntal texture", the contratenor clearly takes the role of fundamentum relationis , as can be seen from its melodic contour in this example:

Example 10. Contratenor of Veni Sancte Spiritus, bars 109 - 115.



[18]

It seems that, gradually, a subtle interaction between voices is starting to take place, in which voices enjoy a certain degree of independence, in so far as phrases for individual voices can vary and overlap, but without losing their cohesion and harmonic value. This is particularly in evidence in Dufay's Nuper rosarum, where the introductory duos that take up the first half of each

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[18] Mus.Brit., no.32, p.88, bars 109-115.

section, with their clearly delineated cadences, stand in contrast to the subtle four-voice textures of the second half of each section. Here the integrated harmonic language creates a texture which leaves enough freedom for individual voices to overlap and diverge, but without threatening the coherence of the whole. This is largely effected by a floating triadic tonality with its balanced regulation of consonances and dissonances.[19]

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[19] Corpus Mensurabilis,no. 16,p.70.

A comparison of tonal and proportional procedures in the motets of Dunstable and Dufay.

This transition is pertinent to this dissertation in so far as proportional patterns may reflect this development. It is relevant to ask whether the developments in the tonal language are paralleled by the developments in proportioning already delineated at the beginning of this chapter. For this purpose, the motets will first be arranged according to their tonal and harmonic characteristics into three categories:

a. Motets in which voices are clearly coupled into separate duos.
b. Motets with a prominent melody in the upper voice.
c. Motets in which all voices are, to a large degree integrated.

Their tonal structures will then be compared to their proportional structure, to see whether there is any correspondence between these three categories and the kind of proportioning that is involved in each example. Classifications of this nature are, of course, arbitrary to a certain extent, since it is impossible to find 'pure' examples to fit each category. It is hoped, however, that this comparison will clarify the link between proportioning methods and tonality to some extent.

Diagram 5, illustrating the tonal characteristics of the motets of Dunstable and Dufay.

Motets in which voices are stratified into duos.and imply an intervallic conception of harmony.

DUNSTABLE	DUFAY
Albanus roseo rutilat	Vasilissa ergo gaude
Ave Regina	O gemma, lux
Dies dignus	Ecclesiae militantis
Gaude felix Anna	Balsamus et munda
Gaude Virgo	O, sancte Sebastiane
Specialis Virgo	Rite majorem
Preco preheminencie	
Salve scema	
Christe Sanctorum	

Motets in which the upper voice has a prominent melody.

DUNSTABLE	DUFAY
Veni Sancte Spiritus	Salve flos
Beata Mater	
Salve Regina miseris	
Speciosa facto es	
Nesciens Mater	
Veni Sancte - Veni Creator	
Beata Dei Genitrix	
Sancta Maria, non est	

Diagram 5,continued.

Motets in which voices are largely integrated.

DUNSTABLE	DUFAY
Sancta Maria succurre	Supremum est
	Fulgens iubar
	Moribus et genere
	Apostolo glorioso
	Rite majorem
	Nuper rosarum flores
	Magnanimae gentis

Diagram 6.

A comparison of proportioning methods and tonal structure in the motets of Dunstable.

A. Method of proportioning	B. Tonal organisation
Motets that are both geometric and arithmetic	
Albanus roseo rutilat	Stratified
Ave Regina	Stratified
Dies dignus	Stratified
Gaude felix Anna	Stratified
Gaude Virgo	Stratified
Specialis Virgo	Stratified
Veni Sancte Spiritus	Melodic
Beata Mater	Melodic
Speciosa facta es	Melodic
Motets that use simple proportions frequently	
Preco preheminencie	Stratified
Salve scema	Stratified
Nesciens Mater	Melodic
Christe sanctorum	Stratified
Alma redemptoris	Integrated
Motets making exclusive use of simple proportions	
Veni Sancte-Veni Creator	Melodic
Beata Dei Genitrix	Melodic
Sancta Maria, non est	Melodic
Sancta Maria succurre	Integrated

Diagram 7.

A comparison of proportioning methods and tonal structure in the motets of Dufay.

A. Method of proportioning	B. Tonal structure	date[20]
Motets that are both geometric and arithmetic		
Vasilissa ergo gaude	Stratified	1420
O gemma,lux	Stratified	1425
Ecclesiae militantis	Stratified	1431
Balsamus et munda	Stratified	1431
Motets that use simple proportions frequently		
Supremum est	Integrated	1433
Fulgens iubar	Integrated	1446
Moribus et genere	Integrated	1446
O, sancte Sebastiane	Stratified	1420
Motets making exclusive use of simple proportions		
Apostolo glorioso	Integrated	1426
Rite majorem	Integrated	1426
Salve flos	Melodic	1435
Nuper rosarum	Integrated	1436
Magnanimae gentis	Integrated	1438

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20. These dates,sometimes accurate and sometimes approximate, are suggested by C.Wright in "Dufay at Cambrai:Discoveries and Revisions.",*Journal of the American Musicological Society* (Philadelphia 1975): 175- 229.

The comparison shows that, in the majority of cases, geometric proportions occur in pieces with a stratified texture. Out of the nine motets by Dunstable that use both geometric and arithmetic methods of proportioning, six employ stratification and three are melodic. The four motets by Dufay that are geometric, are all stratified. Textures that are melodic or integrated make use of simple arithmetic proportions. Out of the five motets by Dunstable that make frequent use of simple proportions three are stratified, one is melodic and one integrated and in those that make exclusive use of simple proportions,three are melodic,one is integrated, but none is stratified. Out of the five motets by Dufay that make use of simple proportions, three are integrated and one is stratified, whereas the five motets that are exclusively arithmetic all are,to a large extent, integrated (Salve flos being the only one that is more melodic).

It follows that the tonal structure and the proportional structure of a piece must be viewed in conjunction, because it is the combination of these two aspects that most clearly reveals the aesthetic ideals reflected in it.

The symbolic implications of proportioning methods and tonal structure, viewed in a general historical context.

One of the distinctive contributions of historical research is that, in resurrecting movements of thought, it makes one aware of the imaginitive and emotional overtones of the ideas that support and pervade artistic achievements. It adds a dimension to our understanding in that it captures the fusion of thought and feeling as it is expressed in a musical language. Changes within a musical language will therefore reflect changes in man's emotive and intellectual response to his environment. Change, however, often implies a degree of conflict, because a change in man's perception of reality often occurs under the influence of experiences that contradict established values or modes of thought. Such a conflict can be traced through various stages, showing the degree of opposition or agreement existing between the old and the new.

Cumpsty[21], in writing on socio-cultural change in religion, has suggested a conceptual framework from which such historical conflicts can be viewed. In his opinion, historical change is often preceded by "a static stage" in which social behaviour follows a regular pattern and in which man's world-view pervades everyday life. This is followed by a period in which internal and external pressures disturb the established social pattern, where one finds escalating tensions in which society protects its tradition against the new("a protective stage"). Then it seeks common ground with the new and tries to incorporate as many new elements into the tradition as possible,

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21. J.S.Cumpsty,"A conceptual framework for socio-cultural change", unpublished paper,University of Cape Town,1979.

without changing its beliefs radically ("a search stage"), and finally escalates into a state of extreme paradox, in which the two traditions co-exist and are in direct opposition ("a paradoxical or irrational stage"). As a result of the gradual weakening of man's sense of belonging to his tradition during these stages, there is a move from a world-view that emphasises immanence to one that emphasises transcendence. This, for instance, will often lead to more mystical and personal religious beliefs, at times even to religious convictions that do not depend on the mediation of organised social institutions and traditions such as those of a church. There will naturally be many reactions against such attempts. Eventually, all this may lead to a stage in which synthesis is attempted ("an integrative stage") and during this stage, socio-cultural ideals that were in conflict are fused into a coherent whole, leading to a new unified tradition containing enough underlying predictability to be called "a static stage".

Scholars who have studied the Gothic period have often remarked on its many contradictions. Huizinga, for example, talks about the "absolute dualism"in the mentality of the medieval mind [22] and Sypher, of the "double experience of reality" [23] that predominated during the Gothic period. This implies a conflict which can possibly be related to Cumpsty's framework.

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[22] J.Huizinga,The Waning of the Middle Ages,p.175.

[23] W.Sypher, Four Stages of Renaissance Style,(New York 1956) p. 54.

The Romanesque epoch (ca. 900 - 1150) can be considered as a relatively "static stage", in which music served the aims and ideals of the church. The universal ideal of Benedictine simplicity, spirituality and asceticism produced a world-view dominated by a specific mode of worship in which Gregorian chant played an important role. The emotive power contained in Gregorian chant lies in its use of specific melody types, each one appropriate for a particular text and designed for a particular part of the liturgy. These melody types would evoke within the listeners familiar with these traditional patterns, certain collective emotions, which would involve them in an objective religious experience.

The unity and cohesion that existed within the framework of this Romanesque world was disrupted by the gradual assertion of secular ideals in subsequent ages. During the Gothic period (ca. 1100 - 1450) there is a mixture of rational and mystical ways of thinking which would have been unimaginable in the Romanesque era. Gothic man, with his Franciscan-mystical conception of reality, emphasised the personal and subjective religious experience. Stress is laid on human emotion and this is expressed in music that emphasises the subjective experience of the listener. This new ideal is, to a large degree, embodied in medieval polyphony. At work within the restless striving lines of Gothic polyphony there is an almost exaggerated expressive nuance and magnificence of colour which is in essential contradiction with the austerity of Gregorian chant. At the same time, however, the introduction of a mensural system (which was the inevitable outcome of a polyphonic art) created musical forms with a highly organised structure. These abstract mathematical structures reveal a strictness which seems to contradict the emotive and expressive qualities of polyphony. In this display of mathematical austerity, linked with a magnificence of colour, is revealed the inherent dualism of Gothic thought.

The Gothic period follows the pattern of change that characterises a period of conflict. First there is a "protective stage", in which musicians and public "enthusiastically favoured the new art of polyphony, while the ecclesiastic leaders and learned scholars opposed it, giving frequent expression to their feelings in their writings, pressaging the decay of musical art and deploring especially the intrusion of a secular spirit into the music of the church." [24] The attempts at fusion that are characteristic of the "search stage", is expressed in the fact that the church, following the path of accommodation, accepted the idea of polyphony, and, by fusing it to Gregorian chant (for example in the Organa), gave new meaning to its own traditions. It is also expressed in musical treatises of the Ars Antiqua, where theorists argue in favour of certain techniques, such as the reasoned use of perfections and imperfections by stating that they should be in accordance with"truth". During the Ars Nova, with its innovations, the two sets of values find themselves in open conflict, expressing a stage that can be called "paradoxical". During this period the importance, and even the mere recognisability

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[24] P.H.Lang , Music in Western Civilisation, New York,1941, p.139.

of Gregorian chant within the polyphonic texture recedes and loses its meaning. At the same time, the increasing flexibility and intensity of the polyphonic lines are coupled with increasingly complex mathematical structures, producing musical forms that truly reflect the tensions of a period of extreme conflict.

The"integrative stage" is expressed in the music of the early Renaissance. Now music is based on simple proportions and employs an expressive tonal language in which consonance and dissonance is regulated and in which polyphonic voices support and complete each other. Between logical orderliness and organic expressiveness a synthesis is created that is entirely in accordance with the ideal classical synthesis of concept and intuition, of thought and experience and of intelligence and sensibility.

In this respect, the motets that are perhaps the most expressive of the difference between Gothic and Renaissance attitudes, are Dunstable's Salve Regina and Dufay's Nuper rosarum. In Salve Regina the proportions reveal an infinity of symbolic relations. Each number may denote a number of distinct ideas and have several symbolic meanings and it is in the subtle interplay of these hidden meanings that the esoteric significance of the piece reveals itself. Thus, for example, the role of the symbolic numbers 888 and 152 in the piece can only be understood in the light of the text, which "is a prayer to Mary(152) to intercede with her Son (888) on behalf of the supplicants."[25] Likewise, the number 35, which is Plutarch's symbol of Harmony, fulfills this function in this piece, because it is the product of 5 (the Pentateuch, the Law of Moses) and 7 (the gifts of the Holy Ghost). The capacity of symbolic numbers such as these to assimilate hidden meanings expresses the truly medieval nature of such symbolism. The medieval mind, almost obsessed with the Absolute, reveals itself in the logic of symbolism, which, according to Huizinga,[26] demands that all meanings be connected to all other meanings and so create a harmony of ideas.

Nuper rosarum, on the other hand, is a concrete musical allegory, in which the measurements of the motet correspond exactly to those of the cathedral and the dome. Here, architectonic allegory and the sonorous polyphonic texture work together to produce an accurate expression of the dimensions of Brunelleschi's dome. In the same way, the use of a double tenor, which, according to Warren,[27] corresponds to

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[25] B. Trowell, "Proportion...", p.117.

[26] J. Huizinga, p. 198.

[27] C. Warren, p. 98.

the double cupola of Brunelleschi's dome, is not merely symbolic, but aesthetic, in the sense that the amplified sonorities that result give an aural impression similar to the visual impression created by the "more magnificent and swelling form" of Brunelleschi's double cupola.

Symbolism, as it is found in Dunstable's Salve Regina, is distinguished by an impeccable order that is in tension with the subtle and restless movement of the melodic lines. In Dufay's Nuper rosarum symbols have become outwardly expressive and it is no longer possible to separate them from the actual musical language. Without a word of caution, this comparison cannot be made into a general principle. Fifteenth century polyphony moves in a world in which the systematic and esoteric idealisation that characterises the Gothic mind, interacts with the tangible and exoteric expressivity that characterises the Renaissance. Long after their inception, these tendencies can be seen to co-exist and inform artistic creation.

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ABSTRACT GEOMETRY IN THE MOTETS OF DUNSTABLE AND DUFAY

VOL. 2

P. ROMMELAERE

Diagrams illustrating the use of Proportioning Methods in the Motets of Dunstable and Dufay

Volume II consists of 37 diagrams illustrating the use of proportioning methods in the motets of Dunstable and Dufay. These illustrations are explained in chapter II of Volume I.

Chronological table.

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Léonin	O 1 Judea et iherusalem	see vol. I
	036 Vir iste	
	M15 Alleluya	
	M22 Alleluya	
	Gloria Patri	
Anonymous.	Excelsus in numine	1
14th century		
Anonymous.	O livor/Inter amenitatis/Revertenti	2
Philippe de Vitry.	Floret/Florens/Neuma	3
	Hugo Princeps	4
	Vos qui admiramini	5
15th century		
Dunstable	Albanus roseo rutilat	6
	Ave Regina	7
	Dies dignus	9
	Gaude felix Anna	10
	Gaude Virgo	11
	Specialis Virgo	14
	Veni Sancte Spiritus	15

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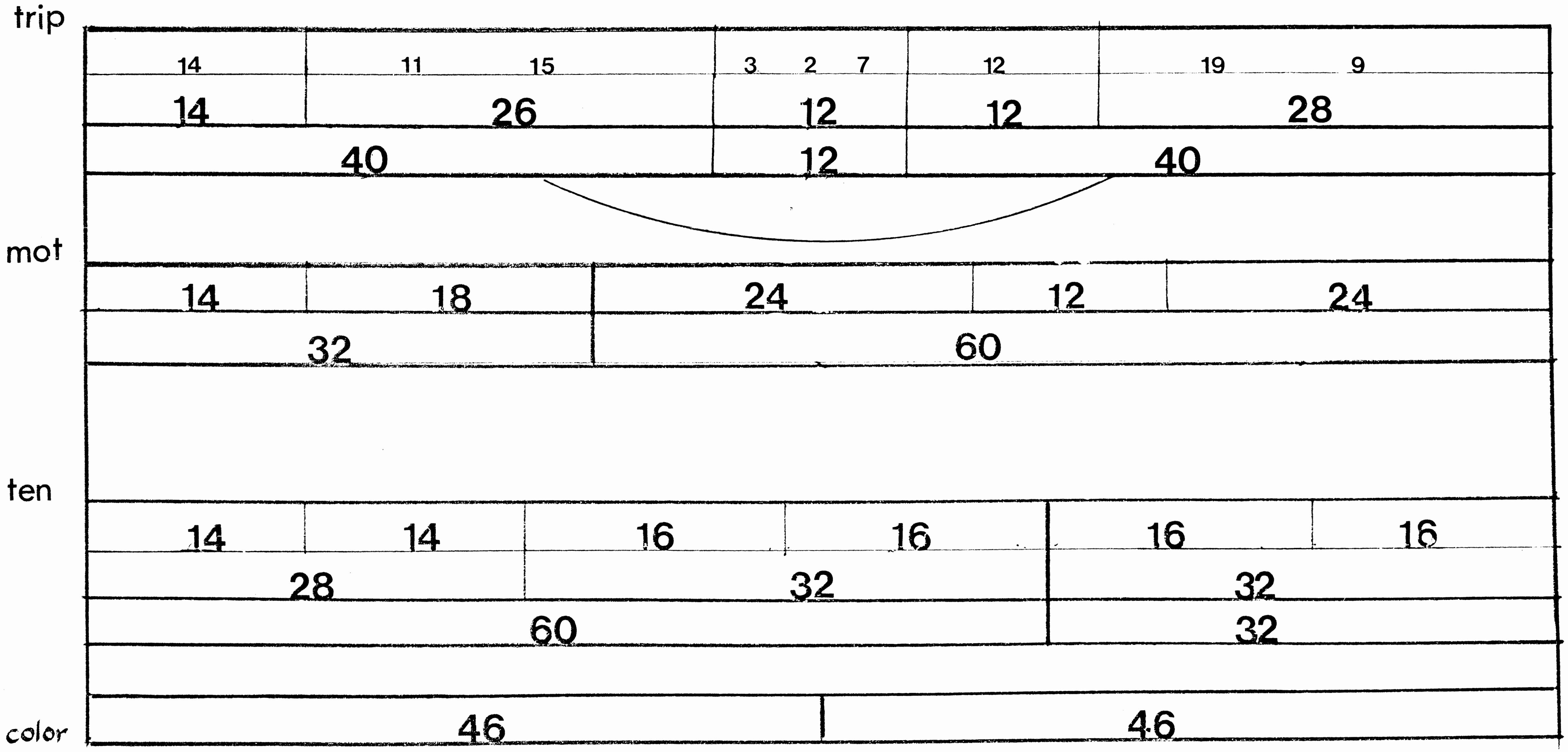
Dufay	Vasilissa ergo gaude	1420	25
	O, sancte Sebastiane	1420	26
	O gemma, lux	1425	27
	Apostolo glorioso	1426	28
	Rite majorem	1426	29
	Ecclesiae militantis	1431	30
	Balsamus et munda	1431	31
	Supremum est	1433	32
	Salve flos	1435	33
	Nuper rosarum	1436	34
	Magnanimae gentis	1438	35
	Fulgens iubar	1446	36
	Moribus et genere	1446	37

* In the case of Dunstable's motets, no accurate dating is as yet possible.

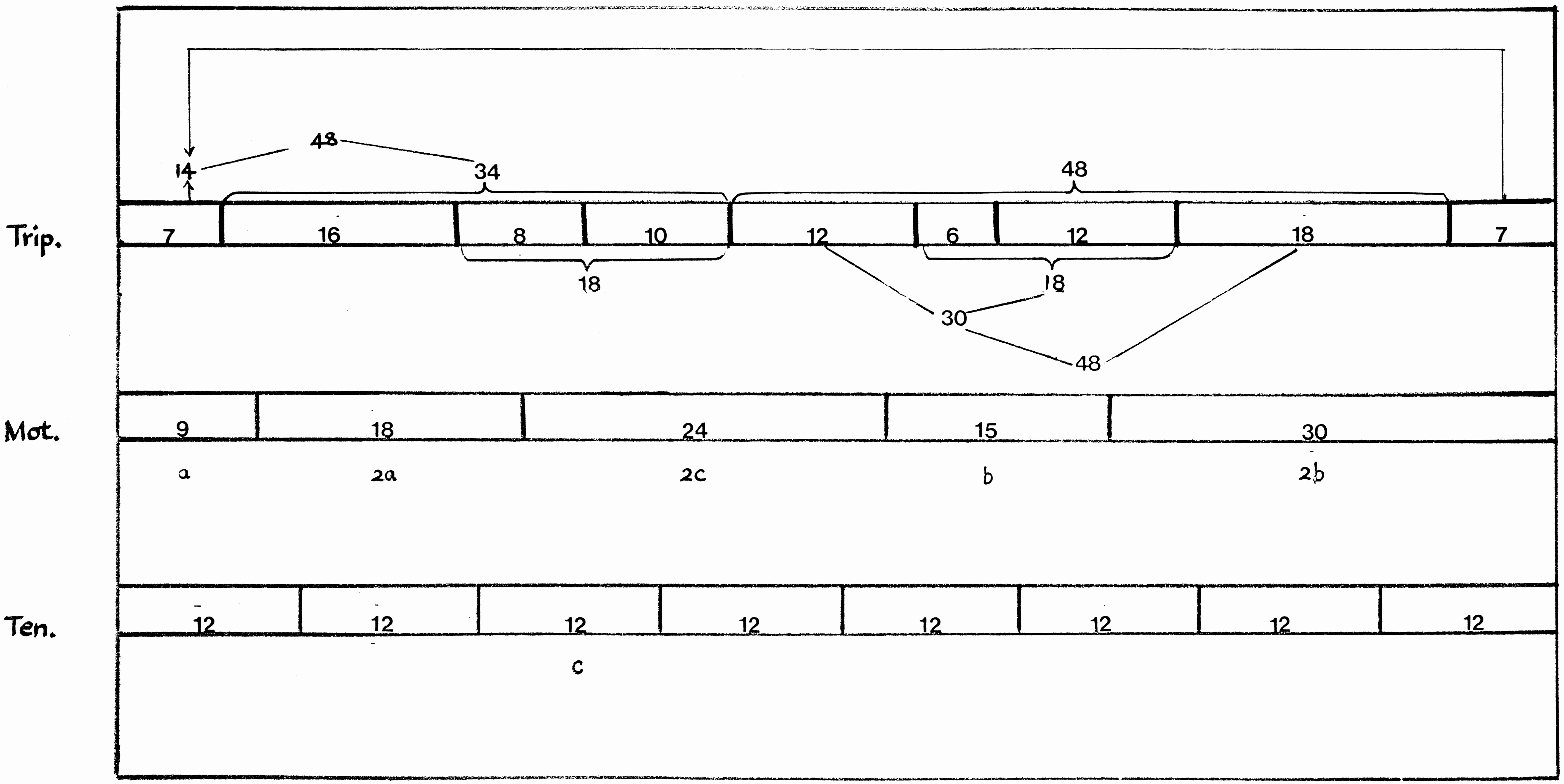
Excelsus in numine / Benedictus Dominus / Tenor

Anonymous



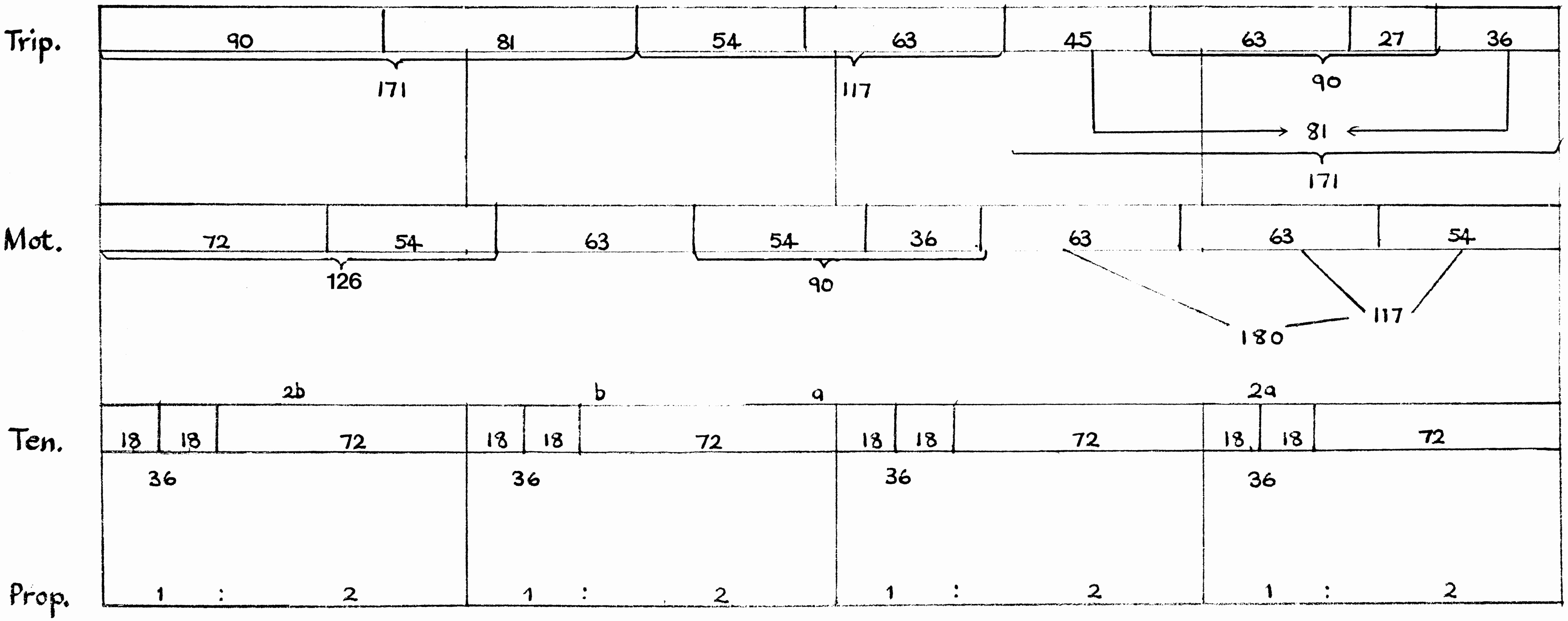
O livor / Inter amenitatis / Revertenti

Anonymous



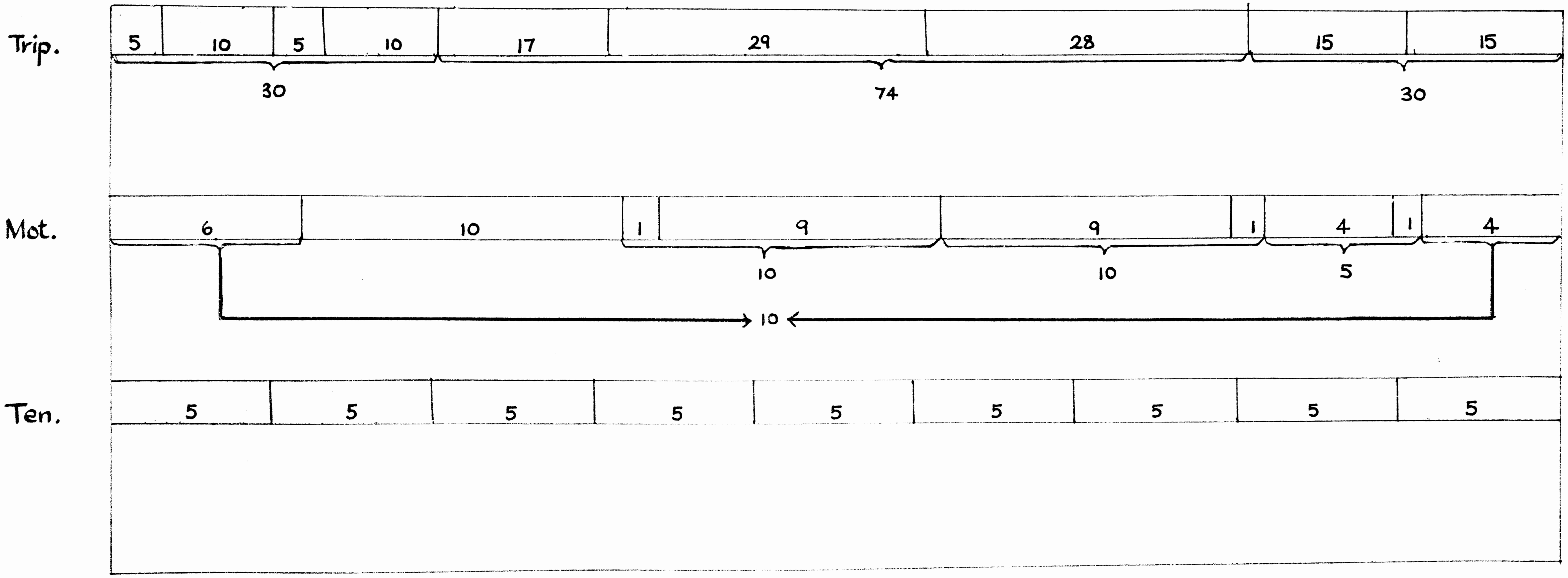
Floret / Florens / Neuma

Philippe de Vitry



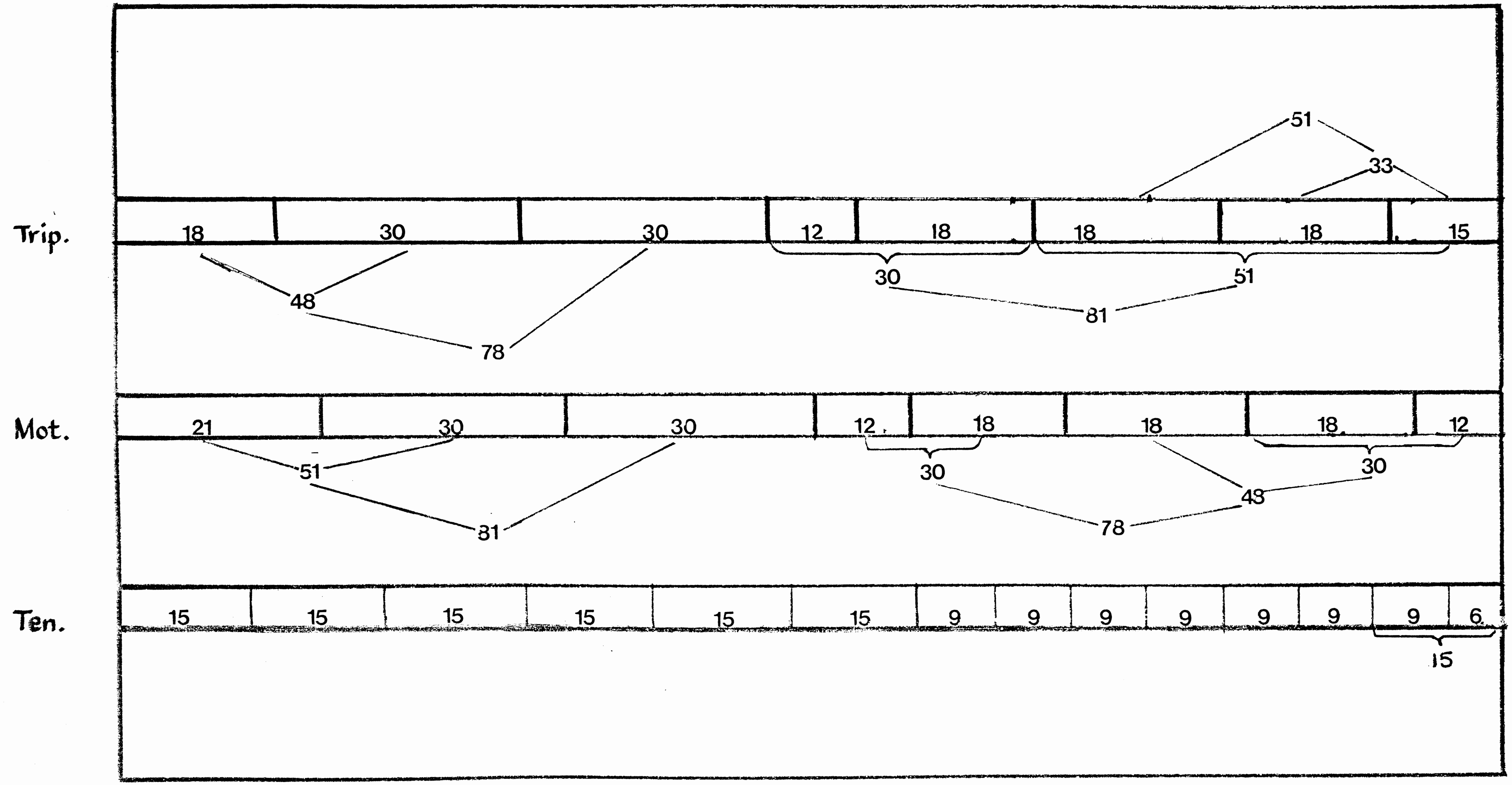
Hugo princeps

Philippe de Vitry

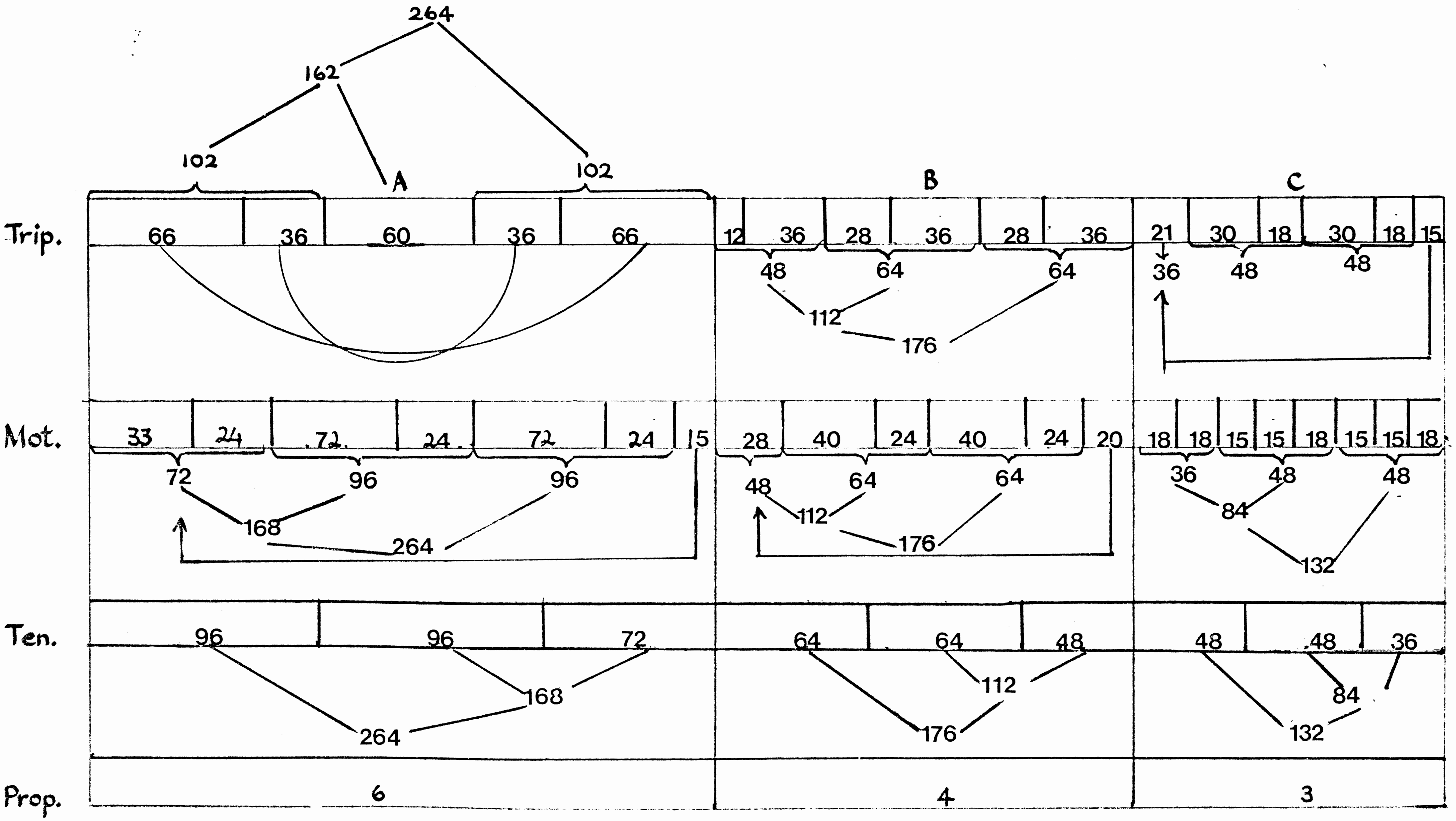


Vos qui admiramini

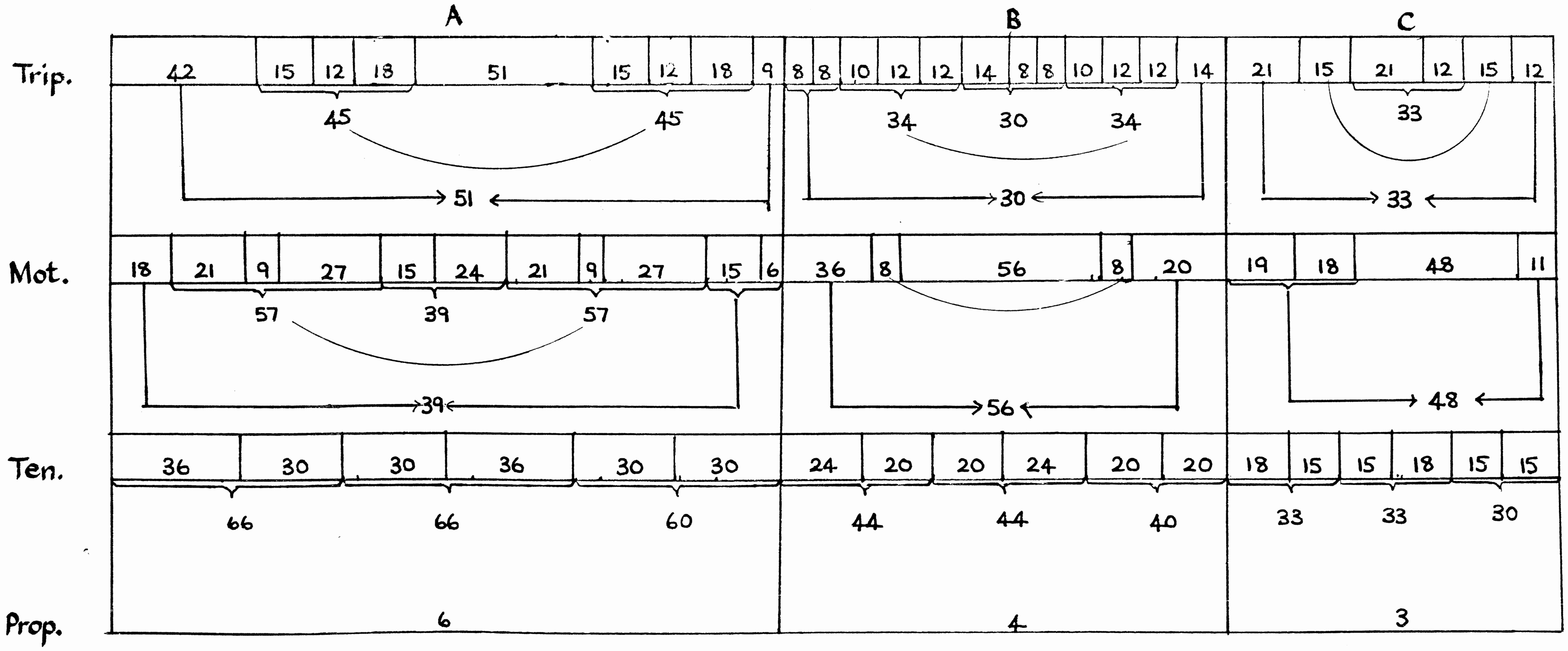
Philippe de Vitry



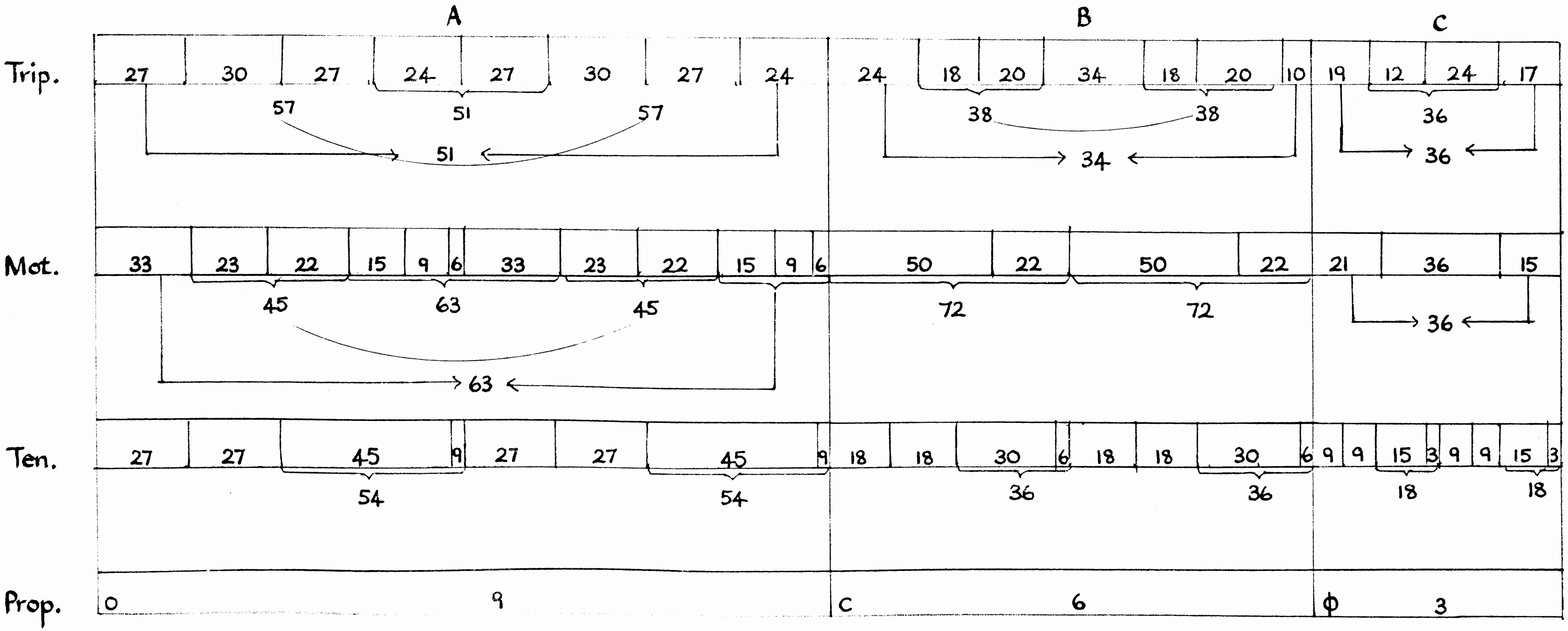
23. Albanus roseo rutilat



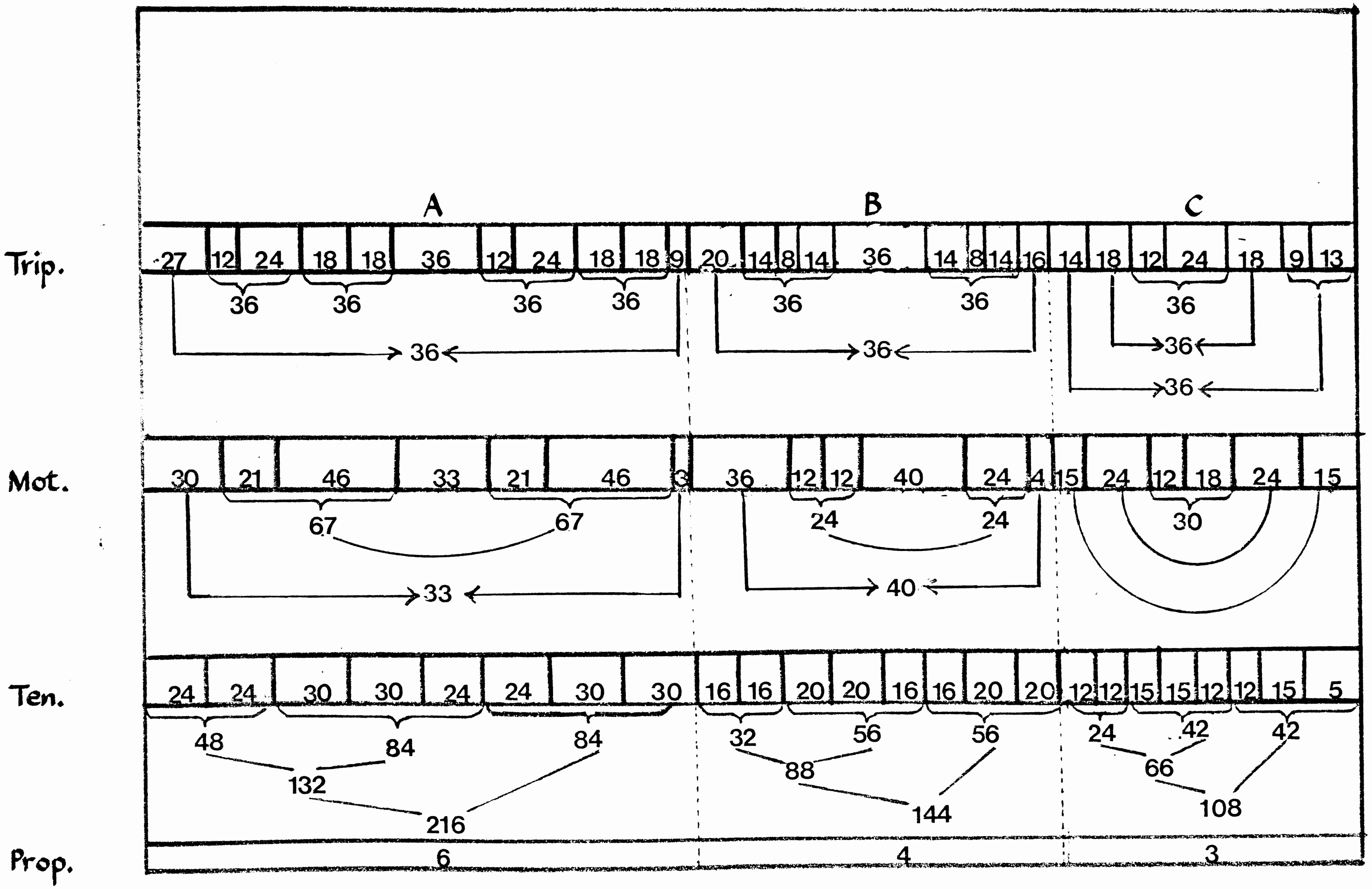
24. Ave Regina celorum



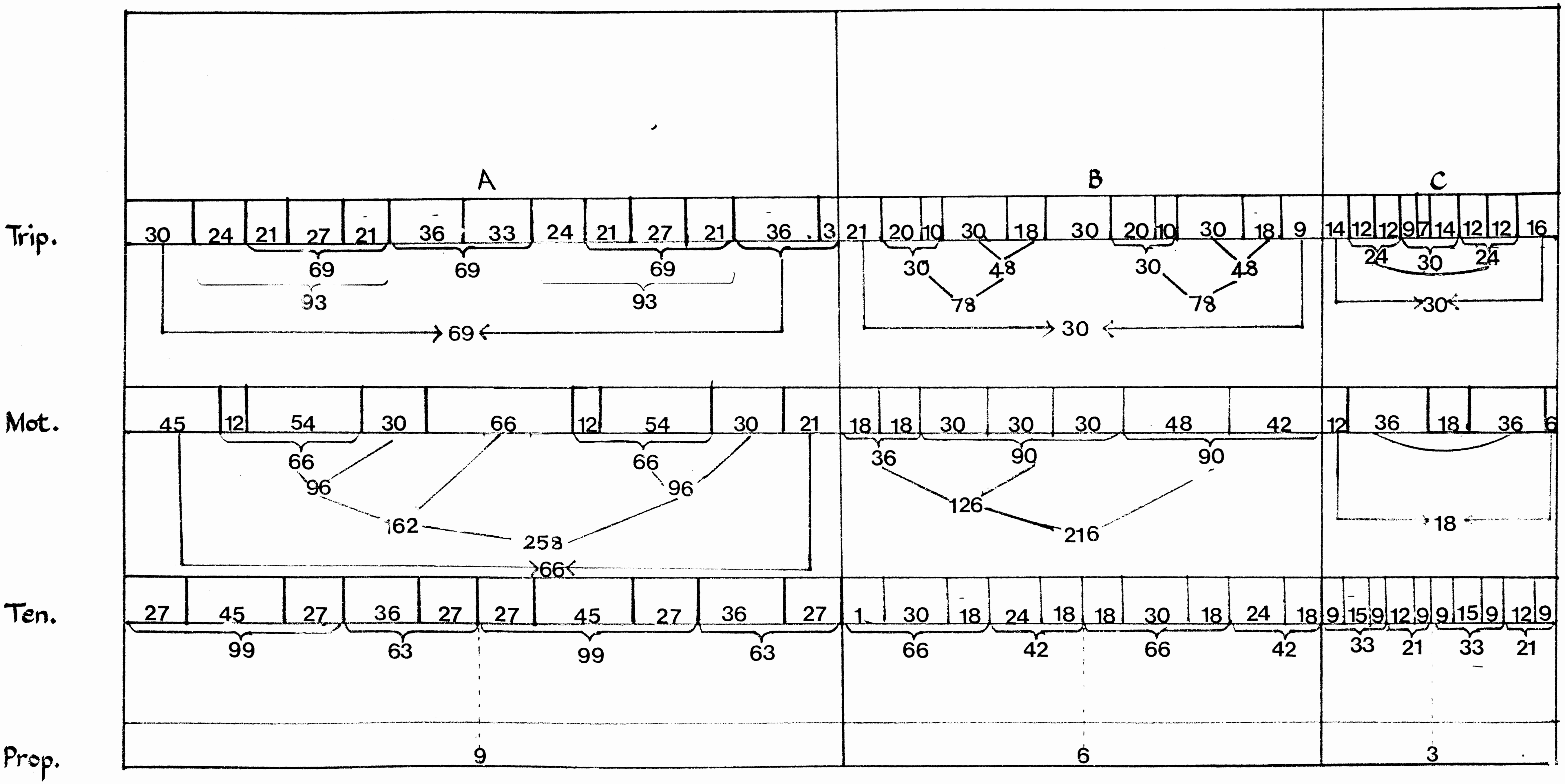
25. Christe sanctorum



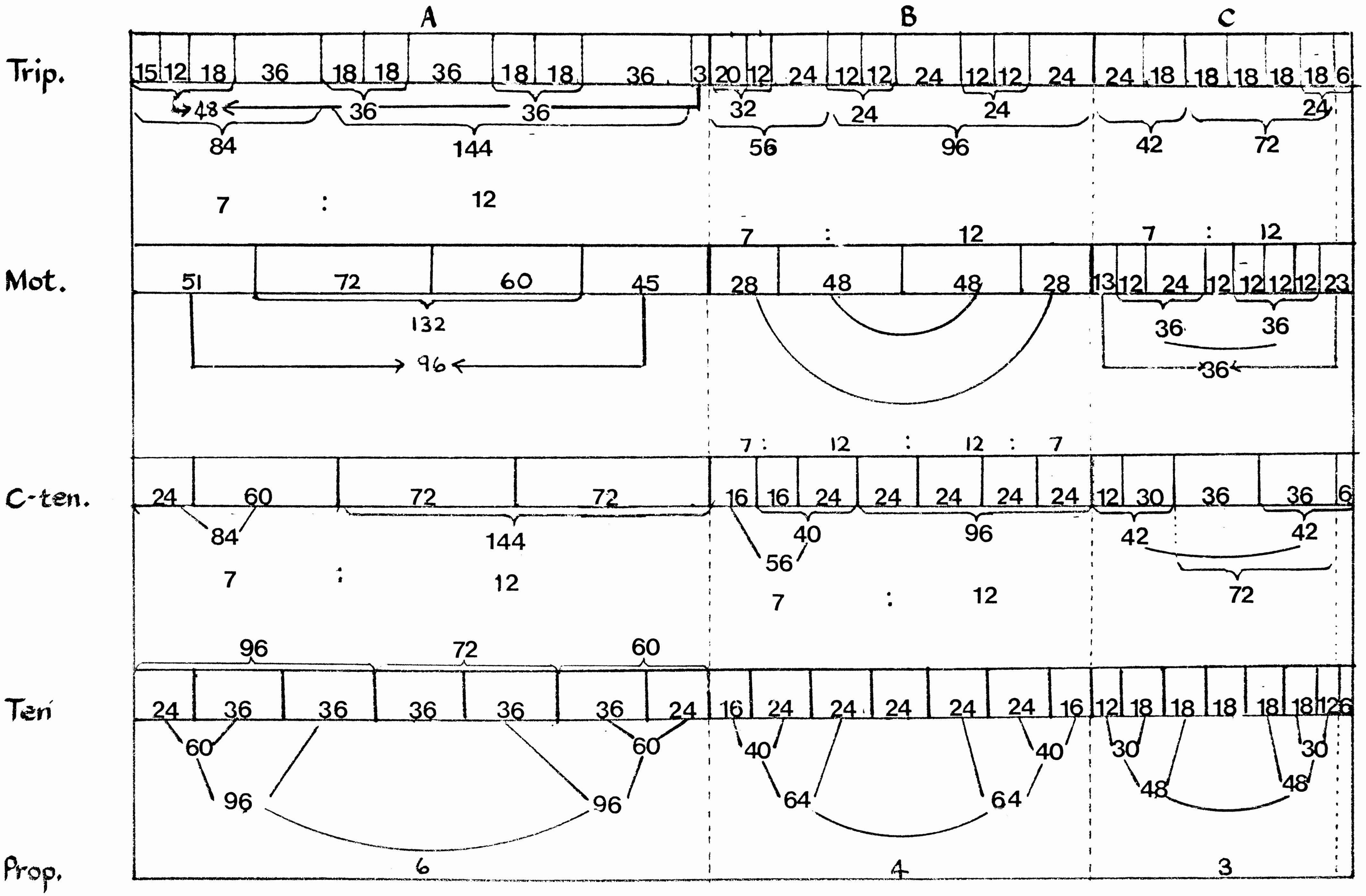
26. Dies dignus decorari



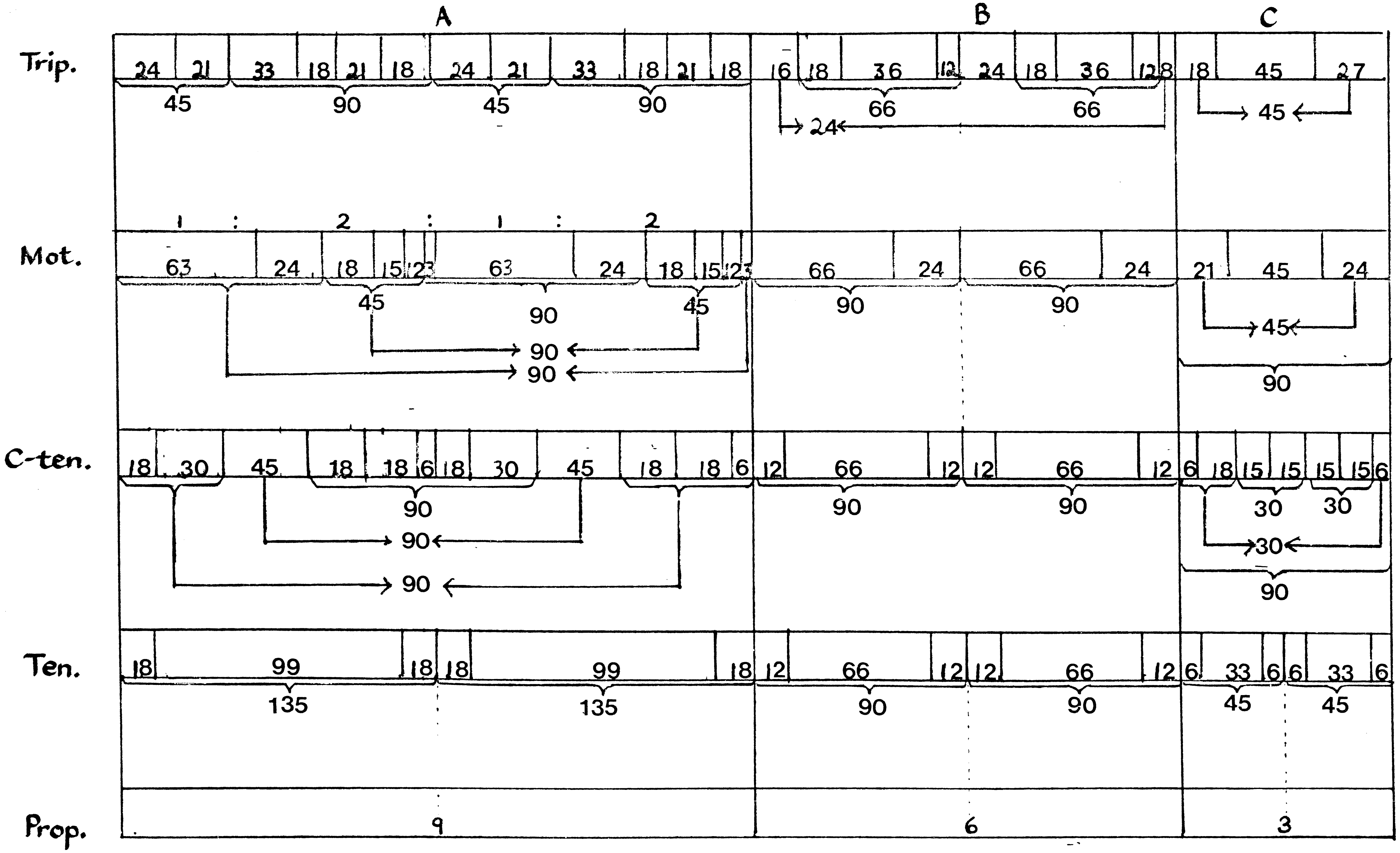
27. Gaude felix Anna



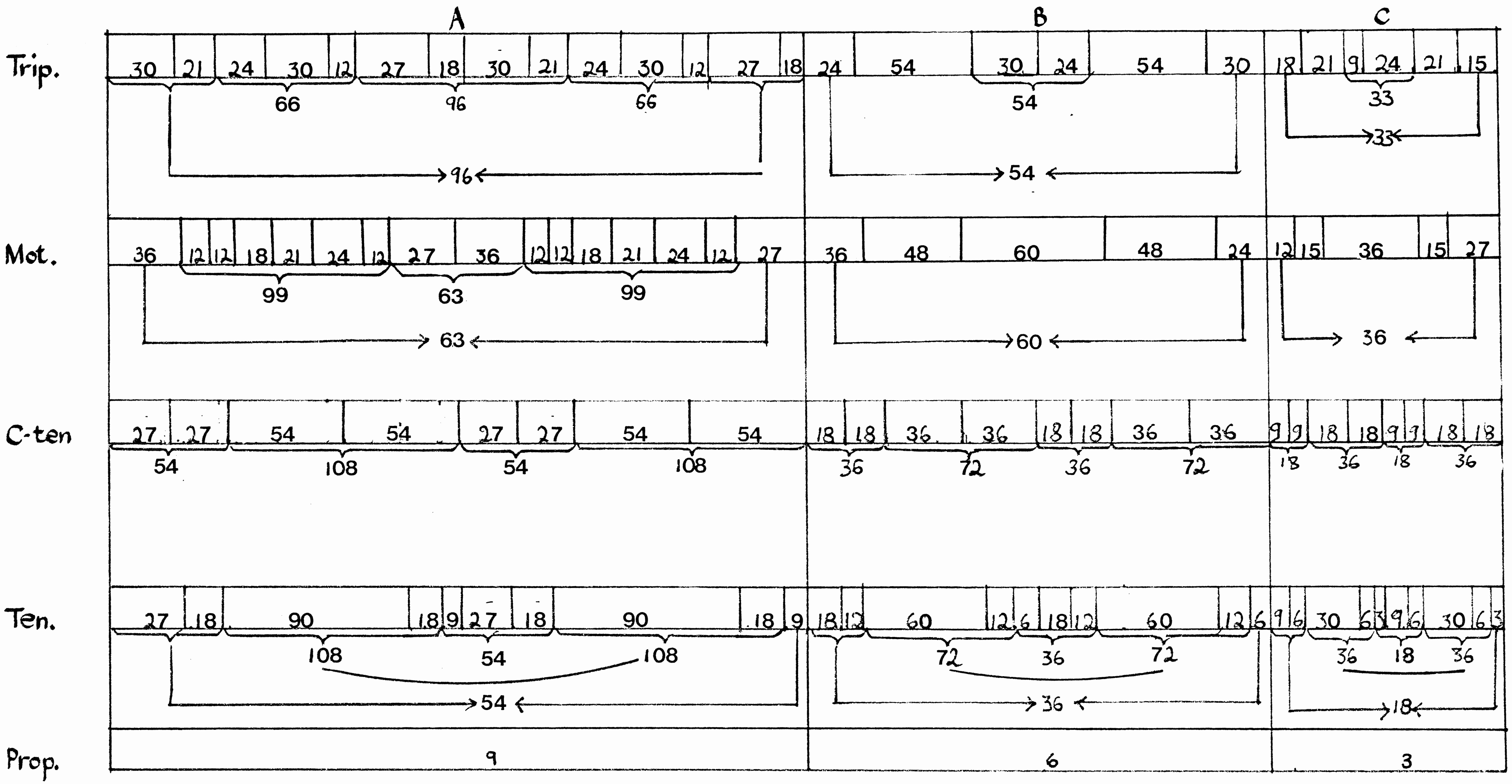
28. Gaude Virgo salutata



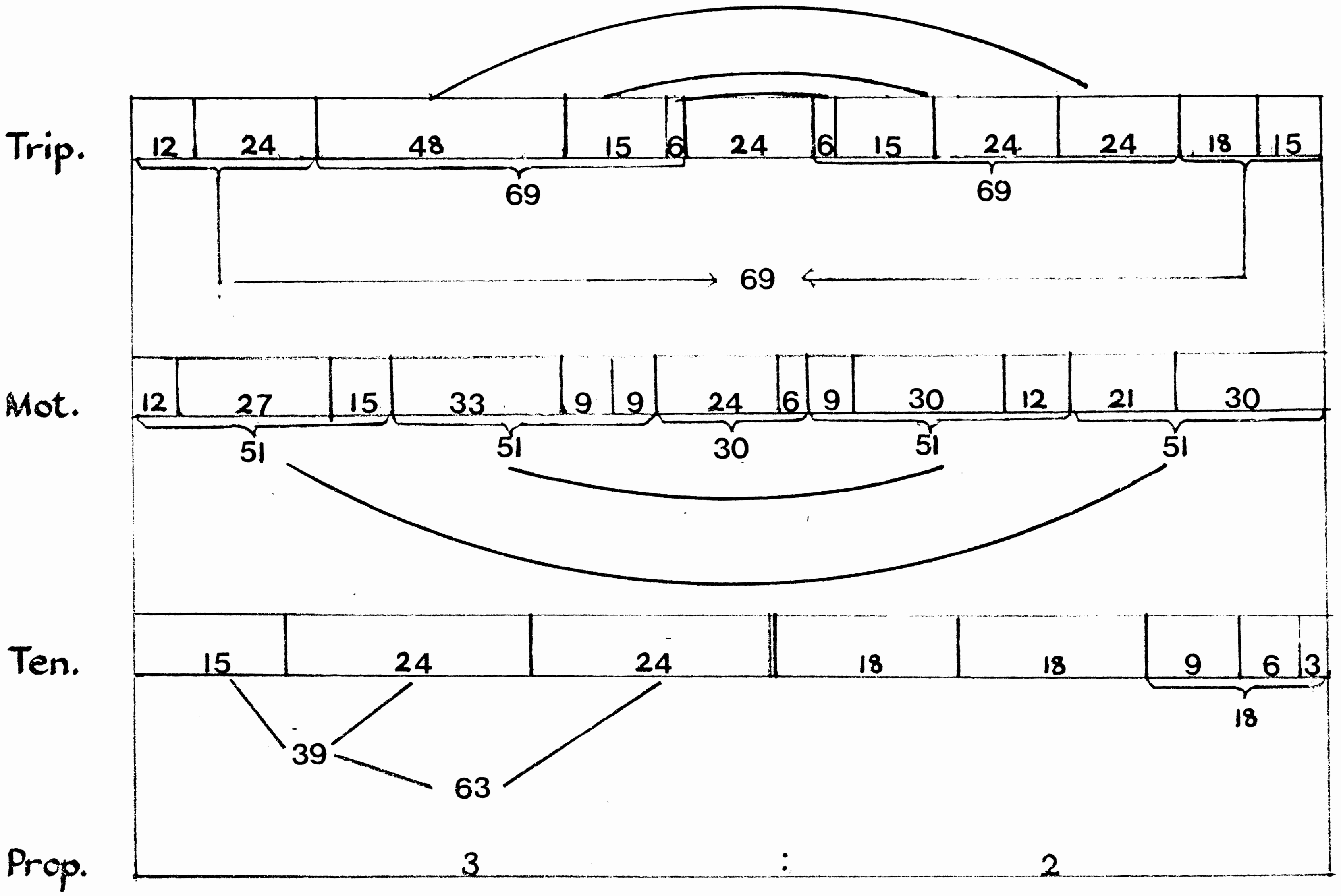
29. Preco preheminencie



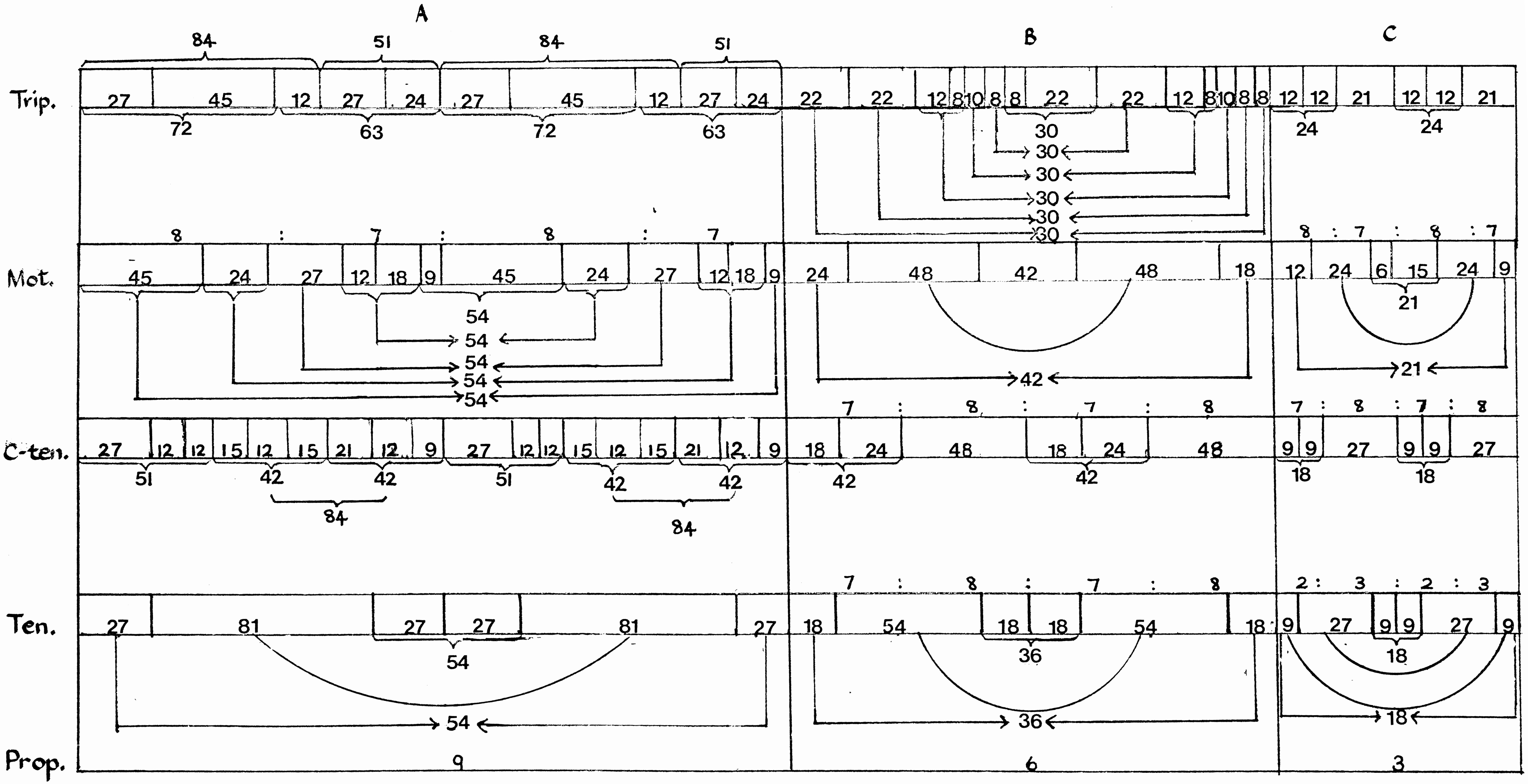
30. Salve scema sanctitatis



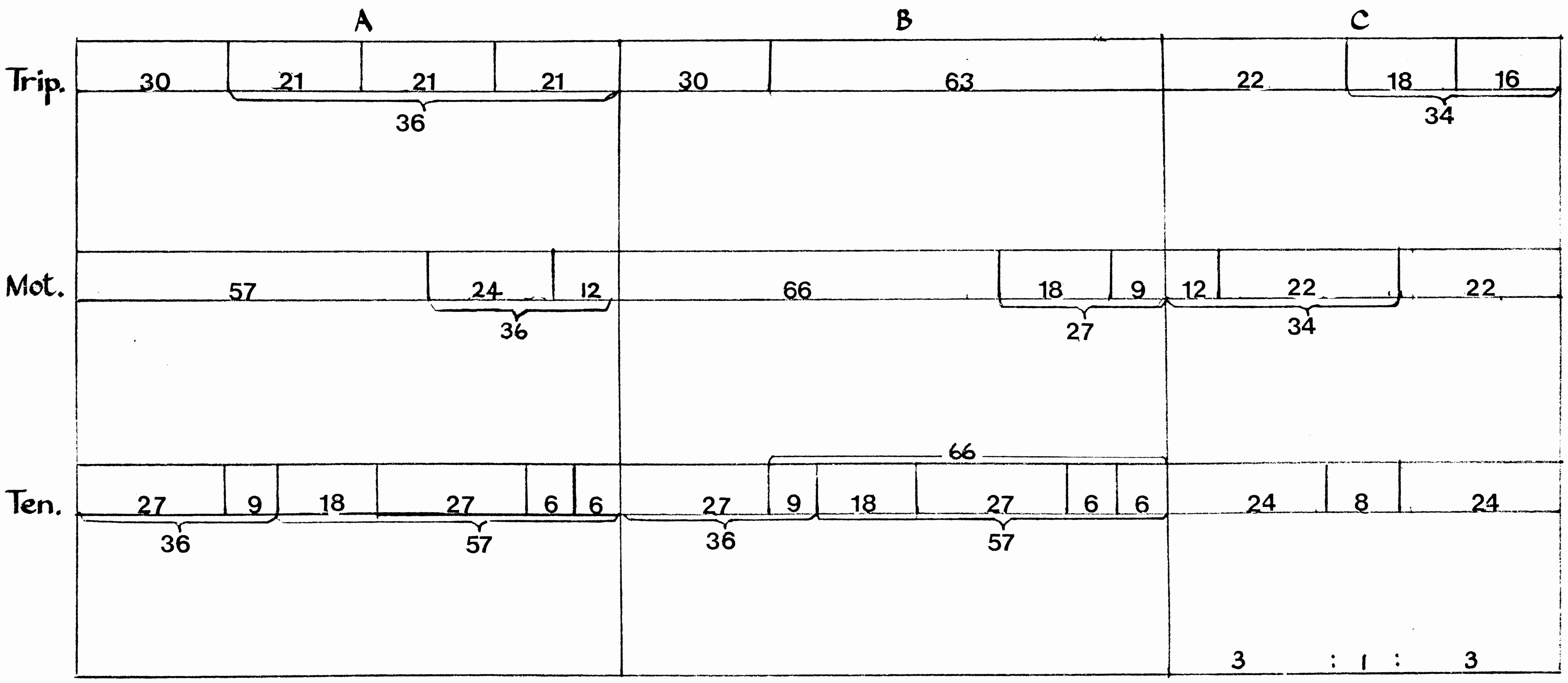
31. Specialis Virgo



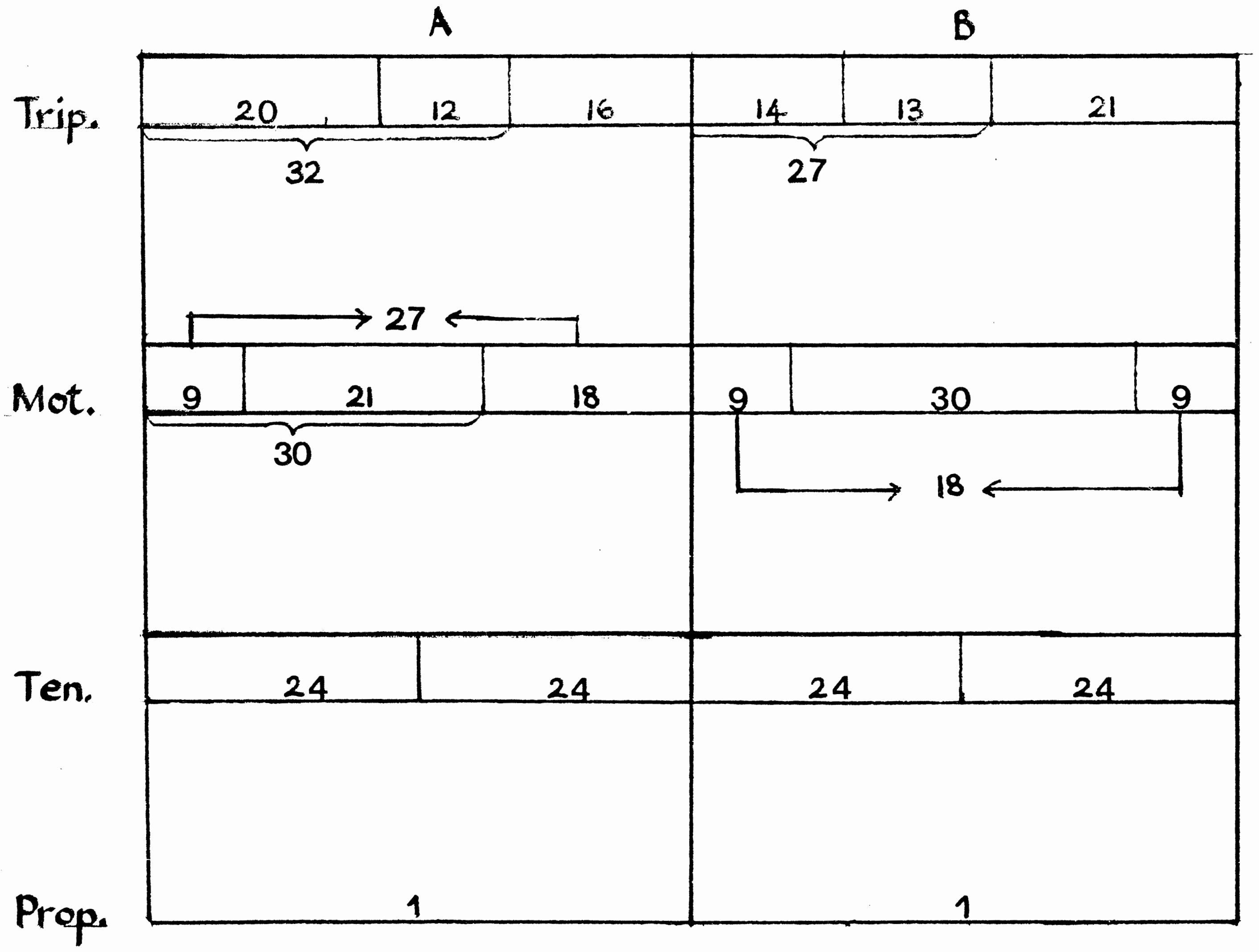
32. Veni Sancte Spiritus



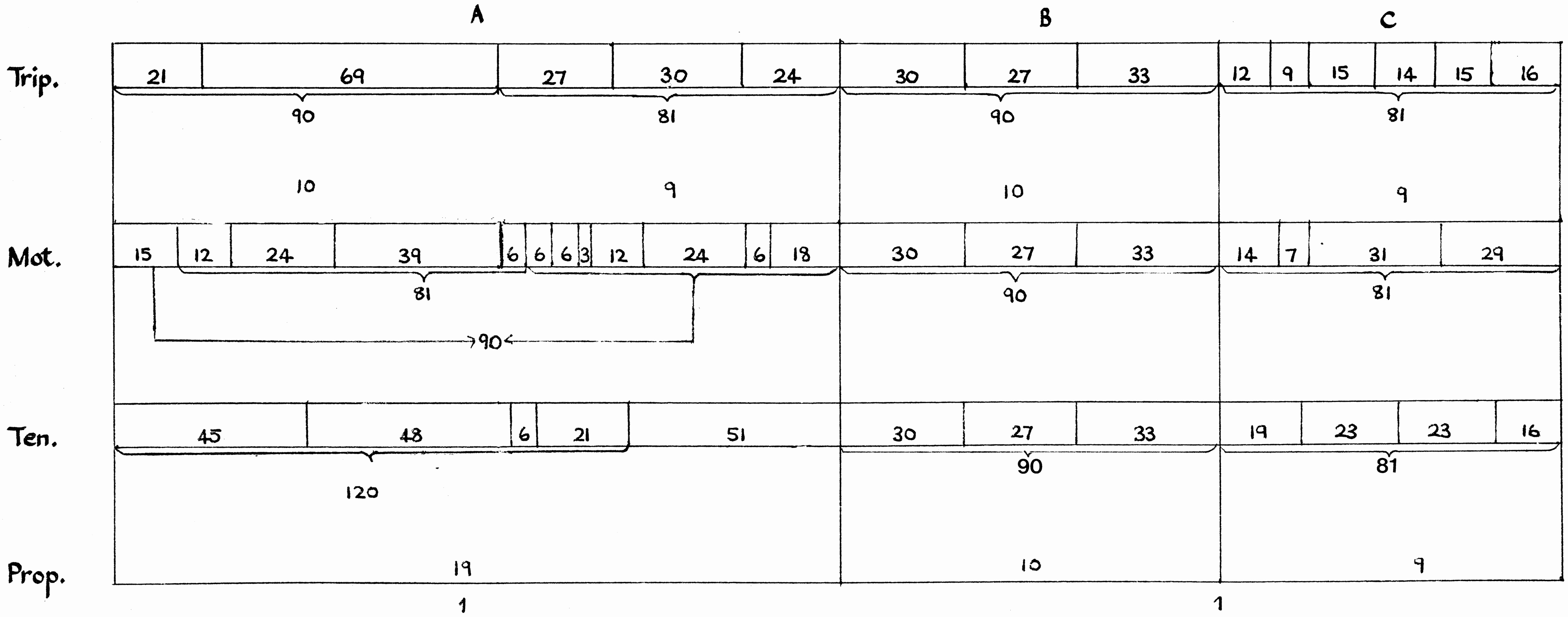
33. Veni Sancte Spiritus



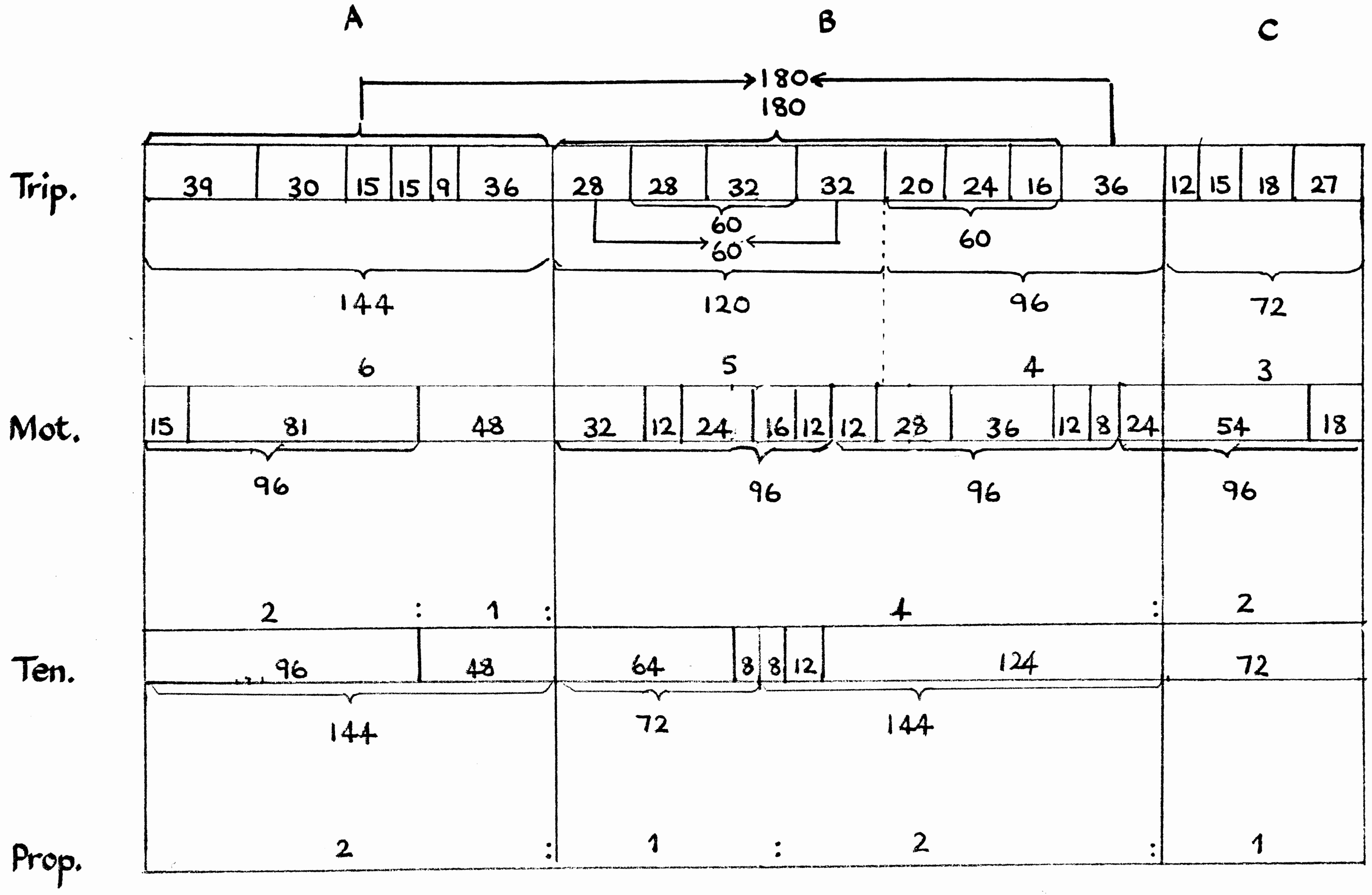
34. Nesciens Mater Virgo



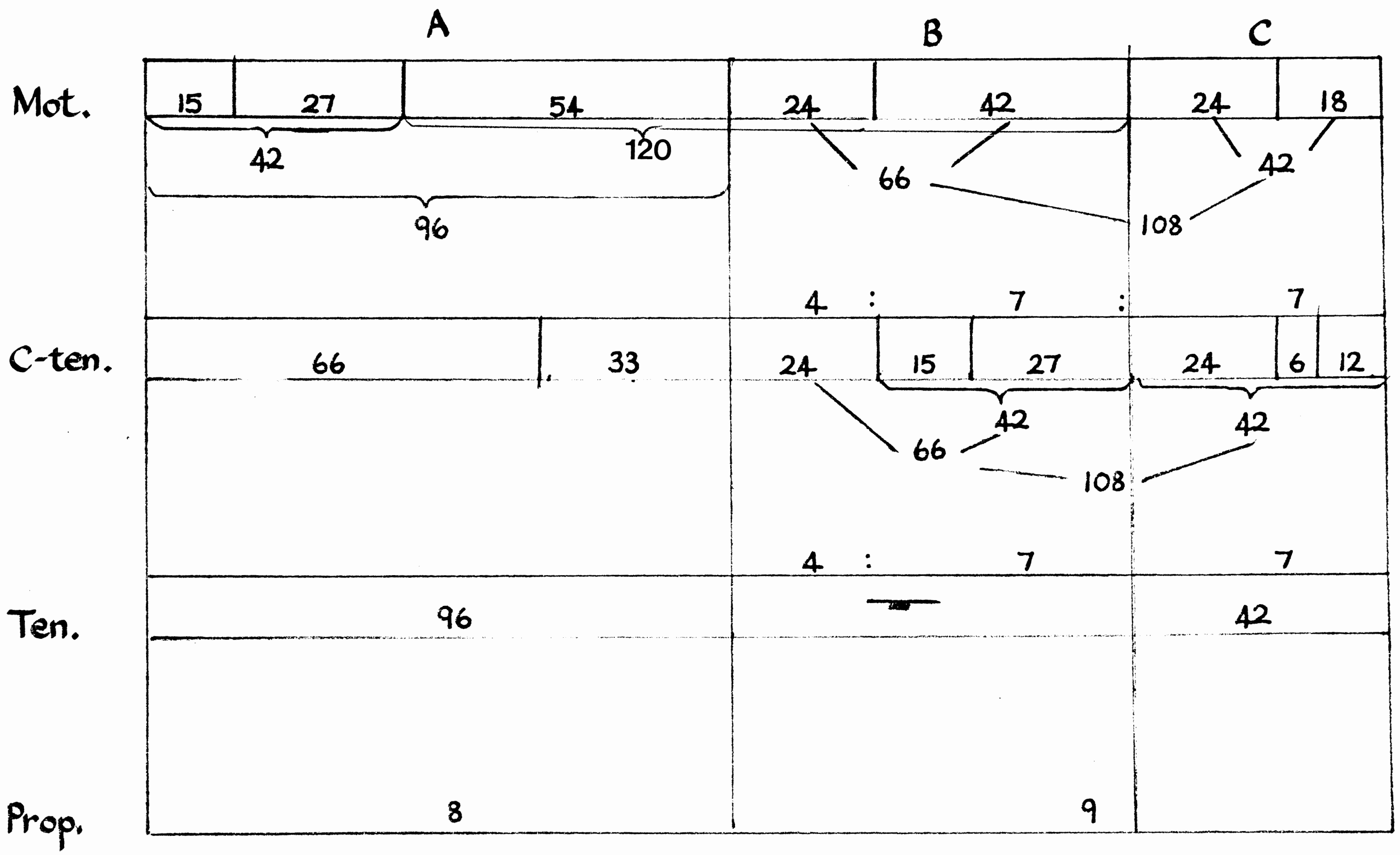
40. Alma redemptoris Mater



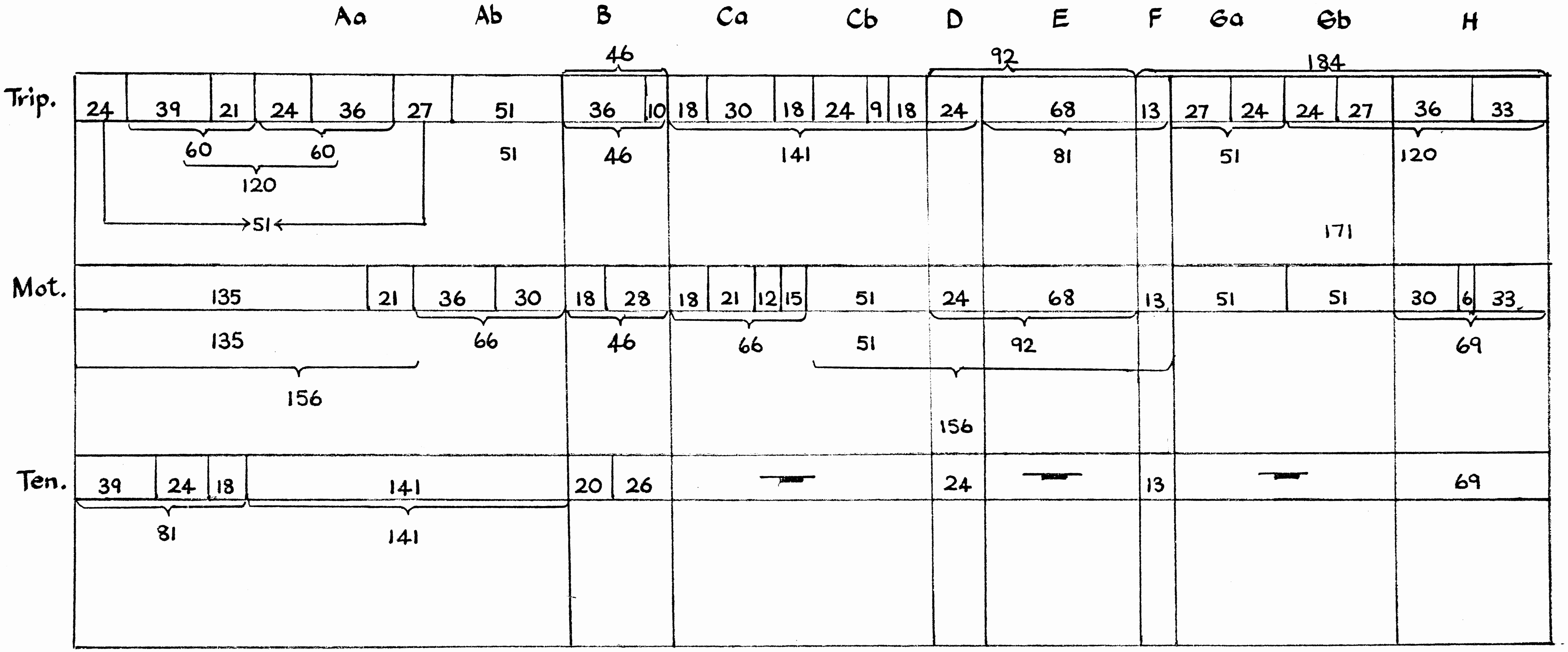
41. Beata Dei Genitrix



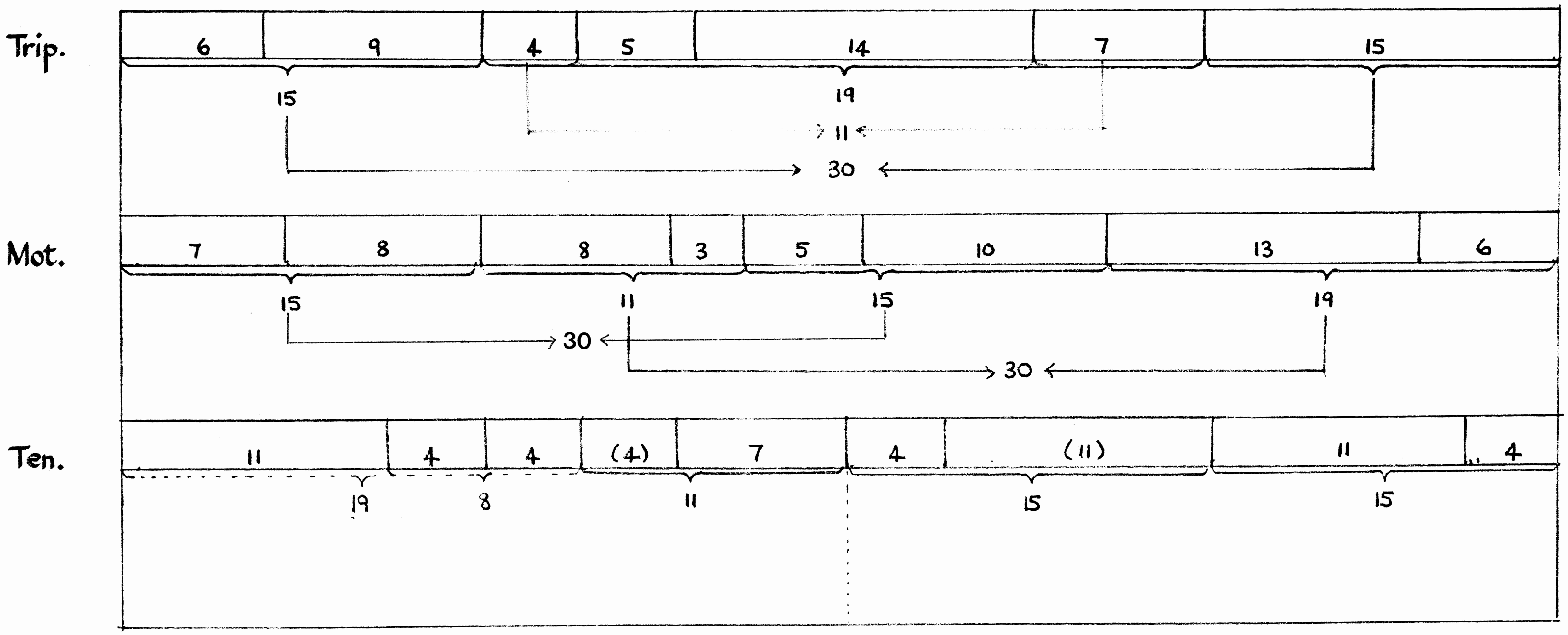
42. Beata Mater



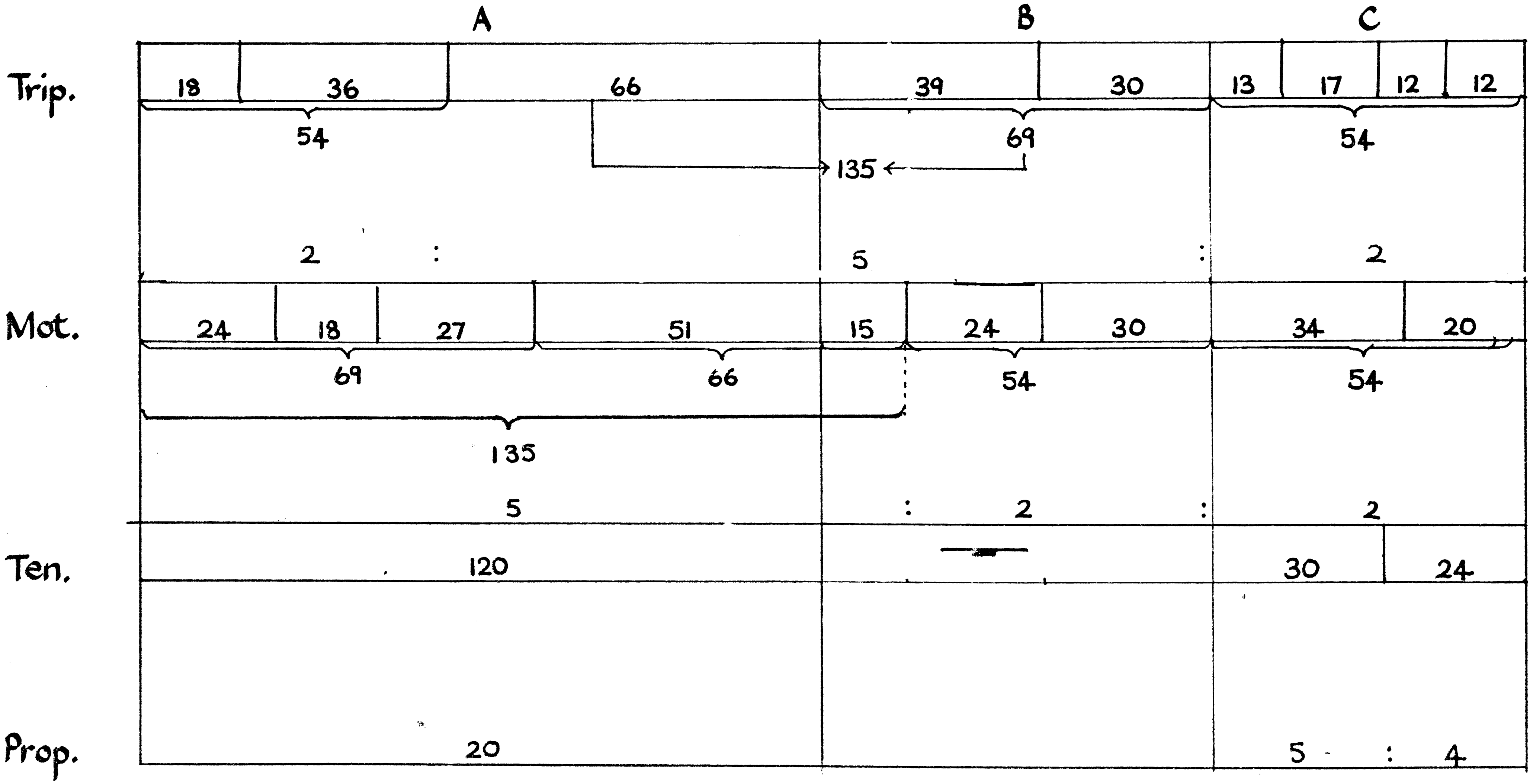
46. Salve Regina misericordi



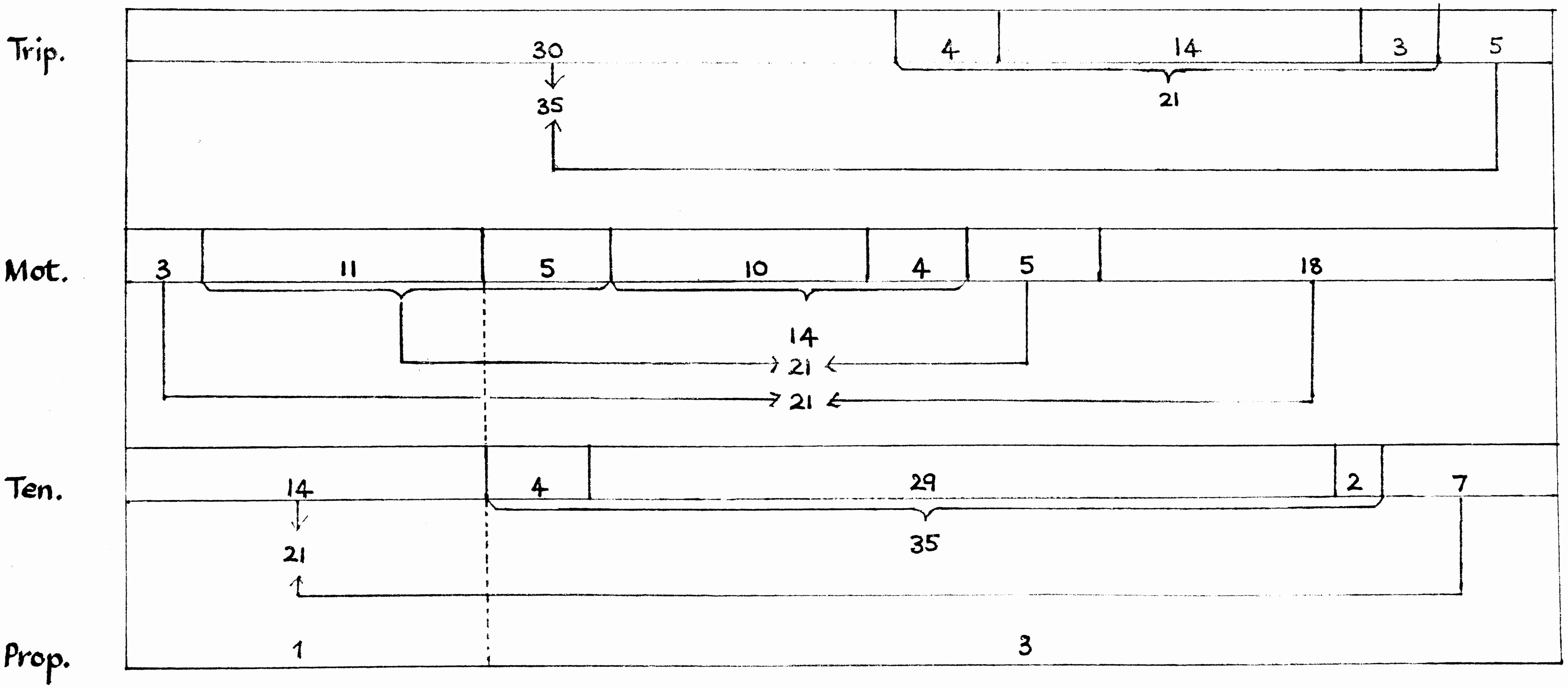
48. Sancta Maria, non est tibi similis



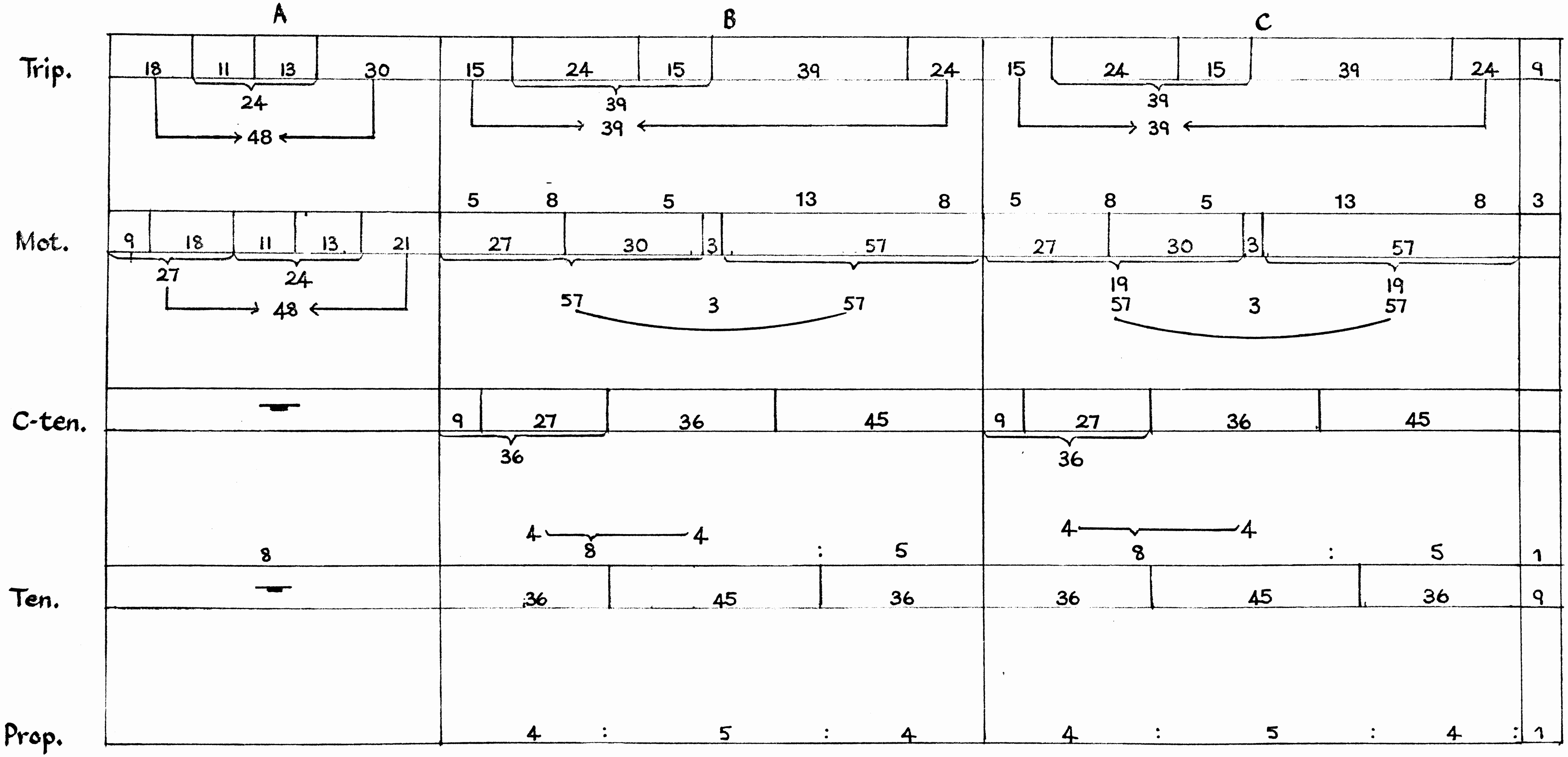
49. Sancta Maria succurre miseris



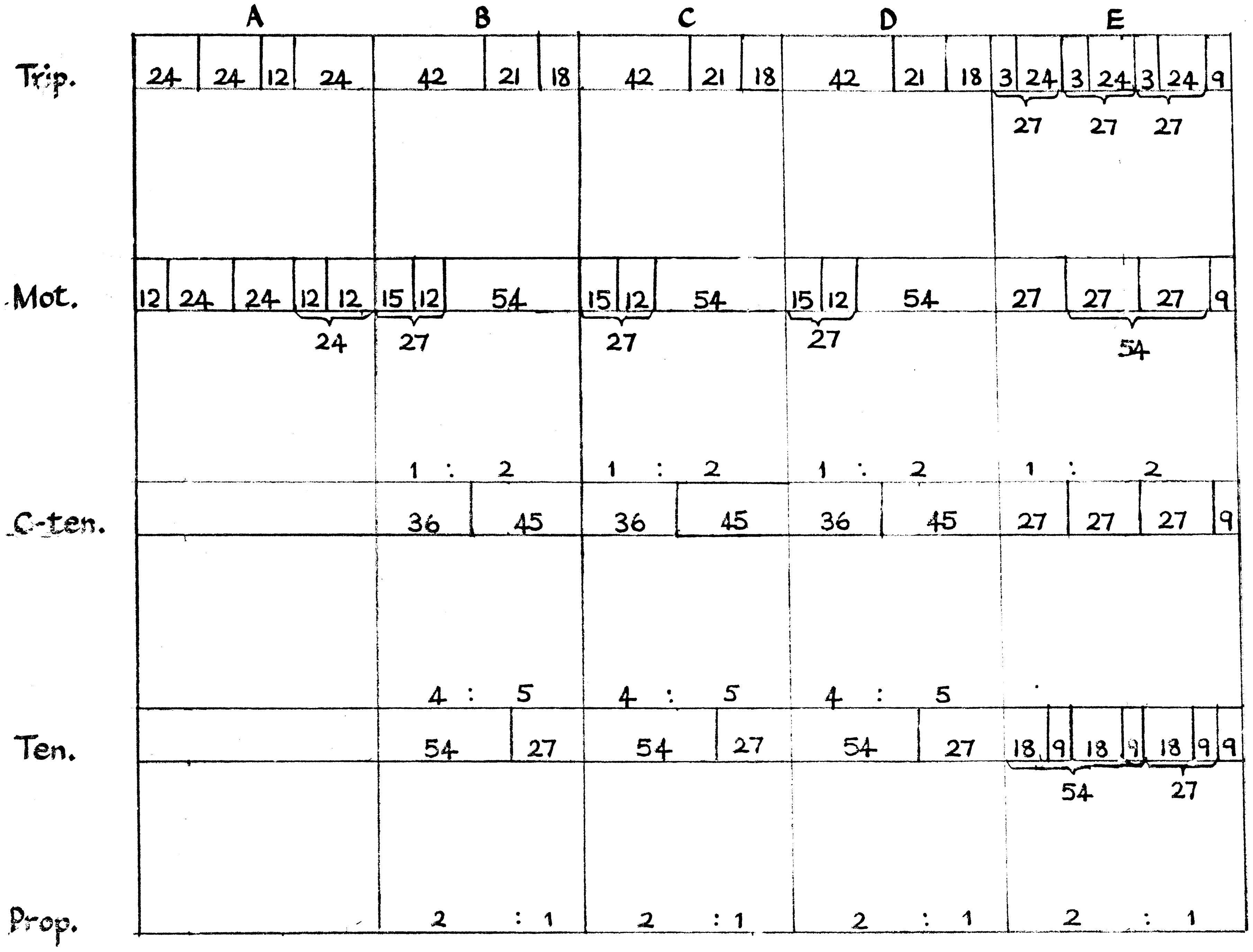
50. Speciosa facta es



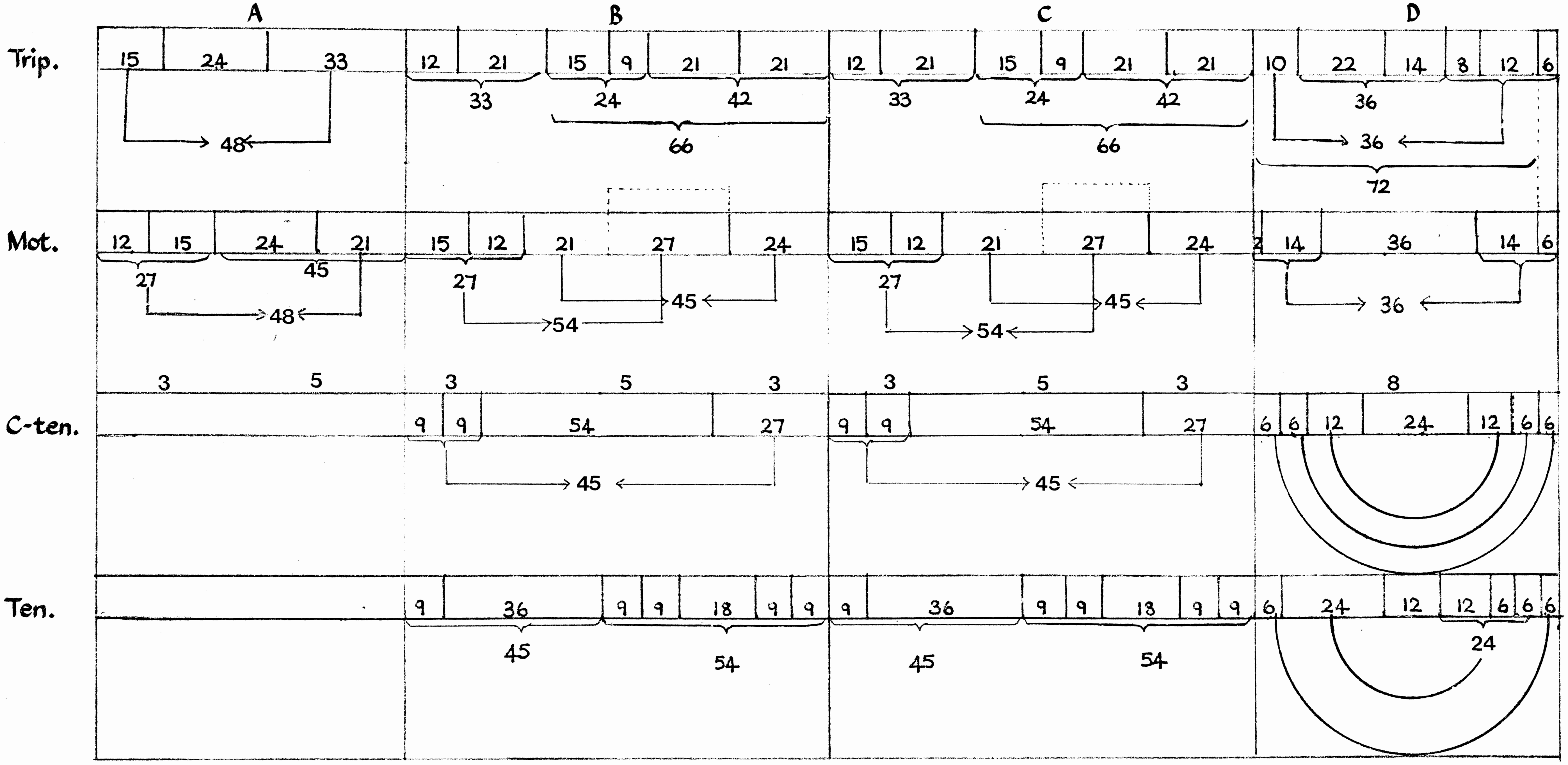
7. Vasilissa, ergo gaude



8. O sancte Sebastiane



9. O gemma, lux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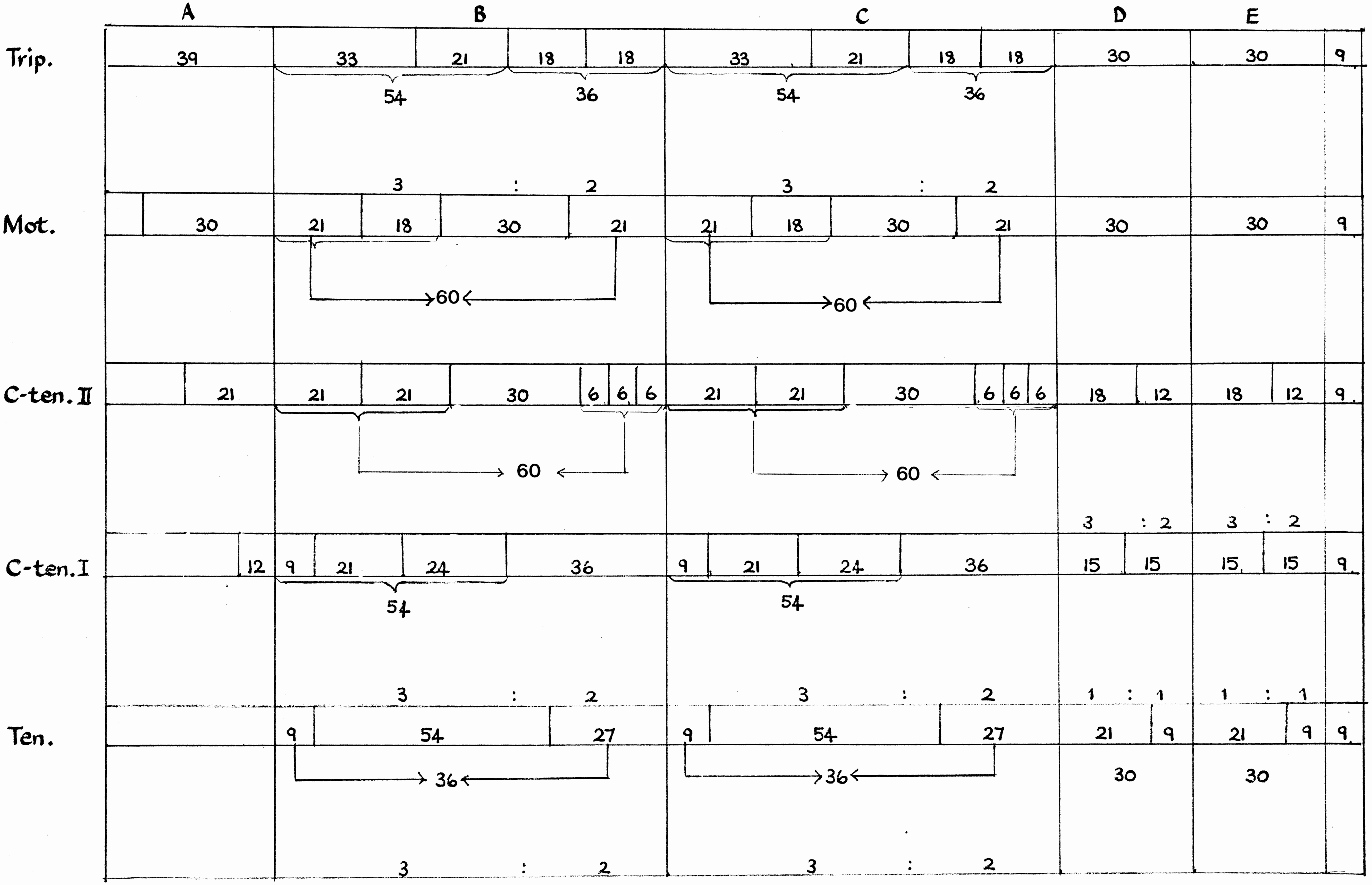
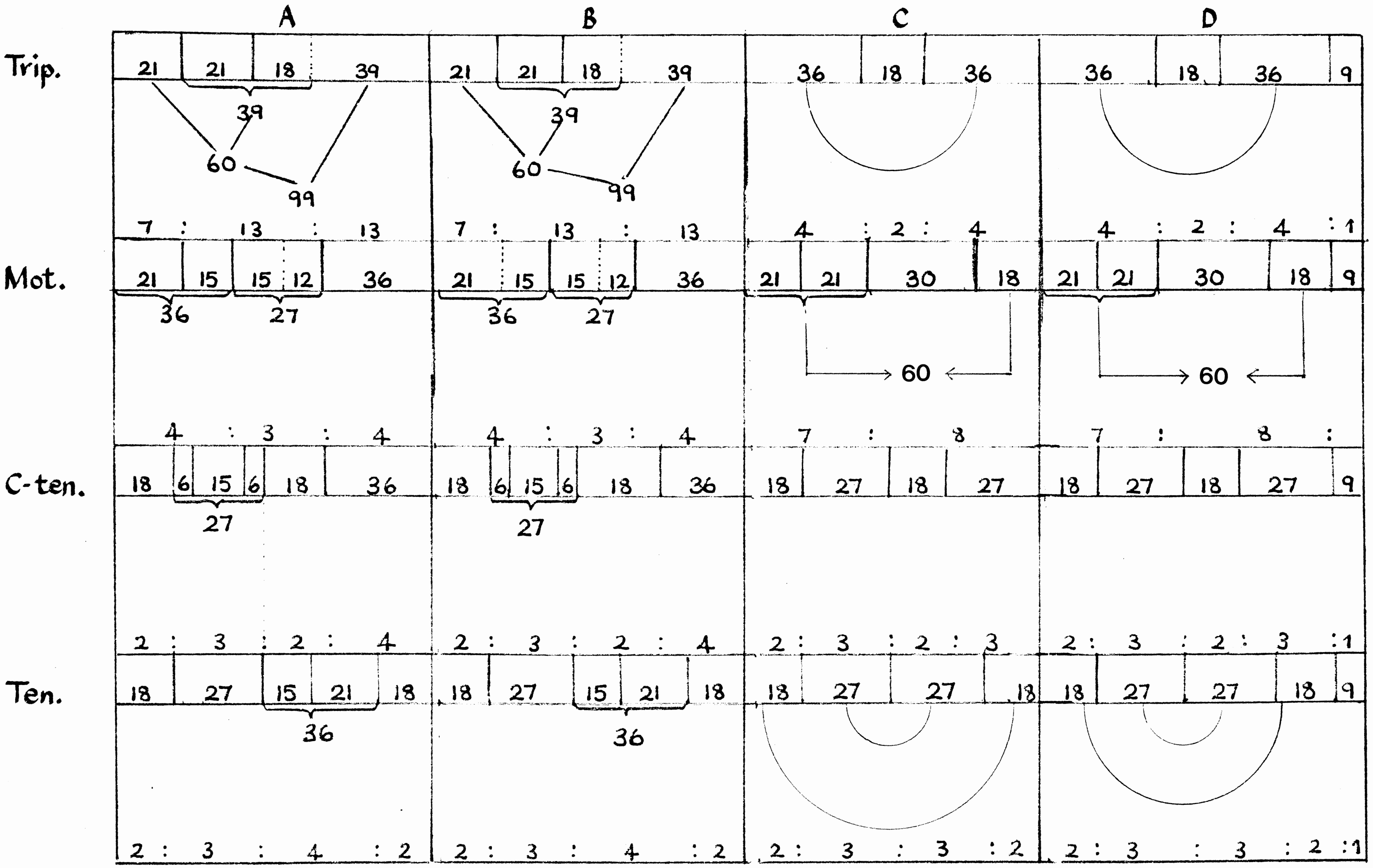
24

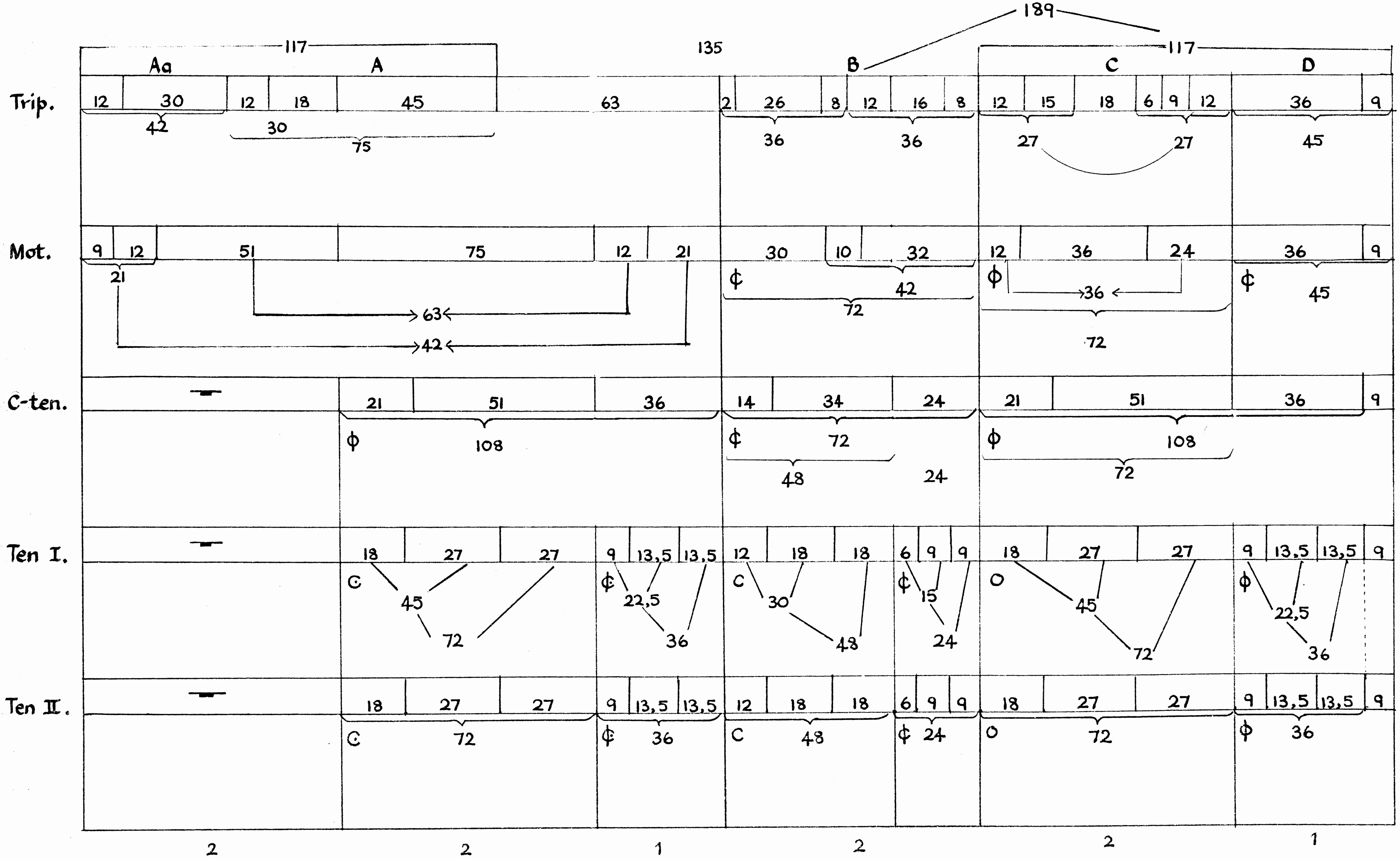
10. Apostolo, glorioso



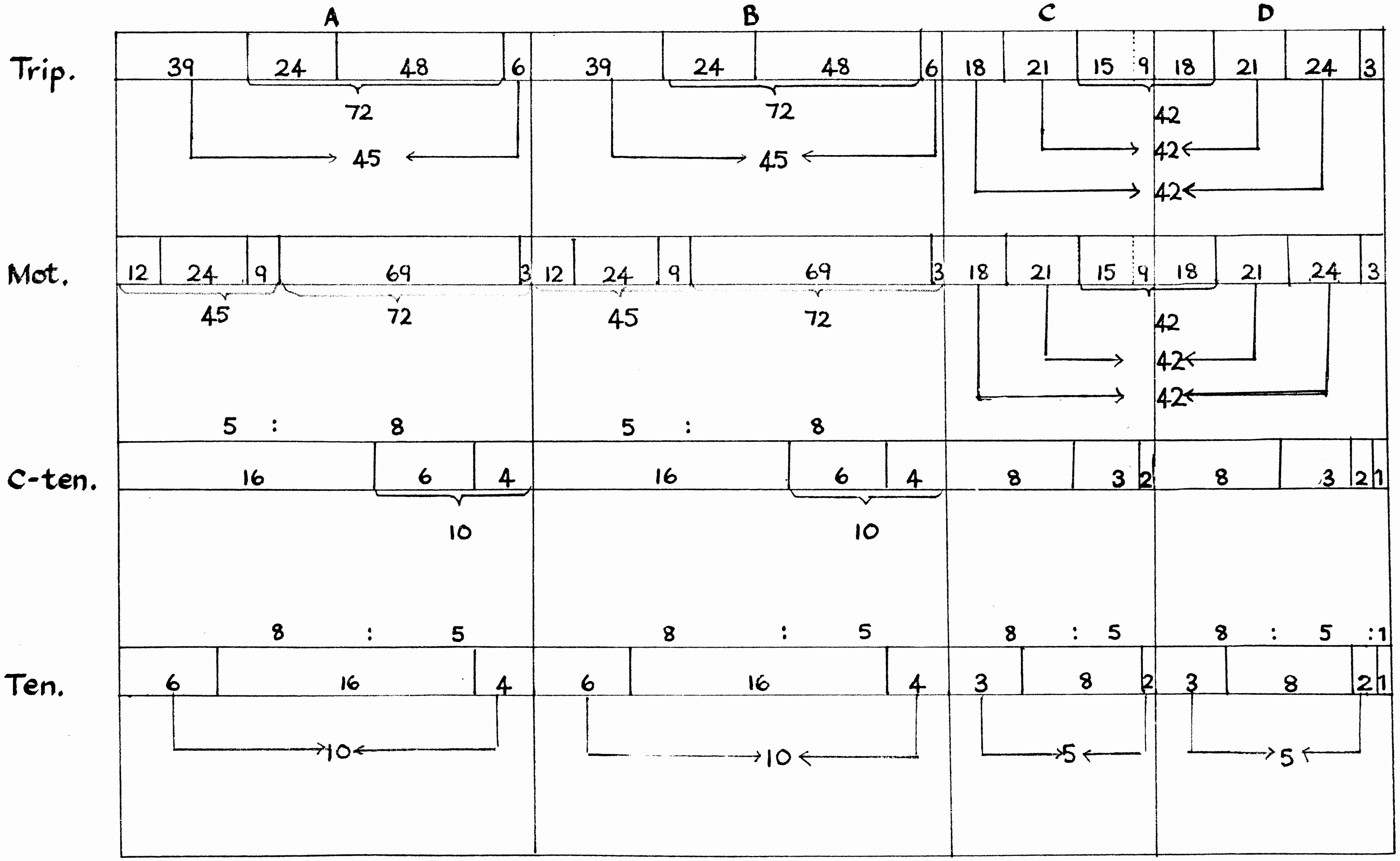
11. Rite majorem



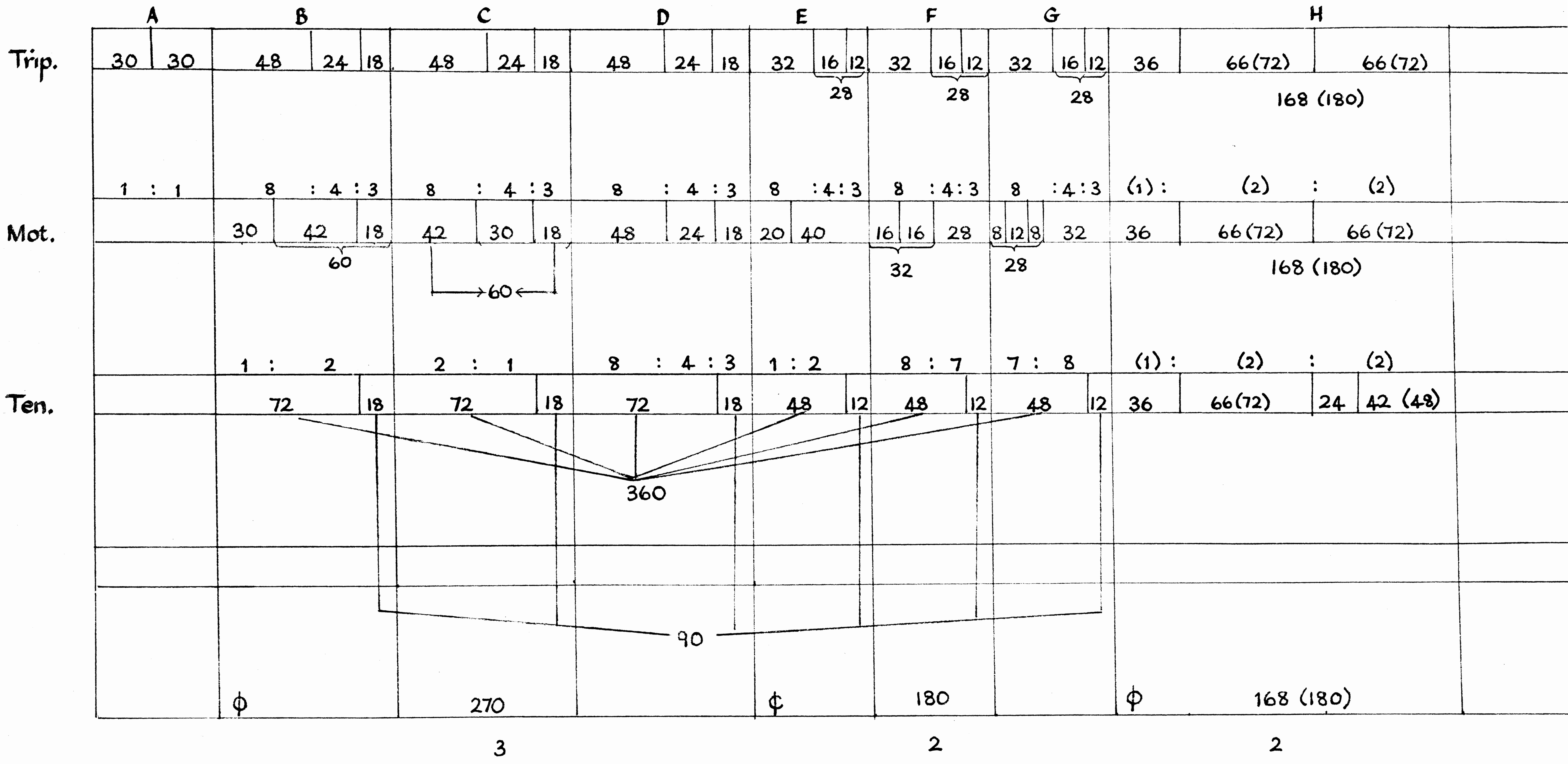
12. Ecclessiæ militantis



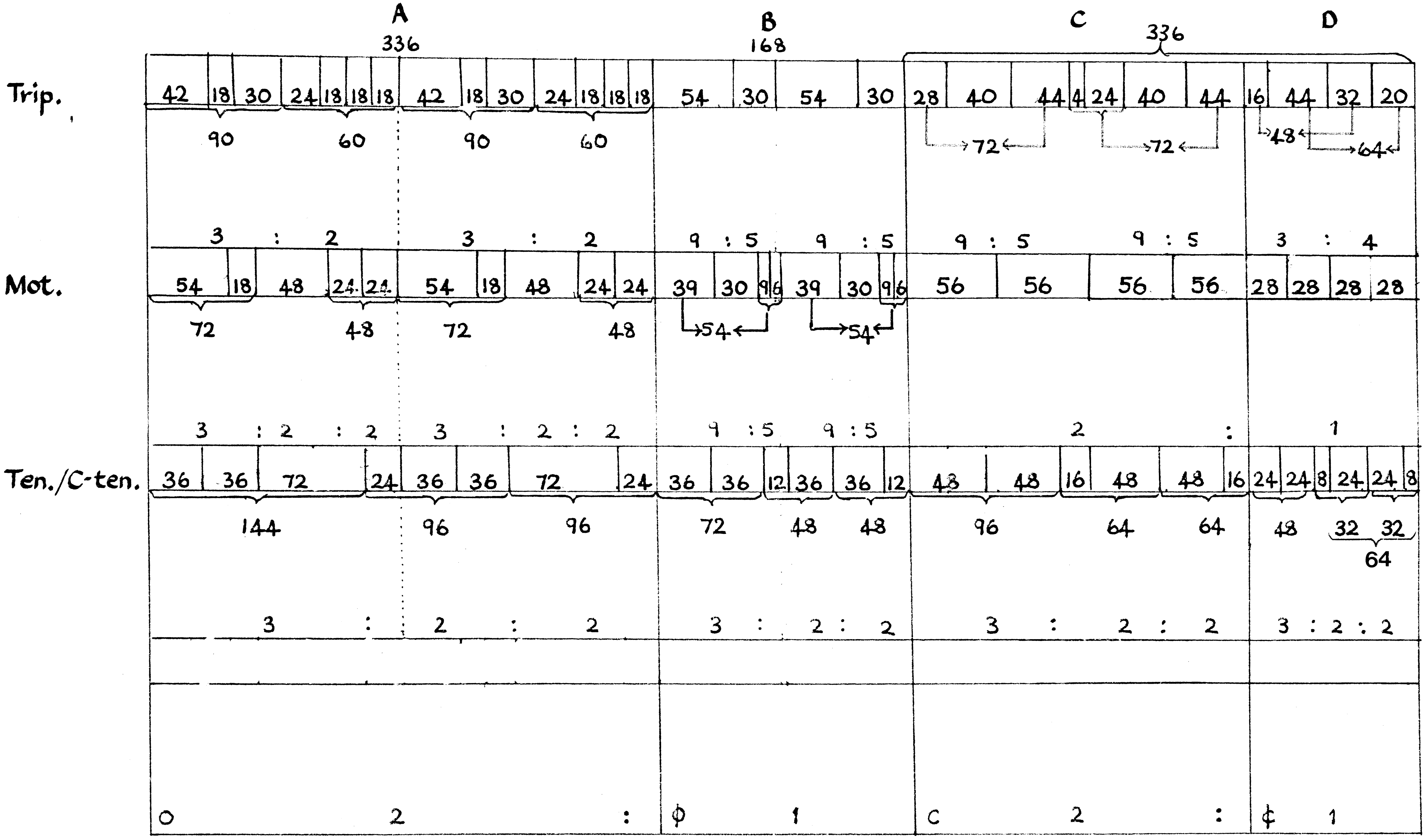
13. Balsamus et munda



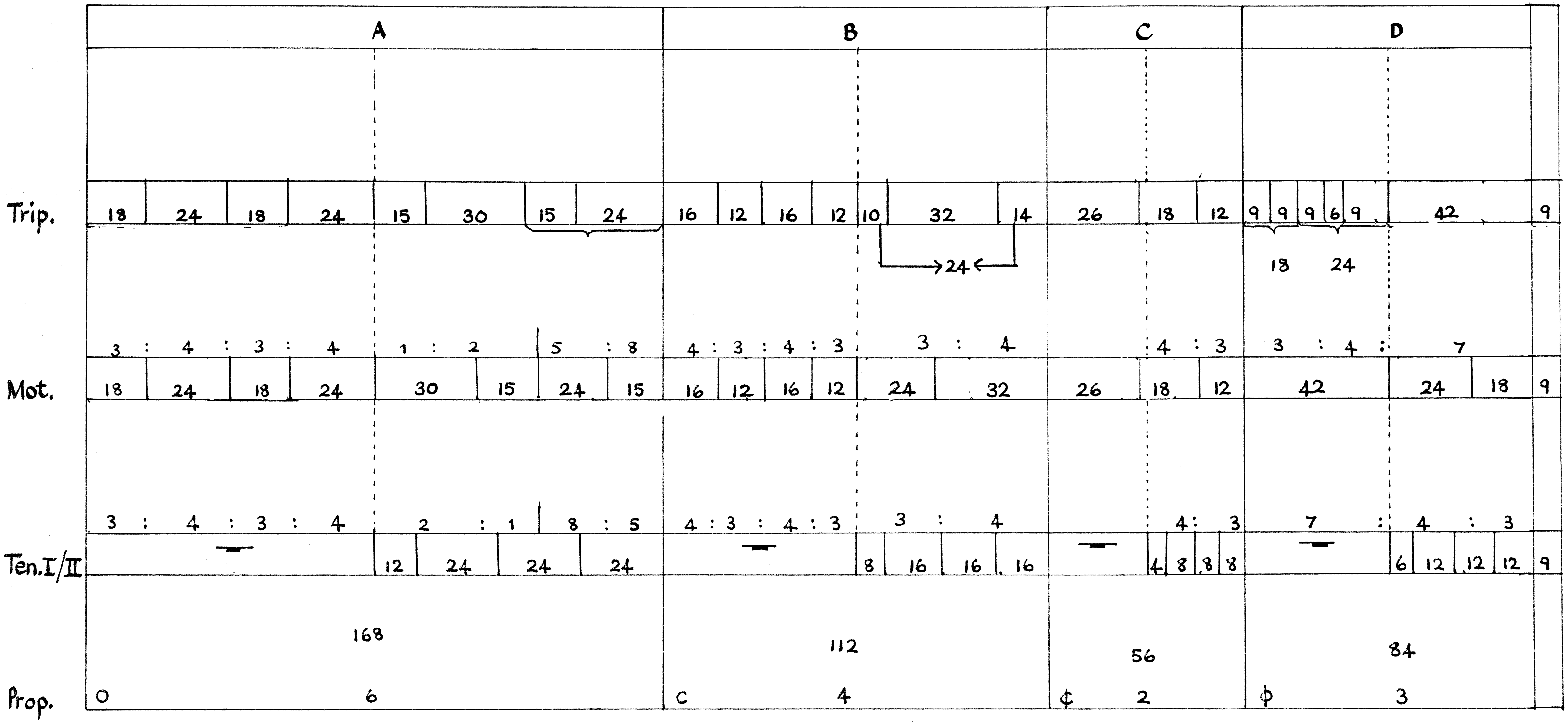
14. Supremum est mortalibus bonum



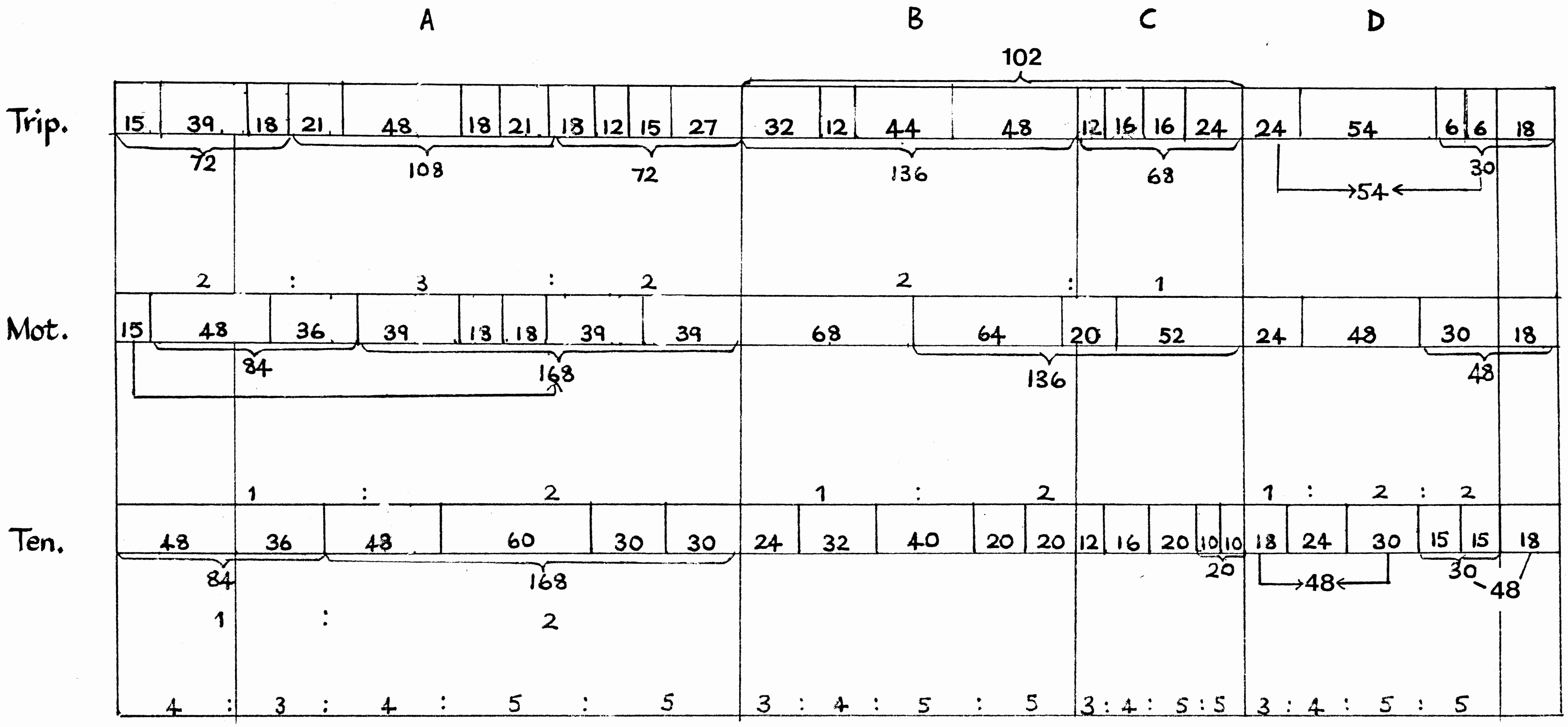
15. Salve flos



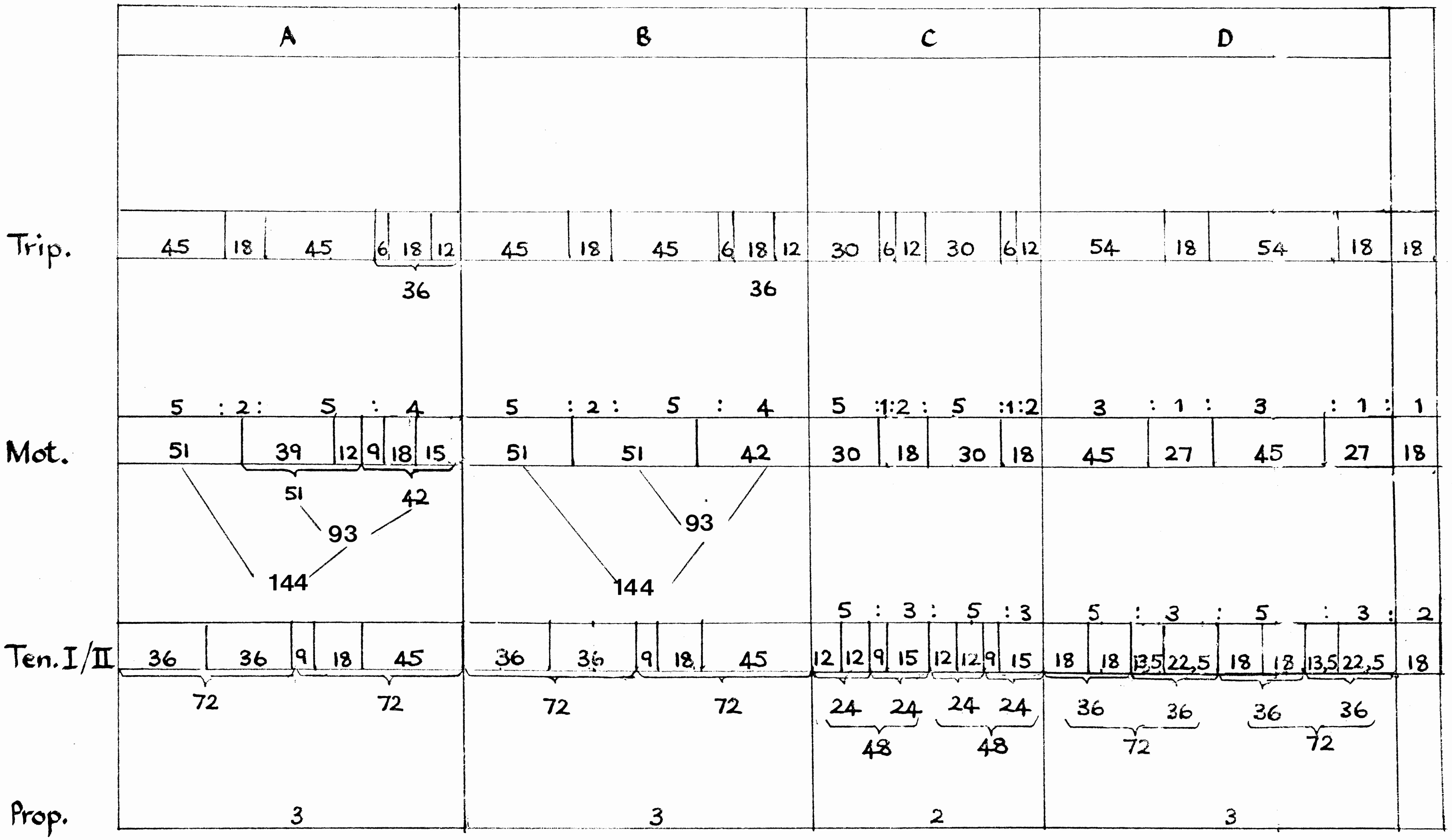
16. Nuper rosarum flores



17. Magnanimæ gentis



18. Fulgens iubar



19. Moribus et genere