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**The simulation of the dynamic hedging of  
Guaranteed Equity Bonds issued by a South African life office**

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## Glossary of terms<sup>1</sup>

<b>ALSI40</b>	The Johannesburg Stock Exchange All Share Top 40 index.
<b>Arbitrage</b>	Instruments that are perfect substitutes should trade at the same price. If they do not, a risk-free profit can be generated by simultaneously selling the higher-priced asset and buying the lower-priced asset. Arbitrage is the identification and exploitation of such price anomalies.
<b>Basis</b>	The difference between the futures price and the price of the underlying asset (the spot price).
<b>Call (option)</b>	The right but not the obligation to buy a pre-agreed amount of a specified underlying at a pre-determined price from the option writer.
<b>Cash-and-carry arbitrage</b>	A trade aimed at profiting from an expensive futures price. It involves shorting a futures contract and borrowing at money market rates to finance a long position in the underlying. The investor either delivers the underlying asset into the futures contract or waits for a narrowing of the basis and closes out the positions. This arbitrage and its opposite, reverse cash-and-carry, ensure that the spot and derivatives markets prices are consistent.
<b>Delta</b>	The rate of change of the value of an option for a given change in the value of the underlying asset. The first partial derivative of the option price with respect to the underlying price.
<b>Delta neutral</b>	A portfolio is delta neutral if an infinitesimal change in the index to which the assets and liabilities are linked will have an equal effect on the assets and liabilities, thus leaving the net asset position unchanged.

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<sup>1</sup> Most of the definitions were obtained from the following comprehensive glossary of terms: *Corporate Finance Risk Management & Derivatives Yearbook* (1995).

<b>Derivative instrument</b>	A security or contract whose value is dependent on or derived from the value of some underlying asset or index.
<b>Dynamic hedging</b>	The replication of an option's delta with a continuously changing portfolio of the risky asset (a market-priced, traded security on which the option is based) and a risk-free zero coupon bond.
<b>Futures contract</b>	An agreement to buy or sell a given quantity of a particular asset at a specified future date at a pre-agreed price.
<b>Implied volatility</b>	The value of the volatility parameter which equates the price of an option based on the assumed valuation formula to the observed option price.
<b>Kurtosis</b>	The kurtosis of a distribution is the extent to which the peak of the distribution is unusually tall and pointed (positive kurtosis, <i>leptokurtic</i> ) or low and flat (negative kurtosis, <i>platykurtic</i> ). A true normal distribution has a kurtosis of zero ( <i>mesokurtic</i> ).
<b>Lognormal distribution</b>	A variable has a lognormal distribution if the logarithm of the variable is normally distributed.
<b>Margin</b>	The collateral derivatives traders must set aside against their exchange-traded positions. The initial margin is an amount of money paid to the clearing house on entering a trade. Variation margin is the amount of money to be paid or received as a result of the daily mark to market.
<b>Mark-to-market</b>	The process of evaluating positions at the prevailing market prices to establish profit or loss and also margin calls.
<b>Notional</b>	The amount of asset exposure underlying a derivative contract. For example, a SAFEX future on the ALSI40 index has a notional of R10 times the ALSI40 index level.
<b>Put (option)</b>	The right but not the obligation to sell a pre-agreed amount of a specified underlying at a pre-determined price and time to the option writer.
<b>SAFEX</b>	The South African Futures Exchange

<b>Skewness</b>	The skewness of a distribution is the extent to which the distribution is asymmetric around its mean. Positive skewness usually means that the right-hand side of the distribution is more widely dispersed from the mean than the left-hand side. Similarly, negative skewness means that the left-hand side of the distribution is more widely dispersed from the mean than the right-hand side. A true normal distribution has a skewness of zero.
<b>Spot (price)</b>	The price for immediate delivery of an asset. The shortest delivery period.
<b>Tranche</b>	A single issue or offering of a product. A specific guaranteed equity bond may be offered in multiple tranches, each with a limited subscription amount.
<b>Variance</b>	The statistical measure of how widely a variable at different points in time is dispersed around its mean value.
<b>Volatility</b>	The measure of a variable's tendency to vary over time. The more volatile the price or price growth, the more it is likely to exceed the option strike price and so the more valuable the option. In the Black-Scholes option pricing formula, it is defined as the annualized standard deviation of the natural logarithm of the price returns of an asset.
<b>Zero (Coupon Bond)</b>	An obligation which contains only initial and final cash flows. There are no intermediate cash flows or coupons, which makes pricing simpler as no assumptions have to be made as to the rate at which coupons would be reinvested. Frequently used to guarantee a minimum value at maturity in a capital guaranteed product.

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## **Abstract**

South African life offices have issued a variety of guaranteed equity bonds (GEBs), but have typically avoided in-house dynamic hedging of these products and opted for static hedges offered by large investment banks. The opposition to in-house dynamic hedging appears to be fueled by the perceived esoteric nature of dynamic hedging, the associated operational and market risks, and recent high profile derivative-related bankruptcies. Despite these objections, there remains significant pressure on life offices to investigate dynamic hedging. An increasingly price-sensitive and sophisticated customer base together with fierce competition among GEB providers force life offices to seek the most cost-effective equity guarantees. Life offices' derivative trading departments are also lobbying for in-house dynamic hedging to help them reach critical mass and justify their investment in human and technological resources.

This dissertation illustrates how in-house dynamic hedging could be evaluated through modeling, simulation and sensitivity testing. The illustration helps to illuminate the financial benefits and associated risks of dynamic hedging, but does not provide a specific conclusion on whether dynamic hedging should be implemented. Ultimately this decision will depend on the unique circumstances and strategic objectives of the life office. The scope of the dissertation is thus limited to an illustration and discussion of the relevant issues only and specifically does not cover the costing of in-house dynamically hedged guarantees.

The evaluation approach is decomposed into three stages: the first stage is the clear definition and specification of the dynamic hedging process and the financial markets within which it will operate. The next stage is the creation of a model of the dynamic hedge and the simulation of the financial effects under various scenarios. Finally, a subjective assessment is necessary to determine whether the potential financial benefit (as reflected in the simulated results) is sufficient to compensate for the associated risks.

The dissertation addresses the first stage by presenting a definition and specification of the dynamic hedging process and the financial markets within which it operates. (For the sake of simplicity, the scope of the dissertation is limited to the South African financial markets.) It presents a specification of the put option liability faced by the issuing life office, the assets used to dynamically hedge the liability, and the market environment (equity index, yield curve, dividend yield, scrip borrowing/lending fee, transaction costs and market impact costs). It describes a dynamic hedging strategy, i.e. when and how the mismatch between assets and liabilities is evaluated, the extent of mismatch allowed before taking corrective action, and the nature of the action taken.

Once the model is specified and coded, Monte Carlo simulations are performed for various levels of market volatility and transaction costs. The dissertation presents the simulation results in terms of the calculated required reserve and the distribution of the accumulated tracking error (as a % of the required reserve). This provides some indication of the potential upside and downside from dynamic hedging.

Sensitivity testing of the simulation results yielded interesting observations regarding the appropriateness of mismatching in the dynamic hedging strategy. In particular, the results appear to be insensitive to small changes in market volatility and transaction costs, at all levels of the allowable delta mismatch. A delta mismatching strategy can thus not be assumed to reduce the sensitivity of results to changes in volatility and transaction costs. A delta mismatch strategy also appears to be sub-optimal since the additional tracking error losses seem to outweigh the transaction cost savings, resulting in overall lower returns.

The final stage of the analysis is addressed through a review of practicalities, issues to consider in costing the dynamically hedged guarantee, and areas for further research that were simplified for the purpose of the simulation. These issues reveal the weaknesses of the model and therefore of the conclusions drawn from the simulation results.

## Chapter 1 Introduction

Since mid-1997, South African life offices have issued a variety of equity index-linked investment products with a guarantee that the maturity value will at least equal the amount invested. These single premium products, commonly referred to as guaranteed equity bonds (GEBs), represent promises by the life office to pay an amount at the end of the investment term. The amount payable is specified in terms of the amount originally invested and growth in an equity index, which could be adjusted by an index growth multiplier (referred to as "gearing") and be subject to upper or lower limits (referred to as a "cap" and "floor").<sup>1</sup>

The life office could provide for the liability in a number of ways. The life office could invest in an asset that exactly mirrors the liability, thereby creating an almost risk-free net asset position that requires no ongoing management (this is referred to as static hedging<sup>2</sup>). Usually an investment bank tailors the static hedge asset<sup>3</sup> according to the life office's needs by combining various financial derivative contracts.

Alternatively, the life office could dynamically manage a portfolio of assets in an attempt to mirror the liability. Since this hedge strategy is imperfect, the net asset position is exposed to the risk of insolvency and capital must be set aside to cover this risk.

A third alternative available to the life office, called risk pooling, does not involve the use of assets to offset the risks introduced by the GEB liability, rather it aims to reduce the risk through diversification of the liability. This involves issuing a variety of GEBs

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<sup>1</sup> Sheldon & Dodia (1994) provide a concise overview and explanation of various GEB products.

<sup>2</sup> Taleb (1997, p.256) defines static hedging as "a risk management strategy that consists in finding a match for an option position that does not require continuous rebalancing".

<sup>3</sup> The investment bank issues a tradable note to the life office. This is an asset and not a reinsurance agreement (which would be treated as a negative liability).

over time (with different maturity dates, different indices, and different forms of index linking) to ensure that the capital guarantees do not become claims all at the same time.

Life offices in South Africa typically hedge their GEB liabilities statically, and have not attempted in-house dynamic hedging. The perceived esoteric nature of dynamic hedging, the associated operational and market risks, and recent high profile derivative-related bankruptcies appear to have motivated this reluctance. As Oldfield *et al* (1997: 35) put it, "if the institution has no comparative advantage in managing attendant risks, it has no reason to absorb or manage such risks rather than transfer them." However, without an in-depth assessment of the benefits and risks of dynamic hedging the life office enters negotiations on static hedges in ignorance. Pressure on life offices to consider alternative, potentially more cost-effective equity guarantees also arises from an increasingly price-sensitive and sophisticated customer base and the fierce competition among GEB providers. Life offices' derivative trading departments are also lobbying for in-house dynamic hedging to help them reach critical mass in transaction volumes and justify their investment in human and technological resources.

The primary purpose of this dissertation is to illustrate how in-house dynamic hedging could be evaluated through modeling, simulation and sensitivity testing. The illustration helps to illuminate the financial benefits and associated risks of dynamic hedging, but does not provide a specific conclusion on whether dynamic hedging should be implemented. Ultimately this decision will depend on the unique circumstances and strategic objectives of the life office. The secondary purpose of this dissertation is to review the practicalities of dynamic hedging and the issues to consider in costing the dynamically hedged guarantees.

The scope of the dissertation is thus limited to an illustration and discussion of the relevant issues only and specifically does not include an attempt to cost the dynamically hedged guarantees. The scope is further limited to the South African financial markets and legislation governing South African registered life offices.

The evaluation approach is decomposed into three stages: the first stage is the clear definition and specification of the dynamic hedging process and the financial markets within which it will operate. The next stage is the creation of a model of the dynamic hedge and the simulation of the financial effects under various scenarios. Finally, a subjective assessment is necessary to determine whether the potential financial benefit (as reflected in the simulated results) is sufficient to compensate for the associated risks.

The dissertation addresses the first stage in Chapters 2 and 3. Chapter 2 presents a definition and specification of the put option liability and the dynamic hedging process. Chapter 3 covers the factors that influence the dynamic hedging payoff and how these factors are incorporated in the simulation model. These factors are the market environment, the nature of the assets and liabilities, the transaction costs and market impact costs, the hedging strategy and the reserve requirement.

Chapter 4 addresses the second stage and presents an analysis of the simulations and sensitivity testing results.

The final stage is addressed through a review of practicalities (Chapter 5) and two specific issues to consider in costing dynamically hedged guarantees, namely model risk and risk-return preferences (Chapter 6).

In Chapter 7 the results are discussed, conclusions are drawn and suggestions for future research are made.

## Chapter 2      Specification of the dynamic hedge

This chapter describes the liability that is to be dynamically hedged by the life office and the dynamic hedging procedure to be followed.

### 2.1 The liability to be dynamically hedged

The life office is assumed to issue a single tranche<sup>4</sup> of a GEB to be offered to individual investors at a particular time. Each contract will require the investment of a single investment amount for a 5-year investment term. The benefit is a maturity value based on the performance of the JSE All Share Top 40 equity price index (ALSI40), which does not include dividends. The maturity value is guaranteed to be at least the initial amount invested. The GEB contract cannot be surrendered before maturity, i.e. no liability payment is due prior to maturity.

Thus the payout at the end of the investment term is:

$$\begin{aligned} & \text{amount invested} \times \max\left(1, \frac{\text{ALSI}_5}{\text{ALSI}_0}\right) \\ = & \text{amount invested} \times \left[1 + \max\left(0, \frac{\text{ALSI}_5 - \text{ALSI}_0}{\text{ALSI}_0}\right)\right] \\ = & \text{amount invested} + \left[\text{amount invested} \times \max\left(0, \frac{\text{ALSI}_5 - \text{ALSI}_0}{\text{ALSI}_0}\right)\right] \\ = & \text{"capital guarantee"} + \text{"upside gain"} \end{aligned}$$

where  $\text{ALSI}_t$  represents the official closing ALSI40 index value of year  $t$

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<sup>4</sup> A "tranche" is a single offering from a series of one or more offerings.

The GEB liability could be viewed as either: (a) an index fund without tracking error<sup>5</sup> plus a long position in 5-year European put option struck at-the-money; or (b) as a zero coupon bond plus a long position in 5-year European call option struck at-the-money.<sup>6</sup>

Life office tax legislation places the latter structure at a disadvantage, since tax is levied on the interest accrued on the zero coupon bond but not on the index fund (equity) growth. Although it is possible to construct a synthetic zero coupon bond which currently does not attract tax, there is always the risk of it being taxed in the future.

Thus for the purpose of this research we consider only the first structure. It is assumed that the life office has set up an acceptable match for the index fund component of the liability. Since the GEB liability excludes dividends, the dividends on the index fund accrues to the life office. The dividends are recognised in the pricing of the GEB, i.e. it is used to meet policy expenses, the cost of the guarantee and the profit margin.

The dissertation focuses only on the dynamic hedging of the European put option (a five-year at-the-money European put option on the ALSI40 index).

## **2.2 The dynamic hedging procedure**

Dynamic hedging is performed through a continuous adjustment to the portfolio's allocation between the risky asset (the option's underlying asset) and the zero coupon bond. The allocation is adjusted so that the option and portfolio values are equally sensitive to the underlying asset price, thereby ensuring that a small change in the underlying asset price will have a similar impact on both the portfolio value and the option value. However, while the portfolio's sensitivity remains fixed, the option's sensitivity changes with changes in the underlying asset price. This necessitates a continuous adjustment to the portfolio to maintain the correct sensitivity.

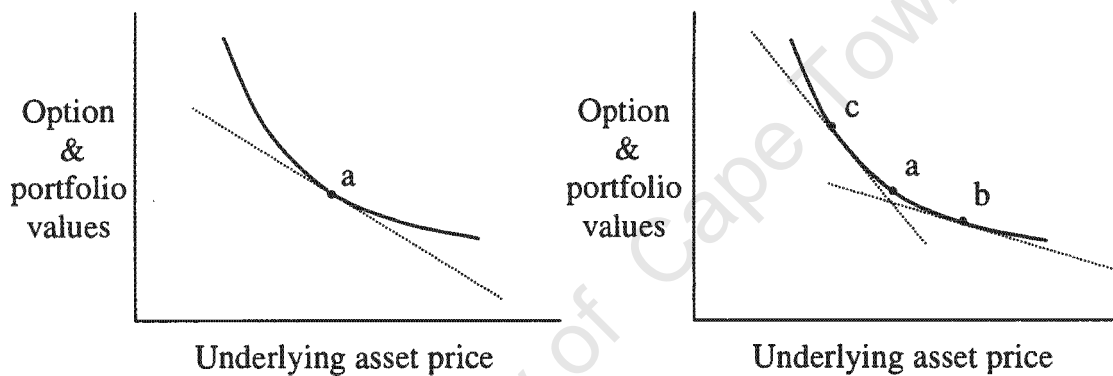
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<sup>5</sup> An example of an exact match for the index fund component is an ALSI40 forward contract with a forward price of zero. This asset will have no dividend yield or tracking error risks. The value of the index fund dividend stream over the term of the contract is recognised as a reduction in the cost of the contract.

<sup>6</sup> This equivalence occurs via put-call parity.

The following figures illustrate how the portfolio must be adjusted according to changes in the underlying asset price. The option value is represented by a solid line and the portfolio value is represented by a dotted line. At the initial price level (assumed to be "a"), the option price sensitivity (the slope of the option value line) is equal to the portfolio's sensitivity (the slope of the portfolio value line). In such a situation the liability is matched by the asset and the effect of an infinitely small change in the price of the underlying asset will have an equal effect on the asset and liability, thus leaving the net asset position unchanged.

**Exhibit 1: Matching portfolio sensitivity to option sensitivity**



An increase in the underlying asset price (to "b") results in a lower option sensitivity, but no change to portfolio sensitivity. The portfolio's sensitivity will have to be reduced. (This is achieved by reducing the portfolio's short position in the underlying asset). Similarly, a decrease in the underlying asset price (to "c") requires the asset portfolio to change to a higher sensitivity. (This is achieved by increasing the short position in the underlying asset.)

In practice, the adjustment to the portfolio cannot be instantaneous (continuous). The portfolio's sensitivity will not continuously match the option's sensitivity, and changes in the underlying asset price will thus lead to deviations between the option value and portfolio value. The potential losses from these mismatches require the provision of capital (reserves), which will add to the option cost.

## Chapter 3 Simulation specification

This chapter describes how the dynamic hedge payoff is calculated using a Monte Carlo simulation of the dynamic hedging cash flows.

### Overview

The calculation of the dynamic hedge payoff is separated into a number of interim steps. Firstly, the liability and the assets used for hedging are defined and their value functions specified. A value function defines the relationship between the asset or liability value (the dependent variable) and the various influencing factors (the independent variables). This is presented in sections 3.1 and 3.2. These influencing factors are modelled as part of the market model which is outlined in section 3.3.

Secondly, the hedging strategy is specified, i.e. the decision rule that determines how the assets are managed to hedge the liability. This is detailed in section 3.4.

Thirdly, following the Monte Carlo simulation approach, 10 000 simulated histories of market data are generated, with each simulated history containing 1300 days (260 trading days  $\times$  5 years). These are used to calculate the delta of the liabilities (i.e. the sensitivity of the liability value to changes in the underlying index) and the mid-market price of the near-dated ALSI40 futures contract. Futures are bought or sold according to the hedging rule, and the associated transaction costs are recognised as cash outflows at the time of the transaction. On a daily basis, the mark-to-market margin adjustment is reflected as a cash flow. The result from this step is 10 000 simulated histories of daily cash flows. Positive cash flows represent cash returned to the policyholder fund. Negative cash flows are payments from the policyholder fund.

Fourthly, the initial reserve requirement is calculated. This is the amount required to reduce the risk of insolvency (ruin) to an acceptable level. The statutory and internal reserve requirements are discussed in section 3.5. The required reserves are funded by a

life office capital injection, which requires sufficient compensation for the risks taken. The risk-return preferences of the capital contributors and the implications for pricing the put option are discussed in section 3.6.

### **3.1 Description and valuation of the liability**

#### **Description of the liability**

As described in section 2.1, the liability to be dynamically hedged is a single 5-year at-the-money European put option on the ALSI40.

Since the option notional (the amount of underlying asset exposure) has no impact on the relative results of the simulation, no specific size of option notional will be assumed. The dynamic hedge payoff will be calculated as a percentage of the option notional.<sup>7</sup>

The simulation focuses only on a single guaranteed equity bond tranche and ignores the risk diversification benefits gained from issuing further tranches on different indices and different guarantee structures. The estimated dynamic hedge payoff is thus not dependent on the issuance of further tranches.

#### **Valuation of the liability**

Dynamic hedging involves matching the sensitivity of asset value to the sensitivity of liability value, where the sensitivity is measured relative to one or more influencing factors. For the purpose of this study, only the sensitivity to changes in the underlying index is dynamically hedged.

The liability value must be clearly distinguished from the reserve and the liability price. The liability value is the realistic best estimate (at time  $t$  for  $0 \leq t < 5$ ) of the future liability (put option) payment. The reserve is the amount set aside (at time  $t$

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<sup>7</sup> Refer to section 5.6 for a discussion about the potential cost implications of different notional amounts.

for  $0 \leq t < 5$ ) to avoid insolvency. Capital is usually required to finance the reserve. The liability price (charged to the client at time 0) is equal to the liability value at time 0 plus a charge to compensate the owners of the capital for being at risk.

The Black-Scholes option pricing formula offers a measure of the sensitivity of option values to underlying asset/index prices. This is discussed in the following section.

### The Black-Scholes Option Pricing Formula

Black and Scholes (B-S) observed that it is possible to replicate an option with a continuously adjusted (dynamic) portfolio of the risky asset (a market-priced, traded security on which the option is based) and a riskless zero coupon bond (Hull 1997: 235).

In explaining the B-S option pricing formula the following notation and definitions are used:

- $O(S,t)$  is the value of an option at time  $t$  with underlying asset price  $S$
- $\sigma$  is the price volatility of the underlying asset
- $K$  is the strike price of the option
- $T$  is the expiry
- $R$  is the risk-free interest rate
- Call option (right to buy asset for  $K$ ) has terminal value  $C(S,t) = \text{Max}(S-K,0)$
- Put option (right to sell asset for  $K$ ) has terminal value  $P(S,t) = \text{Max}(K-S,0)$
- Put-call parity ensures  $S+P-C = K e^{-R(T-t)}$

To simplify the derivation of the B-S pricing formulae, the following assumptions are made:

- a) The underlying asset price (index) follows a stationary diffusion process (log normal random walk process):

$$\frac{dS}{S} = \mu dt + \sigma dW$$

(where  $S$  is the asset price,  $dS$  is a small change in price over time  $dt$ ,  $\mu$  is the drift

per unit of time,  $dt$  is a small time step,  $\sigma$  is the volatility per unit of time for the period  $dt$ , and  $dW$  is a random variable from a standard normal distribution)

- b) The risk-free interest rate and volatility are constant and the same for all maturities.
- c) There are no transaction or market impact costs involved in the hedging exercise (i.e. investment decisions can be implemented at the market price).
- d) No dividends are paid during the life of the option.
- e) There are no arbitrage possibilities.
- f) The underlying asset can be traded continuously.
- g) The underlying asset can be sold short in any size without penalty.

Although some of these assumptions can be relaxed, this usually results in a more complex pricing formula.

Under the above assumptions, a riskless portfolio can be constructed with the option, its underlying asset and a zero coupon bond. Since the option's sensitivity to the price of the underlying asset changes continuously, the composition of the portfolio must be continuously adjusted to remain riskless. According to arbitrage-free pricing theory, the return on this riskless portfolio should be the same as that of any short-term riskless asset. This leads to the derivation of the Black-Scholes partial differential equation (Hull 1997: 237-8):

$$\frac{\partial O}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 O}{\partial S^2} + RS \frac{\partial O}{\partial S} - RO = 0$$

This differential equation is valid for all options and other securities that are dependent on the price of the underlying asset ( $S$ ). The vast range of solutions narrowed down by imposing the boundary conditions applicable to the security of interest. A solution of the above differential equation, with the boundary conditions applicable to a European put option with no dividends, is:

$$P(S, t) = Ke^{-R(T-t)}N(-d_2) - SN(-d_1)$$

Where 
$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{y^2}{2}} dy$$

$$d_1 = \frac{\ln(S/K) + (R + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/K) + (R - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

The partial derivatives or option price sensitivities, referred to as the "Greeks", are:

Delta  $\Delta = \frac{\partial P}{\partial S}$  = the rate of change in option price with respect to the index<sup>8</sup>

Gamma  $\Gamma = \frac{\partial^2 P}{\partial S^2}$  = the rate of change in option delta with respect to the index

Theta  $\Theta = \frac{\partial P}{\partial t}$  = the rate of change in option price with respect to time

Vega  $v = \frac{\partial P}{\partial \sigma}$  = the rate of change in option price with respect to volatility

Rho  $\rho = \frac{\partial P}{\partial R}$  = the rate of change in option price with respect to interest rate

Zhang (1997: 77-79) provides names and definitions for the higher order sensitivities.

### Refinements of the Black-Scholes Option Pricing Formula

The Black-Scholes pricing formula is valid only under the simplifying assumptions. Thus the option delta estimated using these assumptions could differ from the true option delta (calculated without any simplifying assumptions). Since the dynamic hedge portfolio attempts to replicate the option delta, a more accurate estimate of the option delta will reduce the likelihood of a mismatch between the asset delta and the

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<sup>8</sup> Lamda ( $\lambda$ ), measures the percentage change in option price for each percentage change in the index.

$$\lambda = \frac{\partial P}{P} \times \frac{S}{\partial S}$$

option delta. Refinements to the original B-S pricing formula are thus needed to estimate more accurately the option delta.

The following two refinements to the original Black-Scholes formula are made for the purpose of the study:

- Transaction costs

With no transaction costs, continuous rebalancing would be the optimal hedging strategy. With transaction costs, each rebalancing transaction involves a trade-off between the transaction cost and the benefit of reducing the tracking error. More frequent rebalancing may improve the accuracy of the replication and make the option cost more certain, but this certainty would be at a price.

Leland (1985) suggests modifying the B-S pricing formula to allow for transaction costs, by assuming a higher variance. The rationale for this is that transaction costs on the purchase (sale) of the underlying asset increase (decreases) the net purchase (sale) price. This amplifies the movements in the underlying price which is similar in effect to assuming an increased variance of:

$$\sigma^2 \times \left[ 1 + \Psi \times \frac{E[\Delta S]}{S\sigma^2\Delta t} \right],$$

where  $\Psi$  is the buy-sell spread (round-trip transaction cost) expressed as a percentage of the transaction amount,  $\sigma^2$  is the variance of the underlying asset per unit time ignoring transaction costs,  $\Delta t$  is the time period between portfolio revisions, and  $E[\Delta S]$  is the expected change in the underlying asset price between portfolio revisions.

- Continuous dividend yield on the underlying index

The Black-Scholes option pricing formula can be adapted to allow for underlying assets which provide a continuous dividend yield,  $q$  p.a. This adjustment involves splitting the dividend-paying asset into two components: a non-dividend-paying asset and a pooling of the dividends to be paid over the option term. Since a European option is defined in terms of the asset value at expiry and the dividend

pool component is depleted at expiry, we only consider the non-dividend-paying asset. Assuming the dividend-paying asset is priced at  $S$ , then the non-dividend-paying component is priced at  $Se^{-q(T-t)}$ . The asset price is thus reduced from  $S$  to  $Se^{-q(T-t)}$  and the option valued as though the asset pays no dividend (Hull 1997: 261-3):

$$P(S,t) = Ke^{-(T-t)R}N(-d_2) - Se^{-(T-t)q}N(-d_1)$$

$$\text{Where } N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{y^2}{2}} dy$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(R - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(R - q - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

### 3.2 Description and valuation of the assets

#### Description of the assets

The delta of the liability value can be replicated via either the spot, futures or OTC forward markets. All these markets involve uncertainty about transaction costs. The spot and OTC forward markets both suffer from a lack of liquidity while the futures market exposes the portfolio to roll-over risk. However, in terms of administration, the futures market appears to offer the simplest and most efficient approach to replicating the option delta. A more detailed comparison of the spot, futures and OTC forward approaches to dynamic hedging is presented in Appendix A.

Five-year futures contracts would be required to hedge dynamically without roll-over risk. However, the ALSI40 futures contracts traded on the SA Futures Exchange (SAFEX) usually have terms of only up to nine months. Furthermore, liquidity in the nine-month futures contract is low, resulting in greater market impact risk and greater

variability of bid-offer spreads. The near-dated (3-month) ALSI40 future is more liquid but would necessitate more frequent roll-over of the hedge position.

We assume that hedging is carried out only through short positions in the near-dated exchange-traded ALSI40 futures contracts. As each futures cohort reaches expiry, the futures position will be re-established using the new near-dated futures contracts. Detailed specifications of both the actual SAFEX ALSI40 futures contract and the assumed ALSI40 futures contract are included in Appendix B.

It will be assumed the margin deposited with the futures exchange earns the same rate of interest as the other cash deposits of the life office. It will also be assumed that all margin calls are financed from the life office's cash holdings and all margin payouts are invested in cash. The margin deposit is thus merely another cash investment of the life office and it has no impact on the dynamic hedge results.

No ALSI40 OTC forward contracts are used in the asset portfolio due to their lack of liquidity and uncertainty of bid-offer spreads. Exchange-traded and OTC options are not considered since the aim of dynamic hedge assets is to replicate the optionality rather than to buy it from an option writer.<sup>9</sup>

### **Valuation of the hedging assets**

The futures position is revalued daily (marked-to-market) and will produce a variation margin cash flow. To determine the marked-to-market cash flow, we simulate the mid-market futures price. The following paragraphs explain the derivation of this price based on the forward index level.

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<sup>9</sup> To offset the effects of convexity (gamma risk) and changing volatility (vega risk), the life office would have to purchase options. The purchase of these options merely shifts the risk to another option writer. Since the other option writers are not keen to increase their exposure to gamma and vega risks either, these options are likely to be expensive.

The forward index level is bounded by the index arbitrage strategies of "cash-and-carry" and "reverse cash-and-carry". A "cash-and-carry" strategy is the replication of a long futures position through borrowing money and buying the underlying asset. A "reverse cash-and-carry" strategy is the replication of a short futures position through short-selling the underlying asset and investing in a zero coupon bond.

Forward price  $\leq$  Spot  $\times$  (1 + cash borrowing rate - dividend yield - scrip lending rate)

Forward price  $\geq$  Spot  $\times$  (1 + cash lending rate - dividend yield - scrip borrowing rate)

Differences between the cash borrowing and lending rates and the scrip borrowing and lending rates create a band of arbitrage-free forward prices. It is assumed that borrowing and lending rates in the simulation of forward prices are equal.

In practice the futures price may differ from the forward price, because futures contracts are marked to market daily and forward contracts are only settled at expiry. The interest earned or paid on these daily cash flows may differ from the fixed interest rate presumed in the forward price. If the risk-free interest rate were constant and the same for all terms, then the futures price would be the same as the forward price. Since the simulation is based on the near-dated futures contract (with maximum term of three months), the likely difference between the forward and futures prices would be small enough to be ignored.

Futures prices are linked to the spot price of their underlying assets, but this relationship is flexible. This means that the actual relationship between futures price and spot price (the implied cost of carry or the basis) may differ from the expected relationship for a period of time. Basis risk is the risk of unexpected changes in this relationship. Since the dynamic hedge requires the purchase of futures contracts in the future on uncertain terms, there is the risk that the implied cost of carry will differ from that assumed at the inception of the dynamic hedge strategy. A reduction in the cost of carry after inception of the put option will be to the advantage of the put option issuer, since the futures price falls causing a gain on the short futures position held by the put option issuer. An increase in the cost of carry after inception of the put option will be to the disadvantage

of the put option issuer, since the futures price increases causing a loss on the short futures position held by the put option issuer.

### 3.3 The model for the market environment

Since both assets and liabilities follow the same stochastic model for the market environment, there is an offsetting effect in the simulated (asset minus liability) results. This decreases the sensitivity of the results to the type of stochastic investment model used.

The stochastic investment model as described by Thomson (1996) is used to simulate the market environment. The model is detailed in Appendix C.

#### Spot ALSI40 index level

The simulation applies Thomson's autoregressive model to determine the annual growth rate in the index, and then uses this growth rate in a discrete-time (Brownian motion) diffusion process for the daily index prices. The inclusion of a daily volatility component makes the model more realistic for generating daily index prices.

The daily ALSI40 index level thus progresses according to the following formulae:

$$ALSI_{260,t} = \left( e^{EQDG_t + EQDY_{t-1} - EQDY_t} \right) ALSI_{0,t},$$

$$ALSI_{0,t} = ALSI_{260,t-1},$$

$$ALSI_{0,1} = 1000$$

$$ALSI_{s,t} - ALSI_{s-1,t} = \frac{ALSI_{260,t} - ALSI_{s-1,t}}{261-s} + \sigma dz \cdot ALSI_{s-1,t},$$

$$s \in [1, 260] \text{ and } t \in [1, \infty)$$

where

$ALSI_{s,t}$  = official closing ALSI40 index value on day  $s$  of year  $t$

$EQDG_t$  = the average force of dividend growth during year  $t$

$EQDY_t$  = the logarithm of historic dividend yield(%) at end of year  $t$  <sup>10</sup>

$EQDY_0$  = the logarithm of historic dividend yield(%) at start of year 1

$\sigma$  = daily index growth volatility <sup>11</sup>

$dz$  = a random draw from the standardized normal distribution<sup>12</sup>

### Yield curve

The market model generates a money market force of interest and a long-term force of interest on an annual basis (i.e. new forces of interest for each year). The yield curve is assumed to be flat at the level of the money market force of interest.

The yield curve was assumed to remain unchanged during the year. A more sophisticated model could project a smooth progression between the annually projected yield curves. Similarly, the yield curve model could be defined to be arbitrage-free and consistent with the interest rate process.

### Prospective dividend yield on ALSI40 index

The dividend yield assumption only has an impact on the option delta and futures price calculations. The life office does not earn an uncertain dividend stream from the assets backing the GEB. The index fund mentioned in section 2.1 excludes dividends since the underlying dividend stream is reflected in a discounted purchase price for the index fund.

Typically only limited information is available on the prospective dividends of a company. Furthermore, dividend policy is often not certain, making the past dividend history less relevant in predicting future dividends. However, the dividend yield on an equity index consists of many diverse dividend flows, and through this diversification the uncertainty of the future dividend flow is reduced.

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<sup>10</sup> Index growth is determined by both dividend growth and changes in the dividend yield.

<sup>11</sup> Rather than using the volatility implied by the Thomson model, the simulation will assume various levels of index growth volatility to test the sensitivity of the dynamic hedging results to this factor.

<sup>12</sup> A polynomial approximation to the inverse cumulative standard normal distribution function is used to generate the random draws. Refer to Appendix D for further details.

The simulation assumes that the market's implied expectation of future dividends (as reflected by futures prices) is on average correct. The expected future dividend was thus calculated by growing the current dividend at the market model's expected force of equity dividend growth (estimated by Thomson as  $0.093 + 0.076\eta_{EQDG,t-1}$ ).<sup>13</sup> The prospective dividend yield at time  $t$  is thus the actual historic dividend yield  $e^{EQDY_t}$  multiplied by the expected dividend growth rate factor  $e^{0.093 + 0.076\eta_{EQDG,t-1}}$ , to give a result of  $e^{EQDY_t + 0.093 + 0.076\eta_{EQDG,t-1}}$ .

### Scrip borrowing or lending fee

As noted in section 3.2, the uncertainty of this parameter contributes to basis risk. However, since basis risk is already modelled through its stochastic dividend and interest rate components, a fixed scrip borrowing or lending fee of 1.5% p.a. is assumed.

### Transaction cost

Transaction costs impact on all stages of the hedging exercise – the creation of the hedge, the day-to-day rebalancing (especially when deal sizes increase, e.g. when gamma is large) and the removal of the hedge at expiry. The transaction costs are the bid-offer spread, brokerage, value-added tax and marketable securities tax.

The transaction cost incurred on purchase (sale) will be reflected in the simulation as a offer (bid) spread above (below) the futures mid-price. It was assumed that no costs are incurred on the expiry of the futures contract. Various bid and offer spreads will be used in the simulation to test the sensitivity of results.

### Market impact costs

The market impact cost is the additional cost incurred by a buyer when the buyer's

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<sup>13</sup> Refer to Appendix C. According to the Thomson model  $EQDG_t = 0.093 + 0.116\eta_{EQDG,t} + 0.076\eta_{EQDG,t-1}$ .

Since  $\eta_{EQDG,t}$  is unknown at time  $t-1$  the mean of zero is assumed.

action inflates the purchase price. Similarly, the decrease in selling price due to the seller's order is the market impact cost of that sale.

Market impact costs depend on the transaction size relative to the total volume traded, i.e. the liquidity of the market. Investment banks typically aim to keep the transaction volume below 25% of the daily trading volume to avoid a marked impact on prices. The size of the hedge that can be cost-effectively managed thus depends on the liquidity of the market. A liquid market (a market with "depth") has a large volume of price-sensitive buyers and sellers. The market for exchange-traded futures is generally more liquid than the spot market, which in turn is more liquid than the market for long-dated OTC forward contracts.

Market impact can be reduced through defining the option payoff in terms of an average spot index value, and issuing options with a variety of strike prices and expiry dates. Diversity in characteristics avoids a concentration of market trading and thus minimizes the need for making large trades. This is especially important where there are similar options offered by other institutions (life offices, investment banks, etc.) which will all be performing the same hedging activity.

Market impact is also reduced when the exposures to individual risks within a very large portfolio of risks are offset (diversified). Only the net exposure is hedged. The focus in such a situation is on trading only to manage the profile of the overall diversified portfolio, not individual liabilities. The offsetting of the individual liabilities prior to hedging avoids "dealing with oneself" (i.e. using one position to hedge one contract and the opposite position to hedge another contract).

For the purpose of the simulation exercise, market impact costs were ignored since the size of the dynamic hedging transactions are not expected to be significant relative to the market's daily trading volume.

### 3.4 Hedging strategy

The Black-Scholes option pricing methodology assumes the hedging strategy entails instantaneous rebalancing of the asset portfolio to ensure the asset delta matches the liability (option) delta after an underlying price change. The mismatch created by the price change between successive asset rebalancings is infinitesimal.<sup>14</sup>

In practice, the discontinuity in market prices (jumps) prevents continuous trading during the price change. The rebalancing transaction to re-establish the matched position locks in the non-infinitesimal tracking error between the asset and liability. The larger the jumps in the price series, the greater the tracking error.

When a price jump is expected, the asset should be adjusted to match the expected change in the liabilities due to the price jump. In terms of option sensitivities, an "interval delta" (change in option price for a given jump in market price), rather than an instantaneous delta (change in liability for an infinitesimal change in market price) should be used. However, given that it is unreasonable to assume that market jumps can be anticipated, only an instantaneous delta is applied in the simulation.

Tracking errors could be positive or negative. The value of a written put option is convex with respect to the asset price ( $\gamma > 0$ ). If the value of the hedge portfolio is linear with respect to the asset price, with a delta equal to that of the option, then the tracking error will always be negative. Where the asset delta differs from the option delta, the tracking error could be positive if the price movement favours the asset delta.

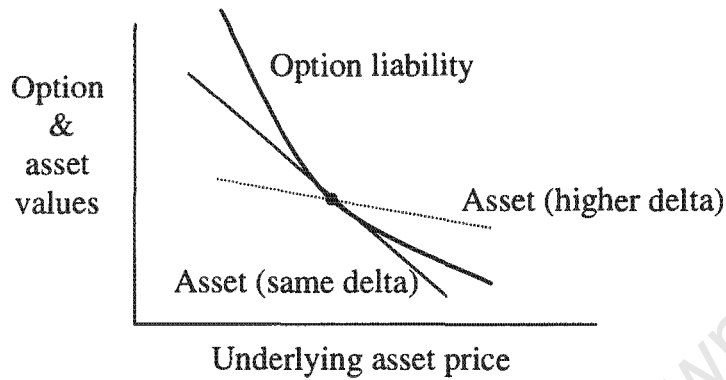
Exhibit 2 provides an illustration of the option liability and two asset alternatives, one with the same delta as the liability, and one with a higher delta. The starting asset price (before the asset price changes) is indicated by the dot. Here the option liability value is

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<sup>14</sup> A portfolio is said to be "delta neutral" if an infinitesimal change in the index to which the assets and liabilities are linked will have an equal effect on the assets and liabilities, thus leaving the net asset position unchanged.

equal to the value of both the asset alternatives. The asset with the higher delta will lead to a mismatch gain when the underlying asset price increases, but a larger potential loss when the underlying asset price decreases.

**Exhibit 2: How a gain may arise from a delta mismatch**



The hedging strategy involves three aspects: (a) how the asset-liability mismatch is evaluated; (b) when the asset-liability mismatch is evaluated; and (c) the decision rule ("trigger") for adjusting assets following an evaluation of the asset-liability position.

**(a) How the asset-liability mismatch is evaluated**

The asset-liability mismatch is evaluated by comparing the delta of the assets to the delta of the liability. The following section describes how these deltas are calculated.

**Calculation of liability delta**

Since the liability (put option) is valued using the Black-Scholes pricing formula, the delta simplifies to:

$$\Delta = e^{-q(T-t)} [N(d_1) - 1],$$

where  $q$  = force of dividend on the ALSI40 index

$T$  = option expiry

$t$  = current time

$N(d_1)$  is defined as for the option pricing formula (Hull 1997: 318)

The delta calculation above is based on the assumption that the underlying asset price follows a log normal random walk process, and not the autoregressive process assumed in this study. Refinement of the delta calculation to recognise the differences in the asset price processes is complicated and falls outside the scope of this study.

The following parameter values have to be specified in the calculation of the liability delta:

- *Time*: The option has an initial duration of 5 years, or 1300 trading days.
- *Zero coupon interest rate for the period until option expiry*: The interest rate used to calculate the liability delta is the interest rate generated by the market model for the outstanding duration of the option (i.e. the time to expiry).
- *Prospective ALSI40 dividend yield*: It is assumed that the hedger's view of prospective dividends is the same as the market's view of prospective dividends (as implied by the prices of futures). The dividend yield used is therefore the prospective dividend yield generated by the market model.
- *Expected future ALSI40 growth rate volatility*: Since the hedger does not have knowledge of the price processes underlying the market, the market model assumed for this analysis cannot be used as the basis for the hedging strategy. Consequently, the expected future ALSI40 growth rate volatility was not derived from the Thomson model.

The overall market view of expected volatility, as revealed in the implied volatility of listed options, can be used as an estimate of expected short term ALSI40 volatility.<sup>15</sup> However, the implied volatility may differ for options with different maturities and for options with the different strike prices. The implied volatility is

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<sup>15</sup> The implied volatility is the value of the volatility parameter which equates the option valuation formula assumed with the observed option price.

generally higher for deep in-the-money or far out-of-the-money options<sup>16</sup>.

A reasonable benchmark estimate of daily volatility over the remaining term of the put option would be the actual historic volatility in daily ALSI40 growth rates. The daily volatility over the 100 trading days until end-March 2001 was 21.61% p.a.<sup>17</sup> Since the purpose of the study is not the accurate pricing of the option, but rather the illustration of the simulation method, the absolute market volatility becomes less important.

Transaction costs must be recognised in the delta to ensure more accurate hedging. Assuming the bid spread and offer spread remain at 0.2%, and the adjustment suggested by Leland (1985) is applied, the assumed volatility of daily asset price returns becomes 21.7% p.a.<sup>18</sup>

In practice, each time the asset-liability mismatch is evaluated, all the information available would be used. Specifically, the estimate of future volatility would be updated to recognise the volatility experienced during the expired term of the option. This will necessitate the calculation of historic volatility each time the asset-liability mismatch is evaluated.

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<sup>16</sup> This so-called "smile effect" exhibited by the implied volatility surface (implied volatility plotted against duration and strike) reflects the incorrect assumptions made in the Black-Scholes option pricing formula.

<sup>17</sup> Volatility per day  $\times \sqrt{\text{no. of trading days per annum}}$  = Volatility per annum

$$\begin{aligned}
 {}^{18} \text{ (New } \sigma^2 \Delta t) &= \sigma^2 \Delta t \times \left[ 1 + \%(\text{buy} + \text{sell spread}) \cdot \frac{E\left[\frac{\Delta S}{S}\right]}{\sigma^2 \Delta t} \right] \\
 &= (0.01340195\dots)^2 \times \left[ 1 + 2 \times 0.002 \cdot \frac{e^{\left[\frac{EQDG}{260}\right]} - 1}{(0.01340195\dots)^2} \right] = 0.0001810433\dots
 \end{aligned}$$

$$\text{New volatility of daily returns} = \sqrt{0.0001810433\dots} = 0.01346\dots$$

- *Strike price*: The strike ALSI40 index level is the starting ALSI40 index level, which is assumed to be 1000.
- *Spot ALSI40 index level*: The index level as generated by the market model is used.

### Calculation of asset delta

The delta of a futures contract differs from that of its underlying asset. This is due to the cost of carry incorporated into the futures price, and the fact that the delta is expressed in terms of absolute and not relative changes. In other words,

$$\begin{aligned}
 \text{futures price} &= \text{spot price} \times (1 + \% \text{ cost of carry}) \\
 &= \text{spot price} \times (1 + \text{interest rate} - \text{dividend yield}) \\
 &= \text{spot price} \times e^{(r-q)(T^*-t)}
 \end{aligned}$$

where  $r$  = force of interest,  $q$  = force of dividend,  $T^*$  = futures expiry and  $t$  = current time.

When the stock price increases by  $\Delta S$ , then the futures price increases by  $\Delta S \times e^{(r-q)(T^*-t)}$ . A positive cost of carry thus means that the absolute change in the spot price has an amplified absolute effect on the futures price (note that the change as a percentage is the same).

The delta of the futures contract is  $e^{(r-q)(T^*-t)}$ . This implies that  $e^{-(r-q)(T^*-t)}$  futures contracts have the same sensitivity to stock price movements as one stock ( $e^{(r-q)(T^*-t)} \times e^{-(r-q)(T^*-t)} = 1$ ). To match asset and liability deltas, futures contracts of  $e^{-(r-q)(T^*-t)}$  times the liability option notional should be held.

The following parameter values have to be specified in the calculation of the asset delta:

- *Time*: The future has an initial duration of 65 trading days.
- *Zero coupon interest rate for the period until expiry of the futures contract*: The interest rate used is the money market interest rate as generated by the market model.

- *Prospective ALSI40 dividend yield*: The prospective dividend yield used is the same as that generated by the market model. The justification for its use in the case of the liability delta calculation also applies here.
- *Spot ALSI40 index level*: The index level as generated by the market model is used.

**(b) Incidence of asset-liability mismatch evaluation**

It is possible to monitor the asset-liability mismatch continuously (e.g. a computer program could calculate the asset and liability deltas continuously based on real-time market prices). Alternatively, a risk manager or trader could look at the position periodically throughout the day.

To simplify the simulation process, the asset-liability mismatch (i.e. the asset and liability deltas) will be calculated on a daily basis. This is also consistent with the accounting process, which involves daily marking-to-market of exchange-traded derivatives.

**(c) Trigger rule for adjusting assets given an asset-liability mismatch evaluation**

The trigger rule should recognise the cost-benefit trade-off involved in a rebalancing exercise, i.e. transaction costs versus potential mismatch losses. The trigger rule could be set to initiate a transaction only when the difference in asset and liability deltas falls outside a specific range. This range represents the deltas where the transaction costs outweigh the gain from avoiding mismatch risk. An increase in the costs per transaction would lead to a larger range of inaction. This range will be set at three levels for simulation purposes: 0, 0.05 and 0.1.

A wider range can be expected to reduce the number of asset adjustments, which in turn will reduce the transaction costs. Note that the transaction costs are avoided only when the change in asset or liability delta narrows the difference between them. When the delta difference widens, even without immediately triggering a transaction, a subsequent trigger would require an asset adjustment for the total movement in the liability delta. The following example illustrates the point more clearly.

### Exhibit 3: Example of transaction cost saving for non-zero delta mismatch

Zero delta mismatch allowed:

Time	Liability delta	Asset delta before adjustment	Asset delta after adjustment	Transaction size (adjustment to asset delta)
0	0.50		0.50	
1	0.54	0.50	0.54	0.04
2	0.52	0.54	0.52	0.02
3	0.53	0.52	0.53	0.01
4	0.57	0.53	0.57	0.04

Total transaction size in terms of asset delta: 0.11

0.05 delta mismatch allowed:

Time	Liability delta	Asset delta before adjustment	Asset delta after adjustment	Transaction size (adjustment to asset delta)
0	0.50		0.50	
1	0.54	0.50	0.50	0
2	0.52	0.50	0.50	0
3	0.53	0.50	0.50	0
4	0.57	0.50	0.57	0.07

Total transaction size in terms of asset delta: 0.07

The asset delta change during the interval between time points was ignored. Provided the time interval is small (e.g. 1 day), the asset delta change will be insignificant.

The 0.05 delta mismatch saved the transaction costs only on part of the first movement in liability delta (the part from 0.52 to 0.54). The saving consists of the part movement (0.02) and its reversal at time 2 (another 0.02).

### 3.5 Reserve requirement

The Long Term Insurance Act requires a South African life office to prove that its admissible asset value exceeds its statutory liability value on an office-wide basis. The life office must, at least, follow these statutory reserving guidelines, and may apply

stricter internal guidelines. The reserving guidelines involve both assets and liabilities and we look at each in turn in the case of the dynamic hedge.

### **Assets**

The hedge asset consists of a futures position that is marked-to-market daily. Since it has no intrinsic market value the futures are not considered for admissibility. The life office will include assets from the policyholder funds and possibly the corporate fund in the statutory asset valuation. These assets comprise a capital injection, the premium received from the client for the put option, and the accumulation of the changes in the margin requirement.

### **Liabilities**

A liability that is linked solely to specific assets held by the life office is "linked" (or perfectly matched) and can be excluded from the statutory valuation. The put option liability is a guarantee issued by the life office. Since it is not linked to specific assets held by the life office, the liability is "non-linked" (as defined by the Long Term Insurance Act) and must be accounted for in the statutory valuation.

The statutory and financial soundness valuations (Actuarial Society of South Africa 1998) are based on deterministic future investment assumptions and therefore do not recognize the stochastic nature of the liability. Specifically, the deterministic future investment assumptions are likely to project an equity index level above the guaranteed minimum index level, and ignore the scenarios where the guarantee becomes effective.

The life office also has to show that its capital holding meets the statutory capital adequacy requirement (CAR). The CAR is calculated as the capital necessary to avoid ruin under the worst of two scenarios: (a) a fall in equity values of 30% on the valuation date; and (b) an equity return of 2% p.a. to maturity. Neither of these scenarios will trigger the capital guarantee of the GEB. Under realistic future investment assumptions, the equity index growth over the 5-year investment term is expected to be sufficient to absorb a 30% decline at any point; and a 2% growth rate will ensure that the index on expiry is above the guarantee level.

There is thus no statutory reserve requirement for the put option liability component of the GEB. The required reserve for the GEB as a whole is thus the value of the equity index exposure ignoring the capital guarantee.

However, the Professional Guidance Note 104 of the Actuarial Society of South Africa (1998) requires the actuary to adopt whatever prudence is necessary, so reserves for the maturity guarantee and other risks (e.g. tracking error, changes in tax legislation, counterparty failure) will be required.

The Maturity Guarantees Working Party of the Institute of Actuaries (1980) (referred to as the MGWP) proposed an approach to calculating these reserves. It suggested that a model office simulation be used to set the reserve requirement for an investment guarantee, i.e. put option. The reserves are set so that at a given confidence level no further capital injections will be required in future to meet the liability.

The MGWP suggested the following **basis**:

- a) no withdrawals
- b) initial ruin probability of  $1/100$
- c) reserves should be strengthened if ruin probability subsequently rises to  $1/50$
- d) reserves may be released if the ruin probability subsequently falls to  $1/1000$

The MGWP suggested the following **process** for calculation of the initial and subsequent reserves:

- a) a simulation program incorporating the stochastic investment model to produce 5000 independent sequences of unit prices for the next  $n$  years (where  $n$  is the time to the guarantee);
- b) for each sequence of unit prices, the maturity guarantee payment for a policy is computed as the guarantee less the projected unit fund (provided the guarantee exceeds the fund);

- c) the sum of the discounted maturity guarantee payments for each sequence is computed and ranked in order so that  ${}_t A_1$  is the largest sum for the valuation at time  $t$  and  ${}_t A_{5000}$  is the smallest;
- d) if  ${}_t V$  is the maturity guarantee reserve at time  $t$  then  ${}_0 V = {}_0 A_{50}$ , i.e. the value corresponding to a ruin probability of 1/100;
- e) for valuations where  $t > 0$  a further calculation is necessary to obtain  ${}_t B_1, \dots, {}_t B_{5000}$  where the  ${}_t B$ 's are calculated in the same way as the  ${}_t A$ 's but excluding policies less than 1 year old. The maturity guarantee reserve for new business  ${}_t N = {}_t A_{50} - {}_t B_{50}$
- f) the total reserve  ${}_t V = {}_{t-1} V(1+i) + ({}_t P - G)(1 + \frac{i}{2}) + {}_t N$ , i.e. the previous reserve plus premiums for the guarantee less claim payments under the guarantee all accumulated at interest plus the new business reserve;
- g) if  ${}_t V < {}_t A_{100}$  the reserve has to be increased and if  ${}_t V > {}_t A_5$  the reserve may be reduced.

The MGWP proposal assumes an asset management strategy that is independent of the liability value, and a corresponding fixed discount rate (as noted in point c. above). The dynamic hedging (option replication) strategy, in contrast, manages the assets so that it mirrors the liability performance. An appropriate discount rate is thus the rate earned on the dynamically managed asset portfolio. When the equity index falls rapidly, leading to the possibility of a large guarantee payment, the assets grow faster to compensate. Similarly, when the index rises, leading to the possibility of a small or no guarantee payment, the assets decline. This dependence between the asset performance and the liability performance is not recognised when a fixed discount rate is used. The discount rate should reflect the asset performance associated with each guarantee payment, i.e. large guarantee payments should be discounted at a higher rate, and smaller guarantee payments at a lower rate.

The simulation model incorporated this effect through the addition of the dynamic hedging cash flows to the assets representing the reserve. The overall growth rate earned on the reserve therefore consists of its investment return (interest rate on cash) combined with the impact of the dynamic hedging cash flows on the assets held as reserves. When the likelihood of a guarantee payment increases, dynamic hedging cash

inflows will arise, thereby increasing the overall growth rate of the assets held as reserve. Similarly, when the likelihood of a guarantee payment decreases, cash outflows will arise, thereby decreasing the overall growth rate of the assets held as reserves.

Roff (1992), in the context of the dependence between the liability valuation basis and asset mix, shows that failure to recognize this ability to change asset mix will result in a significant overstatement of the likelihood of insolvency.

The simulation model in this study set the initial reserve requirement so as to avoid ruin in 99% of cases. The initial reserve requirement was calculated as follows. For each of the 10 000 simulations<sup>19</sup>, the daily cash flows were accumulated, on a day-by-day basis, at the simulated interest rate on cash. Whenever the accumulated asset pool fell below zero, a capital injection was made to maintain the asset pool at or above zero.<sup>20</sup> By avoiding a negative asset pool this approach prevents the recognition of negative reserves. For each simulation run, the capital injections were discounted at the interest rates generated within the simulation run. The 99<sup>th</sup> percentile of this distribution of capital injections was used as the initial reserve requirement.

Guidance Note 25 of the Institute of Actuaries (1996) recommends that reserves established for a derivative liability should be at least sufficient to meet the cost of closing out the derivative position involved. In practice, the life office would thus have to compare the calculated reserve to the cost of the static hedge from various investment banks. Due to the reluctance of investment banks to offer static hedge quotes for this study, it was impossible to subject the reserves to this minimum requirement.

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<sup>19</sup> Although the MGWP suggested 5000 simulations, 10 000 simulations were performed to improve the reliability of results.

<sup>20</sup> No attempt was made to model the future reserve requirements dynamically, i.e. we are not setting the initial reserve to cover all future reserve requirements. In practice, the reserve requirement will be set at the time of each reserve evaluation by running the model with the parameters that reflect the best estimates at that time.

## Chapter 4      Analysis of results and sensitivity testing

This chapter presents an analysis of the simulation results for a variety of scenarios. The sensitivity of the simulation results to changes in the model parameters is also analysed.

Pemberton (1998: 166) notes that "for scenario modeling to be effective, it is important that the range of scenarios should be chosen having regard to the nature of the real situation being investigated, and should be sufficiently broad". In practice, the life office would test a very wide range of scenarios to ensure that it is prepared for most eventualities. However, for the purpose of the dissertation, only a limited set of scenarios were tested to determine whether the hedging strategy (specifically, the delta mismatch allowed) influenced the sensitivity of results to (a) the market volatility and (b) the bid and offer spreads. The other components of the market model were modelled stochastically and therefore not tested for sensitivity.

Three levels of allowed delta mismatch were selected (0, 0.1 and 0.2), each level representing a distinct dynamic hedging strategy (labeled scenario 1, 2, and 3). For each hedging strategy five different combinations of market volatility and transaction costs (labeled sub-scenario a, b, c, d, and e) were investigated.<sup>21</sup>

In total 15 scenarios were investigated, each with the following results:

- a) the reserve required to avoid ruin in 99% of cases,
- b) the distribution of the accumulated tracking error<sup>22</sup> expressed as a fraction of the required reserve<sup>23</sup>, and

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<sup>21</sup> Note that the hedging strategy remains unchanged across all sub-scenarios (specifically, the volatility and bid and offer spreads used to calculate the dynamic hedge ratio remain fixed).

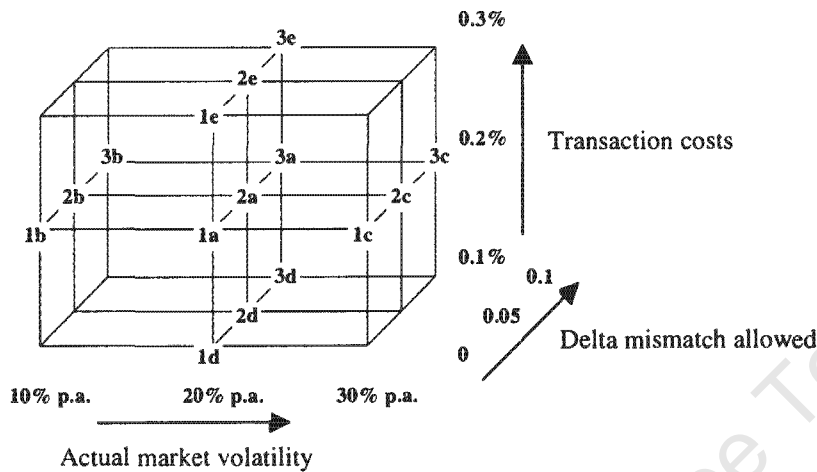
<sup>22</sup> The accumulated tracking error consists of all the dynamic hedging cash flows including all transaction costs and the guarantee payment, if any, on the GEB maturity date.

<sup>23</sup> The accumulated tracking error was expressed as a fraction of the required reserve to normalise the results for difference in reserve requirements between scenarios.

c) the sensitivity of the required reserve and accumulated tracking error to changes in market volatility and the bid and offer spreads.

The following diagram illustrates the relationship between the 15 scenarios.

**Exhibit 4: Spatial representation of the 15 scenarios investigated**

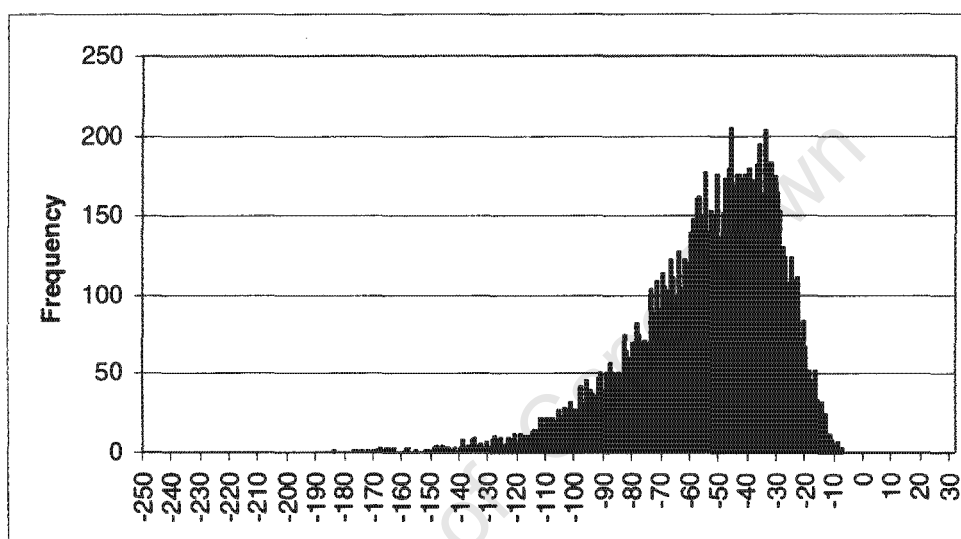


The complete results appear in Appendix E. This chapter highlights the most important findings. The findings are presented for each hedging strategy separately to allow comparison between the strategies.

#### 4.1 No delta mismatch allowed

The hedging strategy investigated here triggers an asset change whenever the asset and liability deltas differ. Simulation of scenario 1(a) produced the frequency distribution shown in Exhibit 5.

**Exhibit 5: Accumulated tracking error as a % of required reserve (scenario 1(a))**



The distribution is negatively skewed (skewness = -0.98).<sup>24</sup> This is the result of different changes in asset and liability values as the underlying index changes. As the underlying index increases, the liability value reduces at a decreasing rate (i.e. gamma is positive), while the asset value reduces at a constant rate (i.e. gamma is zero). Similarly, when the underlying index decreases, the liability value increases at an increasing rate, while the asset value increases at a constant rate. Assuming the delta of assets and liabilities are matched, then any change in the index will lead to a loss (since the asset value will fall faster or rise slower than the liability value).

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<sup>24</sup> The skewness of a distribution is the extent to which the distribution is asymmetric around its mean. A positive skewness usually means that the right-hand side of the distribution is more widely dispersed from the mean than the left-hand side. Similarly, a negative skew means that the left-hand side of the distribution is more widely dispersed from the mean than the right-hand side. A normal distribution has a skewness of zero.

The necessity of a loss on changes in the index also explains why the asset pool distribution is below the level of 0% (maximum is -8.6%). The 0% level is only achieved when there are no tracking error losses or tracking error gains offset tracking error losses. Scenario 1(b) produced a number of projections with positive accumulated tracking errors (maximum is 27.28%). The positive tracking errors were due to differences between the actual liability delta and the estimated liability delta (which is used to set the asset delta). The estimated liability delta was based on a volatility assumption of 21.7% p.a. while the simulation was based on a volatility assumption of 10% p.a. Similarly, all the scenarios involving a deliberate mismatch of asset delta and calculated liability delta produce projections with positive accumulated tracking errors.

Assuming zero volatility (i.e. the underlying index does not move at all over the 5 year period), there are no mismatch losses and the frequency distribution would be very narrow and peaked. A small increase in the volatility will lead to a few mismatch losses which create a negative skewness. As volatility increases further, we expect more mismatch losses and the frequency distribution of the asset pool to shift its weight from the small negative to the larger negative values, resulting in a more negative mean and less negative skewness. This is confirmed by the results.

**Exhibit 6: Accumulated tracking error distributions for change in volatility**

Scenario	Volatility	Mean	Skewness
1(b)	10% p.a.	(39.04%)	(1.38)
1(a)	20% p.a.	(54.95%)	(0.98)
1(c)	30% p.a.	(67.91%)	(0.34)

It is interesting to note that an increase in the transaction costs also results in a more negative mean and a less negative skewness. This is consistent with the argument that transactions costs have the same effect as an increase in volatility.<sup>25</sup>

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<sup>25</sup> Section 3.1 details how the volatility assumption in the Black-Scholes pricing model can be adjusted to incorporate transaction costs in option pricing.

**Exhibit 7: Accumulated tracking error distributions for change in transaction costs**

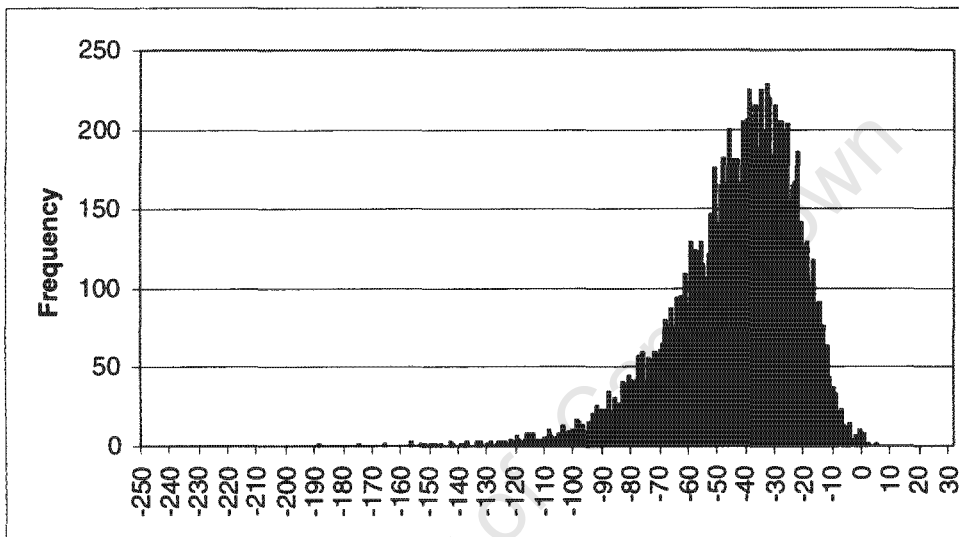
Scenario	Bid & Offer spread	Mean	Skewness
1(d)	0.1%	(54.50%)	(1.03)
1(a)	0.2%	(54.95%)	(0.98)
1(e)	0.3%	(58.49%)	(0.86)

The reserve required was calculated as 25.1% of the option notional, and was not sensitive to small changes in market volatility and bid offer spreads (increased to 25.7% for 1% increase in volatility; increased to 25.3% for 0.05% increase in bid and offer spreads). The insensitivity of the reserve requirement is observed at all the other zero mismatch sub-scenarios (1(a), 1(b), 1(c), 1(d), 1(e))

#### 4.2 Delta mismatch of 0.05 allowed

This hedging strategy triggers an adjustment to the assets whenever the absolute difference between asset and liability delta exceeds 0.05. Simulation of scenario 2(a) produced the frequency distribution shown in Exhibit 8.

**Exhibit 8: Accumulated tracking error as a % of required reserve (scenario 2(a))**

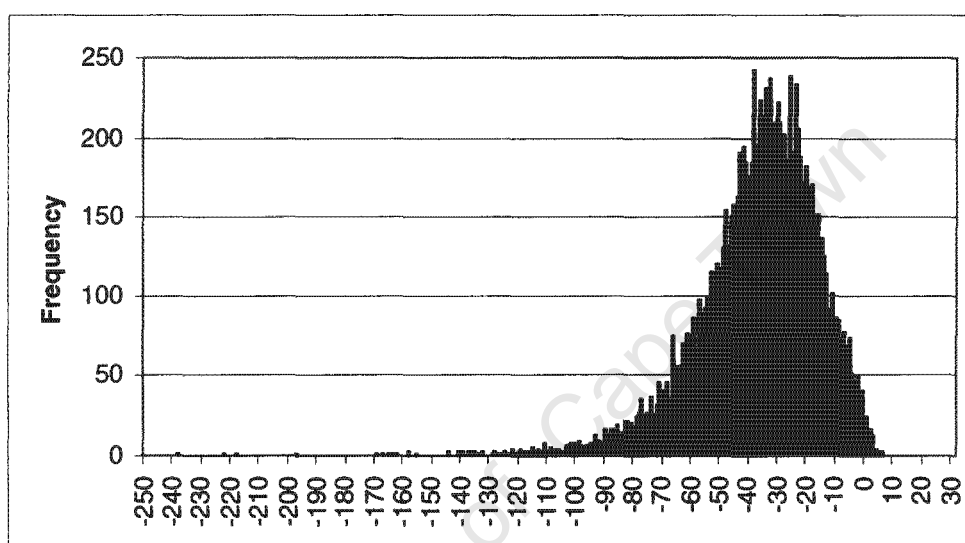


Given the greater leniency in this matching strategy, one would expect a greater variance of results than in the previous scenario (where no delta mismatch was allowed). This is reflected in the higher reserve requirement (31.3% versus 25.1%). Again the reserve requirement was not very sensitive to small changes in market volatility and bid and offer spreads.

### 4.3 Delta mismatch of 0.1 allowed

This hedging strategy triggers an adjustment to assets whenever the absolute difference between asset and liability delta exceeds 0.1. Simulation of scenario 3(a) produced the frequency distribution shown in Exhibit 9.

**Exhibit 9: Accumulated tracking error as a % of required reserve (scenario 3(a))**



The higher level of leniency in the matching strategy results in a significantly higher reserve requirement. In fact, the reserve requirement increases exponentially with a linear increase in the delta mismatch allowed.

The mismatch strategy benefits from mismatch gains on reversals in the underlying index movement. In the extreme this results in a net mismatch gain (refer to maximum accumulated tracking error). The distribution reveals a higher kurtosis, which confirms the benefits from reversals in the underlying index movement. These reversals in the projected index movements are attributable to the mean-reverting Thomson model used. The figures are presented in Exhibit 10 for comparison.

**Exhibit 10: Accumulated tracking error distribution for different delta mismatches**

Scenario	Delta mismatch allowed	Required reserve	Tracking error distribution	
			Skewness	Kurtosis
1(a)	0	25.1%	(0.98)	(1.23)
2(a)	0.05	31.3%	(0.99)	(1.70)
3(a)	0.1	40.8%	(1.29)	(4.41)

A zero delta mismatch strategy effectively “locks in” each tracking error loss and does not attempt to benefit from a reversal in the index. A delta mismatch strategy, on the other hand, involves accepting exposure to a higher potential tracking error loss in return for a potential gain on index reversal. The decision depends on the likelihood of a reversal and the size of the potential tracking error gain relative to the potential tracking error loss.

To determine the relative profitability of the different mismatching strategies, it seems reasonable to compare the mean accumulated tracking errors. This should be expressed as a % of option notional to normalise for the differences in the required reserve between different mismatching strategies. The results are presented in exhibit 11.

**Exhibit 11: Required reserve and tracking error for different delta mismatches**

Scenario	Delta mismatch allowed	Required reserve as % of option notional	Mean tracking error as % of option notional
1(a)	0	25.1%	(13.8%)
2(a)	0.05	31.3%	(13.9%)
3(a)	0.1	40.8%	(15.3%)

The zero delta mismatch strategy has the lowest capital requirement and the smallest negative mean accumulated tracking error, and consequently appears to be the most capital-efficient and profitable strategy. Although not investigated here, it seems likely that intra-day matching would also prove to be more capital-efficient and profitable than daily matching.

## Chapter 5      Practicalities

This chapter will briefly examine the practical issues that fall outside the scope of the dissertation and were either ignored or simplified for the purposes of the simulation. These issues must be considered before the simulation results can be extrapolated to dynamic hedging practice and conclusions made about the feasibility of in-house dynamic hedging.

### 5.1 Counterparty credit risk

Counterparty credit risk refers to the potential non-performance by the counterparty to a contract. Some of the approaches for mitigating counterparty credit risk are discussed below.

Dynamic hedging through exchange-traded derivative contracts involves very little credit risk since the counterparty is the exchange clearing-house rather than another derivative trader. Static and dynamic hedging through over-the-counter (OTC) derivative contracts involve much greater credit risk since these contracts are traded outside recognised exchanges and do not involve a centralised trading facility, clearing-house or margining system. Apart from the high credit risk these contracts involve lower liquidity and greater search costs to offset exposures.

Counterparty credit risk also depends on the extent of the amount at risk over time. In the case of a marking-to-market arrangement, the credit risk is limited to the price movement during a day, which reduces credit exposure and allows better utilization of available credit lines. Where settlement is delayed to the end of the contract there could be a significant build up of credit risk.<sup>26</sup>

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<sup>26</sup> Johnson and Stulz (1987: 267) investigated the prices of options subject to default risk ("vulnerable options"). They found that the value of a vulnerable European option declined as term to expiry increased, interest rates increased and volatility increased. They concluded that this was due to the greater likelihood of counterparty default offsetting the usual increase in the option price.

The credit risk of the OTC contract can be reduced through careful wording of the contract. The International Swaps and Derivatives Association (ISDA) has formulated a standard agreement that can be incorporated in an OTC transaction. This so-called ISDA master agreement includes a "netting agreement" that allows the offset of payments owed to a defaulting counterparty against those owed by the defaulting party. The option writer may also be required to provide collateralisation through depositing some or all of the market value of the option in a suitable escrow account.

The credit risk of a static hedge (OTC contract) could also be mitigated by wording the GEB contract so that this risk of default is carried by the private investor. This is only feasible where the risk is clearly defined. The nature of the underlying assets, the counterparty and their credit quality must be clearly explained to the private investor to avoid subsequent claims that the life office misrepresented the risks.

The assessment of the credit risk associated with a proposed OTC contract requires a projection of the possible future market movements to develop an estimate of the potential credit exposure over the life of the contract. Knowledge of the "worst case" risk exposures and their likelihood would be essential for a reasoned judgement of the credit risk. In contrast to the potential credit risk, the actual credit risk is the amount which the life office would lose if a default occurs at a specific time. For more detail about the assessment of credit risk refer to Banks (1994: 31).

## **5.2 Operational (staff and systems)**

Static hedging and dynamic hedging place emphasis on different operational areas. Static hedging involves very complex and unique OTC contracts that require very flexible back office systems (for maintaining records, custodianship, and settlements). The front office activity (pricing, negotiation and contracting) involves significant manual work due to the uniqueness of each contract: contract terms are non-standard and must thus be cleared by the legal and credit risk divisions.

Dynamic hedging, on the other hand, involves standard contracts (exchange-traded futures) which allow most of the back office and front office activities to be automated. However, much greater emphasis is placed on the real time monitoring of the asset-liability position and the continuous trading to maintain the match between assets and liabilities.

The development costs required for either static or dynamic hedging will depend on the sophistication of the life office's existing systems. The simulation ignores these costs.

The risk of operational failure in the case of static hedging is low since matching is a once-off activity (setting the terms of the static hedge). In the case of dynamic hedging, matching is an ongoing activity of real-time asset and liability monitoring and management. It requires rapid access to the inputs for pricing models (interest rates, dividend prospects and volatility estimates) and significant processing power to model all the (non-linear) sensitivities of the portfolio. Actions to correct for observed mismatches should also be implemented speedily and correctly. A failure in any of these aspects can result in significant mismatch losses.

Appendix F lists potential sources of operational failure and preventative measures that can be taken.

### **5.3 Legal**

Legislation governing derivative contracts in South Africa is in its infancy. New regulations or changes to existing regulations could adversely affect existing derivative positions. The risk is a change in legislation that results in losses to the life office. It is unlikely that the life office would pass the losses on to the private investor, since claims of misrepresentation or general bad publicity could inflict a much higher cost.

## 5.4 Tax

The taxation of life offices differs from that of investment banks. This has important implications for the tax treatments of static and dynamic hedging.

A life office pays tax on behalf of its policyholders according to the four funds tax basis. The policyholder funds, with tax rates in brackets, are: untaxed policyholders (0%), individual policyholders (30%), and corporate policyholders (30%). These funds are taxed on an "I-E" (investment income minus expenses) basis, where income excludes capital gains. Retirement funds are taxed separately at 25% on interest and rental income.

The profit made in the policyholder funds is transferred to the corporate fund (representing the capital of the life office) where it is taxed as the profit of a normal corporate taxpayer. The transfers from the policyholder funds are smoothed over five years to decrease the volatility of life office profits.

In contrast, an investment bank is taxed as a trader. All its gains and losses whether of a capital or income nature are netted off for the tax calculation, and it does not pay tax on behalf of its clients.

### Implications for static hedging

An investment bank offsets the put option payment (made to the holder of the OTC put option) against all net income and capital gains made on the bank's dynamic hedging portfolio. The bank is only taxed on the net result of the dynamic hedge, i.e. its profit.

The life office faces the risk that the proceeds from the static hedge (purchased OTC put option) will be deemed as trading income and not as untaxed capital gains.

It is currently accepted that the put option payoff should be treated as a capital gain. Since the put option is a long-term (5 year) hedge against equity downside, it is used to offset capital losses incurred on the equity investment. Should the put option payoff be

treated as trading income then the loss on the underlying index fund should also be treated as a trading loss. The two results offset one another.

The argument for treating the put option payoff as income is based on the idea that the put option effectively converts some of the equity exposure into cash exposure (a put option is equivalent to a short position in the underlying asset and a cash holding). The cash exposure earns interest income which is subject to income tax.

A reinsurance treaty transfers the tax risk from one life office to the reinsurer. Although the proceeds from a reinsurance agreement are not taxed, the reinsurer will charge the insured life office a fee for the tax risk accepted.

Static hedging thus benefits from a tax loophole, namely, that the investment bank can offset the capital payment made to the life office against all the taxable income and gains, while the life office does not pay tax on the capital receipt. Where a life office recreates this payment dynamically there is no offset allowed against the taxable income in the policyholder fund.

#### Tax implications for dynamic hedging

Dynamic hedging of a put option involves frequent dealing in forward sale agreements (equivalent to borrowing shares, selling the shares and investing the proceeds in a zero). These forward agreements can either be exchanged-traded (as in the dissertation) or over-the-counter.

Although a life office may trade frequently (e.g. sell shares within five years from date of share purchase) it is in practice deemed to be a long-term investor, and is thus not taxed on capital gains. However, the life office that engages in dynamic hedging faces a greater risk of being classified as a trader (which will imply taxation of capital gains) since the hedging contracts have a shorter time horizon and involve frequent trading.

Taxation of interest income in the hands of the life office makes the futures market more attractive to create a short position. Selling shares short and investing the proceeds in

zero coupon bonds results in a tax liability on the accrued interest. An equivalent position in the futures market (shorting futures contracts) results in a synthetic cash holding, which is not taxed. Although these strategies create equivalent exposures, the synthetic cash is not taxed while the actual cash holding is taxed.

Thus in summary, different investment strategies involve different tax treatments and may face different risks of future tax treatment. The future tax treatment is uncertain and retrospective tax changes are not uncommon. Explicit allowance for the tax risk will involve detailed investigation of the applicable tax legislation, case law and the work of current tax committees. This is beyond the scope of the dissertation.<sup>27</sup>

## 5.5 Confidentiality

To obtain a static hedge price quotation, the life office must supply the investment bank with a complete description of the GEB contract. This information could be leaked to a competitor life office, which could attempt to sabotage the planned GEB launch through the launch of a subsidized GEB.

A further problem with putting a GEB contract out to tender amongst various investment banks is the "winner's curse". The banks that lose the static hedge tender will expect the winner to enter the spot or futures markets to hedge the GEB liability. This information can be used to their advantage and to the winner's disadvantage. Ultimately, all the banks will make less attractive tender offers in recognition of this risk.

The competitive edge gained from the in-house development of an innovative, tax-efficient hedging strategy will also be short-lived if communicated to investment banks. The ideas are copied by competitors, thereby reducing margins and increasing the risk of regulatory scrutiny.

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<sup>27</sup> Hutton (1998) provides a comprehensive discussion of the taxation of derivatives in South Africa.

When guarantees are created internally, either dynamically or statically (e.g. where an internal pool underwrites the guarantee), then the details of the contract and the hedging activity can be kept confidential.

## **5.6 Transaction size**

Investment banks expect a minimum absolute profit per transaction. Rather than specifying a profit margin that changes according to deal size, fixed terms and a minimum transaction size will be specified. This avoids the embarrassment of quoting very poor terms on small deal sizes. In general, the option to be hedged statically should have a notional of at least R30 million.

The assets used in creating a dynamic hedge also impose a lower bound on the option notional. The option notional must be large enough to allow the smallest hedge requirement to be hedged by one futures contract. For example, if the minimum delta to be hedged is 0.01, then the option notional should be equivalent to that of at least 100 futures contracts. A delta of 0.01 would then require one futures contract. In the case of an ALSI40 option the notional should then be at least  $100 \times R10 \times \text{index}$ , since each futures contract represents an exposure of  $R10 \times \text{index}$ . Assuming an index level of 6252, this translates into an option notional of R6,252,000.

The simulation assumes that the option notional is sufficiently large to allow effective dynamic hedging with the chunky exposures associated with futures contracts, but not too large to impact on market prices. This assumption seems reasonable given that investment banks, which apply the same transaction size criteria, are underwriting recently issued GEB tranches in South Africa.

## **5.7 Liquidity risk and market jumps**

Liquidity risk is the risk of increased costs associated with liquidating a position in an illiquid market. A normally liquid market (many price-sensitive buyers and sellers) may

lose its liquidity during a major crisis. All buyers or sellers disappear and the market jumps (discontinuously) to a new equilibrium level.

This so-called "liquidity hole" typically appears when a major fall in price triggers a large number of sell orders with very few buyers. Apart from the widening of bid-offer spreads, the market may not have the administrative capacity to handle the large flow of orders. Sell orders, normally executed in minutes, may take hours and the prices obtained be much lower than sellers expected.

The market model could explicitly include the discontinuous asset price jumps. Merton (1976) developed a jump-diffusion model that assumes Poisson-distributed events (the arrival of important asset information) trigger the price jumps. In addition to the five inputs required by the Black-Scholes pricing formula, the model also requires two further inputs: the average size of the jump in the asset price, and the frequency with which such jumps are likely to occur. Since these inputs cannot be accurately estimated, the jump-diffusion model is unlikely to produce more reliable decisions. For more detail refer to Natenberg (1994: 397).

Dowd (1998: 191) notes that "the only solution to this problem is for institutions to address the liquidity risk issue by making worst case assumptions about close-out costs... (and)... to consider what potential bid/ask spreads could be when markets are distressed...." In other words, stress testing scenarios should be investigated using the simulation model and a judgement made about the appropriate addition to reserves.

## **5.8 Goodwill**

Investment banks market their static hedging services through a continuous supply of information and assistance on product design, pricing, and regulatory issues. This is very valuable to a life office that cannot afford to develop its own in-house research capability.

In addition, an investment bank would agree to unusual structures, provided that there is the promise of sufficient follow-up deals to justify the costs incurred. For example, a bank could agree to structure a pilot GEB tranche, below the usual minimum deal size, provided that there is a commitment to do follow-up tranches with the same bank.

The investment banks expect a steady stream of deals for the information and assistance supplied. The life office may wish to consider the outsourcing of some of its hedging activity to maintain goodwill with the investment banks.

## **5.9 Accounting**

Static hedging involves non-standard derivative-based assets. The market price of the asset is either the price at which the original seller is willing to unwind the contract, or the price at which an offsetting contract can be established with another party. In both cases, it requires the office to approach a counterparty. Unless the life office has the intention to transact, the price offered by the counterparty will merely be indicative and thus not an accurate indicator of value.

Dynamic hedging is based on regularly traded assets that have a readily available market price. Since the price is objective and can be acted on, the life office gets a much more accurate view of asset values. This allows the quotation of realistic disinvestment values to the GEB investor.

## **5.10 New business**

The simulation only considered a single guaranteed equity bond tranche and ignored the risk diversification benefits gained from issuing further tranches on different indices and different guarantee structures.

Apart from the general diversification benefits, derivative-backed products may have directly opposite, and thus offsetting, liability profiles. The written put option

embedded in a guaranteed equity bond could be offset against the bought put option embedded in a high income bond.<sup>28</sup>

The marginal (i.e. additional) portfolio hedging activity required for each additional GEB tranche is thus likely to be less than that for the tranche on its own, reducing transaction and market impact costs and tracking error. The dynamic hedging costs should thus be costed on a marginal cost basis to recognize the costs savings from the liability portfolio diversification.

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<sup>28</sup> A high income bond consists of a zero coupon bond plus a written put option, usually with a strike far out-of-the-money. The option premium received for the written put option is added to the interest income earned on the zero coupon bond.

## **Chapter 6            Costing dynamically hedged guarantees**

The preceding chapters illustrate how in-house dynamic hedging could be evaluated through modeling, simulation and sensitivity testing. This helps to illuminate the financial benefits and associated risks of dynamic hedging, but does not provide a specific conclusion on whether the life office should implement dynamic hedging. This requires a cost comparison between the dynamically hedged guarantee and alternative guarantees such as a static hedge from an investment bank.

Although the scope of the dissertation excludes costing of dynamically hedged guarantees, this chapter will review two important issues to consider in this regard:

- 1.) the risk that the simulation model does not represent a reliable proxy of reality, and
- 2.) the importance of appropriate risk-return preferences to ensure arbitrage-free pricing.

### **6.1 Model risk**

The simulation model is a simplified representation of the life office's actual environment and dynamic hedge operation. Model imperfections, specifically inaccurate volatility forecasts, lead to inaccurate costing of the dynamic hedge. The dynamic option cost may be underestimated, the estimated risk exposures may be greater than anticipated, and hedging strategies may be less effective than assumed. Green and Figlewski (1999: 1465) performed a simulation based on the actual historic distribution of returns and found that "imperfect models and inaccurate volatility forecasts create sizeable risk exposures for option writers." Maitland (1996) provides a statistical and economic review of the Thomson model that was used in this study.

Street (1996: 7) confirms that in practice a higher volatility assumption is used to correct for all sources of model risk: "If the market is illiquid and the parameter has to be forecast, any derivative seller is going to want to build in a generous margin of error into his volatility parameter estimation as insurance against getting it wrong and having to

hedge the position over a long time horizon." Note that this addition to the assumed volatility is only used to increase the option price charged to the buyer. Dynamic hedging would still be performed on the best estimates of volatility.

Adjusting the volatility parameter is an indirect and potentially confusing approach to compensate for the model risk. A direct and more transparent way of increasing the option cost is through the required return (cost of capital). In essence, it could be argued that the model risk carried by the life office justifies a higher required return. This leads naturally into the following section, which addresses the issue of the appropriate required return or risk-return preference to be applied in costing the dynamically hedged option.

## **6.2 Risk-return preferences**

Each investment opportunity represents a risk-return tradeoff, which is set by the demand and supply of all potential investors collectively. A specific individual investor's preferences do not impact this risk-return tradeoff and thus should not affect the valuation of the investment opportunity. However, faced with a range of market-priced investment opportunities, the individual investor would choose among these according to his or her own risk-return preference in an effort to maximise personal satisfaction or utility.

Life office shareholders expect the life office management team to produce a return exceeding the market-determined required return for the associated level of risk. This goal is achieved by evaluating all investment opportunities against the market-determined risk-return tradeoff. The management team's personal risk-return preference or utility maximisation goals should not enter the equation. However, in practice, the risk-return preference applied by management is seldom clearly defined and, when defined, is commonly expressed as a hurdle rate based on a subjective assessment of the potential risk.

Other life office stakeholders (e.g. clients, government, and staff) do not affect the life office's risk-return preference directly. Rather, their influence is felt in the risk-return characteristics of each investment alternative. For example, a very risky investment alternative may conflict with a very conservative brand image, resulting in loss of goodwill among clients and staff. Similarly, an investment that involves the exploitation of a tax loophole may tarnish the relationship between the life office and the government, adding to the cost of future tax and regulatory negotiations.

Given an appropriate risk-return preference it seems possible to calculate the net present value (NPV) of the cash flows of a dynamically hedged written put option. A negative NPV would then represent the amount that should be charged for the provision of the put option.

A fundamental obstacle to applying this valuation approach is that the simulation involves the netting of cash flows with different levels of associated risk. Ignorance of the varying levels of risk associated with each of the component cash flows may lead to mispricing, thereby creating arbitrage opportunities for other investors. For example, an equity investment financed by risk-free borrowing should have an NPV of zero (expected net payoff from equity compensates for the equity risk taken). However, at any discount rate, the NPV of the projected cash flows will be positive.

There are two possible approaches to modifying the simulation to avoid this mispricing. One approach involves separating the component cash flows and applying an appropriate risk discount rate to each individual cash flow. Alternatively, the risky cash flows could each be reduced (in absolute terms) according to their riskiness, and these risk-adjusted cash flows are then discounted at the risk-free rate. This is referred to as the risk-neutral valuation approach and is discussed in detail by Hull (1997:194).

## Chapter 7      Conclusions

South African life offices are issuing a steadily increasing amount of guaranteed equity bonds and other equity guarantees, but typically ignore dynamic hedging in favour of static hedging. There are various motivations for considering the benefits, risks and costs of in-house dynamic hedging.

Reluctance to investigate the underlying cost of the static hedges (in other words, the dynamic hedge cost) causes the life office to enter negotiations on static hedges in ignorance. Apart from the risk of paying inflated prices for static hedges, life offices effectively let the investment banks drive the innovation in their product design. If the investment bank is unwilling to create a static hedge for a proposed product, the product is unlikely to be considered any further by the life office. As competition between life offices and other suppliers of investment products increases, the need for a low cost base, efficient capital management and innovative product design will become increasingly important.

The dissertation presented the theory that underpins the dynamic hedging process and incorporated this into a simulation. Various assumptions were made about the market environment, the hedging strategy and reserve requirements. This serves merely as a proposed model of the life office's financial environment, and it would be appropriate for each life office to consider its own circumstances and adapt the model accordingly.

The simulation model was used to investigate various aspects of the dynamic hedging strategy with a number of practical issues either ignored or simplified for the purposes of the simulation. These issues need further investigation, but do not appear to prevent the successful implementation of dynamic hedging.

There appear to be good reasons for life offices to give greater consideration to dynamic hedging. It is based on sound financial theory and does not require anything beyond

suitably qualified staff and readily available computing and information resources. This dissertation could serve as a guide to the first steps in such an investigation.

In interpreting the results of the simulation one should bear in mind Pemberton's words (1998: 153): "A recognition of the limited nature of the regularities available at each point in time, the limited nature of the available data in the light of a complex reality, and the recognition of the change over time of the reality being modelled, all contribute to the actuary's cautious attitude to the accuracy of the model results. It is in the cautious form of the statement of conclusions, in the recognition of ranges of possible outcomes, and in the recognition of the need for on-going empirical checking of the model, that we witness the actuary's strong embrace of approximation."

Each hedging strategy offers a trade-off between transaction cost savings and tracking error losses. A higher delta mismatch requires less frequent adjustment to the asset delta and thus creates transaction cost savings. The delta mismatch increases the exposure to a tracking error loss, but introduces the potential for a tracking error gain. Overall, the results appear to show that **as the allowable delta mismatch is increased linearly, the required reserve increases exponentially.**

The sensitivity of the required reserve to small changes in market volatility and transaction costs appears to be unchanged at different levels of allowable delta mismatch. **Although the reserve requirement increases exponentially with a linear change in the allowable delta mismatch, the reserve requirement seems to be fairly stable for a given delta mismatch strategy. A delta mismatch strategy can also not be assumed to reduce the sensitivity of results to changes in volatility and transaction costs.**

Apart from the greater capital requirement associated with a delta mismatch strategy, the strategy involves exposure to a higher potential tracking error loss in return for a potential tracking error gain on an index reversal. The decision to accept this trade-off depends on the likelihood of a reversal and the size of the potential tracking error gain relative to the potential tracking error loss.

Based on the mean accumulated tracking error as an indicator of profitability, **the zero mismatch strategy appeared to simultaneously be the most profitable and to have the lowest capital requirement.** The transaction cost savings and tracking error gains incorporated in the model were insufficient to offset the amplified tracking error losses introduced by the larger mismatch.

**Finally, since the results show very little sensitivity to small changes in model parameters, differences between the assumed parameter values and their real world values will have little impact on these conclusions.** The simulated hedging results can thus be assumed to be approximately valid in practice. Even if the results from the simulation are not believed to be a true reflection of the real world, the relative results (i.e. the differences in results from different assumptions) obtained from following different hedging strategies are useful in choosing the correct strategy for given market characteristics.

## **Future work in the field**

The concept of dynamic hedging has been extensively investigated since the path-breaking paper by Black and Scholes. However, very little has been written on the issues facing a South African life office in the dynamic hedging of its guaranteed equity liabilities. Within this narrower area of investigation there is scope for much further work. Three major thrusts are:

### **1. Models for the market environment**

There is scope for investigation into reliable models for the market environment, both in the probabilistic structure of historical economic data and in financial economic theory. However, Huber (1999: 393) warns that "difficulties associated with testing models suggest that it is likely to be difficult to demonstrate that one economic model is clearly superior to another. This implies that a model's pragmatic qualities will be more important than they would otherwise have been, and that more than one model may be appropriate, depending on the particular application under consideration."

### **2. Investment decision rules**

Possible improvements in the decision rules used to differentiate alternative uses of capital is another area of investigation that is likely to come strongly to the fore as greater emphasis is placed on the efficient use of life office capital.

### **3. Guaranteed equity product innovations**

Finally, on the product development front there are vast opportunities yet to be exploited. Specifically, the dynamic hedging of guarantees on internally managed unit-linked funds and the dynamic hedging of annuity rate guarantees could be investigated.

## Appendices

### Appendix A: Alternative markets for dynamic hedging assets

The delta of the ALSI40 put option liability can be replicated via the spot, futures or OTC forward markets. This appendix provides a comparison of these alternative markets for dynamically hedging the liability.

#### Spot Market

In the spot market the hedge would involve borrowing a portfolio of shares (representing the composition of the ALSI40 index), selling the shares and buying a zero coupon bond. The problems with this strategy are:

- (i) The zero coupon bond will attract tax on its interest accrual.
- (ii) It may be difficult to find a scrip lender that can provide the required portfolio of shares. An internally managed ALSI40 index fund could supply such a portfolio of shares, or an investor in the index fund could lend its index fund units.
- (iii) The purchase and sale of shares involve trading costs, taxes and possibly market impact.
- (iv) If units in an internally managed ALSI40 index fund are borrowed, then there is the risk that the ALSI40 index fund could deviate from the ALSI40 index.<sup>29</sup> This is especially likely in the case of a small fund that has inadequate funds to purchase the necessary exposure in each of the ALSI40 constituents.
- (v) The share portfolio must be adjusted when the ALSI40 index constituents change or when their weightings change (e.g. due to additional share issues).

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<sup>29</sup> Where the ALSI40 put option liability guarantees a price for the index fund units, rather than an ALSI40 index level, then tracking error within the ALSI40 index fund has no impact on the dynamic hedge.

Provided stock borrowing is done within the life office and not from an outside party this strategy has an important cost advantage. When an investment bank wishes to hedge an OTC put option via the spot market, it would have to borrow shares in the market. The scrip borrowing cost is typically about 150 basis points per annum, and is incorporated in the price of the put option charged to the life office. The internal borrowing of scrip thus avoids this additional expense in replicating the put option.

### **Futures Market**

In the futures market the hedge would involve the sale of exchange-traded futures contracts on the ALSI40 index. The problems with this strategy are:

(i) The short term of futures contracts (usually terms are only up to nine months) requires the regular replacement of expiring future positions. When a short futures position reaches expiry it must be replaced with a new futures position (through the sale of futures), at a price that cannot be fixed in advance. There is the risk that the futures selling price (and therefore the cost-of-carry implied by the price) could be lower than assumed in the costing of the dynamic hedge, causing a loss to the hedger. This risk is referred to as roll-over risk.

(ii) Exchange-traded futures have frequently traded at a discount to fair value (the price at which a futures position can be replicated either through borrowing money and buying the underlying asset, or short-selling the underlying asset and investing in a zero coupon bond). This is due to the difficulty of short-selling the underlying in the spot market (it is particularly difficult for individual investors to borrow ALSI40 shares or units of an index fund). The discount to fair value makes the hedge more expensive to the life office.

### **OTC Forward Market**

In the OTC forward market the hedge would involve the sale of OTC forward contracts on the ALSI40 index.

A five-year OTC forward sale effectively locks in the prices of the equivalent series of short-term futures contracts. There is thus no "roll-over" risk. However, the OTC market is illiquid, resulting in the risk of wide bid-offer spreads when rebalancing purchases or when sales transactions are required. In addition, each rebalancing transaction would require negotiations with the OTC counterparty, which could lead to delays in rebalancing and aggravate the tracking error.

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## Appendix B: Specification of SAFEX and assumed ALSI40 futures contracts

### Details of an actual SAFEX ALSI40 futures contract:

Contract size:	R10 × index level
Expiry dates:	16h00 on 3rd Thursday of March, June, September, December (or previous business day)
Min price movement:	one index point
Expiry valuation:	arithmetic average of index every 2 minutes over final 2 hours of trading on expiry date
Settlement method:	cash settled
Brokerage:	approximately R10 per contract (1 point)
Bid-Offer spread:	approximately 2-20 points for near-dated contract

### Details of the ALSI40 futures contract assumed in the simulation:

Contract size:	R10 × index level
Expiry dates:	every 13 weeks (each year is assumed to have 52 weeks of 5 trading days)
Min price movement:	one index point
Expiry valuation:	ALSI40 index value at close of expiry date
Settlement method:	cash settled
Brokerage:	included in the bid-offer spread
Bid-Offer spread:	bid spread of 0.2% and offer spread of 0.2% <sup>30</sup>

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<sup>30</sup> The bid-offer spread, expressed in terms of index points, is expected to grow as the index grows. The bid-offer spread is thus translated into a percentage for the purposes of the simulation.

## Appendix C: The Thomson stochastic investment model

Thomson (1996) modelled the following variables:

- $INFL_t$  the average force of inflation during year  $t$ ;  
 $EQDG_t$  the average force of dividend growth during year  $t$ ;  
 $EQDY_t$  the logarithm of historic dividend yield(%) at time  $t$ ;  
 $LINT_t$  the force of interest corresponding to the yield to redemption on long-term interest-bearing securities at time  $t$ ;  
 $MINT_t$  the average force of interest on money-market instruments during year  $t$ ;  
 $z_{X,t}$  the carried-forward effect of inflation at time  $t$  in respect of variable  $X$ ;  
 $XZ_t$  the "real" value of variable  $X$  relative to the carried-forward effect of inflation.

Thomson also modelled variables for returns on property and property unit trusts. These variables have no relevance to our investigation and will be ignored.

Thomson defines the variables in terms of autoregressive integrated moving average (ARIMA) processes with transfer functions. The starting conditions for these processes were defined to be the unconditional means.

$$EQDG_t = 0.093 + 0.116\eta_{EQDG,t} + 0.076\eta_{EQDG,t-1}$$

$$EQDY_t = 0.310 + 0.810EQDY_{t-1} + 0.198\eta_{EQDY,t}$$

$$EQDY_0 = 1.63158$$

$$INFL_t = 0.008 + 0.899INFL_{t-1} + 0.088EQDG_t - 0.079EQDG_{t-1} + 0.077EQDG_{t-2} \\ - 0.069EQDG_{t-3} + 0.020\eta_{INFL,t}$$

$$INFL_0 = 0.09486$$

$$LINT_t = z_{LINT,t} + LINTZ_t = z_{LINT,t} + 0.010\eta_{LINT,t} + 0.006\eta_{LINT,t-1}$$

$$z_{LINT,t} = 0.006 + 0.126INFL_t + 0.85z_{LINT,t-1}$$

$$z_{LINT,0} = 0.11968$$

$$MINT_t = z_{MINT,t} + MINTZ_t = z_{MINT,t} + 0.008 - 0.091EQDG_t + 0.885LINTZ_t \\ + 0.019\eta_{MINT,t} + 0.010\eta_{MINT,t-1}$$

$$z_{MINT,t} = 0.004 + 0.141INFL_t + 0.85z_{MINT,t-1}$$

$$z_{MINT,0} = 0.11584$$

$\eta_{X,t}$  denotes a random draw from the standard normal distribution (zero mean and unit variance), where  $\eta_{X,t}$  is independent of  $\eta_{Y,s}$  for  $s \neq t$  and  $Y \neq X$

(Appendix D describes the approach used to generate this random variable.)

$X$  denotes any one of the variables modelled

The dissertation does not attempt to identify the optimal model for the market, and will therefore not attempt to improve on the parameter values estimated by Thomson.

## Appendix D: Generation of random draws from the standard normal distribution

The simulation model requires multiple series of independent random draws from the standard normal distribution. These series are used in the Thomson stochastic investment model (detailed in Appendix C) and in the daily volatility component of the spot ALSI40 index level (detailed in section 3.3).

Each random draw is generated by first calculating a random draw from the uniform distribution over the interval  $[10^{-15}, 1-10^{-15})$  and then inputting this in the inverse cumulative standard normal distribution function.<sup>31</sup> Both zero and 1 must be excluded as inputs since the inverse is undefined at these points.

The uniform random variable was generated using the random number generator function RAND() included in the Microsoft Excel® spreadsheet program. RAND() generates a random draw from the uniform distribution over the interval  $[0,1)$ .<sup>32</sup> The formula  $(1-2 \cdot 10^{-15}) \times \text{RAND}() + 10^{-15}$  thus generates a random draw from the uniform distribution over the interval  $[10^{-15}, 1-10^{-15})$ .

The inverse cumulative standard normal distribution function does not exist in closed form and must either be approximated through a numerical method or a polynomial function. Microsoft Excel® calculates the inverse through repeated numerical integration of the cumulative normal distribution function. This iterative process was found to be very inaccurate at both extremes of the input range.

A polynomial approximation to the inverse cumulative distribution function, suggested by Hastings (1955), was used in the simulation. This function,  $F^{-1}(prob)$ , where *prob* is the cumulative probability, is defined on the following page.

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<sup>31</sup> Since the Pentium computer and Microsoft Excel® software used only recorded 15 significant digits, the largest number smaller than 1 that could be used was 0.999,999,999,999,999 ( $=1 - 10^{-15}$ ).

<sup>32</sup> The interval is not truly open due to finite accuracy.

if  $prob > 0.5$  then

$$F^{-1}(prob) = \frac{-2.515517 + X - 0.802853X - 0.010328X^2}{1 + 1.432788X + 0.189269X^2 + 0.001308X^3}$$

$$\text{where } X = \sqrt{-2 \ln(1 - prob)}$$

otherwise

$$F^{-1}(prob) = \left[ \frac{-2.515517 + X - 0.802853X - 0.010328X^2}{1 + 1.432788X + 0.189269X^2 + 0.001308X^3} \right]$$

$$\text{where } X = \sqrt{-2 \ln(prob)}$$

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## Appendix E: Detailed simulation results

### Scenario 1(a)

Delta mismatch allowed = 0

Bid / offer spreads = 0.2%

Actual market volatility = 20% p.a.

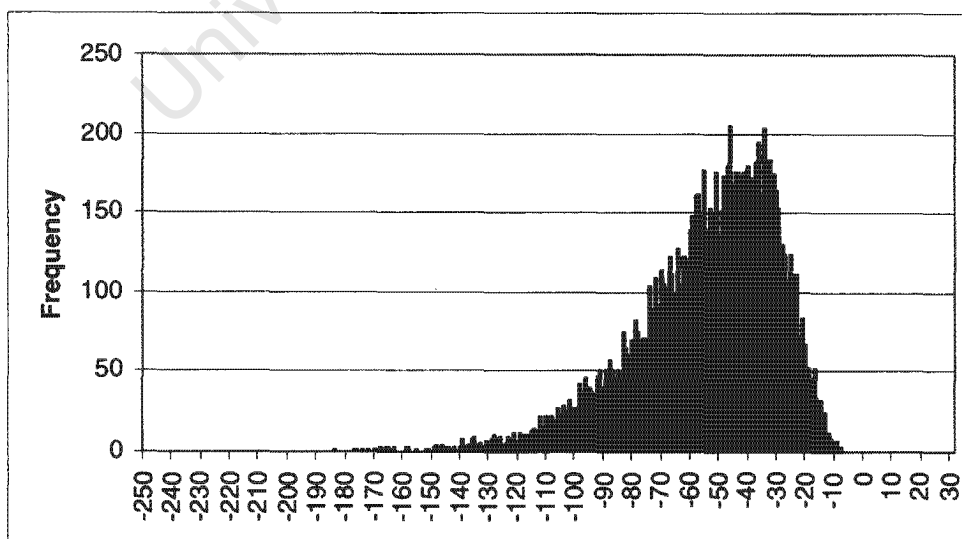
	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve (as % of option notional)</b>	25.1	25.3	25.7

### Distribution of accumulated tracking error (as % of required reserve)

#### a) Descriptive statistics

Mean	(54.95)	(57.32)	(56.78)
Median	(50.67)	(53.20)	(52.49)
Standard Deviation	24.88	25.34	25.18
Kurtosis	1.23	0.79	0.98
Skewness	(0.98)	(0.87)	(0.91)
Range	175.41	191.61	197.78
Minimum	(184.02)	(197.82)	(199.60)
Maximum	(8.60)	(6.22)	(1.82)
99th percentile	(10.31)	(10.17)	(10.29)
1st percentile	(167.01)	(161.29)	(166.28)

#### b) Histogram (benchmark assumptions)



**Scenario 1(b)**  
Delta mismatch allowed = 0  
Bid / offer spreads = 0.2%  
Actual market volatility = 10% p.a.

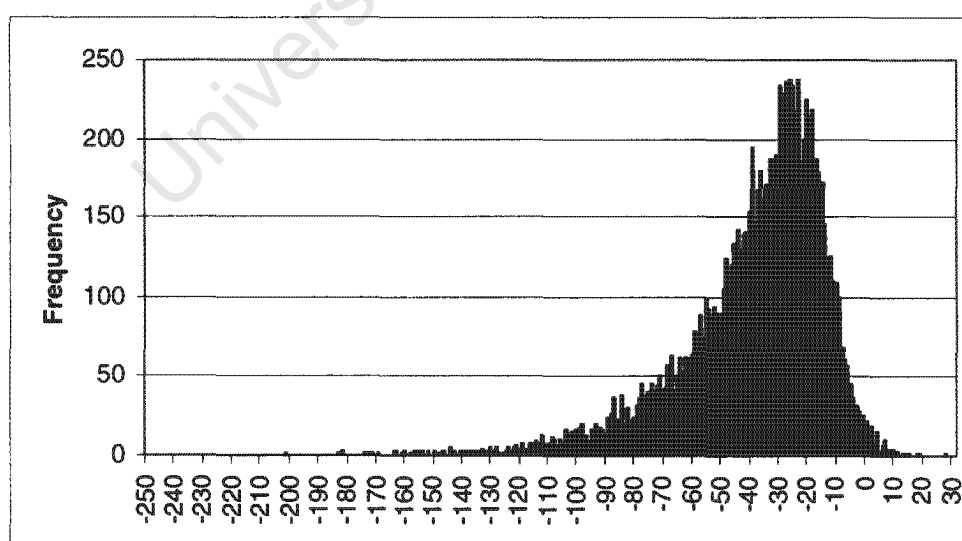
	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve (as % of option notional)</b>	23.0	23.4	24.1

**Distribution of accumulated tracking error (as % of required reserve)**

**a) Descriptive statistics**

Mean	(39.04)	(39.48)	(38.93)
Median	(33.24)	(34.01)	(33.59)
Standard Deviation	25.23	24.44	24.25
Kurtosis	2.86	3.12	2.76
Skewness	(1.38)	(1.40)	(1.35)
Range	228.68	247.96	227.33
Minimum	(201.40)	(231.25)	(201.34)
Maximum	27.28	16.71	25.99
99th percentile	9.73	7.47	7.78
1st percentile	(164.14)	(161.02)	(158.38)

**b) Histogram (benchmark assumptions)**



**Scenario 1(c)**

Delta mismatch allowed = 0

Bid / offer spreads = 0.2%

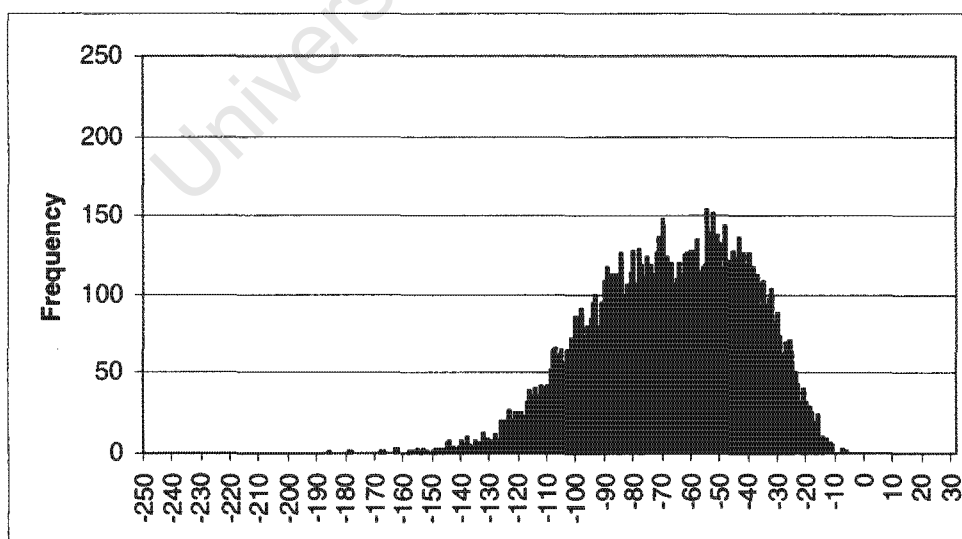
Actual market volatility = 30% p.a.

**Required reserve**  
(as % of option notional)

Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
31.8	34.0	33.2

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(67.91)	(66.13)	(68.29)
Median	(66.21)	(64.06)	(66.45)
Standard Deviation	27.07	26.61	26.99
Kurtosis	(0.37)	(0.34)	(0.41)
Skewness	(0.34)	(0.34)	(0.31)
Range	179.37	177.07	178.04
Minimum	(186.00)	(184.02)	(185.95)
Maximum	(6.63)	(6.94)	(7.91)
99th percentile	(12.14)	(10.84)	(11.40)
1st percentile	(158.94)	(157.57)	(155.21)

**b) Histogram (benchmark assumptions)**

**Scenario 1(d)**

Delta mismatch allowed = 0

Bid / offer spreads = 0.1%

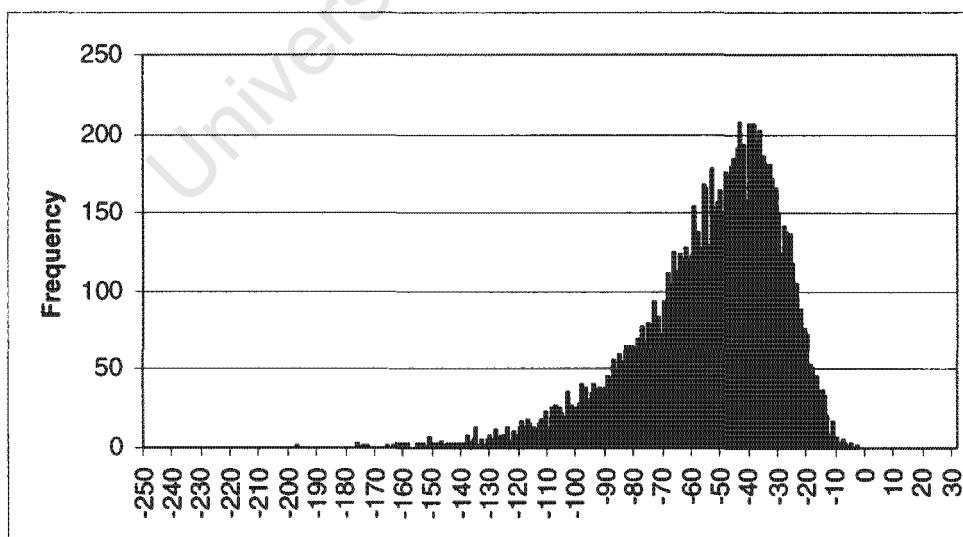
Actual market volatility = 20% p.a.

**Required reserve  
(as % of option notional)**

Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
23.7	25.1	24.8

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(54.50)	(53.62)	(54.46)
Median	(49.74)	(49.08)	(50.15)
Standard Deviation	25.07	24.41	24.46
Kurtosis	1.27	1.65	1.20
Skewness	(1.03)	(1.06)	(0.98)
Range	193.65	208.95	169.99
Minimum	(197.09)	(213.81)	(176.27)
Maximum	(3.44)	(4.86)	(6.29)
99th percentile	(8.74)	(9.36)	(11.02)
1st percentile	(161.78)	(167.61)	(159.96)

**b) Histogram (benchmark assumptions)**

**Scenario 1(e)**

Delta mismatch allowed = 0

Bid / offer spreads = 0.3%

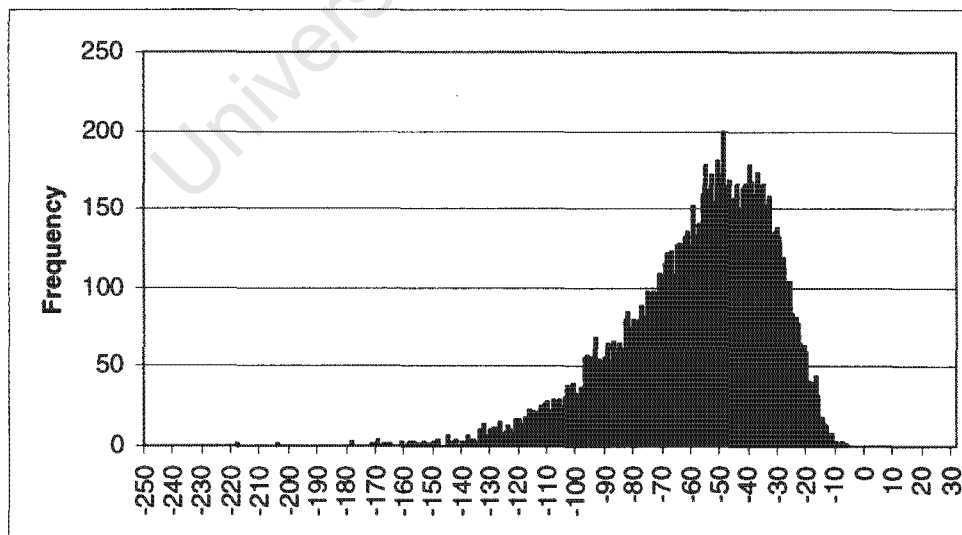
Actual market volatility = 20% p.a.

**Required reserve  
(as % of option notional)**

Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
25.5	26.1	27.0

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(58.49)	(59.46)	(58.35)
Median	(54.09)	(55.51)	(54.28)
Standard Deviation	25.84	25.94	25.55
Kurtosis	0.83	0.48	0.65
Skewness	(0.86)	(0.76)	(0.78)
Range	212.09	177.26	183.95
Minimum	(218.90)	(181.48)	(191.53)
Maximum	(6.81)	(4.22)	(7.57)
99th percentile	(11.20)	(10.44)	(10.22)
1st percentile	(169.34)	(167.40)	(162.26)

**b) Histogram (benchmark assumptions)**

**Scenario 2(a)**  
Delta mismatch allowed = 0.05  
Bid / offer spreads = 0.2%  
Actual market volatility = 20% p.a.

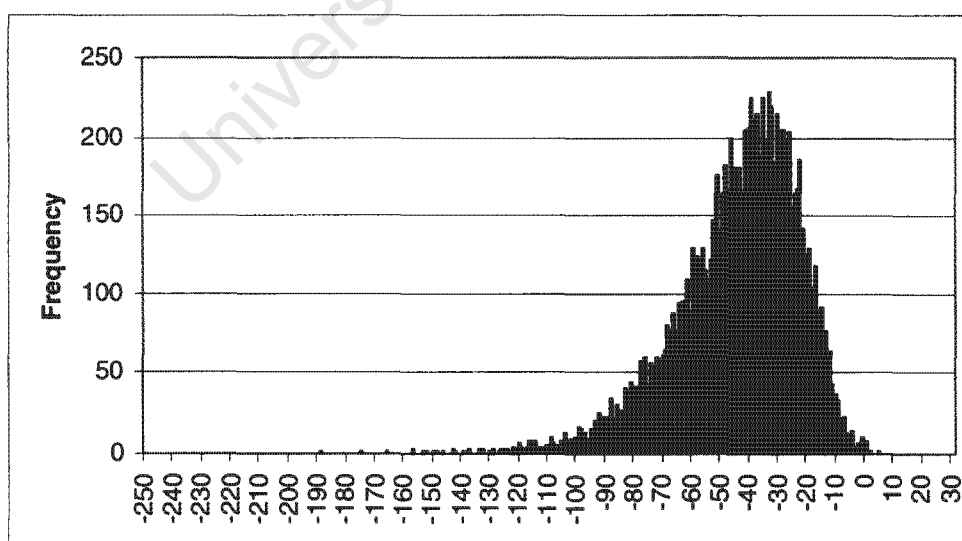
	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve (as % of option notional)</b>	31.3	28.3	28.6

**Distribution of accumulated tracking error (as % of required reserve)**

**a) Descriptive statistics**

Mean	(44.37)	(50.31)	(50.33)
Median	(40.91)	(46.39)	(46.84)
Standard Deviation	21.54	24.05	23.73
Kurtosis	1.70	1.34	1.27
Skewness	(0.99)	(0.96)	(0.90)
Range	193.70	205.19	202.09
Minimum	(189.17)	(203.13)	(197.19)
Maximum	4.53	2.05	4.90
99th percentile	(0.59)	(1.94)	(0.52)
1st percentile	(148.63)	(158.77)	(150.78)

**b) Histogram (benchmark assumptions)**



**Scenario 2(b)**

Delta mismatch allowed = 0.05

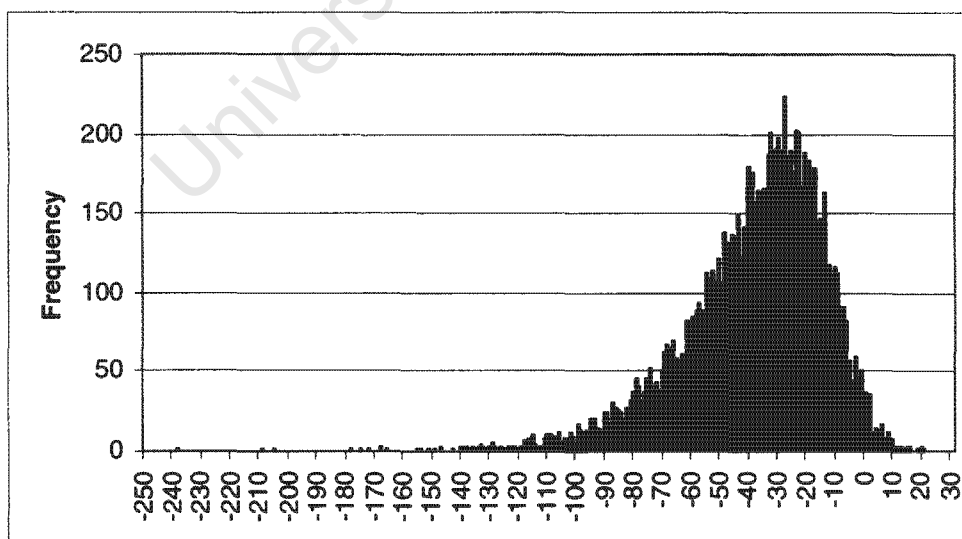
Bid / offer spreads = 0.2%

Actual market volatility = 10% p.a.

	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve (as % of option notional)</b>	27.8	26.8	26.0

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(38.35)	(40.76)	(42.01)
Median	(34.08)	(36.44)	(37.61)
Standard Deviation	24.42	25.49	25.83
Kurtosis	2.39	1.59	1.57
Skewness	(1.10)	(1.04)	(1.01)
Range	259.45	209.08	223.60
Minimum	(238.46)	(191.26)	(202.85)
Maximum	20.98	17.82	20.75
99th percentile	12.31	11.55	9.79
1st percentile	(155.94)	(154.85)	(161.56)

**b) Histogram (benchmark assumptions)**

**Scenario 2(c)**  
Delta mismatch allowed = 0.05  
Bid / offer spreads = 0.2%  
Actual market volatility = 30% p.a.

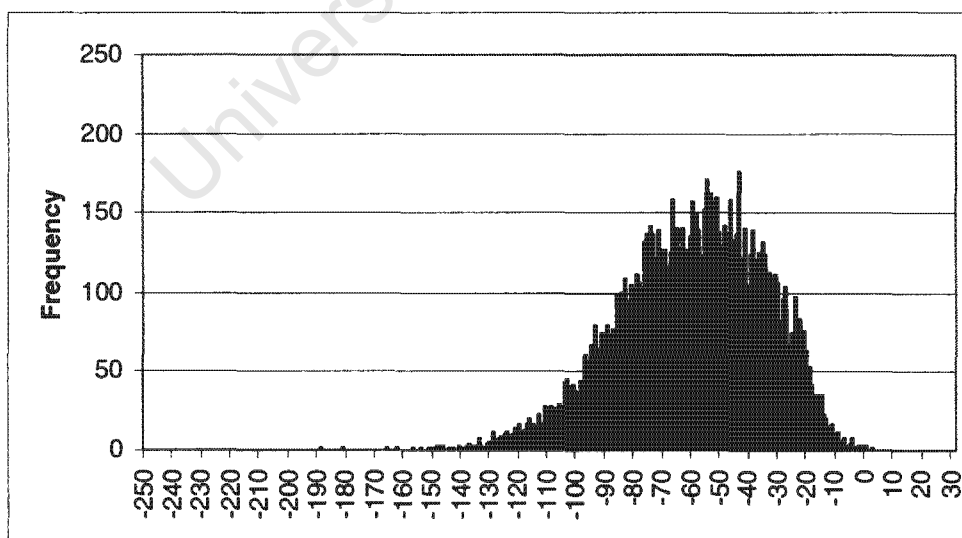
	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve (as % of option notional)</b>	33.6	32.0	33.3

**Distribution of accumulated tracking error (as % of required reserve)**

**a) Descriptive statistics**

Mean	(60.48)	(64.43)	(63.18)
Median	(58.60)	(62.47)	(61.52)
Standard Deviation	25.42	26.89	26.55
Kurtosis	(0.09)	(0.21)	(0.19)
Skewness	(0.40)	(0.37)	(0.36)
Range	191.99	171.46	188.64
Minimum	(189.98)	(173.13)	(187.88)
Maximum	2.01	(1.67)	0.76
99th percentile	(4.45)	(6.15)	(3.78)
1st percentile	(148.78)	(156.78)	(157.61)

**b) Histogram (benchmark assumptions)**



**Scenario 2(d)**

Delta mismatch allowed = 0.05

Bid / offer spreads = 0.1%

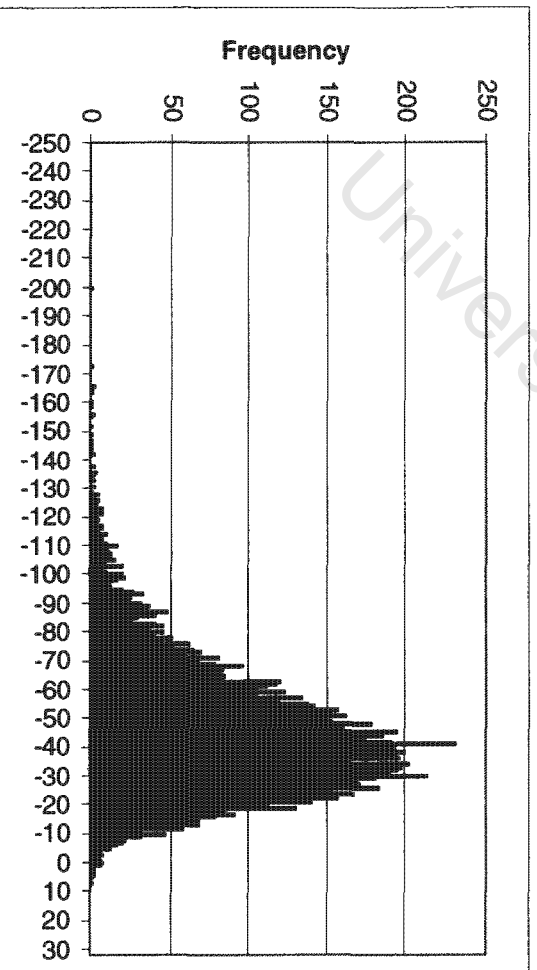
Actual market volatility = 20% p.a.

**Required reserve  
(as % of option notional)**

Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
28.7	27.5	27.9

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(46.82)	(49.22)	(49.95)
Median	(43.03)	(45.47)	(46.28)
Standard Deviation	23.09	23.95	23.87
Kurtosis	1.54	1.22	1.24
Skewness	(0.98)	(0.93)	(0.89)
Range	206.83	186.13	211.73
Minimum	(200.22)	(181.08)	(205.52)
Maximum	6.61	5.06	6.21
99th percentile	0.65	(1.91)	(0.29)
1st percentile	(156.59)	(154.20)	(157.41)

**b) Histogram (benchmark assumptions)**

**Scenario 2(e)**

Delta mismatch allowed = 0.05

Bid / offer spreads = 0.3%

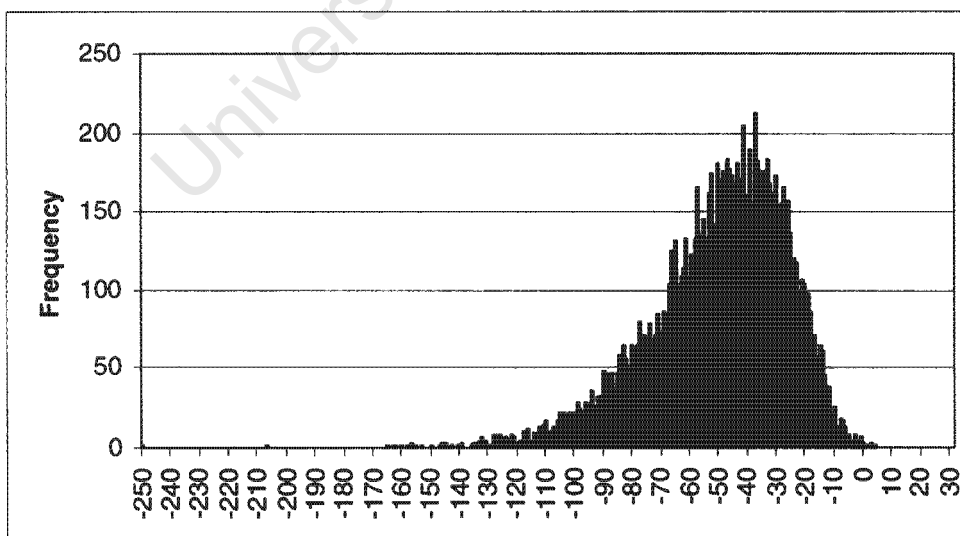
Actual market volatility = 20% p.a.

**Required reserve  
(as % of option notional)**

Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
28.8	27.5	30.2

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(50.55)	(53.73)	(49.68)
Median	(47.12)	(50.12)	(46.34)
Standard Deviation	24.00	25.04	23.35
Kurtosis	2.63	0.90	1.81
Skewness	(0.97)	(0.83)	(0.92)
Range	331.60	218.72	271.75
Minimum	(328.37)	(212.39)	(269.93)
Maximum	3.23	6.33	1.82
99th percentile	(1.45)	(1.11)	(0.82)
1st percentile	(155.24)	(159.95)	(147.98)

**b) Histogram (benchmark assumptions)**

**Scenario 3(a)**

Delta mismatch allowed = 0.1

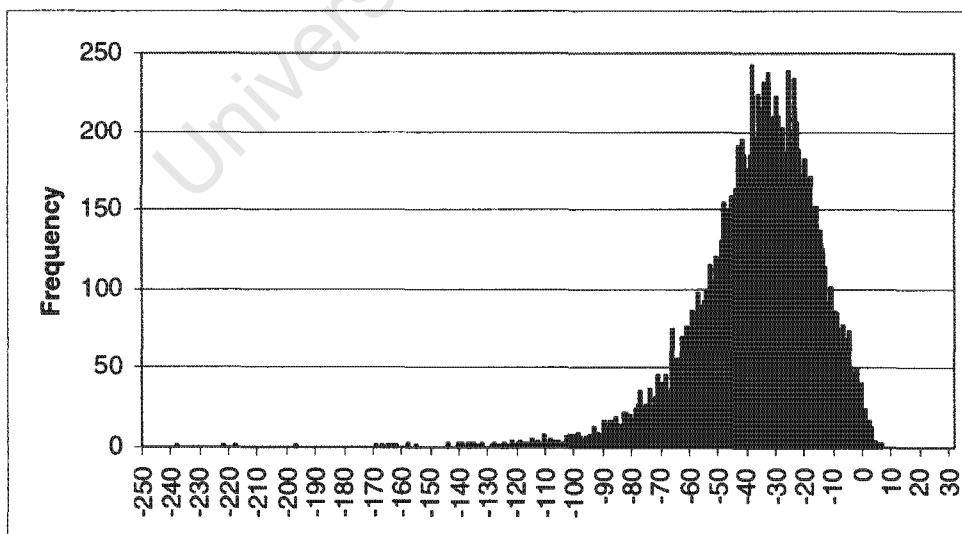
Bid / offer spreads = 0.2%

Actual market volatility = 20% p.a.

	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve</b> (as % of option notional)	40.8	39.3	41.6

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(37.49)	(39.22)	(37.43)
Median	(34.81)	(36.26)	(34.66)
Standard Deviation	21.30	21.79	20.57
Kurtosis	4.41	3.02	3.36
Skewness	(1.29)	(1.11)	(1.16)
Range	244.49	227.60	212.43
Minimum	(238.97)	(220.25)	(205.19)
Maximum	5.52	7.35	7.24
99th percentile	2.79	3.16	3.71
1st percentile	(158.90)	(155.80)	(148.53)

**b) Histogram (benchmark assumptions)**

**Scenario 3(b)**

Delta mismatch allowed = 0.1

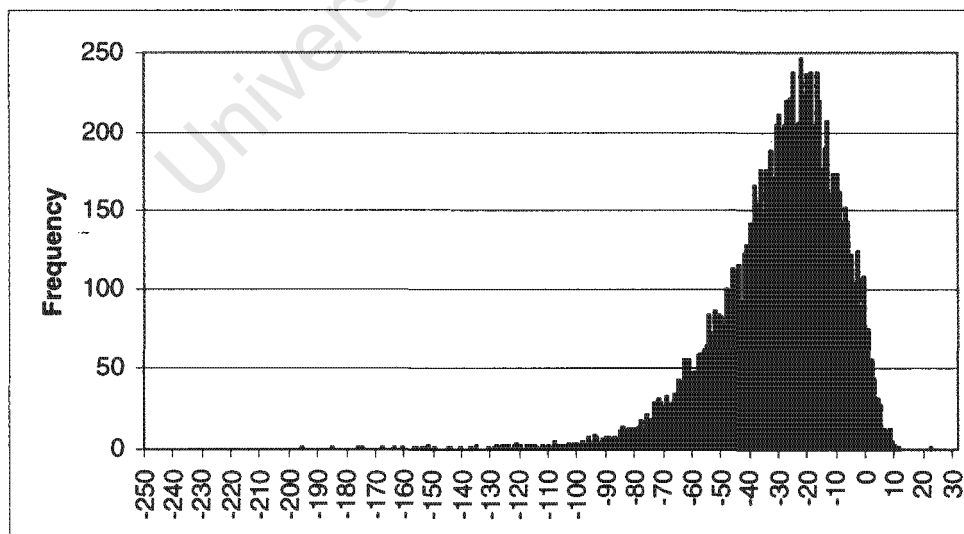
Bid / offer spreads = 0.2%

Actual market volatility = 10% p.a.

	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve (as % of option notional)</b>	42.2	41.4	37.8

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(30.19)	(31.45)	(33.98)
Median	(26.85)	(28.15)	(30.44)
Standard Deviation	21.11	20.95	23.17
Kurtosis	3.88	2.85	2.59
Skewness	(1.33)	(1.16)	(1.17)
Range	218.27	213.89	220.05
Minimum	(196.86)	(198.91)	(200.10)
Maximum	21.40	14.98	19.95
99th percentile	8.03	8.32	9.37
1st percentile	(154.11)	(143.89)	(151.54)

**b) Histogram (benchmark assumptions)**

**Scenario 3(c)**

Delta mismatch allowed = 0.1

Bid / offer spreads = 0.2%

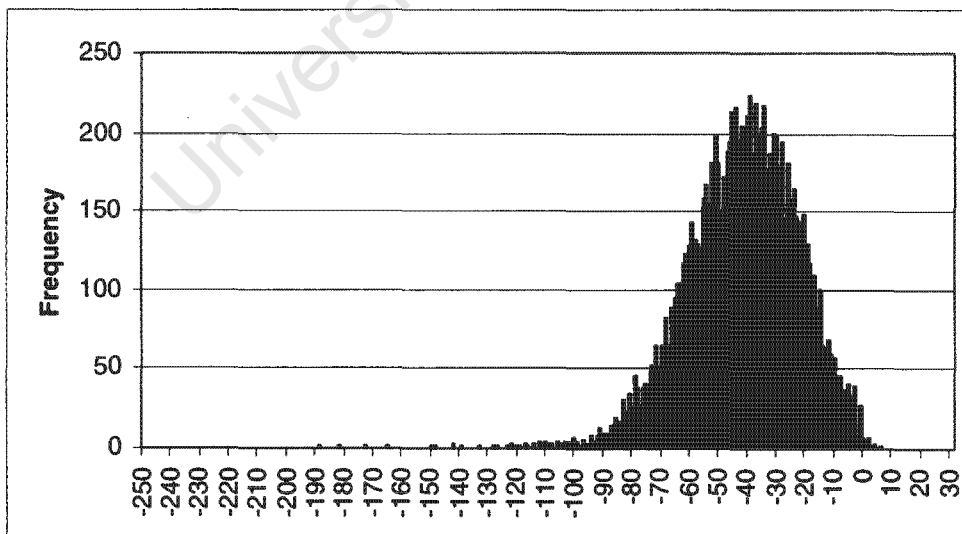
Actual market volatility = 30% p.a.

**Required reserve  
(as % of option notional)**

Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
48.1	46.6	44.1

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(42.29)	(44.34)	(47.19)
Median	(41.15)	(43.03)	(45.82)
Standard Deviation	19.34	20.34	21.75
Kurtosis	1.50	0.76	0.99
Skewness	(0.60)	(0.51)	(0.53)
Range	195.02	173.98	201.45
Minimum	(189.31)	(169.39)	(195.45)
Maximum	5.71	4.59	6.00
99th percentile	1.07	0.62	2.01
1st percentile	(133.41)	(136.41)	(143.22)

**b) Histogram (benchmark assumptions)**

**Scenario 3(d)**

Delta mismatch allowed = 0.1

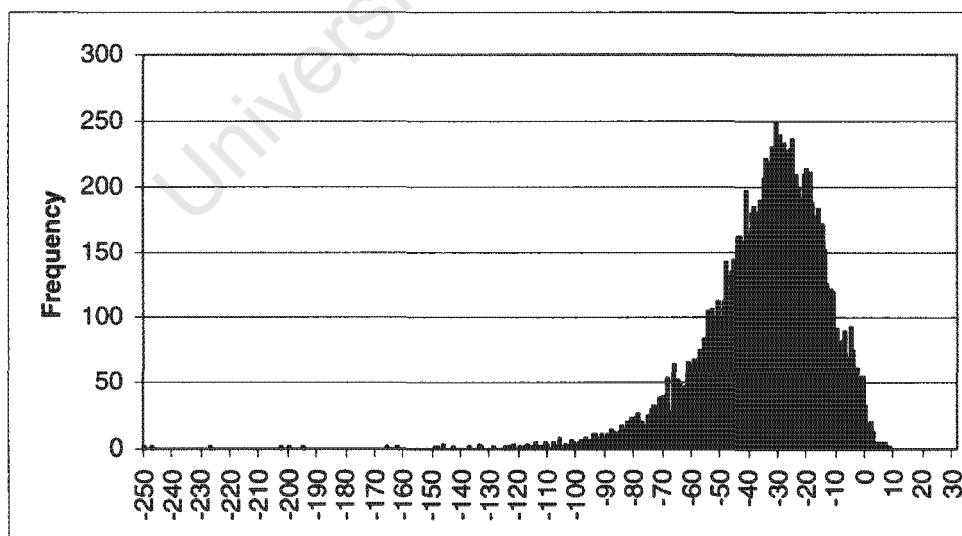
Bid / offer spreads = 0.1%

Actual market volatility = 20% p.a.

	Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
<b>Required reserve (as % of option notional)</b>	41.2	42.4	40.2

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(35.22)	(35.30)	(37.71)
Median	(32.29)	(32.43)	(34.66)
Standard Deviation	20.77	20.25	21.32
Kurtosis	7.93	3.68	3.65
Skewness	(1.52)	(1.21)	(1.21)
Range	324.09	208.70	252.11
Minimum	(316.25)	(200.36)	(243.33)
Maximum	7.84	8.35	8.79
99th percentile	5.04	3.55	4.24
1st percentile	(148.63)	(154.89)	(157.78)

**b) Histogram (benchmark assumptions)**

**Scenario 3(e)**

Delta mismatch allowed = 0.1

Bid / offer spreads = 0.3%

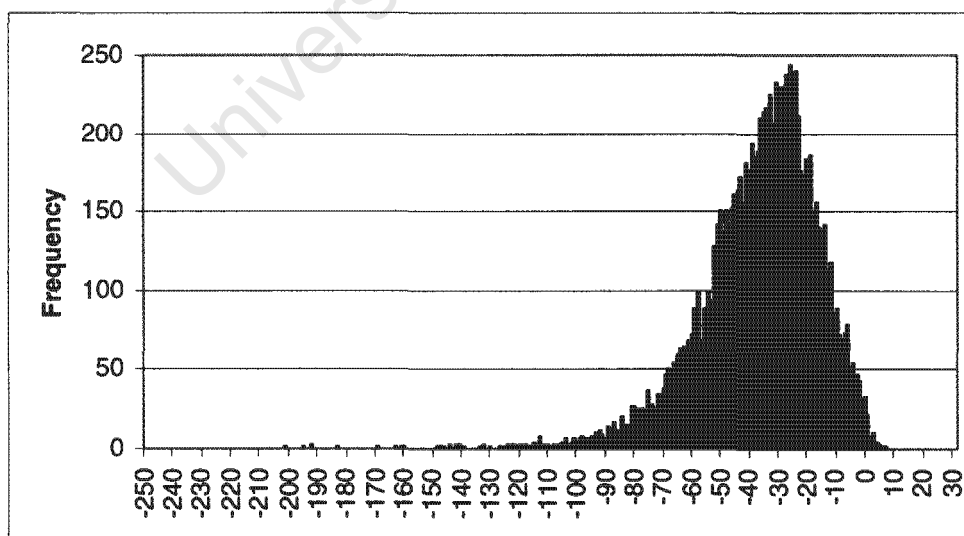
Actual market volatility = 20% p.a.

**Required reserve  
(as % of option notional)**

Benchmark assumptions	0.05% increase in bid and offer spreads	1% increase in market volatility
42.7	41.1	40.8

**Distribution of accumulated tracking error (as % of required reserve)****a) Descriptive statistics**

Mean	(37.10)	(38.83)	(39.17)
Median	(34.15)	(36.13)	(36.69)
Standard Deviation	20.66	21.07	21.08
Kurtosis	3.67	3.09	2.90
Skewness	(1.21)	(1.10)	(1.05)
Range	207.61	227.38	223.30
Minimum	(201.10)	(219.42)	(214.76)
Maximum	6.51	7.96	8.54
99th percentile	2.86	1.86	2.14
1st percentile	(148.44)	(153.06)	(147.76)

**b) Histogram (benchmark assumptions)**

## Appendix F: Potential operational failures and preventative control measures

### Potential sources of operational failure

- The information required for hedging is miscalculated (e.g. the delta of liability). This could be the result of a model error, a programming (software) error, a hardware failure, or a telecommunications failure (breakdown in the live data feed).
- Trading actions do not follow the hedging rule. The trader could attempt to improve on a passively implemented hedging rule. The risks of rogue trading and fraud also exist. The loss of key personnel may also lead to failed implementation of the trading rule if no substitute is available.
- The trade instruction and actual transaction do not correspond. This could be due to any back office failure: execution error, booking error, settlement error, and documentation errors.

### Preventative control measures

Allen (1998, p.6) lists some criteria necessary to control operational risk. Non-compliance with these criteria means a greater chance of adverse operational events:

- qualified personnel with proper contingency/succession planning;
- integrative systems that avoid the manual transfer of data (especially the communication of the liability profile from the product developer to the asset manager)
- market and credit risk functions independently organized from trading functions;
- documented policies and procedures detailing activities, limits, credit controls and reporting requirements;
- internal audits of activities ensuring that the policies, procedures and limits established are being followed;
- an overview of the company's business and operations being undertaken by knowledgeable and involved senior management.

Kemp (1996, Section 11) lists further preventative control procedures.

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