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APPLICATIONS AND EXTENSIONS OF FINANCIAL MODELS TO SMALL MARKETS : THE SOUTH AFRICAN CASE

by
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A thesis prepared under the supervision of
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in fulfilment of the requirements for the degree of
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to my late stepfather **KEN McTAVISH**

CONTENTS

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CHAPTER 1

INTRODUCTION

1.1 HISTORICAL BACKGROUND

Deep in the Middle Ages, in the year 1291, an almost forgotten incident bearing an astonishing modern aspect, was documented. Teodisio d'Ora one of the merchants of Genoa, encouraged by reports filtering through from the mysterious regions of Africa, was responsible for organizing a joint venture by sea round the African Continent. He imagined that a regular sailing route around Southern Africa would have been far more profitable than the overland caravan trail to the East. Backing for the venture, however, required vast sums of money and was therefore obtained from several wealthy merchants. In 1291 the ships sailed through the Straits of Gibralter and headed for the south. Nearly sixty years later in 1350 information was received that they had rounded the Cape and reached the coast of the present day Somalia, where the two ships were ship-wrecked. So ended the very first documented *Joint Stock Company.*

Nearly 300 years were to pass before the Dutch had developed a large shipping trade. These shipping ventures also required joint financing and it did not take long before a keen interest in obtaining shares in these companies was generated. This led to the development of the Amsterdam Stock Exchange which was built in 1613.

Not long after, Dutch ships were venturing to America where they bought an island from the Manhattan Indians and built a wall across its northern boundary. They called the path alongside it, Wall Street. It was here in 1692 after New Amsterdam had been renamed New York that an area under a buttonwood tree became known to the traders as the *Stock Exchange.* In London, share trading was similarly becoming a well established investment avenue at the time.

It was almost 200 years later, in 1886, that two itinerant prospecters stumbled across a rocky outcrop of gold-bearing quartz pebbles on a farm at Langlaagte, (now encorporated in the present-day Johannesburg) in South Africa. The discovery of that small outcrop led to the proclamation of the Witwatersrand Gold Fields in September 1886 and set the stage for the greatest gold rush in history. George Harrison, a handyman prospector, more than anyone else, has been credited with having made the actual discovery of the main reef. He did not realize the full extent of his discovery, and sold his discoverer's claim shortly afterwards for a mere ten pounds. Little did he realize at the time that he was standing on the greatest golden treasure chest ever discovered by man, and that the revenue from this discovery would be the life-blood of South Africa as well as the sub-continent of southern Africa for well over a hundred years. Furthermore gold from this source would also play a prominent role in shaping the world's monetary system.

Thousands of prospectors flocked to this area from various parts of the world, but soon realized that the unique gold deposits were more difficult to extract than the alluvial gold found elsewhere in the world and thus beyond the mining capabilities and resources of small-time prospectors. The problems prospectors faced were associated with the the unique geology of the Witwatersrand system. This system was formed some 2600 million years ago when inlets into a shallow inland sea (approximately 500 km at its widest point) carried gold from surrounding mountains and deposited it in large fan-shaped deltas that was to become the edge of the Witwatersrand basin. The resulting conglomerates formed from these deposits required crushing and sophisticated metallurgical processing to extract the gold. Furthermore the reefs plunged underground at steep angles, out of reach of the ordinary prospector. The discovery, fortunately, attracted the mining magnates from the Kimberley diamond fields, whose capital and mining expertise were to help overcome these problems and hence prove instrumental to the success of the gold mines.

Individual prospectors thus turned their attention to purchasing shares in larger and more successful mining operations. This led to the establishment of the Johannesburg Stock Exchange (JSE) in Symmonds Street in 1887. Trading in the JSE soon became hectic and overflowed into the street which had to be sectioned off by chains for this purpose. Hence in 1889 the building was demolished to make way for a larger double-story exchange on the same site. At the same time investors were becoming concerned about the enormous technological problems facing the mines as they delved deeper into the earth. This awakened the need for co-operative action to solve the massive problem, and in 1889 the Chamber of Mines was formed.

International investors in Europe and America were at first sceptical of reports of the huge profits in the gold mines in South Africa. However knowledge that the Chamber of Mines and names like Cecil John Rhodes and Werhner Beit and Co. (that had already made fortunes for the Rothschilds and other investors in Europe on the diamond mines) were behind the Mining Houses prompted initial interest from abroad. It was not until 1893, however, when experts from France and Germany brought back reports of bonanza returns that the floodgates of international investment were opened and the JSE enjoyed its first major boom.

The turn of the century saw increasing production with the annual value of gold output exceeding £20 million. In 1903, just after the Boer war the JSE was moved to a new building in Hollard Street. The period up to 1930 was one of hardships rather than prosperity and the value of gold declined in real terms. At the time the gold price stood at a little more than the long constant price of \$20 or £4.5s. sterling. The breakdown of the Gold Standard saved the industry with the gold price subsequently reaching U.S.\$35 or £7 sterling. By 1936 fifteen new mines were being established and the JSE boomed right up to the 1939-1945 war when inflation again eroded the profitability. Fortunately in 1949 timely devaluation of sterling lifted the gold price to £12.8s.3d an ounce, enough to provide a further boost to the industry.

In the 1950's more gold mines opened along the Witwatersrand basin, and in the same decade the industry received yet another boost as the world entered the nuclear age, and the uranium present in some gold ores became highly profitable. It was during this period that the character of the JSE began to change with the growth in post-war industrial companies, increasing from 66 listed companies in 1939 to 359 in 1961. This attracted the institutional investors who were now able to diversify their portfolios into less risky ventures. This led to the need for a new exchange, and in 1960 a new building was erected on the site of the old exchange. By 1966 the gold mining industry was once more at a crisis point, suffering under a constant gold price while costs were soaring. The following year the government introduced a state assistance scheme and by 1970 more than a third of the gold mines were receiving state assistance to avoid closing down. This gloomy picture was gradually transformed after the freeing of the gold price in March 1968. By April 1972, gold had edged up to \$49 an ounce after the United States tried unsuccessfully to force the gold price to below $35 US dollars in an attempt to strengthen demand for the dollar. In May 1973 it passed the $100 an ounce mark for the first time in history due partly to rising oil prices.

The long era of fixed prices was at an end and the industry had entered a new era of highly volatile prices. This era would see gold rise above $800 an ounce and briefly touch $875 an ounce in 1980, while falling back later to depths of below $300 an ounce. This era, which is still under way today sees the mining houses continuing their search for new deposits in the Witwatersrand basin system. Today, the JSE, with more than 1500 listed securities, standing at its new home in Diagonal Street, not only reflects the hopes and fears of investors, but also plays a vital rôle in Southern Africa's economy enabling huge capital sums to be raised to expand existing industries and to finance new ones.

1.2 THE EVOLUTION OF CAPITAL MARKET THEORY

It was perhaps inevitable that the unfolding history of stock exchanges throughout the world would go hand in hand with an endeavour to understand price movements in these markets. Thus it is only natural that pursuits into this field have been virile, especially over the last thirty five years where the fruitful marriage between the development of the computer and the quality of financial data, in particular stock market price series, are the envy of many other fields of research.

The modern theory of finance has thus been rooted in empirical analysis, with the major testing grounds being the New York Stock Exchange (NYSE). Considerably less published empirical research has been conducted in the United Kingdom, while virtually nothing by contrast, has been conducted on smaller exchanges, such as the JSE.

The major problem facing investors has always been the maximization of wealth in a world of uncertainty. The problem of choice under uncertainty is characterised by the situation where an investor faces a set of investment alternatives, and the outcomes associated with these alternatives are uncertain. Two main conceptual frameworks have been developed to deal with the problem; the state–preference framework developed by Arrow (1951) and later by Debreu (1959), and the mean–variance or parameter–preference framework developed by Markowitz (1952). Both of these approaches are generalizations to a world of uncertainty based on the work of Fisher (1930) on the theory of interest. The state–preference approach assumes the objects of choice are payoffs offered in different states of nature. While this framework is useful for investigating theoretical issues, it lacks empirical content due to the difficulty in quantifying all the payoffs offered in different states of nature. The pioneering work of Markowitz (1952, 1959) in the mean variance framework on portfolio selection, however, laid the foundations for a great deal of new research into models in the mean–variance framework. Markowitz developed the theory on portfolio selection in a framework where investors define indifference curves in terms of only the mean and standard deviation of asset returns.

A logical consequence of the theory was that diversification amongst assets would result for risk–averse investors desiring to maximize their expected utility. Furthermore that the resulting utility maximizing portfolio for investors would be mean–variance efficient, that is, efficient in the sense that for the period under consideration it provided maximum expected return for a given level of risk and minimum risk for a given level of expected return. Tobin (1958) extended this theory by showing that the investment decision could be separated into two phases: firstly the selection of a unique optimum combination of risky assets; and secondly a separate choice concerning the allocation of funds between the unique optimum combination and a single riskless asset.

Initially the paper by Markowitz (1952) did not receive much attention and it was only after Tobin (1958) independently derived many of Markowitz' results and Markowitz himself published a book (Markowitz (1959)) that interest in this field awakened. Researchers soon realized that the implementation of the theory required a model to select the optimal portfolio. Various algorithms were proposed for the solution of this problem, with the most popular being those of Markowitz (1959), Wolfe (1959), Houthakker (1960) and Sharpe (1963, 1970).

Subsequently, there was a great deal of research interest into various alternative portfolio selection models. Of the earlier contributions Farrar (1962) considered the utility function of an investor in order to find an explicit solution representing the single utility maximizing portfolio for the investor instead of choosing from the entire set of efficient portfolios. Unfortunately Farrar's work found little support amongst practitioners as it depended on the construction of the individual investors utility function which was found to be difficult to determine in practice. Furthermore practitioners were unable to overcome the problem of manipulating the vast amounts of data input required by the model at the time. On the suggestion of Markowitz (1959), Sharpe (1963) made the first major breakthrough in practical portfolio selection by proposing his *diagonal* model which significantly reduced the number of input variables as well as the complexity of the computations. Researchers continued to concentrate on reducing the efficient set in order to make computations manageable. Further contributions here include those of Baumol (1963), Fama (1965b), Sharpe (1967) and Hastie (1967).

As the popularity of single period portfolio selection models in the mean–variance framework grew some researchers questioned the assumptions underlying the models. It was known, for example, that the consumption–investment problem was clearly a multiperiod problem embodying the investors lifetime consumption pattern, and that the lack of a well–developed multiperiod theory at the time had led all of the above–mentioned researchers to assume that the portfolio decision could be treated as single–period decision. This led to the consideration of portfolio revision models (multi–period models) resulting in significant contributions in this area from Smith (1967), Mossin (1968), Hakansson (1971) and Chen, Jen and Zionts (1971). Unfortunately the multiperiod models are regarded as being generally rather impractical due to the vast amounts of input required and the difficulty in obtaining numerical solutions. Furthermore Fama (1968, 1970) has shown that while investors are essentially concerned with a multi–period problem they would behave as though they were single–period utility maximizers. Fama (1970) argued that if preferences and future investment opportunity sets are not state–dependent, then intertemporal portfolio maximization could be treated as if the investor had a single–period utility function. Hence single period models became widely accepted as having considerable advantages over the multi–period models. Thus most of the subsequent work in the mean–variance framework concentrated on single–period models.

Another aspect which was questioned was the fact that the mean–variance approach relied on the existance of finite variances for the distributrion of security returns. Mandelbrott (1963), Fama (1965a) and Fama and Roll (1968) however have produced empirical results which indicate that the distribution of returns on common stocks and bonds appear to belong to the Stable class of distributions for which the mean exists but the variance is undefined (a special case of the family of Stable distributions is the Normal distribution which has a finite variance). Fama (1965b, 1968, 1971), however, has demonstrated that as long as the distributions of returns come from any symmetric member of the class of Stable distributions with finite mean, most of the results consistent with the Normal distribution in the mean–variance framework can be obtained for the Stable class of distributions.

In spite of the growing popularity of single–period models in the mean–variance framework, most researchers in the spirit of Markowitz had developed normative models dealing with asset *choice* under conditions of risk. No one had as yet attempted to extend these models to construct a market equilibrium theory of asset *pricing* under conditions of risk in the mean–variance framework. The work of Markowitz, however, had brought home the idea that through diversification some of the risk inherent in an asset could be avoided so that the assets total risk was obviously not the relevant influence on its price. In the early 1960's little had been said about the relationship between the price of a single asset and its risk, in fact little had been said concerning the particular risk component which was relevant for asset pricing. It was left to Sharpe (1964), Lintner(1965a) and Mossin (1966) to make the first major contributions in developing a theory of asset pricing. This theory has become known as Capital Market Theory and as a result the model is frequently referred to as the Sharpe–Linter–Mossin [1] Capital Asset Pricing Model or more simply as the CAPM.

The most important implication of the model is that investors can only expect to be compensated for bearing systematic or market related risk. Consequently any unsystematic or firm specific risk would not be priced in the market for all assets. Clearly, if valid the CAPM has important implications for both individual firms and investors and as a result, it has been the focus of a vast deluge of empirical research.

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1 Jack Treynor (1961) had also independently developed the model. Unfortunately his work remains unpublished.

In the early 1970's the prominent studies on the CAPM were those of Black, Jensen and Scholes (1972), Blume and Friend (1973) and Fama and Macbeth (1973). Roll (1977) raised several pertinent questions relating to the validity of the empirical tests used to test the CAPM. In particular, one of the issues Roll raised was that any test of the CAPM was in fact a joint test of both the CAPM and the suitability of the index used as a surrogate for the market portfolio. Consequently, rejection of the null hypothesis did not necessarily imply rejection of the CAPM. Stambaugh (1982) addressed this issue by constructing broader market indices which included bonds and real estate. He concluded that the tests did not appear to be very sensitive to the choice of the market proxy.

Other researchers meanwhile have concentrated on the assumptions underlying the CAPM and have developed extensions of the model under relaxed assumptions. For example, Mayers (1972) has considered a model which incorporates the existence of nonmarketable assets such as a human capital. Merton (1973) has derived a version of the CAPM which assumes that trading takes place continuously over time, and that asset returns are distributed lognormally. Brennan (1970) and Litzenberger and Ramaswamy (1979) have considered models which include the effect of differential tax rates on capital gain and dividends.

Recent attention has, however, been focussed on whether additional explanatory factors are relevant in asset pricing models. In particular, factors such as dividend yield market capitalisation (or firm size), price–earnings ratios, January effects and others have all been empirically tested for evidence that may promise the investor consistent excess returns over the market. The methodology used in most of these studies usually follows the traditional univariate testing procedures suggested by Fama and Macbeth (1973) and Black and Scholes (1974).

Among these researchers Black and Scholes (1974) pioneered the empirical testing of the effects of dividend yield on common stock return. They did not, however, find evidence of a significant dividend yield effect. Several other researchers meanwhile continued to sift through the data in search of a dividend yield effect. Among these researchers are Long (1978), Litzenberg and Ramaswamy (1979, 1980, 1982), Rosenberg and Marathe (1979), Stone and Bartter (1979), Blume (1980), Gordon and Bradford (1980), Miller and Scholes (1978, 1982) and Keim (1985). While many of the above researchers have uncovered evidence of a positive significant dividend yield effect, they did not, in general, attribute the significant yield effects to taxes. Rather it has been suggested that other anomalies may be embodied in the dividend yield effect.

Several recent studies on the NYSE have, however, documented evidence of a significant relation between common stock returns and the market value of common stock. This has become known as the 'size effect'. Banz (1981) was one of the first to investigate this relationship. Banz (1981) finds a negative statistical association between returns and firm size which implies that shares of firms with large market values have had smaller returns on average than small firms in similar risk classes. Similar results have been documented by Reinganum (1981), Blume and Stambaugh (1983) and Stoll and Whaley (1983).

Attention subsequently turned towards determining whether this size effect was concentrated in specific seasons. Brown, Kleidon and Marsh (1983), Keim and Stambaugh (1984), Keim (1983), and Roll (1983) find evidence of seasonal effects. In particular, the size effect was found to be concentrated in the month of January. Various differing explanations for the size effect have been put forward by Chan, Chen and Hsieh (1985), Ross (1976), Schultz (1983), Roll (1981), Reinganum (1981) and Blume and Stambaugh (1983). Some of the explanations given include arguments involving thin–trading, non–stationarity of beta, bid–ask bias and absence of additional explanatory risk measures. However, no general consensus concerning the explanation on the size effect has however yet been reached.

Attempts to explain the effect in the month of January have focussed on market frictions that violate the CAPM assumptions. Brown, Keim, Kleidon and Marsh (1983), Constantinides (1984), Roll (1983), Reinganum (1983), Schultz (1985), Chan (1985), De Bondt and Thaler (1985), and Berges, McConnell and Schlarbaum (1984) all present various arguments for and against a year–end tax–loss hypothesis. The lack of general agreement on the tax–loss hypothesis has, however, led to the consideration of other possibilities. Keim (1986), for example, suggested that liquidity constraints on market participants (for example proceeds from bonuses) could influence security returns in a seasonal fashion and hence investor liquidity or payroll effects rather than tax effects could be responsible for the January effect.

The price–earnings (P/E) multiple has long been entrenched in the investment community as an important indicator. While Nicholson (1960) first documented a significant relationship between P/E multiples and subsequent returns it was Basu (1977) who first investigated this relationship in the CAPM framework and found evidence of a significant negative relationship. The results of Basu imply higher returns are associated with low P/E multiple stocks on average. Reinganum (1981) and Peavy and Goodman (1983) also find evidence supporting this result. The interaction between P/E multiplies and the firm size effect was investigated by Basu (1983) and Cooke and Rozeff (1984). Keim (1985) suggests that the implications of these two studies are that if investors select portfolios based on low P/E stocks, further benefit may still be attained by considering the additional dimension of firm size.

Very little empirical evidence concerning tests of the liquidity effect on stock return has, however, been documented to date. In a recent paper by Amihud and Mendelson (1986) empirical evidence was found which indicates that liquidity (the bid–ask apread) is significantly related to return in the CAPM framework. The implication of this finding is that investors should require a higher expected return for less liquid stocks in order to compensate them for the higher cost of trading.

None of the results concerning the CAPM discussed above were obtained using multivariate testing procedures. Recently only a handful of researchers by contrast have considered testing the CAPM in a multivariate framework. Among these Gibbons (1980, 1982) and Shanken (1985, 1986) have made significant contributions. Recently however Gibbons, Ross and Shanken (1986) proposed a multivariate test for which the exact small sample distribution is known. Mackinlay (1987) has subsequently shown that the test has low power. Mackinlay does however argue that power gains are possible by introducing a specific alternative hypothesis. Several modifications of the test have therefore been considered in order to increase the power. Affleck–Graves and McDonald (1987) have considered using a structured covariance matrix in the test, while Gibbons and Shanken (1986) show that the power of the test can be increased by aggregating subperiods.

Although the CAPM has been subjected to a barrage of tests it is still widely felt that the validity of the CAPM has neither been conclusively proved nor disproved.

1.3 ORGANIZATION OF THESIS

In Chapter 2, the important contributions to the development of Capital Market Theory will be discussed. Greater emphasis will be given to the more classical contributions and only a brief outline of the mathematical development will be presented where it is deemed necessary for the ensuing development of the thesis.

In Chapter 3, a modified approach for portfolio selection in thinly–traded environments is proposed. This proposal concentrates on improving estimation of the inputs for practical implementation of the usual Markowitz portfolio selection routine. The estimation procedure adopted makes corrections for thin–trading and also makes use of the CAPM to improve the vector of return inputs.

Chapter 4, basically consists of four sections. The first, gives a brief outline on historical estimation problems associated with the market model. In the second section the extent of thin–trading is investigated on the JSE. Furthermore a suitable beta estimation procedure which corrects for the effects of thin–trading is investigated empirically. In the third section an extended market model is proposed. This model leads to a more detailed, yet tractable structure of the risk components of stocks on smaller markets. An empirical investigation is subsequently conducted to investigate the risk structure of JSE stocks. In the last section of Chapter 4, an example of an empirical study using risk–adjusted returns is presented. The example considers the choice between bullion and South African gold shares from the international diversification perspective.

In Chapter 5, empirical tests of the CAPM are conducted. This chapter consists of 2 main sections, namely, univariate tests and multivariate tests of the CAPM. Both of the tests also consider possible extensions of the CAPM by encorporating additional factors in the tests.

Chapter 6 represents the main focus of this thesis, here the power of the univariate and multivariate tests of the CAPM are investigated using a simulation approach. The power investigation is conducted on simulated data that charaterizes the NYSE, however the JSE parameters are also considered. In the final section of this chapter the power of these tests are compared using various structured residual variance–covariance matrices.

Lastly, some final thoughts and directions for future research are offered.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The ideas of Capital Market Theory play an important role in the methods used by decision makers in the field of modern finance. Modern investment techniques have undoubtedly been greatly influenced by the development of the CAPM, its unrealistic assumptions, and its provocative implications. It is understandable therefore that much keen interest has focussed on the outcome and controversies of tests on the validity of the CAPM. Furthermore arguments involving the consequences of the restrictiveness and the relaxation of the assumptions of the CAPM have been keenly followed by practitioners in the pursuit of greater realism. The more recent deluge of finance literature dealing with the misspecification of the CAPM in the multi-factor context is further evidence of the continuing demand by practitioners for research in the field of Capital Market Theory.

The development of Capital Market Theory has thus gone hand in hand with diverse needs to use quantitative methods in investment decisions. In this chapter the interesting sequence of literature on Capital Market Theory will be traced with detailed emphasis being given where it is deemed necessary for the ensuing development of the thesis.

2.2 THE MARKOWITZ PORTFOLIO SELECTION MODEL

The modern history of models in the mean-variance framework begins with a publication on portfolio selection by Harry Markowitz in 1952. Markowitz's (1952) treatment of the portfolio problem was almost entirely normative and was based on the following assumptions.

(1) Investors can form probability distributions about the future performance of securities.

(2) These distributions have finite means and variances.

(3) There are decreasing returns to risk bearing beyond some point.

(4) An individual's perferences are a function of portfolio return and variance only.

(5) For any given expected return on a portfolio, the portfolio with the smallest variance is preferred to all others; for any given portfolio variance, the portfolio with maximum expected return is preferred to all others.

Assumption (5) above, called the ***mean-variance criterion*** by Markowitz was the significant insight of Markowitz that reduced the portfolio selection problem to a quadratic programming problem. In particular, the problem involved finding the portfolio with minimum portfolio variance subject to a given level of portfolio return. The problem has a quadratic form since the variance of a portfolio has the form:

$$V_p = X'\Sigma X \tag{2.1}$$

where

X is the vector of investment weights in each security

Σ is the variance-covariance matrix of returns of the securities.

The problem is really a parametric quadratic programming as not only does the portfolio variance contain terms in x_i^2 , but due to the various portfolio returns that can be attained a *set* of solutions must be generated. Markowitz called the collection of all possible solutions to this problem the ***efficient frontier.***

Figure 2.1 gives a geometrical summary of the Markowitz mean-variance theory. (The theory is conventionally interpreted in mean-standard deviation space instead which yields equivalent results to the mean-variance framework.)

The horizontal axis represents the standard deviation of the portfolio return, $\sigma(R_p)$, while the vertical axis represents the expected portfolio return $E(R_p)$. The shaded area of Figure 2.1 represents the feasible region of all possible combinations of risk and return positions attainable from investing in risky securities. Portfolios lying on the boundary $ABCD$ represent the set of mean-variance efficient portfolios known as the efficient frontier or efficient boundary. The assumption of normality of security returns and the existance of risk-aversion on the part of investors have been shown by Tobin (1958) to be sufficient to yield a family of positively sloping convex indifference curves in mean-standard deviation space, represented by I_1 , I_2 and I_3 in Figure 2.1.

Markowitz argued that an investor limited to investments in only risky assets

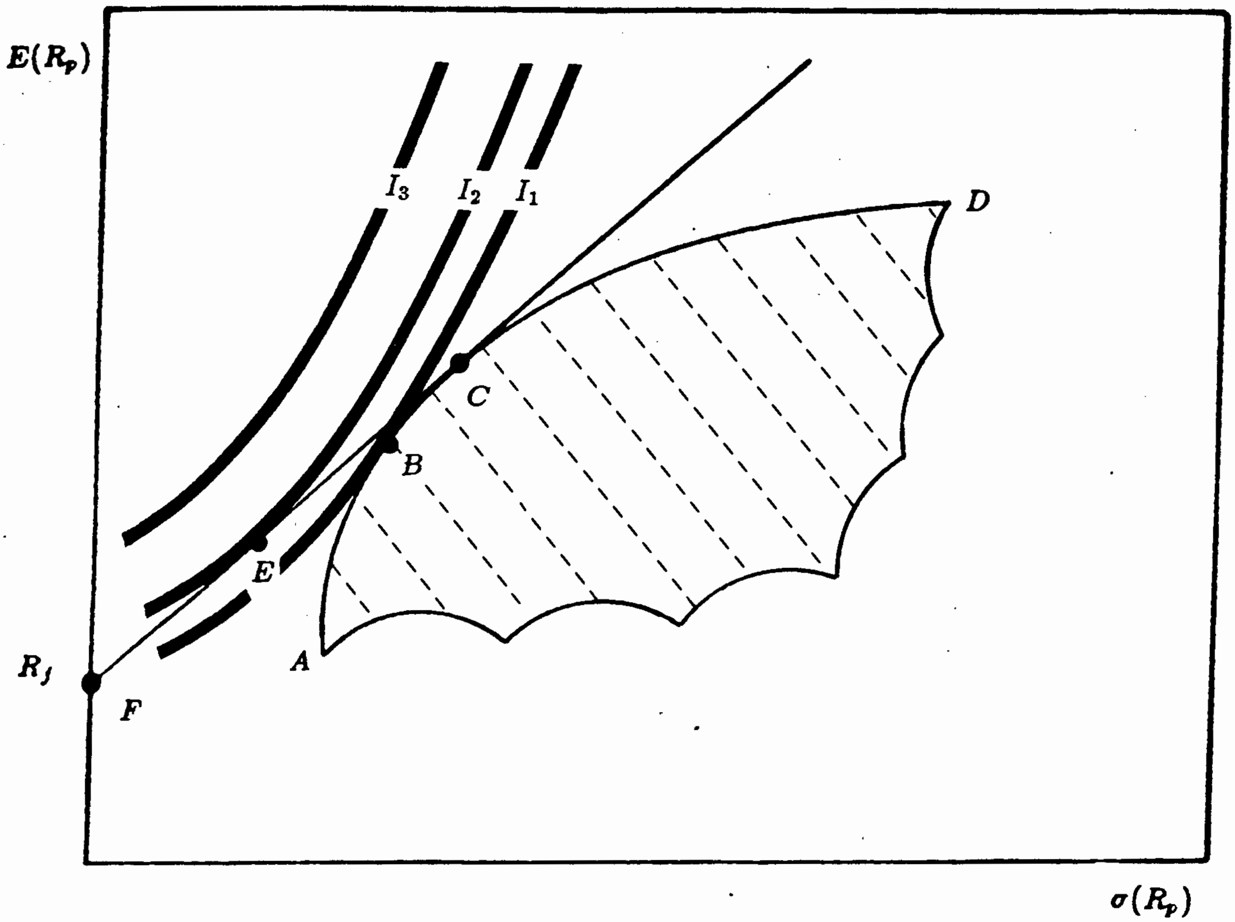

Figure 2.1 Geometrical outline of Markowitz theory

and having the indifference curves shown in Figure 2.1 will maximize his expected future utility by investing in portfolio B with utility I_1. On the other hand Tobin (1958) showed that investors who are able to invest in the risk-free asset as well (represented by F in Figure 2.1) can construct a portfolio consisting of a combination of the risk-free asset and portfolio C. This will enable the investor to reach any combination of risk and return lying along the line joining F to C. The line joining the risk free asset to the efficient frontier has steepest slope at the tangent to the efficient frontier, thus representing the maximum attainable return/risk tradeoff. An investor with the family of hypothetical indifference curves represented in Figure 2.1 will therefore maximize his expected utility by investing in portfolio E with utility I_2.

Algorithms for the computation of the efficient frontier appeared in the literature soon after the publication of Tobin's (1958) paper. Some of the more notable solutions were proposed by Markowitz (1959), Wolfe (1959), Houthakker (1960) and Sharpe (1963, 1970).

The basic portfolio selection problem was typically formulated as follows:

$$\text{Minimize} \quad -\lambda E(R_p) + V_p \tag{2.2}$$

for all possible $\lambda \geq 0$

subject to

(1) $\sum_{i=1}^{n} x_i = 1$

(2) and $x_i \geq 0, \qquad i = 1, 2, \ldots, n$

where

n is the number of securities considered for the portfolio;

x_i is the proportion of funds invested in the ith security and is restricted to be positive to exclude the possibility of short sales; and

$$E(R_p) = \sum_{i=1}^{n} x_i E(R_i) \tag{2.3}$$

where $E(R_p)$ is the expected return on the ith security.

Solution of the above problem generally requires the estimation of n expected returns and $n(n-1)/2$ distinct covariances, consequently the large number of input requirements tended to make computations unmanageable. Attempts were subsequently made to reduce the efficient set in order to simplify computations. Sharpe (1963, 1967), Baumol (1963), Fama (1965b), Hastie (1967) Cohen and Pogue (1967) and Wallingford (1967) all published papers dealing with reduced efficient set computational procedures. Since the details of these procedures were not directly relevant to the development of Capital Market Theory they will not be discussed further in this thesis.

While the development of the Markowitz portfolio selection theory was a significant milestone in Financial Economics, the theory did not directly help to explain the manner in which individual assets would be priced in the market. The theory did, however, provide the framework for Sharpe (1964), Lintner(1965a) and Mossin

(1966) to develop a positive theory of asset pricing. These contributions will be considered in section 2.4.

2.3 THE MARKET MODEL

It seems that the market model was originally considered by Markowitz (1959) whereafter considerable simplifications and advantages were first fully discussed by Sharpe (1963). The original model was commonly referred to as Sharpe's Index Model, and was considered in order to simplify the estimation problems for the practical implementation of Markowitz portfolio selection theory. The intention was to reduce the number of input estimates by relating the co–movement of the opportunity set of shares through a common index. Since the magnitude of estimation problems are no longer a severe problem due to the increased power of computers, the emphasis of the model has shifted to empirical studies where the beta coefficient is extensively used as a measure of systematic risk.

The market model is not supported by any theory, it simply assumes that the slope and intercept terms are constant over the time period during which the model is fitted to the available data. More precisely, the model simply states that returns on some security i , are linearly related to returns on a market index. The model may be written as

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$$

where

R_{it} is the return on security i in time period t

R_{mt} is the return on the market in time period t

α_i and β_i are parameters unique to security i

e_{it} is the disturbance or error term satisfying the following assumptions:

(i) $E(e_{it}) = 0$;

(ii) $\text{cov}(e_{it}, e_{is}) = 0$ for all $t \neq s$;

(iii) $\text{var}(e_{it}) = \sigma^2$ for all t ; and

(iv) e_{it} is independent of R_{mt} for all t .

The β parameter has been used extensively as a measure of risk of the specific security in relation to the market. It can be shown that if the ordinary least squares regression technique is used then

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

where

β_i is the beta coefficient for share i ;

σ_{im} is the covariance between the returns on share i and the market return;

σ_m^2 is the variance of returns on the market index.

The value of β_i indicates the volatility of security i's rate of return by comparison with the market.

Securities having β greater than one therefore are regarded as being more volatile and hence more risky than the market, while securities having β less than one are regarded as being less risky than the market.

It is evident from the market model that the variance of a security's returns stems from two sources:

Firstly, the variance of the return on the market index, R_m ;

and secondly, the variance of the random error term, e_i .

These two elements of risk are commonly known as market or systematic risk, and unique or unsystematic risk respectively. Expressions for the above 2 components of risk can be found by considering

$$\begin{aligned}\text{var}(R_i) &= \text{var}(\alpha_i + \beta_i R_m + e_i) \\ &= \text{var}(\alpha_i) + \text{var}(\beta_i R_m) + \text{var}(e_i)\end{aligned}$$

since α_i is a constant, $\text{var}(\alpha_i) = 0$ therefore

$$\begin{aligned}\text{var}(R_i) &= \text{var}(\beta_i R_m) + \text{var}(e_i) \\ &= \beta_i^2\ \text{var}(R_m) + \text{var}(e_i).\end{aligned}$$

Using a more compact notation the above equation can be written as

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2$$

where

σ_i^2 is the variance of returns on share i

σ_m^2 is the variance of returns on the market index

$\sigma_{e_i}^2$ is the residual variance.

In particular

Total risk = Market risk + Unique risk

Market risk stems from the fact that there are economy wide factors which affect all securities. These might include factors like movement in interest rates, the business cycle, or the inflation rate, which affect almost all firms to a degree.

On the other hand unique risk is the risk embodied in individual companies, for example, local strikes, bad management or setbacks affecting production, etc.

The model has received a great deal of attention in the literature. Most of the empirical research however, has tended to concentrate on the use of the coefficients of the model. Perhaps some of the more important results that have emerged in the literature are:

Firstly that the linearity assumption seems to be fairly well satisfied (Fama, Fisher, Jensen and Roll (1969)).

Secondly the beta coefficients were found to be relatively unstable over time (Fabozzi and Francis (1978), Kon and Jen (1978) and Levy (1971) have researched this aspect). However the beta coefficients were found to be stable over bull and bear market conditions (Fabozzi and Francis (1977) on the NYSE, and Bradfield, Barr and Affleck–Graves (1982) on the JSE).

Thirdly Modigliani and Pogue (1974) found that beta coffficients do give a fairly good measure of risk inherent in a security.

Lastly Beaver, Kettler and Scholes (1970) and Rosenberg and McKibben (1973) confirm that the value of beta in any period can be related to some fundamental characteristic of that firm in that period.

Jacob and Petit (1984) emphasize the important ties between the market model and the CAPM. They argue that if the market index contains all risky assets in proportion to their aggregate equilibrium values, then the security's market model beta will equal its CAPM beta. Furthermore, Jacob and Petit (1984) argue that the link between the two models require that in equilibrium the coefficient α_i must fulfill the requirement

$$\widehat{\alpha}_i = (1 - \widehat{\beta}_i)R_f$$

They assert that the above conclusion ties the market model directly to the CAPM.

2.4 THE CAPITAL ASSET PRICING MODEL

Although Markowitz (1952) was the first to develop a major normative theory of portfolio selection, the theory did not directly help to explain the manner in which individual assets would be priced in the market. However, using the Markowitz portfolio selection approach, Sharpe (1964) and Lintner(1965a) independently developed a positive theory of asset pricing. Many generalizations of the theory followed with Mossin's (1966) contribution being probably the most significant. Subsequently the model is frequently referred to as the Sharpe–Lintner–Mossin (SLM) Capital Asset Pricing model, or simply as the CAPM.

The CAPM can be written as

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

where

$E(R_i) =$ the expected return on the ith security;

$E(R_m) =$ the expected return on the market of all assets;

R_f = the risk–free rate; and

$\beta_i =$ covariance $(R_i; R_m)$/variance (R_m) .

A brief outline of the approaches used by Sharpe, Lintner and Mossin to develop the CAPM follows.

2.4.1 The Sharpe Model

William Sharpe (1964) was the first to publish a paper on asset pricing under conditions of market equilibrium. Sharpe realized that although Markowitz (1952) had developed a theory of asset selection based on the influences of investor preferences, that no equilibrium theory existed at the time which described how these preferences influenced the price of bearing risk.

The Sharpe CAPM is based on the following assumptions:

(1) Investors are risk-averse individuals who maximize the single-period expected utility of their end-of-period wealth, and find it possible to make portfolio decisions solely on the basis of the means and standard deviations of the probability distributions of portfolio returns.

(2) All investors have the same decision horizon.

(3) All assets are infinitely divisible, there are no transaction costs or taxes, information is costless and available to everybody, borrowing and lending rates are equal and are the same for all investors.

(4) All investors have homogeneous expectations of the means, variances and covariances of return among all assets.

The first assumption places the analysis within the same framework as the Markowitz portfolio selection model. Sharpe (1964) pointed out that a single-period return is just a linear transformation of the units in which terminal wealth is measured, hence utility functions can be defined in terms of single period returns instead of terminal wealth. The other assumptions of the Sharpe model standardize the opportunity set available to each investor.

Under the above assumptions the situation facing each investor can be represented in Figure 2.2 which typifies the set of investment alternatives available to investors as formulated by Markowitz (1952, 1959) and extended by Tobin (1958). Sharpe (1964) pointed out that the portfolio an investor choses would depend on his preferences for risk and return, but that the optimum portfolio for all investors would involve some combination of the risk-free asset, F , and the portfolio of risky assets, M , shown in Figure 2.2. Sharpe (1964) further argues that therefore there would be no incentive for investors to hold risky assets not included in M , hence if M did not include all of the risky assets in the market, or if they were not included in their exact market capitalization proportions, then there would be some assets that no one would hold. Sharpe pointed out that this situation would be inconsistent with market equilibrium which requires that all assets be held. Hence in equilibrium portfolio M (the market portfolio) consists of all assets held in their exact market capitalization proportions. [1]

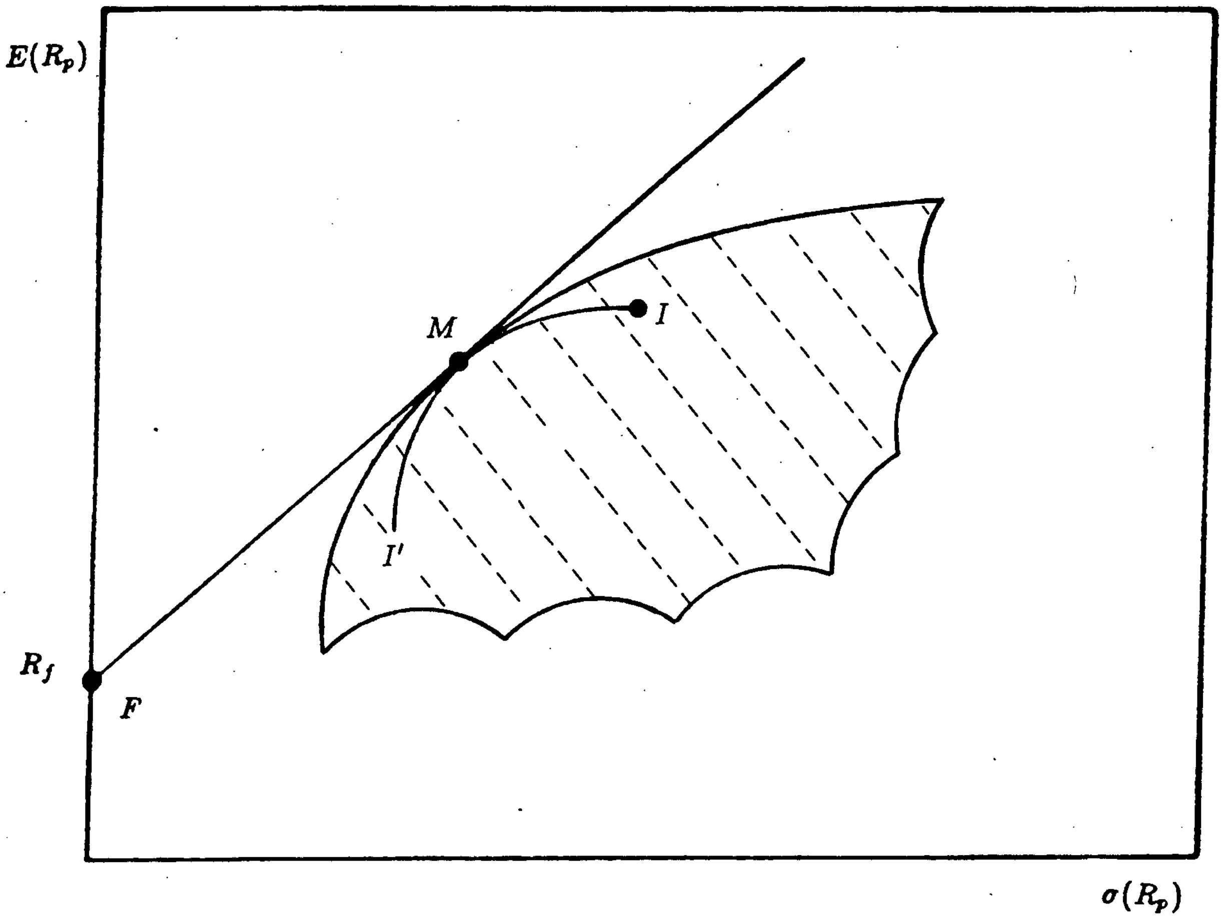

Figure 2.2 Geometry of the Sharpe framework

Sharpe therefore reasoned that for any asset I there would be a curve like IMI' in Figure 2.2 representing all attainable risk–return combinations of asset I and the market portfolio, M . Furthermore that the expected return and standard deviation of a given combination having a proportion, x , of available funds invested

[1] Sharpe (1964) in fact asserts that in equilibrium an entire segment of the efficient frontier may be tangent to the straight line through R_f . Sharpe does, however, note that all portfolios lying on this segment would have to be perfectly correlated. Fama (1968) points out that it is unlikely that investors would expect this to occur *ex ante* as *ex post* returns on portfolios are never perfectly correlated.

in asset I and $1-x$ in the market portfolio can be expressed as

$$E(R_p) = xE(R_i) + (1-x)E(R_m) \tag{2.4}$$

$$\sigma(R_p) = [x^2\sigma_i^2 + (1-x)^2\sigma_m^2 + 2x(1-x)\sigma_{im}]^{\frac{1}{2}} \tag{2.5}$$

where

σ_i^2 is the variance of risky asset I ;

σ_m^2 is the variance of the market portfolio, M ;

σ_{im} is the covariance between asset I and the market portfolio.

Sharpe [2] obtains an expression for the marginal rate of exchange of the expected return for standard deviation in the market portfolio as the proportion of asset I in the market portfolio is changed, by noting that

$$\frac{\delta E(R_p)}{\delta\sigma(R_p)} = \frac{\delta E(R_p)/\delta x}{\delta\sigma(R_p)/\delta x} \tag{2.6}$$

where

$$\frac{\delta E(R_p)}{\delta x} = E(R_i) - E(R_m) \tag{2.7}$$

$$\begin{aligned}\frac{\delta\sigma(R_p)}{\delta x} &= \frac{1}{2}[x^2\sigma_i^2 + (1-x)^2\sigma_m^2 + 2x(1-x)\sigma_{im}]^{-\frac{1}{2}} \\ &\quad\times [2x\sigma_i^2 - 2\sigma_m^2 + 2x\sigma_m^2 + 2\sigma_{im} - 4x\sigma_{im}]\end{aligned} \tag{2.8}$$

Sharpe's insight that enabled him to use the above facts to determine a market equilibrium price for risk was the fact that in equilibrium, the market portfolio already has the market capitalization proportion of asset I invested in it. Therefore the proportion x in the above equations is the excess demand for asset I, which must be zero in equilibrium.

Evaluating equations (2.7) and (2.8) at $x = 0$, Sharpe obtained

$$\frac{\delta E(R_p)/\delta x}{\delta\sigma(R_p)/\delta x} = \frac{E(R_i) - E(R_m)}{(\sigma_{im} - \sigma_m^2)/\sigma_m} \tag{2.9}$$

[2] Sharpe's (1964) notation differs slightly. Furthermore his derivation is given in $(\sigma(R_p)\,;\,E(R_p))$ instead of $(E(R_p);\sigma(R_P))$ space as is assumed here.

Sharpe's final insight was to note that the slope of the opportunity set IMI' provided by the relationship between the risky asset I, and the market portfolio, M, must be equal to the slope of the capital market line $R_f M$ at point M

$$\frac{E(R_i) - E(R_m)}{(\sigma_{im} - \sigma_m^2)/\sigma_m} = \frac{E(R_m - R_f)}{\sigma_m} \tag{2.10}$$

Sharpe (1964) unfortunately puts his major results (that is derivation of (2.10) above) in a footnote of his paper and concentrates on emphasizing this result in the framework of the ***market*** or ***diagonal model*** he proposed in an earlier paper (cf. Sharpe (1963)). The relationship given in equation (2.10) above, however, is easily rearranged to yield an explicit solution for $E(R_i)$, which results in the well known form of the CAPM, that is

$$E(R_i) = R_f + [E(R_m) - R_f]\frac{\sigma_{im}}{\sigma_m^2} \tag{2.11a}$$

or

$$E(R_i) = R_f + \lambda\sigma_{im} \tag{2.11b}$$

where

$$\lambda = \frac{E(R_m) - R_f}{\sigma_m^2} \tag{2.11c}$$

The coefficient λ can be thought of as the market price per unit of risk so that the appropriate measure of risk of asset i is its covariance with the market (i.e. σ_{im}).

2.4.2 The Lintner Model

John Lintner developed the basic relationship of the CAPM independently from Sharpe and comments in a footnote of his first paper in 1965 (cf. Lintner (1965a)) that Sharpe's (1964) paper appeared only after his paper was in its final form and on its way to the printers.

Lintner's (1965a, 1965b) development of the CAPM took a slightly different line of approach to that of Sharpe's (1964). His development was mathematically more rigid although the framework within which the development took place was identical. The assumptions made by Lintner were also essentially the same as those made by Sharpe (1964) given in section 2.4.1 above.

Lintner begins by considering investment in an arbitrary mix of individual stocks and a riskless asset chosen from a total of m stocks. He states that the net return per dollar invested could be formulated [3] as:

$$E(R_T) = (1-y)R_f + yE(R_p) \tag{2.12}$$

where

$$E(R_p) = \sum_{i=1}^{m} x_i E(R_i)$$

the expected return on the portfolio of risky stocks;

$E(R_T)$ is the expected total return of the combined investment;

y is the ratio of gross investment in stocks to total net investment;

x_i is the proportion of y invested in risky stock i.

Furthermore

$$\sigma^2(R_T) = y^2\sigma^2(R_p) \tag{2.13}$$

where $\sigma^2(R_T)$ is the variance of returns on the total investment;

$\sigma^2(R_p)$ is the variance of returns on the portfolio of risky assets.

Lintner then eliminated y from equations (2.12) and (2.13) thus obtaining:

$$E(R_T) = R_f + \theta\sigma(R_T) \tag{2.14}$$

where

$$\theta = \frac{E(R_p) - R_f}{\sigma(R_p)} \tag{2.15}$$

Lintner thereafter concentrates on the above expression for θ and argues that the optimal portfolio of risky stocks will be the one with the highest θ ratio. Although Lintner did not appeal to the geometry directly for the interpretation of θ, Figure 2.3 summarizes the intuition behind Lintner's approach.

In Figure 2.3 θ is recognised as the slope of the lines in $(E(R_p)\ ;\ \sigma(R_p))$ space. Clearly the available investment opportunities, restricted by the curve representing the efficient frontier in the Markowitz sense, imposes a limit on the extent to which θ can be increased. Given the opportunity set represented in Figure

[3] The notation used by Lintner (1965a, 1965b) differs from the above

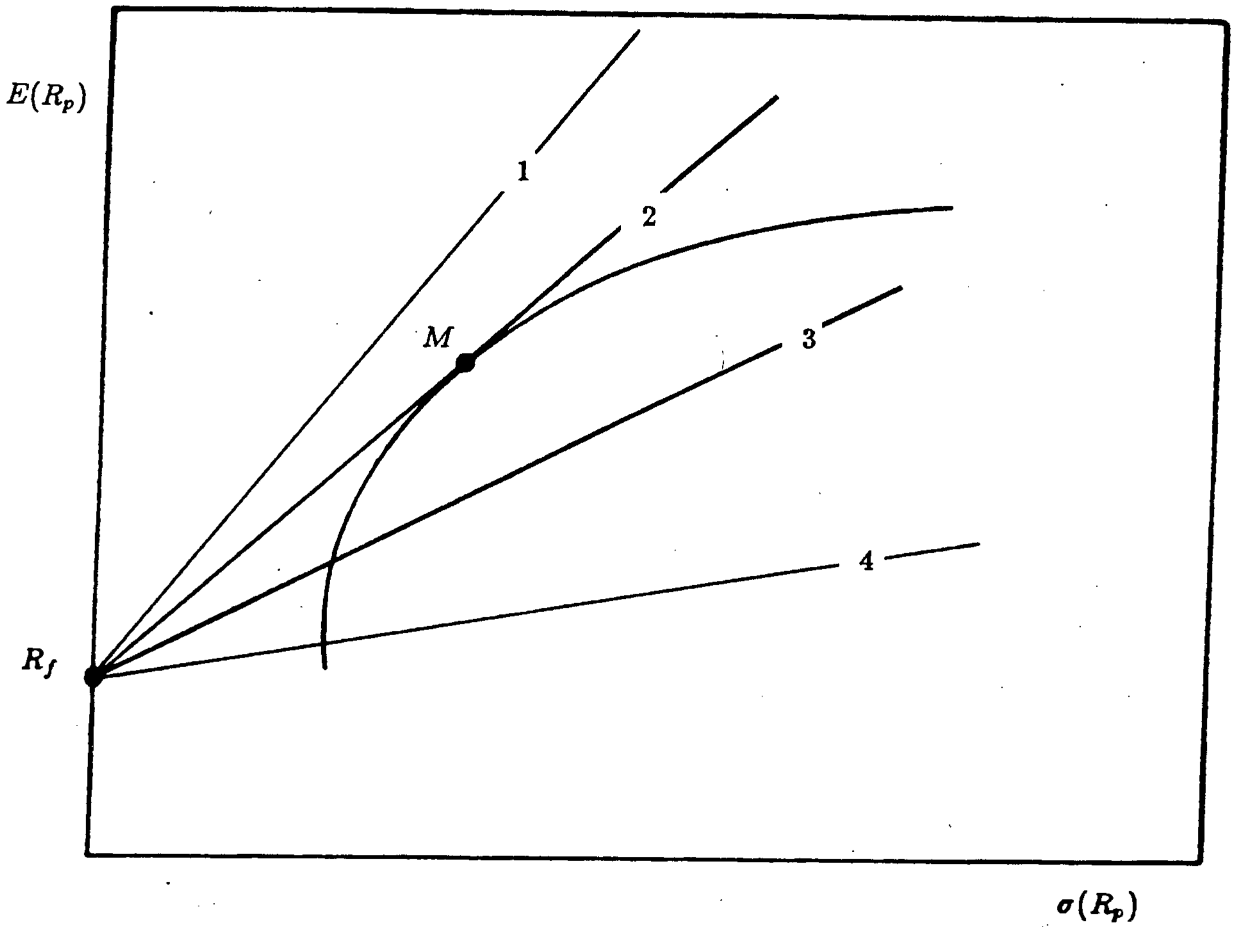

Figure 2.3 Lintner's geometrical framework

2.3 above, line 1 cannot be obtained, and of the lines 2, 3 and 4, line 2 represents the maximum attainable value of θ which occurs where the line is tangent to the efficient frontier. This graphical interpretation suggests the original graphical formulation of the Sharpe model.

Lintner rewrites equation (2.15) as:

$$\theta = \frac{\sum_{i=1}^{m} x_i(E(R_i) - R_f)}{\sqrt{\sum_{i=1}^{m} x_i^2\sigma_i^2 + 2\sum_{i=1}^{m}\sum_{i=j+1}^{m} x_i x_j \sigma_{ij}}} \tag{2.16}$$

where

$$\sigma^2(R_p) = \sum_{i=1}^{m} x_i^2\sigma_i^2 + 2\sum_{i=1}^{m}\sum_{i=j+1}^{m} x_i x_j \sigma_{ij}$$

Lintner further argues that θ is not affected by a proportionate change in the weighting factors x_i . Hence an unconstrained maximum for θ can be obtained whereafter the investment weights can be standardized to sum to unity.

For the maximization of θ , Lintner considered the rate of change θ with respect to the amount invested in stock i, that is

$$\frac{\partial\theta}{\partial x_i} = [E(R_i) - R_f - \lambda(x_i\sigma_i^2 + \sum_{j\neq i} x_j\sigma_{ij})]/\sigma(R_p) \tag{2.17}$$

where

$$\lambda = \frac{E(R_p) - R_f}{\sigma^2(R_p)} \tag{2.18}$$

Setting (2.17) equal to zero and solving yields:

$$E(R_i) = R_f + \lambda(x_i\sigma^2 + \sum_{j\neq i} x_j\sigma_{ij}) \tag{2.19}$$

Lintner (1965a, 1965b) unfortunately stresses the importance of the variance component of stock i (i.e. σ_i^2) in equation (2.19) above as a measure of risk. Fama (1968) however points out that x_i is the market capitalization of stock i relative to the total market capitalization of the entire market and is thus likely to be small, implying that the contribution of the term $x_i\sigma_i^2$ to the risk component in equation (2.19) is likely to be negligible. Furthermore Fama (1968) pointed out that (2.19) is easily rearranged to obtain exactly the same expression obtained by Sharpe (1964) (i.e. equation 2.11a) by noting that the optimal portfolio of risky assets will be the market portfolio, and that:

$$\sigma_{im} = x_i\sigma_i^2 + \sum_{j\neq i} x_j\sigma_{ij} \tag{2.20}$$

2.4.3 The Mossin Model

Jan Mossin's (1966) treatment of asset pricing and market equilibrium followed that of Sharpe and Lintner. Mossin's paper is probably one of the most quantitative of the papers that have dealt with the subject. Due to the quantitative nature of Mossin's treatment, the outline of his paper will not be considered in detail here.

The major distinction between the Mossin treatment and the Sharpe–Lintner treatment is that instead of using return and standard deviation of return as parameters of the utility function, Mossin uses wealth and its variance as parameters.

Furthermore Mossin uses shares as variables rather than relative wealth invested in securities and is thus able to treat prices explicitly. Mossin's formulation is nevertheless based on the same assumptions as those of Sharpe (1964) and Lintner (1965a, 1965b)

2.4.4 Variants of the Sharpe-Lintner-Mossin CAPM

The major variants of the Sharpe-Lintner-Mossin CAPM (SLM-CAPM) arose out of the concern that the underlying assumptions were generally thought to be unrealistic. Several researchers in the earlier seventies considered the model under relaxed assumptions and were able to derive alternative models which were surprisingly simple extensions of the SLM-CAPM.

One of the major variants of the CAPM was derived by Black (1972) who did not assume the existance of a risk-free security.

2.4.4.1 The Black Model

Although the model of Black (1972) is less restrictive in the sense that it does not assume the existence of a risk-free security, it does however assume that short-selling of positive variance securities is unrestricted.

The intuition behind Black's (1972) arguments are illustrated in Figure 2.4.

Black shows that all investors still identify portfolio M as lying on the efficient frontier in Figure 2.4 as the market portfolio. Furthermore the portfolios on the dashed line from B through A can be identified as portfolios which have zero correlation with the market portfolio. Consequently these portfolios will have zero beta's with equal expected returns, $E(R_z)$. Only portfolio B , lying on the opportunity set however, can be identified as the minimum-variance zero-beta portfolio, and is thus unique.

Black thus shows that the expected rate of return on any asset can be written as a linear combination of the expected rate of return of two assets, namely, the market portfolio, and the unique minimum-variance zero-beta portfolio.

This can easily be derived by firstly considering the slope of the line from $E(R_z)$ through M by considering a portfolio with $x\%$ invested in M and $(1-x)\%$ invested in the zero-beta portfolio B .

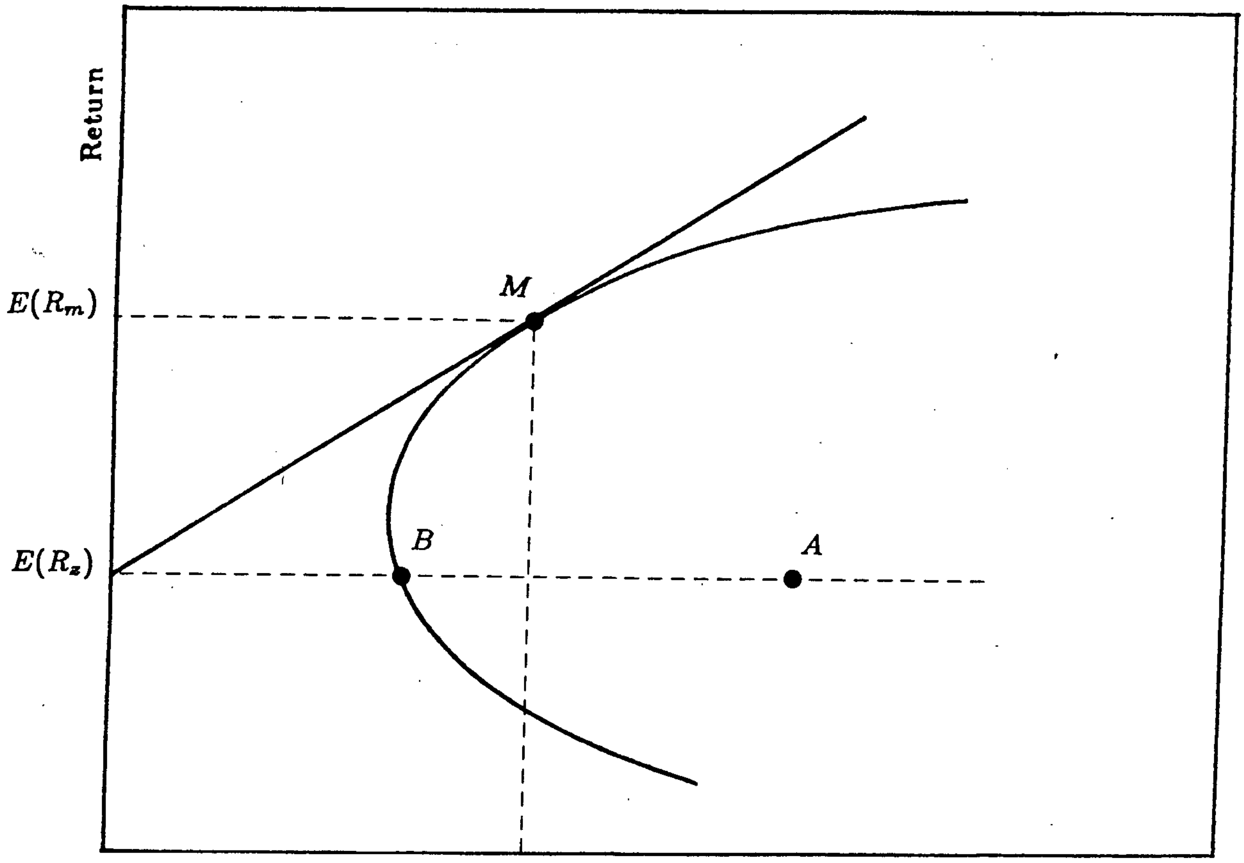

Figure 2.4 The Black model

The mean and standard deviation of this portfolio can be expressed as:

$$E(R_p) = x\,E(R_m) + (1-x)E(R_z) \tag{2.21}$$

$$\sigma(R_p) = [x^2\sigma_m^2 + (1-x)^2\sigma_z^2]^{\frac{1}{2}} \tag{2.22}$$

since the covariance term of equation (2.22) above is zero;

$E(R_z)$ is the expected return on the zero–beta portfolio; and

σ_z^2 is the variance of the minimum variance zero–beta portfolio.

The slope of the line tangent to the efficient set at point M can be found by considering the partial derivatives of (2.21) and (2.22) where 100% of funds are invested in the market portfolio, M .

$$\frac{\delta E(R_p)}{\delta x} = E(R_m) - E(R_z) \tag{2.23}$$

$$\frac{\delta \sigma(R_p)}{\delta x} = \frac{2x\sigma_m^2 - 2\sigma_z^2 + 2x\sigma_z^2}{\frac{1}{2}[x^2\sigma_m^2 + (1-x)^2\sigma_z^2]^{\frac{1}{2}}} \tag{2.24}$$

Evaluating the ratio of (3) and (4) at $x = 1$, yields

$$\frac{\delta E(R_p)/\delta x}{\delta \sigma(R_p)/\delta x} = \frac{E(R_m) - E(R_z)}{\sigma_m} \tag{2.25}$$

Now recall that in equilibrium (2.25) must be equal to (2.9), that is, the slope of a line tangent to a portfolio composed of the market portfolio and any other asset at the point represented by the market portfolio. Equating the two definitions of the slopes:

$$\frac{E(R_m) - E(R_z)}{\sigma_m} = \frac{[E(R_i) - E(R_m)]\sigma_m}{\sigma_{im} - \sigma_m^2} \tag{2.26}$$

From (2.26) the required rate of return on asset i can be written as

$$E(R_i) = (1 - \beta_i)E(R_z) + \beta_i E(R_m) \tag{2.27}$$

where

$$\beta = \sigma_{im}/\sigma_m^2$$

Equation (2.27) has the interpretation that the expected rate of return on any asset can be expressed as the linear combination of the expected rate of return on the unique zero-beta portfolio and the market portfolio, with the weight invested in the market portfolio being the beta of the ith asset.

This can be rearranged, yielding

$$E(R_i) = E(R_z) + \beta_i(E(R_m) - E(R_z)) \tag{2.28}$$

This model is seen to be similar to the SLM-CAPM with the exception that $E(R_z)$ in (2.28) above is the return on any positive variance portfolio whose return is uncorrelated with the return on the market portfolio. Hence the linearity of the CAPM still obtains, and beta is still seen to be the appropriate measure of systematic risk.

Another of the CAPM's unrealistic assumptions is the assumption that transaction costs are zero. Although transaction costs in stock markets are generally a small percentage of the price of assets, some assets are constrained by law to be nonmarketable. This would imply that transaction costs on such assets are infinite. Mayers (1972) considered a model which allows for the inclusion of nonmarketable assets in the investor's portfolio.

2.4.4.2 The Mayers Model

Mayers (1972) cites as examples of nonmarketable assets, human capital, claims to government transfer payments, pensions and trust income, but singles out human capital as the most important nonmarketable asset. Mayers (1972) argues that human capital cannot be physically separated from its owner as society does not accept the purchase or sale of produced human capital, for example slavery. Since direct prices or values of human capital are not observed, Mayers argues that the effects of this type of asset are not captured in the usual derivation of the CAPM. Human capital not only includes the accrual of wealth from investment in the investor in person, but also the endowments received from the investor's parents. This has the effect of introducing a non-diversifiable asset into the investor's portfolio as the investor is constrained to hold a large risky component of his wealth in the form of his own human capital.

Mayers (1972) derives a model which is similar to the SLM-CAPM and is appropriate when investors are constrained to hold nonmarketable assets which have risky rates of return, R_H . In particular

$$E(R_i) = R_f + \lambda \ \mathrm{cov}(R_i, R_m + R_H) \tag{2.29}$$

where

$$\lambda = \frac{E(R_m) - R_f}{\mathrm{cov}(R_m, R_m + R_H)} \tag{2.30}$$

$$= \frac{E(R_m) - R_f}{V_m \sigma_m^2 + \mathrm{cov}(R_m, R_H)} \tag{2.31}$$

where V_m is the market value of all marketable securities.

The interpretation of λ in (2.29) above is that λ is the market price per unit of risk, which includes not only the market variance, but also the covariance of return between marketable and nonmarketable assets.

Contrary to the SLM-CAPM the above model implies that not all rational investors hold the same portfolio of risky assets, however the equilibrium price of a risky asset is still determined independently from the individual's indifference curves, that is two fund separation still obtains. Furthermore although the covariance is still the correct measure of risk of a risky asset, it is the covariance between the risky asset and a portfolio of both marketable and nonmarketable assets which is relevant.

The SML-CAPM has a further restrictive assumption, that investors are single-period utility maximizers. Although Fama (1970) has provided some justification for treating it as if it holds intertemporally, Fama and Miller (1972) do recognise the restrictive nature of the required assumptions.

Merton (1973) derived a version of the CAPM which has the less restrictive assumption that trading takes place continuously over time.

2.4.4.3 The Merton Model

Under the assumption that trading is continuous and that asset returns are distributed lognormally over time, as well as the less restrictive assumption that risk-free rates of interest are stochastic over time, Merton (1973) derives a model which exhibits three-fund separation, that is

$$E(R_i) = R_f + \gamma_1[E(R_m) - R_f] + \gamma_2[E(R_N) - R_f] \tag{2.32}$$

where R_i and R_m are instantaneous rates of return on asset i and the market portfolio respectively;

R_N is the instantaneous rate of return on a portfolio which has perfect negative correlation with the riskless asset;

$$\gamma_1 = \frac{\beta_{im} - \beta_{iN}\beta_{Nm}}{1 - \rho^2_{Nm}} \tag{2.33}$$

$$\gamma_2 = \frac{\beta_{iN} - \beta_{im}\beta_{Nm}}{1 - \rho^2_{Nm}} \tag{2.34}$$

ρ_{Nm} is the correlation between portfolio N and the market portfolio; and

$$\beta_{ik} = \frac{\mathrm{cov}(R_i; R_k)}{\sigma^2_k} \quad \text{for some portfolio, } k. \tag{2.35}$$

Merton (1973) argues that investors are exposed to a further component of risk, that is, the risk of unfavourable shifts in the risk-free asset. Merton (1973) points out that three funds are identified in equation (2.32), and that investors will hold portfolios chosen from these three funds. Firstly, the risk-free asset, secondly, the market portfolio, and thirdly a portfolio chosen so that its returns are perfectly negatively correlated with the risk-free asset. Merton further asserts that the sign of the γ_2 coefficient above will be negative for high beta assets and positive for low beta assets. This assertion is evident by considering the interpretation of the usual beta, that is, β_{im} in equation (2.34).

Merton (1973) also shows that if the risk-free rate of interest is assumed to be constant, that the third term in (2.32) will fall away. Clearly this implies that β_{iN} and β_{Nm} in (2.34) are zero so that γ_2 becomes zero. Hence (2.32) becomes

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{2.36}$$

which is analagous to the SLM–CAPM with the exception that the usual rates of return over discrete intervals of time are replaced by instantaneous rates of return.

2.4.4.4 The Brennan Model

Brennan (1970) was the first to consider the unrealistic assumption of no taxes. Since dividends are generally more heavily taxed than capital gains for individual investors, Brennan (1970) argued that the higher a stock's dividend yield, the higher the pre-tax return an investor requires to compensate for the tax liability incurred. He thus proposes a model which includes an additional term, thereby relating the expected return on an asset to the systematic risk as well as the dividend yield:

$$E(R_i) = \gamma_1 R_f + \gamma_2 \beta_i + \gamma_3 D_i \tag{2.37}$$

where D_i is the dividend yield on asset i.

In the CAPM framework, γ_2 is hypothesized to be equal to $E(R_m) - R_f$, while γ_3 denotes the dividend yield effect.

Several researchers have conducted empirical tests to determine whether dividend yields (and many other factors) have any significant effect on the pricing of stocks. The results will not be discussed here as the details of these tests and their results will be discussed at length in Chapter 5.

2.5 EMPIRICAL TESTS OF THE CAPM

2.5.1 Univariate studies

There have been a multitude of papers presenting empirical results of univariate tests of the CAPM. Unfortunately many of these testing procedures used are fraught with statistical problems. Consequently many of the earlier testing procedures have been seriously criticised in the literature, with the result that their findings have not generally been accepted into the canon of established empirical findings.

The early empirical evidence presented by Lintner (1965b) and Douglas (1968) indicated that average realized returns were more strongly related to the variance, rather than the beta of stocks. This implies that the market for stocks was not dominated by diversifiers, contrary to Capital Market theory which suggests that investors diversify in order to hold efficient portfolios. Miller and Scholes (1972) conducted check tests that detected bias in the procedures used, and subsequently presented a strong argument indicating that the evidence found was an artifact of the testing procedures used by Lintner (1965a) and Douglas (1968). In particular Miller and Scholes (1972) argued that the presence of any error in the estimation of a securities beta would inevitably weaken the association between returns and estimated systematic risk in their second–pass regressions. They argued further that an exaggeration of the apparent significance of residual risk as an explanatory variable arises from the skewness of the underlying distribution of returns. Since Lintner and Douglas both used annualized returns, a skewness to the right of the distribution of these returns would be more apparent than if monthly or weekly returns had been used.

The first empirical study to provide solutions to the above problems was conducted by Black, Jensen and Scholes (1972) on the NYSE. They initially conducted a time series test of the CAPM. Their methodology involved estimating β coefficients on an equally weighted index using monthly data over 5 year periods. These securities were then ranked on the basis of their betas and partitioned into 10 portfolios. The excess return in each of the next 12 months for each of the ten portfolios was then calculated. The entire procedure was repeated moving one year forward, resulting in excess monthly return data on 10 portfolios over the 35 year period 1931 to 1965. Finally Black, Jensen and Scholes regressed the excess return on each of the 10 portfolios on the excess return of their equally weighted market index using the monthly time series data over the 35 years.

On the basis of the resulting estimated coefficients they tentatively concluded that low β securities offer slightly higher returns than predicted by the CAPM, and high β securities offer slightly lower returns than predicted by the model. The intercept on the lowest β portfolio for example, indicated an annual return of 2.4 percent per year more than predicted by the CAPM. By contrast, the highest β portfolio had an annual return of 0.96 percent less than predicted by the CAPM.

Black, Jensen and Scholes allude to the fact that their time series test did not constitute a direct test of the CAPM and therefore proposed a cross–sectional test. Their methodology for the cross–sectional tests initially involved re–estimating betas for each of the 10 portfolios using the portfolio returns constructed for the time series tests. Thereafter average returns on the 10 portfolios were calculated over a testing period subsequent to the beta estimation period. The 10 portfolio returns were then regressed on the 10 portfolio betas using the model below:

$$\overline{R}_p - R_f = \gamma_0 + \gamma_1 \widehat{\beta}_p + e_p$$

where

$$\overline{R}_p = \frac{1}{T} \sum_{t=1}^{T} R_{pt}$$

is the mean portfolio return averaged over the testing period;

e_p is the cross-sectional error term.

The intuition behind the above test is that

$$\gamma_0 = 0$$

and

and $\quad \gamma_1 = \overline{R}_m - r_f$ if the CAPM is valid.

On the basis of the results of their cross–sectional test Black, Jensen and Scholes offer several conclusions. Firstly, that the relationship between return and systematic risk appeared to be linear. Secondly, that the value for $\widehat{\gamma}_0$ over the whole period as well as three out of four subperiods was significantly positive, that is, larger than the CAPM suggests, (the other testing period had a significantly negative $\widehat{\gamma}_0$ value). Lastly the value for $\widehat{\gamma}_1$ was found to be significantly smaller than the average excess market return in all but one of the subperiods.

Fama and Macbeth (1973) extended these results using a similar cross–sectional investigation. In an attempt to investigate nonlinearities as well as diversifiable risk, Fama and Macbeth used the following model for their cross-sectional regressions:

$$R_p = \gamma_0 + \gamma_1\widehat{\beta}_p + \gamma_2\widehat{\beta}_p^2 + \gamma_3\sigma(e_p)$$

They argued that the inclusion of the quadratic term β_i^2 was included to test their linearity proposition and that the diversifiable risk term, $\sigma(e_p)$, was included to test the proposition that β was the only relevant measure of risk. These propositions clearly require that γ_2 and γ_3 be equal to zero. The testing methodology used by Fama and Macbeth is similar in spirit to that of Black, Jensen and Scholes with the exception that cross-sectional regressions were run in each month of the testing periods, whereafter they were summarized within particular subperiods.

The results of Fama and Macbeth were obtained over a variety of subperiods ranging from 1931 to 1968. On the basis of their results they offer several conclusions: Firstly, that the $\widehat{\gamma}_2$ coefficient was close to zero over the entire period as well as the subperiods. They argued that this evidence supported the linearity proposition. Secondly, that $\widehat{\gamma}_3$ was found to be extremely close to zero, implying that residual variability was unimportant in stock pricing, and that only β was the relevant risk measure. Thirdly, their conclusion for the intercept term is similar to that of Black, Jensen and Scholes, that is, that the intercept was significantly larger than the treasury bill rate. Lastly Fama and Macbeth find that the $\widehat{\gamma}_1$ coefficient was positive and significant for most of the subperiods, although it was less than the return on the market portfolio. On the basis of these results, however, Fama and Macbeth concluded that their results supported the important testable implications of the CAPM.

Other major empirical results concerning the validity of the CAPM have focused on whether additional explanatory factors should be in the model. Although a multitude of authors have made contributions in this area, the pioneering work can be attributed to Black and Scholes (1974), who considered a dividend yield factor, Basu (1977), who considered a price/earning component, and Banz (1981) who considered a firm size component.

The most common procedure for considering additional components has been similar to the procedure outlined by Black, Jensen and Scholes (1972). The cross-sectional regression model used generally has the following form:

$$R_p = \gamma_0 + \gamma_1 \widehat{\beta}_p + \gamma_2 \left[\frac{\widehat{\delta}_p - \widehat{\delta}_m}{\widehat{\delta}_m} \right] + e_p$$

where $\widehat{\delta}_p$ is the estimate of the additional effect being investigated for portfolio p; and $\widehat{\delta}_m$ is the estimate of the additional effect for the market portfolio.

Clearly within the CAPM framework γ_2 is hypothesized to be equal to zero. Using the above approach Black and Scholes (1974) find no significant dividend yield effect. Basu (1977) however finds evidence of a price/earnings ratio effect and Banz (1981) finds a significant firm size effect.

More recently Keim (1983) documents evidence which suggests that the size effect is concentrated in January. No consensus has been reached regarding explanations of the latter effect, and consequently debate in this area is still ongoing in the literature.

2.5.2 Multivariate Studies

Multivariate tests of the CAPM by contrast have made a relatively recent appearance in the literature. Although the first multivariate test of the CAPM could be attributed to MacBeth (1975) it was Gibbons (1980, 1982) who presented the first extensive treatment in this field. Several other researchers meanwhile have conducted investigations in this area, namely Stambaugh (1981, 1982), Jobson and Korkie (1982), Shanken (1983, 1985, 1986), Kandel (1984a, 1984b), Amsler and Schmidt (1985) and Roll (1985). In this section however the attention will be focused on a multivariate test recently proposed by Gibbons, Ross and Shanken (1986) and the subsequent literature concerning this test. Although the Gibbons, Ross and Shanken (GRS) test has only been structured as a test of the efficiency of a given portfolio, Roll (1977) has argued that testing the validity of the CAPM is in fact equivalent to testing the assertion that the market portfolio is mean–variance efficient. Hence this test will be referred to as a test of the CAPM in this thesis, although it is essentially only a test of the efficiency of the portfolio used as a proxy for the true market portfolio. Clearly the validity of the CAPM is only testable if the proxy is a valid surrogate for the true market portfolio.

The GRS test statistic is derived under the assumption of the existence of a riskless asset, and, that excess asset returns are independently and identically multivariate normally distributed through time. With these assumptions returns can be described by the excess return market model:

$$\widetilde{R}_{it} = \alpha_{ip} + \beta \widetilde{R}_{pt} + \widetilde{e}_{it} \quad \text{for} \quad i = 2, 3, \ldots, N \ ; \ t = 1, 2, \ldots, T$$

where

$\widetilde{R}_{it}$ is the excess return on asset i in period t ;

$\widetilde{R}_{pt}$ is the excess return on the portfolio whose efficiency is being tested;

$\widetilde{e}_{it}$ is the disturbance term for asset i in period t .

The following assumptions are also implied for the distribution term

$$E(e_t) = 0$$
$$E(e_t, e_s) = \Sigma \qquad \text{for} \quad t = s$$
$$= 0 \qquad \text{for} \quad t \neq s$$

where Σ is the $(N \times N)$ disturbance covariance matrix.

GRS further demonstrate that testing a particular portfolio for mean variance efficiency is equivalent to testing the null hypothesis that the α_{ip} are equal to zero.

If ordinary least squares are used to estimate the α_{ip} then

$$T(1 + \widehat{\theta}_p^2)^{-1} \widehat{\alpha}_p' \Sigma^{-1} \widehat{\alpha}_p \sim \chi_N^2$$

where $\widehat{\alpha}_p = (\widehat{\alpha}_{ip}, \widehat{\alpha}_{2p}, \ldots, \widehat{\alpha}_{Np})$;

$\widehat{\theta}_p = \bar{R}_p / s_p$

$\bar{R}_p =$ the sample mean of $\widehat{R}_{p_t}$; and

$s_p^2 =$ the maximum likelihood estimate of the variance of $\bar{R}_{pt}$

However since Σ is not generally known, GRS show that

$\Gamma_1 = [T/(T-2)][(T-N-1)/N]W \sim F'_{N;T-N-1}$

where $W = (1 + \widehat{\theta}_p^2)^{-1} \widehat{\alpha}_p' \widehat{\Sigma}^{-1} \widehat{\alpha}_p$;

$\widehat{\Sigma}$ is the unbiased sample covariance matrix of the residuals

F' is the noncentral F distribution with noncentrality parameter λ; and

$\lambda = T(1 + \widehat{\theta}_p^2)^{-1} \alpha_p' \Sigma^{-1} \alpha_p$

Under the null hypothesis the non–centrality parameter equals zero and Γ_1 has a central F distribution.

By rearranging the expression for the test statistic, Γ_1 , GRS show that it has a insightful geometric interpretation, in particular that

$$W = \frac{1+\widehat{\theta}^{*2}}{1+\widehat{\theta}_p^2} - 1$$

where

$\widehat{\theta}^*$ is the Sharpe reward–to–variability ratio of the *ex post* optimal portfolio; and

$\widehat{\theta}_p$ is the Sharpe reward–to–variability ratio of the portfolio being tested for efficiency.

Using a data set over the 1931–1965 period GRS are unable to reject the CAPM. GRS comment that the result may occur because the null hypothesis is in fact true, or that the test is not powerful enough to detect economically important deviations from efficiency of the index. Unlike the univariate tests, the exact distribution of the multivariate test statistic under the alternative hypothesis is known, consequently the power of the test can be investigated theoretically.

GRS and Mackinlay (1987) subsequently provide insights into the power of multivariate tests of portfolio efficiency. Mackinlay finds that the multivariate test suffers from low power. Although substantial increases in power can be achieved by increasing the length of the test period, empiricists are usually reluctant to expand the testing period beyond 5 to 10 year periods due to the problem of non–stationarity of parameters. Another way in which the power can be improved is by increasing the number of assets considered. Unfortunately to obtain a non–singular estimate of Σ , N needs to be less than T , hence restricting the number of assets chosen.

In an attempt to increase the power of the test some researchers have endeavoured to overcome the above–mentioned problems. Gibbons and Shanken (1986) show that power can be increased by aggregating testing periods. Affleck–Graves and McDonald (1987) have considered imposing a structure on the residual covariance matrix so that it could still be invertable when the number of assets are increased.

A more detailed discussion of the power of the multivariate test is presented in section 6.1, hence it will not be pursued further here. In conclusion, it is worth noting that the multivariate test does suffer from low power, however this area of research has captured substantial interest recently, and it is expected that a wealth of literature will continue in this area.

CHAPTER 3

PORTFOLIO SELECTION IN THINLY–TRADED ENVIRONMENTS

Since the late 1950's investors have drawn from the findings of Capital Market Theory in an endeavour to form superior portfolios. Initially investors concentrated on the pioneering work of Markowitz (1952) who developed the theory on portfolio selection in a risk–return framework. Consequently attention initially focussed on finding efficient computational procedures for determining optimally diversified portfolios (Sharpe (1963), Baumol (1963) and Fama (1965b) were the first to concentrate on making computations more manageable). In order to employ the portfolio selection model proposed by Markowitz, forecasts of expected security returns, risks, and covariances were required. The composition of the resultant optimal portfolios consequently depended on these forecasts.

The emergence of papers in the mid sixties by Sharpe (1964), Lintner (1965) and Mossin (1966) on an equilibrium theory of asset pricing, gave some new insights in the area of portfolio selection. This theory points to the composition of the portfolio that is expected to be optimal. Under various simplifying assumptions, the theory identifies what is termed the ***true market portfolio*** [1] as the expected tangency portfolio in the Markowitz risk–return framework. Subsequently much attention shifted to the construction of portfolios that were attempts to proxy this ***true market portfolio***. The emergence of unit trusts and mutual funds consisting of component securities chosen to reconstruct the market portfolio were evidence of this.

In this chapter [2] a flexible technique is proposed for selecting portfolios when a number of assets in the opportunity set are thinly–traded. The proposal uses estimators which adjust for infrequent trading to generate the expected inputs. This technique has the advantage of being applicable when the opportunity set includes thinly–traded securities as well as well–traded securities, since the estimators converge to the ordinary least squares estimators as the level of trading increases. In addition it allows portfolio estimation to be carried out under a range of expecta-

[1] Roll (1977) argues that the true market portfolio consists of *all* individual assets.

[2] This section has been published in *Managerial and Decision Economics*, **9**, 1988. The paper is entitled "Portfolio Selection in Thinly–Traded Environments — a Case Study." (see Barr and Bradfield (1988)).

tions regarding the market's performance. An empirical study on the Johannesburg Stock Exchange indicates that the proposed method is superior to traditionally implemented techniques for data analysed over the period 1974 to 1985.

It was noted in section 2.4 of chapter 2 that some of the major assumptions of the SLM–CAPM are, market efficiency, equal borrowing and lending rates and homogeneous expectations of the returns and covariances of securities. In an attempt to consider practical implementation of portfolio selection in *thinly-traded* environments, it should be noted that one of the assumptions of the theory will almost certainly be violated, namely, the assumption of market efficiency. Clearly if shares are not traded in a period in which new information arrives, then recorded prices of these shares will not reflect the arrival of new information in that period. These recorded prices will thus not "fully reflect all available information", and as such, violate the necessary conditions of market efficiency. Dimson (1979) in fact asserts that infrequent trading is a likely explanation for the apparent nonrandomness of the prices of smaller companies' shares. On the JSE in particular, a large proportion of listed shares suffer from the effects of thin–trading, and in some cases this effect is severe. Table 4.1 in chapter 4 shows that at least 20 percent of all JSE stocks can be categorized (using criteria defined in section 4.2) as being infrequently traded. In some cases even companies with relatively large market capitalizations have the majority of their shares tightly held by families or by institutions, resulting in the remainder of the shares being infrequently traded. Furthermore trading in these securities generally occurs in extremely low volumes which, arguably, could result in these shares changing hands at prices which do not always reflect underlying market values.

Features such as those cited above characterise thinly–traded environments and may result in inefficient pricing of these securities. Thus, while traditional portfolio selection techniques may generally be acceptable for well–traded and efficiently functioning environments, they may well be inapplicable to thinly–traded environments. Various procedures for dealing with the problem of thin–trading have been considered in the literature. For example, Scholes and Williams (1977), Dimson (1979) and Cohen *et al* (1983) have considered beta estimation procedures which employ corrections for the effects of thin–trading, while Shanken (1987) considered using a correction procedure for covariance estimation in thinly-traded environments. In the ensuing portfolio selection proposal, some of the above estimation procedures are implemented.

3.1 INTRODUCTION

Although Markowitz theory is entrenched as the standard basis for portfolio construction it leaves several questions unanswered for the practitioner. The theory was developed within a framework where the expected return vector and the matrix of covariances of the component assets are assumed to be known. Practical implementation thus involves the problems of estimating these inputs prior to the estimation of the portfolio weights.

Certain practitioners have considered the problems of how to estimate the inputs prior to portfolio estimation. [3] In particular, the use of historical averages of returns as surrogates for the expected return vector has been widely used. This method of estimating the expected return vector would have some justification with large data sets if the expected return vector was relatively stationary. However, over the medium term securities can exhibit tremendous volatility in return as factors which have a major bearing on the prospects for the company in question do change. Thus, for example, the fact that a windfall rise in asset value pushes up the share price by 100% may well not alter the market's feeling about growth prospects for the firm in the foreseeable future. The expected return for such a company would thus be over-estimated by any historical average of returns.

Jobson and Korkie (1981) have identified the problems of using individual historical averages as surrogates for expected returns. They proposed instead that each expected return be set to the global average of all securities in the data set. On the basis of a simulation study they concluded that the practical implementation of Markowitz-Sharpe portfolio theory is greatly enhanced by using their proposed technique. Although these estimators of return are robust, their procedure cuts across one of the main tenets of financial theory; namely, that in equilibrium higher risk is associated with a higher expected return. [4] Hodges and Brealey (1972) also

[3] See for example McEnally (1986).

[4] Jobson and Korkie (1981) do not allude to the fact that their technique yields negative

address the problem of using crude estimates of return. On the basis of a simulation study they concluded that prior correction of the inputs for estimation bias produces an improvement in portfolio efficiency.

These problems associated with portfolio selection tend to be accentuated in an environment such as the Johannesburg Stock Exchange (JSE) where a large portion of the shares are thinly–traded. In this section a procedural technique which makes adjustments for thin–trading is proposed. This proposal overcomes the criticisms levelled against the use of historical averages for the expected return input. It is shown that over a period of 7 consecutive years on the JSE, portfolios selected in this way show superior performance to traditional techniques in 6 of the years.

3.2 THEORETICAL DISCUSSION

As discussed above portfolio selection techniques which employ the Markowitz procedure require two basic inputs, a vector of expected returns and a variance-covariance matrix of returns.

The Expected Return Vector

In this section the use of the Capital Asset Pricing Model (CAPM) in conjunction with a thinly–traded beta estimator to generate the vector of expected returns is considered. [5] The use of this model is theoretically appealing as several expected market scenarios can be postulated in equation (3.1) below, resulting in a range of scenarios for the return input vector.

The CAPM can be expressed as follows:

$$E(\tilde{R}_i) - R_f = \beta_i(E(\tilde{R}_m) - R_f) \tag{3.1}$$

where

$E(\tilde{R}_i)$ is the expected return on share i,

$E(\tilde{R}_m)$ is the expected return on the market,

portfolio weights. The portfolio weights published for a number of securities were correct in absolute value but of the wrong sign. Negative weights imply short sales which creates problems for the practitioner.

[5] In well–traded markets this approach is, in theory, expected to yield the market index used in the computation of β as the tangency portfolio. This is a consequence of the fact that the efficiency of the market portfolio and the validity of the CAPM are joint hypothesis.

R_f is the risk-free rate of interest,

β_i is the systematic risk component of share i.

In order to derive the expected value of an asset's return, the β of the asset and some expectation of the market's return is required.

Estimation of beta

The use of the CAPM to generate a vector of expected returns of course requires the prior estimation of the associated vector of β's .

In a thinly–traded environment however, price determination of thinly–traded shares is a problem which has long been recognised in the financial literature. [6] The root of the problem stems from the fact that *recorded* asset prices do not necessarily represent *underlying* [7] asset prices. Hence two series of prices are perceived, a series of *recorded* prices and an unknown series of *underlying* prices based on industry and market movements. Thus problems arise when betas are estimated for thinly–traded shares using conventional methods such as Ordinary Least Squares (OLS). In particular (see Dimson (1979)), it has been established that thinly–traded shares have their OLS estimate of β substantially underestimated implying in turn that the expected returns of these shares will be underestimated by the CAPM.

Dimson (1979) established an estimator (see below) that corrects for this bias in the OLS $\widehat{\beta}$ and also has the advantage of producing unbiased estimates of β for well–traded shares. This allows one to apply the Dimson β estimation procedure to all shares which form the opportunity set without having to judge whether they are thinly traded or not.

Techniques for estimating betas of thinly traded shares have been proposed by Scholes and Williams (1977) and Dimson (1979) (later amended by Cohen *et al* (1983)). In this study the recent estimator proposed by Cohen *et al* (1983) will be used. Cohen's proposed estimator of beta has the advantage of converging on the OLS beta for well–traded shares under an assumption of market efficiency. Cohen *et al* (1983) proposed the following estimator of β for thinly–traded shares.

$$\beta_j^* = \frac{b_j + \sum_{n=1}^{N} b_{j \cdot t+n} + \sum_{n=1}^{K} b_{j \cdot t-n}}{1 + \sum_{n=1}^{N} b_{m \cdot t+n} + \sum_{n=1}^{K} b_{m \cdot t-n}} \tag{3.2}$$

[6] Fisher (1966) was one of the first to identify the phenomenon of thin–trading.

[7] An *underlying* price would be the same as a *recorded* price in an efficient market.

where N is the maximum number of lags and K is the maximum number of leads referred to by Dimson.

The components of the above estimator are

b_j is the OLS estimate of β in the regression

$$R_{j,t} = \alpha + \beta R_{m,t} + e_{jt} \quad , \tag{3.3}$$

$b_{j \cdot t+n}$ is the OLS estimate of β in the regression

$$R_{j,t+n} = \alpha + \beta R_{m,t} + e_{jt} \quad , \tag{3.4}$$

$b_{m \cdot t+n}$ is the OLS estimate of β in the regression

$$R_{m,t+n} = \alpha + \beta R_{m,t} + e_{jt} \quad . \tag{3.5}$$

The reader is referred to Cohen *et al* (1983) for suitable justification of the formulation shown in (3.2) above.

As discussed above, in a thinly–traded environment such as the JSE, it is appropriate to use a β estimation procedure that adjusts for thin–trading. A detailed discussion of the estimation procedure is conducted in section 4.2.

The variance-covariance input

In the case of the second input it is more reasonable to use a straightforward ex-post estimate of the variance-covariance matrix as a surrogate for the market's perception of the riskiness of assets and their degree of co-movement. This stems from the assertion that the riskiness/covariability is essentially a fairly stable characteristic and is highly autocorrelated through time. For example, if a firm is in a risky line of business, and exhibits a certain correlation with other securities then unless there is a major structural shift in the risk of that type of business the correlation with the other securities will remain fairly stable. [8] By contrast weak form efficiency ensures that returns themselves are not autocorrelated, and thus past estimates of returns are unhelpful in the estimation of future return.

In the preceding discussion on the estimation of beta, it was argued that in thinly–traded markets the effects of thin–trading would have an influence on the

[8] Assuming of course that the risk structure of the 'other' securities do not change.

OLS estimation of beta. Similarly, it is evident that thin–trading would have an effect on the estimation of the covariance between securities as well. The bias associated with the OLS estimation of beta for thinly–traded shares can be explained by the fact that the covariance between the returns on the security and the returns on the market index is underestimated. This bias occurs because movements of recorded prices of thinly–traded shares are not always synchronized with movements in the market index. Similarly the market related co–movements of two securities will not always be synchronized if either of the securities suffers from thin–trading. Consequently, the covariance between two such securities is similarly likely to be underestimated.

It was only very recently that Shanken (1987) recognised that the derivation of the beta estimator presented by Cohen *et al* (1983) contained an explicit expression for the estimation of the covariance between two securities in a thinly–traded market. This covariance estimator can be written as:

$$\operatorname{cov}(R_{j,t}; R_{k,t}) = \operatorname{cov}(R^0_{j,t}; R^0_{k,t}) + \sum_{n=1}^{N} \operatorname{cov}(R^0_{j,t}; R^0_{k,t-n}) + \sum_{n=1}^{N} \operatorname{cov}(R^0_{j,t-n}; R^0_{k,t}) \tag{3.6}$$

for any pair of securities j and $k, j \neq k$

where

$R_{j,t}$ is the *underlying* but unknown return on security j at time;

$R^0_{k,t}$ is the *recorded* return on security k at time t ; and

$R^0_{k,t-n}$ is the *recorded* return on security k at time $t-n$.

The components of the estimator in (3.6) can be thought of as the contemporaneous covariance between observed returns of security j and k , plus the sum of the serial cross–covariances of their observed returns for all leads and lags up to N periods.

3.3 EMPIRICAL STUDY

The data

In order to investigate the proposed technique on a small scale, a random selection of 15 shares was chosen from the mining sector (representing well-traded shares) and 15 shares were randomly selected from the industrial sector on the JSE (industrial shares on the JSE are, on average, thinly-traded relative to gold shares on the JSE as can be seen from table 3.1).

The data thus consisted of a weekly price series of 30 shares as well as the JSE Actuaries Overall Market Index quoted on the JSE from 4 January 1974 to 4 January 1985. A second series of prices taken at 4 week intervals was constructed from the above data as a surrogate for monthly data. Table 3.1 shows the sample of shares ranked according to their average weekly volumes. The second column indicates the number of weeks that no trades took place over the period of study.

Methodology

The only meaningful way to assess a portfolio selection technique is to estimate the set of portfolio weights using some set of historical data and then apply these estimates to some set of "unseen" (usually "future") data. One can do this to compare different portfolio selection techniques against each other.

The proposed portfolio selection approach was tested over 7 contiguous non-overlapping periods comprising 12 data points in each (i.e. 12 four-weekly intervals). Portfolios were selected for each testing period using the previous 48 data points to obtain the input estimates for the Sharpe (1970) Algorithm. Hence the estimation periods were overlapping although the testing periods were not.

The inputs to the Sharpe portfolio selection algorithm using the proposed technique was estimated as follows:

In each estimation period the 30 β_j^* were computed using equations (3.2) to (3.5). The value of N was chosen to be equal to 2 and the value of K was taken to be equal to 1 in equation (3.2). [9] In order to avoid any risk of "data-mining" no experimentation to determine the value of N and K was entered into. The set of expected returns for the 30 shares was then obtained by substituting the β_j^* into equation (3.1). The required value of R_f in (3.1) was taken to be the 12-month

[9] Dimson (1979 pp.218-219) finds in his study that lagged market coefficients are more important than the leading coefficients. His study also indicates that with monthly UK data one leading and several lagged terms are needed if risk measures are to take account of the effects of infrequent trading.

Table 3.1 Sumarised trading statistics for the sampled securities over the period of study, 4 January 1974 to 4 January 1985 (574 weeks)

Security	Ranked average weekly volume	Number of weeks that no trading occurred
Zandpan	299 602	1
Egoli	149 009	75
Elsburg	147 089	1
Leslie	132 144	1
Loraine	115 958	1
Village	106 999	1
Grootvlei	85 805	1
Vlakfontein	51 309	1
Sallies	47 877	1
Harties	46 070	5
Marievale	40 101	1
East Dagga	35 951	31
Harmony	32 501	1
South Roodepoort	29 713	18
Plate Glass	29 645	14
Curfin	20 798	3
Lonrho	14 106	13
Kloof	13 199	1
Eureka	11 522	374
Ovgroup	9 882	19
Sterns	8 972	16
AVI	8 636	11
Cullinan	6 772	72
OK	6 602	9
Micor	3 452	122
Lefic	3 225	129
Edgars	1 450	126
Boymans	1 143	361
Spitz	1 019	274
Foschini	755	242

fixed deposit rate of major South African commercial banks at the beginning of each testing period. The component $E(R_m) - R_f$ in (3.1) was taken to be the average excess return of the JSE Actuaries Overall Index above the 12 month fixed deposit rate over the previous 15 years. The covariance structures computed in the estimation periods were used as surrogates for the expected covariance structure. [10]

The 7 portfolios obtained using this modified approach were then monitored

[10] Usually the terms "risk" and "expected risk" are synonymous.

over the corresponding testing periods. For comparison purposes 7 additional portfolios were estimated over the same periods using the traditional approach (henceforth referred to as the standard approach). This involved using the average returns for each share over the corresponding estimation periods as surrogates for expected returns. The same covariance structure used for the modified approach was used here.

In addition the performance of the JSE–Actuaries Overall Market index was also monitored over the testing periods for comparison purposes.

The algorithm proposed by Sharpe (1970) was then used to generate the efficient frontiers in risk–return space over the testing periods and the performance of the above mentioned portfolios were then compared in this risk–return framework.

Results

The details of the resulting performance of these portfolios for each of the 7 testing periods is summarized in table 3.2. After viewing the results the following features emerged:

(1) The Sharpe reward to variability ratio (i.e. the slope of the line joining R_f to each portfolio) for the modified approach was larger than that of the standard approach in 6 out of the 7 testing periods.

(2) The Sharpe reward to variability ratio of the modified approach was also superior to that of the portfolio representing the market index in 6 out of 7 of the testing periods.

(3) In all of the testing periods the portfolios estimated using the modified approach consisted of more shares than the portfolio estimated using the standard approach (as well as the portfolios representing the ex–post optimal portfolio). This finding is more in line with theoretical considerations on diversification, that is, that diversification benefits increase as the number of shares included in the portfolio increases. Furthermore the larger number of shares found using the modified approach indicates a stronger tendency towards the composition of the market portfolio than traditional techniques. Recall that under the strict assumptions of the CAPM, the market portfolio is expected to be optimal.

In summary it is seen that the portfolio estimated using this modified approach

Table 3.2 Summarised comparative portfolio performance results over the period January 1978 to January 1985

Period		Modified approach	Standard approach	Market index	ex-post optimal
1	R_p	0.0312	-0.0067	0.0268	0.0283
	σ_p	0.0514	0.0544	0.0436	0.0084
	N_p	12	2		10
	SPI	0.4448[a]	-0.2763	0.4235[b]	2.3770
2	R_p	0.0180	0.0198	0.0163	0.0294
	σ_p	0.0355	0.0482	0.0520	0.0023
	N_p	12	6		10
	SPI	0.2723[a]	0.2379[b]	0.1532	9.1590
3	R_p	0.0474	0.0307	0.0468	0.0697
	σ_p	0.0395	0.0580	0.0557	0.0162
	N_p	11	8		7
	SPI	0.9890[a]	0.3856	0.6906[b]	3.1708
4	R_p	0.0155	0.0087	-0.0098	0.0334
	σ_p	0.0474	0.0612	0.0539	0.0201
	N_p	14	12		5
	SPI	0.1512[a]	0.0060[b]	-0.3364	1,2470
5	R_p	-0.0019	-0.0022	-0.0202	0.0292
	σ_p	0.0712	0.0694	0.0700	0.0474
	N_p	12	11		2
	SPI	-0.1437[a]	-0.1518[b]	-0.4076	0.4402
6	R_p	0.0178	-0.0032	0.0584	0.0625
	σ_p	0.0705	0.0517	0.0933	0.0405
	N_p	9	7		5
	SPI	0.1342[b]	-0.2230	0.5366[a]	1.3374
7	R_p	0.0128	0.0276	0.0069	0.0259
	σ_p	0.0374	0.0535	0.0632	0.0176
	N_p	10	6		8
	SPI	0.1194[b]	0.3601[a]	-0.0227	0.9981

[a] The portfolio that had the best performance in the relevant period.

[b] The portfolio that had the second-best performance in the relevant period.

R_p is the average monthly portfolio return over the period.

σ_p is the monthly portfolio standard deviation over the period.

N_p is the number of shares in the portfolio.

$SPI = \frac{R_p - R_f}{\sigma_p}$ the Sharpe Performance index.

outperformed (on the basis of the Sharpe reward to variability ratio) the market index and the *ex-post* optimal portfolio estimated using the traditional method in 6 out of 7 testing periods. The approach outlined here is thus proposed as a serious alternative to the traditional method.

3.4 CONCLUSION

In this chapter a technique for estimating portfolio weights in a thinly-traded environment has been derived and demonstrated. The method, by contrast to traditional techniques, does not rely on historical returns for its expected return input but uses expected returns corrected for thin-trading. Furthermore a proposal for the use of a thinly-traded correction of the covariance input was suggested. This plausible and flexible method can be used to estimate portfolio weights for a range of prior expectations on the overall market performance. Thus, for example, this technique can be used for the estimation of portfolio weights under various proposed bull and bear market scenarios. The method has been tested over the period from 1974 to 1985 and was shown to yield superior performance portfolios to those yielded by traditional estimation techniques.

CHAPTER 4

RISK, BETA AND THE MARKET MODEL: SOME EXTENSIONS AND APPLICATIONS

4.1 ESTIMATION PROBLEMS IN SMALL MARKETS

The use of the market model and the importance of the beta coefficient as a measure of risk are well established in the literature. The market model, and the implications of using beta as a measure of risk, are discussed in detail in section 2.3. In this section some of the statistical considerations of the market model are discussed with particular reference to smaller markets like the JSE.

The consistent estimation of the parameters α and β of the market model, using ordinary least squares as a criterion, depends on the following assumptions concerning the model:

(i) $E(e_t) = 0$;

(ii) $E(e_t e_s) = 0$ for all $t \neq s$;

(iii) $E(e_t^2) = \sigma^2$ for all t ; and

(iv) e_t is independent of R_{m_t} for all t ..

Fama, Fisher, Jensen and Roll (1969) show that the first two assumptions are upheld for the NYSE, while on the JSE, Affleck–Graves and Money (1975) find similar results. Affleck–Graves (1977) points out that if the third assumption is violated, known as heteroscedasticity, then ordinary least squares is no longer a suitable criterion. However both Fama *et al* (1969) and Martin and Klemkosky (1975) give evidence supporting the third assumption of homoscedasticity on the NYSE.

By contrast, the evidence on smaller exchanges, at first glance, indicates that the nature of variances differs from that of the NYSE. On the Sydney Stock Exchange (SSE) for example, Praetz (1969) found that 95% of the securities sampled on the SSE exhibited significant heteroscedasticity at the one percent level. Similar evidence of heteroscedasticity was found by Belkaouri (1977) on the Canadian Stock Market. On the JSE Affleck–Graves (1977) observed that apparent heteroscedasticity occurred in approximately 30% of the securities sampled, but argues that this effect was largely a consequence of poor model fit, rather than the existence of real heteroscedasticity.

Some of the observations presented by Affleck–Graves (1977) are worth considering in more detail here, as further insights into the estimation problems of the model for smaller markets can be derived. Affleck–Graves (1977) presents empirical results which reveal that 85% of sampled securities on the JSE had beta coefficients less than one (using a market capitalization type index). This result is similar to that obtained by Altman, Jacquillat and Levasseur (1974) for the Paris Bourse over the 1964–71 period. They found that approximately 70% of beta coefficients were less than one. Affleck–Graves (1977) finds further that the market index could explain more than 10% of the variation of the security returns for only 34% of the sampled securities. By contrast, King (1966) found that the market factor on the NYSE accounted for approximately 31% of the movement of securities on average. It is also interesting to note that the average correlation between security returns and the market index $(\bar{R})$, was found to be only 0.259 on the JSE (Affleck–Graves (1977)), which is much lower than the corresponding average of 0.41 found by Altman *et al* (1974) for the Paris Bourse, and the corresponding average of 0.528 found by Blume (1971) for the NYSE. Affleck–Graves (1977) also documents the fact that as $\bar{R}$ increases, so does the beta coefficient and vice versa. This phenomenon is also evident in the statistics presented in table 5.17 of Chapter 5 in this thesis and can be partly ascribed to the problem of thin–trading, which is dealt with in detail in section 4.2.

Affleck–Graves (1977) suggests that a low $\widehat{\beta}_i$ might arise from the fact that the fit of the model is poor (i.e. a low R^2 value). He argues that when the fit is poor, OLS procedure (which minimizes the sum of the squares of the *vertical* distances) would tend to fit a more *horizontal* line to the data regardless of the true beta of the security, that is it would set $\widehat{\beta}$ close to zero. He thus concludes that, as the fit gets poorer, the beta coefficient will get smaller and will almost certainly be less than one if the fit is poor. Carter (1983) considers several beta estimation techniques for JSE stocks, but concludes that the choice of the estimation technique should depend on the purpose for which the beta coefficient is used.

It is felt however, that an assertion made by Dimson (1979) is the most likely cause for the estimation problems evident in small markets. In particular, Dimson asserts that infrequent trading is a likely explanation for this phenomenon in non–United States markets. In thinly traded markets, the major problem is that recorded prices do not essentially represent true prices. For example, the market may move on new relevant information, but if a particular security is not traded over this period, its price is usually *observed* (recorded) at the level of the last trade. Consequently situations arise where the *true* but unrecorded price theoretically reacts to the new information, but the recorded price does not. This results in estimation problems, particularly in markets where infrequent trading is severe. Dimson (1979) in fact shows that infrequent trading results in beta coefficients being underestimated in thinly traded markets. This phenomenon may explain the high percentage of betas observed to be less than one on smaller markets (Brealey and Myers (1981) show that the betas of all assets weighted by their proportion in the index sum to 1).

* β
Find a price adjustment technique
observe P_t
derive P_t^*
↓
derive inde[x]
↓
PC, GP on this to identify meaningful
↓
relationsh[ip]

In section 4.2 the thinly traded problem on the JSE is investigated in detail with a view to improving the estimation of beta coefficients. In section 4.3 a "multi–market" model is proposed. Here the influence of the US market is considered jointly with the market index on the JSE, to determine whether the relatively large unique risk components of JSE stocks may be related to movements in the US market. Lastly in section 4.4 an empirical study showing the typical application of empirical models using beta coefficients is presented.

4.2 BETA ESTIMATION IN THINLY TRADED ENVIRONMENTS

4.2.1 Introduction

The fundamental concept of systematic risk, or beta, of a security is central to Capital Market Theory and consequently much empirical work has focused on the associated estimation problems. One of the more fruitful areas of empirical research arose from seeking the source of the estimation problems. While Fisher (1966) was one of the first to identify the phenomenon of thin trading, it was only relatively recently that Ball (1977) researched the effects of thin trading on the estimation of systematic risk. Scholes and Williams (1977) and Dimson (1979) were among the first to offer serious solutions to the estimation problems by developing a plausible analytical framework for the thinly traded phenomenon.

Several other approaches to the problem have, however, been suggested. For example, Ibbotson (1975), Dimson (1974) and Schwert (1977) introduced lagged market returns as additional independent variables in their market model regressions. Marsh (1979) and Franks, Broyles and Hecht (1977) on the other hand, used returns calculated only over periods when trades occurred, and regressed these returns on the market index over precisely the same periods. Neither of the above approaches takes into account the fact that in thinly traded markets the market index itself suffers from the effect of having component securities that are infrequently traded.

Scholes and Williams (1977) combined these ideas by using both non–synchronous and synchronous market returns as explanatory variables for trade–to–trade returns. Dimson (1979), however, points out that although this method merely requires a record of whether or not a share was traded within a time period, a return is calculated and used only if a transaction is known to have occurred in consecutive time periods. The market index used in the Scholes and Williams (1977) derivation is thus defined to be the mean of all such returns. Dimson (1979) further argues that because the multi–period returns are discarded in the Scholes and Williams (1977) approach, his proposed estimator turns out to be more efficient than the Scholes–Williams method. The approach proposed by Dimson (1979) is similar in spirit but largely overcomes these drawbacks. His approach does not require all components of the market index to be continuously traded, nor does it require information on the transaction dates. Cohen *et al* (1983) later identified some inconsistencies in Dimson's derivation, and therefore proposed a modified version of Dimson's estimator. The estimator proposed by Cohen *et al* (1983) will therefore be used in this investigation, and a detailed discussion of the estimation procedure is provided in section 4.2.3.

Although the focus of the Cohen *et al* (1983) paper is on the derivation of the beta estimator, some of the intermediate steps of their analysis deal with covariance estimation. Shanken (1987) alludes to this work, and uses this covariance estimator to investigate the effects of infrequent trading on estimation of the covariance structure. Shanken (1987) presents evidence which suggests that estimation problems of the covariance structure using standard techniques are substantial in thinly traded markets, and that the problem is most severe in the estimation of the covariance between well–traded and infrequently traded securities. Consequently in thinly traded markets where the proposed beta estimator is needed, it is suggested that a similar technique be used for the estimation of the covariance matrix if it is required. Examples of possible applications of this covariance estimator include, *inter alia*, superior estimation of the covariance input for diversification investigations, for example in the Markowitz portfolio selection algorithm, [1] as well as for the multivariate tests [2] of the CAPM.

In section 4.2.2 the extent of thin trading on the JSE is investigated empirically, whereafter a theoretical discussion on the proposed estimators used for thinly–traded stocks is presented in section 4.2.3. In section 4.2.4 an empirical investigation is conducted with the aim of determining the optimal number of leads and lags required for the general applicability of the estimators over all JSE stocks. Finally some conclusions are offered in section 4.2.5

4.2.2 The extent of thin trading on the JSE

In order to investigate the extent of thin trading on the JSE, the entire population of shares (amounting to 671) currently recorded on the JSE data tape was considered for selection over the period 1 January 1978 to 31 August 1987. With the view of simplifying computations only shares listed over this entire period were selected. This amounted to 360 shares in total. These 360 shares were subsequently ranked according to various criteria relating to their trading frequency. A share having a weekly volume of zero was taken to indicate that the share was not traded during a particular week. Several trading frequency criteria were then used to rank the shares, namely

(i) the total number of times the share was not traded for one week;

(ii) the total number of times the share was not traded for consecutive non–

[1] Although the portfolio selection proposal of section 3.2 only used a modified beta estimator, the use of the covariance estimator may lend greater theoretical appeal to the suggested portfolio selection technique.

[2] Similarly, the multivariate investigations of section 5.3 as well as chapter 6 use standard estimation techniques. An interesting line of future research would be the use of thinly traded estimators for these multivariate investigations.

overlapping two week periods;

(iii) the total number of times the share was not traded for consecutive non–overlapping three week periods;

(iv) the total number of times the share was not traded for consecutive non–overlapping four week periods over the entire period.

These securities were then partitioned into 10 portfolios (with 36 shares in each) on the basis of their ranked trading frequencies, for each of the above criteria. Table 4.1 gives a summary of the resulting trading statistics for the four partitions over the 500 week period 1 January 1978 to 31 August 1987.

Table 4.1 Average frequency of non–trades expressed as a percentage (period January 1978 to August 1987)

	CRITERIA			
Portfolios ranked by trading frequency	1 week untraded	consecutive* 2 week periods untraded	consecutive* 3 week periods untraded	consecutive* 4 week periods untraded
1	1.2	0.0	0.0	0.0
2	1.4	0.0	0.0	0.0
3	1.0	0.1	0.0	0.0
4	2.2	0.6	0.0	0.0
5	4.3	1.4	0.7	0.1
6	8.4	3.2	1.5	0.8
7	15.4	7.6	3.6	2.0
8	24.9	14.6	9.4	6.2
9	39.5	27.6	20.1	15.4
10	68.2	57.3	50.1	44.9

* the consecutive periods were non–overlapping

Inspection of table 4.1 shows that the extent of thin–trading on the JSE is indeed significant. For example, for the criterion "1 week untraded" deciles 8, 9 and 10 have no trades occurring on average, for at least 25% of the 500 weeks, i.e. about one week in every four, on average. Since deciles 8, 9 and 10 comprise one third of the sample, this can be interpreted as meaning, that approximately one third of JSE stocks are not traded on average, for at least one consecutive week in every four week period. Table 4.1 also reveals that the extent of thin–trading in deciles 9 and 10, is fairly extreme. For example, on average deciles 9 and 10 were found to have a frequencies of 19.3 and 56.1 respectively for non–trades occurring for 4 consecutive, nonoverlapping weeks, out of a total of 125 possible nonoverlapping 4 week periods. This implies that at least 20 percent of JSE stocks can be categorized as being very infrequently traded. Consequently estimation procedures using *recorded* security prices are likely to be significantly affected by the thinly–traded phenomenon on the JSE. Since the first criterion representing "1 week untraded" is seen to give the largest spread of trading characteristics across the deciles in table 4.1, this criterion will be used in the ensuing empirical investigation conducted in section 4.2.4.

4.2.3 Theoretical development

The main cause of the bias associated with estimation problems in thinly–traded environments is the fact that *recorded* prices are used to represent true *underlying* prices. For example, when a security has not been traded in the period in question then the *recorded* price of the security remains unchanged, and represents the outcome of a transaction in some previous period. The *underlying* (theoretical) price of the security, by contrast, would reflect the arrival of any new information in the period in question. Hence two series of prices are created, a series of *recorded* prices and an unknown, and more volatile, series of *underlying* prices. Clearly estimation problems arise when the series of *recorded* rather than *underlying* prices are used in the estimation procedures. In particular it is evident that the covariance between *recorded* security returns and the market's return, is likely to be less than the covariance between *underlying* security returns and the market's return for thinly–traded securities. This is clearly due to the fact that *underlying* prices reflect movements in the market instantaneously while recorded prices may remain unchanged. Since the OLS beta estimate embodies this covariance component in the numerator, it is evident that OLS estimates of beta for thinly–traded will be underestimated when recorded prices are used in the estimation process. Even beta estimates for well–traded shares in a thinly–traded environment may be subjected to estimation bias. This occurs because in thinly–traded markets, the market index itself may be comprised of a significant proportion of thinly–traded securities. This implies once more that an *observed* series and a *underlying* series of market index returns exist as well, and that this may cause further estimation problems, even for well–traded securities.

The beta estimator proposed by Dimson (1979) and corrected by Cohen *et al* (1983) is designed to overcome the above problems by incorporating both lags and leads of the relevant return series in the analytical framework. Furthermore Cohen *et al* (1983) argue that the estimator has general applicability as it converges to the usual OLS beta estimate in well–traded markets. This estimator can be written as:

$$\widehat{\beta}_j = \frac{b_j^0 + \sum_{n=1}^{N} b_{j+n}^0 + \sum_{n=1}^{N} b_{j-n}^0}{1 + \sum_{n=1}^{N} b_{M+n}^0 + \sum_{n=1}^{N} b_{M-n}^0} \quad . \tag{4.1}$$

where the " 0 " superscripts denotes the coefficient has been estimated from a series of observed, or recorded prices; and N denotes the number of leads, or lags used.

Furthermore estimates of the right–hand–side components of the above expression can be obtained as follows:

$$b_j^0 = \frac{\text{cov}(R_{j,t}^0;\ R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{M+n}^0 = \frac{\text{cov}(R_{M,t+n}^0;\ R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{M-n} = \frac{\text{cov}(R_{M,t-n}^0;\ R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{j+n}^0 = \frac{\text{cov}(R_{j,t+n}^0;\ R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{j-n}^0 = \frac{\text{cov}(R_{j,t-n}^0;\ R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

where $R_{M,t}^0$ is the *observed* return on the market index at time t; and

$R_{j,t}^0$ is the *observed* return on security j at time t .

In order to justify the use of the proposed estimator, it is worth considering the structure of (4.1) in more detail. Firstly it is evident that in perfectly efficient, and hence well–traded markets, market efficiency ensures that all non–synchronous covariances will in theory be equal to zero. This implies that in (4.1) $\widehat{\beta}_j = b_j^0$, i.e. the proposed estimator converges to the usual OLS beta in efficient markets. Other than the term, b_j^0, the components in the numerator of (4.1) capture the relationship between leads and lags of the security and the contemporaneous market index. The components of the denominator on the other hand, reflect adjustments for autocorrelations induced into the market index by the component thinly–traded securities.

Shanken (1987) pointed out that one of the steps used by Cohen *et al* (1983) yielded a useful estimator for the covariance between 2 securities in thinly–traded markets as well. The covariance estimator in Cohen *et al* (1983) can be written as:

$$\begin{aligned}\operatorname{cov}(R_{j,t};\ R_{k,t}) = \operatorname{cov}(R^0_{j,t};\ R^0_{k,t}) &+ \sum_{n=1}^{N} \operatorname{cov}(R^0_{j,t};\ R^0_{k,t-n}) \\ &+ \sum_{n=1}^{N} \operatorname{cov}(R^0_{j,t-n};\ R^0_{k,t}) \qquad (4.2)\end{aligned}$$

where all the symbols have been previously defined.

Clearly the non–synchronous covariance terms are likely to be largest for covariance estimation between thinly–traded and well–traded securities. The correlation between the synchronous returns of a *thinly–traded* share, in particular, and the lagged returns of a *well–traded* share is likely to be larger than the correlation between the synchronous and lagged returns of two *well–traded* shares. Consequently the covariance bias is greatest for this case, and equation (4.2) adjusts for that bias.

In section 4.2.4 an empirical investigation is conducted to determine the optimum number of leads and lags that need to be included for general application of (4.1) across all JSE stocks.

4.2.4 Suitable beta estimation on the JSE — an empirical investigation

The data

The weekly data base of section 4.2.2 is used here as well. This database consisted of the 360 securities having a complete price history over the period 1 January 1978 to 31 July 1987. Furthermore, two market indices were used in this investigation, the JSE–Actuaries Overall Index (a market capitalization type index), and an equally weighted index constructed to be the equally weighted average price of the 360 securities in each week. Finally, series of returns were constructed for each security and both market indices for use in the analysis.

Methodology

The securities were ranked according to the number of weeks that no trades [3] occurred for each security over the sampled period. The securities were then partitioned into 10 deciles on this basis, each consisting of 36 securities.

The methodology proceeds along the same lines as that of Dimson (1979), with the exception that the final beta estimator used here was the one proposed by Cohen *et al* (1983). In essence, the only major difference between the two estimators is the fact that the non–synchronous coefficients of the Dimson estimator are estimated from a single multivariate regression model, while Cohen *et al* use separate bivariate regression models to estimate each non–synchronous coefficient.

To obtain the component beta coefficients of equation (4.1), the weekly returns for each of the 360 securities were regressed against lagged, leading and matching market index returns. This procedure was repeated using both the JSE–Actuaries Overall Index and the Equally Weighted Index. To investigate which, if any, of the lags and leads are significant for JSE stocks, five lags and five leads were considered in the investigation.

Results using the JSE–Actuaries Overall Index

Table 4.2 shows the resulting component beta coefficients, averaged over each decile at the various lags and leads using the JSE–Actuaries Overall Index as the market index.

The most notable feature of table 4.2 is the fact that the component beta coefficients are largest for the synchronous data, i.e. having a lag of zero, as expected, with the exception of the 8th, 9th and 10th deciles representing the most thinly–traded securities having unexpectedly small synchronous coefficients, with decile 10 having an average coefficient of only 0.05. Furthermore the average beta

[3] This criterion was chosen as it gave the largest spread of trading frequencies across the deciles (see section 4.2.2)

Table 4.2 Average component beta coefficients for the JSE–Actuaries Overall Index

	LAG OR LEAD											Cohen
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5	estimator
1	.11	.07	.05	.18	.17	1.30	.26	.19	.05	.09	.16	2.62
2	.12	.08	.04	.11	.23	.92	.17	.10	.00	.06	.07	1.91
3	.07	.05	.04	.13	.30	.74	.15	.06	.07	.04	.03	1.70
4	.12	.09	.07	.16	.33	.60	.09	.06	.01	.02	.01	1.56
5	.11	.05	.06	.18	.31	.45	.04	-.01	.01	.01	.05	1.26
6	.13	.09	.09	.19	.35	.44	.07	.00	.00	.03	-.01	1.38
7	.11	.11	.12	.18	.27	.29	.05	-.00	.03	.00	-.02	1.14
8	.11	.08	.16	.11	.24	.19	.13	.02	.08	.00	-.01	1.12
9	.13	.12	.09	.15	.15	.11	-.01	-.01	-.02	.00	-.02	.68
10	.05	.04	.06	.06	.06	.05	-.01	-.01	.03	.02	-.01	.34

*Decile 1 consists of the most frequently traded securities whilst decile 10 consists of the most infrequently trades securities.

coefficients for the synchronous data, decrease monotonically from 1.30 for decile 1 to 0.05 for decile 10. Note that these coefficients computed at the lag zero represent the usual, unadjusted beta coefficients. Since there is no plausible reason why the average beta coefficients should decrease systematically to this extent, it is clear that a severe bias due to thin–trading is evident for JSE stocks. The last column of table 4.2 shows the corrected beta estimator proposed by Cohen *et al.* It is clear that the proposed estimator does improve beta estimation for the thinly–traded deciles, however there appears to be an overestimation bias for the well–traded securities. In particular, decile 1 has an average beta estimate of 2.62, and this value appears to be too large while the beta of decile 10 is too small to be economically plausible. Nevertheless, in order to determine which leads and lags should ideally be used with the JSE–Actuaries Overall Index for beta estimation, the associated

t–statistics for the component beta coefficients were averaged for each decile at the various leads and lags. Table 4.3 shows the resulting average t–statistics for the component beta coefficient obtained using the JSE–Actuaries Overall Index.

Table 4.3 Average t–statistics of the component beta coefficients for the JSE–Actuaries Overall Index

	LAG OR LEAD										
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	1.05	.76	.49	1.81	1.86	$\boxed{18.84}$	$\boxed{2.70}$	1.92	.47	.93	1.60
2	1.24	.80	.39	1.21	$\boxed{2.58}$	$\boxed{12.64}$	1.93	1.15	.03	.59	.67
3	1.02	.63	.68	1.52	$\boxed{3.59}$	$\boxed{9.18}$	1.73	.75	.43	.45	.26
4	1.21	.87	.77	1.75	$\boxed{3.86}$	$\boxed{6.72}$	1.09	.52	.04	.26	.09
5	1.48	.72	.65	1.79	$\boxed{3.39}$	$\boxed{4.46}$	.67	-.18	.17	.20	.33
6	1.28	.76	.91	$\boxed{2.01}$	$\boxed{3.31}$	$\boxed{3.66}$	.36	-.02	.25	.27	-.15
7	1.18	1.17	1.23	$\boxed{2.02}$	$\boxed{3.05}$	$\boxed{2.88}$	.42	.05	.288	.02	-.22
8	1.25	1.14	1.33	1.81	$\boxed{2.80}$	$\boxed{2.14}$	.48	.15	.19	.072	-.07
9	1.56	1.48	1.08	1.85	1.83	1.30	.26	-.07	-.35	-.01	-.22
10	.58	.42	.63	.67	.64	.51	-.06	-.15	.44	.18	-.10

*The deciles having significant average t–statistics (at the 5% level) for the component beta coefficients are boxed in.

Inspection of the resulting average t–statistics in table 4.3 reveal results similar in spirit to those found in Dimson (1979). It is evident that all synchronous coefficients (i.e. at lag zero) are significant with the exception of deciles 9 and 10, representing the extreme thinly–traded [4] cases. Furthermore the coefficients at 1 lag are found to be generally significant with the exception of decile 1, and decile 9 and 10. A further notable feature is the fact that several of the leading coefficients for the thinly–traded shares (deciles 6 to 10) are negative.

[4] As indicated in section 4.2.2 the thin–trading in deciles 9 and 10 is so extreme that even 5 lags may be insufficient to capture these effects.

These results are consistent with the intuitive arguments given by Dimson (1979). The major intuition behind Dimson's arguments are that for frequently traded securities the leading coefficients are more important as they "lead" a market index suffering from thin–trading effects. Whilst for infrequently traded securities the lagged coefficients are more important as they generally lag behind the market index.

From the above analysis it is claimed that the inclusion of one lagged and one matching coefficient in equation (4.1) would appear to be sufficient to improve the estimation of beta on the JSE [5] using the JSE–Actuaries Overall Index. Although it was found that the final beta estimator was still not ideal, it is felt that this may be due to the fact that the JSE–Actuaries Overall Index (a market capitalization type index), rather than the estimator (4.1), is not suitable for use in beta estimation procedures. The analysis was thus repeated using the constructed Equally Weighted Index in the proposed estimator.

Results using the Equally Weighted Index

The major distinction between the two indices used here is that the JSE–Actuaries Overall Index is comprised of almost exclusively well–traded securities having relatively large market capitalization proportions. The Equally Weighted Index on the other hand includes all thinly–traded securities as well, all being given the same weight. Consequently the Equally Weighted Index itself is likely to suffer from the effects of thin–trading to a far greater extent than the JSE–Actuaries Overall Index. Furthermore, the usual beta coefficients of the component securities of an equally weighted index will necessarily average out to unity, when the equally weighted index is used as the independent variable. In thinly–traded environments this averaging suggests that betas of well–traded securities are usually overestimated, while those of infrequently traded securities are underestimated.

Table 4.4 shows the resulting component beta coefficients averaged over each decile at the various lags and leads using the Equally Weighted Index as the market index. From table 4.4 it can be seen that the synchronous beta coefficients (at lag zero) again decrease monotonically, from 1.74 for decile 1 to 0.21 for lag 10, again

[5] Dimson (1979) conducted a similar analysis for UK stocks using daily data Dimson found that at least 4 daily lags and 1 lead should be included with a matching coefficient.

indicating that the beta coefficients are overestimated for well–traded securities and underestimated for thinly traded securities, as expected. The final beta estimators shown in the last column of table 4.4 range from 1.52 to 0.43 compared to the equivalent column of table 4.2 which range from as high as 2.62 to 0.34.

Table 4.4 Average component beta coefficients for the Equally Weighted Index

	LAG OR LEAD											
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5	Cohen estimator
1	0.07	0.07	0.09	0.16	0.25	1.74	0.83	0.49	0.21	0.24	0.41	1.52
2	0.13	0.09	0.09	0.09	0.33	1.44	0.70	0.36	0.16	0.17	0.25	1.27
3	0.11	0.06	0.04	0.14	0.45	1.34	0.63	0.33	0.24	0.20	0.20	1.25
4	0.14	0.11	0.10	0.24	0.58	1.19	0.54	0.27	0.16	0.14	0.15	1.21
5	0.18	0.11	0.08	0.29	0.60	1.11	0.48	0.19	0.13	0.11	0.15	1.14
6	0.20	0.14	0.14	0.29	0.67	1.07	0.45	0.21	0.10	0.13	0.13	1.18
7	0.19	0.19	0.25	0.35	0.58	0.77	0.36	0.18	0.14	0.11	0.07	1.06
8	0.19	0.17	0.29	0.31	0.59	0.75	0.37	0.24	0.25	0.09	0.14	1.10
9	0.27	0.27	0.27	0.34	0.41	0.39	0.15	0.10	0.05	0.07	0.05	0.80
10	0.13	0.12	0.19	0.17	0.20	0.21	0.06	0.01	0.08	0.07	0.04	0.43

The final average beta estimators using the Equally Weighted Index thus appear to be more economically plausible than those of table 4.2 as for each of the deciles the betas are closer to 1. Furthermore, for the Equally Weighted Index the final beta estimator is seen to make corrections which are consistent with the theoretical intuition. For example, for decile 1, representing the well–traded securities the coefficient at lag zero, i.e. 1.74 was identified as an overestimate of beta, the final estimate, i.e. 1.52 is seen to be corrected downward, i.e. in the right direction. By contrast the correction for decile 1 in table 4.2 is seen to be counter–intuitive. The corrections for the other deciles in table 4.4 also appear to be consistent with intuition, resulting in final estimations fairly close to one. The final estimators still however appear to decrease down the deciles in table 4.4, although this decrease is not as extreme as the case shown in table 4.2.

Further support for using the equally weighted index with the Cohen estimator can be seen by considering the associated average t–statistics for the component beta coefficients obtained using the Equally Weighted Index. These results are shown in table 4.5 and are similarly seen to be consistent with the theoretical preamble. Here at least one lagged and one leading coefficient, together with the matching coefficient, appear to be generally applicable for JSE stocks. For deciles 1, 2 and 3, representing well–traded securities, two leading leading coefficients are significant, while for deciles 6, 7, 8 and 9, representing the thinly–traded securities, two lagged coefficients are significant (several more are also significant for decile 9). The results for decile 10 in table 4.5 show, by contrast, that none of the component beta coefficients are significant. As mentioned before the incidence of thin–trading is likely to be so extreme for decile 10 that even 5 lags are probably insufficient to capture the desired effects.

These results were intuitively expected, as the well–traded securities are expected to "lead" an equally weighted market index. This is a consequence of the fact that an equally weighted index has a positive autocorrelation induced by its component thinly–traded securities. The thinly–traded securities on the other hand, are themselves expected to "lag" the market index.

In order to determine the effect of the thinly–traded phenomenon on the two indices used, the denominator of equation (4.1) will be considered here. The denominator attempts to capture the extent to which the components of the indices induce a thinly–traded component into the index itself, and is used to correct for this in model (4.1). For the 5 leads and 5 lags, the JSE–Actuaries Overall Index yielded a value of 1.966 for the denominator of equation (4.1). By contrast, the equally weighted index yielded a value of as high as 3.280 for the denominator of (4.1). Clearly the Equally Weighted Index is seen to reflect the significant degree of thin–trading, induced by its component securities on the JSE, to a far greater extent than the JSE–Actuaries Overall Index.

Although the Equally Weighted Index itself does not escape the problem of thin–trading, it does appear to yield more intuitively appealing estimates of beta (when used in conjunction with Cohen's estimator) than does the JSE–Actuaries

Table 4.5 Average t–statistics of the component beta coefficients for the Equally Weighted Index

	LAG OR LEAD										
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	0.42	0.46	0.51	0.99	1.65	$\boxed{13.51}$	$\boxed{5.64}$	$\boxed{3.18}$	1.39	1.52	$\boxed{2.61}$
2	0.90	0.56	0.45	0.55	$\boxed{2.26}$	$\boxed{11.19}$	$\boxed{4.89}$	$\boxed{2.49}$	1.12	1.09	1.60
3	0.82	0.39	0.48	1.18	$\boxed{3.37}$	$\boxed{9.66}$	$\boxed{4.33}$	$\boxed{2.20}$	1.37	0.27	1.23
4	0.96	0.71	0.66	1.61	$\boxed{4.14}$	$\boxed{8.49}$	$\boxed{3.77}$	1.79	1.07	0.96	0.92
5	1.15	0.83	0.62	1.81	$\boxed{3.95}$	$\boxed{6.75}$	$\boxed{3.05}$	1.15	0.94	0.77	0.93
6	1.25	0.69	0.86	$\boxed{1.97}$	$\boxed{4.06}$	$\boxed{5.91}$	$\boxed{2.33}$	1.12	0.81	0.83	0.61
7	1.31	1.29	1.59	$\boxed{2.41}$	$\boxed{3.98}$	$\boxed{4.86}$	$\boxed{2.18}$	1.17	0.92	0.80	0.47
8	1.54	1.43	1.83	$\boxed{2.42}$	$\boxed{3.83}$	$\boxed{4.17}$	$\boxed{1.99}$	1.09	0.89	0.56	0.56
9	$\boxed{2.12}$	$\boxed{2.07}$	$\boxed{2.08}$	$\boxed{2.73}$	$\boxed{3.23}$	$\boxed{3.07}$	1.08	0.83	0.30	0.51	0.28
10	1.00	0.88	1.20	1.09	1.33	1.32	0.32	0.03	0.47	0.47	0.19

*The deciles having significant average t–statistics (at the 5% level) for the component beta coefficients are boxed in.

Overall Index.

4.2.5 Conclusion

One of the initial conclusions in this analysis is that the thinly–traded phenomenon on the JSE is fairly severe. For example, it was found that approximately one third of JSE stocks are not traded on average, for at least one week out of every four week period.

Furthermore, it can be concluded that the effect of thin–trading on the estimation of beta coefficients is substantial on the JSE. The estimation bias was also found to be more severe when a market capitalization index like the JSE–Actuaries Overall Index was used as the independent variable in the market model.

The estimator proposed by Cohen *et al* (1983) was found to yield substantial improvements in the estimation of beta coefficients for thinly traded shares. However it was found that more satisfactory improvements can be achieved when an equally weighted index is used in conjunction with the beta estimator proposed by Cohen *et al.* It was found that if an approach is sought which was generally applicable across all JSE stocks and the JSE–Actuaries Overall Index is used as the independent variable than at least one lagged and the matching coefficient should be included in the estimator. On the other hand if an equally weighted index is used, as is recommended, at least one lagged, one leading and the matching coefficient should be included in the estimator. If however additional information on the extent of thin–trading of the individual security is available to the reader, then tables 4.3 and 4.5 provide a guide to the optimal number of leads and lags required in the estimator.

The results of this section show that there is possible scope for further improvement in the beta estimation procedure. It is felt that a fruitful direction of further research would be to simulate thinly–traded data characteristic of the JSE with known beta's, and investigate estimation procedures which most accurately estimate these betas.

4.3 AN EXTENDED MARKET MODEL FOR SMALL MARKETS

4.3.1 Introduction

The well known market model, in essence, makes a simple statement concerning the relationship between the returns on a given security and the returns on some market index. The coefficients of the model are of considerable importance to financial analysts and researchers alike. Estimation of the coefficients of the market model requires, inter alia, a series of returns of the given security, and a series of returns on some market index. Usually the market index is constructed from some aggregate of value weighted securities of the local stock market in question. It is well known that indices constructed in this manner embody movements caused by factors which influence the market as a whole, for example local interest rates, inflation, the business cycle etc. It is also likely however, that movements of overseas markets may have an impact on local markets. Consider for example the events of October 1987 when the prices of shares listed on the NYSE fell dramatically. Prices on all major exchanges, including Frankfurt, London, Paris, Tokyo, Hong Kong, Sydney and the Johannesburg Stock Exchange fell in unison. This crash illustrated that events occurring on the NYSE can affect stock markets world–wide.

It is evident that the NYSE is probably considered as the most internationally influential stock exchange in the world. In order to investigate this assertion, the correlation between the JSE, the NYSE, the London Stock Exchange (LSE) and the Tokyo Stock Exchange (TSE) was estimated using monthly return data over the period September 1978 to November 1987. [6] The indices used to represent these markets were the JSE–Actuaries Overall Index, the Dow–Jones Index, the Financial Times Index and the Nikkei–Dow (Toyko) Stock Exchange Index respectively. These indices were all converted to dollar denominations via the corresponding exchange rates, for comparison purposes. Table 4.6 shows the correlation coefficients estimates on this basis.

Table 4.6 Correlation [†] coefficients for international markets (1978–1987)

	JSE	LSE	NYSE	TSE
JSE	1.000			
LSE	0.181	1.000		
NYSE	0.268	0.639	1.000	
TSE	0.205	0.370	0.423	1.000

† all of the above correlation coefficients are significant at the 5% level of significance.

Table 4.6 reveals that for all cases the correlations are positive and significant at the 5 percent level of significance, implying that significant relationships between stock markets do exist.

The results also show that the NYSE is the market which is most highly correlated with each of the other markets, thus supporting the assertion that the NYSE is probably the most influential of the stock markets.

[6] The JSE–Actuaries Overall Index was only launched from September 1978, hence the period was selected to commence on this date. Furthermore, November 1987 represented the latest date for which data was available.

Lessard (1974) conducted a study on the world–wide influences on stock returns. His study included 16 major stock exchanges and a constructed "world index". The study examined the international diversification benefits from the viewpoint of the investor with dollars to invest. Lessard (1974) showed that, on average, 22 percent of the variation in the market indices could be explained by the "world index".

One of the characteristics of the JSE is that many of the companies listed on the JSE derive a large proportion of their income from overseas, in particular from the USA. It is thus plausible that some of the securities on the JSE are influenced by movements in the NYSE to a larger extent than others. Consequently a "multi–market" model is proposed in section 4.3.2 in an attempt to identify whether any of the unique risk of JSE stocks could in fact be explained by movements in the NYSE. In section 4.3.3 these components of risk are investigated empirically for a range of shares. It is also plausible that some markets only react to either sharp movements in the NYSE (such as the sharp decline in the NYSE in October 1987), or perhaps react differently in bull and bear phases of the NYSE. These assertions are examined empirically for the JSE in section 4.3.4.

4.3.2 Theoretical discussion

The model proposed in this section, in essence, relates the return of a security listed on the JSE to the return on a local market index, plus a US market index. In order to obtain tractable expressions for the risk components, i.e. local market risk, US market risk and unique risk, the vector of local and US market returns are orthogonalized. This amounts to removing the effect of the US index from the returns of the local index. This can be simply achieved by regressing the returns of the local index on the returns of the US index, and using the resultant residuals to represent the local index, with the effects of the US index removed.

This proposed model, henceforth referred to as the *multi–market model* can be written as

$$R_{it} = \alpha_i + \beta_i^{\mathrm{SA-USA}} R_{mt}^{\mathrm{SA-USA}} + \beta_i^{\mathrm{USA}} R_{mt}^{\mathrm{USA}} + e_{it} \tag{4.3}$$

where

R_{it} is the return on share i on the JSE at time t ;

$\alpha_i, \beta_i^{\mathrm{SA-USA}}$ and β_i^{USA} are coefficients unique to share i ;

R_{mt}^{USA} is the return on the US market index at time t ;

$R_{mt}^{\mathrm{SA-USA}}$ is the residual JSE market index return at time t , obtained by regressing the returns of JSE market index on the US market index returns and;

the following assumptions regarding the e_{it} are made:

$$E(e_{it}) = 0$$

$$\mathrm{Cov}(e_{it} \; ; \; e_{is}) = 0 \qquad \text{for } t \neq s$$

$$\mathrm{Cov}(R_{mt}^{\mathrm{USA}} \; ; \; e_{it}) = 0 \qquad \text{for all } t$$

$$\mathrm{Cov}(R_{mt}^{SA-USA} \; ; \; e_{it}) = 0 \qquad \text{for all } t$$

The components of risk for security i can be obtained by considering the expression for the variance of security i 's returns, i.e.

$$\begin{aligned}\mathrm{Var}(R_{it}) &= \mathrm{Var}(\alpha_i + \beta_i^{\mathrm{SA-USA}} R_{mt}^{\mathrm{SA-USA}} + \beta_i^{\mathrm{USA}} R_{mt}^{\mathrm{USA}} + e_{it}) \\ &= \mathrm{Var}(\alpha_i) + \beta_i^{\mathrm{SA-USA}^2} \mathrm{Var}(R_{mt}^{\mathrm{SA-USA}}) + \beta^{\mathrm{USA}^2} \mathrm{Var}(R_{mt}^{\mathrm{USA}}) \\ &\quad + 2\beta_i^{\mathrm{SA-USA}} \beta_i^{\mathrm{USA}} \mathrm{Cov}(R_{mt}^{\mathrm{SA-USA}} \; ; \; R_{mt}^{\mathrm{USA}}) + \mathrm{Var}(e_{it})\end{aligned}$$

Since α_i is a constant $\mathrm{Var}(\alpha_i) = 0$ and;

by construction $\mathrm{Cov}(R_{mt}^{\mathrm{SA-USA}} \; ; \; R_{mt}^{\mathrm{USA}}) = 0$; the above expression simplifies to:

$$\mathrm{Var}(R_{it}) = \beta_i^{\mathrm{SA-USA}^2} \mathrm{Var}(R_{mt}^{\mathrm{SA-USA}}) + \beta_i^{\mathrm{USA}^2} \mathrm{Var}(R_{mt}^{\mathrm{USA}}) + \mathrm{Var}(e_{it}) \qquad (4.4)$$

Thus the above expression can be interpreted as:

Total risk = SA market risk only + USA market risk + unique risk.

The empirical investigation conducted in section 4.3.3 considers the model from two perspectives. Firstly, the investigation is conducted with the US market index expressed in dollar returns, whilst the other variables are expressed in local currency returns. Thereafter the investigation is conducted with all the variables expressed in local currency returns. The intuition behind the two approaches is that local investors who are concerned about dollar movements of the US market index, are perhaps more likely to be concerned with the aspect of changes in "sentiment" associated with these movements. On the other hand local investors who are concerned

Table 4.7 Estimated coefficients ranked by β^{SA-USA} for case 1

Share name	β^{USA}	p−value* of β^{USA}	β^{SA-USA}	p−value* of β^{SA-USA}
Rusplat	0.91	0.000	1.14	0.000
Rembrandt	0.88	0.000	0.68	0.000
De Beers	0.81	0.000	0.92	0.000
Lorraine	0.74	0.007	1.99	0.000
Dorbyl	0.69	0.000	0.60	0.000
Lyd Plat	0.63	0.002	1.34	0.000
Anglo Am	0.61	0.000	1.17	0.000
Sappi	0.58	0.005	0.76	0.000
Barlows	0.52	0.000	0.79	0.000
Highveld	0.50	0.006	0.41	0.002
Kloof	0.48	0.001	1.20	0.000
Reunert	0.46	0.154	0.46	0.055
Bracken	0.44	0.043	1.62	0.000
SA Eagle	0.44	0.035	0.36	0.019
Wooltru	0.43	0.003	0.64	0.000
SA Brews	0.43	0.001	0.74	0.000
AECI	0.41	0.004	0.68	0.000
Johnnies	0.41	0.004	1.19	0.000
Pick 'n Pay	0.36	0.042	0.79	0.000
Amcoal	0.34	0.056	0.73	0.000
Randfontn	0.28	0.047	1.43	0.000
Fedfund	0.23	0.130	0.55	0.000
Fedfood	0.20	0.261	0.50	0.000
Toyota	0.19	0.516	0.62	0.005
Mcarthy	0.17	0.404	0.57	0.000
Tradegro	0.16	0.534	0.66	0.001
Amaprop	0.15	0.771	0.86	0.022
Rex True	0.14	0.407	0.03	0.829
Barclays	0.14	0.366	0.61	0.000
LTA	0.12	0.573	0.50	0.002

* The p−values are for the hypothesis $H_0 : \beta_i = 0$

of shares that are more influenced by movements in the Dow–Jones (expressed in US dollars). Among the largest $\widehat{\beta}^{USA}$ coefficients are shares like Rembrandt, De Beers and Anglo American which typically do derive a large proportion of their income from overseas markets.

It is clearly not the magnitude of the $\widehat{\beta}^{USA}$ that are of major concern here, but rather the decomposition of total risk into the various components of risk so that the influence of the Dow–Jones on the risk of JSE stocks can be determined. Table 4.8

gives a detailed breakdown of the risk components expressed as a percentage of total risk for the 30 samples shares over the period September 1979 to November 1987. These components are computed separately for the proposed multi–market model (4.3) as well as for the traditional market model. The shares in table 4.8 are ranked in accordance with the percentage of the NYSE market risk component relative to total risk. The risk components shown in table 4.8 reveal several interesting features.

The percentage of US market risk was greater than 5 percent for 12 of the 30 shares, and greater than 10 percent in 5 out of the 30 shares, with Rembrandt having the largest component, making up 22.85 percent of total risk. It is thus evident that for only specific shares on the JSE is the component of risk attributable to US market movements of practical significance.

A further insight that is evident from table 4.8 is that most of the observed US market risk is captured in the JSE market risk component obtained from the traditional market model anyway (recall the JSE index used in the traditional model does not have the US market effect removed). This can be seen by comparing the unique risk component percentages for the proposed model (4.3) and for the traditional market model. In only 1 case, i.e. Rembrandt was the reduction in unique risk greater than 5 percent. Furthermore the average percentage of unique risk using the traditional market model was 67.30 percent, while the corresponding average for the proposed model (4.3) was 66.35 percent. That is on average there was no significant reduction in unique risk.

It can thus be concluded that for case 1, where the Dow–Jones Index was analysed in dollar terms, that:

Although a significant percentage of total risk of a large proportion of securities is associated with dollar movements in the US market, this component is mostly captured by the component of JSE market risk obtained using the traditional market model anyway. In other words, almost none of the unique risk in the traditional market model sense, can be explained away by dollar movements in the US market. The proposed *multi–market model* is still useful however, as it provides a way of identifying the proportion of local market risk that is attributable to dollar movements in US markets for individual securities.

Table 4.8 Percentage of total risk attributable to each of the components of risk for case 1

Share Name	Multi–Market Model (4.3) NYSE Risk [a] Component %	JSE Risk [b] Component %	Unique Risk Component %	Traditional Market Model JSE Risk Component %	Unique Risk Component %
Rembrandt	22.85	24.58	52.57	38.60	61.40
De Beers	18.32	43.05	38.64	57.48	42.53
Rusplat	13.99	40.45	45.56	52.20	47.80
Dorbyl	13.25	18.08	68.67	26.96	73.04
Anglo Am	10.46	68.92	20.62	79.32	20.68
Barlows	9.28	39.70	51.02	48.37	51.63
SA Brews	7.20	38.52	54.28	45.61	54.39
Highveld	7.02	8.83	84.15	13.44	86.56
Wooltru	6.56	15.18	67.26	32.27	67.73
Sappi	6.31	19.90	73.79	25.42	74.58
AECI	5.84	29.02	65.14	34.72	65.28
Lyd Plat	5.49	45.85	48.65	51.60	48.40
Kloof	4.87	57.07	38.06	62.12	37.88
SA Eagle	4.35	5.38	90.27	8.22	91.78
Lorraine	3.75	49.69	46.56	53.57	46.43
Johnnies	3.63	56.73	39.63	60.33	39.67
Pick 'n Pay	3.07	27.84	69.09	31.14	68.86
Amcoal	2.85	24.51	72.64	27.57	72.43
Bracken	2.08	50.52	47.40	52.14	47.86
Reunert	2.03	3.74	94.23	5.26	94.74
Fedfund	1.89	20.41	77.70	22.48	77.52
Randfontn	1.36	66.53	32.11	66.02	33.98
Fedfood	1.15	12.86	85.99	14.13	85.87
Rex True	0.72	0.05	99.23	0.23	99.77
Barclays	0.66	23.02	76.33	23.33	76.67
Mcarthy	0.63	13.18	86.18	13.80	86.20
Toyota	0.41	8.06	91.54	8.47	91.53
Tradegro	0.36	11.34	88.30	11.57	88.43
LTA	0.30	9.27	90.43	9.48	90.52
Amaprop	0.08	5.38	94.53	5.32	94.68
Average	5.36	28.29	66.35	32.70	67.30

[a] These components are obtained from the expression: $\beta^{\mathrm{USA}^2}\,\mathrm{var}(R_{mt}^{\mathrm{USA}})$

[b] These components are obtained from the expression: $\beta^{\mathrm{SA-USA}^2}\,\mathrm{var}(R_{mt}^{\mathrm{SA-USA}})$

To gain further insight into the stability of these risk components over time, a continuous time series of these risk components for selected securities were computed. In essence, this technique involved using moving 4 year estimation periods, that is, estimating the risk components over the first 4 year period, moving one month forward by deleting the first month and recomputing the risk component. This process is performed repeatedly by moving one month forward and recomputing the components over the immediate preceding 4 year period, this resulted in a time series of estimated risk components over the August 1983–November 1987 period. The graphs of these time series of risk components are shown for 3 shares, namely, Rembrandt, De Beers and AECI which are selected because they have differing NYSE risk components. These are shown in figures 4.1, 4.2 and 4.3 respectively.

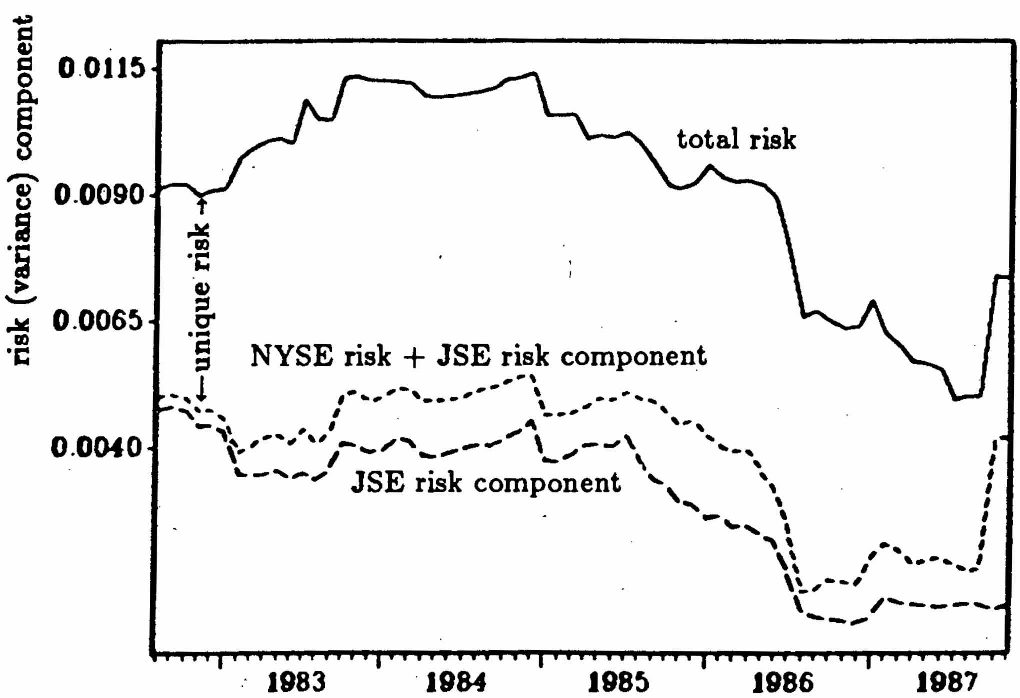

Figure 4.1 Risk components of Rembrandt

With reference to figures 4.1, 4.2 and 4.3 the graphs should be interpreted as follows:

At any given point in time, the "total risk" (indicated by the solid line) can be interpreted as the scaled vertical distance between the $x-$axis and the solid line. The scaled vertical distance between the $x-$axis and the line denoted "JSE risk component" can be interpreted as the risk attributable to movements in the JSE market index with the dollar effect of the Dow–Jones Index removed. The scaled

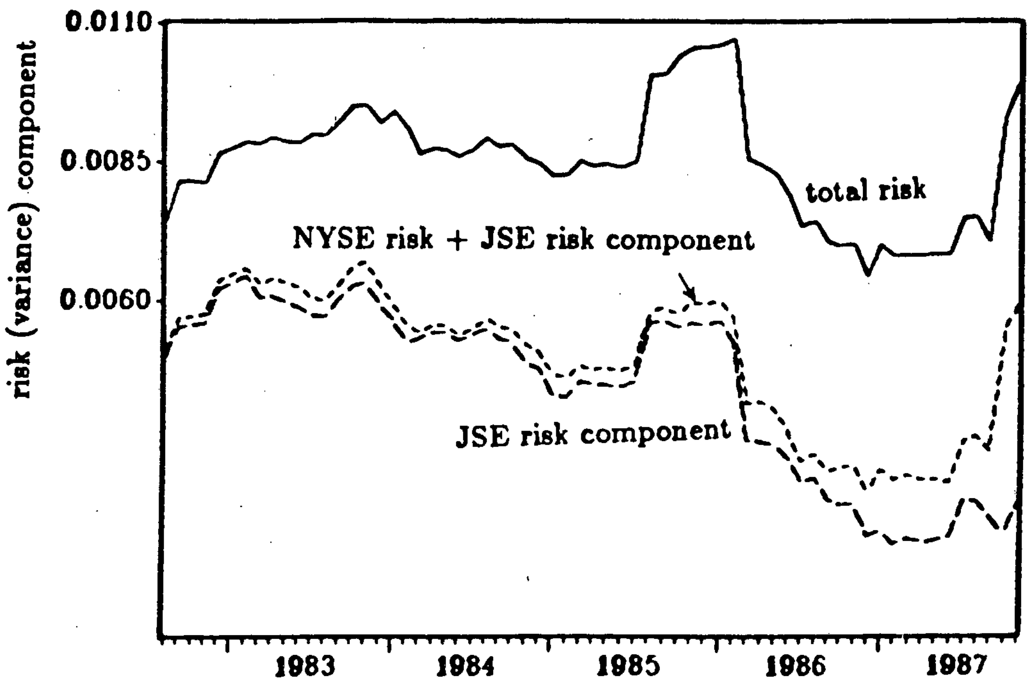

Figure 4.2 Risk components of De Beers

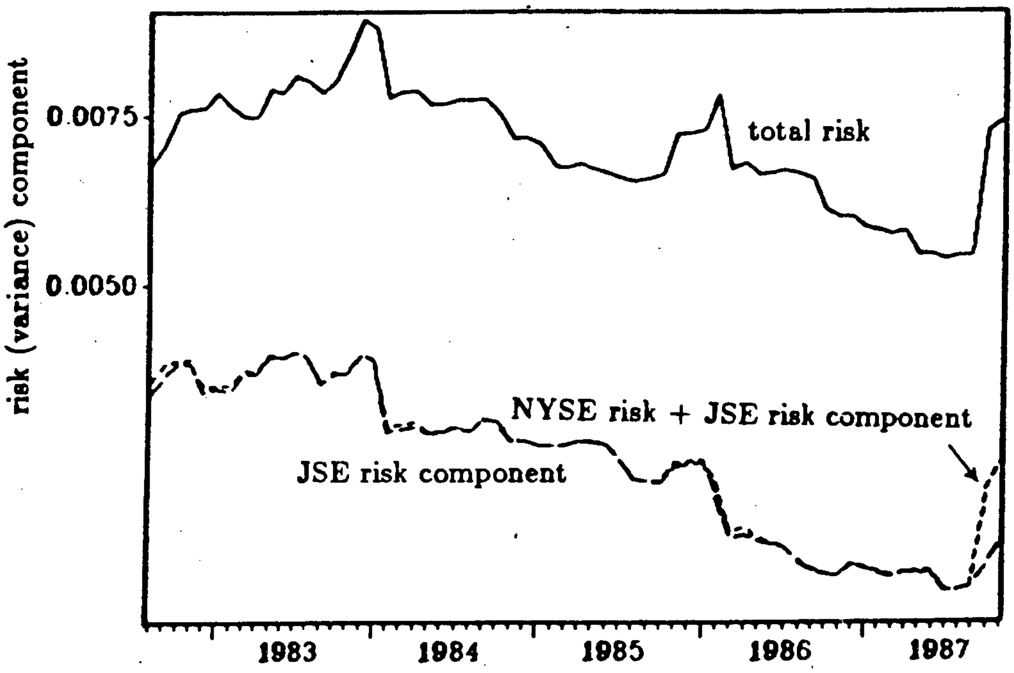

Figure 4.3 Risk components of AECI

vertical distance between this line and the line denoted "NYSE risk + JSE risk component" can be interpreted as the risk attributable to dollar movements in the Dow–Jones Index. Finally the scaled vertical distance between this line and the line representing "total risk" can be interpreted as the unique risk estimated at the given point in time.

Comparison of figures 4.1, 4.2 and 4.3 reveals several interesting insights:

In all three figures, the "JSE risk component" line changes level considerably, and is seen to show a decreasing trend over all three figures. Furthermore the "total risk" appears to track the movements of the "JSE risk component" closely. Consequently both the level of "unique risk" and the "NYSE risk" are seen to remain fairly constant over the entire period for all three figures. It is interesting to note however, that although the "NYSE risk component" is asserted to be fairly constant, during October 1987, the level of this risk component is seen to rise dramatically in all three figures. This rise is clearly associated with the devastating crash on the NYSE in October 1987, which had a dramatic effect on stock exchanges world-wide. This dramatic increase in the "NYSE risk component" naturally leads to the question of whether movements of share prices on the JSE are more sensitive to the NYSE during times of shock movements on the NYSE. In section 4.3.4 answers to this question, and others, which consider whether this relationship is stronger in bull or bear markets, are sought.

Case 2 : The Dow–Jones Index expressed in rand returns

The intuition behind the analysis here, is to assess the risk associated with changes in the fundamental value of SA securities as a consequence of changes in the rand return of the Dow–Jones Index.

The analysis here proceeds in the same way as the analysis for case 1, with the exception that the series, R_{mt}^{USA} , represents the series of *rand* returns on the Dow–Jones Index. Furthermore the $R_{mt}^{\mathrm{SA-USA}}$ represents the series of rand returns on the JSE–Actuaries Overall Index with the *rand* effect of the Dow–Jones Index removed. Table 4.9 shows the estimated β coefficients for the proposed model (4.3) with their associated p–values for the 30 shares using the variables described above.

Table 4.9 Estimated beta coefficients for model 4.3 for case 2

Share name	β^{USA}	p–value of β^{USA}	β^{SA-USA}	p–value of β^{SA-USA}
Rembrandt	0.46	0.000	0.76	0.000
Rustplat	0.40	0.001	1.20	0.000
De Beers	0.37	0.000	0.98	0.000
Highveld	0.33	0.010	0.45	0.000
Lyd Plat	0.23	0.0088	1.34	0.000
Amcoal	0.22	0.082	0.72	0.000
Dorbyl	0.22	0.063	0.68	0.000
Anglo Am	0.20	0.002	1.18	0.000
Bracken	0.19	0.235	1.56	0.000
SA Eagle	0.19	0.196	0.41	0.007
Kloof	0.15	0.113	1.19	0.000
Reunert	0.15	0.499	0.50	0.029
Wooltru	0.09	0.356	0.67	0.000
Fedfund	0.08	0.445	0.54	0.000
Barlows	0.07	0.42	0.83	0.000
Lorraine	0.06	0.766	1.98	0.000
SA Brews	0.05	0.579	0.77	0.000
Rex True	0.04	0.745	0.05	0.670
Pick 'n Pay	0.03	0.796	0.80	0.000
AECI	0.01	0.902	0.71	0.000
Fedfood	-0.02	0.854	0.50	0.000
Barclays	-0.02	0.890	0.59	0.000
Randfontn	-0.03	0.764	1.38	0.000
Johnnies	-0.04	0.689	1.19	0.000
Sappi	-0.05	0.738	0.83	0.000
Amaprop	-0.12	0.725	0.84	0.000
Tradegro	-0.21	0.236	0.67	0.000
Mcarthy	-0.26	0.065	0.59	0.000
Toyota	-0.36	0.075	0.65	0.002
LTA	-0.37	0.013	0.53	0.001

From table 4.9 it can be seen that only 6 out fo 30 shares sampled had significant $\widehat{\beta}^{USA}$ coefficients at the 5 percent level of significance, and of these, only one, (the construction company LTA) had a negative $\widehat{\beta}^{USA}$ coefficient. By contrast for case 1, where the analysis used the Dow–Jones Index in dollar returns, 19 out of 30 shares had significant $\widehat{\beta}^{USA}$ coefficients. It thus appears as if securities on the JSE are generally more responsive to movements in the dollar return, rather than the rand return of the Dow–Jones Index. A plausible reason for this may be associated with the fact that the Dow–Jones Index expressed in rand denominations includes the rand/dollar exchange rate component. Movements in the Dow–Jones index are often linked to compensating movements in the rand/dollar exchange rate, thus weakening the relationship between the Dow–Jones expressed in ***rand*** denominations and rand returns of SA shares. Consequently it is unlikely that any additional insights can be gained by further analysis of the risk components using rand returns on the Dow–Jones Index.

4.3.4 International shock effects — an empirical investigation

The events of October 1987 where the sudden sharp decline on the NYSE had a dramatic effect on stock markets world–wide, leads to the question of whether stock returns on the JSE respond consistently in different market conditions on the NYSE. In particular, the following lines of investigation will be pursued here.

In order to investigate the assertion that JSE stocks respond differently during times of sharp movements on the NYSE, the return data was partitioned according to substantial movements (up and down) in the monthly dollar return of the Dow–Jones Index. This amounted to the selection criterion that return data is used only in months where the absolute value of the dollar return on the Dow–Jones Index exceeded 6 percent per month. The proposed model (4.3) was then run for each of the 30 shares, with their monthly returns partitioned in this manner, and the components of risk were estimated within this framework.

To investigate the risk of JSE stocks during bull and bear phases of the NYSE, returns were partitioned separately according to only positive (bull) and negative (bear) movements of the NYSE respectively. This amounted to matching up returns in months where the dollar return on the Dow–Jones Index was positive, and conducting the analysis for the "bull" market scenario. The "bear" market scenario, involved matching up returns in months where the dollar return on the Dow–Jones Index was negative.

For the above cases the risk components were averaged across the 30 securities. The summarized results of this analysis are shown in table 4.10. The results of the analysis in section 4.3.3 are included for comparison purposes.

The results showing the NYSE risk component in the first column of table 4.10 are of primary importance in this investigation. From table 4.10 it is evident

Table 4.10 Average risk components expressed as a percentage of total risk

Criterion	Average risk components using model (4.3)			Average risk components using the traditional market model	
	NYSE component [a]	JSE component [b]	Unique component	JSE component	Unique component
None	5.36%	28.29%	66.35%	32.70%	67.30%
$\lvert R_{mt}^{USA}\rvert > 6\%$	17.25%	20.01%	62.74%	34.27%	65.73%
$R_{mt}^{USA} > 0\%$	2.65%	28.96%	68.39%	30.87%	69.13%
$R_{mt}^{USA} < 0\%$	9.85%	29.67%	60.47%	38.12%	61.88%

[a] Computed from the expression $\beta^{USA^2}\,\mathrm{var}(R_{mt}^{USA})$

[b] Computed from the expression $\beta^{SA-USA^2}\,\mathrm{var}(R_{mt}^{SA-USA})$

that for the criterion $\lvert R_{mt}^{USA}\rvert > 6\%$, the NYSE risk component is largest, that is 17.25%.

By contrast over the whole period the NYSE risk component was only 5.36%. It thus appears that during times of both significant rises and declines of the NYSE, shares on the JSE tend to be driven to a greater extent by movements in the NYSE than at other times, while over general market conditions on the NYSE, JSE shares virtually ignore the movement on the NYSE. The findings of table 4.10 for NYSE bull and bear markets are; that during times when the NYSE was rising, only 2.65% of the risk of JSE stocks was related to these NYSE movements, while during times when the NYSE was declining, 9.85% of the risk of JSE stocks was related to these NYSE movements. It thus appears as if JSE stock generally respond more strongly to declines rather than rises in the NYSE.

The risk components shown in table 4.10 were average across all 30 securities in the sample, however, it was found in section 4.3.3 that only 19 of these securities had significant $\hat{\beta}^{USA}$ coefficients. In order to determine to what extent these shares, in particular, were influenced by the various NYSE market conditions, the resulting risk components for the criteria listed in table 4.10 were averaged across only these 19 securities. These results are shown in table 4.11.

Table 4.11 Average risk components of shares with significant $\widehat{\beta}^{USA}$ coefficients expressed as a percentage of total risk

Criterion	Average risk components using model (4.3)			Average risk components using the traditional market model	
	NYSE component	JSE component	Unique component	JSE component	Unique component
None	7.88%	37.73%	54.38%	44.18%	55.82%
$\lvert R_{mt}^{USA}\rvert > 6\%$	25.33	24.97%	49.70%	46.72%	53.28%
$R_{mt}^{USA} > 0$	3.25%	37.93%	58.82%	40.62%	59.37%
$R_{mt}^{USA} < 0$	14.26%	37.62%	48.12%	50.36%	49.64%

From table 4.11 it is evident that the percentage of risk explained by the NYSE (shown in the first column) is significantly larger than table 4.10 for the criteria $\lvert R_{mt}^{USA}\rvert > 6\%$ and $R_{mt}^{USA} < 0$, implying that the securities with significant $\widehat{\beta}^{USA}$ coefficients are influenced to a greater extent in these NYSE market conditions. However for the criteria $R_{mt}^{USA} > 0$ the improvement from 2.65% (for 30 shares) to 3.25% (for the 19 shares) is hardly significant. This tends to imply that the negligible relationship between rises in the NYSE and JSE returns shown in table 4.10 were not significantly biased by the 11 shares with insignificant $\widehat{\beta}^{USA}$ coefficients. Hence the conclusion here is that even JSE shares with significant $\widehat{\beta}^{USA}$ coefficients do not respond significantly to upward movements in the NYSE. A further interesting finding that is worth noting, is that the percentage of the JSE risk component for both models in table 4.11 is approximately 10 percent higher than the corresponding columns of table 4.10. This tends to imply that, interestingly, shares with large NYSE risk components also have large JSE risk components.

4.3.5 Conclusion

The major conclusions for investors on the JSE are:

Firstly, a large proportion of shares on the JSE do appear to be influenced by movements in the Dow–Jones Index. The influence of the US dollar return on Dow–Jones Index however, is stronger than that of the Dow–Jones Index expressed in rand returns. This could perhaps be due to the fact that changes in sentiment (assumed to be associated with movements in dollar returns) are viewed as being more important than changes in the fundamental rand value of the Dow–Jones.

Secondly, it can be concluded that, almost all of the risk associated with movements on the NYSE is captured by the component of JSE market risk obtained using the traditional market model anyway. Consequently the traditional market model is appropriate for use on JSE stocks. The *multi-market model* proposed here, is however, useful for providing a framework to assimilate more information about the risk characteristics of individual securities. For example it was found that while the level of local (JSE) market risk tended to vary significantly over time, both the level of risk attributable to the NYSE as well as to unique factors tended to remain fairly constant over time.

Thirdly, it was found that the type of market condition apparent on the NYSE has an effect on the extent to which the JSE stocks were related to movements on the NYSE. In particular, during times when there are sharp movements on the NYSE (both rises and declines), a significantly large effect on JSE stocks can be expected, while over general NYSE conditions, the JSE stocks hardly reflect these movements.

Lastly it was found that JSE stocks were more sensitive to bear market conditions than bull market conditions on the NYSE.

4.4 AN EMPIRICAL APPLICATION — THE CHOICE BETWEEN BULLION OR SA GOLD SHARES FOR INTERNATIONAL DIVERSIFICATION [7]

4.4.1 Introduction

Some form of gold asset has long been recognised as an important component of a balanced portfolio. The issue, which has been the subject of more debate, is what form this gold investment should take. While Bradfield and Barr (1985) proposed a model which can be used to make selection decisions amongst gold shares, the choice between gold shares and bullion has been of long standing concern to international investors. In this section the issue of whether gold bullion or South African gold shares yield the better diversification benefit, is addressed. In particular, this issue will be considered from the perspective of the US investor and the UK investor separately.

4.4.2 Theoretical overview

The movement of gold shares follows that of gold bullion closely, but represent a more highly geared alternative. Gold shares show greater variability of movement and have higher market betas than does gold bullion. We first analyse whether there are any theoretical reasons for an investor to choose between these two gold assets if their movement is linearly related in a deterministic way.

A hypothetical scenario is considered in the well known risk–return optimisation framework of Markowitz (1952). Two gold assets are considered, viz gold bullion, G_B (the return on bullion), and gold shares, G_S (the return on gold shares), and assume

(a) $\rho(G_B, G_S) = 1$

i.e. that the correlation between gold bullion and gold shares is 1.

(b) The total risk associated with G_S is greater than with G_B .

(c) The return associated with G_S is greater than with G_B .

This implies G_S and G_B must lie on a rho–isogram emanating from R_f .

The correlation between returns on bullion and returns on any given portfolio will be equal to the correlation between gold shares and the given portfolio. Thus, neither offers any diversification benefit over the other when included in portfolios. The prior position, therefore, is that more or less highly geared gold assets may only be useful if these gold assets do not move together in a linearly related way. If these assets move completely in tandem, then buying the more highly geared option cannot be more beneficial than buying the less highly geared one from a more highly

[7] This section has been published in *The Investment Analysts Journal*, May 1987 (see Barr and Bradfield (1987)).

geared position, i.e. with borrowed money.

Any advantage of G_S or G_B (or vice versa) must stem from $\rho(G_S, G_B) \neq 1$. Although past data does not necessarily give unbiased estimates of the market's perception on future return, risk or correlation, the two gold assets will be considered using historical data for two overseas markets.

The empirical analysis is conducted for investors in the US market and then separately for investors in the UK market. In each instance, the All–Gold Index (used to proxy SA gold shares) and the gold bullion price are expressed in the currency of the relevant markets (US$ and UK£ respectively). The Dow–Jones Industrial Average is used as a proxy for the overall US market and the UK Actuaries Index is used as a proxy for the overall UK market.

Raw monthly index and price data were extracted for the following series over the period January 1975 to December 1985:

(1) The Johannesburg Stock Exchange (JSE) All–Gold Index (US$ and UK£).

(2) The Dow–Jones Industrial Index.

(3) The UK Actuaries Index.

(4) The gold price.

4.4.3 The US perspective

In order to assess the performance of gold shares and bullion in the framework of the above theoretical discussion, it is necessary firstly to investigate the correlation between returns of gold shares and bullion (expressed in dollars). Figure 4.4 shows the monthly moving correlation of monthly returns of bullion and the All–Gold Index($ s). Four years' data were used in the estimation of each monthly correlation.

Over almost the whole period (1979–1985), the moving correlation can be seen to be above the 0.6 level and remains predominantly in the 0.6 to 0.7 range. Clearly, one cannot practically expect a correlation coefficient of 1 as assumed in the theoretical discussion. However, the issue is whether the correlation is sufficiently large so that the role of bullion can be duplicated by gold shares or vice versa.

The relative riskiness of the three assets are considered in Figure 4.5. Figure 4.5 shows the monthly moving standard deviation of monthly returns of gold shares

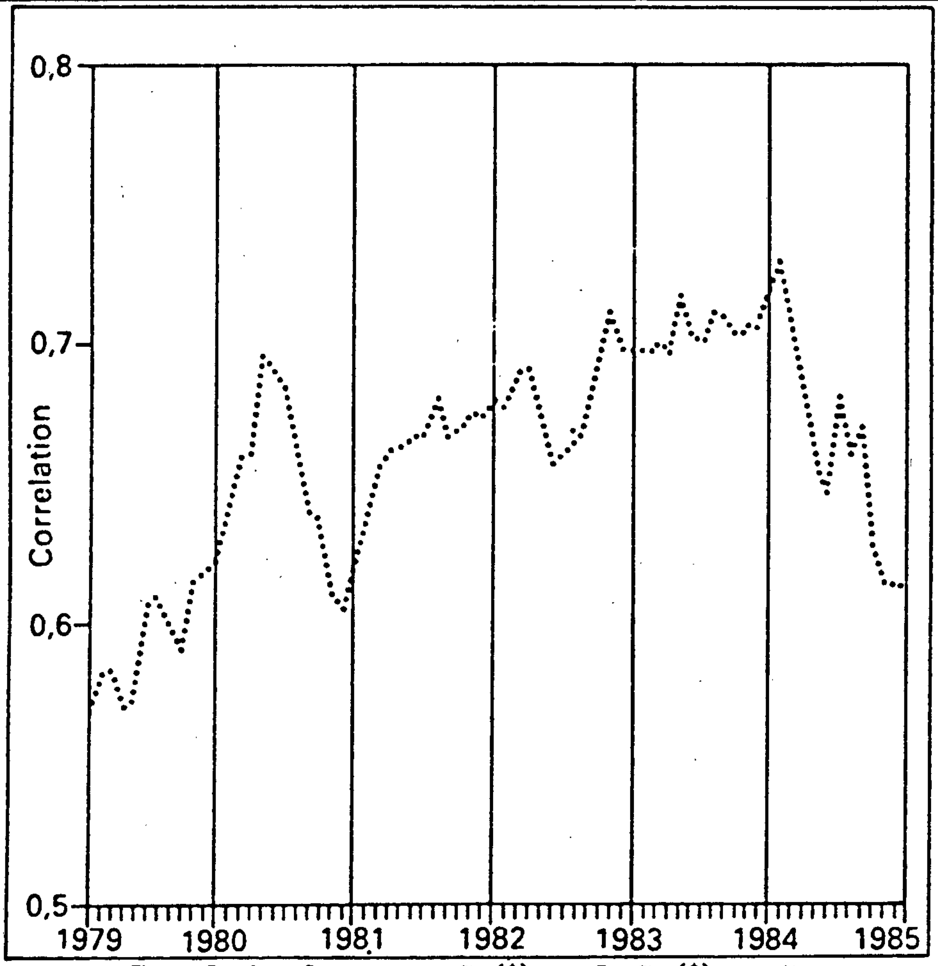

Figure 4.4 Correlation between $G_S(\$)$ and $G_B(\$)$: 1979–1985

(\$), bullion (\$) and the Dow–Jones using a moving 48–month estimation period.

It can be seen that the relative levels of standard deviation are maintained over the six–year period indicating that gold shares have been consistently more volatile than the Dow–Jones (which has maintained a level of about 4% over the six–year period). The standard deviation of gold shares by contrast reached 18% in December 1984. Although it is useful to consider the total risk of assets, the systematic risk of assets as measured by their betas is the correct measure of risk in the context of diversified portfolios.

Figure 4.6 shows the monthly moving betas of gold shares (\$) and bullion relative to the Dow–Jones Index.

It is evident from Figure 4.6 that the betas of gold shares are consistently larger than the betas of bullion but that the relative movements are similar (since

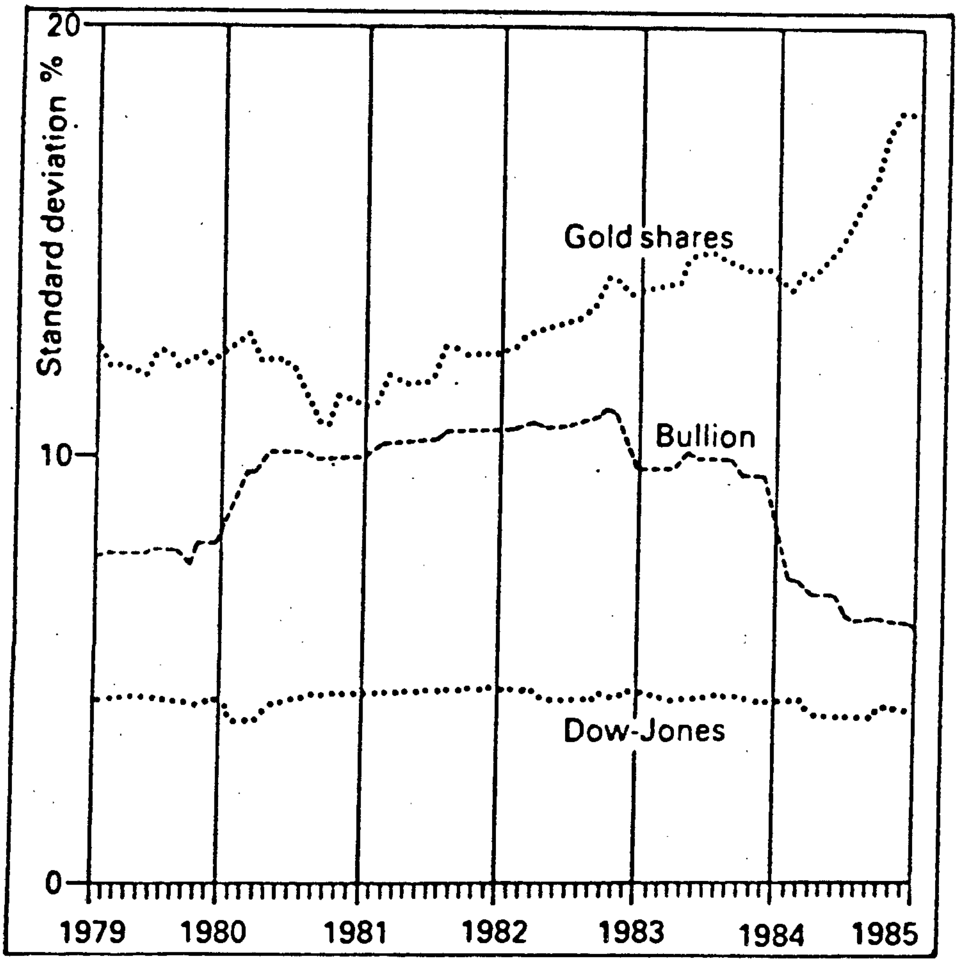

Figure 4.5 Standard deviation of monthly returns of $G_S(\$), G_B(\$)$ and Dow–Jones: 1979–1985

the correlation between the two are seen to be large and positive).[8]

In dollar terms, gold shares are seen to be more risky than bullion. In order to assess whether US investors have been adequately compensated for the risk to which they have been exposed, for holding either gold shares or gold bullion, a risk adjusted measure of return will be examined.

In order to compare bullion and gold shares directly on this basis, a series of adjusted returns was computed for both gold assets using

$$R_t - \beta_{t-1}(R_{mt})$$

where R_t is the annual return on the asset (either gold shares or bullion) during

[8] This can be explained by the fact that the percentage change in profits of gold shares are clearly more sensitive to changes in the bullion price than the percentage change in the bullion price itself. Hypothetically a mine with zero costs will have a beta similar to gold bullion. As cost approach revenue however, a mine's profitability becomes more sensitive to changes in the bullion price resulting in higher betas.

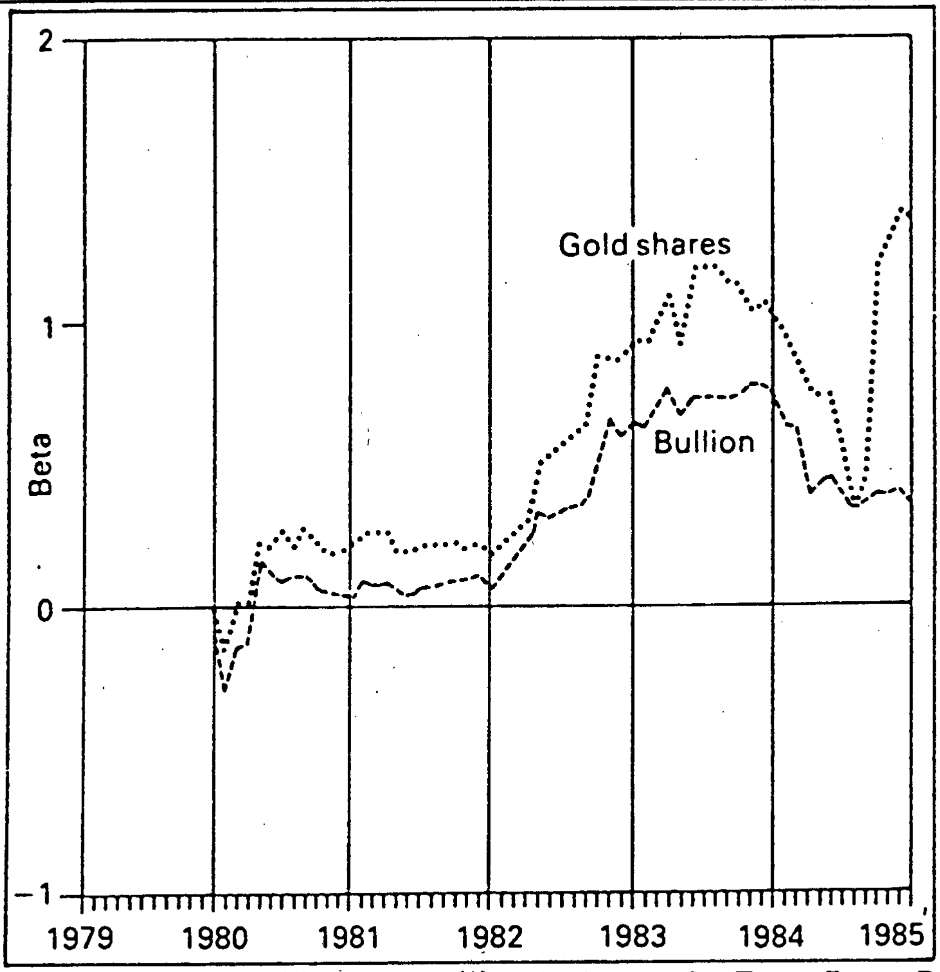

Figure 4.6 Betas of $G_S(\$)$ and $G_B(\$)$ relative to the Dow–Jones Index: 1979–1985

period t;

R_{mt} is the annual return on the market during period t (returns on the Dow–Jones here); and

β_{t-1} is the beta of the asset relative to the market (Dow–Jones) calculated in some previous period.

Figure 4.7 shows the monthly moving excess dollar return for gold shares and bullion using the above adjustment procedure. The beta used in the above expression was estimated one year prior to the estimated return.

It is evident that there is no consistent superiority in either of the series in Figure 4.7. The series for gold shares did however move out of line with that of

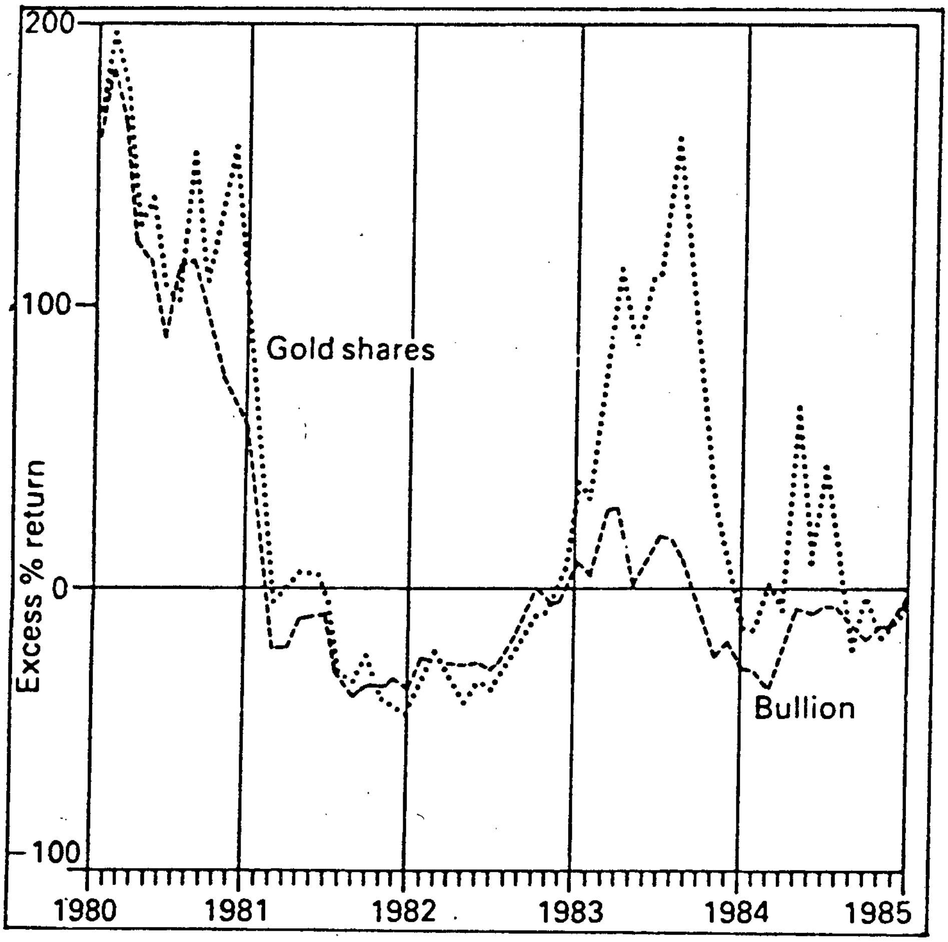

Figure 4.7 Adjusted returns for $G_S(\$)$ and $G_B(\$)$: 1980–1985

bullion during 1983. The major reason for this was the abolition of the financial rand discount in February 1983, causing the return on the All–Gold Index in dollars to rise dramatically.

Table 4.12 shows the resultant summary statistics of the analysis from the USA perspective.

Table 4.12 Summary of year end statistics from USA viewpoint

	Standard deviation (% per month) (4–year estimation period)			Beta using DJIA as the "market" (4–year estimation period)		Excess return	
Year end	All–Gold Index ($)	Bullion ($)	DJIA	All–Gold Index ($)	Bullion ($)	All–Gold Index ($)	Bullion ($)
1980	11.1%	9.9%	4.4%	0.21	0.02	109%	57%
1981	12.4%	10.6%	4.5%	0.18	0.06	-45%	-35%
1982	13.9%	9.7%	4.4%	0.92	0.64	40%	8%
1983	14.1%	8.2%	4.2%	1.04	0.74	15%	-28%
1984	18.0%	5.9%	3.9%	1.35	0.35	- 2%	- 7%

4.4.4 The UK perspective

The same approach used above was used to determine if any empirical evidence exists which indicates whether gold shares or bullion are the superior investment asset for the UK investor.

Figure 4.8 shows the monthly moving correlation between monthly returns on gold shares (expressed in sterling) and gold bullion (expressed in sterling). A moving four–year estimation period was used.

As with the case of the US investors the correlation is seen to remain predominantly in the 0.6 to 0.7 range over the 1979 to 1985 period.

Figure 4.9 shows the monthly moving, standard deviation of monthly returns of the All–Gold Index (£), bullion (£) and the UK Actuaries Index.

It is evident that the relative levels of total risk are maintained over the six years with gold shares remaining above the 10% level over the entire period.

Figure 4.10 shows the monthly moving betas of the All–Gold Index (£) and bullion (£) for a market model which uses the sterling returns of the UK Actuaries

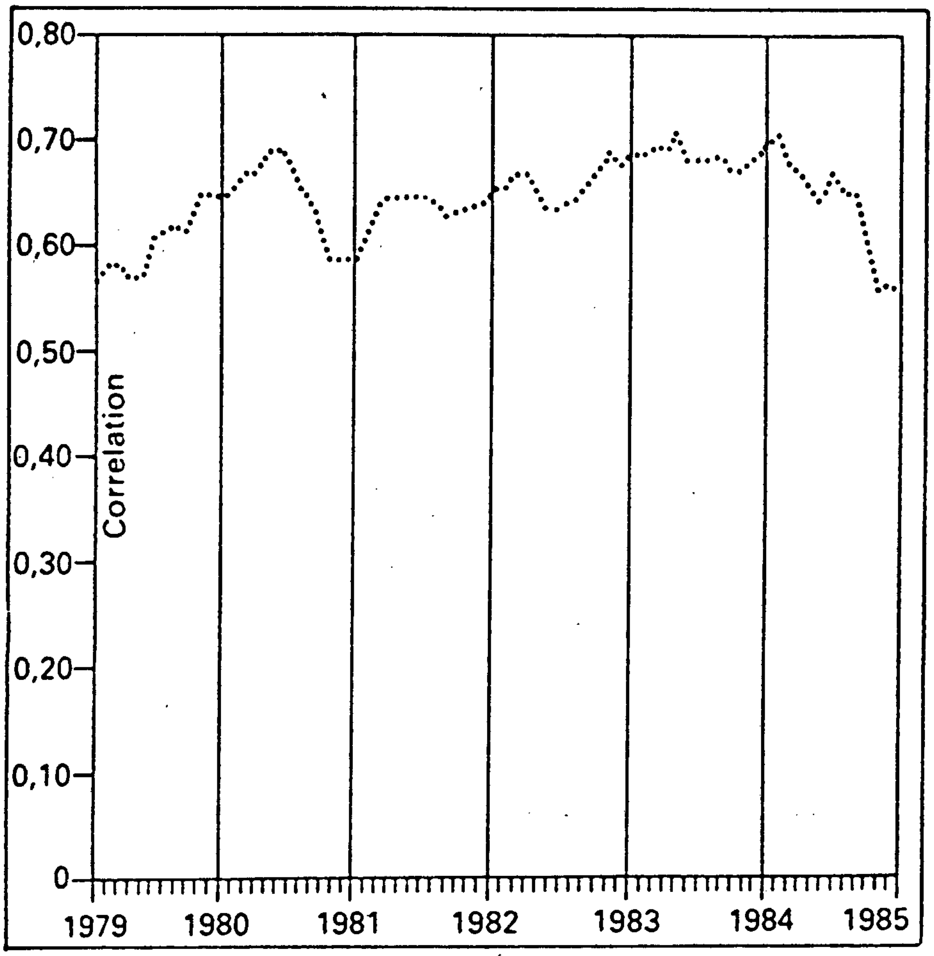

Figure 4.8 Correlation between G_S (£) and G_B (£): 1979–1985

Index.

The betas of gold shares can be seen to be consistently larger than those of bullion. This indicates that gold shares have had more market related risk than bullion for UK investors. Investors would thus expect higher returns from gold shares than bullion to compensate for the additional market risk.

In order to assess whether this compensation was more favourable for gold shares or for bullion, the same risk adjusted process used for the US case is used here. Here R_{mt} is the annual return on the UK Actuaries Index and other components have their usual meaning. Figure 4.11 shows the monthly moving excess return for the All–Gold Index (£) and gold bullion.

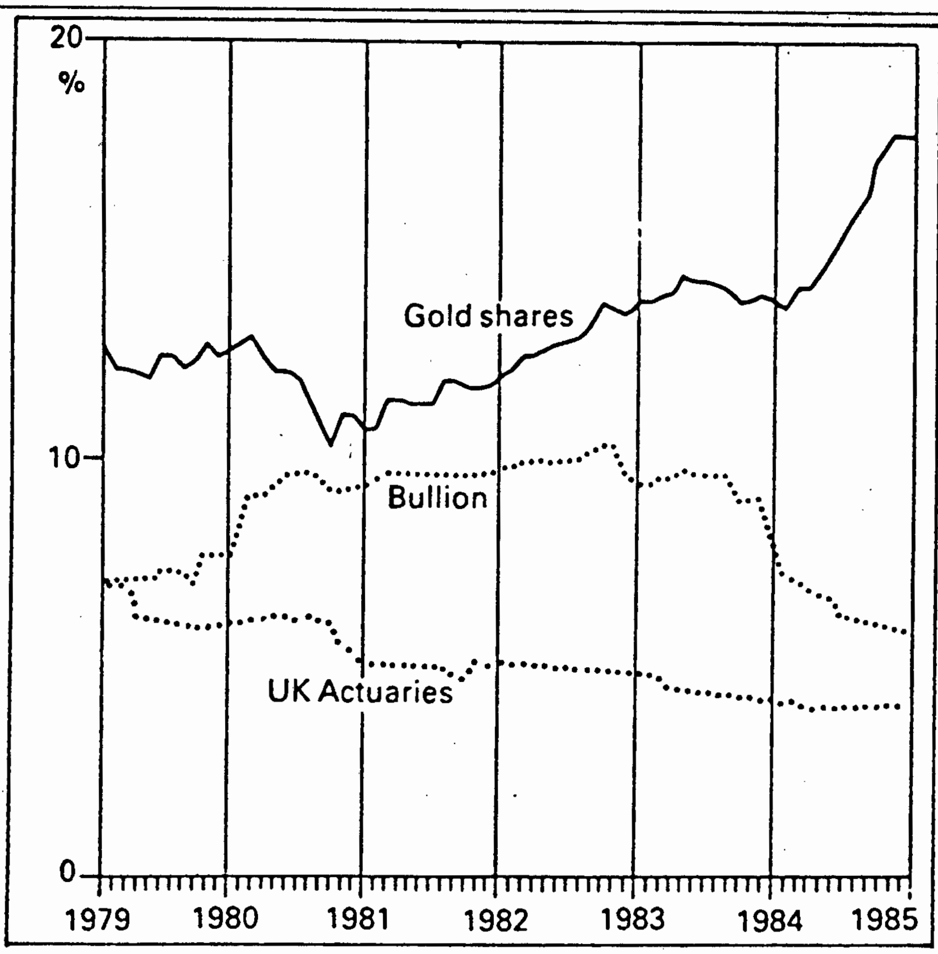

Figure 4.9 Standard deviation of G_S (£), G_B (£) and the UK Actuaries Index: 1979–1985

Again the only major dominance of bullion over gold shares for the UK investor seems to occur during early 1983, when the abolition of the financial rand discount caused one–off increases in SA share prices quoted on foreign markets.

Table 4.13 shows the resultant summary statistics of the analysis from the UK perspective.

4.4.5 Conclusion

The evidence suggests that:

There does not appear to be any consistent superiority over bull and bear markets between either of the series of risk–adjusted foreign currency returns of the two gold assets. There is however evidence of a large difference between the two

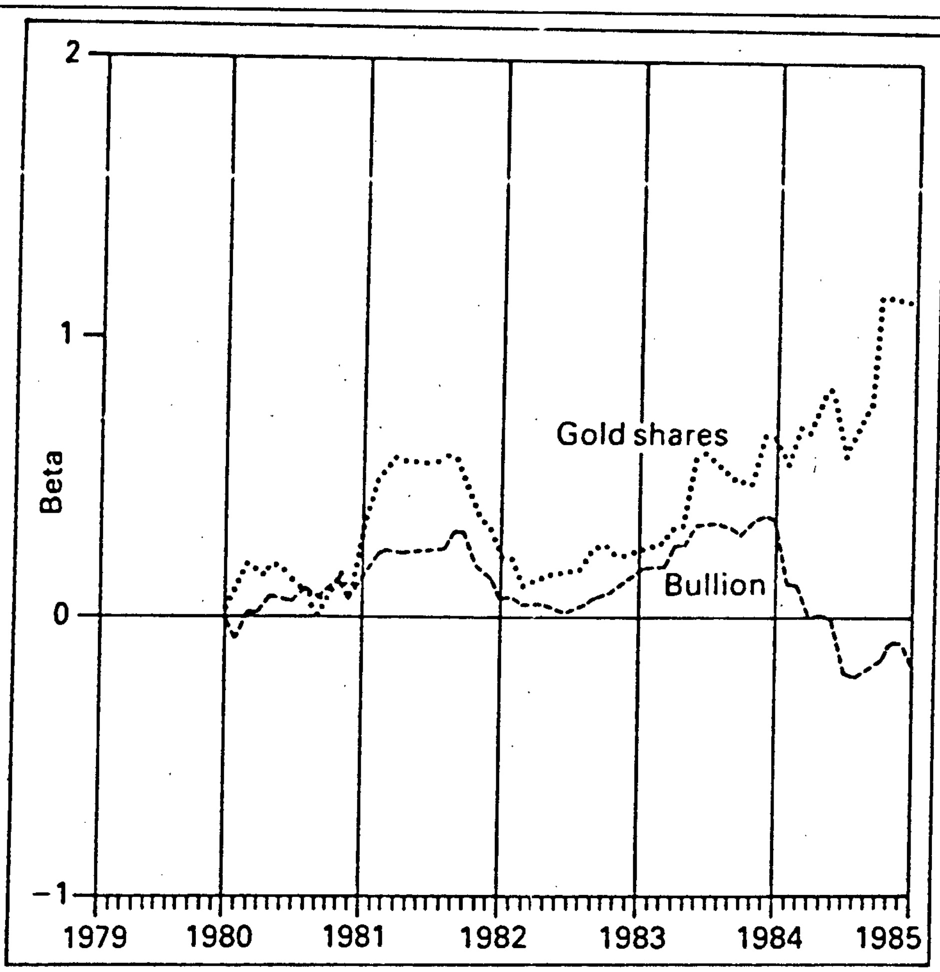

Figure 4.10 Betas of G_S (£), G_B (£) relative to UK Actuaries Index: 1979–1985

series starting in February 1983, and reaching a peak in July, before returning back to previous levels later in 1983. This discrepancy between the series of the two gold assets can easily be explained by the fact that during the first half of 1983 the suspension of the financial rand caused SA share prices in foreign currency terms to move out of line temporarily.

The above analysis therefore does give evidence that lends support to the view of the theoretical preamble that the two gold assets, namely bullion and SA gold shares, are for the most part equivalent, one simply representing a more highly geared version of the other.

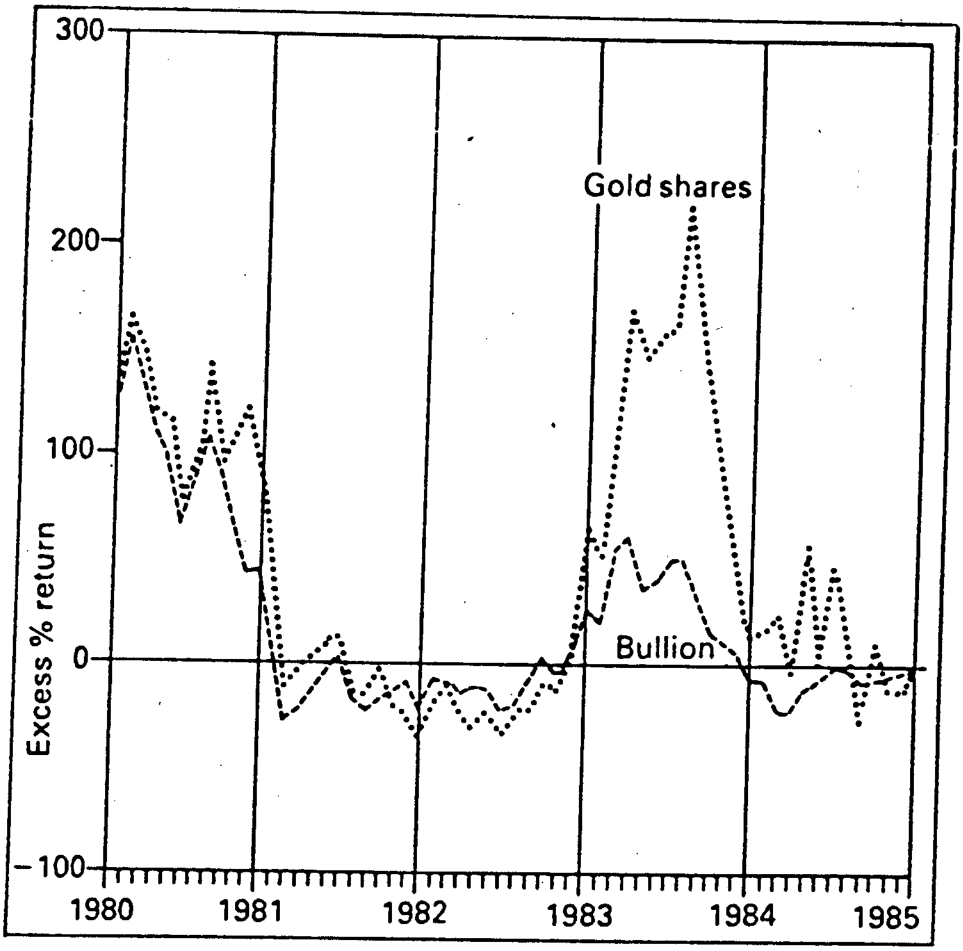

Figure 4.11 Difference between actual and expected (CAPM) returns for G_S (£) and G_B (£): 1980–1985

Table 4.13 Summary of year end statistics from UK viewpoint

	Standard deviation (% per month) (4-year estimation period)			Beta using UK Index as the "market" (4-year estimation period)		Excess return	
Year end	All-Gold Index (£)	Bullion (£)	UK Index (£)	All-Gold Index (£)	Bullion (£)	All-Gold Index (£)	Bullion (£)
1980	10.7%	9.3%	5.1%	0.31	0.13	93%	45%
1981	12.0%	9.9%	5.2%	0.20	0.06	-35%	-21%
1982	13.8%	9.5%	4.9%	0.25	0.17	64%	27%
1983	13.8%	7.9%	4.3%	0.65	0.35	13%	7%
1984	17.8%	5.8%	4.0%	1.13	-0.17	- 2%	- 1%

CHAPTER 5

EMPIRICAL TESTS OF THE CAPM AND EXTENSIONS : THE SA CASE

5.1 INTRODUCTON

Most of the empirical tests of the CAPM have been conducted using data from the major American exchanges, in particular the New York Stock Exchange (NYSE), the American Stock Exchange and the NASDAQ Over–the–counter Markets. Some of these have attempted to test the CAPM directly whilst others have investigated whether additional factors are relevant for asset pricing. In particular, factors such as dividend yield, market capitalization, price–earnings ratios, and January effects have all been empirically tested for evidence that may promise the investor consistent excess returns over the market. (See for example Black and Scholes (1974), Banz (1981), Basu (1977) and Reinganum (1983) respectively.)

Despite considerable efforts to empirically test the CAPM in the American environment, relatively little attention has been paid to smaller exchanges such as the Johannesburg Stock Exchange (JSE). Because the assumptions on which the CAPM is based include, *inter alia*, market efficiency and the absence of transaction costs (Jacob and Pettit (1984)), it is not obvious that the American results are directly translatable from the NYSE to smaller exchanges such as the JSE. Therefore in this chapter the CAPM will be examined in the context of the JSE.

This chapter will consist of two main subsections, that is, univariate tests and multivariate tests of the CAPM. Within these sections, possible extensions, namely the inclusion of dividend yield, market capitalization and liquidity components will also be considered to determine whether they have any significant additional influence on JSE stocks.

In the context of this chapter it should be noted that Roll (1977) has raised several doubts as to whether the CAPM can be legitimately tested. In particular Roll emphasizes that the validity of the CAPM and the efficiency of the market portfolio are joint hypotheses and that insofar as proxies are used for the market portfolio, the Sharpe–Lintner–Mossin theory is not being strictly tested. If the proxy for the market portfolio is not a valid surrogate, then testing the CAPM using existing empirical investigations is somewhat beside the point. If the proxy is valid however, then it may be worthwhile pursuing testing procedures.

For the most part of this chapter the implicit assumption will be made, that the JSE–Actuaries Overall Index, a value–weighted index, is a suitable surrogate for the market portfolio. Hence, in the light of Roll's critique, the mean variance efficiency of the JSE–Actuaries Overall Index is really being tested. Consequently, if the test rejects, it cannot be ascertained whether the CAPM is indeed invalid or whether the JSE–Actuaries Overall Index is simply not the true market portfolio.

5.2 UNIVARIATE TESTS

In this section [1] JSE stocks will be under scrutiny. Initially the one-parameter CAPM will be empirically examined. Thereafter several additional factors, namely dividend yield, market capitalization, seasonality and liquidity will be empirically examined to determine whether they have any significant additional influence on returns of JSE stocks. Traditional univariate tests in the CAPM framework will be conducted for each of the factors. As gold share behaviour is of particular interest in the JSE context this sector will be tested separately.

5.2.1 Past studies — a perspective

The first prominent studies concerning tests of the CAPM were conducted by Black, Jensen and Scholes (1972) and Fama and Macbeth (1973). These studies found general support of the CAPM, in particular they both agreed on the linearity of the model and the significance of the systematic risk component. Both studies however confirm that the evidence suggests a slightly higher intercept than the treasury bill rate which is posited as a proxy for the risk–free rate by the CAPM.

Recent attention however has focused on whether additional explanatory factors are relevant in asset pricing models. Since this area is of central importance in this section it will be instructive to consider some of the previous studies on this topic.

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[1] This section has been published in the *South African Journal of Business Management* in a paper entitled "Asset pricing in small markets — the South African case." (see Bradfield, Affleck–Graves and Barr (1988)).

Dividend yield effects

The controversy surrounding dividend yield effects stems from the effect of differential tax rates on capital gains and dividends. Since dividends are generally more heavily taxed than capital gains, the question arises of whether or not an investor who 'tilts' his portfolio towards low yield securities is increasing or decreasing his expected after tax return. Brennan (1970) and Litzenberger and Ramasmany (1979) argue that the higher a stock's dividend yield, the higher the pre-tax return a taxable investor requires to compensate for the tax liability incurred (holding risk constant). Miller and Scholes (1978) however present a counter argument whereby investors can effectively transform dividend income into capital gains. For example, sufficient leverage of an equity portfolio can create interest expenses that can be used to offset the dividend income entirely. Further any unwanted risk in this levered position can be removed by the purchase of whole life insurance which contains a tax deferred investment component. They therefore argue that if investors are using these or similar tax shelters then the pre-tax rate of return on dividend paying stocks may not differ from the rate of return on stocks paying no dividends at all. This implies that tax exempt institutions that shift their portfolios toward high yielding stocks do not enjoy the benefits of higher pre-tax returns.

Black and Scholes (1974) pioneered the empirical testing of the effects of dividend yield on common stock return. Their test involved the addition of a dividend payout term in the empirical version of the CAPM

$$R_j = \gamma_0 + \gamma_1\beta_j + \gamma_2(\delta_j - \delta_m)/\delta_m + \epsilon_j$$

where,

R_j is the rate of return on the jth portfolio,

γ_0 is an intercept term which should be equal to the risk-free rate, r_f, according to the CAPM,

γ_1 is the expected market premium, according to the CAPM,

β_j is the systematic risk of the jth portfolio,

γ_2 is the additional factor effect coefficient (i.e. the dividend yield effect coefficient here),

δ_j is the additional factor measure of the jth portfolio, (i.e. the dividend yield of the jth portfolio here),

δ_m is the additional factor measure of the market portfolio, (i.e. the dividend yield of the market here),

ϵ_j is the error term.

The results of Black and Scholes (1974) are summarized in table 5.1.

Table 1 Results of the Black-Scholes test for dividend effects

Period	$\alpha_2 = \hat{\gamma}_2$	$t(\alpha_2)$
1936–66	0.0009	0.94
1947–66	0.0009	0.90
1936–46	0.0011	0.54
1947–56	0.0002	0.19
1957–66	0.0016	0.99
1940–45	0.0018	0.34

On the basis of table 5.1 Black and Scholes conclude that over the entire period as well as for every sub-period under study the estimate of γ_2 is not significantly different from zero. This implies that the expected returns on high yielding securities are not significantly different from the expected returns on low yielding securities for the same level of risk. Note that a significantly negative value for γ_2 implies that high yielding stocks have lower expected returns (as their prices are bid up) than low yielding stocks of equal risk. This in turn implies a preference for high yielding stocks on the part of investors. Alternatively a significantly positive value for γ_2 would imply an aversion for dividends on the part of investors.

Several other researchers have continued to sift through the data in the hope of unearthing evidence to reveal why so important a tax penalty on dividends should have left so small a trace in the data. Among these researchers are Long (1978), Litzenberger and Ramaswamy (1979, 1980, 1982), Rosenberg and Marathe (1979), Stone and Bartter (1979), Blume (1980), Gordon and Bradford (1980), Miller and

Scholes (1982) and Keim (1985). While many of the above researchers do uncover evidence of a positive significant dividend yield effect, they do not, in general attribute the significant yield effects to taxes. Rather it is suggested that other anomalies such as the size effect (discussed below) may interact with the dividend yield effect.

The size effect

Several recent studies on the NYSE have documented evidence of a significant relation between common stock returns and the market value of common stock. This has become known as the 'size effect'. Banz (1981) was one of the first to investigate this relationship. His study was similar in spirit to that of Black and Scholes (1974). The empirical form of the CAPM with the additional factor shown in the above section was also used. For the δ_j above, Banz used the market value of security j and for the δ_m he used the average market value. The time series of gammas obtained from the cross-sectional regressions were also regressed on the excess returns of the market index to obtain the final estimators. Table 5.2 shows a summary of the results obtained by Banz (1981).

Table 5.2 Results from the Banz study on size effects

Period	$\widehat{\gamma}_2$	$t(\widehat{\gamma}_2)$
1936–75	-0.00052	-2.92
1936–55	-0.00043	-2.12
1956–75	-0.00062	-2.09
1936–45	-0.00075	-2.32
1946–55	-0.00015	-0.65
1956–65	-0.00039	-1.27
1966–75	-0.00080	-1.55

From table 5.2 it is evident that all the signs of the $\widehat{\gamma}_2$ (i.e. the size factors) are negative and most of these coefficients are significant. Banz (1981) therefore finds a negative statistical association between returns and firm size. This clearly implies that shares of firms with large market values have had smaller returns on average than small firms in similar risk classes.

Reinganum (1981), using a different method over the 1963-77 period also found that portfolios of small firms had higher risk–adjusted returns than portfolios of larger firms. Blume and Stambaugh (1983) however show that the technique used in Reinganum's study produces an upward bias on estimates of small–firm portfolios returns due to a 'bid–ask' bias that is inversely related to firm size. They show that avoidance of this bias results in the size premium being halved in magnitude. Results presented by De Villiers *et al* (1986) interestingly reveal that no size effect exists on the JSE.

The seasonal effect

In the light of the above findings, attempts to explain the size phenomenon on the NYSE have focused on trading in the month of January.

This concern has prompted researchers to investigate seasonal effects on stock returns. In particular, Rozeff and Kinney (1976), Keim (1983) and Roll (1983) find that NYSE stock market returns are, on average higher in January than any other month of the year. Corhay, Hawawini and Michel (1987) summarize results from international stock markets, in particular they examine the New York, the London (LSE) the Paris (PSE) and the Brussels (BSE) stock exchanges. They find seasonality in returns exist in all of these exchanges in January. In addition significant seasonality was found in other months as well. The LSE exhibited additional seasonality in April, the PSE in July and the BSE in February, April, June, July and October as well.

Attempts to explain the January effect have been, at most, partially successful. The most popular hypothesis attributes the effect to year–end tax–loss selling. This hypothesis assumes that shares whose prices decreased during the year would be subject to selling pressure towards the end of the year. Consequently prices are depressed prior to the end of the year and rebound at the beginning of January. Keim (1986) reviews evidence in the literature which attempts to explain the January effect. Keim further suggests that liquidity or payroll effects may influence monthly returns.

The liquidity effect

One may well expect the size effect to be related to a liquidity effect, in that the smaller the firm the less liquid its stock is expected to be. Chi-Cheng, Reilly and Wong (1985) give empirical evidence in support of the above statement. They suggest that liquidity could be one of the missing factors in the CAPM because of the close relation between the size and liquidity effects. An issue of interest therefore is to determine to what extent, if any, the liquidity of a capital asset affects its pricing structure.

While the concept of liquidity is straightforward the problem of finding a suitable measure of liquidity that the practitioner can easily apply is not. Several measures of liquidity have been used by the financial community. These measures are usually formulated as ratios of asset price changes to trading volume changes. The main shortfall of measuring liquidity in this way is that asset price changes not only reflect the liquidity effect but also reflect changes that occur due to the arrival of new information, thus biasing measures of this type. Tinic (1972) suggests the use of the bid-ask spread as a proxy for liquidity. He justifies its use by saying:

"Since specialists supply liquidity (illiquidity) service under conditions of uncertainty, the difference between the bid and the ask prices, twice the specialists price for supplying liquidity, must represent not only the technical efficiency of inventory management but also the extent of the prevailing risks and the ability of the dealer to assume these risks."

Very little empirical evidence concerning tests of the liquidity effect on stock return has been documented to date. In a recent paper by Amihud and Mendelson (1986) empirical evidence was found which indicates that liquidity (the bid-ask spread) is significantly related to return in the CAPM framework. The implication of this finding is that investors should require a higher expected return for less liquid stocks in order to compensate them for the higher cost of trading. That is, the price of less liquid assets should be bid down in order to yield higher returns. Furthermore since the cost associated with the bid-ask spread has to be borne only once over a holding period, investors will tend to hold high spread stocks for longer periods. Investors with shorter holding periods however will be willing to pay more to acquire the low spread securities. Real estate or stamps for example have after-commission returns too low for short-term holding, but may provide superior performance over longer holding periods. This suggests the existence of liquidity 'clienteles' for the various assets. Long term investors for example may prefer more illiquid assets provided they promise higher returns.

5.2.2 Empirical evidence

In this section the applicability of the CAPM for JSE stocks will be empirically investigated. The four additional factors reviewed above, namely, dividend yield, market capitalization, seasonality and liquidity effects will thereafter be tested in the context of the JSE. In addition, due to the unique nature of the gold share market, gold shares will be tested separately under the above headings.

Methodology

The empirical tests used in most of the remainder of this study are based on the well-known testing procedures suggested by Black, Jensen and Scholes (1972), Fama and Macbeth (1973) and Black and Scholes (1974), where expected return is modelled as a linear function of market risk, β , with an additional factor δ representing the effect being investigated. The approach adopted involves splitting the data into three separate time periods. Data in the first time period are used to group the individual securities into portfolios on the basis of the additional factor and the security beta. The relevant parameters of the portfolios are then estimated using data from the second period. Finally cross-sectional regressions are run on data from the third time period.

The empirical analog used is thus

$$R_{pt} = \gamma_{0_t} + \gamma_{1_t}\widehat{\beta}_{p,t-1} + \gamma_{2_t}\left[\frac{\widehat{\delta}_{p,t-1} - \widehat{\delta}_{m,t-1}}{\widehat{\delta}_{m,t-1}}\right] \tag{5.1}$$

The specifics of the approach are as follows:

The securities are assigned to one of 20 portfolios containing a similar number of securities as follows. Firstly the securities are ranked according to their estimates of the additional factor and divided into 4 groups. Thereafter each group is further subdivided into 5 portfolios [2] on the basis of their ranked β . Four years data are used for the initial estimation of β and additional factor measures for the

[2] Due to the smaller number of gold shares available only 12 portfolios were constructed for

construction of the portfolios. The next four years data are used to re-estimate these statistics for the 20 portfolios. The cross sectional regression is then performed at each 4 week period of the subsequent year. Thus 13 cross sectional regressions were performed in each testing period of one year.

In order to help remove the effects of interpreting market induced positive and negative $\hat{\gamma}_{i,t}$ for the series of cross-sectional regressions, the widely used technique suggested by Black and Scholes (1974) of regressing the time series of gammas on the excess return of the market is used. This correction involves running the following time series regression in each testing period

$$\hat{\gamma}_{2,t} = \alpha_2 + \eta_2(R_{m,t} - R_{f,t}) \tag{5.2}$$

where $R_{m,t}$ is the return on the market at time t ; $R_{f,t}$ is the risk free rate: η_2 is the slope coefficient. The $\hat{\alpha}_2$ is then interpreted as the final estimator for $\hat{\gamma}_2$.

The Data

A sample of 100 stocks listed on the JSE was chosen using a systematic sampling technique to ensure that no sector or well traded security was favoured in the selection procedure. The extracted data consisted of weekly prices from 1 January 1973 to 31 December 1984. Only securities having a record of prices for 1973 through to 1984 were considered for selection. A series of returns taken at 4-week intervals was then used for the study. B1.72.

For the tests on gold shares all available gold shares having a series of prices over the period of study were used. The JSE Actuaries Overall Index and the All-Gold Index were used to proxy the 'market', while the 12 month fixed deposit rate at major commercial banks was used to proxy the risk-free rate.

Table 5.3 shows the breakdown of the time periods used in the empirical tests.

5.2.2.1 The one-parameter CAPM

The JSE as a whole:

The one parameter CAPM tests were conducted on the systematic sample of 100 stocks. The model suggests a positive relation between expected stock return

testing the gold share market separately, i.e. the additional factor was ranked and divided into 4 groups whereafter each group was subdivided into 3 portfolios on the basis of their ranked betas.

Table 5.3 Data periods

	Period				
	1	2	3	4	5
Portfolio formation period	1973–75	1973–76	1974–77	1975–78	1976–79
Initial estimation period	1976–79	1977–80	1978–81	1979–82	1980–83
Testing period	1980	1981	1982	1983	1984

and market risk, beta, and hence for these tests the γ_2 term in equation (5.1) is set equal to zero in the cross-sectional regressions of the beta sorted portfolios. Otherwise the methodology is identical to that described above.

The results of the tests that γ_1 is not significantly different from the market premium and that γ_0 is not significantly different from the risk free rate are summarized in table 5.4.

Table 5.4 The JSE as a whole

Period	$\overline{\widehat{\gamma}_0 - R_f}$	$t(\overline{\widehat{\gamma}_0 - R_f})$	$\overline{\widehat{\gamma}_1 - (R_m - R_f)}$	$t(\overline{\widehat{\gamma}_1 - (R_m - R_f)})$	$\overline{R}^2$
1973–80	0.0074	0.699	-0.0042	-0.393	0.234
1973–81	0.0229	1.467	-0.0148	-0.885	0.166
1974–82	-0.0247	-1.259	0.0239	1.025	0.305
1975–83	0.0093	0.581	0.0032	0.212	0.218
1976–84	0.0008	0.030	-0.0052	-0.197	0.249

The periods indicated include the formation, estimation and testing periods.

Columns 2 and 4 show the resulting coefficients minus their hypothesized value averaged over each testing period. The coefficients are obtained from the cross sectional regressions in each month while the subtracted values (i.e. the R_f and $R_m - R_f$) are the actual realized risk free rate and market premiums respectively in each corresponding month. Under the null hypothesis these values are expected to be equal to zero. [3]

Inspection of the t–statistics in table 5.4 show that for all 5 periods none of the t–statistics are statistically significant at the 5% level. This implies that the γ_1 of equation (5.1) is not significantly different from the market premium. Similarly that the γ_0 in (5.1) is not significantly different from the risk-free rate. This evidence gives support to the validity of the one–parameter CAPM for JSE stocks.

The conclusion regarding the JSE slope coefficient (γ_1) is similar to that reached in many of the U.S. studies (e.g. Fama and Macbeth (1973)). However it is worth noting that the results for the intercept term (i.e. γ_0) differ somewhat from those of Banz (1981), who obtained a value of 0.0045 for $\overline{\gamma_0 - R_f}$ with an associated t-value of 2.76 for the 1936-1975 period on the NYSE. He thus concluded that γ_0 was different from the assumed risk-free rate, namely the 3 month U.S. Treasury Bill (T.B.) rate. Black, Jensen and Scholes (1972) and Black and Scholes (1974) also found evidence on the NYSE that the intercept term in the CAPM is different from the U.S. 3 month T.B. rate. However they argue that γ_0 is still an appropriate risk free rate, namely the expected return on a zero beta portfolio, but that this is different from the 3 month T.B. rate.

Further insight surrounding the linear relationship between return and beta posited by the CAPM can be found by considering the regressions of the time series of $\hat{\gamma}_1'$s on the market premium. This widely used technique suggested by Black and Scholes (1974) is similar to the one described in the methodology section, except that the dependent variable in this instance is γ_1 , where

$$\hat{\gamma}_{1,t} = \alpha_1 + \eta_1(R_{m,t} - R_f) \tag{5.3}$$

Under the null hypothesis η_1 is expected to be equal to 1 as the series of $\hat{\gamma}_1$ are theoretically estimates of the market premium.

Table 5.5 shows the results of these corrected regressions.

[3] The t statistics shown in table 5.4 are calculated using

$$t(\overline{\hat{\gamma}_j - r}) = \frac{\overline{\hat{\gamma}_j - r}}{S(\hat{\gamma}_j - r)/\sqrt{n}}$$

where r is equal to R_f in column 2 and $R_m - R_f$ in column 3. Column 6 shows the average R^2 of the 13 cross sectional regressions in each testing period.

Table 5.5 The JSE as a whole

Period	$\hat{\eta}_1$	$t(\hat{\eta}_1)$	R^2
1973–80	1.008	5.191	0.729
1973–81	0.890	2.586	0.378
1974–82	1.028	4.122	0.607
1975–83	1.041	4.591	0.657
1976–84	0.674	1.306	0.134
1973–84	0.988	7.986	0.507

The periods indicated include the formation, estimation and testing periods.

From table 5.5 all the $\hat{\eta}_1$, with the exception of period 5, are seen to be very close to unity and highly significant. Over all the testing periods $\hat{\eta}_1$ was found to be 0,988 with a t-statistic of 7,986. This evidence further supports the validity of the CAPM for the JSE.

The results obtained in this section therefore lead one to conclude that the data from the JSE appears to be consistent with the one-parameter CAPM. In addition, unlike many US studies, the results show that in the JSE context, the 12 month fixed deposit rate at major commercial banks appears to be a reasonable surrogate for the risk free rate.

The Gold Share Market on the JSE

The foregoing tests were repeated using all 45 gold shares available over the same period. For the sake of brevity, only the resulting coefficients of the time series of $\hat{\gamma}_1$ regressed on the market premium from (5.3) will be shown in table 5.6. The average of the coefficients corrected by their hypothesized values and their associated t–statistics for each period, are tabulated in Appendix 1 (table A1).

The tests were repeated using the JSE Actuaries All-Gold Index as well as the JSE Actuaries Overall Index and the results are summarized in table 5.6.

The results using the All Gold Index are essentially the same as those obtained using the Overall Index. However, they differ substantially from the results obtained

Table 5.6 Gold Shares — Rand Returns

	Period	$\hat{\eta}_1$	$t(\hat{\eta}_1)$	R^2
All–Gold Index	1973–80	0.133	0.829	0.059
	1973–81	0.491	1.124	0.103
	1974–82	–0.278	–1.106	0.100
	1975–83	0.754	3.214	0.484
	1976–84	0.517	1.374	0.147
Overall Index	1973–80	0.344	1.431	0.157
	1973–81	0.833	1.792	0.226
	1974–82	–0.451	–1.073	0.095
	1975–83	0.638	1.676	0.203
	1976–84	–0.110	–0.247	0.005

for common stocks. In both cases the $\hat{\eta}_1$ are not as close to 1 as was the case for common stocks, and surprisingly, even negative $\hat{\eta}_1$ coefficients are evident. In fact in only one of the periods (i.e. 1983 for the All-Gold Index) was the $\hat{\eta}_1$ significantly different from zero. It thus appears as if the related market risk, i.e. β has not been highly correlated with return for gold shares. This finding may be due to the fact that gold shares are not priced predominantly by local investors but by foreign investors in the SA gold market.

In order to ascertain whether the influence of USA investment in gold shares is in line with the CAPM when expressed in dollar returns, the study was repeated converting gold share prices and the relevant indices to dollars. (Adjustments were made for the Financial Rand and Securities Rand discount when applicable.)

Results of the cross-sectional regressions using (5.3) with dollar return estimates are shown in table 5.7. (Table A2 in the Appendix A2 contains the details relating to the cross-sectional regressions.)

Comparing table 5.7 (dollar return statistics) with table 5.6 (rand return statistics) it is evident that the $\hat{\eta}_1$ are now closer to the theoretical value of unity with

Table 5.7 Gold Shares — Dollar Returns Study

	Period	$\hat{\eta}_1$	$t(\hat{\eta}_1)$	R^2
All–Gold Index	1973–80	0.223	0.223	0.126
	1973–81	0.680	2.266	0.318
	1974–82	0.091	0.297	0.008
	1975–83	0.671	4.253	0.622
	1976–84	0.536	2.027	0.272
Overall Index	1973–80	0.248	0.817	0.057
	1973–81	0.765	1.226	0.120
	1974–82	0.316	0.883	0.066
	1975–83	0.801	4.276	0.624
	1976–84	0.644	2.547	0.371

higher t-statistics than those obtained using rand returns. The results obtained using both indices are again essentially similar. The hypothesis that η_1 is equal to 1 (i.e. that the one parameter CAPM is valid) is only rejected for the 1980 and 1982 testing periods for both indices. It is interesting to note that these 2 years represent the major bull and bear phases of the gold share market respectively, with the gold price reaching \$800 an ounce in the 1980 testing period while plummeting to a low of \$296 an ounce in June during the 1982 testing period. It is thus evident that during these extreme periods the dollar pricing of gold shares was not consistent with the *ex ante* beta predictors.

Comparing these results to the results obtained for common stocks (table 5.5) it can be seen that, although there is a marked improvement when returns are measured in dollar terms, the results nevertheless indicate that the CAPM is a more appropriate model for common stocks in the JSE context than for gold shares.

The relative weakness of the CAPM in the case of gold stocks may, as mentioned earlier, revolve around the large foreign holding of gold shares. During the period under consideration several exchange rate regimes have been operative and the exchange rate itself has been volatile. These problems have given rise to risk factors for foreign holders of gold shares, over and above those faced by local investors, and have been instrumental in the instability of the risk assessment of S.A. gold shares by foreigners.

5.2.2.2 The dividend yield factor

The empirical test for a dividend yield effect was based on estimating the γ from cross-sectional regressions using equation (5.1) as is explained in the section headed 'methodology'. The *ex ante* dividend yield measures were taken to be the total dividends per share paid during the previous year, divided by the share price at the end of that year. Estimates of the dividend yield of a portfolio (i.e. $\widehat{\delta}_{p,t-1}$) were inflation' computed as the average dividend yield of the components securities. Estimates of the dividend yield of the market i.e. $\widehat{\delta}_{m,t-1}$ were similarly computed.

The JSE as a whole:

The systematic sample of 100 stocks was used to test the dividend yield effect. The average of the $\widehat{\gamma}_2$ and the associated statistics obtained from the 13 cross-sectional regressions using equation (5.1) in each testing period are shown in table A3 in the Appendix A3.

The final estimates of the γ_2 , i.e. the $\widehat{\alpha}_2$ obtained from the time series regression of the γ_2 on the market premium in equation (5.2), are shown in table 5.8.

Table 5.8 Dividend yield effects — JSE as a whole

Period	$\widehat{\alpha}_2$	$t(\widehat{\alpha}_2)$
1973–81	–0.0174	–0.011
1974–82	0.0048	1.265
1975–83	0.0028	0.275
1976–84	0.0031	0.417
1973–84	–0.0004	-0.043

The results shown in table 5.8 can be compared directly to the results obtained by Black and Scholes (1974) on the NYSE shown in table 5.1. The results obtained here are almost identical in magnitude and spirit to the the results obtained by Black and Scholes (1974). The column of $\widehat{\alpha}_2$ indicates the dividend yield effect and none of the t–statistics of the $\widehat{\alpha}_2$ are significantly different from zero.This implies that the expected returns on high dividend yield securities on the JSE are not significantly different from the expected returns on low dividend yield securities on the JSE, other things being equal. For the whole period the $\widehat{\alpha}_2$ is equal to -0.0004, which is approximately 0.5% per year, which is nowhere near the level that would make the tax penalty significant. By comparison Black and Scholes (1974) obtained a value of 0.0009 for $\widehat{\alpha}_2$ with a t-statistic value of 0.94 over their entire period of study on the NYSE. In addition the emergence of some negative estimates of $\widehat{\alpha}_2$ in table 5.8 does not imply that these inferences are any different from the inferences of Black and Scholes on the NYSE.

The implication of these findings for the JSE investor are that higher yielding shares have not had their prices bid down to reflect higher expected returns in compensation for the tax penalty on dividends. In other words the expected returns of high-yielding stocks on the JSE are not essentially different from those of low-yielding stocks for the same level of risk.

The Gold Share Market on the JSE:

The tests were repeated on all gold shares for which there was a complete record available of the relevant information over the 1973-84 period. These amounted to 39 shares. Due to the small sample, only 9 intermediate portfolios were formed. This portfolio formation involved grouping the securities into 3 portfolios on the basis of their ranked dividend yield and dividing each of these portfolios into a further 3 portfolios on the basis of their ranked beta's. The average $\widehat{\gamma}_2$ obtained from the 13 cross–sectional regressions in each testing period are shown with their relevant statistics in the Appendix A4 (table A4).

The final estimates of the dividend yield effect i.e. the $\widehat{\alpha}_2'$s obtained from (5.2) are shown in table 5.9.

All of the $\widehat{\alpha}_2$ in table 5.9 are negative but not significant at the 5% level of

Table 5.9 Dividend yield effects — Gold Shares

Period	$\widehat{\alpha}_2$	$t(\widehat{\alpha}_2)$
1973–81	–0.0135	–0.807
1974–82	–0.0099	–0.550
1975–83	–0.0260	–1.291
1976–84	–0.0102	–1.484
1973–84	–0.0100	–1.010

significance. The fact that the $\widehat{\alpha}_2'$s are consistently negative may imply that gold shares having high dividend yields have lower expected returns than low yielding gold shares with the same level of risk. This would be consistent with the hypothesis that investors prefer higher yielding gold shares. Thus, although these effects are not significant at the 5% level the results do suggest that there may be a slight preference for high dividend yielding stocks in the gold sector of the JSE.

The problem of unstable $\widehat{\beta}$ and the poor performance of $\widehat{\beta}$ as predictors of return for gold shares has already been cited. It is possible that other components of gold mine risk which are not captured by the $\widehat{\beta}$ may be partly manifested in the dividend yield effect. Perhaps factors such as the life of the mine, grade of ore or the working costs are related to dividend payouts. Consequently investors may be paying more for certain stocks in order to avoid these possible components of risk which may not have been fully captured by the shares $\widehat{\beta}$, and this choice is manifesting itself in the dividend yield term. Furthermore the fact the US investors did not have to take dividends out of SA through the financial rand when it was operable could well have caused US investors to migrate towards higher yielding gold shares.

In summary, from the empirical evidence it must be concluded that there does not appear to be a significant dividend yield effect on the JSE. Thus it does not appear that, on average, either high yielding or low yielding stocks trade at a premium. However in the case of the gold shares it is possible that a slight preference for high yielding stocks may exist. Whether this is because of confounding factors such as specific mine characteristics and exchange rate policy or because of a genuine preference for high yield gold stocks, remain an open question.

5.2.2.3 The market capitalization (size) factor

The method used for calculating the ex-ante measures of size was the same as that used by Banz (1981) - that is, the stock price multiplied by the number of shares outstanding at the end of the period. The estimates of the size of the portfolios were computed as the average of the size of all component securities (i.e. $\widehat{\delta}_{p;t-1}$) and the size of the market ($\widehat{\delta}_{m;t-1}$) was similarly calculated.

The JSE as a whole:

The data used here was again the systematic sample of 100 JSE stocks. The 13 cross-sectional regressions using equation(5.1)were again run in each testing period using the series of portfolio estimates of β and δ as discussed in the methodology section.

The resulting average $\widehat{\gamma}_2$ are shown in the Appendix A5 (table A5). Again the final estimators of the γ_2 , i.e. the $\widehat{\alpha}_2$, are obtained by running the usual regression correction using equation (5.2). The results are shown in table 5.10.

Table 5.10 Size effects - JSE as a whole

Period	$\widehat{\alpha}_2$	$t(\widehat{\alpha}_2)$
1973–81	-0.0083	-1.200
1974–82	0.0028	0.936
1975–83	-0.0026	-1.695
1976–84	0.0032	1.109
1973–84	-0.0017	-0.169

Table 5.10 shows a negative value for the size effect $(\hat{\alpha}_2)$ over the whole period - but it is not statistically significant. Further no consistency in sign nor any significant t-statistics for $\widehat{\alpha}_2$ can be found for any of the sub-periods. This

implies that there does not appear to be a small firm effect on the JSE. In other words investors on the JSE do not appear to pay more for large firms than small firms given the same level of risk.

In contrast studies on the NYSE have found a small but significant negative size effect for NYSE stocks. The widely quoted paper by Banz (1981) for example documents a value of -0.00052 with a t–value of -2.92 for $\widehat{\alpha}_2$ for the 1936–1975 period (see table 5.1). While not all of his subperiods have significant $\widehat{\alpha}_2$ they are consistently negative in sign. As was mentioned previously, the issue of whether size *per se* or some anomaly related to size is responsible for this effect on the NYSE has not yet been resolved.

The Gold Share market on the JSE:

The same testing procedure was repeated for the gold shares. The details of the cross-sectional regressions are shown in the Appendix (table A4).

Table 5.11 shows the final estimators of the 'size' effect, i.e. the $\widehat{\alpha}_2$ obtained from the regression correction equation (5.2).

Table 5.11 'Size' effects — Gold shares

Period	$\widehat{\alpha}_2$	$t(\widehat{\alpha}_2)$
1973–81	-0.0020	-0.334
1974–82	0.0004	0.073
1975–83	0.0010	0.126
1976–84	0.0021	0.696
1973–84	-0.0010	-0.122

From inspection of table 5.11 it can be seen that there is clearly no evidence of a size effect measured by the $\widehat{\alpha}_2$ for gold shares. Indeed, if anything the t–statistics are smaller for the gold shares as a group than for the overall sample representing the JSE as a whole.

Thus it is concluded that, unlike the NYSE, there does not appear to be a significant size effect on the JSE. This is true for both the overall market and the gold sector.

5.2.2.4 The seasonal factor

Most factors concerned with seasonality in stock returns consider this concept in conjunction with the size effect. Since no evidence of size effect was found above, the hypothesis of seasonality in JSE stock returns will be investigated separately.

The testing procedure outlined below will concentrate on investigating whether seasonality exists in equally weighted indices constructed for this purpose. Due to the unique nature of the JSE, an equally weighted mining share, industrial share and all–share index was constructed over the period January 1974 to December 1984. All shares that had reliable records over this period were included in the respective indices. This amounted to 112 mining shares, 357 industrial shares and 469 shares in total in the respective indices.

Weekly returns for each of the above–mentioned indices were computed, and subsequently partitioned and averaged within each of the relevant months of the year. This amounted to 11 data points associated with each of the 12 months for each of the three indices. This procedure is similar to the procedure outlined by Corhay, Hawawini and Michel (1987). It should be noted that the framework of this testing procedure is different from the procedure used in the remainder of this section (i.e. section 5.2) and does not constitute a test within the usual CAPM framework, mainly because the entire market indices rather than individual stock returns are examined.

The JSE as a whole:

The results [4] of the equally weighted all–share index constructed from the 469 JSE stocks are shown in table 5.12. Table 5.12 also includes results from various other stock exchanges conducted over the 1970–1983 period quoted in Corhay, Hawawini and Michel (1987) for comparative purposes.

The result for JSE stocks in table 5.12 is indeed surprising. The table shows an insignificant January effect for JSE stocks in the face of overwhelming evidence of a

[4] The average weekly returns in each month were converted to monthly returns for ease of comparison with the international evidence.

Table 5.12

	NYSE	LSE	PSE	BSE	JSE
Number of listed firms	1490	2170	504	197	671
Sample size	782	527	112	170	469
January	5.08*	5.49*	4.10*	3.99*	2.45
	(2.10)	(2.04)	(2.40)	(4.08)	(1.09)
February	0.78	2.21	0.41	1.86*	2.12
	(0.74)	(1.58)	(0.36)	(2.72)	(1.52)
March	1.51	0.73	1.59	0.37	0.41
	(0.94)	(0.42)	(0.75)	(0.49)	(0.22)
April	0.57	4.19*	1.17	1.93*	0.78
	(0.36)	(3.31)	(1.19)	(2.71)	(0.76)
May	–0.70	–0.48	–0.69	–0.08	2.19
	(–0.50)	(–0.41)	(–0.45)	(–0.13)	(1.41)
June	0.70	–1.39	–1.56	1.06*	0.68
	(0.59)	(–0.96)	(–1.11)	(2.06)	(0.32)
July	0.92	1.22	3.92*	1.76*	2.89*
	(0.67)	(1.18)	(2.76)	(2.98)	(2.02)
August	1,06	1.13	1.96	0.28	2.57
	(0.66)	(0.83)	(1.52)	(0.36)	(1.21)
September	0.31	–1.04	–0.40	–0.94	1.39
	(0.23)	(–0.52)	(–0.21)	(–1.17)	(0.67)
October	–0.91	–0.07	–1.76	–1.48*	1.25
	(–0.42)	(–0.06)	(–1.24)	(–2.48)	(1.34)
November	2.25	–0.06	–0.27	–0.70	–0.36
	(1.22)	(–0.03)	(–0.22)	(–0.98)	(–0.26)
December	1.48	1.62	0.37	1.58*	2.78*
	(1.31)	(1.06)	(0.41)	(1.58)	(2.96)

t–statistics are in parenthesis. They are computed as $t(\overline{R}) = \bar{R} \cdot \sqrt{n}/\sqrt{\sigma}(R)$ where n = number of years.

*significant at the 5% level for a one–sided alternative.

January effect in international markets. A significant July and December seasonal effect is however evident for JSE stocks. These results will be compared with results for mining and industrial shares where explanations for these effects are offered.

For the moment, it is worth considering the results from the international perspective. The January effect on the NYSE has received much attention in literature and it is thus also conceivable that the significant January seasonals in the other international stock markets could be explained by similar hypothesis. Furthermore the July seasonal on the PSE is likely to be caused by dividend payments, approximately two–thirds all dividend payments in France are paid in July, according to Hamon (1986). The high incidence of significant monthly seasonals on the BSE, is however difficult to explain. A plausible cause of this may be related to the relative size of the stock markets. Relative to the world market capitalization [5] the BSE comprises 0.32% in contrast to 40%, 6% and 1% for the NYSE, LSE and PSE respectively. Furthermore, substantially fewer shares are quoted on the BSE, namely 197 compared to 1490, 2171 and 504 on the NYSE, LSE and PSE respectively. Issues like non–synchronous and lacklustre trading on smaller markets may therefore influence the outcome on these markets.

The mining and industrial sectors on the JSE:

The results for the equally weighted mining and industrial indices are shown in table 5.13. Results for the all–share index are included for comparative purposes.

From table 5.13 it can be seen that the December seasonal is significant for all three data sets and that the seasonal for July is significant at the 2.5% level for the mining index, while that for October is significant at the 5% level for the industrial index. Strictly speaking the test should have a 2–sided alternative hypothesis to incorporate significantly poor months in the alternative hypothesis. This change would mean that the above significance levels are effectively doubled. [6] Thus the only other significant month at the 5% level besides December, occurs for mining stocks during July.

Since dividends were not included in this study, the statistical significance is unlikely to be caused by dividends. Furthermore dividend announcements are usually made in February or March for midyear dividends and August or September for end of year dividends. Hence the seasonal effect is unlikely to be caused by issues

[5] Source: Corhay, Hawawini and Michel (1987).

[6] The one–sided alternative t–values are used to be consistent with the Corhay *et al* evidence.

Table 5.13 Monthly returns for all–share, mining and industrial indices

	Equally Weighted Indices								
	All–share			Mining			Industrial		
Months	$\overline{R}$%	$t(\overline{R})$	$\sigma(R)$%	$\overline{R}$%	$t(\overline{R})$	$\sigma(R)$%	$\overline{R}(\%)$	$t(R)$	$\sigma(R)$%
January	2.45	1.09	7.45	1.36	0.38	12.00	2.83	1.44	6.54
February	2.12	1.52	4.64	2.34	0.88	8.84	2.10	1.67	4.16
March	0.41	0.22	6.22	0.65	0.24	9.26	0.39	0.23	5.72
April	0.78	0.76	3.43	0.52	0.28	6.09	0.87	0.90	3.20
May	2.19	1.41	5.13	1.86	0.91	6.74	2.28	1.55	4.90
June	0.68	0.32	7.03	0.70	0.22	10.40	0.71	0.38	6.23
July	2.89	*2.02	4.74	5.99	**2.84	7.00	1.88	1.22	5.11
August	2.57	1.21	6.88	2.59	0.66	13.02	2.47	1.43	5.72
September	1.39	0.67	6.85	3.99	0.99	13.38	0.52	0.31	5.64
October	1.25	1.34	3.10	–0.70	–0.35	6.62	1.77	*1.87	3.15
November	–0.36	–0.26	4.66	–1.15	–0.56	6.84	–0.23	–0.16	4.80
December	2.78	**2.96	3.11	4.56	**2.28	7.23	2.01	**2.42	2.77

*significant at the 5% level for a one–sided alternative.

**significant at the 2.5% level for a one–sided alternative.

relating to announcements. It is possible that the July effect evident for mining stocks is a consequence of the particular sample period. However the significant December effect occuring in all 3 indices is unlikely to be explained away as easily.

It is well known that thin and lacklustre trading on the JSE is characteristic of December, which is traditionally the holiday season in South Africa, and that this impacts on the volatility of stocks over this month. In particular, thin trading gives rise to non–synchronous prices being recorded, causing the variance of these returns to be underestimated over these periods. For example, if a particular share has not been traded for several weeks, its price will be recorded daily at the last transaction price, whereas the 'true' price varies according to the economic influences. Hence the 'observed' return series will have substantially less variability than the 'true' series. Although this argument can easily explain why the variance of daily or weekly returns in December, relative to other months, is underestimated, it does not fully explain why the series of monthly returns for each December over the sampled period exhibit relatively less volatility. However, the fact that there is unlikely to be large scale selling or buying pressure over the December holiday period may imply less variability in monthly December returns. From table 5.13 it is seen that for the industrial index, December month had the smallest standard deviation. For the all-share index the second smallest standard deviation occurred in December and for the mining index, the December standard deviation was relatively smaller than most months, although not the smallest. Hence in conclusion, it could be argued that the significant seasonal in December is more likely to be a result of relatively less volatility than substantial return in December.

5.2.2.5 The liquidity factor

The problem of finding a suitable computational measure of liquidity that does not reflect the arrival of new information was previously cited and the bid-ask spread was suggested as being a suitable measure. Unfortunately accurate records of bid-ask spreads of listed shares are rarely found and such spreads are not quoted in many non-specialist markets such as the JSE. Roll (1984) has proposed a method for estimating an implicit bid-ask spread directly from a time series of share prices. Roll shows that the percentage bid-ask spread can be estimated from the covariance of successive returns as follows

$$\text{Spread } \% = 200\sqrt{-\text{Cov}(R_t; R_{t-1})} \tag{5.4}$$

where R_t is the return of the share at time t.

The assumptions necessary for the derivation of equation (5.4) are:

(i) the asset is traded in an informationally efficient market; and

(ii) the probability distribution of observed price changes is stationary.

Unfortunately actual bid-ask spread data is not available for JSE stocks and hence the accuracy of Rolls proposed estimator cannot be assessed in this context. However Amihud and Mendelson (1986) report a correlation of 0.242 between the average return and the logarithm of spread on the NYSE over the 1961-80 period. By comparison Bradfield (1986) finds a similar correlation of 0.2408 between the average return and the average estimate of spread using equation (5.4) for 50 gold shares on the JSE over the 1971-1984 period. [7] This at least indicates that Roll's formula may provide a reasonable estimate of the bid-ask spread implicit in JSE stocks.

Rolls' formula for estimating the bid-ask spread was thus used as a proxy for the liquidity effect (δ) in this study. The time series of returns over the formation period, and again over the estimation period were used to obtain the *ex ante* estimates of spread. The estimate of a portfolio's spread, δ_p was taken to be the average spread of its components. The spread of the market, δ_m, was taken to be the average spread of all shares in the sample.

The JSE as a whole:

Again the same initial testing procedure was repeated here using the sample of 100 JSE stocks. The results of the cross-sectional regressions using equation (5.1) are shown in Appendix A (table A5). The final estimators of the liquidity effects i.e. the $\hat{\alpha}_2$ obtained from (5.2) for each test period are shown in table 5.14.

Table 5.14 Liquidity effects - JSE as a whole

Period	$\hat{\alpha}_2$	$t(\hat{\alpha}_2)$
1973–80	–0.005	–0.461
1973–81	–0.001	–0.110
1974–82	–0.023	-1.551
1975–83	–0.009	–0.757
1976–84	0.006	0.526
1973–84	-0.005	-1.121

[7] Roll (1984) further finds evidence that the estimated bid-ask spread is related to firm size. He documents a rank correlation of -0.226 between estimated bid-ask spread and size taken at 5 day intervals on the NYSE. Bradfield (1986) finds an average annual Kendall's rank correlation over the period (1971-1984) of -0.2323 for gold shares taken at weekly intervals on the JSE.

None of the $\widehat{\alpha}_2$ in table 5.14 are seen to be significant. Thus it must be concluded that there is insufficient statistical evidence to infer any liquidity effect on the JSE as a whole.

By contrast Amihud and Mendelson (1986) found significant evidence that the spread effect is positively related to stock return. Their study was conducted over the 1961–80 period on the NYSE using actual bid–ask spread data. They obtain a value of 0.00375 with t–statistic of 3.23 for the spread effect. This implies that returns on high–spread (illiquid) stocks are higher than returns on low–spread (liquid) stocks. It is interesting to note that in four of the five periods examined on the JSE the sign of $\widehat{\alpha}_2$ was negative, implying that any liquidity effect present was in fact in the opposite direction to that found in the Amihud and Mendelson study. This is counter intuitive and hence we can only conclude that either no liquidity effect exists on the JSE or that the bid–ask spread measure proposed by Roll (1984) does not measure the true implicit bid–ask spread on the JSE with sufficient accuracy to detect the effect.

The Gold Share Market on the JSE

The above testing procedure was again repeated for the complete set of 45 gold shares. Here 12 portfolios were constructed, by dividing the shares into 4 groups on the basis of their ranked bid–ask spread estimates, whereafter each group was further divided into 3 portfolios on the basis of their ranked β . This procedure was repeated using both the All–Gold Index and the Overall Index. Table A5 in the Appendix contains details of the correction on the time series of $\widehat{\gamma}_2$ using equation (5.2).

Again the $\widehat{\alpha}_2$ in table 5.15 show no systematic nor significant statistical behaviour. This is consistent with the results obtained using the sample of 100 stocks and hence the overall conclusion of this section is that a liquidity effect does not exist on the JSE.

5.2.3 Conclusion

The results of our analysis based on traditional testing procedures have several implications for the investor on the JSE. In particular, our results imply that the one–parameter CAPM appears to be a reasonable model for the JSE as a whole.

Table 5.15 Liquidity effects – Gold shares

	Period	$\widehat{\alpha}_2$	$t(\widehat{\alpha}_2)$
All–Gold Index	1973–80	–0.005	–0.430
	1973–81	–0.007	–0.433
	1974–82	0.022	0.706
	1975–83	0.009	0.485
	1976–84	–0.001	–0.032
Overall Index	1973–80	–0.009	–0.847
	1973–81	–0.021	–1.493
	1974–81	–0.021	–1.493
	1974–82	0.043	1.211
	1975–83	0.003	0.186
	1976–84	–0.002	–0.193

This implies that *ex ante* estimates of β are successful in systematically predicting stock returns on the JSE given the level of market premium and the risk free rate.

For gold shares, β's do not enjoy the same level of success as predictors of return. This could be due to the interactive pricing of gold shares by both local SA investors and foreign investors who are exposed to different risk factors. The results show a marked improvement in the predictability of β when assessed in dollar terms. This improvement seems to imply that US investors may have been dominant over local investors in the pricing of gold shares over the 1973–1984 period. It is expected that this situation will change due to the subsequent (November 1986) restriction of US investment in South Africa. It will of course be interesting to monitor to what extent β will improve as predictors of Rand returns (that is, from the SA investors' viewpoint) due to this restriction.

Secondly the evidence on the dividend yield effect for the JSE as a whole does not show any significant differences between the returns on high dividend yield stocks and the returns on low dividend yield stocks. In the analysis no account was taken of tax on dividends or capital gains. Hence the implications are that a tax–exempt investor may not gain significantly by selecting high yield stocks over low yield stocks, other things being equal. The implication for corporations is that a change in dividend policy will not be expected to have a definite effect on its stock price. For gold shares there may be some evidence of a slight systematic shift towards a negative dividend yield effect on expected returns. This implies that investors may have a slight preference for high yielding gold shares. However this effect is not statistically significant. A plausible reason for such an effect could be that foreign investors do not have dividend payouts diluted by the financial rand discount while capital gains on the other hand are diluted by the financial rand. This may well induce foreign investors (especially short term investors) to have a preference for higher yielding gold shares. Again this effect is likely to change due to the subsequent restriction on SA gold share purchases.

The size or market capitalization effect documented on the NYSE does not appear to exist on the JSE, for either industrial or for gold stocks. This claim implies that investors cannot expect to earn consistently higher risk adjusted returns by tilting their portfolios either towards small firms or large firms.

The January effect found on most international exchanges was not apparent for JSE stocks. However a December effect was evident. A likely cause for the statistical significance is possibly due to less volatility, caused by thin or lacklustre trading in December rather than substantial return.

Lastly the liquidity effect measured by a proposed estimate of bid–ask spread was not found to have any significant effect on returns of industrial and gold shares on the JSE. Intuitively this result was expected for gold shares which are highly liquid. It should be noted however, that it is not clear whether the results for the JSE as a whole were insignificant due to the absence of a liquidity effect, or whether the proposed measure of estimating the bid–ask spread was inaccurate.

In conclusion the one–parameter CAPM has stood up well to traditional univariate empirical testing. Moreover hypothesized additional parameters, namely; dividend yield, market capitalization and liquidity were not found to significantly effect return. Consequently the CAPM is accepted as a reasonable model in the context of the JSE.

5.3 MULTIVARIATE TESTS

5.3.1 Past studies — a perspective

Although several researchers have considered multivariate testing procedures, they are relatively few by comparison to the number of studies using univariate testing procedures. Only the more recent results will be quoted here. [8]

Shanken (1985) used a cross-sectional regression having a Hotelling's T-squared distribution to test the efficiency of CRSP equally weighted index. On the basis of this test, he rejects the efficiency of the index at the 0,01 level. Gibbons, Ross and Shanken (1986) [9] used a data set similar to that of Black, Jensen and Scholes (1972). Using monthly returns on 10 beta-sorted portfolios from 1931 through to 1965, they obtain an F statistic of 0.96 which has a p-value of 0.48. They therefore conclude that their test cannot reject the SLM version of the CAPM, and that their findings complement the classical findings of Black, Jensen and Scholes (1972). [10] They do however express reservations about the power of the test.

Gibbons, Ross and Shanken (1986) (henceforth GRS) also consider the size effect within their multivariate framework. They construct 10 portfolios ranked and rebalanced on market capitalization every 5 years over the period 1926 through 1982. They obtain a p-value of 0.301 for their test, and comment that given the existing evidence on the size effect on the NYSE, the fact that they were unable to reject the CRSP Value-weighted Index was somewhat surprising. They argue however that their result is consistent with findings in Brown, Kleidon and Marsh (1983) who suggest that the existence of a size effect is dependent on the methodology used.

5.3.2 Methodology and data

The data consisted of all shares listed on the JSE and having a complete price record over the entire period from 1 January 1973 to 31 December 1984. This constituted a total of 390 listed securities.

A series of 12 beta-sorted portfolios were constructed in the usual manner from the data set as follows: Three years data were initially used to estimate the betas of the 390 shares, which were then ranked according to their β and divided into

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8 See section 2.5.2 for further references.

9 Their test statistic is discussed in section 2.5.2.

10 Black, Jensen and Scholes (1972) used a univariate testing procedure.

12 portfolios, whereafter the returns on these 12 portfolios were computed over the next three years. The entire procedure was repeated 3 times, rebalancing portfolios every three years, with the exception that 4 years data was subsequently used in the estimation of the β. This resulted in 9 years return data for the 12 beta–sorted portfolios. Since this section also focuses on additional explanatory effects, 3 additional data sets were constructed over the same period for this purpose. The construction procedure is identical to that outlined above, with the exception that an estimate of the additional factor replaces the role of beta in the outline above. Firstly a liquidity–sorted data set was constructed using the liquidity measure proposed by Roll (1984). This measure was discussed in section 5.2. The same 390 securities were used in the construction of the liquidity–sorted data set, as were used in the construction of the beta–sorted data set. Market capitalization–sorted and dividend yield–sorted data sets were similarly constructed. Due to the difficulty in obtaining complete records of market capitalization and dividend yields for the 390 securities, a systematic sample of 100 securities was selected for the latter 2 data sets. Table 5.16 shows the periods of construction.

Table 5.16

Formation Period	Return computation period
1973–1975	1976–1978
1975–1978	1979–1981
1978–1981	1982–1984

The JSE–Actuaries Overall index was used as a surrogate for the market portfolio (i.e. portfolio p), while the 12 month fixed deposit rate quoted at major commercial banks was extracted from the South African Quarterly Reserve Bank bulletins, and used as a surrogate for the risk free rate, R_{ft}. Returns were computed at 4 week intervals.

The test statistic proposed by Gibbon, Ross and Shanken (1986) to test the efficiency of the market portfolio was computed for each of the data sets.

Recall from section 2.5.2

$$\Gamma_1 = [T/(T-2)][(T-N-1)/N].W$$

has a central F distribution with degrees of freedom N and $T-N-1$ under the null hypothesis that $\widehat{\alpha}_p$ are jointly zero

$$\text{where} \quad W = (1+\widehat{\theta}_p^2)^{-1}\widehat{\alpha}_p'\widehat{\Sigma}^{-1}\widehat{\alpha}_p$$

$\widehat{\theta}_p$ is the ratio of *ex post* average excess return on portfolio p to its standard deviation.

These statistics were computed using the 9 years of data for each of the data sets.

5.3.3 Empirical Results

The beta–sorted data set

Table 5.17 gives the summary statistics on the beta–sorted portfolios based on 4–weekly return data, from 1976 to 1984. $(T = 117,\ N = 12)$. All simple returns used are excess returns, with the 12 month fixed deposit rate at major commercial banks used as a surrogate for the risk–free rate. The JSE–Actuaries Overall Index is portfolio p. The tabulated parameter estimates are for the regression model

$$\widetilde{R}_{it} = \alpha_{ip} + \beta_{ip}\widetilde{R}_{pt} + \widetilde{e}_{it} \qquad \text{for } i = 1,2,\ldots,12 \quad \text{for } t = 1,\ldots,117 \tag{5.5}$$

Comparing this table to the equivalent table in the GRS study, GRS find systematically negative alphas for high–beta portfolios and positive alphas for low–beta portfolios. They comment that these findings are similar to those of Black, Jensen and Scholes (1972), who argue that the expected excess returns on high–beta assets are lower than the CAPM suggests, and that those of low–beta assets are higher than the CAPM suggests. By contrast the evidence on the JSE presented in table 5.17 shows no systematic relationship between the portfolio alphas and betas. The R–squared values are somewhat lower than those documented by GRS. However this reduction is a well known characteristic of JSE stocks, as was discussed previously. Furthermore it is interesting to note that all the alphas are

Table 5.17 Summary statistics for beta–sorted portfolios

Portfolio number*	$\widehat{\alpha}_{ip}$	$t(\widehat{\alpha}_{ip})$	$\widehat{\beta}_{ip}$	$t(\widehat{\beta}_{ip})$	R_i^2
1	0.00715	1.89	0.33	5.34	0.20
2	0.01045	2.19	0.50	6.47	0.27
3	0.00427	1.16	0.52	8.60	0.39
4	0.00793	1.81	0.47	6.66	0.28
5	0.00396	1.12	0.56	9.67	0.45
6	0.00351	1.09	0.59	11.38	0.53
7	0.00397	1.12	0.70	12.21	0.56
8	0.00250	0.81	0.67	13.30	0.61
9	0.00789	1.84	0.89	12.75	0.59
10	0.00400	1.32	0.98	19.89	0.77
11	0.00652	2.04	1.11	21.43	0.80
12	0.01114	2.36	1.30	17.04	0.72

*Portfolio 1 consists of the low–beta firms while portfolio 12 consists of the high–beta firms.

positive. This point will be discussed in more detail after the discussion of the results presented in table 5.18.

Table 5.18 gives the summary statistics of the multivariate test. The large p–value of the multivariate test in table 5.18 implies that the *ex ante* efficiency of the JSE–Actuaries Overall Index cannot be rejected. In other words, if this Index is taken as the true market portfolio, then the validity of the SLM version of the CAPM cannot be rejected for the South African case. This conclusion supports the conclusions given using the univariate procedure in section 5.2 above.

It is indeed somewhat surprising that all the $\widehat{\alpha}_i$ of table 5.17 are positive, yet the multivariate test is still unable to reject the null hypothesis. Similar results are also found in tables 5.21, 5.22 and 5.23 where the portfolios are constructed according to dividend yield, firm size and liquidity factors respectively.

A reason for the systematic occurrence of positive estimates of alpha is likely

Table 5.18 Summary statistics for the beta–sorted data set

Parameter	Statistic
$\widehat{\theta}_p$	0.0828
$\widehat{\theta}^*$	0.3727
$\widehat{\alpha}_p'\widehat{\Sigma}^{-1}\widehat{\alpha}_p$	0.132
Γ_1	1.155
p–value	0.325

Recall that $\widehat{\theta}^*$ is the maximum excess sample mean return per unit of standard deviation.

to be associated with estimation problems caused by thin trading. Results were presented in section 4.2 which showed that when a value weighted index like the JSE–Actuaries Overall Index is used in conjunction with the OLS regression procedure, then estimates of beta tend to be substantially underestimated. Consequently estimates of the alpha coefficient, in turn, are likely to be overestimated. [11] It was also shown that use of an equally weighted index in a thinly traded environment resulted in estimates of beta that were vastly overstated for well traded shares, and understated for thinly traded shares.

In order to gain further insights as to why the multivariate test fails to reject this evidence, it is worth noting that GRS examine a data set based on size–sorted portfolios, and similarly find that as many as 9 out of the 10 portfolios they examined, had positive alphas, yet the multivariate test also failed to reject their null hypothesis. GRS present their sample correlation matrix of the market model residuals and argue that the estimators for α_{ip} will have the same pattern of correlation. This is evident by considering the distribution of the vector of alphas, i.e.

[11] The aim of this chapter is to present results of traditional testing procedures. However, an interesting direction of further research would be to conduct the multivariate test on estimates of alpha and the residual covariance matrix constructed from estimates of beta, corrected for thin trading.

$\widehat{\alpha}_p$. The distribution is multivariate normal, and can be written as

$$\widehat{\alpha} \sim MVN(\alpha_p \; ; \; \frac{1}{T}(1+\theta_p^2)\Sigma)$$

where all symbols have been previously defined.

GRS were thus able to investigate the observed pattern in the correlation matrix of residuals, and found that the estimation errors among the significantly positive alphas were positively correlated. They thus argued that their evidence was not sufficient for statistical significance in the multivariate setting because of this pattern in the correlation of the residuals.

The sample correlation matrix of the market model residuals based on the regressions summarized in table 5.17 are similarly presented in table 5.19.

Table 5.19 The sample correlation matrix of residuals for the beta–sorted portfolios

Portfolio Number	1	2	3	4	5	6	7	8	9	10	11
2	.40										
3	.51	.45									
3	.46	.39	.58								
5	.45	.40	.60	.52							
6	.53	.52	.63	.63	.64						
7	.51	.50	.70	.53	.58	.67					
8	.50	.40	.66	.54	.60	.68	.73				
9	.32	.25	.36	.29	.33	.35	.43	.41			
10	.40	.30	.45	.35	.28	.34	.39	.42	.18		
11	.11	.05	.08	.03	.10	.01	.07	.09	.08	.52	
12	.16	.01	.02	-.07	.01	-.08	.00	.03	.05	.43	.68

*Portfolio 1 consists of firms with smallest betas while portfolio 12 consists of firms with the largest betas.

Although GRS allude to the fact that these correlation matrices of residuals , are fairly difficult to interpret, it is evident that the pattern of the correlations in table 5.19 are similar to those of the size–sorted data set of GRS, and consequently many of the arguments invoked by GRS are similar here.

The main point to note, is that all of the correlations, with the exception of two with portfolio 12, are positive, and fairly large. It thus appears as if the pattern of positive estimated values of α_{ip} shown in table 5.17 is associated with the correlation in the estimation error of the market model.

In order to understand the impact of these correlation coefficients on the univariate test results, suppose all twelve alphas were *independent*. Then, under the null hypothesis that $\alpha_i = 0$, for all i, the probability of all twelve $\hat{\alpha}_i$ being positive is only $(\frac{1}{2})^{12}$. However, if all twelve $\hat{\alpha}_i$ were perfectly positively correlated, then the probability that all the $\hat{\alpha}_i$ are positive is $\frac{1}{2}$. Similar arguments can be invoked to show that although six out of twelve $\hat{\alpha}_i$ were significant at the 5% level using individual t–tests, the strong positive correlations suggests that the univariate tests mis–state the results. This does help to explain why, even though six out of the twelve portfolios had significant alphas at the 5 percent level (cf. table 5.17), and all of the twelve alphas were positive, this evidence was not sufficient for statistical significance in the multivariate setting.

The results of the multivariate test found here are not unlike those of GRS who tested the efficiency of the CRSP Equal–Weighted Index on the NYSE over the 1931–1965 period. Their resulting test statistic and component statistics are summarized and compared to the results on the JSE in table 5.20.

It is interesting to note that the relative difference between $\hat{\theta}^*$ and $\hat{\theta}_p$ for the JSE is substantially larger than that found on the NYSE, but that the $p-$values perhaps do not reflect this to the same extent. This may suggest that the multivariate test has substantially less power on the JSE than on the NYSE (this issue will be taken up in chapter 6).

Additional effects — Dividend yield, firm size and liquidity

Tables 5.21, 5.22 and 5.23 give the summary statistics for the dividend [12]

[12] In the construction of the dividend yield–sorted data set, estimates of dividend yield were computed as the total dividend paid during the year divided by the share price at the end of that

Table 5.20 Comparison of JSE and NYSE multivariate test results

	JSE	NYSE
N	12	10
T	117	420
$\widehat{\theta}^*$	0.373	0.227
$\widehat{\theta}_p$	0.083	0.166
W	0.131	0.023
Γ_1	1.155	0.958
$p-$value	0.325	0.476

yield, firm size [13] and liquidity–sorted [14] data sets respectively over the 1976–1984 period. The tabulated parameter estimates in these tables are for the regression model (5.5)

Inspection of the summary statistics in tables 5.21, 5.22 and 5.23 show no systematic relationship between the $\widehat{\alpha}_{ip}$ and $\widehat{\beta}_{ip}$. For the size–sorted data set summarized in table 5.22, however, it appears as if the betas are systematically related to firm size. In particular the larger the firm size, the larger the beta coefficient. It is felt however that the beta estimates once again suffer from estimation problems associated with thin–trading. Historically smaller companies are known to suffer from the thinly–traded phenomenon to a greater extent than larger companies. Consequently smaller companies are likely to have their betas underestimated to a larger extent. This phenomenon is especially evident on small markets like the JSE and has already been recognised by Affleck–Graves (1977) and is discussed at length in section 4.2. Furthermore the $\widehat{\alpha}_{ip}$ are found to be consistently positive in tables 5.21, 5.22 and 5.23. Explanations for these positive alphas have been offered previously, and are once more related to the problem of thin trading.

year.

13 Estimates of market capitalization were computed as the year end share price multiplied by the number of shares outstanding.

14 year end liquidity estimates were computed using Roll's(1984) measure on the preceding 4 years data.

Table 5.21 Summary statistics for dividend yield–sorted portfolios

Portfolio number*	$\widehat{\alpha}_{ip}$	$t(\widehat{\alpha}_{ip})$	$\widehat{\beta}_{ip}$	$t(\widehat{\beta}_{ip})$	R_i^2
1	0.0153	2.324	1.025	9.616	0.446
2	0.0155	1.866	0.981	7.282	0.316
3	0.0055	1.415	0.764	12.122	0.561
4	0.0041	1.000	0.679	10.170	0.474
5	0.0129	0.067	0.867	3.993	0.122
5	0.0078	1.712	0.793	10.776	0.502
7	0.0119	2.882	0.457	6.821	0.288
8	0.0023	0.623	1.128	19.020	0.759
9	0.0058	1.783	0.635	11.969	0.555
10	0.0020	0.611	0.732	14.116	0.634
11	0.0032	0.629	0.865	10.377	0.484
12	0.0054	0.774	1.372	12.006	0.556

*Portfolio 1 consists of the low dividend yield firms while portfolio 12 consists of the high dividend yield firms.

The univariate test that the α_{ip} are significantly different from zero reveals that 5, 3 and 7 out of the 12 portfolios for the dividend yield–sorted, size–sorted and liquidity–sorted data sets respectively, have significant alphas at the 5% level. In order to determine whether this evidence is sufficient for statistical significance in the multivariate setting, the multivariate test statistic was computed using each of the above–mentioned data sets.

Table 5.24 gives the summary statistics of the multivariate test for each of the component–sorted data sets. The results of the beta–sorted data set are also included for comparative purposes.

Inspection of table 5.24 shows that although the smallest p–value is 0.12 for the dividend yield–sorted data set, none of the test statistics are significant. Furthermore only the p–value of the dividend yield–sorted data set is smaller than that of the beta sorted data set, i.e. 0.325. This evidence suggests that

Table 5.22 Summary statistics for size–sorted portfolios

Portfolio number*	$\widehat{\alpha}_{ip}$	$t(\widehat{\alpha}_{ip})$	$\widehat{\beta}_{ip}$	$t(\widehat{\beta}_{ip})$	R_i^2
1	0.0149	1.989	0.622	5.119	0.186
2	0.0161	2.012	0.678	5.209	0.191
3	0.0164	1.166	0.788	3.461	0.094
4	0.0056	1.335	0.668	9.802	0.455
5	0.0063	1.413	0.662	9.173	0.423
6	0.0035	0.851	0.692	10.427	0.486
6	0.0069	1.366	0.942	11.459	0.533
8	0.0088	2.569	1.091	19.722	0.772
9	0.0048	1.357	0.859	14.988	0.661
10	0.0029	0.784	1.031	16.921	0.713
11	0.0026	0.534	1.257	16.159	0.694
12	0.0024	1.011	1.064	27.732	0.870

*Portfolio 1 consists of the small firms, while portfolio 12 consists of the large firms.

grouping shares either by dividend yield, size or liquidity does not have significant additional influence on returns to constitute a rejection of the *ex ante* efficiency of the JSE–Actuaries Overall index, and consequently the SLM–CAPM. These results are consistent with those concerning the additional effects found using univariate testing procedures in section 5.2.

GRS offer a geometrical interpretation of the test statistic in the usual risk–return space.

GRS show that [15]

$$W = \left[\frac{\sqrt{1+\widehat{\theta}^{*2}}}{\sqrt{1+\widehat{\theta}_p^2}}\right]^2 - 1 = \Psi^2 - 1$$

[15] In this derivation GRS require that portfolio p be included in the opportunity set, hence in the geometrical interpretation of the optimal slope, $\widehat{\theta}^*$ includes the consideration of the additional asset p.

Table 5.23 Summary statistics for liquidity sorted portfolios

Portfolio number*	$\widehat{\alpha}_{ip}$	$t(\widehat{\alpha}_{ip})$	$\widehat{\beta}_{ip}$	$t(\widehat{\beta}_{ip})$	R_i^2
1	0.0049	1.80	0.59	13.18	0.60
2	0.0058	2.37	0.62	15.52	0.68
3	0.0101	1.83	0.67	7.54	0.33
4	0.0049	1.63	0.71	14.54	0.65
5	0.0055	1.24	0.66	9.23	0.43
6	0.0021	0.71	0.70	14.26	0.63
7	0.0047	1.58	0.66	13.71	0.62
8	0.0019	0.78	0.71	17.71	0.73
9	0.0065	2.06	0.78	15.23	0.67
10	0.0065	2.13	0.87	17.79	0.73
11	0.0065	1.85	0.79	13.87	0.63
12	0.0139	2.49	0.87	9.63	0.45

*Portfolio 1 consists of highly liquid firms while portfolio 12 consists of the most illiquid firms.

Table 5.24 Multivariate test results for the period 1976–1984

	beta–sorted	dividend yield–sorted	size–sorted	liquidity–sorted
$\widehat{\theta}^*$	0.373	0.428	0.356	0.349
W	0.131	0.175	0.119	0.114
$\widehat{\alpha}'\widehat{\Sigma}^{-1}\widehat{\alpha}$	0.132	0.176	0.120	0.115
Γ_1	1.155	1.545	1.051	1.006
$p-$value	0.325	0.120	0.409	0.449

where

$\widehat{\theta}^*$ represents the slope of the lines joining to the ex–post optimal component sorted portfolios;

$\widehat{\theta}_p$ is the slope of the line joined to p, the JSE–Actuaries Overall index, in this case; and

Ψ therefore is the distance along the line from the origin up to any given level of risk σ, divided by the distance to the same level of risk along the line joined to p.

Figure 5.1 shows the geometrical summary of the results for the various component–sorted data sets.

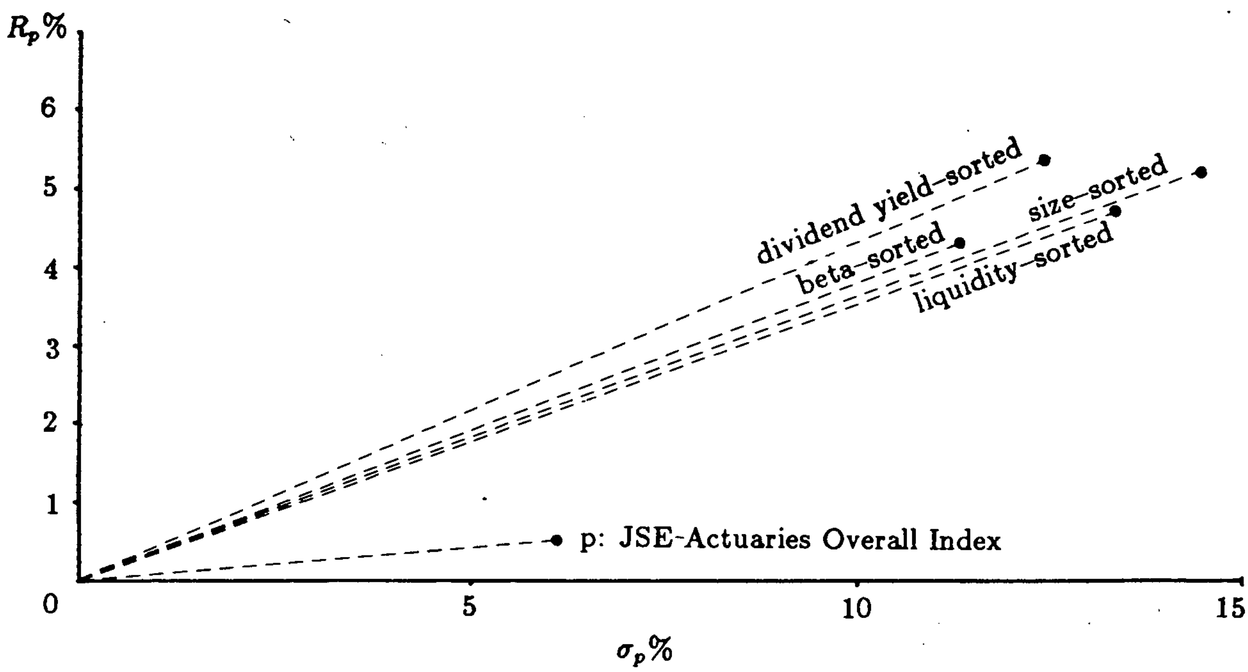

Figure 5.1 Geometrical summary over the period 1976–1984

It is evident from figure 5.1 that sorting the portfolios by either beta, dividend yield, size or liquidity does not have a significant impact on the relative difference in their slopes. Whether or not these slopes are significantly larger than the slope of portfolio p, as incorporated in the test statistic, depends largely on the power of the test. In this case the test result indicates that the slopes, in particular the distances along the slopes to given point, are not significantly different.

A similar size–sorted data set constructed by GRS on the NYSE over the period 1926–1982, where GRS test the efficiency of the CRSP Value–Weighted Index, yields values of 0.109 and 0.172 for $\widehat{\theta}_p$ and $\widehat{\theta}^*$ respectively. These resulted in a value of 1.147 for Γ_1, having a p–value of 0.301. Consequently they also conclude that no size effect was evident. They comment that their result is surprising, given the existing evidence of the size effect documented on the NYSE. They argue however that the result is consistent with findings in Brown, Kleidon and Marsh (1983) who suggest that the existence of a size effect is dependent on the methodology that is used.

5.3.4 Conclusion

The results of this analysis using a multivariate testing procedure as well as traditional estimation procedures has several implications. In particular, the results imply that if the JSE Actuaries Overall Index is used as a surrogate for the market portfolio, the SLM–CAPM appears to be a reasonable model in the South African context.

Furthermore the testing procedure also focused on several possible extensions, namely dividend yield, firm size and liquidity. Neither of the data sets [16] sorted according to the above–mentioned components resulted in the rejection of the *ex ante* efficiency of the JSE–Actuaries Overall Index. Consequently it is concluded that none of these effects have a significant bearing on return in the CAPM framework from the South African perspective.

Of major importance is the fact that all of the above results, and hence conclusions, including those for the additional effects are consistent with the univariate test results and conclusions of section 5.2.

[16] It has not been confirmed whether the absence of a liquidity effect or whether the possible inaccuracy of the estimate of liquidity is responsible for the lack of significance of the liquidity–sorted data set.

CHAPTER 6

POWER OF UNIVARIATE AND MULTIVARIATE CAPM TESTS

6.1 INTRODUCTION

A great deal of attention in modern financial research has been concerned with the CAPM. The emergence of both univariate and multivariate tests and the multitude of papers on these tests are evidence of the importance of the model to the financial community. Much of this research has endeavoured to detect economically plausible deviations from the SLM–CAPM. In this thesis, chapter 5 has already been devoted to this area.

Of particular concern to researchers conducting such tests is the ability of the tests to reject the null hypothesis when it is in fact false, in other words, the power of the test. Financial researchers have long been concerned with cases where tests have been unable to reject the null hypothesis. For example, failure to reject the model may occur because the null hypothesis is in fact true or simply because the test was not powerful enough to detect economically important deviations from the model. In this chapter several comparisons of the power of these tests are made. In particular, the power of the univariate and multivariate tests are compared with each other in different economic environments, namely the JSE (which typifies smaller markets) and the NYSE.

The multivariate test, unlike the univariate test, has the advantage that the exact power of the test can be studied as the distribution of the test statistic under the alternative hypothesis is known. Hence unlike the univariate test several aspects of the power of the multivariate test have been researched. The major contribution in this area has been made by MacKinlay (1987), and to a lesser extent by Gibbons, Ross and Shanken (1986) and Gibbons and Shanken (1986).

MacKinlay (1987) investigated the power of multivariate tests by introducing violations into the model by considering the case where the risk–free rate is measured with error. This was done by writing an expression for the intercept term in the form

$$\alpha = \gamma(\mathbf{1} - \beta)$$

where $\mathbf{1}$ is a $(N \times 1)$ vector of ones. MacKinlay argues that the null hypothesis is true when γ equals zero and that violations can be introduced into the model by increasing the value of γ. Furthermore the power of the test can be obtained by computing the non–centrality parameter of the non–central F–distribution, λ, as

$$\lambda = T\left(1 + \frac{\widehat{\mu}_m^2}{\widehat{\sigma}_m^2}\right)^{-1}(\mathbf{1} - \beta)'\Sigma^{-1}(\mathbf{1} - \beta)\gamma^2$$

and determining the proportion of non–central F above the critical value of the central F-distribution for a given level of significance.

Table 6.1 extracted from MacKinlay (1987) shows the resulting power statistics of the multivariate tests computed in this way. This was conducted over 3 sampled subperiods of 60 months each on the NYSE for 20 beta–ranked portfolios.

From table 6.1 it is evident that if measurement errors of the risk–free rate are 0.4% (or approximately 5% per year) the power of the tests at the five percent significance level ranges from only 0.07 to 0.16 in the test periods. MacKinlay thus concludes that the multivariate test does not appear useful when the error is in the measurement of the risk–free rate. On the basis of further investigation MacKinlay argues however that power gains are possible by introducing a specific alternative hypothesis. An example of such an alternative hypothesis is found in Banz (1981) where the CAPM is rejected by specifying an alternative hypothesis with the deviation related to the market value of the equity.

Gibbons, Ross and Shanken (1986) (henceforth GRS) investigate the power of the multivariate test over differing test period lengths as well as over differing numbers of assets. On the basis of their investigation they conclude that greater power regions can be attained by selecting the number of assets, N, to be roughly a third to a half of T, the length of the test period. Since stationarity concerns usually point to a 5 year testing period of monthly data, i.e. $T = 60$, twenty to thirty assets are recommended by GRS to be appropriate for maximum power benefits.

Gibbons and Shanken (1986) consider the power of aggregating test results from several subperiods. They show that aggregate power is considerably higher than the power for single subperiods.

Table 6.1 Power summary for periods of 60 months with 20 portfolios using the excess return market model as the alternative hypothesis

			$P(\text{reject } \alpha = 0 \mid \alpha = \alpha_0)$	
Period	γ_0	λ	0.05	0.01
1954–58	0.000	0.00	0.05	0.01
	0.002	1.45	0.07	0.02
	0.004	5.81	0.16	0.05
	0.006	13.10	0.37	0.15
	0.008	23.30	0.67	0.38
	0.010	36.30	0.89	0.69
1964–68	0.000	0.00	0.05	0.01
	0.002	0.53	0.06	0.01
	0.004	2.14	0.08	0.02
	0.006	4.81	0.14	0.04
	0.008	8.54	0.24	0.08
	0.010	13.40	0.38	0.16
1974–78	0.000	0.00	0.05	0.01
	0.002	0.31	0.05	0.01
	0.004	1.24	0.07	0.02
	0.006	2.78	0.10	0.02
	0.008	4.95	0.14	0.04
	0.010	7.73	0.21	0.07

It should be noted that the above studies all determined the power of the multivariate test by considering the distribution of the test statistic under the alternative hypothesis, henceforth referred to as the "exact power" in this chapter. For the univariate test, the distribution of the test statistic under the alternative is not known, consequently the "exact power" of this test cannot be determined. It is for this reason that the ensuing power study of sections 6.3 and 6.4 deals with power from an empirical viewpoint. In particular, return data is simulated with violations of the model introduced into the series of simulated return data. The percentage of times the null hypothesis is rejected at a given level of significance is referred to as the "empirical power" of the test in this chapter. The "empirical power" rather than the "exact power" is considered in this chapter so that the power of both the univariate and multivariate tests can be compared directly with each other on identical sets of simulated data. A further advantage of studying the power from the empirical viewpoint is that the power can be investigated under various controlled shifts in parameters that characterize the market setting. Although these shifts can be investigated theoretically for the multivariate test, as demonstrated by MacKinlay (1987), an empirical approach is needed to investigate parameter shifts for the univariate test. The specific details of the power computations are left to the methodology discussion in section 6.3. It is however necessary to firstly consider the precise formulation of the hypotheses commonly used in the univariate and multivariate tests.

6.2 TESTABLE HYPOTHESES

6.2.1 Traditional univariate test hypotheses

The widely quoted and copied hypotheses of Black, Jensen and Scholes (1972) and Fama and Macbeth (1973) will be considered here. This will be done in order to ascertain what the most popular hypothesis for conducting univariate tests of the SLM–CAPM in the ensuing power study should be.

Black, Jensen and Scholes emphasize that a direct test of the CAPM could be obtained by estimating

$$\widetilde{R}_{jt} = \alpha_j + \beta_j \widetilde{R}_{mt} + \widetilde{e}_{jt}$$

for a single security over some time period and testing whether α_j is significantly different from zero. Black *et al* comment on the fact that testing a single security is inefficient and propose instead an aggregated test on a large number of securities. On the basis of their empirical results of the aggregated time series tests, Black *et al* however were clearly concerned about the existence of nonstationarity of the parameters used in the test. They consequently proposed an aggregated cross–

sectional test using the model

$$\overline{R}_p = \gamma_0 + \gamma_1 \widehat{\beta}_p + e_p$$

where $\overline{R}_p$ is the mean excess portfolio return averaged over the test period only; and e_p is the cross–sectional error term.

They state that if appropriate grouping procedures are employed an obvious test of the traditional form (as opposed to the Black (1972) version) of the CAPM is to test the hypothesis that

$$\gamma_0 = 0$$

and

$$\gamma_1 = \overline{R}_m$$

where $\overline{R}_m$ is the mean excess return on the market portfolio over the testing period.

Fama and Macbeth (1973) set up additional hypotheses by proposing the following empirical analog to their model which they refer to as a stochastic generalization of the CAPM:

$$R_{pt} = \widehat{\gamma}_{0t} + \widehat{\gamma}_{1t}\widehat{\beta}_{p,t-1} + \widehat{\gamma}_{2t}\widehat{\beta}^2_{p,t-1} + \widehat{\gamma}_{3t}\overline{S}_{p,t-1}(\widehat{e}_i) + \widehat{\eta}_{pt}$$

where

R_{pt} is the return on portfolio p at time t ;

$\widehat{\beta}_{p,t-1}$ is the beta of portfolio p at time $t-1$;

$\widehat{\beta}^2_{p,t-1}$ is included to test the linearity of the CAPM;

$\overline{S}_{p,t-1}$ is the average of the standard deviations of the OLS market model residuals for the securities in portfolio p at time $t-1$ and is included to test that beta is the only measure of risk; and

$\widehat{\eta}_{pt}$ is the disturbance term and is assumed to have zero mean and to be independent of all other variables.

Fama and Macbeth state that the CAPM has three testable implications and consequently formulate the following three associated hypotheses:

(C1) the relationship between the expected return on a security and its risk in any efficient portfolio m is linear

$$\text{i.e.} \qquad E(\tilde{\gamma}_{2t}) = 0$$

(C2) Beta is a complete measure of the risk of a security in the efficient portfolio m and no other measure of risk is relevant

$$\text{i.e.} \qquad E(\tilde{\gamma}_{3t}) = 0$$

(C3) In a market of risk–averse investors, higher risk should be associated with higher expected return

$$\text{i.e.} \qquad E(\tilde{\gamma}_{1t}) = E(\tilde{R}_{mt}) - E(\tilde{R}_{0t}) > 0$$

where $E(\tilde{R}_{0t})$ is the expected return on a minimum variance zero–beta portfolio at time t .

They also state that for the traditional Sharpe–Lintner model (as opposed to the Black (1972) model) in addition to conditions C1 to C3, one has the hypothesis:

$$E(\tilde{\gamma}_{0t}) = R_{ft}$$

which should hold in a market setting where unrestricted riskless borrowing and lending at a known rate R_{ft} is assumed. It should be noted that if the above hypothesis does not hold, the least squares intercepts, $\hat{\gamma}_{0t}$ can always be interpreted as the return on a zero–β portfolio during month t .

Fama and Macbeth use t–statistics for testing their hypothesis that the $E(\tilde{\gamma}_j) = 0$, and refer to Officer (1971) for suitable justification. These t–statistics are computed using:

$$t(\bar{\hat{\gamma}}_j) = \frac{\bar{\hat{\gamma}}_j}{s(\hat{\gamma}_j)/\sqrt{n}}$$

where n is the number of months in the test period, which is also the number of estimates of $\hat{\gamma}_{jt}$ used to compute $\bar{\hat{\gamma}}_j$ and $s(\hat{\gamma}_j)$.

It should be noted that Fama and Macbeth do not concentrate on testing the hypothesis that $E(\tilde{\gamma}_1) = E(\tilde{R}_m)$ as did Black, Jensen and Scholes, but rather that $E(\tilde{\gamma}_1) > 0$ (i.e. condition C3). Fama and Macbeth refer to C3 as a critical condition and they state:

> *That is, we are not happy with the model unless there is on average a positive tradeoff between risk and return.*

Clearly there is a danger in relying on C3 as a critical hypothesis if testing periods are of relatively short duration. Fama and Macbeth present results which show that the difference between $\bar{\tilde{\gamma}}_1$ and $\overline{R_m - R_f}$ are never statistically large. The occurrence of an insignificant or even negative excess market return over a testing period would thus generaly lead to an insignificant value for $\bar{\tilde{\gamma}}_1$ which would in turn constitute a rejection of the model according to Fama and Macbeth. To emphasize this point results from the Fama and Macbeth (1973) study on the NYSE were extracted and are shown in table 6.2 below:

Table 6.2 Results of Fama and Macbeth study

Period	$\overline{R_m - R_f}$	$\bar{\tilde{\gamma}}_1$	$\bar{\tilde{\gamma}}_0$	$t(\overline{R_m - R_f})$	$t(\bar{\tilde{\gamma}}_1)$	$t(\bar{\tilde{\gamma}}_0)$
1935–6/68	0.0130	0.0085	0.0061	4.28	2.57	3.24
1935–45	0.0195	0.0163	0.0039	**2.54**	**1.92**	0.86
1946–55	0.0103	0.0027	0.0087	**2.60**	**0.70**	3.71
1956–6/68	0.0095	0.0062	0.0060	2.92	1.73	2.45
1935–40	0.0132	0.0109	0.0024	1.04*	0.79*	0.32
1941–45	0.0272	0.0229	0.0056	3.65	2.55	1.27
1946–50	0.0070	0.0029	0.0050	1.05*	0.48*	1.27
1951–55	0.0136	0.0024	0.0123	3.22	0.53	5.06
1956–60	0.0070	-0.0059	0.0148	1.60*	-1.37*	5.68
1961–6/68	0.0111	0.0143	0.0001	2.44	2.81	0.03

From table 6.2 it can be seen that in each testing period where the $t(\overline{R_m - R_f})$ values were not significant at the 5% level, the $t(\bar{\tilde{\gamma}}_1)$ values were also insignificant at the 5% level (indicated by an asterisk in table 6.2). Consequently it can be inferred that if the market index being used in testing C3 does not rise significantly over a testing period, then one is unlikely to obtain a significant $\bar{\bar{\gamma}}_1$ coefficient, subsequently resulting in rejection of the model for these periods. Hence conducting the detailed test on condition C3 is somewhat beside the point in testing periods where an insignificant average excess return on the market index used as a proxy for the market portfolio is observed.

For testing periods where the average excess return on the market portfolio is significant it is uncertain beforehand what the outcome of testing C3 would be. In this instance the power of the test will be important. Clearly large excess market returns and longer testing periods will increase the power of the test, however other factors may also be important. For example the test periods 1941–45 and 1951–55 of table 6.2 have similar $t(\overline{R_m - R_f})$ values of 3.65 and 3.22 respectively with the same test period duration, that is 60 months. The respective $t(\bar{\bar{\gamma}}_1)$ values however, are found to differ substantially, that is 2.55 for the former and 0.53 for the latter period, emphasizing the possible importance of factors other than the test period duration and the magnitude of excess market returns for power considerations (holding the validity of the model constant). These issues will be considered in section 6.3 where a power investigation will be conducted using simulation techniques.

Although Roll (1977) had several pertinent criticisms [1] relating to the hypotheses of Fama and Macbeth, the majority of researchers continued to concentrate on testing condition C3. Hence in the ensuing power study only conditions C3 (henceforth referred to hypothesis 1) [2] and the hypothesis that

$$E(\gamma_0) = R_f$$

(henceforth referred to as hypothesis 2) will be considered.

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[1] In particular Roll (1977) argued that a test of the efficiency of the market portfolio is the single testable hypotheses and that C1 to C3 are not independently testable, but follow automatically from the efficiency of the market portfolio.

[2] Fama and Mcbeth (1973) themselves allude to the fact that using C1 and C2 for testing the traditional Sharpe–Lintner hypothesis yielded ambiguous results after suppressing some of the variables in their model.

6.2.2 The multivariate test hypothesis

The hypothesis considered by Gibbons, Ross and Shanken (1986) in the multivariate setting by contrast, is much less ambiguous than the hypotheses that have been proposed in the univariate setting. They consider the central problem addressed in tests of the CAPM. Based on the assertion that the market portfolio is mean–variance efficient their hypothesis is designed to test whether a particular portfolio (usually some market index) is *ex ante* mean–variance efficient.

Under the assumption of a given riskless rate of interest as well as multivariate normality of excess returns, Gibbons *et al* (1986) show that their null hypothesis can be restated in the form:

$$H_0 : \alpha_{ip} = 0 \quad \forall\, i = 1, \ldots, N$$

where N is the number of assets being tested; and α_{ip} is the intercept of asset i obtained by regressing excess returns of asset i on the excess returns of the portfolio whose efficiency is being tested (denoted portfolio p). The test statistic of Gibbons *et al* (1986) designed to test the significance of the α_{ip} jointly has been discussed in section 2.5.2 as well as in section 5.3. Since this is the only hypothesis presented in the multivariate setting, this hypothesis alone will be considered in the ensuing power investigation of the multivariate test. This hypothesis is essentially similar to hypothesis 2 discussed in section 6.2.1 and henceforth will also be referred to as hypothesis 2.

6.3 SIMULATION METHODOLOGY

The ensuing power study was conducted using return data simulated for 100 securities under various market conditions. Return data was simulated using parameters characteristic of small markets like the JSE, as well as for larger markets like the NYSE for power comparison purposes. Furthermore various scenarios for the relevant parameters were also considered in order to compare the power of univariate and multivariate tests under different market conditions.

The specifics of the grouping procedures and testing methodologies conducted in chapter 5 were also implemented here on the data simulated for this purpose. Furthermore the power of the tests were only considered for the hypotheses discussed in sections 6.2 and 6.3 covering univariate and multivariate tests respectively.

A summary of the simulation procedure is outlined below:

Step 1: *Read in parameters*

The relevant parameters required as inputs for the generation of return data are superscripted by P (to denote parameter), and are listed below:

$\mu^P_{R_m-R_f}$: the mean monthly excess return on the market portfolio;

$\sigma^P_{R_m-R_f}$: the standard deviation of monthly excess returns on the market portfolio;

$\bar{\sigma}^P_e$: the average residual standard deviation of market model regressions (required if return data is generated assuming a univariate error structure as discussed in step 3); or

Σ^P_e : the (100×100) variance–covariance matrix of market model residuals (required if return data is generated assuming a multivariate error structure);

μ^P_{beta} : the mean of the market's beta values, which is usually taken to be equal to unity; and

σ^P_{beta} : the standard deviation of the market's beta values.

Step 2: *Simulate excess market returns for a 14 year period*

This amounts to 168 observations; chosen to be similar to the total length of the formation, estimation and testing periods used by Fama and Macbeth (1973). These excess market returns were simulated assuming excess market returns are normally distributed [3] with mean, $\mu^P_{R_m-R_f}$ and variance, $\sigma^{2^P}_{R_m-R_f}$. More formally:

$$(R_{m_t} - R_{f_t})^S = Z_t\sigma^P_{R_m-R_f} + \mu^P_{R_m-R_f} \qquad t = 1, 2, \ldots, 168$$

where $Z_t \sim N(0;1)$; and the superscript S henceforth denotes that the series has been simulated.

Step 3: *Simulate 100 security beta values*

The beta values are also assumed to be normally distributed, hence were similarly simulated from the normal distribution with mean, μ^P_{beta} and variance $\sigma^{P^2}_{\text{beta}}$,

[3] All random drawings from the normal distribution were drawn from the NAG subroutine GO5DDE.

that is

$$\beta_j^S = Z_j \sigma_{\text{beta}}^P + \mu_{\text{beta}}^P \qquad j = 1, 2, \ldots, 100$$

where $Z_j \sim N(0; 1)$.

Step 4: *Simulate 14 years return data for 100 securities*

A series of monthly return data amounting to 168 observations for each security was simulated for each of 100 securities using the model below:

$$(R_{jt} - R_{ft})^S = C + \beta_j^S (R_{mt} - R_{ft})^S + e_{jt} \qquad t = 1, 2, \ldots, 168; \quad j = 1, 2, \ldots, 100.$$

where 2 cases for the e_{jt} are considered separately in the study : for the assumption of univariate errors, e_{jt} was drawn from $N(0; \bar{\sigma}_e^P)$ (i.e. independence assumed) or for the assumption of a multivariate error structure e_{jt} was drawn from $N(0; \Sigma_e^P)$; and C represents a constant and remains unchanged for each iteration.

Clearly if C is set equal to zero then the model simulates returns according to the SLM–CAPM. Simulating returns with C set at a non–zero value would imply that violations of the SLM–CAPM are introduced into the series of returns. It is within this framework that the power investigation of the major hypothesis that the intercept terms are zero was conducted.

Step 5: *Calculate an Index of observed excess market returns*

An equally weighted market index was constructed from the series of simulated security returns as follows:

$$R_{mt} - R_{ft} = \frac{\sum_{j=1}^{100} (R_{jt} - R_{ft})^S}{100} \qquad t = 1, 2, \ldots, 168$$

Step 6: *Calculate betas over the 4 year formation period*

This step represents the start of the univariate testing procedure. To be consistent with the Fama–Macbeth (1973) methodology the beta of each security was estimated over an initial formation period consisting of the first 48 observations on each security. The simulated excess returns of step 4 were regressed against the market index computed in step 5 to obtain initial estimates of beta to be used in the formation of portfolios. More specifically this amounted to running the usual market model regression for each security:

$$(R_{jt} - R_{ft})^S = \hat{\alpha}_j^F + \hat{\beta}_j^F (R_{mt} - R_{ft}) + \eta_{jt} \qquad t = 1, 2, \ldots, 48; \quad j = 1, 2, \ldots, 100$$

where the superscript F on the coefficients denote that they were estimated over the formation period; and

the η_{jt} has the usual market model error structure.

Step 7: *Form 20 portfolios on the basis of ranked betas*

At this stage the well known procedure of first ranking the $\widehat{\beta}_j^F$ and partitioning them into 20 portfolios was conducted. The identity of the securities in each of the 20 portfolios was subsequently recorded.

Step 8: *Re-estimate portfolio betas over the subsequent 5 year estimation period*

Betas for each security were recomputed over the subsequent 60 observations (i.e. $t = 49, 50, \ldots, 108$) and were averaged for each portfolio to obtain an estimate of the portfolio beta. The 20 portfolios consisted therefore of equally weighted proportions of each of their component securities identified in step 7, that is, 5 securities in each portfolio. These estimates were once again obtained by regression, using:

$$(R_{jt} - R_{ft})^S = \widehat{\alpha}_j^E + \widehat{\beta}_j^E (R_{mt} - R_{ft}) + \eta_{jt} \qquad j = 1, 2, \ldots, 100; \quad t = 49, 50, \ldots, 108$$

where the superscript E on the coefficients denote that they were estimated over the estimation period.

Finally the $\widehat{\beta}_j^E$ were averaged within each portfolio to obtain an estimate of the portfolio beta, i.e. $\widehat{\beta}_p^E$.

Step 9: *Run cross-sectional regressions in the subsequent one year testing subperiod*

Excess portfolio returns were computed as the average excess return of the portfolios component securities in each month of the subsequent 12 month testing subperiod, and denoted $(R_{pt} - R_{ft})^S$ where $p = 1, 2, \ldots, 20$ and $t = 109, 110, \ldots, 120$. The univariate Fama–Macbeth procedure requires that these 20 portfolio returns be regressed against their corresponding portfolio betas in each month, that was

$$(R_{pt} - R_{ft})^S = \widehat{\gamma}_{0t} + \widehat{\gamma}_{1t} \widehat{\beta}_p^E \qquad p = 1, 2, \ldots, 20$$

The above regression was thus run in each month of the one year testing period resulting in 12 $\widehat{\gamma}_{0t}$ and $\widehat{\gamma}_{1t}$ values for $t = 109, 110, \ldots, 120$.

Step 10: *Repeat steps 6 and 9 a further 4 times moving one year forward each time*

The procedure of repeating steps 6 and 9 was done in order to update the portfolio betas at the beginning of each of the 5 testing subperiods. This resulted in 60 cross-sectional regressions being run in each month of the total 5 year test period, consequently a time series of 60 $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$ values were estimated at $t = 109, 110, \ldots, 168$.

Step 11: *Conduct univariate tests on the time series of gamma coefficients*

The t–statistics of the $\hat{\gamma}_0$ and $\hat{\gamma}_1$ series were computed using

$$t(\overline{\hat{\gamma}}_i) = \frac{\overline{\hat{\gamma}}_i}{s(\gamma_i)/\sqrt{n}} \qquad \text{for} \quad i = 0, 1$$

where $s(\hat{\gamma}_i)$ was the standard deviation of the series of $\hat{\gamma}_i$; and n was the number of $\hat{\gamma}_i$; in this case n = 60.

These t–values were subsequently tested for significance at the 1 and 5 percent levels. If the observed $t(\overline{\hat{\gamma}}_i)$ exceeded the associated critical value, a counter was incremented by one. This stage denotes the end of the univariate testing procedure on the 14 year series of simulated data.

Step 12: *Store the returns of beta-ranked portfolios over the 5 year test period for the multivariate test*

At this stage the multivariate test statistic was computed and tested for significance. The multivariate test was usually conducted over a 5 year test period on portfolios constructed in the same way as the univariate methodology. Hence the same series of returns of the 20 beta-ranked portfolios computed over the 5 year test period of step 10 were used here.

Step 13 *Run regressions to estimate the portfolio alphas and the variance-covariance disturbance matrix*

The multivariate test statistic requires, amongst other statistics, a vector of intercepts, $\hat{\alpha}_p$ and a variance-covariance matrix of residuals, $\hat{\Sigma}$.

These were estimated by running regressions of excess portfolio returns on the excess return on the market over the 5 year testing period, i.e:

$$(R_{pt} - R_{ft})^S = \widehat{\alpha}_p + \widehat{\beta}_p(R_{mt} - R_{ft}) + e_{pt} \quad \text{for each} \quad p = 1, 2, \ldots, 20$$
$$\text{over} \quad t = 109, 110 \ldots, 168$$

where e_{pt} were the usual market model residuals.

From these regression the 20×20 matrix $\widehat{\Sigma}$ was estimated. At this stage $\widehat{\Sigma}$ was inverted and $\widehat{\alpha}'_p \widehat{\Sigma}^{-1} \widehat{\alpha}_p$ was computed.

Step 14 *Compute excess market return and variance over the 5 year test period*

This amounts to computing:

$$\widehat{\mu}_{R_m - R_f} = \overline{R_m - R_f} = \sum_{t=109}^{168} (R_{mt} - R_{ft})/60$$

and computing $\widehat{\sigma}^2_{R_m - R_f}$, the variance of the excess market returns over this period.

Step 15 *Compute the multivariate test statistic and test it for significance*

The statistics computed in steps 13 and 14 were used to compute the test statistic:

$$\Gamma_1 = \frac{(60)(60-20-1)}{(60-2)(20)} \left[1 + \frac{\widehat{\mu}^2_{R_m - R_f}}{\widehat{\sigma}^2_{R_m - R_f}} \right]^{-1} \widehat{\alpha}'_p \widehat{\Sigma}^{-1} \widehat{\alpha}_p$$

The observed Γ_1 value obtained above was tested against $F_{20;60-20-1}$ at the 1% and 5% level. If Γ_1 exceeded the relevant critical value a counter was incremented by one.

This point denotes the end of the multivariate testing procedure on the 14 years of simulated data.

Step 16 *Start the entire iteration again, starting from step 4*

Steps 4 to 16 were repeated 500 times and the proportion of occasions when the relevant tests were rejected when violations were introduced (step 4), was interpreted as the empirical power of the respective test. In other words, the proportion of aggregate realizations in the critical region, after 500 iterations, was taken to be the unbiased estimate of the power of the test. As a binomial proportion, the standard error of the estimate of power is at most $\sqrt{(0.5)(0.5)/500} = 0.022$, since the binomial expression for the variance, i.e. $p(1-p)/n$, is maximized at $p = 0.5$.

The results presented in section 6.4 consider the empirical power of the above-mentioned tests using data simulated according to parameters consistent with conditions characteristic of the NYSE. Results of the power investigation using parameters characteristic of the JSE are shown in Appendix B.

Table 6.3 shows the input parameters used in the study to characterise the JSE and the NYSE. The required parameters for the JSE were computed using monthly data on the entire universe of shares quoted on the JSE over the 1974–1984 period. In order to obtain parameters for the NYSE for the ensuing simulation investigation, the parameters of several studies conducted on the NYSE were scrutinized. The values documented in these studies that were felt to be consistent with recent expectations on the NYSE were subjectively selected. In particular, for the NYSE, the input parameters $\sigma^P_{R_m-R_f}$ and $\bar{\sigma}^P_e$ were extracted from the Fama–Macbeth (1973) study ranging from 1935 to 1968. The value for mean excess market return in the Fama–Macbeth study was 15.6 percent per annum, which was uncharacteristically high for the NYSE. Consequently, $\mu^P_{R_m-R_f}$ was extracted from the MacKinlay (1987) study by averaging the excess market return over periods ranging from 1954 to 1983, yielding 0.77 percent per month, or 9.24 percent per annum. This figure is probably more characteristic of recent expectations on the NYSE and compares favourably with the excess market return of 8.8 percent per annum documented by Ibbotson and Sinquefield (1977) over the 1926–1977 period. One could argue that the value for $\mu^P_{R_m-R_f}$ on the JSE is expected to be marginally higher as the variability, i.e. $\sigma^P_{R_m-R_f}$, is seen to be slightly larger than that of the NYSE. In fact Favish and Affleck–Graves (1977) documented an annual excess market return of 12.3% for the JSE over the 1960–1985 period, but forecast a value of 9.1 percent per annum. This value is identical to the value in table 6.3 for the JSE (when expressed in annual terms). Of greater importance is the fact that the parameters for the JSE and the NYSE are very similar, with the exception of $\bar{\sigma}^P_e$. Intuitively one might expect that the amount of unique risk on smaller exchanges like the JSE would be larger than that of the NYSE. This point was emphasized in chapter 4, where it found that the average level of unique risk (the average residual variance) of shares in thinly–traded environments was substantially higher than the average level of unique risk in well-traded environments .

The parameter σ^P_{beta} was extracted from a study by Kim and Zumwalt (1979) on the NYSE where the variance of a sample of 322 security betas was computed over the 1962–1976 estimation period.

In order to gain insights into the sensitivity of the power of the tests to plausible shifts in the parameters, only results obtained using the NYSE parameters shown in table 6.3 will be presented initially. The corresponding results for the JSE are however shown in Appendix B.

Table 6.3 Input parameters for the JSE and the NYSE

Stock	Parameters*				
Exchange	$\mu^P_{R_m - R_f}$	$\sigma^P_{R_m - R_f}$	$\overline{\sigma}^P_e$	μ^P_{beta}	σ^P_{beta}
JSE	0.0076	0.0628	0.1102	1	0.5062
NYSE	0.0077	0.0610	0.0740	1	0.4600

*The parameters were estimated from monthly return data. All of the above symbols are defined in step 1 of Section 6.3

6.4 RESULTS

6.4.1 Hypothesis 1 : Univariate tests

Hypothesis 1 as discussed in section 6.2.1 was proposed only within the univariate framework, hence the multivariate test will be ignored here. Traditionally the test is formulated as follows:

$$H_0 : E(\tilde{\gamma}_{1t}) = 0$$

$$H_a : E(\tilde{\gamma}_{1t}) > 0$$

It should be noted that rejection rather than acceptance of the null hypothesis in fact implies consistency with the SLM–CAPM as argued by Fama and Macbeth. Within this framework the empirical power of the test is considered. This is done by increasing the value of the parameter $\mu^P_{R_m - R_f}$ upwards over a plausible range starting at zero and noting the number of times H_0 is rejected in the simulation. The expectation of $\tilde{\gamma}_{1t}$ is simply the excess return on the market; consequently violations of the null hypothesis are introduced into the series of returns by introducing a non-zero $\mu^P_{R_m - R_f}$ value. The results of this analysis are shown in table 6.4 for the typical NYSE parameters listed in table 6.3

Table 6.4 Power results for hypothesis 1 under different excess market return scenarios using NYSE parameters

$\mu^P_{R_m-R_f}$%		Results[a]					
		No. of	$\overline{\hat{\gamma}_1}$	$\overline{t(\hat{\gamma}_1)}$	$\overline{R}^2$	Power	
monthly	annual	runs	percent per month			.05	.01
0.00	0.0	500	0.05	0.07	.30	.05	.01
0.50	6.0	500	0.54	0.70	.30	.17	.04
0.77[b]	**9.2**	**500**	**0.80**	**1.03**	**.30**	**.27**	**.09**
1.00	12.0	500	1.03	1.32	.31	.40	.15
1.30[c]	**15.6**	**500**	**1.30**	**1.66**	**.31**	**.51**	**.24**
1.50	18.0	500	1.51	1.94	.31	.59	.35
2.00	24.0	500	2.00	2.56	.32	.79	.56

[a] These results were obtained from simulated data using the NYSE parameters of table 6.3. Furthermore a univariate error structure was used in the simulation procedure as outlined in step 4 of section 6.3.

[b] The actual average monthly excess return on the NYSE over the period 1954–1983.

[c] The actual average monthly excess return documented by Fama and Macbeth (1973) over the period 1935–1968.

The results shown in table 6.4 reveal several interesting insights:

Firstly, for the assumed NYSE parameters the power of the test of hypothesis 1 at the five percent level is only 0.27. This of course implies that the probability of rejecting H_0 when it is false, that is when the SLM–CAPM is in fact valid, is only 0.27 (recall rejection of H_0 implies consistency with the SLM–CAPM). Hence it does not appear as if conducting this test on the NYSE is very useful, as there is only a 27% chance of obtaining a result consistent with the SLM–CAPM when the model is in fact valid. In testing periods when the monthly excess market return on the NYSE is higher than 0.77 percent per month (approximately 9.2% percent per annum) significant power gains can be expected. However from table 6.4 it can be seen that for an excess market return of as much as 1.5% per month (or about 18% per annum) the power at the 5 percent level, although significantly greater, is only 0.59. This implies that there is only about a 59% chance of obtaining results consistent with the SLM–CAPM when the excess market return is as high as 18 percent per annum and the model is in fact valid for NYSE stocks.

The above results reveal that the test does not have a great deal of power. These findings can easily explain why Fama and Macbeth document such weak evidence in support of the SLM–CAPM using tests of hypothesis 1 (assuming the model is valid). Furthermore the above findings can also explain why Lakonishok and Shapiro (1986), Tinic and West (1984) and many other researchers have arrived at counter intuitive conclusions that suggest that, "taking higher risks does not lead to higher returns" or more simply put, that "beta is unrelated to return". Clearly these conclusions cut across one of the main tenets of financial theory, that is, higher risks are associated with higher returns. Consequently this should have led them, instead, to offer conclusions relating to reservations about the power of the test.

It is worth noting that the overall test period of Fama and Macbeth had an average excess market return of 1.3% per month, which suggests that the power of their test was about 0.51. This level of power is consistent with the weak results found by Fama and Macbeth if the SLM–CAPM is valid. Lakonishok and Shapiro (1986) by contrast conducted the test over a period having an average excess return of 0.62% per month which suggests that the power of their test at the 5 percent level was unlikely to have been much greater than about 0.22 according to the results of table 6.4. Hence the results (but not the conclusions) of Lakonishok and Shapiro and others are consistent with the evidence suggested by the power analysis of the test, and therefore cannot deny support of the SLM–CAPM.

The $\overline{R}^2$ values of table 6.4 are very similar to the $\overline{R}^2$ values obtained from tests of actual data on the NYSE. For the NYSE the tests conducted by Fama and Macbeth on actual data yielded an average $\overline{R}^2$ value of 0.295 (across 6 subperiods). This value is also very similar to the $\overline{R}^2$ value of 0.307 obtained for tests on data simulated according to the NYSE parameters. Bearing in mind the sensitivity of the $\overline{R}^2$ values to plausible shifts in the input parameters (as is evident in tables 6.5 and 6.6), the final $\overline{R}^2$ values obtained from the tests on simulated data compare favourably to the $\overline{R}^2$ obtained from tests on actual data. This comparison in some sense indicates that the power study represents a good representation of reality.

An obvious parameter that is likely to influence the power of the test is the variability of the excess market returns. This is due to the fact that the denominator of the test statistic (i.e. the t−statistic of the time series of $\hat{\gamma}_{1t}$) is comprised of the standard deviation of the $\hat{\gamma}_{1t}$, which in theory is expected to be equal to the standard deviation of the expected excess market returns. Consequently a reduction in the parameter $\sigma^P_{R_m-R_f}$ is likely to be associated with an increase in power, and vice versa.

Table 6.5 shows the resulting power statistics for various plausible shifts in the parameter $\sigma^P_{R_m-R_f}$. From table 6.5 an increase in $\sigma^P_{R_m-R_f}$ is seen to be associated with a reduction in power. More interesting however, is the range of power on the NYSE over the plausible [4] range of $\sigma^P_{R_m-R_f}$ scenarios. The scenarios for $\sigma^P_{R_m-R_f}$ start at 0.025 and are incremented by 0.025 to a maximum of 0.075. For the NYSE the power at the 5 percent level ranges from 0.22 to as high as 0.56 as $\sigma^P_{R_m-R_f}$ is reduced from 0.075 to 0.025 respectively.

Besides the parameters $\mu_{R_m-R_f}$ and $\sigma_{R_m-R_f}$, other characteristics may also be important for power considerations. There is evidence that this may be the case in the empirical results of Fama and Macbeth (1973). Their results for the 1951–55 test period reveal an insignificant value for $t(\overline{\hat{\gamma}}_1)$ of only 0.53 in the face of a large average monthly excess return of 0.0136 (the second largest out of six test periods) with a standard deviation of 0.33 (the smallest in the six test periods). The other important market characteristics which may be worth considering are the average variability of the market model residuals and the variability of the betas of the shares listed on the market.

Table 6.6 gives the resulting power statistics associated with various scenarios for the average variability of the market model residuals, $\overline{\sigma}^P_\epsilon$, of the component

[4] In the Fama and Macbeth study the standard deviation of excess market returns ranged from a minimum of 0.033 to a maximum of 0.108 over the 6 test periods studied.

Table 6.5 Power results for hypothesis 1 under different $\sigma^P_{R_m-R_f}$ scenarios using NYSE parameters

$\sigma^P_{R_m-R_f}$	No. of runs	Results: $\overline{\hat{\gamma}_1}$ percent per month	$\overline{t(\hat{\gamma}_1)}$	$\overline{R}^2$	Power .05	Power .01
.025	500	0.65	1.83	.10	.56	.32
.050	500	0.78	1.21	.24	.36	.13
.061 [a]	**500**	**0.80**	**1.03**	**.30**	**.27**	**.09**
.075	500	0.82	0.86	.37	.22	.07

[a] The actual standard deviation of the monthly excess return on the NYSE over the period 1935–1968.

shares of the market. Inspection of table 6.6 shows that although there is a decrease in power associated with an increase in $\overline{\sigma}^P_e$, the rate of decrease is not substantial over the range of plausible [5] $\overline{\sigma}^P_e$ scenarios. For the NYSE the power decreases from 0.27 to only 0.23 (at the 5% level) for the $\overline{\sigma}^P_e$ scenarios given in table 6.6. This reduction in power, however, is only apparent for $\overline{\sigma}^P_e$ greater than 0.1. It can thus be concluded that a reduction in the power of hypothesis 1 can be expected if the average standard deviation of the market model residuals exceeds 10 percent per month.

Table 6.7 considers the standard deviation of the population of beta coefficients of listed securities. Although there is an increase in power from 0.21 to 0.27 at the 5 percent level associated with a respective shift in σ^P_{beta} from 0.2 to 0.4, no further power gains are evident as σ^P_{beta} increases to 0.6. The resulting power statistics of table 6.7 show that an increase in the variability of share beta's up to a particular level, $\sigma^P_{\text{beta}} = 0.4$, is associated with an increase in power. However beyond this level the power of hypothesis 1 appears to be insensitive to changes in σ^P_{beta} .

[5] In the Fama and Macbeth (1973) study the average standard deviation of market model residuals ranged from a minimum of 0.061 to a maximum of 0.112.

Table 6.6 Power results for hypothesis 1 under different $\bar{\sigma}_e^P$ scenarios using NYSE parameters

		Results				
$\bar{\sigma}_e^P$	No of runs	$\bar{\hat{\gamma}}_1$ percent per month	$t(\bar{\hat{\gamma}}_1)$	$\bar{R}^2$	Power .05	Power .01
.050	500	0.81	1.05	.43	.27	.09
.074[a]	**500**	**0.80**	**1.03**	**.30**	**.27**	**.09**
.100	500	.079	1.00	.22	.27	.08
.150	500	0.74	0.89	.13	.23	.06

[a]The actual average standard deviation of market model residuals estimated over 1935–68 of the NYSE.

Table 6.7 Power results for hypothesis 1 under different σ_{beta}^P scenarios using NYSE parameters

		Results				
σ_{beta}^P	No of runs	$\bar{\hat{\gamma}}_1$ percent per month	$t(\bar{\hat{\gamma}}_1)$	$\bar{R}^2$	Power .05	Power .01
.200	500	0.69	0.81	.11	.21	.04
.400	500	0.80	1.02	.26	.27	.09
.460[a]	**500**	**0.80**	**1.03**	**.30**	**.27**	**.09**
.600	500	0.81	1.04	.39	.27	.09

[a]The actual standard deviation of beta estimated over 1962–76 on the NYSE.

To summarize the results of the power investigation for hypothesis 1:

It is clear that the power of the test of hypothesis 1 on the NYSE is fairly weak. Substantial gains in power are found to be associated with an increase in

Table 6.8 Power results of hypothesis 1 for $\alpha = 0.006$ using JSE parameters

	Results				
				power	
No of runs	$\overline{\hat{\gamma}_1}$ percent per month	$t(\overline{\hat{\gamma}_1})$	$\overline{R}^2$	.05	.01
500	**0.75**	**.93**	**.22**	**.24**	**.08**

The parameters for the JSE were set at the levels shown in table 6.3.

with the inferences for the JSE made above.[6] In particular, it was asserted that the power on the JSE can be expected to be marginally lower than on the NYSE, due solely to the larger $\overline{\sigma}_e^P$ parameter for the JSE. This is indeed the case. It was found that at the 5 percent and 1 percent levels of significance, the associated power of hypothesis 1 on the NYSE was 27 percent and 9 percent respectively, while the corresponding power on the JSE (shown in table 6.8) was marginally lower, at 24 percent and 8 percent respectively.

It is worth noting that in chapter 4 it was argued that the average residual standard deviation of shares in thinly–traded environments are likely to be substantially larger than the average residual standard deviation of shares in well–traded environments. It can thus be concluded that in markets which suffer from the effects of thin–trading, the power of hypothesis 1 is likely to be marginally lower than that of the NYSE (assuming all other relevant parameters are similar).

[6] The $\overline{R}^2$ value of 0.22 shown in table 6.8 when compared to the $\overline{R}^2$ values obtained from tests of actual data on the JSE, are very similar. The results of the tests of hypothesis 1 on actual data on the JSE are presented in table 5.4 of chapter 5. These tests yielded an average $\overline{R}^2$ value (across 5 subperiods) of 0.234, which is very close to the value of 0.22 obtained for the tests conducted on the simulated data shown in table 6.8. Again, as with the NYSE, the consistency of the final $\overline{R}^2$ values to some extent indicates that the power study represents a good representation of reality.

6.4.2 Hypothesis 2. Univariate versus multivariate tests

Hypothesis 2 deals with testing the intercept term of the excess return SLM–CAPM and is discussed in sections 6.2.1 and 6.2.2. The framework within which violations of this hypothesis are introduced into the series is outlined in step 4 of section 6.3. The empirical power here can be interpreted as the ability of the test to identify departures in the intercept term from zero. In particular the hypotheses can be formulated as:

$$H_0 : \alpha_{ip} = 0 \quad \text{for} \quad i = 1, 2, \ldots, N.$$

Although the exact distribution of the multivariate test statistic under the alternative hypothesis is known (which implies the exact power rather than the empirical power can be determined), as mentioned previously, it is the empirical power of the multivariate test that is considered in this study. The advantages of this approach are apparent from section 6.4.1, where the power of hypothesis 1 was investigated by considering set shifts of certain crucial parameters. The other advantage of this empirical approach is that the outcome of the univariate and multivariate can be compared on the identical sets of simulated data.

The power of the test of hypothesis 2 is investigated here by shifting alpha in increments of 0.002, starting from zero. These alpha shifts are chosen to be the same as those used in the power study by MacKinlay (1987). Initially, the analysis will, as before, be conducted from the NYSE viewpoint, i.e. the data will be simulated according to NYSE parameters, (the corresponding results for the JSE however are also shown in Appendix B). Table 6.9 shows the resulting NYSE power statistics for the univariate and multivariate test for comparison purposes.

The most notable feature to emerge from table 6.9 is the large difference in power of tests of hypothesis 2 between the univariate and multivariate test. In particular, the univariate test is seen to be significantly more powerful than the multivariate test. For the NYSE parameters shown in table 6.9 the power of the univariate test is seen to increase to 0.99 for α fixed at 0.010 at the 5% level, while the power of multivariate increases to only 0.33 for the same α at the 5% level. Furthermore the results indicate there is very little evidence to suggest that the multivariate test is able to pick up violations in α of anything less than 0.006 per

Table 6.9 Power results for hypothesis 2 under shifts in the alpha coefficient using NYSE parameters

α^a	No. of runs	Univariate test $t(\overline{\hat{\gamma}_0})$	Univariate Power .05	Univariate Power .01	Multivariate test $\overline{\Gamma}_1$	Multivariate Power .05	Multivariate Power .01	Agreement[b] % at the .05 level
.000	500	0.15	.06	.01	0.99	.04	.01	6
.002	500	0.95	.25	.09	1.10	.04	.01	52
.004	500	1.78	.53	.29	1.11	.07	.01	71
.006	500	2.53	.77	.53	1.24	.13	.04	94
.008	500	3.43	.96	.81	1.43	.22	.07	99
.010	500	4.25	.99	.97	1.68	.33	.15	100

[a]Non-zero α coefficients represent violations of the SLM-CAPM. In step 4 of section 6.3, where the procedure is explained in detail, the α is denoted by C.

[b]This column shows the percentage of times that both the univariate and multivariate rejected the null hypothesis for a give test period of simulated data, i.e. were in agreement.

month (or 7.2% per year). For example, the power of the univariate test is 0.53 (at the 5 percent level) for α equal to 0.004 per month (or 4.8% per year), whilst the corresponding level of power for the multivariate test is only 0.07.

As a yardstick, it is worth noting that Fama and Macbeth found a measurement error in the risk-free rate of return of 0.0061 per month over the 1935–68 period, and are consequently able to reject the null hypothesis. Around this level of α (i.e. $\alpha = 0.006$) the power of the univariate test on the NYSE is seen to be 0.77, while the corresponding power of the multivariate test on the NYSE is only 0.13 at the 5 percent level. Hence it can be concluded that while the power of the univariate test for detecting plausible misspecifications in the risk-free rate is reasonable on the NYSE, the same cannot be said for the multivariate test.

The last column of table 6.9 indicates the percentage of times both the univariate and multivariate test were in joint agreement about the rejection of hypothesis 2 for a particular simulated test period. It is clear from this column that the two tests were not always in agreement as to whether violations of the null hypothesis were evident in the data.

The empirical power statistics obtained for the multivariate test on the NYSE shown in table 6.9 compare very favourably with the theoretical power statistics documented by MacKinlay (1987) and presented in table 6.1, especially for the 1965–68 and 1974–78 test periods. It is evident from both the simulated "empirical power" study presented here and the "theoretical power" study of MacKinlay that the multivariate test is unable to detect violations in the model of anything less than α equal to 0.006 per month. Even at α equal to 0.010 per month the multivariate test is seen to have low power. The consistency of the results presented here with the results of MacKinlay, as before, tend to confirm that the simulation study is an accurate representation of the markets under study.

Furthermore it is worth considering whether the fact that the return data was simulated from a univariate error structure rather than a multivariate error structure, had any bearing on the results of the power investigation. Accordingly, the variance–covariance matrix of market model residuals of 100 randomly selected securities on the JSE was estimated over the 1974–1984 period. Step 4 of section 6.3 describes how this matrix is used to simulate [7] returns having a multivariate error structure. The resulting power statistics (for both tests) obtained from data simulated with this multivariate error structure is shown in table B5 of Appendix B. For comparison purposes, the tests were repeated on data simulated from a univariate structure (using the same data set to estimate the univariate error parameter). The resulting power statistics are shown in table B5 of Appendix B. Comparison of the power of both tests in table B5 reveal that the power statistics are indeed similar. In particular, there is no systematic bias evident in either the univariate or the multivariate power statistics. In addition the levels of power for both tests are similar for the associated shifts in α. On the basis of this consistency and the consistency of the results of table 6.9 for the NYSE with those of MacKinlay, it does not appear as if simulating returns from a univariate error structure as

[7] It should be noted that simulating returns having a multivariate error structure of this magnitude was highly inefficient in terms of time and costs of computer runs.

opposed to a multivariate error structure will have any significant bearing on the results presented here. Consequently, for the remainder of this section the return data will be simulated from a univariate error structure in order to avoid the highly computationally inefficient multivariate alternative. [8]

In order to investigate the power of the test of hypothesis 2 under changing market characteristics a fixed plausible violation of the null hypotheses [9] had to be introduced into the series of simulated return data. Fama and Macbeth found a measurement error in the risk–free rate of return of 0.0061 per month. Using this again as a yardstick, the value of α equal to 0.006 per month (or 7.2 percent per year) will henceforth be introduced into the series of return data in order to investigate the power of the tests under shifts of other market parameters.

A parameter which is likely to influence the power is the average variance of the market model residuals, or more simply put, the average level of unique risk. The importance of this parameter in the multivariate test is clear since the test–statistic is a function of the inverse of the variance–covariance matrix of residuals. Hence the larger the variance of the residuals, the smaller the impact of the inverse of the variance–covariance matrix, consequently yielding a smaller test statistic, which in turn implies less power.

Table 6.10 shows the results of the power investigation (for $\alpha = 0.006$) under various shifts in the parameter $\overline{\sigma}_e^P$. With reference to table 6.10 it is evident that an increase in the average residual standard deviation for both the univariate and multivariate tests is associated with a substantial reduction in power. Comparing the range of the power of the test at the 5% level for the univariate and multivariate test, it can be seen that the levels of observed power associated with shifts in $\overline{\sigma}_e^P$ from 0.50 to 0.150, range from 0.97 to 0.36 for the univariate test, and from only 0.30 to as low as 0.04 for the multivariate test. Unfortunately, as documented previously, the multivariate test has low power at $\alpha = 0.006$. Hence even for the plausible decreases in $\overline{\sigma}_e^P$ shown in table 6.10, the corresponding increase in power

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8 In section 6.4.3, where various structured multivariate error structures are considered, the multivariate alternative will be implemented.

9 By definition the power of a test is only meaningful for cases where the null hypothesis is violated.

to 0.30 at the 5 percent level of significance is not even sufficient to conclude that the multivariate test has reasonable power at low levels of $\bar{\sigma}_e^P$. It should however be noted, that the relative increase in power of both the tests are, nevertheless, substantial. It can thus be inferred that the average variability of the market model residuals has a major influence on the power on these tests. This is an important point since, as explained previously, many of the smaller exchanges characteristically suffer from the effects of thin-trading, and consequently have larger unique risk components. Clearly the power of the test of hypothesis 2 is thus likely to be weaker in these markets.

Table 6.10 Power results for hypothesis 2 for $\alpha = 0.006$ under different $\bar{\sigma}_e^P$ scenarios using the NYSE parameters

	No.	Univariate test			Multivariate test			Agreement %
	of		Power			Power		at .05
$\bar{\sigma}_e^P$	runs	$t(\bar{\bar{\gamma}}_0)$	.05	.01	$\bar{\Gamma}_1$	.05	.01	level
.050	500	3.78	.97	.88	1.64	.30	.11	100
.074[a]	**500**	**2.53**	**.77**	**.53**	**1.24**	**.13**	**.04**	**94**
.100	500	1.85	.55	.32	1.13	.08	.02	80
.150	500	1.26	.36	.16	1.03	.04	.01	78

[a] The actual average standard deviation of market model residuals estimated over 1935–68 on the NYSE.

Another less obvious parameter that has an effect on the power of hypothesis 2 is the standard deviation of the population of beta coefficients of the shares listed on a stock market. Table 6.11 gives the resulting power statistics for various σ_{beta}^P scenarios for α set at 0.006. Table 6.11 shows that there is an increase in power associated with an increase in σ_{beta}^P for both the univariate and multivariate tests. The influence of changes in σ_{beta}^P is more dramatic for the univariate test where the power increases from 0.34 to as high as 0.92 at the 5% level. By contrast the power of the multivariate test increases from 0.05 to only 0.20 over the corresponding range at the 5% level.

Table 6.11 Power results for hypothesis 2 for $\alpha = 0.006$ under different σ^P_{beta} scenarios

	No.	Univariate test			Multivariate test			Agreement %
	of		Power			Power		at .05
σ^P_{beta}	runs	$t(\bar{\bar{\gamma}}_0)$	.05	.01	$\bar{\Gamma}_1$	.05	.01	level
.200	500	1.24	.34	.15	1.05	.05	.01	47
.400	500	2.24	.66	.44	1.18	.11	.01	90
.460[a]	**500**	**2.53**	**.77**	**.53**	**1.24**	**.13**	**.04**	**94**
.600	500	3.15	.92	.73	1.43	.20	.04	97

[a] The actual standard deviation of beta estimated over 1962–76 on the NYSE.

This evidence can be used to draw some insights for the portfolio grouping procedures that have been traditionally used. In particular, it is evident that the larger the variance of a population of individual share betas, the larger will be the variance of the betas of the beta–sorted portfolios used in the testing procedure. The results presented here show that an increase in the variance of share betas, or consequently an increase in the variance of the portfolio betas, leads to greater power for both tests. Hence it can be asserted, that the traditional procedure of constructing portfolios on the basis of a vector of ranked betas, is worthwhile, as this leads to an increase in the spread (variance) of the portfolio betas which has been shown above to lead to an increase in the power of both tests.

Lastly, plausible shifts in the parameters $\mu^P_{R_m-R_f}$ (the mean excess return on the market), and $\sigma^P_{R_m-R_f}$ (the variability of excess returns on the market) are considered below. Tables 6.12 and 6.13 show the resulting power statistics (assuming $\alpha = 0.006$) for plausible shifts in the parameters $\mu^P_{R_m-R_f}$ and $\sigma^P_{R_m-R_f}$ respectively.

Table 6.12 shows that an increase in $\mu^P_{R_m-R_f}$ is associated with a marginal increase in the power of the univariate test (for alpha set at 0.006 per month). In particular, the power of the univariate test is seen to increase from 0.75 to 0.80 at the 5% level as the parameter $\mu^P_{R_m-R_f}$ increases from 0 to 24 percent per

Table 6.12 Power results for hypothesis 2 for $\alpha = 0.006$ under different $\mu^P_{R_m - R_f}$ scenarios

$\mu^P_{R_m-R_f}$ % monthly	annual	No. of runs	Univariate test $t(\bar{\hat{\gamma}}_0)$	Power .05	Power .01	Multivariate test $\bar{\Gamma}_1$	Power .05	Power .01	Agreement % at .05 level
0.00	0.0	500	2.45	.75	.51	1.27	.13	.04	94
0.50	6.0	500	2.50	.75	.52	1.25	.13	.04	94
0.77[a]	**9.2**	**500**	**2.53**	**.77**	**.53**	**1.24**	**.13**	**.04**	**94**
1.00	12.0	500	2.55	.77	.54	1.23	.12	.04	96
1.50	18.0	500	2.60	.79	.55	1.21	.10	.04	95
2.00	24.0	500	2.65	.80	.56	1.19	.09	.03	94

[a] The actual monthly average excess market return on the NYSE over the period 1954–1983.

annum, respectively. By contrast, the power of the multivariate test at the 5% level *decreased* very slightly from 0.13 to 0.09 as $\mu^P_{R_m - R_f}$ increased from 0 to 24 percent per annum respectively. Again, this finding is consistent with inferences that can be drawn by inspection of the computational expression for Γ_1, shown in step 15 of the simulation outline. The expression for Γ_1 is a function of the inverse of $\hat{\mu}_{R_m - R_f}$. Hence an increase in $\mu^P_{R_m - R_f}$ (holding the other parameters constant) would lead to a decrease in Γ_1, which would in turn lead to a reduction in power. Empirically however, it is evident that over the plausible range of parameters investigated, this reduction in power is marginal.

A summary of the resulting power statistics obtained by investigating shifts in $\sigma^P_{R_m - R_f}$ (the standard deviation of excess returns on the market) is shown in table 6.13. For the univariate test, the power at the 5% level remains at 0.77 for the various $\sigma^P_{R_m - R_f}$ scenarios, with the exception of the smallest scenario for $\sigma^P_{R_m - R_f}$ (i.e. $\sigma^P_{R_m - R_f} = 2.5$ percent per month), here the power increased marginally to 0.82. By contrast, the results for the multivariate test shown in table 6.13 indicate that an increase in $\sigma^P_{R_m - R_f}$ is associated with a slight increase in power. Again, this is intuitive, and can be established by considering the term $\hat{\sigma}^{R_m - R_f}$ in the expression for Γ_1 (shown in step 15 of the simulation outline). From this expression it is evident that Γ_1 and $\hat{\sigma}_{R_m-R_f}$ are positively related, consequently an increase in $\sigma^P_{R_m-R_f}$, other things being equal, is likely to be associated with an increase in power. However it is worth noting that the results for the multivariate test shown in table 6.13 suggest that the multivariate test is nevertheless fairly insensitive to plausible changes in $\sigma_{R_m-R_f}$.

Table 6.13 Power results for hypothesis 2 for $\alpha = 0.006$ under different $\sigma^P_{R_m-R_f}$ scenarios

	No.	Univariate test			Multivariate test			Agreement %
$\sigma^P_{R_m-R_f}$%	of		Power			Power		at .05
monthly	runs	$t(\bar{\bar{\gamma}}_0)$	.05	.01	$\bar{\Gamma}_1$	.05	.01	level
2.5	500	2.75	.82	.59	1.11	.07	.02	89
5.0	500	2.54	.77	.54	1.21	.10	.02	97
6.1[a]	**500**	**2.53**	**.77**	**.53**	**1.24**	**.13**	**.04**	**.94**
7.5	500	2.52	.77	.53	1.28	.13	.04	96

[a] The actual standard deviation of the monthly excess return on the NYSE over the period 1935–1968.

To summarize, there are several important inferences that can be drawn from the resulting power statistics of hypothesis 2:

Firstly univariate tests of hypothesis 2 appear to have reasonable power, but the multivariate test, by comparison, has very low power. In particular, assuming a fixed violation in the measurement of the risk-free rate of 0.6 percent per month, and using typical NYSE parameters, the power of the univariate test was found to be 0.77 at the 5% significance level, compared to only 0.13 documented for the multivariate test using the same set of parameters.

Secondly, a significant reduction in the power of both tests was found to be associated with plausible increases in the parameter $\bar{\sigma}_e$, representing the average standard deviation of market model residuals. This does have important implications for researchers conducting similar tests on smaller markets. As mentioned previously smaller markets generally have relatively larger average σ_e components.[10] This would imply that both univariate and multivariate tests can be expected to have substantially less power when conducted on smaller markets.

Thirdly, an increase in the spread/variability of the beta coefficients was found to lead to an increase in the power of both the univariate and multivariate test, although the increase in the power of the univariate test however was more marked than for the multivariate test. As explained previously these findings do confirm that forming portfolios to maximize the spread of the beta, does appear to be worthwhile.

Finally influences of parameters $\mu_{R_m-R_f}$ and $\sigma_{R_m-R_f}$ on the power of both tests appear to be minimal. However it is interesting to note that the investigation reveals that the resulting power statistics of the univariate test moves in opposite directions to those of the multivariate test as parameter shifts in both $\mu^P_{R_m-R_f}$ and $\sigma^P_{R_m-R_f}$ are investigated. In particular, the power of the univariate test *increases* marginally with increases in $\mu^P_{R_m-R_f}$, while the respective power of the multivariate test *decreases* marginally. Furthermore, it could be argued that the power of the univariate test *decreases* (over a specific range) with increases in $\sigma^P_{R_m-R_f}$, but *increases* marginally for the multivariate test as $\sigma^P_{R_m-R_f}$ increases.

In order to draw inferences relating to the power of tests of hypothesis 2 for smaller markets, it is important to note that the parameter $\overline{\sigma}_e$, as discussed previously, was the one parameter that is expected to be characteristically larger for smaller markets. In chapter 4 evidence was presented which suggests that thin-trading may be a likely cause of the larger average standard deviation of market model residuals documented on smaller markets. Unlike the results of hypothesis 1 presented in section 6.4.1, the results presented in this section indicate that the power of both the univariate and multivariate tests of hypothesis 2 are indeed sensitive to changes in the parameter $\overline{\sigma}_e$.

The results presented here can be used to make some inferences concerning the power of tests of hypothesis 2 on smaller markets like the JSE, for example. Considering the magnitude of the parameter $\overline{\sigma}_e$, for the JSE shown in table 6.3, it

10 As discussed previously, thin-trading is one of the likely causes of this phenomenon.

is evident that the $\overline{\sigma}_e^P$ value computed for the JSE, of 0.110, is indeed larger than the corresponding value of 0.074 documented on the NYSE. Inspection of table 6.10 reveals that the scenario for the $\overline{\sigma}_e^P$ value that is closest to that of the JSE, is $\overline{\sigma}_e^P$ set at 0.100. At this level the power (for $\alpha = 0.006$) of the univariate test was found to be 0.55 and 0.32 at the 5 percent and 1 percent significant levels respectively, which is substantially lower than the corresponding power statistics of 0.77 and 0.53 documented for the NYSE parameters respectively. Similarly, for the multivariate test, at $\overline{\sigma}_e^P$ set at 0.100, the power was found to be only 0.08 and 0.02 for the 5 and 1 percent significance levels respectively. Again this is somewhat lower than the corresponding power statistics of 0.13 and 0.04 documented for the NYSE parameters respectively. Hence it can be concluded that, due to the large $\overline{\sigma}_e$ parameter, both the univariate and the multivariate tests of hypothesis 2 can be expected to have lower power on the JSE, and that the levels of power on the JSE could be expected to be similar to the levels suggested above.

The entire investigation of this section was repeated for the parameters characteristic of the JSE, using the same range of plausible parameter shifts. These results are shown in Appendix B. The resulting power statistics of hypothesis 2 for all JSE parameters set at the levels shown in table 6.3, were extracted from these results and are shown in table 6.14.

Table 6.14 Power results of hypothesis 2 for $\alpha = 0.006$ using JSE and NYSE parameters

		Univariate tests			Multivariate test			
Market	No of runs	$t(\overline{\gamma}_0)$	Power .05	Power .01	$\overline{\Gamma}_1$	Power .05	Power .01	Agreement % at .05 level
JSE	500	1.85	.55	.31	1.11	.06	.02	73
NYSE	500	2.53	.77	.53	1.24	.13	.04	94

The power statistics for the JSE shown in table 6.14 appear to be consistent with the inferences for the JSE, made above. In particular it was inferred that the power (for $\alpha = .006$) would be approximately 0.55 at the 5 percent level, and 0.32

at the 1 percent level for the univariate test. The results shown in table 6.14 show that the power documented for the JSE was in fact 0.55 and 0.31 at the respective levels of significance. Similarly the power of the multivariate test was found to be 0.06 and 0.02 for the 5 and 1 percent levels of significance respectively, which is similar to those inferred from the NYSE results.

Of greater importance is the very low power of the multivariate test on the JSE, while the power of the univariate test, although substantially larger than the multivariate test on the JSE, is substantially less than the power documented on the NYSE.

6.4.3 Conclusions

In this chapter the power of the tests of the most popular hypotheses concerning tests of the SLM–CAPM have been investigated. The major implication of the findings are that the power of the univariate tests (of hypothesis 2) is substantially higher than that of the multivariate test, which was found to have low power. This conclusion holds good even in the face of plausible shifts in the parameters which are characteristic of market settings. Secondly the power of the tests of hypothesis 1 and 2 are substantially higher for the NYSE than for the JSE. The relative reduction in power can be mainly attributed to the fact that smaller markets, like the JSE, are comprised of shares having relatively larger residual standard deviations, on average. In the preceding power investigation an increase in the average residual standard deviation was found to be associated with a corresponding reduction in power. This was consistent for both hypothesis 1 and hypothesis 2 (although the effect was more substantial for hypothesis 2) as well as for both the univariate and multivariate tests. It could thus be inferred that these SLM–CAPM tests on smaller markets are likely to suffer from low power.

In particular, tests of hypothesis 1 which are concerned with testing for a positive tradeoff between systematic risk and return were found to be weak, even on the NYSE. In the framework of hypothesis 1 rejection of the null hypothesis implies consistency with the SLM–CAPM, hence the low power of this test may explain why so many researchers have arrived at the alarming conclusion that investors were not being compensated for bearing additional systematic risk.

In the context of hypothesis 2 the multivariate tests were found to have substantially less power than the univariate tests. The evidence shows that the multivariate test is unlikely to detect any measurement error in the risk free rate of 0.006 per month or less. By contrast the univariate test has a probability of 0.82 of detecting this measurement error at the 5 percent level on the NYSE.

Lastly, it was found that any grouping procedure which led to an increase in the spread of beta would lead to an associated gain in power, hence forming portfolios on the basis of ranked beta, appears to be worthwhile.

The final section of this chapter investigates the effects of various fixed residual correlation structures on the power of both tests.

6.4.4 Power comparisons assuming structured residual variance–covariance matrices

Univariate testing procedures have been extensively used in applied empirical stock market research over the past 15 years. Recently however, some researchers have had reservations about using these univariate procedures for testing the CAPM. Gibbons, Ross and Shanken (GRS) (1986), in particular, questioned the validity of summarizing results across a number of univariate tests that were not independent. In order to highlight this problem, GRS presented summary statistics for data sets which yielded conflicting results for the univariate and multivariate testing procedures. To gain further insights into the cause of this problem, GRS presented the resultant residual correlation matrix of a 'size–sorted' data set. By investigating the pattern of the correlations in this matrix, GRS found that all the portfolios that had significant univariate alphas, also had positively correlated estimation errors among these alphas. GRS were led to claim that

> *drawing a proper joint inference across a number of univariate tests is difficult at best, for the statistics may be highly dependent.*

In order to gain additional insights into the reason for these conflicting results when dependencies (positive correlations) are apparent in the estimation errors, a further simulation study investigating the power of these tests is presented below. The investigation aims at *comparing* the power of univariate and multivariate testing procedures when these dependencies exist in the estimation errors.

Methodology

The analysis was conducted by inducing a range of constant residual correlation scenarios into the series of simulated returns and conducting the tests on these simulated returns. The simulation methodology used here is identical to the simulation outline discussed at length in section 6.3, with the exception that *structured* residual variance–covariance matrices are used in the return generating procedure. The return generation procedure corresponds to that outlined in detail in step 4 of section 6.3.

Details of the construction of the structured residual variance–covariance matrices are outlined below:

Step 1. *Simulate for the diagonal elements 100 residual variances*

Residual standard deviations were assumed to be normally distributed, hence the 100 $\sigma^S_{e_j}$ $(j = 1, 2, \ldots, 100)$ were drawn from the normal distribution, having mean, $\overline{\sigma}^P_e$, and variance, $\mathrm{Var}(\sigma_e)$. As before, the superscript S denotes that the series was simulated; $\overline{\sigma}^P_e$ is the fixed (chosen) mean residual standard deviation; and $\mathrm{Var}(\sigma_e)$ is the fixed (chosen) variance of the residual standard deviations.

In the analysis the value for $\mathrm{Var}(\sigma_e)$ was assumed to be $(0.03)^2$. This value was derived from the summary statistics presented by Fama and Macbeth (1973). The value for $\overline{\sigma}^P_e$ was assumed to be 0.074, i.e. the estimated value for the NYSE shown in table 6.3. Finally the $\sigma^{S^2}_{e_j}$ $(j = 1, 2, \ldots, 100)$ represent the simulated[11] diagonal elements of the residual variance–covariance matrix, Σ_e.

Step 2. *Assume a constant cross-correlation of the residuals, ρ_e, and compute the off-diagonal elements covariances*

Assuming a constant value for the cross-correlations, ρ_e, amongst the residuals, each off-diagonal covariance term, $\sigma_{e_{ij}}$, was computed using

$$\sigma_{e_{ij}} = \rho_e \sigma^S_{e_i} \sigma^S_{e_j} \qquad i \neq j;\ i = 1, \ldots, 100;\ j = 1, 100.$$

The resultant structured residual variance–covariance matrix, Σ^S_e, was used as the input to the simulation procedure outlined in step 4 of section 6.3. As before a

[11] If a negative value for $\sigma^S_{e_j}$ was drawn, the absolute value was taken.

series of returns for a 100 shares is simulated, and the shares are grouped into 20 beta–sorted portfolios, whereafter the portfolio betas are re–estimated in a further estimation period prior to the implementation of the tests. The estimate of the power of the test was taken to be the proportion of aggregate realizations in the critical region, in this case, after 200 iterations. As a binomial proportion, the standard error of the estimate of the power of the tests is at most 0.035[12] here.

Various residual correlation scenarios were considered by changing the value of ρ_e, and repeating steps 1 and 2. The range of scenarios considered, began with $\rho_e = 0$, and was incremented by 0.1 to a maximum of $\rho_e = 0.9$.

Results

The violation of the null hypothesis was introduced into the series of simulated returns, by setting alpha equal to 0.006 per month, as before. Table 6.15 shows a summary of the resultant power statistics obtained for the univariate and multivariate tests over the range of ρ_e scenarios.

From the resultant power statistics shown in table 6.15, it is evident that for moderate ρ_e (i.e. for $\rho_e = 0.1$ and $\rho_e = 0.2$), the apparent power of the *univariate test* initially decreases (from 0.695 for $\rho_e = 0$, to 0.460 for $\rho_e = 0.2$, at the 5 percent level of significance) but begins to increase as ρ_e is increased from 0.2. The results for the *multivariate test* show no increase in power for ρ_e less than 0.3, after which the power increases as ρ_e is increased. Although the results of table 6.15 show that the *relative* increase in the power of the multivariate test is larger than that of the univariate test over the higher range of ρ_e scenarios, it is only at the scenario, $\rho_e =$ 0.9, that the power of the multivariate test marginally exceeds the apparent power of the univariate test.

At this point it should be noted that the quoted significance levels (i.e. .05 and .01), assumed in table 6.15, may not be appropriate for the observed levels of power obtained for the univariate test. Summarizing results across a number of univariate statistics could lead to a lack of control over the significance level when dependencies are evident amongst these statistics. By construction, dependencies have been

[12] The actual standard deviation is $\sqrt{p(1-p)/200}$, where p is the true (population) proportion. This expression is maximized at $p = 0.5$, resulting in a 0.035 maximum.

Table 6.15 Apparent results of hypothesis 2 for alpha = 0.006 using structured residual variance–covariance matrices

	No.	Univariate test				Multivariate test		
	of			power[a]			power	
ρ_e	runs	$t(\hat{\gamma}_0)$	$\overline{R}^2$	.05	.01	$\overline{\Gamma_1}$	.05	.01
.0	200	2.29	.31	**.695**	.475	1.26	**.120**	.030
.1	200	1.56	.32	**.460**	.255	1.21	**.110**	.025
.2	200	1.46	.33	**.460**	.210	1.23	**.110**	.055
.3	200	1.57	.35	**.475**	.255	1.29	**.135**	.030
.4	200	1.83	.37	**.540**	.350	1.39	**.220**	.070
.5	200	2.16	.40	**.645**	.455	1.56	**.265**	.080
.6	200	2.46	.43	**.755**	.540	1.72	**.400**	.145
.7	200	2.75	.48	**.840**	.645	2.07	**.565**	.335
.8	200	3.18	.53	**.895**	.755	2.65	**.745**	.565
.9	200	3.93	.60	**.960**	.875	4.25	**.965**	.885

[a] The significance levels chosen, i.e. .05 and .01 may be inappropriate for the univariate test due to lack of control over the size of the test.

introduced into the estimation errors via the structured residual correlation matrix in this investigation. GRS have argued that the estimates of alpha would have the same pattern of correlation, which in turn would result in the univariate t–statistics for these alphas exhibiting a similar pattern. It is important to note that the results of table 6.15 should thus **not** be used to make valid power comparisons between the univariate and multivariate tests, as the ***size of the test***, i.e. the significance level, is not controlled by the univariate test.

In an attempt to control for the ***size of the univariate*** test, the analysis was repeated, setting $\alpha = 0$ (i.e. the null hypothesis), and incrementing ρ_e in units of 0.1, as before. This procedure thus gives an indication of the ***actual significance levels*** appropriate for the univariate test over the range of ρ_e scenarios. More specifically, this procedure confirms whether only 5 and 1 percent are empirically rejected as is expected under the null hypothesis. If this is not the case, the increase in "power" obtained empirically is thus likely to be caused by a lack of control over the size of the univariate test. Consequently, the resultant "power" of the univariate test can be interpreted as the observed significance level for a given value of ρ_e.

Table 6.16 shows the summary statistics obtained (for alpha set at a zero) over the range of ρ_e scenarios for both the univariate and multivariate tests.

Table 6.16 Power (size) results of hypothesis 2 for alpha = 0 using structured variance–covariance matrices

		Univariate test				Multivariate test		
	No of			Power			Power	
ρ_e	runs	$t(\widehat{\gamma}_0)$	$\overline{R}^2$	.05	.01	$\overline{\Gamma}_1$	.05	.01
.0	200	0.15	.31	**.050**	.010	1.01	.050	.015
.1	200	-0.11	.32	**.060**	.010	1.05	.070	.020
.2	200	-0.15	.33	**.065**	.010	1.03	.070	.010
.3	200	-0.14	.35	**.065**	.010	1.02	.050	.000
.4	200	-0.03	.37	**.065**	.010	.99	.050	.010
.5	200	0.14	.40	**.095**	.020	1.01	.055	.010
.6	200	0.31	.43	**.110**	.025	1.01	.030	.005
.7	200	0.47	.48	**.145**	.030	1.05	.045	.015
.8	200	0.60	.53	**.155**	.030	1.08	.055	.020
.9	200	0.70	.60	**.180**	.030	1.07	.045	.010

Inspection of the power statistics for the univariate tests shown in table 6.16, reveals that there is indeed a problem with the control of the size of the univariate test. The "power" does not remain in the region of 5 percent, as expected under the null hypothesis, but in fact is seen to increase with increasing ρ_e. By contrast, the "power" of the multivariate test is seen to be approximately consistent with the quoted levels (at the null hypothesis) for all values of ρ_e.

To test if the empirically observed significance levels are greater than sampling theory suggests, a Poisson approximation to the binomial process can be assumed in order to construct approximate critical regions for $p = 0.05$ and $p = 0.01$. Testing at an approximate[13] 5 percent signifcance level, the critical values for $p = 0.05$ and $p = 0.01$ after 200 iterations are approximately 0.075 and 0.020 respectively. From table 6.16 it can be seen that the observed levels found for the *univariate* testing procedure assuming $p = 0.05$, exceeds the critical value at $\rho_e = 0.5$. For the case where $p = 0.01$ the critical value is exceeded at $\rho_e = 0.6$. None of the observed levels of the multivariate test shown in table 6.16, by contrast, exceed the approximate critical values of 0.075 and 0.020 for $p = 0.05$ and $p = 0.01$ respectively, implying that none of the observed levels found for the multivariate test are greater than sampling theory suggests. Hence it is evident that the power statistics of the univariate and multivariate test shown in table 6.15 are not directly comparable, since the apparent power statistics for the univariate test at given ρ_e, are not consistent with the quoted levels (i.e. .05 and .01). It is however more reasonable to assume that the actual significance levels are similar to the levels estimated empirically for the univariate test, and shown in table 6.16.

In an attempt to compare the power of the univariate and multivariate tests using the same set of significance levels, the empirical power of the multivariate test was determined at the same significance level (as observed for the univariate test in table 6.16) for each corresponding ρ_e scenario. These power statistics are shown in table 6.17. These results show that *although* there is an obvious improvement in the power of the multivariate test at the higher ρ_e scenarios, due to the higher significance used, the power of the multivariate test still only dominates the univariate test at the highest ρ_e scenario, i.e. $\rho_e = 0.9$.

Conclusion

The results presented in this section may help to throw some light on the interpretation of cases where conflicting results between univariate and multivariate

[13] As a binomial process with $n = 200$, a test statistic can be constructed by using a Poisson approximation with $\lambda = np$. For $p = .05, \lambda = 10$, yielding a critical value of 15 in 200 interations (or a critical proportion of 0.075) at the 0.049 significance level. For $p = 0.01, \lambda = 2$, yielding a critical value of 4 in 200 iterations (or a critical proportion of 0.02) at the 0.053 significance level.

Table 6.17 Power results for hypothesis 2 with alpha = 0.006 for various levels of significance

ρ_e	No of runs	Significance level	Power of Univariate test	Power of Multivariate test
.0	200	.050	.695	.120
.1	200	.060	.460	.110
.2	200	.065	.460	.170
.3	200	.065	.475	.185
.4	200	.065	.540	.270
.5	200	.095	.645	.390
.6	200	.110	.755	.540
.7	200	.145	.840	.735
.8	200	.155	.895	.890
.9	200	.180	.960	.990

tests are obtained. Knowledge of the residual correlation structures are useful for interpreting such conflicting results. For example, GRS were able to construct a market capitalization–sorted data set which the multivariate test is unable to reject, yet appears to be rejected by the univariate test. The correlation matrix presented by GRS contain elements of which the maximum correlation is 0.75, with the average correlation being 0.30. The simulation results presented here suggest that for an average residual correlation less than 0.8, the power of the univariate test will still dominate (assuming alpha = 0.006). Thus the results of GRS appear to be consistent with the power statistics presented here, for the levels of ρ_e documented by GRS.

A major implication of this study is that there is a need to control for the size of the univariate test, when large positive correlations dominate the residual correlation matrix. In particular, the results suggest that for average residual correlations of 0.5 or higher, the actual size (significance level) of the univariate test could be at least twice as large as when the residuals are truly independent.

FINAL THOUGHTS

The major portion of empirical work which has addressed aspects of Capital Market Theory has used data from the New York Stock Exchange. In this thesis many issues relating to Capital Market Theory have been examined on the JSE. The JSE typifies in many ways the smaller stock exchanges [1] worldwide, differing markedly from the three major international markets [2] in terms of the number of listings, the value of securities traded and the scope of traded options and futures contracts. As such the JSE provides a testing ground for Capital Market Theory in an environment which characterises these smaller exchanges. In particular, it was shown (in Chapter 4) that a large proportion of JSE listed securities are thinly-traded, a phenomenon which is characteristic of smaller markets. It is perhaps of interest that research into the effects of thin-trading on risk estimation has been rather limited [3] and confined to studies on the well-traded London and New York stock exchanges.

Although some of the suggested estimation corrections for thin-trading were empirically researched in this thesis for the JSE, it is felt that with the technical advances that have been made in the field of data storage and analysis, a fruitful direction of further research would be to further investigate possible improvements in these procedures in thinly-traded environments. With better estimation procedures in thinly-traded environments, there is much scope to improve, not only the testing procedures employed in these environments, but also many of the more practical aspects of the theory for implementation purposes. For example the improvement of beta estimation and covariance estimation would be an important contribution to analysts and investors who are active in thinly-traded environments.

In addition evidence was presented which suggests that for JSE stocks the validity of the CAPM could not be disputed. In particular, no trace of a 'size effect' or a 'January effect' could be found in the record of trading statistics of JSE

[1] Some of the smaller stock markets include: Australia, Belgium, Canada, France, West Germany, Holland, Italy, Singapore, Spain, Sweden and Switzerland.

[2] The three major stock markets are in: New York, London and Tokyo.

[3] With Scholes and Williams (1977), Dimson (1979) and Cohen *et al* (1983) making the major contributions.

stocks. This finding is in contrast to the NYSE, as well as several other exchanges, where overwhelming evidence of these effects have been documented. It is important to bear in mind, however, that due to the relatively larger residual variances for securities on the JSE, the power of these tests is considerably lower on the JSE than on the NYSE. The results of the simulation study presented in Chapter 6 of this thesis suggest that the power of the univariate test is fairly low on the JSE, while the power of the multivariate test is very low (even for the NYSE).

The evidence presented in Chapter 6 also indicates that the traditional univariate test of the null hypothesis (originally proposed by Fama and Macbeth (1973)) namely that: '*a positive trade-off between risk and return exists*', has fairly low power. It was noted that several researchers documented results that were unable to reject this null hypothesis. Consequently some of these researchers arrived at the counter–intuitive conclusion that '*bearing risk was not worthwhile*' (over the period analyzed). The results of the simulation study presented in section 6.4.1 reveal that tests of this hypothesis can be expected to have low power over the periods analysed by these researchers (assuming the CAPM is true). Hence the results (but not the conclusions) were found to be consistent with the evidence suggested by the empirical power of the test documented in this thesis (assuming the CAPM is true). Clearly researchers have been unaware that the test is generally not powerful enough to detect economically plausible deviations from this null hypothesis.

Due to the inherent multivariate structure of stock returns, one would expect multivariate testing procedures to have greater theoretical appeal over univariate testing procedures. Results presented in this thesis suggest that the multivariate test (of portfolio efficiency) unfortunately has considerably less power than the corresponding univariate test in plausible stock market settings. In support of the multivariate test however, evidence was presented indicating that when dependencies are evident in the estimation errors of the market model, there is a lack of control over the significance level of the traditional univariate test. Hence the results of traditional univariate tests presented at the usual fixed significance levels (usually 5 percent), are dubious when the market model residuals are cross–correlated. It is felt that a fruitful direction of future research would be to investigate in more detail how various residual cross–correlation structures/distributions influence the comparative power of these tests over a range of violations of the null hypothesis.

APPENDIX A

Table A1 Gold shares (rand returns) — corrected regression coefficients

	Period	$\overline{\hat{\gamma}_0 - R_f}$	$t(\overline{\hat{\gamma}_0 - R_f})$	$\overline{\hat{\gamma}_1 - (R_m - R_F)}$	$t(\overline{\hat{\gamma}_1 - (R_m - R_f)})$
All–Gold Index	1973–80	0.0478	1.44	-0.0453	-1.66
	1973–81	-0.0360	-1.13	0.0321	1.00
	1974–82	0.0464	0.62	-0.0348	-0.58
	1975–83	0.0136	0.37	-0.0081	-0.26
	1976–84	0.0199	0.60	-0.0246	0.77
Overall Index	1973–80	0.0230	0.86	-0.0148	-0.77
	1973–81	-0.0549	-2.66	0.0342	1.70
	1974–82	0.0567	0.57	-0.0131	-0.23
	1975–83	-0.0429	-0.84	0.0244	0.77
	1976–84	0.0019	0.04	0.0022	0.08

Table A2 Gold shares (dollar returns) — corrected regression coefficients

	Period	$\overline{\hat{\gamma}_0 - R_f}$	$t(\overline{\hat{\gamma}_0 - R_f})$	$\overline{\hat{\gamma}_1 - (R_m - R_F)}$	$t(\overline{\hat{\gamma}_1 - (R_m - R_f)})$
All–Gold Index	1973–80	0.0455	1.29	-0.0426	-1.31
	1973–81	-0.0243	-1.23	0.0217	0.96
	1974–82	0.0388	0.53	-0.0283	-0.48
	1975–83	0.0087	0.34	-0.0037	-0.16
	1976–84	-0.0183	-0.52	0.0136	0.41
Overall Index	1973–80	0.0024	0.06	0.0026	0.08
	1973–81	-0.0820	-2.03	0.0498	1.53
	1974–82	0.0271	0.40	0.0068	0.16
	1975–83	-0.0169	-0.66	0.0069	0.40
	1976–84	-0.0386	-1.14	0.0411	1.82

Table A3 Average $\hat{\gamma}_2$ coefficient measuring the dividend yield effect

	The JSE as a whole		Gold shares	
Period	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$
1973–81	-0.1543	-0.96	-0.0075	-0.45
1974–82	0.0048	1.33	-0.0121	-0.56
1975–83	0.0021	0.19	-0.0296	-1.19
1976–84	0.4057	0.45	-0.0101	-1.53

Table A4 Average $\hat{\gamma}_2$ coefficient measuring the firm size effect

	The JSE as a whole		Gold shares	
Period	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$
1973–81	-0.0076	-1.18	0.0040	0.06
1974–82	0.0030	1.01	-0.0001	-0.01
1975–83	-0.0025	-1.43	0.0002	0.20
1976–84	0.0026	0.80	0.0023	0.78

Table A5 Average $\hat{\gamma}_2$ coefficient measuring the liquidity effect

			Gold shares				
	JSE as a whole		All–Gold Index		Overall Index		
Period	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\gamma}_2$
1973–80	-0.0067	-0.73	-0.0042	-0.40	-0.0104	-0.96	
1973–81	-0.0016	-0.21	-0.0083	-0.58	-0.0191	-1.46	
1974–82	-0.0190	-1.44	0.0378	1.08	0.0489	1.06	
1975–83	-0.0117	-1.09	0.0085	0.50	0.0048	0.28	
1976–84	0.0069	0.56	-0.0020	-0.12	-0.0013	-0.10	

APPENDIX B

Table B1 Power results for hypothesis 1 under different excess market return scenarios using JSE parameters

		Results				
	No. of				Power	
$\mu^P_{R_m-R_f}$	runs	$\overline{\hat{\gamma}_1}$	$\overline{t(\hat{\gamma}_1)}$	$\overline{R}^2$	.05	.01
.0000	500	0.06	0.08	.22	.05	.01
.0050	500	0.53	0.67	.22	.17	.04
.0076[a]	**500**	**0.75**	**0.93**	**.22**	**.24**	**.08**
.0100	500	1.02	.125	.22	.35	.14
.0150	500	1.49	1.83	.23	.58	.31
.0200	500	1.97	2.41	.23	.76	.52

[a]the actual monthly excess return on the JSE over the period 1974–1984.

Table B2 Power results for hypothesis 1 under different $\sigma^P_{R_m-R_f}$ scenarios using JSE parameters

				Results		
					Power	
$\sigma^P_{R_m-R_f}$	No. of runs	$\overline{\hat{\gamma}_1}$ percent per month	$\overline{t(\hat{\gamma}_1)}$	$\overline{R}^2$	.05	.01
.025	500	0.52	1.33	.07	.36	.14
.050	500	0.72	1.09	.17	.26	.12
.063[a]	**500**	**0.75**	**0.93**	**.22**	**.24**	**.08**
.075	500	0.78	0.81	.28	.22	.07

[a]The actual standard deviation of the monthly excess return on the JSE over the period 1974–1984.

Table B3 Power results for hypothesis 1 under different $\overline{\sigma}_e^P$ scenarios using JSE parameters

	No of	Results				
					Power	
$\overline{\sigma}_e^P$	runs	$\overline{\hat{\gamma}}_1$	$t(\overline{\hat{\gamma}}_1)$	$\overline{R}^2$	.05	.01
.050	500	.0079	0.99	.47	.25	.09
.100	500	.0077	0.95	.25	.24	.08
.110[a]	**500**	**.0075**	**0.93**	**.22**	**.24**	**.08**
.150	500	.0072	0.86	.22	.21	.09

[a] The actual average standard deviation of market model residuals estimates over 1974–84 on the JSE.

Table B4 Power results for hypothesis 1 under different $\overline{\sigma}_{\text{beta}}^P$ scenarios using JSE parameters

	No of	Results				
					Power	
$\overline{\sigma}_{\text{beta}}^P$	runs	$\overline{\hat{\gamma}}_1$	$t(\overline{\hat{\gamma}}_1)$	$\overline{R}^2$	.05	.01
.200	500	.0053	.54	.07	.13	.02
.400	500	.0073	.87	.17	.21	.08
.506[a]	**500**	**.0075**	**.93**	**.22**	**.24**	**.08**
.600	500	.0076	.95	.27	.24	.08

[a] The actual standard deviation of beta estimated over 1974–84 on the JSE

Table B5 Power comparisons for hypothesis 2 on the JSE for alpha shifts where data is simulated from a univariate as well as a multivariate error structure

Assumed error structure	α[a]	No. of runs	Univariate test $t(\overline{\gamma}_0)$	Power .05	Power .01	Multivariate test $\overline{\Gamma}_1$	Power .05	Power .01	Agreement[b] % at the .05 level
Univariate	.000	500	0.09	.05	.01	0.98	.04	.01	0
	.002	500	0.67	.17	.05	0.99	.04	.01	25
	.004	500	1.26	.36	.15	1.03	.05	.01	43
	.006	500	1.85	.55	.31	1.11	.06	.02	73
	.008	500	2.43	.76	.50	1.21	.11	.02	90
	.010	500	3.02	.89	.72	1.33	.18	.04	99
Multivariate	.000	500	0.15	.06	.01	0.99	.04	.01	17
	.002	500	0.67	.16	.05	1.01	.04	.01	33
	.004	500	1.18	.33	.13	1.07	.06	.01	43
	.006	500	1.71	.54	.25	1.16	.07	.02	71
	.008	500	2.24	.71	.46	1.29	.12	.04	88
	.010	500	2.77	.84	.64	1.46	.19	.06	91

[a] Non–zero α coefficients represent violations of the SLM–CAPM. In step 4 of section 6.3 where the procedure is explained in detail the α is denoted by C.

[b] This column shows the percentage of times that the univariate and multivariate rejected the null hypothesis simultaneously for a given test period of simulated data, i.e. were in agreement.

Table B6 Power results for hypothesis 2 for $\alpha = 0.006$ under different $\bar{\sigma}_e^P$ scenarios using JSE parameters

	No.	Univariate test			Multivariate test			Agreement %
	of		Power			Power		at .05
$\bar{\sigma}_e^P$	runs	$t(\bar{\bar{\gamma}}_0)$	.05	.01	$\bar{\Gamma}_1$	.05	.01	level
.50	500	4.05	.99	.95	1.67	.33	.12	100
.100	500	2.02	.60	.36	1.16	.10	.02	74
.110[a]	**500**	**1.85**	**.55**	**.31**	**1.11**	**.06**	**.02**	**73**
.150	500	1.40	.39	.17	1.04	.04	.01	57

[a] The actual average standard deviation of market model residuals over 1974–84 on the JSE.

Table B7 Power results for hypothesis 2 for $\alpha = 0.006$ under different σ_{beta}^P scenarios using JSE parameters

	No.	Univariate test			Multivariate test			Agreement %
	of		Power			Power		at .05
σ_{beta}^P	runs	$t(\bar{\bar{\gamma}}_0)$	.05	.01	$\bar{\Gamma}_1$	.05	.01	level
.200	500	0.98	.24	.10	0.99	.04	.00	26
.400	500	1.55	.44	.22	1.07	.05	.01	80
.506[a]	**500**	**1.85**	**.55**	**.31**	**1.11**	**.06**	**.02**	**73**
.600	500	2.11	.63	.39	1.54	.09	.02	76

[a] The actual standard deviation of beta estimated over 1974–84 on the JSE.

Table B8 Power results for hypothesis 2 for $\alpha = .006$ under different $\mu^P_{R_m-R_f}$ scenarios using JSE parameters

		Univariate test			Multivariate test			Agreement %
	No of		Power			Power		at .05
$\mu^P_{R_m-R_f}$%	runs	$\overline{t(\hat{\gamma}_0)}$	.05	.01	$\overline{\Gamma}_1$	.05	.01	level
0.00	500	1.71	.50	.28	1.15	.09	.02	80
0.50	500	1.77	.53	.30	1.14	.09	.02	85
0.76[a]	**500**	**1.85**	**.55**	**.31**	**1.11**	**.06**	**.02**	**73**
1.00	500	1.85	.55	.32	1.13	.06	.02	83
1.50	500	1.19	.56	.33	1.12	.06	.01	81
2.00	500	1.98	.57	.35	1.11	.06	.01	81

[a] The actual monthly excess return on the JSE over the period 1974–84.

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